RURAL ECONOMY

A PREDATOR-PREY MODEL FOR SHARES OF WEALTH BETWEEN INDUSTRY AND AGRICULTURE

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STAFF PAPER

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1. Introduction to the Problem

The problem is to explore the possible coordinates and stability conditions for equilibria between agricultural wealth and industrial wealth generation and distribution processes. Instability of the equilibria may establish the possibility of moving along trajectories to preferred positions. The behaviour of the trajectories approaching such positions, as oscillations periodic or otherwise, may be explored by studying the operation of bifurcation parameters to determine whether such positions are attractors or repellors. The problem extends to determining qualitative change in the dynamic properties of equilibrium shares of wealth under varying conditions of ecospheric sustainability.

This paper is prompted by curiosity about the long future of agriculture as the ecosphere is pressured, and dependency on government supports and off-farm income is put in jeopardy by government deficits and economic recession. The problem definition and approach stem from the second generation of systems theory (Bertalanffy, 1980; Emery, 1969; Lange, 1965; Le Moigne, 1980, 1983; Morin, 1977; Osty, 1974; and Prigogine, 1979).

A predator prey paradigm is used to model the relationship of agriculture to the rest of the economy and the ecosphere. In the event that the ecosphere continues to degrade and productivity differentials between agriculture and industry expand, could chaotic processes emerge to jeopardize the persistence of both agriculture and industry (Goldberger et al., 1990; Goldberger and West, 1987)?

The transfer of wealth between agriculture and industry is a central theme in development economics. The transfer takes place through shifts of value added attributable to labour, intellectual property, land and capital. Wealth is understood in Adam Smith's terms of the purchasing power for necessities, conveniences and enjoyments of human life (Smith, 1901).

The term agriculture could be generalized to denote all natural resource sectors. These are usually located in rural economies. Industry refers to manufacturing and tertiary sectors, usually associated with urban economies. Debate is ongoing in development economics around sectoral investment priorities based on leading sector hypotheses, dualism and strategies for balanced and unbalanced growth.

Comparative statics and market equilibrium modelling, premised on perfect competition, have driven many of the prescriptions for markets structures, wages, productivity, extension, training and capital allocations between agriculture and industry. Debate over the prescriptions regularly spills over into trade friction when governments modify competitive advantages of one sector relative to the other through domestic support programs and border policies.

Predator prey modelling appears to offer new insight into a number of persistent questions left unresolved by agricultural economists (Freedman and Moson, 1990; Freedman and Waltman, 1984; and Hirshleifer, 1978; and Samuelson, 1971). Are there substantive economic reasons for government support of agriculture and industry? Government accompanies or directs agricultural development in many jurisdictions globally. Is their reluctance to disengage and allow competitive forces to determine development outcomes based on more than opportunistic lobbying of vested interests?

What are the equilibrium and stability conditions underlying the relationships of agricultural to industrial wealth in the long run? What is the effect of ecospheric degradation on the properties of these relationships which could shed light on the sustainability question? Can human determinism influence attainment of these conditions? Are the conditions leading to and away from chaos predictable?

The solution of the model presented in this paper suggests that three parametric relationships among economic variables determine the long run tendency either to impoverish or to enrich agriculture and industry. The first is that the terms of trade between agriculture and industry should at least favour agriculture to enable trajectories to move away from both axes in the long run. Second, a scaling parameter representing the productivity for agriculture relative to industry should be near to the value one. Third, the ratio of the growth rate to the rate of degradation of agricultural wealth should be greater than a threshold level of agricultural wealth determined as a composite parameter. This composite is calculated from a fixed industrial cost expressed in terms of a decline in industrial wealth in the absence of agriculture, the price index for industry, and the economic recovery rates for agriculture and industry. All possible trajectories are bounded by parametric asymptotes to the isoclines.

The model is formulated mathematically so as to explore the predictability of effects of stability of equilibria on changes in agricultural wealth (Coddington, 1955; Hurwicz, 1958; and Marsden and McCracken, 1976). This formulation provides for oscillations or "equilibrium chases" along trajectories. The outcomes suggest that for economies where industry buys and sells to agriculture, levels of wealth are indeed limited and that the fortunes of industry are favoured by a wealthy sustained agriculture. The alternative, subsistence agriculture, is associated with near zero levels of industrial wealth. Finally, the model demonstrates that agricultural and industrial wealth, while complementing each other in growth processes, may also rise and fall in sustained oscillation relative to each other on a periodic or near periodic basis.

2. The Model

Let $E(t)$, $A(t)$, and $I(t)$ represent environmental quality (condition of the ecosphere), agricultural wealth and industrial wealth, respectively, by whatever units they are measured. Agriculture derives its wealth mainly from the quality of the environment. The process of agricultural wealth creation may be enhanced by industrial input such as machinery and pesticides, at some cost. Industry derives its wealth from agriculture. Both industry and agriculture will in general need to replenish the environmental quality at the cost of foregone and deferred wealth accretion.

This reasoning motivates the model given by the system of three ordinary differential equations

(1a) $$\frac{dE}{dt} = -f_1(A)E + f_2(E, A, I)$$

(1b) $$\frac{dA}{dt} = \alpha^\circ \left(\frac{E}{e+E}\right) A - \beta^\circ A^2 + (\gamma - \delta)\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right)$$

(1c) $$\frac{dI}{dt} = (-\xi - \eta I)I + \delta\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right),$$

$$E(0) \geq 0,\ A(0) \geq 0,\ I(0) \geq 0.$$

The functions f_1 and f_2 represent the specific transformation of environmental quality into agricultural wealth and the restoration of the environment by both agriculture and industry, respectively.

The term $\alpha^\circ\left(\frac{E}{e+E}\right)A$ represents the rate of growth in agricultural wealth. α° is the maximal rate of increase, however propitious the condition of the ecosphere. It is assumed that as a function of environmental quality, the process of agricultural wealth creation will reach a point of diminishing returns. This possibility is represented by the negative term $-\beta^\circ A^2$.

The term $(\gamma-\delta)\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right)$ represents the net transfer of wealth between industry and agriculture. There is a net gain by agriculture if $\gamma > \delta$ and a net loss if $\gamma < \delta$. The ratio $\frac{\gamma}{\delta}$ represents agricultural terms of trade. It is assumed that transfers of wealth reach a satiation level no matter how large the wealth of either component. Hence the form of this term using two Michaelis Menton-type constants, a and b.

The model is a heuristic representation of three adjoining systems. They are an industrial system, an agricultural system and the ecospheric system. Together they encompass all the possible states of development of wealth in an economy, no matter whether industrialized, commercial economic systems fundamentally similar to those of OECD countries, or subsistence impoverished economies in less developed countries.

The principal attribute is wealth measured as asset value, in keeping with the fundamental purpose of enrichment in economic systems. The three systems are related by mutual interest of the facultative type. The industrial system is modelled as the predator relative to the agricultural system, extracting asset value. The agricultural system in turn is modelled as a predator on the ecosphere, using the ecosphere as a source of energy. The ecosphere is not considered to be a feedstock because of the facultative mutualism inherent in its relationship with agriculture.

Asset value is selected as the single attribute because it represents the capitalization of the productivity of assets rewarded through exchange processes within and among the systems. Asset value is considered to reflect both current income streams and the expectation of changes in these streams. For purposes of this model, assets may include depreciable capital stock and a composite bundle of property rights, of which land is the main focus for agriculture.

It is assumed that human migration between agriculture and industry is costly, as is the transformation of agricultural enterprises into industrial enterprises. This simplifying assumption allows exploration of thresholds of predation. In reality, migration takes place and agricultural households are changing their enterprises, which is part of structural change.

Industrial growth faces both fixed and variable expenses independent of agriculture. In the absence of agriculture, industrial wealth diminishes at both a constant rate, ξ, and linear rate η. It is assumed that industrial input into the environment is indirectly, through taxes, embedded within this cost structure. Finally the term $\delta(\frac{A}{a+A})(\frac{I}{b+I})$ represents the transfer of agricultural wealth into industrial wealth.

In this paper we assume that the environment is in a state of equilibrium, that is uptake from the ecosphere by agriculture is balanced by input from the other components. In that case $\frac{dE}{dt} = 0$ and E is a constant. Model (1) reduces to the form

(2a) $$\frac{dA}{dt} = \alpha^\circ \left(\frac{E}{e+E}\right) A - \beta^\circ A^2 + (\gamma - \delta)\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right)$$

(2b) $$\frac{dI}{dt} = (-\xi - \eta I)I + \delta\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right),$$

$$A(0) \geq 0,\ I(0) \geq 0.$$

We can simplify this form by defining $\alpha = \frac{\alpha^\circ E}{e+E}$, $\beta = \beta^\circ$. α may be understood as the conditional growth rate of agriculture. Then (2) becomes

(3a) $$\frac{dA}{dt} = \alpha A - \beta A^2 + (\gamma - \delta)\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right), \quad A(0) \geq 0$$

(3b) $$\frac{dI}{dt} = (-\xi - \eta I)I + \delta\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right), \quad I(0) \geq 0.$$

Finally we introduce a parameter $\mu > 0$ which represents a scaling of the rate of change in agricultural wealth as compared to the rate of change of industrial wealth. This parameter is viewed as a relative efficiency measure relating to the relative technical capacities to generate wealth expressed as the relative ratios of total factor productivities in agriculture and industry. μ operates as a bifurcation parameter. The complete specification of the model is:

(4a) $$\frac{dA}{dt} = \mu\left[\alpha A - \beta A^2 + (\gamma - \delta)\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right)\right], \quad A(0) \geq 0$$

(4b) $$\frac{dI}{dt} = (-\xi - \eta I)I + \delta\left(\frac{A}{a+A}\right)\left(\frac{I}{b+I}\right), \quad I(0) \geq 0.$$

Note that μ does not affect the equilibrium values of the model.

For small values of A and I, these constants may be thought of as inverse economic recovery rates. The smaller the constant the faster is the rate of recovery. These constants are system specific and may be observed by experimentation.

3. Equilibria

Model (4) has at least two and at most three nonnegative equilibria, denoted by $F_0(0,0)$, $F_1(\alpha/\beta, 0)$ and $\widehat{F}(\widehat{A}, \widehat{I})$. F_0 and F_1 always exist. $\widehat{F_1}$ corresponds to a subsistence rural economy with a predominance of family farms.

$\widehat{F}$ exists if and only if the isoclines

(5a) $$(\gamma - \delta)\left(\frac{1}{a+A}\right)\left(\frac{I}{b+I}\right) = \beta A - \alpha$$

(5b) $$\delta\left(\frac{A}{a+A}\right)\left(\frac{1}{b+I}\right) = \xi + \eta I$$

intersect at positive values of A and I. Call the coordinates of the intersection $(\widehat{A}, \widehat{I})$.

The solution curve of (5b) is shown in Fig. 1. The trajectory starts at $\overline{A}$ and progresses upward to the right approaching $\widetilde{I} = -\frac{1}{2}\left(\frac{\xi}{\eta} + b\right)\sqrt{\frac{1}{4}\left(\frac{\xi}{\eta} - b\right)^2 + \frac{\delta}{\eta}}$ as A becomes large. Condition 6, derived from (5b), is necessary for $\widehat{F}$ to exist,

$$(6) \qquad b\xi < \delta.$$

$\overline{A} = \frac{ab\xi}{\delta - b\xi}$ is also derived from (5b) for $I = 0$.

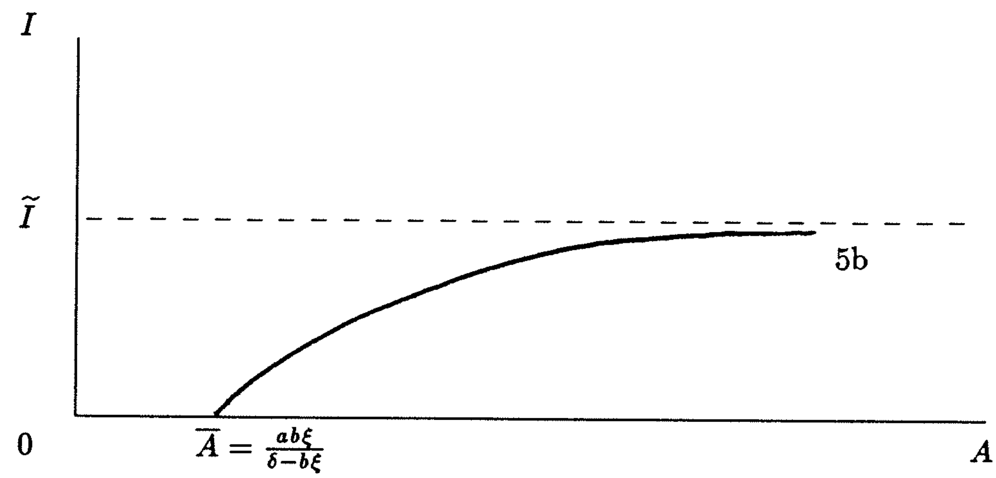

Fig. 1

Turn now to (5a). Four different isoclines are possible from the same starting point on the A axis. The starting point is $A^* = \alpha/\beta$. In the case of unfavourable agricultural terms of trade, $\gamma/\delta < 1$ corresponding to a net transfer of agricultural wealth to industry, two of the isoclines 5a1 and 5a2 move upwards to the left approaching the I axis (Figure 2). Isocline 5a1 intersects the I axis under the condition that $\frac{\delta - \gamma}{a\alpha} > 1$. The other approaches the I axis as I approaches infinity. This isocline exists when $\frac{\delta - \gamma}{a\alpha} \leq 1$.

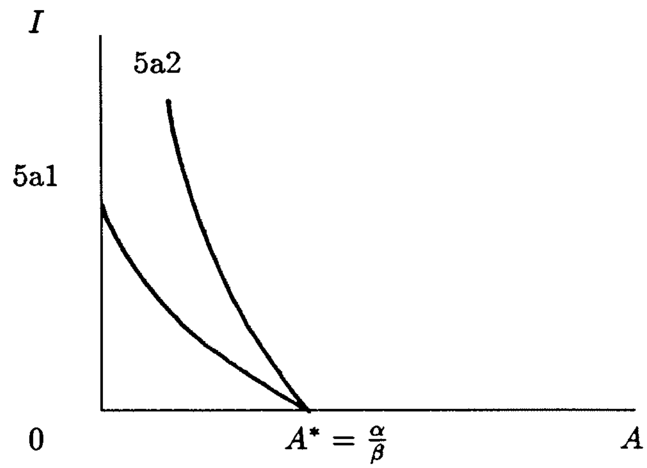

Fig. 2

Hence under the assumptions (6) and $\delta \geq \gamma$, a necessary and sufficient condition for $\widehat{F}$ to exist is

(7) $$A^* > \overline{A}.$$

The third possible isocline 5a3, corresponding to the case when $\delta = \gamma$, is a vertical straight line for $A = A^*$. This isocline is unlikely, existing only under conditions too complicated and trivial to demonstrate.

We now consider the case where $\gamma > \delta$ (net gain in agricultural wealth). Then the curve given by (5a) is given as in Figure 3. This fourth possible isocline 5a4 corresponds to favourable terms of trade for agriculture, $\gamma/\delta > 1$ (Figure 3). The isocline curves upward from $A^* = \alpha/\beta$ exponentially to the right approaching $\widetilde{A}$ derived from (8) as I goes to infinity.

(8) $$(\widetilde{A} + a)(b\widetilde{A} - \alpha) = \gamma - \delta.$$

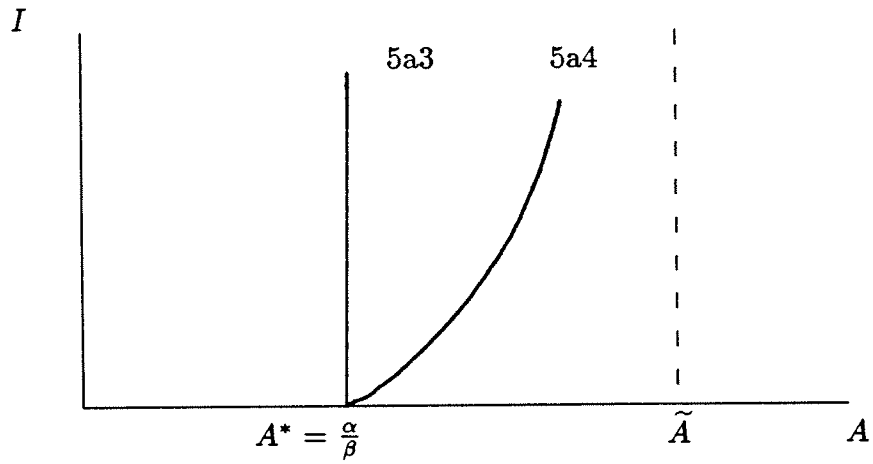

Fig. 3

$\widetilde{A}$ is the maximal value at which agricultural wealth can increase, no matter how large the available industrial wealth. Again one can see that condition (7) is a sufficient condition for $\widehat{F}$ to exist. Note, however, that in this case even if $\frac{\alpha}{\beta} < \overline{A}$, it may be possible for two intersection points to occur, but the algebraic criteria for that to happen are very complicated and in general, one would not expect that to happen.

Intersection of the four possible isoclines given by (5a) with the single possible isocline (5b) define the possible equilibria for the wealth relationships among these two systems (Figure 4). As mentioned, if (7) holds, all intersections are guaranteed to exist,

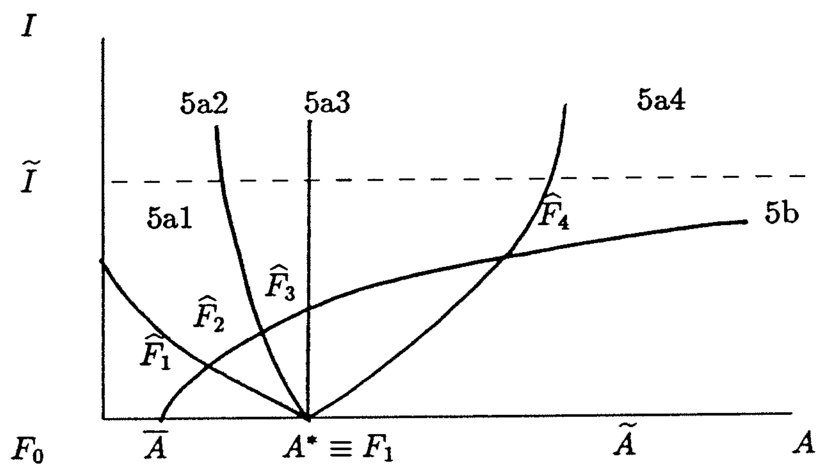

Fig. 4

4. Stability

The local stability of each equilibrium may be determined from the variational matrix about the respective equilibria (Coddington and Levinson 1955, Hurewicz 1958). Let V be the variational matrix, i.e., $V(A,I) = (m_{ij}) = \begin{pmatrix} \frac{\partial \dot{A}}{\partial A} & \frac{\partial \dot{A}}{\partial I} \\ \frac{\partial \dot{I}}{\partial A} & \frac{\partial \dot{I}}{\partial I} \end{pmatrix}$, $V_0 = V(0,0)$, $V_1 = V(\frac{\alpha}{\beta}, 0)$, $\widehat{V} = V(\widehat{A}, \widehat{I})$. Then

$$V_0 = \begin{pmatrix} \mu\alpha & 0 \\ 0 & -\xi \end{pmatrix}, \qquad V_1 = \begin{pmatrix} -\mu\alpha & \frac{\mu\alpha(\gamma-\delta)}{(\alpha+a\beta)b} \\ 0 & -\xi + \frac{\alpha\delta}{(\alpha+a\beta)b} \end{pmatrix},$$

$$\widehat{V} = \begin{pmatrix} \mu\left[-\beta\widehat{A} + \frac{(\delta-\gamma)\widehat{A}\widehat{I}}{(a+\widehat{A})^2(b+\widehat{I})}\right] & \frac{\mu(\gamma-\delta)b\widehat{A}}{(a+\widehat{A})(b+\widehat{I})^2} \\ \frac{\delta a\widehat{I}}{(a+\widehat{A})^2(b+\widehat{I})} & -\eta\widehat{I} - \frac{\delta\widehat{A}\widehat{I}}{(a+\widehat{A})(b+\widehat{I})^2} \end{pmatrix}.$$

The stability depends on the real parts of the eigenvalues of these variational matrices. For V_0, the eigenvalues are $\mu\alpha > 0$ and $-\xi < 0$. Hence F_0 is a saddle point, locally stable in the I direction and locally unstable in the A direction. This may be interpreted as stating that for small values of agricultural and industrial wealth, agricultural wealth will grow and industrial wealth will decline. This interpretation of F_0 could be considered to apply to the stage of development immediately following economic collapse. F_0 will always tend to become F_1 with subsistence agriculture and no industry.

The eigenvalues of V_1 are $-\mu\alpha < 0$ and $-\xi + \frac{\alpha\delta}{(\alpha+a\beta)b} = \lambda_{12}$. If $\lambda_{12} < 0$, i.e. $\alpha\delta < (\alpha + a\beta)b\xi$, then F_1 is asymptotically stable and industrial wealth will locally go extinct. In the absence of $\widehat{F}$, this local extinction will be global extinction. If $\lambda_{12} > 0$, then F_1 is locally unstable in the I direction, which implies the possibility for survival and growth of industrial wealth. This outcome can be proved rigorously and is a consequence of persistence-type theorems (Freedman and Waltman 1984, Freedman and Moson 1990).

The eigenvalues of $\widehat{F}$ are given by solutions of the equation in λ,

$$
\begin{aligned}
(9) \qquad & \lambda^2 + \left[\mu\beta\widehat{A} + \eta\widehat{I} + \frac{\mu\widehat{A}\widehat{I}((\gamma-\delta)(b+\widehat{I}) + \delta(a+\widehat{A}))}{(a+\widehat{A})^2(b+\widehat{I})^2}\right]_1 \lambda \\
& + \mu\left[\beta\eta\widehat{A}\widehat{I} + \frac{\beta\delta\widehat{A}^2\widehat{I}}{(a+\widehat{A})(b+\widehat{I})^2} - \frac{(\delta-\gamma)\eta\widehat{A}\widehat{I}^2}{(a+\widehat{A})^2(b+\widehat{I})} + \frac{\delta(\delta-\gamma)\widehat{A}\widehat{I}(ab-\widehat{A}\widehat{I})}{(a+\widehat{A})^3(b+\widehat{I})^3}\right]_2 \\
& = 0
\end{aligned}
$$

The solution for λ is quite complicated, and may have both negative and positive real parts, which implies that $\widehat{F}$ could be either stable or unstable. This possibility will be discussed further below.

What can be noted from the isoclines when $\widehat{F}$ exists, by comparing the cases $\gamma > \delta$, $\gamma = \delta$, $\gamma < \delta$ is that if $\gamma > \delta$ then $\widehat{A} > \frac{\alpha}{\beta}$, if $\gamma = \delta$ then $\widehat{A} = \frac{\alpha}{\beta}$, if $\gamma < \delta$ then $\widehat{A} < \frac{\alpha}{\beta}$. These cases exactly coincide with the equilibrium value of agricultural wealth being larger, the same as, or less than, respectively, the value if no industry were present.

Asymmetric stability is an important quality of the outcome of the model. It is precisely the instability in the I dimension for (5a) that enables the trajectories leading to possible $\widehat{F}$'s. Stability in the A dimension signifies that a human population may subsist on the land under conditions of an ecospheric equilibrium in a poverty trap not unlike Nelson's low level equilibrium trap, for initial improvements in industrial wealth (Nelson, 1956). However, predictability of the paths of development from F_1 to possible $\widehat{F}$ raises the question of the dynamics of wealth transfer and consideration of bifurcation phenomena leading to oscillations.

5. Trajectories and Oscillations

In the case that $\delta > \gamma$ (industry extracts more wealth from agriculture than it inputs) it may be possible for sustained oscillations in the form of periodic solutions to occur. Evidence of the possibility of oscillatory behaviour was first presented by Ruud

(1988) and Apedaile and Rudd (1988). These will arise via a Hopf bifurcation (Marsden and McCracken 1976) associated with values of the μ parameter. The bifurcations emanate from $\widehat{F}$.

In the case of our system, a Hopf bifurcation will occur if there is a value $\mu_0 > 0$ such that (i) $[\ ]_1|_{\mu=\mu_0} = 0$, (ii) $[\ ]_2 > 0$, and (iii) $\frac{d[\ ]_1}{d\mu}\big|_{\mu=\mu_0} \neq 0$. In that case, there will exist periodic solutions either for $\mu < \mu_0$, $\mu > \mu_0$, or $\mu = \mu_0$, in a neighbourhood of μ_0.

We now get criteria for (i) to hold. $[\ ]_1|_{\mu=0} = \eta\widehat{I} + \frac{\delta(a+\widehat{A})}{(a+\widehat{A})^2(b+\widehat{I})} > 0$. As $\mu \to +\infty$, $[\ ]_1 \to -\infty$ provided $\beta\widehat{A} + \frac{\widehat{A}\widehat{I}(\gamma-\delta)(b+\widehat{I})}{(a+\widehat{A})^2(b+\widehat{I})^2} < 0$, i.e. $\frac{(\delta-\gamma)\widehat{I}}{(a+\widehat{A})^2(b+\widehat{I})} > \beta$. But from the definitions of $\widehat{A}$ and $\widehat{I}$, $\frac{(\delta-\gamma)\widehat{I}}{(a+\widehat{A})(b+\widehat{I})} = \alpha - \beta\widehat{A}$. Hence our last inequality reduces to $\frac{\alpha-\beta\widehat{A}}{a+\widehat{A}} > \beta$, or $\alpha - \beta\widehat{A} > a\beta + \beta\widehat{A}$, or $\alpha - a\beta > 2\beta\widehat{A}$, i.e.

$$\widehat{A} < \frac{\alpha - a\beta}{2\beta}. \tag{10}$$

Hence if (10) holds there exists $0 < \mu_0 < +\infty$ such that (i) holds. Note that condition (10) automatically implies (iii) as well.

For criterion (ii), we see that for $\eta = 0$, $[\ ]_2$ consists of two positive terms. Hence there are possible values of parameters such that $[\ ]_2 > 0$.

The above analysis shows that sustained oscillations in our model can indeed occur, provided that industry is sufficiently "greedy" and the parameters lie in an appropriate range of values.

The observable trajectories for wealth obey the attractors determined by the unobservable isoclines and their intersections. The trajectories have initial points arbitrarily defined by circumstances of the systems analysis or by time. The trajectories constitute the image of the search process for the systems in predatory interaction as they seek an equilibrium.

The equilibria have been defined above. They are themselves dynamics with variable parameters subject to their own transformation functions. To simplify this presentation these parameters are assumed constant or subject to very slow change.

In general, the trajectories may become limit cycles for part of their lives in time. They may also display monotonicity during other periods of time. The oscillation behaviour is important because of the locking-in effect it has on the performance of industry and agriculture. The relationships of the parameters determine this behaviour.

6. Examples

The model is examined for the robustness and versatility of its dynamics using the two general cases of less developed and industrialized countries. The cases are distinguished by different sets of values for the parameters. The procedure is to find numerical values for all possible $\widehat{F}$ corresponding to the given values for the parameters. The stability in the vicinity of each equilibrium was defined as that of the linearized system in accordance with the Grobman-Hartman Theorem (Hartman, 1964). The trajectories and bifurcation values for the parameters were determined experimentally using the PHASER software developed by the Koçak team (Koçak, 1986).

6.1. Less developed country (LDC)

The LDC version of the model has two possible outcomes, a stable node, illustrated by Figure 5a, and a stable focus by 5b. Both equilibria are comparable to Nelson's low level equilibrium trap (Nelson, 1956). These two outcomes are insensitive to the starting points for the trajectories. The type of equilibrium is sensitive to productivity differentials. The stable node occurs for $\mu < \mu_0 \cong 1.22$. The focus occurs for $\mu > \mu_0$.

This LDC version incorporates adverse agricultural terms of trade, diminishing carrying capacity of the ecosphere for agriculture, identical low economic recovery rates for agriculture and industry and finally, a linear depreciation rate for industrial wealth. The dynamics are limited by, but not sensitive to these values, only to the value of μ. The equilibria are arbitrarily scaled in this example.

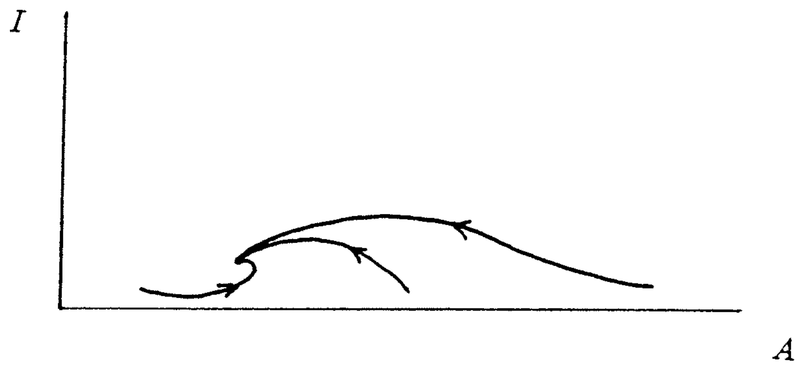

Fig. 5(a) $\mu = 0.5 < \mu_0$

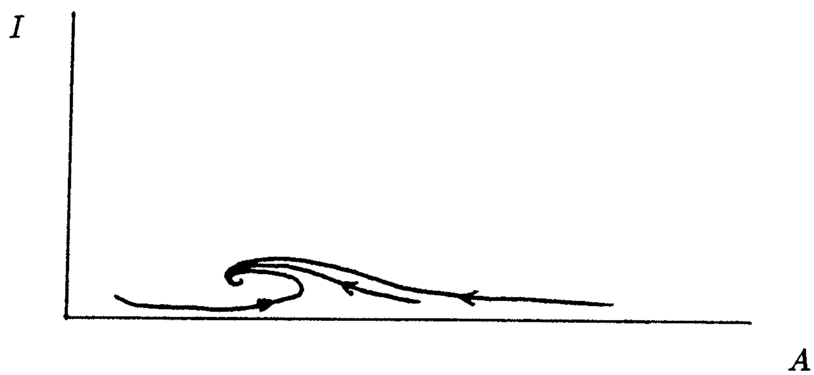

Fig. 5(b) $\mu = 2.0 > \mu_0$

$\alpha = 2;\quad \beta = 1;\quad \gamma = 10;\quad \delta = 20;\quad a = 1;\quad b = 1;\quad \xi = 8;\quad \eta = 0$

Fig. 5

The result points to a two-part development strategy for LDCs. The first is to alter the stability of the equilibria in at least one dimension or possibly alternating between agriculture and industry. For example, the major transitions in Chinese development policy since 1949 feature alternating destabilization of agriculture and industry (Zhu and Apedaile, 1994). The second is to change the values of the parameters so as to create an opportunity to shift equilibria $\widehat{F}$ towards increasing agricultural and industrial wealth.

6.2. Industrialized country scenario (ICS)

The ICS demonstrates extremely rich dynamic properties. Four cases illustrate the versatility of the model. The industrial country example is differentiated from the LDC example by a scaling technique to change the relationships among carrying capacity of the ecosphere for agriculture, agricultural terms of trade and economic recovery rates for agriculture and industry. Agricultural terms of trade remain adverse. The dynamics are not sensitive to marginal changes in these values. The dynamics are, however, very sensitive to the rates of depreciation and use of industrial capital, and to differentials in productivity.

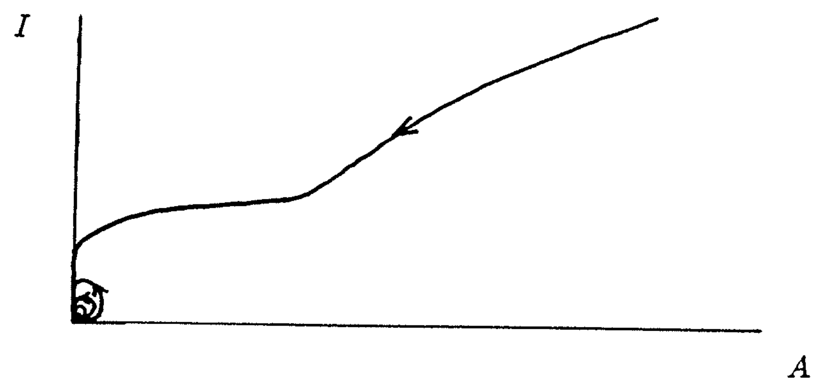

Fig. 6a. $\xi = 1.75 < \xi_0$; $\mu = 1 < \mu_0 \approx 1.62$.

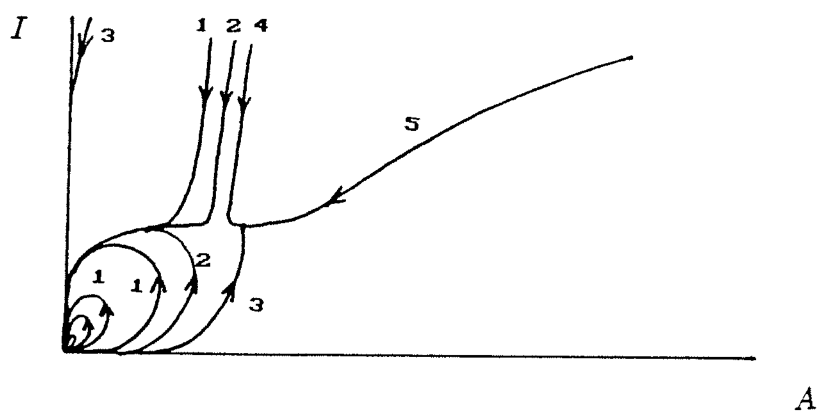

Fig. 6b. $\xi = 1.80 > \xi_0$; $\mu = 1 < \mu_0$.

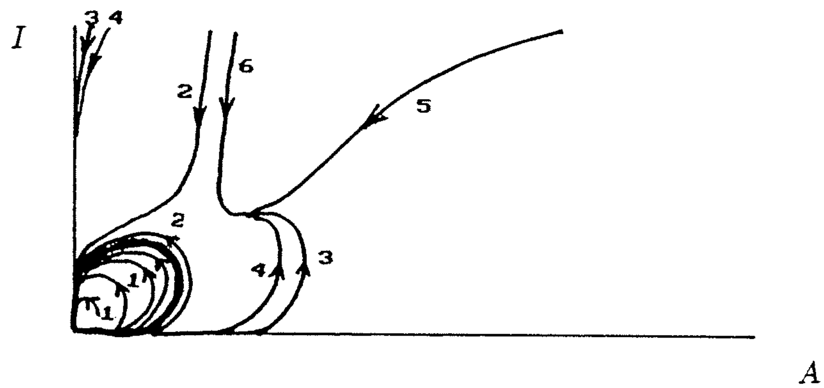

Fig. 6c. $\xi = 1.80 > \xi_0$; $\mu = 1.8 > \mu_0 \approx 1.62$.

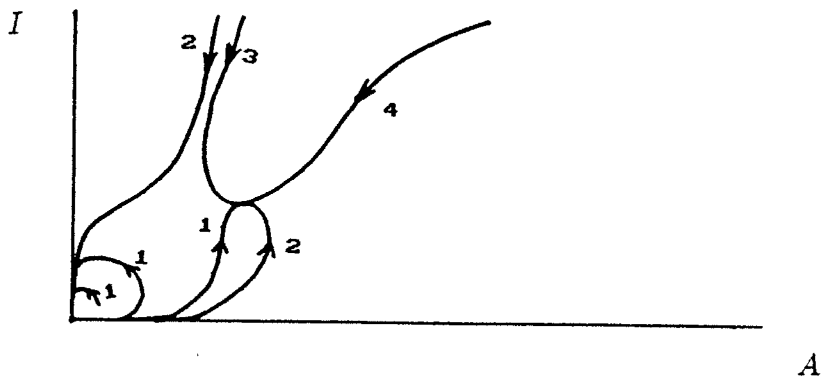

Fig. 6d. $\xi = 1.8 > \xi_0$; $\mu = 2.5$.

Fig. 6

The first case, illustrated in Figure 6a, is a stable focus located close to the origin. All trajectories end in this focus. This is an unsatisfactory situation for an industrialized country. There is no endogenous capability to expand total wealth or to change the shares held by agriculture and industry. This case corresponds to balanced productivity change between the two sectors $\mu = 1$, combined with a rate of linear capital depreciation less than the bifurcation value. Any benefits from a favourable external shock to industrial and agricultural wealth would be short lived, unless the relationships among values for parameters were changed or rescaled.

The second case shown in Figure 6b contains complex dynamics. When ξ surpasses the bifurcation point, a saddle-node bifurcation takes place. Two equilibria additional to the stable focus become possible, a saddle point and a stable node. Consequently, some trajectories aren't attracted to the stable focus any longer. Trajectories 3, 4 and 5 end at the new stable node at much higher levels of agricultural and industrial wealth. The conditions for this improvement are that ξ exceeds the bifurcation value compared to case 6a where ξ is less than this value. Both the dynamics and the equilibrium attractor may be made more consistent with objectives for economic development by slightly improving the rate of use of capital stock beyond the bifurcation value, for cases of balanced improvements to productivity.

The third case in the ICS produces a Hopf bifurcation and the consequent limit cycle replaces the stable focus (Figure 6c). The saddle point and node remain. The formation of a limit cycle replaces the stability of the focus with instability. Now some trajectories, 1 and 2, are attracted to this limit cycle, while the other four, 3, 4, 5 and 6, are attracted to the stable node. This third case is only different from case 6b because productivity of agriculture increases relative to that of industry, thereby improving the predatory gains for industry.

The fourth case produces a textbook result. The limit cycle expands with growing μ until it breaks and all trajectories end in the stable node at much higher levels of wealth (Figure 6d). This result, conditional upon the given relationships among the values of the other parameters, stems from further increasing agricultural productivity relative to that of industry. The imprisoning nature of the limit cycle collapses as μ passes beyond a two to one relationship between agriculture and industry productivities.

6.3. Discussion of the examples

These examples are our first exploration of the explanatory power of the model. The examples confirm the importance of understanding the predator prey relationship between agriculture and industry in advancing the fortunes of both sectors. The restrictive assumptions of perfect competition are unnecessary to gaining this understanding. The focus for productivity gains should be on the prey system.

The principal distinction between the industrial and less developed economies in these examples lies in the higher economic recovery rates for the former, relative to carrying capacity. This scenario highlights the importance of the tradeoff between recovery rates and adverse terms of trade for the prey system. Recovery rates are learnable and therefore endogenous to the system.

The dynamics of this complex predator prey model lie in the stability of the attractors. Instability appears to be more important to the development of agriculture and industry than is the equilibrium per se between agricultural and industrial wealth. Thus the normative preference for stable equilibria in computable general equilibrium (CGE) modelling may preclude useful predictions arising from explorations of the dynamic behaviour of economic systems.

7. Intepretation and Discussion

7.1. Relaxing the ecospheric equilibrium assumption

The recovery rate of the ecosphere for given economies of size determines the possibility for equilibria at positive levels of industrial and agricultural wealth. The actual rate of growth for agriculture approaches the maximum as the recovery rate is faster.

The recovery rate is a composite recovery response to reversible and almost irreversible damage. Disasters such as flooding, pestilence and industrial pollution, are usually reversible with finite cost and short periods of deferred outputs. Other damage may be termed almost irreversible with much slower recovery rates. Soil erosion and degradation, aquifer depletion, and atmospheric change engender slower recoveries. Near irreversible damage also includes social degradation such as feudal property rights, ethnic cleansing and tribalism.

The existence of an interior equilibrium depends on the juxtapositioning of $\overline{A}$ and A^*. $\overline{A}$ must normally be less than A^*. A^* is ecosphere dependent. When the ecosphere is degraded or slow to recover because of the irreversible qualities of the damage, A^* would move to the left, lowering the equilibrium values of $\widehat{A}$ and $\widehat{I}$. Eventually the isoclines may not intersect anywhere.

Recall that wealth trajectories chase equilibria determined by the possible intersections of isoclines (5a) and (5b). In the absence of an intersection, trajectories 'caught' out somewhere in AI space could behave chaotically without the orderly influence of attractors. The only possible prediction could be a deterioration of the economy until an F_1 or F_0 equilibrium were attained.

First industry wealth would disappear, then agricultural wealth would decline until the stability conditions attenuated the ecospheric degradation allowing F_1 to be regained. Industrial wealth in our model appears to be in greater jeopardy than agricultural wealth as the ecosphere degrades or is slow to recover.

7.2. Recovery rates and cost structures

The industry fixed cost factor, ξ, and the inverse recovery rates, a and b play important roles in determining whether the equilibrium levels of wealth depart from both axes. All three parameters should be smaller rather than larger to favour an interior equilibrium. Both industry and agriculture should be able to recover quickly in the event of lost wealth, occuring for example, during periods of persistently adverse terms of trade.

The coefficient of diminishing returns for agriculture, β, should also be small or even negative denoting increasing returns, to enable the industry coordinate for equilibrium solutions to be interior to the two axes. Increasing returns are associated with the industrialization and commercialization of agriculture.

7.3. Bifurcation parameter; efficiency

The interpretation given to the bifurcation parameter in this model is one of efficiency. The position taken is that efficiency is sought by a system, but rarely attained. If it is attained, the state is transitory. Thus, μ is unlikely to be equal to $\mu_0 = 1$. For nearly efficient systems, the μ fulfills the role of an attractor.

The amplitude of the oscillation is of the order of the square root of the absolute value of the departure of μ from efficiency given by $\mu_0 = 1$. As the difference increases, then the oscillation pattern likely takes the form of a limit cycle with a repeller of the trajectory from the point of perfectly efficient wealth accretion plus a peripheral attractor. The form of the oscillations of the trajectories represents the flexibility of the systems.

Differential rates of learning appear to be the underlying reasons for the productivity differences between agriculture and industry. The solutions constitute an argument for balanced learning in the two systems. This balance is more likely as agriculture becomes industrialized and subject to the same learning processes as industry. Conversely, a dual economy with an artisan agriculture and a science driven industrial sector would be predictably subject to a limit cycle type attractor with complex and frequent returns to stages of development which threaten persistence.

7.4. Limits to growth

The upper limits for equilibria shares of wealth are defined by $\widetilde{A}$ and $\widetilde{I}$ for isoclines (5a4) and (5b) respectively. The economic implication is limits to growth. There is no possibility of sustained growth to wealth in the long run. The limit for agriculture is defined by condition (8) involving the recovery rates for agriculture and industry, the conditional growth rate of agriculture and terms of trade. Recall that the conditional rate of growth for agriculture is a function of the qualitative value for the ecosphere and its rate of recovery.

The upper limit for industry wealth is a constant representing the ratio of the price index for industrial outputs to the constant component of the depreciation of industrial wealth with time, less the inverse recovery rate for industry. This upper limit is more constraining the faster the recovery rate and the larger the constant rate of depreciation of industrial wealth for any given price level. The price level is a predatory instrument which may be increased by shortages of industrial goods caused by deliberately shorting the market or alternatively by a slow down in learning through reduced investment in research and development. Obviously increases to the industrial price level relative to the agricultural price level increase industrial wealth.

7.5. Instability in agriculture

The behaviour of instability has already been mentioned as requiring further research. It appears that the speed of learning and therefore the way the systems program information expressed in the form of emerging properties, and how this information is processed into wealth influence stability. The model would predict that an agricultural economy ages by slowing down its learning thereby becoming less flexible. In the process it seeks and gains greater stability. Enhanced stability for agriculture seems to translate into stronger trapping at lower levels of wealth for both industry and agriculture.

Stabilization policies for agriculture may be counter-productive, in that they attempt to deny inherent oscillation behaviour. The focus should instead be on strengthening long run terms of trade in the context of a full accounting for the wealth generated by agriculture.

7.6. Agricultural terms of trade

The most interesting isocline is (5a3) (Figure 4). This isocline is only possible under chronically favourable agricultural terms of trade. This result would seem to raise interesting questions about financing industry from agriculture on a sustained basis, whether by taxation or price policy. Similar questions are raised about persistent extraction of value added from agriculture through policy and market measures.

Agricultural terms of trade should be viewed in a broader context than competitive market prices. Predation by means of monopolistic and monopsonistic competition influence terms of trade. Over or under compensation for public goods and services produced by industry and agriculture also affect terms of trade. For example taxation, uncompensated amenity and option value, territorial management services and pollution disposal by agriculture on behalf of industry all contribute to unfavourable agricultural terms of trade.

Agricultural support programs may be viewed as a blunt correctional instrument for terms of trade. OECD countries are already heavily committed politically and financially to these programs. Other countries with different views on human rights and standards and with weaker treasuries allow their agricultural populations to take on the bulk of the cost of economic adjustment in the form of poverty and malnutrition. Generally these countries, with some exceptions like China, have weak industrial sectors, just as our model would predict under long run adverse agricultural terms of trade.

This model suggests that it would be useful to determine the impacts on terms of trade from deregulation, removal of subsidies, or dismantlement of supply management as part of the design of the agriculture components of international trade agreements and debt management policies. In principle, public contractual supply management alliances may be the instrument most amenable to fine tuning terms of trade in domestic markets and the most amenable to practical international border relationships among those markets.

7.7. Improvements to the model

The model is simplified to two dimensions by assuming an equilibrium within the ecosphere. An improved version of the model would seek to relax this assumption to incorporate this dimension into the analysis of the equilibrium and stability properties. Similarly, the model should be able to cope with alternative cost structures for agriculture and industry. In particular, diminishing returns would correspond to F_1 type equilibria, but constant or increasing returns likely apply as $\widehat{F}$ $(\widehat{A}, \widehat{I})$ moves away from the origin.

Perhaps the greatest interest in improvement lies in modelling the dynamics of self organization including impulse analysis and learning. There is a need to predict the asymmetric instability behaviour of the equilibrium as trajectories are attracted or repelled with the various possible configurations of oscillation. The familiar issues of estimation based upon cross-sectional and longitudinal data arise.

8. Conclusions

This model is designed for investigation and learning about agricultural economic development. It explores predator prey behaviour among systems. The principal difference from competitive models is the presence of mutualism among the three systems. This is a more general case than competition. The emphasis is on wealth creation, rather than on profit or utility maximization. Wealth is valuable for the various consumptive and uncertainty averting properties it has in the present and the future.

The solutions for equilibria and stability properties are instructive. First, it seems that design concepts drawn from ecology and the theory of complex dynamical systems can shed light on relevant economic problems. Getting the prices right seems to be less important than getting capacity and information generating processes right.

Instability about an equilibrium may be more desirable than stability. An equilibrium may involve a lot of chasing around in the form of oscillations of wealth sharing arrangements. That is to say, distributive issues are important to persistence of both the agricultural and the industrial systems. Government policy should be centrally focussed upon the rules of the chase, especially in identifying and enforcing mutualism to the benefit of predator and prey alike.

Degeneration of the ecosphere by agricultural predation is probably a more critical issue for urban people obtaining wealth directly from the industrial sector than for agricultural people. Thus environmental concern among urban populations relating to agricultural practices may not be misplaced or ill-advised.

There are also indications of a need to re-assess apparently conventional wisdom as regards agricultural support programs and international trade. Transactions, whether market or nonmarket are the principal venue for predator prey behaviour. It little matters the reason why agricultural terms of trade turn for or against agriculture. Rather, favourability of terms of trade should alternate regularly and undramatically between agriculture and industry. In the long run, up to now undefined, persistently adverse agricultural terms of trade would lead to Pareto worse solutions for wealth distribution in society.

The examples point to the nature of economic restructuring needed for successful economic development. Development requires changing the stability and dynamic properties of the equilibrium structure of shares of economic wealth for agriculture and industry to a saddle point node combination from a stable node. The progression may be visualized again by scanning Figures 5a through 6d. The underlying process of economic restructuring resides in the reproportioning of the values of economic and ecospheric recovery rates relative to the structure of carrying capacity, terms of trade and industry cost structure.

Exogenous impulses to trajectories to force open a system cannot generate sustainable changes to the dynamics and form of equilibria unless the conjunction of parameter values permit a saddle point. Exogenous impulses on an economic system through new institutional arrangements for liberalized trade or foreign aid at any and all stages of development would require complementary endogenous reproportioning of parameter values to avoid regression to stable nodes, foci or unstable limit cycles. This restructuring process should be the focus for research on sustainable agricultural and rural development.

These conclusions amount to little more than flags on a play. Nevertheless they seem to merit some time out to consider the need for a paradigm shift in agricultural economics and public policy.

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