University of Alberta

Controlling IER, EER, and FDR In Replicated Regular Two-Level Factorial Designs

by

Oludotun Joseph Akinlawon

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Science
in
Statistics

Department of Mathematical and Statistical Sciences

Spring 2012
Edmonton, Alberta

TO MY WIFE

Abstract

Replicated regular two-level factorial experiments are very useful for industry. The basic purpose of this type of experiments is to identify active effects that affect the mean and variance of the response. Hypothesis testing procedures are widely used for this purpose. However, the existing methods give results that are either too liberal or too conservative in controlling the individual and experimentwise error rates (IER and EER respectively). In this thesis, we propose a resampling procedure and an exact-variance method for identifying active effects for the mean and variance of the response, respectively. Monte Carlo studies show that our proposed methods perform extremely well in terms of controlling the IER and EER. We also extend our proposed methods to control the false discovery rate. Two real data sets were used as case study to illustrate the performance of the proposed methods.

Acknowledgements

First and foremost, I would like to thank my supervisors Dr. Pengfei Li and Dr. Rohana Karunamuni for their time, ideas and financial support to make my M.Sc experience productive and successful. In particular, my sincere gratitude to Dr. Pengfei Li for his patience, motivation, enthusiasm and insightful comments. It has been an honor to be his first M.Sc student. His guidance helped me a lot during the research and writing of this thesis.

I would also like to thank the chair and the other examiner of my oral defense committee, Dr. Peng Zhang and Dr. Yan Yuan, respectively, for their time and insightful questions.

To my parents and my younger ones for their spiritual support throughout my graduate years.

Lastly, a special thank to my wife for her love, support and encouragement throughout my graduate years. I love you very much. Thank you all and God Bless.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 A brief Review of Two-Level Factorial Experiments

Experimental Designs involve laying out of detailed experimental plans in advance of doing experiments. They are widely used in industries to control and improve the quality of outputs. In robust designs, experimentation is used to determine the factor levels so that the product or production process is insensitive to potential variations in operating, environmental, and market conditions (Tagushi, 1986). A factor in this case is a variable that is studied in the experiment while levels refer to the values of the factor. To study the effect of a factor on the response, two or more values of the factor are used. A combination of factor levels is called a treatment or a run.

The most important class of these designs is two-level factorial designs. They are easy to design, efficient to run, straightforward to analyze and full of useful information (Mee, 2009).

In two-level factorial designs, each factor is investigated at only two levels. These designs are widely used in industrial (Box, Hunter and Hunter, 1978) and agricultural (Kempthorne, 1952; Cochran and Cox, 1957) experiments to identify the most important factor(s) affecting a process.

Two-level factorial designs can be un-replicated. That is, a single experiment is carried out at each run. This type of design is often performed when runs are expensive. The other case is the replicated experimental design. In this case, the experiment is replicated more than one time at each run.

This thesis concentrates on replicated two-level factorial designs.

1.1.1 Two-Level Full Factorial Designs

Suppose that a two-level factorial design has k experimental factors each having two levels. The experimental factors can be numerical variables such as Temperature (e.g. $low\ level = 100^0 F$, $high\ level = 180^0 F$) or categorical such as Deposition time (e.g $low\ level = Low, high\ level = High$).

A full factorial experiment consists of every combination of the levels of factors in the experiment. Thus, for the k factors each with two levels, a *full two-level factorial design* consists of $2 \times 2 \times 2 \times \cdots \times 2 = 2^k$ treatments/runs. We denote it as 2^k factorial designs. We shall use $-$ and $+$ to represent the low level and high level of the factors respectively in this thesis.

The following gives an example of a full factorial design.

Example 1.1. *This example is taken from Montgomery (2009, p. 267). The experiment is a 2^4 factorial design. Four experimental factors: length of putt (A), types of putter (B), break of putt (C) and slope of putt (D) were investigated each at two levels. The primary response in this experiment is the distance from the ball to the center of the cup after the ball comes to rest. The experiment is replicated seven times for each run. The purpose of this experiment is to improve the golfer's scores (putting accuracy). That is, minimize the putting variability while maintaining the distance from the ball to the center of the cup closest to zero. Tables 1.1 and 1.2 below show the data, and factors and levels respectively for this example.*

Table 1.1: Design Matrix and Distance Data for Example 1.1

Runs	A	B	C	D	Distance from Cup (replicates) y							$\bar{y}$	s^2	$\log_e s^2$
1	-	-	-	-	10.0	18.0	14.0	12.5	19.0	16.0	18.5	15.429	11.536	2.445
2	+	-	-	-	0.0	16.5	4.5	17.5	20.5	17.5	33.0	15.643	116.893	4.761
3	-	+	-	-	4.0	6.0	1.0	14.5	12.0	14.0	5.0	8.071	28.702	3.357
4	+	+	-	-	0.0	10.0	34.0	11.0	25.5	21.5	0.0	14.571	167.202	5.119
5	-	-	+	-	0.0	0.0	18.5	19.5	16.0	15.0	11.0	11.429	68.369	4.225
6	+	-	+	-	5.0	20.5	18.0	20.0	29.5	19.0	10.0	17.429	62.369	4.113
7	-	+	+	-	6.5	18.5	7.5	6.0	0.0	10.0	0.0	6.929	40.119	3.692
8	+	+	+	-	16.5	4.5	0.0	23.5	8.0	8.0	8.0	9.789	61.071	4.112
9	-	-	-	+	4.5	18.0	14.5	10.0	0.0	17.5	6.0	10.071	47.786	3.867
10	+	-	-	+	19.5	18.0	16.0	5.5	10.0	7.0	36.0	16.000	107.250	4.675
11	-	+	-	+	15.0	16.0	8.5	0.0	0.5	9.0	3.0	7.429	42.869	3.758
12	+	+	-	+	41.5	39.0	6.5	3.5	7.0	8.5	36.0	20.286	305.738	5.723
13	-	-	+	+	8.0	4.5	6.5	10.0	13.0	41.0	14.0	13.857	154.726	5.042
14	+	-	+	+	21.5	10.5	6.5	0.0	15.5	24.0	16.0	13.429	70.786	4.260
15	-	+	+	+	0.0	0.0	0.0	4.5	1.0	4.0	6.5	2.286	7.155	1.968
16	+	+	+	+	18.0	5.0	7.0	10.0	22.5	18.5	8.0	14.214	94.071	4.544

(Columns A–D are headed "Factors".)

Table 1.2: Factors and Levels for Example 1.1

	Factor	Level $-$	Level $+$
A	Length of put (ft)	10	30
B	Types of putter	Mallet	Cavity back
C	Break of putt	Straight	Breaking
D	Slope of putt	Level	Downhill

1.1.2 Two-level Fractional Factorial Designs

When the number of factor k is large, the number of runs required for the 2^k factorial designs also increases. For economic reasons and/or in the absence of enough resources to carry out the full factorial experiment, a fraction (say 2^{-p}) of 2^k factorial design is often used. This type of two-level factorial designs is called 2^{k-p} factorial design, where p is any positive integer less than k.

Two-level fractional factorial designs can be regular and non-regular fractional factorial designs (Wu and Hamada, 2000). A regular fractional factorial design is formed through defining relations among factors. That is, the design is constructed by assigning p of the k factors to the interaction columns of the 2^{k-p} full factorial design. Fractional factorial designs that are not regular are non-regular designs.

This thesis focuses on regular two-level fractional factorial designs. The following is an example of a regular 2^{k-p} factorial designs.

Example 1.2. *This example is taken from Montgomery (2001, p. 352). This experiment is a regular 2^{6-3} factorial design with 3 replicates for each run. The experiment is based on the use of carbon anodes in a smelting process baked in a ring furnace. The six factors used are: Pitch/Fines ratio (A), Packing material type (B), Packing material temperature (C), Flue location (D), Pit temperature (E) and Delay time before packing (F). The response recorded is the weight of packing material stuck to the anodes measured in grams. The purpose of this experiment is to minimize the variability in the weight of the packing material while maintaining the weight of the material to a certain nominal level. Tables 1.3 and 1.4 below contain the data, and Factors and Levels for this example.*

Table 1.3: Design Matrix and Weight of Packing Material for Example 1.2

	Factors						Weigth of Material					
Runs	A	B	C	D(=AB)	E(=AC)	F(=BC)		y		$\bar{y}$	s^2	$\log_e s^2$
1	+	+	-	+	-	-	984	826	936	915.333	6561.333	8.789
2	+	+	+	+	+	+	1275	976	1457	1236.000	58981.000	10.985
3	-	+	-	-	+	-	1217	1201	890	1102.667	33984.333	10.434
4	+	-	-	-	-	+	1474	1164	1541	1393.000	40453.000	10.608
5	-	-	-	+	+	+	1320	1156	913	1129.667	41932.333	10.644
6	-	-	+	+	-	-	765	705	821	763.667	3365.333	8.121
7	+	-	+	-	+	-	1338	1254	1294	1295.333	1765.333	7.476
8	-	+	+	-	-	+	1325	1299	1253	1292.333	1329.333	7.192

Table 1.4: Factors and Levels for Example 1.2

	Factor	Level $-$	Level $+$
A	Pitch/Fines ratio	0.45	0.55
B	Packing material type	1	2
C	Packing material temperature	Ambient	325^0C
D	Flue location	Inside	Outside
E	Pit temperature	Ambient	195^0C
F	Delay time before packing	Zero	24 hours

In this example, $k = 6$ and $p = 3$. The three factors A, B and C generate a 2^3 factorial design. Factors D, E and F are defined through the following relationship:

$$D = AB,\ E = AC,\ F = BC.$$

That is, the column D is the interaction between column A and column B; column E is the interaction between column A and column C, and column F is the interaction between column B and column C.

1.2 Types of Responses and Two-Step Procedures

In general, responses can be classified according to the stated objective of the experiment. The three broad categories are: nominal-the-best, smaller-the better and larger-the-better responses.

1.2.1 Nominal-The-Best Response

A nominal-the-best response is a measured response or characteristic with a specific target (nominal) value. The response in Example 1.2 is a good example of the nominal-the-best response. For this case, one would like to minimize

$$E\{(Y-t)^2\} = Var(Y) + \{E(Y) - t\}^2,$$

where Y is the response and t is the nominal value. For the nominal-the-best response, a two-step procedure was introduced in Wu and Hamada (2000, 2009) as follows to select the levels of factors:

Step 1. Find the levels of the some factors to minimize the dispersion of response;

Step 2. Find the levels of some factors that are not in Step 1 to move the location of response closer to t.

For the above two two-step procedures, any factor appears in Step 2 is called an adjustment factor.

1.2.2 Smaller-The-Better Response

A smaller-the-better response is a measured characteristic with an ideal target value of 0. That is, as the value for this type of response decreases, quality increases. The response in Example 1.1 is a good example of the smaller-the-better response. The interest here would be to minimize

$$E(Y^2) = Var(Y) + E^2(Y).$$

For the smaller-the-better response, a two-step procedure is outlined in Wu and Hamada (2000, 2009) as follows:

Step 1. Find some factor levels that minimize the location of response;

Step 2. Find some factor levels that are not in Step 1 to minimize the dispersion of response.

Again, any factor appears in Step 2 is called an adjustment factor.

For the larger-the-better response, a similar two-step procedure can be found in Wu and Hamada (2000, 2009).

1.3 Model and Parameter Estimation

As we can see in Section 1.2, identifying the factors which have significant effects on the mean of response and variance of response is the first step before applying two-step procedures. In this section, we introduce some notations and setup the model.

1.3.1 Model Setup

Let y_{ij} be the response for the ith treatment and jth replication in regular two-level factorial experiments, $i = 1, \cdots, m$; $j = 1, \cdots, n_i$. Here $m = 2^k$ or 2^{k-p}. For the convenience of presentation, we consider the case when $n_1 = \cdots = n_m = n$ as in Examples 1.1 and 1.2. Suppose there are I effects that we are interested in (in most cases $I = m - 1$). These effects can be main effects or interaction effects. Let $x_{i1}, x_{i2}, \cdots, x_{iI}$ denote the corresponding covariates values of these interested I effects for the ith treatment, $i = 1, \cdots, m$.

The popular model for modelling the mean and variance of the response simultaneously is the normal model with a linear regression for the mean and a log-linear model for the variance (Harvey, 1976; Cook and Weisberg, 1983; Nair and Pregibon, 1988; Wang, 1989; Brenneman and Nair, 2001; Variyath et al., 2005; Wu and Hamada, 2000 and 2009; Loughin and Rodríquez, 2011; and others). That is,

$$y_{ij} \sim N\left(\mu_i, \sigma_i^2\right) \tag{1.1}$$

where

$$\mu_i = \alpha_0 + \alpha_1 x_{i1} + ... + \alpha_I x_{iI} \tag{1.2}$$

and

$$\log_e \sigma_i^2 = \gamma_0 + \gamma_1 x_{i1} + ... + \gamma_I x_{iI}. \tag{1.3}$$

Further, $y_{ij}'s$ are independent. Here $\alpha_1, \cdots, \alpha_I$ are the interested effects for the location or mean of response; $\gamma_1, \cdots, \gamma_I$ are the interested effects for the dispersion or the variance of the response. The models in (1.2) and (1.3) are called the location model and the dispersion model, respectively.

Before we end this subsection, we use Example 1.2 to illustrate the notations above. In Example 1.2, $m = 8$, $n = 3$ and suppose that we are interested in $I = 8 - 1 = 7$ factorial effects: six main effects and one two-factor interaction between factors A and F. The covariate values for the 7 factorial effects are:

$$\begin{pmatrix} A & B & C & D & E & F & AF \\ 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 & -1 & 1 & -1 \end{pmatrix}.$$

1.3.2 Parameter Estimation

To fit model (1.1), we first summarize y_{ij} to

$$\bar{y}_i = \sum_{j=1}^{n} y_{ij}/n \quad \text{and} \quad s_i^2 = \sum_{j=1}^{n} (y_{ij} - \bar{y}_i)^2/(n-1).$$

Next, we regress $\bar{y}_i$ over $\{x_{i1}, \cdots, x_{iI}\}$, $i = 1, \ldots, n$, to obtain least square estimates of $\alpha_1, \cdots, \alpha_I$ (denoted by $\hat{\alpha}_1, \cdots, \hat{\alpha}_I$). We also regress $\log_e s_i^2$ over $\{x_{i1}, \cdots, x_{iI}\}$

to obtain least square estimates of $\gamma_1, \cdots, \gamma_I$ (denoted by $\hat{\gamma}_1, \cdots, \hat{\gamma}_I$). Let

$$\mathbf{X} \;=\; \begin{pmatrix} x_{11} & \cdots & x_{1I} \\ \cdots & \cdots & \cdots \\ x_{m1} & \cdots & x_{mI} \end{pmatrix}$$

be the matrix consisting of columns corresponding to all the I effects that we are interested in. In two-level full factorial designs and two-level regular fractional factorial designs, $\mathbf{X}$ is an orthogonal matrix such that $\mathbf{X}^T\mathbf{X} = m\mathbf{I}$, where $\mathbf{I}$ is an identity matrix. Further, for each column of $\mathbf{X}$, $m/2$ elements equal -1 and other $m/2$ elements equal 1. Then,

$$\begin{pmatrix} \hat{\alpha}_1 \\ \cdots \\ \hat{\alpha}_I \end{pmatrix} = \frac{1}{m}\mathbf{X}^T\mathbf{Z}_L \quad \text{and} \quad \begin{pmatrix} \hat{\gamma}_1 \\ \cdots \\ \hat{\gamma}_I \end{pmatrix} = \frac{1}{m}\mathbf{X}^T\mathbf{Z}_D, \tag{1.4}$$

where $\mathbf{Z}_L = (\bar{y}_1, \cdots, \bar{y}_m)^T$ and $\mathbf{Z}_D = (\log_e s_1^2, \cdots, \log_e s_m^2)^T$.

Let $\mathbf{x}_l$ denote the column for lth interested effects in $\mathbf{X}$, $l = 1, \cdots, I$. Then, from (1.4), we have

$$\hat{\alpha}_l = \frac{1}{m}\mathbf{x}_l^T\mathbf{Z}_L \quad \text{and} \quad \hat{\gamma}_l = \frac{1}{m}\mathbf{x}_l^T\mathbf{Z}_D, \quad \text{respectively.} \tag{1.5}$$

1.4 Review of Existing Methods

After we obtain the estimates of $\alpha_l's$ and $\gamma_l's$, we can use them to construct hypothesis testing procedures to identify the significant effects for both location and dispersion models. We introduce two important concepts, namely individual error rate (IER) and experiment-wise error rate (EER). Generally speaking, IER is the probability of making an error for a single hypothesis, while EER is the probability of making at least one error for all I hypotheses.

In the literature, there are three well-known hypothesis testing methods for identifying the significant effects for the location and dispersion models in replicated two-level factorial designs: Wu and Hamada (2000, 2009) methods, Variyath et al. (2005) method and Lenth (1989) method.

1.4.1 Wu and Hamada's Methods

To introduce Wu and Hamada's methods, we need to investigate the expectation and variance of $\hat{\alpha}_l' s$, and $\hat{\gamma}_l' s$. By using (1.5), Wu and Hamada showed that

$$E(\hat{\alpha}_l) = \alpha_l \tag{1.6}$$

and

$$\begin{aligned} \mathrm{Var}(\hat{\alpha}_l) &= \frac{\mathbf{x}_l^T Var(\mathbf{Z}_L)\mathbf{x}_l}{m^2} = \frac{1}{m^2}\mathbf{x}_l^T Var\{(\bar{y}_1, \ldots, \bar{y}_m)^T\}\mathbf{x}_l \\ &= \frac{1}{m^2 n}\mathbf{x}_l^T \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_m^2\}\mathbf{x}_l = \frac{1}{m^2 n}\sum_{i=1}^{m}\sigma_i^2. \end{aligned} \tag{1.7}$$

The last step in (1.7) is from the fact that the elements of $\mathbf{x}_l$ are either -1 or 1 in two-level experiments. Similarly, they showed that

$$E(\hat{\gamma}_l) = \gamma_l \tag{1.8}$$

and

$$\begin{aligned} \mathrm{Var}(\hat{\gamma}_l) &= \frac{\mathbf{x}_l^T \mathrm{Var}(\mathbf{Z}_D)\mathbf{x}_l}{m^2} = \frac{1}{m^2}\mathbf{x}_l^T \mathrm{Var}\{(\log_e s_1^2, \ldots, \log_e s_m^2)^T\}\mathbf{x}_l \\ &= \frac{1}{m^2}\mathbf{x}_l^T \mathrm{diag}\{\mathrm{Var}(\log_e s_1^2), \ldots, \mathrm{Var}(\log_e s_m^2)\}\mathbf{x}_l. \end{aligned}$$

Again using the fact that the elements of $\mathbf{x}_l$ are either -1 or 1 in two-level experiments, we further have

$$\mathrm{Var}(\hat{\gamma}_l) = \frac{1}{m^2} \sum_{i=1}^{m} \mathrm{Var}(\log_e s_i^2). \tag{1.9}$$

Dispersion Model

Wu and Hamada noted that

$$(n-1)s_i^2 = \sum_{j=1}^{n} (y_{ij} - \bar{y}_i)^2 \sim \sigma_i^2 \chi_{n-1}^2,$$

where χ_v^2 is the chi-squared distribution with v degrees of freedom. Then taking natural logarithm yields

$$\log_e(s_i^2) \sim \log_e(\sigma_i^2) + \log_e\{\chi_{n-1}^2/(n-1)\}.$$

Using the first-order Taylor expansion, they argued that approximately

$$\log_e(s_i^2) \sim N(\log_e(\sigma_i^2), \frac{2}{n-1}). \tag{1.10}$$

By using (1.8), (1.9) and (1.10), they obtained that approximately $\hat{\gamma}_l$ has the following distribution

$$\hat{\gamma}_l \sim N(\gamma_l, \frac{2}{m(n-1)}).$$

A z-type test statistic

$$z_l = \frac{\hat{\gamma}_l}{\sqrt{\frac{2}{m(n-1)}}}$$

was constructed to test the hypothesis $H_0 : \gamma_l = 0$. To control IER, the $N(0,1)$ distribution is used to calculate the critical value of the z-type test statistic. To control EER, they used the studentized maximum modulus distribution with two parameters I and ∞ to calculate the critical value.

Location Model

In this case, Wu and Hamada (2000, 2009) via (1.6) and (1.7), obtained the distribution of $\hat{\alpha}_l$ as

$$\hat{\alpha}_l \sim N(\alpha_l, \frac{1}{m^2 n} \sum_{i=1}^{m} \sigma_i^2).$$

They construct a t-type test statistic

$$t_l = \frac{\hat{\alpha}_l}{\sqrt{\frac{1}{m^2 n} \sum_{i=1}^{m} s_i^2}}$$

to test the hypothesis $H_0 : \alpha_l = 0$. Further, to control IER, the $t_{m(n-1)}$ distribution is suggested to calculate the critical value. To control EER, they used the studentized maximum modulus distribution with two parameters I and $m(n-1)$ to calculate the critical value.

1.4.2 Variyath et al.'s Method

Variyath et al. (2005) suggested a Jackknife method on the replicated responses to provide an estimate of variance of the performance measures such as the $\bar{y}$ and $\log_e s^2$ of the replicated responses at each run. The variance estimate of the performance measure is then used to estimate the variance of the estimated factorial effects. Their method was applied to control only the IER. To describe their method, we have adopted the notations used in their paper. Let $y_i = (y_{i1}, \ldots, y_{ij}, \ldots, y_{in})$ be the random sample of size n for each run. Let $c(y_i)$ be the performance measure of interest. Then by deleting y_{ij} from y_i for $j = 1, \ldots, n$, n delete-one Jackknife replicates of size $(n-1)$: $y_i(j)$, $i = 1$ ..., m was obtained. Hence, n Jackknife replications of the performance measure $c(y_i(j))$ were obtained. The Jackknife variance estimate of

$c(y_i)$ was given as

$$\hat{V}_{ja}(c(y_i)) = \frac{n-1}{n}\sum_{j=1}^{n}(c(y_i(j)) - c(y_{i.}))^2,$$

where $c(y_{i.}) = \frac{1}{n}\sum_{j=1}^{n} c(y_i(j))$. A pooled estimate of the variance of $c(y_i)$ was given as

$$\hat{V}_{pja}(c(y)) = \frac{1}{m}\sum_{i=1}^{m}\hat{V}_{ja}(c(y_i)).$$

They construct a F-statistic

$$F = \frac{\text{Mean Square (MS) for the factorial effect}}{\hat{V}_{pja}(c(y))}$$

to test the null hypothesis of interest. They provide some theoretical explanations to show that the mean square of the factorial effect and $\hat{V}_{pja}(c(y))$ are independent. The F-distribution with degrees of freedom 1 and $m(n-1)$ is used to calculate the critical value of the above F-statistic.

Dispersion Model

Here, $c(y_i) = \log_e s_i^2$ and $\hat{V}_{pja}(c(y)) = \frac{1}{m}\sum_{i=1}^{m}\hat{V}_{ja}(\log_e s_i^2)$. The F-statistic is given as

$$F_l = \frac{\text{MS}(\hat{\gamma}_l)}{\frac{1}{m}\sum_{i=1}^{m}\hat{V}_{ja}(\log_e s_i^2)}$$

to test the hypothesis $H_0 : \gamma_l = 0$, where $\text{MS}(\hat{\gamma}_l) = \hat{\gamma}_l^2(\mathbf{x_l}^T\mathbf{x_l}) = m\hat{\gamma}_l^2$. Based on their simulation results for consistency, the Jacknife variance estimate for $\log_e(s_i^2)$ is consistent when $n = 50$. For small n, they considered an adjustment factor for the variance estimate of $\log_e s_i^2$.

Location Model

Here, $c(y_i) = \bar{y}_i$ and $\hat{V}_{pja}(c(y)) = \frac{1}{m}\sum_{i=1}^{m}\hat{V}_{ja}(\bar{y}_i)$. The F-statistic to test the hypothesis $H_0 : \alpha_l = 0$ can be written as

$$F_l = \frac{\mathrm{MS}(\hat{\alpha}_l)}{\frac{1}{m}\sum_{i=1}^{m} \hat{V}_{ja}(\bar{y}_i)}, \tag{1.11}$$

where $\mathrm{MS}(\hat{\alpha}_l) = \hat{\alpha}_l^2(\mathbf{x_l}^T\mathbf{x_l}) = m\hat{\alpha}_l^2$ in this case.

1.4.3 Lenth's Method

Lenth (1989) proposed a robust estimator of the standard deviation of the factorial effects of interest. His approach is the same for both the dispersion and location models. We describe his method for dispersion model only. Suppose $\hat{\gamma}_1, \ldots, \hat{\gamma}_I$ are the least square estimates of factorial effects $(\gamma_1, \ldots, \gamma_I)$ of interest in the dispersion model.

Lenth (1989) proposed a pseudo standard error (PSE) for the standard deviation of $\hat{\gamma}_l$ as

$$\mathrm{PSE} = 1.5\ \mathrm{Median}_{\{|\hat{\gamma}_l| < 2.5 s_0\}}|\hat{\gamma}_l|. \tag{1.12}$$

Here the median is computed among the $|\hat{\gamma}_l' s|$ with $|\hat{\gamma}_l| < 2.5 s_0$ and $s_0 = 1.5\mathrm{Median}|\hat{\gamma}_l|$. He defined a t-type statistic

$$t_{\mathrm{Lenth},l} = \frac{\hat{\gamma}_l}{PSE}$$

to test the hypothesis $H_0 : \gamma_l = 0$.

Lenth's method does not require an unbiased estimate of variance of response. For this reason, researchers have used his method for both un-replicated and replicated factorial experiments. The critical values for controlling the IER and EER are given in the Tables of Appendix H of Wu and Hamada (2000, 2009).

1.5 Motivation and Organization of The Thesis

In this section, we first state the motivation of the thesis and then give the outline of the thesis. Our motivation is from the following observations for the above mentioned methods.

1. For dispersion model, the validity of the $N(0,1)$ distribution for the z-type statistic suggested by Wu and Hamada depends on the approximation of the variance of $\log_e(s_i^2)$. When n is large, the approximation $2/(n-1)$ to the $\text{Var}\{\log_e(s_i^2)\}$ is reasonable. If n is small, which is the common situation in practice (such as in Examples 1.1 and 1.2), then the approximation may not be good. For example, when $n = 3$, the actual variance of $\log_e(s_i^2)$ is around 1.64, but the approximate variance is $2/(3-1) = 1$. If we use the approximate variance, the statistic will be inflated. Also, the Variyath et al. Jacknife variance estimate for $\log_e(s_i^2)$ is not consistent for small n and this may also inflate their suggested F statistic.

2. For location model, the validity of the t-distribution suggested by Wu and Hamada for the t-type statistic depends on the homogeneity of σ_i^2's. But in some practical situations, heterogeneity of σ_i^2's is a real possibility. If they are not the same, the t-distribution may not be true. Also the connection between Wu and Hamada's method and Variyath et al.'s method is not clear so far.

In this thesis, we will follow the line of Wu and Hamada's methods. Our purpose is two-fold.

1. We intend to identify the distribution for the z-type statistic under the null hypothesis for the dispersion model. This distribution should work well for the small n case. Based on this distribution, we develop corresponding procedures to control the IER and EER in the dispersion model. This distribution will

also be used to calculate the P-values of the I z-type statistic and those P-values, in turn, are used to control the false discovery rate (FDR; Benjamini and Hochberg, 1995) in the dispersion model.

2. We intend to identify the distribution for the t-type statistic under the null hypothesis for the location model. This distribution should work well whether σ_i^2's are homogeneous or not. Based on this distribution, we develop corresponding procedures to control the IER and EER in the location model. This distribution will also be used to calculate the P-values of the I t-type statistic and those P-values, in turn, are used to control the FDR in the location model.

The thesis is organized as follows. In Chapter 2, we present the new distributions for the z-type and t-type statistics, respectively, and propose new procedures for controlling the IER and EER. Also, we perform some simulation studies to compare the performance of the methods in controlling IER and EER. Further, we apply the methods to real data sets given in Examples 1.1 and 1.2. In Chapter 3, the new distributions are used to calculate the P-values of z-type and t-type statistics, respectively. Those P-values are used to control the FDR using some existing procedures. Simulation studies are used to examine the performance of all the methods in controlling the FDR. We also apply the methods to real data set given in Examples 1.1 and 1.2. Chapter 4 is the closing chapter of this thesis. It contains the summary and conclusion on the performance of the methods. We provide some directions for further research.

Chapter 2

Controlling IER and EER in Location and Dispersion Models

2.1 Dispersion Model

2.1.1 New Distribution of z-type statistic

Recall that for testing $H_0 : \gamma_l = 0$ in the dispersion model, the z-type statistic is defined as

$$z_l = \frac{\hat{\gamma}_l}{\sqrt{\frac{2}{m(n-1)}}}.$$

The actual variance of $\hat{\gamma}_l$ is given in (1.9) as

$$\text{Var}(\hat{\gamma}_l) = \frac{1}{m^2} \sum_{i=1}^{m} \text{Var}(\log_e s_i^2).$$

For the small n case, instead of using the approximation $2/(n-1)$ to $\text{Var}(\log_e s_i^2)$, we suggest using the exact variance of the $\log_e(s_i^2)$. Note that

$$\log_e(s_i^2) \sim \log_e(\sigma_i^2) + \log_e(\chi^2_{n-1}/(n-1)).$$

Therefore

$$\mathrm{Var}\{\log_e(s_i^2)\} = \mathrm{Var}\{\log_e(\frac{\chi^2_{n-1}}{n-1})\} = \mathrm{Var}\{\log_e(\chi^2_{n-1})\}$$

and

$$\mathrm{Var}(\hat{\gamma}_l) = \frac{1}{m}\mathrm{Var}\{\log_e(\chi^2_{n-1})\}.$$

Here $\mathrm{Var}\{\log_e(\chi^2_{n-1})\}$ means the variance of the logarithm of a random variable from the χ^2_{n-1} distribution.

Let

$$a_n = \sqrt{\frac{\mathrm{Var}\{\log_e(\chi^2_{n-1})\}}{2/(n-1)}},$$

which is just the square root of the ratio of true variance over the approximate variance for $\log_e(s_i^2)$. The values of a_n for some small n values are tabulated in Table 2.1.

We observe that the z-type test statistic can be written as

$$z_l = \frac{\hat{\gamma}_l}{\sqrt{\frac{1}{m}\mathrm{Var}\{\log_e(\chi^2_{n-1})\}}}\sqrt{\frac{\mathrm{Var}\{\log_e(\chi^2_{n-1})\}}{2/(n-1)}} = a_n\frac{\hat{\gamma}_l}{\sqrt{\mathrm{Var}(\hat{\gamma}_l)}}.$$

Motivated from the above form of the z-type statistic, we suggest using $N(0, a_n^2)$ to approximate the true distribution of the z-type statistic under the null hypothesis $H_0 : \gamma_l = 0$.

Table 2.1: Comparison of the Exact and Approximate variances of $\log_e(s_i^2)$

	n							
Variance	3	4	5	6	7	8	9	10
Exact	1.644	0.935	0.645	0.490	0.395	0.330	0.284	0.249
Approximate	1.000	0.667	0.500	0.400	0.333	0.286	0.250	0.222
a_n	1.282	1.184	1.140	1.107	1.089	1.075	1.065	1.058

We should point out here that the true distribution of $\hat{\gamma}_l$ may not be normal. However, since $\hat{\gamma}_l$ as given in (1.5) is a linear combination of m independent identically distributed log-transformed random variables, central limit theorem implies that the distribution of $\hat{\gamma}_l$ may be well approximated by the normal distribution. Simulation studies show that the normal approximation works very well even for small n and m, for example $m = 8$ and $n = 3$.

From Table 2.1, our suggested distribution $N(0, a_n^2)$ and the suggested distribution $N(0, 1)$ by Wu and Hamada (2000, 2009) can be quite different for small n. The two distributions become close to each other as n becomes large. However, the suggested distribution based on the exact variance of $\log_e(s_i^2)$ may be preferable for small n in practical applications. This has been verified in the simulation study.

2.1.2 Controlling IER and EER in Dispersion Model

With the suggested distribution $N(0, a_n^2)$, if we would like to control the IER at the given α level, we can set the critical value C_{IER} to be the upper $1 - \alpha/2$ quantile of the $N(0, a_n^2)$. That is,

$$C_{IER} = a_n \Phi^{-1}(1 - \alpha/2).$$

Here $\Phi(\cdot)$ is the cumulative distribution function of $N(0, 1)$.

For controlling EER, we note that for

$$\begin{aligned} EER &= Pr(\max_{1 \le l \le I} |z_l| \ge C_{EER} | H_0 : \gamma_1 = \cdots = \gamma_I = 0) \\ &= 1 - Pr(\max_{1 \le l \le I} |z_l/a_n| < C_{EER}/a_n | H_0 : \gamma_1 = \cdots = \gamma_I = 0) \\ &= 1 - \{\Phi(C_{EER}/a_n) - \Phi(-C_{EER}/a_n)\}^I \\ &= 1 - \{2\Phi(C_{EER}/a_n) - 1\}^I. \end{aligned}$$

Here C_{EER} is the critical value for controlling EER. Therefore if we need to control the EER at the given α level, then

$$C_{EER} = a_n \Phi^{-1}\left(0.5 + 0.5(1 - \alpha)^{1/I}\right).$$

2.2 Location Model

2.2.1 New distribution for the t-type Statistic

Recall that the t-type test statistic for testing $H_0 : \alpha_l = 0$ in the location model is given as

$$t_l = \frac{\hat{\alpha}_l}{\sqrt{\frac{1}{m^2 n}\sum_{i=1}^m s_i^2}}$$

and the variance of $\hat{\alpha}_l$ is $\mathrm{Var}(\hat{\alpha}_l) = \frac{1}{m^2 n}\sum_{i=1}^m \sigma_i^2$. Next we try to find the distribution of t_l under the null hypothesis.

Note that the t-type test statistic can be rewritten as

$$t_l = \frac{\hat{\alpha}_l \Big/ \sqrt{\frac{1}{m^2 n}\sum_{i=1}^m \sigma_i^2}}{\sqrt{\frac{1}{m^2 n}\sum_{i=1}^m s_i^2} \Big/ \sqrt{\frac{1}{m^2 n}\sum_{i=1}^m \sigma_i^2}}. \tag{2.1}$$

The classical theory of normal distribution implies that the numerator of (2.1) is independent of the denominator. Further, under the null hypothesis of $\alpha_l = 0$, it follows the $N(0,1)$. That is

$$\hat{\alpha}_l \Big/ \sqrt{\frac{1}{m^2 n}\sum_{i=1}^m \sigma_i^2} \sim N(0,1).$$

The denominator of (2.1) can be further expressed as

$$\frac{\sum_{i=1}^m s_i^2}{\sum_{i=1}^m \sigma_i^2} = \frac{1}{n-1}\sum_{i=1}^m \left[\frac{(n-1)s_i^2}{\sigma_i^2}\left(\frac{\sigma_i^2}{\sum_{i=1}^m \sigma_i^2}\right)\right].$$

Let

$$\rho_i^2 = \frac{\sigma_i^2}{\sum_{i=1}^m \sigma_i^2}.$$

Note that for $i = 1, \ldots, m$

$$\frac{(n-1)s_i^2}{\sigma_i^2} \sim \chi^2_{n-1}$$

and they are independent. Therefore the denominator of (2.1) follows a weighted sum of m independent χ^2_{n-1} distribution. For the convenience of presentation, we write in the following way:

$$\frac{\sum_{i=1}^m s_i^2}{\sum_{i=1}^m \sigma_i^2} \sim \sum_{i=1}^m \rho_i^2 \chi^2_{n-1}/(n-1).$$

Therefore the distribution of the t-type statistic under the null hypothesis of $\alpha_l = 0$ is

$$t_l \sim \frac{N(0,1)}{\sqrt{\sum_{i=1}^m \rho_i^2 \chi^2_{n-1}/(n-1)}}. \tag{2.2}$$

In the above form, the $N(0,1)$ and m χ^2_{n-1} distributions are independent.

Remark 2.1. *If the σ_i^2's are homogeneous, then*

$$\frac{\sigma_i^2}{\sum_{i=1}^m \sigma_i^2} = \frac{\sigma^2}{m\sigma^2} = \frac{1}{m}.$$

Therefore,

$$\frac{\sum_{i=1}^m s_i^2}{\sum_{i=1}^m \sigma_i^2} \sim \frac{\chi^2_{m(n-1)}}{m(n-1)}.$$

Thus, under $H_0 : \alpha_l = 0$, we obtain

$$t_l \sim \frac{N(0,1)}{\sqrt{\frac{\chi^2_{m(n-1)}}{m(n-1)}}} = t_{m(n-1)}.$$

But if the σ_i^2 are not homogeneous, then the distribution of t_l may not be t-distribution under the null hypothesis.

2.2.2 Controlling IER in Location Model

The explicit form of the cumulative distribution function of t_l in (2.2) is unknown if σ_i^2's are not homogeneous. But it suggests a way to generate the random sample from this distribution, which can be used to calculate the critical value for controlling

IER. In the following steps, we propose a resampling procedure to generate random samples from the distribution in (2.2). Since ρ_i^2's are unknown, we estimate it from the given data by $\hat{\rho}_i^2 = s_i^2 / \sum_{i=1}^m s_i^2$.

Step 1: Compute $\hat{\rho}_i^2$, for $i = 1, 2, \ldots, m$, from the given data set.

Step 2: For $b = 1, \ldots, M$,

Step 2.1 Generate one $N(0, 1)$ random variable U_b.

Step 2.2: Generate m independent χ^2_{n-1} random variables $V_{b1}, \ldots, V_{bm}$.

Step 2.3: Compute $t^{(b)} = \frac{U_b}{\sqrt{\sum_{i=1}^m V_{bi}\hat{\rho}_i^2/(n-1)}}$.

Step 3: The critical value C_{IER} for controlling IER in the location model at the given α value is set to be the $1-\alpha/2$ upper quantile of $\{t^{(b)}, b = 1, 2, ..., M\}$.

In R, it is very fast to calculate the critical value C_{IER}. With our R function, it takes several seconds to get the C_{IER} for $M = 1,000,000$. The R function will be provided in the Appendix.

2.2.3 Controlling EER in Location Model

Here, we are interested in controlling

$$\text{EER} = Pr(\max_l |t_l| \geq C_{EER} | H_0 : \alpha_1 = \cdots = \alpha_I = 0)$$

at a given α level.

We first use the result in Section 2.2.1 to investigate the distribution of $\max_{1 \leq l \leq I} |t_l|$.

Note that

$$\begin{aligned}
\max_{1\le l\le I}|t_l| &= \max_{1\le l\le I}\frac{\left|\hat{\alpha}_l\Big/\sqrt{\frac{1}{m^2n}\sum_{i=1}^m\sigma_i^2}\right|}{\sqrt{\frac{1}{m^2n}\sum_{i=1}^m s_i^2}\Big/\sqrt{\frac{1}{m^2n}\sum_{i=1}^m\sigma_i^2}} \\
&= \frac{\max_{1\le l\le I}\left|\hat{\alpha}_l\Big/\sqrt{\frac{1}{m^2n}\sum_{i=1}^m\sigma_i^2}\right|}{\sqrt{\frac{1}{m^2n}\sum_{i=1}^m s_i^2}\Big/\sqrt{\frac{1}{m^2n}\sum_{i=1}^m\sigma_i^2}}. \qquad (2.3)
\end{aligned}$$

The distribution of the denominator of (2.3) has been investigated in Section 2.2.1. We now study the distribution of the numerator. Note that

$$(\hat{\alpha}_1,\cdots,\hat{\alpha}_I)^T = \frac{1}{m}\mathbf{X}^T\mathbf{Z}_L$$

with $\mathbf{Z}_L = (\bar{y}_1,\cdot\cdot\cdot,\bar{y}_m)^T$. Under the normal assumption in (1.1) on y_{ij}, we have that $\mathbf{Z}_L$ follows a multivariate normal distribution. Due to the properties of multivariate normal distribution, we also have that $(\hat{\alpha}_1,\cdots,\hat{\alpha}_I)^T$ is multivariate normally distributed. Under the null hypothesis $H_0:\alpha_1=\cdots=\alpha_I=0$,

$$(\hat{\alpha}_1,\cdots,\hat{\alpha}_I)^T \sim \mathbf{MVN}(\mathbf{0},\frac{1}{m^2}\mathbf{X}^T\mathrm{Var}(\mathbf{Z}_L)\mathbf{X}).$$

With the fact that $\mathrm{Var}(\mathbf{Z}_L)=\mathrm{diag}\{\sigma_1^2/n,\ldots,\sigma_m^2/n\}$, we have

$$\left(\frac{\hat{\alpha}_1}{\sqrt{\frac{1}{m^2n}\sum_{i=1}^m\sigma_i^2}},\ldots,\frac{\hat{\alpha}_I}{\sqrt{\frac{1}{m^2n}\sum_{i=1}^m\sigma_i^2}}\right)^T \sim \mathbf{MVN}(\mathbf{0},\mathbf{X}^T\mathrm{diag}\{\rho_1^2,\ldots,\rho_m^2\}\mathbf{X})$$

Here, "**MVN**" stands for the multivariate normal distribution.

Combining the distributions of the numerator and denominator of (2.3), we get that $\max_{1\le l\le L}|t_l|$ has the same distribution as the ratio U/V such that

(1) U and V are independent;

(2) U has the same distribution as the maximum of the absolute values of a I-

dimensional multivariate normal random vector with mean vector 0 and variance-covariance matrix $\mathbf{X}^T\text{diag}\{\rho_1^2, \ldots, \rho_m^2\}\mathbf{X}$;

(3) V follows the weighted sum of m independent χ^2_{n-1} distributions, $\sum_{i=1}^m \rho_i^2 \chi^2_{n-1}/(n-1)$.

The explicit form of the cumulative distribution function of $\max_{1\leq l\leq L}|t_l|$ may be unknown. But, it suggests a way to generate the random sample from the distribution as follows.

Step 1: Compute $\hat{\rho}_i^2$, for $i = 1, 2, \ldots, m$, from the given data set.

Step 2: For $b = 1, \ldots, M$,

Step 2.1 Generate a I-dimensional random vector $(U_{b1}, \cdots, U_{bI})^T$ from the multivariate normal distribution with mean vector 0 and variance-covariance matrix $\mathbf{X}^T\text{diag}\{\hat{\rho}_1^2, \ldots, \hat{\rho}_m^2\}\mathbf{X}$.

Step 2.2: Generate m independent χ^2_{n-1} random variables $V_{b1}, \ldots, V_{bm}$.

Step 2.3: Compute $t_l^{(b)} = \frac{U_{bl}}{\sqrt{\sum_{i=1}^m V_{bi}\hat{\rho}_i^2/(n-1)}}$, $l = 1, \ldots, I$.

Step 2.4: Compute $\max_{1\leq l\leq I}|t_l^{(b)}|$.

Step 3: The critical value C_{EER} for controlling EER in the location model at the given α value is set to be the $1-\alpha/2$ upper quantile of $\{\max_{1\leq l\leq I}|t_l^{(b)}|, b = 1, 2, ..., M\}$.

We give two remarks here. First, we can show that if σ_i^2's are homogeneous, $\max_{1\leq l\leq I}|t_l|$ follows a studentized maximum modulus distribution with two parameters I and $m(n-1)$, which is suggested by Wu and Hamada to control the EER in location model. However, if σ_i^2's are not homogeneous, $\max_{1\leq l\leq I}|t_l|$ no longer follows this distribution. Our method can be applied to both situations. Second, the computation cost again is very cheap. Our R function in the Appendix only take several seconds to obtain C_{EER} when $M = 10,000$.

2.2.4 Connection between Wu and Hamada's method and Variyath et al.'s method

Both Wu and Hamada's method and Variyath et al.'s method can be applied to control the IER in the location model. In this subsection, we present the connection between these two methods in the following proposition.

Proposition 2.1. *For testing $H_0 : \alpha_l = 0$, we have*

$$t_l^2 = F_l.$$

Here F_l is defined in (1.11). Therefore the two methods are equivalent for controlling the IER.

Proof. Recall that the Jackknife variance estimate of performance measure of interest, $c(y_i)$, is given as

$$\hat{V}_{ja}(c(y_i)) = \frac{n-1}{n}\sum_{j=1}^{n}(c(y_i(j)) - c(y_{i.}))^2,$$

where $c(y_{i.}) = \frac{1}{n}\sum_{j=1}^{n} c(y_i(j))$.

For the location model, we have $c(y_i) = \bar{y}_i$,

$$c(y_i(j)) = \frac{\sum_{k\neq j} y_{ik}}{n-1} = \frac{\sum_{k=1}^{n} y_{ik} - y_{ij}}{n-1} = \frac{n\bar{y}_i - y_{ij}}{n-1},$$

and

$$c(y_{i.}) = \frac{1}{n}\sum_{j=1}^{n} c(y_i(j)) = \frac{1}{n}\sum_{j=1}^{n}\left(\frac{n\bar{y}_i - y_{ij}}{n-1}\right) = \bar{y}_i$$

Therefore

$$\hat{V}_{ja}(c(y_i)) = \frac{n-1}{n}\sum_{j=1}^{n}\left[\frac{n\bar{y}_i - y_{ij}}{n-1} - \bar{y}_i\right]^2 = \frac{n-1}{n}\sum_{j=1}^{n}\left[\frac{\bar{y}_i - y_{ij}}{n-1}\right]^2 = \frac{\sum_{j=1}^{n}\left[y_{ij} - \bar{y}_i\right]^2}{(n-1)n}.$$

Thus, the Jackknife variance estimate for $c(y_i)$ now becomes

$$\hat{V}_{ja}(\bar{y}_i) = \frac{s_i^2}{n}.$$

Then, a pooled estimate of $c(y_i)$ is

$$\hat{V}_{pja}(c(y)) = \frac{1}{m}\sum_{i=1}^{m}\hat{V}_{ja}(c(y_i)) = \frac{1}{mn}\sum_{i=1}^{m}s_i^2.$$

Therefore, the F-statistic to test the hypothesis $H_0 : \alpha_l = 0$ can be written as

$$F_l = \frac{\mathrm{MS}(\hat{\alpha}_l)}{\frac{1}{m}\sum_{i=1}^{m}\hat{V}_{ja}(\bar{y}_i)} = \frac{\hat{\alpha}_l^2 m}{\frac{1}{mn}\sum_{i=1}^{m}s_i^2} = t_l^2. \tag{2.4}$$

The preceding expression on the right-hand side of (2.4) implies that the F-statistic proposed by Variyath et al. (2005) for the location model is the same as the square of t-type statistic of Wu and Hamada (2000, 2009). □

Since Wu and Hamada's method and Variyath et al.'s method are equivalent for controlling the IER in location model, Variyath et al.'s method shares the same problem as Wu and Hamada's method. That is, if σ_i^2's are not homogeneous, the true distribution of F_l may not be F-distribution.

2.3 Simulation Study

Here, a simulation study is carried out to compare the performance of the proposed methods for IER and EER with the three existing methods for both dispersion and location models.

2.3.1 Simulation Results in Dispersion Model

Results for IER

We compare the performance of our new method, Wu and Hamada's method, Variyath et al.'s method, and Lenth's method for controlling IER in the dispersion model.

In the simulation, we considered 2^3 and 2^4 factorial experiments. For a 2^3 experiment with three two-level factors A, B and C, we generate the data using the models

$$y_{ij} \sim N\left(0, \exp(0.35A + 0.3C + 0.3AC)\right)$$

and

$$y_{ij} \sim N(0, \exp(0.35A + 0.3C + 0.3BC))$$

where A, C, AC and BC take values ± 1 depending on the combination of factors levels. Since the mean of the response does not affect the procedures mentioned above, it is set to be 0 for each run. Then, we test the significance of the $I = 2^3 - 1 = 7$ factorial effects of interest at 5% level based on the above mentioned procedures. For $l = 1, \ldots, I$, the percentage of rejecting the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level by each method is calculated based on $N = 20,000$ repetitions. The results are summarized in Tables 2.2 and 2.3.

For a 2^4 factorial experiments with four two-level factors A, B, C, and D, we used the models

$$y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AC))$$

and

$$y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AD))$$

where A, B, C, AC and AD take values ± 1 depending on the combination of factors levels. We test the significance of each of $I = 2^4 - 1 = 15$ effects at 5% level based on all the four methods. The simulation is also repeated for $N = 20,000$ times. Then, the percentage of each factorial effect being declared significant at 5% level is recorded

Table 2.2: Percentage of rejecting the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level for model: $y_{ij} \sim N(0,\ \exp(0.35A + 0.3C + 0.3AC))$ in replicated 2^3 experiments

Effects	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
	Wu and Hamada's method				Our method			
A	50.6	65.7	77.6	85.9	33.5	53.8	70.2	81.2
B	12.7	9.5	8.1	7.5	5.3	5.0	4.9	5.1
C	42.2	54.1	65.2	74.8	25.8	41.6	56.6	68.6
AB	13.0	9.8	8.3	7.7	5.4	5.2	4.8	5.0
AC	41.3	54.8	65.2	74.8	25.5	42.3	56.2	68.3
BC	12.5	9.7	8.3	8.0	5.4	5.1	5.1	5.4
ABC	12.4	10.2	8.3	7.8	5.3	5.4	5.0	5.3
	Lenth's method				Variyath et al.'s method			
A	16.5	21.3	25.3	29.6	33.3	52.0	67.8	78.5
B	1.9	1.1	0.6	0.5	6.5	7.3	6.7	6.3
C	12.3	15.3	18.1	21.9	26.6	41.4	55.1	66.8
AB	2.0	0.9	0.7	0.4	6.7	7.0	6.6	6.7
AC	12.0	15.2	18.1	21.6	26.2	41.5	55.2	66.4
BC	2.0	1.0	0.7	0.5	6.7	6.9	7.0	6.6
ABC	2.0	1.2	0.6	0.5	6.6	7.1	6.9	6.8

Table 2.3: Percentage of rejecting the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level for model: $y_{ij} \sim N(0, \exp(0.35A + 0.3C + 0.3BC))$ in replicated 2^3 experiments

Effects	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
	Wu and Hamada's method				Our method			
A	51.2	65.4	77.8	85.8	33.7	53.5	70.0	80.8
B	12.7	9.8	8.3	7.8	5.3	5.1	4.9	5.1
C	41.9	53.9	65.7	74.6	25.6	41.5	56.6	67.9
AB	12.2	9.7	8.5	7.6	5.1	5.2	5.0	5.1
AC	12.4	9.5	8.6	8.2	5.4	5.1	5.1	5.2
BC	42.2	54.0	65.5	74.0	25.7	41.8	56.4	67.4
ABC	12.7	9.8	8.3	7.6	5.4	5.1	5.1	5.0
	Lenth's method				Variyath et al.'s method			
A	16.4	20.7	25.5	29.5	33.1	52.1	67.7	78.9
B	2.3	1.3	0.8	0.6	7.0	7.0	6.9	7.0
C	11.9	14.7	18.6	21.6	26.4	42.0	54.3	67.1
AB	1.9	1.1	0.7	0.4	6.7	7.1	6.9	6.9
AC	2.3	1.3	0.8	0.6	6.8	6.7	6.9	6.9
BC	11.9	15.0	18.5	21.5	26.4	41.6	55.4	66.9
ABC	2.2	1.4	0.8	0.5	7.0	7.0	6.9	6.7

in Tables 2.4 and 2.5.

From Tables 2.2, 2.3, 2.4 and 2.5, the simulated IERs for the factorial effects not in the models are quite close to the 5% nominal level by our new method. Wu and Hamada's method inflates the IER especially for small n. It becomes better as n increases. Lenth's method is quite conservative for controlling IER whether n is large or small. Variyath et al.'s method is also liberal for controlling IER. The performance is the same for all the n's we considered.

We emphasize that the z-type statistics are the same for our method and Wu and Hamada's method. The difference between the two methods are the suggested distributions for the z-type statistics. Therefore the simulation results above suggest that our suggested distribution is more accurate than the one suggested by Wu and Hamada.

Results for EER

Here, we also compare the performance of our new method, Wu and Hamada's method, and Lenth's method for controlling EER in the dispersion model. Since Variyath et al. (2005) does not have a procedure for controlling EER, then it is not included in the comparison.

In the simulation, we considered 2^3 and 2^4 factorial experiments. For each experiment, the model under the null hypothesis $H_0 : \gamma_1 = \ldots = \gamma_I = 0$ is

$$y_{ij} \sim N(0, 1).$$

We set the mean of response to be 0 since it does not affect the above mentioned three methods. For each experiment, we are interested in $I = m - 1$ experiments. For example, for the 2^3 experiment, we are interested in $I = 2^3 - 1 = 7$ effects.

The simulated EER at the 5% level in the dispersion model is calculated based on $N = 20,000$ repetitions. The results are presented in Table 2.6.

Table 2.4: Percentage of rejecting the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level for model: $y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AC))$ in replicated 2^4 experiments

Effects	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Wu and Hamada's method				Our method			
A	63.8	79.7	89.6	95.0	46.4	70.3	85.0	92.7
B	63.5	80.0	89.6	95.2	46.4	70.3	85.1	93.0
C	63.8	80.2	89.9	95.1	46.2	70.7	85.2	93.0
D	12.8	9.7	8.6	7.5	5.0	4.9	5.1	5.0
AB	12.2	9.9	8.5	7.8	5.0	5.1	5.1	5.2
AC	51.6	66.1	77.3	86.0	34.6	54.2	69.7	81.6
AD	12.3	9.7	8.5	7.4	5.1	5.2	5.1	5.0
BC	12.6	9.4	8.5	7.9	5.1	5.0	5.3	5.2
BD	12.3	9.6	8.3	7.7	4.9	4.9	5.0	4.9
CD	12.3	9.7	8.4	7.8	4.8	5.1	4.9	5.2
ABC	12.2	9.7	8.3	7.5	5.0	5.0	5.1	5.0
ABD	12.7	9.8	8.4	7.8	5.1	5.1	5.1	5.0
ACD	12.8	10.2	8.5	7.8	5.4	5.4	5.2	5.1
BCD	12.5	9.5	8.4	7.7	5.0	5.0	5.2	4.9
ABCD	13.1	9.7	8.3	7.3	5.4	5.0	5.1	4.8
	Lenth's method				Variyath et al.'s method			
A	23.4	37.3	50.3	63.1	46.0	68.0	83.6	92.1
B	23.7	37.4	50.2	63.1	45.8	68.0	83.1	92.4
C	23.4	37.5	50.5	62.5	45.2	68.2	83.7	92.2
D	1.7	1.5	1.8	2.1	6.2	6.2	5.9	6.2
AB	1.6	1.7	1.7	2.0	6.2	6.5	6.1	5.9
AC	16.1	25.9	36.8	49.1	34.1	53.5	69.3	80.4
AD	1.6	1.6	1.7	2.2	6.2	6.1	6.1	6.0
BC	1.7	1.5	1.9	2.3	6.1	6.3	5.7	6.0
BD	1.7	1.5	1.7	1.8	6.4	6.4	5.9	5.8
CD	1.7	1.7	1.7	2.2	6.0	6.5	5.8	5.9
ABC	1.6	1.6	1.6	2.1	6.1	6.4	6.3	5.8
ABD	1.7	1.6	1.8	2.3	6.2	6.2	6.1	5.9
ACD	1.8	1.7	1.8	2.1	6.1	6.1	6.3	6.2
BCD	1.7	1.6	1.6	1.9	6.2	6.1	6.2	6.0
ABCD	1.9	1.6	1.8	2.1	6.1	6.2	6.1	6.0

Table 2.5: Percentage of rejecting the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level for model: $y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AD))$ in replicated 2^4 experiments

Effects	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Wu and Hamada's method				Our method			
A	63.2	80.1	89.5	95.0	46.0	70.5	84.5	92.8
B	63.6	79.8	89.7	95.3	46.2	70.3	84.7	93.0
C	63.6	79.5	89.9	95.1	46.2	69.7	85.4	92.9
D	12.8	9.6	8.5	7.7	5.3	5.0	5.3	4.9
AB	12.3	9.8	8.3	7.6	5.2	5.0	5.1	5.0
AC	12.5	9.8	8.4	7.3	5.1	5.1	5.0	4.8
AD	51.2	65.8	78.1	86.0	33.8	54.3	70.7	81.3
BC	12.8	9.5	8.1	7.8	5.4	4.8	4.7	5.1
BD	12.0	9.9	8.5	7.7	4.9	5.2	4.9	5.2
CD	12.7	9.7	8.3	7.5	5.1	5.2	5.1	5.0
ABC	12.7	9.8	8.6	7.7	5.0	5.1	5.2	5.0
ABD	12.6	9.7	8.4	7.8	5.4	5.1	5.1	5.2
ACD	12.0	10.0	8.6	7.9	4.8	5.3	5.1	5.2
BCD	12.8	10.0	8.3	7.8	5.4	5.3	5.0	5.1
ABCD	12.6	9.9	8.3	7.7	5.1	5.2	5.0	5.2
	Lenth's method				Variyath et al.'s method			
A	23.7	36.8	50.8	62.7	45.9	67.9	83.2	91.9
B	23.5	36.8	50.9	62.8	45.7	68.6	83.6	91.7
C	24.0	37.1	50.8	62.8	45.4	67.5	83.6	91.7
D	1.7	1.6	1.8	2.1	6.1	6.3	6.0	6.1
AB	1.9	1.6	1.9	2.0	6.0	6.1	5.9	6.0
AC	1.7	1.5	1.7	2.1	6.3	6.3	6.2	5.9
AD	16.4	25.7	37.2	48.6	33.1	53.6	68.9	80.9
BC	1.8	1.7	1.6	2.0	6.1	6.0	5.9	5.9
BD	1.7	1.6	1.7	2.1	6.2	6.2	6.0	5.8
CD	1.8	1.5	1.7	2.1	6.2	6.2	6.2	6.1
ABC	1.7	1.7	1.7	2.0	6.0	6.1	6.0	5.7
ABD	1.7	1.6	1.7	2.0	6.3	6.1	6.1	6.2
ACD	1.7	1.8	1.8	2.2	6.0	6.1	6.1	5.9
BCD	1.9	1.6	1.8	1.9	6.3	6.2	5.8	5.9
ABCD	1.8	1.6	1.8	2.1	6.1	6.3	6.0	5.9

Table 2.6: Percentage of rejecting the null hypothesis $H_0 : \gamma_1 = \ldots = \gamma_I = 0$ at the 5% level for model: $y_{ij} \sim N(0, 1)$ in replicated 2^3 and 2^4 factorial experiments

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Wu and Hamada's method				Our method				Lenth's method			
7	0.216	0.149	0.119	0.109	0.055	0.054	0.054	0.054	0.045	0.045	0.047	0.048
15	0.264	0.175	0.140	0.119	0.055	0.054	0.053	0.051	0.043	0.045	0.043	0.043

From Table 2.6, it is observed that the values for the EER based on our new method are around 0.05 (5%). This is an evidence that our proposed method can accurately control the EER in the dispersion model. The method suggested by Wu and Hamada gives results that are far more than the 5% nominal level. Therefore Wu and Hamada's method can not tightly control the EER in the dispersion model. The EER based on the Lenth's method is quite close to the nominal level. Therefore Lenth's method can also tightly control the EER.

2.3.2 Simulation Results in Location Model

Results for IER

As we discussed in Section 2.2.4, Wu and Hamada's method and Variyath et al.'s method are equivalent for controlling IER in the location model. Therefore, we only compare the performance of our new method, Wu and Hamada's method, and Lenth's method for controlling IER in the location model. We consider two cases: σ_i^2's are homogeneous and σ_i^2's are not homogeneous.

Case I: σ_i^2's are homogeneous

In this case, we performed simulations for 2^3 and 2^4 factorial experiments. For the 2^3 experiment with three two-level factors A, B, and C, we used the model

$$y_{ij} \sim N(10 + 0.5A + 0.5B + 0.4AB, 1)$$

where A, B and AB take values ± 1 depending on the combination of factors levels. For the 2^4 experiment with four two-level factors A, B, C and D, we used the model

$$y_{ij} \sim N(5 + 0.3A + 0.3B + 0.3D + 0.25BD, 1)$$

where A, B, D and BD take values ± 1 depending on the combination of factors levels. Then, we test the significance of the factorial effects of interest for each model at 5% level based on the aforementioned methods. The simulation is repeated for $N = 20,000$ times for each model. We compute the percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0,\ l = 1, \ldots, I$. The results are summarized in Tables 2.7 and 2.8.

From the simulated results in Tables 2.7 and 2.8, it is seen that both our proposed method and Wu and Hamada's method can tightly control IER at the 5% nominal level when the σ_i^2's are homogeneous. However, Lenth's method can not tightly control the IER. In terms of the power, our method almost has the same power as Wu and Hamada's method in all the situations except for the 2^3 experiment with $n = 3$. In that situation, our method is a little bit less powerful.

Table 2.7: Percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model: $y_{ij} \sim N(10 + 0.5A + 0.5B + 0.4AB, 1)$ in replicated 2^3 experiments

Effects	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				Wu and Hamada's method				Lenth's method			
A	58.7	75.8	86.0	92.7	63.3	77.0	86.4	92.1	22.9	27.5	33.1	37.4
B	58.8	76.6	85.8	92.3	64.3	77.8	86.0	92.0	23.7	28.3	33.1	37.6
C	4.5	4.6	4.7	5.3	5.2	5.1	4.8	5.3	0.7	0.5	0.4	0.5
AB	41.3	57.0	68.4	77.3	45.1	57.8	69.0	77.8	13.9	17.6	21.4	25.2
AC	4.6	4.8	5.0	5.4	5.0	5.0	4.9	4.8	0.8	0.5	0.5	0.4
BC	4.3	4.8	4.5	5.4	4.7	4.9	5.0	5.1	0.6	0.5	0.5	0.4
ABC	4.7	4.5	4.7	4.9	5.1	4.9	4.9	4.7	0.6	0.6	0.4	0.4

Case II: σ_i^2's are not homogeneous

In this case, we used the models

$$y_{ij} \sim N(10 + A + B + 0.5AB, \exp(A + C + 0.5AC))$$

and

$$y_{ij} \sim N(5 + 0.65A + 0.65B + 0.45AB, \exp(A + C + 0.5AC))$$

for the 2^3 factorial experiment with three two-level factors A, B, and C. For the 2^4

Table 2.8: Percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model: $y_{ij} \sim N(5 + 0.3A + 0.3B + 0.3D + 0.25BD, 1)$ in replicated 2^4 experiments

Effects	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
	Our method				Wu and Hamada's method				Lenth's method			
A	51.2	63.8	74.1	81.7	52.6	65.2	75.2	82.8	26.2	33.0	40.7	48.1
B	49.6	63.4	73.6	81.2	51.3	64.9	74.8	82.9	26.5	32.1	41.4	47.4
C	4.8	4.6	4.5	4.5	5.1	5.0	4.8	4.7	1.9	1.6	1.7	1.8
D	51.0	63.6	74.2	81.9	52.6	65.3	75.2	83.4	25.5	32.8	41.1	47.0
AB	4.6	4.5	4.4	4.4	5.0	4.8	4.8	5.0	1.7	1.8	1.7	2.0
AC	4.8	4.6	4.5	4.5	5.2	5.0	4.7	5.0	1.8	1.7	1.7	1.8
AD	4.5	4.5	4.6	4.5	4.9	5.1	5.0	4.8	1.8	1.6	1.8	1.8
BC	4.8	4.6	5.1	4.7	5.1	5.1	5.4	4.9	1.7	1.6	1.7	1.7
BD	37.7	47.9	58.3	65.8	39.1	49.6	59.4	67.5	18.7	22.5	28.7	34.2
CD	4.5	4.8	4.5	4.5	4.8	5.2	4.9	5.1	1.6	1.7	2.0	1.7
ABC	5.0	4.5	4.6	4.5	5.4	4.9	4.9	5.1	1.9	1.7	1.7	1.9
ABD	4.6	4.5	4.5	4.4	4.8	4.9	4.6	4.5	1.7	1.5	1.7	1.7
ACD	4.6	4.6	4.5	4.6	4.6	4.9	4.7	5.1	1.8	1.8	1.5	1.8
BCD	4.6	4.7	4.8	4.5	5.1	5.1	5.3	4.4	1.7	1.6	1.8	1.6
ABCD	4.7	4.5	4.7	5.0	5.1	4.8	4.9	4.8	2.1	1.7	1.7	1.8

experiment with four two-level factors A, B, C and D, we used the models

$$y_{ij} \sim N(10 + 0.5A + 0.45B + 0.5D + 0.4AD, \exp(A + B + D + 0.5AD))$$

and

$$y_{ij} \sim N(5 + 0.75A + 0.65B + 0.55C + 0.5AD, \exp(A + B + C + 0.5BD)).$$

Also, we test the significance of the I factorial effects of interest at the 5% level based on the above mentioned methods. For $l = 1, \ldots, I$, the percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level by each method is calculated based on $N = 20,000$ repetitions. The results are summarized in Tables 2.9, 2.10, 2.11 and 2.12 respectively.

From the simulated results in Tables 2.9, 2.10, 2.11 and 2.12, only our proposed method can tightly control the IER for all the models. These results support our argument that the t-distribution with degrees of freedom $m(n-1)$ suggested by Wu and Hamada may not be true and fail to control IER when σ_i^2's are not the same. Again, Lenth's method can not accurately control the IER in the location model.

Table 2.9: Percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model: $y_{ij} \sim N(10+A+B+0.5AB, \exp(A+C+0.5AC))$ in replicated 2^3 experiment

Effects	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				Wu and Hamada's method				Lenth's method			
A	57.5	74.3	86.6	93.1	71.0	82.3	90.5	94.8	31.2	38.6	44.4	49.9
B	57.0	74.1	87.0	93.3	71.3	82.2	90.3	94.9	27.2	35.1	41.4	47.4
C	4.7	4.6	5.2	5.3	8.3	7.4	7.1	7.0	0.2	0.3	0.2	0.3
AB	18.3	24.6	33.8	41.6	28.5	33.3	40.6	46.2	9.8	12.4	16.3	20.1
AC	4.5	4.6	5.1	5.3	8.5	7.4	7.0	6.9	0.3	0.2	0.2	0.3
BC	4.6	4.5	5.3	5.2	8.2	7.1	7.4	6.9	0.5	0.4	0.3	0.4
ABC	4.7	4.6	5.1	5.0	8.5	6.6	7.0	6.4	0.5	0.4	0.4	0.4

Table 2.10: Percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model: $y_{ij} \sim N(5+0.65A+0.65B+0.45AB, \exp(A+C+0.5AC))$ in replicated 2^3 experiment

Effects	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				Wu and Hamada's method				Lenth's method			
A	28.6	40.4	51.1	60.0	41.2	50.6	58.5	66.0	15.2	17.4	20.8	24.6
B	28.1	40.0	51.2	59.9	40.2	50.0	59.2	65.7	10.6	13.8	16.6	20.8
C	5.0	5.2	5.0	5.0	8.1	7.6	7.0	6.4	0.3	0.3	0.2	0.3
AB	16.5	21.8	28.3	33.4	24.8	28.7	34.4	38.9	5.1	7.0	9.0	12.0
AC	5.0	5.2	5.3	5.0	8.0	7.6	7.1	6.3	0.4	0.3	0.2	0.2
BC	5.3	5.1	4.7	5.2	8.6	7.6	6.9	6.5	1.5	1.1	0.9	0.8
ABC	5.4	4.9	4.8	4.8	8.6	7.3	6.8	6.5	1.4	1.0	0.9	0.8

Results for EER

We compare the performance of our proposed method, Wu and Hamada's method, and Lenth's method for controlling the EER in the location model. We still consider two cases: σ_i^2's are homogeneous and σ_i^2's are not homogeneous.

Case I: σ_i^2's are homogeneous

Here, we considered 2^3 and 2^4 factorial experiments. We used the model

$$y_{ij} \sim N(0,1)$$

Table 2.11: Percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model: $y_{ij} \sim N(10 + 0.5A + 0.45B + 0.5D + 0.4AD, \exp(A + B + D + 0.5AD))$ in replicated 2^4 experiment

Effects	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				Wu and Hamada's method				Lenth's method			
A	25.7	34.3	43.3	51.2	35.1	42.1	49.9	56.5	18.4	22.7	27.3	31.4
B	21.6	29.4	36.8	44.2	30.5	36.7	43.4	49.1	16.3	21.0	25.1	29.0
C	5.0	5.5	5.0	5.1	7.7	7.7	6.6	6.5	0.5	0.4	0.5	0.6
D	25.7	34.3	43.0	51.2	35.5	42.3	49.8	56.5	18.5	23.2	27.0	31.4
AB	5.1	5.2	5.2	4.9	8.0	7.5	7.0	6.2	0.5	0.5	0.4	0.6
AC	5.1	5.3	5.2	5.2	7.8	7.5	7.0	6.5	0.6	0.4	0.5	0.6
AD	18.6	24.3	29.9	36.2	26.1	30.9	35.7	41.0	13.5	16.8	19.9	22.7
BC	4.9	5.6	5.1	5.0	8.0	7.8	6.7	6.4	0.6	0.6	0.5	0.4
BD	4.9	4.9	5.1	5.1	8.0	7.4	6.8	6.4	0.6	0.7	0.5	0.4
CD	4.9	5.2	4.9	5.4	7.9	7.4	6.6	6.8	0.5	0.5	0.5	0.5
ABC	5.2	5.5	5.4	4.9	8.4	7.4	7.1	6.3	0.5	0.5	0.5	0.5
ABD	5.0	5.1	5.1	5.0	8.0	7.6	6.8	6.2	0.7	0.5	0.5	0.5
ACD	5.2	5.2	5.1	5.3	8.5	7.4	6.8	6.9	0.7	0.5	0.5	0.4
BCD	5.1	5.4	4.8	5.1	8.1	7.6	6.7	6.3	0.7	0.5	0.5	0.5
ABCD	5.1	5.3	5.1	5.0	7.9	7.6	6.9	6.4	0.7	0.5	0.5	0.5

Table 2.12: Percentage of rejecting the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model: $y_{ij} \sim N(5 + 0.75A + 0.65B + 0.55C + 0.5AD, \exp(A + B + C + 0.5BD))$ in replicated 2^4 experiment

Effects	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				Wu and Hamada's method				Lenth's method			
A	59.1	75.9	85.9	91.6	70.7	82.1	89.4	93.8	42.1	51.6	59.5	65.1
B	48.4	62.7	75.4	83.5	59.3	70.2	80.4	86.7	32.7	42.2	50.5	56.3
C	36.9	49.9	61.2	70.7	47.5	57.1	67.3	75.0	24.9	33.2	41.2	46.8
D	5.2	5.1	4.9	5.3	7.7	6.8	6.2	6.4	1.0	1.1	1.1	1.3
AB	5.1	4.9	4.9	5.2	7.4	6.2	6.2	6.4	2.3	1.8	2.1	2.1
AC	4.8	5.1	5.1	5.5	6.9	6.9	6.5	6.6	1.7	1.7	1.9	2.1
AD	31.4	43.0	52.9	61.6	40.4	50.2	59.0	66.1	20.5	28.9	34.8	40.6
BC	5.2	5.3	4.7	5.2	7.8	6.8	6.0	6.3	1.7	1.7	2.0	1.7
BD	5.0	5.0	5.2	5.5	7.6	6.7	6.3	6.7	0.6	0.9	0.9	1.0
CD	5.0	5.2	5.0	5.3	7.5	6.8	6.3	6.2	0.5	0.6	0.7	0.8
ABC	5.2	5.0	5.0	5.2	7.5	6.6	6.4	6.7	1.1	1.0	1.4	1.1
ABD	5.0	5.2	4.9	5.4	7.3	7.0	6.4	6.5	0.8	1.1	1.1	1.2
ACD	4.8	4.9	5.0	5.0	7.4	6.5	6.4	6.2	1.0	1.0	1.0	1.1
BCD	5.2	5.1	5.3	5.4	7.6	6.9	6.6	6.7	0.6	0.7	0.7	0.8
ABCD	5.1	5.3	5.2	5.4	7.3	7.4	6.7	6.7	0.5	0.7	0.7	0.9

for the simulations for both factorial experiments. The simulated EER at the 5% level by each method is calculated based on $N = 20,000$ repetitions. The results are shown in Table 2.13. As we can see from this table, all the three methods can accurately control the EER at the 5% level.

Table 2.13: Percentage of rejecting the null hypothesis $H_0 : \alpha_1 = \ldots = \alpha_I = 0$ at the 5% level for model: $y_{ij} \sim N(0, 1)$ in replicated 2^3 and 2^4 experiments

I	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
	Our method				Wu and Hamada's method				Lenth's method			
7	0.045	0.046	0.046	0.048	0.053	0.051	0.044	0.049	0.053	0.050	0.051	0.050
15	0.046	0.045	0.047	0.051	0.049	0.048	0.045	0.054	0.049	0.050	0.052	0.050

Case II: σ_i^2's are not homogeneous

Here, we used the models

$$y_{ij} \sim N(0, \exp(A + C + 0.5AC))$$

and

$$y_{ij} \sim N(0, \exp(A + C + D + 0.5CD))$$

for the 2^3 and 2^4 factorial experiments respectively. The simulation is repeated for 20,000 times and the EER is computed using the above procedures. The results are summarized in Table 2.14.

From Table 2.14, it is seen that only our proposed method controls the EER at 5% nominal level. The results given by Wu and Hamada's method are too liberal while those of Lenth's method are conservative. Again, these results also support our argument that the studentized maximum modulus distribution with parameters I and $m(n-1)$ suggested by Wu and Hamada may not be true and fail to control the EER when σ_i^2's are not homogeneous.

Table 2.14: Percentage of rejecting the null hypothesis $H_0 : \alpha_1 = \ldots = \alpha_I = 0$ at the 5% level for model: $y_{ij} \sim N(0, \exp(A+C+0.5AC))$ and $y_{ij} \sim N(0, \exp(A+C+D+0.5CD))$ in replicated 2^3 and 2^4 experiments respectively

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				Wu and Hamada's method				Lenth's method			
7	0.054	0.052	0.052	0.049	0.087	0.068	0.066	0.063	0.030	0.027	0.030	0.030
15	0.052	0.050	0.049	0.049	0.081	0.066	0.063	0.060	0.023	0.023	0.025	0.026

2.4 Application To Real Examples

In this Section, we illustrate the above methods by two real data sets given in Examples 1.1 and 1.2 above. In each set, we examine the performance of the proposed methods.

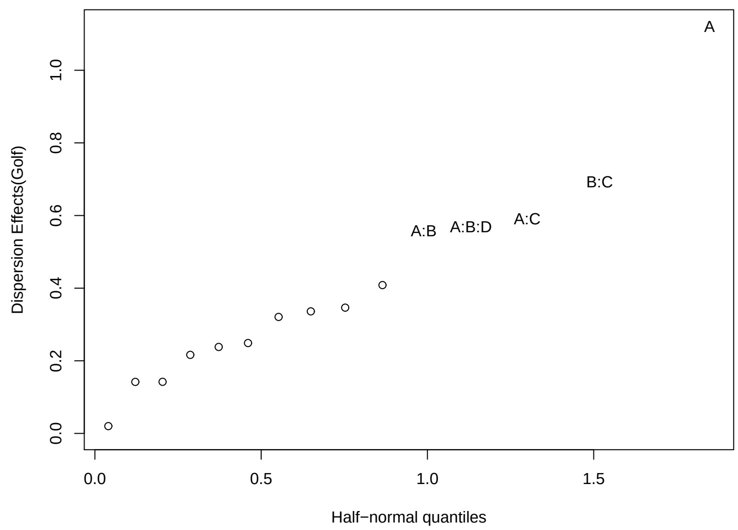
2.4.1 Example 1.1

Traditionally, half-normal plots developed by Daniel (1959, 1976) are used to identify the active effects in factorial experiments. We first consider the data set of Example 1.1. The half-normal plot for the interested 15 effects in the dispersion model is shown in Figure 2.1. Clearly, effect A is significant, and probably BC, AC, ABD and AB are also significant for the dispersion model.

If we control the IER of each effect in the dispersion model to be 5%, Wu and Hamada's method declared that effects A, AC, BC and ABD are found significant; both our method and Variyath et al.'s method claimed that effects A and BC are significant; Lenth's method found that only effect A is significant. If we control the EER in the dispersion model to be 5%, both the Wu and Hamada and our method declared that only effect A is significant, while Lenth's method declared that no effect is significant for the dispersion model.

For the location model, the half-normal plot of 15 interested effects as shown in Figure 2.2 declared effect A and probably B and AB as significant. If we control the IER of each effect in the location model to be 5% we found that effects A and B are declared significant by the Wu and Hamada and our methods while no effect is declared significant by the Lenth's method. If we control the EER in the location model to be 5%, both our method and Wu and Hamada's method found that only effect A is significant, again Lenth's method does not identify any significant effect.

Figure 2.1: Half-Normal Plot for the interested 15 effects in the dispersion model, Example 1.1.

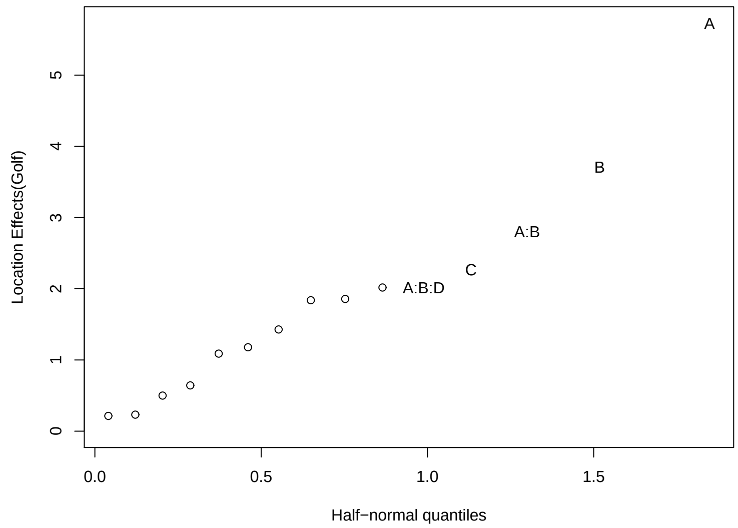


2.4.2 Example 1.2

Figure 2.3 gives the half-normal plot of the interested seven effects in the dispersion model. It is quite hard to tell which effects are significant from the plot. If we control the IER for each effect at 5% in the dispersion model, we found out that effects C and AF are declared significant by Wu and Hamada and Variyath et al. methods, while no effect is declared significant by our proposed method and Lenth's method. If we control the EER for the dispersion model to be 5%, effect C is declared significant by the Wu and Hamada's method. Again, no effect is found significant by both our method and Lenth's method.

For the location model, the half-normal plot as shown in Figure 2.4, declared

Figure 2.2: Half-Normal Plot for the interested 15 effects in the location model, Example 1.1.



effects D and F as significant. If we control the IER for each effect in the location model to be 5%, Wu and Hamada's method declared effects A, D and F as significant. Effects D and F are declared significant by our method and no effect is declared significant by Lenth's method. If we control the EER to be 5% in the location model, effects D and F are are declared significant by Wu and Hamada's method. Effect D is found significant by our method and no effect is found significant by Lenth's method.

Figure 2.3: Half-Normal Plot for the interested 7 effects in the dispersion model, Example 1.2.

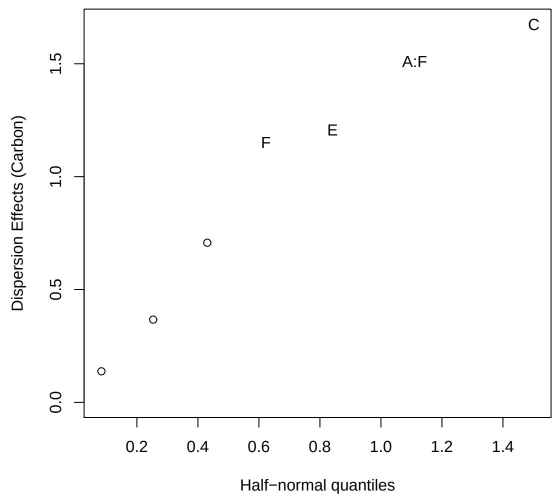
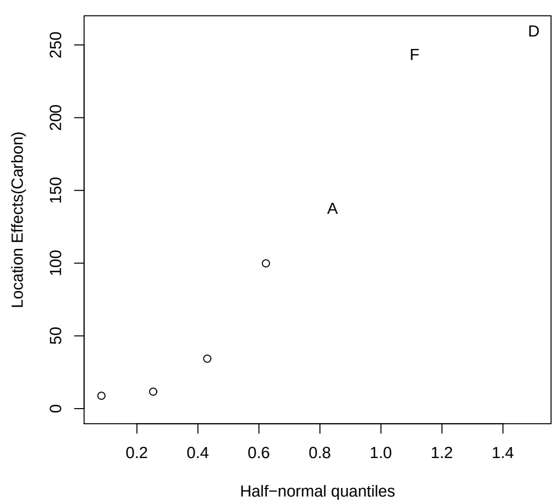


Figure 2.4: Half-Normal Plot for the interested 7 effects in the location model, Example 1.2.



Chapter 3

Controlling the FDR in Location and Dispersion models

3.1 False Discovery Rate

False discovery rate (FDR) was first introduced by Benjamini and Hochberg (1995, hereinafter BH) as an error rate to control in many multiple hypotheses testing problems. It provides an alternative for the EER. Since then, this error rate has been given much attention by various researchers in different settings (Weller et al., 1998; Troendle, 1999; Benjamini and Hochberg, 2000; Benjamini and Yekutieli, 2001; Efron et al., 2001; Mosig et al., 2001; Genovese and Wasserman, 2002a, b; Storey, 2002; Sarkar, 2002; Storey and Tibshirani, 2003a, b; Benjamini, Krieger and Yekutieli, 2006; Kimel et al., 2008; and others). Kimel, Benjamini and Steinberg (2008) first applied this error rate in two-level unreplicated regular factorial experiments.

Suppose we are interested in testing simultaneously I null hypotheses of interest. Suppose that m_0 of these hypotheses are true and $m_1 = (I - m_0)$ are false. Let V be the number of false discoveries and R be the total number of discoveries. BH (1995) defined FDR as the expected proportion of false discoveries among the total discoveries. That is

$$FDR = E\left(\frac{V}{R}\right) \text{ if } R > 0.$$

When $R = 0$, they defined the FDR to be 0, since no error of false discovery can be committed.

The following two important properties about FDR were shown by BH (1995):

(a) If all the null hypotheses are true, then the FDR is equivalent to the EER.

(b) When $m_0 < I$, the FDR is less than or equal to EER. This implies that any procedure that controls the EER also controls the FDR as well at the same nominal level but the converse is not true.

3.1.1 Benjamini and Hochberg's Procedure for controlling the FDR

Suppose we are interested in I factorial effects. We use P_l to denote the P-value of the test statistic for the lth hypothesis, $l = 1, \ldots, I$. Let $P_{(1)} \leq P_{(2)} \leq \ldots \leq P_{(I)}$ be the ordered observed P-values. For the given q value, let

$$h = \max\left\{l : P_{(l)} \leq \frac{l}{I}q\right\},$$

then we declare the h largest effects active or significant.

We call the above procedure BH procedure. When the test statistics are independent, BH (1995) showed that the foregoing procedure controls the FDR at level $(m_0/I)q$. Benjamini and Yekutieli (2001) later proved that the preceding procedure also controls the FDR at a level less than or equal to q for positively dependent test statistics.

3.1.2 Adaptive Procedure for controlling FDR

Recall that the Benjamini and Hochberg (1995) FDR procedure controls the FDR at the desired level q for independent and positively dependent test statistics. When all the null hypotheses are true, and the test statistics are independent and continuous, the bound is sharp (Bejamini et al., 2006). When some of the null hypotheses are not true (that is, when $m_0 < I$), the BH procedure becomes conservative by a factor of m_0/I (Benjamini and Hochberg, 2000). To remedy this conservativeness, Benjamini and Hochberg (2000) suggested an adaptive version of Benjamini and Hochberg (1995) procedure that controls FDR by first estimating m_0. The estimate is then used to adjust the BH procedure to control the FDR at precisely the desired level q. Since then, the problem of estimating m_0 has received wide attention (Efron et al., 2001; Mosig et al., 2001; Storey, 2002, 2003; Black, 2004; Storey et al., 2004; Benjamini, Krieger and Yekutieli, 2006; and others). Of all the aforementioned methods for estimating m_0, Benjamini and Hochberg (2000)'s adaptive FDR control procedure is found most effective for analyzing factorial experiments (Kimel et al., 2008). The following is the adaptive procedure for controlling FDR. We call this procedure ABH procedure.

The ABH FDR Controlling Procedure

1. Use the BH procedure described in Section 3.1 at level q. If no significant effect is found, then stop; otherwise, proceed.

2. Calculate the slopes $S_l = \frac{(1-P_{(l)})}{(I+1-l)}$, which is the l-th slope estimate of the line passing through the points $(I+1,\ 1)$ and $(l,\ P_{(l)})$ on the quantile plot of the P-values.

3. Starting with $l = 1$, proceed as long as $S_l \geq S_{l-1}$. When for the first time $S_l < S_{l-1}$, stop.

4. Set $\hat{m}_0 = \min([1/S_l + 1], I)$. Here $[x]$ is the largest integer which is smaller than or equal to x.

5. Use the BH procedure described in Section 3.1 again at level $q(I/\hat{m}_0)$.

Remark 3.1. *Here are some remarks about Steps 2, 3, and 4. In Step 2, we compute the slopes S_l's since they contain the information for estimating m_0. If all the null hypotheses are true, that is $m_0 = I$, and the test statistics are independent, $P_{(l)}$'s can be considered as a realization of ordered sample from the uniform distribution over the interval [0,1]. The expected value of the l-th P-value is $E(P_{(l)}) = l/(m_0+1)$. A plot of $P_{(l)}$ against l should therefore indicate a straight line with the slope $1/(m_0+1)$ passing through the origin and the point $(I+1, 1)$. When $m_0 < I$, the P-values corresponding to the alternative hypotheses tend to be smaller than the P-values corresponding to the null hypotheses, so they concentrate on the left-hand side of the plot. The relationship over the right-hand side of the plot should be approximately linear with slope $1/(m_0 + 1)$. Therefore the slopes S_l's for large l contain the useful information for estimating m_0.*

The condition $S_l \geq S_{l-1}$ in Step 3 is equivalent to

$$(P_{(l)} - P_{(l-1)})/(1 - P_{(l-1)}) \leq 1/(I + 1 - (l - 1)).$$

The value $1/(I + 1 - (l - 1))$ is the expected value of the normalized gap on the left-hand side under the assumption that all P-values greater or equal to $P_{(l-1)}$ correspond to true null hypotheses. Therefore the stopping rule is equivalent to dropping a small P-value if its gap to its larger neighborhood is smaller than its expected value.

The estimate of m_0 in Step 4 is called the lowest slope estimate of m_0. This is the most satisfactory estimate found by Benjamini and Hochberg (2000). See Benjamini and Hochberg (2000) for more details.

3.2 FDR in Replicated Regular Two-Level Experiments

As we can see in the last section, finding the P-values of the test statistic for testing a given null hypothesis for each method is the first step before using the BH and ABH procedures to control the FDR. In the following subsections, we present how to obtain the P-value of the test statistic for the existing and the proposed methods. For each method, once the I P-values are obtained in the location model or dispersion model, the BH and ABH procedures can be applied to control the FDR.

3.2.1 Wu and Hamada's Methods

Recall that for the dispersion model, Wu and Hamada constructed a z-type statistic

$$z_l = \frac{\hat{\gamma}_l}{\sqrt{\frac{2}{m(n-1)}}}$$

to test the hypothesis $H_0 : \gamma_l = 0$. They used $N(0,1)$ distribution to approximate the distribution of the z-type test statistic under the null hypothesis. Therefore the P-values of the z-type test statistic can be calculated as

$$P_l = Pr(|N(0,1)| > \text{ observed } |z_l|).$$

For the location model, the t-type test statistic

$$t_l = \frac{\hat{\alpha}_l}{\sqrt{\frac{1}{m^2 n}\sum_{i=1}^{m} s_i^2}}$$

is used to test the hypothesis $H_0 : \alpha_l = 0$. The P-value of the t-type statistic can be calculated based on the $t_{m(n-1)}$ distribution as

$$P_l = Pr(|t_{m(n-1)}| > \text{ observed } |t_l|).$$

3.2.2 Variyath et al.'s Method

Variyath et al. (2005) argued that the F statistic, F_l, follows the $F_{1,m(n-1)}$ distribution. Therefore, the P-value of the F statistic, F_l, can be calculated as

$$P_l = Pr(F_{1,m(n-1)} > \text{ observed } F_l).$$

3.2.3 Lenth's Method

Here, we adopt the method proposed by Edwards and Mee (2008) to compute the P-value for Lenth's test statistic. Their method is the same for both the dispersion and location models. To describe their method for the dispersion model, we have adopted some notations used in their paper. Suppose $\hat{\gamma}_1, \ldots, \hat{\gamma}_I$ are the least square estimates for factorial effects $(\gamma_1, \ldots, \gamma_I)$ of interest in dispersion model. In order to handle the cases where the diagonal elements of $\text{Var}(\hat{\gamma}) = \text{Var}(\hat{\gamma}_1, \ldots, \hat{\gamma}_I)^T$ are not equal, they standardized the estimated factorial effects of interest as

$$c_l = \frac{\hat{\gamma}_l}{\sqrt{\upsilon_{ll}}}, \tag{3.1}$$

where υ_{ll} is the diagonal element of $\mathbf{V} = (\mathbf{X}^{\mathbf{T}}\mathbf{X})^{-1}$ corresponding to $\hat{\gamma}_l$. Here, $\mathbf{X}$ is the matrix corresponding to the columns of the I factorial effects of interest only. Therefore, for a given set of least square estimates $\hat{\gamma}_1, \ldots, \hat{\gamma}_I$, they compute the P-values for the Lenth t-type statistic through Monte Carlo simulation given by the following steps.

Step 1: Compute the standardized coefficients using (3.1), the PSE using

(1.12), and the I Lenth t statistics, $t_{\text{Lenth},l}$, $l = 1, \ldots, I$.

Step 2: Compute the $I \times I$ matrix $\mathbf{R}$, where the (l, p)th element of $\mathbf{R}$ is given by $\upsilon_{lp}/(\upsilon_{ll}\upsilon_{pp})^{1/2}$.

Step 3: Generate M sets of I random variables from a multivariate normal distribution with mean $\mathbf{0}$ and covariance $\mathbf{R}$.

Step 4: For the bth set of random variables, compute PSE and the Lenth t statistics, $t^{(b)}_{\text{Lenth},l}$, $l = 1, \ldots, I$, $b = 1, \ldots, M$.

Step 5: Compute the P-value of $\hat{\gamma}_l$ as

$$P_l = \frac{\#\left\{b : \left|t^{(b)}_{\text{Lenth},l}\right| \geq \left|t_{\text{Lenth},l}\right|\right\}}{M},$$

where $'\#'$ stands for “the number of”.

3.2.4 Our Proposed Methods

For the dispersion model, we proposed using $N(0, a_n^2)$ to approximate the distribution of z_l in Section 2.1.1. Therefore it can also be used to calculate the P-value of z_l as

$$P_l = Pr(|N(0, a_n^2)| > \text{observed } |z_l|).$$

The values of a_n can be found in Table 2.1.

For the location model, we argued that under the null hypothesis of $H_0 : \alpha_l = 0$,

$$t_l \sim \frac{N(0,1)}{\sqrt{\sum_{i=1}^{m} \rho_i^2 \chi^2_{n-1}/(n-1)}}$$

and proposed a method to approximately generate M random observations, $t_l^{(b)}$, $b = 1, \ldots, M$, from the above distribution. The details can be found in Section 2.2.2.

Then the P-value of t_l can be calculated as

$$P_l = \frac{\#\left\{b : \left|t_l^{(b)}\right| \geq \text{observed } |t_l|\right\}}{M}.$$

3.2.5 Simulation Study

Now, we perform a simulation study to compare all the four methods in terms of controlling FDR in replicated regular two-level factorial experiments.

Dispersion Model

For the simulations, we considered 2^3 and 2^4 factorial experiments. Two different cases are considered. First, when no effect is active, we used the model

$$y_{ij} \sim N(0, 1)$$

for both the 2^3 and 2^4 factorial experiments. Second, when there are some active effects, we used the models

$$y_{ij} \sim N\left(0, \exp(0.35A + 0.3C + 0.3AC)\right)$$

and

$$y_{ij} \sim N(0, \exp(0.35A + 0.3C + 0.3BC))$$

for 2^3 experiment. For 2^4 experiments, we used the models

$$y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AC))$$

and

$$y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AD)).$$

The simulation is repeated for 10,000 times except for Lenth's method. The simulation is repeated only 5,000 times for Lenth's method since it is quite computationally extensive to run the simulations for Lenth's method. The simulated FDRs are then calculated and are summarized in Tables 3.1, 3.2, 3.3, 3.4, and 3.5.

Table 3.1: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(0,1)$ in 2^3 and 2^4 experiments

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method BH				Our method ABH				Wu and Hamada BH				Wu and Hamada ABH			
7	0.057	0.054	0.054	0.056	0.057	0.054	0.054	0.056	0.210	0.155	0.124	0.104	0.210	0.155	0.124	0.104
15	0.060	0.055	0.054	0.054	0.060	0.055	0.054	0.054	0.280	0.187	0.141	0.122	0.280	0.187	0.141	0.122
	Variyath et al. BH				Variyath et al. ABH				Lenth BH				Lenth ABH			
7	0.085	0.097	0.092	0.091	0.085	0.097	0.092	0.091	0.034	0.033	0.031	0.039	0.034	0.033	0.031	0.039
15	0.087	0.088	0.084	0.089	0.087	0.088	0.084	0.089	0.035	0.031	0.037	0.032	0.035	0.031	0.037	0.032

Table 3.2: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(0,\ \exp(0.35A + 0.3C + 0.3AC))$ in 2^3 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	7	0.033	0.031	0.031	0.030	0.040	0.041	0.043	0.045
Wu and Hamada's method	7	0.109	0.073	0.059	0.051	0.122	0.088	0.077	0.071
Variyath et al.'s method	7	0.044	0.046	0.046	0.044	0.056	0.059	0.060	0.062
Lenth's method	7	0.004	0.005	0.002	0.002	0.004	0.005	0.003	0.002

Table 3.3: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(0, \exp(0.35A + 0.3C + 0.3BC))$ in 2^3 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	7	0.034	0.030	0.030	0.029	0.040	0.041	0.044	0.045
Wu and Hamada's method	7	0.107	0.074	0.059	0.052	0.119	0.091	0.078	0.073
Variyath et al.'s method	7	0.044	0.047	0.044	0.041	0.058	0.058	0.060	0.060
Lenth's method	7	0.004	0.004	0.003	0.002	0.004	0.004	0.003	0.002

From Tables 3.1, 3.2, 3.3, 3.4, and 3.5, it is seen that our proposed method coupled with ABH procedure can accurately controls FDR at the $q = 0.05$ for both the cases of no active effect and when there are some active effects. Lenth's method is quite conservative especially when there are some active effects. Wu and Hamada's method is quite liberal for both cases no matter BH or ABH procedures are used. Variyath

Table 3.4: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AC))$ in 2^4 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	15	0.039	0.038	0.038	0.038	0.050	0.048	0.049	0.049
Wu and Hamada's method	15	0.160	0.103	0.080	0.068	0.180	0.126	0.102	0.088
Variyath et al.'s method	15	0.054	0.056	0.051	0.052	0.062	0.070	0.067	0.068
Lenth's method	15	0.006	0.008	0.010	0.010	0.007	0.008	0.012	0.013

Table 3.5: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AD))$ in 2^4 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	15	0.040	0.040	0.040	0.040	0.046	0.050	0.050	0.050
Wu and Hamada's method	15	0.160	0.106	0.083	0.069	0.180	0.128	0.105	0.090
Variyath et al.'s method	15	0.055	0.056	0.051	0.052	0.064	0.068	0.066	0.069
Lenth's method	15	0.009	0.008	0.009	0.010	0.010	0.011	0.012	0.013

et al.'s method is quite liberal for the case of no active effect whether BH or ABH procedure is used. It works quite well for the case when there are some active methods if BH procedure is used.

Location Model

We considered two situations here: σ_i^2's are homogeneous and σ_i^2's are not homogeneous.

Case I: σ_i^2's are homogeneous

We performed simulations for 2^3 and 2^4 factorial experiments in this case. For the 2^3 and 2^4 experiments when no effect is active, we used the model

$$y_{ij} \sim N(0, 1).$$

When there are some active effects, we used the models

$$y_{ij} \sim N(10 + 0.5A + 0.5B + 0.4AB, 1)$$

and

$$y_{ij} \sim N(5 + 0.3A + 0.3B + 0.3D + 0.25BD, 1)$$

for 2^3 and 2^4 experiments, respectively. The simulated FDRs are presented in Tables 3.6, 3.7 and 3.8.

Table 3.6: Simulated FDRs under the 5% using BH and ABH Procedures for the model: $y_{ij} \sim N(0, 1)$ in 2^3 and 2^4 experiments

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method BH				Our method ABH			
7	0.030	0.035	0.043	0.045	0.030	0.035	0.043	0.045
15	0.032	0.040	0.044	0.046	0.032	0.040	0.044	0.046
	Wu and Hamada BH				Wu and Hamada ABH			
7	0.050	0.050	0.048	0.050	0.050	0.050	0.048	0.050
15	0.048	0.047	0.047	0.046	0.048	0.047	0.047	0.046
	Lenth BH				Lenth ABH			
7	0.033	0.036	0.035	0.035	0.033	0.036	0.035	0.033
15	0.040	0.035	0.033	0.034	0.040	0.035	0.033	0.034

Table 3.7: Simulated FDRs under the 5% using BH and ABH Procedures for the model: $y_{ij} \sim N(10 + 0.5A + 0.5B + 0.3C + 0.4AB, 1)$ in 2^3 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	7	0.021	0.023	0.026	0.027	0.035	0.037	0.040	0.044
Wu and Hamada	7	0.029	0.026	0.029	0.030	0.041	0.041	0.044	0.048
Lenth	7	0.003	0.002	0.001	0.001	0.002	0.002	0.001	0.002

Table 3.8: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(5 + 0.3A + 0.3B + 0.3D + 0.25BD, 1)$ in 2^4 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	15	0.029	0.030	0.032	0.034	0.034	0.036	0.040	0.044
Wu and Hamada	15	0.033	0.037	0.037	0.035	0.039	0.044	0.046	0.045
Lenth	15	0.008	0.009	0.009	0.010	0.009	0.010	0.010	0.012

From the simulated results in Tables 3.6, 3.7 and 3.8, it is seen that all the methods can control FDR when no factorial effect is active. In this situation, Wu and Hamada's method works slightly better than our method and Lenth's method.

When some effects are active, both our proposed method and the Wu and Hamada's method control FDR at 5% nominal level especially when the ABH procedure is used for both method. The performance of both methods are similar. The Lenth's method is quite conservative whether BH or ABH procedure is used.

Case II: σ_i^2's are not homogeneous

For the case of no active effect, we used the models

$$y_{ij} \sim N(0, \exp(A + C + 0.5AC))$$

and

$$y_{ij} \sim N(0, \exp(A + C + D + 0.5CD))$$

for 2^3 and 2^4 experiments, respectively.

In the presence of some active effects, we used the models

$$y_{ij} \sim N(10 + A + B + 0.5AB, \exp(A + C + 0.5AC))$$

and

$$y_{ij} \sim N(5 + 0.65A + 0.65B + 0.45C, \exp(A + C + 0.5AC))$$

for 2^3 experiment. For 2^4 experiments, we used the models

$$y_{ij} \sim N(5 + 0.5A + 0.45B + 0.5D + 0.4AD, \exp(A + B + D + 0.5AD))$$

and

$$y_{ij} \sim N(5 + 0.75A + 0.65B + 0.55C + 0.5AD, \exp(A + B + C + 0.5BD)).$$

The simulated FDRs are summarized in Tables 3.9, 3.10, 3.11, 3.12 and 3.13.

From the simulation results in Tables 3.9, 3.10, 3.11, 3.12 and 3.13, it is clearly seen

Table 3.9: Simulated FDRs under the 5% using BH and ABH procedures for the models: $y_{ij} \sim N(0, \exp(A+C+0.5AC))$, $y_{ij} \sim N(0, \exp(A+C+D+0.5CD))$ in 2^3 and 2^4 experiments respectively

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method BH				Our method ABH			
7	0.046	0.039	0.037	0.037	0.046	0.039	0.037	0.037
15	0.045	0.038	0.035	0.035	0.045	0.038	0.035	0.035
	Wu and Hamada BH				Wu and Hamada ABH			
7	0.087	0.071	0.071	0.062	0.087	0.071	0.071	0.062
15	0.098	0.075	0.066	0.061	0.098	0.075	0.066	0.061
	Lenth BH				Lenth ABH			
7	0.024	0.026	0.028	0.029	0.024	0.026	0.028	0.029
15	0.020	0.018	0.021	0.022	0.020	0.018	0.021	0.022

Table 3.10: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(10+A+B+0.5AB, \exp(A+C+0.5AC))$ in 2^3 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	7	0.029	0.026	0.025	0.024	0.049	0.045	0.049	0.046
Wu and Hamada's method	7	0.048	0.047	0.039	0.038	0.072	0.072	0.064	0.065
Lenth's method	7	0.002	0.001	0.001	0.001	0.002	0.002	0.002	0.002

Table 3.11: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(5+0.65A+0.65B+0.45C, \exp(A+C+0.5AC))$ in 2^3 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	7	0.029	0.026	0.025	0.023	0.042	0.040	0.042	0.042
Wu and Hamada's method	7	0.054	0.045	0.042	0.039	0.071	0.063	0.061	0.058
Lenth's method	7	0.003	0.003	0.003	0.002	0.003	0.004	0.003	0.003

Table 3.12: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(5+0.5A+0.45B+0.5D+0.4AD, \exp(A+B+D+0.5AD))$ in 2^4 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	15	0.031	0.028	0.027	0.028	0.041	0.042	0.042	0.046
Wu and Hamada's method	15	0.065	0.055	0.049	0.042	0.080	0.071	0.066	0.064
Lenth's method	15	0.011	0.012	0.012	0.012	0.012	0.013	0.014	0.013

Table 3.13: Simulated FDRs under the 5% using BH and ABH procedures for the model: $y_{ij} \sim N(5+0.75A+0.65B+0.55C+0.5AD, \exp(A+B+C+0.5BD))$ in 2^4 experiment

Method	I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
		BH				ABH			
Our method	15	0.033	0.033	0.034	0.032	0.051	0.052	0.054	0.054
Wu and Hamada's method	15	0.066	0.052	0.050	0.048	0.085	0.075	0.074	0.073
Lenth's method	15	0.007	0.006	0.004	0.007	0.008	0.007	0.007	0.009

that our proposed method can also control FDR both in the absence of active effect and in the presence of some active effects, especially when ABH procedure is used. Lenth's method also controls FDR in both cases, but the results are more conservative compared to our proposed method. Wu and Hamada's method is quite liberal in the absence of active effect whether BH or ABH procedure is used. Interestingly, Wu and Hamada's method coupled with ABH procedure is quite liberal in the presence of active effects, while Wu and Hamada's method coupled with BH procedure works well in the presence of active effect.

3.3 Application To Real Examples

3.3.1 Example 1.1

Dispersion Model

Here the P-values for the $I = 15$ effects of interest are shown in Table 3.14. Suppose that we would like to control the FDR to be $q = 5\%$ in the dispersion model. Then, using the BH procedure, effect A is declared active by all the methods except Lenth's method that declared no active effect. Using the ABH procedure, we found that $\hat{m}_0 = 13$ for our method, $\hat{m}_0 = 12$ for Wu and Hamada's method, $\hat{m}_0 = 14$ for Variyath et al.'s method and $\hat{m}_0 = 15$ for Lenth's method. In this case, effect A is also declared active by our method, Wu and Hamada's methhod, and Variyath et al.'s method. Again, no effect is declared active by Lenth's method.

Table 3.14: P-values for the fifteen effects in the dispersion model: Example 1.1

		Sorted P-values			
l	Effects	Our method	Wu and Hamada	Variyath et al.	Lenth
1	A	0.0004	0.0001	0.0014	0.0447
2	BC	0.0272	0.0162	0.0438	0.1644
3	AC	0.0560	0.0406	0.0850	0.2283
4	ABD	0.0699	0.0485	0.0968	0.2414
5	AB	0.0752	0.0528	0.1030	0.2507
6	ABC	0.1936	0.1571	0.2321	0.3871
7	ACD	0.2703	0.2302	0.3104	0.4678
8	CD	0.2848	0.2443	0.3249	0.5404
9	BD	0.3074	0.2665	0.3474	0.5576
10	D	0.4283	0.3886	0.4655	0.6515
11	BCD	0.4486	0.4095	0.4849	0.6651
12	C	0.4912	0.4536	0.5257	0.6915
13	ABCD	0.6512	0.6226	0.6767	0.7946
14	B	0.6516	0.6231	0.6771	0.7965
15	AD	0.9490	0.9444	0.9529	0.9708

Location Model

For each method, the P-values for the $I = 15$ effects of interest are given in Table 3.15. Suppose that we would also like to control the FDR to be $q = 5\%$ in the location model. By using BH procedure, effect A is declared active by our method and Wu and Hamada's method. Lenth's method declared no active effect in this case. Using the ABH procedure, we found that $\hat{m}_0 = 15$ for all the three methods. Again, effect A is declared active by our method and Wu and Hamada's method. No effect is declared active by Lenth's method.

3.3.2 Example 1.2

Dispersion Model

For each method, the P-values for the seven effects of interest are shown in Table 3.16. Suppose that we would like to control the FDR to be $q = 5\%$ in the dispersion model. Then, no effect is declared active by all the methods using BH procudure. Using the ABH procedure, we found that $\hat{m}_0 = 7$ for all the methods. Again, no

Table 3.15: P-values for the fifteen effects in the location model: Example 1.1

l	Effects	Sorted P-values Our method	Wu and Hamada	Lenth
1	A	0.0020	0.0016	0.0559
2	B	0.0391	0.0374	0.1735
3	AB	0.1163	0.1144	0.2862
4	C	0.2024	0.2005	0.3817
5	BC	0.2556	0.2542	0.4390
6	ABD	0.2558	0.2542	0.4406
7	ABCD	0.2952	0.2940	0.4766
8	AD	0.2998	0.2984	0.5026
9	BD	0.4193	0.4188	0.6326
10	ACD	0.5045	0.5045	0.6942
11	BCD	0.5370	0.5373	0.7162
12	AC	0.7151	0.7156	0.8297
13	ABC	0.7762	0.7769	0.8658
14	CD	0.8951	0.8953	0.9374
15	D	0.9031	0.9033	0.9431

effect is declared active by all the methods.

Table 3.16: P-values for the seven effects in the dispersion model: Example 1.2

l	Effects	Sorted P-values Our method	Wu and Hamada	Variyath et al.	Lenth
1	C	0.0648	0.0179	0.0302	0.2821
2	AF	0.0956	0.0325	0.0475	0.3256
3	E	0.1832	0.0878	0.1059	0.4325
4	F	0.2039	0.1032	0.1213	0.4966
5	D	0.4355	0.3172	0.3302	0.7322
6	A	0.6860	0.6041	0.6097	0.8578
7	B	0.8793	0.8456	0.8474	0.9460

Location Model

In this case, we also examined the performance of all the methods considered under location model. The P-values for the seven effects of interest for each method are summarized in Table 3.17. Again, suppose that we would like to control the FDR at $q = 5\%$. Then, two effects D and F would be identified as active by both our method and Wu and Hamada's method using the BH procedure. No effect is declared active by Lenth' method using the BH procedure. Using the ABH procedure, we found that

$\hat{m}_0 = 7$ for all three methods. Again, two effects D and F are declared active by both our method and Wu and Hamada's method. No effect is declared active by the Lenth's method.

Table 3.17: P-values for the seven effects in the location model: Example 1.2

		Sorted P-values		
l	Effects	Our method	Wu and Hamada	Lenth
1	D	0.0022	0.0008	0.0970
2	F	0.0034	0.0013	0.1117
3	A	0.0549	0.0428	0.3032
4	E	0.1458	0.1306	0.4958
5	AF	0.5970	0.5912	0.8460
6	C	0.8562	0.8546	0.9473
7	B	0.8907	0.8896	0.9601

Chapter 4

Summary and Future Work

4.1 Summary

In this thesis, we focused on controlling the IER, EER and FDR in the location and dispersion models of the response. More specifically, our methods are based on the z-type test statistic (Wu and Hamada, 2000, 2009) for the factorial effects in the dispersion model and the t-type test statistic (Wu and Hamada, 2000, 2009) for the factorial effects in the location model.

In Chapter 2, we re-investigated the distribution of the the z-type statistic and proposed a new distribution for this test statistic. Based on this new distribution, new procedures have been suggested to control the IER and EER in the dispersion model. Our simulation studies suggest that the new procedures work well in terms of controlling the IER and EER in the dispersion model. Other existing methods are either too liberal or too conservative for controlling either or both of the IER and EER in the dispersion model. We also identified the distribution of the t-type statistic and suggested a resampling method to generate random samples from this distribution. Based on the generated random samples, we suggested some new procedures to control the IER and EER in the location model. Our simulation results showed that our method works well in terms of controlling the IER and EER in the location model whether σ_i^2's are homogeneous or heterogeneous among the m runs. However, Wu and Hamada's method works well when the σ_i^2's are homogeneous.

In Chapter 3, we suggested using the new distributions we found in Chapter 2 to calculate the P-values of the z-type statistic and t-type statistic, respectively. Coupled with the ABH procedure, our methods can also accurately control the FDR in the location and dispersion models. On the other hand, Wu and Hamada's method only works in the location model for some situations, for example, when σ_i^2's are homogeneous.

Our proposed methods have also been applied to two real examples to control the IER, EER and FDR in the location and dispersion models.

4.2 Recommendations for Future Work

The followings are some recommendations for future research based on this thesis:

- As stated earlier, replicated two-level factorial designs are full of useful information. This is due to the fact that they allow reliable estimation of the location and dispersion of the response. In this thesis, we considered only the case when the number of replications in each run is the same. Though, there may be some practical situations where the number of replications is not the same. However, further research could be conducted on controlling these error rates when the number of replications is not the same for each run.

- There may be some practical situations that require to study factors with more than two levels or mixed levels. See Wu and Hamda (2000, 2009) for some real examples. Further research could also be carried out to see what will happen by extending the current setup to other three-level or mixed-level factorial experiments.

- Since the main goal of this thesis is to control the IER, EER and FDR at any nominal level, little consideration was given to the power of the proposed methods in identifying the active effects in replicated two-level factorial experiments. Further study could be carried out to develop methods or modify the proposed methods to simultaneously control the error rates and achieve higher power in identifying active effects.

Bibliography

Benjamini, Y. and Hochberg, Y. (1995), "Controlling the False Discovery Rate: A Practical and Powerful Approach to Multiple Testing," *Journal of the Royal Statistical Society*, Ser.B, **57**, 289-300.

Benjamini, Y. and Hochberg, Y. (2000), "On the Adaptive Control of the False Discovery Rate in Multiple Testing with Independent Statistics," *J. Behav. Educ. Statist.*, **25**, 60-83.

Benjamini, Y. and Yekutieli, D. (2001), "The Control of the False Discovery Rate in Multiple Testing Under Dependency," *Ann. Statist.*, **29**, 1165-88.

Benjamini, Y., Krieger, A.B. and Yekutieli, D. (2006), "Adaptive Linear Step-Up Procedures That Control the False Discovery Rate," *Biometrika*, **3**, 491-507.

Black, M.A. (2004), "A Note on the Adaptive Control of False Discovery Rates," *J. Royal Statist. Soc.*, B, **66**, 297-304.

Box, G. E. P., Hunter, W. G., and Hunter, J. S. (1978), *Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building*, John Wiley and Sons.

Brenneman, W.A. and Nair, V.N. (2001), "Methods for Identifying Dispersion Effects in Unreplicated Factorial Experiments: A Critical Analysis and Proposed Strategies," *Technometrics*, **43(4)**, 388-405.

Cochran, W.G. and Cox, G.M. (1957), *Experimental Designs, 2nd ed*, Wiley, New York.

Cook, R. D. and Weisberg, S. (1983), "Diagnostics for Heteroscedasticity in Regression," *Biometrika*, **70**, 1-10.

Daniel, C. (1959), "Use of half-normal Plots in Interpreting Factorial Two-Level Experiments," *Technometrics*, **1**, 311-341.

Daniel, C. (1976), *Applications of Statistics to Industrial Experimentation*, Wiley, New York.

Edwards, D.J. and Mee, R.W. (2008), "Empirically Determined p-Values for Lenth t-Statistics," *Journal of Quality Tech.*, **40**, 368-380.

Efron, B., Tibshirani, R.J., Storey, J.D. and Tusher, V. (2001), "Empirical Bayes Analysis of Microarray Experiments" *J. Amer.Statist. Assoc.*, **96**, 1151-60.

Genovese, C. and Wasserman, L. (2002a), "Operating Characteristics and Extension of the False Discovery rate Procedure," *J. Royal Statist. Soc.*, B, **64**, 499-517.

Genovese, C. and Wasserman, L. (2002b), "A Stochastic Process Approach to False Discovery Control," *Ann Statist.*, **32**, 1035-61.

Harvey, A.C. (1976), "Regression Models with Multiplicity Heteroscedasticity," *Econometrica*, **44**, 461-465.

Kempthorne, O. (1952), *The Design and Analysis of Experiments*, Wiley, New York.

Kimel, M.T., Benjamini, Y. and Steinberg, M. (2008), "The False Discovery Rate for Multiple Testing in Factorial Experiments," *Technometrics*, **50**, 32-39.

Lenth, R.V. (1989), "Quick and Easy Analysis of Unreplicated Factorials," *Technometrics*, **31**, 469-473.

Loughin, T.M. and Rodríquez, J.E. (2011), “Computational Issues with Fitting Joint Location/Dispersion Models in Unreplicated 2^k Factorials,” *Computational Statistics and Data Analysis*, **55**, 491497.

Mee, R.W. (2009), *A Comprehensive Guide to Factorial Two-Level Experimentation*, Springer, New York.

Montgomery, D.C. (2001), *Design and Analysis of Experiments*, Wiley, New York.

Montgomery, D.C. (2009), *Design and Analysis of Experiments*, Wiley, New York.

Mosig, M.O., Lipkin, E., Khutoreskaya, G., Tchourzyna, E., Soller, M. and Friedman, A.A. (2001), “Whole Genome Scan for Quantitative Trait Loci Affecting Milk Protein Percentage in Israeli-Holstein Cattle, by Means of Selective Milk DNA Pooling in a Daughter Design, Using an Adjusted False Discovery Rate Criterion,” *Genetics*, **157**, 1683-98.

Nair, V.N. and Pregibon, D. (1988), “Analyzing Dispersion Effects from Replicated Factorial Experiments,” *Technometrics*, **30**, 247-257.

Sarka, S. (2002), “Some results on the False Discovery Rate in Stepwise Multiple Testing Procedures,” *Ann. Statist.*, **30**, 239-57.

Storey, J.D. (2002), “A Direct Approach to False Discovery Rate,” *J. Royal Statist. Soc.*, B, **64**, 479-98.

Storey, J.D. (2003), “The Positive False Discovery Rate: A Bayesian Interpretation and the q-value,” *Ann. Statist.*, **31**, 2013-35.

Storey, J.D. and Tibshirani, R. (2003a), “Statistical Significance for Genome-wide Studies,” *Proc. Nat. Acad. Sci.*, **100**, 9440-45.

Storey, J.D. and Tibshirani, R. (2003b), “SAM Thresholding and False Discovery Rates for Detecting Differential Gene Expression in DNA Microarrays. In

Parmigiani, G., Garrett, E.S., Irizarry, R.A. and Zeger, S.L. (eds.)," *The Analysis of Gene Expression Data: Methods and Software*, Springer, New York.

Storey, J.D., Taylor, J.E. and Siegmund, D. (2004), "Strong Control, Conservative Point Estimation, and Simultaneous Conservative Consistency of False Discovery Rates: A Unified Approach," *J. Royal Statist. Soc.*, B, **66**, 187-205.

Tagushi, G. (1986), "Introduction to Quality Engineering: Designing Quality into Products and Processes," *Asian Productivity Organization, Tokyo.*

Tronendle, J. (1999), "A Permutational Step-Up Method for Testing Multiple Outcomes," *J. Statist. Plann. Inf.*, **84**, 139-58.

Variyath, A.M., Abraham, B. and Chen, J. (2005), "Analysis of Performance Measures in Experimental Designs Using Jackknife," *J. Quality Tech.*, **37**, 91-100.

Wang, P.C. (1989), "Tests for Dispersion Effects from Orthogonal Arrays," *Comp. Statist. Data Anal*, **8**, 109-117.

Weller, J. I., Song, J.Z., Heyen, D.W., Lewin, H.A. and Ron, M. (1998), "A new approach to the Problem of Multiple Comparisons in the Genetics Dissection of Complex Traits," *Genetics*, **150**, 1699-1706.

Wu, J.C.F. and Hamada, M. (2000), *Experiments: Planning, Analysis and Parameter Design Optimization*, Wiley, New York.

Wu, J.C.F. and Hamada, M. (2009), *Experiments: Planning, Analysis and Parameter Design Optimization, Second Edition*, Wiley, New York.

Appendix

Appendix A: R code to Calculate a_n

```
### Our Method ###
f1=function(x,n)
{
### log(x/(n-1)) times the density function of chisq with (n-1)
 degress of freedom###
### x: a random variable that follows a chisquare distribution
### n: number of replicates
log(x/(n-1))*dchisq(x,df=n-1)
}
f2=function(x,n)
{
### log(x/(n-1)) times the density function of chisq with (n-1)
degress of freedom###
### x: a random variable that follows a chisquare distribution
### n: number of replicates

( log(x/(n-1)) )^2*dchisq(x,df=n-1)
}
exact.var.dispersion<-function(n)
```

```
{
### Variance of log(chisq/(n-1)) with (n-1)
degress of freedom###
mom1=integrate(f1,0,Inf,n=n)
mom2=integrate(f2,0,Inf,n=n)
mom2$value-mom1$value^2
}
## Wu and Hamada's Method ##
approx.var.dispersion<-function(n)
{
### Variance of the approximation method ###
## n: number of replicate ##
2/(n-1)
}
## Computing a_{n} ##
a<-function(n)
{
## n: number of replicate ##
sqrt(exact.var.dispersion(n)/approx.var.dispersion(n))
}
```

Appendix B: R code to Calculate the IER critical value for our Proposed Method in Location Model

```
## Critical value (IER)##
simul.t<-function(rho,n)
{
### rho is estimated from the given data ###
## n is the number of replication ##
```

```
m=length(rho)
w=rnorm(1000000,0,1)
x=matrix(rchisq(1000000*m,n-1),ncol=m)
y=(x%*%rho)/(n-1)
z=abs(w/sqrt(y))
quantile(z,0.95)
}
```

Appendix C: R code to Calculate the EER critical value for our Proposed Method in Location Model

```
## Critical Value (EER) ##
quantile.eer<-function(rho,v,n)
{
m=length(rho)
## m is the number of run ##
## rho is estimated from the given data ##
V=matrix(v,nrow=m)
## V is the matrix with columns corresponding to
the factorial effects of interest ##
P=diag(rho,m,m)
A=t(V)%*%P%*%V
## A is the variance-covariance matrix ##
W=mvrnorm(10000,rep(0,nrow(A)),A)
R=apply(abs(W),1,max)
Q=matrix(rchisq(10000*m,n-1),ncol=m)
Y=(Q%*%rho)/(n-1)
Z=R/sqrt(Y)
q=quantile(Z,0.95)
```

```
return(q)}
```

Appendix D: R code to Calculate the FDR P- value for our Proposed Method in Location Model

```
simul.pvalue<-function(rho,n,statistic)
{
## rho is estimated from the data##
##n is the number of replicate##
m=length(rho)
## m is the number of run ##
I=m-1
w=rnorm(1000000,0,1)
x=matrix(rchisq(1000000*m,n-1),ncol=m)
y=(x%*%rho)/(n-1)
z=abs(w/sqrt(y))
pvalue=rep(0,I)
for(i in 1:I)
{
pvalue[i]=mean(z>=abs(statistic[i]))
}
pvalue
}
```