University of Alberta

Inversion Use Among Students in Grades 2 - 4

by

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ABSTRACT

Well beyond the time at which children become competent at addition and subtraction, many children fail to use the principle of *inversion,* that $a + b - b$ *must* equal a, in solving arithmetic problems. Application of this fundamental mathematical concept renders computationally difficult problems easy and demonstrates an understanding of the inverse relation between addition and subtraction. Although some children show some understanding of this principle prior to formal schooling, many children fail to apply it in symbolic contexts through Grade 4. In this study we successfully employed a novel testing task to identify children who use inversion and those who do not. We also investigated the roles of attentional skills, general conceptual understanding, and computational skills in the use of inversion. To further explore the profiles of children who use inversion and those who do not, we examined cluster patterns but found no direct link between inversion use and these skills.

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INTRODUCTION

The study of children's development of arithmetic skill informs both instructional practices and our understanding of cognitive development. An important aspect of arithmetic that many children have difficulty applying is the principle of *inversion*: that $a + b - b$ *must* equal a. To fully understand arithmetic, one must understand the additive composition of number (Bryant et al., 1999), and a key component of that understanding is the principle of inversion. The ability to apply the inversion principle to arithmetic problems can also facilitate the solution of computationally difficult problems (Bisanz & LeFevre, 1990). To apply this principle to the solving of arithmetic equations, one must understand the inverse relation between addition and subtraction, and also recognize that no calculation is required. Although many children demonstrate some level of understanding this principle even before they enter school (Klein & Bisanz, 2000; Rasmussen, Ho, & Bisanz, 2003; Sherman & Bisanz, in press), low levels of application of this principle in solving arithmetic equations persist well into elementary school (Bisanz & LeFevre, 1990; Bryant, Christie, & Rendu, 1999; Stern, 1992). Inversion is a prime example of children not applying a fundamental concept that is, most likely, available to them. The study of the development of inversion use can therefore be used as a vehicle to further understand how children begin to apply concepts in problem solving more generally. In this study we explore how the application of the inversion concept is related to other cognitive skills.

If the principle of inversion is applied to solve an inversion problem such as $8 + 4 - 4$, no computation is required, and it should therefore be solved, on average, more quickly and more accurately than a *standard* problem on which the principle of inversion is not as easily applied, such as 7 + 4 – 5. By comparing performance on inversion and standard problems, several researchers have found that at least some children have some sensitivity to inversion at a very young age. Rasmussen et al. (2003) found that when problems were presented through the addition and subtraction of blocks instead of in symbolic (numeric) form, both 4-year old and 6-year old children showed evidence of applying the inversion principle. Sherman and Bisanz (in press) found that even some 3-year-olds used this principle in solving inversion problems presented in a similar block format. In contrast, many children do not use inversion to solve symbolic problems (i.e., problems presented with numbers rather than in block form) through the early years of elementary school. Bisanz and LeFevre (1990) found that the proportion of children who used conceptually based shortcuts (40%) did not increase between the ages of 6 to 9 years. Stern (1992) found that children under 10 years of age were more likely to use the shortcut when they were presented inversion problems alone (61%), as opposed to intermixed with standard problems (30%) where the shortcut strategy had to compete with the more familiar strategy of computing from left to right.

The studies demonstrating that children can apply the principle of inversion to block problems were all conducted with young children, whereas the studies showing children do not consistently apply the principle to symbolic problems were all conducted with older children. Although block and symbolic performance have not been included simultaneously in a single study to date, clearly older children do not consistently use whatever underlying conceptual understanding younger children appear to demonstrate in their success on block problems. The question of why some children do not use inversion when problems are presented in symbolic form remains unanswered. One hypothesis is that children develop an understanding of the principle of inversion through their experience with addition and subtraction. That is, as children gain experience correctly solving addition and subtraction problems, they are more likely to recognize the pattern that adding and subtracting the same quantity results in no change. However, evidence for early understanding of inversion may partially contradict this hypothesis. Even when counting skills are not well developed, Sherman and Bisanz (in press) found that 3-year-old children were better at inversion problems (presented in block form) than they were at standard problems. Bisanz and LeFevre (1990) noted inversion use remained low among 6- to 9-year-olds while computational skill increased, as measured by performance on control problems. Bryant et al. (1999) also found that inversion did not relate to simple addition and subtraction skills among 6- to 8-year-old children. However, Canobi (2005) found that, among children 5 to 7 years old, those who were classified as likely using inversion also solved addition and subtraction problems faster and reported high use of retrieval. Thus, contradictory findings have left unanswered the question of whether inversion use is related to calculational skill.

One potential reason for the apparent contradiction is that the relation between inversion use and calculational skill may not be linear. Gilmore (2005) found that 5- to 10-year-old children clustered into three groups: those who had poor calculational skills and did not use inversion, those who had good calculational skills and used inversion and, most interestingly, a final group that had poor calculational skills but used inversion. Inversion use related to calculational skill overall, $r(47) = 0.58$, $p < 0.01$, but children with poor calculational skills could have either high or low use of inversion. Although this finding is interesting, replication is essential to verify the generalizability of these clusters. Replication is an important component of inferential studies and all the more so in studies using clustering techniques, where significance testing is not possible. Therefore, further exploration of the relation between calculational skill and use of the inversion shortcut is warranted.

A second hypothesis is that use of inversion is linked to the development of other forms of conceptual understanding in mathematics, growth that is not necessarily related to addition and subtraction. Resnick (1982) proposed that children may gradually move from seeing school-based math as a procedural domain (syntax) to a conceptual domain (semantics). If so, as children come to appreciate a variety of math concepts more fully, they would begin to use those concepts more consistently in their solutions. Thus, young children may not use inversion when solving symbolic problems because they search for procedures to solve school-based math, rather than searching for underlying concepts. As children begin to see school-based math as a conceptual domain, gains are seen in many areas where conceptual understanding can be applied.

A third hypothesis is that use of inversion requires not only an understanding of the inverse relation between addition and subtraction, but also the ability to sustain attention and process the three terms and required actions simultaneously. If a child understands the concept of inversion but approaches the problem from left to right without attending to both the addition and subtraction and to all three terms involved, he or she is less likely to use inversion to solve the problem. The idea of *cognitive inertia,* in which children continue to use a tried-and-true algorithm (such as computing from left to right) even when a shortcut is available, has also been proposed as a reason why children fail to use inversion (Bisanz & LeFevre, 1990). Even for children who understand the concept of inversion, to use it they also have to (a) notice the potential to apply the concept in the problem presented, and (b) inhibit the prepotent response of calculating from left to right. Thus, these two attentional skills may be related to children's use of inversion.

We theorize that children generally follow a developmental path in which they begin with an early quantitative understanding of inversion in non-symbolic form [as demonstrated in Rasmussen et al. (2003) and in Sherman and Bisanz (in press)]. Children then develop familiarity with symbolic notation for quantities through schooling and gain computational skill. This experience, however, may lead to a syntactic (procedural) approach to symbolic problems, hindering performance on symbolic inversion problems. The difficulty persists until such time as they begin to see school-based math as a conceptual domain, and have sufficient attentional skills to (a) process all terms and required actions in the problem simultaneously, and (b) inhibit the prepotent response of using a familiar calculating procedure.

Through probes of children's use of inversion across three grades we addressed two main questions. First, are there age- or gender-related differences in the use of inversion? Although no gender effects have been observed in inversion use to date, such effects have been found in other areas of mathematical development including complex arithmetic problems and problem solving (Geary, 1994; Hyde, Fennema, & Lamon, 1990), and thus examination of potential gender differences is warranted. Our second question is how calculational skill, conceptual understanding, and attentional skills are related to the use of inversion, and whether these relations provide possible reasons for any age-related differences we might observe? Because children's use of inversion can be inconsistent and the relations to calculational, conceptual, and attentional skills may not be overwhelmingly large, it is useful to test a large number of children to address these questions. Prior to this study inversion tasks had only been administered in experimental forms that would be difficult and expensive to run with large numbers of children. In this study a shorter test of inversion use was employed to explore the three hypotheses so that large numbers of children could be tested and so that other data could be gathered from these children at the same time. Another objective of the present study is to attempt to replicate the interesting cluster pattern identified by Gilmore (2005). No one has investigated inversion use in relation to the development of other mathematical concepts or in relation to attentional skill. Thus, this study is designed to shed light on the cognitive characteristics of those children who use inversion and those who do not.

METHOD

The data for this research were collected during the third year of a longitudinal project on children's early mathematical development. The nearly 500 participants involved in the project, from pre-kindergarten through Grade 4, complete a wide range of measures each year. These tests include measures of literary skills (e.g., vocabulary, phonemic awareness, and word reading), cognitive skills (e.g., fine motor ability, spatial skills, and processing speed), and many math-related skills (e.g., counting, digit recognition, addition, subtraction, multiplication and place value). The cross-sectional study presented here focuses on a subset of tasks completed as a part of the larger study. Children's use of inversion in Grades 2-4 was investigated by comparing performance on inversion and standard problems, and performance was related to measures of computational skill, conceptual understanding, and attentional skill.

Participants

Two hundred one children, 111 girls and 90 boys, in Grades 2 through 4 from two Canadian cities participated. The sample included 32 boys and 34 girls in Grade 2 (median age, in yr;mo, 7;10, ranging from 7;4 to 8;11); 29 boys and 43 girls in Grade 3 (median 8;10, range 7;6 to 10;1); and 29 boys and 34 girls in Grade 4 (median 9;10, range 9;5 to 10;4).

Materials and Procedures

Inversion use. Each child was presented a set of 14 three-term addition and subtraction questions. The first two trials were used for practice, followed by 12 test trials presented in one of two pseudo-random orders. Each version consisted of four easy inversion problems, four hard inversion problems, and four standard problems. Response times were recorded and children had a maximum of 15 seconds to respond. Following Siegler and Stern (1998), the hard inversion ($a + b - b$) and standard problems ($a + b - c$) were designed to be nearly impossible to solve within the time limit using successive addition and subtraction (i.e., adding $a + b$ and then subtracting b or c). Thus all children were expected to perform poorly on standard problems, and accurate performance on hard inversion problems was expected only for children who used the principle of inversion to solve inversion problems. The easy inversion problems (with terms from 1 through 4) presumably could be solved using either calculation or the inversion principle. They were included only to ensure some level of success for children and to cue possible use of inversion on other problems (Stern, 1992).

The problems used in the study are presented in the Appendix A. In all cases, the difficulty of the standard problems was comparable to that of the hard inversion problems, assuming left-to-right computation. That is, the sums of $a + b$ and $b + c$ were very similar for hard inversion and standard problems.

To verify whether our behavioural measures accurately captured children's use of inversion, we asked for verbal self-reports on one additional inversion problem. Following the 2 practice and 12 test trials, the child was presented a new hard inversion problem (17 + 28 – 28) and asked "How would you solve this problem?" (primary solution strategy). The child was then asked whether there were any other ways he or she could solve it (alternate solution strategy), until the child could not think of any additional solution strategies. Each response was coded as one of three solution strategies found in prior studies: inversion (reporting that the answer is *a* without computing at all), computation (adding $a + b$, and then subtracting *b*), or other (Bisanz & LeFevre, 1990). Children were then asked which solution strategy they believed to be the best, and which they believed to be the fastest. A subset of 50 children were asked to justify their selection of the best strategy.

Computational skill. To assess computational skill, children completed the Woodcock-Johnson-Revised Calculation subtest (Woodcock & Johnson, 1989). Initial items required the child to compute single-digit addition, and later items involved single-digit subtraction, multi-digit addition and subtraction, multiplication, division, and fractions. Children were stopped after six incorrect answers, or when they indicated to the experimenter that they did not know how to solve any of the remaining questions. Later problems involved mathematics more advanced than the Grade 4 curriculum, and thus ceiling effects were avoided. This test has a median reliability of .85 and a one-year test-retest correlation of .89 for Grades 2 through 4 (Woodcock & Johnson, 1989). Administration of the test took approximately 5 minutes.

Mathematical conceptual knowledge. Measures of conceptual understanding were assessed with the Canadian version of the KeyMath Numeration subtest (Connolly, 2000). Although this task does not measure understanding of inversion, it covers other important mathematical concepts such as quantity, order, and place value. In the development of the KeyMath test, extensive research was conducted to ensure the content was representative of the mathematics content in Kindergarten through Grade 9. Some slight modifications were made for the Canadian version, maintaining difficulty progression as established in the American version. The Numeration subtest correlates highly with the Mathematics Concepts subtests of the Iowa Tests of Basic Skills (.69) and the Comprehensive Tests of Basics Skills (.57). The median split-half reliability correlation is .84 for Grades 2 through 4, with a range of .81 to .90 (Connolly, 2000).

Attention. The Children's Color Trails Test (Williams et al., 1995) was administered as a measure of the child's ability to sustain attention and process multiple sources of information simultaneously. This test can be used with children unfamiliar with the English language, those who have reading and language disorders, or have limited educational experiences. The Children's Color Trails Test has similar discriminant validity as other trails tests with established reliability (Williams et al., 1995). The children were asked to follow a "trail" of numbers on a printed page. Each number was presented in either a pink or yellow circle such that the colour of the circle always switched from the current number to the next number, for example, pink 1, yellow 2, pink 3, and so on. The child was timed from the moment he or she began movement toward the first circle until he or she reached the last circle. In the first section of the test there was only one instance of each number, so the colour of the circle was irrelevant. In the second section, every number after 1 was presented in both a yellow and a pink circle. The child had to take note of both the numbers and colours, requiring the child to inhibit the prepotent response of drawing the line to the first circle he or she saw with the correct number. Similar inhibiting may be required in the inversion task if a child must notice the potential for using the inversion shortcut, and overcome the prepotent response of calculating from left to right. Thus, this test taps both components of attention that may be required in the use of inversion, (a) noticing potential alternative solution options in the problem presented; and (b) inhibiting the prepotent response. An interference index was calculated as the difference between the time the child took to complete the first section, which does not require attending to the colour of the circles, and the time the child took to complete the second section, where alternating between colours is required, divided by the completion time of the first section. Thus, an interference index of 1.00 indicates the child took twice as long to complete the second section when inhibition was required. Interference indices were calculated for children in Grades 2 and 4. Time constraints of the larger project prohibited administration of the Color Trails Test to the Grade 3 children and one Grade 2 boy.

RESULTS AND DISCUSSION

Validity of Inversion Scores

If answering hard inversion problems correctly is likely to reflect use of the inversion principle, inversion use can be indexed on a scale from 0 (no inversion problems solved correctly) to 4 (all inversion problems solved correctly). Recall that the "easy inversion" problems were included mainly to provide the child with a sense of success rather than as a test of inversion use. Therefore scores on these problems are not included in the analyses.[1] To assess whether children could solve the hard inversion problems (hereafter referred to simply as *inversion* problems) by left-to-right calculation, we examined performance on the standard problems. Because standard and inversion problems were matched for difficulty, children who use calculation to solve both types of problems should perform equally well on standard and inversion problems. Ninety-seven children (48%) solved one or more inversion problem correctly but did not solve any standard problems correctly, and thus it is unlikely that they used calculation to obtain correct answers to the inversion problems. For these children, use of inversion was indexed by the number of inversion problems solved correctly. Thirty-six children (18%) answered some standard problems correctly, making the interpretation of their inversion scores more challenging. However, most (25 of 36) were accurate more often on inversion problems than on standard problems, suggesting that they used something other than calculational skills alone to solve the inversion problems. For these children, the number of inversion problems solved correctly was taken as a measure of inversion use. Four children solved the same number of inversion problems and standard problems

[1] Although performance on the small inversion problems correlated highly with performance on the large inversion problems, $r(199) = .38$, $p < .001$, it was impossible to determine whether answers to the small inversion problems were obtained by calculating and/or by using the inversion shortcut. If correct answers to the small inversion problems were obtained by calculating the answer, we would expect calculation scores to be predictive of the number of small inversion problems solved correctly. If correct answers to the small inversion problems were obtained by using the inversion shortcut, we would expect performance on the large inversion problems to be predictive of the number of small inversion problems solved correctly. To determine whether children calculated the small inversion answers or used the inversion shortcut, we conducted a regression using Woodcock-Johnson calculation scores and performance on the large inversion problems as predictors of performance on the small inversion problems. Both predictors independently accounted for a significant amount of the variance in small inversion scores (Woodcock-Johnson $\beta = 0.128$, $p < .001$; Large inversion score $\beta = 0.252$, $p < .001$). Thus, children may have calculated the answers to some small inversion problems, providing further reason not to include these items in the analyses.

correctly, making their inversion scores uninterpretable. These children's scores were eliminated from the remaining analyses, thus reducing the sample size to 197. Among the 7 children who had more standard than inversion problems correct, only one had any inversion problems correct. On the assumption that this child may have solved the inversion problem by some method other than the inversion shortcut, the inversion score of that child was adjusted to zero.

If children who answered some standard problems correctly used their computational skills to solve the standard problems, they might have used their computational ability to solve some of the inversion problems. If this were the case, we would expect to see a correlation between the number of standard problems answered correctly and the number of inversion problems answered correctly, among the children who answered any standard problems correctly. This correlation was negligible, however, $r(34) = .11, p = .54$. We can therefore be confident that children who were credited with solving inversion problems correctly most likely did not calculate answers using left-to-right computation.

All problems were presented in the form "$a + b - c = answer$". Therefore the a term was always the correct answer for the inversion problems. Some children may have answered with the a term for some of the problems they could not solve, thus obtaining correct answers to some inversion problems without applying the appropriate concept. If a child provided the a term as the response for standard problems more frequently than inversion problems, it is reasonable to assume that he or she may not have used inversion to solve the inversion problems. Three inversion scores (1.5%) were adjusted to zero where the children answered with the a term more frequently for the standard problems than for inversion problems. If, however, a child provided the *a* term as the response for inversion problems more frequently than standard problems, it would appear that he or she was using a more sophisticated strategy to solve the inversion problems. Therefore the inversion scores of these children were not adjusted.

Inversion use was distributed bimodally (see Figure 1), such that children tended to either answer all or nearly all of the problems correctly (score of 3 or 4), or all or nearly all incorrectly (score of 0 or 1). We therefore categorized the children as non-inverters (0-2 correct) or inverters (3 or 4 correct).

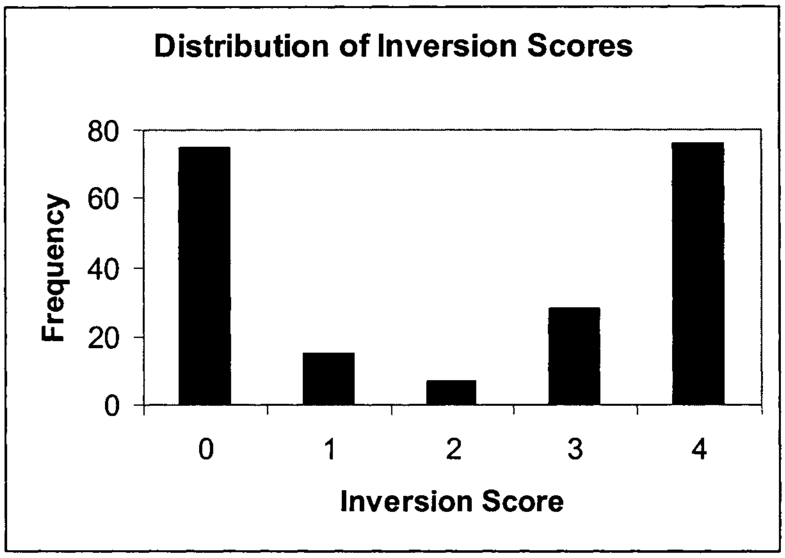

Figure 1. Distribution of students as a function of inversion scores.

To verify whether our categorization of children as inverters or non-inverters was appropriate, we turned to the verbal reports each child provided about how he or she would solve an inversion problem. Verbal reports for the child's primary solution method (the method he or she would use to solve the problem), alternate solution method (any other method the child knows), and the method he or she thought was best and fastest were coded as an inversion response, a computation response, or other (which included guessing). Interrater reliability was .96. (See Appendix B for further information on coding procedures.)

Reactivity can pose a threat to the validity of responses when participants are asked to explain how they solve problems, because the task of verbal reporting can cause participants to alter how they perform the task under investigation. In this study, however, children completed all 12 inversion trials prior to being asked to describe how they would solve an inversion problem. Therefore, performance on the inversion trials should not have been affected by this probe.

We examined whether children's verbal reports appeared to correspond with their apparent inversion use based on number of problems correct. Reports of solution method for each level of inversion problems solved correctly are presented in Table 1. The majority of children (83%) who were categorized as inverters also reported inversion as their primary solution method. The majority of children (87 %) who were categorized as non-inverters reported computation as their primary solution method. These findings are consistent with the view that the inverter and non-inverter categorizations were generally appropriate. (For more information pertaining to the verbal reports, please see Appendix C.)

Table 1

Number of Students Reporting Inversion, Computation, or Other as Primary or Alternate Method as a Function of Number of Inversion Problems Solved Correctly

	Number of Inversion Problems Correct					
Reported Method	0 $n = 75$	1 $n = 12$	2 $n = 7$	3 $n = 28$	4 $n = 75$	Total $N = 197$
Primary						
Inversion	9	5	4	21	70	109
Computation	62	7	3	6	5	83
Other	4	0	0	1	0	5
Alternate						
Inversion	17	3	1	5	7	34
Computation	29	4	2	8	35	78
Other	29	5	4	15	33	86

Note. Some alternate solution method responses were simply different descriptions of the primary method (e.g., a different method of computation). Therefore the same child may be counted in the same category under both the primary and alternate method headings.

Inversion Use

We first examined how grade and gender were related to the distribution of inverters and non-inverters (see Table 2). The distribution of non-inverters and inverters was not related to grade, $\chi^2(2, N = 197) = 2.51, p = .29$, or gender, $\chi^2(1, N = 197) = 1.68$,

$p = .20$. In Grade 2, however, the number of boy inverters exceeded the number of girl inverters, $\chi^2(1, N = 65) = 8.13, p < .01$. No such gender difference was observed in the other two grades (Grade 3: $\chi^2(1, N = 71) = .003, p > .90$; Grade 4: $\chi^2(1, N = 61) = .53, p = .47$). For girls, the number of inverters increased with age, $\chi^2(2, N = 110) = 7.14, p = .028$, but for boys performance did not vary across grades, $\chi^2(2, N = 87) = 2.38, p = .30$.[2] Therefore, although there were no overall effects of grade or gender, inversion use increased with grade for girls but not for boys.

Table 2

Percentage of Inverters and Non Inverters as a Function of Grade and Gender

	N	% Inverters	% Non-Inverters
Grade 2	65	49.2 %	50.8 %
Boys	31	67.7 %	32.3 %
Girls	34	32.4 %	67.6 %
Grade 3	71	47.9 %	52.1 %
Boys	29	48.3 %	51.7 %
Girls	42	47.6 %	52.4 %
Grade 4	61	60.7 %	39.3 %
Boys	27	55.6 %	44.4 %
Girls	34	64.7 %	35.3 %

Note. Inverters answered 3 or 4 inversion problems correctly, whereas non-inverters answered 0-2 problems correctly.

[2] A 2(Gender) X 3(Grade) ANOVA on the inversion scores essentially replicated this finding. The ANOVA revealed a grade by gender interaction, $F(2,191) = 3.61, p = .029$, such that girls (*M*s = 1.35, 1.98, 2.47 for Grades 2, 3, and 4 respectively) showed increased use of inversion with grade but boys (*M*s = 2.65, 1.86, 2.26) did not. In Grade 2 boys used inversion much more than did girls, $F(1, 191) = 17.18, p < .05$, but in Grades 3 and 4, there was no difference between genders (*F*s < 1).

For girls, inversion use is correlated not only with grade, $r(108) = .25, p < .01$, but also with computational skill, $r(108) = .25, p < .01$, and attentional skill, $r(108) = .19, p = .045$, whereas none of these factors correlate significantly with inversion use in boys. Because there were also significant intercorrelations among grade, computational skill, and conceptual skill for girls, *p*s < .001, we conducted a regression using each of these variables as predictors of inversion use to identify the independent contributions of each factor. Together the factors accounted for a significant amount of the variability in inversion scores, $R^2 = .08, p = .031$, but none of the variables uniquely predicted inversion scores, *p*s > .23. Thus, the observed effect of grade in girls could be due to improvement in computational and/or conceptual skill, but we were unable to draw such a conclusion from the data. For boys, computational and conceptual skills correlated with grade, and each other, *p*s < .001, but none of these variables correlated with inversion scores, *p*s > .40.

Relations of Calculational Skill, Conceptual Understanding, and Attentional Skills to Inversion Use

To test for grade- and gender- related changes in calculational skill, conceptual understanding, and attentional skills, we first conducted grade by gender ANOVAs on each predictor variable. As expected, Woodcock-Johnson Calculation raw scores increased with grade, $F(2, 191) = 57.74, p < .01$, as did KeyMath ceiling scores, $F(2, 191) = 21.28, p < .01$. Recall that Color Trails performance measures were only collected for students in Grades 2 and 4. No effects of grade or gender were found, Fs$(1, 121) < 1$.

To determine how calculational skill, conceptual understanding, and attentional skills relate to each other and to inversion use, we examined how the Woodcock-Johnson

Calculation raw score, the Key Math Numeration subtest ceiling item, and the Colour Trails Interference Index correlated with each other and with the inversion score in each grade (see Tables 3-5). To better understand the relative contributions of each skill to inversion use, we then conducted a multiple regression analysis using gender, computational skill, attention, and conceptual knowledge to predict inversion use in each grade. The only regression predicting a significant amount of the variance in inversion scores was in Grade 2, $R^2 = .21, p < .01$, where gender was the only reliable predictor, $t = 2.98, p < .01$. No effects were found in Grade 3, $R^2 = .06$, or in Grade 4, $R^2 = .05$.

Table 3

Correlations Among Gender, Measures of Skill, and Inversion Scores in Grade 2

Measures	Gender	Woodcock-Johnson Calculation raw score	KeyMath ceiling item	Colour Trails Interference Index	Inversion Score
Woodcock-Johnson Calculation raw score	.196	1			
KeyMath ceiling item	.194	.485**	1		
Colour Trails Interference Index	.101	-.103	.100	1	
Inversion Score	.354**	.125	.274*	-.079	1

* p < .05. ** p < .01. *n* = 65.

Table 4

Correlations Among Gender, Measures of Skill, and Inversion Scores in Grade 3

Measures	Gender	Woodcock-Johnson Calculation raw score	KeyMath ceiling item	Inversion Score
Woodcock-Johnson Calculation raw score	-.107	1		
KeyMath ceiling item	-.029	.446**	1	
Inversion Score	-.031	.233	.089	1

** $p < .01$; $n = 71$.

Table 5

Correlations Among Gender, Measures of Skill, and Inversion Scores in Grade 4

Measures	Gender	Woodcock-Johnson Calculation raw score	KeyMath ceiling item	Colour Trails Interference Index	Inversion Score
Woodcock-Johnson Calculation raw score	.167	1			
KeyMath ceiling item	.299*	.487**	1		
Colour Trails Interference Index	.005	.149	.117	1	
Inversion Score	-.061	-.035	-.024	-.220	1

* $p < .05$. ** $p < .01$. $n = 61$.

One explanation for why calculational, conceptual, and attentional skills did not predict inversion use could be that the relation between inversion use and the predictor skills is nonlinear. Groups of individuals may be identified by characteristic skill profiles. Indeed, Gilmore (2005) found three clusters of inversion use based on calculational skill. One group had high calculational skill and high inversion use, a second group had low calculational skill and low inversion use, and a third group had low calculational skill but high inversion use. Gilmore's measure of computational skill consisted of addition and subtraction problems of similar difficulty to the inversion problems. We wished to investigate whether this noteworthy pattern of computational skill and inversion use held when using a standardized measure of computational skill, and what the profiles of conceptual and attentional skill would be for these groups. Inversion scores were standardized to ease interpretability of the results, but similar patterns resulted when raw scores were used instead. Ward's (1963) hierarchical agglomerative method was first employed to investigate potential solutions. A large increase in fusion coefficients indicates that the clusters being merged are dissimilar. All increases in fusion coefficients were quite small until five clusters were merged into four (95.01 to 145.43). A second reasonably large increase in fusion coefficients occurred when four clusters were merged into three (145.43 to 207.20). When three clusters were merged into two a much larger jump (from 207.20 to 392.77) occurred, reflecting in a substantial loss of information and resulting in clusters that were uninformative. We therefore discounted the two-cluster solution. Thus examination of three-, four-, and five-cluster solutions was warranted.

Using squared Euclidean distance seed values from Ward's (1963) method, we conducted a *k*-means cluster analysis based on each of the possible cluster solutions (k = 5, 4, and 3 means). *K*-means cluster analysis is an iterative partitioning method where seed values estimate the initial centroids of the *k* clusters, but as each member is assigned to a cluster the centroid is recalculated. Iterations through the dataset continue re-assigning each case to its nearest cluster centroid until no further iterations can provide a more optimal assignment of cases to clusters (Aldenderfer & Blashfield, 1984).

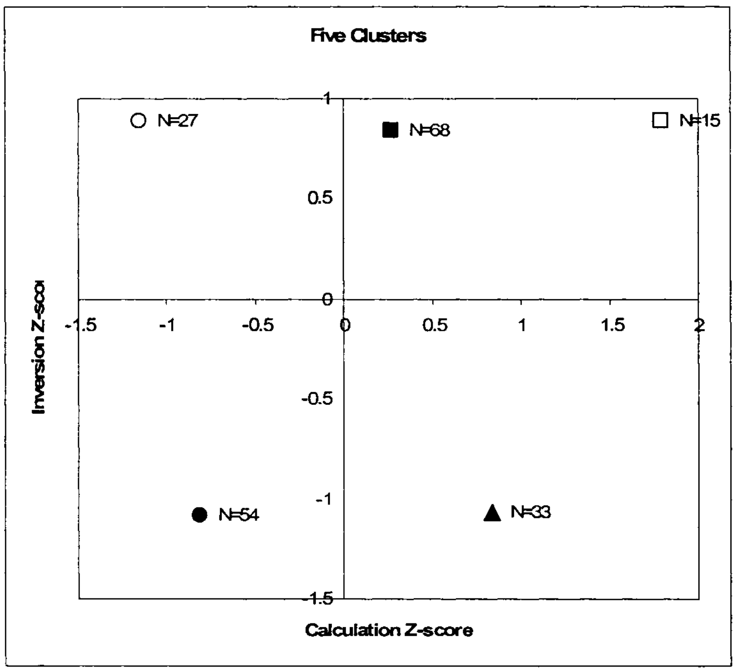

Figure 2. Five-cluster group means on calculational skill and inversion use.

In Figure 2 the five-cluster solution is displayed. Along the top of the graph there are three clusters of children who use inversion. One cluster has relatively low calculational skill, another had a medium level of skill, and a third cluster has a high level of calculational skill. We can clearly see that children across the spectrum of calculational skill use inversion. This finding matches that of Gilmore (2005), who found that children with both high and low calculational skill can use inversion. A fourth cluster of children, with low inversion use and low calculational skill ($n = 54$), also matches that found by Gilmore (2005). Contrary to Gilmore's findings, however, we found an additional cluster of children displaying high calculational skill yet low inversion use ($n = 34$).

In Figures 3 and 4 the four- and three-cluster solutions are displayed, respectively. The only difference among all three solutions is how the children who use inversion consistently are clustered by their level of calculational skill. The two clusters in the low inversion use range remain virtually constant across all three cluster solutions. The high calculational skill and low inversion use group that was found in the cluster analysis conducted with all three predictors and inversion use was also consistently found in this cluster analysis. Due to small numbers of participants in some clusters in the five-cluster solution, we decided to proceed with the four-cluster solution.

The distribution of grades and gender across clusters is provided in Table 6. Overall, the distribution of boys and girls did not differ among the clusters, $\chi^2(3, N = 197) = 4.14, p = .25$, but there was a gender difference between some clusters in Grade 2, $\chi^2(3, N = 65) = 10.61, p = .014$. Females outnumbered males significantly in the low-calculation/low-inversion cluster but the distribution was even or reversed in the remaining clusters. The distribution of grades was not uniform across clusters, $\chi^2(6, N = 197) = 47.88, p < .01$. The low calculational skill clusters (high and low inversion use) did not differ from each other in grade distribution, $\chi^2(2, N = 88) = 2.45, p = .33$, and the high calculational skill clusters did not differ from each other, $\chi^2(2, N = 109) = 1.71, p = .43$, but both low calculational skill clusters differed from each of the high calculational skill clusters ($ps < .001$). Children in higher grades were more likely to be in the high calculational skill clusters.

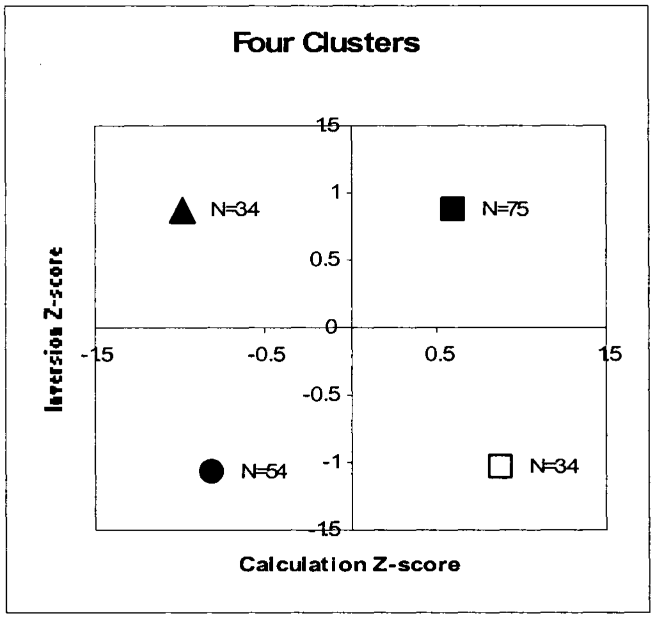

Figure 3. Four-cluster group means on calculational skill and inversion use.

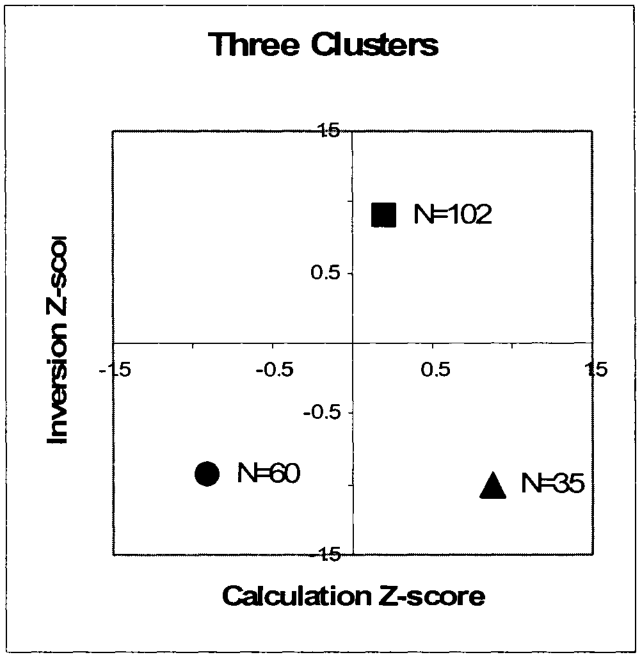

Figure 4. Three-cluster group means on calculational skill and inversion use.

Table 6

Number of Children in Each Cluster Across Grades and Gender

Cluster		Grade 2			Grade 3			Grade 4			All Grades		
Calculation	Inversion	F	M	Total	F	M	Total	F	M	Total	F	M	Total
Low	Low	20	6	26	15	10	25	1	2	3	36	18	54
Low	High	7	12	19	5	6	11	4	0	4	16	18	34
High	Low	2	3	5	4	5	9	11	9	20	17	17	34
High	High	5	10	15	18	8	26	18	16	34	41	34	75

Note. *n*s = 65, 71, and 61 for Grades 2, 3, and 4 respectively.

We wanted to determine whether the conceptual and attentional skills of the clusters provided further insight into why some children use inversion and others do not. See Table 7 for the mean KeyMath Numeration Subtask ceiling item, Color Trails Interference Index, and Woodcock-Johnson calculation raw score across clusters. Recall that attentional scores were not collected for Grade 3 students. Thus, comparisons of attentional skill between clusters include only students in Grades 2 and 4. A 2(Inversion Use) x 2(Calculational skill) ANOVA on conceptual skill showed a positive relation between conceptual skill and calculational skill, $F(1, 193) = 52.90, p < .01$, as expected, but no effect of inversion use, $F(1, 193) < 1$, and no interaction, $F(1, 193) < 1$. Thus, the clusters differ in conceptual skill corresponding to their differences in calculational skill, but the high and low inversion use clusters do not differ from each other in conceptual skill. A second 2(Inversion Use) x 2(Calculational skill) ANOVA performed on Grade 2 and 4 attentional skills revealed no differences in attentional skill based on calculational skill, $F(1, 121) < 1$, but inversion users had somewhat higher levels of attention than non-users, $F(1, 121) = 3.36$, $p = .069$. Although the interaction was not significant, $F(1, 121) < 1$, the difference in attentional skill between children who use inversion and those who do not was particularly pronounced between the high calculational skill clusters, $t(71) = 1.94$, $p = .056$. Thus attentional skill may play a role in inversion use, particularly among high calculational skill children, but this interpretation is not strongly supported and its validity requires replication.

Table 7

Mean Attentional, Conceptual, and Calculational Scores for Each Cluster

Cluster		Colour Trails Interference index (Grades 2 and 4)	KeyMath Ceiling Item	Woodcock-Johnson Calculation Raw Score
Calculation	Inversion			
Low	Low	1.33 (.59)	19.09 (3.66)	11.09 (2.12)
Low	High	1.21 (.67)	18.88 (2.99)	10.41 (2.32)
High	Low	1.48 (.58)	22.15 (2.23)	17.82 (2.49)
High	High	1.15 (.73)	22.23 (2.55)	16.72 (2.67)

Note. Low interference indices suggest better attentional skills. A high interference index reflects more difficulty inhibiting. Standard deviations are presented in parentheses.

GENERAL DISCUSSION

Why do many children not use inversion when solving arithmetic problems? In this large-scale study we examined children's use of inversion across three grades. This study is the first in which calculational, conceptual, and attentional skills were simultaneously examined in relation to inversion use. We explored the contributions each of these skills to the use of inversion, and how these relations differ between grades. We first examined whether there were age- or gender- related differences in the use of inversion. Boys outperformed girls in Grade 2, but across grades girls showed an improvement and boys did not. Although it is unclear why this gender difference might exist, we can speculate that girls may be more likely than boys to attend closely to their teachers' instructions at a younger age. For inversion problems adhering to the procedure of calculating from left to right, as most children are instructed to do in the classroom, decreases performance.

We then examined how calculational skill, conceptual understanding, and attentional skills are related to the use of inversion, and whether these relations provide possible reasons for the observed inconsistencies in inversion use. One hypothesis was that inversion use might develop as a result of experience with addition and subtraction. If this hypothesis were true, then we would expect inversion use to be related to computational skill, which generally increases across grade. The results were not so clear. Investigation into the profiles of children who use inversion and those who do not revealed the three groups Gilmore (2005) discovered (high-calculation/high-inversion, low-calculation/low-inversion, and low-calculation/high inversion) and an additional cluster with children who were high in calculational skill but low in use of inversion. This finding demonstrates that even among children who have developed high levels of calculational skill, some do not use the inversion shortcut.

A second hypothesis was that inversion use might result from the child increasingly seeing school-based mathematics as a conceptually based domain rather than procedurally based. We would then expect to see a link between inversion use and other conceptually based mathematical skills. We did not find such a relation in this in this study, but it is possible that our measure of conceptual knowledge was not appropriate to capture this particular transition. A test focusing on children's syntactic versus semantic views of mathematics (with age-appropriate material) may tap this hypothesis more directly.

A third hypothesis was that inversion use may develop as a result of the child being able to sustain attention to process both the addition and subtraction, and all three terms involved, and overcome the prepotent response of left-to-right calculation. If so, then we would expect inversion use to be related to an increase in attention and inhibition. This overall relation was not apparent in this study, but, there was some suggestion that inversion use among children with high calculational skill may be related to attentional skill. High calculational skill children who use inversion have marginally better attentional skills than those who do not use inversion. One explanation for this tentative finding is that children with high calculational skill may have to inhibit left-to-right calculation to use the inversion shortcut. This marginally significant difference suggests additional testing with different measures of attention may be appropriate in future studies.

This study provides further evidence that inversion use does not relate to calculational skill, and provides preliminary evidence suggesting use may not relate to general conceptual understanding in mathematics. It also provides preliminary evidence suggesting that attentional skill may play a role in inversion use among children with high calculational skill.

Because use of inversion did not consistently increase across grades for both genders, it may be appropriate to expand the age range examined to capture the development of the use of inversion. As a next step we suggest identifying factors prior to Grade 2—such as counting and adherence to canonical procedures—that may be critical for discovering inversion and predicting later use of inversion in Grades 2-4.

Although we did not find overwhelming evidence relating inversion use to the cognitive skills we thought might be predictive, an important contribution of this study is that we were able to successfully identify inverters and non-inverters with a novel testing task. This approach can facilitate collection of larger data sets that will prove useful in future studies of mathematical development. Large-scale studies of inversion use may contribute to our understanding of why some children fail to use procedural shortcuts that could facilitate their tasks, and how children begin to apply concepts in problem solving. Such discoveries would inform both instructional practices and our understanding of cognitive development.

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APPENDIX A

Version 1

a	+	*b*	-	*c*	=	*answer*
3	+	2	-	1	=	4
19	+	25	-	25	=	19
14	+	27	-	19	=	22
4	+	2	-	2	=	4
12	+	31	-	31	=	12
5	+	1	-	1	=	5
18	+	23	-	23	=	18
19	+	24	-	17	=	26
17	+	26	-	26	=	17
13	+	29	-	32	=	10
3	+	2	-	2	=	3
16	+	26	-	31	=	11
15	+	32	-	32	=	15
2	+	1	-	1	=	2

Version 2

a		*b*		*c*		*answer*
3	+	2	-	1	=	4
19	+	25	-	25	=	19
2	+	1	-	1	=	2
15	+	32	-	32	=	15
16	+	26	-	31	=	11
3	+	2	-	2	=	3
13	+	29	-	32	=	10
17	+	26	-	26	=	17
19	+	24	-	17	=	26
18	+	23	-	23	=	18
5	+	1	-	1	=	5
12	+	31	-	31	=	12
4	+	2	-	2	=	4
14	+	27	-	19	=	22

APPENDIX B

Self-reports were initially coded in one of 13 categories (see Table B1). For a sample of 100 children two coders reached 83% agreement at this level. We found the 13 categories fit well into three main groups that succinctly describe the solution method: computation, inversion, and computation-inversion. The two coders reached 89% agreement at this level, and most disagreements were between the coding of inversion and computation-inversion, suggesting that this distinction is difficult to detect and may reflect slightly different ways of describing the same procedure. Upon collapsing these two categories, the coders were in agreement on 96% of the cases. The remaining disagreements were resolved by reviewing the self-reports together and discussing which coding would be more appropriate. In three of the four cases the first coder's coding was retained. All additional coding was performed by the first coder.

Table B1

Sample problem: 17 + 28 – 28 = ?

Final Coding	Initial Coding	Description
Inversion	Inversion	Uses inversion shortcut. E.g., "I would take 28 from 28 because they are the same and then I have 17 left."
	Computation-inversion	Explains computation, but gives correct answer (17). E.g. "I would do 17 plus 28 and then subtract 28 and then I would know that it was 17."
Computation	Computation	Describes a computational strategy without providing further details of how it would be accomplished. E.g., "Add the 17 and the 28 then find the answer then minus the other 28."
	Computation-digits	Explanation of adding ones, then tens, etc. (or putting vertically) E.g., "I would look at the ones and add them and then look at the tens and add them and subtract the 28."
	Computation-decomposition	Explanation of attempt at decomposition method. E.g., "I would add 10 to make 38 and then add 7 more and then take away 28."
	Computation-counting	Describes counting to solve the problem. E.g., "Use my fingers to count them all up and figure the answer out."
	Computation-reverse	Describes changing the order of actions performed. E.g., "You could do 28 plus 17; and then you could minus the 28." (As opposed to 17 plus 28).
	Computation-wrong	Provides an incorrect computational method. E.g., "Do 28 + 28 and then subtract 17."
	Computation-other	Other computational method described. E.g., "Figure out the problem the mental math way."
Other	Other	E.g., "Put first number."
	Guess	E.g., "I would just guess."
	Don't know	E.g., "I don't know how to solve it."
	None	No response (or "Nope").

APPENDIX C

Validity of Verbal Reports

A small number of discrepancies existed between our categorization of inverters and the children's verbal reports. Eleven children stated that computation was their primary method but answered 3 or 4 inversion problems correctly. Because we designed the problems to be too difficult to solve within the time limit by computation, these verbal reports appear suspect. Of the 11 children, 5 gave inversion as an alternate method, thus demonstrating inversion was indeed in their procedural repertoire. It appears likely that the remaining 6 also had inversion in their repertoire but did not verbally report it. It is not uncommon for young children to have difficulty reporting mental procedures relating to mathematical thinking (Ginsburg, Kossan, Schwartz, & Swanson, 1983). Of the 83 children who stated that computation was their primary method, only 22 (27%) gave inversion as another possible solution method. In contrast, of the 109 who stated inversion was their primary method, 48 (44%) gave computation as an alternate solution method. We expected that even more of the children who provided inversion as their primary solution method to provide computation as an alternate method. We suspect that many of the children may have realized they would not have been able to solve such large problems correctly within the time limit using computation and therefore did not consider it to be a possible solution method.

It is important that we have as accurate a measure as possible of which children have some understanding or use of inversion to relate inversion use to other skills. To account for children who may have used inversion on some trials but not reported it as their primary method, we considered whether inversion was reported as an alternate method and refer to children who reported inversion to either probe as having inversion in their repertoire.

Grade- and Gender-Related Changes in Verbal Reports

Reports of solution procedures for each grade are presented in Table C1, and for each gender in Table C2. Because only a small number of children reported "Other" as their primary solution method (n = 5), those children were eliminated from the analyses, leaving 192 children who reported inversion or computation as their primary method. A chi-square test revealed no difference in self-reports of primary solution method across grades, $\chi^2(2, N = 192) = 2.19, p = .34$, or between genders, $\chi^2(1, N = 192) = .61, p = .66$ Primary solution methods also did not differ between genders within any of the grades, or between grades within either of the genders. These results appear not to match the results based on inversion scores, but recall that children may have had inversion in their repertoire but not reported it as their primary method. We therefore also conducted chi-square tests using counts of whether or not the children had inversion in their repertoire. Reports of inversion in their repertoire increased with grade, $\chi^2(2, N = 197) = 8.93$, p = .011, but overall boys and girls were equally likely to have inversion in their repertoire, $\chi^2(1, N = 197) = 0.92, p = .34$. In Grade 2 boys reported inversion in their repertoire marginally more often than girls, $\chi^2(1, N = 65) = 3.66$, $p = .056$. In Grades 3 and 4 boys and girls were equally likely to have inversion in their repertoire. Girls were more likely to have inversion in their repertoire as a function of grade, $\chi^2(2, N = 110) = 9.42$, $p < .01$, but among boys reports of inversion in their repertoire remained similar across grades, $\chi^2(2, N = 87) = 2.64, p = .27$. Therefore, children's verbal reports of inversion use increased across grades for girls, but not for boys.

Table C1

Percentage of Students Reporting Solution Procedures as a Function of Grade

Criterion Reported Procedure	Grade 2 *n* = 65	 3 *n* = 71	 4 *n* = 61	Total *N* = 197
Primary				
Inversion	47.7 %	54.9 %	63.9 %	55.3 %
Computation	46.2 %	43.7 %	36.1 %	42.1 %
Other	6.2 %	1.4 %	0.0 %	2.5 %
Alternate				
Inversion	10.8 %	18.3 %	21.3 %	16.8 %
Computation	27.7 %	39.4 %	52.5 %	39.6 %
Other	61.5%	42.3 %	26.2 %	43.7 %
In Repertoire				
Inversion	55.4 %	64.8 %	80.3 %	66.5 %
Computation	60.0 %	64.8 %	77.0 %	67.0 %
Other	63.1 %	42.3 %	26.2 %	44.2 %

Note. Primary solution method is the method the child reported that he or she would use to solve the problem, Alternate solution methods are other ways the child said he or she could solve the problem, and In Repertoire is the percentage of children who reported that solution method as either primary or alternate.

Table C2

Percentage of Students Reporting Solution Procedures as a Function of Gender

Criterion Reported Procedure	Gender Girls $n = 110$	 Boys $n = 87$	 Total $N = 197$
Primary			
Inversion	53.6 %	57.5 %	55.3 %
Computation	43.6 %	40.2 %	42.1 %
Other	2.7 %	2.3 %	2.5 %
Alternate			
Inversion	18.2 %	14.9 %	16.8 %
Computation	39.1 %	40.2 %	39.6 %
Other	42.7 %	44.8 %	43.7 %
In Repertoire			
Inversion	63.6 %	70.1 %	66.5 %
Computation	66.4 %	67.8 %	67.0 %
Other	43.6 %	44.8 %	44.2 %

See Note to Table C1.

Reports of the best and fastest solution methods across grades are presented in Table C3. These reports were highly correlated such that those who judged computation as the best method were highly likely to judge computation as the fastest method, $r(195)$

= .77, and those who judged inversion as the best method were highly likely to judge inversion as the fastest method, $r(195) = .86$. We therefore chose to report only the findings related to the reports of the best solution. Children in all grades tended to identify inversion as the best method more frequently than computation, and the frequency with which children provided an inversion response increased across grades, $\chi^2(2, N = 197) = 8.30, p = .02$. Reports of the best and fastest solution methods by gender are presented in Table C4. Both genders responded similarly to the best solution method probe, $\chi^2(1, N = 197) = .57, p = .45$, identifying inversion as the best method more frequently than computation. A subset of 50 children were asked why they selected the method they provided as the best method. The majority of these children (84%) stated either their selection was the easiest or the fastest method (regardless of whether their response was inversion or calculation).

Table C3

Percentage of Students Reporting Best and Fastest Solution Procedures as a Function of Grade

Criterion Reported Procedure	Grade 2 $n = 65$	 3 $n = 71$	 4 $n = 61$	Total $N = 197$
Best				
Inversion	50.8 %	59.23 %	75.4 %	61.4 %
Computation	38.5 %	31.0 %	21.3 %	30.5 %
Other	10.8 %	9.9 %	3.3 %	8.1 %
Fastest				
Inversion	53.8 %	54.9 %	72.1 %	59.9 %
Computation	29.2 %	32.4 %	24.6 %	28.9 %
Other	16.9 %	12.7 %	3.3 %	11.2 %

Table C4

Percentage of Students Reporting Best and Fastest Solution Procedures as a Function of Gender

Criterion / Reported Procedure	Gender: Girls $n = 110$	Boys $n = 87$	Total $N = 197$
Best			
Inversion	59.1 %	64.4 %	61.4 %
Computation	30.9 %	29.9 %	30.5 %
Other	10.0 %	5.7 %	8.1 %
Fastest			
Inversion	55.5 %	65.5 %	59.9 %
Computation	32.7 %	24.1 %	28.9 %
Other	11.8 %	10.3 %	11.2 %

Interestingly, although inversion use did not clearly increase across grades with behavioural measures, we found increases across grades in the number of children reporting inversion as the best and fastest way to solve inversion problems, and surprisingly even an increase in the number who report that the inversion shortcut is the method they *would* use. This finding may suggest that children are more aware of inversion as a solution strategy as they get older, yet are somehow inhibited from using it under some circumstances. Alternatively, an increase in verbal expressiveness skills might explain the increase in ability to report inversion solution method. However, most children who were categorized as inverters based on inversion scores also reported inversion use verbally.