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ELASTIC-PLASTIC AND CREEP ANALYSIS OF CASINGS FOR THERMAL WELLS

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ABSTRACT

This report considers an idealized model of a horizontal slice through a thermal recovery well subject to the constraint of zero strain in the longitudinal direction, develops the governing equations for the stress analysis for a number of different types of responses, presents a special-purpose computer code for the analysis of the problem, and gives numerical results for some example problems.

The model idealizes the casing as a thin cylinder supported laterally by the geological formulation. The loading consists of temperature and internal pressure. In Chapter 1 the elastic case is considered and closed form expressions are derived for all components of casing stress and strain. This basic approach is then modified in subsequent chapters to predict:

(a) Combinations of pressure and temperature that initiate yielding.

(b) Stress and strain histories for elastic-plastic strain hardening material response.

(c) Stress and strain histories for elastic-plastic strain hardening material response, with a Bailey-Norton creep relationship, in which the thermal coefficient, modulus of elasticity, yield strength and strain hardening moduli are all temperature dependent.

These formulations and solutions were developed as the initial phase for a study directed at predicting creep and relaxation effects on casings in thermal recovery wells. Funding for the study was provided by the Center for Frontier Engineering Research. However, funding was not available to permit a study of the application of these developments to realistic time-history situations.

Contents

Page

Notation

$p(\Delta p)$	:	internal pressure of the casing
$T(\Delta T)$	:	temperature
$t(\Delta t)$	:	time
σ_θ	:	tangential stress
σ_r	:	radial stress
σ_z	:	axial stress
σ_{ij}	:	stress tensor
$\sigma_1, \sigma_2, \sigma_3$	:	principal stresses
$\bar{\sigma}$	:	effective stress
$\sigma_Y{}^\circ$	:	initial yield limit
ε_θ	:	tangential strain (total)
ε_r	:	radial strain (total)
ε_z	:	axial strain (total)
ε_{ij}	:	strain tensor
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	:	principal strains
$\varepsilon_\theta{}^p$	:	plastic tangential strain
$\varepsilon_r{}^p$	:	plastic radial strain
$\varepsilon_z{}^p$	:	plastic axial strain
$\varepsilon_{ij}{}^p$	:	plastic strain tensor
$\varepsilon_{ij}{}^T$	:	thermal strain tensor
$\varepsilon_\theta{}^c$	:	creep tangential strain
$\varepsilon_r{}^c$	:	creep radial strain
$\varepsilon_z{}^c$	:	creep axial strain
$\varepsilon_{ij}{}^c$	:	creep strain tensor
$\dot{\varepsilon}_{ij}{}^c$	:	tensor of the creep strain rates
$\bar{\varepsilon}$	:	effective strain

S_{ij}	:	deviatoric stress tensor
S_1, S_2, S_3	:	principal-stress deviators
J_2	:	second invariant of the deviator of the stress tensor
$d\lambda$	:	a positive scale factor in flow rule
R	:	fraction of total strain increment in elastic region
u	:	radial displacement
v	:	tangential displacement
w	:	axial displacement
r	:	internal radius of the casing
t	:	wall thickness of the casing
E	:	elastic modulus of the casing
E^t	:	plastic modulus of the casing (tangent modulus)
μ	:	Poisson ratio of the casing
G	:	shear modulus of the casing ($G = E/2(1+\mu)$)
α	:	thermal expansion coefficient of the casing
$[\alpha]$	:	$[\alpha] = (\alpha, \alpha, \alpha)^T$
$D(D^E)$	:	elastic stiffness matrix
D^P	:	plastic stiffness matrix
D^{EP}	:	elastic-plastic stiffness matrix ($D^{EP} = D^E - D^P$)
$\overline{E}$	:	elastic modulus of rock
$\overline{\mu}$	:	Poisson ratio of rock
C_{ijkl}	:	4th order stiffness tensor of material
δ_{ij}	:	Kronecker delta
$\underline{\underline{\nabla}}$	:	expression that denotes some formula is abridged

CHAPTER 1 INTRODUCTION

1.1 Background

In order to investigate relaxation effects on the stress in the casing of a thermal recovery well it is necessary to be able to model a casing in such a way that the elastic-plastic-temperature-stress-strain response of the casing is properly simulated. Subsequently, the creep can be developed. The model should be capable of determining all stress and strain components for time-dependent problems for thermoplastic materials in a simple and convenient manner. In defining a tractable study which might contribute in a meaningful way to useful predictions for casing analysis the following constraints were adopted for the model.

1. The simplest possible model which might predict a reasonable simulation of stress-strain history in the casing has been adopted.

2. No attempt has been made to predict temperature or pressure variation along the casing. It has been assumed that the temperature and pressure at the point of interest would be available as input quantities.

3. It has been assumed that a reasonable simulation for this limited study can be obtained from an axisymmetric horizontal slice.

4. The response of the slice can be adequately represented as

   i) a thin cylinder, for which

   ii) thermal expansion of surrounding rock can be neglected,

   iii) extensional strains in the axial direction are zero.

Within the above constraints a simple analysis for an elastic-strain hardening plasticity solution, including thermal effects in the casing, has been developed. The simple analysis is based directly on the equilibrium equation for rock-casing interaction rather than the FEM which would require a more complex computer program. In this way simple numerial solutions can be obtained which are "exact" within the above constraints.

This report embraces both elastic-plastic temperature dependent problems and the time dependent creep problem. Computation procedure, programming and numerical results are presented as well as the creation of the model.

1.2 Idealized Model

In polar coordinates the general strain-dependent equations are as follows.

$$\varepsilon_r = \frac{\partial u}{\partial r} \; ; \quad \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r} \; ; \quad \varepsilon_z = \frac{\partial w}{\partial z} \; ; \tag{1.1a-1.1c}$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{u}{r}\right) \; ; \tag{1.1d}$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \; ; \tag{1.1e}$$

$$\varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right) \; ; \tag{1.1f}$$

where u represents the displacement in the radial direction, v in the tangential direction and w in the axial direction. All notation is defined in the list following the Table of Contents.

Using the assumptions of Section 1.1, we isolate a segment from the casing system in the well and model it as a hollow cylinder, with internal and external pressure, which is inextensible in the longitudinal direction, as shown in Fig. 1.1. The internal pressure is the pressure of the injected steam. The external pressure comes from the surrounding rock which can be regarded as a spring.

Thus, we have the following constraints

i) $w = 0, \ \varepsilon_z = 0$; (1.2a-b)

ii) $\frac{\partial v}{\partial \theta} = 0, \ \varepsilon_\theta = \frac{u}{r}$; (1.2c-d)

iii) There are not any shearing stresses.

Obviously this is a plane strain problem for which the normal stresses, σ_r, σ_θ and σ_z, are principal stresses (Fig. 1.2). The matrices of the stress and strain state are

$$\begin{bmatrix} \sigma_\theta \\ \sigma_r \\ \sigma_z \end{bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu \\ \mu & 1-\mu & \mu \\ \mu & \mu & 1-\mu \end{bmatrix} \begin{bmatrix} \varepsilon_\theta \\ \varepsilon_r \\ \varepsilon_z \end{bmatrix} \quad (1.3)$$

and

$$\begin{bmatrix} \varepsilon_\theta \\ \varepsilon_r \\ \varepsilon_z \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\mu & -\mu \\ -\mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{bmatrix} \begin{bmatrix} \sigma_\theta \\ \sigma_r \\ \sigma_z \end{bmatrix} \quad (1.4)$$

in which E is elastic modulus of the casing and μ is the Poisson ratio of the casing.

If we consider the internal surface, $\sigma_r = -p$, and the top and bottom boundary conditions require that $\varepsilon_z = 0$. It follows that a closed form solution can be obtained for (1.3) and (1.4) for the elastic case.

Before proceeding further we first need to define the spring constant of supporting rock. For thick cylinders (Fig. 1.3) the displacement along the radial direction is (Timoshenko, 1956)

$$u = \frac{(1+\mu)(1-2\mu)}{E} \frac{p_i a^2 - p_o b^2}{b^2 - a^2} r + \frac{1}{r} \frac{1+\mu}{E} \frac{(p_i - p_o) a^2 b^2}{b^2 - a^2} \tag{1.5}$$

For a hole in an infinite slice the displacement at the internal surface is obtained by setting $r = a$, $p_o = 0$ and letting $b \rightarrow \infty$. The expression then becomes

$$u = \frac{1+\mu}{E} a \, p_i \tag{1.6}$$

Introducing $\bar{E}$ and $\bar{\mu}$ for rock properties

$$p_i = \frac{\bar{E}}{(1+\bar{\mu}) \, r} u \tag{1.7a}$$

Now we define $\bar{K}$ for the supporting rock as follows

$$\bar{K} = \frac{\bar{E}}{(1+\bar{\mu})} \tag{1.7b}$$

Thus, for the casing

$$p_o = \bar{K} \frac{u}{r_o} = K \, \varepsilon_\theta \tag{1.8}$$

in which r_o is the external radius of the casing and ε_θ is the circumferential strain in the casing.

Returning to the basic problem of achieving a closed form solution for the casing system we consider a thin cylinder as shown in Fig. 1.4. When the wall thickness is small, so that normal stresses are distributed uniformly throughout the thickness, the equilibrium equation for the system can be written as (Popov, 1976)

$$2\sigma_\theta t = 2pr_i - 2p_o r_o \tag{1.9a}$$

or

$$\sigma_\theta = \left(pr_i - p_o r_o\right)/t = \left(p - \overline{K}\frac{r_o}{r_i}\varepsilon_o\right)\frac{r_i}{t} \tag{1.9b}$$

where r_i is the internal radius. Now we define afresh the spring coefficient for the supporting rock in the following form

$$K = \overline{K}\frac{r_o}{r_i} \underline{\underline{\nabla}} \alpha_m \overline{K} = \alpha_m \frac{\overline{E}}{(1+\overline{\mu})} \tag{1.9c}$$

where $\alpha_m = r_o/r_i$. In addition, for simplicity, r is used for the internal radius of the casing rather than r_i in what follows.

Since displacement is constrained in the vertical direction

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \mu(\sigma_r + \sigma_\theta)\right) = 0$$

Thus $\quad \sigma_z = \mu\left(\sigma_r + \sigma_\theta\right) \quad (1.9b)$

and $\quad \sigma_r = -p \quad (1.9c)$

The displacement u can be obtained by substituting the above relations into the first equation in (1.4) leading to

$$\varepsilon_\theta = \frac{1-\mu^2}{E}\left(\sigma_\theta - \frac{\mu}{1-\mu}\sigma_r\right) = \frac{u}{r}$$

in which the latter equality arises from (1.2d).

Solving for u,

$$u = \frac{\left(\frac{r}{t} + \frac{\mu}{1-\mu}\right)}{\frac{E}{r(1-\mu^2)} + \frac{K}{r}} \; p \; \underset{=}{\nabla} \; a \; p \qquad (1.10a)$$

in which

$$a = \frac{[r(1-\mu) + \mu t] \; r \; (1+\mu)}{Et + Kr(1-\mu^2)} \qquad (1.10b)$$

Substituting (1.10a) into (1.9a) yields

$$\sigma_\theta = \frac{1 - \frac{\mu(1+\mu)K}{E}}{1 + \frac{r}{t}\frac{(1-\mu^2)}{E}K} \frac{r}{t} \; p \; \underset{=}{\nabla} \; b \; p \qquad (1.11a)$$

in which $b = \frac{r}{t}\left(1 - \frac{K}{r} a\right)$ (1.11b)

When there is temperature change, but the physical properties of the casing system are independent of temperature, equations (1.3) and (1.4) are replaced by

$$\begin{bmatrix} \sigma_\theta \\ \sigma_r \\ \sigma_z \end{bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu \\ \mu & 1-\mu & \mu \\ \mu & \mu & 1-\mu \end{bmatrix} \begin{bmatrix} \varepsilon_\theta - \alpha\Delta T \\ \varepsilon_r - \alpha\Delta T \\ \varepsilon_z - \alpha\Delta T \end{bmatrix} \qquad (1.12)$$

and

$$\begin{bmatrix} \varepsilon_\theta \\ \varepsilon_r \\ \varepsilon_z \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\mu & -\mu \\ -\mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{bmatrix} \begin{bmatrix} \sigma_\theta & & \alpha\Delta T \\ \sigma_r & + & \alpha\Delta T \\ \sigma_z & & \alpha\Delta T \end{bmatrix} \qquad (1.13)$$

The radial displacement becomes

$$u = a \; p + a^o \; \Delta T \qquad (1.14a)$$

where $$a^{o} = \frac{r(1+\mu)E\alpha t}{Et + Kr(1-\mu^{2})} \tag{1.14b}$$

Then the tangential stress σ_θ is

$$\sigma_\theta = b\ p + b^{0}\ \Delta T \tag{1.15a}$$

in which $b^{0} = -\frac{K}{t}\ a^{0}$ (1.15b)

It is clear that as long as the pressure is given all stress and strain components will follow. The concise computing formulas of the elastic solution are listed in the following

$$u = pa + \Delta T\ a^{0} \qquad \text{(a)}$$
$$\sigma_\theta = pb + \Delta T\ b^{0} \qquad \text{(b)}$$
$$\sigma_r = -\ p \qquad \text{(c)}$$
$$\sigma_z = p\mu(b-1) + \Delta T\ (\mu b^{0} - E\alpha) \qquad \text{(d)} \quad (1.16)$$
$$\varepsilon_\theta = pc + \Delta T\ c^{0} \qquad \text{(e)}$$
$$\varepsilon_r = -\left(pd + \Delta T\ d^{0}\right) \qquad \text{(f)}$$
$$\varepsilon_z = 0 \qquad \text{(g)}$$

where $$a = \frac{[r(1-\mu) + \mu t]\ r(1+\mu)}{Et + Kr(1-\mu)^{2}}$$

$$b = r\left(1 - \frac{K}{r}\ a\right) / \ t$$

$$c = (1+\mu)\ [(1-\mu)b+\mu]\ /\ E$$

$$d = (1+\mu)\ [(b-1)\mu+1]\ /\ E$$

$$K = \alpha_m\ \overline{E}\ /\ (1+\overline{\mu}) \tag{1.17}$$

$$a^0 = \frac{r(1+\mu)E\alpha t}{Et + Kr(1-\mu^2)}$$

$$b^0 = -Ka^0/t$$

$$c^0 = [(1-\mu^2)b^0 / E] + (1+\mu)\alpha$$

$$d^0 = [(1+\mu)\mu b^0/E] - (1+\mu)\alpha$$

The above process is summarized as follows. We first establish the stress-strain relations like (1.3) and (1.4). Then the equilibrium equation is combined with them to yield the closed form solution to the simple model. In the later chapters, all formulations that we will discuss are based on this simple procedure and the elastic-plastic-temperature-stress-strain formulation of the casing system is obtained in the same way.

CHAPTER 2 TEMPERATURE INDEPENDENT ANALYSIS

2.1 Prediction of Initial Yield

This chapter considers solution for elastic-plastic problems where the material properties are not dependent upon temperature. Because σ_θ, σ_r and σ_z are principal stresses the Von-Mises condition for initial yield of the casing can be written as

$$\bar{\sigma} \equiv \frac{1}{\sqrt{2}} \left[(\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2 \right]^{1/2} = \sigma_Y \tag{2.1}$$

where σ_Y represents the yield strength and $\bar{\sigma}$ is defined as the 'effective stress'.

Substituting (1.16b,c,d) into (2.1) it follows that the pressure at the onset of the yield of the casing under different temperatures satisfies a second-degree equation

$$A p_Y^2 + B p_Y + C = 0 \tag{2.2a}$$

where
$$A = b(b+1) + \mu(\mu-1)(b-1)^2 + 1 \tag{2.2b}$$

$$B = \Delta T\ b^0(2b+1) + \Delta T(\mu b^0 - E\alpha)\left[(2\mu-1)(b-1)\right] - \Delta T\ b^0 \mu(b-1) \tag{2.2c}$$

$$C = (\Delta T\ b^0)^2 + (\Delta T)^2(\mu b^0 - E\alpha)(\mu b^0 - E\alpha - b^0) - \sigma_Y^2 \tag{2.2d}$$

Obviously, only the positive root of Eq. (2.2a) is meaningful, i.e.,

$$p_Y = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{2.2e}$$

Then the stresses and strains under the critical yield condition will come from the pressure p_y using (1.16) from (h) to (g). So, we have

$$\sigma_\theta = p_y b + \Delta T\, b^0 \tag{2.3a}$$

$$\sigma_r = - p_y \tag{2.3b}$$

$$\sigma_z = p_y \mu (b-1) + \Delta T(\mu b^0 - E\alpha) \tag{2.3c}$$

$$\varepsilon_\theta = p_y c + \Delta T c^0 \tag{2.3d}$$

$$\varepsilon_r = - (p_y d + \Delta T\, d^0) \tag{2.3e}$$

$$\varepsilon_z = 0 \tag{2.3f}$$

To use simple nondimensional ratios and present the results in a systematic manner we define p_y as the pressure in an unconstrained casing which produces the yield stress σ_Y, i.e.,

$$p_y = \frac{t}{r} \sigma_Y \tag{2.4}$$

Define ΔT_y as the temperature increment that causes the longitudinally constrained pipe to yield in compression

$$- E\alpha\, \Delta T_y = - \sigma_Y$$

So, $$\Delta T_y = \frac{\sigma_Y}{E\alpha} \tag{2.5}$$

We also introduce a nondimensional ratio for the rock stiffness, which one may call the foundation coefficient, defined as

$$c_f = \frac{K}{E/(1+\mu)} = \frac{\overline{E}(1+\mu)}{E(1+\overline{\mu})} \tag{2.6}$$

Now we can use p_y, ΔT_y and c_f to nondimensionalize equations (2.3)

$$\tilde{\sigma}_\theta = \frac{\sigma_\theta}{\sigma_Y} = \frac{1-\mu\ c_f}{1+(1-\mu)c_f\ \frac{r}{t}}\ \frac{r}{t}\ \frac{p}{\sigma_Y} - \frac{E\ \alpha\ c_f\ \frac{r}{t}}{1+(1-\mu)c_f\ \frac{r}{t}}\ \frac{1}{\sigma_Y}\ \Delta T \tag{2.7a}$$

$$= a_p\ \tilde{p} - a_T \Delta\tilde{T} \tag{2.7b}$$

in which $$\tilde{p} = p/p_y \tag{2.8a}$$

$$\Delta\tilde{T} = \Delta T/\Delta T_y \tag{2.8b}$$

and

$$a_p = \left(1-\mu c_f\right) / \left[1+(1-\mu)c_f\ \frac{r}{t}\right] \tag{2.9a}$$

$$a_T = c_f / \left[\frac{t}{r} + (1-\mu)c_f\right] \tag{2.9b}$$

Similarly,

$$\tilde{\sigma}_r = \frac{\sigma_r}{\sigma_Y} = -\frac{t}{r}\ \tilde{p} \tag{2.10a}$$

$$\tilde{\sigma}_z = \frac{\sigma_z}{\sigma_Y} = b_p \tilde{p} - b_T \Delta\tilde{T} \tag{2.10b}$$

in which

$$b_p = \mu\left(1-c_f-\frac{t}{r}\right) \Big/ \left[1+(1-\mu)c_f\,\frac{r}{t}\right] \tag{2.11a}$$

$$b_T = \left(1+c_f\,\frac{r}{t}\right) \Big/ \left[1+(1-\mu)c_f\,\frac{r}{t}\right] \tag{2.11b}$$

The yield condition (2.1) can be written as

$$2\,\frac{\bar{\sigma}_Y^{\,2}}{\sigma_Y^{\,2}} = (\tilde{\sigma}_\theta - \tilde{\sigma}_r)^2 + (\tilde{\sigma}_r - \tilde{\sigma}_z)^2 + (\tilde{\sigma}_z - \tilde{\sigma}_\theta)^2 \equiv 2 \tag{2.12}$$

Substituting (2.7b), (2.10a) and (2.10b) into (2.12), a second-degree equation in terms of nondimensional ratio $\tilde{p}$ (when $\Delta\tilde{T}$ is kept constant) yields

$$A(\tilde{p})^2 + B\tilde{p} + C = 0 \tag{2.13a}$$

where

$$A = a_p^{\,2} + b_p^{\,2} + (a_p+b_p)\,\frac{r}{t} - a_p b_p \tag{2.13b}$$

$$B = \left[(b_T-2a_T)a_p + (a_T-2b_T)b_p - (a_T+b_T)\,\frac{r}{t}\right]\Delta\tilde{T} \tag{2.13c}$$

$$C = \left(a_T^{\,2} + b_T^{\,2} - a_T b_T\right)(\Delta\tilde{T})^2 - 1 \tag{2.13d}$$

Because for a given casing r and t are fixed, and the constants E, μ and α don't vary for any casing, we can consider the nondimensional ratios $\tilde{p}$ and $\Delta\tilde{T}$ to be the independent variables and the nondimensional ratio c_f as a parameter which affects the yield condition expressed in terms of $\tilde{p}$ and $\Delta\tilde{T}$.

From Eq. (2.13a) the ratio $\tilde{p}$ under initial yield limit is a function of $\Delta\tilde{T}$ and c_f, i.e.,

$$\tilde{p} = f\left(\Delta\tilde{T},\ c_f\right) \quad (2.13e)$$

With this relation it is convenient to plot the initial yield condition against the nondimensional ratios $\Delta\tilde{T}$ and c_f, as shown in Chapter 6.

2.2 Derivation of Governing Equations for Plastic Behavior

For this chapter, elastic-plastic behavior is defined as time independent but loading history dependent. The constitutive equations are written in incremental form. To simplify later derivations tensor notation is introduced. The stress increment tensor and strain increment tensor can be written, respectively, as

$$d\sigma_{ij} \quad \text{and} \quad d\varepsilon_{ij} \qquad i,j = 1,2,3$$

Now we may derive the constitutive law for plastic behavior. It is usual for isotropic hardening materials to employ the Von-Mises yield condition (Chen 1982 and Wong 1982), i.e.,

$$f\left(\sigma_{ij},\sigma_y\right) = \bar{\sigma} - \sigma_Y = 0 \quad (2.14)$$

where $\bar{\sigma} = \sqrt{3J_2}$ is the effective stress. J_2 represents the second invariant of the deviatoric stress tensor S_{ij}. We have

$$J_2 = \frac{1}{2} S_{ij}\ S_{ij} \quad (2.15)$$

in which $S_{ij} = \sigma_{ij} - \frac{1}{3}\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right) \delta_{ij}$ (2.16)

The usual form for the flow rule (Normality principle) is

$$d\varepsilon^{p}_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \tag{2.17}$$

where $d\lambda$ is a positive scale factor.

The total strain $d\varepsilon$ can be expressed as the sum of elastic, plastic and thermal strains (Bushnell 1973)

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p} + d\varepsilon^{T} \tag{2.18a}$$

so $$d\varepsilon^{e} = d\varepsilon - d\varepsilon^{p} - d\varepsilon^{T} \tag{2.18b}$$

in which $d\varepsilon$ is total strain vector $\quad d\varepsilon = [d\varepsilon_\theta, d\varepsilon_r, d\varepsilon_z]^T$

$d\varepsilon^e$ is elastic strain vector $\quad d\varepsilon^e = [d\varepsilon_\theta{}^e, d\varepsilon_r{}^p, d\varepsilon_z{}^e]^T$

$d\varepsilon^p$ is plastic strain vector $\quad d\varepsilon^p = [d\varepsilon_\theta{}^p, d\varepsilon_r{}^p, d\varepsilon_z{}^p]^T$

$d\varepsilon^T$ is thermal strain vector $\quad d\varepsilon^T = [\alpha]\, dT$

$[\alpha] = [\alpha, \alpha, \alpha]^T$

The stress increments are expressed as

$$d\sigma_{ij} = C_{ijk\ell}\, d\varepsilon_{k\ell}{}^{e}$$

$$= C_{ijk\ell} \left(d\varepsilon_{k\ell} - d\varepsilon_{k\ell}{}^{p} - d\varepsilon_{k\ell}{}^{T}\right) \tag{2.19}$$

where $c_{ijk\ell}$ is a 4th-order symmetrical tensor (Chen 1982)

$$C_{ijk\ell} = 2G\ \delta_{ik}\delta_{j\ell} + \lambda_E\ \delta_{ij}\delta_{k\ell} \tag{2.20}$$

We take Taylor expansion for the function (2.14) in the neighbourhood of point $f_o\ (\sigma_{ij}, \sigma_y)$

$$f_1 = f_o + \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \sigma_Y} \sigma_Y + 0\left[(d\sigma_{ij})^2 + (d\sigma_Y)^2\right]$$

Because the yield surface f_1 and f_0 are zero, and we can neglect the second-order term, Prager's consistency condition is

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \sigma_Y} d\sigma_Y = 0 \tag{2.21}$$

Substituting (2.19) into (2.21) we have

$$\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}\left(d\varepsilon_{k\ell} - d\varepsilon_{k\ell}{}^{p} - d\varepsilon_{k\ell}{}^{T}\right) + \frac{\partial f}{\partial \sigma_Y}\frac{d\sigma_Y}{d\bar{\varepsilon}p} d\bar{\varepsilon}p$$

$$= \frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}\left(d\varepsilon_{k\ell} - d\varepsilon^{T}_{k\ell}\right) - \frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}\ d\varepsilon^{p}_{k\ell} + (-1)H' d\bar{\varepsilon}^{p}$$

$$= \frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}\left(d\varepsilon_{k\ell} - d\varepsilon_{k\ell}{}^{T}\right) - \left(\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}\frac{\partial f}{\partial \sigma_{k\ell}} + H'\frac{d\bar{\varepsilon}^{p}}{d\lambda}\right) d\lambda$$

Solving for $d\lambda$

$$d\lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}\left(d\varepsilon_{k\ell} - d\varepsilon_{k\ell}{}^{T}\right)}{\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}\frac{\partial f}{\partial \sigma_{k\ell}} + H'} \tag{2.22a}$$

in which

$$H' = \frac{d\sigma_Y}{d\bar{\varepsilon}^{p}} = \frac{E\ E^{t}}{\left(E - E^{t}\right)} \tag{2.22b}$$

E^t is plastic modulus of the casing, and $d\bar{\varepsilon}^p$ is effective plastic strain defined in (2.35).

Now substituting (2.22a) into the flow rule (2.17) we have

$$d\varepsilon_{mn}^{\ p} = \frac{\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}}{\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell} \frac{\partial f}{\partial \sigma_{k\ell}} + H'} \left(d\varepsilon_{k\ell} - d\varepsilon_{k\ell}^{\ T}\right)$$

$$\underline{\underline{\nabla}}\ P_{mnk\ell} \left(d\varepsilon_{k\ell} - d\varepsilon_{k\ell}^{\ T}\right) \tag{2.23}$$

in which

$$P_{mnk\ell} = \frac{\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}}{\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell} \frac{\partial f}{\partial \sigma_{k\ell}} + H'} \tag{2.24a}$$

Evaluating the factors in (2.22a) and (2.24a), we have

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}} = \frac{\sqrt{3}}{2\sqrt{J_2}} \frac{\partial (J_2)}{\partial \sigma_{ij}} = \frac{\sqrt{3}}{2} \frac{S_{ij}}{\sqrt{J_2}} = \frac{3}{2} \frac{S_{ij}}{\bar{\sigma}}$$

$$\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell} = \frac{3}{2} \frac{S_{ij}}{\bar{\sigma}} \left(2G\delta_{ik}\delta_{j\ell} + \lambda_E \delta_{ij}\delta_{k\ell}\right) = \frac{3G}{\bar{\sigma}} S_{k\ell} \tag{2.24b}$$

$$\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell} \frac{\partial f}{\partial \sigma_{k\ell}} = \frac{3G}{\bar{\sigma}} S_{k\ell} \frac{3}{2} \frac{S_{k\ell}}{\bar{\sigma}} = \frac{9GJ_2}{\bar{\sigma}^2} = 3G$$

So that (2.24a) becomes

$$P_{mnk\ell} = \frac{\frac{3}{2} \frac{S_{mn}}{\bar{\sigma}} S_{k\ell}}{\bar{\sigma}\left(1 + \frac{H'}{3G}\right)} = \frac{3\, S_{mn}\, S_{k\ell}}{2\left(1 + \frac{H'}{3G}\right) \bar{\sigma}^2} \tag{2.25a}$$

and (2.22a) becomes

$$d\lambda = \frac{S_{k\ell}}{\bar{\sigma}\left(1 + \frac{H'}{3G}\right)} \left(d\varepsilon_{k\ell} - d\varepsilon_{k\ell}{}^{T}\right) \tag{2.25b}$$

Substituting (2.23) into (2.19) we get $d\sigma_{ij}$ as

$$d\sigma_{ij} = C_{ijmn}\left[d\varepsilon_{mn} - P_{mnk\ell}\left(d\varepsilon_{k\ell} - d\varepsilon^{T}_{k\ell}\right) - d\varepsilon^{T}_{mn}\right]$$

$$= C_{ijmn}\left(\delta_{km}\delta_{\ell n} - P_{mnk\ell}\right)\left(d\varepsilon_{k\ell} - d\varepsilon^{T}_{k\ell}\right)$$

We can write this as

$$d\sigma_{ij} = C^{EP}_{ijk\ell}\left(d\varepsilon_{k\ell} - d\varepsilon^{T}_{k\ell}\right) \tag{2.26a}$$

in which $C^{EP}_{ijk\ell} = C_{ijmn}\left(\delta_{km}\delta_{\ell n} - P_{mnk\ell}\right)$

$$= \left(2G\delta_{im}\delta_{jn} + \lambda_E\delta_{ij}\delta_{mn}\right)\left(\delta_{km}\delta_{\ell n} - \frac{3S_{mn}S_{k\ell}}{2\left(1 + \frac{H'}{3G}\right)\bar{\sigma}^2}\right)$$

$$= \left(2G\delta_{ik}\delta_{j\ell} + \lambda_E\delta_{ij}\delta_{k\ell}\right) - \frac{3}{2\bar{\sigma}^2\left(1+\frac{H'}{3G}\right)}\left(2GS_{ij}S_{k\ell} + \lambda_E\delta_{ij}S_{mn}S_{k\ell}\right)$$

$$= \left(2G\delta_{ik}\delta_{j\ell} + \lambda_E\delta_{ij}\delta_{k\ell}\right) - \frac{3G}{\bar{\sigma}^2\left(1 + \frac{H'}{3G}\right)} S_{ij}S_{k\ell} \tag{2.26b}$$

The first term in (2.26b) is the elastic part and the second is the plastic part. Eq. (2.26a) is the constitutive equation for elastic-plastic behavior. In order to express it in matrix form we expand $S_{ij}S_{k\ell}$. In general,

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right)\delta_{ij}$$

As there are principal stresses only we have

$$S_{11} = \sigma_\theta - \frac{1}{3}\left(\sigma_\theta - \sigma_r - \sigma_z\right) = \frac{1}{3}\left(2\sigma_\theta - \sigma_r - \sigma_z\right) \underline{\underline{\nabla}} \bar{S}_\theta$$

$$S_{22} = \frac{1}{3}\left(2\sigma_r - \sigma_\theta - \sigma_z\right) \underline{\underline{\nabla}} \bar{S}_r \qquad (2.27)$$

$$S_{33} = \frac{1}{3}\left(2\sigma_z - \sigma_r - \sigma_\theta\right) \underline{\underline{\nabla}} \bar{S}_z$$

So,

$$S_{ij}S_{k\ell} = \begin{bmatrix} \bar{S}_\theta^{\,2} & \bar{S}_\theta \bar{S}_r & \bar{S}_\theta \bar{S}_z \\ & \bar{S}_r^{\,2} & \bar{S}_r \bar{S}_z \\ (\text{Sym}) & & \bar{S}_z^{\,2} \end{bmatrix}$$

and

$$2G\delta_{ij}\delta_{j\ell} + \lambda_E \delta_{ij}\delta_{k\ell} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu \\ \mu & 1-\mu & \mu \\ \mu & \mu & 1-\mu \end{bmatrix}$$

In addition, using (2.22b), the coefficient of the last term in (2.26b) can be expressed as

$$\frac{3G}{\bar{\sigma}^2\left(1 + \frac{H'}{3G}\right)} = \frac{9E(E-E^t)}{2(1+\mu)\,\bar{\sigma}^2\left[3(E-E^t) + 2E^t(1+\mu)\right]}$$

Now we can write (2.26a) in matrix form as

$$d\sigma = D^{EP}\left(d\varepsilon - d\varepsilon^T\right) \qquad (2.28a)$$

in which

$$d\sigma = \left(d\sigma_\theta, d\sigma_r, d\sigma_z\right)^T \tag{2.28b}$$

$$d\varepsilon = \left(d\varepsilon_\theta, d\varepsilon_r, d\varepsilon_z\right)^T \tag{2.28c}$$

$$d\varepsilon^T = \left(d\varepsilon^T_\theta , d\varepsilon^T_r, d\varepsilon^T_z\right)^T \tag{2.28d}$$

and $D^{EP} = D^E - D^p$

$$= \begin{bmatrix} \lambda' & \lambda & \lambda \\ \lambda & \lambda' & \lambda \\ \lambda & \lambda & \lambda' \end{bmatrix} - \begin{bmatrix} S_{\theta\theta} & S_{\theta r} & S_{\theta z} \\ S_{r\theta} & S_{rr} & S_{rz} \\ S_{z\theta} & S_{zr} & S_{zz} \end{bmatrix} \tag{2.28e}$$

Consequently, (2.28a) may be written in expanded form as

$$\begin{bmatrix} d\sigma_\theta \\ d\sigma_r \\ d\sigma_z \end{bmatrix} = \begin{bmatrix} \lambda' & \lambda & \lambda \\ \lambda & \lambda' & \lambda \\ \lambda & \lambda & \lambda' \end{bmatrix} \begin{bmatrix} d\varepsilon_\theta - \Delta T\alpha \\ d\varepsilon_r - \Delta T\alpha \\ d\varepsilon_z - \Delta T\alpha \end{bmatrix} - \begin{bmatrix} S_{\theta\theta} & S\theta r & S_{\theta z} \\ S_{r\theta} & S_{rr} & S_{rz} \\ S_{z\theta} & S_{zr} & S_{zz} \end{bmatrix} \begin{bmatrix} d\varepsilon_\theta - \Delta T\alpha \\ d\varepsilon_r - \Delta T\alpha \\ d\varepsilon_z - \Delta T\alpha \end{bmatrix} \tag{2.28f}$$

in which

$$\lambda = \frac{E\mu}{(1+\mu)(1-2\mu)}$$

$$\lambda' = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)}$$

$$S_{\theta\theta} = \frac{E(E-E^t)(2\sigma_\theta-\sigma_r-\sigma_z)^2}{2(1+\mu)\ \bar{\sigma}^2\left(3(E-E^t) + 2(1+\mu)E^t\right)}$$

$$S_{rr} = \frac{E(E-E^t)\ (2\sigma_r-\sigma_\theta-\sigma_z)^2}{2(1+\mu)\ \bar{\sigma}^2\ \left(3(E-E^t) + 2(1+\mu)E^t\right)}$$

$$S_{zz} = \frac{E(E-E^T)\ (2\sigma_z-\sigma_r-\sigma_\theta)^2}{2(1+\mu)\ \bar{\sigma}^2\ \left(3(E-E^t) + 2(1+\mu)E^t\right)} \tag{2.28g}$$

$$S_{\theta r} = \frac{E(E-E^t)\ (2\sigma_\theta-\sigma_r-\sigma_z)(2\alpha_z-\sigma_\theta-\sigma_r)}{2(1+\mu)\ \bar{\sigma}^2\ \left(3(E-E^t) + 2(1+\mu)\ E^t\right)} = S_{r\theta}$$

$$S_{\theta z} = \frac{E(E-E^t)(2\alpha_\theta-\sigma_r-\sigma_z)(2\sigma_z-\sigma_\theta-\sigma_r)}{\alpha(1+\mu)\ \bar{\sigma}^2\ \left(3(E-E^t) + 2(1+\mu)E^t\right)} = S_{z\theta}$$

$$S_{rz} = \frac{E(E-E^t)\ (2\sigma_r-\sigma_\theta-\sigma_z)(2\sigma_z-\sigma_\theta-\sigma_r)}{2(1+\mu)\ \bar{\sigma}^2\ \left(3(E-E^t) + 2(1+\mu)\ E^t\right)} = S_{zr}$$

Now tht the constiutive matrix is completely defined, solutions for stress and strain may be obtained by proceeding in the same way as in Chapter One. If the loading increment is dp, we can write the equilibrium equations in increment form. These are (see equation (1.9))

$$d\sigma_\theta = \left(dp - kd\varepsilon_\theta\right)\frac{r}{t} \tag{2.29a}$$

$$d\sigma_r = -\ dp \tag{2.29b}$$

and
$$d\varepsilon_z = 0 \tag{2.29c}$$

Using (2.28b), (2.29) becomes

$$-dp = (\lambda-S_{r\theta})(d\varepsilon_\theta-\alpha dT) + (\lambda'-S_{rr})(d\varepsilon_r-\alpha dT) - (\lambda-S_{rz})\ \alpha dT$$

Solving for $(d\varepsilon_r - \alpha dT)$

$$d\varepsilon_r - \alpha dT = \frac{1}{(\lambda'-S_{rr})}\left[(\lambda-S_{r\theta})(d\varepsilon_\theta-\alpha dT) + dp - (\lambda-S_{rz})\ \alpha dT\right]$$

Substituting $d\varepsilon_r$ into the first line of (2.28b) yields

$$d\sigma_\theta = (\lambda'-S_{\theta\theta})(d\varepsilon_\theta-\alpha dT) + \frac{\lambda-S_{\theta r}}{\lambda'-S_{rr}}\left[(\lambda-S_{r\theta})(d\varepsilon_\theta-\alpha dT) + dp - (\lambda-S_{rz})\ \alpha dT\right]$$

Collecting the above terms and solving for $d\varepsilon_\theta$ we obtain

$$d\varepsilon_\theta = \frac{\lambda' - S_{rr}}{(\lambda'-S_{\theta\theta})(\lambda'-S_{rr})-(\lambda-S_{\theta r})^2}\, d\sigma_\theta + \frac{(\lambda'-S_{\theta\theta})(\lambda'-S_{rr})-(\lambda-S_{r\theta})^2-(\lambda-S_{\theta r})(\lambda-S_{rz})}{(\lambda'-S_{\theta\theta})(\lambda'-S_{rr}) - (\lambda-S_{\theta r})^2}\alpha dT$$
$$+ \frac{\lambda - S_{\theta r}}{(\lambda'-S_{\theta\theta})(\lambda'-S_{rr}) - (\lambda-S_{\theta r})^2}\, dp \qquad (2.30)$$

Finally the tangential stress increment $d\sigma_\theta$ is obtained by substituting (2.30) into the equilibrium equation (2.29a)

$$d\sigma_\theta = \frac{\left(1 - \frac{(\lambda-S_{\theta r})}{B}\, k\right)}{1 + \frac{r}{t}\, k\, \frac{(\lambda'-S_{rr})}{B}}\, \frac{r}{t}\, dp \qquad (2.31)$$

$$- \frac{(\lambda'-S_{rr})\left((\lambda'-S\theta\theta)-(\lambda-S_{\theta z})\right)-(\lambda-S_{r\theta})\left((\lambda-S_{r\theta})+(\lambda-S_{rz})\right)}{B + (\lambda' - S_{rr})\ k\ \frac{r}{t}}\, \frac{kr}{t}\, \alpha dT$$

in which $B = (\lambda'-S_{\theta\theta})(\lambda'-S_{rr}) - (\lambda-S_{\theta r})^2$.

Now it follows immediately that we can solve all stress and strain increments in a similar way as Chapter One. For simplicity, to compute the concise formulas for plastic behavior, when the pressure increment and temperature change are prescribed, we have

$$d\sigma_\theta = \left(\frac{1 + ka_2^0}{1 + ka_1^0 \frac{r}{t}}\right) \frac{r}{t}\, dp + \left(\frac{a_T^0 k \frac{r}{t}}{1+a_1^0 k \frac{r}{t}}\right) \alpha dT \qquad (2.32a)$$

$$d\sigma_r = -\, dp \qquad (2.32b)$$

$$d\sigma_z = c_1 d\sigma_\theta + c_2 d\sigma_r + c_T \alpha dT \qquad (2.32c)$$

and

$$d\varepsilon_\theta = a_1^0 d\sigma_\theta + a_2^0 d\sigma_r + a_T^0 \alpha dT \qquad (2.33a)$$

$$d\varepsilon_r = b_1^0 d\sigma_\theta + b_2^0 d\sigma_r - b_T^0 \alpha dT \qquad (2.33b)$$

$$d\varepsilon_z = 0 \qquad (2.33c)$$

in which

$$a_1 = \lambda' - S_{\theta\theta} \qquad a_2 = \lambda - S_{\theta r}$$

$$b_1 = \lambda - S_{r\theta} = a_2 \qquad b_2 = \lambda' - S_{rr}$$

$$c_1' = \lambda - S_{z\theta} \qquad c_2' = \lambda - S_{zr}$$

$$c_1 = c_1' a_1^0 + c_2' b_1^0 \qquad c_2 = c_1' a_2^0 + c_2' b_2^0 \qquad (2.34)$$

$$a_1^0 = b_2/(a_1b_2-b_1a_2) \qquad a_2^0 = -a_2/(a_1b_2 - b_1a_2)$$

$$b_1^0 = -b_1/(a_1b_2-b_1a_2) \qquad b_2^0 = a_1/(a_1b_2-b_1a_2)$$

$$a_T = (S_{\theta\theta} + S_{\theta r} + S_{\theta z} - 2\lambda - \lambda')$$

$$b_T = (S_{r\theta} + S_{rr} + S_{rz}) - 2\lambda - \lambda')$$

$$c'_T = (S_{z\theta} + S_{zr} + S_{zz} - 2\lambda - \lambda')$$

$$a_T^0 = a_1^0 a_T + a_2^0 a_T$$

$$b_T^0 = b_1^0 a_T + b_2^0 b_T \qquad (2.34)$$

$$c_T = c'_T - c'_1 a_T^0 - c'_2 b_T^0$$

$$k = \alpha_m \overline{E} / (1+\overline{\mu})$$

$$H' = E\,E^t/(E - E^t)$$

$$\overline{\sigma} = \frac{1}{\sqrt{2}} \left[(\sigma_r-\sigma_\theta)^2 + (\sigma_\theta-\sigma_z)^2 + (\sigma_z-\sigma_r)^2 \right]^{1/2}$$

and

λ, λ', $S_{\theta\theta}$, $S_{\theta r}$, $S_{\theta z}$, $S_{r\theta}$, S_{rr}, S_{rz}, $S_{z\theta}$, S_{zr}, S_{zz} are defined in (2.28g).

Once we have determined the plastic solution from (2.32) to (2.34) we may determine the plastic strains as follows. First we note that the effective plastic strain can be defined as

$$d\overline{\varepsilon}^p = c\sqrt{d\varepsilon^p_{ij}d\varepsilon^p_{ij}} = c\,d\lambda\sqrt{\frac{\partial f}{\partial \sigma_{ij}}\frac{\partial f}{\partial \sigma_{ij}}}$$

in which the flow rule of (2.17) has been used.

For $d\overline{\varepsilon}^p$ to identify with $d\varepsilon^p_1$ in uniaxial test

$$d\overline{\varepsilon}^p = c\sqrt{(d\varepsilon^p_1)^2 + 2\left(-\frac{d\varepsilon^p_1}{2}\right)^2} = c\sqrt{\frac{3}{2}}\,d\varepsilon^p_1 \equiv d\varepsilon^p_1$$

Therefore $c = \sqrt{\frac{2}{3}}$

Then $$d\overline{\varepsilon}^p = d\lambda\sqrt{\frac{2}{3}\frac{\partial f}{\partial \sigma_{ij}}\frac{\partial f}{\partial \sigma_{ij}}} \tag{2.35}$$

From Eq. (2.24)b

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{3}{2}\frac{S_{ij}}{\overline{\sigma}}$$

Substituting this into (2.35) yields

$$d\overline{\varepsilon}^p = d\lambda\sqrt{\frac{2}{3}\frac{9}{4}\frac{S_{ij}S_{ij}}{\overline{\sigma}^2}} = d\lambda \tag{2.36a}$$

Then, using (2.36a) in flow rule we have

$$d\varepsilon^p_{ij} = d\lambda\frac{\partial f}{\partial \sigma_{ij}} = d\overline{\varepsilon}^p\frac{\partial f}{\partial \sigma_{ij}} = \frac{3}{2}\frac{S_{ij}}{\overline{\sigma}}\,d\overline{\varepsilon}^p \tag{2.36b}$$

From (2.25b) and (2.36a) we have

$$d\overline{\varepsilon^p} = d\lambda = \frac{S_{k\ell}}{\overline{\sigma}\ (1 + \frac{H'}{3G})} (d\varepsilon_{k\ell} - d\varepsilon^T_{k\ell})$$

Because only principal stresses exist

$$S_{ij} = 0 \quad \text{when } i \neq j$$

and

$$S_{11} = \frac{1}{3}\ (2\sigma_\theta - \sigma_r - \sigma_z) \underline{\underline{\nabla}} \overline{S}_\theta$$

$$S_{22} = \frac{1}{3}\ (2\sigma_r - \sigma_\theta - \sigma_z) \underline{\underline{\nabla}} \overline{S}_r$$

$$S_{33} = \frac{1}{3}\ (2\sigma_z - \sigma_\theta - \sigma_r) \underline{\underline{\nabla}} \overline{S}_z$$

So $d\overline{\varepsilon^p}$ becomes

$$d\overline{\varepsilon^p} = \frac{1}{\overline{\sigma}\ (1 + \frac{H'}{3G})} \left[\overline{S}_\theta\ (d\varepsilon_\theta - \alpha dT) + \overline{S}_r (d\varepsilon_r - \alpha dT) + \overline{S}_z (d\varepsilon_z - \alpha dT)\right] \tag{2.37a}$$

Similarly, (2.36b) becomes

$$d\varepsilon^p_{ij} = \frac{3}{2\overline{\sigma}^2 (1 + \frac{H'}{3G})} \left[\overline{S}_\theta (d\varepsilon_\theta - \alpha dT) + \overline{S}_r (d\varepsilon_r - \alpha dT) + \overline{S}_z (d\varepsilon_z - \alpha dT)\right] S_{ij} \tag{2.37b}$$

2.3 Elastic-Plastic Solution - Computational Procedure

We can first find the condition at which yield occurs from (2.2) and (2.3) for any given temperature. Then, the prediction of initial yield is obtained.

Then, suppose that the computer has just completed the calculations for a certain loading or unloading state and the load has just been increased by increments ΔT and Δp. For generalization we also suppose that the initial point doesn't fall on the yield surface. Refering to Fig. 2.1, the initial conditions are in the following

i) σ_{Yo} is known; σ_{Y1} is known too, but $f(\sigma_{ij}^{(1)}, \sigma_{Y1}) < 0$; $\sigma_{ij}^{(1)}$ are known, say point A is given; Δp and ΔT are given.

ii) $\Delta\sigma_{ij}$ are unknown. That is to say the point B or point C needs to be determined.

The subsequent computational steps follow.

1. Solve for $\Delta\sigma_{ij}^{(e)}$ and $\Delta\varepsilon_{ij}^{(e)}$ using the computational formulas of elastic solution from (b) to (g) of (1.16), in which Δp replaces p.

$$\Delta\sigma_\theta = \Delta pb + \Delta Tb^0 \tag{2.38a}$$

$$\Delta\sigma_r = -\Delta p \tag{2.38b}$$

$$\Delta\sigma_z = \Delta p\mu(b-1) + \Delta T(\mu b^0 - E\alpha) \tag{2.38c}$$

$$\Delta\varepsilon_\theta = \Delta pc + \Delta Tc^0 \tag{2.38d}$$

$$\Delta\varepsilon_r = -(-\Delta pd + \Delta Td^0) \tag{2.38e}$$

$$\Delta\varepsilon_z = 0 \tag{2.38f}$$

in which all parameters are defined in (1.17) of Chapter 1.

2. For stress increments $\Delta\sigma_{ij}$ calculate

$$\sigma_{ij}^{(e)} = \sigma_{ij}^{(1)} + \Delta\sigma_{ij}^{(e)} \tag{2.39}$$

and

$$f(\sigma_{ij}^{(e)}, \sigma_{Y1}) = \bar{\sigma} - \sigma_{Y1} \tag{2.40}$$

3. Check if $f(\sigma_{ij}^{(e)}, \sigma_{Y1}) \leqslant 0$. If so, the values of $\Delta\sigma_{ij}^{(e)}$ are correct and it is an elastic loading or unloading state. Therefore

$$\sigma_{ij}^{(2)} = \sigma_{ij}^{(e)} = \sigma_{ij}^{(1)} + \Delta\sigma_{ij}^{(e)}$$

Then, go to next load increment, i.e., return to step 1.

4. If $f(\sigma_{ij}^{(e)}, \sigma_{Y1}) > 0$ it indicates a plastic loading state. It shows that the stress increments $\Delta\sigma_{ij}^{(e)}$ should include two portions which are the elastic and the plastic. We can express the elastic portion with a scale factor "R", i.e.,

$$R\ \Delta\sigma_{ij}^{(e)} \text{ and } R\ \Delta\varepsilon_{ij}^{(e)}$$

To find the scalar R we directly employ the yield condition (Wong 1982)

$$f(\sigma_{ij}^{(1)} + R\Delta\sigma_{ij}^{(e)}, \sigma_{Y1}) = 0 \tag{2.41a}$$

The expansion can be written as

$$\frac{1}{2}(S_{ij}^{(1)} + R\Delta S_{ij}^{(e)})(S_{ij}^{(1)} + R\Delta S_{ij}^{(e)}) - \frac{1}{3}\sigma_{Y1}^2 = 0$$

or

$$(\Delta S_{ij}^{(e)}\ \Delta S_{ij}^{(e)})\ R^2 + 2S_{ij}^{(1)}\ \Delta S_{ij}^{(e)}\ R + S_{ij}^{(1)} S_{ij}^{(1)} - \frac{2}{3}\sigma_{Y1}^2 = 0 \qquad (2.41b)$$

When only principal stresses exist we have

$$S_{ij}S_{ij} = S_{ii}S_{jj} = S_{11}^2 + S_{22}^2 + S_{33}^2 = S_\theta^2 + S_r^2 + S_z^2 \qquad (2.41c)$$

$$\Delta S_{ij}\Delta S_{ij} = \Delta S_\theta^2 + \Delta S_r^2 + \Delta S_z^2 \qquad (2.41d)$$

$$S_{ij}\Delta S_{ij} = S_\theta \Delta S_\theta + S_r \Delta S_r + S_z \Delta S_z \qquad (2.41e)$$

where S_θ, S_r, S_z and ΔS_θ, ΔS_r, ΔS_z are defined in (2.27). Thus, Eq. (2.41b) can be written as a second-degree algebra equation

$$AR^2 + BR + C = 0 \qquad (2.42a)$$

where

$$A = \Delta S_{ij}^{(e)}\ \Delta S_{ij}^{(e)} = (\Delta S_\theta^{(e)})^2 + (\Delta S_r^{(e)})^2 + (\Delta S_z^{(e)})^2 \qquad (2.42b)$$

$$B = 2S_{ij}^{(1)}\ \Delta S_{ij}^{(e)} = 2(S_\theta^{(1)}\Delta S_\theta^{(e)} + S_r^{(1)}\Delta S_r^{(e)} + S_z^{(1)}\Delta S_z^{(e)}) \qquad (2.42c)$$

$$C = S_{ij}^{(1)}S_{ij}^{(1)} - \frac{2}{3}\sigma_{Y1}^2 = (S_\theta^{(1)})^2 + (S_r^{(1)})^2 + (S_z^{(1)})^2 - \frac{2}{3}\sigma_{Y1}{}^2 \qquad (2.42d)$$

Obviously, just the positive roof of Eq. (2.42a) is applicable and it follows that if

$R = 0$ the increment is entirely plastic;

$R = 1$ the increment is entirely elastic;

$0 < R < 1$ the increment is elastic-plastic.

5. Now supply the increments

$$\sigma_{ij}^{(1)} \rightarrow \sigma_{ij}^{(1)} + R\Delta\sigma_{ij}^{(e)}$$
$$\varepsilon_{ij}^{(1)} \rightarrow \varepsilon_{ij}^{(1)} + R\Delta\varepsilon_{ij}^{(1)} \qquad (2.43)$$

to get to the yield surface.

6. Let

$$\Delta p^* = (1-R)\Delta p$$
$$\Delta T^* = (1-R)\Delta T \qquad (2.44)$$

7. Solve for $\Delta\sigma_{ij}$ and $\Delta\varepsilon_{ij}$ using the formulas of plastic solution (2.32) and (2.33), in which Δp^* and ΔT^* replace Δp and ΔT respectively.

Note that the $\Delta\sigma_{ij}$ and $\Delta\varepsilon_{ij}$ here should be consistent with the constitutive relations (2.28b). Consequently

$$\Delta\sigma_\theta = \left(\frac{1+ka_2^0}{1+ka_1^0\frac{r}{t}}\right)\frac{r}{t}\,\Delta p^* + \left(\frac{a_T^0 k\frac{r}{t}}{1+a_1^0 k\frac{r}{t}}\right)\alpha\Delta T^* \qquad (2.45a)$$

$$\Delta\sigma_r = -\Delta p^* \qquad (2.45b)$$

$$\Delta\sigma_z = c_1\Delta\sigma_\theta + c_2\Delta\sigma_r + c_T\alpha\Delta T^* \qquad (2.45c)$$

and

$$\Delta\varepsilon_\theta = a_1^0 \, \Delta\sigma_\theta + a_2^0 \, \Delta\sigma_r - a_T^0 \alpha \Delta T^* \tag{2.46a}$$

$$\Delta\varepsilon_r = b_1^0 \, \Delta\sigma_\theta + b_2^0 \Delta\sigma_r - b_T^0 \alpha \, \Delta T^* \tag{2.46b}$$

$$\Delta\varepsilon_z = 0 \tag{2.46c}$$

8. Compute effective plastic strain from (2.37a) and (2.37b) as

$$\Delta\bar{\varepsilon}^p = \frac{1}{\bar{\sigma}(1+\frac{H'}{3G})} \left[\bar{S}_\theta(\Delta\varepsilon_\theta - \alpha\Delta T^*) + \bar{S}_r(\Delta\varepsilon_r - \alpha\Delta T^*) + \bar{S}_z(\Delta\varepsilon_z - \alpha\Delta T^*)\right]$$

$$\Delta\varepsilon_{ij}^p = \frac{1}{2\bar{\sigma}^2(1+\frac{H'}{3G})} \left[\bar{S}_\theta(\Delta\varepsilon_\theta - \alpha\Delta T^*) + \bar{S}_r(\Delta\varepsilon_r - \alpha\Delta T^*) + \bar{S}_z(\Delta\varepsilon_z - \alpha\Delta T^*)\right] S_{ij}$$

9. Use above results to update the values of the previous step.

$$\sigma_{ij}^{(2)} = \sigma_{ij}^{(1)} + \Delta\sigma_{ij} \tag{2.47a}$$

$$\varepsilon_{ij}^{(2)} = \varepsilon_{ij}^{(1)} + \Delta\varepsilon_{ij} \tag{2.47b}$$

$$\varepsilon_{ij}^{p(2)} = \varepsilon_{ij}^{p(1)} + \Delta\varepsilon_{ij}^p \tag{2.47c}$$

$$\bar{\varepsilon}^{p(2)} = \bar{\varepsilon}^{p(1)} + \Delta\bar{\varepsilon}^p \tag{2.47d}$$

$$\sigma_y^{(2)} = \sigma_y^{(1)} + H' \Delta\bar{\varepsilon}^{p(2)} \tag{2.47e}$$

10. Go back to Step 1 for the next load increment.

11. Use the following relations to check the consistency of the results after each loading cycle

$$\sigma_\theta^c = \frac{E}{(1+\mu)(1-2\mu)}\left[(1-\mu)(\varepsilon_\theta-\varepsilon_\theta^p-\alpha\Delta T) + \mu(\varepsilon_r-\varepsilon_r^p-\alpha\Delta T) + \mu(\varepsilon_z-\varepsilon_z^p-\alpha\Delta T)\right]$$

$$\sigma_z^c = \frac{E}{(1+\mu)(1-2\mu)}\left[(1-\mu)(\varepsilon_z-\varepsilon_z^p-\alpha\Delta T) + \mu(\varepsilon_\theta-\varepsilon_\theta^p-\alpha\Delta T) + \mu(\varepsilon_r-\varepsilon_r^p-\alpha\Delta T)\right]$$

(2.48)

If σ_{ij}^c are consistent with computational values σ_{ij}, the results are correct.

CHAPTER 3 TEMPERATURE DEPENDENT ANALYSIS

3.1 Modification to Governing Equations

In the previous chapter we have been concerned with a solution which took no account of the change of material properties as the temperature varied. In fact, physical properties of Algoma Steel MN-80 which is representative of steels from which Canadian casings for thermal recovery wells are made, show that initial yield limit σ_{Yo}, modulus of elasticity E, tangent modulus E^t, and thermal expansion coefficient α, all vary with the temperature. Adding these variations of properties to the computation is necessary.

We first consider the elastic phase in which σ_{Yo}, E and α are functions of temperature, while Poisson ratio μ is temperature independent.

The total strain can be expressed with the differential

$$d\varepsilon = d\varepsilon^e + d\varepsilon^T = d\varepsilon^e + [\alpha]dT \tag{3.1}$$

From Hooke's Law

$$\sigma = D\varepsilon^e$$

and $$\varepsilon^e = D^{-1}\sigma \ .$$

Differentiating with respect to temperature T

$$d\varepsilon^e = \frac{dD^{-1}}{dT}\sigma\, dT + D^{-1}\, d\sigma \tag{3.2}$$

Substituting (3.2) into (3.1) and solving for $d\sigma$ we have

$$d\sigma = D\left[d\varepsilon - d\varepsilon^T - \frac{dD^{-1}}{dT}\,\sigma\, dT\right]$$

This can be written as

$$d\sigma = D\left[d\varepsilon - [\alpha]^* \, dT\right] = D\left[d\varepsilon - d\varepsilon^{*T}\right] \tag{3.3}$$

where $[\alpha]^*$ is a matrix which has the following increment form and

$$d\varepsilon^{*T} \equiv [\alpha]^* dT. \tag{3.4a}$$

$$[\alpha]^* = \begin{bmatrix} \alpha_\theta^* \\ \alpha_r^* \\ \alpha_z^* \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha \\ \alpha \end{bmatrix} + \frac{1}{\Delta T \Delta E} \begin{bmatrix} 1 & -\mu & -\mu \\ -\mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{bmatrix} \begin{bmatrix} \sigma_\theta \\ \sigma_r \\ \sigma_z \end{bmatrix} \tag{3.4b}$$

in which $\Delta E = E^{T+\Delta T} - E^T$.

Now the constitutive relations (1.12) of Chapter 1 are replaced by the following

$$\begin{bmatrix} \Delta\sigma_\theta \\ \Delta\sigma_r \\ \Delta\sigma_z \end{bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu \\ \mu & 1-\mu & \mu \\ \mu & \mu & 1-\mu \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_\theta - \alpha_\theta^* \, \Delta T \\ \Delta\varepsilon_r - \alpha_r^* \, \Delta T \\ \Delta\varepsilon_z - \alpha_z^* \, \Delta T \end{bmatrix} \tag{3.5}$$

For elastic solution and prediction of initial yield the computational procedure of using the temperature dependent model can obviously follow the procedure described previously. That is, formulas (1.16) of Chapter 1 and formulas (2.2) and (2.3) of Chapter 2 are still applicable except that the thermal expansion coefficient $[\alpha]$ should be replaced by $[\alpha]^*$ of (3.4b), and it is necessary to use an increment form in the computation.

For the plastic temperature dependent problem we modify Eq. (2.19), namely,

$$d\sigma_{ij} = C_{ijk\ell}\left(d\varepsilon_{k\ell} - d\varepsilon^{p}_{k\ell} - d\varepsilon^{T}_{k\ell}\right)$$

as follows. First, $d\varepsilon^{*T}_{k\ell}$ of (3.4a) should replace $d\varepsilon^{T}_{k\ell}$, with $d\varepsilon^{*T}_{k\ell}$ defined in (3.4)

$$d\varepsilon^{*T}_{k\ell} = \alpha^{*}_{k\ell}\, dT \tag{3.6}$$

As yield now depends on the temperature, the Mises hardening rule should be written as

$$\bar{\sigma} = \sigma_Y\left(\int d\bar{\varepsilon}_p,\ T\right) \tag{3.7}$$

It follows that the consistency condition (2.21) needs to be modified such that

$$\frac{\partial f}{\partial \sigma_{ij}}\, d\sigma_{ij} + \frac{\partial f}{\partial \sigma_Y}\, d\sigma_Y = 0$$

in which $$d\sigma_Y = \frac{\partial \sigma_Y}{\partial \bar{\varepsilon}_p}\, d\bar{\varepsilon}_p + \frac{\partial \sigma_Y}{\partial T}\, dT \tag{3.8}$$

Then

$$\frac{\partial f}{\partial \sigma_{ij}}\, d\sigma_{ij} + \frac{\partial f}{\partial \sigma_Y}\left(\frac{\partial \sigma_Y}{\partial \bar{\varepsilon}_p}\, d\bar{\varepsilon}_p + \frac{\partial \sigma_Y}{\partial T}\, dT\right) = 0$$

Following the same procedure as from (2.21) to (2.23) of Chapter 2 the plastic strain is given as

$$d\varepsilon^{P}_{mn} = \frac{\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell}}{\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell} \frac{\partial f}{\partial \sigma_{k\ell}} + H'} \left(d\varepsilon_{k\ell} - d\varepsilon^{*T}_{k\ell}\right) - \frac{\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial \sigma_Y}{\partial T} dT}{\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell} \frac{\partial f}{\partial \sigma_{k\ell}} + H'}$$

$$\stackrel{\nabla}{=} P_{mnk\ell} \left(d\varepsilon_{k\ell} - d\varepsilon^{*T}_{k\ell}\right) - \frac{\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial \sigma_Y}{\partial T} dT}{\frac{\partial f}{\partial \sigma_{ij}} C_{ijk\ell} \frac{\partial f}{\partial \sigma_{k\ell}} + H'} \qquad (3.9)$$

The expressions from (2.24a) to (2.28a) of Chapter 2 are still applicable and we obtain the constitutive relations with thermoplastic material as follows

$$d\sigma = D^{EP} \left(d\varepsilon - d\varepsilon^{*T}\right) + d\sigma^{*T} \qquad (3.10)$$

in which $d\sigma^{*T}_{ij} = \dfrac{3 \frac{\partial \sigma_Y}{\partial T} dT}{2\left(1 + \frac{H'}{3G}\right) \bar{\sigma}^2} S_{ij}$

or $$\begin{bmatrix} d\sigma^{*T}_{\theta} \\ d\sigma^{*T}_{r} \\ d\sigma^{*T}_{z} \end{bmatrix} = \frac{3 \frac{\partial \sigma_Y}{\partial T} dT}{2\left(1 + \frac{H'}{3G}\right) \bar{\sigma}^2} \begin{bmatrix} \bar{S}_{\theta} \\ \bar{S}_{r} \\ \bar{S}_{z} \end{bmatrix}$$

(For $\bar{S}_{\theta}$, $\bar{S}_{r}$, $\bar{S}_{z}$ see (2.27))

$$d\varepsilon^{*T}_{ij} = \alpha^{*}_{ij}\, dT \qquad (3.11)$$

or
$$\begin{bmatrix} d\varepsilon_\theta^{*T} \\ d\varepsilon_r^{*T} \\ d\varepsilon_z^{*T} \end{bmatrix} = dT \begin{bmatrix} \alpha_\theta^{*} \\ \alpha_r^{*} \\ \alpha_z^{*} \end{bmatrix}$$

in which the α^* are specified by (3.4b). The elastic-plastic matrix D^{EP} now has the same form as in (2.28), namely,

$$D^{EP} = D^{E} - D^{P} = \begin{bmatrix} \lambda' & \lambda & \lambda \\ \lambda & \lambda' & \lambda \\ \lambda & \lambda & \lambda' \end{bmatrix} - \begin{bmatrix} S_{\theta\theta} & S_{\theta r} & S_{\theta z} \\ S_{r\theta} & S_{rr} & S_{rz} \\ S_{z\theta} & S_{zr} & S_{zz} \end{bmatrix}$$

Once the constitutive relations (3.10) are established it is clear that the formulas (2.32), (2.33) and (2.34) with the computational procedure in 2.3 of Chapter 2 can be exactly used to solve for temperature dependent problems as long as we define $(d\sigma - d\sigma^{*T})$ as an unknown. The only difference is to consider $\sigma_{Y\circ}$, E, E^t and α to be the functions of temperature. As a consequence, the final results will come from adding $d\sigma^{*T}$ which can be determined without any difficulty.

3.2 Solution Strategy

In the above section the method for temperature dependent problems was presented. The temperature effect is treated by replacing the thermal expansion coefficient α by an "effective thermal expansion coefficient α_{ij}^{*}" and the stress response $d\sigma$ is replaced by "effective thermal stress $(d\sigma - d\sigma^{*T})$. For each temperature increment we have

$$\begin{bmatrix} \alpha_\theta^* \\ \alpha_r^* \\ \alpha_z^* \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha \\ \alpha \end{bmatrix} + \frac{1}{\Delta T \ \Delta E} \begin{bmatrix} 1 & -\mu & -\mu \\ \mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{bmatrix} \begin{bmatrix} \sigma_\theta \\ \sigma_r \\ \sigma_z \end{bmatrix} \tag{3.12}$$

$$\begin{bmatrix} d\sigma_\theta^{*T} \\ d\sigma_r^{*T} \\ d\alpha_z^{*T} \end{bmatrix} = \frac{3 \ \frac{\partial \sigma_Y}{\partial T} \ \Delta T}{2\left(1 + \frac{H'}{3G}\right) \bar{\sigma}^2} \begin{bmatrix} \bar{s}\theta \\ \bar{s}_r \\ \bar{s}_z \end{bmatrix} \tag{3.13}$$

However, there exists a simple alternative way to deal with the same problem. If the change of physical properties with temperature is gradual and, in addition, the step size of the temperature increment is small enough, we can drop the derivative terms dD^{-1}/dT in (3.2) and $\partial\sigma_Y/\partial T$ in (3.8). Instead, we may use directly a Lagrangian interpolation of σ_{Yo}, E, E^T and α, which are tabulated functions of temperature. The error caused from this procedure would be acceptable as long as a small step size can be guaranteed. This simple method is adopted in the current program CASEPT-1 of temperature dependent problems while the former, in which the derivative terms are taken into account, will be employed in the creep model, i.e., the time dependent problem to be addressed subsequently.

The strategy employed with Lagrangian interpolation is shown in Fig. 3.1. The available data for MN80 steel, for σ_{Yo}, E, E^t and α, have been tabulated. We use one-dimensional interpolation for E and σ_{Yo}, curve fitting for α, and two-dimensional interpolation for E^t. This is because the plastic modulus E^t is a bivariate function of temperature T and total strain ε (or effective strain $\bar{\varepsilon}$), i.e.,

$$E^t = E^t\ (T,\ \varepsilon) \tag{3.14}$$

The two-dimensional interplation scheme for E^t will be given in Chapter 5.

CHAPTER 4 TIME DEPENDENT ANALYSIS - CREEP

4.1 Creep Phenomena and Their Mathematical Modeling

As we know, the creep problem refers to time dependent deformation and (or) failure of components and structures. Time depedence is the chief characteristic of creep. A tyical creep curve consists of three regimes, designated as primary, secondary and tertiary, as shown in Fig. 4.1. The instantaneous response to ε_o, at t=0, depends on the magnitude of the stress and can be elastic or elastic-plastic. The creep curve is strongly influenced by stress and temperature. The influence of stress at constant temperature is shown in Fig. 4.2 while the influence of temperature at constant stress is shown in Fig. 4.3. We note that both show thresholds below which no noticeable creep is observed.

For casings for thermal wells, typical working conditions are T = 350°C and P = 16.5 MPa, and the operating period, in general, lasts about three months. So the primary and secondary creep are important in such problems.

To model the creep process in one dimension, most work is based on the equation of state approach, which is popular within the engineering community. An expression of the form called the Bailey-Norton Law (Harry, K., 1980) is

$$\varepsilon^c = A\,\sigma^m\,t^n \qquad (4.1)$$

Here ε^c is the creep strain, and A, m and n are constants that are a function of temperature. This law is intended to model primary and secondary creep only. If we ignore the time derivative of the stress the strain rate can be obtained by differentiating Eq. (4.1) with respect to time. This may result in two forms (Harry, K., 1980). The time hardening formulation is

$$\dot{\varepsilon}^c = \frac{\partial \varepsilon^c}{\partial t} = A\ \sigma^m\ n\ t^{n-1} \tag{4.2}$$

and the strain hardening formulation is

$$\dot{\varepsilon}^c = A^{1/n}\ n\ \sigma^{m/n}\ \left(\varepsilon^c\right)^{(n-1)/n} \tag{4.3}$$

For multiaxial stress conditions it is assumed that the tensor of the creep strain rates can be written in terms of the stress deviator tensor as follows

$$\varepsilon^c_{ij} = \lambda\ S_{ij} \qquad i,j = 1,2,3 \tag{4.4}$$

in which

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\ \sigma_{kk}\ \delta_{ij}$$

and λ is a factor of proportionality, which is determined as (Harry, K., 1980)

$$\lambda = \frac{3}{2\bar{\sigma}}\ \frac{d\bar{\varepsilon}^c}{dt} \tag{4.5}$$

Here $\bar{\sigma}$ is the effective stress and $\bar{\varepsilon}^c$ is an effective creep strain, extended from the uniaxial creep model to the multiaxial case, and expressed as

$$\bar{\varepsilon}^c = A\ \bar{\sigma}^m\ t^n \tag{4.6}$$

So, using (4.2) and (4.6) in (4.5), now we may write

$$\lambda = \frac{3}{2\bar{\sigma}} \frac{d\overline{\varepsilon^c}}{dt} = \frac{3}{2} A \, n \, \bar{\sigma}^{m-1} \, t^{n-1} \tag{4.7}$$

Then the time hardening formation is represented by

$$\dot{\varepsilon}_{ij}{}^{c} = \frac{3}{2} A \, n \, \bar{\sigma}^{m-1} \, t^{n-1} \, S_{ij} \tag{4.8}$$

and the strain hardening formation is rewritten, similarly, as

$$\dot{\varepsilon}_{ij}{}^{c} = \frac{3}{2} A^{\frac{1}{n}} \, n \, \bar{\sigma}^{\left(\frac{m}{n} - 1\right)} \, \varepsilon^{c\left(\frac{n-1}{n}\right)} \, S_{ij} \tag{4.9}$$

Both the time hardening and strain hardening rules can be used for computation, however, the time hardening formation is easier to use than is the strain hardening formation. In this report we prefer to adopt the time hardening rule. In practice, we make use of another incremental form for computations, which stem directly from Eq. (4.4), in the form

$$\dot{\varepsilon}_{ij}{}^{c} = \lambda \, S_{ij} = \frac{3}{2\bar{\sigma}} \frac{d\overline{\varepsilon^c}}{dt} S_{ij}$$

This incremental form is approximately

$$\Delta\varepsilon_{ij}{}^{c} = \frac{3}{2\bar{\sigma}} \frac{\Delta\overline{\varepsilon^c}}{\Delta t} S_{ij} \, \Delta t = \frac{3}{2\bar{\sigma}} \Delta\overline{\varepsilon^c} \, S_{ij} \tag{4.10}$$

For small changes of stress the formula is valid to obtain creep strain increments using an iterative process.

4.2 Formulation of the Creep Problem for Casings

We note first that the total strain, in tensor form, can be decomposed as

$$\varepsilon_{ij} = \varepsilon_{ij}{}^{e} + \varepsilon_{ij}{}^{p} + \varepsilon_{ij}{}^{c} + \varepsilon_{ij}{}^{T} \qquad (4.11a)$$

or, in vector form,

$$\varepsilon = \varepsilon^{e} + \varepsilon^{p} + \varepsilon^{c} + \varepsilon^{T} \qquad (4.11b)$$

in which $\varepsilon_{ij}{}^{T} = \alpha \; \Delta T \; \sigma_{ij}$

The incremental form is

$$\Delta\varepsilon = \Delta\varepsilon^{e} + \Delta\varepsilon^{p} + \Delta\varepsilon^{c} + \Delta\varepsilon^{T} \qquad (4.11c)$$

where the superscripts e and p refer to the elastic strain and plastic strain, respectively, which have been obtained from Chapter 3. The superscript c refers to the creep strain which is unknown. The constant, α, is the coefficient of linear thermal expansion and ΔT is the temperature change. From (4.11c) we have

$$\Delta\varepsilon^{e} = \Delta\varepsilon - \Delta\varepsilon^{p} - \Delta\varepsilon^{c} - \Delta\varepsilon^{T} \qquad (4.12a)$$

and

$$\Delta\sigma = D\Delta\varepsilon^{e} = D\left[\Delta\varepsilon - \Delta\varepsilon^{p} - \Delta\varepsilon^{c} - \Delta\varepsilon^{T}\right] \qquad (4.12b)$$

Expressing (4.12b) in matrix form, yields

$$\begin{bmatrix} \Delta\sigma_\theta \\ \Delta\sigma_r \\ \Delta\sigma_z \end{bmatrix} = \frac{E(T)}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu \\ \mu & 1-\mu & \mu \\ \mu & \mu & 1-\mu \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_\theta - \Delta\varepsilon_\theta{}^p - \Delta\varepsilon_\theta{}^c - \alpha\ \Delta T \\ \Delta\varepsilon_r - \Delta\varepsilon_r{}^p - \Delta\varepsilon_r{}^c - \alpha\ \Delta T \\ \Delta\varepsilon_z - \Delta\varepsilon_z{}^p - \Delta\varepsilon_z{}^c - \alpha\ \Delta T \end{bmatrix} \qquad (4.13)$$

in which E(T) is the temperature dependent elastic modulus.

As in Chpaters 1 to 3, we assume that the total axial strain $\varepsilon_z = 0$. The components of stress in the matrix (4.13) can then be expressed as

$$\Delta\sigma_\theta = \lambda'\Delta\varepsilon_\theta + \lambda\Delta\varepsilon_r + a^c \qquad (4.14a)$$

$$\Delta\sigma_r = \lambda\ \Delta\varepsilon_\theta + \lambda'\Delta\varepsilon_r + b^c \qquad (4.14b)$$

$$\Delta\sigma_z = \lambda\ \Delta\varepsilon_\theta + \lambda\Delta\varepsilon_r + c^c \qquad (4.14c)$$

The the inverse form is

$$\Delta\varepsilon_\theta = a_1^c\Delta\sigma_\theta + a_2^c\Delta\sigma_r + a_T^c \qquad (4.15a)$$

$$\Delta\varepsilon_r = b_1^c\Delta\sigma_\theta + b_2^c\Delta\sigma_r + b_T^c \qquad (4.15b)$$

$$\Delta\varepsilon_z = 0 \qquad (4.15c)$$

in which

$$\lambda = E(T)\ \mu\ /\ (1+\mu)(1-2\mu) \tag{4.16a}$$

$$\lambda' = E(T)(1-\mu)\ /\ (1+\mu)(1-2\mu) \tag{4.16b}$$

$$a^c = -\lambda'\left(\Delta\varepsilon_\theta{}^c+\Delta\varepsilon_\theta{}^p+\alpha\Delta T\right) - \lambda\left(\Delta\varepsilon_r{}^c+\Delta\varepsilon_z{}^c+\Delta\varepsilon_r{}^p+\Delta\varepsilon_z{}^p + 2\alpha\Delta T\right) \tag{4.16c}$$

$$b^c = -\lambda'\left(\Delta\varepsilon_r{}^c+\Delta\varepsilon_r{}^p+\alpha\Delta T\right) - \lambda\left(\Delta\varepsilon_\theta{}^c+\Delta\varepsilon_z{}^c+\Delta\varepsilon_\theta{}^p+\Delta\varepsilon_z{}^p + 2\alpha\Delta T\right) \tag{4.16d}$$

$$c^c = -\lambda'\left(\Delta\varepsilon_z{}^c+\Delta\varepsilon_z{}^p+ \alpha\Delta T\right) - \lambda\left(\Delta\varepsilon_\theta{}^c+\Delta\varepsilon_r{}^c+\Delta\varepsilon_\theta{}^p+\Delta\varepsilon_r{}^p+ 2\alpha\Delta T\right) \tag{4.16e}$$

and

$$a_1^c = \frac{\lambda'}{\lambda'^2 - \lambda^2} = b_2^c \tag{4.17a}$$

$$a_2^c = \frac{-\lambda}{\lambda'^2 - \lambda^2} = b_1^c \tag{4.17b}$$

$$a_T^c = -a_1^c a^c - a_2^c b^c \tag{4.17c}$$

$$b_T^c = -\ b_1^c a^c - b_2^c b^c \tag{4.17d}$$

In the above coefficients, just the creep strains $\Delta\varepsilon_\theta{}^c$, $\Delta\varepsilon_r{}^c$ and $\Delta\varepsilon_z{}^c$ are unknown.

Recalling the equilibrium equations for the casing system (i.e. Eqn. (1.9)) in incremental form,

$$\Delta\sigma_\theta = \left(\Delta p - K\ \Delta\varepsilon_\theta\right)\frac{r}{t} \tag{4.18a}$$

$$\Delta\sigma_r = - \Delta p \tag{4.18b}$$

Substituting the expression of $\Delta\varepsilon_\theta$ in (4.15) into Eq. (4.18) yields

$$\Delta\sigma_\theta = \left(\frac{1 + K a_2^c}{1 + K \frac{r}{t} a_1^c}\right) \frac{r}{t} \Delta p - \frac{K \frac{r}{t} a_T^c}{1 + K \frac{r}{t} a_1^c} \tag{4.19a}$$

$$\Delta\sigma_r = - \Delta p \tag{4.19b}$$

$$\Delta\sigma_z = \lambda \left(a_1^c + b_1^c\right) \Delta\sigma_\theta + \lambda\left(a_2^c + b_2^c\right) \Delta\sigma_r + \lambda\left(a_T^c + b_T^c\right) + c^c \tag{4.19c}$$

It is clear that as long as $\Delta\varepsilon_{ij}^c$ is formulated properly the problem can be solved.

4.3 Calculation Procedure for Creep Prediction

Since the creep strains involved in the computation formulas (4.19) depend on the stresses, through the correspondent coefficients, the process in Sect. 4.3 is nonlinear. To solve the problem the widely accepted method of initial strains is employed.

The elastic-plastic responses, without regard for creep are first computed. That is to say, the loading is applied at the instant, t=0, when time-dependent creep has not yet occurred. It then becomes possible to solve for the first time interval. The size of this interval depends on the particular problem and the principal considerations are that the computation be convergent and that the stress change be kept small in any one interval.

Next, the procedure for computing creep begins with some initial estimates of the creep increments during the first interval. The sequential computation steps are indicated below.

1. Chose an appropriate time increment Δt in hours. The initial accumulated creep strains ε_θ^c, ε_r^c and ε_z^c are taken as zero, namely

$$\varepsilon_\theta^c = \varepsilon_r^c = \varepsilon_z^c = 0$$

while the initial estimates of creep strain increments are prescribed. For example, assume

$$\Delta\varepsilon_\theta^{c_o} = 2.5E - 3$$

$$\Delta\varepsilon_r^{c_o} = 4.0E - 4$$

$$\Delta\varepsilon_z^{c_o} = -1.0E - 3$$

These estimates are not critical since subsequent iterations quickly produce the correct values. However, it should be noted that, if the estimates for the three components are equal in magnitude to one another, the computation will fail.

To make the estimates in the first time period be reasonable, the following procedure may be used. Assume that the stresses do not change during the time interval. Then σ_θ, σ_r, σ_z and $\bar{\sigma}$, are known from the previous elastic-plastic computation. The value of $\bar{\sigma}$ is used together with Δt in the incremental form of Eq. (4.6), which is

$$\Delta\bar{\varepsilon}^c = A\,\bar{\sigma}^m\, n\, t^{n-1}\,\Delta t \qquad (4.20)$$

to obtain the effective creep strain increment $\Delta\bar{\varepsilon}^c$. Eq. (4.10) is then used, together with this effective creep strain and the stresses, to obtain estimates of incremental creep strain components, namely, $\Delta\varepsilon_{\theta}^{c_o}$, $\Delta\varepsilon_{r}^{c_o}$ and $\Delta\varepsilon_{z}^{c_o}$. These initial estimates of the creep strain increments are now used to recompute

$$\Delta\overline{\varepsilon^c} = \frac{\sqrt{2}}{3}\sqrt{\left(\Delta\varepsilon_{\theta}^{c} - \Delta\varepsilon_{r}^{c}\right)^2 + \left(\Delta\varepsilon_{r}^{c} - \Delta\varepsilon_{z}^{c}\right)^2 + \left(\Delta\varepsilon_{z}^{c} - \Delta\varepsilon_{\theta}^{c}\right)^2} \qquad (4.21)$$

2. Solve for $\Delta\sigma_\theta$, $\Delta\sigma_r$ and $\Delta\sigma_z$ from Eqs. (4.15), then

$$\sigma_\theta = \sigma_\theta^{(o)} + \Delta\sigma_\theta$$

$$\sigma_r = \sigma_r^{(o)} + \Delta\sigma_r$$

$$\sigma_z = \sigma_z^{(o)} + \Delta\sigma_z$$

Here $\sigma_\theta^{(o)}$, $\sigma_r^{(o)}$ and $\sigma_z^{(o)}$ refer to the instantaneous stresses at t=0. The values of σ_{ij} are used to obtain the deviatoric stress tensor

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{KK}\delta_{ij}$$

3. Compute the effective $\overline{\sigma}$ using a creep law like Eq. (4.20). That is,

$$\overline{\sigma} = \left(\frac{\Delta\overline{\varepsilon^c}}{A\ n\ t^{n-1}\Delta t}\right)^{\frac{1}{m}} \qquad (4.22)$$

Note that one doesn't use the values from step 2 for computing $\overline{\sigma}$.

4. Using the values of $\overline{\sigma}$, $\Delta\overline{\varepsilon^c}$ and S_{ij}, compute a new effective creep strian incrment

$$\Delta\varepsilon_{ij}^{c} = \frac{3}{2\bar{\sigma}} \overline{\Delta\varepsilon^{c}} \, S_{ij}$$

5. Repeat step 1 through step 4 using the new values of $\Delta\varepsilon_{ij}^{c}$ as a starting point until the components of the creep strain increments converge. The convergence criterion is

$$\left| \Delta\varepsilon_{ij}^{C_o} - \Delta\varepsilon_{ij}^{c} \right| < \varepsilon \qquad (4.23)$$

If the iteration process fails to converge, it is necessary to change the intial estimates of the creep strain increments or shorten the time interval.

After convergent solutions are reached we have

$$\sigma_{ij}^{(1)} = \sigma_{ij}^{(0)} + \Delta\sigma_{ij} \qquad (4.24a)$$

$$\varepsilon_{ij}^{(1)} = \varepsilon_{ij}^{(0)} + \Delta\varepsilon_{ij}^{c} \qquad (4.24b)$$

in which $\sigma_{ij}^{(o)}$ and $\varepsilon_{ij}^{(o)}$ are elastic-plastic responses.

6. Another time increment is added and step 1 is repeated. But this time ε_θ^{c}, ε_r^{c} and ε_z^{c} were no longer zero and the initial values for iteration $\Delta\varepsilon_\theta^{c}$, $\Delta\varepsilon_r^{c}$ and ε_z^{c} should be taken to be equal to those obtained in the previous interval, while the $\sigma_\theta^{(o)}$, $\sigma_r^{(o)}$ and $\sigma_z^{(o)}$ in Step 2 are replaced by those from convergent solution of Step 5.

7. Continue until the required time span for solution has been completed.

The above procedure is shown in the block diagram of Fig. 4.4.

Since creep data are not available for casing steel, we have selected the five creep expressions from the monographs and papers which are listed in Table 4.1. Preliminary calculations for each indicate that just two among them are adaptable to casing systems. These are: formula (2) in Table 4.1,

$$\bar{\varepsilon}^c = 1.5 * 10^{-32} * \bar{\sigma}^{6.0} * t^{2/3}$$

and, formula (4) in Table 4.1

$$\bar{\varepsilon}^c = 2.72 * 10^{-18} * \bar{\sigma}^{2.99} * t^{0.45}.$$

In these equations stress is in psi and time is in hours.

The results using these two formulas in the computer program for the creep prediction do not show much difference.

Table 4.1 Listing of Creep Laws

	(1) 1980 H. Kraus	(2) 1980 H. Kraus	(3) 1983 "Design for Creep"	(4) 1982 "Design for Creep"	(5) 1980 D. Snyder, K. Bathe
Test Objective	Pressure vessel with an ellipsoidal head Internal pressure=3 MPa	Stress redistribution in a rotating disc	Creep of a beam in bending	Creep of an internally pressurized thick tube	A thick-wall cylinder Internal pressure=25.5 MPa
Test Material	Poisson's Ratio = 0.3	12% Chrome steel	Commercially pure aluminum	0.19% Carbon steel	Yield stress = 70 Mpa
Young's Modulus	140 GPa	126 GPa	70 GPa	170 GPa	157 GPa
Test Temperature	Room temperature	530°C	Room temperature	450°C	550°C
Creep Law $\bar{\varepsilon}^c =$	$19.8*10^{-16}\bar{\sigma}^{3.61} * t^{1.06}$	$15*10^{-32}*\bar{\sigma}^{6.0} * t^{2/3}$	$2.21*10^{-15}*\bar{\sigma}^{3.2} * t^{0.3}$	$2.72*10^{-18}\bar{\sigma}^{2.99} * t^{0.45}$	$F*(1-e^{-Rt}) + Gt$ $F=1.608*10^{-10}*\bar{\sigma}^{1.843}$ $R=5.929*10^{-5} * e^{2.029*10^{-4}\bar{\sigma}}$ $G=6.73*10^{-9} *[\sinh(1.479 *10^{-4}*\bar{\sigma})]^3$
Comment	Applied to small load short time	Applied to casing system		Applied to casing system	

CHAPTER 5 STRUCTURE OF PROGRAM CASEPC

5.1 Flow of Operation

In this section we shall consider some aspects of the computer program, called CASEPC for CAsing Elastic-Plastic and Creep analysis, which can be utilized to predict creep response and the elastic-plastic stresses and strains for both loading and unloading. As mentioned in the previous chapters, the program has two main functions. The first function is to determine elastic-plastic solutions, while the second is to predict creep effects in combination with elastic-plastic response. Both of these types of solutions are temperature dependent.

A schematic chart of the program is shown in Fig. 5.1.

5.2 Structure of Program

Program CASEPC is arranged in 27 subprogram segments. The main program contains just eight subroutines which are supported by the other subroutines. A flow chart of CASEPC is given in Fig. 5.2.

In the program CASEPC the two most important segments are subroutine EPSOL and subroutine CRSOL. The first is specially designed to deal with elastic-plastic stress analysis including loading and unloading. The flow chart of this subroutine is shown as in Fig. 5.3. The second is responsible for creep computation which is shown in the block diagram of Fig. 5.4.

5.3 Additional Subroutines for Temperature Dependent Solutions

To take account of the effect of temperature on material properties curve fitting is utilized for E and α. One-dimensional interpolation is used to find $\sigma_{Y\circ}$. As we have noted the tangent modulus E^t is a bivariate function of temperature and strain, and two-dimensional interpolation is employed. Here we shall give some explanation.

For the initial yield limit of MN80 steel, σ_{Y°, the interpolation nodes are given as in Table 5.1.

Table 5.1 Temperature Dependence of Yield Point for MN80 Steel

T°C	0°	21°	93°	204°	316°	427°
σ_{Y° (MPa)	622	610	570	500	420	355

The computational formula used in program is two-dimensional Lagrangian interpolation through three points at unequal intervals. That is

$$\sigma_Y(T) = \sum_{k=i}^{i+2} \left(\prod_{\substack{j=i \\ (j\neq k)}}^{i+2} \frac{T - T_j}{T_K - T_j} \right) \sigma\left(T_K\right) \tag{5.1}$$

in which

$$i = \begin{cases} 1 & T<T_2 \\ S & T_s<T<T_{s+1},\ T-T_s>T_{s+1}-T,\ s = 2,3,\ldots n-2 \\ S-1 & T_s<T<T_{s+1},\ T-T_s>T_{s+1}-T,\ s = 2,3,\ldots n-2 \\ n-2 & T>T_{n-1} \end{cases} \tag{5.2}$$

In the program we take n=6.

For the tangent modulus E^t, stress values provided by C-FER are tabulated in Table A.6.2.1 of their Phase 1 report on THERWELL, as a function of strain and temperature. The tangent modulus is defined as the slope of the $\bar{\sigma}-\bar{\varepsilon}$ diagram in the plastic region. So we can calculate E^t for each strain increment, under different temperatures, using Table A6.2.1, as

$$E^t = \frac{\Delta\sigma}{\Delta\varepsilon} \tag{5.3}$$

The interpolation values are listed in Table 5.2.

The corresponding formula for computation is two dimensional Lagrangian interpolation through three points at unequal intervals for bivariate functions, namely,

$$E^t(T,\varepsilon) = \sum_{r=i}^{i+2} \sum_{s=j}^{j+2} \left(\prod_{\substack{k=i \\ (k\neq r)}}^{i+2} \frac{T-T_k}{T_r-T_k} \right) \left(\prod_{\substack{\ell=j \\ (\ell\neq s)}}^{j+2} \frac{\varepsilon-\varepsilon_\ell}{\varepsilon_s-\sigma_\ell} \right) E^t \left(T_r, S_r\right) \tag{5.4}$$

in which $r = 1,2,\ldots N$ $N=6$

$s = 1,2,\ldots M$ $M=8$

N denotes the number of interpolation nodes of the first argument, T. M denotes the number of interpolation nodes of the second argument, ε. Values of i and j are the same as in (5.2).

The interpolation subroutines are named respectively LAG13 and LAG23. The latter is for two-dimensional interpolation of E^t.

Table 5.2 Stress-Strain Relation for MN80 Steel at Different Temperatures

	0°		21°		93°		204°		316°		427°	
ε %	E^t (GPa)	σ (MPa)	E^t (GPa)	σ (MPa)	E^t (GPa)	σ (MPa)	E^t (GPa)	σ (MPa)	E^t (GPa)	σ (MPa)	E^t (GPa)	σ (MPa)
0.3	2.5	622	5.0	610	12.5	570	25.0	500	45.0	420	45.0	355
0.5	3.0	627	3.3	620	5.0	595	10.0	550	15.0	510	9.3	445
0.8	2.58	636	2.5	630	2.08	610	4.17	580	4.58	555	3.92	473
2.0	1.55	667	1.75	660	2.5	635	3.10	630	3.0	610	0.75	520
4.0	1.0	698	1.0	695	1.0	685	1.25	690	1.75	670	-3.5	535
6.0	0.45	718	0.25	715	-0.5	705	-1.25	715	0.25	705	-3.15	471
10.0	-0.9	725	-1.6	710	-4.0	660	-6.5	640	-1.75	705	-3.15	471
12.0	707		678		580		510		670		408	

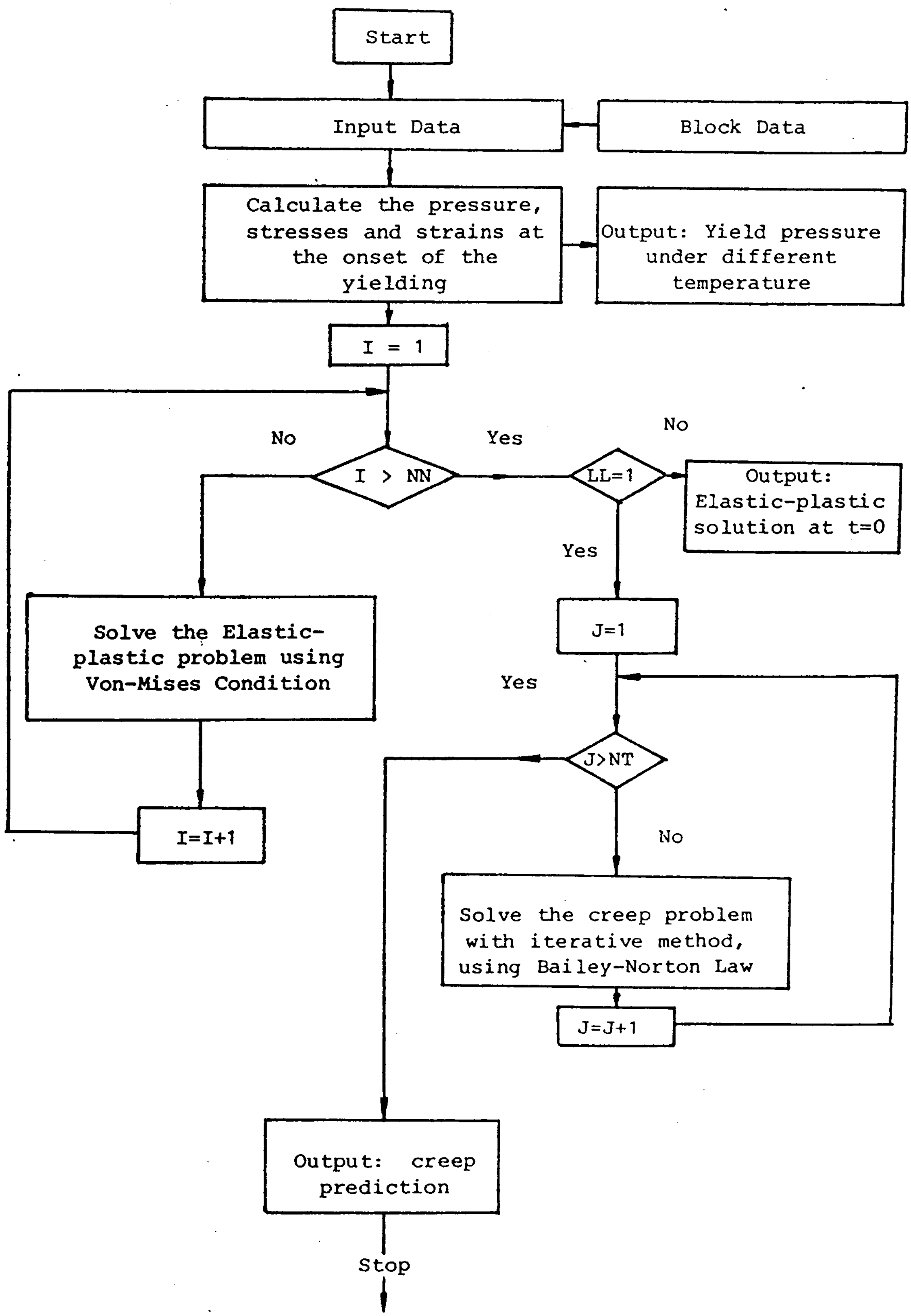

Fig. 5.1 Schematic of CASEPC

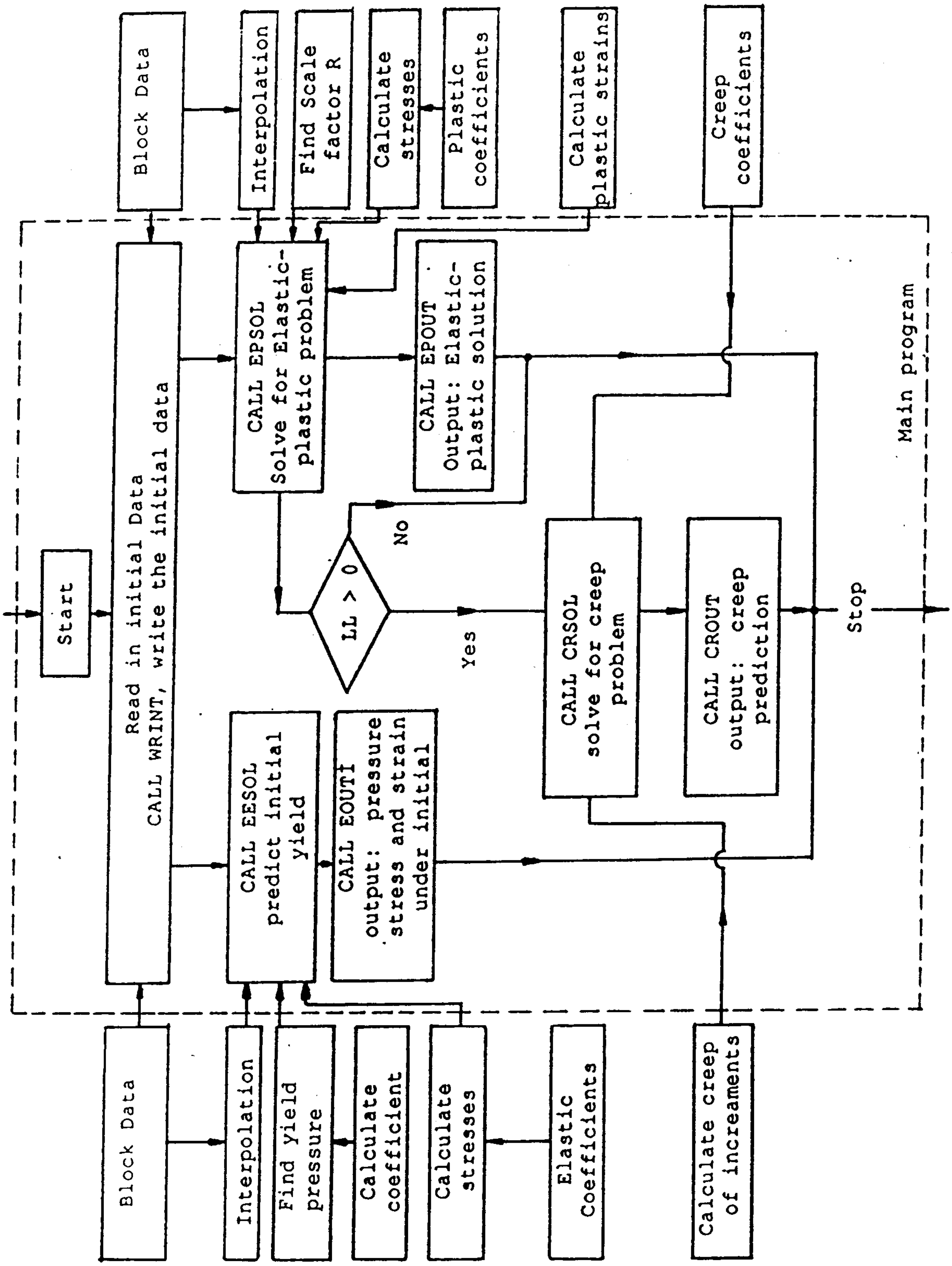

Fig. 5.2 Flow chart of interaction of subroutines

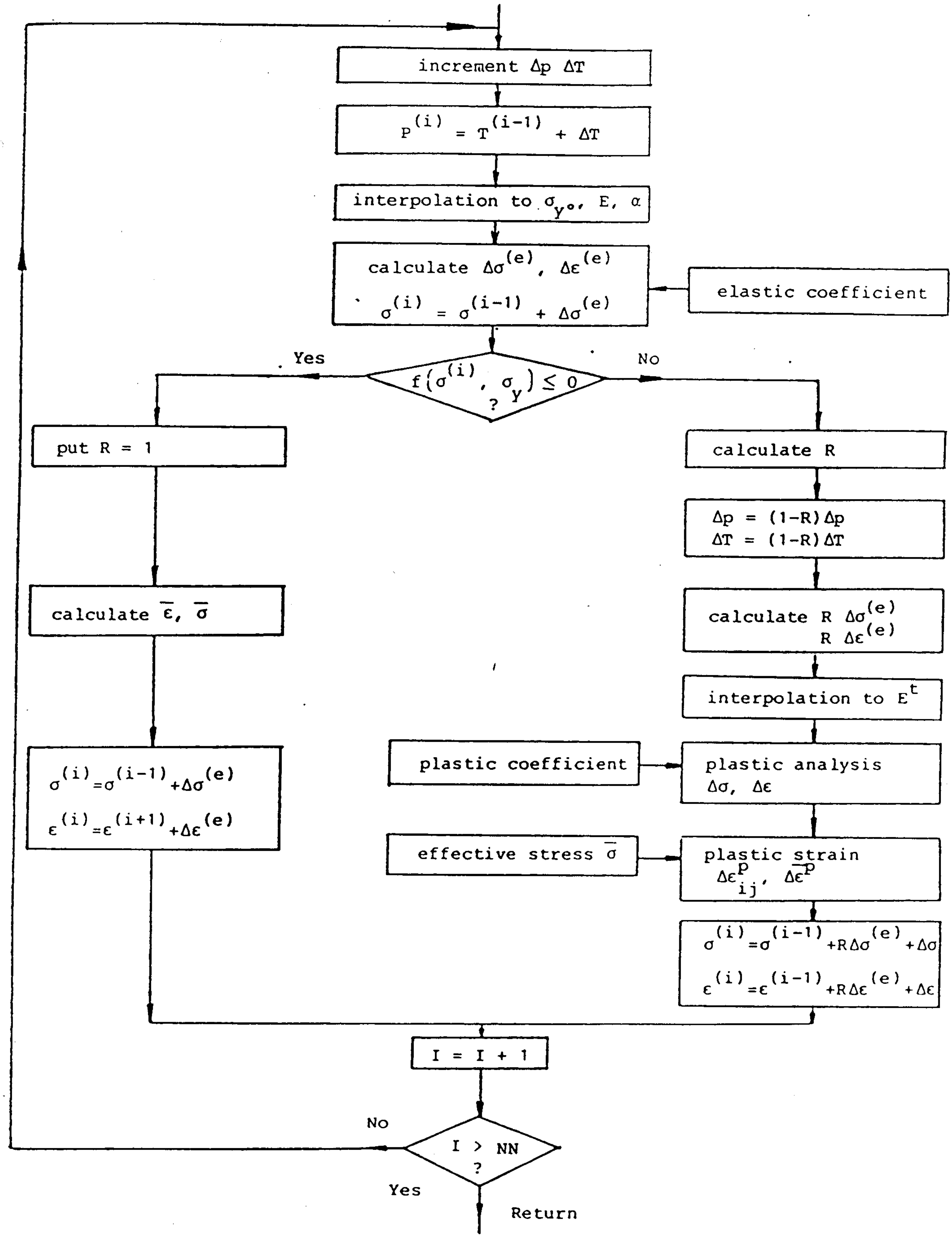

Fig. 5.3 Flow Chart of Subroutine EPSOL

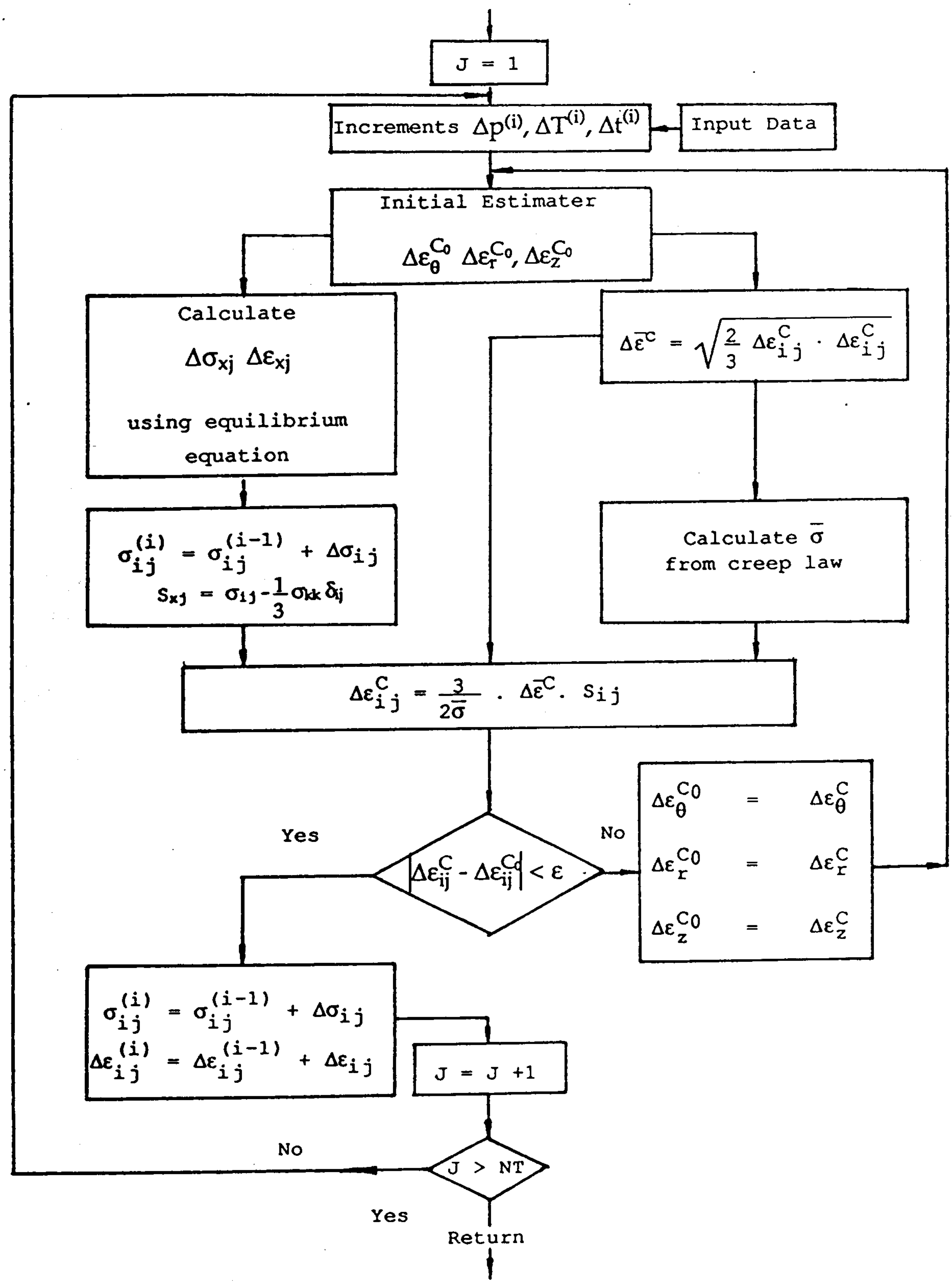

Fig. 5.4 Flow Chart of subroutine CRSOL

CHAPTER 6 SOME NUMERICAL SOLUTIONS

6.1 Numerical Results

Prediction of initial yield using nondimensional ratios $\tilde{p}_y$ and $\tilde{T}_{y0}$ from (2.13a,b) is made for the following examples.

a. 4 1/2" casing

b. For oil sand, the data set is

$C_f = 0$ indicates no foundation

$(C_f)_{max} = 10.9 \times 10^{-3}$ indicates $\overline{E}_{rock} = 2{,}200$ MPa

$(C_f)_{max} = 7.5 \times 10^{-3}$ indicates $\overline{E}_{rock} = 1{,}600$ MPa

c. For shale the data set is

$C_f = 0$ indicates no foundation

$(C_f)_{max} = 20.6 \times 10^{-4}$ indicates $\overline{E}_{rock} = 400$ MPa

$(C_f)_{max} = 4.5 \times 10^{-4}$ indicates $\overline{E}_{rock} = 88$ MPa

Fig. 6.1 shows yield pressure ratio vs temperature ratio for oil sand and temperature independent properties. Fig. 6.2 is similar to Fig. 6.1 but for shale.

The prediction of initial yield for temperature dependent material properties has been carried out for the oil sand and shale properties summarized above. Fig. 6.3 shows the diagram of yield pressure vs temperature for a shale formation and 4 1/2" casing with temperature dependent properties. Fig. 6.4 is similar with an oil sand formation and 4 1/2" casing. These curves are lines of demarcation between the elastic and plastic regions.

6.2 Elastic-plastic solution with thermoplastic material

Data set:

4 1/2" casing   outer diameter of pipe 114.3 mm

	thickness of wall	5.2 mm
Oil sand formation	elastic modulus	(temperature dependent)
	Poisson ratio	0.28

This example has been solved for two loading conditions:

A: Keeping the pressure constant at p = 15 MPa

B: Allowing both pressure and temperature to increase simultaneously.

As this case is a temperature dependent problem, all properties of thermoplastic material are determined from Tables 4.1 and 4.2. The results are listed in Table 6.1.

The following for test problems were investigated.

1. A comparison between the results of an ADINA analysis by C-FER and the results which are computed by program CASEPC is made in Table 6.2 for the conditions where the loading path holds the pressure constant at p = 16.5 MPa, while the temperature increases from 0°C to 350°C. The formation was an oil-sands formation.

2. A comparison between a temperature dependent and temperature independent solutions is shown in Fig. 6.5, using 4 1/2" casing and oil sand formation.

3. For loading consisting of temperature cycling and keeping the pressure constant at p = 15 MPa the results are shown in Fig. 6.6.

4. The results of the elastic-plastic analysis with temperature dependent properties are shown as Fig. 6.7 and Fig. 6.8. In Fig. 6.7 the diagrams of effective stress vs pressure under different temperature are given and for Fig. 6.8 the diagram shows the results of effective st ess vs temperature under different pressures.

Both of the above are taken from oil sand formation and 4.5" casing.

6.3 Creep Prediction

The same data set as in the elastic-plastic computation is used as an example for a creep solution. The temperature and pressure were held constant (at 16.5 MPa and 350°C) for 1000 hours. The results are listed in Table 6.3 and the results of the first 300 hours are shown in Figs. 6.9 to 6.12.

For comparison, ADINA was used to compute creep effects for the first 12 hours, using the same data used in CASEPC. The solution contrasts between both are shown in Figs. 6.13 to 6.16.

6.4 Conclusion

It is evident that the model we adopted and the program CASEPC are of many advantages, such as the reasonable simplized and theoretical basis, simple construct and convenient use for the program. Especially, the convincing results have been obtained throught he comparison with the results from ADINA. So, the model along with the program CASEPC could be recommended to use in practice.

Table 6.1 Comparison of Effect of Variation in Sequence of Temperature Pressure Increments for Time Independent Problem

Loading	A				B
Temperature / Results	3.5°	200°	300°	350°	350°
σ_θ (MPa)	134.5	82.9	50.18	34.2	50.58
σ_r	-15.0	-15.0	-15.0	-15.0	-15.0
σ_z	27.09	-434.8	-521.8	-531.3	-546.2
$\varepsilon_\theta \times 10^{-2}$	0.668	3.48	5.236	6.278	6.335
$\varepsilon_r \times 10^{-2}$	-0.2625	2.907	4.73	5.729	5.634
$\varepsilon_z \times 10^{-2}$	0	0	0	0	0
$\varepsilon_\theta^p \times 10^{-3}$	0	0	0.504	0.832	0.7633
$\varepsilon_r^p \times 10^{-3}$	0	0	0.309	0.6519	0.5575
$\varepsilon_z^p \times 10^{-3}$	0	0	-0.813	-1.482	-1.321

A = Constant pressure of 15 MPa.

B = Pressure increases linearly to 15 MPa while temperature increases linearly to 350°C.

Table 6.2 Comparison of Time Independent Solutions by CASEPC and ADINA*

	CASEPC	ADINA	DIFFRENCE
σ_θ (MPa)	41.84	35.30	+18.5%
σ_r (MPa)	-16.5	-16.5	
σ_z (MPa)	-487.3	-415.2	+17.3%
ε_θ (1.E-2)	0.6354	0.7252	-12.3%
ε_r (1.E-2)	0.5634	0.6413	-12.1%
ε_z (1.E-2)	0.0	0.0	
ε_θ^p (1.E-3)	1.028	1.089	-5.6%
ε_r^p (1.E-3)	0.6893	0.7509	-8.2%
ε_z^p (1.E-3)	-1.717	-1.839	-6.8%
$\bar{\sigma}$ (MPa)	502.7	426.8	+17.7%
$\bar{\varepsilon}$ (1.E-2)	0.4018	0.4581	-12.3%

* p = constant = 16.5 MPa

T = increases from 0°C to 350°C

Table 6.3 Time Dependent Solutions for p = 16.5 MPa and T = 350°C

t (hr)	σ_θ (MPa)	σ_r (MPa)	σ_z (MPa)	ε_θ (1.E-2)	ε_r (1.E-2)	ε_z (1.E-2)	$\bar{\sigma}$ (MPa)	$\bar{\varepsilon}$ (1.E-2)	$\dot{\varepsilon}^c$
0	44.23	-16.5	-545.0	0.6235	0.5597	0.0	561.4	0.3961	
2	37.34		-393.9	0.6545	0.5757	0.0	407.0	0.4126	0.3367E-3
10	33.09		-305.5	0.6735	0.5840	0.0	316.7	0.4224	0.4391E-4
20	31.52		-274.2	0.6806	0.5867	0.0	284.7	0.4259	0.1818E-4
50	30.07		-246.1	0.6871	0.5883	0.0	256.1	0.4291	0.7135E-5
100	28.91		-224.4	0.6923	0.5903	0.0	233.9	0.4316	0.3301E-5
200	27.81		-204.5	0.7003	0.5922	0.0	201.2	0.4353	0.9299E-6
400	26.65		-183.8	0.7024	0.5926	0.0	192.5	0.4363	0.6503E-6
600	25.98		-172.3	0.7054	0.5931	0.0	180.8	0.4377	0.3916E-6
800	25.53		-164.5	0.7075	0.5935	0.0	172.9	0.4386	0.2741E-6
1000	25.18	-16.5	-158.7	0.7090	0.5937	0.0	167.0	0.4393	0.208E-6

Note: The results come from the program CASEPC under the following conditions.

a. No pressure increment and temperature increment in the time period. That is to say, the pressure and temperature were kept constant, p = 16.5 MPa and ΔT = 350°C.

b. For the tangent modulus E^t, two dimensional Lagrangian interpolation is employed. This differs from ADINA where just one dimensional interpolation is available.

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FIGURES

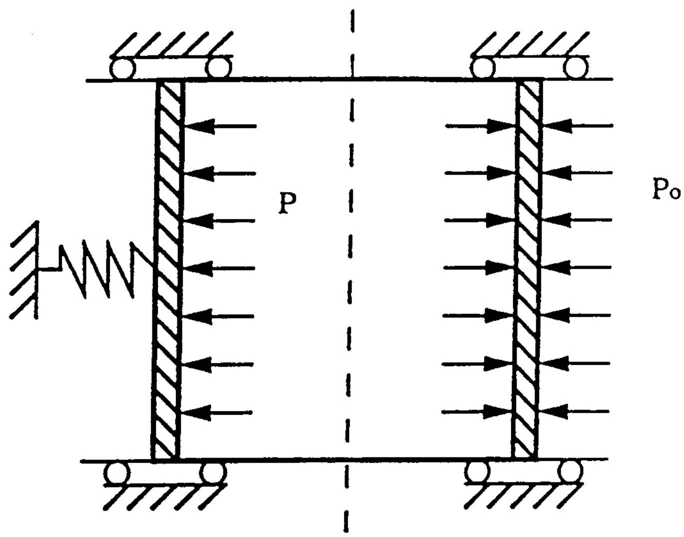


Fig. 1.1-- Casing System

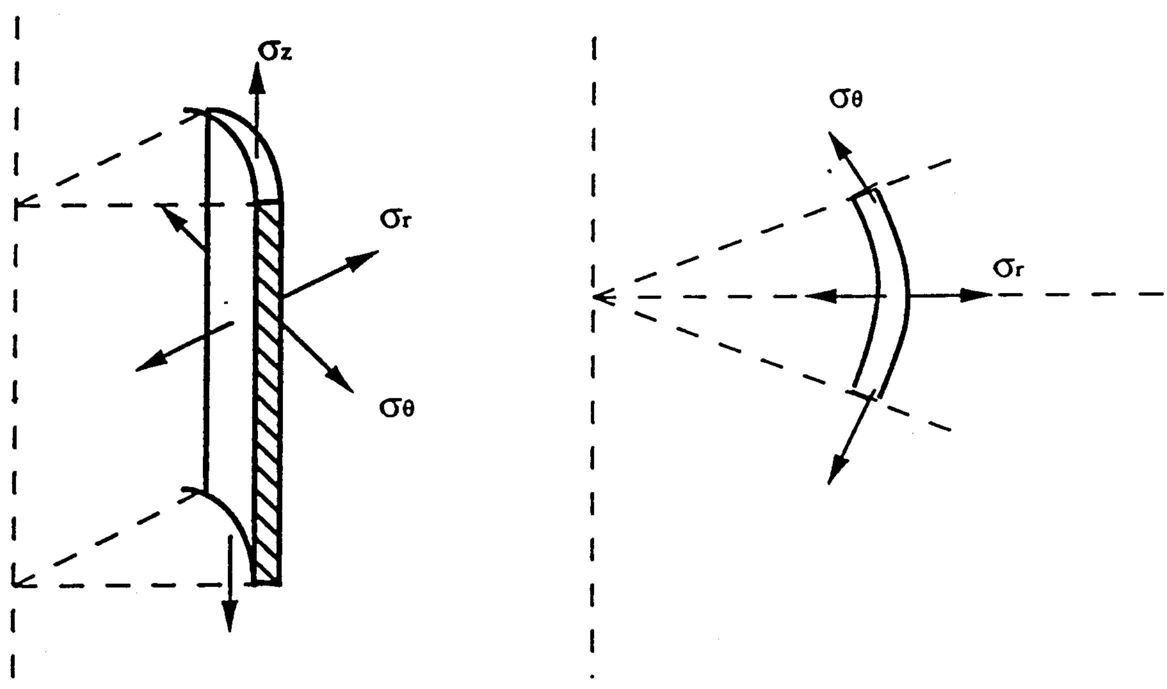


Fig. 1.2

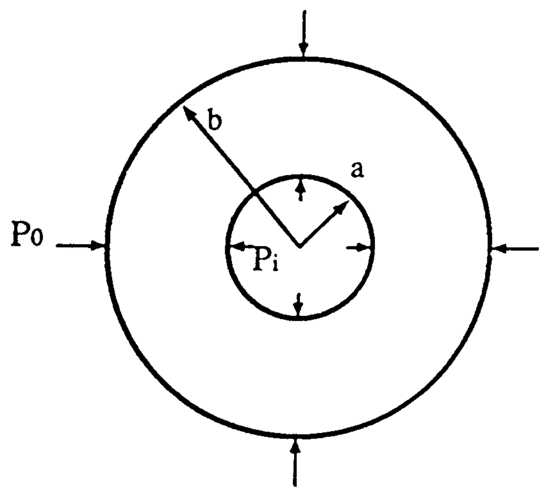


Fig. 1.3

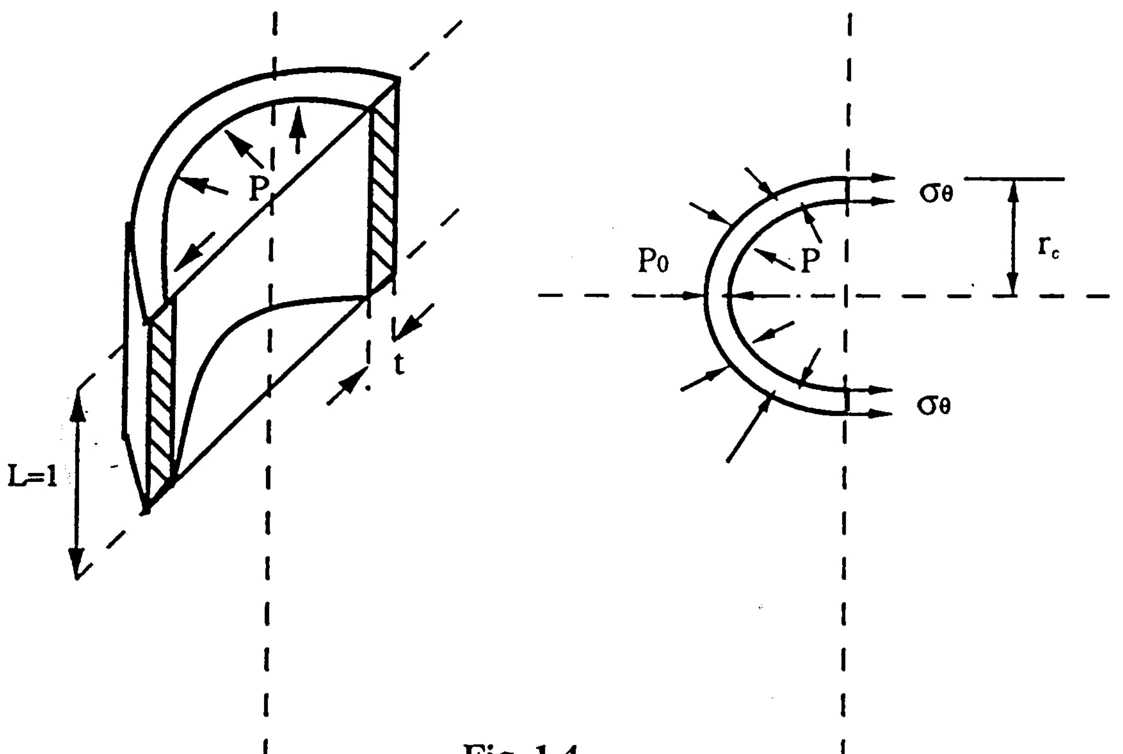
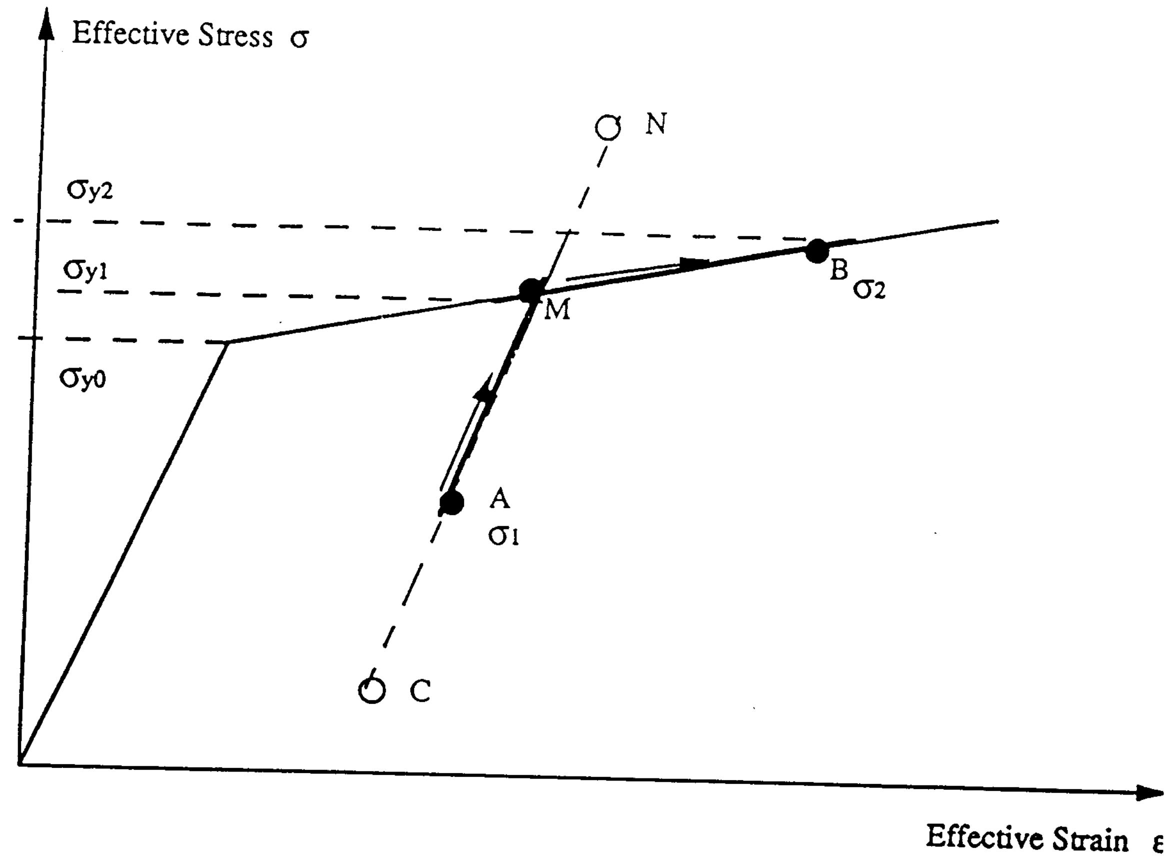


Fig. 1.4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N
σy2
σy1
B
σ2
M
σy0
A
σ1
C
Effective Strain ε

Fig. 2.1

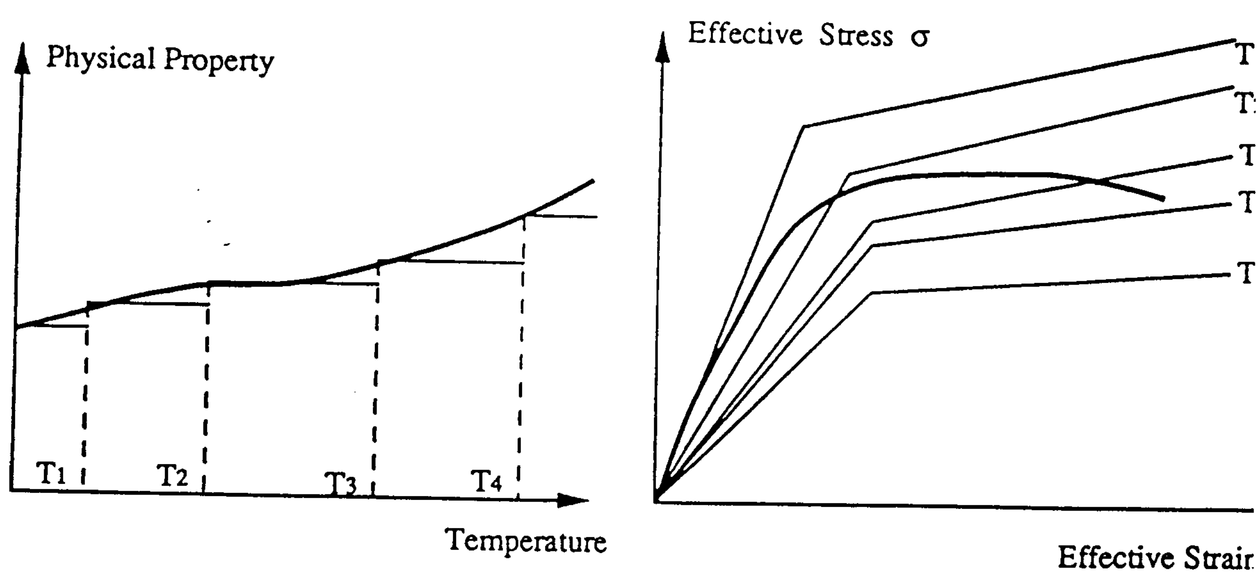
Physical Property
T1
T2
T3
T4
Temperature
Effective Stress σ
Effective Strair

Fig. 3.1

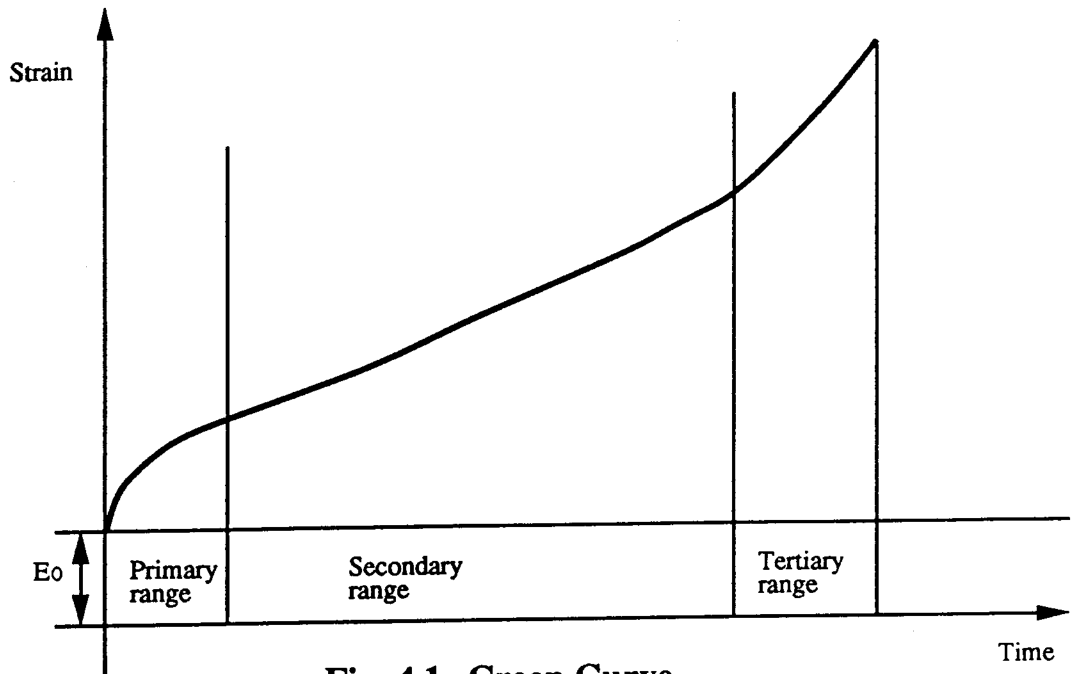

Fig. 4.1--Creep Curve

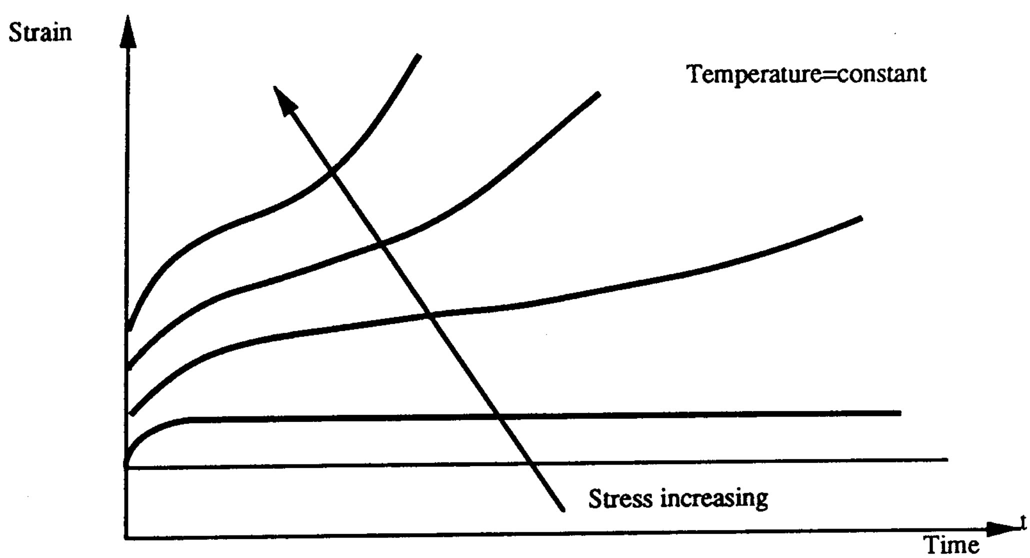

Fig. 4.2-Effect of stress on the creep curve at constant temperature

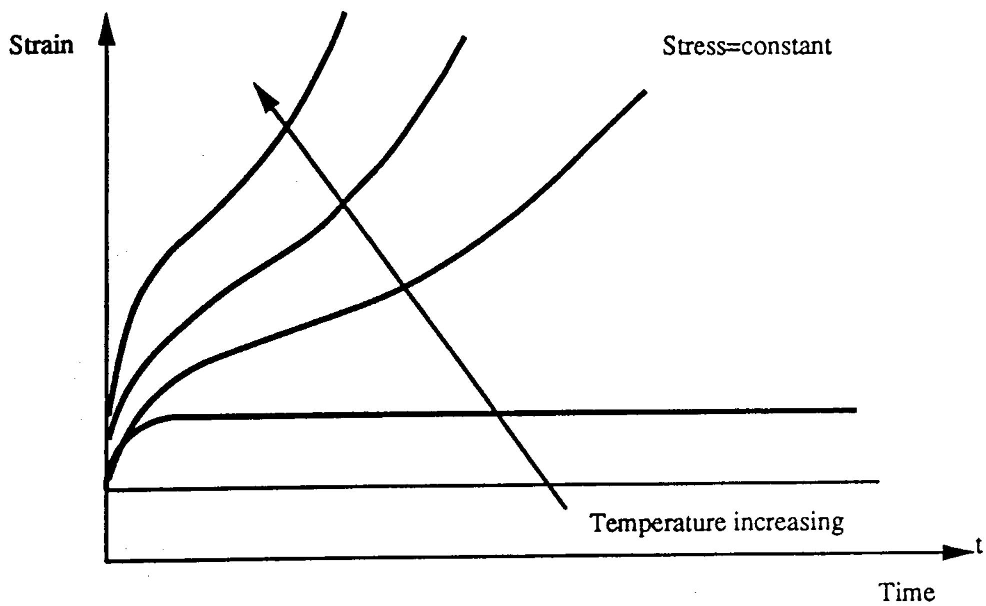

Fig. 4.3- Effect of temperature on the creep curve at constant stress

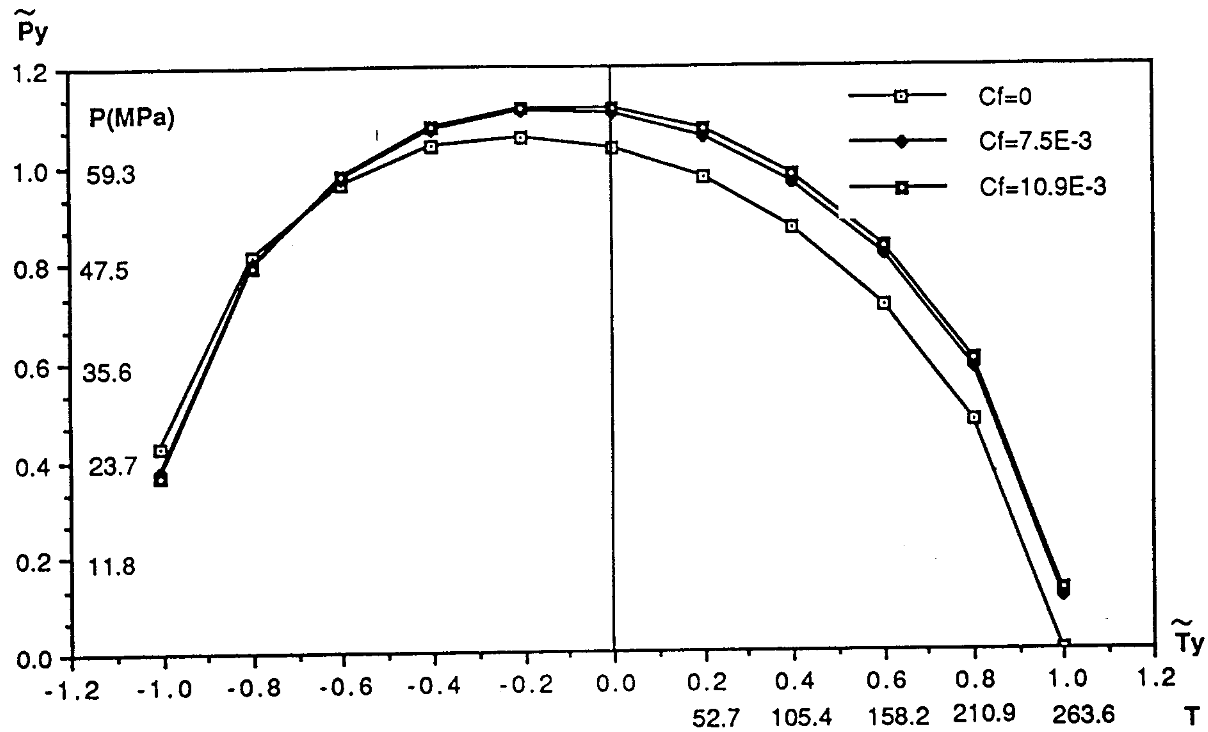

Fig. 6.1--Yield Pressure Ratio Vs Temperature Ratio
(Oil sand, 4.5" Casing)

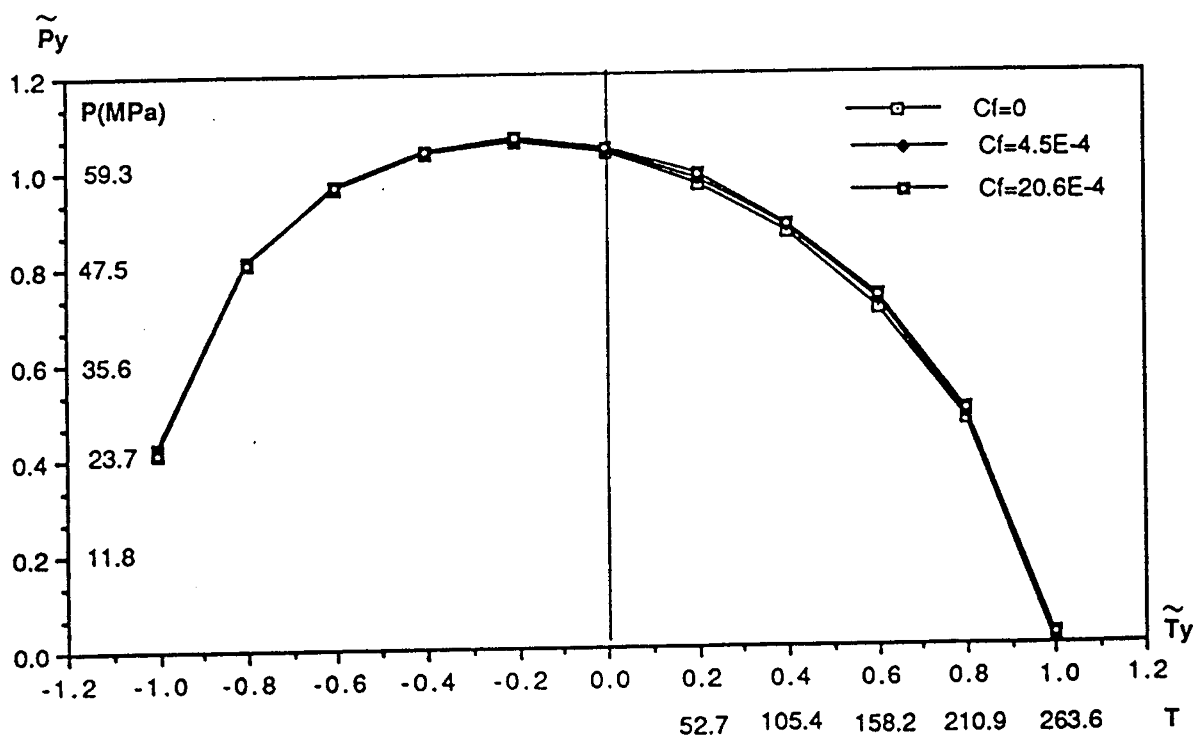

Fig. 6.2--Yield Pressure Ratio Vs Temperature Ratio
(Shale Formation, 4.5" Casing)

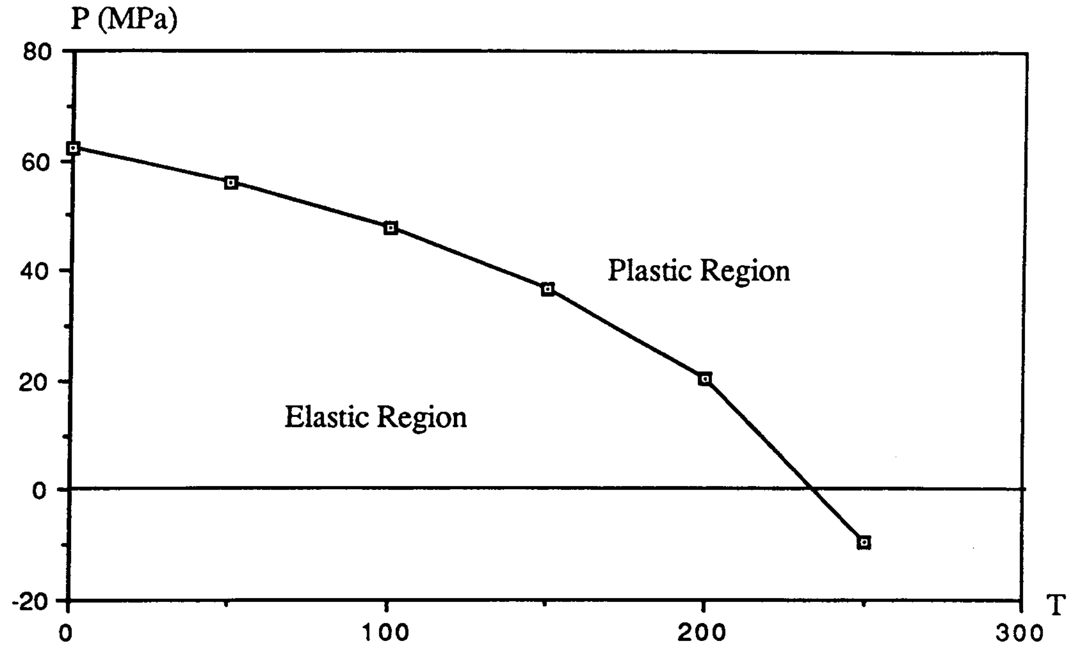

Fig. 6.3--Yield Pressure Vs Temperature (Shale Formation, 4.5" Casing)

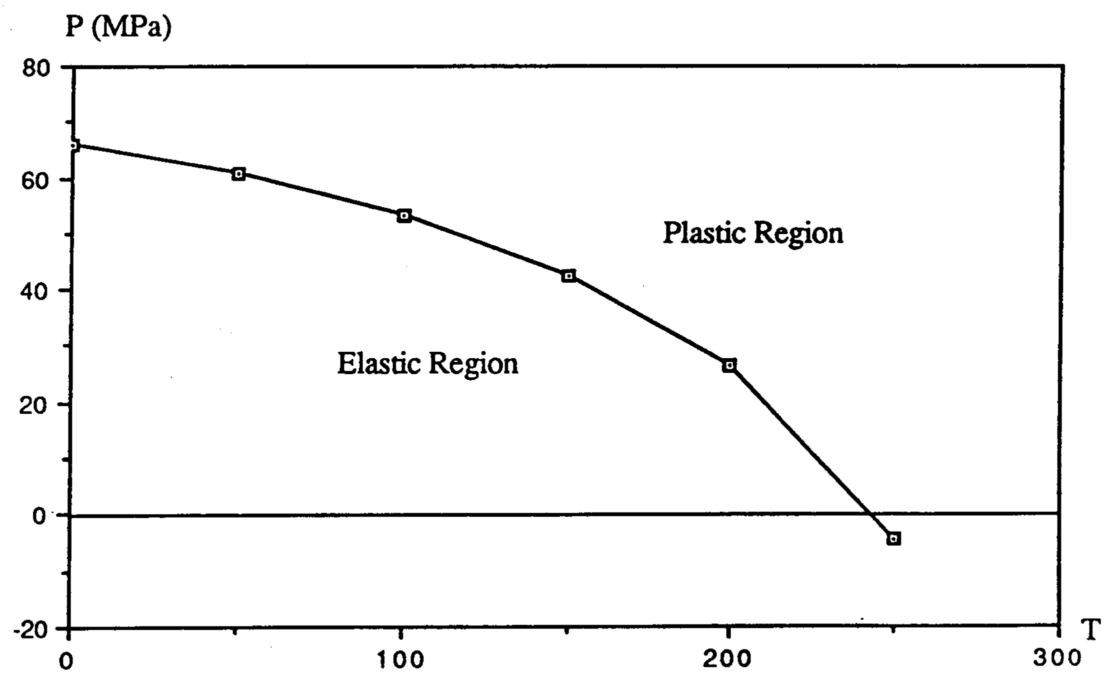

Fig. 6.4--Yield Pressure Vs Temperature (Oil Sand, 4.5"Casing)

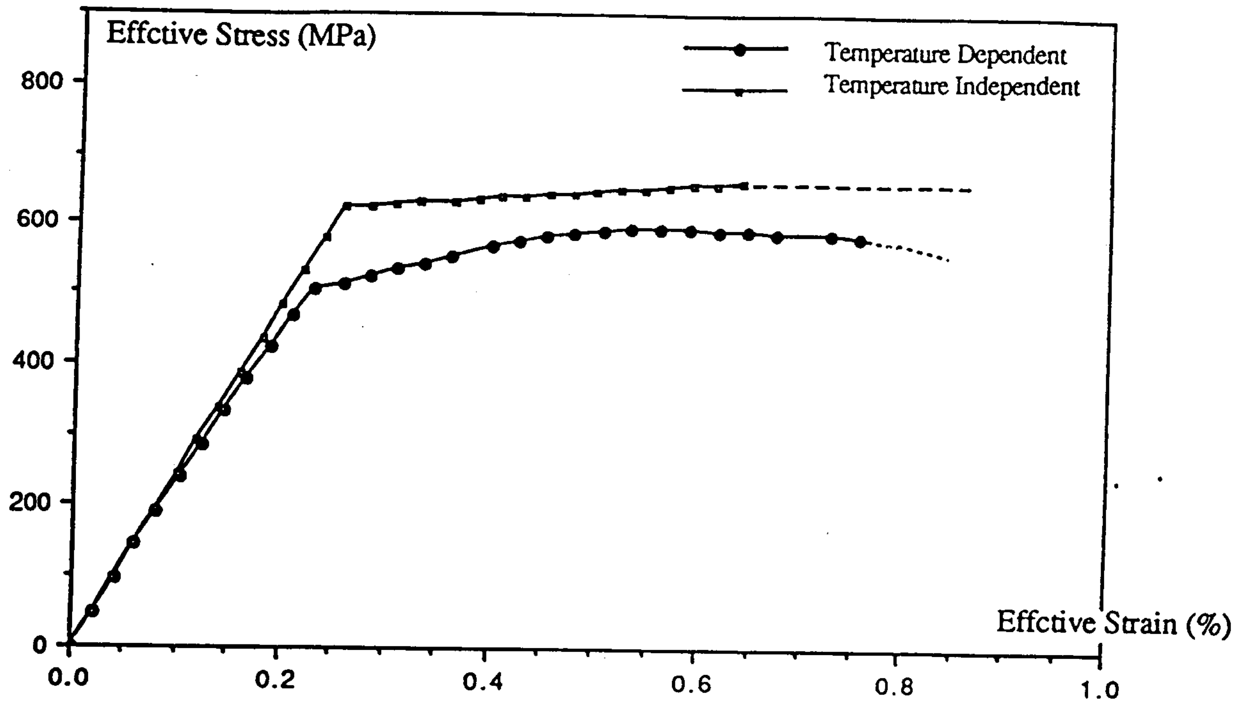

Fig. 6.5--Temperature Dependent
(Oil Sand, 4.5" Casing)

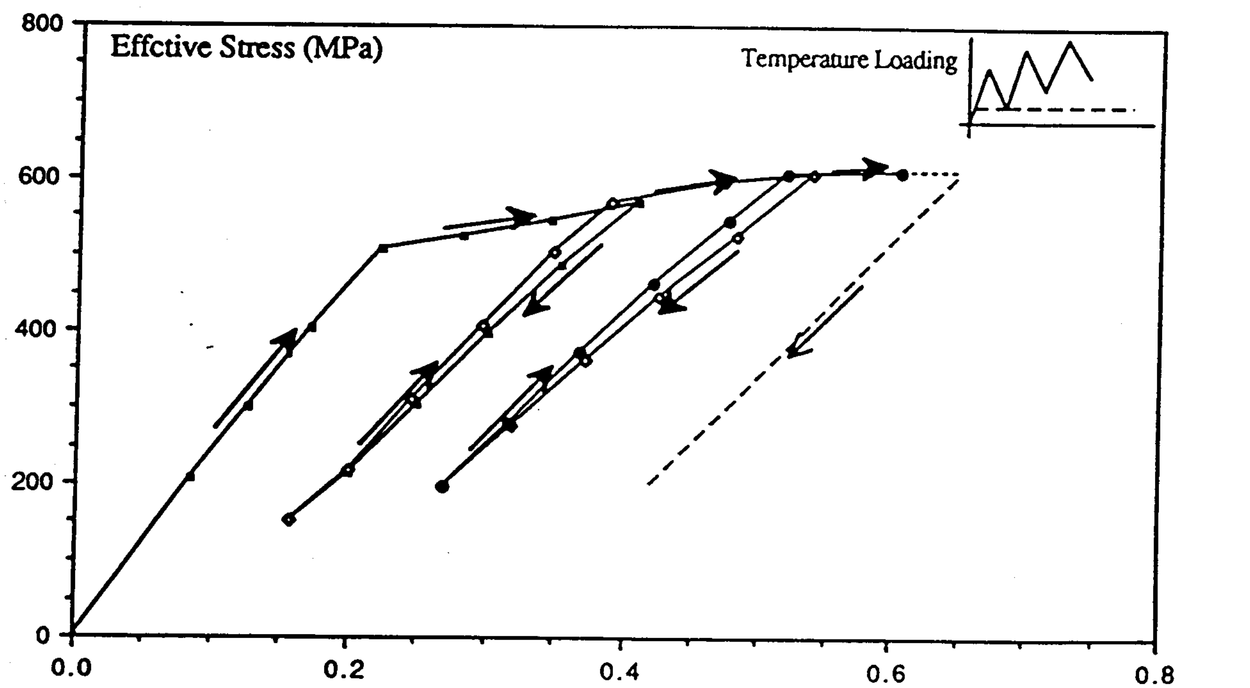

Fig. 6.6-- Cycle Loading of Temperature
(Oil Sand, 4.5" Casing, P=15 MPa)

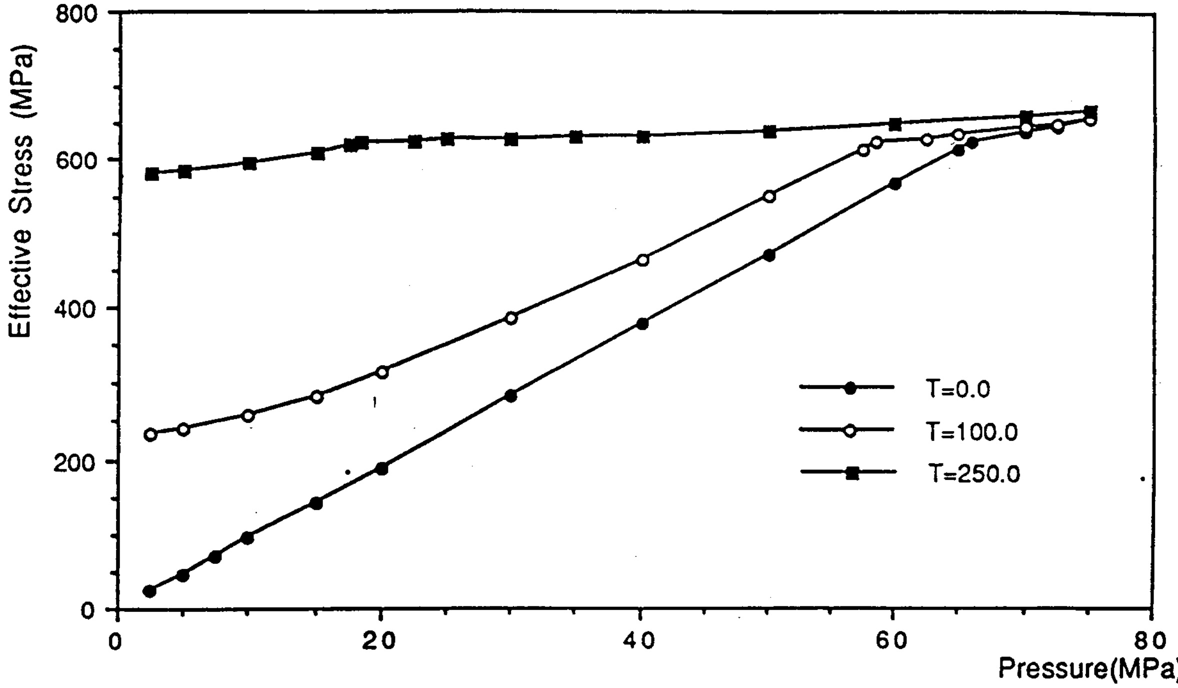

Fig. 6.7--Effective Stress Vs Pressure (Oil Sand,4.5"Casing,Under Different Temperature)

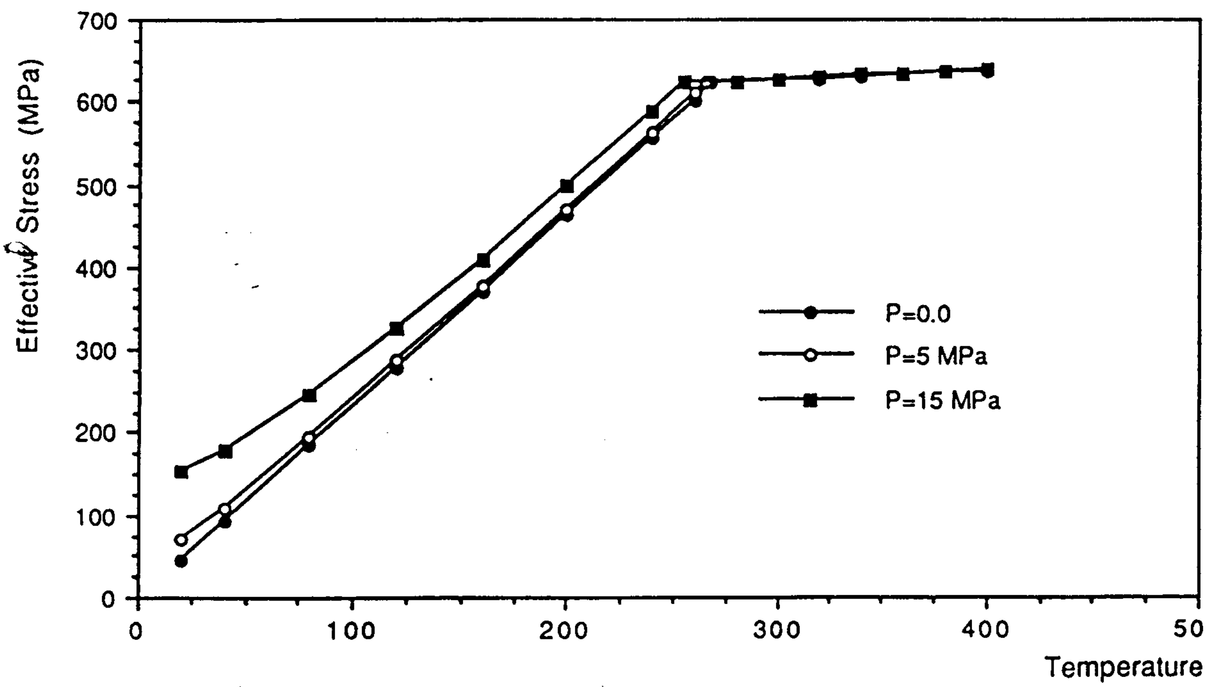

Fig. 6.8--Effective Stress Vs Temperature (Oil Sand,4.5"Casing,Under Different Pressure)

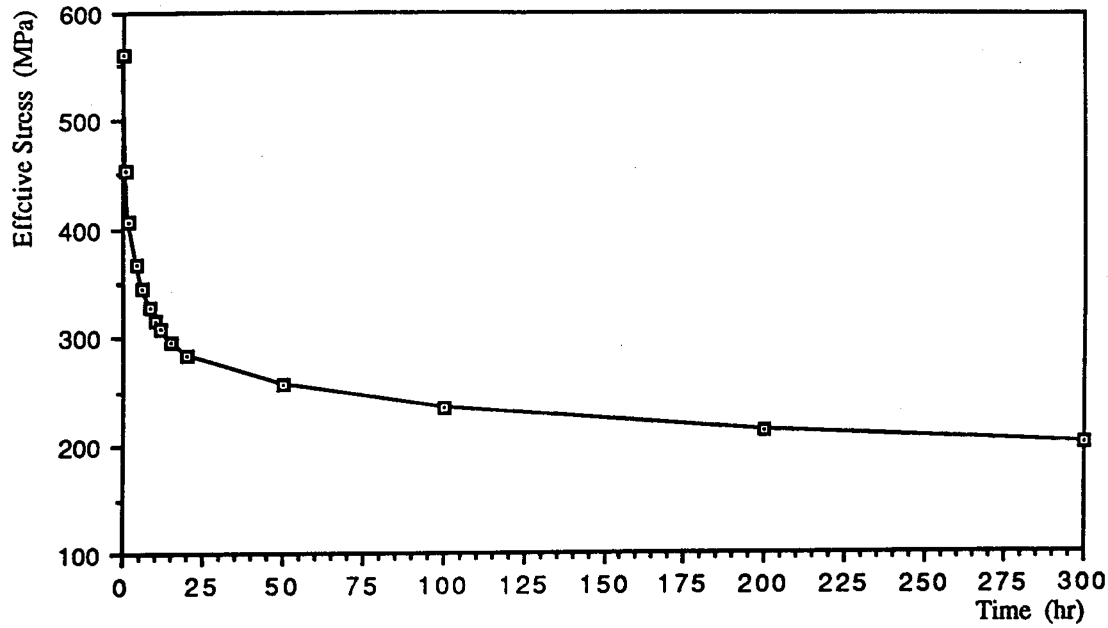

Fig. 6.9--Time Dependent Solution: Effective Stress (Oil Sand, 4.5" Casing, P=16.5 MPa,T=350)

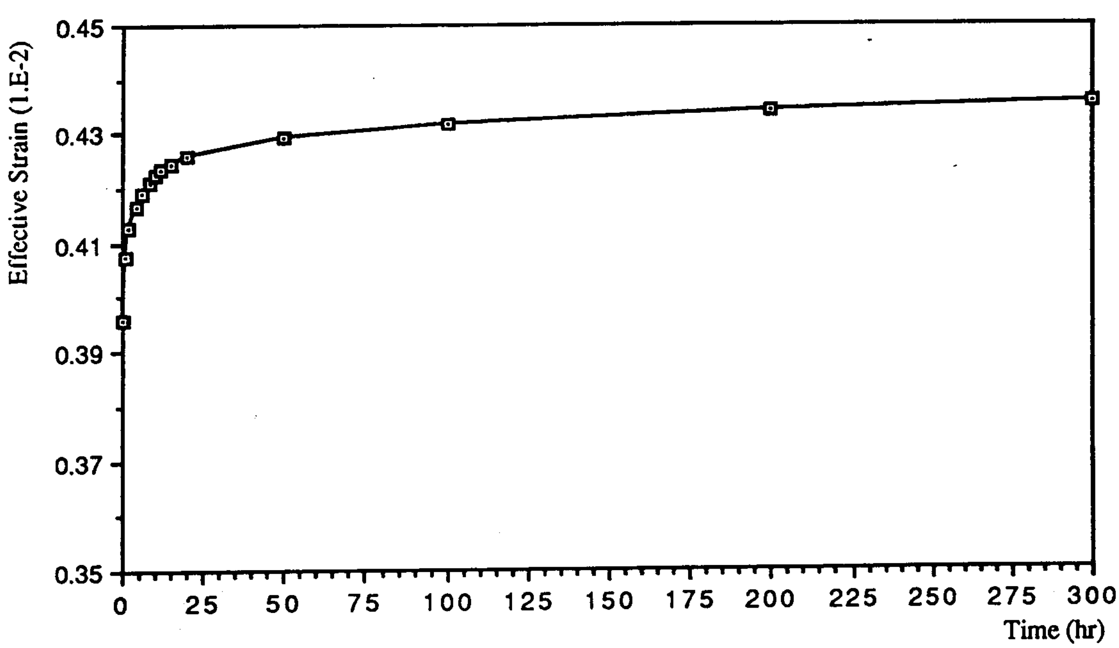

Fig. 6.10--Time Dependent Solution: Effective Strain (Oil sand, 4.5" Casing, P=16.5 MPa,T=350)

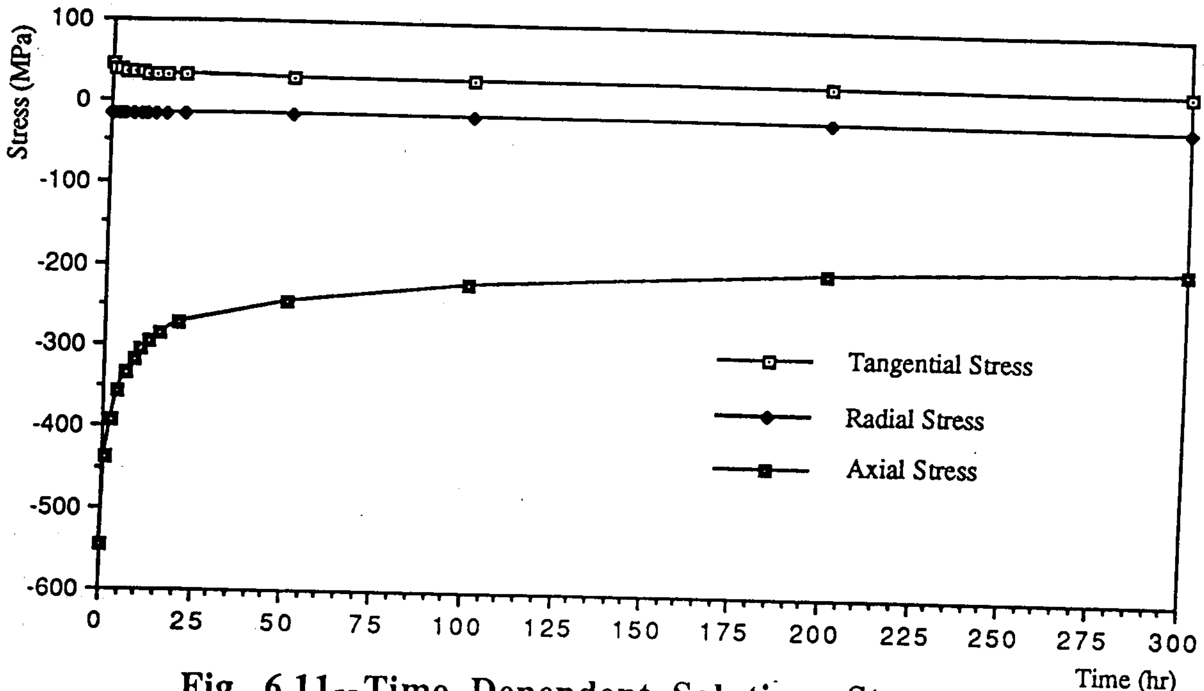

Fig. 6.11--Time Dependent Solution: Stresses (Oil Sand, 4.5"Casing, P=16.5MPa, T=350)

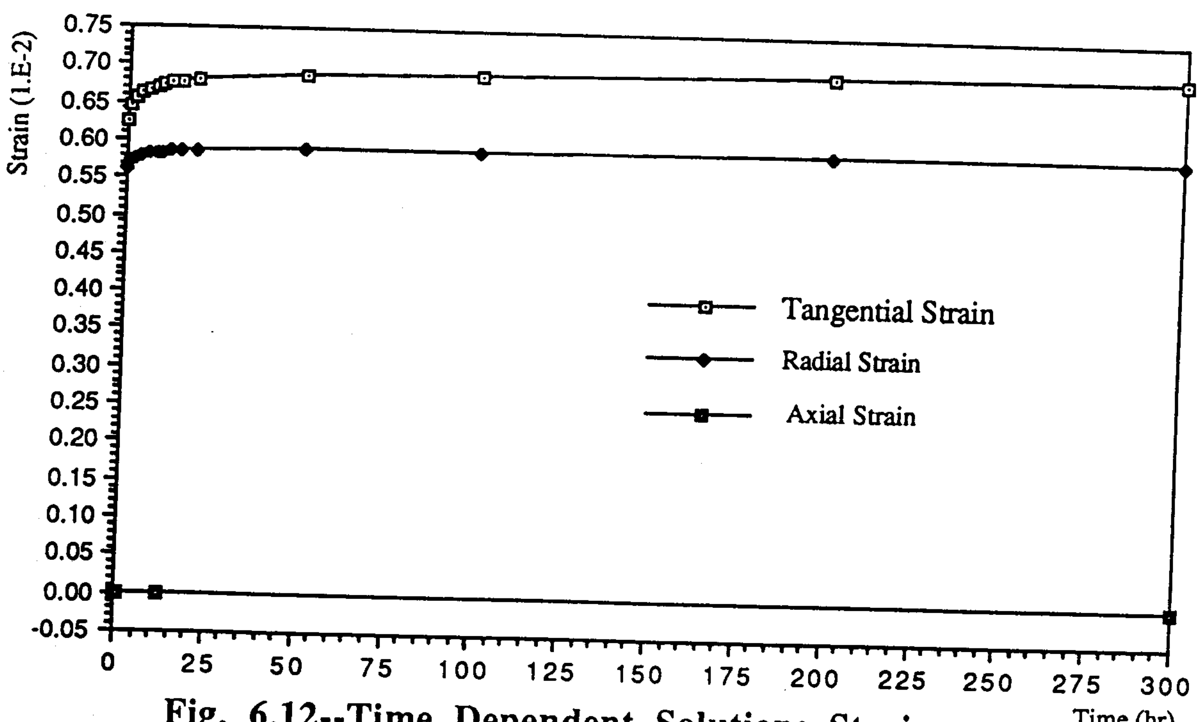

Fig. 6.12--Time Dependent Solution: Strain (Oil Sand, 4.5"Casing, P=16.5MPa,T=350)

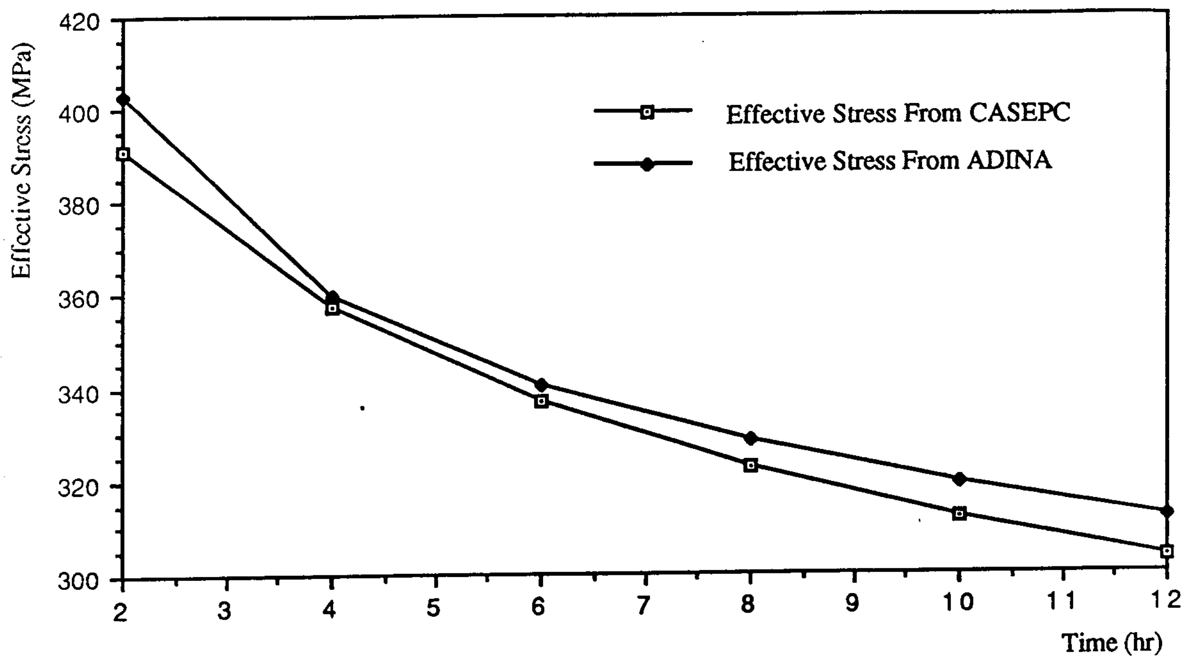

Fig. 6.13--Creep Solution Contrast Between CASEPC and ADINA (Oil Sand, 4.5" Casing, P=16.5MPa, T=350)

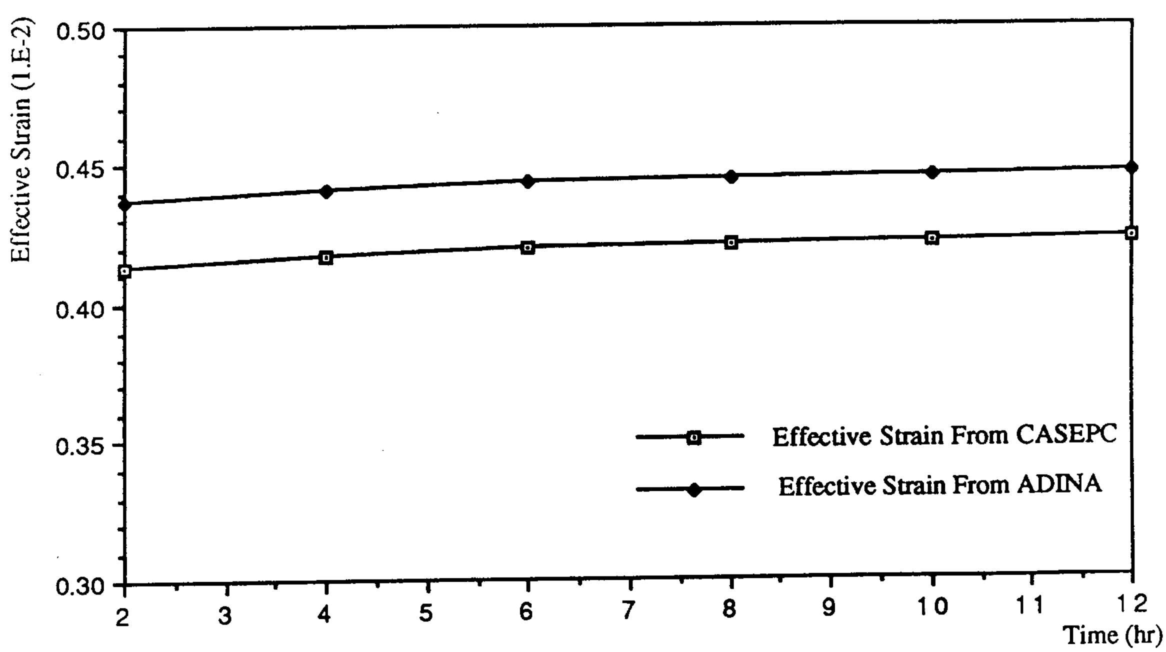

Fig. 6.14--Creep Solution Contrast Between CASEPC and ADINA (Oil Sand, 4.5" Casing, P=16.5MPa, T=350)

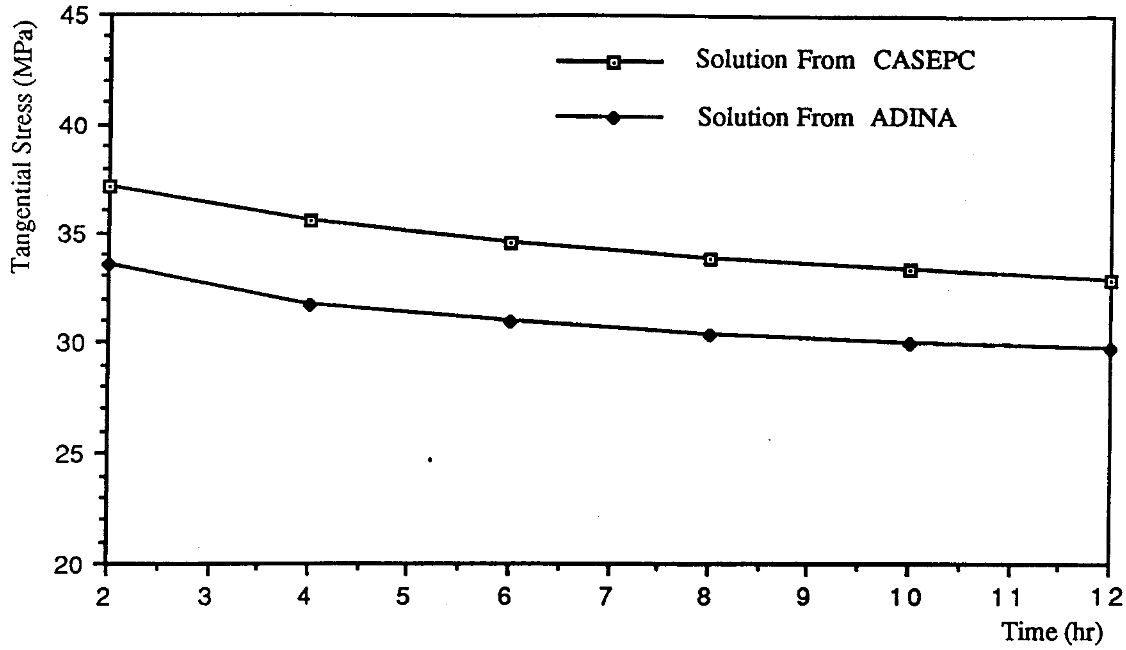

Fig. 6.15--Creep Solution Contrast: Tangential Stress (Oil Sand, 4.5"Casing, P=16.5MPa,T=350)

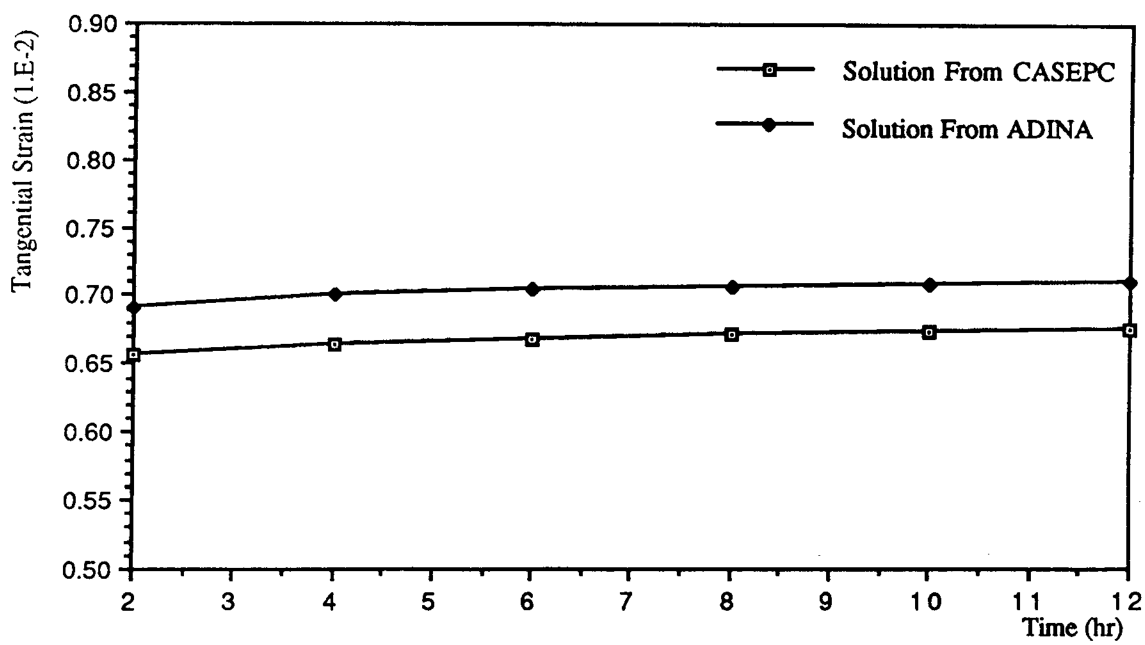

Fig. 6.16--Creep Solution Contrast: Tangential Strain (Oil Sand, 4.5"Casing, P=16.5MPa,T=350)

APPENDIX A

USER'S MANUAL

USER'S MANUAL FOR PROGRAM CASEPC

Program CASEPC is written in Fortran 77 language. The program uses an incremental isotropic elastic-plastic temperature dependent material model based on the Von-Mises yield condition to analyze a horizontal slice of the casing for which the vertical strain is assumed to be zero, and the supporting rock is assumed linear elastic. It can be used for both elastic-plastic loading and unloading to solve the temperature dependent and time dependent problems in which the material properties of the steel are temperature dependent. The current file name for the object deck is WEACC.

A1. LISTING OF SUBROUTINES

A1.1 Main Program

The main program calls eight subroutines: INPUT, WRINT, EESOL, EOUT1, EPSOL, EPOUT1, CRSOL, CROUT. The function of each is discussed in the following.

A1.2 Subroutine INPUT

Function: Reads the initial data, which includes the control parameters, casing size, properties of materials and loading history, which consists of increments of pressure, temperature and time.

A1.3 Block Data WDAT (See also Sect. A6)

Function: Prescribes initial data which includes the following:

XX1(2,9), XX2(2,9): data points for determining E and α of the casing as a function of temperature.

XO(6), YO(6): data points for one-dimensional interpolation of $\overset{\circ}{\sigma}_Y$ as a function of temperature.

XO(6), Y1(8), ZO(6,8,1): data points for two-dimensional interpolation of E^t as a function of temperature and time.

TM(6): temperatures for the calculation of the initial yield under different temperature.

A1.4 Subroutine WRINT

Function: Prints (i.e., echo checks) all initial data.

A1.5 Subroutine EESOL

Function: Calculates the pressure, stresses and strains at the onset of yield under different temperatures. To do so, some supplementary subroutines which perform operations, such as, curve fitting and interpolation, are used to determine the yield pressure and the elastic response.

Input: N, the number of temperatures at which the yield value is to be computed. In the program the maximum value of N is prescribed as 6.

Output: SC(N), SR(N), SZ(N) denote tangential, radial and axial stress respectively.

EC(N), ER(N), EZ(N) denote tangential, radial and axial strain respectively.

ED(N) and EE(N) are effective stress and strain respectively.

BY(N) are the yield values.

A1.6 Subroutine YEN

Function: Calculates the yield pressure at the onset of initial yield under different temperatures.

Input: N

Output: PY(N)

A1.7 Subroutine ECOEF

Function: Finds elastic response. The coefficients of (2.34) are computed in this subroutine.

A1.8 Subroutine EOUT1

Function: Outputs elastic stresses and strains at the onset of yield.

A1.9 Subroutine EPSOL

All arrays in this subroutine are adjustable. NNC = 50 is maximum upper limit.

Function: Finds the elastic-plastic solution for thermally dependent properties using Von-Mises condition.

Input: NN, the number of computing steps which, in turn, determines the number of increments of ΔP or ΔT. NN $\leq$ NNC

Output:
- stresses PSC(J), PSR(J), PSZ(J)
- total strains PEC(J), PER(J), PEZ(J)
- plastic strains ECP(J), ERP(J), EZP(J)
- effective stress PED(J)
- effective strain PEE(J)
- effective plastic strain EEP(J)

Here the number of J varies from 1 to NN.

A1.10 Subroutine ELASE

Function: Calculates elastic stresses and strains using (1.16) of this report. The subroutine ELASE is called by the subroutine EPSOL.

Input: Pressure P (or ΔP), temperature T (or ΔT) and material properties of casing E, α (ALPHA), $\sigma_Y°$ (BYO).

Output: Elastic response.

A1.11 Subroutine QUAD

Function: Calculates the real roots of second degree algebric equations. The subroutine is commonly used.

A1.12 Subroutine EDSTS

Function: Calculates effective stress.

A1.13 Subroutine EESTN

Function: Calculates effective strain.

A1.14 Subroutine PCOEF

Function: Presents the coefficients, as shown in (2.34) of this report, for computing plastic response.

A1.15 Subroutine PLASE

Function: Finds the increment of plastic response using (2.32) and (2.33) of this report. The subroutine PCOEF is called.

Input: Pressure increment ΔP, temperature increment ΔT, material properties E, α, $\sigma_Y°$, E^P.

Output: - Increment of stresses DPSC, DPSR, DPSZ (i.e., $\Delta\sigma_\theta$, $\Delta\sigma_r$, $\Delta\sigma_r$).

- Increment of strains DPEC, DPER, DPEZ (i.e., $\Delta\varepsilon_\theta$, $\Delta\varepsilon_r$, $\Delta\varepsilon_z$).

A1.16 Subroutine RCOEF

Function: Calculates the scale factor for elastic response.

A1.17 Subroutine FPOUT

Function: Outputs the elastic-plastic solution. In this subroutine some checking formulas, as shown in Eq. (2.48), are used.

Input: N1, N2, the integer numbers which control the range of load incements for the output.

Output:
- CTE(J) the temperature before current step.
- CDP(J), CDT(J) the increment of pressure and temperature in current step.
- BYE(J), EEE(J), ALE(J), EPE(J): material properties σ_Y, E, α, E^t for each step.
- RC(J), scale factor.
- SC1, SR, SZ, SC2 are check values for σ_θ, σ_v, σ_z. Others are all elastic-plastic response.
- Here the number of J varies from N1 to N2.

A1.18 Subroutine PSRAN

Function: Calculates plastic strains using (2.44).

A1.19 Subroutine LAG13

Function: One-dimensional Lagrangian interpolation for σ_Y^o. This subroutine is commonly used.

A1.20 Subroutine LAG23

Function: Two-dimensional Lagrangian interpolation for E^t.

A1.21 Subroutine FIT13

Function: Calculates the coefficients of fitting a curve with a 5th-degree polynomial.

Input: Data points of the fitting X(2,N).

Output:
- A(M1) the coefficents of the fitting polynomial
- M1 the order of fitting polynomial
- M1=M+1, M2=M1+1
- S(M1,M2) working array

A1.22 Subroutine GS

Function: Solves for the system of linear algebra equations using the direct method.

A1.23 Subroutine FF

Function: Fits a 5th-degree polynomial to a set of points.

A1.24 Subroutine CRSOL

All array sizes in this subroutine are adjustable. The integer NTC is maximum upper limit of the adjustable array. (NTC = 50)

Function: Solves the creep problem for thermally dependent properties with the Bailoy-Norton Law using iteration.

Input: NT, the number of computing steps which determines the number of increments of Δt, Δp and ΔT. NT $<$ NTC. The results at t=0, that come from subroutine EPSOL, are the initial point for creep computation.

Output: Total stresses, total strains, creep strains and creep strain rate for each time increment.

A1.25 Subroutine CROUT

Function: Outputs the final results for the creep problem.

Input: NT1, NT2, the integer which is to contorl the amount of output.

Output: Final results for creep.

A1.26 Subroutine CCDEF

Function: Determines the coefficients as shown in Eqs. (4.16) and (4.17) of this report for computing creep.

A1.27 Subroutine CSTRS

Function: Calculates the increments of total stress and strain within one time step.

A2. INPUT INSTRUCTIONS

Program CASEPC reads its primary input on unit 5 and prints the results on units 9 and 10. When the user runs the program the input will be read from an input file called INDATA which should be assigned to connect with unit 5.

The data are input through the following data cards (lines) and each card (line) must start with Column 1.

A2.1. Card 1 Format (*)

| LL, CDPM, CDTM |

LL: Integer. Control parameter indicating solution mode required.

EQ.0, Elastic-plastic solution only.

EQ.1, Includes elastic-plastic and creep behavior.

CDPM: If LL=0 put CDPM=0.0.

If LL=1 enter maximum pressure for the loading condition (in MPa).

CDTM: If LL=0 put CDTM=0.0.

If LL=1 enter maximum temperature for the loading condition.

A2.2 Card 2 Format (*)

| BM, CNEFO, CNBMF |

BM: Poisson ratio of casing

CNEFO: Modulus of the supporting rock

CNBMF: Poisson ratio of the supporting rock

A2.3 Card 3 Format (*)

| DI, DO |

DI: Internal diameter of casign (in meters)

DO: Outer diameter of casing (in meters)

A2.4 Card 4 Format (*)

| NN, N1, N2 |

NN: Integer number of increments of pressure and temperature for elastic-plastic solution. Since the upper limit of the adjustable array is 50, NN $\leq$ 50.

N1: Integer to indicate the load step at which to begin output of the results.

N2: Integer to indicate load step at which to end output of the results.

Notes: a. N1 $\leq$ N2 $\leq$ NN.

b. N1 and N2 control the output mode for solution results. If you don't want to print the elastic-plastic results, just put N1=N2=0. In that case the following message will appear on the screen:

"(NO OUTPUT FOR ELASTIC-PLASTIC SOLUTION. IF YOU WANT, PUT: N2 $\geq$ N1 > 0)".

A2.5 Card 5 Format (*)

| CDP(1), CDP(2), ... CDP(NN) |

CDP(J), (J=1,NN): Pressure increments (in MPa)

Notes: a. For LL=0, that is, for elastic-plastic solution only, the input CDP(J) is necessary.

If LL=1 skip this card. The program generates automatically all required pressure increments in the following way.

CDP(J) = CDPM/NN   J=1, NN.

b. The number of values specified must be equal to NN, when LL=0. If less than NN values are input, the unspecified fields should be set zero.

A2.6 Card 6   Format (*)

CDT(1), CDT(2), ... CDT(NN)

CDT(J), (J=1, NN): Temperature increments

Notes: a. For LL=0, that is for elastic-plastic solution only, the input CDT(J) is necessary.

If LL=1 skip the card. The program generates automatically all required temperature increments in the following way.

CDT(J) = CDTM/NN   J=1, NN

b. See the previous note b. for Card 5.

Note for cards 5 and 6:

A special strategy is designed in this program to guarantee that after passing the transition from elasticity to plasticity the output values of temperature and pressure still remain integers. However, to do so the increments of pressure and temperature should be taken in a definite way. Both must be either increasing or decreasing at the same time. However, one of them may be kept constant, but if one is increased and the other is decreased it will bring confusion. In fact, it is impossible for the latter case to arise in engineering practice. So the above restriction on the input of pressure and temperature increments is reasonable.

Example: If NN=30, the user might input the following

ΔP: 20 * 0.8E6, 10 * 0.0

ΔT: 15 * 20, 5 * 10, 10 * 0.0

or ΔP: 16.E6, 19 * 0.0, 10 * -0.8E6

ΔT: 20 * 20, 10 * 0.0

But what is shown below is not permissible

ΔP: 10 * 0.8E6, 10 * -0.8E6, 10 * 0.0

ΔT: 20 * 20, 10 * 0.0

A2.7 Card 7 **Format (*)** If LL=0, skip this card.

| NT, NT1, NT2 |

NT: Integer = the number of increments of the time step-length for creep computation. NT $\leq$ 50.

NT1: Integer = beginning point for output of the results.

NT2: Integer = ending point for output of the results.

Notes: a. NT1 $\leq$ NT2 $\leq$NT

b. NT1 and NT2 control the output mode for solution results. If you don't want to print the creep results, just put NT1=NT2=0. In this case the following message will appear on the screen:

"(NO OUTPUT FOR CREEP RESULTS. IF YOU WANT, PUT: NT2 $\geq$ NT1 > 0)".

A2.8 Card 8 **Format (*)** If LL=0, skip this card.

CTI(1), CTI(2), ... CT1(NT)

CTI(J), (J=1,NT): Time increments (in hours)

Note: It is important that the value specified for each time increment should be more than zero. A zero increment is absolutely forbidden. Otherwise the computation will fail to proceed and the following message will appear on the screen:

"TIME INCREMENT SHOULD BE GREATER ZERO, IMPORTANT!"

A2.9 Card 9 **Format (*)** If LL=0, skip this card.

CCDP(1), CCDP(2), ... CCDP(NT)

CCDP(J), (J=1,NT): Pressure increments within the correspondent time interval as t > 0.

A2.10 Card 10 **Format (*)** If LL=0, skip this card.

CCDT(1), CCDT(2), ... CCDT(NT)

CCDT(J), (J=1,NT): Temperature increments within the correspondent time interval as t > 0.

Note: For cards 9 and 10, the number of values specified must be equal to NT, when LL=1. If less than NT values are input, or if no increment occurs, the unspecified fields should be set to zero.

A2.11 Card 11 **Format (*)** If LL=0, skip this card.

| EPS, CO(1), CO(2), CO(3) |

EPS: Convergence criterion for iteration. In general, values between 1.E-4 and 1.E-7 are recommended.

CO(I), (I=1,2,3): Initial estimates of creep strain increments (i.e. ε_θ^{Co}, ε_r^{Co} and ε_z^{Co}, respectively). The CO(3) = ε_z^{Co} should be negative and the three estimated values equal to each other are strictly prohibited.

A2.12 Card 12 **Format (*)** If LL=0, skip this card.

| CAC, CMC, CNC |

CAC: Coefficient A in the creep law shown below.

CMC: Coefficient m in the creep law shown below.

CNC: Coefficient n in the creep law shown below.

The creep law employed is of the following form

$$\bar{\varepsilon}^c = A\,\bar{\sigma}^m\,t^n$$

A3. Output

The results of this program are printed on units 9 and 10. Unit 9 contains all main results for the elastic-plastic solution and creep predictions. Unit 10 includes the prediction of initial yield only. To obtain the results the user needs to create two files, for instance, RESULT1 and RESULT2, which are to be connected with unit 9 and unit 10, respectively.

A4. Operation

In MTS the user first compiles the program using the Fortran compiler (if needed):

```
$RUN *FORTRANVS SCARDS=WEACC SPUNCH=OBJ
```

Then the program is invoked with a $RUN command:

```
$RUN OBJ 5=INDATA 9=RESULT1 10=RESULT2 T=5S
```

Then the command LIST RESULT1 or LIST RESULT2 presents the results of the solution.

A5. Examples

Four examples of input and output forms are listed below. All examples assume a 4.5" casing, Poisson ratio = 0.28, and an oil sand geological formation under the loading conditions P = 16.5 MPa and T = 350°C.

Example Case 1

Elastic-plastic solution only.

<Input>

Card 1	LL, CDPM, CDTM :	0, 0.0, 0.0
Card 2	BM, CNEF, CNBMF :	0.28, 2200.E6, 0.25

Card 3 DI, DO : 0.1039, 0.1143

Card 4 NN, N1, N2 : 3.0, 14, 21

Card 5 CDP(I), I=1,30 : 1.6 * 1.E6, 14 * 0.5E6

Card 6 CDT(I), I=1,30 : 16 * 20.0, 30.0, 13 * 0.0

Card 7 - Card 12 are omitted.

<Output> RESULT1 : Elastic-plastic solution from Steps 14-21. No creep output.

RESULT2 : Prediction of initial yield.

Example Case 2

Elastic-plastic solution and creep prediction.

<Input>

Card 1 LL, CDPM, CDTM : 1, 16.5E6, 350.

Card 2 BM, CNEF, CNBMF : 0.28, 2200.E6, 0.25

Card 3 DI, DO : 0.1039, 0.1143

Card 4 NN, N1, N2 : 20, 18, 20

Card 5 Omitted

Card 6 Omitted

Card 7 NT, NT1, NT2 : 25, 1, 10

Card 8 CTI(I), I=1,25 : 2*1.0, 4*2.0, 10., 30., 7*50., 10*100.

Card 9 CCDP(I), I=1,25 : 25 * 0.0

Card 10 CCDT(I), I=1,25 : 25 * 0.0

Card 11 EPS, CO(I), I=1,3 : 1.E-7, 1.E-3, 5.E-4, -2.E-3

Card 12 CAC, CMC, CNC : 1.5E-32, 6.0, 0.667

<Output> RESULT1 : Elastic-plastic solution from Steps 18-20. Creep prediction from time Steps 1-10. (Maximum time interval is 1400 hours, no

increments of pressure and temperature as t>0).

RESULT2 : No change from Case 1.

Example Case 3

The solution is the same as the Case 2, except that no elastic-plastic results are printed.

In this case only Card 4 needs to be changed to the following.

Card 4	NN, N1, N2 :	20, 0, 0
<Output>	RESULT1 :	Only creep results are printed.
	RESULT2 :	No change.

Example Case 4

The same as Case 2 except that neither elastic-plastic nor creep solution is printed.

In this case Card 4 and Card 7 need to be changed.

Card 4	NN, N1, N2 :	20, 0, 0
Card 7	NT, NT1, NT2 :	25, 0, 0
<Output>	RESULT1 :	Only original input is printed.
	RESULT2 :	No change.

The output forms for the four cases listed above are shown sequentially on the following pages.

A6. TEMPERATURE DEPENDENT PROPERTIES

Note that temperature dependent properties for NM80 steel, as supplied by C-FER and displayed in Table 4.2, are contained in the BLOCK DATA subroutine.

Reference to the BLOCK DATA subroutine in the program listing of Appendix B shows how the data of Table 4.2 has been assigned to the arrays defined in Sect. A1.3. If different temperature dependent material property data is supplied, the data in the BLOCK DATA subroutine can be altered and the program recompiled.

<CASE1 OUTPUT>

```
                              CASEPC   RESULTS
                          **************************
                            ( FILE NAME : WEACC )
===========================================================================
  <INPUT DATA>
 ---------------
   OUTER DIAMETER     WALL THICKNESS    POISSON RATIO
      0.1143000          0.0052000         0.2800

   MODULUS OF ROCK    POISSON RATIO OF ROCK
      0.2200E+10         0.2500

 INCREMENT OF PRESSURE AT t=0:
 0.1000E+07  0.1000E+07  0.1000E+07  0.1000E+07  0.1000E+07  0.1000E+07
 0.1000E+07  0.1000E+07  0.1000E+07  0.1000E+07  0.1000E+07  0.1000E+07
 0.1000E+07  0.1000E+07  0.1000E+07  0.1000E+07  0.5000E+06  0.5000E+06
 0.5000E+06  0.5000E+06  0.5000E+06  0.5000E+06  0.5000E+06  0.5000E+06
 0.5000E+06  0.5000E+06  0.5000E+06  0.5000E+06  0.5000E+06  0.5000E+06

 INCREMENT OF TEMPERATURE AT t=0:
    20.0000     20.0000     20.0000     20.0000     20.0000     20.0000
    20.0000     20.0000     20.0000     20.0000     20.0000     20.0000
    20.0000     20.0000     20.0000     20.0000     30.0000      0.0000
     0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
     0.0000      0.0000      0.0000      0.0000      0.0000      0.0000

 INCREMENT OF TIME:
     0.0000      0.0000      0.0000      0.0000      0.0000      0.0000

 INCREMENT OF PRESSURE AS t>0:
 0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00

 INCREMENT OF TEMPERATURE AS t>0:
     0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
===========================================================================
 <PHYSICAL PROPERTIES OF THE CASING UNDER DIFFERENTE TEMPERATURE>:

POINT- 1:   0.00    BYO= 0.6220E+09    E= 0.2075E+12    ALFA= 0.1138E-04
POINT- 2:  50.00    BYO= 0.5937E+09    E= 0.1995E+12    ALFA= 0.1173E-04
POINT- 3: 100.00    BYO= 0.5659E+09    E= 0.1885E+12    ALFA= 0.1205E-04
POINT- 4: 150.00    BYO= 0.5352E+09    E= 0.1780E+12    ALFA= 0.1236E-04
POINT- 5: 200.00    BYO= 0.5027E+09    E= 0.1680E+12    ALFA= 0.1267E-04
POINT- 6: 250.00    BYO= 0.4683E+09    E= 0.1572E+12    ALFA= 0.1299E-04
===========================================================================
```

```
                    <ELASTIC-PLASTIC SOLUTION>
                    **********************************
================================================================================
 CT   = 280.00
 BY   =     0.4441E+09  E     =   0.1497E+12   ALFA   =   0.1318E-04 EP   =   0.3907E+10

   STEP ( 15 ):         DP    =   0.1000E+07   DT     =   20.00       FACTOR--RR= -.000

 PRES-R= 0.1500E+08   TEMP-T= 300.00                              SGMA--Y= 0.5068E+09
 SGMA-C= 0.4725E+08   SGMA-R=-0.1500E+08   SGMA-Z=-0.4878E+09   SGMA-BR= 0.5068E+09
 EPS--C= 0.5304E-02   EPS--R= 0.4693E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.3351E-02
 EPS-CP= 0.6086E-03   EPS-RP= 0.4362E-03   EPS-ZP=-0.1045E-02   EPS-BRP= 0.1050E-02
--------------------------------------------------------------------------------
    <CHECK THE NUMBER VALUE OF STRESS>:
 SGMC-1= 0.4725E+08   SGMR-1=-0.1500E+08   SGMZ-1=-0.4878E+09
 SGMC-2= 0.4725E+08
================================================================================
 CT   = 300.00
 BY   =     0.4305E+09  E     =   0.1444E+12   ALFA   =   0.1330E-04 EP   =   0.3864E+10

   STEP ( 16 ):         DP    =   0.1000E+07   DT     =   20.00       FACTOR--RR= -.000

 PRES-R= 0.1600E+08   TEMP-T= 320.00                              SGMA--Y= 0.5078E+09
 SGMA-C= 0.4883E+08   SGMA-R=-0.1600E+08   SGMA-Z=-0.4883E+09   SGMA-BR= 0.5078E+09
 EPS--C= 0.5739E-02   EPS--R= 0.5057E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.3620E-02
 EPS-CP= 0.7634E-03   EPS-RP= 0.5426E-03   EPS-ZP=-0.1306E-02   EPS-BRP= 0.1312E-02
--------------------------------------------------------------------------------
    <CHECK THE NUMBER VALUE OF STRESS>:
 SGMC-1= 0.4883E+08   SGMR-1=-0.1600E+08   SGMZ-1=-0.4883E+09
 SGMC-2= 0.4884E+08
================================================================================
 CT   = 320.00
 BY   =     0.4174E+09  E     =   0.1388E+12   ALFA   =   0.1342E-04 EP   =   0.3766E+10

   STEP ( 17 ):         DP    =   0.5000E+06   DT     =   30.00       FACTOR--RR= 0.000

 PRES-R= 0.1650E+08   TEMP-T= 350.00                              SGMA--Y= 0.5093E+09
 SGMA-C= 0.4236E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.4938E+09   SGMA-BR= 0.5093E+09
 EPS--C= 0.6332E-02   EPS--R= 0.5632E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4008E-02
 EPS-CP= 0.9882E-03   EPS-RP= 0.6948E-03   EPS-ZP=-0.1683E-02   EPS-BRP= 0.1692E-02
--------------------------------------------------------------------------------
    <CHECK THE NUMBER VALUE OF STRESS>:
 SGMC-1= 0.4236E+08   SGMR-1=-0.1650E+08   SGMZ-1=-0.4938E+09
 SGMC-2= 0.4236E+08
================================================================================
```

```
CT  = 350.00
BY  =    0.3986E+09  E    =  0.1304E+12   ALFA  =  0.1360E-04 EP  =  0.3519E+10

  STEP ( 18 ):       DP   =  0.5000E+06   DT    =   0.00      FACTOR--RR= 0.000

PRES-R= 0.1700E+08  TEMP-T= 350.00                          SGMA--Y= 0.5093E+09
SGMA-C= 0.4668E+08  SGMA-R=-0.1700E+08  SGMA-Z=-0.4915E+09  SGMA-BR= 0.5094E+09
EPS--C= 0.6367E-02  EPS--R= 0.5618E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4018E-02
EPS-CP= 0.9937E-03  EPS-RP= 0.6986E-03  EPS-ZP=-0.1692E-02  EPS-BRP= 0.1701E-02
-------------------------------------------------------------------------------
   <CHECK THE NUMBER VALUE OF STRESS>:
SGMC-1= 0.4668E+08  SGMR-1=-0.1700E+08  SGMZ-1=-0.4915E+09
SGMC-2= 0.4668E+08
===============================================================================
CT  = 350.00
BY  =    0.3986E+09  E    =  0.1304E+12   ALFA  =  0.1360E-04 EP  =  0.3519E+10

  STEP ( 19 ):       DP   =  0.5000E+06   DT    =   0.00      FACTOR--RR= -.000

PRES-R= 0.1750E+08  TEMP-T= 350.00                          SGMA--Y= 0.5093E+09
SGMA-C= 0.5100E+08  SGMA-R=-0.1750E+08  SGMA-Z=-0.4892E+09  SGMA-BR= 0.5094E+09
EPS--C= 0.6402E-02  EPS--R= 0.5604E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4028E-02
EPS-CP= 0.9995E-03  EPS-RP= 0.7026E-03  EPS-ZP=-0.1702E-02  EPS-BRP= 0.1711E-02
-------------------------------------------------------------------------------
   <CHECK THE NUMBER VALUE OF STRESS>:
SGMC-1= 0.5100E+08  SGMR-1=-0.1750E+08  SGMZ-1=-0.4892E+09
SGMC-2= 0.5100E+08
===============================================================================
CT  = 350.00
BY  =    0.3986E+09  E    =  0.1304E+12   ALFA  =  0.1360E-04 EP  =  0.3519E+10

  STEP ( 20 ):       DP   =  0.5000E+06   DT    =   0.00      FACTOR--RR= -.000

PRES-R= 0.1800E+08  TEMP-T= 350.00                          SGMA--Y= 0.5094E+09
SGMA-C= 0.5532E+08  SGMA-R=-0.1800E+08  SGMA-Z=-0.4869E+09  SGMA-BR= 0.5095E+09
EPS--C= 0.6437E-02  EPS--R= 0.5590E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4039E-02
EPS-CP= 0.1005E-02  EPS-RP= 0.7065E-03  EPS-ZP=-0.1712E-02  EPS-BRP= 0.1721E-02
-------------------------------------------------------------------------------
   <CHECK THE NUMBER VALUE OF STRESS>:
SGMC-1= 0.5532E+08  SGMR-1=-0.1800E+08  SGMZ-1=-0.4869E+09
SGMC-2= 0.5532E+08
===============================================================================
CT  = 350.00
BY  =    0.3986E+09  E    =  0.1304E+12   ALFA  =  0.1360E-04 EP  =  0.3519E+10

  STEP ( 21 ):       DP   =  0.5000E+06   DT    =   0.00      FACTOR--RR= -.000

PRES-R= 0.1850E+08  TEMP-T= 350.00                          SGMA--Y= 0.5094E+09
SGMA-C= 0.5963E+08  SGMA-R=-0.1850E+08  SGMA-Z=-0.4845E+09  SGMA-BR= 0.5095E+09
EPS--C= 0.6472E-02  EPS--R= 0.5575E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4049E-02
EPS-CP= 0.1012E-02  EPS-RP= 0.7105E-03  EPS-ZP=-0.1722E-02  EPS-BRP= 0.1731E-02
-------------------------------------------------------------------------------
   <CHECK THE NUMBER VALUE OF STRESS>:
SGMC-1= 0.5963E+08  SGMR-1=-0.1850E+08  SGMZ-1=-0.4845E+09
SGMC-2= 0.5963E+08
===============================================================================
  ( NO OUTPUT FOR CREEP SOLUTION. IF YOU WANT, PUT:   LL=1 )
```

```
C
C       <CASE2 OUTPUT>
C
C
                                  CASEPC   RESULTS
                              *************************
                                ( FILE NAME : WEACC )
=============================================================================
   <INPUT DATA>
  ---------------
     OUTER DIAMETER     WALL THICKNESS    POISSON RATIO
        0.1143000          0.0052000          0.2800

     MODULUS OF ROCK    POISSON RATIO OF ROCK
        0.2200E+10         0.2500

  INCREMENT OF PRESSURE AT t=0:
  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06
  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06
  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06
  0.8250E+06  0.8250E+06

  INCREMENT OF TEMPERATURE AT t=0:
     17.5000     17.5000     17.5000     17.5000     17.5000     17.5000
     17.5000     17.5000     17.5000     17.5000     17.5000     17.5000
     17.5000     17.5000     17.5000     17.5000     17.5000     17.5000
     17.5000     17.5000

  INCREMENT OF TIME:
      1.0000      1.0000      2.0000      2.0000      2.0000      2.0000
     10.0000     30.0000     50.0000     50.0000     50.0000     50.0000
     50.0000     50.0000     50.0000    100.0000    100.0000    100.0000
    100.0000    100.0000    100.0000    100.0000    100.0000    100.0000
    100.0000

  INCREMENT OF PRESSURE AS t>0:
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00

  INCREMENT OF TEMPERATURE AS t>0:
      0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
      0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
      0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
      0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
      0.0000
=============================================================================
 <PHYSICAL PROPERTIES OF THE CASING UNDER DIFFERENTE TEMPERATURE>:

POINT- 1:   0.00    BYO= 0.6220E+09    E= 0.2075E+12    ALFA= 0.1138E-04
POINT- 2:  50.00    BYO= 0.5937E+09    E= 0.1995E+12    ALFA= 0.1173E-04
POINT- 3: 100.00    BYO= 0.5659E+09    E= 0.1885E+12    ALFA= 0.1205E-04
POINT- 4: 150.00    BYO= 0.5352E+09    E= 0.1780E+12    ALFA= 0.1236E-04
POINT- 5: 200.00    BYO= 0.5027E+09    E= 0.1680E+12    ALFA= 0.1267E-04
POINT- 6: 250.00    BYO= 0.4683E+09    E= 0.1572E+12    ALFA= 0.1299E-04
=============================================================================
```

```
                         <ELASTIC-PLASTIC SOLUTION>
                      *********************************

================================================================================
 CT   = 297.50
 BY   =    0.4322E+09  E     =   0.1451E+12  ALFA   =   0.1329E-04 EP   =   0.3872E+10

   STEP ( 18 ):        DP    =   0.8250E+06  DT     =   17.50      FACTOR--RR= -.000

 PRES-R= 0.1485E+08  TEMP-T= 315.00                              SGMA--Y= 0.5009E+09
 SGMA-C= 0.4038E+08  SGMA-R=-0.1485E+08  SGMA-Z=-0.4859E+09  SGMA-BR= 0.5009E+09
 EPS--C= 0.5582E-02  EPS--R= 0.5005E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.3545E-02
 EPS-CP= 0.7248E-03  EPS-RP= 0.5390E-03  EPS-ZP=-0.1264E-02  EPS-BRP= 0.1268E-02
--------------------------------------------------------------------------------
    <CHECK THE NUMBER VALUE OF STRESS>:
 SGMC-1= 0.4038E+08  SGMR-1=-0.1485E+08  SGMZ-1=-0.4859E+09
 SGMC-2= 0.4038E+08
================================================================================
 CT   = 315.00
 BY   =    0.4206E+09  E     =   0.1402E+12  ALFA   =   0.1339E-04 EP   =   0.3796E+10

   STEP ( 19 ):        DP    =   0.8250E+06  DT     =   17.50      FACTOR--RR= -.000

 PRES-R= 0.1567E+08  TEMP-T= 332.50                              SGMA--Y= 0.5018E+09
 SGMA-C= 0.4131E+08  SGMA-R=-0.1567E+08  SGMA-Z=-0.4866E+09  SGMA-BR= 0.5018E+09
 EPS--C= 0.5960E-02  EPS--R= 0.5329E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.3781E-02
 EPS-CP= 0.8585E-03  EPS-RP= 0.6346E-03  EPS-ZP=-0.1493E-02  EPS-BRP= 0.1499E-02
--------------------------------------------------------------------------------
    <CHECK THE NUMBER VALUE OF STRESS>:
 SGMC-1= 0.4131E+08  SGMR-1=-0.1567E+08  SGMZ-1=-0.4866E+09
 SGMC-2= 0.4131E+08
================================================================================
 CT   = 332.50
 BY   =    0.4094E+09  E     =   0.1352E+12  ALFA   =   0.1350E-04 EP   =   0.3678E+10

   STEP ( 20 ):        DP    =   0.8250E+06  DT     =   17.50      FACTOR--RR= -.000

 PRES-R= 0.1650E+08  TEMP-T= 350.00                              SGMA--Y= 0.5027E+09
 SGMA-C= 0.4218E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.4873E+09  SGMA-BR= 0.5027E+09
 EPS--C= 0.6341E-02  EPS--R= 0.5654E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4018E-02
 EPS-CP= 0.9937E-03  EPS-RP= 0.7302E-03  EPS-ZP=-0.1724E-02  EPS-BRP= 0.1731E-02
--------------------------------------------------------------------------------
    <CHECK THE NUMBER VALUE OF STRESS>:
 SGMC-1= 0.4218E+08  SGMR-1=-0.1650E+08  SGMZ-1=-0.4873E+09
 SGMC-2= 0.4218E+08
================================================================================
```

<CREEP-TIME DEPENDENT SOLUTION>
****

```
==============================================================================

 <ORIGINAL STATE>:  INITIAL TIME t= 0.0
                    PRES-R= 0.1650E+08  TEMP-T= 350.00
 SGMA-C= 0.4218E+08 SGMA-R=-0.1650E+08  SGMA-Z=-0.4873E+09  SGMA-BR= 0.5027E+09
 EPS--C= 0.6341E-02 EPS--R= 0.5654E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4018E-02
==============================================================================

 STEP( 1 ):  CUMULATIVE TIME=    1.00  C-RATE= 0.5615E-03  CGMA=BR= 0.4266E+09

 SGMA-C= 0.3875E+08 SGMA-R=-0.1650E+08  SGMA-Z=-0.4128E+09  SGMA-BR= 0.4266E+09
 EPS--C= 0.6495E-02 EPS--R= 0.5732E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4099E-02
 C-EPSC= 0.3335E-03 C-EPSR= 0.2244E-03  C-EPSZ=-0.5579E-03  C-EPSBR= 0.5615E-03
------------------------------------------------------------------------------

 STEP( 2 ):  CUMULATIVE TIME=    2.00  C-RATE= 0.2640E-03  CGMA=BR= 0.3910E+09

 SGMA-C= 0.3711E+08 SGMA-R=-0.1650E+08  SGMA-Z=-0.3778E+09  SGMA-BR= 0.3909E+09
 EPS--C= 0.6569E-02 EPS--R= 0.5766E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4138E-02
 C-EPSC= 0.4917E-03 C-EPSR= 0.3283E-03  C-EPSZ=-0.8200E-03  C-EPSBR= 0.8254E-03
------------------------------------------------------------------------------

 STEP( 3 ):  CUMULATIVE TIME=    4.00  C-RATE= 0.1230E-03  CGMA=BR= 0.3577E+09

 SGMA-C= 0.3555E+08 SGMA-R=-0.1650E+08  SGMA-Z=-0.3452E+09  SGMA-BR= 0.3576E+09
 EPS--C= 0.6639E-02 EPS--R= 0.5797E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4174E-02
 C-EPSC= 0.6405E-03 C-EPSR= 0.4234E-03  C-EPSZ=-0.1064E-02  C-EPSBR= 0.1071E-02
------------------------------------------------------------------------------

 STEP( 4 ):  CUMULATIVE TIME=    6.00  C-RATE= 0.7559E-04  CGMA=BR= 0.3374E+09

 SGMA-C= 0.3458E+08 SGMA-R=-0.1650E+08  SGMA-Z=-0.3252E+09  SGMA-BR= 0.3372E+09
 EPS--C= 0.6682E-02 EPS--R= 0.5816E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4196E-02
 C-EPSC= 0.7325E-03 C-EPSR= 0.4811E-03  C-EPSZ=-0.1214E-02  C-EPSBR= 0.1222E-02
------------------------------------------------------------------------------

 STEP( 5 ):  CUMULATIVE TIME=    8.00  C-RATE= 0.5302E-04  CGMA=BR= 0.3231E+09

 SGMA-C= 0.3390E+08 SGMA-R=-0.1650E+08  SGMA-Z=-0.3112E+09  SGMA-BR= 0.3229E+09
 EPS--C= 0.6713E-02 EPS--R= 0.5828E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4211E-02
 C-EPSC= 0.7974E-03 C-EPSR= 0.5212E-03  C-EPSZ=-0.1319E-02  C-EPSBR= 0.1328E-02
------------------------------------------------------------------------------

 STEP( 6 ):  CUMULATIVE TIME=   10.00  C-RATE= 0.4020E-04  CGMA=BR= 0.3124E+09

 SGMA-C= 0.3337E+08 SGMA-R=-0.1650E+08  SGMA-Z=-0.3006E+09  SGMA-BR= 0.3120E+09
 EPS--C= 0.6737E-02 EPS--R= 0.5838E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4223E-02
 C-EPSC= 0.8468E-03 C-EPSR= 0.5514E-03  C-EPSZ=-0.1398E-02  C-EPSBR= 0.1409E-02
------------------------------------------------------------------------------
```

```
 STEP( 7 ):  CUMULATIVE TIME=    20.00  C-RATE= 0.1894E-04  CGMA=BR= 0.2864E+09

SGMA-C= 0.3211E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2756E+09  SGMA-BR= 0.2865E+09
EPS--C= 0.6793E-02  EPS--R= 0.5858E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4252E-02
C-EPSC= 0.9647E-03  C-EPSR= 0.6210E-03  C-EPSZ=-0.1586E-02  C-EPSBR= 0.1598E-02
--------------------------------------------------------------------------------

 STEP( 8 ):  CUMULATIVE TIME=    50.00  C-RATE= 0.7291E-05  CGMA=BR= 0.2570E+09

SGMA-C= 0.3061E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2468E+09  SGMA-BR= 0.2571E+09
EPS--C= 0.6861E-02  EPS--R= 0.5879E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4284E-02
C-EPSC= 0.1103E-02  C-EPSR= 0.6989E-03  C-EPSZ=-0.1802E-02  C-EPSBR= 0.1817E-02
--------------------------------------------------------------------------------

 STEP( 9 ):  CUMULATIVE TIME=   100.00  C-RATE= 0.3359E-05  CGMA=BR= 0.2347E+09

SGMA-C= 0.2943E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2247E+09  SGMA-BR= 0.2346E+09
EPS--C= 0.6914E-02  EPS--R= 0.5894E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4310E-02
C-EPSC= 0.1210E-02  C-EPSR= 0.7570E-03  C-EPSZ=-0.1967E-02  C-EPSBR= 0.1984E-02
--------------------------------------------------------------------------------

 STEP( 10 ): CUMULATIVE TIME=   150.00  C-RATE= 0.2051E-05  CGMA=BR= 0.2211E+09

SGMA-C= 0.2869E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2113E+09  SGMA-BR= 0.2209E+09
EPS--C= 0.6947E-02  EPS--R= 0.5903E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4325E-02
C-EPSC= 0.1276E-02  C-EPSR= 0.7917E-03  C-EPSZ=-0.2068E-02  C-EPSBR= 0.2087E-02
--------------------------------------------------------------------------------
```

```
C
C
C        <CASE3 OUTPUT>
C
                                 CASEPC   RESULTS
                            **************************
                              ( FILE NAME : WEACC )
===========================================================================
    <INPUT DATA>
   ---------------
      OUTER DIAMETER     WALL THICKNESS    POISSON RATIO
         0.1143000         0.0052000          0.2800

      MODULUS OF ROCK    POISSON RATIO OF ROCK
         0.2200E+10         0.2500

   INCREMENT OF PRESSURE AT t=0:
   0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06
   0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06
   0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06  0.8250E+06
   0.8250E+06  0.8250E+06

   INCREMENT OF TEMPERATURE AT t=0:
      17.5000     17.5000     17.5000     17.5000     17.5000     17.5000
      17.5000     17.5000     17.5000     17.5000     17.5000     17.5000
      17.5000     17.5000     17.5000     17.5000     17.5000     17.5000
      17.5000     17.5000

   INCREMENT OF TIME:
       1.0000      1.0000      2.0000      2.0000      2.0000      2.0000
      10.0000     30.0000     50.0000     50.0000     50.0000     50.0000
      50.0000     50.0000     50.0000    100.0000    100.0000    100.0000
     100.0000    100.0000    100.0000    100.0000    100.0000    100.0000
     100.0000

   INCREMENT OF PRESSURE AS t>0:
   0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
   0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
   0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
   0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
   0.0000E+00

   INCREMENT OF TEMPERATURE AS t>0:
       0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
       0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
       0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
       0.0000      0.0000      0.0000      0.0000      0.0000      0.0000
       0.0000
===========================================================================
  <PHYSICAL PROPERTIES OF THE CASING UNDER DIFFERENTE TEMPERATURE>:

 POINT- 1:   0.00    BYO= 0.6220E+09    E= 0.2075E+12    ALFA= 0.1138E-04
 POINT- 2:  50.00    BYO= 0.5937E+09    E= 0.1995E+12    ALFA= 0.1173E-04
 POINT- 3: 100.00    BYO= 0.5659E+09    E= 0.1885E+12    ALFA= 0.1205E-04
 POINT- 4: 150.00    BYO= 0.5352E+09    E= 0.1780E+12    ALFA= 0.1236E-04
 POINT- 5: 200.00    BYO= 0.5027E+09    E= 0.1680E+12    ALFA= 0.1267E-04
 POINT- 6: 250.00    BYO= 0.4683E+09    E= 0.1572E+12    ALFA= 0.1299E-04
===========================================================================
  ( NO OUTPUT FOR ELASTIC-PLASTIC SOLUTION. IF YOU WANT, PUT: N2 > N1 > 0 )
```

<CREEP-TIME DEPENDENT SOLUTION>
****

```
===============================================================================

 <ORIGINAL STATE>:   INITIAL TIME t= 0.0
                     PRES-R= 0.1650E+08   TEMP-T= 350.00
SGMA-C= 0.4218E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.4873E+09   SGMA-BR= 0.5027E+09
EPS--C= 0.6341E-02   EPS--R= 0.5654E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4018E-02
===============================================================================

 STEP( 1 ):  CUMULATIVE TIME=     1.00   C-RATE= 0.5615E-03   CGMA=BR= 0.4266E+09

SGMA-C= 0.3875E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.4128E+09   SGMA-BR= 0.4266E+09
EPS--C= 0.6495E-02   EPS--R= 0.5732E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4099E-02
C-EPSC= 0.3335E-03   C-EPSR= 0.2244E-03   C-EPSZ=-0.5579E-03   C-EPSBR= 0.5615E-03
-------------------------------------------------------------------------------

 STEP( 2 ):  CUMULATIVE TIME=     2.00   C-RATE= 0.2640E-03   CGMA=BR= 0.3910E+09

SGMA-C= 0.3711E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.3778E+09   SGMA-BR= 0.3909E+09
EPS--C= 0.6569E-02   EPS--R= 0.5766E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4138E-02
C-EPSC= 0.4917E-03   C-EPSR= 0.3283E-03   C-EPSZ=-0.8200E-03   C-EPSBR= 0.8254E-03
-------------------------------------------------------------------------------

 STEP( 3 ):  CUMULATIVE TIME=     4.00   C-RATE= 0.1230E-03   CGMA=BR= 0.3577E+09

SGMA-C= 0.3555E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.3452E+09   SGMA-BR= 0.3576E+09
EPS--C= 0.6639E-02   EPS--R= 0.5797E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4174E-02
C-EPSC= 0.6405E-03   C-EPSR= 0.4234E-03   C-EPSZ=-0.1064E-02   C-EPSBR= 0.1071E-02
-------------------------------------------------------------------------------

 STEP( 4 ):  CUMULATIVE TIME=     6.00   C-RATE= 0.7559E-04   CGMA=BR= 0.3374E+09

SGMA-C= 0.3458E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.3252E+09   SGMA-BR= 0.3372E+09
EPS--C= 0.6682E-02   EPS--R= 0.5816E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4196E-02
C-EPSC= 0.7325E-03   C-EPSR= 0.4811E-03   C-EPSZ=-0.1214E-02   C-EPSBR= 0.1222E-02
-------------------------------------------------------------------------------

 STEP( 5 ):  CUMULATIVE TIME=     8.00   C-RATE= 0.5302E-04   CGMA=BR= 0.3231E+09

SGMA-C= 0.3390E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.3112E+09   SGMA-BR= 0.3229E+09
EPS--C= 0.6713E-02   EPS--R= 0.5828E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4211E-02
C-EPSC= 0.7974E-03   C-EPSR= 0.5212E-03   C-EPSZ=-0.1319E-02   C-EPSBR= 0.1328E-02
-------------------------------------------------------------------------------

 STEP( 6 ):  CUMULATIVE TIME=    10.00   C-RATE= 0.4020E-04   CGMA=BR= 0.3124E+09

SGMA-C= 0.3337E+08   SGMA-R=-0.1650E+08   SGMA-Z=-0.3006E+09   SGMA-BR= 0.3120E+09
EPS--C= 0.6737E-02   EPS--R= 0.5838E-02   EPS--Z= 0.0000E+00   EPS--BR= 0.4223E-02
C-EPSC= 0.8468E-03   C-EPSR= 0.5514E-03   C-EPSZ=-0.1398E-02   C-EPSBR= 0.1409E-02
-------------------------------------------------------------------------------
```

```
 STEP(  7 ):  CUMULATIVE TIME=    20.00  C-RATE= 0.1894E-04  CGMA=BR= 0.2864E+09

SGMA-C= 0.3211E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2756E+09  SGMA-BR= 0.2865E+09
EPS--C= 0.6793E-02  EPS--R= 0.5858E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4252E-02
C-EPSC= 0.9647E-03  C-EPSR= 0.6210E-03  C-EPSZ=-0.1586E-02  C-EPSBR= 0.1598E-02
--------------------------------------------------------------------------------

 STEP(  8 ):  CUMULATIVE TIME=    50.00  C-RATE= 0.7291E-05  CGMA=BR= 0.2570E+09

SGMA-C= 0.3061E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2468E+09  SGMA-BR= 0.2571E+09
EPS--C= 0.6861E-02  EPS--R= 0.5879E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4284E-02
C-EPSC= 0.1103E-02  C-EPSR= 0.6989E-03  C-EPSZ=-0.1802E-02  C-EPSBR= 0.1817E-02
--------------------------------------------------------------------------------

 STEP(  9 ):  CUMULATIVE TIME=   100.00  C-RATE= 0.3359E-05  CGMA=BR= 0.2347E+09

SGMA-C= 0.2943E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2247E+09  SGMA-BR= 0.2346E+09
EPS--C= 0.6914E-02  EPS--R= 0.5894E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4310E-02
C-EPSC= 0.1210E-02  C-EPSR= 0.7570E-03  C-EPSZ=-0.1967E-02  C-EPSBR= 0.1984E-02
--------------------------------------------------------------------------------

 STEP( 10 ):  CUMULATIVE TIME=   150.00  C-RATE= 0.2051E-05  CGMA=BR= 0.2211E+09

SGMA-C= 0.2869E+08  SGMA-R=-0.1650E+08  SGMA-Z=-0.2113E+09  SGMA-BR= 0.2209E+09
EPS--C= 0.6947E-02  EPS--R= 0.5903E-02  EPS--Z= 0.0000E+00  EPS--BR= 0.4325E-02
C-EPSC= 0.1276E-02  C-EPSR= 0.7917E-03  C-EPSZ=-0.2068E-02  C-EPSBR= 0.2087E-02
--------------------------------------------------------------------------------
```

<CASE4 OUTPUT>

CASEPC RESULTS
****
(FILE NAME : WEACC)

<INPUT DATA>

OUTER DIAMETER	WALL THICKNESS	POISSON RATIO
0.1143000	0.0052000	0.2800

MODULUS OF ROCK	POISSON RATIO OF ROCK
0.2200E+10	0.2500

INCREMENT OF PRESSURE AT t=0:

0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06
0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06
0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06	0.8250E+06
0.8250E+06	0.8250E+06				

INCREMENT OF TEMPERATURE AT t=0:

17.5000	17.5000	17.5000	17.5000	17.5000	17.5000
17.5000	17.5000	17.5000	17.5000	17.5000	17.5000
17.5000	17.5000	17.5000	17.5000	17.5000	17.5000
17.5000	17.5000				

INCREMENT OF TIME:

1.0000	1.0000	2.0000	2.0000	2.0000	2.0000
10.0000	30.0000	50.0000	50.0000	50.0000	50.0000
50.0000	50.0000	50.0000	100.0000	100.0000	100.0000
100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
100.0000					

INCREMENT OF PRESSURE AS t>0:

0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00					

INCREMENT OF TEMPERATURE AS t>0:

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000					

<PHYSICAL PROPERTIES OF THE CASING UNDER DIFFERENTE TEMPERATURE>:

POINT- 1:	0.00	BYO= 0.6220E+09	E= 0.2075E+12	ALFA= 0.1138E-04
POINT- 2:	50.00	BYO= 0.5937E+09	E= 0.1995E+12	ALFA= 0.1173E-04
POINT- 3:	100.00	BYO= 0.5659E+09	E= 0.1885E+12	ALFA= 0.1205E-04
POINT- 4:	150.00	BYO= 0.5352E+09	E= 0.1780E+12	ALFA= 0.1236E-04
POINT- 5:	200.00	BYO= 0.5027E+09	E= 0.1680E+12	ALFA= 0.1267E-04
POINT- 6:	250.00	BYO= 0.4683E+09	E= 0.1572E+12	ALFA= 0.1299E-04

(NO OUTPUT FOR ELASTIC-PLASTIC SOLUTION. IF YOU WANT, PUT: N2 > N1 > 0)
(NO OUTPUT FOR CREEP RESULTS. IF YOU WANT, PUT: NT2 > NT1 > 0)

```
C
C
C
C       <OUTPUT RESULT2>
C
C
C
C
```

<YIELD PRESSURE UNDER DIFFERENT TEMPERATURES>

POINT	TEMPERATURE	YIELD PRESSURE
1	0.00	0.6920E+08
2	50.00	0.6374E+08
3	100.00	0.5602E+08
4	150.00	0.4498E+08
5	200.00	0.2829E+08
6	250.00	-0.4219E+07

YIELD PRESSURE AND INITIAL STRESS AND STRAIN

POINT	SGMA-C	SGMA-R	SGMA-Z	SGMA-BR	SGMA-Y
1	0.6345E+09	-0.6920E+08	0.1583E+09	0.6220E+09	0.6220E+09
2	0.5691E+09	-0.6374E+08	0.2458E+08	0.5937E+09	0.5937E+09
3	0.4822E+09	-0.5602E+08	-0.1077E+09	0.5659E+09	0.5659E+09
4	0.3651E+09	-0.4498E+08	-0.2403E+09	0.5352E+09	0.5352E+09
5	0.1977E+09	-0.2829E+08	-0.3783E+09	0.5027E+09	0.5027E+09
6	-0.1099E+09	0.4219E+07	-0.5399E+09	0.4970E+09	0.4683E+09

POINT	EPS-C	EPS-R	EPS-Z	EPS-BR
1	0.2938E-02	-0.1403E-02	0.0000E+00	0.2558E-02
2	0.3495E-02	-0.5667E-03	0.0000E+00	0.2540E-02
3	0.4006E-02	0.3512E-03	0.0000E+00	0.2561E-02
4	0.4354E-02	0.1405E-02	0.0000E+00	0.2566E-02
5	0.4389E-02	0.2667E-02	0.0000E+00	0.2553E-02
6	0.3502E-02	0.4431E-02	0.0000E+00	0.2698E-02

APPENDIX B

PROGRAM LISTING FOR CASEPC

```
C
C
C
C
C                                      CASEPC
C                                *******************
C                                (FILE NAME: WEACC)
C
C  =====================================================================
C
C     THIS PROPRAM IS DESIGNED TO MAKE  THE CREEP PREDICTION  AS WELL AS
C     TO FIND OUT THE ELASTIC-PLASTIC SOLUTION WITH  THERMOPLASTIC MATER
C     -IALS FOR CASING PIPE IN THE  OIL-WELL BOTTOM OF CANADA THERMAL RE
C     -COVERY WELL.
C     IT IS BASED ON  VON-MISES  CONDITION USING  A SIMPLIFIED MODEL AND
C     THE  BAILOY-NORTON LAW IS ADOPTED  AS THE CURRENT CREEP LAW IN THE
C     PROGRAM,SO THIS PROGRAM CAN BE VERY EASILY EMPLOYED TO PRESENT THE
C     CREEP STRAINS AND ELASTIC -PLASTIC RESPONS OF BOTH LOADING AND UNL
C     -OADING.
C
C                                          BY    S.P.WEN  AND   D.W.MURRAY
C
C                                          DEPARTMENT OF MATHEMATICS
C                                          DEPARTMENT OF CIVIL ENGINEERING
C                                          UNIVERSITY OF ALBERTA, EDMONTON
C
C                                                            DECEMBER 1989
C
C  =====================================================================
C
C               < NOTATIONS IN THE PROGRAM >
C
C      PSC AND SC:  TANGENTIAL STRESS
C      PSR AND SR:  RADIAL STRESS
C      PSZ AND SZ:  AXIAL STRESS
C      PED AND ED:  EFFECTIVE STRESS
C      PEC AND EC:  TANGENTIAL STRAIN
C      PER AND ER:  RADIAL STRAIN
C      PEZ AND EZ:  AXIAL STRAIN
C      PEE AND EE:  EFFECTIVE STRAIN
C      BYO AND BY:  YIELD LIMITATION OF CASING UNDER DIFFERENT TEMPERATURE
C
C      PBY:   YIELD SURFACE
C      ECP:   PLASTIC OR CREEP TENGENTIAL STRAIN
C      ERP:   PLASTIC OR CREEP RADIAL STRAIN
C      EZP:   PLASTIC OR CREEP AXIAL STRAIN
C      EEP:   PLASTIC OR CREEP EFFECTIVE STRAIN
C
C      TM:    TEMPERATURE
C      CT:    CUMULATIVE TEMPERATURE
C      CTE:   CUMULATIVE TEMPERATURE
C      CDP:   INCREMENT OF PRESSURE AT t=0
C      CDT:   INCREMENT OF TEMPERATURE AT t=0
C      CCT:   CUMULATIVE TIME
C      CCTE:  CUMULATIVE TIME
C      CCDP:  INCREMENT OF PRESSURE AS t>0
C      CCDT:  INCREMENT OF TEMPERATURE AS t>0
```

```
C
C
C
C
C      CNEFO: MODULUS OF ROCK
C      CNBMF: POSSION RATIO OF ROCK
C      ALFA:  THERMAL EXPANSION COEFFICINT OF CASING
C      E:     ELASTIC MODULUS OF CASING
C      EP:    PLASATIC MODULUS OF CASING
C      BM:    POSSION RATIO OF CASING
C      DI:    INTERNAL DIAMETER OF CASING
C      DO:    OUTER DIAMETER OF CASING
C      RR:    SCALE FACTOR TO DTTERMINE THE ELASTIC PART OF THE STRESS
C
C      CO:    INITIAL ESTIMATES FOR ITERATION
C      EPS:   ERROR CRITERION
C      CAC:   THE COEFFICIENT (A) IN CEEP LAW
C      CMC:   THE COEFFICIENT (M) IN CEEP LAW
C      CNC:   THE COEFFICIENT (N) IN CEEP LAW
C      CDPM:  MAXIMUM WORKING PRESSURE
C      CDTM:  MAXIMUM WORKING TEMPERATURE
C      LL:    CONTROL PARAMETER TO DETERMINE WHETHER TO CALCULATE ELAST
C             IC-PLASTIC PROBLEM ONLY OR TO COMPUTE CREEP TOGETHER
C
C ======================================================================
```

```
C
C
C
C             ( 1 ) ---- MAIN PROGRAM
C
      PROGRAM MAIN
C
      DIMENSION PSZ(50),PED(50),PEC(50),PER(50),PEZ(50),PEE(50),BY(10)
      DIMENSION ECP(50),ERP(50),EZP(50),EEP(50),BYE(50),EEE(50),ED(10)
      DIMENSION SC(10),SR(10),SZ(10),EC(10),ER(10),EZ(10),EE(10),CO(3)
      DIMENSION ALE(50),EPE(50),CTE(50),CCTE(50),CCKE(50),PPED(50)
      DIMENSION PPY(50),PTM(50),PBY(50),RC(50),PSC(50),PSR(50)
      DIMENSION CDP(50),CDT(50),CCDP(50),CCDT(50),CTI(50)
      COMMON/CN1/NN,NT,N1,N2,NT1,NT2/CN2/LL/A7/CO
C
C         ****        ****        ****        ****        ****
C
C   **** INPUT DATA
      CALL INPUT(CDP,CDT,CTI,CCDP,CCDT)
C
C   **** PRINT THE ORIGINAL DATA
      CALL WRINT(50,NN,CDP,CDT,CCDP,CCDT,40,NT,CTI)
C
C   **** CALCULATE THE PRESSURE ,STRESS AND STRAIN ON THE YIELDT LIMIT-
C          ATION UNDER DIFFERENT TEMPERATURE
      CALL EESOL(6,SC,SR,SZ,EC,ER,EZ,ED,EE,BY)
C
C   **** OUTPUT OF THE STRESSES AND THE STRAINS AT THE PLASTIC ONSET ****
      CALL EOUT1(6,SC,SR,SZ,EC,ER,EZ,ED,EE,BY)
C
C   **** FIND OUT THE ELASTIC-PLASTIC SOLUTION WITH THERMOPLASTIC MATER-
C          IALS USING VON-MISES CONDITION
      CALL EPSOL(50,NN,CDP,CDT,CTE,PPY,PTM,PBY,RC,PSC,PSR,PSZ,PED,PEC,PE
     $R,PEZ,PEE,ECP,ERP,EZP,EEP,BYE,EEE,ALE,EPE)
C
C   **** OUTPUT OF THE ELASTIC-PLASTIC SOLUTION
      CALL EPOUT(50,NN,CDP,CDT,CTE,N1,N2,PPY,PTM,PBY,RC,PSC,PSR,PSZ,PED,
     $PEC,PER,PEZ,PEE,ECP,ERP,EZP,EEP,BYE,EEE,ALE,EPE)
C
      IF(LL.GT.0) THEN
C   **** SOLVE THE CREEP PROBLEM
      CALL CRSOL(50,NT,CCDP,CCDT,CTI,CCTE,PSC,PSR,PSZ,PED,PEC,PER,PEZ,PE
     $E,ECP,ERP,EZP,EEP,CCKE,PPED)
C
C   **** OUTPUT OF THE RESULTS OF TIME DEPENDENT PROBLEM
      CALL CROUT(50,NT,CCTE,NT1,NT2,PSC,PSR,PSZ,PEC,PER,PEZ,ECP,ERP,EZP,
     $PED,PEE,EEP,CCKE,PPED)
         ELSE
      WRITE(6,*) '   ( NO OUTPUT FOR CREEP SOLUTION. IF YOU WANT, PUT:
     $LL=1 )'
      WRITE(9,*) '   ( NO OUTPUT FOR CREEP SOLUTION. IF YOU WANT, PUT:
     $LL=1 )'
         END IF
C
      STOP 'THE END OF THE JOB'
      END
C   ---------------------------------------------------------------------
```

```
C
C
C
C           ( 2 ) ---- INPUT DATA
C
      SUBROUTINE INPUT(CDP,CDT,CTI,CCDP,CCDT)
C
      DIMENSION CDP(50),CDT(50),CTI(50),CCDP(50),CCDT(50),CO(3)
      COMMON/C1/CNEF,CNBMF,CNR,CNT/D1/BM/C3/DO,CNEFO/C4/EPS
      COMMON/CN1/NN,NT,N1,N2,NT1,NT2/C2/CAC,CMC,CNC/CN2/LL/A7/CO
C
C         ****         ****         ****         ****         ****
C
C  **** PRESENT SOME INITIAL DATA ****
C
C  ENTER CONTROL PARAMETERS
C                             LL:     LL=0, ONLY ELASTIC-PLASTIC SOLUTION
C                                     LL=1, INCLUDE CREEP PREDICTION
C                             CDPM:   =MAXIMUM PRESSURE WHEN LL=1
C                                     =0.0  WHEN LL=0
C                             CDTM:   =MAXIMUM TEMPERATURE WHEN LL=1
C                                     =0.0  WHEN LL=0
      READ(5,*) LL,CDPM,CDTM
C
C  ENTER THE ORIGINAL PARAMETERS
C                             BM:     POISSON RATIO OF CASING
C                             CNEFO: MODULUS OF THE SUPPORTING ROCK
C                             CNBMF: POISSON RATIO OF THE SUPPORTING ROCK
      READ(5,*) BM,CNEFO,CNBMF
C
C  ENTER THE CASING SIZE
C                             DI: INTERNAL DIAMETER OF CASING
C                             DO: OUTER DIAMETER OF CASING
      READ(5,*) DI,DO
C
      IF(LL.LT.1) THEN
C  ENTER THE SOME INTEGER NUMBERS
C                             NN: THE NUMBER OF  INCREMENTS OF PRESSURE
C                                 AND TEMPERATURE
C                             N1: BIGINING POINT OF THE RESULT OUTPUT
C                             N2: ENDING POINT OF THE RESULT OUTPUT
      READ(5,*) NN,N1,N2
C
C  ENTER THE PRESSURE INCREMENT
      READ(5,*) (CDP(I),I=1,NN)
C
C  ENTER THE TEMPERATURE INCREMENT
      READ(5,*) (CDT(I),I=1,NN)
         ELSE
      READ(5,*) NN,N1,N2
            PN=CDPM/NN
            TN=CDTM/NN
          DO 2 I=1,NN
            CDP(I)=PN
            CDT(I)=TN
  2       CONTINUE
       END IF
```

```
C
C
C
C
      IF(LL.LT.1) THEN
         NT=6
         DO 4 I=1,NT
         CTI(I)=0.0
         CCDP(I)=0.0
         CCDT(I)=0.0
  4   CONTINUE
         GO TO 5
C
        ELSE
C  ENTER THE FOLLOWING INTEGER NUMBERS
C                        NT:  THE NUMBER OF INCREMENTS OF TIME STEP-LENGTH
C                        NT1: BIGINIG POINT OF THE RESULT OUTPUT
C                        NT2: ENDING POINT OF THE RESULT OUTPUT
      READ(5,*) NT,NT1,NT2
C
C  ENTER THE TIME INCREMENT
      READ(5,*) (CTI(I),I=1,NT)
C  CHECK IF THE VALUE OF TIME INCREMENT IS MORE THAN ZERO
      DO 1 I= 1,NT
      IF(CTI(I).EQ.0.0) THEN
       STOP 'TIME INCREMENT MUST BE GREATER ZERO, IMPORTANT!'
      END IF
  1   CONTINUE
C  ENTER THE PRESSURE INCREMENT AS t>0
      READ(5,*) (CCDP(I),I=1,NT)
C  ENTER THE TEMPERATURE INCREMENT AS t>0
      READ(5,*) (CCDT(I),I=1,NT)
C
C  ENTER THE ERROR LIMIT AND INITIAL ESTIMATES FOR ITERATION
      READ(5,*) EPS,CO
C
C  ENTER THE COEFFICIENTS OF CREEP LAW
      READ(5,*) CAC,CMC,CNC
        END IF
C
C        ****        ****        ****        ****        ****
C
  5   CNR=0.5*DI
      CNT=0.5*(DO-DI)
      CNEF=DO*CNEFO/DI
C
      RETURN
      END
C  -------------------------------------------------------------------
```

```
C
C
C
C           ( 3 ) ---- CREATE DATA BLOCK
C                    THIS SEGMENT INITIALIZES SOME VARIABLES
C
C       XX1:   FITTING POINTS OF THE ELASTIC MODULUS OF CASING
C       XX2:   FITTING POINTS OF THE THERMAL EXPANSION COEFFCIENT OF CASING
C       X0:    INTERPOLATION POINTS OF TEMPERATURE
C       Y0:    INTERPOLATION POINTS OF YIELD LIMITATION
C       Y1:    INTERPOLATION POINTS OF EFFECTIVE STRAIN
C       Z0:    INTERPOLATION POINTS OF PLASTIC MODULUS OF CASING
C
C          ****        ****        ****        ****        ****
C
      BLOCK DATA WDAT
C
      DIMENSION TM(6),XX1(2,9),XX2(2,9),X0(6),Y0(6),Y1(8),Z0(6,8,1)
      COMMON/A1/XX1/A2/XX2/A3/X0/A4/Y0/A5/Y1/A6/Z0/DM/TM
C
      DATA TM/0.,50.,100.,150.,200.,250./
      DATA XX1/0.,207.E9,10.,206.5E9,21.,206.E9,50.,198.5E9,93.,190.E9,
     $150.,178.7E9,204.,166.7E9,316.,140.E9,427.,118.33E9/
      DATA XX2/0.,11.4E-6,10.,11.45E-6,21.,11.5E-6,50.,11.75E-6,93.,
     $12.E-6,150.,12.35E-6,204.,12.7E-6,316.,13.4E-6,427.,14.E-6/
      DATA X0/0.,21.,93.,204.,316.,427./
      DATA Y0/622.E6,610.E6,570.E6,500.E6,420.E6,355.E6/
      DATA Y1/0.3E-2,0.5E-2,0.8E-2,2.0E-2,4.0E-2,6.0E-2,8.0E-2,10.0E-2/
      DATA Z0/2.5E9,5.E9,12.5E9,25.E9,45.E9,45.E9,3.E9,3.3E9
     $,5.E9,10.E9,15.E9,9.3E9,2.58E9,2.5E9,2.08E9,4.17E9,4.58E9,3.92E9,
     $1.55E9,1.75E9,2.5E9,3.E9,3.E9,.75E9,1.E9,1.E9,1.E9,1.25E9,1.75E9,
     $-3.5E9,.45E9,.25E9,-.5E9,-1.25E9,.25E9,-1.2E9,-0.1E9,-0.5E9,-1.75
     $E9,-2.5E9,-.25E9,-1.65E9,-0.9E9,-1.6E9,-4.0E9,-6.5E9,-1.75E9,-3.15
     $E9/
C     DATA Z0/2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9,
C    $2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9,
C    $2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9,
C    $2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9,
C    $2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9,
C    $2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9,
C    $2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9,
C    $2.06E9,2.12E9,2.29E9,3.58E9,3.79E9,2.33E9/
      END
C ----------------------------------------------------------------------
```

```
C
C
C
C          ( 4 ) ---- WRITE ORIGINAL DATA
C
      SUBROUTINE WRINT(NNC,NN,CDP,CDT,CCDP,CCDT,NTC,NT,CTI)
C
      DIMENSION CDP(NNC),CDT(NNC),CTI(NTC),CCDP(NTC),CCDT(NTC)
      COMMON /D1/BM/C1/CNEF,CNBMF,CNR,CNT/C3/DO,CNEFO
C
      WRITE(6,90)
      WRITE(9,90)
      WRITE(6,100) DO,CNT,BM
      WRITE(9,100) DO,CNT,BM
      WRITE(6,120) CNEFO,CNBMF
      WRITE(9,120) CNEFO,CNBMF
      WRITE(6,130) (CDP(I),I=1,NN)
      WRITE(9,130) (CDP(I),I=1,NN)
      WRITE(6,140) (CDT(I),I=1,NN)
      WRITE(9,140) (CDT(I),I=1,NN)
      WRITE(6,141) (CTI(I),I=1,NT)
      WRITE(9,141) (CTI(I),I=1,NT)
      WRITE(6,142) (CCDP(I),I=1,NT)
      WRITE(9,142) (CCDP(I),I=1,NT)
      WRITE(6,143) (CCDT(I),I=1,NT)
      WRITE(9,143) (CCDT(I),I=1,NT)
      WRITE(6,145)
      WRITE(9,145)
C
   90 FORMAT(30X,'CASEPC   RESULTS',/,25X,26('*'),/,28X,'( FILE NAME : W
     $EACC )',/,74('='),/,4X,'<INPUT DATA>',/,3X,15('-'),/)
  100 FORMAT(5X,'OUTER DIAMETER',4X,'WALL THICKNESS',3X,'POISSON RATIO'
     $,/,7X,F10.7,8X,F10.7,7X,F7.4,/)
  120 FORMAT(5X,'MODULUS OF ROCK',3X,'POISSON RATIO OF ROCK',/,
     $7X,E11.4,7X,F7.4,/)
  130 FORMAT(2X,'INCREMENT OF PRESSURE AT t=0: ',/,6(E12.4))
  140 FORMAT(/,2X,'INCREMENT OF TEMPERATURE AT t=0: ',/,6(F12.4))
  141 FORMAT(/,2X,'INCREMENT OF TIME: ',/,6(F12.4))
  142 FORMAT(/,2X,'INCREMENT OF PRESSURE AS t>0: ',/,6(E12.4))
  143 FORMAT(/,2X,'INCREMENT OF TEMPERATURE AS t>0: ',/,6(F12.4))
  145 FORMAT(1X,73('='))
      RETURN
      END
C -----------------------------------------------------------------------
```

```
C
C
C
C            ( 5 ) ---- CALCULATE THE PRESSURE, STRESSES AND STRAINS
C                    AT THE INITIAL YIELDT LIMITATION UNDER DIFFERE
C                    -NT TEMPERATURE
C
      SUBROUTINE EESOL(N,SC,SR,SZ,EC,ER,EZ,ED,EE,BY)
C
      DIMENSION TM(6),PY(10),X0(6),Y0(6),XX1(2,9)
      DIMENSION SC(10),SR(10),SZ(10),BY(10),EC(10),ER(10),EZ(10),EE(10)
      DIMENSION ED(10),A(6),CK1(6),CK2(6),AX(6),S(6,7),XX2(2,9)
      COMMON /A1/XX1/A2/XX2/A3/X0/A4/Y0/B1/CK1,CK2/D1/BM/DM/TM
C
      DO 13 I=1,N
   13 A(I)=0.0
C
C  **** CALCULATE THE COEFICIENTS OF CURVE FITTING FOR ELASTIC MODULUS
C         OF CASING
      CALL FIT13(9,6,7,XX1,A,S,FATL)
C
      DO 14 I=1,N
   14 CK1(I)=A(I)
C
C  **** CALCULATE THE COEFICIENTS OF CURVE FITTING FOR THERMAL EXPANSION
C         OF CASING
      CALL FIT13(9,6,7,XX2,A,S,FATL)
C
      DO 16 I=1,N
   16 CK2(I)=A(I)
C
C  **** FIND OUT THE YIELD PRESSURE UNDER DIFFERENT TEMPERATURE
        CALL YEP(N,PY)
C
      DO 20 I=1,N
C
      E=FF(TM(I),CK1)
      ALFA=FF(TM(I),CK2)
C
C  **** TAKE LAGRANGE INTERPOLATION FOR YIELD LIMIT OF CASING
      CALL LAG13(X0,Y0,6,TM(I),BY(I))
C
C  **** CALCULATE THE STRESSES AND THE STRAINS AT THE PLASTIC ONSET
      CALL ELASE(PY(I),TM(I),SC(I),SR(I),SZ(I),EC(I),ER(I),EZ(I),E,ALFA,
     $BY(I))
C
C  **** CALCULATE THE EFFECTIVE STRESS
      CALL EDSTS(SC(I),SR(I),SZ(I),ED(I))
C
C  **** CALCULATE THE EFFECTIVE STRAIN
      CALL EESTN(EC(I),ER(I),EZ(I),EE(I))
C
   20 CONTINUE
      RETURN
      END
C -----------------------------------------------------------------------
```

```
C
C
C
C          ( 6 ) ---- CALCULATE THE YIELD PRESSURE UNDER DIFFERENT
C                     TEMPERATURE
C
      SUBROUTINE YEP(N,PY)
C
      DIMENSION PY(10),TM(6),CK1(6),CK2(6),X0(6),Y0(6)
      COMMON /D1/BM/B1/CK1,CK2/DM/TM
      COMMON/C1/CNEF,CNBMF,CNR,CNT/A3/X0/A4/Y0
C
      WRITE(6,116)
      WRITE(9,116)
  116 FORMAT(2X,'<PHYSICAL PROPERTIES OF THE CASING UNDER DIFFERENTE TEM
     $PERATURE>:',/)
C
      DO 10 I=1,N
      CALL LAG13(X0,Y0,6,TM(I),BY0)
      E=FF(TM(I),CK1)
      ALFA=FF(TM(I),CK2)
      WRITE(6,123) I,TM(I),BY0,E,ALFA
      WRITE(9,123) I,TM(I),BY0,E,ALFA
  123 FORMAT(1X,'POINT-',I2,':',F7.2,4X,'BY0=',E11.4,4X,'E=',E11.4,4X,
     $'ALFA=',E11.4)
C
      CALL ECOEF(AA,BB,CC,DD,AA0,BB0,CC0,DD0,E,ALFA,BY0)
      BM1=BM-1.
      BB1=BB-1.
      T0=(BM*BB0-E*ALFA)*TM(I)
      X=BB*(BB+1.)+BM*BM1*BB1*BB1+1.
      Y=T0*(2.*BM*BB1+1.-BB)+BB0*(2.*BB+1.-BM*BB1)*TM(I)
      Z=BB0*BB0*TM(I)*TM(I)+T0*(T0-BB0*TM(I))-BY0*BY0
      IF((ABS(X)+ABS(Y)+ABS(Z)).EQ.0.0) STOP
C
      CALL QUAD(X,Y,Z,C1,C2,D,KEY)
      IF(KEY) 22,23,23
 22   WRITE(6,*) 'DATA ERROR'
      WRITE(9,*) 'DATA ERROR'
      GO TO 10
 23   PY(I)=C1
 10   CONTINUE
C
      WRITE(6,101)
      WRITE(9,101)
      WRITE(6,100)
      WRITE(10,100)
      WRITE(6,200) (I,TM(I),PY(I),I=1,N)
      WRITE(10,200) (I,TM(I),PY(I),I=1,N)
      WRITE(6,101)
      WRITE(10,101)
 100  FORMAT(8X,'<YIELD PRESSURE UNDER DIFFERENT TEMPERATURES>',/,1X,
     $73('='),//,1X,'POINT',12X,'TEMPERATURE',12X,'YIELD PRESSURE',/)
 200  FORMAT(1X,I3,14X,F7.2,14X,E12.4)
 101  FORMAT(1X,73('='))
      RETURN
      END
C   ------------------------------------------------------------
```

```
C
C
C
C           ( 7 ) ---- PRESENTE THE COEFFIENTS FOR ELASTIC COMPUTATION
C
      SUBROUTINE ECOEF(AA,BB,CC,DD,AA0,BB0,CC0,DD0,E,ALFA,BY0)
C
      COMMON/C1/CNEF,CNBMF,CNR,CNT/D1/BM/CK/AKK
C
      AKK=CNEF/(1.+CNBMF)
      B1M=1.+BM
      B0M=1.-BM
      AA=(CNR*B0M+BM*CNT)*CNR*B1M/(E*CNT+AKK*CNR*(1.-BM*BM))
      BB=CNR*(1.-(AKK*AA/CNR))/CNT
      CC=B1M*(B0M*BB+BM)/E
      DD=B1M*((BB-1.)*BM+1.)/E
      AA0=CNR*B1M*E*ALFA*CNT/(E*CNT+AKK*CNR*(1.-BM*BM))
      BB0=-(AKK*AA0/CNT)
      CC0=((1.-BM*BM)*BB0/E)+(B1M*ALFA)
      DD0=(B1M*BM*BB0/E)-(B1M*ALFA)
      RETURN
      END
C    ------------------------------------------------------------
C
C           ( 8 ) ---- FIRST OUTPUT: ELASTIC STRESS AND STRAIN AT
C                      THE ONSET OF YIELD
C
      SUBROUTINE EOUT1(N,SC,SR,SZ,EC,ER,EZ,ED,EE,BY)
C
      DIMENSION SC(10),SR(10),SZ(10),ED(10),BY(10)
      DIMENSION EC(10),ER(10),EZ(10),EE(10)
C
      WRITE(6,110)
      WRITE(10,110)
      WRITE(6,210) (I,SC(I),SR(I),SZ(I),ED(I),BY(I),I=1,N)
      WRITE(10,210) (I,SC(I),SR(I),SZ(I),ED(I),BY(I),I=1,N)
      WRITE(6,105)
      WRITE(10,105)
      WRITE(6,120)
      WRITE(10,120)
      WRITE(6,220) (I,EC(I),ER(I),EZ(I),EE(I),I=1,N)
      WRITE(10,220) (I,EC(I),ER(I),EZ(I),EE(I),I=1,N)
      WRITE(6,106)
      WRITE(10,106)
  105 FORMAT(1X,79('-'))
  106 FORMAT(1X,79('='))
  110 FORMAT(//,6X,'YIELD PRESSURE AND INITIAL STRESS AND STRAIN',/,1X
     $,79('='),//,1X,'POINT',4X,'SGMA-C',9X,'SGMA-R',9X,'SGMA-Z'
     $,9X,'SGMA-BR',9X,'SGMA-Y',/)
  210 FORMAT(1X,I3,3X,E12.4,3X,E12.4,3X,E12.4,3X,E12.4,3X,E12.4)
  120 FORMAT(1X,'POINT',4X,'EPS-C',10X,'EPS-R',10X,'EPS-Z',10X
     $,'EPS-BR',/)
  220 FORMAT(1X,I3,3X,E12.4,3X,E12.4,3X,E12.4,3X,E12.4)
      RETURN
      END
C    ------------------------------------------------------------
```

```
C
C
C
C           ( 9 ) ---- CALCULATE THE INITIAL VALUES OF STRESS AND STRAIN
C
      SUBROUTINE ELASE(DP,DT,DSC,DSR,DSZ,DEC,DER,DEZ,E,ALFA,BY0)
C
      COMMON/C1/CNEF,CNBMF,CNR,CNT/D1/BM
C
      CALL ECOEF(AA,BB,CC,DD,AA0,BB0,CC0,DD0,E,ALFA,BY0)
C
      DU=DP*AA+AA0*DT
      DSC=DP*BB+BB0*DT
      DSR=-DP
      DSZ=DP*BM*(BB-1.)+(BM*BB0-E*ALFA)*DT
      DEC=DP*CC+CC0*DT
      DER=-(DP*DD+DD0*DT)
      DEZ=0.0
      RETURN
      END
C -----------------------------------------------------------------
C
C           ( 10 ) ---- FIND OUT THE ELASTIC-PLASTIC SOLUTION WITH
C                       THERMOPLASTIC MATERIALS USING VON-MISES CONDITION
C
      SUBROUTINE EPSOL(NNC,NN,CDP,CDT,CTE,PPY,PTM,PBY,RC,PSC,PSR,PSZ,PED
     $,PEC,PER,PEZ,PEE,ECP,ERP,EZP,EEP,BYE,EEE,ALE,EPE)
C
      DIMENSION CDP(NNC),CDT(NNC),CTE(NNC)
      DIMENSION X0(6),Y0(6),Y1(8),XX1(2,9),XX2(2,9),Z(1),BYE(NNC)
      DIMENSION A(6),CK1(6),CK2(6),AX(6),S(6,7),Z0(6,8,1),RC(NNC)
      DIMENSION PSC(NNC),PSZ(NNC),PSR(NNC),PED(NNC),PBY(NNC),EEE(NNC)
      DIMENSION PEC(NNC),PER(NNC),PEZ(NNC),PEE(NNC),PPY(NNC),ALE(NNC)
      DIMENSION ECP(NNC),ERP(NNC),EZP(NNC),EEP(NNC),PTM(NNC),EPE(NNC)
      COMMON/CD1/DPSC,DPSR,DPSZ,DPEC,DPER,DPEZ/DC/CNSC,CNSR,CNSZ
      COMMON/AC3/AE,AL/AC4/ATM,APP,AED,APE/AC5/ASC,ASR,ASZ,AEC,AER,AEZ
      COMMON/A1/XX1/A2/XX2/A3/X0/A4/Y0/A5/Y1/A6/Z0/B1/CK1,CK2
      DATA CNEE,CNPY,CNTM,CNECP,CNERP,CNEZP,CNEEP/7*0.0/
      DATA KK,RR,CT,DDP,DDT/1,1.,3*0.0/
      DATA CNEC,CNER,CNEZ,DDP1,DDT1/5*0.0/
C
      CNSC=0.
      CNSR=0.
      CNSZ=0.
      DO 1 J=1,NN
C
         IF(CDT(J).LT.0.0) THEN
             CNDT=CDT(J)-DDT1
             CNDP=CDP(J)-DDP1
           ELSE IF(CDP(J).LT.0.0) THEN
                CNDP=CDP(J)-DDP1
                CNDT=CDT(J)-DDT1
             ELSE
                  CNDP=CDP(J)+DDP
                  CNDT=CDT(J)+DDT
         END IF
C
```

```
C
C
C
C
         IF(J.EQ.1) THEN
             CTE(J)=0.0
           ELSE
               J1=J-1
               CT=CT+CDT(J1)
               CTE(J)=CT
         END IF
C
      DDP=0.0
      DDT=0.0
      DDP1=0.0
      DDT1=0.0
C
C  TAKE LAGRANGE INTERPOLATION FOR YIELD LIMIT OF CASING UNDER DIFFERENT
C  TEMPERATURE
      CALL LAG13(X0,Y0,6,CT,BY0)
      E=FF(CT,CK1)
      ALFA=FF(CT,CK2)
      BYE(J)=BY0
      EEE(J)=E
      ALE(J)=ALFA
C
          IF(KK.GT.1) THEN
              CNED=PBY(J1)
              CNBY=PBY(J1)
            ELSE
                CNED=BY0
                CNBY=BY0
          END IF
C
      IF(RR.LT.0.99) CNED=PED(J1)
C
C  CLCULATE THE INCREMENT OF STRESS AND STRALN UNDER ELASTIC CONDITION
      CALL ELASE(CNDP,CNDT,DSC,DSR,DSZ,DEC,DER,DEZ,E,ALFA,BY0)
C
      PSC(J)=CNSC+DSC
      PSR(J)=CNSR+DSR
      PSZ(J)=CNSZ+DSZ
      PEC(J)=CNEC+DEC
      PER(J)=CNER+DER
      PEZ(J)=CNEZ+DEZ
C
C  CALCULATE EFFECTIVE STRESS
      CALL EDSTS(PSC(J),PSR(J),PSZ(J),DCNED)
C
      CF=DCNED-CNED
      IF(CF.LT.0.0) GO TO 800
C
          IF(CF.EQ.0.0) THEN
              RR=0.0
            ELSE
C
```

```
C
C
C
C  WHEN THE LOADING POINT EXCEEDS THE YIELD SURFACE, CALCULATE THE SCALE
C  FACTOR OF ELASTIC PART
             CALL RCOEF(CNSC,CNSR,CNSZ,DSC,DSR,DSZ,CNED,RR)
C
           END IF
C
      RC(J)=RR
      IF(RR.LT.0.99) KK=KK+1
C
      DSC=RR*DSC
      DSR=RR*DSR
      DSZ=RR*DSZ
      DEC=RR*DEC
      DER=RR*DER
      DEZ=RR*DEZ
C
          IF(RR.LT.1.E-2) THEN
              DDP=0.0
              DDT=0.0
              DDP1=0.0
              DDT1=0.0
            ELSE
                DDP=(1.-RR)*CNDP
                DDT=(1.-RR)*CNDT
                DDP1=RR*CNDP
                DDT1=RR*CNDT
          END IF
C
      CNSC=CNSC+DSC
      CNSR=CNSR+DSR
      CNSZ=CNSZ+DSZ
      CNEC=CNEC+DEC
      CNER=CNER+DER
      CNEZ=CNEZ+DEZ
C
C  CALCULATE EFFACTIVE STRAIN
      CALL EESTN(CNEC,CNER,CNEZ,CNEE)
C
          IF(RR.GT.1.E-2) THEN
              PSC(J)=CNSC
              PSR(J)=CNSR
              PSZ(J)=CNSZ
              PEC(J)=CNEC
              PER(J)=CNER
              PEZ(J)=CNEZ
C
              CALL EDSTS(PSC(J),PSR(J),PSZ(J),CNED)
C
              CNDP=RR*CNDP
              CNDT=RR*CNDT
C
              GO TO 801
          END IF
C
```

```
C
C
C
C  TAKE TWO-DIMENSIONAL INTERPOLATION FOR PLASTIC MODULUS AGAIST DIFFERENT
C  TEMPERATURE AND TOTAL STRAIN
      CALL LAG23(X0,Y1,Z0,6,8,1,CT,CNEE,EPE(J))
      EP=EPE(J)
C
C  CALCULATE THE STRESSES AND TOTAL STRAINS UNDER PLASTIC CONDITION
      CALL PLASE(CNDP,CNDT,E,ALFA,BY0,EP)
C
      PSC(J)=CNSC+DPSC
      PSR(J)=CNSR+DPSR
      PSZ(J)=CNSZ+DPSZ
      PEC(J)=CNEC+DPEC
      PER(J)=CNER+DPER
      PEZ(J)=CNEZ+DPEZ
C
C  CALCULATE THE PLASTIC STRAINS
      CALL PSRAN(CNDT,DDPEE,DDPEC,DDPER,DDPEZ,E,ALFA,BY0,EP)
C
          IF(ABS(E-EP).LT.1.E-2) THEN
              PCNBY=CNBY
            ELSE
                PCNBY=CNBY+(((E*EP)/(E-EP))*DDPEE)
          END IF
C
      CALL EDSTS(PSC(J),PSR(J),PSZ(J),CNED)
C
      CALL EESTN(PEC(J),PER(J),PEZ(J),CNPEE)
C
      PED(J)=CNED
      PBY(J)=PCNBY
      PEE(J)=CNPEE
      ECP(J)=CNECP+DDPEC
      ERP(J)=CNERP+DDPER
      EZP(J)=CNEZP+DDPEZ
      EEP(J)=CNEEP+DDPEE
      GO TO 400
C
 800  RR=1.
      RC(J)=RR
 801  EPE(J)=0.0
      CALL EESTN(PEC(J),PER(J),PEZ(J),CNEE)
C
      PED(J)=DCNED
      IF(DCNED.GT.CNED) PED(J)=CNED
      PBY(J)=CNED
      PEE(J)=CNEE
      ECP(J)=CNECP
      ERP(J)=CNERP
      EZP(J)=CNEZP
      EEP(J)=CNEEP
C
```

```
C
C
C
 400   PPY(J)=CNPY+CNDP
       PTM(J)=CNTM+CNDT
       CNECP=ECP(J)
       CNERP=ERP(J)
       CNEZP=EZP(J)
       CNEEP=EEP(J)
       CNSC=PSC(J)
       CNSR=PSR(J)
       CNSZ=PSZ(J)
       CNEC=PEC(J)
       CNER=PER(J)
       CNEZ=PEZ(J)
       CNEE=PEE(J)
       CNBY=PBY(J)
       CNPY=PPY(J)
       CNTM=PTM(J)
C
           IF(RR.LT.1.E-2) THEN
               CDT(J)=CNDT
               CDP(J)=CNDP
              ELSE IF(CDT(J).LT.0.0) THEN
                  CDT(J)=CNDT
                  CDP(J)=CNDP
                ELSE IF(CDP(J).LT.0.0) THEN
                    CDT(J)=CNDT
                    CDP(J)=CNDP
                  ELSE
                      CDP(J)=RR*CDP(J)
                      CDT(J)=RR*CDT(J)
           END IF
C
   1   CONTINUE
C
C  PUT THE ELASTIC-PLASTIC SOLUTION AT THE WORKING POINT INTO THE INITIAL
C  POINT FOR CREEP COMPUTE
       ASC=PSC(NN)
       ASR=PSR(NN)
       ASZ=PSZ(NN)
       AEC=PEC(NN)
       AER=PER(NN)
       AEZ=PEZ(NN)
       ATM=PTM(NN)
       APP=PPY(NN)
       AED=PED(NN)
       APE=PEE(NN)
       AE=EEE(NN)
       AL=ALE(NN)
C
       RETURN
       END
C   ------------------------------------------------------------------
```

```
C
C
C
C
C            ( 11 ) ---- THE REAL ROOTS OF SECOND DEGREE ALGEBRA EQUATION
C
      SUBROUTINE QUAD(A,B,C,X1,X2,D,ID)
C
      COMPLEX XX1,XX2
C
      XX1=(0.0,0.0)
      XX2=(0.0,0.0)
      ID=1
      IF(A.EQ.0.) GO TO 4
      D=B*B-4.*A*C
      IF(D) 1,2,3
C
    1 XX1=.5*CMPLX(-B,SQRT(-D))/A
      XX2=CONJG(XX1)
      X1=REAL(XX1)
      X2=REAL(XX2)
      RETURN
C
    2 XX1=CMPLX(-.5*B/A,0.0)
      XX2=XX1
      X1=REAL(XX1)
      X2=REAL(XX2)
      RETURN
C
    3 XX1=CMPLX(.5*(-B+SQRT(D))/A,0.0)
      XX2=CMPLX(-B/A,0.0)-XX1
      X1=REAL(XX1)
      X2=REAL(XX2)
      RETURN
C
    4 IF(B.EQ.0.) GO TO 5
      XX1=CMPLX(-C/B,0.0)
      X1=REAL(XX1)
      ID=0
      RETURN
C
    5 ID=-1
      RETURN
      END
C    -------------------------------------------------------------------
```

```
C
C
C
C         ( 12 ) ---- CALCULATE THE EFFECT STRESS
C
      SUBROUTINE EDSTS(A,B,C,CNED)
C
      AA=(A-B)*(A-B)
      BB=(B-C)*(B-C)
      CC=(C-A)*(C-A)
      ABC=(AA+BB+CC)/2.
      CNED=SQRT(ABC)
      RETURN
      END
C   -----------------------------------------------------------------
C
C
C         ( 13 ) ---- CALCULATE THE EFFECT STRAIN
C
      SUBROUTINE EESTN(A,B,C,CNEE)
C
      AB=(A-B)*(A-B)
      BC=(B-C)*(B-C)
      CA=(C-A)*(C-A)
      ABC=AB+BC+CA
      CNEE=(SQRT(ABC))*1.4142/3.
      RETURN
      END
C   -----------------------------------------------------------------
```

```
C
C
C
C         ( 14 ) ---- PRESENTE THE COEFFCINTS FOR PLASTIC COMPUTATION
C
      SUBROUTINE PCOEF(E,ALFA,BY0,EP)
C
      COMMON/DA/A10,A20,AT0,B10,B20,BT0,C11,C21,CT1
      COMMON/DC/CNSC,CNSR,CNSZ/D1/BM
C
      ALAM=E*BM/((1.+BM)*(1.-2.*BM))
      ALAM1=ALAM*(1.-BM)/BM
C
      CALL EDSTS(CNSC,CNSR,CNSZ,ED)
C
      A1=3.*E/(2.*(1.+BM))
C
         IF(ABS(E-EP).LT.1.E-2) THEN
             A2=0.0
             AE=0.0
           ELSE
               A2=E*EP/(E-EP)
               AE=(A1*A1)/((A1+A2)*ED*ED)
         END IF
C
      ASC=(2.*CNSC-CNSR-CNSZ)/3.
      ASR=(2.*CNSR-CNSC-CNSZ)/3.
      ASZ=(2.*CNSZ-CNSC-CNSR)/3.
      CCR=AE*ASC*ASR
      CCZ=AE*ASC*ASZ
      CRZ=AE*ASR*ASZ
      CCC=AE*ASC*ASC
      CRR=AE*ASR*ASR
      CZZ=AE*ASZ*ASZ
      AA1=ALAM1-CCC
      AA2=ALAM-CCR
      BB1=AA2
      BB2=ALAM1-CRR
      CC1=ALAM-CCZ
      CC2=ALAM-CRZ
      AB12=(AA1*BB2)-(BB1*AA2)
      A10=BB2/AB12
      A20=-AA2/AB12
      B10=-BB1/AB12
      B20=AA1/AB12
      C11=CC1*A10+CC2*B10
      C21=CC1*A20+CC2*B20
      A2A=2.*ALAM+ALAM1
      AAT=CCC+CCR+CCZ-A2A
      BBT=CCR+CRR+CRZ-A2A
      CCT=CCZ+CRZ+CZZ-A2A
      AT0=A10*AAT+A20*BBT
      BT0=B10*AAT+B20*BBT
      CT1=CCT-CC1*AT0-CC2*BT0
      RETURN
      END
C    -----------------------------------------------------------------
```

```
C
C
C
C             ( 15 ) ---- CALCULATE PLASTIC STRESS AND TOTAL STRAIN
C
      SUBROUTINE PLASE(DP,DT,E,ALFA,BY0,EP)
C
      COMMON/DA/A10,A20,AT0,B10,B20,BT0,C11,C21,CT1
      COMMON/C1/CNEF,CNBMF,CNR,CNT/CK/AKK
      COMMON /CD1/DPSC,DPSR,DPSZ,DPEC,DPER,DPEZ
C
      CALL PCOEF(E,ALFA,BY0,EP)
C
      RCT=CNR/CNT
      RCK=1.+AKK*A10*RCT
      DPSC=(((1.+AKK*A20)/RCK)*RCT*DP)+((AT0*AKK*RCT/RCK)*ALFA*DT)
      DPSR=-DP
      ALT=ALFA*DT
      DPSZ=C11*DPSC+C21*DPSR+CT1*ALT
      DPEC=A10*DPSC+A20*DPSR-AT0*ALT
      DPER=B10*DPSC+B20*DPSR-BT0*ALT
      DPEZ=0.
      RETURN
      END
C   -----------------------------------------------------------------
C
C             ( 16 ) ---- CALCULATE THE SCALE FCTOR OF ELASTIC PART: <R>
C
      SUBROUTINE RCOEF(AC,AR,AZ,DSC,DSR,DSZ,BY,RR)
C
      A0=(AC+AR+AZ)/3.
      EA0=(DSC+DSR+DSZ)/3.
      AA=(DSC-EA0)*(DSC-EA0)
      BB=(DSR-EA0)*(DSR-EA0)
      CC=(DSZ-EA0)*(DSZ-EA0)
      A=AA+BB+CC
      AA=(AC-A0)*(DSC-EA0)
      BB=(AR-A0)*(DSR-EA0)
      CC=(AZ-A0)*(DSZ-EA0)
      B=(AA+BB+CC)*2.
      AA=(AC-A0)*(AC-A0)
      BB=(AR-A0)*(AR-A0)
      CC=(AZ-A0)*(AZ-A0)
      C=(AA+BB+CC)-(BY*BY*2./3.)
C
      CALL QUAD(A,B,C,C1,C2,D,KEY)
C
      IF(KEY) 2,3,3
    2 WRITE(6,*) 'DATA ERROR'
      WRITE(9,*) 'DATA ERROR'
      GO TO 5
    3 RR=C1
    5 RETURN
      END
C   -----------------------------------------------------------------
```

```
C
C
C
C          ( 17 ) ---- OUTPUT OF THE ELASTIC-PLASTIC SOLUTION
C
      SUBROUTINE EPOUT(NNC,NN,CDP,CDT,CTE,N1,N2,PPY,PTM,PBY,RC,PSC,PSR,P
     $SZ,PED,PEC,PER,PEZ,PEE,ECP,ERP,EZP,EEP,BYE,EEE,ALE,EPE)
C
      DIMENSION PSC(NNC),PSZ(NNC),PSR(NNC),PED(NNC),PBY(NNC),BYE(NNC)
      DIMENSION PEC(NNC),PER(NNC),PEZ(NNC),PEE(NNC),PPY(NNC),EEE(NNC)
      DIMENSION ECP(NNC),ERP(NNC),EZP(NNC),EEP(NNC),PTM(NNC),ALE(NNC)
      DIMENSION CDP(NNC),CDT(NNC),RC(NNC),EPE(NNC),CTE(NNC)
      COMMON/C1/CNEF,CNBMF,CNR,CNT/D1/BM/CN2/LL/CK/AKK
C
      IF(N1.GT.0) THEN
C
      WRITE(6,5)
      WRITE(9,5)
    5 FORMAT(///,26X,'<ELASTIC-PLASTIC SOLUTION>',/,22X,34('*'),//,
     $80('='))
C
      AM=1.-BM
      DO 10 J=N1,N2
C
      J1=J-1
      AB=EEE(J)/((1.+BM)*(1.-2.*BM))
      ALD=ALE(J)*CDT(J)
C
C   CALCULATE CHECK VALUES FOR COMPARISON
            IF(J1.EQ.0) THEN
                AC=PEC(J)-ECP(J)-ALD
                AR=PER(J)-ERP(J)-ALD
                AZ=PEZ(J)-EZP(J)-ALD
                SC1=AB*(AM*AC+BM*(AR+AZ))
                SZ=AB*(BM*(AC+AR)+AM*AZ)
              ELSE
                  AC=(PEC(J)-PEC(J1))-(ECP(J)-ECP(J1))-ALD
                  AR=(PER(J)-PER(J1))-(ERP(J)-ERP(J1))-ALD
                  AZ=(PEZ(J)-PEZ(J1))-(EZP(J)-EZP(J1))-ALD
                  SC1=PSC(J1)+(AB*(AM*AC+BM*(AR+AZ)))
                  SZ=PSZ(J1)+(AB*(BM*(AC+AR)+AM*AZ))
            END IF
C
C   CALCULATE CHECK VALUES USING ANOTHER FORMULA
      SR=-PPY(J)
      SC2=(PPY(J)-AKK*PEC(J))*CNR/CNT
C
```

```
C
C
C
C  EXPORT THE ELASTIC-PLASTIC SOLUTION
      WRITE(6,30) CTE(J)
      WRITE(9,30) CTE(J)
      WRITE(6,50) BYE(J),EEE(J),ALE(J),EPE(J)
      WRITE(9,50) BYE(J),EEE(J),ALE(J),EPE(J)
      WRITE(6,100) J,CDP(J),CDT(J),RC(J)
      WRITE(9,100) J,CDP(J),CDT(J),RC(J)
      WRITE(6,120) PPY(J),PTM(J),PBY(J)
      WRITE(9,120) PPY(J),PTM(J),PBY(J)
      WRITE(6,140) PSC(J),PSR(J),PSZ(J),PED(J)
      WRITE(9,140) PSC(J),PSR(J),PSZ(J),PED(J)
      WRITE(6,160) PEC(J),PER(J),PEZ(J),PEE(J)
      WRITE(9,160) PEC(J),PER(J),PEZ(J),PEE(J)
      WRITE(6,180) ECP(J),ERP(J),EZP(J),EEP(J)
      WRITE(9,180) ECP(J),ERP(J),EZP(J),EEP(J)
      WRITE(6,190) SC1,SR,SZ,SC2
      WRITE(9,190) SC1,SR,SZ,SC2
   10 CONTINUE
C
          ELSE
      WRITE(6,200)
      WRITE(9,200)
         END IF
C
   30 FORMAT(1X,'CT   =',F7.2)
   50 FORMAT(1X,'BY   =',E13.4,2X,'E    =',E12.4,3X,'ALFA   =',E12.4,1X,
     $'EP   =',E12.4)
  100 FORMAT(/,3X,'STEP (',I3,' ):',6X,'DP  =',E12.4E2,3X,'DT     =',F7.2
     $,6X,'FACTOR--RR= ',F5.3,/)
  120 FORMAT(1X,'PRES-R=',E11.4,2X,'TEMP-T=',F7.2,26X,'SGMA--Y=',E11.4)
  140 FORMAT(1X,'SGMA-C=',E11.4,2X,'SGMA-R=',E11.4,2X,'SGMA-Z=',E11.4,
     $2X,'SGMA-BR=',E11.4)
  160 FORMAT(1X,'EPS--C=',E11.4,2X,'EPS--R=',E11.4,2X,'EPS--Z=',E11.4,
     $2X,'EPS--BR=',E11.4)
  180 FORMAT(1X,'EPS-CP=',E11.4,2X,'EPS-RP=',E11.4,2X,'EPS-ZP=',E11.4,
     $2X,'EPS-BRP=',E11.4,/,83('-'))
  190 FORMAT(4X,'<CHECK THE NUMBER VALUE OF STRESS>:',/,1X,
     $'SGMC-1=',E11.4,2X,'SGMR-1=',E11.4,2X,'SGMZ-1=',E11.4,/,1X,
     $'SGMC-2=',E11.4,/,83('='))
  200 FORMAT(/,2X,'( NO OUTPUT FOR ELASTIC-PLASTIC SOLUTION. IF YOU WANT
     $, PUT: N2 > N1 > 0 )',/)
C
  301 RETURN
      END
C   -----------------------------------------------------------------
```

```
C
C
C
C          ( 18 ) ---- CACULATE PLASTIC STRAIN
C
      SUBROUTINE PSRAN(DDT,CNEE,DDPEC,DDPER,DDPEZ,E,ALFA,BYO,EP)
C
      COMMON /DC/CNSC,CNSR,CNSZ/D1/BM
      COMMON /CD1/DPSC,DPSR,DPSZ,DPEC,DPER,DPEZ
C
      CNEE=0.0
      DDPEC=0.0
      DDPER=0.0
      DDPEZ=0.0
C
      CALL EDSTS(CNSC,CNSR,CNSZ,PD)
C
          IF(ABS(E-EP).LT.1.E-2) THEN
              HH=0.0
            ELSE
                HH=E*EP/(E-EP)
          END IF
C
      GG=.5*E/(1.+BM)
      AT=PD*(1.+(HH/(3.*GG)))
      AC=1./AT
      AS=(3.*AC)/(2.*PD)
      EA=(CNSC+CNSR+CNSZ)/3.
      SC=CNSC-EA
      SR=CNSR-EA
      SZ=CNSZ-EA
      ASD=SC*DPEC+SR*DPER+SZ*DPEZ
      XE=AC*(ASD-ALFA*DDT*(SC+SR+SZ))
      XC=AS*ASD*SC
      XR=AS*ASD*SR
      XZ=AS*ASD*SZ
C
      CNEE=CNEE+XE
      DDPEC=DDPEC+XC
      DDPER=DDPER+XR
      DDPEZ=DDPEZ+XZ
      RETURN
      END
C    -------------------------------------------------------------------
```

```
C
C
C
C
C           ( 19 ) ---- ONE-DIMENSIONAL LAGRANGE INTERPOLATION FOR YIELD
C                            LIMITATION
C
      SUBROUTINE LAG13(XX0,YX0,N,X,YX)
C
      DIMENSION XX0(N),YX0(N)
C
      I=1
   10 IF(X.LT.0.5*(XX0(I+1)+XX0(I+2))) GO TO 30
      IF(X.GE.0.5*(XX0(N-2)+XX0(N-1))) GO TO 20
      I=I+1
      GO TO 10
   20 I=N-2
   30 M=I+2
      YX=0.0
      DO 60 J=I,M
      P=1.0
      DO 50 K=I,M
      IF(J-K) 40,50,40
   40 P=P*(X-XX0(K))/(XX0(J)-XX0(K))
   50 CONTINUE
   60 YX=YX+P*YX0(J)
      RETURN
      END
C   ---------------------------------------------------------------------
```

```
C
C
C
C          ( 20 ) ---- TWO-DIMENSIONAL LAGRENGE INTERPOLATION FOR
C                      PLASTIC MODULUS
C
      SUBROUTINE LAG23(XX0,YY0,ZZ0,N,M,L,X,Y,Z)
C
      DIMENSION XX0(N),YY0(M),ZZ0(N,M,L),Z(L),V(3),U(3)
C
      N1=N-2
      M1=M-2
      DO 1,I=1,N1
      IF(X.LE.XX0(I+1)) GO TO 2
    1 CONTINUE
      I=N-2
    2 DO 3 J=1,M1
      IF(Y.LE.YY0(J+1)) GO TO 4
    3 CONTINUE
      J=M-2
    4 IF(I.EQ.1) GO TO 5
      IF((X-XX0(I)).GE.(XX0(I+1)-X)) GO TO 5
      I=I-1
    5 IF(J.EQ.1) GO TO 6
      IF((Y-YY0(J)).GE.(YY0(J+1)-Y)) GO TO 6
      J=J-1
    6 A1=XX0(I)
      A2=XX0(I+1)
      A3=XX0(I+2)
      B1=YY0(J)
      B2=YY0(J+1)
      B3=YY0(J+2)
      U(1)=(X-A2)*(X-A3)/((A1-A2)*(A1-A3))
      U(2)=(X-A1)*(X-A3)/((A2-A1)*(A2-A3))
      U(3)=(X-A1)*(X-A2)/((A3-A1)*(A3-A2))
      V(1)=(Y-B2)*(Y-B3)/((B1-B2)*(B1-B3))
      V(2)=(Y-B1)*(Y-B3)/((B2-B1)*(B2-B3))
      V(3)=(Y-B1)*(Y-B2)/((B3-B1)*(B3-B2))
      DO 8 K=1,L
      W=0.0
      DO 7 II=1,3
      DO 7 JJ=1,3
      I1=I+II-1
      J1=J+JJ-1
    7 W=W+U(II)*V(JJ)*ZZ0(I1,J1,K)
    8 Z(K)=W
      RETURN
      END
C ----------------------------------------------------------------------
```

```
C
C
C
C           ( 21 ) ---- CALCULATE THE COEFFICIENTS OF CURVE FITTING
C                       USING A 5-DEGREE POLYNOMIL
C
      SUBROUTINE FIT13(N,M1,M2,X,A,S,FAIL)
C
      DIMENSION X(2,N),A(M1),S(M1,M2)
C
      M=M1-1
      DO 5 I=1,M1
      DO 5 J=1,M2
    5 S(I,J)=0.0
      S(1,1)=N
      DO 6 J=1,N
    6 S(1,M2)=S(1,M2)+X(2,J)
      DO 7 I=2,M1
      I1=I-1
      MI=M1+I-2
      DO 7 J=1,N
      S(I,1)=S(I,1)+X(1,J)**I1
      S(M1,I)=S(M1,I)+X(1,J)**MI
    7 S(I,M2)=S(I,M2)+X(1,J)**I1*X(2,J)
      DO 8 J=2,M
      DO 8 I=J,M
    8 S(I,J)=S(I+1,J-1)
      DO 9 I=1,M
      I1=I+1
      DO 9 J=I1,M1
    9 S(I,J)=S(J,I)
      CALL GS(M1,S,A,M2,FAIL)
      RETURN
      END
C   -----------------------------------------------------------------
C
```

```
C
C
C
C           ( 22 ) ---- SOLVE FOR THE SYSTEM OF LINEAR ALGEBRA
C                     EQUATIONS USING DIRECT METHOD
C
      SUBROUTINE GS(N,A,X,N1,FAIL)
C
      DIMENSION A(N,N1),X(N)
C
      DO 11 K=1,N
      DO 6 I=K,N
      IF (A(I,K).EQ.0.0) GO TO 6
      I0=I
      GO TO 7
   6  CONTINUE
      FAIL=-1
      RETURN
   7  IF(I0.EQ.K) GO TO 9
      DO 8 J=K,N1
      T=A(K,J)
      A(K,J)=A(I0,J)
   8  A(I0,J)=T
   9  K1=K+1
      DO 10 J=K1,N1
  10  A(K,J)=A(K,J)/A(K,K)
      DO 11 I=K1,N
      DO 11 J=K1,N1
  11  A(I,J)=A(I,J)-A(I,K)*A(K,J)
      X(N)=A(N,N1)
      NI=N-1
      DO 12 K=1,N1
      K1=NI-K+1
      X(K1)=A(K1,N1)
      K2=K1+1
      DO 12 J=K2,N
  12  X(K1)=X(K1)-A(K1,J)*X(J)
      FAIL=0.
      RETURN
      END
C ----------------------------------------------------------------------
C
C          ( 23 ) ---- FITTING FUNCTION: IT IS A 5-DEGREE POLYNOMIAL FOR
C                      ELASTIC MODULUS AND THERMAL EXPANSION COEFFICIENT
C
      FUNCTION FF(X,AX)
C
      DIMENSION AX(6)
C
      FF=AX(1)+X*(AX(2)+X*(AX(3)+X*(AX(4)+X*(AX(5)+X*AX(6)))))
      RETURN
      END
C ----------------------------------------------------------------------
```

```
C
C
C
C           ( 24 ) ----SOLVE THE CREEP PROBLEM WITH ITERATIVE METHOD
C                      USING THE BAILOY-NORTON LAW
C
      SUBROUTINE CRSOL(NTC,NT,CCDP,CCDT,CTI,CCTE,PSC,PSR,PSZ,PED,PEC,PER
     $,PEZ,PEE,ECP,ERP,EZP,EEP,CCKE,PPED)
C
      DIMENSION PSC(NTC),PSR(NTC),PSZ(NTC),PEC(NTC),PER(NTC),PEZ(NTC)
      DIMENSION ECP(NTC),ERP(NTC),EZP(NTC),PED(NTC),PEE(NTC),EEP(NTC)
      DIMENSION CTI(NTC),CCTE(NTC),CCKE(NTC),PPED(NTC),CO(3)
      DIMENSION CCDP(NTC),CCDT(NTC)
      COMMON /A7/CO/C2/CAC,CMC,CNC/C4/EPS/AC5/AASC,AASR,AASZ,AAEC,AAER,A
     $AEZ
      DATA CTE,CEC,CER,CZE,DPEC,DPER,DPEZ/7*0.0/
C
C  OFFER THE INITIAL VALUES AT t=0
      ASC=AASC
      ASR=AASR
      ASZ=AASZ
      AEC=AAEC
      AER=AAER
      AEZ=AAEZ
C
C  OFFER THE INITIAL ESTIMATES FOR ITERATION
      AA=CO(1)
      BB=CO(2)
      CC=CO(3)
C
C  SOLVE THE CREEP PROBLEM FOR EACH TIME INTERVAL
      DO 11 I=1,NT
      CNDP=CCDP(I)
      CNDT=CCDT(I)
      CTE=CTE+CTI(I)
      CCTE(I)=CTE
      M=0
C
C  ITERATIVE PROCESS IN ONE TIME INTERVAL
   10 M=M+1
      IF(M.GT.100) THEN
           STOP 'FAIL IN DIVERGENCE'
        END IF
C
C  CALCULATE THE COEFFICIENTS FOR COMPUTING CREEP
      CALL CCOEF(CNDT,DPEC,DPER,DPEZ,AA,BB,CC)
C
C  CALCULATE THE INCREMENTS OF TOTAL STRESSES AND STRAINS
      CALL CSTRS(CNDP,DSC,DSR,DSZ,DEC,DER,DEZ)
C
      PSCN=ASC+DSC
      PSRN=ASR+DSR
      PSZN=ASZ+DSZ
C
C  CALCULATE THE DEVIATORIC STRESS TENSOR
      SSC=(2.*PSCN-PSRN-PSZN)/3.
      SSR=(2.*PSRN-PSCN-PSZN)/3.
      SSZ=(2.*PSZN-PSCN-PSRN)/3.
```

```
C
C
C
      CALL EESTN(AA,BB,CC,CDEE)
      CMCD=1./CMC
      CKE=CAC*CNC*(CTE**(CNC-1.))*CTI(I)
C  COMPUTE THE EFFECTIVE STRESS FROM CREEP LAW
      CPED=((CDEE/CKE)**CMCD)*(6.895*1.E3)
C  COMPUTE THE CREEP STRAIN INCREMENTS
      CCS=1.5*CDEE/CPED
      CDEC=CCS*SSC
      CDER=CCS*SSR
      CDEZ=CCS*SSZ
C  CHECK THE DIFFERENCE BETWEEN TWO COMPUTATIONS
      CR1=ABS(AA-CDEC)
      CR2=ABS(BB-CDER)
      CR3=ABS(CC-CDEZ)
      CR=AMAX1(CR1,CR2,CR3)
C  CHECK THE CONVERGENCE USING THE DESINGED CRITERION EPS
      IF(CR.GT.EPS) THEN
          AA=CDEC
          BB=CDER
          CC=CDEZ
            GO TO 10
       END IF
C  CALCULATE THE CREEP STRAIN RATE
      CCKE(I)=CAC*CNC*(CTE**(CNC-1.))*((CPED/(6.895*1.E3))**CMC)
C
      PEC(I)=AEC+DEC
      PER(I)=AER+DER
      PEZ(I)=AEZ+DEZ
      ECP(I)=CEC+CDEC
      ERP(I)=CER+CDER
      EZP(I)=CEZ+CDEZ
      PPED(I)=CPED
      PSC(I)=PSCN
      PSR(I)=PSRN
      PSZ(I)=PSZN
C
      CALL EDSTS(PSC(I),PSR(I),PSZ(I),PED(I))
      CALL EESTN(PEC(I),PER(I),PEZ(I),PEE(I))
      CALL EESTN(ECP(I),ERP(I),EZP(I),EEP(I))
      AA=CDEC
      BB=CDER
      CC=CDEZ
      ASC=PSC(I)
      ASR=PSR(I)
      ASZ=PSZ(I)
      AEC=PEC(I)
      AER=PER(I)
      AEZ=PEZ(I)
      CEC=ECP(I)
      CER=ERP(I)
      CEZ=EZP(I)
  11  CONTINUE
      RETURN
      END
C ----------------------------------------------------------------------
```

```
C
C
C
C          ( 25 ) ---- OUTPUT OF THE FINAL RESULTS FOR CREEP PROBLEM
C
      SUBROUTINE CROUT(NTC,NT,CCTE,NT1,NT2,PSC,PSR,PSZ,PEC,PER,PEZ,ECP,E
     $RP,EZP,PED,PEE,EEP,CCKE,PPED)
C
      DIMENSION PSC(NTC),PSR(NTC),PSZ(NTC),PEC(NTC),PER(NTC),PEZ(NTC)
      DIMENSION ECP(NTC),ERP(NTC),EZP(NTC),PED(NTC),PEE(NTC),EEP(NTC)
      DIMENSION CCTE(NTC),CCKE(NTC),PPED(NTC)
      COMMON /AC4/ATM,APP,AED,APE/AC5/ASC,ASR,ASZ,AEC,AER,AEZ
C
      IF(NT1.GT.0) THEN
C  INDICATE THE ORIGINAL STATE WITHOUT CREEP OCCURANCE AT t=0
      CTT=0.0
      WRITE(6,5) CTT
      WRITE(9,5) CTT
    5 FORMAT(///,22X,'<CREEP-TIME DEPENDENT SOLUTION>',/,20X,35('*'),//,
     $80('-'),//,2X,'<ORIGINAL STATE>:',2X,'INITIAL TIME t=',F4.1)
      WRITE(6,20) APP,ATM
      WRITE(9,20) APP,ATM
      WRITE(6,100) ASC,ASR,ASZ,AED
      WRITE(9,100) ASC,ASR,ASZ,AED
      WRITE(6,120) AEC,AER,AEZ,APE
      WRITE(9,120) AEC,AER,AEZ,APE
   20 FORMAT(21X,'PRES-R=',E11.4,2X,'TEMP-T=',F7.2)
      WRITE(6,25)
      WRITE(9,25)
   25 FORMAT(80('='))
C  EXPORT THE FINAL RESULTS INCLUDING CREEP
      DO 10 I=NT1,NT2
      WRITE(6,30) I,CCTE(I),CCKE(I),PPED(I)
      WRITE(9,30) I,CCTE(I),CCKE(I),PPED(I)
 30   FORMAT(/,2X,'STEP(',I3,' ):',2X,'CUMULATIVE TIME=',F8.2,2X,'C-RATE
     $=',E11.4,2X,'CGMA=BR=',E11.4,/)
      WRITE(6,100) PSC(I),PSR(I),PSZ(I),PED(I)
      WRITE(9,100) PSC(I),PSR(I),PSZ(I),PED(I)
      WRITE(6,120) PEC(I),PER(I),PEZ(I),PEE(I)
      WRITE(9,120) PEC(I),PER(I),PEZ(I),PEE(I)
      WRITE(6,140) ECP(I),ERP(I),EZP(I),EEP(I)
      WRITE(9,140) ECP(I),ERP(I),EZP(I),EEP(I)
   10 CONTINUE
        ELSE
      WRITE(6,150)
      WRITE(9,150)
        END IF
  100 FORMAT(1X,'SGMA-C=',E11.4,2X,'SGMA-R=',E11.4,2X,'SGMA-Z=',E11.4,
     $2X,'SGMA-BR=',E11.4)
  120 FORMAT(1X,'EPS--C=',E11.4,2X,'EPS--R=',E11.4,2X,'EPS--Z=',E11.4,
     $2X,'EPS--BR=',E11.4)
  140 FORMAT(1X,'C-EPSC=',E11.4,2X,'C-EPSR=',E11.4,2X,'C-EPSZ=',E11.4,
     $2X,'C-EPSBR=',E11.4,/,80('-'))
  150 FORMAT(/,2X,'( NO OUTPUT FOR CREEP RESULTS. IF YOU WANT, PUT:
     $ NT2 > NT1 > 0 )',/)
C
      RETURN
      END
C ------------------------------------------------------------
```

```
C
C
C
C          ( 26 ) ---- CALCULATE THE COEFFICIENTS FOR COMPUTING CREEP
C
      SUBROUTINE CCOEF(CNDT,DPEC,DPER,DPEZ,AA,BB,CC)
C
      COMMON /D1/BM/ACO/A1,B1,C1,CA10,CA20,CB10,CB20,CA0T,CB0T
      COMMON /AC3/AE,AL/ACL/CLEM
C
      CLEM1=AE*(1.-BM)/((1.+BM)*(1.-2.*BM))
      CLEM=CLEM1*BM/(1.-BM)
      ALCT=AL*CNDT
      AT=DPEC+ALCT
      BT=DPER+ALCT
      CT=DPEZ+ALCT
      A1=-(CLEM1*(AA+AT)+CLEM*(BB+CC+BT+CT))
      B1=-(CLEM1*(BB+BT)+CLEM*(AA+CC+AT+CT))
      C1=-(CLEM1*(CC+CT)+CLEM*(AA+BB+AT+BT))
      CCMM=(CLEM1*CLEM1)-(CLEM*CLEM)
      CA10=CLEM1/CCMM
      CA20=-CLEM/CCMM
      CB10=CA20
      CB20=CA10
      CA0T=-(CA10*A1+CA20*B1)
      CB0T=-(CB10*A1+CB20*B1)
C
      RETURN
      END
C -----------------------------------------------------------------------
C
C
C          ( 27 ) ---- CALCULATE THE INCREMENTS OF TOTAL STRESSES AND
C                      STRAINS INCLUDING CREEP STRAINS IN A TIME INTERVAL
C
      SUBROUTINE CSTRS(CNDP,DSC,DSR,DSZ,DEC,DER,DEZ)
C
      COMMON /C1/CNEF,CNBMF,CNR,CNT/CK/AKK
      COMMON /ACO/A1,B1,C1,CA10,CA20,CB10,CB20,CA0T,CB0T/ACL/CLEM
C
      KA=1.+(AKK*CNR*CA10)/CNT
      DSC=(((1.+AKK*CA20)*CNR*CNDP)/(KA*CNT))-(AKK*CNR*CA0T)/(KA*CNT)
      DSR=-CNDP
      DSZ=CLEM*((CA10+CB10)*DSC+(CA20+CB20)*DSR+CA0T+CB0T)+C1
      DEC=CA10*DSC+CA20*DSR+CA0T
      DER=CB10*DSC+CB20*DSR+CB0T
      DEZ=0.0
C
      RETURN
      END
```

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