Letters

Transient Analysis of Systems With Multiple Nonlinear Elements Using TLM

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***Abstract*—This letter compares the performance of conventional Newton-Raphson (N-R) algorithm with that of the transmission line modeling (TLM) method for the simulation of systems containing multiple nonlinear elements. It is shown that the TLM method offers significant advantages in terms of convergence, accuracy and computational speed.**

***Index Terms*—Newton-Raphson method, nonlinear circuits, simulation.**

I. Introduction

Traditional methods for transient analysis of nonlinear elements in EMTP-type programs include the Compensation Method and the Network Equivalent's Method [1]. The advantage of these methods is that they separate the nonlinear element(s) from the linear part of the network thereby reducing the computational burden by confining the nonlinear solution process to incident nodes only. However, one of their limitations is that in the presence of multiple nonlinear elements, represented analytically, a simultaneous solution of nonlinear equations using Newton-Raphson (N-R) is required. This approach creates a bottleneck in the efficiency as well as the accuracy of simulation. Another drawback is that proper initial conditions are needed to ensure convergence to ac steady-state solution. The speed of convergence is slower if the simulation is started from zero initial conditions.

An alternate approach, transmission line modeling (TLM), for modeling lumped networks containing both linear and nonlinear elements was first proposed by Johns and O'Brien [2]. This method was later used for transient analysis by Hui and Christopoulos [3], [4]. More recently the TLM technique has been used to solve finite element problems [5]. This letter compares the performance (convergence, accuracy and CPU time requirement) of the TLM method with the conventional N-R method for the simulation of systems with multiple nonlinear elements.

II. TLM Model for a Nonlinear Element

Fig. 1(a) shows a nonlinear element R connected by a loss-less transmission line, with surge impedance Z_0 and travel time $\Delta t/2$, to the network. The simulation time-step is Δt. At the nth iteration, the network launches a pulse ${}_nV^i$ into the nonlinear branch, which becomes an incident pulses V_R^i on the nonlinear element at $\Delta t/2$. A reflected pulse produced by the nonlinear element V_R^r becomes the next incident pulse ${}_{n+1}V^r$ on the network at Δt. Let the nonlinear element be described as follows:

$$i_R = f(v_R). \tag{1}$$

Since

$$i_R = \frac{1}{Z_0}\left(V_R^i - V_R^r\right) \quad \text{and} \quad v_R = V_R^i + V_R^r \tag{2}$$

the reflected pulse from the nonlinear element can be obtained from the following equation:

$$V_R^i - V_R^r = Z_0 * f\left(V_R^i + V_R^r\right). \tag{3}$$

Equation (3) is a single nonlinear equation which is independent of the rest of the network and it can be solved by N-R. Similar equations can be developed for other nonlinear elements in the network. A linear inductor and a capacitor can be modeled using a loss-less line with a short-circuit and an open-circuit termination respectively. The node voltage vector ${}_n\mathbf{V}$ at the nth iteration can be obtained by solving

$$\mathbf{Y}_\mathbf{n}\mathbf{V} =_\mathbf{n} \mathbf{J} \tag{4}$$

where $\mathbf{Y}$ is the nodal admittance matrix and ${}_n\mathbf{J}$ is the nodal source vector at the nth iteration. Equation (4) can be solved by a direct matrix inversion of $\mathbf{Y}$; since the nonlinearities in the network have been isolated, the matrix inversion needs to be performed only once at the beginning of the simulation.

III. Case Study

Fig. 1(b) shows a bridge circuit [6] containing four nonlinear resistors and other linear elements. This circuit has been used to compare the performance of the full N-R method with that of the TLM method. The comparison of the two methods was made on the criteria of convergence (number of iterations), accuracy (root-mean-square (rms) error in fundamental current and voltages) and CPU time requirement under two starting conditions: zero initial conditions and matched initial conditions. The two methods were coded in MATLAB and run on a AMD Athlon XP 1.8-GHz processor. Resistors R_3 and R_5 are characterized by the equation $i_R = I_s\left[e^{v_R/V_T} - 1\right]$ whereas the resistors R_2 and R_6 are characterized by $i_R = -I_s\left[e^{-v_R/V_T} - 1\right]$, where $I_s = 10^{-15}$ A and $V_T = 26\mathrm{e}^{-3}$ V. The other circuit parameters are $R_1 = R_4 = 1\ \mathrm{k}\Omega$, $L = 1\ \mathrm{mH}$, $\varepsilon = 50\cos(\omega t)$ and $j = 10^{-3}\cos(\omega t - 90^\circ)$.

Under dc conditions with $\varepsilon = 10$ V, $j = 1$ mA and zero initial conditions, the TLM method converged within three iterations to the final solution of $[v_a\ v_b\ v_c]^T = [6.15\ 0.69\ 5.45]^T$ with an absolute error tolerance of 10^{-5}, whereas the full N-R method failed to converge to the specified tolerance.

Under ac steady-state conditions, Fig. 2(a) and (b) show the percentage rms error in current i_1 and CPU time, for a simulation of 0.03 s, for the two methods starting with zero initial conditions, as the simulation time step Δt is varied from 5 to 600 μs. The high error and high CPU time of the full N-R is obvious from these figures; the y-axis is plotted in log scale to underscore the large difference in values for the two methods. Furthermore, in Fig. 2(a), discontinuities can be observed at certain points on the curve for the full N-R method; these points denote the time-steps where the method did not converge. The TLM method, on the other hand, showed no convergence problems at any time-step. The full N-R method failed to converge beyond $\Delta t = 200\ \mu$s whereas the TLM was convergent all the way up to $\Delta t = 600\ \mu$s.

Fig. 2(c) and (d) show the percentage rms error in i_1 and CPU time for the two methods starting with initial conditions $[v_a\ v_b\ v_c]^T = [6\ 1\ 5]^T$. The full N-R method's error started increasing over 5% around $\Delta t = 10\ \mu$s, however, the TLM method's error stayed less than 3%

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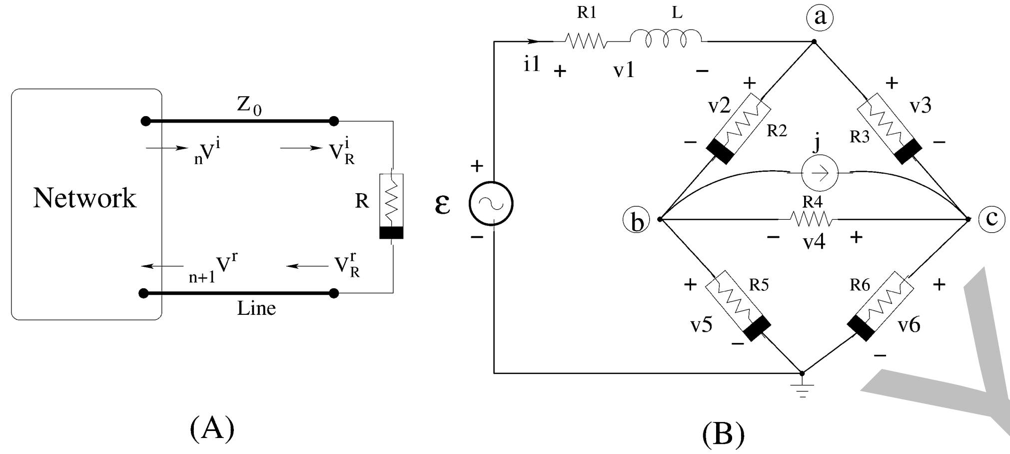

Fig. 1. (a) TLM model of a nonlinear element. (b) Nonlinear resistor bridge.

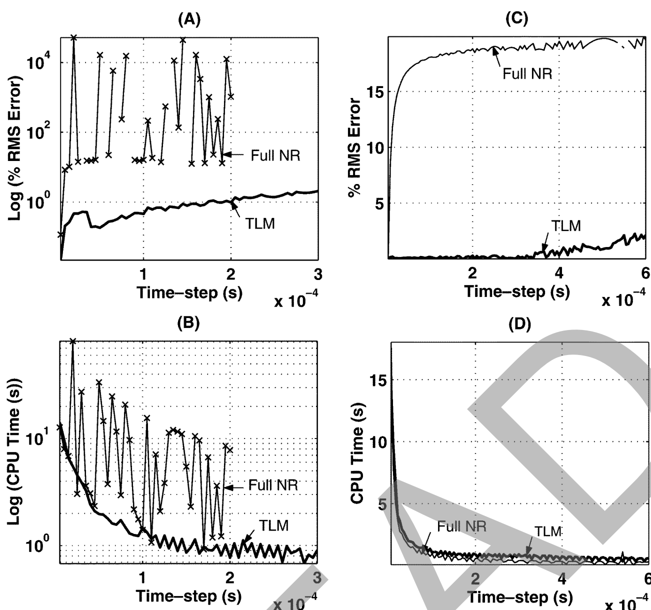

Fig. 2. Comparison of accuracy and cpu time requirement of full N-R and TLM at varying simulation time-step. (a), (b) Zero initial conditions. (c), (d) Matched initial conditions.

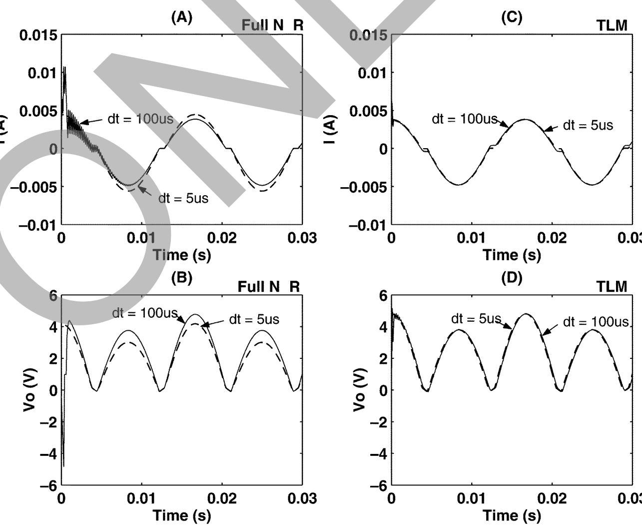

Fig. 3. Comparison of time-domain results. (a), (b) Full N-R. (c), (d) TLM.

up to $\Delta t = 600\ \mu\text{s}$. The large difference in error in the two methods can be seen from Fig. 2(c). The CPU time requirement for the full N-R method under matched initial conditions was found to be fairly close [Fig. 2(d)] to that of the TLM method. Fig. 3 shows the simulation results: input current i_1 and output voltage $v_o (= v_c - v_b)$ -of the bridge circuit for the two methods using two different time steps $\Delta t = 5\ \mu\text{s}$ and $\Delta t = 100\ \mu\text{s}$. The full N-R method [Fig. 3(a) and (b)] at 100 μs exhibits numerical oscillations up to 0.005 s and a steady-state error with respect to the curve obtained at $\Delta t = 5\ \mu\text{s}$. Using the TLM method [Fig. 3(c) and (d)], the results for the two time-steps are almost coincident and numerical oscillations are not noticeable. These time domain results were verified using SPICE and PSCAD/EMTDC.

IV. Conclusions

The TLM method is a stable and accurate modeling method which offers numerous advantages over the conventional N-R method of simultaneously solving nonlinear equations for systems containing multiple nonlinear elements. Among its benefits are faster convergence, higher accuracy, and less CPU time requirement compared to the full N-R method. One of the promising applications of the TLM approach is real-time transient simulation.

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