The *Ten Mathematical Classics*: The Mathematics and Mathematical Education of Pre-Modern China

by

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Abstract

The *Ten Mathematical Classics* were the only imperially prescribed mathematics textbooks in pre-modern China. They were used during the Sui (581 – 618), Tang (618 – 906), and Northern Song (960 – 1127) dynasties at the imperial academy to structure the mathematical training of students. This dissertation explores the contents as well as information about the writers and commentators of these texts in order to arrive at a better understanding of Chinese mathematics and mathematical writers. It also analyzes how mathematical education actually took place, presenting a new perspective on why state-run mathematical education only existed at specific times. Lastly, this dissertation examines the circulation and transmission of the *Ten Mathematical Classics*. My thesis consists of three central points. Firstly, the history of Chinese mathematics should take into serious account the entire corpus of knowledge and endeavours, such as divination, that historical actors associated indivisibly with what we would consider pure mathematical knowledge. Secondly, the known writers, commentators, and readers of the *Ten Mathematical Classics* were all highly educated, many of whom were also government officials, so it is over-simplistic to attribute the lack of long-term state-run mathematical education to a general disdain among the Chinese literati for technical subjects. Thirdly, I argue that the state's decisions to institute or drop mathematical education should be understood within the broader context of the state's needs at the time, and were directly related to the availability of suitable mathematically skilled candidates who could be recruited into the bureaucracy.

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Table of Contents

List of Tables

List of Figures

List of the *Ten Mathematical Classics* of the Tang Dynasty

1. *Zhoubi suanjing* 周髀算經 (Mathematical classic of the gnomon of the Zhou dynasty)
2. *Jiuzhang suanshu* 九章算術 (Nine chapters on the mathematical art)
3. *Haidao suanjing* 海島算經 (Sea island mathematical classic)
4. *Sunzi suanjing* 孫子算經 (Master Sun's mathematical classic)
5. *Zhuishu* 綴術 (Techniques for calculation by combination) – later lost and replaced as one of the ten classics by the *Shushu jiyi* 數術記遺 (Writings on the mathematical art)
6. *Xiahou Yang suanjing* 夏侯陽算經 (Xiahou Yang's mathematical classic)
7. *Zhang Qiujian suanjing* 張丘建算經 (Zhang Qiujian's mathematical classic)
8. *Wucao suanjing* 五曹算經 (Mathematical classic of the five bureaucratic departments)
9. *Wujing suanshu* 五經算術 (Mathematical art of the five classics)
10. *Jigu suanjing* 緝古算經 (Classic on the continuation of ancient mathematics)

Introduction

When Liu Hui 劉徽 (fl. 263 CE) first sat down to write a commentary on a mathematical text that he had been studying since his early youth, it is difficult to imagine that he could have foreseen that he would earn an aristocratic title for his work, or that this title would only come eight centuries after his death. While this belated honour could not have brought Liu Hui himself any material benefit, it demonstrates the long-lasting influence of his contribution to mathematical knowledge in China. That contribution and the mathematical text that he commented upon will play a central role in this narrative.

This dissertation is about the mathematics and mathematical education of China between the sixth and the twelfth century, the only period in pre-modern Chinese history during which competence in mathematics provided a direct entryway into the civil service, regardless of the person's birth. The focus is on the *Ten Mathematical Classics* (*Shibu suanjing* 十部算經), which is the only set of imperially approved mathematics textbooks throughout pre-modern Chinese history, and which structured the state-run mathematical education of this period.[1]

The *Ten Mathematical Classics* refer to a set of ten mathematical treatises that were annotated in the seventh century by the director of the Imperial Bureau of Astronomy and Astrology (*taishi ling* 太史令), Li Chunfeng 李淳風 (602 – 670), and his team of government officials. The initial impetus for such an endeavour came in

[1] From the eighteenth century onwards, it became common to refer to this collection of treatises as *Suanjing shishu* 算經十書 (Mathematical classics in ten books), a name probably first coined by Kong Jihan 孔繼涵 (1739 – 1783), who printed the collection in 1777 under that title. See Guo Shuchun 郭書春 and Liu Dun 劉鈍, *Suanjing shishu* 算經十書 (The Ten Mathematical Classics) (Shenyang: Liaoning jiaoyu chubanshe, 1998), preface – no page number. The name "*Shibu suanjing*" is the term used in the *Tang shu* 唐書 (Standard history of the Tang dynasty) to refer to the collection, and is the earliest known reference to the collection as such contained in official sources.

the year 656 in the form of a decree from the emperor to produce a set of error-free mathematics textbooks, which were subsequently approved for use by the instructors and students of the imperial academy (*guozijian* 國子監). The contents of this collection of textbooks were decidedly eclectic. No effort was made at the time by Li Chunfeng and his team to write new treatises that would address the specific needs of the government, instructors, or the students. Instead, they limited themselves to the mathematical treatises that were already in existence, most of which had been used at the imperial academy under the previous dynasty, in order to meet their needs and expectations of what the mathematics curriculum at the imperial academy should look like. It was this vision that would guide state-sponsored mathematical education for the rest of the Tang 唐 dynasty (618 – 906). When state-run mathematical education was re-instated under the Northern Song 北宋 dynasty (960 – 1127) in the early twelfth century, the *Ten Mathematical Classics* were supplemented with other textbooks, but continued to be the main texts from which students of the imperial academy learnt mathematics.

Among the ten textbooks approved for official use in 656, a few were relatively recent treatises, while others were already centuries old. Together, they covered a formidable range of topics, ranging from elementary astronomy, surveying, taxation, grain conversion, and problems aimed at honing skills in performing arithmetic operations. In general, the questions in these treatises were presented as word problems. Each question was couched in a plausible real-life scenario that required the reader to first think about what method they would have to use to solve it, which is not always obvious to modern eyes. One such example is the problem that gives the reader a woman's age and the number of months she is pregnant, and asks the reader to determine the sex of the unborn baby based on this information.[2] Each question is then followed by the answer and instructions on how to obtain the answer. These instructions tend to be very brief, even abrupt, with no explanation of why the problem should be solved in this particular way or why this method works.

The *Ten Mathematical Classics* were aimed more at teaching a set of calculation techniques than at building a body of mathematical theory in the fashion of the Platonic tradition of ancient Greece. These calculation techniques include square and cube root extraction, area and volume calculations, and application of what is commonly known today as the Pythagorean theorem. In general, pre-modern Chinese mathematics did tend to be more pragmatic and grounded in the real world, but this is not to say that there was absolutely no theoretical aspect to it. If similarity to European traditions must be drawn, pre-modern Chinese mathematics shared similarities with Enlightenment mathematics where Jean-Baptiste le Rond d'Alembert (1717 – 1783) and others thought of nature's structure as fundamentally mathematical, and did mathematics that was tied to the world so as to delve into nature's mysteries.[3]

In the case of pre-modern Chinese mathematics, this is seen most clearly when the authors of mathematical treatises explained in their prefaces what they were trying to achieve with their work, or when other writers explained why the learning of mathematics was essential. Such writings, which will be discussed in detail in the following chapters, generally present a belief that numbers provided a sense of order to natural phenomena and human endeavours, hence a proficiency in the manipulation of numbers would lead to a better understanding of the natural

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[2] *Sunzi suanjing* 孫子算經 (Master Sun's mathematical classic) (Beijing: Zhonghua shuju, 1985), 26.

[3] Amir Alexander, *Duel at Dawn: Heroes, Martyrs, and the Rise of Modern Mathematics*, New Histories of Science, Technology, and Medicine, ed. Margaret C. Jacob and Spencer R. Weart (Cambridge, MA: Harvard University Press, 2010), 8.

world as well as greater competence in the human realm. Furthermore, within the Neo-Confucian tradition that came to dominate Chinese philosophical sensibilities from the thirteenth century onwards, there was a fundamental belief that there existed one universal principle underlying everything in the physical and the social world, a principle that every scholar should strive to understand and then reflect in his every act. Because numbers were part of this world, they were of course under the mantle of the universal principle as well. Therefore, the study of numbers and their manipulation was, for the mathematically inclined, a perfectly legitimate way to attempt to grasp the universal principle. Of course, not all pre-modern Chinese scholars shared this view. In fact, many over the centuries looked down upon mathematics as inferior to the study of the Confucian classics.

Presumably, by studying the questions and solutions in the *Ten Mathematical Classics*, and with the guidance of proficient instructors, the student would be able to generalize what he had learnt to similar problems that he would later encounter and know what to do to solve them. Like the study of the Confucian classics, a certain degree of memorization was required. The text in the *Ten Mathematical Classics* does not enlighten the student on why the methods work, only that they should. The role of the instructor at the imperial academy was most likely to fill in that gap. Self-learning without the help of a teacher would have been difficult, though perhaps not impossible for the gifted or for those who already possessed a certain degree of mathematical proficiency learnt from other texts. In fact, these classics often served as objects of interest for people who were already proficient in mathematics. Even after mathematics was dropped from the official curriculum in 1120, most of the *Ten Mathematical Classics* continued to circulate and remained part of the mathematical discourse of China. Even as late as the nineteenth century, we see learned scholars and accomplished mathematicians like Li Shanlan 李善蘭 (1810 – 1882) arduously studying them in order to learn what previous generations of Chinese scholars had known.[4] When state-run mathematical education was finally re-instituted in the late 1860's, the *Ten Mathematical Classics* once again came to be used as textbooks.

HISTORICAL BACKGROUND

From the above, it is clear that the study of the *Ten Mathematical Classics* cannot be confined to any one century or dynasty if the goal is to be thorough. The simple matter of when the classics first came into being spans centuries. The earliest is believed to date from the first century BCE, while the latest is from the seventh century CE. However, mathematical education at the state level began perhaps much earlier with the Zhou 周 dynasty (1046 – 256 BCE). At the time, there were officials responsible for teaching the six gentlemanly arts (*liu yi* 六藝) to the children of the nobility, including the art of *shu* 數,[5] which could be interpreted variously as mathematics, numerology, divination, astronomy, astrology, and geomancy.[6] The Zhou dynasty was a time when the nobility monopolized government positions, so at least some of them must have needed to acquire some skills in numeracy and calculation in order to perform the necessary administrative tasks such as accounting and surveying. Unfortunately, no record of the mathematical texts used for instruction at the time has survived. Since the Zhou dynasty is believed to pre-date even the earliest of the *Ten Mathematical Classics*, it

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[4] Jean-Claude Martzloff, *A History of Chinese Mathematics*, trans. Stephen S. Wilson (Berlin: Springer, 1997), 173-175.

[5] The other five arts were ritual (*li* 禮), music (*yue* 樂), archery (*she* 射), horsemanship (*yu* 御), and writing (*shu* 書). See Li Yan, and Du Shiran. *Chinese Mathematics: A Concise History*, translated by John N. Crossley and Anthony W.C. Lun (Oxford: Clarendon Press, 1987), 22.

[6] Ho Peng Yoke, *Chinese Mathematical Astrology: Reaching Out to the Stars*, Needham Research Institute Series, ed. Christopher Cullen (London: Routledge Curzon, 2003), 2.

must be assumed that the *Ten Mathematical Classics*, as they are known today, played no role in mathematical education during that period. However, this does not rule out the possibility that earlier versions of several of the ten classics might have been used at the time since those classics with no clear authorship might not have been the work of a single author, but the end result of generations of practitioners adding to and deleting from the same treatise.

Records from this early period remain scarce. During the dynasties that followed, it is not clear how or by whom officials were trained to handle the state's finances. The Sui 隋 dynasty (581 – 618) gives us the earliest substantial record of the teaching and learning of mathematics as part of the state's endeavours. The *Sui shu* 隋書 (Standard history of the Sui) tells us that mathematics was one of the subjects that students of the imperial academy could choose to learn. At one point, there were two teachers, two teaching assistants, and eighty students involved in the teaching and learning of mathematics at the academy.[7]

In 656, Emperor Gaozong 高宗 (r. 650 – 683) of the succeeding Tang dynasty issued the unprecedented decree that called for the production of a standardized and error-free version of useful treatises for the teaching of the students of mathematics at the imperial academy. The ten treatises that Li Chunfeng and his team edited and annotated became the first and only set of officially prescribed mathematics textbooks used at the imperial academy until mathematics was dropped from the curriculum in 1120. In other words, the *Ten Mathematical Classics* formed a complete mathematics curriculum that was aimed at training young men to perform various calculations, most of which are related to the daily administrative tasks performed by the bureaucracy. A separate civil service

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[7] *Sui shu* 隋書 (Standard history of the Sui dynasty) (Beijing: Zhonghua shuju, 1973), 28, 777.

examination was created to test individuals' mathematical abilities, and this examination was open to both the graduates of the state-run institutions and to men who had trained elsewhere. Successful examination candidates could then enter the bureaucracy as the lowest-ranking government officials.[8]

Despite this early commitment to mathematics training, the rest of the Tang dynasty would see frequent interruptions to its state-run mathematics education as this subject was sometimes dropped from the curriculum and then re-instated again. Mathematical training became even more intermittent during the next major dynasty, the Northern Song, as mathematical education was only offered very sporadically at the imperial academy towards the very end of the dynasty. However, the Northern Song greatly expanded state-funded education by opening a large number of prefectural and county schools (*zhouxue* 州學 and *xianxue* 縣學).[9] Access to state-funded education was thus no longer limited to the imperial capital. The Northern Song government also had the *Ten Mathematical Classics* printed by woodblock technology in 1084, and copies might have been distributed to prefectural and county schools.

The time period from the late sixth to the early twelfth century is thus the focus of my research. Although dynasties are usually thought to be discrete entities, the use of the *Ten Mathematical Classics* as textbooks was an example of continuity across dynasties. Therefore, to not investigate this entire period would be putting arbitrary time boundaries on a practice that spanned centuries. It was during this period that the *Ten Mathematical Classics* were known to have formed a complete mathematics curriculum for the students of the imperial academy. This is also the

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[8] Li and Du, *Chinese Mathematics*, 106.

[9] Richard L. Davis, "Custodians of Education and Endowment at the State Schools of the Southern Sung," *Journal of Song-Yuan Studies* 25 (1995): 97-98.

period in pre-modern Chinese history that was unique for the state's demonstrated interest in mathematics education and its willingness to take real action to see it implemented. What prompted this interest and what ended it are questions that have very much to do with what the *Ten Mathematical Classics* were perceived as being able to offer to the state.

PURPOSE AND QUESTIONS

The contents of the *Ten Mathematical Classics* are therefore of particular interest to this study. What knowledge and skills did they impart? Why were these ten particular treatises chosen to be the official textbooks? Why were they never replaced by newer or more sophisticated treatises? How did people teach and learn from them?

The *Ten Mathematical Classics* are almost the only Chinese mathematical treatises from before the mid-seventh century to have survived to the present day. The overwhelming majority of early mathematical texts have been irretrievably lost largely due to wars and neglect. The only exceptions to this for the period up to 656 are the *Ten Mathematical Classics* and a handful of short mathematical texts that pre-date the *Ten Mathematical Classics*.[10] Among these shorter texts, the one that has garnered the most scholarly attention so far is the *Suanshu shu* 算數書 (Writings on reckoning). The *Suanshu shu* is the earliest known Chinese text that contains substantial mathematical content to have survived. No mention of the *Suanshu shu* has been found in any historical record, and this text became known to the academic world only in 1983 when one copy in the form of bamboo strips was excavated from

[10] Endymion Wilkinson, *Chinese History: A New Manual*, 4th ed., Harvard-Yenching Institute Monograph Series, vol. 100 (Cambridge, MA: Harvard University Asia Centre, 2015), 491.

a second-century BCE tomb at Zhangjiashan 張家山 in the province of Hubei.[11] No other copy of the *Suanshu shu* has been found to date, suggesting that the usage and circulation of this text were probably quite limited, thus leaving the *Ten Mathematical Classics* as our main portal into early Chinese mathematics.

Because the *Ten Mathematical Classics* were closely tied to the training of officials, investigating their contents will give us insight into what officials were expected to be able to do during that period in Chinese history. What did an education in mathematics based on the *Ten Mathematical Classics* enable people to accomplish? What kinds of problem were perceived as being particularly necessary to the training of officials? How were these problems linked to the actual needs of society at the time? What can we say about the society in which the *Ten Mathematical Classics* were first written? Furthermore, what can we say about the society that saw them incorporated into state-run education, and also the one that finally gave up on state-funded mathematics education?

Another part of this study concerns the people whose lives were touched by the *Ten Mathematical Classics*. Who wrote these treatises, and why did they do it? Who read these treatises, and what motivated them to seek the knowledge contained within? Who wrote the mathematics civil service examinations, and what

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[11] Ma Li, "Should the History of Chinese Mathematics be Rewritten?", *Asian and African Studies* 14, no. 3 (2010): 78-79; Bai Shangshu 白尚恕, "Jinnian lai Zhongguo xuezhe dui Zhongguo shuxue shi yanjiu de gaikuang yu zhanwang 近年來中國學者對中國數學史研究的概況與展望" (A summary of recent Chinese research in the history of mathematics and future prospects), in *Zhongguo shuxue shi yanjiu: Bai Shangshu wenji* 中國數學史研究：白尚恕文集 (Research in the history of Chinese mathematics: Bai Shangshu's writings), ed. Li Zhonglai 李仲來 (Beijing: Beijing shifan daxue chubanshe, 2008), 3-4. For translations of the *Suanshu shu* into English, see Christopher Cullen, *The Suan shu shu* 算數書 *"Writings on Reckoning": A Translation of a Chinese Mathematical Collection of the Second Century BCE, with Explanatory Commentary*, Needham Research Institute Working Papers (Cambridge: Needham Research Institute, 2004), and Joseph W. Dauben, "算數書 *Suan Shu Shu* A Book on Numbers and Computations: English Translation with Commentary", *Archive for History of Exact Sciences* 62, no. 2 (2008): 91-178.

opportunities did examination success open up to them? How did the unique presence of such an examination during the Tang dynasty affect the lives of the educated?

The third part of this study involves the actual production and circulation of the *Ten Mathematical Classics*. Where and by whom were the classics printed and copied? How widely and by what routes did the classics circulate both within China and across the rest of East Asia? How did their circulation impact the shape of mathematical discourse and the legacy of pre-modern Chinese mathematics?

The above questions outline a research programme that pushes the study of the *Ten Mathematical Classics* beyond their technical aspects. The aim here is to achieve a thorough understanding of these texts that is not merely limited to the words contained within them, but will encompass the human actors associated with them and the context in which they were embedded. While I am not arguing that the mathematical facts and knowledge contained in the *Ten Mathematical Classics* are a direct product of Chinese culture, the sorts of questions and the type of mathematics in them are certainly influenced by the society and culture in which they arose.

WORKING ASSUMPTIONS AND THESIS

This approach to the history of mathematics follows a recent trend both within the field and in the broader history of science, which sees the sciences as an inseparable part of the continuum of human activity, and thus seeks to provide a more integrated narrative to reflect this. This new trend is probably best encapsulated in the title of Steven Shapin's book that argues against thinking about science as a purely objective phenomenon: *Never Pure: Historical Studies of*

Science as if It was Produced by People with Bodies, Situated in Time, Space, Culture, and Society, and Struggling for Credibility and Authority.[12]

More traditional histories of the sciences tend to focus exclusively on discoveries and the people who made them, producing a perspective that the only matters of consequence are new ideas and the extraordinary people who discovered them. The resulting picture is that of scientific ideas and scientific practitioners operating in isolation of the people and the environment surrounding them. To most historians of science up to the 1960's, the fact that the sciences were done differently in different parts of the world seemed merely incidental, being an accident of history that required no explanation because science was viewed as an objective field of knowledge that was free of human influence. Their focus was on the history of ideas, and their work tends to be accounts of when and by whom elements of modern science were discovered.

Throughout the twentieth century, more and more historians of science began to more systematically include so-called external factors into consideration. They began to see scientific activity as a response to the need to solve problems in the real world. Therefore, the notion of context became a recurrent theme of historical narratives as scholars explored how scientific practitioners were affected by their political, social, and cultural surroundings, and how this effect was reflected in their work. The context and the science became linked in a cause-and-effect relationship, with the two understood to be easily identifiable and clearly distinct from each other.

An early proponent of this approach is Soviet physicist Boris Hessen (1893 – 1936), whose seminal paper "The Social and Economic Roots of Newton's *Principia*", presented at the Second International Congress of the History of Science in London

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[12] Steven Shapin, *Never Pure: Historical Studies of Science as if It was Produced by People with Bodies, Situated in Time, Space, Culture, and Society, and Struggling for Credibility and Authority* (Baltimore: Johns Hopkins University Press, 2010).

in 1931, argued that the mathematics and physics problems that Isaac Newton (1642 – 1727) chose to address arose out of the economic and technical needs of his time, such as the needs of navigation and ballistics.[13] Recent advocates of more sophisticated examples of this approach include Barry Barnes, David Bloor, and Bruno Latour, who apply sociological methods to analyze the history of science and the production of scientific knowledge.[14]

At the same time, other scholars have become inclined to think that the context and the science are actually so intertwined that it is impossible to separate them into two distinct wholes. Representative of this type of approach is Geoffrey E.R. Lloyd and Nathan Sivin's *The Way and the Word: Science and Medicine in Early China and Greece*. In their comparative study of how the sciences developed in early China and Greece, Lloyd and Sivin are primarily concerned with exploring what they call a "cultural manifold", by which they mean all the intellectual, social, and institutional dimensions from which a scientific problem arises and in which practitioners work.

> [They] do not think of social factors as determining thought nor of ideas as changing society. These are not external causes. Thinkers respond to, but also influence, institutions and prevalent values. Thus [they] do not speak of inquiry in context. Context is not an autonomous setting that may or may not be connected to inquiry.[15]

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13 For Hessen's paper, see Boris Hessen, "The Social and Economic Roots of Newton's 'Principia'", in *Science at the Cross Roads*, ed. Nicolai Bukharin (London: Frank Cass, 1971 reprint). For an analysis of Hessen's paper, see Loren R. Graham, "The Socio-Political Roots of Boris Hessen: Soviet Marxism and the History of Science", *Social Studies of Science* 15, no. 4 (Nov 1985): 705-722.

14 For Barnes and Bloor's Strong Programme of the sociology of knowledge, see David Bloor, *Knowledge and Social Imagery*, 2nd ed. (Chicago: University of Chicago Press, 1991), and Zheng-Feng Li, Ruey-Chyi Hwang, and Chih-Tung Huang, "Go Strong or Go Home: An Interview with David Bloor", *East Asian Science, Technology and Society: An International Journal* 4, no. 3 (2010): 419-432. For Latour's Actor-Network-Theory, see Bruno Latour, *Reassembling the Social: An Introduction to Actor-Network-Theory* (Oxford: Oxford University Press, 2005).

15 Geoffrey E.R. Lloyd and Nathan Sivin, *The Way and the Word: Science and Medicines in Early China and Greece* (New Haven: Yale University Press, 2002), 3.

In other words, the relationship between scientific ideas and their context is much more complex than mere cause-and-effect. Lloyd and Sivin argue that the way individuals think and approach inquiries into natural phenomena is ultimately a product of the society in which they live and work. However, these individuals in turn influence and shape society and the institutions within it. Therefore, the question of *why* the sciences developed in a certain way in a particular society becomes of utmost interest to historians taking this type of approach.

This approach is not new to the field of the history of mathematics. Eleanor Robson's prize-winning book, *Mathematics in Ancient Iraq: A Social History*, already leads the way. In her study of ancient Mesopotamian mathematics, Robson emphasizes the need to view mathematical ideas and practitioners as being situated within their respective societies because "the ways in which mathematics is conceptualized, described, and discussed are culturally bounded."[16] She points out that "although mathematics is most immediately the product of individuals, those individuals are shaped and constrained by the society in which they live, think, and write."[17] Therefore, in her work, there is also a sense that different pre-modern societies developed mathematics differently because their people have been shaped to think differently and to pursue different types of goals. This is the fundamental assumption that guides the present study.

This dissertation, however, is not a history of how mathematical ideas and concepts developed in China. My goal is not to point out how sophisticated early Chinese mathematics was, or to highlight all the instances when the Chinese discovered an idea earlier than everyone else in the world. That has already been

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[16] Eleanor Robson, *Mathematics in Ancient Iraq: A Social History* (Princeton: Princeton University Press, 2008), 45.

[17] Ibid, 8.

thoroughly and expertly done by previous scholarship. Instead, the emphasis here is on exploring why the contents of the *Ten Mathematical Classics* are such as they are, why they were chosen in the year 656 to train future officials, and why they were deemed to have outlived their usefulness by 1120. What was it about the society and the people in it that saw such a fate for the ten classics? The answers to these questions will yield a more enlightened conceptualization of early Chinese mathematics and its place in society up to the twelfth century that goes beyond a chronology of Chinese mathematical discoveries and their discoverers.

It must be acknowledged, however, that this conceptualization can only be incomplete because it is based only on the *Ten Mathematical Classics*, which consist only a small fraction of the vast corpus of mathematical texts written in China. It must also be acknowledged that the *Ten Mathematical Classics* count among the privileged texts in the history of Chinese mathematics. More effort had been expended on preserving them over the centuries than any other mathematical text in pre-modern China,[18] and there is no reliable way of ascertaining how representative the contents of the *Ten Mathematical Classics* were of those of their contemporary mathematical treatises. Nevertheless, because they were chosen to form a complete mathematics curriculum at the imperial academy during the Tang dynasty, the *Ten Mathematical Classics* represented a complete corpus of useful mathematics to the state and the students who relied upon these texts to enter the civil service. Therefore, it is fair to say that an analysis of the *Ten Mathematical Classics'* contents and usage in the state-run mathematical education system will

[18] Karine Chemla, "A Chinese Canon in Mathematics and Its Two Layers of Commentaries: Reading a Collection of Texts as Shaped by Actors", in *Looking at it from Asia: The Processes that Shaped the Sources of History of Science*, Boston Studies in the Philosophy of Science, vol. 265, ed. Florence Bretelle-Establet (New York: Springer, 2010), 183.

yield a valid picture of what mathematics meant to the writers and readers of these texts, as well as mathematics' role in the socio-political world of mid-imperial China.

The goals of this dissertation are three-fold. Firstly, it examines the contents as well as the writers and commentators of the *Ten Mathematical Classics*. This examination is aimed at uncovering the type of mathematics that was practised by these particular writers on the one hand, and the mathematical knowledge and skills that were considered relevant by the state to the training of necessary personnel on the other. Furthermore, it will reveal the types of people who became mathematical writers in pre-modern China up to the mid-seventh century.

Secondly, this dissertation explores the evolution of mathematical education and the role played by the *Ten Mathematical Classics* in that evolution. As noted earlier, little is known about how mathematical education in China took place before the Sui dynasty. During intermittent periods of the Sui, Tang, and Northern Song dynasties, the government created a specialized school within the imperial academy to teach mathematics. Decisions to create and abolish this school, as well as the choice of textbooks to use, should not be seen as random or natural decisions that demand no explanation. While the lack of relevant primary sources makes it difficult to pinpoint the motivations of the historical actors, contextualizing the history of state-run mathematical education within the broader history of state-run education in general, in addition to the broader history of mathematical education within the whole of Chinese society, will shed light on the impetus and rationale behind the way state-run mathematical education developed in China.

Thirdly, this dissertation traces the circulation and transmission of the *Ten Mathematical Classics* within China and across the rest of East Asia up to modern times. It expands beyond mathematical history's traditional preoccupation with only the intellectual aspects by following the history of book production and the circulation of the *Ten Mathematical Classics*.

As the next section will demonstrate, traditional narratives of pre-modern Chinese mathematics have focused on the knowledge and skills contained in extant mathematical texts including the *Ten Mathematical Classics*, and expounded eloquently on what the Chinese knew and were able to do. Previous scholarship that discusses the history of mathematical education in pre-modern China tends to focus exclusively on the state-run system during the Sui, Tang, and Northern Song, and explain that disdain among the literati towards technical subjects meant that mathematical education was considered unimportant, and therefore state efforts to sustain it never lasted for long. However, this type of explanation fails to account for why mathematical education *was* in fact offered by the state at particular points in time.

This dissertation therefore presents a reassessment of the available evidence and a new interpretation of the history of mathematics and of mathematical education in China. My thesis consists of three central points:

1. The history of mathematics can and should be expanded beyond the modern perspective of what counts as mathematics, and take into serious account the entire corpus of knowledge and endeavours that historical actors associated indivisibly with what we would consider pure mathematical knowledge. In the case of mid-imperial China, this includes the predictive capabilities made possible by various calculation techniques, as exemplified in the problem discussed earlier about predicting the gender of an unborn child.
2. An examination of the known writers, commentators, and readers of the *Ten Mathematical Classics* both within China and the rest of the East Asia will show very clearly that they were all highly educated, many of whom were

also government officials. Therefore, it is over-simplistic to attribute the lack of long-term state-run mathematical education to a general disdain among the literati for technical subjects.

3. Whereas state-run mathematical education is ubiquitous in modern times, our views about its necessity and perhaps even inevitability should not be projected onto pre-modern times. Therefore, the presence of state-run mathematical education in pre-modern China deserves just as much explanation as its absence. I argue that the state's decisions to institute or drop mathematical education should be understood within the broader context of the state's needs at the time, and were directly related to the availability of suitable mathematically skilled candidates who could be recruited into the bureaucracy.

STATE OF THE FIELD

In 2002, Chinese historian of mathematics Qu Anjing 曲安京 presented a paper to the International Congress of Mathematicians in which he identified two approaches that have dominated the field in China. The first approach focused on discovering *what* mathematics had been done during the pre-modern period, and was exemplified in the works of Li Yan 李儼 (1892 – 1963) and Qian Baocong 錢寶琮 (1892 – 1974), the pioneer historians of mathematics in China.[19] The second approach, in contrast, aimed at recovering *how* mathematics had been done in the

[19] Qu Anjing, "The Third Approach to the History of Mathematics in China", in *Proceedings of the International Congress of Mathematicians, Beijing, Aug 20 – 28, 2002*, vol. 3, 948.

pre-modern period, and was first proposed by Wu Wenjun 吳文俊 (1919 – 2017) in the 1980's.[20]

In the paper, Qu advocated a third approach to the history of Chinese mathematics: explaining *why* mathematics was done in the pre-modern period. He further stated that "once the three approaches have merged into a single whole, the research paradigm will shift from *mathematics in history* to the *history of mathematics*"[21] (emphasis added). By this, Qu probably meant that by limiting the historical focus on what and how mathematics was done, a historian's objects of study would only be the mathematical texts and instruments that have survived from the past, and the history he or she writes would inevitably be episodic, or what Jacqueline Stedall calls "stepping-stone history".[22] Stedall is similarly critical of this type of historical narrative, which she does not consider true history:

> Without the attempt [to trace the ground between the stepping stones], there is no history, only the series of anecdotes on which much of the popular history of mathematics is still too often based.[23]

For the history of Chinese mathematics, while it remains difficult due to the lack of sources to embed all the so-called "stepping stones" in a fully integrated and complete narrative, much progress has certainly been made since the field's earliest days. Within China, Li Yan and Qian Baocong made lasting contributions to the field by uncovering what mathematics had been done in pre-modern China. They both wrote broad-sweeping histories of pre-modern Chinese mathematics, thus opening up the field for later scholars to pursue different questions and more in-depth

[20] Qu, "The Third Approach", 950. For a study in English of Wu Wenjun's life and work, see Jiri Hudecek, *Reviving Ancient Chinese Mathematics: Mathematics, History and Politics in the Work of Wu Wen-Tsun* (New York: Routledge, 2014).

[21] Qu, "The Third Approach", 954.

[22] Jacqueline Stedall, *The History of Mathematics: A Very Short Introduction*, Very Short Introductions Series (Oxford: Oxford University Press, 2012), 9-12.

[23] Ibid, 11-12.

research.[24] Qian had also compiled a critical and punctuated edition of the *Ten Mathematical Classics* in 1963.[25]

Around the same time, Yan Dunjie 嚴敦傑 (1917 – 1988), another first-generation Chinese historian of mathematics, published an essay on the history of mathematical education in China in which he followed the type of periodization that had become standard in China by that time:

- Ancient period (Spring and Autumn 春秋 [722 – 476 BCE] – 1840)
- Early modern period (1840 – 1949), which is further subdivided into two segments:
    - 1840 – 1919
    - 1919 – 1949
- Modern period (1949 – present)

For the ancient period, Yan argued that whereas state-run mathematical education was aimed at training a very small minority to do the basic calculations that were necessary for administrative tasks, private education in mathematics had a long tradition in China, and that many of China's most brilliant mathematical achievements were made by people who were privately trained.[26]

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[24] For Li Yan's work, see for example Li Yan, *Zhongguo suanxue shi* 中國算學史 (History of Chinese mathematics) (Shanghai: Shanghai shudian, 1938), and Li Yan and Du Shiran. *Chinese Mathematics: A Concise History*, trans. John N. Crossley and Anthony W.C. Lun (Oxford: Clarendon Press, 1987). For Qian Baocong's work, see for example Qian Baocong, *Zhongguo suanxue shi* 中國算學史 (A history of Chinese mathematics) (Beijing: Kexue chubanshe, 1964).

[25] Qian Baocong, *Suanjing shishu* 算經十書 (The Ten Mathematical Classics) (Beijing: Zhonghua shuju, 1963).

[26] Yan Dunjie, "Zhongguo shuxue jiaoyu jianshi 中國數學教育簡史" (A short history of Chinese mathematical education), *Shuxue tongbao* 數學通報 (Bulletin of Mathematics) 8 (1965): 45-46.

Following a hiatus during the Cultural Revolution (1966 – 1976), Chinese publications on the history of mathematics, and specfically on a few of *the Ten Mathematical Classics*, burgeoned. In the 1980's, Wu Wenjun famously diverged from Qian Baocong's interpretation of how certain problems were solved in the *Ten Mathematical Classics*.[27] A new critical edition of the *Ten Mathematical Classics* by Guo Shuchun 郭書春 and Liu Dun 劉鈍 was published in 1998.[28]

One of the *Ten Mathematical Classics*, the *Jiuzhang suanshu* 九章算術 (Nine chapters on the mathematical art), has generated especial interest among Chinese historians because of its influence of later mathematical writers and because it is considered the most sophisticated of all extant early Chinese mathematical texts.[29] The *Jiuzhang* contains 246 problems that deal with topics like field measurement, calculation of comodity prices, and division of labour. Its current form is most likely the result of cumulative effort by several writers, but it had assumed definite shape by the third century since extensive annotations are known to have been made to it in the year 263 by Liu Hui, who is mentioned at the beginning of this Introduction. The *Jiuzhang* is perhaps the most influential treatise in the history of Chinese mathematics. Its format provided the model for many of the later mathematical treatises. At least five translations of the *Jiuzhang* into modern Chinese have been published by Chinese historians of mathematics like Bai Shangshu 白尚恕 and Shen

[27] The major sticking point was whether or not pre-modern Chinese had ever used the notion of similar triangles and its related properties. This will be discussed in more detail in Chapter One.

[28] Guo Shuchun and Liu Dun, *Suanjing shishu* 算經十書 (The Ten Mathematical Classics) (Shenyang: Liaoning jiaoyu chubanshe, 1998).

[29] For the *Jiuzhang suanshu's* influence on later writers, see Chapters Four and Five. For Chinese historians' assessment of the *Jiuzhang* representing the pinnacle of early Chinese mathematics, see for example Guo Shuchun, *Gudai shijie shuxue taidou Liu Hui* 古代世界數學泰斗劉徽 (The Pinnacle of Ancient World Mathematics: Liu Hui) (Jinan: Shandong kexue jishu chubanshe, 1992), 4.

Kangshen 沈康身.[30] In-depth studies of the mathematics contained in the *Jiuzhang*'s original text as well as Liu Hui's commentary have been done by scholars like Guo Shuchun.[31]

Several of the rest of the *Ten Mathematical Classics* have also been studied by Chinese scholars as individual texts. These include Qu Anjing's translation of the *Zhoubi suanjing* 周髀算經 (Mathematical classic of the gnomon of the Zhou dynasty) into modern Chinese,[32] as well as Ji Zhigang's 紀志剛 annotations on the *Sunzi suanjing* 孫子算經 (Master Sun's mathematical classic), *Zhang Qiujian suanjing* 張丘建算經 (Zhang Qiujian's mathematical classic), and *Xiahou Yang suanjing* 夏侯陽算經 (Xiahou Yang's mathematical classic).[33]

Outside China, the historiography of Chinese mathematics has developed along very similar patterns as within China. Several broad surveys have been written by scholars including Mikami Yoshio (1875 – 1950), Joseph Needham (1900 – 1995), and Jean-Claude Martzloff. Mikami did his research during the opening decades of the twentieth century, and he experienced "considerable hardship in procuring [reference] materials" for writing a history of Chinese and Japanese mathematics.[34] Therefore, he refrained from calling his book a "history".[35] His book

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30 Chemla, "A Chinese Canon", 186. For Bai's translation, see Bai Shangshu, *Jiuzhang suanshu jin yi* 九章算術今譯 (A translation of the Jiuzhang suanshu into modern vernacular) (Jinan: Shandong jiaoyu chubanshe, 1990). For Shen's translation, see Shen Kangshen, *Jiuzhang suanshu dao du* 九章算術導讀 (A guide to reading the Jiuzhang suanshu) (Hankou: Hubei jiaoyu chubanshe, 1997).

31 See Guo Shuchun, *Gudai shijie shuxue taidou Liu Hui*.

32 Qu Anjing, *Zhoubi suanjing xin yi* 周髀算經新議 (A new discussion of the *Zhoubi suanjing*) (Xi'an: Shaanxi renmin chubanshe, 2002).

33 Ji Zhigang, *Sunzi suanjing, Zhang Qiujian suanjing, Xiahou Yang suanjing dao du* 孫子算經，張邱建算經，夏侯陽算經導讀 (A guide to reading *Sunzi suanjing, Zhang Qiujian suanjing,* and *Xiahou Yang suanjing*) (Wuhan: Hubei jiaoyu chubanshe, 1999).

34 Mikami Yoshio, *The Development of Mathematics in China and Japan* (New York: Chelsea Publishing Company, 1913), vi-vii.

35 Ibid, vi.

consists largely of separate discussions of individual texts and writers, including most of the *Ten Mathematical Classics* as well as many of the best-known mathematical works from after the eleventh century.

In contrast, Joseph Needham and his earlier collaborators' work on the monumental *Science and Civilization in China* series sought to embed the history of Chinese sciences, including mathematics, into a single coherent narrative that highlighted what they saw as Chinese contributions to modern science with the aim of answering the famous Needham question of why modern science did not arise in China.[36] Not only did the *Science and Civilization in China* project show the world that science was not a uniquely Western product,[37] the Needham question very much set the tone for many historians' approach to the history of Chinese sciences for several decades. Whereas Needham had suggested factors like China's "bureaucratic feudalism" and lack of a fully developed money economy as inhibitors to the rise of modern science in China,[38] scholars like Ho Peng Yoke have sought other answers to the Needham question,[39] while others like Nathan Sivin see the question as having heuristic value for the exploration of the history of Chinese sciences in general, but not as an ultimate question for historical inquiry.[40] The thirteenth volume of the history of science journal *Osiris* published in 1998 was entitled "Beyond Joseph Needham", serving simultaneously as a tribute to

[36] Joseph Needham et al., *Science and Civilization in China* (Cambridge: Cambridge University Press, 1954 -. The first part of Volume 3 is devoted to the history of Chinese mathematics.

[37] Morris Low, "Beyond Joseph Needham: Science, Technology, and Medicine in East and Southeast Asia", *Osiris* 13 (1998): 1.

[38] See Joseph Needham, Kenneth Girdwood Robinson, and Ray Huang, *General Conclusions and Reflections*, vol. 7, pt. 2 of *Science and Civilization in China*, ed. Kenneth Girdwood Robinson (Cambridge: Cambridge University Press, 2004).

[39] See for example Ho Peng Yoke, "Chinese Science: The Traditional Chinese View", *Bulletin of the School of Oriental and African Studies* 54, no. 3 (Oct 1991): 506-519.

[40] See for example Nathan Sivin, "Why the Scientific Revolution did not Take Place in China – or Didn't It?", *Chinese Science* 5 (June 1982): 45-66.

Needham's unparalleled contribution to the history of Chinese sciences, as well as a signal that the field has by and large moved beyond considerations of the Needham question.

Jean-Claude Martzloff's more recent history of Chinese mathematics provides an updated version of the topic in comparison to the mathematics section of the *Science and Civilization in China*.[41] In Martzloff's estimation, the *Ten Mathematical Classics'* level of difficulty is low, especially in relation to later works from the thirteenth and fourteenth centuries.[42]

Like the situation in China, few scholars outside China have undertaken to focus their study on the *Ten Mathematical Classics* as a whole. A notable exception to this is Robert Schrimpf's doctoral dissertation in French, *La collection mathematique "Souan king che chou"* 算經十書, which provides translations of all the problem statements contained in the *Ten Mathematical Classics*.[43]

Nevertheless, several of the *Ten Mathematical Classics* have garnered a great deal of scholarly attention. The *Zhoubi suanjing* was translated into French by Edouard Biot (1803 – 1850) in 1841.[44] A translation and detailed study in English has been done by Christopher Cullen, which points out that the *Zhoubi* is "the only rationally based and fully mathematized account of a flat earth cosmos" known around the world.[45]

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41 See Martzloff, *A History of Chinese Mathematics*.

42 Ibid, 16-17.

43 See Robert Schrimpf, La collection mathematique "Souan king che chou" 算經十書. PhD diss. Rennes: Université de Rennes, 1963.

44 Chemla, "A Chinese Canon", 184.

45 Christopher Cullen, *Astronomy and Mathematics in Ancient China: The Zhou bi suan jing* (Cambridge: Cambridge University Press, 1996), xi.

Translations of the *Jiuzhang suanshu* have been done by Kurt Vogel into German in 1968,[46] into Japanese by Kawahara Hideki in 1980,[47] jointly by Shen Kangshen, John Crossley, and Anthony Lun into English in 1999,[48] and jointly by Karine Chemla and Guo Shuchun into French in 2004.[49] Another notable English-language study is Joseph W. Dauben's article "Ancient Chinese Mathematics: The *Jiu Zhang Suan Shu* vs. Euclid's *Elements*. Aspects of Proof and the Linguistic Limits of Knowledge", which employs the comparative approach to analyze why Chinese mathematics developed the way that it did, concluding that linguistic limitations were part of the reason the Chinese never developed a highly theoretical axiomatic mathematics like the Greeks.[50]

A close study of the *Haidao suanjing* 海島算經 (Sea island mathematical classic), which focuses on surveying problems and was written by Liu Hui, the first known commentator of the *Jiuzhang suanshu*, has been done by Frank J. Swetz.[51] Liu Hui had originally appended it to the *Jiuzhang* as a supplement, but the *Haidao suanjing* had become a stand-alone text by the Tang dynasty. Another of the *Ten Mathematical Classics*, the *Sunzi suanjing*, has been translated into English by Lam

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46 See Kurt Vogel, *Neun Bücher arithmetischer Technik* (Braunschweig: Friedr. Vieweg & Sohn, 1968).

47 See Kawahara Hideki, "Ryuki chu kyusho sanjutsu" (Liu Hui's commentary on the *Jiuzhang suanshu*), in *Chogoku tenmon gaku sugaku shu* (Collection of Chinese astronomical and mathematical texts) (Tokyo: Asahi shuppansha, 1980).

48 See Shen Kangshen, John N. Crossley, and Anthony W.C. Lun, *The Nine Chapters on the Mathematical Art: Companion and Commentary* (Oxford: Oxford University Press, 1999).

49 See Karine Chemla and Guo Shuchun, *Les neuf chapitres. Le classique mathématique de la Chine ancienne et ses commentaires* (Paris: Dunod, 2004).

50 See Joseph W. Dauben, "Ancient Chinese Mathematics: the *Jiu Zhang Suan Shu* vs. Euclid's *Elements*. Aspects of Proof and the Linguistic Limits of Knowledge", *International Journal of Engineering Science* 36, no. 12/14 (1998): 1339-1360.

51 See Frank J. Swetz, *The Sea Island Mathematical Manual: Surveying and Mathematics in Ancient China* (University Park: Pennsylvania State University Press, 1992).

Lay Yong and Ang Tian Se.[52] Six problems from the *Jigu suanjing* 緝古算經 (Classic on the continuation of ancient mathematics) have been translated into English by Tina Su-lyn Lim and Donald B. Wagner.[53]

My dissertation offers the first comprehensive study in English of the *Ten Mathematical Classics* as a complete collection. While the significance of these texts to the intellectual history of mathematics has been extensively analyzed, a systematic attempt to situate them and their authors, commentators, and readers within the intertwined histories of mathematics, mathematical education, and book production has not yet been done. Therefore, instead of approaching the ten classics as just works of mathematics to be examined in a technical manner, I will treat them in a way that will bring into focus how these texts as well as their authors, commentators, and readers fit into the world around them.

OUTLINE OF CHAPTERS

This dissertation is organized into five chapters. Chapter One lays out the contents of each of the *Ten Mathematical Classics*. It analyzes the range of topics covered, the specific knowledge and skills that they impart, and the commentaries that were included in the official textbook version of the Tang dynasty. A selection of problems is explained in detail using modern terminology and symbols to highlight what and how problems were solved by the writers of the *Ten Mathematical Classics*. The use of modern mathematical terminology and symbols is admittedly anachronistic. However, the method used closely follows the one given in the

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[52] See Lam Lay Yong and Ang Tian Se, *Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China* (River Edge: World Scientific, 2004).

[53] See Tina Su-lyn Lim and Donald B. Wagner, "Wang Xiaotong on Right Triangles: Six Problems from 'Continuation of Ancient Mathematics' (Seventh Century AD)", *East Asian Science, Technology, and Medicine* 37 (2013): 12-35.

original source, and the reader's ease of understanding outweighs considerations to be authentic here. Chapter One's goal is to arrive at a picture of how the authors and commentators of the *Ten Mathematical Classics* conceptualized mathematics and what they felt was the function of mathematics.

Chapter Two focuses on the writers, compilers, and commentators of the *Ten Mathematical Classics* whose work became part of the officially prescribed textbooks in the year 656. While the authorship of several of these classics remains unknown, the others are known to have been written by men about whom limited biographical information can be gleaned from the official records. This exposition of these writers' lives and careers is meant to uncover the types of people who wrote about mathematics in pre-modern China up to the mid-seventh century, thus answering the bigger question of how we should characterize early Chinese mathematical writers.

Chapter Three explores how the *Ten Mathematical Classics* were actually used in the process of teaching at the imperial academy during the Sui, Tang, and Northern Song dynasties. The purpose of the chapter is to investigage how state-run mathematical education was structured, how it related to the actual recruitment of civil servants, and how it compared with other types of education offered by the state.

The end of state-run mathematical education in 1120 did not stop new mathematical treatises from being written. Nor did it result in the collapse of civil administration due to a lack of mathematically adept personnel to handle the finances of the state. All these point to the existence of private mathematical education in pre-modern China. Chapter Four outlines the history of mathematical education after the Northern Song up to the time state-run mathematical education was again re-instated during the late Qing 清 dynasty (1644 – 1911). While much of the evidence for details about private mathematical education remains circumstantial, it will become obvious that the need for and presence of mathematically skilled people in Chinese society were continuous through pre-modern Chinese history. By analyzing the exact points in time when state-run and private mathematical education co-existed with each other, I will argue that the institution of state-run mathematical education must be understood in the larger context of the state's political needs and ambitions at the time.

Chapter Five discusses other aspects of the history of the *Ten Mathematical Classics* relating to the copying, printing, circulation, and transmission of these texts across East Asia. There was no evidence of printing technology in China before the seventh century, so the fact that treatises that date as far back in time as the first century BCE were able to survive up to the Tang dynasty and beyond is a matter of great historical interest. In addition, the role played by the *Ten Mathematical Classics* in contributing to a cohesive mathematical discourse in East Asia will be analyzed.

NOTE ON SOURCES AND TRANSLATIONS

Neither the Tang nor the Northern Song editions of the *Ten Mathematical Classics* are extant. The earliest surviving copies of seven of the classics date from the thirteenth century. The eighteenth-century editions prepared by Dai Zhen 戴震 (1724 – 1777),[54] one of the leading mathematics and evidential scholars of the Qing dynasty, for the imperial library collection are believed to be near-complete versions of all ten classics as well as their most important commentaries that have survived to that time. A recent reprint edition was produced in 1985 by a major Chinese

[54] For a study in English on Dai Zhen and his philosophy, see Minghui Hu, *China's Transition to Modernity: The New Classical Vision of Dai Zhen* (Seattle: University of Washington Press, 2015).

publisher, the Zhonghua shuju 中華書局, based on Dai Zhen's work. Unless otherwise stated, this is the edition to which I will refer throughout this dissertation.

Although this is a history of mathematics and mathematical education, I refrain from using the term "mathematician" to refer to the writers and commentators discussed in this dissertation. This restraint is in line with a recent trend in the historiography of science that strives for a more nuanced understanding of historical actors by avoiding the use of terms that might suggest that people of the past were simply earlier versions of today's professional scientists and mathematicians. As we shall see in Chapters One and Two, pre-modern Chinese did not share our modern mental landscape regarding clear boundaries between scientific pursuits and everything else. Furthermore, the historical actors in question did not refer to themselves with a special term that could be taken as the equivalent of "mathematician". The term "*chouren*" 疇人 was sometimes used in the historical sources to refer to people who did mathematics, but the same term was also applied to people who did astronomy, astrology, and calendrical calculations without the modern sense that what these people did was fundamentally different from one another. In the place of the term "mathematician", I use "mathematical writer" or "mathematical practitioner" to more accurately describe the relevant historical actors based on what they have done in terms of mathematics rather than project our modern categories on the past.

Because this is a work in English about Chinese texts and institutions, decisions about whether to use transliterations or translations of Chinese titles and names must be made. Currently, there is no consensus among the leading historians in the field who have published in English. For instance, Christopher

Cullen tends to use transliterations,[55] while Karine Chemla prefers to use translations.[56] Both methods pose problems, especially for non-specialist readers. Transliterations obscure the meaning of terms for readers who cannot read the original language. Translations, on the other hand, can become very wordy and can differ significantly from scholar to scholar, especially for titles whose meanings are not immediately obvious. A classic example in the history of the *Ten Mathematical Classics* is the title of the *Zhoubi suanjing*, which has variously been translated as the "Arithmetic Classic of the Gnomon and the Circular Paths of Heaven", and the "Mathematical Classic of the Gnomon of the Zhou Dynasty" among other variations.

In this dissertation, I have made these decisions primarily with a non-specialist reader's ease of reading and understanding foremost in mind. For the titles of individual Chinese texts, I use transliterations accompanied by a translation at first mention, mainly to avoid having long and cumbersome titles scattered throughout the text, but also because I do not feel translated titles are necessarily better than transliterated ones at helping readers identify and understand what is contained in the text.

For the names of government departments and positions, however, I use translations, most of which follow Charles O. Hucker's *Dictionary of Official Titles in Imperial China*,[57] because translations in such cases easily and directly convey to the reader the function of the department or position in question.

Lastly, all other translations of Chinese textual material are my own unless otherwise indicated.

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[55] See for example Cullen, *Astronomy and Mathematics in Ancient China*.

[56] See for example Chemla, "A Chinese Canon".

[57] See Charles O. Hucker, *A Dictionary of Official Titles in Imperial China* (Stanford: Stanford University Press, 1985).

Chapter One

Putting Thought into Writing: The Formation of the *Ten Mathematical Classics*

In the year 656, an unsual memorial reached Emperor Gaozong of the Tang dynasty. The memorial was writting by Wang Sibian 王思辯 (fl. 656), a low-ranking astronomical observer (*taishi jianhou* 太史監候) of the Imperial Bureau of Astronomy and Astrology, to complain that the *Ten Mathematical Classics* contained a multitude of errors.[1] The precise circumstances that gave rise to Wang Sibian's frustration with the texts remain a mystery. Nor do we know if the *Ten Mathematical Classics* had any direct relevance for his official duties at the bureau. The results of his actions, however, are well documented and momentous for the history of mathematical education. Wang's memorial prompted Gaozong to issue an imperial decree to correct and annotate the *Ten Mathematical Classics*. The officials appointed for this task were Li Chunfeng, director of the Bureau of Astronomy and Astrology; Liang Shu 梁述 (fl. 656), mathematics instructor at the imperial academy (*guozijian suanxue boshi* 國子監算學博士); and Wang Zhenru 王真儒 (fl. 656), assistant instructor at the imperial university (*taixue zhujiao* 太學助教). Once their work was completed, Gaozong decreed that these classics should be used at the imperial academy,[2] thus formalizing for the first time in Chinese history the mathematics textbooks of state-run institutions.

[1] *Jiu Tang shu* 舊唐書 (Old standard history of the Tang dynasty) (Beijing: Zhonghua shuju, 1975), 79, 2719.

[2] Ibid.

No copy of the *Ten Mathematical Classics* from the Tang dynasty has survived, but their titles are recorded in the official history:[3]

- *Zhoubi suanjing* 周髀算經 (Mathematical classic of the gnomon of the Zhou dynasty)
- *Jiuzhang suanshu* 九章算術 (Nine chapters on the mathematical art)
- *Haidao suanjing* 海島算經 (Sea island mathematical classic)
- *Sunzi suanjing* 孫子算經 (Master Sun's mathematical classic)
- *Zhuishu* 綴術 (Techniques for calculation by combination)
- *Xiahou Yang suanjing* 夏侯陽算經 (Xiahou Yang's mathematical classic)
- *Zhang Qiujian suanjing* 張丘建算經 (Zhang Qiujian's mathematical classic)[4]
- *Wucao suanjing* 五曹算經 (Mathematical classic of the five bureaucratic departments)
- *Wujing suanshu* 五經算術 (Mathematical art of the five classics)
- *Jigu suanjing* 緝古算經 (Classic on the continuation of ancient mathematics)

Two other mathematical treatises are recorded as having been studied at the imperial academy during the Tang dynasty, but were not originally considered part of the *Ten Mathematical Classics*:

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[3] *Xin Tang shu* 新唐書 (New standard history of the Tang dynasty), 44, 1160-1161. The order in which the texts are listed here follows the chronological order in which they were written.

[4] The 1776 copy of this treatise prepared by Dai Zhen for the imperial collection underwent a slight change to its title. The second character 丘 became 邱 so as to avoid transgressing on the taboo on Confucius' personal name. See Guo Shuchun, "Guanyu suanjing shishu de ji ge wenti" "關於《算經十書》的幾個問題" (A Few Questions Concerning the Ten Mathematical Classics), *Zhonghua keji shi xuehui huikan* 中華科技史學會會刊 (Bulletin of Association for the History of Science) 10 (Dec 2006): 66.

- *Shushu jiyi* 數術記遺 (Writings on the mathematical art)
- *Sandeng shu* 三等術 (Art of the three degrees)

However, the *Zhuishu* fell into disuse and was replaced as one of the *Ten Mathematical Classics* by the *Shushu jiyi*.[5] This replacement may have occurred quite early in the Tang dynasty. When Li Chunfeng wrote the harmonics and calendar systems (*lüli* 律曆) section in the standard history of the Sui dynasty, he indicated that the mathematics instructors at the Sui imperial academy had access to the *Zhuishu*, but never made much use of it due to difficulties understanding its contents.[6] Even had the Tang-dynasty instructors managed to fare better, no copy of the *Zhuishu* seems to have survived into the Song 宋 dynasty (960 – 1279). Certainly, the earliest extant copy of the *Ten Mathematical Classics* dating from 1213 includes the *Shushu jiyi* but not the *Zhuishu*. Like the *Zhuishu*, the *Sandeng shu* seems to have become lost by the Song dynasty. Furthermore, it never became one of the *Ten Mathematical Classics*.[7] The contents of both the *Zhuishu* and the *Sandeng shu* can only be speculated at this time.

Interestingly, several of the classics as they have been passed down to the present still include identifiable commentaries that had been added to the original texts sometimes centuries after their initial completion. By the Tang, these commentaries were considered essential parts of the classics and continued to be included in the official textbook version.[8] A complete list of the *Ten Mathematical*

[5] Guo, "Guanyu suanjing shishu de ji ge wenti", 65.

[6] *Sui shu*, 16, 388.

[7] Benjamin Elman, *On Their Own Terms: Science in China 1550 – 1900* (Cambridge, MA: Harvard University Press, 2005), 423-424.

[8] Chemla makes this point specifically for the *Jiuzhang suanshu* in "A Chinese Canon", 169-170.

Classics in chronological order, along with their authors, date of compilation, and known commentators up to the year 656 is given in Table 1.1.

This chapter deals with the technical aspects of the *Ten Mathematical Classics*. It describes the contents of each classic and of the extant commentaries that were included in the official textbook version of the Tang dynasty. It also highlights representative problems and solutions from the *Ten Mathematical Classics* in order to present the types of problems solved in these texts and the mathematical skills used to solve them. We shall see that these problems were couched in practical, often realistic but sometimes fantastical, situations rather than in abstract drills that are common in today's mathematics textbooks. We shall also see that each treatise and commentary constituted a specific facet of mathematics and its related view of the world that each author wished to convey to his readers. When considered as a single collection of textbooks, the *Ten Mathematical Classics* formed a diverse curriculum that was not simply teaching mathematical skills, but also other topics like conceptualizations of the universe, the importance of numbers, and the uses to which mathematical skills could be applied.

Table 1.1 The *Ten Mathematical Classics*[9]

	Title	Author	Date of Compilation	Commentators
1.	*Zhoubi suanjing*	unknown	ca. 1st century BCE	Zhao Shuang 趙爽 (fl. 206 CE) Zhen Luan 甄鸞 (fl. 565 – 570 CE) Li Chunfeng et al.
2.	*Jiuzhang suanshu*	unknown	ca. 1st century CE	Liu Hui Li Chunfeng et al.
3.	*Shushu jiyi*	Xu Yue 徐岳	3rd century	Zhen Luan
4.	*Haidao suanjing*	Liu Hui	263	Li Chunfeng et al.
5.	*Sunzi suanjing*	unknown	ca. 400	Zhen Luan (lost) Li Chunfeng et al. (lost)
	Zhuishu (lost)	Zu Chongzhi 祖沖之 (429 – 500)	5th century	Li Chunfeng et al. (lost)
6.	*Xiahou Yang suanjing*	Xiahou Yang	5th century	Zhen Luan (lost) Li Chunfeng et al. (lost)
7.	*Zhang Qiujian suanjing*	Zhang Qiujian	ca. 466 – 485	Li Chunfeng et al.
8.	*Wucao suanjing*	unknown	ca. 500	Zhen Luan (lost) Li Chunfeng et al. (lost)
9.	*Wujing suanshu*	unknown	ca. 6th century	Li Chunfeng et al.
10.	*Jigu suanjing*	Wang Xiaotong 王孝通 (fl. 618 – ca. 630)	7th century	Wang Xiaotong

[9] Yong Rong 永瑢 et al., *Siku quanshu jianming mulu* 四庫全書簡明目錄 (Shanghai: Shanghai guji chubanshe, 1985), 400 and 409-411; Guo Shuchun, "Guanyu suanjing shishu de ji ge wenti", 65; Martzloff, *A History of Chinese Mathematics*, 124-125.

1. ZHOUBI SUANJING

The *Zhoubi* is believed to be the oldest of the *Ten Mathematical Classics*, and is primarily a text on astronomy. The standard history of the Sui dynasty is the earliest dynastic history to contain a record of the *Zhoubi*,[10] which is listed among the titles of works on celestial patterns or astrology (*tianwen* 天文).[11] The *Zhoubi* does not appear to be one coherent work written by a single author, but a jumble of various texts that deal with very similar topics. None of the original authors or compilers are known by name, but the contents suggest that this text probably first came together during the first century BCE.[12]

It opens with a short dialogue between the Duke of Zhou 周公 and Shang Gao 商高. The Duke of Zhou is known in history as an extremely wise and capable political figure who served as his nephew's regent from ca. 1020 to 1012 BCE. Shang Gao, on the other hand, is unknown in history, even though Zhao Shuang's commentary describes him as a Zhou-dynasty (ca. 1046 – 256 BCE) official who was skilled in *suan* 算, which can be interpreted as either calculating or foretelling:

> "商高。周時賢大夫。善算者也。" [13]
> (Shang Gao, a grand master of the Zhou dynasty, was a person skilled in *suan*.)

The authenticity of this dialogue cannot be ascertained, but it appears to be a rhetorical device aimed at persuading the reader of the importance of numbers to establishing order in this world.

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[10] Qu, *Zhoubi suanjing xin yi*, 8.

[11] *Sui shu*, 34, 1018.

[12] Cullen, *Astronomy and Mathematics in Ancient China*, xi; Qu, *Zhoubi suanjing xin yi*, 7-8.

[13] *Zhoubi suanjing*, 1. However, Cullen suggests that Zhao Shuang might have misread the name Shang Rong 商容 as Shang Gao. Shang Rong was an official of the last Shang 商 (ca. 1600 – ca. 1046 BCE) ruler before the dynasty was conquered by the Zhou. See Cullen, *Astronomy and Mathematics in Ancient China*, 172-173.

In the dialogue, the Duke of Zhou appealed to Shang Gao as an expert in the art of numbers and sought explanations for the division of what was believed to be a circular heaven into quantifiable measurements. Shang Gao then proceeded with great success to convince the duke that order in the world could be imposed by the use of a try-square (*ju* 矩) because of its embodiment of unchanging numerical rules.

> "夫矩之於數。其裁制萬物。惟所為耳。" [14]
> ("Through its relations to numbers, what the try-square does is simply to settle and regulate everything there is.")[15]

In fact, the try-square is often associated with the legendary figure Fuxi 伏羲, while the compass (*gui* 規) with his sister Nüwa 女媧, both instruments said to have been invented by Fu Xi (see Figure 1.1).[16] This is similar to the association in Europe of the compass with God as well as mathematicians. In other words, in both China and Europe, certain mathematical instruments came to assume highly symbolic functions. In China, the try-square and the compass were regarded as tools with which humans could impose order on the natural world. In Europe where it was believed that the cosmos had been designed by a divine creator, the compass was associated with the art of design and was seen as the means through which mathematicians like Johannes Kepler (1571 – 1630) could divine God's "blueprint" for the cosmos (see Figure 1.2).

[14] *Zhoubi suanjing*, 12.
[15] See translation in Cullen, *Astronomy and Mathematics in Ancient China*, 174.
[16] Li and Du, *Chinese Mathematics*, 3.

Figure 1.1 A pictorial representation of Fuxi and Nüwa dating from the Tang dynasty, with Fuxi holding a try-square and Nüwa holding a compass[17]

[17] Anonymous (Chinese Wikipedia.) [Public domain], via Wikimedia Commons <https://commons.wikimedia.org/wiki/File:Fuxi_et_N%C3%BCwa.jpg>

Figure 1.2 Johannes Kepler holding a compass[18]

Perhaps more significant for the history of mathematics is Shang Gao's mention of the *gougu* 勾股, now commonly known as the Pythagorean relationship of a 3-4-5 right-angled triangle. The $a^2 + b^2 = c^2$ relationship is used in several subsequent problems within the *Zhoubi*, and is essentially the most complicated mathematics in the text. In other words, the *Zhoubi* is not a mathematically sophisticated treatise, and the majority of its contents is actually a discussion of astronomical questions. The fact that it would be used as a mathematical textbook at the imperial academy has special implications for the Chinese conception of a complete mathematical education.

[18] https://en.wikipedia.org/wiki/File:Johannes_Kepler_1610.jpg#filelinks

The next section of the *Zhoubi* is again a dialogue, but in a different style compared to the previous section. Here, instead of a royal prince asking a subject to provide explanations, we have a disciple, Rong Fang 榮方, asking his teacher, Master Chen 陳子, to teach him to understand various aspects of heaven and earth:

> 日之高大。光之所照。一日所行。遠近之數。人所望見。四極之窮。列星之宿。天地之廣袤。[19]
> (The height and size of the sun, the area of its radiance, the distance it travels in a day, its distance from a terrestrial observer, the limits of human vision, the boundaries of the four poles, the lodges of the stars, the length and breadth of the sky and the earth.)

This dialogue is reminiscent of the Buddhist sutras that would circulate in China later, in which the teacher is a figure of profound wisdom who appears to be waiting for the student to ask the right questions, at which point the teacher would then reveal all that he knows. Much of the content is built around making observations of celestial bodies using a gnomon (*bi* 髀), hence the title of the treatise. The vision of the cosmos presented by the text follows the *gaitian* 蓋天 ("canopy heaven") doctrine, where heaven and earth were believed to be two parallel planes, with the heaven rotating around a celestial axis like an umbrella over the earth, thus explaining the daily motion of celestial bodies. The heaven was said to be in the shape of a rain hat, while the earth was in the shape of an inverted plate.[20] Heaven and earth were uniformly 80 000 *li* 里 apart.[21] Because of the near impossibility of

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[19] *Zhoubi suanjing*, 13-14.

[20] Ibid, 54. In other literature from early China, such as the *Kao gong ji* 考工記 (Artificers' Record) in *Zhou li* 周禮 (Rites of Zhou), the earth is said to be shaped like a chariot.

[21] *Zhoubi suanjing*, 54. The *li* was defined slightly differently by different dynasties. During the early dynasties like the Zhou, Qin 秦 (221 – 206 BCE), and Han 漢 (202 BCE – 220 CE), one *li* was approximately equal to 415 metres. See Wen Renjun 聞人軍, "Lidai li mu zhidu zong kao" 歷代里畝制度綜考 (Studies of Past Dynastic Systems about li and mu in China), paper presented at the Conference of the 740th Anniversary of Qin Jiushao's *Shushu jiuzhang* 秦九韶《數書九章》成書 740 週年紀念暨學術研討國際會議, Beijing, May 1987, 2-4.

reconciling actual observations with the *gaitian* doctrine, another doctrine gained dominance among imperial astronomical and astrological officials by the Han dynasty. This was the *huntian* 渾天 ("murky heaven") doctrine, where heaven was believed to be a sphere rotating around a flat earth. This means that the *gaitian* doctrine had become very much out of date by the Tang dynasty, and yet the *Zhoubi* was still considered to be a relevant text for study at the imperial academy.

The vision of the world presented by Master Chen to Rong Fang adheres to the *gaitian* doctrine and consists of these main points:

- The sun is 80 000 *li* directly above the ground, and its diameter is 1250 *li*.[22]
- Its radiance reaches 167 000 *li* on all sides, and the extent of human vision does the same.
- On the summer solstice, the sun travels in a circular path that has a diameter of 238 000 *li* and a circumference of 714 000 *li*, which implies that the value of π (ratio of a circle's circumference to its diameter) was assumed to be 3.
- On the winter solstice, the sun is further south of an observer in China, implying it is travelling in a larger circular path in the sky. The diameter of this path 476 000 *li*, and the circumference is 1 428 000 *li*.
- On both the spring and autumn equinoxes, the sun follows a path that is exactly halfway in between its two solstice orbits. Therefore, the diameter of the equinox orbits is 357 000 *li*, and the circumference is 1 071 000 *li*.
- For every thousand *li* travelled on earth in the north-south direction, the shadow of an eight-*chi* 尺 gnomon at noon would differ by one *cun* 寸.[23]

[22] 1250 *li* is around 518.75 kilometres. For the sake of comparison, the sun's actual diameter is approximately 1 391 400 kilometres.

[23] One *chi* is around 23.1 centimetres. One *cun* is one-tenth of a *chi*.

The interest of this section does not lie in how well these numerical values adhere to reality, but in the fact that the Chinese had such an elaborately articulated vision of the world whose dimensions were assumed to be either known or calculable.

The rest of the *Zhoubi* provides further details about the world. The heaven was divided into seven concentric circles marking the solar orbit at different times of the year, with the smallest or innermost circle being the solar orbit at the summer solstice, and the outermost circle being the orbit at the winter solstice. Each of the seven concentric circles was divided into 365 $\frac{1}{4}$ *du* 度 because a solar year was believed to be 365 $\frac{1}{4}$ days long, or that it took the sun 365 $\frac{1}{4}$ days to return to its original position against the background of the stars. Therefore, the astronomical system described in the *Zhoubi* is known as an example of a quarter-remainder system (*sifen li* 四分曆). Here, *du* should not be understood as an angular measure like the way it is used today. Instead, it was used as a measure of spatial displacement that could be observed in the sky. No circle in any other context was ever divided into 365 $\frac{1}{4}$ *du*. In fact, the Chinese never seemed to have made use of any concept that could count as an angular measure. This point will be brought up again in our discussion of the *Haidao suanjing* below.

Further into the *Zhoubi*, we find evidence that the various sections were most likely not the work of a single author because we begin to see contradictory pieces of information presented in the text. For example, in the second half of the *Zhoubi*, the extent of the sun's radiance is said to have a diameter of 810 000 *li* and a circumference of 2 430 000 *li*. This is different from Master Chen's value of 167 000 *li* for the radius, which would have meant a diameter of 334 000 *li*. Another example is the length of the lunar orbit. One section of the *Zhoubi* states that the number of days between two consecutive new moons to be 30 $\frac{7}{16}$ days, while the last section states the value to be 29 $\frac{499}{940}$ days.

In addition to these values, the *Zhoubi* also lists several other numerical constants that were probably meant to be useful to calendrical experts and trainees at the time, such as the rule that there should be seven intercalary months every 19 years in order for the calendar to keep up with the seasons. Another well-known example is the list of the shadow lengths of an eight-*chi* gnomon at noon measured at 24 different times of the year.[24]

<u>Commentaries on the *Zhoubi*</u>

Zhao Shuang's preface and commentary were most likely added to the *Zhoubi* during the third century. In one instance, Zhao's commentary notes changes he had made to the original text: he wrote that he had found errors in the list of shadow lengths mentioned above and corrected them accordingly.[25] In some cases, the commentary states Zhao's own interpretation of the original text. For example, the *Zhoubi* describes the shape of the world as *tian yuan di fang* 天圓地方 (heaven is circular and earth is square).[26] Immediately following this phrase in the text, Zhao Shuang commented that it was impossible to know the shapes of heaven and earth because one could never see them in their entirety. Therefore, he argued that the phrase was only used as a metaphor to contrast heaven and earth. Just as there were odd vs. even numbers, *yang* vs. *yin*, heaven and earth were also associated with different shapes. Heaven moved, so it was said to be round and was

[24] Pre-modern Chinese divided the solar year into 24 equal parts known as *jieqi* 節氣.
[25] *Zhoubi suanjing*, 72.
[26] Ibid, 11.

associated with odd numbers. Earth was stationary, so it was said to be square and was associated with even numbers:

> 物有圓方。數有奇耦。天動為圓。其數奇。地靜為方。其數耦。此配陰陽之義。非實天地之體也。天不可窮而見。地不可盡而觀。豈能定其方圓乎。[27]
> (Objects may be round or square. Numbers may be odd or even. Heaven moves and is round. Its numbers are odd. Earth is stationary and is square. Its numbers are even. This is in accordance to the meaning of *yin* and *yang*, not the actual shapes of Heaven and Earth. Heaven cannot be seen completely. Earth cannot be viewed in its entirety. How can one designate them as round or square?)

In other cases, Zhao's commentary provides the reader with very useful explanations of an otherwise extremely terse text. For example, he gave background information on whom the Duke of Zhou and Shang Gao were. He also added a few diagrams to illustrate the meaning of several passages in the *Zhoubi*, one of which was originally a colour diagram about the *gougu* or Pythagorean relationship of a 3-4-5 right-angled triangle. This part of Zhao's commentary has sometimes been seen by modern scholars as an example of mathematical proof in pre-modern Chinese mathematics. However, because Zhao Shuang himself did not use any term that could correspond to our concept of "proof", and because "proof" has acquired a very specific meaning in the context of modern mathematics, which tends to be different from the definition applied by historians when studying the mathematics of pre-modern non-Western societies, this dissertation shall avoid its use and bypass the ongoing debate on whether or not pre-modern Chinese did any mathematical proof. What is important to assert here is that Zhao Shuang and

[27] *Zhoubi suanjing*, 11.

Chinese mathematical writers in general thought a diagram useful and sufficient to demonstrate and explain the *gougu* relationship.[28]

Zhen Luan added another layer of commentary during the sixth century. His contribution to the text largely consists of elaborating step by step the calculations found in both the original *Zhoubi* text and Zhao Shuang's commentary. This shows that the version of the *Zhoubi* that was passed into Zhen Luan's hands had Zhao Shuang's commentary preserved in it.

Li Chunfeng and his team read both Zhao Shuang's and Zhen Luan's commentaries, meaning that the palace copy of the Tang dynasty contained both. The comments added by Li Chunfeng's team probably proved quite useful to later students of the *Zhoubi* because they pointed out all the errors that they found in the text, many of which were actually in Zhen Luan's explanations. It is interesting that Li's team did not choose to correct these errors directly or to simply leave out the incorrect parts of Zhen Luan's commentary. Instead, they chose to preserve the errors as they were and to add another layer of commentary to point them out. Notable errors that they found include the one-*cun*-for-every-thousand-*li* shadow principle mentioned in the dialogue between Chenzi and Rong Fang. Li's team pointed out that the principle does not work in reality because the ground was not flat and because the calculations did not concur with actual observations.[29] Another error that they found was in the list of shadow lengths of the eight-*chi* gnomon at noon at each of the 24 *jieqi*. The shadow lengths recorded in the *Zhoubi* were not all obtained from actual observations. Only the shadows on the two solstices were

[28] For a study on the development of mathematical proofs in the Greek tradition, see Reviel Netz, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*, Ideas in Context, vol. 51, ed. Quentin Skinner et al. (Cambridge: Cambridge University Press, 1999).

[29] *Zhoubi suanjing*, 24.

measured. All the other values were obtained through interpolation, which did not correspond to actual observed shadow lengths.[30]

2. JIUZHANG SUANSHU

Out of the *Ten Mathematical Classics*, the *Jiuzhang suanshu* is the one with the greatest length, fame, and influence. It is also one of the better organized, giving the impression that it was either written by a single author, or was meticulously arranged by a single compiler. The earliest extant commentary on the *Jiuzhang* was written by Liu Hui in the year 263. He also added a preface that suggests that the *Jiuzhang* had very early origins. However, after the book burnings in 213 BCE under the Qin dynasty,[31] the text was damaged. Zhang Cang 張倉 (253 – 152 BCE) and Geng Shouchang 耿壽昌 (fl. 75 – 49 BCE) of the Han dynasty later restored the text, but the result may be somewhat different from the original.

> 周公制禮。而有九數。九數之流。則九章是矣。往者暴秦焚書。經術散壞。自時厥後。漢北平侯張倉大司農中丞耿壽昌。皆以善算命世。倉等因舊文之遺殘。各稱刪補。故校其目。則與古或異。而所論者多近語也。[32]
> (The Duke of Zhou systematized the rites, from which arose the nine numbers. Encapsulating the nine numbers is the *Jiuzhang*. In the past, the tyrannical Qin caused books to be burnt and classics to be damaged and lost. The Han dynasty's Marquis of Beiping Zhang Cang and Palace Aide to the Chamberlain of the National Treasury Geng Shouchang are both known for being skilled in suan. Because the original texts were incomplete, they made deletions and additions to restore the texts. Therefore, the texts may be different from the ancient ones, and may contain many contemporary terms.)

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30 *Zhoubi suanjing*, 72-73.

31 The First Emperor (*Shi huangdi* 始皇帝) (r. 247 – 210 BCE) of the Qin dynasty ordered the burning of all texts except for practical manuals on subjects like agriculture and medicine in an attempt to suppress references to past writings to criticize his policies. See Patricia Buckley Ebrey, *Cambridge Illustrated History of China*, 2nd ed. (Cambridge: Cambridge University Press, 2010), 61-63.

32 *Jiuzhang suanshu* - preface, 1.

The version of the *Jiuzhang* that has survived to today has 246 problems divided into nine chapters:

1. *Fang tian* 方田 (field measurement)
2. *Li mi* 粟米 (millet and rice)
3. *Shuai fen* 衰分 (distribution by proportion)
4. *Shao guang* 少廣 (short width)
5. *Shang gong* 商功 (construction consultations)
6. *Jun shu* 均輸 (fair levies)
7. *Ying bu zu* 盈不足 (excess and deficit)
8. *Fangcheng* 方程 (rectangular arrays)
9. *Gougu* 勾股 (right-angled triangles)

The style and format of all the chapters are very consistent, generally following the question-answer-method format. Sometimes, two or three sets of questions and answers are stated one after another, followed by a general method that can be used to solve each one of them. The questions within each chapter tend to become progressively more difficult, with the beginning questions aimed at having the reader practise and master a few basic techniques before presenting the more complicated questions.

The first chapter, *Fang tian*, mainly has to do with calculating the area of fields of various shapes. It begins with several questions on calculating the area of rectangular fields whose lengths and widths are all whole numbers. The method sections tell the reader to multiply the length by the width to obtain the area of each field, and they also instruct the reader on how to convert between different units of area measurements. Then follow several questions on working with fractions because this skill would be required to solve questions that would come later in the chapter and in the rest of the text. The Chinese did not use any special notation for fractions, but expressed them in words. For example, $\frac{1}{3}$ was written as 三分之一 or "one part out of three". It is interesting to note that even though their calculation tool, the counting rod system, was fully capable of handling decimals, the Chinese never made use of such a concept until the fifth century CE at the earliest.

The counting rod system is believed to have been in use in China since at least the Warring States 戰國 period (475 – 221 BCE). This calculation tool was completely replaced by the abacus during the fifteenth and sixteenth centuries, after which very few people knew anything about the counting rod system.[33] The counting rod system involved arranging sticks in various formations to represent each digit of a number, and then manipulating them for calculation. Therefore, place value was easily expressed by the placement and arrangement of the sticks, so the system could very well accommodate the decimal system.

[33] Lam Lay Yong, "Why did Traditional Chinese Mathematics Decline?", paper presented at the International Conference on the History of Chinese in China, San Diego, Aug 1988, 2-3.

Figure 1.3 Replica of Han-dynasty counting rods made from animal bones[34]

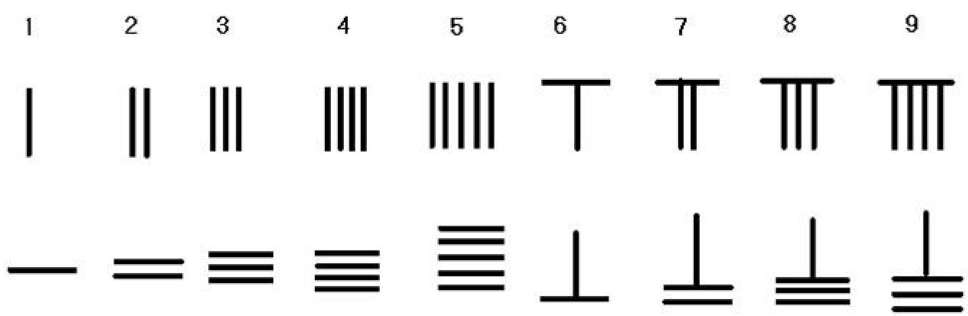

Figure 1.4 Counting rods display digits by using these very specific placement patterns.[35] Place value is indicated by alternating between the vertical and the horizontal orientations. The digit zero is marked with a blank space.

[34] SSR2000 [CC BY-SA 3.0 (https://creativecommons.org/licenses/by-sa/3.0) or GFDL (http://www.gnu.org/copyleft/fdl.html)], from Wikimedia Commons <https://commons.wikimedia.org/wiki/File:Replica_of_Han_Dynasty_Counting_rods_in_National_Museum_of_Natural_Science_in_Taiwan.JPG>

The rest of the *Jiuzhang's* first chapter deals exclusively with calculating the areas of fields with increasingly complex shapes, progressing from triangular, trapezoidal, and circular fields to fields that were shaped like an annulus, a segment of a circle, or a bowl. Therefore, this chapter would have been a very useful manual for officials charged with calculating the area of fields for taxation purpose.

The second chapter of the *Jiuzhang* begins by listing the market value exchange rates of different types of agricultural products, including millet, wheat, beans, sesame seed, rice, and malt. Then follow three types of questions. The first two types are quite straightforward:

- Type 1: There is a given amount of agricultural product A that must be exchanged for product B. How much of product B would one get?
- Type 2: A given amount of money has been spent on buying a given amount of product C. Calculate the unit price of product C.

The first type of problems requires one to be able to do calculations with ratios. The second type is exercises for division often involving four- or five-digit numbers as well as fractions. The third type of problems combines the techniques of the first two types and are in this form:

- Type 3: A given amount of money has been spent on buying a given amount of product D, which is of two different qualities. Calculate the unit price of each quality of the product.

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[35] I, Gisling [GFDL (http://www.gnu.org/copyleft/fdl.html), CC-BY-SA-3.0 (http://creativecommons.org/licenses/by-sa/3.0/) or CC BY 2.5 (https://creativecommons.org/licenses/by/2.5)], from Wikimedia Commons <https://commons.wikimedia.org/wiki/File:Chounumerals.jpg>

The examples and techniques presented in this chapter of the *Jiuzhang* would have been of tremendous practical value to both officials in charge of acquiring goods for the government, and to common people who were involved in the everyday trade of goods in the market.

The third chapter of the *Jiuzhang* deals with calculations with proportions and rates. For example, the first question has five men of different ranks sharing five deer in proportion to their rank, and asking how much deer each man should get:

> 今有大夫、不更、簪裹、上造、公士，凡五人。共獵得五鹿。欲以爵次分之。問各得幾何。[36]
> (There is a grand master, a grandee of the fourth order, a grandee of the third order, a grandee of the second order, and a grandee of the first order, totalling five people. They caught five deer in total. If they divide the deer in accordance to their differences in rank, how much does each receive?)

Several other questions in the chapter run in the same vein: a given number of people or villages are dividing a certain monetary obligation or tax proportionately among themselves; calculate the contribution of each. The rest of the questions in the chapter involve calculation with rates. For example, one question asks to calculate the total worth of a certain amount of silk given its unit price:

> 今有絲一斤。價值三百四十五。今有絲七兩一十二銖。問得錢幾何。[37]
> (There is one *jin* of silk worth 345. With 7 *liang* and 12 *zhu* of silk, how much money would be received?)[38]

In this chapter, we find the *Jiuzhang's* first reference to gender roles within Chinese society. Whereas a large proportion of the *Jiuzhang's* questions do not mention human actors at all, most of the remaining questions use gender-neutral

[36] *Jiuzhang suanshu*, 37.
[37] Ibid, 42.
[38] One *jin* = 16 *liang*; one *liang* = 24 *zhu*.

terms like *ren* 人 (person or people) and *jia yi bing* 甲乙丙 (person A, person B, and person C). However, in this chapter, we have one question that asks:

> 今有女子善織。日自倍。五日織五尺。問日織幾何。[39]
> (There is a woman who is adept at weaving. Her productivity doubles each day. If she wove 5 *chi* in 5 days, how much did she weave in each of the 5 days?)

Instead of using another gender-neutral term, the author of the question specified that it was a woman doing the weaving which accords with the traditional Chinese view of how labour should be divided between men and women.

In the *Jiuzhang's* fourth chapter, the mathematics begins to become more complicated or, at the very least, more cumbersome. Instead of calculating areas of fields like in the first chapter, in this chapter, the area and the width of the field are already known, so the length is the sought value and can be found by a straightforward division. However, the widths given in the problems require the reader to first perform some arithmetic acrobatics before the necessary division can be done. For example, one question has a field whose area is one *mu* 畝 and whose width is given as, using modern notation:

$$1\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5}+\tfrac{1}{6}+\tfrac{1}{7}+\tfrac{1}{8}+\tfrac{1}{9}+\tfrac{1}{10}+\tfrac{1}{11}+\tfrac{1}{12}$$ *bu* 步.[40]

To find the length of the field, the reader would first have needed to find a number that would divide evenly into all the denominators above and calculate the actual value of the width before proceeding with the division to find the length. Even

[39] *Jiuzhang suanshu*, 39.

[40] Ibid, 51. One *bu* = six *chi*, and one *mu* = 240 *bu*. Note that *bu* was used both as a length unit and as an area unit. It was left up to the reader to use the context of a statement to discern how *bu* was being used.

though the questions in Chinese mathematical treatises are typically described as practical problems, a question like the one mentioned here could not have come from a real-life situation since it is inconceivable that anyone measuring the width of a rectangular field would have found it necessary to report the measurement as the combination of so many fractional sections. Questions like this are more likely to be deliberately designed skill exercises couched in quasi-realistic situations.

Chapter four of the *Jiuzhang* also contains the methods for extracting square and cube roots. These methods discuss very specifically how counting rods should be laid out and manipulated in order to perform the calculation. In all early Chinese mathematical treatises, only the positive roots were sought.

The questions in the *Jiuzhang's* fifth chapter mainly cover how to calculate the volume and dimensions of earthworks, the number of labourers required to complete a construction project, and the volume of piles of grain or beans. The types of shapes involved include cubes, cylinders, square pyramids and cones, as well as more irregular shapes like the one in Figure 1.5:

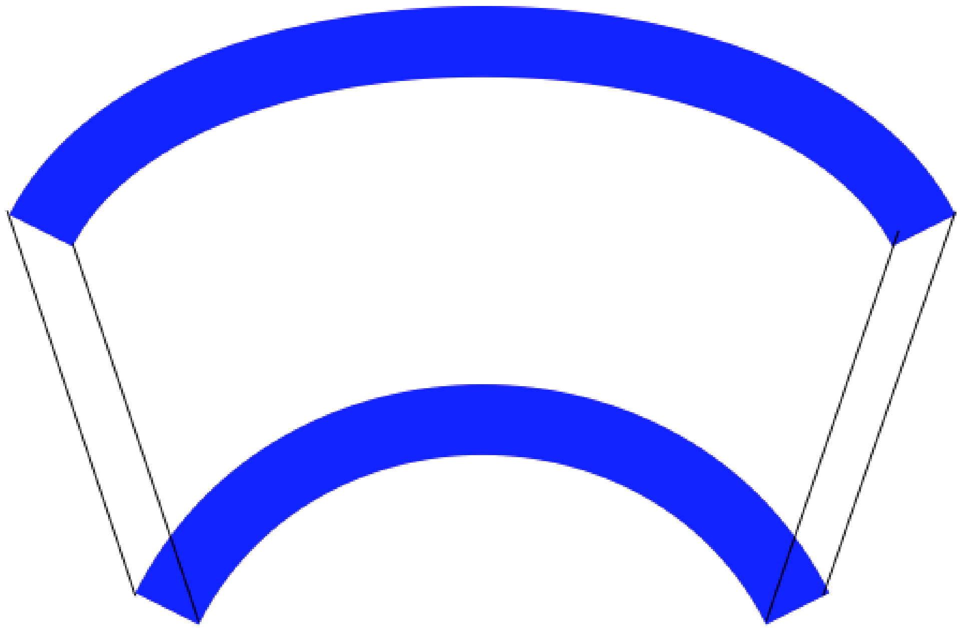

Figure 1.5 An irregular shape from the *Jiuzhang suanshu*

Mastery of this chapter would have been useful for people designing storage facilities and for those who had to calculate the amount of labour required to complete construction projects.

The title of the *Jiuzhang's* sixth chapter, *Jun shu*, refers to the problems within the chapter on how to divide tax levies fairly among a given number of counties based on certain conditions like their individual population and distance from the tax bureau. However, the chapter also contains several kinematics problems. For example, one question has a faster walker chasing after a slower walker after the latter has had a head start, and the reader is asked to calculate when the faster walker would catch up with the slower:

> 今有善行者行一百步。不善行者行六十步。今不善行者先行一百步。善行者追之。問幾何步及之。[41]
> (There is a fast walker who walks 100 *bu* for every 60 *bu* walked by a slow walker. If the slow walker walks 100 *bu* before the fast walker starts chasing after the slow walker, at how many *bu* would the fast walker catch up?)

Two other problems of the same type adopt a more whimsical tone: one has a dog chasing after a rabbit, while the other has a duck flying north and a goose flying south towards each other. While the problems in this chapter deal with very diverse situations, the reader must have been expected to recognize that they all belong in the same broad category because all of them must be solved with the use of rates and/or proportions, but in a way that is more complicated than the problems in the third chapter. In fact, generally speaking, the mathematics in each chapter of the *Jiuzhang* appears to be more complicated than the chapters before it, suggesting that the reader was expected to work through the text from chapter one progressively onwards rather than skip chapters or work on multiple chapters simultaneously.

The problems in the *Jiuzhang's* seventh and eighth chapters are quite similar. Generally, each problem presents a situation where there are multiple unknowns that need to be calculated, and in modern times would very likely all be solved with the same method using systems of equations or rectangular arrays (*fangcheng* 方程), which is the title of the eighth chapter. However, the problems in chapter seven were treated by the Chinese as belonging to a completely different category: excess and deficit (*ying bu zu* 盈不足). An example from the chapter is:

> 今有共買物。人出八。盈三。人出七。不足四。問人數物價各幾何。答曰。七人。物價五十三。[42]
> (There is a jointly purchased item. If each person contributes 8, the excess is 3. If each person

[41] *Jiuzhang suanshu*, 98.
[42] Ibid, 111.

> contributes 7, the deficit is 4. Find the number of people who purchased the item and the price of the item. Answer: seven people; item's price is 53)

Nowadays, a student would probably be expected to translate this problem into the following system of equations and solve it:

Let x = the number of people

y = price of item

(i) $8x = y + 3$

(ii) $7x = y - 4$

Subtract (ii) from (i), we get $x = 7$

To find y:

$$y = 8x - 3 = 8(7) - 3 = 56 - 3 = 53$$

However, the method given in the *Jiuzhang* gives the reader a series of arithmetic operations to follow, which yield the same answer as above:

Lay down the amounts contributed and then the excess and deficit below them	8 7 3 4
Cross-multiply and add the results; this is the dividend	$8 \times 4 = 32$ $3 \times 7 = 21$ $32 + 21 = 53$
Add the excess and deficit; this is the divisor	$3 + 4 = 7$
Find the difference between the excess and the deficit	$4 - 3 = 1$

Divide the dividend by the difference to find the cost of the item	$53 \div 1 = 53$
Divide the divisor by the difference to find the number of people	$7 \div 1 = 7$

A typical problem from chapter eight is as follows:

> 今有牛五。羊二。值金十兩。牛二。羊五。值金八兩。
> 問牛羊各值金幾何。答曰。牛一、值金一兩二十一分兩之一十三。羊一、值金二十一分兩之二十。[43]
> (Five cattle and two sheep are worth 10 *liang* 兩 of gold.
> Two cattle and five sheep are worth 8 *liang* of gold.
> Find the worth in gold of one cattle and of one sheep.
> Answer: one cattle is worth $1\frac{13}{21}$ *liang* of gold; one sheep is worth $\frac{20}{21}$ *liang* of gold.)

In modern notation, this problem translates into:

Let x = worth of one cattle in gold

y = worth of one sheep in gold

(i) $5x + 2y = 10$

(ii) $2x + 5y = 8$

The method in the *Jiuzhang* instructs the reader to arrange counting rods in an array that looks very similar to a modern matrix, except that the terms of each equation were arranged vertically rather than horizontally.

[43] *Jiuzhang suanshu*, 132. Sixteen *liang* were approximately equal to 250 grams. See Shen Kangshen, John N. Crossley, and Anthony W.C. Lun, *The Nine Chapters on the Mathematical Art: Companion and Commentary* (Oxford: Oxford University Press, 1999), 9.

	(ii)	(i)
Cattle	2	5
Sheep	5	2
Gold	8	10

To solve this by elimination, the reader is told to multiply all the terms in (i) by 2 and those in (ii) by 5, yielding:

	(ii)	(i)
Cattle	10	10
Sheep	25	4
Gold	40	20

Subtracting (i) from (ii), we are left with:

Cattle	0
Sheep	21
Gold	20

Therefore, $21y = 20$, which means $y = \frac{20}{21}$ *liang* of gold. To find x:

$$5x + 2\left(\frac{20}{21}\right) = 10$$

$$5x + \frac{40}{21} = 10$$

$$5x = \frac{170}{21}$$

$$x = \frac{170}{105} = 1\frac{65}{105} = 1\frac{13}{21}$$ *liang* of gold

Chapter nine of the *Jiuzhang* is on the *gougu* principle $a^2 + b^2 = c^2$ for right-angled triangles. *Gou* and *gu* refer to the two sides of the triangle that form the right angle, with *gou* usually being the shorter side. *Xian* 弦 refers to the hypotenuse.

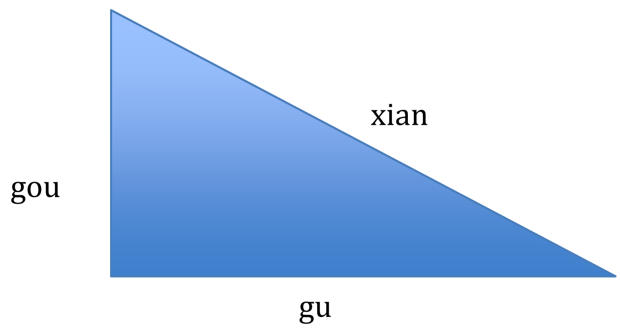

Figure 1.6 The three sides of a right-angled triangle

Like in the *Zhoubi*, the *Gougu* chapter in the *Jiuzhang* begins its discussion with a basic 3-4-5 right-angled triangle to demonstrate how the *gougu* principle could be used to find an unknown side when the other two sides are known.

$$(gou)^2 + (gu)^2 = (xian)^2$$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

To find one side:

$$gou = \sqrt{(xian)^2 - (gu)^2}$$

$$gu = \sqrt{(xian)^2 - (gou)^2}$$

$$xian = \sqrt{(gou)^2 + (gu)^2}$$

Therefore, the technique used for finding square roots introduced in chapter four again comes into active play here. Most of the questions involve small, easy-to-handle numbers. For example, immediately following the discussion of the 3-4-5 triangle, we have the following two questions:

> 今有圓材。徑二尺五寸。欲為方版。令厚七寸。問廣幾何。答曰。二尺四寸。[44]
> (There is a circular log. Its diameter is 2 *chi* 5 *cun*. It has to be made into a rectangular plank with a 7-*cun* thickness. What would be the width of the plank? Answer: 2 *chi* 4 *cun*)

> 今有木長二丈。圍之三尺。葛生其下。纏木七周。上與木齊。問葛長幾何。答曰。二丈九尺。[45]
> (There is a tree that is 2 *zhang* 丈 tall.[46] Its circumference is 3 *chi*. A vine grows from the bottom of the tree and circles it seven times before reaching the top of the tree. How long is the vine? Answer: 2 *zhang* 9 *chi*)

Neither question specifically mentions any right angles or triangles, and is not accompanied by pictorial representation of the problem. Therefore, the reader would have required an exercise of the imagination to visualize the situations described:

[44] *Jiuzhang suanshu*, 146.
[45] Ibid.
[46] One *zhang* = 10 *chi*

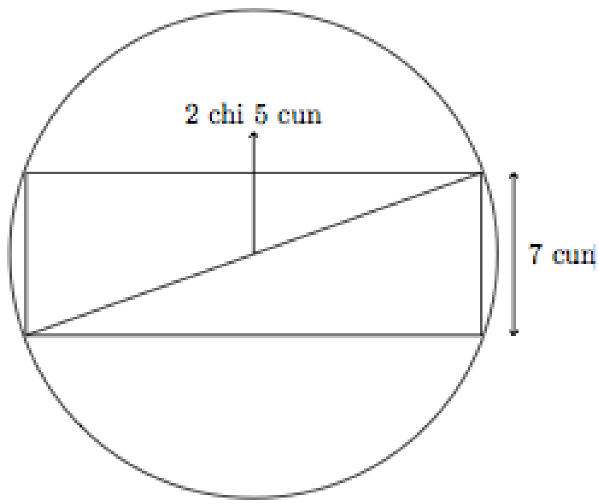

Figure 1.7 Representation of the log-and-plank question from the *Jiuzhang suanshu*

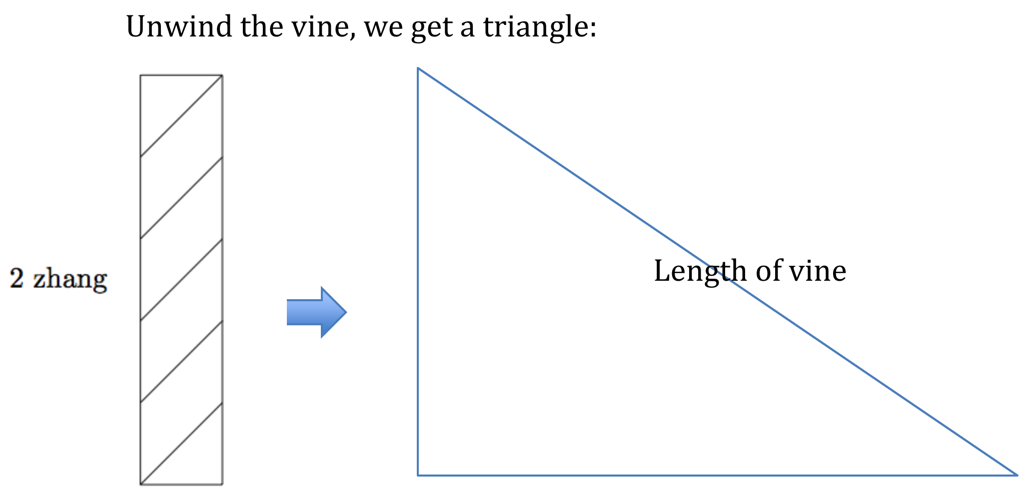

Figure 1.8 Representation of the tree-and-vine question from the *Jiuzhang suanshu*

Both answers can be found with a straightforward application of the *gougu* principle.

The chapter also contains questions involving the distances of far-away objects, thus paving the way for the surveying questions that Liu Hui felt it was appropriate to add to the *Jiuzhang*. These questions do not use the *gougu* principle, but still concern right-angled triangles. For example:

> 今有邑方二百步。各中開門。出東門一十五步有木。問出南門幾何步而見木。答曰。六百六十六步大半步。[47]
> (There is a square walled city with sides 200 *bu*. Each side has a gate in the middle. Fifteen *bu* out of the east gate is a tree. How far does one have to walk out of the south gate to see the tree? Answer: 666 *bu* and a bit over half a *bu*)

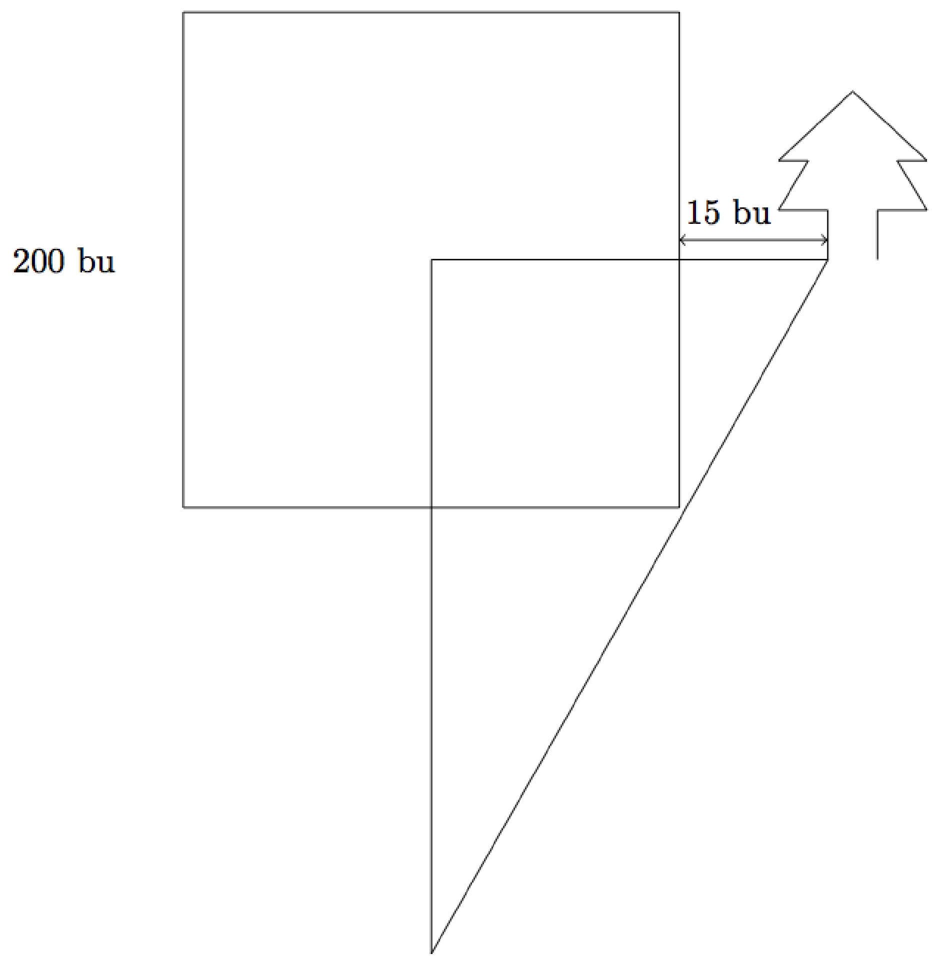

Figure 1.9 Representation of the city-and-tree question from the *Jiuzhang suanshu*

[47] *Jiuzhang suanshu*, 161.

Commentaries on the *Jiuzhang*:

Liu Hui added substantial annotations to the *Jiuzhang* in the year 263.[48] Like in the *Zhoubi*, the original method sections in the *Jiuzhang* are very terse statements of what one had to do to obtain the answers. Liu Hui's commentary serves to make the *Jiuzhang* much more accessible to a non-expert reader. Not only did Liu Hui provide explanations for the technical terms, he also demonstrated the step-by-step procedures to obtain the answers. Furthermore, Liu Hui's commentary displays a level of mathematical competence that went far beyond the confines of the *Jiuzhang*, earning him a place among the most famous and respected figures in the history of Chinese mathematics. Like Zhao Shuang in the *Zhoubi*, Liu Hui also believed a diagram to be useful and sufficient to demonstrate the *gougu* principle. Although Liu's original diagrams have all been lost, Dai Zhen attempted to restore them in the eighteenth century based on what Liu Hui had written. One of the diagrams was to illustrate in colour that $3^2 + 4^2 = 5^2$:[49]

[48] Donald B. Wagner has argued that the commentary usually attributed solely to Liu Hui may actually be a conflation of commentaries by two or more individuals. See Donald Blackmore Wagner, "Doubts Concerning the Attribution of Liu Hui's Commentary on the Chiu-chang suan-shu", *Acta Orientalia* 39 (1978): 199-212.

[49] *Jiuzhang suanshu*, 144.

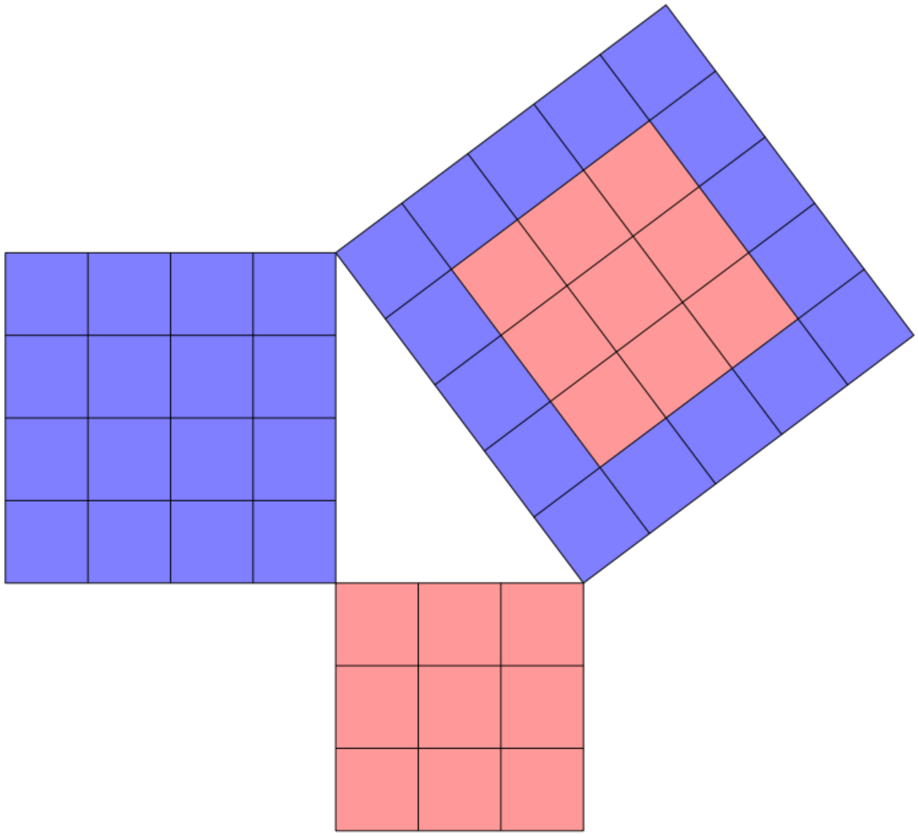

Figure 1.10 Representation of Dai Zhen's reconstruction of one of Liu Hui's original diagrams

A different interpretation of Liu Hui's comments was offered by Li Huang 李潢 (d. 1811):[50]

[50] Guo Shuchun, *Gudai shijie shuxue taidou,* 187.

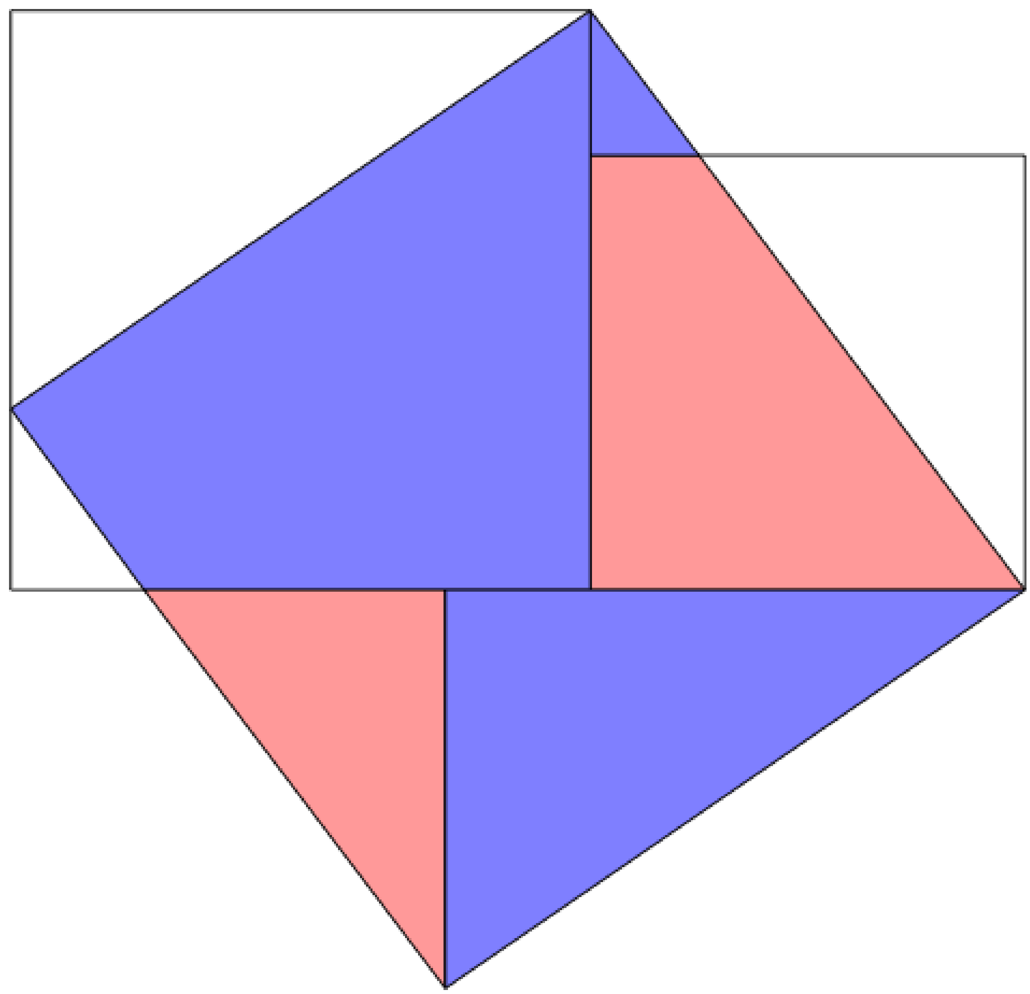

Figure 1.11 Representation of Li Huang's reconstruction of one of Liu Hui's original diagrams

Liu Hui's commentary shows that he was not simply a blind follower of the methods given in the *Jiuzhang*. In chapter one, he pointed out that the formula for finding the surface area of a bowl-like field $A = p \times q \div 4$ was incorrect.[51]

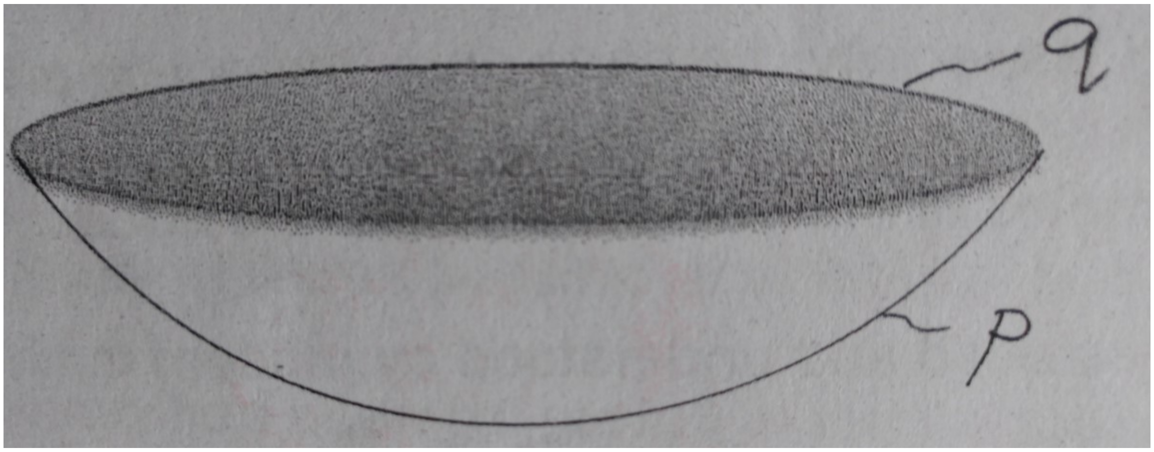

Figure 1.12 A bowl-like field

In chapter four, Liu Hui again pointed out an incorrect formula, this time for the volume of a sphere, which the *Jiuzhang* implied was $V = D^3 \times 9 \div 16$, where $D =$ diameter of the sphere.[52]

Liu Hui also realized that instead of using 3 as the value for π like in the main text of the *Jiuzhang*, a more precise value could be found. He reasoned that he could approximate the value of π by using regular polygons inscribed in a circle. As the number of sides of the polygons increased, the size and shape of the inscribed polygon would get closer and closer to the circle. He began by using a regular hexagon and gradually doubled the number of sides until he ended the procedure

[51] *Jiuzhang suanshu*, 18.
[52] Ibid, 61.

with a polygon of 192 sides. Liu Hui came up with two approximations of π: $\frac{157}{50}$ $(= 3.14)$ to keep calculations simpler, and $\frac{3927}{1250}(= 3.1416)$ as a more precise value.[53]

Liu Hui's mathematical expertise is apparent from his rigorous arguments and sound reasoning. In any discussion of early Chinese mathematical sophistication, Liu Hui's work is always accorded much praise. However, his work has also become one of the most contentious features of Chinese mathematics. While traditional historiography sees Chinese mathematics as totally devoid of proof, a new view has been promoted in recent decades by scholars like Guo Shuchun and Karine Chemla, who point to Liu Hui's commentary in the *Jiuzhang*, especially the parts on π, the *gougu* principle, and the volume of a sphere, as examples of mathematical proof.[54] As discussed earlier in this chapter, the notion of proof as used and understood currently by modern mathematicians is perhaps too narrowly defined to accommodate non-Western mathematics that were not built upon axioms, lemmas, and theorems.[55] Therefore, just as with Zhao Shuang, what is perhaps more important to emphasize in historical studies is not whether or not we think Liu Hui's work constitutes mathematical proof. Instead, historians should attempt to reconstruct what Liu Hui thought he was doing,[56] and why he was doing it. A preliminary step towards achieving that goal is attempted here. This sort of discussion may never bring us any closer to understanding Liu Hui and his work

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[53] *Jiuzhang suanshu*, 11-14. The precise value of π is 3.141592654…

[54] See for example Karine Chemla, "What is at Stake in Mathematical Proofs from Third-Century China?", *Science in Context* 10, no. 2 (1997): 227-251.

[55] It must be noted that standards of mathematical proof in the West have never been absolute, but have repeatedly been changed and re-defined. See Michael J. Crowe, "Ten Misconceptions about Mathematics and Its History", in *History and Philosophy of Modern Mathematics*, ed. William Aspray and Philip Kitcher, Minnesota Studies in the Philosophy of Science, vol. 11 (Minneapolis: University of Minnesota Press, 1988), 267-269.

[56] Christopher Cullen urges historians to take this approach in his article, "How Can We Do the Comparative History of Mathematics? Proof in Liu Hui 劉徽 and the Zhou bi 周髀", *Philosophy and the History of Science: A Taiwanese Journal* 4, no. 1 (April 1995): 59-94.

because it is partly built upon speculation. However, it is important and worthwhile to attempt because it is a discussion of Chinese mathematics that uses Chinese points of reference rather than Western or modern ones.

Just as with Zhao Shuang, Liu Hui did not use any label to describe his commentary or to tell the reader how his commentary should be interpreted. He did not use any term that would indicate that he was offering irrefutable proof of his findings and conclusions. When he pointed out that the two formulae in the *Jiuzhang* were incorrect, he stated reasons to convince the reader that they could not possibly be correct. When approximating the value of π, Liu Hui laid out all the steps he did to arrive at the new values in order to show the reader his reasoning and to offer the reader the chance to cross-check his calculations. When explaining the *gougu* principle, he offered the reader a visual illustration that showed that the principle worked for the 3-4-5 triangle, perhaps hoping or expecting the reader to then infer that the principle would work for any right-angled triangle.

Li Chunfeng and his team later added another layer of commentary to the *Jiuzhang*. Much of it consists of adding explanations for the methods that Liu Hui had not commented on. Some of it was targeted at what Liu Hui had written, so it is clear that Liu Hui's commentary had been preserved in the palace copy that Li Chunfeng's team worked with. For example, Li's team appended further comments to Liu Hui's discussion of π, stating that a much more precise value, $\frac{355}{113}$ ($\cong 3.1415929$), had been found by Zu Chongzhi, but a value of $\frac{22}{7}$ ($\cong 3.1428571$) could be used instead for ease of calculation. In some other instances, Li's team found fault with Liu Hui's comments, but modern scholars tend to agree that their objections were either unnecessary or unfounded.[57]

3. SHUSHU JIYI

The *Shushu jiyi* has been attributed since at least the Tang dynasty to Xu Yue. Because Zhen Luan's name appears in several places in the last part of the text, it was believed that the commentary found in the text was added by him. However, the authenticity of both the text and the commentary have been called into question since at least the eighteenth century. When the *Shushu jiyi* was added to the *Siku quanshu*, Dai Zhen added a preface to the text in which he raised doubts that the text could have been written during the Han dynasty. The author of the *Shushu jiyi* wrote that he had acquired the knowledge recorded in the text from Liu Hong 劉洪, who was serving as an official at Kuaiji 會稽 at the time of their meeting.[58] However, Dai Zhen pointed out that, according to official historical records, Liu Hong had never served at Kuaiji, suggesting that the origin of the *Shushu jiyi* must have been fabricated. Dai Zhen also made the point that the title of the *Shushu jiyi* had not appeared in any imperial catalogue until the Tang dynasty. Therefore, he believed that both the main text and the commentary were a forgery made by an unidentified individual during the Tang dynasty who used Xu Yue's and Zhen Luan's names for the sake of passing it off as an ancient mathematical treatise.

Writing in the 1960's, historian Qian Baocong also questioned the attribution of the *Shushu jiyi* to Xu Yue because of the various references to Buddhist sutras in the text. From these, Qian inferred that the *Shushu jiyi* could not have been written

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[57] See for example Guo and Liu, *Suanjing shi shu*; and Shen, Crossley, and Lun, *The Nine Chapters on the Mathematical Art*.

[58] *Shushu jiyi*, 2-3.

as early as the Han dynasty. Instead, he suggested that both the text and the commentary were written by Zhen Luan, and that it was Zhen Luan who falsely attributed the main text to Xu Yue.[59] However, Qian did not offer any possible motive for Zhen Luan, who is known to have both written and annotated several mathematical treatises in his own name, to ascribe his own work to someone else. Using the Buddhist references in the *Shushu jiyi* to question the dating of the text has also failed to convince some scholars, who pointed out that there were nearly 300 Buddhist sutras translated into Chinese during the last 154 years of the Han dynasty, so it was entirely conceivable for Xu Yue to have written the original text.[60]

Regardless of who the real author was, his or her mathematical and writing capabilities have never won much praise. It may be that large tracts of the original text have been irretrievably and unknowingly lost, but the version of the *Shushu jiyi* that has survived is a short and, at times, incoherent discussion of various topics that the author had supposedly learnt from Liu Hong. Some of the discussion takes a more philosophical bent, such as the question: if one cannot understand the minute, how could one begin to comprehend the infinite:

> 未識刹那之賒促安知麻姑之桑田。不識積微之為量詎曉百億與大千。[61]
> (Without recognizing the haste of an instant, how can one comprehend the tale of Magu's mulberry plantation? Without recognizing that the infinitesimal has magnitude, how can one know the billions and the thousands?)

Other parts of the *Shushu jiyi* have more direct relevance for the history of Chinese mathematics. They point to a time when the number system in China was in flux. For example, the *Shushu jiyi* explains three different systems used for naming large numbers:

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59 Qian, *Suanjing shi shu*, 531.
60 Guo and Liu, *Suanjing shi shu*, preface (no page numbers).
61 *Shushu jiyi*, 8-9.

	System I	System II	System III
wan 萬	10^4	10^4	10^4
yi 億	$10 \times 10^4 = 10^5$	$10^4 \times 10^4 = 10^8$	$10^4 \times 10^4 = 10^8$
zhao 兆	$10 \times 10^5 = 10^6$	$10^8 \times 10^8 = 10^{16}$	$10^8 \times 10^8 = 10^{16}$
jing 京	$10 \times 10^6 = 10^7$	$10^8 \times 10^{12} = 10^{20}$	$10^{16} \times 10^{16}$ $= 10^{32}$

The *Shushu jiyi* also describes fourteen different systems for recording numbers. The first system is the counting rods system, which was familiar to all early Chinese mathematical practitioners and usually required the use of many rods. The other thirteen systems were probably developed to simplify the process of doing calculations, but none of them actually came into widespread use.[62]

Commentary on the *Shushu jiyi*

The subtext under the title of the *Shushu jiyi* names Zhen Luan as the author of the commentary included in the text. From the current version, it is unclear whether Zhen Luan really wrote the commentary. Unlike in the *Zhoubi suanjing* where each line of Zhen Luan's commentary is clearly marked by the phrase "Your subject Luan says" (*chen Luan yue* 臣鸞曰), no such markers can be found in the *Shushu jiyi*, which is a suspicious departure in style. Regardless of who wrote the commentary, we can say for certain that this individual was very familiar with Buddhist sutras and was able to use them to explain the main text's allusions to ideas of very large numbers and long eons of time that seem to have accompanied

[62] Qian, *Suanjing shi shu*, 532.

the transmission of Buddhism to China from India along the Silk Road.[63] The commentary also has a few mathematics questions, including two versions of the famous "Hundred Fowls" problem:

> 今有雞翁一隻值五文。雞母一隻值四文。雞兒一文得四隻。合有錢一百文。買雞大小一百隻。問各幾何。答曰。雞翁十五隻。雞母一隻。雞兒八十四隻。[64]
> (A rooster is worth 5 coins, a hen 4 coins, and four chicks are worth 1 coin. If 100 coins were used to buy 100 fowls, how many roosters, hens, and chicks would one get? Answer: 15 roosters, 1 hen, 84 chicks)

> 今有雞翁一隻值四文。雞母一隻值三文。雞兒三隻值一文。合有錢一百文。還買雞大小一百隻。問各幾何。答曰。雞翁八隻。雞母十四隻。雞兒七十八隻。[65]
> (A rooster is worth 4 coins, a hen 3 coins, and 3 chicks are worth 1 coin. If 100 coins were used to buy 100 fowls, how many roosters, hens, and chicks would one get? Answer: 8 roosters, 14 hens, 78 chicks)

However, no method was attached to any of the problems to indicate how they were solved. In other words, the mathematical content of the *Shushu jiyi* is actually very little and quite rudimentary. Qian Baocong was of the opinion that, if not for the fact that the *Shushu jiyi* had been used at the Tang imperial academy, it would not have been worthwhile to pass the text onto posterity.[66] However, the fact that such a text was thought worth studying during the Tang dynasty means it offered something that was considered worthwhile at the time.

There is no indication that Li Chunfeng and his team ever added annotations to the *Shushu jiyi*.

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[63] Martzloff, *A History of Chinese Mathematics*, 96-99.
[64] *Shushu jiyi*, 31.
[65] Ibid, 31-32.
[66] Qian, *Suanjing shi shu*, 532.

4. HAIDAO SUANJING

Liu Hui wrote the *Haidao suanjing* around the year 263. He had originally appended it to the *Jiuzhang suanshu* as the tenth chapter. By the Tang dynasty, it had become an independent treatise, and was named after its first question in which there is an island in the sea ("*hai dao*" or "sea island"). There is no way to ascertain how many questions had been in Liu Hui's original version, but the current one is a short text with only nine questions that deal with calculating the distance or size of far-away landmarks. Therefore, the *Haidao suanjing* would have been useful to people engaged in surveying and cartography.[67]

	Content of the Question
Question 1	Calculate the height of an island and its distance away
Question 2	Calculate the height of a tree and its distance away
Question 3	Calculate the size of a walled city and its distance away
Question 4	Calculate the depth of a ravine
Question 5	Calculate the height of a building
Question 6	Calculate the width of the mouth of a river
Question 7	Calculate the depth of a pool
Question 8	Calculate the width of a river
Question 9	Calculate the size of a walled city

[67] Swetz, *Sea Island Mathematical Manual*, x.

The layout of this text is very consistent with that of the *Jiuzhang suanshu*, but it involves a separate category of problems known as *chongci* 重差 (multiple differences). In each problem of the *Haidao*, at least two gnomons are set up by the observer, and the multiple differences come from the observations made with the variously situated gnomons. For example, the first problem is as follows:

> 今有望海島。立兩表。齊高三丈。前後相去千步。令後表與前表參相直。從前表卻行一百二十三步。人目著地。取望島峰。與表末參合。從後表卻行一百二十七步。人目著地。取望島峰。亦與表末參合。問島高及去表各幾何。答曰。島高四里五十五步。去表一百二里一百五十步。
>
> 術曰。以表高乘表間。為實。相多為法。除之。所得。加表高。即得島高。求前表去島遠近者。以前表卻行。乘表間。為實。相多為法。除之。得島去表數。[68]
> (There is an island in the sea. Set up two gnomons, both 3 *zhang* tall, and 1000 *bu* apart. The two gnomons are in a straight line with the island. Walking backwards 123 *bu* from the front gnomon, an observer whose eye level is level with the ground would see that the peak of the island is in line with the top of the front gnomon. Walking backwards 127 *bu* from the back gnomon, an observer whose eye level is level with the ground would see that the peak of the island is in line with the top of the back gnomon. What is the height of the island and how far is the island from the front gnomon? Answer: height of island is 4 *li* 55 *bu*; distance away is 102 *li* 150 *bu*)
>
> Method: Multiply the height of the gnomons by the distance between them, yielding the dividend. Take the difference between 123 and 127 as the divisor. Divide the dividend by the divisor. Then add the height of the gnomon to the answer to get the height of the island. To get the distance between the front gnomon and the island, take 123 and multiply it by the distance between the two gnomons. This is the dividend. The divisor is the difference between 123 and 127. Divide the dividend by the divisor to get the distance)

---

[68] *Haidao suanjing*, 1.

The cryptic quality of the given method is typical of all the *Ten Mathematical Classics*. Why the method works is not explained in the text, and is therefore open to scholars' interpretation and speculation. Many believe that Liu Hui must have been using similar triangles to solve this question:[69]

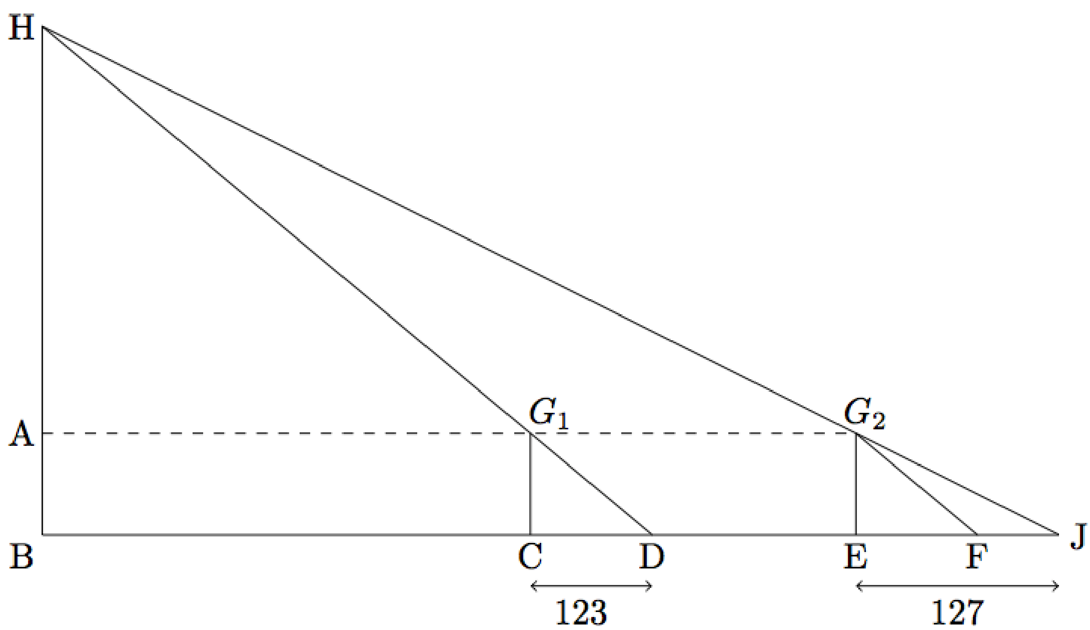

Figure 1.13 Representation of the first problem of the *Haidao suanjing*

In the above diagram, we have:

$$CD = EF = 123 \text{ } bu$$

$$AB = G_1C = G_2E = 3 \text{ } zhang$$

$$HAG_1 \text{ is similar to } G_2EF, \text{so } \frac{HA}{G_2E} = \frac{AG_1}{EF} = \frac{HG_1}{G_2F}$$

$$HG_1G_2 \text{ is similar to } G_2FJ, \text{so } \frac{HG_1}{G_2F} = \frac{G_1G_2}{FJ} = \frac{HG_2}{G_2J}$$

Noting that $\frac{HG_1}{G_2F}$ appears in both equations above, we therefore have:

[69] Guo Shuchun, *Gudai shijie shuxue taidou Liu Hui*, 178-179.

$$\frac{HA}{G_2E} = \frac{G_1G_2}{FJ} \; and \; \frac{AG_1}{EF} = \frac{G_1G_2}{FJ}$$

Height of the island $HB = HA + AB = \frac{G_2E \times G_1G_2}{FJ} + G_1C$

$$= \frac{\text{(height of gnomon)} \times \text{(distance between gnomons)}}{\text{difference between 127 } and \text{ 123}} + \text{height of gnomon}$$

$$Distance\ AG_1 = \frac{G_1G_2 \times EF}{FJ}$$

$$= \frac{\text{(distance between gnomons)} \times 123}{\text{difference between 127 } and \text{ 123}}$$

Solving the question in this way, we can see that the two equations above correspond exactly to the steps given in the method, so it appears as though we have correctly inferred Liu Hui's rationale for the solution. However, a significant problem emerges when one realizes that the notion of similar triangles cannot be found in any of Liu Hui's work. This is the problem where Wu Wenjun famously diverged from Qian Baocong's earlier method of solution using similar triangles. Because Liu Hui had used the term *chu ru xiang bu* 出入相補 ("complementing discrepancies") in his commentary in chapter nine of the *Jiuzhang*, Wu believed the same principle was used again here. Instead of similar triangles, Liu Hui might have used rectangles and areas to solve the problem:[70]

[70] Guo Shuchun, *Gudai shijie shuxue taidou Liu Hui*, 201-202.

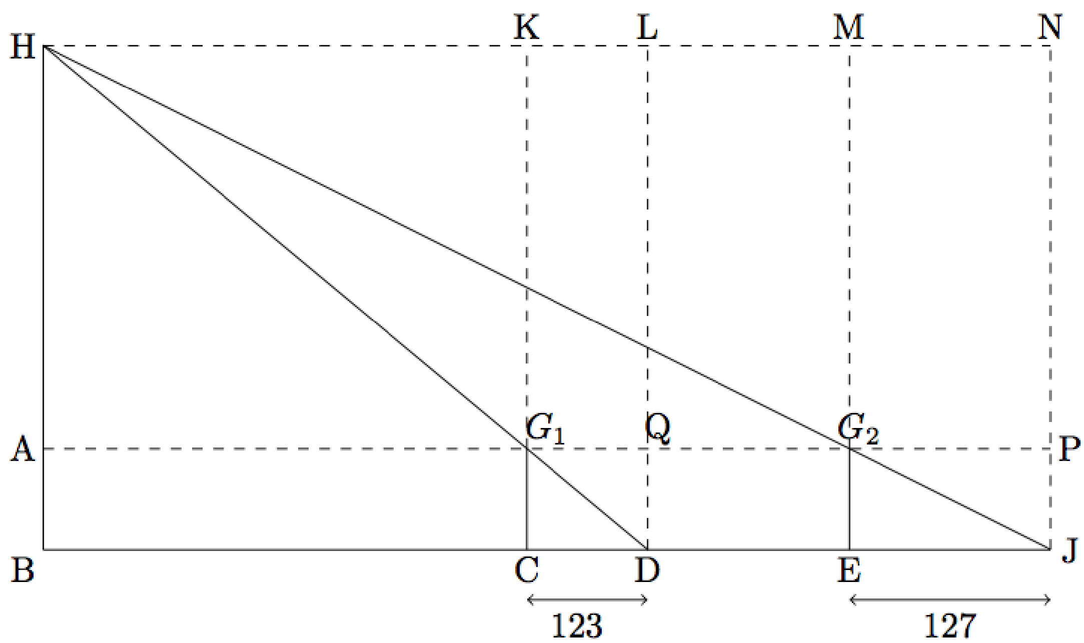

Figure 1.14 A different representation of *Haidao suanjing's* first problem

Given that the diagonal of a rectangle always bisects its area,

$$\square AE = \square G_2N \quad \text{and} \quad \square AC = \square G_1L$$

$$\square G_2N - \square G_1L = \square AE - \square AC = \square G_1E$$

$$(G_2P \times PN) - (G_1Q \times QL) = G_1G_2 \times CG_1$$

127(island's height – gnomon's height) – 123(island's height – gnomon's height)

= distance between gnomons × gnomon's height

(127 – 123)(island's height – gnomon's height)

= distance between gnomons × gnomon's height island's height = (distance between gnomons × gnomon's height)

÷ (difference between 123 and 127)

+ gnomon's height

From the above, it is clear that using rectangles and their areas would also result in the same steps that Liu Hui prescribed in his method. Furthermore, this solution utilizes only the mathematical concepts that we are certain the Chinese were familiar with, making it a far more persuasive conjecture of how Liu Hui himself had solved all the questions in the *Haidao*.

Commentary on the *Haidao suanjing*

Given the nature of its questions, the *Haidao* is striking today for its total lack of diagrams. However, this may be the consequence of human negligence rather than of any inconsiderateness of Liu Hui. His preface to the *Jiuzhang suanshu* states clearly that not only had he added an extra chapter to the *Jiuzhang*, he had also added commentary to the chapter to explain it. However, the current version of the *Haidao* only has Li Chunfeng's team's commentary, which consists largely of elaborating the step-by-step procedures given in the methods. In fact, the commentary added by Li's team only refers to the main text of the *Haidao*, so it may be that they had never seen any copy of Liu Hui's commentary, and that it had been lost before the Tang dynasty.

5. SUNZI SUANJING

Like the *Shushu jiyi*, the dating of the *Sunzi suanjing* has posed problems for historians. It has sometimes been thought to have been written by Sun Wu 孫武 (ca.

544 – 496 BCE), who is famous for his *Sunzi bingfa* 孫子兵法 (Master Sun's Military Tactics, or more commonly known as the Art of War). However, Dai Zhen pointed out that the *Sunzi suanjing* contained references to the cities Chang'an and Luoyang, as well as a question on calculating the number of words in a Buddhist sutra. Therefore, he believed it could not have originated with Sun Wu, but must have been written after the reign of Emperor Ming 明 (r. 58 – 75) of the Han dynasty. Qian Baocong noted that the units of measurement listed in the first section of the *Sunzi suanjing* correspond to Tang regulations, but not to the Sui, but he dated the text to around the year 400 partly because the Tang imperial catalogue stated that Zhen Luan had written a commentary for the *Sunzi suanjing*. Therefore, the first section is taken as evidence of Tang-dynasty individuals altering the original text.[71]

The *Sunzi suanjing* comes with a preface that appears to have been written by the original author. The encouraging tone of the preface very much suits the function of this treatise as a text for beginners. In the preface, the author states that *suan* 算 (calculating or predicting) can allow people to understand practically everything in the universe. Anyone who is willing to devote effort into it can learn it and would thereby be vastly enriched by it:

> 孫子曰。夫算者。天地之經緯。群生之元首。五常之本末。陰陽之父母。星辰之建號。三光之表裏。五行之準平。四時之終始。萬物之祖宗。六藝之綱紀。稽群倫之聚散。考二氣之降升。推寒暑之迭運。步遠近之殊同。觀天道精微之兆基。察地理從横之長短。采神祇之所在。極成敗之符驗。窮道德之理。究性命之情。立規矩。準方圓。謹法度。約尺丈。立權衡。平重輕。剖毫釐。析黍絫。歷億載而不朽。施八極而無疆。。。嚮之者富有餘。背之者貧且窶。。。夫欲學之者。必務量能揆已。志在所專。如是則焉有不成者哉。[72]

[71] Qian, *Suanjing shi shu*, 275.

[72] *Sunzi suanjing xu* 孫子算經序 (Master Sun's Mathematical Classic – preface), 1.

> (Master Sun says, with *suan*, one can grasp the length and breadth of Heaven and Earth, the origin of living things, the fundamentals of the five constant virtues, the parent of *yin* and *yang*, the establishment of the stars, the outer and inner workings of the three lights, the balance of the five phases, the beginning and end of the four seasons, the progenitor of all objects, and the regulations of the six gentlemanly arts. One can examine the comings and goings of relationships, investigate the rise and fall of the two energies, predict the change of cold and heat, measure the distance between near and far, see the precise foundation of the Heavenly Way, find horizontal and vertical distances on Earth, locate heavenly beings, predict success and failure, comprehend the principle of virtue, investigate the situation of life, establish rules, regulate the round and the square, treat the laws with care, control and chi and the zhang, establish balance, balance differences, and distinguish between small differences. Suan will not deteriorate despite eons, and can be used in the eight directions without encountering boundaries... People who follow it will be rich. People who turn their backs on it will be poor... People who wish to learn it should apply themselves with discipline. If one concentrates on learning, there is no reason not to have success.)

The *Sunzi suanjing* is divided into three sections. The first section was probably meant to serve as a useful reference guide as well as an introduction to some basic mathematical skills. It begins by listing the units of measurement of length, area, and volume. It also discusses how to name large numbers, and the rules described correspond to the second system mentioned in the *Shushu jiyi*. Then follows a series of handy items for the reader's reference, such as the conversion rates of various grains, the weight of one square *cun* of various metals, and useful, though imprecise, ratios of $\pi = 3$ and 5:7 for a square's side in comparison to its diagonal. This section also discusses how to work with fractions and how to use counting rods to perform basic arithmetic operations, which is why historians are convinced the *Sunzi suanjing* was meant for beginners. The rest of the section is a long list of multiplication and division facts, which might have been meant to be memorized by the reader or meant as exercises for the reader to practice doing multiplication and division with counting rods.

The second and third sections of the *Sunzi suanjing* follow the question-answer-method format like in the *Jiuzhang*, but the *Sunzi suanjing* is a much shorter text in comparison, with only 28 questions in the second section and 35 in the third. Also the questions in the *Sunzi suanjing* do not appear to be organized in terms of any theme, but are a mix of multiplication and division problems, area and volume problems, proportion questions, and questions that require the exertion of logic and reasoning. The most famous question from the *Sunzi suanjing* is sometimes called the Chinese Remainder Theorem:

> 今有物。不知其數。三三數之。賸二。五五數之。賸三。七七數之。賸二。問物幾何。答曰。二十三。[73]
> (There is an unknown number of items. If they are counted by threes, there is a remainder of two. If counted by fives, the remainder is 3. If counted by sevens, the remainder is 2. How many items are there? Answer: 23)

There are also a few somewhat more imaginative questions, such as the following:

> 今有獸六首四足。禽二首二足。上有七十六首。下有四十六足。問禽獸各幾何。答曰。八獸七禽。[74]
> (There is a beast with six heads and four legs. There is a bird with two heads and two legs. If there are 76 heads and 46 legs in total, how many beasts and birds are there? Answer: 8 beasts and 7 birds)

Perhaps the most interesting question is the last one:

> 今有孕婦。行年二十九歲。難九月。未知所生。答曰。生男。
> 術曰。置四十九。加難月。減行年。所餘。以天除一。地除二。人除三。四時除四。五行除五。六律除六。七

[73] *Sunzi suanjing*, 23.
[74] Ibid.

> 星除七。八風除八。九州除九。其不盡者。奇則為男。耦則為女。[75]
> (There is a woman, aged 29 *sui*, who gives birth after nine months of pregnancy. What is the gender of her child? Answer: Male.
> Method: Take 49. Add the number of months of pregnancy. Minus the woman's age. Then take away 1 for the heaven, 2 for the earth, 3 for humans, 4 for the four seasons, 5 for the five phases, 6 for the six musical notes, 7 for the seven stars, 8 for the eight winds, 9 for the nine regions. If the result is odd, the child is male. If the result is even, the child is female.)

Though often dismissed by historians as totally absurd, this problem is interesting because it is significantly different from all the other questions in the *Sunzi suanjing*, and it actually ties in very well with the author's claim in the preface that *suan* is the basis for comprehending everything in this world. While the rest of the questions had involved *suan* as a calculation tool, in the last question it was used for its predictive function. That the author felt it was appropriate to include such a question in a treatise that is otherwise exclusively mathematical will have bearing on our discussion of how the Chinese conceptualized the subject and function of mathematics.

Another interesting thing to note about the *Sunzi suanjing* is that nine questions in its second section are exactly the same as in the *Jiuzhang suanshu*.[76] It might have been that the author of the *Sunzi suanjing* had simply copied the said questions from the *Jiuzhang*. In fact, the layout of the *Sunzi suanjing* in the question-answer-method format suggests that the influence of the *Jiuzhang* on its author might be quite strong. Alternatively, the authors of both the *Jiuzhang* and the *Sunzi* might have copied the questions from another mathematical treatise that is no longer extant.

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[75] *Sunzi suanjing*, 26.
[76] Guo and Liu, *Suanjing shi shu*, preface (no page numbers).

<u>Commentary on the *Sunzi suanjing*</u>

Both Zhen Luan and Li Chunfeng's team are believed to have added commentaries to the *Sunzi suanjing*. Unfortunately, neither of these has survived.

6. XIAHOU YANG SUANJING

The *Xiahou Yang suanjing* is one of the more problematic treatises among the *Ten Mathematical Classics* because it appears to be suffering from a case of mistaken identity. The original *Xiahou Yang suanjing* is known to have been written before the *Zhang Qiujian suanjing* because the latter's preface refers to it. The *Zhang Qiujian suanjing* has been dated with a high degree of certainty to some time between the year 466 and 485, or at most shortly after, because the thirteenth question of its second section invokes a law on tax distribution that was in effect during that period.[77] Therefore, the *Xiahou Yang suanjing* should have been written even earlier. However, the present version contains several references to laws that were passed in the Tang dynasty, so Qian Baocong advanced the view that it was written sometime between 763 and 779.[78] Because these references fit very well with the internal coherence of the text, it does seem unlikely that they were added to the text centuries after its initial compilation. Why it became mistaken for the real *Xiahou Yang suanjing* is due to the fact that it begins with the phrase "Xiahou Yang says" (*Xiahou Yang yue* 夏侯陽曰) to mark a quote from that treatise. During the Song dynasty, when compiling the Tang imperial catalogue as part of the standard dynastic history, this later treatise became identified as the *Xiahou Yang suanjing* while the real one had most likely been lost. Therefore, when the Song government

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[77] Guo and Liu, *Suanjing shi shu*, preface (no page number).
[78] Qian, *Suanjing shi shu*, 551.

printed the *Ten Mathematical Classics* in 1084, this was the version that was printed and distributed. Because this is the *Xiahou Yang suanjing* recognized by the Chinese from the Song dynasty onwards, we shall continue to refer to it by this name.

It is unclear who Xiahou Yang actually was. The author of the present *Xiahou Yang suanjing* remains unknown too. However, the treatise is accompanied by a preface written by the author, which shows that this individual had read both the *Wucao suanjing* and the *Sunzi suanjing*, and he greatly admired both Liu Hui's and Zhen Luan's commentaries on them:

> 五曹孫子。述作滋多。甄鸞、劉徽。為之詳釋。稽之往古。妙絕其能。儲校今時。少有聞見。[79]
> (Works that relate to the *Wucao* and the *Sunzi* are many. Zhen Luan and Liu Hui have made detailed commentaries on them. Looking at the past, their work is outstanding. Comparing with the present, it is rare to meet their equal.)

The *Xiahou Yang suanjing* appears to be designed as a practical handbook. Unlike the *Wucao suanjing*, however, which is comprised entirely of questions, the *Xiahou Yang suanjing* consists of a mixture of both questions and passages meant to explain certain calculation techniques or relevant regulations of the period. The treatise begins with a passage on simplifying calculations, doing calculations with counting rods, and converting between different units of measurement, all of which is probably a quote from the original *Xiahou Yang suanjing*. It then goes on to explain units of measurement for length, area, volume, and weight as set out by imperial regulations. This first section of the treatise also gives formulae for finding the areas of various shapes, but without using any numbers or examples. It also discusses grain conversion rates and finding square roots. The second and third sections are comprised entirely of questions in the traditional question-answer-method format.

[79] *Xiahou Yang suanjing xu* (Xiahou Yang's Mathematical Classic – preface), 1.

These questions, 73 in total, cover topics that can also be found in earlier treatises, such as calculating the amount of tax grain and dividing a certain number of items among a certain number of people. Therefore, it would have served as a good exercise book for practising various calculation techniques. The fact that it was mistaken as one of the *Ten Mathematical Classics* was very likely a strong contributing factor to its preservation, making it the only mathematical treatise from the eighth century to have survived until today.[80]

Commentaries on the *Xiahou Yang suanjing*

The original *Xiahou Yang suanjing* should have commentaries by Zhen Luan and Li Chunfeng's team. The present version did not contain any commentary when it was reprinted during the Song dynasty.

7. ZHANG QIUJIAN SUANJING

The *Zhang Qiujian suanjing* was named after its author, but very few details are known about him. From his preface, it is clear that he had read both the *Sunzi suanjing* and the original *Xiahou Yang suanjing*, but found them deficient in certain respects, so he wrote a new treatise to address that:

> 其夏侯陽之方倉。孫子之蕩杯。此等之術。皆未得其妙。故更造新術。推盡其理。附之於此。[81]
> (The granary question in the *Xiahou Yang* and the cup-washing question in the *Sunzi* both use methods that have not attained true finesse, so I have constructed new methods and extract all their principles, which are attached here.)

In general, the questions in the *Zhang Qiujian suanjing* are more complex than the *Sunzi suanjing*, and were probably meant for intermediate learners who had

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[80] Qian, *Suanjing shi shu*, 553.
[81] *Zhang Qiujian suanjing xu* (Zhang Qiujian's Mathematical Classic – preface), 1.

already mastered the most basic techniques. However, the author recognized that working with fractions could be a difficult task, so he devoted a large part of his preface to explain the relevant rules, and began the first section of the main text with questions on the multiplication and division of fractions.

The *Zhang Qiujian suanjing* has almost 100 questions divided unevenly into three sections. However, Dai Zhen noted that the copy he was working with was missing several pages, and the contents of these pages still have not been recovered, so it is unclear exactly how many questions were in the original version. The questions are not organized thematically, but cover a wide range of topics. In addition to area and dimensions questions, the *Zhang Qiujian suanjing* also has questions like calculating the height or width of objects at a distance, and calculating distances based on the travelling speeds of two or three people. One type of question interspersed throughout the entire text is on the concept of squaring the circle and vice versa, where a given circle has to be turned into a square of the same area, or vice versa, and the reader is asked to calculate the dimensions of the new shape. This type of question was also extended to three dimensions with spheres and cubes. The *Zhang Qiujian suanjing* also contains a variation of the weaving question seen in earlier treatises:

> 今有女子善織。日益功疾。初日織五尺。今一月。日織九匹三丈。問日益幾何。[82]
> (There is a woman adept at weaving. She increases her productivity at a constant rate everyday. On the first day, she wove 5 *chi*. After a month, she is weaving 9 *pi* 3 *zhang* in one day. How much did her productivity increase per day?)

The most famous question of the *Zhang Qiujian suanjing* is the Hundred Fowls question, different versions of which are also contained in the *Shushu jiyi*:

[82] *Zhang Qiujian suanjing*, 13.

> 今有雞翁一值錢五。雞母一值錢三。雞雛三值錢一。凡百錢。買雞百隻。問雞翁、母、雛、各幾何。[83]
> (A rooster is worth 5 *qian*, a hen 3 *qian*, and 3 chicks are worth 1 *qian*. If 100 fowls were bought with 100 *qian*, how many roosters, hens, and chicks were bought?)

Three solutions were given for this question:

- 4 roosters, 18 hens, 78 chicks
- 8 roosters, 11 hens, 81 chicks
- 12 roosters, 4 hens, 84 chicks

The method given to explain the solution is rather cryptic and leaves the reader wondering why it works:

> 術曰。雞翁每增四。雞母每減七。雞雛每益三。即得。[84]
> (Method: Every time the number of roosters is increased by 4, decrease the number of hens by 7 and increase the number of chicks by 3.)

However, it demonstrates an awareness that multiple solutions are possible for this type of indeterminate problem.

<u>Commentaries on the *Zhang Qiujian suanjing*</u>

The title page of the *Zhang Qiujian suanjing* states that commentaries had been added by Zhen Luan and by Li Chunfeng's team. Zhen Luan's commentary is no longer extant. Li's team's commentary consists largely of explaining some of the methods given by Zhang Qiujian and of supplying the methods where none had existed before.

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[83] *Zhang Qiujian suanjing*, 54.
[84] Ibid, 55.

8. WUCAO SUANJING

The *Wucao suanjing* appears to have been written specifically for the benefit of officials and government clerks because the contents are tailored to address the sorts of mathematical tasks involved in day-to-day administration. This treatise first appeared in the Tang imperial catalogue. Its original author and its date of compilation remain unknown, but Zhen Luan is said to have written a commentary on it, so the *Wucao* must have been written by the sixth century at the latest.

The *Wucao* has 67 questions unevenly divided into five sections, with each section named after a government department and with each section's questions dealing with the sorts of tasks associated with that particular department. The *Wucao* is not a treatise for beginners. Multiplication and division with large numbers are assumed to have been mastered by the reader. However, the mathematics has been kept as simple as possible by avoiding the use of fractions.[85]

The first section, *Tian cao* 田曹 (department of fields), contains 19 questions on calculating the area of fields of various shapes, but Dai Zhen and modern historians have noted that the formulae given for the more complex shapes are all wrong. For example:

> 今有腰鼓田。從八十二步。兩頭各廣三十步。中央廣十二步。問為田幾何。答曰。八畝奇四十八步。
> 術曰。并三廣得七十二步。以三除之。得二十四步。以從八十二步乘之。得一千九百六十八步。以畝法除之。即得。[86]
> (There is a field in the shape of a waist-drum. Its length is 82 *bu*. Its width at either end is 30 *bu*. The width at the middle is 12 *bu*. What is the area? Answer: 8 *mu*, remainder 48.

[85] Qian, *Suanjing shi shu*, 409.
[86] *Wucao suanjing*, 1. One mu = 240 bu.

Method: Combine the three widths to get 72 *bu*. Divide it by 3 to get 24 *bu*. Multiply it by 82 to get 1968 *bu*. Convert it to *mu*.)

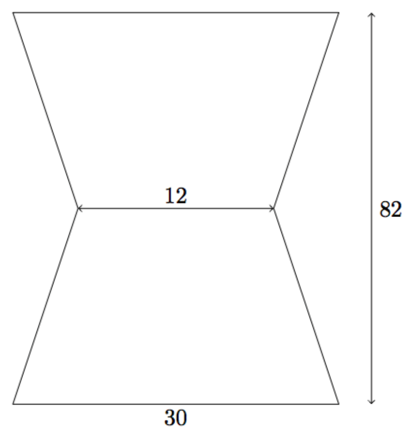

Figure 1.15 Field in the shape of a waist-drum

The original author seemed to have assumed that the formula for trapezoidal fields could simply be extended to include one more side. However, as the formula given by Dai Zhen suggests, this shape should be treated as the combination of two trapezoids.

The second section, *Bing cao* 兵曹 (department of military affairs), contains 12 questions on calculating military personnel and supplies, as well as one question on calculating the area of land needed to park a given number of carts.

The 14 questions in the third section, *Ji cao* 集曹 (department of markets), mainly deal with the trading or bartering of goods, including questions on calculating the amount of a certain type of grain that can be exchanged for another, as well as calculating the number of items that can be purchased with a certain amount of money.

The fourth section, *Cang cao* 倉曹 (department of granaries), mainly concerns the calculation of tax grain, payment required for the transportation of grain, and the volume of both granaries and piles of grain on the floor. The last section, *Jin cao* 金曹 (department of finance), contains questions that generally deal with dividing a certain amount of money or goods among a given number of households or people.

From the above, we can see that the *Wucao suanjing* does not contain any mathematical technique that has not been covered by the *Jiuzhang suanshu*. However, the way the questions are organized probably made the treatise a very useful handbook for officials and clerks as they went about their daily duties, and a very good exercise book for everyone else to test their skill in solving these particular sorts of questions.

Commentary on the *Wucao suanjing*

According to the Tang imperial catalogue, commentaries had been added by Zhen Luan and Li Chunfeng's team. Unfortunately, neither commentary seems to have survived past the Song dynasty. Therefore, we have no way to tell if the erroneous area formulae given in the *Wucao's* first section had ever been pointed out to the students of the Tang imperial academy, or if generations of the academy's graduates were trained to muddle the actual size of the Chinese empire.

9. WUJING SUANSHU

The *Wujing suanshu* is believed to have been written by Zhen Luan in the later half of the sixth century. It is significantly different from all of the other *Ten Mathematical Classics* in that it is comprised entirely of passages copied from the Confucian classics that are concerned with mathematics, calendrical calculations, and music, accompanied by Zhen Luan's annotations. It is not certain what the primary purpose of this treatise was, and not many historians have devoted much attention to it. It is possible that it was designed, like the *Wucao suanjing*, to be a useful reference for officials and clerks. An example of the passages contained in the *Wujing suanshu* is as follows:

> 求十九年七閏法
> 置一年閏十日。以十九年乘之。得一百九十日。又以八百二十七分。以十九年乘之。得一萬五千七百一十三。以日法九百四十除之。得十六日。餘六百七十三。以十六加上日。得二百六日。以二十九除之。得七月。餘三日。以法九百四十乘之。得二千八百二十。以前分六百七十三加之。得三千四百九十三。以四百九十九命七月。分之適盡。是謂十九年。得七閏月。月各二十九日九百四十分日之四百九十九。[87]
> (Calculating the rule of having seven intercalary months for every 19 years
> Take 10 for the number of intercalary days per year. Multiplying it by 19 years, one gets 190 days. Take 827. Multiplying it by 19 years, one gets 15 713. Dividing it by the constant 940, one gets 16 days, with a remainder of 673. Adding 16 to the number of days above, one gets 206 days. Dividing it by 29, one gets seven months, with a remainder of three days. Multiplying it by 940, one gets 2820. Adding the 673 from above to it, one gets 3493. Dividing it by 499, one gets seven months without any remainder. Hence for every 19 years, there are seven intercalary months, and each month has 29 $\frac{499}{940}$ days.)

<u>Commentary on the *Wujing suanshu*</u>

The commentary added by Li Chunfeng's team has survived, but it does not contribute much towards our understanding of the text because it consists mainly of posing some of the information presented in the text as questions that can be used for practice.

[87] *Wujing suanshu*, 2-3.

10. JIGU SUANJING

The *Jigu suanjing* was written and annotated by Wang Xiaotong, a calendrical expert of the early Tang dynasty. It has 20 questions in total. From the title, which translates into the "continuation of ancient mathematics", and from Wang's preface, it is clear that this treatise was meant to build upon the mathematics of the *Jiuzhang suanshu* and push the level of sophistication further. Indeed, the complexity of its mathematics is higher than any of the earlier extant treatises.[88] The first question has to do with calculating the path of the moon. Then follow thirteen questions on construction problems and six questions on the application of the *gougu* principle. Although Wang Xiaotong annotated the treatise himself, the text remains terse, obscure, and difficult to understand.[89] For example, historians have puzzled over the question on building an observatory that is comprised of two structures joined together. While the actual question is asking to calculate the dimensions of the observatory, one of the most interesting questions for historians has actually been about the shapes of the two structures and how they joined together. Scholars have put forward different theories. For example, Qian Baocong believed the observatory might have looked like this:[90]

[88] Guo Shuchun et al., *Zhongguo kexue jishu shi – shuxue juan* 中國科學技術史 —— 數學卷 (History of Chinese Science and Technology – Mathematics) (Beijing: Kexue chubanshe, 2010), 202.

[89] Tina Su-lyn Lim and Donald B. Wagner argue that differences in terminology in the main text and the commentary may actually indicate that not all the commentaries were made by Wang Xiaotong. See Lim and Wagner, "Wang Xiaotong on Right Triangles", 13.

[90] Guo Shirong 郭世榮, "*Jigu suanjing* zao yangguan tai ti xin jie 緝古算經造仰觀台題新解" (A new interpretation of the construction of an observatory question in the *Jigu suanjing*), *Ziran kexue shi yanjiu* 自然科學史研究 (Studies in the History of Natural Sciences) 13, no. 2 (1994): 108.

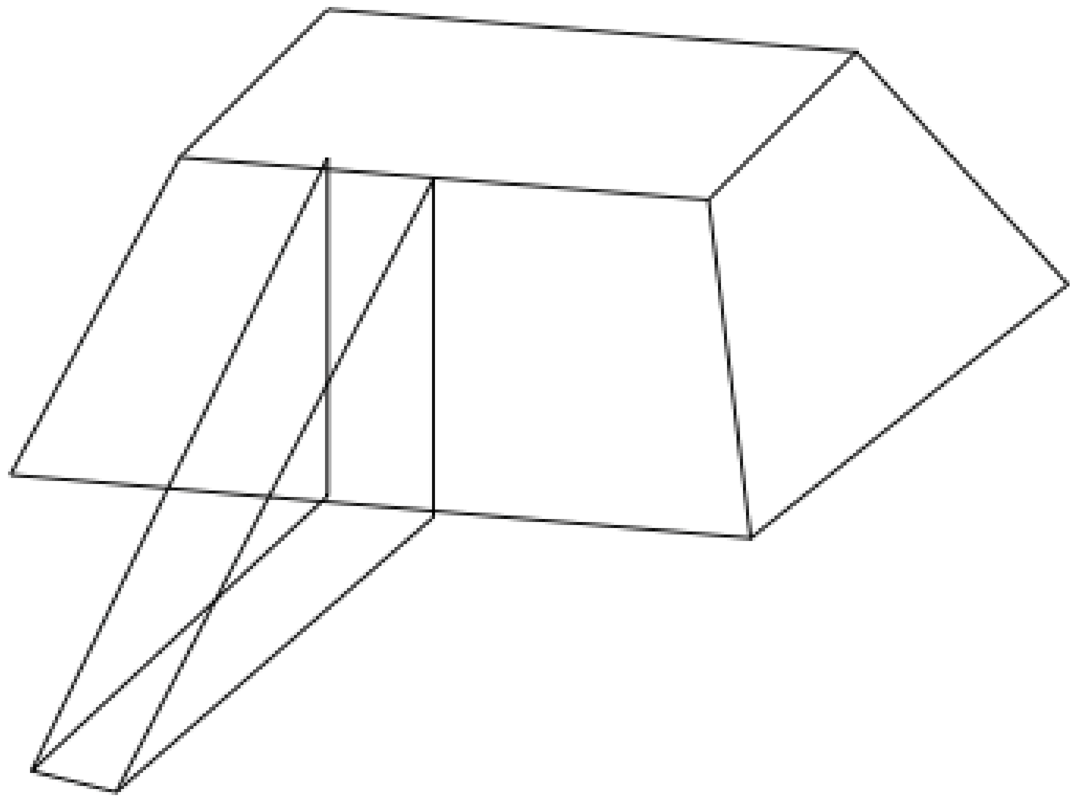

Figure 1.16 Representation of Qian Baocong's conceptualization of *Jigu suanjing*'s observatory

Guo Shirong came up with a different shape:[91]

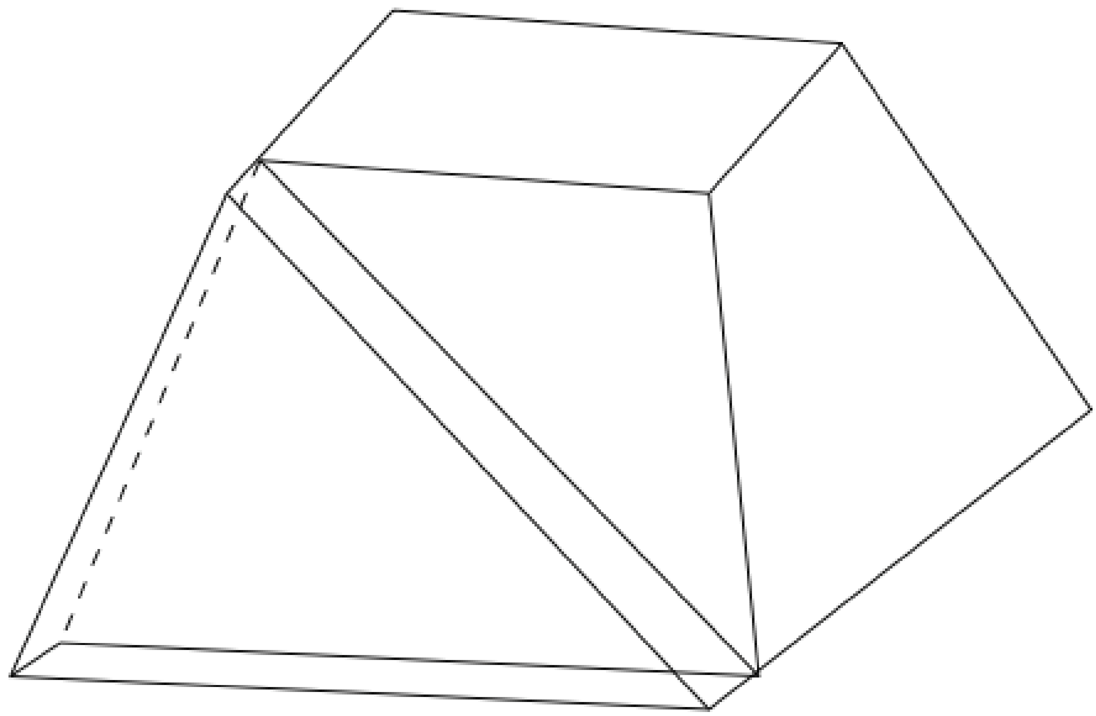

Figure 1.17 Representation of Guo Shirong's conceptualization of *Jigu suanjing*'s observatory

However, without actual models of observatories that had existed during the Tang dynasty, it is impossible to determine what Wang Xiaotong's observatory must have looked like.

CONCLUSION

Although the *Ten Mathematical Classics* were written in very different periods, common themes can be found among them that reflect the type of society from

[91] Guo Shirong, "*Jigu suanjing* zao yangguan tai ti xin jie", 108.

which they came. Repeatedly, we see questions concerning grain conversion, storage of grain, transportation, and land measurements. These were also some of the central concerns of the state, which had to manage the calculation, collection, and transportation of taxes. Taken together, the *Ten Mathematical Classics* would have trained students to master the skills needed to fulfil the tasks of government administration, including local adminstrative tasks that required the accurate surveying of land or the correct calculation of money. However, that is not to say that these treatises were all written specifically to address the practical administrative problems of the government. Many of the authors discussed in this chapter were not simply concerned with promoting technical competence to solve day-to-day problems, but also held a strong belief in the power of numbers and of the art of *suan* to help humans comprehend the world in which they lived. These people who projected such a view of the universe will be the subject of the next chapter.

Chapter Two

Writing About Mathematics in Pre-Modern China:

The Writers and Commentators of the *Ten Mathematical Classics*

"...mathematics should be defined by what the subject is for those who practise, learn, and teach it..."[1]

~ Joan L. Richards

In 1109 CE, a commemorative ritual was held during the reign of Emperor Huizong 徽宗 (r. 1100 – 1126), who is known for his fondness of lavish Daoist rituals,[2] to honour and award posthumous noble titles to men who were considered to have made a significant contribution to *suan* 算.[3] In total, sixty-six men were thus honoured. Among them were many of the authors, compilers, and commentators of the *Ten Mathematical Classics*. In the ritual, mathematical writers were honoured alongside men whose work dealt primarily with calendar-making, astronomy, and astrology, which reflects the broad definition of the concept of *suan* throughout most of China's imperial history.

The people of interest to us in this chapter, namely all the known writers, compilers, and commentators of the *Ten Mathematical Classics*, can be divided into three groups:

1. Men who are not known to have held any official position within the government

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[1] Joan L. Richards, "The History of Mathematics and L'esprit humain: A Critical Reappraisal", *Osiris* 10 (1995): 129.

[2] Patricia Buckley Ebrey, *Emperor Huizong* (Cambridge, MA: Harvard University Press, 2014), 148-151.

[3] *Song shi*, 105, 2551-2552.

2. Men who held official positions that were outside the Bureau of Astronomy and Astrology
3. High-ranking officials within the Bureau of Astronomy and Astrology

The first group includes men of uncertain historicity, whose existence has not been confirmed by sources other than the mathematical treatises in which their names appear. Our purpose, however, is not to question whether or not these men were real historical figures. What is important here is that the Chinese believed them to have been real and to have been examples of men who had contributed to the development of mathematics. This can be seen from the fact that they were included in the commemorative ritual of 1109, and that their biographies, however insubstantial, were compiled by Ruan Yuan 阮元 (1764 – 1849) around the year 1800 in his *Chouren zhuan* 疇人傳 (Biographies of Mathematical and Astronomical Practitioners), a work that was meant to serve as a biographical dictionary of all noteworthy mathematical and astronomical practitioners from both China's and Europe's past.[4] In the *Chouren zhuan*, Ruan Yuan explicitly rejected the inclusion of foretelling and astrological activities in the biographies, explaining that these were not and ought not be within the purview of *suanshu* 算術, or the art of *suan*. Therefore, Ruan's notion of *suan*, and likely most of his readers' at the start of the nineteenth century, already reflected the modern concept of *suan* as well as the modern distinction between the mathematical and the non-mathematical, the astronomical and the non-astronomical.

It must be noted here that all the writers, compilers, and commentators discussed in this chapter are men. In this narrative, and indeed in the existing

[4] Ruan Yuan, *Chouren zhuan* (Beijing: Zhonghua shuju, 1991), front matter, 2-3.

narrative of all of China's mathematical and astronomical history, there is no figure like Caroline Herschel (1750 – 1848), who famously assisted her brother William Herschel (1738 – 1822) in his astronomical observations, and was recognized as an accomplished astronomer herself.[5] That is not to say that there were no learned women in early and mid-imperial China. The most famous Chinese brother-and-sister team is probably Ban Gu 班固 (32 – 92) and Ban Zhao 班昭 (ca. 49 – ca. 120), who together compiled a history of the first half of the Han 漢 dynasty (202 BCE – 220 CE). Although their history includes monographs on astronomy and calendar-making, the Ban siblings are not known to have been particularly knowledgeable in these fields. Related more directly to the history of the sciences, another example of a learned woman is the mother of He Chengtian 何承天 (370 – 447), an astronomical writer of the Northern and Southern dynasties 南北朝 period (420 – 581). However, unlike Caroline Herschel, there is no evidence that He's mother ever participated in his astronomical activities.[6] As we examine the lives of the *Ten Mathematical Classics'* writers, compilers, and commentators in this chapter, while we can certainly be hopeful that they had received the intellectual support of their mothers, sisters, wives, or daughters, there is at present nothing we can say about these women because they are completely absent from our available sources.

The questions that we explore in this chapter are not only about the lives of these mathematical writers and details about the China in which they lived. Where possible, we also examine how these writers themselves viewed their work on the *Ten Mathematical Classics*. Another goal of this chapter is to find a meaningful way to characterize the mathematical writers of early and mid-imperial China. Given the

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[5] See Michael Hoskin, *Discoverers of the Universe: William and Caroline Herschel* (Princeton: Princeton University Press, 2011).

[6] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 198-199.

scarcity of sources, it is difficult to create an all-encompassing characterization that would fit all the writers of the period. Still, it is vitally important to make the effort because, in the popular imagination and in the minds of many modern scholars, these writers are often thought of as belonging to the same group of people as modern mathematicians, but perhaps as earlier, less advanced versions of the latter. However, as we will see throughout the rest of this chapter, the popular image that we have of mathematicians devoting their lives entirely to mathematics to the exclusion of other activities does not map well onto these early Chinese mathematical writers. While the scarcity of relevant sources on the actual readers of the Ten Mathematical Classics hampers our effort to discuss the readership of these texts, it must be kept in mind that the commentators were simultaneously readers. While they may not necessarily be representative of all the people who read the Ten Mathematical Classics, they and the later mathematical writers to be discussed in Chapter Four constitute our only available samples.

The next section explores the lives of these men. Rather than list them in chronological order, they are divided into the three groups described earlier in this chapter:

Table 2.1 Known Authors and Commentators of the *Ten Mathematical Classics*

Men with No Official Position	Men with Official Positions Outside the Bureau of Astronomy and Astrology	Men with Official Positions Within the Bureau of Astronomy and Astrology
Xu Yue 徐岳	Zhang Cang 張蒼	Wang Xiaotong 王孝通
Zhao Shuang 趙爽	Geng Shouchang 耿壽昌	Li Chunfeng 李淳風
Liu Hui 劉徽	Zu Chongzhi 祖沖之	
Sunzi 孫子	Zhen Luan 甄鸞	
Xiahou Yang 夏侯陽		
Zhang Qiujian 張丘建		

Perhaps surprisingly, only two out of the 12 writers of interest to this thesis belonged to the Bureau of Astronomy and Astrology, the government apparatus that is often thought of as the primary site for the creation of mathematical knowledge in order to serve the purpose of calendar-making and predicting heavenly phenomena. Close examination of the lives of these early creators and disseminators of mathematical knowledge will therefore call into serious doubt the traditional view that mathematics' importance to the pre-modern Chinese lay primarily in its ability to serve the loftier sciences of the heavens.[7] This older view suggests that the history of Chinese mathematics is a simple, unilinear story, and is primarily preoccupied with perhaps the most obvious and high-profile use of mathematics by the state, which was in the Bureau of Astronomy and Astrology. However, as will be discussed in this chapter and Chapter Four, mathematics was also written about and practised outside the bureau as well as outside any state apparatus. Therefore, the rich and complex

[7] See for example Ulrich Libbrecht, *Chinese Mathematics in the Thirteenth-Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, MIT East Asian Science Series, vol. 1, ed. Nathan Sivin (Cambridge, MA: MIT Press, 1973), 4.

nature of the history of Chinese mathematics should be recognized as a multi-strand story, some of which were inevitably tied to state uses while others were concerned with other purposes.

WRITERS, COMPILERS, AND COMMENTATORS WHO ARE NOT KNOWN TO HAVE HELD ANY OFFICIAL POSITION

This is the group about whom the available sources hold the least information. This is typical of the situation in pre-modern China: the records and histories maintained by the government tend to be the most well-preserved historical source. People who had no cause to be mentioned in these texts usually become unknowable over time unless they or their relatives or associates had left a significant amount of writing about their lives. For the people of interest in this section, very little is known beyond their names and what can be speculated from circumstantial evidence. Of course, the official histories by no means contain comprehensive lists of everyone who served within the bureaucracy. However, from the fact that information about these men was not included in the biographical section of the dynastic histories about imperial family members and noteworthy officials and individuals, it is unlikely that they had high political status during their lifetime.

Xu Yue and Zhao Shuang

Xu Yue (fl. ca. 220 – 226), possibly the author of the *Shushu jiyi*, hailed from Donglai 東萊 in modern Shandong province. He is recorded in the dynastic histories as having debated with Han Yi 韓翊 (fl. ca. 220 – 226) over the merits of Liu Hong's 劉洪 (ca. 130 – ca. 210) *Qianxiang* 乾象 ("Uranic manifestation") calendar system in the early years of the Northern Wei 北魏 dynasty (220 – 265). It is not clear if Xu

Yue had any official function within the Northern Wei government, but the contents of the *Shushu jiyi* suggests that he was a student of Liu Hong,[8] so he probably learnt the *Qianxiang* calendar system directly from him. The *Shushu jiyi* does not come with a preface, but the title suggests that the author wanted to put into writing various components of the mathematical art that he had learnt from Liu Hong, thus suggesting that Xu Yue had learnt both mathematics and astronomy from Liu Hong.

Zhao Shuang is the composer of the earliest extant mathematical commentary in Chinese history. His name is known from the preface he added to the *Zhoubi suanjing*. However, the imperial catalogues included in the dynastic histories of the Sui and Tang list his name as Zhao Ying 趙嬰. Scholars are divided over whether this was merely a copyist error or if this was in fact a literary name (*hao* 號) that Zhao Shuang had used. In any case, he is most commonly referred to as Zhao Shuang in the existing literature. In his own preface, Zhao Shuang signed off as Zhao Junqing 趙君卿, which is usually interpreted as his courtesy name (*zi* 字).[9]

No other biographical detail is mentioned in Zhao Shuang's writings. His commentary on the *Zhoubi suanjing* demonstrates the deep familiarity with the Confucian canon that was typical of all highly educated men in pre-modern China since Emperor Wu 武 (r. 141 – 87 BCE) of the Han dynasty instituted the policy of banning the hundred schools of thought and honouring only Confucianism (ba chu bai jia, du zun ru shu 罷黜百家, 獨尊儒術). What was less typical was Zhao Shuang's familiarity with astronomical texts including the *Qianxiang* calendar system,

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[8] *Shushu jiyi*, 2-3.

[9] Both men and women of higher social status from the Zhou dynasty onwards were usually given a courtesy name when they came of age. However, while men's courtesy names were to be used outside of their families, women's given names were rarely revealed to outsiders. See Endymion Wilkinso, *Chinese History: A New Manual,* 4th ed. (Cambridge, MA: Harvard University Asia Centre, 2015), 113-114, 173-174.

which was also known to Xu Yue, since astronomy had never been considered a mandatory component of the Confucian tradition.[10] The *Qianxiang* calendar system was completed by Liu Hong around 206 CE under the auspices of the Han dynasty, but was adopted for official use only by the state of Wu 吳 (222 – 280) during the Three Kingdoms 三國 period (220 – 265).[11] This has led many scholars to believe that Zhao Shuang must have lived within the short-lived state of Wu,[12] although it is also conceivable that he, like Xu Yue, might have been a close associate of Liu Hong and had acquired knowledge about the *Qianxiang* calendar system directly from its creator.

Knowing so little about Zhao Shuang's life, it is impossible to connect his work on the *Zhoubi suanjing* with the other activities he had engaged in. However, his motivation for writing a commentary on the *Zhoubi* is clear from his preface: he considered the *Zhoubi* a valuable text for understanding the heavens, but was terse and difficult to understand. Therefore, he wanted to add explanations to it lest it be abandoned by people who studied the heavens.

> "聊觀周髀。其旨約而遠。其言曲（或作典）而中。將恐廢替。濡滯不通。使談天者無所取則。"[13]
> (Looking at the *Zhoubi*, its instructions are simple but far-reaching. Its words are confusing but right. I fear that it may become neglected and misunderstood,

[10] The six classics singled out by Confucius for study were the *Shi* 詩 (the songs), the *Shu* 書 (the documents), the *Li* 禮 (the rites), the *Yi* 易 (the changes), the *Chunqiu* 春秋 (the Spring and Autumn Annals), and the *Yue* 樂 (the music). The classic on music was lost during the Qin 秦 dynasty (221 – 206 BCE). However, several other works, like the *Classic of Filial Piety* 孝經 and the *Analects* 論語 were later added to what has traditionally been called the Confucian canon. See Wilkinson, *Chinese History*, 369.

[11] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 173; Christopher Cullen, *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems* (London: Routledge, 2017), 235-236.

[12] Cullen, *Astronomy and Mathematics in Ancient China*, 69; Qu, *Zhoubi suanjing xin yi*, 27.

[13] *Zhoubi suanjing* - preface, 1.

> leading people who study the sky to acquire nothing of use out of it.)

That is not to say that Zhao Shuang considered the *gaitian* ("canopy heaven") cosmology described in the *Zhoubi* as being more accurate than the *huntian* ("murky heaven"). In fact, he stated that it was necessary to learn both in order to better penetrate the mysteries of heaven and earth. While the *gaitian* doctrine was explained in the *Zhoubi*, the *huntian* was explained in the *Lingxian* 靈憲 [written by Zhang Heng 張衡 (78 – 139)].[14] Both texts had been preserved through generations and had enabled the officials to bestow the seasons on (ie. construct a calendar for) the people:

> 遂有渾天蓋天。兼而並之。故能彌綸天地之道。有以見天地之賾。則渾天有靈憲之文。蓋天有周髀之法。累代存之。官司是掌。所以欽若昊天。恭授民時。[15]
> (Only by learning both the *huntian* and the *gaitian* and combining them can one understand the Way of Heaven and Earth, and see their profound secrets. The huntian doctrine has the text of the *Lingxian*. The gaitian doctrine has the methods of the *Zhoubi*. Both have been preserved through generations in government departments, enabling them to bestow the seasons on the people.)

This suggests that Zhao Shuang himself was familiar with both texts, but it is not clear why he had cause to read them in the first place. While it could be supposed that Zhao Shuang might have played a role in the calendar-making endeavour of the government and thus in "bestowing the seasons on the people", he definitely did not see the *Zhoubi* as a text that should only be read by astronomical officials. It was his hope that all gentlemen of wide learning (*bowu junzi* 博物君子),

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[14] Zhang Heng was a Han-dynasty official best known in history for having invented and built a seismoscope that could indicate the cardinal direction where there was an earthquake occurring.

[15] *Zhoubi suanjing* – preface, 1.

presumably men of similar education as himself, would come to read it, and thus gain a better understanding of the physical world.[16] This is in line with Nathan Sivin's observation that there had always been private astronomical practitioners throughout pre-modern Chinese history because the occasional laws prohibiting the practice outside the government were seldom enforced.[17] Zhao Shuang's preface gives the impression that both the *Lingxian* and the *Zhoubi* were in fact texts that were commonly attainable, though perhaps not commonly read, and that by the third century, there were still people like Zhao Shuang who had not been completely won over by the *huntian* doctrine, and who still saw validity in the *gaitian* cosmology. However, it did not appear as if Zhao Shuang meant for his readers to do anything practical with the knowledge gained from the *Zhoubi*, such as creating or fine-tuning a calendar, because neither the *Zhoubi* itself nor Zhao's commentary contains sufficient astronomical information for the reader to do so.

Liu Hui

Among all the known writers and commentators of the *Ten Mathematical Classics*, Liu Hui stands out as the most prominent figure in historiography. There is a record of Liu Hui in the official histories in reference to one part of his commentary on the *Jiuzhang suanshu* that dealt with different standards of weights and measures used in different periods. The record states that Liu Hui wrote his commentary in the fourth year of the Jingyuan reign period of the last ruler of the Northern Wei,[18] which corresponds to the year 263 or early 264 CE, giving us the only definite biographical

[16] *Zhoubi suanjing* – preface, 1. For a complete English translation of Zhao Shuang's preface, see Cullen, *Astronomy and Mathematics in Ancient China*, 171.

[17] Nathan Sivin, *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records: Shou shih li cong kao* (New York: Springer, 2009), 26 and 58-59.

[18] *Sui shu* (Beijing: Zhonghua shuju, 1973), 16, 404.

detail about him. In Guo Shuchun's book about Liu Hui, despite its promising title *Gudai shijie shuxue taidou Liu Hui* 古代世界數學泰斗劉徽 (The pinnacle of ancient world mathematics: Liu Hui), Guo cautions the reader in his preface that the book does not actually contain a biography of the title character.[19] Karine Chemla has rightly noted that the historiography about Liu Hui mainly discuss only his mathematical achievements rather than his life.[20] This is because historians have not been able to uncover any other confirmed biographical information about him.

Liu Hui's name is not accompanied by any official title in the record, so it is usually assumed that he did not hold any official post or bear any aristocratic title during this lifetime. However, he was posthumously awarded the title of the Baron of Zixiang (Zixiang *nan* 淄鄉男) during the commemorative ritual of 1109.[21] Because most of these titles were chosen according to the individuals' place of origin or of residence, this suggests that Liu Hui probably hailed from Zixiang, which can be traced to a region in the southwestern part of Shandong province.[22] Zixiang had become the estate of one of the Han imperial princes in 38 BCE. Since Liu Hui also shares the same surname as the Han imperial house, some scholars believe that he might have been a direct descendent of the early Han emperors.[23]

Like Zhao Shuang, the available sources do not contain enough information about Liu Hui for us to be able to connect his work on the *Jiuzhang suanshu* and the

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[19] Guo Shuchun, *Gudai shijie shuxue taidou*, 1.

[20] Chemla, "A Chinese Canon", 188.

[21] *Nan* 男, usually translated as "baron", was the most junior of the five levels of noble titles: *gong* 公 (duke), *hou* 侯 (marquis), *bo* 伯 (earl), *zi* 子 (viscount), *nan* (baron). See Hucker, *A Dictionary of Official Titles in Imperial China*, 338.

[22] Li Di 李迪, "Liu Hui zhuan suo kao 劉徽傳瑣考" (Investigations into Liu Hui's biography), in *Liu Hui yanjiu* 劉徽研究, ed. Wu Wenjun et al. (Xi'an: Shaanxi renmin jiaoyu chubanshe, 1993), 54.

[23] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 179.

Haidao suanjing to his other activities. Therefore, we can only attempt to glean information from his written words.

Liu Hui's preface and various parts of his commentary refer not only to passages within the Confucian canon, but also to the Mohist canon and the Daoist text *Zhuangzi* 莊子.[24] Therefore, it is evident that Liu Hui, like Zhao Shuang, had also received a high level of education. In his preface, Liu Hui stated that he had studied the *Jiuzhang suanshu* during his childhood. Having re-visited the text as an adult and gained a better understanding of the fundamentals of mathematics, he undertook to write a commentary on it.

> "徽幼習九章。長再詳覽。觀陰陽之割裂。總算術之根源。探賾之暇。遂悟其意。是以敢竭頑魯。采其所見。為之作注。"[25]
>
> (I studied the *Jiuzhang* in my youth. In adulthood, I looked over it again in detail, found the division between yin and yang, and came to a conclusion about the origins of the art of suan. While looking for its profound secrets, I suddenly understood them. Therefore, I dare to use my inadequate intellect to write a commentary on it.)

However, the *Jiuzhang* was apparently not a commonly studied text because Liu Hui also lamented that so few people were interested in mathematics during his time that even people of great talent and learning might not be any good at it.

> "當今好之者寡。故世雖多通才達學。而未必能綜於此耳。"[26]
>
> (There are very people interested in mathematics today. Therefore, even though there are many talented and learned people, they may not necessarily be good at it.)

[24] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 169.

[25] *Jiuzhang suanshu* - preface, 1.

[26] Ibid.

Again like Zhao Shuang, Liu Hui was most likely expecting his commentary on the *Jiuzhang* and his *Haidao suanjing* to benefit men of similar education as himself since he also addressed his preface particularly to gentlemen of wide learning (*bowu junzi*).[27] After all, such men were the ones who were most likely to understand his numerous references to the canonical texts.

Furthermore, like Zhao Shuang, there is no evidence that Liu Hui expected his readers to then apply the skills learnt from the *Jiuzhang* and the *Haidao* to any practical endeavour like construction or cartography. Instead, his preface conveys the sense that the content of the *Jiuzhang* had been passed down through the ages since the Zhou dynasty, and that he was merely passing on the tradition while also expanding the section on long-distance surveying and making the text more accessible to the average scholar.[28]

<u>*Sunzi, Xiahou Yang, and Zhang Qiujian*</u>

As noted in the previous chapter, the authors of the *Sunzi suanjing*, *Xiahou Yang suanjing*, and *Zhang Qiujian suanjing* are all of uncertain dates and identities. Unlike the *Zhoubi* and the *Jiuzhang*, the contents of these three treatises are almost entirely mathematical, thus yielding even fewer clues about the background of the people who brought them into being. While some scholars in the past have put forward the view that the *Sunzi suanjing* must have been written by Sun Wu of the Spring and Autumn 春秋 period (722 - 481 BCE), the mainstream view remains that

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[27] *Jiuzhang suanshu* – preface, 2.

[28] For an English translation of Liu Hui's preface, see Shen, Crossley, and Lun, *The Nine Chapters on the Mathematical Art*, 52-54.

the text must have been written by someone else much later, probably around 400 CE.[29]

The author of the *Sunzi suanjing* wrote a preface that placed the art of *suan* in a highly exalted position, portraying it as the root of all things in being, and also the source of knowledge about everything both natural and supernatural. At the same time, he described it as an art that could be mastered by anyone who wanted to learn it.[30] In other words, the author was not laying claim to the art of *suan* as specialist knowledge, but as a field that was open to all. Therefore, the author is not likely to have been a professional fortune-teller or an official within the Bureau of Astronomy and Astrology since such persons would probably have viewed their skill in *suan* as an ability that was reserved for the very few. The fact that the *Sunzi suanjing* is a text for beginners suggests that the author might have been engaged in mathematical instruction of some form. If this was indeed the case, then the preface would have served as an advertisement of *suan's* wonderful capabilities as well as the author's apparent expertise.

As discussed in the previous chapter, the original *Xiahou Yang suanjing* seems to have been lost and was replaced by another treatise. The original is usually dated to around the fifth century CE, slightly later than the *Sunzi suanjing*.[31] Everything about Xiahou Yang, presumably its author, can only be speculated. The Xiahou family name originated shortly after 445 BCE and belonged to one of the aristocratic families who traced their ancestry to the imperial family of the legendary Xia 夏 dynasty (ca. 21st – ca. 16th century BCE).[32] Members of the Xiahou family are

[29] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 185-186.
[30] *Sunzi suanjing* - preface, 1.
[31] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 188.
[32] Chen Ruisong, *Baijia xing su yuan* 百家姓溯源 (The origins of Chinese surnames) (Beijing: Zhongguo huaqiao chuban gongsi, 1993), 211.

known to have served as either civil or military officials during the Han and the Three Kingdoms period, the most famous of whom was the general Cao Cao 曹操 (ca. 155 – 220), who was originally from the Xiahou family but used the Cao family name because his father became the adopted son of the eunuch Cao Teng 曹騰 (fl. 120's – 150's). Whether Xiahou Yang wrote his treatise as a gentleman of leisure, a bureaucrat, or as someone dependent on it to make a living, cannot be ascertained at this time.

Just as little is known about the author of the replacement *Xiahou Yang suanjing*, except that he probably lived in the eighth century.[33] From his preface, it can be inferred that the author had studied mathematics at some length, having read both the *Wucao suanjing* and the *Sunzi suanjing*. Since mathematical skill provided a pathway into the bureaucracy via the civil service examinations during that time, it would not be far-fetched to speculate that the author might have availed himself of this opportunity. Furthermore, given the author's apparent concern with detailing imperial regulations on weights and measures, he could possibly have been a lower-ranking official or a clerk whose duties involved the accounting of goods or grain.

While both Zhao Shuang and Liu Hui had seemed to address their work to men of equal status as themselves, the author of the *Zhang Qiujian suanjing* assumed more of the air of an instructor hoping to benefit future generations of new learners. He signed his preface as "Qinghe Zhang Qiujian" 清河張丘建. In 1109, he was posthumously awarded the title Baron of Xincheng 信城. Both Qinghe and Xincheng can be traced to an area on the border between Shandong province and Hebei province. Because he referred to both the *Sunzi suanjing* and presumably the

[33] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 188.

original *Xiahou Yang suanjing*, *Zhang Qiujian* is usually dated to the late fifth century.[34]

A significant portion of the preface is devoted to explaining the rules of working with fractions, no doubt meant to be a convenient aid for readers. In the preface, Zhang Qiujian presented his treatise as an advancement on the *Xiahou Yang* and the *Sunzi*, saying that he had improved a particular technique contained in them,[35] which is evidence against the old stereotype that pre-modern Chinese had been reluctant to innovate and contradict received wisdom.

WRITERS, COMPILERS, AND COMMENTATORS WHO WERE OFFICIALS OUTSIDE THE BUREAU OF ASTRONOMY AND ASTROLOGY

Out of the three groups of men discussed in this chapter, the lives of this particular group perhaps least fit the image of mathematicians in the popular imagination. The sources contain much more information about these men than the previous group discussed above, and we are able to see these mathematical writers engaging in activities that are not usually associated with mathematical practitioners or discussed within most histories of mathematics. However, it is precisely these activities that will allow us to paint a more complete picture of early Chinese mathematical writers.

It must be noted here that Liang Shu and Wang Zhenru, who assisted Li Chunfeng with editing and annotating the *Ten Mathematical Classics* in the seventh century, also belong to this group, both being instructors at state-run institutions. Unfortunately, not enough is known about their lives and their work to initiate a meaningful discussion of their particular thoughts and actions. From this, we can

[34] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 189.
[35] *Zhang Qiujian suanjing* – preface, 1.

see that the project to edit and annotate the *Ten Mathematical Classics* in 656 was much like the many other communal literary efforts that occurred within the bureaucracy, the most notable being the compilation of dynastic histories: not all the people who worked on such projects are mentioned by name in the sources, and even fewer have biographical details on record.

Zhang Cang and Geng Shouchang

According to Liu Hui's preface to the *Jiuzhang suanshu*, the contents of the *Jiuzhang* were partially destroyed after the First Emperor 始皇帝 (r. 221 – 210 BCE) of the Qin dynasty ordered the burning of all texts except useful agricultural treatises and other practical manuals. The *Jiuzhang* was saved during the Han after Zhang Cang and Geng Shouchang managed separately to re-compile it, even though parts of the text may not be the same as the original.[36] Looking at the *Jiuzhang* today, it is impossible to tell how exactly Zhang Cang and Geng Shouchang contributed to the original text or how far their mathematical skill extended. However, according to the historical record, both of them had illustrious political careers.

Zhang Cang was born in Yangwu 陽武 in Henan province. He was originally an imperial scribe (*yushi* 御史) within the Qin government and was in charge of maintaining the imperial library. However, in 207 BCE, he joined the rebel army of Liu Bang 劉邦 (r. 202 – 195 BCE), who would soon found the Han dynasty. In 202 BCE, Zhang was awarded the title Marquis of Beiping 北平侯 in recognition of his contribution to Liu Bang's military successes. Under the Han, Zhang Cang continued to be active in various roles within the bureaucracy. He helped stabilize the new dynasty by managing its finances, selecting a calendar system for official use, and

[36] *Jiuzhang suanshu* – preface, 1.

regulating the standard weights and measures for the country. After the death of Liu Bang, Zhang Cang continued to enjoy high political status, first as Censor-in-chief (*yushi dafu* 御史大夫), and then as Counsellor-in-chief (*chengxiang* 丞相). In 162 BCE, Zhang Cang resigned from his post, citing ill health.[37]

From the historical record, it is impossible to say how Zhang Cang and Geng Shouchang acquired their mathematical skills or when exactly they worked on the *Jiuzhang suanshu*. Geng Shouchang lived a few decades after Zhang Cang, but presumably was able to continue Zhang Cang's work on re-compiling the *Jiuzhang* into the form that Liu Hui came to know. Geng Shouchang's precise dates are unknown, but the historical record states that he served as an aide to the Chamberlain for the National Treasury (*da si nong zhong cheng* 大司農中丞) during the reign of Emperor Xuan 宣帝 (r. 73 – 49 BCE). The office of the Chamberlain was mainly in charge of collecting taxes and managing the state's grain supplies and monopolies on commodities like salt.[38] Therefore, it is not surprising that the *Jiuzhang* was of interest, and likely much utility, to a man like Geng Shouchang. He was awarded the title Marquis of Guannei 關內侯 for his creation of the "ever-normal" granaries (*chang ping cang* 常平倉) scheme, where the state was able to regulate the price of grain by buying large quantities of it when the price was low, and then releasing it back into the market when supply was low but demand was high. In the commemorative ceremony of 1109, even though Zhang Cang was not recognized, Geng Shouchang was posthumously given the title Earl of Anding 安定伯.[39] Geng

[37] Ruan, *Chouren zhuan*, 2, 9; Guo et al., *Zhongguo kexue jishu shi*, 93-94.
[38] Hucker, *Dictionary of Official Titles*, 471.
[39] It is interesting that the Song government demoted him from marquis to earl.

Shouchang is also known to have dabbled in astronomy and written treatises about the motion of the moon.[40]

Zu Chongzhi

Zu Chongzhi, courtesy name Wenyuan 文遠, was from a family of officials originally from Fanyang 範陽 in modern Hebei province. Therefore, the posthumous title he was given in 1109 was Viscount of Fanyang 範陽子. At the beginning of the fourth century, north China was ravaged by civil wars among the imperial princes of the Western Jin 西晉 dynasty (265 – 317) as well as rebellions by non-Chinese chieftains,[41] so Zu Chongzhi's great-grandfather moved his family southwards across the Yangzi River. His grandfather served the Eastern Jin 東晉(317 – 420) government as Chief Minister for the Palace Buildings (*da jiang qing* 大匠卿).[42] After the Eastern Jin gave way to the Liu-Song 劉宋 dynasty (420 – 479) followed by the Qi 齊 (479 – 502), Zu Chongzhi was able to retain a place within the officialdom, even though he never achieved very high status. Under the Liu-Song, he first served as a retainer (*congshi* 從事) and military aide (*canjun* 參軍) in Xuzhou 徐州, then as magistrate (*xianling* 縣令) of Lou 婁 County, before becoming the supervisor of court attendants (*ye zhe pu she* 謁者僕射). Under the Qi, Zu Chongzhi was a military commandant (*xiaowei* 校尉) at Changshui 長水. He appears to have been particularly multi-talented. Not only was he able to hold both civil and military positions, he also wrote the mathematical treatise *Zhuishu*, created the Daming 大明 calendar system, and annotated Confucian classics like the *Analects*. Moreover, his

40 Ruan, *Chouren zhuan*, 2, 16; Guo Shuchun et al., *Zhongguo kexue jishu shi*, 94.
41 Ebrey, *Cambridge Illustrated History of China*, 89.
42 Guo Shuchun et al., *Zhongguo kexue jishu shi*, 191.

talents also extended to craftsmanship, since he successfully repaired a broken south-pointing chariot 指南車 (ie. chariot holding a magnetic compass) for one of the Liu-Song emperors.[43] Because the *Zhuishu* is no longer extant, it is impossible to say how Zu Chongzhi himself might have connected his work in mathematics with his other pursuits. However, like Zhen Luan discussed below, Zu Chongzhi is an example of a highly skilled mathematical and astronomical writer who had very varied concerns and pursuits in life. Apart from writing the *Zhuishu*, Zu Chongzhi is best known in historiography for having calculated the value of π to a precision of six decimal places (see page 66). His son Zu Geng 祖暅 (fl. 5th century) is also said to have been an accomplished mathematical practitioner and to have helped Zu Chongzhi with his work in mathematics.[44]

<u>*Zhen Luan*</u>

Zhen Luan, whose courtesy name was Shuzun 叔遵, was from Wuji 無極 in modern Hebei province. In the commemorative ceremony of 1109, he was posthumously given the title Baron of Wuji 無極男. He was an unusually prolific writer, compiler, and commentator of both mathematical and astronomical treatises. Out of the *Ten Mathematical Classics*, both the *Wucao suanjing* and the *Wujing suanshu* are believed to have been compiled by him. According to the dynastic histories, he wrote commentaries for seven of the remaining *Ten Mathematical Classics*. Furthermore, as well as other mathematical treatises, he also created the Tianhe 天和 calendar system in 566,[45] and wrote a mathematical treatise named after himself and an astronomical treatise called *Qiyao suanshu* 七曜算術

[43] Guo Shuchun et al., *Zhongguo kexue jishu shi*, 191-192.
[44] Martzloff, *A History of Chinese Mathematics*, 80.
[45] Qu, *Zhoubi suanjing xin yi*, 27.

(Mathematical Art of the Seven Celestial Bodies). Unfortunately, the great majority of his works have not survived to the present time. Given the prodigious amount of work Zhen Luan seemed to have devoted to mathematics and astronomy, it is perhaps surprising that it was completely unrelated to his duties as a government official for the Northern Zhou 北周 dynasty (557 – 581). According to information contained in the *Shushu jiyi*, Zhen Luan annotated it while serving as the commandery governor of Hanzhong (Hanzhong *junshou* 漢中郡守) after being transferred from the post of metropolitan commandant (*si li xiaowei* 司隸校尉), who was in charge of supervising officials in the capital region.[46] From this, we can see that Zhen Luan was not prevented by his political duties to pursue more technical hobbies. It is also evidence that officials serving outside the state-run schools and the Bureau of Astronomy and Astrology could possess expertise on both mathematics and astronomy.

WRITERS AND COMMENTATORS WHO SERVED WITHIN THE BUREAU OF ASTRONOMY AND ASTROLOGY

Out of the three groups of men discussed in this chapter, one would have imagined that this particular group would best fit the older view that pre-modern Chinese valued mathematics mainly for the sake of doing astronomy and astrology. However, between the two men discussed in this section, only Li Chunfeng seems to actually fit the bill, while Wang Xiaotong's *Jigu suanjing* has questionable astronomical or astrological value.

All governments throughout China's imperial history had personnel who were well-versed in doing mathematical astronomy.[47] This is because the Chinese saw

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[46] Ruan, *Chouren zhuan*, 11, 130; Hucker, *Dictionary of Official Titles*, 451.
[47] Cullen, *Astronomy and Mathematics in Ancient China*, 4.

the emperor as the central figure in upholding the cosmic balance between heaven, earth, and man.[48] The accurate prediction of celestial events like eclipses was understood as a sign that all was in order and that the emperor truly held the Mandate of Heaven to rule, while unexpected or unusual occurrences were taken as warning signs that something was amiss.[49] The Bureau of Astronomy and Astrology was thus an apparatus that was meant to bolser imperial authority by observing, predicting, and interpreting celestial events, as well as creating an accurate calendar that would be promulgated by imperial authority across the empire. Interpreting celestial as well as terrestrial events and advising the emperor accordingly were the main duties of the director of the bureau.

Wang Xiaotong

Wang Xiaotong served within the early Tang Bureau of Astronomy and Astrology as the assistant director (*taishi cheng* 太史丞) and as an instructor in calendar-making (*li suan boshi* 歷算博士). The official calendar in use since the very beginning of the Tang dynasty failed to accurately predict solar and lunar eclipses, and Wang was one of the officials charged with correcting it in the year 627.[50]

Around the year 630 CE, Wang Xiaotong wrote the *Jigu suanjing* as well as a preface that was addressed to the emperor. In both, there was no mention of either astronomy or astrology. Instead, he discussed his treatise as purely a work of mathematics, presenting it as the supplement to the parts of the *Jiuzhang suanshu* on construction and division of labour, as well as an improvement on Zu Chongzhi's

48 Cullen, *Astronomy and Mathematics in Ancient China*, 2.
49 Sivin, *Granting the Seasons*, 35-36 and 56.
50 Guo Shuchun et al., *Zhongguo kexue jishu shi*, 201.

techniques of volume calculations in the *Zhuishu*.[51] Therefore, in the mind of Wang Xiaotong, who was as close to a professional astronomer as could be found in Tang China, mathematics appeared to have been its own independent area of study, and not necessarily linked to either astronomy or astrology.

Li Chunfeng

Li Chunfeng became the director of the Bureau of Astronomy and Astrology in 627. Therefore, he would have been Wang Xiaotong's superior within the bureau for a number of years. Li Chunfeng's son and grandson would later succeed him as the director of the bureau.[52] Li Chunfeng was from modern Shaanxi province. Although he was the appointed leader of the project to edit and annotate the *Ten Mathematical Classics*, it is difficult to say what exactly he did in the project. In fact, modern scholars tend to rank his mathematical abilities as being lower than earlier writers like Liu Hui and Zu Chongzhi because the comments Li Chunfeng and his team added to the ten classics are not always mathematically sound. However, his abilities in astronomy are unquestionable. He constructed an armillary sphere for the Tang government and created the *Linde* 麟德 calendar system to replace the previous one in 665 CE. The armillary sphere designed by Li Chunfeng had three concentric rings, and was an improvement on previous models in terms of observational accuracy.[53] He also participated in the writing of the dynastic histories of the Jin and the Sui dynasties, where he wrote the monographs on astronomy, the calendar, and the five phases 五行. He was subsequently awarded the title Baron of

51 *Jigu suanjing* – preface, 1.

52 *Jiu Tang shu*, 79, 2719.

53 Jennifer W. Jay, "Li Chunfeng", in *Encyclopedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, 2nd ed., ed. Helaine Selin (Dordrecht: Springer, 2008), 1222.

Changyue 昌樂男 for his work on the dynastic histories. In 1109, the Song government posthumously promoted him to become the Viscount of Changyue 昌樂子.[54]

Even though modern historians of science and mathematics tend not to discuss it, Li Chunfeng was believed to have been very skilled in astrology and divination, which were necessary skills for the director of the bureau because one of his main functions was to interpret omens and advise the emperor accordingly. In his official biography in the dynastic history of the Tang dynasty, Li Chunfeng was said to have accurately foretold the coming of Wu Zetian 武則天 (r. 684 – 705 CE), the only female emperor in Chinese history, who would soon cause the death of many members of the Tang imperial clan.[55] Li also wrote divination texts that set out specific rules on how to observe and interpret esoteric phenomena as omens to guide decision-making. A sample of his writings on this topic is as follows:

> 凡候氣之法，常以平旦寅時及日晡夜半，或戊巳之日看候敵上。敵在東，日出候之，敵在南，日中候之，敵在西，日入候之，敵在北，夜半候之，乃知敵人動靜盛衰之兆。
> The general method of observing the qi [cosmic forces] is one takes observations above the enemy always at the yin hour at dawn, in the afternoon [from 3 to 5 p.m.] or at midnight, or on the wusi day. When the enemy is to the east, observe it when the sun comes up; when they are to the south, observe it at mid-day; when they are to the west, when the sun sets; when they are to the north, at midnight. Then you will know the omens concerning the enemy's movement or rest, their rise or decline.[56]

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[54] Ruan, *Chouren zhuan*, 13, 157-158; Guo Shuchun et al., *Zhongguo kexue jishu shi*, 203.

[55] *Jiu Tang shu*, 79, 2717-2719.

[56] Quoted from Robin D.S. Yates, "The History of Military Divination in China", *East Asian Science, Technology, and Medicine* 24 (2005): 28.

CONCLUSION

From the above, it is evident that there does not exist a single mould into which all the writers discussed in this chapter can comfortably fit. They led very different lives and played different roles in the society in which they lived. One similarity that does stand out from this analysis is that, among the people whose places of origin are known, the great majority hailed from the provinces in northeast China. It is beyond the scope of this dissertation to investigate why this is the case, but it does suggest that that particular region was unique either in the preservation of sources or in the dissemination of mathematical and astronomical knowledge.

If we look solely at the activities and concerns of the state, then it is perhaps reasonable to say that mathematics was made to serve the purposes of astronomy and astrology. After all, the person chosen to head the project to edit and annotate the *Ten Mathematical Classics* was the director of the Bureau of Astronomy and Astrology, while the two mathematics instructors had to work under his supervision. However, once we look beyond the state and really scrutinize the words of mathematical writers and commentators like Zhao Shuang and Liu Hui, they seemed to have been less interested in mathematics as a practical function than as an intellectual pursuit. Moreover, it was a pursuit not for the sake of mental or philosophical exercise, but aimed at gaining a true understanding of the world in which they lived.

In addition, the above analysis suggests that many of the *Ten Mathematical Classics* were not meant to be practical manuals designed for the benefit of common people engaged in crafts or trades, but for people of a higher social status. How the *Ten Mathematical Classics* were actually used to train future officials will be examined in the next chapter.

Chapter Three

State-Run Mathematical Education

and the *Ten Mathematical Classics* as Textbooks

"The mathematical equations may be universal, but the allocation of human effort among the possibilities of natural knowledge is not."[1]

~ Nathan Sivin

Despite the fact the mathematics was considered one of the six gentlemanly arts, people in pre-modern China did not always agree on how essential it was for scholars to learn mathematics. The two quotes below illustrate two very different views of the importance of *suan* in the education of scholars. While both writers agreed that the art of *suan* was a useful form of knowledge, they diverged significantly on the point of for whom it was important as well as its priority relative to other forms of knowledge.

> "算術亦是六藝要事，自古儒士論天道，定律歷者皆學通之。然可以兼明，不可以專業。" [2]
> [The art of *suan* is also an important aspect of the six gentlemanly arts. Since ancient times, scholars who discuss the Moral Way and who construct calendars are all proficient in it. However, one can learn it in combination with other knowledge. One ought not specialize in it.]
>
> ~ Yan Zhitui 顏之推 of the state of Northern Qi 北齊 (550 – 577), "Admonitions to the Scions of the Yan Family" (*Yan shi jiaxun* 顏氏家訓)

[1] Sivin, "Why the Scientific Revolution did not Take Place in China", 52.
[2] Quoted in Yan Dunjie, "Zhongguo shuxue jiaoyu jianshi", 45.

> “數為六藝之一，似緩而實急。凡天文、律歷、水利、兵法、農田之類，皆須用算。學者不知算，雖知算而不精，未可云用世也。” [3]
> [Mathematics is one of the six gentlemanly arts. It appears unimportant, but is actually critical. Matters relating to the study of the heavens, calendar-making, waterworks, military affairs, and agricultural fields, all require the use of *suan*. Scholars who do not understand *suan*, or who understand it only superficially, cannot yet say that they are doing practical good for the world.”]

~ Lu Shiyi 陸世儀 (1611 – 1672)

The history of Chinese state-run mathematical education reflects a similar kind of contention. While there was no question that people with knowledge of mathematics were necessary for effective governing, official policy regarding matters like who, if anybody, should receive state-sponsored mathematical training, what the training then qualified them to do, and whether or not people skilled in mathematics deserved official positions within the bureaucracy, was in a state of flux from dynasty to dynasty, from emperor to emperor, and sometimes even from year to year within the reign of a single emperor.

This chapter traces the use of the *Ten Mathematical Classics* as textbooks in state-run institutions from the Sui (581 – 618) to the Northern Song dynasty (960 – 1127), a period that marks the flourishing of mathematical education that was sponsored by the state, which had a clearly defined curriculum and fully developed mechanisms for the mathematics graduates to join the ranks of the bureaucracy. The end of the Northern Song dynasty signalled the end of large-scale, systematic investment by the central government in mathematics education. It also marked the beginning of the fading of the *Ten Mathematical Classics* into oblivion, to the point

[3] Quoted in Yan Dunjie, “Zhongguo shuxue jiaoyu jianshi”, 45.

where, by the Qing dynasty (1644 – 1911), they had become obscure ancient texts that had to be re-discovered.

This chapter explores how the *Ten Mathematical Classics* were used to structure state-run mathematics education, and what that education then enabled its graduates to do. Here, the focus is on the School of Mathematics (*suan xue* 算學) established by the government within the capital. It is not to be conflated with the training of personnel for the Bureau of Astronomy and Astrology, which naturally also involved the teaching of some mathematics, but was usually, though not always as will be seen later in this chapter, run by the bureau itself and independent of the School of Mathematics.[4]

This chapter seeks to illuminate the evolution of state policy regarding mathematics education in the context of the broader history of education in general, particularly as it relates to state investment in the transmission of various types of knowledge, which included Confucian philosophy as well as legal and medical knowledge during this period. It must be kept in mind that the main purpose of the imperial academy was to prepare young men for assuming office and serving the state. We shall see that the historical development of mathematics education was intimately intertwined with the state's evolving perspective on the types of people who should be recruited into the bureaucracy, and how they ought to be recruited. The overall trend tended towards the narrowing of the state's definition of the ideal recruit. On the other hand, the abandonment of mathematics education by the state is an indication that mathematics was also part of the broader shift towards the

[4] Guo Shirong 郭世榮, "Lun Zhongguo gudai de guojia tian suan jiaoyu 論中國古代的國家天算教育" (A Discussion of Pre-modern China's State-Run Astronomical Education), in *Shuxue shi yanjiu wenji* 數學史研究文集 (Research papers on the History of Mathematics), vol. 2, ed. Li Di 李迪 (Hohhot, Inner Mongolia: Nei Menggu daxue chubanshe, 1991), 27.

dominance of private education during the Northern and Southern Song 南宋 (1127 – 1279) dynasties, as will be demonstrated in this chapter and the next.

MATHEMATICS EDUCATION PRIOR TO THE SUI DYNASTY

The earliest Chinese records concerning schools and education can be found in the oracle bones of the Shang 商 dynasty (ca. 1600 – 1046 BCE),[5] which refer to the animal bones and turtle shells that were used for royal divinations during the Shang. These divinations involved posing a question to the spirits, putting a heated object into contact with a piece of animal bone or turtle shell, and divining the spirits' answer by interpreting the cracks caused by the heated object.[6] Both the question and the result of the divination were then inscribed on the bone or shell. Among the inscriptions uncovered to date are the characters *jiao* 教 (teach), *xue* 學 (school or learn), and *shi* 師 (teacher).[7] Unfortunately, very little is known about the modes and content of education of that period.[8]

Under the Zhou dynasty, the state maintained schools to teach the sons of the nobility, with the goal of preparing them for serving in the government.[9] During that time, the concept of the six gentlemanly arts appears to have become a well-established curriculum.[10] In the sources, the six gentlemanly arts are usually listed in the following order: *li* 禮 (ritual), *yue* 樂 (music), *she* 射 (archery), *yu* 御

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[5] Thomas H.C. Lee, *Education in Traditional China, a History* (Leiden: Brill, 2000), 41; Yan Binghai 顏秉海, and Wen Xiaoyu 文曉宇, "Zhongguo shuxue jiaoyu shi jianlun 中國數學教育史簡論" (A Brief Discussion of the History of Chinese Mathematics Education), paper presented at the Conference of the 740th Anniversary of Qin Jiushao's Shushu jiuzhang 秦九韶《數書九章》成書 740 週年紀念暨學術研討國際會議 (Beijing, May 1984), 4.

[6] See Wilkinson, *Chinese History*, 681-683. Wilkinson notes that some oracle bones that date from before the Shang were made of human bones.

[7] Yan and Wen, "Zhongguo shuxue jiaoyu shi", 4.

[8] Lee, *Education in Traditional China*, 41.

[9] Ibid; Li and Du, *Chinese Mathematics*, 22.

[10] Yan and Wen, "Zhongguo shuxue jiaoyu shi", 4.

(horsemanship or charioteering), *shu* 書 (writing), *shu* 數 (most likely mathematics, but it might also have included numerology, divination, astronomy, astrology, and geomancy).[11] This particular order with the art of mathematics as the last of the six arts would lead countless emperors, policy makers, and scholars over the subsequent centuries to view it as also the least important of the six arts,[12] thus justifying their low regard for or complete negligence of it.

Records from the Zhou dynasty are too scarce for us to formulate a detailed discussion of how the art of mathematics might have been taught. However, although the period pre-dates all known versions of the *Ten Mathematical Classics*, there might have been earlier versions of the *Zhoubi suanjing*, *Jiuzhang suanshu*, *Sunzi suanjing*, and *Wucao suanjing*, which are all of uncertain authorship, in circulation at the time that might have been used for teaching. In addition to the schools run by the state, private education appears to have been in a flourishing state at the time, with many important philosophers of the period, including Confucius (551 – 479 BCE), teaching disciples in a private capacity and establishing their own schools of thought,[13] marking this period as one of immense intellectual creativity and cultural vibrancy. Other schools of thought that emerged during this time included Daoism, Mohism, and Legalism. However, it was Confucian philosophy that would come to dominate Chinese political thought from the reign of Emperor Wu 武帝 (r. 141 – 87 BCE) of the Han dynasty to the end of imperial China in 1911.

How much the art of mathematics might have featured in private education during the Zhou cannot be ascertained, but the tradition of private teaching would be

[11] Ho, *Chinese Mathematical Astrology*, 2.
[12] Guo Shirong, "Lun Zhongguo gudai de guojia tian suan jiaoyu", 29.
[13] Lee, *Education in Traditional China*, 43.

a long-standing practice in the history of mathematics education,[14] eventually becoming mathematics' sole channel of knowledge transmission after the Northern Song dynasty.

THE SUI DYNASTY: THE CREATION OF THE SCHOOL OF MATHEMATICS

Following the Zhou, records from the subsequent dynasties leading up to the Sui cannot further illuminate how mathematical education might have proceeded either in state-run institutions or in privately established schools, except for a passing reference to a *suan sheng boshi* 算生博士 (instructor, often translated as "erudite", of mathematics) during the Northern Wei 北魏 dynasty (386 – 534), but no further remarks on where or how he carried out his duties.[15]

The Sui is the earliest dynasty on record to provide systematic mathematics education through its imperial academy, which was in fact a government institution that had begun during the Han dynasty as a school reserved for instilling the Confucian philosophy into the sons of officials, thus preparing them for a life of serving the state in the Confucian Way, which promoted the benevolence of the ruler, the loyalty of his subjects, and the moral cultivation of all.[16] The term "imperial academy" is used here to refer to the main independent educational institution set up by the Chinese central government within the primary, and sometimes also the secondary, capital. However, it must be noted here that its name had not always been the same throughout history. It has been known at various times as *tai xue* 太

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[14] Yan Dunjie, "Zhongguo shuxue jiaoyu jianshi", 46.

[15] Ibid, 45. Also, see Huang Benji 黃本驥, *Lidai zhi guan biao* 歷代職官表 (Shanghai: Shanghai guji chubanshe, 1984), 3, 160-161.

[16] Feng Xiaolin 馮曉林, "Zhongguo Sui Tang Wudai jiaoyu shi 中國隋唐五代教育史", in *Bai juan ben Zhongguo quan shi* 百卷本中國全史, ed. Shi Zhongwen 史仲文 and Hu Xiaolin 胡曉林, vol. 10 (Beijing: Renmin chubanshe, 1994), 40 and 56; Thomas H.C. Lee, *Government Education and Examinations in Sung China* (Hong Kong: Chinese University Press, 1985), 11.

學 (Imperial University), *guozi si* 國子寺 (Court for Education), *guozi jian* 國子監 (Directorate of Education), *guozi xue* 國子學 (School for the Sons of the State, often also translated as Directorate School), among several other names. During the Sui, Tang, and Northern Song, it was usually called the *guozi jian*, and comprised of as many as seven different schools, some of which were devoted to teaching the Confucian canon, and the rest to more technical subjects. For the sake of consistency and to avoid confusion, the term "imperial academy" will be used instead of its specific name at different points in time.

By Chinese standards, the Sui was a short-lived dynasty. Yet many of its policies and accomplishments would have long-lasting effects on China's subsequent development. In 589, the Sui founder, Emperor Wen 文帝 (r. 581 – 604) completed the reunification of China, which had been politically fragmented for close to four centuries. In its efforts to rebuild central authority and government institutions, the Sui dynasty set two important precedents: the institution of the civil service examinations to select men of merit to fill bureaucratic positions,[17] and the expansion of the imperial academy to include one school for the teaching of mathematics.[18]

Previously, the bureaucracy had largely been staffed by members of the aristocracy, the relatives of officials, or talented or especially virtuous individuals who had been recommended by local officials. The civil service examinations gave the central government a greater degree of direct control over who would be selected to serve in the bureaucracy because the examinations were open to men of all backgrounds, and the questions could be tailored so that only candidates who met very specific requirements would pass the examinations and be selected. From their

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[17] Wang Kaixuan 王凱旋, *Zhongguo keju zhidu shi* 中國科舉制度史 (History of the Chinese Civil Service Examination System) (Shenyang: Wanjuan chuban gongsi, 2012), 34.

[18] Feng, "Zhongguo Sui Tang Wudai jiaoyu shi", 41.

inception until their abolition in 1905, the vast majority of civil service examinations ever held tested candidates' familiarity with the Confucian canon, which was interpreted as having a direct correlation with their degree of moral cultivation. However, when the civil service examinations were first created, they did not replace the other methods of recruitment mentioned above, but provided merely one more possible pathway into officialdom. It was only during the course of the Song dynasty that it would come to play a much more crucial role in dictating the fate of men aspiring to become officials.[19]

During the Sui, however, the number of officials recruited via the civil service examinations remained very small.[20] Moreover, the Sui civil service examinations did not yet appear to have provided for the testing of more technical subjects like mathematics,[21] implying that the graduates of the newly established School of Mathematics had other means of becoming civil officials. It is only during the Tang dynasty that the civil service examinations would come to play a greater role in the history of mathematics.

The School of Mathematics was founded along with a School of Writing (*shu xue* 書學), which taught students the proper form and meaning of Chinese characters, setting a precedent for specialist training by the government in these two subjects for the first time in Chinese history. Both schools were placed within the imperial academy, joining the three schools that were devoted to teaching the Confucian canon: the School for the Sons of the State (*guozi xue*), the Imperial University (*tai xue*), and the School of the Four Gates (*si men xue* 四門學). While the standard

19 John W. Chaffee, *The Thorny Gates of Learning in Sung China: A Social History of Examinations*, 2nd ed. (Albany: State University of New York Press, 1995), 4.

20 Ibid, 15.

21 *Tongdian* 通典 (Encyclopedic History of the Institutions of Government) (Hangzhou: Zhejiang guji chubanshe, 2000), 14, 81.

history of the Sui dynasty does not specifically explain why three different schools were necessary for the teaching of the Confucian canon, records about the Tang dynasty, its immediate successor, state that the students of the imperial academy were placed in different schools according to their birth and the official rank held by their fathers, grandfathers, and great-grandfathers, which will be discussed in detail in the next section. This suggests that the Sui government might have done the same. Certainly, the varied status of the instructors of the five different schools of the imperial academy is a clear indication that the schools were not held to be equal in the eyes of the government, which lends support to the hypothesis that individual schools, and by extension their student bodies, were ranked hierarchically as well.

Out of the five schools of the imperial academy, the instructors at the School for the Sons of the State were ranked the highest. Under the nine-rank system in use at the time to rank officials hierarchically, with one being the highest rank and nine being the lowest, each rank was further subdivided into two levels: upper class (*zheng* 正) and lower class (*cong* 從), denoted "a" and "b" respectively, with "a" being higher in rank. Table 3.1 shows the rank of the erudites and teaching assistants (*zhujiao* 助教) at the five schools:[22]

[22] *Sui shu*, 28, 786-789. The order of the schools in this list follows the order they are usually presented in the *Sui shu*.

Table 3.1 Rank of Erudites and Teaching Assistants During the Sui Dynasty

School	Rank of Erudite	Rank of Teaching Assistants
Sons of the State	5a	7b
Imperial University	7b	9a
Four Gates	8b	9b
Writing	9b	Unranked
Mathematics	9b	Unranked

From the table, it is clear that the instructors of both the School of Writing and the School of Mathematics were ranked especially low within the official hierarchy compared with their colleagues in the other schools, and their teaching assistants were not even considered to be actual officials. Therefore, their students were likely to have been youth whose fathers were either commoners or on the lower echelons of the official hierarchy, just like during the Tang dynasty.

The respective numbers of erudites, teaching assistants, and students at the five schools are shown in the following table:[23]

[23] *Sui shu*, 28, 777.

Table 3.2 Number of Erudites, Teaching Assistants, and Students During the Sui Dynasty

School	Number of Erudites	Number of Teaching Assistants	Number of Students
Sons of the State	5	5	140
Imperial University	5	5	360
Four Gates	5	5	360
Writing	2	2	40
Mathematics	2	2	80

From the above, we can surmise that the focus of the imperial academy leant heavily towards teaching the Confucian canon, with the vast majority of the teaching staff and students concentrated in that area. Why, then, did the Sui rulers deem it necessary to create the School of Writing and the School of Mathematics? The answer probably lies in the fact that the Sui dynasty did not inherit a unified empire, but had to create one after conquering a cluster of independent political bodies, each with its own set of governing practices. In order to build an efficient bureaucracy to run the empire and large-scale construction projects like building the Grand Canal to link the economies of the north and south, the Sui rulers must have needed well-trained personnel who could handle the day-to-day administrative tasks of government, including scribes in charge of record-keeping, communications between different government bodies, surveying land, as well as calculating taxes and labour. Creating an institution that could be directly overseen by the central government to train such personnel would have been the most efficient way to ensure that the said personnel would follow a common set of practices. With that view in mind, the emergence of a School of Mathematics in the imperial academy at this particular point in Chinese history cannot be seen as coincidental, but as part of the Sui rulers' effort to build an effective government. We shall see in the next section that, by inheriting the empire and the bureaucracy directly from the hands of the Sui rulers, the Tang dynasty would continue and further develop many of the Sui's policies on education and personnel-selection.

As for how the curriculum of the School of Mathematics might have been structured, there were no officially prescribed mathematics textbooks. However, a glance through the imperial catalogue (*jingji zhi* 經籍志) in the standard history of the Sui dynasty suggests that up to eight out of the original *Ten Mathematical Classics* might have been used to teach the students of the School of Mathematics. The *Zhoubi suanjing* is listed among the works on celestial patterns (*tianwen*).[24] In the section on calendar-making, we find the other classics:[25]

- *Jiuzhang suanshu* in ten chapters, so the tenth must have been the *Haidao suanjing*
- *Zhuishu*
- *Sunzi suanjing*
- *Xiahou Yang suanjing*
- *Zhang Qiujian suanjing*
- *Wujing suanshu*

In the imperial catalogue, there is no separate section on mathematical texts. However, that is not to imply that the School of Mathematics was used solely for the purpose of training personnel for the Bureau of Astronomy and Astrology. On the

[24] *Sui shu*, 34, 1018.
[25] Ibid, 34. 1025.

contrary, the bureau operated its own school and had its own teaching staff for the following subjects: calendar-making, celestial patterns, clock-making and time-keeping (*louke* 漏刻), and identifying evil influences (*shijin* 視祲).[26]

To further contextualize state-run mathematics education in the broader picture of state-run education in general, it must be noted that the imperial academy and the Bureau of Astronomy and Astrology were not the only government institutions to provide specialist education and training at this time. For example, the Imperial Medical Office (*taiyi shu* 太醫署) had erudites and teaching assistants for the subjects of medical treatment (*yi* 醫), massage (*anmo* 按摩), and some form of spiritual healing (*zhujin* 祝禁).[27] The Imperial Divination Office (*taibu shu* 太卜署) had erudites and teaching assistants in the subjects of divination (*bu* 卜) and fortune-telling (*xiang* 相).[28] The Court of the Imperial Stud (*taipu si* 太僕寺) had erudites for veterinary medicine (*shouyi* 獸醫), while the Court of Judicial Review (*dali si* 大理寺) had erudites for teaching law.[29] In other words, the investments of the Sui government in education were actually quite varied, all aiming at producing well-trained personnel to meet the diverse needs of the state.

However, even though it had completed the reunification of China, the Sui was by no means a peaceful and stable dynasty. The reign of the second emperor, Emperor Yang 煬帝 (r. 604 – 617), was marked by onerous labour demands on the general population to construct the Grand Canal and to rebuild the Great Wall, as well as ambitious military campaigns to conquer Champa (in central Vietnam) and to subdue Koguryo (in northern Korea and southern Manchuria). Discontent among

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26 *Sui shu*, 28, 775.
27 Ibid, 28, 776.
28 Ibid.
29 Ibid.

both the common people and members of the political elite resulted in the rise of as many as two hundred rebel groups.[30] Therefore, the imperial academy must have had tremendous difficulty functioning normally, especially during the last years of the Sui.[31]

THE TANG DYNASTY: THE INTERTWINED RELATIONSHIP BETWEEN THE SCHOOL OF MATHEMATICS AND THE CIVIL SERVICE EXAMINATION SYSTEM

Widespread rebellions against the Sui rulers eventually forced the dynasty's third emperor, Emperor Gong 恭帝 (r. 617 – 618), to abdicate in favour of the general Li Yuan 李淵 (r. 618 – 627) in the year 618, before dying at the age of 15 the following year.[32] Li Yuan declared a new dynasty, the Tang, which would nevertheless continue many of the policies of its predecessor, including the ones on education. Even though the new dynasty had to overcome the ongoing rebellions to re-establish political stability, putting the imperial academy back into operation seemed to have been a priority. An imperial decree was passed in 619 for the School for the Sons of the State, the Imperial University, and the School of the Four Gates to begin admitting students from the families of officials.[33] Another was passed as early as 632 for the re-establishment of the School of Mathematics.[34] However, the fact that the same decree had to be issued again in 656 by the third emperor, Gaozong, either suggests that the earlier one had not achieved its desired objective, or is indicative of the vacillating attitude of the Chinese government towards the necessity of having a School of Mathematics. Even Emperor Gaozong

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30 Charles Holcombe, *A History of East Asia: From the Origins of Civilization to the Twenty-First Century* (New York: Cambridge University Press, 2011), 93.

31 Feng, "Zhongguo Sui Tang Wudai jiaoyu shi", 44.

32 *Sui shu*, 5, 99-102.

33 Feng, "Zhongguo Sui Tang Wudai jiaoyu shi", 45.

34 *Tang huiyao* 唐會要 (Essential Documents and Regulations of the Tang) (Beijing: Zhonghua shuju, 1985), 35, 633.

himself, who gave the School of Mathematics an official set of textbooks for the first time, was not free of such doubt. After successfully establishing the School of Mathematics in 656, he abolished it and transferred its staff and students to the Bureau of Astronomy and Astrology in 658, only to re-establish it again within the imperial academy in 662.[35]

The Tang imperial academy consisted of six main schools, all of which were clustered within one of the walled wards in central Chang'an 長安, the primary capital. These schools lay immediately south of the imperial ancestral shrine, and were surrounded by several Daoist abbeys and Buddhist monasteries in neighbouring wards.[36]

The Tang imperial academy's main difference from its predecessor lies in its inclusion of the School of Law, which had been run by the Court of Judicial Review during the Sui dynasty. The number of erudites and teaching assistants of each school as well as their respective rank are listed below.[37] The Tang bureaucracy was divided into nine ranks like the Sui. However, in addition to separating each rank into two classes, the Tang further subdivided each class into two grades (*shang* 上 and *xia* 下). Here, "i" denotes the higher grade and "ii" the lower grade:

[35] *Xin Tang shu*, 48, 1268.

[36] Charles Benn, *China's Golden Age: Everyday Life in the Tang Dynasty* (Oxford: Oxford University Press, 2002), xiii-xvi.

[37] *Xin Tang shu*, 48, 1266-1268. The order of the schools presented here follows the order in the *Xin Tang shu*.

Table 3.3 Number and Rank of Erudites and Teaching Assistants During the Tang Dynasty

School	Number of Erudites	Rank of Erudites	Number of Teaching Assistants	Rank of Teaching Assistants
Sons of the State	10	5ai	5	6bi
Imperial University	6	6ai	6	7bi
Four Gates	6	7ai	6	8bi
Law	3	8bii	1	9bii
Writing	2	9bii	1	Unranked
Mathematics	2	9bii	1	Unranked

The records from the Tang dynasty offers an extremely comprehensive view of how these schools operated. Like the Sui, the purpose of the School for the Sons of the State, the Imperial University, and the School of the Four Gates was to teach students the Confucian canon. Admittance into all six schools was based on birth, with the School for the Sons of the State reserved for young men from the most elite background (see table below).[38]

[38] *Xin Tang shu*, 44, 1160; 48, 1266-1268.

Table 3.4 Student Composition of the Six Schools During the Tang Dynasty

<table>
<tr><th>School</th><th>Ages of Students First Admitted (in years)</th><th>Background of Students Admitted</th></tr>
<tr><td>Sons of the State</td><td>14 to 19</td><td>• Sons and grandsons of officials with rank 3 or above
• Sons and grandsons of dukes of state (guo gong 國公)
• Great-grandsons of officials with rank 2b or above</td></tr>
<tr><td>Imperial University</td><td>14 to 19</td><td>• Sons and grandsons of officials of rank 5 or above
• Sons and grandsons of dukes (jun xian gong 郡縣公)
• Great-grandsons of officials of rank 3b or above</td></tr>
<tr><td>Four Gates</td><td>14 to 19</td><td>• Sons of officials of rank 7 or above
• Sons of marquis, earls, viscounts, and barons
• Especially talented commoners (jun shi 俊士)</td></tr>
<tr><td>Law</td><td>18 to 25</td><td rowspan="3">• Sons of officials with rank 8 or below
• Commoners</td></tr>
<tr><td>Writing</td><td>14 to 19</td></tr>
<tr><td>Mathematics</td><td>14 to 19</td></tr>
</table>

The School of Mathematics, along with the Schools of Law and of Writing, was reserved for commoners and the sons of the lowest-ranking officials, thus reflecting the belief that such specialist training could not be put on the same par as learning the Confucian classics. These modes of knowledge were considered inferior to learning the Way of the sages in order to cultivate oneself and to provide good government. A thorough understanding of the law, a precise knowledge of the written language, and an expert grasp of mathematical skills were not considered necessary for becoming a good official of the highest ranks. Only virtue, as reflected in one's knowledge of the Confucian classics, was the sufficient condition to distinguish men who were suitable for the top positions in government. This belief would be the determining factor in shaping how the civil service examination system developed during the Tang dynasty and beyond.

The number of students at the imperial academy fluctuated greatly throughout the Tang dynasty. It probably peaked during the reign of Taizong 太宗 (r. 627 – 650), with 3260 Chinese staff and students, as well as students from Japan, the Korean states, and the Inner Asian states, bringing the total number of people at the imperial academy to over 8000.[39] Foreign students within the Tang capitals must have been a common sight, contributing to the Tang's reputation as a very cosmopolitan empire. It seemed to have been a favourite education destination of people from the Korean kingdoms, but the Tang government set quotas for the number of students each kingdom could send. In the year 837, the Korean kingdom of Parhae sent sixteen students to China, but only six were permitted entry. The remaining ten were made to turn back at Qingzhou 青州. The same year, 209 students were sent back to Silla.[40]

All students studying at the imperial academy, including the foreign students from neighbouring states, were provided with free room and board.[41] No tuition fees were necessary during the Tang, but regulations were passed in 706 requiring students entering the imperial academy for their first year of study to present bolts of fine cloth as gifts to their instructors as a symbolic gesture to express both respect and gratitude. Each student of the School for the Sons of the State and the Imperial

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[39] *Tang huiyao*, 35, 633.
[40] Ibid, 36, 668.
[41] Feng, "Zhongguo Sui Tang Wudai jiaoyu shi", 83.

University were required to give three bolts of fine cloth; the School of Four Gates, two bolts; the Schools of Law, Writing, and Mathematics, just one bolt. Of all the bolts of cloth collected from the students, 60 percent went to the erudites and 40 percent to the teaching assistants.[42]

The numbers recorded in the standard history probably reflects official capacities of the six schools during the first half of the dynasty:[43]

Table 3.5 Official Capacities of the Six Schools During the Early Tang Dynasty

School	Number of Students
Sons of the State	300
Imperial University	500
Four Gates	1300 (800 of whom were commoners)
Law	50
Writing	30
Mathematics	30
	Total: 2210

However, these schools did not always operate at full capacity due to either the lack of students or the lack of government resources to support so many students, especially after the devastating An Lushan Rebellion (755 – 762) that resulted in the Tang central government losing control over much of the frontier regions. Student numbers were sharply reduced thereafter. Around the year 765, the official capacities for the six schools were set at the following levels:[44]

[42] *Tang huiyao*, 35, 634.
[43] *Xin Tang shu*, 34, 1159-1160.
[44] Ibid, 44, 1165.

Table 3.6 Official Capacities Around the Year 765

School	Number of Students at the Primary Capital (Chang'an 長安)	Number of Students at the Secondary Capital (Luoyang 洛陽)
Sons of the State	80	10
Imperial University	70	15
Four Gates	300	50
Law	20	10
Writing	10	3
Mathematics	10	2
	Total: 490	Total: 90

During the school year, the students were given oral examinations on what they had learnt every ten days and at the end of the year. The examinations that took place every ten days consisted of three questions. Each student had to answer at least two of them adequately in order to pass. Students who could not pass received some form of punishment. The year-end exam consisted of ten questions. Students had to answer at least six of them adequately in order to pass. Those who failed the year-end exams three times or who could not pass the civil service examinations by the end of their time limit at the imperial academy were expelled. The time limit was six years for the School of Law and nine years for all the other five schools.[45]

This time limit should have been sufficient for most students of the School of Mathematics to complete their studies. The students of the school appear to have been divided into two streams to study, with each stream designed to be completed within seven years.

[45] *Xin Tang shu*, 44, 1161.

Stream A:

- *Sunzi suanjing* and *Wucao suanjing* in one year
- *Jiuzhang suanshu* and *Haidao suanjing* in three years
- *Zhang Qiujian suanjing* in one year
- *Xiahou Yang suanjing* in one year
- *Zhoubi suanjing* and *Wujing suanshu* in one year

Stream B

- *Zhuishu* in four years
- *Jigu suanjing* in three years,[46] although some records from the Tang state that the *Jigu* was meant to be studied in one year.[47]

Both streams had to study the *Sandeng shu* and the *Shushu jiyi* as supplementary texts. Historians have long been split on whether these two streams were parallel or consecutive. This dissertation takes the view that they must have been parallel, meaning that each student of the School of Mathematics engaged in only one stream of study rather than both, one after the other. This is because even if we were to make allowances for the possibility that the *Jigu suanjing* might have occupied only one year of study, Stream B would still have taken five years to complete. The combined amount of time to complete both streams consecutively would have added up to fourteen years, which is over the nine-year time limit. Furthermore, there were two different types of civil service examinations in mathematics, each type designed specifically to ask questions about the texts

46 *Xin Tang shu*, 44, 1160-1161.
47 See Guo Shuchun et al., *Zhongguo kexue jishu shi*, 207.

studied in each stream. Studying both streams consecutively would have required the mathematics students to write and pass the civil service examinations twice before being allowed to take office, which is a requirement that none of the students of the other schools at the imperial academy were expected to fulfil. Therefore, it is more reasonable to suppose that the students of the School of Mathematics were split into two specializations according either to ability, personal inclination, or the needs of the state. Each student would then write the civil service examination for his particular specialization at the end of his course of study. However, given that the *Zhuishu* was described by Li Chunfeng as a particularly difficult and neglected text, Stream B might not have had many students, and the majority of the students of the School of Mathematics might have been concentrated in Stream A.

In addition to getting one day of holiday every ten days, the students of the imperial academy also had various days off school throughout the year:[48]

- Fifteen days in the fifth lunar month and again in the ninth lunar month
- A total of six days during the first lunar month to celebrate the new year: three at the beginning of the month and three in the middle
- The Qingming 清明 festival for paying respects to the spirits of the ancestors
- Birthday of the Sakyamuni Buddha (ca. 5th century BCE)
- Birthday of Laozi 老子 (ca. 6th century BCE), a Daoist philosopher whom the Tang imperial family claimed as their ancestor
- From 746 onwards, the birthday of the founding Tang emperor Li Yuan became a holiday
- Birthday of the reigning emperor

[48] Feng, "Zhongguo Sui Tang Wudai jiaoyu shi", 65.

- Students who had to care for sick family members could receive up to 200 days off

Students who did not report back to the imperial academy after the permitted days off were expelled. However, the sons and grandsons of officials with rank 5 or above, if expelled, could be sent to the Ministry of Military Affairs (*bing bu* 兵部) and be given a position by virtue of their birth.[49]

The Tang civil service examinations took place every year in the capital and was intimately linked with the structure of the education system. Unlike the civil service examinations of the Sui and of the late imperial period, the Tang system offered a wide variety of examinations and degrees. The six main degrees very closely matched the subjects taught at the six schools of the imperial academy:

- Cultivated talent (*xiucai* 秀才)
- Presented scholar (*jinshi* 進士)
- Understanding the classics (*ming jing* 明經)
- Understanding law (*ming fa* 明法)
- Understanding writing (*ming shu* 明書)
- Understanding mathematics (*ming suan* 明算)

However, these examinations could also be taken by people who had not attended the imperial academy.[50] The cultivated talent, presented scholar, and understanding classics examinations were for scholars who had concentrated on studying the Confucian classics. By the end of the Song dynasty, the civil service

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[49] *Xin Tang shu*, 44, 1161.
[50] Chaffee, *Thorny Gates of Learning*, 15.

examination system would consist of only the presented scholar degree, signifying both the simplification of the examination system and the narrowing of the state's requirements when searching for the ideal bureaucrat.

There were two types of civil service examinations in mathematics aimed at candidates who had studied different sets of texts. The first type specifically took questions from the texts studied in Stream A:

- Ten calculation questions
  - Three questions based on the *Jiuzhang suanshu*
  - One question based on each of the *Haidao*, *Sunzi*, *Wucao*, *Zhang Qiujian*, *Xiahou Yang*, *Zhoubi*, and *Wujing*
- Ten memorization questions based on the *Shushu jiyi* and the *Sandeng shu*

The second type of examination was based on the texts studied in Stream B:

- Ten calculation questions
  - Seven questions based on the *Zhuishu*
  - Three questions based on the *Jigu suanjing*
- Ten memorization questions based on the *Shushu jiyi* and the *Sandeng shu*

In order to pass, one had to correctly answer at least six calculation questions and nine memorization questions.[51] No examination questions or papers in mathematics from the Tang dynasty have survived. However, Siu Man-Keung and Alexei Volkov have suggested that the calculation problems were probably structured

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[51] *Xin Tang shu*, 44, 1162.

like the problems from the respective textbooks, but with modified numerical parameters.[52] Volkov later further suggested that the examination candidates likely had to use counting rods during the examination in order to solve the calculation questions and record their calculation procedures following the model set by the commentaries found in the *Ten Mathematical Classics*.[53] It was likely that the candidates had to bring their own counting rods into the examination, just as they were required to bring their own stationery, food, water, and up to three candles if they planned on writing into the night on the day of the examination.[54]

The memorization questions were known as "strip reading" (*tie du* 帖讀). The examination candidates were shown ten different lines of text from the *Shushu jiyi* and the *Sandeng shu*, each with a few characters blacked out or removed, and the candidates had to answer what those characters were. The entire civil service examination in mathematics was most likely written, with no oral component, unlike the examinations that took place regularly in the School of Mathematics. In contrast the civil service examinations in classics and writing had both written and oral components.[55]

After the examinations, each successful candidate was awarded an official rank ranging from 8ai to 9bii based on his examination results and became eligible to hold government positions of that particular rank. All holders of the mathematics

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52 Siu Man-Keung and Alexei Volkov, "Official Curriculum in Traditional Chinese Mathematics: How did Candidates Pass the Examinations?", *Historia Scientiarum* 9-1 (1999): 96-97.

53 Alexei Volkov, "Argumentation for State Examinations: Demonstration in Traditional Chinese and Vietnamese Mathematics", in *The History of Mathematical Proof in Ancient Traditions*, ed. Karine Chemla (Cambridge: Cambridge University Press, 2012), 530-531.

54 Feng, "Zhongguo Sui Tang Wudai jiaoyu shi", 129.

55 *Xin Tang shu*, 44, 1161-1162.

degree were given the rank of 9bii, just like those of the writing degree and the lower achievers of the classics, presented scholar, and law degrees.[56]

The ages of the School of Mathematics students writing the civil service examinations must have ranged from 21 to 28 years, given that they were 14 to 19 years of age when they first entered the school. As for the candidates who were not students of the School of Mathematics, there is no extant data on their ages, family background, and numbers.

It must be noted that the civil service examination system remained a very minor method of official recruitment throughout the Sui, Tang, and much of the Song. It has been estimated that only 6 to 16 percent of the Sui-Tang civil service had been recruited through the examination system.[57] The majority of the rest would have been recruited through the traditional methods of recommendation by officials, or protection privileges where the relatives of higher-ranking officials could be recruited directly into the civil service.

To further contextualize mathematical education during the Tang dynasty, attention must be paid to areas other than the imperial academy where education took place. Just like the Sui dynasty, the Tang imperial academy was not the only government department that employed erudites to teach students. Other departments that also had erudites include the Bureau of Astronomy and Astrology, the Imperial Medical Office, the Imperial Divination Office, and the Court of the Imperial Stud. Within the palace, there were also erudites to teach palace servants reading, writing, and mathematics.[58]

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[56] *Xin Tang shu*, 45, 1173.
[57] Chaffee, *Thorny Gates of Learning*, 15.
[58] *Xin Tang shu*, 44-48, 1215-1253.

Outside the capitals, even though imperial decrees had been passed to establish state-run schools in all prefectures and counties,[59] there is great uncertainty over how thoroughly these decrees were implemented,[60] especially given the fact that the Tang government at times even had trouble keeping the imperial academy in operation after the devastating An Lushan Rebellion (755 – 763),[61] which is usually seen as *the* key event that altered the course of Tang history.[62] The rebellion caused the central government to permanently lose control over the northeast regions of the country, thus diminishing its tax base and putting strains on its finances.[63]

Education at the other government departments probably also became difficult to maintain. In 767, it was recorded that the Bureau of Astronomy and Astrology did not have enough staff, and an imperial invitation was issued across the country for people knowledgeable in the study of the heavens to join the bureau.[64]

As for privately run schools, a decree was passed in 733 to permit their operation,[65] but records concerning their situations are scarce. However, it is likely that the private teaching of mathematics occurred during this time since the civil service examinations in mathematics were open to anyone who wished to take them.

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59 *Xin Tang shu*, 44, 1160; *Tang huiyao*, 35, 635.

60 Feng, "Zhongguo Sui Tang Wudai jiaoyu shi", 74-77.

61 *Xin Tang shu*, 44, 1165.

62 Mark Edward Lewis, *China's Cosmopolitan Empire: The Tang Dynasty* (Cambridge, MA: The Belknap Press of Harvard University Press, 2009), 2.

63 Lewis, *China's Cosmopolitan Empire*, 12.

64 *Tang huiyao*, 44, 796-797.

65 Ibid, 35, 634-635.

THE NORTHERN SONG DYNASTY: THE PARTING OF WAYS BETWEEN MATHEMATICS EDUCATION AND THE CIVIL SERVICE EXAMINATIONS

Tang China underwent a slow process of disintegration as the central government failed to reassert political and military control over much of its territory following the An Lushan Rebellion. This created the opportunity for the rise of regional warlords who kept China politically fragmented during the Five Dynasties and Ten Kingdoms period 五代十國 (907 – 960). Even though warfare was common during this period, especially in the north, efforts were made by the various governments to maintain the institutions inherited from the Tang, such as the imperial academy and the civil service examination system, in spite of their limited resources. For example, in 909, officials of the Later Liang 後梁 (907 – 923) imperial academy implored the emperor to take 1.5 percent from all officials' salaries and use that money to rebuild one of the academy's temples.[66] The civil service examinations continued to take place almost every year, but mainly for the classics and presented scholar degrees only.[67]

It was the Song rulers who would come to reunite China, but on a smaller scale than Tang China at its height, due to the emergence of powerful neighbouring states founded by non-Chinese peoples. In fact, in 1127, the Song would lose all of its northern territories and 35 percent of its population to the Jurchens,[68] and be forced to move their capital southwards from Bianjing 汴京 (modern Kaifeng 開封) to Lin'an 臨安 (modern Hangzhou 杭州). Therefore, historians regularly distinguish between the Northern Song, which refers to the earlier period, and the Southern Song (1127 – 1279), the later period.

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66 *Wudai huiyao* 五代會要 (Essential Documents and Regulations of the Five Dynasties) (Beijing: Zhonghua shuju, 1985), 16, 211.

67 Wang Kaixuan, *Zhongguo keju zhidu shi*, 66-67.

68 Chaffee, *Thorny Gates of Learning*, 26.

Much like the Sui dynasty, the Song founders had to reconstruct an effective central government after a period of disunity. Rather than merely following Sui and Tang precedent, however, the Song also experimented with various alternatives, so that the Song dynasty became a period of great transformation for both mathematics education and the civil service examination system. While the two had been closely intertwined during the Tang, mathematics would become permanently dissociated from the imperial academy and the civil service examinations by the end of the Northern Song.

Earlier during the Five Dynasties and Ten Kingdoms period, the civil service examination in mathematics did not occur as regularly as the examinations in the Confucian canon, indicating that mathematics was already being seen as a non-essential part of the curriculum. While the imperial academy and the presented scholar examination were re-established by the Song as early as 961,[69] mathematics was not taught at the imperial academy until 1104, and no civil service examination in mathematics was ever held throughout the Song.

The Northern Song period can best be understood as a time of experimentation in both education and the recruitment of officials. There were three waves of reform affecting education policy and official recruitment during the Song, all of which ultimately failed, but had the result of encouraging the great proliferation of both public and private schools across China.

The first wave of reform, known as the Qingli 慶曆 reforms, started in 1043 under the guidance of officials like Fan Zhongyan 范仲淹 (989 – 1052), Ouyang Xiu 歐陽修 (1007 – 1072), and Song Qi 宋祁 (998 – 1061). The reform proposals

[69] *Song shi* (Beijing: Zhonghua shuju, 1990), 1, 8-10.

covered areas like land reclamation, local militias, and the building of dikes,[70] as well as schools and the civil service examinations. Prefectural and county governments were ordered to open schools in order to provide more education opportunities for the masses. Also, all candidates had to have had attended a recognized educational institution for at least 300 days in order to be eligible to take the civil service examinations.[71]

The examinations themselves were also targeted by the reformers. Earlier, the Song government's treatment of the presented scholar examinations had not been very systematic, betraying an uncertainty of what exactly they should be testing for. One of the earliest examinations during the Song had required the candidates to box one another.[72] Over time, poetic ability became the main criterion for determining whether candidates passed or failed. The reformers proposed to change the presented scholar examination by shifting the emphasis to political acumen, which would be tested through an essay discussing government policies. While the Qingli reform policies were revoked the very next year, they had the long-lasting effect of increasing the number of state-run schools throughout the country, so that towards the end of the Northern Song, there were approximately 200 000 students in all the state-run schools combined.[73]

The next reform movement that took place in the 1070's under the initiatives of Emperor Shenzong 神宗 (r. 1068 – 1085) and Wang Anshi 王安石 (1021 – 1086) had the further effect of making these schools by and large financially self-sufficient

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70 Dieter Kuhn, *The Age of Confucian Rule: The Song Transformation of China*, History of Imperial China, ed. Timothy Brook (Cambridge, MA: The Belknap Press of Harvard University Press, 2009), 52.

71 Qiao Weiping 喬衛平, "Zhongguo Song Liao Jin Xia jiaoyu shi 中國宋遼金夏教育史", in *Bai juan ben Zhongguo quan shi*, ed. Shi Zhongwen and Hu Xiaolin (Beijing: Renmin chubanshe, 1994), 27-28.

72 Chaffee, *Thorny Gates of Learning*, 48.

73 Ibid, 6.

by ceding plots of government-owned land to the schools that could then be sub-let to tenant farmers.[74] While the new policies promoted the study of the Confucian classics and the study of law in the training of future officials, mathematics did not seem to have come into the purview of the reform programme until 1084 when Emperor Shenzong issued a decree to have the *Ten Mathematical Classics* printed by the imperial academy, and then distributed to all prefectural and county schools. This was a curious initiative because mathematics education did not appear to have been offered in any government institution at the time. It is possible that the decree indicated Shenzong's wish to revive mathematics education in state-run schools. However, he passed away shortly after, and the School of Mathematics had to wait until the third reform movement under Emperor Huizong 徽宗 (r. 1100 – 1126) and his Chief Councillor Cai Jing 蔡京 (1046 – 1126) to be established.

In 1104, the Schools of Writing and of Mathematics were founded within the imperial academy for the first time during the Song dynasty. A School of Painting (*hua xue* 畫學) was also established at this time to reflect the personal interest of Emperor Huizong, who was himself an accomplished painter. However, Huizong's support for such a diversified curriculum can only be described as sporadic at best. During his reign, these schools were repeatedly abolished before being re-established again, mirroring the tumultuous career of Chief Councillor Cai Jing who supported the creation of these schools, but was repeatedly ousted from power before being re-instated again. Specifically for the School of Mathematics, it was abolished in 1106, restored in 1107, abolished again in 1110, restored in 1113, and then permanently abolished in 1120.[75]

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[74] Yang et al., *Liang Song wenhua shi*, 339.
[75] Lee, *Government Education and Examinations*, 102.

During the school's brief bouts of existence, its capacity was set at 210 students, and the Three-Hall system (*san she fa* 三舍法) proposed earlier by Wang Anshi was implemented. Students began their studies in the Outer Hall (*wai she* 外舍), and could be promoted to the Inner Hall (*nei she* 内舍), and then the Upper Hall (*shang she* 上舍) by passing examinations. They graduated from the Upper Hall by passing an internal examination rather than the civil service examination, and were then awarded the official rank of 8b, 9a, or 9b.[76] In other words, for a man to enter the bureaucracy on the basis of mathematical skills during this period, his only option was to become a student of the School of Mathematics within the imperial academy. Compared with the Tang dynasty, the Song policy spelt a tremendous reduction in opportunity for the mathematically inclined to become officials. Also, Cai Jing's reforms in effect dissociated the recruitment of officials from the civil service examination system, and tied it directly to enrolment within the imperial academy.[77]

Furthermore, the curriculum of the Song School of Mathematics was very different from the Tang. In addition to the *Ten Mathematical Classics*, students also had to study calendar-making, astrology, divination, and geomancy.[78] This suggests that the School of Mathematics was intended as the single training ground for all officials of all departments, including the Bureau of Astronomy and Astrology, who had to use mathematics to fulfil their duties. According to the standard history of the Song, the staff of the Bureau of Astronomy and Astrology did not include any erudites throughout the dynasty.[79]

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[76] Lee, *Government Education and Examinations*, 95, fn 146; Hucker, *Dictionary of Official Titles*, 140, 490, and 555.

[77] Chaffee, *Thorny Gates of Learning*, 77.

[78] Lee, *Government Education and Examinations*, 95-96.

[79] See *Song shi,* 165, 3923.

CONCLUSION

The imperial academy's School of Mathematics was never revived after the Northern Song. This is what makes the period from the Sui to the Northern Song so unique in the history of mathematics and of education. This period saw the creation of both the School of Mathematics and the civil service examination system. Both institutions were intended to help supply capable and knowledgeable men to serve the state. However, the type of men who would make good and useful officials was defined differently by different people occupying positions of power. Therefore, this period can best be understood as a time of experimenting with various alternatives to produce an effective bureaucracy. The Sui-Tang period saw the creation of specialized schools to train young men to fill a wide variety of positions within the bureaucracy. These initiatives began to lag amid the wars and their associated social and financial strains of the Five Dynasties and Ten Kingdoms period. Following the experiences of its predecessors and trying more experiments of its own, the Northern Song government eventually reached the conclusion that the ideal official only had to be well-versed in the Confucian classics and in literary skills. All other types of knowledge, including mathematics, were considered secondary, which was why the civil service examination system conferred only one degree, the presented scholar degree, by the end of the Song. This situation would continue in subsequent dynasties until the civil service examinations were abolished in 1905. However, an efficient and effective bureaucracy cannot consist of only men who could expound eloquently on morality and benevolent rule. An in-depth knowledge of the Confucian classics may perhaps prepare one to devise good policies and adjudicate legal matters fairly, but it cannot teach one to actually solve the mathematical problems associated with tax collection, surveying, calculation of labour and resources, and calendar-making. Who, then, were fulfilling these latter duties? How and where were they trained? If the government was no longer providing such training, then other parties must be supplying that need. The answer must lie in private educational institutions, and indeed the flourishing of such institutions is one of the key distinguishing features of the Song dynasty. The next chapter explores the growing dominance of private education to help explain why mathematics was dropped from the curriculum of state-run institutions.

Chapter Four

Mathematical Education in China:

The Complementary Roles of State-Run and Private Schools

In many ways, the year 1127 marks a crucial turning point in Chinese history as the Song lost all its northern territories to the Jurchen Jin 金 (1115 – 1234), and blame was subsequently heaped upon the policies of the recently deceased former Chief Councillor Cai Jing and his faction for the diminished state of the empire.[1] The reforms instituted during the last years of the Northern Song became reviled, and mathematics would never again be formally taught in government schools until the 1860's. Yet this deprivation did not have any drastic effect on mathematical education in general or on the supply of skilled individuals who could handle the mathematics of government administration. This is because the government had only ever played a secondary role in mathematical education. In contrast, it was private education, including education within families, that was the primary means through which mathematics was taught throughout pre-modern Chinese history.

The writers and compilers of the *Ten Mathematical Classics* discussed in Chapter Two are not known to have learnt mathematics in any government school, even though this does not rule out the high possibility of on-the-job training in government departments that required the use of mathematics. For example, the *Wucao suanjing* seems to have been designed specifically as a convenient learning tool and reference handbook for government personnel who had to calculate land area, military provisions, taxes, and the trading of goods. When we look at a text like the *Sunzi suanjing*, however, which contains problems of a more recreational nature,

[1] Chaffee, *Thorny Gates of Learning*, 81.

such as the one discussed in Chapter One about fantastical beasts and birds with more than the usual number of heads, we cannot help but wonder if the treatise arose in a private educational setting. Moreover, the fact that the Tang civil service examination in mathematics could be taken by people who were not enrolled in the imperial academy implies that there were people learning mathematics outside government schools, and that the government was aware of it.

Throughout pre-modern Chinese history, we find individuals who were said to be learned in *suan*, or whose writings proved them to be so. As discussed in Chapter Two, such people include high-ranking officials like Zhang Cang and Geng Shouchang, educated men like Zhao Shuang and Liu Hui, as well as officials of the Bureau of Astronomy and Astrology like Li Chunfeng. From the Song until the 1860's, in spite of the lack of government-run mathematical education and mathematics civil service examinations, men who were proficient in mathematics continued to appear in the historical record, usually as people who were recommended to the court based on their technical skills. As will be demonstrated in this chapter, some of these men took the trouble of learning mathematics despite already having bureaucratic careers that did not require mathematical skills. Therefore, while it is true that mathematics conferred a lower status than the Confucian classics both within the Tang imperial academy and in the minds of many literati, it would be a gross oversimplification to think that all officials and literati looked down upon mathematics as a scholarly pursuit. In other words, opinion had always been divided among educated Chinese on the value of learning mathematics, just as the ancient Greek philosophers could not reach a consensus on whether mathematics should be studied for practical reasons or as a purely intellectual pursuit.[2]

On the other hand, the question of whether or not mathematics should be taught in government schools seemed resolved for good within the highest political circles during the transition between the Northern and the Southern Song. There would be no mathematical education in government schools from then until 1867, and the civil service examinations would no longer test for any technical proficiency. Yet the government could not possibly be run without the use of mathematics. Administrative tasks like tax collection and the organization of labour and military supplies required personnel who were proficient at calculating. If the government felt it was not necessary to teach such skills, or to test for them through the examinations, to what should we attribute the continued functioning of the Chinese government after the Northern Song?

The answer most likely lies in the fact that the Chinese bureaucracy was not composed solely of men who had passed the civil service examinations. There were also a large number of men who were employed by individual government offices as clerks without having to take the civil service examinations. Moreover, men who were skilled in mathematics and astronomy are sometimes mentioned in the historical record as having been recommended to the court or been summoned to enter government service on account of these skills. Many of these men must have attended privately run schools, whether formal or informal.

The particular pattern of development of mathematical education in China has profoundly affected how its history has been studied. Previous scholars noted the lack of documentation of state-run mathematical education before the Sui

[2] Geoffrey E.R. Lloyd, “The Pluralism of Greek ‘Mathematics’”, in *The History of Mathematical Proof in Ancient Traditions*, ed. Karine Chemla (Cambridge: Cambridge University Press, 2012), 298.

dynasty, the establishment of both the School of Mathematics and the mathematics civil service examination during the Tang, and the absence of state-run mathematical education after the Northern Song until the late 1860's. They then came to the conclusion that pre-modern Chinese governments and society in general looked down upon technical knowledge like mathematics, which was the cause of the government not taking the trouble to teach it for most of written history. In such a narrative, the Tang dynasty is presented as a particularly enlightened period for having a School of Mathematics and mathematics examinations to recruit skilled individuals into the bureaucracy. Including mathematics in the curriculum of state-run schools is seen as a perfectly natural and right thing to do, and so no explanation is required as to why the School of Mathematics and the mathematics examinations were established at this particular period in Chinese history. This type of narrative, however, is a reflection of the biases of modern times as mathematics is now widely accepted as an indispensable subject to teach in schools. From such a perspective, decisions to teach mathematics seem to require no explanation or justification, whereas causes have to be found for not teaching it. This type of approach is problematic because it does not acknowledge the anomalous nature of the existence of the School of Mathematics and the mathematics civil service examinations for such a short period of time. It also fails to explain why they were suddenly deemed necessary and just as suddenly dismissed as superfluous to government schools and recruitment.

I argue that a different approach is needed for the history of Chinese mathematical education. Instead of asking why the government no longer offered mathematical education after the Northern Song, I seek to explain *why* the government brought mathematics into its schools at particular points in history. My thesis stands on the premise that the main purpose of pre-modern Chinese government schools was to produce useful candidates to fill the ranks of the bureaucracy, rather than a more altruistic purpose like increasing literacy and mathematical skills among the general population. I argue that the mathematics curriculum was instituted at the imperial academy during the Sui-Tang period because the government needed to rebuild a newly unified empire and felt it would be expedient to train men in specific mathematical skills in order to ensure that an effective bureaucracy could be built quickly. The *Ten Mathematical Classics* were used as the official textbooks because they taught useful calculation skills, were regarded as respectable canons from the past, and instilled a sense of culture and learning. This reading means that the ideal of a low-ranking bureaucrat at the time was a person who could handle administrative tasks while still having some degree of cultivation.

Even without the government actively molding such men, however, people who fit these criteria were by no means lacking in society, thanks to the robust private education system in existence. Therefore, government-run mathematical education proved to be intermittent at best, especially when other needs, such as the quelling of rebellions, became more pressing. This was why the School of Mathematics and the mathematics civil service examination were not in regular existence even during the Tang.

During the Northern Song, attempts were made by Cai Jing to eliminate the civil service examinations with the aim of tying government recruitment directly to government schools.[3] This is why the mathematics curriculum was briefly revived in the imperial academy towards the end of the Northern Song. After Cai Jing fell out of favour, his reforms were summarily overthrown.

3 Chaffee, *Thorny Gates of Learning*, 77.

After the arrival of Europeans in China starting in the 1500's, the Chinese government came to the realization that there was a "new" kind of mathematics that could better serve its needs. The use of the mathematics from Europe was initially largely confined to producing an accurate calendar, but later expanded to include industrialization projects from the 1860's onwards. This was when government-run mathematical education was once again revived.

The above summary exposes the utilitarian nature of state-run education. Mathematics was brought into government schools when people in power perceived a need to supervise the training of necessary personnel in specific skills. When the need proved not to be pressing because suitable people could be recruited even without government investment in mathematics schools and examinations, the state dropped mathematical education. When suitable candidates could no longer be found in sufficient numbers, the state again intervened to train them itself. In other words, the history of Chinese mathematical education is not simply the story of government negligence and literati's disdain for mathematics following a "golden age" during the Sui, Tang, and Northern Song. Instead, it is best understood as a multi-faceted story, with private education being acknowledged as the primary means through which mathematical knowledge was passed down through the generations, and with state-run education becoming involved whenever mathematics was perceived as necessary to further political goals.

MATHEMATICAL EDUCATION AFTER THE NORTHERN SONG

As I demonstrated in the previous chapter, the Sui, Tang, and Northern Song governments established the School of Mathematics with the goal of training skilled individuals who could then be recruited to help build an efficient empire-wide bureaucracy. The intermittent operation of the School of Mathematics as well as its usually low student capacities throughout this period suggests that recruiting such individuals in sufficient numbers to fulfil the necessary duties of administration posed no great difficulty. This is probably because men with mathematical skills were readily available within society to be employed for administrative work. When the civil service examination in mathematics was held during the Tang dynasty, these men could achieve official status in the government's nine-rank system by passing the examination. During times when the mathematics examination was not available, it is likely that most of the people who handled the mathematics of administration and governing were in clerical rather than official positions. During the Tang dynasty, it was relatively easy for clerks to be directly promoted to become minor officials.[4] Over the course of the Song dynasty, that became more difficult as distinctions between officials (*liu nei* 流内, or "within the flow") and clerks (*liu wai* 流外, or "outside the flow") hardened, even though this did not stop the clerical service from continuing to grow in size.[5]

Detailed records about clerks and how they acquired their mathematical knowledge are difficult to find, but the fact that there existed a vigorous, though fragmented, private education system in mathematics is unquestionable. This can be deduced from the greater body of extant mathematical writings that have survived from the Song onwards compared to the earlier period. Some of these were written by scholar-officials, while others were by men with no apparent links to the bureaucracy.

The best known mathematical writer of the Northern Song period is Shen Gua 沈括 (1031 – 1095), who had a varied official career after passing the civil service examination. He became ambassador to the Khitan Liao 遼 (907 – 1126),

[4] Chaffee, *Thorny Gates of Learning*, 55-56.
[5] Ibid, 21.

held editorials posts within several institutes of the central government, served as supervisor of the Bureau of Astronomy and Astrology (*ti ju sitian jian shi* 提舉司天監事), and was put in charge of a project to create an accurate map of the Song empire.[6] In his later years, Shen Gua wrote the *Mengxi bitan* 夢溪筆談 (Dream pool essays), a diary-like treatise of which about a third of the entries are related to scientific, technological, and medical subjects, including mathematics, astronomy, physics, chemistry, biology, and geography.[7] The rest of the treatise includes discussions of astrology, cosmology, divination, architecture, and various “strange occurrences” (*yi shi* 異事).[8] Of course, Shen Gua did not group these topics using our modern categories, and neither did he evince any thought that what we would consider as scientific content was fundamentally different from the non-scientific. Astronomy was grouped together with astrology, cosmology, and divination under “regularities underlying phenomena” (*xiang shu* 象數), while mathematics was grouped with medicine, engineering, architecture, and games under “technical skills” (*ji yi* 技藝).[9]

Among the entries relating to mathematics, Shen Gua displays a recreational interest in topics like calculating the length of a circular arc, the sum of a finite series, and the total number of possible outcomes of *weiqi* 圍棋 or *go*, a popular board game among the literati.[10]

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[6] See biography of Shen Gua in *Song shi*, 331, 10653-10657; Yang et al, *Liang Song wenhua shi*, 751-752.

[7] Yang et al, *Liang Song wenhua shi*, 746-747.

[8] Sivin, “Why the Scientific Revolution did not Take Place in China”, 48.

[9] Ibid, 48 and 50.

[10] Martzloff, *A History of Chinese Mathematics*, 16; Guo Zhimeng 郭志猛, *Zhongguo Song Liao Jin Xia keji shi* 中國宋遼金夏科技史, *Bai juan ben Zhongguo quan shi* 百卷本中國全史, vol. 12, ed. Shi Zhongwen 史仲文 and Hu Xiaolin 胡曉林 (Beijing: Renmin chubanshe, 1994), 189.

Figure 4.1 A *weiqi* or *go* game in progress.[11]

The *weiqi* board is an 18 x 18 square, and the game pieces are placed on the intersections between the lines. Each intersection can hold a black piece, a white piece, or be left empty, thereby having three possible outcomes. The total number of possible outcomes for the entire board is then $3^{19\times19} = 3^{361}$. The answer approximated by Shen Gua, however, is significantly too large. Shen recognized that the answer would be a number too large to be conveniently expressed in any notation available to him, so he gave an approximation of 10 000 multiplied by itself 52 times. However, the real answer is closer to 10 000 multiplied by itself 43 times.[12]

[11] Zizou man (Flickr: Go game) [CC BY 2.0 (https://creativecommons.org/licenses/by/2.0)], via Wikimedia Commons.

[12] Mark Elvin, "Personal Luck: Why Pre-Modern China – Probably – did not Develop Probabilistic Thinking", in *Concepts of Nature: A Chinese-European Cross-Cultural Perspective*, Conceptual History and Chinese Linguisitics, vol. 1, ed. Hans Ulrich Vogel and Günter Dux (Leiden: Brill, 2010), 407-408.

Shen Gua is an example of a literatus who acquired mathematical as well as other technical knowledge in the absence of state-run education in such subjects. His interests might have been more broad-ranging than most other people's, but were certainly not unique to him among the educated segment of society. Throughout the rest of pre-modern Chinese history, there would be many other highly educated men like him who turned to mathematics as an area of study in addition to the traditional Confucian curriculum. Some of them took up teaching mathematics as a profession, while others were very much like the early mathematicians of seventeenth- and eighteenth-century Europe, like René Descartes (1596 – 1650), who did not depend on mathematics to make a living, but pursued and wrote about it as a matter of intellectual interest.

During the thirteenth and turn of the fourteenth century, there were a group of men collectively known in historiography as the four Song-Yuan masters: Qin Jiushao 秦九韶 (ca. 1202 – ca. 1261), Yang Hui 楊輝 (fl. 1261 – 1275), Li Zhi 李治 (1192 – 1279), and Zhu Shijie 朱世傑 (fl. 1280 – 1303). They led different lives and do not seem to have known each other personally, but have all left writings on sophisticated mathematical breakthroughs, some of which were apparently built upon the contents of some of the *Ten Mathematical Classics*. Both Qin Jiushao and Yang Hui lived within the Southern Song empire. Qin had a life-long, though tumultuous, career as an official in both civil and military positions at the local level, none of which absolutely required knowledge of mathematics. Yet he completed a mathematical treatise, the *Shushu jiuzhang* 數書九章 (Mathematical treatise in nine sections), in 1247 during the mandatory three-year leave he had to take to mourn for his recently deceased mother.[13]

In his preface, Qin wrote:

> 早歲侍親中都，因得訪習於太史，又嘗從隱君子受數學。[14]
> [In my youth, I attended my parents at the capital, so I had the chance to study under an official of the Bureau of Astronomy and Astrology. I also learnt mathematics from a recluse.]

Qin also referred to the *Jiuzhang suanshu*, and explained that his treatise contained the *dayan* 大衍 method (indeterminate analysis), which was not covered in the *Jiuzhang*.[15]

The influence of the *Jiuzhang suanshu* is also evident in the works of Yang Hui, another Southern Song mathematical writer, but less is known about him than Qin Jiushao. Yang Hui might have been a government official as well.[16] He wrote several mathematical treatises, one of which was called *Xiang jie Jiuzhang suanfa* 詳解九章算法 (Detailed explanation of the *Jiuzhang*) and was a discussion of 80 of the *Jiuzhang's* 246 problems.[17]

Another of Yang Hui's mathematical works, the *Yang Hui suanfa* 楊輝算法 (Yang Hui's calculation methods), contained references to three others of the *Ten*

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[13] For studies in English on Qin Jiushao and his mathematical treatise, see Ulrich Libbrecht, *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, MIT East Asian Science series, ed. Nathan Sivin (Cambridge, MA: The MIT Press, 1973), and Ke-Xin Au Yong, "Qin Jiushao and His *Mathematical Treatise in Nine Sections* in Thirteenth-Century China" (Master's thesis, University of Alberta, 2011).

[14] Qin Jiushao, preface to the *Shushu jiuzhang*, as reproduced in Chen Xinchuan 陳信傳, Zhang Wencai 張文材, and Zhou Guanwen 周冠文, *Shushu jiuzhang jin yi ji yanjiu* 數書九章今譯及研究 (Vernacular translation and study of the *Shushu jiuzhang*) (Guiyang: Guizhou jiaoyu chubanshe, 1993), 1-8.

[15] Ibid.

[16] Ho Peng Yoke, "Yang Hui", in *Dictionary of Scientific Biography*, vol. 14, ed. Charles Coulston Gillispie (New York: Charles Scribner's Sons, 1976), 539.

[17] Ibid.

Mathematical Classics: the *Haidao suanjing*, the *Sunzi suanjing*, and the *Zhang Qiujian suanjing*.[18] In this work, Yang Hui also laid out a curriculum for the study of mathematics, which was much simpler than the Tang and Northern Song imperial academies' official curriculum. The only textbooks that he thought necessary to use were the *Wucao suanjing*, his own *Xiang jie Jiuzhang suanfa*, and a treatise called *Ying yong suanfa* 應用算法 (Practical calculation methods).[19] This suggests that Yang Hui might have taught mathematics himself. Indeed, the Southern Song period is noted for a great proliferation of private schools, especially academies (*shuyuan* 書院) that were geared primarily towards the study of the Confucian classics.[20] By one count, Song China had 234 prefectural schools, 516 county schools, and 464 private schools.[21] This last number probably does not take into account the more informal schools where the teaching took place in the home of the teacher or the student, and where the student may or may not be related to the teacher. This particular type of less formal private school was a common means through which mathematics was taught. Both Li Zhi and Zhu Shijie, the two other Song-Yuan masters, seem to have taught mathematics at such schools.

Li Zhi and his father both served the Jurchen Jin dynasty. Li Zhi had passed the civil service examination in 1230, and was quickly appointed to a post in the province of Henan 河南. His bureaucratic career proved very short, however, as the Jurchen empire was conquered by the Mongols in 1234. Li Zhi appears to have devoted himself to mathematical study from then on, and wrote two mathematical

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18 Wang Qingjian 王青建, "Suanjing shishu yu shuxue shi jiaoyu 算經十書與數學史教育" [The Ten Mathematical Classics and the teaching of the history of mathematics], *Neimenggu shifan daxue xuebao* 內蒙古師範大學學報（自然科學漢文版）[Journal of Inner Mongolia Normal University (Natural Science Edition)] 38, no. 5 (Sept 2009): 584.

19 Ibid.

20 Chaffee, *Thorny Gates of Learning*, 90.

21 Ibid, 136-137.

treatises between 1248 and 1261: the *Ceyuan haijing* 測圓海鏡 (Sea mirror of the circle measurements) and the *Yigu yanduan* 益古演段 (New steps in computation). These two treatises probably gained him some renown as he is said to have had many students during his old age.[22]

The same was also said of Zhu Shijie, who never served in any bureaucratic position. Zhu apparently did a considerable amount of travelling within the newly unified China under the Mongol Yuan 元 dynasty (1271 – 1368), and taught many students during his travels. He wrote two mathematical treatises: the *Suanxue qimeng* 算學啟蒙 (Introduction to mathematical studies) in 1299, and the *Siyuan yujian* 四元玉鑑 (Jade mirror of the four elements) in 1303.[23]

Notable figures in the history of mathematical astronomy who were active at around the same time also studied astronomy either at home or under a master outside of any formal school. Two such examples are Wang Xun 王恂 (1235 – 1281) and Guo Shoujing 郭守敬 (1231 – 1316), two of the officials involved in the astronomical reforms of the 1270's and in the creation of the *Shoushi* 授時 ("granting the seasons") calendar system promulgated in 1280.

Wang Xun was born shortly after the Mongol conquest of northern China. His father had been an official under the Jurchen Jin dynasty. Wang Xun started learning mathematics at a very young age, and was already a master in the subject by his early teens. He then studied under Liu Bingzhong 劉秉忠 (1216 – 1274), a

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[22] Martzloff, *A History of Chinese Mathematics*, 143; Ho Peng Yoke, "Li Chih", in *Dictionary of Scientific Biography*, vol. 8, ed. Charles Coulston Gillispie (New York: Charles Scribner's Sons, 1973), 313-314.

[23] Ho Peng Yoke, "Chu Shih-chieh", in *Dictionary of Scientific Biography*, vol. 3, ed. Charles Coulston Gillispie (New York: Charles Scribner's Sons, 1971), 265; Lam Lay Yong, "Chu Shih-chieh's Suan-hsüeh ch'i-meng (Introduction to Mathematical Studies)", *Archives for History of Exact Sciences* 21 (1979): 1.

close advisor of Khubilai (r. 1260 – 1294), Great Khan of the Mongol empire from 1260 onwards and founder of the Yuan dynasty in China. Wang Xun was recommended to Khubilai in 1253, and was later appointed as the director of the Chinese Bureau of Astronomy and Astrology.[24]

The Mongols saw great value in prognostication for guiding decision-making. The vast extent of their empire and the plurality of their subjects allowed the Yuan rulers to also establish an Islamic Bureau of Astronomy and Astrology operating in parallet to the Chinese one, so that they were routinely supplied with multiple prognostications that they could then compare before making important decisions.[25] There was probably very little exchange of ideas between the Chinese and the Islamic bureaus since Khubilai expressly discouraged their personnel from sharing information with one another.[26]

It should also be noted that it was actually not unusual to have more than one Bureau of Astronomy and Astrology. During the Northern Song, Emperor Renzong created a second, but smaller, bureau in 1027 that could be put in charge of testing proposed calendrical reforms.[27] Also, the Ming dynasty, which overthrew the Mongol Yuan in 1368, continued its predecessor's policy of maintaining parallel Chinese and Islamic bureaus.[28]

Guo Shoujing was another one of Liu Bingzhong's students. Guo learnt astronomy, astrology, and divination among other skills from his grandfather before going to study under Liu. He was recommended to Khubilai in 1262, and became

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[24] Sivin, *Granting the Seasons*, 156-157.

[25] Ibid, 22-23.

[26] Ibid, 30.

[27] Sun Xiaochun and Han Yi, "The Northern Song State's Financial Support for Astronomy", *East Asian Science, Technology, and Medicine* 38 (2014): 19-20.

[28] Thatcher E. Deane, "Instruments and Observation at the Imperial Astronomical Bureau during the Ming Dynasty", *Osiris* 9 (1994): 130.

the director of the Chinese Bureau of Astronomy and Astrology in 1286, five years after the death of Wang Xun.[29]

This tradition of literati taking a scholarly interest in mathematics and studying it through private channels continued well into the nineteenth century. Famous examples from the Ming and the Qing dynasties include Zhu Zaiyu 朱載堉 (1536 – 1611), Xu Guangqi 徐光啟 (1562 – 1633), Mei Wending 梅文鼎 (1633 – 1721) and his family, Li Guangdi 李光地 (1642 – 1718), and Li Shanlan 李善蘭 (1811 – 1882).

Zhu Zaiyu was a royal prince of the Ming dynasty. Although it is not known how he learnt mathematics, he is famous in history for being accomplished in the subject as well as in mathematical harmonics and astronomy. By the late 1500's, the official astronomical system in use could no longer accurately predict solstices and eclipses, and Zhu Zaiyu proposed astronomical reforms to remedy the situation, but his proposal was ultimately rejected.[30]

Xu Guangqi was a high-profile Chinese Christian whom the Jesuit missionaries converted very early during their mission in China. The Jesuits began settling in China in 1583. One of them, Matteo Ricci (1552 – 1610), known in historiography as the founding father of the Jesuit mission in China, eventually reached the Ming capital Beijing in 1601. While he was there, Ricci was able to convert several government officials, including Xu Guangqi, who would later become Vice-Minister of Rites (*libu zuo shilang* 禮部左侍郎). Xu was interested not just in the new religion to which Ricci introduced him, but also in the mathematics that the latter brought from Europe. Therefore, Xu took mathematics lessons from Ricci, and

[29] Sivin, *Granting the Seasons*, 158-160.
[30] Martzloff, *A History of Chinese Mathematics*, 21.

together, they translated into Chinese the first six books of the Jesuit Christoph Clavius's (1538 – 1612) reworked edition of Euclid's *Elements*.[31]

The Manchu conquest of China in 1644 was followed by an era of increasing contact with Westerners and their sciences and technologies, eventually triggering a response on the part of the Qing government to establish a formal school for the teaching of both Chinese and Western mathematics in 1867. Before this revival of state-run mathematical education, however, an interest in "practical learning" (*shixue* 實學) was already evident in the actions of many literati during the early Qing dynasty.[32] These literati included Mei Wending and his family, as well as Li Guangdi.

Like the vast majority of candidates who aspired to become government officials by passing the civil service examinations, Mei Wending and two of his brothers were not able to pass all three levels of the examinations.[33] Mei had been interested in observing the stars with his father and his family tutor since he was a child. At the age of 27, he began devoting himself to the study of mathematics and astronomy.[34] Mei had heard of the *Jiuzhang suanshu*, but was never able to acquire a complete copy of this text for his studies.[35] His brother Wennai 文鼐 (1637 -1671) and Wenmi 文鼏 (1642 – 1716), son Yiyan 以燕 (1654 – 1705), grandson Juecheng 瑴成 (1681 – 1763), and two great-grandsons are also noted in history as being

[31] Catherine Jami, *The Emperor's New Mathematics: Western Learning and Imperial Authority During the Kangxi Reign (1662 – 1722)* (Oxford: Oxford University Press, 2012), 23.

[32] Catherine Jami, "Scholars and Mathematical Knowledge during the Late Ming and Early Qing", *Historia Scientiarum* 42 (1991): 99.

[33] Limin Bai, "Mathematical Study and Intellectual Transition in the Early and Mid-Qing", *Late Imperial China* 16, no. 2 (Dec 1995): 30.

[34] Jami, "Scholars and Mathematical Knowledge", 103.

[35] Li Di, "Jiuzhang suanshu yanjiu shi gang 九章算術研究史綱" (Summary of research on the Jiuzhang suanshu), in *Liu Hui yanjiu* 劉徽研究 (Research on Liu Hui), ed. Wu Wenjun et al. (Xi'an: Shaanxi renmin jiaoyu chubanshe, 1993), 29.

accomplished in mathematics and astronomy.[36] By 1702, Mei Wending had written 62 treatises on astronomy and 26 on mathematics.[37] That same year, his work was first introduced to the Kangxi 康熙 emperor (r. 1662 – 1722) by Li Guangdi.

Li and Mei first met in 1689.[38] At the time, Li Guangdi was the chancellor of the Hanlin Academy (*Hanlin yuan zhang yuan xueshi* 翰林院掌院學士),[39] a central government body that was mainly responsible for the drafting and editing of imperial edicts and imperially sponsored compilations.[40] Impressed with Mei Wending's technical expertise, Li Guangdi asked Mei to teach him, his brothers, his son, and his students mathematics and astronomy.[41]

The Kangxi emperor also showed great appreciation for Mei Wending's work, and summoned him to the court in 1705 to discuss mathematics.[42] Kangxi was quite exceptional among rulers of pre-modern China in his keen interest in and mastery of mathematics and astronomy, which he primarily learnt from Jesuit tutors. He insisted that his sons also become knowledgeable in these subjects.[43] Furthermore, he established an Office of Mathematics (*Suanxue guan* 算學館) in 1713 to compile vast compendia on mathematics, astronomy, calendar-making, and harmonics. Some of its staff were recruited through a special examination in mathematics held in 1712. This examination was only held once, and had no connection to the regular civil service examinations.[44] Among the people recruited to work at the Office of

36 Martzloff, *A History of Chinese Mathematics*, 80; Jami, *The Emperor's New Mathematics*, 83.
37 Jami, *The Emperor's New Mathematics*, 82.
38 Bai, "Mathematical Study and Intellectual Transition", 30.
39 Jami, *The Emperor's New Mathematics*, 121.
40 Hucker, *Dictionary of Official Titles*, 223.
41 Bai, "Mathematical Study and Intellectual Transition", 30.
42 Ibid, 29.
43 Jami, *The Emperor's New Mathematics*, 275.
44 Ibid, 262-263.

Mathematics was Mei Juecheng, Mei Wending's grandson, who was awarded a special presented scholar degree for mathematics and astronomy (*chouren jinshi* 疇人進士) in 1715.[45] While some earlier scholarship had characterized the Office of Mathematics as a state-run institution for the teaching of mathematics, it seems that no actual teaching occurred at the Office except in a very informal sense where the Kangxi emperor was regarded as the teacher of the staff working at the Office, and the staff had the privilege of asking the emperor questions about the material that they had to compile.[46] The actual re-institution of state-run mathematical education would only come in the late 1860's, and would be closely connected to the career of Li Shanlan.

Li Shanlan is said to have been a child prodigy who chanced upon a copy of the *Jiuzhang suanshu* at the private school he was attending when he was eight years old. Despite his young age and the lack of guidance, mastering the contents of the *Jiuzhang* apparently proved to be no difficulty at all. At fourteen years old, he mastered Matteo Ricci and Xu Guangqi's translation of the first six books of the *Elements*, again without instruction by a teacher. Throughout his life, Li Shanlan had an avid interest in mathematics, which he primarily learnt from books rather than teachers. In the 1850's, he was recruited by Alexander Wylie (1815 – 1887) to translate European scientific and technological works into Chinese for the London Missionary School. The works that Li helped to translate include John F.W. Herschel's *Outlines of Astronomy*, William Whewell's *An Elementary Treatise on Mechanics*, Augustus De Morgan's *Elements of Algebra*, and the last nine books of Euclid's *Elements*.[47] These translations earned Li Shanlan a reputation of being

[45] Bai, "Mathematical Study and Intellectual Transition", 33.
[46] Jami, *The Emperor's New Mathematics*, 272.
[47] Martzloff, *A History of Chinese Mathematics*, 173-175.

conversant with "Western learning" (*xixue* 西學), leading to his appointment in 1869 as the head of the new school for mathematics and astronomy created by the Qing government.

RE-INSTITUTION OF STATE-RUN MATHEMATICAL EDUCATION

Increasing interactions between the Chinese and Westerners did not just involve benign exchanges of religions, science, and technology. Tensions built during the eighteenth and nineteenth centuries over foreigners' trading rights within China and the ever-growing influx of opium through foreign traders that was aimed at countering the trade imbalance that had seen the flow of silver into China increase five-fold from the 1760's to the 1780's.[48] These tensions culminated in the First and Second Opium Wars (1839 – 1842, 1856 – 1860), both of which China lost. China's inability to win armed conflicts against foreign powers during this period forced the Qing government to repeatedly concede to foreigners' demands for indemnities and additional rights. At the same time, the Chinese became aware that their military technology lagged behind the foreign powers', and that there existed a system of international diplomacy that was completely different from the one built on tributary relations to which they had been used. Training and acquiring personnel with the necessary knowledge and skills to remedy these problems became an imperative for the Qing government. Accordingly, the College for Combined Learning (*Tongwen guan* 同文館) was opened by the central government in Beijing in 1862. Initially, the college was designed solely for the teaching of foreign languages, including English, French, Russian, and German. In 1867, the School of Astronomy and Mathematics

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[48] Jonathan D. Spence, *The Search for Modern China*, 2nd ed. (New York: W.W. Norton & Company, 1999), 129.

(*Tianwen suanxue guan* 天文算學館) was added to the college.[49] This is because there was a strong feeling among the Chinese at the time that the Westerners' navigation and military technologies were derived from their astronomical and mathematical knowledge. Initially, it was Baron Johannes von Gumpach who was appointed to head the new school, "but he considered the level of teaching expected of him beneath his dignity and refused to teach during the two years he lived in [Beijing]."[50] Li Shanlan was eventually appointed in 1869 to fill the vacant post. At the School of Astronomy and Mathematics, there were no officially prescribed textbooks, and Li and his colleagues were by and large given a free hand to establish a curriculum. Guided by the principle "Chinese learning as the essence, Western learning as application" (*zhongxue wei ti, xixue wei yong* 中學為體，西學為用), the textbooks that were used were a mix of Chinese mathematical treatises including the *Ten Mathematical Classics*, translated European mathematical works, and treatises written by the teachers themselves.[51]

MATHEMATICS IN PRACTICE

Besides the corpus of extant mathematical works discussed so far, mathematics was for certain taught and practised in other spheres within Chinese society. When writing about ancient Greek mathematics, Jacqueline Stedall rightly observes that

> both Euclid and Diophantus belonged to tiny mathematical elites. A moment's reflection is enough to

[49] Li Zhaohui 李朝暉, and Zhang Wei 張偉, "Qing mo de shuxue jiaokeshu 清末的數學教科書" (Mathematics textbooks of the Late Qing), *Neimenggu shifan daxue xuebao* 內蒙古師範大學學報（自然科學漢文版）[Journal of Inner Mongolia Normal University (Natural Science Edition)] 38, no. 5 (Sept 2009): 586.

[50] Martzloff, *A History of Chinese Mathematics*, 84.

[51] Li and Zhang, "Qing mo de shuxue jiaokeshu", 586-587.

> show how much more mathematics must have been going on than the mathematics they wrote about. Greek society, like every other, had its shopkeepers and housekeepers, farmers and builders, and many others who routinely measured and calculated. We know almost nothing about their methods because such people would have learned and taught mostly by example and word of mouth... When mathematical historians speak of 'Greek mathematics', as they frequently do, they are almost always speaking of the sophisticated written texts that have come down to us from Euclid, Archimedes, Diophantus, and others, not of the common or garden mathematics of the *hoi polloi*.[52]

For China, the situation was much the same. It too "had its shopkeepers and housekeepers, farmers and builders", as well as others whose work or daily activities necessitated some mathematical skills that they had to acquire either through texts or teachers. These people include those who handled hydraulic and flood control projects for the state. Because Chinese civilization was centred along the banks of the Yellow and Yangzi Rivers throughout much of its history, building effective flood control projects was a major concern for Chinese governments. It has been estimated that the Yellow River broke through its dikes over 1500 times over the course of China's imperial history.[53] Flood control was an especially urgent concern during the Northern Song because the capital sat at the conjunction of four rivers, and over a hundred floods occurred in the vicinity over the course of the dynasty.[54] People who designed these dikes and managed the building projects must have had some mathematical skills in order to accomplish the necessary calculations.

[52] Stedall, *The History of Mathematics*, 12-13.

[53] Mark Edward Lewis, *The Early Chinese Empires: Qin and Han*, History of Imperial China, ed. Timothy Brook (Cambridge, MA: The Belknap Press of Harvard University Press, 2007), 7.

[54] Heping Liu, "Picturing Yu Controlling the Flood: Technology, Ecology, and Emperorship in Northern Song China", in *Cultures of Knowledge: Technology in Chinese History*, ed. Dagmar Schafer (Leiden: Brill, 2012), 91.

Surveying and map-making were also important concerns of the state for the purposes of tax policies, military engagements, and designing the overall administrative structure. It has been estimated that just the Song dynasty alone re-drew the boundaries of various counties and prefectures over a thousand times.[55] Therefore, the state must have needed skilled surveyors and map-makers to be in constant supply.

Outside the state apparatus, there were traders, merchants, carpenters, and many others who would have required the mathematical skills that were contained in texts like the *Jiuzhang suanshu* and the *Sunzi suanjing*. How many of them actually learnt their skills from the *Ten Mathematical Classics* instead of other texts about which we know nothing? The answer cannot be ascertained. What is certain is that the mathematics done in pre-modern China encompassed more than what we see in the extant mathematical treatises.

CONCLUSION

From the above, it is clear that government decisions to implement mathematical education throughout pre-modern Chinese history were always tied to political goals. Therefore, the history of Chinese state-run mathematical education is not so much a tale of the government neglecting it to the detriment of society in general, but can best be understood as the government playing a dynamic supplementary role to private mathematical education.

The opinion of the literati towards mathematics also cannot be generalized as being always one of disdain. As we have seen in the second and the current chapter, the writers and compilers of all the extant mathematical treatises discussed in this

[55] Ruth Mostern, *"Dividing the Realm in order to Govern": The Spatial Organization of the Song State (960 – 1276 CE)* (Cambridge, MA: Harvard University Asia Centre, 2011), 2.

dissertation were literati who thought mathematics a worthwhile area of study even if mastery of it would not bring any material benefit.

Historians have long noted a renewal of interest among the literati in practical learning during the late Ming and early Qing. This renewal came after a lull in scientific and technological writings by the literati during the fourteenth and fifteenth centuries. Whether this lull is in fact real or is merely the result of an error of perception caused by the loss of relevant sources from that period is a matter of speculation at present. The themes of loss and preservation of sources, however, will be explored in next chapter's discussion of the circulation and transmission of the *Ten Mathematical Classics*.

Chapter Five

Mathematics in Motion:

The Circulation and Transmission of the *Ten Mathematical Classics*

The Jurchen invasion of Song China's capital, Kaifeng, in 1126 – 1127 resulted in their conquest of the region and the capture of Emperor Qinzong (r. 1126 – 1127), his father the retired emperor Huizong, and his son the crown prince, along with thousands of imperial family members, court officials, palace servants, entertainers, and craftsmen, all of whom were marched north into the heart of Jurchen territory as hostages. The Jurchens also claimed as their spoils of war imperial treasures, ritual paraphernalia, musical instruments, an armillary sphere, the elaborate clock tower built by Su Song 蘇頌 (1020 – 1101) , and the book and carved woodblock collections of several government libraries.[1] Just these books and woodblocks required 8000 soldiers to haul into the camps of the triumphant Jurchens.[2] The rest of the government library collections, including books and carved woodblocks, were either salvaged by fleeing Song officials or destroyed in the fires that swept through the libraries during the invasion.

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[1] *Song shi*, 23, 436; Dagmar Schaefer, "Science in the Pre-Modern East", in *The Oxford Illustrated History of Science*, ed. Iwan Rhys Morus (Oxford: Oxford University Press, 2017), 108.

[2] Zhang Xiumin 張秀民, *Zhongguo yinshua shi* 中國印刷史 (History of Chinese printing) (Hangzhou: Zhejiang guji chubanshe, 2006), 760.

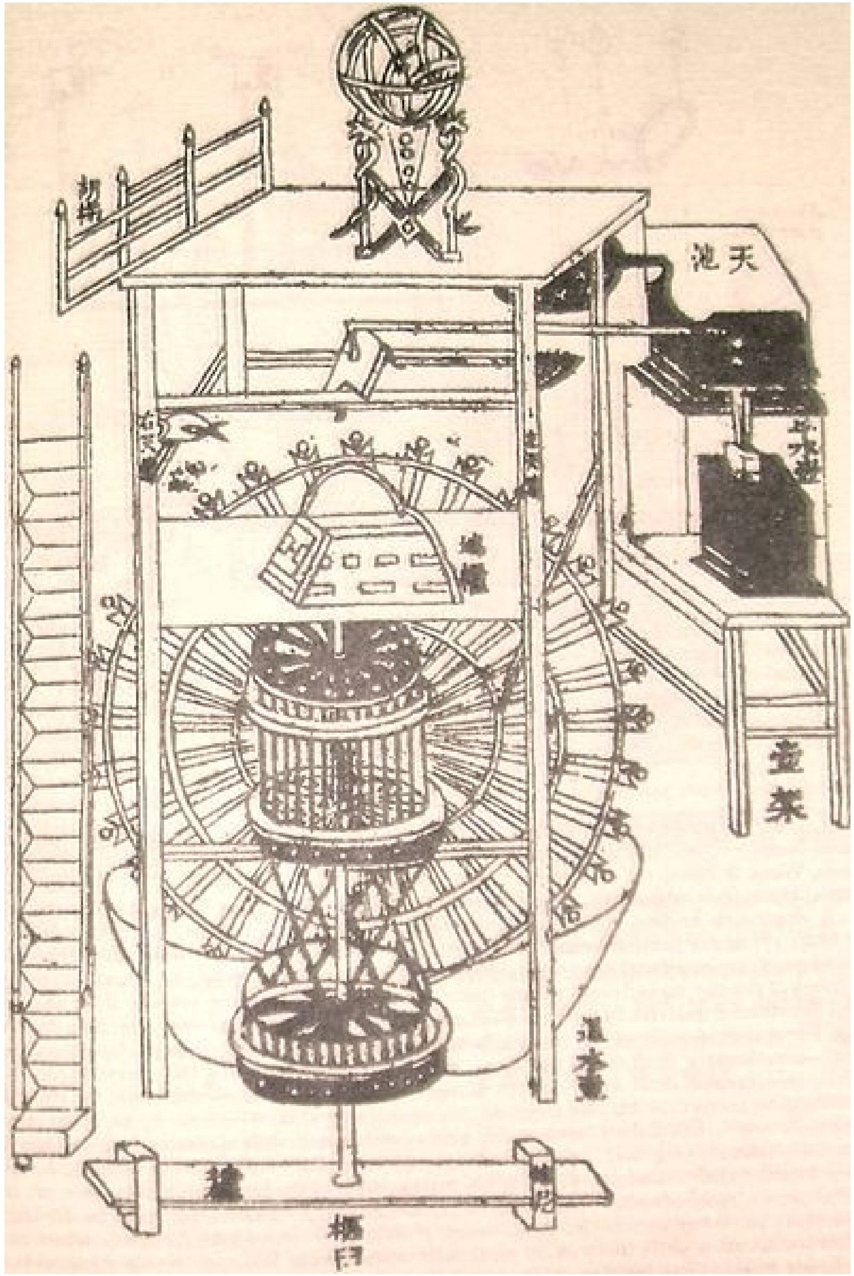

Figure 5.1 An image of Su Song's clock tower.[3]
Su Song presented a model of this tower to the emperor in 1088 to apply for funds for its construction so as to enable him to better observe celestial phenomena. After the clock tower was built, the model was subsequently discarded. During the invasion of 1126 – 1127, the Jurchens dismantled the tower and took the parts back to their capital. Unfortunately, they were unable to re-assemble it for use.[4]

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[3] Su Song [Public domain], via Wikimedia Commons

[4] Schaefer, "Science in the Pre-Modern East", 108.

How the Song central government's copies of the *Ten Mathematical Classics* fared amidst this chaos cannot be ascertained. No detailed inventory exists of what the Jurchens took. There is also no catalogue of the government library holdings in the standard history of the Jin dynasty. What is clear is that by the time the Song court re-established itself in Lin'an (modern Hangzhou) by rallying around the ninth son of Huizong as the new emperor, thus beginning the era known as the Southern Song, the collections of the central government's libraries had been greatly diminished, and it was only with great difficulty that Bao Huanzhi 鮑澣之 (fl. 1200 – 1213), prefect of Tingzhou 汀州 (modern Changting county 長汀縣 in the province of Fujian 福建), managed to locate copies of the *Ten Mathematical Classics* for re-printing in 1213.

Figure 5.2 Southern Song and Jurchen Jin after 1127[5]

How a local prefect came to oversee a project to re-print these mathematical textbooks, at a time when mathematics had already been sidelined from state-run education for nearly a century, is one of the questions that will be explored here. The focus of this chapter is on the production and circulation of the *Ten Mathematical Classics*, as well as their transmission to the rest of East Asia. The aim here is to widen the scope of this dissertation by approaching the history of the *Ten Mathematical Classics* not just from the perspective of intellectual history, but

[5] China - Southern Song Dynasty - cs.svg: User:Mozzan derivative work: Kanguole [CC BY-SA 3.0 (https://creativecommons.org/licenses/by-sa/3.0)], via Wikimedia Commons <https://commons.wikimedia.org/wiki/File%3AChina_-_Southern_Song_Dynasty-en.svg > – with modifications.

also of the history of the book. In this chapter, we trace the history of what is concrete and tangible in the history of mathematics by following the history of copies of the actual texts. In the case of the *Ten Mathematical Classics*, this history encompasses both manuscript copies and imprints, hence it also comprises part of the history of printing in China. In other words, in this chapter, we situate the *Ten Mathematical Classics* within the broader history of manuscript and imprint production. We highlight the critical role of government offices in making new copies of the *Ten Mathematical Classics*, and how essential private collectors were to preserving these copies.

The circulation of the *Ten Mathematical Classics* was not limited to only China, but extended across much of East Asia. Therefore, the last section of this chapter discusses the transmission of the *Ten Mathematical Classics* to Korea, Japan, and possibly Vietnam, as well as their usage in these societies respectively. We shall see that the *Ten Mathematical Classics* played a very similar role in these countries as they did in China, and were used to structure state-run mathematical education. Therefore, the *Ten Mathematical Classics* contributed towards unifying the state-sponsored mathematics of what is known as the East Asian cultural zone. Contrary to common perceptions, the East Asian cultural zone was not created merely through common philosophies, religions, and writing systems, but incorporated shared technical aspects as well.

At the end of our survey of the transmission and usage of the *Ten Mathematical Classics* across East Asia, it will become clear that these texts, though venerated today as undeniable evidence of pre-modern Chinese mathematical achievements, were by no means what could be called popular books. In other words, the *Ten Mathematical Classics* only circulated among a certain type of readers. While these readers were the educated elite and occupied the higher rungs of the social ladder, their numbers were small and could only have been a small subset of the people who needed to acquire mathematical skills and knowledge. Therefore, even though the *Ten Mathematical Classics* are almost the only mathematics texts dating from China's early imperial period to have survived to this day, it remains very difficult to determine how well they represent or encapsulate the entire body of mathematics that was done in China and the rest of East Asia during the pre-modern era.

EARLIEST COPIES

The earliest extant copies of the *Ten Mathematical Classics* date from 1213, around 550 years after they became the official textbooks of the Tang imperial academy, and over 1000 years after the earliest of the ten classics had come into existence. Such limitations of available sources place us in the position of having to speculate about copies that pre-date 1213.

Paper was invented in China in the year 105 and became the dominant writing medium by the third or fourth century.[6] Before that, the Chinese mainly wrote on bamboo strips that were then strung together, or on pieces of silk.[7] Since both the *Zhoubi suanjing* and the *Jiuzhang suanshu* were likely compiled before the invention of paper, their earliest forms must have been in one or both of these materials.

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[6] Zhang Xiumin, *Zhongguo yinshua shi*, 6-7.
[7] Ibid, 3-4.

Figure 5.3 Example of an early Chinese text (fifth to third century BCE) written on bamboo strips[8]

From the third century onwards, as the rest of the *Ten Mathematical Classics* came to be written, and as commentaries were added to both the *Zhoubi* and the *Jiuzhang*, one can say with some confidence that most, if not all, of these writers were already working with paper, and that these texts circulated in manuscript form in relatively small numbers.

Woodblock printing technology was invented in China perhaps as early as the seventh century.[9] However, early use of this technology appears to have been limited to Buddhist institutions and commercial printers, while government bodies would only come to employ it starting in the tenth century.[10] The earliest printed

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[8] Shanghai Museum [Public domain], via Wikimedia Commons https://commons.wikimedia.org/wiki/File%3AManuscript_from_Shanghai_Museum_1.jpg.

[9] Cynthia Brokaw and Peter Kornicki, "Introduction", in *The History of the Book in East Asia*, ed. Cynthia Brokaw and Peter Kornicki, The History of the Book in the East series, ed. Peter Kornicki (Farnham, Surrey: Ashgate, 2013), xvii.

[10] Xiao Dongfa 肖東發 and Yang Hu 楊虎, *Chatu ben Zhongguo tushu shi* 插圖本中國圖書史 (Illustrated history of the book in China) (Guilin: Guangxi Normal University Press, 2005), 12 and 14.

works in China include Buddhist sutras, almanacs, elementary books to teach writing, and divination texts.[11] These texts might not have ever made it into the Tang imperial collection, which was largely inherited from the Sui dynasty, and is believed to have consisted of only manuscripts.[12] In other words, the copies of the *Ten Mathematical Classics* that Li Chunfeng and his team worked with in 656 must still have been in manuscript. After their commentary had been added, it must have been necessary to have scribes then produce enough copes for use by the erudites, teaching assistants, and students of the Schools of Mathematics in the imperial academies in the two Tang capitals.

Elsewhere in China around the same time, other mathematical works are known to have circulated. Among the 21 183 Chinese texts dating from the early fifth to the early eleventh century discovered in 1900 in a previously sealed temple cave at Dunhuang,[13] a very small number, all of which have only survived in fragments, have been positively identified as having mathematical contents. Because the Dunhuang collection is currently scattered around the world, however, it is difficult to determine an exact number.[14] While none corresponds exactly to the *Ten Mathematical Classics* as they are known today, there are at least three manuscript fragments dating from the Tang dynasty whose contents closely resemble the first section of the *Sunzi suanjing*.[15] Two of these fragments also contain area calculation problems, some of which were solved using the erroneous methods given in the

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11 Xiao and Yang, *Chatu ben Zhongguo tushu shi*, 12.

12 Brokaw and Kornicki, "Introduction", xxi.

13 Ibid.

14 Wang Jinyu 王進玉, "Dunhuang yishu zhong de shuxue shiliao ji qi yanjiu 敦煌遺書中的數學史料及其研究" (Mathematical sources in the Dunhuang collection and their research), in Shuxue shi yanjiu wenji 數學史研究文集 (Research papers on the history of mathematics) 2 (1991): 58-59. Only a handful of the mathematical sources have been or are in the process of being digitized. See International Dunhuang Project <http://idp.bl.uk>.

15 Wang Jinyu, "Dunhuang yishu", 59.

Wucao suanjing.[16] From the above, we can infer that some version of the *Sunzi* and the *Wucao* might have circulated in the Dunhuang area during the Tang dynasty. Whether or not these could have been the official versions compiled by Li Chunfeng and his team is a question that awaits the availability of more sources.

FIRST IMPRINTS OF THE *TEN MATHEMATICAL CLASSICS*

The Chinese state started adopting woodblock printing technology only during the tenth century, a few hundred years after the emergence of commercial and religious printers in parts of China. During the Five Dynasties and Ten Kingdoms period (906 – 960), four out of the five dynasties that ruled northern China in succession sustained a continuous effort to print a standardized version of the Confucian classics. The project was proposed and supervised by a small group of officials led by Feng Dao 馮道 (882 – 954), who served all four dynasties as chancellor, which probably played a crucial role in protecting the project from being abandoned during the abrupt changes of government. The compilation of the standardized version, carving of the woodblocks, and eventual printing of the classics took over 21 years to complete.[17]

In this example as in subsequent printing projects undertaken by the Chinese government, as well as those of private and commercial printers, it is extremely difficult for historians to determine the size of the print runs. The problems stems from the fact that the available primary sources tend not to specify the actual number of copies printed. Furthermore, the nature of woodblock printing technology makes even educated guesses of actual print runs difficult. Whereas in the history of the book in Europe, one can generally safely assume that any title printed by commercial

[16] Wang Jinyu, “Dunhuang yishu”, 61.
[17] Xiao and Yang, *Chatu ben Zhongguo tushu shi*, 120-121.

printers using movable type technology must have run into at least several hundred copies in order to make it possible for the printer to earn a profit from the run, the same assumptions cannot be applied to China.[18] Even though movable type printing was invented in China during the 1040's by a commoner named Bi Sheng 畢昇 (990 – 1051), it never came into widespread use to challenge woodblock printing as the dominant printing technology. A few of the advantages woodblock printing had over movable type were that it required comparatively little investment by the printer in tools and equipment, and that printers ran a lower risk of over-stretching their resources and bankrupting themselves by investing a substantial amount of capital in large print runs of titles that would never find sufficient demand to turn a profit. With woodblock printing, publishers could safely print a small number of copies initially to test consumers' reception of new works, and re-use the same woodblocks to print more if necessary.[19] Therefore, even though a set of relatively well-made woodblocks could generally be used to print 15 000 to 30 000 clear copies before the wood or the carvings deteriorated,[20] it cannot be assumed that all woodblocks were indeed used to the end of their useful lifespan, thus making it difficult for historians to determine how many copies of any particular work were made.

This difficulty is relevant to our study of the *Ten Mathematical Classics* because even though we know that the central government of the Northern Song (960 – 1127) made imprints of the *Ten Matheatical Classics* in 1084, it is impossible to ascertain how many copies were made at the time. What is certain is that the

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[18] Ann Blair, "Afterword: Rethinking Western Printing with Chinese Comparisons", in *Knowledge and Text Production in an Age of Print: China, 900 – 1400*, ed. Lucille Chia and Hilde de Weerdt, Sinica Leidensia series, vol. 100, ed. Barend J. ter Haar and Maghiel van Crevel (Leiden: Brill, 2011), 351-352.

[19] Ibid, 351.

[20] Joseph P. McDermott, *A Social History of the Chinese Book: Books and Literati Culture in Late Imperial China*, Understanding China: New Viewpoints on History and Culture series (Hong Kong: Hong Kong University Press, 2006), 21.

printing of the *Ten Mathematical Classics* was only one of the many printing projects undertaken by the Northern Song government, and can be understood as part of the government's ongoing efforts to preserve existing texts and to produce standardized editions of these texts.

Throughout history, the central government usually, if not always, owned the largest collection of books in the empire. The main Tang imperial library had around 89 000 *juan* 卷, or chapters, in the year 731.[21] Various government offices and the imperial academies would also have held separate book collections, though probably with many duplicate copies, making the total number of books owned by the central government even larger. This was at a time when one of the largest Buddhist monastery book collections held around 10 000 *juan*, while private libraries might only have at most several thousand *juan* by the tenth century.[22] The dissolution of Tang central authority from the eighth century onwards, resulting in the violence and political upheaval that lasted through the Five Dynasties, meant that much of the Tang imperial collection became destroyed amidst the turmoil.

At the beginning of the Northern Song, the main imperial library collection had inherited a mere 13 000 *juan* from its predecessors.[23] A powerful desire to replenish the imperial library with what had been lost prompted the Northern Song government to decree the transfer of sought-after books from local government offices and to entice private collectors with money and examination degrees to hand over such books. These strategies eventually allowed the Northern Song government to

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[21] McDermott, *Social History of the Chinese Book*, 50.

[22] Joseph P. McDermott, "Book Collecting in Jiangxi during the Song Dynasty", in *Knowledge and Text Production in an Age of Print: China, 900 – 1400*, ed. Lucille Chia and Hilde de Weerdt, Sinica Leidensia series, vol. 100, ed. Barend J. ter Haar and Maghiel van Crevel (Leiden: Brill, 2011), 66 & 69.

[23] Ronald Egan, "To Count Grains of Sand on the Ocean Floor: Changing Perceptions of Books and Learning in the Song Dynasty", in *Knowledge and Text Production in an Age of Print: China, 900 – 1400*, ed. Lucille Chia and Hilde de Weerdt, Sinica Leidensia series, vol. 100, ed. Barend J. ter Haar and Maghiel van Crevel (Leiden: Brill, 2011), 43.

rebuild its main library collection to 73 877 *juan* by 1127 before much of it was taken or destroyed during the Jurchen invasion.[24]

Among the 73 877 *juan* must have been printed copies of the *Ten Mathematical Classics* that the Palace Library (*bishu sheng* 祕書省) made in 1084. Even though the Northern Song government had embraced printing technology very early on and used it to print a wide variety of texts, the main library collection and the imperial archives were still dominated by manuscripts, with only 8.5% of their contents as imprints in the year 1177.[25] Besides the Palace Library, various government offices, including the imperial academy and local government schools, also operated their own printing presses.[26] Between them, they printed a wide variety of works: Confucian classics, histories, literary and religious works, medical texts, among others.[27] The *Ten Mathematical Classics* appear to have been the only mathematical works printed by all levels of government throughout the entire Song dynasty. While it is known that copies from the 1084 print run survived into the early thirteenth century, none are extant today. This suggests that the print run might have been too small to generate a great proliferation of the *Ten Mathematical Classics*. As noted earlier in this chapter, however, it is impossible to ascertain the exact number of copies printed at the time.

After the Southern Song court had established itself in Lin'an, its imperial library holdings had been greatly diminished. Efforts to rebuild the collection again

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[24] Egan, "To Count Grains of Sand", 43.

[25] McDermott, *Social History of the Chinese Book*, 54.

[26] Yang et al, *Liang Song wenhua shi*, 421.

[27] TJ Hinrichs, "Governance Through Medical Texts and the Role of Print", in *Knowledge and Text Production in an Age of Print: China, 900 – 1400,* ed. Lucille Chia and Hilde de Weerdt, Sinica Leidensia series, vol. 100, ed. Barend J. ter Haar and Maghiel van Crevel (Leiden: Brill, 2011), 218.

brought the numbers of holdings to only 59 429 *juan* by 1220.[28] It was within such a context that the earliest extant imprints of the *Ten Mathematical Classics* came to be made by Bao Huanzhi, prefect of Tingzhou. Whereas during the Northern Song, state-financed publishing had been concentrated within the capital, the Southern Song central government no longer had the resources to continue it on the same scale as before. Therefore, local levels of government came to dominate the state-run sector of the publishing industry.[29] State-run prefectural and county schools often had their own printing presses, which could be financed by the rents collected from their lands. A wide range of local government offices also had their own printing presses, including prefectural offices, offices for regulating tea and salt (*yan cha si* 鹽茶司), fiscal offices (*zhuan yun si* 轉運司), and judicial offices (*ti xing si* 提刑司). These offices had special allocations within their administrative budgets that could be used for printing books and documents.[30] This practice was to help ensure that local government offices had the means to print the books and documents necessary for their own use, for distributing to the local population, and for selling to raise revenue.[31] Since at least the early years of the Northern Song, local officials increasingly used administrative funds to print books that they would then present to other officials as personal gifts in exchange for some immediate or distant favour.[32]

Which of these motivations prompted Bao Huanzhi to print the *Ten Mathematical Classics* in the year 1213 is unclear at this time. Unlike the printing of 1084, which was directed and accomplished by the central government, the 1213 printing was definitely a local effort, which tended to be more limited in quantity and

[28] McDermott, *Social History of the Chinese Book*, 50.
[29] Yang et al, *Liang Song wenhua shi*, 421.
[30] Ibid, 422.
[31] Egan, "To Count Grains of Sand", 36.
[32] McDermott, *Social History of the Chinese Book*, 90.

circulation.[33] At the time, Bao Huanzhi had only been recently appointed to the position of Tingzhou's prefect. Tingzhou was a military prefecture in Fujian, a southeastern coastal province. His assignment there also included taking charge of local agricultural affairs (*nong shi* 農事) and mining (*keng ye* 坑冶). Prior to this appointment, Bao had served within the central government in Lin'an as a case reviewer for the Court of Judicial Affairs (*dali ping shi* 大理評事). Bao was, however, also knowledgeable in the calendrical sciences, and contributed to the promulgation of the new Kaixi 開禧 calendar system in 1208.[34]

From the above, we can speculate two possible reasons for Bao Huanzhi to feel the need to organize a re-print of the *Ten Mathematical Classics*: he might have thought these texts useful for his new duties at Tingzhou,[35] or his personal interests in mathematics, which can be deduced from his engagement in the calendrical sciences, might have compelled him to wish for the preservation of the *Ten Mathematical Classics*.

By all accounts, by the Southern Song, copies of the *Ten Mathematical Classics*, either in manuscript or printed form, had become extremely rare. In the appendix Bao Huanzhi added to the *Shushu jiyi* before it went into print, he noted that the *Ten Mathematical Classics* had been used as textbooks in the Tang School of Mathematics, but the *Zhuishu* and the *Sandeng shu* had both been lost before his time.[36] From this, we can infer that he was able to locate all the remaining ten works

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33 Egan, "To Count Grains of Sand", 36.

34 "Chuban shuoming 出版說明" (Publishing notes), *Song ke suanjing liu zhong, fu yi zhong* 宋刻算經六種，附一種 (Facsimile of six mathematical classics printed during the Song dynasty, with another text appended) (Beijing: Wenwu chubanshe, 1980), 1-2.

35 Ibid, 2.

36 Appendix in *Shushu jiyi*, in *Song ke suanjing liu zhong, fu yi zhong* 宋刻算經六種, 附一種 (Facsimile of six mathematical classics printed during the Song dynasty, with another text appended) (Beijing: Wenwu chubanshe, 1980), no page number.

that had been used at the Tang School of Mathematics. However, only eight of Bao's re-printed titles survived into the Qing dynasty:

- *Zhoubi suanjing*
- *Jiuzhang suanshu* – only the first five chapters
- *Shushu jiyi*
- *Sunzi suanjing*
- *Wucao suanjing*
- *Xiahou Yang suanjing*
- *Zhang Qiujian suanjing*
- *Jigu suanjing*

Figure 5.4 Bao Huanzhi's re-print edition of the *Zhoubi suanjing*

Moreover, Qing scholars were able to locate only a single Southern Song copy of each of these titles.[37] The whereabouts of the copies of the *Xiahou Yang suanjing* and the *Jigu suanjing*, however, are no longer known. The remaining six are currently stored at the Shanghai Library (Shanghai tushuguan 上海圖書館) and the Peking University Library. Facsimiles were made of all six in 1980 by the publisher Wenwu chubanshe 文物出版社. These six treatises are among the approximately 1500 extant titles from the Song dynasty.[38]

37 "Chuban shuoming", *Song ke suanjing liu zhong*, 2.

38 Lucille Chia, "The Uses of Print in Early Quanzhen Daoist Texts", in *Knowledge and Text Production in an Age of Print: China, 900 – 1400*, ed. Lucille Chia and Hilde de

Figure 5.5 The facsimile copy of six of the Southern Song edition of the *Ten Mathematical Classics*

Traditionally, scholars have believed that, except for the *Shushu jiyi*, the copies of the *Ten Mathematical Classics* that Bao Huanzhi used to produce his re-prints were all from the 1084 edition printed by the Northern Song Palace Library. Looking at the 1980 facsimiles, it is certainly true that the *Zhoubi*, *Sunzi*, and *Wucao* all bear a note on their last two pages indicating that these texts had been prepared in the seventh year of the Yuanfeng 元豐 reign period (1084) by the following officials of the Palace Library:

Weerdt, Sinica Leidensia series, vol. 100, ed. Barend J. ter Haar and Maghiel van Crevel (Leiden: Brill, 2011), 168.

- Zhao Yanruo 趙彥若, director (*bishu jian* 祕書監)
- Sun Jue 孫覺, vice-director (*bishu shao jian* 祕書少監)
- Han Zonggu 韓宗古, assistant director (*bishu cheng* 祕書丞)
- Ye Zuqia 葉祖洽, editor (*jiao shu lang* 校書郎)
- Wang Zhongxiu 王仲脩, editor
- Qian Changqing 錢長卿, editor

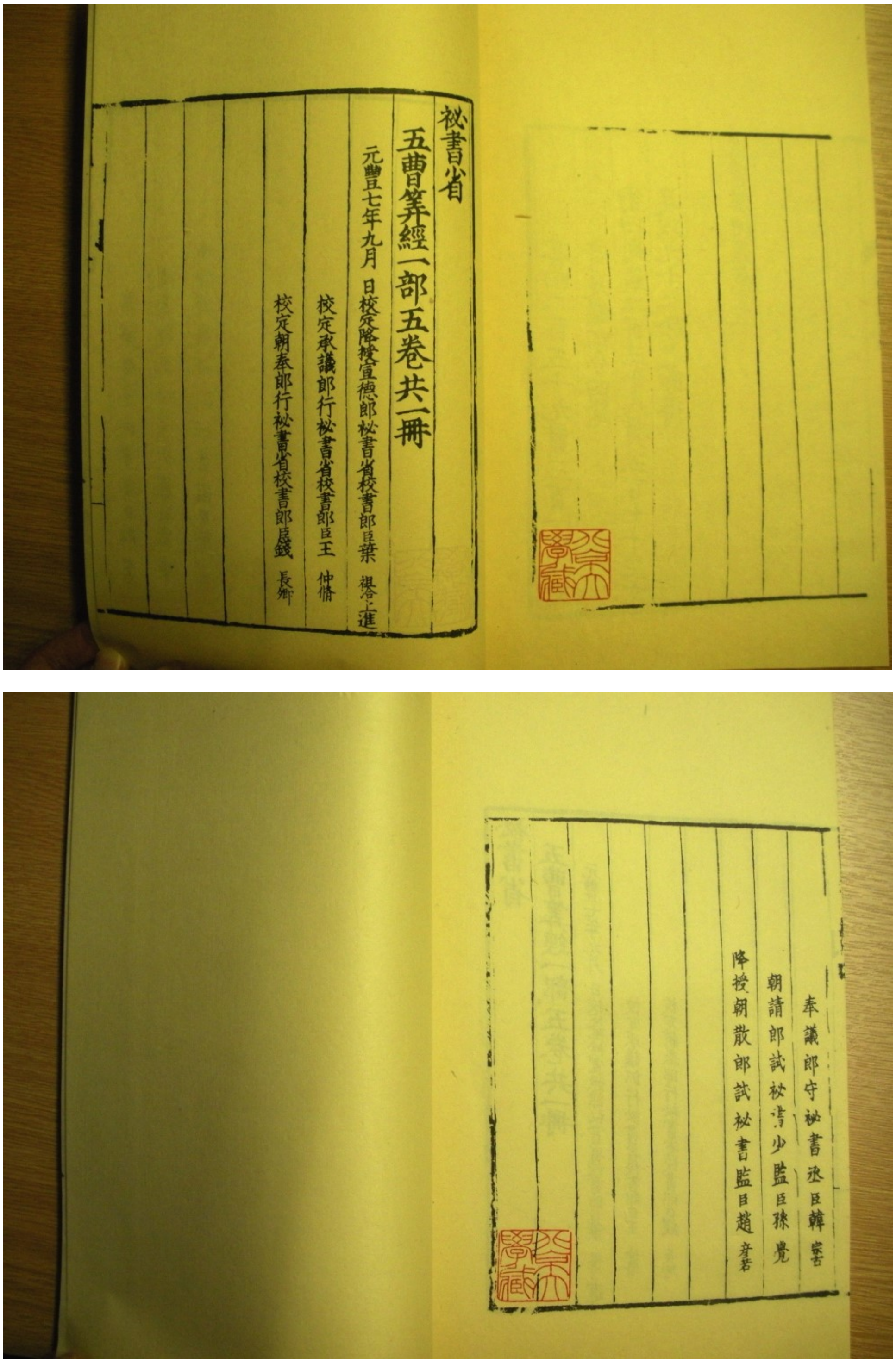


Figure 5.6 The last two pages of Bao Huanzhi’s re-print edition of the *Wucao suanjing*, with the names of the Palace Library officials who prepared the 1084 edition

Therefore, it is clear that Bao Huanzhi did indeed find copies of the 1084 Palace Library edition of these three treatises, and then had them copied onto woodblocks to be carved and printed, along with the original statements of provenance. For the *Jiuzhang suanshu* and the *Zhang Qiujian suanjing*, however, things are not so clear. Bao's edition of the *Jiuzhang suanshu* probably had all nine chapters, or he would have noted otherwise. The only known extant copy, unfortunately, is missing the last four chapters and does not contain the same note regarding the Palace Library as the three treatises discussed above. Bao's postface to the *Jiuzhang suanshu* indicates that he had found a copy of this text in a colleague's library, and that the Palace Library edition had been based on this same copy,[39] suggesting that the Tingzhou edition and the Palace Library edition were likely quite similar.

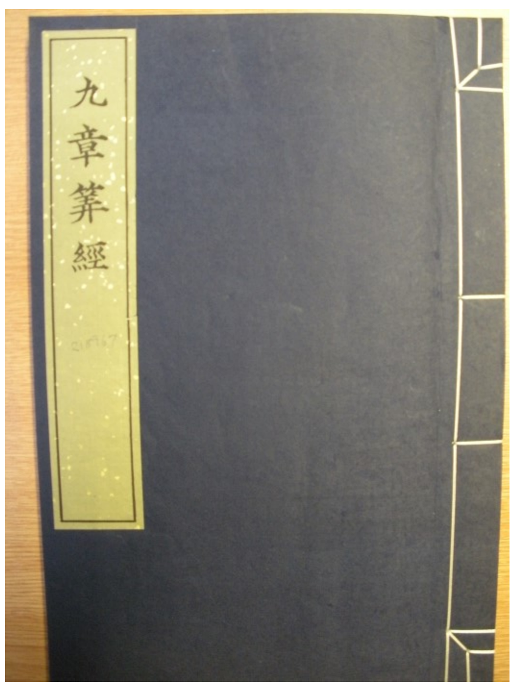


Figure 5.7 Bao Huanzhi's re-print edition of the *Jiuzhang suanshu*

[39] *Jiuzhang suanshu*, 189.

Figure 5.8 Bao Huanzhi’s re-print edition of the *Zhang Qiujian suanjing*

The *Zhang Qiujian suanjing* does not contain any note either about having been collated by Northern Song Palace Library officials. Therefore, the possibility exists that the copy of the *Zhang Qiujian* that Bao Huanzhi was able to find might not have been the 1084 Palace Library edition at all, but was from other editions circulating in southern China. All the five classics discussed above have a statement at the beginning of every chapter indicating that commentaries had been added by Li Chunfeng and his team, suggesting that their history can all be traced to the 656 imperial academy edition produced by Li Chunfeng. The possibility that there might be editions of the *Ten Mathematical Classics* that originate from the Tang imperial collection but not the Northern Song collection stems partly from the common and longstanding practice among officials, especially those working in departments with large book collections like the Palace Library, of copying, borrowing, or outright pilfering books in the government collections.[40]

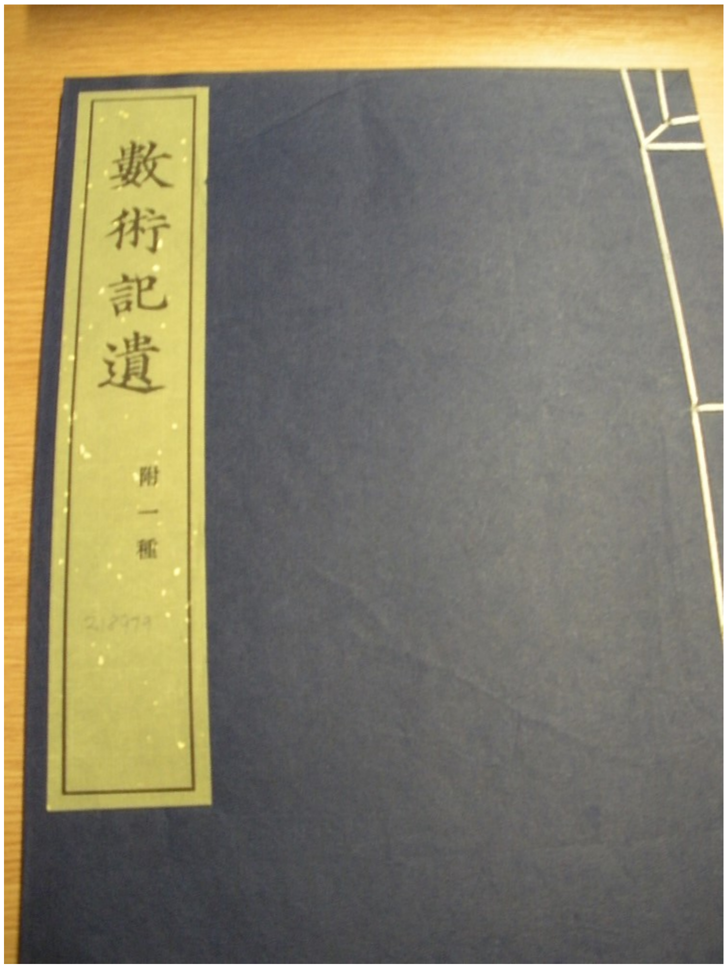


Figure 5.9 Bao Huanzhi's re-print edition of the *Shushu jiyi*

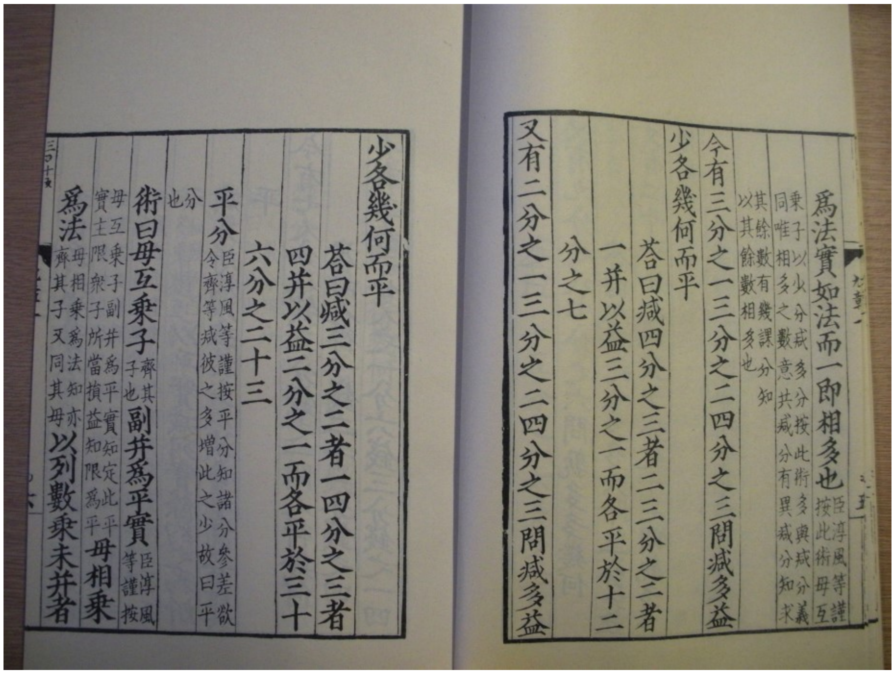
[40] Hilde de Weerdt, "Byways in the Imperial Chinese Information Order: The Dissemination and Commercial Publication of State Documents", in *The History of the Book in East Asia*, ed. Cynthia Brokaw and Peter Kornicki, The History of the Book in the East series, ed. Peter Kornicki (Farnham, Surrey: Ashgate, 2013), 90 & 115; McDermott, "Book Collecting in Jiangxi", 74-76.

The extant 1213 edition of the *Shushu jiyi*, on the other hand, does not contain any reference to Li Chunfeng or the Palace Library. Bao Huanzhi wrote in his appendix that he had not been able to locate a copy of the *Shushu jiyi* anywhere until he chanced upon a manuscript copy in the collection of a Daoist temple.[41] Who had copied it in the first place or where the copy came from is unfortunately left unexplained, but this is further evidence that the circulation of the *Ten Mathematical Classics* might have been much wider than the historical record suggests.

PRINTING OF THE 1213 TINGZHOU EDITION

Judging from the extant copies of the 1213 Tingzhou edition of the *Ten Mathematical Classics*, all the treatises were carved using the same format. Each page had nine lines, and each line could fit eighteen characters of the main text. The text of the solution that follows a problem is indented. There are also explanatory notes that are carved in smaller characters.

[41] Appendix in *Shushu jiyi*, in *Song ke suanjing liu zhong*, no page number.



爲法實如法而一即相多也臣淳風等謹按此術母互
乘子以少分減多分按此術多與減分義同唯相多之數意共減分有異減分知求
其餘數有幾課分知以其餘數相多也
今有三分之一三分之二四分之三問減多益
少各幾何而平
荅曰減四分之三者二三分之二者
一并以益三分之一而各平於十二
分之七
又有二分之一三分之二四分之三問減多益

少各幾何而平
荅曰減三分之二者一四分之三者
四并以益二分之一而各平於三十
六分之二十三
平分臣淳風等謹按平分知諸分參差欲令齊等減彼之多增此之少故曰平
分也
術曰母互乘子齊其子也副并爲平實臣淳風等謹按
母互乘子副并爲平實知定此平實主限衆子所當損益知限爲平母相乘
爲法母相乘爲法知亦齊其子又同其母以列數乘未并者

Figure 5.10 Two pages from the first chapter of the *Jiuzhang suanshu* (1213 Tingzhou edition)

Even though Tingzhou was very close to Jianyang 建陽, one of the major commercial publishing centres of Southern Song, which was famous for producing cheap imprints,[42] Bao Huanzhi chose to print the *Ten Mathematical Classics* using the printing press of the prefectural office. Therefore, it is likely that the printing was funded by the administrative budget of the prefecture.

The woodblock printing process required scribes to write the text and draw the illustrations on the woodblocks, carvers to carve the characters and illustrations into relief, printers to print the carved woodblocks onto paper, and binders to bind the pages together to form a book, which was usually done with glue during the Song

[42] For a detailed study of Jianyang's publishing industry, see Lucille Chia, *Printing for Profit: The Commercial Publishers of Jianyang, Fujian (11th – 17th Centuries)* (Cambridge, MA: Harvard University Asia Centre, 2002).

dynasty. The process had become very specialized by the Song, so it was rare for a single worker to participate in more than one stage of the process.[43] For government-financed print runs, it was common for these workers to be hired temporarily on commission. Once the project was completed, the workers would then disperse and move onto their next commission, or many would return to their primary occupation as farmers.[44] Therefore, the carvers hired for government projects often had their names and the number of characters they had carved on each page carved into each block so that this information would appear on the inside edge of each page to facilitate the calculating of wages.[45] As a result, we know the names of some 3000 carvers of the Song dynasty, but mainly of the Southern Song period. Scribes, printers, and binders, on the other hand, most likely had some other method of tallying their work, so only a few dozen of their names have survived in the sources.[46]

The Tingzhou edition of the *Ten Mathematical Classics* contains the names of the carvers who had worked on them. A few of the names only have a single character, suggesting that these carvers were only using their family name, given name, or nickname to differentiate themselves from the others.

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[43] Zhang Xiumin, *Zhongguo yinshua shi*, 655-656 & 709.
[44] Ibid, 657; Brokaw and Kornicki, "Introduction", xxii-xxiii.
[45] Zhang Xiumin, *Zhongguo yinshua shi*, 656.
[46] Ibid, 657-661.

Table 5.1 Names of the Carvers Who Worked on the Tingzhou Edition of the *Ten Mathematical Classics*[47]

Treatise	Carvers
Zhoubi	He Quan 何全, **Fu Wen 傅汶**, **Wu Xian 吳顯**, Cai Zheng 蔡政, Cai Wen 蔡文, **Ye Ding 葉定**，Ye Cai 葉才, Ye Quan 葉全, Kui Fu 媿甫, Kui Cai 媿才，**Chen Wen 陳文**
Jiuzhang	**Wei Xin 魏信**, You Min 游旻, Xu Ding 徐定, Xu Cheng 徐成, Xu Zicheng 徐子成，Yu Fu 余夫，Quan 全，Yu 俞
Shushu jiyi	Weng Sui 翁遂
Sunzi	Ding Yong 丁用，Fu Zhang 傅璋，Chen Gui 陳圭
Wucao	**Wu Xian 吳顯**
Xiahou Yang	He Zheng 何正，**Wei Xin 魏信**，**Wu Xian 吳顯**，Xiao Zi 蕭子，**Ye Ding 葉定**, Yu Zuo 俞左
Zhang Qiujian	**Fu Wen 傅汶**，Kui Yuan 媿元，Kui Mao 媿茂，Kui Zhong 媿中，Yu Zhongcheng 俞仲成，Zheng 正
Jigu	Wang Ding 王定，**Chen Wen 陳文**

From the table, we see that several of the carvers (names in bold) worked on two, or in one case, even three classics. Also, the surnames of Ye and Kui seem especially common among this group of carvers, which supports historians'

47 "Chuban shuoming", *Song ke suanjing liu zhong*, 4; Wang Zhaowen 王肇文, *Guji Song Yuan kangong xingming suoyin* 古籍宋元刊工姓名索引 (Index of the carvers of Song-Yuan imprints) (Shanghai: Shanghai guji chubanshe, 1990), 357-358.

observation that woodblock carving seemed to be a skill that was commonly passed down within families.[48]

While we do not have data on how much the carvers of the *Ten Mathematical Classics* were paid, information about several other printing projects undertaken in the thirteenth century suggests that carvers' wages varied quite widely. Between 1233 and 1241, wages ranged from about four coins (*wen* 文) to 16 coins per character, but a sharp increase appear to occur shortly after, raising to range to 30 to 65 coins per character between 1243 and 1253.[49] If we assume that carvers' wages during the thirteenth century maintained an upward trend, then the wages of the carvers of the *Ten Mathematical Classics* in 1213 must have been very low, perhaps less than ten coins per character.

THE *TEN MATHEMATICAL CLASSICS* AFTER THE SONG DYNASTY

The circulation of Bao Huanzhi's Tingzhou edition of the *Ten Mathematical Classics* is difficult to trace. Known copies that have survived into the eighteenth century were all in the hands of private collectors, so it is likely that the Tingzhou edition never entered the Palace Library collection.

No notable effort appeared to have been made by any government official or private individual to collate or print the *Ten Mathematical Classics* during the Yuan dynasty (1279 – 1368). Early in the Ming dynasty (1368 – 1644), however, the *Yongle dadian* 永樂大典 (Great encyclopedia of the Yongle reign period) was compiled under the auspices of the Yongle emperor (r. 1402 – 1425) to contain all

48 Luo Shubao 羅樹寶, *Zhongguo gudai yinshua shi* 中國古代印刷史 (The history of publishing in pre-modern China) (Beijing: Yinshua gongye chubanshe, 1993), 178.

49 Ibid, 181.

existing books. Only 370 of the original 11 095 volumes are currently extant,[50] the rest being destroyed during the violent transition between the Ming and the Qing dynasty.[51] Chapters 16 329 to 16 365 of the Yongle dadian were dedicated to mathematical and astronomical works, but only Chapters 16 343 and 16344 have survived to the present.[52]

The text of at least seven of the *Ten Mathematical Classics* were copied into the Yongle dadian.[53] The *Shushu jiyi*, *Zhang Qiujian suanjing*, and *Jigu suanjing* appear to have been left out probably because copies of these could not be located in time before the project came to a close in 1408. Scholars who have compared the *Yongle dadian* version of the *Ten Mathematical Classics* with the Tingzhou edition agree that while both versions are very similar for most of the classics, the copy of the *Jiuzhang suanshu* used by the compilers of the *Yongle dadian* seems to be different from the one used by Bao Huanzhi to produce the Tingzhou edition.[54] This is further evidence that the Tingzhou edition probably never achieved widespread circulation. In general, even though government officials and offices frequently copied or printed works either for the sake of standardization or of preservation, that motivation did not necessarily translate into a wish for widespread dissemination of the works in question. Taking the *Yongle dadian* as an example, this monumental encyclopedia was clearly compiled with the aim of preserving all existing works. Yet only two manuscript copies were ever made of it, and both were kept within the

[50] Libbrecht, *Chinese Mathematics in the Thirteenth Century*, 38.
[51] McDermott, *Social History of the Chinese Book*, 128.
[52] Libbrecht, *Chinese Mathematics in the Thirteenth Century*, 38.
[53] Guo and Liu, "Ben shu shuoming 本書說明", *Suanjing shi shu*, no page number.
[54] Guo Shuchun, "Jiuzhang suanshu tiyao 九章算術提要" (Synopsis of the Jiuzhang suanshu), *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 (Chinese scientific and technological texts), ed. Guo Shuchun (Zhengzhou: Henan jiaoyu chubanshe, 1993), 1:86.

imperial library and archives collection, where even government officials had very limited access to them.[55]

Similar efforts were undertaken during the Qing dynasty to produce complete collections of existing works. The largest of these projects was the *Siku quanshu* 四庫全書 (Complete Collection of the Four Treasuries) (1773 – 1782), which took over 360 scholars and 3826 scribes ten years to complete.[56] Only seven manuscript copies were eventually made of the *Siku quanshu* and stored in libraries throughout the country. When the compilation process for the *Siku quanshu* was still in its early stages, Dai Zhen, the scholar responsible for putting together the mathematical works, was only able to gather seven of the *Ten Mathematical Classics*, which he did by copying them out himself from the relevant sections of the *Yongle dadian*. The *Ten Mathematical Classics* had evidently become very rare books by the eighteenth century. In 1774, these seven were printed by the Qing government with movable type as part of the *Wuying dian ju zhen ban congshu* 武英殿聚珍版叢書 (Collectanea of the Wuying Court):

- *Zhoubi suanjing*
- *Jiuzhang suanshu*
- *Haidao suanjing*
- *Sunzi suanjing*
- *Wucao suanjing*
- *Xiahou Yang suanjing*
- *Wujing suanshu*

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55 McDermott, *Social History of the Chinese Book*, 127-128.

56 Brokaw and Kornicki, “Introduction”, xxvii.

Shortly after, Bao Huanzhi's Tingzhou edition of the *Ten Mathematical Classics* resurfaced in the historical record. A single copy of each of eight of the treatises was found to have meandered its way through the hands of private collectors. By the eighteenth century, they were in the possession of Mao Yi 毛扆, who then hired someone to produce a manuscript facsimile of these books.[57] This facsimile copy later became part of the imperial collection, and all ten of the surviving mathematics textbooks of the Tang dynasty came to be preserved within the *Siku quanshu*.

From the above, we come to the following conclusion: while government offices made the most contributions to making new copies of the *Ten Mathematical Classics*, private book collectors played a very crucial role in preserving copies, thus ensuring the survival of texts like the *Ten Mathematical Classics* that probably did not enjoy wide appeal.

Commercial printers, on the other hand, might not have made any significant contribution to the production and dissemination of the *Ten Mathematical Classics*. Based on research by Lucille Chia on commercial publishing in Jianyang, the most prolific printing centre in China from the Song to the end of the Ming dynasty,[58] commercial publishers there did not seem to have printed any mathematical or astronomical works during the Song and the Yuan dynasty. For the Ming, Chia counted 13 mathematical or astronomical works printed by the Jianyang commercial publishers, which accounted for only 0.8% of all the works they published during the

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57 Qian Baocong, "Zhoubi suanjing tiyao 周髀算經提要" (Synopsis of the Zhoubi suanjing), *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 (Chinese scientific and technological texts), ed. Guo Shuchun (Zhengzhou: Henan jiaoyu chubanshe, 1993), 1:3.

58 Lucille Chia, "Mashaben: Commercial Publishing in Jianyang from the Song to the Ming", in *The History of the Book in East Asia*, ed. Cynthia Brokaw and Peter Kornicki, The History of the Book in the East series, ed. Peter Kornicki (Farnham, Surrey: Ashgate, 2013), 118.

period.[59] If such numbers are representative of the overall commercial publishing across China, then few mathematical works ever had the chance to become bestsellers on the commercial book market.

TRANSMISSION TO KOREA, JAPAN, AND VIETNAM

The four countries of East Asia have had longstanding relations that encompass cultural transmission and trade throughout most of recorded history. The region shared common political systems, philosophies, schools of Buddhism, and writing systems, most of which were transmitted from China to the other countries. Many Chinese institutions and practices, such as its law codes and units of measurement, were held to be the model for the region. The spread of the *Ten Mathematical Classics* from China to Korea, Japan, and probably Vietnam is one component of this cultural transmission. Once there, the circulation of the *Ten Mathematical Classics* also became very much associated with court circles, government schools, and the selection of officials through examinations, which will be discussed below. The treatment of these three countries, however, can only be uneven because less research on Vietnam is available either in English or Chinese compared to the other two countries.

The transmission of the *Ten Mathematical Classics* to the Korean peninsula began very early. The Western Han set up four colonies in the Korean peninsula in 108 BCE, the last of which was taken over in 313 CE by the emerging Korean kingdom of Koguryo. During the four centuries of Chinese rule, the *Zhoubi suanjing*, *Jiuzhang suanshu*, Liu Hui's commentary of the *Jiuzhang*, and the *Haidao suanjing* made their way to the Korean peninsula. Officials working on astronomy and on administrative affairs in the Korean kingdoms of Koguryo and Paekche used these

[59] Chia, "Mashaben", 124.

texts to train themselves to fulfil their duties.[60] The speed of transmission appeared to have been very fast. Liu Hui's commentary and the *Haidao* were written in 263, and had reached the Korean kingdoms by around 273. Also, the *Zhuishu*, which was written sometime in the fifth century, had been transmitted to the Koreans by the beginning of the sixth.[61]

After the Korean peninsula was unified by the kingdom of Silla in 668, an education system modelled on Tang China's imperial academy was established in 682, including a School of Mathematics.[62] Unlike the Tang, however, Silla's School of Mathematics only had four official textbooks: *Jiuzhang suanshu* (Kujang in Korean), *Zhuishu* (Cholsul in Korean), and two other treatises, the *Yukchang* 六章 (Six chapters) and the *Samgae* 三開 (Three extractions), that probably originated within Korea itself.[63]

The students of the Korean School of Mathematics ranged in age from 19 to 30, and included commoners. The normal time limit for completion was nine years. Successful completion of the programme earned the graduates bureaucratic positions.[64]

Even though the rest of the *Ten Mathematical Classics* never became designated as official textbooks, that is not to say that they never reached Korea. As previously discussed in Chapter Three, Silla regularly sent students to study at the

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60 Jin Hujun 金虎俊, "Jiuzhang suanshu, Zhuishu he Chaoxian bandao gudai shuxue jiaoyu《九章算術》、《綴術》和朝鮮半島古代數學教育" (The Jiuzhang suanshu, Zhuishu, and pre-modern mathematical education on the Korean peninsula), *Shuxue shi yanjiu wenji* 數學史研究文集 (Research papers on the history of mathematics) 4 (1993): 64.

61 Ibid.

62 Kim Yong Woon, "Pan-Paradigm and Korean Mathematics in the Choson Dynasty", *Korea Journal* (March 1986): 28.

63 Ibid, 65; Na Risu 那日蘇, "Zhongguo chuantong shuxue dui Riben hesuan de yingxiang 中國傳統數學對日本和算的影響" (The influence of traditional Chinese mathematics on Japanese mathematics), *Shuxue shi yanjiu wenji* 數學史研究文集 (Research papers on the history of mathematics) 3 (1992): 17.

64 Kim, "Pan-Paradigm and Korean Mathematics", 28-29.

Tang imperial academy. Along with the students went official envoys and Buddhist monks, all of whom were given money by Silla's central government to be used for acquiring books in China that would then be brought back to Korea.[65] Therefore, it is certainly conceivable that the other mathematical classics, together with Li Chunfeng's commentaries, might also have been brought over.

A similar mathematics curriculum and a civil service examination in mathematics were maintained under the Koryo dynasty (918 – 1392).[66] The Koryo dynasty shared Silla's preference for a narrower mathematics curriculum compared to the Chinese. It also used only four mathematics textbooks, but had replaced the *Yukchang* with another text known as the *Saga* 謝家.[67]

Under the Choson dynasty (1392 – 1910), state-run mathematical education and mathematics civil service examinations continued largely unabated.[68] However, the *Jiuzhang suanshu* had become lost in Korea by the middle of the seventeenth century, and was only re-introduced into Korea in the mid-nineteenth century from China.[69] A commentary on the *Jiuzhang* was subsequently written in 1864 by Nam Pyong-Gil 南秉吉 (1820 – 1869), one of the best-known mathematical writers of the late Choson period.

Nam Pyong-Gil belonged to the ruling *yangban* 兩班 elite. He held a variety of high-ranking positions within the Choson government, including supervisor of the National Observatory, deputy minister of personnel, and minister of punishments.[70]

65 Yang et al, *Liang Song wenhua shi*, 436.

66 Jin Hujun, "Jiuzhang suanshu, Zhuishu he Chaoxian bandao gudai shuxue jiaoyu", 65.

67 Kim, "Pan-Paradigm and Korean Mathematics", 29-30.

68 Ibid, 30-31 and 34.

69 Jia-Ming Ying, "The Kujang sulhae: Nam Pyong-Gil's Reinterpretation of the Mathematical Methods of the Jiuzhang suanshu", *Historia Mathematica* (2010): 3-4.

70 Ying, "The Kujang sulhae", 8.

In the *Kujang sulhae*, Nam deliberately left out Liu Hui's and Li Chunfeng's commentaries on the *Jiuzhang*, and replaced them with his own because he felt the previous commentaries were not sufficiently illuminating to help people understand the original text.[71]

In the history of cultural transmission in East Asia, Korea often acted as a conduit that transferred knowledge and texts from China to Japan. For mathematics, interactions between the Korean kingdoms and Japan since at least the fourth century brought Chinese mathematical texts from Korea to Japan.[72] From the seventh century onwards, the transmission became much more direct as Japan also regularly sent envoys, students, and monks to Tang China. The latter half of the century saw the Japanese central government also establishing a School of Mathematics modelled on the Tang. The official mathematics textbooks selected for use, however, showed both Chinese and Korean influences. Even though a catalogue of existing works in Japan compiled in the ninth century suggests that all 12 of the mathematical treatises used at the Tang School of Mathematics had been transmitted to Japan, only six were chosen from the *Ten Mathematical Classics*: *Zhoubi*, *Jiuzhang*, *Haidao*, *Sunzi*, *Zhuishu*, and *Wucao*. Another two were from Korea and were the same texts used in Silla's School of Mathematics: *Yukhang* and *Samgae*. The ninth text, the *Jiusi* 九司 (Nine bureaus), is still of unknown origins.[73]

Much less is known about the situation in Vietnam. It was administered as a Chinese province until it gained independence in 939. Therefore, the *Ten Mathematical Classics* probably circulated there as well. The 22 extant Vietnamese

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71 Ying, "The Kujang sulhae", 12.

72 Na, "Zhongguo chuantong shuxue dui Riben hesuan de yingxiang", 16.

73 Ibid, 17-18; Li Bochun 李伯春, "Hanyu qu de shuxue jiaoliu 漢語區的數學交流" (Mathematical exchanges in the region that uses the Chinese language), *Shuxue shi yanjiu wenji* 數學史研究文集 (Research papers on the history of mathematics) 4 (1993): 69-70.

mathematical treatises written between the fifteenth and the early twentieth century appear very similar to Chinese mathematical treatises like the *Ten Mathematical Classics*.[74] Civil service examinations in mathematics were held between the eleventh and the eighteenth century, albeit very sporadically,[75] which is evidence of the influence of Tang institutions and practices.

From the above, we see that the Tang School of Mathematics and its textbooks, the *Ten Mathematical Classics*, were an integral part of the package of Chinese institutions that Korea, Japan, and possibly Vietnam adopted to structure their own state-run mathematical education.

CONCLUSION

This chapter's survey of the circulation of the *Ten Mathematical Classics* within China and their transmission to the rest of East Asia has focused almost exclusively on the production and movement of these texts within government circles. This is partly due to the nature of the available sources: the governments of East Asia generally kept extensive records and histories. Therefore, we know more about what went on in government offices than about the lives and affairs of the common people. We are positive that the *Ten Mathematical Classics* were used to structure mathematical education in government-run schools across much of East Asia, but we are much less certain about how people learnt mathematics outside of these schools. Through Southern Song mathematical writers discussed in the previous chapter like Qin Jiushao and Yang Hui, both of whom referred to having studied the *Jiuzhang suanshu* during a time when mathematics was no longer taught in any government school in China, we can deduce that the use and circulation of one or

[74] Volkov, "Argumentation for State Examinations", 523.
[75] Ibid, 523-524; Li Bochun, "Hanyu qu de shuxue jiaoliu", 70.

more of the *Ten Mathematical Classics* extended beyond what is mentioned explicitly in the historical record. Yet the dismal survival rate of copies of the *Ten Mathematical Classics* within China makes one question how wide that circulation could have been. All the known writers, commentators, and people who organized the copying or printing of the *Ten Mathematical Classics* belonged to a certain social group: highly educated and almost always with ties to political authority. Therefore, it is difficult to say how well the mathematics of the *Ten Mathematical Classics* corresponded to the mathematics learnt and practised by the common people, or how representative the *Ten Mathematical Classics* actually were of the mathematics done in China from the earliest period up to the end of the Northern Song. However, because the *Ten Mathematical Classics* are almost the only mathematical texts among their contemporaries to have survived to this day, they have indelibly shaped our perception and understanding of pre-modern Chinese mathematics.

Concluding Remarks

"Exploring various approaches to mathematical history will of course lead us to draw multiple images of mathematics from them. This diversity can create many unsettling divisions... But it need not be this way; the cacophony is less a threat than a testimony to the richness of the mathematical tradition."[1]

~ Joan L. Richards

My main contribution to scholarship is to offer a new perspective with which to understand the mathematics and mathematical education of mid-imperial China. In the above chapters, I have sought to present a more varied account of the history of Chinese mathematics than has traditionally been attempted. In the popular mind as well as in the minds of historians, mathematics seems to possess unique qualities that make it appear objective and completely free of human influence, which is why the history of mathematics is usually a history of ideas, focusing only on the intellectual aspects of the subject.

However, mathematics, like all other scientific enterprises, is fundamentally a human endeavour, and therefore can be subjected to analyses of how it fits into and is also shaped by the world around it. This dissertation has explored the history of Chinese mathematics through multiple lenses on the premise that the history of mathematics need not be limited to the mathematical knowledge and ideas that have emerged during the past. The people who wrote or read about mathematics, the context in which mathematical treatises were written, how and where mathematics was learnt, as well as the practice of mathematics in society are all avenues for

[1] Richards, "The History of Mathematics and L'esprit humain", 134.

exploration that will enrich our understanding of the past and how society and its attitude towards mathematics have evolved over time.

Of course, mathematics is also fundamentally a technical enterprise, so Chapter One explored the *Ten Mathematical Classics* from the technical point of view, revealing the types of problems favoured by the writers, as well as the specific mathematical skills that the state valued in its civil servants. These skills ranged from simple multiplication to the solution of systems of equations with multiple linear variables. In today's public education system here in Canada, most of these skills are taught at the elementary and junior high levels, while the most complex techniques are taught in high school mathematics. In view of this comparison, the mathematical education offered at the imperial academy during the mid-imperial period certainly did not teach very sophisticated mathematics. For the training of personnel to handle day-to-day administrative tasks, however, the techniques taught at the imperial academy must have been sufficient for the graduates to accomplish their duties within the bureaucracy. While the mathematics contained in the *Ten Mathematical Classics* is not overly complex, these treatises and their commentaries are not sufficiently explicit for a true beginner to learn and understand the contents on his or her own without the aid of a tutor.

The problems contained in the *Ten Mathematical Classics* tend to be practically oriented and are usually couched in realistic situations. In this, these early Chinese mathematical texts are very similar to those of ancient Egypt and Mesopotamia, as agricultural and administrative themes can be found in many of these texts. Nevertheless, at least in the Chinese case, that does not mean that these treatises were only aimed at promoting technical competence. The authors of some of these classics also promoted the idea that proficiency in mathematics would ultimately lead to a more thorough comprehension of the world. In other words, they did not conceptualize mathematics with the same intellectual boundaries as the ones we have today, but saw it as part of a continuum of activities that allowed them to better understand and impose order on the world.

Chapter Two discussed the mathematical writers of pre-modern China as exemplified by those of the *Ten Mathematical Classics*. These writers all came from the highly educated segment of society, but it is not always clear how mathematics fit into their daily activities and worldviews. What motivated them to write about mathematics in the first place also remains very much a mystery. What stands out is that most of these writers did not have to depend on mathematical skills to earn a living. For them, mathematics was more an intellectual pursuit than a means of livelihood. Their intended audience was probably men like themselves who had undergone a similar education and would understand their references to the Confucian canon. Therefore, the *Ten Mathematical Classics* might not have been the type of practical manuals used by craftsmen and builders to learn their trade, and their circulation might have been limited to people who were higher on the social ladder.

Chapter Three mapped out the evolution of state-run mathematical education and contextualized it within the other types of education simultaneously run by the state. We saw that mathematical education was not always considered a priority for the state even though the state could not have functioned effectively without people who were proficient enough in mathematics to handle its finances and adminstration. By further contextualizing state-run mathematical education within the history of mathematical education in China in general in Chapter Four, we saw the utilitarian nature of state-run education, and how the state was able to co-opt privately trained mathematical practitioners to work for it. One of the central arguments of this dissertation is that state-run mathematical education is not a phenomenon whose creation should be considered natural or inevitable. Instead, both its creation and absence should be seen as the result of conscious decisions that were inextricably linked to what the state wanted to accomplish and whom it wanted to recruit into the bureaucracy.

Chapter Five discussed the material aspects of the history of Chinese mathematics. We saw that the *Ten Mathematical Classics* were preserved to the present largely due to efforts by the central government and by people with links to political authority to reproduce copies. At the same time, the dismal survival rate of copies of the *Ten Mathematical Classics*, as well as of pre-modern Chinese mathematical treatises in general, suggests that only preservation, rather than widespread distribution, was the goal. While issues of transmission are often mentioned in the history of mathematics, the physical books that facilitated that transmission are seldom investigated in any depth. In other words, the history of mathematics and the history of book production have very seldom been linked together. It is hoped that this dissertation has made a contribution towards associating one with the other.

In the history of pre-modern Chinese mathematics, women are very much absent from the narrative. This is because we have no evidence that they ever wrote mathematical treatises. However, women and their activities are very much present in the treatises themselves. Questions in the *Ten Mathematical Classics* that refer to cloth-weaving or to domestic chores invariably describe women rather than men performing these tasks, revealing the writers' attitudes about gender roles within the society in which they lived.

In this dissertation, I have explored the contents, writers and commentators, educational use, and production of the *Ten Mathematical Classics*. However, these topics are by no means exhausted. More research can certainly still be done on them. I have relied primarily on the *Ten Mathematical Classics* themselves and the standard histories here to re-construct the circumstances in which they were produced and used. The incorporation of more sources on local histories into the narrative presented here will very likely shed more light on the writers, readers, copiers, and printers of the *Ten Mathematical Classics*, thereby yielding a more complete picture of how these treatises fit into society and history. A systematic comparison of the Chinese mathematical tradition with those of India and the Arabic world will also yield a better understanding of the development and use of mathematics in these non-Western traditions. For the history of mathematics in general, further investigation into how mathematical skills were put into practice in activities such as navigation will definitely provide much enrichment and historical interest.

For a work on the history of Chinese mathematics, this dissertation has been remarkably silent on issues of precedence and priorities of mathematical ideas. I have not endeavoured to point out every instance when someone in China came up with a mathematical idea before anyone else in the rest of the world. While many Chinese historians have insisted on the importance of staking these claims, I do not see these claims leading to anything fruitful in historical exploration. As Christopher Cullen once put it rather bluntly, "Why [should one] consider it worth while to ask who did what before whom when issues of transmission are not at stake?"[2] This attitude towards historical research is certainly not aimed at belittling the mathematical achievements of China. However, I think it is possible to acknowledge the brilliance of Chinese mathematics without having to use claims of priorities as proof of that brilliance.

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[2] Cullen, "How can We Do the Comparative History of Mathematics", 60.

One question for reflection at this point is the relevance of the *Ten Mathematical Classics* for present times and for modern mathematical education. I do not think that it is either realistic or desirable to use the *Ten Mathematical Classics* as the only official mathematics textbooks for elementary and secondary education because these texts are too terse and lack the explanatory power of modern textbooks. However, I believe these treatises would make excellent supplementary textbooks to both enrich the types of problems students are asked to solve and make students aware of the wide variety of mathematical traditions that existed in the past.[3]

Another question for reflection is whether or not there is validity in the claim that early Chinese mathematics arose in a bureaucratic setting, as is sometimes claimed for other ancient mathematical traditions like Egypt's.[4] The earliest extant Chinese mathematical sources certainly make it tempting to conclude that their contents were designed by bureaucrats for bureaucrats. However, the history of Chinese mathematics is very much like a jigsaw puzzle with more than half of its pieces missing. Focusing on the available pieces helps us gain an understanding of what was present, yet one must never lose sight of the empty spaces in the big picture and how different a picture the missing pieces can potentially create.

[3] Zhu Zaiyu's work on mathematical harmonics is mentioned in this year's *gaokao* 高考 examination in mathematics. The *gaokao* examinations take place in China in June every year, and are taken by graduating high school students to determine which universities they will be eligible to enter. This year's mathematics examination contains no reference to the *Ten Mathematical Classics.*

[4] See Annette Imhausen, *Mathematics in Ancient Egypt: A Contextual History* (Princeton: Princeton University Press, 2016), 2.

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Appendix

A Selection of Problems and Passages from the *Ten Mathematical Classics*

Zhoubi suanjing, Chapter One, Part One:

"商高。周時賢大夫。善算者也。"
(Shang Gao, a grand master of the Zhou dynasty, was a person skilled in *suan*.)

物有圓方。數有奇耦。天動為圓。其數奇。地靜為方。其數耦。此配陰陽之義。非實天地之體也。天不可窮而見。地不可盡而觀。豈能定其方圓乎。
(Objects may be round or square. Numbers may be odd or even. Heaven moves and is round. Its numbers are odd. Earth is stationary and is square. Its numbers are even. This is in accordance to the meaning of yin and yang, not the actual shapes of Heaven and Earth. Heaven cannot be seen completely. Earth cannot be viewed in its entirety. How can one designate them as round or square?)

Zhoubi suanjing, Chapter One, Part Two:

日之高大。光之所照。一日所行。遠近之數。人所望見。四極之窮。列星之宿。天地之廣袤。
(The height and size of the sun, the area of its radiance, the distance it travels in a day, its distance from a terrestrial observer, the limits of human vision, the boundaries of the four poles, the lodges of the stars, the length and breadth of the sky and the earth.)

Jiuzhang suanshu, Liu Hui's preface:

周公制禮。而有九數。九數之流。則九章是矣。往者暴秦焚書。經術散壞。自時厥後。漢北平侯張倉大司農中丞耿壽昌。皆以善算命世。倉等因舊文之遺殘。各稱刪補。故校其目。則與古或異。而所論者多近語也。
(The Duke of Zhou systematized the rites, from which arose the nine numbers. Encapsulating the nine numbers is the *Jiuzhang*. In the past, the tyrannical Qin caused books to be burnt and classics to be damaged and lost. The Han dynasty's Marquis of Beiping Zhang Cang and Palace Aide to the Chamberlain of the National Treasury Geng Shouchang are both known for being skilled in suan. Because the original texts were

incomplete, they made deletions and additions to restore the texts. Therefore, the texts may be different from the ancient ones, and may contain many contemporary terms.)

Jiuzhang suanshu, Chapter Three, Problem 1:

今有大夫、不更、簪裹、上造、公士，凡五人。共獵得五鹿。欲以爵次分之。問各得幾何。
(There is a grand master, a grandee of the fourth order, a grandee of the third order, a grandee of the second order, and a grandee of the first order, totalling five people. They caught five deer in total. If they divide the deer in accordance to their differences in rank, how much does each receive?)

Jiuzhang suanshu, Chapter Three, Problem 4:

今有女子善織。日自倍。五日織五尺。問日織幾何。
(There is a woman who is adept at weaving. Her productivity doubles each day. If she wove 5 *chi* in 5 days, how much did she weave in each of the 5 days?)

Jiuzhang suanshu, Chapter Three, Problem 11:

今有絲一斤。價值三百四十五。今有絲七兩一十二銖。問得錢幾何。
(There is one *jin* of silk worth 345. With 7 *liang* and 12 *zhu* of silk, how much money would be received?)

Jiuzhang suanshu, Chapter Six, Problem 13:

今有善行者行一百步。不善行者行六十步。今不善行者先行一百步。善行者追之。問幾何步及之。
(There is a fast walker who walks 100 *bu* for every 60 *bu* walked by a slow walker. If the slow walker walks 100 *bu* before the fast walker starts chasing after the slow walker, at how many *bu* would the fast walker catch up?)

Jiuzhang suanshu, Chapter Seven, Problem 1:

今有共買物。人出八。盈三。人出七。不足四。問人數物價各幾何。答曰。七人。物價五十三。

(There is a jointly purchased item. If each person contributes 8, the excess is 3. If each person contributes 7, the deficit is 4. Find the number of people who purchased the item and the price of the item. Answer: seven people; item's price is 53)

Jiuzhang suanshu, Chapter Eight, Problem 7:

今有牛五。羊二。值金十兩。牛二。羊五。值金八兩。問牛羊各值金幾何。答曰。牛一、值金一兩二十一分兩之一十三。羊一、值金二十一分兩之二十。
(Five cattle and two sheep are worth 10 *liang* 兩 of gold. Two cattle and five sheep are worth 8 *liang* of gold. Find the worth in gold of one cattle and of one sheep. Answer: one cattle is worth $1\frac{13}{21}$ *liang* of gold; one sheep is worth $\frac{20}{21}$ *liang* of gold.)

Jiuzhang suanshu, Chapter Nine, Problem 4:

今有圓材。徑二尺五寸。欲為方版。令厚七寸。問廣幾何。答曰。二尺四寸。
(There is a circular log. Its diameter is 2 *chi* 5 *cun*. It has to be made into a rectangular plank with a 7-*cun* thickness. What would be the width of the plank? Answer: 2 *chi* 4 *cun*)

Jiuzhang suanshu, Chapter Nine, Problem 5:

今有木長二丈。圍之三尺。葛生其下。纏木七周。上與木齊。問葛長幾何。答曰。二丈九尺。
(There is a tree that is 2 *zhang* tall. Its circumference is 3 *chi*. A vine grows from the bottom of the tree and circles it seven times before reaching the top of the tree. How long is the vine? Answer: 2 *zhang* 9 *chi*)

Jiuzhang suanshu, Chapter Nine, Problem 16:

今有邑方二百步。各中開門。出東門一十五步有木。問出南門幾何步而見木。答曰。六百六十六步大半步。
(There is a square walled city with sides 200 *bu*. Each side has a gate in the middle. Fifteen *bu* out of the east gate is a tree. How far does one have to walk out of the south gate to see the tree? Answer: 666 *bu* and a bit over half a *bu*)

Shushu jiyi:

未識刹那之賒促安知麻姑之桑田。不識積微之為量詎曉百億與大千。
(Without recognizing the haste of an instant, how can one comprehend the tale of Magu's mulberry plantation? Without recognizing that the infinitesimal has magnitude, how can one know the billions and the thousands?)

今有雞翁一隻值五文。雞母一隻值四文。雞兒一文得四隻。合有錢一百文。買雞大小一百隻。問各幾何。答曰。雞翁十五隻。雞母一隻。雞兒八十四隻。
(A rooster is worth 5 coins, a hen 4 coins, and four chicks are worth 1 coin. If 100 coins were used to buy 100 fowls, how many roosters, hens, and chicks would one get? Answer: 15 roosters, 1 hen, 84 chicks)

今有雞翁一隻值四文。雞母一隻值三文。雞兒三隻值一文。合有錢一百文。還買雞大小一百隻。問各幾何。答曰。雞翁八隻。雞母十四隻。雞兒七十八隻。
(A rooster is worth 4 coins, a hen 3 coins, and 3 chicks are worth 1 coin. If 100 coins were used to buy 100 fowls, how many roosters, hens, and chicks would one get? Answer: 8 roosters, 14 hens, 78 chicks)

Haidao suanjing, Problem 1:

今有望海島。立兩表。齊高三丈。前後相去千步。令後表與前表參相直。從前表卻行一百二十三步。人目著地。取望島峰。與表末參合。從後表卻行一百二十七步。人目著地。取望島峰。亦與表末參合。問島高及去表各幾何。答曰。島高四里五十五步。去表一百二里一百五十步。

術曰。以表高乘表間。為實。相多為法。除之。所得。加表高。即得島高。求前表去島遠近者。以前表卻行。乘表間。為實。相多為法。除之。得島去表數。
(There is an island in the sea. Set up two gnomons, both 3 *zhang* tall, and 1000 *bu* apart. The two gnomons are in a straight line with the island. Walking backwards 123 *bu* from the front gnomon, an observer whose eye level is level with the ground would see that the peak of the island is in line with the top of the front

gnomon. Walking backwards 127 *bu* from the back gnomon, an observer whose eye level is level with the ground would see that the peak of the island is in line with the top of the back gnomon. What is the height of the island and how far is the island from the front gnomon? Answer: height of island is 4 *li* 55 *bu*; distance away is 102 *li* 150 *bu*)

Sunzi suanjing, preface:

孫子曰。夫算者。天地之經緯。群生之元首。五常之本末。陰陽之父母。星辰之建號。三光之表裏。五行之準平。四時之終始。萬物之祖宗。六藝之綱紀。稽群倫之聚散。考二氣之降升。推寒暑之迭運。步遠近之殊同。觀天道精微之兆基。察地理從横之長短。采神祇之所在。極成敗之符驗。窮道德之理。究性命之情。立規矩。準方圓。謹法度。約尺丈。立權衡。平重輕。剖毫釐。析黍絫。歷億載而不朽。施八極而無疆。。。嚮之者富有餘。背之者貧且窶。。。夫欲學之者。必務量能揆已。志在所專。如是則焉有不成者哉。
(Master Sun says, with suan, one can grasp the length and breadth of Heaven and Earth, the origin of living things, the fundamentals of the five constant virtues, the parent of yin and yang, the establishment of the stars, the outer and inner workings of the three lights, the balance of the five phases, the beginning and end of the four seasons, the progenitor of all objects, and the regulations of the six gentlemanly arts. One can examine the comings and goings of relationships, investigate the rise and fall of the two energies, predict the change of cold and heat, measure the distance between near and far, see the precise foundation of the Heavenly Way, find horizontal and vertical distances on Earth, locate heavenly beings, predict success and failure, comprehend the principle of virtue, investigate the situation of life, establish rules, regulate the round and the square, treat the laws with care, control and *chi* and the *zhang*, establish balance, balance differences, and distinguish between small differences. *Suan* will not deteriorate despite eons, and can be used in the eight directions without encountering boundaries... People who follow it will be rich. People who turn their backs on it will be poor... People who wish to learn it should apply themselves with discipline. If one concentrates on learning, there is no reason not to have success.)

Sunzi suanjing, Chapter Three, Problem 25:

今有物。不知其數。三三數之。賸二。五五數之。賸三。七七數之。賸二。問物幾何。答曰。二十三。
(There is an unknown number of items. If they are counted by threes, there is a remainder of two. If counted by fives, the remainder is 3. If counted by sevens, the remainder is 2. How many items are there? Answer: 23)

Sunzi suanjing, Chapter Three, Problem 26:

今有獸六首四足。禽二首二足。上有七十六首。下有四十六足。問禽獸各幾何。答曰。八獸七禽。
(There is a beast with six heads and four legs. There is a bird with two heads and two legs. If there are 76 heads and 46 legs in total, how many beasts and birds are there? Answer: 8 beasts and 7 birds)

Sunzi suanjing, Chapter Three, Problem 35:

今有孕婦。行年二十九歲。難九月。未知所生。答曰。生男。
術曰。置四十九。加難月。減行年。所餘。以天除一。地除二。人除三。四時除四。五行除五。六律除六。七星除七。八風除八。九州除九。其不盡者。奇則為男。耦則為女。
(There is a woman, aged 29 *sui*, who gives birth after nine months of pregnancy. What is the gender of her child? Answer: Male.
Method: Take 49. Add the number of months of pregnancy. Minus the woman's age. Then take away 1 for the heaven, 2 for the earth, 3 for humans, 4 for the four seasons, 5 for the five phases, 6 for the six musical notes, 7 for the seven stars, 8 for the eight winds, 9 for the nine regions. If the result is odd, the child is male. If the result is even, the child is female.)

Xiahou Yang suanjing, preface:

五曹孫子。述作滋多。甄鸞、劉徽。為之詳釋。稽之往古。妙絕其能。儲校今時。少有聞見。
(Works that relate to the Wucao and the Sunzi are many. Zhen Luan and Liu Hui have made detailed commentaries on them. Looking at the past, their work

is outstanding. Comparing with the present, it is rare to meet their equal.)

Zhang Qiujian suanjing, preface:

其夏侯陽之方倉。孫子之蕩杯。此等之術。皆未得其妙。故更造新術。推盡其理。附之於此。
(The granary question in the *Xiahou Yang* and the cup-washing question in the *Sunzi* both use methods that have not attained true finesse, so I have constructed new methods and extract all their principles, which are attached here.)

Zhang Qiujian suanjing, Chapter One, Problem 22:

今有女子善織。日益功疾。初日織五尺。今一月。日織九匹三丈。問日益幾何。
(There is a woman adept at weaving. She increases her productivity at a constant rate everyday. On the first day, she wove 5 *chi*. After a month, she is weaving 9 *pi* 3 *zhang* in one day. How much did her productivity increase per day?)

Zhang Qiujian suanjing, Chapter Three, Problem 38:

今有雞翁一值錢五。雞母一值錢三。雞雛三值錢一。凡百錢。買雞百隻。問雞翁、母、雛、各幾何。
(A rooster is worth 5 *qian*, a hen 3 *qian*, and 3 chicks are worth 1 *qian*. If 100 fowls were bought with 100 *qian*, how many roosters, hens, and chicks were bought?)

Wucao suanjing, Chapter One, Problem 5:

今有腰鼓田。從八十二步。兩頭各廣三十步。中央廣十二步。問為田幾何。答曰。八畝奇四十八步。
術曰。并三廣得七十二步。以三除之。得二十四步。以從八十二步乘之。得一千九百六十八步。以畝法除之。即得。
(There is a field in the shape of a waist-drum. Its length is 82 *bu*. Its width at either end is 30 *bu*. The width at the middle is 12 *bu*. What is the area? Answer: 8 *mu*, remainder 48.

Method: Combine the three widths to get 72 *bu*. Divide it by 3 to get 24 *bu*. Multiply it by 82 to get 1968 *bu*. Convert it to *mu*.)

Wujing suanshu, Chapter One:

求十九年七閏法
置一年閏十日。以十九年乘之。得一百九十日。又以八百二十七分。以十九年乘之。得一萬五千七百一十三。以日法九百四十除之。得十六日。餘六百七十三。以十六加上日。得二百六日。以二十九除之。得七月。餘三日。以法九百四十乘之。得二千八百二十。以前分六百七十三加之。得三千四百九十三。以四百九十九命七月。分之適盡。是謂十九年。得七閏月。月各二十九日九百四十分日之四百九十九。

(Calculating the rule of having seven intercalary months for every 19 years
Take 10 for the number of intercalary days per year. Multiplying it by 19 years, one gets 190 days. Take 827. Multiplying it by 19 years, one gets 15 713. Dividing it by the constant 940, one gets 16 days, with a remainder of 673. Adding 16 to the number of days above, one gets 206 days. Dividing it by 29, one gets seven months, with a remainder of three days. Multiplying it by 940, one gets 2820. Adding the 673 from above to it, one gets 3493. Dividing it by 499, one gets seven months without any remainder. Hence for every 19 years, there are seven intercalary months, and each month has 29 $\frac{499}{940}$ days.)