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A BIOMECHANICAL INVESTIGATION OF MUSCULAR WORK

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled....

A BIOMECHANICAL INVESTIGATION OF MUSCULAR WORK

submitted by Allen G. Elliott....

in partial fulfilment of the requirements for the degree of Master of Science.

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Date.. December 20 ..1976

ABSTRACT

Strength, in varying degrees, is generally recognized as a major component of successful participation in almost every sport. How best to train for maximum strength benefits has been a sugject of controversy and the lack of agreement has stemmed, in part, from a lack of knowledge about the nature of muscular contraction.

The purpose of the study was to re-examine the nature of muscular contraction from a biomechanical point of view because it was believed that the differences found in the strength levels for the three modes of contraction were not real differences but were, reflections of the combined influence of angular motion and inertia upon the contraction. The hypothesis tested was that the maximum torques produced at a specified angle of flexion by the forearm flexor muscles would be the same regardless of the mode of contraction after the apparent torque produced had been adjusted for the peculiarities of angular motion and inertia.

Six male and female subjects performed isometric, concentric and eccentric contractions about the elbow joint using the forearm flexor muscles. The fundamental premise underlying the experimental method was that the force, which imparted motion to an object, could be calculated if the time and distance characteristics of the motion

were known. Eccentric and concentric contractions against a resistance in the form of a dumbell were recorded photographically using a high speed motion picture camera. The isometric contraction was also performed using a dumbell but the recording of this event photographically was not necessary since its exact motion was known. Special precautions were taken in order to ensure stabilization of the shoulder and elbow joints while the contractions were being performed. Since most of the strength data in existance had been accumulated as the result of experiments performed on dynamometers, an isometric reading was taken on the dynamometer designed by Karpovich and Karpovich. The torque required to produce the movement was calculated by summing moments about the elbow joint.

Where accelerated motion was involved, the results tended to show no differences. There was no statistical difference between isometric and eccentric contractions. The concentric contractions exhibited a tendency to decelerate as the angle under review was approached. This caused large differences between the concentric and isometric torques. It was reasoned that the mathematical treatment of the concentric motion was erroneous because the torque producing the motion could not have decreased. Therefore, a conclusion regarding the concentric contractions and their relationship to isometric contractions was postponed until the experiment could be re-run using modified equations. equations. There was no difference in isometric torque values arising out of the dynamometer readings and the value when performed with a dumbell.

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CHAPTER I

Introduction

Strength, in varying degrees, is generally recognized as a major component of successful participation in almost every sport. It is important, therefore, that athletes, coaches and all those interested in strength training have knowledge of how to best train for maximum benefits. Ironically, this type of information is lacking or it is at least subject to controversy. This lack of agreement stems, in part, from a lack of knowledge about the nature of muscular contraction.

There are three ways in which a muscle can contract, namely, isometrically, concentrically and eccentrically. As reported in detail in Chapter II, the literature has stated that the force developed by a muscle was least concentrically and maximal eccentrically. Conversely, it has also stated that heat production and oxygen consumption by muscle was maximal concentrically and least eccentrically. It appeared, therefore, that the relationship between the force exerted on the one hand and the oxygen uptake and heat production associated with it on the other was contrary to that which one would normally expect. We had, therefore, an apparent contradiction. An eccentric contraction had been reported to be the strongest while at the same time, it required the least oxygen and produced the least heat. This phenomenon has been explained in terms of reversible chemical reactions with regard to the eccentric contraction.

The purpose of this study was to re-examine the nature of muscular contraction from a biomechanical point of view because it was believed that the differences found in the strength levels for the various modes of contraction were not real differences but were reflections of the combined influence of gravity and angular motion upon the contraction. It was believed that there was no difference in the way a muscle contracts whether it was eccentrically, isometrically or concentrically.

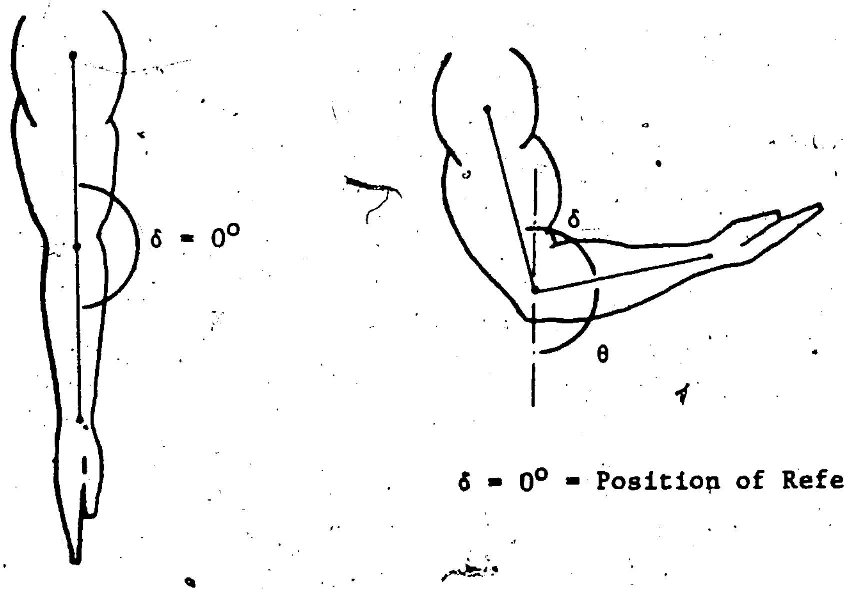
The hypothesis to be tested was that the maximum torque produced at a specified angle of flexion by the forearm flexor muscles would be the same regardless of the mode of contraction after the apparent torque produced had been adjusted for the influence of gravity, the peculiarities of angular motion and the method of measurement.

The following definitions were used in this paper. A concentric contraction was the generation of force by a shortening muscle against a resistance. An eccentric contraction was the generation of force by a muscle against a resistance which caused the muscle to lengthen while it contracted. An isometric contraction was the generation of force by a muscle which did not change length while contracting against a resistance.

The word "tension" deserved special attention because it had been used liberally in the literature while, at the same time, it did not appear to have a precise meaning. In general, it appeared to refer to the force generated by a muscle.

Negative and positive work referred to eccentric and concentric contractions respectively. The phrase "angle of flexion" referred to the angle between the anterior surfaces of the upper arm and the forearm. It was measured from the "position of reference" which has been defined as "the position achieved when the axis of the arm and forearm are in a straight line"(Kapandji:1974). An angle which was used extensively in this study, was the angle θ which referred to the angle between the axis of the forearm and an imaginary vertical axis through the elbow joint in the sagittal plane. The two angles referred to have been demonstrated in Figure 1 below.

FIGURE 1
Position of Reference



δ = 0° = Position of Reference

Lateral Views

CHAPTER II

Review of Literature

There has been general agreement in the literature that the tension developed by a muscle was least concentrically, greatest eccentrically and middling isometrically. In 1965, Doss and Karpovich conducted an experiment using a manually operated dynamometer in order to compare the strength levels of concentric, eccentric and isometric contractions of the elbow flexors. The results were similar to those found in later studies (Singh and Karpovich:1966, Singh and Karpovich: 1968) where concentric strength was found to be approximately twenty (20) percent lower than isometric strength. Other researchers have found similar relationships although their studies may not have been designed specifically for comparison purposes.

Not only did muscle develop maximum tension eccentrically, it also produced the least energy expenditure and heat. Abbott, Bigland et al (1952) studied the physiological cost of negative work using two bicycle ergometers placed back to back and coupled by a chain. While one subject pedalled in the conventional forward direction, the other resisted in the opposite direction. It was reasoned that the cyclist, who pedalled forward, pedalled concentrically while the cyclist, who resisted, had his legs driven backwards eccentrically. The results confirmed the findings of other studies, namely, that the oxygen cost of eccentric work was less than that which was characteristic of concentric work. Other researchers have obtained similar results

(Asmussen: 1953, Abbott & Bigland: 1953, Kamon: 1970, Knuttgen et al: 1971).

The significance of the aforementioned experimental results and others was not lost on researchers. An eccentric contraction could be as high as seven times more efficient than concentric work. Abbott and Aubert (1952) referred to studies which have shown that heat production during eccentric work was less than that produced during concentric contractions. The researchers went one step further by observing that some work performed on a muscle during an eccentric contraction disappeared and it did not reappear as heat or mechanical energy. They concluded by speculating that the missing work "might have been used to stop or reverse some chemical processes normally providing energy in the muscle."

Abbott and Aubert (1952) studied the absorption of work by muscle forced to stretch. In their summary of literature, they asserted that the heat produced during stretching (eccentric contraction) was more than that expected during relaxation but was too small to account for the work done. The authors also stated:

> "It is concluded that the missing work, about half of the whole, is absorbed, presumably as chemical energy . . . it appears that the physical system responsible for mechanical work is reversibly coupled."

As a result of the foregoing experiments and others, the notion of a fundamental difference between eccentric and other forms of contraction became deeply imbedded in the theory of strength physiology. Unfortunately, the data did not fit into a convenient although the work performed was apparently equal, the strain while lifting a mass felt less than when it was lowered; that is, there was less strain associated with eccentric work than with concentric work.

The every day experience of most people and personal experience in the gym appeared to contradict the conclusions reached by researchers regarding the relationships which exist between the force outputs of the three modes of contraction. Further, oxygen uptake and heat production studies, which have already been referred to, provided concrete evidence that the work loads were not equal in these studies if one disregards the notion of reversible chemical reactions. the foregoing comments are limited to the special case of motion in a vertical plane against the force of gravity.

In view of the foregoing, a critical review of the literature was undertaken in order to establish the basis for the present state of knowledge. As was indicated in Chapter I, the use of the word "tension" appears to refer to force output of a muscle. Webster's New Collegiate Dictionary (1956 edition) defines the word tension thus:

> 6.MECH. a A force (either of two balancing forces) causing, or tending to cause extension. b The stress or condition due to these forces.

Clearly, the apparent forces tending or causing extension varied according to the mode of contraction. It is known that muscle consistent model which could be used to unify the knowledge available about strength and endurance.

Those who are experienced in weight training are generally aware of the fact that, with the proper amount of resistance applied to a limb moving against gravity, the initial concentric contraction will cease and become an isometric one. This phenomenon demonstrated the intimate relationship which appeared to exist between isometric, eccentric and concentric contractions. In view of this experience, it made no sense to say that the force generating capacity of muscle during the concentric contraction was twenty (20) percent less than its isometric value at the instant the concentric contraction became isometric. Intuitively, it appeared to be an error to make the statement that the force generating capacity of concentrically contracting muscle was eighty (80) percent of an isometric contraction.

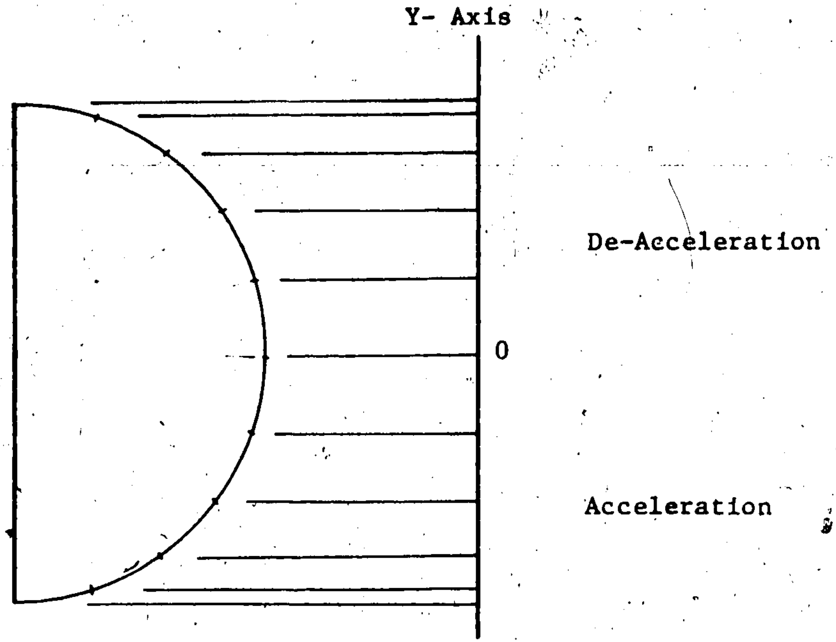
The same line of reasoning was used when comparing eccentric and isometric contractions. A resistance could be moved downward with gravity voluntarily until a peculiar angle was reached where a maximal contraction could be made isometrically without changing the resistance but beyond the said angle, the resistance forced the muscle to contract involuntarily in an eccentric manner. At the instant after the contraction became eccentric, it again did not seem reasonable to state that muscle could eccentrically contract twenty (20) percent more forcefully than it could isometrically.

Finally, it has been the experience of most people that, can support more weight than it can lift, for example. To make the logical jump from tension (resistance) to force output by the muscle, however, was unwarranted because the equivalence of tension to the strength generation required to overcome a resistance had not been established. None of the Singh and Doss studies, for example, considered the weight of the forearm or the characteristics of angular motion. In addition, it was assumed that the constant angular velocity involved in their studies was devoid of acceleration. Normally, this assumption would probably be a reasonable one, but the particular motions studied in the above referred to studies involved angular motion of the forearm about the elbow joint in the sagittal plane where the acceleration due to gravity operated vertically. It is a peculiar property of the said angular motion that a point on the forearm accelerates in the vertical plane from an angle θ of zero (0) to ninety (90) degrees and it de-accelerates from ninety (90) to one hundred and eighty (180) degrees (see Figure 2).

Figure 2 demonstrates vertical acceleration of a point on an arm which is rotating at a constant angular velocity. An arc has been drawn which represents the path of the point. Equal distances on the arc have been marked off and the distances between these points represent equal distances travelled per unit time. These points have been projected to a Y-axis where the distances between points are clearly not equal although the time interval between these same points has not changed.

FIGURE 2

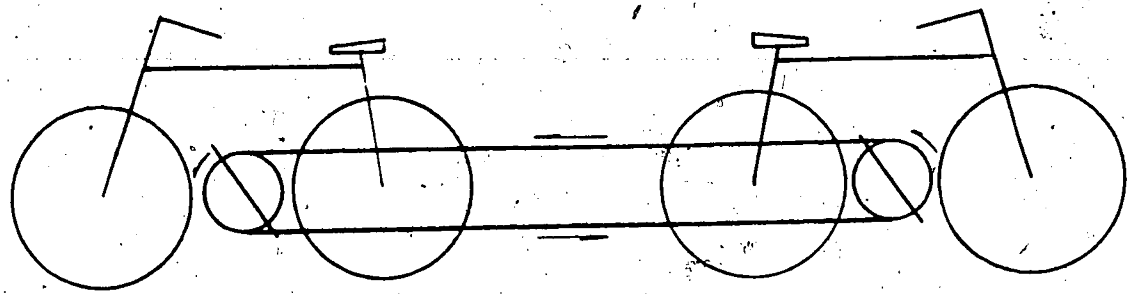
Acceleration in Uniform Angular Motion



In the study of the physiological cost of negative work by Abbott, Bigland (1952), two bicycle ergometers were placed back to back. A single chain coupled the bikes such that when one subject pedalled in the normal manner, the legs of the other were driven backwards, that is, he was forced to pedal eccentrically. The authors of the experiment stated that the forces applied by the cyclists against each other were equal because the pedalling was performed at a constant velocity. A reconsideration of the experiment indicated that the assumption of equal and opposite forces was unwarranted. The subject who pedalled concentrically had to constantly lift his own legs and those of the resister, overcome the inertia of the system and overcome the resistance applied by the resister. The resister on the other hand needed only to apply resistance. Clearly, the work performed by the two cyclists was unequal. The apparatus has been schematically illustrated in Figure 3.

FIGURE 3

Schematic Representation of Abbott's Negative Work Experiment



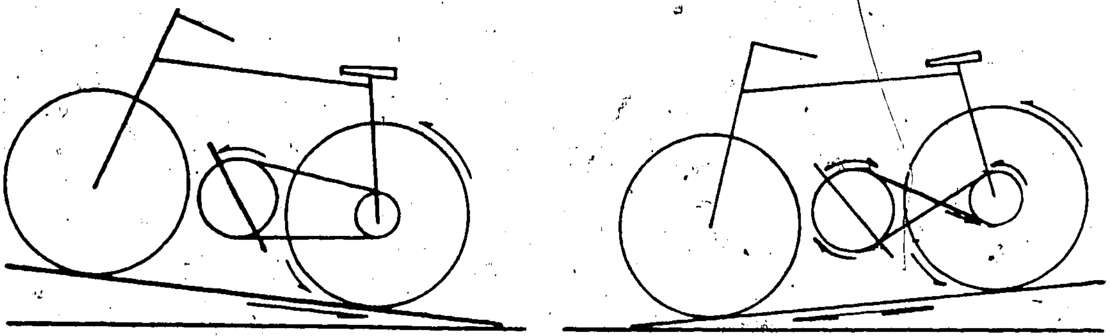
Concentric Work

Eccentric Work

Asmussen (1953) studied the energy cost of positive and negative work using a bike ridden uphill and downhill on a motorized treadmill. Although the experiment was incompletely described, a detailed examination suggested that both up and downhill work were concentrically performed. In order to perform positive (concentric work uphill) work, the cyclist had to overcome the component of the combined force (weight) of the rider and bike which was parallel to the treadmill as well as the force generated by the treadmill causing the bike to move backwards. (see Figure 4) With regard to the negative work, it appeared that the treadmill was running and that the bike faced in the downhill position. If the treadmill was moving in the downhill direction, the cyclist had to pedal backwards in order to maintain his position on the treadmill and the muscular contraction had to have been concentrically performed. If the treadmill was moving in the uphill direction, then concentric contractions must have been performed also. (see Figure 4) The only way eccentric work could have taken place was to have held the treadmill motionless and to have inclined the platform at a steep angle so that the cyclist was forced to resist the downhill movement of the combined bike and rider weight by contracting eccentrically against the bicycle pedals. The chain would have had to have been in its normal position.

FIGURE 4

Schematic Representation of Asmussen's Negative Work Experiment



If the foregoing reduced ones confidence in the current state of knowledge about muscular contraction, the experiment of Infanti et al (1964) caused profound doubts. Referring to the foregoing experiments and others, he stated in his review of literature that these studies

> "leave little doubt that the chemical reactions which normally occur during contraction can be reversed by stretch under the influence of the mechanical work supplied."

The purpose of his study was to see if ATP was re-synthesized under such conditions. He found that, although the ATP breakdown was about one-half of that which would have occurred concentrically, the re-synthesis of ATP did not occur. The researcher remarked that his results were in apparent contradiction to the current theory. He then concluded that ATP was not the final source of energy for muscular work.

Knuttgen et al (1971) studied oxygen debt in short term exercise with regard to eccentric and concentric work. The results indicated no significant difference in recovery patterns.

If the notion of reversible chemical reactions has been found wanting, the notion of absorption of heat was found to be equally suspect. The experiment of Abbott and Aubert (1952), which studied the absorption of work by muscle, calculated the heat produced eccentrically by subtracting the heat which would have been produced had no stretch occurred from the actual heat produced. This procedure may be criticized by stating that the work performed during positive and negative work had to be equal in order to make such a statement. Movement in the human body is normally angular whereas the motion of the resistance in the experiment was linear. As has been explained in Chapter IV, the resistive forces in angular motion in the vertical plane are not the same when moving up as when moving down.

It was considered possible that the differences in the production of force by muscle as a function of the contraction could have been the result of the series elastic component in the muscle tissue. Wilkie (1956) stated in his study of the Mechanical Properties of Muscle, that the series elastic component of muscle smooths out the rapid changes in tension in the muscle. He correctly pointed out that the series elastic component has no mechanical effect upon a contraction when the contractions are performed isotonically where the term "isotonic" refers to equal tension in the muscle. Much of the strength data produced and all of the strength data referred to in this report has been the result of experiments performed upon machines which purport to allow uniform motion only. The dynamometer referred to in this report was a case in point and it was the machine used in the studies of Singh and Karpovich (1968 and 1969). Since these studies involved non accelerated motion, the series elastic component could not have accounted for the spread in strength values found for the various modes of contraction.

In summary, the literature has been found both contradictory and suspect with regard to some of the conclusions reached. It is contradictory in that there was evidence which did not support the reversible chemical reaction theory. In fact, the experiments of Infanti and Knuttgen et al suggested that there was no difference in the muscle physiology of concentrically and eccentrically contracting muscle. The experiments of Abbott and Asmussen, which found differences in work efficiency, have been found suspect because it was not clear that the work loads were equal on the one hand and on the other hand, there was doubt as to whether or not eccentric and concentric work were, in fact, being performed.

CHAPTER III

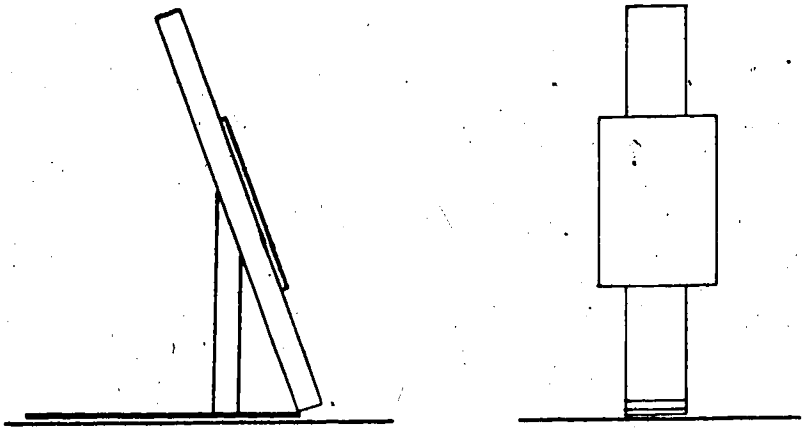
Method and Proceedure

The fundamental premise underlying the experimental method and proceedure was that the force which imparted motion to an object could be calculated if the time and distance characteristics of the motion were known. The satisfaction of two criteria followed from the foregoing; first, a simple, uncomplicated movement was needed about a joint which was geometrically simple and second, a method of accurately recording the motion in sufficient detail for analysis was required. The movement, which best fulfilled the first criteria, was contraction of the flexors of the forearm about the elbow joint. Eccentric and concentric contractions against a resistance in the form of a dumbell were recorded photographically using a high speed motion picture camera. An isometric contraction was also performed using a dumbell but the recording of this event on film was not necessary because its exact motion was known. In view of the large amount of existing dynamometer data, it was decided to include an isometric measurement from the dynamometer described by Karpovich and Karpovich (1969).

In order to ensure that the forearm flexors were producing movement without assistance, a method of stabilizing the upper arm had to be devised. Those experienced in weight lifting are aware of an inclined exercise board used by "body builders" to stabilize the upper arm while performing "curls", that is, concentric contractions. An inclined bench (Figure 5) was constructed for the experiment.

FIGURE 5

Incline Board



The subjects were instructed to stand on the base and to lean into the board such that the top was securely placed into the axilla and the extended arm was lying on the board. A foreward lean was necessary in order to both position the axilla properly and to stabilize the body by using the bench as support. Those experienced with handling weights assumed the proper position without instruction, and the elbow and shoulder joints remained fixed while the muscles contracted. The inexperienced, however, transferred support of their upper body from the incline board to their front leg during maximal work and thereby, freed the shoulder from its position of contact with the board. As a result, the subject was able to displace the shoulder and elbow joints. This movement affected the contraction by improving the mechanical advantage of the lever system. This resulted in gross deviations in the resistance required to produce an isometric contraction. As a result, restraints had to be introduced to prevent the shoulder and elbow joints from moving laterally and medially.

A photographer from the photographic services department at the University of Alberta was used for the actual filming. The film was taken at right angles to the movement of the forearm around the elbow joint. Black and white film was used and the camera setting specifications were one hundred (100) frames per second, thirty six (36) degree shutter angle and one-thousandths (1000) of a second shutter speed. The camera was located twelve (12) feet from the subject.

A group of six male and female subjects were used for the study. Pertinent data for each has been summarized below.

Subject	Sex	Age	Comments
1	f	35	Mother & housewife; average physique & fitness
2	f	19	First year university student; average fitness
3	m	15	Junior high school student; non athletic
4	m	30	Masters degree student in physical education
5	m	36	Masters degree student in physical education; experienced with weights; athletic background
6	m	21	Clothing salesman; average fitness

Three testing sessions were conducted. The first session consisted of familiarizing the subjects with the apparatus and the movements to be performed on it. It was also held for the purpose of determining the amount of dumbell resistance required to produce an isometric contraction at an angle θ of eighty (80) degrees. Each subject was given progressively heavier weights while performing concentric contractions on the inclined bench. At the correct amount of resistance, the subject was able to move the forearm concentrically upwards until the angle was reached at which time movement ceased and the weight was held isometrically at that angle. Four or five trials were required to determine the correct poundage. Weight was then added and subtracted from the isometric value to fine those resistances which would allow eccentric and concentric contractions through the angle. The resistances used and their relationships to one another have been outlined on Tables 1 and 2 .

TABLE 1

Resistances Used For The Concentric
Isometric And Eccentric Contractions
(pounds)

	$\theta = 80^\circ$			$\theta = 110^\circ$		
Subject	Concentric	Isometric	Eccentric	Concentric	Isometric	Eccentric
1	14.5	17.3	32.7	16.9	28.5	35.1
2	10.7	12.9	16.7	16.8	20.4	27.7
3	16.8	22.5	29.9	22.5	30.0	37.5
4	29.1	39.4	46.2	32.4	40.4	53.6
5	27.1	32.3	52.5	27.2	45.4	52.5
6	32.2	37.9	53.4	32.2	40.4	58.0

TABLE 2

Resistances As A Proportion Of Isometric

	$\theta = 80^\circ$			$\theta = 110^\circ$		
Subject	Concentric	Isometric	Eccentric	Concentric	Isometric	Eccentric
1	0.84	1.00	1.89	0.59	1.00	1.23
2	0.83	1.00	1.29	0.82	1.00	1.36
3	0.75	1.00	1.33	0.75	1.00	1.25
4	0.74	1.00	1.17	0.80	1.00	1.33
5	0.84	1.00	1.63	0.60	1.00	1.16
6	0.85	1.00	1.41	0.80	1.00	1.44

The second session was divided into two parts. The first part consisted of taking isometric measurements on the dynamometer at two angles of θ equalling eighty (80) degrees and one-hundred and ten (110) degrees. These two angles were chosen because they were above and below the horizontal at points where gravity acted with almost full force. The subjects were placed in the dynamometer so that the shoulder was placed twenty degrees posterior to the elbow joint. This was necessitated by the fact that the upper arm lay at an angle of twenty (20) degrees to the vertical when it was lying on the incline board. The lateral epicondyle of the humerus was placed opposite the axis of rotation of the lever arm. The contact plate for the distal forearm was positioned such that the distal border of the plate lay at the styloid process of the radius. The distance from the axis to the distal border of the plate was measured after each contraction. Each subject was taken in order and each performed three contractions at each angle.

θ = 80°			θ = 110°		
Concentric	Isometric	Eccentric	Concentric	Isometric	Eccentric
0.84	1.00	1.89	0.59	1.00	1.23
0.83	1.00	1.29	0.82	1.00	1.36
0.75	1.00	1.33	0.75	1.00	1.25
0.74	1.00	1.17	0.80	1.00	1.33
0.84	1.00	1.63	0.60	1.00	1.16
0.85	1.00	1.41	0.80	1.00	1.44

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stal border of the plate was measured after each contraction.
ubject was taken in order and each performed three contractions
h angle.

to the four raw data scores. Statistically speaking, this study was designed as a single factor experiment where the adjusted scores were considered to be repeated measures. Treatment effects were measured relative to the individual who thus was his own control. The difference between two observations on the same person depended, in part, upon uncontrolled or residual sources of variation. The appropriate test used was an F ratio which compared the mean square of the treatment effects to the mean square of the residual effects. F tests the hypotheses that $T_1 = T_2 = T_3 = T_4$ and in this study, T represented the treatment effects. The verbal hypotheses restated was that there would be no difference in T.

CHAPTER IV

Treatment

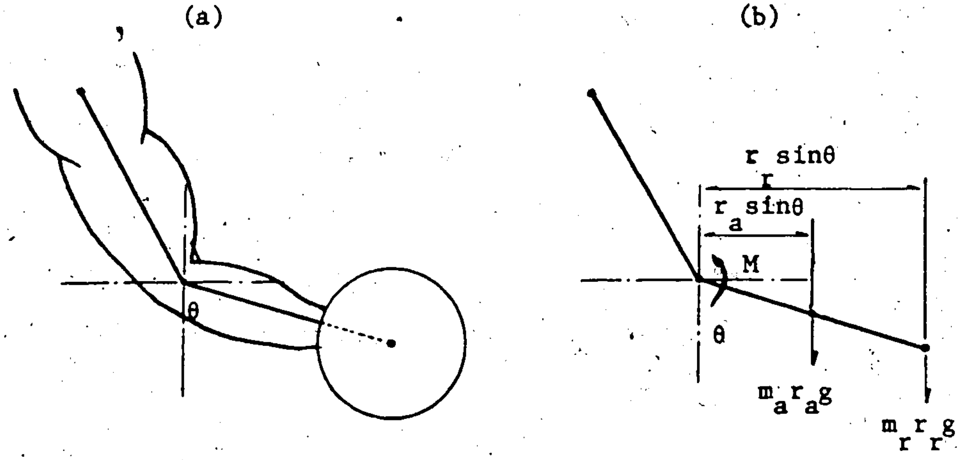
Natural phenomenon and even the research data quoted suggested a more plausible model of muscle contraction and a model which was more consistent with the physical laws of nature. It hypothesized no difference in the ability of muscle to contract, that is, muscle would contract in the same way regardless of the method employed. If this were the case, then there were only two factors which could account for the apparent differences in work output, namely, the quantity of motion and the effect of inertia. Since all limb movement in this experiment was angular, all motion was necessarily accompanied by acceleration even if the angular velocity was uniform in the vertical plane. In addition, all movement would be accompanied by a moment of inertia whose effect would increase exponentially as the resistance moved away from the axis of rotation.

It followed from the model proposed that the effective strength output of a concentrically contracting muscle had to be its maximal strength potential less that which was needed to provide acceleration and that which was needed to overcome the inertial effect of the system. During an isometric contraction, the inertial component became zero while during an eccentric contraction, the inertial influence assisted the muscular effort by resisting the downward motion of the resistance. Thus the amount of force which a muscle could sustain eccentrically under identical velocity criteria would be more than it would be able to concentrically sustain.

Since a study of this kind had not been conducted before, the mathematical expressions to be used had to be developed. Meriam (1971), Plagenhoef (1971), and Slote et al (1963) were useful in this regard. The total torque required for each contraction was calculated by summing moments of force about the elbow joint. An isometric contraction (Figure 6) using a resistance in the form of a dumbell held in the hand proved illustrative. Figure 6a illustrated the position of the arm while Figure 6b summarized the forces involved. The shoulder and elbow joints were assumed to be fixed and the only movement possible about the elbow joint "o" was rotation of the forearm and resistance in the vertical plane.

FIGURE 6

Isometric Force Diagram



The weight of the arm was considered to act at the centre of gravity of the limb and the summation of moments of force about the axis "o" was:

$$M = Wt_a r_a \sin\theta + Wt_r r_r \sin\theta$$

$$\text{but } Wt_a = m_a g$$

$$Wt_r = m_r g$$

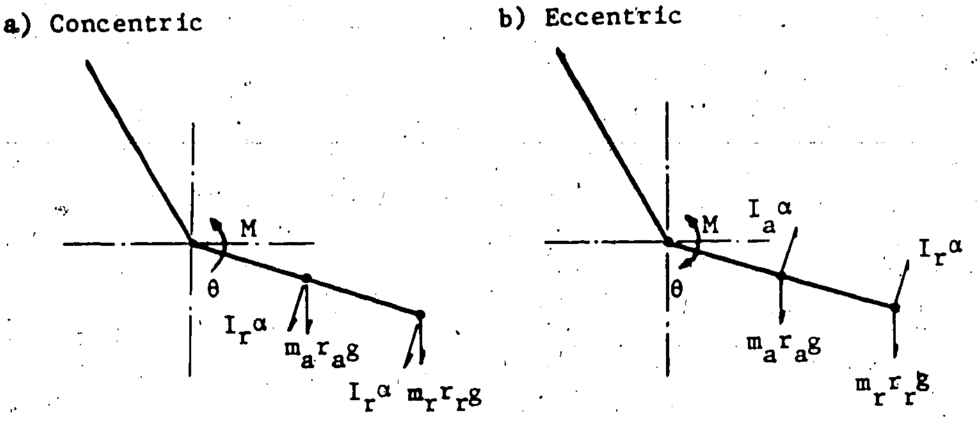
$$\text{therefore } M = (m_a r_a + m_r r_r) 32.17 \sin\theta$$

The isometric contraction was the simplest to deal with as it did not involve motion. The same isometric equation used for calculating moments of force where dumbells were utilized was used to analyze the dynamometer readings. A small adjustment was necessary, however, because the dynamometer recorded the force applied at right angles to the lever arm regardless of the angle of flexion of the arm. Therefore, the moment of force of the dynamometer reading was the reading "R" times its perpendicular distance from the axis which was the distance from the axis of rotation to the point of application of torque to the dynamometer. The point of application was considered to be the mid point of the plate which received the distal end of the forearm. The modified equation was written as follows:

$$M = m_a r_a (32.17) \sin\theta + Rr$$

Concentric and eccentric contractions were analyzed with the aid of their respective resistive force diagrams contained in Figures 7a and 7b. In the case of a concentric contraction, two additional forces were added to basic isometric drawing. These two were the inertial forces associated with the masses and they were labelled $I_a\alpha$ and $I_r\alpha$. These same two forces were present in the eccentric contractions but their role was reversed; that is, they opposed the motion due to gravity rather than the motion due to muscle.

FIGURE 7
Concentric and Eccentric Force Diagrams


a) Concentric
b) Eccentric

Meriam (240:1971) has shown that the resultant sum of all moments about the centre of mass of an object was the same as $I_o\alpha$ where I_o was the moment of inertia about the centroid and α was the angular acceleration. It was also subsequently pointed out that the resultant sum of all moments about a point called the centre of percussion was Iα where I was the moment of inertia associated with that point and α was the angular acceleration. It was more convenient to choose the centre of percussion as the point of application of the force mrα. Since the centre of percussion was a unique point where the resultant moment about the axis of rotation was preserved, it was chosen because consideration of the force mrα was eliminated. Thus, only two pieces of information were required in order to solve for the torque producing a movement, namely, the angular acceleration of the movement and the moment of inertia associated with the centre of percussion of the mass.

Since two masses were involved in each contraction (the arm and resistance), two moments of inertia were required. The details of their calculation have been included in the appendices of this report.

The equation used to solve for the resultant torque of a concentric contraction was:

$$M = m_a r_a g + m_r r_r g + (I_a + I_r)\alpha$$

The equation used to solve for the resultant torque of an eccentric contraction was:

$$M = m_a r_a g + m_r r_r g - (I_a + I_r)\alpha$$

CHAPTER V

Results and Analysis

The raw and adjusted scores obtained during the testing process have been presented in Table 3. Inspection of the data indicated adjusted concentric scores which were significantly below the scores of the other treatment groups. The adjusted eccentric scores tended to be greater than their corresponding isometric scores. The adjusted scores were subjected to an F test in order to determine whether or not the apparent trends were statistically significant.

The statistical foremat used was that found in Winer (268:1971). The detailed calculations have been omitted but the significant calculations have been included in Table 4. The computational procedures used have been listed below as has the variance formulae.

Summary of Computational Procedures

$(1) = \frac{G^2}{kn}$

$(2) = \Sigma\Sigma X^2$

$(3) = \frac{\Sigma T^2}{n}$

$(4) = \frac{\Sigma P^2}{k}$

Computation of Variance

$SS_{btwn} = (4) - (1)$

$SS_{within} = (2) - (4)$

$SS_{treatments} = (3) - (1)$

$SS_{residual} = (2)-(3)-(4)+(1)$

$SS_{total} = (2) - (1)$

TABLE 3

Raw and Adjusted Scores
(pound feet)

		RAW SCORES				ADJUSTED SCORES			
θ	Subject	Concentric	Isometric	Eccentric	Dynamometer Eccentric	Concentric	Isometric	Eccentric	Dynamometer Eccentric
80°	1	14.5	17.3	32.7	27.5	15.2	17.2	19.6	17.2
80°	2	10.7	12.9	16.7	21.3	10.6	13.3	13.4	14.2
80°	3	16.8	22.5	29.9	34.6	23.3	24.2	27.9	25.7
80°	4	29.1	39.4	46.2	53.2	26.5	39.5	44.0	44.5
80°	5	27.1	32.3	52.6	53.7	31.0	33.9	43.2	38.5
80°	6	32.2	37.9	53.4	47.0	35.9	40.0	43.3	37.5
110°	1	16.8	28.5	35.1	42.3	17.7	26.3	29.6	29.3
110°	2	16.8	20.4	27.7	26.5	17.7	19.3	16.5	19.4
110°	3	22.5	30.0	37.5	40.8	22.6	30.2	36.0	31.1
110°	4	32.4	40.4	53.6	57.6	28.3	38.6	38.1	47.9
110°	5	27.2	45.4	52.5	59.0	29.	44.7	44.6	42.4
110°	6	32.2	40.4	58.0	51.4	27.4	40.6	46.5	42.4

TABLE 4

Statistical Analysis of Adjusted Scores
(pound feet)

Subject	1 (Conc.)	2 (Iso)	3 (Ecc.)	4 (Dyn.)	TOTAL	MEAN
1	15.2	17.2	19.6	17.2	69.2	17.3
2	10.6	13.3	13.4	14.2	51.5	12.9
3	23.3	24.2	27.9	25.7	101.1	25.3
4	26.5	39.5	44.0	44.5	154.5	38.6
5	31.0	33.9	43.2	38.5	146.6	36.6
6	35.9	40.0	43.3	37.5	156.7	39.2
1	17.7	26.3	29.6	29.3	102.9	25.7
2	17.7	19.3	16.5	19.4	72.9	18.2
3	22.6	30.2	36.0	31.1	119.9	30.0
4	28.8	38.6	38.1	47.9	153.4	38.3
5	29.6	44.7	44.6	42.4	161.3	40.3
6	27.4	40.6	46.5	42.4	156.9	39.2
TOTAL	286.3	367.8	402.7	390.1	1446.9=G	
MEAN	23.9	30.6	33.6	32.5		30.1

The data compiled in Table 4 was used in the computational formulae to compute variances. These calculations have been assembled in Table 5 under the heading of Analysis of Variance.

TABLE 5

Analysis of Variance

Source of Variation		SS		d.f.	M.S.
Between subjects		4391		11	
Within subjects		1088		36	
Treatment	684		3		228.0
Residual	404		33		12.2
Total		5479		47	

The interpolated critical value for the F ratio was:

$$F_{.05}(3,33) = 2.896$$

$$F = \frac{MS_{treatment}}{MS_{residual}} = \frac{228.0}{12.2} = 18.7$$

As a result of the analysis of variance of the treatment scores (Table 5), it was found that the value of the F ratio exceeded the critical value required for no difference between the treatments. An inspection of the data suggested that the difference lay between treatment groups (1) and the others. The hypothesis that $T_2 = T_3 = T_4$ was then tested using the following procedure outlined by Winer (268:1971).

$$SS_{234} = \frac{T_2^2 + T_3^2 + T_4^2}{n} - \frac{(T_2 + T_3 + T_4)^2}{3n} = 26.0$$

$$MS_{234} = \frac{SS_{234}}{d.f.} = \frac{26}{2} = 13.0$$

The critical value for the F ratio was : $F_{.05}(2,33) = 3.293$

$$F = \frac{MS_{treatment}}{MS_{residual}} = \frac{13.0}{12.2} = 1.100$$

The statistical test confirmed that the difference between treatments lay between (1) and the remainding groups and that there was no statistical difference between groups (2), (3) and (4) at the .05 confidence level. Thus, the hypothesis of this study was confirmed in part.

The data in group (1) was examined more closely in an attempt to determine the reason why the results did not follow the pattern of the other groups. A simple t-test was run for each subject at the two angles studied. The mean and standard deviation were computed for treatments (2), (3) and (4). The score for (1) was then tested to see if it was statistically different from the mean previously computed. It was found that five (5) of the twelve items in group (1) were statistically different and of these, four (4) were at an angle θ of one hundred and ten (110) degrees. A chi square test of independance using Yates correction for continuity, did not indicate a significant difference between the two angles. The probability associated with the distribution of differences found in (1) of statistically significant scores was $.20 < p < .30$.

The statistically significant scores were examined for a common factor which would explain their presence. The scores revealed no preference for age, sex or fitness level. However, four (4) of the five (5) deviant scores were characterized by negative acceleration.

Score (2) was also characterized by negative acceleration but it was not found to be statistically significant.

It was reasoned that a contraction at $\theta = 80^{\circ}$ should experience decreasing acceleration if not de-acceleration. However, it was not clear why negative acceleration should occur at where acceleration should have been the rule as the elbow joint provided an increasing percentage of the force required to support the mass and where the resistance lever arm shortened.

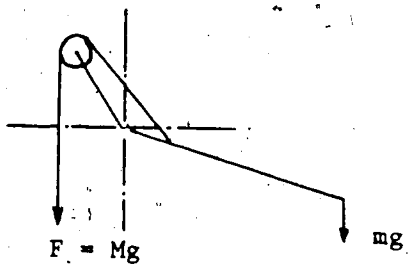
The equation used to adjust the raw scores was then re-examined.

$$\Sigma M_o: \quad M = m_a r_a g + m_r r_r g + (I_a + I_r)\alpha$$

The equation indicated that the difference between the torque of a static contraction and a dynamic one, was the interaction of inertia and motion. When "r" was changed, I_r also changed. For a given resistance, α should, depending upon the angle examined, experience positive or negative acceleration. Thus the equation confirmed that which was expected. However, what was not expected was the fact that the equation would yield a concentric torque which was less than the isometric. This situation could not be unless the resistance was moving faster than the arm. This result did not make sense because, in the case case of the example previously cited of a concentric contraction ending as an isometric one, it appeared to be nonsense to state that the concentric contraction was produced by less torque than the subsequent isometric contraction.

If the foregoing argument was cogent, the reason for the low concentric values was either in the mathematical expression of the movement or in the application of that expression. In an attempt to find a solution to the problem, a model was constructed to simulate the movements examined in this study. The apparatus has been illustrated in Figure 8.

FIGURE 8
Mechanical Analogue of The Forearm
And It's Flexor Muscles



Using a constant weight at F, the transformation of a concentric contraction into an isometric one was accomplished. A concentric contraction, which de-accelerated as it moved through the angle and then accelerated, was also duplicated using a constant force F.

In view of the foregoing, it was concluded that the application of the moment equation and/or its form was incorrect.

In summary, the statistical results did not indicate a significant difference between the adjusted eccentric and isometric values. On the other hand, a difference was found between concentric and isometric values. The first result supported the hypothesis while the second did not. With regard to the second, it was pointed out that the interaction of the data and the mathematical treatment of it produced a suspicious result when, during a concentric contraction, the movement decelerated. A judgement regarding the results arising out of the concentric contractions must wait for a retest at which time a refined methodology will take deceleration into account.

The results in general should be viewed with caution due to the small number of subjects. However, the data does suggest that motion plays a significant role in the ability of muscle to contract against a resistance.

In conclusion, the results of this study are inconclusive. The lack of a significant difference with regard to the eccentric contraction and the large differences found with regard to the concentric contractions may be due to chance alone. However, other explanations have been put foreward and a repeat of the experiment is warrented.

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APPENDIX A

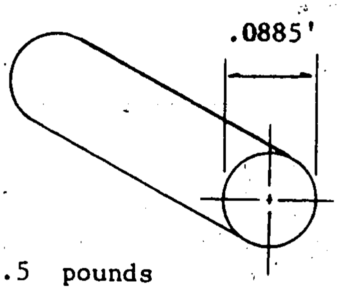
CENTROIDAL MOMENTS OF INERTIA
OF THE RESISTANCES

The resistance used by an individual for a particular contraction was an assemblage of seperate resistances which collectively formed the dumbell. Each part had its own peculiar mass and shape and thus, had its own peculiar moment of inertia about its centre of mass.

In order to calculate the moment of force which imparted momentum to a particular resistance, it was necessary to know the moment of inertia of the dumbell about its axis of rotation (the elbow joint of the subject performing the contraction). The first step in this computation was to calculate the moment of inertia of the masses about their centroids. Standard moment of inertia formula were used (Meriam:1971).

Each calculation has been presented in three parts. A sectional drawing of each part of the dumbell, the calculation of the moment of inertia (I_o) and the computations leading up to the inertia calculation.

BAR



$$I_o = 1/2\ mr^2$$

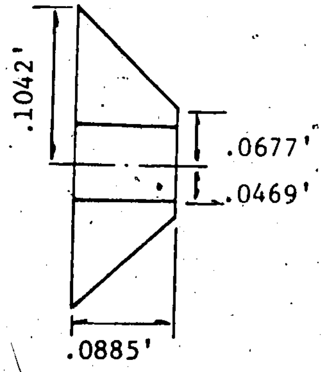
$$= 1/2\ (.1088)(.0442)^2$$

$$= .0001 \text{ slug feet squared}$$

Weight: 3.5 pounds

Mass: .1088 slugs

COLLAR



Weight: .6875 pounds

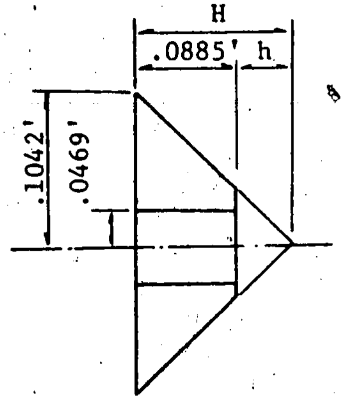
Mass: .0214 slugs

$$I_o = 3/10MR^2 - 3/10mr^2 - 1/2m_c r_c^2$$

$$= .3(.0352)(.1042)^2 - .3(.0096)(.0677)^2 - .5(.0042)(.0469)^2$$

$$= .001 \text{ slug feet squared}$$

VOLUME CALCULATION



$$\frac{H}{.1042} = \frac{H-.0885}{.0677}$$

$$H = .2526 \text{ feet}$$

Volume (collar) $= 3/5\ R^2H - 3/5\ r^2h - r^2(H-h)$

$= 3/5\ (.1042)^2(.2526) - 3/5\ (.0677)^2(.1641) - (.0469)^2(.0885)$

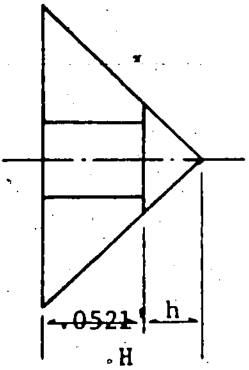
$= .003141$ cu. ft.

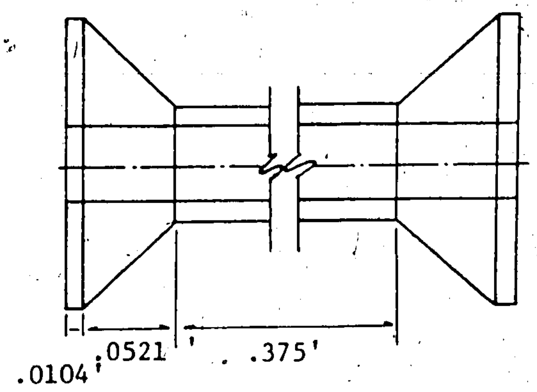
Mass density: $\frac{\text{mass}}{\text{volume}} = 6.8131$ slugs per cu. ft.

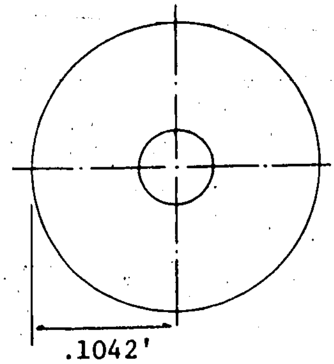
MASS CALCULATION

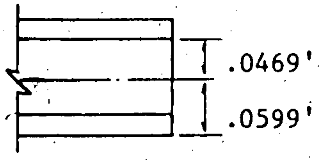
Object	Volume	x	Mass Density	=	Mass		
Total Cone	.005169	x	6.8131	=	M	=	.0352
Small Cone	.001417	x	6.8131	=	m	=	.0096
Cylinder	.000611	x	6.8131	=	m_c	=	.0042
Mass of Collar $(M-m-m_c)$							.0214 slugs

GRIP









$$\frac{.1042}{H} = \frac{.0599}{H-.0521}$$

$$H = \frac{.005429}{.0443} = .1225 \text{ ft.}$$

Weight = 1.625 pounds

Mass = .0505 slugs

VOLUME CALCULATION

$$V \text{ cylinder} = 2(R^2h - r^2h)$$

$$= 2[(.1042)^2(.0104) - (.0469)^2(.0104)] = .000566$$

$$V \text{ cone} = 2[3/5\ R^2H - 3/5\ r^2h - r^2(H-h)]$$

$$= 2[3/5\ (.1042)^2(.1225) - 3/5\ (.0599)^2(.0704) - (.0469)^2(.0521)]$$

$$= .003342$$

$$V \text{ cylinder} = r_1^2 - r_2^2$$

$$= (.375)(.0599^2 - .0469^2) = \underline{.001636}$$

Total Volume .005544 cu.ft.

$$\text{Mass Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{.0505}{.005544} = 9.1089 \text{ slugs per cu. ft.}$$

Mass Calculation

Object	Volume	x	Mass density	=	Mass		
Cylinder	.000566	x	9.1089	=	M	=	.0052
Cone	.003342	x	9.1089	=	m	=	.0304
Cylinder	.001636	x	9.1089	=	m_c	=	.0149
Mass of grip $(M-m-m_c)$							.0505 slugs

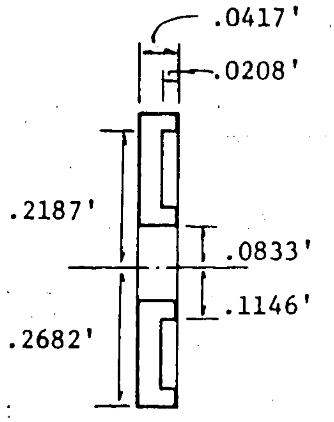
I = Cylinder + Cone + Cylinder

$$I = [1/2(MR^2-mr^2)] + [3/10mR^2-3/10mr^2-1/2mr^2] + [1/2m_1r_1^2-1/2m_2r_2^2]$$

$$=1/2(.0064)(.1042^2)-1/2(.0012)(.0469^2)+.3(.0456)(.1042^2)-$$
$$.3(.0087)(.0599^2)-.5(.0033)(.0469^2)+$$
$$.5(.0385)(.0599^2)-.5(.0236)(.0469^2)$$

= .0002 slug feet squared

SILVER 2 1/2



Weight: 2.85 pounds

Mass : .0886 slugs

$$I_o = 1/2MR^2 - 1/2m_c r^2 - 1/2m(r_2^2 + r_1^2$$

$$= 1/2(.1336)(.2682)^2 - 1/2(.0129)(.0833)^2 - 1/2(.0321)(.2187^2 + .1146^2)$$

$$= .0038 \text{ slug feet squared}$$

VOLUME CALCULATION

$$\text{Volume} = R^2h - r^2h - d(r_2^2 - r_1^2)$$

$$= (.2682^2)(.0417) - (.0833^2)(.0417) - (.0208)(.2187^2 - .1146^2)$$

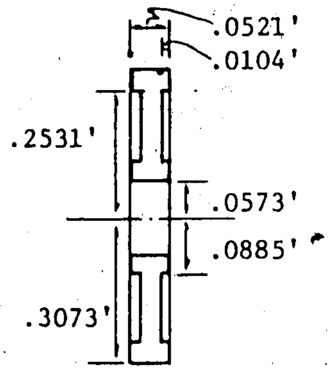
$$= .006247 \text{ cu.ft.}$$

$$\text{Mass density} : \frac{\text{mass}}{\text{Volume}} + \frac{.0886}{.006247} = 14.1828 \text{ slugs per cubic foot}$$

MASS CALCULATION

Volume	x	Mass density	=	Mass		
.009423	x	14.1828	=	M	=	.1336
.000909	x	14.1828	=	m_c	=	.0129
.002267	x	14.1828	=	m	=	.0321
Mass of plate $(M-m_c-m)$						.0886 slugs

GREEN & BLACK 5



Weight: 5.1 pounds

Mass: .1585 slugs

$$I = 1/2MR^2 - 1/2M(r_2^2 + r_1^2) - 1/2m_c r^2$$

$$= 1/2(.2178)(.3073^2) - 1/2(.0517)(.2531^2 + .0885^2) - 1/2(.0076)(.0573^2)$$

$$= .0084 \text{ slug feet squared}$$

VOLUME CALCULATION

$$\text{Volume} = R^2h - 2\ d(r_2^2 - r_1^2) - r^2h$$

$$= (.3073^2)(.0521) - 2\ (.0104)(.2531^2 - .0885^2) - (.0573^2)(.0521)$$

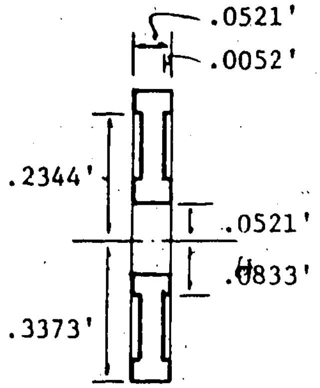
$$= .011245 \text{ cu. ft.}$$

MASS CALCULATION

$$\text{Mass density} = \frac{\text{mass}}{\text{volume}} = \frac{.1585}{.011245} = 14.0951 \text{ slugs per cubic foot}$$

Volume	x	Mass density	=	Mass		
.015456	x	14.0951	=	M	=	.2178
.003674	x	14.0951	=	m	=	.0517
.000537	x	14.0951	=	m_c	=	.0076
Mass of plate $(M-m-m_c)$						.1585 slugs

GOLD 5



Weight: 5.175 pounds

Mass: .1609 slugs

$$I = 1/2MR^2 - 1/2m_c r^2 - 1/2m(r_2^2 + r_1^2)$$

$$= 1/2(.1914)(.3073^2)-1/2(.0055)(.0521^2)-1/2(.0250)(.2344^2+.0833^2)$$

$$= .0082 \text{ slug feet squared}$$

VOLUME CALCULATION

$$\text{Volume} = R^2h - r^2h - 2\,d(r_2^2-r_1)$$

$$= (.3073^2)(.0521)-(.0521^2)(.0521)-2(.0052)(.2344^2+.0833^2)$$

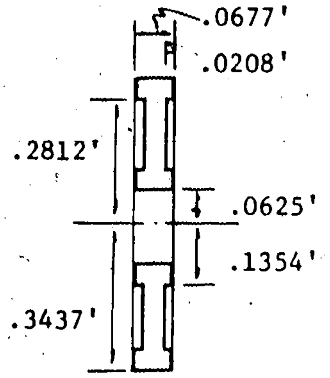
$$= .01299 \text{ cu. ft.}$$

$$\text{Mass density} = \frac{\text{mass}}{\text{volume}} = \frac{.1609}{.01299} = 12.3864 \text{ slugs per cu. ft.}$$

MASS CALCULATION

Volume	x	Mass density	=	Mass		
.015456	x	12.3864	=	M	=	.1914
.000444	x	12.3864	=	m_c	=	.0055
.002022	x	12.3864	=	m	=	.0250
Mass of Plate $(M-m_c-m)$						.1609 slugs

BLACK 7 1/2



Weight: 7.7562 pounds

Mass: .2411 slugs

$$I = 1/2MR^2 - 1/2mr^2 - 1/2m(r_2^2 + r_1^2$$

$$= 1/2(.3285)(.3437^2)-1/2(.0109)(.0625^2)-1/2(.0765)(.2812^2+.1354^2)$$

$$= .0156 \text{ slug feet squared}$$

VOLUME CALCULATION

$$\text{Volume} = R^2h - r^2h - 2\,d(r_2^2 - r_1^2)$$

$$= (.3437^2)(.0677)- (.0625^2)(.0677)-2\ (.0208)(.2812^2-.1354^2)$$

$$= .018442 \text{ cu. ft.}$$

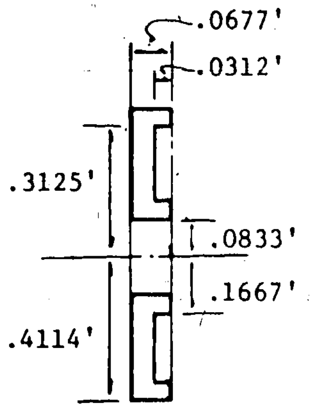
$$\text{Mass density: } \frac{\text{mass}}{\text{volume}} = \frac{.2411}{.018442} = 13.0734 \text{ slugs per cu. ft.}$$

MASS CALCULATION

Volume	x	Mass density	=	Mass		
.025124	x	13.0734	=	M	=	.3285
.000831	x	13.0734	=	m_c	=	.0109
.005851	x	13.0734	=	m	=	.0765

Mass of Plate $(M-m_c-m)$ .2411 slugs

SILVER 10



Weight: 10.1 pounds

Mass: .3139 slugs

$$I = 1/2MR^2 - 1/2m_c r^2 - 1/2M(r_2^2 + r_1^2)$$

$$= 1/2(.4083)(.4114^2) - 1/2(.0167)(.0833^2) - 1/2(.0777)(.3125^2 + .1667^2)$$

= .0296 slug feet squared

VOLUME CALCULATION

$$\text{Volume} = R^2h - r^2h - d(r_2^2 - r_1^2)$$

$$= (.4114^2)(.0677) - (.0833^2)(.0677) - (.0312)(.3125^2 - .1667^2)$$

= .027673 cu. ft.

Mass density: $\frac{\text{mass}}{\text{volume}} = \frac{.3139}{.027673} \doteq 11.3432$ slugs per cu. ft.

MASS CALCULATION

Volume	x	Mass density	=	Mass	
.035997	x	11.3432	=	M	= .4083
.001476	x	11.3432	=	m_c	= .0167
.006848	x	11.3432	=	m	= .0777
Mass of Plate ($M-m_c-m$)			=		.3139 slugs

APPENDIX B

PERSONAL DATA

Two bits of information were collected from each subject. The individuals body weight was taken in pound measure and from this information, the mass of the forearm, hand and arm were calculated using percentage tables published from the research of Dempster (Duggar:1962). The measurement was taken with the individual in his or her street clothes but without the shoes.

The second bit of information taken was the length of the arm measured in feet. This measurement actually consisted of the addition of two measurements, namely, the length of the forearm (from the lateral epicondyle of the humerus to the styloid process of the radius) and the length of the clenched hand (from the styloid process of the radius to the carpal/metacarpal joint of the clenched fist). The hand had to be clenched because that was the position it approximated when wrapped around the handle of a dumbell.

Knowing the length of the forearm and hand, the location of the centroid or centre of mass of each limb segment from the axis of rotation (the elbow joint) was computed using percentage tables published from the research of Dempster (Duggar:1962). The assumption had to be made, however, that the location of the centre of mass of the clenched fist was the same as when the hand was in the anatomical position.

Moments of force about the elbow joint were calculated where the only two forces acting about the joint were the result of the masses of the hand and forearm under the influence of gravity. Dividing the result by the total mass of the arm, the location of the centre of mass of the arm as a whole was computed. This calculation has been shown for each subject. The computation of the systems (forearm and hand) centre of mass was necessary because the two limbs were considered to be one mass in the experimental treatments.

The moment of inertia of the arm (I_a) was required as well as the radius of gyration because, as explained in the body of the report, the resistance was moved from its normal centre of mass location to a parallel position through a point called the centre of percussion. The radius of gyration (k) about this same point was also necessary in order to compute I_a.

The relationship between the moment of inertia of a body and its radius of gyration was found to be;

$$k = \sqrt{\frac{I}{m}} \qquad (1)$$

The arm was considered to be a long slender rod whose moment of inertia about the centroid was given by the expression;

$$I_o = \frac{1}{12} m \ell^2 \qquad (2)$$

Substituting I_o into equation (1), the following centroidal relationship was derived;

$$k_o = \sqrt{\frac{\ell^2}{12}}$$

The radius of gyration k associated with the centre of percussion was calculated from the expression;

$$k = \sqrt{k_o^2 + r^2}$$

The moment of inertia of the arm associated with the centre of percussion was found using the following expression;

$$I_a = m_a k^2$$

PERSONAL DATA

Subject 1

BODY WEIGHT			105.0000 pounds
MASS	Body	100.00%	3.2639 slugs
	Forearm	1.57%	.0512
	Hand	0.66%	.0215
	Arm	2.23%	.0728
LENGTH	Forearm		.7917 ft.
	Hand (clenched)		.2917
	Arm (ℓ)		1.0834
CENTROID	Forearm (r_f)	43.00% of length	.3404 ft.
	Hand (r_h)	51.00% of length plus forearm	.9405
	Arm (r_a)	by summing moments	.5172

$$\Sigma M_o = M_a r_a = m_f r_f + m_h r_h$$

$$r_a = \frac{.0512(.3404) + .0215(.9405)}{.0728}$$

$$r_a = .5172 \text{ feet}$$

RADIUS OF GYRATION $k_o = \sqrt{\frac{\ell^2}{12}} = \sqrt{\frac{1.0834^2}{12}} =$ .3127 ft.

$$k = \sqrt{k_o^2 + r^2}$$

$$= \sqrt{.3127^2 + .5172^2}$$ .6044 ft.

MOMENT OF INERTIA OF ARM

$$I_a = m_a k^2 = .0728(.6044^2)$$

$$= .0266 \text{ slug feet squared}$$

PERSONAL DATA

Subject 2

BODY WEIGHT			120.5000 lbs.
MASS	Body	100.00%	3.7457 slugs
	Forearm	1.57%	.0588
	Hand	0.66%	.0247
	Arm	2.23%	.0835
LENGTH	Forearm		.7917 ft.
	Hand (clenched)		.2917
	Arm (ℓ)		1.0834
CENTROID	Forearm	(r_f) 43.00% of length	.3404 ft.
	Hand	(r_h) 51.00% of length plus forearm	.9405
	Arm	(r_a) by summing moments	.5179

$$\Sigma M_o = M_a r_a = m_f r_f + m_h r_h$$

$$r_a = \frac{.0588(.3404) + .0247(.9405)}{.0835}$$

$$r_a = .5179 \text{ feet}$$

RADIUS OF GYRATION $k_o = \sqrt{\frac{\ell^2}{12}} = \sqrt{\frac{1.0834^2}{12}} = .3128$ ft.

$$k = \sqrt{k_o^2 + r^2}$$

$$= \sqrt{.3128^2 + .5179^2} = .6050 \text{ ft.}$$

MOMENT OF INERTIA OF ARM

$$I_a = m_a k^2 = .0835(.6050^2)$$

$$= .0306 \text{ slug feet squared}$$

PERSONAL DATA

Subject 3

BODY WEIGHT			116.0000 pounds
MASS	Body	100.00%	3.6058 slugs
	Forearm	1.57%	.0566
	Hand	0.66%	.0238
	Arm	2.23%	.0804
LENGTH	Forearm		.8333 ft.
	Hand (clenched)		.3750
	Arm (ℓ)		1.2083
CENTROID	Forearm (r_f)	43.00% of length	.3583 ft.
	Hand (r_h)	51.00% of length plus forearm	1.0245
	Arm (r_a)	by summing moments	.5555

$$\Sigma M_o = M_a r_a = m_f r_f + m_h r_h$$

$$r_a = \frac{.0566(.3583) + .0238(1.0245)}{.0804}$$

$$r_a = .5555 \text{ feet}$$

RADIUS OF GYRATION $k_o = \sqrt{\frac{\ell^2}{12}} = \sqrt{\frac{1.2083^2}{12}} = .3488$ ft.

$$k = \sqrt{k_o^2 + r^2}$$

$$= \sqrt{.3488^2 + .5555^2} = .6559 \text{ ft.}$$

MOMENT OF INERTIA OF ARM

$$I_a = m_a k^2 = .0804(.6559^2)$$

$$= .0346 \text{ slug feet squared}$$

PERSONAL DATA

Subject 4

BODY WEIGHT			157.0000 pounds
MASS	Body	100.00%	4.8803 slugs
	Forearm	1.57%	.0766
	Hand	0.66%	.0322
	Arm	2.23%	.1088
LENGTH	Forearm		.8021 ft.
	Hand(clenched)		.3333
	Arm (ℓ)		1.1354
CENTROID	Forearm (r_f)	43.00% of length	.3449 ft.
	Hand (r_h)	51.00% of length plus forearm	.9721
	Arm (r_a)	by summing moments	.5305

$$\Sigma M_o = M_a r_a = m_f r_f + m_h r_h$$

$$r_a = \frac{.0766(.3449) + .0322(.9721)}{.1088}$$

$$r_a = .5305 \text{ feet}$$

RADIUS OF GYRATION $k_o = \sqrt{\frac{\ell^2}{12}} = \sqrt{\frac{1.1354^2}{12}} =$ .3278 ft.

$$k = \sqrt{k^2 + r^2}$$

$$= \sqrt{.3278^2 + .5305^2}$$ .6236 ft.

MOMENT OF INERTIA OF ARM

$$I_a = m_a k^2 = .1088(.6236^2)$$

$$= .0423 \text{ slug feet squared}$$

PERSONAL DATA

Subject 5

BODY WEIGHT				160.0000 pounds
MASS	Body		100.00%	4.9736 slugs
	Forearm		1.57%	.0781
	Hand		0.66%	.0328
	Arm		2.23%	.1109
LENGTH	Forearm			.8132 ft.
	Hand (clenched)			.3750
	Arm (ℓ)			1.1882
CENTROID	Forearm	(r_f)	43.00% of length	.3497 ft.
	Hand	(r_h)	51.00% of length plus forearm	1.0044
	Arm	(r_a)	by summing moments	.5433

$$\Sigma M_o = M_a r_a = m_f r_f + m_h r_h$$

$$r_a = \frac{.0781(.3497) + .0328(1.0044)}{.1109}$$

$$r_a = .5433 \text{ feet}$$

RADIUS OF GYRATION $k_o = \sqrt{\frac{\ell^2}{12}} = \sqrt{\frac{1.1882^2}{12}} =$ .3430 ft.

$$k = \sqrt{k_o^2 + r^2}$$

$$= \sqrt{.3430^2 + .5433^2} = .6425 \text{ ft.}$$

MOMENT OF INERTIA OF ARM

$$I_a = m_a k^2 = .1109(.6425^2)$$

$$= .0458 \text{ slug feet squared}$$

PERSONAL DATA

Subject 6

BODY WEIGHT			145.0000 pounds
MASS	Body	100.00%	4.5073 slugs
	Forearm	1.57%	.0708
	Hand	0.66%	.0297
	Arm	2.23%	.1005
LENGTH	Forearm		.8542 ft.
	Hand (clenched)		.3333
	Arm (ℓ)		1.1875
CENTROID	Forearm (r_f)	43.00% of length	.3673 ft.
	Hand (r_h)	51.00% of length plus forearm	1.0242
	Arm (r_a)	by summing moments	.5614

$$\Sigma M_o = M_a r_a = m_f r_f + m_h r_h$$

$$r_a = \frac{.0708(.3673) + .0297(1.0242)}{.1005}$$

$$r_a = .5614 \text{ feet}$$

RADIUS OF GYRATION $$k_o = \sqrt{\frac{\ell^2}{12}} = \sqrt{\frac{1.1875^2}{12}} = .3428 \text{ ft.}$$

$$k = \sqrt{k_o^2 + r^2}$$

$$= \sqrt{.3428^2 + .5614^2} = .6578 \text{ ft.}$$

MOMENT OF INERTIA OF ARM

$$I_a = m_a k^2 = .1005(.6578^2)$$

$$= .0435 \text{ slug feet squared}$$

APPENDIX C

RESISTANCE DATA

Subject 1
(contractions at $\theta = 80^{o}$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
One green 5	5.125		.0084
One silver 2 1/2	2.850		.0038
Total	14.475	.4499	.0127
plus: mk^2 $(.4499 \times .9554^2)$			.4107
I_r			.4234 slug ft^2

Isometric Dumbell			
Bar; grip; two collars	6.500		
One green 5	5.125		
Two silver 2 1/2	5.700		
Total	17.325	.5385	

Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
One green 5	5.125		.0084
One gold 5	5.175		.0082
Two black 5	10.200		.0168
Two silver 2 1/2	5.700		.0076
Total	32.700	1.0165	.0415
plus: mk^2 $(1.0165 \times .9619^2)$			.9405
I_r			.9820 slug ft^2

RESISTANCE DATA

Subject 1
(contractions at $\theta = 110^{\circ}$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two gold 5	10.350		.0164
Total	16.850	.5238	.0169
plus: mk^2 (.5238x.9410^2)			.4638
I_r			.4807 slug ft^2

Isometric Dumbell			
Bar; grip; two collars	6.500		
One green 5	5.125		
Three gold 5	15.525		
Two collars	1.375		
Total	28.525	.8867	

Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two black 5	10.200		.0168
One silver 2 1/2	2.850		.0038
Total	35.062	1.0899	.0523
plus: mk^2 (1.0899x.9657^2)			1.0525
I_r			1.1048 slug ft^2

RESISTANCE DATA

Subject 2
(contractions at $\theta = 80^{o}$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
One silver 2 1/2	2.850		.0038
Two collars	1.375		.0002
Total	10.725	.3334	.0045
plus: mk^2 $(.3334 \times .9476^2)$			.2994
I_r			.3039 slug ft^2

Isometric Dumbell		
Bar; grip; two collars	6.500	
Two silver 2 1/2	5.700	
One collar	.687	
Total	12.887	.4006

Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
Two black 5	10.200		.0168
Total	16.700	.5191	.0173
plus: mk^2 $(.5191 \times .9581^2)$			.4764
I_r			.4937 slug ft^2

RESISTANCE DATA

Subject 2
(contractions at $\theta = 110^{o}$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two gold 5	10.350		.0164
Total	16.850	.5238	.0169
plus: mk^2 (.5238x.9575^2)			.4802
I_r			.4971 slug ft^2

Isometric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		
Two gold 5	10.350		
One silver 2 1/2	2.850		
One collar	.687		
Total	20.387	.6337	

Eccentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two silver 2 1/2	5.700		.0076
Total	27.712	.8614	.0393
plus: mk^2 (.8614x.9644^2)			.8012
I_r			.8405 slug ft^2

RESISTANCE DATA

Subject 3
(contractions at $\theta = 80^o$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two gold 5	10.350		.0164
Total	16.850	.5238	.0169
plus: mk^2 $(.5238x1.0401^2)$			.5666
I_r			.5835 slug ft^2

Isometric Dumbell	Weight	Mass	
Bar; grip; two collars	6.500		
Two gold 5	10.350		
Two silver 2 1/2	5.700		
Total	22.550	.7010	

Eccentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two gold	10.350		.0164
Two black	10.200		.0168
One silver 2 1/2	2.850		.0038
Total	29.900	.9294	.0375
plus: mk^2 $(.9294x1.0440^2)$			1.0130
I_r			1.0505 slug ft^2

RESISTANCE DATA

Subject 3
(contractions at $\theta = 110^{\circ}$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two gold 5	10.350		.0164
Two silver 2 1/2	5.700		.0076
Total	22.550	.7010	.0245
plus: mk^2 $(.7010 \times 1.0414^2)$			.7602
I_r			.7847 slug ft^2

Isometric Dumbell		
Bar; grip; two collars	6.500	
Three gold 5	15.525	
One green 5	5.125	
One silver 2 1/2	2.850	
Total	30.000	.9325

Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two gold 5	10.350		.0164
One green 5	5.125		.0084
Total	37.487	1.1653	.0565
plus: mk^2 (1.1653×1.0479^2)			1.2796
I_r			1.3361 slug ft^2

RESISTANCE DATA

Subject 4
(contractions at θ = 80^o)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two silver 2 1/2	5.700		.0076
Two collars	1.375		.0002
Total	29.087	.9042	.0395
plus: mk^2 $(.9042x.9943^2)$			.8939
I_r			.9334 slug ft^2

Isometric Dumbell	Weight	Mass
Bar; grip; two collars	6.500	
Two black 7 1/2	15.512	
One green 5	5.125	
One gold 5	5.175	
Two silver 2 1/2	5.700	
Two collars	1.375	
Total	39.387	1.2243

Eccentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Four black 7 1/2	31.024		.0624
One gold 5	5.175		.0082
One silver 2 1/2	2.850		.0038
One collar	.687		.0001
Total	46.236	1.4372	.0750
plus: mk^2 $(1.4372x.9971^2)$			1.4289
I_r			1.5039 slug ft^2

RESISTANCE DATA

Subject 4
(contractions at $\theta = 110^\circ$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two gold 5	10.350		.0164
Total	32.362	1.0060	.0481
plus: mk^2 ($1.0060 \times .9955^2$)			.9970
I_r			1.0451 slug ft^2

Isometric Dumbell			
Bar; grip; two collars	6.500		
Four black 7 1/2	31.024		
One silver 2 1/2	2.850		
Total	40.374	1.2550	

Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
Four black 7 1/2	31.024		.0624
Two gold 5	10.350		.0164
Two silver 2 1/2	5.700		.0076
Total	53.574	1.6653	.0869
plus: mk^2 ($1.6653 \times .9977^2$)			1.6576
I_r			1.7445 slug ft^2

RESISTANCE DATA

Subject 5
(contractions at $\theta = 80^{\circ}$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
One green 5	5.125		.0084
Total	27.137	.8435	.0401
plus: mk^2 $(.8435 \times 1.0278^2)$			.8910
I_r			.9311 slug ft^2

Isometric Dumbell			
Bar; grip; two collars	6.500		
Two black 7 1/2	15.512		
One green 5	5.125		
One gold 5	5.175		
Total	32.312	1.0044	

Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two silver 10	20.200		.0592
One gold 5	5.175		.0082
One green 5	5.125		.0084
Total	52.572	1.6323	.1075
plus: mk^2 (1.6323×1.0367^2)			1.7543
I_r			1.8618 slug ft^2

RESISTANCE DATA

Subject 5
(contractions at $\theta = 110^\circ$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
One gold 5	5.175		.0082
Total	27.187	.8451	.0399
plus: mk^2 $(.8451x1.0276^2)$			.8924
I_r			.9323 slug ft^2

Isometric Dumbell	Weight	Mass
Bar; grip; two collars	6.500	
Two black 7 1/2	15.512	
Two gold 5	10.350	
One green 5	5.125	
One black 5	5.100	
One silver 2 1/2	2.850	
Total	45.437	1.4124

Eccentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two silver 10	20.200		.0592
One green 5	5.125		.0084
One gold 5	5.175		.0082
Total	52.512	1.6323	.1075
plus: mk^2 $(1.6323x1.0367^2)$			1.7395
I_r			1.8618 slug ft^2

RESISTANCE DATA

Subject 6
(contractions at $\theta = 80^o$)

Concentric Dumbell	Weight	Mass	I_o
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
Two black 5	10.200		.0168
Total	32.212	1.0013	.0485
plus: mk^2 (1.0013×1.0476^2)			1.0989
I_r			1.1474 slug ft^2

Isometric Dumbell			
Bar; grip; two collars	6.500		
Two black 7 1/2	15.512		
Two black 5	10.200		
Two silver 2 1/2	5.700		
Total	37.912	1.1785	

Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
Four black 7 1/2	31.024		.0624
Two black 5	10.200		.0168
Two silver 2 1/2	5.700		.0076
Total	53.424	1.6607	.0873
plus: mk^2 (1.6607×1.0495^2)			1.8292
I_r			1.9165 slug ft^2

RESISTANCE DATA

Subject 6
(contractions at $\theta = 110^{\circ}$)

	Weight	Mass	I_o
Concentric Dumbell			
Bar; grip; two collars	6.500		.0005
Two black 7 1/2	15.512		.0312
One green 5	5.125		.0084
One black 5	5.100		.0084
Total	32.237	1.0021	.0485
plus: mk^2 (1.0021×1.0457^2)			1.0958
I_r			1.1443 slug ft^2
Isometric Dumbell			
Bar; grip; two collars	6.500		
Four black 7 1/2	31.024		
One silver 2 1/2	2.850		
Total	40.374	1.2550	
Eccentric Dumbell			
Bar; grip; two collars	6.500		.0005
Four black 7 1/2	31.024		.0624
Two black 5	10.200		.0168
Two silver 2 1/2	5.700		.0076
Total	58.024	1.6607	.0873
plus: mk^2 (1.6607×1.0495^2)			1.8292
I_r			1.9165 slug ft^2

APPENDIX D

ANGULAR ACCELERATION DETERMINATION

Two resistances were moved through two angles of flexion. A Vangard film analyzer containing an X and Y axis was used to record Y co-ordinates every five frames as the resistance moved through an angle under review. Seven measurements were taken and by computing the differences between them, the distance ($\overline{Y}$) traversed by the centre of the resistance was computed. Since each frame equalled a time span of point one (.01) seconds, the velocity (v_y) of the centre along the Y axis could be calculated. In order to minimize errors in measurement. $\overline{Y}$ was converted to a moving average (Y) of three successive points and it was this distance which was used to compute v_y.

Acceleration (a_y) along the Y co-ordinate was computed by dividing the change of time into the change of velocity over a ten (10) frame period. The acceleration tangent to the arm (a_t) was calculated by dividing a_y by angle which the forearm made with the angle of reference.

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 80^{\circ}$)

Subject 1

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
1959			
2013	54		
2074	61	61	1220
2143	69	69	1380
2219	76	76	1520
2293	74		
2361			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1520 - 1220}{.10}$$

$$= 3000 \text{ units} = 1.290 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{1.290}{\sin 80^{\circ}} = 1.310 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} =$ 1.393 rad/sec^2

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2653			
2553	100		
2395	158	158	3160
2180	215	216	4320
1905	275	281	5620
1552	353		

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{5640 - 3160}{.10}$$

$$= 24{,}800 \text{ units} = 10.664 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{10.664}{\sin 80^{\circ}} = 10.828 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} =$ 11.513 rad/sec^2

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 110^\circ$)

Subject 1

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2370			
2373	103		
2506	133	128	2560
2653	147	147	2940
2805	162	162	3240
2963	158		
3118			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{3240 - 2560}{.10}$$

$$= 6800 \text{ units} = 2.924 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{2.924}{\sin 70^\circ} = 3.116 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} =$ 3.313 rad/sec^2

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2987			
2884	103		
2768	116	114	2280
2646	122	126	2520
2507	139	141	2820
2346	161		
2171			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{2820 - 2280}{.10}$$

$$= 5400 \text{ units} = 2.322 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{2.322}{\sin 70^\circ} = 2.471 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} =$ 2.627 rad/sec^2

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 80^\circ$)

Subject 2

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
1319			
1447	123		
1590	143	134	2680
1720	130	128	2560
1880	112	113	2260
1930	100		
2013			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{2260 - 2680}{.10}$$

$$= -4200 \text{ units} = -1.806 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{-1.806}{\sin 80^\circ} = -1.834 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = -1.950 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2435			
2341	94		
2219	122	119	2380
2078	141	143	2860
1912	166	163	3260
1730	182		
1578			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{3260 - 2380}{.10}$$

$$= 8800 \text{ units} = 3.784 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{3.784}{\sin 80^\circ} = 3.842 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 4.085 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 110^o$

Subject 2

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2768			
2823	55		
2880	57	57	1140
2940	60	62	1240
3010	70	71	1420
3092	82		
3167			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1420 - 1140}{.10}$$

$$= 2800 \text{ units} = 1.204 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{1.204}{\sin 70^o} = 1.281$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 1.280 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
3553			
3474	79		
3351	123	123	2460
3185	166	173	3460
2956	229	233	4660
2652	304		
2266			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{4660 - 2460}{.10}$$

$$= 22000 \text{ units} = 9.46 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{9.46}{\sin 70^o} = 10.067 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 10.704 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 80^{o}$)

Subject 3

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
1995			
2062	67		
2153	91		
2160	7		140
2259	99		1980
2366	107		
2474			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1980 - 140}{.10}$$

$$= 18{,}000 \text{ units} = 7.912 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{7.912}{\sin 80^{o}} = 8.034 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 7.842 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2282			
2140	142		
1978	162	159	3180
1804	174	181	3620
1598	206	199	3980
1380	218		
1134			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{3980 - 3180}{.10}$$

$$= 8000 \text{ units} = 3.440 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{3.440}{\sin 80^{o}} = 3.493 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 3.409 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 110^\circ$)

Subject 3

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
3142			
3213	71		
3295	82	75	1500
3366	71	70	1400
3423	57	69	1380
3501	78		
3579			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1380 - 1500}{.10}$$

$$= 1200 \text{ units} = -.516 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{-.516}{\sin 70^\circ} = .549 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = -.536 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
3344			
[illegible]	26		
[illegible]	49	42	840
3219	50	52	1040
3161	58	54	1081
3108	53		
3055			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1080 - 840}{.10}$$

$$= 2400 \text{ units} = 1.032 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{1.032}{\sin 70^\circ} = 1.098 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 1.072 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 80^{\circ}$)

Subject 4

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
1212			
1420	208		
1613	193	193	3860
1791	178	176	3520
1949	158	157	3140
2084	135		
2200			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{3140 - 3860}{.10}$$

$$= -7200 \text{ units} = -3.096 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{-3.096}{\sin 80^{\circ}} = -3.144 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = -3.234 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
1956			
1818	138		
1665	153	150	3000
1506	159	158	3160
1343	163	165	3300
1169	174		
990			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{3300 - 3000}{.10}$$

$$= 3000 \text{ units} = 1.290 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{1.290}{\sin 80^{\circ}} = 1.310 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 1.348 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 110^{\circ}$)

Subject 4

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2715			
2814	99		
2900	86	84	1680
2967	67	68	1360
3018	51	60	1200
3079	61		
3144			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1200 - 1680}{.10}$$

$$= -4800 \text{ units} = -2.064 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{-2.064}{\sin 70^{\circ}} = -2.196 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = -2.259 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
3521			
3381	140		
3198	183	180	3600
2980	218	214	4280
2738	242	255	5100
2433	305		
2106			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{5100 - 3600}{.10}$$

$$= 15000 \text{ units} = 6.450 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{6.450}{\sin 70^{\circ}} = 6.864 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 7.061 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 80^\circ$)

Subject 5

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
843			
1065	222		
1296	231	232	4640
1538	242	241	4820
1789	251	295	5180
2073	284		
2340			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{5180 - 4640}{.10}$$

$$= 5400 \text{ units} = 2.322 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{2.322}{\sin 80^\circ} = 2.358 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 2.348 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2242			
2113	129		
1971	142	139	4633
1824	147	151	5033
1659	165	162	5400
1484	175		
1300			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{5400 - 4600}{.06}$$

$$= 12{,}783 \text{ units} = 5.497 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{5.497}{\sin 80^\circ} = 5.582 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 5.557 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 110^\circ$)

Subject 5

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2850			
3027	177		
3230	203	200	4000
3449	219	217	4340
3679	230	224	4480
3902	223		
4132			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{4480 - 4000}{.10}$$

$$= 4800 \text{ units} = 2.064 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{2.064}{\sin 70^\circ} = 2.196 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 2.186 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
3261			
3228	33		
3190	38	37	1233
3150	40	43	1433
3100	50	51	1700
3036	64		
2976			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1700 - 1233}{.06}$$

$$= 7783 \text{ units} = 3.347 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{3.347}{\sin 70^\circ} = 3.562 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 3.546 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 80^{\circ}$)

Subject 6

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
1404			
1504	100		
1608	104	110	2200
1735	127	119	2380
1861	126	126	2520
1985	124		
2105			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{2520 - 2200}{.10}$$

$$= 3200 \text{ units} = 1.376 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{1.376}{\sin 80^{\circ}} = 1.397 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 1.364 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2710			
2596	114		
2451	145	142	2840
2284	167	176	3520
2067	217	216	4320
1804	263		
1486			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{4320 - 2840}{.10}$$

$$= 14,800 \text{ units} = 6.364 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{6.364}{\sin 80^{\circ}} = 6.462 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 6.309 \text{ rad/sec}^2$

ANGULAR ACCELERATION DETERMINATION
(contractions at $\theta = 110^{o}$)

Subject 6

CONCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
2705			
2919	214		
3116	197	200	4000
3305	189	179	3580
3456	151	150	3000
3566	110		
3661			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{3000 - 4000}{.10}$$

$$= -10{,}000 \text{ units} = 4.300 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{-4.300}{\sin 70^{o}} = 4.576 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = -4.468 \text{ rad/sec}^2$

ECCENTRIC DUMBELL

Y	ΔY	$\overline{\Delta Y}$	v_y
3180			
3151	29		1450
3118	33		1650
3083	35		1750
3047			

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{1750 - 1450}{.04}$$

$$= 7500 \text{ units} = 3.225 \text{ ft/sec}^2$$

$$a_t = \frac{a_y}{\sin\theta} = \frac{3.225}{\sin 70^{o}} = 3.432 \text{ ft/sec}^2$$

Angular acceleration: $\alpha = \frac{a_t}{r} = 3.351 \text{ rad/sec}^2$

APPENDIX E

MOMENT CALCULATIONS
(contractions at $\theta = 80^\circ$)

Subject 1

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.0728(.5172) + .4499(.9405)]32.17\sin 80^\circ + (.4713)1.393$$

$$= 15.2 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.0728(.5172) + .5385(.9405)]32.17\sin 80^\circ$$

$$= 17.2 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta - (I_a + I_r)\alpha$$

$$= [.0728(.5172) + 1.0165(.9405)]32.17\sin 80^\circ - (1.0299)11.513$$

$$= 19.6 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .0728(.5172)\sin 80^\circ(32.17) + 27.474(.5833)$$

$$= 17.2 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 110^\circ$)

Subject 1

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.0728(.5172) + .5238(.9405)]32.17\sin 70^\circ + (.5073)3.313$$

$$= 17.7 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [(.0728)(.5172) + .8867(.9405)]32.17\sin 70^\circ$$

$$= 26.3 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.0728(.5172) + 1.1048(.9405)]32.17\sin 70^\circ - (1.1314)2.627$$

$$= 29.6 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .0728(.5172)\sin 70^\circ(32.17) + 42.260(.6667)$$

$$= 29.3 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 80^\circ$)

Subject 2

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.0835(.5179) + .3334(.9405)]32.17\sin 80^\circ + (.3345)1.950$$

$$= 10.6 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\ \sin\theta$$

$$= [.0835(.5179) + .4006(.9405)]32.17\sin 80^\circ$$

$$= 13.3 \text{ pound feet}$$

[illegible]CENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta - (I_a + I_r)\alpha$$

$$= [.0835(.5179) + .5191(.9405)]32.17\sin 80^\circ - (.8418)4.085$$

$$= 13.4 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .0835(.5179)\sin 80^\circ(32.17) + 21.270(.6042)$$

$$= 14.2 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 110^{\circ}$)

Subject 2

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.0835(.5179) + .5238(.9405)]32.17\sin 70^{\circ} + (.5277)1.362$$

$$= 16.9 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.0835(.5179) + .6337(.9405)]32.17\sin 70^{\circ}$$

$$= 19.3 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta - (I_a + I_r)\alpha$$

$$= [.0835(.5179) + .8614(.9405)]32.17\sin 70^{\circ} - (.8711)10.704$$

$$= 16.5 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .0835(.5179)\sin 70^{\circ}(32.17) + 26.527(.6823)$$

$$= 19.4 \text{ pound feet}$$

MOMENT CALCULATIONS

(contractions at θ= 80°)

Subject 3

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.0804(.5555) + .5238(1.0245)]32.17\sin 80^\circ + (.6181)7.842$$

$$= 23.3 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.0804(.5555) + .7010(1.0245)]32.17\sin 80^\circ$$

$$= 24.2 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta - (I_a + I_r)\alpha$$

$$= [.0804(.5555) + .9294(1.0245)]32.17\sin 80^\circ - (1.0851)3.409$$

$$= 27.9 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .0804(.5555)\sin 80^\circ(32.17) + 34.564(.7033)$$

$$= 25.7 \text{ pound feet}$$

D

MOMENT CALCULATIONS
(contractions at $\theta = 110^{\circ}$)

Subject 3

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.0804(.5555) + .7010(1.0245)]32.17\sin 70^{\circ} + (.\text{[illegible]}93).536$$

$$= 22.6 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.0804(.5555) + .9325(1.0245)]32.17\sin 70^{\circ}$$

$$= 30.2 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta - (I_a + I_r)\alpha$$

$$= [.0804(.5555) + 1.1653(1.0245)]32.17\sin 70^{\circ} - (1.3707)1.072$$

$$= 36.0 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .0804(.5555)\sin 70^{\circ}(32.17) + 40.767(.7292)$$

$$= 31.1 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 80^\circ$)

Subject 4

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta + (I_a + I_r)\alpha$$

$$= [.1088(.5305) + .9042(.9721)]32.17\sin 80^\circ + (.9757)3.234$$

$$= 26.5 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.1088(.5305) + 1.2243(.9721)]\ 32.17\sin 80^\circ$$

$$= 39.5 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)32.17\sin\theta - (I_a + I_r)\alpha$$

$$= [.1088(.5305) + 1.4372(.9721)]32.17\sin 80^\circ - (1.5462)1.348$$

$$= 44.0 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .1088(.5305)\sin 80^\circ(32.17) + 53.175(.8021)$$

$$= 44.5 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 110^\circ$)

Subject 4

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r) 32.17 \sin\theta + (I_a + I_r)\alpha$$

$$= [.1088(.5305) + 1.0060(.9721)] 32.17 \sin 70^\circ + (1.0874) 2.259$$

$$= 28.8 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.1088(.5305) + 1.255(.9721)] 32.17 \sin 70^\circ$$

$$= 38.6 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r) 32.17 \sin\theta - (I_a + I_r)\alpha$$

$$= [.1088(.5305) + 1.6653(.9721)] 32.17 \sin 70^\circ - (1.7868) 7.061$$

$$= 38.1 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .1088(.5305) \sin 70^\circ (32.17) + 57.606(.8021)$$

$$= 47.9 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 80^\circ$)

Subject 5

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta + (I_a + I_r)\alpha$$

$$= [.1109(.5433) + .8435(1.0044)]32.17\sin 80^\circ + (.9769)2.348$$

$$= 31.0 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\, 32.17 \sin\theta$$

$$= [.1109(.5433) + 1.0044(1.0044)]32.17\sin 80^\circ$$

$$= 33.9 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta \text{ [illegible] } (I_a + I_r)\alpha$$

$$= [.1109(.5433) + 1.6323(1.0044)]32.17\sin 80^\circ - (1.9076)5.557$$

$$= 43.2 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\, (32.17) + Rr$$

$$= .1109(.5433)\sin 80^\circ(32.17) + 53.685(.6823)$$

$$= 38.5 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 110^\circ$)

Subject 5

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta + (I_a + I_r)\alpha$$

$$= [.1109(.5433) + .8451(1.0044)]32.17\sin 70^\circ + (.0458 + .9323)2.186$$

$$= 29.6 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.1109(.5433) + 1.4124(1.0044)]32.17\sin 70^\circ$$

$$= 44.7 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta - (I_a + I_r)\alpha$$

$$= [.1109(.5433) + 1.6323(1.0044)]32.17\sin 70^\circ - (1.9076)3.546$$

$$= 44.6 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .1109(.5433)\sin 70^\circ(32.17) + 59.053(.6875)$$

$$= 42.4 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at θ= 80°)

Subject 6

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta + (I_a + I_r)\alpha$$

$$= [.1005(.5614) + 1.0013(1.0242)]32.17\sin 80^\circ + (.0435 + 1.1474)1.364$$

$$= 35.9 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.1005(.5614) + 1.1785(1.0242)]32.17\sin 80^\circ$$

$$= 40.0 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta - (I_a + I_r)\alpha$$

$$= [.1005(.5614) + 1.6607(1.0242)]32.17\sin 80^\circ - (1.96)6.309$$

$$= 43.3 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .1005(.5641)\sin 80^\circ(32.17) + 46.971(.7604)$$

$$= 37.5 \text{ pound feet}$$

MOMENT CALCULATIONS
(contractions at $\theta = 110^\circ$)

Subject 6

CONCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta + (I_a + I_r)\alpha$$

$$= [.1005(.5614) + 1.0021(1.0242)]32.17\sin 70^\circ - (.0435 + 1.1443)4.468$$

$$= 27.4 \text{ pound feet}$$

ISOMETRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\ 32.17\ \sin\theta$$

$$= [.1005(.5614) + 1.2550(1.0242)]32.17\sin 70^\circ$$

$$= 40.6 \text{ pound feet}$$

ECCENTRIC DUMBELL

$$\Sigma M_o = (m_a r_a + m_r r_r)\sin\theta - (I_a + I_r)\alpha$$

$$= [.1005(.5614) + 1.6607(1.0242)]32.17\sin 70^\circ - (1.96)(3.351)$$

$$= 46.6 \text{ pound feet}$$

ISOMETRIC WITH DYNAMOMETER

$$\Sigma M_o = m_a r_a \sin\theta\ (32.17) + Rr$$

$$= .1005(.5641)\sin 70^\circ(32.17) + 51.402(.7917)$$

$$= 42.4 \text{ pound feet}$$

APPENDIX F

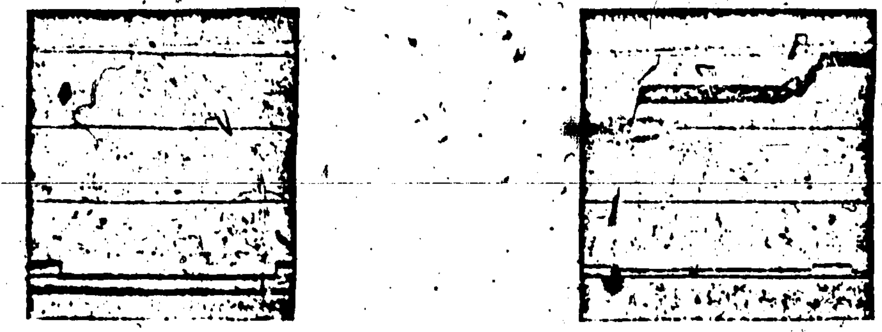
DYNAMOMETER DATA

Two maximal isometric contractions performed on the dynamometer described by Karpovich and Karpovich (1969) were recorded and included in this report as an additional isometric strength measurement. The two contractions were performed at an angle of eighty (80) and one-hundred and ten (110) degrees. This section of the report contains the results of the tests.

A problem was encountered with the recording apparatus which had the potential to seriously affect the results. Exhibit 1 displays two parts of the tracing used to calibrate the dynamometer output. Part "a" shows two horizontal lines. The uppermost line was the base line and a close examination of same revealed that it had shifted first downward and then upward. Part "b" was the actual tracing used in the calibration process and it was noticed that the base line had shifted several times there also. Another student who was using the equipment for his thesis was instructed by Dr. Singh, who was in charge of the machine, to use the level of the line which predominated before the testing began. The lower portion of the line was therefore chosen as the reference point.

Forty-six point zero nine (46.09) pounds was loaded onto the lever arm of the dynamometer. The tracing showed a deflection of thirteen (13) units. The calibration process indicated three point five four five (3.545) pounds per unit.

EXHIBIT 1



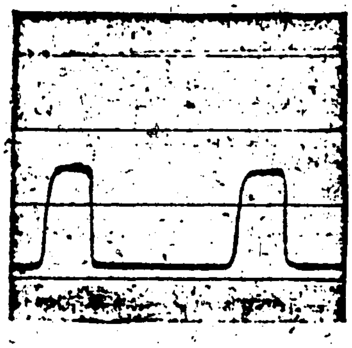
(a) (b)

Exhibit 2 displays the tracings for the various subjects which were tested the same day that the above calibration took place.

EXHIBIT 2

$\theta = 80^{o}$ $\theta = 110^{o}$

Subject 1



Subject 2

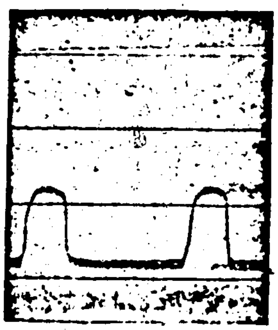
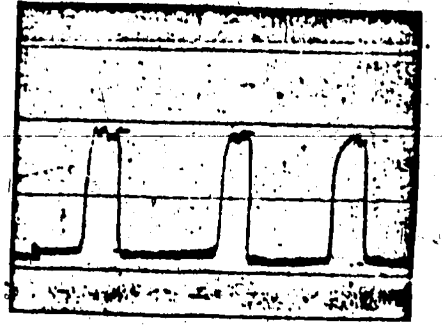
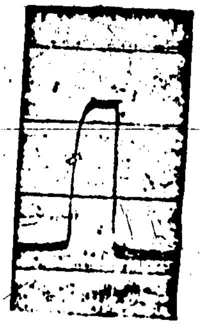


EXHIBIT 2 cont'd

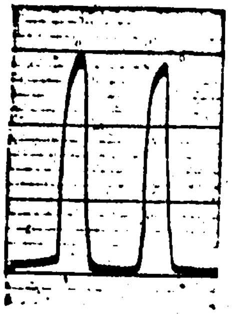
$\theta = 80^\circ$ $\theta = 110^\circ$

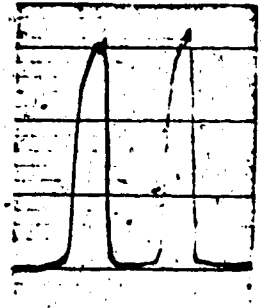
Subject 3



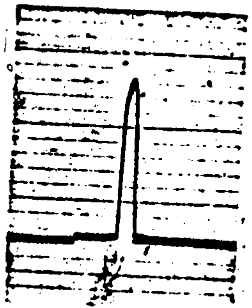


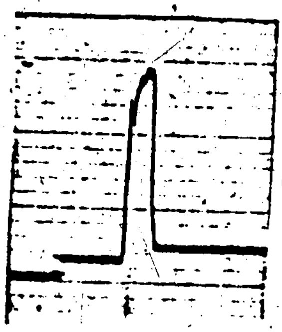
Subject 4





Subject 6

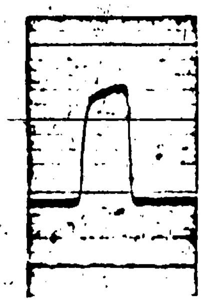




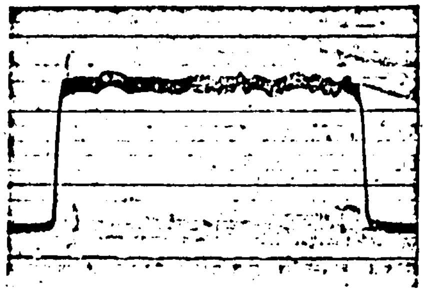
The data displayed in Exhibit 2 was deficient with regard to the contractions of subjects one, two and five. For a variety of reasons, a retest of each subject was performed. In the case of subjects one and two the retest was conducted after it was suspected that a maximal output had not been given. In the case of subject five, the original results were lost. The results of the retest have been listed in Exhibit 3 and both the test and calibration recording have been included.

EXHIBIT 3

$\theta = 110^\circ$



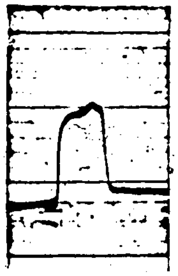
Calibration



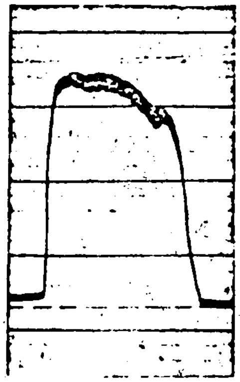
Subject 1

Forty-two point two six (42.26) pounds was loaded onto the lever arm of the dynamometer. The tracing produced a deflection of twelve (12) units. The calibration process indicated three point five two two (3.522) pounds per unit.

$\theta = 110^\circ$



Calibration



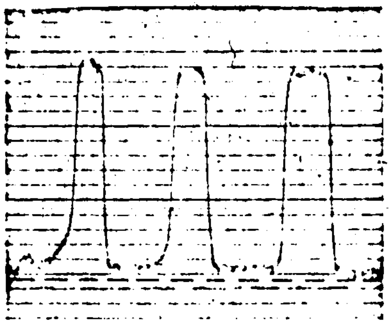
Subject 2

Fifty-three point zero six two (53.062) pounds was loaded onto the arm of the dynamometer. The tracing indicated a deflection of fifteen (15) units. The calibration process indicated three point five three seven (3.537) pounds per unit.

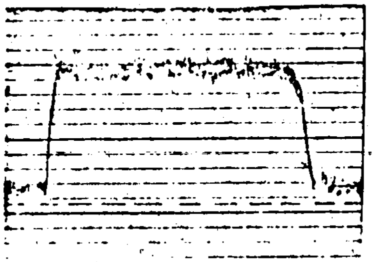
EXHIBIT 3 cont'd

Subject 5

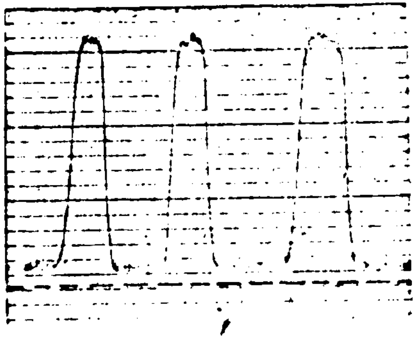
θ = 80°



Calibration



θ = 110°



Thirty-four (34) pounds was loaded onto the arm of the dynamometer. The tracing indicated a deflection of Nine point five (9.5) units. The calibration process indicated three point five four five (3.545) pounds per unit.

Table 10 contains the data used to compute the R reading for each individual.

TABLE 10

The Determination of R

(the dynamometer torque reading)

	Subject	Deflection (units)	Calibration (pounds/unit)	R (pounds)
80°	1	7.75	3.545	27.474
80°	2	6.00	3.545	21.270
80°	3	9.75	3.545	34.564
80°	4	15.00	3.545	53.175
80°	5	15.00	3.579	53.685
80°	6	13.25	3.545	46.971
110°	1	12.00	3.522	42.264
110°	2	7.50	3.537	26.527
110°	3	11.50	3.545	40.767
110°	4	16.25	3.545	57.606
110°	5	16.50	3.579	59.053
110°	6	14.50	3.545	51.402