THE UNIVERSITY OF ALBERTA

APPLICATION OF THE SMITH PREDICTOR TO MULTIVARIABLE SYSTEMS WITH TIME DELAYS

BY

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A THESIS

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ABSTRACT

The classical Smith Predictor method for single variable systems with time delays is extended to a class of linear multivariable systems. Derivations of the multivariable Smith Predictor are presented for both continuous time and discrete time systems which contain time delays in the control variables and/or output variables. As in the classical method, use of the multivariable Smith Predictor eliminates the time delays from the characteristic equation of the closed-loop system. The multivariable Smith Predictor is applied to a double effect evaporator pilot plant in both simulation and experimental runs and the effect of modelling errors on the predictor response is also examined in the simulation runs.

Finally, algorithms are derived for more complex cases such as systems with only some delayed control variables or with time varying or inaccurately determined delays.

The simulated and experimental results demonstrate the ability of the multivariable Smith Predictor method to efficiently handle delays in the control and/or output variables while maintaining freedom in the design of the feedback control matrix.

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CHAPTER ONE

INTRODUCTION

The existence of time delays is of common occurrence in many process control systems. In fact, classical methods for process modelling often include a time delay in the system transfer function.

Time delays often occur due to transportation lines for fluids, pneumatic instrument signals, etc. or due to the time delays associated with measurements (e.g. time required to analyze chemical composition).

The detrimental effects of time delays on system stability and control are well-known to both control system designers and personnel responsible for plant operation. From the classical viewpoint, the phase lag introduced by a time delay tends to reduce system stability and make satisfactory control more difficult to achieve. Furthermore, time delays greatly complicate control system design for multivariable systems since many design approaches are either not applicable to time delay systems or the resulting control system may be unduly complicated and difficult to apply.

1.1 Literature Survey

Several methods for the control of multivariable systems with time delays have been proposed in the literature. For example, optimal control design methods have been extended to systems with time delays [1-4], but except for special cases, the derivation of the control law is often difficult, relies heavily on complicated control algorithms and consequently, actual implementation of these optimal control algorithms in process control systems is seldom attractive.

On the other hand, various methods for single variable control systems have been reported in the literature that utilize a predictive approach for compensation of the time delays, but it appears that these methods have been applied to actual processes in only very few cases [5-12]. These predictive methods involve the use of a time delay in a feedback loop around the controller since a controller which uses the information that the process has a delay should be able to make more intelligent corrections than a controller that receives only the delayed error signal.

An efficient continuous time predictor, (the Smith Linear Predictor), was derived by Smith [5,6], for single variable systems. Applications of this predictor scheme and comparative studies with classical I, PI and PID controllers were reported in the literature [7-10], for continuous and sampled data systems. The conclusions resulting from these studies do not agree in all the cases especially as far as the effectiveness of the Smith Linear Predictor for regulatory control is concerned. Shinskey [7], and Nielsen [8] reported that the Linear Predictor and the similar "complementary feedback technique" are of questionable value for regulatory control of time delay processes while Buckley [9,10] and Lupfer and Oglesby [11,12] applied the Smith Linear Predictor on actual processes with excellent results for regulatory control. Single variable, sampled data controllers utilizing the Smith Predictor approach have been designed and simulated results have been reported in the literature [13,14]. Moore [15] derived a digital algorithm for single variable discrete-time systems with time delays that is conceptually similar to the Smith Linear Predictor, the difference being mainly a more extensive use of the mathematical model of the process than in the Smith Predictor scheme. Results for actual processes with this predictor indicate a satisfactory regulatory control and better performance than the classical methods of I, PI and PID control [15].

Although many studies have considered predictors for single variable systems, no suitable predictor has been reported for multivariable systems with time delays. The analysis required presents more difficulties and complexities than in the single variable case since, typically, combinations of delayed and undelayed process variables and/or measurements occur in multivariable control systems.

Jacobson [16] proposed an extension of the digital predictor algorithm derived by Moore [15] to compensate for the measurement delay introduced by the digital interface in computer control systems. He applied this scheme to a double effect evaporator pilot plant at the University of Alberta with satisfactory results; however, no general method has been reported for the more general case of time delays in multivariable control systems.

1.2 Objectives of the Study

This study is concerned with the development of a suitable predictor scheme for the control of multivariable systems that contain time delays. The Smith Linear Predictor was extended to the case of multivariable systems. Since the Smith Linear Predictor permits design of the control by using methods suitable for undelayed systems, this method was considered particularly suitable for multivariable systems.

The derivation of the control algorithm had to conform to the following requirements:

(i) Satisfactory control for linear multivariable systems with time delays in the manipulated and/or the measurement variables.

(ii) Ability to handle the cases when only some of the control variables or some of the measurement variables are delayed.

(iii) Easy implementation of the control algorithm on a digital control computer.

1.3 Structure of the Thesis

The thesis is concerned with the derivation and evaluation of a predictor algorithm for several classes of multivariable systems with time delays.

In Chapter Two, the theoretical derivation is presented for certain continuous and discrete time systems together with a theoretical investigation of more complex cases and techniques for situations encountered often in practice. In Chapter Three, simulated results of the multivariable predictor are given for the double effect evaporator model and several combinations of measurement and process time delays. In Chapter Four, the results of an experimental application of the multivariable predictor on the evaporator pilot plant, using the IBM 1800 process control computer are given and in Chapter Five, the overall conclusions are given for the multivariable predictor control scheme developed in this thesis.

CHAPTER TWO

THEORETICAL DEVELOPMENT

2.1 Introduction

In 1957, O.J.M. Smith (Smith 1957, 1959) proposed a control technique for single variable control systems which contain time delays. This technique, which became known as the Smith Predictor (or Smith Linear Predictor), is illustrated in Figures 2.1 and 2.2. The chief advantage of the Smith Predictor method is that time delays are eliminated from the characteristic equation of the closed-loop system. This is achieved by including a mathematical model of the process in the feedback loop around the controller.

The output of the predictor block in Figure 2.2 is the difference between two model responses: the response of the system without the time delay minus the response of the system with the time delay. If the process models were perfect, then the actual process response, y(s), would be cancelled by the model response and the control action would be based on the response of the model without time delay. For the control system in Figure 2.2, the closed-loop transfer function for load changes is

$$\frac{x(s)}{d(s)} = G_L(s) - \frac{G_c(s)G_p(s)G_L(s)e^{-as}}{1 + G_c(s)G_p(s)H(s)} \qquad (2.1)$$

The characteristic equation for the closed-loop system in Figure 2.2 is given by

$$1 + G_c(s)G_p(s)H(s) = 0 \qquad (2.2)$$

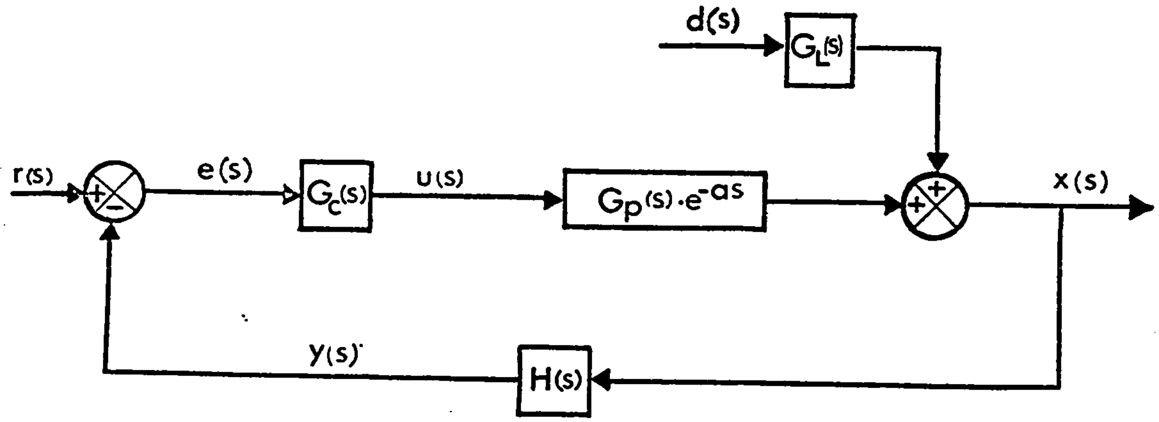

Figure 2.1 Single Variable Control System with a Time Delay in the Process Transfer Function

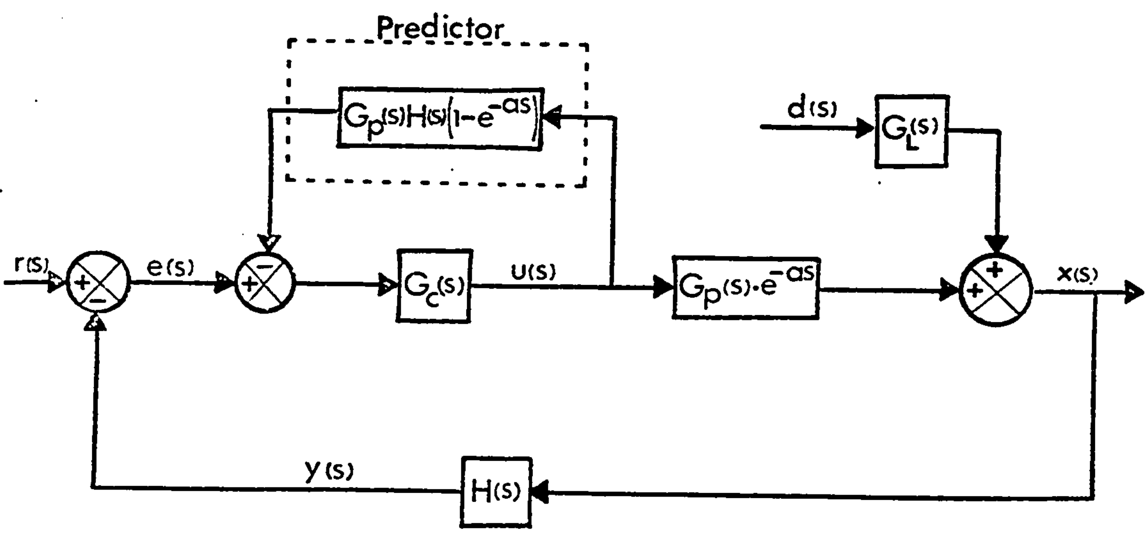

Figure 2.2 Smith Predictor for the Feedback Control of Figure 2.1

which is also the characteristic equation for the system in Figure 2.1 when the time delay is zero. Thus the Smith Predictor has successfully eliminated the time delay from the characteristic equation and the design of a suitable control matrix can be done with any method appropriate for undelayed systems.

2.2 Multivariable Continuous Time Systems

Consider the following linear, stationary, state-space model with time delays in the output variables and the control variables:

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\ \underline{x}(t) + \underline{\underline{B}}\ \underline{u}(t - a) + \underline{\underline{D}}\ \underline{d}(t) \tag{2.3}$$

$$\underline{y}(t) = \underline{\underline{C}}_1\ \underline{x}(t) + \underline{\underline{C}}_2\ \underline{x}(t - b) \tag{2.4}$$

where: $\underline{x}(t)$ = state vector of dimension n,

$\underline{u}(t)$ = control vector of dimension m,

$\underline{d}(t)$ = disturbance vector of dimension p,

$\underline{y}(t)$ = output vector of dimension r,

a,b = constant time delays

$\underline{\underline{A}}, \underline{\underline{B}}, \underline{\underline{C}}_1, \underline{\underline{C}}_2$ and $\underline{\underline{D}}$ are constant, real matrices of appropriate dimensions.

The time delays in this state-space model can be given the following physical interpretation. Time delay a, is associated with the calculation and implementation of control, that is, a time delay in all control variables. The more general case of some control variables delayed and some not is theoretically examined in § 2.4. In many physical systems, time delays are also associated with the measurement of certain state variables. A notable example in process control systems is chemical composition. This variable is, in general, difficult to measure and often requires a period of time to carry out the analysis, i.e. a time delay. The inclusion of time delay, b, in Equation (2.4) provides a general model for systems in which some state variables can be measured instantaneously but time delays are involved in the measurement of other state variables. Apparently, the output equation in Equation (2.4) has not been widely used in previous investigations of time delay systems despite its obvious practical importance.

2.2.1 Time Delay in Output Variables

If no time delay is present in the control variables (i.e. a = 0 in Equation (2.3)), the state-space model in Equations (2.3) and (2.4) reduces to

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{\underline{B}}\,\underline{u}(t) + \underline{\underline{D}}\,\underline{d}(t) \tag{2.5}$$

$$\underline{y}(t) = \underline{\underline{C}}_1\,\underline{x}(t) + \underline{\underline{C}}_2\,\underline{x}(t - b) \tag{2.6}$$

Assuming zero initial conditions and taking the Laplace transform of Equations (2.5) and (2.6) gives, after rearrangement,

$$\underline{x}(s) = \underline{\underline{G}}_p(s)\,\underline{u}(s) + \underline{\underline{G}}_L(s)\,\underline{d}(s) \tag{2.7}$$

$$\underline{y}(s) = \underline{\underline{C}}_1\,\underline{x}(s) + \underline{\underline{C}}_2\,e^{-bs}\,\underline{x}(s) \tag{2.8}$$

where $\underline{\underline{I}}$ = nxn identity matrix

$\underline{\underline{G}}_p(s) \equiv (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{\underline{B}}$ = process transfer function matrix

$\underline{\underline{G}}_L(s) \equiv (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{\underline{D}}$ = load transfer function matrix

and it is implicitly assumed throughout this thesis that matrix inverses exist.

Suppose an output feedback control law of the form

$$\underline{u}(s) = -\underline{\underline{G}}_c(s)\,\underline{y}(s) \tag{2.9}$$

is assumed where $\underline{\underline{G}}_c(s)$ is the matrix of feedback controller transfer functions. Then combining Equations (2.7) - (2.9) gives the following characteristic equation:

$$\left|\, \underline{\underline{I}}_n + \underline{\underline{G}}_p(s)\underline{\underline{G}}_c(s)(\underline{\underline{C}}_1 + \underline{\underline{C}}_2 e^{-bs}) \,\right| = 0 \tag{2.10}$$

where $\underline{\underline{I}}_n$ is an n x n identity matrix and the symbol | | denotes the determinant.

Next, it will be demonstrated that the Smith Predictor method can be used to eliminate the time delay from the characteristic equation. Consider the block diagram shown in Figure 2.3. As for the single variable system in § 2.1, the feedback loop around the controller contains a mathematical model of the process, both with and without time delays. From the block diagram in Figure 2.3, it follows that for $\underline{r}(s) = \underline{0}$,

$$\underline{u}(s) = -\underline{\underline{G}}_c\,\underline{y}(s) - \underline{\underline{G}}_c\,\underline{\underline{C}}_2\,\underline{\underline{G}}_p(1 - e^{-bs})\,\underline{u}(s) \tag{2.11}$$

where $\underline{\underline{G}}_c$ and $\underline{\underline{G}}_p$ denote $\underline{\underline{G}}_c(s)$ and $\underline{\underline{G}}_p(s)$, respectively. Rearranging Equation (2.11) gives the following control algorithm

$$\underline{u}(s) = -[\underline{\underline{I}}_r+\underline{\underline{G}}_c\underline{\underline{C}}_2\underline{\underline{G}}_p]^{-1}\,\underline{\underline{G}}_c[\underline{y}(s)-\underline{\underline{C}}_2\underline{\underline{G}}_p e^{-bs}\,\underline{u}(s)] \tag{2.12}$$

Combining Equations (2.11), (2.7) and (2.8) gives

$$\underline{x}(s) = \underline{\underline{G}}_L\underline{d}(s) - \underline{\underline{G}}_p\underline{\underline{G}}_c\underline{\underline{C}}_1\underline{x}(s) - \underline{\underline{G}}_p\underline{\underline{G}}_c\underline{\underline{C}}_2(\underline{x}(s)-\underline{\underline{G}}_L\underline{d}(s))$$

$$- \underline{\underline{G}}_p\underline{\underline{G}}_c\underline{\underline{C}}_2 e^{-bs}\underline{\underline{G}}_L\underline{d}(s) \tag{2.13}$$

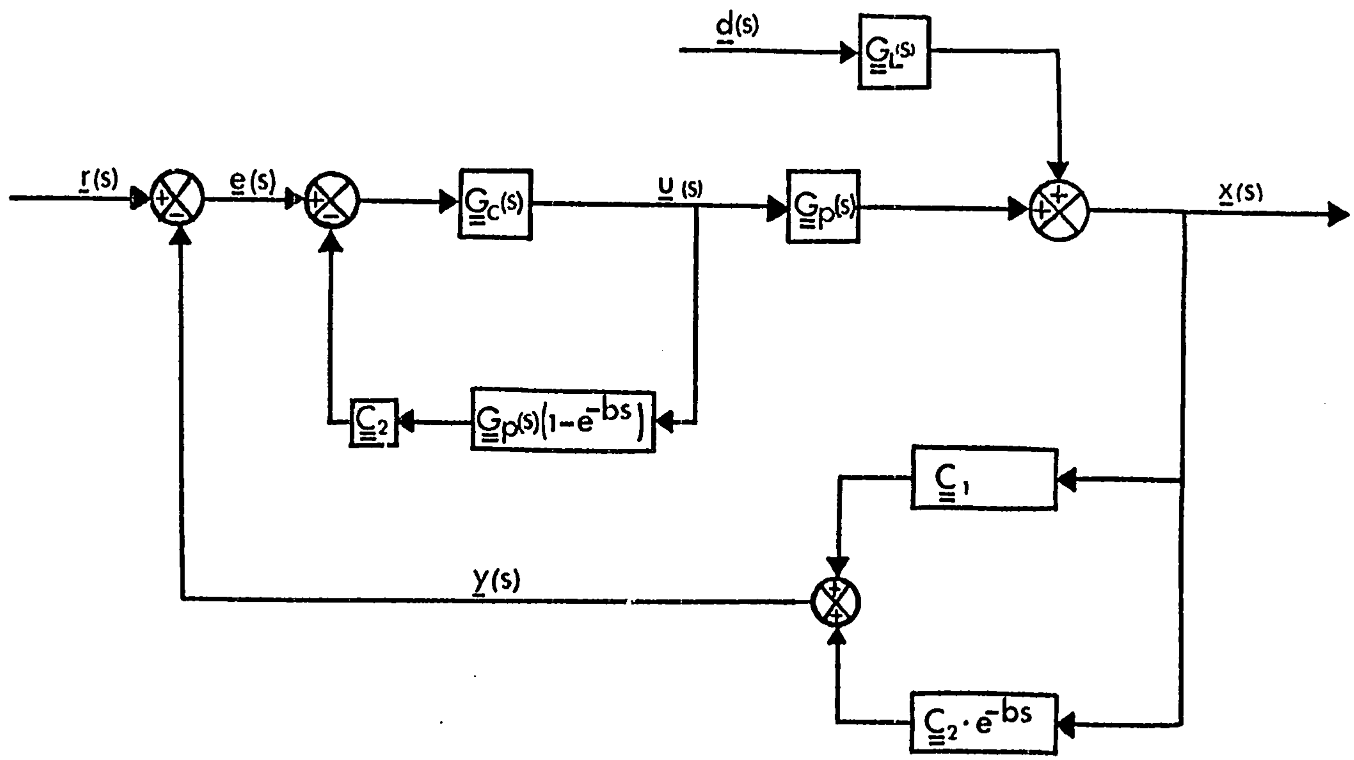

Figure 2.3 Smith Predictor for a Multivariable System with a Time Delay in the Output Variables

or rearranging gives

$$\underline{x}(s) = [\underline{\underline{I}}_n+\underline{\underline{G}}_p\underline{\underline{G}}_c(\underline{\underline{C}}_1+\underline{\underline{C}}_2)]^{-1}[\underline{\underline{I}}_n+\underline{\underline{G}}_p\underline{\underline{G}}_c\underline{\underline{C}}_2(1-e^{-bs})]\underline{\underline{G}}_L\underline{d}(s) \qquad (2.14)$$

From Equation (2.14) it is apparent that the characteristic equation for the closed loop system is

$$|\underline{\underline{I}}_n + \underline{\underline{G}}_p\underline{\underline{G}}_c(\underline{\underline{C}}_1 + \underline{\underline{C}}_2)| = 0 \qquad (2.15)$$

which is also the characteristic equation for the system without time delays (i.e. b = 0 in Equation (2.6)). Thus the multivariable Smith Predictor has eliminated the time delay from the characteristic equation. The design of the controller transfer function matrix $\underline{\underline{G}}_c(s)$ can then proceed using design techniques developed for systems without time delays.

2.2.2 Time Delays in Both Control Variables and Output Variables

The case is considered where constant time delays occur in both the control variables and the measured outputs. The system of interest is given in Equations (2.3) and (2.4). Assuming zero initial conditions and taking the Laplace transform of Equations (2.3) and (2.4) gives

$$\underline{x}(s) = \underline{\underline{G}}_p\, e^{-as}\, \underline{u}(s) + \underline{\underline{G}}_L\, \underline{d}(s) \qquad (2.16)$$

$$\underline{y}(s) = \underline{\underline{C}}_1\, \underline{x}(s) + \underline{\underline{C}}_2\, e^{-bs}\, \underline{x}(s) \qquad (2.17)$$

The multivariable Smith Predictor for this system consists of two feedback loops around the controller as shown in Figure 2.4. From Figure 2.4 it follows that for $\underline{r}(s) = \underline{0}$,

$$\underline{u}(s) = -\underline{\underline{G}}_c\underline{y}(s) - \underline{\underline{G}}_c(\underline{\underline{C}}_1+\underline{\underline{C}}_2)\underline{\underline{G}}_p\underline{u}(s) + \underline{\underline{G}}_c\underline{\underline{C}}_1\underline{\underline{G}}_pe^{-as}\underline{u}(s) + \underline{\underline{G}}_c\underline{\underline{C}}_2\underline{\underline{G}}_pe^{-(a+b)s}\underline{u}(s) \qquad (2.18)$$

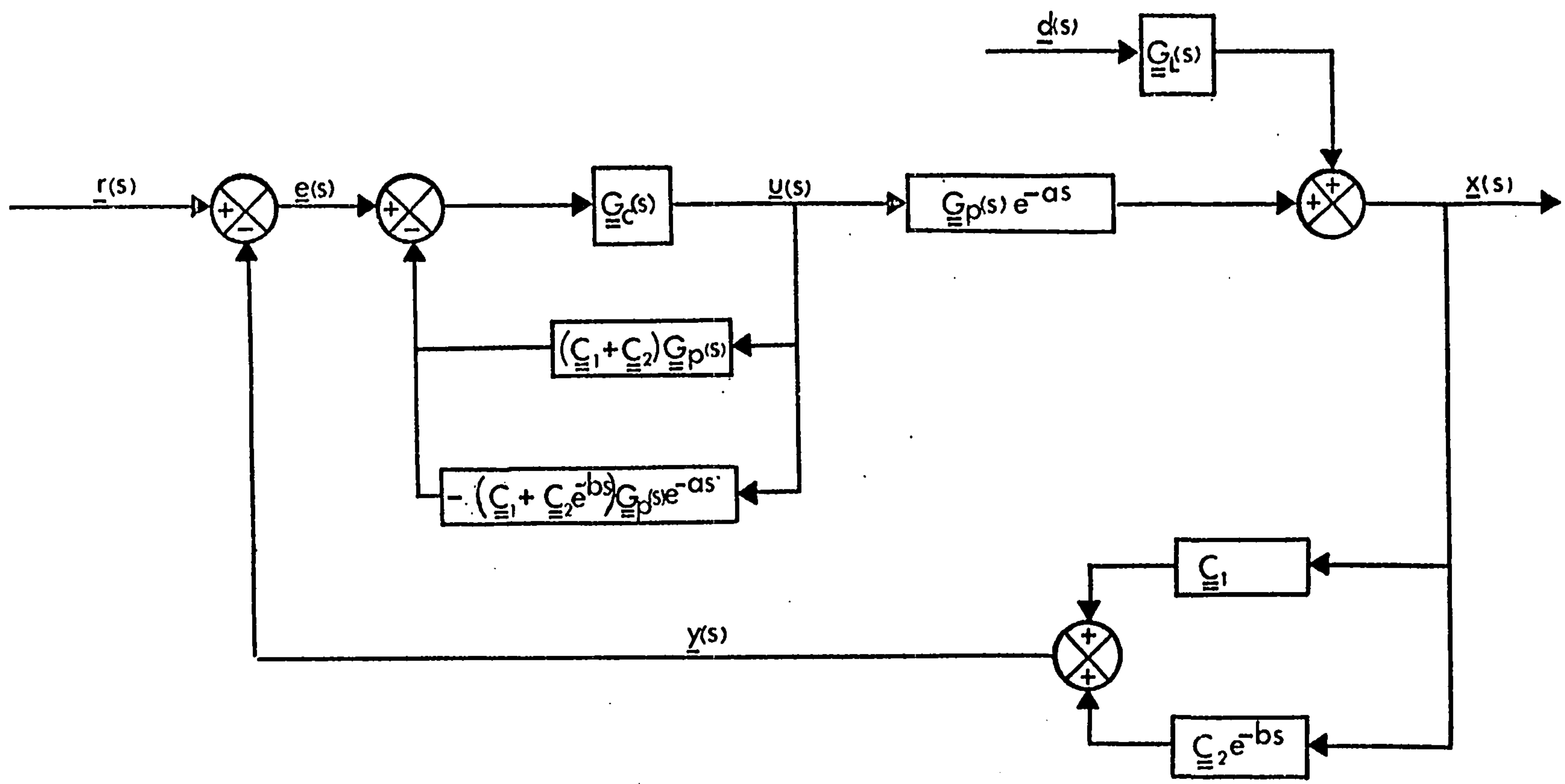

Figure 2.4 Smith Predictor for a Multivariable System with Time Delays in Both Control and Output Variables

or rearranging gives the following control algorithm

$$\underline{u}(s) = -[\underline{\underline{I}}_m+\underline{\underline{G}}_c(\underline{\underline{C}}_1+\underline{\underline{C}}_2)\underline{\underline{G}}_p]^{-1}\ \underline{\underline{G}}_c[\underline{y}(s)-(\underline{\underline{C}}_1-\underline{\underline{C}}_2\ e^{-bs})e^{-as}\ \underline{\underline{G}}_p\underline{u}(s)] \tag{2.19}$$

Combining Equations (2.16), (2.17) and (2.18) gives the result

$$\underline{x}(s) = [\underline{\underline{I}}_n+\underline{\underline{G}}_p\underline{\underline{G}}_c(\underline{\underline{C}}_1+\underline{\underline{C}}_2)]^{-1}[\underline{\underline{I}}_n+\underline{\underline{G}}_p\underline{\underline{G}}_c[(1-e^{-as})\underline{\underline{C}}_1+(1-e^{-(a+b)s})\underline{\underline{C}}_2]]\underline{\underline{G}}_L\underline{d}(s) \tag{2.20}$$

From Equation (2.20) it follows that the characteristic equation for the closed loop system is

$$|\underline{\underline{I}}_n + \underline{\underline{G}}_p\underline{\underline{G}}_c(\underline{\underline{C}}_1+\underline{\underline{C}}_2)| = 0 \tag{2.21}$$

Thus, the Smith Predictor is again successful in eliminating time delays from the characteristic equation. Furthermore, the resulting characteristic equation in Equation (2.21) can easily be shown to be the characteristic equation for the system without time delays (i.e. the system in Equations (2.3) and (2.4) with a = b = 0).

2.3 Multivariable Discrete-Time Systems

A multivariable Smith Predictor can be developed in an analogous fashion for discrete-time systems which contain time delays. Consider the following stationary state-space model

$$\underline{x}(n+1) = \underline{\underline{\phi}}\ \underline{x}(n) + \underline{\underline{\theta}}\ \underline{u}(n-a) + \underline{\underline{\Delta}}\ \underline{d}(n) \tag{2.22}$$

$$\underline{y}(n) = \underline{\underline{C}}_1\ \underline{x}(n) + \underline{\underline{C}}_2\ \underline{x}(n-b) \tag{2.23}$$

where $\underline{x}(n)$ = state vector of dimension n,

$\underline{u}(n)$ = manipulated vector of dimension m,

$\underline{d}(n)$ = disturbance vector of dimension p,

$\underline{y}(n)$ = output vector of dimension r,

a,b = constant time delays which are integers (i.e. positive integer multiples of the sampling period, T)

$\underline{\underline{\phi}}, \underline{\underline{\theta}}, \underline{\underline{\Delta}}, \underline{\underline{C}}_1$ and $\underline{\underline{C}}_2$ are constant matrices of appropriate dimensions.

As in the continuous time case, a multivariable Smith Predictor algorithm can be developed by considering feedback loops around the controller. Suppose a proportional feedback control law is assumed of the form

$$\underline{u}(n) = - \underline{\underline{K}}_c \, \underline{y}(n) \tag{2.24}$$

where $\underline{\underline{K}}_c$ is a constant m x r matrix. Then a suitable algorithm for the Smith Predictor is

$$\underline{u}(n) = - \underline{\underline{K}}_c \, \underline{y}(n) - \underline{\underline{K}}_c \underline{p}(n) \tag{2.25}$$

$$\underline{p}(n) = \underline{\underline{C}}_1 \underline{p}_1(n) + \underline{\underline{C}}_2 \underline{p}_2(n) \tag{2.26}$$

where

$$\underline{p}_1(n) = \underline{\underline{\phi}} \, \underline{p}_1(n-1) + \underline{\underline{\theta}} \, \underline{u}(n-1) - \underline{\underline{\theta}} \, \underline{u}(n-a-1) \text{ for } n \geq 1 \tag{2.27}$$

$$\underline{p}_2(n) = \underline{\underline{\phi}} \, \underline{p}_2(n-1) + \underline{\underline{\theta}} \, \underline{u}(n-1) - \underline{\underline{\theta}} \, \underline{u}(n-a-b-1) \text{ for } n \geq 1 \tag{2.28}$$

and

$$\underline{p}_1(0) = \underline{p}_2(0) = \underline{0}$$

This formulation of the predictor algorithm eliminates the time delays from the characteristic equation of the closed-loop system as it is shown below. Taking the z-transform of Equations (2.22), (2.23) and (2.25) - (2.28) gives

$$z\,\underline{X}(z) = \underline{\underline{M}}\underline{\underline{\theta}} z^{-a}\,\underline{U}(z) + \underline{\underline{M}}\underline{\underline{\Delta}}\underline{D}(z) \quad (2.29)$$

$$\underline{Y}(z) = \underline{\underline{C}}_1\underline{X}(z) + \underline{\underline{C}}_2 z^{-b}\underline{X}(z) \quad (2.30)$$

$$\underline{U}(z) = \underline{\underline{K}}_c\underline{Y}(z) - \underline{\underline{K}}_c\underline{P}(z) \quad (2.31)$$

$$\underline{P}(z) = \underline{\underline{C}}_1\underline{P}_1(z) + \underline{\underline{C}}_2\underline{P}_2(z) \quad (2.32)$$

$$\underline{P}_1(z) = z^{-1}[\underline{\underline{\phi}}\underline{P}_1(z) + \underline{\underline{\theta}}\underline{U}(z) - \underline{\underline{\theta}} z^{-a}\underline{U}(z)] \quad (2.33)$$

$$\underline{P}_2(z) = z^{-1}[\underline{\underline{\phi}}\underline{P}_2(z) + \underline{\underline{\theta}}\underline{U}(z) - z^{-(a+b)}\underline{\underline{\theta}}\underline{U}(z)] \quad (2.34)$$

where, the notation, $\underline{X}(z)$ = z-transform of $\underline{x}(n)$, is used. Equations (2.29) - (2.34) can be combined and rearranged to give the following expression:

$$\underline{X}(z) = [z\underline{\underline{I}}_n + \underline{\underline{M}}\underline{\underline{\theta}}\underline{\underline{K}}_c(\underline{\underline{C}}_1+\underline{\underline{C}}_2)]^{-1}[\underline{\underline{I}}_n + \underline{\underline{M}}\underline{\underline{\theta}}\underline{\underline{K}}_c\underline{\underline{C}}_1(1-z^{-a})\,z^{-1} + \underline{\underline{M}}\underline{\underline{\theta}}\underline{\underline{K}}_c\underline{\underline{C}}_2(1-z^{-(a+b)})z^{-1}]\,\underline{\underline{M}}\underline{\underline{\Delta}}\underline{D}(z) \quad (2.35)$$

where $\underline{\underline{M}}(z)$ is defined as

$$\underline{\underline{M}}(z) \equiv (\underline{\underline{I}}_n - z^{-1}\underline{\underline{\phi}})^{-1} \quad (2.36)$$

Equation (2.35) implies that the characteristic equation for the system with the Smith Predictor is

$$\left| z\,\underline{\underline{I}}_n + \underline{\underline{M}}\underline{\underline{\theta}}\underline{\underline{K}}_c(\underline{\underline{C}}_1 + \underline{\underline{C}}_2)\right| = 0 \quad (2.37)$$

It can easily be shown that Equation (2.37) is also the characteristic equation for the original system in Equations (2.22) and (2.23) when the time delays are zero (i.e. a = b = 0). Thus the Smith Predictor algorithm in Equations (2.25) - (2.28) can be used to eliminate time delays from the characteristic equation. The design of the feedback controller matrix $\underline{\underline{K}}_c$ can then proceed using the wide variety of design techniques developed for systems without time delays. The above analysis could also be extended to other types of multivariable control techniques such as proportional plus integral control.

2.4 Application of Predictor Algorithm to Systems With Delay Only in Some Manipulated Variables

2.4.1 Introduction

The case of systems with some control variables delayed and some not can be considered a generalization of the previously considered case of a delay in all of the control variables. From a practical point of view this situation is more commonly found in practice where usually it is expected that the delays in the various control variables will be different or zero (different length of pneumatic transmission lines, etc.).

Assume that a system can be described by the following state-space model:

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{\underline{B}}_1{}'\,\underline{u}_1(t) + \underline{\underline{B}}_2{}'\,\underline{u}_2(t-a) \tag{2.38}$$

$$\underline{y}(t) = \underline{x}(t) \qquad \text{(for simplicity)} \tag{2.39}$$

Here, the control vector has been partitioned as

$$\underline{u}(t) = \begin{bmatrix} \underline{u}_1(t) \\ ----- \\ \underline{u}_2(t) \end{bmatrix} \tag{2.40}$$

where $\underline{u}_1$ denotes the ℓ undelayed control variables and $\underline{u}_2$ the $(m-\ell)$

control variables which are delayed. An equivalent representation of the system in Equations (2.38) and (2.39) is given by

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{\underline{B}}_1\underline{u}(t) + \underline{\underline{B}}_2\underline{u}(t-a) \tag{2.41}$$

$$\underline{y}(t) = \underline{x}(t) \tag{2.42}$$

where $\underline{\underline{B}}_1$ and $\underline{\underline{B}}_2$ are defined by

$$\underline{\underline{B}}_1 \equiv \left[\underline{\underline{B}}_1{}' \;\vdots\; \underline{\underline{0}} \right] \tag{2.43}$$

$$\underline{\underline{B}}_2 = \left[\underline{\underline{B}}_2{}' \;\vdots\; \underline{\underline{0}} \right] \tag{2.44}$$

where $\underline{\underline{B}}_1$ and $\underline{\underline{B}}_2$ are n x m matrices and the notation is the same as in § 2.2 for the other matrices and vectors.

Taking the Laplace transform of Equations (2.41) and (2.42) gives

$$\underline{x}(s) = \underline{\underline{G}}_1(s)\underline{u}(s) + \underline{\underline{G}}_2(s)e^{-as}\underline{u}(s) + \underline{\underline{G}}_2(s)\underline{d}(s) \tag{2.45}$$

$$\underline{y}(s) = \underline{x}(s) \tag{2.46}$$

where

$$\underline{\underline{G}}_1(s) \equiv (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\,\underline{\underline{B}}_1 \tag{2.47}$$

$$\underline{\underline{G}}_2(s) \equiv (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\,\underline{\underline{B}}_2 \tag{2.48}$$

In applying the predictor scheme on a system described by Equations (2.45) and (2.46) a difficulty is encountered due to the interaction between the delayed and undelayed parts of the system.

The search for a suitable predictor scheme for this system was directed towards finding an algorithm that will in effect decouple the system in the sense that any predictive action on the delayed manipulated variables will not affect the undelayed portion of the system.

2.4.2 Theoretical Development

For the system represented by Equation (2.45), the following notation is used.

1. Due to the definition of $\underline{\underline{B}}_1$ and $\underline{\underline{B}}_2$ in Equations (2.43) and (2.44), matrices $\underline{\underline{G}}_1(s)$ and $\underline{\underline{G}}_2(s)$ are of the form

$$\underline{\underline{G}}_1 = \left[\begin{array}{c|c} \underline{\underline{G}}_{11} & \underline{\underline{0}} \\ \hline \underline{\underline{G}}_{12} & \underline{\underline{0}} \end{array}\right] , \quad \underline{\underline{G}}_2 = \left[\begin{array}{c|c} \underline{\underline{0}} & \underline{\underline{G}}_{21} \\ \hline \underline{\underline{0}} & \underline{\underline{G}}_{22} \end{array}\right]$$

where the partitioning serves the purpose of separating the undelayed and delayed control variables.

$\underline{\underline{G}}_1$ and $\underline{\underline{G}}_2$ are n x m matrices and if the number of undelayed manipulated variables is ℓ, the dimensions of the partitions are, $\underline{\underline{G}}_{11} = \ell x \ell$, $\underline{\underline{G}}_{12} = (n-\ell)x\ell$, $\underline{\underline{G}}_{21} = \ell x(m-\ell)$, $\underline{\underline{G}}_{22} = (n-\ell)x(m-\ell)$.

2. Define a matrix $\underline{\underline{K}}$ as

$$K = \left[\begin{array}{c|c} \underline{\underline{0}} & \underline{\underline{0}} \\ \hline \underline{\underline{K}}_1 & \underline{\underline{K}}_2 \end{array}\right]$$

where

$\underline{\underline{K}}$ = mxm matrix

$\underline{\underline{K}}_1 = (m-\ell)x\ell$ matrix

$\underline{\underline{K}}_2 = (m-\ell)x(m-\ell)$ matrix

3. The control matrix $\underline{\underline{G}}_c(s)$ is written as

$$\underline{\underline{G}}_c = \left[\begin{array}{c|c} \underline{\underline{G}}_{c1} & \underline{\underline{G}}_{c2} \\ \hline \underline{\underline{G}}_{c3} & \underline{\underline{G}}_{c4} \end{array}\right]$$

where the dimensions of the partitions are

$\underline{\underline{G}}_c = mxn$, $\underline{\underline{G}}_{c1} = \ell x\ell$, $\underline{\underline{G}}_{c2} = \ell x(n-\ell)$, $\underline{\underline{G}}_{c3} = (m-\ell)x\ell$, $\underline{\underline{G}}_{c4} = (m-\ell)x(n-\ell)$

The assumption is made that all states are measurable.

A satisfactory predictor technique was devised, based on the block diagram in Figure 2.5 and by placing the following two conditions on matrices $\underline{\underline{K}}$ and $\underline{\underline{G}}_c$:

<u>Condition 1</u>

Matrix $\underline{\underline{K}}$ is selected such that

$$\underline{\underline{K}}_1\underline{\underline{G}}_{c1} = (\underline{\underline{I}} - \underline{\underline{K}}_2)\underline{\underline{G}}_{c3} \qquad (2.49)$$

$$\underline{\underline{K}}_1\underline{\underline{G}}_{c2} = (\underline{\underline{I}} - \underline{\underline{K}}_2)\underline{\underline{G}}_{c4} \qquad (2.50)$$

where $\underline{\underline{I}}$ is the $(m-\ell)x(m-\ell)$ unit matrix. This conditions is easily satisfied by selecting $\underline{\underline{K}}_1 = \underline{\underline{0}}$, $\underline{\underline{K}}_2 = \underline{\underline{I}}$.

<u>Condition 2</u>

The restriction is imposed on the partitions $\underline{\underline{G}}_{c3}$ and $\underline{\underline{G}}_{c4}$ of the control matrix that they must satisfy the following relation:

$$\underline{\underline{G}}_{c3}\,\underline{\underline{G}}_{11} + \underline{\underline{G}}_{c4}\,\underline{\underline{G}}_{12} = \underline{\underline{0}} \qquad (2.51)$$

It can easily be shown by direct substitution that if conditions 1 and 2 are satisfied, the following equations hold

$$\underline{\underline{K}}\ \underline{\underline{G}}_c\ \underline{\underline{G}}_1 = \underline{\underline{0}} \tag{2.52}$$

$$\underline{\underline{G}}_2\ \underline{\underline{K}}\ \underline{\underline{G}}_c = \underline{\underline{G}}_2\ \underline{\underline{G}}_c \tag{2.53}$$

$$\underline{\underline{G}}_1\ \underline{\underline{K}}\ \underline{\underline{G}}_c = \underline{\underline{0}} \tag{2.54}$$

By virtue of Equations (2.52) - (2.54) a satisfactory predictor control law can be derived as follows.

Assume that a control law is desired of the form

$$\underline{u}(s) = -\underline{\underline{G}}_c\ \underline{x}(s) - \underline{w}(s) \tag{2.55}$$

where

$\underline{x}(s)$ = measurement vector of dimension n

$\underline{w}(s)$ = predictor output vector of dimension m

It will be shown that if $\underline{w}(s)$ is selected as

$$\underline{w}(s) = \underline{\underline{K}}\ \underline{\underline{G}}_c\ \underline{p}(s) \tag{2.56}$$

where

$\underline{p}(s)$ = predictor output of dimension n

then the effect of the delayed control action can be compensated by a prediction with no effect on the undelayed part of the control action. Substitution of Equation (2.56) into (2.55) gives the following control law

$$\underline{u}(s) = -\underline{\underline{G}}_c\underline{x}(s) - \underline{\underline{K}}\underline{\underline{G}}_c\underline{p}(s) \tag{2.57}$$

Substitution of the expression for $\underline{u}(s)$ from Equation (2.57) into Equation (2.47) gives

$$\underline{x}(s) = - \underline{\underline{G}}_1\underline{\underline{G}}_c\underline{x}(s) - \underline{\underline{G}}_1\underline{\underline{K}}\underline{\underline{G}}_c\underline{p}(s) - \underline{\underline{G}}_2\underline{\underline{G}}_c e^{-as}\underline{x}(s) - \underline{\underline{G}}_2\underline{\underline{K}}\underline{\underline{G}}_c e^{-as}\underline{p}(s) + \underline{\underline{G}}_L\underline{d}(s) \quad (2.58)$$

Combining Equations (2.53), (2.54) and (2.58) gives

$$\underline{x}(s) = - \underline{\underline{G}}_1\underline{\underline{G}}_c\underline{x}(s) - \underline{\underline{G}}_2\underline{\underline{G}}_c e^{-as}\underline{x}(s) - \underline{\underline{G}}_2\underline{\underline{G}}_c e^{-as}\underline{p}(s) + \underline{\underline{G}}_L\underline{d}(s) \quad (2.59)$$

and combining Equations (2.59), (2.52), (2.53) and (2.45) gives

$$\underline{x}(s) = - \underline{\underline{G}}_1\underline{\underline{G}}_c\underline{x}(s) + \underline{\underline{G}}_L\underline{d}(s) - \underline{\underline{G}}_2\underline{\underline{G}}_c\underline{\underline{G}}_L e^{-as}\underline{d}(s) - \underline{\underline{G}}_2\underline{\underline{G}}_c\underline{\underline{G}}_2 e^{-2as}\underline{u}(s) - \underline{\underline{G}}_2\underline{\underline{G}}_L e^{-as}\underline{p}(s) \quad (2.60)$$

or

$$\underline{x}(s) = - \underline{\underline{G}}_1\underline{\underline{G}}_c\underline{x}(s) + [\underline{\underline{I}}_n - \underline{\underline{G}}_2\underline{\underline{G}}_c e^{-as}]\underline{\underline{G}}_L\underline{d}(s) - \underline{\underline{G}}_2\underline{\underline{G}}_c e^{-as}[\underline{\underline{G}}_2 e^{-as}\underline{u}(s) + \underline{p}(s)] \quad (2.61)$$

Application of the Smith Predictor in the feedback loop around the controller will result in the following expression for $\underline{p}(s)$

$$\underline{p}(s) = - \underline{\underline{G}}_2 e^{-as}\underline{u}(s) + \underline{\underline{G}}_2\underline{u}(s) \quad (2.62)$$

Combining Equations (2.62), (2.61) and utilizing Equations (2.52) - (2.54) gives

$$\underline{x}(s) = - (\underline{\underline{G}}_1 + \underline{\underline{G}}_2)\underline{\underline{G}}_c\underline{x}(s) + [\underline{\underline{I}}_n + \underline{\underline{G}}_2\underline{\underline{G}}_c(1 - e^{-as})]\underline{\underline{G}}_L\underline{d}(s) \quad (2.63)$$

or rearranging

$$\underline{x}(s) = [\underline{\underline{I}}_n + (\underline{\underline{G}}_1 + \underline{\underline{G}}_2)\underline{\underline{G}}_c]^{-1} [\underline{\underline{I}}_n + \underline{\underline{G}}_2\underline{\underline{G}}_c(1 - e^{-as})]\underline{\underline{G}}_L\underline{d}(s) \quad (2.64)$$

which gives the closed loop expression for the state vector. From

Equation (2.64) it is obvious that the characteristic equation of the closed loop system is given by

$$|\underline{\underline{I}}_n + (\underline{\underline{G}}_1 + \underline{\underline{G}}_2)\underline{\underline{G}}_c| = 0 \qquad (2.65)$$

which is also the characteristic equation of the system without time delays. It should be noted that Equation (2.65) can be derived from Equation (2.21) by assuming

$$\underline{\underline{G}}_p = \underline{\underline{G}}_1 + \underline{\underline{G}}_2$$

$$\underline{\underline{C}}_1 + \underline{\underline{C}}_2 = \underline{\underline{I}}$$

The case of incomplete state measurement that is when

$$\underline{y}(s) = \underline{\underline{C}}\,\underline{x}(s) \qquad (2.66)$$

where

$\underline{\underline{C}}$ = rxn measurement matrix

can also be included in this scheme by modifying Equation (2.52) as follows

$$\underline{\underline{K}}\,\underline{\underline{G}}_c\,\underline{\underline{C}}\,\underline{\underline{G}}_1 = \underline{\underline{0}} \qquad (2.67)$$

No other modification of Equations (2.52) - (2.54) is needed and the resulting closed loop expression for $\underline{x}(s)$ is

$$\underline{x}(s) = [\underline{\underline{I}}_n + (\underline{\underline{G}}_1+\underline{\underline{G}}_2)\underline{\underline{G}}_c\underline{\underline{C}}]^{-1}\,[\underline{\underline{I}}_n + \underline{\underline{G}}_2\underline{\underline{G}}_c\underline{\underline{C}}(1 - e^{-as})]\underline{\underline{G}}_L\underline{d}(s) \qquad (2.68)$$

2.4.3 Discrete-Time Systems

The discrete time case follows along the same lines. Assume that the state-space equation for the system is given by

$$\underline{x}(n+1) = \underline{\underline{\phi}}\underline{x}(n) + \underline{\underline{\theta}}_1{}'\underline{u}_1(n) + \underline{\underline{\theta}}_2{}'\underline{u}_2(n-a) + \underline{\underline{\Delta}}\underline{d}(n) \qquad (2.69)$$

$$\underline{y}(n) = \underline{x}(n) \qquad (2.70)$$

where the control vector is partitioned as

$$\underline{u}(n) = \begin{bmatrix} \underline{u}_1(n) \\ \text{-------} \\ \underline{u}_2(n-a) \end{bmatrix} \qquad (2.71)$$

An equivalent expression for Equation (2.69) is given by

$$\underline{x}(n+1) = \underline{\underline{\phi}}\underline{x}(n) + \underline{\underline{\theta}}_1\underline{u}(n) + \underline{\underline{\theta}}_2\underline{u}(n-a) + \underline{\underline{\Delta}}\underline{d}(n) \qquad (2.72)$$

where $\underline{\underline{\theta}}_1$ and $\underline{\underline{\theta}}_2$ are defined by

$$\underline{\underline{\theta}}_1 = [\underline{\underline{\theta}}_1{}' \;\vdots\; \underline{\underline{0}}] \qquad (2.73)$$

$$\underline{\underline{\theta}}_2 = [\underline{\underline{0}} \;\vdots\; \underline{\underline{\theta}}_2{}'] \qquad (2.74)$$

and the dimensions are the same as for the continuous time case.

Taking the Z-transformation of Equation (2.72) gives

$$z\,\underline{X}(z) = \underline{\underline{M}}\underline{\underline{\theta}}_1\underline{U}(z) + \underline{\underline{M}}\underline{\underline{\theta}}_2 z^{-a}\underline{U}(z) + \underline{\underline{M}}\underline{\underline{\Delta}}\underline{D}(z) \qquad (2.73)$$

where $\underline{\underline{M}}$ is defined as in Equation (2.36).

If we denote

$$\underline{\underline{M}}\underline{\underline{\theta}}_1 \equiv \underline{\underline{\theta}}_1{}'' \quad , \quad \underline{\underline{M}}\underline{\underline{\theta}}_2 \equiv \underline{\underline{\theta}}_2{}'' \quad , \quad \underline{\underline{M}}\underline{\underline{\Delta}} \equiv \underline{\underline{\Delta}}'' \qquad (2.74)$$

then the conditions described by Equations (2.51) - (2.54) take the following form in discrete time notation

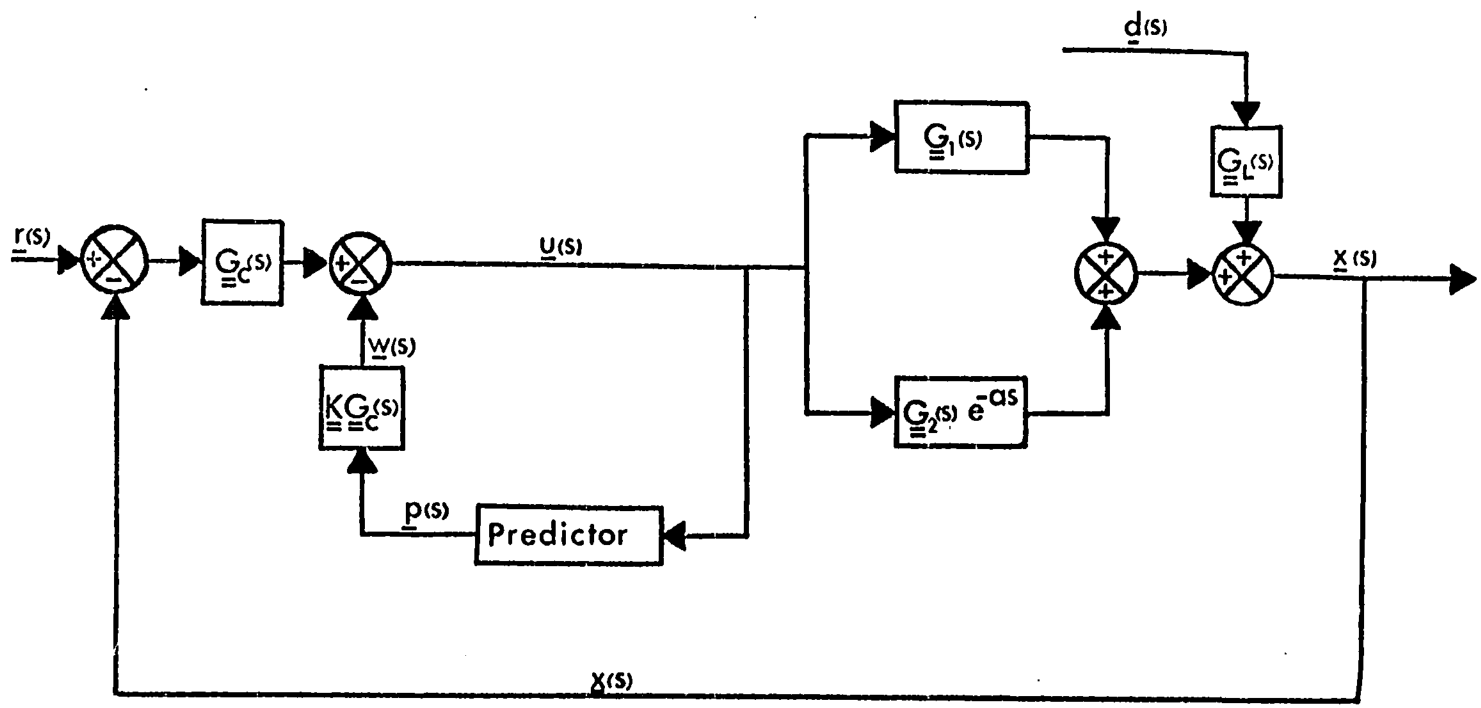

Figure 2.5 Smith Predictor Scheme for the Case of Delays in Some Control Variables

$$\underline{\underline{K}}_{c3}\underline{\underline{\theta}}_{11}'' + \underline{\underline{K}}_{c4}\underline{\underline{\theta}}_{12}'' = \underline{\underline{0}} \tag{2.75}$$

$$\underline{\underline{K}}\,\underline{\underline{K}}_{c}\underline{\underline{\theta}}_{1}'' = \underline{\underline{0}} \tag{2.76}$$

$$\underline{\underline{\theta}}_{2}''\underline{\underline{K}}\,\underline{\underline{K}}_{c} = \underline{\underline{\theta}}_{2}''\underline{\underline{K}}_{c} \tag{2.77}$$

$$\underline{\underline{\theta}}_{1}''\underline{\underline{K}}\,\underline{\underline{K}}_{c} = \underline{\underline{0}} \tag{2.78}$$

where $\underline{\underline{K}}_c$ is the discrete time control matrix, $\underline{\underline{K}}$ is the same matrix as in Equations (2.49), (2.50) and the partitioning of matrices $\underline{\underline{K}}$, $\underline{\underline{K}}_c$, $\underline{\underline{\theta}}_1''$, $\underline{\underline{\theta}}_2''$ is done in the same way as in the continuous time case.

The derivation of the closed loop expression for $\underline{X}(z)$ is easily done by following the similar continuous time procedure and $\underline{X}(z)$ is given by

$$\underline{X}(z) = [z\,\underline{\underline{I}}_n + (\underline{\underline{\theta}}_1'' + \underline{\underline{\theta}}_2'')\underline{\underline{K}}_c]^{-1}\,[\underline{\underline{I}}_n + \underline{\underline{\theta}}_2'\underline{\underline{K}}_c(1-z^{-a})z^{-1}]\underline{\underline{\Delta}}'\underline{D}(z) \tag{2.79}$$

with characteristic equation given by

$$|z\underline{\underline{I}}_n + (\underline{\underline{\theta}}_1'' + \underline{\underline{\theta}}_2'')\underline{\underline{K}}_c| = 0 \tag{2.80}$$

which is also the characteristic equation for the system without time delay.

2.5 Modification of the Predictor Scheme for Compensation of Inaccurately Determined or Time Varying Delays

2.5.1 General

An inaccurate representation of the delay in the predictor loop can occur for the following reasons:

(1) Inaccurate determination of the delay.

(2) Use of an approximation (Padé etc.) for the time delay transfer function.

(3) Time varying delay.

(4) Time delay which is not an integer multiple of the sampling interval in the discrete time case.

The effect of approximating the actual delay by various methods (Padé, etc.) on the response of the closed loop predictor system has been examined by Buckley [9,10] for single variable, continuous time systems. In the multivariable discrete time case, a time delay which is not an exact integer multiple of the sampling interval can be handled by the method used by Jacobson [16] in compensating for a delay equal to half the sampling interval which is a consequence of the digital interface, [15].

2.5.2 Digital Algorithm for the Determination of Inaccurately Known or Time Varying Delays

In this section an algorithm is derived to improve the predictor performance when time varying or inaccurately determined time delays occur. This digital algorithm is realizable if an accurate discrete time model of the process without delays is available.

The derivation concerns the case of a single delay in all the control variables. Measurement delays are not considered since usually there is less uncertainty concerning these values.

Analysis of the digital algorithm

In the discrete time Smith Predictor, the time delay in the predictor loop is realized by storing the value of the control vector in a table and then selecting from this table of past vectors the appropriate delayed control vector. In the case of inaccurate determination of the delay, a wrong selection of the control vector will adversely affect the control, especially in cases of large sampling intervals.

This method of representing the time delay in the predictor loop has the advantage of making possible compensation for uncertain or time varying delays according to the following reasoning.

Consider a system with a delay in all the manipulated variables equal to a sampling interval and which can be represented by an accurate, discrete-time mathematical model as

$$\underline{x}(n) = \underline{\underline{\phi}}\ \underline{x}(n-1) + \underline{\underline{\theta}}\ \underline{u}(n-a-1) + \underline{\underline{\Delta}}\ \underline{d}(n-1) \tag{2.81}$$

where the notation is the same as for Equation (2.22). For the system of Equation (2.81) all state variables are assumed to be measured. The basic function of the Smith Predictor is the cancellation of the effect of the delayed control action and the replacement of it by a prediction of the system response for the system without a time delay. In cases of inaccurate representation of the delay in the predictor loop, the cancellation will be incomplete. However, if an accurate model of the undelayed system is available, the following calculations are feasible:

1. Calculation of changes in $\underline{x}$ due to an unmeasured load variable.
2. Calculation of the state vector for the case where the effect of the delayed control action is completely cancelled.

The calculation of the state vector if complete cancellation of the control action is assumed can be accomplished by using the model and the measurement and forming the following vector:

$$\underline{S}(n) = \underline{\underline{\phi}}\ \underline{x}(n-1) + \underline{\underline{\Delta}}\ \underline{d}(n-1) \tag{2.82}$$

where

$\underline{x}(n-1)$ = previous state measurement

$\underline{d}(n-1)$ = load variable vector

The load change can be calculated from the first deviation of $\underline{x}$ from the steady state value. Any change in the measurement can be interpreted as a load change, assuming that the system is already at steady state. The measurement in perturbed form will then be given by

$$\underline{x}(n) = \underline{\underline{\Delta}}\ \underline{d}(n)$$

It should be noted that only the product $\underline{\underline{\Delta}}\ \underline{d}(n)$ needs to be calculated and that the actual value of $\underline{d}(n)$ is not required. Under steady state conditions, inaccuracies in the predictor do not affect the response since the output of the predictor will be equal to zero. Combining Equations (2.81) and (2.82), the following equation can be written

$$\underline{x}(n) - \underline{\underline{\theta}}\ \underline{u}(n-a-1) = \underline{S}(n) \qquad (2.83)$$

According to the scheme given in Figure 2.6, the following procedure is followed:

(1) An approximate value for the actual delay a is assumed and denoted as a'.

(2) The vector $\underline{S}(n)$ is calculated from Equation (2.82) using the previous measurement, $\underline{x}(n-1)$ and estimated or measured values of the disturbance.

(3) The delayed control vector $\underline{u}(n-a'-1)$ is obtained from the stored past values of the control vector.

(4) The product $\underline{\underline{\theta}}\ \underline{u}(n-a'-1)$ is calculated and subtracted from the present measurement $\underline{x}(n)$ and the result compared to $\underline{S}(n)$. As seen from Equation (2.83), the result of the forementioned subtraction should be equal to $\underline{S}(n)$ when $a' = a$.

If this comparison is satisfactory, then $a' = a$ and the vector $\underline{u}(n-a'-1)$ is sent to the predictor loop. In the opposite case, the search of the table of past values continues sequentially by assuming larger values of a'. In the case of initially unknown but constant time delays, the search can be stopped once the value of the delay has been established. Assuming that the system was initially at steady state, determination of the actual delay will be possible only after a time interval equal to the actual time delay, since until that time the response of the system is open loop due to the time delay.

In the case of slowly varying time delays, the previously described search will be continued but the algorithm has to be modified to include the ability of distinguishing between load changes and changes in the magnitude of the time delay. This is necessary since although the predictor control scheme does not require knowledge of load changes, the determination of the time delay does require knowledge of any load changes.

The algorithm proceeds as follows. It is initially assumed that any unexpected change in the state vector, $\underline{x}(n)$, is due to a change in the value of the time delay, a. Furthermore, it is also assumed that the time delay is restricted to a known range of values. This assumption restricts the search of the table of past control vectors to a certain predetermined range. If no control vector is found inside this range that satisfies the requirements of Equation (2.83), then a change in the load is assumed to have occurred. Then, using the best available estimate of the time delay, the respective control vector is selected from the table and the new load change is calculated from the model. This calculation is done by subtracting from the current measurement, a calculated value of the current measurement which is formed as follows:

$$\underline{X}_{CAL}(n) = \underline{S}(n) + \underline{\underline{\theta}}\ \underline{u}(n-a'-1) \tag{2.84}$$

where $\underline{S}(n)$ is given by Equation (2.82).

Subtraction of $\underline{X}_{CAL}(n)$ from the present measurement will give

$$\underline{X}(n) - \underline{X}_{CAL}(n) = \underline{\underline{\Delta}}\ [\underline{d}(n-1) - \underline{d}_{EST}(n-1)] \tag{2.85}$$

which can be solved for $\underline{\underline{\Delta}}\ \underline{d}_{EST}(n-1)$ where $\underline{d}_{EST}(n-1)$ is the estimated value of the load vector. Since calculation of $\underline{S}(n)$ requires the value of the product $\underline{\underline{\Delta}}\ \underline{d}_{EST}(n-1)$, no calculation of the load vector $\underline{d}_{EST}(n-1)$ is required for the subsequent calculations. This procedure is valid as long as a change in the load does not happen at the same time as a change in the time delay.

This algorithm can also be applied to the case of delays in only some of the manipulated variables with no serious modifications required.

In Chapter 3, simulated results are presented, that demonstrate the ability of the algorithm to estimate a delay in the process as well as distinguishing between load changes and changes in the delay, if an accurate mathematical model of the system (without time delays) is available. Simulated responses for a discrete-time system

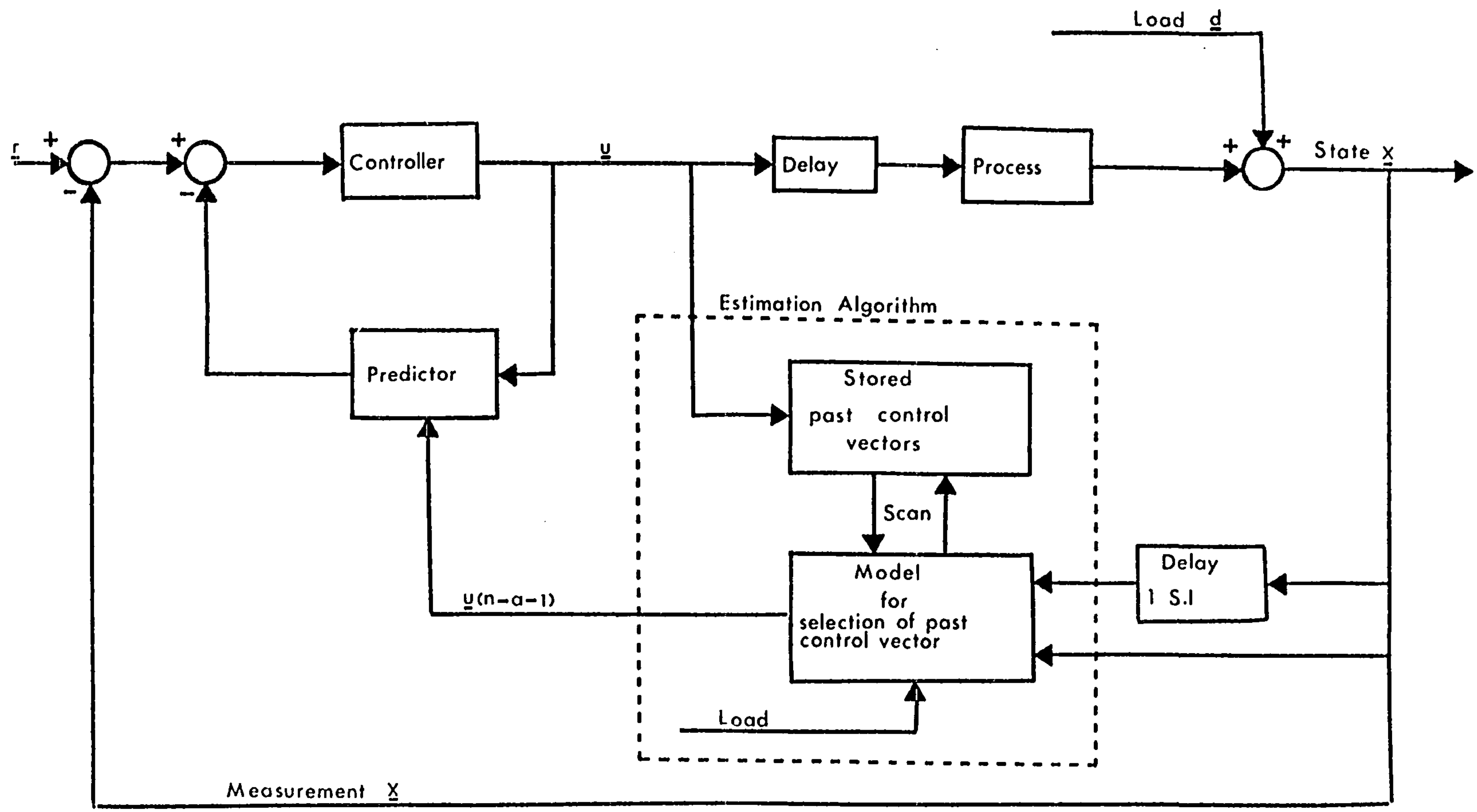

Figure 2.6 Predictor Algorithm for Inaccurate or Time Varying Delay in the Control Variables

are also given in Chapter 3, to illustrate the effect of an inaccurate representation of the time delay in the predictor loop.

2.6 A Numerical Example on the Multivariable Predictor Control

To illustrate the operation of the multivariable Smith Predictor, time delays were introduced into the third order system considered by Takahashi et al. [17]. Two cases will be considered:

(a) Time delay in the control variables.

(b) Time delays in both the control and output variables.

The system equations for this example are:

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{\underline{B}}\,\underline{u}(t-a) + \underline{\underline{D}}\,\underline{d}(t) \tag{2.86}$$

$$\underline{y}(t) = \begin{cases} \underline{\underline{C}}\,\underline{x}(t) & \text{for case (a)} \\ \underline{\underline{C}}_1\,\underline{x}(t) + \underline{\underline{C}}_2\,\underline{x}(t-b) & \text{for case (b)} \end{cases} \tag{2.87}$$

where:

$$\underline{\underline{A}} = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -3 & 2 \\ 0 & 1 & -3 \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\underline{\underline{C}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{C}}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \underline{\underline{C}}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The corresponding discrete time model for T = 0.5 can be obtained in the standard manner, using the analytical solution to Equation (2.86) (Lapidus and Luus, [18]) and is given in Equations (2.88) and (2.89).

$$\underline{x}(n+1) = \underline{\underline{\phi}}\ \underline{x}(n) + \underline{\underline{\theta}}\ \underline{u}(n-a) + \underline{\underline{\Delta}}\ \underline{d}(n) \tag{2.88}$$

$$\underline{y}(n) = \begin{cases} \underline{\underline{C}}\ \underline{x}(n) & \text{for case (a)} \\ \underline{\underline{C}}_1\ \underline{x}(n) + \underline{\underline{C}}_2\ \underline{x}(n-b) & \text{for case (b)} \end{cases} \tag{2.89}$$

where

$$\underline{\underline{\phi}} = \begin{bmatrix} 0.2840 & 0.1310 & 0.0606 \\ 0.2620 & 0.3440 & 0.2620 \\ 0.0606 & 0.1310 & 0.2840 \end{bmatrix} \qquad \underline{\underline{\theta}} = \begin{bmatrix} 0.2760 & 0.0148 \\ 0.1030 & 0.1030 \\ 0.0148 & 0.2760 \end{bmatrix}$$

$$\underline{\underline{\Delta}} = \begin{bmatrix} 0.2760 & 0.0516 \\ 0.1030 & 0.2900 \\ 0.0148 & 0.0516 \end{bmatrix}$$

A feedback control law of the form

$$\underline{u}(n) = -\underline{\underline{K}}_c\ \underline{y}(n) \tag{2.90}$$

is desired. Since the Smith Predictor algorithm will be used, $\underline{\underline{K}}_c$ can be designed using conventional design techniques for systems without time delays, and the state-space model in Equations (2.86) and (2.87) with a = b = 0. For example, a modification of the direct synthesis method of Porter and Crossley [19], can be used to design $\underline{\underline{K}}_c$ so that a specified closed-loop system matrix, $\underline{\underline{T}}$, results.

This approach was used to design $\underline{\underline{K}}_c$ as

$$\underline{\underline{K}}_c = \begin{bmatrix} 0.0257 & 0.0556 \\ -0.4770 & -1.0300 \end{bmatrix} \tag{2.91}$$

corresponding to the closed loop system matrix, $\underline{\underline{T}} = \underline{\underline{\phi}} + \underline{\underline{\theta}}\underline{\underline{K}}_c\underline{\underline{C}}$ where $\underline{\underline{T}}$

was selected to be

$$\underline{\underline{T}} = \begin{bmatrix} 0.2840 & 0.1310 & 0.0606 \\ 0.2620 & 0.2970 & 0.2000 \\ 0.0606 & 0 & 0 \end{bmatrix} \qquad (2.92)$$

Transient responses of the closed-loop system including the Smith Predictor are shown in Figures 2.7 and 2.8 for cases (a) and (b), respectively. All responses are for the feedback control matrix in Equation (2.91), the Smith Predictor algorithm of Equations (2.31) - (2.34) and a unit step change in the disturbance vector, $\underline{d}$. Figure 2.7 illustrates the effect of a delay in the control variables on the system response. The open and closed-loop responses for the system without time delays are given by curves 5 and 1, respectively. When the control variables are delayed as in curves 2-4, the system response initially follows the open-loop response, then changes direction and eventually approaches the closed-loop response for the undelayed system.

Thus, by using the multivariable Smith Predictor and a feedback control matrix designed for the system without time delays, a satisfactory load response is obtained with no offset between the predictor response and the response of the undelayed system.

In Figure 2.8 time delays in both the control variables and output variables are considered for the same disturbance and control scheme that were used in Figure 2.7. The responses are qualitatively similar to those in Figure 2.7 and are judged to give a satisfactory degree of control.

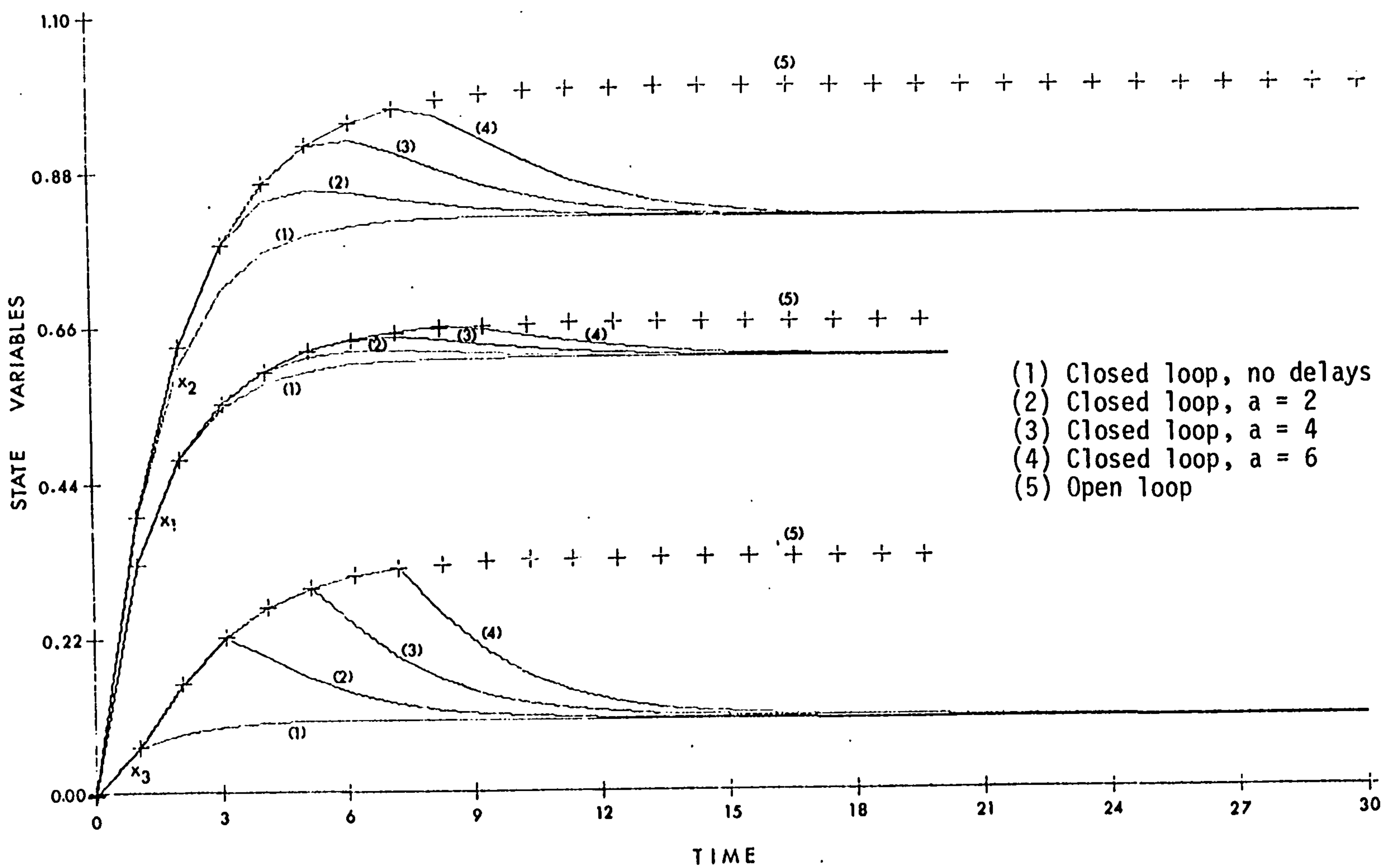

Figure 2.7 Smith Predictor for System with Delay in the Control Variables

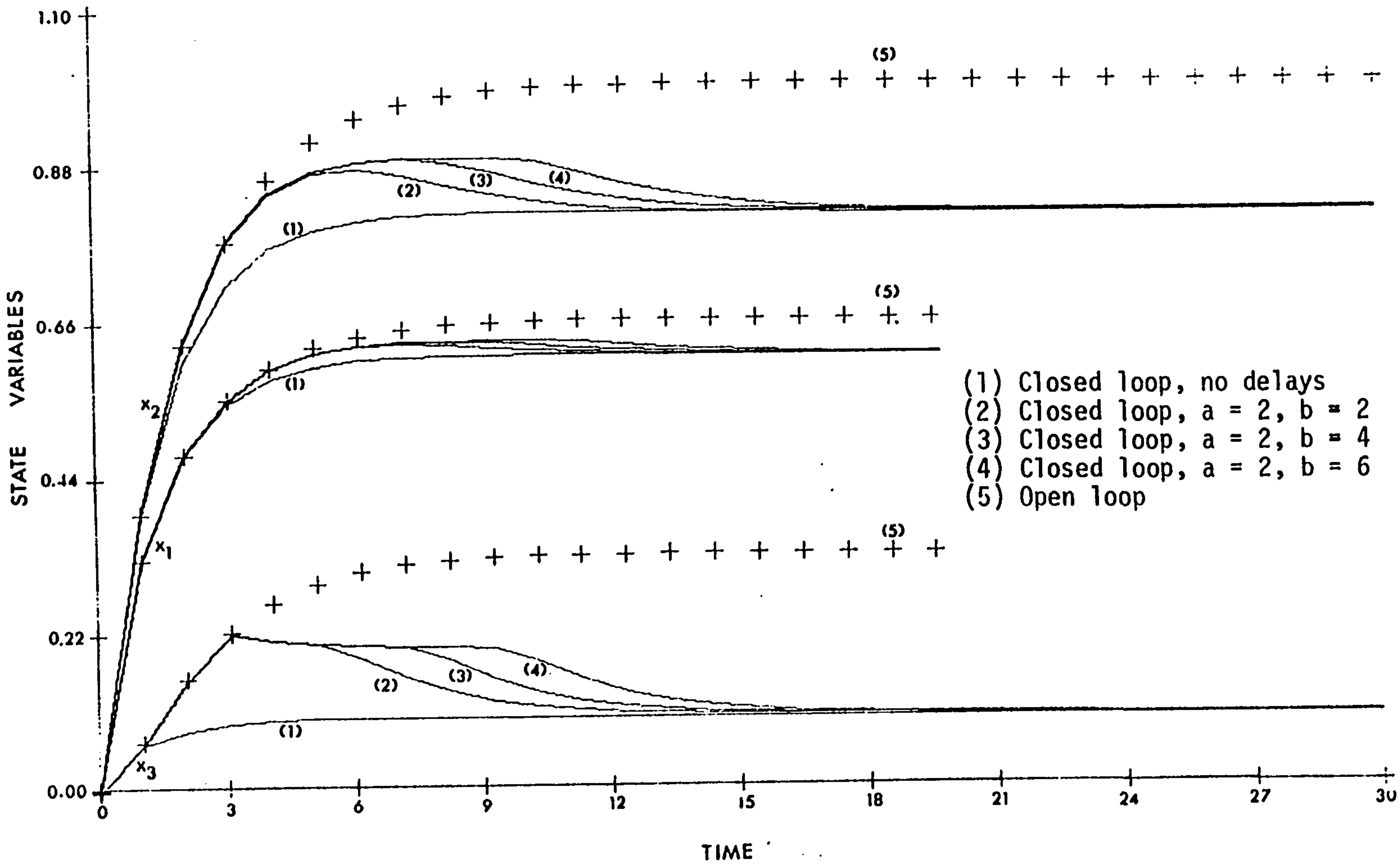

Figure 2.8 Smith Predictor for System with Delays in the Control and Measurement Variables

CHAPTER THREE

SIMULATION RESULTS FOR THE PREDICTOR SCHEME APPLIED TO A DOUBLE EFFECT EVAPORATOR

3.1 Introduction

The multivariable predictor algorithm developed in Chapter Two was simulated on an IBM 1800 Digital Computer using a 5^{th} order state space model of the pilot scale, double effect evaporator in the Department of Chemical and Petroleum Engineering at the University of Alberta. The control algorithm used in the simulation was later easily incorporated in the multivariable control program developed by Newell [21] for experimental verification of the simulated results on the actual evaporator.

The use of a digital computer in the simulation and experimental studies of the predictor algorithm was ideal since the representation of the time delays could be made exact without approximation.

3.2 Mathematical Model

The pilot plant scale, double effect evaporator is represented in the simplified schematic diagram of Figure 3.1.

The first effect is a natural circulation calandria type unit, heated with a nominal 2 lb./min. of fresh steam and fed with a nominal 5 lb./min. of three percent triethylene glycol by weight. First effect vapour is used to heat the second effect, an externally forced-circulation long tube vertical unit, which concentrates first effect product to about ten percent. The second effect is kept under tight pressure control by a vacuum system and condenser. The evaporator has been extensively modelled by André [22], Newell [21] and Wilson [20].

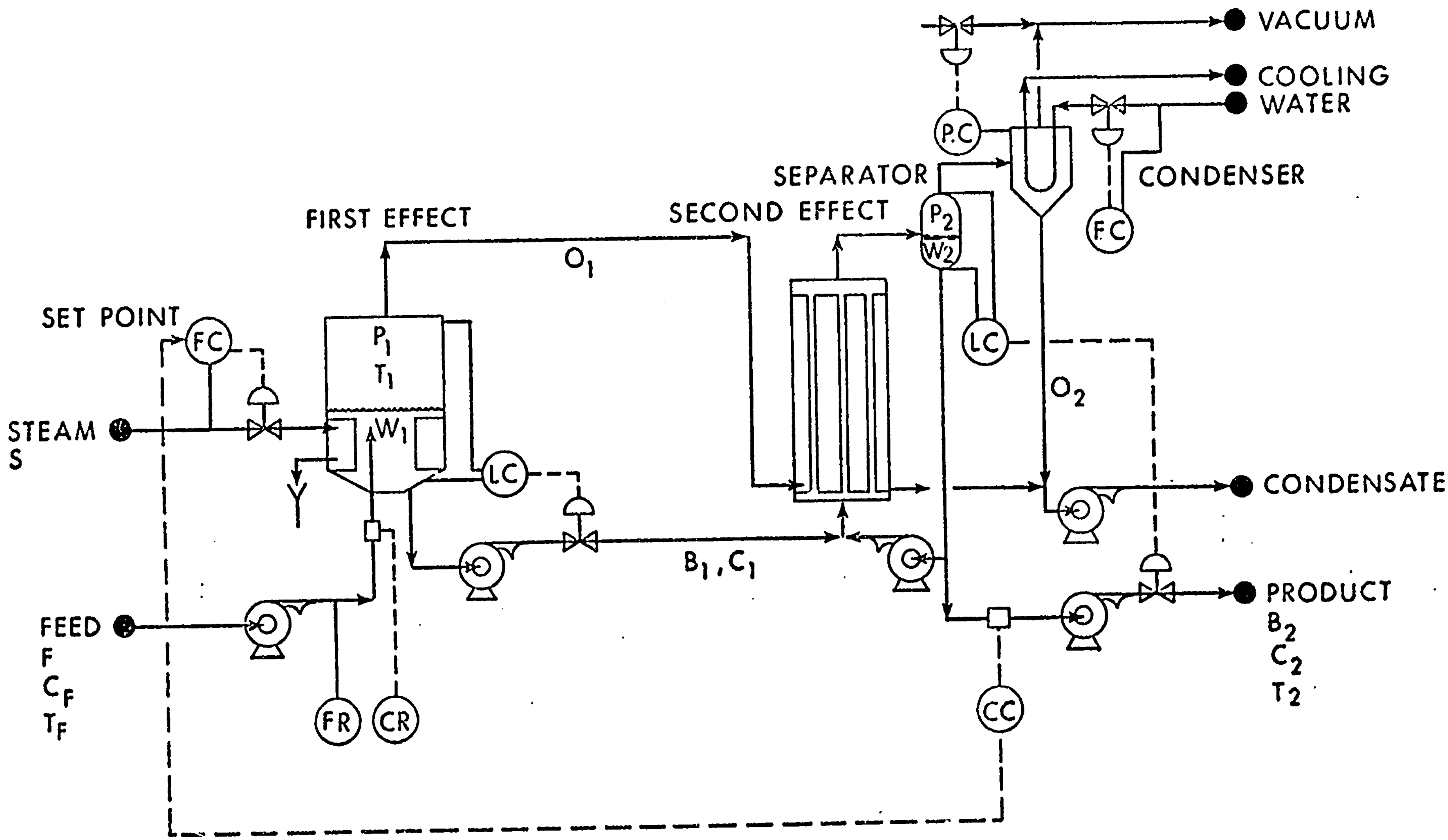

FIGURE 3.1 Schematic Diagram of Double Effect Evaporator and Multiloop Control Scheme

The mathematical model used in the simulation and experimental runs is due to Wilson [20]. The nonlinear model was linearized, with the variables in normalized perturbation form and is expressed in state space notation by:

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\ \underline{x}(t) + \underline{\underline{B}}\ \underline{u}(t) + \underline{\underline{D}}\ \underline{d}(t) \tag{3.1}$$

The model used here is a five state, linear time invariant model with matrices $\underline{\underline{A}}$, $\underline{\underline{B}}$ and $\underline{\underline{D}}$ having the numerical values given in Table 3.1 where the state, disturbance and control vectors are denoted as

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} W_1' \\ C_1' \\ h_1' \\ W_2' \\ C_2' \end{bmatrix}$$

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} S' \\ B_1' \\ B_2' \end{bmatrix}$$

$$\underline{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} F' \\ C_F' \\ h_F' \end{bmatrix}$$

where W_1', C_1', S', etc. denote normalized perturbation variables.

$$\text{e.g.} \qquad W_1' = \frac{W_1 - W_{1ss}}{W_{1ss}} \qquad (ss \equiv \text{steady state})$$

TABLE 3.1

Five State , Continuous Time Linearized Model

$$\begin{bmatrix} \dot{W}_1' \\ \dot{C}_1' \\ \dot{h}_1' \\ \dot{W}_2' \\ \dot{C}_2' \end{bmatrix} = \begin{bmatrix} 0. & -0.0011 & -0.1254 & 0. & 0. \\ 0. & -0.0755 & 0.1254 & 0. & 0. \\ 0. & -0.00604 & -0.7740 & 0. & 0. \\ 0. & -0.0012 & -0.1447 & 0. & 0.0001 \\ 0. & -0.0393 & 0.1447 & 0. & -0.0379 \end{bmatrix} \begin{bmatrix} W_1' \\ C_1' \\ h_1' \\ W_2' \\ C_2' \end{bmatrix} + \begin{bmatrix} 0. & -0.0766 & 0. \\ 0. & 0. & 0. \\ 0.2159 & 0. & 0. \\ 0. & 0.0795 & -0.0381 \\ 0. & -0.0413 & 0. \end{bmatrix} \begin{bmatrix} S' \\ B_1' \\ B_2' \end{bmatrix} + \begin{bmatrix} 0.1097 & 0. & 0. \\ -0.0333 & 0.0766 & 0. \\ -0.0188 & 0. & 0.0911 \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \begin{bmatrix} F' \\ C_F' \\ h_F' \end{bmatrix}$$

For the digital algorithm used in the simulation and experimental application, the state difference equation was obtained from the continuous time equation of Table 3.1 by using the analytical solution, [20], to Equation (3.1) and a time base of 64 secs. The resulting discrete time model is given by:

$$\underline{x}(n+1) = \underline{\underline{\phi}}\, \underline{x}(n) + \underline{\underline{\theta}}\, \underline{u}(n) + \underline{\underline{\Delta}}\, \underline{d}(n) \qquad (3.2)$$

with matrices $\underline{\underline{\phi}}$, $\underline{\underline{\theta}}$ and $\underline{\underline{\Delta}}$ having the numerical values given in Table 3.2. For the purpose of simulating measurement delays, the following output equation was used

$$\underline{y}(n) = \underline{\underline{C}}_1\, \underline{x}(n) + \underline{\underline{C}}_2\, \underline{x}(n-b) \qquad (3.3)$$

where

$\underline{y}(n)$ = output vector of dimension r

b = constant measurement delay

$\underline{\underline{C}}_1, \underline{\underline{C}}_2$ = constant matrices of dimension rxn

The assumption was made that all state variables are available. For the case of delay in the product concentration, (C_2), measurement matrices $\underline{\underline{C}}_1$, $\underline{\underline{C}}_2$ would be

$$\underline{\underline{C}}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \underline{\underline{C}}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

while for the case of delay in the control variables only,

TABLE 3.2

Discrete Time Evaporator Model (T = 64 sec.)

$$\underline{\underline{\Phi}} = \begin{bmatrix} 1. & -0.0008 & -0.0912 & 0. & 0. \\ 0. & 0.9223 & 0.0871 & 0. & 0. \\ 0. & -0.0042 & 0.4376 & 0. & 0. \\ 0. & -0.0009 & -0.1052 & 1. & 0.0001 \\ 0. & 0.0391 & 0.1048 & 0. & 0.9603 \end{bmatrix} \qquad \underline{\underline{\theta}} = \begin{bmatrix} -0.0119 & -0.0817 & 0. \\ 0.0116 & 0. & 0. \\ 0.1569 & 0. & 0. \\ -0.0138 & 0.0848 & -0.0406 \\ 0.0137 & -0.0432 & 0. \end{bmatrix}$$

$$\underline{\underline{\Delta}} = \begin{bmatrix} 0.1182 & 0. & -0.0050 \\ -0.0351 & 0.0785 & 0.0049 \\ -0.0136 & -0.0002 & 0.0662 \\ 0.0012 & 0. & -0.0058 \\ -0.0019 & 0.0016 & 0.0058 \end{bmatrix}$$

$$\underline{\underline{C}}_1 = \underline{\underline{I}}, \quad \underline{\underline{C}}_2 = \underline{\underline{0}}$$

where $\underline{\underline{I}}$ is the 5x5 identity matrix.

3.3 Predictor Algorithm

For the simulated application of the predictor, the equations developed in Chapter Two for the Multivariable Discrete-Time case were used, namely, Equations (2.22) - (2.28).

The simulation of the time delay in the control variables was accomplished by delaying the control action, and the measurement delay by using past values of the stored state variable vector. The time delay used in the predictor algorithm (Equation (2.28)) was achieved in a similar fashion by forming a table of past control variable vectors and selecting the appropriate one according to the value of the time delay. In all of the simulation runs it was assumed that the time delay is an integer multiple of the sampling time. Throughout the simulation and experimental runs a control law of the form:

$$\underline{u}(n) = \underline{\underline{K}}(\underline{x}(n) + \underline{p}(n)) \tag{3.4}$$

was used where $\underline{\underline{K}}$ is a constant proportional control matrix and $\underline{p}(n)$ is the predictor output. This control law implies a change in the sign of the right hand side of Equation (2.25) and this is the only change made to the Equations (2.22) - (2.28).

3.4 Multiloop Control

Multiloop control has been extensively applied to the double effect evaporator at the University of Alberta, and the theoretical considerations involved along with further details can be found in the theses by Newell [21] and Jacobson [16].

A multiloop control scheme consists of several single variable control systems, i.e. each control variable is affected by the changes in only one of the state variables. In the evaporator application the usual pairing of manipulated and controlled variables is [16,21,23],

Control Variable	State Variable
S	C_2
B_1	W_1
B_2	W_2

The control matrix for multiloop control was derived by Oliver [23] and is given by

$$\underset{=}{K}_{FB} = \begin{bmatrix} 0 & 0 & 0 & 0 & -4.89 \\ 3.52 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15.8 & 0 \end{bmatrix} \tag{3.5}$$

A comparison of simulated open and closed loop responses for the model in Table 3.1 and the multiloop control scheme of Equation (3.5) is presented in Figure 3.2, where horizontal arrows denote the initial steady state values. As can be seen in Figure 3.2, the uncontrolled (open loop) system is not self-regulatory due to the integrating nature of the two holdups. The final steady state values (i.e. offsets) after the +20% step change in feed flow are given in Table 3.3.

3.4.1 Multiloop Control of System With Time Delays

The actual double effect evaporator does not have any important time delays. Hence, time delays were arbitrarily introduced into the mathematical model, and transient responses were calculated for multiloop control and different values of the delays. In the

TABLE 3.3

Offsets Resulting from Simulated Multiloop Control After a Step Change of +20% in Feed Flow

Variable	Offset (%)
W_1'	5.84
C_1'	-0.58
h_1'	4.97
W_2'	1.52
C_2'	-3.40

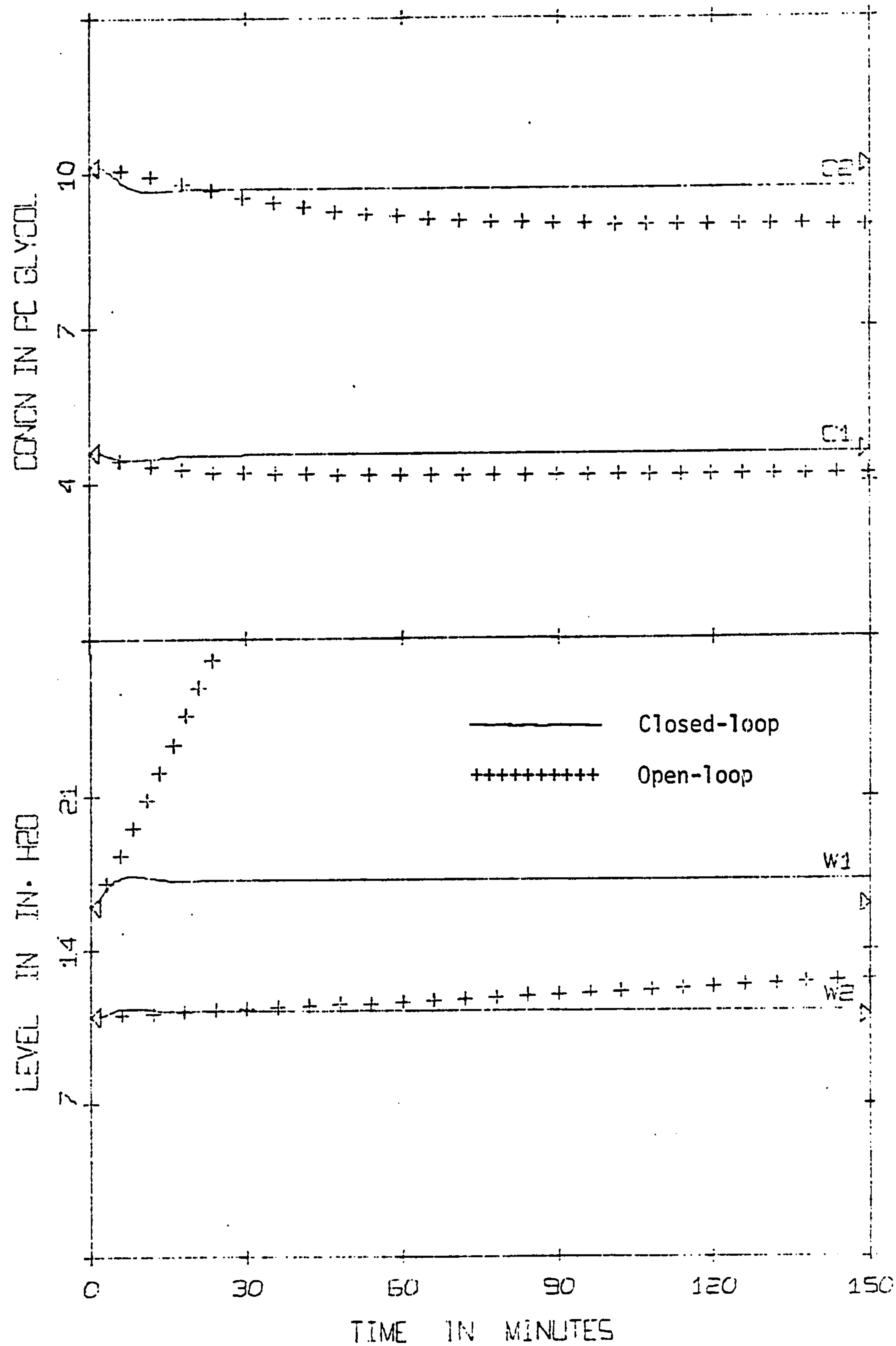

FIGURE 3.2 Simulated Response for Multiloop Control and a Step Change of +20% in Feed Flowrate

simulation runs all states are assumed measurable and the load change consists of a 20% step increase in the total feed flow starting at the first sampling interval (i.e. at t = 64 sec.).

Results were obtained for two cases:

1) Time delay in all the manipulated variables, (Process Delay).
2) A measurement delay in some of the state variables.

In the first case, a single time delay was introduced in all the manipulated variables. Such a case could arise in practice from the existence of long pneumatic lines for the actuation of pneumatic valves [24]. The case of a delay in some manipulated variables was not examined in the simulated or experimental runs. However, a predictor scheme for this case was theoretically derived and the necessary conditions were presented in Chapter Two.

The measurement delay case is restricted to a time delay in product concentration measurement. The behavior of the system for delays in the measurement of other state variables is examined in connection with multivariable control in § 3.5. Due to the fact that in the double effect evaporator an on-line refractometer is used for continuous product concentration measurement, no significant delay exists. Therefore, a delay is introduced in the model for purposes of study, since delays in the measurement of concentration are of common occurrence in other processes and tend to deteriorate the control in a dramatic fashion. Examples of the detrimental effects of the delay in the concentration measurement can be found in the study of distillation column control at the University of Alberta by McGinnis [25].

Delay in the Manipulated Variables

The transient response of the system was obtained under multiloop control for process delays of 64 and 128 sec. As shown in Figure 3.3, the responses are underdamped for a delay of 64 sec. and remain underdamped for a delay of 128 sec. for all states except the second effect holdup (W_2) which becomes unstable. The unstable behavior of W_2 is due to the high controller gain in the W_2-B_2 control loop (i.e. K_{34} = 15.8). Decreasing this gain will result in less oscillations but the offset will increase. Furthermore, the stable closed loop response would still tend to become unstable for larger values of the time delay. The effect of lowering the controller gain is considered in conjunction with measurement delays in Figure 3.8.

Measurement Delay

The transient response of the system for delays 0, 192 and 256 sec. in the measurement of the product concentration with multiloop control is given in Figure 3.4. The response remains underdamped for a delay of 192 sec. with larger oscillations in C_2 than in the other state variables. For a delay of 256 sec. the responses become unstable for all the state variables. Improvement of the responses can again be accomplished by lowering the gains of the control matrix to a certain extent.

Discussion of Results

A series of simulation runs were conducted in order to evaluate the ability of multiloop proportional feedback control in compensating for the detrimental effects of time delays in the double effect evaporator model.

For both cases of delays in the control or measurement variables, the multiloop control scheme in Equation (3.5) proved unsatisfactory. For example, the system is unstable for a delay in all the

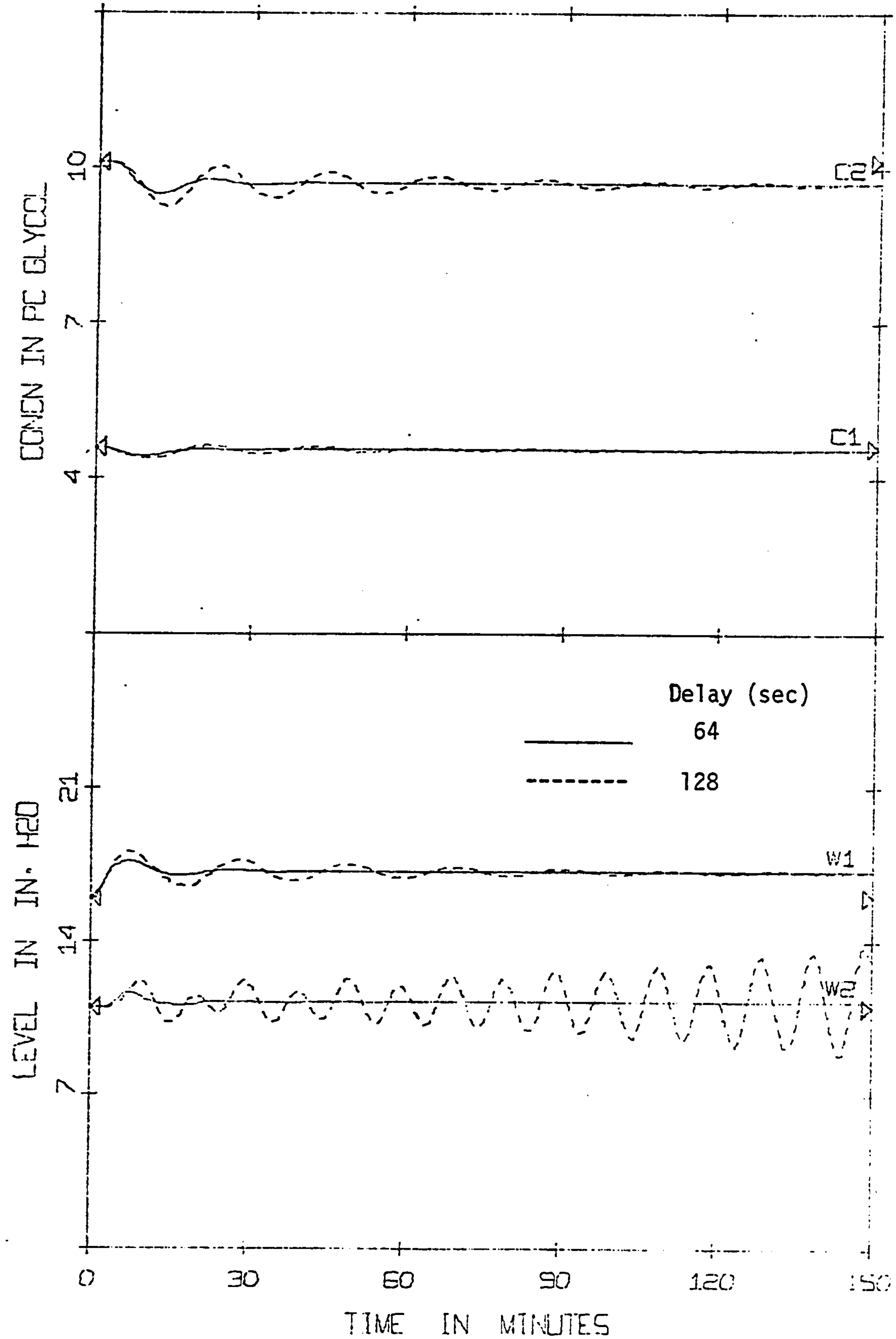

FIGURE 3.3 Simulated Response for Multiloop Control and Delays in the Control Variables (+20% Step Change in Feed Flow)

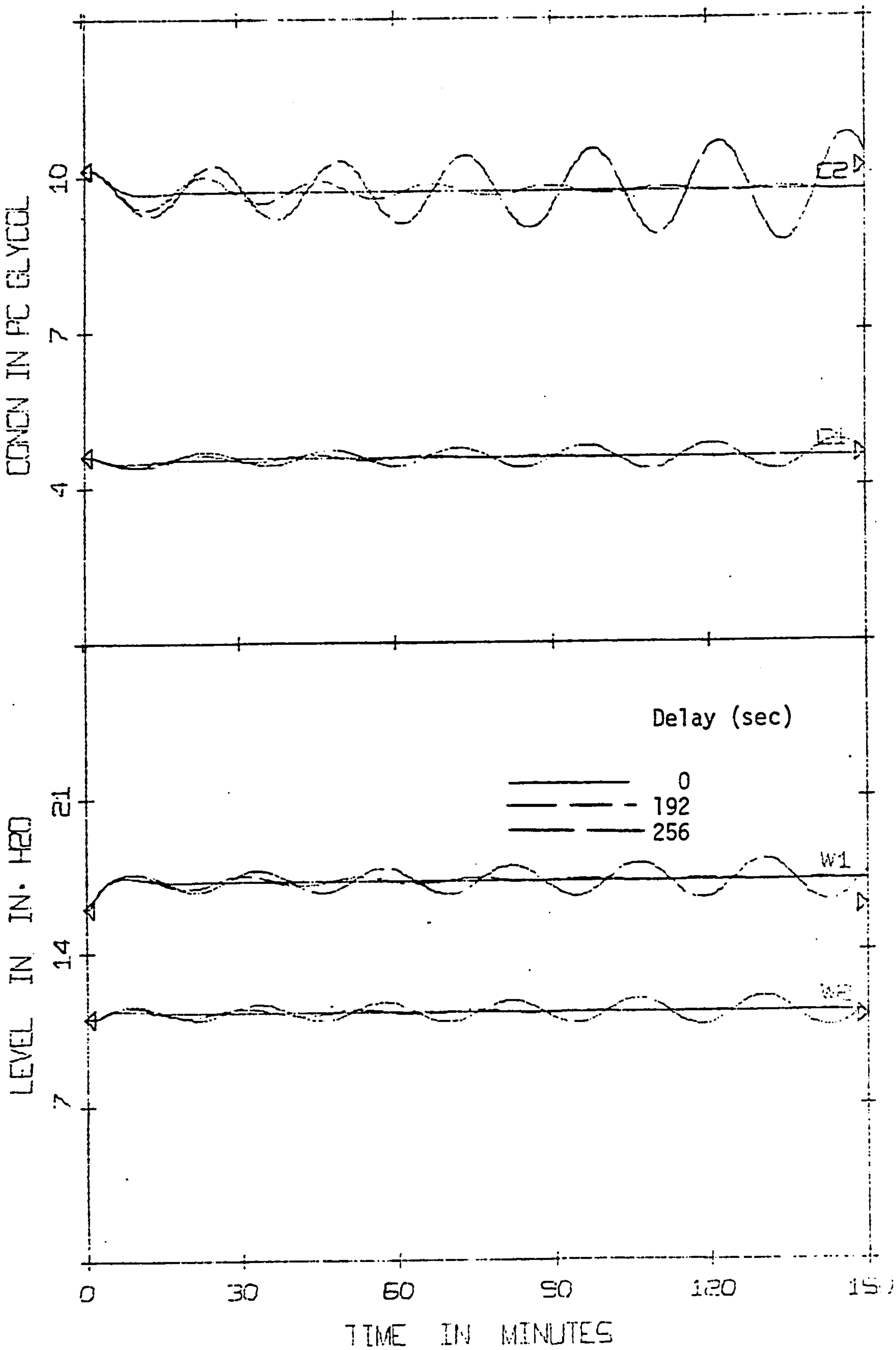

FIGURE 3.4 Simulated Response for Multiloop Control and a Measurement Delay (+20% Step Change in Feed Flow)

control variables of 128 sec. or a measurement delay in C_2 of 192 sec. Further improvement of the response can be accomplished by lowering the gains of the feedback control matrix but this has the well known disadvantage of increased offsets and more sluggish response.

3.4.2 Multiloop Control with a Predictor

Introduction

The predictor algorithm, derived in Chapter Two, was simulated as discussed in § 3.2.2 and used together with the multiloop control matrix given by Equation (3.5).

Theoretical considerations imply that the predictor response will have the same stability characteristics and follow the response of the undelayed system with the same control matrix. The same two cases are considered as in the previous section.

Multiloop Predictor with Delay in the Manipulated Variables

The simulated response of the evaporator model was obtained under multiloop proportional predictor control for delays of 128, 256 and 384 sec. in all of the manipulated variables. The responses are given in Figure 3.5 together with the response of the system without time delays. As it is shown in Figure 3.5, the predictor response is stable and drives the response to the one of the undelayed system. Comparison of the responses in Figures 3.3 and 3.5 shows the improvement on the response when the predictor is used, since the system without the predictor becomes unstable for a delay of 128 sec. The predictor response results in an increase of the offset in the first effect holdup (W_1), due to the small gain of the respective control loop (W_1-B_1), which eventually disappears.

<u>Multiloop predictor with measurement delay</u>

The simulated response of the evaporator model was obtained under multiloop proportional predictor control for delays of 128, 384 and 640 sec. in the measurement of the product concentration. The responses are given in Figure 3.6 together with the response of the system without time delays and the same control matrix. The stabilizing effect of the predictor is obvious by comparing the responses of Figure 3.6 and Figure 3.4. The predictor response closely follows the response of the undelayed system with no increase in the offset. The responses of W_1 and W_2 are hardly affected since the delay is only in C_2 and multiloop control is used (i.e. value of S only has a small effect on W_1 and W_2). The improvement in the response due to the use of the predictor is more dramatically depicted in Figure 3.7.

As mentioned in Section 3.3.1, the effect of the time delays can be compensated to a certain extent by lowering the gains in the proportional feedback control matrix, when a predictor is not used. In Figure 3.8 a comparison is made between the predictor response with the initial multiloop proportional control matrix and the responses of the system without predictor but using smaller gain in the C_2 - S control loop (i.e. K_{15}). The simulated responses correspond to the evaporator model with a measurement delay of 192 sec. in C_2. Since the responses of W_1, W_2 and C_1 are satisfactory for this case, the gains of the other control loops (W_1-B_1 and W_2-B_2) were left unchanged. As expected, the result of the lower gain is stabilization of the response. At the same time, the offset increases and the response is slower in reaching the steady state. Comparison of the predictor response (with the initial control matrix) with the low gain response

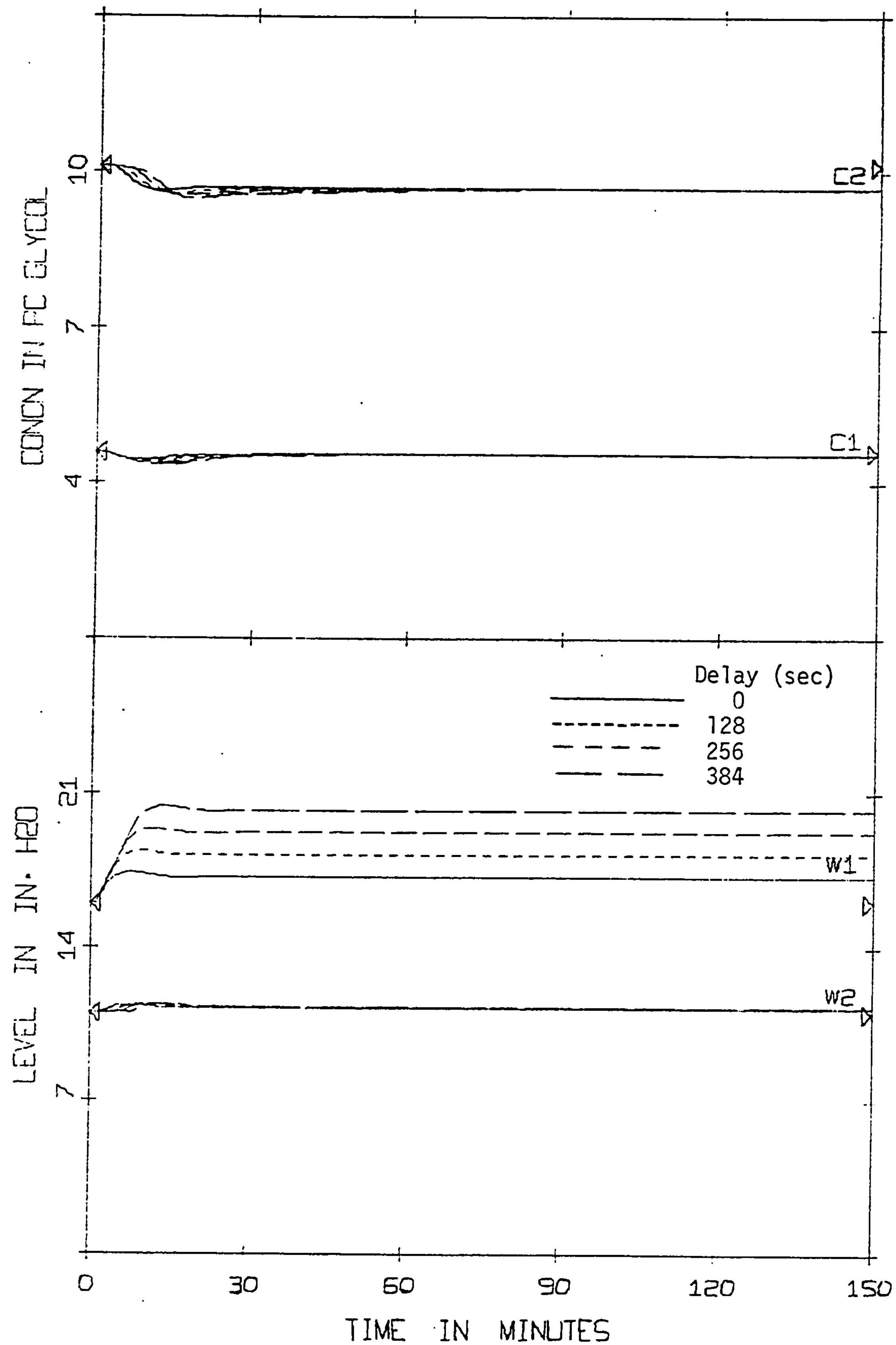

FIGURE 3.5 Simulated Multiloop Predictor Response with Delays in the Control Variables (+20% Step Change in Feed Flow)

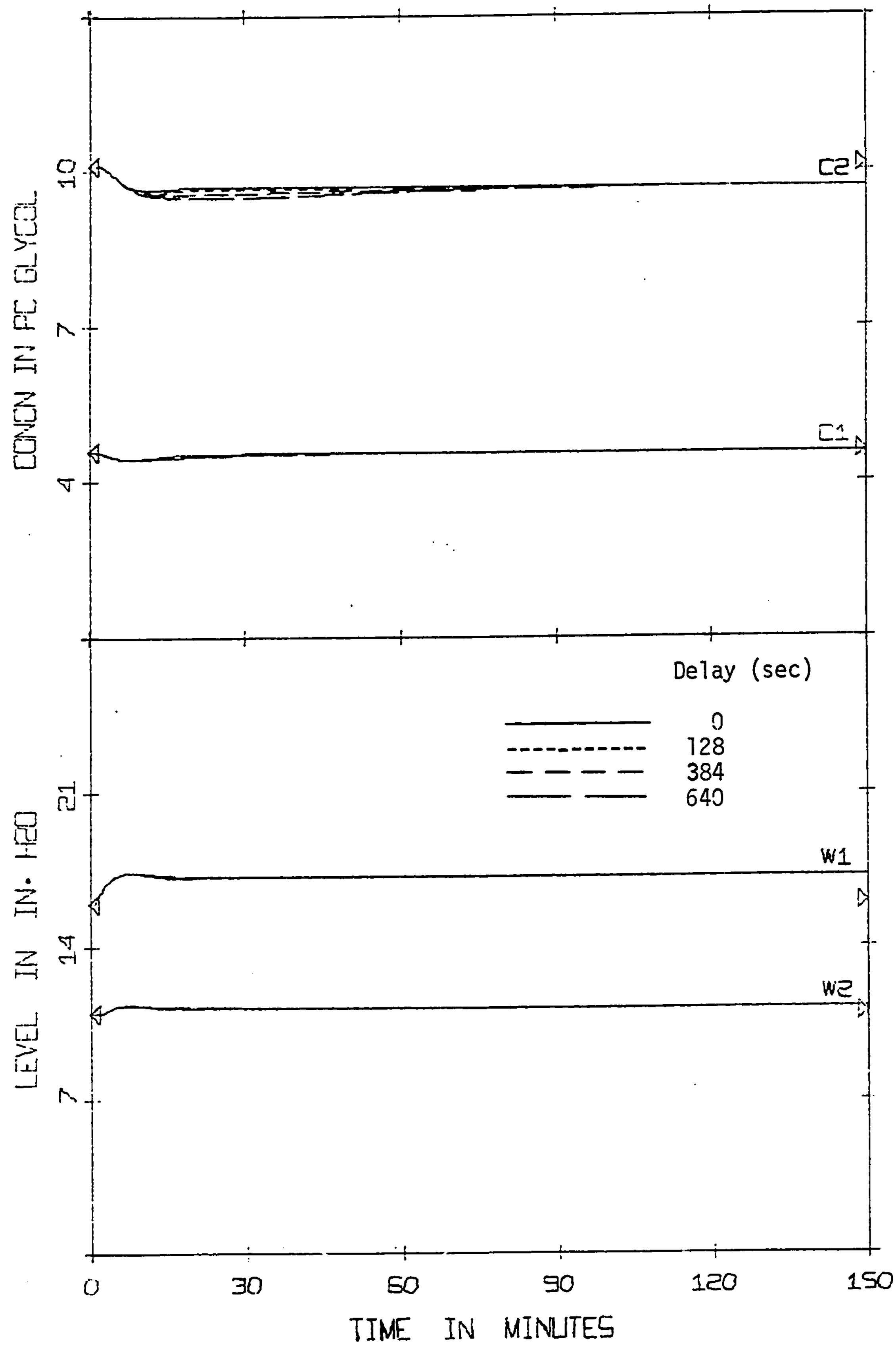

FIGURE 3.6 Simulated Multiloop Predictor Response with Measurement Delay (+20% Step Change in Feed Flow)

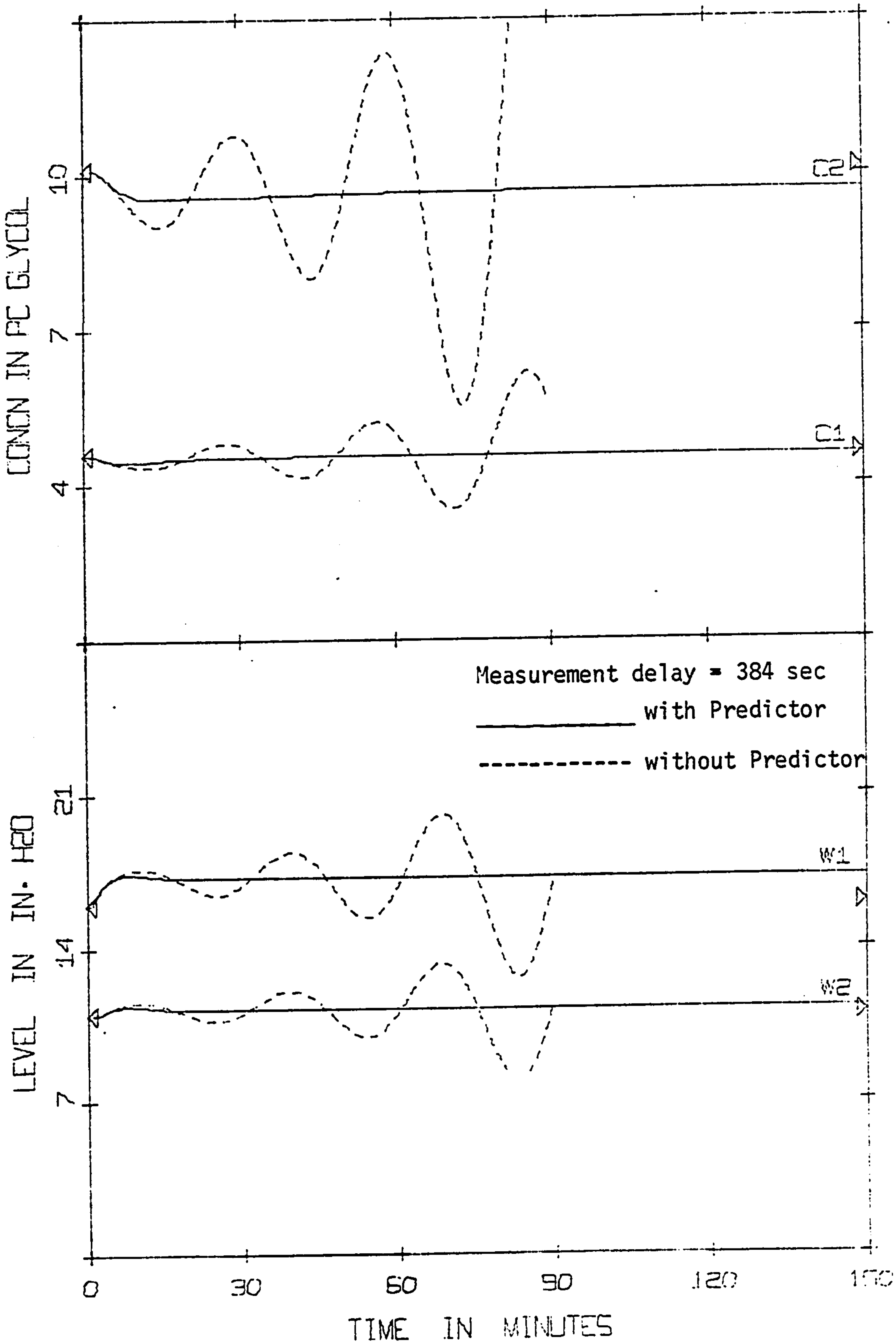

FIGURE 3.7 Simulated Multiloop Response With and Without Predictor and a Measurement Delay in C_2 (+20% Step Change in Feed Flow)

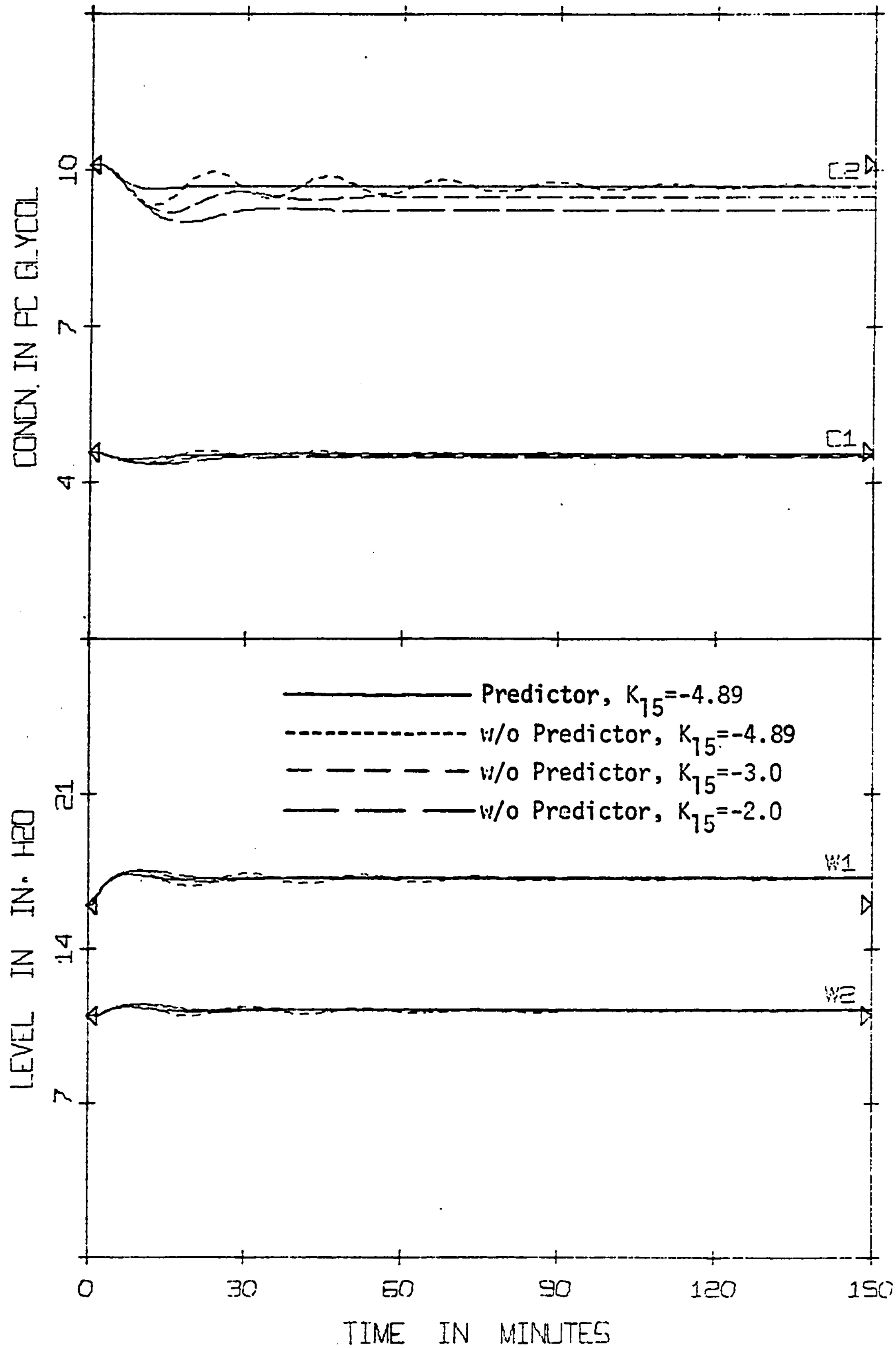

FIGURE 3.8 Simulated Multiloop Responses With and W/O Predictor for Various Controller Gains and Measurement Delay of 384 sec. in C_2 (+20% Step Change in Feed Flow)

shows the advantages of the predictor scheme. If a larger delay is used, these advantages will be even more dramatic.

3.4.3 Effect of Model Inaccuracy on the Predictor Response

The use of an inaccurate mathematical model for the controlled process was examined for the case of the multiloop predictor control. The effect of inaccuracies in the mathematical model of the double effect evaporator was examined by simulating an inaccurate delay in the predictor feedback loop around the controller. Due to the fact that the sampling interval used was 64 sec. with delays taken as integer multiples of this sampling time, the inaccuracies introduced in the simulation were severe. In the case of delay in the manipulated variables, the response was obtained for a delay of 256 sec. in all control variables while the delay in the predictor loop was assumed to be 320 sec. As shown in Figure 3.9, the response is still satisfactory for C_2, C_1 and W_1 while W_2 became unstable. This is due primarily to the high gain in the control loop for W_2 (K_{34} = 15.8). Lowering of this gain from 15.8 to 5.0 resulted in a stable response for W_2 and the response of the other state variables became even better. The effect of using inaccurate estimates of the time delay in the predictor is shown in Figure 3.10. The low gain (for W_2) control matrix was used with an actual delay of 256 sec. and assumed delays in the predictor loop of 192 and 320 sec., respectively. As it is shown, the response is worse for a low estimate of the delay.

The case of measurement delay was examined for a measurement delay of 384 sec. in the product concentration. The initial feedback control matrix in Equation (3.5) was used as well as a predictor with an estimated delay of 320 and 448 sec., respectively. The

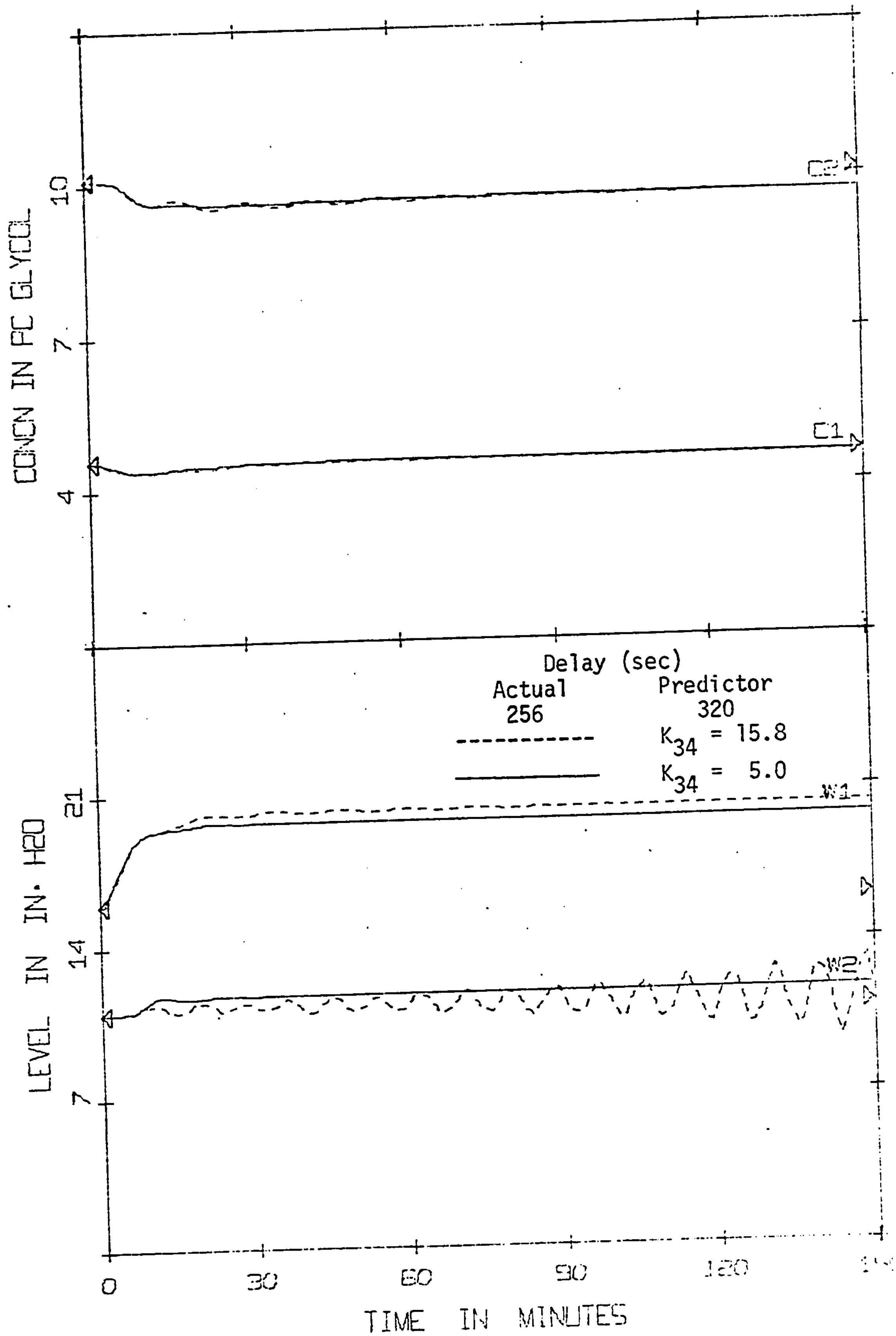

FIGURE 3.9 Effect of Model Inaccuracy. Simulated Multiloop Predictor Response with a Delay in the Control Variable (+20% Step Change in Feed Flow)

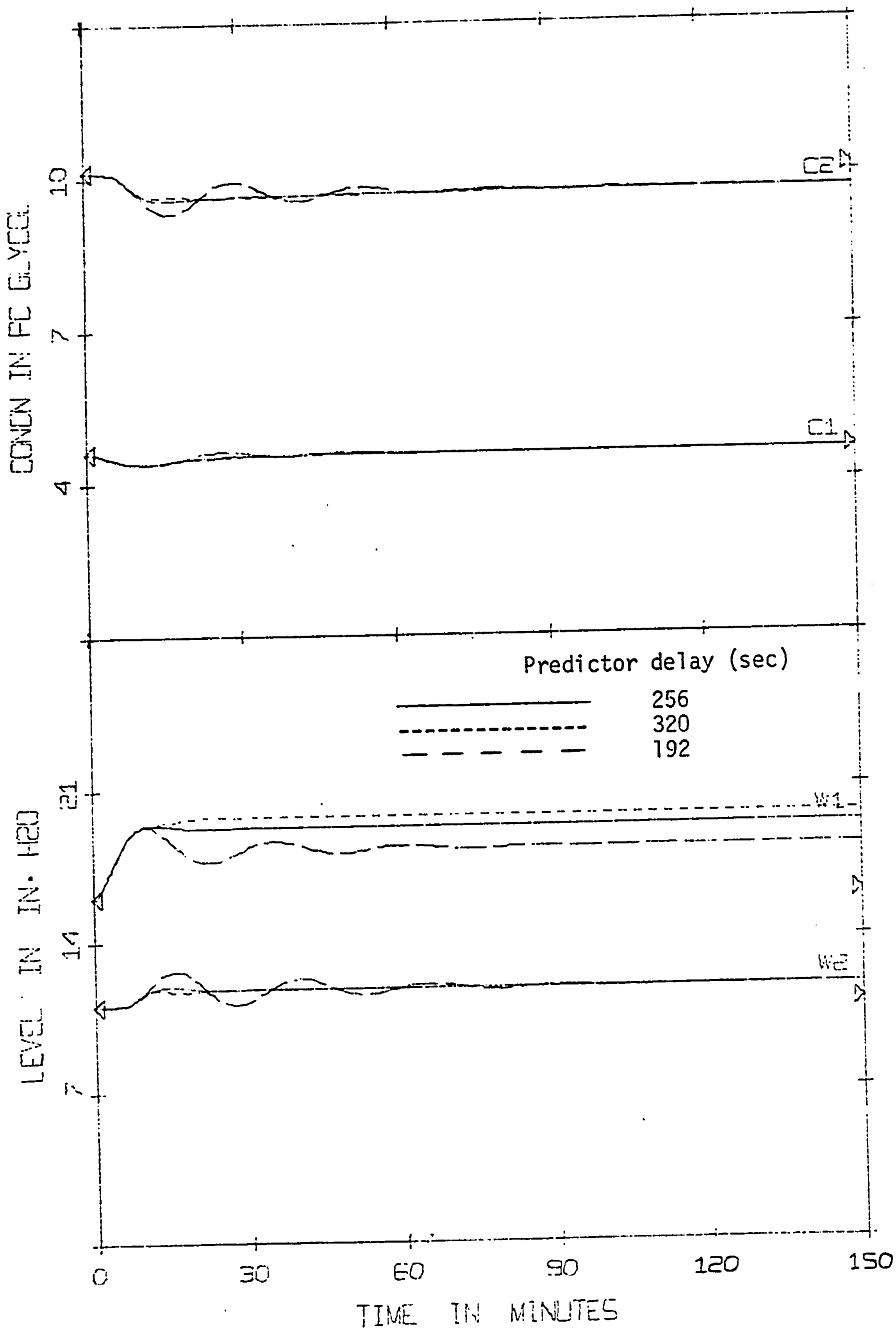

FIGURE 3.10 Simulated Multiloop Response With an Inaccurate Predictor and a Delay 256 sec. in the Control Variables (+20% Step Change in Feed Flow, K_{34} = 5.0)

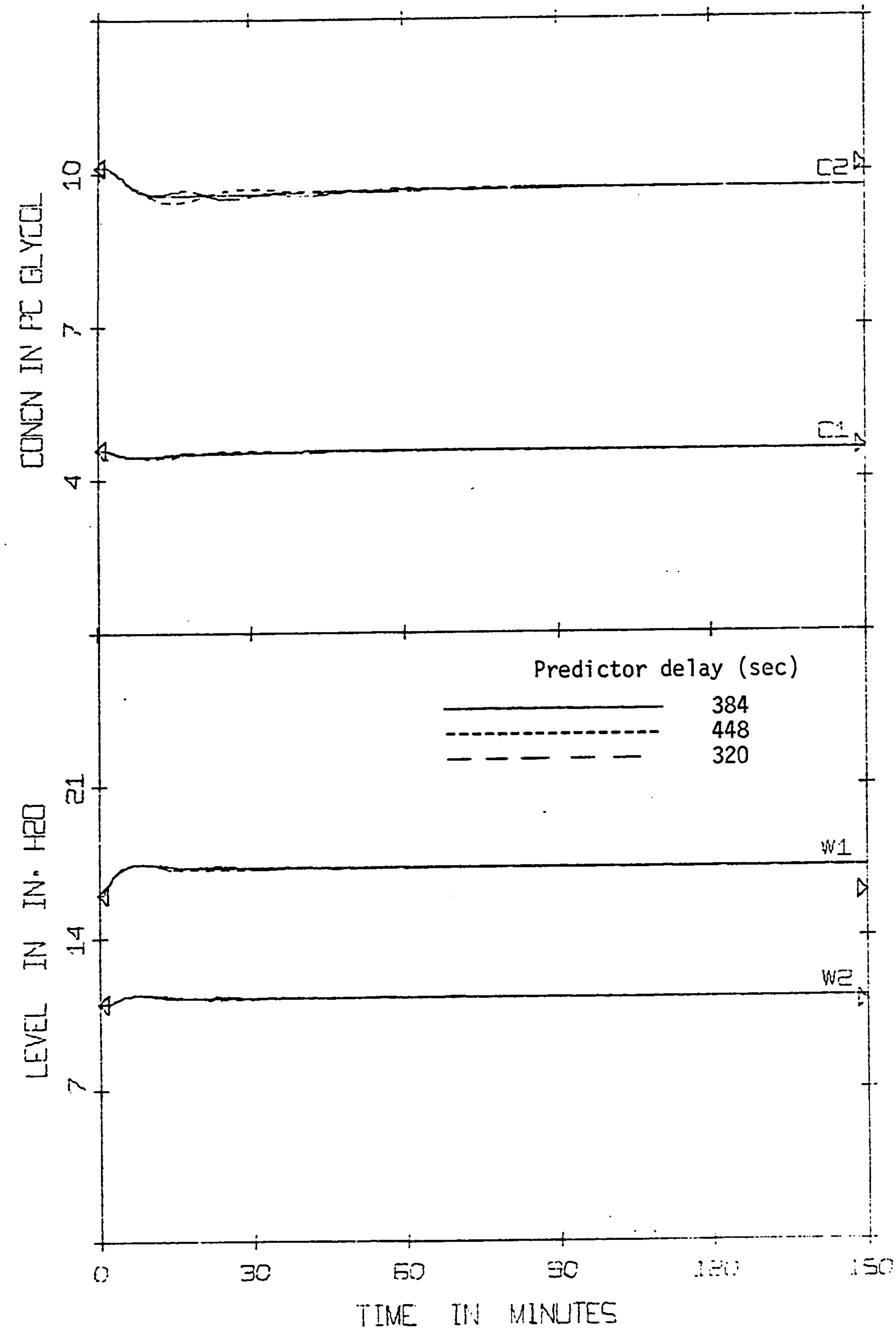

FIGURE 3.11 Simulated Multiloop Response with Inaccurate Predictor and Measurement Delay of 384 sec. in C_2 (+20% Step Change in Feed Flow)

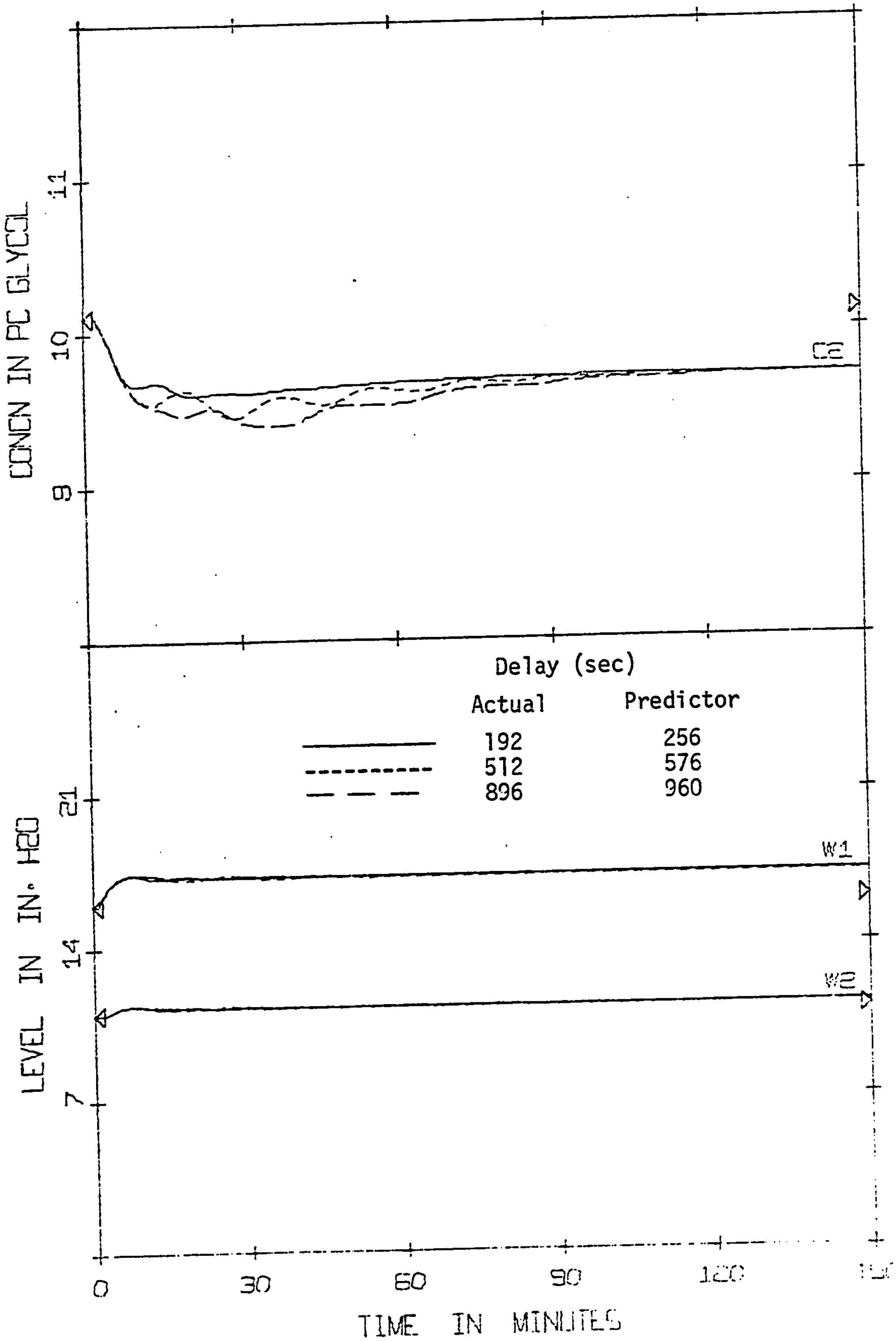

FIGURE 3.12 Simulated Multiloop Response with Inaccurate Predictor and Measurement Delay in C_2 (+20% Step Change in Feed Flow)

simulated responses in Figure 3.11 indicate that the predictor response remains essentially unaffected for both cases of over- and underestimation of the delay in the predictor loop. The effect of the magnitude of the actual delay for the same degree of inaccuracy is shown in Figure 3.12, where the actual delays are 192, 512 and 896 sec. while the assumed predictor delays are 256, 576 and 960 sec. As can be seen in Figure 3.12, the response is far better for small values of the actual delay.

3.5 Multivariable Control

Multivariable control of the double effect evaporator pilot plant has been studied and applied extensively by Newell [21], Jacobson [16], Wilson [20], Hamilton [26], and Oliver [23]. The generation of an appropriate proportional feedback control matrix is done by applying discrete dynamic programming techniques to a formulation based on a quadratic performance index, and the standard linear, time invariant, state space model. The model used in the simulation of multivariable control is the discrete fifth order state space model given in Section 3.2. The proportional feedback control matrix used is due to Wilson [20] and is derived for heavy weighting of the product concentration in the performance index, it is given in Table 3.4.

All state variables are assumed to be measured as in the multiloop case. Examination of this proportional feedback control matrix indicates that steam flowrate (S) depends mainly on product concentration, (C_2), and the product flowrate (B_2) depends mainly on C_2 and the second effect liquid holdup, W_2.

The simulated closed loop response for this control configuration is given in Figure 3.13 together with the open loop response for a

TABLE 3.4

Multivariable Control Scheme and Offsets

Multivariable proportional feedback control matrix

$$\underline{\underline{K}}_{FB} = \begin{bmatrix} 5.09 & -1.48 & -2.68 & 0.0 & -14.6 \\ 3.95 & 0.36 & 0.21 & 0.0 & 7.39 \\ 5.31 & 1.19 & -0.11 & 15.8 & 18.8 \end{bmatrix}$$

Steady state offsets for step change of +20% in feed flow

Variable	Offset (%)
W_1'	5.65
C_1'	0.242
h_1'	5.44
W_2'	0.028
C_2'	-0.511

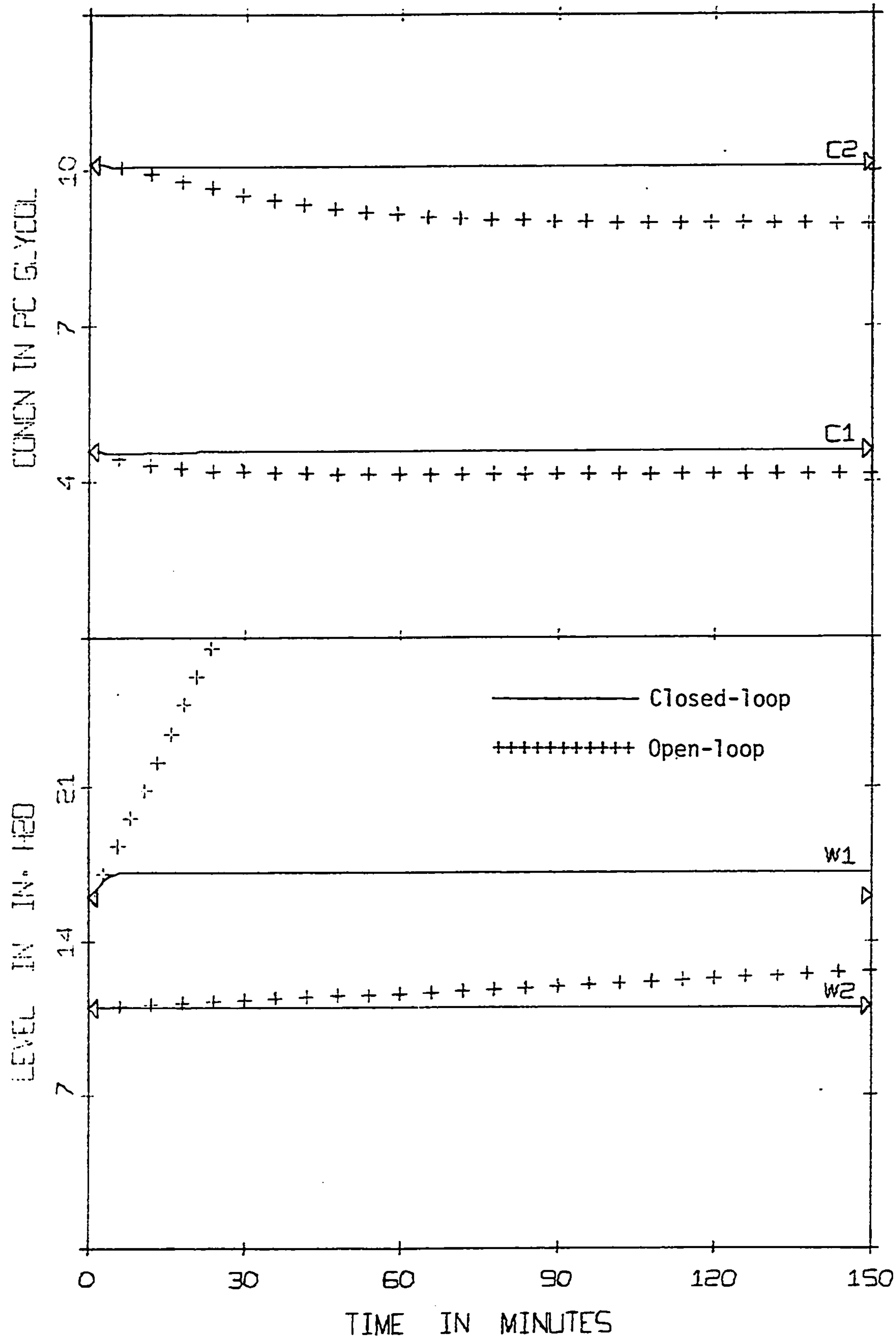

FIGURE 3.13 Simulated Multivariable Response for a +20% Step Change in Feed Flowrate

step change in feed flowrate of +20%, introduced on the second sampling interval. The final steady state values of the perturbation variables are also given in Table 3.4.

3.5.1 Multivariable Control of System with Time Delays

As mentioned in Section 3.4.1, the double effect evaporator does not have any important time delays. For the simulation runs under multivariable control process, measurement and combination of process and measurement delays were assumed and the load responses were for a step change of +20% in the feed flow, introduced on the second sampling interval.

Delay in the Manipulated Variables

The simulated response of the double effect evaporator is given in Figure 3.14 for delays of 0, 64 and 128 sec. in all the manipulated variables. As can be seen from Figure 3.14, the system becomes unstable for a delay of 128 sec. Comparison of Figures 3.14 and 3.3 shows that the multiloop control scheme gives a more stable response due to the lower controller gains.

Measurement Delay

The simulated response of the system in the presence of measurement delays is given in Figure 3.15. A delay of 64, 128 and 192 sec. in the measurement of the product concentration, (C_2), was considered. The system response gradually becomes unstable with a delay of 128 sec. and is obviously unstable for a delay of 192 sec. As in the case of delays in the manipulated variables, the response of the system with a delay is better for multiloop control than it is for multivariable control (cf. Figures 3.15 and 3.4). In Figure 3.16 the simulated response is given for the case of a delay of 192 sec. in the

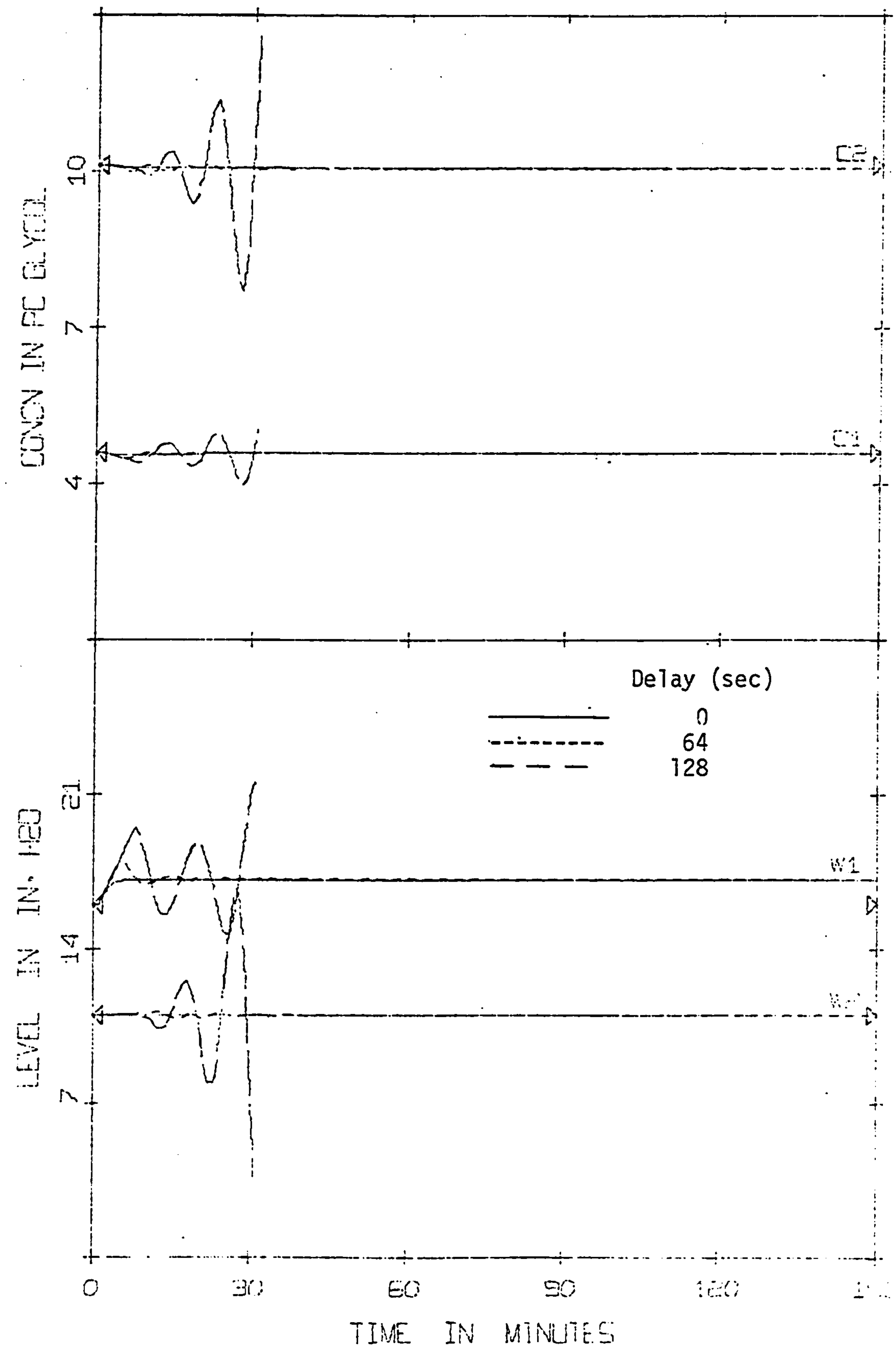

FIGURE 3.14 Simulated Multivariable Response with a Delay in the Control Variables (+20% Step Change in Feed Flow)

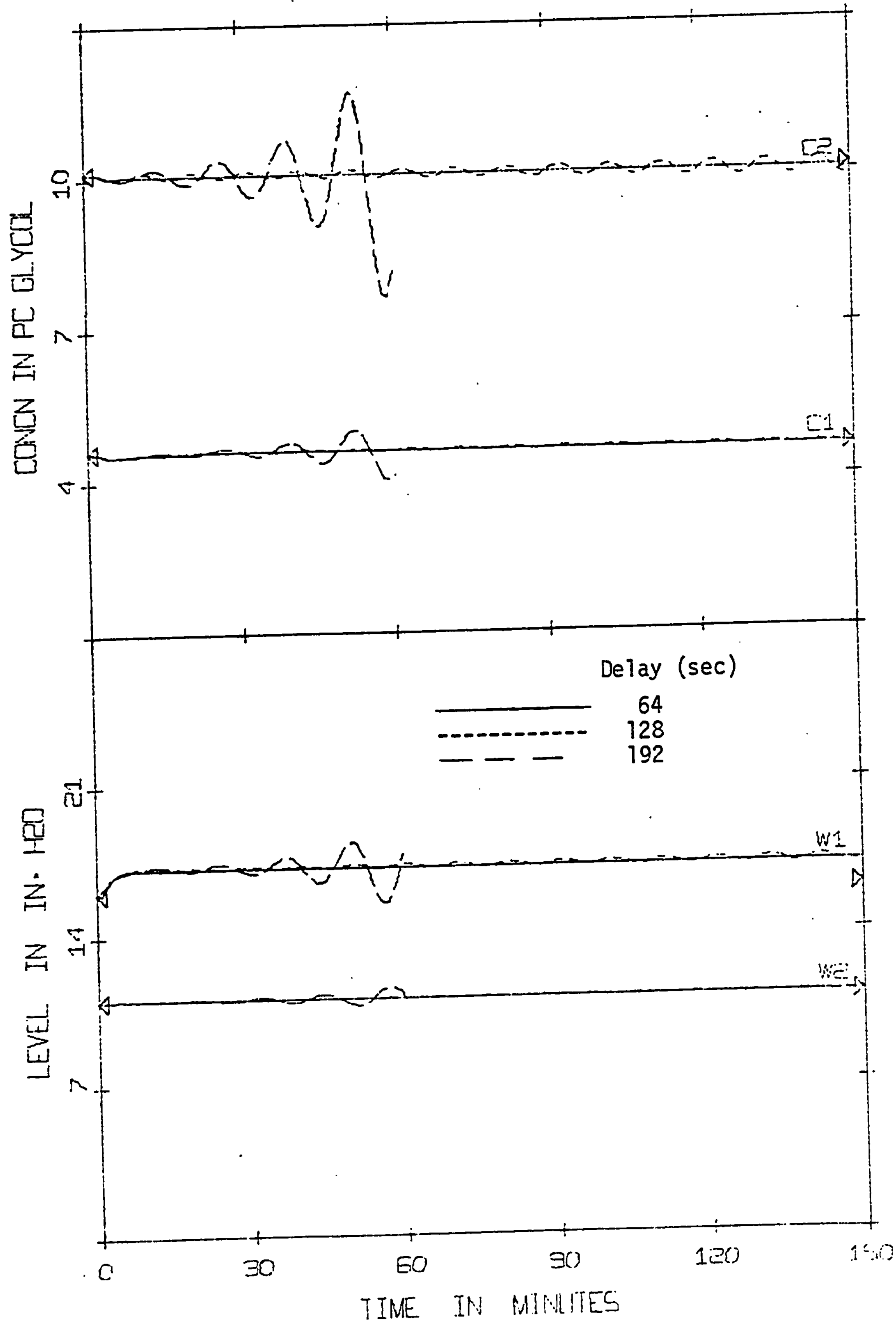

FIGURE 3.15 Simulated Multivariable Response With Measurement Delay in C_2 (+20% Step Change in Feed Flow)

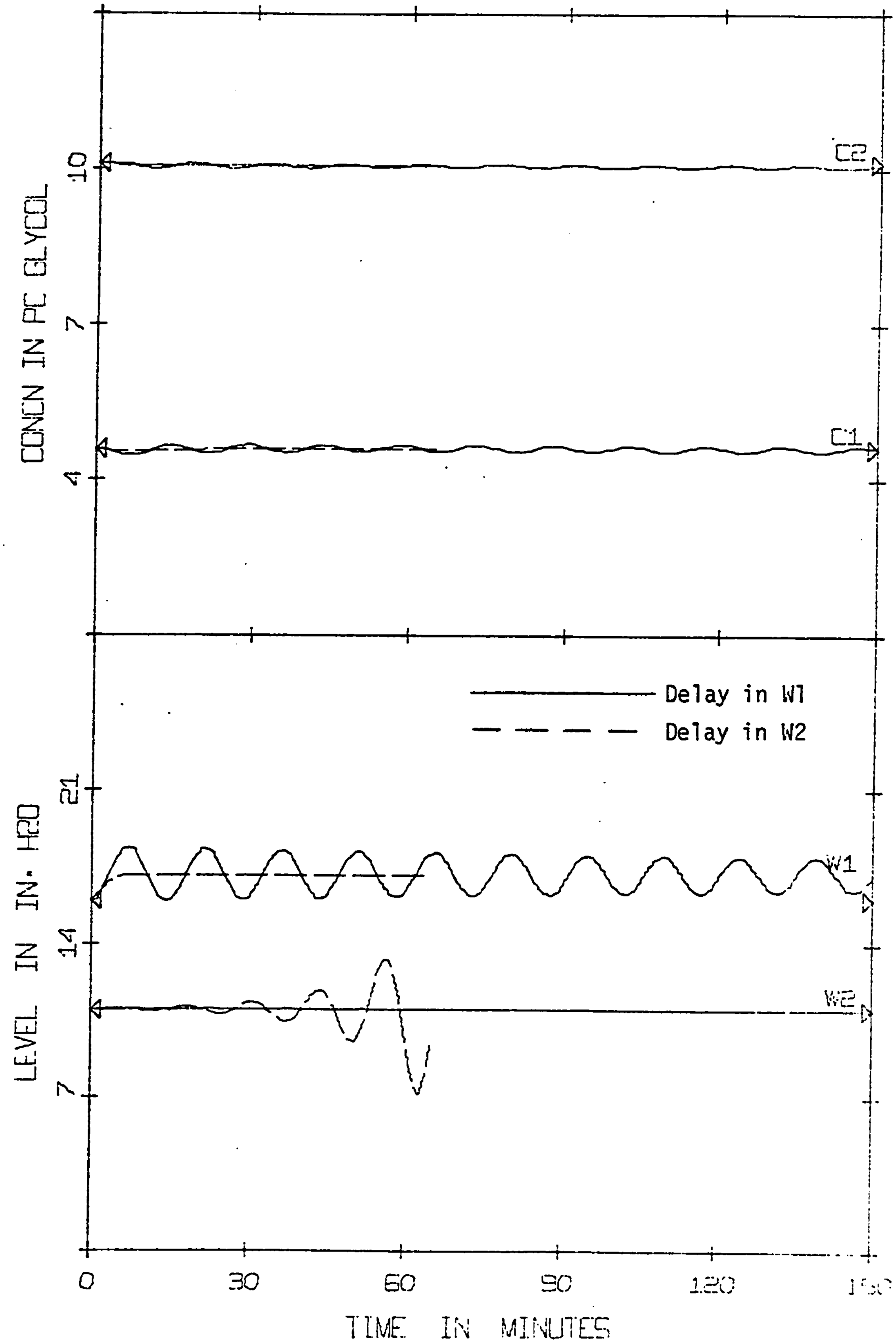

FIGURE 3.16 Simulated Multivariable Response with Measurement Delay 192 sec. in W_1 and W_2 (+20% Step Change in Feed Flow)

measurement of W_1. The overall response is better than for the case of delay of the same magnitude in concentration measurement due to the larger weighting of C2 in the derivation of the optimal feedback control matrix. As is shown in Figure 3.16, the measurement delay in the first effect holdup (W_1) affects the concentration more than a delay in the measurement of W_2, as would be expected from physical intuition (i.e. B_1 affects C_2 while B_2 does not).

3.5.2 Multivariable Predictor Control

The same predictor algorithm as in the multiloop case is used, the only difference being the use of the proportional feedback control matrix given in Table 3.4. Three cases are considered: delay in the manipulated variables, measurement delays and a combination of both types of delays.

Multivariable Predictor with Delays in the Manipulated Variables

The simulated response of the evaporator model under multivariable predictor control was obtained for process delays of 64, 128 and 256 sec. As is shown in Figure 3.17, the response is stable, and eventually approaches the response of the undelayed system except for the first effect holdup (W_1) that shows the same apparent offset as was observed in the multiloop simulated runs.

Multivariable Predictor with Delays in the Measurements and/or Manipulated Variables

The simulated response for the evaporator with multivariable predictor control and delays of 128, 384 and 640 sec. in the measurement of the product concentration, (C_2), is given in Figure 3.18 together with the response of the system without delays. For predictor control the response is stable and follows closely the response of the

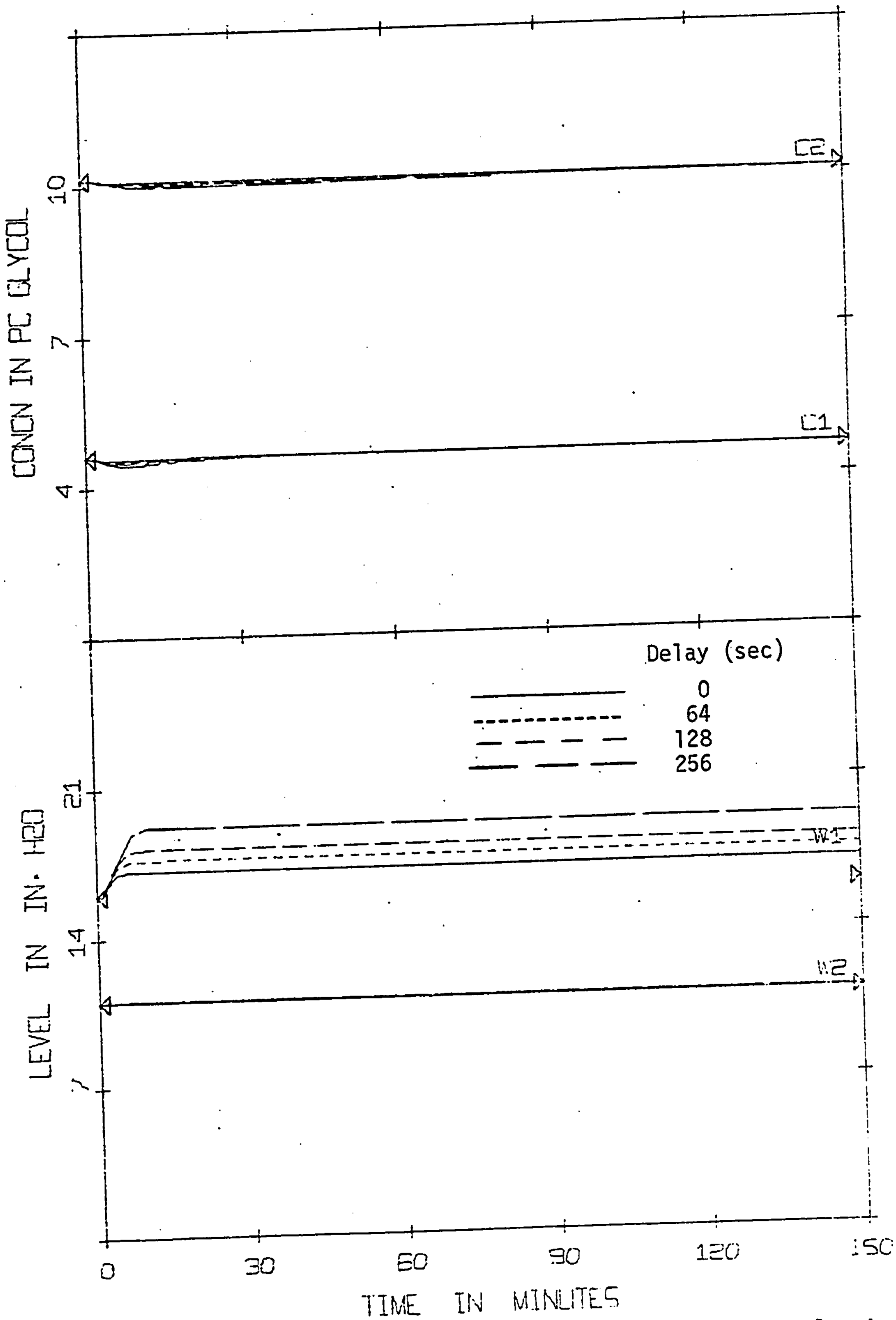

FIGURE 3.17 Simulated Multivariable Predictor Response with Delay in the Control Variables (+20% Step Change in Feed Flow)

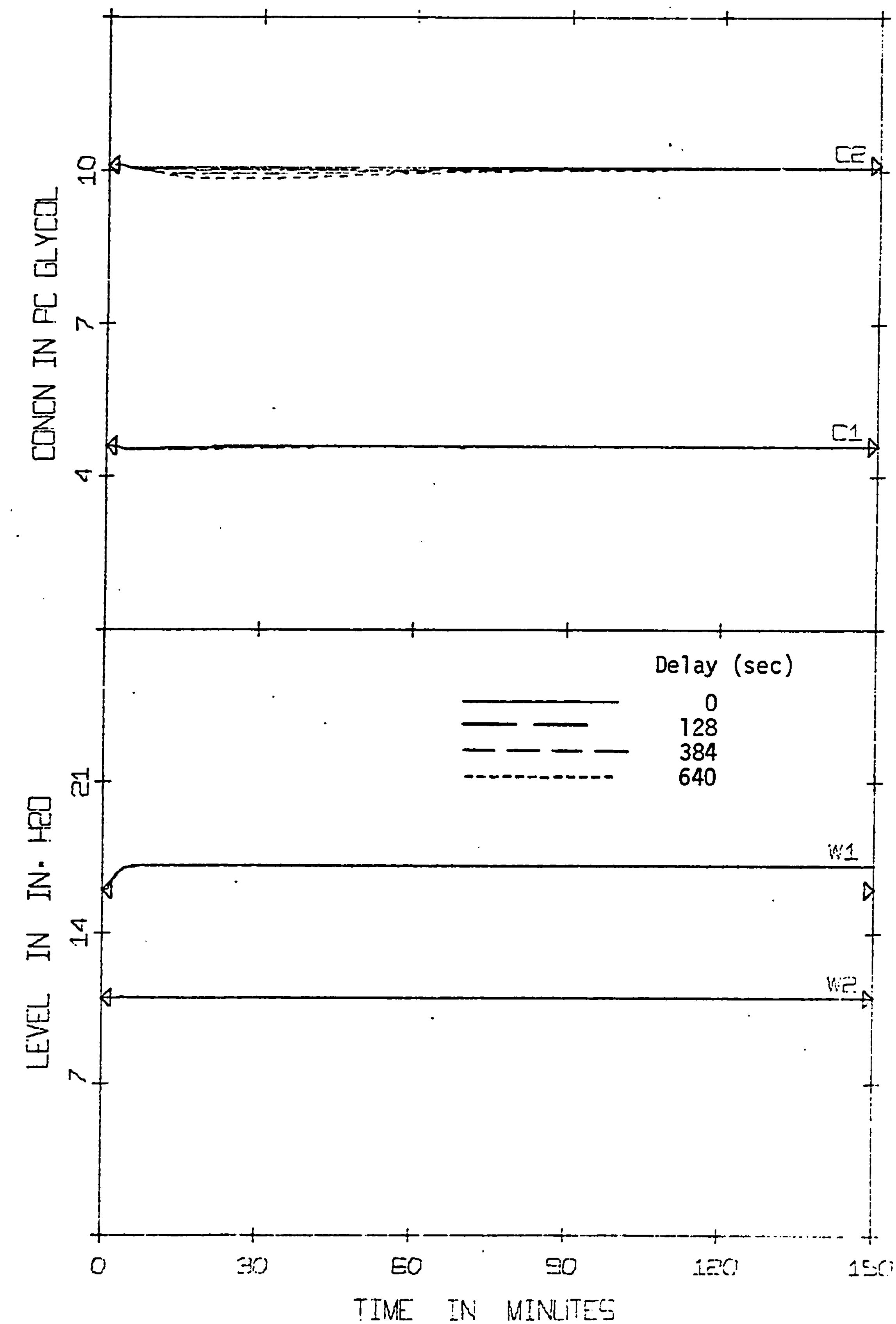

FIGURE 3.18 Simulated Multivariable Predictor Response with Measurement Delay in C_2 (+20% Step Change in Feed Flow)

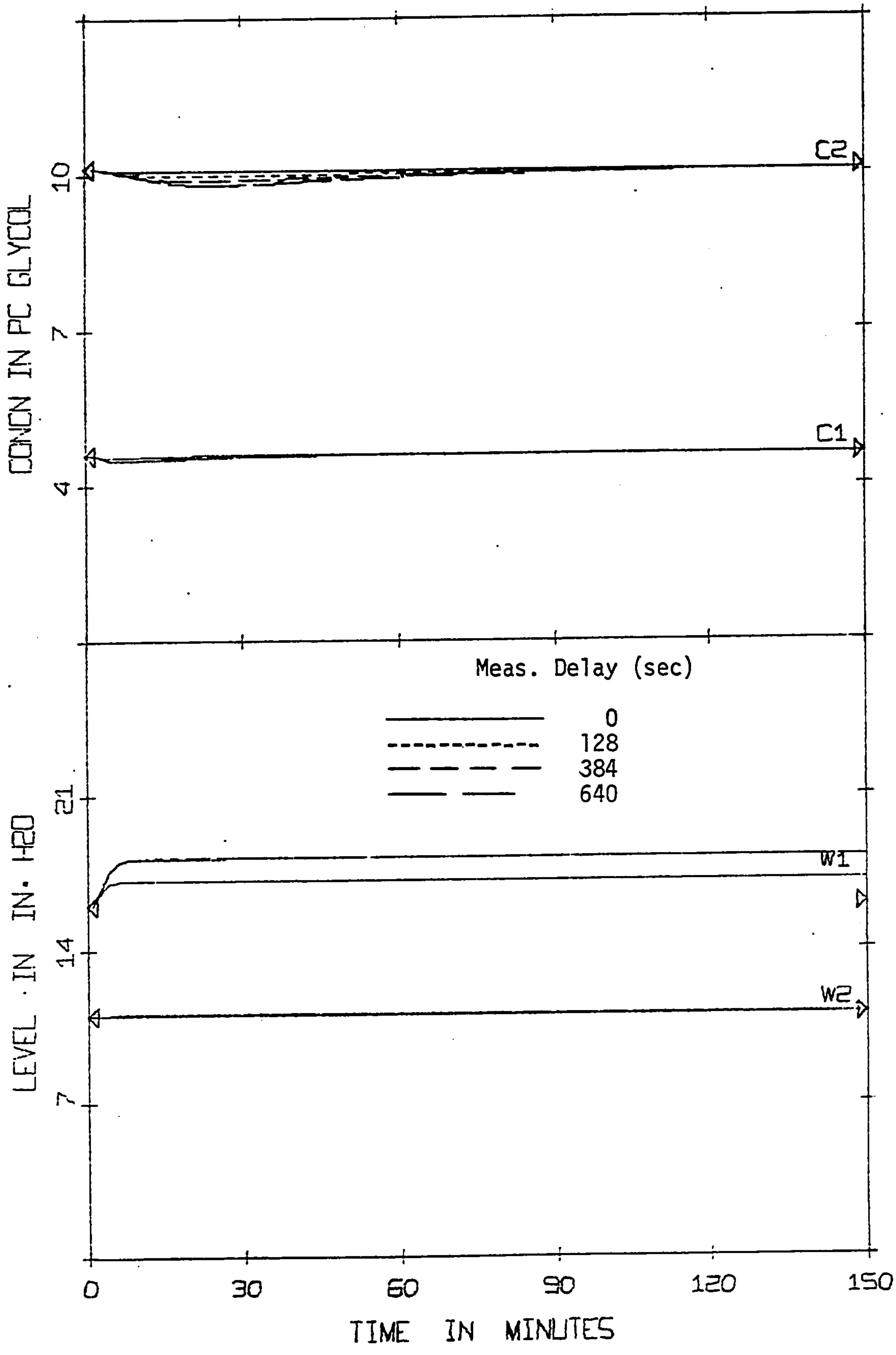

FIGURE 3.19 Simulated Multivariable Predictor Response with a Delay of 128 sec in the Control Variables and a Measurement Delay in C_2 (+20% Step Change in Feed Flow)

system without delays. No difference in offsets is obtained between the predictor response and the response of the undelayed system. In Figure 3.19 the response is given for delays in both manipulated and measured variables. The introduction of the delay in the manipulated variables resulted in increased offset in W_1 similar to that obtained for delay in only the manipulated variables, (Figure 3.5); otherwise, the response is similar to the one given in Figure 3.18 for measurement delays.

Effect of Model Inaccuracy on the Predictor Response

The effect of model inaccuracy was simulated (as in the multiloop case) by using inaccurate representation of the time delay in the predictor loop. The case of delayed control variables proved to be extremely sensitive to inaccurate estimates of the delay. The simulated response is given in Figure 3.20 for an actual delay of 512 sec. with inaccuracy of $\pm$ 64 sec. in the assumed predictor loop delay. The response is unstable and is generally worse for a low estimate (448 sec.) than a high estimate (576 sec.) of the delay.

Similarly for a measurement delay in C_2, the effect of an inaccuracy of $\pm$ 64 sec. in the assumed predictor loop delay results in unstable response (not shown). Improvement of the response is obtained for larger values of the time delay and the same degree of inaccuracy as shown in Figure 3.21.

It should be noted that conclusions drawn from the simulated responses of Figures 3.20 and 3.21 should not be generalized, since the dependence on sampling time and delay is an important factor; eg. faster sampling time will result in less sensitivity for the same inaccuracy of $\pm$ 1 sampling interval in the predictor loop delay,

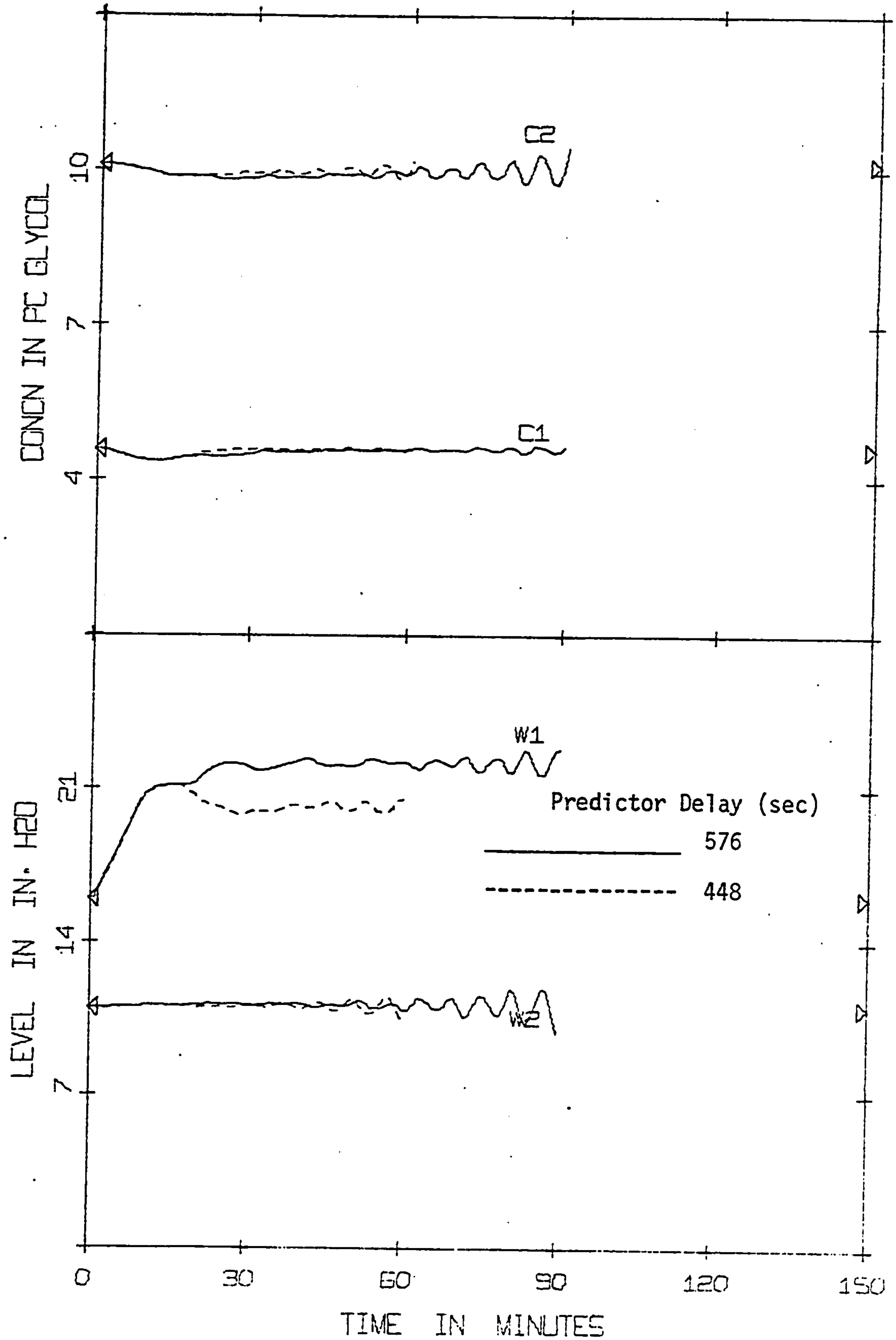

FIGURE 3.20 Simulated Multivariable Response with a Delay in the Control Variables of 512 sec and an Inaccurate Predictor (+20% Step Change in Feed Flow)

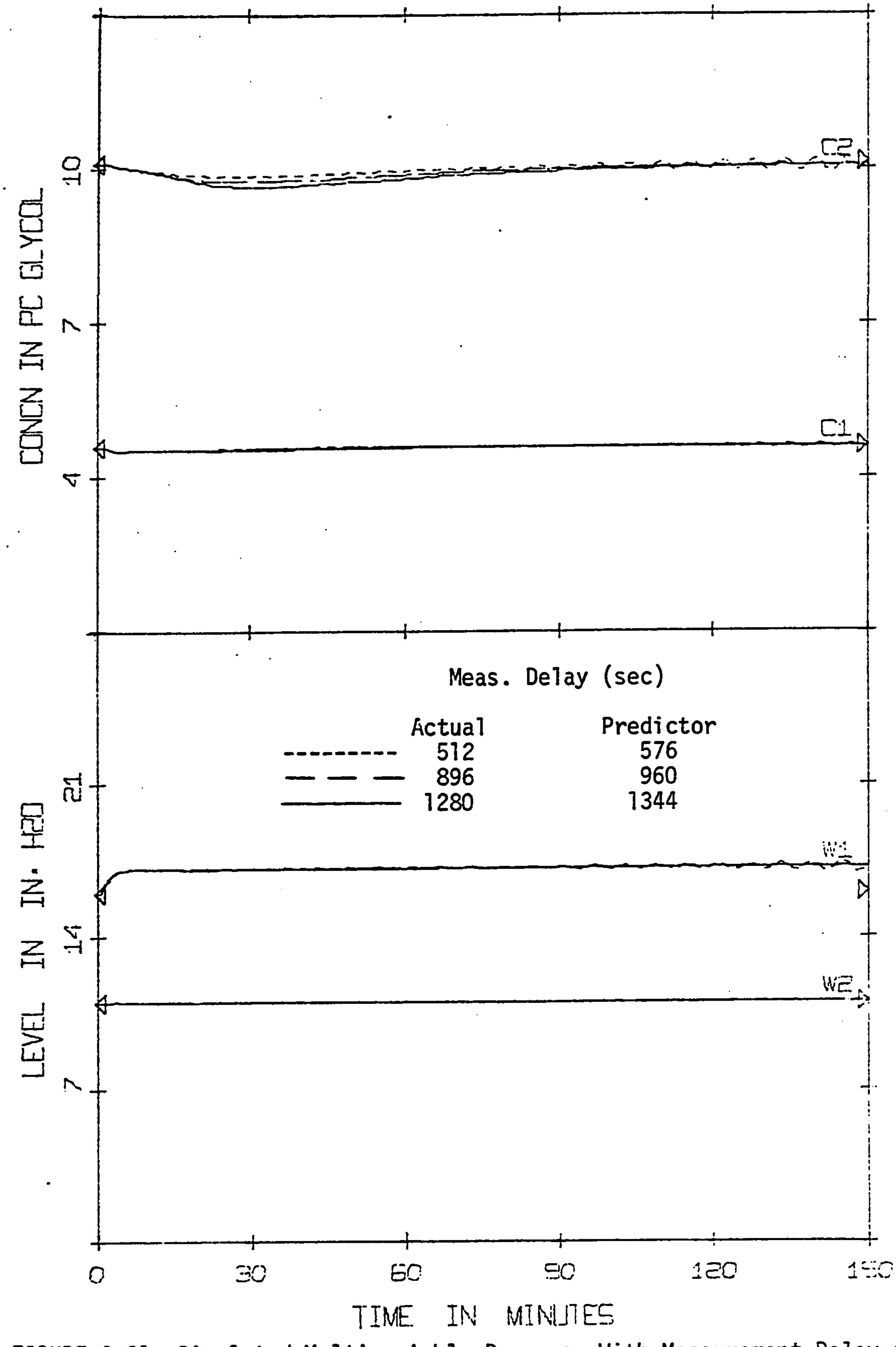

FIGURE 3.21 Simulated Multivariable Response With Measurement Delay in C_2 and Inaccurate Predictor (+20% Step Change in Feed Flow)

assuming that the delay is taken as an integer multiple of the sampling time. For instance, the sensitivity will be different for an error of ± 50% in the representation of the delay, in the predictor loop, according to the sampling time used. As a general conclusion, though, it can be stated that in the evaporator application the multivariable predictor is more sensitive to inaccuracies in the predictor loop than the multiloop predictor due to the large controller gains in multivariable control (cf. Equation (3.5) and Table 3.4).

3.6 The Effect of an Inaccurate Time Delay in the Predictor Loop and a Compensating Algorithm

In this section the compensating algorithm for inaccurate or time varying delays (see Section 2.5) is applied to the double effect evaporator model under multiloop predictor control. A delay of 192 sec. is introduced in all the manipulated variables while the predictor is initially using a delay of 128 sec. According to the theory developed in § 2.5, the compensating algorithm will start a search of the stored past control values according to the algorithm developed in § 2.5. The search of the past control values starts from the initial assumption of a delay of 128 sec. and continues for larger values of the delay. To demonstrate the effect of an unmeasured change in the load variable, the search of the past control values continues at every sampling interval, although the delay is assumed constant, and the initial disturbance of +20% in feed flow is changed after 80 sampling intervals to +10%.

The simulated response is given in Figure 3.22 together with the responses for the ideal case where the predictor used the actual value of the time delay, (i.e. 192 sec.) and is aware of the feed flowrate. As is shown in Figure 3.22, the response follows closely the

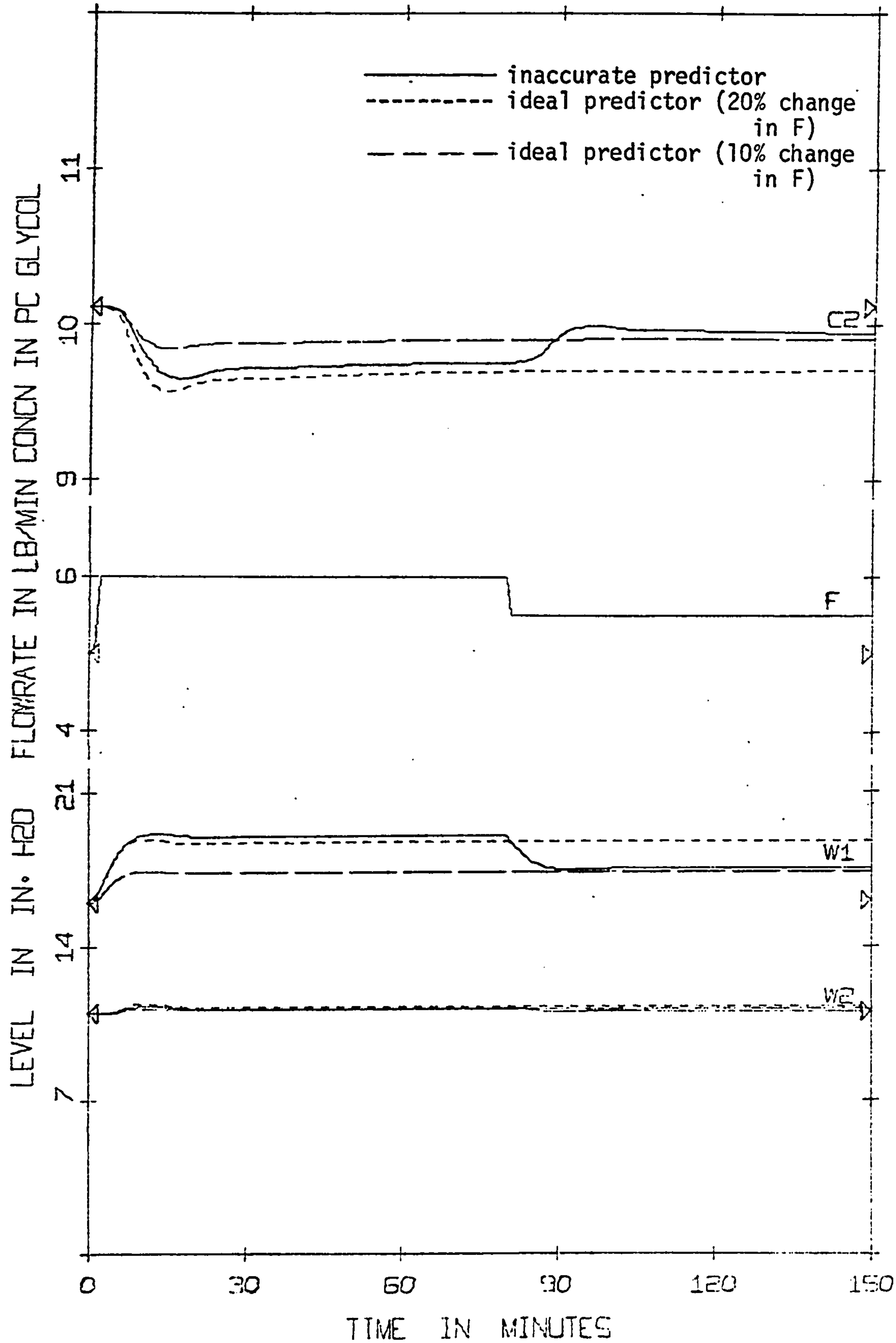

FIGURE 3.22 Simulated Response of Compensating Algorithm for Actual Delay 192 sec and Predictor Delay of 128 sec

responses for the ideal predictor case. The actual value of the delay is determined after a time interval approximately equal to the time delay. The predictor has knowledge of the initial disturbance of +20% but not of the load change from +20% to +10%. In the example of Figure 3.22, since the actual delay is constant, the search should stop as soon as the exact delay was found and this value used in the predictor loop. The predictor then would function according to the original predictor scheme without further use of the compensating algorithm. However, the search was continued at each sampling instant in order to demonstrate the ability of the algorithm to sense the change in the disturbance according to the logic developed in § 2.5.

It should be noted that under steady state conditions any value of the delay in the predictor loop will not affect the response. Also, the continuous search is necessary only if time varying delays occur.

3.7 Discussion of Simulation Results

The effect of the time delays on the simulated response of the double effect evaporator was proved to result in very bad response for both conventional multiloop and multivariable feedback control schemes. The system becomes unstable for small values of the time delay. Compensation for the time delay effect can be accomplished by lowering the controller gains but, inevitably, results in larger offsets and slower responses. This procedure does not always result in a satisfactory response especially when large delays are involved. The response of the delayed system was generally better with multiloop control than with multivariable control for the control matrices used in the simulation runs.

Introduction of the Smith Predictor into the control scheme significantly improved the response for delays in the measurement and/or the control variables. Two control schemes were examined in the Smith Predictor case, multiloop and multivariable predictor control. In both cases, the detrimental effects of the time delay were eliminated, even for very large delays. The response of the system follows the response of the undelayed system with a small increase in the offset of the first effect level in the case of delays in the manipulated variables. This increase in the offset is due to the small controller gains and eventually decreases to the respective value for the undelayed system. The Smith Predictor does not introduce any increase in offset and this can be seen in the numerical example in § 2.6.

Inaccuracies in the process model, expressed as an inaccurate representation of the time delay in the predictor loop badly affected the response when multivariable predictor control was used, while the multiloop predictor scheme demonstrated less sensitivity to modelling errors. Finally, use of smaller controller gains improves the predictor response in the case of large modelling errors.

CHAPTER FOUR

THE MULTIVARIABLE SMITH PREDICTOR APPLIED TO A DOUBLE EFFECT EVAPORATOR: EXPERIMENTAL RESULTS

4.1 Introduction

In this chapter an experimental application of the multivariable Smith Predictor algorithm is presented. The experimental system consisted of a pilot scale double effect evaporator, interfaced to an IBM 1800 data acquisition and control computer. The theoretical development of the predictor algorithm as well as simulated results have been presented in the previous two chapters. This chapter is concerned with the implementation of the predictor algorithm on the experimental system and with the results obtained.

4.2 Control System

The implementation of the predictor algorithm was achieved through a modification of the multivariable control program developed by Newell [21]. The required steps for the implementation of multivariable control are given in the block diagram in Figure 4.1. An IBM 1800 data acquisition and control computer is interfaced to a pilot plant double effect evaporator, shown diagramatically in Figure 4.2, through analog to digital converters, digital to analog converters, and required transducers. Detailed descriptions of the equipment as well as the required electronic instruments etc. can be found in the theses of Newell [21], Fehr [27] and Andre [22].

The multivariable control program developed by Newell utilizes the standard Direct Digital Control (DDC) package [28] for assessing measurements of the state variables and implementing control through

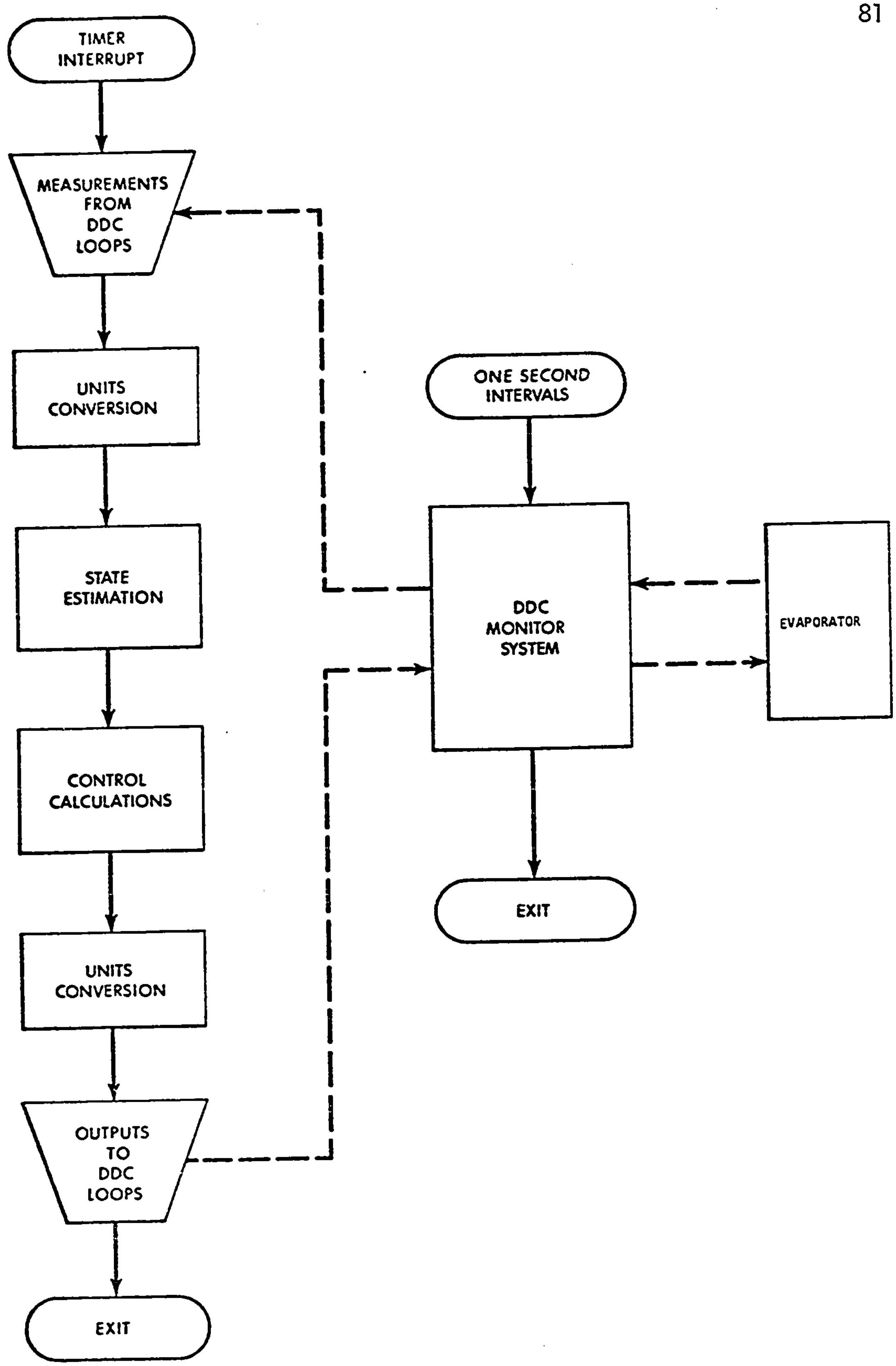

FIGURE 4.1 Schematic Diagram of Multivariable Control

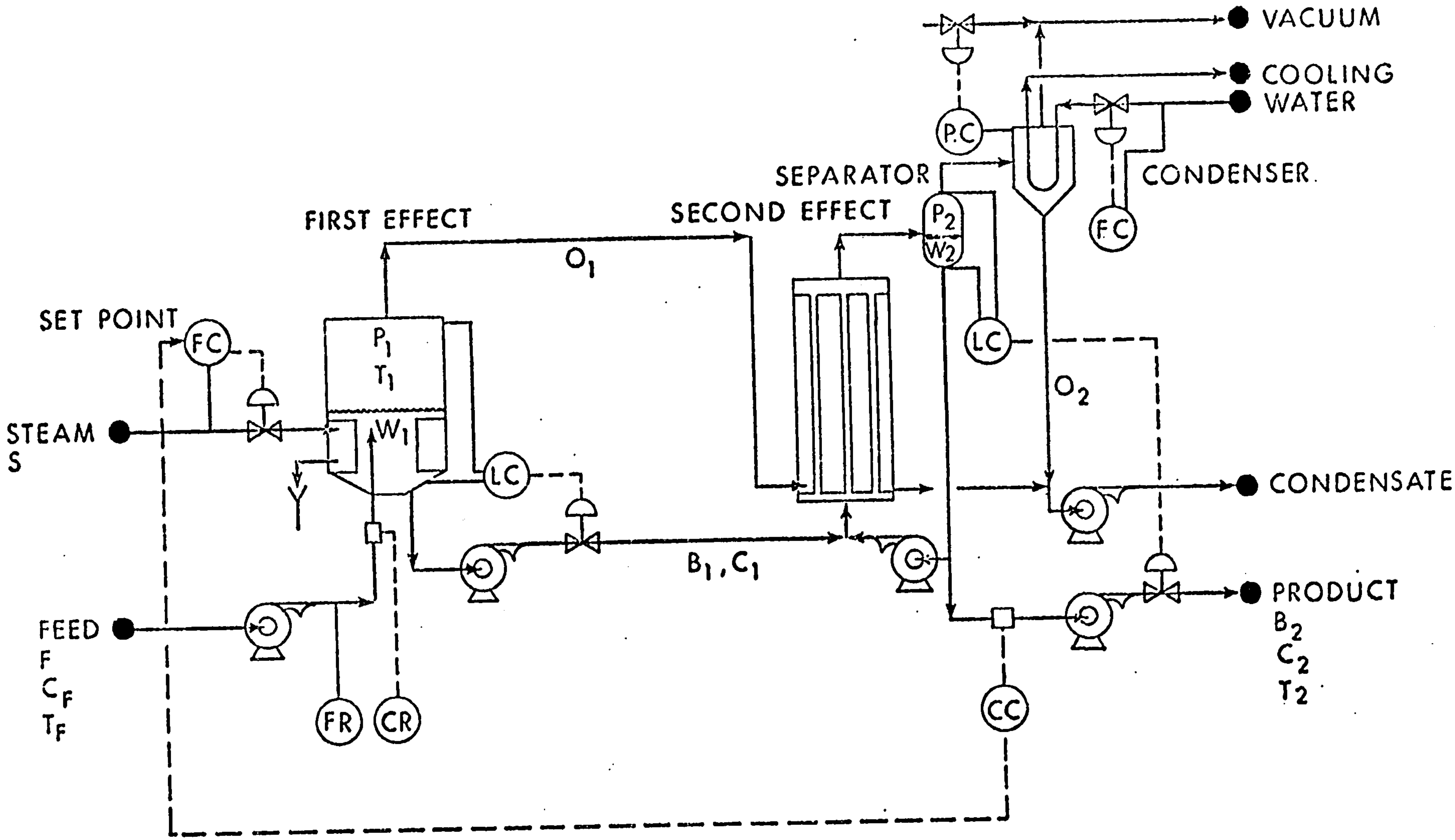

FIGURE 4.2 Schematic Diagram of Double Effect Evaporator and Multiloop Control Scheme

adjustments of setpoints in the DDC loops.

The standard multivariable control program shown in block diagram in Figure 4.1 performs model calculations for estimating the unmeasured first effect concentration (C_1) and performs all the necessary conversions of analog signals to engineering units and vice versa. The control program which is written in FORTRAN except for assembler subroutines used for communications with DDC, is transferred from disk storage to core for execution at specified time intervals designated by a high priority timer interrupt. In the experimental runs presented in this chapter, the control interval was 64 seconds. Although the standard multivariable program requires from 4 to 7 seconds execution time, the modifications introduced for the predictor scheme considerably increased the size of the computer program. This resulted in an increase of the execution time since core storage limitations necessitated an increase in disk swapping.

The modifications for the predictor application consisted of the following parts:

1. The evaporator pilot plant does not have any significant time delays and no provision for such a case was included in the multivariable program. The time delay was easily simulated in the multivariable program by disk storage, and subsequent retrieval, of the measured states. This procedure refers to simulation of measurement delays; the model calculations for the first effect concentration proceeded as in the standard multivariable control program.
2. The program included the discrete-time Smith Predictor algorithm that was used in the simulation study. This was easily incorporated

since the standard multivariable program stores values of the past control vectors. Consequently, these values could be retrieved from disk, converted to perturbation form, and inserted in the calculations of the predictor algorithm.

The case of delays in the manipulated variables can also be easily included in the multivariable control program by sending a delayed control signal to the process.

4.3 Experimental Runs

The predictor control scheme was experimentally applied for the case of a measurement delay in product concentration. All experimental runs utilized a step change of +20% in feed flow at the initiation of the run. The initial steady states were between 9 and 10% in product concentration. Two values for the measurement delay were used, 256 and 512 sec. Due to equipment difficulties, multivariable control runs were not made.

The behavior of the undelayed system under multiloop control is shown in Figure 4.3. The control matrix used in all the multiloop experimental runs is the one given in Table 3.1 for the simulation studies. The initial and final steady state values of the four measured states are shown in Table 4.1. As indicated in Figure 4.3, the +20% step change in the feed flow results in significant offsets in W_1 and C_2 since only proportional feedback control is being used. In Figure 4.4 the response is given for multiloop predictor control with a time delay of 256 sec. in the measurement of product concentration. This delay was introduced as described in Section 4.2 and the same control matrix was used as for the undelayed case of Figure 4.3.

Comparison of the state and the control variables in Figures 4.3 and 4.4 shows that the predictor response is more oscillatory but follows the response of the undelayed system quite well with no increase in the offset. This is more clearly shown in Figure 4.5 where an overplot is given of the experimental responses corresponding to Figures 4.3 and 4.4. In Figure 4.6 the simulated and experimental multiloop predictor responses are compared for a measurement delay of 256 sec. in C_2. Figure 4.6 indicates that the two responses agree quite clearly. In Figure 4.7 the experimental multiloop predictor response is given for a measurement delay of 512 sec. in C_2. The response of C_2 is again satisfactory and only W_2 shows an increase in the degree of oscillations. In Figure 4.8 the simulated and experimental multiloop predictor responses are compared for a measurement delay of 516 sec. in C2.

The experimental responses for B_2 shown in Figures 4.3, 4.4 and 4.7 are somewhat noisier than in previous studies [21,23]. This was due to a noisy flow transmitter which was corrected later.

4.4 Discussion of Results

The experimental results confirm the conclusions drawn from the simulation studies. The predictor performed satisfactorily for delays in the measurement of product concentration as large as 512 sec. The predictor response follows closely the response of the system without delays without undue oscillations or increase in offsets. Furthermore, the experimental runs demonstrate one of the main advantages of the Smith Predictor method, namely, that measurement of the disturbance is not required.

Another conclusion drawn from the experimental runs is that modelling errors that inevitably exist, did not significantly affect the predictor response and this, again, is in agreement with the simulation studies.

Finally, physical constraints on the control variables which were not incorporated in the simulation studies did not have adverse effects on the experimental predictor response.

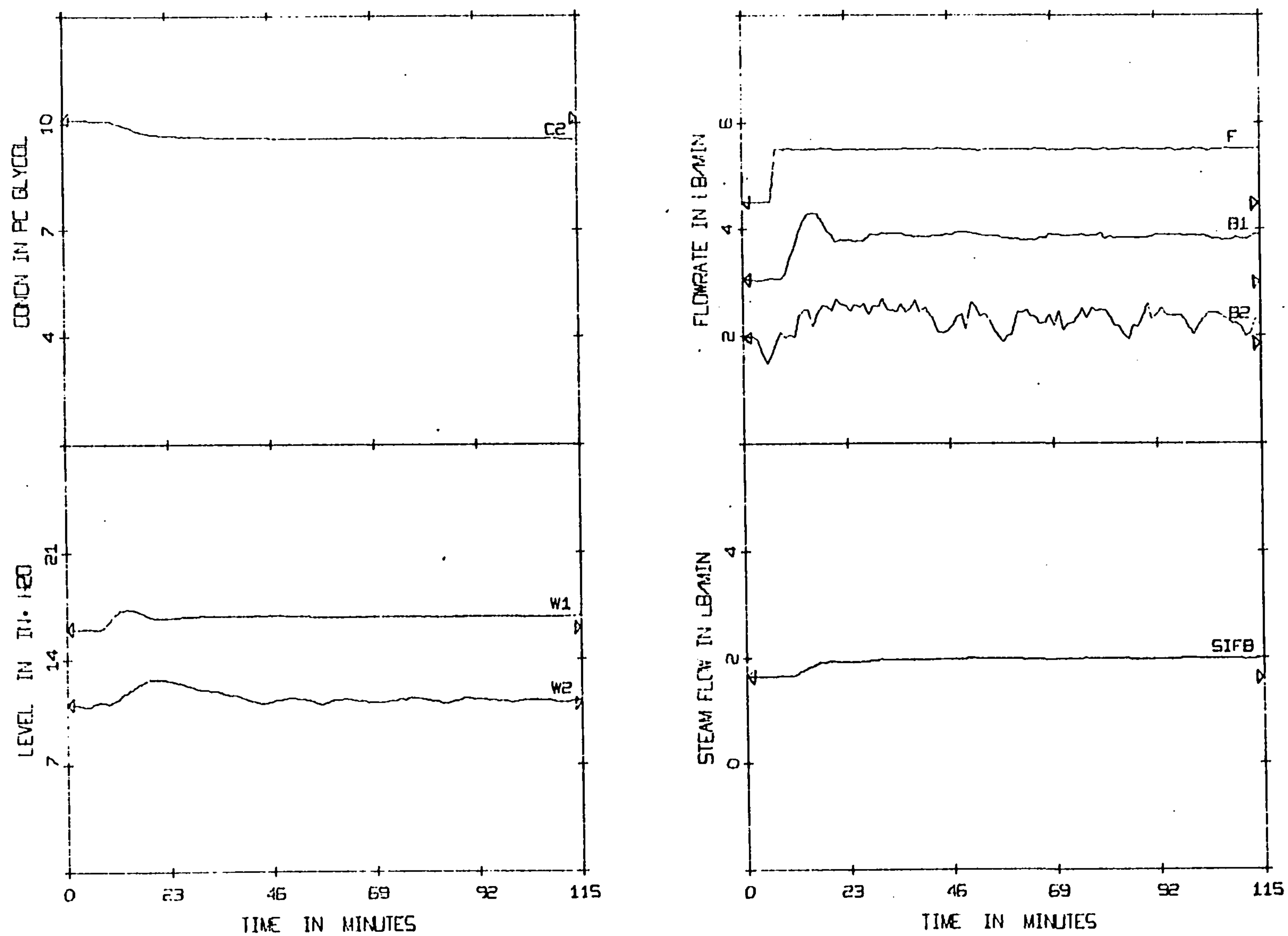

FIGURE 4.3 Experimental Multiloop Response for the Undelayed System (+20% Step Change in Feed Flow)

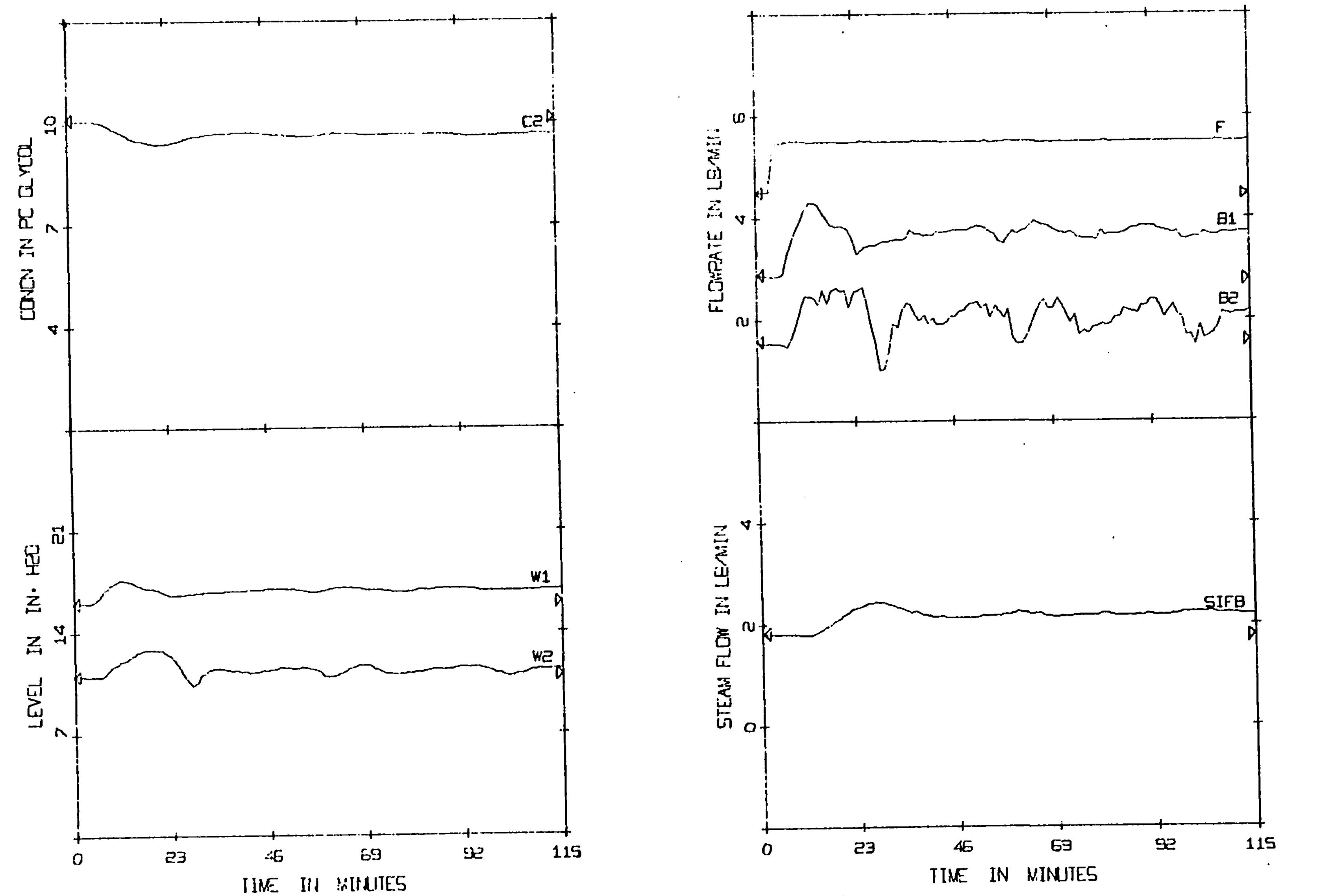

FIGURE 4.4 Experimental Multiloop Predictor Response with Measurement Delay of 256 sec. in C_2 (+20% Step Change in Feed Flow)

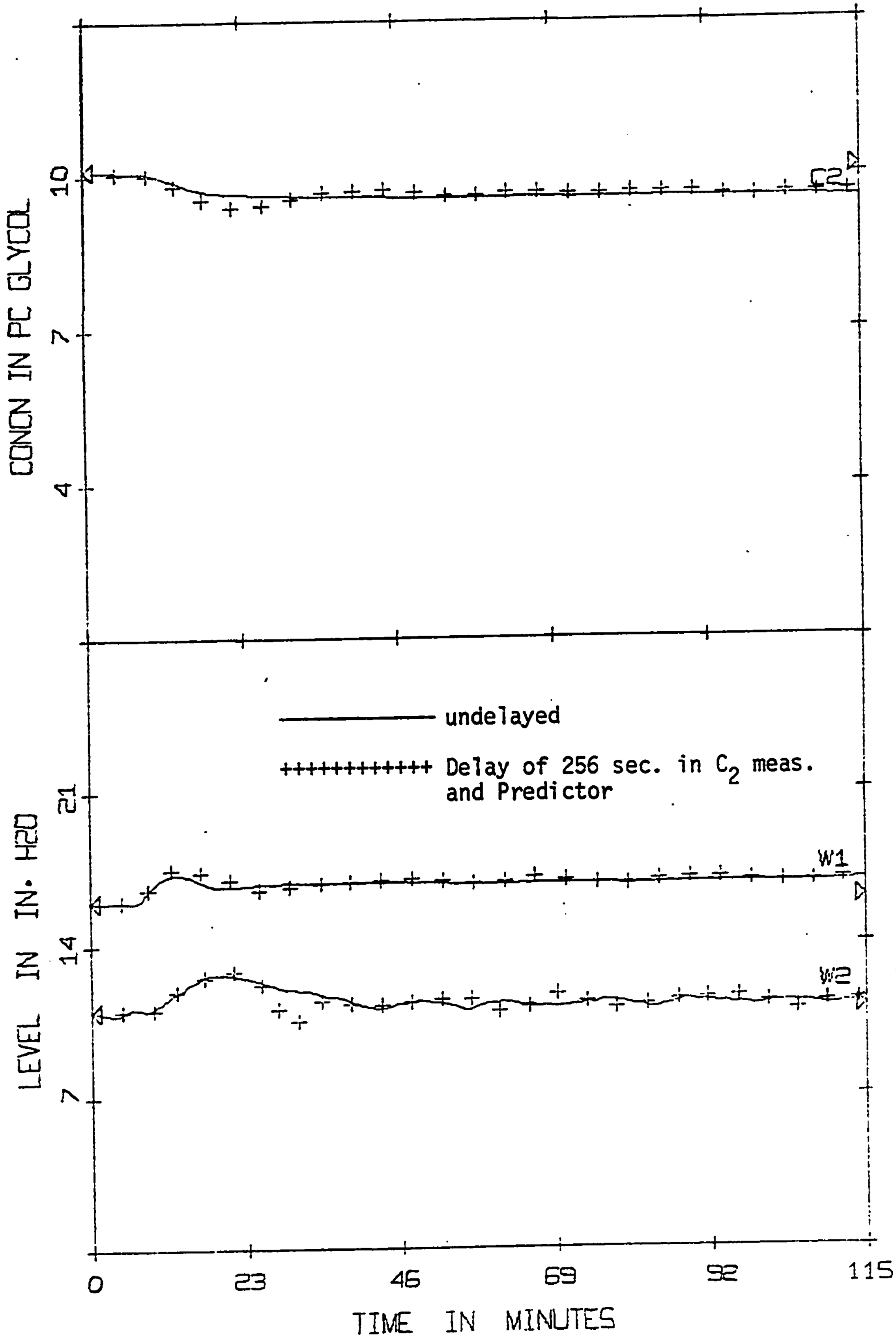

FIGURE 4.5 Comparison of the Experimental Multiloop Response for Predictor and Delay with the Response of the Undelayed System (+20% Step Change in Feed Flow)

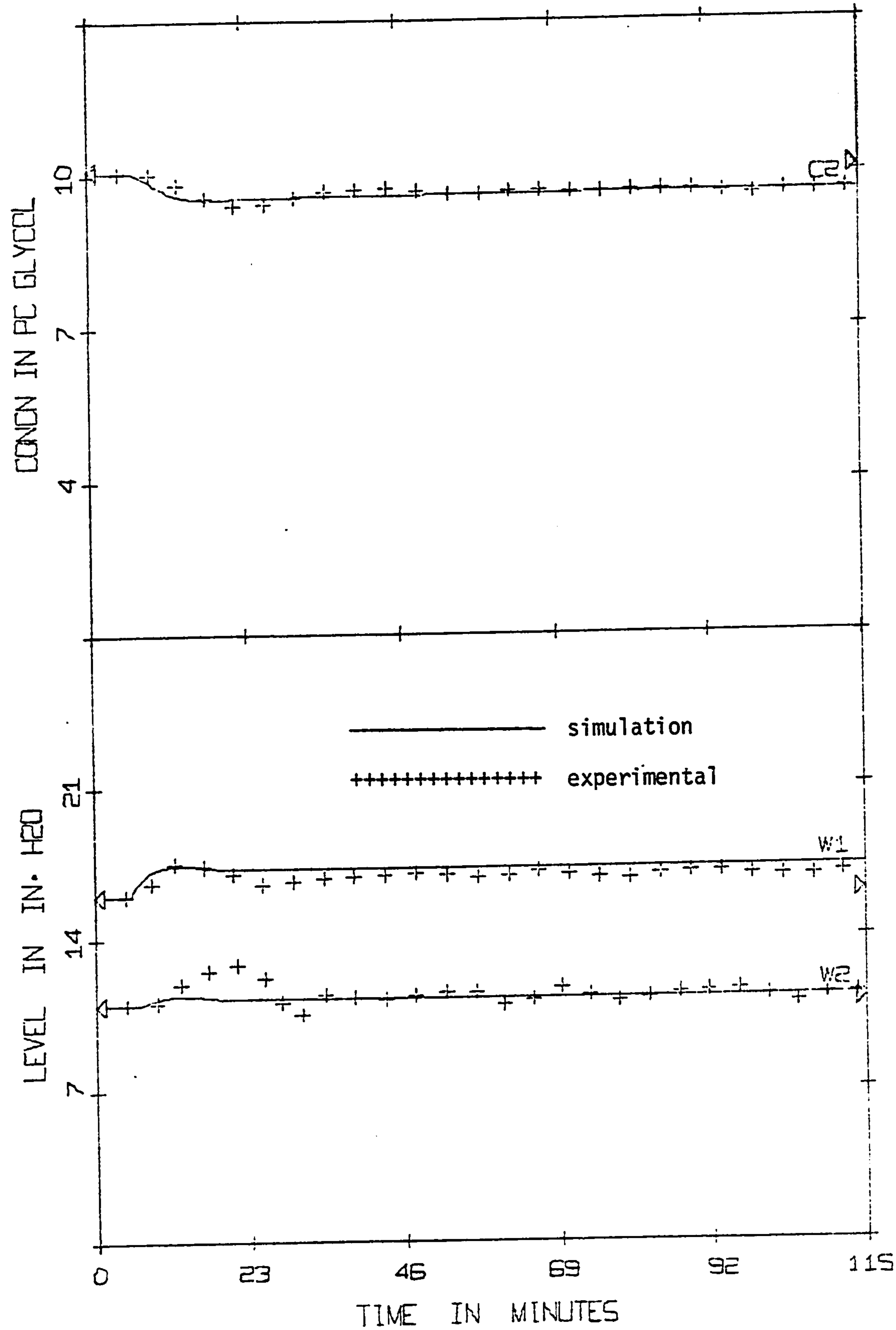

FIGURE 4.6 Comparison of Experimental and Simulated Multiloop Predictor Responses for Measurement Delay of 256 sec. in C_2 (+20% Step Change in Feed Flow)

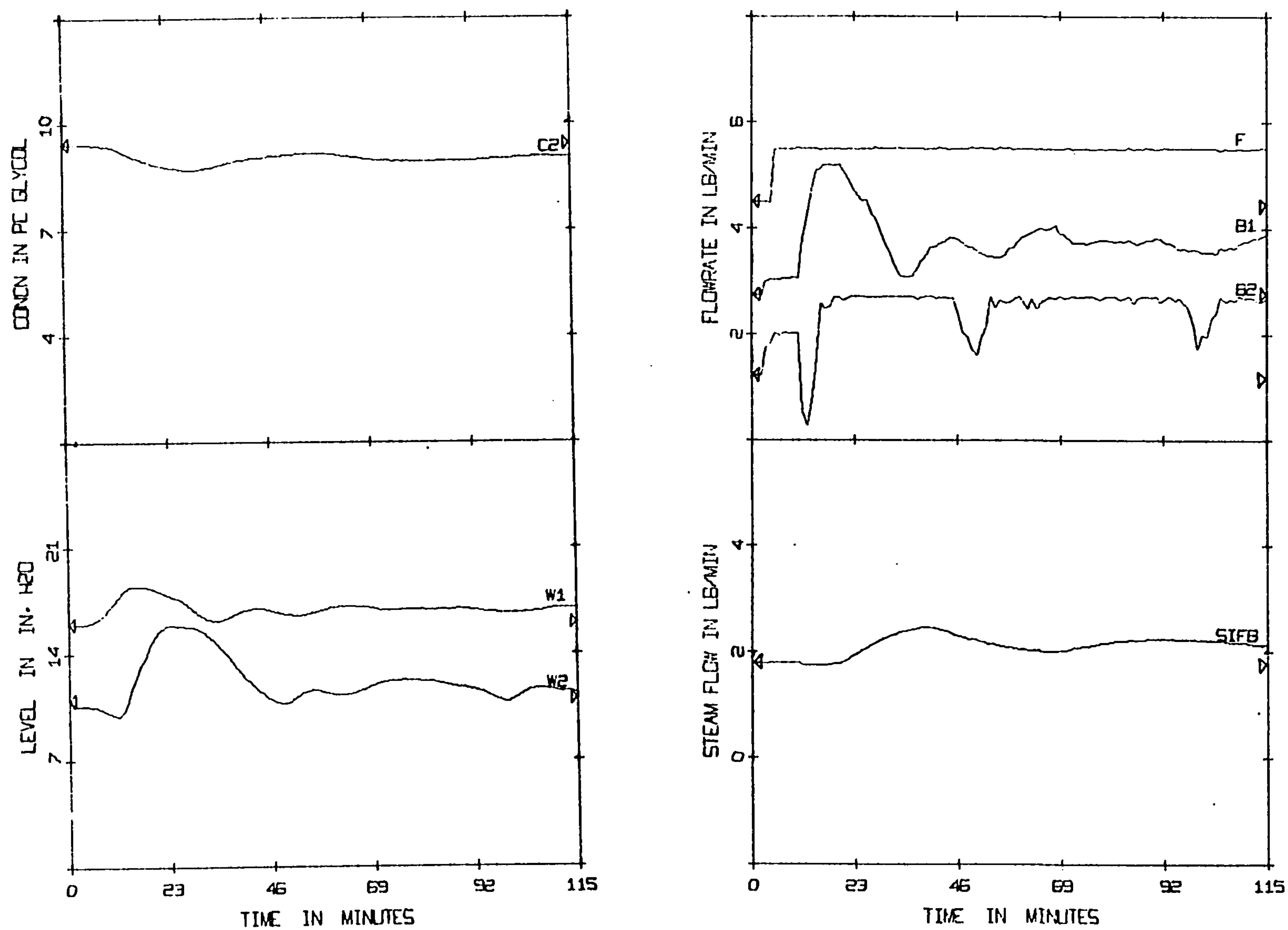

FIGURE 4.7 Experimental Multiloop Predictor Response with Measurement Delay of 512 sec. in C_2 (+20% Step Change in Feed Flow)

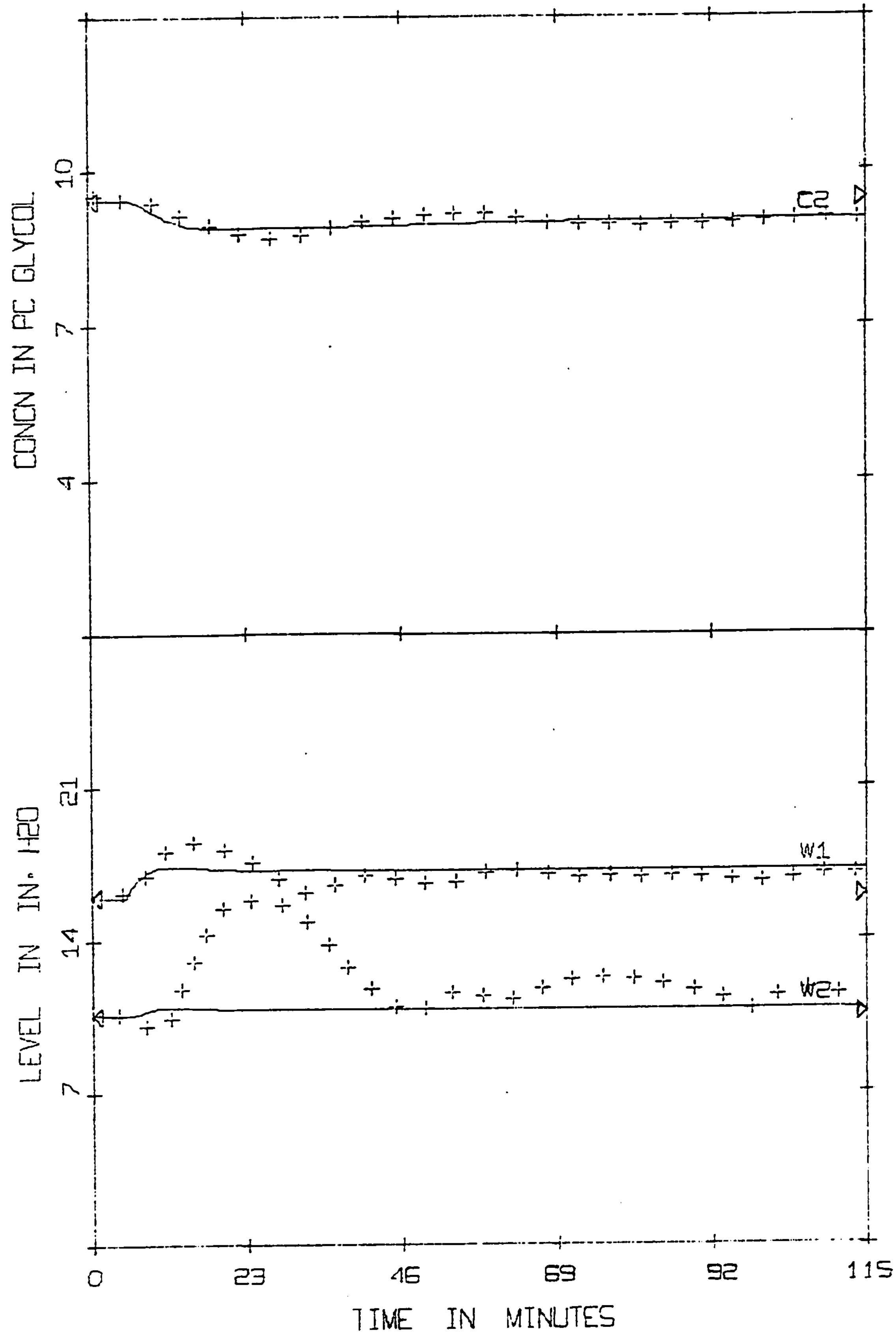

FIGURE 4.8 Comparison of Experimental and Simulated Multiloop Predictor Responses for Measurement Delay of 516 sec. in C_2 (+20% Step Change in Feed Flow)

Table 4.1

Steady States for Experimental Multiloop Runs

Variable	Units	Figure 4.3		Figure 4.4		Figure 4.7	
		Initial s.s.	Final s.s.	Initial s.s.	Final s.s.	Initial s.s.	Final s.s.
W_1	in. H_2O	16.0	16.8	16.0	16.8	15.9	17.0
h_1	deg. F	215.4	220.3	219.5	228.0	218.3	227.0
W_2	in. H_2O	11.0	11.2	11.0	11.2	10.6	11.4
C_2	% Glycol	10.09	9.54	10.05	9.62	9.42	9.01

CHAPTER FIVE

CONCLUSIONS

The classical Smith Predictor Method (Smith, [5,6]) for single variable control systems has been extended to a class of linear multivariable systems. Multivariable Smith Predictors have been derived for both continuous-time and discrete-time systems which contain time delays in either the control variables and/or the output variables. An important advantage of the multivariable Smith Predictor is that it eliminates time delays from the characteristic equation of the closed loop system. This allows the control system designer to choose from the wide variety of synthesis techniques available for systems without time delays as opposed to the much smaller number of techniques which are applicable to systems containing time delays. Another advantage of the multivariable Smith Predictor is that the discrete-time algorithm in Equations (2.25) - (2.28) can be easily implemented on a real-time digital computer.

Simulation and experimental runs were carried out to investigate the performance of the predictor algorithm on a double effect evaporator pilot plant. The simulation runs considered multiloop and multivariable control scheme while the experimental runs with the evaporator/IBM 1800 control configuration confirmed the simulation results for multiloop control. Simulation runs also examined the effect of modelling errors on the predictor response with multiloop and multivariable control.

Furthermore, systems with time delays in some of the control variables and time varying or inaccurately determined delays were theoretically investigated and algorithms were developed for handling these more complex systems.

NOMENCLATURE

Alphabetic

a	time delay in control variables
$\underline{\underline{A}}$	state coefficient matrix
b	time delay in output variables
$\underline{\underline{B}}$	control coefficient matrix
B_1	first effect bottoms flow
B_2	second effect bottoms flow
$\underline{\underline{C}}$	output coefficient matrix
C_1	first effect concentration
C_2	second effect concentration
C_F	feed concentration
d	load variable
$\underline{d}$	load vector
$\underline{D}$	z-transform load vector
$\underline{\underline{D}}$	disturbance coefficient matrix
e	error
$\underline{e}$	error vector
F	feed flow
G	transfer function
$\underline{\underline{G}}$	transfer function matrix
h_1	first effect enthalpy
h_F	feed enthalpy
H	output transfer function
I	integral control
$\underline{\underline{I}}$	identity matrix
k_{ij}	elements of control matrix

Nomenclature (continued)

$\underline{\underline{K}}_c$	control matrix
$\underline{\underline{K}}$	constant matrix in Figure 2.5
ℓ	undelayed control vector dimension
m	control vector dimension
$\underline{\underline{M}}$	matrix defined in Equation (2.36)
n	state vector dimension
p	load vector dimension
$\underline{p}$	predictor output vector
$\underline{P}$	z-transform of predictor output vector
PI	proportional-integral control
PID	proportional-integral-derivative control
r	output vector dimension
$\underline{r}$	setpoint vector
s	Laplace operator
S	steam flow
$\underline{S}$	vector defined in Equation (2.82)
SI	sampling interval
t	time
$\underline{\underline{T}}$	discrete state coefficient matrix
u	control variable
$\underline{u}$	control vector
$\underline{U}$	z-transform of control vector
$\underline{w}$	vector defined in Equation (2.55)
W_1	first effect holdup
W_2	second effect holdup
x	state variable

Nomenclature (continued)

$\underline{x}$	state vector
$\underline{X}$	z-transform of state vector
y	output variable
$\underline{y}$	output vector
$\underline{Y}$	z-transform of output vector
z	z-transform operator

Greek

$\underline{\underline{\Delta}}$	discrete disturbance coefficient matrix
$\underline{\underline{\theta}}$	discrete control coefficient matrix
$\underline{\underline{\phi}}$	discrete state coefficient matrix

Subscripts

c	controller
cj	partitions of control matrix
CAL	calculated
EST	estimated
FB	feedback
L	load
p	process
ss	steady state

Superscripts

'	perturbation variable
•	time derivative

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