Definitions and Conversion Equations for Oil Sands Tailings

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Table of Contents

1. INTRODUCTION 2

2. TAILINGS PHASE DIAGRAM 5

3. INDEX OF TAILINGS PARAMETERS 6

4. DEFINITION OF TAILINGS PARAMETERS 8

5. VISUAL COMPARISON BETWEEN GEOTECHNICAL AND MINING DEFINTIONS 12

6. CONVERSION RELATIONSHIPS FOR TAILINGS PARAMETERS 13

7. CONVERSION RELATIONSHIPS FOR MASSES AND VOLUMES 17

8. SAMPLE CALCULATIONS FOR A LABORATORY SAMPLE OF OIL SANDS TAILINGS 19

1. INTRODUCTION

This survey is written to document a standard protocol for identifying mass-volume relationships for tailings materials. The need for establishing a standard protocol has arisen in the oil sands mining industry as different engineering disciplines; geotechnical, mining, chemical and mechanical, and the geologic discipline work in this industry. Various definitions and means of describing the same phenomena have developed which have caused confusion and difficulties in communication.

In the mining industry at large, tailings materials are usually described following geotechnical engineering practice. However, different definitions also exist here so this survey is generic for all tailings materials.

As geotechnical engineering definitions and parameters dominate in practice, these have been identified as such and all other definitions have been labeled as "mining" definitions. The mass-volume phase diagram is always used in geotechnical engineering to aid in defining parameters and symbols and to keep track of mass-volume calculations to ensure a material balance. This practice is followed here. Section 2 is a phase diagram where symbols and subscripts are defined.

The definitions, symbols and equations were mainly initially derived in a series of technical papers published in the early 1980's by myself and co-workers Dr. M.B. Dusseault and Dr. W.D. Carrier III. Mr. Dennis J. Boratynec, M.Sc. graduate in geoenvironmental engineering, has rederived the conversion equations to ensure that they are current and has formatted much of this work.

What makes oil sands tailings unique is the occurrence of bitumen. This organic material has a significant influence on the behaviour of the clays in the tailings and its presence complicates the definition of terms for oil sand tailings materials. As the bitumen mainly remains with the fines in the tailings and its high viscosity (10^6 times more viscous than water at 20°C) prevents it from readily flowing, the geotechnical practice is to assume it is a solid and part of the fines. As it is necessary sometimes for material balance calculations to consider the bitumen separately, both methods are given. The definition of one parameter, geotechnical bitumen content, has been changed from previous work to now include the mass of bitumen in the mass of solids. This change was done to agree with other definitions and as it simplified many of the conversion equations. Previous calculated geotechnical bitumen contents will now be in error but the differences are small and usually not significant.

Sections 3 and 4 provide an index and definitions respectively of tailings parameters used in the various engineering disciplines and equations used to calculate the parameters once the tailings phase diagram is calculated. Section 5 shows a number of phase diagrams comparing geotechnical and mining (read all other) definitions. The main difference in practice is that mining engineering uses total mass in the denominator while geotechnical engineering uses dry mass. The usual geotechnical practice is to not

include the same mass or volume in both the numerator and denominator of an equation. This practice has been found to give better and simpler correlations between parameters. Void ratio instead of porosity and water content on a dry basis instead of on a wet basis are the prime examples. For those not familiar with geotechnical practice, some surprise exists when these parameters are calculated to be greater than 1.00 or 100%.

Section 6 is of most value, containing conversion equations between parameters. Great pains have been taken to include the specific gravity or density of water in the equations to ensure that they are mathematically and dimensionally correct. Many definitions in the literature leave these out as they are usually, in engineering practice, not significantly different from unity. The degree of saturation has been included in all equations so the equations are fully generic. It should be noted that calculations involving the specific gravity of fines-bitumen material must ensure that the correct specific gravity or density of this material is used as the low density of bitumen can reduce the combined specific gravity considerably below that for fines alone.

Nearly all of the conversion equations in Section 6 require that the specific gravity of the pore water, mineral or solids and the degree of saturation of the mass be known. These conversion relationships have been derived without any assumptions regarding the density or specific gravity of the water, the density of standard water or the degree of saturation. As a result, the appearance of the equations may appear complicated. However, since the density or specific gravity of the pore water and the density of standard water can usually be taken as unity and for most tailings the degree of saturation is 100%, simplifications to the equations can readily be made. In the Table, columns one and two in each row list the mass-volume symbol and name, respectively, while the remaining columns contain conversion equations, which can be used to calculate the mass-volume relationship of interest.

All parameters are written as decimals, not percentages, to avoid confusion in the calculations. As many parameters are discussed as percentages, care must be taken to keep this in mind.

The definition of what constitutes "fines" has not been included. Generally, in geotechnical engineering, fines are the silt size and clay size material. Various definitions exist in civil engineering for the maximum size of silts, such as 60 μm or 75 μm (U.S. No. 200 sieve) or 80 μm (Metric No. 80 sieve). In the oil sand industry, fines have been defined as less than 45 μm (U.S. No. 325 sieve) or less than 50 μm (Metric No. 50 sieve). For special purposes in oil sand tailings the definition has been less than 44 μm or 22μm for mature fine tails.

The definition of clay size is also contentious with the maximum size taken to be 2 μm or based on the Atterberg Limits (typical geotechnical definitions) or 5 μm or more recently 5.5 μm. It should be noted that these definitions are for the amount of clay size material present not the amount of clay. This is especially true for the oil sand fines where large booklets of kaolinite clay occur up to 20 μm or larger in diameter.

The amount of active clay material in oil sands tailings is often measured by the methylene blue adsorption test. This test measures the exposed surface area of the clays and using an empirical relationship developed for typical oil sand tailings, the quantity of exposed clay mineral in a sample is calculated. It should be noted that the empirical relationship was developed for typical oil sands clay mineralogy. For tailings with different clay mineralogy, the relationship is not valid.

It is apparent that a definition for the maximum size of fines (and therefore the minimum size of sand) and for clay size must always be given when discussing tailings or oil sands to avoid misinterpretation. The laboratory procedures for determination of these sizes also vary greatly and should also be noted.

Section 7 gives conversion relationships for masses and volumes to aid in the completion of the phase diagram and in calculations.

Section 8 is a sample calculation which shows the calculations for all the parameters of a sample of mature fine tails.

I would like to thank Dr. Rick J. Chalaturnyk a colleague in oil sand tailings research, for reviewing this document. The entire document is available electronically.

Users of this document are requested to report all errors and questionable symbols and parameters. Methods to simplify this work and to make it more comprehensive will be appreciated.

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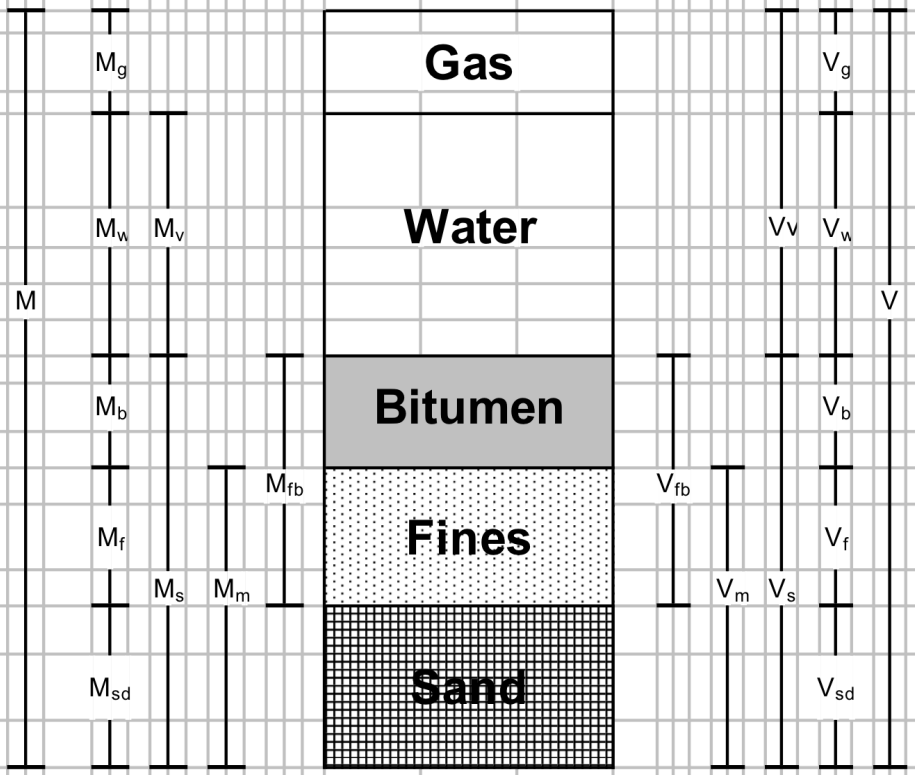
2. Tailings Phase Diagram



Index of subscripts	
b =	bitumen
f =	fines
fb =	fines and bitumen**
g =	gas
m =	mineral
sd =	sand
s =	solids
sw =	standard water
v =	voids
w =	water

Where:

M	= total mass of tailings	V	= total volume of tailings
M_g	= mass of gas*	V_g	= volume of gas
M_w	= mass of water	V_w	= volume of water
M_b	= mass of bitumen	V_b	= volume of bitumen
M_f	= mass of fines	V_f	= volume of fines
M_{sd}	= mass of sand	V_{sd}	= volume of sand
M_v	= mass of voids*	V_v	= volume of voids
M_s	= mass of solids [bitumen (M_b), fines (M_f) and sand (M_{sd})]	V_s	= volume of solids [bitumen (V_b), fines (V_f) and sand (V_{sd})]
M_m	= mass of minerals (fines and sand)	V_m	= volume of minerals (fines and sand)
M_{fb}	= mass of fines and bitumen**	V_{fb}	= volume of fines and bitumen**

Notes:

* Mass of gas is taken as zero. Therefore, by definition the mass of voids and mass of water are equivalent

** Prior to 1991 fines + bitumen were designated as sludge or sludge solids. Since 1991 the term fine tails or mature fine tails has been used.

3. Index of Tailings Parameters

		Definition	Conversion Equation
Bitumen Content	*- Geotechnical (b)*	*8*	*13*
	- Mining (b_m)	*8*	*13*
Degree of Saturation (S_r)		*8*	*13*
Density			
	- bulk(ρ)	*8*	*13*
	- bitumen (ρ_b)	*8*	
	- dry (ρ_d)	*8*	*13*
	- fines(ρ_f)	*8*	
	- fines + bitumen (ρ_{fb})	*8*	
	- mineral (ρ_m)	*8*	
	- sand (ρ_{sd})	*8*	
	- solids(ρ_s)	*8*	
	- water(ρ_w)	*8*	
	- standard water(ρ_{sw})	*8*	
Fines Content	*- Geotechnical I (f)*	*9*	*13*
	- Geotechnical II (fb)	*9*	*13*
	- Mining (f_m)	*9*	*14*
Fines-Water Ratio (FWR)		*9*	*14*
Gas Content (A)		*9*	*14*
Porosity (n)		*9*	*14*
Sand Content	*- Geotechnical (sd)*	*9*	*14*
	- Mining (sd_m)	*9*	*14*
Sand-Fines Ratio (SFR)		*9*	*14*
Solids Concentration (η)		*9*	*14*
Solids Content	*- Geotechnical (s)*	*10*	*14*
	- Mining (s_m)	*10*	*15*
Specific Gravity			
	- bitumen (G_b)	*10*	
	- fines (G_f)	*10*	
	- fines + bitumen (G_{fb})	*10*	*15*
	- mineral (G_m)	*10*	
	- sand (G_{sd})	*10*	
	- solids (G_s)	*10*	*15*
	- water (G_w)	*10*	

Unit Weight

- bulk (γ) ..... 10..... 15
- bitumen (γ_b) 10
- dry (γ_d) ..... 10..... 15
- fines (γ_f) 10
- fines + bitumen(γ_{fb}) 10
- mineral (γ_m) 11
- sand (γ_{sd}) 11
- solids (γ_s) 11
- water (γ_w) 11

Void Ratio

- Geotechnical Void Ratio (e) ..... 11..... 15
- Fines Void Ratio (e_f)..... 11..... 16
- Fines-Bitumen Void Ratio (e_{fb}) ..... 11..... 16
- Sand Void Ratio (e_{sd}) ..... 11..... 16

Volume Concentration (ϕ_c) ..... 11..... 16

Water Content - Geotechnical (w)..... 11..... 16
- Mining (w_m)..... 11..... 16

4. Definition of Tailings Parameters

Bitumen Content - Geotechnical (b)

$$b = \frac{\text{mass of bitumen}}{\text{mass of bitumen, fines and sand}} = \frac{\text{mass of bitumen}}{\text{mass of solids}} = \frac{M_b}{M_s}$$

Bitumen Content - Mining (b_m)

$$b_m = \frac{\text{mass of bitumen}}{\text{total mass of tailings}} = \frac{M_b}{M}$$

Degree of Saturation (S_r)

$$S_r = \frac{\text{volume of water}}{\text{volume of water + volume of gas}} = \frac{\text{volume of water}}{\text{volume of voids}} = \frac{V_w}{V_v}$$

Density (ρ)

$$\rho_{bulk} = \rho = \frac{\text{total mass of tailings}}{\text{total volume of tailings}} = \frac{M}{V}$$

$$\rho_{bitumen} = \rho_b = \frac{\text{mass of bitumen}}{\text{volume of bitumen}} = \frac{M_b}{V_b}$$

$$\rho_{dry} = \rho_d = \frac{\text{mass of solids}}{\text{total volume of tailings}} = \frac{M_s}{V}$$

$$\rho_{fines} = \rho_f = \frac{\text{mass of fines}}{\text{volume of fines}} = \frac{M_f}{V_f}$$

$$\rho_{fines+bitumen} = \rho_{fb} = \frac{\text{mass of fines + mass of bitumen}}{\text{volume of fines + volume of bitumen}} = \frac{M_f + M_b}{V_f + V_b} = \frac{M_{fb}}{V_{fb}}$$

$$\rho_{mineral} = \rho_m = \frac{\text{mass of fines} + \text{mass of sand}}{\text{volume of fines} + \text{volume of sand}} = \frac{M_f + M_{sd}}{V_f + V_{sd}} = \frac{M_m}{V_m}$$

$$\rho_{sand} = \rho_{sd} = \frac{\text{mass of sand}}{\text{volume of sand}} = \frac{M_{sd}}{V_{sd}}$$

$$\rho_{solids} = \rho_s = \frac{\text{mass of bitumen, fines and sand}}{\text{volume of bitumen, fines and sand}} = \frac{\text{mass of solids}}{\text{volume of solids}} = \frac{M_s}{V_s}$$

$$\rho_{water} = \rho_w = \frac{\text{mass of water}}{\text{volume of water}} = \frac{M_w}{V_w}$$

Notes: 1. Units of density are typically kg/m^3 or g/cm^3.

2. If $\rho_{sd} = \rho_f$ then $\rho_m = \rho_{sd} = \rho_f$ as there is no bitumen component to ρ_m.

3. $\rho_{\text{standard water}} = \rho_{sw} = 1000$ kg/m^3 at 1 atmosphere and 4^0C.

Fines Content – Geotechnical I (f)

$$f = \frac{\text{mass of fines}}{\text{mass of bitumen, fines and sand}} = \frac{\text{mass of fines}}{\text{mass of solids}} = \frac{M_f}{M_s}$$

Fines Content – Geotechnical II (fb)

$$fb = \frac{\text{mass of fines} + \text{mass of bitumen}}{\text{mass of bitumen, fines and sand}} = \frac{\text{mass of fines + mass of bitumen}}{\text{mass of solids}} = \frac{M_{fb}}{M_s}$$

Fines Content - Mining (f_m)

$$f_m = \frac{\text{mass of fines}}{\text{total mass of tailings}} = \frac{M_f}{M}$$

Fines-Water Ratio (FWR)

$$FWR = \frac{\text{mass of fines + mass of bitumen}}{\text{mass of fines + mass of bitumen + mass of water}} = \frac{M_f + M_b}{M_f + M_b + M_w} = \frac{M_{fb}}{M_{fb} + M_w}$$

Gas Content (A)

$$A = \frac{\text{volume of gas}}{\text{total volume of tailings}} = \frac{V_g}{V}$$

Porosity (n)

$$n = \frac{\text{volume of water + volume of gas}}{\text{total volume of tailings}} = \frac{\text{volume of voids}}{\text{total volume of tailings}} = \frac{V_v}{V}$$

Sand Content (sd) – Geotechnical

$$sd = \frac{\text{mass of sand}}{\text{mass of bitumen, fines and sand}} = \frac{M_{sd}}{M_s}$$

Sand Content (sd_m) – Mining

$$sd_m = \frac{\text{mass of sand}}{\text{total mass of tailings}} = \frac{M_{sd}}{M}$$

Sand Fines Ratio (SFR)

$$SFR = \frac{\text{mass of sand}}{\text{mass of fines + mass of bitumen}} = \frac{M_{sd}}{M_f + M_b} = \frac{M_{sd}}{M_{fb}}$$

Solids Concentration (η)

$$\eta = \frac{\text{mass of bitumen, fines and sand}}{\text{total volume of tailings}} = \frac{\text{mass of solids}}{\text{total volume of tailings}} = \frac{M_s}{V}$$

Note: Units of solids concentration are typically kg/m^3 or g/cm^3.

Solids Content - Geotechnical (s)

$$s = \frac{\text{mass of bitumen, fines and sand}}{\text{total mass of tailings}} = \frac{\text{mass of solids}}{\text{total mass of tailings}} = \frac{M_s}{M}$$

Note: Due to the viscosity of the bitumen (10^6 times that of water) it is not mobile with respect to the water phase and therefore is treated as a solid.

Solids Content – Mining (s_m)

$$s_m = \frac{\text{mass of fines + mass of sand}}{\text{total mass of tailings}} = \frac{\text{mass of minerals}}{\text{total mass of tailings}} = \frac{M_m}{M}$$

Note: This definition of solids content is often used in material balance calculations and it should always be noted which definition is being used.

Specific Gravity (G)

$$G_{bitumen} = G_b = \frac{\text{density of bitumen}}{\text{density of standard water}} = \frac{\rho_b}{\rho_{sw}}$$

$$G_{fines} = G_f = \frac{\text{density of fines}}{\text{density of standard water}} = \frac{\rho_f}{\rho_{sw}}$$

$$G_{fines+bitumen} = G_{fb} = \frac{\text{density of (fines + bitumen)}}{\text{density of standard water}} = \frac{\rho_{fb}}{\rho_{sw}}$$

$$G_{mineral} = G_m = \frac{\text{density of mineral}}{\text{density of standard water}} = \frac{\rho_m}{\rho_{sw}}$$

$$G_{sand} = G_{sd} = \frac{\text{density of sand}}{\text{density of standard water}} = \frac{\rho_{sd}}{\rho_{sw}}$$

$$G_{solids} = G_s = \frac{\text{density of solids}}{\text{density of standard water}} = \frac{\rho_s}{\rho_{sw}}$$

$$G_{water} = G_w = \frac{\text{density of water}}{\text{density of standard water}} = \frac{\rho_w}{\rho_{sw}}$$

Unit Weight (γ)

$$\gamma_{bulk} = \gamma = \rho \times \mathrm{g}$$

$$\gamma_{bitumen} = \gamma_{\mathrm{b}} = \rho_b \times \mathrm{g}$$

$$\gamma_{dry} = \gamma_{\mathrm{d}} = \rho_d \times \mathrm{g}$$

$$\gamma_{fines} = \gamma_f = \rho_f \times \mathrm{g}$$

$$\gamma_{fines+bitumen} = \gamma_{\mathrm{fb}} = \rho_{fb} \times \mathrm{g}$$

$$\gamma_{mineral} = \gamma_{\mathrm{m}} = \rho_m \times \mathrm{g}$$

$$\gamma_{sand} = \gamma_{sd} = \rho_{sd} \times g$$

$$\gamma_{solids} = \gamma_{s} = \rho_{s} \times g$$

$$\gamma_{water} = \gamma_{w} = \rho_{w} \times g$$

Note: g is the acceleration due to gravity and is taken as 9.81 m/s^2.

Geotechnical Void Ratio (e)

$$e = \frac{\text{volume of water + volume of gas}}{\text{volume of bitumen, fines and sand}} = \frac{\text{volume of voids}}{\text{volume of solids}} = \frac{V_v}{V_s}$$

Fines Void Ratio (e_f)

$$e_f = \frac{\text{volume of water + volume of gas}}{\text{volume of fines}} = \frac{\text{volume of voids}}{\text{volume of fines}} = \frac{V_v}{V_f}$$

Fines-Bitumen Void Ratio (e_{fb})

$$e_{fb} = \frac{\text{volume of water + volume of gas}}{\text{volume of fines + volume of bitumen}} = \frac{\text{volume of voids}}{\text{volume of fines + volume of bitumen}} = \frac{V_v}{V_{fb}}$$

Note: In e_f and e_{fb} the presence of sand is not considered.

Sand Void Ratio (e_{sd})

$$e_{sd} = \frac{\text{volume of voids}}{\text{volume of sand}} = \frac{V_v}{V_{sd}}$$

Note: In e_{sd} the presence of fines and bitumen is not considered.

Volume Concentration (ϕ_c)

$$\phi_c = \frac{\text{volume of bitumen, fines and sand}}{\text{total volume of tailings}} = \frac{\text{volume of solids}}{\text{total volume of tailings}} = \frac{V_s}{V}$$

Water Content (w) - Geotechnical

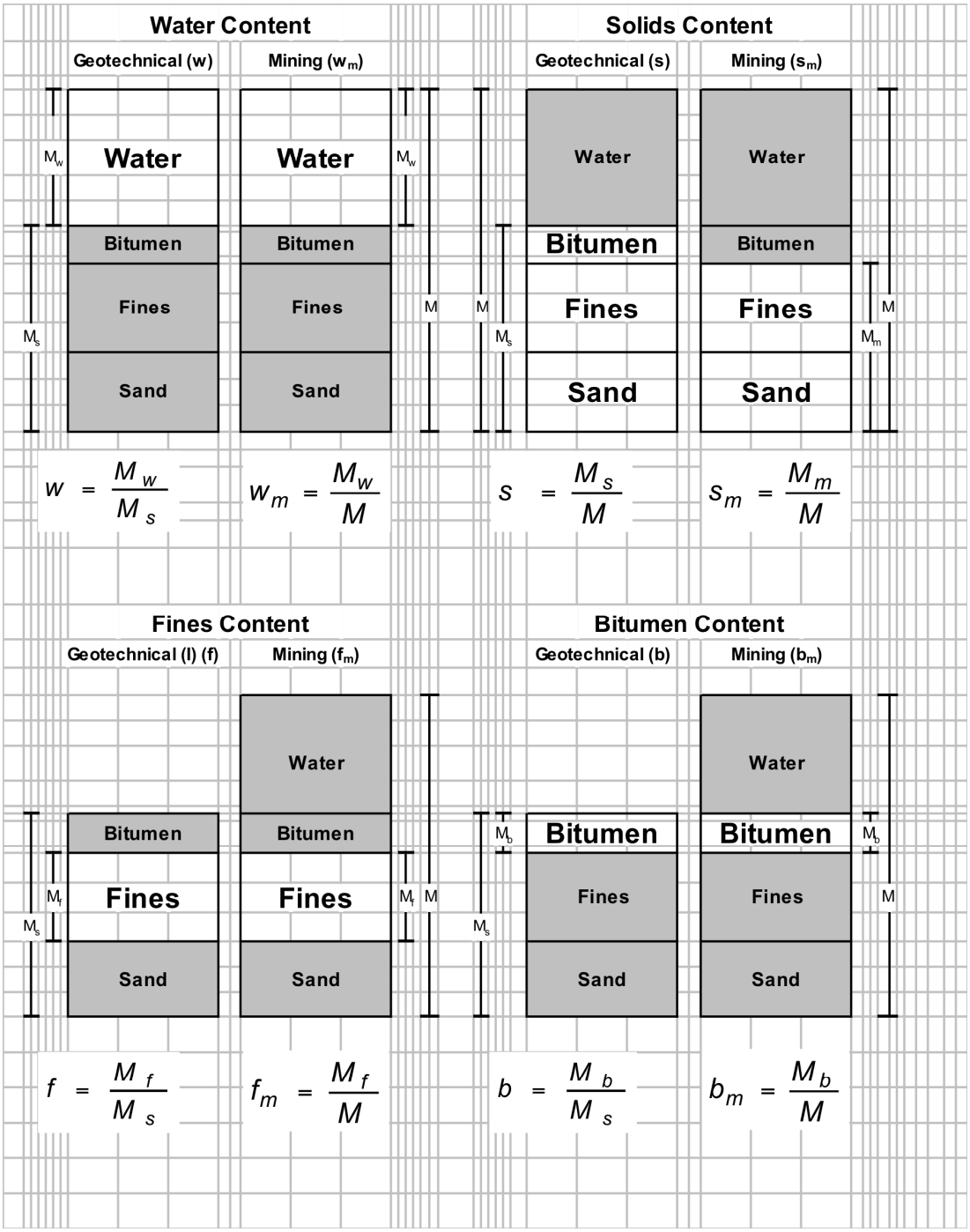
$$w = \frac{\text{mass of water}}{\text{mass of bitumen, fines and sand}} = \frac{\text{mass of water}}{\text{mass of solids}} = \frac{M_w}{M_s}$$

Water Content (w_m) - Mining

$$w_m = \frac{\text{mass of water}}{\text{total mass of tailings}} = \frac{M_w}{M}$$

Note: By definition summation of the Mining solids content, water content and bitumen content equals 100 % while the summation of the Geotechnical solids content, water content and bitumen content does not equal 100 %. See section 5 for visual comparison between Mining and Geotechnical definitions and section 6 for conversion relationships.

5. Visual Comparison Between Geotechnical and Mining Defintions



6. Conversion Relationships for Tailings Parameters

Parameter		Conversion Equations			
b	**Bitumen Content - Geotechnical**	$b = 1 - \frac{s_m}{s}$	$b = \frac{b_m}{s}$	$b = b_m(1+w)$	
b_m	**Bitumen Content - Mining**	$b_m = 1 - w_m - s_m$	$b_m = \frac{b}{1+w}$	$b_m = b \times s$	
S_r	**Degree of Saturation**	$S_r = \frac{w \times G_s}{e \times G_w}$	$S_r = \frac{G_s \times \left(\frac{1}{s} - 1\right)}{e \times G_w}$	$S_r = 1 - \left(\frac{A}{n}\right)$	
ρ	**Bulk Density**	$\rho = \frac{(1+w) \times \rho_w}{\left(\frac{1}{G_s} + \frac{w}{S_r \times G_w}\right)}$	$\rho = \frac{G_s + (G_w \times S_r \times e)}{1+e}\rho_w$	$\rho = \frac{\rho_w}{\left(\frac{s}{G_s} + \frac{1-s}{S_r \times G_w}\right)}$	
ρ_d	**Dry Density**	$\rho_d = \frac{\rho_w}{\left(\frac{1}{G_s} + \frac{w}{G_w \times S_r}\right)}$	$\rho_d = \frac{\rho_s}{1+e}$	$\rho_d = \frac{\rho_w}{\left(\frac{1}{G_s} + \frac{\left(\frac{1}{s} - 1\right)}{G_w \times S_r}\right)}$	
f	**Fines Content - Geotechnical (I)**	$f = \frac{w \times G_f}{e_f \times G_w \times S_r}$	$f = \frac{\left(\frac{1}{s} - 1\right) \times G_f}{e_f \times G_w \times S_r}$	$f = \frac{e \times G_f}{e_f \times G_s}$	$f = \frac{w}{\frac{1}{FWR} - 1} - b$
fb	**Fines Content- Geotechnical (II)**	$fb = \frac{e \times G_{fb}}{e_{fb} \times G_s}$	$fb = \frac{1}{SFR + 1}$	$fb = \frac{FWR \times w}{1 - FWR}$	$fb = \frac{f_m + b_m}{1 - w_m}$
f_m	**Fines Content Mining**	$f_m = fb \times \left(1 - \frac{w}{1+w}\right) - b_m$	$f_m = fb \times (1 - w_m) - b_m$		

FWR	Fines-Water Ratio	$FWR = \frac{fb}{fb + w}$	$FWR = \frac{fb}{fb + \frac{S_r \times e \times G_w}{G_s}}$	$FWR = \frac{fb}{fb + \left(\frac{1}{s} - 1\right)}$	$FWR = \frac{1}{1 + e_f \times \left(\frac{G_w}{G_f}\right)}$
A	Gas Content	$A = \frac{G_s \times w}{(S_r \times G_w) + (G_s \times w)} \times (1 - S_r)$	$A = \frac{e}{1+e} \times (1 - S_r)$	$A = \frac{G_s \times \left(\frac{1}{s} - 1\right)}{(S_r \times G_w) + G_s \times \left(\frac{1}{s} - 1\right)} \times (1 - S_r)$	$A = n \times (1 - S_r)$
n	Porosity	$n = \frac{1}{\left(\frac{S_r \times G_w}{w \times G_s} + 1\right)}$	$n = \frac{e}{1+e}$	$n = \frac{1}{\left(\frac{S_r \times G_w}{\left(\frac{1}{s} - 1\right) \times G_s} + 1\right)}$	
sd	Sand Content - Geotechnical	$sd = 1 - fb$	$sd = 1 - f - b$		
sd_m	Sand Content-Mining	$sd_m = 1 - w_m - f_m - b_m$			
SFR	Sand-Fines Ratio	$SFR = \frac{1}{fb} - 1$			
η	Solids Concentration	$\eta = \frac{\rho}{1 + w}$	$\eta = \left(1 - \frac{e}{e+1}\right)\rho_s$	$\eta = \rho \times s$	$\eta = (1 - n)\rho_s$
s	Solids Content - Geotechnical	$s = \frac{1}{1 + w}$	$s = \frac{G_s}{G_s + (e \times S_r \times G_w)}$	$s = \frac{1}{\left(\frac{\rho_{sw}}{\rho_{dry}} - \frac{1}{G_s}\right) \times G_w \times S_r + 1}$	$s = \frac{1}{1 + \left(\frac{1 - \frac{\rho}{\rho_{sw} \times G_s}}{\frac{\rho}{G_w \times S_r \times \rho_{sw}} - 1}\right)}$
s_m	Solids Content - Mining	$s_m = \frac{(1 - b)}{(1 + w)}$	$s_m = 1 - b_m - w_m$		
Note: All mass-volume ratios are expressed as decimals, not percentages. See Section 4 for definition of parameters.					

G_{fb}	**Specific Gravity of Fines + Bitumen**	$G_{fb} = \frac{fb}{\left(\frac{1}{G_s} - \frac{(1-fb)}{G_{sd}}\right)}$	$G_{fb} = \frac{e_{fb} \times G_s \times fb}{e}$		
G_s	**Specific Gravity of Solids**	$G_s = \frac{S_r \times e \times G_w}{w}$	$G_s = \frac{S_r \times e \times G_w}{\left(\frac{1}{s} - 1\right)}$	$G_s = \frac{e \times G_{fb}}{fb \times e_{fb}}$	$G_s = \frac{\frac{b}{1-b} + 1}{\left(\frac{b}{G_b(1-b)} + \frac{1}{G_m}\right)}$
γ	**Bulk Unit Weight**	$\gamma = \frac{1+w}{\left(\frac{G_w}{G_s} + \frac{w}{S_r}\right)}\gamma_w$	$\gamma = \frac{\frac{G_s}{G_w} + (S_r \times e)}{1+e}\gamma_w$	$\gamma = \frac{\gamma_w}{\left(\frac{G_w \; x \; s}{G_s} + \frac{1-s}{S_r}\right)}$	
γ_d	**Dry Unit Weight**	$\gamma_d = \frac{\gamma_w}{\left(\frac{G_w}{G_s} + \frac{w}{S_r}\right)}$	$\gamma_d = \frac{G_s \; x \; \gamma_w}{(1+e)G_w}$	$\gamma_d = \frac{\gamma_w}{\left(\frac{G_w}{G_s} + \frac{\left(\frac{1}{s} - 1\right)}{S_r}\right)}$	
e	**Void Ratio (Geotechnical)**	$e = \frac{n}{1-n}$	$e = \frac{w \times G_s}{S_r \times G_w}$	$e = \frac{\left(\frac{1}{s} - 1\right) \times G_s}{S_r \times G_w}$	$e = \frac{e_{fb} \times G_s \times fb}{G_{fb}}$
		$e = \frac{\rho_w \times G_s}{\rho_d} - 1$			
e_{fb}	**Fines-Bitumen Void Ratio**	$e_{fb} = \frac{w \times G_{fb}}{S_r \times G_w \times fb}$	$e_{fb} = \frac{e \times G_{fb}}{G_s \times fb}$	$e_{fb} = \frac{\left(\frac{1}{s} - 1\right) \times G_{fb}}{S_r \times G_w \times fb}$	
e_f	**Fines Void Ratio**	$e_f = \frac{w \times G_f}{S_r \times G_w \times f}$	$e_f = \frac{e \times G_f}{G_s \times f}$	$e_f = \frac{\left(\frac{1}{s} - 1\right) \times G_f}{S_r \times G_w \times f}$	
Note: All mass-volume ratios are expressed as decimals, not percentages. See Section 4 for definition of parameters.					

e_{sd}	**Sand Void Ratio**	$e_{sd} = \dfrac{e + \dfrac{G_s}{G_{fb}} \times fb}{\dfrac{G_s}{G_{sd}} \times (1 - fb)}$	$e_{sd} = \dfrac{1 + \dfrac{1}{e_{fb}}}{\dfrac{G_{fb}(1 - fb)}{G_{sd} \times e_{fb} \times fb}}$	$e_{sd} = \dfrac{\dfrac{G_{sd}}{S_r \times G_w} \times \left(\dfrac{1}{s} - 1\right) + \dfrac{G_{sd}}{G_{fb}} \times fb}{1 - fb}$	
ϕ_c	**Volume Concentration**	$\phi_c = \dfrac{\rho}{\rho_s (1 + w)}$	$\phi_c = \dfrac{\rho}{\rho_s \left(1 + \dfrac{S_r \times e \times G_w}{G_s}\right)}$	$\phi_c = \dfrac{s \times \rho}{\rho_s}$	$\phi_c = \dfrac{\eta}{\rho_s}$
w	**Water Content - Geotechnical**	$w = \dfrac{1}{s} - 1$	$w = \dfrac{S_r \times e \times G_w}{G_s}$	$w = \left(\dfrac{\rho_w}{\rho_d} - \dfrac{1}{G_s}\right) \times G_w \times S_r$	$w = \dfrac{\left(1 - \dfrac{\rho}{\rho_w \times G_s}\right)}{\left(\dfrac{\rho}{G_w \times S_r \times \rho_w} - 1\right)}$
w_m	**Water Content - Mining**	$w_m = \dfrac{w}{1 + w}$	$w_m = \dfrac{S_r \times e \times G_w}{G_s + (S_r \times e \times G_w)}$	$w_m = \dfrac{\left(\dfrac{1}{s} - 1\right)}{1 + \left(\dfrac{1}{s} - 1\right)}$	$w_m = 1 - b_m - s_m$
Note: All mass-volume ratios are expressed as decimals, not percentages. See Section 4 for definition of parameters.					

7. Conversion Relationships for Masses and Volumes

Component		Conversion Equations			
M	**Total Mass of tailings**	$M = M_s \times (1 + w)$	$M = \frac{M_s}{s}$		
M_w	**Mass of Water**	$M_w = w \times M_s$	$M_w = w_m \times M$		
M_b	**Mass of Bitumen**	$M_b = b \times M_s$	$M_b = M_m \times \left(\frac{b}{1-b}\right)$	$M_b = b_m \times M$	
M_f	**Mass of Fines**	$M_f = (1 - sd - b) \times M_s$	$M_f = (fb - b) \times M_s$	$M_f = f_m \times M$	
M_{fb}	**Mass of Fines + Bitumen**	$M_{fb} = fb \times M_s$	$M_{fb} = (1 - sd) \times M_s$		
M_{sd}	**Mass of Sand**	$M_{sd} = sd \times M_s$	$M_{sd} = (1 - fb) \times M_s$	$M_{sd} = SFR \times M_{fb}$	
M_s	**Mass of Solids**	$M_s = \frac{M}{(1+w)}$	$M_s = V \times \eta$		
M_m	**Mass of Mineral**	$M_m = M_b \times \left(\frac{1-b}{b}\right)$	$M_m = M \times s_m$		
V	**Total Volume of Tailings**	$V = \frac{V_v}{n}$	$V = \frac{V_v}{\left(\frac{e}{1+e}\right)}$	$V = \frac{V_s}{1-n}$	$V = \frac{M_s}{\eta}$
V_g	**Volume of Gas**	$V_g = V_v(1 - S_r)$	$V_g = A \times V$		
V_w	**Volume of Water**	$V_w = S_r \times V_v$	$V_w = \frac{w \times M_s}{\rho_w}$	$V_w = S_r \times n \times V$	
Note: All mass-volume ratios are expressed as decimals, not percentages. See Section 4 for definition of parameters.					

V_b	**Volume of Bitumen**	$V_b = \frac{b \times M_s}{\rho_b}$	$V_b = \frac{M_m}{\rho_b} \times \left(\frac{b}{1-b}\right)$	$V_b = \frac{M \times b_m}{\rho_b}$	
V_f	**Volume of Fines**	$V_f = \frac{f_m \times M}{G_f}$	$V_f = \frac{(1 - sd - b)M_s}{\rho_f}$	$V_f = \frac{(fb - b)M_s}{\rho_f}$	
V_{fb}	**Volume of Fines + Bitumen**	$V_{fb} = \frac{fb \times M_s}{\rho_{fb}}$	$V_{fb} = \frac{(1 - sd) \times M_s}{\rho_{fb}}$	$V_{fb} = \frac{V_v}{e_f}$	
V_{sd}	**Volume of Sand**	$V_{sd} = \frac{sd \times M_s}{\rho_{sd}}$	$V_{sd} = \frac{(1 - fb) \times M_s}{\rho_{sd}}$	$V_{sd} = \frac{SFR \times M_{fb}}{\rho_{sd}}$	
V_v	**Volume of Voids**	$V_v = V \times \left(\frac{e}{1+e}\right)$	$V_v = n \times V$	$V_v = \frac{V_s}{\left(\frac{1}{n} - 1\right)}$	$V_v = \frac{V_w}{S_r}$
V_s	**Volume of Solids**	$V_s = V(1-n)$	$V_s = \frac{V_v}{e}$	$V_s = \frac{V}{1+e}$	$V_s = \frac{M}{\rho_s(1+w)}$
V_m	**Volume of Mineral**	$V_m = \frac{M_b \times \left(\frac{1-b}{b}\right)}{\rho_m}$	$V_m = \frac{M \times s_m}{\rho_m}$		
Note: All mass-volume ratios are expressed as decimals, not percentages. See Section 4 for definition of parameters.					

8. Sample Calculations for a Laboratory Sample of Oil Sands Tailings

Total mass of sample prior to drying (M) **105.00 g**

Total mass of sample after drying (M_s) **33.00 g**

Total volume of sample prior to drying (V) **86.0 cm^3**

Calculations

1. Use the following known values for oil sand tailings.

ρ_{sd} = 2650 kg/m^3 = 2.650 g/cm^3 G_{sd} = 2.65

ρ_f = 2650 kg/m^3 = 2.650 g/cm^3 G_f = 2.65

ρ_b = 1030 kg/m^3 = 1.030 g/cm^3 G_b = 1.03

ρ_w = 1000 kg/m^3 = 1.000 g/cm^3 G_w = 1.00

2. Measure the following using standard laboratory techniques.

M_b = 1.10 g

M_{sd} = 3.90 g

M_f = 28.00 g

3. The following phase components are calculated using the appropriat phase relationships.

$M_w = M - M_s$ = 105.00 g - 33.00 g = 72.00 g

$M_m = M_s - M_b$ = 33.00 g - 1.10 g = 31.90 g

$V_b = M_b / \rho_b$ = 1.10 g / 1.03 g/cm^3 = 1.07 cm^3

$V_{sd} = M_{sd} / \rho_{sd}$ = 3.90 g / 2.65 g/cm^3 = 1.47 cm^3

$V_f = M_f / \rho_f$ = 28.00 g / 2.65 g/cm^3 = 10.6 cm^3

$V_{fb} = V_f + V_b$ = 10.6 cm^3 + 1.07 cm3 = 11.7 cm^3

$V_w = M_w / \rho_w$ = 72.00 g / 1.00 g/cm^3 = 72.0 cm^3

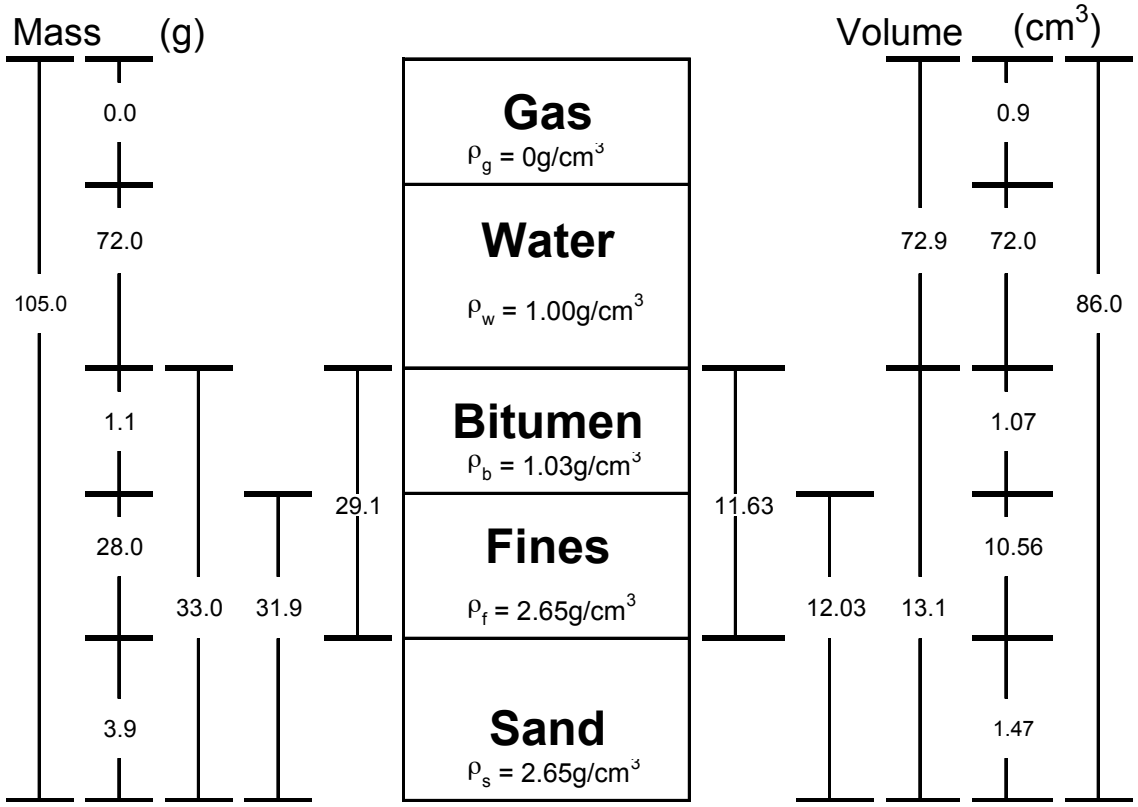
$V_s = V_b + V_{sd} + V_f$ = 1.07 cm^3 + 1.47 + 10.6 = 13.1 cm^3

$V_m = V_{sd} + V_f$ = 1.47 + 10.6 = 12.1 cm^3

$V_v = V - V_s$ = 86.0 cm^3 - 13.1 = 72.9 cm^3

$V_g = V_v - V_w$ = 72.9 cm^3 - 72.00 cm^3 = 0.9 cm^3

Phase Diagram



4. The relevant parameters are calculated using the appropriate definition.

Bitumen Content – Geotechnical (b)

$$b = \frac{\text{mass of bitumen}}{\text{mass of bitumen, fines and sand}} = \frac{\text{mass of bitumen}}{\text{mass of solids}} = \frac{M_b}{M_s} = \frac{1.10\ \text{g}}{33.00\ \text{g}} = 0.0333 \text{ or } 3.33\ \%$$

Degree of Saturation (S_r)

$$S_r = \frac{\text{volume of water}}{\text{volume of water + volume of gas}} = \frac{\text{volume of water}}{\text{volume of voids}} = \frac{V_w}{V_v} = \frac{72.0\ \text{cm}^3}{72.9\ \text{cm}^3} = 0.989 \text{ or } 98.9\ \%$$

Density (ρ)

$$\rho = \frac{\text{total mass of tailings}}{\text{total volume of tailings}} = \frac{M}{V} = \frac{105.00\ \text{g}}{86.0\ \text{cm}^3} = 1.22\ g/cm^3$$

$$\rho_{fines+bitumen} = \rho_{fb} = \frac{\text{mass of fines + mass of bitumen}}{\text{volume of fines + volume of bitumen}} = \frac{M_f + M_b}{V_f + V_b} = \frac{M_{fb}}{V_{fb}} = \frac{29.10\ g}{11.7\ \text{cm}^3} = 2.49\ \text{g/cm}^3$$

$$\rho_{solids} = \rho_s = \frac{\text{mass of bitumen, fines and sand}}{\text{volume of bitumen, fines and sand}} = \frac{\text{mass of solids}}{\text{volume of solids}} = \frac{M_s}{V_s} = \frac{33.00\ \text{g}}{13.1\ \text{cm}^3} = 2.52\ \text{g/cm}^3$$

$$\rho_{dry} = \rho_d = \frac{\text{mass of solids}}{\text{total volume of tailings}} = \frac{M_s}{V} = \frac{33.00\,\text{g}}{86.0\,\text{cm}^3} = 0.384\,\text{g/cm}^3$$

Fines Content – Geotechnical I (f)

$$f = \frac{\text{mass of fines}}{\text{mass of bitumen, fines and sand}} = \frac{\text{mass of fines}}{\text{mass of solids}} = \frac{M_f}{M_s} = \frac{28.00\,\text{g}}{33.00\,\text{g}} = 0.848 \text{ or } 84.8\,\%$$

Fines Content – Geotechnical II (fb)

$$fb = \frac{\text{mass of fines + mass of bitumen}}{\text{mass of solids}} = \frac{M_{fb}}{M_s} = \frac{29.10\text{ g}}{33.00\text{ g}} = 0.882 \text{ or } 88.2\,\%$$

Fines – Water Ratio (FWR)

$$FWR = \frac{\text{mass of fines} + \text{mass of bitumen}}{\text{mass of fines} + \text{mass of bitumen} + \text{mass of water}} = \frac{M_f + M_b}{M_f + M_b + M_w} = \frac{29.1\,\text{g}}{101.1\,\text{g}} = 0.288 \text{ or } 28.8\,\%$$

Gas Content (A)

$$A = \frac{\text{volume of gas}}{\text{total volume of tailings}} = \frac{V_g}{V} = \frac{0.9\text{ cm}^3}{86.0\text{ cm}^3} = 0.01 \text{ or } 1.0\,\%$$

Porosity (n)

$$n = \frac{\text{volume of water + volume of gas}}{\text{total volume of tailings}} = \frac{\text{volume of voids}}{\text{total volume of tailings}} = \frac{V_V}{V} = \frac{72.9\text{ cm}^3}{86.0\text{ cm}^3} = 0.848 \text{ or } 84.8\%$$

Sand Content - Geotechnical (sd)

$$sd = \frac{\text{mass of sand}}{\text{mass of bitumen, fines and sand}} = \frac{M_{sd}}{M_s} = \frac{3.90\text{ g}}{33.00\text{ g}} = 0.118 \text{ or } 11.8\,\%$$

Solids Concentration (η)

$$\eta = \frac{\text{mass of bitumen, fines and sand}}{\text{total volume of tailings}} = \frac{\text{mass of solids}}{\text{total volume of tailings}} = \frac{M_s}{V} = \frac{33.00\text{ g}}{86.0\text{ cm}^3} = 0.384 \text{ or } 38.4\,\%$$

Solids Content – Geotechnical (s)

$$s = \frac{\text{mass of bitumen, fines and sand}}{\text{total mass of tailings}} = \frac{\text{mass of solids}}{\text{total mass of tailings}} = \frac{M_s}{M} = \frac{33.00}{105.00} = 0.314 \text{ or } 31.4\,\%$$

Specific Gravity (G)

$$G_{fines+bitumen} = G_{fb} = \frac{\text{density of (fines + bitumen)}}{\text{density of standard water}} = \frac{\rho_{fb}}{\rho_{sw}} = \frac{2.49\ \text{g/cm}^3}{1.00\ \text{g/cm}^3} = 2.49$$

$$G_{solids} = G_s = \frac{\text{density of solids}}{\text{density of standard water}} = \frac{\rho_s}{\rho_{sw}} = \frac{2.52\ \text{g/cm}^3}{1.00\ \text{g/cm}^3} = 2.52$$

Void Ratio (e) - Geotechnical

$$e = \frac{\text{volume of water + volume of gas}}{\text{volume of bitumen, fines \& sand}} = \frac{\text{volume of voids}}{\text{volume of solids}} = \frac{V_V}{V_S} = \frac{72.9\ \text{cm}^3}{13.1\ \text{cm}^3} = 5.56$$

Fines Void Ratio (e_f)

$$e_f = \frac{\text{volume of water + volume of gas}}{\text{volume of fines}} = \frac{\text{volume of voids}}{\text{volume of fines}} = \frac{V_V}{V_f} = \frac{72.9\ \text{cm}^3}{10.6\ \text{cm}^3} = 6.90$$

Fines-Bitumen Void Ratio (e_{fb})

$$e_{fb} = \frac{\text{volume of voids}}{\text{volume of fines + volume of bitumen}} = \frac{V_v}{V_{fb}} = \frac{72.9\ \text{cm}^3}{11.7\ \text{cm}^3} = 6.23$$

Volume Concentration (ϕ_c)

$$\phi_c = \frac{\text{volume of solids}}{\text{total volume of tailings}} = \frac{V_S}{V} = \frac{13.1\ \text{cm}^3}{86.0\ \text{cm}^3} = 0.152 \text{ or } 15.2\ \%$$

Water Content (w) - Geotechnical

$$w = \frac{\text{mass of water}}{\text{mass of bitumen, fines and sand}} = \frac{\text{mass of water}}{\text{mass of solids}} = \frac{M_w}{M_s} = \frac{72.00\ \text{g}}{33.00\ \text{g}} = 2.182 \text{ or } 218.2\ \%$$