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Stability of Braced Frames

by

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ABSTRACT

This report presents a method of analysis to predict the complete load-displacement response for large mulit-story steel frames, with provision for diagonal bracing members and shear wall elements. The member response is assumed to be elastic - perfectly plastic. The influence of axial load on the stiffness and carry-over factors is considered for the columns, but neglected for the girders. The effects of finite column width and girder hinge reversal on the behavior is also considered. Diagonal bracing members are assumed to be subjected to axial loads only. The frame analysis is second order, that is to say, the story shear equilibrium is formulated on the deformed structure. Axial shortening of the columns is considered. The equilibrium equations are solved by a modified Gauss elimination procedure. A number of comparative studies are described which are used to verify the method of analysis.

An extensive behavioral study is performed on a number of frames subjected to vertical loads alone, and to combined vertical and lateral loads. Comparisons are made between the behavior of unbraced and braced-frames, with particular emphasis on the P-Δ effects. Coupled unbraced-supported and braced-supported frames are also considered. A design procedure, based on the results of the behavioral studies, is recommended for multi-story structures. The method results in a more uniform factor of safety than does present design practice.

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LIST OF SYMBOLS

A	area of column, brace
[A]	coefficient matrix of equilibrium equations
$B_{m,n,i}$	brace m,n,i
{b}	vector of loading terms
C	stability function
C_m	equivalent moment factor
$C_{m,n}$	$column_{m,n}$
$C_{A1}, C_{A2}, C_{A3}, C_{A4}, C_{A5}$ $C_{B1}, C_{B2}, C_{B3}, C_{B4}, C_{B5}$	coefficients of girder slope-deflection equations
$C_{D1}, C_{D2}, C_{D3}, C_{D4}, C_{D5}$	coefficients of bracing axial load-deformation equations
$C_{L1}, C_{L2}, C_{L3}, C_{L4}$ $C_{U1}, C_{U2}, C_{U3}, C_{U4}$	coefficients of column slope-deflection equations
E	modulus of elasticity
F	applied lateral load
F_a	allowable axial stress
F_b	allowable bending stress
F_e'	Euler stress
F_x	horizontal component of bracing force
F_y	vertical component of bracing force
f_a	computer axial stress

f_b	computed bending stress
$G_{m,n}$	$\text{girder}_{m,n}$
h	column height
I	moment of inertia
K	effective length factor; spring stiffness at column base, $\sqrt{P/EI}$
L	length of girder, brace
M	moment
M_{AB}, M_{BA}	moments at left and right ends of the girder
M_{ABP}, M_{BAP}	plastic moment capacity at left and right ends of the girder
M_{FAB}, M_{FBA}	fixed end moments at the left and right ends of the girder
M_{LU}, M_{UL}	moments at the lower and upper ends of the column
M_{MAX}	maximum interior column moment
M_p	plastic moment capacity of girder
M_{PC}	plastic moment capacity of column
$\overline{M_{FAB}}, \overline{M_{FBA}}$	fixed end moments at left and right ends of the girder at the instant of hinge reversal
P	column axial load
P_{CR}	critical axial load of bracing member
P_y	yield load of column
r	radius of gyration of bracing member

S	stability function
V	applied joint load
V_{AB}, V_{BA}	shears at left and right ends of the girder
V_{WA}, V_{WB}	shears at left and right ends of the girders due to applied loads
W_c	width of column
W_{DL}	girder dead load
W_{LL}	girder live load
$\{X\}$	displacement vector
α	percentage of vertical load applied in the horizontal direction; amplification factor
Δ	column sway
Δ_L, Δ_U	sway deflection at lower and upper ends of the column
δ	deflection of column measured from the chord
δ_A, δ_B	deflections of left and right ends of the girder
$\overline{\delta_{AA}}, \overline{\delta_{AB}}$	deflections of left and right ends of the girder at the instant of hinge reversal at A
$\overline{\delta_{BA}}, \overline{\delta_{BB}}$	deflections of left and right ends of the girder at the instant of hinge reversal at B
λ	load factor
θ_A, θ_B	rotations of the joints at the left and right ends of the girder

θ_{AB}, θ_{BA}	rotations of the left and right ends of the girder
θ_L, θ_U	rotations of the joints at the lower and upper ends of the columns
θ_{LU}, θ_{UL}	rotations of the lower and upper ends of the column
$\overline{\theta_{AA}}, \overline{\theta_{AB}}$	rotations of the joints at the left and right ends of the girder at the instant of hinge reversal at A
$\overline{\theta_{BA}}, \overline{\theta_{BB}}$	rotations of the joints at the left and right ends of the girder at the instant of hinge reversal at B
$\overline{\theta_{ABA}}$	rotation of right end of girder at the instant of hinge reversal at A
$\overline{\theta_{BAB}}$	rotation of left end of girder at the instant of hinge reversal at B
$\overline{\theta_{ABP}}$	plastic hinge rotation at the left end of the girder at the instant of hinge reversal at A
$\overline{\theta_{BAP}}$	plastic hinge rotation of the right end of the girder at the instant of hinge reversal at B
ρ	story sway rotation

CHAPTER I

INTRODUCTION

In recent years the number of tall commercial and residential buildings has increased rapidly. As buildings increase in height the need to ensure adequate lateral stiffness and strength becomes more acute. The structure must provide the strength to resist combined lateral and vertical loads, and must provide adequate stiffness to prevent frame buckling under vertical loads alone. In addition the structure must have the stiffness to limit sway deflections (at working load) to a reasonable amount.

The resistance of multi-story framed structures to lateral sway is developed through the flexural resistance of the beams, columns and shear walls in the structure, and the extensional stiffness and strength provided by the bracing members. The architectural requirements of modern buildings often relegate major bracing elements to selected bents and core areas. Thus different member arrangements occur in the frames of a given structure. These frames are normally classified with respect to their contribution to the overall lateral stiffness of the building. An "unbraced" frame, such as that illustrated in FIGURE 1.1a, develops its lateral stiffness solely through the flexural resistance of its columns and girders. The "braced" frame, shown in FIGURE 1.1b, derives its lateral rigidity primarily from an added bracing system consisting of diagonal bracing members, K-bracing members or shear walls. A "supported" bent depends on adjacent braced or unbraced bents for resistance to lateral forces. A supported bent is illustrated in FIGURE 1.1c, coupled with a braced bent.

The columns in a multi-story frame are designed to support the loads from the adjacent girders and the column above, and in many cases to provide lateral stiffness for the frame. Columns may be subjected either to axial forces or to axial forces in combination with bending moments (beam-columns). Beam-columns are commonly designed on the basis of interaction equations, based on the ultimate strength of the member. Moments and axial forces from a first order analysis are used in these empirical equations to guard against local overstressing and overall instability. These equations compute the ratios of actual axial stress to allowable axial stress, and actual bending stress to allowable bending stress, and limit the combined quantities to provide acceptable factors of safety. The local strength equation is independent on the effective length (or buckling load) of the member. The effective length enters into the computation of allowable axial stress, and the amplification and equivalent moment factors. All columns must be checked for both local overstressing and overall stability.

A knowledge of the effective length of an individual member is necessary only because of the way in which the empirical estimate of the ultimate strength is formulated. To determine the effective length factor for a column, the usual procedure is to first classify the column as either "prevented from sway" or "permitted to sway," then to solve the appropriate differential equation, entering with ratios of the column to girder stiffnesses at either end of the column, thus obtaining the critical load, and so select the effective length.

The behavior of columns prevented from sway, as shown in FIGURE 1.2a, and those permitted to sway, shown in FIGURE 1.2b, is significantly different because of the presence of secondary moments induced by the story sway deflections. The moment at a distance x from the upper end of either column is given by:

$$M = M_u - \frac{x}{h}(M_u + M_L) = P\delta \qquad (1.1)$$

where M_u = moment at the upper end of the column

M_L = moment at the lower end of the column

h = column height

P = column axial force

δ = deflection of the column measured from the chord

For the column prevented from sway, the sum of the end moments, $M_u + M_L$, is:

$$M_u + M_L = Vh \qquad (1.2)$$

For the column permitted to sway, the end moments are increased due to the column axial load acting through the relative story sway displacements:

$$M_u + M_L = Vh + P\Delta \qquad (1.3)$$

where Δ = sway deflection of the column.

This additional moment, known as the P-Δ moment, is assumed to be accounted for by using the effective length factor for a column permitted to sway, and is neglected when the effective length factor for a column prevented from sway is used.

Common design practice is to consider the columns in unbraced bents as permitted to sway, and those in braced or supported bents as prevented from sway. This implies that columns in a braced or supported bent are free from significant increases in moment due to sway deflection.

It is the purpose of this dissertation to study the effect of various lateral bracing systems on the behavior of planar steel frames subjected to vertical loads, and to combined vertical and lateral loads. A comparison of the behavior of braced and unbraced frames designed under similar conditions is made. Of particular interest is the additional lateral stiffness required to remove secondary P-Δ moments from frames whose columns are designed assuming sidesway is prevented. The behavior of supported bents is also investigated.

Extensive research has been performed to study the behavior of multi-story planar frames. A brief review of the methods of analysis of multi-story frames, which are able to trace the behavior of tall structures up to their ultimate loads, is presented in CHAPTER 2. The features of particular significance to this dissertation will be discussed in detail. The experimental work, pertinent to the design of multi-story frames will also be reviewed.

The next portion of the dissertation is concerned with the development of a computer program capable of analyzing large planar frames, containing bracing and shear wall elements. The responses of the columns, girders and diagonal bracing members are described in CHAPTER 3. The standard slope-deflection equations, modified for plastic hinging, are used to describe the behavior of columns and girders. The influence of axial load on the stiffness and carry-over factors is considered for the columns, but neglected for the girders. The effects of finite column width and hinge reversal on the behavior of the girders are also considered. A method is presented to describe the behavior of diagonal bracing members subjected to tensile or compressive loads.

The frame analysis program is outlined in CHAPTER 4. Equilibrium equations are formulated for moment and vertical forces at each joint, using the member slope-deflection equations. A story shear equilibrium equation is formulated on the deformed structure at each floor level. The equilibrium equations are solved using a modified Gauss elimination procedure.

Comparative studies of various frames subjected to combined vertical and lateral loads, and to vertical loads only are described in CHAPTER V. These studies are used to verify the present analysis.

The second major part of the dissertation describes behavioral studies of planar frames containing various lateral bracing systems. Comparisons between the behavior of braced and unbraced frames are made in CHAPTER VI. A twenty-four story, plastically designed, three bay frame and a corresponding single story subassemblage frame are considered.

The effect of coupling a supported frame with an unbraced and braced frame in turn is also investigated. The results of the behavioral studies are discussed in CHAPTER VII, and the proposed design method is presented. A summary of the investigation and the conclusions reached are presented in CHAPTER VIII.

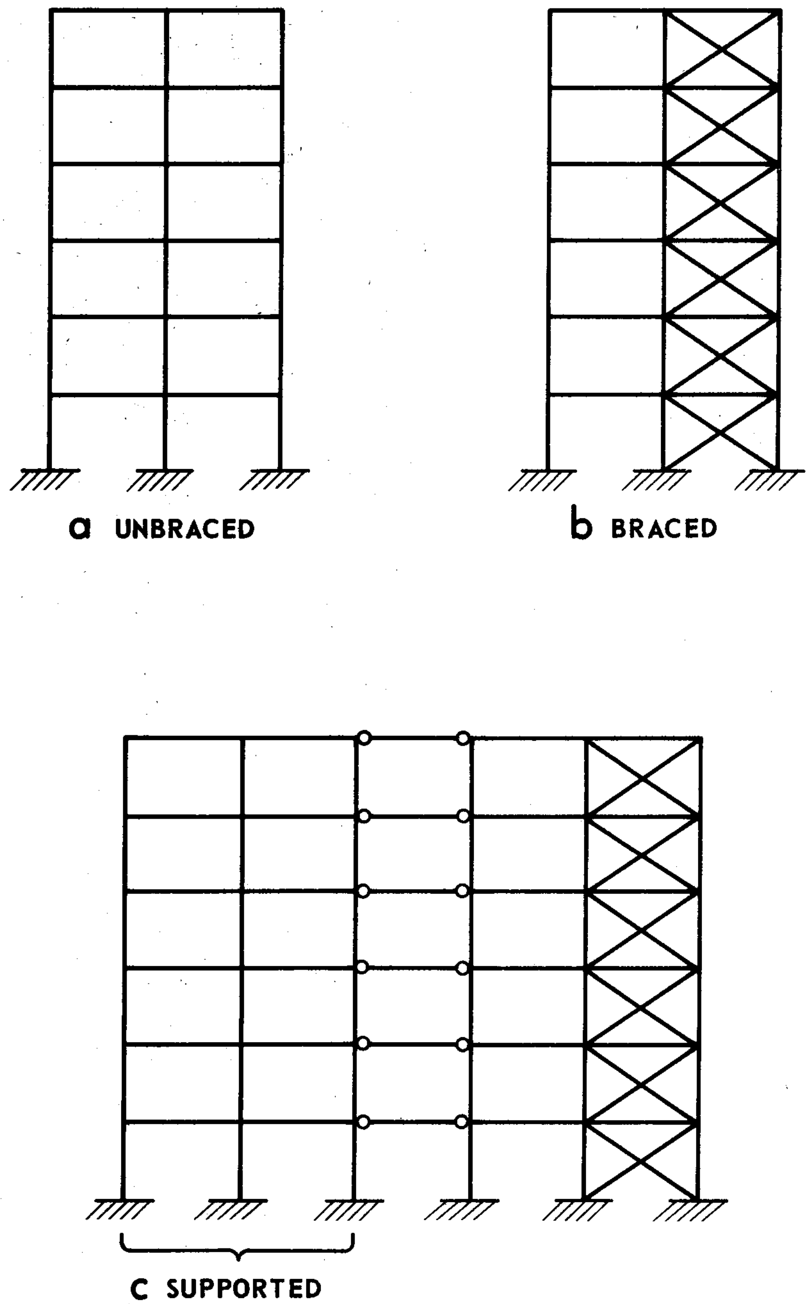


FIGURE 1.1

UNBRACED, BRACED AND SUPPORTED FRAMES

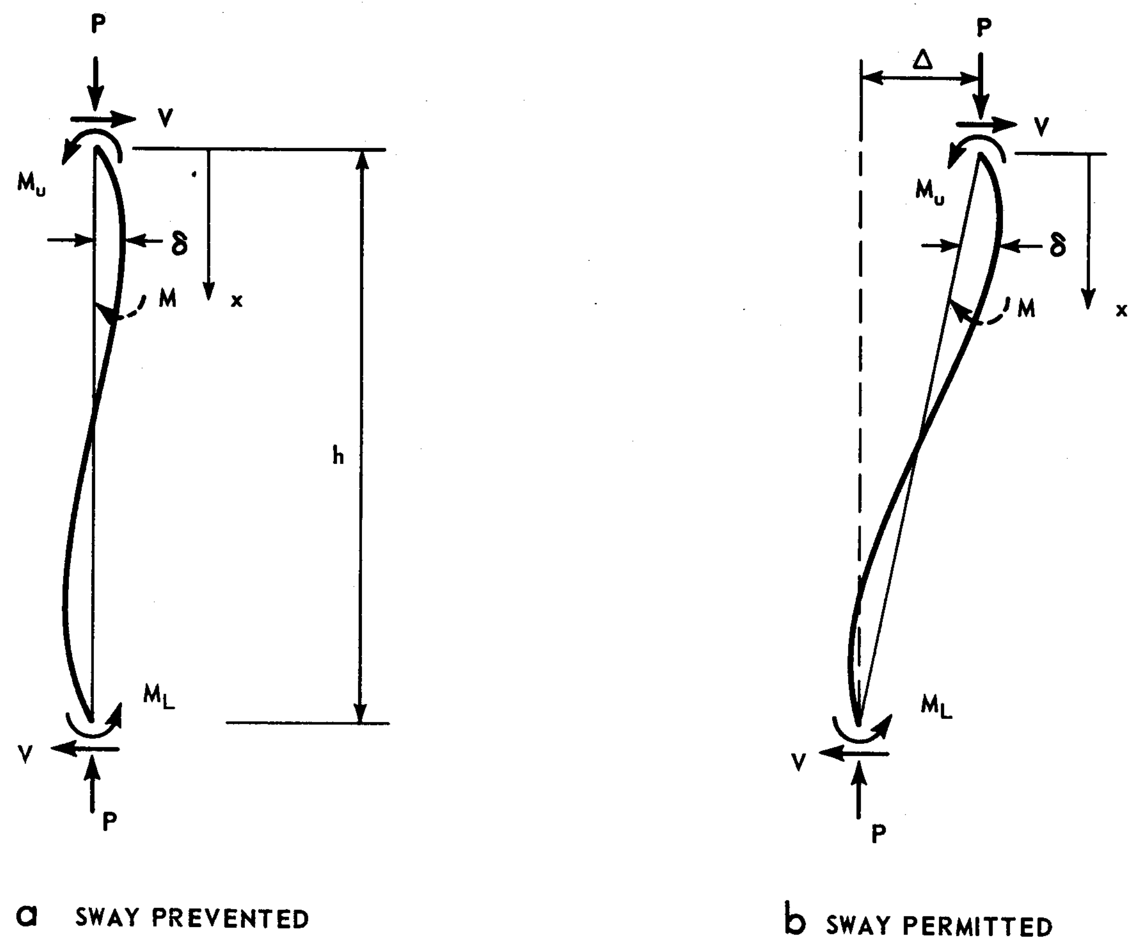


FIGURE 1.2

SWAY PREVENTED AND SWAY PERMITTED COLUMNS

CHAPTER II

REVIEW OF PREVIOUS RESEARCH

2.1 Introduction

The review of previous research, presented in this chapter, is limited to those investigations directly concerned with the development of a procedure for the second order, elastic-plastic analysis of multi-story structures. The work reviewed in the following two sections deals with frames subjected to combined lateral and vertical loading, and frames subjected only to vertical loading, (frame buckling), respectively. Pertinent test results are discussed in each section. For a more complete survey, the reader is referred to References 3, 4, and 5.

2.2 Frames Subjected to Combined Vertical and Lateral Loads

Using large computers the behavior of the multi-story structure can be traced throughout a given loading history. In a second order analysis, equilibrium is formulated on the deformed structure, thus taking into account the secondary (or P-Δ) moments induced by the gravity loads acting on the swayed structure. First order elastic, second order elastic, and second order elastic-plastic analyses of a portal frame subjected to constant vertical load and increasing horizontal load are compared in FIGURE 2.1. The horizontal load, H, is plotted as a function of the sway displacement, Δ. The "true" behavior, (experimental test curve), is also shown. The elastic and inelastic frame buckling loads, and the simple plastic load, are indicated. The difference in slope of the first order elastic and the second order elastic curves is caused by the P-Δ moments. The second order elastic-plastic analysis coincides with the second order elastic analysis until the plastic moment capacity is reached at some location in the frame. Subsequently no further moment increase is permitted at this location and the stiffness of the overall frame decreases. When four such plastic hinges have developed, a mechanism is formed, and the frame can no longer take additional horizontal load. As additional sway occurs the horizontal load must decrease due to the increased P-Δ moments. The second order elastic-plastic analysis closely approximates the true frame behavior. In addition to P-Δ moments, a second order analysis may include the influence of the axial load on the stiffness and carry-over factors of columns; as well as the influence of axial shortening, residual stress, the spread of inelastic zones along the length of a member, the finite column width, and strain reversal.

A number of second order analysis procedures have been developed for multi-story frames subjected to combined vertical and horizontal loading (4,6,7,8,9,10,11 and 12). References 6, 7 and 9 consider the effect of axial load on the stiffness and carry-over factors, (stability functions), of columns and girders, while References 4, 8 and 10 consider stability functions for columns only. The axial forces in the girders of a multi-story frame are usually small and thus the change in girder stiffness due to axial load may be neglected.

References 4, 6, 7, 8, 9 and 10 consider the effect of the axial shortening of the column members on the force distribution and deflections of the frame. In References 4 and 10 it was reported that axial column shortening increased the sway deflections of slender multi-story frames by as much as 30% at working load, however, the ultimate capacity of the frames was little affected by axial shortening. Thus axial shortening is an important consideration in frames designed to limit sways at working load.

The influence of the residual stresses produced by the rolling and cooling process is considered in References 8 and 9. Parikh (8), modified the column moment-curvature relationships to compensate for the decrease in bending stiffness due to the yielded condition of the cross-section, for axial loads greater than 0.7 P_y, where P_y represents the yield load of the column. Burnstiel, (9), formulated the member stiffness matrix so that it accounted for the gradual penetration of yielding, including the presence of residual stresses, the spread of inelastic zones along the member length, and strain reversal in previously yielded fibres. With this rigorous treatment, however, only relatively small structures have been analyzed.

References 10, 11 and 12, consider the effect of finite column width on the lateral stiffness of the structure. Considering the width of vertical members reduces the clear span of the girders, thus increasing the bending stiffness and decreasing the fixed end moments. End hinges are forced to form at the column face. As well, the rotation of a column of finite width introduces a relative displacement of the ends of the connected girders. The total stiffening effect is significant in shear wall structures with stiff connecting beams.

Except for Burnstiel, (9), who used stiffness matrices to describe member behavior, all other investigators mentioned in this chapter assumed the moment-rotation response of frame members to be elastic-plastic. An increment of load is applied to the structure and the resulting joint rotations and displacements are determined. The column and girder moments throughout the structure are then calculated, and, if the plastic moment capacity of a section is exceeded, a hinge is inserted into the structure and the deteriorated structure reanalyzed. Two basic methods have been used to account for plastic hinge formation.

The first approach, developed by Jennings and Majid, (6), involves adding one unknown to the displacement vector, (the plastic hinge rotation), each time a new hinge is formed. This approach was extended by Davies, (7), to include hinge reversal, (unloading), by replacing a closing hinge by a "locked" hinge with a rotational discontinuity.

The second method of including the influence of the plastic hinging regions was developed by Parikh, (8). Slope-deflection equations, expressing the end moment of a column or girder in terms of the end rotations and displacements, are first developed. Joint moment and shear equilibrium equations are derived from the member slope-deflection equations. The member slope-deflection equations are modified for the particular hinging configuration in the member, resulting in changes in the coefficients of the related equilibrium equations.

No additional unknowns are introduced on the formation of a plastic hinge. This method has been used in References 4, 8, 10, 11 and 12.

Majumdar, MacGregor and Adams, (11), have presented a method of analyses in which the structure is lumped into equivalent frame and shear wall systems. The advantage of the lumping procedure is the reduction in the number of unknown rotations and displacements that is achieved. Wynhoven and Adams, (12), have extended this procedure to develop a three dimensional analysis for frame - shear wall structures.

Extensive experimental work has been performed as part of the process of developing rational and economical design methods for multi-story braced and unbraced steel frames. Results of combined loading tests on large scale, multi-story frames are reported in References 11, 19 and 20. Yarimci, (20), and Yura, (19), present the results of tests on three-story unbraced and braced steel frames respectively. Majumdar, MacGregor and Adams, (11), describe the results of tests on four-story frame - shear wall structures, designed to simulate the behavior of the lower stories of large frames. In all three investigations, the actual test behavior was accurately predicted by a second order, elastic-plastic analysis.

2.3 Frames Subjected to Vertical Loads Only

The studies outlined in SECTION 2.2 were concerned with the behavior of frames under combined gravity and horizontal loads. The results of investigations into the behavior of planar frames subjected to gravity load only are discussed in this section.

When a symmetrical unbraced frame is subjected to symmetrically applied load, its deflection configuration will also be symmetrical. However as the applied load reaches its critical value, the structure may buckle into an asymmetrical (or sway) configuration, and large lateral displacements may develop. At this instant the frame has lost its resistance to any imposed lateral force, and buckling has thus terminated the load carrying capacity.

The elastic and inelastic buckling loads of the portal frame, shown in FIGURE 2.1, are illustrated in the figure. Whether or not such a frame will buckle with its members remaining elastic depends largely on the slenderness ratio of its columns. A frame with very slender columns will buckle elastically, while a frame with more stocky columns will undergo some yielding prior to buckling. The buckling load of such a frame may be above or below its simple plastic beam mechanism load.

Two basic approaches have been developed to study the frame buckling problem. The bifurcation approach attempts to determine the load at which frame buckling will occur by computing the vertical load level corresponding to the existence of both a straight and a deformed equilibrium position. Numerous studies have been reported on the inelastic buckling strength of single story frames using this approach (13, 16, 17 and 21).

The second approach, known as the small-lateral-load approach, is illustrated in FIGURE 2.2. A number of trial gravity loads, (w_1,w_2), are selected. For each trial load, the response of the frame to a gradually increasing lateral force, (H), applied at each floor level, is analyzed (FIGURE 2.2a). The response is represented by a load - sway deflection, (H-Δ), curve (FIGURE 2.2b). On each curve the lateral load reaches a maximum, H_{max}, at a certain sway deflection. The value of H_{max} becomes smaller as the gravity load, w, increases. A curve relating H_{max} and w, as shown in FIGURE 2.2c, may be obtained. The critical load, w_{CR}, is reached when the curve intersects with the w-axis. This implies that, at the critical load, no lateral force is required to produce a sway deflection.

McNamee, (14), approximated the small-lateral-load approach by considering the behavior of one and two bay frames under proportional loading, with successive decreases in the percentage of lateral load. A second order, elastic-plastic slope-deflection formulation, including the effect of the stability functions on the column stiffness, was used. Frame buckling tests were performed on three, three-story, pinned base frames. The second order, elastic-plastic analysis accurately predicted the behavior of the test frames. These tests represent the only large-scale, multi-story, frame buckling tests reported in the literature.

The discussion presented above pertains only to symmetrical frames under symmetrically applied loads. For unsymmetrical frames, or symmetrical frames subjected to unsymmetrical loads, sidesway deflections develop from the initial application of load. The situation is therefore similar to the case of combined loading. No analytical or experimental studies have been reported for this general case of an unbraced frame.

In a braced frame, the bracing must be designed to prevent frame instability under vertical load only. The behavior of braced frames subjected to vertical load only is similar to that of unbraced frames. No studies have been reported on braced frames under vertical load only.

2.4 Summary

A brief review of available second order, elastic-plastic analyses for frames subjected to combined vertical and horizontal loads and vertical loads only is presented in the preceding sections. On the basis of previous studies, the displacement method of analysis using slope-deflection equations for members, is chosen for the present investigation. The slope-deflection equations are modified to include hinge reversal, so that a greater range of loading sequence possibilities can be studied.

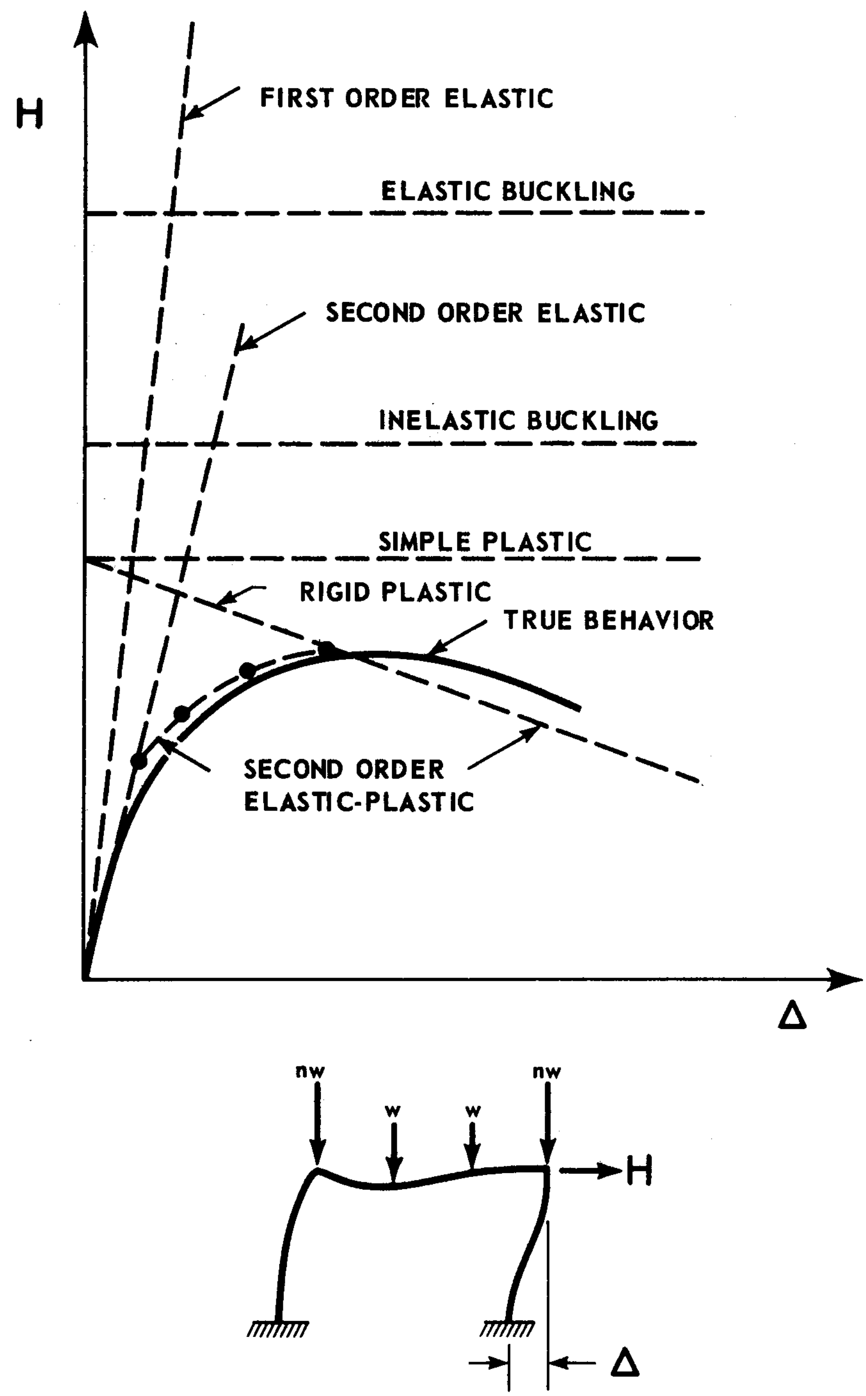


FIGURE 2.1

LOAD-DEFLECTION RELATIONSHIPS

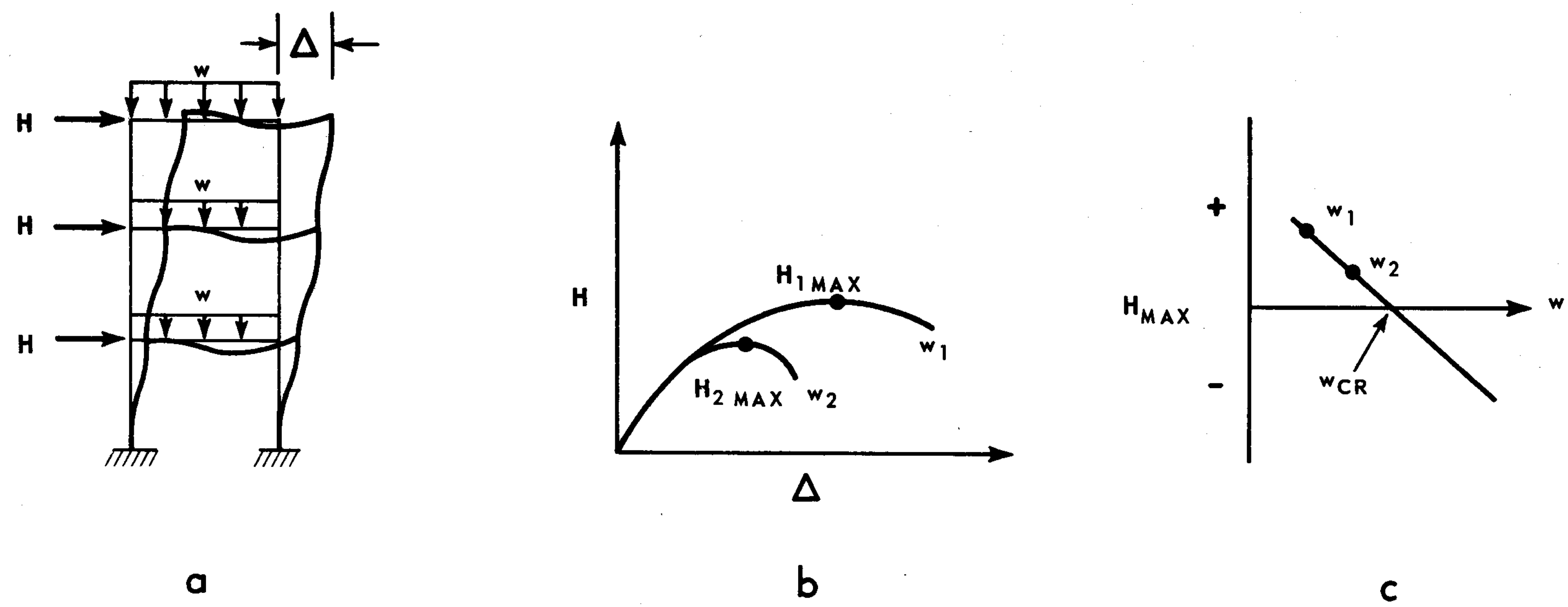


FIGURE 2.2

SMALL-LATERAL-LOAD APPROACH FOR DETERMINING BUCKLING LOAD

CHAPTER III

MEMBER ANALYSIS

3.1 Introduction

The response of a member to an applied load or moment involves many factors. In the elastic range, the response depends on the loading condition, the member length, the boundary conditions and the cross-section properties. As yielding occurs, the member properties are influenced by the gradual penetration of yielded zones from the extremities into the web, and the properties then change along the member length due to the different strain conditions existing in each segment of the member. The advent of yielding is itself complicated by the residual strains in various fibres of the member as well as the differences in material properties that exist in the various plates comprising the cross-section. Although analyses have been performed (on very simple structures) which do account for some or all of the above factors, it is at this date considered impractical to attempt such an analysis for large planar frames.

The basic assumption made in the present analysis is that the behavior of any frame member can be defined in terms of the elastic-plastic, moment-rotation, (M-θ), relationship shown in FIGURE 3.1. End moment is plotted as a function of end rotation for a member subjected to a particular set of boundary conditions. M_P, (or M_{PC} if reduced for axial load), is the plastic moment capacity of the cross-section;

θ_P, (or θ_{PC}), is the rotation corresponding to the attainment of the plastic moment capacity; and P is the axial load on the member. The member is assumed to remain elastic until the plastic moment capacity of the section is reached. For rotations greater than θ_P, (or θ_{PC}), the section is assumed to remain at the plastic moment capacity, (except in the case of hinge reversal at the ends of a girder, where the section is assumed to unload elastically). The slope of the initial portion of the M-θ relationship is proportional to the elastic flexural stiffness, (EI), of the cross-section. No change is made in the value of EI due to yielding of the cross-section under high axial loads.

The "exact" response of a member to an applied moment can be determined from the material stress-strain relationship, obtained from steel coupon tests, combined with a knowledge of the residual strain distribution. The moment-thrust-curvature relationship at a particular section is determined by integrating the resulting stresses in each fibre over the member cross-section. The moment-rotation curves for the member are then obtained by integrating the section moment-thrust-curvature relationships along the length of the member. Such moment-rotation curves, accounting for the residual stress distribution, are presented in Reference 15 for various end moment and axial load ratios. Typical examples of such curves are shown in FIGURE 3.2. In all cases except that of symmetrical single curvature, (FIGURE 3.2d), the assumption of elastic-plastic moment-rotation member behavior, (represented by the dashed lines), is excellent.

For a column in symmetric single curvature the maximum column moment occurs at midheight due to the secondary $P\delta$ moments. Thus the first hinge to form is at this location. After the formation of the first hinge, each half of the column is treated separately. Since no moment increase is permitted at the hinge, the column end moment must unload as the midheight lateral deflection increases. The end moment-end rotation relationship derived in this manner is shown as the dashed curve in FIGURE 3.2d. Agreement with the "exact" curve is good (although the peak of the curve is overestimated). Thus the symmetrical single curvature case can be handled by an elastic-plastic, moment-rotation relationship as long as the secondary $P\delta$ moments are accounted for when the maximum column moment is determined.

Inherent in the assumption of an elastic-plastic moment rotation relationship is the neglect of the spread of inelastic zones along the length of the member; plastic hinges must form at discrete points. The elastic-plastic member response is conveniently incorporated into the standard slope-deflection equations. Such a formulation is used in the present analysis.

A number of further assumptions are made: it is assumed that all members are prismatic; shear deformations are neglected; and local buckling and out of plane behavior are assumed not to effect the member response. The influence of the axial shortening of the girders on the force distribution in the frame is neglected, as is the influence of the axial force on the stiffness and carry-over factors of the girders. Diagonal bracing members are assumed to span between the geometric centers of diagonally opposite joints, and are assumed to resist only axial tension and compression.

The slope-deflection equations for the girders and columns are developed in SECTIONS 3.2 and 3.3 respectively. An analogous approach for diagonal bracing members is developed in SECTION 3.4.

3.2 Girders

3.2.1 Girder Response

Girder response is assumed to be elastic-plastic with elastic unloading, as illustrated in FIGURE 3.3a. In this figure, M_{AB} is the moment at end A of the girder; θ_{AB} is the rotation of end A of the girder; θ_A is the rotation of the joint A; $\overline{\theta_{ABP}}$ is the plastic hinge rotation at A, at the instant of hinge reversal at A; $\Delta\theta_{AB}$ and $\Delta\theta_A$ are the changes in θ_{AB} and θ_A, respectively, after hinge reversal at A; and M_P is the plastic moment capacity of the girder. The plastic hinge rotation at A is the difference between the rotation of the end of the girder, θ_{AB}, and the joint rotation, θ_A. In the loading sequence shown in FIGURE 3.3b, the numbers 1, 2, 3 and 4 refer to the loading stages corresponding to FIGURE 3.3a. From stage 1 to 2 the moment increases linearly with increase in rotation. The slope of this portion of the moment-rotation curve is a function of the girder section properties, (elastic), and the boundary conditions. At stage 2, a plastic hinge develops at end A. Between stages 2 and 3 the moment remains constant at its plastic value; the hinge at A continues to rotate. At stage 3, strain reversal occurs in the plastic hinge as the hinge angle attempts to decrease. At this stage the hinge becomes locked with a rotational discontinuity, θ_{ABP}, and the girder moment-rotation curve again becomes elastic. Between stages 3 and 4 the hinge rotation remains constant and the moment at end A decreases elastically.

3.2.2 Girder Slope-Deflection Equations

The girder notation and sign convention are shown in FIGURE 3.4. "A" refers to the column-girder joint at the left end of the girder; "B" to the right end. w_A and w_B are the widths of the columns at joints A and B respectively. L is the clear span of the girder. Moments are clockwise positive acting on the girder; end shears are positive upwards. An interior hinge is shown at C, a distance x from the left end of the girder, and y from the right end. Joint rotations are positive clockwise, and joint deflections are positive downwards. The inelastic hinge rotation at A, θ_{ABP}, is equal to the difference between the rotation of the left end of the girder, θ_{AB}, and the rotation of the joint A, θ_A. Similarly θ_{BAP} is equal to $\theta_{BA}-\theta_B$. The hinge rotation at C is equal to the rotation of end C of the girder segment CB, minus the rotation of end C of the girder segment AC. Doubly subscripted quantities refer to the girders, and singly subscripted quantities to the joints.

The rotation of a column of finite width introduces a relative displacement of the ends of the connected girders, as illustrated in FIGURE 3.5. If joint A rotates an amount θ_A, the left end of the girder, in addition to an increase in rotation, is displaced by an amount $\Delta = w_A\theta_A/2$. The slope-deflection equations have been modified by Clark, (10), to include this effect, and are:

$$M_{AB} = \frac{4EI}{L}\,\theta_{AB} + \frac{3EI}{L}\,\frac{w_A}{L}\,\theta_A + \frac{2EI}{L}\,\theta_{BA} + \frac{3EI}{L}\,\frac{w_B}{L}\,\theta_B$$

$$+ \frac{6EI}{L^2}\,(\delta_A - \delta_B) + M_{FAB} \qquad (3.1)$$

$$M_{BA} = \frac{2EI}{L}\,\theta_{AB} + \frac{3EI}{L}\,\frac{w_A}{L}\,\theta_A + \frac{4EI}{L}\,\theta_{BA} + \frac{3EI}{L}\,\frac{w_B}{L}\,\theta_B$$

$$+ \frac{6EI}{L^2}\,(\delta_A - \delta_B) + M_{FBA} \qquad (3.2)$$

where M_{AB}, M_{BA} = moments at the left and right end of the girder, respectively,

θ_{AB}, θ_{BA} = rotations of the left and right ends of the girder,

θ_A, θ_B = rotations of joints A and B,

δ_A, δ_B = vertical displacements of joints A and B, and

M_{FAB},M_{FBA} = fixed end moments at the left and right ends of the girder.

3.2.3 Hinge Reversal

Two loading sequence possibilities require the consideration of hinge reversal at the ends of the girders. FIGURE 3.6a shows a frame which has hinged under vertical load only. When the frame sways, due either to frame buckling under increasing vertical loads, or to the application of a horizontal force, the windward hinge angle, at A, attempts to decrease in magnitude, as illustrated in FIGURE 3.6b, while the hinge rotation at B, the leeward side, continues to increase. Since the plastic hinge rotation cannot decrease, and the girder moment at A decreases elastically, the hinge is effectively "locked". The resulting frame, with a rotational discontinuity at the left end of the girder, is shown in FIGURE 3.6c.

The action of the hinge at A can be related to the girder moment-rotation relationship shown in FIGURE 3.3a. Hinge reversal occurs at the left end of the girder, (joint A), at stage 3. The plastic hinge rotation is locked at a value $\overline{\theta_{ABP}}$. At stage 4, the moment, M_{AB}, is given by the expression:

$$M_{AB} = M_{ABP} - \frac{4EI}{L}\Delta\theta_{AB} - \frac{3EI}{L}\frac{w_A}{L}\Delta\theta_A - \frac{2EI}{L}\Delta\theta_{BA}$$

$$- \frac{3EI}{L}\frac{w_B}{L}\Delta\theta_B - \frac{6EI}{L^2}(\Delta\delta_A - \Delta\delta_B) - \Delta M_{FAB} \qquad (3.3)$$

where M_{ABP} = plastic moment capacity at the left end of the girder. The prefix Δ before specific quantities indicates the change in the particular quantity between stages 3 and 4, (that is to say, from the time of hinge reversal).

$$\Delta\theta_{AB} = \Delta\theta_A = \overline{\theta_{AA}} - \theta_A \qquad (3.4a)$$

$$\Delta\theta_{BA} = \overline{\theta_{ABA}} - \theta_{BA} \qquad (3.4b)$$

$$\Delta\theta_B = \overline{\theta_{AB}} - \theta_B \qquad (3.4c)$$

$$\Delta\delta_A = \overline{\delta_{AA}} - \delta_A \qquad (3.4d)$$

$$\Delta\delta_B = \overline{\delta_{AB}} - \delta_B \qquad (3.4e)$$

$$\Delta M_{FAB} = \overline{M_{FAB}} - M_{FAB} \qquad (3.4f)$$

where $\overline{\theta_{ABA}}$ = rotation at the right end of the girder, at the instant of hinge reversal at A,

$\overline{\theta_{AA}}$, $\overline{\theta_{AB}}$ = rotations of joints A and B, respectively, at the instant of hinge reversal at A,

$\overline{\delta_{AA}}$, $\overline{\delta_{AB}}$ = vertical displacements of joints A and B, at the instant of hinge reversal at A, and,

$\overline{M_{FAB}}$ = fixed end moment at the left end of the girder, at the instant of hinge reversal at A.

Substituting EQUATIONS 3.4 into 3.3 and rearranging:

$$M_{AB} = (4+3\,\frac{w_A}{L})\,\frac{EI}{L}\,\theta_A + \frac{2EI}{L}\,\theta_{BA} + \frac{3EI}{L}\,\frac{w_B}{L}\,\theta_B + \frac{6EI}{L^2}\,(\delta_A - \delta_B) + M_{FAB} + M_{ABP} - (4+3\,\frac{w_A}{L})\,\frac{EI}{L}\,\overline{\theta_{AA}} - \frac{2EI}{L}\,\overline{\theta_{ABA}} - \frac{3EI}{L}\,\frac{w_B}{L}\,\overline{\theta_{AB}} - \frac{6EI}{L^2}\,(\overline{\delta_{AA}} - \overline{\delta_{AB}}) - \overline{M_{FAB}} \qquad (3.5)$$

After hinge reversal at A, the girder rotation at A is given by:

$$\theta_{AB} = \theta_A + \overline{\theta_{ABP}} \qquad (3.6)$$

Substituting EQUATION 3.6 into 3.2 and rearranging:

$$M_{BA} = (2+3\,\frac{w_A}{L})\,\frac{EI}{L}\,\theta_A + \frac{4EI}{L}\,\theta_{BA} + \frac{3EI}{L}\,\frac{w_B}{L}\,\theta_B + \frac{6EI}{L^2}\,(\delta_A - \delta_B) + M_{FBA} + \frac{2EI}{L}\,\overline{\theta_{ABP}} \qquad (3.7)$$

Similarly, after hinge reversal at B, M_{AB} and M_{BA} are given by the following expressions:

$$M_{AB} = \frac{4EI}{L}\theta_{AB} + \frac{3EI}{L}\frac{w_A}{L}\theta_A + (2+3\frac{w_B}{L})\frac{EI}{L}\theta_B$$

$$+ \frac{6EI}{L^2}(\delta_A-\delta_B) + M_{FAB} + \frac{2EI}{L}\overline{\theta_{BAP}} \qquad (3.8)$$

$$M_{BA} = \frac{2EI}{L}\theta_{AB} + \frac{3EI}{L}\frac{w_A}{L}\theta_A + (4+3\frac{w_B}{L})\frac{EI}{L}\theta_B$$

$$+ \frac{6EI}{L^2}(\delta_A-\delta_B) + M_{FBA} + M_{BAP} - \frac{2EI}{L}\overline{\theta_{BAB}} - \frac{3EI}{L}\frac{w_B}{L}\overline{\theta_{BA}} \qquad (3.9)$$

$$- (4+3\frac{w_B}{L})\frac{EI}{L}\overline{\theta_{BB}} - \frac{6EI}{L^2}(\overline{\delta}_{BA}-\overline{\delta}_{BB}) - \overline{M_{FBA}}$$

where $\overline{\theta_{BAP}}$ = plastic hinge rotation at the right end of the girder, at the instant of hinge reversal at B,

$\overline{\theta_{BAB}}$ = rotation of the left end of the girder, at the instant of hinge reversal at B,

$\overline{\theta_{BA}}$, $\overline{\theta_{BB}}$ = rotations of joints A and B, respectively, at the instant of hinge reversal at B,

$\overline{\delta}_{BA}$, $\overline{\delta}_{BB}$ = vertical displacements of joints A and B, at the instant of hinge reversal at B,

$\overline{M_{FBA}}$ = fixed end moment at the right end of the girder, at the instant of hinge reversal at B, and,

M_{BAP} = plastic moment capacity at the right end of the girder.

Comparison of EQUATIONS 3.5, 3.7, 3.8 and 3.9, with EQUATIONS 3.1 and 3.2 shows that the expressions for M_{AB} and M_{BA}, after hinge reversal at either A or B, are similar to the corresponding expressions prior to hinge reversal. The difference is a single constant term, (or combination of terms), that can be evaluated at the instant of hinge reversal.

Two assumptions are made regarding hinge reversal in the girders. The first is that hinge reversal will not occur at an interior hinge. In the situation where an interior hinge would form under vertical load, a superimposed sway motion would cause the hinge rotation to increase. The second assumption is that hinge reversal can occur in only one end of any girder. For the loading sequence possibilities considered in this dissertation, this would normally be the case.

3.2.4 Girder Hinge Configurations

In this section the different girder hinging possibilities are discussed, and the modifications to the standard slope-deflection equations necessary to account for the inelastic action, are presented.

A plastic hinge may develop at either end of a girder, or within its length. Once formed, a hinge is assumed to remain in its original position. The different hinge configurations considered in the analysis are illustrated in FIGURE 3.7. Girders in each of the sixteen hinge patterns are shown. The possible sequences of hinge formation and reversal are indicated by arrows. In configuration 1, the girder is elastic. Suppose, under increasing load, the plastic moment capacity of the section is exceeded at the left end. A plastic hinge is inserted and the girder hinge configuration changes from 1 to 2. If a hinge then forms at the right end, the girder is in hinge configuration 5. Suppose a lateral load is applied to the frame causing the frame to sway to the right. The hinge at the left end of the girder reverses. The girder, with a rotational discontinuity at the left end, is now in configuration 13. If a further hinge develops in the interior, the girder is finally in configuration 20. Many other sequences of hinge formation are possible.

The slope-deflection equations are modified to include plastic hinging under the following conditions:

1. At the end of a member which does not contain a plastic hinge, the rotation of the end of the girder is equal to the joint rotation. The moment at such an end is dependent on the girder rotations, displacements and loading.
2. The moment at a hinge is equal to the plastic moment capacity of the girder. The appropriate girder slope-deflection equation can then be used to express the inelastic hinge rotation in terms of the member loading condition and the joint rotations and displacements at its ends.

The slope-deflection equations may be expressed in the form:

$$M_{AB} = C_{A1}\theta_A + C_{A2}\theta_B + C_{A3}(\delta_A - \delta_B) + C_{A4} + C_{A5} \qquad (3.10)$$

$$M_{BA} = C_{B1}\theta_A + C_{B2}\theta_B + C_{B3}(\delta_A - \delta_B) + C_{B4} + C_{B5}$$

The values of the coefficients C_{A1}, C_{A2}, C_{A3} and C_{A4}, and C_{B1}, C_{B2}, C_{B3} and C_{B4}, are summarized in TABLES 3.1 and 3.2 respectively. C_{A5} and C_{B5} are constants computed at the instant of hinge reversal. They are given in TABLE 3.3. Expressions for the plastic hinge rotations for each of the hinge configurations are summarized in TABLE 3.4.

3.3 Columns and Shear Walls

3.3.1 Introduction

The primary function of a column in a multi-story structure is to carry the moments and forces from the members framing into it. It may also be required to contribute to the lateral stiffness of the structure. A shear wall, on the other hand, is designed primarily to resist lateral forces. Shear walls are normally many times stiffer than columns, and are usually much greater in width. It is not uncommon for a shear wall in a service core area to be one million times stiffer than a typical column, and have a width equal to the story height. In the present analysis columns and shear walls are treated identically.

The behavior of a structure can be considerably influenced by the width of columns or shear walls. The clear span of the girders is reduced and the girder hinges are forces to form away from the column centrelines, increasing both the strength and lateral stiffness of the frame. The lateral stiffness of the frame is also increased due to the greater rotational restraint afforded by the girders as they undergo a relative displacement due to column rotation, (as discussed in SECTION 3.2.2).

3.3.2 Column Response

The response of a column in the structure is assumed to be elastic-perfectly plastic, as illustrated in FIGURE 3.8. The moment at the lower end of the column, M_{LU}, is plotted as a function of the rotation of the lower end of the column, θ_{LU}. For given boundary conditions, it is assumed that an increase in end rotation will result in an increase in moment, until the plastic moment capacity of the column is reached. The end moment is thereafter held at the plastic moment capacity of the section, regardless of any change in end rotation.

The effect of axial load on the stiffness and carry-over factors for a column is considered by using the stability functions, C and S, as tabulated in Reference 23. Also considered, is the effect of axial load on the plastic moment capacity of the cross-section. The plastic moment capacity of a column is given by the equations (23):

$$M_{PC} = M_P \qquad , \quad \frac{P}{P_y} \leq 0.15 \qquad (3.12)$$

$$M_{PC} = 1.18\, M_P \left(1 - \frac{P}{P_y}\right) \qquad , \quad \frac{P}{P_y} > 0.15 \qquad (3.13)$$

where M_{PC} = plastic moment capacity of the column, reduced for axial load,

M_P = plastic moment capacity for the column without axial load,

P = axial load in the column, and,

P_y = yield axial load in the column.

If the column axial load changes after the formation of a plastic hinge, the hinge is maintained and the moment at the hinge is adjusted according to EQUATIONS 3.12 and 3.13.

Not considered in the analysis is the effect of residual stress on the column stiffness. Assuming a maximum compressive residual stress of 0.3 σ_y, (where σ_y is the yield stress of the column), yielding of portions of the column cross-section, for axial load ratios, (P/P_y), greater than 0.7, will reduce the effective moment of inertia of the column. This effect is ignored.

3.3.3 Column Slope-Deflection Equations

The column notation and sign convention are shown in FIGURE 3.9. "L" refers to the column-girder joint at the lower end of the column; "U" to the upper end. P is the column axial load, positive in compression, and h represents the column height. Bending moments are clockwise positive acting on the column ends; end shears are positive to the left. An interior hinge is shown at D, a distance x from the lower end of the column, and y from the upper end. Joint rotations are positive clockwise, and the vertical deflections of the joints are positive downwards. Story sway is positive to the right. The plastic hinge rotation at the end of the column, (θ_{LUP} or θ_{ULP}), is equal to the difference between the member end rotation, (θ_{LU} or θ_{UL}), and the corresponding joint rotation, (θ_L or θ_U). The inelastic rotation at D is equal to the difference in slopes of the column segments LD and DU at D. Doubly subscripted quantities refer to the columns; singly subscripted quantities to the joints.

The basic slope-deflection equations for columns are:

$$M_{LU} = \frac{CEI}{h}\theta_{LU} + \frac{SEI}{h}\theta_{UL} + \frac{(C+S)EI}{h^2}(\Delta_L - \Delta_U) \qquad (3.14)$$

$$M_{UL} = \frac{SEI}{h} \theta_{LU} + \frac{CEI}{h} \theta_{UL} + \frac{(C+S)EI}{h^2} (\Delta_L - \Delta_U) \quad (3.15)$$

where M_{LU}, M_{UL} = moments at the lower and upper ends of the column, respectively,

θ_{LU}, θ_{UL} = rotations of the lower and upper ends of the column,

Δ_L, Δ_U = sways of the upper and lower ends of the column,

E = modulus of elasticity

I = moment of inertia, and

h = column height.

C and S are the stability functions discussed in SECTION 3.3.2. For columns subjected to a tensile axial load, C is taken as 4, S as 2.

The maximum interior column moment, including the effect of axial load, is given by the expression (10):

$$M_{MAX} = \frac{\sqrt{M_{UL}^2 + M_{LU}^2 + 2M_{UL}M_{LU} \cos Kh}}{\sin Kh} \quad (3.16)$$

where $K = \sqrt{\frac{P}{EI}}$

M_{MAX} occurs a distance x from the lower end of the column, given by:

$$x = \frac{1}{K} \tan^{-1} \left[- \frac{M_{UL} + M_{LU} \cos Kh}{M_{LU} \sin Kh}\right] \quad (3.17)$$

3.3.4 Column Hinge Configurations

Since hinge reversal in the columns is not considered, only eight hinge configurations are required. These are illustrated in FIGURE 3.10. The possible sequences of hinge formation are indicated by arrows.

The slope-deflection equations are modified to include plastic hinging in the same manner as for the girders. The equations may be expressed in the following form:

$$M_{LU} = C_{L1}\theta_L + C_{L2}\theta_U + C_{L3}\,(\Delta_L - \Delta_U) + C_{L4} \qquad (3.18)$$

$$M_{UL} = C_{U1}\theta_L + C_{U2}\theta_U + C_{U3}\,(\Delta_L - \Delta_U) + C_{U4} \qquad (3.19)$$

The values of the coefficients C_{L1}, C_{L2}, C_{L3} and C_{L4}, and C_{U1}, C_{U2}, C_{U3} and C_{U4} are summarized in TABLES 3.5 and 3.6 respectively. Expressions for the plastic hinge rotations for each of the hinging configurations are given in TABLE 3.7.

3.3.5 Axial Shortening

The present analysis considers the effects of axial column shortening on the force distribution in the structure. A column subjected to axial load is assumed to behave elastically. The axial shortening, δ, may be expressed as:

$$\delta = \frac{Ph}{AE} \qquad (3.20)$$

where P = axial force in the column,

h = column height,

A = column area, and,

E = modulus of elasticity.

EQUATION 3.20 may be rearranged into a more suitable form:

$$P = \frac{AE}{h}\,(\delta_U - \delta_L) \qquad (3.21)$$

where δ_U, δ_L = vertical deflections of the joints at the upper and lower ends of the column, respectively.

3.4 Diagonal Bracing

3.4.1 Member Response

Diagonal bracing members are considered capable of transmitting only axial forces. The assumed load-deformation relationship for a typical bracing member is shown in FIGURE 3.11. The axial load in the brace, P, is plotted as a function of the brace elongation, e. P_y is the yield load of the brace in tension, (equal to the area of the brace multiplied by the yield stress). P_{CR} is the critical axial load of the brace in compression. Since the brace is not assumed to transmit moment, it can be considered as pinned at each end. The critical axial load is defined as the Euler load:

$$P_{CR} = \frac{\pi^2 EAr^2}{L^2} \tag{3.22}$$

where E = modulus of elasticity,

A = area of the brace,

r = minimum radius of gyration of the brace, and,

L = length of the brace.

The Euler load is the critical axial load for slender bracing members, (such as light angles). However the radius of gyration of the brace is assumed to be independent of the other properties of the bracing member, so that the critical axial load in compression can be adjusted by specifying a ficticious value of r. Thus the critical axial load of more stocky bracing members, (which would buckle inelastically), can be accounted for, as well as the critical axial load of diagonal members which are fastened at midlength. Braces acting in compression can be neglected entirely by setting r = 0.

3.4.2 Diagonal Bracing Load-Deformation Equations

The diagonal bracing notation and sign convention are shown in FIGURE 3.12. Two different diagonal braces are considered: TYPE 1 which slopes upward to the left, and TYPE 2 which slopes upward to the right. These are illustrated in FIGURES 3.12a and 3.12b respectively. "U" refers to the joint at the upper end of the brace, "L" to the lower end. Δ_U and Δ_L are the sways of the joints at the upper and lower ends of the brace, respectively. δ_U and δ_L are the vertical deflections of the joints. L is the length of the diagonal brace. In FIGURES 3.12c and 3.12d, the change in length of the brace, e, due to a relative sway of the ends of the brace, Δ, is illustrated for a TYPE 1 and TYPE 2 brace respectively. In FIGURES 3.12e and 3.12f, the change in length of the brace, due to a relative vertical displacement of the ends of the brace, δ, is illustrated for the two types of braces. The axial load in the brace is considered positive in tension. Sway is positive to the right and vertical deflection is positive downwards.

The axial load-deformation relationships for the diagonal bracing members are:

TYPE 1:

$$F_X = \frac{AE}{L} \sin\alpha \cos\alpha \ (\delta_L - \delta_U) + \frac{AE}{L} \cos^2\alpha \ (\Delta_L - \Delta_U) \qquad (3.23)$$

$$F_Y = \frac{AE}{L} \sin^2\alpha \; (\delta_L - \delta_U) + \frac{AE}{L} \sin\alpha \cos\alpha \; (\Delta_L - \Delta_U) \qquad (3.24)$$

TYPE 2:

$$F_X = \frac{AE}{L} \sin\alpha \cos\alpha \; (\delta_L - \delta_U) - \frac{AE}{L} \cos^2\alpha \; (\Delta_L - \Delta_U) \qquad (3.25)$$

$$F_Y = \frac{AE}{L} \sin^2\alpha \; (\delta_L - \delta_U) - \frac{AE}{L} \cos\alpha \sin\alpha \; (\Delta_L - \Delta_U) \qquad (3.26)$$

where F_X = horizontal component of axial force in the brace,

F_Y = vertical component of axial force in the brace,

A = area of the brace,

E = modulus of elasticity,

L = length of the brace,

δ_L, δ_U = vertical deflection of the joints at the lower and upper ends of the brace, respectively,

Δ_L, Δ_U = horizontal deflection of the joints at the lower and upper ends of the brace, and,

α = angle between the brace and the horizontal.

3.4.3 Yield Configurations

Three yield configurations are considered for diagonal bracing members. In yield configuration 1, the brace is elastic. In yield configuration 2, the brace has yielded in tension. In yield configuration 3, the brace has buckled in compression. The axial load-deformation relationships for each of the yield configurations are expressed in the form:

TYPE 1:

$$F_X = C_{D1}\delta_L - C_{D1}\delta_U + C_{D2}\Delta_L - C_{D2}\Delta_U + C_{D5} \qquad (3.27)$$

$$F_Y = C_{D3}\delta_L - C_{D3}\delta_U + C_{D1}\Delta_L - C_{D1}\Delta_U + C_{D4} \qquad (3.28)$$

TYPE 2:

$$F_X = C_{D1}\delta_L - C_{D1}\delta_U - C_{D2}\Delta_L + C_{D2}\Delta_U + C_{D5} \qquad (3.29)$$

$$F_Y = C_{D3}\delta_L - C_{D3}\delta_U - C_{D1}\Delta_L + C_{D1}\Delta_U + C_{D4} \qquad (3.30)$$

The coefficients C_{D1}, C_{D2}, C_{D3}, C_{D4} and C_{D5} are summarized for the different yield configurations in TABLE 3.8.

A bracing member in a yielded or buckled configuration is assumed capable of again becoming elastic if the axial extension becomes less than the yield or buckling values, respectively. No modification to the elastic load-deformation equations are necessary in either case.

HINGE CONFIGURATION	C_{A1}	C_{A2}	C_{A3}	C_{A4}
1,10,11	$(4+3\frac{w_A}{L})\frac{EI}{L}$	$(2+3\frac{w_B}{L})\frac{EI}{L}$	$\frac{6EI}{L^2}$	M_{FAB}
2,14	0	0	0	M_{ABP}
3,13	$(3+1.5\frac{w_A}{L})\frac{EI}{L}$	$(1.5\frac{w_B}{L})\frac{EI}{L}$	$\frac{3EI}{L^2}$	$M_{FAB}-0.5M_{FBA}+0.5M_{BAP}$
4,15,17	$\frac{x^2}{x^3+y^3}(3+1.5\frac{w_A}{x})EI$	$\frac{xy}{x^3+y^3}(3+1.5\frac{w_B}{y})EI$	$\frac{x}{x^3+y^3}3EI$	$\frac{1}{x^3+y^3}[x^3(M_{FAC}-0.5M_{FCA})+(0.5x^3-y^3)M_{CAP}$ $-y^3M_{AX}+xy^2(M_{FBC}-0.5M_{FCB}+1.5M_{CBP}-M_{By})]$
5	0	0	0	M_{ABP}
6,21	0	0	0	M_{ABP}
7,20	0	0	0	$\frac{x}{y}(M_{CBP}+M_{BAP}-M_{By}) - M_{CAP}-M_{AX}$
8	0	0	0	M_{ABP}

TABLE 3.1 COEFFICIENTS OF GIRDER SLOPE-DEFLECTION EQUATIONS

HINGE CONFIGURATION	C_{B1}	C_{B2}	C_{B3}	C_{B4}
1,10,11	$(2+3\frac{w_A}{L})\frac{EI}{L}$	$(4+3\frac{w_B}{L})\frac{EI}{L}$	$\frac{6EI}{L^2}$	M_{FBA}
2,14	$1.5\frac{w_A}{L}\frac{EI}{L}$	$(3+1.5\frac{w_B}{L})\frac{EI}{L}$	$\frac{3EI}{L^2}$	$M_{FBA}-0.5M_{FAB}+0.5M_{ABP}$
3,13	0	0	0	M_{BAP}
4,15,17	$\frac{xy}{x^3+y^3}(3+1.5\frac{w_A}{x})EI$	$\frac{y^2}{x^3+y^3}(3+1.5\frac{w_B}{y})EI$	$\frac{y}{x^3+y^3}3EI$	$\frac{1}{x^3+y^3}[y^3(M_{FBC}-0.5M_{FCB})+(0.5y^3-x^3)M_{CBP}$ $+x^3M_{By}+x^2y(M_{FAC}-0.5M_{FCA}+1.5M_{CAP}+M_{AX})]$
5	0	0	0	M_{BAP}
6,21	0	0	0	$\frac{y}{x}(M_{ACP}+M_{CAP}+M_{AX})+M_{By}-M_{CBP}$
7,20	0	0	0	M_{BAP}
8	0	0	0	M_{BAP}

TABLE 3.2 COEFFICIENTS OF GIRDER SLOPE-DEFLECTION EQUATIONS

HINGE CONFIGURATION	C_{A5}	C_{B5}
10	$M_{ABP}-(4+3\frac{w_A}{L})\frac{EI}{L}\overline{\theta_{AA}}-(2+3\frac{w_B}{L})\frac{EI}{L}\overline{\theta_{AB}}$ $-\frac{6EI}{L^2}(\overline{\delta_{AA}}-\overline{\delta_{AB}})-\overline{M_{FAB}}$	$\frac{2EI}{L}\overline{\theta_{ABP}}$
11	$\frac{2EI}{L}\overline{\theta_{BAP}}$	$M_{BAP}-(2+3\frac{w_A}{L})\frac{EI}{L}\overline{\theta_{BA}}-(4+3\frac{w_B}{L})\frac{EI}{L}\overline{\theta_{BB}}$ $-\frac{6EI}{L^2}(\overline{\delta_{BA}}-\overline{\delta_{BB}})-\overline{M_{FBA}}$
13	$M_{ABP}-0.5M_{BAP}-(3+1.5\frac{w_A}{L})\frac{EI}{L}\overline{\theta_{AA}}$ $-1.5\frac{w_B}{L}\frac{EI}{L}\overline{\theta_{AB}}-\frac{3EI}{L^2}(\overline{\delta_{AA}}-\overline{\delta_{AB}})-\overline{M_{FAB}}+0.5\overline{M_{FBA}}$	0
14	0	$M_{BAP}-0.5M_{ABP}-1.5\frac{w_A}{L}\frac{EI}{L}\overline{\theta_{BA}}-(3+1.5\frac{w_B}{L})\frac{EI}{L}\overline{\theta_{BB}}$ $-\frac{3EI}{L^2}(\overline{\delta_{BA}}-\overline{\delta_{BB}})-M_{FBA}+0.5M_{FAB}$
15	$\frac{x^3}{x^3+y^3}[M_{ABP}-0.5M_{CAP}-(3+1.5\frac{w_A}{x})\frac{EI}{x}\overline{\theta_{AA}}$ $-\frac{3EI}{x^2}(\overline{\delta_{AA}}-\overline{\delta_C})-\overline{M_{FAC}}+0.5\overline{M_{FCA}}]$	$\frac{x^2y}{x^3+y^3}[\quad]$ Note: δ_C is given in TABLE 3.4

TABLE 3.3 COEFFICIENTS OF GIRDER SLOPE-DEFLECTION EQUATIONS

... continued

HINGE CONFIGURATION	C_{A5}	C_{B5}
17	$\frac{xy^2}{x^3+y^3}[M_{BAP}-0.5M_{CBP}-(3+1.5\frac{w_B}{y})\frac{EI}{y}\overline{\theta_{BB}}$ $-\frac{3EI}{y^2}(\overline{\delta_C}-\overline{\delta_{BB}})-\overline{M_{FBC}}+0.5\overline{M_{FCB}}]$	$\frac{y^3}{x^3+y^3}[\quad]$ Note: δ_C is given in TABLE 3.4
20	0	0
21	0	0

TABLE 3.3 COEFFICIENTS OF GIRDER SLOPE-DEFLECTION EQUATIONS

HINGE CONFIGURATION	$\theta_{ABP}, \theta_{BAP}, \theta_{CP}, \delta_C$
1	
2	$\theta_{ABP} = \frac{L}{4EI}(M_{ABP}-M_{FAB})-(1+0.75\frac{w_A}{L})\theta_A-(0.5+0.75\frac{w_B}{L})\theta_B - 1.5\frac{\delta_A-\delta_B}{L}$
3	$\theta_{BAP} = \frac{L}{4EI}(M_{BAP}-M_{FBA})-(0.5+0.75\frac{w_A}{L})\theta_A-(1+0.75\frac{w_B}{L})\theta_B - 1.5\frac{\delta_A-\delta_B}{L}$
4	$\theta_{CP} = \frac{x}{4EI}(M_{CAP}-M_{FCA})-(0.5+0.75\frac{w_A}{x})\theta_A-1.5\frac{\delta_A-\delta_C}{x} - \frac{y}{4EI}(M_{CBP}-M_{FCB})+(0.5+0.75\frac{w_B}{y})\theta_B+1.5\frac{\delta_C-\delta_B}{y}$
	$\delta_C = \frac{x^2y^2}{x^3+y^3}\{y[\frac{1}{3EI}(M_{FAC}-0.5M_{FCA}+1.5M_{CAP}+M_{Ax}) +(1+0.5\frac{w_A}{x})\frac{1}{x}\theta_A+\frac{1}{x^2}\delta_A] - x[\frac{1}{3EI}(M_{FBC}-0.5M_{FCB}+1.5M_{CBP}-M_{By}) +(1+0.5\frac{w_B}{y})\frac{1}{y}\theta_B+\frac{1}{y^2}\delta_B]\}$
5	$\theta_{ABP} = \frac{L}{6EI}(2M_{ABP}-M_{BAP}-2M_{FAB}+M_{FBA})-(1+0.5\frac{w_A}{L})\theta_A - 0.5\frac{w_B}{L}\theta_B - \frac{\delta_A-\delta_B}{L}$
	$\theta_{BAP} = \frac{L}{6EI}(2M_{BAP}-M_{ABP}-2M_{FBA}+M_{FAB})-0.5\frac{w_A}{L}\theta_A -(1+0.5\frac{w_B}{L})\theta_B - \frac{\delta_A-\delta_B}{L}$

... continued

TABLE 3.4 INELASTIC HINGE ROTATIONS

HINGE CONFIGURATION	$\theta_{ABP}, \theta_{BAP}, \theta_{CP}, \delta_C$
6	$\theta_{ABP} = \frac{x}{6EI}(2M_{ACP}-M_{CAP}-2M_{FAC}+M_{FCA})-(1+0.5\frac{w_A}{x})\theta_A - \frac{\delta_A-\delta_C}{x}$
	$\theta_{CP} = \frac{x}{6EI}(2M_{CAP}-M_{ACP}-2M_{FCA}+M_{FAC}) - \frac{y}{4EI}(M_{CBP}-M_{FCB}) - 0.5\frac{w_A}{x}\theta_A+(0.5+0.75\frac{w_B}{y})\theta_B - \frac{\delta_A-\delta_C}{x} + 1.5\frac{\delta_C-\delta_B}{y}$
	$\delta_C = \frac{y^2}{3EI}[\frac{y}{x}(M_{ACP}+M_{CAP}+M_{Ax})+M_{By}-1.5M_{CBP}-M_{FBC} + 0.5M_{FCB}]-(1+0.5\frac{w_B}{y})y\theta_B + \delta_B$
7	$\theta_{BCP} = \frac{y}{6EI}(2M_{BCP}-M_{CBP}-2M_{FBC}+M_{FCB})-(1+0.5\frac{w_B}{y})\theta_B - \frac{\delta_C-\delta_B}{y}$
	$\theta_{CP} = \frac{x}{4EI}(M_{CAP}-M_{FCA}) - \frac{y}{6EI}(2M_{CBP}-M_{BCP}-2M_{FCB}+M_{FBC}) -(0.5+0.75\frac{w_A}{x})\theta_A+0.5\frac{w_B}{y}\theta_B-1.5\frac{\delta_A-\delta_C}{x} + \frac{\delta_C-\delta_B}{y}$
	$\delta_C = \frac{x^2}{3EI}[\frac{x}{y}(M_{By}-M_{CBP}-M_{BAP})+1.5M_{CAP}+M_{Ax}+M_{FAC}-0.5M_{FCA}] +(1+0.5\frac{w_A}{x})x\theta_A + \delta_A$
8	$\theta_{ABP}, \theta_{BAP}, \theta_{CP}$ and δ_C are indeterminant
10	$\theta_{ABP} = \overline{\theta_{ABP}}$

... continued

HINGE CONFIGURATION	$\theta_{ABP}, \theta_{BAP}, \theta_{CP}, \delta_C$
11	$\theta_{BAP} = \overline{\theta_{BAP}}$
13	$\theta_{ABP} = \overline{\theta_{ABP}}$
	$\theta_{BAP} = \frac{L}{4EI}(M_{BAP}-M_{FBA})-(0.5+0.75\ \frac{w_A}{L})\theta_A$ $-(1+0.75\ \frac{w_B}{L})\theta_B - \frac{1.5}{L}(\delta_A-\delta_B)-0.5\overline{\theta_{ABP}}$
14	$\theta_{ABP} = \frac{L}{4EI}(M_{ABP}-M_{FAB})-(1+0.75\ \frac{w_A}{L})\theta_A$ $-(0.5+0.75\ \frac{w_B}{L})\theta_B - \frac{1.5}{L}(\delta_A-\delta_B)-0.5\ \overline{\theta_{BAP}}$
	$\theta_{BAP} = \overline{\theta_{BAP}}$
15	$\theta_{ABP} = \overline{\theta_{ABP}}$
	$\theta_{CP} = \frac{x}{4EI}(M_{CAP}-M_{FCA}) - \frac{y}{4EI}(M_{CBP}-M_{FCB})-(0.5+0.75\ \frac{w_A}{x})\theta_A$ $-(0.5+0.75\ \frac{w_B}{y})\theta_B - \frac{1.5}{x}\ (\delta_A-\delta_C)+\ \frac{1.5}{y}\ (\delta_C-\delta_B)-0.5\ \overline{\theta_{ABP}}$
	$\delta_C = \frac{x^2y^2}{x^3+y^3}\ \{\frac{y}{x}(1+0.5\ \frac{w_A}{x})\theta_A - \frac{x}{y}(1+0.5\ \frac{w_B}{y})\theta_B + \frac{y}{x^2}\ \delta_A$ $+\ \frac{x}{y^2}\ \delta_B + \frac{x}{3EI}[M_{By}-1.5M_{CBP}-M_{FBC}+0.5M_{FCB}]$ $+\frac{y}{3EI}[M_{Ax}+M_{CAP}+M_{FAC}-0.5M_{FCA}+M_{ACP}+0.5\overline{M_{FCA}}$ $-\overline{M_{FAC}}] - \frac{y}{x}(1+0.5\ \frac{w_A}{x})\overline{\theta_{AA}} - \frac{y}{x^2}(\overline{\delta_{AA}}-\overline{\delta_{AC}})\}$

... continued

HINGE CONFIGURATION	$\theta_{ABP}, \theta_{BAP}, \theta_{CP}, \delta_C$
17	$\theta_{BAP} = \overline{\theta_{BAP}}$

$$\theta_{CP} = \frac{x}{4EI}(M_{CAP}-M_{FCA}) - \frac{y}{4EI}(M_{CBP}-M_{FCB}) - (0.5+0.75\ \frac{w_A}{x})\theta_A$$

$$+(0.5+0.75\ \frac{w_B}{y})\theta_B - \frac{1.5}{x}(\delta_A-\delta_C) + \frac{1.5}{y}(\delta_C-\delta_B) + 0.5\overline{\theta_{BAP}}$$

$$\delta_C = \frac{x^2y^2}{x^3+y^3}\{\frac{y}{x}(1+0.5\ \frac{w_A}{x})\theta_A - \frac{x}{y}(1+0.5\ \frac{w_B}{y})\theta_B + \frac{y}{x^2}\ \delta_A + \frac{x}{y^2}\ \delta_B$$

$$+\frac{x}{3EI}[M_{By}-1.5M_{CBP}-M_{FBC}+0.5M_{FCB}-M_{BAP}+0.5M_{CBP}]$$

$$+\frac{y}{3EI}[M_{Ax}+1.5M_{CAP}+M_{FAC}-0.5M_{FCA}]$$

$$+\ \frac{x}{y}(1+0.5\ \frac{w_B}{y})\overline{\theta_{BB}} + \frac{x}{y^2}(\overline{\delta_{BC}}-\overline{\delta_{BB}})\}$$

20

$$\theta_{ABP} = \overline{\theta_{ABP}}$$

$$\theta_{BAP} = \frac{y}{6EI}(2M_{BAP}-M_{CBP}-2M_{FBA}+M_{FCB}) - (1+0.5\ \frac{w_B}{y})\theta_B - \frac{\delta_C-\delta_B}{y}$$

$$\theta_{CP} = \frac{x}{4EI}(M_{BAP}-M_{FCA}) - \frac{y}{6EI}(2M_{CBP}-M_{BAP}-2M_{FCB}+M_{FBA})$$

$$-(0.5+0.75\ \frac{w_A}{x})\theta_A + 0.5\ \frac{w_B}{y}\ \theta_B - \frac{1.5}{x}(\delta_A-\delta_C) + \frac{\delta_C-\delta_B}{y}\ -0.5\overline{\theta_{ABP}}$$

$$\delta_C = \frac{x^2}{3EI}[1.5M_{CAP}+M_{FAB}-0.5M_{FCA}+M_{ABP}-0.5M_{CAP}+M_{Ax}$$

$$-\ \frac{x}{y}(M_{CBP}+M_{BAP}-M_{By})+0.5\overline{M_{FCA}}-\overline{M_{FAB}}]$$

$$+\ x(1+0.5\ \frac{w_A}{x})\theta_A+\delta_A-x(1+0.5\ \frac{w_A}{x})\overline{\theta_{AA}}-\overline{\delta_{AA}}+\overline{\delta_{AC}}$$

... continued

HINGE CONFIGURATION	$\theta_{ABP}, \theta_{BAP}, \theta_{CP}, \delta_C$
21	$\theta_{ABP} = \frac{x}{6EI}(2M_{ABP} - M_{CAP} - 2M_{FAB} + M_{FCA}) - (1 + 0.5\frac{w_A}{x})\theta_A - \frac{\delta_A - \delta_C}{x}$
	$\theta_{BAP} = \overline{\theta_{BAP}}$
	$\theta_{CP} = \frac{x}{6EI}(2M_{CAP} - M_{ABP} - 2M_{FCA} + M_{FAB}) - \frac{y}{4EI}(M_{CBP} - M_{FCB})$
	$-0.5\frac{w_A}{x}\theta_A + (0.5 + 0.75\frac{w_B}{y})\theta_B - \frac{\delta_A - \delta_C}{x} + \frac{\delta_C - \delta_B}{y} - 0.5\overline{\theta_{BAP}}$
	$\delta_C = \frac{y^2}{3EI}[-M_{FBA} + 0.5M_{FCB} - 0.5M_{CBP} - M_{BAP} + 0.5M_{CBP} + \overline{M_{FBC}} - 0.5\overline{M_{FCB}}$
	$+M_{By} - M_{CBP} + \frac{y}{x}(M_{ABP} + M_{CAP} + M_{Ax})] - y(1 + 0.5\frac{w_B}{y})\theta_B + \delta_B$
	$+y(1 + 0.5\frac{w_B}{y})\overline{\theta_{BB}} - \overline{\delta_{BB}} + \overline{\delta_{BC}}$

HINGE CONFIGURATION	C_{L1}	C_{L2}	C_{L3}	C_{L4}
1	$\frac{CEI}{h}$	$\frac{SEI}{h}$	$\frac{(C+S)\ EI}{h^2}$	0
2	0	0	0	M_{LUP}
3	$\frac{C^2-S^2}{C}\frac{EI}{h}$	0	$\frac{C^2-S^2}{C}\frac{EI}{h^2}$	$\frac{S}{C}M_{ULP}$
4	$[1-By]\frac{C_x^{\,2}-S_x^{\,2}}{C_x}\frac{EI}{x}$	$Bx\frac{C_y^{\,2}-S_y^{\,2}}{C_y}\frac{EI}{y}$	$Bx[\frac{C_y^{\,2}-S_y^{\,2}}{C_y}\frac{EI}{y}-P]$	$[\frac{S_x}{C_x}-By(1+\frac{S_x}{C_x})]M_{DLP}+Bx(1+\frac{S_y}{C_y})M_{DUP}$
5	0	0	0	M_{LUP}
6	0	0	0	M_{LUP}
7	$G\frac{C_x^{\,2}-S_x^{\,2}}{C_x}\frac{EI}{x}$	0	$[G-1]\frac{x}{y}P$	$[1-G][\frac{x}{y}(M_{ULP}+M_{DUP})-M_{DLP}]+G\frac{S_x}{C_x}M_{DLP}$
8	0	0	0	M_{LUP}

$$B = \frac{-\frac{C_x^{\,2}-S_x^{\,2}}{C_x}\frac{EI}{x^2}}{(P-\frac{C_y^{\,2}-S_y^{\,2}}{C_y}\frac{EI}{y^2})x+(P-\frac{C_x^{\,2}-S_x^{\,2}}{C_x}\frac{EI}{x^2})y} \qquad G = \frac{P(\frac{x}{y}+1)}{P(\frac{x}{y}+1)-\frac{C_x^{\,2}-S_x^{\,2}}{C_x}\frac{EI}{x^2}}$$

TABLE 3.5 COEFFICIENTS OF COLUMN SLOPE-DEFLECTION EQUATIONS

HINGE CONFIGURATION	C_{U1}	C_{U2}	C_{U3}	C_{U4}
1	$\frac{SEI}{h}$	$\frac{CEI}{h}$	$\frac{(C+S)\ EI}{h^2}$	0
2	0	$\frac{C^2-S^2}{C}\frac{EI}{h}$	$\frac{C^2-S^2}{C}\frac{EI}{h}$	$\frac{S}{C}\ M_{LUP}$
3	0	0	0	M_{ULP}
4	$Ay\ \frac{C_x^2-S_x^2}{C_x}\frac{EI}{x}$	$[1-A_x]\ \frac{C_y^2-S_y^2}{C_y}\frac{EI}{y}$	$Ay[\frac{C_x^2-S_x^2}{C_x}\frac{EI}{x^2}-P]$	$[\frac{S_y}{C_y}-Ax(\frac{S_y}{C_y}+1)]M_{DUP}+Ay(\frac{S_x}{C_x}+1)M_{DLP}$
5	0	0	0	M_{ULP}
6	0	$D\ \frac{C_y^2-S_y^2}{C_y}\frac{EI}{y}$	$[D-1]\ \frac{y}{x}\ P$	$[1-D][\frac{y}{x}(M_{DLP}+M_{LUP})-M_{DUP}]+D\ \frac{S_y}{C_y}\ M_{DUP}$
7	0	0	0	M_{ULP}
8	0	0	0	M_{ULP}

$$A = \frac{\frac{C_y^2-S_y^2}{C_y}\frac{EI}{y^2}}{(\frac{C_y^2-S_y^2}{C_y}\frac{EI}{y^2}-P)x+(\frac{C_x^2-S_x^2}{C_x}\frac{EI}{x^2}-P)y}$$

$$D = \frac{P(1+\frac{y}{x})}{P(1+\frac{y}{x})-\frac{C_y^2-S_y^2}{C_y}\frac{EI}{y^2}}$$

TABLE 3.6 COEFFICIENTS OF COLUMN SLOPE-DEFLECTION EQUATIONS

HINGE CONFIGURATION	$\theta_{LUP}, \theta_{ULP}, \theta_{DP}, \Delta_D$
1	
2	$\theta_{LUP} = \frac{h}{CEI} M_{LUP} - \theta_L - \frac{S}{C}\theta_u - \frac{C+S}{C}\frac{\Delta_L-\Delta_u}{h}$
3	$\theta_{ULP} = \frac{h}{CEI} M_{ULP} - \frac{S}{C}\theta_L - \theta_u - \frac{C+S}{C}\frac{\Delta_L-\Delta_u}{h}$
4	$\theta_{DP} = \frac{x}{C_x EI} M_{DLP} - \frac{y}{C_y EI} M_{DUP} - \frac{S_x}{C_x}\theta_L + \frac{S_y}{C_y}\theta_u - \frac{C_x+S_x}{C_x}\frac{\Delta_L-\Delta_D}{x} + \frac{C_y+S_y}{C_y}\frac{\Delta_D-\Delta_u}{y}$ $\Delta_D = \frac{1}{(P-\frac{C_y^2-S_y^2}{C_y}\frac{EI}{y^2})x + (P-\frac{C_x^2-S_x^2}{C_x}\frac{EI}{x^2})y}\{x[M_{DUP} + \frac{S_y}{C_y}M_{DUP} + \frac{C_y^2-S_y^2}{C_y}\frac{EI}{y}\theta_u + (P-\frac{C_y^2-S_y^2}{C_y}\frac{EI}{y^2})\Delta_u] - y[M_{DLP} + \frac{S_x}{C_x}M_{DLP} + \frac{C_x^2-S_x^2}{C_x}\frac{EI}{x}\theta_L - (P-\frac{C_x^2-S_x^2}{C_x}\frac{EI}{x^2})\Delta_L]\}$
5	$\theta_{LUP} = \frac{h}{(C^2-S^2)EI}[C\,M_{LUP} - S\,M_{ULP}] - \theta_L - \frac{\Delta_L-\Delta_u}{h}$ $\theta_{ULP} = \frac{h}{(C^2-S^2)EI}[C\,M_{ULP} - S\,M_{LUP}] - \theta_u - \frac{\Delta_L-\Delta_u}{h}$

TABLE 3.7 INELASTIC HINGE ROTATIONS

HINGE CONFIGURATION	$\theta_{LUP}, \theta_{ULP}, \theta_{DP}, \Delta_D$
6	$$\theta_{LUP} = \frac{x}{(C_x^2 - S_x^2)EI}[C_x M_{LUP} - S_x M_{DLP}] - \theta_L - \frac{\Delta_L - \Delta_D}{x}$$ $$\theta_{DP} = \frac{x}{(C_x^2 - S_x^2)EI}[C_x M_{ULP} - S_x M_{LUP}] - \frac{y}{C_y EI} M_{DUP} + \frac{S_y}{C_y}\theta_u - \frac{\Delta_L - \Delta_D}{x} + \frac{C_y + S_y}{C_y}\frac{\Delta_D - \Delta_u}{y}$$ $$\delta_D = \frac{1}{(P(1+\frac{y}{x}) - \frac{C_y^2 + S_y^2}{C_y}\frac{EI}{y^2}}\{(1+\frac{S_y}{C_y}) M_{DUP} - \frac{y}{x}[M_{DLP} + M_{LUP}] + \frac{C_y^2 - S_y^2}{C_y}\frac{EI}{y}\theta_u + P\frac{y}{x}\Delta_L + [P - \frac{C_y^2 - S_y^2}{C_y}\frac{EI}{y^2}]\Delta_u\}$$
7	$$\theta_{ULP} = \frac{y}{(C_y^2 - S_y^2)EI}[C_y M_{ULP} - S_y M_{DUP}] - \theta_u - \frac{\Delta_D - \Delta_u}{y}$$ $$\theta_{DP} = \frac{x}{C_x EI} M_{ULP} - \frac{y}{(C_y^2 - S_y^2)EI}[C_y M_{DUP} - S_y M_{ULP}] - \frac{S_x}{C_x}\theta_L + (\frac{C_x + S_x}{C_x}\frac{1}{x} + \frac{1}{y})\Delta_D - \frac{C_x + S_x}{C_x}\frac{1}{x}\Delta_L - \frac{1}{y}\Delta_u$$ $$\Delta_D = \frac{1}{P(\frac{y}{x} + 1) - \frac{C_y^2 - S_y^2}{C_x}\frac{EI}{x^2}}\{\frac{x}{y}[M_{ULP} + M_{DUP}] - (1 + \frac{S_x}{C_x}) M_{DLP} - \frac{C_x^2 - S_x^2}{C_x}\frac{EI}{x}\theta_L + \frac{x}{y}P\Delta_u + [P - \frac{C_x^2 - S_x^2}{C_x}\frac{EI}{x^2}]\Delta_L\}$$

... continued

HINGE CONFIGURATION	$\theta_{LUP}, \theta_{ULP}, \theta_{DP}, \Delta_D$
8	$\theta_{LUP} = \frac{y}{(C_y^2 - S_y^2)EI}[C_y\ M_{LUP} - S_y\ M_{DLP}] - \theta_L - \frac{\Delta_L - \Delta_D}{y}$
	$\theta_{ULP} = \frac{x}{(C_x^2 - S_x^2)EI}[C_x\ M_{ULP} - S_x\ M_{DUP}] - \theta_u - \frac{\Delta_D - \Delta_u}{x}$
	$\theta_{DP} = \frac{y}{(C_y^2 - S_y^2)EI}[C_y\ M_{DLP} - S_y\ M_{LUP}] - \frac{x}{(C_x^2 - S_x^2)EI}[C_x\ M_{DUP} - S_x\ M_{ULP}] - \frac{\Delta_L - \Delta_D}{y} + \frac{\Delta_u - \Delta_D}{x}$
	$\Delta_D = \frac{1}{Ph}\{x[M_{ULP} + M_{DUP}] - y[M_{DUP} + M_{LUP}]\} + \frac{x}{h}\Delta_u + \frac{y}{h}\Delta_L$

YIELD CONFIGURATION	C_{D1}	C_{D2}	C_{D3}	C_{D4}	C_{D5}
1	$\frac{AE}{L} \sin\alpha\cos\alpha$	$\frac{AE}{L} \cos^2\alpha$	$\frac{AE}{L} \sin^2\alpha$	0	0
2	0	0	0	$\sin\alpha \; P_y$	$\cos\alpha \; P_y$
3	0	0	0	$-\sin\alpha \; P_{CR}$	$-\cos\alpha \; P_{CR}$

TABLE 3.8 COEFFICIENTS OF DIAGONAL BRACING AXIAL LOAD-DEFORMATION EQUATIONS

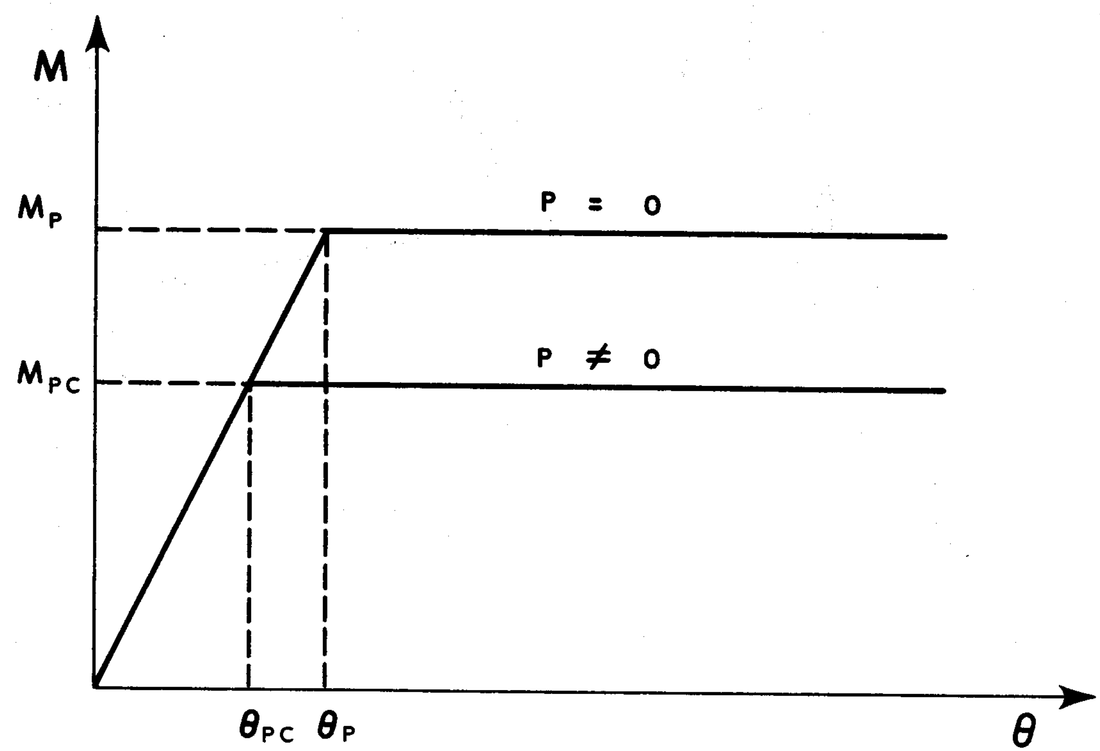
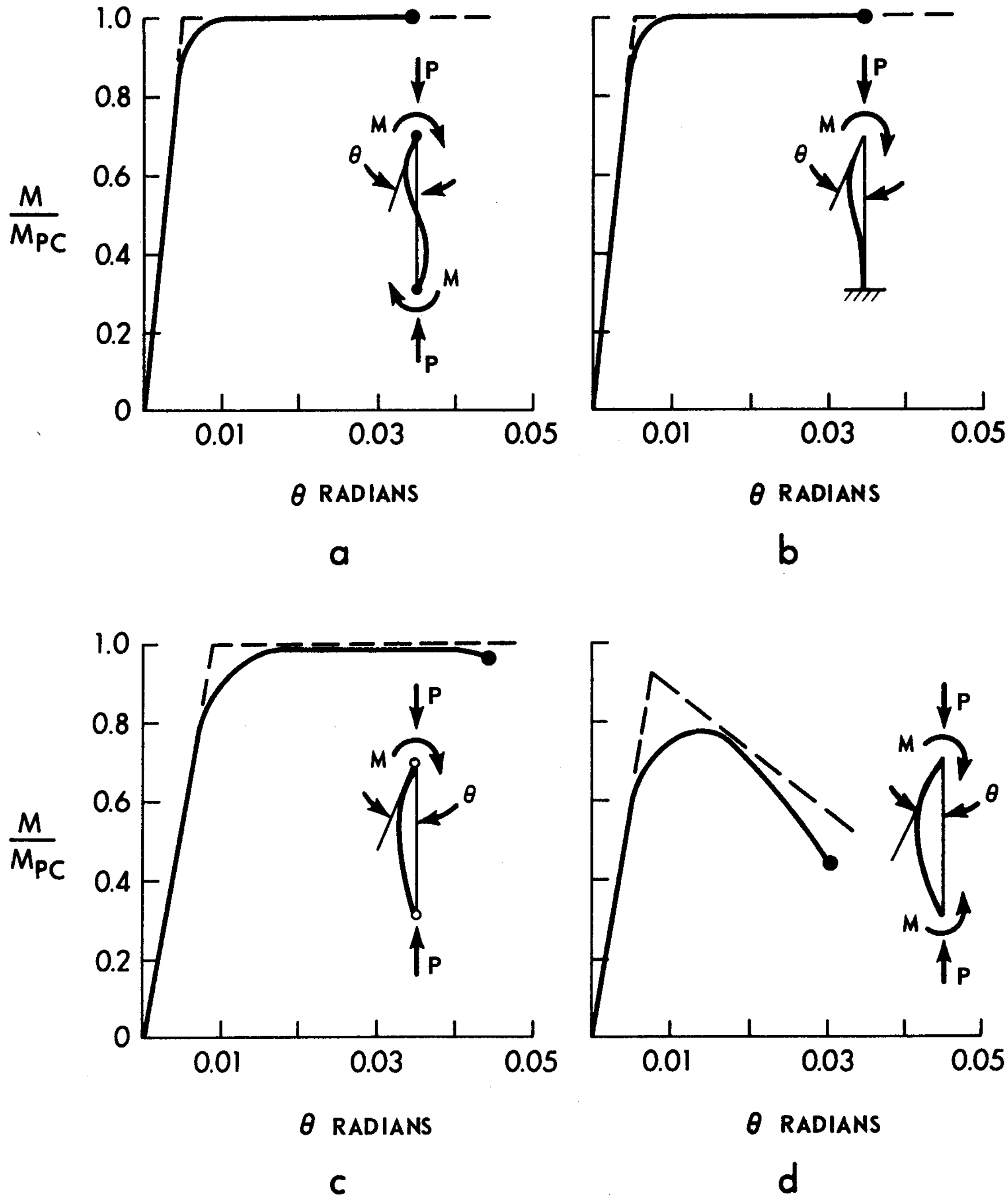


FIGURE 3.1

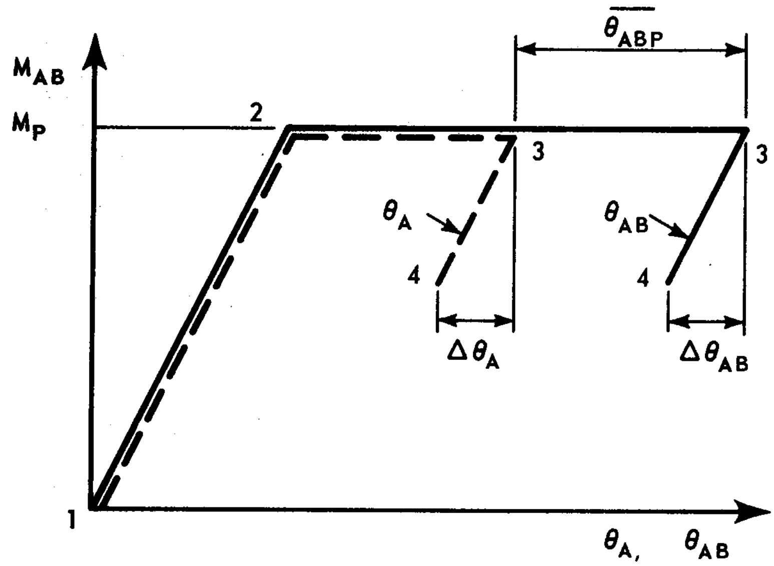
ASSUMED MOMENT-ROTATION RELATIONSHIPS



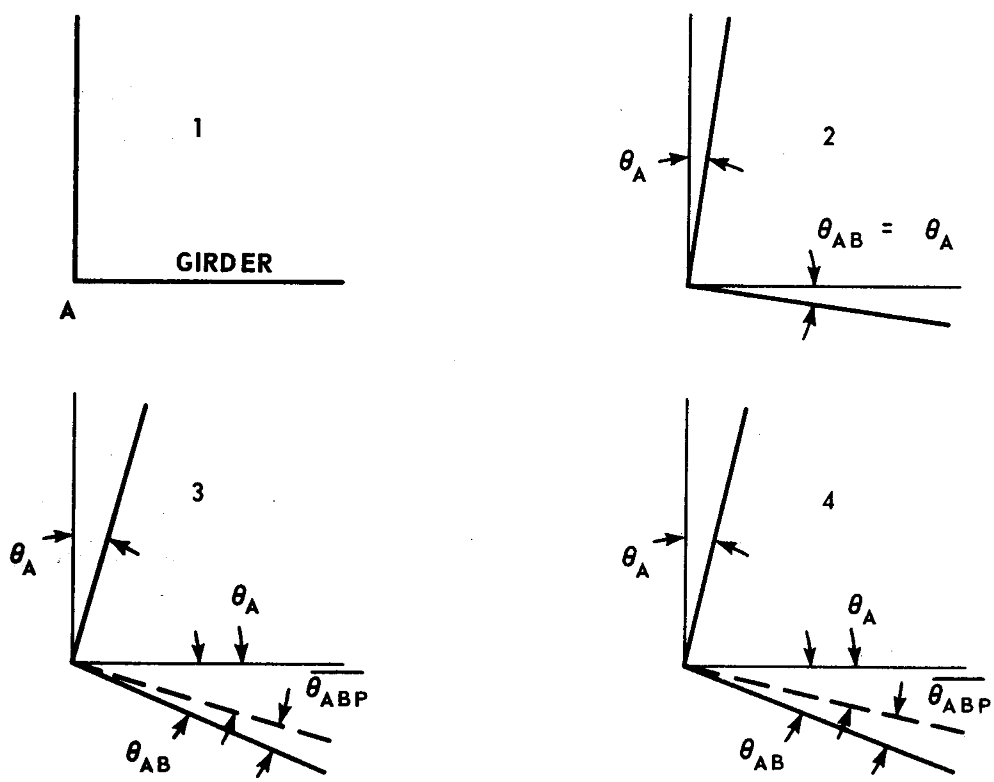
$\frac{h}{r_x} = 30$ $\frac{P}{P_y} = 0.6$

FIGURE 3.2

MOMENT-ROTATION CURVES OF A BEAM-COLUMN



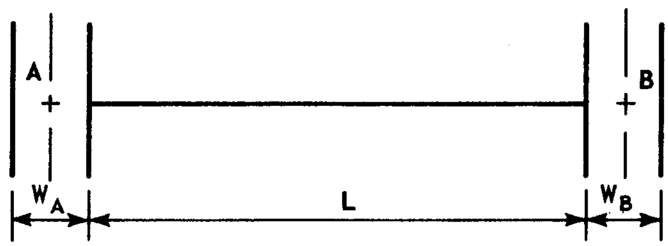
a GIRDER MOMENT - ROTATION RELATIONSHIP



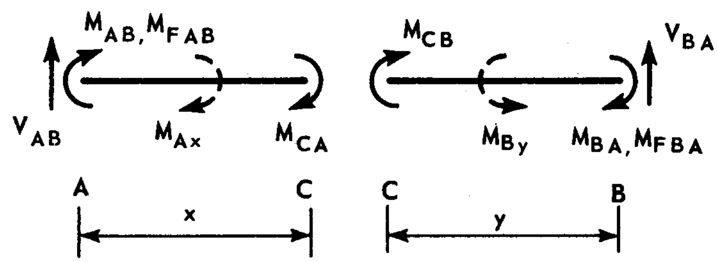
b LOADING SEQUENCE

FIGURE 3.3

GIRDER RESPONSE

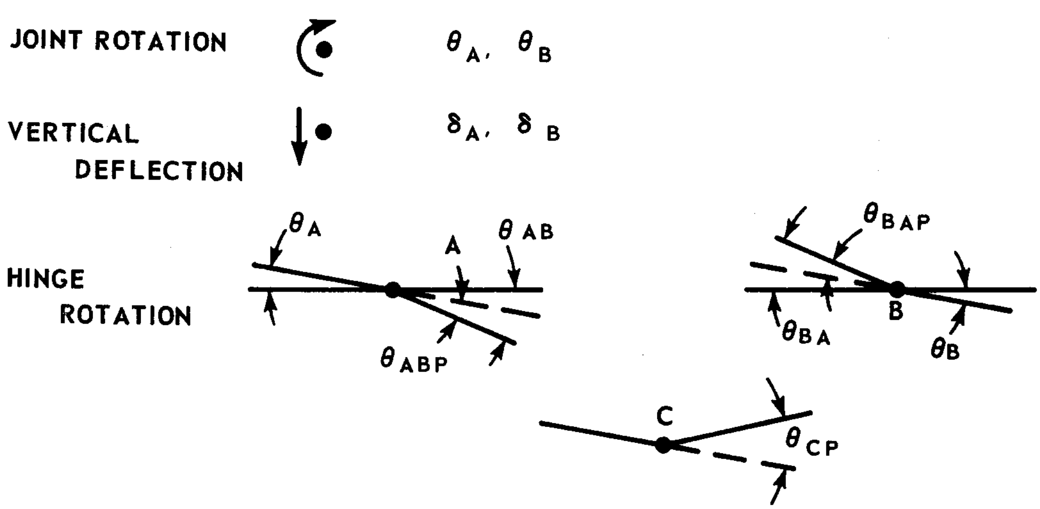


POSITIVE SIGN CONVENTION
(MOMENT AND SHEAR ACTING ON GIRDER)



M_{Ax} MOMENT OF APPLIED LOAD ON SPAN AC ABOUT A

M_{By} MOMENT OF APPLIED LOAD ON SPAN CB ABOUT B



NOTE: DOUBLE SUBSCRIPTS REFER TO MEMBERS, SINGLE SUBSCRIPTS TO JOINTS.

FIGURE 3.4

GIRDER NOTATION AND SIGN CONVENTION

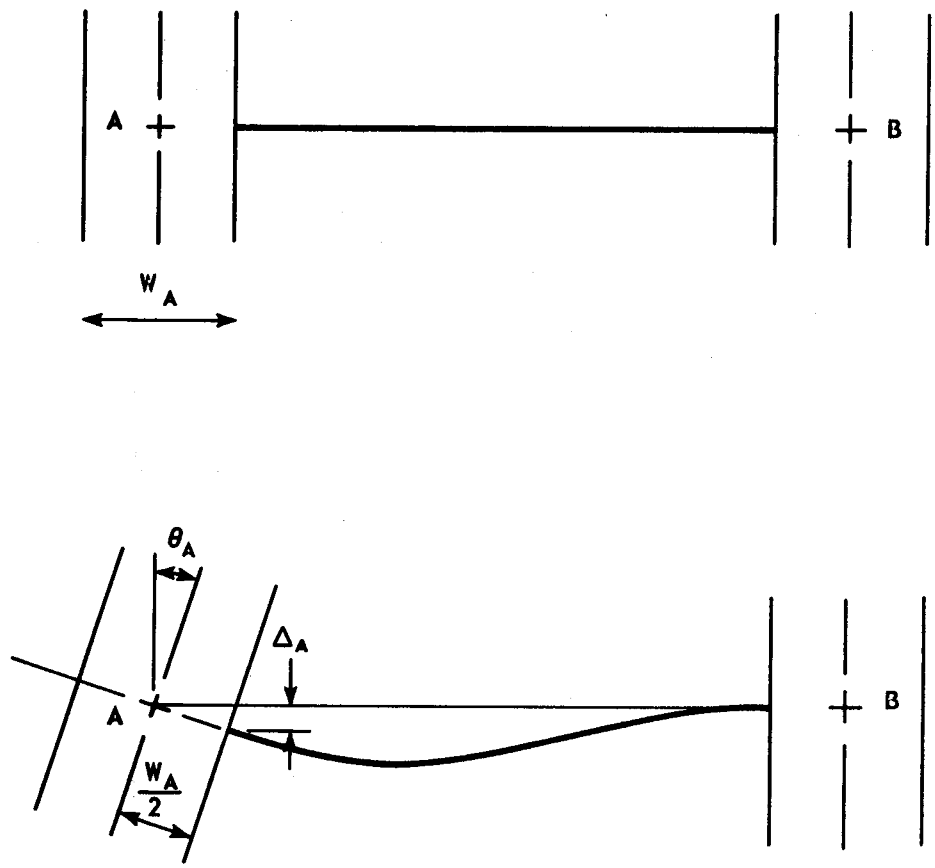


FIGURE 3.5

EFFECT OF COLUMN ROTATION

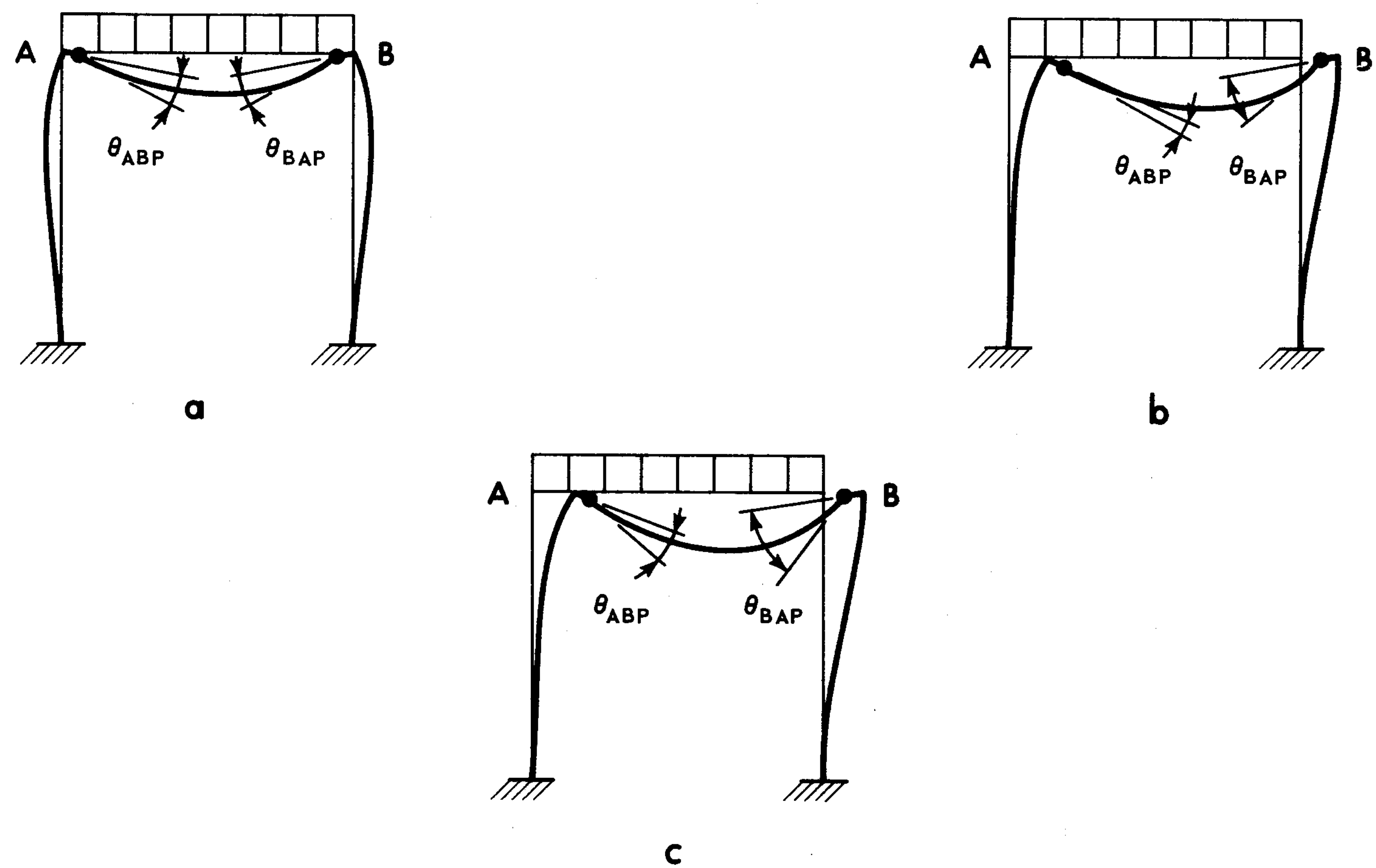


FIGURE 3.6

HINGE REVERSAL

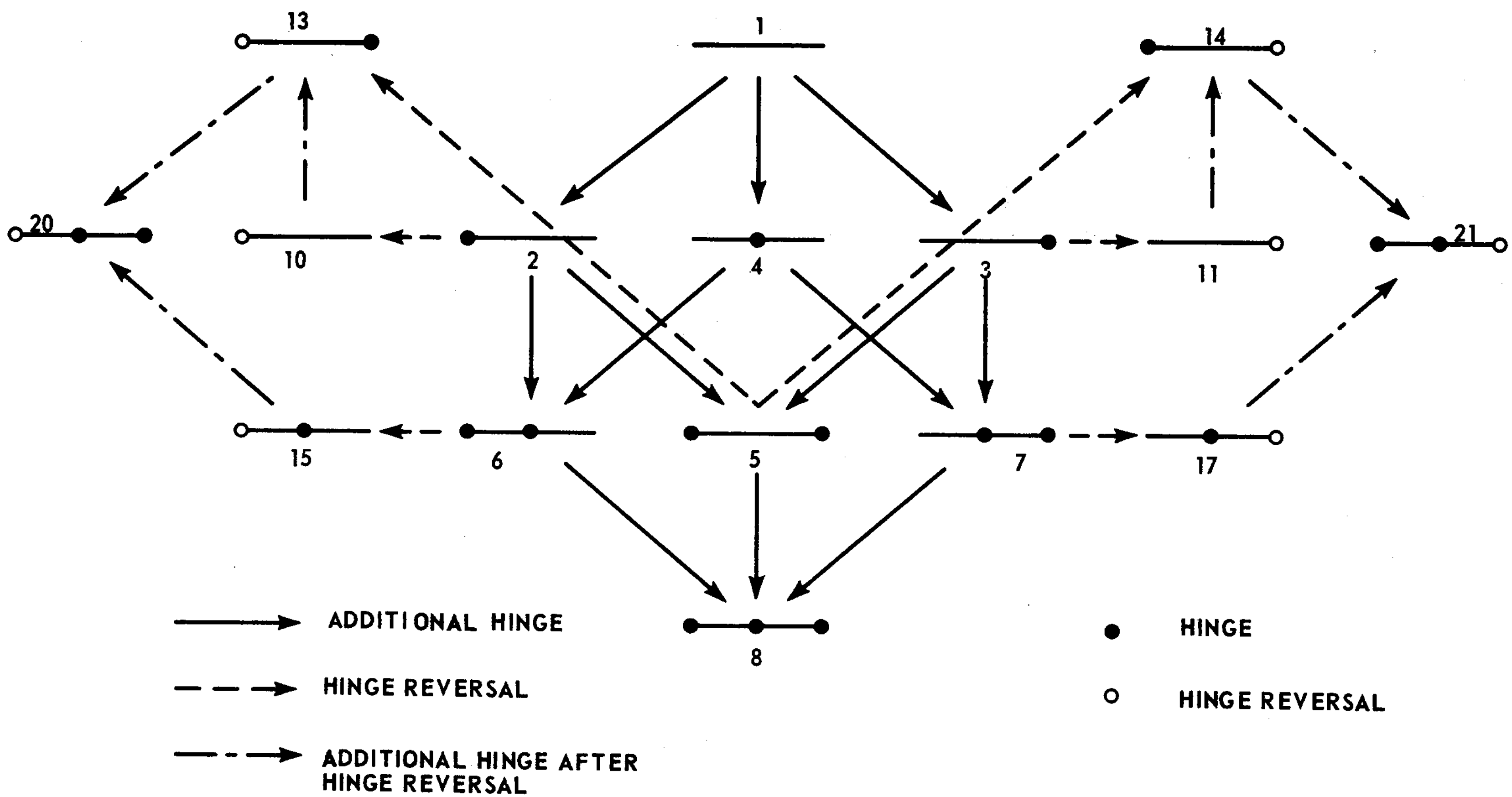


FIGURE 3.7

GIRDER HINGING CONFIGURATIONS

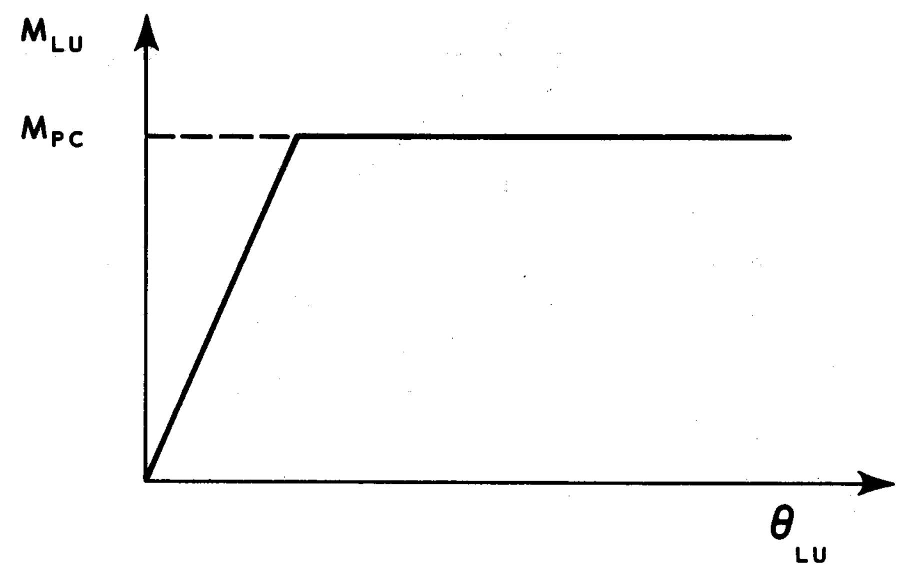


FIGURE 3.8

COLUMN RESPONSE

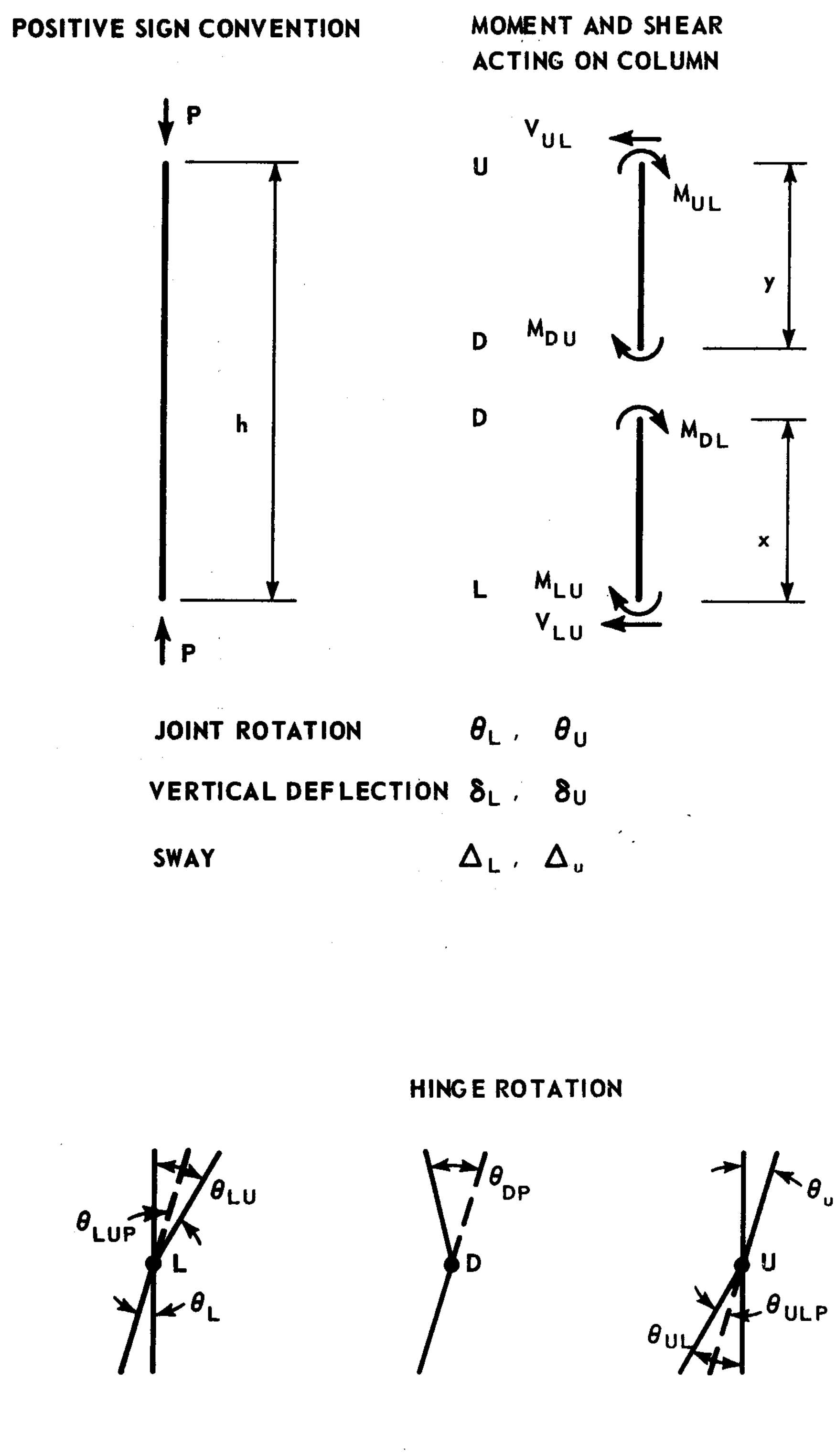


FIGURE 3.9

COLUMN NOTATION AND SIGN CONVENTION

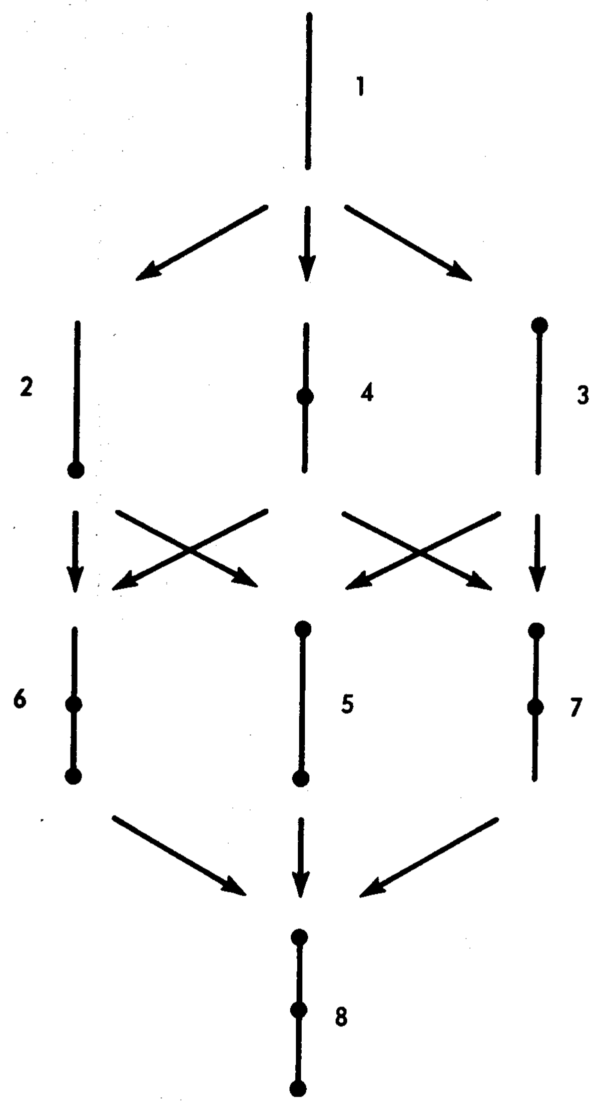


FIGURE 3.10

COLUMN HINGE CONFIGURATIONS

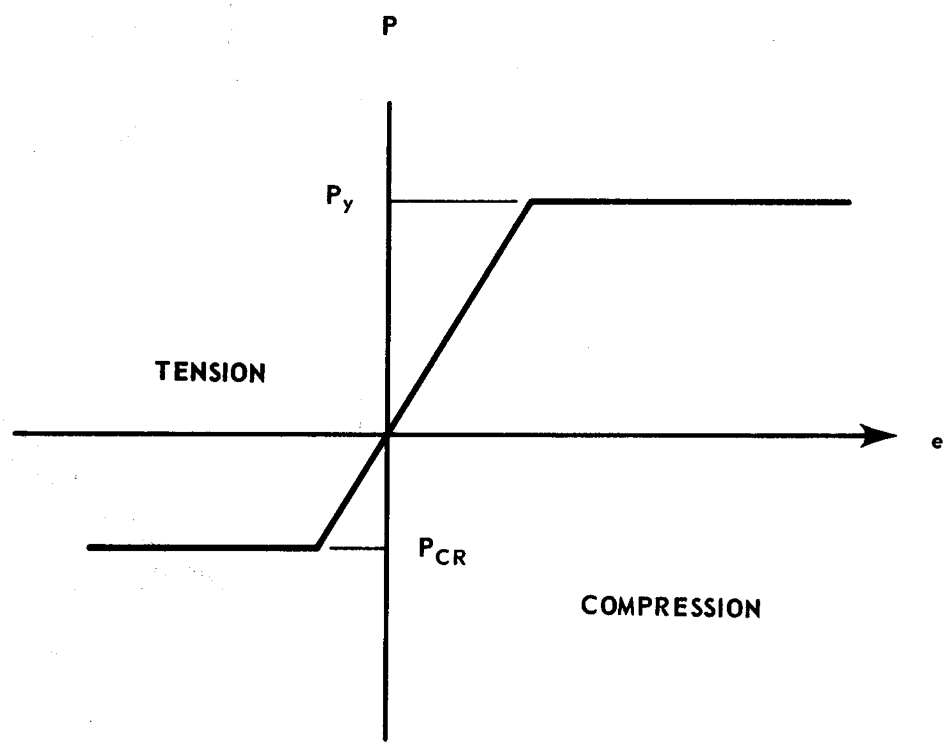


FIGURE 3.11

DIAGONAL BRACING AXIAL LOAD - DEFORMATION RESPONSE

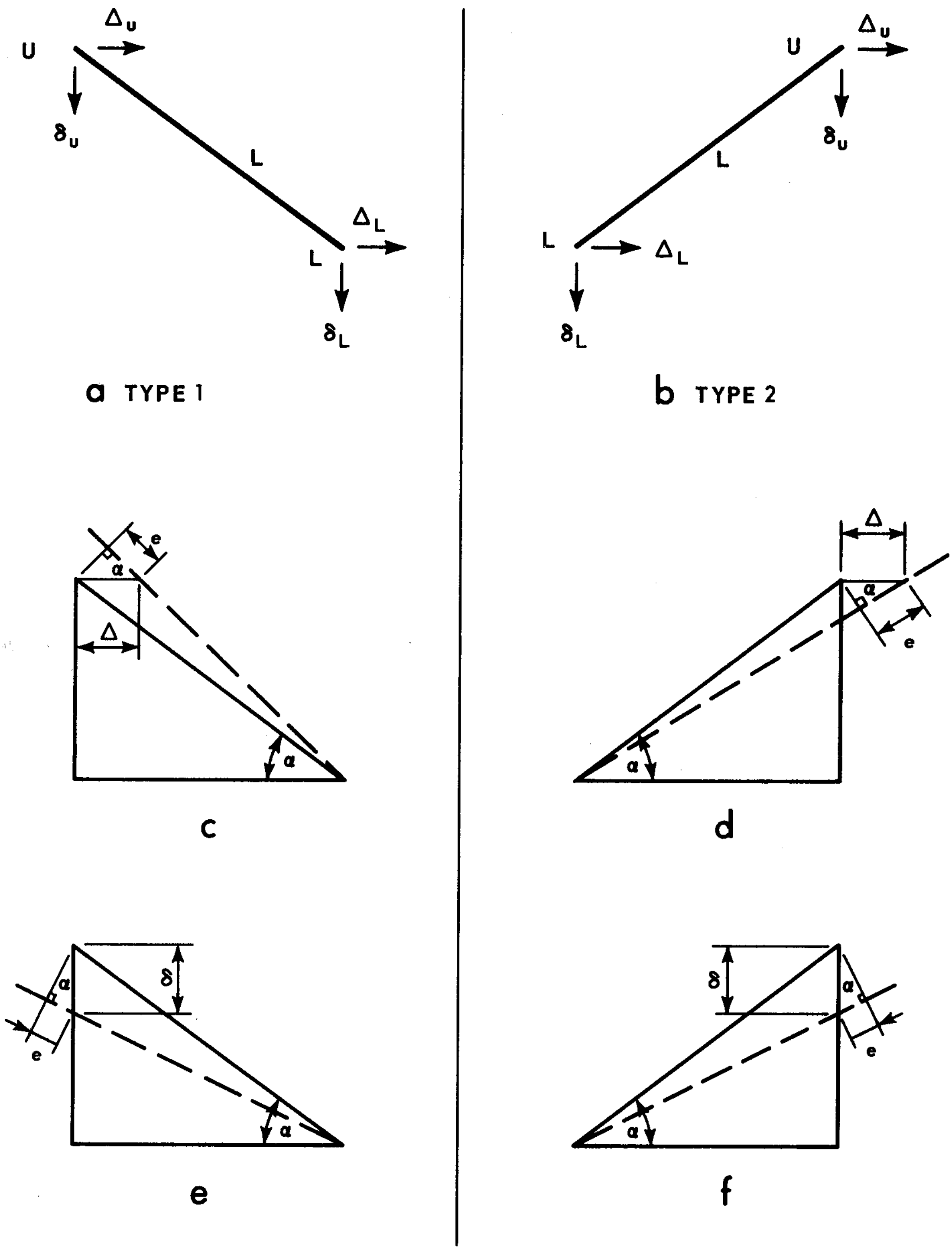


FIGURE 3.12

DIAGONAL BRACING NOTATION AND SIGN CONVENTION

CHAPTER IV

FRAME ANALYSIS

4.1 Introduction

In the previous chapter the behavior of individual frame members, as characterized by their M-θ relationships, was presented. This chapter deals with the analysis of the complete structural frame. In the following sections, the frame arrangement and loading pattern, the formulation of the equilibrium equations, and the method of solution are discussed. The development of a computer program to trace the response of the framework is outlined.

4.2 Frame Arrangement and Loading Pattern

4.2.1 Frame Arrangement

The type of frame considered in the analysis is shown in FIGURE 4.1. The structure is a regular, rectangular, multi-story, multi-bay planar frame. The columns and the girders are rigidly connected to one another, and the bottom story columns are assumed to be connected to the foundations by springs providing a specified degree of rotational stiffness. Pin-ended diagonal bracing members can be included in any bay or story. The effect of the width of all vertical members is considered in the analysis. Plastic hinges in the girders are assumed to occur at the face of the columns. However, since the finite depth of the girders is not considered, the column hinges occur at the intersection of the centrelines of the columns and girders. The size of the frame which can be considered is limited only by computer capacity.

The column lines are numbered from 1 to M. Floor levels are numbered from 1 to N; level one is taken at the foundation. The stories are numbered with reference to the level below. A structural joint, (column-girder intersection), is designated as $JOINT_{m,n}$, where m is the column line, and n the floor level. The structural members, and the joint rotations and displacements, in the vicinity of $JOINT_{m,n}$ are shown in FIGURE 4.2. A girder is designated as $G_{m,n}$, where m and n refer to the joint at the girder left end. A column is designated as $C_{m,n}$, corresponding to the joint at its lower end. A diagonal bracing member is designated as $B_{m,n,i}$, where m and n refer to the joint at its lower end, and i refers to the type of brace. A type 1 brace slopes upward to the left, a type 2 brace upward to the right. Other bracing systems, such as the K-bracing system illustrated in FIGURE 4.3, can be considered by including a "fictitious" column stack with zero area and moment of inertia.

4.2.2 Loading Pattern

In order to consider various loading sequence possibilities, five different loading systems are assumed to act on the frame, as shown in FIGURE 4.4. The live load, $w_{LL}(m,n)$, and the dead load, $w_{DL}(m,n)$, acting on the girders are considered separately. Both are assumed to be distributed uniformly over the length of the span, however, the live load may be incremented while the dead load remains fixed in magnitude throughout the analysis. Concentrated vertical loads, V(m,n), are assumed to act at each joint. These loads may include allowances for the weights of walls, partitions and columns. In addition, the vertical joint loads may be used to adjust column axial loads if different live load reduction factors are used for girders and columns (15). In order to simulate the behavior of the lower stories of a multi-story frame, column top vertical loads, P(m), are included. Lateral loads, F(m), which simulate the action of wind or earthquake forces, are assumed to be concentrated at each floor level. All load systems, (except the dead load on the girders), may be incremented independently, however, all loads within a particular system must increase proportionately. The use of the various loading systems permits consideration of the behavior of structures subjected to vertical loads only, to constant vertical loads and increasing horizontal loads, or to vertical and horizontal loads which are in proportion to one another.

4.3 Equilibrium Equations

4.3.1 Introduction

The frame analysis technique is based on a displacement formulation in which moment and force equilibrium equations are satisfied at each joint and for each story of the structure. The equilibrium equations are expressed in terms of the joint rotations and displacements, using the member force-displacement equations developed in CHAPTER III. The resulting system of equations is solved for the joint rotations and displacements, which are then used to determine the member forces and moments throughout the structure.

In the following sections, the moment equilibrium equations, the vertical force equilibrium equations, and the story shear equilibrium equations, are developed.

4.3.2 Equilibrium of Moments

The moments and forces at the ends of the members framing into $JOINT_{m,n}$ are shown in FIGURE 4.5. The forces act in the positive directions, as assumed in the derivation of the member force-displacement equations. For moment equilibrium, the sum of the moments at any given joint must be zero; for $JOINT_{m,n}$ this may be expressed as:

$$- M_{AB}(m,n) - M_{BA}(m-1,n) - M_{LU}(m,n) - M_{UL}(m,n-1) + V_{AB}(m,n)\,\frac{wc_{m,n-1}}{2} - V_{BA}(m-1,n)\,\frac{wc_{m,n-1}}{2} = 0 \qquad (4.1)$$

where $M_{AB}(m,n)$, $M_{BA}(m-1,n)$, $M_{LU}(m,n)$ and $M_{UL}(m,n-1)$ are the moments at the ends of the girders and columns framing into $JOINT_{m,n}$, and are given by EQUATIONS 3.10, 3.11, 3.18 and 3.19 respectively. The terms $V_{AB}(m,n)\,\frac{wc_{m,n-1}}{2}$ and $V_{BA}(m-1,n)\,\frac{wc_{m,n-1}}{2}$ are the moments produced by the shears at the ends of the girders acting at the face of the column. Referring to FIGURE 4.6, $V_{AB}(m,n)$, the shear at the left end of $GIRDER_{m,n}$, is given by:

$$V_{AB}(m,n) = V_{wA}(m,n) - \frac{M_{AB}(m,n)+M_{BA}(m,n)}{L_{m,n}} \qquad (4.2)$$

where $V_{wA}(m,n)$ is the shear at the left end of $GIRDER_{m,n}$ due to the applied load on the girder, (equal to $\frac{wL}{2}$ for a uniformly distributed load w). Similarly,

$$V_{BA}(m-1,n) = V_{wB}(m-1,n) + \frac{M_{AB}(m-1,n)+M_{BA}(m-1,n)}{L_{m-1,n}} \qquad (4.3)$$

where $V_{wB}(m-1,n)$ is the shear at the right end of $\text{GIRDER}_{m-1,n}$ due to the applied load on the girder. Similar terms do not arise due to the column shears, because they are assumed to act at the centreline of the joint (the effect of the girder depth is ignored).

Substituting EQUATIONS 4.2 and 4.3 into EQUATION 4.1 and rearranging:

$$- M_{AB}(m,n)\left[1 + \frac{wc_{m,n-1}}{2L_{m,n}}\right] - M_{BA}(m,n)\left[\frac{wc_{m,n-1}}{2L_{m,n}}\right]$$

$$- M_{AB}(m-1,n)\left[\frac{wc_{m,n-1}}{2L_{m-1,n}}\right] - M_{BA}(m-1,n)\left[1 + \frac{wc_{m,n-1}}{2L_{m-1,n}}\right]$$

$$- M_{LU}(m,n) - M_{UL}(m,n-1) + \frac{wc_{m,n-1}}{2}[V_{wA}(m,n)-V_{wB}(m-1,n)] = 0 \qquad (4.4)$$

Substituting EQUATIONS 3.10, 3.11, 3.18 and 3.19 into EQUATION 4.4, and defining

$$a = 1 + \frac{wc_{m,n-1}}{2L_{m,n}}$$

$$b = \frac{wc_{m,n-1}}{2L_{m,n}}$$

$$c = \frac{wc_{m,n-1}}{2L_{m-1,n}}, \text{ and}$$

$$d = 1 + \frac{wc_{m,n-1}}{2L_{m-1,n}} ,$$

the moment equilibrium equation may be expressed as:

$$-[a\ C_{A1}(m,n)+b\ C_{B1}(m,n)+c\ C_{A2}(m-1,n)+d\ C_{B2}(m-1,n)$$

$$+ C_{L1}(m,n)+C_{U2}(m,n-1)]\theta_{m,n}-[a\ C_{A2}(m,n)$$

$$+ b\ C_{B2}(m,n)]\theta_{m+1,n}-[c\ C_{A1}(m-1,n)+d\ C_{B1}(m-1,n)]$$

$$\theta_{m-1,n} - [C_{L2}(m,n)]\theta_{m,n+1}-[C_{U1}(m,n-1)]\theta_{m,n-1}$$

$$- [a\ C_{A3}(m,n)+b\ C_{B3}(m,n)-c\ C_{A3}(m-1,n)-d\ C_{B3}(m-1,n)]\ \delta_{m,n}$$

$$- [-a\ C_{A3}(m,n)-b\ C_{B3}(m,n)]\ \delta_{m+1,n}$$

$$- [c\ C_{A3}(m-1,n)+d\ C_{B3}(m-1,n)]\ \delta_{m-1,n}-[C_{L3}(m,n)-C_{U3}(m,n-1)]$$

$$\Delta_n-[-\ C_{L3}(m,n)]\Delta_{n+1} - [C_{U3}(m,n-1)]\Delta_{n-1}$$

$$= [a\ C_{A4}(m,n) +b\ C_{B4}(m,n) + c\ C_{A4}(m-1,n) + d\ C_{B4}(m-1,n)$$

$$+ a\ C_{A5}(m,n) + b\ C_{B5}(m,n) + c\ C_{A5}(m-1,n) + d\ C_{B5}(m-1,n)$$

$$+ C_{L4}(m,n) + C_{U4}(m,n-1) + \frac{wc_{m,n-1}}{2}(V_{wB}(m-1,n)-V_{wA}(m,n))] \quad (4.5)$$

EQUATION 4.5 is the general equation for the moment equilibrium of $JOINT_{m,n}$. It may be specialized for an exterior joint by dropping terms corresponding to a girder or column which does not exist. The moment equilibrium at the base of a column cannot be handled by EQUATION 4.5. Consider the column-support joint illustrated in FIGURE 4.7. The column, $C_{m,1}$, is assumed pinned at its base, with a rotational restraint, K_m, offered by the support. The moment equilibrium equation at the support is:

$$- M_{Lu} - K_m \theta_{m,1} = 0 \qquad (4.6)$$

where $\theta_{m,1}$ is the rotation of the column base.
Substituting EQUATION 3.18 into EQUATION 4.6:

$$- [C_{L1}(m,1)+K_m]\theta_{m,1} - [C_{L2}(m,1)]\theta_{m,2}$$

$$+ [C_{L3}(m,1)]\Delta_2 - [C_{L4}(m,1)] = 0 \qquad (4.7)$$

EQUATIONS 4.5 and 4.7 express the moment equilibrium relationships of the structural frame.

4.3.3 Equilibrium of Vertical Forces

Vertical force equilibrium equations are written at each joint in the frame. Again referring to FIGURE 4.5, the sum of the vertical forces at $JOINT_{m,n}$ must be zero:

$$V_{m,n} + P(m,n) - P(m,n-1) + V_{BA}(m-1,n) + V_{AB}(m,n)$$

$$+ F_y(m+1,n-1,1) + F_y(m-1,n-1,2) - F_y(m,n,1) - F_y(m,n,2) = 0$$

$$(4.8)$$

where $V_{m,n}$ is the applied vertical joint load,

P(m,n) and P(m,n-1) are the axial forces in the columns above and below $JOINT_{m,n}$, respectively,

$V_{BA}(m-1,n)$ and $V_{AB}(m,n)$ are the shears at the ends of the framing girders, and,

$F_y(m+1,n-1,1)$, $F_y(m-1,n-1,2)$, $F_y(m,n,1)$ and $F_y(m,n,2)$ are the vertical components of the axial forces in the diagonal bracing members framing into the joint.

Substituting EQUATIONS 4.2 and 4.3 into EQUATION 4.8 results in:

$$V_{m,n} + P(m,n) - P(m,n-1) + V_{wB}(m-1,n)$$

$$+ \frac{M_{AB}(m-1,n) + M_{BA}(m-1,n)}{L_{m-1,n}} + V_{wA}(m,n) - \frac{M_{AB}(m,n) + M_{BA}(m,n)}{L_{m,n}}$$

$$+ F_y(m+1,n-1,1) + F_y(m-1,n-1,2) - F_y(m,n,1) - F_y(m,n,2) = 0 \tag{4.9}$$

and substituting the member force-displacement relationships, (EQUATIONS 3.10, 3.11, 3.28 and 3.30), into EQUATION 4.9 produces the vertical force equilibrium equation in terms of the joint displacements:

$$\left[\frac{C_{A2}(m-1,n) + C_{B2}(m-1,n)}{L_{m-1,n}} - \frac{C_{A1}(m,n) + C_{B1}(m,n)}{L_{m-1,n}}\right] \theta_{m,n}$$

$$+ \left[\frac{C_{A1}(m-1,n) + C_{B1}(m-1,n)}{L_{m-1,n}}\right]\theta_{m-1,n} - \left[\frac{C_{A2}(m,n) + C_{B2}(m,n)}{L_{m,n}}\right]\theta_{m+1,n}$$

$$- [\frac{C_{A3}(m-1,n) + C_{B3}(m-1,n)}{L_{m-1,n}} + \frac{C_{A3}(m,n) + C_{B3}(m,n)}{L_{m-1,n}}$$

$$+ \frac{A_C(m,n)\ E_C(m,n)}{h_n} + \frac{A_C(m,n-1)\ E_C(m,n-1)}{h_{n-1}} + C_{D3}(m+1,n-1,1)$$

$$+ C_{D3}(m-1,n-1,2) + C_{D3}(m,n,1) + C_{D3}(m,n,2)]\ \delta_{m,n}$$

$$+ [\frac{C_{A3}(m-1,n) + C_{B3}(m-1,n)}{L_{m-1,n}}]\delta_{m-1,n} + [\frac{C_{A3}(m,n) + C_{B3}(m,n)}{L_{m,n}}]\delta_{m+1,n}$$

$$+ [\frac{A_C(m,n)\ E_C(m,n)}{h_n}]\delta_{m,n+1} + [\frac{A_C(m,n-1)\ E_C(m,n-1)}{h_{n-1}}]\delta_{m,n-1}$$

$$+ [C_{D3}(m+1,n-1,1)]\delta_{m+1,n-1} + [C_{D3}(m-1,n-1,2)]\delta_{m-1,n-1}$$

$$+ [C_{D3}(m,n,1)]\delta_{m-1,n+1} + [C_{D3}(m,n,2)]\delta_{m+1,n+1}$$

$$+ [C_{D1}(m-1,n-1,2) - C_{D1}(m+1,n-1,1) + C_{D1}(m,n,2) - C_{D1}(m,n,1)]\Delta_n$$

$$+ [C_{D1}(m+1,n-1,1) - C_{D1}(m-1,n-1,2)]\Delta_{n-1}$$

$$+ [C_{D1}(m,n,1) - C_{D1}(m,n,2)]\Delta_{n+1}$$

$$= 1\ [\frac{C_{A4}(m-1,n) + C_{B4}(m-1,n) + C_{A5}(m-1,n) + C_{B5}(m-1,n)}{L_{m-1,n}}$$

$$- \frac{C_{A4}(m,n) + C_{B4}(m,n) + C_{A5}(m,n) + C_{B5}(m,n)}{L_{m,n}}$$

$$+ C_{D4}(m+1,n-1,1) + C_{D4}(m-1,n-1,2) - C_{D4}(m,n,1) - C_{D4}(m,n,2)$$

$$+ V_{WB}(m-1,n) + V_{wA}(m,n) + V_{m,n}] \quad (4.10)$$

In the top story the external applied axial load, P(m), is included within the brackets on the right hand side of the equation.

4.3.4 Equilibrium of Horizontal Forces

Since axial shortening of the girders is not considered, all columns in a given story will undergo the same sway displacement. It is therefore necessary to write only one horizontal force equilibrium equation per story. In order to include the P-Δ moments, the story shear equations must be formulated on the deformed structure. A section of a multi-story structure is shown in FIGURE 4.8. Floor level n is given a horizontal displacement relative to floor levels n-1 and n+1. The forces acting on the girder at level n are shown in their positive direction. F_x represents the horizontal component of the axial forces in the diagonal bracing members. V_{Lu} and V_{uL} are the horizontal forces at the lower and upper ends of the columns. F_n is the applied story horizontal load. The axial forces in the columns are also shown. The sum of the horizontal forces acting on the girder must be zero:

$$F_n + \sum_{m=1}^{M} V_{LU}(m,n) + \sum_{m=1}^{M} V_{UL}(m,n-1) + \sum_{m=1}^{M-1} F_x(m,n,2)$$

$$- \sum_{m=2}^{M} F_x(m,n,1) + \sum_{m=2}^{M} F_x(m,n-1) - \sum_{m=1}^{M-1} F_x(m,n-1,2) = 0 \quad (4.11)$$

Consider the columns in story n and story n-1, as shown in FIGURES 4.9a and 4.9b, respectively. The moments, shears and axial forces are shown in their assumed positive directions. Taking moments about the upper end of column m,n:

$$V_{LU}(m,n) = \frac{P(m,n)(\Delta_n-\Delta_{n+1})-M_{LU}(m,n)-M_{UL}(m,n)}{h_n} \qquad (4.12)$$

and about the lower end of column m, n-1:

$$V_{UL}(m,n-1) = \frac{P(m,n-1)(\Delta_n-\Delta_{n-1})+M_{UL}(m,n-1)+M_{LU}(m,n-1)}{h_{n-1}} \qquad (4.13)$$

Substituting these values of $V_{LU}(m,n)$ and $V_{UL}(m,n-1)$ into EQUATION 4.11, results in

$$F_n + \frac{1}{h_n} \sum_{m=1}^{M} [P(m,n)\Delta_n-P(m,n)\Delta_{n+1}-M_{LU}(m,n)-M_{UL}(m,n)]$$

$$+ \frac{1}{h_{n-1}} \sum_{m=1}^{M} [P(m,n-1)\Delta_n-P(m,n-1)\Delta_{n-1}+M_{LU}(m,n-1)+M_{UL}(m,n-1)]$$

$$+ \sum_{m=1}^{M-1} F_x(m,n,2) - \sum_{m=2}^{M} F_x(m,n,1) + \sum_{m=2}^{M} F_x(m,n-1,1)$$

$$- \sum_{m=1}^{M-1} F_x(m,n-1,2) = 0 \qquad (4.14)$$

Substitution of EQUATIONS 3.18, 3.19, 3.27 and 3.29, into EQUATION 4.14 produces the story shear equilibrium equation:

$$\sum_{m=1}^{M} \{[\frac{C_{U2}(m,n-1)+C_{L2}(m,n-1)}{h_{n-1}} - \frac{C_{L1}(m,n)+C_{U1}(m,n)}{h_n}]\theta_{m,n}\}$$

$$- \sum_{m=1}^{M} \{\frac{C_{L2}(m,n)+C_{U2}(m,n)}{h_n} \theta_{m,n+1}\}$$

$$+ \sum_{m=1}^{M} \{\frac{C_{U1}(m,n-1)+C_{L1}(m,n-1)}{h_{n-1}} \theta_{m,n-1}\}$$

$$+ \sum_{m=1}^{M-1} \{C_{D1}(m,n,2)\delta_{m,n}\} - \sum_{m=2}^{M} \{C_{D1}(m,n,1)\delta_{m,n}\}$$

$$- \sum_{m=1}^{M-1} \{C_{D1}(m,n,2)\delta_{m+1,n+1}\} + \sum_{m=2}^{M} \{C_{D1}(m,n,1)\delta_{m-1,n+1}\}$$

$$+ \sum_{m=2}^{M} \{C_{D1}(m,n-1,1)\delta_{m,n-1}\} - \sum_{m=1}^{M-1} \{C_{D1}(m,n-1,2)\delta_{m,n-1}\}$$

$$- \sum_{m=2}^{M} \{C_{D1}(m,n-1,1)\ \delta_{m-1,n}\} + \sum_{m=1}^{M-1} \{C_{D1}(m,n-1,2)\delta_{m+1,n}\}$$

$$+ \{ \sum_{m=1}^{M} [\frac{P(m,n)-C_{L3}(m,n)-C_{U3}(m,n)}{h_n} + \frac{P(m,n-1)-C_{L3}(m,n-1)-C_{U3}(m,n-1)}{h_{n-1}}]$$

$$- \sum_{m=1}^{M-1} [C_{D2}(m,n,2) + C_{D2}(m,n-1,2)] - \sum_{m=2}^{M} [C_{D2}(m,n,1)$$

$$+ C_{D2}(m,n-1,1)]\} \Delta_n$$

$$+ \{- \sum_{m=1}^{M} [\frac{P(m,n)-C_{L3}(m,n)-C_{U3}(m,n)}{h_n}] + \sum_{m=1}^{M-1} C_{D2}(m,n,2)$$

$$+ \sum_{m=2}^{M} C_{D2}(m,n,1)\}\Delta_{n+1}$$

$$+ \{\sum_{m=1}^{M} [\frac{-P(m,n+1)+C_{L3}(m,n-1)+C_{U3}(m,n-1)}{h_{n-1}}] + \sum_{m=1}^{M-1} C_{D2}(m,n-1,2)$$

$$+ \sum_{m=2}^{M} C_{D2}(m,n-1,1)\}\Delta_{n-1}$$

$$= - \{F_n + \sum_{m=1}^{M} [\frac{C_{L4}(m,n)+C_{U4}(m,n)}{h_n} + \frac{C_{U4}(m,n-1)+C_{L4}(m,n-1)}{h_{n-1}}]$$

$$+ \sum_{m=1}^{M-1} [C_{D5}(m,n,2)-C_{D5}(m,n-1,2)]$$

$$+ \sum_{m=2}^{M}[C_{D5}(m,n-1,1)-C_{D5}(m,n,1)]\} \quad (4.15)$$

4.4 Solution of the Equilibrium Equations

4.4.1 Introduction

The moment and vertical force equilibrium equations at each joint, and the shear equilibrium equations for each story may be expressed in the form:

$$[A] \{x\} = \{b\} \quad (4.16)$$

where $\{x\}$ is the vector of the unknown joint displacements,

$\{b\}$ is the load vector, corresponding to the vector $\{x\}$, and,

[A] is the coefficient matrix relating {x} and {b}. The total number of unknown joint rotations and displacements in the structure is equal to (2M+1)(N-1)+M, where M is the number of column stacks and N is the number of floor levels. Thus the square matrix [A] is of order (2M+1)(N-1)+M. However for large structures many of the terms of [A] are zero.

Each of the equilibrium equations at a given floor level will only include the unknown rotations and displacements at that level and the levels immediately above and below. By numbering the unknowns across each story, the band width in the coefficient matrix is restricted to three times the total number of unknowns per story. Because the vertical load in the columns is assumed to be known for each cycle within a given load increment, and the structure is linearly elastic for this same cycle, the coefficient matrix is symmetric. Thus only those terms on or above the major diagonal must be stored. For a forty-story, three-bay frame, (N=41,M=4), this technique permits a reduction in the required storage for the coefficient matrix from 364 x 364 elements to 364 x 18. The procedure used to number the unknown rotations and displacements is illustrated in FIGURE 4.10 for a five-story, two-bay frame. The only unknowns at the ground level are the column base rotations, which are numbered consecutively from left to right. For each subsequent floor level, the story sway deflection is first numbered, followed by the joint rotation and vertical displacement at each successive column across the story.

4.4.2 Method of Solution

The method adopted for solving EQUATION 4.16 is a modified Gauss Elimination procedure. A direct solution technique was selected so that the computation time could be determined for a particular size of structure. For large matrices the use of an iterative solution, such as Gauss Seidel, may take considerable time to converge to an acceptable result (24). The accuracy of a direct method is more easily influenced by error propagation during solution, and for large systems of equations this aspect must be considered before accepting the answers obtained.

In the direct solution technique used, the terms of the coefficient matrix, [A], below the major diagonal are transformed, row by row, to zero. The vector of unknown displacements is determined by back substitution, using the resulting triangular coefficient matrix and the load vector.

Each time EQUATION 4.16 is solved, the determinant of the coefficient matrix is calculated. The magnitude of the determinant decreases as the structure enters the inelastic range and plastic hinges form. This reflects a decrease in the structural stiffness. As the slope of the load-displacement curve approaches the horizontal, the coefficient matrix approaches a singular condition. In the analysis, the structure is linearly elastic between the formation of successive hinges (except for the effect of changes in the column axial forces on the stiffness of the structure). Thus the determinant changes by a discrete amount each time a new hinge forms. In this procedure the possibility of the determinant actually becoming zero is remote. When the maximum load-carrying capacity of the structure is reached the determinant changes sign and the load must be decreased to achieve equilibrium under increasing deformation.

4.5 Computer Program

The method of analyses described in the preceding chapters was programmed in FORTRAN IV for the IBM 360/67 system. In this section the logic of the program, and the function of each of its subroutines, is briefly described. The nomenclature used in the computer program is given in APPENDIX A.1. The flow diagrams for the individual subroutines are presented in APPENDIX A.2. The necessary input data is outlined in APPENDIX A.3, and the program listing is given in APPENDIX A.4. The accuracy in the computer solution is checked in APPENDIX A.5.

The frame analysis proceeds in the following manner:

1. The frame geometry, member properties, design loads and loading sequence information are read into the program.
2. The plastic moment capacities of the girders and the critical axial loads of the bracing members are determined.
3. The member stiffness coefficients, which are independent of load, are calculated.
4. For the particular load increment in question, the loads to be applied to the frame are determined.
5. The terms of the equilibrium equations dependent on the applied load on the girders are calculated.
6. The axial load in the columns is estimated, based on the known axial load from the previous load increment, and any

additional applied vertical load.

7. The plastic moment capacity and stiffness coefficients of the columns are determined.
8. The equilibrium equations are formulated and solved for the unknown rotations and displacements. The determinant of the coefficient matrix is also calculated at this stage. If the determinant has changed sign the program returns to step 4, and the load on the structure is decreased.
9. The column axial loads are calculated. If the axial loads differ by more than 1% from those assumed in step 6, the program returns to step 7 with the new values of axial load.
10. Using the member force-displacement equations, the member moments and forces are determined.
11. The inelastic hinge rotation at each plastic hinge is calculated.
12. The girders and columns are checked for additional plastic hinging and the bracing members for the attainment of their critical axial loads. If any new member hinge or yield configurations are detected, the program returns to step 3, and the structure is re-analyzed for the given loads.
13. The story shear forces are summed to check the accuracy of the solution.
14. The inelastic hinge rotations in the girders are compared with those of the previous load increment. If hinge reversal is detected the program returns to step 3, and the structure is re-analyzed for the given loads.

15. At this stage the program returns to step 4, to analyze the frame for the next increment of load.

The MAIN program controls the frame analysis. Its primary function is to call the subroutines which perform the necessary calculations at different stages of the analysis. Subroutine READ performs steps 1 and 2 of the analysis. Subroutine COEFF calculates the stiffness coefficients of the girders, columns and bracing members which are independent of the loads on the frame (step 3). Steps 4, 5 and 6 are performed by subroutine LOAD, and step 7 by subroutine STAB. Subroutine EE1 formulates the moment, vertical force and horizontal shear equilibrium equations for the frame. Subroutine SOLVE solves the equilibrium equations for the unknown displacement vector, and calculates the value of the determinant of the coefficient matrix. Subroutine SUB1 converts the unknown displacement vector to the joint rotations and displacements. Step 9 of the analysis is also performed in SUB1. The girder and column moments are calculated in subroutine CHECK1, and the members checked for additional plastic hinging. In subroutine CHECK2 the inelastic hinge rotations are determined. The axial forces in the bracing members are determined in subroutine CHECK3, and the yield condition checked. In addition the horizontal shear check is performed. Subroutine HREV performs step 14 in the analysis.

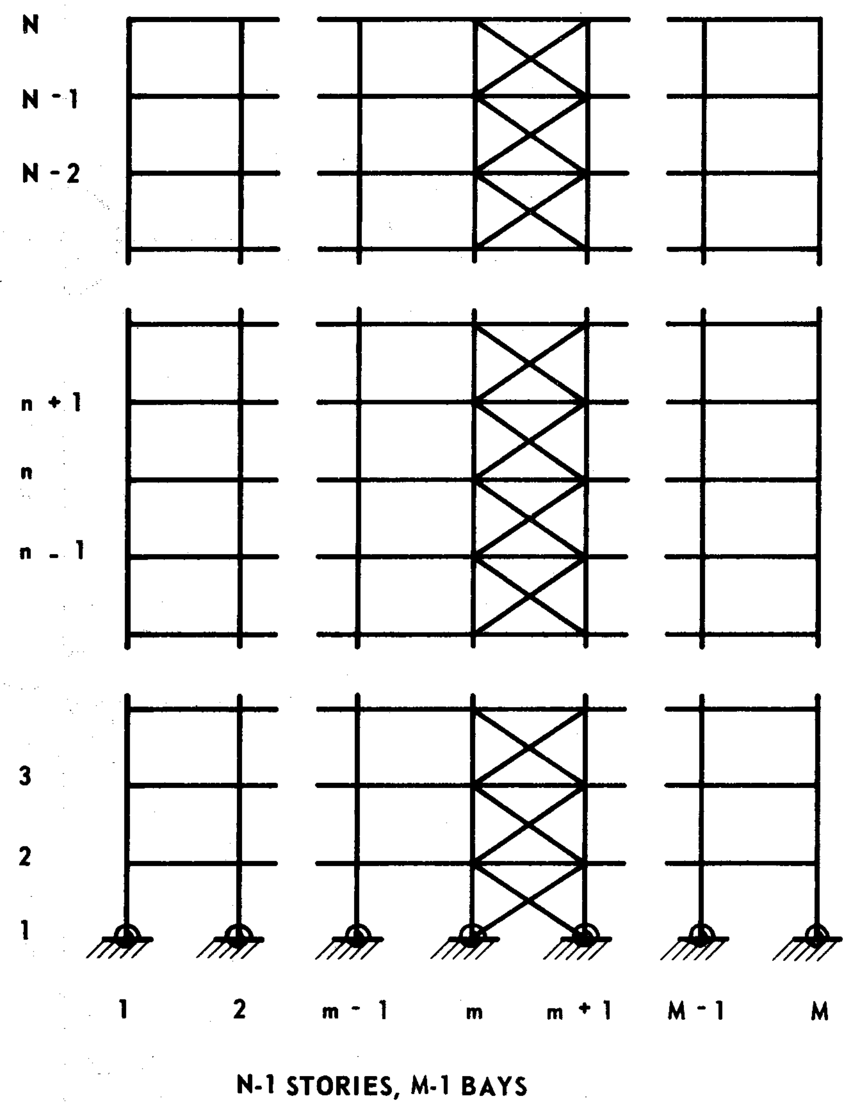


FIGURE 4.1

FRAME ARRANGEMENT

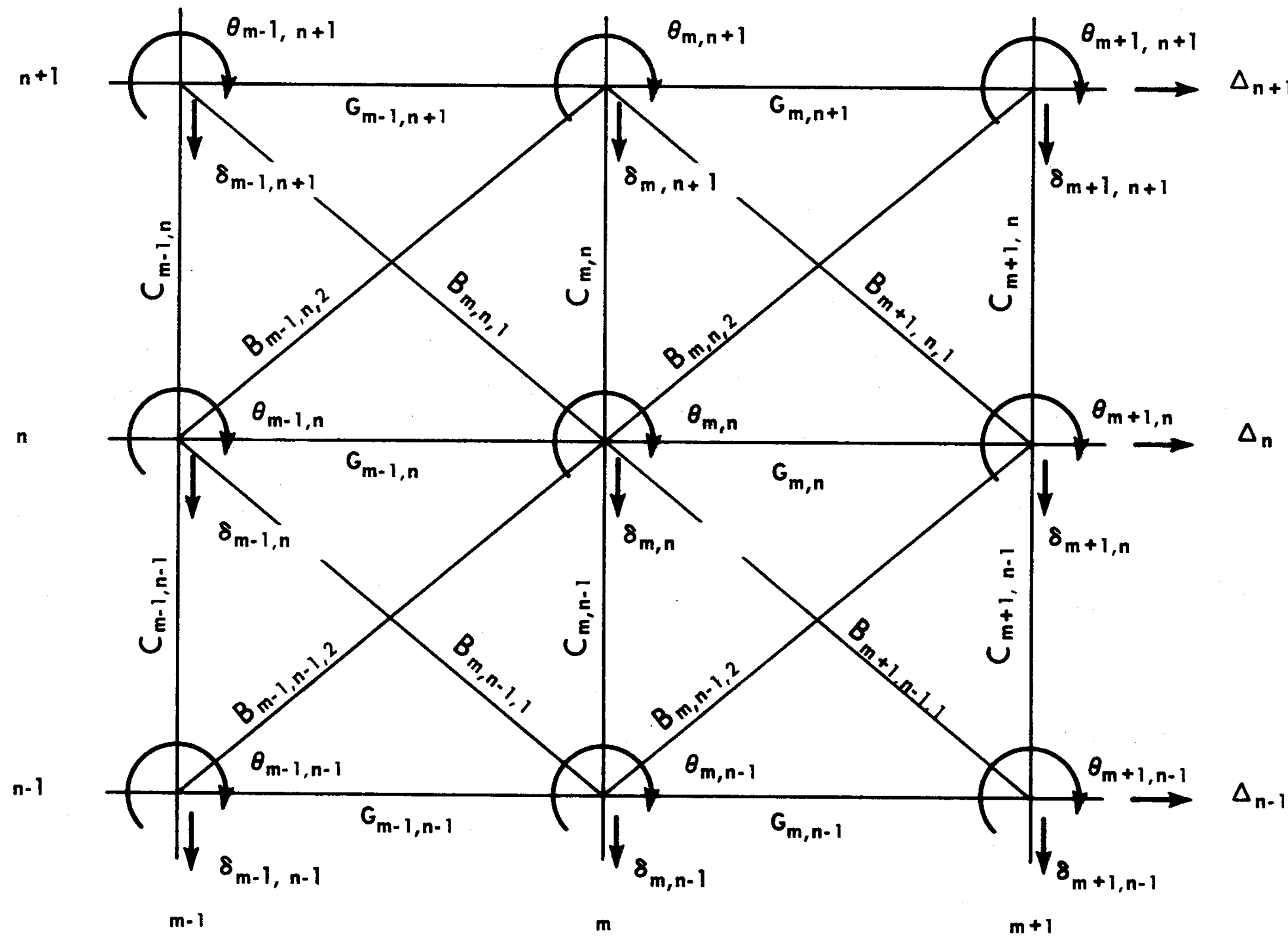


FIGURE 4.2

MEMBER AND DISPLACEMENTS IN THE VICINITY OF JOINT$_{m,n}$

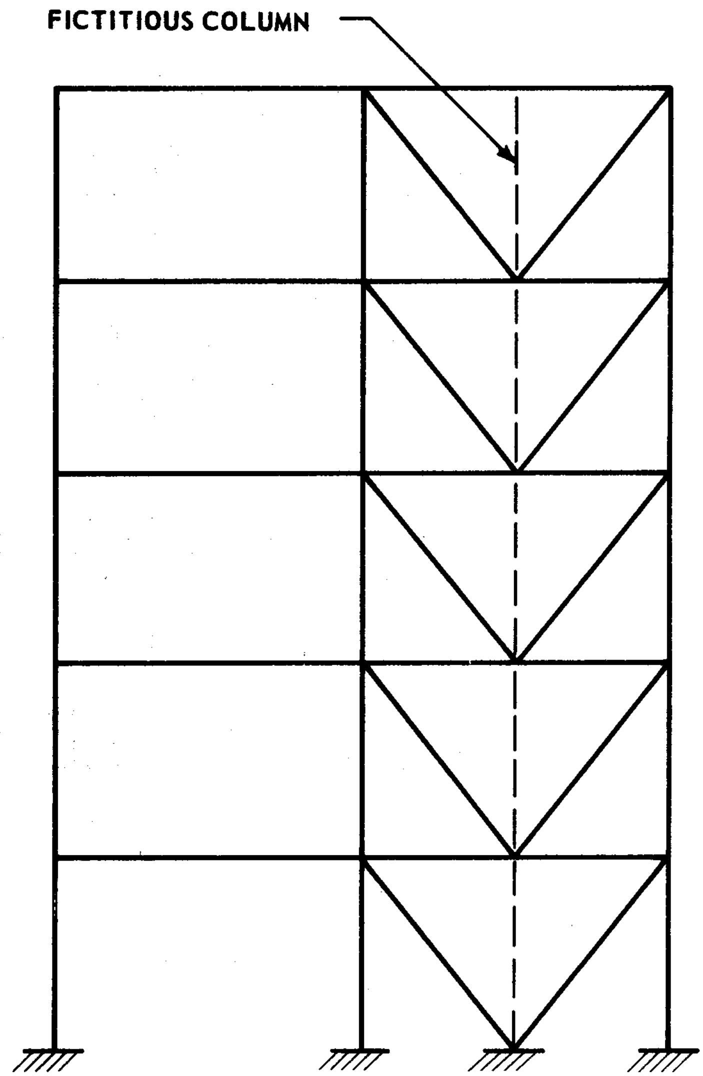


FIGURE 4.3

K-BRACING SYSTEM

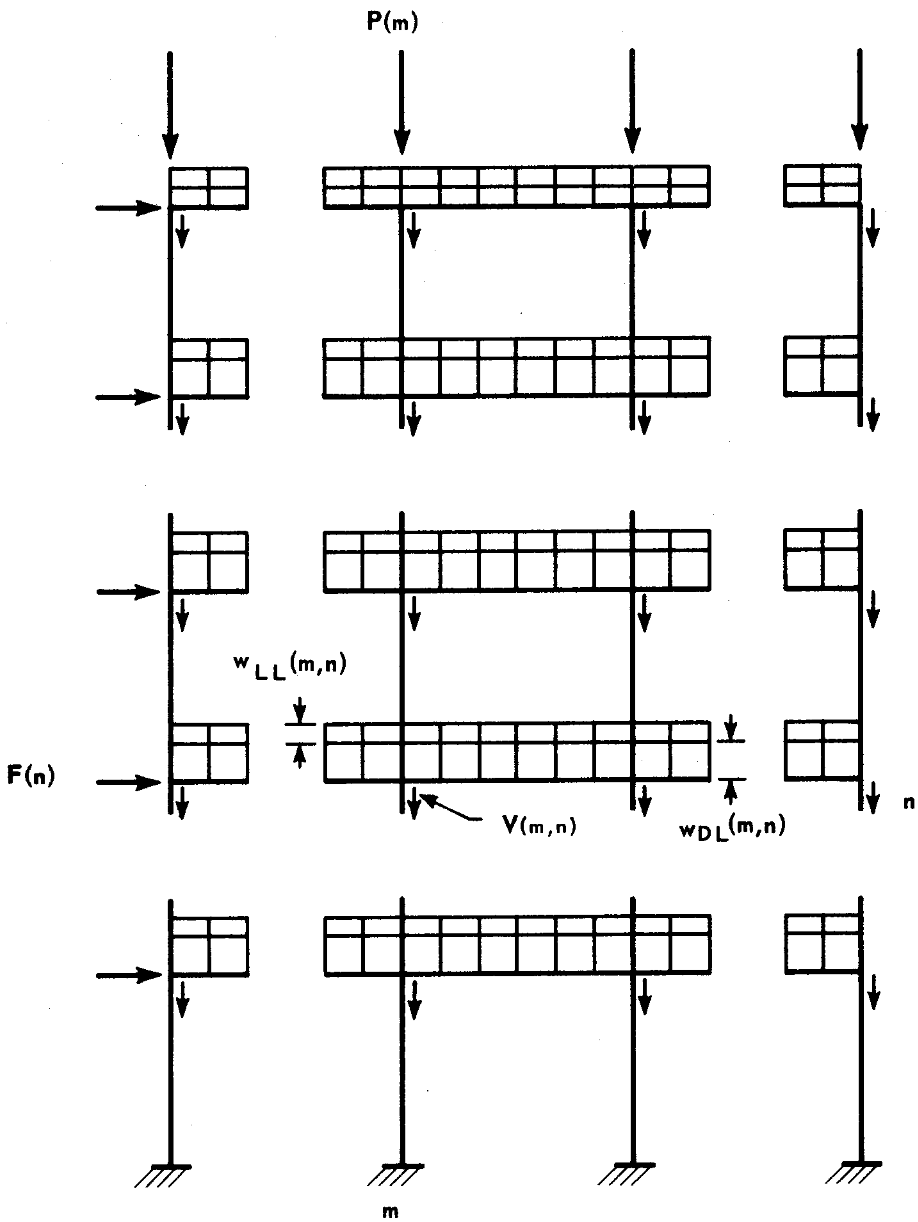


FIGURE 4.4

LOADING PATTERN

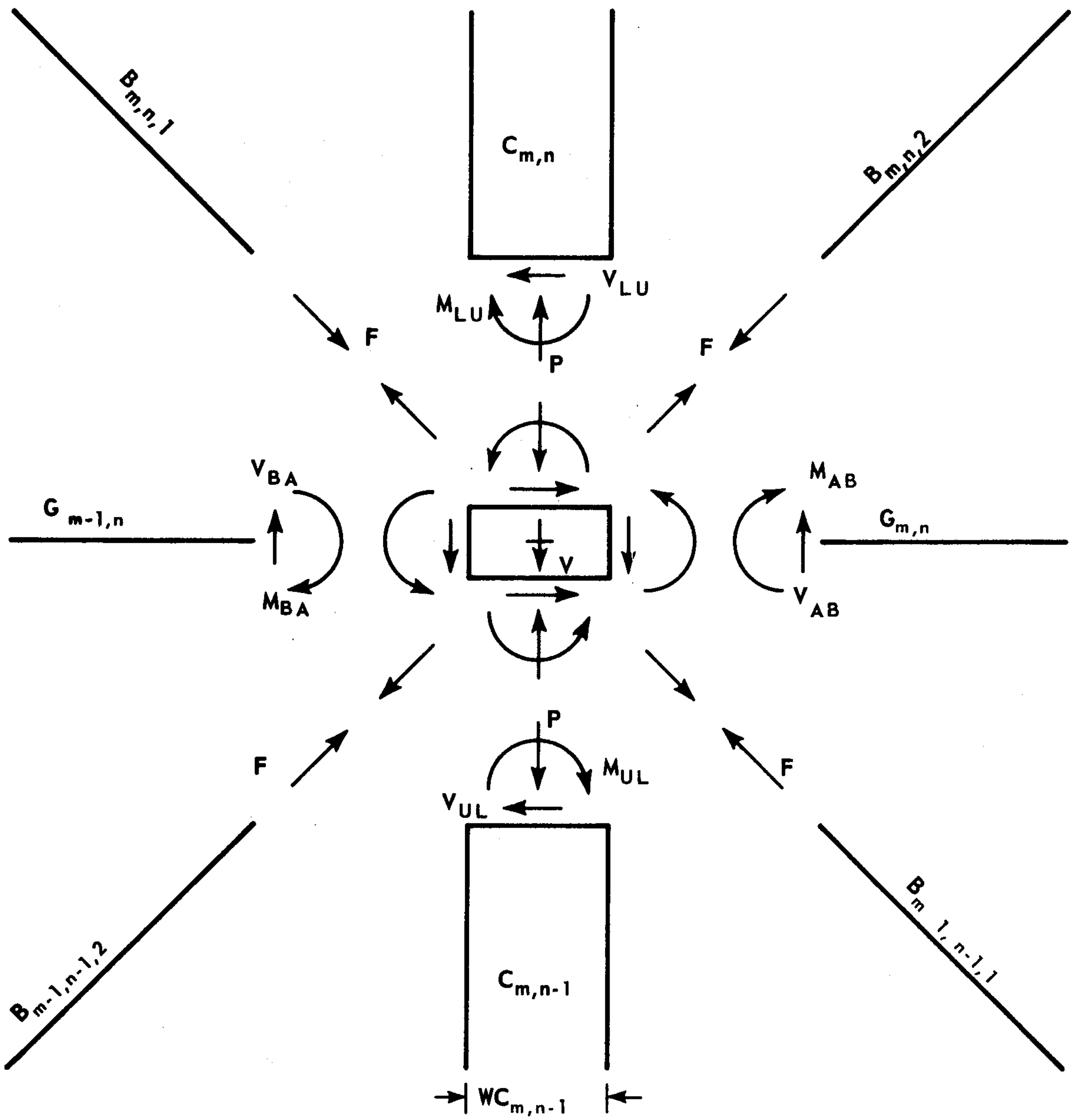


FIGURE 4.5

MOMENTS AND FORCES AT JOINT$_{m,n}$

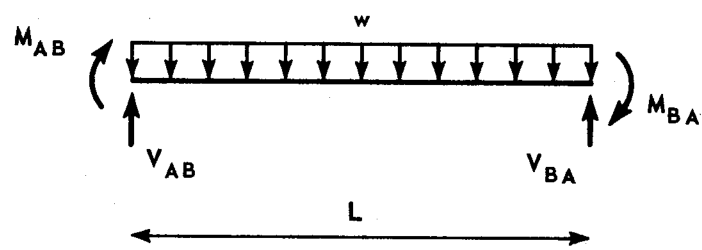


FIGURE 4.6

GIRDER SHEAR

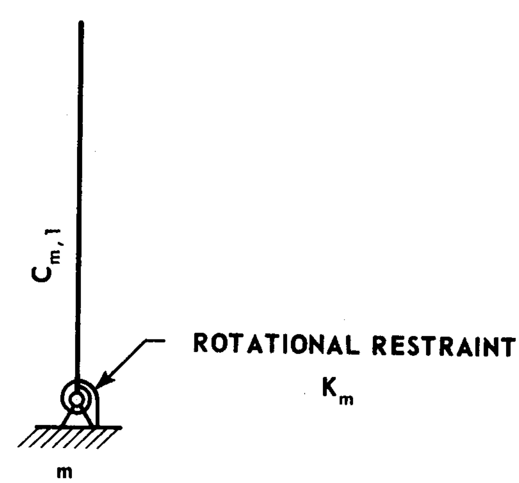
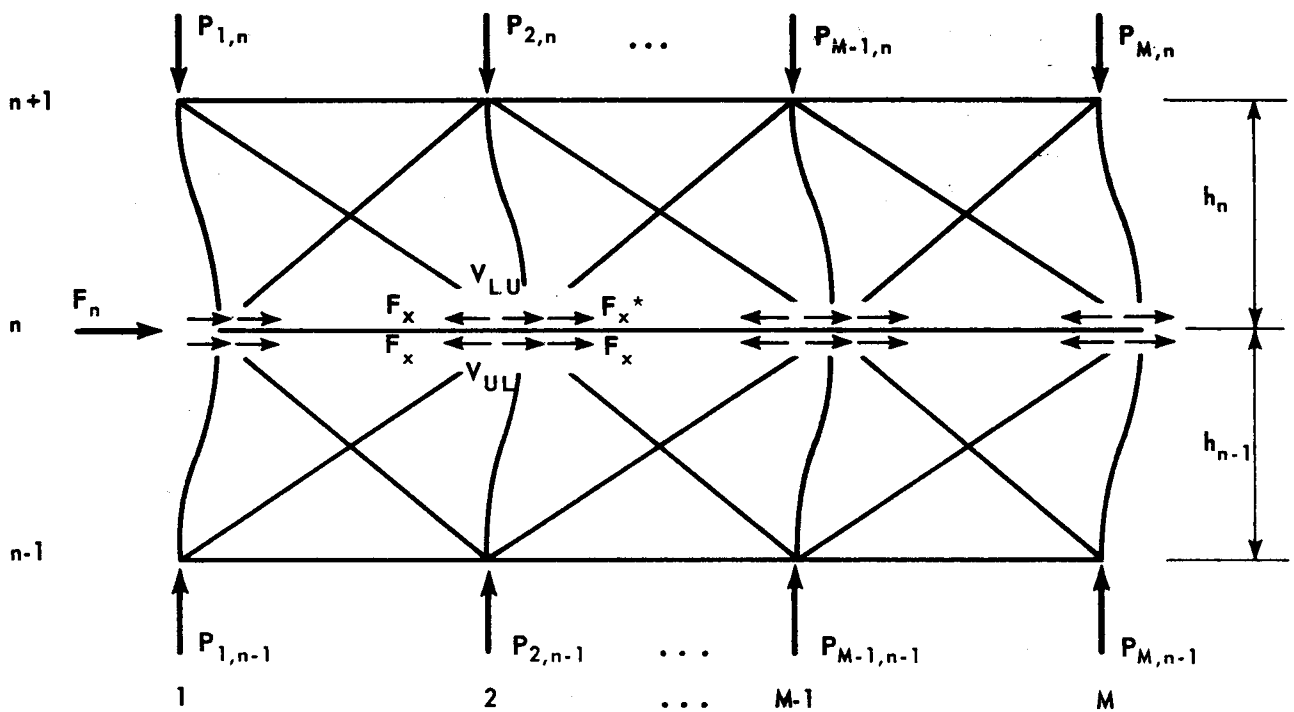


FIGURE 4.7

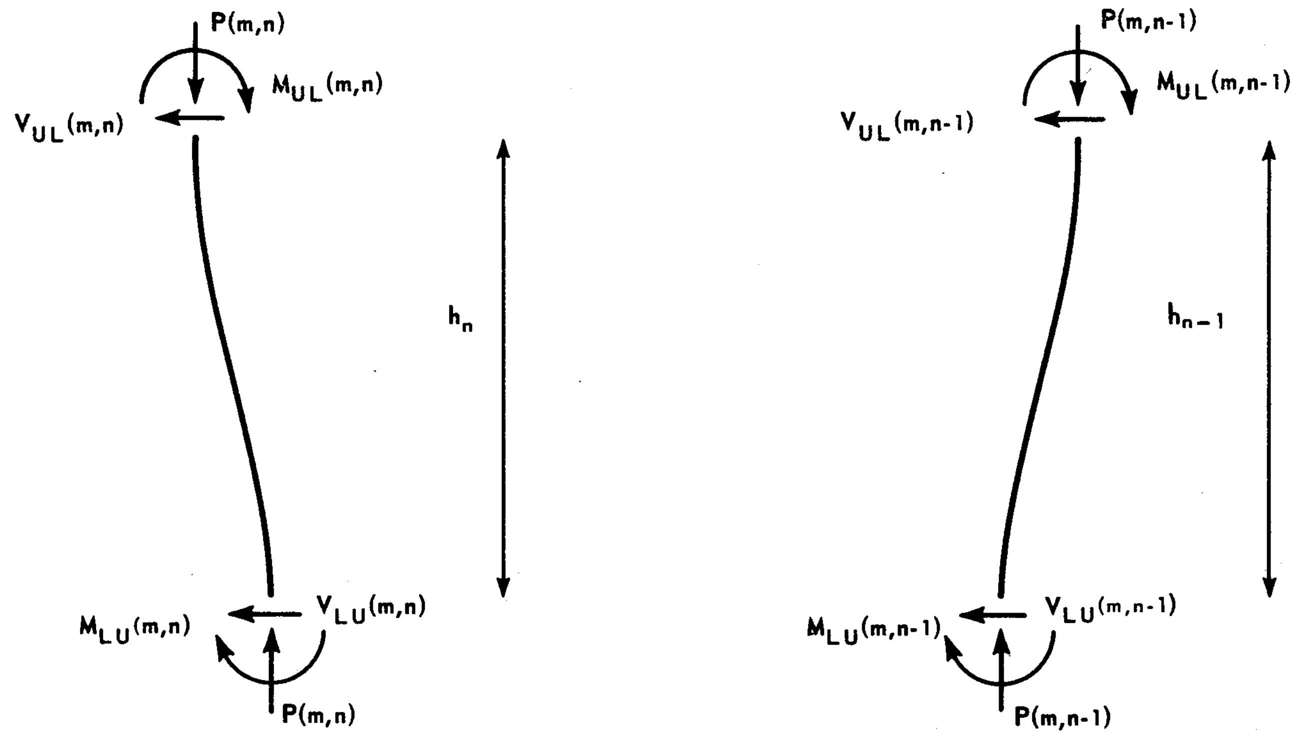
COLUMN BASE



* FORCES ARE SHOWN ACTING ON THE GIRDER

FIGURE 4.8

STORY SHEAR EQUILIBRIUM



a COLUMN m,n   b COLUMN m,n-1

FIGURE 4.9

COLUMN MOMENTS AND FORCES

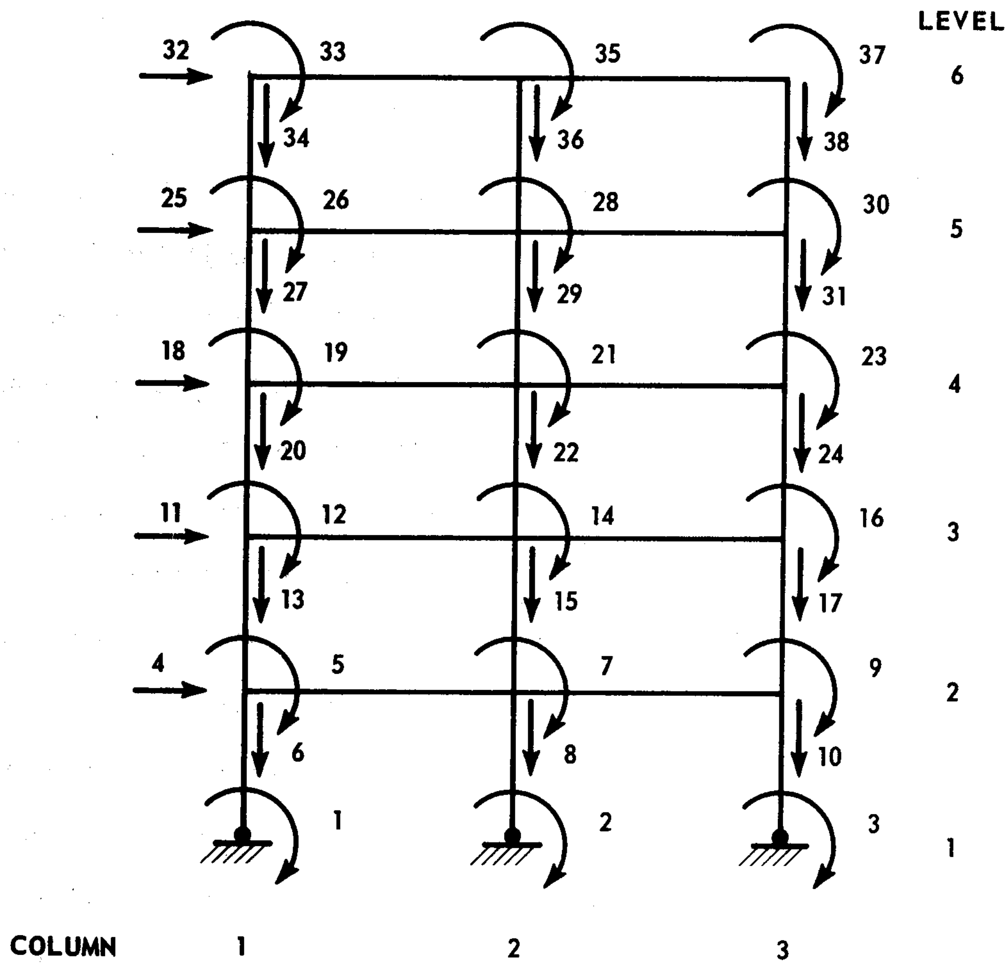


FIGURE 4.10

NUMBERING OF UNKNOWN ROTATIONS AND DISPLACEMENTS

CHAPTER V

COMPARATIVE ANALYSES

5.1 Introduction

The procedure developed in Chapters III and IV is intended for the analysis of large multi-story structures subjected to vertical loads, alone, or in combination with horizontal loads. The program developed for the analysis is able to perform a second order, elastic-plastic analysis of an individual planar frame, or a series of linked planar frames, all of which undergo the same sway displacements at each floor level. A bent may include bracing members or shear wall elements. The changes in column stiffness and plastic moment capacity produced by changes in axial load are considered, as are the effects of the finite width of the shear walls and the axial shortening of the columns. The effect of hinge reversals at the ends of the girders on the force distribution throughout the frame is also included.

The methods of analysis available to date cannot be used to check the present method in its most general form. Therefore, it is necessary to specialize the analysis and compare the influence of each aspect separately.

5.2 Frames Subjected to Combined Vertical and Lateral Loads

The first order, elastic response predicted for a number of small frames was compared with the results obtained from a STRUDL analysis, (25). The structures analyzed included unbraced rigid frames, frames with diagonal bracing, frames with a K-bracing system, and frames with shear walls of various widths. The results agreed in all cases.

The second order elastic-plastic response of a large unbraced frame was next compared with the results predicted by Parikh's program, (8). The twenty-four story, three bay frame, shown schematically in the inset of FIGURE 5.1, was analyzed under the action of proportionally increasing vertical and lateral loads. The frame member sizes and the design level lateral and vertical loads are tabled in Reference 15. The influence of axial force on the column stiffnesses and plastic moment capacities is accounted for in both procedures. The finite widths of the columns are neglected and the live load reduction factors are not considered. The results of the analyses are shown in FIGURE 5.1, where the load factor, λ, is plotted as a function of the top story sway displacement, Δ. The solid curves represent the results of Parikh, which terminate at the ultimate load; the dash curves represent the results obtained from the present analysis. Results were obtained both considering and neglecting axial shortening of the column members. The present method accurately reproduces the results reported by Parikh in both cases.

The twenty-four story frame - shear wall model used in a behavioral study by Guha Majumdar et al, (26), was analyzed to investigate the ability of the present analysis to assess the behavior of a structure containing a shear wall element. The frame is shown in the inset of FIGURE 5.2. Wynhoven's results, (12), are used for the comparison since these included the unloading branch of the load-deflection curve. The member properties and vertical loads for the structure are reported in Reference 12. The flexural stiffness of the shear wall is approximately fifty times that of the column at all levels in the structure. The shear wall width is eight feet, compared with girder spans of twenty-eight and eight feet, (the latter representing an average girder span of sixteen feet in the actual structure). To simulate the roller at the end of the frame girder, (remote from the wall), in the model, the present analysis introduces a ficticious column at the end of the frame girder, having a moment of inertia and plastic moment capacity equal to zero. The effects of axial shortening and the width of the columns were ignored. The stability functions, C and S, were assumed to be equal to four and two, respectively. Constant vertical loads were applied to the frame and the lateral loads were incremented proportionately. The resulting load-deflection curves are shown in FIGURE 5.2, where the concentrated lateral load at each floor level, H, is plotted as a function of the top story sway deflection, Δ. The results obtained from the present analysis and those reported by Wynhoven coincide throughout the complete range of loading.

In order to verify the behavior predicted for a frame containing diagonal bracing, the results of the present analysis were compared with those obtained by Galambos and Lay from a second order, elastic-plastic analysis, (18). The single story, three bay frame shown in the inset of FIGURE 5.3 was analyzed for the comparison. The member properties are given in Reference 18. The column bases are pinned and concentrated axial loads, equal to 0.3 P_y, are applied to each column top. The vertical loads are held constant while the horizontal load, H, is increased. The diagonal braces are assumed to transmit only axial tension. The results of first and second order analyses of the frame without bracing members, and a second order analysis of the frame with bracing members, are plotted in FIGURE 5.3. The applied horizontal force, H, non-dimensionalized by P_y, the yield load of the column, is plotted against the horizontal displacement Δ, divided by the story height, L. The solid curves represent the results of Galambos and Lay; the dashed curves were obtained by the present analysis. The agreement in all three cases is excellent.

The results of an analysis of a series of linked single story frames by Springfield and Adams, (28), were compared with those obtained by the present analysis. The three frames, shown in FIGURE 5.4, represent a one story slice from the lower stories of a tall office building, (28). Frame A is the main stiffening frame of the structure. Frame B is composed of frame A and an additional single bay rigid frame. Frame C represents the complete arrangement of vertical members in the story and consists of the main stiffening frame, the auxiliary rigid frame, and the remaining simply connected columns in the structure. The member properties and frame loads are given in TABLE 5.1. The total lateral load resisted by the frames, H, is plotted against the ratio of story sway to column height, Δ/h, in FIGURE 5.5. The results reported by Springfield and Adams, (represented by the solid curves), were obtained using the subassemblage program described in Reference 27. The results obtained from the present method are shown as the dashed curves. In the subassemblage program the girder hinges are assumed to occur at the column faces, but the girder stiffnesses are based on the centre-to-centre column distances (27). The column width was included in the present analysis and the girder stiffnesses reduced to duplicate the assumption of the subassemblage program. The trends shown by both analyses agree throughout the loading range. The present analysis, however, predicts slightly lower values of the ultimate lateral load, because of the influence of the column width on the bending moment distribution in the girders. The present analysis based on the clear girder spans, predicts greater values of negative moment at the face of the right hand columns than does the subassemblage program, which is based on the centre-to-centre column distances. Thus the right hand girder hinges in the present analysis develop prior to those in Springfield and Adam's analysis, and the predicted ultimate load is reduced.

5.3 Frames Subjected to Vertical Loads Only

Although considerable data is available on the behavior of multi-story frames subjected to combined lateral and gravity loads, published data on frames subjected to gravity loads only is extremely limited. Therefore it is impossible to completely verify each aspect of the present analysis as applied to structures subjected to vertical loads only.

The elastic frame buckling loads predicted for a number of symmetrical one and two story, single bay frames were compared with the results reported by Galambos (29). The agreement was excellent in all cases. The inelastic frame buckling loads predicted for a series of one story, one bay frames were compared with the results reported by Lu (30). Lu's computations were based on the rounded column moment-rotation relationships, which account for gradual yielding of the cross-section, while the present analysis is based on an elastic-plastic M-θ relationship. The agreement between the results of the two analyses was excellent in the high and low ranges of column slenderness, (where the frame buckling loads approach the elastic buckling loads and the sway mechamism loads respectively), but was less accurate in the intermediate range, where partial yielding of the column cross-section has a marked effect on the resistance of the frame to the buckling motion. The maximum error in this region was about 17%.

The only available data on the buckling capacity of frames, having elastic-plastic member moment-rotation relationships, is contained in the investigation by McNamee, (14). McNamee determined the frame buckling load by analyzing the structure subjected to vertical loads in combination with small lateral loads, as described in SECTION 2.3. The lateral load applied at each floor level is a fixed percentage, α, of the total gravity load applied at that level. The ultimate strength of the frame is determined for different values of α, and the frame buckling load extrapolated from the results. The three story, single bay frame, shown in FIGURE 5.6, was analyzed using the present method, neglecting the finite column widths and axial shortening. The section properties are listed in TABLE 5.2a. Uniformly distributed loads were applied in the present analysis. These loads produced fixed end girder moments equal to those produced by the concentrated load, P, used in McNamee's analysis.

Vertical joint loads were also applied in the present analysis in order to adjust the column axial loads to their original values. The results of analyses with α = 1/2% and 1% are shown in FIGURE 5.7a. One half of the vertical load applied at the first floor level, P, is plotted as a function of Δ, the sway displacement at the first level. McNamee's analytical results are shown as the solid curves; those obtained by the present analysis as the dashed curves. The trends shown by both analyses agree throughout the entire loading range. However, the present analysis predicts lower ultimate loads than does McNamee's, due to the differences in the bending moment diagrams produced by the two different loading systems. The uniformly distributed loads in the present analysis produce a greater maximum positive bending moment in the girders than do the concentrated loads in McNamee's analysis. Thus the interior girder hinge, which is last to form in both cases, develops at a lower load factor in the present analysis, resulting in a lower predicted ultimate load.

The results obtained from a test on a similar frame, performed at Lehigh University, are shown in FIGURE 5.7b. Curves obtained from the present analysis are also shown, both neglecting and considering the effect of the finite width of the columns. The ultimate loads obtained from the different analyses are presented in TABLE 5.2b. In FIGURE 5.7b, two unloading branches are plotted for the curve with α = 1/4% neglecting the finite width of the columns, one considering hinge reversal, and the other ignoring this effect. The hinging configuration for the structure at ultimate load is shown in the inset of the figure. As the sidesway motion increases (at the ultimate load) the hinge at the left end of the middle girder reverses, and this region again behaves elastically. The result is an increase in the unloading strength of the structure. The slope of the unloading branch of the analysis considering hinge reversal agrees with that of the test curve.

COLUMN DATA

COLUMN	AXIAL LOAD (Kips)	M_{PC} (Ft.-Kips)	P_y (Kips)	r (in.)	I ($in.^4$)	DEPTH (in.)
1	4,140	17,900	20,088	12.01	80,923	60
2	14,110	12,600	23,935	11.77	92,117	42
3	14,400	12,600	23,935	11.77	92,117	42
4	13,240	10,000	20,353	12.03	81,801	42
5	17,810	4,000	20,088	12.01	80,923	60
6	23,350	16,530	37,980	11.31	121,830	42
7	20,050	11,620	27,900	11.88	109,294	42
8	68,300	99,999	99,999	99.99	999,999	42

GIRDER DATA

GIRDER	UDL (Kips/Ft.)	LENGTH (Ft.)	I ($in.^4$)	M_p (Ft.-Kips)
1-2	4.38	56	36,420	5,919
2-3	4.62	56	36,420	5,919
3-4	4.00	56	36,420	5,919
4-5	4.03	56	36,420	5,919
5-6				
6-7	6.20	56	25,150	9,656
7-8				

TABLE 5.1 MEMBER PROPERTIES - LINKED SINGLE STORY FRAMES

MEMBER PROPERTIES

	4 WF 13	6 B 16
d	4.145	6.232
r_x	1.717	2.56
I_x	11.316	31.10
Z_x	6.265	11.525
σ_y	50.3	34.7

TABLE 5.2a MEASURED MEMBER PROPERTIES -

THREE STORY, SINGLE BAY FRAME

FRAME BUCKLING RESULTS

α	McNAMEE	PRESENT ANALYSIS
0	24.2	23.2
1/2%	22.7	21.7
1%	21.4	20.8
TEST VALUE		24.8
PRESENT ANALYSIS INCLUDING COLUMN WIDTH EFFECT		24.6

TABLE 5.2b SUMMARY OF FRAME BUCKLING ANALYSES -

THREE STORY, SINGLE BAY FRAME

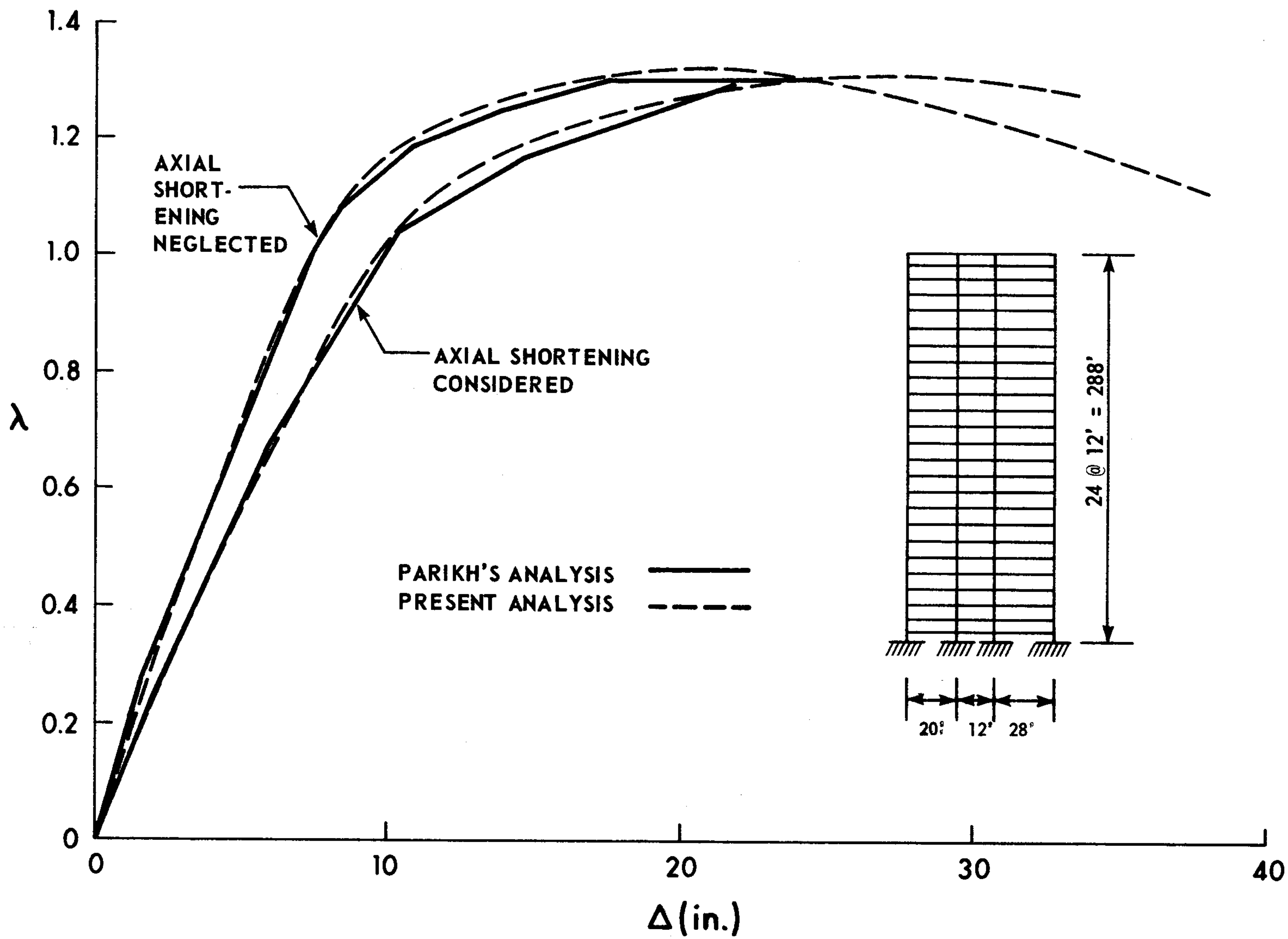


FIGURE 5.1

LOAD-DEFLECTION RELATIONSHIP - 24 STORY FRAME

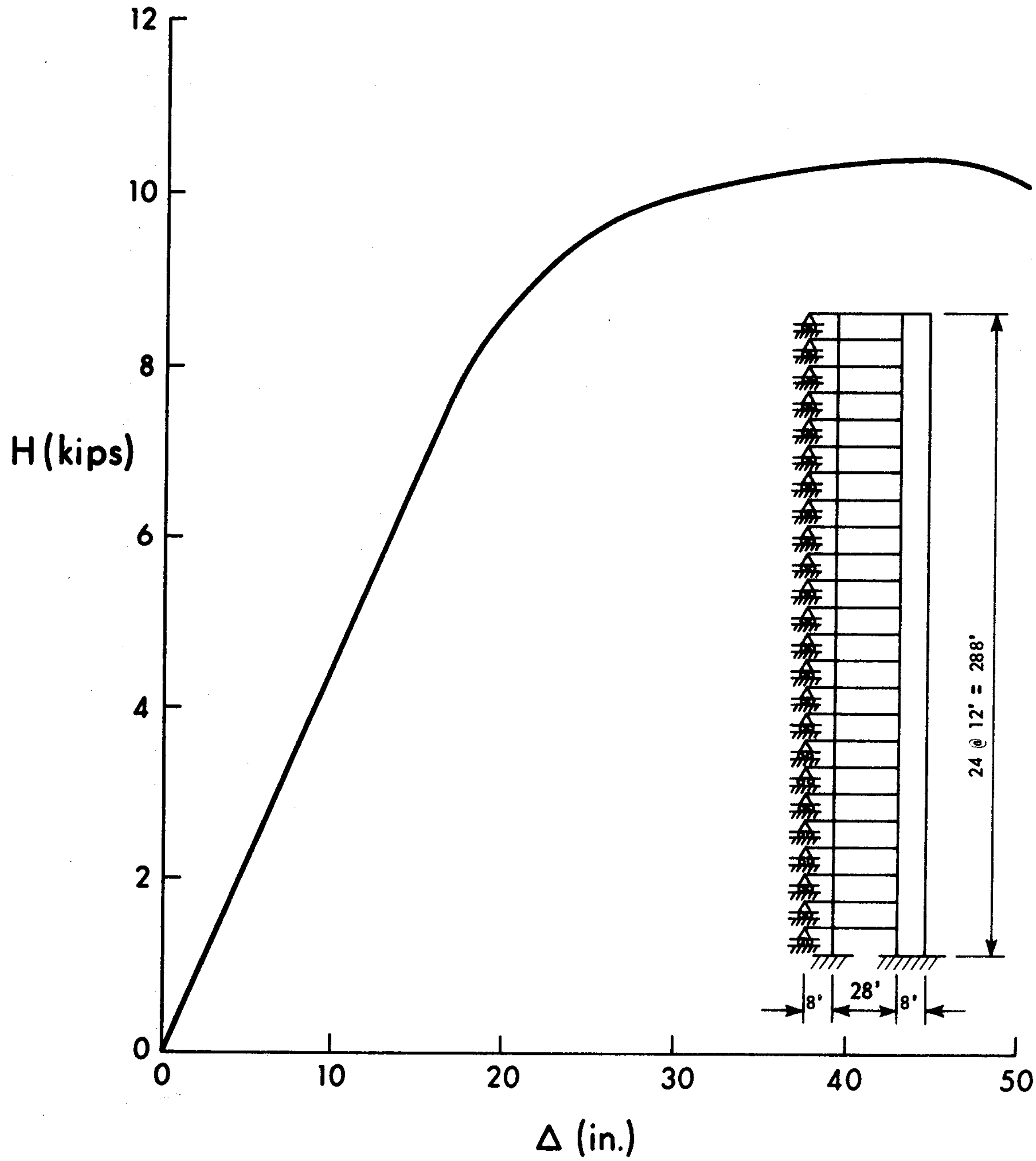


FIGURE 5.2

LOAD-DEFLECTION RELATIONSHIP - FRAME SHEAR WALL STRUCTURE

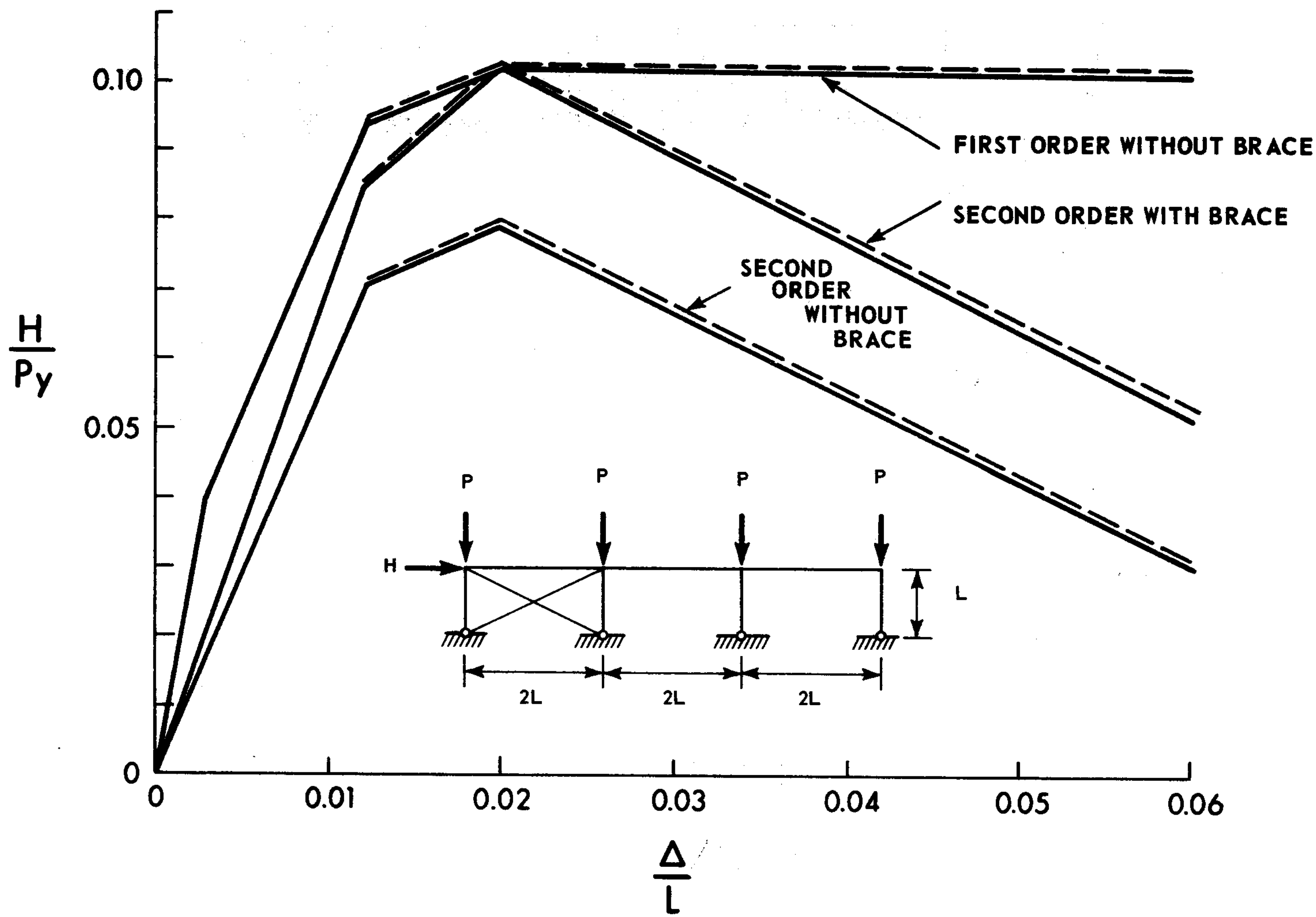


FIGURE 5.3

LOAD-DEFLECTION RELATIONSHIP - BRACED FRAME

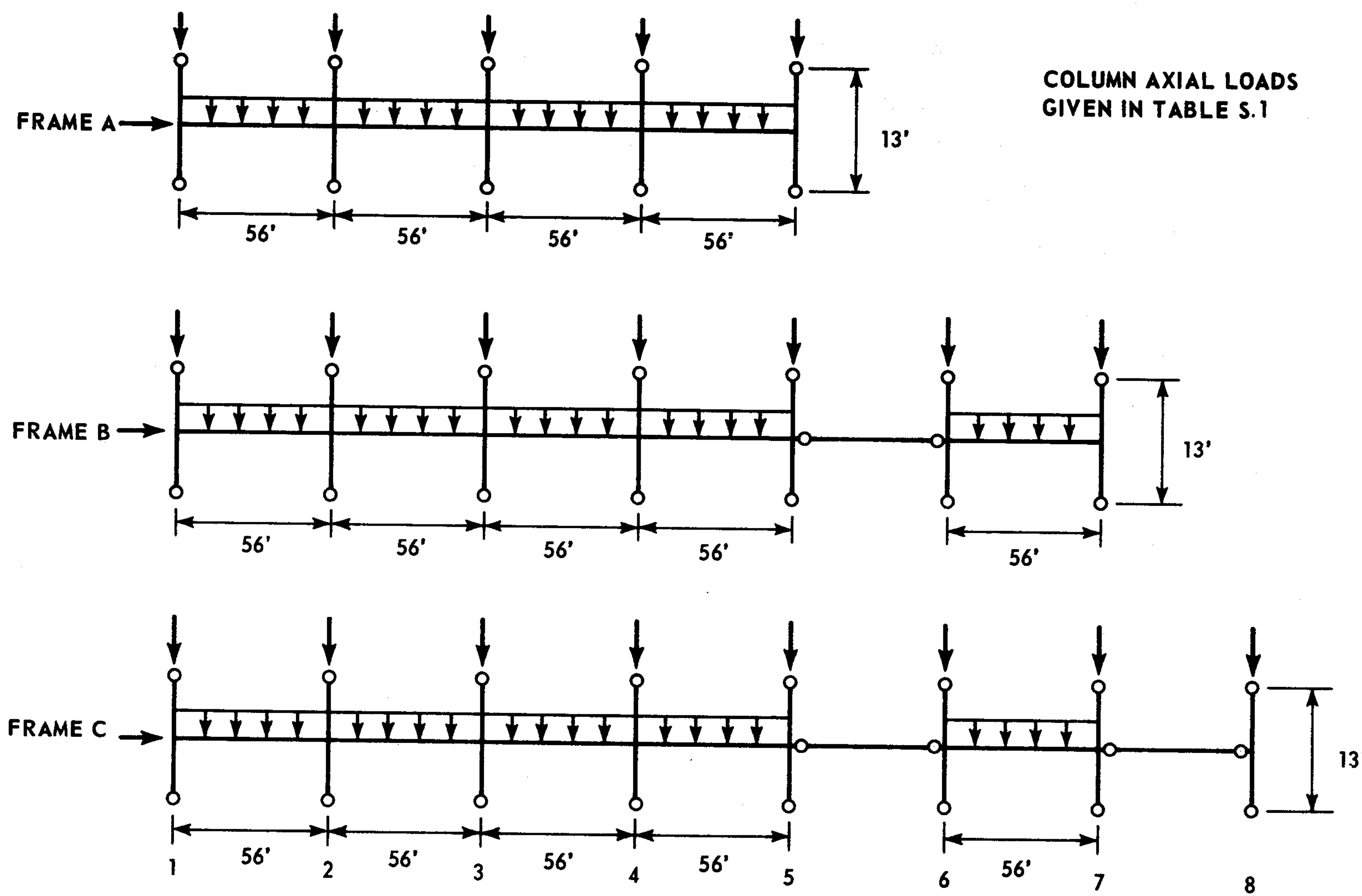


FIGURE 5.4

LINKED SINGLE STORY FRAMES

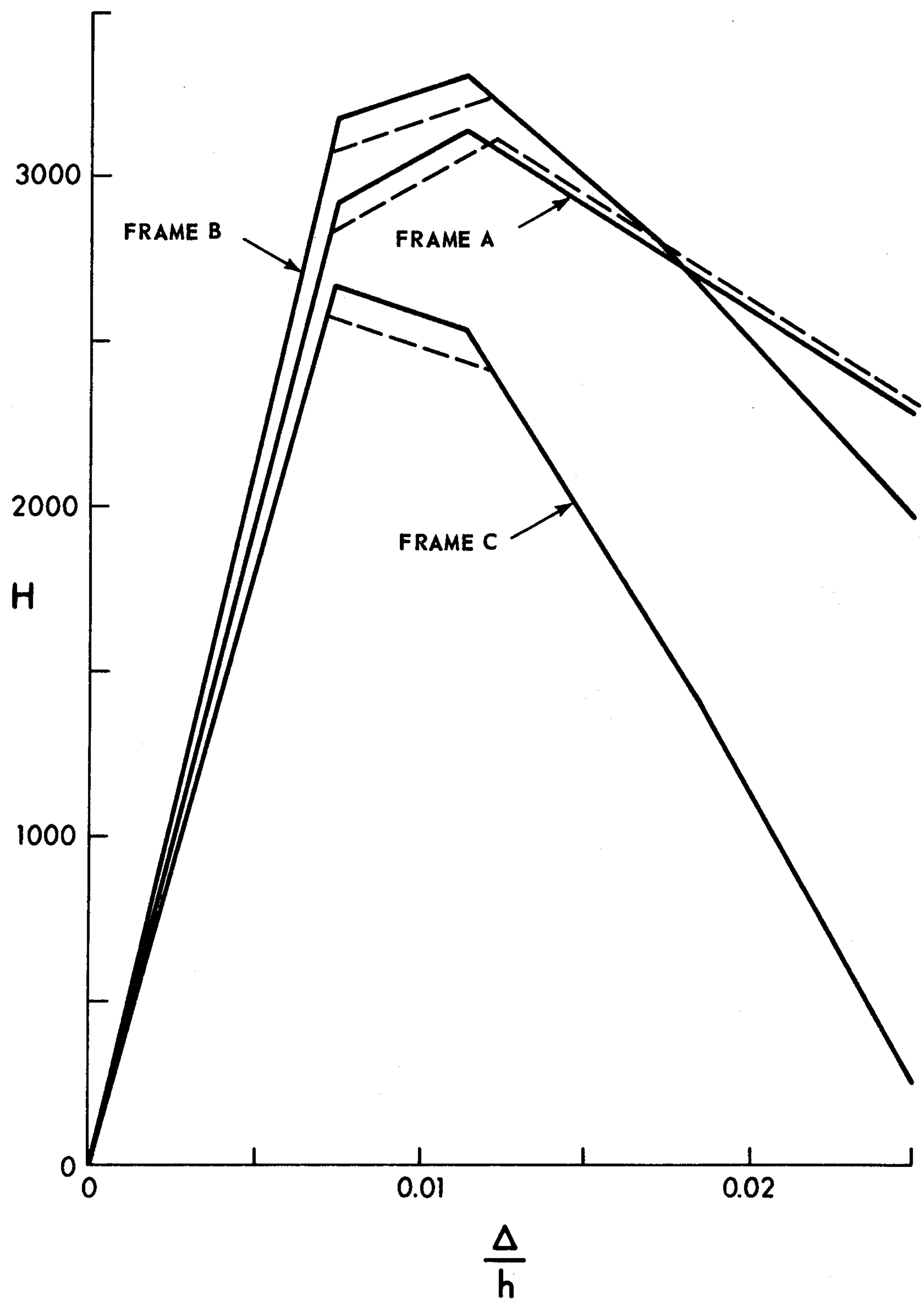


FIGURE 5.5

LOAD-DEFLECTION RELATIONSHIP - LINKED SINGLE STORY FRAMES

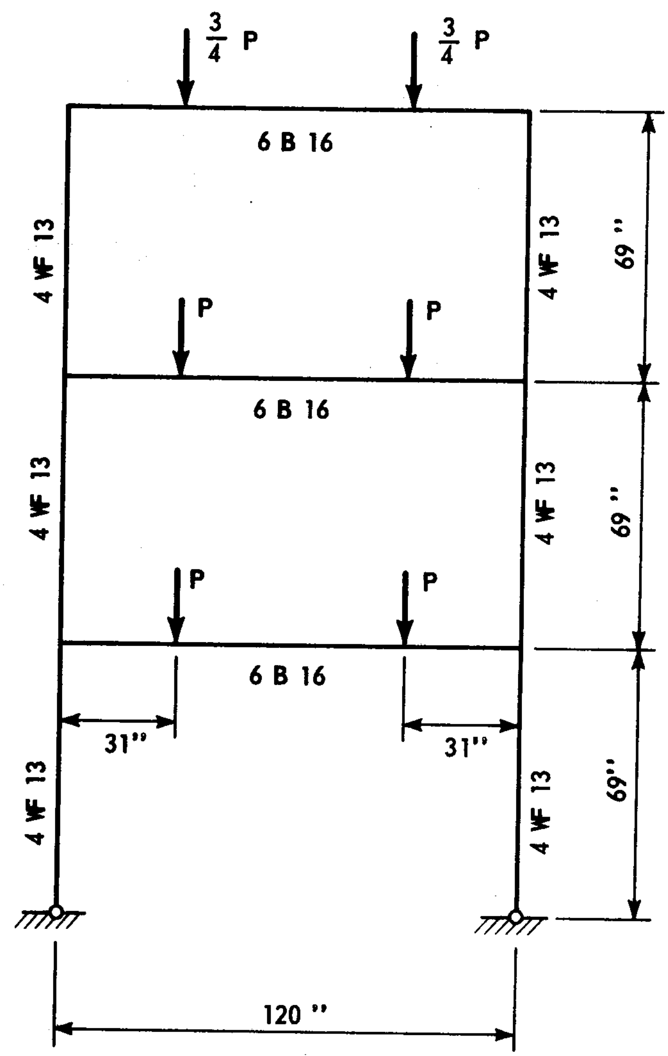


FIGURE 5.6

THREE STORY, SINGLE BAY FRAME

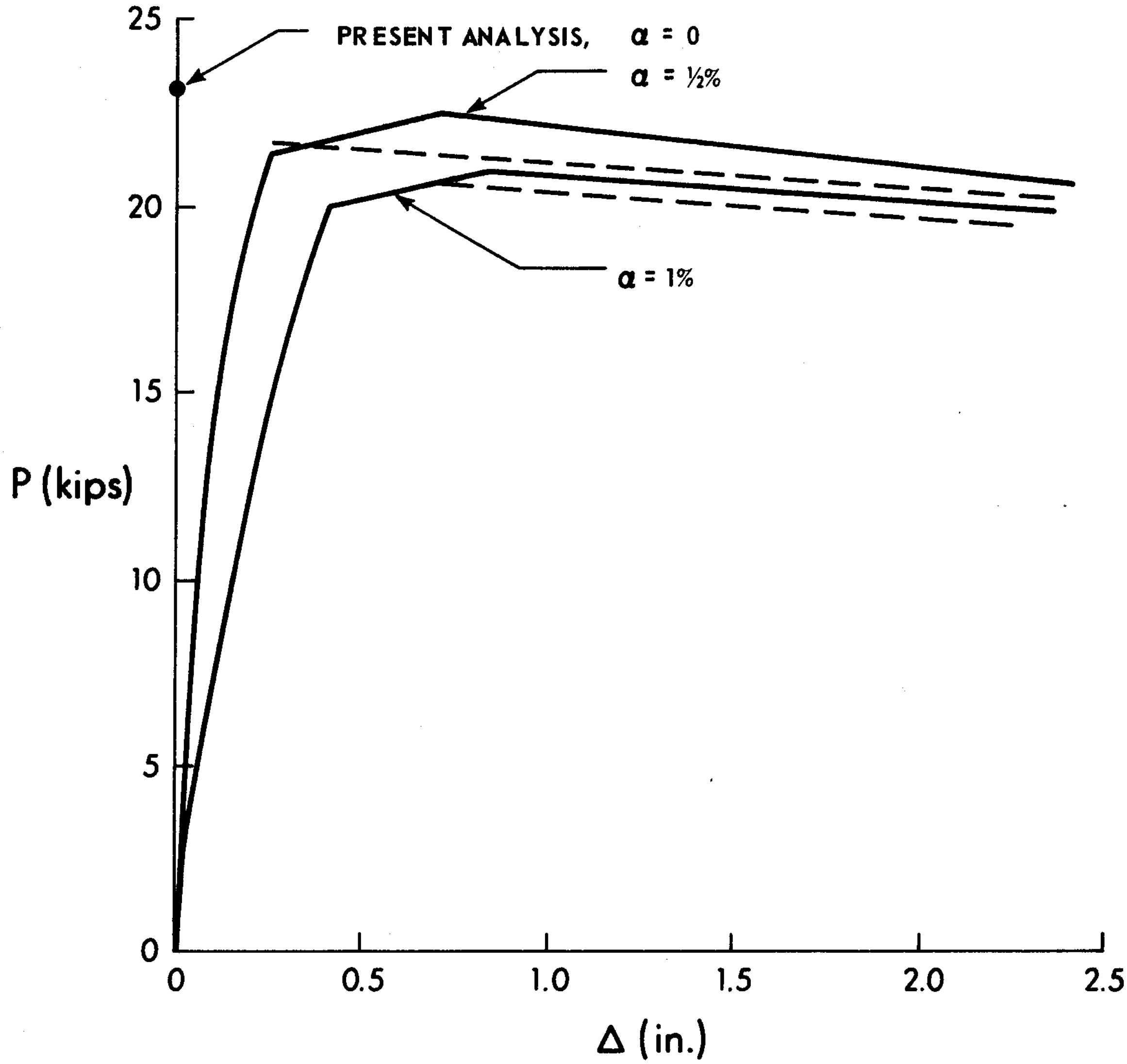


FIGURE 5.7a

LOAD-DEFLECTION RELATIONSHIP, THREE STORY SINGLE BY FRAME

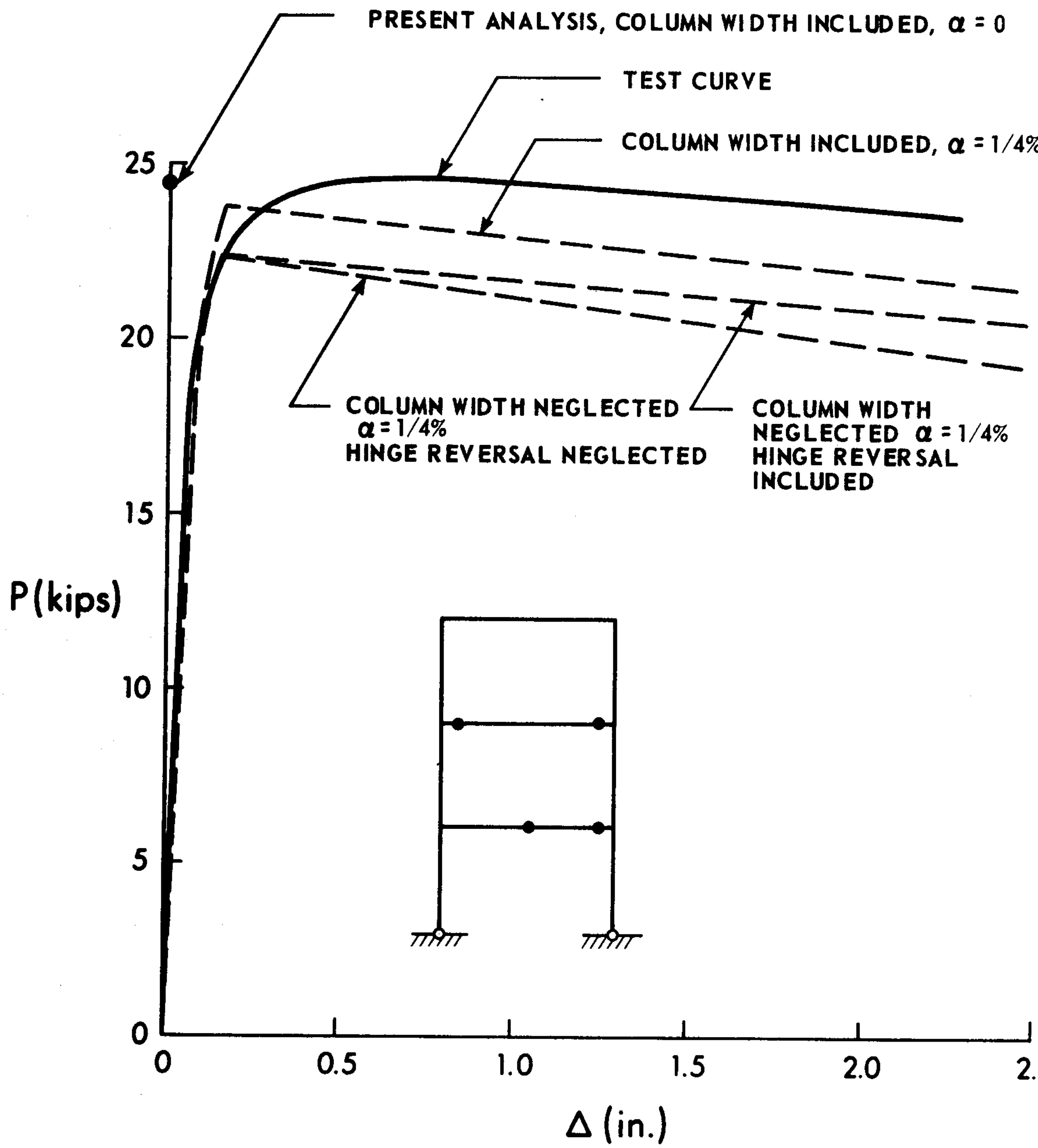


FIGURE 5.7b

LOAD-DEFLECTION RELATIONSHIP, THREE STORY SINGLE BAY FRAME

CHAPTER VI

BEHAVIORAL STUDIES

6.1 Introduction

The primary objectives of this portion of the investigation are to compare the behavior of unbraced and braced frames, and to examine the design of columns in both types of structures. The twenty-four story, plastically designed frames described in Reference 15, and the corresponding subassemblage frames, form the basic structures studied in the investigation. The program of investigation is divided into two sections, the first dealing with the behavior of frames subjected to combined vertical and horizontal loads, and the second dealing with frame action under vertical loads alone. In both sections comparisons are made between the behavior of the unbraced and braced frames designed under the same conditions. In addition, results of first and second order analyses of the braced frames are presented, and the additional bracing required to increase the ultimate load capacities predicted by the second order analyses, to those resulting from the corresponding first order analyses, are determined.

In addition to the basic studies described above, the relationship between bracing strength and stiffness and frame behavior is investigated, and a comparison made between the behavior of diagonally braced frames and those containing a K-bracing system. The effect of supporting a flexible frame by a stiffer braced or unbraced frame, is also investigated.

6.2 Basic Structures

6.2.1 Series A

The two twenty-four story, three bay structures described in Reference 15, form the basis of the present investigation. The frame configuration and design loads are shown in FIGURE 6.1. The structure is a regular, rectangular frame with rigid connections and fixed column bases. The bent spacing is assumed to be 24 feet. The design vertical loads are based on a uniform live load of 100 psf and a deal load of 120 psf, (30 psf and 95 psf respectively for the roof). The exterior wall cladding is assumed to produce a dead load of 85 psf, and the horizontal wind load is 20 psf. Live load reduction factors, as specified in Reference 31, are applied to the girders and columns separately. The resulting design girder loads are listed in TABLE 6.1, and the design column axial loads, based on tributary floor areas, are listed in TABLE 6.2.

The unbraced frame, A-1, and the braced frame, A-2, are shown in FIGURES 6.2 and 6.3, respectively. Both frames were designed plastically on the basis of ultimate strength. Two loading conditions were considered: vertical loads alone, with a load factor of 1.70, and combined vertical and horizontal loads, with a load factor of 1.30. For the unbraced frame the P-Δ effect was accounted for using an estimated deflection index of 0.02 at ultimate load. For the braced frame the P-Δ effect was again included. For this case, the ultimate strength of the frame is assumed to coincide with brace yielding, corresponding to a deflection index of 0.003. Although the secondary effects were computed under different assumptions for the two frames, the resulting designs are consistent with current engineering practice.

6.2.2 Series B

The two subassemblage frames, frames B-1 and B-2, shown in FIGURES 6.4 and 6.5, represent horizontal slices from frames A-1 and A-2 respectively, at the sixth floor level. It is assumed that points of inflection occur at midheight of each column of the original frames. The behavior of the subassemblage frame is not necessarily indicative of the behavior of the corresponding twenty-four story frame, however, the results of the subassemblage studies will be compared with those obtained from analyses of the complete frames.

The member properties for frame B-1, (the unbraced subassemblage frame), are identical to those of the fifth and sixth stories of frame A-1. A fictitious girder, having a moment of inertia and plastic moment capacity equal to zero is inserted across the tops of the columns, so that the subassemblage can be analyzed using the computer program described in Chapter IV. The design loads are shown in FIGURE 6.4. The applied column axial loads are based on the area tributary to each column, with the appropriate live load reduction factor. The horizontal shear at the top of the subassemblage frame is equal to the total horizontal shear at the sixth level of frame A-2.

The columns and girders of frame B-2, (the braced subassemblage frame), are identical to those of the fifth and sixth stories of frame A-2. However, because of the artificial brace arrangement in frame B-2, the diagonal bracing members are re-proportioned to produce the same stiffness (horizontal), critical axial load, and yield load in tension, as the corresponding members of frame A-2. The properties of the bracing members are given in FIGURE 6.5, along with the design loads on the frame.

Frames B-3 and B-4, shown in FIGURE 6.6, are similar to frame B-2, but have the diagonal braces replaced by a K-bracing system. The bracing system of frame B-4 is inverted with respect to that of frame B-3. The members of the K-bracing system are proportioned to provide the same (horizontal) stiffness and capacities in compression and tension as the diagonal bracing system. A fictitious column having values of stiffness, area and plastic moment capacity equal to zero, is inserted at midspan of the braced bay so that the computer analysis can be performed using the program described previously.

Four additional frames, each representing a different structural framing system, are shown in FIGURES 6.7 and 6.8. Frame B-5 consists of the unbraced frame, frame B-1, coupled with a rigid frame designed to resist vertical loads only, (frame B-2 without the diagonal bracing members). Frame B-6 consists of the braced frame, frame B-2, coupled with the above rigid frame. Frame B-7 represents a framing system in which a select unbraced bent, frame B-1, is coupled with a number of bents whose column-girder connections are non-rigid. The supported frames are represented by a single column with an axial load, P_1, equal to the total vertical load on the non-rigid frames. Frame B-8 consists of the braced frame, frame B-2, coupled with such a series of frames. In all of the above cases, the stiffer frame would be expected to resist most of the applied horizontal loads and P-Δ shears for the entire structure.

6.3 Presentation of Results

6.3.1 Combined Vertical and Lateral Loads

6.3.1.1 Loading Procedure

The procedure for the combined load studies was first to apply the factored vertical loads, (1.30 times the design values), and then to increment the lateral loads to trace the response of the structure. The lateral load increment was 2% of the design load level.

6.3.1.2 Twenty-Four Story Frames

The first phase of the study compared the behavior of the unbraced, twenty-four story frame, frame A-1, and the corresponding braced frame, frame A-2. The load-deflection relationships for the two structures are shown in FIGURE 6.9, (solid curves), where the lateral load factor, λ, is plotted as a function of the top story sway, Δ. Throughout most of the loading range the unbraced frame was stiffer than the braced frame. The unbraced frame reached an ultimate load factor of 1.90 at a sway of 30 inches; the braced frame reached a load factor of 1.50 at a sway of 17 inches. The hinging conditions in frames A-1 and A-2 at failure are shown in FIGURES 6.10 and 6.11 respectively. The solid circles represent plastic hinge locations at failure; the open circles, locations where hinges originally formed under vertical load, but reversed when lateral load was applied to the structure. The numbers associated with the hinges indicate the stage on the load-deflection relationship, (FIGURE 6.9), at which the hinges formed. A hinge designated "1" developed when the vertical load was applied to the structure. A hinge designated "2" formed between a load factor of zero and a load factor of 0.20, "3" between 0.20 and 0.40, "4" between 0.40 and 0.60, "5" between 0.60 and 0.80, "6" between 0.80 and 1.00, "7" between 1.00 and 1.20, "8" between 1.20 and 1.40, "9" between 1.40 and 1.60, "10" between 1.60 and 1.80, and "11" at a load factor greater than 1.80. Failure in the unbraced frame was initiated by extensive column hinging in the top stories; yielding of the tension braces in stories 7 to 13 inclusive initiated failure in the braced frame. (All the compression braces in frame A-2, except the one in the top story, had reached their critical axial loads at failure). The bracing members which had yielded or buckled at failure are indicated in FIGURE 6.11.

The load-deflection relationships for frame A-2 with 110, 120, 130, 140 and 150% bracing are also shown in FIGURE 6.9, (the dashed curves). The design bracing members, as given in FIGURE 6.3, represent 100% bracing. An increase in the amount of bracing indicates that the bracing stiffness, compressive capacity and tensile capacity have all been adjusted in the same ratio. The stiffness of the structure varied directly with the amount of bracing. However, the ultimate capacity of the frame increased only for percentages of bracing less than 140. Beyond 140%, the ultimate load capacity decreased with increased bracing capacity. The maximum lateral load factor obtained was 1.84. Failure of the frames with increased bracing occurred when the lower member in the leeward column stack reached its yield axial load. It was not possible to unload the structure under this condition.

6.3.1.3 Twenty-Four Story Frame - P-Δ Effect

The second phase of the study compared first and second order analyses of the braced, twenty-four story frame, frame A-2. The results of the analyses are shown in FIGURE 6.12, where the lateral load factor, λ, is plotted as a function of the top story sway, Δ. The ultimate load factor for the frame, predicted by the first order analysis, was 2.36, as compared with 1.50 predicted by the second order analysis. Since the maximum load factor that could be achieved with additional bracing was 1.84, it was not possible to raise the capacity predicted by the second order analysis to that predicted by the first order analysis by simply adding bracing to the frame.

6.3.1.4 Subassemblage Frames

The third phase of the study compared the behavior of the unbraced and braced subassemblage frames, frames B-1 and B-2. Results of the analyses are shown in FIGURE 6.13, where the lateral load factor, λ, is plotted as a function of the story sway rotation, ρ, (defined as the story sway deflection divided by the story height). The sequences of plastic hinge formation in the two frames are shown in FIGURES 6.14 and 6.15. The solid circles represent the hinge locations; the numbers correspond to the stages on the load-deflection curves, (FIGURE 6.13), at which the hinges formed. Plastic hinges did not develop under gravity loads alone in either structure, and, therefore, no hinge reversal was observed.

The unbraced frame, frame B-1, exhibited increased sway deflections after the formation of plastic hinges in the leeward ends of all three girders. The ultimate load factor of 2.02 was achieved at a rotation of 0.0085 radians, after two additional girder hinges had developed. The final girder hinge formed on the descending portion of the load-deflection curve.

The braced frame, frame B-2, was slightly stiffer than the unbraced frame in the initial stages of loading. Plastic hinges developed first at the leeward ends of the centre and left girders. However, the stiffness of the frame was not reduced significantly until the two compression braces buckled, at a load factor of 0.48. Beyond this point the load carrying capacity of the braced frame was less than that of the unbraced frame at a given sway rotation. The ultimate lateral load factor for the braced frame, 1.92, which was achieved at a rotation of 0.0067 radians, corresponded to yielding of the tension braces.

Also shown in FIGURE 6.13 are the load-deflection relationships for frame B-2 with various percentages of bracing, (dashed curves). The stiffness and load carrying capacity varied directly with the amount of bracing over the range investigated, 90, 105, 110 and 120% bracing. The braced frame with 105% bracing reached an ultimate load factor of 2.02, the same as that of the unbraced frame.

6.3.1.5 Subassemblage Frame - P-Δ Effect

The fourth phase of the study compared the results of the first and second order analyses of the braced subassemblage frame, frame B-2. The results of the analyses are shown in FIGURE 6.16, where the lateral load factor, λ, is plotted as a function of the story sway rotation, ρ. In the first order analysis the tension braces yielded at a load factor of 2.40; the ultimate load factor for the frame was 2.66. The ultimate load factor as predicted by the second order analysis was 1.92. Two dashed curves, representing second order analyses of the frame with 123% and 136% bracing, are also shown. The structure with 123% bracing reached a maximum load factor of 2.40, the same as that predicted by the first order analysis for yielding of the tension braces; the structure with 136% bracing reached a maximum load factor of 2.66, the same as that predicted by the first order analysis.

6.3.1.6 Subassemblage Frame - Bracing Strength and Stiffness

The fifth phase of the study was concerned with the relative effect of varying the bracing strength and stiffness independently. Frame B-2 was analyzed with the bracing stiffness increased 36%; with the bracing strength increased 36%; and with both stiffness and strength increased by 36%. The load-deflection curves are shown in FIGURE 6.17. The response of the original braced frame is shown for comparison.

Increasing the bracing stiffness without increasing its strength had little effect on the ultimate strength of the frame. Conversely, increasing the bracing strength without increasing its extensional stiffness, although increasing the ultimate load factor, had little effect on the stiffness of the frame. It was necessary to increase both stiffness and strength to achieve a significant increase in the ultimate load capacity of the frame, at the sway rotation corresponding to the ultimate strength of the original frame.

6.3.1.7 Subassemblage Frame - Bracing Systems

The sixth phase of the study compared the behavior of the frame containing a diagonal bracing system, with those containing equivalent K-bracing, designed as outlined in SECTION 6.2.2. The results of the analyses are shown in FIGURE 6.18. The behavior of frame B-2 is represented by the solid curve, the relationships for frames B-3 and B-4 by the dashed curves. The frames are shown schematically in the insets to the figure.

The compression braces in the K-bracing system of frame B-3 buckled at a load factor of 1.04, and the tension braces yielded at a load factor of 2.18, corresponding to the attainment of the ultimate load. Thus frame B-3 was able to achieve a slightly higher ultimate load than frame B-2.

However, the K-bracing arrangement used for frame B-4 was vastly inferior to both that used for frame B-3 and the diagonal system of frame B-2. In frame B-4 the bracing members buckled in compression on the application of the vertical loads to the frame. A sway rotation of 0.0011 radians was necessary before the eventual tension braces recovered the deformations associated with the buckling motion. Extensive girder hinging developed at low loads due to the large sway rotations. The ultimate load factor for the frame, 0.78, was achieved without yielding the tension braces.

An additional analysis was performed on frame B-4 after adjusting the critical axial loads of the bracing members so that buckling did not occur on the application of the initial vertical loads.

The results of the analysis are shown in FIGURE 6.18. The improvement in the load capacity of the frame was not substantial.

6.3.1.8 Coupled Subassemblage Frames

In the seventh phase of the study, the behavior of a number of coupled frame arrangements was investigated. In the first series, the unbraced subassemblage frame was used as the principal stiffening element. The results of the analyses are shown in FIGURE 6.19. The solid curves represent the behavior of the unbraced frame, (frame B-1), the unbraced frame coupled with a frame designed to resist vertical loads only, (frame B-5), and the unbraced frame coupled with a single pinned column, (frame B-7). The pinned column is used, in turn, to represent a single non-rigid frame with vertical loads equal to those of the unbraced frame, (P_1 = 6030 kips), two such non-rigid frames, (P_1 = 12060 kips), and three such non-rigid frames, (P_1 = 18090 kips). The ultimate load factor for frame B-1 was 2.02. Frame B-5 reached an ultimate load factor of 1.84, representing a 9% reduction in the load carrying capacity. Frame B-7, with P_1 equal to 6030 kips, reached a load factor of 1.62, (a 20% reduction); with P_1 equal to 12060 kips, 1.30, (a 36% reduction); and with P_1 equal to 18090 kips, 1.00, (a 50% reduction).

The P/P_y ratios of the columns, under a vertical load factor of 1.30, were approximately 0.56 in the unbraced frame and 0.62 in the frame designed for vertical loads only. These values correspond to axial load ratios of 0.74 and 0.81 under a vertical load factor of 1.70, and, thus, might be typical of the lower stories of multi-story frames. The dashed curves shown in FIGURE 6.19 represent analyses of frames B-1 and B-5 with the applied column axial loads reduced to one-half of their original values (P/P_y ratios equal to 0.28 and 0.31, respectively). Such axial loads might be typical of columns near the top of multi-story frames. Frame B-1 with half the applied column axial loads, reached an ultimate load factor of 2.32, an increase of 15% above the load factor of the original frame. Frame B-5, with half the applied column axial loads, reached a load factor of 2.36, an increase of 28% above that of the original unbraced-supported frame. The ultimate load factor for frame B-5, (with $P/P_y = 0.28$), was greater than that of frame B-1, indicating that under these conditions the supported portion of frame B-5 was, indeed, capable of resisting applied lateral loads in addition to its own P-Δ shears.

In the second series, the braced subassemblage frame was used as the principal stiffening element. Results of the analyses are shown in FIGURE 6.20. The behavior of frames B-2, (the braced frame), B-6, (the braced-supported frame), and B-8 (the braced-pinned column frame), are represented by the solid curves. The ultimate load factor for the braced frame was 1.92. Frame B-6 reached an ultimate load factor of 1.72, representing a reduction from the load carrying capacity of frame B-2, of 10%. Frame B-8, with P_1 equal to 6030 kips, 12060 kips and 18090 kips, reached load factors of 1.46, 0.98 and 0.52, respectively, (corresponding to reductions of 24, 49 and 73%). The dashed curves in FIGURE 6.20 represent the responses of frames B-2 and B-6 with the P/P_y ratios of the columns approximately equal to 0.31. Frame B-2, under one-half of the original applied column axial loads, reached an ultimate load factor of 2.16, an increase of 12% above the load factor of the original frame. Frame B-6, with half the applied column axial loads, reached a load factor of 2.20, an increase of 28% above that of the original braced-supported frame. The load factor reached by frame B-6 in this situation was greater than that of the braced frame along.

6.3.2 Vertical Loads Only

6.3.2.1 Loading Procedure

In the case of a frame subjected to vertical loads only, the vertical loads were incremented proportionately from zero to failure. Each load increment was 2% of the corresponding design value.

6.3.2.2 Twenty-Four Story Frames

The eight phase of the study was concerned with the response of frames A-1 and A-2 to increasing vertical loads, and in particular, with the amount of bracing required to prevent frame buckling of frame A-2 until the beam-mechanism load was reached. However, in frame A-2 many of the bracing members in the middle stories buckled in compression between load factors of 0.92 and 1.40, due to axial shortening of the columns. When the braces in a particular story were both in a buckled condition, they were unable to provide any net component of horizontal force to resist the P-Δ shears in the structure. Thus at a load factor of 1.42, the lateral stiffness of the structure, in the absence of any contribution from the buckled compression braces, was reduced so that the determinant of the coefficient matrix became negative, and the structure was unloaded. In actual fact, the frame was capable of resisting additional vertical load, because, as the sidesway motion increased, one of each pair of buckled compression braces would return to the elastic range, and provide additional lateral stiffness to the structure. To eliminate this complexity, the bracing members were prevented from initial buckling, by neglecting axial shortening of the columns in the analysis.

The unbraced frame, frame A-1, reached a load factor of 1.66 before the frame became unstable. The hinging pattern in the frame at this stage is shown in FIGURE 6.21. Hinging is concentrated in the top six stories. The girders did not reach the mechanism condition, although the load factor attained, 1.66, was close to that used in design, 1.70. When axial shortening was considered, the unbraced frame reached a load factor of 1.70 before the frame became unstable.

The braced frame, frame A-2, reached its beam mechanism load without any indication of overall frame instability. The beam mechanism load factor was 1.68. The hinging pattern in the structure at this stage is shown in FIGURE 6.22. Hinging was extensive throughout the structure, but the bracing members remained elastic. To check the effect of column axial shortening on the frame instability load, the braced frame was analyzed with 0.1% of the girder loads applied in the horizontal direction at each floor level. The results agreed with the case where axial shortening was not considered.

In order to determine the amount of bracing necessary to prevent overall frame instability, the braced frame was re-analyzed without bracing members. Frame instability occurred at a load factor of 1.40, however 1% bracing was sufficient to enable the frame to reach its beam mechanism load. These results were obtained for the case where column axial shortening was not considered, and therefore sways under vertical loads were small.

6.3.2.3 Subassemblage Frames

The ninth phase of the study was concerned with the behavior of the subassemblage frames under vertical loads only. The results of the investigation are shown in FIGURE 6.23, where the vertical load factor, λ, is plotted versus the story sway, Δ. The sequences of plastic hinge formation in frames B-1 and B-2 are shown in FIGURES 6.24 and 6.25, respectively. The numbers associated with the hinges indicate the stages on the load deflection diagrams where the hinges developed. The first hinge to form in the unbraced frame developed at a load factor of 1.84. Subsequently, the sway deformations increased under increasing vertical load, as did the number of hinges. The ultimate load factor for the frame was 2.06, at a sway deflection of 0.35 inches. The hinge which formed at stage 3, at the left end of the right hand girder, began to close at stage 6 as the frame buckled sideways. The first hinge to form in the braced frame developed at a load factor of 1.32. The ultimate load factor for the frame was 1.74, corresponding to the formation of a beam mechanism in the left hand girder. The diagonal bracing members in the subassemblage frame remained elastic, as the sixth level was below the portion of the structure where the braces buckled due to axial shortening of the columns. Frame B-2 exhibited only small sways under the vertical loads.

As in the prior phase of the study, the behavior of the braced frame without bracing members was also investigated. The load-deflection curve for the frame is shown in FIGURE 6.26. The load factor corresponding to frame instability was 1.36. The minimum bracing, which would prevent frame instability until after the beam-mechanism load had been attained, was 18% of the original design bracing.

The results of the present analysis, which applied only vertical loads to the frame, were compared with those obtained using the small-lateral-load approach (14). Dashed curves, representing the responses of the braced frame, without bracing members and with 18% bracing, are shown for α equal to 1/2 and 1%, (where α is the fraction of the total vertical load on the frame, applied in the horizontal direction). The results of the small-lateral-load analysis approached those of the present analysis as α approached zero.

In order to investigate the consequences of the buckling of the bracing members due to axial shortening, the subassemblage frame was re-analyzed with the compressive capacity of the braces reduced. The results of the analysis are shown in FIGURE 6.27, where the vertical load factor is plotted versus an expanded sway deflection scale. The relationship for the braced frame without bracing is shown for comparison. The sequence of formation of plastic hinges in the braced frame is shown in FIGURE 6.28. The numbers associated with the hinges correspond to the stages on the load-deflection diagram where the hinges developed. At stages 1, 2, 3 and 4 respectively, the diagonal bracing members reached their critical axial loads in compression. Between stages 4 and 5, the response of the braced frame corresponded to that of the braced frame without bracing, since the bracing in the buckled state was unable to provide any lateral stiffness to the structure. At stage 5, one of the braces in each story returned to the elastic range, and the frame was able to resist additional load. The ultimate load corresponded to the beam-mechanism load for the structure.

Thus in the case where the bracing members buckle due to axial shortening of the columns, the frame instability load is not necessarily impaired. In the twenty-four story frame the small lateral load approach verified that the beam mechanism load could still be reached. The lateral loads can be thought of as the equivalent story P-Δ shears that result from alignment imperfections. A value of .001 times the total applied vertical load in a given story corresponds to an erection tolerance of .001 in the columns. In the subassemblage frame it was found that the sway motion increased rapidly enough to return the braces to the elastic range before the frame instability load was reached. Under additional vertical load the frame remained stable, and the beam mechanism load was achieved.

6.4 Summary

In this chapter the general outline of the behavioral studies has been presented. The basic structures to be investigated were described in detail, and the results of each phase of the study were presented. The behavioral studies are discussed in the next chapter, as are the resulting design implications.

		Working Load
Roof Girders	Live	0.72
	Dead	2.28
	Total	3.00
Floor Girders AB	Live	1.48
	Dead	2.88
	Total	4.36
Floor Girders BC	Live	1.85
	Dead	2.88
	Total	4.73
Floor Girders CD	Live	1.18
	Dead	2.88
	Total	4.06

TABLE 6.1

SERIES A FRAMES - DESIGN GIRDER LOADS

Story	Column A	Column B	Column C	Column D
24	45.7	55.5	67.5	57.7
23	125.9	135.7	162.2	154.5
22	196.9	200.4	244.8	235.3
21	263.4	272.7	333.5	324.1
20	336.0	345.1	422.2	412.9
19	408.6	417.1	510.2	501.7
18	481.2	489.9	599.6	590.5
17	553.8	562.3	688.3	679.3
16	626.4	634.7	777.0	768.1
15	699.0	707.1	865.7	856.9
14	771.6	779.5	954.4	945.7
13	844.2	851.9	1043.1	1034.5
12	916.8	924.3	1131.8	1123.3
11	989.4	996.7	1220.5	1212.1
10	1062.0	1069.1	1309.2	1300.9
9	1134.6	1141.5	1397.9	1389.7
8	1207.2	1213.9	1986.6	1478.5
7	1279.8	1286.3	1575.3	1567.3
6	1352.4	1358.7	1664.0	1656.1
5	1425.0	1431.1	1752.7	1744.9
4	1497.6	1503.5	1841.4	1833.7
3	1570.2	1575.9	1930.1	1922.5
2	1642.8	1648.3	2018.8	2011.3
1	1715.4	1720.2	2107.5	2100.1

TABLE 6.2

SERIES A FRAMES - DESIGN COLUMN AXIAL LOADS

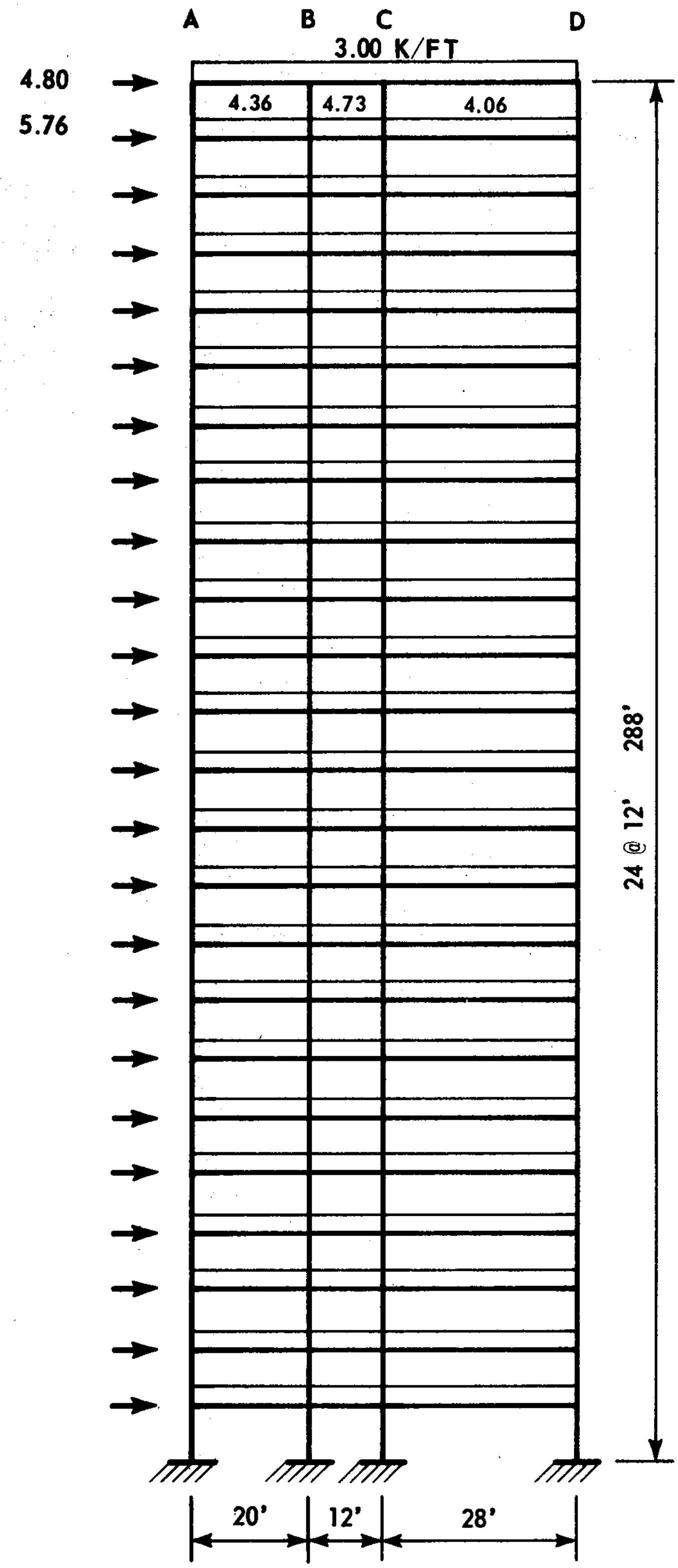
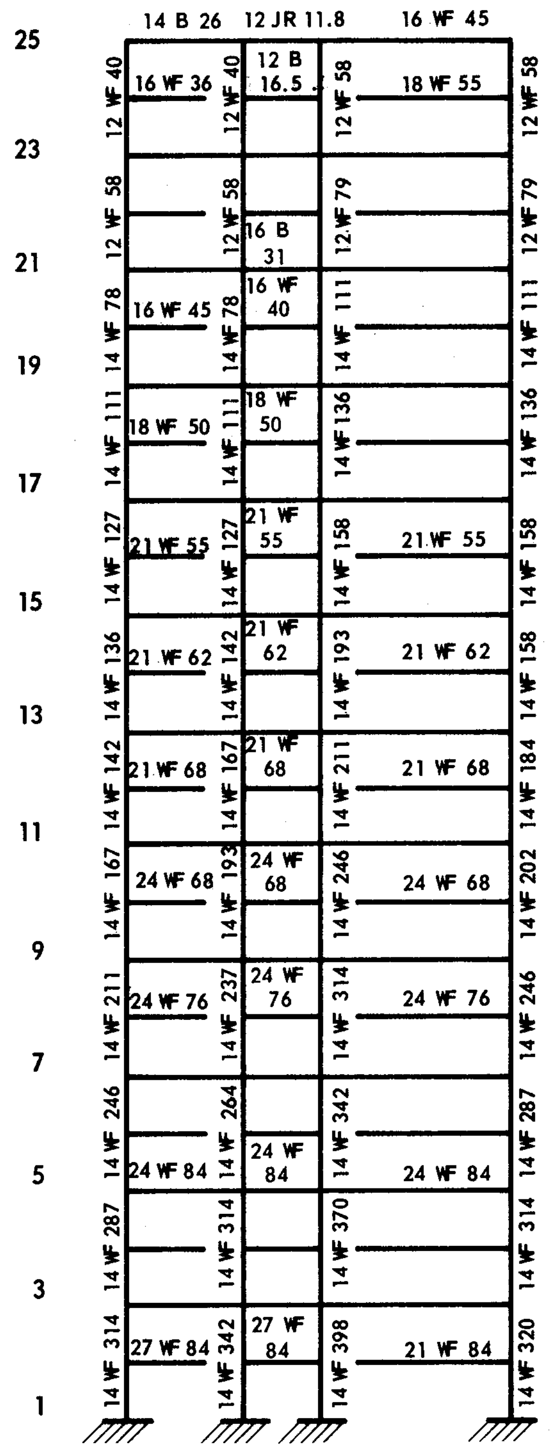


FIGURE 6.1

FRAME CONFIGURATION AND DESIGN LOADS - SERIES A

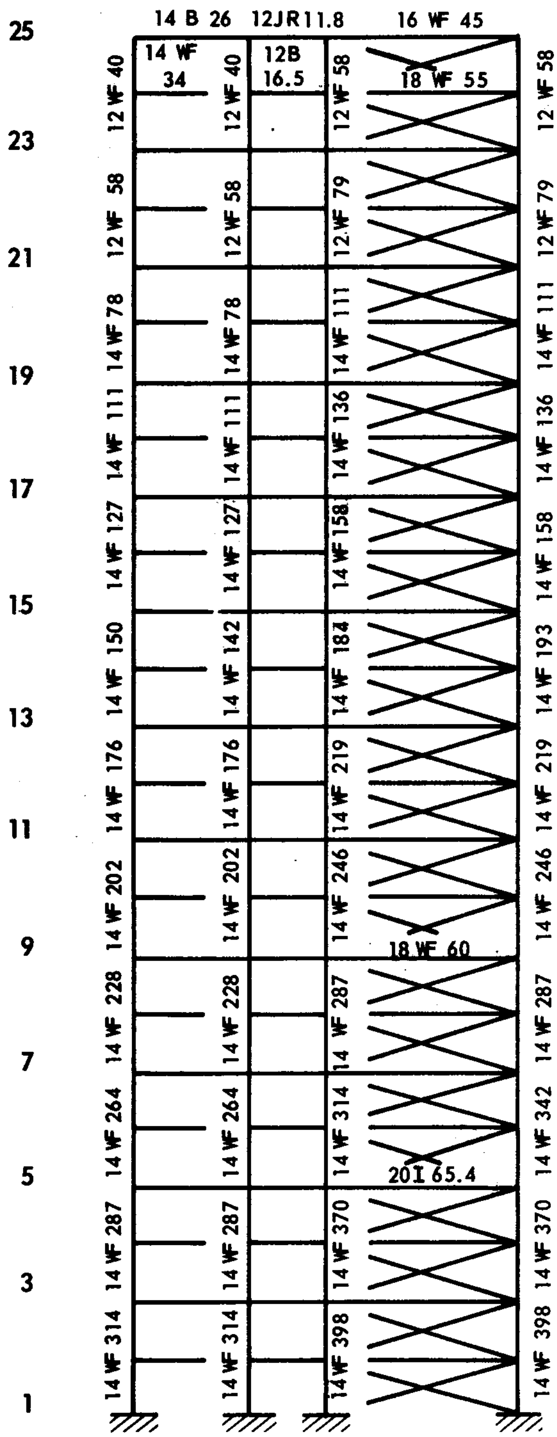


COLUMNS BELOW LEVEL 15 ARE A 441 STEEL

COLUMNS ABOVE LEVEL 15 AND ALL GIRDERS ARE A 36 STEEL

FIGURE 6.2

FRAME A-1



LEVEL 1 TO 3 2 L 5x3x7/16
LEVEL 3 TO 7 2 L 5x3 1/2 x 3/8
LEVEL 7 TO 9 2 L 5x3x5/16
LEVEL 9 TO 13 2 L 4x3 1/2x 1/4
LEVEL 13 TO 14 2 L 4x3x 1/4
LEVEL 14 TO 16 2 L 3 1/2x3x1/4
LEVEL 16 TO 25 2 L 3x2 1/2x1/4

ALL MATERIAL A36 STEEL

FIGURE 6.3

FRAME A-2

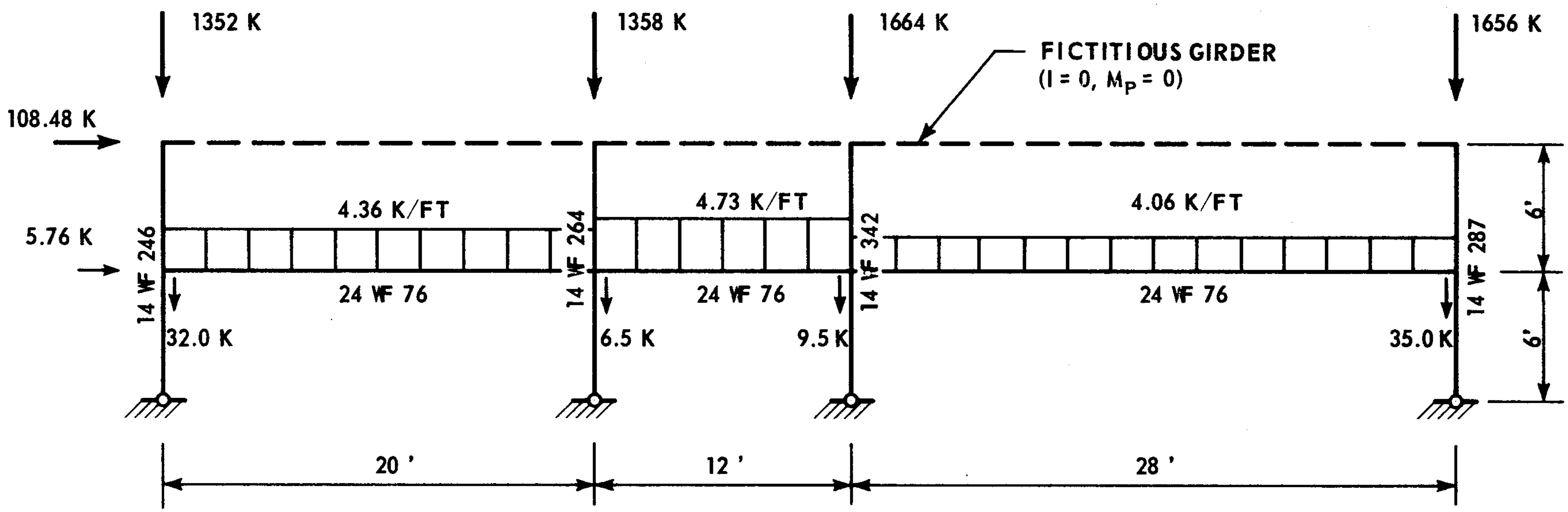
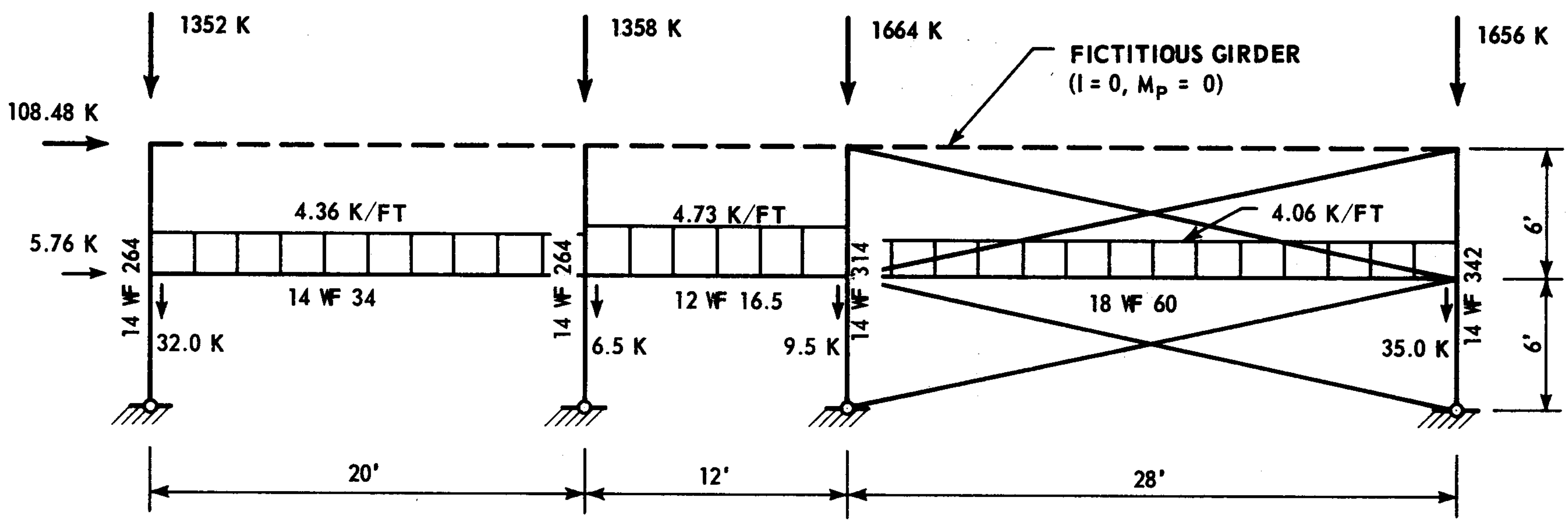


FIGURE 6.4

FRAME B-1



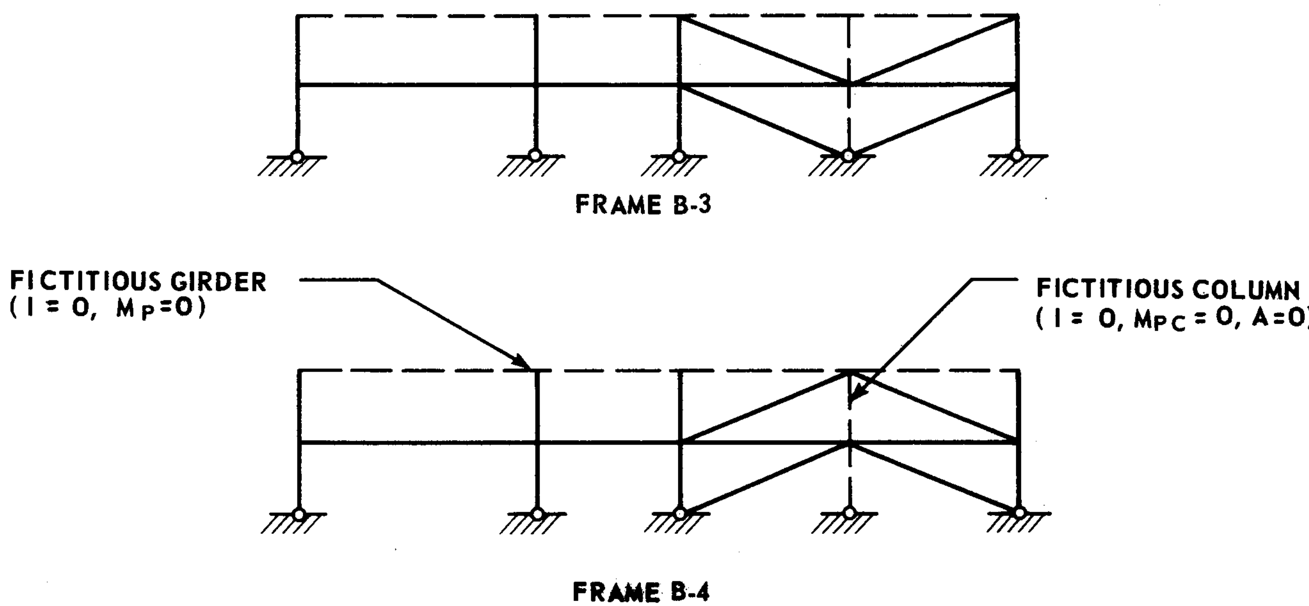
BRACING MEMBERS

$A = 5.39\ IN^2$ $\quad F_y = 38.30$ KSI

$r = 1.55$ IN $\quad E = 2900$ KSI

FIGURE 6.5

FRAME B-2



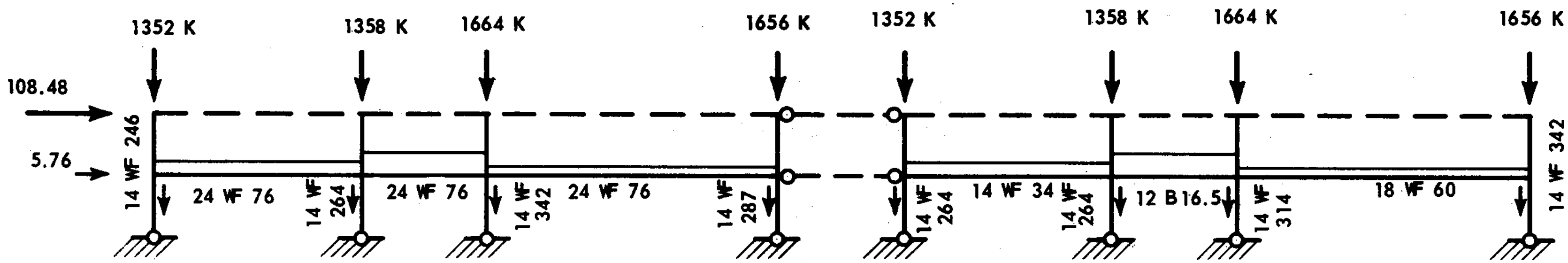
BRACING MEMBERS

$A = 3.05\ IN^2$ $F_O = 72.0$ KSI

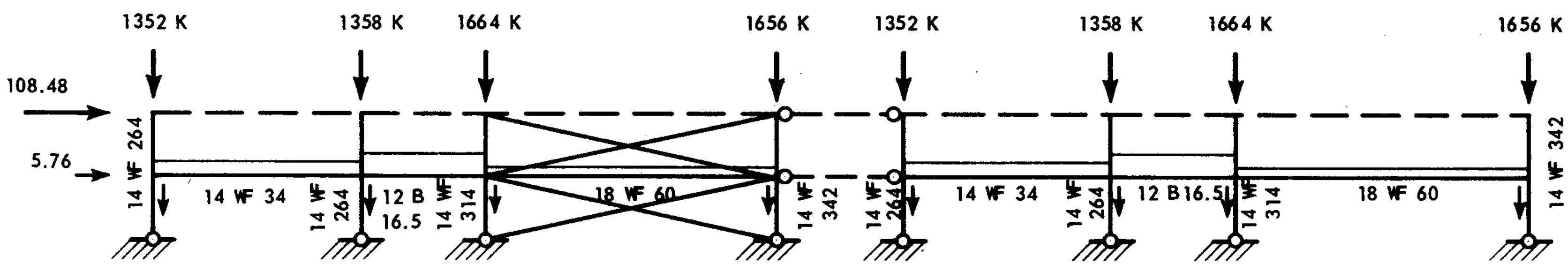
$r = 1.13$ IN $E = 29000$ KSI

FIGURE 6.6

FRAMES B-3 AND B-4



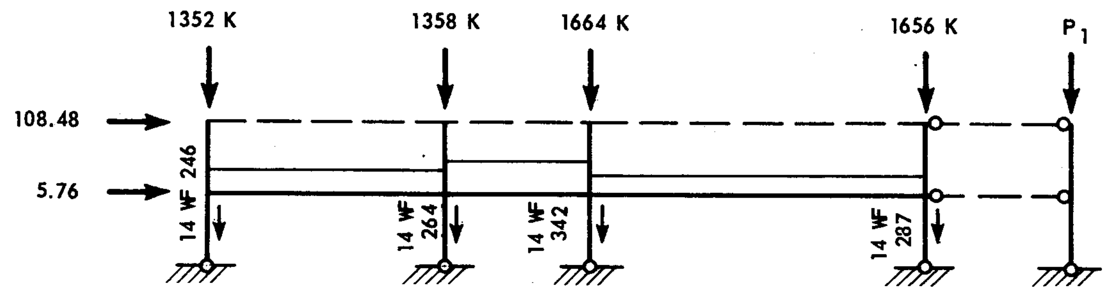
FRAME B-5 UNBRACED-SUPPORTED



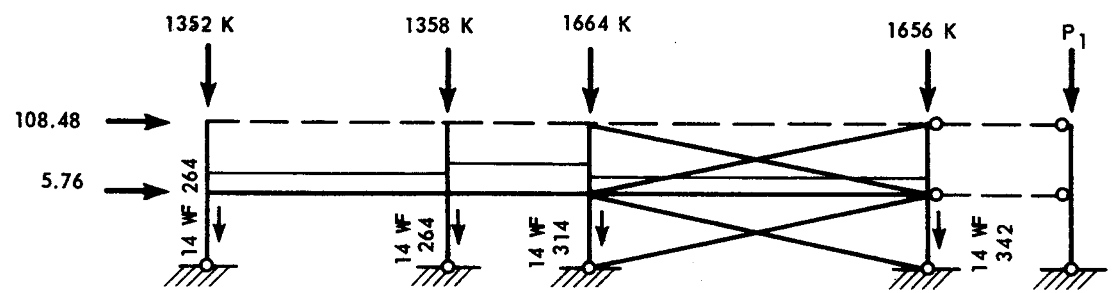
FRAME B-6 BRACED-SUPPORTED

FIGURE 6.7

FRAMES B-5 AND B-6



FRAME B-7 UNBRACED - PINNED COLUMN



FRAME B-8 BRACED - PINNED COLUMN

FIGURE 6.8

FRAMES B-7 AND B-8

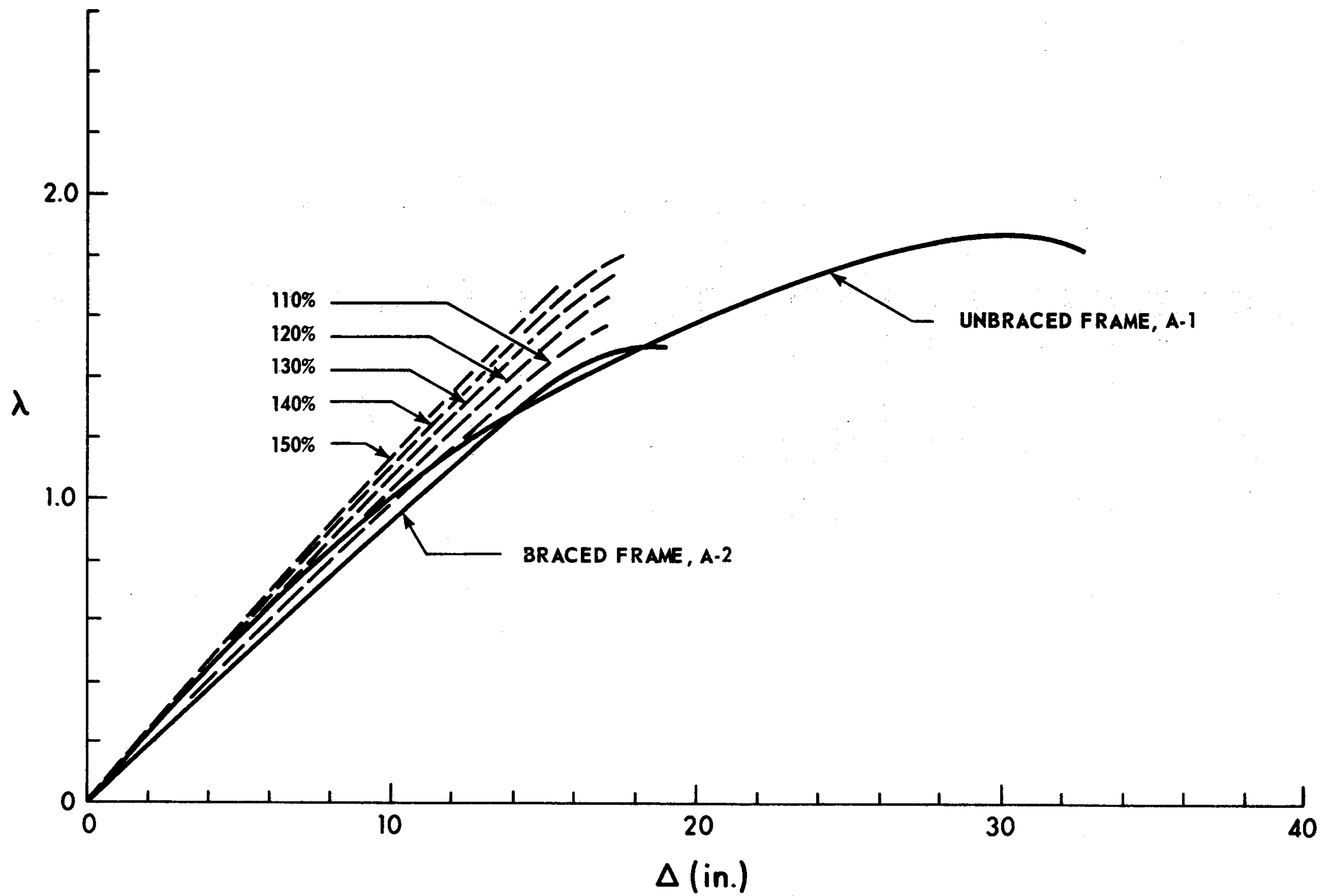


FIGURE 6.9

LOAD-DEFLECTION RELATIONSHIPS

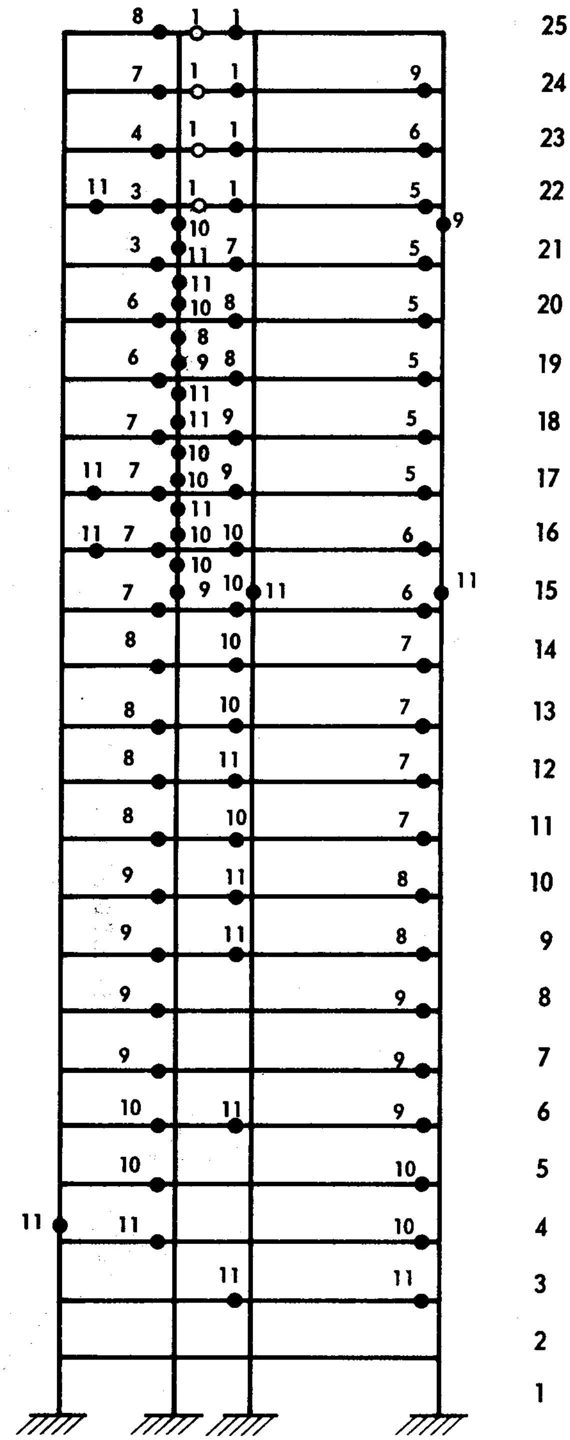


FIGURE 6.10

FRAME A-1 - HINGING CONDITION AT FAILURE

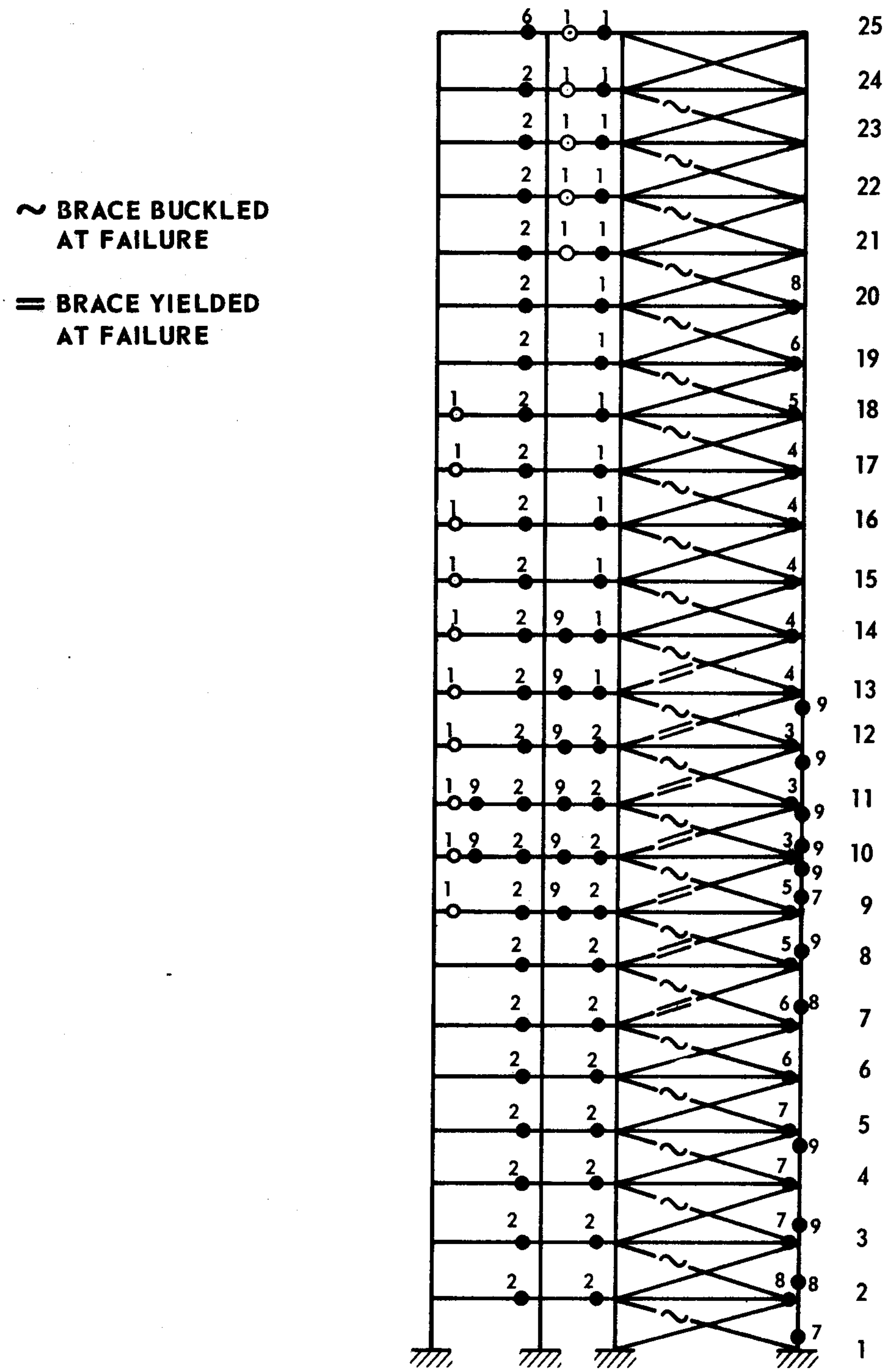


FIGURE 6.11

FRAME A-2 - HINGING CONDITION AT FAILURE

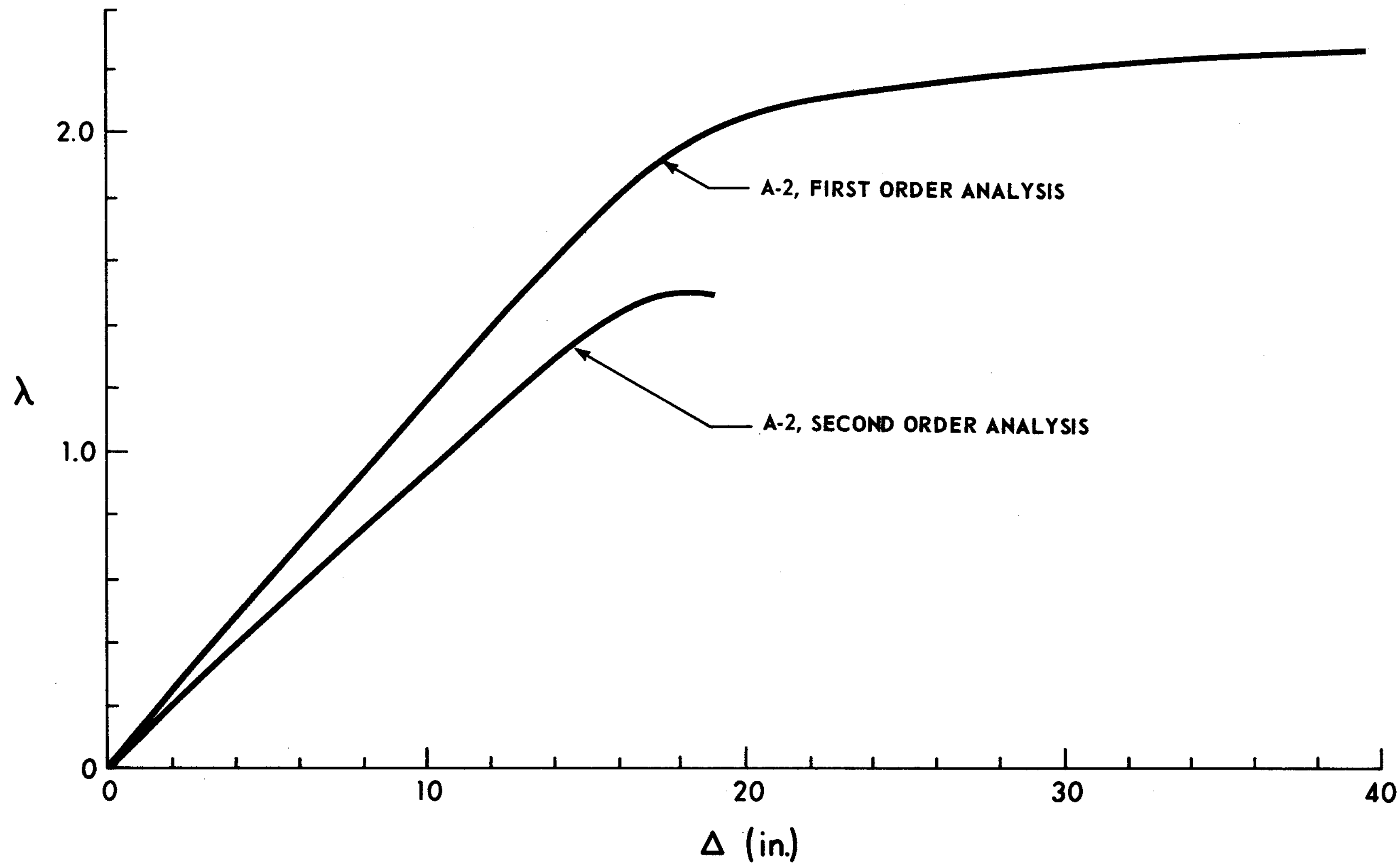


FIGURE 6.12
LOAD-DEFLECTION RELATIONSHIPS

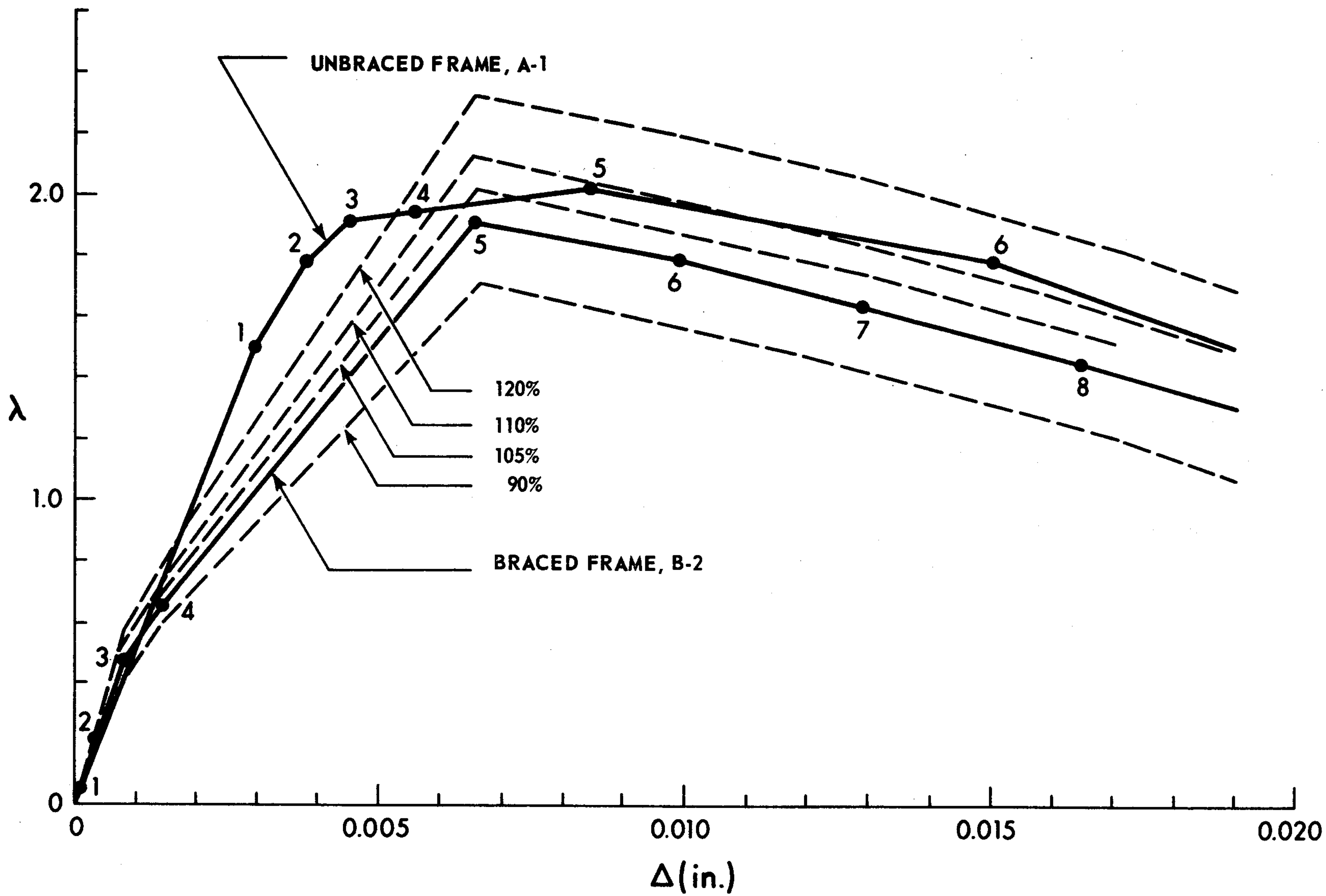


FIGURE 6.13

LOAD-DEFLECTION RELATIONSHIPS

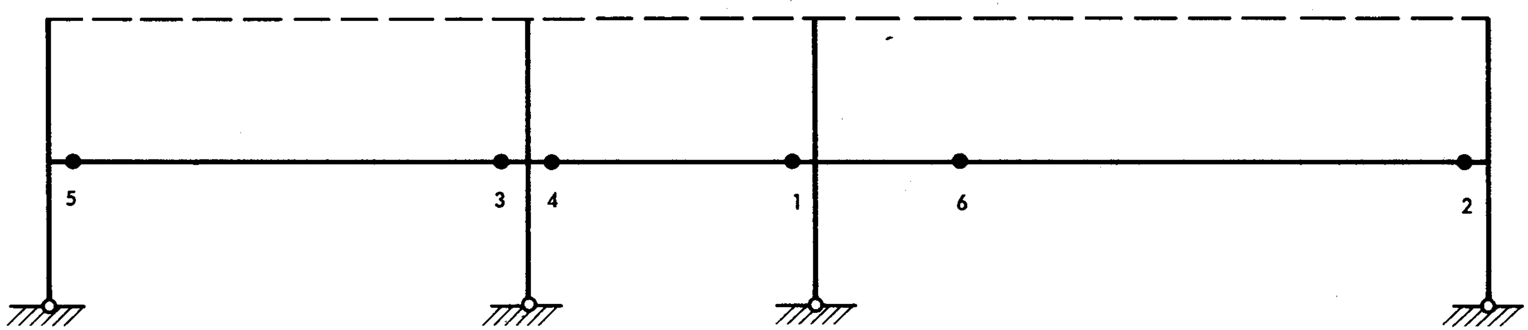


FIGURE 6.14

FRAME B-1 SEQUENCE OF PLASTIC HINGE FORMATION

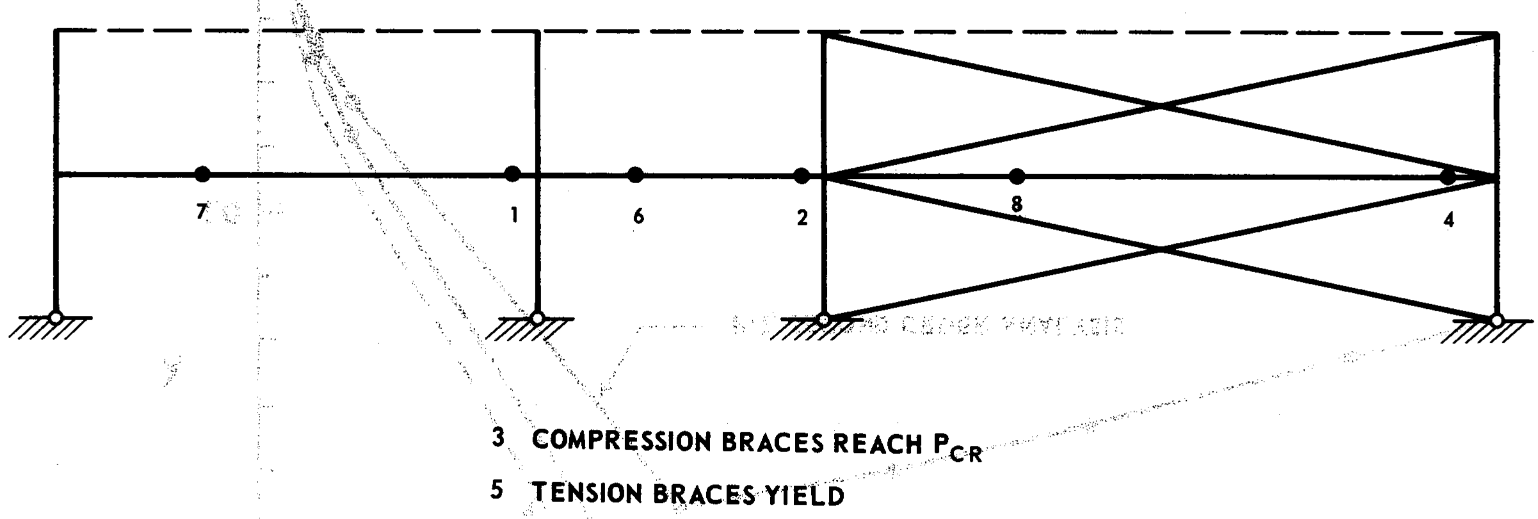


FIGURE 6.15

FRAME B-2 SEQUENCE OF PLASTIC HINGE FORMATION

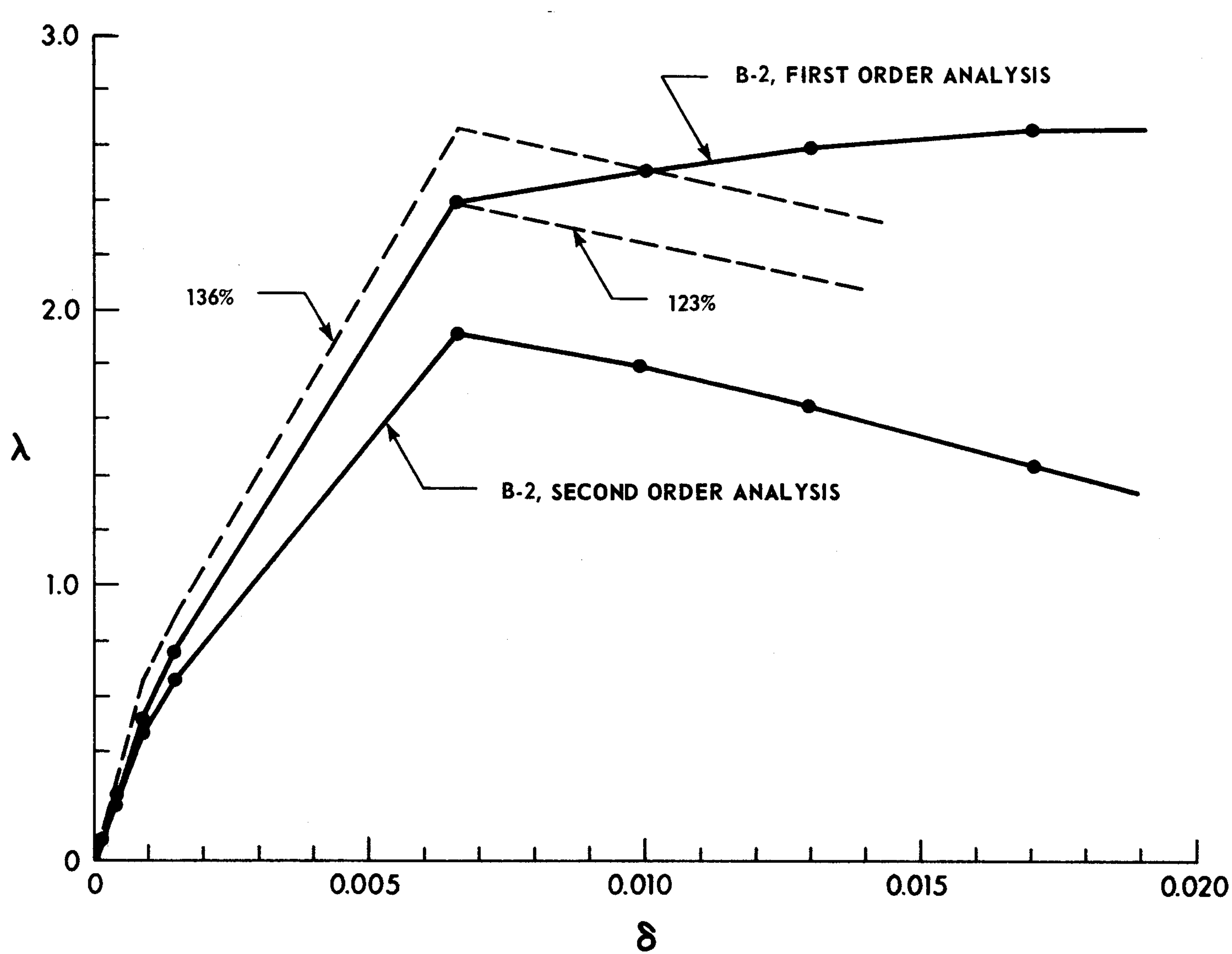


FIGURE 6.16
LOAD-DEFLECTION RELATIONSHIPS

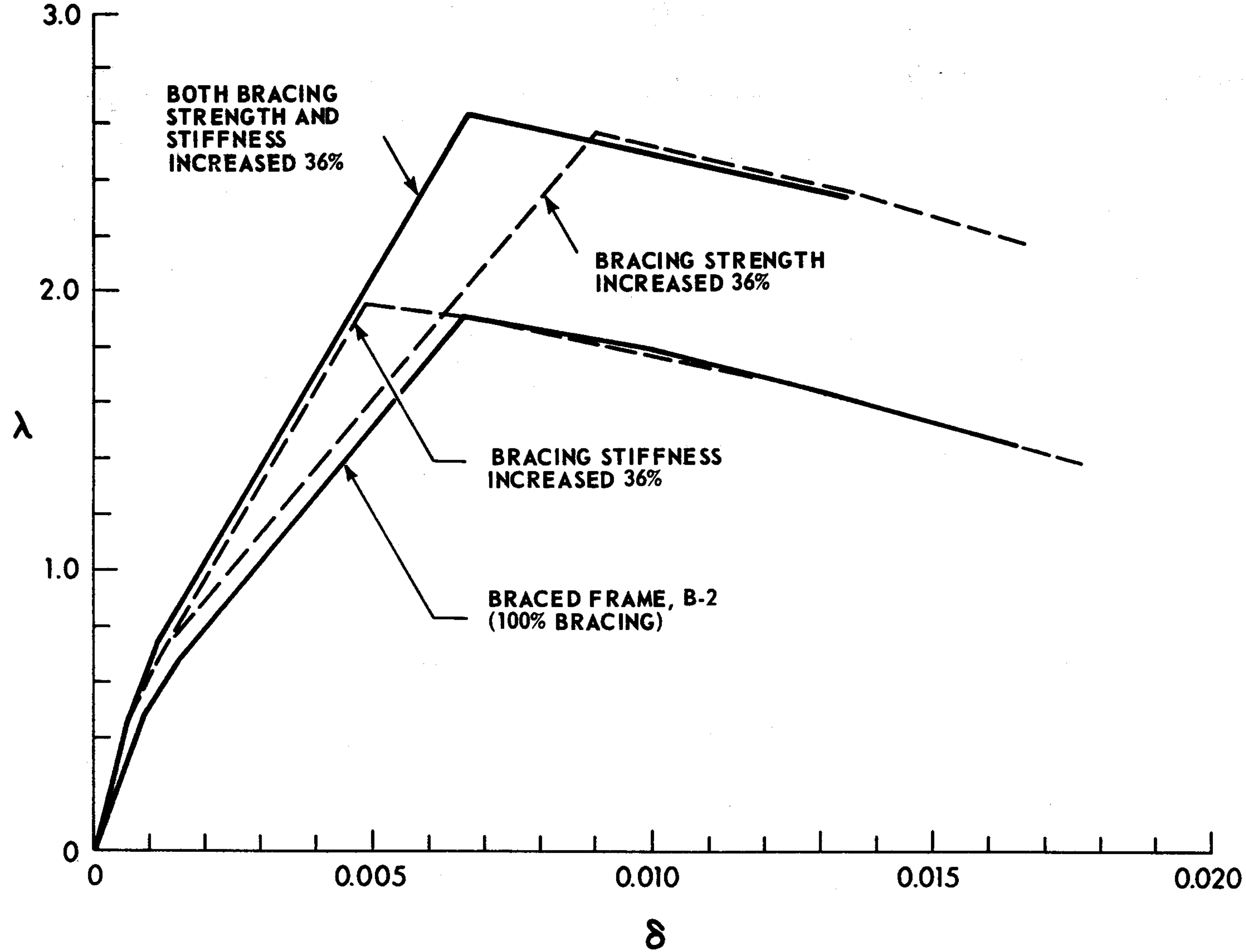


FIGURE 6.17

LOAD-DEFLECTION RELATIONSHIPS

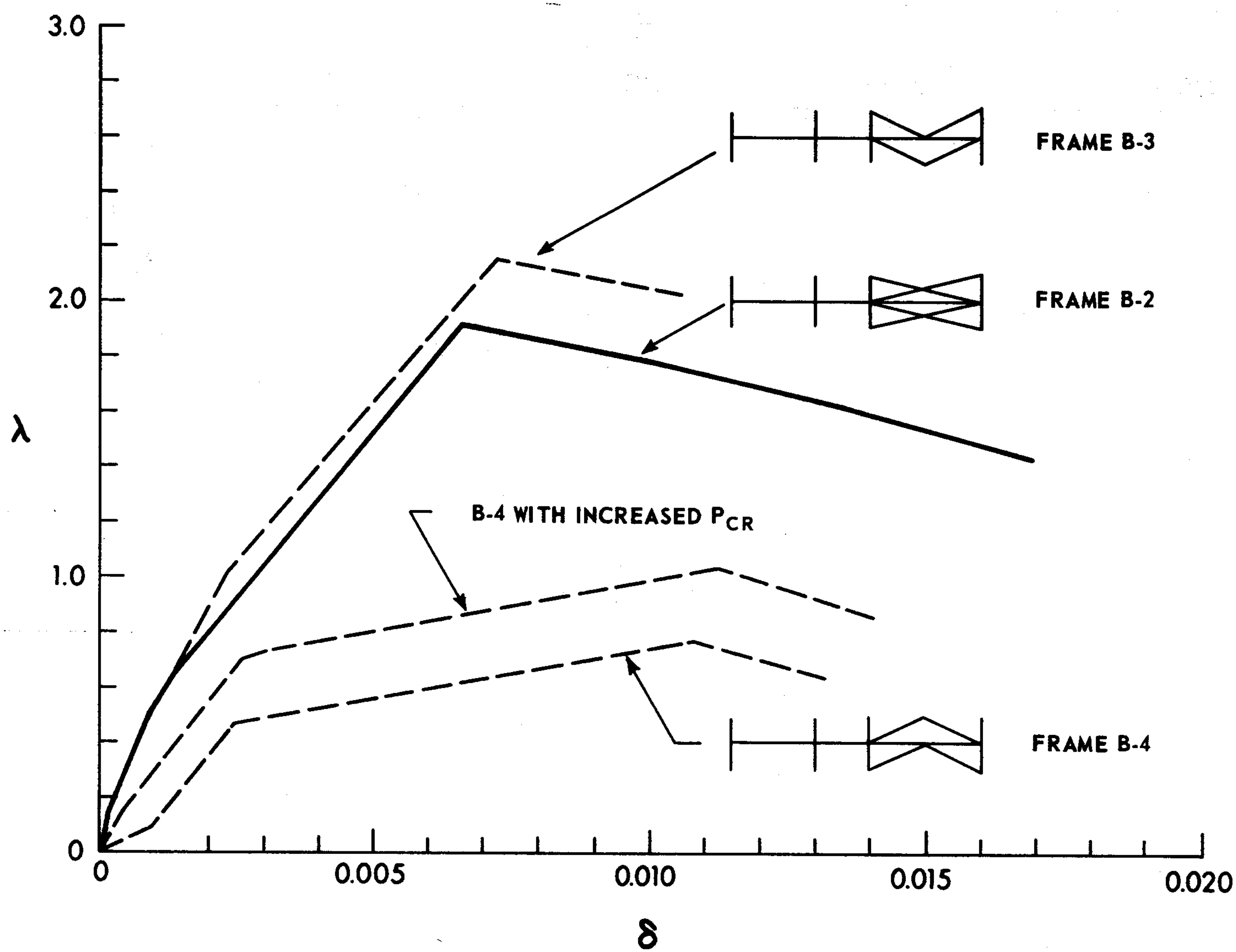


FIGURE 6.18
LOAD-DEFLECTION RELATIONSHIPS

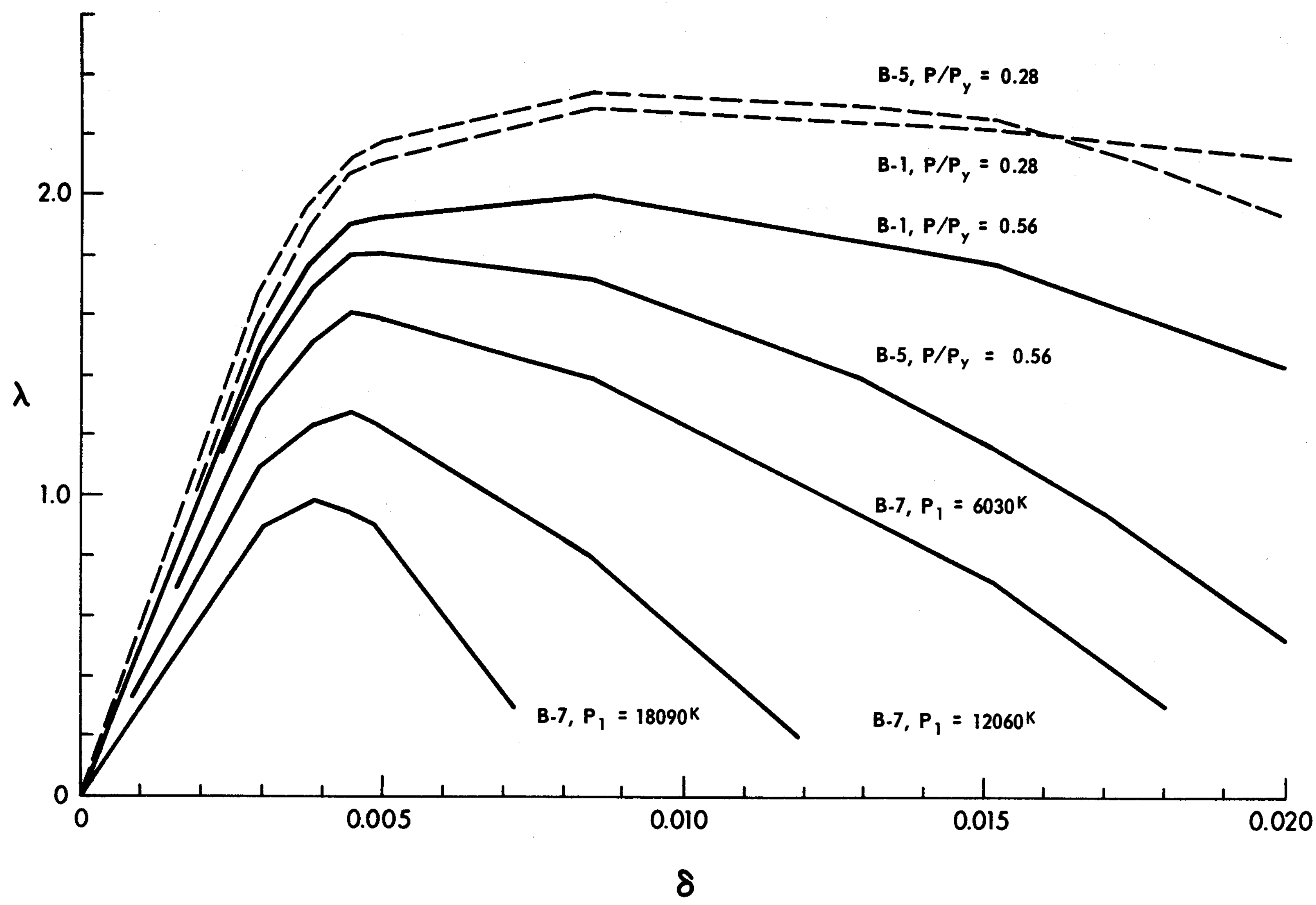


FIGURE 6.19

LOAD-DEFLECTION RELATIONSHIPS

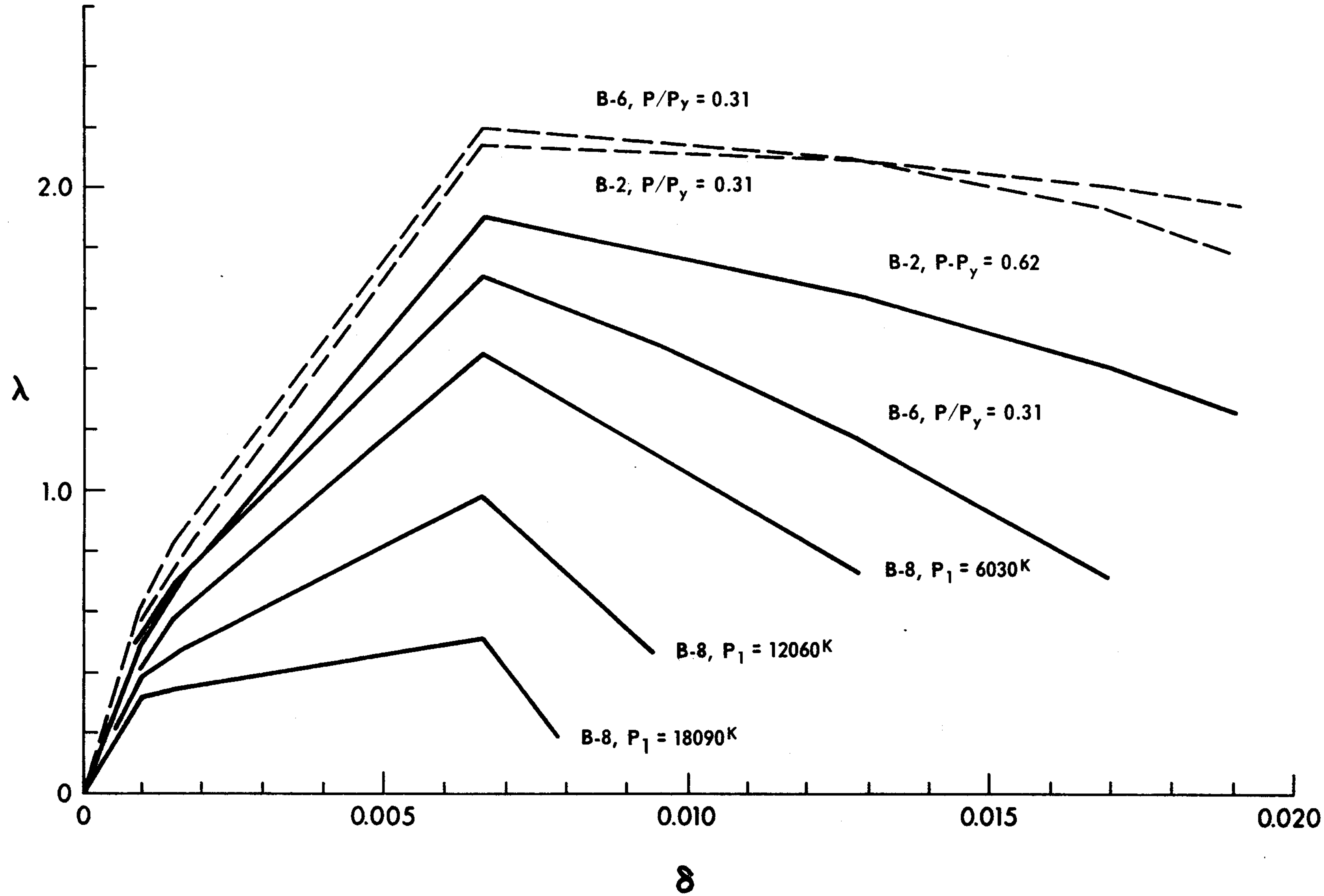


FIGURE 6.20
LOAD-DEFLECTION RELATIONSHIPS

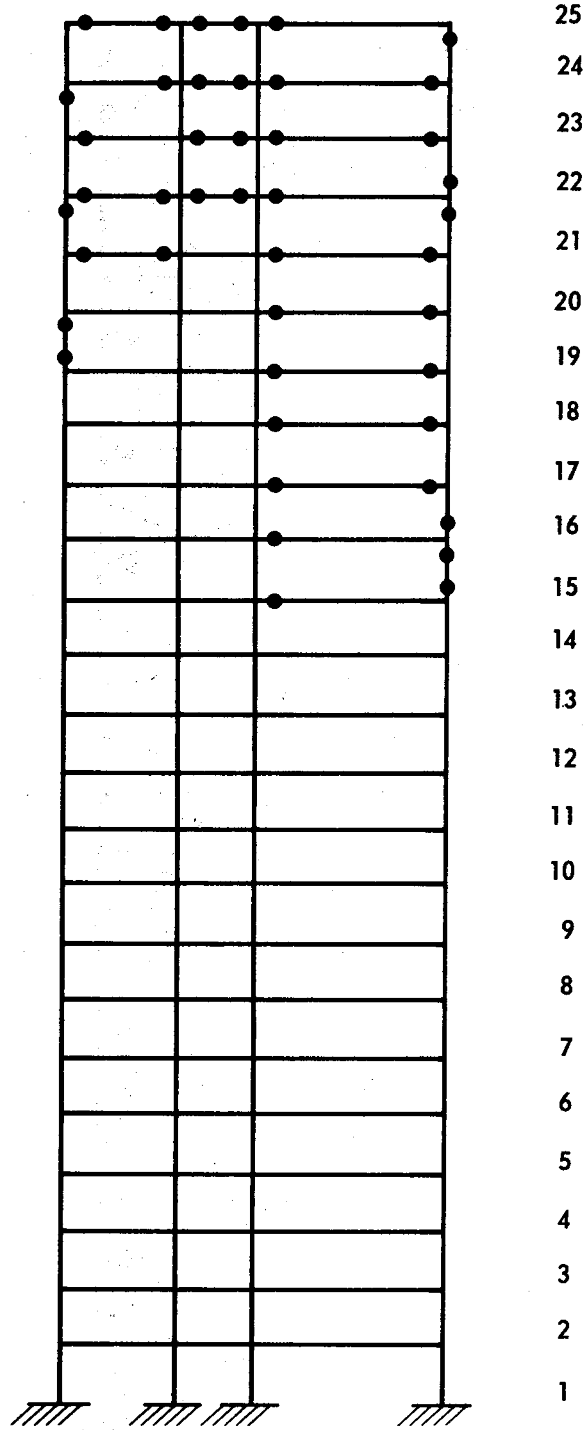


FIGURE 6.21

FRAME A-1 HINGING CONDITION AT FAILURE

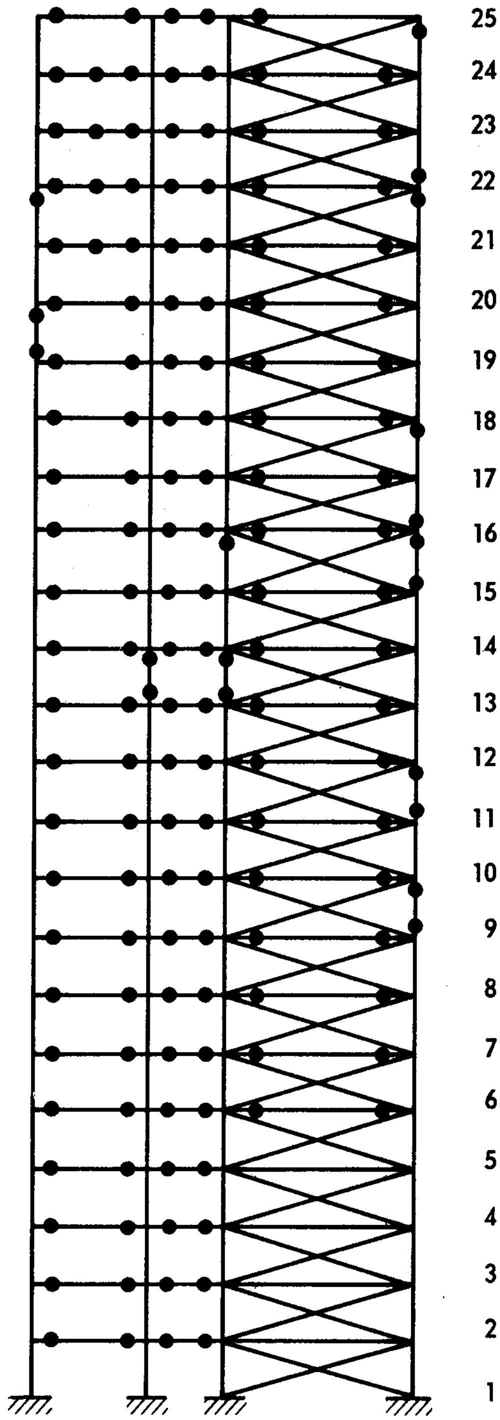


FIGURE 6.22

FRAME B-2 SEQUENCE OF PLASTIC HINGE FORMATION

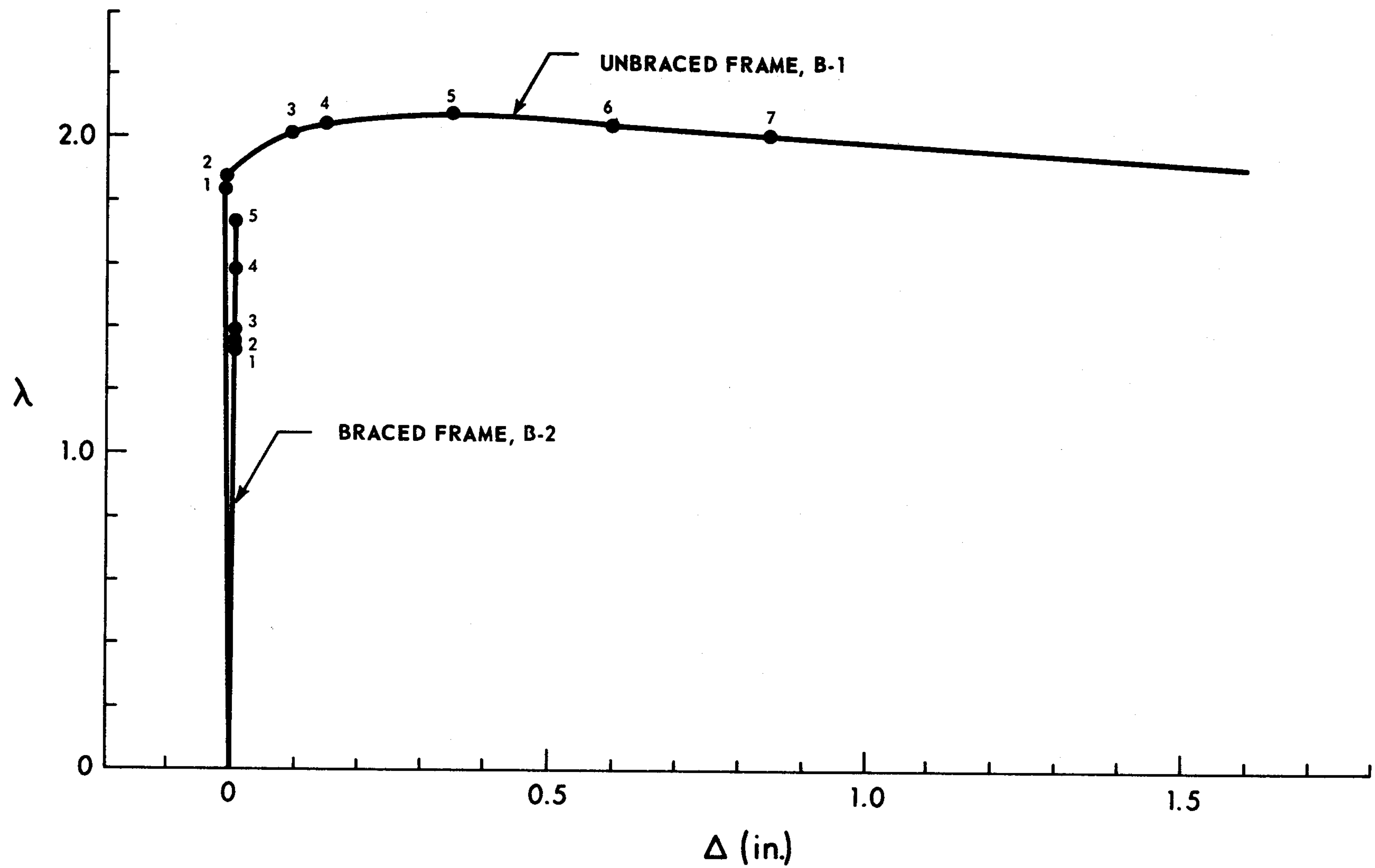


FIGURE 6.23

LOAD-DEFLECTION RELATIONSHIPS

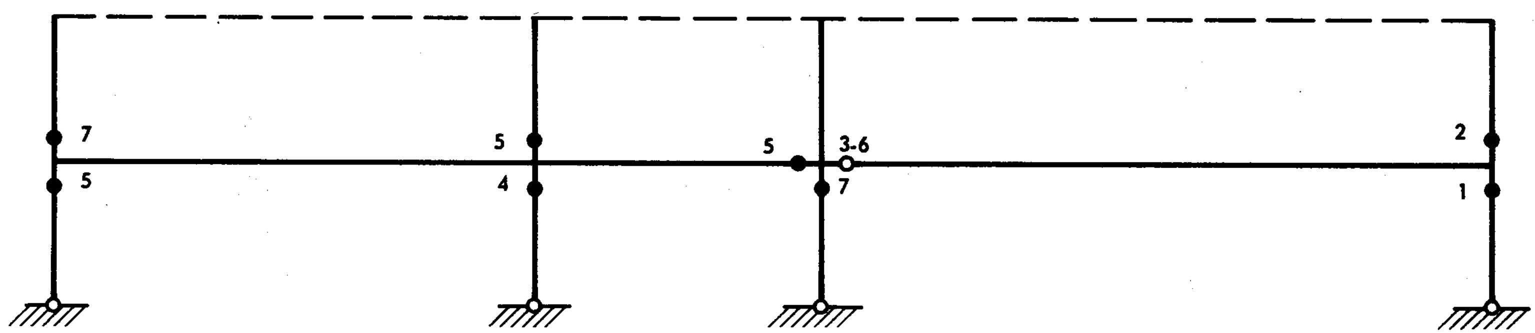


FIGURE 6.24

FRAME B-1 SEQUENCE OF PLASTIC HINGE FORMATION

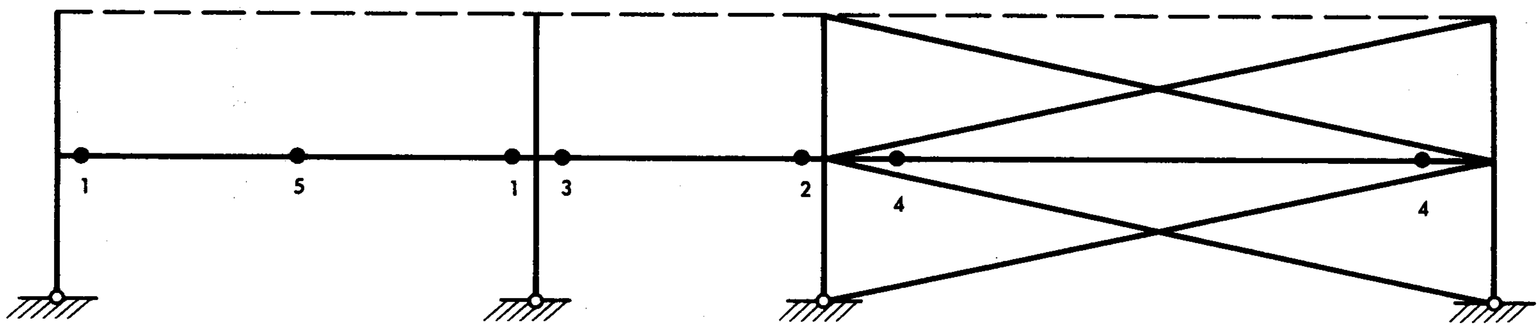


FIGURE 6.25

FRAME B-2 SEQUENCE OF PLASTIC HINGE FORMATION

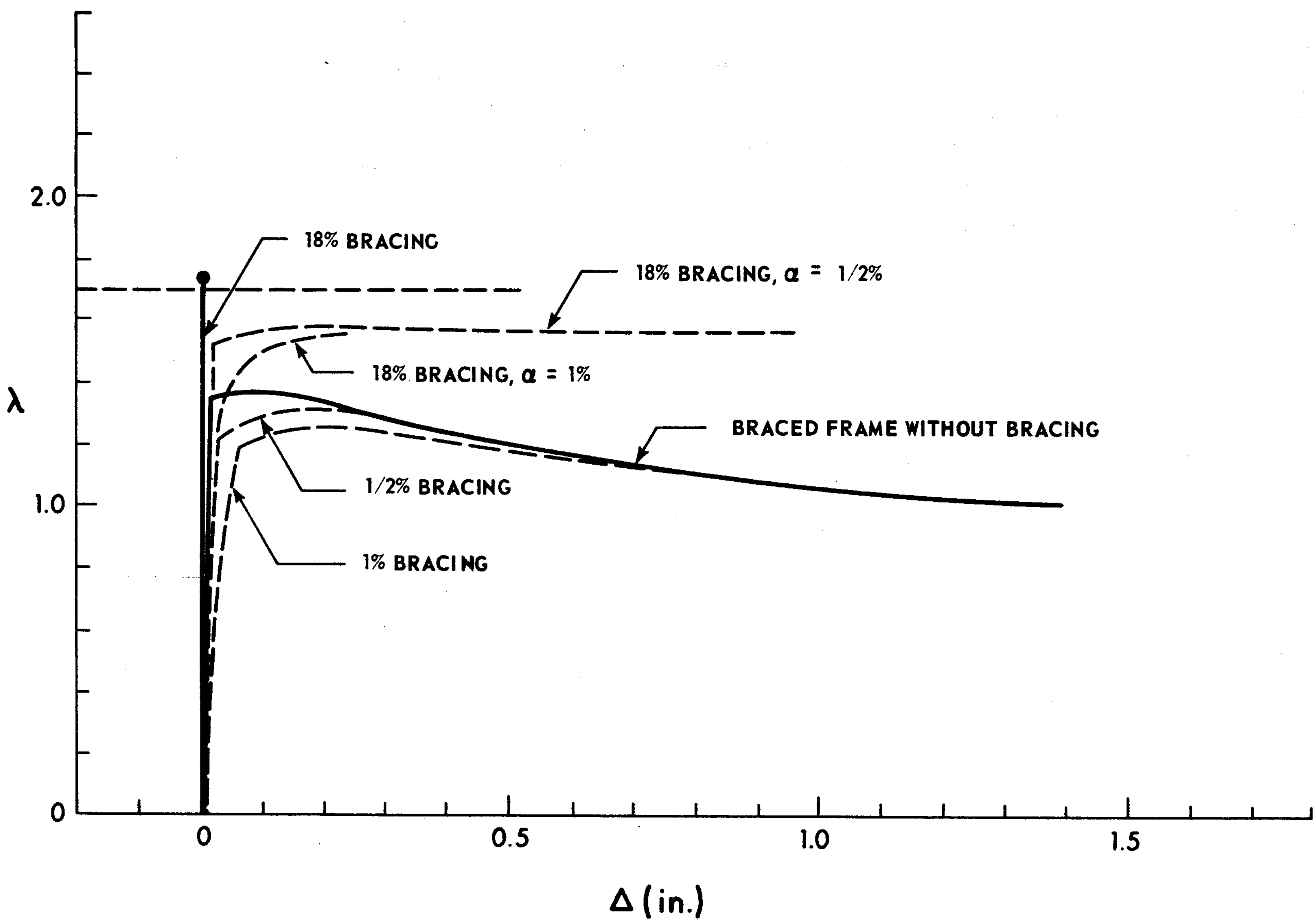


FIGURE 6.26

LOAD-DEFLECTION RELATIONSHIPS

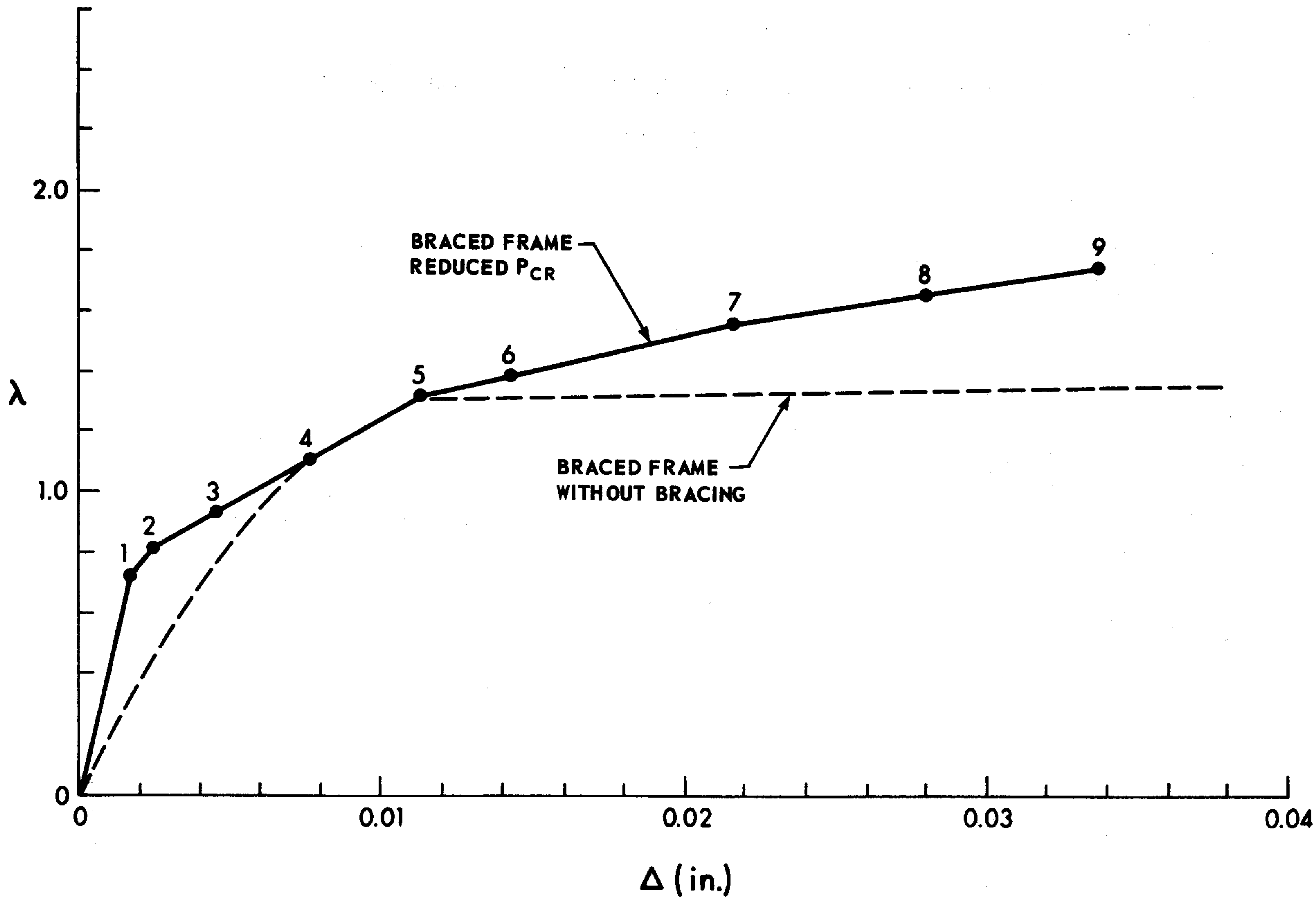


FIGURE 6.27

LOAD-DEFLECTION RELATIONSHIPS

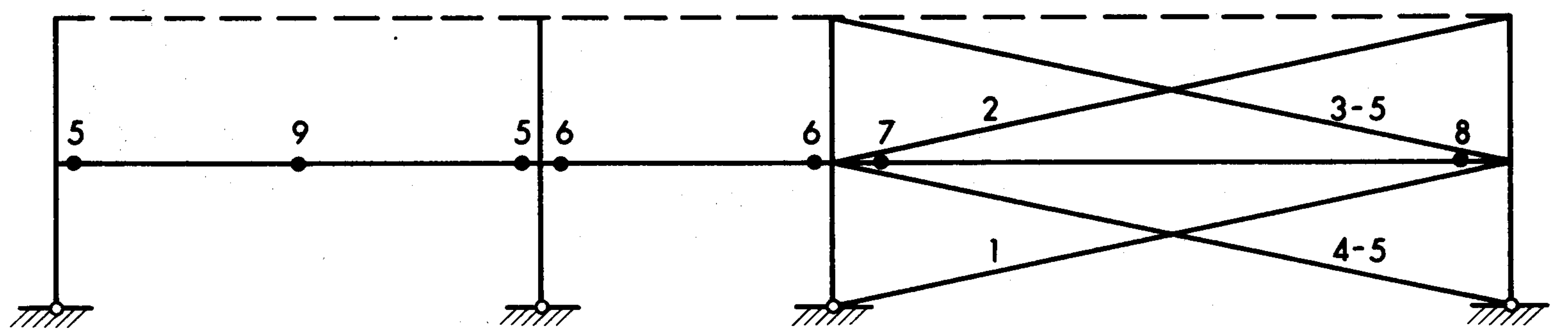


FIGURE 6.28

FRAME B-2 WITH REDUCED P_{CR} - SEQUENCE OF PLASTIC HINGE FORMATION

CHAPTER VII

DISCUSSION OF RESULTS AND DESIGN RECOMMENDATIONS

7.1 Introduction

The behavioral studies described in the previous chapter were performed to study the response of a number of frames to applied loads, and to develop rational design procedures for the girders, columns and bracing systems. In the following section the results of these studies are discussed, and the behavior of each subassemblage frame is compared with that of the corresponding twenty-four story frame. In the final section of the chapter, the implications of the results on current design practices are discussed, and recommendations are made for design.

7.2 Discussion of the Results of the Behavioral Studies

7.2.1 Combined Vertical and Lateral Loads

A comparison of the behavior of the unbraced twenty-four story frame and the corresponding braced frame, subjected to combined vertical and lateral loads, indicates that the unbraced frame reached a higher ultimate load factor than the braced frame, and that the deformation at ultimate load was greater. The ultimate load factor for both frames exceeded the design value of 1.30 by a considerable margin. Of greater significance, however, is the fact that the braced frame deflected more than the unbraced frame at all loads up to the factored design load. Thus the P-Δ effects were more significant for the braced frame than for the unbraced frame.

The unbraced and braced subassemblage frames exhibited the same characteristics as the twenty-four story frames. Both subassemblage frames reached a higher ultimate load factor than the corresponding twenty-four story frames, due to the fact that the sixth level did not participate in the failure mechanism in either of the twenty-four story structures. The difference in stiffness between the braced and unbraced frames (the unbraced frame was stiffer) was more pronounced for the subassemblage frames.

The comparison between the first and second order analyses of the braced twenty-four story frame, showed that the ultimate load factor was reduced 36% by the P-Δ effects; the corresponding reduction for the braced subassemblage frame was 28%. In the latter frame an additional 23% bracing was required to achieve the first order load factor at the yield of the tension braces, and an additional 36% to achieve the first order load factor at ultimate load.

A study of the relative magnitudes of the P-Δ effects in a series of linked frames was performed using the unbraced and braced subassemblage frames alternately as the principal lateral stiffening element. Where the unbraced bent was coupled with the supported bent, the ultimate load factor for the frame was reduced 9% below that of the unbraced bent alone. The corresponding reduction for the braced bent coupled with the supported bent, was 10%. Where the unbraced and braced bents were coupled with a single column, used to represent a given number of non-rigid bents, the reductions in the ultimate load factors were more severe. When the column carried an axial load equal to that of the unbraced or braced bent, the reductions were 20% for the unbraced bent and 24% for the braced bent. With twice the axial load applied to the column, the reductions were 36% and 49%; and with three times the axial load, 50% and 73%. Thus in all cases the P-Δ effects for the coupled braced frames were greater than those for the corresponding unbraced frames.

7.2.2 Vertical Loads Only

In the study of the behavior of the braced twenty-four story frame subjected to vertical loads only, it was found that a minimal amount of bracing was necessary to prevent frame instability before the beam mechanism load for the structure was attained. The results were substantiated by the study of the braced subassemblage frame. In the twenty-four story frame many of the bracing members buckled prior to the attainment of the factored design vertical load. However, the small-lateral-load approach confirmed that the bracing members were capable of returning to the elastic range on the buckling motion, without adversely affecting the frame instability load. In the study of the braced subassemblage frame, the small-lateral-load approach was shown to give a lower bound solution to the frame instability load.

7.3 Design Recommendations

7.3.1 Interaction Equations

As discussed in CHAPTER I, the columns in a multi-story frame are commonly designed on the basis of interaction equations, based on the ultimate strength of the member. Each column must be checked against local overstressing, (EQUATION 7.1), and overall instability, (EQUATION 7.2). The form of Equations 7.1 and 7.2 is that used in allowable stress design:

$$\frac{f_a}{0.60\ F_y} + \frac{f_b}{F_b} \leq 1.0 \qquad (7.1)$$

$$\frac{f_a}{F_a} + \frac{C_m f_b \alpha}{F_b} \leq 1.0 \qquad (7.2)$$

where f_a = axial stress,

f_b = bending stress,

F_a = allowable axial stress in the absence of bending,

F_b = allowable bending stress,

F_y = yield stress of the steel,

C_m = coefficient used to determine the equivalent uniform bending moment,

$$\alpha = \frac{1}{1 - \frac{f_a}{F_e'}}, \text{ where } F_e' = \frac{149{,}000}{(\frac{KL}{r})^2}, \text{ and}$$

$\frac{KL}{r}$ = effective slenderness ratio in the plane of bending

The amplification factor, α, modifies the bending stress to account for the secondary ($P\delta$) moments caused by the deflection of the column from its chord, as shown in FIGURE 1.2. The effective length factor, K, and the equivalent moment factor, C_m, are used to modify both the axial and bending stresses, to account for the P-Δ moments produced by the sway deflection of the column. The P-Δ moments are assumed to be accounted for by using the effective length factor for a column permitted to sway, $(K > 1)$, and are neglected when the effective length factor for a column prevented from sway is used, $(K < 1)$.

Common design practice is to consider the columns in unbraced bents as permitted to sway, and those in braced or supported bents as prevented from sway. However, the results of the behavioral studies in the last chapter indicated that the braced frames swayed more than the corresponding unbraced frames at the same load level. In addition, the comparisons of the first and second order analyses of the braced frames showed that the ultimate load capacities were appreciably reduced by the P-Δ effects. The study of the coupled frames, indeed, showed that the braced bent was more susceptible to a reduction in its load carrying capacity due to P-Δ effects, than was the unbraced bent. Thus a braced or supported bent cannot be considered "prevented from sway".

The important concept is not whether one considers a frame as "prevented from sway" or "permitted to sway", but, rather, the way in which the designer accounts for the P-Δ effects. It follows that if a structure is to be designed using moments and forces from a first order analysis, in combination with the interaction equations, then the effective length must be computed by assuming the frame free to translate, since the sway forces have not been accounted for. On the other hand, if the structure is analyzed under the action of the applied loads and the sway forces, the sway forces have been included in the basic analysis, and need not be considered a second time. Under these conditions, the interaction equations are used to compensate for the neglect of the secondary moments in the columns, $(P\delta)$, and the effective lengths are computed assuming that translation is prevented. Thus the choice of the nomograph to use in computing the effective length factor does not depend on whether the structure does or does not contain a bracing system or shear wall, but rather whether the sway forces are to be accounted for in the basic analysis procedure or within the interaction equations.

If it is desired to resist all the lateral forces in a particular portion of the structure, such as shear wall or vertical truss, this portion must be designed for the sway forces as well as the applied lateral loads. The effective length factors for the columns may then be computed assuming translation is prevented.

If the bracing is designed to resist lateral loads only, without consideration of the sway forces, no guarantee exists that the system is capable of resisting the sway forces, and therefore, in computing the effective length factors for the columns in the structure, the conservative assumption would be that the columns were permitted to sway.

The moments and forces from a first order analysis are also used in the strength interaction equation for combined axial load and bending, EQUATION 7.1. No provision is made for the P-Δ effects. Thus the actual factor of safety against local overstressing obtained using EQUATION 7.1 is dependent on the relative magnitude of the P-Δ moments and forces. To produce a uniform factor of safety against local overstressing, the axial and bending stresses must be based on a second order analysis.

When the moments and forces from a first order analysis are used in conjunction with the interaction equations, only the columns are designed for the P-Δ effects. In most practical cases the strength of the girders controls the strength of the frame. Thus the use of the interaction equations does not generally account for the influence of the P-Δ effects on the ultimate strength of the frame. For the unbraced subassemblage frame, based on the moment capacity of the columns, the interaction equations predict a story shear capacity of 439 kips including the P-Δ effects, and a capacity of 667 kips neglecting the P-Δ effects, (lateral load factors of 4.05 and 6.15 respectively). The actual second order load factor was 2.02. Girder hinging controlled the strength of the frame. The columns were stressed to only half their design capacity at failure.

7.3.2 Computation of Sway Forces

The computation of sway forces is relatively simple. In the combined load case the lateral and vertical loads are applied to the structure and the lateral displacement at each floor level is computed using a first order analysis. These displacements are denoted as Δ_i in Figure 7.1, where i refers to the floor level. The additional story shears due to vertical loads are computed:

$$\Delta V_i = \frac{\Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i) \qquad (7.3)$$

where ΔV_i = additional shear in story i due to the sway forces,

ΣP_i = sum of the column axial loads in story i,

h_i = height of story i, and

Δ_{i+1}, Δ_i = displacements of levels i+1 and i, respectively. The sway forces due to the vertical loads, ΔF_i, are then computed as the difference between the additional story shears at each level:

$$\Delta F_i = \Delta V_i - \Delta V_{i-1} \tag{7.4}$$

The sway forces, ΔF_i are added to the applied lateral loads, and the structure re-analyzed. When the Δ_i values at the end of a cycle are nearly equal to those of the previous cycle, the method has converged, and the resulting moments and forces are, in fact, second order.

The method described above is equally applicable to the vertical load only case. The deflected shape under the applied vertical loads is determined and the sway forces calculated using EQUATIONS 7.3 and 7.4. The sway forces are then applied to the structure and the deflected shape again determined. After only a few cycles it should be evident whether or not the method is converging. If the story deflections do not converge the frame is unstable.

The above method of computing sway forces is illustrated for the braced subassemblage frame in FIGURE 7.2. The first and second order load deflection curves for the frame subjected to 1.30 times the design vertical loads are shown although normally the frame would be analyzed at λ = 1.00. The first line in the table under the figure indicates the first order deflection for the frame. The sway forces are computed and added to the lateral loads to begin the second cycle. The method converges to the second order deflection in three cycles. Comparison with the second order load-deflection curve for the structure shows this sway to be correct.

The sway due to the P-Δ shears was 34% of the sway due to the applied loads alone. At the end of the first cycle in the iterative process 78% of the P-Δ sway had been accounted for.

An interesting feature of the method is that the relative magnitudes of the applied lateral loads and sway forces are known. Thus, if bracing strength controlled the original first order design, the increase in the amount of bracing to provide the same factor of safety against yielding of the bracing including the P-Δ effects should be equal to the ratio of the sway forces to the applied lateral loads. This was indeed the case for the subassemblage frame, as, in the behavioral studies, 23% additional bracing was found necessary to raise the second order load factor at the yield of the bracing members to the first order level.

7.3.3 Proposed Design Method

The following procedure is recommended for the design of multi-story steel frames, regardless of whether or not the frame contains a vertical truss or shear wall:

1. Proportion the columns, girder and bracing members, if any, on the basis of a first order analysis for vertical loads only, at a load factor of 1.70; and for combined vertical and lateral loads, at a load factor of 1.30. Use effective length factors for the columns assuming translation is prevented, ($K < 1$).
2. Perform a second order analysis under vertical loads only, and modify the frame members if necessary.

3. Perform a second order analysis under combined vertical and horizontal loads, and modify the frame members if necessary.

If the bent in question is required to support a number of additional bents, it must be designed for the total P-Δ effects from the coupled frames in steps 2 and 3.

The principal advantage of such a design procedure is that it requires a rational assessment of the P-Δ effects in a given structure. Design economies may result in frames where the P-Δ effects are negligible, but, more important, unsafe designs due to neglect of the P-Δ effects, (especially from supported bents), will be avoided.

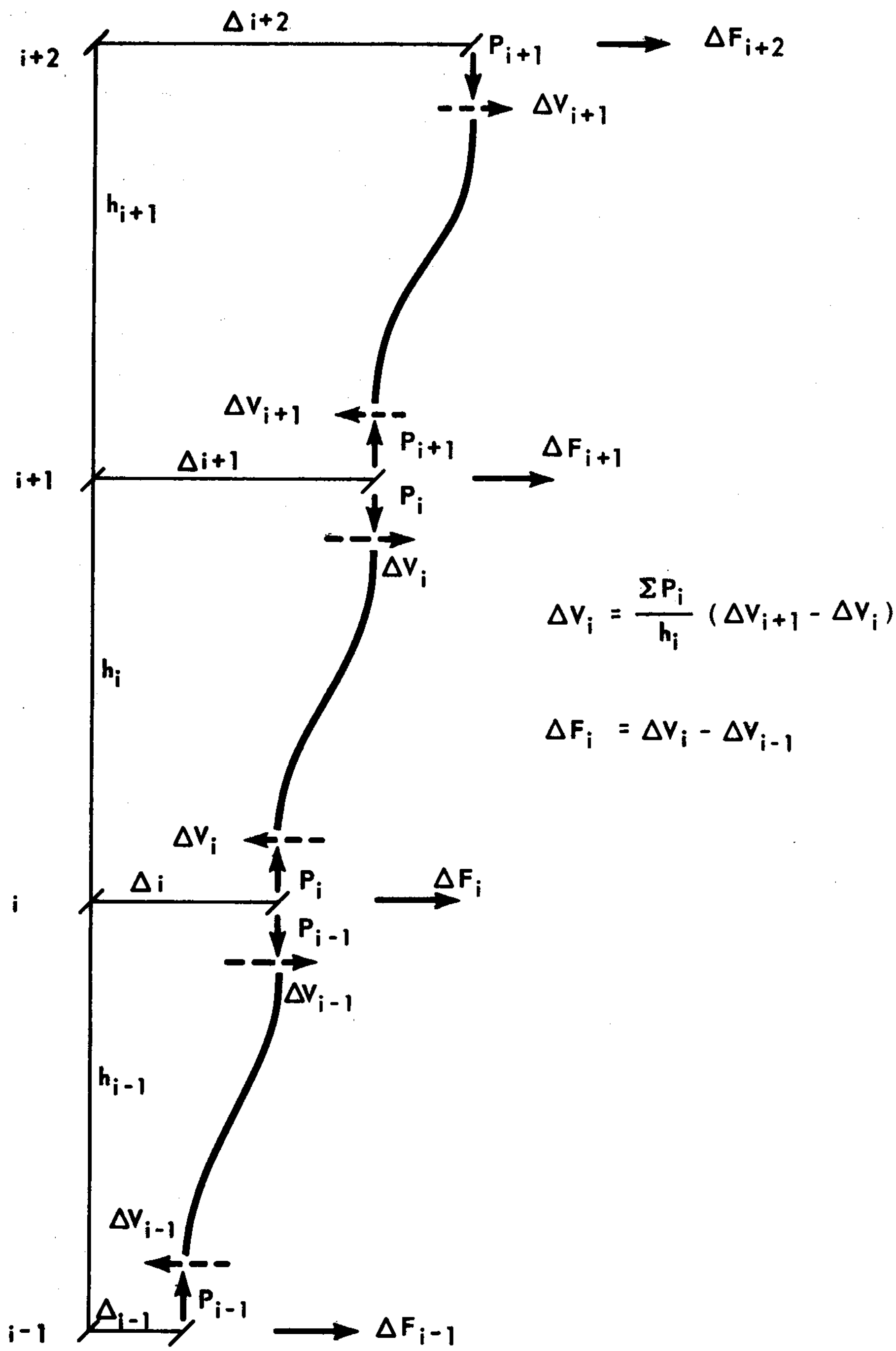
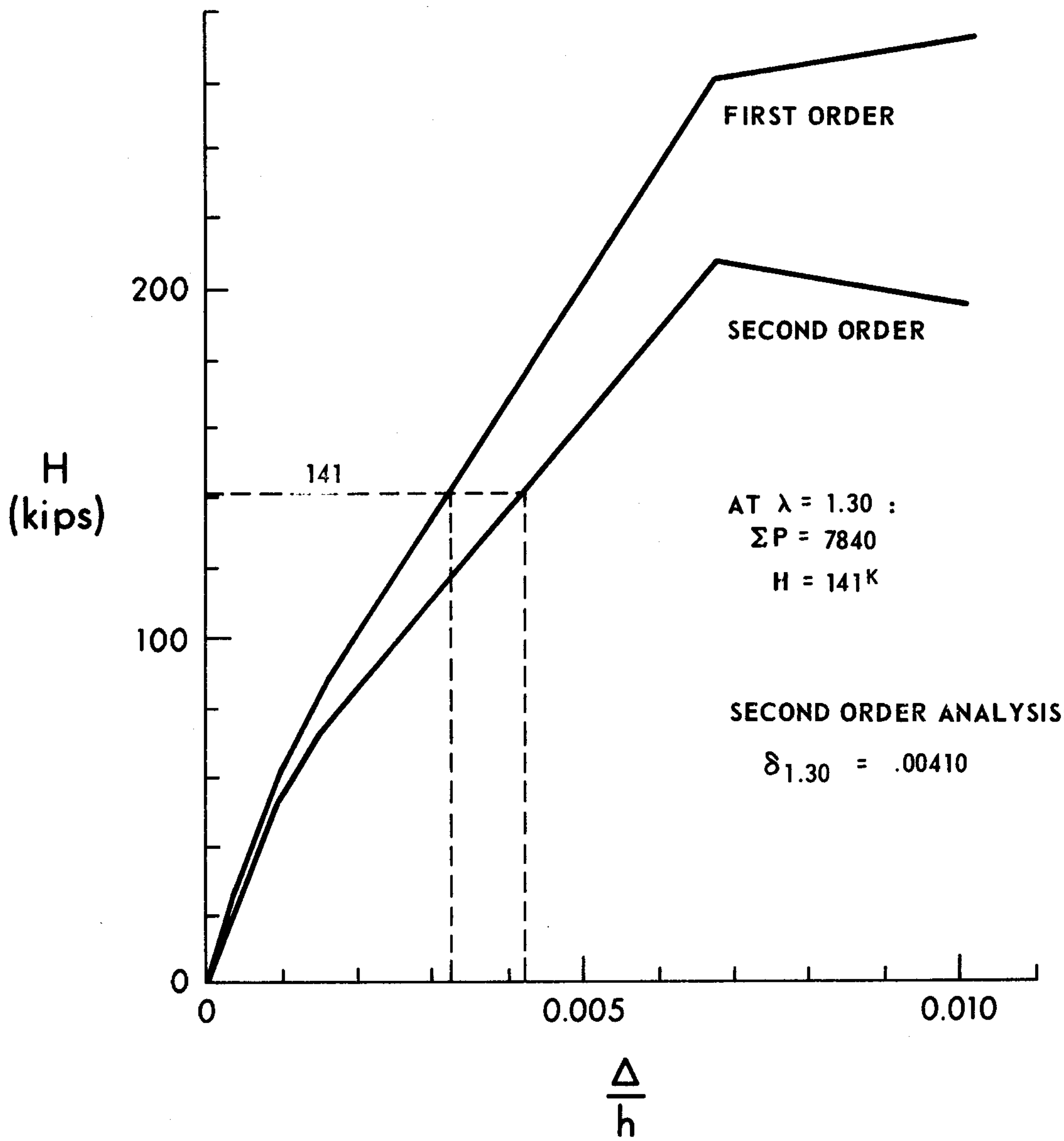


FIGURE 7.1

SWAY FORCES DUE TO VERTICAL LOADS



CYCLE	Δ/h	$\Sigma P\ \Delta/h = \Delta F$
1	.00315	25
2	.00398	31
3	.00416	33
4	.00422	33

$$\frac{33}{141} = 0.23$$

FIGURE 7.2
COMPUTATION OF SECOND ORDER DEFLECTION

CHAPTER VIII

SUMMARY AND CONCLUSIONS

8.1 Summary

The first four chapters of the dissertation describe the formulation of a computer program to analyze multi-story steel frames with provision for diagonal bracing members and shear wall elements. The member response is assumed to be elastic-perfectly plastic. The influence of axial load on the stiffness and carry-over factors is considered for the columns, but neglected for the girders. The effects of finite column width and hinge reversal on the behavior of the girders are also considered. Diagonal bracing members are assumed to be subjected to axial loads only. The frame analysis is second order, that is to say, the story shear equilibrium is formulated on the deformed structure. Axial shortening of the columns is considered. The equilibrium equations are solved by a modified Gauss elimination procedure.

In the fifth chapter, a number of comparative studies are described, which are used to verify the present method of analysis.

The sixth and seventh chapters are concerned with the behavior of a number of frames subjected to vertical loads alone, and to combined vertical and lateral loads. Comparisons are made between the behavior of unbraced and braced frames, with particular emphasis on the P-Δ effects. Coupled unbraced-supported and braced-supported frames are also considered. A design procedure, based on the behavioral studies is recommended for multi-story frames. The method results in a more uniform factor of safety than does present design practice.

8.2 Conclusions

The behavioral studies of the plastically designed Lehigh frames, (15), resulted in two major conclusions, which led to the proposed design procedure. The first was that the braced frames swayed more than the corresponding unbraced frames at a given load factor. Thus the amount of relative story deflection necessary to develop the resisting forces in the bracing members was greater than that for the unbraced columns and girders. The extensional stiffness of the bracing members is therefore an important design consideration.

The second conclusion was that the P-Δ effects significantly reduced the load carrying capacity of the braced frames, especially in the situation where the braced bents were required to provide additional lateral stiffness for a number of supported bents. The braced frames were more susceptible to a reduction in the load carrying capacity than were the corresponding unbraced frames. Thus the P-Δ effects are an important design consideration for braced frames, as well as for unbraced frames.

Based on these findings, a design procedure was recommended which makes no distinction between so-called "braced" or "unbraced" structures. The P-Δ effects are included in the basic analysis, and the designer is therefore justified in using an effective length factor less than one in the interaction equations. The method provides a uniform factor of safety for structures with varying P-Δ

effects, and does not necessitate the artificial distinction between frames which derive their lateral stiffness through the flexural action of their columns and girders only, and those which have a vertical truss or shear wall system.

In addition a number of conclusions regarding the basic behavior of structures arose from the studies of the braced frames. In the combined load case, the rotation capacity of the twenty-four story braced frame was terminated when the leeward column in the bottom story reached its yield load in compression. Thus the increased axial load in the leeward column stack due to the accumulative effects of the vertical components of force from the bracing members is an important design consideration.

Equally important in the vertical loads only case, is the critical axial load of the bracing members. If the bracing members are permitted to buckle due to axial shortening of the columns, they are ineffective in resisting a sidesway motion of the structure until they return to the elastic range. In this case the designer has three alternatives:

1. Design the frame to resist the buckling motion under factored gravity loads without the aid of the bracing members.
2. Calculate the sway necessary to return the bracing members to the elastic range, and determine if the structure is stable in this deflected position.
3. Design the bracing members to remain elastic under the influence of axial shortening.

Alternative 1 is probably uneconomical, and 2 is difficult to assess. Alternative 3 is therefore recommended to ensure adequate structural stiffness.

The type of bracing system chosen influences the behavior of the structure. The K-bracing system has the advantage of supporting the girders at midspan, and therefore permits the use of lighter girder sections. However, when one of the bracing members buckles as the load on the frame is increased, the girder must be capable of resisting the difference between the vertical components of force in the braces. If the girder is too flexible, the effectiveness of the brace will be reduced. In addition, if the bracing members are used in a configuration in which the gravity loads on the girders subjects them to compression, the stiffness of the frame may be drastically reduced if the braces are not both designed to remain elastic for a considerable portion of the loading history.

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APPENDIX A

COMPUTER PROGRAM

THE AUTHORS AND THE UNIVERSITY OF ALBERTA DISCLAIM RESPONSIBILITY FOR THE MISUSE OF THE FOLLOWING PROGRAM, NOR WILL THEY BE RESPONSIBLE FOR ERRORS IN THE LISTING.

A.1 Nomenclature for Computer Program

A	- coefficient matrix of equilibrium equations
AB	- brace area (in^2)
AC	- column area (in^2)
AG	- girder area (in^2)
B	- load vector of equilibrium equations
CA1,CA2,CA3,CA5,CA6, CB1,CB2,CB3,CB5,CB6	- coefficients of girder slope-deflection equations
CC,CCX,CCY	- column stability function (x and y refer to the column segments if an interior column hinge exists)
CC2	- column axial stiffness (KSI)
CD1,CD2,CD3,CD4,CD5	- coefficients of brace load-deformation equations
CL1,CL2,CL3,CL5 CU1,CU2,CU3,CU5	- coefficients of column slope-deflection equations
C7,C8,C9,C10	- dummy variables used in connection with the slope-deflection equations
DEL	- vertical displacement of a joint (in)
DELC	- deflection at an interior girder hinge (in)
DELD	- deflection at an interior column hinge (in)
DET	- determinant of the coefficient matrix
DLDES	- design dead load (KLF)
EB	- brace modulus of elasticity (KSI)
EC	- column modulus of elasticity (KSI)
EG	- girder modulus of elasticity (KSI)
F	- applied lateral load (K)
FB	- axial force in brace (K)

FBP	- plastic capacity of brace (K)
FBX	- horizontal component of force in brace (K)
FDES	- design lateral load (K)
FYB	- brace yield stress (KSI)
FYC	- column yield stress (KSI)
FYG	- girder yield stress (KSI)
H	- story height (column height) (in)
IAREA	- indicator for neglecting axial shortening
IELAST	- indicator for elastic analysis
IL	- indicator for load increment
IND	- indicator of new hinges formed
IND2	- indicator of hinge at column base
IND3	- indicator of negative determinant
IND6	- indicator for print control
IREV	- indicator of hinge reversal
IREV2	- indicator for ignoring hinge reversal
ISTAB	- indicator for neglecting stability functions
IXC	- column moment of inertia (in^4)
IXG	- girder moment of inertia (in^4)
I10	- indicator for ignoring hinge reversal
I11	- maximum number of hinges at failure
I12	- indicator for neglecting P-Δ effects
JB	- brace hinge condition
JBL	- brace hinge condition at beginning of load increment
JC	- column hinge condition

JCL	- column hinge condition at beginning of load increment
JG	- girder hinge condition
JGL	- girder hinge condition at beginning of load increment
K	- column base spring constant (KSI)
L	- bay width (in)
LB	- brace length (in)
LG	- girder length (in)
LLDES	- design live load (KLF)
LLL	- indicator of column axial loads
MA,MB,MC	- girder moments at A, B and C (in-k)
MABP,MBAP,MCAP,MCBP	- girder plastic moment capacity (in-k)
MFAB,MFBA,MFCA,MFCB	- girder fixed end moments (in-k)
MAX,MBY	- moments due to applied girder loads (in-k)
ML,MU,MD	- column moments at L, U and D (in-k)
MLUP,MULP,MDLP,MDUP	- column plastic moment capacity (in-k)
MM	- number of column stacks
MMM	- MM-1
NA	- total number of unknown deformations
NB	- matrix half band width
NCYC	- number of permissible cycles of iteration
NL	- number of load increments
NLAST	- number of last load increment
NN	- number of floor levels
NNN	- NN-1
P	- column axial load (k)

PAPP	- assumed column axial load (k)
PCR	- critical axial load of brace (k)
PDES	- design column axial load (k)
RABP,RBAP,RCAP	- girder plastic hinge rotations at A, B and C (radians)
RABPL,RBAPL,RCAPL	- girder plastic hinge rotations at the end of the last load increment (radians)
RLUP,RULP,RDLP	- column plastic hinge rotations at L, U and D (radians)
RINLL,RINF,RINP,RINV	- live load increment, lateral load increment, column axial load increment, joint load increment
RLL,RF,RP,RV	- live load factor, lateral load factor, column axial load factor, joint load factor.
ROT	- joint rotation (radians)
RXB	- brace radius of gyration (in)
RXC	- column radius of gyration (in)
SS,SSX,SSY	- column stability functions
SUMV	- variable to cetermine column axial loads
SWAY	- lateral deflection (in)
V	- variable used in equation solver
VA,VB	- girder shear at A and B (k)
VDES	- design joint load (k)
VJT	- joint load (k)
VJTL	- joint load in previous increment (k)
VL,VU	- column shear at L and U (k)

VWA,VWB	- girder shear due to applied load (k)
VWAL,VWBL	- girder shear due to applied load in previous increment (k)
W	- girder load (KLI)
WC	- column width (in)
XC	- distance from lower end of column to interior hinge (in)
XG	- distance from left end of girder to interior hinge (in)
XMP	- plastic moment capacity of column (in-k)
YC	- H - XC (in)
YG	- LG - XG (in)
ZXC	- column plastic section modulus (in^3)
ZXG	- girder plastic section modulus (in^3)

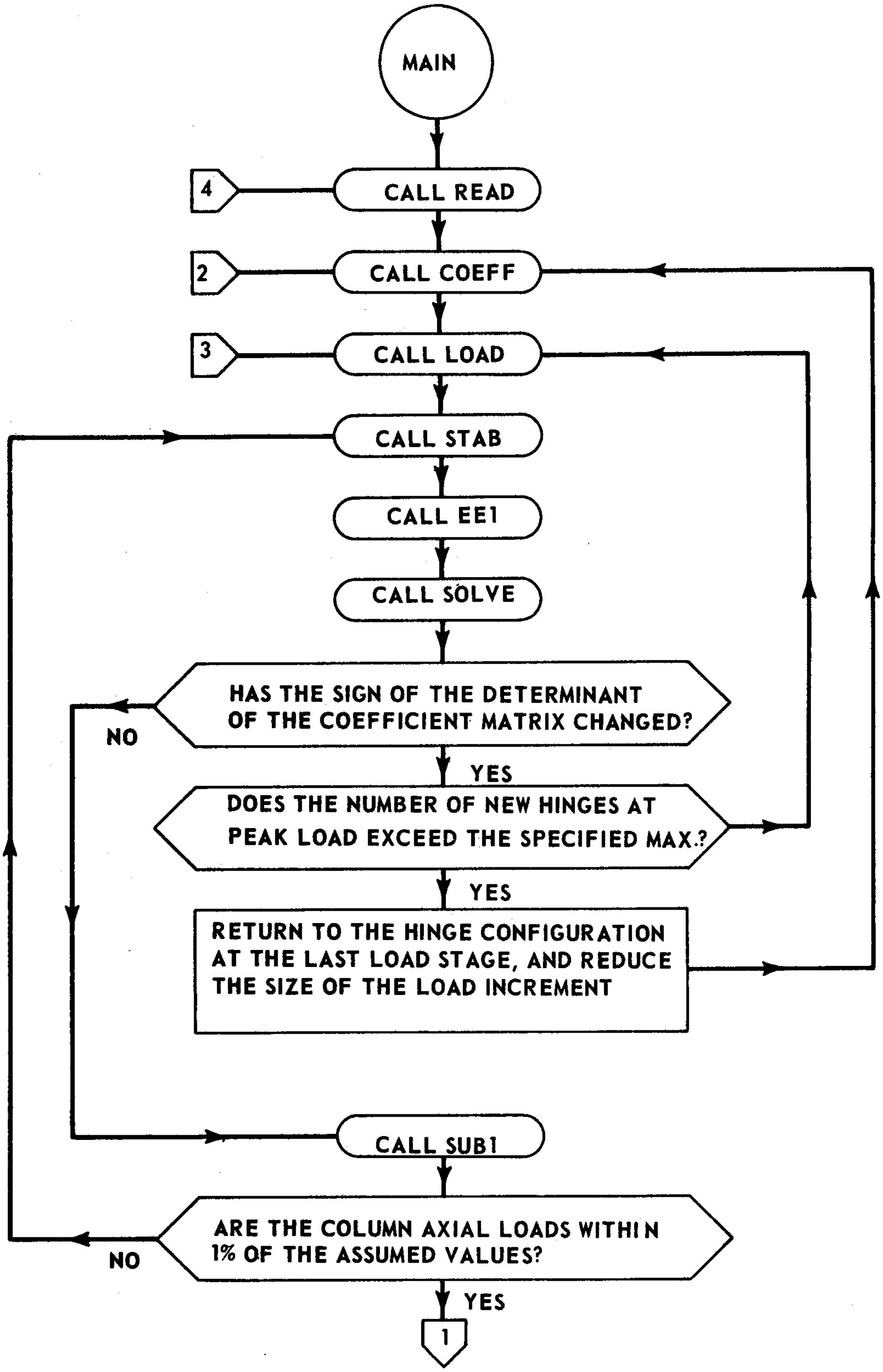

FIGURE A.1

FLOW DIAGRAM MAIN PROGRAM

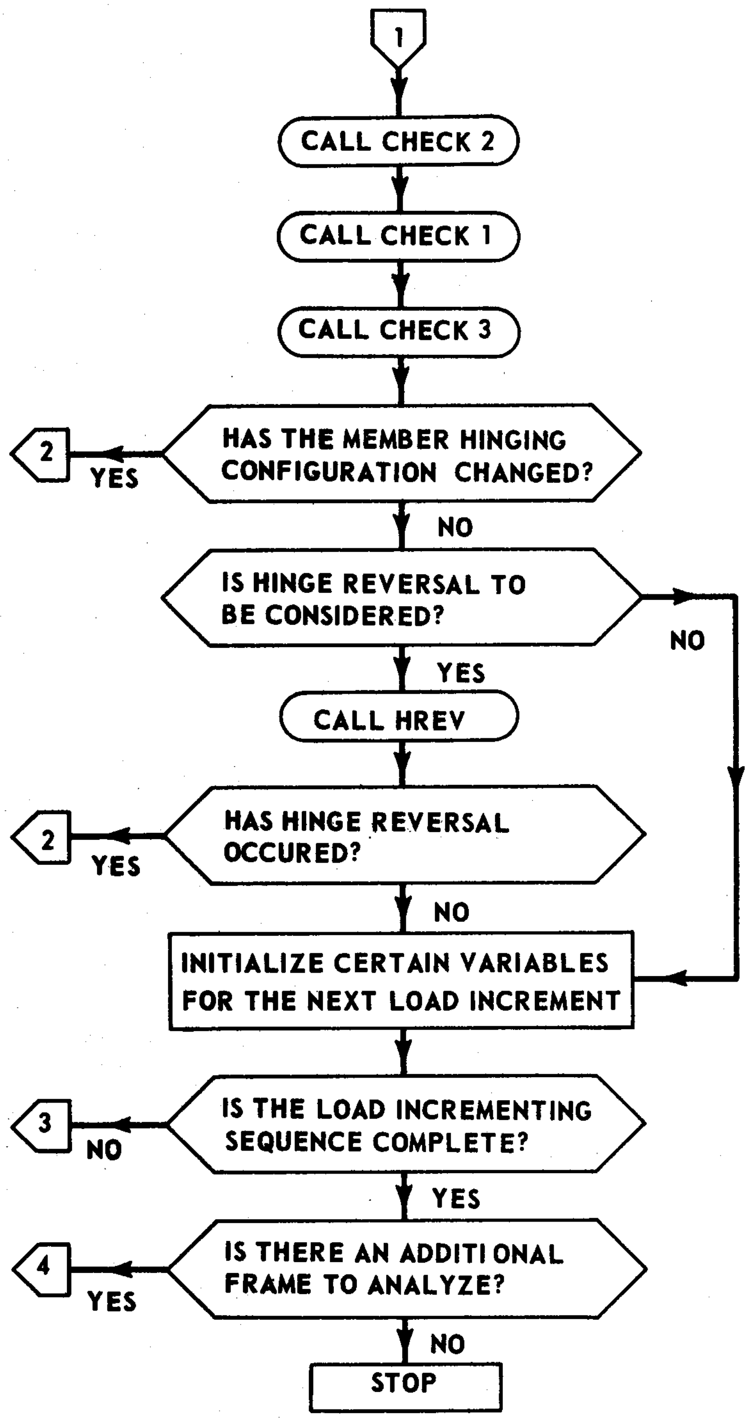

FIGURE A.1

FLOW DIAGRAM MAIN PROGRAM (continued)

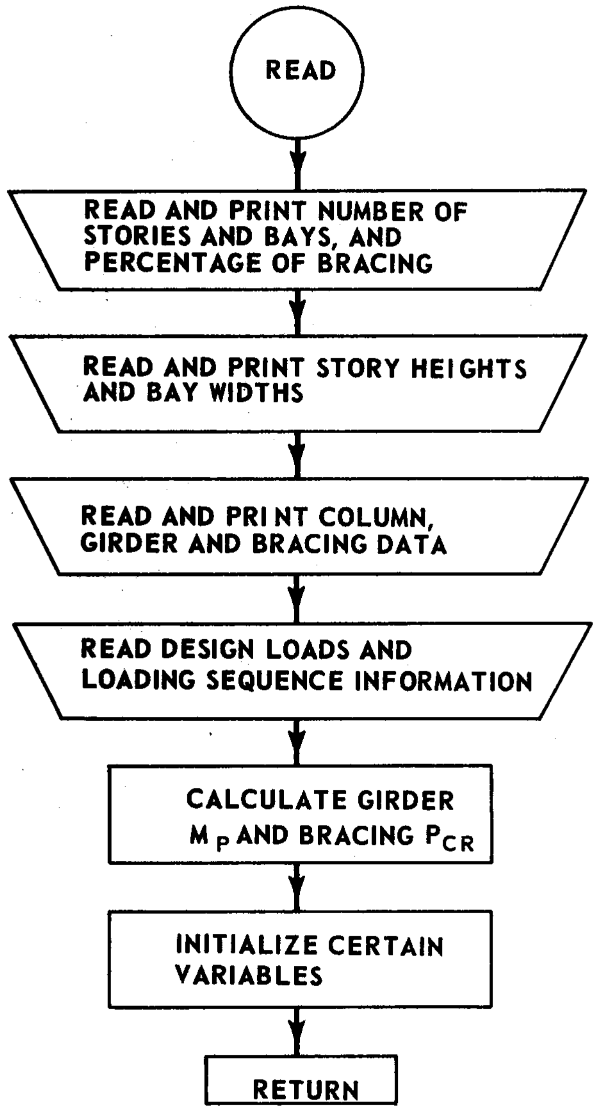

FIGURE A.2

FLOW DIAGRAM SUBROUTINE READ

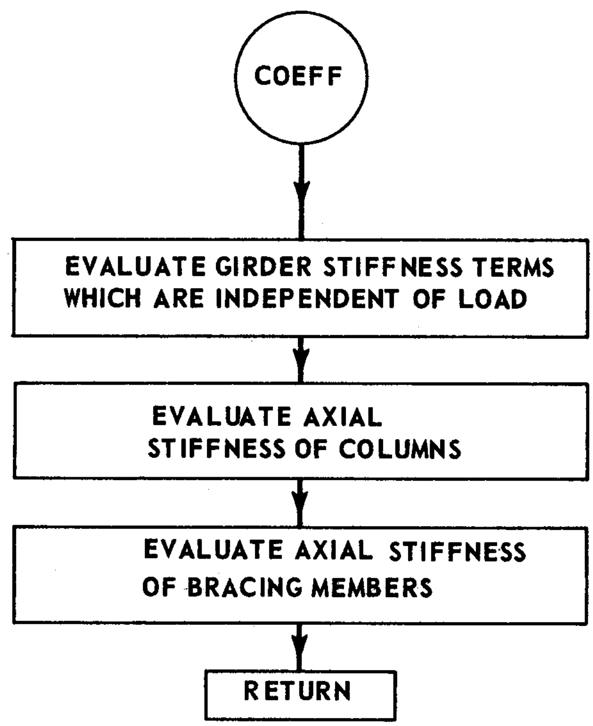

FIGURE A.3

FLOW DIAGRAM SUBROUTINE COEFF

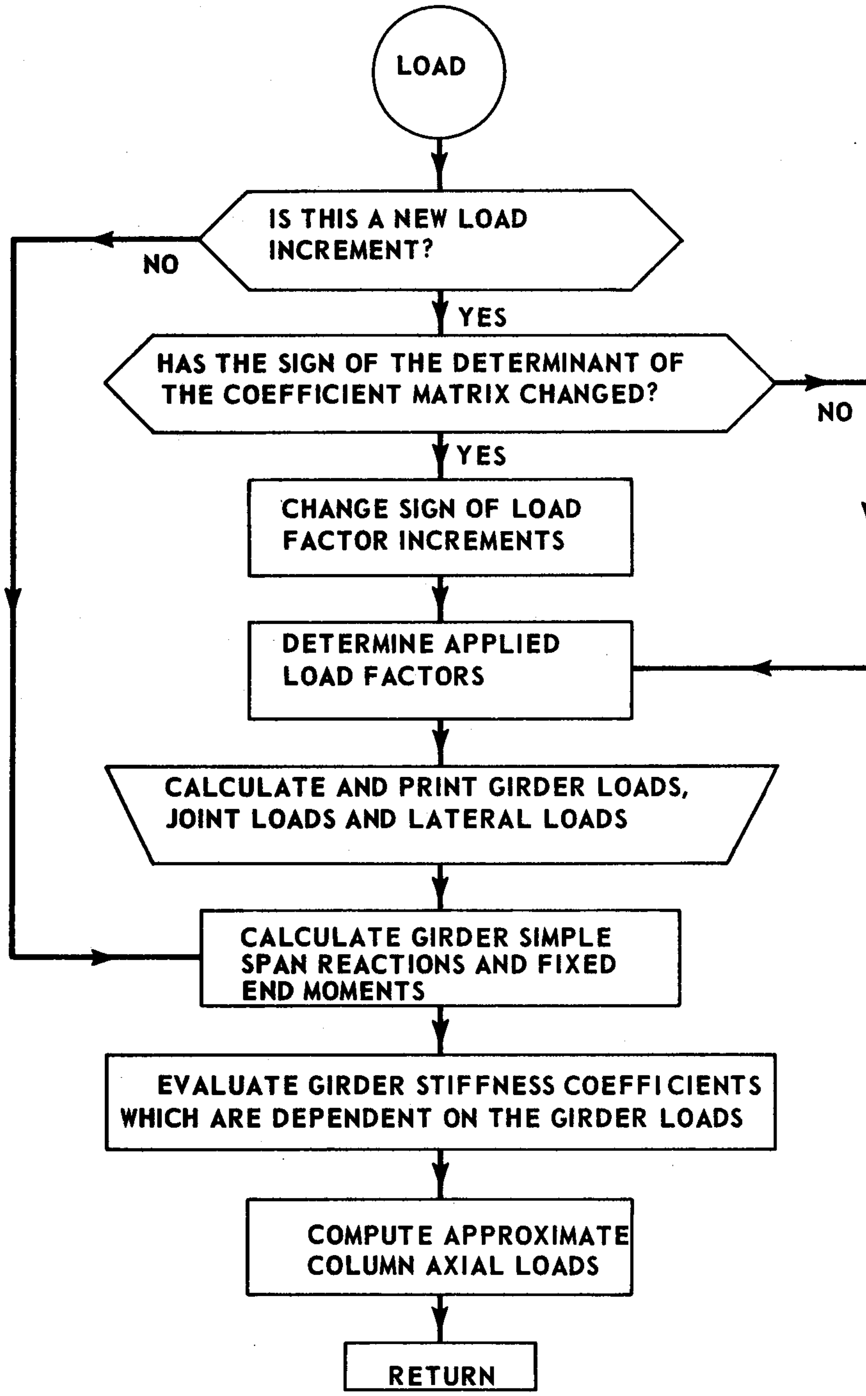

FIGURE A.4

FLOW DIAGRAM SUBROUTINE LOAD

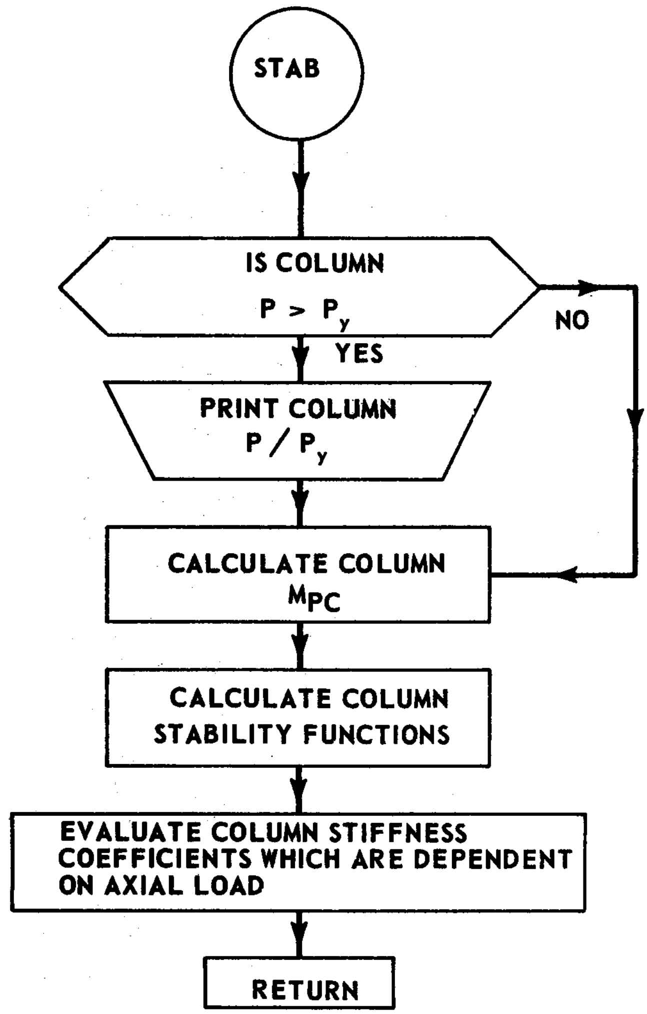

FIGURE A.5

FLOW DIAGRAM SUBROUTINE STAB

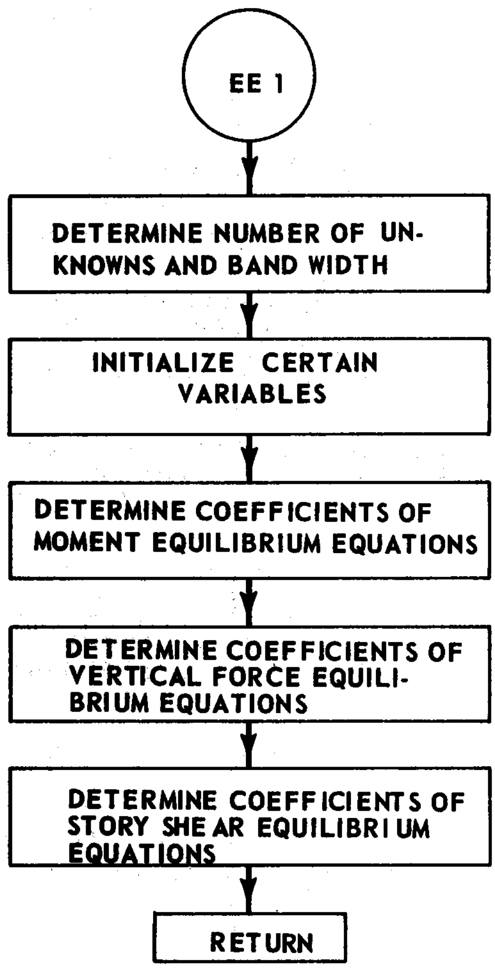

FIGURE A.6

FLOW DIAGRAM SUBROUTINE EE1

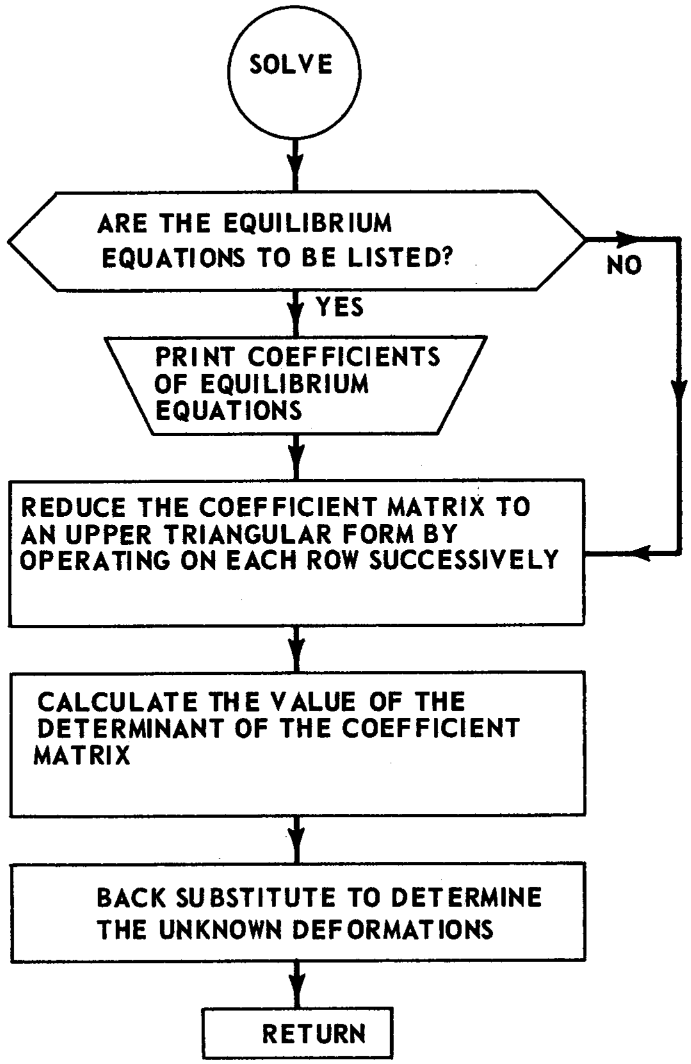

FIGURE A.7

FLOW DIAGRAM SUBROUTINE SOLVE

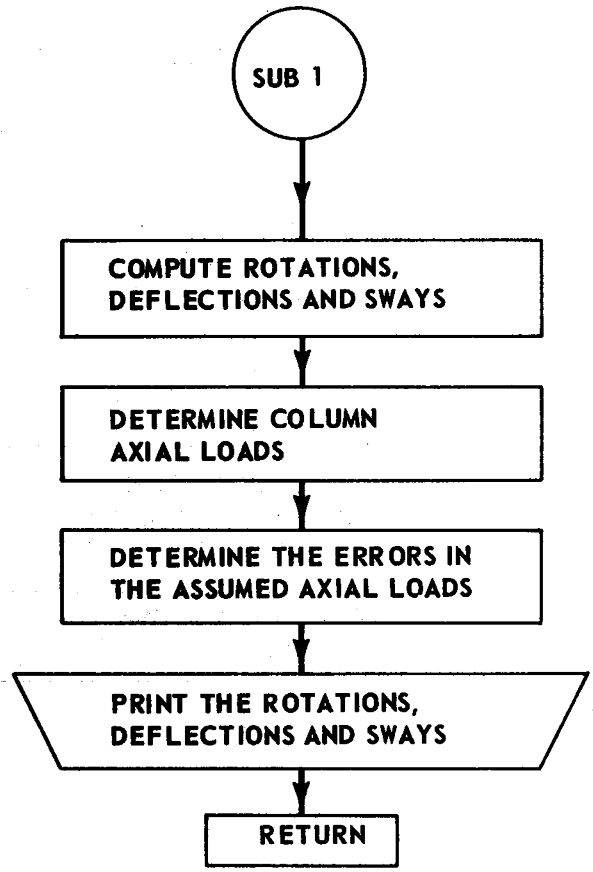

FIGURE A.8

FLOW DIAGRAM SUBROUTINE SUB1

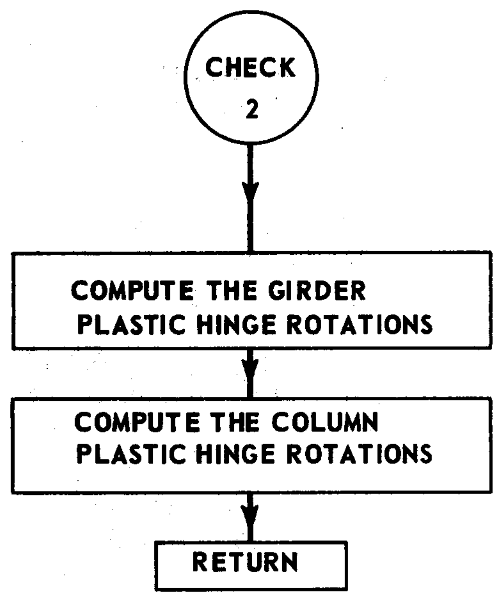

FIGURE A.9

FLOW DIAGRAM SUBROUTINE CHECK 2

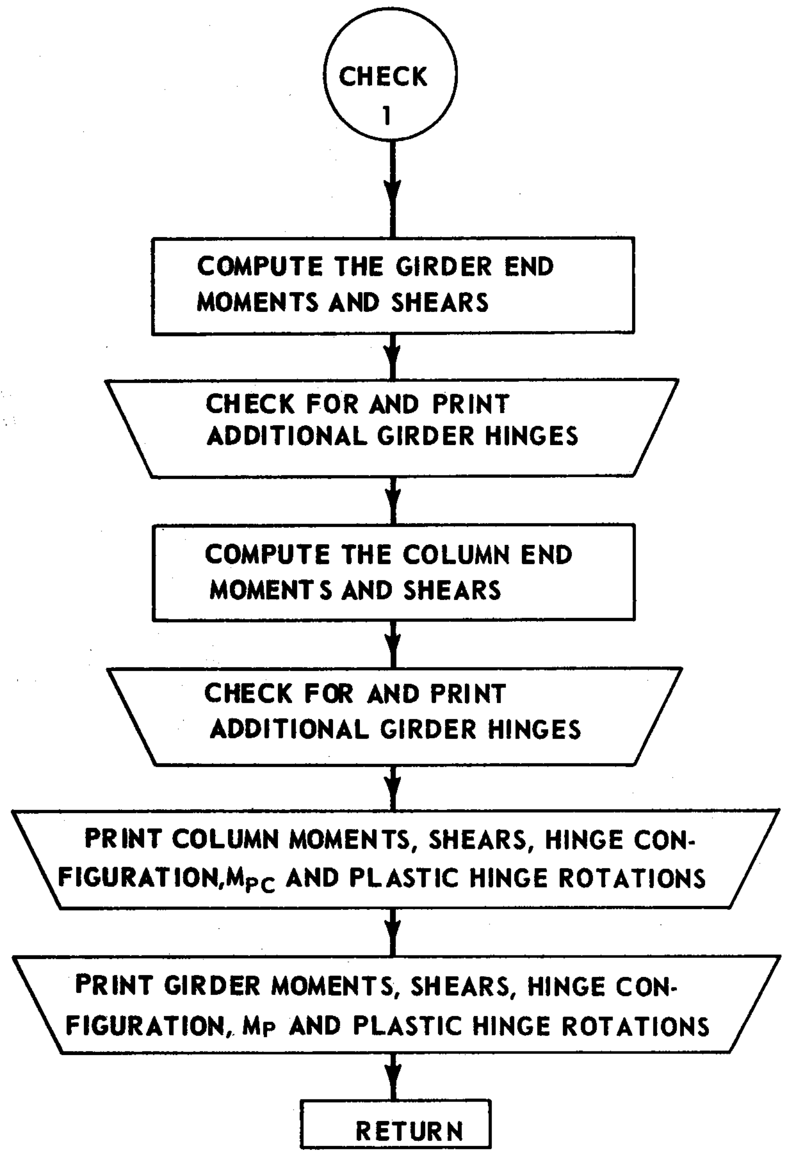

FIGURE A.10

FLOW DIAGRAM SUBROUTINE CHECK 1

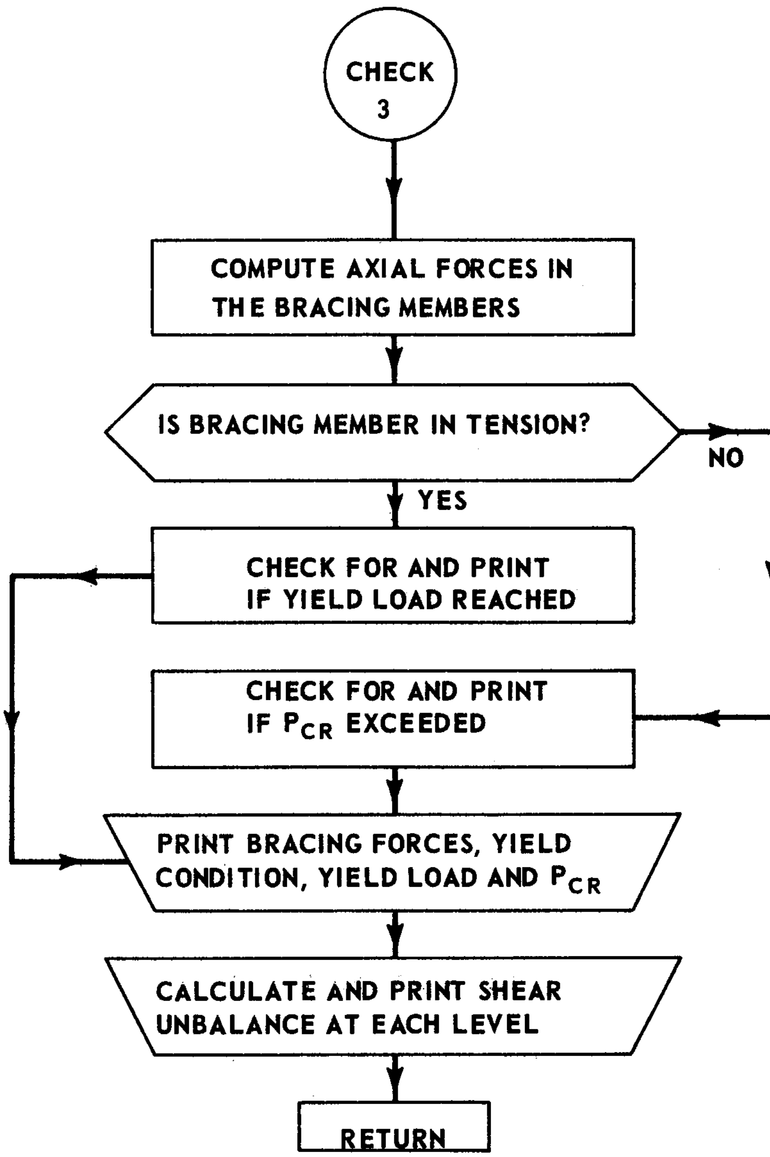

FIGURE A.11

FLOW DIAGRAM SUBROUTINE CHECK 3

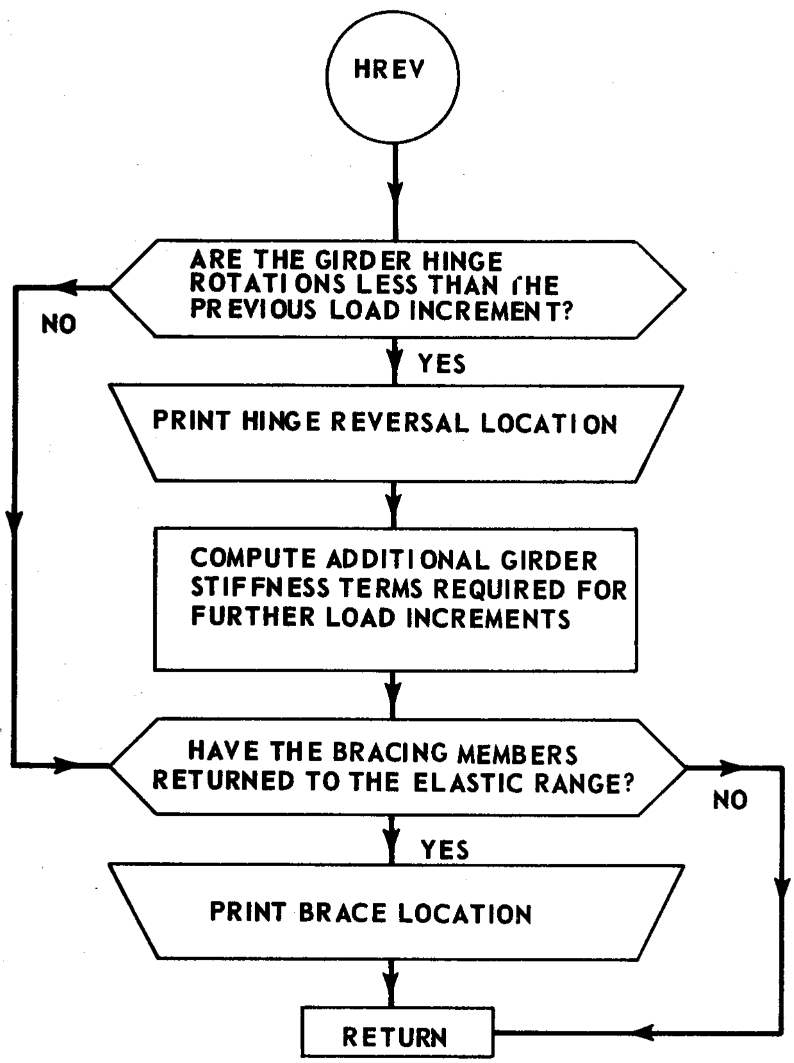

FIGURE A.12

FLOW DIAGRAM SUBROUTINE HREV

A.3 Data Cards

The data cards for the program are read in the following order:

1. the number of data sets to follow (I5)
2. an identification card (reproduces first 40 characters at top of output)
3. number of column stacks, number of stories (counting ground level as story one), and percent bracing (2I3, F9.5)
4. story heights (8F10.0)
5. bay widths (8F10.5)
6. column properties, (one card for each column - read across each floor level in turn),
   IDENTIFICATION, AREA, I_X, Z_X, r_X, WIDTH, F_y, E
   (A8, 2F9.2, F8.2, 3F7.2, F7.0)
7. girder properties, (one card for each girder - read across each floor level in turn),
   IDENTIFICATION, AREA, I_X, Z_X, F_y, E, DLDES, LLDES
   (A8, F7.2, F9.2, F8.2, F7.2, F7.0, 2F7.2)
8. bracing properties, (one card for each possible bracing location - read bracing members sloping upward to the left across a floor level, then members sloping upward to the right, then proceed to the next floor level)
   IDENTIFICATION, AREA, r_X, F_y, E (A8, 3F7.2, F7.0)
9. column base fixity, (one card for each column stack)(E11.4)

10. design loads

FDES (8F10.5)

VDES, (left to right then to next level)(8F10.5)

PDES (8F10.5)

11. loading sequence information

NL, IELAST, IAREA, IND6, IREV2, ISTAB, I10, I11, I12 (9I5)

NL: number of load increments if a particular arbitrary sequence is to follow (if NL = 0, a regular load incrementing procedure will be used)

IELAST: if IELAST = 1, analysis will be elastic

IAREA: if IAREA = 1, axial shortening of the columns will be suppressed

IND6: if IND6 = 1, equilibrium equations will be listed if IND6 = 2, output will include only the sways at each load increment

IREV2: if IREV2 = 1, hinge reversal will not be considered

ISTAB: if ISTAB = 1, C = 4 and S = 2

I10: number of load increments before leeward hinge reversal considered

I11: maximum number of new hinges at peak load

I12: if I12 = - 5, analysis will be first order

12. if NL = 0:

read initial values of RLL, RF, RP, RV (4F10.5)

read RINLL, RINF, RINP, RINV (4F10.5)

Read number of load increments (I5)

if NL $\neq$ 0:

read values of RLL, RF, RP, RV for each load increment

(4F10.5)

```
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 ULDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,C4),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(3C0),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NF,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
      READ(5,1006) NFRAME
 1006 FORMAT(I5)
      DO 550 II=1,NFRAME
      IND=0
      CALL READ
      IF(I11.EQ.0) GO TO 77
      WRITE(6,78) I11
   78 FORMAT(1H0,'MAXIMUM NUMBER OF NEW HINGES AT PEAK LOAD =',I5)
   77 IF(IAREA.EQ.0) GO TO 500
      WRITE(6,1007)
 1007 FORMAT(1H0,24HAXIAL SHORTENING IGNORED)
  500 IF(IELAST.EQ.C) GO TO 420
      WRITE(6,1008)
 1008 FORMAT(1H0,16HELASTIC ANALYSIS)
  420 IF(ISTAB.EQ.0) GO TO 38
      WRITE(6,421)
  421 FORMAT(1H0,'C=4, S=2')
   38 IF(I1C.EQ.0) GO TO 701
      WRITE(6,1091) I1C
 1091 FORMAT(1H0,'HINGE REVERSAL AT LEEWARD END OF GIRDERS NOT CONSIDERE
     1D PRIOR TO LOAD INCREMENT',I5)
  701 IF(IREV.EQ.0) GO TO 702
      WRITE(6,751)
  751 FORMAT(1H0,'HINGE REVERSAL IGNORED')
  702 IF(I12.NE.-5) GO TO 501
      WRITE(6,752)
  752 FORMAT(1H0,'NO P-DELTA MOMENTS')
  501 CALL CCEFF
```

```
 502  CALL LOAD
      NCYC=0
 504  CALL STAB
      CALL FE1
      CALL SOLVE
      IF(I12.EQ.-5) GO TO 66
      I12=0
  66  IF(IND3.NE.1) GO TO 45
      IF(I11.EQ.0) GO TO 502
      IF(IND.LE.I11) GO TO 502
      RINF=0.1*RINF
      RINP=0.1*RINP
      RINV=0.1*RINV
      RINLL=0.1*RINLL
      IND3=0
      I12=1
      I14=IL+1
      DO 60 N=1,NN
      DO 59 M=1,MM
      JC(M,N)=JCL(M,N)
      JB(M,N,1)=JBL(M,N,1)
      JB(M,N,2)=JBL(M,N,2)
      JG(M,N)=JGL(M,N)
      IF(C9(M,N).EQ.1.0) MABP(M,N)=-MABP(M,N)
      IF(C9(M,N).EQ.2.0) MBAP(M,N)=-MBAP(M,N)
      IF(C9(M,N).EQ.3.0) MCAP(M,N)=-MCAP(M,N)
      IF(C9(M,N).EQ.4.0) MCBP(M,N)=-MCBP(M,N)
      IF(C10(M,N).EQ.1.0) MLUP(M,N)=-MLUP(M,N)
      IF(C10(M,N).EQ.2.0) MULP(M,N)=-MULP(M,N)
      IF(C10(M,N).EQ.3.0) MDUP(M,N)=-MDUP(M,N)
      IF(C10(M,N).EQ.4.0) MDLP(M,N)=-MDLP(M,N)
  59  CONTINUE
  60  CONTINUE
      WRITE(6,50) IND
  50  FORMAT(1H0,I5,'  NEW HINGES FOUND AND LOAD INCREMENT DECREASED')
      IND=0
      GO TO 501
  45  CALL SUB1
      IF(NCYC.GE.10) GO TO 508
      IF(LLL.EQ.1) GO TO 504
      IF(NCYC.LT.10) GO TO 509
 508  WRITE(6,999)
 999  FORMAT(1H0,50HPERMISSIBLE NUMBER OF CYCLES OF ITERATION EXCEEDED)
 509  DO 520 N=1,NN
      DO 519 M=1,MM
      P(M,N)=PAPP(M,N)
 519  CONTINUE
 520  CONTINUE
      CALL CHECK2
      CALL CHECK1
      CALL CHECK3
      IF(IND.GE.1) GO TO 501
      IF(IREV2.EQ.1) GO TO 400
      IF(IL.EQ.I14) GO TO 400
      CALL HREV
      IF(IHEV.GE.1) GO TO 501
 400  DO 450 N=2,NN
      DO 449 M=1,MMM
      RABPL(M,N)=RABP(M,N)
      RBAPL(M,N)=RBAP(M,N)
```

```
      RCAPL(M,N)=RCAP(M,N)
  449 CONTINUE
  450 CONTINUE
      DO 65 N=1,NN
      DO 64 M=1,MM
      JCL(M,N)=JC(M,N)
      JGL(M,N)=JG(M,N)
      JBL(M,N,1)=JB(M,N,1)
      JBL(M,N,2)=JB(M,N,2)
      C9(M,N)=0.0
      C10(M,N)=0.0
   64 CONTINUE
   65 CONTINUE
      IF(NL.EQ.0) GO TO 540
      IF(IL.LT.NL) GO TO 502
      GO TO 550
  540 IF(IL.LT.NLAST) GO TO 502
  550 CONTINUE
      STOP
      END
      SUBROUTINE READ
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLLP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,INC,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLLP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     READ FRAME GEOMETRY
C
      WRITE(6,2000)
      WRITE(6,2001)
      READ(5,1060) IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
      WRITE(6,1061) IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
      WRITE(6,2002)
      WRITE(6,2001)
      READ(5,1000) MM,NN,RERBR
      MMM=MM-1
```

```
      NNN=NN-1
      WRITE(6,1062)NNN,MMM
      WRITE(6,1063)
      READ(5,1001) (H(N),N=1,NNN)
      DO 8 N=1,NNN
      WRITE(6,1066) N,H(N)
    8 CONTINUE
      WRITE(6,1064)
      READ(5,1001) (L(M),M=1,MMM)
      DO 7 M=1,MMM
      WRITE(6,1066) M,L(M)
    7 CONTINUE
C
C     READ MEMBER PROPERTIES
C
      WRITE(6,1075)
      WRITE(6,1067)
      DO 10 N=1,NNN
      DO 11 M=1,MM
      READ(5,1002) COLUMN      ,AC(M,N),IXC(M,N),ZXC(M,N),RXC(M,N),WC(M,N
     1),FYC(M,N),EC(M,N)
      WRITE(6,1068) M,N,COLUMN      ,AC(M,N),IXC(M,N),ZXC(M,N),RXC(M,N),W
     1C(M,N),FYC(M,N),EC(M,N)
      JC(M,N)=1
   11 CONTINUE
   10 CONTINUE
      DO 9 M=1,MM
      WC(M,NN)=WC(M,NN-1)
    9 CONTINUE
      WRITE(6,1076)
      WRITE(6,1069)
      DO 12 N=2,NN
      DO 13 M=1,MMM
      READ(5,1003) GIRDER      ,AG(M,N),IXG(M,N),ZXG(M,N),FYG(M,N),EG(M,N
     1),DLDES(M,N),LLDES(M,N)
      WRITE(6,1070) M,N,GIRDER      ,AG(M,N),IXG(M,N),ZXG(M,N),FYG(M,N),E
     1G(M,N),DLDES(M,N),LLDES(M,N)
   13 CONTINUE
   12 CONTINUE
      WRITE(6,1077) PERBR
      WRITE(6,1071)
      DO 14 N=1,NNN
      DO 15 M=2,MM
      READ(5,1004) BRACE       ,AB(M,N,1),RXB(M,N,1),FYB(M,N,1),EB(M,N,1
     1)
      AB(M,N,1)=PERBR*AB(M,N,1)
      WRITE(6,1072) M,N,BRACE       ,AB(M,N,1),RXB(M,N,1),FYB(M,N,1),EB(
     1M,N,1)
   15 CONTINUE
      DO 16 M=1,MMM
      READ(5,1004) BRACE       ,AB(M,N,2),RXB(M,N,2),FYB(M,N,2),EB(M,N,2
     1)
      AB(M,N,2)=PERBR*AB(M,N,2)
      WRITE(6,1073) M,N,BRACE       ,AB(M,N,2),RXB(M,N,2),FYB(M,N,2),EB(
     1M,N,2)
   16 CONTINUE
   14 CONTINUE
C
C     READ COLUMN FIXITY
C
```

```
      WRITE(6,1080)
      DO 47 M=1,MM
      READ(5,1030) K(M)
      WRITE(6,1081) M,K(M)
   47 CONTINUE
C
C     CALCULATE GIRDER & BRACING LENGTHS
C
      DO 21 N=2,NN
      DO 20 M=1,MMM
      LG(M,N)=L(M)-0.5*(WC(M,N)+WC(M+1,N))
      JG(M,N)=1
   20 CONTINUE
   21 CONTINUE
      DO 24 N=1,NNN
      I=1
      DO 25 M=2,MM
      LB(M,N,1)=SQRT(H(N)*H(N)+L(M-1)*L(M-1))
      JB(M,N,1)=1
      FB(M,N,I)=0.0
      FBX(M,N,I)=0.0
      FBP(M,N,I)=AB(M,N,I)*FYB(M,N,I)
   25 CONTINUE
      I=2
      DO 26 M=1,MMM
      LB(M,N,2)=SQRT(H(N)*H(N)+L(M)*L(M))
      JB(M,N,2)=1
      FB(M,N,I)=0.0
      FBX(M,N,I)=0.0
      FBP(M,N,I)=AB(M,N,I)*FYB(M,N,I)
   26 CONTINUE
   24 CONTINUE
C
C     READ DESIGN LOADS
C
      WRITE(6,1082)
      READ(5,1011) (FDES(N),N=2,NN)
      DO 30 N=2,NN
      WRITE(6,1083) N,FDES(N)
   30 CONTINUE
      WRITE(6,1084)
      READ(5,1011) ((VDES(M,N),M=1,MM),N=2,NN)
      DO 32 N=2,NN
      DO 31 M=1,MM
      WRITE(6,1085) M,N,VDES(M,N)
   31 CONTINUE
   32 CONTINUE
      WRITE(6,1086)
      READ(5,1011) (PDES(M),M=1,MM)
      DO 33 M=1,MM
      WRITE(6,1087) M,PDES(M)
   33 CONTINUE
C
C     READ LOADING SEQUENCE INFORMATION
C
      WRITE(6,1088)
      READ(5,1014) NL,IELAST,IAREA,IND6,IREV2,ISTAB,I10,I11,I12
      IF(NL.EQ.0) GO TO 42
      DO 40 I=1,NL
      READ(5,1015) RLL(I),RF(I),RP(I),RV(I)
```

```
      WRITE(6,1089) I,RLL(I),RF(I),RP(I),RV(I)
   40 CONTINUE
      GO TO 50
   42 READ(5,1015) RLL(1),RF(1),RP(1),RV(1)
      I=1
      WRITE(6,1089) I,RLL(I),RF(I),RP(I),RV(I)
      READ(5,1015) RINLL,RINF,RINP,RINV
      WRITE(6,1090) RINLL,RINF,RINP,RINV
      READ(5,1016)NLAST
C
C     CALCULATE PLASTIC MOMENT CAPACITY OF GIRDERS
C
   50 DO 340 N=2,NN
      DO 339 M=1,MMM
      MBAP(M,N)=FYG(M,N)*ZXG(M,N)
      MABP(M,N)=MBAP(M,N)
      MCAP(M,N)=MBAP(M,N)
      MCBP(M,N)=MBAP(M,N)
  339 CONTINUE
  340 CONTINUE
C
C     CALCULATE CRITICAL AXIAL LOAD OF BRACING
C
      DO 350 N=1,NNN
      DO 348 M=2,MM
      PCR(M,N,1)=9.8696*RXB(M,N,1)*RXB(M,N,1)*EB(M,N,1)*AB(M,N,1)/(LB(M,
     1N,1)*LB(M,N,1))
      IF(PCR(M,N,1).LE.FBP(M,N,1)) GO TO 348
      PCR(M,N,1)=FBP(M,N,1)
  348 CONTINUE
      DO 349 M=1,MMM
      PCR(M,N,2)=9.8696*RXB(M,N,2)*RXB(M,N,2)*EB(M,N,2)*AB(M,N,2)/(LB(M,
     1N,2)*LB(M,N,2))
      IF(PCR(M,N,2).LE.FBP(M,N,2)) GO TO 349
      PCR(M,N,2)=FBP(M,N,2)
  349 CONTINUE
  350 CONTINUE
C
C     INITIALIZE CERTAIN VARIABLES
C
      DET=100.0
      DO 370 N=2,NN
      DO 369 M=1,MMM
      C9(M,N)=0.0
      MA(M,N)=0.0
      MB(M,N)=0.0
      MC(M,N)=0.0
      VA(M,N)=0.0
      VB(M,N)=0.0
      CA6(M,N)=0.0
      CB6(M,N)=0.0
      XG(M,N)=120.0
      YG(M,N)=120.0
      VWAL(M,N)=0.0
      VWBL(M,N)=0.0
      RABPL(M,N)=0.0
      RBAPL(M,N)=0.0
      RCAPL(M,N)=0.0
      DELC(M,N)=0.0
  369 CONTINUE
```

```
 370  CCNTINUE
      DC 372 N=1,NNN
      DC 371 M=1,MM
      C10(M,N)=0.0
      ML(M,N)=0.0
      MU(M,N)=0.0
      MC(M,N)=0.0
      VL(M,N)=0.0
      VU(M,N)=0.0
      PAPP(M,N)=0.0
      P(M,N)=0.0
      MULP(M,N)=FYC(M,N)*ZXC(M,N)
      MLUP(M,N)=MULP(M,N)
      MDLP(M,N)=MULP(M,N)
      MDUP(M,N)=MULP(M,N)
      XMP(M,N)=MULP(M,N)
      XC(M,N)=120.0
      YC(M,N)=120.0
      DELD(M,N)=0.0
      VJTL(M,N)=0.0
 371  CCNTINUE
 372  CCNTINUE
      DC 380 M=1,MM
      P(M,NN)=0.0
      VJTL(M,NN)=0.0
      VJT(M,1)=0.0
      PAPP(M,NN)=0.0
 380  CCNTINUE
      IL=0
      IND3=0
      IREV=0
1000  FCRMAT(2I3,F9.5)
1001  FCRMAT(8F10.0)
1002  FCRMAT(A8,F9.2,F9.2,F8.2,3F7.2,F7.0)
1003  FORMAT(A8,F7.2,F9.2,F8.2,F7.2,F7.0,2F7.2)
1004  FCRMAT(A8,3F7.2,F7.0)
1011  FCRMAT(8F10.5)
1014  FCRMAT(9I5)
1015  FCRMAT(4F10.5)
1016  FCRMAT(I5)
1030  FCRMAT(E11.4)
1060  FCRMAT(5A8)
1061  FORMAT (1H0,5A8)
1062  FCRMAT(1H0,I3,9H STCRIES,,I3,5H BAYS)
1063  FCRMAT(1H0,17HSTCRY       HEIGHT)
1064  FORMAT(1H0,15H BAY       WIDTH)
1066  FCRMAT(1H ,I3,6X,F6.0)
1067  FORMAT(1H0,104HLCCATICN        TYPE           AREA             IX
     1    ZX              RX          WIDTH        FY             E)
1068  FCRMAT(1H ,I3,1H,,I3,5X,A8,2(5X,F9.2),5X,F8.2,3(5X,F7.2),5X,F7.0)
1069  FCRMAT(1H0,106HLCCATICN        TYPE           AREA             IX
     1    ZX             FY            E        DEAD LOAD    LIVE LOAD)
1070  FCRMAT(1H ,I3,1H,,I3,5X,A8,5X,F7.2,5X,F9.2,5X,F8.2,5X,F7.2,5X,F7.0
     1,5X,F7.2,5X,F7.2)
1071  FCRMAT(1H0,68H LCCATICN          TYPE           AREA             RX
     1  FY              E)
1072  FCRMAT(1H ,I3,1H,,I3,3H, 1,5X,A8,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.0)
1073  FCRMAT(1H ,I3,1H,,I3,3H, 2,5X,A8,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.0)
1075  FCRMAT(1H0,17HCCLUMN PRCPERTIES)
1076  FCRMAT(1H0,17HGIRDER PRCPERTIES)
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1077  FORMAT(1H0,'PROPERTIES OF DIAGONAL BRACING,',F10.5,' TENSILE & COM
     1PRESSIVE CAPACITY')
1080  FORMAT(1H0,31HCOLUMN     BASE SPRING CONSTANT)
1081  FORMAT(1H ,1X,I3,12X,E11.4)
1082  FORMAT(1H0,35HFLOOR LEVEL      DESIGN LATERAL LOAD)
1083  FORMAT(1H ,4X,I3,14X,F10.5)
1084  FORMAT(1H0,30HJOINT      DESIGN VERTICAL LOAD)
1085  FORMAT(1H ,I2,1H,,I3,6X,F10.5)
1086  FORMAT(1H0,42HCOLUMN     DESIGN AXIAL LOAD AT COLUMN TOP)
1087  FORMAT(1H ,1X,I3,15X,F10.5)
1088  FORMAT(1H0,'LOAD SEQUENCE      LIVE LOAD FACTOR      LATERAL LOAD FA
     1CTOR      AXIAL LOAD FACTOR      JOINT LOAD FACTOR')
1089  FORMAT(1H ,5X,I3,13X,F10.5,12X,F10.5,13X,F10.5,12X,F10.5)
1090  FORMAT(1H0,'LOAD INCREMENT'7X,F10.5,12X,F10.5,13X,F10.5,12X,F10.5)
2000  FORMAT(1H1,'*******************************************')
2001  FORMAT(1H ,'*******************************************')
2002  FORMAT(1H0,'*******************************************')
      RETURN
      END
      SUBROUTINE COEFF
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     EVALUATE COEFFICIENTS OF SLOPE-DEFLECTION EQUATIONS WHICH ARE INDEPENDENT
C     OF LOAD
C
C
C     GIRDERS
C
      DO 129 M=1,MMM
      DO 119 N=2,NN
      IF(JG(M,N).EQ.1) GO TO 110
      IF(JG(M,N).EQ.2) GO TO 111
      IF(JG(M,N).EQ.3) GO TO 112
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      IF(JG(M,N).EQ.4) GO TO 113
      IF(JG(M,N).EQ.5) GO TO 114
      IF(JG(M,N).EQ.6) GO TO 115
      IF(JG(M,N).EQ.7) GO TO 116
      IF(JG(M,N).EQ.8) GO TO 117
      IF(JG(M,N).EQ.10) GO TO 110
      IF(JG(M,N).EQ.11) GO TO 110
      IF(JG(M,N).EQ.12) GO TO 110
      IF(JG(M,N).EQ.13) GO TO 112
      IF(JG(M,N).EQ.14) GO TO 111
      IF(JG(M,N).EQ.15) GO TO 113
      IF(JG(M,N).EQ.16) GO TO 111
      IF(JG(M,N).EQ.17) GO TO 113
      IF(JG(M,N).EQ.18) GO TO 112
      IF(JG(M,N).EQ.20) GO TO 116
      IF(JG(M,N).EQ.21) GO TO 115
110   D1=WC(M,N)/LG(M,N)
      D2=WC(M+1,N)/LG(M,N)
      D3=EG(M,N)*IXG(M,N)/LG(M,N)
      D4=D3/LG(M,N)
      CA1(M,N)=(4.0+3.0*D1)*D3
      CA2(M,N)=(2.0+3.0*D2)*D3
      CA3(M,N)=6.0*D4
      CB1(M,N)=(2.0+3.0*D1)*D3
      CB2(M,N)=(4.0+3.0*D2)*D3
      CB3(M,N)=CA3(M,N)
      GO TO 119
111   D1=WC(M,N)/LG(M,N)
      D2=WC(M+1,N)/LG(M,N)
      D3=EG(M,N)*IXG(M,N)/LG(M,N)
      D4=D3/LG(M,N)
      CA1(M,N)=0.0
      CA2(M,N)=0.0
      CA3(M,N)=0.0
      CB1(M,N)=1.5*D1*D3
      CB2(M,N)=(3.0+1.5*D2)*D3
      CB3(M,N)=3.0*D4
      GO TO 119
112   D1=WC(M,N)/LG(M,N)
      D2=WC(M+1,N)/LG(M,N)
      D3=EG(M,N)*IXG(M,N)/LG(M,N)
      D4=D3/LG(M,N)
      CA1(M,N)=(3.0+1.5*D1)*D3
      CA2(M,N)=1.5*D2*D3
      CA3(M,N)=3.0*D4
      CB1(M,N)=0.0
      CB2(M,N)=0.0
      CB3(M,N)=0.0
      GO TO 119
113   D5=XG(M,N)**3
      D6=YG(M,N)**3
      D1=1.0/(D5+D6)
      D2=EG(M,N)*IXG(M,N)
      D3=WC(M,N)/XG(M,N)
      D4=WC(M+1,N)/YG(M,N)
      CA1(M,N)=D1*XG(M,N)*XG(M,N)*(3.0+1.5*D3)*D2
      CA2(M,N)=D1*XG(M,N)*YG(M,N)*(3.0+1.5*D4)*D2
      CA3(M,N)=D1*XG(M,N)*3.0*D2
      CB1(M,N)=D1*XG(M,N)*YG(M,N)*(3.0+1.5*D3)*D2
      CB2(M,N)=D1*YG(M,N)*YG(M,N)*(3.0+1.5*D4)*D2
```

```
      CB3(M,N)=D1*YG(M,N)*3.0*D2
      MC(M,N)=MCAP(M,N)
      GO TO 119
  114 CA1(M,N)=0.0
      CA2(M,N)=0.0
      CA3(M,N)=0.0
      CB1(M,N)=0.0
      CB2(M,N)=0.0
      CB3(M,N)=0.0
      GO TO 119
  115 CA1(M,N)=0.0
      CA2(M,N)=0.0
      CA3(M,N)=0.0
      CB1(M,N)=0.0
      CB2(M,N)=0.0
      CB3(M,N)=0.0
      MC(M,N)=MCAP(M,N)
      GO TO 119
  116 CA1(M,N)=0.0
      CA2(M,N)=0.0
      CA3(M,N)=0.0
      CB1(M,N)=0.0
      CB2(M,N)=0.0
      CB3(M,N)=0.0
      MC(M,N)=MCAP(M,N)
      GO TO 119
  117 CA1(M,N)=0.0
      CA2(M,N)=0.0
      CA3(M,N)=0.0
      CB1(M,N)=0.0
      CB2(M,N)=0.0
      CB3(M,N)=0.0
      MC(M,N)=MCAP(M,N)
  119 CONTINUE
  129 CONTINUE
C
C     COLUMNS
C
      DO 210 N=1,NNN
      DO 205 M=1,MM
      IF(IAREA.EQ.1) GO TO 202
      CC2(M,N)=AC(M,N)*EC(M,N)/H(N)
      GO TO 205
  202 CC2(M,N)=10000*AC(M,N)*EC(M,N)/H(N)
  205 CONTINUE
  210 CONTINUE
C
C     DIAGONAL BRACING
C
      DO 159 I=1,2
      DO 158 N=1,NNN
      IF(I.EQ.2) GO TO 155
      DO 154 M=2,MM
      IF(AB(M,N,I))154,153,154
  153 JB(M,N,I)=9
  154 CONTINUE
      GO TO 158
  155 DO 157 M=1,MMM
      IF(AB(M,N,I))157,156,157
  156 JB(M,N,I)=9
```

```
157   CONTINUE
158   CONTINUE
159   CONTINUE
      DO 180 I=1,2
      DO 179 N=1,NNN
      IF(I.EQ.2) GO TO 170
      DO 169 M=2,MM
      D1=AB(M,N,I)*EB(M,N,I)/LB(M,N,I)
      D2=L(M-1)/LB(M,N,I)
      D3=H(N)/LB(M,N,I)
      IF(JB(M,N,I).EQ.1) GO TO 160
      IF(JB(M,N,I).EQ.11) GO TO 160
      CD1(M,N,I)=0.0
      CD2(M,N,I)=0.0
      CD4(M,N,I)=0.0
      IF(JB(M,N,I).EQ.2) GO TO 161
      IF(JB(M,N,I).EQ.3) GO TO 162
      IF(JB(M,N,I).EQ.9) GO TO 163
160   CD1(M,N,I)=D1*D2*D3
      CD2(M,N,I)=D1*D2*D2
      CD4(M,N,I)=D1*D3*D3
      CD3(M,N,I)=0.0
      CD5(M,N,I)=0.0
      GO TO 169
161   CD3(M,N,I)=D3*FBP(M,N,I)
      CD5(M,N,I)=D2*FBP(M,N,I)
      GO TO 169
162   CD3(M,N,I)=-D3*PCR(M,N,I)
      CD5(M,N,I)=-D2*PCR(M,N,I)
      GO TO 169
163   CD3(M,N,I)=0.0
      CD5(M,N,I)=0.0
169   CONTINUE
      GO TO 179
170   DO 178 M=1,MMM
      D1=AB(M,N,I)*EB(M,N,I)/LB(M,N,I)
      D2=L(M)/LB(M,N,I)
      D3=H(N)/LB(M,N,I)
      IF(JB(M,N,I).EQ.1) GO TO 171
      IF(JB(M,N,I).EQ.11) GO TO 171
      CD1(M,N,I)=0.0
      CD2(M,N,I)=0.0
      CD4(M,N,I)=0.0
      IF(JB(M,N,I).EQ.2) GO TO 172
      IF(JB(M,N,I).EQ.3) GO TO 173
      IF(JB(M,N,I).EQ.9) GO TO 174
171   CD1(M,N,I)=D1*D2*D3
      CD2(M,N,I)=D1*D2*D2
      CD4(M,N,I)=D1*D3*D3
      CD3(M,N,I)=0.0
      CD5(M,N,I)=0.0
      GO TO 178
172   CD3(M,N,I)=D3*FBP(M,N,I)
      CD5(M,N,I)=D2*FBP(M,N,I)
      GO TO 178
173   CD3(M,N,I)=-D3*PCR(M,N,I)
      CD5(M,N,I)=-D2*PCR(M,N,I)
      GO TO 178
174   CD3(M,N,I)=0.0
      CD5(M,N,I)=0.0
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```
178   CONTINUE
179   CONTINUE
180   CONTINUE
      RETURN
      END
      SUBROUTINE LOAD
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 OLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     DETERMINE APPLIED LOADS ON FRAME & COMPUTE FIXED END MOMENTS
C
      IF(IND3.EQ.1) GO TO 893
      IF(I12.EQ.1) GO TO 893
      IF(IND.GE.1) GO TO 895
      IF(IREV.GE.1) GO TO 895
893   IL=IL+1
      WRITE(6,1309) IL
      IF(I12.EQ.1) GO TO 77
895   IF(NL.GT.0) GO TO 308
      IF(DET.GE.0.0) GO TO 900
      IF(IND3.NE.1) GO TO 900
      RINF=-RINF
      RINP=-RINP
      RINLL=-RINLL
      RINV=-RINV
      GO TO 901
900   IF(IND.GE.1) GO TO 308
      IF(IREV.GE.1) GO TO 308
901   CONTINUE
      IF(IL.EQ.1) GO TO 308
      IL1=IL-1
      RF(IL)=RF(IL1)+RINF
      RLL(IL)=RLL(IL1)+RINLL
      RP(IL)=RP(IL1)+RINP
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```
      RV(IL)=RV(IL1)+RINV
      GO TO 308
   77 IL2=IL-2
      RP(IL)=RP(IL2)
      RF(IL)=RF(IL2)
      RV(IL)=RV(IL2)
      RLL(IL)=RLL(IL2)
  308 DO 310 N=2,NN
      F(N)=RF(IL)*FDES(N)
      DO 309 M=1,MMM
      W(M,N)=DLDES(M,N)+RLL(IL)*LLDES(M,N)
      VWA(M,N)=W(M,N)*LG(M,N)/24.0
      VWB(M,N)=VWA(M,N)
      IF(JG(M,N).EQ.4) GO TO 311
      IF(JG(M,N).EQ.6) GO TO 311
      IF(JG(M,N).EQ.7) GO TO 311
      IF(JG(M,N).EQ.8) GO TO 311
      IF(JG(M,N).EQ.15) GO TO 311
      IF(JG(M,N).EQ.17) GO TO 311
      IF(JG(M,N).EQ.20) GO TO 311
      IF(JG(M,N).EQ.21) GO TO 311
      MFBA(M,N)=W(M,N)*LG(M,N)*LG(M,N)/144.0
      MFAB(M,N)=-MFBA(M,N)
      GO TO 309
  311 MFCA(M,N)=W(M,N)*XG(M,N)*XG(M,N)/144.0
      MFAB(M,N)=-MFCA(M,N)
      MAX(M,N)=6.0*MFCA(M,N)
      MFBA(M,N)=W(M,N)*YG(M,N)*YG(M,N)/144.0
      MFCB(M,N)=-MFBA(M,N)
      MBY(M,N)=6.0*MFBA(M,N)
  309 CONTINUE
      DO 608 M=1,MM
      VJT(M,N)=VDES(M,N)*RV(IL)
  608 CONTINUE
  310 CONTINUE
      IF(IND3.EQ.1) GO TO 693
      IF(I12.EQ.1) GO TO 693
      IF(IND.GE.1) GO TO 700
      IF(IREV.GE.1) GO TO 700
  693 IF(IND6.EQ.2) GO TO 950
      WRITE(6,1300)
      DO 420 N=2,NN
      WRITE(6,1301)N,F(N),RF(IL)
  420 CONTINUE
      WRITE(6,1302)
      DO 325 N=2,NN
      DO 324 M=1,MMM
      WRITE(6,1303)M,N,W(M,N),RLL(IL)
  324 CONTINUE
  325 CONTINUE
 1299 FORMAT(1H0,44HCOLUMN            APPLIED LOAD       LOAD FACTOR)
 1309 FORMAT(1H1,15HLOAD INCREMENT ,I4)
 1300 FORMAT(1H0,44HFLOOR LEVEL       LATERAL LOAD       LOAD FACTOR)
 1301 FORMAT(1H ,I6,10X,F10.5,7X,F10.5)
 1302 FORMAT(1H0,44HGIRDER              UDL(KLF)         LOAD FACTOR)
 1303 FORMAT(1H ,I3,I3,10X,F10.5,7X,F10.5)
C
C     EVALUATE COEFFICIENTS OF SLOPE-DEFLECTION EQUATIONS WHICH ARE DEPENDENT
C     ON THE LOADS ON THE GIRDERS
C
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      GO TO 700
 950  WRITE(6,1225) RF(IL),RLL(IL),RP(IL)
1225  FORMAT(1H0,'LOAD FACTOR  -  LATERAL LOAD  ',F10.5,'  ,  LIVE LOAD
     1 ',F10.5,'  ,  AXIAL LOAD  ',F10.5)
 700  DO 135 N=2,NN
      DO 134 M=1,MMM
      IF(JG(M,N).EQ.1) GO TO 120
      IF(JG(M,N).EQ.2) GO TO 121
      IF(JG(M,N).EQ.3) GO TO 122
      IF(JG(M,N).EQ.4) GO TO 123
      IF(JG(M,N).EQ.5) GO TO 124
      IF(JG(M,N).EQ.6) GO TO 125
      IF(JG(M,N).EQ.7) GO TO 126
      IF(JG(M,N).EQ.8) GO TO 127
      IF(JG(M,N).EQ.10) GO TO 120
      IF(JG(M,N).EQ.11) GO TO 120
      IF(JG(M,N).EQ.12) GO TO 120
      IF(JG(M,N).EQ.13) GO TO 122
      IF(JG(M,N).EQ.14) GO TO 121
      IF(JG(M,N).EQ.15) GO TO 123
      IF(JG(M,N).EQ.16) GO TO 121
      IF(JG(M,N).EQ.17) GO TO 123
      IF(JG(M,N).EQ.18) GO TO 122
      IF(JG(M,N).EQ.20) GO TO 126
      IF(JG(M,N).EQ.21) GO TO 125
 120  CA5(M,N)=MFAB(M,N)
      CB5(M,N)=MFBA(M,N)
      IF(JG(M,N).EQ.1)  GO TO 130
      IF(JG(M,N).EQ.12) GO TO 130
      CA5(M,N)=CA5(M,N)+CA6(M,N)
      CB5(M,N)=CB5(M,N)+CB6(M,N)
      GO TO 130
 121  CA5(M,N)=MABP(M,N)
      CB5(M,N)=MFBA(M,N)-0.5*MFAB(M,N)+0.5*MABP(M,N)
      IF(JG(M,N).EQ.2) GO TO 130
      IF(JG(M,N).EQ.16) GO TO 130
      CA5(M,N)=CA5(M,N)+CA6(M,N)
      CB5(M,N)=CB5(M,N)+CB6(M,N)
      GO TO 130
 122  CA5(M,N)=MFAB(M,N)-0.5*MFBA(M,N)+0.5*MBAP(M,N)
      CB5(M,N)=MBAP(M,N)
      IF(JG(M,N).EQ. 3) GO TO 130
      IF(JG(M,N).EQ.18) GO TO 130
      CA5(M,N)=CA5(M,N)+CA6(M,N)
      CB5(M,N)=CB5(M,N)+CB6(M,N)
      GO TO 130
 123  D5=XG(M,N)**3
      D6=YG(M,N)**3
      D1=1.0/(D5+D6)
      CA5(M,N)=D1*(D5*(MFAB(M,N)-0.5*MFCA(M,N))+(0.5*D5-D6)*MCAP(M,N)-(D
     16*MAX(M,N))+(XG(M,N)*YG(M,N)*YG(M,N))*(MFBA(M,N)-0.5*MFCB(M,N)+1.5
     2*MCBP(M,N)-MBY(M,N)))
      CB5(M,N)=D1*(D6*(MFBA(M,N)-0.5*MFCB(M,N))+(0.5*D6-D5)*MCBP(M,N)+(D
     15*MBY(M,N))+(XG(M,N)*XG(M,N)*YG(M,N))*(MFAB(M,N)-0.5*MFCA(M,N)+1.5
     2*MCAP(M,N)+MAX(M,N)))
      IF(JG(M,N).EQ.4) GO TO 130
      CA5(M,N)=CA5(M,N)+CA6(M,N)
      CB5(M,N)=CB5(M,N)+CB6(M,N)
      GO TO 130
 124  CA5(M,N)=MABP(M,N)
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```
      CB5(M,N)=MBAP(M,N)
      GO TO 130
  125 CA5(M,N)=MABP(M,N)
      CB5(M,N)=(YG(M,N)/XG(M,N))*(MABP(M,N)+MCAP(M,N)-MAX(M,N))+MBY(M,N)
     1-MCBP(M,N)
      GO TO 130
  126 CA5(M,N)=(XG(M,N)/YG(M,N))*(MCBP(M,N)+MBAP(M,N)-MBY(M,N))-MCAP(M,N
     1)-MAX(M,N)
      CB5(M,N)=MBAP(M,N)
      GO TO 130
  127 CA5(M,N)=MABP(M,N)
      CB5(M,N)=MBAP(M,N)
  130 CONTINUE
  134 CONTINUE
  135 CONTINUE
C
C     COMPUTE APPROXIMATE AXIAL LOADS IN COLUMNS
C
      IF(IND6.EQ.2) GO TO 919
      WRITE(6,1299)
  919 IF(NL.GT.0) GO TO 307
      IF(IL.GT.1) GO TO 500
  307 DO 312 M=1,MM
      P(M,NN)=RP(IL)*PDES(M)
      IF(IND6.EQ.2) GO TO 920
      WRITE(6,1301)M,P(M,NN),RP(IL)
  920 SUMV(M)=P(M,NN)
  312 CONTINUE
      GO TO 503
  500 DO 502 M=1,MM
      P(M,NN)=RP(IL)*PDES(M)
      IF(IND6.EQ.2) GO TO 921
      WRITE(6,1301)M,P(M,NN),RP(IL)
  921 P(M,NN)=(RP(IL)-RP(IL1))*PDES(M)
      SUMV(M)=P(M,NN)
  502 CONTINUE
  503 IF(I12.EQ.1) GO TO 504
      IF(IND.GE.1) GO TO 701
      IF(IREV.GE.1) GO TO 701
  504 DO 314 N=2,NN
      VINCR(1,N)=VWA(1,N)-VWAL(1,N)+VJT(1,N)-VJTL(1,N)
      VINCR(MM,N)=VWB(MMM,N)-VWBL(MMM,N)+VJT(MM,N)-VJTL(MM,N)
      IF(MM.EQ.2) GO TO 314
      DO 315 M=2,MMM
      VINCR(M,N)=VWA(M,N)-VWAL(M,N)+VWB(M-1,N)-VWBL(M-1,N)+VJT(M,N)-VJTL
     1 (M,N)
  315 CONTINUE
  314 CONTINUE
      J=1
  316 N=NN-J
      IF(N.EQ.0) GO TO 320
      DO 317 M=1,MM
      SUMV(M)=SUMV(M)+VINCR(M,N+1)
      PAPP(M,N)=P(M,N)+SUMV(M)
  317 CONTINUE
      J=J+1
      GO TO 316
  320 CONTINUE
      DO 323 N=2,NN
      DO 322 M=1,MMM
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      VWAL(M,N)=VWA(M,N)
      VWBL(M,N)=VWB(M,N)
 322  CONTINUE
      DO 628 M=1,MM
      VJTL(M,N)=VJT(M,N)
 628  CONTINUE
 323  CONTINUE
 701  CONTINUE
      RETURN
      END
      SUBROUTINE STAB
      COMMON/AREA1/ AC(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JE(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     CALCULATE PLASTIC MOMENT CAPACITY OF COLUMNS
C
      DO 345 N=1,NNN
      DO 344 M=1,MM
      IF(AC(M,N).EQ.0.0.OR.FYC(M,N).EQ.0.0) GO TO 344
      D1=PAPP(M,N)/(AC(M,N)*FYC(M,N))
      IF(D1.LE.0.15) GO TO 710
      D2=1.18*(1.0-D1)
      IF(D1.LE.1.00) GO TO 711
      WRITE(6,222) D1,M,N
 222  FORMAT(1H0,'P/PY=',F10.5,'COLUMN',2I5)
      GO TO 711
 710  D2=1.0
 711  IF(MULP(M,N).LT.0.0) GO TO 720
      MULP(M,N)=XMP(M,N)*D2
      GO TO 721
 720  MULP(M,N)=-XMP(M,N)*D2
 721  IF(MLUP(M,N).LT.0.0) GO TO 730
      MLUP(M,N)=XMP(M,N)*D2
      GO TO 731
```

```
730   MLUP(M,N)=-XMF(M,N)*D2
731   IF(MDLP(M,N).LT.0.0) GC TC 740
      MDLP(M,N)=XMP(M,N)*D2
      GC TO 741
740   MDLP(M,N)=-XMP(M,N)*D2
741   IF(MDUP(M,N).LT.0.0) GC TO 750
      MDUP(M,N)=XMP(M,N)*D2
      GC TO 751
750   MDUP(M,N)=-XMP(M,N)*D2
751   CCNTINUE
344   CCNTINUE
345   CCNTINUE
C
C     CALCULATE "C" & "S" FACTORS
C
      DC 330 N=1,NNN
      DO 329 M=1,MM
      IF(EC(M,N).EQ.0.0.OR.IXC(M,N).EQ.0.0) GO TO 410
      IF(JC(M,N).EQ.4) GO TC 327
      IF(JC(M,N).GE.6) GO TO 327
      IF(PAPP(M,N).LE.0.0) GC TO 328
      IF(ISTAB.EQ.1) GC TO 328
      D1=(SQRT(PAPP(M,N)/(EC(M,N)*IXC(M,N))))*H(N)
      IF(D1.LT.0.5000) GO TO 351
      IF(D1.LT.1.0000.AND.D1.GE.0.5000) GO TC 352
      IF(D1.LT.1.5000.AND.D1.GE.1.0000) GO TO 353
      IF(D1.LT.2.0000.AND.D1.GE.1.5000) GO TO 354
      IF(D1.LT.2.5000.AND.D1.GE.2.0000) GO TO 355
      IF(D1.LT.3.0000.AND.D1.GE.2.5000) GO TO 356
      IF(D1.LT.3.5000.AND.D1.GE.3.0000) GO TO 357
      IF(D1.LT.4.0000.AND.D1.GE.3.5000) GO TO 358
      IF(D1.LT.4.5000.AND.D1.GE.4.0000) GO TO 359
      WRITE(6,2010)M,N
      IF(D1.LT.5.0000.AND.D1.GE.4.5000) GO TO 160
      IF(D1.LT.5.5000.AND.D1.GE.5.0000) GO TO 161
      IF(D1.LT.6.0000.AND.D1.GE.5.5000) GO TC 162
      IF(D1.LT.6.2750.AND.D1.GE.6.0000) GO TO 163
      WRITE(6,2011)M,N
      GC TO 329
351   CC(M,N)=4.0000-0.0668*D1
      SS(M,N)=2.0000+0.0168*D1
      GC TO 329
352   CC(M,N)=3.9666-0.2034*(D1-0.5000)
      SS(M,N)=2.0084+0.0520*(D1-0.5000)
      GC TC 329
353   CC(M,N)=3.8649-0.3484*(D1-1.0000)
      SS(M,N)=2.0344+0.0924*(D1-1.0000)
      GC TO 329
354   CC(M,N)=3.6907-0.5092*(D1-1.5000)
      SS(M,N)=2.0806+0.1426*(D1-1.5000)
      GC TO 329
355   CC(M,N)=3.4361-0.6966*(D1-2.0000)
      SS(M,N)=2.1519+0.2106*(D1-2.0000)
      GC TO 329
356   CC(M,N)=3.0878-0.9272*(D1-2.5000)
      SS(M,N)=2.2572+0.3086*(D1-2.5000)
      GC TC 329
357   CC(M,N)=2.6242-1.2318*(D1-3.0000)
      SS(M,N)=2.4115+0.4638*(D1-3.0000)
      GC TO 329
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 358  CC(M,N)=2.0083-1.6704*(D1-3.5000)
      SS(M,N)=2.6424+0.7226*(D1-3.5000)
      GO TO 329
 359  CC(M,N) =1.1731-2.3844*(D1-4.0000)
      SS(M,N)=3.0037+1.2206*(D1-4.0000)
      GC TO 329
 160  CC(M,N) =-0.0191-3.7890*(D1-4.5000)
      SS(M,N) =3.6140+2.3432*(D1-4.5000)
      GC TO 329
 161  CC(M,N) =-1.9136-7.5076*(D1-5.0000)
      SS(M,N) =4.7856+5.7146*(D1-5.0000)
      GC TO 329
 162  CC(M,N) =-5.6674-29.938*(D1-5.5000)
      SS(M,N) =7.6429+27.620*(D1-5.5000)
      GC TO 329
 163  CC(M,N) =-20.636-2827.6*(D1-6.0000)
      SS(M,N) =21.453+2824.8*(D1-6.0000)
      GC TO 329
 328  CC(M,N)=4.0
      SS(M,N)=2.0
      GC TO 329
 327  IF(PAPP(M,N).LE.0.0) GO TO 331
      IF(ISTAB.EQ.1) GC TO 331
      D1=(SCRT(PAPP(M,N)/(EC(M,N)*IXC(M,N))))*XC(M,N)
      IF(D1.LT.0.5000) GO TO 361
      IF(D1.LT.1.0000.AND.D1.GE.0.5000) GO TO 362
      IF(D1.LT.1.5000.AND.D1.GE.1.0000) GO TC 363
      IF(D1.LT.2.0000.AND.D1.GE.1.5000) GO TO 364
      IF(D1.LT.2.5000.AND.D1.GE.2.0000) GO TO 365
      IF(D1.LT.3.0000.AND.D1.GE.2.5000) GO TC 366
      IF(D1.LT.3.5000.AND.D1.GE.3.0000) GO TO 457
      IF(D1.LT.4.0000.AND.D1.GE.3.5000) GO TO 458
      IF(D1.LT.4.5000.AND.D1.GE.4.0000) GO TC 459
      WRITE(6,2010)M,N
      IF(D1.LT.5.0000.AND.D1.GE.4.5000) GO TO 170
      IF(D1.LT.5.5000.AND.D1.GE.5.0000) GO TO 171
      IF(D1.LT.6.0000.AND.D1.GE.5.5000) GO TO 172
      IF(D1.LT.6.2750.AND.D1.GE.6.0000) GO TO 173
      WRITE(6,2011)M,N
      GC TO 369
 361  CCX(M,N)=4.0000-C.0668*D1
      SSX(M,N)=2.0000+0.0168*D1
      GC TO 369
 362  CCX(M,N)=3.9666-0.2034*(D1-0.5000)
      SSX(M,N)=2.0084+0.0520*(D1-0.5000)
      GC TO 369
 363  CCX(M,N)=3.8649-C.3484*(D1-1.0000)
      SSX(M,N)=2.0344+0.0924*(D1-1.0000)
      GC TO 369
 364  CCX(M,N)=3.6907-0.5092*(D1-1.5000)
      SSX(M,N)=2.0806+C.1426*(D1-1.5000)
      GC TO 369
 365  CCX(M,N)=3.4361-0.6966*(D1-2.0000)
      SSX(M,N)=2.1519+C.2106*(D1-2.0000)
      GC TO 369
 366  CCX(M,N)=3.0878-C.9272*(D1-2.5000)
      SSX(M,N)=2.2572+C.3086*(D1-2.5000)
      GC TC 369
 457  CCX(M,N)=2.6242-1.2318*(D1-3.0000)
      SSX(M,N)=2.4115+0.4638*(D1-3.0000)
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      GC TC 369
458   CCX(M,N)=2.0083-1.6704*(D1-3.5000)
      SSX(M,N)=2.6424+0.7226*(D1-3.5000)
      GC TO 369
459   CCX(M,N)=1.1731-2.3844*(D1-4.0000)
      SSX(M,N)=3.0037+1.2206*(D1-4.0000)
      GC TO 369
170   CCX(M,N)=-0.0191-3.7890*(D1-4.5000)
      SSX(M,N)=3.6140+2.3432*(D1-4.5000)
      GO TO 369
171   CCX(M,N) =-1.9136-7.5076*(D1-5.0000)
      SSX(M,N)=4.7856+5.7146*(D1-5.0000)
      GO TO 369
172   CCX(M,N)=-5.6674-29.938*(D1-5.5000)
      SSX(M,N)=7.6429+27.620*(D1-5.5000)
      GO TO 369
173   CCX(M,N)=-20.636-2827.6*(D1-6.0000)
      SSX(M,N)=21.453+2824.8*(D1-6.0000)
369   CCNTINUE
      D1=(SCRT(PAPP(M,N)/(EC(M,N)*IXC(M,N))))*YC(M,N)
      IF(D1.LT.0.5000) GO TO 371
      IF(D1.LT.1.0000.AND.D1.GE.0.5000) GO TO 372
      IF(D1.LT.1.5000.AND.D1.GE.1.0000) GO TO 373
      IF(D1.LT.2.0000.AND.D1.GE.1.5000) GO TO 374
      IF(D1.LT.2.5000.AND.D1.GE.2.0000) GO TO 375
      IF(D1.LT.3.0000.AND.D1.GE.2.5000) GO TC 376
      IF(D1.LT.3.5000.AND.D1.GE.3.0000) GO TO 557
      IF(D1.LT.4.0000.AND.D1.GE.3.5000) GO TC 558
      IF(D1.LT.4.5000.AND.D1.GE.4.0000) GO TO 559
      WRITE(6,2010)M,N
      IF(D1.LT.5.0000.AND.D1.GE.4.5000) GO TO 180
      IF(D1.LT.5.5000.AND.D1.GE.5.0000) GO TO 181
      IF(D1.LT.6.0000.AND.D1.GE.5.5000) GO TO 182
      IF(D1.LT.6.2750.AND.D1.GE.6.0000) GO TC 183
      WRITE(6,2011)M,N
      GC TC 329
371   CCY(M,N)=4.0000-0.0668*D1
      SSY(M,N)=2.0000+0.0168*D1
      GC TO 329
372   CCY(M,N)=3.9666-0.2034*(D1-0.5000)
      SSY(M,N)=2.0084+0.0520*(D1-0.5000)
      GC TO 329
373   CCY(M,N)=3.8649-0.3484*(D1-1.0000)
      SSY(M,N)=2.0344+0.0924*(D1-1.0000)
      GC TO 329
374   CCY(M,N)=3.6907-0.5092*(D1-1.5000)
      SSY(M,N)=2.0806+0.1426*(D1-1.5000)
      GC TC 329
375   CCY(M,N)=3.4361-0.6966*(D1-2.0000)
      SSY(M,N)=2.1519+0.2106*(D1-2.0000)
      GC TO 329
376   CCY(M,N)=3.0878-0.9272*(D1-2.5000)
      SSY(M,N)=2.2572+0.3086*(D1-2.5000)
      GO TO 329
557   CCY(M,N)=2.6242-1.2318*(D1-3.0000)
      SSY(M,N)=2.4115+0.4638*(D1-3.0000)
      GC TO 329
558   CCY(M,N)=2.0083-1.6704*(D1-3.5000)
      SSY(M,N)=2.6424+0.7226*(D1-3.5000)
      GC TO 329
```

```
 559  CCY(M,N)=1.1731-2.3844*(D1-4.0000)
      SSY(M,N)=3.0037+1.2206*(D1-4.0000)
      GC TO 329
 180  CCY(M,N)=-0.0191-3.7890*(D1-4.5000)
      SSY(M,N)=3.6140+2.3432*(D1-4.5000)
      GO TO 329
 181  CCY(M,N) =-1.9136-7.5076*(D1-5.0000)
      SSY(M,N)=4.7856+5.7146*(D1-5.0000)
      GO TO 329
 182  CCY(M,N)=-5.6674-29.938*(D1-5.5000)
      SSY(M,N)=7.6429+27.620*(D1-5.5000)
      GC TO 329
 183  CCY(M,N)=-20.636-2827.6*(D1-6.0000)
      SSY(M,N)=21.453+2824.8*(D1-6.0000)
      GC TO 329
 331  CCX(M,N)=4.0
      SSX(M,N)=2.0
      SSY(M,N)=2.0
      CCY(M,N)=4.0
      GC TO 329
 410  CC(M,N)=0.0
      SS(M,N)=0.0
 329  CONTINUE
 330  CONTINUE
2010  FORMAT(1H0,30HNEGATIVE STIFFNESS     COLUMN,I3,1H,,I3)
2011  FORMAT(1H0,30HINFINITE STIFFNESS     COLUMN,I3,1H,,I3)
C
C     EVALUATE COEFFICIENTS OF SLOPE-DEFLECTION EQUATIONS WHICH ARE DEPENDENT
C     ON AXIAL LOAD
C
      DC 149 M=1,MM
      DO 150 N=1,NNN
      IF(JC(M,N).EQ.1) GO TO 140
      IF(JC(M,N).EQ.2) GO TO 141
      IF(JC(M,N).EQ.3) GO TO 142
      IF(JC(M,N).EQ.4) GO TO 143
      IF(JC(M,N).EQ.5) GO TO 144
      IF(JC(M,N).EQ.6) GC TC 145
      IF(JC(M,N).EQ.7) GC TO 146
      IF(JC(M,N).EQ.8) GO TO 147
 140  D1=EC(M,N)*IXC(M,N)/H(N)
      D2=D1/H(N)
      CL1(M,N)=CC(M,N)*D1
      CL2(M,N)=SS(M,N)*D1
      CL3(M,N)=(CC(M,N)+SS(M,N))*D2
      CL5(M,N)=0.0
      CU1(M,N)=CL2(M,N)
      CU2(M,N)=CL1(M,N)
      CU3(M,N)=CL3(M,N)
      CU5(M,N)=0.0
      GC TO 150
 141  D1=EC(M,N)*IXC(M,N)/H(N)
      D2=D1/H(N)
      D3=(CC(M,N)*CC(M,N)-SS(M,N)*SS(M,N))/CC(M,N)
      CL1(M,N)=0.0
      CL2(M,N)=0.0
      CL3(M,N)=0.0
      CL5(M,N)=MLUP(M,N)
      CU1(M,N)=0.0
      CU2(M,N)=D3*D1
```

```
      CU3(M,N)=D3*D2
      CU5(M,N)=(SS(M,N)/CC(M,N))*MLUP(M,N)
      GC TO 150
142   D1=EC(M,N)*IXC(M,N)/H(N)
      D2=D1/H(N)
      D3=(CC(M,N)*CC(M,N)-SS(M,N)*SS(M,N))/CC(M,N)
      CL1(M,N)=D3*D1
      CL2(M,N)=0.0
      CL3(M,N)=D3*D2
      CL5(M,N)=(SS(M,N)/CC(M,N))*MULP(M,N)
      CU1(M,N)=0.0
      CU2(M,N)=0.0
      CU3(M,N)=0.0
      CU5(M,N)=MULP(M,N)
      GC TO 150
143   D1=EC(M,N)*IXC(M,N)/XC(M,N)
      D2=D1/XC(M,N)
      D3=EC(M,N)*IXC(M,N)/YC(M,N)
      D4=D3/YC(M,N)
      D5=(CCX(M,N)*CCX(M,N)-SSX(M,N)*SSX(M,N))/CCX(M,N)
      D6=(CCY(M,N)*CCY(M,N)-SSY(M,N)*SSY(M,N))/CCY(M,N)
      D7=D5*D2
      D8=D6*D4
      B1=D8/((D8-PAPP(M,N))*XC(M,N)+(D7-PAPP(M,N))*YC(M,N))
      B2=D7/((D8-PAPP(M,N))*XC(M,N)+(D7-PAPP(M,N))*YC(M,N))
      B3=1.0-B1*XC(M,N)
      B4=1.0-B2*YC(M,N)
      CL1(M,N)=B4*D5*D1
      CL2(M,N)=B2*XC(M,N)*D6*D3
      CL3(M,N)=B2*XC(M,N)*(D8-PAPP(M,N))
      CL5(M,N)=B4*(SSX(M,N)/CCX(M,N))*MDLP(M,N)+B2*XC(M,N)*(SSY(M,N)/CCY
     1(M,N))*MDUP(M,N)+B2*XC(M,N)*MDUP(M,N)-B2*YC(M,N)*MDLP(M,N)
      CU1(M,N)=B1*YC(M,N)*D5*D1
      CU2(M,N)=B3*D6*D3
      CU3(M,N)=B1*YC(M,N)*(D7-PAPP(M,N))
      CU5(M,N)=B3*(SSY(M,N)/CCY(M,N))*MDUP(M,N)+B1*YC(M,N)*(SSX(M,N)/CCX
     1(M,N))*MDLP(M,N)-B1*XC(M,N)*MDUP(M,N)+B1*YC(M,N)*MDLP(M,N)
      MD(M,N)=MDLP(M,N)
      GC TO 150
144   CL1(M,N)=0.0
      CL2(M,N)=0.0
      CL3(M,N)=0.0
      CL5(M,N)=MLUP(M,N)
      CU1(M,N)=0.0
      CU2(M,N)=0.0
      CU3(M,N)=0.0
      CU5(M,N)=MULP(M,N)
      GC TO 150
145   D1=((CCY(M,N)*CCY(M,N)-SSY(M,N)*SSY(M,N))/CCY(M,N))*EC(M,N)*IXC(M,
     1N)/YC(M,N)
      D2=PAPP(M,N)*(1.C+YC(M,N)/XC(M,N))
      D3=D1/YC(M,N)
      B5=D2/(D2-D3)
      B6=1.0-B5
      CL1(M,N)=0.0
      CL2(M,N)=0.0
      CL3(M,N)=0.0
      CL5(M,N)=MLUP(M,N)
      CU1(M,N)=0.0
      CU2(M,N)=B5*D1
```

```
      CL3(M,N)=-B6*P(M,N)*YC(M,N)/XC(M,N)
      CU5(M,N)=B6*((YC(M,N)/XC(M,N))*(MDLP(M,N)+MLUP(M,N))-MDUP(M,N))+B5
     1*(SSY(M,N)/CCY(M,N))*MDUP(M,N)
      MD(M,N)=MDLP(M,N)
      GO TO 150
146   D1=((CCX(M,N)*CCX(M,N)-SSX(M,N)*SSX(M,N))/CCX(M,N))*EC(M,N)*IXC(M,
     1N)/XC(M,N)
      D2=D1/XC(M,N)
      D3=PAPP(M,N)*(1.0+XC(M,N)/YC(M,N))
      B7=D3/(D3-D2)
      B8=1.0-B7
      CL1(M,N)=B7*D1
      CL2(M,N)=0.0
      CL3(M,N)=-B8*(XC(M,N)/YC(M,N))*PAPP(M,N)
      CL5(M,N)=B8*((XC(M,N)/YC(M,N))*(MULP(M,N)+MDUP(M,N))-MDLP(M,N))+B7
     1*(SSX(M,N)/CCX(M,N))*MDLP(M,N)
      CU1(M,N)=0.0
      CU2(M,N)=0.0
      CU3(M,N)=0.0
      MD(M,N)=MDLP(M,N)
      GO TO 150
147   CL1(M,N)=0.0
      CL2(M,N)=0.0
      CL3(M,N)=0.0
      CL5(M,N)=MLUP(M,N)
      CU1(M,N)=0.0
      CU2(M,N)=0.0
      CU3(M,N)=0.0
      CU5(M,N)=MULP(M,N)
      MD(M,N)=MDLP(M,N)
150   CONTINUE
149   CONTINUE
      RETURN
      END
      SUBROUTINE EE1
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NE,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
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```
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     FORMULATE EQUILIBRIUM EQUATIONS
C
      IF(IND6.EQ.2) GO TO 304
      WRITE(6,3001)
 3001 FORMAT(1H0,'COLUMN/JOINT      COLUMN AXIAL LOAD      APPLIED JOINT L
     1OAD      LOAD FACTOR')
      DO 303 M=1,MM
      DO 302 N=1,NN
      WRITE(6,3000) M,N,PAPP(M,N),VJT(M,N),RV(IL)
 3000 FORMAT(1H ,I5,I5,10X,E11.4,12X,F10.5,10X,F10.5)
  302 CONTINUE
  303 CONTINUE
  304 JI=2*MM+1
      JJ=3*MM+1
      JK=JJ+1
      NA=NNN*JI+MM
      NB=2*JI
      DO 306 J=1,NB
      DO 305 I=1,NA
      A(I,J)=0.0
  305 CONTINUE
  306 CONTINUE
C
C     MOMENT EQUATIONS
C
      DO 110 M=1,MM
      A(M,1)=-CL1(M,1)-K(M)
      A(M,MM+2-M)=CL3(M,1)
      A(M,MM+M+1)=-CL2(M,1)
      B(M)=CL5(M,1)
  110 CONTINUE
      IF(NN.EQ.2) GO TO 131
      DO 130 N=2,NNN
      IF(MM.EQ.2) GO TO 121
      DO 120 M=2,MMM
      I=N*JI-JK+2*M
      BB=0.5*WC(M,N)/LG(M,N)
      CZ=0.5*WC(M,N)/LG(M-1,N)
      AA=1.0+BB
      DD=1.0+CZ
      A(I,1)=-(AA*CA1(M,N)+BB*CB1(M,N)+CZ*CA2(M-1,N)+DD*CB2(M-1,N)
     1 +CL1(M,N)+CU2(M,N-1))
      A(I,2)=-(AA*CA3(M,N)+BB*CB3(M,N)-CZ*CA3(M-1,N)-DD*CB3(M-1,N))
      A(I,3)=-(AA*CA2(M,N)+BB*CB2(M,N))
      A(I,4)=AA*CA3(M,N)+BB*CB3(M,N)
      A(I,2*(MM-M)+3)=CL3(M,N)
      A(I,2*MM+2)=-CL2(M,N)
      B(I)=AA*CA5(M,N)+BB*CB5(M,N)+CZ*CA5(M-1,N)+DD*CB5(M-1,N)
     1 +CL5(M,N)+CU5(M,N-1)+0.5*WC(M,N)*(VWB(M-1,N)-VWA(M,N))
  120 CONTINUE
  121 I=N*JI-JK+2
      BB=0.5*WC(1,N)/LG(1,N)
      AA=1.0+BB
      A(I,1)=-AA*CA1(1,N)-BB*CB1(1,N)-CL1(1,N)-CU2(1,N-1)
      A(I,3)=-AA*CA2(1,N)-BB*CB2(1,N)
      A(I,4)=AA*CA3(1,N)+BB*CB3(1,N)
```

```
      A(I,2)=-A(I,4)
      A(I,2*MM+1)=CL3(1,N)
      A(I,2*MM+2)=-CL2(1,N)
      B(I)=AA*CA5(1,N)+BB*CB5(1,N)+CL5(1,N)+CU5(1,N-1)
     1 -0.5*WC(1,N)*VWA(1,N)
      I=N*JI-JK+2*MM
      CZ=0.5*WC(MM,N)/LG(MMM,N)
      DD=1.0+CZ
      A(I,1)=-CZ*CA2(MM-1,N)-DD*CB2(MM-1,N)-CL1(MM,N)-CU2(MM,N-1)
      A(I,2)=CZ*CA3(MM-1,N)+DD*CB3(MM-1,N)
      A(I,3)=CL3(MM,N)
      A(I,2*MM+2)=-CL2(MM,N)
      B(I)=CZ*CA5(MM-1,N)+DD*CB5(MM-1,N)+CL5(MM,N)+CU5(MM,N-1)
     1 +0.5*WC(MM,N)*VWB(MM-1,N)
130   CONTINUE
131   IF(MM.EQ.2) GO TO 141
      DO 140 M=2,MMM
      I=NN*JI-JK+2*M
      BB=0.5*WC(M,NN)/LG(M,NN)
      CZ=0.5*WC(M,NN)/LG(M-1,NN)
      AA=1.0+BB
      DD=1.0+CZ
      A(I,1)=-AA*CA1(M,NN)-BB*CB1(M,NN)-CZ*CA2(M-1,NN)-DD*CB2(M-1,NN)
     1 -CU2(M,NNN)
      A(I,2)=-AA*CA3(M,NN)-BB*CB3(M,NN)+CZ*CA3(M-1,NN)+DD*CB3(M-1,NN)
      A(I,3)=-AA*CA2(M,NN)-BB*CB2(M,NN)
      A(I,4)=AA*CA3(M,NN)+BB*CB3(M,NN)
      B(I)=AA*CA5(M,NN)+BB*CB5(M,NN)+CZ*CA5(M-1,NN)+DD*CB5(M-1,NN)
     1 +CU5(M,NN-1)+0.5*WC(M,NN)*(VWB(M-1,NN)-VWA(M,NN))
140   CONTINUE
141   I=NN*JI-JK+2
      BB=0.5*WC(1,NN)/LG(1,NN)
      AA=1.0+BB
      A(I,1)=-AA*CA1(1,NN)-BB*CB1(1,NN)-CU2(1,NNN)
      A(I,3)=-AA*CA2(1,NN)-BB*CB2(1,NN)
      A(I,4)=AA*CA3(1,NN)+BB*CB3(1,NN)
      A(I,2)=-A(I,4)
      B(I)=AA*CA5(1,NN)+BB*CB5(1,NN)+CU5(1,NNN)-0.5*WC(1,NN)*VWA(1,NN)
      I=NN*JI-JK+2*MM
      CZ=0.5*WC(MM,NN)/LG(MMM,NN)
      DD=1.0+CZ
      A(I,1)=-CZ*CA2(MMM,NN)-DD*CB2(MMM,NN)-CU2(MM,NNN)
      A(I,2)=CZ*CA3(MMM,NN)+DD*CB3(MMM,NN)
      B(I)=CZ*CA5(MMM,NN)+DD*CB5(MMM,NN)+CU5(MM,NNN)+0.5*WC(MM,NN)*VWB(M
     1MM,NN)
C
C     VERTICAL FORCE EQUATIONS
C
      IF(NN.EQ.2) GO TO 171
      DO 170 N=2,NNN
      IF(MM.EQ.2) GO TO 161
      DO 160 M=2,MMM
      I=N*JI-JJ+2*M
      A(I,1)=-(CA3(M-1,N)+CB3(M-1,N))/LG(M-1,N)-(CA3(M,N)+CB3(M,N))/LG(M
     1,N)-CC2(M,N)-CC2(M,N-1)-CD4(M+1,N-1,1)-CD4(M-1,N-1,2)-CD4(M,N,1)
     2-CD4(M,N,2)
      A(I,2)=-(CA2(M,N)+CB2(M,N))/LG(M,N)
      A(I,3)=(CA3(M,N)+CB3(M,N))/LG(M,N)
      A(I,2*(MM-M)+2)=CD1(M,N,1)-CD1(M,N,2)
      A(I,2*MM+2)=CC2(M,N)
```

```
      A(I,2*MM)=CD4(M,N,1)
      A(I,2*MM+4)=CD4(M,N,2)
      B(I)=-(CA5(M-1,N)+CB5(M-1,N))/LG(M-1,N)+(CA5(M,N)+CB5(M,N))/LG(M,N
     1)-CD3(M+1,N-1,1)-CD3(M-1,N-1,2)+CD3(M,N,1)+CD3(M,N,2)
     2        -VJT(M,N) -VWA(M,N)-VWB(M-1,N)
 160  CONTINUE
 161  I=N*JI-JJ+2
      A(I,1)=-(CA3(1,N)+CB3(1,N))/LG(1,N)-CC2(1,N)-CC2(1,N-1)
     1-CD4(2,N-1,1)-CD4(1,N,2)
      A(I,2)=-(CA2(1,N)+CB2(1,N))/LG(1,N)
      A(I,3)=(CA3(1,N)+CB3(1,N))/LG(1,N)
      A(I,2*MM)=-CD1(1,N,2)
      A(I,2*MM+2)=CC2(1,N)
      A(I,2*MM+4)=CD4(1,N,2)
      B(I)=(CA5(1,N)+CB5(1,N))/LG(1,N)-CD3(2,N-1,1)+CD3(1,N,2)
     1          -VJT(1,N)  -VWA(1,N)
      I=N*JI-JJ+2*MM
      A(I,1)=-(CA3(MMM,N)+CB3(MMM,N))/LG(MMM,N)-CC2(MM,N)-CC2(MM,N-1)
     1-CD4(MMM,N-1,2)-CD4(MM,N,1)
      A(I,2)=CD1(MM,N,1)
      A(I,2*MM+2)=CC2(MM,N)
      A(I,2*MM)=CD4(MM,N,1)
      B(I)=-(CA5(MMM,N)+CB5(MMM,N))/LG(MMM,N)-CD3(MMM,N-1,2)+CD3(MM,N,1)
     1          -VJT(MM,N)             -VWB(MMM,N)
 170  CONTINUE
 171  IF(MM.EQ.2) GO TO 181
      DO 180 M=2,MMM
      I=NN*JI-JJ+2*M
      A(I,1)=-(CA3(M-1,NN)+CB3(M-1,NN))/LG(M-1,NN)-(CA3(M,NN)+CB3(M,NN))
     1/LG(M,NN)-CC2(M,NNN)-CD4(M+1,NNN,1)-CD4(M-1,NNN,2)
      A(I,2)=-(CA2(M,NN)+CB2(M,NN))/LG(M,NN)
      A(I,3)=(CA3(M,NN)+CB3(M,NN))/LG(M,NN)
      B(I)=-(CA5(M-1,NN)+CB5(M-1,NN))/LG(M-1,NN)+(CA5(M,NN)+CB5(M,NN))/L
     1G(M,NN)-CD3(M+1,NNN,1)-CD3(M-1,NNN,2)    -VJT(M,NN)     -VWA(M,NN)
     2-VWB(M-1,NN)-RP(IL)*PDES(M)
 180  CONTINUE
 181  I=NN*JI-JJ+2
      A(I,1)=-(CA3(1,NN)+CB3(1,NN))/LG(1,NN)-CC2(1,NNN)-CD4(2,NNN,1)
      A(I,2)=-(CA2(1,NN)+CB2(1,NN))/LG(1,NN)
      A(I,3)=(CA3(1,NN)+CB3(1,NN))/LG(1,NN)
      B(I)=(CA5(1,NN)+CB5(1,NN))/LG(1,NN)-CD3(2,NNN,1)
     1          -VWA(1,NN)-RP(IL)*PDES(1)  -VJT(1,NN)
      I=NN*JI-JJ+2*MM
      A(I,1)=-(CA3(MMM,NN)+CB3(MMM,NN))/LG(MMM,NN)-CC2(MM,NNN)-CD4(MMM,
     1 NNN,2)
      B(I)=-(CA5(MMM,NN)+CB5(MMM,NN))/LG(MMM,NN)-CD3(MMM,NNN,2)
     1      -VJT(MM,NN)       -VWB(MMM,NN)-RP(IL)*PDES(MM)
C
C     SHEAR EQUATIONS
C
      IF(NN.EQ.2) GO TO 241
      DO 240 N=2,NNN
      I=N*JI-JJ
      Z1=0.0
      Z4=0.0
      Z7=0.0
      IF(I12.EQ.-5) GO TO 524
      DO 210 M=1,MM
      Z1=Z1+(PAPP(M,N)-CL3(M,N)-CU3(M,N))/H(N)+(PAPP(M,N-1)-CL3(M,N-1)
     1 -CU3(M,N-1))/H(N-1)
```

```
      Z4=Z4+(CL3(M,N)+CU3(M,N)-PAPP(M,N))/H(N)
      Z7=Z7+(CL5(M,N)+CU5(M,N))/H(N)-(CL5(M,N-1)+CU5(M,N-1))/H(N-1)
210   CONTINUE
      GO TO 525
524   DO 526 M=1,MM
      Z1=Z1-(CL3(M,N)+CU3(M,N))/H(N)-(CL3(M,N-1)+CU3(M,N-1))/H(N)
      Z4=Z4+(CL3(M,N)+CU3(M,N))/H(N)
      Z7=Z7+(CL5(M,N)+CU5(M,N))/H(N)-(CL5(M,N-1)+CU5(M,N-1))/H(N-1)
526   CONTINUE
525   Z2=0.0
      Z5=0.C
      Z8=0.0
      DO 211 M=1,MMM
      Z2=Z2-CD2(M,N,2)-CD2(M,N-1,2)
      Z5=Z5+CD2(M,N,2)
      Z8=Z8-CD5(M,N,2)+CD5(M,N-1,2)
211   CONTINUE
      Z3=0.0
      Z6=0.0
      Z9=0.0
      DO 212 M=2,MM
      Z3=Z3-CD2(M,N,1)-CD2(M,N-1,1)
      Z6=Z6+CD2(M,N,1)
      Z9=Z9-CD5(M,N-1,1)+CD5(M,N,1)
212   CONTINUE
      A(I,1)=Z1+Z2+Z3
      A(I,2*MM+2)=Z4+Z5+Z6
      B(I)=Z7+Z8+Z9-F(N)
      DO 220 M=1,MM
      A(I,2*M)=(CU2(M,N-1)+CL2(M,N-1))/H(N-1)-(CL1(M,N)+CU1(M,N))/H(N)
      A(I,2*(MM+M)+1)=-(CL2(M,N)+CU2(M,N))/H(N)
220   CONTINUE
      IF(MM.EQ.2) GO TO 231
      DO 230 M=2,MMM
      A(I,2*M+1)=CD1(M,N,2)-CD1(M,N,1)-CD1(M+1,N-1,1)+CD1(M-1,N-1,2)
      A(I,2*(MM+M)+2)=CD1(M+1,N,1)-CD1(M-1,N,2)
230   CONTINUE
231   A(I,3)=CD1(1,N,2)-CD1(2,N-1,1)
      A(I,2*MM+1)=CD1(MMM,N-1,2)-CD1(MM,N,1)
      A(I,2*MM+4)=CD1(2,N,1)
      A(I,4*MM+2)=-CD1(MMM,N,2)
240   CONTINUE
241   I=NN*JI-JJ
      Z1=0.0
      Z4=0.0
      IF(I12.EQ.-5) GO TO 534
      DO 250 M=1,MM
      Z1=Z1+(PAPP(M,NNN)-CL3(M,NNN)-CU3(M,NNN))/H(NNN)
      Z4=Z4-(CL5(M,NNN)+CU5(M,NNN))/H(NNN)
250   CONTINUE
      GO TO 535
534   DO 536 M=1,MM
      Z1=Z1-(CL3(M,NNN)+CU3(M,NNN))/H(NNN)
      Z4=Z4-(CL5(M,NNN)+CU5(M,NNN))/H(NNN)
536   CONTINUE
535   Z2=0.0
      Z5=0.C
      DO 252 M=1,MMM
      Z2=Z2-CD2(M,NNN,2)
      Z5=Z5+CD5(M,NNN,2)
```

```
252   CONTINUE
      Z3=0.0
      Z6=0.0
      DO 254 M=2,MM
      Z3=Z3-CD2(M,NNN,1)
      Z6=Z6-CD5(M,NNN,1)
254   CONTINUE
      A(I,1)=Z1+Z2+Z3
      B(I)=Z4+Z5+Z6-F(NN)
      DO 260 M=1,MM
      A(I,2*M)=(CU2(M,NNN)+CL2(M,NNN))/H(NNN)
260   CONTINUE
      IF(MM.EQ.2) GO TO 266
      DO 265 M=2,MMM
      A(I,2*M+1)=CD1(M-1,NNN,2)-CD1(M+1,NNN,1)
265   CONTINUE
266   A(I,3)=-CD1(2,NNN,1)
      A(I,2*MM+1)=CD1(MMM,NNN,2)
      RETURN
      END
      SUBROUTINE SOLVE
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),RUT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     EQUATION SOLVER - SYMMETRICAL BANDED MATRIX
C
      IF(IND6.NE.1) GO TO 95
      WRITE(6,90)
 90   FORMAT(1H0,21HEQUILIBRIUM EQUATIONS)
      DO 60 I=1,NA
      WRITE(6,62) I
      WRITE(6,61) (A(I,J),J=1,NB),B(I)
 60   CONTINUE
 61   FORMAT((10(2X,F10.0)))
```

```
 62   FORMAT(1H0,I3)
 55   NAA=NA/2
      NAA=2*NAA
      IF(NAA.EQ.NA) GO TO 100
      RRR=-1.0
      GO TO 101
100   RRR=1.0
101   CONTINUE
      NJ=0
      KX=1
      IF(NB.EQ.1) GO TO 601
      N1=NB-1
      N2=NB-2
      DO 600 I=1,NA
      I1=I-1
      I2=I-2
      I3=I-3
      IF(NB.EQ.2) GO TO 609
      DO 610 N=1,N2
      V(1,N)=A(I,N+1)
610   CONTINUE
609   IF(I.GE.NB) GO TO 620
      IF(I.EQ.1) GO TO 648
      IF(A(1,I).EQ.0.0) GO TO 616
      R=A(1,I)/A(1,1)
      NY=NB+1-I
      DO 611 N=1,NY
      A(I,N)=A(I,N)-A(1,N+I1)*R
611   CONTINUE
      B(I)=B(I)-B(1)*R
      IF(I.EQ.2) GO TO 648
      DO 612 N=1,I2
      V(N+1,N)=V(N+1,N)-A(1,I-N)*R
612   CONTINUE
616   IF(I.EQ.2) GO TO 648
      KK=1
      KX=KX+1
613   IF(V(I-KK,I1-KK).EQ.0.0) GO TO 617
      R=V(I-KK,I1-KK)/A(KK+1,1)
      NZ=NB-KX+KK
      DO 614 N=1,NZ
      A(I,N)=A(I,N)-A(KK+1,N+I1-KK)*R
614   CONTINUE
      B(I)=B(I)-B(KK+1)*R
617   IF(I.EQ.3) GO TO 648
      IX=I2-KK
      IF(IX.LE.0) GO TO 648
      IF(V(I-KK,I1-KK).EQ.0.0) GO TO 618
      DO 615 N=1,IX
      V(N+1,N)=V(N+1,N)-A(KK+1,I-N-KK)*R
615   CONTINUE
618   KK=KK+1
      GO TO 613
620   IF(A(I-N1,NB).EQ.0.0) GO TO 630
      R=A(I-N1,NB)/A(I-N1,1)
      B(I)=B(I)-B(I-N1)*R
      A(I,1)=A(I,1)-A(I-N1,NB)*R
      IF(NB.EQ.2) GO TO 600
      DO 625 N=1,N2
      V(N+1,N)=V(N+1,N)-A(I-N1,NB-N)*R
```

```
 625  CCNTINLE
 630  IF(NB.EQ.2) GC TC 600
      DC 640 M=2,N1
      IF(V(NB+1-M,NB-M).EQ.0.0) GC TC 640
      R=V(NB+1-M,NB-M)/A(I-NB+M,1)
      B(I)=B(I)-B(I-NB+M)*R
      DC 645 J=1,M
      A(I,J)=A(I,J)-A(I-NB+M,NB+J-M)*R
 645  CCNTINUE
      NX=N1-M
      IF(NX.LE.0) GC TO 640
      DC 650 N=1,NX
      V(N+1,N)=V(N+1,N)-A(I-NB+M,NB+1-M-N)*R
 650  CCNTINUE
 640  CCNTINLE
 648  N=N1
      IF(NB.EQ.2) GC TC 600
 649  DO 655 J=1,N2
      V(N,J)=V(N-1,J)
 655  CCNTINUE
      N=N-1
      IF(N.EQ.1) GO TC 600
      GC TO 649
 600  CCNTINUE
      DET=RRR
      DC 750 I=1,NA
      ZZ=0.0
      DC 749 J=1,NB
      ZZ=ZZ+A(I,J)*A(I,J)
 749  CCNTINUE
      IF(ZZ.NE.0.0) GC TO 751
      WRITE(6,808)I
      DET=-DET
      GC TO 750
 751  ZZZ=SQRT(ZZ)
      IF( ABS(A(I,1)).GT.0.50E-20) GO TO 752
      WRITE(6,803) I
 803  FCRMAT(1H0,'A(',I4,',1)=0')
      DET=0.0
      GO TO 750
 752  DET=DET*A(I,1)/ZZZ
      DDD=ABS(DET)
      IF(DDD.GT.0.10E-51) GO TC 750
      DET=DET*0.10E+50
 750  CCNTINLE
 808  FCRMAT(1H0,'ALL TERMS CF ROW',I4,' ARE = 0')
      WRITE(6,800)DET
 800  FCRMAT(1H0,12HDETERMINANT=,E11.4)
      IF(IELAST.EQ.1) GC TO 805
      IF(IND3.EQ.1) GC TC 109
      IF(DET.GE.0.0) GC TC 805
 109  IND3=IND3+1
 805  CCNTINLE
 601  I=NA
 602  IF(I.LE.(NA-NB)) GC TC 621
      NJ=NJ+1
      IF(NJ.EQ.1) GC TC 624
      GC TC 622
 621  NJ=NB
 622  SUMA=0.0
```

```
      DO 623 J=2,NJ
      SUMA=SUMA+A(I,J)*X(I+J-1)
623   CONTINUE
      GO TO 203
624   SUMA=0.0
203   IF(A(I,1).NE.0.0) GO TO 626
200   FORMAT(1H0,'REDUCED A(',I3,',1)=0')
      X(I)=0.0
      GO TO 201
626   X(I)=(B(I)-SUMA)/A(I,1)
201   IF(I.EQ.1) GO TO 627
      I=I-1
      GO TO 602
627   CONTINUE
      RETURN
      END
      SUBROUTINE SUB1
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     COMPUTE ROTATIONS, DEFLECTIONS & SWAYS
C
      NCYC=NCYC+1
      JI=2*MM+1
      JJ=3*MM+1
      JK=JJ+1
      SWAY(1)=0.0
      DO 100 M=1,MM
      DEL(M,1)=0.0
      ROT(M,1)=X(M)
100   CONTINUE
      DO 104 N=2,NN
      SWAY(N)=X(N*JI-JJ)
      DO 103 M=1,MM
```

```
      RCT(M.N)=X(N*JI-JK+2*M)
      DEL(M.N)=X(N*JI-JJ+2*M)
 103  CONTINUE
 104  CONTINUE
      LLL=0
      DO 120 N=1.NNN
      DO 115 M=1.MM
      PCAL=(DEL(M.N+1)-DEL(M.N))*AC(M.N)*EC(M.N)/H(N)
      IF(IAREA.NE.1) GO TO 220
      PCAL=10000.*PCAL
 220  CONTINUE
      ERR=ABS(PCAL-PAPP(M.N))
      PABS=ABS(PCAL)
      IF(ERR.LE.(0.01*PABS)) GO TO 130
      LLL=1
 130  PAPP(M.N)=PCAL
 115  CONTINUE
 120  CONTINUE
      IF(IND6.EQ.2) GO TO 320
      WRITE(6.300)
      DO 301 M=1.MM
      DO 302 N=1.NN
      WRITE(6.303)M.N.RCT(M.N).DEL(M.N)
 302  CONTINUE
 301  CONTINUE
 320  WRITE(6.304)
      DO 305 N=2.NN
      WRITE(6.306)N.SWAY(N)
 305  CONTINUE
 300  FORMAT(1H0.50HJOINT              ROTATION     VERTICAL DISPLACEMENT)
 303  FORMAT(1H .I3.I3.9X.E11.4.9X.E11.4)
 304  FORMAT(1H0.20HFLOOR LEVEL      SWAY)
 306  FORMAT(1H .I6.8X.E11.4)
      RETURN
      END
      SUBROUTINE CHECK1
      COMMON/AREA1/ AG(9.04).IXG(9.04).ZXG(9.04).FYG(9.04).EG(9.04).
     1 DLDES(9.04).LLDES(9.04).LG(9.04).W(9.04).VWA(9.04).VWB(9.04).
     2 MFAB(9.04).MFBA(9.04).MFCA(9.04).MFCB(9.04).MAX(9.04).MBY(9.04).
     3 VWAL(9.04).VWBL(9.04).CA1(9.04).CA2(9.04).CA3(9.04).CA5(9.04).
     4 CA6(9.04).CB1(9.04).CB2(9.04).CB3(9.04).CB5(9.04).CB6(9.04).
     5 MBAP(9.04).MABP(9.04).MCAP(9.04).MCBP(9.04).XG(9.04).YG(9.04).
     6 VA(9.04).VB(9.04).MA(9.04).MB(9.04).RABP(9.04).RBAP(9.04).
     7 RCAP(9.04).DELC(9.04).RBAPL(9.04).RCAPL(9.04).RABPL(9.04)
      COMMON/AREA2/ AC(9.04).IXC(9.04).ZXC(9.04).RXC(9.04).WC(9.04).
     1 FYC(9.04).EC(9.04).CC(9.04).SS(9.04).CCX(9.04).SSX(9.04).
     2 CCY(9.04).SSY(9.04).CL1(9.04).CL2(9.04).CL3(9.04).CL5(9.04).
     3 CU1(9.04).CU2(9.04).CU3(9.04).CU5(9.04).MD(9.04).MC(9.04).
     4 MULP(9.04).MLUP(9.04).MDLP(9.04).MDUP(9.04).XC(9.04).YC(9.04).
     5 XMP(9.04).VL(9.04).VU(9.04).ML(9.04).MU(9.04).RLUP(9.04).
     6 RULP(9.04).RDLP(9.04).DELD(9.04).L(9).H(04).K(9).SUMV(9).B(100)
      COMMON/AREA3/ AB(9.04.2).RXB(9.04.2).FYB(9.04.2).EB(9.04.2).
     1 LB(9.04.2).CD1(9.04.2).CD2(9.04.2).CD3(9.04.2).CD4(9.04.2).
     2 CC2(9.04).PCR(9.04.2).FB(9.04.2).FBP(9.04.2).FBX(9.04.2).X(100).
     3 FDES(04).VDES(9.04).PDES(9).F(04).P(9.04).PAPP(9.04).VINCR(9.04).
     4 RF(300).RLL(300).PF(300).A(100.45).V(45.45).DET.RINLL.RINF.RINP.
     5 SWAY(04).ROT(9.04).DFL(9.04).CD5(9.04.2).C7(9.04).C8(9.04).RINV.
     6 RV(300).VJT(9.04).VJTL(9.04).C9(9.04).C10(9.04)
      COMMON/AREA4/ JC(9.04).JG(9.04).JB(9.04.2).IND6.MM.NN.MMM.NNN.IL.
     1 NA.NP.NCYC.IND3.LLL.NL.IND2(9).IFLAST.NLAST.IAREA.IND.IREV.IREV2.
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     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MEY,MEAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
      IND=0
C
C     GIRDER MOMENTS
C
      DO 200 N=2,NN
      DO 199 M=1,MMM
      IF(IXG(M,N).EQ.0.0) GO TO 199
      MA(M,N)=CA1(M,N)*ROT(M,N)+CA2(M,N)*ROT(M+1,N)+CA3(M,N)*(DEL(M,N)-D
     1EL(M+1,N))+CA5(M,N)
      MB(M,N)=CB1(M,N)*ROT(M,N)+CB2(M,N)*ROT(M+1,N)+CB3(M,N)*(DEL(M,N)-D
     1EL(M+1,N))+CB5(M,N)
      D1=(MA(M,N)+MB(M,N))/LG(M,N)
      VA(M,N)=VWA(M,N)-D1
      VB(M,N)=VWB(M,N)+D1
      IF(IELAST.EQ.1) GO TO 199
      IF(JG(M,N).EQ.2) GO TO 260
      IF(JG(M,N).EQ.5) GO TO 260
      IF(JG(M,N).EQ.6) GO TO 260
      IF(JG(M,N).EQ.8) GO TO 260
      IF(JG(M,N).EQ.14) GO TO 260
      IF(JG(M,N).EQ.16) GO TO 260
      IF(JG(M,N).EQ.21) GO TO 260
      D2=ABS(MA(M,N))
      IF(D2.LT.ABS(MABP(M,N))) GO TO 120
      IF(JG(M,N).EQ.10) GO TO 118
      IF(JG(M,N).EQ.13) GO TO 118
      IF(JG(M,N).EQ.15) GO TO 118
      IF(JG(M,N).EQ.20) GO TO 118
      WRITE(6,201)M,N
 201  FORMAT(1H0,27HHINGE AT LEFT END OF GIRDER,2I3)
      IND=IND+1
      IF(MA(M,N).GE.0.0) GO TO 520
      MABP(M,N)=-MABP(M,N)
      CS(M,N)=1.0
 520  IF(JG(M,N).EQ.1) GO TO 111
      IF(JG(M,N).EQ.3) GO TO 113
      IF(JG(M,N).EQ.4) GO TO 114
      IF(JG(M,N).EQ.7) GO TO 117
      IF(JG(M,N).EQ.11) GO TO 115
      IF(JG(M,N).EQ.17) GO TO 116
      GO TO 118
 111  JG(M,N)=2
      GO TO 120
 113  JG(M,N)=5
      GO TO 120
 114  JG(M,N)=6
      GO TO 120
 117  JG(M,N)=8
      GO TO 120
 115  JG(M,N)=14
      CA6(M,N)=0.0
      CB6(M,N)=C7(M,N)
      GO TO 120
 116  JG(M,N)=21
      GO TO 120
 118  IF(D2.LT.ABS(MABP(M,N))) GO TO 120
```

```
      WRITE(6,112) M,N
  112 FORMAT(1H0,'MOMENT VIOLATION LEFT END GIRDER',2I4)
  120 CONTINUE
  260 IF(JG(M,N).EQ.3) GO TO 261
      IF(JG(M,N).EQ.5) GO TO 261
      IF(JG(M,N).EQ.7) GO TO 261
      IF(JG(M,N).EQ.8) GO TO 261
      IF(JG(M,N).EQ.13) GO TO 261
      IF(JG(M,N).EQ.18) GO TO 261
      IF(JG(M,N).EQ.20) GO TO 261
      D2=ABS(MB(M,N))
      IF(D2.LT.ABS(MBAP(M,N))) GO TO 130
      IF(JG(M,N).EQ.11) GO TO 128
      IF(JG(M,N).EQ.14) GO TO 128
      IF(JG(M,N).EQ.17) GO TO 128
      IF(JG(M,N).EQ.21) GO TO 128
      WRITE(6,202)M,N
  202 FORMAT(1H0,28HHINGE AT RIGHT END OF GIRDER,2I3)
      IND=IND+1
      IF(MB(M,N).GE.0.0) GO TO 521
      MBAP(M,N)=-MBAP(M,N)
      CS(M,N)=2.0
  521 IF(JG(M,N).EQ.1) GO TO 121
      IF(JG(M,N).EQ.2) GO TO 122
      IF(JG(M,N).EQ.4) GO TO 124
      IF(JG(M,N).EQ.6) GO TO 126
      IF(JG(M,N).EQ.10) GO TO 123
      IF(JG(M,N).EQ.15) GO TO 125
      GO TO 128
  121 JG(M,N)=3
      GO TO 130
  122 JG(M,N)=5
      GO TO 130
  124 JG(M,N)=7
      GO TO 130
  126 JG(M,N)=8
      GO TO 130
  123 JG(M,N)=13
      CA6(M,N)=C7(M,N)
      CB6(M,N)=0.0
      GO TO 130
  125 JG(M,N)=20
      GO TO 130
  128 IF(D2.LT.ABS(MBAP(M,N))) GO TO 130
      WRITE(6,105) M,N
  105 FORMAT(1H0,'MOMENT VIOLATION RIGHT END GIRDER',2I4)
  130 CONTINUE
  261 IF(JG(M,N).EQ.4) GO TO 140
      IF(JG(M,N).EQ. 6) GO TO 140
      IF(JG(M,N).EQ. 7) GO TO 140
      IF(JG(M,N).EQ. 8) GO TO 140
      IF(JG(M,N).EQ.15) GO TO 140
      IF(JG(M,N).EQ.17) GO TO 140
      IF(JG(M,N).EQ.20) GO TO 140
      IF(JG(M,N).EQ.21) GO TO 140
      IF(W(M,N).EQ.0.0) GO TO 140
      IF(VA(M,N).LT.0.0) GO TO 140
      IF(VB(M,N).LT.0.0) GO TO 140
      D2=ABS(MA(M,N)+VA(M,N)*VA(M,N)*6.0/W(M,N))
      MC(M,N)=D2
```

```
      IF(D2.LT.ABS(MCAF(M,N))) GO TO 140
      IF(JG(M,N).EQ.12) GO TO 137
      IF(JG(M,N).EQ.16) GO TO 137
      IF(JG(M,N).EQ.18) GO TO 137
      IF(JG(M,N).EQ.10) GO TO 137
      IF(JG(M,N).EQ.11) GO TO 137
      XG(M,N)=12.0*VA(M,N)/W(M,N)
      YG(M,N)=LG(M,N)-XG(M,N)
      WRITE(6,203)XG(M,N),M,N
  203 FORMAT(1H0,6HHINGE ,F8.2,31H INCHES FROM LEFT END OF GIRDER,2I3)
      IND=IND+1
      IF(DELC(M,N).LT.0.0) GO TO 204
      MCAP(M,N)=-MCAP(M,N)
      C9(M,N)=3.0
      GO TO 205
  204 MCBP(M,N)=-MCBP(M,N)
      C9(M,N)=4.0
  205 IF(JG(M,N).EQ.1) GO TO 131
      IF(JG(M,N).EQ.2) GO TO 132
      IF(JG(M,N).EQ.3) GO TO 133
      IF(JG(M,N).EQ.5) GO TO 135
      IF(JG(M,N).EQ.13) GO TO 142
      IF(JG(M,N).EQ.14) GO TO 143
      GO TO 137
  131 JG(M,N)=4
      GO TO 140
  132 JG(M,N)=6
      GO TO 140
  133 JG(M,N)=7
      GO TO 140
  135 JG(M,N)=8
      GO TO 140
  142 JG(M,N)=20
      WRITE(6,139) M,N
      GO TO 140
  143 JG(M,N)=21
      WRITE(6,139) M,N
      GO TO 140
  139 FORMAT(1H0,'HINGE ROTATIONS GIRDER',2I4,'  NO LONGER VALID')
  137 IF(D2.LT.ABS(MCAF(M,N))) GO TO 140
      WRITE(6,138) M,N
  138 FORMAT(1H0,'MOMENT VIOLATION INTERIOR GIRDER',2I4)
  140 CONTINUE
  199 CONTINUE
  200 CONTINUE
C
C     COLUMN MOMENTS
C
      DO 480 M=1,MM
      IF(IXC(M,1).EQ.0.0) GO TO 480
      IND2(M)=0
      IF(JC(M,1).EQ.2) GO TO 485
      IF(JC(M,1).EQ.5) GO TO 485
      IF(JC(M,1).EQ.6) GO TO 485
      IF(JC(M,1).EQ.8) GO TO 485
      ML(M,1)=-K(M)*ROT(M,1)
      IF(IELAST.EQ.1) GO TO 490
      D1=ABS(ML(M,1))
      IF(D1.LE.ABS(MLUF(M,1))) GO TO 490
      IND2(M)=1
```

```
      IF(ML(M,1).GE.0.0) GO TO 485
      ML(M,1)=-MLUP(M,1)
      GO TO 490
485   ML(M,1)=MLUP(M,1)
490   MU(M,1)=CU1(M,1)*ROT(M,1)+CU2(M,1)*ROT(M,2)-CU3(M,1)*SWAY(2)+CU5(M
     1,1)
      VU(M,1)=(MU(M,1)+ML(M,1)+P(M,1)*SWAY(2))/H(1)
      VL(M,1)=-VU(M,1)
480   CONTINUE
      DO 400 N=1,NNN
      DO 399 M=1,MM
      IF(IXC(M,N).EQ.0.0) GO TO 399
      IF(N.EQ.1) GO TO 800
      ML(M,N)=CL1(M,N)*ROT(M,N)+CL2(M,N)*ROT(M,N+1)+CL3(M,N)*(SWAY(N)-SW
     1AY(N+1))+CL5(M,N)
      MU(M,N)=CU1(M,N)*ROT(M,N)+CU2(M,N)*ROT(M,N+1)+CU3(M,N)*(SWAY(N)-SW
     1AY(N+1))+CU5(M,N)
      VU(M,N)=(MU(M,N)+ML(M,N)+P(M,N)*(SWAY(N+1)-SWAY(N)))/H(N)
      VL(M,N)=-VU(M,N)
      IF(IELAST.EQ.1) GO TO 399
      GO TO 810
800   IF(IFLAST.EQ.1) GO TO 399
      IF(IND2(M).EQ.1) GO TO 820
      GO TO 360
810   IF(JC(M,N).EQ.2) GO TO 360
      IF(JC(M,N).EQ.5) GO TO 360
      IF(JC(M,N).EQ.6) GO TO 360
      IF(JC(M,N).EQ.8) GO TO 360
      D1=ABS(ML(M,N))
      IF(D1.LT.ABS(MLUP(M,N))) GO TO 320
820   WRITE(6,401)M,N
401   FORMAT(1H0,28HHINGE AT LOWER END OF COLUMN,2I3)
      IND=IND+1
      IF(ML(M,N).GE.0.0) GO TO 501
      MLUP(M,N)=-MLUP(M,N)
      C10(M,N)=1.0
501   IF(JC(M,N).EQ.1) GO TO 311
      IF(JC(M,N).EQ.3) GO TO 313
      IF(JC(M,N).EQ.4) GO TO 314
      IF(JC(M,N).EQ.7) GO TO 317
      GO TO 320
311   JC(M,N)=2
      GO TO 320
313   JC(M,N)=5
      GO TO 320
314   JC(M,N)=6
      GO TO 320
317   JC(M,N)=8
320   CONTINUE
360   IF(JC(M,N).EQ.3) GO TO 361
      IF(JC(M,N).EQ.5) GO TO 361
      IF(JC(M,N).EQ.7) GO TO 361
      IF(JC(M,N).EQ.8) GO TO 361
      D1=ABS(MU(M,N))
      IF(D1.LT.ABS(MULP(M,N))) GO TO 330
      WRITE(6,402)M,N
402   FORMAT(1H0,28HHINGE AT UPPER END OF COLUMN,2I3)
      IND=IND+1
      IF(MU(M,N).GE.0.0) GO TO 502
      MULP(M,N)=-MULP(M,N)
```

```
      C10(M,N)=2.0
 502  IF(JC(M,N).EQ.1) GO TO 321
      IF(JC(M,N).EQ.2) GO TC 322
      IF(JC(M,N).EQ.4) GC TO 324
      IF(JC(M,N).EQ.6) GO TO 326
      GC TO 330
 321  JC(M,N)=3
      GC TO 330
 322  JC(M,N)=5
      GO TO 330
 324  JC(M,N)=7
      GO TO 330
 326  JC(M,N)=8
 330  CONTINUE
 361  IF(JC(M,N).EQ.4) GO TC 350
      IF(JC(M,N).GE.6) GO TC 350
      IF(P(M,N).LE.0.0) GO TC 350
      D1=SQRT(P(M,N)/(EC(M,N)*IXC(M,N)))
      D2=SIN(D1*H(N))
      D3=COS(D1*H(N))
      IF(ML(M,N).EQ.0.0) GO TC 758
      D6=(ML(M,N)*D3+MU(M,N))/(ML(M,N)*D2)
      XC(M,N)=ATAN(-D6)/D1
      GC TO 759
 758  XC(M,N)=1.5708/D1
 759  IF(XC(M,N).LT.0.10*H(N)) GC TO 350
      IF(XC(M,N).GT.0.90*H(N)) GC TO 350
      D4=ML(M,N)*ML(M,N)+2.0*D3*ML(M,N)*MU(M,N)+MU(M,N)*MU(M,N)
      IF(D4.GE.0.0)GO TO 331
      WRITE(6,403)M,N
 403  FORMAT(1H0,53HNEGATIVE SQUARE RCCT - MAXIMUM INTERIOR MOMENT COLUM
     1N,2I3)
      GC TO 350
 331  D5=SQRT(D4)/D2
      MD(M,N)=D5
      IF(D5.LT.ABS(MDLP(M,N))) GO TO 350
      YC(M,N)=H(N)-XC(M,N)
      WRITE(6,404)XC(M,N),M,N
 404  FORMAT(1H0,6HHINGE ,F8.2,32H INCHES FROM LOWER END OF COLUMN,2I3)
      IND=IND+1
      IF(ML(M,N).GE.0.0) GO TC 570
      MDUP(M,N)=-MDUP(M,N)
      C10(M,N)=3.0
 570  IF(MU(M,N).GE.0.0) GO TC 571
      MDLP(M,N)=-MDLP(M,N)
      C10(M,N)=4.0
 571  IF(ML(M,N)*MU(M,N).LT.0.0) GO TO 510
      D7=ABS(ML(M,N))
      D8=ABS(MU(M,N))
      IF(D7.GE.D8) GC TC 572
      MDUP(M,N)=-MDUP(M,N)
      C10(M,N)=3.0
      GC TO 510
 572  MDLP(M,N)=-MDLP(M,N)
      C10(M,N)=4.0
 510  IF(JC(M,N).EQ.1) GC TC 341
      IF(JC(M,N).EQ.2) GC TC 342
      IF(JC(M,N).EQ.3) GC TC 343
      IF(JC(M,N).EQ.5) GC TC 345
      GC TC 350
```

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341   JC(M,N)=4
      GO TO 350
342   JC(M,N)=6
      GO TO 350
343   JC(M,N)=7
      GO TO 350
345   JC(M,N)=8
350   CONTINUE
399   CONTINUE
400   CONTINUE
      IF(IND6.EQ.2) GO TO 435
      WRITE(6,410)
      DO 420 N=2,NN
      DO 419 M=1,MMM
      WRITE(6,411)M,N,MA(M,N),MC(M,N),MB(M,N),VA(M,N),VB(M,N),JG(M,N),MA
     1BP(M,N),RABP(M,N),RCAP(M,N),RBAP(M,N)
419   CONTINUE
420   CONTINUE
      WRITE(6,412)
      DO 430 N=1,NNN
      DO 429 M=1,MM
      WRITE(6,411)M,N,ML(M,N),MD(M,N),MU(M,N),VL(M,N),VU(M,N),JC(M,N),ML
     1UP(M,N),RLUP(M,N),RDLP(M,N),RULP(M,N)
429   CONTINUE
430   CONTINUE
435   CONTINUE
410   FORMAT(1H0,'GIRDER          MA               MC               MB              VA
     1          VB          HINGE          MP               AP               CP
     2BP')
411   FORMAT(1H ,2I3,3(2X,F11.4),2(2X,F9.4),I6,2X,F11.4,3(3X,E11.4))
412   FORMAT(1H0,'COLUMN          ML               MD               MU              VL
     1          VU          HINGE          MPC              LP               DP
     2UP')
      RETURN
      END
      SUBROUTINE CHECK2
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCH(9,04,2),FB(9,04,2),FBP(9,04,2),FEX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,INC,IREV,IREV2,
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     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MC,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     HINGE ROTATIONS
C
C
C     GIRDERS
C
      DO 651 N=2,NN
      DO 649 M=1,MMM
      IF(JG(M,N).EQ.1) GO TO 611
      IF(JG(M,N).EQ.2) GO TO 612
      IF(JG(M,N).EQ.3) GO TO 613
      IF(JG(M,N).EQ.4) GO TO 614
      IF(JG(M,N).EQ.5) GO TO 615
      IF(JG(M,N).EQ.6) GO TO 616
      IF(JG(M,N).EQ.7) GO TO 617
      IF(JG(M,N).EQ.8) GO TO 618
      IF(JG(M,N).EQ.13) GO TO 613
      IF(JG(M,N).EQ.14) GO TO 612
      IF(JG(M,N).EQ.15) GO TO 614
      IF(JG(M,N).EQ.16) GO TO 612
      IF(JG(M,N).EQ.17) GO TO 614
      IF(JG(M,N).EQ.18) GO TO 613
      IF(JG(M,N).EQ.20) GO TO 617
      IF(JG(M,N).EQ.21) GO TO 616
      GO TO 649
  611 RABP(M,N)=0.0
      RBAP(M,N)=0.0
      RCAP(M,N)=0.0
      GO TO 649
  612 RABP(M,N)=(0.25*LG(M,N)/(EG(M,N)*IXG(M,N)))*(MABP(M,N)-MFAB(M,N))-
     1(1.0+0.75*WC(M,N)/LG(M,N))*ROT(M,N)-(0.5+0.75*WC(M+1,N)/LG(M,N))*R
     2OT(M+1,N)+1.5*(DEL(M+1,N)-DEL(M,N))/LG(M,N)
      RCAP(M,N)=0.0
      IF(JG(M,N).EQ.14) GO TO 661
      RBAP(M,N)=0.0
      GO TO 649
  661 RABP(M,N)=RABP(M,N)-0.5*RBAP(M,N)
      GO TO 649
  613 RBAP(M,N)=(0.25*LG(M,N)/(EG(M,N)*IXG(M,N)))*(MBAP(M,N)-MFBA(M,N))-
     1(0.5+0.75*WC(M,N)/LG(M,N))*ROT(M,N)-(1.0+0.75*WC(M+1,N)/LG(M,N))*R
     2OT(M+1,N)+1.5*(DEL(M+1,N)-DEL(M,N))/LG(M,N)
      RCAP(M,N)=0.0
      IF(JG(M,N).EQ.13) GO TO 660
      RABP(M,N)=0.0
      GO TO 649
  660 RBAP(M,N)=RBAP(M,N)-0.5*RABP(M,N)
      GO TO 649
  614 D1=XG(M,N)*XG(M,N)
      D2=YG(M,N)*YG(M,N)
      D3=D1*XG(M,N)
      D4=D2*YG(M,N)
      D5=D1*D2/(D3+D4)
      DELC(M,N)=D5*(YG(M,N)*((MFAB(M,N)-0.5*MFCA(M,N)+1.5*MCAP(M,N)+MAX(
     1M,N))/(3.0*EG(M,N)*IXG(M,N))+(1.0+0.5*WC(M,N)/XG(M,N))*ROT(M,N)/XG
     2(M,N)+DEL(M,N)/D1)+XG(M,N)*((0.5*MFCB(M,N)-MFBA(M,N)-1.5*MCBP(M,N)
     3+MBY(M,N))/(3.0*EG(M,N)*IXG(M,N))-(1.0+0.5*WC(M+1,N)/YG(M,N))*ROT(
```

```
     4M+1,N)/YG(M,N)+DEL(M+1,N)/D2))
      IF(JG(M,N).NE.15) GO TO 662
      DELC(M,N)=DELC(M,N)+CB6(M,N)*D2/(3.0*EG(M,N)*IXG(M,N))
 662  IF(JG(M,N).NE.17) GO TO 663
      DELC(M,N)=DELC(M,N)+CB6(M,N)*D1/(3.0*EG(M,N)*IXG(M,N))
 663  RCAP(M,N)=(MCAP(M,N)-MFCA(M,N))*XG(M,N)*0.25/(EG(M,N)*IXG(M,N))-(0
     1.5+0.75*WC(M,N)/XG(M,N))*ROT(M,N)+1.5*(DELC(M,N)-DEL(M,N))/XG(M,N)
     2-(MCBP(M,N)-MFCB(M,N))*0.25*YG(M,N)/(EG(M,N)*IXG(M,N))+(0.5+0.75*W
     3C(M+1,N)/YG(M,N))*ROT(M+1,N)-1.5*(DEL(M+1,N)-DELC(M,N))/YG(M,N)
      IF(JG(M,N).EQ.15) GO TO 664
      IF(JG(M,N).EQ.17) GO TO 665
      RABP(M,N)=0.0
      RBAP(M,N)=0.0
      GO TO 649
 664  RCAP(M,N)=RCAP(M,N)-0.5*RABP(M,N)
      RBAP(M,N)=0.0
      GO TO 649
 665  RCAP(M,N)=RCAP(M,N)-0.5*RBAP(M,N)
      RABP(M,N)=0.0
      GO TO 649
 615  RABP(M,N)=(LG(M,N)/(6.0*EG(M,N)*IXG(M,N)))*(2.0*MABP(M,N)-MBAP(M,N
     1)-2.0*MFAB(M,N)+MFBA(M,N))-(1.0+0.5*WC(M,N)/LG(M,N))*ROT(M,N)-(0.5
     2*WC(M+1,N)/LG(M,N))*ROT(M+1,N)+(DEL(M+1,N)-DEL(M,N))/LG(M,N)
      RBAP(M,N)=(LG(M,N)/(6.0*EG(M,N)*IXG(M,N)))*(2.0*MBAP(M,N)-MABP(M,N
     1)-2.0*MFBA(M,N)+MFAB(M,N))-(0.5*WC(M,N)/LG(M,N))*ROT(M,N)-(1.0+0.5
     2*WC(M+1,N)/LG(M,N))*ROT(M+1,N)+(DEL(M+1,N)-DEL(M,N))/LG(M,N)
      RCAP(M,N)=0.0
      GO TO 649
 616  DELC(M,N)=(YG(M,N)*YG(M,N)/(3.0*EG(M,N)*IXG(M,N)))*((YG(M,N)/XG(M,
     1N))*(MABP(M,N)+MCAP(M,N)+MAX(M,N))+MBY(M,N)-1.5*MCBP(M,N)-MFBA(M,N
     2)+0.5*MFCB(M,N))-(1.0+0.5*WC(M+1,N)/YG(M,N))*YG(M,N)*ROT(M+1,N)+DE
     3L(M+1,N)
      IF(JG(M,N).NE.21) GO TO 622
      DELC(M,N)=DELC(M,N)-(XG(M,N)**3+YG(M,N)**3)*CA6(M,N)/(3.0*EG(M,N)
     1*IXG(M,N)*XG(M,N))
 622  RABP(M,N)=(XG(M,N)/(6.0*EG(M,N)*IXG(M,N)))*(2.0*MABP(M,N)-MCAP(M,N
     1)-2.0*MFAB(M,N)+MFCA(M,N))-(1.0+0.5*WC(M,N)/XG(M,N))*ROT(M,N)+(DEL
     2C(M,N)-DEL(M,N))/XG(M,N)
      RCAP(M,N)=(XG(M,N)/(6.0*EG(M,N)*IXG(M,N)))*(2.0*MCAP(M,N)-MABP(M,N
     1)-2.0*MFCA(M,N)+MFAB(M,N))-(YG(M,N)/(4.0*EG(M,N)*IXG(M,N)))*(MCBP(
     2M,N)-MFCB(M,N))-(0.5*WC(M,N)/XG(M,N))*ROT(M,N)+(0.5+0.75*WC(M+1,N)
     3/YG(M,N))*ROT(M+1,N)+(DELC(M,N)-DEL(M,N))/XG(M,N)-1.5*(DEL(M+1,N)-
     4DELC(M,N))/YG(M,N)
      IF(JG(M,N).EQ.21) GO TO 623
      RBAP(M,N)=0.0
      GO TO 649
 623  RCAP(M,N)=RCAP(M,N)-0.5*RBAP(M,N)
      GO TO 649
 617  DELC(M,N)=(XG(M,N)*XG(M,N)/(3.0*EG(M,N)*IXG(M,N)))*((XG(M,N)/YG(M,
     1N))*(MBY(M,N)-MCBP(M,N)-MBAP(M,N))+1.5*MCAP(M,N)+MAX(M,N)+MFAB(M,N
     2)-0.5*MFCA(M,N))+(1.0+0.5*WC(M,N)/XG(M,N))*XG(M,N)*ROT(M,N)+DEL(M,
     3N)
      IF(JG(M,N).NE.20) GO TO 624
      DELC(M,N)=DELC(M,N)+CA6(M,N)*(XG(M,N)**3+YG(M,N)**3)/(3.0*EG(M,N)
     1 *IXG(M,N)*XG(M,N))
 624  RBAP(M,N)=(XG(M,N)/(6.0*EG(M,N)*IXG(M,N)))*(2.0*MBAP(M,N)-MCBP(M,N
     1)-2.0*MFBA(M,N)+MFCB(M,N))-(1.0+0.5*WC(M+1,N)/YG(M,N))*ROT(M+1,N)+
     2(DEL(M+1,N)-DELC(M,N))/YG(M,N)
      RCAP(M,N)=(XG(M,N)/(4.0*EG(M,N)*IXG(M,N)))*(MCAP(M,N)-MFCA(M,N))-(
     1YG(M,N)/(6.0*EG(M,N)*IXG(M,N)))*(2.0*MCBP(M,N)-MBAP(M,N)-2.0*MFCB(
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     2M,N)+MFBA(M,N))-(0.5+0.75*WC(M,N)/XG(M,N))*ROT(M,N)+(0.5*WC(M+1,N)
     3/YG(M,N))*ROT(M+1,N)+1.5*(DELC(M,N)-DEL(M,N))/XG(M,N)-(DEL(M+1,N)-
     4DELC(M,N))/YG(M,N)
      IF(JG(M,N).EQ.20) GO TO 625
      RABP(M,N)=0.0
      GO TO 649
 625  RCAP(M,N)=RCAP(M,N)-0.5*RABP(M,N)
      GO TO 649
 618  RABP(M,N)=9999.99
      REAP(M,N)=9999.99
      RCAP(M,N)=9999.99
 649  CONTINUE
 651  CONTINUE
C
C     COLUMNS
C
      DO 750 N=1,NNN
      DO 749 M=1,MM
      IF(JC(M,N).EQ.1) GO TO 711
      IF(JC(M,N).EQ.2) GO TO 712
      IF(JC(M,N).EQ.3) GO TO 713
      IF(JC(M,N).EQ.4) GO TO 714
      IF(JC(M,N).EQ.5) GO TO 715
      IF(JC(M,N).EQ.6) GO TO 716
      IF(JC(M,N).EQ.7) GO TO 717
      IF(JC(M,N).EQ.8) GO TO 718
 711  RULP(M,N)=0.0
      RLUP(M,N)=0.0
      RDLP(M,N)=0.0
      GO TO 749
 712  RLUP(M,N)=(H(N)/(CC(M,N)*EC(M,N)*IXC(M,N)))*MLUP(M,N)-(SS(M,N)/CC(
     1M,N))*ROT(M,N+1)-ROT(M,N)+((CC(M,N)+SS(M,N))/(CC(M,N)*H(N)))*(SWAY
     2(N+1)-SWAY(N))
      RULP(M,N)=0.0
      RDLP(M,N)=0.0
      GO TO 749
 713  RLUP(M,N)=0.0
      RULP(M,N)=(H(N)/(CC(M,N)*EC(M,N)*IXC(M,N)))*MULP(M,N)-(SS(M,N)/CC(
     1M,N))*ROT(M,N)-ROT(M,N+1)+((CC(M,N)+SS(M,N))/(CC(M,N)*H(N)))*(SWAY
     2(N+1)-SWAY(N))
      RDLP(M,N)=0.0
      GO TO 749
 714  D1=CCX(M,N)*CCX(M,N)
      D2=SSX(M,N)*SSX(M,N)
      D3=CCY(M,N)*CCY(M,N)
      D4=SSY(M,N)*SSY(M,N)
      D5=(D1-D2)/CCX(M,N)
      D6=(D3-D4)/CCY(M,N)
      D7=EC(M,N)*IXC(M,N)
      D8=D7/XC(M,N)
      D9=D7/YC(M,N)
      D10=D8/XC(M,N)
      D11=D9/YC(M,N)
      D12=PAPP(M,N)-D6*D11
      D13=PAPP(M,N)-D5*D10
      DELD(M,N)=(XC(M,N)*(MDUP(M,N)+(SSY(M,N)/CCY(M,N))*MDLP(M,N)+D6*D9*
     1ROT(M,N+1)+D12*SWAY(N+1))-YC(M,N)*(MDLP(M,N)+(SSX(M,N)/CCX(M,N))*M
     2DLP(M,N)+D5*D6*ROT(M,N)-D13*SWAY(N)))/(D12*XC(M,N)+D13*YC(M,N))
      RDLP(M,N)=(XC(M,N)/(CCX(M,N)*D7))*MDLP(M,N)-(YC(M,N)/(CCY(M,N)*D7)
     1)*MDUP(M,N)-(SSX(M,N)/CCX(M,N))*ROT(M,N)+(SSY(M,N)/CCY(M,N))*ROT(M
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     2,N+1)+(CCX(M,N)+SSX(M,N))/(CCX(M,N)*XC(M,N))*(DELD(M,N)-SWAY(N))-(
     3CCY(M,N)+SSY(M,N))/(CCY(M,N)*YC(M,N))*(SWAY(N+1)-DELD(M,N))
      RLLP(M,N)=0.0
      RLUP(M,N)=0.0
      GO TO 749
 715  RULP(M,N)=H(N)*(SS(M,N)*MLUP(M,N)-CC(M,N)*MULP(M,N))/((SS(M,N)*SS(
     1M,N)-CC(M,N)*CC(M,N))*EC(M,N)*IXC(M,N))-ROT(M,N+1)+(SWAY(N+1)-SWAY
     2(N))/H(N)
      RLUP(M,N)=H(N)*(CC(M,N)*MLUP(M,N)-SS(M,N)*MULP(M,N))/((CC(M,N)*CC(
     1M,N)-SS(M,N)*SS(M,N))*EC(M,N)*IXC(M,N))-ROT(M,N)+(SWAY(N+1)-SWAY(N
     2))/H(N)
      RDLP(M,N)=0.0
      GO TO 749
 716  D1=(CCY(M,N)*CCY(M,N)-SSY(M,N)*SSY(M,N))*EC(M,N)*IXC(M,N)/(YC(M,N)
     1*CCY(M,N))
      DELD(M,N)=((1.0+(SSY(M,N)/CCY(M,N)))*MDUP(M,N)-(YC(M,N)/XC(M,N))*(
     1MDLP(M,N)+MLUP(M,N))+D1*ROT(M,N+1)+PAPP(M,N)*(YC(M,N)/XC(M,N))*SWA
     2Y(N)+(PAPP(M,N)-D1/YC(M,N))*SWAY(N+1))/(PAPP(M,N)*(1.0+YC(M,N)/XC(
     3M,N))-D1/YC(M,N))
      RDLP(M,N)=(XC(M,N)/((SSX(M,N)*SSX(M,N)-CCX(M,N)*CCX(M,N))*EC(M,N)*
     1IXC(M,N)))*(SSX(M,N)*MULP(M,N)-CCX(M,N)*MULP(M,N))-(YC(M,N)/(CCY(M
     2,N)*EC(M,N)*IXC(M,N)))*MDUP(M,N)+(SSY(M,N)/CCY(M,N))*ROT(M,N+1)+(D
     3ELD(M,N)-SWAY(N))/XC(M,N)-(1.0+SSY(M,N)/CCY(M,N))*(SWAY(N+1)-DELD(
     4M,N))/YC(M,N)
      RLUP(M,N)=XC(M,N)*(CCX(M,N)*MLUP(M,N)-SSX(M,N)*MDLP(M,N))/((CCX(M,
     1N)*CCX(M,N)-SSX(M,N)*SSX(M,N))*EC(M,N)*IXC(M,N))-ROT(M,N)+(DELD(M,
     2N)-SWAY(N))/XC(M,N)
      RULP(M,N)=0.0
      GO TO 749
 717  D1=(CCX(M,N)*CCX(M,N)-SSX(M,N)*SSX(M,N))*EC(M,N)*IXC(M,N)/(XC(M,N)
     1*CCX(M,N))
      DELD(M,N)=((XC(M,N)/YC(M,N))*(MULP(M,N)+MDUP(M,N)+PAPP(M,N)*SWAY(N
     1+1))-(1.0+SSX(M,N)/CCX(M,N))*MDLP(M,N)-D1*ROT(M,N)+(PAPP(M,N)-D1/X
     2C(M,N))*SWAY(N))/(PAPP(M,N)*(XC(M,N)/YC(M,N)+1.0)-D1/XC(M,N))
      RULP(M,N)=(SSY(M,N)*MDUP(M,N)-CCY(M,N)*MULP(M,N))*YC(M,N)/((SSY(M,
     1N)*SSY(M,N)-CCY(M,N)*CCY(M,N))*EC(M,N)*IXC(M,N))
      RDLP(M,N)=XC(M,N)*MULP(M,N)/(CCX(M,N)*EC(M,N)*IXC(M,N))-(CCY(M,N)*
     1MDUP(M,N)-SSY(M,N)*MULP(M,N))*YC(M,N)/((CCY(M,N)*CCY(M,N)-SSY(M,N)
     2*SSY(M,N))*EC(M,N)*IXC(M,N))-(SSX(M,N)/CCX(M,N))*ROT(M,N)+((CCX(M,
     3N)+SSX(M,N))/(CCX(M,N)*XC(M,N)))*(DELD(M,N)-SWAY(N))+(DELD(M,N)-SW
     4AY(N+1))/YC(M,N)
      RLUP(M,N)=0.0
      GO TO 749
 718  RDLP(M,N)=9999.99
      RLUP(M,N)=9999.99
      RULP(M,N)=9999.99
 749  CONTINUE
 750  CONTINUE
      RETURN
      END
      SUBROUTINE CHECK3
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 CLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
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     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLUF(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,C4),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMCN/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMCN/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NB,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLUP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     BRACING FORCES
C
      DC 690 N=1,NNN
      I=1
      DC 640 M=2,MM
      IF(JB(M,N,I).EQ.9) GO TO 640
      IF(JB(M,N,I).EQ.2) GO TO 620
      IF(JB(M,N,I).EQ.3) GO TO 630
      D1=AB(M,N,I)*EB(M,N,I)/LB(M,N,I)
      D2=H(N)/LB(M,N,I)
      D3=L(M-1)/LB(M,N,I)
      FB(M,N,I)=D1*(D2*(DEL(M,N)-DEL(M-1,N+1))+D3*(SWAY(N)-SWAY(N+1)))
      FBX(M,N,I)=FB(M,N,I)*D3
      IF(IELAST.EQ.1) GO TO 64C
       IF(IREV.EQ.0) GC TC 520
      IF(JE(M,N,I).EQ.11) GO TO 640
  520 IF(FB(M,N,I).GE.0.0) GO TO 610
      IF(FB(M,N,I)+PCR(M,N,I)) 605,640,640
  605 JE(M,N,I)=3
      IND=IND+1
      WRITE(6,600) M,N,I
      GC TO 640
  610 IF(FB(M,N,I)-FBP(M,N,I)) 640,640,603
  603 JE(M,N,I)=2
      INC=INC+1
      WRITE(6,601) M,N,I
      GC TO 640
  620 FB(M,N,I)=FBP(M,N,I)
      D3=L(M-1)/LB(M,N,I)
      FBX(M,N,I)=FB(M,N,I)*D3
      GC TO 640
  630 FB(M,N,I)=-PCR(M,N,I)
      D3=L(M-1)/LB(M,N,I)
      FBX(M,N,I)=FB(M,N,I)*D3
  640 CCNTINUE
      I=2
      DC 68C M=1,MMM
      IF(JB(M,N,I).EQ.9) GO TC 680
      IF(JB(M,N,I).EQ.2) GO TC 660
      IF(JB(M,N,I).EQ.3) GC TC 670
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      D1=AB(M,N,I)*EB(M,N,I)/LB(M,N,I)
      D2=H(N)/LB(M,N,I)
      D3=L(M)/LB(M,N,I)
      FB(M,N,I)=D1*(D2*(DEL(M,N)-DEL(M+1,N+1))-D3*(SWAY(N)-SWAY(N+1)))
      FBX(M,N,I)=FB(M,N,I)*D3
      IF(IELAST.EQ.1) GC TO 680
       IF(IREV.EQ.0) GC TC 521
      IF(JB(M,N,I).EQ.11) GC TC 680
  521 IF(FB(M,N,I).GE.0.0) GC TC 650
      IF(FB(M,N,I)+PCR(M,N,I)) 645,680,680
  645 JB(M,N,I)=3
      INC=INC+1
      WRITE(6,600) M,N,I
      GC TO 680
  650 IF(FB(M,N,I)-FBP(M,N,I)) 680,680,652
  652 JB(M,N,I)=2
      INC=INC+1
      WRITE(6,601) M,N,I
      GC TO 680
  660 FB(M,N,I)=FBP(M,N,I)
      D3=L(M)/LB(M,N,I)
      FBX(M,N,I)=FB(M,N,I)*D3
      GC TO 680
  670 FB(M,N,I)=-PCR(M,N,I)
      D3=L(M)/LB(M,N,I)
      FBX(M,N,I)=FB(M,N,I)*D3
  680 CONTINUE
  690 CONTINUE
      IF(INC6.EQ.2) GC TO 708
  600 FORMAT(1H0,5HBRACE,3I3,11HREACHED PCR)
  601 FORMAT(1H0,5HBRACE,3I3,16HYIELD IN TENSION)
      WRITE(6,914)
      DC 940 N=1,NNN
      DO 935 M=2,MM
      I=1
      IF(JB(M,N,I).EQ.9) GO TC 935
      WRITE(6,915) M,N,I,FB(M,N,I),JB(M,N,I),FBP(M,N,I),PCR(M,N,I),FBX(M
     1,N,I)
  935 CONTINUE
      DC 939 M=1,MMM
      I=2
      IF(JB(M,N,I).EQ.9) GO TC 939
      WRITE(6,915) M,N,I,FB(M,N,I),JB(M,N,I),FBP(M,N,I),PCR(M,N,I),FBX(M
     1,N,I)
  939 CONTINUE
  940 CONTINUE
  708 CONTINUE
  914 FORMAT(1H0,64HBRACE               FORCE     HINGE       FPL              PCR
     1           FX)
  915 FORMAT(1H ,3I3,5X,F8.2,I6,3(5X,F8.2))
C
C     SHEAR CHECK
C
      WRITE(6,60)
      DC 80 N=2,NN
      SB=0.0
      DC 64 M=1,MM
      IF(N.EQ.NN) GC TC 62
      SB=SB+VU(M,N-1)+VL(M,N)
      GO TO 64
```

```
62    SB=SB+VU(M,N-1)
64    CONTINUE
      DO 75 M=2,MM
      IF(N.EQ.NN) GO TO 72
      SB=SB-FBX(M,N,1)+FBX(M,N-1,1)
      GO TO 75
72    SB=SB+FBX(M,N-1,1)
75    CONTINUE
      DO 76 M=1,MMM
      IF(N.EQ.NN) GO TO 73
      SB=SB+FBX(M,N,2)-FBX(M,N-1,2)
      GO TO 76
73    SB=SB-FBX(M,N-1,2)
76    CONTINUE
      SB=SB+F(N)
      WRITE(6,66) N,SB
80    CONTINUE
60    FORMAT(1H0,29HFLOOR LEVEL      SHEAR BALANCE)
66    FORMAT(1H ,I6,11X,E11.4)
      RETURN
      END
      SUBROUTINE HREV
      COMMON/AREA1/ AG(9,04),IXG(9,04),ZXG(9,04),FYG(9,04),EG(9,04),
     1 DLDES(9,04),LLDES(9,04),LG(9,04),W(9,04),VWA(9,04),VWB(9,04),
     2 MFAB(9,04),MFBA(9,04),MFCA(9,04),MFCB(9,04),MAX(9,04),MBY(9,04),
     3 VWAL(9,04),VWBL(9,04),CA1(9,04),CA2(9,04),CA3(9,04),CA5(9,04),
     4 CA6(9,04),CB1(9,04),CB2(9,04),CB3(9,04),CB5(9,04),CB6(9,04),
     5 MBAP(9,04),MABP(9,04),MCAP(9,04),MCBP(9,04),XG(9,04),YG(9,04),
     6 VA(9,04),VB(9,04),MA(9,04),MB(9,04),RABP(9,04),RBAP(9,04),
     7 RCAP(9,04),DELC(9,04),RBAPL(9,04),RCAPL(9,04),RABPL(9,04)
      COMMON/AREA2/ AC(9,04),IXC(9,04),ZXC(9,04),RXC(9,04),WC(9,04),
     1 FYC(9,04),EC(9,04),CC(9,04),SS(9,04),CCX(9,04),SSX(9,04),
     2 CCY(9,04),SSY(9,04),CL1(9,04),CL2(9,04),CL3(9,04),CL5(9,04),
     3 CU1(9,04),CU2(9,04),CU3(9,04),CU5(9,04),MD(9,04),MC(9,04),
     4 MULP(9,04),MLLP(9,04),MDLP(9,04),MDUP(9,04),XC(9,04),YC(9,04),
     5 XMP(9,04),VL(9,04),VU(9,04),ML(9,04),MU(9,04),RLUP(9,04),
     6 RULP(9,04),RDLP(9,04),DELD(9,04),L(9),H(04),K(9),SUMV(9),B(100)
      COMMON/AREA3/ AB(9,04,2),RXB(9,04,2),FYB(9,04,2),EB(9,04,2),
     1 LB(9,04,2),CD1(9,04,2),CD2(9,04,2),CD3(9,04,2),CD4(9,04,2),
     2 CC2(9,04),PCR(9,04,2),FB(9,04,2),FBP(9,04,2),FBX(9,04,2),X(100),
     3 FDES(04),VDES(9,04),PDES(9),F(04),P(9,04),PAPP(9,04),VINCR(9,04),
     4 RF(300),RLL(300),RP(300),A(100,45),V(45,45),DET,RINLL,RINF,RINP,
     5 SWAY(04),ROT(9,04),DEL(9,04),CD5(9,04,2),C7(9,04),C8(9,04),RINV,
     6 RV(300),VJT(9,04),VJTL(9,04),C9(9,04),C10(9,04)
      COMMON/AREA4/ JC(9,04),JG(9,04),JB(9,04,2),IND6,MM,NN,MMM,NNN,IL,
     1 NA,NE,NCYC,IND3,LLL,NL,IND2(9),IELAST,NLAST,IAREA,IND,IREV,IREV2,
     2 ISTAB,I10,I11,I12,JGL(9,04),JCL(9,04),JBL(9,04,2),I13,I14,I15
      REAL MA,MB,ML,MU,L,IXC,IXG,LLDES,LG,LB,MFBA,MFAB,MFCA,MFCB,MAX,
     1MBY,MBAP,MABP,MCAP,MCBP,MULP,MLLP,MDLP,MDUP,K,MC,MD
      REAL*8 COLUMN,BRACE,GIRDER,IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
C
C     GIRDERS
C
      IREV=0
      DO 200 N=2,NN
      DO 199 M=1,MMM
      IF(JG(M,N).EQ.1) GO TO 199
      IF(JG(M,N).EQ.10) GO TO 199
      IF(JG(M,N).EQ.11) GO TO 199
      IF(JG(M,N).EQ.12) GO TO 199
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```
      IF(JG(M,N).EQ.2) GO TO 115
      IF(JG(M,N).EQ.5) GO TO 115
      IF(JG(M,N).EQ.6) GO TO 115
      IF(JG(M,N).EQ.14) GO TO 115
      IF(JG(M,N).EQ.16) GO TO 115
      GO TO 120
115   IF(AG(M,N).EQ.-1.0) GO TO 120
      IF(F(N).GE.0.0) GO TO 70
      IF(IL.LE.I10) GO TO 120
 70   IF(RABP(M,N)*RABPL(M,N).LT.0.0) GO TO 110
      D1=ABS(RABP(M,N))
      D2=ABS(RABPL(M,N))
      IF(D1.GE.D2) GO TO 120
110   WRITE(6,99) M,N
      IREV=IREV+1
      IF(JG(M,N).EQ.2) GO TO 116
      IF(JG(M,N).EQ.5) GO TO 117
      IF(JG(M,N).EQ.6) GO TO 118
      IF(JG(M,N).EQ.14) GO TO 119
      GO TO 120
116   JG(M,N)=10
      D1=EG(M,N)*IXG(M,N)/LG(M,N)
      CA6(M,N)=MABP(M,N)-MFAB(M,N)-D1*((4.0+3.0*WC(M,N)/LG(M,N))*ROT(M,N
     1)+(2.0+3.0*WC(M+1,N)/LG(M,N))*ROT(M+1,N)-6.0*(DEL(M+1,N)-DEL(M,N))
     2 /LG(M,N))
      CB6(M,N)=2.0*D1*RABP(M,N)
      C7(M,N) =MABP(M,N)-0.5*MBAP(M,N)+0.5*MFBA(M,N)-MFAB(M,N)+D1*(3.0*(
     1DEL(M+1,N)-DEL(M,N))/LG(M,N)-3.0*(1.0+0.5*WC(M,N)/LG(M,N))*ROT(M,N
     2)-1.5*(WC(M+1,N)/LG(M,N))*ROT(M+1,N))
      GO TO 120
117   JG(M,N)=13
      D1=EG(M,N)*IXG(M,N)/LG(M,N)
      CA6(M,N)=MABP(M,N)-0.5*MBAP(M,N)+0.5*MFBA(M,N)-MFAB(M,N)+D1*(3.0*(
     1DEL(M+1,N)-DEL(M,N))/LG(M,N)-3.0*(1.0+0.5*WC(M,N)/LG(M,N))*ROT(M,N
     2)-1.5*(WC(M+1,N)/LG(M,N))*ROT(M+1,N))
      CB6(M,N)=0.0
      GO TO 120
118   JG(M,N)=15
      D1=EG(M,N)*IXG(M,N)/XG(M,N)
      D2=1.0/(XG(M,N)**3+YG(M,N)**3)
      D3=MABP(M,N)-0.5*MCAP(M,N)+0.5*MFCA(M,N)-MFAB(M,N)-D1*((3.0+1.5*WC
     1(M,N)/XG(M,N))*ROT(M,N)-3.0*(DELC(M,N) -DEL(M,N))/XG(M,N))
      CA6(M,N)=XG(M,N)**3*D2*D3
      CB6(M,N)=XG(M,N)*XG(M,N)*YG(M,N)*D2*D3
      GO TO 120
119   WRITE(6,88)
 88   FORMAT(1H ,'SECOND HINGE REVERSAL DETECTED AND WILL BE IGNORED')
      IREV=IREV-1
120   IF(JG(M,N).EQ.3) GO TO 125
      IF(JG(M,N).EQ.5) GO TO 125
      IF(JG(M,N).EQ.7) GO TO 125
      IF(JG(M,N).EQ.13) GO TO 125
      IF(JG(M,N).EQ.18) GO TO 125
      GO TO 140
125   IF(AG(M,N).EQ.-1.0) GO TO 140
      IF(F(N).LE.0.0) GO TO 75
      IF(IL.LE.I10) GO TO 140
 75   IF(RBAP(M,N)*RBAPL(M,N).LT.0.0) GO TO 130
      D1=ABS(RBAP(M,N))
      D2=ABS(RBAPL(M,N))
```

```
      IF(D1.GE.D2) GO TO 140
  130 WRITE(6,98) M,N
      IREV=IREV+1
      IF(JG(M,N).EQ.3) GO TO 136
      IF(JG(M,N).EQ.5) GO TO 137
      IF(JG(M,N).EQ.7) GO TO 138
      IF(JG(M,N).EQ.13) GO TO 139
      GO TO 140
  136 JG(M,N)=11
      D1=EG(M,N)*IXG(M,N)/LG(M,N)
      CA6(M,N)=2.0*D1*RBAP(M,N)
      CB6(M,N)=MBAP(M,N)-MFBA(M,N)-D1*((2.0+3.0*WC(M,N)/LG(M,N))*ROT(M,N
     1)+(4.0+3.0*WC(M+1,N)/LG(M,N))*ROT(M+1,N)-6.0*(DEL(M+1,N)-DEL(M,N))
     2/LG(M,N))
      C7(M,N) =MBAP(M,N)-0.5*MABP(M,N)+0.5*MFAB(M,N)-MFBA(M,N)+D1*(3.0*(
     1DEL(M+1,N)-DEL(M,N))/LG(M,N)-3.0*(1.0+0.5*WC(M+1,N)/LG(M,N))*ROT(M
     2+1,N)-1.5*(WC(M,N)/LG(M,N))*ROT(M,N))
      GO TO 140
  137 JG(M,N)=14
      D1=EG(M,N)*IXG(M,N)/LG(M,N)
      CA6(M,N)=0.0
      CB6(M,N)=MBAP(M,N)-0.5*MABP(M,N)+0.5*MFAB(M,N)-MFBA(M,N)+D1*(3.0*(
     1DEL(M+1,N)-DEL(M,N))/LG(M,N)-3.0*(1.0+0.5*WC(M+1,N)/LG(M,N))*ROT(M
     2+1,N)-1.5*(WC(M,N)/LG(M,N))*ROT(M,N))
      GO TO 140
  138 JG(M,N)=17
      D1=EG(M,N)*IXG(M,N)/YG(M,N)
      D2=1.0/(XG(M,N)**3+YG(M,N)**3)
      D3=MBAP(M,N)-0.5*MCBP(M,N)-MFBA(M,N)+0.5*MFCB(M,N)-3.0*D1*((1.0+0.
     15*WC(M+1,N)/YG(M,N))*ROT(M+1,N)-(DEL(M+1,N)-DELC(M,N))/YG(M,N))
      CA6(M,N)=XG(M,N)*YG(M,N)*YG(M,N)*D2*D3
      CB6(M,N)=YG(M,N)**3*D2*D3
      GO TO 140
  139 WRITE(6,88)
      IREV=IREV-1
  140 IF(JG(M,N).EQ.4) GO TO 145
      IF(JG(M,N).EQ.6) GO TO 145
      IF(JG(M,N).EQ.7) GO TO 145
      IF(JG(M,N).EQ.15) GO TO 145
      IF(JG(M,N).EQ.17) GO TO 145
      GO TO 199
  145 IF(RCAP(M,N)*RCAPL(M,N).LT.0.0) GO TO 150
      D1=ABS(RCAP(M,N))
      D2=ABS(RCAPL(M,N))
      IF(D1.GE.D2) GO TO 199
  150 WRITE(6,97) M,N
      IF(JG(M,N).EQ.4) JG(M,N)=12
      IF(JG(M,N).EQ.6) JG(M,N)=16
      IF(JG(M,N).EQ.7) JG(M,N)=18
  199 CONTINUE
  200 CONTINUE
   99 FORMAT(1H0,'HINGE REVERSAL LEFT END GIRDER,'2I5)
   98 FORMAT(1H0,'HINGE REVERSAL RIGHT END GIRDER,'2I5)
   97 FORMAT(1H0,'HINGE REVERSAL INTERIOR GIRDER,'2I5)
C
C     DIAGONAL BRACING
C
      DO 300 N=1,NNN
      DO 249 M=2,MM
      IF(JE(M,N,1).EQ.3) GO TO 205
```

```
      GC TC 249
205   D1=AB(M,N,1)*EB(M,N,1)/LB(M,N,1)
      D2=H(N)/LB(M,N,1)
      D3=L(M-1)/LB(M,N,1)
      D4=D1*(D2*(DEL(M,N)-DEL(M-1,N+1))+D3*(SWAY(N)-SWAY(N+1)))
      IF(D4+PCR(M,N,1)) 249,249,210
210   WRITE(6,91) M,N
      IREV=IREV+1
      JB(M,N,1)=11
249   CCNTINUE
      DC 299 M=1,MMM
      IF(JB(M,N,2).EQ.3) GC TC 255
      GC TO 299
255   D1=AB(M,N,2)*EB(M,N,2)/LB(M,N,2)
      D2=H(N)/LB(M,N,2)
      D3=L(M)/LB(M,N,2)
      D4=D1*(D2*(DEL(M,N)-DEL(M+1,N+1))-D3*(SWAY(N)-SWAY(N+1)))
      IF(D4+PCR(M,N,2)) 299,299,260
260   WRITE(6,92) M,N
      IREV=IREV+1
      JB(M,N,2)=11
299   CCNTINUE
300   CCNTINUE
 91   FCRMAT(1H0,'BRACE ELASTIC',2I4,' 1')
 92   FCRMAT(1H0,'BRACE ELASTIC',2I4,' 2')
      RETURN
      END
FILE
```

A.5 Accuracy of the Computer Solution

Each time the simultaneous equations were solved and the moments and forces determined, a check was made on the accuracy of the results. The horizontal forces in each story were summed, and the resulting errors determined. For the twenty-four story braced frame, the maximum error in the shear balance in any story at a load factor of 1.30 was 0.16% of the applied horizontal load using single precision arithmetic, (6 significant figures), and 0.03% using double precision arithmetic, (15 significant figures). These errors are well within acceptable tolerances.

In addition, the deflected shapes of the structures were examined at various stages in their respective loading histories, to check the "reasonableness" of the solution. In all cases individual displacement quantities increased in a regular fashion, and agreed with known structural behavior.