UNIVERSITY OF ALBERTA

INTERACTIVE TEACHING

by

RICHARD B. KABAROFF

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ABSTRACT

The purpose of this study was to determine the appropriateness of an instructional model designed for effective senior-high mathematics instruction. The study involved several phases: the development of the model, the model's implementation and interpretation by classroom teachers, the testing for achievement gains, and, an analysis of student attitudes towards mathematics. The research was conducted in four large senior-high schools located in a large metropolitan area in Western Canada.

The design of the model was based both on a set of key instructional strategies proposed by Good, Grouws, and Ebmeier (1983) and elements of cooperative learning. Lessons taught involved whole group instruction followed by group practice.

The model was used by four senior-high mathematics teachers to teach two consecutive units - systems of equations and geometry- to four grade-eleven classes of average mathematical ability. Four classes taught by three mathematics teachers served as the control group.

Data were collected through classroom observations, teacher and student interviews, teacher journals, pre- and post- tests, and a student attitude questionnaire.

Three of the four treatment teachers satisfactorily implemented the model in their classrooms. The study reports these three teachers' interpretations of the model and those aspects of the model which they considered appropriate to the senior-high mathematics teaching. In particular, approaches to homework, oral work, review, lesson development, teaching for meaning, and group work were

cited as effective. Students interviewed supported their teachers' claims as to the effectiveness of group work.

The results of the achievement tests indicated that the treatment group significantly outperformed the control group on both post - tests. On the student attitude questionnaire, administered at the end of the treatment, there was no significant difference in the attitudes towards mathematics between the control and treatment groups.

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A. Introduction

In the last decade there has been growing public concern over the state of mathematics education on this continent and abroad. Alarmed that American students' academic achievement compared unfavourably with that of students in Europe and Japan, the American government created the National Commission On Excellence in Education. The Commission's report, A Nation at Risk (1983), recommended that teachers should meet high academic standards and demonstrate both an aptitude for and competence in teaching their discipline. Similarly, in Britain, the Report of the Committee of Inquiry into the Teaching of Mathematics in Schools (1982) under the chairmanship of Dr. W. H. Cockcroft outlined recommendations for the improvement of the teaching of mathematics. Although not advocating a particular instructional style, the Committee suggested that effective mathematics teaching involves clarity of explanation and provides opportunities for discussion between the teacher and students and among students, practical work, problem solving and investigation, and the practice of skills and routines.

In reaction to the criticism that mathematics teaching is not meeting the needs of students in an increasingly technological and information-based society, the National Council of Teachers of Mathematics published its Professional Standards for Teaching Mathematics (1989). The Standards document assumes that the improvement of mathematics teaching is primarily the responsibility of the classroom teacher. However, the teacher can not be expected to initiate changes that will enhance student learning without a rationale provided by the research findings from mathematics education and cognitive psychology. As well, Goodlad (1983) argues that teachers will respond to and adopt alternate instructional approaches if they are encouraged, given support and allowed to experiment in a risk-free environment.

B. Purpose of the Study

The primary purpose of this study is to investigate teacher and student perceptions of an Interactive Teaching Model designed for effective senior-high mathematics instruction. This model is an adaptation of the Missouri Mathematics Program described by Good, Grouws, and Ebmeier, in Active Mathematics Teaching (1983), and found effective in elementary and junior- high mathematics classrooms. In addition, each lesson incorporates group activities in the monitored seatwork segment. A secondary purpose of the study is to test the suitability of the model in the classroom and assess the achievement and attitude gains, if any, of the students taught using this treatment.

This study is not predicated on the principle that there is a single or best system for teaching mathematics, but rather that experienced mathematics teachers should consider the findings of teacher-effectiveness research for the enhancement of the classroom environment.

C. Significance of the Study

In the first half of the century, studies in mathematics education were conducted primarily by educational psychologists such as Thorndike, Judd and Brownell (Kilpatrick and Greeno,1989). By the 1950s the popular topics included drill and practice, teaching approaches, diagnostic testing and prediction of achievement, with little emphasis on how the subject matter itself was approached.

Early research relied on statistical methods adopted from experimental biology and psychology. In the 1960s and 1970s with growth in funding, a more systematic approach to research problems evolved with greater ties and collaboration among researchers, and greater attention to a theoretical rationale. Disappointing results of the behaviorist tradition have lead to a greater emphasis on qualitative research.

Garrison and Macmillan (1987) discuss the problems of converting the findings of educational research into practical knowledge. They refer to and expand on the writings of N. L. Gage and the analogy Gage draws between medical and educational research. Garrison and Macmillan argue that in the medical profession both researchers and practitioners share the same world view, facilitating the application of research into practice; whereas, in education, both teachers' and students' interpretations of the educational experience often differ from researchers'. Garrison and Macmillan appeal for research based on well-defined theoretical positions enabling practitioners to judge the applicability of the findings of that research to their own situations.

Regardless of how experienced mathematics teachers are, most are interested in enhancing their classroom practice. The teaching and learning of mathematics is not a sterile endeavour, but rather an active pursuit of excellence. The theoretical positions of both educational researchers and practitioners must be similar if the results of educational research is not to be either discounted or ignored (Garrison and Macmillan, 1987). If we accept the premise that the value of educational research depends on the extent to which its findings speak to educational practice, we can not ignore teachers' views of the relevance of that research to their personal situations.

An increasing acceptance by mathematics educators of constructivism as an epistemology and its implementation in the classroom can be traced directly to Piaget and many contemporary researchers. However, teachers burdened by increasing demands in the workplace often feel distanced from the theoretical and question the practicality of suggestions advanced. This study addresses some of the concerns classroom teachers voice about innovative practice and its viability. For example, research reveals the importance of worked examples, and the use of non-goal specific problems in developing students' mathematical thinking (Silver, 1990). In order to investigate how this would translate into classroom practice, continuing collaboration between researchers and practitioners is essential. If research in the teaching and learning of mathematics is to remain relevant and valued. a continual interchange of ideas and perspectives must exist. Researchers can not ignore the needs of the classroom, and teachers should not dismiss the findings of research that takes those needs into consideration.

Considerable research in both the elementary and junior-high classrooms exists; however, few studies have been reported in a senior-high-school setting. In this study, it is possible to abstract from the participants' views those elements of a teacher-effectiveness model which are appropriate in a senior-high context, not only in their ease of implementation but also their effectiveness in promoting student achievement and a positive attitude towards mathematics.

Chapter II - Review of Related Literature

A. Introduction

The purpose of this study is twofold: to investigate teacher and student perceptions towards a system of effective teaching designed for a senior-high mathematics classroom, and to determine the achievement and attitude gains of students taught using this treatment. The model is an adaptation of the Missouri Model described by Good, Grouws and Ebmeier (1983) modified to incorporate elements of cooperative learning in the lesson design.

This chapter includes an overview of the Missouri Model, a summary of related research in effective mathematics teaching, a rationale for including cooperative-learning structures, the development of mathematical understanding, and the constructivist view on direct instruction.

B. The Missouri Model

Process-product research of the 1970s is now considered unreliable because investigators limited their studies to collecting data to support preconceived notions of effective teaching, chose small samples based on convenience, and ignored the subject-matter context to evaluate teacher effects (Good, Grouws and Ebmeier, 1983). To address those concerns and to identify appropriate teacher behaviours in mathematics classrooms, from 1973 through 1975 Good and Grouws conducted a large naturalistic study in the American Midwest involving over one hundred third and fourth-grade teachers. The school district chosen was uniform socioeconomically and had a stable student population. Achievement tests were administered each fall to target teachers whose students' academic gains were either consistently high or low. Nine effective and nine ineffective teachers were selected for further study. Based on the analysis of the data, high achievement gains were correlated to high teacher expectations, task orientation, clarity of explanations, positive feedback and large-group instruction.

Based on these results, Good and Grouws conducted three pre- and post-test experimental studies - two at the elementary level (1977 -1978) and one at junior high (1979-1981). These treatment studies helped shape current ideas about effective mathematics instruction. Described in Active Mathematics Teaching (Good, Grouws and Ebmeier, 1986), these studies employed a set of key instructional behaviours incorporated into an instructional model. This lesson format, known as the Missouri Model, involved daily homework reviews, active whole-class lesson development and monitored seatwork. The instructional model used in the second elementary and the junior-high treatment studies included a daily ten-minute problem-solving component. Teacher observations, pre- and post-treatment achievement scores, and student attitude scales were the primary means of data collection. In the first experimental study, significant gains were realized. In the grade eight experimental study, the treatment had little impact on students' computational scores; however, there were significant gains in student problem solving scores.

C. Research in Effective Mathematics Teaching

Process-product research in teacher effectiveness is concerned with the identification of teacher behaviors that are associated with positive student attitudes and gains in student achievement. Brophy and Good (1986) describe this research as focussing on the teacher in normal school settings sampled from uniform and well-described populations. Bourke (1984) extends these criteria to include the contexts of teaching and learning. Historically, these studies have followed an observational-correlational-experimental loop. Initially, observational studies are used to describe appropriate teacher behaviors, then follow-up research correlates these behaviors either singly or in clusters to specific student outcomes. Finally teaching models based on these results are tested, usually in pretest-treatment-postest experiments. This cycle is repeated to further refine knowledge in this area.

Rosenshine (1971) in his review of fifty-one early process-product studies helped define the field. He outlined four steps usually followed in this genre: (1) developing an instrument to record teacher behaviors, (2) recording the behaviors of teachers and their students, (3) comparing the classes on the basis of student achievement corrected for initial differences, and (4) identifying teacher behaviors that are related to student achievement. The observational instruments used are either low-inference category systems in which the observer or students record the occurrence of behaviors, or a high-inference rating system in which the observer or students infer the level of behavior on a scale.

Rosenshine (1971) criticized high-inference rating systems as lacking specificity and requiring a substantial degree of observer judgement. While not as subjective, even low-inference systems were questioned. In reviewing observational and correlational studies, he noted the difficulty of assessing the consistency among different observers' ratings, the tendency to define measured variables differently in different studies, and the differing methods of grouping or categorizing variables.

Two procedures commonly used in correlational studies are univariate and multivariate analysis. Univariate procedures compare a single teacher behavior with an outcome measure; in multivariate procedures, several related behaviors are combined into a single measure which is then correlated to an outcome measure. Since various studies use different constructs, the results are often not directly comparable.

Nevertheless, the results of effective teaching research have helped define good classroom practice. Good and Biddle (1988) argue that much still can be learned from observational research in developing better teaching models, particularly in mathematics. Typical of variables investigated are classroom organization, problem-solving processes, teacher and student backgrounds and perceptions of mathematics (Taylor, 1988), lesson structure and presentation styles (Smith, 1985) and the role of meaning in teacher explanations (Sigurdson and Olson, 1988).

Brophy and Good (1986) in their review of research on teacher behavior and student achievement cite the works of Flanders as landmark studies in process-product research prior to the 1970s. Flanders' correlational studies examined the effects of direct and indirect teaching on student attitudes and achievement. Direct teaching being defined as lecturing and giving directions; whereas indirect teaching, which Flanders favoured, included encouragement and praise, questioning and clarifying ideas. In addition to administering student questionnaires and testing the students' initial achievement levels, the studies involved observers working in pairs to alternately code classroom interaction. This use of observer pairs influenced subsequent research methodology.

Bourke (1984) in the Teaching and Learning of Mathematics described the IEA Classroom Environment Study conducted in grade five mathematics classrooms in Melbourne, Australia. What differentiated this study from the mainstream of process-product research was the inclusion of classroom context and presage variables (i,e,. descriptive of teacher background). This study cut across the descriptive-correlational-experimental loop. The effects of both the classroom context and teacher behaviours were assessed in terms of both cognitive and affective outcomes. A teaching model was created by grouping the independent variables into constructs; this model was tested by correlating these constructs to student outcomes. The results were consistent with those of Good and Grouws. Additional recommendations included some small-group work, the use of concrete materials, and individually prescribed homework.

Process-product research has declined in the last decade. This type of research describes classroom practices using abstract categories that speak little about how specific content matter is to be taught. Even the results of experimental studies that advance specific instructional models can not be interpreted as the final words on presentation styles. In addition, this type of research does not adequately address the issues of how students learn, what students actually do in the classroom, and what is worth teaching or learning. Observational research, in general, must not be viewed in isolation, but must be integrated into the larger picture that involves the subjective assessment of what educational experiences are worthwhile and significant for the student.

Berliner (1986) argues that the research community needs to identify experienced and expert teachers and compare those teachers with the novice to refine the information about teacher behaviours that have been found effective in other studies. Expert teachers employ exemplary schema, process information selectively and efficiently, and utilise scripts and well-practised and automated routines that may assist us in identifying behaviors which are less effective in novice or ordinary teachers. Leinhardt (1988) describes expert teachers as both managing classroom discussions, demonstrations, seatwork and independent practice effectively and presenting concepts clearly. All of these teacher behaviours enrich the classroom experiences of their students. Expert-novice studies will establish guidelines for beginning teachers to consider in planning lessons and improving their own instruction. The experienced cooperating teacher often serves as a role model for the student teacher which affects the student teacher's professional development; expert-novice studies may assist the cooperating teacher to better articulate their expertise. Apart from the practical considerations of teacher training and the nature of pedagogy itself, both expert and beginning teachers make interesting subjects to study when researching effective practices.

Expert-novice studies in mathematics teaching have employed differing methodologies. This research may involve observing both experienced and novice teachers in naturalistic settings, teaching routine lessons in ordinary classrooms. Lessons are observed and often recorded using audio or videotapes. Pre- and post-lesson interviews are used to supplement the data base. The different constructs that experienced and novice teachers employ in setting lesson agendas, presenting the concepts, and reflecting on the lesson as taught are analyzed. Leinhardt (1985, 1988) has pioneered the use of semantic nets - pictorial representations illustrating how subjects identify concepts and form relationships among them - in comparing novice and expert teachers' lesson plans and presentations. Although difficult for the reader to follow, these semantic nets reveal qualitative differences in the ways teachers conceive of and explain the material presented to students.

In other studies, Carter et al. (1987) and Berliner et al., (1988) have posed educational problems for experts, novices and postulants - people from industry or business with an interest in mathematics or the sciences- to resolve. The differences in how the various groups approach these tasks were then analyzed.

Problems in the selection of expert teachers arise in expert-novice research. If teachers are selected solely on the basis of experience, are they necessarily expert teachers? Experience is not necessarily equated with expertise. Research must address the problem why experience in teaching does not consistently lead to expertise (Berliner, 1986) or why some experienced teachers exhibit expertise in some instances but not in others. Knowing what is best to do and actually performing those actions in a teaching situation are not synonymous. This clouds the findings of research that contrasts student teachers and their cooperating teachers (Borko and Livingston, 1987; 1990).

At the elementary level, the academic achievement gains of a teacher's students may be a useful criterion for identifying a teacher's expertise (Leinhardt, 1985, 1990); but, at the secondary level students often have several teachers, and the same group of students is not taught by the same teacher in consecutive years. However, resorting to reputation, nominations for excellence in teaching, or results on standardized tests is inherently suspect (Berliner, 1986).

Related to the expert-novice research described above are the eleven interpretive case studies conducted as part of the Exemplary Practice in Science and

Mathematics study in Australia (Tobin and Fraser, 1987). This study compared exemplary and non-exemplary teachers. The data collected were obtained primarily through participant observation - a minimum of eight classroom visits- and student and teacher questionnaires and interviews. The internal validity of the study was established through triangulation - the use of multiple researchers and several sources of confirming data, repeated observations - and meetings of the entire research team to discuss preliminary assertions on the basis of confirming and refuting evidence. Cross-case comparisons supported the external validity of the findings: that exemplary teachers have good classroom control, encourage student participation, emphasize meaning, and maintain a nurturing student environment.

In spite of the researchers' definite bias towards selecting exemplary teachers, not only on the basis of peer and key educators recommendations but also on the basis of a constructivist epistemology, the study identified salient aspects of effective teaching. What the study did not address was how teachers with ordinary skills can construct knowledge about mathematics pedagogy to enhance their own teaching.

Effective teaching research can not ignore the academic tasks that students perform in the classroom; without that context, knowledge of instructional behaviors is of limited value (Doyle,1983). The tasks teachers set direct the learning of students. How teachers structure those tasks, the cognitive level of those tasks, and the level of meaning imbedded within those tasks were the central issues of the case-study research conducted in 1983 in Austin (Doyle and Sanford, 1985). Ten classes, including two classes in secondary mathematics, participated. The mathematics classes were observed for a six-week period. In addition to the observational records of the classroom events and the processes related to the work assigned (i.e. resources used, teacher directions, the students' final products and student accountability) data were obtained through student and teacher interviews.

Based on their findings, Doyle and Sanford suggest that teachers clearly define tasks for their students and how those tasks will be evaluated, consider their students' past experience the context of the tasks within the curriculum, monitor individual and group work, encourage novel tasks and minimize the risk in performing those novel tasks.

D. Cooperative Learning

The Curriculum Branch of Alberta Education in its Interim Teacher Resource Manual (1990, Alberta Education) for senior-high mathematics emphasizes that, when planning the instructional experiences for students, the teacher must consider alternate strategies, such as cooperative learning and discussion, to accommodate student differences and enhance learning. Similarly, The Curriculum and Evaluation Standards for School Mathematics (1989. NCTM) proposes not only content changes to reform school mathematics, but also identifies new societal goals for schooling. Schools, a product of the industrial age, are not addressing today's needs. The new societal goals for education should provide for

1. mathematically literate workers
2. lifelong learning
3. opportunity for all, and
4. an informed electorate.

In expanding the definition of mathematical literacy, the Standards emphasizes developing the student's ability to work with others when solving problems. Their goal is not simply to meet the needs of an increasingly technological and information-based society. The Standards is a constructivist document, subscribing to the view that learning is an active process in which students bring prior knowledge to the new situation and construct their own meanings. This is necessarily a social process. To this end, therefore,

> instruction should vary and include opportunities for ... group and individual assignments; [and] discussion between teacher and students and among students.

Neil Davidson (1990), in his article, Small-Group Cooperative Learning in Mathematics, argues that the NCTM Standards of mathematical communication, logical reasoning, problem solving, and making mathematical connections are enhanced by cooperative structures. In addition, cooperative learning provides

1. social support for the discussion of mathematical ideas
2. opportunities for all students to succeed
3. a forum for group discussion and the resolution of mathematical problems.
4. a vehicle for the exploration of alternate approaches.
5. an atmosphere in which students learn through discussing ideas, listening, and teaching others.
6. opportunities for creative and critical thinking through the exploration of nonroutine problems that may be beyond a single individual's ability. and
7. opportunities for the mastery of basic skills in a novel context.

Johnson, Johnson and Holubec (1986) distinguish among competitive, individualistic, and cooperative structures of classroom instruction. They emphasize that a cooperative learning lesson to be effective in enhancing learning should contain the following five elements: positive interdependence established through student discussion enabling students to assist each other to understand the material and encouraging each other to work hard , face-to-face interaction, individual accountability, interpersonal and small group skills, and group processing of those social skills.

The importance of peer relationships in enhancing learning is supported by Doyle and Sanford (1985) , Skemp (1987) and Prawat (1989). Skemp maintains that communicating ideas helps to clarify them and that the explanation of ideas to others allows them to assimilate those ideas into their schema. Discussion engenders new ideas, and the sharing of different standpoints promotes understanding. Prawat (1989) uses the term "negotiation" to describe the process in which learners through the discussion of differing or parallel ideas reach consensus, constructing knowledge consistent with the accepted view in the discipline. Doyle and Sanford (1985) in their studies of academic work argue that group work reduces the anxiety and risks individual students experience in tackling novel tasks; and anxiety diminishes understanding and limits higher mental activity (Skemp, 1987).

The positive effects of cooperative learning have been well documented in research. Johnson, Johnson and Holubec (1987) report a meta-analysis of 122 studies in which cooperative learning is shown to result in higher academic achievement, enhanced problem solving, and greater retention of the material. These studies were conducted across all ages and in all subject areas. Slavin (1990) analyzed 60 studies that compared cooperative learning strategies and more traditional methods of instruction. Positive gains in achievement occurred when the teacher emphasized both individual and group goals. Students gained in mutual respect, self esteem, time on task, and attendance.

The model of cooperative learning most similar to the one proposed for this study is Student Teams Achievement Divisions (Slavin, 1990). In STAD, after the teacher presents the material, the students work in preassigned four-member learning teams that are composed heterogeneously in achievement level, sex, and ethnic background. The students are examined individually on the material taught. Students' performances are compared to their own previous achievement, and points are awarded on the basis of improvement. The teams earn certificates or awards based on the total scores of the individual members which comprise that team. The primary purpose of STAD is to motivate students to assist each other master material presented by the teacher.

E. Mathematical Understanding

Any system of teaching mathematics must, necessarily, consider both how students learn and whether that system enhances mathematical understanding. Among many mathematics educators, constructivism as an epistemology has gained currency.

From a constructivist standpoint, human beings are active learners, their behaviour is purposive and they organize concepts within structures or schema based on past experiences (Skemp, 1987). All knowledge including mathematics is personally constructed; no two people have the same knowledge (Goldin, 1990). Reality is part of our experiential world and our adapation to it; and, we define rules and perceive regularities based on experience (von Glaserfeld, 1990). Mathematics is a human endeavour constructed to accommodate human purposes (Confrey,1990) rather than an eternal body of truth (Goldin,1990).

If knowledge is personally constructed and cannot be imparted directly from the teacher to the student (Confrey, 1990; Goldin,1990; Steffe, 1990), then the mathematics teacher must provide opportunities for students to develop as accurate and complete mathematical understandings as possible (Baroody, 1990). According to Goldin (1990), the teacher should allow for guided discovery, meaningful applications, problem solving, and a positive learning atmosphere in the classroom. The learner should assume an active role in his/her learning (Confrey,1990).

Meaningful learning must assume a greater role in classroom instruction than rote learning (Baroody and Ginsburg, 1990). The concepts taught should be connected to the learner's existing knowledge base (Baroody and Ginsburg,1990) and placed in context with other material taught (Steffe, 1990). Prawat(1989) insists that student understanding is enhanced when formal knowledge is supported by the informal knowledge students bring to the classroom. Through every-day applications we can make the student aware about the mathematical structures that underlie the physical world (Confrey,1990). The classroom should be organized to compel students to think, by teachers' modelling and active questioning eliciting students' responses and conjectures (Noddings, 1990). The onus is on students to decide the appropriateness of their constructions (Confrey, 1990), since none of us can know with certitude what another's constructs are.

Students should be made aware of their constructions, and through reflection modify them to accommodate new concepts (Confrey,1990; Mason,1987). In addition teachers should reflect on their own mathematical constructions to determine novel approaches to presenting material (Steffe,1990).

Because knowledge must be tested to determine its fit or appropriateness (von Glaserfeld, 1990), active discussion between teacher and students and among students is essential to confirm or reject tentative constructs (Prawat, 1989;Confrey,1990); problem solving in mathematics is necessarily a social process (Steffe, 1990).

If constructivism is assumed, then we may also assume mathematics is a creative activity not only for mathematicians but children as well (Davis and Maher, 1990). However, students may think that mathematics helps them think creatively and logically, on the one hand, but maintain that the subject is best learned by memorization (Schoenfeld, 1989). Educating students in mathematics may be viewed as a process of transforming novices into experts (Blais, 1988). The difficulty teachers experience in attempting to have their students bridge that gap is that they may give too much explanation and guidance with the belief that the students can "escape the struggle inherent in the process of learning to reason" (Blais,1988; p. 628). Expertise, as well, depends on domain specific knowledge; problem solving skills in mathematics may not be dependent on heuristics as much as a firm knowledge base in the subject matter, that is, the appropriate schema which empower the student to tackle the non routine (Owen and Sweller, 1989).

Sigurdson and Olson (1988) conducted an experimental study that focused on the role of meaning in teaching mathematics. Fifty-four grade eight mathematics teachers taught for a six-month period using one of four approaches: (1) conventional textbook instruction, (2) direct instruction using the Missouri Model, (3) direct instruction with an emphasis on meaning, and (4) direct instruction emphasizing meaning and problem solving. The results of the study supported the effectiveness of the Missouri Model and a focus on meaning within that framework.

F. Constructivist Views on Direct Instruction

> There is no recipe-like method that can supplant the individual teacher working skillfully to establish a mathematical environment (Davis, Maher & Noddings, 1990).

Educators with a strict constructivist perspective tend to view direct instruction as either too prescriptive or failing to provide an adequate basis for "higher-cognitive skills" (Confrey,1990). Direct instruction is criticized as failing to connect symbols and their manipulation to existing student knowledge, overlooking student differences, individual needs and readiness, and presenting content too quickly to be understood (Baroody and Ginsburg, 1990).

However, even constructivists concede that the Active Teaching strategies of Good, Grouws and Ebmeier (1983) need not be entirely abandoned, especially when routine practice may be advantageous in acquiring facts and skills that may later be used in problem solving.

The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), a constructivist document, criticizes classroom instruction during which students "passively absorb information" presented in bits and pieces to be mastered through drill and practice. The Standards advocates classroom instruction that ensures students are active participants in their learning, and are given opportunities for group work and discussion, project work and individual assignments, practice with methodology, and appropriate teacher explanation and interaction.

G. Summary

According to Lampert (1988) research on effective mathematics instruction indicates that good teachers are confident enough in their subject to direct student inquiry without being too specific; deliver a curriculum that addresses student questions and is true to the discipline; work from the premise that students are actively forming their representations of mathematical ideas, provide appropriate experiences and motivate students to learn.

Mathematics researchers cannot concentrate solely on the curriculum and ignore instructional practices (Brophy, 1986). The reality of the classroom situation dictates that teachers must move entire classes through prescribed curricula. For implementation within that context, systematic approaches to instruction are required (Brophy, 1986). Teachers learn by experience that certain techniques are more successful than others and avidly adopt suggestions, such as those outlined in Every Minute Counts: Making Your Math Class Work (Johnson, 1982). He suggests approachesfor effectively using the time in classrooms for active instruction, in presenting material, having students practice, monitoring progress with appropriate questioning, providing feedback and trying to get improved responses and performance from students.

Chapter III - Methodology

A. Overview

The purpose of this study was to determine the appropriateness of an instructional model designed for effective senior-high mathematics instruction. The study involved several phases:

1. The development of the model;
2. The model's implementation and interpretation by classroom teachers;
3. The testing for achievement gains; and,
4. The analysis of student attitudes towards mathematics.

The preparatory phase of the study involved the development of a model for effective mathematics teaching. The design of the Interactive Teaching Model was based both on the set of key instructional strategies proposed by Good, Grouws and Ebmeier (1983) and elements of cooperative learning.

The model was used by four senior-high mathematics teachers to teach two consecutive units - systems of equations and geometry in Mathematics 23, a grade-eleven course of studies intended for students of average mathematical ability. Three teachers agreed to participate as part of the control group. A total of eight classes, four in the treatment group and four in the control group were observed over a four-month period.

Over the course of the study, the treatment teachers were asked to keep a log, recording their perceptions of the model; in particular, noting those elements which they considered successful, and those they would modify or discontinue after completion of the study. Included in the logs were the teachers' daily lesson plans and homework assignments.

A pretest, and two unit tests were developed and administered to the classes to determine differences in achievement, if any, between the treatment and control groups. In addition, a questionnaire to assess the students' attitude towards mathematics was designed and administered to both groups to evaluate the students' perception of the learning and teaching of mathematics, and whether attitudes differed for students in the treatment group.

Prior to the study, both the treatment and control teachers were observed to determine which elements, if any, of the model they used in their daily teaching routine. During the study, the treatment teachers' classes were observed to determine the degree to which those teachers implemented the model. A Classroom Observation Scale was developed to supplement and in aid the analysis of field notes. In addition, the teachers and students were interviewed to ascertain their perception of the efficacy and appropriateness of the model.

B. Design of the Interactive Teaching Model

The following is an outline of the lesson format that was used in this model. This model was structured to accommodate a class period varying between sixty-four and sixty-seven minutes in length.

LESSON FORMAT

DAILY REVIEW AND HOMEWORK CHECK - 10 minutes

* Begin with oral work
* Review previous lesson's skills and knowledge
* Deal with homework

DEVELOPMENT - 25 minutes

* Place concept to be taught in context of past knowledge and future problems

* Emphasize meaning
* Monitor student understanding through active questioning
* Reinforce concept through controlled practice

COOPERATIVE PRACTICE - 25 minutes
* Provide opportunity for successful practice
* Include word problems and applications related to the lesson
* Encourage active group discussion
* Keep individuals accountable

HOMEWORK
* Assign homework relevant to lesson
* Ensure that the questions assigned can be completed successfully by the majority of students working independently
* Include a review question.

C. Discussion of the Model

The lesson structure is an adaptation of a set of key instructional strategies for effective mathematics instruction proposed by Good, Grouws and Ebmeier (1983). The Good and Grouws approach, originally based on teacher- centred, whole-class instruction, has been modified to encourage student-student interaction and discussion about the mathematics presented, and to focus on mathematical meaning. Throughout the lesson the teacher is expected to provide opportunities to actively engage the students in their own learning. As much as possible in the development segment and during group work, student understanding is developed through real-world applications, references to the students' own experiences, mathematics modelling, concrete materials, and process-problem solving.

Oral work at the beginning of the lesson is intended to emphasize the importance of the first few minutes of the class. Rather than focussing on routine clerical matters, all students will engage in relevant and meaningful mathematical activities that will set the tone for the rest of the period.

Less time than teachers routinely spend is devoted to addressing problems with the previous day's homework. Problems from assigned work, dealt with in a whole class setting, are limited to those which concern the majority of the class. By restricting the time spent on taking up homework, more time can be devoted to the presentation and discussion of new material.

The development segment of the lesson is that part of the period devoted to actively involving students in developing their understanding of skills and concepts. Meaning is established by relating the content to previous knowledge, by placing the concepts in the context of the students' own experience, by modelling everyday situations, by focussing on applications, and by using concrete materials when applicable. Through questioning, the teacher monitors student understanding, and ensures that students are held accountable.

The controlled practice portion of the development, when the students work on one or two problems followed by a class discussion, is designed to provide additional feedback to the teacher, and to enhance the students' proficiency with the material.

In the cooperative-practice segment, students work in groups of four. These groups are to be heterogeneous by sex and achievement. The intent of the groups is to provide student support when they are working on questions assigned based on the material presented in the lesson development. In this phase of the class, both group and individuals are held accountable for the work to be completed in class. As well, in this portion of the lesson, novel problems are discussed by the class as a whole and within the groups. These problems are integrated into every second lesson. The intent of these problems is to develop students' problem solving skills and to motivate students.

Homework is to provide students with an opportunity for successful practice. Because of the time spent in the group-practice phase of the lesson, the number of questions assigned should be fewer than what normally might have been assigned.

D. Sample Lesson

The following lesson illustrates the Interactive Teaching Model used by the treatment teachers during the study. It is representative of the six lessons- the first three in each unit- which were prepared for the treatment teachers and included in the inservice package. The textual references are from <u>MathMatters: Book Three</u> (Ebos, Zolis and Morrison, 1991).

Lesson 1: Solve systems of linear equations in two unknowns graphically.

Daily Review and Homework Check - 10 minutes

* Oral Work

  Ask the students to respond to the following questions.
  Have the students justify their own or their peers' answers.

  1. Describe the graph of $x = 1$? Explain.
  2. What is the equation of the horizontal line which passes through the point (1, 2)? Why?
  3. At what point do the lines $x = 3$, and $y = 4$ intersect?
  4. What are the equations of the horizontal and vertical lines which pass through the point (-1, 5)?
  5. What is the slope of the line $y = 3x - 1$? Why?
  6. What are the slopes of the lines $y = 2x$ and $y = 2x + 1$? How are these lines related? Why? Would these lines intersect?

* Review: none
* Homework: none

Development - 25 minutes

* Define systems of equations.
* Explain that this unit involves using many of the principles learned in the previous chapter on linear equations and their graphs. Mention that the systems of equations that the students will be solving consist of a pair of linear equations, that each linear equation represents a straight line, and that the objective is to determine if and where the lines intersect.
* Example 1

  Two students, X and Y, who live 10 km apart decide to cycle towards each other to meet one afternoon. How far has each travelled when they met?

The following questions should be addressed.

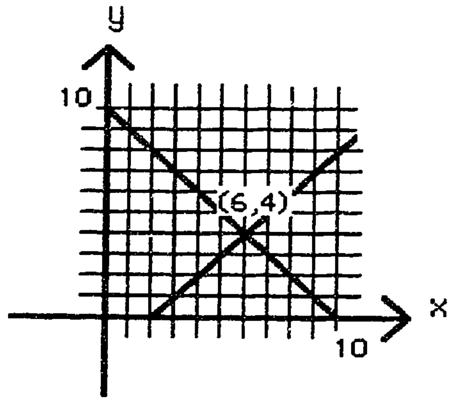
1. What distances are possible? Emphasize that a number of answers are possible.
2. If x represents how far Student X has travelled, and y represents how far Student Y has travelled, what equation represents the relationship between x and y ?

   $x + y = 10$
3. Ask students to suggest ordered pairs which satisfy the equation. Point out that there are restrictions on the two variables.

   $0 \leq x \leq 10$ and $0 \leq y \leq 10$
4. Have the students graph the equation.
5. Discuss the reason why the information given in the original question was nadequate to find unique values for x and y.
6. Provide an additional constraint to the original problem. If Student X cycled 2 km farther than Student Y, how far did each travel?
   The answer should now be obvious: $x = 6$ km and $y = 4$ km.
7. Discuss the equation representing the second statement, $x - y = 2$, and its graph.
8. Explain that the two equations

   $$x + y = 10 \ldots (1)$$
   $$x - y = 2 \ldots (2)$$

   form a system of equations, and that the point of intersection of the graphs of those equations is the required solution.



* Example 2

Solve the following system graphically, and verify the solution by substitution. Discuss both tables of values and intercepts.

$2x - y = 7$...(1)

$x + 2y = 1$...(2)

* Use controlled practice to emphasize concepts. At least two additional examples should be completed; however, do not exceed 25 minutes for the development segment. One example of a system to be solved graphically follows.

$3x + y = 6$... (1)

$x = y - 2$... (2)

Cooperative Practice - 15 minutes

* Assign pp. 199-200, questions 1-4. Indicate that at the end of the period, the assignment will be selected, at random, from one of the students from each group. These assignments will be graded, and each student in the group will receive the mark based on that assignment.

Problem Solving - 10 minutes

* Spend the time with the entire class discussing the following problem.

How many different pairs of positive integers can be found to solve the following equations? For the purposes of this question, the pairs 3 and 2, and 2 and 3 are considered the same.
Look for patterns.

1. $x + y = 2$
2. $x + y = 3$
3. $x + y = 4$
4. $x + y = 5$
5. $x + y = 1001$

Homework

* Assign p. 200, questions 6 and 7.

E. Sample

Seven experienced senior-high mathematics teachers volunteered to participate in the study. The model was used in four non-semestered Mathematics 23 classes; four non-semestered Mathematics 23 classes served as the control group. Both the teachers who participated in the treatment group and those in the control group taught the same two units - systems of equations and geometry - both of which are outlined in the Alberta Education Mathematics 23 Course of Studies (1990). All seven teachers used the same text: MathMatters: Book 3 (Ebos, Zolis and Morrison; 1991).

F. Choice of Units

The two units - systems of equations and geometry - taught for the duration of the study were chosen for convenience. In the majority of senior high schools these units are the first two that are covered in the second semester. By limiting the study to these two units, there was a minimum disruption to the teachers' yearly plans. Furthermore, both topics lend themselves to process problems and teaching for understanding.

G. Inservice

Two three-hour inservice sessions were arranged during the last week in January, 1991. The treatment teachers were provided with an overview of the study, the lesson format to be employed, and suggestions on implementation. The first three classes in each unit were prepared for the teachers prior to the inservices. The prepared material included problem-solving activities for each unit and assisted the teachers in planning the remaining lessons.

The two units were not be prepared for the teachers in their entirety. This permitted the teachers to interpret the model for themselves and allow for flexibility in the classroom to address the unique requirements of each class. It was not the intention of the study to script each lesson.

During the first month of implementation, each treatment teacher was visited twice to monitor the program and offer assistance.

H. Pretest

A pretest comprised of thirty multiple-choice questions was administered in each of the eight classes prior to the first unit taught. This test was a survey of the three units of study in Mathematics 23 covered before systems of equations. The number of items from each unit was based on the unit weightings suggested in the Interim Teacher Resource Manual for Mathematics 23 (Alberta Education, 1990). The emphasis is on the application of the concepts in a problem-solving context.

This test was piloted and revised before implementation.

I. Unit Tests

A unit test was administered at the end of each unit. Each test consisted of thirty problem oriented, multiple-choice items based on the learner expectations listed in the Mathematics 23 Course of Studies (Alberta Education, 1990). Both unit tests were piloted and revised.

The purpose in administering these tests was determine the differences, if any, between the achievement of students in the treatment group and students in the control group.

J. Student Questionnaire and Interview

Twenty-five closed-ended items which comprise the Attitude to Mathematics student survey (see attached) were piloted and administered at the end of the study to determine the students' perception of both the relevance of mathematics to their daily lives and the appropriateness of the instructional model. In particular, the attitude survey focused on six factors:

1. Interest in lesson
2. Class cohesiveness
3. Teacher- student relationship
4. Organization of learning environment
5. Problem solving
6. Homework

Three students from each of the treatment teachers' classes were interviewed. The teachers assisted in selecting from those who volunteered, students who were representative of high, low and average students in their classes. The students were asked to give their impressions of the lesson format, the style of presentation and group work.

K. Classroom Observation

Each of the teachers' classes was observed during the course of the study according to the following schedule.

Week	Classes
1	treatment and control
2	treatment
4	treatment
6	treatment and control
8	treatment

The first two visits to the treatment classes served two purposes: to observe whether the teaching format was being followed and to coach the teachers in the implementation of the model. In addition, both the control and treatment classrooms were observed to confirm that neither the treatment r control classes were being taught according to the lesson format to be used in the study.

The classroom observation scale detailed in Chapter IV was used to rate the degree the teaching format was implemented.

L. Interviews and Journals

Each treatment teacher was interviewed during each of the two units and was asked to maintain a journal. The intention was to determine teacher reactions regarding the effectiveness and appropriateness of the model in teaching senior-high mathematics. A listing of the interview questions is given in Chapter IV.

Chapter IV - The Instruments

A. Introduction

This chapter details the instruments used during the study to gather data; specifically, the pre- and post-tests designed to assess achievement differences between the treatment and control groups, the teacher and student interviews developed to determine teacher and student impressions of the effectiveness and appropriateness of the Interactive Teaching Model, the student questionnaires administered to measure attitude differences between students in the treatment and control groups, and the Classroom Observation Scale used during classroom visits.

B. Pre- and Post-tests

Three thirty-item multiple-choice tests based on the Mathematics 23 Course of Studies (Alberta Education, 1990) were prepared, one pre-test and two post-tests.

The pre-test surveyed the program objectives students covered prior to the unit on systems of equations. Ten items were constructed for each of the following units: Powers and Radicals, Algebra, and Linear Relations

The items for the each of the two post-tests, Systems of Equations and Geometry, were distributed equitably across the objectives for those two units.

Both the text, MathMatters: Book 3 (Ebos, Zolis, and Morrison, 1991) and the Senior High Mathematics 20/23/24 Interim Teacher Resource Manual (Alberta Education, 1990) were referenced during item construction.

For all three tests, the items tested student comprehension, applications, and problem solving. As a result, these tests were more difficult than the usual teacher-made tests which emphasized knowledge items. The following selected from each of the three tests are representative of these items. The complete tests are included in the appendix.

1. A packing case is cube shaped. If its volume is 6.7 m^3, then the area of the top of the case, correct to one decimal place is

   A 3.4 m^2 B. 3.5 m^2

   B. 3.6 m^2 D. 3.7 m^2

2. What is the value of "a" so that $ax + 3y = 2$ and $4x - 7y - 8 = 0$ cross the x-axis at the same point?

   A. 1 B. 2

   C. -1 D. -2

3. How many cubes 2 cm on a side can be placed in a box 6 cm long, 6 cm wide, and 6 cm high?

   A. 27 B. 18

   B. 36 D. 54

Each test was piloted prior to the implementation of the model. The tests were piloted in a semestered Mathematics 23 class of twenty-four students who had completed the course and were reviewing for their final exam. Items were revised based on the following criteria:

1. At least 5% of the students chose each distractor.
2. At least 30% of the students answered each question correctly.

As well, the tests were discussed with the seven teachers who participated in the study, the teacher who piloted the tests, and a former test-construction expert with the provincial department of education.

As a result the following items were revised or replaced.

Test	Items Revised
Survey (Pre-test)	#1, 2, 3, 5, 8, 9 10,13, 15, 16, 18, 20, 21, 23, 27, 29
Systems of Equations	#1, 8, 10, 14, 16, 21, 24, 29
Geometry	#2, 6, 15, 29, 30

The majority of changes to the pre-test were made to ensure that all the items reflected what the seven teachers had taught prior to implementation of the model.

C. Teacher Interviews

Each of the four teachers who used the Interactive Teaching Model in their classrooms was interviewed twice, at the end of each of the two units of study. The first interview examined their initial impressions of the model, their observations of student reaction, and the elements of the treatment program they felt they may continue at the conclusion of the study.

Interview 1

1. How does the Interactive Teaching Model differ from how you have traditionally taught?
2. What are the advantages, if any, with beginning the lesson with oral work?
3. How have you been dealing with homework during this study?
4. Have you been satisfied with this approach to homework? Why?
5. Have you been spending more time in preparing lessons? Why.
6. Is teaching for "meaning" difficult? Why?
7. How do you reconcile teaching for meaning with teaching for skills?
8. How did you select the groups for the cooperative-practice segment?
9. How have the students accepted this arrangement?

10. What difficulties or advantages do you see in "cooperative practice"?
11. How does the homework you have been assigning differ from what you had normally done in the past?
12. What are the students' impressions of the Interactive Teaching Model?
13. Do you feel the students are learning more or less mathematics under this system? Why?
14. How do you think students learn mathematics?
15. What are important considerations a teacher should take into account to enhance her students' learning mathematics?
16. What are your impressions of the strengths of Active Teaching?
17. What are your impressions of the weaknesses of Interactive Teaching?
18. Do you feel your teaching will change after the completion of the study. Why?

After the first round of interviews the teacher responses were reviewed. Based on the teachers' initial reactions, the second interview focussed on the strengths and weakness of the model and the possible change in the teachers' classroom practice following the study.

<u>Interview 2</u>

1. What aspects of interactive teaching will you continue after the completion of the project? Why?
2. What aspects will you not continue? Why?
3. What have been the students' reaction to the Interactive Teaching Model?
4. Do you feel the students learned more or less mathematics under this system? Why?
5. What are your views on the success or failure of students' working together?
6. What are your views on the times allotted to the various segments of the lesson format?
7. Have the students been active participants throughout the course of the study?
8. Did it become easier to plan lessons as the study progressed?
9. Concluding statements?

D. Student Interviews

At the end of the treatment program, as a result of informal conversation with students during my classroom visits, I was interested in students' impressions of

1. How the Interactive Teaching Model differed from their teachers' previous practice,
2. Whether they felt those changes helped them to learn mathematics,

3. Their attitude towards cooperative learning activities,

4. Their task orientation both to classroom work and homework,

5. Their attitude towards mathematics and its application, and,

6. What classroom routines they preferred.

Three students from each treatment class were interviewed towards the end of the study. The classroom teachers selected a high achiever, an average student, and a low achiever from those students who volunteered to be interviewed. The same questions were asked of each student.

<u>Student Interview</u>

During the last two units you have participated in a study that involves a lesson design that may or may not differ from the style of instruction to which you are used to in mathematics. I will be asking you several questions which relate to the approach to instruction your teacher has used in the last two units: system of equations and geometry. Your answers are important to me in deciding the appropriateness of this approach to teaching mathematics. Your answers will be kept in strict confidence. Please answer each question as accurately as possible.

1. What were the main differences you noticed in the way the lessons were taught during the last two units from what was done before?
2. Do you think those differences have helped you learn mathematics? Explain.
3. What are your impressions about working together with other students on mathematics?
4. Have you been doing more mathematics in class? Why or why not?
5. Has it been easier to understand the material you have been taught? Why or why not?
6. How useful to you do you think the mathematics you have been taught in the last two units?
7. Have you been completing your homework assignments?
8. What things can a teacher do to make it easier for you to learn mathematics?

E. Student Questionnaires

Originally, thirty closed-ended items were constructed to determine students' perception of both the relevance of mathematics to their daily lives and the appropriateness of the instructional model. In particular, the attitude survey focused on six factors:

1. Interest in lesson
2. Class cohesiveness
3. Student-teacher relationship
4. Organization of learning environment
5. Problem solving
6. Homework

Each factor consisted of five items. A number of these items were selected from published attitude surveys (Tobin and Fraser, 1987; Sigurdson and Olson, 1989).

The questionnaire was piloted in two semestered Mathematics 23 classes consisting of a total of forty-seven students . A varimax factor analysis (see appendix) was performed on the data, and, as a result, the items clustered into nine factors based on inter-item correlations. Twenty-five items that correlated most strongly to these factors were retained; five items were deleted. The revised questionnaire which was administered to the treatment and control groups appears below.

ATTITUDE TO MATHEMATICS: MATHEMATICS 23

Answer all questions on the answer sheet. Fill in the circle that best represents your answer. Use HB pencil.

A B C D

0 0 0 0

A = always B = often C = seldom D = never

Example:

I like watching hockey on television. A B C D

0 0 0 0

THERE ARE NO RIGHT ANSWERS; PLEASE INDICATE WHAT YOU THINK.

1. I can see how the mathematics in this class can be applied outside the classroom.
2. I am interested in the work we do in this mathematics class.
3. It is important to learn the material taught in this class.
4. What I learn in mathematics will help me when I have a job.
5. I find mathematics class interesting.
6. I like working with the other students in this class on mathematics.
7. Students help each other learn mathematics in this class.
8. I like to discuss problems in mathematics with other students in this class.
9. I like it when my teacher does a few examples before I am asked to do a question on my own.
10. I need my teacher to learn mathematics.
11. When I have difficulty with a question in mathematics, I ask my teacher.
12. I prefer to have my teacher help me with a difficult problem rather than try it on my own.
13. This class is well organized.
14. I like the way the material is presented in this class.
15. I spend most of the time in each class doing mathematics.
16. Getting a certain amount of class work done is very important in this class.
17. Doing challenging, thinking questions is an important part of mathematics.
18. There is more than one way to do most mathematics problems.
19. I like to do mathematics problems my own way.
20. Mathematics problems are interesting.
21. Doing mathematics problems is a good way to learn mathematics.
22. It is important to complete my homework in this class.
23. I can do most of the questions assigned for homework in this class.
24. Doing homework makes it easier to get better marks in this class.
25. Doing homework for this class makes it easier to learn mathematics.

The nine factors were interpreted as follows:

Factor	Items
1. Class Cohesiveness	6, 7, 8
2. Homework	21, 24, 25
3. Goal orientation	3, 12, 15
4. Interest in lesson	2, 5, 17, 20

5. Independence	19, 23
6. Class organization	10,13, 14
7. Relevance of content	1, 4
8. Task orientation	9, 16, 22
9. Teacher dependence	11, 18

F. Classroom Observation Scale

A classroom observation scale was developed to determine the extent to which the treatment teachers adhered to the model, to assess the differences in instuctional practices between the control and treatment teachers, and to identify the changes in classroom practice of the treatment teachers arising from the implementation of the model.

The classroom observation scale was adapted from a similar instrument developed by Sigurdson and Olson (1990). The purpose of this scale was twofold.

1. To record the times the teachers spent on oral work, review, homework, lesson development, and cooperative practice; and, at the end of each lesson, the length of time, based on teacher estimates, students would be required to complete homework assigned

2. To score each of the lesson segment on a high-inference four-point implementation scale, according to the extent each matched the expectations of the Interactive Teaching Model.

As the lesson was observed, each lesson segment was assigned a score from 0 through 3 based on the level to which the teacher implemented the model. At the end of the lesson the extent to which the teacher taught for meaning rather than merely emphasizing an algorithmic approach was rated on a Meaning Scale. This scale, like the Implementation Scale ranged from 0 to 3, where 0 represented no attempt on the teacher's part to teach for understanding and 3 represented a sincere and effective approach to teaching for meaning, such as: utilizing real-world examples, indicating alternate approaches, using models and concrete materials, relating new concepts to previously taught material, engaging the students in dialogue through effective questioning techniques, and drawing on the students' own experiences.

The problem solving scale was used to rate from 0 to 3 the extent to which the non routine problems were approached interactively through teacher-student discussion. As non routine problems were to be introduced every other lesson during the cooperative practice, not every lesson was rated.

As well as using the Classroom observation Scale, extensive field notes were taken, detailing the development of each lesson observed.

Figure 1. Classroom observation Scale

Teacher No: ______ Date: ______ Topic: ______________

	Daily Review					
	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Home-work
Time						Teacher Est
(I)						

Implementation (i)
0 Not attempted
1 Some implementation
2 Implementation incomplete
3 Fully implemented

Cooperative Practice: Page ______ Question ______

Homework: Page ______ Question ______

Problem Solving Scale
0 No PS activity
1 PS given
2 Teacher solution
3 Solved interactively

Meaning Scale
0 Not attempted
1 Some attempt
2. Attempted, but not complete
3. Completed Effectively

Comments

Chapter V - Qualitative Results

A. Introduction

This chapter contains the qualitative results of the study and addresses the treatment teachers' and their students' perceptions of the effectiveness and appropriateness of the Interactive Teaching model. Data were gathered and verified through classroom observation, personal interviews with the participating teachers at the end of each unit, each treatment teacher's daily journal, and the student interviews conducted at the conclusion of the study.

Three of the four teachers who volunteered to participate in the treatment, fully implemented the Interactive Teaching Model. The fourth teacher, who due to personal and work-related factors, did not carry out the model to that teacher's satisfaction. Quantitative data supporting the exclusion of the fourth teacher is contained in Chapter VI. As a result, this chapter includes only the views of those teachers who fully implemented the Interactive Teaching Model and the views of their students. A discussion of the fourth teacher's involvement and views may be found in the appendix.

B. Personal Background and Possible Sources of Bias

This study was conducted while I was on sabbatical leave from my school district. Prior to the leave, I had taught senior-high-school mathematics for twenty-three years, the last three years as a department head of mathematics in a large, urban high school. Two years before that assignment, I was employed as a mathematics consultant. I have known the participating teachers a number of years, and I respect their dedication to their profession, their competence, and their honesty of opinion

C. Carol

During the study, Carol was teaching mathematics part time in a large senior-high school located in a largely working-class neighbourhood. Carol had taught high school for twelve years and had been at the same school for the past six. She has completed five years of post-secondary education, a four-year Arts degree and one year of teacher training. Her temperament and educational background are well suited for teaching mathematics. She takes an active interest in her students' welfare and strives continually to enhance her classroom climate and improve her instruction. Carol's teaching assignments during the study included both academic and remedial classes at the grade ten and eleven levels.

Carol's Mathematics 23 class, in which she implemented the Interactive Teaching Model, consisted of twenty-eight students. The class met three times per week, each class period being sixty-four minutes in length. The treatment began with the introduction of the unit on systems of equations and ended three months later with the completion of the geometry unit. Throughout the study she made a concerted effort to apply both the lesson format and the spirit of the treatment.

During her first interview two weeks after the study began, Carol acknowledged that the Interactive Teaching Model was a departure from her regular classroom practice.

> I used to spend more time on homework; maybe more than I wanted to in taking up student questions at the beginning of the class. I am better

> organized now and I don't spend more than ten minutes. I used fewer examples with meaning; now I make a conscious effort to include meaning. I did not use cooperative practice. I used to review at the beginning of the class, but now I am more conscious of what I will be reviewing (with the class).

As part of the treatment, she divided her students into seven groups of four. Carol used the students' grades which reflected their achievement from the beginning of the school year. For each group she chose a high-achieving student, a low achiever, and two students with average marks. As well as using student grades, Carol identified those students whose attendance had been unsatisfactory. Every effort was made to avoid placing more than one po[illegible] attender in a given group. In addition, Carol balanced the groups [illegible] personality. Students who had a history of being off task were split u[illegible] groups remained the same throughout the study.

Immediately prior to implementing the model, Carol discussed her expectations with her class. She asked the students to maintain orderly notebooks, with assignments clearly dated and referenced to the text. Homework assignments were to be revised by the students with corrected solutions written next to their original work. The students were cautioned that they would be held accountable for their homework. On Fridays, several questions were to be chosen randomly from the week's assignments, and the students were to copy their solutions to those questions directly from their notebooks and hand them in for grading . Students were told that their group work was to be graded as well. Each member of the group was to be responsible for recording the work the group was asked to complete. The class was instructed that any member of the group could be called on to hand in his/her work and that all members of the group would receive the same mark. These expectations were summarized on a handout and the students were asked to retain it for their reference.

For the first three lessons taught in each unit, Carol followed the suggestions outlined in the inservice package. I observed the first lesson she taught. After the lesson, she admitted that because the format differed from her regular classroom routine, she concentrated too much on adhering to the time limits and did not focus on whole-group interaction during the lesson development, at least to the extent she would have wished. During the first interview, conducted after she had taught the first unit, Carol still was concerned with the time constraints.

> I was hurried using the model. I've been watching the clock and perhaps I don't cover all the material during the development. Also I don't have enough time just to talk to the students. Just talking to the students is motivating.

Another of Carol's initial concerns was the time taken in lesson preparation; however, she and Robert, the other teacher involved in the project at the school, began planning together. Working cooperatively significantly reduced her work load and both teachers benefited through sharing ideas.

During all five lessons I observed, Carol began her lessons with a review of the homework assigned from the previous lesson. Complete solutions to the homework were prepared on overhead transparencies before the lesson. Questions with which the students had difficulty were discussed, and time was allowed for the students to correct their written work and ask questions. As well, Carol would circulate throughout the class to assist individual students with their work and to monitor whether they had completed their assignments. Regularly, students were asked to hand in questions from their assigned work for grading.

Carol limited the amount of homework she assigned to what she estimated that the majority of her students could complete successfully in fifteen minutes. She did not include difficult questions in the homework; non routine and challenging questions were reserved for group discussion. She found that even though she was assigning less daily homework than she had in the past, students were in fact doing more work outside of class. This approach to homework, though successful, she limited to this class, since Carol felt her workload would have been too onerous otherwise.

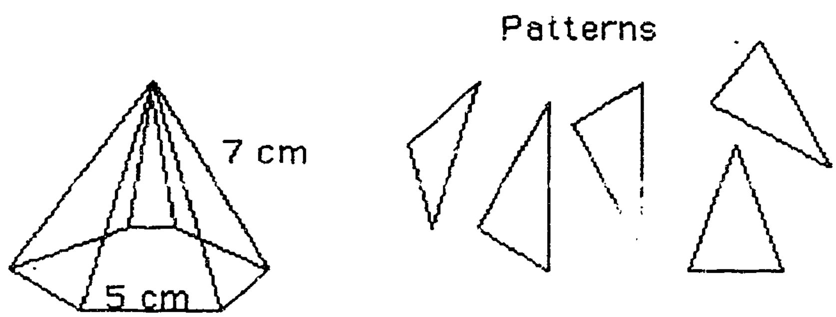
During the lessons observed, Carol followed the homework review with oral exercises. For Carol, these oral activities served a dual role. Not only did she design them to set the scene for the day's lesson but also to review key concepts and procedures. For instance, in the lesson she taught on algebraic solutions to systems of equations, the oral work dealt with solving linear equations for "x" or "y" as subject and simplifying algebraic expressions by removing parentheses. Both of these procedures were prerequisite skills for the lesson which followed. When interviewed about her impressions of oral work, Carol replied.

> It gets the students started more quickly and prepares them for what they are going to do for the day. It gets them thinking more about math. If I had started this from the beginning of the year I think I might have gotten more positive results. The students would not have wasted as much time getting started.

Within the development segment of the lessons, Carol incorporated real-world examples, hands-on materials, particularly in the geometry unit, and approached problem situations from alternate perspectives. As she developed the material she used a Socratic approach to engage the students in discussion.

Typical of her approach was a lesson on polyhedra. She brought a deflated soccer ball into class and had the students count the flat surfaces sewn together to form the three-dimensional object- specifically a truncated icosahedron. The class discussed several other polyhedra illustrated on posters as well. During the cooperative practice segment of this lesson, Carol had the groups use "Zaks"- interlocking geometric shapes- to construct regular tetrehedra, hexahedra, octahedra, and icosahedra. The homework assignment was to cut out combinations of the patterns below to draw a net and form the shell for Figure 2.

Figure 2 . Polyhedron and its net



The following represents her impressions on teaching for meaning and for student understanding.

> Teaching for understanding is easier than I thought it would be. However, I think some students make and see connections more readily than others - the high achievers.

She feels that a balance must be struck between the teacher striving for student understanding and having students master skills and procedures through drill and practice. Activities emphasizing meaning should be followed with practice exercises for skill development. Once the skills are in place the teacher should provide students with the opportunity to explore the material further to foster a deeper understanding of the concepts. Students, she believes, who are weak in skills may initially have difficulty in abstracting meaning from the lesson content when first presented, and that for these students meaning may be developed more fully once algorithmic proficiency is in place. Also, if students are to be motivated to make the effort to understand mathematics, exams should emphasize understanding as well as algorithmic expertise .

After the development segment of each lesson, the students moved their desks into the groups of four to which they were assigned. In all lessons observed, her students worked well together, discussing with others the questions assigned for practice. Seldom were students off task. Carol monitored the groups and assisted students when requested. Assigned work was regularly handed in for marking. At the conclusion of the study, Carol asked her students for their reactions. They affirmed that they liked working together. In particular, group work provided them an opportunity to discuss mathematics with their peers. Carol commented that the class as a whole developed a greater interest in mathematics. Student work habits improved and assignments were completed more regularly. As well, some of her weaker students showed marked improvement in achievement.

At the end of the study, during the second interview, Carol summarized her impressions of Interactive Teaching and the aspects she would continue using.

> I like the idea of the review at the beginning of the class since I only see my students every other day. I like using concrete materials and teaching mathematics with meaning. If anything else, students are more interested in hands-on materials. I will spend less time on taking up homework than I did in the past. I like cooperative practice; the students worked more efficiently, and liked mathematics more. In the past I did a lot of talking; I will restrict the time I spend on lesson development and give my students more practice time. I did enjoy using the model. Even though I spent a

long time using it, you really need a full year to get a better idea of its effectiveness.

Carol was troubled by the time constraints within the lesson format. Even though she felt the guidelines were reasonable, she considered them restrictive and limited her spontaneity, particularly in interacting with students. The non-routine problems she discussed with her students every other lesson seldom took only ten minutes to address. These problems, introduced during the cooperative practice phase, she felt detracted from the time the students had to discuss mathematics among themselves. In part, the difficulty Carol experienced with the time required to address the non-routine problems was a function of the problems chosen. Some problems were too large or to complex to limit to a fifteen-minute discussion.

D. Robert

During his participation in the study, Robert taught mathematics in the same senior- high school as Carol. Robert had taught for nineteen years, the last seven of which were at that school, teaching mathematics. Prior to his current assignment, he had taught industrial education for twelve years. He has his B. Ed. with specialization in industrial arts.

Robert has a reputation as an excellent mathematics teacher and has earned the respect of his colleagues and students. He is conscientious and is demanding of himself and his students. His lessons are consistently well prepared, and draw on his experience in industrial arts to make mathematics relevant, particularly to those students whose strengths lie outside mathematics. Students relate well to his style. His classes are disciplined and hard working.

Robert began using the Interactive Teaching Model with the introduction of systems of equations and ended three months later with the completion of the geometry unit.

Interactive Teaching was a significant departure from his regular classroom routine. Before he implemented the model, Robert would routinely spend most of each sixty-four minute period reviewing past work, taking up homework and presenting new material.

> It's very, very different. It's quite a departure for me. I tend to teach structured lessons spending a lot of time discussing material, giving notes, working through examples and taking up homework. I tend to spend a lot of time in developing concepts.

Robert prepared his Mathematics 23 students for the study prior to the first lesson. Each student received a photocopied sheet outlining the expectations for homework assignments, group work and evaluation. Homework was to be dated, referenced, and corrected in ink next to the students' written work. The marks students received for both units, systems of equations, and geometry, were a blend of the unit-test results, grades from selected homework questions , and in-class group assignments.

At the start of the first lesson, students were assigned to their support groups which remained the same throughout the study. Robert divided his class of twenty students into five groups of four. Besides using the students' achievement scores, groups were balanced by sex and personality. Students who were natural leaders were placed in different groups, and no group had two students with attendance problems.

Initially, Robert had reservations about the times recommended for the lesson segments. After having taught the second lesson, he recorded the following in his journal.

> In order to thoroughly cover the content, and have active participation from the class, fewer examples would have to be covered. Again, I had difficulty covering the content in 25 minutes. Should I be giving more of the content and asking fewer questions in the lesson?

However, by the fifth lesson, he wrote,

> I thought this lesson went very well. My timing was much better. The group practice seemed to go well with only one individual who seemed to hold back somewhat on group participation. Everyone seemed on task throughout the class and there were no interruptions or distractions.

Each class began with oral work that related directly to the day's lesson. Robert encouraged students to volunteer answers, and held individuals accountable through directed questioning. By way of illustration, the following dialogue was selected from a lesson dealing with surface area, volume and the application of geometric formulas.

Robert writes 2A + 2B + 2C on the board.

Robert: "Is it 6ABC ?"

Student A: "No, the terms are added, not multiplied."

Robert: "Is there any other way of writing the expression?"

Student B. "Two times A plus B plus C 2(A + B + C)"

Robert: "What did he do?"

Student C: "He factored"

Robert writes: If A = 7, B = 3 and C= 2, evaluate"

Student D: "24"

Robert: "How would we key this into our calculators?

Each of the preceding concepts was key to students successfully dealing with the material to be introduced that lesson.

During the first interview, at the completion of the first unit, Robert made the following comments about starting his lessons with oral work.

> I do like that quick start. I think at this point that's a beneficial thing. This particular class doesn't have a problem with arriving on time. They're there and they're ready to work. Oral work sets the tone for the class. The review component of it is good. It's a bit of work for the teacher, I think, especially finding questions relevant to the lesson. I don't have any test results to prove it's beneficial, but it's very logical to begin a lesson this way.

Homework was handled efficiently in a variety of ways. Solutions to the assigned work were regularly presented on the overhead or discussed from the chalkboard. Occasionally, students checked their work from the solution key in the textbook. Whenever possible student difficulties were dealt with at that time, Individual student problems, peculiar to one or two students, that could not be addressed quickly were resolved on a one-to-one basis during the cooperative practice phase of the lesson. Students were held accountable for correcting their written work. Homework assignments were collected for grading.

Robert, during the study, assigned less homework than he had previously, in part because students worked efficiently during the cooperative practice segment. Because he assigned less homework, he was more selective in the questions he assigned. Students were assigned more of the "typical" type to do on their own; he challenged students by assigning the more difficult questions during group work when students could collaborate on obtaining the solutions. As a result, homework was completed successfully by most students. The only concern he expressed about the "homework review" was the limited time he had for addressing student problems, since few students asked questions even when the opportunity arose later in the class period.

Initially, Robert spent considerable time in preparing lessons, in part because the lesson structure was unfamiliar but primarily because he wanted to to do the best job possible.

> Definitely. The content is no problem; I am so familiar with the content that I can teach it off the top of my head. It's a lot more work, there's no doubt about it.
>
> It's so structured and formal. My teaching background is in an informal setting.
>
> I now spend a long time in preparing examples, making sure that that each example works out just right. Possibly because of my concern for the job, I've been extra cautious.

Robert has a facility for teaching for meaning that reveals his philosophy towards teaching and how students learn. Almost without exception, the examples he chose for the lesson development related to the students' experience or prior knowledge. Also, he displayed a natural ability in using concrete materials to developing mathematical concepts. The following comments reflect his attitudes towards developing student understanding through making mathematics relevant.

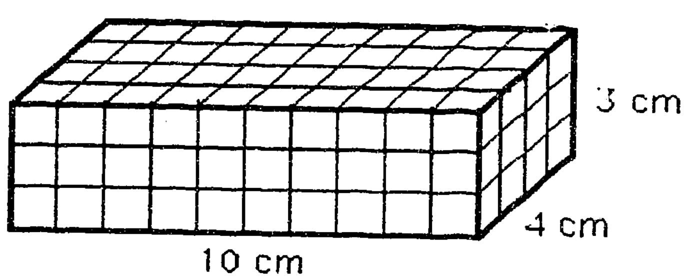
> I have always taught for meaning; I guess it's my personal philosophy. I've probably told every class I've ever taught that if I just wanted them to spit back the information, I might as well be teaching a bunch of robots. If they weren't asking 'why' then something's wrong. Maybe it's because my background is teaching in a technical area or because it's the way I learn. I am blessed with a real good memory, but I've found if I don't understand something it's been to my detriment.
>
> I would like to think that ultimately each and every student will acquire an understanding of the reason why something works. I think that understanding comes to different people in different ways. Some might have to develop the skill first, then meaning. I think the ideal is if we would understand the procedure first, and then acquire the skill to do it. But meaning may come to some of these kids at a later date.

His enthusiasm for teaching for mastery through understanding is tempered by the realization that, for some students, full understanding of mathematical concepts or procedures develops over time. A mature perspective is dependent on the student's mathematical background and skill level, and it may not be either appropriate or possible to teach for understanding in all instances.

The following vignette illustrates Robert's approach to meaning. The lesson in question dealt with the development of the formula for the volume of a prism: V = Area of the base x height.

Wooden blocks, all of the same size (10 cm x 4 cm x 3 cm), were distributed to each of the groups of four students. The groups were asked to draw grid lines on each of the six sides of the block, illustrating the surface area of each face. See Figure 3.

Figure 3. Surface area of a rectangular prism



The students were then asked for the volume of the block. All the groups obtained the same result, 120 cm^3. When asked how they knew, the majority of the class responded, " V = lwh = 10 x 4 x 3". When questioned why that formula worked, one of the students suggested that that there were three layers of 10 x 4 or 40 cm^3.

The fact that 10 x 4 or 40 was the area of the base of the block, led naturally to the formula,

V = Area of the base x height.

$V = 10 \text{ cm} \times 4 \text{ cm} \times 3 \text{ cm}$

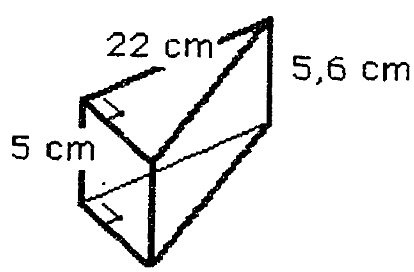
$= (10 \text{ cm} \times 4 \text{ cm}) \times 3 \text{ cm}$

$= 40 \text{ cm}^2 \times 3 \text{ cm}$

= Area of base x height

This formula was then applied to find the volume of a triangular prism, the doorstop that was wedged under the classroom door! The class then discussed how the formula may be applied whenever the base of the prism is not rectangular.

Figure 4. Triangular prism



$$\begin{aligned} V &= Bh \\ &= \frac{b \times a}{2} \times h \\ &= \frac{3.6 \times 22}{2} \times 5 \\ &= 198 \text{ cm}^3 \end{aligned}$$

Except when non-routine problems were discussed, after the lesson development was concluded and the students had an opportunity to resolve difficulties during guided practice, the last twenty-five minutes of each class were spent in cooperative practice. During this time period the students worked cooperatively in groups of four on exercises assigned from their texts. The composition of the groups remained the same throughout the two units. In each of the five classes I observed the students were involved in their work, the discussion was on topic and the students were supportive.

After the first month Robert made the follow observations about his students' and his impressions on group activity.

> They have accepted this arrangement amazingly well; they did not question it at all. I don't know whether it is blind faith or trust. I don't know whether it's personality or what, but for the most part the kids have been great; they seem to be giving it an honest effort.
>
> There is the benefit of different approaches to problem solving. There is a certain amount of reinforcement that the kids get from one another. They seem to be willing to help one another. They don't seem to see themselves as one of the skilled individuals or one of the unskilled individuals. They work together as a comfortable group. There hasn't been any resentment from the skilled individuals, that they are carrying the others; if there has I certainly haven't heard about it. They seem to get a fair bit done n that time period. I find it a problem if we are doing a word problem that day; I find the time period just too short.

By the end of the study, Robert's impressions of his students' reaction to both the cooperative learning activities, and the model in general, didn't change.

> I thought the students were super about it, much to my surprize. Students at this age are often reluctant participants, but they were positive, willing to go along with it. I heard only one negative comment from a student working together in the two months they worked on it. They seemed to enjoy Math 23 as much as Math 23 students can enjoy learning mathematics.

As reported in his journal, there were several occasions when the ten-minute process problems, which were to be discussed every second class, were dropped because Robert felt it would be preferable to extend the group work rather than curtail it to accommodate the problems. Commenting at the conclusion of the study, Robert felt that one of the difficulties with the model were the times allotted for the lesson segments, and that inevitable distractions would throw him off schedule, and to his regret, the one thing that was sacrificed was the problem-solving activity.

Robert's overall impression of the Interactive teaching Model was positive. He commented that his students were "much more active participants than in a conventional lecture". In particular, he felt that he would continue the ten -minute oral review and the strategies for handling homework, although he would not necessarily limit, as rigidly, the time allocated for addressing student difficulties on assignments.

He intends to use group activities regularly, but not necessarily on a daily basis because of content and curriculum constraints and the preparation time necessary. He attributed the success of the group work to the unique nature of the class. He considered his class was unusually receptive and positive.

Even though Robert considered the times recommended for the lesson segments reasonable, he considered them confining, He would prefer more flexibility in setting his own agenda for the class. Rather than spending ten-minutes on a regular basis for problem solving, he felt that the problems should be specific to the content being covered and integrated within the lesson development.

The experience of having planned several lessons, made lesson preparation easier, but it was still time consuming.

> Trying to make it a cooperative lesson, with lots of examples, applications, models, both in the lesson and in the practice takes time to do.

At the end of the study his concluding comments follow.

> Just the very fact that I went into this and said that I would do it, and because I had every intention of following through, caused me to look at some of the things I had been doing in teaching math. I am going to look at how I prepare lessons. I did benefit personally. It was a lot of work but I was gratified to see that the students gained from it. I learned a lot from it - a good return on the investment.

E. Joan

Joan teaches mathematics in a large metropolitan senior-high school with over two-thousand students from both moderate and high socioeconomic communities. At the time of the study, Joan was completing her twenty-ninth year of teaching, the last seventeen of which were in mathematics. She has taught English, music, science, and mathematics to both junior and senior-high-school students.

With an undergraduate degree in science and a graduate degree in education, Joan is well qualified to teach both average and academically-oriented classes. Joan is conservative in outlook, and she demands a high standard of performance from her students. Her classes are well-disciplined and strive to meet her expectations. She teaches in a large classroom, decorated with numerous posters and well supplied with additional textual and hands-on resources.

Joan's class that participated in the study consisted of twenty-five students typical of Mathematics 23: cooperative and polite, but having to be reminded about work habits and attendance. Before the start of each class, she stood outside the classroom door to greet her students as they entered, and to caution particular students on punctuality and attendance.

She delayed implementing the treatment because her students had taken longer than expected to complete the preceding unit on linear equations, material that is prerequisite to understanding systems of linear equations - the first unit in the study.

For Joan, the treatment was different from her regular classroom practice. Prior to the study, she had not used oral work, often took much more time in handling homework depending on the nature of student difficulties, did not follow a set lesson format, and used group work only occasionally and then primarily for student review.

Group work during cooperative practice proved the greatest hurdle for Joan. Joan divided her class into four groups of four and one group of five. The groups formed were heterogeneous by

1. Achievement: two average students, one high achiever and one low;
2. Sex; and,
3. Attendance factors.

The following entries from Joan's journal are representative of her observations of the slow progress her class made in cooperative activities, both in staying on task and actively contributing to the group. Joan made a concerted effort to hold students accountable for group work by grading assignments selected from one individual from each group. She intended to vary from whom she selected those assignments from lesson to lesson. However, she was frustrated by what she viewed as less than satisfactory attendance. During cooperative practice, Joan had wished to incorporate those elements of cooperative learning which foster cooperation and mutual assistance and formulate pro-social skills- ensuring that everyone participated and did not proceed until there was agreement and mutual understanding.

Week 2: Speed of effort in each group was better. Some extremely weak- so confused they remain isolates. Two of the groups work extremely well together, a third reasonably well. Some are too independent- know their work and want to get it done; some reticent- too shy and unwilling to reveal self. Feel they need teaching in group behaviour. There is enough to do, so some with weak attendance go into themselves or are ignored by others. I have had to encourage the good students to see that the weakest get going. I am finding that those that need a lot of information want me, and yet in some groups there is a definite free flow of information

Week 3: Reasonably good group work, Attendance still a factor in order to get daily continuity. I am concerned that some are not concerned about their group marks. Every lesson I try to take in assigned work from different group members. Attendance again is a consideration. Some are not even there to share their results when it is given back.

By the end of the first month of the study, she was more positive in her perception of cooperative activities. When interviewed about students' working together, she commented,

I can see if you give them the proper training and skills it will work, but you must guide them to work together. Students give each other quick feedback; some kids are very good at it, some aren't. I guess it depends on their personalities. Sometimes in a larger group a student with really weak skills is reluctant to speak out, because other will hear and judge, whereas in a small group that student is not scared to reveal himself. One girl has really opened up in her group.

Disadvantages? Sometimes a kid who is really out of it- lack of attendance or really weak skills- the group is under pressure to get work done and doesn't want to spend the time with that individual; so I have to spend the time with him. However, they have been using the time really effectively.

After the end of the second month, although her overall reaction to group work was positive, Joan remained somewhat skeptical about its effectiveness for all students. She wished to continue with group structures, but not on a daily basis. Joan intended to alternate them with exercises the students would practice independently, and would only use group activities for variety in those lessons which she felt lent themselves naturally to that strategy. In those instances she would insist on group accountability.

> The success of group work lies in those instances when you could see that if they were stumped, they would pool their resources. There are always individuals who don't respond to a group, and I think it's not for all students. If they are at all a bit shy, or intimidated, or because of a lack of knowledge, they don't want to expose themselves; a lot of teenagers are like that. Both the students and the teacher need a lot of guidelines as to how to use it effectively.

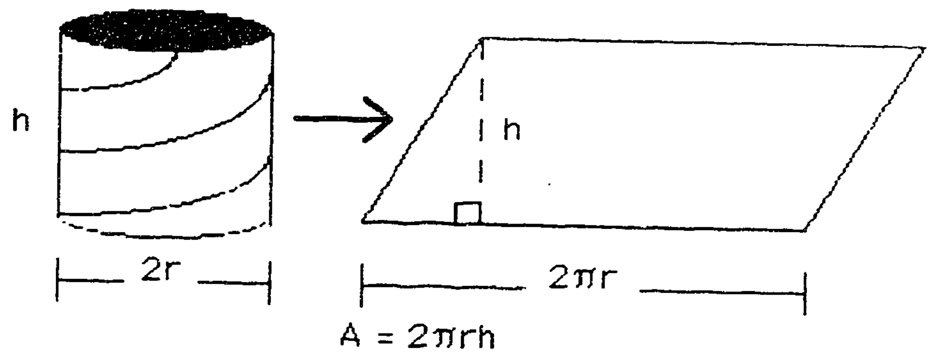
Throughout the study, Joan devoted more time in preparing lessons than was her habit.

> I definitely spend more time, because I want to make sure it fits the model the best that I can. Because of the limited time I have, sometimes I say to myself, if I weren't doing it for someone else, I would cut more corners. I always get a guilty feeling if I don't handle it the way it should be handled. I try to follow it closely all the time.

The oral work, which Joan introduced at the beginning of each of her lessons, served two purposes: a review of concepts from the previous day's lesson, and an introduction to the new material to be taught that day. Occasionally, she would try to introduce a motivating puzzle relevant to the unit under discussion, or a novel approach to an old problem. As much as practical, within the limited time, she involved the entire class in the discussion by distributing questions or calling for a group response.

During the last lesson I observed her teach, she reviewed, during the oral exercise, the formulas for the area of a cylinder and a parallelogram. She began by holding up the cardboard cylinder from a roll of paper toweling. The class discussed the formula for determining the area; measurements were made, and the area calculated. Joan then unwound the roll revealing that it had been, in the early stages of its manufacture, a parallelogram.

Figure 5. The surface area of a cylinder



Next the area of the roll was calculated using measurements of the base and height of the parallelogram.

This exercise served not only as a review of formulas previously introduced but also as a convincing argument illustrating an alternate geometrical representation of a cylindrical surface.

Joan viewed oral work as one of the strengths of the Interactive Teaching and intended to continue with this strategy after the conclusion of the study.

> I have been doing some thinking of how I can improve my oral work. I can see where you could review previous concepts, just backtrack anything you felt like or even introduce a little puzzle. The fact is it gets them just doing something immediately; it gets their attention and focused on math a little easier.

Immediately after the oral work, Joan reviewed the students' homework from the previous lesson. Initially, she displayed the worked solutions to the assignments on the overhead. Because of the physical arrangement of the room, she soon changed to writing the solutions on the side chalkboards before class. Only the solutions to one or two questions would be handled by whole-group discussion. These question she identified either by a show of hands or during the time the students were correcting their work and she was monitoring their progress.

Because Joan was concerned that some of her students were not completing their homework, she began to regularly take in assignments for grading. The students' marks on their assignments were included in their unit results.

During her second interview, she remarked the approach to homework she used in this class was effective but time consuming. She did consider it possible to handle homework this way in all seven of her classes.

Before her participation in the study, she allowed time for students to work on "homework" in class. Depending on how efficiently the students worked, many did not have any work to complete after class. After introducing the model, her assignments were shorter, but, in fact, her students were accomplishing more outside of class. The questions Joan, as did the other treatment teachers, assigned for homework were of a straight-forward type that students should have had little difficulty completing, since questions of a more exploratory nature were dealt with during cooperative practice.

During the development segment of the lesson, Joan taught in a whole-group setting. Joan used student questioning effectively to monitor understanding and to foster discussion. Whenever possible, she tried to incorporate real-world applications, mathematical models to represent everyday situations , hands-on materials, and problem situations. Because her students were accustomed to a more algorithmic and skills approach, and she found it difficult to come up with examples appropriate to students of varying abilities, this strategy met with occasional resistance, even after the students' first month on the program.

> The amount of time (for my students) to think through (the material presented) is phenomenal. We have to sacrifice some time on drill and practice- a confidence builder for some, even if it does not necessarily develop student understanding. I am not sure that some can or want to analyze, or that the emphasis on understanding gives them enough confidence to produce results. Some of the weak students just sit back. Analysis has not been a way of life for most, it would appear.

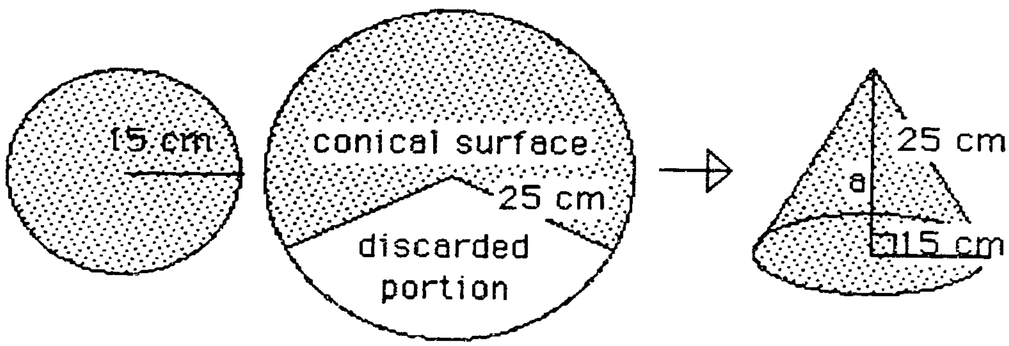
During our first interview Joan emphasized the importance of selecting appropriate examples that match the skill level of the class, and the difficulty she experienced in using them to develop mathematical understanding, particularly at the beginning of a unit.

> I am not really convinced that (real-world) applications should be given immediate attention (in the unit). They're fine if you can (use them to) illustrate a fact with some simplicity, You have to pick your spots for applications very carefully. The tendency (i. e., before the study) is to introduce them at the end of a unit. I am not totally convinced (of the appropriateness of applications) with this type of kid, especially with the limited time factor. I find it takes so much time in the development section of the lesson, that honestly, it has to be the focus for that day. I find that the applications are fairly sophisticated in this unit (systems of equations) that the students don't have sufficient general knowledge.

In spite of Joan's misgivings about applications, in each of her lesson's she made a concerted effort to emphasize meaning before developing skills though guided practice. In the geometry unit, even the formula for the surface area of a cone was developed and discussed, rather than simply presented and practiced. The lesson was presented as follows.

Joan suggested to the class that it was possible to construct a a cone using two circles cut from light card. The small circle, to be used to form the base of the cone, was 15 cm in radius; the larger circle, to be used to form the conical surface, was 25 cm in radius.

Figure 6. The surface of a cone.



Initially, the classroom discussion centred on how high the cone would be, and how the larger paper circle should be cut. To answer those questions Joan sketched the cone on the chalkboard as in Figure 6. The students suggested that the vertical height could be determined by applying the Pythagorean theorem to the right triangle formed by the slant height,s, the radius of the base, b, and the vertical height, a. The result, 20 cm, was calculated as follows.

$$a^2 + b^2 = s^2$$

$$a^2 + 15^2 = 25^2$$

$$a^2 + 225 = 625$$

$$a^2 = 400,$$

and $a = 20$.

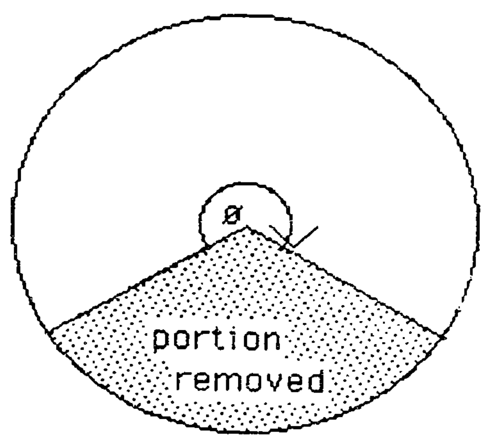
The class then determined the size of the sector which had to be cut from the larger circle in order to form the cone. Students volunteered that the size of that sector depended on the size of the smaller circle; the remaining arc of the larger circle had to complete the smaller circle's circumference. Next, Joan directed the discussion by asking the students to calculate the circumference of each circle.

Circumference of the smaller circle = $2\pi r = 2 \times 3.14 \times 15 = 94.2$ cm

Circumference of the larger circle = $2\pi R = 2 \times 3.14 \times 25 = 157$ cm

Once the students had calculated the circumference of each circle, Joan explained that the arc remaining on the larger circle, after the sector was cut out, must be 94.2 cm in length. The next task for the class was to calculate the central angle of that arc. Joan explained that if an arc length of 94.2 cm remained, because the circumference of large circle of was 157 cm, the measure of the central angle forming that arc length must be the same fraction of the degree measure of the circle, 360°.

Figure 7. The sector of a circle which determines a cone.



$$\frac{94.2 \text{ cm}}{157 \text{ cm}} = \frac{ø}{360°}$$, where, ø = the central angle

$$ø = 216°$$

The students determined that the sector that must be cut out is 360°-216° or 144°. Joan then measured, cut and removed the required portion. The parts of the cone were then taped together.

After the construction of the cone, the class discussed how the area of the conical surface is related to the original area of the larger circle. The conical surface must be related to the area of the large circle according to the same ratio formed by comparing the arc of the large circle which now forms the circumference of the cone to the original circumference of the large circle.

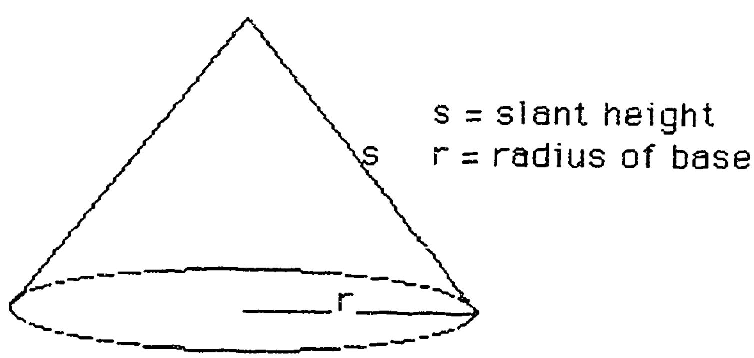
$$\frac{\text{Area of cone}}{\text{Area of large circle}} = \frac{\text{Circumference of cone}}{\text{Circumference of large circle}}$$

$$\frac{\text{Area of cone}}{\pi(25)^2} = \frac{94.2 \text{ cm}}{157 \text{ cm}}$$

$$\text{Area of cone} = 1177.5 \text{ cm}^2$$

Finally, Joan generalized from this example and developed the formula for the surface area of any cone. Students supplied the steps in the process.

Figure 8. The slant height and radius of a cone



$$\frac{\text{Area of cone}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

Area of the conical surface = πrs

The formula was verified by recalculating the surface area for the cone given in the example.

Area = πrs

$= 3.14 \times 15 \times 25$

$= 1177.5 \text{ cm}^2$

Joan spent ten minutes on process problems every other lesson. These problems were introduced between the lesson development and the group work work during cooperative practice, and in some instances at the beginning of the development. Occasionally, if she felt she would be pressed for time in a lesson, the problem solving component was postponed until the following class.

At the conclusion of the study, Joan expressed her reservations about handling problem solving within this framework. She also felt problem solving should relate to the content being taught and should be handled during the development itself.

> I find that really hard. I takes a long time to handle properly, It is not just a two or three minute thing, it takes ten or fifteen minutes. Some of the problems I used weren't the best either. The ideal would be to find the most appropriate problem, putting it in the context of the unit. But problem solving for a technique, some of students felt a bit dry and it wasn't always obvious to them why I was doing it. Consequently, there were days I taught problem solving as the introductory part of the lesson.

Joan found the times recommended for the lesson segments constraining.

> I find if the lesson goes well, you can pretty well stick to it. But I find if they have troubles with homework, and some days it would just show up - it wasn't that many days- it really upsets the timeline. When it came to the actual group work, well the development first of all... I find it takes a lot of time and energy to get what you want out of them, and to try to put ideas

> together. Not that it couldn't be done, but I find the times constraining, and you have to finish the course.
>
> When it came to the group work, and depending on the groupings, some could whip through it quite readily, but others didn't. Some group members, the 'non attenders' who couldn't participate that well, were a drag. Some students felt they just had to get it done without waiting for anybody.

In summary, Joan's perceptions of the Interactive Teaching were positive. She likes the general format, the variety of instructional strategies, the introductory activities - the oral work and the review of background material, and the emphasis on group work. However, she does not intend to use cooperative practice on a daily basis, but will restrict it to those lessons which lend themselves more readily to its use.

Joan felt that some of the strategies for dealing with homework were unrealistic - that taking in homework for grading limits the time she has for preparation. Finally, the recommended times for the lesson should be guidelines only, because individual student needs can not be addressed adequately if the teacher were to adhere to them rigidly.

F. Student Reactions

At the end of the study, three students from each class were selected, from those who volunteered, to be interviewed. The teachers identified a high achiever, an average student, and a low achiever, to obtain a wider range of opinion. The following summarizes the reactions to the treatment of the nine students chosen from Carol's, Robert's and Joan's classes.

The first question asked for the differences they observed in the way the lessons were taught during the treatment as opposed to what had been done in class prior to the study.

Table 1

Numbers of Students Identifying Differences in Treatment from Regular Classroom Routine

Difference	Carol's class	Robert's Class	Joan's Class
Lesson Organization and Presentation	3	3	3
Group work	3	3	3
Homework	2	0	0
Review	2	1	0

The following excerpts are representative of students' comments.

From Carol's class , a male student remarked,

> Homework was checked so the students are motivated to do it. I find it hard to do homework if it's not checked. It's nice to have a few minutes to work on math in class. Group work was different.

From the same class a female student observed,

> I felt I learned more because it was presented in more depth. More time was spent on previous work; the review is more extensive now. The teacher spent more time in answering questions students had. She didn't use group work in the past.

Two of Robert's students identified the following differences.

> Every class we review. We work in partners which makes it easier to learn and work out questions.

> I noticed more group work, and it's nice to see the class is on a schedule. We do his stuff till 10:00, then we work on our own stuff in groups.

One of the boy's from Joan's class remarked,

> They are more organized than anything else. There is a better plan of attack, the class works more efficiently. She did not use group work before.

When asked if those differences helped them learn mathematics, seven of the nine agreed. Those seven students identified group activities and increased teacher-student interaction as the primary reasons.

From Robert's class, a female student observed that group activities reduced the feeling of risk and encouraged her to participate in mathematical discussion.

> It helped me in a way to understand mathematics more. In my group I'm not afraid to answer questions. The whole class won't laugh at me, just the other members of the group.

From Carol's class students observed both an increase in student-student and student-teacher interaction.

> Yes, I came from Math 20. Here the teacher takes more time to explain mathematics. She assists us more when we need the help. Not as much time was taken on homework; so now there's more time on teaching.

> It helped me. I learn more when my friends show me, than when the teacher does sometimes.

The seven students who felt they learned more mathematics during the study, were all enthusiastic about group activities. They cited as reasons: reduced risk, self esteem, motivation to do mathematics, immediate feedback, the desire to assist others and exposure to different approaches to mathematics.

Two of Joan's students felt more comfortable in seeking help from their peers.

> I think group work is better. Different students have different ways of doing mathematics, so they can explain it to you. And I like to help other students sometimes. We do quite a bit of work in our group.

Carol's students identified the following benefits of group work: an increased interest in and appreciation of mathematics, immediate feedback, exposure to alternate approaches and an enhanced classroom climate.

> At first it's embarrassing. It was hard to get to know the other people, but then it got easier. Some of the students get better marks, so they can help you. It's easier to relate on a student-to-student basis.

> I always used to like to work on my own, but now it's nice to lean over and talk to others about mathematics without getting yelled at. And you don't have to run up to the teacher all the time; usually someone knows the answer.
>
> I find it an advantage. You have another person's approach to a problem; you have another person's knowledge. I felt really good working with other students.

> I enjoy it. It's fun. You can help other people when they're stuck. I like group work; it makes it easier for me to learn. It makes math more interesting. It's nice to have a mix of kids in the groups. I love math now.

A girl from Robert's class summarized it well.

> I feel more comfortable. I don't feel as inferior as I would in front of a teacher. I feel really dumb raising my hand all the time. I believe other students feel the same way. We are learning from each other, it's just easier. It feels wonderful to help them, because you know something. We do each question at the same time and help each other when we're stuck.

The two students who were not as positive identified lack of discussion and inadequate attendance of group members as detracting from the effectiveness.

From Joan's class a better student had misgivings about the extra work he felt he had to do because some group members were poor attenders.

> Group work isn't too bad, but you should be able to chose your groups. I'm stuck with people who aren't there all the time. I end up doing the brunt of the work. It should be a group of two or you shouldn't be just in your own group all the time. You need to interact with everyone in the class.

Seven of the students were convinced they were accomplishing more in class; however, two of Joan's students reacted negatively.

> The work itself is harder. We have been doing basically the same amount; it's just that it's harder so we have been doing more thinking.

> We are getting more thrown at us.

From Carol and Robert's classes the following reactions were typical.

> Yes, but I don't know why. Now I want to do my work. I feel more motivated to get things done.

Yes, I have been doing more in class and at home. The material is easier to understand. I enjoy math now.

When the students were asked whether they were completing their homework assignments, all the students indicated they were either completing their homework or doing more than they had done in the past. The principal reasons given were either because they wished to improve or maintain their grades or because they found the course content easier to understand.

> Since we started this, yes. I don't want to get busted. My marks started going down when we started, then I thought I'd better start doing it.

> Yes I have. Basically because I know what I'm doing now. It doesn't cause me any stress, so I get down to it right away.

Five of the nine students felt the material taught in the two units of the study was easier to understand than the content taught previously. These students gave the following reasons:

1. The greater depth of treatment of content,
2. Immediate feedback in group work,
3. Quality of teacher explanations,
4. Consistent review, and,
5. Real-world examples.

Disappointingly, in spite of the teachers' emphasis on applications, examples drawn from daily experience, and the use of concrete materials, students did not change their views on the immediate usefulness of mathematics. Consensus was that mathematics is relevant only in terms of possible future careers or in post-secondary education.

G. Summary

For all three teachers who fully implemented Interactive Teaching in their classes, the model represented a marked departure from their classroom routines. Typically, prior to implementation, lessons would begin with a discussion of difficulties arising from that part of the assigned work students had not completed in class. The time spent on this activity varied considerably, depending on the number and types of questions which arose. The teachers approached the homework discussion through a question-and-answer approach, first querying the students who raised the question, then seeking resolution by asking those students who successfully completed the assignment to supply a partial or complete solution. This approach was often unsatisfactory, since it did not emphasize student accountability and encouraged students to neglect homework as solutions to uncompleted questions were provided the following lesson. As well, the productive time remaining in the lesson to discuss new concepts was limited. It is a moot point if the majority benefited, as better students were bored by the process, and weaker students were occasionally reticent in admitting in front of the class that they could not cope with the material or did not understand the explanations.

During implementation, the treatment teachers restricted their discussion of the previous day's homework , as they began their lessons with oral work, a review of the previous lesson's skills and knowledge, and a homework check which were limited to a total of ten minutes. Strategies the teachers used in dealing with homework included displaying worked solutions on the overhead or on a side board and having the students revise their own work, having the students correct their own work from the textbook answer key, collecting assigned work, and homework quizzes comprised of questions selected at random from previous assignments and with the solutions transcribed directly by the students from their workbooks. Regardless of the approach the teacher used, the students were held accountable, with homework graded and included in unit grades. The treatment teachers reported that their students were now completing assignments, and that they preferred this approach in spite of the additional preparation, although they were not certain they would have the energy to continue it indefinitely and with all their classes; and, that it worked, in part, because they restricted assignments to what they felt students could successfully complete in fifteen minutes, and to questions of a routine nature. More difficult questions were addressed during cooperative practice.

The oral work during the introductory segment of the lesson evolved during the course of the study. The questions the treatment teachers asked their students became a combination of those which reviewed prerequisite concepts and lead naturally to an introduction to the day's lesson. The teachers found beginning the lesson with oral work quickly focused the students' attention on mathematics, and by distributing the questions throughout the room enhanced the active participation of the class.

The lesson development was conducted in a whole class setting. After the lesson's objective was stated, the teacher lead discussion of the new content through active questioning and skill development was monitored and reinforced through controlled practice. The treatment teachers interpreted teaching for meaning using the following strategies:

1. Incorporating real-world examples and examples derived from the students' every-day experiences
2. Approaching problem situations from alternate perspectives;
3. Using concrete materials; and,
4. Mathematical proof.

This segment of the lesson took the most preparation, since the examples the treatment teachers designed were not typical of those that could be thought of extemporaneously. It was the time guideline of twenty-five minutes for this segment that the teachers found most confining. Non-routine or non-algorithmic examples, by their nature, are time consuming. Furthermore, in their effort to foster student understanding, they would have preferred that the lesson development was longer. Also, some topics in senior-high mathematics simply require more time to address adequately.

Immediately after the lesson development, the students moved into groups of four for the cooperative practice phase of the lesson. The treatment teachers kept the same groups throughout the study, except in the cases where new students were added to the classes. The criteria the teachers used in assigning students to groups follow.

1. Achievement. Each group consisted of one high achiever, one low achiever, and two "average" students.

2. Sex. The groups were balanced between male and female students.

3. Attendance. Attempts were [illegible] that no group contained more than one poor at[illegible]

4. [illegible]

All three treatment teachers remarked that implementing cooperative practice was a positive experience both for themselves and their students. Students readily took to group activity; they actively participated in group discussion, exchanged approaches, offered assistance to their peers, and accomplished more in this setting than when working individually.

In particular, students identified student-to-student interaction as motivating, reducing risk, enhancing self esteem, providing immediate feedback, providing the opportunity to positively interact with others, and to view alternate perspectives.

The success of group work, in part, was due to the classroom management of the teachers. They continually monitored the groups insisting students stay on task, collected and graded work, and offered assistance and encouragement when required.

In spite of the apparent success of cooperative practice, the treatment teachers felt that at the conclusion of the study, they would prefer alternating group and individual activities. Their reservations arose primarily because of the preparation time required to structure suitable group activity, the effect of absenteeism of several students on the continuity of the groups, the feeling that individual effort must also be encouraged and rewarded, the lack of time if the development segment was lengthy, and the fact that it was difficult to set aside traditional classroom practice.

The non-routine problem solving was the least successfully implemented feature of the model. The original intent was to introduce during every other lesson, a novel problem , preferably related to the content, to be solved within ten minutes interactively through teacher-lead discussion immediately prior to the cooperative segment. Teachers either spent more time than suggested on the problem, or simply omitted it altogether feeling that more would be gained by having students practice the skills and concepts taught during the lesson. Also, because of its position within the lesson, it interfered with group work. Other considerations were the sense of artificiality the teachers experienced in introducing a problem after the lesson was taught, and the difficulty in selecting a problem relevant to the lesson that could be satisfactorily discused in ten minutes.

Based on teacher comments, it may be better to imbed problem-solving situations within the development segment, introducing problems when appropriate, and those which arise directly from the lesson content. In this situation, teacher-lead discussion can focus on process, applicable strategies, and alternate points of view.

H. Modified Model

As a result of the teachers' and students' critiques, comments and experiences outlined in this chapter, a modified lesson format is now proposed.

PRINCIPLES OF INTERACTIVE TEACHING

DAILY REVIEW AND HOMEWORK CHECK - 10 minutes

* Begin with oral work that reviews and reinforces previous lesson's skills and knowledge and sets the scene for the day's lesson
* Deal with homework

DEVELOPMENT - 25 minutes

* Place concept to be taught in context of past knowledge and future problems
* Emphasize meaning
* Introduce novel problems, when appropriate, to underscore process.
* Monitor student understanding through active questioning
* Reinforce concept through controlled practice

COOPERATIVE PRACTICE/INDIVIDUAL PRACTICE- 25 minutes

* Provide opportunities in alternate lessons for group and individual practice.
* Provide opportunity for successful practice
* Include word problems and applications related to the lesson.
* Assign more difficult questions in this segment rather than for homework to take advantage of peer and teacher support.
* Encourage active group discussion
* Keep individuals accountable

HOMEWORK

* Assign homework relevant to lesson
* Ensure that the questions assigned can be completed successfully by the majority of students working independently
* Include a review question.

Begin each lesson with the ten-minute Daily Review and Homework Check to focus students' attention immediately on mathematics. Use oral work to

1. Engage the majority of students in dialogue,
2. Review the previous lesson,
3. Reinforce prerequisite skills, and

4. Introduce the lesson topic.

Limit the time spent in addressing problems from the previous lesson's homework assignment to maintain student interest and lesson momentum. Vary strategies for taking up assignments. Restrict discussion to problems experienced by the majority of the class. Keep students accountable for assigned work.

During the lesson development, emphasize meaning through the use of real-world examples, alternate perspectives, concrete materials, and mathematical proof. Problem solving strategies and process problems should be integrated within the lesson content, and should be related to the lesson's theme.

Because the lesson content is developed through whole-group discussion, students should be kept accountable through timely questioning. During the controlled practice phase, whole group discussion and individual practice should alternate to optimize individual effort and to enhance the effectiveness of teacher-student interaction.

The last twenty-five minutes of the lesson is devoted to developing skills and understanding through either individual or cooperative practice. As much as practical, the teacher should provide opportunities [illegible]perative practice every other lesson. Certain topics lend themseves to group work.

Groups of four should be maintained for at least the duration of the unit under discussion, to promote positive group dynamics and foster productive student-student interaction. Encourage active participation by all all group members; hold both groups and individuals accountable for work assigned. As well, when assigning students to their groups, keep the groups heterogeneous by ability, personality, sex, and ethnic or racial background.

More difficult questions should be assigned during the cooperative practice segment, allowing students to assist each other in resolving problems or to seek help from the teacher. Homework assignments should be kept short and the questions assigned simple, to reduce student frustration when neither teacher nor peer help is available.

The times for each lesson segment are recommendations only and the teacher should remain flexible, depending on student or lesson needs.

Chapter VI - Quantitative Results

A. Overview

The quantitative data for the study were obtained over a sixteen week period during the second semester of the school year. The results presented in this chapter are based on classroom observations, pre- and post- testing, and attitude questionnaires from eight Mathematics 23 classes- four in the treatment group and four in the control group. These classes were taught by four teachers who volunteered to use the Interactive Teaching Model in their classes, and by three teachers who continued to use their classroom routines. Each of the teachers, as mentioned previously, taught mathematics in large urban senior-high schools serving largely middle-class nei[illegible]oods. All were experienced teachers with at least one undergraduate degree.

Three research questions will be addressed in this chapter:

1. To what extent did the treatment teachers implement the Interactive Teaching Model, and were they doing something different from the control teachers?
2. Did the treatment classes achieve higher on the post-tests than the control classes?
3. Did the Interactive Teaching Model affect students' attitude to learning mathematics?

B. Classroom Observation

Before the study, each of the seven participating teachers was observed twice while they were teaching their Mathematics 23 classes. The purpose of those observations was to verify that neither the four treatment teachers nor the three control teachers were using the elements of the Interactive Teaching Model in their classroom instruction prior to the implementation of the study.

In the implementation phase of the study, each of the treatment classes was observed five times, during weeks one, two, and four of the first unit - systems of equations - and during weeks two and four of the second unit - geometry. Each of the control teachers was observed a further two times, once teaching systems of equations and once teaching geometry. Because not all the teachers had covered the introductory material for the unit on systems of equations by the end of the first semester, in order to adhere to the above schedule, the classroom observation phase of the study was extended from mid February through mid May.

As well as taking notes on the classroom instruction, during each of the classroom observations, also recorded were the times the classes spent on oral work, review, homework, lesson development, and cooperative practice, and, at the end of each lesson, the length of time, based on teacher estimates, students would be required to complete homework assigned.

Each of the lesson segments was scored on a four-point implementation scale, according to the extent each matched the expectations of the Interactive Teaching Model.

Implementation Scale

0 Not attempted

1 Some implementation

2 Implementation Incomplete

3 Fully implemented

At the end of each lesson, the extent to which the teacher had attempted to develop student understanding through combinations of worked examples, real-world applications, concrete materials, and alternate interpretations was scored on a meaning scale.

Meaning Scale

0 Not attempted

1 Some attempt

2 Attempted but not complete

3 Completed effectively

Tables 2 and 3 summarize the data obtained from the two preliminary observations of the treatment teachers. None of the treatment teachers used either oral work or cooperative practice in their classrooms. Both in terms of the times spent on lesson segments and the results from the implementation scales, the Interactive Teaching Model represented a clear departure from their traditional classroom practice.

For all but Teacher D, the total time spent on review and dealing with homework exceeded the ten-minute guideline in the Interactive Teaching Model. Teacher A averaged 29 minutes; Teacher B, 12 minutes; and Teacher C, 14.5.

On the Implementation Scales for Review, Homework, and Lesson Development, before implementing the Interactive Teaching Model in their classrooms, all the treatment teachers received average ratings between "Not attempted" and "Attempted but not complete". This was also true for the Meaning Scale, except for Teacher A, who structured his lessons around the development of student understanding and meaning activities.

Table 2

Time (in Minutes) Spent on Each Lesson Segment by Treatment Teachers before Implementation of Model - Two Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Homework Assigned
A						
X	0	12.5	16.5	20.0	0	25.0
Rge	0	25.0	13.0	10.0	0	35.0
B						
X	0	12.0	0	29.5	0	30.0
Rge	0	20.0	0	1.0	0	30.0
C						
X	0	9.0	5.5	22.0	0	0
Rge	0	18.0	11.0	10.0	0	0
D						
X X	0	12.0	0	20.0	0	7.5
Rge	0	5.0	0	40.0	0	15.0

Table 3

Implementation Scales for Each Lesson Segment for Treatment Teachers before Implementation of Model - Two Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Meaning
A						
X	0	0	1	1.5	0	2.5
Rge	0	0	0	1	0	1
B						
X	0	1	0	1	0	1
Rge	0	2	0	0	0	0
C						
X	0	0.5	0.5	1	0	1
Rge	0	1	1	0	0	0
D						
X	0	0.5	0	0.5	0	0.5
Rge	0	1	0	1	0	1

Tables 4 and 5 illustrate the classroom practice of the three control teachers based on all four classroom visits to each teacher. Even though the control teachers did not use the Interactive Teaching Model in their classrooms, each teacher was competent, experienced, respected, and well qualified to teach mathematics.

The control teachers varied from day-to-day in their classroom activities; however, most classes they taught began with a discussion of previously assigned work, followed by an algorithmic approach to the lesson development, and ended with time for in-class independent practice. Those students who did not finish their assignments within the class were expected to complete them as homework.

None of the control teachers used oral work or cooperative practice in their teaching. On the Implementation Scales for Review, Homework, Development, and Meaning the control teachers were rated between "Not attempted" and "Attempted but not complete".

Table 4

Time (in Minutes) Spent on Each Lesson Segment by Control Teachers - Four Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Homework Assigned
E						
X	0	0.25	8	20.75	0	10
Rge	0	1.00	7	7.00	0	30
F						
X	0	17.5	0	47.50	0	20
Rge	0	9.0	0	22.00	0	30
G						
X	0	9.0	5	23.25	0	7.5
Rge	0	18.0	10	27.00	0	30.0

Table 5

Implementation Scale for Each Lesson Segment for Control Teachers - Four Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Meaning
E						
X	0	0.5	1.5	1.25	0	1.5
Rge	0	2	1	2	0	1
F						
X	0	0.5	0	0.75	0	1
Rge	0	1	0	1	0	1
G						
X	0	0.5	1.25	1.0	0	1
Rge	0	1	2	0	0	0

Tables 6 and 7 illustrate the classroom practice of the four treatment teachers during the implementation phase of the study. Three of the four treatment teachers fully implemented the Interactive Teaching Model.

Table 6

Times (in Minutes) Spent on Each Lesson Segment by Treatment Teachers - Five Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Homework Assigned
A						
X	6.8	3.6	2.8	23.6	18.6	16
Rge	7	10	5	15	11	5
B						
X	4.4	2.6	3.6	28	23	15
Rge	7	5	7	10	4	30
C						
X	5.2	2.8	4	22	27.7	16
Rge	7	6	5	17	11	10
D						
X	6.8	3.6	1.8	17.8	22.6	11
Rge	16	10	6	30	22	20

Table 7

Implementation Scale for Each Lesson Segment for Treatment Teachers - Five Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Meaning
A*						
X	2.6	2.2	2.4	2.8	2.4	3.0
Rge	1	3	3	1	1	0
B*						
X	1.8	1.8	2.4	2.6	2.4	2.0
Rge	3	3	3	1	1	0
C*						
X	2.4	2.4	2.0	2.8	2.4	2.4
Rge	2	3	0	1	1	0
D						
X	1.4	1.0	0.4	0.8	2.0	0.8
Rge	2	2	1	2	2	1

Note: * represents the treatment teachers considered implementers.

All four treatment teachers structured their lessons closely to the recommended times for the Daily Review and Homework Check (10 minutes), Development (25 minutes) and Cooperative Practice (25 minutes). However, on the Implementation Scales for on Homework, Development, Meaning, and Cooperative Practice, only Teachers A, B and C were rated between "Attempted but not complete" and "Completed effectively". Teacher D on all but the Cooperative Practice Implementation Scale lay between "Not attempted" and "Attempted but not complete". Therefore only Teachers A, B, and C are considered as having implemented the Interactive Teaching Model.

Table 8 demonstrates the differences among the treatment teachers, implementers, and control teachers based on the Implementation Scales.

Table 8

Comparison between Treatment and Control Teachers on Implementation Scales

Group	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Meaning
Treatment (4)						
X	2.05	1.85	1.80	2.10	2.35	2.05
S. D.	0.95	1.23	1.24	0.97	0.59	0.89
Implementers (3)						
X	2.27	2.13	2.27	2.53	2.47	2.47
S. D.	0.88	1.25	1.03	0.56	0.51	0.51
Control (3)						
X	0.00	0.50	0.92	1.00	0.00	1.17
S. D.	0.00	0.67	0.90	0.60	0.00	0.12

Teachers A, B, and C implemented the Interactive Teaching Model. Prior to using the treatment in their classes, on the Implementation Scales for Oral Work, Review, Homework, Development, Meaning, and Cooperative Practice, those teachers' ratings ranged between "Not attempted" and "Attempted but not complete". After implementing the treatment, Teachers A, B, and C were rated on the same scales between "Attempted but not complete" and "Completed effectively". The implementers had not used cooperative practice or oral work as part of their daily routine prior to using the model.

The control teachers did not change their classroom practice during the study. None used oral work or cooperative practice as defined in the model. On the Implementation Scales for Review, Homework, Development, and Meaning, their classroom practice was rated, on average, between "Not attempted" and "Attempted but not complete"

C. Test Results

This section addresses the question of achievement differences between students in the treatment and control groups.

At the beginning of the study, a thirty-item multiple-choice survey test was administered to the four treatment and the four control classes. The intent of this pre test was to determine the equivalence, if any, of the control and treatment groups. The items on the pretest were based on the Mathematics 23 course of studies (Alberta Education, 1990), and reflected the material the students were taught prior to systems of equations and geometry - the two units taught using the Interactive Teaching Model.

At the end of each of those units, thirty-item multiple-choice post tests were administered to both groups. Again all items were based on the Mathematics 23 course of studies (Alberta Education 1990).

To ensure that students who withdrew from or were added to the teachers' classes did not bias the results, only students who wrote all three exams were included in the results.

Tables 9 and 10 compare the performance of the treatment group, the group of students whose teachers (Teachers A, B, and C) implemented the model, and the control group on three tests: a survey test, the test on systems of equations and the geometry test. To balance numbers of students between the two groups, two classes of control Teacher F were tested.

Table 9

Performance of Treatment and Control Classes on Pre and Post Tests

Class	N	Survey X	Survey S. D.	Systems X	Systems S. D.	Geometry X	Geometry S. D.
Treatment							
Teacher A	18	13.44	5.03	16.17	3.76	19.67	4.56
Teacher B	20	12.20	4.02	14.45	2.56	15.95	4.86
Teacher C	24	10.54	3.40	11.33	2.20	11.96	3.22
Teacher D	22	11.05	3.81	11.14	3.81	14.18	3.71
Control							
Teacher E	23	14.22	4.36	13.48	4.10	14.70	4.19
Teacher F	16	10.94	4.84	11.25	3.61	13.63	4.61
Teacher F	11	11.91	5.21	9.55	3.21	16.00	6.26
Teacher G	16	9.19	4.15	10.25	2.96	11.25	3.00

Note: Teacher F had two classes

Table 10

Performance of Treatment and Control Groups on Pre and Post Tests

Group	N	Survey X	Survey S. D.	Systems X	Systems S. D.	Geometry X	Geometry S. D.
Treatment	84	11.69	4.12	13.06	3.42	15.14	4.88
Implementers	62	11.92	4.23	13.74	3.46	15.48	5.21
Control	66	11.82	4.89	11.50	3.84	13.82	4.65

Note: Implementers = Teachers A, B, and C.

Unpaired t-tests were employed to determine the significance of the differences between the means of the implementer and control groups on all three tests. Table 11 supports the null hypothesis that, on the basis of the pretest scores, there was no difference between the mean performance of those two groups of students.

Table 11

A Comparison of Survey (Pre-test) Scores for Implementer and Control Groups

Group	N	Mean	S. D.	t	Prob (2 tail)
Implementer	62	11.92	4.23	0.125	0.9009
Control	66	11.82	4.89		

The students in the classes of the three teachers who implemented the treatment outperformed the control students on both post-tests, as shown in Tables 12 and 13. On the systems exam, the difference between the means is significant beyond the 0.01 level. However, for the geometry exam, based on a two-tailed t-test of significance, the null hypothesis can not be rejected at the 0.05 level. The probability that the differences between the two means is due to chance is 5.86%. If the reader accepts, on the basis of the first post-test results, that the treatment does not adversely affect student performance, then a one-tailed t-test of significance would be justified. In this instance the difference between the means of the two groups would be significant at the 0.05 level ($p < 0.0253$).

Table 12

A Comparison of Systems (Post-test) Scores for Implementer and Control Groups

Group	N	Mean	S. D.	t	Prob (2 tail)
Implementer	62	13.74	3.46	3.463	0.0007
Control	66	11.50	3.84		

Table 13

A Comparison of Geometry (Post-test) Scores for Implementer and Control Groups

Group	N	Mean	S. D.	t	Prob (2 tail)
Implementer	62	15.48	5.22	1.909	0.0586
Control	66	13.82	4.64		

In summary, the Implementers' students outperformed the Controls' students on both systems of equations and geometry. Two control teachers, representing three classes of students, spent at least two weeks longer in teaching the geometry unit than did the treatment teachers. It is possible that the performance of their students would not have been as high had the length of time they had spent in teaching that unit been the same as for the treatment teachers. If this study were to be repeated, the time allocations for both groups should be the same.

D. Attitude Questionnaire

This section addresses the issue of attitude differences to mathematics, if any,between students from the control and treatment groups.

At the end of the study, a twenty-five item Likert-style student questionnaire was administered to all students in both the treatment and control classes. Each item consisted of a statement followed by four possible levels of agreement. Each item was scored from 1 to 4 on the following basis:

Never = 1, Seldom = 2, Often = 3, and Always = 4.

A factor analysis on the total set of data was carried out; the orthogonal transformation solution appears in the appendix. Eight factors were obtained from combinations of those items on the questionnaire which were most closely related. Table 14 outlines those factors, the items which comprise them, and, from those items, the themes which were inferred.

Table 14

Factors Obtained from Attitude Questionnaire

Factor	Description	Questionnaire Items				
1	Interest in mathematics	2	5	13	14	20
2	Cooperation	6	7	8		
3	Teacher Dependence	11	12			
4	Homework and Relevance of Mathematics	1	23			
5	Task Orientation	15	22	24	25	
6	Teacher Assistance	9	10			
7	Independence	18	19			
8	Perception towards Problem Solving and Doing Mathematics	3	4	16	17	21

A two-tailed t-test was performed on each factor to identify those factors for which the means between the implementer and control groups differed significantly at the 0.05 level. For each of the eight factors there was no significant difference between the two groups. That there were no differences between the two groups might be attributed to the students answering the questionnaire accordingto whether they liked the subject or teacher rather than the method of instruction.

Figure 9 and Table 15 summarize the results of the questionnaire.

Figure 9 . Student attitude results

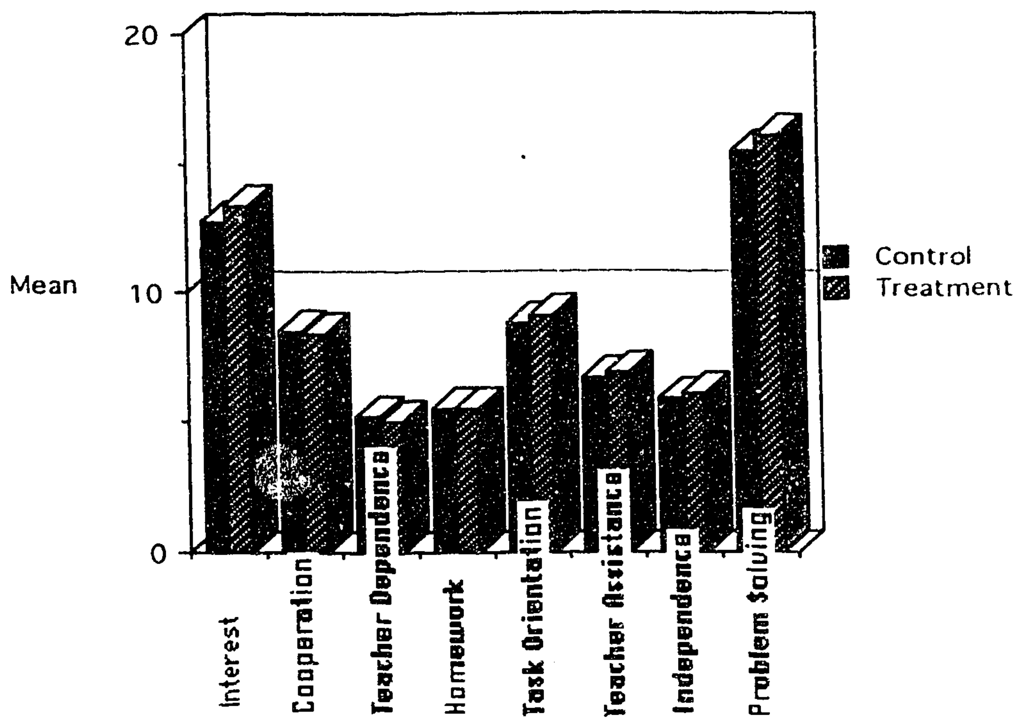


Table 15

Mean Student Attitude Scores for Implementer and Control Groups

Factor	Implementer (N = 60)	Control (N = 69)	Total Possible Score
1. Interest in Mathematics	13.40	12.80	20
2. Cooperation	8.43	8.49	12
3. Teacher Dependence	4.98	5.20	8
4. Homework and Relevance of Math	5.48	5.52	8
5. Task Orientation	9.17	8.88	16
6. Teacher Assistance	6.90	6.75	8
7. Independence	6.11	5.93	8
8. Problem Solving and Doing Mathematic	16.03	15.49	20

The implementers' students responded more positively towards being on task, being assisted by the teacher, working independently, seeing mathematics and problem solving as important, and showed a higher interest in mathematics. The control teachers' students, on the other hand, indicated a higher importance on working cooperatively with other students, dependence on the teacher, and the importance of homework. These differences, however, can not be held as significant, and we can only speculate at best whether these differences can be ascribed to the treatment.

E. Summary

Based on the analysis in this chapter, three of the four treatment teachers teachers implemented the Interactive Teaching Model. Their implementation was a significant departure from their regular classroom routine and differed as well from the control teachers in the areas of cooperative practice, oral work, homework, review, development and teaching for meaning.

On the post-tests, the treatment favoured the implementation groups for both units- systems of equations and geometry . The difference between the implementer group and the control group was significant at the 0.05 level.

On the attitude scale there was no significant difference between the two groups at the 0.05 level. However, the means for the following factors - Interest in Mathematics, Task Orientation, Teacher Assistance, Independence, and Problem Solving were higher for the implementer group. The control teachers students were more positive towards Homework, Teacher Dependence, and Cooperation among students.

Chapter VII - Conclusions

A. Review of the Problem

The purpose of the study was to determine the appropriateness of an instructional model designed for effective senior-high mathematics instruction. The study involved four phases.

1. The development of the model;
2. The model's implementation and interpretation by classroom teachers;
3. The testing for achievement gains; and,
4. The analysis of student attitudes towards mathematics..

This model is an adaptation of the Missouri Mathematics Program described by Good, Grouws, and Ebmeier in Active Mathematics Teaching (1983). The Interactive Teaching Model also incorporates cooperative-learning activities in the monitored seatwork segment of the lesson.

The study was conducted over a four and one-half month period in five large senior-high schools, serving middle class neighbourhoods located in a large metropolitan area in Western Canada. Four experienced senior-high mathematics teachers participated in the treatment group and three teachers in the control group. Two units of study, systems of equations and geometry, based on the provincial course of studies, were taught to a total sample of eight Mathematics 23 classes, involving over one hundred fifty students. Four classes participated in the treatment group, and four in the control group. Mathematics 23 is a grade-eleven mathematics program of studies designed for students of average mathematical ability.

Data were collected through classroom observation, teacher and student interviews, teacher journals, pre- and post-testing, and student questionnaires.

B. The Interactive Teaching Model

The Model, designed for lessons taught during a sixty-four minute period involved a ten-minute daily review and homework check, a twenty-five minute lesson development, and twenty-five minutes of group work (cooperative practice) during the monitored practice phase.

The lesson development, taught in a whole-group setting, emphasized developing student understanding through active questioning, and the use of concrete materials, real-world examples, and mathematical models. Controlled practice reinforced the skills and concepts taught.

During the cooperative practice segment, students worked in groups of four on practice exercises.

Every second lesson, non-routine process problems were introduced, to be discussed either in whole group setting or during cooperative practice.

C. Principles of Interactive Teaching

The Interactive Teaching Model represented a marked departure from the regular classroom routines of all four treatment teachers. Prior to implementing the Model, the treatment teachers' classroom practice varied from lesson to lesson. The general format that the treatment teachers followed and control teachers continued to use throughout the study, was to deal at the beginning of the lesson, in a whole-group setting, with individual student concerns arising from the previous lesson's assignment. The length of time spent in homework review depended on the number of student questions which arose. The lesson development was algorithmic in focus, with notes given and examples discussed to ensure satisfactory skill development. During the last part of each lesson, students were given exercises to practice. Questions the students did not complete were expected to be done as homework, and were discussed at the beginning of the next class. None of the seven teachers used cooperative-learning structures on a regular basis.

After the inservice for the treatment teachers, three of the four treatment teachers implemented both the lesson format and the spirit of the Interactive Teaching Model. In particular, they taught for meaning- using throughout their presentations mathematical models, hands-on materials, real-world examples, and mathematical explanations to illustrate concepts taught, and an active style of instruction in an attempt to engage all their students in whole-class discussion. Based on the implementing teachers' interpretation of the model and comments on its appropriateness in their classroom situations, an amended model, titled Principles of Interactive follow. A detailed discussion appears in Chapter 5.

PRINCIPLES OF INTERACTIVE TEACHING

DAILY REVIEW AND HOMEWORK CHECK - 10 minutes
* Begin with oral work that reviews and reinforces previous lesson's skills and knowledge and sets the scene for the day's lesson
* Deal with homework

DEVELOPMENT - 25 minutes
* Place concept to be taught in context of past knowledge and future problems
* Emphasize meaning
* Introduce novel problems, when appropriate, to underscore process
* Monitor student understanding through active questioning
* Reinforce concept through controlled practice

COOPERATIVE PRACTICE/INDIVIDUAL PRACTICE- 25 minutes

* Provide opportunities in alternate lessons for group and individual practice.
* Provide opportunity for successful practice
* Include word problems and applications related to the lesson.
* Assign more difficult questions in this segment rather than for homework to take advantage of peer and teacher support.
* Encourage active group discussion
* Keep individuals accountable

HOMEWORK

* Assign homework relevant to lesson
* Ensure that the questions assigned can be completed successfully by the majority of students working independently
* Include a review question.

D. Summary of Teacher Reactions.

Homework Review

The teachers' began their lessons by quickly dealing with the previous day's homework using a variety of techniques:

1. By writing worked solutions to the assignments on the side boards before the class started or displaying solutions on the overhead. The students would revise their work immediately on coming into the classroom. During this time, the teachers would address questions of a general nature and monitor students' completion of the assignments.

2. By having students copy and hand in for grading selected homework questions directly from their notebooks.

3. By collecting and grading individual assignments. The teachers carefully selected the questions assigned for homework, limiting the number and difficulty of the problems to those that could be successfully practised in fifteen minutes, assigning more difficult questions during cooperative practice when students had ready access to peer and teacher assistance.

All the teachers believed these practices were efficient and effective allowing more time for lesson development; however, they felt those practices were time consuming to prepare and prohibitive if used in all their classes. Because the students were held accountable for completing their homework, the teachers observed that more students were completing their assignments than they had in prior to the study.

Oral work

Teachers tied the oral work at the beginning of the class into the review, interpreting oral work as an opportunity to review prerequisite concepts and to set the scene for the upcoming lesson. All the teachers, felt oral exercises provided a vehicle for engaging the class quickly, getting them to think about mathematics, and providing an effective transition to the development segment of the lesson. All intended to incorporate this element of the Model into their daily practice.

Development

Initially, the lesson development presented the greatest hurdle. Even though the treatment teachers believed that developing student understanding is the essence of teaching, they found time consuming the detailed preparation necessary to design appropriate lessons.

The success of this approach depends on the philosophy of individual teachers, their beliefs about how students learn, and how they perceive the balance between teaching for meaning and teaching for skills. They also felt that student skills are what is valued by the "system" - diploma exams, for example, still demand student acquisition of factual knowledge.

Difficulties with this approach might be overcome through joint planning among teachers, or the provision of appropriate textual and teacher support materials.

Cooperative practice

Cooperative practice was handled well. The students were assigned to heterogeneous groups of four on the basis of sex, achievement- one high achiever, one low, and two average students- personality and attendance. Groups remained the same during the study, except in those instances when new students arrived or when students withdrew from school.

Throughout the study, the teachers were pleased with the level of student participation, effort, and cooperation. The teachers observed that the students enjoyed the opportunity to work together, to discuss, encounter alternate perspectives, and to share expertise. Few students were observed to be off task during this segment of the lesson.

Teachers held students accountable by interacting with the groups, offering suggestions, or posing questions to direct students' thinking. Teachers regularly graded group assignments - either group products or an individual student's work selected at random from each group.

The only concerns teachers had were in designing appropriate cooperative activities and that some groups were adversely affected by absenteeism. In addition, the treatment teachers suggested they would be more comfortable in either planning group work for lessons which lent themselves to that process, or alternating cooperative practice with individual practice.

Time guidelines

Teachers felt the suggested times for the lesson segments were appropriate, and were effective in balancing the lesson format; especially by limiting the time spent in taking up homework, more class time is freed for teaching new concepts and providing practice time. However, all three teachers considered the guidelines restrictive and difficult to adhere to when introducing a more involved concept.

Process problems

Teachers experienced difficulty integrating the ten-minute problem solving component into the lesson structure every second lesson, not only in terms of restricting discussion to that time period, but also in finding appropriate problems that would be relevant to the lesson topic.

Problem solving may be better integrated within the lesson development. Problem solving situations should relate to the lesson content being presented, with class discussion of heuristics and alternate perspectives imbedded within the lesson.

E. Teacher Change

Elements that teachers intend to continue after the conclusion of the study include:

1. Teaching for meaning
2, Strategies for dealing with homework
3 Oral work
4. Cooperative practice on a regular basis

5. Giving more time to practice concepts independently and within groups
6. Lesson sequence.

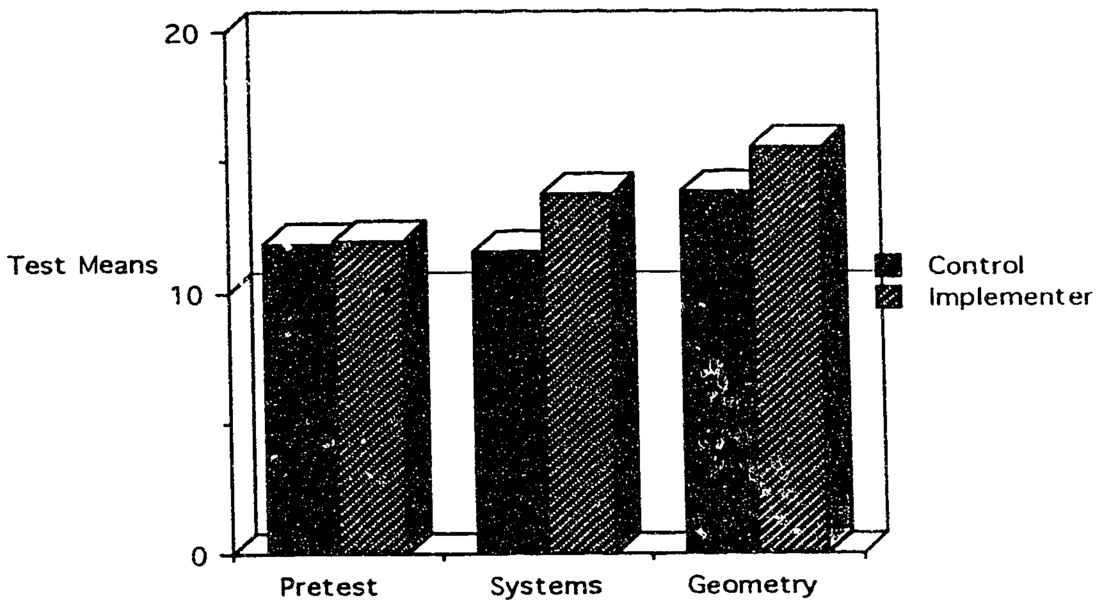
The Model has provided an opportunity for teachers to reflect on their teaching. It has been a catalyst for discovering strategies for enhancing their personal effectiveness.

F. Student Achievement

Three tests were designed to assess student achievement- a thirty-item pre-test surveying the Mathematics 23 course of studies covered prior to the implementation of the treatment program, and two thirty-item multiple choice post-tests based on the unit objectives for systems of equations and geometry.

Based on the survey pre-test, there was no significant difference, initially, between the control group and the students of the three teachers who implemented the Model. The post-test results of the same groups favoured the students in the Implementers' classes compared to the control group at the 0.05 level of confidence.

Figure 10. Comparison of control and implementer groups



G. Student Perceptions

Seven of the nine students interviewed felt the program enhanced their mathematical understanding and helped them learn mathematics, citing increased teacher interaction and cooperative activities as contributing factors. The students were particularly enthusiastic about group work; the reasons included:

1. A more relaxed learning environment and the ease of relating to other students - reduced risk.
2. Learning through sharing different student perspectives.
3. Immediate feedback.

4. Increased self esteem and an enhanced feeling of competence.
5. Pride in being in a position to offer assistance to others.

Seven of the nine students felt they were accomplishing more in class, and all nine felt they were doing more homework because they were being held accountable. The student consensus was positive and favoured the Model compared to previous teacher practice.

H. Implications for Research

A second experiment involving more teachers and students to determine if the results can be replicated is warranted.

Because so many variables were involved in the Interactive Teaching Model, it is improbable that those factors which favoured the treatment group could be readily identified. Nevertheless, it may be of interest to investigate whether the achievement gains for the two units of study and the positive student perception of the Model would be sustained over the entire course.

A follow-up study is indicated to ascertain those elements of the model the treatment teachers continue to use regularly as part of their teaching repertoire and what implications does this have for teacher inservice? To what extent do teachers change their teaching practice when presented with alternatives?

Also, the treatment was applied in a Mathematics 23 context. Students in this stream are less mathematically motivated and competitive than in more academically oriented classes. Would the results be replicated in all senior-high classrooms and grade levels, specifically highly motivated and mark-conscious classes? In particular, Sigurdson and Olson (1989) suggest that more academically able students relate more favourably to instructional strategies which emphasize mathematical meaning.

The teachers who participated in the study were all seasoned teachers who had the benefit of years of successful classroom practice. Each brought to the model a mature perspective that facilitated its implementation. Would less experienced, competent, or novice teachers implement this program as successfully?

Of continuing interest to teachers and school administrations are instructional strategies for enhancing student performance and attitude towards mathematics. It is possible that this research could serve as a vehicle for discussing action research toward these ends.

Finally, one of the weaknesses of this study was the unsatisfactory attitude questionnaire which detracted from a definitive determination of student attitudes towards the salient elements of the model. If the research were repeated, a more carefully designed instrument would be desirable.

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APPENDIX 1

INTERACTIVE TEACHING INSERVICE

LESSON FORMAT

DAILY REVIEW AND HOMEWORK CHECK - 10 minutes
* Begin with oral work
* Review previous lesson's skills and knowledge
* Deal with homework

DEVELOPMENT - 25 minutes
* Place concept to be taught in context of past knowledge and future problems
* Emphasize meaning
* Monitor student understanding through active questioning
* Reinforce concept through controlled practice

COOPERATIVE PRACTICE - 25 minutes
* Provide opportunity for successful practice
* Include word problems and applications related to the lesson
* Encourage active group discussion
* Keep individuals accountable

HOMEWORK
* Assign homework relevant to lesson
* Ensure that the questions assigned can be completed successfully by the majority of students working independently
* Include a review question.

DAILY REVIEW AND HOMEWORK CHECK

The oral work at the beginning of the lesson is intended to emphasize the importance of the first few minutes of the class. Rather than focussing on routine clerical matters, all students will engage in relevant and meaningful mathematical activities that will set the tone for the rest of the period. Questions selected for the oral drill should pertain to the prerequisite skills necessary for the concepts to be presented in the development segment of the lesson. The oral work may be integrated with he review of the previous lesson's skills and knowledge.

Less time is to be devoted to addressing problems with the previous day's homework than is routinely the case. Problems from assigned work, to be dealt with in a whole class setting, should be limited to those which concern the majority of the class. By restricting the time spent on taking up homework, more time can be devoted to the presentation of new material. Some suggestions for limiting the time usually spent in correcting the assignments from the previous lesson follow. These suggestions have been adapted from Every Minute Counts: Making Your Math Class Work (Johnson, 1982).

1. Display the answers on an overhead transparency or on the chalkboard. Have the students record the correct answers next to their own work, so that they can review their steps later to determine where they made their mistakes.

2. While the class is checking their assignments, the teacher can circulate among the students to check to see who has completed their assigned work, whether there are problems of a general nature that require class discussion. At this time the student scores on the homework may be recorded.

3. The teacher should not spend class time to discuss questions that only a few students were not able to do. Individual assistance may be provided later, during the seatwork portion of the lesson, or arrangements may be made to meet the student outside of class, perhaps during a tutorial period, to resolve the difficulty.

4. It is sometimes advisable to have the students to redo one of the questions with which they had difficulty once they are given the correct answer. This question would be turned in to the teacher for additional advice or assistance if the second attempt is also incorrect.

5. Another method for determining which questions should be addressed in a whole group discussion is to ask for a show of hands or have the students as they

enter the room check off on a tally sheet next to the number of the questions assigned those that presented problems.

6. Homework quizzes may also motivate students to complete homework and to revise their solutions. For instance, at the end of the week, a question or two may be selected at random from their week's assignments. The students are asked to copy, directly from their notebooks, their written solutions to the items chosen. They would not have access to their textbooks, and only enough time would be allowed for them to copy their answers, not to work out the questions from scratch. These quizzes would be marked by the teacher and used in part in determining their unit grades. This method also encourages students to keep orderly notebooks, and emphasizes the importance the teacher places on assigned work.

Students should be held accountable for the homework assigned. However, regardless of the method used in dealing with homework, the time traditionally taken in dealing with this aspect should be restricted to no more than the ten minutes allotted. This allows for more effort to be expended on the development segment of the lesson.

DEVELOPMENT

The development segment of the lesson is that part of the period devoted to actively involving students in developing their understanding of skills and concepts. Meaning is to be established by relating the content to previous knowledge, by placing the concepts in the context of the students' own experience, by modelling everyday situations, by focussing on applications, and using concrete materials when applicable. Though questioning, the teacher will monitor student understanding, and ensure students are participating.

The controlled practice portion of the development, when the students work on one or two problems followed by a class discussion, is designed to provide additional feedback to the teacher, and to enhance the students' proficiency with the material.

Questioning is an important aspect of actively engaging the students in the learning process. The following represent suggestions included in Active Mathematics Teaching (Good, Grouws and Ebmeier, 1983) and Every Minute Counts (Johnson, 1982).

1. Give time for students to formulate an answer. A pause will allow time for all students to think of an answer to the question posed. Avoid naming a student before a question is asked; calling on a student before the question is framed permits the rest of the class to sit back and allow someone else to do the thinking.

2. The questions asked should involve more than a yes or no answer or questions that are limited to simple recall. Avoid asking questions that contain the answer . Ask open-ended questions. Probe for understanding and encourage discussion.

3. The teacher should avoid answering his own questions. The class must be held accountable and as much as possible each student must be encouraged to participate actively

4. When a student answers a question, do not repeat the answer. Encourage other students to clarify or expand on the responses given by others. This ensures the rest of the class will listen to those giving responses and will reflect on what was said. Insist that students are attentive to each other and to the teacher.

5. Foster a classroom environment in which students will risk an answer. Don't label questions as trivial or difficult, or comment on the quality of the student's answer.

6. Try to ask questions that guide the students towards the development of the concept.

COOPERATIVE PRACTICE

In the cooperative-practice segment, students will be assigned to groups of four. These groups are to be heterogeneous by sex and achievement. Groups should be formed by selecting one high achiever, one low achiever, and two students of average mathematical ability. These groups are to be maintained during the two units of the study. Over time the students should improve their interpersonal skills and grow to appreciate their responsibility to the other members of the group.

The intent of the groups is to provide student support when they are working on questions assigned based on the material presented in the lesson development. In this phase of the class, both group and individuals are to be held accountable for the work to be completed in class. As well, in this portion of the lesson, process problems will be discussed by the class as a whole and within the groups. These problems should be integrated into the lesson at least every other day. The intent of these problems is to develop students' problem solving skills and to motivate.

Suggestions for group work follow.

1. The groups should be held accountable for completing assigned work. To be successful the students must expect each member to make a contribution. Individuals can not sit back and and let the others carry the load.

2. To ensure individual and group accountability, the teacher must monitor students' progress and provide assistance whenever necessary. However, the teacher must also encourage the group members to exchange ideas and information, and to support each other, clarifying ideas and often explaining concepts to other within the group experiencing difficulty. Everyone is expected to master the material and help others achieve the same goal.

3. The cooperative activities should be structured for success. To ensure students are on task, randomly select a student to justify the group's solution or select a student's paper for a group mark. A single group product is to be turned in for grading.

4. The group must work together in a close seating arrangement to encourage discussion. Keep the groups far enough apart to avoid interference.

5. On occasion it is necessary to discuss with individual groups or the class as a whole the social skills requisite for effective group interaction. Group members should be sensitive to others and avoid being judgmental or critical of their peers.

6. To enhance participation, roles may be assigned to the members of a group in the completion of tasks. For instance, the teacher may have one student clarify a question, another suggest a strategy or encourage participation, another record the responses, and the fourth verify the results. Roles assigned would depend on the task set, and various roles may be assumed by a student over time.

HOMEWORK

Homework is to provide students with an opportunity for successful practice. The assigned questions should be within the ability of the students to do; questions the students may do incorrectly fostering misconceptions or improper technique should be avoided.

Because of the time spent in the cooperative-practice phase of the lesson, the number of questions assigned should be fewer than are normally given.

The homework must be worth doing; and, if worth doing well it should be used to determine, in part, a student's unit mark.

PROBLEM SOLVING

Every second lesson process problems are to be discussed for ten minutes, between the development of the lesson and cooperative practice. A number of problems which may be used follow.

SUGGESTED PROBLEMS

1. A sign in a store window advertises 20% off all items. Which is better, to receive the discount first and then pay the 7% GST on the discounted price, or pay the GST first and then obtain a discount on the total?

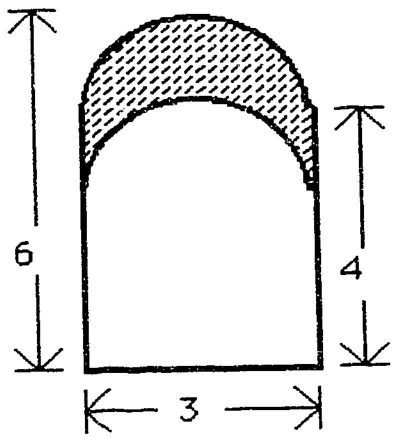
2. A spiral spring is hung up and weights are hung from its lower end. Its length is 35 cm when 20 grams are added, and its length is 45 cm when 40 grams are added. What is its length when all the weights are removed?

3. What is the smallest number of cubical blocks that can be spread out to form a square or stacked to form a cube?

4. Take three consecutive numbers, say 7, 8, and 9 and square the middle number. How is your result related to the product of the other two? Does this always work? Why?

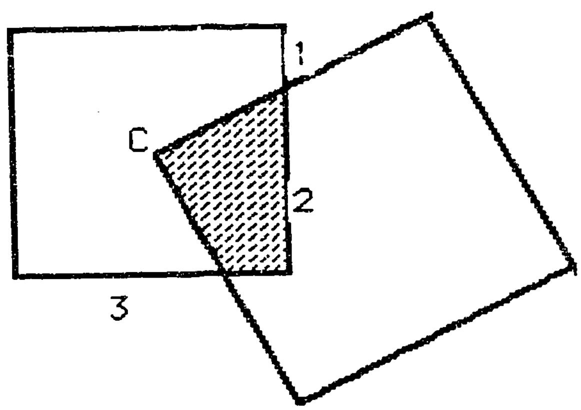
5. Assume that the earth is a perfect sphere and a wire is stretched around it at the equator. If the wire is cut and three metres added to its length so that it now forms a ring the same distance from the equator at all points, what is the distance between the wire and the earth?

6. What is the area of the shaded region?



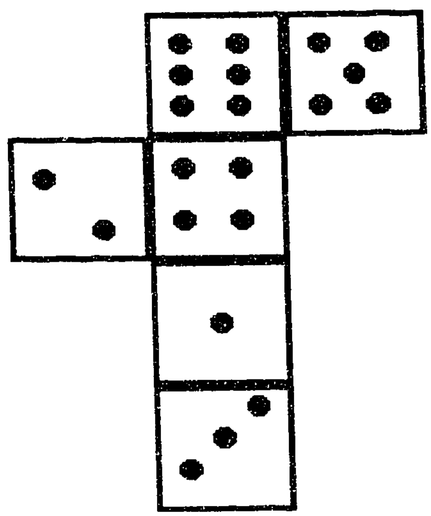
7. A basketball player scored 48 points in a game, raising his average from 25 to 26. How many points would he have had to score to ring his average up to 27?

8. What is the area of the overlap of the two squares? C is the centre of the small square.



9. If 5 frogs can catch 5 flies in 5 minutes, how long does it take 1000 frogs to catch 1000 flies?

10. Draw the die from its net. If this die is rolled 5 times and the total on the top faces is 25, what is the sum of the opposite (bottom) faces?



APPENDIX 2

SAMPLE LESSONS

SYSTEMS OF EQUAIONS

Lesson 1: Solve systems of linear equations in two unknowns graphically.

Daily Review and Homework Check - 10 minutes

* Oral Work

Ask the students to respond to the following questions. Have the students justify their own or their peers' answers.

1. Describe the graph of $x = 1$? Explain.
2. What is the equation of the horizontal line which passes through the point (1, 2)? Why?
3. At what point do the lines $x = 3$, and $y = 4$ intersect?
4. What are the equations of the horizontal and vertical lines which pass through the point (-1, 5)?
5. What is the slope of the line $y = 3x - 1$? Why?
6. What are the slopes of the lines $y = 2x$ and $y = 2x + 1$? How are these lines related? Why? Would these lines intersect?

* Review: none
* Homework: none

Development - 25 minutes

* Define systems of equations.
* Explain that this unit involves using many of the principles learned in the previous chapter on linear equations and their graphs. Mention that the systems of equations that the students will be solving consist of a pair of linear equations, that each linear equation represents a straight line, and that the objective is to determine if and where the lines intersect.
* Example 1

Two students, X and Y, who live 10 km apart decide to cycle towards each other to meet one afternoon. How far has each travelled when they met?

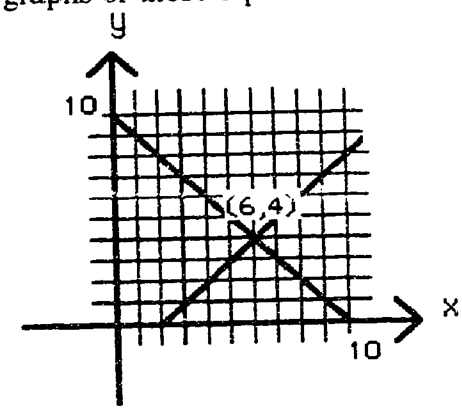
The following questions should be addressed.

1. What distances are possible? Emphasize that a number of answers are possible.
2. If x represents how far Student X has travelled, and y represents how far Student Y has travelled, what equation represents the relationship between x and y ?

   $x + y = 10$
3. Ask students to suggest ordered pairs which satisfy the equation. Point out that there are restrictions on the two variables.

   $0 \leq x \leq 10$ and $0 \leq y \leq 10$
4. Have the students graph the equation.
5. Discuss the reason why the information given in the original question was inadequate to find unique values for x and y.

6. Provide an additional constraint to the original problem. If Student X cycled 2 km farther than Student Y, how far did each travel? The answer should now be obvious: x = 6 km and y = 4 km.
7. Discuss the equation representing the second statement and its graph.
   $x - y = 2$
8. Explain that the two equations $x + y = 10$ ... (1)
   $x - y = 2$ ... (2)
   form a system of equations, and that the point of intersection of the graphs of those equations is the required solution.



* Example 2
Solve the following system graphically, and verify the solution by substitution. Discuss both tables of values and intercepts.
$2x - y = 7$...(1)
$x + 2y = 1$...(2)

* Use controlled practice to emphasize concepts. At least two additional examples should be completed; however, do not exceed 25 minutes for the development segment. One example of a system to be solved graphically is:
$3x + y = 6$... (1)
$x = y - 2$... (2)

Interactive Practice - 15 minutes

* Assign pp. 199-200, questions 1-4. Indicate that at the end of the period, the assignment will be selected, at random, from one of the students from each group. These assignments will be graded, and each student in the group will receive the mark based on that assignment.

Problem Solving - 10 minutes

* Spend the time with the entire class discussing the following problem. How many different pairs of positive integers can be found to solve the following equations? For the purposes of this question the pairs 3 and 2, and 2 and 3 are considered the same.

Look for patterns.
1. $x + y = 2$
2. $x + y = 3$

3. x + y = 4
4. x + y = 5
5. x + y = 1001

Homework

* Assign p. 200, questions 6 and 7.

Lesson 2: Solve systems of linear equations in two unknowns graphically. Analyze the three possible solutions to linear systems.

Daily Review and Homework Check - 10 minutes

* Oral Work
  1. What is the relationship between the slopes of two parallel lines?
  2. State the slope of the line y = 4x + 2.
  3. Give the equation of a line parallel to y = 4x + 2
  4. Give the equation of a line which intersects y = 4x + 2 at exactly one point.
  5. What is the slope of x + y = 2 ?
* Review: Solving a linear system graphically.
* Homework: p. 200, questions 6 and 7.

Development - 25 minutes

* Example 1

  Solve the system x - y = -4 ... (1) graphically.
  2x + y = 7 ... (2)

  Use either a short table of values for each equation or their intercepts to sketch the lines in the plane.

  The following questions should be addressed.
  1. What is the solution of this linear system.
  2. Why is there exactly one point of intersection?
  3. What is the slope-y-intercept form of the first equation?
  4. What is the slope of the first line?
  5. What is the slope-y-intercept form of the second equation?
  6. What is the slope of the second line?

  Emphasize that if the slopes of the two lines differ, they must intersect at exactly one point. The terms *consistent* or *independent* may or may not be introduced at this point.

  7. What would happen if the slopes of the lines were the same?
* Example 2

  Solve the system 2x + y = 6 ... (1)
  2x + y = -4 ...(2)

  Ask the students to sketch the lines from their intercepts.

  Discuss the graph in terms of the slope of each line.

  Stress that this system of equations has no solution. The term inconsistent may or may not be introduced.

* Example 3
Show that each ordered pair that satisfies $x + 2y = 3$ also satisfies $3x + 6y = 9$. This may be done using tables of values.
Graph the equations, showing that they represent the same line in the plane.
Ask the students how the second equation was formed from the first equation.
Emphasize that there are infinitely many solutions; the term *dependent system* may be introduced.
* Controlled practice. At least three more examples, one of each type should be discussed if time permits.

Interactive Practice - 25 minu
* Have each group dete...ine the number of solutions to each system and have them justify their answers in their own words. If the system represents a pair of lines which intersect at exactly one point, have the students determine the coordinates of that point and check their answer by substitution. To ensure accountability, at the end of the period have each student, working independently, solve and hand in for grading one of the questions to be chosen at random from thegroup assignment.

1. $2x - 3y = 7$
   $4x = 6y + 14$
2. $3x - y = 4$
   $x + y = 0$
3. $y = 5x + 7$
   $2y = 10x - 21$
4. $x - y = 3$
   $2y + x + 9 = 0$

* In addition, assign page 200, questions 7, 11, 13

Homework: pp. 199-200, questions 5, 8, and 12.

Lesson 3: Solve systems of linear equations by substitution

Daily Review and Homework Check - 10 minutes
* Oral Work: Write the questions on the board.
  1. Simplify $4 - (3 - x)$
  2. Simplify $3x - 2(x - 7)$
  3. Solve $3x - 4 = 13$
  4. Find the value of y if $y = 2x - 3$ and $x = 5$

* Review: Solving systems graphically.
* Home ork: pp. 199-200, questions 5, 8 and 12.

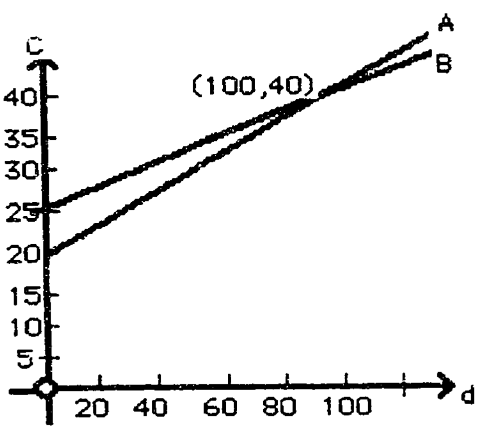
Development - 25 minutes
* Discuss why graphical methods of solving linear systems are inadequate.

* Example 1

Two car rental firms use different formulas for charging their customers. Company A charges $25.00 per day and $0.15 per kilometre. Company B charges $20.00 per day and $0.20 per kilometre. How far would you have to drive each day, before Company A is less expensive?

Discuss this example from at least three perspective.

1. Because Company A charges $0.05/km less than Company B, the $5.00 difference in the daily charges would be made up in 100 km.
   100 km ($0.05/km) = $5.00
2. Solve the problem by setting up a system of equations, and solving that system graphically.
   Let C be the total cost, and d be the distance driven.
   For Company A: $C = 25 + 0.15d$ ... (1)
   For Company B: $C = 20 + 0.20d$ ... (2)



3. Solve the system by substitution.
   For Company A: $C = 25 + 0.15d$ ... (1)
   For Company B: $C = 20 + 0.20d$ ... (2)
   Therefore, $25 + 0.15d = 20 + 0.20d$
   $5 = 0.05d$
   $d = 100$ km and $C = \$40.00$
   Verify by substitution

* Through controlled practice, develop substitution further.

Example 2
$2x + y = 3$ ... (1)
$y = x$ ... (2)

Example 3
$3x - y = 7$ ... (1)
$2x + y = 3$ ...(2)

This method will take two lessons to complete.

Interactive Practice - 15 minutes

* p. 202, questions 1-5

* Have students complete the questions in their home groups on a single sheet to be turned in at the end of the exercise. Each of the four students will do **one** of parts (a) through (d) of each question.

Problem Solving - 10 minutes

* Spend the time with the entire class discussing the following problem. What happens when we try to solve $x + 2y = 4$, and $x + 2y = 16$ by substitution? Why?

Homework: p. 202, questions 6 and 7.

GEOMETRY

Lesson 1: Working with perimeter.

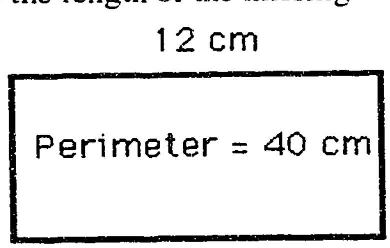
Daily Review and Homework Check - 10 minutes

* Oral Work
  1. What is two times the sum of 4 cm and 3.5 cm?
  2. What is the cost of 25 m of fencing if 20 m cost $40.00?
  3. Determine the area of a square with a perimeter of 20 cm.
  4. How many times longer is the perimeter of a square 4 cm on a side than an equilateral triangle 4 cm on a side?
  5. How many sides does a pentagon have?
  6. What is 3 (5.25 cm + 4.75 cm)?
* Review: Classify polygons by their number of sides.
* Homework: none

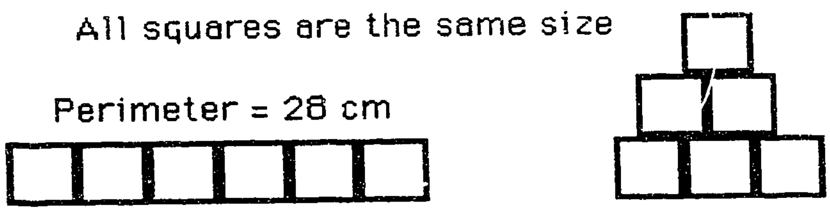
Development - 25 minutes

* Discuss the meaning of perimeter.
* Have the students estimate the cost of installing baseboards in their classroom if 1 m costs $2.00.
* Review possible formulas for the perimeters of triangles, squares, rectangles, parallelograms, quadrilaterals, and regular polygons. Have the student supply the answers whenever possible.
* Example 1

  What is the length of the missing side?



* Example 2
What is the perimeter of the figure on the right?



* Controlled Practice: Questions 6 and 8 from page 225.

Problem solving - 10 minutes
* Six people enter a room. Each person shakes hands with every other person once. How many different handshakes were there?
Simplify the problem. Try the question with just 2 people, then three. What is the pattern? It may help to represent the people as points on a circle and the handshakes as sides and diagonals of polygons.

Cooperative Practice - 15 minutes
* pp. 222 -225, questions 1.a., 3.c., 4, 5.d., and10

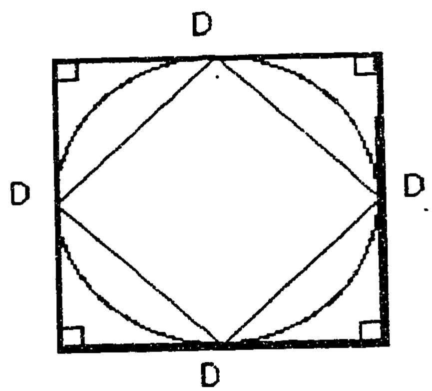
Homework: pp. 223 - 225, questions 5.b., 5.c., 8, and11

Lesson 2: Circumference of a circle

Daily Review and Homework Check - 10 minutes
* Oral Work
1. What is the radius of a circle 16.5 m in diameter?
2. State the term for the distance around a circle.
3. What is the diameter of a circle (x + 3) units inr adius?
4. What is the value of π correct to two decimal places?
5. Which is longer, a circle of diameter 5 cm, or a square 5 cm on a side? Why?

* Review the concept of perimeter.
* Homework: pp. 223 - 225, questions 5.b., 5.c., 8, and11.

Development - 25 minutes
* Define *circumference*.
* Demonstrate that the circumference of a circle should be approximately 3 times the diameter of the circle. The following questions may be asked.



1. How is the circumference of the circle related to the perimeters of the two squares?
2. What is the perimeter of the large square?
3. Is it possible to find the perimeter of the small square? How?

Perimeter of larger square = 4D
Perimeter of smaller square?
Using Pythagoras's Theorem, find the length of each side.

$$(D/2)^2 + (D/2)^2 = (\text{side})^2$$

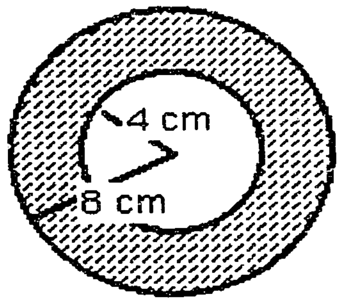
$$(\text{side})^2 = \frac{2D^2}{4}$$

$$\text{side} = \frac{\sqrt{2}\,D}{2}$$

Therefore, the perimeter is 4(side) = $2\sqrt{2}$ D or about 2.8 D

4. Discuss that the circumference of the circle must be smaller than 4D and larger than 2.8 D

* Using a paper cup demonstrate that the circumference of a circle is a little more than three times the diameter. Roll the lip of the cup along the edge of a sheet of paper, making the staring point and end point for one complete rotation. How many times does the top fit in that distance? Does the ratio of the circumference to the diameter of a circle depend on the size of the circle? What is that ratio?
* Discuss the formulas $C = \pi D$ and $C = 2\pi r$.
* Example
  What is the difference in the circumferences of the inner and outer circles of the washer below?



* Use controlled practice to discuss the following.
  1. What is the radius of a circle of circumference 45 cm?
  2. How many times does a tire of radius 40 cm rotate in from Edmonton to Calgary, a distance of 300 km?
  3. Question 16, p. 229.

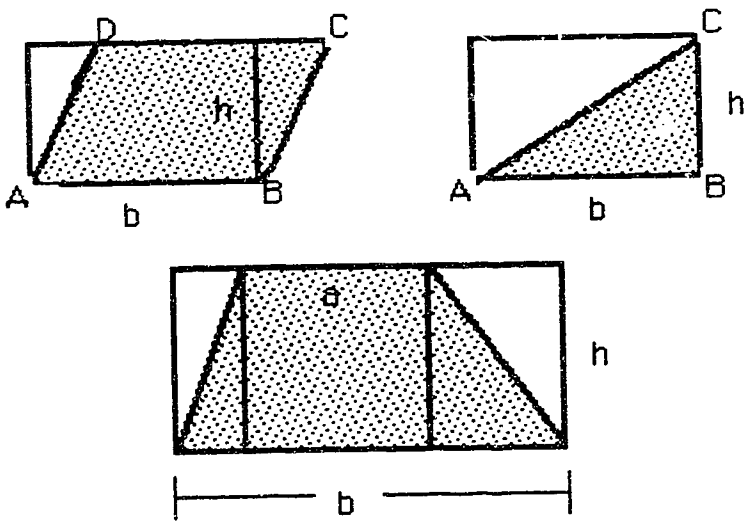
Cooperative Practice - 25 minutes
pp. 227 -229; questions 1.a., 3, 6, 7, 9, 13 and 14

Homework: pp. 227 -229; questions 5, 8, 10, 14 and 18.

Lesson 3: Formulas for the areas of rectangles, squares, parallelograms, triangles and trapezoids.

Daily Review and Homework Check - 10 minutes
* Oral Work
  1. How many square centimetres are there in a square metre?
  2. How many times larger is a square centimetre than a square millimetre?
  3. Evaluate 1/2 (4 + 5) 20
  4. How much larger is 9^2 than 8^2?
  5. Factor $\frac{ah}{2} + \frac{bh}{2}$
* Review: Circumference of a circle
* Homework: pp. 227 -229; questions 5, 8, 10, 14 and 18.

Development - 25 minutes
* Review the formulas for the areas of rectangles and squares. Stress the importance of units in your examples.
* Discuss the formulas for areas of parallelograms, triangles and trapezoids in terms of related rectangles.



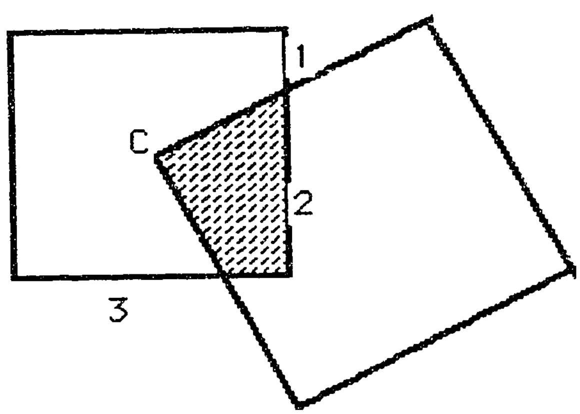
* For the trapezoid, its area is the average of two rectangles of area ah and bh. Use a numerical example first, then develop the formula:

$$A = \frac{ah + bh}{2} = 1/2\ h\ (a + b)$$

* Use controlled practice to discuss real-world applications.
  pp. 232 - 233; questions 8,11, and 15.

Problem solving - 10 minutes

* What is the area of the overlap of the two squares? C is the centre of the small square.



Cooperative Practice - 15 minutes

* pp. 232 - 233; questions 1.a., 2.a., 3.a., 4.a., 12, 13.

Homework: pp. 232 - 233; questions 6 and 7.

APPENDIX 3

PRE- AND POST- TESTS

SURVEY TEST - PRE-TEST

1. The value of the expression 3^{-2} is

A. 1/9 B. -9
C. 9 D. -1/9

2. The greatest common factor of $12x^3y^4 - 8xy^2$ is

A. $4xy^2$ B. xy^2
C. 4 D. $24x^3y^4$

3. The value of $-36^{1/2}$ is

A -1/6 B. -6
C. -18 D. undefined

4. The number of integers between $-\sqrt{10}$ and $\sqrt{10}$ is

A. 3 B. 6
C. 7 D. 9

5. A packing case is cube shaped. If its volume is 27 m^3, then the area of the top of the case, correct to one decimal place, is

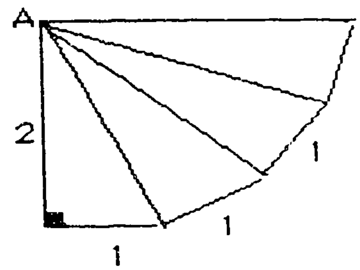
A. 3.0 m^2 B. 5.2 m^2
C. 9.0 m^2 D. 6.0 m^2

6. Which of the following radical expressions is the largest in value?

I. $\sqrt{50}$
II. $\sqrt{33} + \sqrt{17}$
III. $\sqrt{10} + \sqrt{40}$
IV. $\sqrt{26} + \sqrt{24}$

A. I B. II
C. III D. IV

7. In the diagram on the right,



the length of segment AB is

A. 3
B. $\sqrt{6}$
C. 4
D $\sqrt{8}$

8. The value of 6^{-2} (6^{2}) is

A. 0
B. 1/36
C. 1
D. 36

9. 2(2x - 1)(3x + 2) is the same as

A. $24x^2 - 4x - 8$
B. $24x^2 + 4x - 8$
C. $12x^2 + 2x - 4$
D. $12x^2 - 2x - 4$

10. $(2x - 1)^2$ is equivalent to

A. $4x^2 - 4x - 1$
B. $4x^2 - 4x + 1$
C. $4x^2 - 1$
D. $4x^2 + 1$

11. The solution of the equation

$3^2(5x - 7) = 15$ is a(n)

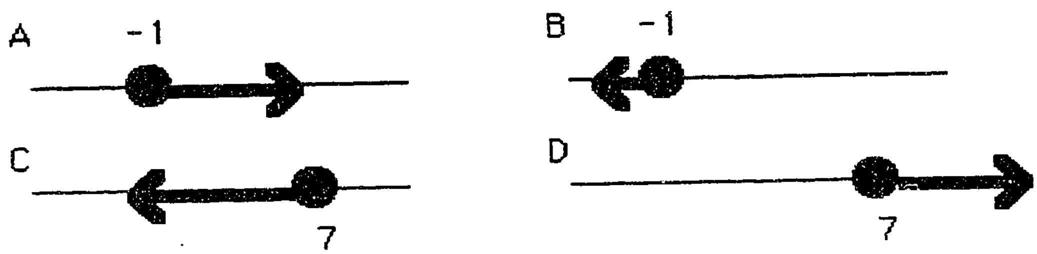
A. whole number
B. integer
C. fraction
D. radical

12. The distance, d, in metres, an object falls from rest after t seconds is given by $d = \frac{1}{2}gt^2$ where g is the acceleration due to gravity. Ann wants to calculate how long it takes a rock to fall down a 20m well. The formula she can use to calculate t is

A. $t = \frac{4d^2}{g^2}$ B. $t = \frac{2d}{g}$

C. $t = \frac{4d}{g}$ D. $t = \frac{4d^2}{g}$

13. The graph of the inequality $3 - x \leq 4$ is



14. A factor of $12x^2 - 5x - 3$ is

A. $3x + 1$ B. $3x - 1$
C. $4x + 3$ D. $4x + 1$

15. If the trinomial $2x^2 + bx + 1$ can be factored, which of the following is a possible value of b ?

A. -4 B. -3
C. -2 D. -1

16. The solutions of the equation $2x^2 - 5x + 2 = 0$ are

A. 1/2 and 2 B. -1/2 and 2
C. -1/2 and -2 D. 1/2 and -2

17. Which of the following equations has -1 as a solution?

A. $124x^2 - 4x - 120$ B. $124x^2 + 4x - 120$
C. $124x^2 + 4x - 128$ D. $124x^2 + 4x + 128$

18. The solutions to $4x^2 = 25$ are

A. $\pm 2/5$ B. 25/4
C. 4/25 D. $\pm 5/2$

19. If a = -2, b = 7 and x = -69, the value of $x^2 - 10abx + 25a^2b^2$ is

A. 19 321 B. -19 321
C. -1 D. 1

20. The solution of the equation $3x - 5 = -1$ is

A. 2 B. 4/3
C. -2 D. 3/4

21. The line with an x-intercept of 2 and a y-intercept of -2 is

A. $x + y - 2 = 0$ B. $x - y - 2 = 0$
C. $x + y + 2 = 0$ D. $x - y + 2 = 0$

22. The line passing through the (0,0) and (3, 4) is

A. $4x - 3y = 0$ B. $3x + 4y = 0$
C. $4x + 3y = 0$ D. $3x - 4y = 0$

23. The equation of the vertical line through (-2, 3) is

A. $y - 3 = 0$ B. $x - 2 = 0$
C. $y + 3 = 0$ D. $x + 2 = 0$

24. The equation of the line passing through (1, 1) and (2, 3) is

A. $2x - y - 1 = 0$ B. $2x + y - 3 = 0$
C. $x - 2y + 1 = 0$ D. $x + 2y - 3 = 0$

25. The line passing through (1, -1) and (-1, 1) also passes through

A. (3, 2) B. (3, 3)
C. (3, -3) D. (3, -2)

26. Which of the following lines are parallel?

I $3x - y - 6 = 0$
II $x + 3y - 6 = 0$
III $6x - 2y - 6 = 0$
IV $3x + y - 6 = 0$

A. I and II B. I and III
C. II and IV C. III and IV

27. The equation of the horizontal line through (1, 2) is

A. $x = 1$ B. $y + 2 = 0$
C. $x + 1 = 0$ D. $y = 2$

28. The y-intercept of the line $3x + 4y + 12 = 0$ is

A. -4 B. -3
C. 3 D. 4

29. If the line with an x-intercept of -1 and a y-intercept of 1 passes through (7, y), then the value of y is

A. 6 B. 7
C. 8 D. 9

30. The equation of the line parallel to $y = 4x$ and which passes through (3, 0) is

A. $x - 4y - 3 = 0$ B. $4x + y - 12 = 0$
C. $x + 4y - 3 = 0$ D. $4x - y - 12 = 0$

SYSTEMS OF EQUATIONS - POST-TEST

1. The graph of the equations which form the system $x + 2y = 3$, $4y = -2x$ is

   A. a pair of intersecting lines
   B. a pair of parallel lines
   C. a pair of perpendicular lines
   D. a single line

2. If the system $y = mx + b$, $y = nx + c$ represents lines which intersect at exactly one point, then

   A. $m = n$
   B. $b = c$
   C. $m \neq n$
   D. $b \neq c$

3. The point of intersection of the lines $2x - 6 = 0$ and $2y + 6 = 0$ is

   A. (3, -3)
   B. (-3, 3)
   C. (6, -6)
   D. (-6, 6)

4. The system equivalent to $3(x - y) = 4x - 1$, $1 + 2(3x - 1) = 3 - 2(y - 1)$ is

   A. $x + 3y = 1$, $3x + y = 3$
   B. $x + 3y = 1$, $9x - y = 2$
   C. $x - 3y = 1$, $6x + 2y = 2$
   D. $x - 3y = 1$, $9x - 2y = 2$

5. The coordinates of one of the vertices of the triangle formed by the lines $x + 2y = 11$, $x - y = 4$ and $7x - 4y = 23$ is

   A. (9, 1)
   B. (8, 4)
   C. (7, 2)
   D. (5, 3)

6. Partial tables of values of two linear equations are given below.

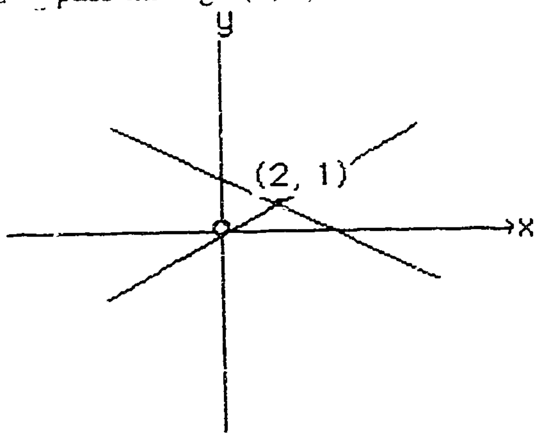
x	-1	0	1
y	6	4	2

x	0	1	2
y	10	11	12

   The solution of this system is

   A. (2, 0)
   B. (-2, 8)
   C. (3, 13)
   D. (-1, 9)

7. Lines l_1 and _ pass through (2, 1).



The line formed by adding the equations of the two lines, must pass through

A. (4, 2)
B. (2, 1)
C. (2, 2)
D. (4, 1)

8. The sum of two numbers is 19. The larger number exceeds the smaller number by 5. If x and y represent the larger and smaller numbers respectively, the equations that could be used to solve for x and y are

A. $x + y = 19$
$x - y = -5$

B. $x + y = 19$
$y = -5x$

C. $x + y = 19$
$x - y = 5$

D. $x + y = 19$
$y = 5x$

9. Norma has 37 nickels and quarters. If she had half as many quarters and and seven more nickels she would still have 37 coins. The total value of the coins she has is

A. \$2.25
B. \$4.65
C. \$6.45
D. \$7.85

10. To solve the system $4x - 5y = 14$ ..... (1) for x , before adding the
$3x - 10y = 4$ ..... (2)
equations, first multiply equations (1) and (2) respectively, by

A. 2 and 1
B. 2 and -1
C. 3 and -4
D. 3 and 4

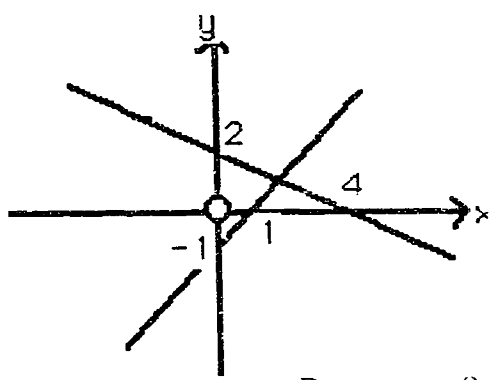
11. Which of the following lines passes through the same point as
$2x + y = 4$ and $x - y = -7$?

A. $2x - y = 11$
B. $2x - y = -10$
C. $3x + y = 3$
D. $3x + y = 17$

12. What is the value of "a" so that $ax + 3y = 2$ and $4x - 7y - 8 = 0$ cross the x-axis at the same point ?

A. 1
B. 2
C. -1
D. -2

13. The system of equations graphed below is



A. $x + y = 0$
$x - y = 8$

B. $x + y = 0$
$x - y = 2$

C. $x - y = 1$
$2x + y = 8$

D. $x - y = 1$
$x + 2y = 4$

14. The solution to question 13 is

A. (1.5 , 1)
B. (2, 1)
C. (2, 1.5)
D. impossible to determine

15. The solution to the system $y + 11 = 2x$
$2x - y = 11$ contains

A. a single point
B. no points
C. many points
D. every point in the plane

16. Consider the following question.

The sum of two positive integers is 9. What are the numbers?

If that was all the information you were given, how many solutions would that question have?

A. 1
B. 4
C. 8
D. an infinite number

17. The local community league charges a both a fixed fee, F, for renting its hall and D-dollars per person. If it charges $320 for 100 people and $430 for 150 people, then the system of equations that can be used to find F and D is

A. $F + 320D = 100$, $F + 430D = 150$
B. $F + 430D = 100$, $F + 320D = 150$
C. $F + 100D = 430$, $F + 150D = 320$
D. $F + 100D = 320$, $F + 150D = 430$

18. The perimeter of a garden plot is 40 m. If it is twice as long as it is wide, then its area, correct to one decimal place, is

A. 88.7 m^2
B. 88.8 m^2
C. 88.9 m^2
D. 89.0 m^2

19. John has $2000. He invests one part, x, at 12% and the second part, y, at 10%. If his total interest for the year is $230, the amount invested at each rate may be found by solving the system

A. $12x + 10y = 2000$, $0.12x + 0.10y = 230$
B. $12x + 10y = 2000$, $12x + 10y = 230$
C. $x + y = 2000$, $12x + 10y = 230$
D. $x + y = 2000$, $12x + 10y = 23000$

20. If $y = 4x + 8$ and $y = 2x - 3$ then y equals

A. -14
B. -5.5
C. -2.5
D. -2

21. The two equations x + 3y = 4 and x + y = 3 may be combined into a single equation by substitution. If y is replaced in the first equation, the resulting equation in x is

A. x + 3(x - 3) = 4
B. x + 3(3 - x) = 4
C. x + 3(x + 3) = 4
D. x + 3 (-x - 3) = 4

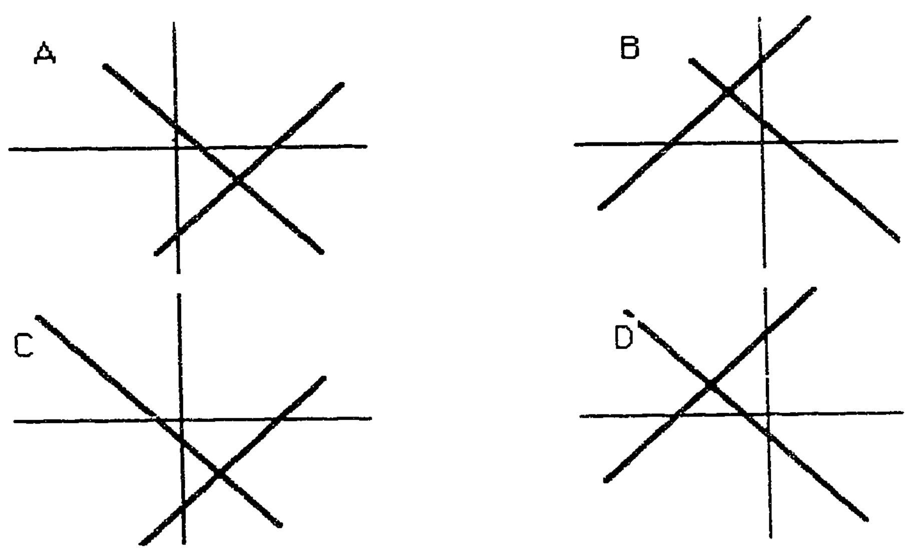
22. If the graphs of y = mx + b and y = nx + c do **not** intersect, then

A. b ≠ c, and m = n
B. b = c and m = n
C. b ≠ c and m ≠ n
D. b = c and m ≠ n

23. If two lines intersect at exactly one point, then their

A. slopes must be equal
B. y-intercepts must be equal
C. slopes must be different
D. y-intercepts must be different

24. Which of the following represents the graph of the linear system
x + y = -1.........(1) ?
x - y = 4 ....(2)



25. One car rental company charges $50 a day and 5 cents a kilometre. A second charges $55 a day and 4 cents a kilometre. How far would you have to drive a car rented from the second company before it becomes less expensive to rent than a car from the first company?

A. 50 km
B. 100 km
C. 125 km
D. 500 km

26. The solution of the system $x - 4y = 7$
$x = 2 - y$ is

A. (3, -1)
B. (-3, 1)
C. (3, 1)
D. (-3, - 1)

27. If x is Tom's age and y is Ann's age then $x - 4 = 5(y - 4)$ means

A. 4 years ago Tom was 5 times as old as Ann is now.
B. In 4 years Tom will be 5 times as old as Ann is now.
C. 4 years ago Tom was 5 times as old as Ann was then.
D. In 4 years Tom will be 5 times as old as Ann.

28. The sum of two numbers is 11. The larger number is three times as large as the smaller. One of the numbers is

A. 2.25
B. 2.75
C. 3.67
D. 6.33

29. Peanuts cost $0.50 a package and chocolate bars are $0.75 each. The equations $50p + 75c = 500$
$p = c$ may mean

A. 2 bars were purchased
B. 6 bars were purchased
C. 4 bars were purchased
D all of the above

30. If the system $y = mx + b$ and $y = nx + c$ represents a pair of perpendicular lines then

A. m = n
B. m = -
C. m = 1/n
D. m = -1/n

GEOMETRY - POST-TEST

1. The sum of the interior angles of a pentagon is

   A. 360° B. 540°
   C. 720° D. 900°

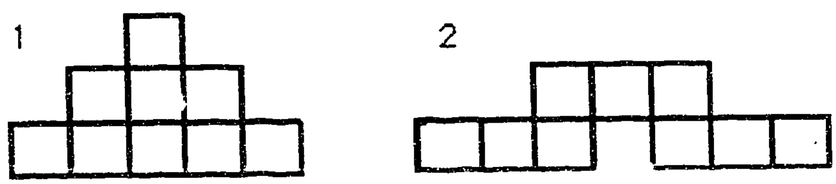
2. The measure of each interior angle of a octagon is

   A. 135° B. 145°
   C. 150° D 155°

3. How many more diagonals does an octagon have than a hexagon?

   A. 7 B. 9
   C. 11 D. 13

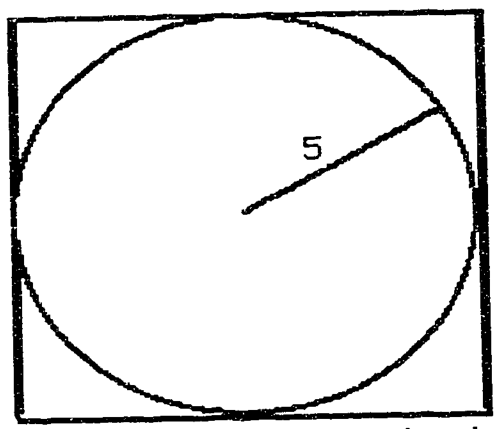
4. Figures 1 and 2 below a.. made up of congruent squares.



If the perimeter of figure 1 is 24 cm, then the perimeter of figure 2 is

   A. 20 cm B. 24 cm
   C. 30 cm D. 36 cm

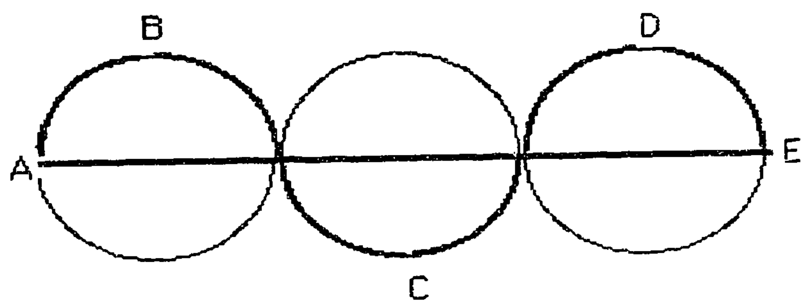
5. A circle of radius 5 cm is drawn inside a square and just touches each side.



How much longer is the perimeter of the square than the circumference of the circle, correct to one decimal place?
Use $\pi = 3.14159$

A. 4.2 cm
B. 4.3 cm
C. 8.5 cm
D. 8.6 cm

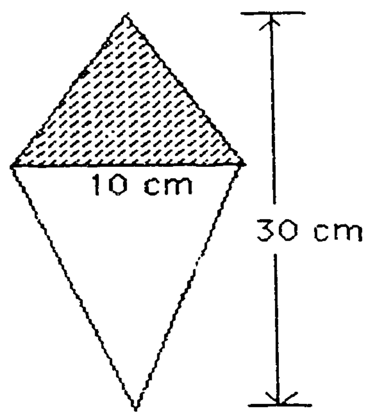
6. The three tangent circles in the figure below are the same size and lie in a straight line. The length of segment AE is 36 cm.



The length of the curved path ABCDE, correct to the nearest centimetre is

A. 56 cm
B. 57 cm
C. 76 cm
D. 77 cm

7. The total area of the two isosceles triangles shown below is



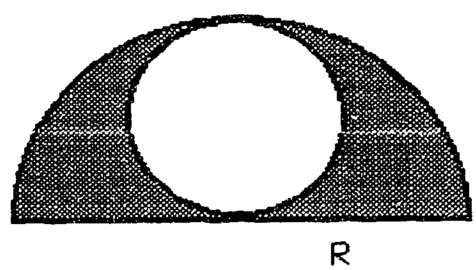
A. 150 cm^2

B. 187.5 cm^2

C. 225 cm^2

D. 300 cm^2

8. A small circle is drawn within a larger semicircle as shown below.



If the radius of the semicircle is R units, then the area of the shaded region is

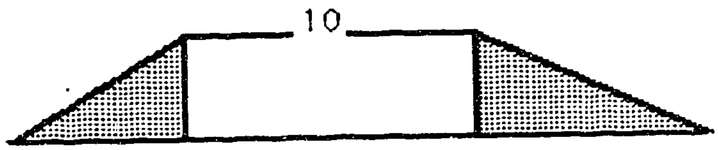
A. $\frac{\pi R^2}{4}$

B. $\frac{\pi R^2}{2}$

C. $\frac{3\pi R^2}{4}$

D. $\frac{3\pi R^2}{2}$

Use the following diagram to answer questions 9 and 10.



9. If the area of the trapezoid is 90 square units, then the height of the trapezoid is

A. 4 B. 5
C. 6 D. 7

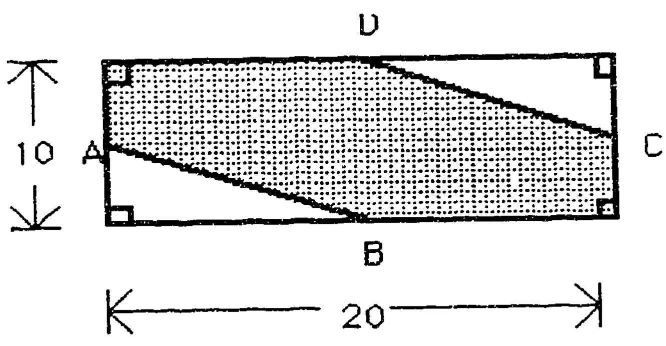
10. What fraction of total area of the trapezoid is the sum of the areas of the two shaded right triangles?

A. 1/3 B. 1/2
C. 2/3 D. 3/4

11. The perimeter of a rectangular patio is 44 m. If each side of the patio is at least 8 m long, what is the difference in area between the largest and smallest of two such patios?

A. 4 m^2 B. 6 m^2
C. 8 m^2 D. 9 m^2

12. In the diagram below, A, B, C, and D are the midpoints of the sides of the rectangle.



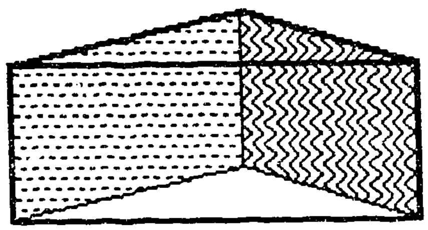
The area of the shaded region is

A. 100 B. 125
C. 150 D. 175

13. A certain polyhedron has 5 vertices and 5 faces. The number of its edges is

A. 5 B. 6
C. 7 D. 8

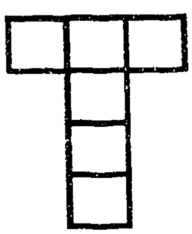
14. Consider the polyhedron in the figure below.

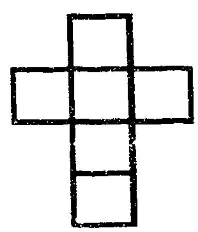


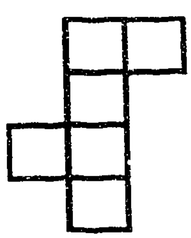
The total number of diagonals is

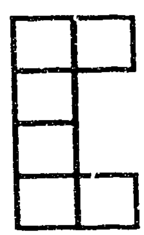
A. 2 B. 4
C. 6 D. 8

15. Which of the following nets can NOT be folded into a cube?

A. 

B. 

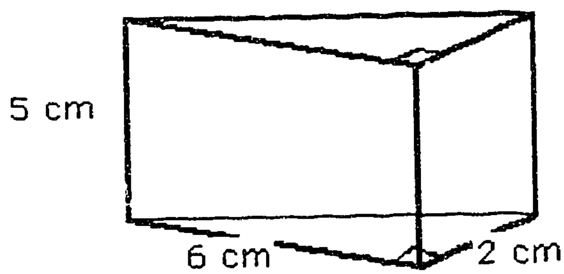
C. 

D. 

16. The surface area of a sphere of diameter 10 cm, correct to one decimal place, is

A. 78.5 cm^2 B. 314.2 cm^2
C. 628. 8 cm^2 D. 1256.6 cm^2

17. The volume of the figure below is

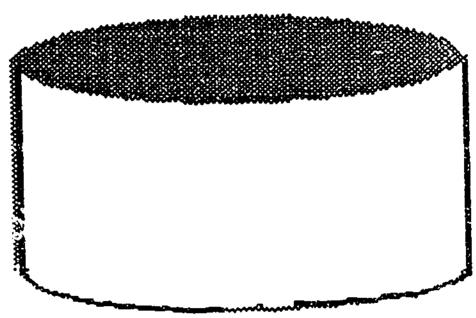


A. 30 cm^3
B. 40 cm^3
C. 50 cm^3
D. 60 cm^3

18. $\pi r\ (r + s)$ is the formula for the surface area of a

A. tetrahedron
B. cylinder
C. cone
D. sphere

19. A cylindrical granery has a diameter of 6 m and a height of 5 m,as shown below.



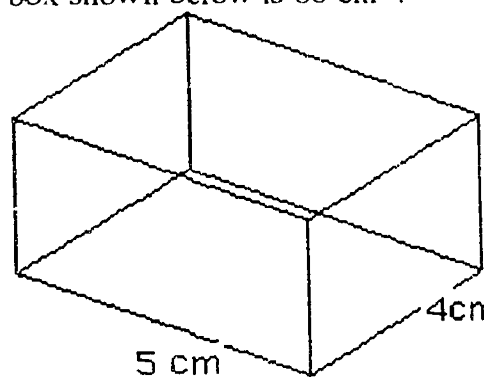
If one can of paint can cover 25 m^2, the number of cans of paint needed to cover only the curved surface is

A. 3
B. 4
C. 5
D. 6

20. The volume of the granery in Question 19 , correct to the nearest unit is

A. 47 m^3
B. 94 m^3
C. 141 m^3
D. 283 m^3

21. The volume of the box shown below is 60 cm^3.



The surface area is

A.. 188 cm^2
B. 120 cm^2
C. 60 cm^2
D. 94 cm^2

22. How many times larger is the surface area of a sphere than a circle of the same diameter?

A. π
B. 4
C. 2
D. 4π

23. How many cubes 2 cm on a side can be placed in a box 6 cm long, 6 cm wide, and 6 cm high?

A. 27
B. 18
C. 36
D. 54

24. Two cylindrical cans have the same height but the diameter of one can is 6 cm and the diameter of the other is 18 cm. How many times larger in volume is the larger can than the smaller?

A. 3
B. 3π
C. 9π
D. 9

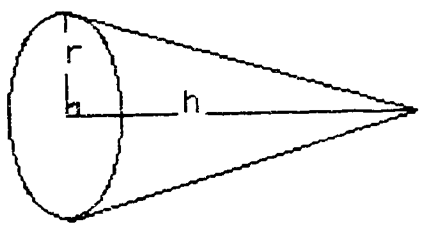
25. The volume of a sphere is gven by $V = \frac{4\pi r^3}{3}$. If the volume of a certain sphere is 288π cm^3, then its surface area is

A. 36π cm^2
B. 72π cm^2
C. 108π cm^2
D. 144π cm^2

26. The ratio of the volume of a pyramid with a square base to a cube with the same base is

A. 1: 3 B. 1: 2
C. 2 : 1 D. 3 : 1

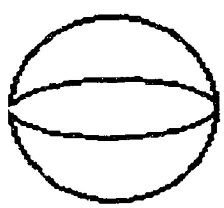
27. What formula should you use to calculate the volume of the shape in the figure below?



A. $\pi r^2 h$ B. $\frac{1}{3}\pi r^2 h$

C. $4\pi r^2 h$ D. $\frac{4\pi r^2 h}{3}$

28. Which of the figures below are NOT polyhedra?

1

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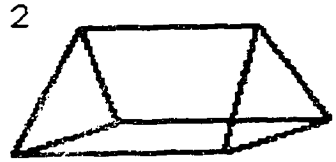
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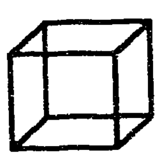
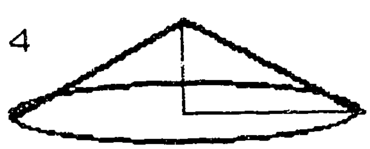
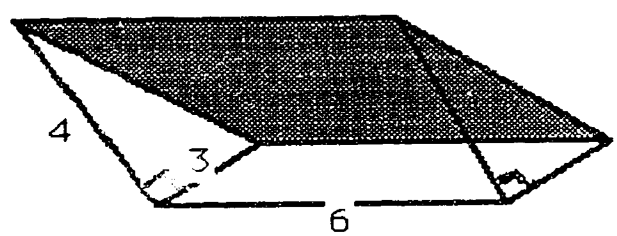

4


A. 2 and 4 B. 1 and 4
C. 1 and 3 D. 2 and 3

29. What is the area of the shaded side of the figure below?



A. 30
B. 24
C. 18
D. 36

30. Which of the following is Euler's formula?

A. F - V + E = 2
B. V - F + E = 2
C. V + F - E = 2
D. F - V - E = 2

TEST KEYS

SURVEY TEST		SYSTEMS		GEOMETRY	
1.	A	1.	B	1.	B
2.	A	2.	C	2.	A
3.	B	3.	A	3.	B
4.	C	4.	A	4.	C
5.	C	5.	D	5.	D
6.	D	6.	B	6.	B
7.	D	7.	B	7.	A
8.	C	8.	C	8.	A
9.	C	9.	B	9.	C
10.	B	10.	B	10.	A
11.	C	11.	C	11.	D
12.	B	12.	A	12.	C
13.	A	13.	D	13.	D
14.	A	14.	B	14.	C
15.	B	15.	C	15.	D
16.	A	16.	B	16.	B
17.	B	17.	D	17.	A
18.	D	18.	C	18.	C
19.	D	19.	D	19.	B
20.	B	20.	A	20.	C
21.	B	21.	B	21.	D
22.	A	22.	A	22.	B
23.	D	23.	C	23.	A
24.	A	24.	C	24.	D
25.	C	25.	D	25.	D
26.	B	26.	A	26.	A
27.	D	27.	C	27.	B
28.	B	28.	B	28.	B
29.	C	29.	B	29.	A
30.	D	30.	D	30.	C

APPENDIX 4

FACTOR ANALYSIS

STUDENT ATTITUDE QUESTIONNAIRE

FACTOR ANALYSIS OF ATTITUDE QUESTIONNAIRE

Eight factors were identified from the data obtained from the twenty-five item student-attitude questionnaire. Table 16 summarizes the factor analysis for the data obtained from 129 students, 60 from the classes of the teachers who implemented the treatment and 69 from control teachers classes.

Table 16

Factor Analysis of the Twenty-five Item Attitude Questionnaire

Oblique Solution Primary Pattern Matrix-Orthotran/Varimax

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Q1	.061	.007	-.035	.794	-.099	-1.77E-5	-.026	.253
Q2	.588	.159	.062	.362	.085	.005	.02	.056
Q3	.041	.053	-.093	.164	.358	.028	-.16	.452
Q4	-.053	-.006	.182	.266	.086	-.086	-.056	.669
Q5	.697	.086	.09	.175	.09	-.109	.134	-.029
Q6	.075	.775	-.016	.05	-3.69E-5	.253	.001	-.097
Q7	-.046	.764	-.004	-.042	.237	-.182	.025	-.063
Q8	.092	.796	-.054	-.08	-.106	-.03	-.045	.218
Q9	.153	.055	-.169	-.174	.157	.763	-.07	-.037
Q10	-.104	-.011	.269	.162	-.154	.767	.179	.075
Q11	.238	-.165	.719	.012	-.032	-.07	-.134	.122
Q12	-.016	.002	.778	-.11	.119	.102	.063	-.095
Q13	.766	.082	.239	-.136	-.066	.084	-.137	.041
Q14	.789	-.083	.001	-.052	.118	.075	-.015	-.025
Q15	.305	-.032	-.157	.053	.547	.188	-.385	.03
Q16	.343	-.018	-.189	-.299	.392	.045	-.097	.452

Orthogonal Transformation Solution-Varimax

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Q17	.395	.033	-.11	.035	-.076	-.144	.185	.623
Q18	.046	.18	.155	.187	.135	.104	.573	.122
Q19	.098	-.174	-.285	-.142	.08	.018	.724	.029
Q20	.624	.057	-.146	.338	.087	.001	.252	.255
Q21	.011	.072	.026	.097	.109	.137	.118	.724
Q22	.288	.2	.275	-.072	.595	-.068	.087	.375
Q23	.24	-.142	-.145	.538	.353	-.01	.03	.032
Q24	.01	.046	.074	.145	.8	.006	.221	.009
Q25	.267	.153	.285	.038	.675	.031	.141	.272

APPENDIX 5

TEACHER D

TEACHER D

Teacher D taught in a medium-sized senior-high school in a middle-class suburban neighbourhood. With both an undergraduate teaching degree, and a graduate degree in science, he was well qualified academically to teach both mathematics and science.

However, during the study, Teacher D was actively involved in coaching, and could not devote the time he would have liked to more thoroughly prepare lessons according to the model. Initially, he felt the lack of preparation time would prevent him from continuing in the study. As a result, there was limited evidence, during classroom observations, of his implementing the strategies for dealing with homework, using oral work, or teaching for meaning during lesson development.

Although not implementing all the features of the model in his Mathematics 23 class, when interviewed, at the conclusion of the study, one of the elements of the Interactive Teaching Model he had used and intended to continue was cooperative practice.

> Peer tutoring is effective. I find my students help each other cope with material they could not do on their own. They worked well during cooperative practice, accomplishing more than they had in the past.

Other factors which mitigated against his implementing the model were his having to cope with a number of new students who transferred into his class during the study, and student absenteeism and tardiness.

In spite of these limiting factors, he handled his students competently and effectively. It is unfortunate he did not have more time to devote to the model.