

Increasing transparency and accessibility of a slurry consolidation model in GoldSim

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ABSTRACT

Reclamation of slurry materials such as mine tailings requires effective management of consolidation-induced water release in the long-term. This is particularly challenging for flocculated or thickened tailings which require excessive time to fully consolidate to the final landform due to their low permeability and high clay content. Most software that predicts magnitude and rate of slurry consolidation are proprietary with a black-box implementation of the theory of one-dimensional consolidation of saturated clays by Gibson et al. This paper discusses the development of TMSim-Consol, an open-source slurry consolidation model implemented in GoldSim, a system-based simulation environment widely used by the mining and hydropower industry. The purpose of the paper is to demonstrate the feasibility of coding an explicit finite difference solution scheme in GoldSim. All variables and relationships in TMSim-Consol are explicitly modelled and visible to users in the free version of GoldSim (i.e. GoldSim Player). One of the model outputs include consolidation-induced water flow which can be added as an input parameter to existing water balance and geochemical models. The capability of the model is demonstrated by benchmarking against several validation case studies from current commercial software. This paper also discusses best practices and lessons learned during the modelling process.

RÉSUMÉ

La récupération des boues telles que les résidus miniers nécessite une gestion efficace de la libération d'eau induite par la consolidation à long terme. Ceci est particulièrement difficile pour les résidus flocculés ou épaissis qui nécessitent un temps excessif pour se consolider complètement jusqu'au relief final en raison de leur faible perméabilité et de leur forte teneur en argile. La plupart des logiciels qui prédisent l'ampleur et le taux de consolidation du lisier sont propriétaires avec une mise en œuvre en boîte noire de la théorie de la consolidation unidimensionnelle des argiles saturées par Gibson et al. Cet article traite du développement de TMSim-Consol, un modèle de consolidation de boues open-source implémenté dans GoldSim, un environnement de simulation basé sur un système largement utilisé par l'industrie minière et hydroélectrique. Le but de cet article est de démontrer la faisabilité du codage d'un schéma explicite de solution aux différences finies dans GoldSim. Toutes les variables et relations dans TMSim-Consol sont explicitement modélisées et visibles pour les utilisateurs dans la version gratuite de GoldSim (c'est-à-dire GoldSim Player). L'un des résultats du modèle comprend le débit d'eau induit par la consolidation qui peut être ajouté comme paramètre d'entrée aux modèles de bilan hydrique et géochimiques existants. La capacité du modèle est démontrée par une comparaison avec plusieurs études de cas de validation à partir de logiciels commerciaux actuels. Ce document traite également des meilleures pratiques et des leçons apprises au cours du processus de modélisation.

1 INTRODUCTION

Oil sands mining operations in the Athabasca region of Northern Alberta, Canada, generates large quantities of fluid fine tailings (FFT), a slurry waste by-product of the bitumen extraction process that currently occupies a total volume of more than 1.4 billion cubic metres (Alberta Energy Regulator 2021). If left untreated, these FFT will take decades to settle due to extremely low permeability and clay-rich composition (Hyndman et al 2018). To accelerate the dewatering of FFT, some mining operators started injecting high-molecular weight polyacrylamide polymers into FFT to bind clay particles together which produced a paste-like, high-density tailings, known as thickened tailings (TT) (Abdulnabi et al. 2021). TT is known to have higher permeability which facilitates dewatering after deposition but the effectiveness of thickening on enhancing undrained shear strength is less clear (Wilson et al. 2018). During active filling of tailings ponds, the high rate of production and low permeability limit the dissipation of excess pore pressure and development of effective stress in TT. Therefore, the dominant consolidation mechanism in TT is self-weight which only starts when no

new tailings are deposited at the end of mine life (Bonin et al 2014; COSIA 2012). Experimental studies and numerical simulations have shown that self-weight consolidation in TT can take decades to complete (COSIA 2012). Though the desiccated surface crust may have achieved sufficient strength to support reclamation activities, self-weight consolidation over a much longer period can still transport highly saline pore water upward to the vegetation cover, and depending on the magnitude of settlement, alter the final closure landscape (Hyndman et al. 2018; Wilson et al. 2018; Kabwe et al. 2019). Root uptake of the consolidation-induced saline water to the surface by deep-rooted vegetation poses ecological risks to the chemical stability of the closure landform (Kempenaar et al., 2017). Long-term vegetation survival is highly sensitive to the salinity of pore water (Cilia, 2018; Kessler et al. 2010). Final settlement greater than 2m may also transform the closure landscape from a marsh area to a lake (Hyndman et al. 2018), creating uncertainties in how vegetation and wildlife will adapt and respond to this transition.

The implication of self-weight consolidation in tailings slurry extends beyond the geotechnical discipline since the typical output parameters, namely the amount of

settlement and upward water flux, becomes the input for contaminant transport and geomorphic models. Tailings operations are multi-disciplinary in nature. Shared understanding across disciplinary boundaries is essential for successful tailings planning during and after the life of a mine. However, each discipline often uses different software platforms for numerical modelling purposes. The purpose of this paper is to demonstrate the potential to incorporate a large-strain consolidation (LSC) model called TMSim-Consol in GoldSim, a system-modelling simulation environment which has been extensively used in site-wide water balance evaluation, geochemical and contaminant transport modelling (Kossik and Miller 2004). Based on the author's experience, GoldSim is often used as a system-integration tool that performs first-order, time-dependent calculations. Most oil sands mining operators and their consultants have been using GoldSim in mine planning and surface-groundwater management practices. More complex models that require solutions to partial differential equations are typically run by commercial software which takes input from GoldSim and feeds output back to GoldSim (Cooper et al. 2018; Beier et al. 2020).

This paper will briefly discuss the theoretical background behind LSC models, conduct a literature review of the current state of practice and discuss the modelling philosophy in implementing LSC models directly in GoldSim. Another objective of this paper is to serve as a companion paper to TMSim-Consol, highlighting key features of implementing finite difference solutions in GoldSim and major improvement to the prototype model developed in Zheng (2019), all of which are too lengthy to demonstrate in textboxes in the actual model.

1.1 Theoretical Background

TMSim-Consol simulates self-weight settlement and settlement-induced surface flux in one dimension (i.e. the vertical direction), which are primarily driven by the large-strain consolidation process in the slurry under its self-weight forces.

The governing equations, presented by Equation 1. are derived from the large-strain consolidation theory developed by Gibson et al. (1967, 1981) who added the effect of self-weight to the original formulation by McNabb (1960). To solve the governing equation, two experimentally acquired constitutive relationships are needed: the void ratio-effective stress ($e-\sigma'$) relationship which can be obtained by performing a variety of strain-controlled and stress-controlled LSC tests (Suthaker 1995; Abu-Hejleh et al. 1996; Islam and Williams 2018; Burden 2021); and the void ratio-saturated hydraulic conductivity ($e-k$) relationship which can be obtained by constant head test with upward flow after each consolidation load step is completed (Suthaker and Scott 1996).

$$\pm \left(\frac{\rho_s}{\rho_f} - 1 \right) \frac{d \left[\frac{k(e)}{1+e} \right]}{de} \frac{\partial e}{\partial z} + \frac{\partial}{\partial z} \left[\frac{k(e)}{\rho_f(1+e)} \frac{d\sigma'}{de} \frac{\partial e}{\partial z} \right] + \frac{\partial e}{\partial t} = 0 \quad [1]$$

Where ρ_s is the solids density; ρ_f is the fluid density; e is the void ratio; k is the hydraulic conductivity expressed as function of void ratio; σ' is the effective stress

also expressed as function of void ratio; t is the time step; and z is the material coordinate.

1.2 Current State of Practice in LSC Modelling

A wide range of software using various numerical and analytical methods have been developed at research institutions and subsequently commercialized to solve Equation 1. Table 1 listed existing models with respective numerical schemes and key features. While other tools exist for small-strain consolidation with non-linear treatment of saturated permeability (K_s) and consolidation coefficient (C_v) (e.g. Settle3D), the summary in Table 1 is restricted to large-strain numerical models that specifically target soft tailings slurry. Without commercial licenses, accessibility is generally limited except the water-balance based GoldSim model from Stantec as any downloads and troubleshooting must be requested by users without publicly available web links.

Over the past decade, three-dimensional LSC modelling has been gaining ground in both research and commercial space (Jeeravipoolvarn et al. 2008; Fredlund et al. 2009; Zhou et al. 2019; Amodio et al. 2020). Most three-dimensional LSC tools are extensions of their one-dimensional counterpart (i.e. pseudo-3D) spatially superimposing several one-dimensional models (Fredlund et al. 2015). Due to challenges in validating three-dimensional models against experimental data, this paper will limit the discussion to the one-dimensional scenario.

Table 1. Summary of existing software and models for large-strain consolidation of tailings slurry.

Name	Numerical Scheme	Reference
CONDES0	Finite Difference Implicit	Yao and Znidarcic (1997)
FSCONSOL	Finite Difference Implicit	Pollock (1988)
CS2	Piecewise-Linear	Fox and Berles (1997)
FSCA	Finite Difference Implicit	Jeeravipolvaarn (2010)
UNSATCON	Piecewise-Linear	Qi et al. (2017)
PLAXIS-LE ¹	Finite Element	Fredlund et al (2009)
FLAC	Finite Difference Implicit	Zhou et al. (2019)
GoldSim (Stantec)	Water Balance	Brink (2019)

¹Formerly SVFlux developed by SoilVision

2 MODEL DEVELOPMENT

2.1 Solution to the Governing Equation

An analytical solution to Equation 1 is not possible due to the non-linearity of its coefficients. Finite difference

methods based on either explicit or implicit scheme are used to solve Equation 1. The numerical solution to Equation 1 is based on an explicit finite-difference scheme Cargill (1982). Unlike the implicit scheme described by Somogyi (1980) which was formulated in terms of excess pore pressure and adopted by most of the software listed in Table 1, the explicit theme in TMSim-Consol was formulated in terms of void ratio without the need to iteratively solve a system of linear equations. This is advantageous if the objective is to implement the solution entirely inside GoldSim since iteratively solving a matrix of equations require coupling GoldSim to external PDE solvers.

Zheng and Beier (2021) provided detailed description of the explicit finite difference solution to the partial differential equations in Cargill (1982). For the sake of completeness, a summary of key equations will be described here. Section 2.2 will demonstrate how TMSim-Consol implemented some of the solution equations in GoldSim.

At each time step, the explicit numerical scheme in Equation 2 updates void ratio at the current time step based on void ratio from the previous time step and the alpha and beta terms which are primarily functions of the slopes of the $K_s - e$ and $e - \sigma'$ relationships. Spatially, void ratio at the current layer also depends on the spacing and void ratios of the layer immediately above and below it.

$$e_{i,j+1} = e_{i,j} - \left(\frac{\Delta t}{\gamma_w}\right) \left[\left(\gamma_b \beta(e_{i,j}) + \frac{a(e_{i+1,j}) - a(e_{i-1,j})}{2\Delta z} \right) \left(\frac{e_{i+1,j} - e_{i-1,j}}{2\Delta z} \right) + a(e_{i,j}) \left(\frac{e_{i+1,j} - 2e_{i,j} + e_{i-1,j}}{(\Delta z)^2} \right) \right] \quad [2]$$

Where γ_w is the unit weight of water; γ_b is the buoyant unit weight of solids; γ_s is the unit weight of solids; i represents a specific discretized layer as part of spatial increment and j is the time increment; Δz is the mesh discretization in material coordinate and Δt is the time step. $a(e_{i,j})$ and $\beta(e_{i,j})$ depend on void ratio from previous time steps and slopes of the two constitutive relationships with respect to void ratio as described in the previous section:

$$a(e_{i,j}) = \frac{k(e_{i,j-1})}{1+e_{i,j-1}} \frac{d\sigma'}{de} \quad [3]$$

$$\beta(e_{i,j}) = \frac{d}{de} \left(\frac{k(e_{i,j-1})}{1+e_{i,j-1}} \right) \quad [4]$$

$$\beta(e_{i,j}) = \frac{dk(e_{i,j-1})}{de_{i,j-1}} \left(\frac{1}{1+e_{i,j-1}} \right) - \left(\frac{k(e_{i,j-1})}{(1+e_{i,j-1})^2} \right) \quad [5]$$

Where $k(e_{i,j-1})$ is the saturated hydraulic conductivity from the previous time step interpolated from the $K_s - e$ relationship; $d\sigma'/de$ is the slope of the $e - \sigma'$ relationship; and dk/de is the slope of the $e - K$ relationship. Equation 5 is the expanded derivation of Equation 4 using chain-rule.

2.2 Implementation of The Numerical Solution in GoldSim

2.2.1 Typical Layer Implementation

This section demonstrates how Equation 2 is implemented in GoldSim. A total of ten layers was chosen as a manageable discretization scheme. The ability to march time forward at different time step discretization is a built-in function in the software therefore no additional coding on the Δt term and the j term for each variable is necessary. On the input panel, users can enter the duration of the simulation and the basic time step (i.e. Δt) in the unit of choice.

The discretization of layers is implemented through the “container” concept in GoldSim. The “container” element looks like a box which is akin to a folder or directory on the typical computer where a group of elements and functions are housed inside. Each container named Layer# represents a single discretized layer (Figure 1). Arrows linking each container indicate that elements in those containers are related mathematically as shown by Equation 2. For example, the void ratio in Layer 5 is determined in part by the void ratio and the alpha terms in Layer 4 and Layer 6. Void ratio in Layer 5 along with other output is then fed into the “Results” container as indicated by the arrow in Figure 1.

In each layer “container”, Equation 2 is broken down in terms of its variables into GoldSim elements. Since time is inherently built-in by the software, no time element is explicitly shown in Figure 2 but instead time step Δt can be directly referenced as `TimestepLength |day|` as shown in Figure 3. The purpose of “|day|” is to remove the unit of choice for ease of computation. This unit removal practice is acceptable as long as all other elements adopt the same unit (i.e. on a metre per day basis for hydraulic conductivity). The unit removal maneuver has also been done for the unit weight (i.e. Y_b , Y_w) and spatial discretization term (i.e. Δz) as shown in Figure 3.

Elements with a PV in front of their names reference to values from the previous time (i.e. $j-1$). For the PV type element to work, users must specify an initial value for those elements since at time zero, there are no more values to reference from the previous time. For the alpha and beta terms in Equation 2, the initial value elements are represented by `a_e_initial`, `B_e_initial` and `dstress_de_initial` (see Figure 2). The initial value for the void ratio term PV_e has already been input by the user and is therefore not explicitly shown.

Two types of “containers” are used in the model: open-type such as `Consol_Input` and `Results` with a missing top; closed-type such as `Layer` with a closed top (see Figure 1). In closed-type containers, variables can only be referenced inside the container unless the developer specifies which variable can be exposed and made accessible from outside (see Figure 4). The advantage of using a closed-type container for each layer is that the same naming convention for each variable can be repeated in each layer without conflict, facilitating future addition or removal of layers. To reference variables from the above and underlying layers (i.e. closed-type

containers) as required by Equation 2, syntax typical of object-oriented programming languages is used (see Figure 3).

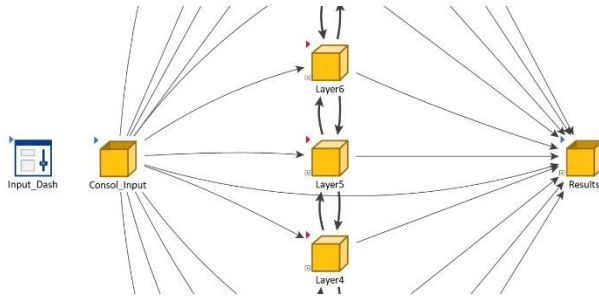


Figure 1. Layer organization using the “container” concept

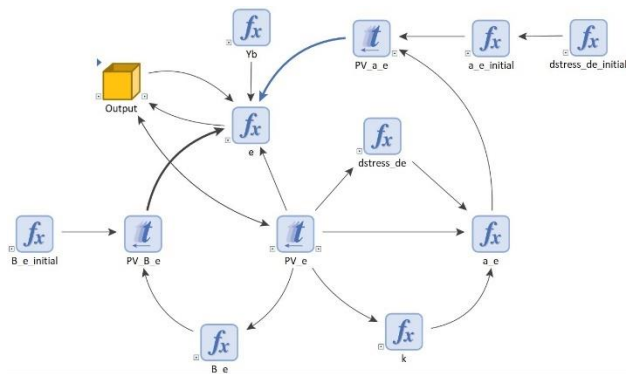


Figure 2. Implementation of Equation 2 to 5 in a typical layer “container”

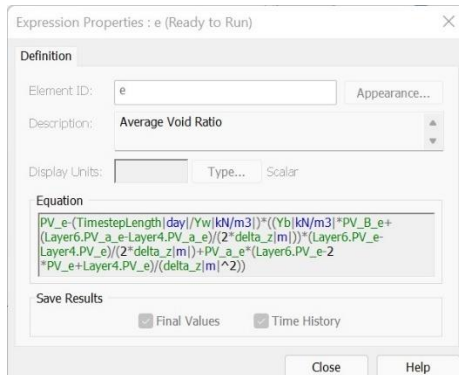


Figure 3. Details of coding and calculation for the “e” void ratio element in Layer5 container.

Container Properties : Layer2 (Ready to Run)

General Graphics Information Exposed Outputs

Output	Alias
\Layer2\ie	e
\Layer2\PV_e	PV_e
\Layer2\PV_a_e	PV_a_e
\Layer2\..Dissipation_	Dissipation_Percent
\Layer2\..settlement_h	settlement_h

Figure 4. Variables in the Layer2 “container” accessible to variables from other “containers” outside.

2.2.2 Bottom Imaginary Layer

A key feature of the explicit scheme is the use of an additional layer below the bottom boundary in the form of an imaginary mesh point. The bottom boundary is assumed to be impermeable since deep deposits of thickened tailings form a low permeability layer at the bottom, keeping downward seepage at a minimum (Jeeravipoolvarn et al. 2014). The void ratio in this imaginary layer, denoted as Layer0, at any time steps primarily depends on void ratios of the layer above the bottom layer (i.e. Layer 2) and the slope of the $e-\sigma'$ relationship:

$$e_{0,j} = e_{2,j} + 2\Delta z \left(\frac{de}{d\sigma'} \right)_{e_{i,j}} * \gamma_b \quad [6]$$

Where $\frac{de}{d\sigma'}$ is the slope of the curve determined from the $e-\sigma'$ relationship at $e_{0,j-1}$ (i.e. imaginary void ratio from the previous time step). Once the fictitious $e_{0,j}$ is calculated at the current time step j , then $e_{1,j+1}$ in Layer 1 (i.e. void ratio of the layer above the imaginary layer) at the next time step can be determined from Equation 2. The entire process is repeated at the next time step.

At the beginning of the consolidation process, the slope term $de/d\sigma'$ can be highly negative due to the large amount of settlement typically observed in soft tailings, potentially driving $e_{0,j}$ to negative values and subsequently affecting void ratios in Layer 1 and above. To fix this issue, the minimum value that $e_{0,j}$ can take at time step j is restricted to the minimum final void ratio derived from the end-of consolidation effective stress experienced at the bottom of the tailings deposit. This fix was implemented by an IF statement shown in Figure 5 where δ_{z_j} , Y_s , Y_b correspond to the definitions in Equation 2; PV_e is the void ratio from the previous time step and $de_dstress$ is the inverse of the slope in the $e-\sigma'$ relationship; $Final_e$ is void ratio interpolated from the $e-\sigma'$ relationship where the corresponding effective stress is the buoyant unit weight times the thickness of the entire deposit.

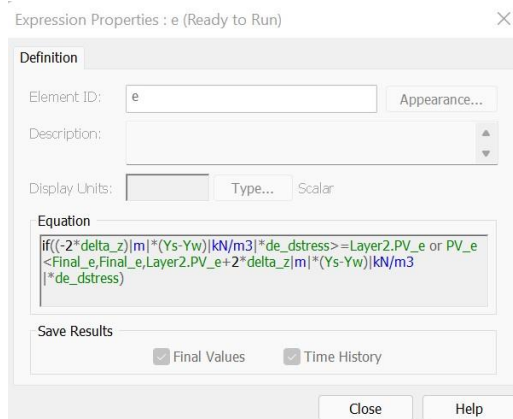


Figure 5. Details of coding and calculation for the “e” void ratio element in the bottom imaginary layer.

2.2.3 Top Imaginary Layer

The function of the top imaginary layer is identical to that of the bottom imaginary layer. Void ratio, hydraulic conductivity, the alpha, and beta terms in the top imaginary layer are based on their initial values at time zero and throughout the simulation. Therefore, the top imaginary layer does not contribute to the total settlement and consolidation-induced flux of the entire deposit.

2.3 User Input

A graphical user input panel is shown in Figure 6. The design uses a step-wise approach to clarify the workflow required to run the simulation with access to the model structure placed on the right side. Under the Step 1 panel, initial solids content is used instead of initial water content due to common practices of using solids content in the oil sands industry.

The e-K and e- σ' relationships are typically input in the form of a curve-fitting function (e.g. power law, Weibull) in the commercial and academic software listed in Table 1 with slope terms in Equation 2 calculated analytically as a derivative with respect to void ratio. For maximum flexibility, these curve-fitted relationships must be entered as tabulated values in this model. The slope terms in Equation 2 are calculated between two points in the relationship using interpolated table entries and the user-specified data step which has a default value of 0.001. One disadvantage of using tabulated values is that users must generate a table of values themselves in a spreadsheet then copy the values in the e_Sigma and e_Ksat tab (see Figure 6).

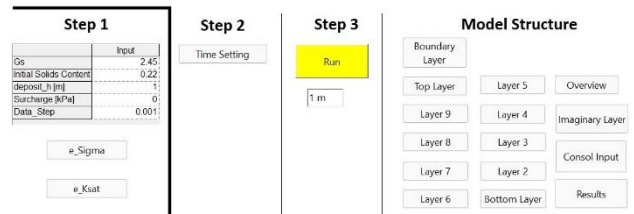


Figure 6. User input panels and access to the model structure.

2.4 Modelling Philosophy

A top-down approach was adopted during the development of TMSim-Consol, starting from simple models consisting of only three layers and gradually adding more layer “containers” after calibration. The closed type “container” element allows for rapidly adding more layers (i.e. finer discretization) without the need to rename variables inside the container. However, finer discretization must be done manually by adding layer “containers” where slight modifications of Equation 2 are required. The placement of influence arrows and function elements is carefully designed to avoid visual congestion and enhance transparency of the model (Figure 2).

A key consideration in the visualization of the numerical solution is to minimize the amount of coding inside each element. For example, in Figure 2, it is possible to embed the calculation for the a_e and B_e terms inside the void ratio element. However, doing so would make coding in the void ratio element appear congested. Therefore, as in the case of a_e and B_e, separate elements are created for each component in the numerical solution to maximize the exposure of these components in the user panel.

3 BENCHMARKING AND VALIDATION

Three types of tailings material (phosphate, caustic and non-caustic MFT) and one type of natural material (river sediment) with different experimental setup were used to benchmark and validate TMSim-Consol against measured data and commercial software (FSCA and PLAXIS-LE/SoilVision). Figure 7 showed good agreement between TMSim-Consol and the validation dataset. For clarity, experimentally measured data were omitted in Figure 7; however, further discussion on validation can be found in Zheng (2019) for the prototype model, and Zheng and Beier (2021) for the updated model. Due to new experimental data and improved algorithms from commercial software, validation should be an on-going, continual process (Fredlund et al. 2009). Therefore, future improvement in the boundary condition handling, the number of layer discretization and numerical stability are expected.

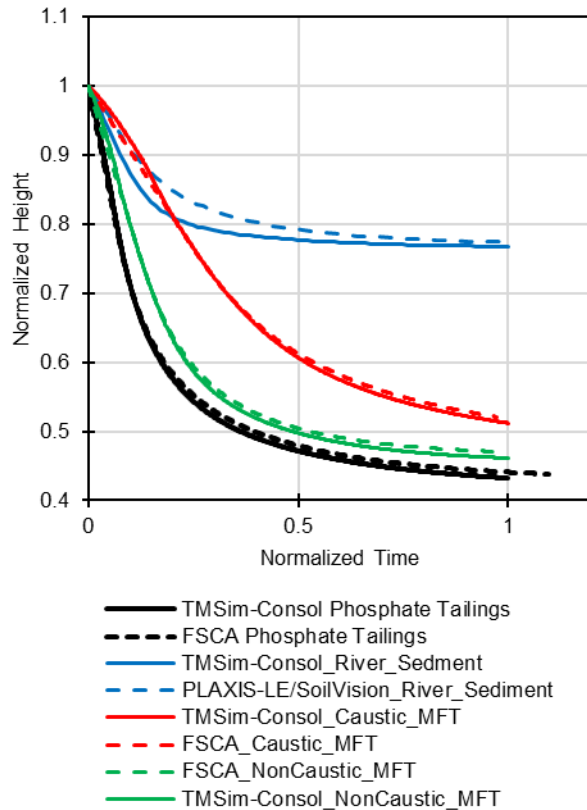


Figure 7. Normalized benchmark results of TMSim-Consol against commercial software

4 LIMITATION

While TMSim-Consol can simulate the self-weight consolidation process in tailings slurry, the following limitations require improvement of the model in future versions:

- Additional option to allow for double-drainage and hydrogeological boundary conditions at the top and bottom layer.
- Ability to correctly model the effect of surcharge on the consolidation process; currently the surcharge option has not been validated.
- Ability to model continuous and intermittent filling during operation
- Additional layer discretization to better handle deep deposits (i.e. for deposit thickness greater than 10m).
- Ability to input spatially variable material properties as initial values.
- Ability to couple with other numerical models (i.e. incorporate the effect of desiccation and rewetting at the top boundary).

5. CONCLUSION

A physically based numerical model for large-strain consolidation of slurry materials was developed in the GoldSim simulation environment, where all variables and mathematical relationships in the finite difference solution were viewable to end users. The GoldSim software is an effective tool for bringing a shared understanding of the mathematical relationships governing the large-strain consolidation process. The ability to run a one-dimensional self-weight consolidation model entirely in GoldSim reduces the need for additional coding to link external software and facilitates integration with water balance and geochemical models used in operations and mine planning.

6. DATA AVAILABILITY

All the numerical codes can be viewed by the user with a free, downloadable software called “GoldSim Player” which allows the model to be distributed as an open-source simulation tool. In the player file, users can choose input parameters and access the numerical codes through a centralized interface (Figure 6). A full license of GoldSim is required for modifying the model.

The player file for TMSim-Consol is currently available for download under an Attribution Non-Commercial International 4.0 (CC BY-NC 4.0) license at <https://doi.org/10.7939/r3-vqbb-va13> on the ERA website of University of Alberta.

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