

# Modelling and simulation of three-phase transformers for inrush current studies

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**Abstract:** A fast and stable approach for the simulation of transformer nonlinearities during transient and unbalanced operating conditions is given. The proposed scheme implements separate magnetic and electric equivalent circuits. The solution of the transformer nonlinear mathematical model is carried out using the Newton–Raphson iterative method. Introducing the magnetic circuit nonlinearities into the model as either continuous or piecewise functions eliminated the difficulty of the Jacobian formation. The method was verified against experimental and computed results. The application of the Jacobian iterative method increased the stability and convergence of the solution as compared to a predictor corrector scheme. The proposed method was able to accurately simulate the transformer behaviour under switching and no-load conditions.

## 1 Introduction

The modelling and simulation of transformer and reactor nonlinearities is especially important for studying inrush currents, ferroresonance, harmonics and subharmonics. Some of the transformer models are based on the analysis of the electromagnetic field in the transformer [1–6]. Another approach is to model the transformer using lumped circuit models [7–9]. The first approach tends to represent the transformer using two separate equivalent circuits, one for the magnetic circuit and the other representing the transformer electric circuit. For the second approach, the magnetic behaviour and characteristics are modelled in a single electrical circuit with nonlinear inductors with the nonlinear characteristics being a function of each nonlinear inductor current.

From the implementation point of view, the main difficulties of using lumped circuit models are: first, the equivalent circuit will become quite complicated [9] and secondly, the resulting impedance matrices might become ill-conditioned for certain network connections. For the case of single-phase transformers, models have been proposed using the classical transformer equivalent circuit with a single nonlinear inductor representing each phase of the 3-phase transformers [7]. The approximation of the transformer model using three single-phase transformers can lead to serious errors, especially if the transformer behaviour under transient and/or unbalanced conditions is to be studied [10]. Another problem associated with transformer transient simulation is related to the solution algorithm of the mathematical model. For example, the EMTP uses the classical compensation method to solve for the system solution [11, 12]. The system is first solved for Thevenin equivalents ignoring the nonlinear elements. Next,

the intersection of the system solution curve with the nonlinear element characteristic represents the current operating point. The problem with this method is that the Thevenin equivalent cannot always be determined due to possible floating network formations.

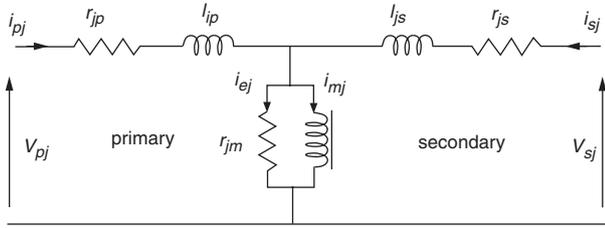
This paper presents a mathematical modelling technique and a solution algorithm based on the dual magnetic–electric circuit. The proposed method was found to be stable and accurate for transformer transient simulations. The method can use either piecewise or fitted nonlinear characteristic curves to formulate the Jacobian matrix. The proposed method has two more major advantages: first, since the model is linearised using the trapezoidal integration rule it can be easily implemented in EMTP like programs. Secondly, the formation of the Jacobian is only done once using the continuous curve fitting technique or a maximum of two times if the piecewise characteristic curves are used, making the proposed approach fast as compared to other simulation schemes.

## 2 Transformer model description

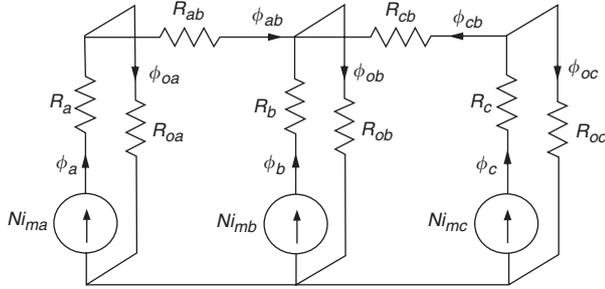
Many nonlinear transformer models have been proposed [1, 4, 6–9]. After evaluating a number of published models, we found that a dual electric–magnetic circuit model proposed in [1, 4] has a good balance between accuracy and complexity. The model is also general enough to simulate various transformer transient and/or nonlinear phenomena. In addition, our experiments independently verified the validity of the model.

The transformer model implemented applies two separate equivalent circuits to model the transformer. One is the electric circuit model and the other is the magnetic circuit model. Generally, for a three-limb transformer with neutral wire, the equivalent electric circuit for each phase is as shown in Fig. 1. The magnetic circuit of the transformer can be described by considering the transformer fluxes and equivalent magnetic circuit shown in Fig. 2.

In Fig. 1,  $r_{ap}$ ,  $r_{bp}$ ,  $r_{cp}$  and  $l_{ap}$ ,  $l_{bp}$ ,  $l_{cp}$  represent the resistances and leakage inductances respectively, per phase of transformer primary winding. These resistances and leakage inductances can be assumed to be linear. The shunt resistances  $r_{ma}$ ,  $r_{mb}$  and  $r_{mc}$  represent the combined eddy



**Fig. 1** Transformer electrical equivalent circuit referred to the primary side



**Fig. 2** Equivalent magnetic circuit and flux distribution for three-limb, three-phase transformer

and hysteresis losses (iron losses) of the respective phases. These magnetising resistances are nonlinear in nature and can be approximated with a constant value calculated at the fundamental frequency of the induced voltage [7, 13]. The nonlinear magnetising inductances are represented as an induced voltage depending on the respective limb flux ( $N \cdot d\phi_j/dt$  where  $j=a, b$  or  $c$ ). For the transformer secondary windings, only the winding resistances  $r_{as}, r_{bs}, r_{cs}$  and the leakage inductances  $l_{as}, l_{bs}$  and  $l_{cs}$  are represented. The system of equations describing the electric equivalent circuit can be written as follows:

$$\begin{aligned}
 V_j &= r_{jp} \cdot i_{jp} + l_{jp} \cdot di_{jp}/dt + N \cdot d\phi_l/dt + V_n \\
 V_n &= r_n \cdot (i_a + i_b + i_c) \\
 N \cdot \sum_{j=a,b,c} d\phi_j/dt &= \sum_{j=a,b,c} r_{js} \cdot i_{js} + \sum_{j=a,b,c} l_{js} \cdot di_{js}/dt \quad (1) \\
 i_{mj} &= i_{jp} - \frac{N}{r_{mj}} d\phi_j/dt - i_{js}
 \end{aligned}$$

$i_{as} = i_{bs} = i_{cs} = i_\Delta$  for an unloaded secondary side operation

From the flux distribution and the equivalent magnetic circuit shown in Fig. 2, the system of equations describing the magnetic circuit behaviour at any instant of time is given by (2) and (3).

$$\begin{aligned}
 \mathfrak{R}_{Lj} \cdot \phi_j + \mathfrak{R}_{Oj} \cdot \phi_{Oj} &= N \cdot i_{mj} \\
 \mathfrak{R}_{La} \cdot \phi_a + \mathfrak{R}_{Ya} \cdot \phi_{ab} - \mathfrak{R}_{Lb} \cdot \phi_b &= N \cdot i_{ma} - N \cdot i_{mb} \quad (2) \\
 \mathfrak{R}_{Lc} \cdot \phi_c + \mathfrak{R}_{Yc} \cdot \phi_{cb} - \mathfrak{R}_{Lb} \cdot \phi_b &= N \cdot i_{mc} - N \cdot i_{mb}
 \end{aligned}$$

$$\begin{aligned}
 \phi_a &= \phi_{oa} + \phi_{ab} \\
 \phi_b &= \phi_{ob} - \phi_{ab} - \phi_{cb} \\
 \phi_c &= \phi_{oc} + \phi_{cb}
 \end{aligned} \quad (3)$$

where

- $N$  number of turns per phase
- $i_{ml}$  primary side phase magnetising currents ( $i_{ma}, i_{mb}$  and  $i_{mc}$ )
- $\phi_j$  magnetic fluxes which link different phases and follow the magnetic circuit ( $\phi_a, \phi_b, \phi_c, \phi_{ab}, \phi_{cb}$ )

- $\phi_{oj}$  phase leakage fluxes PLF which link all the windings of one phase, and partially go through air
- $\mathfrak{R}_{Lj,Yj}$  nonlinear reluctances seen by the associated fluxes ( $\phi_a, \phi_b, \phi_c, \phi_{ab}, \phi_{cb}$ )
- $\mathfrak{R}_{ol}$  constant reluctances seen by the associated phase leakage fluxes PLF ( $\phi_{ab}, \phi_{bc}, \phi_{ca}$ ) for a three-limb three-phase transformer

### 3 Transformer model implementation

#### 3.1 Model linearisation

The previously described system of equations consists of eight equations in eight unknowns. The system consists of five algebraic nonlinear equations and three linear ordinary differential equations. To solve the complete set of equations at the same time, the differential equations need to be discretised and implemented as difference equations in the model. The resulting system will consist of eight discrete time equations, which are sufficient to solve for the unknown fluxes and currents at each instant of time  $t$ .

Due to its stability and proven accuracy, the trapezoidal rule of integration will be applied on the nonlinear system of equations. By applying the trapezoidal rule of integration on the nonlinear ordinary differential equations (1) with an integration time step of  $\Delta t = t - t_o$ , the linearised difference equations (4) and (5) can be obtained for a numerical solution at any instant of time  $t$ .

$$\begin{aligned}
 V_j \cdot dt &= r_{jp} \cdot i_j \cdot dt + l_{jp} \cdot di_j + N \cdot d\phi_j + V_n \cdot dt \\
 \int_{t_o}^t V_l \cdot dt &= r_{jp} \cdot \int_{t_o}^t i_l \cdot dt + l_{jp} \cdot \int_{t_o}^t di_l + N \cdot \int_{t_o}^t d\phi_j \\
 &+ V_n \cdot \int_{t_o}^t dt \\
 [V_j(t) + V_l(t_o)] \frac{\Delta t}{2} &= r_{lp} [i_l(t) + i_l(t_o)] \frac{\Delta t}{2} + l_{lp} [i_l(t) - i_l(t_o)] \\
 &+ N [\phi_l(t) - \phi_l(t_o)] + [V_n(t) + V_n(t_o)] \frac{\Delta t}{2} \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 [i_{ml}(t) + i_{mj}(t_o)] \frac{\Delta t}{2} &= [i_{mj}(t) + i_{mj}(t_o)] \frac{\Delta t}{2} \\
 &- \frac{N}{r_{mj}} [\phi_j(t) - \phi_j(t_o)] \quad (5) \\
 V_n(t) &= r_n \cdot [i_a(t) + i_b(t) + i_c(t)]
 \end{aligned}$$

where

- $j$  a, b and c
- $t$  present sampling step
- $t_o$  preceding sampling step
- $\Delta t$  sampling time step

Extending the above equation for three phases, the equations will become a function in eight unknowns, i.e. the magnetic fluxes ( $\phi_a, \phi_b, \phi_c, \phi_{ab}, \phi_{cb}$ ) and the phase currents ( $i_a, i_b, i_c$ ) and ten known quantities: the history values of fluxes  $\phi(t_o)$ , currents  $i(t_o)$ , present and history values of phase voltages  $V(t)$  and  $V(t_o)$  and present and history values of neutral point voltage  $V_n(t)$  and  $V_n(t_o)$ .

At the start of the simulation, the initial values for the fluxes and currents are required. An initial value of zero for all fluxes and currents is typical for a relaxed initial condition. It also can be stated that the case of remnant flux

can be easily implemented by setting a proper initial value for the fluxes. The accuracy of the solution depends on the proper choice of  $\Delta t$ .

Rearranging (4) for the unknowns on the left-hand side and the known quantities on the right-hand side

$$\begin{aligned} & \left( r_{lp} \cdot \frac{\Delta t}{2} + l_{jp} \right) \cdot i_{jp}(t) + N \cdot \phi_l(t) + v_n(t) \cdot \frac{\Delta t}{2} \\ & = (v_{jp}(t) + v_{jp}(t - \Delta t)) \cdot \frac{\Delta t}{2} - \left( r_{jp} \cdot \frac{\Delta t}{2} - l_{lp} \right) \\ & \cdot i_{jp}(t - \Delta t) + N \cdot \phi_l(t - \Delta t) + v_n(t - \Delta t) \cdot \frac{\Delta t}{2} \end{aligned}$$

The system of equations can be described in matrix form as

$$[\mathfrak{R}, Z]_{8 \times 8} \cdot [\phi, i]_{8 \times 1} = [\phi_o, i_o, v, v_o]_{8 \times 1} \quad (6)$$

where

$[\mathfrak{R}, Z]$  reluctance and impedance matrix

$[\phi, i]$  vector of unknowns given by

$(\phi_a \ \phi_b \ \phi_c \ \phi_{ab} \ \phi_{cb} \ i_{ap} \ i_{bp} \ i_{cp})^T$

$[\phi_o, i_o, v, v_o]$  vector of history values and current voltage values given by  $(0 \ 0 \ 0 \ 0 \ 0 \ v_{xa} \ v_{xb} \ v_{xc})^T$

$$\begin{aligned} v_{xt}(t) &= (v_{jp}(t) + v_{jp}(t - \Delta t)) \cdot \frac{\Delta t}{2} \\ &- \left( (r_{jp} + r_n) \cdot \frac{\Delta t}{2} - l_{jp} \right) \cdot i_{jp}(t - \Delta t) \\ &+ N \cdot \phi_i(t - \Delta t) - v_n(t - \Delta t) \cdot \frac{\Delta t}{2} \end{aligned}$$

### 3.2 Representation of the reluctance curves

The nonlinear reluctance-flux characteristics will be presented as a nonlinear function of the respective flux. The leakage path reluctances  $\mathfrak{R}_{aj}$  are constant reluctances and can be obtained by experimental tests [1]. The limb and yoke reluctances ( $\mathfrak{R}_a, \mathfrak{R}_b, \mathfrak{R}_c, \mathfrak{R}_{ab}, \mathfrak{R}_{cb}$ ) are nonlinear in nature and can be computed from the actual transformer design dimensions and material parameters from the manufacturer or through experimental tests [1, 4]. The  $\mathfrak{R} - \phi$  curve can then be fitted using a high-order polynomial and hence the limb and yoke reluctance at any step can be easily calculated using the fitted curve.

$$\mathfrak{R}_L(\phi) = 10 \sum_{i=0}^N C_{Li} \cdot \phi^i, \quad \mathfrak{R}_Y(\phi) = 10 \sum_{i=0}^N C_{Yi} \cdot \phi^i \quad (7)$$

$C_{Li}$  and  $C_{Yi}$  are the polynomial parameters of the curve fitting for the limb and yoke reluctances, respectively, and  $N$  is the polynomial order. It was found that a suitable polynomial order for these curves is in the range 9th–20th degree. The more the details are required to be presented in the fitted curve, the higher will be the polynomial order. For example, the fitted yoke reluctance–flux curve shown in Fig. 3 was generated using a 12th order polynomial while for the limb, due to the slowly varying nature of the curve for high saturation levels, the polynomial order was raised to 16th. For low and extreme values of flux the reluctance–flux curve was assumed to be constant and the fitting was applied only to the nonlinear portion in between.

## 4 Solution methodology

The objective of the simulation algorithm becomes the solution of (6) at each time step. The difficulty encountered

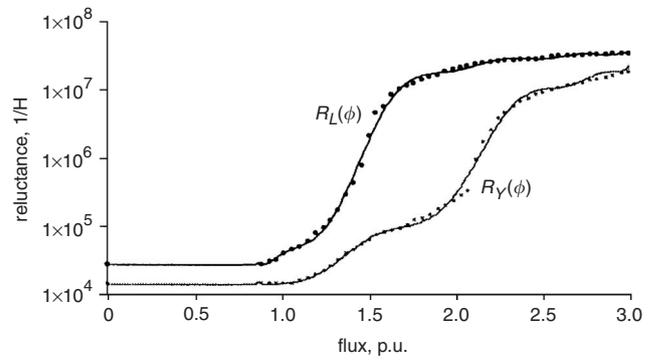


Fig. 3 Fitted  $R$ - $\phi$  curve for a typical three-limb transformer

is that the  $[\mathfrak{R}, Z]$  matrix is a nonlinear function of the solution results vector  $[\phi, i]$ . Accordingly, an iterative method has to be implemented to solve the equation. Three solution algorithms for solving the set of discrete time equations (6) were studied. Two of them are described here in detail: the predictor–corrector solution algorithm and the Newton–Raphson method. The choice of the predictor–corrector method and the Newton–Raphson method to be studied in detail was based on the fact that both methods can be described in matrix form and both can be easily implemented as an automated solution algorithm in EMT-type programs.

### 4.1 Predictor–corrector method

A predictor–corrector scheme for solving the model is now described. Generally, the solution for the unknown flux and current values can be obtained using (6). Since the matrix  $[\mathfrak{R}, Z]$  is nonlinear and has elements depending on the ‘present’ solution of  $\phi_i$  and  $i_j$ , a predictor–corrector method is used within each time step to calculate the correct  $[\mathfrak{R}, Z]$  matrix.

The predictor–corrector method can be applied as follows:

- (1) Matrix  $[\mathfrak{R}, Z]$  can be calculated initially from  $\phi$  and  $i$  values at  $(t - \Delta t)$  or from the initial conditions at the first time step i.e.  $t = 0$ .
- (2) The calculated value of  $[\phi, i]$  using the initial  $[\mathfrak{R}, Z]$  matrix calculated in (1) are used as a *predictor* for the solution.
- (3) Since the exact solution of  $[\phi, i]$  can only be found by using the matrix  $[\mathfrak{R}, Z]$  at the current solution of  $[\phi, i]$  which is not yet available, the predictor found in (2) is again used to calculate a new matrix  $[\mathfrak{R}, Z]$ .
- (4) New  $[\mathfrak{R}, Z]$  matrix calculated in (3) is used to calculate a *corrector* value of  $[\mathfrak{R}, Z]$ .
- (5) If the difference between the predictor and corrector values of the solution matrix  $[\phi, i]$  calculated in steps (2) and (4) are within a predefined tolerance  $\epsilon$ , proceed to the next time step.
- (6) If the error is larger than  $\epsilon$ , go to step 2 again.

### 4.2 Newton–Raphson method

The predictor–corrector scheme described earlier was found unsuitable for the problem of transformer transient simulation mainly due to the highly nonlinear behaviour of transformers especially during transient and saturation operating modes.

The original system model is described by (6). The system can be solved using the Newton–Raphson method by reformulating the system solution algorithm. For a solution at any time step  $t$  and an iteration  $k$ , the solution can be obtained as given by (8).

$$[\phi, i]_k = [\phi_o, i_o] - J^{-1}([\mathfrak{R}, Z]_{k-1} - [\phi_o, i_o, v, v_o]) \quad (8)$$

where  $J$  the Jacobian matrix for the system and is generated for each iteration  $k$ . The formulation of the Jacobian matrix is based on the total differential of function  $f$  at a solution  $x$  using the Taylor expansion and hence, the solution of  $f(x + \Delta x)f(x) + df(x)$  [14, 15]. The  $J_{m,n}$  element of the Jacobian matrix can be calculated using  $J_{m,n} = \partial f_m / \partial x_n$  where  $f_m$  is the system equation representing row  $m$  in matrix  $[\mathfrak{R}, Z]$  and  $x_n$  is the variable represented in column  $n$ . Although reluctances were not being considered as independent variables in the mathematical model, the partial derivatives of the reluctances were possible since they are dependent variables of respective flux variables and hence the terms  $R_i \phi_i$  can be mathematically expressed as two multiplied functions in one variable  $\phi_i$ . The partial derivative of  $R_i$  will become a constant ‘slope’ if the reluctances are implemented using piecewise segments and will become a continuous variable in  $\phi_i$  if the reluctances are represented as a continuous function of  $\phi_i$ . The reluctance derivatives can be evaluated by differentiating the fitted reluctance flux function with respect to flux as given by (9).

$$\partial \mathfrak{R}(\phi) / \partial \phi = 10 \sum_{j=0}^N C_j \phi^j \cdot \sum_{j=1}^N j \cdot C_j \cdot \phi^{j-1} \cdot \ln(10) \quad (9)$$

where

$[\phi, i]_k$	solution vector of system at iteration $k$
$[\phi_o, i_o]$	previous solution vector of system at time $(t - \Delta t)$
$[\mathfrak{R}, Z]_{k-1}$	reluctance–impedance matrix of system (described earlier) at previous iteration
$[\phi_o, i_o, v, v_o]$	vector of history variable values at $(t - \Delta t)$ and current voltage values at time $t$ .

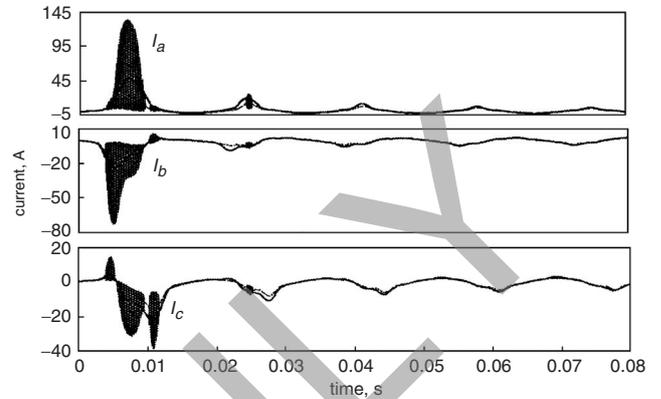
The proposed Newton–Raphson solution scheme can be summarised as follows:

- (1) Calculate the vector of history variable values  $[\phi_o, i_o, v, v_o]$  from the solution at the previous time step and the value of the current phase voltages. This is only done once in each time step
- (2) Generate matrix  $[\mathfrak{R}, Z]$  using  $\phi$  values at  $(t - \Delta t)$  or from initial conditions at the first time step  $t = 0$
- (3) The Jacobian matrix  $J$  is generated using the values of fluxes  $\phi$  at  $(t - \Delta t)$  to calculate the reluctances and reluctances partial derivatives. For next iterations the newer values of  $\phi$  will be used
- (4) The mismatch solution vector is calculated as:  $[\Delta \phi, \Delta i] = [J]^{-1} \cdot ([\mathfrak{R}, Z], [\phi, i] - [\phi_o, i_o, v, v_o])$
- (5) New solution vector is calculated by:  $[\phi_1, i_1] = [\phi_o, i_o] - [\Delta \phi, \Delta i] = [J]^{-1}$
- (6) If the largest number in the modulus of the mismatch vector  $[|\Delta \phi|, |\Delta i|]$  is greater than  $\varepsilon$  then; (i) set the initial values vector  $[\phi_o, i_o] = [\phi_1, i_1]$ ; (ii) return to step 2.
- (7) If the largest number in, the modulus of the mismatch vector  $[|\Delta \phi|, |\Delta i|]$  is less than or equal  $\varepsilon$  then; (i) set the solution vector  $[\phi_1, i_1] = [\phi_1, i_1]$ ; (ii) proceed to the next time step.

## 5 Results and discussion

### 5.1 Predictor–corrector scheme

The predictor–corrector scheme was implemented to simulate the transformer described in [1, 4]. The method was found to be suitable and convergent, for the simulation of single-phase switching. For three-phase switching, the method failed to converge and the solution contained heavy numerical oscillations regardless of how small the time step was (Fig. 4).



**Fig. 4** Phase inrush currents for instantaneous three-phase switching for transformer given in [1, 4]

--- predictor–corrector scheme  
— Newton–Raphson scheme

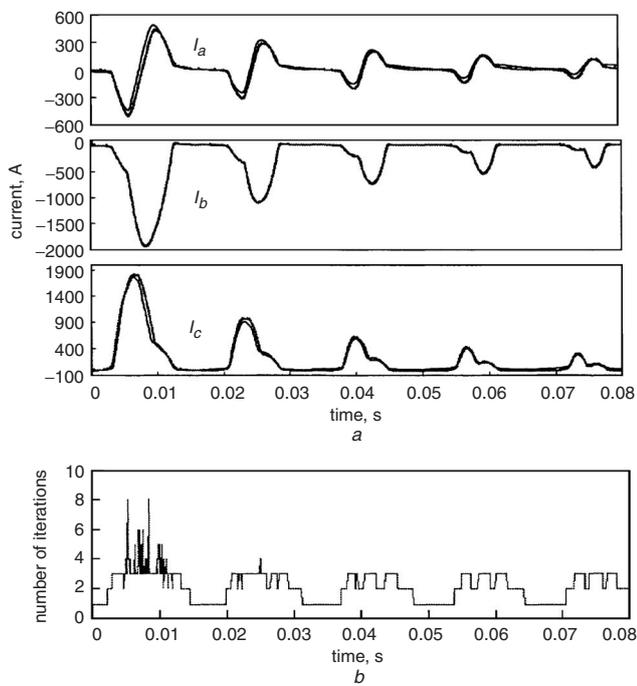
The reason for the oscillations shown in Fig. 4 is that, the transformer model is being driven more heavily into saturation during instantaneous three-phase switching and thus, increasing the nonlinearity of the system operating in the nonlinear portion of the reluctance curves. The method oscillates between two operating points for an even number of iterations while for an odd number of iterations, it only picks up one part of the solution.

### 5.2 Instantaneous three-phase switching using Newton–Raphson scheme

The proposed Newton–Raphson method was tested on single-phase, instantaneous three-phase and sequential three-phase switching cases. In all simulated cases, the model was stable and converging with a time step of no less than  $50 \mu\text{s}$ . Figure 5 shows a comparison with the experimental results on the transformer described in the appendix for instantaneous three-phase switching using the Yg–D connection.

Fig. 5 clearly shows the simulated and measured phase current values are in close agreement. The simulation procedure requires three iterations for saturation region operation where the current experiences sharp magnitude rises and a single iteration for low saturation levels. For extreme saturation operation conditions, the number of iterations rises to eight to achieve the required maximum converging tolerance of  $10^{-6}$ .

The measured terminal voltage was fed into the simulation program with the recorded resolution. The sampling frequency was 256 samples/cycles, which dictates a simulation time step of about  $65.1 \mu\text{s}$ . Such a relatively high simulation time step reflects the rigidity and the convergence nature of the developed algorithm. The time required for the simulation can be effectively reduced



**Fig. 5** Instantaneous three phase switching of  $Yg-\Delta$  transformer described in Appendix

a — simulation

--- measured though currents

b maximum number of iterations at each time step

through the application of a variable time step in lower saturation levels which represent about 50% of the total simulation time. Due to the nature of the implemented discretisation method, the variable time step can be easily implemented using dual multiplications of the original time step, i.e.  $\Delta t = 2n \cdot (\Delta t_o)$

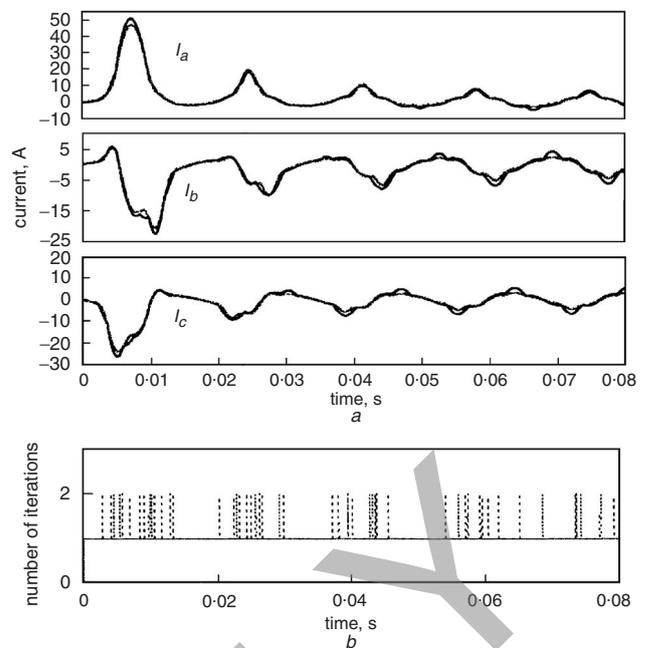
### 5.3 Piecewise curve fitting technique

To verify the capability of the proposed method to using the classical piecewise fitting technique for the transformer nonlinear reluctances, simulation cases were carried out for different numbers of piecewise segments. Figure 6 shows the simulation results using a nonlinear reluctance curve with nine equal-length segments. The results showed that the proposed method also converges for the piecewise curve implementation. This is an important finding since in some cases it is neither simple nor possible to have a best-fitted curve for the nonlinear characteristics.

As can be shown in Fig. 6, the piecewise technique results in a maximum error of 5% during the first cycle and 30% as the transformer inrush current decays. Increasing the number of segments will enhance the simulation results. In the other hand, the number of iterations is increased due to the introduction of more segments which will introduce more breakpoints and hence require more iterations.

## 6 Conclusions

A new application of the Newton–Raphson iterative method for solving three-phase nonlinear transformer models has been presented. The application of the Newton–Raphson method was based on the introduction of the nonlinear reluctances as a function of system variables using either polynomial curve fitting techniques or piecewise segments. A comparison of the proposed method against



**Fig. 6** Instantaneous switching of the transformer described in [4] using nine-segment piecewise reluctance–flux curves

a computed phase current

--- piecewise reluctance curve

— continuous reluctance curve

b maximum number of iterations at each time step

the predictor–corrector method was carried out showing that the former method is not suitable for transformer inrush current modelling and simulation. The formulation of the Jacobian matrix using direct derivatives of the fitted reluctance curves or constant slopes of piecewise segments was shown to achieve a convergence error of less than  $10^{-7}$  with a  $10 \mu\text{s}$  time step. The simulation results using piecewise segments of equal length for the nonlinear reluctances against the polynomial fitted method showed a tolerable difference between both results. If a suitable fitted curve cannot be obtained, increasing the number of points of the piecewise curve will lead to the desired accuracy. The proposed simulation scheme can be easily applied for different applications, for example, other nonlinear system components such as power electronic devices and variable speed drives.

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## 8 Appendix

### *Test transformer data*

The experimental and simulation results presented in this paper has been obtained and investigated on a laboratory transformer having the following data:

Y- $\Delta$ , 30 kVA 3-phase 3-limb 208 V/208 V.

Primary winding resistance  $r = 0.0096 \Omega$  and leakage inductance  $l = 5.29 \times 10^{-2} \text{mH}$ .

Number of primary turns  $N = 60$

Zero sequence tests results in an air path reluctance  $R_o = 5.7 \times 10^{+5} / \text{H}^{-1}$ .

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