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YOUNG CHILDREN'S BEHAVIOR IN SOLVING PROBLEMS IN TWO AND THREE DIMENSIONAL SPACE

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Young Children's Behavior in Solving Problems in Two and Three Dimensional Space submitted by Patricia Dianne Locke Fenrich in partial fulfilment of the requirements for the degree of Master of Education.

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ABSTRACT

The development of problem-solving ability has always been a basic aim in mathematics instruction at the elementary school level. Nelson and Sawada (1974) undertook an extensive project in which they traced the development of responses of young children to oral problems.

In the present study, the behavior of children, aged three to nine, was examined as they attempted to solve two mathematically equivalent problems involving a coordinate reference system. The problem situations consisted of a parking lot and a theatre. Problems of location and selection were presented to each subject.

The subjects were interviewed twice. In the first interviews 60 subjects were given the parking lot problem and 30 of these also did the theatre problem. The second interviews were conducted one year later. In this part of the investigation, 48 of the original 60 subjects were presented the theatre problem and 24 of these also attempted the parking lot problem. The behaviors of the subjects as they did the problems were recorded on half inch videotape.

The apparatus used with each problem permitted different levels of solution. The subjects exhibited a variety of procedures in attempting the tasks. Some subjects used a random approach, others were systematic in their procedure. Counting, matching or a combination

thereof were the techniques used to solve the location problems. As the age of the subjects increased the amount of manipulation of the materials decreased.

Comparison of solution results and procedures employed seemed to indicate that children work as well in two-dimensional space as they do in three-dimensional space. However the younger subjects were more successful in working with a coded triple in three-dimensional space than a numeral pair in two-dimensional space.

The use of the videotape recorder proved to be a very effective means for recording the behaviors of young children as they solved problems. Behavioral trends were catalogued after a team of investigators had extensively viewed the tapes.

It was suggested by the study that further research be undertaken in certain areas of problem solving, the most important being the development of a child's concept of space in such learning situations.

ACKNOWLEDGEMENTS

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The research project, "Nature and Development of Problem Solving Behavior in Early Childhood" directed by Dr. L. D. Nelson with assistant Dr. D. Sawada, which was supported by a Canada Council Research Grant, provided the data on which this study was based.

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INTRODUCTION AND STATEMENT OF THE PROBLEM

The development of problem-solving ability has always been a basic aim in mathematics instruction at the elementary school level. Considerable research has been conducted to determine the way children solve problems. Unfortunately the research has been concerned mainly with written answers to problems given orally or which children had to read. Not much has been done to determine the factors which affect children's ability to apply mathematical knowledge in the process of solving problems of a practical nature or problems which have significance in the life of the child.

Nelson and Sawada (1974) undertook an extensive project in which they traced the development of responses of young children to oral problems. In the project the verbal aspect was kept to a minimum and the child was not required to respond in symbols.. The data for this study has come from part of the Nelson and Sawada investigation.

The domain of their study was to:

a. Identify, select, and refine criteria for the specification and construction of mathematical problems

which could be used effectively to study the problem-solving behavior of children.

b. Construct, on the basis of the refined criteria, a selection of problem situations.

c. Engage children, under carefully controlled conditions, in interacting with the problem situations.

d. Observe, record and categorize the behaviors manifested by the children while interacting with the problem situations.

e. Trace the scope and sequence of problem-solving behaviors over the age range of three to eight.

Although criteria for the construction of problems can be formulated, there was little information available to indicate how children would react to different aspects of the problem at different age levels. Therefore the first step in the investigation was to establish guidelines for the designing of problem situations that would be suitable to explore children's behavior. The characteristics of a "good" problem outlined by Nelson and Kirkpatrick (1975) were used. They are:

1. The problem should be of significance mathematically.

2. The situations in which the problem occurs should involve real objects or obvious simulations of real objects.

3. The problem situation should capture the

interest of the child.

4. The problem should require the child himself to move, transform, or modify the materials.

5. The problem should offer opportunities for different levels of solution.

6. The problem situation should have many physical embodiments.

7. The child should be convinced that he can solve the problem, and he should know when he has a solution for it.

Six pairs of problems were designed to investigate concepts such as factoring, seriation, the operation of division, reflection, geometrical shapes, and a coordinate reference system. Although the physical aspect of the problem pairs differed sharply they were designed to be equivalent mathematically. Solving problems which involved a coordinate reference system formed the basis for this particular study. A simulated parking lot and theatre were designed to observe young children as they solved problems in two and three dimensional space.

BACKGROUND TO THE STUDY

1. Problem Solving

In today's world of modern technology and rapid change, the ability to adjust and to solve one's problems is of paramount importance. Many studies investigating how

children solve problems have been done in the past (Kilpatrick, 1969; Riedesel, 1969; Suydam, 1967, 1974). Suydam, in categorizing the research in elementary mathematics, found that most studies were concerned with children's verbal or written responses to problems. The method frequently employed in these studies was to present the child with a verbally stated problem which was to be solved usually by using mathematical symbols. The responses alone did not yield sufficient evidence to indicate the thought processes that were involved. Kilpatrick (1969) indicated that problem solving had not been investigated systematically. He urged that problem-solving studies build on previous research and that more clinical investigations be undertaken.

Attempts in other studies have been made to understand the development of the thought processes of children by analyzing the procedures or strategies they have employed while involved in problem-solving situations (Affolter, 1970; Flaherty, 1973; Kilpatrick, 1969; Menchinskaya, 1969; Stern, 1967; Yaroschuck, 1969). Having subjects describe what they are thinking does not appear to be an effective means for studying problem-solving behavior. Flaherty (1973), using the 'thinking aloud' technique, found there were significantly more errors made by those who were required to verbalize while doing computations compared to those who worked silently.

Menchinskaya (1969) describes the inability of young children to verbalize the course of their solution even though they had successfully completed the problem.

If children are unable to describe their course of thought as they solve a problem, other means are required to trace the development of problem-solving abilities. Some mathematics educators are beginning to use techniques similar to those revealed in psychological studies of higher cognitive processes. The work of Piaget has had great impact on the method of conducting such investigations. He observed and recorded the actions and verbalizations of individual children as they proceeded to solve problems accompanied with physical objects found in a child's environment. Experiences with physical objects is considered by Piaget (1964) as an important factor in development. He wrote:

> Experience of objects, of physical reality, is obviously a basic factor in the development of cognitive structure. (p. 1)

Piaget's theory of cognitive development has led to investigations in mathematics education which try to devise meaningful learning experiences for young children. Yaroschuck (1969) devised problems in pairs which were characterized by the same mathematical structure. It was assumed that the same process of solution would be required to solve both the "subject problem" which was supplemented with concrete material and the "numerical problem" which

consisted of numerical content. He found that the children were more successful in solving the "subject problems" than the "numerical problems."

A few investigations have been conducted where solutions to problems have been expressed in the arrangement of physical objects rather than in verbal or written responses (Sawada, 1966; Little, 1974). Little found that young boys would attempt to solve division problems through the manipulation of physical objects.

Nelson and Sawada (1974), recognizing the merits of Piaget's method of investigation and a clinical setting, conducted a longitudinal study whereby young children's problem-solving behavior was recorded on videotape. The problems involved simulated materials of real life situations that would be familiar to a child and that would enable a solution to be expressed without the use of numerical symbols. The behaviors manifested by the children while interacting with the problem situations were categorized. Their scope and sequence were traced over the age range of three to eight.

If the relationship between problem solving and mathematics learning is to be studied systematically, appropriate problem-solving situations must be carefully devised. Information must be obtained on how children behave at different age levels when attempting to solve problems. This would enable the construction of problem-

solving situations which would present mathematical concepts in a form that would be most comprehensible to young children.

2. The Development of a Concept of Space

How does a child develop his concept of space when space is thought of in terms of a frame of reference? It is the space involved in an orientation process which enables movement from one specified place to another. Mastery of space is essential for man and seems to be achieved without difficulty. We have all experienced an occasional upset when emerging from an unfamiliar exit of a building. It takes only a moment to orient oneself. How does a child interpret space? What concepts does he have at his disposal? Little research has been done to shed light on these issues. Piaget and his colleagues, Inhelder and Szeminska, have probably made the largest contribution to a theory of how children interpret space and acquire the various geometrical concepts to orient themselves in their environment.

Geometry is a mathematics concerned with position or location in space. Topology, projective geometry and Euclidean geometry are most closely related to children's experiences. Piaget and Inhelder (1967) were intrigued by the fact that the historical and logical sequences of geometry were in conflict. In elementary schools the

fundamental principles introduced were based on Euclidean concepts such as straight line and angles. In contrast they have successfully demonstrated that a young child interprets space with such concepts as proximity, separation, order and enclosure which are topological types of relationships.

Piaget maintains that the evolution of spatial relations proceeds at two distinct levels: perceptual and conceptual. Children's perceptual space has reached projective and quasi-metric levels during the first year of life when their conceptual space has barely begun. He also maintains that the child's interpretation of space is developmental in nature proceeding from topological space which deals with the internal relations of isolated objects, to projective space which deals with the relation of the object from the subject's viewpoint, and finally to Euclidean space which deals with relations of objects to objects.

Laurendeau and Pinard (1970) conducted an in-depth study on how a child develops a concept of space. Their evidence supports Piaget's theory that the acquisition of representational space is developmental in nature with the initial concepts being topological.

Euclidean geometry is concerned with the relations or coordination between different objects in space. One Euclidean relation between objects is their position

relative to each other. A vertical and horizontal axes are used as a frame of reference. Piaget proposed that before a child is able to develop the abstract idea of horizontal and vertical axes as a frame of reference, certain topological and projective concepts need to be attained. A child needs to be able to orient objects in relation to himself. The topological concepts of proximity, separation, and order enable him to establish the necessary relationships for orientation. Vertical and horizontal directions, developed through the notions of above-below and left-right respectively, are concepts of projective geometry which are necessary in the development of a frame of reference. In the development of Euclidean space which contains concepts of parallels, angles, and proportion, the conservation of distance together with some notion of displacement (the congruent transformation of spatial figures) is required to coordinate relative distances and true positions of objects. The findings of Shantz (1966) support Piaget's hypothesis that distance conservation is a prerequisite for the attainment of the coordinate concept. The representation by a scale drawing would provide a workable format.

The simplest and most natural frame of reference available to a child is probably that provided by the physical world. The floor or ground represent his horizontal axis while trees and buildings provide him with a

vertical reference. Piaget's study of the construction of horizontal and vertical axes provides a way to understand the construction of a coordinate system in Euclidean space.

Piaget (1956) designed various tasks which were devised to investigate a child's concept of Euclidean space. The task for studying horizontality uses two narrow-necked bottles: one with straight, parallel sides and the other with rounded sides. Each is partially filled with colored water and the children are asked to "guess" the position of the water when the bottle is tilted. The study of the vertical involves floating a cork with a matchstick placed vertically in it on the surface of the water in the jars. The child is asked to draw the position of the "mast" of this "ship" at different inclinations of the jar. In addition, the child is shown a mountain of sand and asked to plant posts "nice and straight" on the summit, on the ground nearby, and on the slopes of the mountain.

The results show a developmental trend in that four and five-year-olds are unable to represent either the water or mountain as a plane surface, whereas as the age increases the capabilities increase to the point where an eight or nine-year-old is able to predict the horizontal and vertical in all cases.

Lovell (1971) studied children's concept of the horizontal axes by placing a partially-filled bottle in

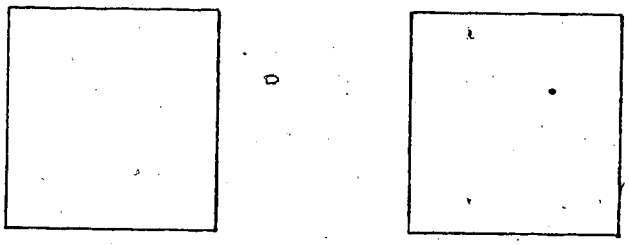
prone position, a car with a horizontal line drawn on it, and another partially-filled bottle, the same size but in an upside-position, in front of the children. The subjects were then asked to draw the bottles and their contents. Lovell concluded that a Euclidean frame of reference develops through incidental experience and action. The development arises through the evolution of schemes.

Olson (1969) studied children's conceptual framework of space by having the subjects perform various tasks while blindfolded to prevent the use of perceptual concepts. He found that preschool children possess a concept of spatial relationships on a horizontal plane. The children could invalidate all external cues other than those necessary to make directional judgments. They could function without knowing the labels left and right. Olson's research favors Piaget's view that language is not a necessary condition for conceptual differentiation, at least for the directional aspect of space.

A young child seldom uses the vertical and horizontal system provided by the physical environment in which he lives. To study the ability to locate several objects correctly in relation to each other Piaget used a set of blocks arranged in arbitrary fashion and asked children to make another pattern like it with another set of blocks. After his first attempt the child was given paper strips to serve as a set of axes and asked if they would help him

in positioning the objects correctly. Young children failed to see the value of the strips of paper. Only at age nine did children begin to make use of the strips as a reference system.

Piaget (1960) has devised tasks to show how young children interpret the ideas of horizontal and vertical. A process was designed to analyze how they would use a set of axes. The next question to be answered would be how do children proceed to locate a point in two-dimensional space without an instrument to measure angles. To locate a point requires a coordinate reference system composed of horizontal and vertical axes or rectangular coordinates. A child is given as tools a ruler, stick, strips of paper, and a piece of string and asked to represent on a blank piece of paper a point in the same position as a point shown on another piece of paper.



The results revealed that children at four and five years of age do not attempt to use the tools but simply guess the position of the dot. In the next stage a child would use a ruler to measure the distance from the corner to the point and guess the appropriate angle. At the

approximate age of nine, the child is able to coordinate horizontal and vertical measures to locate a point. The ability to locate a point in three-dimensional space was found to occur almost simultaneously with that of two-dimensional space.

It has been theorized that as a child grows, the incidental experiences in his daily living enable him to develop a concept of space which incorporates a reference system. After he is able to distinguish objects which are near or far from him, he gradually develops the concepts of horizontal plane and a vertical plane through relationships such as: above and below, left and right. When a child is able to acknowledge that the relationship among objects remains constant even though his position has changed, the final phase for developing a reference system has been achieved.

PURPOSE OF THE STUDY

At the University of Alberta a research project was initiated to study the behaviors of young children in solving non-verbal problems (Nelson, 1976a; Nelson and Sawada, 1975). The data from one set of parallel problems, which encompasses a cross-sectional sampling and a longitudinal sampling, formed the basis of this investigation. In this study an attempt was made to observe, record and describe the behaviors of children, age three to nine,

while engaged in problems involving a coordinate reference system. The behaviors analyzed consist of manipulative actions and verbalizations. The procedures used to solve the problems were catalogued and their differences were compared between age groups for the various tasks.

ASSUMPTIONS

1. It was assumed that each of the two problems devised for the study met the criteria for a good problem.

2. It was assumed that the problem situations would elicit a wide range of non-verbal solution responses.

3. It was assumed that the solution responses made by the subjects in a clinical situation were indicative of the way they would respond to similar problems in their everyday experience.

4. It was assumed that the range of problem-solving behaviors observed were dictated by the problems and the materials.

5. It was assumed that from the recorded data it would be possible to trace the nature and development of behaviors of children in problems requiring a frame of reference.

LIMITATIONS OF THE STUDY

1. The problems designed for this study limit the range of problem-solving behaviors displayed by the

subjects.

2. No background information on the subjects was obtained for consideration of its implications.

3. The behavior observed will be characteristic only to the subjects involved in this study.

4. When the data were taped the subjects were in an unfamiliar setting.

CLARIFICATION OF TERMS

Location Problem: A location problem is one in which a position on a grid is to be found from a description. The location problem consisted of performing three tasks with the parking lot apparatus and four tasks with the theatre apparatus.

Selection Problem: A selection problem is one in which a given position is described by the selection of the appropriate ticket.

Ordered Pair: The description of a position is given by two numerals. The first numeral refers to the horizontal direction and the second numeral refers to the vertical direction.

Coded Triple: The description of a position is given by three different elements: color refers to the plane, number to the horizontal direction, and letter refers to the vertical direction.

Problem: A verbal statement about the apparatus requiring

a response from the subject.

Problem Situation: All aspects of the apparatus designed for the problem.

Problem-solving Behavior: Any act or reaction manifested by the child's interaction with the material.

SIGNIFICANCE OF THE STUDY

There has been a great deal of research on children's problem-solving abilities. In the past very few studies were undertaken with young children using the non-verbal approach in studying problem-solving behaviors. By assuming that non-verbal responses are indicative of the way a child perceives a task this study should reveal information on the kinds of processes available to a child when he attempts to solve a problem. The categorization of these responses should lead to the identification of the strategies available to children in problem solving at the various age levels.

The analysis of the behaviors exhibited by the subjects in using a coordinate reference system will be combined with the analysis of the behaviors exhibited in the other problems studied in the project. The findings should serve as a guide in devising problem situations to be researched in the study of children's thinking through the medium of non-verbal problem solving.

OUTLINE OF THE STUDY

The following chapter contains a detailed explanation of the two coordinate reference problems investigated and their mathematical basis. The sampling, the data collection and analysis procedures are also explained.

Chapter III contains the findings of the study.

Chapter IV provides a summary of the study, a discussion of the findings, implications of the study, and recommendations for further research.

Chapter II

DESIGN OF THE INVESTIGATION

The purposes for Nelson and Sawada (1975) in their research project "The Nature and Development of Problem Solving Behavior in Early Childhood" were to create good problems, and to record and analyze behaviors of young children engaged in the problems. Six problems and a set of six different but mathematically equivalent problems were designed having as their basis either numeration or spatial relationships. Six of the twelve problems were presented individually to each child who had at his disposal the apparatus specific to each. The behaviors of the subjects while attempting to solve the problems were recorded on videotape.

PURPOSE OF THIS STUDY

The basic purpose of this study is to investigate the behaviors of children when they are engaged in problem solving tasks.

The specific purposes are:

1. To encode the behaviors exhibited by the children while they were engaged in solving problems in

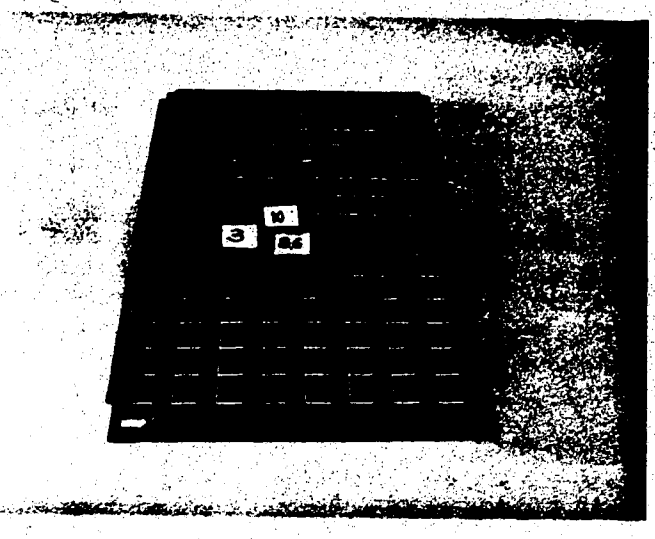
   (i) two dimensional space

   (ii) three dimensional space.

2. To analyze and classify these behaviors.

THE PROBLEMS

The Parking Lot Problem



The apparatus consisted of a masonite base 76.2 cm x 106.7 cm x 0.6 cm painted black. Eight rows of parking spaces, each containing 15 bays were constructed from 0.3 cm x 0.3 cm balsa wood glued to the baseboard. Each row was 95 cm long and each bay was 5 cm x 6 cm. The first row was 2.5 cm from the left hand side of the baseboard and all rows started 7.5 cm from the bottom of the baseboard. An arrow indicated the entrance to the parking lot.

Along with the masonite base were two masonite scales which when fitted alongside the baseboard gave the

coordinates to each bay. The horizontal scale measured 70 cm x 3.8 cm x 0.6 cm and was labelled with the numerals 1 to 8 inclusive while the vertical scale measured 95 cm x 3.8 cm x 0.6 cm and was labelled with the numerals 1 to 15 inclusive.

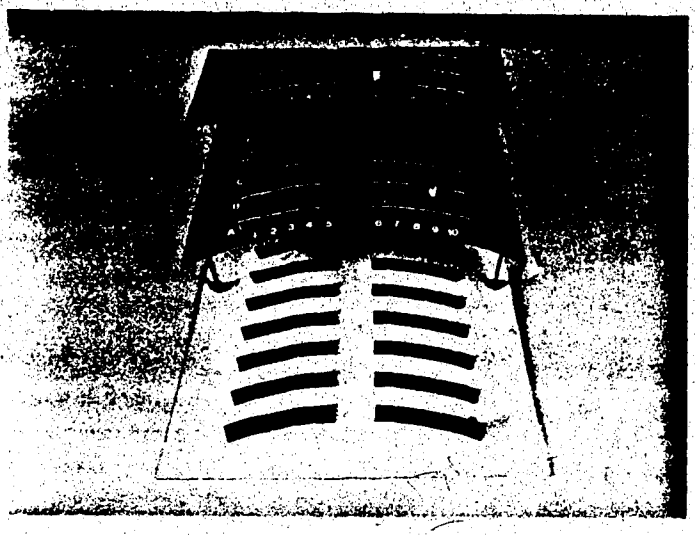
The apparatus also consisted of 12 plastic toy cars of assorted colors and nine laminated cardboard tickets designating the various parking spots.

The subject was given an opportunity to familiarize himself with the apparatus. This was followed by instruction and working with the coordinate system one dimension at a time. The final phase of the parking lot problem was the interviewer demonstrating how to use the information given in a number pair. The subject was presented with a car to park and a ticket designating the location. After parking three cars the subject was to select the ticket that described the location of a specified parked car. (See Appendix A for detailed protocol.)

The Theatre Problem

The apparatus consisted of three levels of seats constructed out of masonite and balsa wood. Each level had two blocks of seats 10.8 cm from each side and a center aisle 7.5 cm wide. There were five seats in each row on both sides of the aisle. A row of seats in each block was 23 cm long, 3.8 cm wide, and was slightly concave towards the front. The individual seats in each row were marked

out with 0.3 cm x 0.3 cm balsa wood glued to the baseboard.



Letters were used to label the rows and numerals were used to indicate the seat. The rows of seats were painted black and their labelling was done in white. The background color of the three levels was different with the floor level being painted yellow, the second red, and the top level blue. The rows of level one were labelled A to K, level two rows were labelled A to F, and level three rows were labelled A to C. In front of row A on each level white numerals of 1 to 10 inclusive identified each seat in the row.

Level 2 was supported by six dowels 0.9 cm in diameter and 15 cm long. These dowels were fitted into sockets made on the upper surface of level one and the

lower surface of level two. Each socket was made from 2.5 cm dowelling, 1.3 cm thick, with 0.9 cm diameter hole through the center. Level three was supported on top of level two by four dowels 0.9 cm diameter and 15 cm long, which fit into sockets made on the upper surface of level two and the lower surface of level three. Ten toy 'men' and some tickets completed the apparatus. The toy 'men' were commercial toys similar to the Fisher-Price little people. Their bodies consist of a head and a cylindrical torso with no arms and legs. The heads have painted faces and hair. Some wear fixed hats. The cardboard tickets were colored; some yellow, some red, and some blue to indicate the various levels. On each ticket was printed a letter and a number to describe a particular seat.

As in the parking lot problem an opportunity was given to the subject to familiarize himself with the apparatus. This was followed by instruction and working with the coordinate system one dimension at a time. The final phase of the theatre problem was the interviewer demonstrating how to use the information given in a triple consisting of color, letter and numeral. The subject was presented with a 'man' to seat and a ticket designating the location. After seating four 'men' the subject had the task of selecting the ticket which corresponded to the seat of the identified 'man.' (See Appendix A for detailed protocol.)

THE MATHEMATICS OF THE PROBLEMS

Each problem consisted of location and selection tasks in a coordinate reference system. The parking lot grid was intended for investigating concepts of the coordinate plane, including the recognition of functional relationships. The theatre grid, like the parking lot, was designed to facilitate the investigation of concepts such as correspondence in a coordinate plane. It also enabled subjects to experience working with coordinates other than number and with axes which were not rectangular. It will be noted that in the description of the theatre, it was stated that the rows were slightly concave.

The location problem involved interpreting the code printed on a ticket and relating it to a position on a grid. The selection problem, the reversal of the location one, consisted of identifying a position on the grid and selecting a ticket which described that position. In the parking lot the position referred to was in a two-dimensional space. The code was written as a numeral pair; the first numeral referring to the horizontal axis or the row and the second referring to the vertical axis which indicated the space. The theatre involved locating a position in three-dimensional space. The code for the theatre consisted of three different elements, each specifying a direction. The letter indicated the vertical axis, i.e., the row and the numeral indicated the horizontal

axis which was the seat. The color of the ticket indicated the third dimension which was the plane or tier where the seat was located.

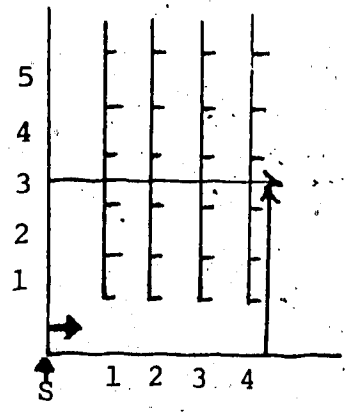
Theoretically it would be expected that the thinking process in locating a position using a code consisting of a single element would differ from locating a position using a code made up of several different attributes. When one element such as number was used to describe a given position the order of presentation became important. The number pairs in the parking lot problem had the first numeral referring to the horizontal dimension and the second numeral referring to the vertical one. The number pair (4,3) and (3,4) would describe two different positions. A process was inferred to attain a solution. To work with an ordered pair the subjects needed to formulate a rule in their own minds. An example of a rule would be "the first number tells how far to go this way and the second tells how far to go that way." Theoretically speaking, working with the rule and subsequently finding the correct location would reinforce the rule. The ordered pair then would become a functional tool. On the other hand when the code was made up of differing attributes such as color, letter and number the order of presentation was not important. Yellow E4 described the same seat as 4E on the yellow tier. The triple used in the theatre problem involved order in a different way than the number pair in

the parking lot problem. The triple was ordered to the extent that color referred to plane, letter to row, and numeral to the seat. With the triple the formulating of a rule was still involved. Each element of the triple referred to a specific direction but the subject could consider the elements in the order of his choosing each time he attempted a task. In the first task he could focus his attention first on the row and space, then decide on which plane it should be. Doing the second task he may identify the plane and then proceed to find the designated row and space. It would be the individual who would formulate the order in which the elements of the triple were to be considered.

The parking lot tasks were performed in a two-dimensional field consisting of a horizontal and vertical axis. A three-dimensional field was encountered in the theatre problem. Each of the three levels of the theatre was a two-dimensional plane like that of the parking lot. The subject not only had to locate the particular spot but had to decide on which plane it should be. For example, the seat B8 was found to be on all three levels. It was up to the subject to interpret the third element of the triple and select the appropriate plane.

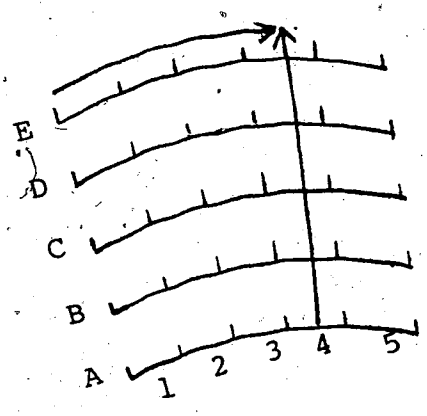
The rows of parking spaces in the parking lot were parallel to each other and to the horizontal axis. They were also perpendicular to the vertical axis forming a

rectangular grid. The scales labelling the rows and spaces corresponded in such a way that the movement of locating a position followed straight lines which were perpendicular to each other. The place of intersection would be the position described by the number pair and found by coordinating the vertical element with the horizontal one. In locating (4,3) as an example, there would be movement up row 4 parallel to the vertical axis and movement over the rows in space 3 parallel to the horizontal axis. The coordinated action would result with an intersection at position (4,3).



(4,3)

In contrast the rows in the theatre were not straight but slightly concave. The rows themselves were parallel but they were not parallel to the horizontal axis and not perpendicular to the vertical axis. In locating the seating space E4, if subjects tried to coordinate the position on the vertical axis with the position on the horizontal axis in the same manner as in the parking lot problem, they



(E4)

would not locate the appropriate position because of the theatre's rows being slightly concave. Instead of moving in straight lines at right angles to each other, the movement had a slightly curved path. The focus would change from that of coordinating the movement with the labelling on the axis to that of finding the fourth seat in each row until row E was reached.

The parking lot enabled the children to work in a two-dimensional space while using a number pair to describe a position. The equivalent problem involving a coordinated reference system was the theatre. The subjects encountered a three-dimensional plane and symbols other than number to describe a position. Both problems involved remembering what each element in the code stood for and coordinating their movements to locate a position or to identify a description of a given location.

Methods of Solving the Problem

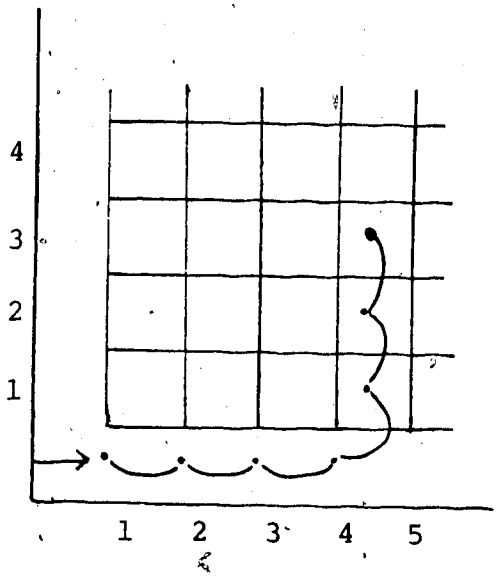
The problems involving both the parking lot and the theatre were designed to be mathematically equivalent. Both involved locating a point in a plane. The methods of solving the problems involved basically two different procedures and a combination thereof. A point could be located by:

1. counting
2. coordinating
3. a combination of counting and coordinating.

Counting involved designating the first element of

the number pair to name the row, moving down the horizontal axis and pausing to count each row as it was passed until the desired row was reached. The second element resulted in indicating the space, therefore when the desired row was reached a turn into the row was made. Moving in a

Counting



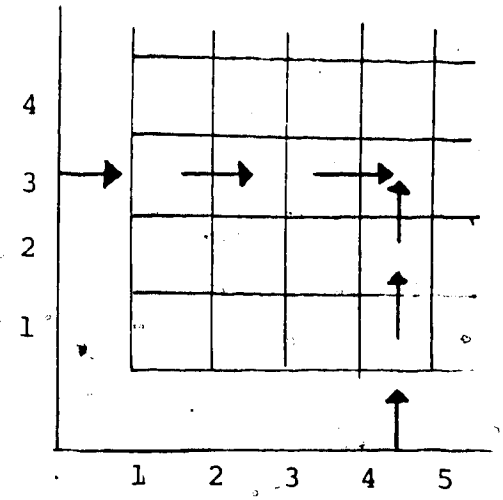
Parking Lot
Number Pair (4,3)

vertical direction the spaces were then counted. When the count corresponded to the second element the specified space should have been reached and a unique point located in the plane.

Coordinating involved matching the first element of the number pair with the horizontal axis and the second element with the vertical axis. After the row and space had been identified, with hand or eye movement, a move was made down the specified row and up the specified column,

coordinating the two moves to meet at a single space. A point had been located in a plane.

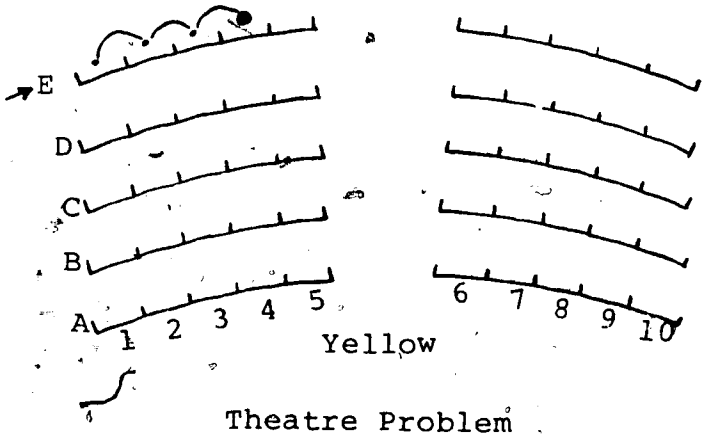
Coordinating



Parking Lot
Number Pair (4,3)

An example of using a combination of matching and counting was given when locating the point yellow E4 in the theatre problem. The plane was identified by

Combination of Coordinating and Counting



Theatre Problem

Triple
(Yellow E4)

matching the color of the ticket with the color of the tier. In the pair E4, the E designated the vertical axis (row) and was located with a matching process. Next a move occurred down the row in the horizontal direction accompanied by the counting of each seat until seat 4 was located. The location of point yellow E4 had been identified.

RESEARCH PROCEDURES

A detailed description of the research procedures has been given by Little (1976), therefore a summary has been allocated.

The Sample

Six hundred letters, describing the project and inviting the child's participation, were distributed to parents of children ranging in age from three to eight and attending a daycare or school in the vicinity of the university. Fifteen children from each age level, making a total of 90, were chosen to satisfy the requirements of the study. The selection of the children was done by random sampling. A detailed account of the method of selection has been given by Little (1976).

The twelve problems were arranged in 15 sets, each consisting of six problems. The problem set was comprised of four different problems and two equivalent forms. In the cross-sectional sampling each of the 15 problem sets

was randomly assigned to one subject in each age group. The outcome of this procedure was that six different problems were presented to 10 subjects in each age group and six equivalent problems were presented to five subjects in each age group. A schedule was devised for the presentation of the problems with 60 subjects doing the parking lot problem and 30 doing the theatre problem. The interviews began on June 24 and terminated July 23, 1974.

In the longitudinal sampling, theoretically each subject was to be presented the equivalent to each problem done in the previous year. This would provide for six equivalent problems to be presented to 10 subjects in each age group and six different problems to be presented to five subjects in each age group. The distribution of subjects doing both a different problem and its equivalent in both the cross-sectional and longitudinal sampling was five subjects in each age group. Seventy-four of the original 90 subjects returned with 24 doing the parking lot problem and 48 doing the theatre problem. The interviews for the longitudinal sampling took place from July 3 to July 25, 1975.

Pilot Study

A pilot study was conducted prior to the data collection schedule with a number of children in the age range of three to eight years. Little (1976) has outlined the purposes of the pilot study.

A second pilot study was conducted in June 1975. Its purpose was to enable the new support staff to familiarize themselves with the protocols and the apparatus for the longitudinal data collection. At this time protocols and recording techniques were refined. Minor changes in the protocols guaranteed greater consistency between a problem and its equivalent (see Appendix A).

Interviews

Five interviews were conducted each day, three in the morning session and two in the afternoon session. An effort was made to have the younger subjects interviewed in the morning sessions because of the fatigue factor. During an interview there were usually seven adults present—two cameramen, the interviewer, two assistants, the director or the associate director of the project and a parent. The subjects did not appear to be distracted by the recording equipment or the adults present once they became involved in the problem.

Each problem was presented to the child individually. The material was placed on a low table in front of the child. The interviewer sat to the subject's right and presented the problem adhering to the protocol. Each problem had an introductory phase which enabled the subject to familiarize himself with the apparatus. At the completion of a problem, the apparatus was removed and new materials were placed on the table as the interviewer

proceeded to the next problem. The taping sessions lasted from 1/2 to 1 hour. The younger subjects tended to take more time in completing the various tasks than the older ones.

For the cross-sectional sampling there were two persons interviewing while in the longitudinal sampling there was one interviewer.

The Recording

For the recording of the interviews a Javelin videotape recorder and two Sony video cameras were used. The behaviors of the subjects were recorded on half inch (new format) magnetic tape. The two cameras were positioned to enable the capturing of the behavior of the child. A split screen technique was used at times to capture the action of the subjects from two different angles. A microphone was suspended from the ceiling providing for the recording of the verbalizations of the subject and the interviewer.

In the cross-sectional sampling, the data collected from the 90 subjects were recorded on 54 one hour tapes while 45 one hour tapes held the data collected from the 74 subjects in the longitudinal sampling.

Coding Procedures

A system was required that would enable cataloguing and analyzing the behaviors generated while the subjects

worked on the problems. A team of research assistants viewed many tapes of the various problems and of the various age groups. Recurring behaviors were identified. The presentation of the problem by the interviewer, the verbalizations by both the interviewer and the subject, and the hand movements of the subject were the categories deemed relevant to the study. A common code was devised for all the problems to enable the transcribing of behaviors as seen on the videotape. Each problem and its equivalent required certain modifications of the common code to permit the recording of behaviors unique to the problems involved. (See Appendix B).

The symbols employed in the code provided for any action or sequence of actions on the part of the subjects and the interviewer to be recorded. In order to avoid losing important information all verbalizations, except for those provided for in the protocol, were recorded verbatim.

The encoding of the behaviors required the tapes to be played and replayed several times. The time required to accomplish this for each problem ranged from ten minutes for some of the older subjects to one half hour for some of the younger subjects. Sample sheets of the recordings may be found in Appendix C.

A check on reliability was done during the coding of the behaviors. Several tapes were viewed again to

assure correct interpretation of the data. A further check was provided by the comparison of the experimenter's observation to one other observer. The procedure employed was to select one subject randomly from each age group of the longitudinal sampling. After the coding system had been explained the observer encoded the behaviors. The recording sheets were compared with those of the experimenter. Little disagreement was found.

THE ANALYSIS OF DATA

The behaviors of the subjects as they interacted with the material in the various problem-solving situations were recorded on videotape. In the present study the data from two of the 12 problems were encoded and analyzed. The two mathematically equivalent problems were the parking lot and the theatre problems. The behaviors exhibited by the subjects consisted of physical manipulations and verbalizations.

In the cross-sectional sampling the behaviors were encoded, the verbalizations were transcribed verbatim. In the analysis of the data solution procedures were catalogued and comparisons were done between the various aspects of the two problems and across the age range. The verbalizations were classified and a discussion followed concerning the verbalizations used by the various age groups while performing the various tasks.

The data from the longitudinal sampling were analyzed in the same manner as those of the cross-sectional sampling. The procedures employed by the subjects and the verbalizations were compared with those of the first sample.

Chapter III

RESULTS OF THE INVESTIGATION

The main purpose of this study was to observe, record and describe behaviors of young children while they were engaged in problem-solving tasks. The tasks done in the parking lot problem involved locating positions in two-dimensional space. The theatre problem involved locating positions in three-dimensional space. In both the two- and three-dimensional tasks tickets were presented to the child. A pair of numerals written on a ticket indicated a position for the child to find in two-dimensional space. The first numeral indicated the row and the second numeral indicated the space in the row. The position designated in three-dimensional space was indicated with a triple consisting of color, letter and numeral. The color indicated the tier, the letter indicated the row and the numeral indicated the seat or space in the row.

There were 60 subjects in the cross-sectional sampling who performed the parking lot problem, 30 of these also performed the theatre problem. For the longitudinal sampling 48 of the 60 subjects in the cross-sectional sampling returned to do the theatre problem. Twenty-four of these 48 subjects also worked on the parking lot problem. Each situation included a location problem and a selection

problem. The location problem was comprised of giving the subject an object to place and a ticket which indicated the position to be found. The selection problem consisted of the interviewer pointing to an object already placed with the subject having to select the ticket describing the position. The findings of the investigation are reported in this chapter.

The chapter is composed of three major parts with subsections described as follows:

I. The Cross-sectional Sampling
   1. The Parking Lot
      a. Procedures used in solving the three problems of locating positions using a number pair.
      b. Procedures used in making the selection of a number pair.
      c. The comparison of the location and selection tasks.
   2. The Theatre
      a. Procedures used in solving the four problems of locating positions using a triple.
      b. Procedures used in making the selection of a triple.
      c. The comparison of the location and selection tasks.
   3. Comparison of
      a. the location problems of the parking lot and

the theatre;

b. the selection problems of the parking lot and the theatre.

4. Verbalizations.

II. The Longitudinal Sampling

1. The Parking Lot

a. Procedures used in solving the three problems of locating positions using a number pair.

b. Procedures used in making the selection of a number pair.

c. Comparison of the location and selection problems.

2. The Theatre

a. Procedures used in solving the four problems of locating positions using a triple.

b. Procedures used in making the selection of a triple.

c. Comparison of the location and selection problems.

3. Comparison of

a. the location problems of the parking lot and the theatre;

b. the selection problems of the parking lot the theatre.

4. Verbalizations.

III. Comparison of the Cross-Sectional and Longitudinal Data.

PART I. CROSS-SECTIONAL SAMPLING

Section 1. The Parking Lot

a. Location Problem

Ten subjects at each age level were presented the location problem which consisted of three tasks. Each task involved locating a position in two-dimensional space. Before a subject began the first task he was given instructions explaining the meaning of the numerals written on the ticket. The first task, involving the number pair (4,3) was demonstrated as driving up four rows, turning, and moving in three spaces. With the interviewer's assistance the subjects at the various age levels attempted to park the car in the space (4,3). The two remaining tasks were to be done without further instructions from the interviewer. The second task was to locate the position (2,5) and the third task was to locate the position (8,6). (See Appendix A for protocol.)

The behaviors of the subjects as they worked at solving the problems were recorded on videotape. Analysis of the videotapes established procedures used by the children which could be classified as follows:

I. The subject did not attempt the task or the task was abandoned.

II. The subject parked the car, with or without assistance, but did so incorrectly.

III. The subject parked the car having one element of the

number pair correct.

IV. The subject parked the car correctly but with explicit directions from the interviewer.

V. The subject parked the car by counting or matching the numeral on the row axis with the first element of the number pair and by counting the number of spaces which corresponded to the second element of the number pair.

VI. The subject parked the car correctly by coordinating both the row and the space.

Task 1 (number pair 4,3) was an instructional task therefore the distribution of procedures used was not included in a table. The protocol provided for a demonstration on how to use a number pair. If a subject indicated the need for assistance while trying to locate (4,3) it was given. The three-year-olds either did not attempt to do the instructional task, or if they did, they did so incorrectly. Eight four-year-olds and eight five-year-olds parked correctly with the interviewer's assistance. The four remaining subjects in these two age groups parked the cars incorrectly. The six-year-olds were the first age group to have some subjects park correctly receiving no further assistance once the instructions had been given. Three of the ten six-year-olds did this. Four parked correctly when given further assistance and the remaining three were incorrect in their parking. All

the seven-year-olds and all the eight-year-olds parked the cars correctly, with only one requiring assistance.

The distribution of the procedures outlined on page 40, according to age and sex, are given in Table 1 for task 2 and Table 2 for task 3. All the subjects who were unable to solve the problem were classified in categories I, II and III (N = 25 for task 2, N = 28 for task 3). The protocol did not call for assistance to be given for task 2 and 3 but some assistance was given. The interviewer may have either repeated the instructions on how to use a number pair or else prompted the subject when a solution was being attempted. Those subjects who attained a solution when given assistance were classified in category IV (N = 8 for task 2, N = 3 for task 3). Subjects who successfully solved the problems unassisted were classified in categories V and VI (N = 27 for task 2, N = 29 for task 3). Let it be noted that in the following discussions a correct response will be assumed to be unassisted unless stated otherwise.

If the classification of the responses does represent solutions which would suggest increasingly higher levels of organization on the part of the subjects, then the younger subjects tended to use the lower levels and the older subjects the higher levels. In analyzing the procedures used for locating the position (2,5) (see Table 1) it was observed that the three-year-olds fell into the

Table 1

Distribution of Procedures for the Location Problem of the Parking Lot
Task 2 (2,5)
(N = 60)

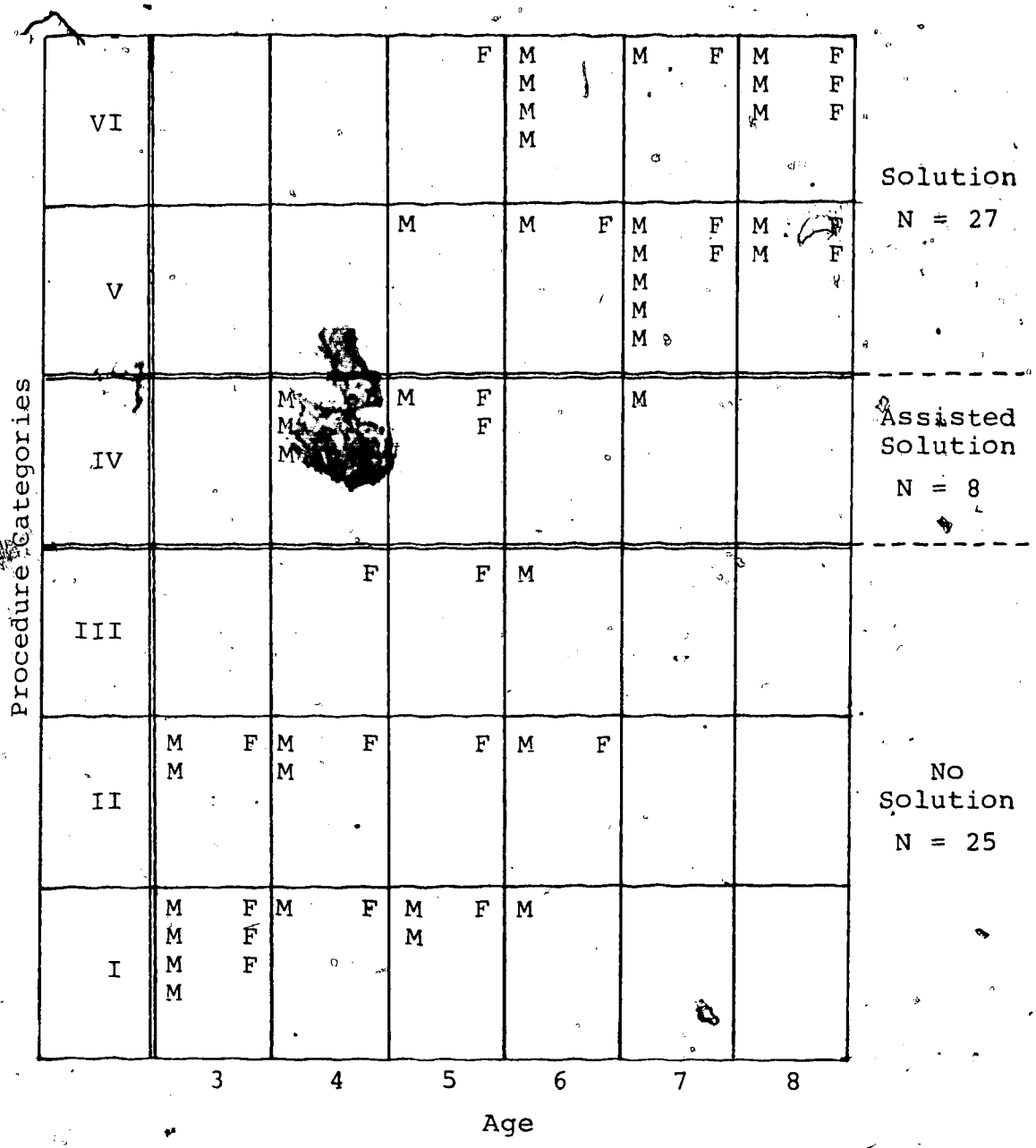
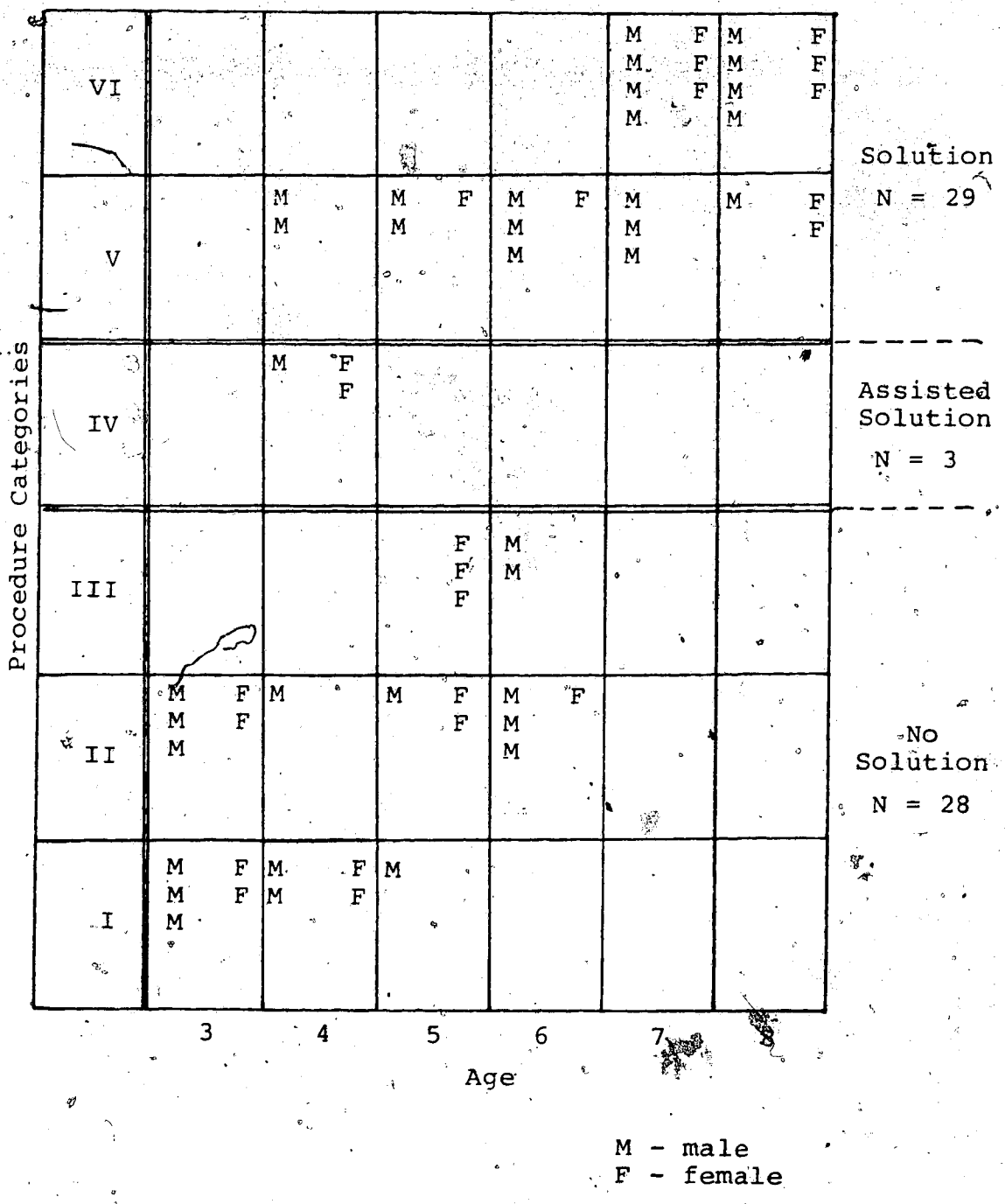


Table 2

Distribution of Procedures for the Location Problem of the Parking Lot
Task 3 (8,6)
(N = 60)



first two categories indicating incorrect responses. Seven of these subjects did not attempt to do the task. The four, five and six-year-olds appeared to be in a stage of transition because of the wide variances in responses. Each of these age groups had some subjects who did not attempt the task, some who gave incorrect responses and some who were able to get one element of the pair correct. The five-year-old and six-year-old group had subjects in each of the six procedure categories. Four four-year-olds were able to park the car in the space (2,5) when assisted by the examiner. The five-year-olds were the first age group to have subjects locate the position without assistance. Of the two who were correct one used category V and the other used category VI. There was a sharp increase in the number of correct responses from two in the five-year-old group to six in the six-year-old group. In the six-year-old group, four of the subjects who made correct responses used the procedure of coordinating the row and the space and two used the procedure stated in category V—counting or matching the first element and counting to locate the second element. Of the nine seven-year-olds who parked correctly seven fell into the category of matching and counting. About half the eight-year-olds fell into this category. The remainder fell into the coordinating category.

In examining the procedures used in task 3 (number

pair 8,6) the three-year-olds again fell into the first two categories of incorrect responses. The four and five-year-olds exhibited the widest range of behaviors. Both of these age groups had some subjects who did not attempt to do the task. There was a marked decrease from four in the four-year-old group to one in the five-year-old group. The four-year-olds were the only age group who required assistance. This age group was the youngest age group to give correct responses. Three of the five who made correct placements were assisted. The five and six-year-olds had seven subjects who gave totally incorrect responses, five who had one element correct, and seven who were correct. All the five-year-olds and six-year-olds who found the position (8,6) used a strategy of matching and counting. A sharp increase in the number of correct responses occurred between the six-year-olds and the seven-year-olds. No seven or eight-year-olds made errors. In examining the correct response categories, category V was the one more widely used. Subjects ranging in age from four to eight used the strategy of matching and counting. Category VI which involves a coordinating strategy was the one preferred by the seven and eight-year-olds.

It should be noted that (2,5) in task 2 is a space closer to the origin than (8,6) in task 3. In comparing the performance between the two tasks it was observed that the three-year-olds performed no differently

in locating a space close to the origin than in locating a distant space. They failed to do either task. There was a wider age range of subjects who did not attempt to find (2,5), the second task and the space closer to the origin, than (8,6) the third task. Children, ranging in age from three to six years, did not try to find (2,5) while locating (8,6) was not attempted by children in the age range of three to five years. The age range of subjects giving incorrect responses was three to six years in both tasks. The number of subjects falling into Category III, the category indicating that the subjects located correctly one of the two elements of the number pair, increased from task 2 to task 3. One five-year-old and one six-year-old had one element correct in finding (2,5) whereas three five-year-olds and two six-year-olds had one element correct in locating (8,6).

The correct responses were placed in either category V or category VI. These two categories differ in that there had to be evidence of some counting being done in locating a position if it was to be a category V response. In category VI there was no evidence of counting but the child appeared to be matching and coordinating the row and column. The four-year-olds were the only subjects given assistance in task 3. Examining the correct responses, more subjects found (8,6) than (2,5). Five out of twenty four-year-olds and five-year-olds were correct in locating (8,6) compared

to one five-year-old finding (2,5). A striking increase in the number of correct responses occurred between the five and six-year-olds in locating (2,5) and the six and seven-year-olds in locating (8,6). The six-year-olds were the only age group in which the number of correct responses decreased from task 2 to task 3. In task 2, locating (2,5), four of the six making correct responses used a coordinating strategy. In contrast the four correct responses of the six-year-olds fell into the matching and counting strategy in locating (8,6). Unlike the six-year-olds, the seven-year-olds more frequently used a matching and counting strategy in locating (2,5) and a coordination strategy in locating (8,6). More than half of the eight-year-olds used the coordinating strategy in solving both task 2 and task 3.

In each of the tasks the process of matching and counting was used by slightly more than half of the subjects who gave correct responses. A wider age range of subjects used matching and counting in locating (8,6) than in locating (2,5). The process of coordinating the axes was used more by the younger subjects in locating the closer point (2,5) than (8,6). Half of the seven and eight-year-olds used matching and counting in locating (2,5). For the more distant position (8,6) the seven-year-olds and the eight-year-olds used a coordination process in fourteen of twenty cases.

b. The Selection Problem

The completion of the location problem was followed by the selection problem if the requirements were met. The protocol stated that at least one car had to be in a correct position whether it was placed by the subject in completing the locations tasks or by the interviewer in task one, the instructional task. The selection problem consisted of the examiner pointing to a correctly parked car and asking the subject to select the ticket with the number pair which described the car's position. (See Appendix A for protocol.)

The observed solution procedures were categorized as follows:

I. The subject did not attempt the problem or the problem was abandoned.

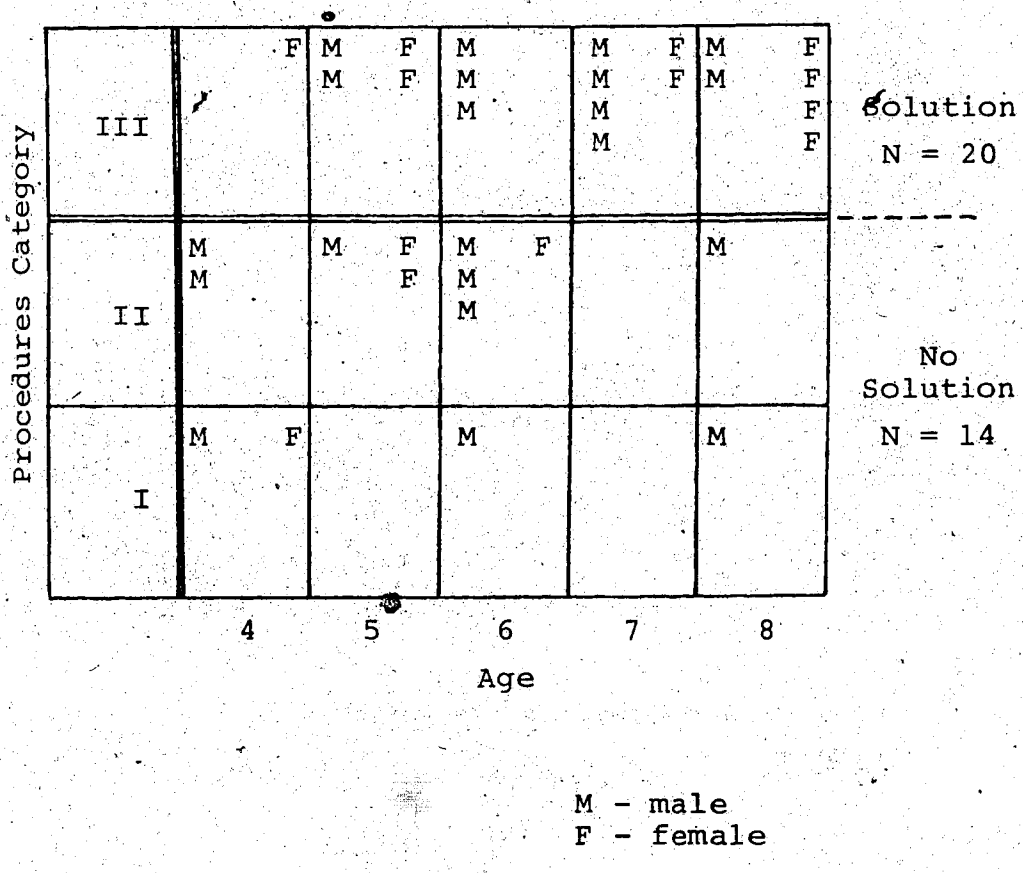
II. The subject chose an incorrect card.

III. The subject chose the correct card.

In 19 cases protocol was not followed in the presentation of the parking lot problem. For this reason the procedures they employed could not be classified. Only 41 of the 60 subjects were presented the selection problem in the manner specified by protocol. The procedures employed by these subjects have been classified as belonging to one of three categories. The subjects who made incorrect selections have been classified in categories I and II. Those who made correct selections have been classified in

Table 3

Distribution of Procedures for the Selection Problem of the Parking Lot (N = 41)*



* Three-year-olds were not included because in all cases the requirements were not met.

category III. The distribution of the different procedures used is shown in Table 3.

The three-year-olds did not meet the requirements so no three-year-old did this problem. In each of the remaining age groups there were subjects in the three categories, except for age seven. All the seven-year-olds made correct selections. The number of correct responses in a given age group increased as age increased with age six being the exception. The proportion of correct responses in the six-year-old group was smaller than that of the five-year-old group. There was a marked difference between the performance of the six-year-olds and the seven-year-olds. Three out of eight six-year-olds were correct and all the seven-year-olds were correct. Prior to age seven, 12 out of 20 subjects failed to select the correct ticket whereas all but two out of the 14 seven and eight-year-old group selected the correct ticket.

c. Comparison between the Location and Selection of a Number Pair

A comparison of the solution results for the location and selection tasks should reveal whether the children in the sample were as successful with the tasks involving one process as with those involving the other. The distribution of the correct solutions for the 60 subjects who performed both the location and the selection problem using the parking lot is given in Table 4. As stated in

Table 4

Distributions of Solutions for Parking Lot Tasks
(N = 60)

Subject	Age 3		Age 4		Age 5		Age 6		Age 7		Age 8	
	L	S	L	S	L	S	L	S	L	S	L	S
S_1		TA	3	PA	2,3	PA		PA	2,3	✓	2,3	*
S_2		TA		TA		✓	2,3	✓	2,3	PA	2,3	✓
S_3	TA	TA						✓	2,3	PA	2,3	**
S_4	TA	TA		PA		✓			2,3	✓	2,3	✓
S_5		PA		✓		✓	2,3	✓	2,3	✓	2,3	PA
S_6		PA		PA	2,3	PA	2,3	PA	2,3	✓	2,3	✓
S_7	TA	TA		PA		✓	2		3	PA	2,3	PA
S_8	TA	TA		TA	3		2,3		2,3	✓	2,3	✓
S_9	TA	TA					2		2,3	✓	2,3	✓
S_{10}		PA	3	PA		PA		TA	2,3	PA	2,3	✓

L - Location Task

☐ - empty space: the correctly parked car was placed in the instructional task
2 - location task 2 was performed correctly
3 - location task 3 was performed corfectly

S - Selection Task

✓ - selection correct

TA - Task abandoned
PA - Protocol abandoned
* - Task was not presented
** - Video trouble

Section b, 19 subjects were presented the selection problem in a manner that did not follow the protocol. This has been denoted with a "PA" in the selection column. When the responses indicated to the interviewer that the subject had not grasped the problem the task was abandoned. This occurred primarily in the three-year-old group. The protocol stated that at least one car had to be parked correctly before the selection task could be presented. Because in attempting the location problem no cars were correctly parked, the selection problem was not presented to all three-year-olds, two four-year-olds and one six-year-old. This has been denoted by a 'TA' in the selection column.

In the instructional phase of the parking lot problem the interviewer used the number pair occurring in task 1 for his demonstration. Either the interviewer parked the car or assisted the subject in doing so. This enabled the requirement to be met so the selection problem could be presented but the responses for task 1 have not been included in Table 4. In keeping with the format only the correct and unassisted responses have been included. An empty space in Table 4 indicates that the tasks were attempted but were either incorrect or assisted.

In the three-year-old group if the tasks were not abandoned the responses were incorrect. On the other hand, the seven and eight-year-olds completed successfully both

the location and selection problems, with the exception of one seven-year-old who failed to do task 2 correctly. Therefore the remainder of the discussion of Table 4 will refer only to those subjects in the age range of four to six.

It is interesting to note that four five-year-olds were able to make a correct selection when they were unable to do a location task when no assistance was given. Other than these four five-year-olds, only one four-year-old and one six-year-old performed in this manner.

Fifteen of the 20 children in the four and five-year-old group were unable to complete even one location task. Two four-year-olds and one five-year-old did one location task correctly. In these instances it was the third location task. In contrast the two six-year-olds who were successful in only one location task, located (2,5) task 2. Not one subject in the four to six year age range did only one location task and the selection task. The two five-year-olds, who were the youngest subjects to do both task 2 and 3 in the location problem, were not presented the selection problem according to the protocol. The six-year-olds were the youngest ones to do both the location and selection problems. Four of the ten six-year-old boys found the positions stated in task 2 and 3. Two of these four also did the selection problem correctly.

Age six was the age group with the greatest variance in performance. Two subjects completed all aspects of the parking lot problem correctly, while two others did not do one. Of the six subjects who did correct locating all did the second task which was locating number pair (2,5). Four of these six also found the position (8,6) which was task 3. One child was successful only in the selection problem.

Generally speaking, the subjects who were able to perform successfully all three location tasks were almost always successful in completing the selection problem.

Section 2. The Theatre

a. Location Problem

Five of the subjects at each age level who performed the parking lot problem also performed the theatre problem. The location problem in the theatre consisted of four tasks. Each task involved locating a position in three-dimensional space. The tickets in the theatre problem differed from the number pair in the parking lot problem in that they contained three different symbols—color, letter, and numeral. Before the subject began the first task he was given instructions explaining the meaning of what was written on the ticket. The first task, involving the triple yellow B8, was demonstrated as being on the yellow tier because the letter was colored yellow, in row B and seat 8. With the interviewer's assistance the

subjects at the various age levels attempted to place the 'man' in seat—yellow B8. The three remaining tasks were to be done without any further assistance from the interviewer (see Appendix A for protocol).

Analysis of the videotapes on which were recorded the behaviors of the subjects, established the procedures which could be classified as follows:

I. The subject did not attempt the task or the task was abandoned.

II. The subject placed the 'man' having either no element or one element of the triple correct.

III. The subject placed the 'man' having two elements of the triple correct.

IV. The subject placed the 'man' correctly but with assistance from the interviewer.

V. The subject placed the 'man' correctly.

As in the parking lot problem, a distribution of procedures table for task 1 has not been included because task 1 was an instructional task. The three remaining tasks as outlined in the protocol are:

Task 2—placing a 'man' in red A5, the red level being the middle tier.

Task 3—placing a 'man' in blue B8; the blue level being the top tier.

Task 4—placing a 'man' in yellow E4; the yellow level being the bottom tier.

The distributions of the procedures according to age and sex are given in Table 5 for task 2, Table 6 for task 3, and Table 7 for task 4. All subjects who were unable to solve the problem were classified in categories I, II, and III (N = 11 for task 2, N = 12 for task 3, N = 12 for task 4). Those subjects who attained a solution when given assistance by the interviewer were classified in category IV (N = 2 for task 2, N = 8 for task 3, N = 2 for task 4). Subjects who successfully solved the problem were classified in category V (N = 17 for task 2, N = 10 for task 3, N = 16 for task 4).

Tables 5, 6, and 7 tend to suggest that if the classification of responses does represent solutions which would require increasingly higher levels of organization on the part of the subjects, the older subjects used these procedures. The younger subjects were classified in the lower categories while the older subjects were classified in the upper categories. Three three-year-olds and one four-year-old did not attempt to do either task 2 or task 4. All the children except two three-year-olds attempted to do task 3. The results of the performances of task 2 (red A5) and task 4 (yellow E4) were very similar. The three-year-olds failed to do either task. The only children given assistance were two four-year-olds. There was one four-year-old on each of the two tasks who gave a correct response. On the other hand, on each of these same tasks

Table 5

Distribution of Procedures for the Location Problem of the Theatre
Task 2 (Red A5)
(N = 30)

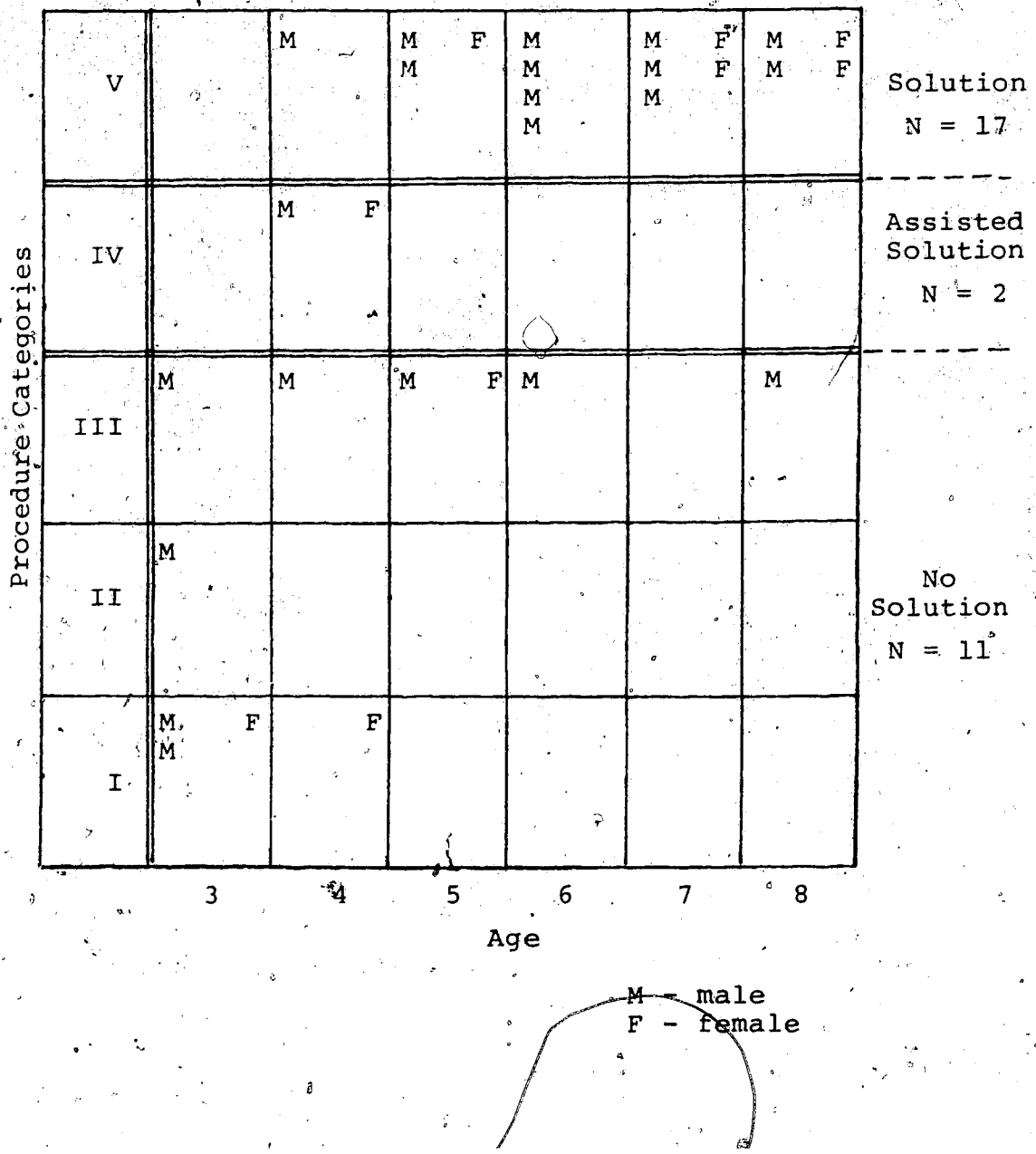


Table 6

Distribution of Procedures for the Location Problem of the Theatre Task 3 (Blue B8) (N = 30)

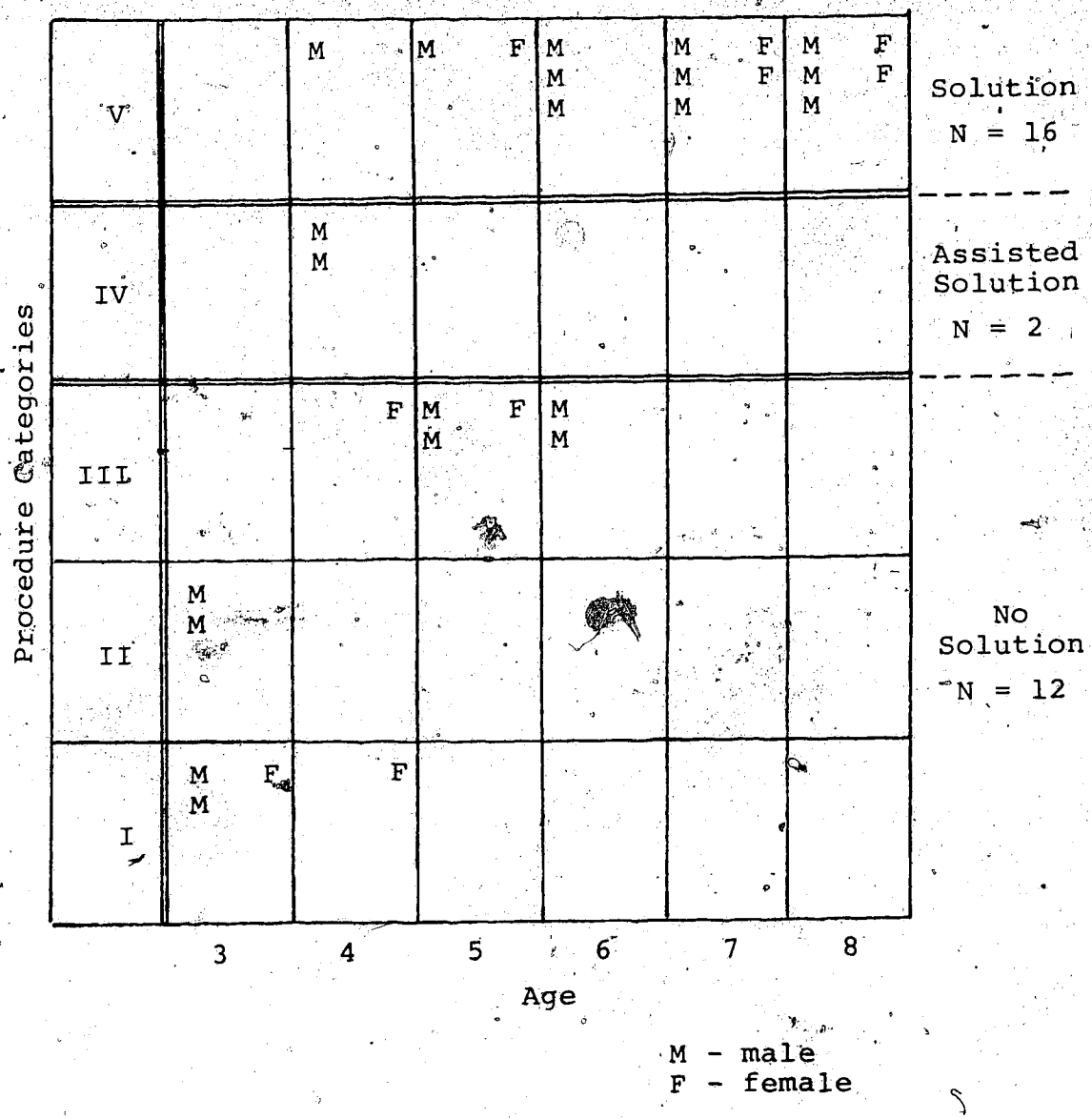
Procedure Categories	3	4	5	6	7	8	
V			M F	M	M F M F	M F F	Solution N = 10
IV	M	M	M F M	M M M			Assisted Solution N = 8
III	M	M F		M	M	M M	
II	F	M F					No Solution N = 12
I	M M						

Age

M - male
F - female

Table 7

Distribution of Procedures for the Location Problem of the Theatre
Task 4 (Yellow E4)
(N = 30)



there was one four-year-old who did not attempt to solve the problem. The solutions of the five-year-olds and the six-year-olds fell into one of two categories. They either had two elements of the triple correct or were entirely correct. Three of the five and six-year-old group failed to complete task 2 while five of them failed to do task 4. All seven-year-olds and eight-year-olds performed both task 2 and 4 successfully with the exception of one eight-year-old who failed to locate red A5, i.e. task 2.

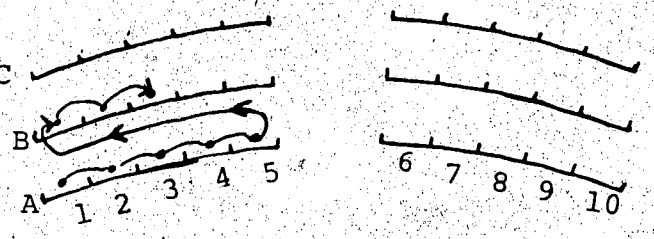
Task 3 was locating blue B8. More subjects were given assistance in performing this task than in task 2 and task 4. Subjects ranging in age from three years to six years inclusive made correct responses when given assistance. Task 3 was the only task where a three-year-old made a successful response and that occurred when assistance was given. In comparing the successful responses of task 3 with those of tasks 2 and 4, the youngest subjects to locate blue B8 (task 3) were five-year-olds while the youngest subjects locating red A5 (task 2) and yellow E4 (task 4) were four-year-olds. Only one six-year-old located blue B8 compared to four six-year-olds locating red A5 and three six-year-olds locating yellow E4. The seven and eight-year-olds made more errors in task 3 than in tasks 2 and 4 put together. Three errors were made by this age group in task 3 compared to one error in tasks 2 and 4 inclusive.

Only 10 subjects gave correct responses when performing task 3 compared to 17 correct responses on task 2 and 18 correct responses on task 4. In task 3 (blue B8) there was a marked difference in the number of correct responses between the six-year-olds and the seven-year-olds. Only one out of five six-year-olds found the seat blue B8 compared to four out of the five seven-year-olds. In the other two tasks the increase in correct responses was gradual. As age increased the number of correct responses increased.

When considering variance in the responses on the three tasks, the seven-year-olds varied the least. All but one of the responses were correct. The four-year-olds were the age group with the greatest variance in behavior. In performing the three location tasks the four-year-olds displayed responses that had at least one subject being placed in each category. Some four-year-olds would not attempt a task while others in the same age group not only attempted the task but were correct.

As in the parking lot problem the subjects used what appeared to be counting procedures, matching procedures, or a combination of both. The layout of the apparatus had an aisle between the block of seats 1 to 5 and the block of seats 6-10. The rows were labelled with letters. Each of the three tiers was a different color (see Appendix B). Of the children who used what appeared to be a combination

of matching and counting, two subjects counted to locate the seat then checked the selected position by matching the seat to the numerals. A four-year-old who used what appeared to be counting as her strategy in locating seat blue B8 started counting at A1, then proceeded systematically to A5. Instead of moving across the aisle to count the sixth seat, the count of six was done at B1, the count of seven at B2, and the count of eight at B3. She ended up in the wrong seat but did indicate that she had a notion of how to use a coded triple.



b. The Selection Problem

When a subject completed the location tasks of the theatre problem, four 'men' had been placed with a minimum of one being in its correct place. If this situation existed the subject was asked to solve a selection problem (see Appendix A for protocol). The selection problem with the theatre consisted of the interviewer pointing to a correctly placed 'man' and asking the subject to select the ticket with the triple which described the 'man's' position.

As in the parking lot selection problem, a number of subjects were presented the problem in a manner that did not follow protocol. In the theatre problem nine out

of the 30 subjects were presented the selection problem according to the protocol. For this reason the responses can not be interpreted. Five of the nine subjects were three-year-olds who did not attempt the problem. The remaining four subjects were in the age range of four to six. No seven-year-olds and no eight-year-olds were presented the selection problem according to protocol. The protocol for the selection problem in the longitudinal sampling has been followed. Therefore a discussion of the procedures used in making a selection while using the theatre apparatus will be discussed in a later section.

c. Comparison between the Location and Selection of a Triple

Because the data on the selection section of the theatre was not interpreted for the reason previously stated, i.e., the protocol was not followed, it follows that there can be no comparison of behaviors on the locating of a triple on the theatre with that of selecting a ticket which describes a specified seat.

Section 3. Comparisons

a. The Comparison of the Location Problems

In the parking lot situation the sampling had been presented tasks of locating a position in two-dimensional space using a number pair such as (4,3). As the subjects worked on the problems they had to keep in mind that the first numeral of the pair indicated the horizontal

and the second numeral indicated the vertical direction. The theatre situation involved tasks of locating a seating position in three-dimensional space using a triple comprised of color, letter and number, such as yellow B8. The subjects had to remember that the color of the lettering referred to the tier, the letter referred to the row, and that the numeral referred to a seat in any given row.

The responses of the subjects have been classified into three categories:

I. The response was correct.

II. The response was correct but made with assistance from the interviewer.

III. The response was incorrect.

According to the protocols two location tasks were to be done following the instructional task in the parking lot problem and three were to be done in the theatre problem. The parking lot had 10 subjects in each age group attempting to solve the problem whereas the theatre had five subjects in each age group. For these reasons the responses have been recorded as percentages to enable a comparison. Category II has been included because assistance was given in several instances even though the protocol did not call for it.

Examining the percentage of the correct responses, the three-year-olds had no successful responses in either

Table 8

Percentage Responses in Each Category at Each Age Level

Parking Lot

Procedure Categories	Age 3	4	5	6	7	8
I	0	10	25	50	95	100
II	0	35	15	0	5	0
III	100	55	60	50	0	0

Theatre

Procedure Categories	Age 3	4	5	6	7	8
I	0	13	47	54	93	80
II	7	33	20	20	0	0
III	93	54	33	26	7	20

problem. The four-year-olds had 10 percent of the responses correct in the parking lot and 13 percent correct responses in the theatre. The performance for this age level is very similar whether the subjects were using a number pair or a coded triple. The five-year-olds had the largest variance in the percentage of correct responses. Twenty-five percent of the responses were correct in the parking lot while 47 percent were correct in the theatre. The eight-year-old group performed the parking lot tasks perfectly. Their performance on the theatre tasks dropped to an 80 percent rate of success.

In the parking lot problem where the subjects were using a number pair in two-dimensional space to solve the problem the number of correct responses increased as the age of the subjects increased. This same tendency occurred in the theatre problem with the exception being the eight-year-olds. Subjects working with a triple in three-dimensional space met with success more frequently as age increased.

Table 8 does not indicate that the subjects performed better on one task than the other. Number pair tasks in two-dimensional space tend to be developmental as do the coded triple tasks in three-dimensional space.

b. Comparison of the Selection Tasks

A comparison of the selection tasks on the parking lot and theatre was not done because there were not

sufficient data on the theatre selection task for reasons stated in section 2b.

Section 4. Verbalizations

Thus far the analysis of the data has revealed that the majority of the students interacted freely with the apparatus. Some of the children in the sample made spontaneous verbalizations while attempting to solve the problems. Some of these verbalizations, such as counting, appeared to be closely related to the tasks. It will be recalled that the subjects were asked questions pertaining to the problems. The responses to such questions have not been included, only the subjects' spontaneous verbalizations are discussed.

Table 9 gives the number of subjects in each age group who made some form of verbalization during the parking lot and the theatre problems. The table reveals that 32 out of 60 subjects spoke while attempting to solve

Table 9

Number of Subjects Verbalizing

		Age Level					
Task		3	4	5	6	7	8
Parking Lot	(N = 60)	4	5	7	5	6	5
Theatre	(N = 30)	2	3	5	5	4	5

the parking lot problems while 24 out of 30 subjects spoke when attempting to solve the theatre problem. It also shows that more subjects at the five-year-old level made verbalizations than at any other age level.

It appears from the table that a greater proportion of subjects verbalized during the theatre problem than during the parking lot problem. Of the 30 subjects who performed only the parking lot problem nine did not verbalize. Thirty subjects attempted both the parking lot and the theatre problems and only six of them made no verbalizations.

All the observed verbalizations in the parking lot and the theatre problems were classified into five categories. They were the following:

a. Asking for clarification or repetition of the question.
b. Repetition of the question.
c. Comments pertaining to the task or the material (e.g. counting).
d. Nonverbal sounds (e.g. car noises, sighs).
e. Comments not pertaining directly to the task.

Examples of the different categories reveal the type of verbalizations the subjects made while they engaged in the tasks. Some subjects asked questions pertaining to a task. In asking for clarification in the location tasks

of either the parking lot problem or the theatre problem, the children asked questions like "This one?", "Where to?", "Which one goes first?", "Where does it tell you?". When the subject repeated the question to himself he would make comments like "Three and a four. I have to find a three and a four." Most of the verbalizations made by the subjects while performing either the parking lot tasks or the theatre tasks fell into category c—comments pertaining to the task.

When working on the parking lot, subjects would verbalize the numeral of the ordered pair on which they appeared to be concentrating. One example would be, while working on task (2,5), the audible comment "Two," then the subject would move the car to row 2, followed by the audible comment "Five" and the moving of the car to the fifth space in the second row. Subjects would count the spaces as they moved the car down the selected row. As in the parking lot problem most of the comments made in the theatre problem pertained to the task. The subjects tended to verbalize the element of the triple on which they were concentrating. "Yellow," "E," "Four" would be verbalized as they attempted to find the appropriate seating place.

Some of the irrelevant comments made were:

"I'll put the card on the roof of the car because it's raining."

"I'm parked backwards."

"That man is wearing a hat."

"That's a Dodge."

Table 10 shows the frequency for the five categories of verbalizations. It reveals that verbalizations involving comments pertaining to the tasks occurred more often than any other form of verbalizations. Comments pertaining to the task were most common in the location problems of the theatre. However it was in the location tasks of the parking lot problem where most of the counting was done. Audible counting did not occur as often as one might expect. Most of the counting done was inaudible but implied by lip and hand movements of the subjects during the completion of the tasks. Only one subject, a six-year-old, counted audibly the number of spaces as he moved down the row, then overtly checked his position by moving his hand down the column to the numerical markings on the grid.

Table 10

Frequencies for the Five Verbalizations Categories

	Verbalization Category				
Problem	a	b	c	d	e
Parking Lot (N = 60)	19	9	38	21	10
Theatre (N = 30)	12	1	24	6	9
Total	31	10	62	27	19

problems made no comment as they made their selection.

PART II. THE LONGITUDINAL SAMPLING

For the longitudinal sampling theoretically the distribution of the problem sets provided for five subjects at each age level to be given the parking lot problem and 10 subjects in each age group to be given the theatre problem. Twelve of the original subjects did not participate in this phase of the investigation thus reducing the number of subjects in each age group, except age seven, who attempted the problems in 1975 (see Appendix D). In order to check the validity of the procedures and behaviors identified in the analysis of the cross-sectional data, the analysis of the behaviors for the longitudinal data is treated in the same manner.

Section 1. The Parking Lot

a. The Location Problems

Twenty-four of the 60 subjects who attempted the parking lot problem in 1974 were again given the problem in the longitudinal sampling. The procedures employed by the subjects in attempting to solve the location tasks were observed to be the same as those used in the cross-sectional sampling. Therefore these procedures

have been classified according to the same categories (see page 40).

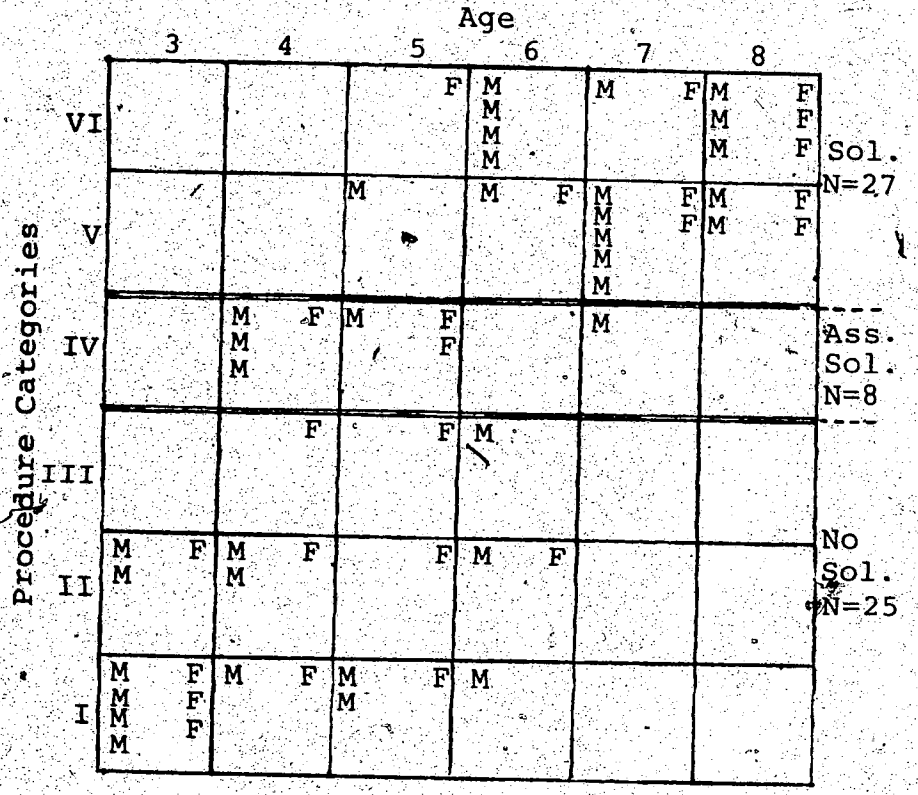
For the longitudinal sampling a change of emphasis was made. However, this did not require a change in the procedure classifications. Whereas the subjects were assisted in more than the first task in the cross-sectional interview, they were not given assistance after the first car was parked in the longitudinal sampling. A subject who may have been classified in category IV, the category where a correct response was made with the assistance of the interviewer in the cross-sectional sampling, now would be classified in any one of the remaining categories depending on his solution. In the longitudinal study for task 2 and 3, category IV would be expected to be an unused category. As in the cross-sectional sampling a table of the distribution of the solution categories for task 1 has not been included because task 1 was an instructional task. The distribution of the procedures used in the cross-sectional and the longitudinal study is given in Table 11 for task 2 and Table 12 for task 3.

The overall distribution remained unchanged although fewer subjects attempted the task in 1975. In the longitudinal sampling the proportion of successful solutions has increased but this is to be expected as there were fewer younger subjects and more older subjects. In the longitudinal sampling no four-year-olds were

Table 11

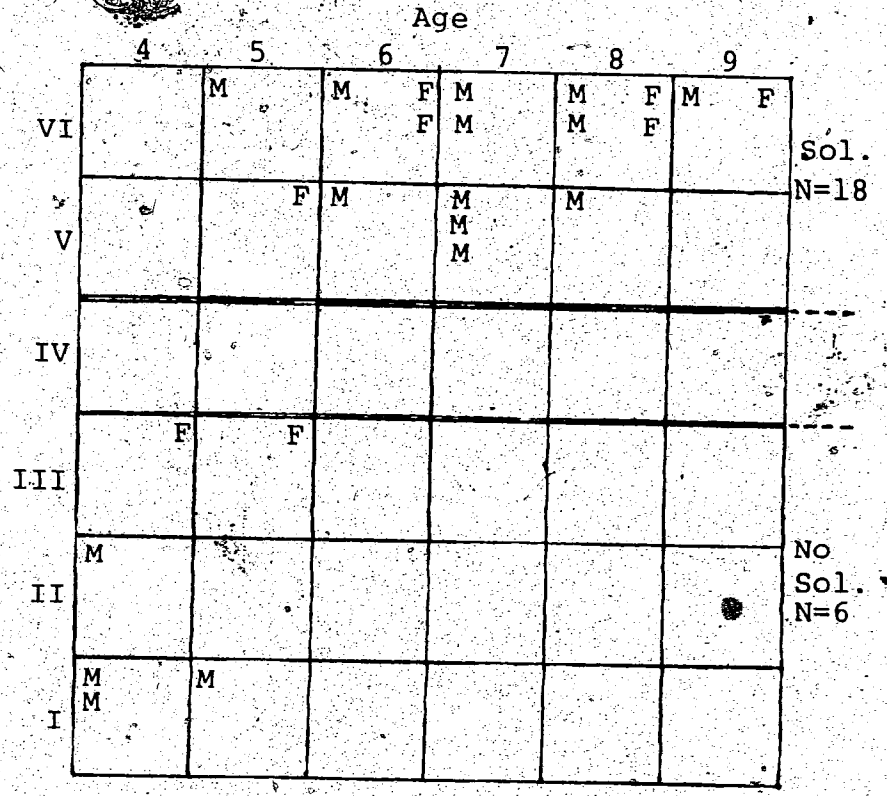
Distribution of Procedures for the Location Problems of the Parking Lot
Task 2 (2,5)

Cross-sectional



M - Male
F - Female

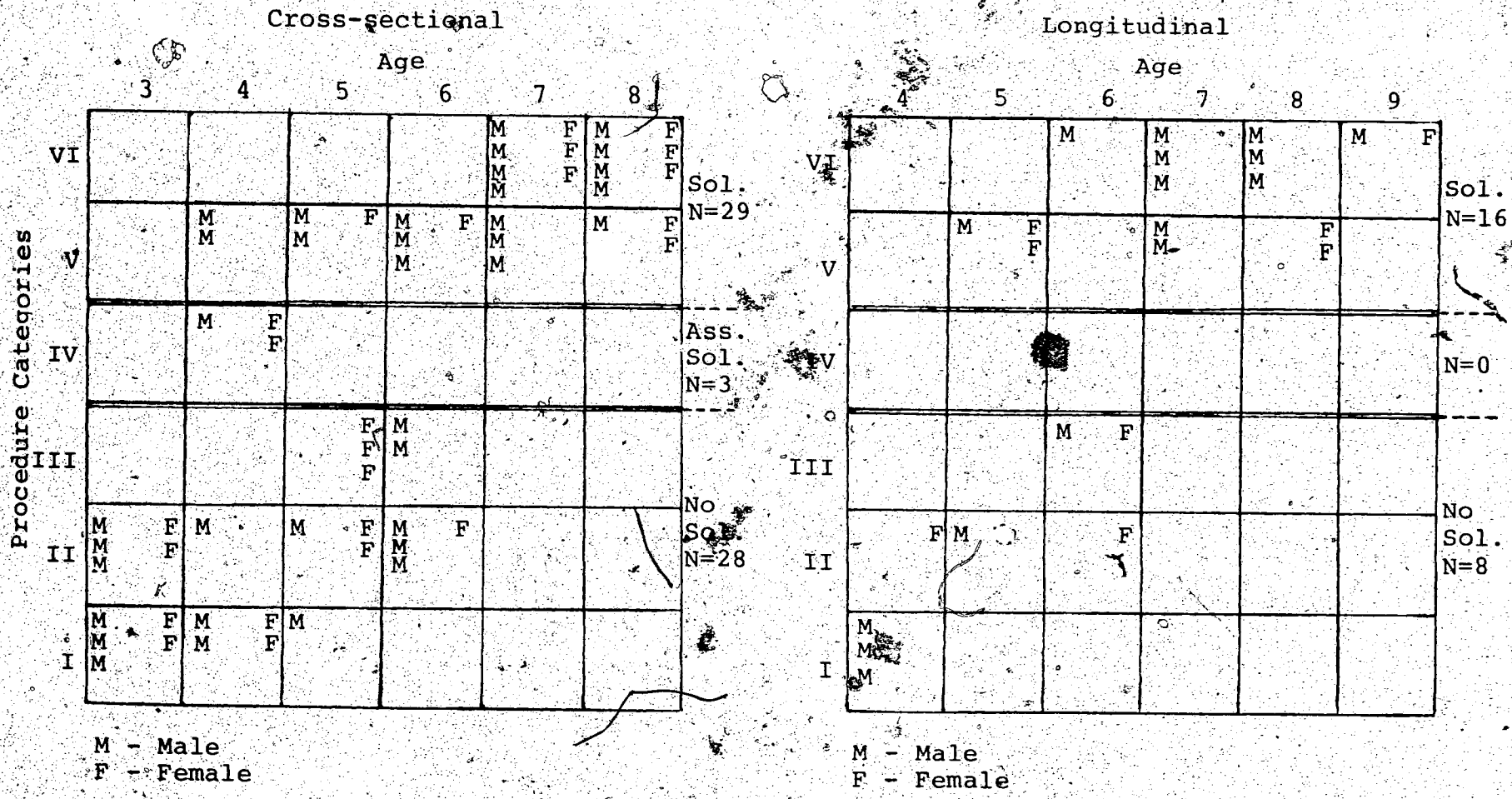
Longitudinal



M - Male
F - Female

Table 12

Distribution of Procedures for the Location Problems of the Parking Lot
Task 3 (8,6)



successful in performing task 2 or task 3 whereas two four-year-olds gave correct solutions to the third task in the cross-sectional sampling. The five-year-olds in the longitudinal sampling markedly improved in their performance with at least half of the subjects completing both task 2 and 3 successfully compared to one out of ten and three out of ten respectively in the five-year-old group of the previous year. In the 1975 sampling the six-year-olds were inconsistent in their performance. Locating the number pair (2,5) was completed successfully by all the subjects in this age group while task 3, locating the number pair (8,6), was correctly located by only one six-year-old.

There was not such a wide variance of successful performance between task 2 and task 3 in the cross-sectional sampling, yet, like the longitudinal sampling, the six-year-olds performed better on task 2 than on task 3. Mastery of the tasks occurred with the children who were seven and older in both the cross-sectional sampling and the longitudinal sampling. The age of the youngest child to make a correct response was four years in the cross-sectional sampling and five years in the longitudinal one. In both samplings the oldest subjects to make incorrect responses were the six-year-olds. A six-year-old in the cross-sectional sampling and a five-

year-old in the longitudinal sampling were the oldest subjects who made no attempt to solve the problem. The greatest change in behavior occurred between the six and seven-year-olds in both samplings. In the longitudinal sampling the performance of the six-year-old group is noteworthy. In performing task 2, which is locating (2,5), all were correct. In contrast only one was correct in locating (8,6) in task 3. In both the cross-sectional sampling and the longitudinal sampling the proportion of correct responses in task 2 increased as age increased. This occurred also in the cross-sectional sampling in task 3. Task 3 of the longitudinal sampling was completed more successfully by the five-year-olds than the six-year-olds. Three out of four five-year-olds located (8,6) whereas only one out of four six-year-olds did the same.

In the longitudinal sampling, the strategies employed by the subjects were observed to be the same as those used in the cross-sectional sampling. It was observed in the cross-sectional sampling that most subjects used what appeared to be the strategy of counting or matching the first element of the number pair to the row axis. Once the row axis was selected the subject would count the number of spaces corresponding to the second element of the number pair. However in the longitudinal sampling the strategy most frequently used was what appeared

to be matching for each element of the number pair. The subject would match the first element of the number pair with the row axis and the second element with the column axis, moving from the column axis, over the rows until the intersecting space was found. Another matching strategy employed was such that the subject surveyed the parking lot with his eyes and then placed the car in the appropriate place.

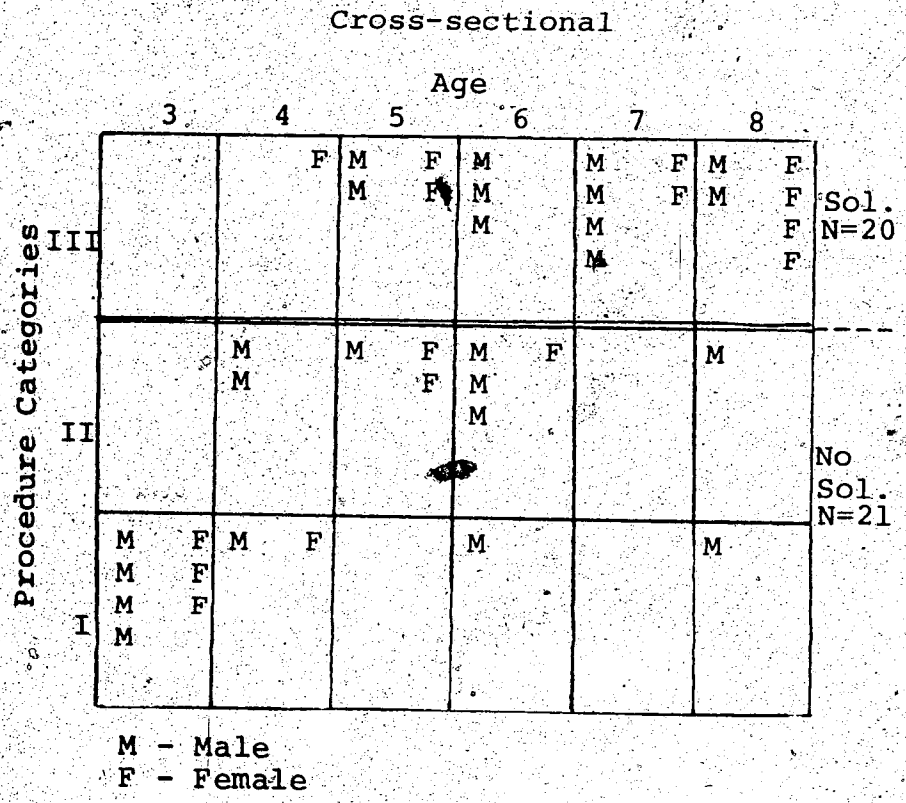
b. The Selection Problem

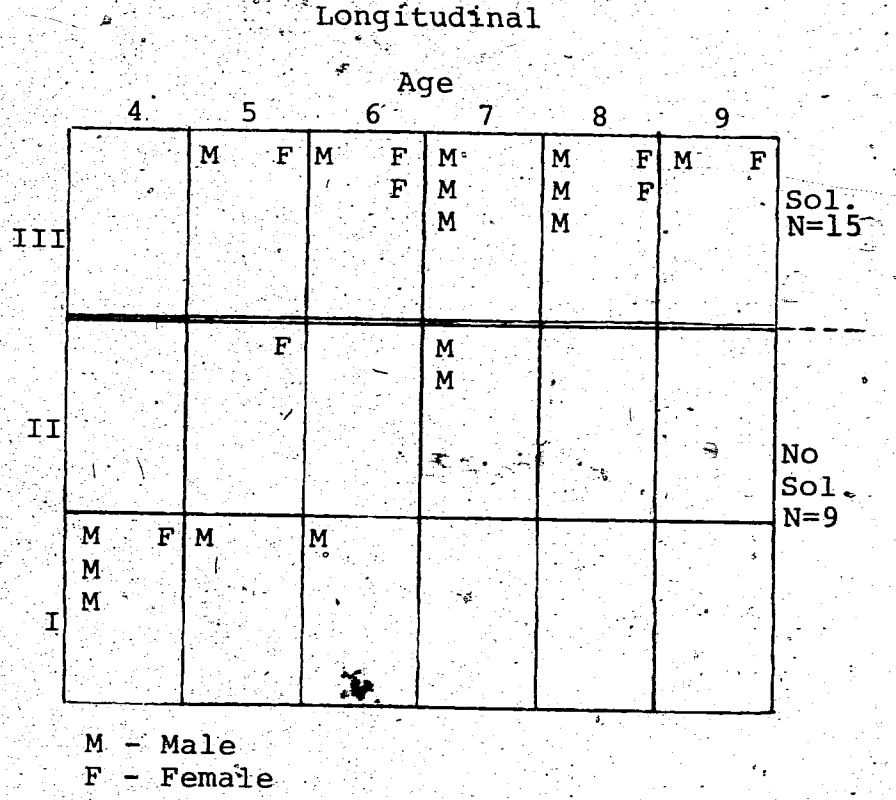
As in the cross-sectional sampling, after completing the location of three number pairs on the parking lot, each subject was asked to solve a selection problem if there was at least one car parked correctly. It should be noted that the car parked in the instructional task is considered even though the data have not been included in this study for reasons previously stated. The selection task consisted of the interviewer pointing to a correctly parked car and asking the subject to select the ticket with the number pair which described the car's position.

The procedure categories employed by the 24 subjects attempting to solve the selection task were observed to be the same as those used in the cross-sectional sampling. The observed procedures of the longitudinal sampling have been classified according to the same categories (see page 49). Table 13 shows the distribution of procedures utilized by the 24 subjects who were presented the selection problem.

Table 13

Distributions of Procedures for the Parking Lot Selection Problem





Each subject has been classified as belonging to one of the three categories according to the procedure used. The distribution of the solution procedures employed in the cross-sectional sampling is also given in Table 13.

The overall distribution remained unchanged even though fewer subjects attempted the task in 1975. The proportion of successful selections increased in the longitudinal sampling. Fifteen of the 24 subjects in the longitudinal sampling made correct selections whereas 20 of the 41 subjects in the cross-sectional sampling were successful. In the longitudinal sampling no four-year-olds attempted a solution. Three of the five four-year-olds in the cross-sectional sampling attempted a solution with one of them being successful. The proportion of successful performances increased as age increased with two exceptions: the six-year-olds in the cross-sectional sampling and the seven-year-olds in the longitudinal sampling.

In examining the location performances of the subjects who made correct selections in the cross-sectional sampling and the longitudinal sampling, it was observed that at the age of seven subjects completed all the location tasks and selection tasks successfully. The variant behaviors occurred in the younger age groups. In the longitudinal sampling one age group was successful in the selection problem but was not successful in both location tasks. This was the six-year-old group. All the six-year-

olds who made correct selections were able to park the car in space (2,5), but were unable to park the car in space (8,6). The five-year-olds in the same sampling who could make a correct selection could also park the cars correctly in both task 2 and task 3.

In the cross-sectional sampling (1974) there was a larger number of young subjects who made correct selections. This was expected because there were more younger participants in the cross-sectional sampling. They were also given more assistance in completing the location tasks. Of the four-year-olds and five-year-olds who made correct selections, they completed only one two location tasks successfully.

The six-year-olds performed differently in the cross-sectional sampling compared to the longitudinal sampling. In the cross-sectional sampling the six-year-olds who completed the selection task successfully, also completed correctly both of the location tasks. In the longitudinal sampling, the six-year-olds who made a correct selection were successful only on the first location task. The last location task was using number pair (8,6). In the longitudinal sampling all the six-year-olds selected row eight but made an error in locating space six. All the subjects in this age group used a matching process in locating the second element of the number pair (8,6) and were not accurate in tracing the path

from the column axis to row eight.

c. Comparison between the Location and Selection of a Number Pair

A comparison of the solution results for the location tasks and selection problem should reveal whether the subjects in the sample were as successful with the tasks involving one process as with those involving the other. As in the cross-sectional examination, a table of distributions has been devised to record the correct responses of the twenty-four subjects who performed both the location and selection tasks in the parking lot problem. These results are given in Table 14.

In the longitudinal sampling, according to protocol, the subject was not presented the selection problem unless at least one location task had been completed correctly. No four-year-olds were successful in the location tasks and therefore did not do the selection problem. Three of the four five-year-olds completed at least one of the location tasks. These five-year-olds were all successful in locating (8,6). Two of the three that were eligible for the selection task did the task correctly. In the six-year-old group all four participants performed task 2 correctly but only one was correct in task 3. Three of the four six-year-olds made correct selections. All the [illegible] year-olds did both location problems correctly. Errors were made in the selection problem. Twenty percent of the

Table 14

Distributions of Solutions for Parking Lot Tasks: Longitudinal Sampling (N = 24)

Subject	Age 4		Age 5		Age 6		Age 7		Age 8		Age 9	
	L	S	L	S	L	S	L	S	L	S	L	S
S_1		TA	2,3	✓	2	✓	2,3		2,3	✓	2,3	✓
S_2		TA	3	✓	2	✓	2,3	✓	2,3	✓	2,3	✓
S_3		TA			2,3		2,3	✓	2,3	✓		
S_4		TA	2,3		2	✓	2,3	✓	2,3	✓		
S_5							2,3		2,3	✓		

L - Location Problem

□ - empty space: the correctly parked car was placed in the instructional task

2 - solution for task 2

3 - solution for task 3

S - Selection Task

✓ - solution for selection task

TA - Task Abandoned

selection responses were incorrect. All eight-year-olds and all nine-year-olds completed both the location and the selection problem without error. As age increased the proportion of correct responses increased indicating that the ability to solve problems improves as age increases when observing children.

Section 2. The Theatre

a. Location Problem

Of the 48 children who attempted the theatre problem in the longitudinal sampling, 30 had been given the problem in the cross-sectional sampling (see Appendix D). The location problem in the theatre consisted of four tasks involving the use of a coded triple. The subjects were presented a 'man' to place and a ticket designating his seating space. The ticket was coded with color, letter and number. The color of the ticket designated the tier, the letter stated the row and the number indicated the seat (see Appendix A for protocol).

As in the parking lot problem, the longitudinal study (1975) of the theatre problem had the revision that the subjects would not be assisted by the interviewer after performing the initial location task. This slight revision of the protocol did not necessitate any change in the procedure categories (see page 56). The distributions of the procedures employed by the subjects in the cross-sectional and in the longitudinal sampling are given

in Table 15 for task 2, Table 16 for task 3, and Table 17 for task 4.

Due to the modification in the protocol of no assistance being given after the initial location task, category IV would be expected to be an empty category in the longitudinal sampling (1975). The subjects who made incorrect responses on the various location tasks have been placed in categories I, II or III. Those who made correct responses to the location tasks in the theatre problem have been placed in category V. The proportion of successful solutions is greater for the longitudinal sampling (36/48 for task 2, 27/48 for task 3, 31/48 for task 4) than it is for the cross-sectional sampling (17/30 for task 2, 10/30 for task 3, 16/30 for task 4). When the total number of correct solutions is compared to the number of subjects who performed the tasks, it is noted that 65 percent of the longitudinal tasks were performed correctly while only 48 percent of the cross-sectional tasks were correct. This was expected, as has been previously stated, because there were no three-year-olds in the longitudinal sampling and there were more older subjects.

In comparing the results of the cross-sectional sampling and the longitudinal sampling it was observed that the subjects at the same age level behaved in a similar manner in the two studies. The subjects who were four and older met with a degree of success in locating the triples.

Table 15

Distribution of Procedures for the Location Problems of the Theatre Task 2 (Red A5)

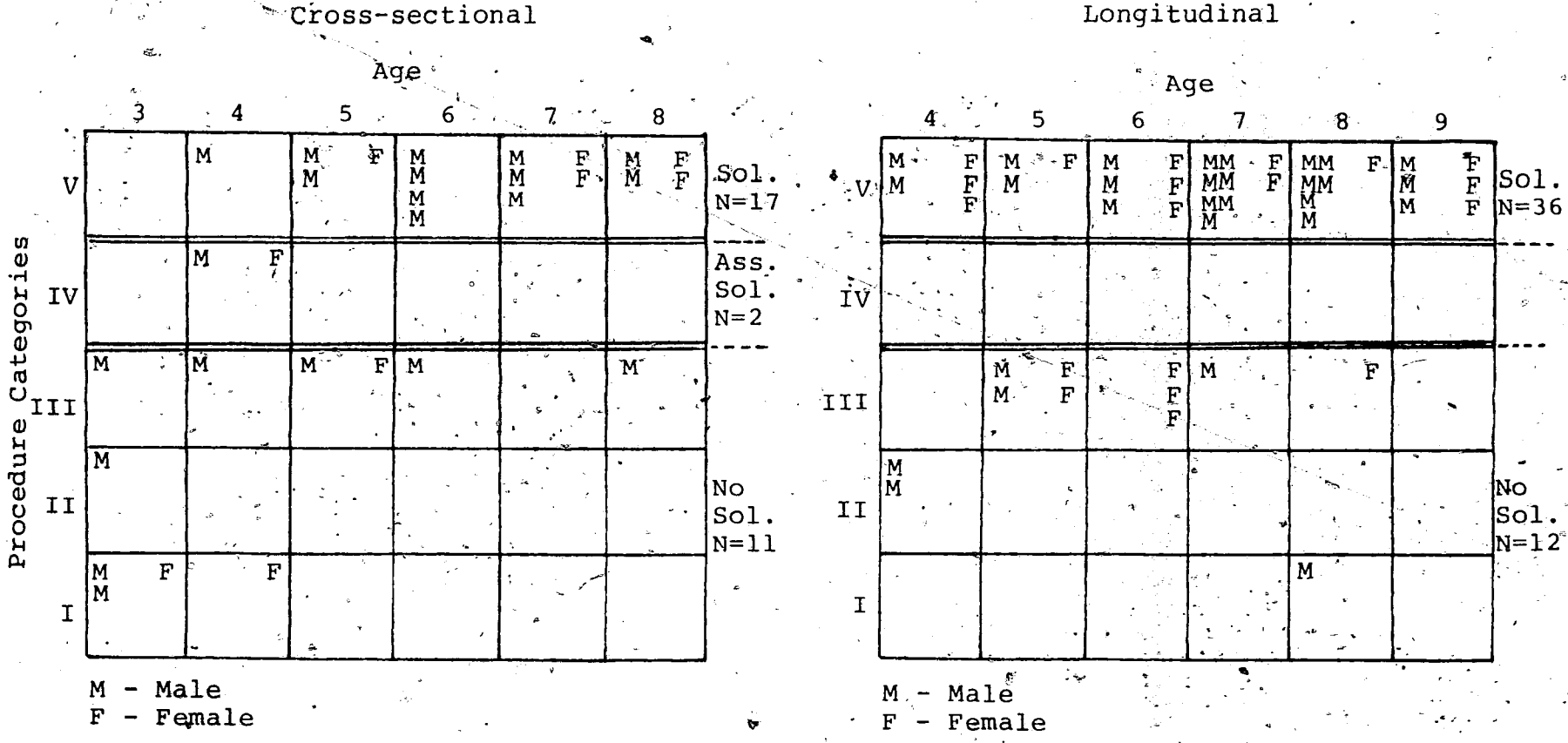
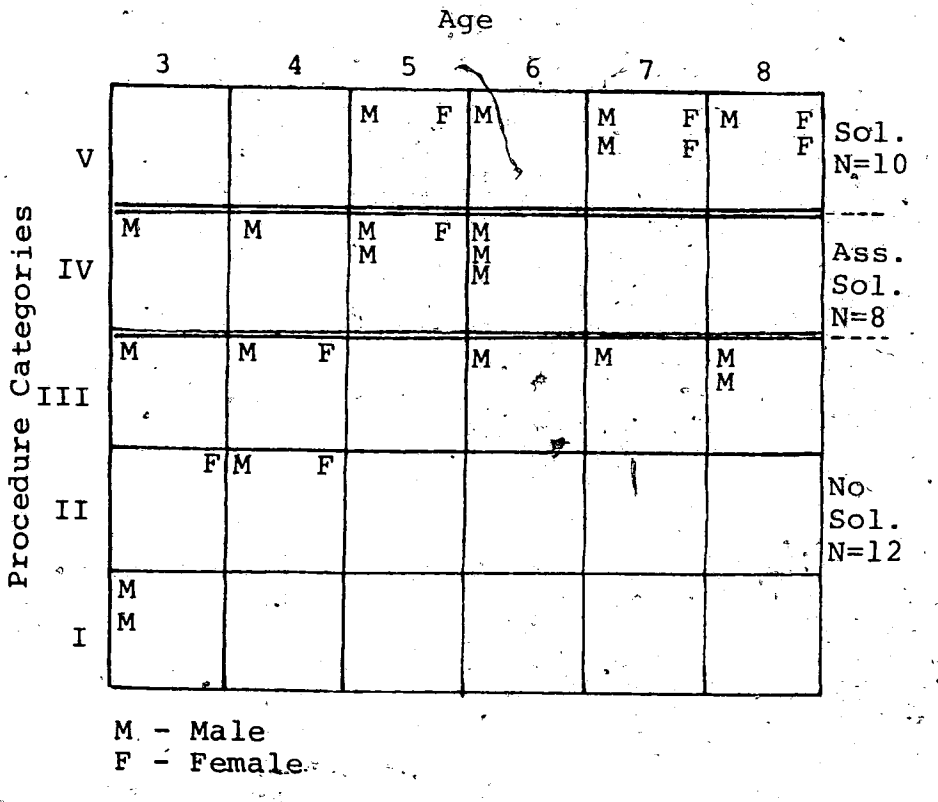


Table 16

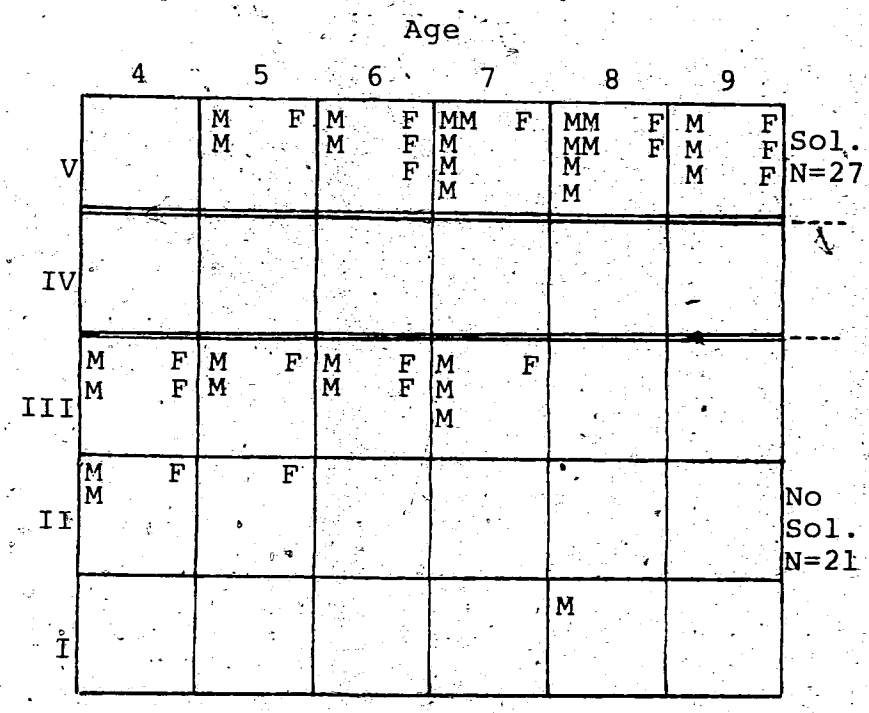
Distribution of Procedures for the Location Problems of the Theatre Task 3 (Blue B8)

Cross-sectional



M - Male
F - Female

Longitudinal

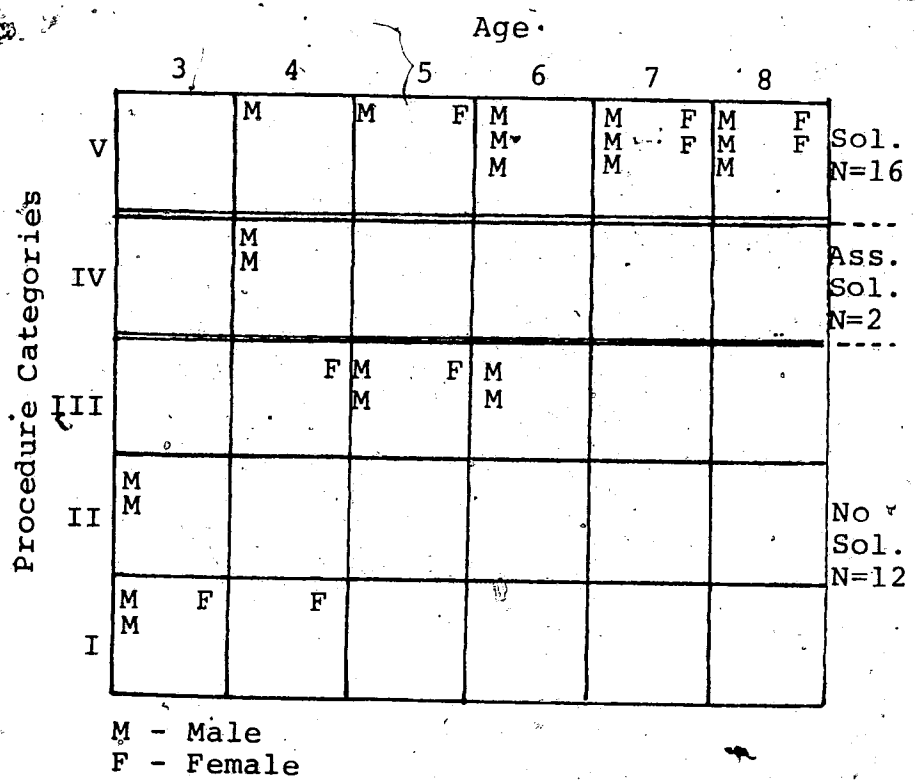


M - Male
F - Female

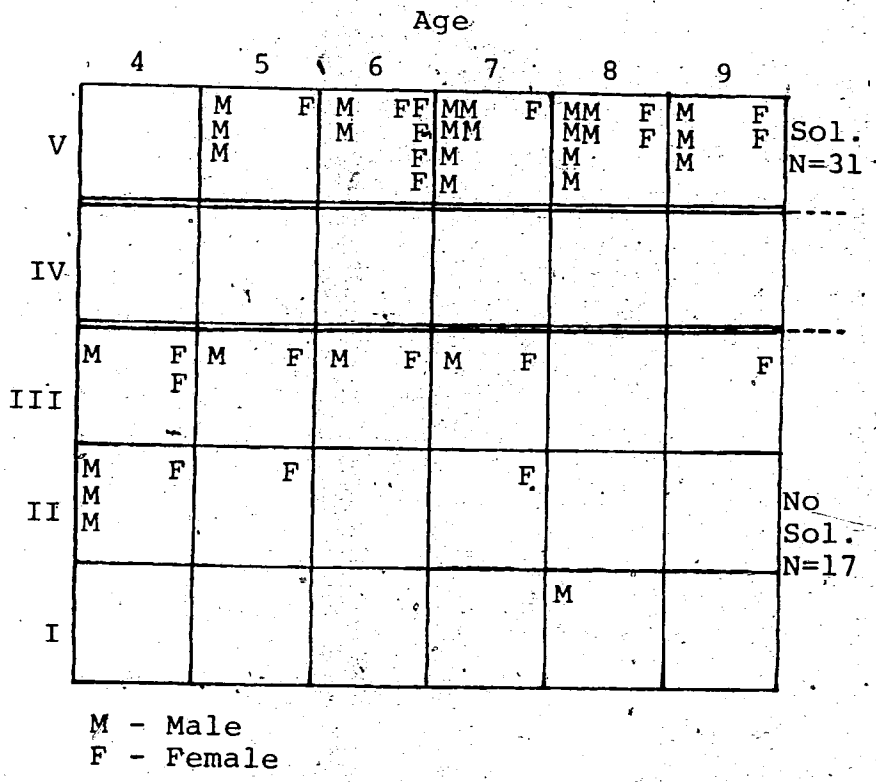
Table 17

Distribution of Procedures for the Location Problems of the Theatre Task 4 (Yellow E4)

Cross-sectional



Longitudinal



The number of correct locations increased as the age of the subjects increased. In the cross-sectional sampling one four-year-old performed the location tasks of Red A5 and Yellow E4 correctly. In the longitudinal sampling the correct responses made by the four-year-olds were on task 2—Red A5. The performance of the five-year-olds was consistent in the two samplings. In each sampling about half of the subjects performed each of the location tasks correctly. The performance of the six-year-olds was better in the longitudinal study than in the cross-sectional sampling. Two-thirds of the responses were correct for the six-year-olds when engaged in the location tasks in the longitudinal sampling compared to about half in the cross-sectional sampling. The seven-year-olds, eight-year-olds and nine-year-olds performed all the location tasks with near perfection.

For both the cross-sectional sampling and the longitudinal sampling task 3—Blue B8 was the task with the fewest correct responses. The five-year-olds were the youngest age group to correctly locate Blue B8. The youngest subjects to perform successfully on the other two tasks were four years old with one exception. In the longitudinal sampling no four-year-olds located Yellow E4. In each of the age categories having correct responses, the number of correct responses in task 3—Blue B8 was either the same as in the other location tasks or fewer.

Not one age group performed better on task 3 than on the other two location tasks. At least half of the errors made in the location problems of the theatre in both the cross-sectional and longitudinal sampling were category III type errors. The subjects who were five and older had two of the three elements in the triple correct. In the longitudinal study the four-year-olds attempted the location problems but were only successful in locating Red A5. In contrast to the subjects of five years and older, they made mostly category II type errors. Of the 12 category II type errors in the longitudinal sampling, the four-year-olds made nine. The four-year-olds were the age group with the greatest variance in behavior in both samplings. Their behavior ranged from having one element of the triple correct to having the correct location with all elements correct.

b. The Selection Problem

As in the cross-sectional sampling, after the completion of the location problem each subject was asked to solve a selection problem if the requirements were met. The protocol stated that at least one man had to be correctly seated whether it was placed by the subject in completing the location tasks or by the interviewer in the instructional task, i.e., task 1. It will be recalled that the selection problem with the theatre consisted of the interviewer pointing to a correctly placed 'man' and asking

the subject to select the ticket with the triple describing his position (see Appendix A for protocol).

As in the selection problem in the cross-sectional sampling, some subjects [illegible] the task in a manner which did not follow the protocol. In the longitudinal sampling 43 of the 48 subjects who were presented the theatre grid problem were presented the selection problem according to protocol. The observed solution procedures were categorized as follows:

I. The subject did not attempt the task or the task is abandoned.

II. The subject chose the incorrect ticket.

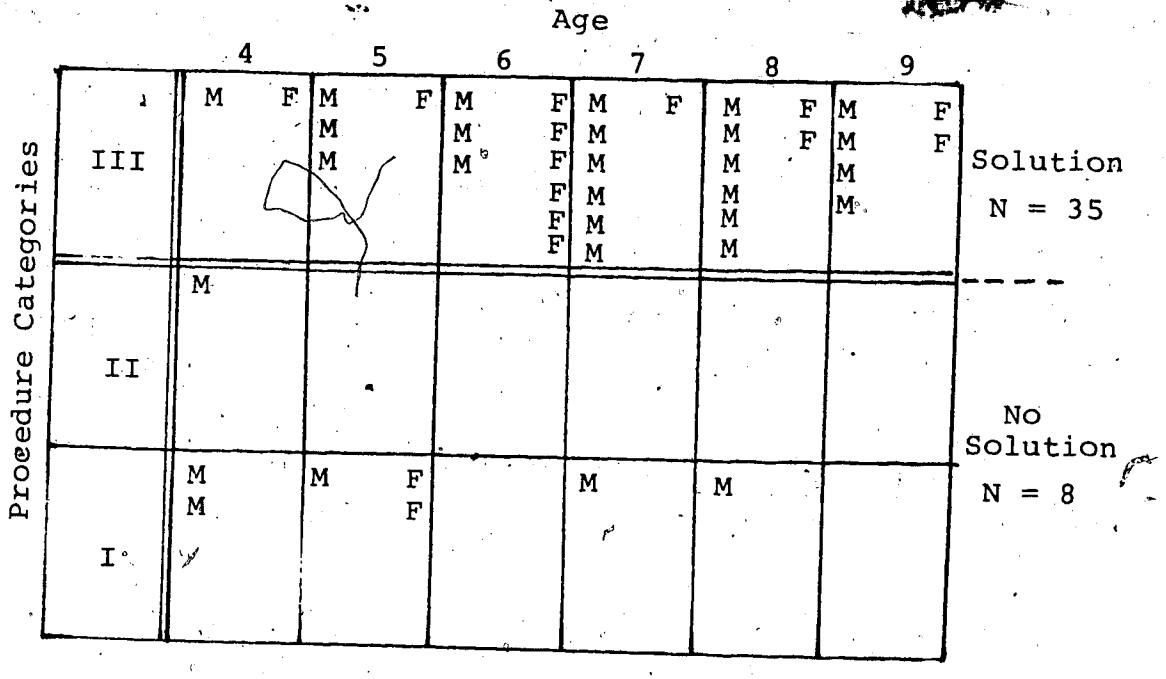
III. The subject chose the correct ticket.

Table 18 shows the distribution of procedures used by the 43 subjects who were presented the selection problem. Each subject has been classified as belonging to one of the three categories. This table does not include the results of the selection problem in the cross-sectional sampling because there was not sufficient data to enable interpretation.

Of the 43 subjects in the longitudinal sampling who attempted the selection problem, 35 made a correct selection. Each age group had some subjects that were successful. The older age groups had a larger proportion of successful subjects than the younger age groups. Two out of five four-year-olds made correct selections. This

Table 18

Distribution of Procedures for the Selection Problem of the Theatre—Longitudinal (N = 43)*



* Protocol was abandoned in five cases.

age group was the only one which made an incorrect response as in the other age groups the responses were either correct or the task was abandoned. Of the seven five-year-olds, four made correct selections. In each of the age groups from six to nine inclusive all but two selections were correct. The two selections which did not fall into category III were category I responses. The task was abandoned in these instances. The reason for this was two-fold. The protocol stated that if there was no 'man' seated correctly the task was to be abandoned. The task was also to be abandoned if the only 'man' correctly placed was in Red A5 and there was no other 'man' on the red level.

c. Comparison between the Location and Selection of a Coded Triple

A comparison of the solution results for the two major problems should reveal whether the subjects were as successful with the tasks involving one process as with those involving the other. A table of distributions has been devised to record the correct responses of the 48 subjects who performed both the location and selection problems in the theatre. These results are given in Table 19.

As in the cross-sectional sampling the subject was not to be presented the selection problem unless the requirements concerning the location tasks were met. Five of the seven four-year-olds completed only the second location task

Table 19

Distributions of Solutions for Theatre Tasks
(N = 48)

	Age 4		Age 5		Age 6		Age 7		Age 8		Age 9	
Subject	L	S	L	S	L	S	L	S	L	S	L	S
S_1	2	PA	4	✓	2,3	✓	2,3,4	✓	2,3,4	✓	2,3,4	✓
S_2	2	✓		TA	4	✓	2	PA	2,3,4	✓	2,3,4	✓
S_3	2	PA	2,3,4	✓	2,3	✓	2,4	✓		*	2,3,4	✓
S_4		TA	2,3,4	✓	2,4	✓	2,3,4	✓	2,3,4	✓	2,3	✓
S_5		TA		TA	2,4	✓	2,3,4	✓	2,3,4	✓	2,3,4	✓
S_6	2	PA	2,3,4	✓	3,4	✓		TA	2,3,4	✓	2,3,4	✓
S_7	2	✓		TA	4	✓	2,3,4	✓	2,3,4	✓		
S_8					2,3,4	✓	2,4	PA	2,3,4	✓		
S_9					2,3,4	✓	2,3	✓	3,4	✓		
S_{10}							234	✓				

L - Location Problem

2 - location task 2 was correct
3 - location task 3 was correct
4 - location task 4 was correct

S - Selection Problem

✓ - selection task correct

TA - Task abandoned
PA - Protocol abandoned
* - Technical difficulties

successfully, i.e., Red A5. Of these five two made correct selections. No four-year-old located Blue B8 or Yellow E4. Unlike the four-year-olds, the five-year-olds were most successful in locating Yellow E4, the fourth and final task. Each of the four five-year-olds who located Yellow E4 also made a correct selection. This age group was the youngest one to have subjects do all location tasks plus the selection task correctly. Three of the four five-year-olds who made some correct responses, were correct in all location tasks and the selection task. All of the six-year-olds completed at least one location task plus the selection task successfully. Two of the nine in this age group completed all tasks successfully. One seven-year-old did all the location tasks incorrectly. Therefore the selection task was abandoned. Four seven-year-olds completed both location and selection parts of the theatre problem correctly. The remaining five subjects in this age group completed a minimum of one location task correctly. The performance of one eight-year-old was not recorded due to technical problems. The remaining eight eight-year-olds completed a minimum of two location tasks. Five of the eight performed all aspects of the theatre problem correctly. One subject in the nine-year-old group made an error in a location task. Five of the six nine-year-olds had a totally successful performance. There is the least variability in performance in the older

age groups. The five-year-olds had the most variant behavior for this problem. Their behaviors ranged from having no aspect of the problem correct to having all location tasks done correctly plus the selection task.

Section 3. Comparison

a. Comparison of the Location Problems of the Longitudinal Sampling

As was previously stated in the discussion of the cross-sectional sampling, the location tasks in the parking lot problem involve using a number pair in two-dimensional space. In the theatre problem a triple consisting of color, letter and number was used to locate a position in three-dimensional space.

In the longitudinal sampling 24 subjects did the parking lot problem and 48 did the theatre problem. The number of subjects who performed in each age group varies because not all those in the cross-sectional sampling returned to do the problems in the longitudinal sampling. For these reasons the total number of correct responses for the two locations tasks in the parking lot problem have been combined for each age group and recorded as percentages. The same treatment was given to the three location tasks in the theatre situation. Because no additional assistance was given the responses were classified either as correct or incorrect. The percentage of correct responses in each age group is given in the following table:

Table 20

Percentage of Correct Responses in Location Problems

Situation	Age 4	5	6	7	8	9
Parking Lot	0	62.5	62.5	100	100	100
Theatre	24	43	67	73	89	89

Examining the percentage of correct responses, the four-year-olds were the only ones who performed better on the location problems of the theatre. The proportion of correct responses for the six-year-olds was about the same in the location tasks of the two problems. The five-year-olds and the seven years and older age groups performed better on the parking lot problem. In the longitudinal sampling, with the exception of the four-year-old group, it appears that the subjects were more successful with the number pair in two-dimensional space than with the code triple in three-dimensional space.

b. Comparison of the Selection Problems of the Longitudinal Sampling

In both the parking lot and the theatre situation of the longitudinal sampling, the selection problem was not done unless at least one location task was done correctly according to the protocol.

In the longitudinal sampling 24 subjects were presented the parking lot problem and 48 subjects were

presented the theatre problem. The number of correct responses have been converted to percentages to enable a comparison of responses. In converting the proportion of correct responses in each age group to percentages the following ratio was used:

$$\frac{\text{the number of correct responses in the age group}}{\text{the total number in the age who were presented task according to protocol*}}$$

*Protocol was abandoned in five cases of the selection problem in the theatre situation.

The results of the correct responses on the two problems can be seen in the following table:

Table 21

Percentage of Correct Responses in the Selection Problems (Longitudinal)

Situation	Age 4	5	6	7	8	9
Parking Lot	0	50	75	60	100	100
Theatre	50	57	100	87.5	100	100

On the whole the selection task in the theatre problem had a larger proportion of correct responses than the parking lot selection task. Each of the age groups from four to seven years had a larger proportion of correct responses in the selection task of the theatre than in the parking lot. All eight and nine-year-olds completed both selection tasks successfully. In both the problems the six-year-old group performed better than its adjacent age

groups. There was a marked increase in the proportion of correct responses between the five and six-year-old group. The proportion of correct responses in the six-year-old group was slightly greater than the seven-year-old group. In every case the selection task of the parking lot was abandoned in the four-year-old group. Their performance on the theatre selection task markedly improved as 50 percent of the four-year-olds made correct selections. Overall the performance on the theatre selection task was better than the parking lot selection as was indicated by the proportion of correct responses.

Section 4. Verbalizations

Verbalizations were again studied in the longitudinal data in order to compare the types and frequencies with those of the cross-sectional sampling. Analysis of the first sampling revealed that the subjects exhibited many spontaneous verbalizations while attempting to solve the various tasks. Examination of the data given in Table 22 below revealed that the proportion of subjects exhibiting verbalizations during the longitudinal data collection was greater than in the cross-sectional sampling. Thirty-five out of the 48 subjects made comments while attempting the tasks in the longitudinal sampling while only 32 out of 60 made comments in the first sampling.

Table 22

Number of Subjects Verbalizing

Task	Age Level					
	4	5	6	7	8	9
Parking Lot (N = 24)	4	4	2	5	4	1
Theatre (N = 48)	5	6	7	8	5	4

In the longitudinal sampling the nine-year-olds did not exhibit many verbalizations but the seven-year-olds did. The frequency of verbalizations expressed by the subjects in these age groups did not appear to depend as much on age as on the sample.

Table 23 gives the frequencies for the five categories of verbalizations exhibited by the subjects in the longitudinal sample. The findings in the longitudinal sampling were similar to those of the cross-sectional sampling. Verbalizations involving comments pertaining to the tasks occurred more often than any other form of verbalizations. In contrast to the study of 1975, the longitudinal sampling had as its second most frequently used category, category b versus category a in the cross-sectional study. In the cross-sectional sampling category b—the repetition of the question—was the least used category. In the longitudinal sampling subjects frequently verbalized the aspect of the task on which they were

Table 23

Frequencies for the Five Verbalization Categories

Problem	Verbalization Category				
	a	b	c	d	e
Parking Lot (N = 24)	2	8	19	5	1
Theatre (N = 48)	4	21	31	3	4
Total	6	29	50	8	5

a - Asking for clarification or repetition of the question.
b - Repetition of question to self.
c - Comments pertaining to task or material (e.g., counting).
d - Nonverbal sounds (car noises, sighs).
e - Comments not pertaining directly to problem.

concentrating, especially when working with the theatre. The parking lot problem still elicited the most animated noises. There was a decrease from the cross-sectional data in the number of subjects asking for clarification of the problem. This decrease can be explained in part by the demonstration given by the interviewer before the tasks were attempted. The subjects were encouraged to watch and listen to the demonstration. Even though it was not expressed, the subjects had prior experience with the apparatus and tasks involved.

There were very few irrelevant comments made in the longitudinal sampling. When the children attempted the tasks the first time, the laboratory set-up, which was unfamiliar, may have prompted them to exhibit this behavior more often.

As had been observed in the first sampling, audible counting was not too evident in the longitudinal sampling. It appeared that most younger subjects in the parking lot problem did inaudible counting as indicated by lip and hand movements. Only two five-year-olds and one six-year-old overtly counted.

All verbalizations examined were exhibited by the subjects while attempting to solve the problems. Even though the frequency of verbalizations was higher in the longitudinal sampling, the comments made were usually pertinent to the task and very brief—usually one word

or a short phrase. None of the subjects indicated that they remembered the problems from the previous year.

PART III. COMPARISON OF THE DATA OF THE CROSS-SECTIONAL SAMPLING AND THE LONGITUDINAL SAMPLING

In 1974 when the first phase of the investigation in non-verbal problem-solving was conducted, there were 30 subjects who did both the parking lot problem and the theatre problem. These subjects ranged in age from three-year-olds to eight-year-olds. The following year the second phase of the study was conducted with 24 of the 30 subjects returning to do both the parking lot problem and the theatre problem. These subjects were then in the age range of four to nine years. The distribution of the solutions for the 24 subjects who attempted the parking lot problems and the theatre problems in both samplings is given in Table 24. Comparing the performance of the cross-sectional sampling and the longitudinal sampling it was observed that for the younger subjects the location tasks in the theatre problem were the first tasks performed with a degree of success. The location tasks in the theatre problem involved working with a coded triple in three-dimensional space. The triple consisted of a code written with color designating one of the three tiers, a letter telling the row in which the seating place is to be found, and a number which gave the seat. In

Table 24

Distribution of Solutions for Subjects who Attempted Parking Lot and Theatre Problems in Both Samplings

Age	Subject	Cross-sectional				Longitudinal			
		PL	PS	TL	TS	PL	PS	TL	TS
		2 3		2 3 4		2 3		2 3 4	
3-4	S_1							*	
	S_2								
	S_3							*	*
	S_4							*	*
4-5	S_1								
	S_2					* *	*	* * *	*
	S_3					* *		* * *	*
	S_4					*	*		
5-6	S_1					*	*	*	*
	S_2	* *		* * *		*	*	* * *	*
	S_3		*	* * *		* *		* *	*
	S_4	*				*	*	* * *	*
6-7	S_1	* *	*	* * *		* *	*	* *	*
	S_2		*			* *		* * *	
	S_3	* *	*	* * *		* *	*	* * *	*
	S_4	*		*		* *		* * *	*
	S_5	*		* *		* *	*	* * *	*
7-8	S_1	* *		* *		* *	*	* * *	*
	S_2	* *	*	* * *		* *	*	* *	*
	S_3	* *	*	* * *		* *	*	* * *	
	S_4	* *	*	* * *		* *	*	* * *	*
	S_5	* *		* * *		* *	*	* * *	
8-9	S_1	* *		* * *		* *	*	* * *	*
	S_2	* *	*	* * *		* *	*	* * *	*

* - solution
PL - Parking Lot Location
PS - Parking Lot Selection
TL - Theatre Location
TS - Theatre Selection

contrast the location tasks of the parking lot problem involved tasks using a number pair in two-dimensional space. The number pair consisted of two numerals. The first designated the row in which the car was to be parked and the second numeral gave the parking space.

The developmental trend in problem-solving was in evidence when the performance of the subjects who attempted the parking lot problem and the theatre problem in both samplings was examined. Three of the four three-year-olds who experienced no success in the cross-sectional sampling, a year later, were still not successful in attempting the parking lot problem. However they had a degree of success when attempting the theatre problem. These three subjects performed the location task Red A5 correctly. Two of these three three to four-year-old subjects were also successful in the selection problem. The performance of the four subjects in the four to five year age group had a wide variance between the cross-sectional sampling and the longitudinal sampling. The four-year-olds of the cross-sectional sampling were not successful in any part of either the parking lot problem or the theatre problem. Their performance as five-year-olds was different. One subject as a four-year-old attempted both the parking lot and theatre problems and met with no success. A year later, in the longitudinal sampling, the same subject performed every task of both the parking lot problem and the theatre

problem correctly. Another subject in this age group had almost the same experience going from performing no tasks correctly as a four-year-old to a near perfect performance as a five-year-old. This subject failed in his attempt to make the correct selection in the parking lot problem.

Of the four subjects in the five to six year age group, there was more variance in the performance in the cross-sectional sampling than in the longitudinal sampling. In the cross-sectional sampling one subject did not perform any part of the parking lot problem or the theatre problem correctly. Another subject was successful in one location task in the parking lot and not in any part of the theatre problem. A third subject in the five to six age group performed successfully all the location tasks in the theatre problem but no part of parking lot problem was completed successfully. The fourth subject in this particular age group performed all the location tasks in both the parking lot and the theatre correctly. In the longitudinal sampling the same four subjects in the five to six age group displayed less variance in their solution performances of the parking lot problem and the theatre problem. Each subject was successful in at least one of the two location tasks of the parking lot problem plus one of the three location tasks and the selection task in the theatre problem. The subject who was the least successful in the cross-sectional sampling was the least successful in the

longitudinal sampling. The two subjects who were successful in one part of the parking lot problem or one part of the theatre problem in the cross-sectional sampling, were now successful in one location task in the parking lot problem and two location and selection tasks in the theatre problem. No subjects in this five to six year age group completed every aspect of both the parking lot and the theatre problem correctly.

The six to seven year age group was the youngest age group to have a subject perform all tasks in the cross-sectional sampling and all tasks in the longitudinal sampling correctly. One subject in this group performed only the selection task in the parking lot problem correctly as a six-year-old. A year later, his age now seven, he performed all location tasks of both the problems correctly but neither selection task. The remaining three subjects in this age group in 1974 performed a minimum of one location task in the parking lot and one in the theatre problem. In 1975 these same subjects were correct in the two location tasks of the parking lot problem and two of the three location tasks of the theatre problem. Two of the three made correct selections in the parking lot task and all three made correct selections in the theatre problem.

In the seven to eight age group and the eight to nine age group there was little variance in the performance

in the cross-sectional sampling and the longitudinal sampling. The subjects performed the location and the selection problems of the parking lot and the theatre with very little error in either cross-sectional sampling or the longitudinal sampling.

Going through the sampling, comparing the performance of a subject in the cross-sectional sampling to his performance a year later in the longitudinal sampling, the subject's performance either had the same number or more correct responses. For the subjects who were seven and older in both the 1974 and 1975 samplings, most of the responses were correct. In the three to four and four to five age groups the performance of two subjects remained unchanged in the two samplings. The remaining six subjects' performance had more correct responses in the longitudinal sampling. The experience of the five to six and the six to seven age groups was more correct responses in the longitudinal sampling than the cross-sectional sampling indicating that the ability to solve problems has a developmental trend.

CONCLUDING STATEMENTS

In this chapter the findings from the cross-sectional and longitudinal phases of the investigation have been reported. A videotape recorder was used to record the behaviors of the children as they did the problems. An

indepth analysis of its usefulness was not done but from the results of this investigation it appears to be effective. The longitudinal data support many of the cross-sectional findings. Most of the procedures employed by the subjects in the first sampling were again observed in the second sampling.

The following general statements summarize the specific findings which are considered important.

1. Videotaping the behaviors of young children appeared to be an effective technique for collecting data in a clinical situation.

2. The majority of subjects used a systematic procedure in their attempt to do the location problems.

3. In the location problem of the theatre, the position to be found governed the order in which the elements of the triple were interpreted.

4. The younger subjects experienced a larger degree of success with the location tasks utilizing a code triple compared to using a number pair.

5. The children were just as successful with the location problems associated with the parking lot as with those associated with the theatre.

6. The children's procedures changed and became more systematic as they got older.

7. The younger subjects were more successful with the selection tasks than the location tasks in both the

parking lot and the theatre problem.

8. In the location problems of the theatre the use of colored letters to indicate the tier was just as effective as the use of colored tickets.

9. The children displayed a variety of verbalizations when attempting the problems.

Chapter IV

SUMMARY, DISCUSSION AND IMPLICATION OF RESULTS, CONCLUDING STATEMENTS, RECOMMENDATIONS FOR FURTHER RESEARCH

SUMMARY

Much research on problem solving in mathematics has been reported with the majority of studies being concerned with children's verbal responses to problems. Nelson and Sawada (1974) undertook a longitudinal study into young children's behavior in non-verbal problem-solving situations. They designed six pairs of problems on the basis of 'derived criteria' as outlined by Nelson and Kirkpatrick (1975). Because little is known at present on how to devise appropriate problem-solving situations for children their ultimate purpose was to study the responses of young children to these problem situations in an attempt to include non-verbal problem solving in a model for mathematics learning.

This present study is based on data collected by Nelson and Sawada for one pair of problems. The data were collected in a clinical situation and comprised a cross-sectional and a longitudinal sampling. The cross-sectional interviews were conducted in the summer of 1974 with a sample composed of 90 children ranging in age from three to eight years. The longitudinal interviews were conducted

the following summer with a sample of 76 of the original subjects who were available. The children thus were in the age range four to nine.

The aim of the present study was to observe and describe behaviors of young children while attempting to solve problems associated with two spatial situations: one in two dimensions, which involved a simulated parking lot, and the other in three dimensions, where a simulated theatre was used. There were 60 subjects, 10 at each age level, who were presented with problems involving a parking lot during the cross-sectional interviews. Thirty of the subjects, five at each age level, also attempted the problems associated with the theatre. In the longitudinal sampling, 24 of the available subjects tried to solve problems involving the parking lot and 48 of the subjects attempted the theatre problems. Each situation consisted of a location problem and a selection problem.

An interviewer presented the problems to the subjects individually. As they worked at solving the problems, their behaviors were recorded on videotape. Analysis of the videotapes suggests the following general conclusions:

1. Videotaping the behaviors of young children appeared to be an effective technique for collecting data in a clinical situation.

2. A systematic procedure was incorporated by many of the children as they attempted the problems.

3. For the pre-school child the physical structure of mathematically equivalent problems can make some more difficult to solve than others.

DISCUSSION AND IMPLICATION OF RESULTS

A summary of the findings was made at the conclusion of Chapter III. A discussion on each of these follows.

Videotaping the behaviors of young children appeared to be an effective technique for collecting data in a clinical situation.

Using the videotape recorder as a data-gathering device meant that a permanent record of the children's actual behaviors and verbalizations has been made for every problem situation. One advantage in having a permanent record is that the problem of reliability in interpreting the data is lessened. A tape can be viewed any number of times and by any number of individuals. In other methods of observation, the one gathering the data has some preconceived notions and is often trained to record specific behaviors. The unexpected behaviors may be disregarded. The bias created by these factors would be assumed to be eliminated in the data gathering process when using a videotape recorder because all the actions of the children are to be recorded. In this study the behaviors that were deemed important were isolated after the data had been collected and the tapes had been viewed many times by a team of investigators.

An outcome of using the videotape recorder for collecting data is that the information has to be converted into a workable format. A coding system can be devised by observing the actual tapes. Unlike the situations where the data are being coded while the behavior is going on, the code does not have to be developed beforehand. If ever a particular coding is found to be inadequate a revised system can be developed without the loss of data. If on-the-spot coding is done and the code proved to be defective, all the data recorded would be lost. The ones using the code do not have to be fully trained before transcribing the data but can develop their skill and efficiency as they record. A technician who is unable to retrieve the information from the tapes in a reliable manner can easily be replaced without interfering with the research.

A general coding system was devised to record the behaviors in all the problem situations. Even though modifications and additional symbols were necessary for each problem the similiarity of the codes has given continuity to the recording system. The codes were easy to read and included those behaviors which were stimulated by the problem. Patterns of behavior could often be detected just by looking at the coded material. The one disadvantage in using taped data was that the task of transcribing the information into a useable format was very time consuming.

Another advantage in having the data recorded on videotape is that it is easy to detect inconsistencies in the use of the protocols. Slight variations may occur unnoticed if behaviors are being coded as they occur.

Using a videotape recorder in a clinical setting means introducing extra people and equipment into the situation which may distract the child. This could be a shortcoming of this system. Fortunately in this project the problems and their accompanying apparatus were sufficiently attractive that such distractions were minimal. Another difficulty is that the behaviors have to occur in a relatively small space if they are to be totally recorded on tape. Problems requiring a child to move about a great deal may encounter some technical difficulties.

The majority of subjects used a systematic procedure in their attempt to do the location problems.

The systematic procedures used by the subjects performing the various location tasks involved counting, matching or a combination of the two. In the parking lot problem the design of the apparatus encouraged the use of a strategy combining matching and counting. Two numeral scales when fitted to the baseboard gave the coordinates to each bay. An arrow indicated the entrance or starting point. The majority of subjects appeared to use matching for the first element of the number pair as they would slide the car along the scale and pause opposite the scale

number which matched the first numeral of the ordered pair. Turning into the row the spaces were then usually counted. A check was often made by a glance or by moving a hand from the designated space, across the rows, to the scale indicating the space coordinate. The older subjects often used a coordinating strategy. They would be presented the task, scan the grid and then place the car in the correct space.

The design of the theatre apparatus seemed to encourage a strategy of matching. Two elements of the triple, namely color and letter, were found by a matching process. Finding the seat could involve counting. With the seats being labelled by numerals in front of the first row, matching seemed to be the more obvious strategy. The subjects had more flexibility in their procedures for finding the given seat. Unlike the parking lot there was no obvious starting point. The subjects were systematic in that the tier was usually identified first and then depending on the task, the row or the seat was identified next.

Both sets of apparatus encouraged the subjects to use a systematic procedure in locating a position in either two-dimensional or three-dimensional space. The subjects easily adopted a systematic approach and tried to complete the location tasks.

In the location problem of the theatre, the position to be found governed the order in which the elements of the triple were interpreted.

In performing the tasks of the location problem involving the theatre, a position was to be found on each of the three tiers. Of the three elements of the triple to be interpreted, the tier was the first element identified in the majority of cases. The seat was the next item to be indicated in tasks Red A5 and Blue B8. The tendency to single out the seat in task Red A5 could have resulted from the fact that the subject stood in front of the apparatus. The numerical labels of the seats were very evident as they were in front of the first row on each tier while the letters labelling the rows were off to one side. The '5' being directly in front of the seat was the more obvious element on which to focus.

In the case of Blue B8 most of the subjects could not reach the position from where they were sitting. Many walked around to the back of the theatre and found the '8' and then the row. In this task the letter was probably disregarded until the end because an aisle separated the seats 1 to 5 and seats 6 to 10. The lettering was beside the first seat in each row and therefore quite a distance from the eighth seat. When the subjects walked to the back of the theatre they stood closer to the numerical labels. The resulting response in the majority of cases was to focus on the seat and then locate the particular row in

which it was situated.

The procedure for locating Yellow E4 was different from the preceding tasks. In this case the row was indicated before the seat. Because the seats were in a curvilinear arrangement and the label '4' was far from the seat, the row was usually found before the seat.

The triple that named the position was composed of three different elements. The subjects could use these elements in any order. The location of the position influenced their choice when they attempted the various tasks.

The younger subjects experienced a larger degree of success with the location tasks utilizing a coded triple compared to using a number pair.

When the children were doing location problems the description of the position for the car in the parking lot situation and the 'man' in the theatre situation were given by two different means. The parking lot being bounded by a horizontal and vertical axes consisted of rows of parking spaces which were identified by a pair of numerals. The first numeral referred to the horizontal direction and the second one indicated the vertical. The intersection of the two coordinates indicated the desired parking position. The location problem involving the theatre was completed in a three-dimensional space. The theatre consisted of three tiers of seats. Each seat could be described by using a triple consisting of color

to indicate the tier, letter to indicate the row, and numeral to indicate the seat. As in the parking lot, the point of intersection of the three elements in the triple identified the seat.

The location problems of the parking lot and the theatre were assumed to be mathematically equivalent. However it appears that the younger subjects experienced more success with the location problem of the theatre than with the parking lot. It appears that children in the age range of three to five years find that information made up of color, letter, and numeral is easier to interpret than information given by a numerical code. The three-year-olds were unable to make any correct responses in either of the location problems. Their responses fell into one of two 'no-solution' categories in the parking lot situation. In the majority of the cases the task was abandoned. In the instances where the task was not abandoned, an attempt was made but the car was incorrectly placed (refer to Tables 11 and 12). In contrast the range of responses made by the three-year-olds in the theatre situation were broader. There were responses categorized in each of the three 'no-solution' categories. Also a correct response was made when additional assistance was provided by the interviewer (refer to Tables 15, 16 and 17). Compared to the performance on the parking lot where cars were randomly placed with complete disregard for the information on the

ticket, two three-year-olds found seats where two of the three elements stated were correct.

The four-year-olds are perhaps the ones whose responses are most indicative to the belief that the location tasks using a coded triple were found to be easier than those using a number pair. In the parking lot problem all but two responses fell into the categories of incorrect solution, and in half of those cases the task was abandoned. The remaining responses were classified as incorrectly placed or parking so that one of the two elements was correct. Only in the cross-sectional sampling did two four-year-olds perform the final location task correctly. Perhaps the reason why correct responses were made in this task is because additional assistance was given in performing task 2.

In the theatre problem the task was abandoned only in two cases, both of which occurred in the cross-sectional sampling. The majority of responses fell into the categories where the responses had some aspect correct. Unlike the parking lot, the task was abandoned only in two instances for this age group. The reason for this could be attributed to the fact that in the cross-sectional sampling the parking lot problem was presented first. However in the longitudinal sampling the theatre problem was presented first and all four-year-olds made an effort to solve the location tasks. As in the cross-sectional

sampling the task was abandoned for about half the four-year-olds in the parking lot situation. From this it appears that the setting of the theatre had more appeal and the subjects found the tasks easier.

The responses of the five-year-olds fell into a wider range on the parking lot problem than the theatre problem. The task was abandoned for several of the subjects in the parking lot situation whereas all five-year-olds attempted to do the theatre location tasks. The remaining responses on both problems were very similar in that there were correct responses made on both location tasks of the parking lot and on all three location tasks of the theatre. The majority of errors made in both situations were one element being incorrect.

The subjects ranging in age from 3 to 5 years were more successful in completing the location problems using a coded triple in three-dimensional space than in using a numeral pair in two-dimensional space. A larger proportion of the subjects not only attempted the tasks involving the theatre but were more successful in giving correct responses.

The children were just as successful with the location problems associated with the parking lot as with those associated with the theatre.

The location problem in each situation was to find the position which corresponded to the description on a given ticket. The parking lot tasks were done in a space bounded by two dimensions with the position being described

by a pair of numerals. Three-dimensional space was involved in the theatre tasks with the description of the position being given by three elements: color, letter and numeral. The information given to the subjects in both situations followed the same sequence (see Appendix A for protocols). The solution procedures used with each apparatus were the same. The problems were solved using strategies of what appeared to be counting, matching, or a combination thereof.

In both situations as the age of the subjects increased the number of correct responses increased. All of the three-year-olds were unable to grasp the notion of how to locate the designated positions on both sets of apparatus. The youngest subjects to complete successfully the location problem in both the parking lot and the theatre were the four-year-olds. In the cross-sectional and longitudinal samplings the eight-year-olds were the only subjects to do all the tasks correctly. This occurred only with the parking lot apparatus. The few eight-year-olds who were not totally successful in the theatre location tasks had two of the three elements of the triple correct. For each of the six and seven-year-old age groups the proportion of correct responses was very similar for the parking lot and theatre situations. The subjects did not appear to perform better using a number pair in two-dimensional space than using a coded triple in three-dimensional space.

The children's procedures changed and became more systematic as they got older.

The responses of the subjects were classified in a hierarchy of solution procedures developed from the observations of the behaviors displayed while attempting to solve the problems. The levels of responses ranged from the lowest level where the task was abandoned to the highest level where a correct response was made.

Most three and four-year-olds were unable to follow instructions. For them the task presented was ignored and they proceeded to place the objects in a line. The four-year-olds were distracted by the apparatus in that their responses involved more play-like behavior.

The five and six-year-olds were in a transitional stage in that these two age groups had the widest range of responses. Some of the subjects attempted the task but were incorrect. Other subjects in the same age groups gave responses that were partially correct and a few made responses which were correct.

The subjects who were seven or older solved the problems with considerable ease. Very few errors were made by these subjects. Having better eye and hand coordination, many would perform the tasks with little effort. Their actions were quick and decisive. Some would view the apparatus with their eyes and then place the object emphasizing the fact that a parking lot and theatre were ignored and only the task of locating a position seemed important.

The findings seem to indicate that children use more sophisticated procedures as they get older.

The younger subjects were more successful with the selection tasks than the location tasks in both the parking lot and the theatre problem.

According to the protocol, the selection task was not presented unless a minimum of one location task was completed correctly. The placement made by the interviewer in his instructions often enabled the requirement to be met, especially in the situations involving the younger subjects. It is felt that the young subjects who could not do any of the locations tasks and were able to make a correct selection, did so not because they were able to interpret the information more readily but because they remembered the object placed by the interviewer and what card was used in the instructional task. This was especially evident in the parking lot problem.

In the theatre problem the 'man' designated was usually red A5. This enabled an easy selection for two reasons. Firstly, there usually was only one 'man' sitting on the red floor and red would be the only element requiring consideration. The second reason is that the 'man' sat beside the numeral '5' and only one ticket contained a '5.' Again a correct selection could be made by isolating only one element of the triple.

In the location problems of the theatre the use of colored letters to indicate the tier was just as effective as the use of colored tickets.

In the cross-sectional sampling the color of the tier was indicated by the color of the symbols on the ticket. However in the longitudinal sampling the color of the ticket designated the tier. The manner in which the information was conveyed did not seem to have an effect on the number of correct responses. In the longitudinal sampling there were more older subjects who made correct responses. Therefore only the subjects ranging in age from four to eight years were considered. The proportion of correct responses made in both samplings was very similar. With the use of colored tickets or colored letters the children's behavior was not noticeably different.

The children displayed a variety of verbalizations when attempting the problems.

Many spontaneous verbalizations were exhibited by the subjects as they attempted the various tasks. Repeating the question to oneself or making comments such as counting which pertained to the task were the most frequent types of verbalizations. The young children seemed to be distracted by the apparatus and made comments which were irrelevant to the task. The parking lot encouraged more irrelevant comments than the theatre. Some of the younger subjects were interested in the type of car they were to

park. Others asked if the car should be parked frontwards or backwards. It seems that they impose their own rules upon certain irrelevant aspects of a task and this distracts them from the problem.

CONCLUDING STATEMENTS

The findings of the present study are not conclusive but they do reveal many things that young children do as they go about solving problems. The data appear to support the following concluding statements but they may need to be confirmed through further research.

1. Using the videotape recorder as a data gathering device is an effective means in studying children's behavior in problem-solving situations.

2. The procedures used by the children become more systematic and sophisticated as they get older.

3. For the pre-school child the physical structure of mathematically equivalent problems can make some more difficult to solve than others.

4. For his initial experience, a young child should be introduced to problems involving a frame of reference with axes indicated by different elements such as letter and numeral.

5. Spontaneous verbalizations expressed by the children while solving problems appear to be elicited mostly by children while they are young and seem to be

more task related when they are older.

An implication that can be made from the observation that young children develop more sophisticated procedures in solving problems is that clear problems with few distractions need to be designed for the very young. Four-year-olds appear to be able to follow instructions more readily than three-year-olds. Problems designed for three-year-olds need to consider both the mental and physical capabilities of this age group. The verbalizations made by the young subjects reveal that they are more distracted by some settings than others. More research needs to be conducted into what constitutes a 'good problem' for the various age levels.

RECOMMENDATIONS FOR FURTHER RESEARCH

The results of this study have created an awareness that children of different ages respond to problems in different ways. In the guidelines devised by Nelson and Kirkpatrick (1975) it was suggested that the problem situations should involve real objects or simulations of real objects. In this investigation it appeared that the older subjects appeared to ignore the apparatus and to concentrate on the problem. After the problem was presented the grid was surveyed and the object was placed. The younger subjects were the ones who responded to the apparatus. Further research is required to establish the criteria to be met in

the construction of the manipulative materials. The parking lot and the theatre seemed to be well designed. The children were very willing to park the cars or to place the 'people' in the theatre. Only a few of the younger subjects were distracted by the situations to the extent that they lost focus of the problem.

Another area that needs further research is how to use the manipulative materials most effectively. The presentation of the problem will influence the way in which the subjects will respond in their attempt at a solution. In the parking lot and theatre situations the subjects were given an opportunity to play with the materials. The play period was followed by activities which involved working with each dimension of the apparatus separately. Next a demonstration was given on how to use a numeral pair or a coded triple. After these preliminaries were completed the problems were presented. The results have indicated that the older subjects would not require such extensive activities before the problems. For the younger subjects it appeared to be a good technique. When a subject could not cope with the preliminary activities the interviewer abandoned the problem. Most subjects were willing to attempt the problems. More research is required into what should be involved in the presentation of a problem when the child is required to move or modify objects to express his solution.

Little research has been conducted on how young children interpret space or how they develop spatial concepts. The results of this investigation have indicated that the ability to do problems in two and three-dimensional space has a developmental trend. What spatial concepts does a seven-year-old possess that enable him to survey a grid and locate a given position? What skills are lacking in a four-year-old which prevent him from solving the same problem? Further investigation is necessary to answer such questions.

A means should be devised which would determine the spatial concepts that a child possesses. With the evidence of the research already completed there has been a change in the mathematics curriculum. In the area of geometry concepts of topology are being introduced at earlier age. Euclidean concepts are studied later. By being made more aware of the manner in which a child interprets space more effective learning situations can be devised.

In transcribing the information from the videotape into a workable format, the winding and rewinding of tapes was very time consuming. In the Nelson and Sawada study the tapes were arranged in such a manner that all the problems done by a particular child were recorded on one tape. In this study two of twelve problems were investigated. This meant that a list of children who did the problems involving the parking lot and the theatre had to

be prepared. Every subject did six problems with the order of presentation being different in each case. The information to be coded could be found in any one of six possible places on the tape. The time of retrieval could be lessened if the tapes were arranged according to the problem. All the behaviors of the subjects doing the parking lot problem could be recorded on one set of tapes. The actions of those doing the theatre problem could be recorded on another set. There would be a set of tapes for each particular problem. In transcribing the information the investigator could begin at the start of the tape and work through to its completion with the amount of winding and rewinding being minimal.

In this investigation it was concluded that the videotape recorder was an effective device in studying the behaviors of young children as they did problems. In the theatre situation color was one element of the triple which was used to describe a particular position. The design of the theatre enabled the investigator, while viewing the black and white videotape, to interpret responses to color with little difficulty. Color is frequently involved in the problems designed for young children. The manipulative materials are most often colored. It is one of the attributes they frequently use in distinguishing objects in their environment. An outcome of this observation is to recommend the use of colored videotape in further

investigations. The design of the apparatus does not necessarily lend itself to the interpretation of how children respond to color. Incorporating colored videotape could increase the possibility that all important behavior trends are being observed.

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APPENDICES

APPENDIX A

PROTOCOLS

1. The Parking Lot:
   a. For Both Samplings
2. The Theatre:
   a. For the Cross-sectional Sampling
   b. For the Longitudinal Sampling

CROSS-SECTIONAL AND LONGITUDINAL SAMPLING

THE PARKING LOT

The grid is placed on a low table in front of the child, the long axis pointing away from him. Between the grid and the child are placed twelve plastic cars in random fashion. The child is told that what he sees is a parking lot and that the cars in front of him are to be placed in the parking places.

He is then instructed to choose one of the cars and to park it wherever he wishes. The arrow showing where he is to enter is pointed out to him. Once he has parked the first one, he is asked to choose two more cars and to park them, in turn, wherever he wishes. These cars are then removed from the board.

A piece of cardboard is used to cover all of the grid except the first horizontal row. The child is given a card with the numeral 3 written on it and told that it is a slip which tells him in which place to park a car. He is asked then to choose a car and park it in the correct space. If necessary, he is assisted in parking the first car. He is then given another card with the numeral 5 written on it and asked to park a car of his choice. This procedure is repeated once more, using a card marked with the numeral 2.

All cars are then removed and the covering cardboard

adjusted until the row of parking spaces are covered. The child is given a card with the numeral 7 on it, asked to choose a car, and to park it on the correct space. He is helped with the first car if necessary. He is given two more cards, one with 4 on it, the other with 10 on it, and asked to park two cars in the correct spaces. No help is given in parking these cars. The child who can park no cars correctly (up to this point), will not proceed further with the problem. For those who park at least one car correctly, the cardboard cover is now removed from the grid.

The child is then shown a card with the number pair 4,3 written on it. He is told that the first numeral tells how far he goes in the direction of the arrow. (This is made as clear as possible by gestures and/or additional directions.) He is told that the second numeral tells how many spaces he moves up. These, and any additional directions, are accompanied by gestures. Assistance is given if necessary in parking the first car. He is given another card with the pair 2,5 written on it and asked to choose a car and park it. Finally, he is given a card with the pair 8,6 written on it and asked to choose another car and park it. No help is given with the last two cars.

The three cards he used to park the last three cars are placed in front of him. One of the cars is pointed out and he is asked to find the card that tells where that car is parked.

CROSS-SECTIONAL SAMPLING

THE THEATRE

The Theatre Grid is placed on a low table in front of the child. On the table, between the grid and the child, are placed ten objects representing people to be seated in the theatre.

The three colored levels of the theatre are pointed out as are the letters and numerals which designate the rows and seats of the grid.

The child is asked to choose one of the people and to place him in any seat he wishes. Once he has done this, he is asked to choose another person and to place him in a seat on the blue level. The child is then shown a card with an A on it and asked to choose a person and place him in any seat in the row marked the same way as the card. Finally, he is shown a card with a 2 on it and asked to place a person in a seat marked the same as the card. Assistance is given with any or all of the moves above. All the objects representing persons are left on the grid.

The child is then shown a card with the symbols B-8 written in yellow. He is asked to choose a person and told that the card indicates that person's seat in the theatre. He is reminded that the color of the marks on the card indicates what floor the seat is on, the letter shows what row the seat is in and the numeral shows which seat in the

row. The child is assisted in seating the person correctly.

Then the child is given a card with A-5 on it in red, asked to choose a person, and to seat him in that seat. No assistance is given but when the child seats the person to his satisfaction, he is given a second card and asked to choose another person. The card has B-8 written on it in blue. Once that one is seated, a third card is presented with E-4 on it in yellow. Again the child is to choose a person and place him in the correct seat. All objects are left on the grid.

The cards for the last three persons are placed in front of the child and one of the last three persons seated pointed to at random. The child is asked to indicate which card belongs to that person. However, if the first person seated is the only one seated correctly that person must be selected as the one whose card the child is asked to find.

LONGITUDINAL SAMPLING

THE THEATRE

The Theatre Grid is placed on a low table in front of the child. On the table, between the grid and the child, are placed ten objects representing people to be seated in the theatre.

The three colored levels of the theatre are pointed out as are the letters and numerals which designate the rows and seats of the grid.

The child is asked to choose one of the people and to place him in any seat he wishes. Once he has done this, he is asked to choose another person and to place him in a seat on the blue level. The child is then shown a card with an A on it and asked to choose a person and place him in any seat in the row marked the same way as the card. Finally, he is shown a card with a 2 on it and asked to place a person in a seat marked the same as the card. Assistance is given with any or all of the moves above. All the objects representing persons are left on the grid.

The child is then shown a yellow card with the symbols B-8 written on it. He is asked to choose a person and told that the card indicates that person's seat in the theatre. He is reminded that the color of the card indicates what floor the seat is on, the letter shows what row the seat is in and the numeral shows which seat in the row.

The child is assisted in seating the person correctly.

Then the child is given a red card with A-5 on it, asked to choose a person, and to seat him in that seat. No assistance is given but when the child seats the person to his satisfaction, he is given a second card and asked to choose another person. The card is blue and has B-8 written on it. Once that one is seated, a third card which is yellow is presented with E-4 on it. Again the child is to choose a person and place him in the correct seat. All objects are left on the grid.

The cards for the last three persons are placed in front of the child and one of the last three persons seated pointed to at random. The child is asked to indicate which card belongs to that person. However, if the first person seated is the only one seated correctly that person must be selected as the one whose card the child is asked to find.

APPENDIX B

CODE USED FOR RECORDING DATA FROM VIDEOTAPE

CODE FOR RECORDING DATA

Demonstration: a verbal explanation accompanied by gestures such as tracing over rows, or matching numerals and spaces.

DEM (1) Physically moving object down the row, then into the column.

DEM (2) Indicating how the object is to move by tracing a path with a finger.

DEM (2A) Trace over exposed row (column).

DEM (3) Interviewer points to numeral (letter) on ticket, then to letter (numeral) on apparatus.

DEM (4) Interviewer points to numeral (letter, color) on axis and matches with corresponding space.

DEM (5_1) With 2 or 3 dimension tickets, the interviewer matches with left hand the one axis, the right hand on the other.

DEM (5) With 2 or 3 dimension tickets, the interviewer matches as in DEM 5_1, then moves hands together to the point of intersection.

Verbal Explanation (VE): no gestures except pointing on a ticket or in a general direction.

VE (1) ". . . space that has the same number as the card."

VE (2) "first number tells you to go this way and the second number tells how far to go that way."

VE (3) ". . . the color tells you the floor (balcony), letter—the row, and the number—the seat."

Setting

1. Cars set at starting point (↑) by interviewer
2. Car (man) handed to subject
3. Other car (man) placement: _____
4. Cards given to the subject
5. Cards placed on board:
   5_E - by interviewer 5_S - by subject
6. Cards placed on board opposite corresponding numeral (letter)
   6_E - by interviewer 6_S - by subject
7. Other: _____

After parking car:

8. Cards handed back to examiner

8a. Cards held by subject

9. Cards placed on board opposite numeral (letter)

10. Cards piled on board
    10_E - by interviewer 10_S - by subject

11. Other card placement: _____

Pause	P_L	- pause looking at a specific place
	P_S	- pause searching - eye movement over apparatus
	$P_{S(cd \leftrightarrow r.\ axis)}$	- pause searching - card to row axis and vs.
	$P_{S(cd \leftrightarrow c.\ axis)}$	- pause searching - card to column axis and vs.
Interviewer	E	- poses question
	E_1	- poses question with a gesture
	E_2	- prompts
	E_3	- praise
	E_4	- comment pertaining to task
	E_5	- irrelevant comment
Subject's Vocalizations	V_1	- subject asks for CLARIFICATION or REPETITION of question
	V_2	- subject repeats question to self
	V_3	- expression involving task or material
	V_4	- car noises, grunts, groans, giggles, sighs, . . .
	V_5	- irrelevant comments (not pertaining to problem)
	Y	- nods "yes"
	V_Y	- verbal yes
	N	- shakes head "no"
	V_N	- verbal no.

Subject's Hands

B - both hands

R - right hand

L - left hand

L↔R - shift from left to right

↓ - removed object from space

- turns car around (rotates)

----- moving (lifting) object to location

--.-- lifting, pausing in process

—•—• sliding car (object) with slight pauses

Subscripts

pt - pointing
m - moving
t - touching
s - sliding

Color code - Parking lot: blue - task one

green - task two

red - task three

Number code - Theatre:

(1) task one

(2) task two

(3) task three

(4) task four

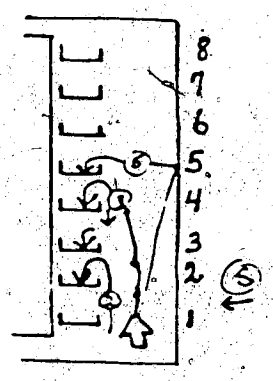
APPENDIX C

SAMPLE OF RECORD SHEETS

1. Parking Lot
2. Theatre

Name _____ CA ____ Tape ____ Task ____ Parking Lot

Row

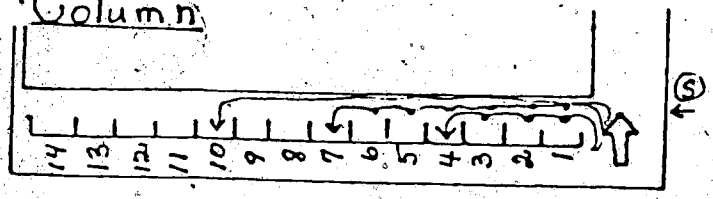


1 4 8 3 E (DEM (2A), (4)) E $P_{L(cd)}$ $P_{L(cns)}$ $P_{L(cd)}$ L_S (2,-) (3,-) (4,1) V_3 "3" E_2 V_4 "Oh" R↓ (4,1) E R_m (3,1) E_3

2 E V_4 $P_{S(cd \leftrightarrow \lambda.)}$ $P_{L(cns)}$ L_S (2,1) E_3

5 E L_S (5,0) → (5,1) $P_{L(cd)}$ E_3

Column

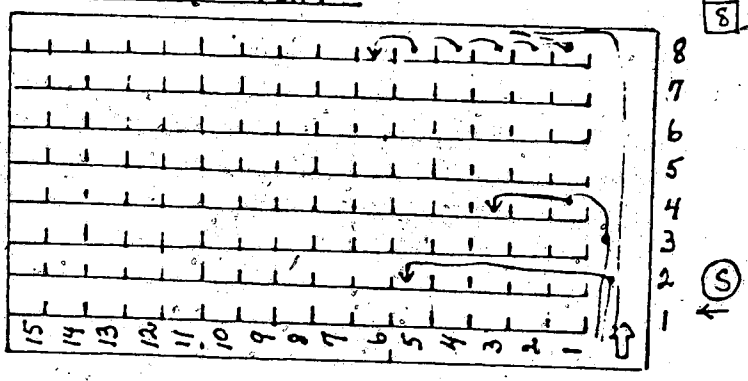


1 4 8 4 E (DEM (2A)) V_5 "See that over there?" E E (DEM (5)) E $P_{S(cd \leftrightarrow c.)}$ $P_{L(cns)}$ L_S (1,1) (1,2) ... (1,4) V_3 1, 2, 3, 4

7 E $L_{T(cn)}$ $P_{L(c.)}$ L_S (1,1) ... (1,7) V_3 "1, 2, ... 6"

10 E $P_{L(c.)}$ $L_{T(cn)}$ $P_{L(c.)}$ L_S (1,10) E_3

Ordered Pair



1 4 8 E (DEM (5)) V_3 "I know."

(4,3) E $P_{S(\lambda. \leftrightarrow cn.)}$ L_S (3,-) $P_{S(cd \leftrightarrow app.)}$ $P_{S(app.)}$ L_T λ. 1,2,3,4 L_T λ. 1,2,3,4 L_S (4,1) $P_{S(cd \leftrightarrow c.)}$ L_S (4,3) L_T (4,3) (3,3) (2,3) (1,3) E_3

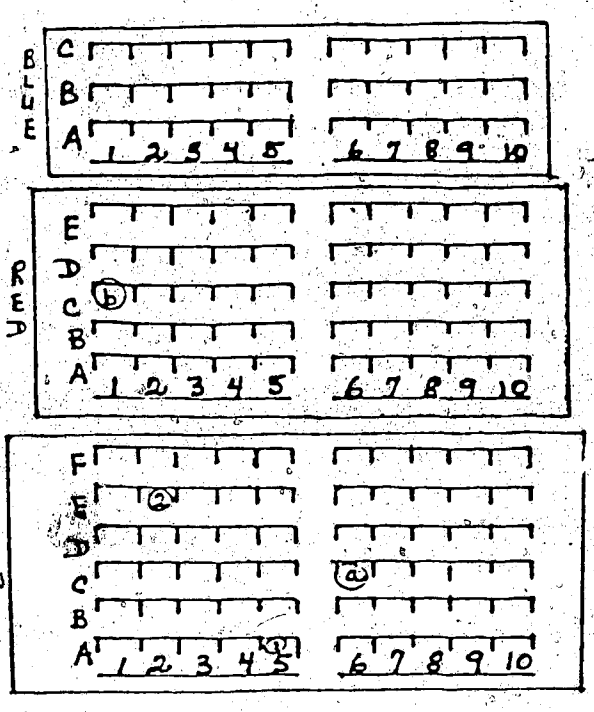
(2,5) E $P_{L(cd \leftrightarrow \lambda.)}$ L_S (2,1) S_L at E V_1 "Should 5" E $P_{S(cd \leftrightarrow c.)}$ L_S (2,5)

(8,6) E $P_{L(cd)}$ L_S (6,-) $P_{S(cd \leftrightarrow c.)}$ $P_{S(cd \leftrightarrow \lambda.)}$ L_S (8,1) L_S (8,3) R↓ (8,3) R_m (8,1) (8,2) ... (8,6) $P_{L(c.)}$ E_3

Selection

Car (8,6) → Card (8,6)

Name _____ CA ____ Tape ___ Task ____ Theatre



Own @ E R YC6

ⓑ Lm RC1 E5 E3

One dimension cards

[2/4] A E1 V4 "O.K." P3(cd↔app.) Rm YA1 ... YA5 Rm YA5

[2/5E] 2 ≡ E PL(y.let) Rm YE2 E3

Coded Triple

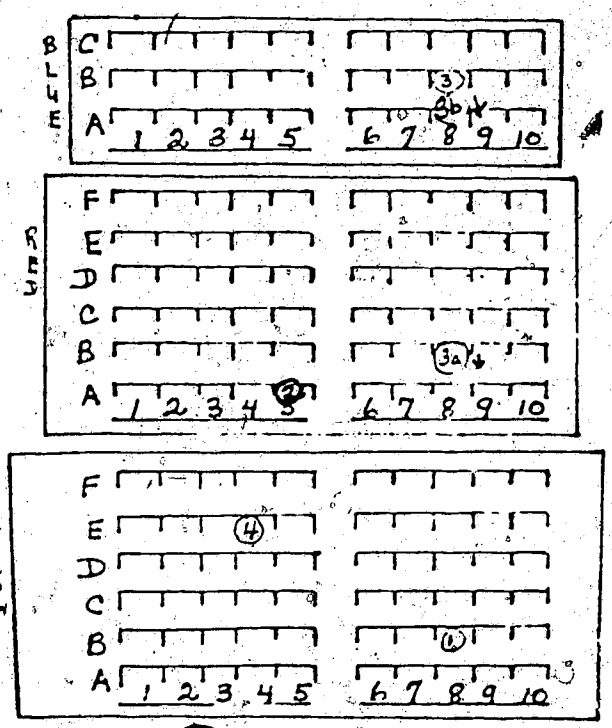
E1 (VE③) Y

[YB8] E PL(cd) V3 "Yellow" PL(cd) V3 assisted. "9n 1.8" PL(y.nos.) LT YA8 PL(y.let) Lm YB8 V3 "There."

[RA5] E PL(cd→1.lev.) PL(cd→1.low.) V3 "A" PL(cd→1.nos.) Lm RA5.

[Bl B8] E PL(1. let.) V3 "B" PL(1st. nos.) V3 "8" Lm R[redacted] E4 "level?" PL (1st→bL) V2 L↓ AB8 Lm BlA8 E2 "new?" V3 "8" E2 "seat 8, new?" PL(cd.) V3 "B" PL BlA8 V4 "4" L↓ ElA8 LT ElB6 E4 Lm ElB6 E3

[YE4] E PL(cd) PL(cd→y.let.) V3 "E" PL (1st⇒1.E) Lm V3 "4, E4." PL(y. nos.) PL(1.E) LT YA4 ... YE4 Lm YE4 PL(1st.↔1.E↔cd.) E3



⑤

Selection: Man YB8 → Ticket ?

APPENDIX D

ALLOCATION OF PROBLEM SETS TO SUBJECTS FOR EACH AGE GROUP

PROBLEMS ATTEMPTED BY EACH SUBJECT IN THE CROSS-SECTIONAL AND LONGITUDINAL SAMPLINGS

Age	Subj.	1	2	3	4
3-4	1	x	x		
	2	x			x
	3	x	x	x	x
	4	x			
	5	x			
	6	x	x	x	x
	7	x	x	x	x
	8	x			x
	9	x	x	x	x
	10	x			x
Total		10	5	4	7

Age	Subj.	1	2	3	4
4-5	1	x	x	x	x
	2	x			
	3	x	x		
	4	x	x	x	x
	5	x			
	6	x			x
	7	x	x	x	x
	8	x	x	x	x
	9	x			x
	10	x			x
Total		10	5	4	7

Age	Subj.	1	2	3	4
5-6	1	x	x		
	2	x			x
	3	x			x
	4	x			x
	5	x	x	x	x
	6	x	x	x	x
	7	x	x	x	x
	8	x			x
	9	x			x
	10	x	x	x	x
Total		10	5	4	9

Age	Subj.	1	2	3	4
6-7	1	x			x
	2	x	x	x	x
	3	x	x	x	x
	4	x			x
	5	x	x	x	x
	6	x			x
	7	x	x	x	x
	8	x			x
	9	x	x	x	x
	10	x			x
Total		10	5	5	10

Age	Subj.	1	2	3	4
7-8	1	x			x
	2	x	x	x	x
	3	x			
	4	x	x	x	x
	5	x			x
	6	x	x	x	x
	7	x			x
	8	x			x
	9	x	x	x	x
	10	x	x	x	x
Total		10	5	5	9

Age	Subj.	1	2	3	4
8-9	1	x			x
	2	x			x
	3	x			
	4	x	x		
	5	x	x		
	6	x	x	x	x
	7	x			x
	8	x	x	x	x
	9	x			x
	10	x	x		
Total		10	5	2	6

1. Parking lot problem - cross-sectional
2. Theatre grid problem - cross-sectional
3. Parking lot problem - longitudinal
4. Theatre grid problem - longitudinal

ALLOCATION OR PROBLEM SETS TO SUBJECTS
FOR EACH AGE GROUP:

CROSS-SECTIONAL SAMPLING

Problem Set	Problems					
1	1	7	5	3	2	4
2	5	3	1	9	6	2
3	3	11	1	5	12	2
4	3	9	7	1	10	2
5	1	7	3	11	2	8
6	9	1	3	11	4	12
7	7	9	1	5	8	6
8	11	7	5	1	12	6
9	1	5	11	9	6	10
10	9	1	11	7	12	8
11	5	3	9	7	4	8
12	11	5	7	3	6	4
13	9	3	5	11	10	4
14	7	11	9	3	8	10
15	5	11	9	7	12	10

1. Cargo groups
2. Animal groups
*3. Parking lot
#4. Theatre grid
5. Linear sequence
6. Circular sequence
7. Object reflection
8. Mirror reflection
9. Factor platform
10. Factor board
11. Fold-up shapes
12. Projected shapes

*Problems in this study

ALLOCATION OF PROBLEM SETS TO SUBJECTS
FOR EACH AGE GROUP:

LONGITUDINAL SAMPLING

Problem Set	Problems					
1	2	8	6	4	1	3
2	6	4	2	10	5	1
3	4	12	2	6	11	1
4	4	10	8	2	9	1
5	2	8	4	12	1	7
6	10	2	4	12	3	11
7	8	10	2	6	7	5
8	12	8	6	2	11	5
9	2	6	12	10	5	9
10	10	2	12	8	11	7
11	6	4	10	8	3	7
12	12	6	8	4	5	3
13	10	4	6	12	9	3
14	8	12	10	4	7	9
15	6	12	10	8	11	9

1. Cargo groups
2. Animal groups
*3. Parking lot
*4. Theatre grid
5. Linear sequence
6. Circular sequence
7. Object reflection
8. Mirror reflection
9. Factor platform
10. Factor board
11. Fold-up shapes
12. Projected shapes

*Problems in this study

APPENDIX E

TABLES

Table E-1

Distribution of Solutions for Subjects who Attempted the Four Tasks in the Cross-Sectional Sampling (N = 30)

Age	Subject	Parking Lot			Theatre			
		Location		Selection	Location			Selection
		2	3		2	3	4	
3	S_1							
	S_2							
	S_3							
	S_4							
	S_5							
4	S_1							
	S_2		*		*		*	
	S_3							
	S_4							
	S_5							
5	S_1	*	*		*			*
	S_2	*	*		*	*	*	
	S_3			*	*	*	*	
	S_4		*					
	S_5							
6	S_1	*	*	*	*	*	*	
	S_2			*				
	S_3	*	*	*	*	*	*	
	S_4	*			*			
	S_5	*			*		*	
7	S_1	*	*		*		*	
	S_2	*	*	*	*	*	*	
	S_3	*	*	*	*	*	*	
	S_4	*	*	*	*	*	*	
	S_5	*	*		*	*	*	
8	S_1	*	*	*	*	*	*	
	S_2	*	*				*	
	S_3	*	*		*	*	*	
	S_4	*	*	*	*	*	*	
	S_5	*	*		*		*	

* - Solution

Table E-2

Distribution of Solutions for Subjects who Attempted the Four Tasks in the Longitudinal Sampling (N = 24)

Age	Subject	Parking Lot			Theatre			
		Location		Selection	Location			Selection
		2	3		2	3	4	
4	S_1				*			
	S_2							
	S_3				*			*
	S_4				*			*
5	S_1							
	S_2	*	*	*	*	*	*	*
	S_3	*	*		*	*	*	*
	S_4		*	*				
6	S_1	*		*			*	*
	S_2	*		*	*	*	*	*
	S_3	*	*		*		*	*
	S_4	*		*	*	*	*	*
7	S_1	*	*	*	*		*	*
	S_2	*	*		*	*	*	
	S_3	*	*	*	*	*	*	*
	S_4	*	*		*	*	*	*
	S_5	*	*	*	*	*	*	*
8	S_1	*	*	*	*	*	*	*
	S_2	*	*	*		*	*	*
	S_3	*	*	*	*	*	*	
	S_4	*	*	*	*	*	*	*
	S_5	*	*	*	*	*	*	
9	S_1	*	*	*	*	*	*	*
	S_2	*	*	*	*	*	*	*

* - Solution