University of Alberta
Department of Civil Engineering

Structural Engineering Report No. 58

A Statistical Study of Variables Affecting the Strength of Reinforced Normal Weight Concrete Members

by

S.A. Mirza

and

J.G. MacGregor

December, 1976

A STATISTICAL STUDY OF VARIABLES AFFECTING THE STRENGTH OF REINFORCED NORMAL-WEIGHT CONCRETE MEMBERS

by

Sher Ali Mirza

James G. MacGregor

The University of Alberta
Department of Civil Engineering
Edmonton, Alberta, Canada

August, 1976

TABLE OF CONTENTS

PREFACE

Introduction

In 1963 the ACI Building Code adopted a load and resistance factor design format based on the design equation:

$$\phi R \leq \lambda_D D + \lambda_L L$$

where R = member strength computed by the designer

D = specified dead load

L = specified live load

ϕ = capacity reduction factor

λ_D = load factor on dead load

λ_L = load factor on live load

The derivation of the ϕ and λ values in the 1963 ACI code are discussed by MacGregor (1975). This design procedure was adopted for reinforced concrete design in Canada soon after.

During the past five years work has been underway in the United States and Canada to introduce "limit states design" for steel structures involving the basic design equation presented above. In Canada, it is hoped that the load factors from these new studies will eventually be adopted for use with steel, concrete, wood, masonry and other technologies. When this occurs, however, it will probably be necessary to revise the ϕ values currently used in reinforced concrete design. As a first step in revising the ϕ factors it is necessary to gather data on the variability of the strength of reinforced concrete members in flexure, shear, bond, etc. This report is the first in a series studying this problem and is a compilation of data on the variability of the material strengths and geometrical quantities which affect the strength of reinforced concrete members. This is based primarily on data obtained from a number of published sources and involves no additional experimental work.

Scope and findings

Chapter 1 of this report describes the variability of the compressive and tensile strengths and the modulus of elasticity of normal-weight concrete. The principal findings are summarized in the following sections:

Concrete compressive strength - Section 1.1.6 - Page 17

Concrete tensile strength - Section 1.2.6 - Page 29

Modulus of elasticity of concrete - Section 1.3.6 - Page 36

Chapter 2 documents the variability inherent in the reinforcing steel. The principal findings are summarized in the following sections:

Yield strength - Section 2.1.5 - Page 49

Ultimate strength - Section 2.2.3 - Page 54

Modulus of elasticity - Section 2.3.1 - Page 56

Chapter 3 reviews the literature on geometric imperfections. The principal findings are summarized in the following sections:

Slab dimensions - Section 3.2.4 - Page 64

Beam dimensions - Section 3.3.7 - Page 70

Column dimensions - Section 3.4.3 - Page 73

Effects of discrete bar sizes - Section 3.5.1 - Page 76

The distributions presented in the sections listed above will be used in studies of the variability of member strengths.

Acknowledgements

This report was prepared as part of the project "Limit States Design for Concrete" financed by the National Research Council of Canada and the Reinforced Concrete Research Council. The writers wish to acknowledge with thanks the assistance of L.H. Grant in collecting much of the data and W.C. Siu in critically reading the manuscript.

Chapter 1

VARIABILITY OF CONCRETE STRENGTH

1.1 CONCRETE STRENGTH IN COMPRESSION

Concrete, like all other construction materials, is variable. Research has shown that under current design, production, testing and quality control procedures, the strength of concrete in a structure differs from its specified design strength. Furthermore, strength of concrete in a structure, is not uniform. The major sources of variations in concrete strength are:

1. Variations in material properties and proportions,
2. Variations in mixing, transporting, placing and curing methods,
3. Variations in testing procedures.

Since concrete is a heterogeneous mixture of cement, water, sand, aggregate, entrained air, and in some cases admixtures, variations in proportions and/or properties of any one of these ingredients or combinations of variations in more than one ingredient will lead to a variation in the final strength of concrete. Similarly, the methods of mixing, transporting, placing, curing, and testing will also result in variability of concrete strength. Finally, the strength of the concrete in a structure and the strength measured by control specimens will differ somewhat due to differences in the shape, size, placing, curing and testing conditions.

1.1.1 Degree of control

The variability of concrete strength depends on the quality control of concreting operation. Depending on these controls, the coefficient of variation may range from 5% for laboratory conditions to as high as 30% for uncontrolled conditions. The 30% value is unacceptable under present construction techniques and a 5% value is not practical for field conditions. In the construction of the Skylon Tower at Niagara Falls (Lauer and Rigby, 1966), coefficients of variation ranging from 7% to 10% were achieved using exceptional control methods. This suggests a minimum value for site conditions. The Bureau of Reclamation (ACI Comm. 214, 1973) consistently achieves a coefficient of variation of about 15% which suggests a value for average control. Table 1.1 indicates that the coefficient of variation in many cases is between 15% to 20% which suggests that 20% is a reasonable maximum value. Therefore, ACI Committee 214 (1965) has suggested that the level of quality control of concrete can be divided into three classes, based on overall coefficient of variation of the control cylinder as follows:

(1). 10 to 15% for good control,

(2). 15 to 20% for average control,

(3). Above 20% for poor control.

The total variation in concrete strength must include the variation in concrete strength within a single batch. This in-batch test variation may be considered as a variation in testing procedures, mixer inefficiencies and variation in actual concrete strength. Thus, the in-test coefficients of variation given in column 1 of Table 1.2 vary from 0.5 to 8.1 percent. Based on available information (ACI Comm 214, 1965) the level of control for within-batch tests can be divided into three classes with corresponding coefficients of variations as follows:

(1). 4 to 5% for good control,

(2). 5 to 6% for average control,

(3). Above 6% for poor control.

An examination of previous investigations indicates that the standard deviation and the coefficient of variation are not constant for different strength levels. The relationship between the mean strength and the standard deviation shown in Fig. 1.1 was developed using data from several sources (Murdock, 1953; Entroy, 1960; Rüsch, 1964; ACI Comm. 214, 1965). The differences in values from different sources in Fig. 1.1 may be partially explained by the type of data used. The specimens of Entroy (1960) and Murdock (1953) were 6 in. cubes while the ACI Committee 214 (1965) specimens were standard 6 x 12 in. cylinders. The data reported by Rüsch (1964) contained test specimens of both types. On the basis of this test data it appears that the mean coefficient of variation is roughly constant for strength levels below 3000 to 4000 psi while for concrete with an average strength above 4000 psi the standard deviation remains approximately constant with values 400, 600 and 800 psi for the three levels of control listed above. This is expected since the greater control required for the production of higher strength concrete contributes to smaller variability.

1.1.2 <u>Distribution of the strength of concrete specimens in compression</u>

Previous researchers have shown that for most purposes the variability of concrete strength in compression can be represented by a normal distribution if the coefficient of variation does not exceed 15 to 20 percent, although a slight skewness is generally present. In cases where the coefficient of variation is very high the skewness tends to become considerable and a lognormal function represents the tail areas of the distribution more reasonably. Table 1.1 is a collection of data from a number of statistical studies of concrete strength. The majority of researchers have used a normal distribution due to its simplicity and because the central area of the curve is most important in concrete control. At the same time, however, Fruendenthal (1956), Julian (1955), and Shalon and Reintz (1955) have shown that the lognormal distribution gives a better fit for concrete strength in which the control is poorer than average and should be used where extreme values are important.

Shalon and Reintz (1955) observed no discrepancy between the actual distribution and a normal distribution for concrete with a coefficient of variation of 14%. Conversely, for concrete with a coefficient of variation of 23%, no discrepency was observed when actual distribution was tested against lognormal distribution (Shalon and Reintz, 1955).

In establishing understrength factors for members to reflect the probability of the material strength being lower than the specified strength, the lower strength tails of the distribution curves are important. For this reason it seems reasonable to assume a normal distribution for concrete strength if the coefficient of variation is 15% or less. For concrete strengths with coefficients of variation greater than 15% the lognormal distribution should be used to increase accuracy in the tail areas of the curve.

1.1.3 <u>In-situ strength vs control cylinder strength</u>

The strength of concrete in a structure is somewhat lower than the strength of control cylinders moulded from the same concrete. This difference is due to the effects of different placing and curing procedures, the effects of vertical migration of water during placing of concrete in deep members, the effects of difference in size and shape, and the effects of different stress regimes in the structure and the specimen. Petersons (1968) reviewed the available data on core strengths as compared to standard control cylinder strength. He concluded that the most important factors affecting the concrete strength in a structure are the curing conditions of the concrete, the strength level of the concrete and the location of the concrete in the structure.

(a) Effects of degree of control

Bloem (1968) observed that the strength of concrete cores from well-cured slabs was 90% of the strength of well-cured cylinder, moulded from the same concrete. The ratio reduced to 79% for slab cores and cylinders, poured from the same concrete, when slab and cylinders were subjected to lower standards of curing nearly typical of usual field practice. This indicates a 12% reduction in the in-situ strength of concrete for minimum acceptable field curing conditions. Petersons (1968) also concluded that the difference in in-situ strengths for minimum acceptable and good curing standards can be approximated by a factor of 0.9. No significant difference in variability of in-situ strength of concretes, cured under different controls, was observed by Bloem (1968).

(b) Effects of in-situ concrete placement

The concrete in higher portions of deep members tends to be weaker than the concrete in the lower parts. Thus, for example, the concrete in the top one-foot portion of a column is weaker than the concrete in the remainder of column. This can be explained by the increased water-cement ratio at the top due to water migration after concrete has been placed, and by the greater compaction of concrete near the bottom of a column due to weight of the concrete higher in the form. Petersons (1968) concluded that this reduction in strength was about 15%. He also reports a similar weak layer at the top surface of beams or slabs.

The average ratios of core strengths to cylinder strengths from various studies (Bloem, 1965, 1968; Campbell and Tobin, 1967; Petersons, 1968) varied from 0.74 to 0.96 with an overall average from all studies of 0.87. Petersons (1968) shows, however, that the ratio of the strength of concrete in a structure to the strength of the same concrete in a standard cylinder decreases as the strength level increases.

Allen (1970) suggested the following expression for the relationship between compressive strengths of cores and cylinders, based on analysis of Petersons (1968) data:

$$f_{ccore} = 0.7\ f_{ccyl} + 600\ \text{psi} \qquad \text{(Eq. 1.1)}$$

Bloem (1968) observed that the strengths of cores drilled from slabs were 93% of the strength of the push-out cylinders from the same slab. This was essentially independent of the type of cement used, curing conditions, and age of concrete at testing. If the strength of concrete in a structure is assumed to be represented by the push-out cylinder strength, which seems to be a reasonable assumption, then:

$$f_{cstructure} = 0.75\ f_{ccyl} + 650 \text{ psi} \quad \text{(Eq. 1.2)}$$

The reduction in the in-situ strength of concrete is partially offset by the requirement that the average cylinder strength must be about 700 to 900 psi greater than the design strength to meet the existing design codes. Based on this observation and on equations and data from Allen, Bloem and Petersons, MacGregor (1975) suggested that the mean 28-day strength of concrete in a structure for minimum acceptable curing can be expressed as:

$$\overline{f}_{cstructure} = 0.675\ f'_c + 1100 \leq 1.15\ f'_c \text{ psi} \quad \text{(Eq. 1.3)}$$

where f'_c is design compressive strength of concrete in psi.

(c) Effects of volume

The in-situ strength of concrete is affected by the difference in the volumes of material under stress. Since the ratio of the volumes of the control cylinder and the in-situ concrete vary substantially, the influence of size should be carefully examined.

Concrete is neither a perfectly brittle nor a perfectly plastic material; its behavior is somewhere in between. For this reason, the classical statistical theories of strength of materials are not truly applicable to concrete. Nevertheless, the statistical theory of brittle solids (Bolotin, 1969) gives good estimates for the influence of size in geometrically similar specimens. According to this theory the dependence of mean strength on the volume can be represented by the expression:

$$\overline{X} = \overline{X}_o \{\beta + (1-\beta) (\frac{v_o}{v})^{1/\alpha}\} \qquad \text{(Eq. 1.4)}$$

where $\overline{X}$, v are the mean strength and volume of a specimen of a given size,

$\overline{X}_o$, v_o are the mean strength and volume of standard specimen,

α, β are constants.

Eq. 1.4 shows clearly that as the volume increases the mean strength decreases. When volume tends to infinity, the mean strength tends to the strength of the weakest constituent element of the material and is equal to $\beta\overline{X}_o$. The same theory estimates the dependence of the variability on volume by the formula:

$$V_x = \frac{(1-\beta) (\frac{v_o}{v})^{1/\alpha} \{\frac{\pi}{\alpha\sqrt{6}} - \frac{0.7}{\alpha^2}\}}{\beta + (1-\beta) (\frac{v_o}{v})^{1/\alpha}} \qquad \text{(Eq. 1.5)}$$

where V_x = coefficient of variation of a specimen of volume v associated with the mean value $\overline{X}$ in Eq. 1.4.

From Eq. 1.5 it is clear that as the volume increases the coefficient of variation decreases, the coefficient of variation tending to zero when volume tends to infinity.

Using v_o as the volume of a 4 x 4 x 4 in. cube, Bolotin (1969) has empirically established the values of constants "α" and "β" to be 3 and 0.58, respectively. Substituting, Eqs. 1.4 and 1.5 become:

$$\overline{X} = \overline{X}_o \{0.58 + 0.42 (\frac{v_o}{v})^{1/3}\} \qquad \text{(Eq. 1.6)}$$

$$V_x = \frac{0.147 \left(\frac{V_o}{V}\right)^{1/3}}{0.58 + 0.42 \left(\frac{V_o}{V}\right)^{1/3}} \qquad \text{(Eq. 1.7)}$$

Assuming a normal distribution for the strength, the minimum strength and maximum strength can be easily estimated by the usual rule of "three standard deviations". Based on this observation and Eqs. 1.6 and 1.7, the effects of volume on the mean, the maximum and the minimum strengths are shown in Fig. 1.2. Also plotted in Fig. 1.2 is a curve representing 0.58 $\overline{X}_0$, which represents the minimum strength when volume tends to infinity. It is interesting to note from Fig. 1.2 that the values of minimum strength obtained from these two different methods are in close agreement. In spite of the fact that the mean strengths are substantially dependent on the volume, the influence of size on minimum strength seems to be very small. In the study of understrength factors for concrete members, it is the lower strength tail that is more important. For this reason it is not unsafe to neglect the effect of volume in probalistic studies of understrength.

1.1.4 Speed of loading

The observed strength of concrete is considerably affected by the rate of application of the load, the lower the rate of loading the lower the apparent strength. This is probably due to the increase in strain with time owing to creep and micro-cracking if it is assumed that failure takes place independent of the level of applied stress when a limiting compressive strain is reached. The normal rate of loading for the standard cylinder test is approximately 25 to 40 psi/sec (test duration about 2 minutes). Compared with this rate of loading, the loading at 1 psi/sec reduces the apparent strength of concrete by approximately 12%, whereas loading at 100 psi/sec increases the strength by about 12%. Concrete is capable of withstanding indefinitely the stresses only up to 70% of the strength under loads applied at 35 psi/sec.

Based on standard compression cylinder tests, Jones and Richart (1936) suggested the following relation between compressive strength of concrete and the rate of loading:

$$f_{cR} = f_{c1} (1 + K \log_{10} R) \quad (0.1 < R < 10{,}000) \qquad \text{(Eq. 1.8)}$$

where f_{cR} = strength at a given rate of loading R psi/sec,
f_{c1} = strength at a rate of 1 psi/sec,
K = a constant, roughly equal to 0.07 for a 7-day strength and 0.08 for 28-day strength.

In order to make Eq. 1.8 ready-to-use it is desirable to relate the 28-day strengths of concrete to the nominal testing speed at which cylinders are generally tested, i.e., roughly 35 psi/sec. Thus for 28-day concrete:

$$f_{cR} = 0.89 \, f_{c35} (1 + 0.08 \log_{10} R) \qquad \text{(Eq. 1.9)}$$

where f_{c35} = strength at a loading rate of 35 psi/sec.

Allen (1970) suggested that the dispersion of concrete strength remained unaffected by the speed of testing. Although no concrete information is available on variability of the concrete strength due to speed effects, a small dispersion was observed by Jones and Richart (1936). For this reason it is assumed here that the coefficient of variation of concrete strength obtained from Eq. 1.9 is approximately 5%.

1.1.5 Model for in-situ concrete strength in compression

Based on discussions in the preceding paragraphs, the model of concrete compressive strength in a structure can be constructed as follows:

$$f_{cstrR} = f'_c \cdot r_{creal} \cdot r_{in\text{-}situ} \cdot r_{vol} \cdot r_R \qquad \text{(Eq. 1.10)}$$

where f_{cstrR} = compressive strength of in-situ concrete at a given rate of loading R psi/sec

f'_c = design compressive strength of concrete

r_{creal} = random variable relating real cylinder strength with design strength, f_{creal}/f'_c

$r_{in\text{-}situ}$ = random variable relating in-situ strength with real cylinder strength, $f_{cstructure}/f_{creal}$

r_{vol} = random variable relating volume effect

r_R = random variable relating rate of loading effect, f_{cR}/f_{c35}

Because the influence of volume on minimum strength is neglible, Eq. 1.10 can be revised to be:

$$f_{cstrR} = f'_c \cdot r_{creal} \cdot r_{in\text{-}situ} \cdot r_R \qquad \text{(Eq. 1.10a)}$$

Substituting values from Eqs. 1.3 and 1.9, the mean value for in-situ compressive strength of concrete at a given rate of loading R psi/sec is found to be:

$$\overline{f}_{cstrR} = 0.89\ (1 + 0.08\ \log_{10} R) \left\{0.675\ f'_c + 1100\right\} \text{psi}$$

$$(\left\{0.675\ f'_c + 1100\right\} \leq 1.15\ f'_c) \qquad \text{(Eq. 1.11)}$$

At a loading rate equal to 35 psi/sec, Eq. 1.11 becomes:

$$\overline{f}_{cstr35} = 0.675\ f'_c + 1100 \leq 1.15\ f'_c \qquad \text{(Eq. 1.3a)}$$

where $\overline{f}_{cstr35}$ = mean compressive strength of in-situ concrete at rate of loading equal to 35 psi/sec.

Similarly, variation for in-situ compression strength of concrete at a given rate of loading is calculated using the model of Eq. 1.10a:

$$V^2_{cstrR} = V^2_{creal} + V^2_{in\text{-}situ} + V^2_R \qquad \text{(Eq. 1.12)}$$

where V_{creal} = coefficient of variation of real cylinder strength

$V_{in\text{-}situ}$ = coefficient of variation of concrete strength in structure relative to cylinder strength

V_R = coefficient of variation for rate of loading effect

The strength of concrete measured by control cylinders includes variations in the "real" concrete strength, and the so-called "in-test" variations due to testing procedure. Thus,

$$V^2_{ccyl} = V^2_{creal} + V^2_{in\text{-}test} \qquad \text{(Eq. 1.13)}$$

or

$$V^2_{creal} = V^2_{ccyl} - V^2_{in\text{-}test} \qquad \text{(Eq. 1.13a)}$$

where V_{ccyl} = variation of compressive strength of cylinders.

Substituting, Eq. 1.13a in Eq. 1.12, we get:

$$V^2_{cstrR} = (V^2_{ccyl} - V^2_{in-test}) + V^2_{in-situ} + V^2_R \qquad \text{(Eq. 1.14)}$$

Assuming a 10% variation of concrete strength in a structure with respect to compressive strength of control cylinders and an in-test variation of 4% and variation due to rate of loading effect to be 5% as described in previous sections, the coefficient of variation of in-situ strength of concrete at a given rate of loading will be taken as:

$$V_{cstrR} = V^2_{ccyl} - 0.04^2 + 0.10^2 + 0.05^2 \qquad \text{(Eq. 1.14a)}$$

when coefficient of variation of in-situ concrete strength at rate of loading similar to cylinder test (35 psi/sec) is required, it will be taken as:

$$V_{cstr35} = V^2_{ccyl} - 0.04^2 + 0.10^2 \qquad \text{(Eq. 1.14b)}$$

The equations for mean value and coefficient of variation presented above are based on a multiplicative model of in-situ concrete strength and thus would theoretically tend towards a log-normal distribution. The discussions in Section 1.1.2, however, lead to a normal probability distribution for concrete cylinder strengths if the coefficient of variation is 15% or less. Therefore, the probability distribution of in-situ strength of concrete should be taken normal if coefficient of variation is 15% or less, otherwise it should be considered log-normal.

1.1.6 Summary and recommended distributions of concrete compressive strength

For loading rates similar to that of a cylinder test, the probability distribution of the in-situ strength of concrete in a structure can be described by a normal curve with mean and dispersion calculated using Eqs. 1.3a and 1.14b:

$$\bar{f}_{cstr35} = (0.675\ f'_c + 1100) \leq 1.15\ f'_c \text{ psi} \qquad \text{(Eq. 1.3a)}$$

$$V_{cstr35} = \sqrt{0.10^2 - 0.04^2 + V_{ccyl}^2} \qquad \text{(Eq. 1.14b)}$$

For concrete loaded at other rates the mean and coefficient of variation can be computed using Eq. 1.11 and Eq. 1.14a:

$$\bar{f}_{cstrR} = 0.89\ (1 + 0.08 \log_{10} R)\ (0.675\ f'_c + 1100) \text{ psi}$$
$$\left(\left[0.675\ f'_c + 1100\right] \leq 1.15\ f'_c\right) \qquad \text{(Eq. 1.11)}$$

$$V_{cstrR} = \sqrt{0.10^2 - 0.04^2 + V_{ccyl}^2 + 0.05^2} \qquad \text{(Eq. 1.14a)}$$

1.2 CONCRETE STRENGTH IN TENSION

There are many types of that can be used for determining tensile strength of concrete: direct tension, flexure, split cylinder and ring tests. Of these, the split cylinder test seems to be gaining popularity because of convenience of testing it offers. In this study tensile strength of concrete refers to splitting strength unless mentioned otherwise.

Very limited information is available for the variability of concrete strength in tension and most of it is in relation to its compressive strength. Available information for the probability model of tensile strength is even more meager. For this reason, this study was conducted to relate the probability model of tensile strength with the model of its compressive strength in the structure. This is consistent with the current design and control practices of concrete strength in North America, since design strengths as well as control cylinder strengths are both specified in terms of compressive strengths.

1.2.1 Relation of compressive and tensile strengths of concrete

The tensile strength of concrete is related to its compressive strength but they are not proportional to one another. The ratio of tensile to compressive strengths decreases with an increase in the level of strength. The general forms of the relation are:

$$f_t = a\, f_c + n \quad \text{for linear relation} \qquad \text{(Eq. 1.15)}$$

$$f_t = a\, f_c^n \quad \text{for power relation} \qquad \text{(Eq. 1.16)}$$

$$f_t = a \cdot \exp(n f_c) \quad \text{for exponential relation} \qquad \text{(Eq. 1.17)}$$

where f_c = compressive strength in psi,

f_t = tensile strength in psi,

exp = 2.718282,

and a,n are empirical constants.

(a) Splitting tension

Regression analyses of the test data of 671 sets of compression cylinder and split cylinder strengths collected from recent Ph.D. dissertations and reports of the Universities of Alberta, Calgary and Texas give the following least-square-fit equations:

$$f_{sp} = 0.056\ f_c + 168 \text{ psi } (r\ \ 0.76,\ cov = 12.3\%) \qquad \text{(Eq. 1.18)}$$

$$f_{sp} = 4.14\ f_c^{0.55} \text{ psi } (r = 0.75,\ cov = 12.5\%) \qquad \text{(Eq. 1.19)}$$

$$f_{sp} = 237 \cdot \exp\ \frac{0.124\ f_c}{1000} \text{ psi } (r = 0.74,\ cov = 12.4\%) \qquad \text{(Eq. 1.20)}$$

where r = regression coefficient,

cov = coefficient of variation,

and f_{sp} = splitting tension strength.

These equations are plotted in Fig. 1.3 along with the data points. As can be expected, all three equations represent the data fairly well and show a fair degree of correlation between the splitting tensile and compressive strengths of concrete. Eq. 1.19, however, seems more promising than the other two for its simplicity and the fact that it passes through the zero origin. Based on the available test data other investigators (Malhotra, 1969, Rüsch, 1975) have also suggested equations similar to Eq. 1.19 with somewhat different values of the coefficient "a" and "n".

In studies of the strength of concrete elements it is common practice to express tensile strength as a function of square-root (ACI Code, 1971) or cube-root (Zsutty, 1968) of compressive strength. It should be noted that the power 0.55 in Eq. 1.19 is close to a square root. A statistical analysis of the data shown in Fig. 1.3 was carried out for different values of coefficient "n" in Eq. 1.16. The values of coefficient "a" in Eq. 1.16 were found to be 6.4 and 26.2 for "n" equal to 1/2 and 1/3, respectively:

$$f_{sp} = 6.4\ f_c^{1/2} \qquad \text{(Eq. 1.21)}$$

$$f_{sp} = 26.2\ f_c^{1/3} \qquad \text{(Eq. 1.22)}$$

Eqs. 1.19, 1.21 and 1.22 are plotted in Fig. 1.4 along with the data points, and are compared to the data in Table 1.3. Equations 1.23 and 1.24, used to represent the tensile strength in the CEB Recommendations (converted to psi) and the ACI Code are also plotted in Fig. 1.4:

$$\text{CEB:}\quad f_{sp} = 1.43\ f_c^{2/3} \qquad \text{(Eq. 1.23)}$$

$$\text{ACI:}\quad f_{sp} = 6.0\ f_c^{1/2} \qquad \text{(Eq. 1.24)}$$

A comparison of all curves in Fig. 1.4 indicates that Eqs. 1.21 and 1.22 are reasonably close to the regression equation except that Eq. 1.22 loses accuracy in the very high and very low strength regions. Nonetheless, any of Eqs. 1.19, 1.21 and 1.22 can be used with reasonable accuracy to calculate the tensile strength of concrete from its compressive strength in the range 2000 to 9000 psi. The CEB (Rüsch, 1975) and ACI (1971) equations give somewhat conservative estimates of the tensile strength of concrete. A comparison of some empirical equations to representative test data quoted by Neville (1973) is shown in Table 1.4.

The relationship between tensile and compressive strengths of concrete depends not only on the level of strength as described above but also on many other factors. The size and type of aggregates, air entrainment, curing conditions, water-cement ratio, cement content and age at the time of loading also affect this relation. Due to variations contributed by these factors formulation of a unique relation between the two strengths does not seem possible at this time. Until further knowledge is acquired, an empirically derived equation that represents the available test data with acceptable accuracy, such as Eq. 1.19, 1.21 or 1.22, seems to be a reasonable solution.

(b) Flexural tension

The available test results of standard modulus of rupture beams with third-point loading (342 tests) were analyzed to develop equations for the strength of concrete in flexural tension. The following equations were obtained:

$$f_r = 0.0455\ f_c + 322\ \text{psi}\ (r = 0.51,\ cov = 19.9\%) \qquad \text{(Eq. 1.25)}$$

$$f_r = 12.23\ f_c^{0.447}\ \text{psi}\ (r = 0.55,\ cov = 19.7\%) \qquad \text{(Eq. 1.26)}$$

$$f_r = 338 \cdot \exp \frac{0.093\ f_c}{1000}\ \text{psi}\ (r = 0.51,\ cov = 20.2\%) \qquad \text{(Eq. 1.27)}$$

where f_r = flexural tension strength.

Of these equations, Eq. 1.26 shows the highest degree of correlation, lowest degree of dispersion and is the only equation passing through zero origin, as shown in Fig. 1.5. Using statistical analysis Eqs. 1.28 and 1.29 were developed for square and cubic relationships between flexural tension and compressive strengths:

$$f_r = 8.0\ f_c^{1/2}\ \text{psi} \qquad \text{(Eq. 1.28)}$$

$$f_r = 32.0\ f_c^{1/3}\ \text{psi} \qquad \text{(Eq. 1.29)}$$

Equations 1.26, 1.28 and 1.29 are compared to the tests data in Fig. 1.6 and Table 1.3. Equations 1.30 and 1.31, used to represent flexural tensile strength in the CEB Recommendations (Rüsch, 1975) and the ACI Code (1971) are also plotted in Fig. 1.6:

$$\text{CEB:}\quad f_r = 2.37\ f_c^{2/3}\ \text{psi} \qquad \text{(Eq. 1.30)}$$

$$\text{ACI:}\quad f_r = 7.5\ f_c^{1/2}\ \text{psi} \qquad \text{(Eq. 1.31)}$$

A comparison of all the curves in Fig. 1.6 indicates that any of Eqs. 1.26, 1.28 and 1.29 represent the data fairly well. Equation 1.31 is slightly conservative whereas Eq. 1.30 seems to be the upper bound for the available data.

1.2.2 Distribution of the strength of concrete specimens in tension

The same factors that influence the concrete strength in compression also influence the tensile strength, although the degree of effect may not be the same. For example, crushed coarse aggregate tends to improve both the tensile and compressive strengths, but the effect is greater for the tensile strength with the result that the ratio of tensile to compressive strengths is higher with crushed stone than with gravel. Similarly, better curing conditions improve the tensile strength relatively more than they improve the compressive strength. On the other hand, air-entrainment appears to affect the compressive strength more than it affects the tensile strength of concrete.

A statistical analysis of the test data for compression and tension strengths of concrete cylinders and beams used earlier in this section, was carried out using Eqs. 1.19, 1.21 and 1.22, and Eqs. 1.26, 1.28 and 1.29. The results from this analysis are presented in Table 1.3. The detailed distribution diagrams for the commonly used form of equations for splitting tension and flexural tension, Eqs. 1.21 and 1.28 respectively, are shown in Figs. 1.7 through 1.10. An examination of Table 1.3 and Figs. 1.7 and 1.8 reveals that the distribution of the deviations in observed splitting tensile strength relative to predicted tensile strength is normal with a mean value close to unity. A Chi-square test carried out on the grouped data of the histogram shown in Fig. 1.7 did not reject this hypothesis up to 21.5% level of significance, a much higher value than the usual acceptable level of 5%. This, however, does not seem to be valid for flexural tension strength, where the hypothesis of normality was rejected at much lower than 5% level of significance. Table 1.3 and Figs. 1.9 and 1.10 also subscribe to this fact. This contradictory behaviour of observed flexural tension strength cannot be explained in definite terms at the present time. It is, probably, due to the high degree of dispersion demonstrated by the available test data. Nonetheless, the normal distribution approximates flexural tension strength reasonably well except in the low probability regions where it overestimates the number of low tests as is evidenced by Figs. 1.9 and 1.10. Until further information is available it seems reasonable to assume that the probability distribution of tensile strength of concrete follows the same distribution as its compressive strength.

To determine whether the type of distribution for tensile strength was affected by the value "n" in Eq. 1.16, one thousand values of compressive strength were generated based on a normal distribution of f_c with a mean value of 4000 psi and cov 17.5%. These values were transformed into four populations of tensile strength for four sets of coefficients "a" and "n" with "n" varying from 0.33 to 1.0. As the power coefficient "n" decreased from unity the tensile strength distribution deviated from normality, but only slightly. This seems to indicate that the probability density functions of tensile strengths will not be affected significantly by using different values of coefficient "n" when Eq. 1.16 is used to compute the tensile strength.

1.2.3 Dispersion of tensile strength of concrete

A limited amount of data is available for variability of tensile strength. A comparison of variability of concrete strengths in compression and different tension tests from previous studies (Wright, 1955; Ramesh and Chopra, 1960; Malhotra, 1969; Orr, 1970; Komloš, 1970; AASHO, 1962) is shown in Table 1.5 These data were taken mostly from laboratory tests (where high controls result in low variability of concrete strength) and do not represent the field conditions. Nonetheless, the data indicates that for laboratory conditions the dispersion of tensile strength tests tends to be slightly higher than the dispersion of compressive strength tests.

Similarly, an examination of the in-batch coefficients of variation of compression and tensile strengths in Table 1.2 indicates similar trends as those for overall dispersions. The differences in the in-batch dispersions of different strengths of concrete are, however, neglegible. Therefore, within-batch variability of tensile strength can be assumed to be the same as that for the compression strength of concrete.

When tensile strength is not directly controlled but obtained from a relation with the compressive strength, considerably larger dispersions in the tensile strength should be expected. This increase in dispersion of tensile strength is attributed to the variability of observed tensile strength of concrete with respect to its computed strength.

The ratio of observed and calculated tensile strengths can be calculated from:

$$A = \frac{f_t \text{ (observed)}}{a f_c^n} \qquad \text{(Eq. 1.32)}$$

If the distribution properties of the ratio A are determined from test data of f_c and f_t (shown in Figs. 1.3 to 1.6), (then the dispersion of A will include in-test variation of 4% in f_c and f_t (for laboratory controlled concrete) and, using Taylor's expansion, the coefficient of variation for the observed real tensile strength of concrete with respect to its computed strength is:

$$V_A = \sqrt{V_{Atest}^2 - (0.04)^2 - (0.04n)^2} \qquad \text{(Eq. 1.33)}$$

where V_{Atest} = coefficient of variation of the ratio A determined from experimental data of f_c and f_t,

and V_A = coefficient of variation of observed real tensile strength with respect to its calculated strength.

The results of Eq. 1.33 for different equations of splitting and flexural tension strengths of concrete are presented in Table 1.6. When the splitting tensile strength is obtained from Eqs. 1.19, 1.21 or 1.22, the variability of observed tensile strength relative to its calculated strength will be approximately 12%. Similarly, if the flexural tension strength is calculated from Eqs. 1.26, 1.28 or 1.29, the dispersion of observed flexural tension strength relative to its computed strength will be roughly 19%.

1.2.4 In-situ strength vs control specimens

There are three main sources of variation in in-situ strength of concrete compared to its control specimens:

(1) Effect of volume,

(2) Effect of concrete in-situ rather than in cylinders, and

(3) Effect of speed of loading.

According to the statistical theory of brittle solids (Bolotin, 1969), the dependence of the mean value and variation of geometrically similar specimens on the volume can be represented by Eqs. 1.4 and 1.5, respectively, as described in Section 1.1.3(c). Using data from the modulus of rupture strength of prismatic specimens, Bolotin (1969) has empirically estimated the values of coefficients "α" and "β" in Eqs. 1.4 and 1.5 to be 3 and 0.43, respectively. Substituting these values in the appropriate equations, we get:

$$\overline{X} = \overline{X}_o \left\{0.43 + 0.57 \left(\frac{V_o}{V}\right)^{1/3}\right\} \qquad \text{(Eq. 1.34)}$$

and

$$V_X = \frac{0.199 \left(\frac{V_o}{V}\right)^{1/3}}{0.43 + 0.57 \left(\frac{V_o}{V}\right)^{1/3}} \qquad \text{(Eq. 1.35)}$$

As has been explained in Section 1.1.3(c), the influence of volume on minimum strength turns out to be negligible, although mean strength is substantially affected. For this reason the effect of volume on tensile strength may be neglected for studies of understrength, where lower tails of the probability distributions are more important than the mean values.

Hardly any information is available on the effects of the remaining two parameters causing variability in the tensile strength of concrete. Until further knowledge is acquired, it seems reasonable to consider the effects of in-situ casting and speed of loading on tensile strength of concrete through its compressive strength. For example, if tensile strength of concrete in a structure loaded at 3 psi/sec is desired, the distribution of in-situ strength in compression at 3 psi/sec should be defined first as per Section 1.1.6; the distribution of tensile strength can be obtained easily using equations in Section 1.2.1 through 1.2.3.

1.2.5 Model for in-situ concrete strength in tension

Based on discussions in the preceding sections, the model of tensile strength of concrete in a structure, loaded at a given rate of loading R psi/sec, can be constructed as follows:

$$f_{tstrR} = A \cdot \left\{ g\ (f_{cstrR}) \right\} \qquad \text{(Eq. 1.36)}$$

where $g\ (f_{cstrR}) = a\ f^{n}_{cstrR}$

The mean value of tensile strength is calculated from the above model to be:

$$\overline{f}_{tstrR} = a\ \overline{f}^{n}_{cstrR} \qquad \text{(Eq. 1.16a)}$$

Similarly, variation of concrete tensile strength is calculated using model of Eq. 1.36:

$$V^2_{tstrR} = V^2_A + V^2_g$$

where V_{tstrR} = coefficient of variation of tensile strength of in-situ concrete loaded at R psi/sec.

For $g\ (f_{cstrR}) = a\ f^{n}_{cstrR}$, $V_g = n\ V_{cstrR}$, and substituting in the above equation, we get:

$$V^2_{tstrR} = (n\ V_{cstrR})^2 + V^2_A \qquad \text{(Eq. 1.37)}$$

From the general model of tensile strength shown above, specific models for splitting tensile and flexural tensile strengths can be easily constructed using the appropriate equations for tensile strength. For example, using square-root relationship for splitting tensile strength, the mean value and coefficient of variation of splitting tensile strength of in-situ concrete will be calculated from:

$$\overline{f}_{sp} = 6.4\ \overline{f}_c^{\ 1/2}\ \text{psi} \qquad \text{(Eq. 1.21a)}$$

and

$$V_{f_{sp}} = \sqrt{(V^2_c/4) + (0.12)^2} \qquad \text{(Eq. 1.37a)}$$

where $\overline{f}_c$ and V_c are values of $\overline{f}_{cstrR}$ or $\overline{f}_{cstr35}$ and V_{cstrR} or V_{cstr35}, respectively, and are calculated according to Section 1.1.6.

Similarly, the mean value and coefficient of variation of flexural tension strength of in-situ concrete for the square-root relationship between compressive and tensile strengths will be calculated from:

$$\overline{f}_r = 8.0\ \overline{f_c}^{\,-1/2} \text{ psi} \qquad \text{(Eq. 1.28a)}$$

and

$$V_{f_r} = \sqrt{(V_c^2/4) + (0.19)^2} \qquad \text{(Eq. 1.37b)}$$

The values of the ratio A are normally distributed as shown in Section 1.2.3. Since the model of tensile strength is also dependent on g (f_{cstrR}), departure from normality might be expected for the distribution of tensile strength of concrete. The discussions in Section 1.2.2, however, lead to the same probability distribution for tensile strength of concrete as for concrete compressive strength. Therefore, the model of tensile strength of concrete should be assumed to follow the same distribution as its compressive strength and may be approximated by a normal distribution.

1.2.6 Summary and recommended distributions of concrete tensile strength

With the discussion in previous sections, it seems reasonable to assume that the probability model of tensile strength of concrete in a structure can be described with a normal distribution with:

Splitting Tension:

$$\bar{f}_{sp} = 6.4\ \bar{f}_c^{1/2} \text{ psi} \qquad \text{(Eq. 1.21a)}$$

and

$$V_{f_{sp}} = \sqrt{V_c^2/4 + (0.12)^2} \qquad \text{(Eq. 1.37a)}$$

and Modulus of Rupture:

$$\bar{f}_r = 8.0\ \bar{f}_c^{1/2} \text{psi} \qquad \text{(Eq. 1.28a)}$$

and

$$V_{f_r} = \sqrt{V_c^2/4 + (0.19)^2} \qquad \text{(Eq. 1.37b)}$$

where $\bar{f}_c$ and V_c are calculated according to Section 1.1.6.

1.3 MODULUS OF ELASTICITY

Although some five or six equations are present in the literature to estimate the static modulus of elasticity of concrete, the available data on the variability of this parameter is limited. This study was conducted to relate the probability models of static modulus of elasticity and compressive strength of concrete. Two different moduli of concrete are estimated in the succeeding sections:

(1) Initial tangent modulus, and

(2) Secant modulus at specified stress levels

Only normal weight concrete and compressive loads will be considered.

1.3.1 Initial tangent modulus

An analysis of available test data for 139 tests of standard cylinders (Richart et. al, 1931, 1934, 1938) gave the following linear, power and exponential regression equations for the relationship between the initial tangent modulus and the compressive strength of concrete:

$$E_{ci} = 1{,}918{,}000 + 453\ f_c \text{ psi } (r = 0.88,\ cov = 8.2\%) \qquad \text{(Eq. 1.38)}$$

$$E_{ci} = 92{,}000\ f_c^{0.448} \text{ psi } (r = 0.91,\ cov = 7.4\%) \qquad \text{(Eq. 1.39)}$$

$$E_{ci} = 2{,}130{,}000 \exp \frac{0.138\ f_c}{1000} \text{ psi } (r = 0.85,\ cov = 8.9\%) \qquad \text{(Eq. 1.40)}$$

where E_{ci} = initial tangent modulus.

A high degree of correlation existed between initial tangent modulus and compressive strength as indicated by correlation coefficients shown with the above equations. The data used for the regression analysis are plotted in Fig. 1.11 with Eqs. 1.38, 1.39 and 1.40. A comparison of these equations with the data in Fig. 1.11 indicates that any of these three equations may be used for estimation of initial tangent modulus of concrete, although Eq. 1.39 seems to be the most promising because it has the highest correlation factor and the lowest coefficient of variation. In studies of stiffness and deflection of concrete members it is sometimes desirable to express the modulus of elasticity as a function of square-root of compressive strength (ACI, 1971). Based on a statistical study of the available data the following relation was obtained:

$$E_{ci} = 60{,}400\ f_c^{1/2} \text{ psi} \qquad \text{(Eq. 1.41)}$$

and is also plotted in Fig. 1.11 for comparison.

Using Eq. 1.39 and 1.41 a statistical analysis of the ratios of observed to calculated moduli was carried out in order to study the distribution properties of initial tangent modulus of concrete; results from this study are presented in Table 1.7. These results suggest that the distribution of observed initial tangent modulus of concrete relative to its calculated value can be approximated by a normal distribution with a mean value of the ratio of observed to calculated moduli equal to 1. In the above comparison the observed initial tangent modulus was determined experimentally by dividing initial stress by initial strain. Assuming an in-test variation of 2% in measurements of initial stress and strain and 4% in-test variation in concrete strength, the experimentally determined coefficient of variation of observed initial tangent modulus from Eq. 1.41, shown in Table 1.7, reduces to:

$$\sqrt{(0.07695)^2 - 2(0.02)^2 - (0.04 \times 0.5)^2} = 0.07, \text{ or } 7\%$$

A 7% coefficient of variation for observed initial tangent modulus of concrete relative to the computed value was also found to be valid for the relation represented by Eq. 1.39.

When the initial tangent modulus is estimated from Eq. 1.39 or Eq. 1.41, the variability of E_{ci} relative to the calculated value is approximately 7%.

1.3.2 Secant modulus

An extremely limited amount of data from standard cylinder tests (Shideler, 1957; Hanson, 1958) was available for use in establishing the relation between compressive strength of concrete and its secant modulus at 30% of the maximum stress. The regression equations based on 45 data points are:

$$E_{cs,0.3f_c} = 2{,}365{,}000 + 300\ f_c \text{ psi } (r = 0.86,\ cov = 13.8\%) \quad \text{(Eq. 1.42)}$$

$$E_{cs,0.3f_c} = 72{,}100\ f_c^{0.469} \text{ psi } (r = 0.89,\ cov = 12.2\%) \quad \text{(Eq. 1.43)}$$

$$E_{cs,0.3f_c} = 2{,}554{,}000 \exp\left(\frac{0.076\ f_c}{1000}\right) \text{psi } (r = 0.82,\ cov = 14.8\%) \quad \text{(Eq. 1.44)}$$

where $E_{cs,0.3f_c}$ = secant modulus of concrete at 30% of maximum stress. The square-root relation was found to be:

$$E_{cs,0.3f_c} = 55{,}400\ f_c^{1/2} \text{ psi} \quad \text{(Eq. 1.45)}$$

Equations 1.42 through 1.45 are compared in Fig. 1.12 with the data also plotted in the diagram. Equation 1.43 and 1.45 seem to be the best representation of the available data. The ACI Equation (1971) is also plotted on this figure.

Although the results of a statistical analysis, presented in Table 1.7, indicate a deviation from normal distribution for the ratio of observed to calculated secant moduli, normality will be assumed in this report, particularly in view of the small data population used in the study. Considering in-test variations in data measurements to be the same as those for initial tangent modulus, the real variation of observed secant modulus relative to calculated secant modulus of concrete was found to be approximately 12%. This variation is somewhat higher than the variation of initial tangent modulus of concrete.

Using modulus of elasticity data reported by Hognestad (1951), the modulus of elasticity of concrete at 90 to 95 percent of the ultimate strength was found to be 37,500 $f_c^{1/2}$ psi.

If the stress-strain curve for concrete is assumed to be parabolic up to the ultimate stress, the secant moduli corresponding to the initial tangent modulus given by Eq. 1.41 would be:

At 0.3 f_c:

$$E_{cs,0.3f_c} = 55{,}500\ f_c^{1/2}\ \text{psi:}$$

At 0.925 f_c:

$$E_{cs,0.925f_c} = 38{,}500\ f_c^{1/2}\ \text{psi.}$$

Because these values are so close to the statistically derived values, a parabolic stress-strain relationship can be assumed to exist and can be used to compute secant modulus values at various stress levels.

1.3.3 Effects of rate of loading

Like compressive strength, the mean value and dispersion of modulus of elasticity of concrete are subject to a rate of loading effect. This effect on the modulus of elasticity can be considered using Eq. 1.46 (Allen, 1970):

$$E_{cR} = \lambda \, E_c \qquad \text{(Eq. 1.46)}$$

where E_{cR} = modulus of elasticity at a required rate of loading,

E_c = modulus of elasticity under laboratory test conditions, such as those obtained from Eqs. 1.39, 1.41, 1.43 or 1.45.

$\lambda = 0.0036/\varepsilon_{uR}$, ratio of ultimate strain under test conditions, 0.0036, to the ultimate strain at the required rate of loading, ε_{uR}.

The data given by Allen on the effect on λ of the loading duration (time to failure in a cylinder test) can be expressed using the following equation:

$$\lambda = 1.20 - 0.08 \log_{10} t \qquad \text{(Eq. 1.47)}$$

where t = loading duration in seconds.

Although no information is available for the effect of rate of loading on dispersion of modulus of elasticity of concrete, it is assumed here to be 5%.

1.3.4 Effect of in-situ casting

No information is available on the effect of in-situ casting on modulus of elasticity of concrete in a structure compared to that of control specimens. For this reason modulus of elasticity of in-situ concrete will be calculated from compressive strength of cast-in-place concrete. All equations developed in the preceding sections to relate modulus of elasticity and compressive strength of standard cylinders will be considered valid for in-situ concrete as well.

1.3.5 Model for modulus of elasticity of in-situ concrete

Based on discussions in previous sections, a general model for modulus of elasticity of in-situ concrete can be constructed. This model will be similar to the one developed for tensile strength of concrete (Section 1.2.5). From the general model specific models for initial tangent modulus and secant modulus at a desired stress level can be easily constructed.

The distribution of modulus of elasticity of concrete will be assumed to follow normal probability curve.

1.3.6 Summary and recommended distributions of modulus of elasticity of normal-weight concrete

The probability distribution of the initial tangent modulus of elasticity of normal-weight in-situ concrete in compression can be described with a normal curve with:

$$\bar{E}_{ci} = 60{,}400\ \bar{f}^{1/2}_{cstr35}\ \text{psi} \qquad \text{(Eq. 1.41a)}$$

and

$$V_{E_{ci}} = \sqrt{\frac{V^2_{cstr35}}{4} + 0.07^2} \qquad \text{(Eq. 1.48)}$$

where $\bar{f}_{cstr35}$ and V_{cstr35} are calculated according to Section 1.1.6.

The secant modulus at any stress level can be computed from the initial tangent modulus assuming that the initial portion of the stress-strain curve is a parabola with a horizontal tangent at maximum stress. The coefficient of variation of the secant modulus will be taken as:

$$V_{E_{cs}} = \sqrt{\frac{V^2_{cstr35}}{4} + 0.12^2} \quad \text{(Eq. 1.49)}$$

In order to consider the rate of loading effect for loads other than laboratory testing conditions, these equations should be modified as:

$$\overline{E}_{cr} = (1.20 - 0.08 \log_{10} t)\, \overline{E}_c \quad \text{(Eq. 1.46a)}$$

and

$$V_{E_{cR}} = \sqrt{V^2_{E_c} + 0.05^2} \quad \text{(Eq. 1.50)}$$

where $\overline{E}_{cR}$ = mean value of modulus of elasticity at required rate of loading,

$V_{E_{cR}}$ = coefficient of variation at the required rate of loading,

$\overline{E}_c$ = mean value from Eq. 1.41a,

V_{E_c} = coefficient of variation from Eq. 1.48 or Eq. 1.49.

Chapter 2

VARIABILITY OF REINFORCEMENT STRENGTH

2.1 YIELD STRENGTH OF REINFORCING STEEL

There are three main sources of variation in steel yield strength:

(1). variation in the strength of material,

(2). variation in the area of the cross-section of the bar, and

(3). variation in the rate of loading.

The variablity of yield strength depends on the source and the nature of the population. The variation in strength within a single bar is relatively small, while the in-batch variation for a given heat is slightly larger. However, the variability of samples derived from different batches and sources may be high. This is expected since rolling practices and quality measures vary for different countries, different manufacturers and different bar sizes. Furthermore, the cross-sectional areas vary due to differences in the setting of the rolls, and this adds to the variation. Mill tests are generally carried out at a rapid rate of loading (ASTM corresponds to 1040 micro-in/in/sec) and have the tendency of reporting the unstable high yield point rather than the stable low yield point. Since the strains in a structure are generally induced at a much lower rate than the mill tests, mill tests tend to over-estimate the strength of reinforcement, hence another source of variation.

An examination of the test data revealed that the bars of large diameter tended to develop less yield strength (Allen, 1972; Gamble, 1973; Robles, 1972) than #3 to #11 bars. Thus, for purpose of statistical evaluation of yield strength, the #14 and #18 bars were studied separately from other sizes. Also, the #2 bars were not included in this study because of their rare use for structural concrete.

In this study the terms Grade 40, Grade 50 and Grade 60 refer to reinforcing bars with minimum specified yield strength of 40, 50 and 60 ksi, respectively, even though the bars in question may not have been produced according to ASTM or CSA specifications. Only data for deformed bars have been included. In some cases data for cold-worked bars have been considered but most of the data are for hot-rolled bars.

2.1.1 Variation in strength of material

Different values for the yield strength of steel can be obtained depending how it is defined. The static yield strength based on nominal area seems to be desirable because the strain rate is similar to what is expected in a structure, and designers use the nominal areas in their calculations. Most mill tests, however, are conducted with a rapid rate of loading, and the strength is generally referred to actual areas. For these reasons yield strength corresponding to rapid strain rate and measured area is discussed in this section, and the effects on this strength of variations in cross-sectional area and rate of loading are dealt with in the succeeding sections.

A review of literature on yield strength of reinforcing bars showed that the coefficient of variation was in general in the order of 1 to 4% for individual bar sizes and 4 to 7% overall for data derived from one source. When data was taken from many sources the coefficient of variation increased to 5 to 8% for individual sizes and 10 to 12% overall. A summary of selected studies from literature (Allen, 1972; Baker, 1970: Bannister, 1968; Julian, 1957; Wiss, Janney, Elstner and Assoc., 1976; Narayanaswamy, 1972) is shown in Table 2.1.

The data reported by Allen (1972), Julian (1957) and Wiss, Janney, Elstner and Assoc. (1976) on Grade 40 and 60 steel bars showed close agreement with normal distribution (with respective mean and standard deviation) in the range from about the 5th to the 95th percentile but differed from the normal distribution outside this range. Some authors have suggested other types of distributions such as skewed (Bannister, 1968; Roberts, 1967), truncated normal (Johnson, 1956) and Beta distributions (Costello, 1969). Similarly, for yield strength of structural steel shapes and plates, Alpsten (1972) has suggested an extreme-value distribution Type I or a log-normal distribution. These suggestions were, however, based on a particular set of data and only approximated the distribution of the population from which the data were drawn. Nonetheless, they suggest that yield strength is a phenomenon that can be described by a particular theoretical distribution with certain limitations.

The normal distribution seems to correlate very well in the vicinity of mean values for different populations of yield strength, but it is a crude approximation at low and high probabilities where the steel strength distribution curves tend to have a certain minimum and maximum value instead of following the theoretical tails. This is expected since the manufacturing of steel is not truly a random process. There are always some quality controls that are used to attain a certain minimum yield strength. Because much of the data indicated a positive skewness, particularly when derived from different sources and mixed together, a log normal distribution should be a better fit for this case than a normal distribution since it takes into account the skew nature of the data. However, the log-normally distributed values of yield strength at low and high probabilites did not show a prominent improvement over normally distributed values of available data. Therefore, it was decided to empirically establish a distribution that would correlate with the North American data on yield strength. For this purpose modified log-normal, Beta and Pearson system distribution curves were tried.

(a) Grade 40 steel

Values of $\log_{10}(f_y - 34 \text{ ksi})$ were plotted on a normal probability paper for the data from 249 tests reported by Julian (1957) and Allen (1972) for Grade 40 reinforcing bars. These values were found to be in good agreement with a normal distribution in the range from the 0.01 percentile to the 99th percentile. The modification constant of 34 ksi was established by trial and error. The probability density functions of normal and modified log-normal distributions for Grade 40 steel can be obtained from Eqs. 2.1 and 2.2, respectively:

$$\text{PDF} = \frac{1}{5.218\sqrt{2\pi}} \cdot \exp -\left\{1/2 \left(\frac{f_y - 48.8}{5.218}\right)^2\right\} \qquad \text{(Eq. 2.1)}$$

$$\text{PDF} = \frac{0.43429}{0.14866\sqrt{2\pi}\,(f_y - 34)} \cdot \exp\left[-1/2\left\{\frac{\log_{10}(f_y - 34) - 1.14482}{0.14866}\right\}^2\right] \text{(Eq. 2.2)}$$

$(f_y > 34 \text{ ksi})$

where f_y = steel strength in ksi.

Using the Pearson system of frequency curves (Elderton and Johnson, 1969) it was found that the above data can be represented fairly, even in the high and low probability regions, by Pearson Main Type I distribution. The probability density function of the distribution can be calculated from:

$$PDF = \frac{7.42}{100} \left(1 + \frac{f_y - 47.86}{15.47}\right)^{5.13} \left(1 - \frac{f_y - 47.86}{32.04}\right)^{10.62}$$

$$(32.39 \leq f_y \leq 79.90 \text{ ksi}) \qquad \text{(Eq. 2.3)}$$

Similary, a Beta distribution with the range from 36 ksi to 68 ksi seems to represent the available data for Grade 40 steel quite well. The probability density function can be represented by the equation:

$$PDF = 3.7138 \left(\frac{f_y - 36}{32}\right)^{2.2105} \left(\frac{68 - f_y}{32}\right)^{3.8157} \qquad \text{(Eq. 2.4)}$$

$$(36 \leq f_y \leq 68)$$

All the theorteical distributions described in the preceding paragraphs were empirically derived using the grouped data. The mean value of the data was found to be 48.8 ksi and the coefficient of variation was 10.7%. The theoretical frequency curves for normal, modified log-normal, Pearson Main Type I and Beta distributions are shown in Fig. 2.1. The cumulative frequencies obtained from the data are also shown in Fig. 2.1 for the purpose of comparison. The corresponding density functions and the histogram of the grouped data are plotted in Fig. 2.2. An examination of these figures clearly indicates that the central region can be represented by any of the four distributions. The modified log-normal distribution fits the lower tail as well but considerably overestimates the strengths in the upper tail region. Pearson Type I and Beta distributions, however, fit the entire region. It is, therefore, proposed that the β-distribution or Pearson Main Type I distribution, shown in Figs. 2.1 and 2.2 and represented by Eqs. 2.3 and 2.4, should be used to represent the distribution of yield strength of Grade 40 steel.

(b) Grade 60 steel

The values of $\log_{10}(f_y - 54 \text{ ksi})$ for Grade 60 reinforcing bars from 273 mill tests, reported by Allen (1972) and Wiss, Janney, Elstner and Assoc. (1976) grouped together, were found to be normally distributed in the range from the 0.01 percentile to the 99th percentile. The probability density functions of normal and modified log-normal distributions for Grade 60 steel can be calculated from the equations:

$$\text{PDF} = \frac{1}{6.582\sqrt{2\pi}} \cdot \exp\left\{- 1/2 \left(\frac{f_y - 71.04}{6.582}\right)^2\right\} \quad \text{(Eq. 2.5)}$$

$$\text{PDF} = \frac{0.43429}{0.16200\,(f_y - 54)\sqrt{2\pi}} \cdot \exp\left[- 1/2\left\{\frac{\log_{10}(f_y - 54) - 1.20115}{0.16200}\right\}^2\right] \quad \text{(Eq. 2.6)}$$

$(f_y > 54 \text{ ksi})$

where f_y = yield strength in ksi.

As in the case of Grade 40 reinforcement, Pearson and Beta distributions were also found to represent the yield strength data for Grade 60 steel. The probability density functions of Pearson Type VI and Beta distributions for Grade 60 reinforcement can be obtained from Eqs. 2.7 and 2.8, respectively:

$$\text{PDF} = 18.923 \times 10^{28}\,(f_y - 51.320)^{24.8022}\,(f_y - 39.798)^{-41.8811} \quad \text{(Eq. 2.7)}$$

$(f_y > 51.320 \text{ ksi})$

$$PDF = 7.1562 \left(\frac{f_y - 57}{51}\right)^{2.0204} \left(\frac{108 - f_y}{51}\right)^{6.9545} \qquad \text{(Eq. 2.8)}$$

$$(57 \leq f_y \leq 108 \text{ ksi})$$

The theoretical frequency curves and the grouped data for Grade 60 steel are plotted in Figs. 2.3 and 2.4. The mean value of the data was found to be 71.04 ksi and the coefficient of variation 9.3%.

For Grade 60 steel the modified log-normal and Pearson Type VI distribution curves are better approximations at the lower end of the curve and the normal curve is better at the high end of the curve, while Beta distribution curve approximates the entire range of distribution.

2.1.2 Variations in the area of bar cross-section

The actual areas of reinforcing bars tend to deviate from the nominal areas due to the rolling process. The designers do not have this information readily available to them, and hence use the nominal areas in their calculations. For this reason, this variation should be incorporated in the strength of steel.

The variation in the ratio of measured/nominal areas (A_m/A_n) was studied as a measure of variation in cross-sectional area of reinforcing bars. The values of A_m/A_n are reproduced from available literature (Allen, 1972; Baker, 1970; Narayanaswamy, 1972) in Table 2.1. Table 2.1 indicates that the data reported by Baker (1970) for Grade 60 steel demonstrates high mean value and coefficient of variation. Such values cannot be explained in definite terms. It is possible that this data contained a good percentage of values from mills with old rolls that increased the mean and coefficient of variation. Furthermore, British rolling practice may differ from Canadian practice. For these reasons, these values were not included in the analysis.

The ratios of A_m/A_n from tests on Grade 40 and 60 reinforcing bars, manufactured in Canada (Studies No. 1 and 6 in Table 2.1), were plotted on normal probability paper. These values exhibited close agreement in the range from 5th to 95th percentile for Grade 40 steel and from 2nd to 98th percentile for Grade 60 bars with normal distribution. When values for both studies were combined, they resulted in a normal distribution in the range between the 3rd and the 99th percentiles with a mean value of 0.988 and coefficient of variation 2.43% as shown in Fig. 2.5. Allen (1972) has suggested a mean value of 0.97 for A_m/A_n. This seems to be a conservative estimate of the average values of A_m/A_n shown in Table 2.1, and close to ASTM rolling tolerances that allow an average ratio as low as 0.965 and a minimum single value down to 0.94.

From the above discussion and Fig. 2.5 it seems reasonable to assume a normal distribution truncated at 0.94 and 1.06 with a mean value 0.988 and coefficient of variation of 2.4% for the ratio of measured nominal areas. Where the effect of bar areas is relatively unimportant, a single value of 0.97 for the ratio A_m/A_n may be used in lieu of the distribution shown in Fig. 2.5.

2.1.3 Effects of rate of loading

The apparent yield strength of a test specimen increases as the strain rate or the rate of loading increases. Since mill tests on steel specimens are generally carried out at much greater strain rates (approximately 1040 micro-in/in/sec) than encountered in a structure, they tend to over-estimate the yield strength. A strain rate of 1 in/in/sec may increase the yield strength of Grade 40 steel as much as 50% over the static yield point (Keenan, 1960).

Tests conducted on steel coupons of A36, A441 and A514 steel (Rao et al., 1966) demonstrated a yield strength reduction of more or less the same value for all types of steel with decreases in the rate of strain. The regression equation developed by Rao on the basis of these tests gives values of static yield strength that are 4.2 ksi and 3.4 ksi less than the yield strengths obtained at cross-head speed of 1000 and 200 micro-in/in/sec, respectively. Rao's (1966) equation is:

$$df_{ys} = f_y - f_{ys} = 3.2 + 0.001\varepsilon \text{ ksi} \qquad \text{(Eq. 2.9)}$$

$$(200 \leq \varepsilon \leq 1600)$$

where ε = strain in micro-in/in/sec.

NRC tests on Grade 40 bars (Allen, 1972) showed a reduction of approximately 3 ksi in the mean yield strength when speed of testing machine was dropped from 208 micro-in/in/sec to static. This value correlates well with the one obtained from Rao's equation (1966). Similarly, for Grade 40 bars, it has been shown at University of Illinois (Keenan, 1960) that the difference between yield strength at strain rate 1040 micro-in/in/sec and strength at strain rate 20 micro-in/in/sec is about 9% or 4 ksi. Swiss tests (Lampert et al., 1967) for high strength reinforcement demonstrated a reduction of 3 ksi for static condition. Similar results were obtained elsewhere (Wiss, Janney, Elstner and Assoc., 1976).

A statistical analysis of NRC data (Allen, 1972) for Grade 40 reinforcing bars suggests that the distribution of the difference in yield strength at machine speed of 208 micro-in/in/sec and static yield strength (df_{ys}) has the following properties:

Mean Value	=	2.8359
Coefficient of Variation	=	0.1340
Coefficient of Skewness	=	-0.1030
Coefficient of Kurtosis	=	2.9543

These properties strongly indicate a normal distribution for df_{ys}. The grouped data for df_{ys} and the corresponding theoretical normal distribution are plotted in Fig. 2.6. These values clearly exhibit a close agreement in the range from 0.01 percentile to 98th percentile with the normal distribution. It should be noted that these data are for lab tests. For evaluation of static yield strength from mill tests, Allen (1972) has suggested a decrease of 4 ksi. This value seems to be a conservative estimate for the available test data.

Based on Eq. 2.9 and Allen's (1972) tests it will be assumed that the distribution of df_{ys} can be considered normal for Grade 40 and Grade 60 reinforcing bars with a mean value 3.5 ksi and a coefficient of variation of 13.4%. In cases where the effect of variability of df_{ys} is considered negligible, a single value of 4 ksi may be used.

2.1.4 Effects of bar diameter

The strength of steel tends to vary across the cross-section of a reinforcing bar with highest strength near the outside of the bar. This is probably due to cold-working of circumferential sections of bars during rolling process. Thus the mean yield strength is expected to decrease with the increase in diameter. In addition, rolling mill practice is different for large bars than for small bars. The variation of the mean yield strength with size is plotted in Figs. 2.7 and 2.8. The data shown in the figures were taken from several test series for Grade

40 and Grade 60 reinforcement (Allen, 1972; AASHO, 1962; Baker, 1970; Bannister, 1968; Gamble, 1973; Wiss, Janney, Elstner and Assoc., 1976). For bars with relatively small diameter the effect of this variation is small and not clearly established. For large diameter bars such as #14 and #18 this effect becomes more prominent. In addition, the ASTM specifications allow the use of small specimens machined from samples of large diameter bars for testing purposes. A specimen machined to a smaller diameter from a quarter-piece of a full size bar tends to show higher yield strength than the bar itself (Gamble, 1973). Since some manufacturers may use these tests as a measure of quality control, there is a higher probability that #14 and #18 bars may satisfy the quality controls without developing the required strength.

An extremely limited amount of data is available for #14 and #18 bars. Tests on Grade 40, #14 bars carried out by Allen (1972) showed that the mean yield strength of #14 bars was 44 ksi, a 15% decrease from strength of #3 to #11 bars produced by the same manufacturer. Some data has been reported by Gamble (1973) for #14 and #18 bars of Grade 60 steel. The mean yield strengths were 60 ksi for #14 and 55 ksi for #18 bars. These strengths were referred to the nominal areas. Using the mean yield strength of Grade 60, #3 to #11 bars as 71.5 ksi (as per Study No. 6 in Table 2.1) and a 3% adjustment for deviation from nominal area, the reduction in strength is approximately 13% for #14 bars and 21% for #18 bars. This comparison is, however, not truly justified since the data for both studies were not drawn from the same source. Nonetheless, it strongly indicates the potential under-strength of #14 and #18 bars. On the other hand, however, the data reported by Wiss, Janney, Elstner and Associates (1976) for Grade 60, #5, #8, #11 and #18 reinforcing bars did not indicate any significant influence of bar size, as shown in Fig. 2.8. This is in contradiction to other studies (Allen, 1972; Gamble, 1973). Until more data are available, it seems reasonable to assume that the yield strength of #14 and #18 bars will be at least 15% below that of reinforcing bars with small diameter.

2.1.5 Summary and recommended distributions of the yield strength of reinforcement

The Beta distribution and Pearson distribution curves, shown in Figs. 2.1 through 2.4, seem to correlate well, particularly the Beta distribution curve, with the entire distribution range of the available North American data for yield strength of Grade 40 and Grade 60 reinforcing bars. The mean values and coefficients of variation for the selected data were found to be 48.8 ksi and 10.7% for Grade 40 and 71.04 ksi and 9.3% for Grade 60 bars.

Based on the Beta Distribution the PDF of yield strength of Grade 40 reinforcement can be calculated from:

$$PDF = 3.7138 \left(\frac{f_y - 36}{32}\right)^{2.2105} \left(\frac{68 - f_y}{32}\right)^{3.8157}$$

$$(36 \leq f_y \leq 68 \text{ ksi}) \qquad \text{(Eq. 2.4)}$$

Similarly, using the Beta Distribution the PDF of f_y of Grade 60 steel bars can be calculated from:

$$PDF = 7.1562 \left(\frac{f_y - 57}{51}\right)^{2.0204} \left(\frac{108 - f_y}{51}\right)^{6.9545}$$

$$(57 \leq f_y \leq 108 \text{ ksi}) \qquad \text{(Eq. 2.8)}$$

The distribution curve of the ratio measured/nominal areas can be approximated with a normal function truncated at 0.94 and 1.06 with a mean value 0.988 and coefficient of variation 2.4%. Similarly, for evaluation of static yield strength from mill tests corresponding to ASTM specifications, the distribution curve for df_{ys} should be assumed to be normal with a mean value 3.5 ksi and coefficient of variation 13.4%. The distribution functions of A_m/A_n and df_{ys} are assumed to be the same for both Grade 40 and Grade 60 steel.

When calculating the yield strength of #14 and #18 reinforcing bars from the strength of bars of smaller sizes a reduction of at least 15% should be used to account for the effect of large diameter. In doing this the mean values should be adjusted to account for deviations due to bar size, but the standard deviations must be kept constant.

If a single distribution for steel yield strength based on nominal area for static loading is desired, it may be crudely approximated by assuming a normal distribution with:

$$\text{Mean yield strength} = \left(\overline{\frac{A_m}{A_n}}\right) \left\{\mu\bar{f}_y - \overline{df}_{ys}\right\} \text{ksi} \qquad \text{(Eq. 2.10)}$$

$$\text{Coefficient of variation} = \sqrt{V^2_{A_m/A_n} + \left\{\frac{\sqrt{\sigma^2_{f_y} + \sigma^2_{df_{ys}}}}{\mu\bar{f}_y - \overline{df}_{ys}}\right\}^2} \qquad \text{(Eq. 2.11)}$$

where $\bar{f}_y$, σ_{f_y} are mean yield strength and standard deviation based on measured area from mill tests in ksi,

$\overline{df}_{ys}$, $\sigma_{df_{ys}}$ are mean correction and standard deviation for static condition in ksi,

$\left(\overline{\frac{A_m}{A_n}}\right)$, V_{A_m/A_n} are mean value and coefficient of variation of the ratio of measured to nominal areas of bars, and μ is a factor for diameter effect, and is equal to 1.00 for #3 to #11 bars and 0.85 for #14 and #18 bars.

Thus Eqs. 2.10 and 2.11 will give a mean strength 44.76 ksi and cov 11.82% for #3 to #11 reinforcing bars with mill mean yield strength 48.8 ksi and cov 10.7%.

2.2 ULTIMATE STRENGTH OF REINFORCING STEEL

The sources of variation of ultimate strength of reinforcing bars are same as those causing variation of the yield strength, except that the ultimate strength does not appear to be influenced by the bar size. Consequently, for this study no distinction was made between #14 and #18 bars and bars of smaller sizes for the purpose of studying ultimate strength.

2.2.1 Variations in strength of material

Results from statistical analyses of ultimate and yield strengths of Grade 40 and Grade 60 reinforcing bars (Allen, 1972; Wiss, Janney, Elstner & Assoc., 1976) are compared in Table 2.2. These results indicate an increase in average ultimate strength of steel in the order of roughly 55% over the yield strength, with other distribution properties of steel strengths, particularly the coefficient of variation, remaining approximately unchanged in most cases. From this observation it can be reasonably assumed that the probability distribution of ultimate strength of steel follows Beta distribution, which was found to be the best fit for yield strength data and the equations of the probability density functions for yield strength will be used after modification for the higher values of the ultimate strengths. This will be done here because of the lack of data for Grade 40 bars.

The modified version of Eq. 2.8 for the Beta distribution of the ultimate strength of Grade 60 bars is:

$$PDF = 4.6169 \left(\frac{f_u - 88.35}{79.05}\right)^{2.0204} \left(\frac{167.40 - f_u}{79.05}\right)^{6.9545} \qquad \text{(Eq. 2.12)}$$

$(88.35 \leq f_u \leq 167.40 \text{ ksi}; \ f_u \geq f_y)$

Frequency distribution curves from Eq. 2.12 for the Beta distribution of ultimate strength of Grade 60 reinforcement are plotted in Figs. 2.9 and 2.10. The data from 274 mill tests reported by Allen (1972) and Wiss, Janney, Elstner & Associates (1976) grouped together are also shown in the figures. The Beta distribution correlates well with the data, and thus confirms its validity for the ultimate strength of Grade 60 bars. The average and coefficient of variation of the ultimate strength of Grade 60 steel were found to be 110.8 ksi and 7.9%, respectively.

A normal curve is also plotted in Figs. 2.9 and 2.10 for the purpose of comparison. The normal distribution also indicates a good correlation with the data in the range from the 2nd percentile to the 99th percentile, but does not correlate in the regions of the lowest and highest probabilities. With the comparison of both curves, it seems that the normal distribution is more representative of the actual distribution in the central region, while Beta distribution is more acceptable for the tail areas. The probability density function for normal distribution of Grade 60 steel can be calculated from:

$$PDF = \frac{1}{8.735\sqrt{2\pi}} \cdot \exp\left[-1/2\left(\frac{f_u - 110.8}{8.735}\right)^2\right] \qquad \text{(Eq. 2.13)}$$

Unfortunately, insufficient data is available for the ultimate strength of Grade 40 steel to allow one to establish an ultimate strength distribution. However, due to the acceptable degree of correlation demonstrated by Grade 60 reinforcement with the corresponding Beta distribution equation, Eq. 2.12, it seems logical that the ultimate strength distributions for both grades of steel should be based on similar arguments, and Beta distribution should be adequate for Grade 40 steel as well. From Table 2.2 and the yield strength distributions in Figs. 2.1 and 2.2, the mean ultimate strength and coefficient of variation of Grade 40 steel should be taken as 79.3 ksi and 10%, respectively. The following Beta distribution has been arbitrarily selected to represent the ultimate strength of Grade 40 reinforcement:

$$PDF = 2.3960\left(\frac{f_u - 55.8}{49.6}\right)^{2.2105}\left(\frac{105.4 - f_u}{49.6}\right)^{3.8157} \qquad \text{(Eq. 2.14)}$$

$$(55.8 \leq f_u \leq 105.4 \text{ ksi}, \; f_u \geq f_y)$$

2.2.2 Effects of rate of loading

Like yield strength, the apparent ultimate strength of a test specimen is influenced by the rate of loading, the higher the strain rate, the higher the ultimate strength. Mill tests tend to over-estimate the ultimate strength due to very high strain rate at which these tests are conducted. This increase in strength due to rapid loading must be deducted from mill tests in order to estimate the ultimate strength for the strain rate at which loads are applied in a structure, which in most cases, is close to the static condition.

Some data is available from NRC (Allen, 1972) for effect of rate of loading on strength of Grade 40 reinforcing bars with a strain rate drop from 208 micro-in/in/sec to static condition. Results obtained from the statistical evaluation of the data are given below:

	$df_{us} = f_u - f_{us}$	$df_{ys} = f_y - f_{ys}$
Mean Value (ksi)	4.124	2.8359
Coefficient of variation	0.1333	0.1340
Coefficient of skewness	+0.7804	-0.1030
Kurtosis	2.7979	2.9543

Although the mean value for df_{us} increased by approximately 50% over df_{ys}, the coefficient of variation remained constant. Other statistical properties indicate a distribution of df_{us} somewhat skewed towards the higher tail, but a close normal distribution for df_{ys}.

Until further information is available the distribution of df_{us} may be assumed to be normal with mean value 4.5 ksi and coefficient of variation 13.4%. In cases where the effect of variability of df_{us} is considered negligible, a single value of 5 ksi may be used instead of above-described distribution.

2.2.3 Summary and recommended distributions of ultimate strength of reinforcing steel

The distribution of ultimate strength of reinforcing steel can be represented by Beta distributions. The probability density function for these distributions can be calculated from Eqs. 2.14 and 2.12 for Grade 40 and Grade 60 bars, respectively.

The distribution curve for the ratio measured/nominal areas, A_m/A_n, should be taken as normal with average 0.988 and coefficient of variation 2.4%. Similarly, the distribution of the difference between ultimate strength from mill tests and the static ultimate strength, df_{us}, can be assumed to be normal with the average 4.5 ksi and coefficient of variation 13.4%. The effects of rate of loading and the deviations from the nominal areas must be incorporated in the ultimate strength. The distribution functions of A_m/A_n and df_{us} are same for both grades of steel.

If a single distribution for ultimate strength based on nominal area for static loading condition is desired, it may be crudely approximated by assuming a Gaussian curve with:

$$\text{Mean ultimate strength} = \overline{\left(\frac{A_m}{A_n}\right)} \left\{ \overline{f}_u - \overline{df}_{us} \right\} \quad \text{ksi} \qquad \text{(Eq. 2.15)}$$

$$\text{Coefficient of variation} = \sqrt{V^2_{A_m/A_n} + \left\{ \frac{\sqrt{\sigma^2_{f_u} + \sigma^2_{df_{us}}}}{\overline{f}_u - \overline{df}_{us}} \right\}^2} \qquad \text{(Eq. 2.16)}$$

where $\overline{f}_u$, σ_{f_u} are average ultimate strength and standard deviation from mill tests,

$\overline{df}_{us}$, $\sigma_{df_{us}}$ are mean correction in ultimate strength and standard deviation for static condition,

and $\overline{\left(\frac{A_m}{A_n}\right)}$, V_{A_m/A_n} are mean and coefficient of variation of the ratio measured/nominal area of bars.

2.3 MODULUS OF ELASTICITY OF STEEL

Modulus of elasticity of steel is an easily predictable parameter, which has a small dispersion and is more or less insensitive to the rate of loading (Allen, 1970). Also, like ultimate strength, the modulus of elasticity seems to be unaffected by the bar size (Allen, 1972).

Based on Robertson's data (1931), Allen (1970) has proposed a normal distribution with mean value 29,200 ksi and a coefficient of variation of 3% for the modulus of elasticity of steel. A statistical analysis of the NRC data (Allen, 1972) of #3 to #14 bars of Grade 40 reinforcing steel gave following properties of distribution:

Mean value (ksi)	= 29,250
Coefficient of variation	= 0.0154
Coefficient of skewness	= -0.3582
Kurtosis	= 3.2662

These properties indicate a slight skewness toward the lower values, although the distribution seems to be close to normal curve.

2.3.1 Summary and recommended distribution for modulus of elasticity of reinforcement

The probability distribution of modulus of elasticity of reinforcing bars should be considered normal with a mean value of 29,200 ksi and a coefficient of variation of 2.4 percent. This coefficient of variation was obtained by combining with equal weightage the coefficients of variation of Allen's (1972) and Robertson's (Allen, 1970) data.

Chapter 3

GEOMETRIC IMPERFECTIONS

3.1 INTRODUCTION

Geometric imperfections in reinforced concrete elements are caused due to deviations from the specified values of the cross-sectional shape and dimensions, the position of reinforcing bars, ties and stirrups, the horizontality and verticality of concrete lines, the alignment of columns and beams, and the grades and surfaces of the constructed structures. Geometric imperfections arise during different phases of the construction process. For example, variations in size and shape are mainly dependent on the size, shape and quality of forms used, and to some extent on concreting and vibrating operations. For these reasons geometric imperfections vary from country to country, region to region, and even from structure to structure, depending on the quality of construction techniques, equipment, and the training of site personnel.

Unfortunately, the process of collecting and reporting data for geometric imperfections has not yet been standardized. Without a reasonable degree of uniformity it is difficult to compare the results of measurements reported by various researchers for quantitative conclusions. Furthermore, no data were available for certain imperfections, such as stirrup spacing, shear span, etc. and probability models for these imperfections must be extrapolated on the basis of distribution properties of the related variables. As a result, the probability models of geometric imperfections suggested in this study are the consequence of rational interpretation of the qualitative trends of the available data. These models should be considered preliminary as such, and should be modified when more complete data are available.

While most researchers have recommended the use of a normal distribution (van den Berg, 1971; Johnson, 1953; Connolly, 1975) for the probability models of the majority of dimensions, probably because of its simplicity and versatility, others have preferred the use of a log-normal distribution (Connolly, 1975; Tichý and Vorlíček, 1972) for certain dimensions. In order to determine the form of the probability distribution of a dimension it must be established whether the variations in the dimension are caused by accidental or systematic errors. Examination of the data presented in this chapter shows that at least in some cases part of the variation is due to systematic errors. In most cases, however, the whole or at least the greater part of the variation can be considered due to accidental errors. If these variations can be attributed to a great number of mutually independent causes that produce additive effects, then the variations, according to the central limit theorem, will tend toward the normal distribution. If the effects are multiplicative, then the distribution will tend to approach log-normal. From this discussion it follows that either normal or log-normal distribution can be used to represent the dispersion of geometric imperfections of reinforced concrete members. Due to simplicity offered by normal distribution, the normal distribution will be used here unless specifically mentioned otherwise for geometric imperfections related to reinforced concrete elements.

In this chapter dimensional variations of in-situ and precast concrete members are treated. The variations in the dimensions of precast elements should be expected to be smaller than those for in-situ structures, because the higher degree of dimensional controls and smaller specified tolerances contribute to lower dispersions.

The areas and spacings of reinforcing steel actually furnished in concrete elements will differ somewhat from those calculated on the basis of design strength equations. This is due to the fact that the designer's option is limited to bars of standard sizes. This difference in required and specified areas of reinforcement introduces further variations in member capacities. For this reason the variability of the ratios of actual to computed areas of longitudinal and transverse steel for beams as well as vertical steel for columns is also discussed in this chapter.

Certain original references used in the text, namely Jacobson and Widmark (1970), Klingberg (1970), van den Berg (1971), Johansson and Warris (1968) and Bishop (1963), were not available, and the data attributed to these references were taken from the excellent review by Fiorato (1973).

3.2 SLAB DIMENSIONS

3.2.1 Slab thickness

Variations in slab thicknesses are important from the view point of serviceability as well as strength. High dispersions in the slab thickness can result in poorly finished floor surfaces. These dispersions also affect the effective depth of slabs, and thus influence their strength capacities. Variations in slab thickness generally depend on finishing operations of concrete surfaces, and to some extent on the supporting conditions of the forms.

Results from various studies (AASHO, 1962; Johnson, 1953; Johansson and Warris, 1968; Bishop, 1963; Hernandez and Martinez, 1974) of the distribution properties of cast-in-place and precast slabs are given in Table 3.1. The average measured thicknesses in Table 3.1 are slightly higher than the specified values for in-situ slabs in almost all cases. The standard deviations for these studies are nearly constant, except for the AASHO results for the AASHO Road Test bridge decks where the standard deviation is much smaller than in the other studies. Lower values of standard deviation for the AASHO data is probably due to higher degree of controls employed in bridge construction and possibly due to the experimental nature of the bridges considered. As expected for precast construction, the mean deviations from nominal values seem to be negligible and the standard deviations are small too.

If the results from individual studies shown in Table 3.1 are assumed to be samples from population of the slab thickness data with the means and variances of these samples being independent, then the weighted mean and standard deviation of the total sample can be statistically calculated from Eqs. 3.1 and 3.2, respectively.

$$\overline{X} = (\sum_{i=1}^{k} N_i \overline{X}_i)/(\sum_{i=1}^{k} N_i); \; i = 1, 2, \cdots, k \qquad \text{(Eq. 3.1)}$$

$$\sigma = \sqrt{\left\{\sum_{i=1}^{k} N_i \sigma_i^2 + \sum_{i=1}^{k} N_i (\overline{X}_i - \overline{X})^2\right\} / (\sum_{i=1}^{k} N_i)} \qquad \text{(Eq. 3.2)}$$

where $\overline{X}_i$, σ_i are mean and standard deviation of sample size N_i, and $\overline{X}$, σ are mean and standard deviation of total sample size N. Weighted means and standard deviations calculated from these equations for deviations of thickness from nominal values are shown at the bottom of Table 3.1.

Comparing the weighted values it is evident that the standard deviation of precast units is roughly 40% of the standard deviation of in-situ slabs. But, the mean deviation of in-situ slabs is only slightly higher than one for the precast units. Based on the observations in Table 3.1, the recommended properties of distribution for slab thickness are given in Table 3.3.

3.2.2 Effective depth of slab reinforcement

Variations in effective depth of steel are very important from point of view of strength. These variations are caused during different phases of construction, although most of the inaccuracies are induced during steel fabrication process. These deviations vary depending on the dimensional inaccuracies involved, locations of supporting chairs with the possibility of sag of bars between them, and the congestion of reinforcement that, sometimes, necessitates forcing the bars out of position during fabrication and vibration processes. In addition top steel of in-situ concrete is affected also by the methods of placing concrete. For example, lower effective depths for top steel should be expected due to practice of construction personnel walking on the reinforcement during fabrication and concreting operations. This is particularly true when concrete is transported on planks placed over reinforcement.

Available data on the effective depths of top and bottom reinforcement of slabs are presented in Table 3.2 (Johnson, 1953; Johansson and Warris, 1968). For all studies on in-situ slabs the average measured depths of top as well as bottom reinforcements were smaller than the nominal dimensions, although top reinforcement was more affected than the bottom steel. This does not seem to be valid for precast units, however, where the differences in average measured and nominal dimensions are marginal. This should be expected due to greater degree of controls employed in precast construction. The standard deviations among individual studies of in-situ as well as precast slabs varied only slightly.

The weighted means and standard deviations of all studies for in-situ as well as precast slabs were calculated using Eqs. 3.1 and 3.2, and are also shown in Table 3.2. A comparison of these values for in-situ slabs indicates that the mean deviation of top reinforcement from nominal effective depth is approximately 2 1/2 times the mean deviation of bottom steel, but the standard deviations for both steels are about the same. No data were available for the variability of effective depths of top reinforcement of precast slabs. Since the variabilities in the positions of top and bottom steels of precast slabs arise from the same sources, it seems reasonable to assume that the probability models for top and bottom steel placements are nearly identical. Based on this observation and on weighted values in Table 3.2, the recommended distributions for effective depths are given in Table 3.3.

3.2.3 Concrete cover for slab steel

Distribution properties of concrete cover for top and bottom reinforcement of in-situ as well as precast slabs were established from the recommended distributions of slab thickness and effective depths of reinforcement. Assuming that the variability in the diameter of reinforcing bars is negligible, the distribution of concrete cover is the difference between distributions of slab thickness and effective depth.

The positions of top and bottom reinforcement in an in-situ slab are controlled by placing chairs on the formwork. Therefore, concrete cover of bottom steel and effective depth of top bars can be assumed to be independent random variables. Since, slab thickness is also an independent random variable, the concrete cover for the top reinforcement can be calculated from:

$$c_t = t - d_t$$

and its dispersion can be calculated from:

$$\sigma^2_{ct} = \sigma^2_t + \sigma^2_{dt} \qquad \text{(Eq. 3.3)}$$

$$\text{or} \quad \sigma^2_{dt} = \sigma^2_{ct} - \sigma^2_t \qquad \text{(Eq. 3.3a)}$$

where σ_t = standard deviation of slab thickness,

σ_{dt} = standard deviation of effective depth of top steel,

σ_{ct} = standard deviation of concrete cover of top steel.

Similarly, the effective depth of bottom reinforcement in an in-situ slab can be expressed by:

$$d_b = t - c_b$$

where the cover c_b, and the thickness, t, are independent variables. Hence the variability of the effective depth is:

$$\sigma^2_{db} = \sigma^2_t + \sigma^2_{cb} \qquad \text{(Eq. 3.4)}$$

$$\text{or} \quad \sigma^2_{cb} = \sigma^2_{db} - \sigma^2_t \qquad \text{(Eq. 3.4a)}$$

where σ_{db} = standard deviation of effective depth of bottom bars,

σ_{cb} = standard deviation of concrete cover of bottom bars.

Based on these observations and Eqs. 3.3 and 3.4a, the distribution properties of concrete cover in the in-situ slabs are shown in Table 3.3. The recommended mean and variances of concrete cover are somewhat higher than the following values found by van Daveer (1975):

Nominal Cover (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)
1 1/2	+ 3/8	3/8
2	+ 1/8	3/8
1 7/8	- 1/8	1/4

However, van Daveer's data (1975) were drawn from studies of bridge deck construction where lower variability should be expected due to higher degree of controls. To avoid inaccuracies resulting from the combination of standard deviations, calculations should be based on distributions of thickness and effective depth rather than thickness and concrete cover, etc.

For variability of concrete cover of precast slabs, Eq. 3.3 was used for top as well as for bottom reinforcement because of the different forming and placing practices.

3.2.4 Summary and recommended distributions of slab dimensions

Dimensional variations in thickness, effective depths and the concrete cover of top and bottom reinforcement of in-situ as well as precast slabs were studied. Based on available information, Gaussian distributions are recommended to represent the probability models of slab dimensions with means and standard deviations as shown in Table 3.3.

3.3 BEAM DIMENSIONS

3.3.1 Beam width

Results from various investigators (van den Berg, 1971; Hernandez and Martinez, 1974; Connolly, 1975; Jacobson and Widmark, 1970; AASHO, 1962) for widths of in-situ and precast beam stems as well as precast beam flanges are shown in Table 3.4. The weighted means and standard deviations of all data are also shown in the table. Contrary to what is expected, the data on width of stems of precast beams indicate a higher dispersion than the comparative data for in-situ beams. This is possibly due to a higher than usual degree of controls employed at the construction sites reported by Connolly (1975) and AASHO (1962). Until further information is available, the standard deviations of the widths of in-situ beam stems should be taken at least as large as those for precast beams. Comparison of the results of precast beam ribs and flanges indicates somewhat higher dispersion as well as higher mean deviations from nominal dimensions for flange widths than for web widths of precast beams. This is consistent with the trend since the more complex formwork required for T-beams than for rectangular beams would be expected to lead to larger dispersion in dimensions of T-sections (Fiorato, 1973).

Based on Table 3.4 and the observations mentioned in the above paragraph the recommended distribution properties of rib and flange widths of beams are given in Table 3.7.

3.3.2 Overall depth of beams

Variations in the overall depth of beams affect the effective depth of reinforcement, and thus influence the strength capacity. Results from studies of beam depths of in-situ and precast concrete by various investigators (AASHO, 1962; Jacobson and Widmark, 1970; van den Berg, 1971; Johansson and Warris, 1968; Connolly, 1975) are shown in Table 3.5. Weighted mean deviations from nominal dimensions and standard deviations are also shown in the table. Data for in-situ concrete joists poured in steel forms are not included in the weighted mean and standard deviation of in-situ beams, since the metal forms result in better controls for joist construction than for ordinary in-situ beam-and-column structures. Based on the weighted values the recommended distribution properties of the overall depth of beams are given in Table 3.7.

3.3.3 Concrete covering for beam reinforcement

Dispersions of concrete covering of reinforcing bars in concrete elements are important due to their substantial influence on the effective depth of reinforcing bars. The variability of the concrete covering is affected by the same factors as the variability of the effective depth, although the degree of influence may not be the same. In many cases the variability of effective depth and of concrete covering may be regarded as mutually dependent; if the concrete cover increases the effective depth decreases provided that the total thickness remains unchanged.

Available data (Hernandez and Martinez, 1974; Connolly, 1975) on concrete covering of top and bottom reinforcement of in-situ beams and joists are shown in Table 3.6. Weighted mean values and standard deviations of the available data are also given in the table. Data reported by Hernandez and Martinez (1974) for in-situ beams were not included in the weighted values. It was felt that this data did not conform to the general trend due to very high mean value and very low standard deviation it exhibited. This data was obtained from buildings built in Mexico and may reflect different construction practices.

As expected, the top reinforcement of in-situ beams exhibited higher mean and standard deviation than did the bottom steel. In-situ joists poured in steel forms showed much lower dispersion than the in-situ beams. No information was available for the concrete cover in precast beams, but its distribution properties can be assumed to be the same as the properties of in-situ joists poured in steel forms. Based on this assumption and weighted values in Table 3.6, recommended distribution properties of concrete covering of in-situ and precast beams are given in Table 3.7.

3.3.4 Effective depth of beam reinforcement

No data was available for effective depths of beam reinforcement. Therefore, distribution properties of depth of top and bottom bars for in-situ as well as precast beams were calculated from the recommended distributions of beam thickness and concrete cover. Variability of bar diameters were considered negligible for this purpose. Recommended distributions are shown in Table 3.7.

The position of steel in reinforced concrete beams are affected by the same factors and during the same phases of construction as the position of reinforcement in slabs. This is particularly true for top bars. However, the position of beam reinforcement is less severely affected than the slab steel due to increased rigidity of the steel cage and the formwork for beams as can be seen by comparing Tables 3.3 and 3.7.

Depth of reinforcement in concrete elements is not only a very important dimension from strength point of view, it also represents one of the most unpredictable dimensions. Unfortunately, no data is available on direct measurements of effective depths of beam reinforcement. Until further information is available it seems reasonable to use distribution properties of effective depth established from other related dimensions such as those given in Table 3.7.

3.3.5 Stirrup spacing

There is absolutely no data available for variations of stirrup spacing in the literature searched. Ellingwood and Ang (1972) have suggested that the uncertainties in stirrup spacing are about the same as those in the effective depth. This seems logical, since the uncertainties in stirrup spacing are mostly caused during the fabrication of steel cage.

Current construction practices of cage fabrication in North America require that the specified number of stirrups should be placed around the longitudinal bars before tying the stirrups at proper spacings. Therefore, it seems reasonable to assume that the average spacing of all stirrups in a beam will not deviate significantly from the nominal spacing provided that the mean length of the beam is equal to the specified length. Even if beam length deviates slightly from the nominal dimension, the influence on average stirrup spacing will be negligible. Consequently, mean deviation from nominal for average stirrup spacing will be assumed to be zero. The standard deviation of stirrup spacing will arbitrarily be taken as the average of standard deviations of effective depths of top and bottom steel in beams. The probability distribution of the stirrup spacing will be assumed to be normal.

Based on the above discussion the recommended distribution properties of stirrup spacing are given in Table 3.7.

3.3.6 Beam spacing

Dispersions in lateral spacing of secondary beams framing into main girders influence the strength capacity of main girders and, therefore, should be incorporated, wherever warranted, in study of under-strength of reinforced concrete beams. Since no information is available on direct measurements of the beam spacing in the literature searched, it is assumed that the beam spacing will follow the same probability distribution as does the spacing of walls and columns.

Fiorato (1973) has reported following summary of data on deviations from specified location of prefabricated walls and columns:

Nominal spacing		= 130 - 300	in.
Mean deviation:	minimum	= 0	in.
	maximum	= + 0.47	in.
Standard deviation:	minimum	= 0.146	in.
	maximum	= 0.512	in.

It should be realized that the summary shown above is based on a very limited amount of field data and should not be considered conclusive. However, it can serve as a guide until more information is available.

Birkeland and Westhoff (1971) have reported measurements of spacing between column lines of a hull-core type high rise office building representing high quality of construction. The hull was comprised of cast-in-place columns and spandrel beams and the floor area was framed with precast prestressed single tees spanning from the hull to the core. The reported deviations of the spacing between column lines as well as the deviations of individual columns from their lines were usually in the order of ±1 in., but the extreme deviations reached to ±2 in. Assuming a normal distribution with mean deviation equal to zero and the usual rule of "three standard deviations" for extreme values, the standard deviation of column spacing is estimated to be 2/3 in. This means that about 68% of the deviations will lie within the range of ±2/3 in., and about 86% within the range ±1 in.

From the data presented above it seems reasonable to assume that the probability model of spacing of in-situ beams can be represented by a normal distribution with mean deviation from nominal value to be zero and standard deviation 0.67 in. The standard deviation of spacing of precast beams should be assumed to be average of minimum and maximum values of standard deviation of the data on precast walls and columns reported by Fiorato (1973). The standard deviation thus calculated is 0.33 in., or 50% of the standard deviation of in-situ beam spacings. The mean deviation for precast beams should be taken as zero. Recommended properties of distributions are shown in Table 3.7.

3.3.7 Summary and recommended distributions of beam dimensions

Recommended probability models of dimensional uncertainties of cast-in-place and precast beams are shown in Table 3.7. It is emphasized that this table is based on the very limited information available and as such should be considered preliminary. As further information becomes available the recommended values will probably change. For the sake of simplicity a Gaussian distribution is recommended for all dimensional variations.

3.4 COLUMN DIMENSIONS

3.4.1 Cross section dimensions

Results from various studies of the variation of the dimensions of column cross-sections are shown in Table 3.8(a). With the exception of van den Berg's (1971) study of circular precast columns all data shown in the table pertain to rectangular cross section. Weighted mean and standard deviations of all the data on rectangular columns are also shown in Table 3.8(a). It should be noted that the measurements reported by Tso and Zelman (1970) for in-situ rectangular columns were made to the nearest 1/4 inch. Inaccuracies due to this are largely offset by the large number of measurements made.

No data are available for dispersion of cross-sectional dimensions of in-situ circular columns. However, a comparison of precast and in-situ rectangular columns indicates that the dispersion of precast columns is about 50% of the dispersion of in-situ columns. Therefore, it seems reasonable to assume similar relationships between circular precast and in-situ columns.

Based on the above observations and Table 3.8(a), the recommended distribution properties of column cross sectional dimensions are presented in Table 3.8(b). Normal probability distributions are recommended mainly because of their simplicity.

3.4.2 Reinforcing steel placement

Errors in placement of reinforcing steel of columns are very important from strength point of view. Redkop (1971) has reported data on measurement of steel placement errors in columns. His measurements were taken on 14 in-situ concrete buildings of various size and use in the Toronto-Hamilton area.

Redkop's (1971) data on concrete cover of exterior steel, described in Fig. 3.1, shows that the actual cover on the average is 0.32 in. larger than the specified cover, with the standard deviation 0.166 in. Similar results were reported by Hernandez and Martinez (1974) who found the mean deviation from nominal +0.47 inch, and the standard deviation 0.130 inch.

Based on Redkop's (1971) measurements Grant (1976) has suggested that the error in placement of the interior steel can be described by a linear regression equation:

$$\overline{C}_a = C_{sp} + 0.04$$

$$\sigma_{ca} = 0.2035 + 0.329h \qquad \text{(Eq. 3.5)}$$

The placement of the steel in the exterior layers can be described with the regression equation:

$$\overline{C}_a = C_{sp} + 0.250 + 0.0039h$$

$$\sigma_{ca} = 0.166 \qquad \text{(Eq. 3.6)}$$

where $\overline{C}_a$ = Average actual cover or distance from face of column, in.,

C_{sp} = Specified cover or distance from face of column, in.,

σ_{ca} = Standard deviation, in.,

h = Column dimension perpendicular to the neutral axis, in.

For variability of interior as well as exterior steel Grant (1976) has suggested a normal distribution.

3.4.3 Summary and recommended distributions of column dimensions

Recommended probability distributions of cross sectional dimensions of precast as well as in-situ columns are given in Table 3.8. Probability models of reinforcing bar placements in in-situ columns may be represented by Gaussian curves, with mean values and standard deviations calculated from Eq. 3.5 or 3.6. For probability models of steel placement in precast columns the mean values of deviations from nominal dimensions and standard deviations obtained from Eqs. 3.5 and 3.6 should be reduced by 50%.

3.5 EFFECT OF DISCRETE BAR SIZES

Because the reinforcement in a beam or column must be some combination of whole bars, the area of steel actually provided in a reinforced concrete element may differ from that found to be necessary in the calculations. A comparison of provided and required areas of reinforcement for a column is shown in Fig. 3.2. This effect of discrete bar sizes on choice of reinforcement was studied in terms of the ratio furnished area/calculated area (A_f/A_c) of steel. This study was conducted for longitudinal and transverse steel of beams and vertical reinforcement of columns. Four different sizes of tied square columns and three different sizes of rectangular beams were studied with 4000 psi concrete, Grade 40 stirrups, and Grade 60 longitudinal and vertical reinforcement. These strengths were chosen because of their frequent use in design offices in North America.

For each beam or column a number of practical steel areas were chosen from actual bar sizes that met all ACI Code (1971) limitations on spacing, size and number of bars as well as amount of reinforcement. It was assumed that when the calculated area of reinforcement was less than 90% of the lowest or more than 105% of the highest practical value for the cross-section under consideration the size of the concrete cross section would be revised. This limitation is necessary in order to avoid unrealistically large differences between calculated and furnished areas of steel near minimum and maximum practical values.

Assuming a uniform distribution (with a range as described above) for calculated steel area, a 500-point population of the ratio A_f/A_c was simulated for each beam or column, using a random number generator. The distribution properties thus obtained are shown in Table 3.9. The histograms are plotted in Figs. 3.3, 3.4 and 3.5 for flexural tension steel in beams, beam stirrups and vertical reinforcement in columns, respectively. The histograms shown in these figures are highly skewed due to the fact that a maximum 5% under-design of the reinforcement area was allowed, while no restriction was made for over-design of steel. Modified log-normal distributions with a modification constant (Mean Value - 0.10) are also shown with the histograms shown in Figs. 3.3 through 3.5. The probability density functions of these distributions can be calculated from the equations:

$$PDF = \frac{0.43429}{\sqrt{2\pi}\,(A_f/A_c - c)\sigma_{1g}} \cdot \exp\left[-1/2\left\{\frac{\log_{10}(A_f/A_c - c)-\overline{X}_{1g}}{\sigma_{1g}}\right\}^2\right] \qquad \text{(Eq. 3.7)}$$

$$\overline{X}_{1g} = \log_{10}\left[(\overline{X} - c)\left\{1 + \left(\frac{\sigma_X}{\overline{X} - c}\right)^2\right\}^{-1/2}\right] \qquad \text{(Eq. 3.8)}$$

$$\sigma_{1g} = \sqrt{0.43429 \log_{10}\left\{1 + \left(\frac{\sigma_X}{\overline{X} - c}\right)^2\right\}} \qquad \text{(Eq. 3.9)}$$

where c is modification constant and is equal to $\overline{X} - 0.10$,

and $\overline{X}$, σ_X are mean value and standard deviation of the ratio A_f/A_c.

To investigate the influence of size of cross section on distribution of the ratio A_f/A_c the cumulative frequency functions for different sizes of concrete elements are compared in Fig. 3.6. Even though the mean strength and coefficient of variation are affected due to the size of cross section (Table 3.9), the effect of size on distribution of the ratio A_f/A_c seems to be insignificant in the lower 10% probability region for all three types of steel investigated. An examination of the distribution properties shown in Table 3.9 clearly indicates that as the mean value reduces so does the dispersion of the ratio A_f/A_c. This reduces the differences between lower probability regions of the distributions obtained for different sizes of concrete elements. In the study of understrength factors it is the lower tail of the distribution that is most important. For this reason it is not unsafe to neglect the influence of size on distribution of the ratio A_f/A_c.

3.5.1 Summary and recommended distribution of effects of discrete bar sizes

Based on above observations it is suggested that the probability distribution of the ratio A_f/A_c similated for a 36 in. x 36 in. column and shown in Figs. 3.5 and 3.6 should be used to represent the effects of selection of discrete bar sizes on flexural tension steel in beams, beam stirrups and vertical reinforcement in columns for all sizes of member cross section. This distribution will give same effects in the low probability region as other simulated distributions, but the effects in the high probability region will be on the conservative side. If a continuous curve is desired for the ratio A_f/A_c it may be approximated by a modified log-normal distribution with mean value 1.01, coefficient of variation 0.04 and modification constant 0.91. The probability density function for this curve can be calculated from Eqs. 3.7 to 3.9.

REFERENCES

AASHO, 1962: THE AASHO ROAD TEST - REPORT 2 - MATERIALS AND CONSTRUCTION, Highway Research Board of the NAS-NRC Division of Engineering and Industrial Research, Special Report 61B, Publication No. 951, Washington, D.C.

ACI Committee 214, 1965: RECOMMENDED PRACTICE FOR EVALUATION OF COMPRESSION TEST RESULTS OF CONCRETE, American Concrete Institute, Detroit.

ACI Committee 214, 1973: REALISM IN THE APPLICATION OF STANDARD 214-65, American Concrete Institute, Publication SP-37, Detroit.

ACI Committee 318, 1971: BUILDING CODE REQUIREMENTS FOR REINFORCED CONCRETE (ACI 318-71), ACI Standard 318-71, American Concrete Institute, Detroit.

Allen, D.E., 1970: PROBABILISTIC STUDY OF REINFORCED CONCRETE IN BENDING, Technical Paper NRC 11139, National Research Council of Canada, Ottawa, January.

Allen, D.E., 1972: STATISTICAL STUDY OF THE MECHANICAL PROPERTIES OF REINFORCING BARS, Building Research Note No. 85, Division of Building Research, National Research Council, Ottawa, April.

Alpsten, G.A., 1972: VARIATIONS IN MECHANICAL AND CROSS-SECTIONAL PROPERTIES OF STEEL, Proceedings, International Conference on Planning and Design of Tall Buildings, v. 1b, Tall Building Criteria and Loading, Lehigh University, Bethlehem, August 21-26, pp. 775-805.

BAKER, M.J., 1970: THE EVALUATION OF SAFETY FACTORS IN STRUCTURES: VARIATION IN THE STRENGTH OF STRUCTURAL MATERIALS AND THEIR EFFECT ON STRUCTURAL SAFETY, Report, Department of Civil Engineering, Imperial College, London, July.

Bannister, J.L., 1968: STEEL REINFORCEMENT AND TENDONS FOR STRUCTURAL CONCRETE, PART 1: STEEL FOR REINFORCED CONCRETE, Concrete, v. 2, n. 7, July, pp. 295-306.

Birkeland, P.W. and Westhoff, L.J., 1971: DIMENSIONAL TOLERANCES IN A TALL CONCRETE BUILDING, American Concrete Institute, Proceedings, v. 68, n. 8, August, pp. 600-607.

Bishop, D., 1963: DIMENSIONAL ACCURACY IN BUILDINGS AND BUILDING COMPONENTS - FACTORY PRODUCED PRECAST CONCRETE FLOOR PANELS, Work Report 8:1963, Large Concrete Elements, The Swedish Council for Building Research, Stockholm, pp. 329-343.

Bloem, D.L., 1955: STUDIES OF UNIFORMITY OF COMPRESSION STRENGTH TESTS OF READY MIXED CONCRETE, American Society for Testing and Materials, Bulletin No. 206, May, pp. 65-70.

Bloem, D.L., 1965: CONCRETE STRENGTH MEASUREMENT - CORES VERSUS CYLINDERS, American Society for Testing and Materials, Proceedings, v. 65 pp. 668-686.

Bloem, D.L., 1968: CONCRETE STRENGTH IN STRUCTURES, American Concrete Institute, Proceedings, v. 65, n. 3, March, pp. 176-187.

Bolotin, V.V., 1969: STATISTICAL METHODS IN STRUCTURAL MECHANICS, Translated by Samuel Aroni, Holden-Day Inc., San Francisco.

Bureau of Public Roads, 1965: THE STATISTICAL APPROACH TO QUALITY CONTROL IN HIGHWAY CONSTRUCTION - RESEARCH GUIDES, April.

Campbell, R.H. and Tobin, R.E., 1967: CORE AND CYLINDER STRENGTHS OF NATURAL AND LIGHTWEIGHT CONCRETE, American Concrete Institute, Proceedings, v. 64, n. 4, April, pp. 190-195.

Connolly, J.P., 1975: CONSTRUCTION TOLERANCES IN REINFORCED CONCRETE, Research Report Submitted to ACI Committee 117: Tolerances, Auburn University, Auburn, Alabama.

Costello, J.F. and Chu, K., 1969: FAILURE PROBABILITIES OF REINFORCED CONCRETE BEAMS, American Society of Civil Engineers, Journal of the Structural Division, v. 95, n. ST 10, October, pp. 2281-2304.

Cummings, A.E., 1955: STRENGTH VARIATIONS IN READY MIXED CONCRETE, American Concrete Institute, Proceedings, v. 51, n. 4, April, pp. 765-772.

Elderton, W.P. and Johnson, N.L., 1969: SYSTEMS OF FREQUENCY CURVES, Cambridge University Press, England.

Ellingwood, B.R. and Ang, A.H.-S., 1972: A PROBABILISTIC STUDY OF SAFETY CRITERIA FOR DESIGN, Structural Research Series No. 387, Department of Civil Engineering, University of Illinois, Urbana, Illinois, June.

Erntroy, H.C., 1960: THE VARIATION OF WORKS TEST CUBES, Research Report 10, Cement and Concrete Association, London, November.

Fiorato, A.E., 1973: GEOMETRIC IMPERFECTIONS IN CONCRETE STRUCTURES, National Swedish Building Research Document Number 5, Chalmers University of Technology, Gothenberg, Sweden.

Fruedenthal, A.M., 1956: SAFETY AND THE PROBABILITY OF STRUCTURAL FAILURE, American Society of Civil Engineers, Transactions, v. 121, pp. 1337-1375.

Gamble, W.L., 1973: SOME OBSERVATIONS OF THE STRENGTH OF LARGE REINFORCING BARS, Proceedings, American Concrete Institute, v. 70, n. 1, January, pp. 31-35.

Grant, L.H., 1976: A MONTE CARLO STUDY OF THE STRENGTH VARIABILITY OF RECTANGULAR TIED REINFORCED CONCRETE COLUMNS, M.Sc. Thesis, Department of Civil Engineering, The University of Alberta, Edmonton, Alberta.

Hanson, J.A., 1958: SHEAR STRENGTH OF LIGHTWEIGHT REINFORCED CONCRETE BEAMS, Proceedings, American Concrete Institute, v. 55, n. 3, September.

Hernandez, A.A. and Martinez, F.V., 1974: VARIACIONES DE LAS DEMENSIONES Y DE LA POSICION DEL ACERO DE REFUERZO EN LAS ESTRUCTURES DE CONCRETO, Universidad Nacional Autonoma de Mexico, Facultad de Ingenieria, Mexico.

Hognestad, E., 1951: A STUDY OF COMBINED BENDING AND AXIAL LOAD IN REINFORCED CONCRETE MEMBERS, Bulletin No. 399, Engineering Experiment Station, University of Illinois, Urbana, November.

Jacobson and Widmark, and Skånska Cementgjuteriet, 1970: MÅTTAVVIKELSER HOS BETONGELEMENT (DIMENSIONAL VARIATION OF CONCRETE ELEMENTS), February, Stockholm.

Johansson, A. and Warris, B., 1968: AVVIKELSER I ARMERINGSSTÄNGERNAS LÄGE (DEVIATIONS IN POSITION OF BARS), Report 30/68, Statens institute för byggnadsforskning, Stockholm.

Johnson, A.I., 1953: STRENGTH, SAFETY AND ECONOMICAL DIMENSIONS OF STRUCTURES, Report Nr. 22, Statens Kommittee För Byggnadsforskning, Stockholm.

Johnson, A.I., 1956: THE DETERMINATION OF THE DESIGN FACTOR FOR REINFORCED CONCRETE STRUCTURES, Symposium on the Strength of Concrete Structures, Institution of Civil Engineers - Cement and Concrete Associaton, London, May.

Jones, P.G. and Richart, F.E., 1936: THE EFFECT OF TESTING SPEED ON STRENGTH AND ELASTIC PROPERTIES OF CONCRETE, Proceedings, ASTM, v. 36, Part II, pp. 380-391.

Julian, O.G., 1955: DISCUSSION OF REFERENCE BY CUMMINGS, A.E., 1955, Proceedings, American Concrete Institute, v. 51, n. 12, December, pp. 772-4 to 772-8.

Julian, O.G., 1957: SYNOPSIS OF FIRST PROGRESS REPORT OF COMMITTEE ON FACTORS OF SAFETY, American Society of Civil Engineers, Journal of the Structural Division, v. 83, n. ST 4, July, pp. 1316-1 to 1316-22.

Kawasaki, N. and Rahulan, G., 1974: CONSTRUCTION OF DBS BUILDINGS, SINGAPORE, Proceedings, Conference on Tall Buildings, Kuala Lumpur, Malaysia, December, pp. 6-37 to 6-43.

Keenan, W.A. and Feldman, A., 1960: THE YIELD STRENGTH OF INTERMEDIATE REINFORCING BARS UNDER RAPID LOADING, University of Illinois, Civil Engineering Studies Structural Research Series, No. 197, Urbana, Illinois, March.

Klingberg, L., 1970: MÅTTNOGGRANNHETS-STUDIER PÅ PELAR-BALKSTOMME (STUDIES OF THE DIMENSIONAL ACCURACY OF A COLUMN AND BEAM FRAMEWORK), Report R38:1970, Statens institute för byggnadsforskning, Stockholm.

Komloš, K., 1970: COMMENTS ON THE LONG-TERM TENSILE STRENGTH OF PLAIN CONCRETE, Mag. of Concrete REsearch, v. 22, n. 73, December, pp. 232-238.

Lampert, P. Wegmuller, A. and Thurlman, B., 1967: EINFLUSS DER DEHNGESCHWINDIGKEIT AUF FESTIGKEIT-SWERTE VON ARMIERUNGSSTAHLEN, Institut fur Banstatik ETH Zurich, Bericht Nr. 10, April.

Lauer, L.R. and Rigby, R.J., 1966: CONCRETE PROPORTIONING AND CONTROL FOR THE "SKYLON", Proceedings, American Concrete Institute, v. 63, n. 9, September, pp. 897-909.

MacGregor, J.G., 1975: SAFETY AND LIMIT STATES DESIGN FOR REINFORCED CONCRETE, The 1975-76 Visiting Lectureship, Canadian Society of Civil Engineers, University of Alberta, Edmonton, Alberta. (also published in Canadian Journal of Civil Engineering, v. 3, n. 4, December 1976.

Malhotra, V.M., 1962: MATURITY OF CONCRETE AND ITS EFFECT ON STANDARD DEVIATION AND COEFFICIENT OF VARIATION, Proceedings, American Concrete Institute, v. 59, n. 5, May, pp. 729-730.

Malhotra, V.M., 1969: RELATIONS BETWEEN SPLITTING-TENSILE AND COMPRESSIVE STRENGTH OF CONCRETE, Engineering Journal, v. 52, n. 5, May, pp. 11-17.

Murdock, L.J., 1953: THE CONTROL OF CONCRETE QUALITY, Proceedings, Institution of Civil Engineers, Part 1, v. 2, n. 4, July.

Narayanaswamy, V.P. and Gadh, A.D., 1972: CHARACTERISTIC STRENGTH OF REINFORCING STEEL, Journal of the Institute of Engineers (India), Civil Engineering Divions, v. 53, Part CI2, November, pp. 85-88.

Neville, A.M., 1973: PROPERTIES OF CONCRETE, Second Ed., John Wiley and Sons, New York.

Newlon, H.H., 1966: VARIABILITY OF PORTLAND CEMENT CONCRETE, National Conference on Statistical Quality Control Methodology in Highway and Airfield Construction, Proceedings, University of Virginia, Charlottesville, Virginia, pp. 259-289.

Orr, D.M.F., 1970: DEFORMATION AND STRENGTH OF CONCRETE IN TENSION, Institution of Engineers, Australia: Civil Engineering Transactions, v. CE 12, n. 1, April, pp. 26-29.

Petersons, N., 1968: SHOULD STANDARD CUBE TEST SPECIMENS BE REPLACED BY TEST SPECIMENS TAKEN FROM STRUCTURES, Materiaux et Constructions, RILEM, Paris, v. 1, n. 5, pp. 425-435.

Ramesh, C.K. and Chopra, S.K., 1960: DETERMINATION OF TENSILE STRENGTH OF CONCRETE AND MORTAR BY THE SPLIT TEST, Indian Concrete Journal, v. 34, n. 9, September, pp. 354-357.

Rao, N.R.N., Lohrmann, M. and Tall, L., 1966: THE EFFECT OF STRAIN ON THE YIELD STRESS OF STRUCTURAL STEELS, American Society for Testing and Materials, v. 1. n. 1, March, pp. 241-262.

Redekop, D., 1971: A STUDY OF REINFORCED CONCRETE COLUMNS IN EXISTING BUILDINGS, Master of Engineering Thesis, McMaster University, August.

Richart, F.E. and Brown, R.L., 1934: AN INVESTIGATION OF REINFORCED CONCRETE COLUMNS, University of Illinois Engineering Experiment Station, Bulletin No. 267, June.

Richart, F.E., Dolan, T.J. and Olson, T.A., 1938: AN INVESTIGATION OF RIGID FRAME BRIDGES - PART I - TESTS OF REINFORCED CONCRETE KNEE FRAMES AND BAKELITE MODELS, University of Illinois Engineering Experiment Station, Bulletin No. 23, November.

Richart, F.E. and Jensen, V.P., 1931: TESTS OF PLAIN AND REINFORCED CONCRETE MADE WITH HAYDITE AGGREGATES, University of Illinois Engineering Experiment Station, Bulletin No. 237, October.

Riley, O. and Cooper, S.B., 1971: CONCRETE CONTROL ON A MAJOR PROJECT, American Concrete Institute, Proceedings, v. 68, n. 2, February, pp. 107-114.

Roberts, N.P., 1967: THE CHARACTERISTIC STRENGTH OF STEEL FOR REINFORCING AND PRESTRESSING CONCRETE, Concrete, v. 1, n. 8, August, pp. 273-275.

Robertson, A., 1931: THE USE OF SMALL MACHINED SPECIMENS IN TESTING OF MILD STEEL, First Report of Steel Structures Research Committee, Department of Scientifc and Industrial Research, HMSO, London, pp. 194-210.

Robles, F., 1972: STRENGTH FACTORS: MATERIAL AND GEOMETRICAL ASPECTS, ASCE-IABSE International Conference on Tall Buildings, Lehigh University, Proceedings, v. III, pp. 907-921.

Rüsch, H., 1964: ZUR STATISTISCHEN QUALITATSKONTROLLE DES BETONS (ON THE STATISTICAL QUALITY CONTROL OF CONCRETE), Materialpruf, November.

Rüsch, H.H., 1975: DIE ABLEITUNG DER CHARAKTERISTIISCHEN WERTE DER BETONZUGFESTIGKEIT (DERIVATION OF CHARACTERISTIC VALUES OF THE TENSILE STRENGTH OF CONCRETE), Beton Herstellung verwend, v. 25, n. 2, February, pp. 55-58.

Shalon, R. and Reintz, R.C., 1955: INTERPRETATION OF STRENGTHS DISTRIBUTION AS A FACTOR IN QUALITY CONTROL OF CONCRETE, RILEM Symposium on the Observation of Structures, V. 2, Lisbon Laboratorio Naciano de Engenharia Civil, pp. 100-116.

Shideler, J.J., 1957: LIGHTWEIGHT-AGGREGATE CONCRETE FOR STRUCTURAL USE, Proceedings, American Concrete Institute, v. 29, n. 4, October.

Soroka, I., 1968: AN APPLICATION OF STATISTICAL PROCEDURES TO QUALITY CONTROL OF CONCRETE, Materiaux et Constructions, RILEM, Paris, v. 1, n. 5, pp. 437-441.

Tichý, M. and Vorlíček, M., 1972: STATISTICAL THEORY OF CONCRETE STRUCTURES WITH SPECIAL REFERENCE TO ULTIMATE DESIGN, Czeckoslovak Academy of Sciences, Irish University Press, Shannon; Academia, Prague.

Tso, W.K. and Zelman, I.M., 1970: CONCRETE STRENGTH VARIATION IN ACTUAL STRUCTURES, American Concrete Institute, Proceedings, v. 67, n. 12, December, pp. 981-988.

van Daveer, J.R., 1975: TECHNIQUES FOR EVALUATING REINFORCED CONCRETE BRIDGE DECKS, American Concrete Institute, Proceedings, v. 72, n. 12, December, pp. 697-704.

van den Berg, J., 1971: MÅTTNOGGRANNHETEN VID BYGGNADSSTYRELSENS NYA BYGGSYSTEM (THE DIMENSIONAL ACCURACY OF THE BUILDING COUNCIL'S NEW BUILDING SYSTEMS), Report R18, Statens institut för byggnadsforskning, Stockholm.

Wagner, W.K., 1955: DISCUSSION OF REFERENCE BY CUMMINGS, A.E., 1955, American Concrete Institute, Proceedings, v. 51, n. 12, December, pp. 772-14 to 772-16.

Wagner, W.K., 1963: EFFECT OF SAMPLING AND JOB CURING PROCEDURES ON COMPRESSIVE STRENGTH OF CONCRETE, Materials and Research Standards, August.

Wiss, Janney, Elstner and Associates, Inc., 1976: BENDING AND TENSILE TEST OF REINFORCING BARS, Draft Report, Submitted to Technical Committee, Associated Reinforcing Bar Producers.

Wright, P.J.F., 1955: COMMENTS ON AN INDIRECT TENSILE TEST ON CONCRETE CYLINDERS, Mag. of Concrete Research, v. 7, n. 20, July, pp. 87-96.

Zsutty, T.C., 1968: BEAM SHEAR STRENGTH PREDICTION BY ANALYSIS OF EXISTING DATA, Proceedings, American Concrete Institute, v. 65, n. 11, November, pp. 943-951.

NOTATION

A	= ratio of observed and calculated tensile strength.
A_f/A_c	= ratio of the furnished area/calculated area of steel.
A_m/A_n	= ratio of the measured area/nominal area of a reinforcing bar.
$(\overline{\frac{A_m}{A_n}})$	= mean value of the ratio A_m/A_n.
A_s	= total area of vertical steel in a column.
A_v	= area of both legs of stirrups.
a	= a constant for relationship between compressive and tensile strengths of concrete.
b	= width of beam stem.
$\overline{C}_a$	= mean value of actual cover or distance from column face.
CFF	= cumulative frequency function.
cov	= coefficient of variation.
C_{sp}	= specified cover or distance from face of column.
c	= modification constant for modified log-normal distribution.
c_b	= concrete cover of bottom reinforcement.
c_t	= concrete cover of top reinforcement.
D	= dead load effect.
d_b	= effective depth of bottom reinforcement.
d_t	= effective depth of top reinforcement.
df_{us}	= $f_u - f_{us}$.
$\overline{df}_{us}$	= mean value of df_{us}.
df_{ys}	= $f_y - f_{ys}$.
$\overline{df}_{ys}$	= mean value of df_{ys}.
E_{cr}	= modulus of elasticity of concrete at required rate of loading.
$\overline{E}_{cR}$	= mean value of E_{cR}.
E_c	= modulus of elasticity of concrete under laboratory loading conditions.

$\overline{E}_c$ = mean value of E_c.

E_{ci} = initial tangent modulus of concrete.

$\overline{E}_{ci}$ = mean value of E_{ci}.

$E_{cs,0.3f_c}$ = secant modulus of concrete at 30% of maximum stress.

$\overline{E}_{cs,0.925f_c}$ = secant modulus of concrete at 92.5% of maximum stress.

f_c = compressive strength of concrete.

$\overline{f}_c$ = mean compressive strength of concrete.

f'_c = design compressive strength of concrete.

f_{ccore} = compressive strength of cores taken from structure.

f_{ccyl} = compressive strength of standard cylinders.

f_{cR} = compressive strength of concrete at a given rate of loading R psi/sec.

$\overline{f}_{cR}$ = mean compressive strength of in-situ concrete at a given rate of loading R psi/sec.

$\overline{f}_{cstructure}$ = mean compressive strength of concrete in structure.

f_{cstrR} = compressive strength of in-situ concrete at a given rate of loading R psi/sec.

$\overline{f}_{cstrR}$ = mean value of f_{cstrR}.

$\overline{f}_{cstr35}$ = mean compressive strength of in-situ concrete at rate of loading equal to 35 psi/sec.

f_{c1} = compressive strength of concrete at rate of loading of 1 psi/sec.

f_{c35} = compressive strength of concrete at loading rate of 35 psi/sec.

$\overline{f}_{c35}$ = mean compressive strength of in-situ concrete for loading rate similar to that of a cylinder test.

f_r = modulus of rupture of concrete.

$\overline{f}_r$ = mean value of modulus of rupture of concrete.

f_{sp} = splitting tensile strength of concrete.

$\overline{f}_{sp}$ = mean splitting tensile strength of concrete.

f_t = tensile strength of concrete.

f_{tstrR} = tensile strength of in-situ concrete at loading rate of R psi/sec.

$\bar{f}_{tstrR}$ = mean value of f_{tstrR}.

f_u = ultimate strength of reinforcing steel from mill tests.

$\bar{f}_u$ = mean value of f_u.

f_{us} = static ultimate strength of reinforcing steel based on measured area.

f_y = yield strength of reinforcing steel from mill tests.

$\bar{f}_y$ = mean value of f_y.

f_{ys} = static yield strength of reinforcing steel based on measured area.

h = column dimension perpendicular to neutral axis.

K = a constant for rate of loading effect on compressive strength of concrete.

L = live load effect.

N = sample size.

n = a constant for relationship between compressive and tensile strengths of concrete.

PDF = probability density function.

R = rate of loading in psi/sec corresponding to f_c.

R = strength.

r = regression coefficient.

r_{creal} = random variable relating real cylinder strength with design strength, f_{creal}/f'_c.

rfy(max) = maximum value of the ratio A_v f_y/bs.

rfy(min) = minimum value of the ratio A_v f_y/bs.

$r_{in\text{-}situ}$ = random variable relating in-situ strength with real cylinder strength, $f_{cstructure}/f_{creal}$.

r_R = random variable relating rate of loading effect, f_{cR}/f_{c35}.

r_{vol} = random variable relating volume effect.

s = stirrup spacing.

t = loading duration in seconds.

t = slab thickness.

V_A = coefficient of variation of observed tensile strength with respect to calculated strength of concrete after accounting for in-test errors.

V_{A_m/A_n} = coefficient of variation of ratio A_m/A_n.

V_{Atest} = coefficient of variation of the ratio A determined from experimental data for f_c and f_t.

V_c = coefficient of variation of compressive strength of in-situ concrete.

V_{ccyl} = coefficient of variation of compressive strength of concrete cylinders.

V_{creal} = coefficient of variation of real cylinder strength.

V_{cstrR} = coefficient of variation associated with $\overline{f}_{cstrR}$.

V_{cstr35} = coefficient of variation associated with $\overline{f}_{cstr35}$.

V_{E_c} = coefficient of variation corresponding to $\overline{E}_c$.

$V_{E_{cR}}$ = coefficient of variation corresponding to $\overline{E}_{cR}$.

$V_{E_{ci}}$ = coefficient of variation of initial tangent modulus of concrete.

$V_{E_{cs}}$ = coefficient of variation of secant modulus of concrete.

V_{f_r} = coefficient of variation of modulus of rupture of concrete.

$V_{f_{sp}}$ = coefficient of variation of splitting tensile strength of concrete.

$V_{in-situ}$ = coefficient of variation of compressive strength of in-situ concrete relative to cylinder strength.

$V_{in-test}$ = coefficient of variation representing in-test variations.

V_R = coefficient of variation for rate of loading effect.

V_{tstrR} = coefficient of variation associated with $\overline{f}_{tstrR}$.

V_x = coefficient of variation corresponding to $\overline{X}$.

v = volume of a specimen.

v_o = volume of a standard specimen.

$\overline{X}$ = mean value or mean strength of a specimen.

$\overline{X}_{lg}$ = mean value of $\log_{10}(A_f/A_c - c)$.

$\overline{X}_o$ = mean strength of a standard specimen.

α = a constant for effect of volume on strength.

β = a constant for effect of volume on strength.

ε = strain rate in micro-in/in/sec.

ε_{ur} = ultimate strain at a required rate of loading.

λ = ratio of ultimate strain under test conditions to ultimate strain at the required rate of loading.

λ = load factor.

λ_D = dead load factor.

λ_L = live load factor.

μ = a factor for diameter effect on yield strength of reinforcing bars.

ρ_{max} = maximum longitudinal steel percentage.

ρ_{min} = minimum longitudinal steel percentage.

σ_{ca} = standard deviation corresponding to $\overline{C}_a$.

σ_{cb} = standard deviation of concrete cover of bottom reinforcement.

σ_{ct} = standard deviation of concrete cover of top reinforcement.

σ_{db} = standard deviation of effective depth of bottom reinforcement.

$\sigma_{df_{us}}$ = standard deviation of df_{us}.

$\sigma_{df_{ys}}$ = standard deviation of df_{ys}.

σ_{dt} = standard deviation of effective depth of top reinforcement.

σ_{f_u} = standard deviation of f_u.

σ_{f_y} = standard deviation of f_y.

σ_{lg} = standard deviation corresponding to $\overline{X}_{lg}$.

σ_t = standard deviation of overall depth.

σ_x = standard deviation corresponding to $\overline{X}$.

ϕ = capacity reduction factor.

Table 1.1

Concrete Strength Variability

Source	Test		Type of Distribution	Coefficient of Variation (%)
	Type	No.		
Julian (1955)	cyl.	861	Normal	10.4
Cummings (1955)	cyl.	208	Normal	9.3
Shalon (1955)	cube	---	Normal	14.2
Shalon (1955)	cube	---	Log-normal	23.6
Bloem (1955)	cyl.	1429	Normal	11.4
Bloem (1955)	cyl.	354	Normal	16.4
Wagner (1955)	cyl.	613	Normal	11.8
Erntroy (1960)	cube	4000	----	20.0
Malhotra (1962)	cyl.	68	----	13.5
Wagner (1963)	cyl.	688	Normal	12.4
Wagner (1963)	cyl.	688	Normal	15.2
BPR (1965)	cyl.	975	Normal	12.4
BPR (1965)	cyl.	200	Normal	10.9
Virginia Hwy.(Newlon,1966)	cyl.	210	Normal	7.2
Soroka (1968)	cyl.	68	Normal	15.2
Riley (1971)	cyl.	50,000	Normal	13.6
Kawasaki (1974)	cube	742	Normal	13.2

Table 1.2

In-Test Coefficients of Variation, cov, of Concrete Strength

Source	Compression		Splitting Tension		Flexural Tension		Direct Tension		Number of Batches
	Individual Batches cov%	Avge cov%	Individual Batches cov%	Avge cov%	Individual Batches cov%	Avge cov%	Individual Batches cov%	Avge cov%	
Komloš (1970)*	3.03-8.11 8x8x8 in.cubes	5.51	3.81-8.64 6x12 in.cyls	6.50	2.18-8.63 4x4x16 in.bms	5.92	2.63-8.60 4x4x8 in.prisms	5.97	15
Ramesh and Chopra(1960)**	0.49-1.72 6x6x6 in.cubes	1.05	1.19-6.85 6x12 in.cyls	3.26	1.73-8.30 4x4x20 in.bms	4.30	---	---	11-16
Both Studies	0.49-8.11	3.62	1.19-8.64	4.83	1.73-8.63	5.24	2.63-8.60	5.97	26-31

* Total 15 batches; 6 specimens per batch; 5 age groups at time of testing: 7, 28, 90, 180, 360 days; 3 batches in each age group; 3 types of concrete mixes for each age group.

** Total 16 batches. From 11 batches compression, splitting tension and flexural tension specimens were made and from remaining 5 batches specimens for only splitting tension were casted; 3 specimens per batch; 3 age groups at time of testing: 3, 7, 28 days; different number of batches in each age group; only one type of concrete mix for all batches.

Table 1.3

Results from Statistical Analysis of Available Data Using Observed Tensile Strength/Calculated Tensile Strength

Stressing Condition	Equation No.	Equation Used $f_t =$	Mean Value	Std. Deviation	Coeff. of Variation	Coeff. of Skewness*	Coeff. of Kurtosis*
Splitting Tension	1.19	$4.14\ f_c^{0.55}$	1.00726	0.12605	0.12514	-0.27960	3.15321
	1.21	$6.4\ f_c^{1/2}$	0.99562	0.12622	0.12678	-0.21493	3.11710
	1.22	$26.2\ f_c^{1/3}$	1.00051	0.14064	0.14057	0.08001	2.91130
Flexural Tension	1.26	$12.23\ f_c^{0.447}$	1.01802	0.20024	0.19669	0.22553	2.19862
	1.28	$8.0\ f_c^{1/2}$	1.00144	0.19827	0.19799	0.26070	2.20271
	1.29	$32.0\ f_c^{1/3}$	1.00232	0.19805	0.19760	0.14992	2.23715

* The Coefficients of Skewness and Kurtosis are 0 and 3.0, respectively, for a normal distribution.

Table 1.4

Comparison of Splitting Tensile Strength Obtained from Empirical Equations and Representative Tests

Compressive Strength	Splitting Tensile Strength f_{sp} (psi)							
	Representative Tests (Neville, 1973)		Canada (Malhotra, 1969)	Germany (Rüsch, 1975)	U.S.A. (ACI, 1971)	Eq. 1.19	Eq. 1.21	Eq. 1.22
f_c (psi)	Crushed Stone	Gravel	$3.67(f_c)^{.565}$	$1.43(f_c)^{2/3}$	$6.0\ (f_c)^{1/2}$	$4.14(f_c)^{0.55}$	$6.4(f_c)^{1/2}$	$26.2(f_c)^{1/3}$
2500	316	300	305	263	300	306	320	356
3000	360	338	338	297	329	338	351	378
3500	407	368	369	330	355	368	379	398
4000	445	400	398	360	379	396	405	416
4500	473	428	425	390	402	423	429	433
5000	506	462	451	418	424	448	453	448
5500	529	502	476	446	445	472	475	462
6000	563	548	500	472	465	495	496	476

Table 1.5

Comparison of Average Coefficients of Variation, cov, of Concrete Strength Under Different Tests*

Source	Compression		Splitting Tension		Flexural Tension		Direct Tension	
	cov	Specimen	cov	Specimen	cov	Specimen	cov	Specimen
Wright (1955)	3.5	4x4x4 cubes	5.0	6x12 cyl	6.0	4x4x16 bms	7.0	4x18 cyl
Efsen & Glarbo (Malhotra, 1969)	4.9	6x12 cyl	6.3	6x12 cyl	6.8	4x4x20 bms	-	
Rüsch & Vigerust (Malhotra, 1969)	3.5	6x12 cyl	6.0	6x16 cyl	4.6	4x4x20 bms	-	
Ramesh & Chopra (1960)	1.0	6x6x6 cubes	3.3	6x12 cyl	4.3	4x4x20 bms	-	
Kenis (Malhotra, 1969)	5.6	6x12 cyl	8.5	6x12 cyl	7.4	6x6x21 bms	-	
Malhotra (1969)	3.9	6x12 cyl	7.3	6x12 cyl	6.0	3 1/2x4x16 bms	-	
Orr (1970)	10.4	6x12 cyl	10.6	6x6 cyl	8.5	**	-	
Komološ (1970)	5.5	8x8x8 cubes	6.5	6x12 cyl	5.9	4x4x16 bms	6.0	4x4x8 prisms
AASHO (1962)	12.6	6x12 cyl	-		6.9	6x6x30 bms	-	
Avge. of all studies	5.7		6.7		6.3		6.5	

* All values in %age. The values reported for each author were obtained by averaging the coefficients of variation reported for several series of tests with different sample sizes representing different mix design and age in a given study. Most investigators reported coefficients of variation of laboratory test data from several batches.

** Not specified.

Table 1.6

Coefficient of Variation of Real Observed Tensile Strength of Concrete Relative to its Calculated Strength

Stressing Condition	Equation No.	Equation Used $f_t =$	V_{Atest}	V_A
Splitting	1.19	$4.14\ f_c^{0.55}$	0.12514	0.11652
Tension	1.21	$6.4\ f_c^{1/2}$	0.12678	0.11863
	1.22	$26.2\ f_c^{1/3}$	0.14057	0.13410
Flexural	1.26	$12.23\ f_c^{0.447}$	0.19669	0.19175
Tension	1.28	$8.0\ f_c^{1/2}$	0.19799	0.19287
	1.29	$32.0\ f_c^{1/3}$	0.19760	0.19305

Table 1.7

Results from Statistical Analysis of Available Data
Using Observed Modulus/Calculated Modulus of Elasticity

Type	Equation No.	Equation Used E_c =	Mean Value	Standard Deviation	Coeff. of Variation	Coeff. of Skewness*	Coeff. of Kurtosis*
Initial Tangent Modulus	1.39	$92,000\ f_c^{0.448}$	0.99926	0.07414	0.07420	-0.05648	3.51425
	1.41	$60,400\ f_c^{1/2}$	1.00000	0.07695	0.07695	-0.05648	3.72328
Secant Modulus @ $0.3f_c$	1.43	$72,100\ f_c^{0.469}$	1.00461	0.12246	0.12190	0.42413	2.72359
	1.45	$55,400\ f_c^{1/2}$	0.99951	0.12369	0.12375	0.48082	2.57026

* The coefficients of Skewness and Kurtosis are 0 and 3.0, respectively, for a normal distribution.

Table 2.1

Summary of Selected Studies on Steel Strength

Study No.	Grade	Size	Total No. of Samples	Number of Sources	Testing House	Manufacturing Country	Yield Strength (f_y) Individual bar sizes Mean (ksi)	Yield Strength (f_y) Individual bar sizes cov (%)	Yield Strength (f_y) Overall Mean (ksi)	Yield Strength (f_y) Overall cov (%)	A_m/A_n Individual bar sizes Mean	A_m/A_n Individual bar sizes cov (%)	A_m/A_n Overall Mean	A_m/A_n Overall cov (%)	Reported by
1	40	#3,#5,#8,#11	78	One	Lab	Canada	49.0 - 54.0	0.5 - 3.0	51.2	4.1	0.958 - 0.985	0.2 - 1.5	0.967	1.4	Allen (1972)
2	40	#3,#5,#8,#11	--	One	Mill	Canada	48.4 - 58.4	--	52.4	-	0.962 - 1.016	--	0.987	-	Allen (1972)
3	40	#3 to #10	171	-	--	U.S.A.	--	--	47.7	12.4	--	--	-	-	Julian (1957)
4	50	#8	35	One	Lab	India	60.8	1.2	60.8	1.2	0.986	0.5	0.986	0.5	Narayanaswamy (1972)
5	50	#4 to #10	656	Unknown	Lab	England	62.1 - 69.1	5.6 -11.9	67.1	9.30	0.988 - 1.008	1.0 - 3.1	1.001	2.1	Baker (1970)
6	60	#5 to #14	132	One	Mill	Canada	--	--	71.5	7.7	--	--	1.000	1.93	Allen (1972)
7	60	#8	34	One	Lab	India	73.4	2.5	73.4	2.5	0.992	1.6	0.992	1.6	Narayanaswamy (1972)
8	60	#3 to #10	1173	-	-	England	64.9 - 70.2	2.8 - 4.5	66.9	4.8	--	--	-	-	Bannister (1968)
9	60	#3 to #10	381	Unknown	Lab	England	64.0 - 68.2	5.1 - 7.7	66.3	6.9	1.115 - 1.212	4.8 - 9.0	1.168	7.4	Baker (1970)
10	60	#5,#8,#10, #11,#18	141	Nineteen	Mill	U.S.A.	69.1 - 70.8	--	70.6* (70.8)	10.5* (10.8)	--	--	-	-	Wiss, Janney, Elstner (1976)
11	60	#5,#8,#10, #11,#18	141**	Nineteen	Lab	U.S.A.	66.6 - 68.6	--	67.5* (67.6)	7.3* (7.4)	--	--	-	-	Wiss, Janney, Elstner (1976)

* Mean value and cov in parentheses represent data without #18 bars.

** Each sample is average of 2 to 8 tests. All tests were conducted at a loading rate of 26 micro-in/in/sec.

Table 2.2

Comparison of Statistical Properties of Ultimate and Yield Strength Distributions

Grade of Steel	Testing House	Strength Type	Mean Value (ksi)	Coef. of Variation	Coef. of Skewness	Coef. of Kurtosis
Grade 40	NRC Lab Tests (Allen, 1972)	f_u	83.13	0.0239	-0.0557	2.4831
		f_y	51.24	0.0409	-0.0117	2.0851
	Mill Tests (Allen, 1972)	f_u	79.26	--	--	--
		f_y	52.43	--	--	--
Grade 60	WJE Lab Tests (Wiss, Janney, Elstner, 1976)	f_u	108.89	0.0748	0.2236	2.5070
		f_y	67.48	0.0731	1.1619	5.9755
	Mill Tests (Wiss, Janney, Elstner, 1976)	f_u	109.55	0.0826	0.8221	4.3023
		f_y	70.60	0.1052	1.5854	6.6273
	Mill Tests (Allen, 1972)	f_u	112.10	0.0730	-0.3000	--
		f_y	71.50	0.0770	+0.3000	--

Table 3.1

Distribution Properties of Slab Thickness

In-situ Slabs					Precast Slabs					Investigator
Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	
90	6.50	+0.03	0.142	Normal	-	-	-	-	-	AASHO (1962)
196 201 1736 155 423	5.51 5.91 6.30 6.69 7.87	+0.20 +0.08 +0.01 +0.55 -0.19	0.315 0.315 0.469 0.433 0.543	Normal Normal Normal Normal Normal	-	-	-	-	-	Johnson (1953)
633	6.30	+0.06	0.341	*	1600 630	7.87 8.66	-0.08 +0.12	0.039 0.079	* *	Johansson & Warris (1968)
-	-	-	-	-	3383	5.98	+0.01	0.212	*	Bishop (1963)
20	3.94	+0.23	0.366	*	-	-	-	-	-	Hernandez & Martinez (1974)
Weighted values of all data shown above assuming normal distribution:										
3454	3.94-7.87	+0.04	0.458	Normal	5613	5.98-8.66	+0.00	0.178	Normal	-

* Not Reported

Table 3.2

Distribution Properties of Distance of Slab Reinforcement from Compression Face

In-Situ Slabs					Precast Slabs					Investigator
Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	
(a) Top Reinforcement										
111	4.96	-0.43	0.472	Normal	-	-	-	-	-	Johnson (1953)
77	5.31	-0.74	0.630	Normal						
975	5.71	-0.71	0.565	Normal						
287	6.02-6.14	-0.77	0.602	Normal						
246	7.28	-1.16	0.775	Normal						
Weighted values of above data for top reinforcement assuming normal distribution:										
1696	4.96-7.28	-0.77	0.630	Normal	-	-	-	-	-	-
(b) Bottom Reinforcement										
181	4.80	-0.04	0.354	Normal	-	-	-	-	-	Johnson (1953)
199	5.12	-0.16	0.394	Normal						
1708	5.55-5.59	-0.32	0.628	Normal						
153	5.91	-0.39	0.512	Normal						
420	7.09-7.17	-0.51	0.626	Normal						
144	5.67	-0.13	0.343	*	80	7.09	-0.08	0.039	*	Johansson & Warris (1968)
					70	7.72	+0.08	0.079	*	
Weighted values of above data for bottom reinforcement assuming normal distribution:										
2805	4.80-7.17	-0.31	0.594	Normal	150	7.09-7.72	-0.01	0.101	Normal	-

* Not reported

Table 3.3

Recommended Distribution Properties
of Slab Dimension

Dimension Description		In-Situ Slabs				Precast Slabs			
		Nominal Range (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Distribution	Nominal Range (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Distribution
Thickness		4-8	+1/32	15/32	Normal	6-9	0	3/16	Normal
Top Reinforcement	Effective Depth Conc.Cover	4-8	-3/4 +25/32	5/8 25/32	Normal Normal	4-8	0 0	3/32 7/32	Normal Normal
Bottom Reinforcement	Effective Depth Conc.Cover	4-8	-5/16 +11/32	5/8 13/32	Normal Normal	4-8	0 0	3/32 7/32	Normal Normal

Table 3.4

Distribution Properties of Beam Widths

In-Situ Beams					Precast Beams					Investigator
Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	
(a) Width of Beam Rib										
60	11.50	+0.05	0.073	Normal	-	-	-	-	-	AASHO (1962)
-	-	-	-	-	474 123	13.78 13.78	+0.06 -0.15	0.126 0.280	Normal -	van den Berg (1971)
195	11.81	+0.14	0.159	-	-	-	-	-	-	Hernandez & Martinez (1975)
60	12.00	+0.00	0.063	Lognormal	-	-	-	-	-	Connolly (1975)
Weighted values of above data for beam width:										
315	11.50-12.00	+0.10	0.144	Normal	597	13.78	+0.02	0.190	Normal	-
(b) Flange Width										
-	-	-	-	-	119	19.69	+0.09	0.165	-	Jacobson & Widmark (1970)
-	-	-	-	-	101	23.62	+0.24	0.268	-	van den Berg (1971)
Weighted values of above data for flange width:										
-	-	-	-	-	220	19.69-23.62	+0.16	0.231	Normal	-

Table 3.5

Distribution Properties of Overall Depth of Beams

In-Situ Beams					Precast Beams					Investigator
Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	
60	26.50	+0.01	0.155	Normal	-	-	-	-	-	AASHO (1962)
-	-	-	-	-	119	21.65	+0.09	0.165	-	Jacobson & Widmark (1970)
-	-	-	-	-	516	23.62	+0.18	0.138	Normal	van den Berg (1971)
-	-	-	-	-	48 54	29.53 38.39	+0.09 -0.02	0.087 0.181	- -	Johansson and Warris (1968)
48 *196	18.00 16.50	-0.25 -0.63	0.188 0.250	Lognormal Lognormal	-	-	-	-	-	Connolly (1975)
Weighted values of all data shown above except in-situ joists poured in steel forms:										
108	18.00-26.50	-0.11	0.214	Normal	737	21.65-38.39	+0.14	0.156	Normal	-

* In-situ concrete joists poured in steel forms.

Table 3.6

Distribution Properties of Concrete Covering of In-Situ Beams

Top Reinforcement					Bottom Reinforcement					Investigator
Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	
-	-	-	-	-	188	0.98	+0.51	0.110	-	Hernandez & Martinez (1974)
48	1.50	+0.11	0.620	Normal	48 48	0.75 1.00	+0.13 +0.04	0.400 0.470	Normal Normal	Connolly (1975)
Weighted values of above data reported by Connolly:										
48	1.50	+0.11	0.620	Normal	66	0.75-1.00	+0.06	0.454	Normal	-
Insitu Joists Poured in Steel Forms										
66 82	2.00 2.50	-0.01 +0.04	0.334 0.360	Normal Normal	196	0.75	+0.01	0.291	Normal	Connolly (1975)
Weighted values of above data for concrete joists:										
148	2.00-2.50	+0.02	0.350	Normal	196	0.75	+0.01	0.291	Normal	-

Table 3.7

Recommended Distribution Properties of Beam Dimensions

Dimension Description		In-Situ Beams				Precast Beams			
		Nominal Range (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Distribution	Nominal Range (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Distribution
Width	Rib Flange	11-12 -	+3/32 -	3/16 -	Normal -	14 19-24	0 +5/32	3/16 1/4	Normal Normal
Overall Depth		18-27	-1/8	1/4	Normal	21-39	+1/8	5/32	Normal
Top Reinforcement	Conc. Cover Effective Depth	1 1/2	+1/8 -1/4	5/8 9/16	Normal Normal	2-2 1/2	0 +1/8	5/16 11/32	Normal
Bottom Reinforcement	Conc. Cover Effective Depth	3/4-1	+1/16 -3/16	7/16 1/2	Normal Normal	3/4	0 +1/8	5/16 11/32	Normal Normal
Stirrup Spacing			0	17/32	Normal		0	11/32	Normal
Beam Spacing			0	11/16	Normal		0	11/32	Normal

Table 3.8

(a) Distribution Properties of Column Cross Section Dimensions

In-Situ Columns					Precast Columns					Investigator
Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	Number of Samples	Nominal Specified (in.)	Mean Deviation from Nominal (in.)	Standard Deviation (in.)	Recommended Distribution	
1844	12.00-30.00	+0.06	0.280	Normal	-	-	-	-	-	Tso & Zelman (1970)
510	19.69x11.81	+0.15	0.165	Normal	-	-	-	-	-	Hernandez & Martinez (1974)
-	-	-	-	-	136	7.87-15.75	+0.06	0.154	-	Jacobson & Widmark (1970)
-	-	-	-	-	60 60	13.78 13.78	-0.07 +0.08	0.087 0.063	-	Klingberg (1970)
-	-	-	-	-	*433	11.81-12.99	-0.01	0.091	-	van den Berg (1971)
Weighted values of data shown above except circular columns:										
2354	11.81-30.00	+0.08	0.262	Normal	256	7.87-15.75	+0.03	0.137	-	-

* Circular columns: Overall diameter measurements.

(b) Recommended Distribution Properties of Column Dimensions

Dimension Description	In-Situ Columns				Precast Columns			
	Nominal Range (in.)	Mean Dev. from Nom. (in.)	Std. Dev. (in.)	Distr. Type	Nominal (in.)	Mean Dev. from Nom. (in.)	Std. Dev. (in.)	Distr. Type
Rect. Col.: Width, thickness	11 - 30	+1/16	1/4	Normal	7 - 16	+1/32	1/8	Normal
Circular Col.: Diameter	11 - 13	0	3/16	Normal	11 - 13	0	3/32	Normal

Table 3.9

Distribution Properties of Furnished/Calculated Area of Steel, A_f/A_c

Member Type	Reinforcement Type	Size of Cross Section (in. x in.)	Distribution Properties				Steel Limits
			Mean Value	Coefficient of Variation	Coefficient of Skewness	Kurtosis	
Beams*	Flexural Tension Steel	10x15	1.04057	0.06876	1.30684	5.13642	ρ_{min} = 0.37%, ρ_{max} = 2.11%
		16x18	1.00653	0.04872	1.70895	7.29346	ρ_{min} = 0.38%, ρ_{max} = 1.83%
		20x30	0.99308	0.03493	1.21241	4.39505	ρ_{min} = 0.33%, ρ_{max} = 1.37%
	Transverse Steel (Stirrups)**	10x15	1.02843	0.06353	1.19353	3.82222	rf_y(min) = 106 psi, rf_y(max) = 529 psi
		16x18	1.00724	0.03912	0.81899	3.54933	rf_y(min) = 55 psi, rf_y(max) = 526 psi
		20x30	1.02672	0.06181	1.45499	5.22768	rf_y(min) = 48 psi, rf_y(max) = 522 psi
Columns	Vertical Steel	12x12	1.03193	0.05982	0.75437	2.81726	ρ_{min} = 1.10%, ρ_{max} = 4.56%
		18x18	1.02275	0.05576	0.81092	2.82809	ρ_{min} = 0.98%, ρ_{max} = 5.20%
		24x24	1.01274	0.04478	0.88552	3.34513	ρ_{min} = 0.99%, ρ_{max} = 5.85%
		36x36	1.00559	0.03768	0.58494	2.48764	ρ_{min} = 0.97%, ρ_{max} = 4.39%

* All Beams: f'_c = 4000 psi, f_y = 60,000 psi (Longitudinal steel), f_y = 40,000 psi (stirrups).

** For transverse steel the ratio $\frac{A_v}{s}$(furnished)/$\frac{A_v}{s}$(calculated) was studied, where A_v, s are area and spacing of stirrups.

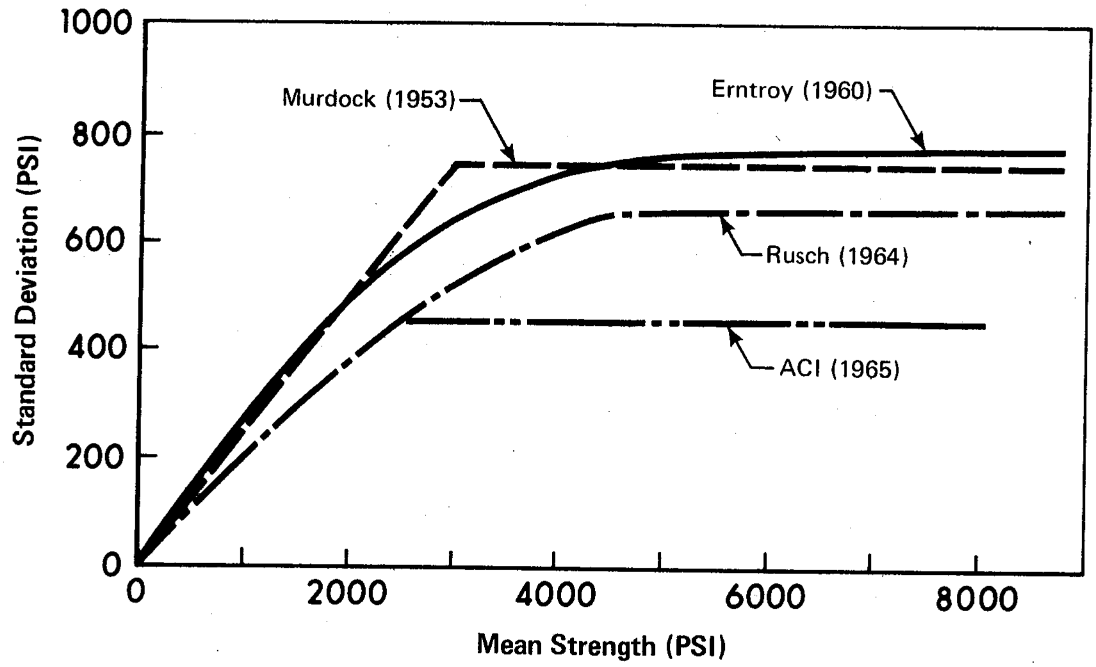


Figure 1.1 Relationship Between Standard Deviation and Mean Strength of Concrete

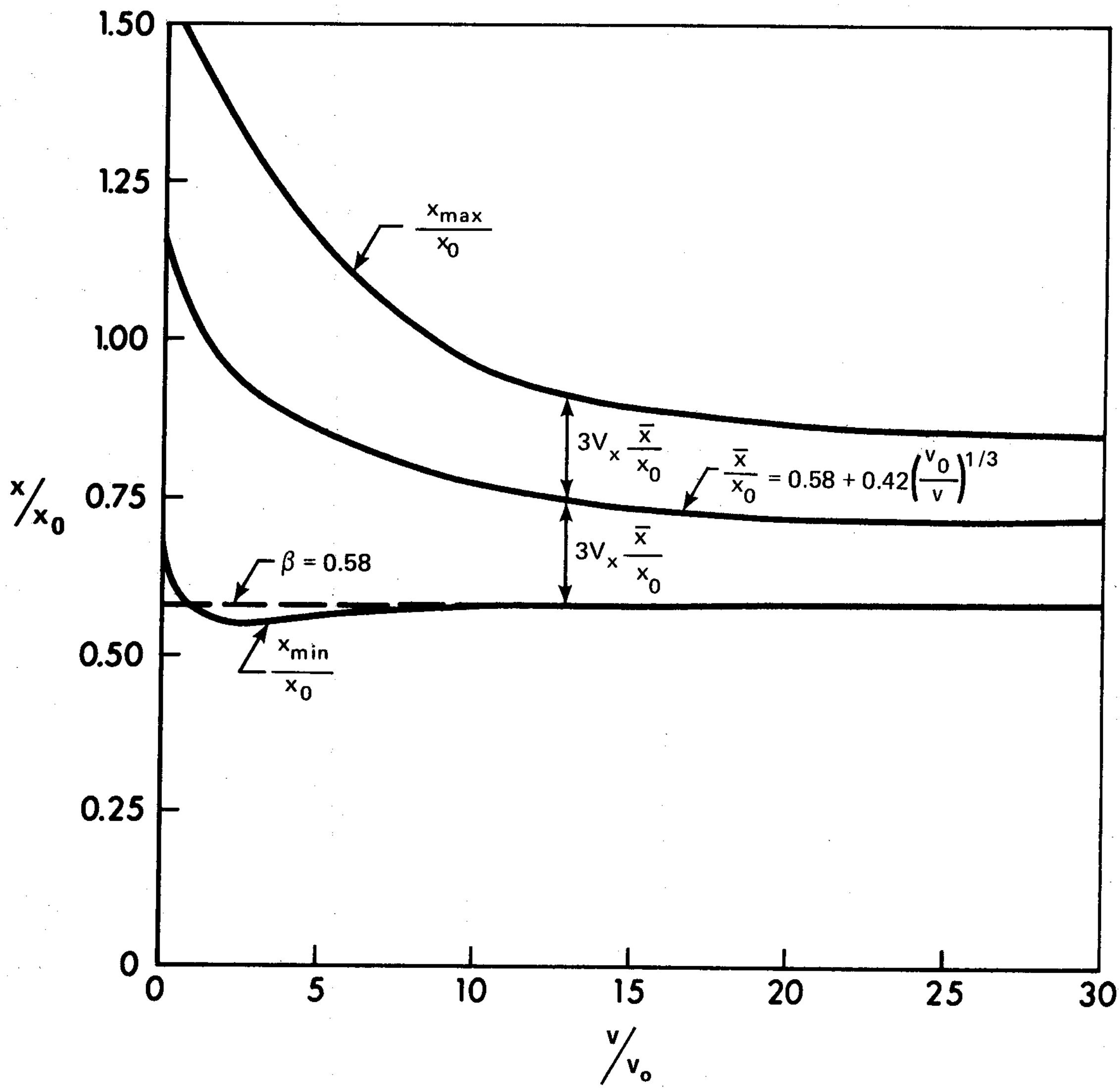


Fig. 1.2 Effect of Volume on Mean, Minimum and Maximum Strengths

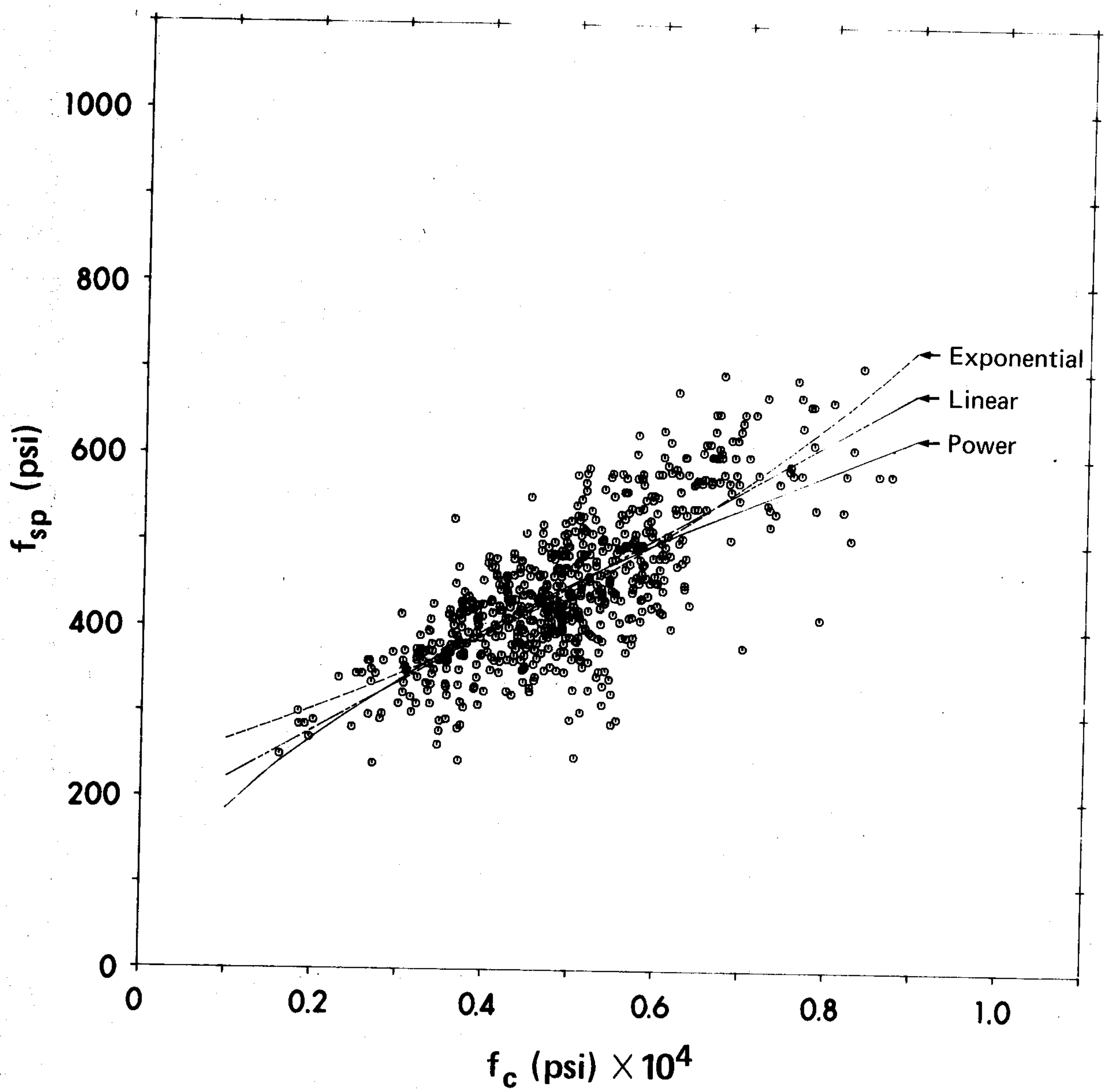


Figure 1.3 Relation Between f_c and f_{sp}

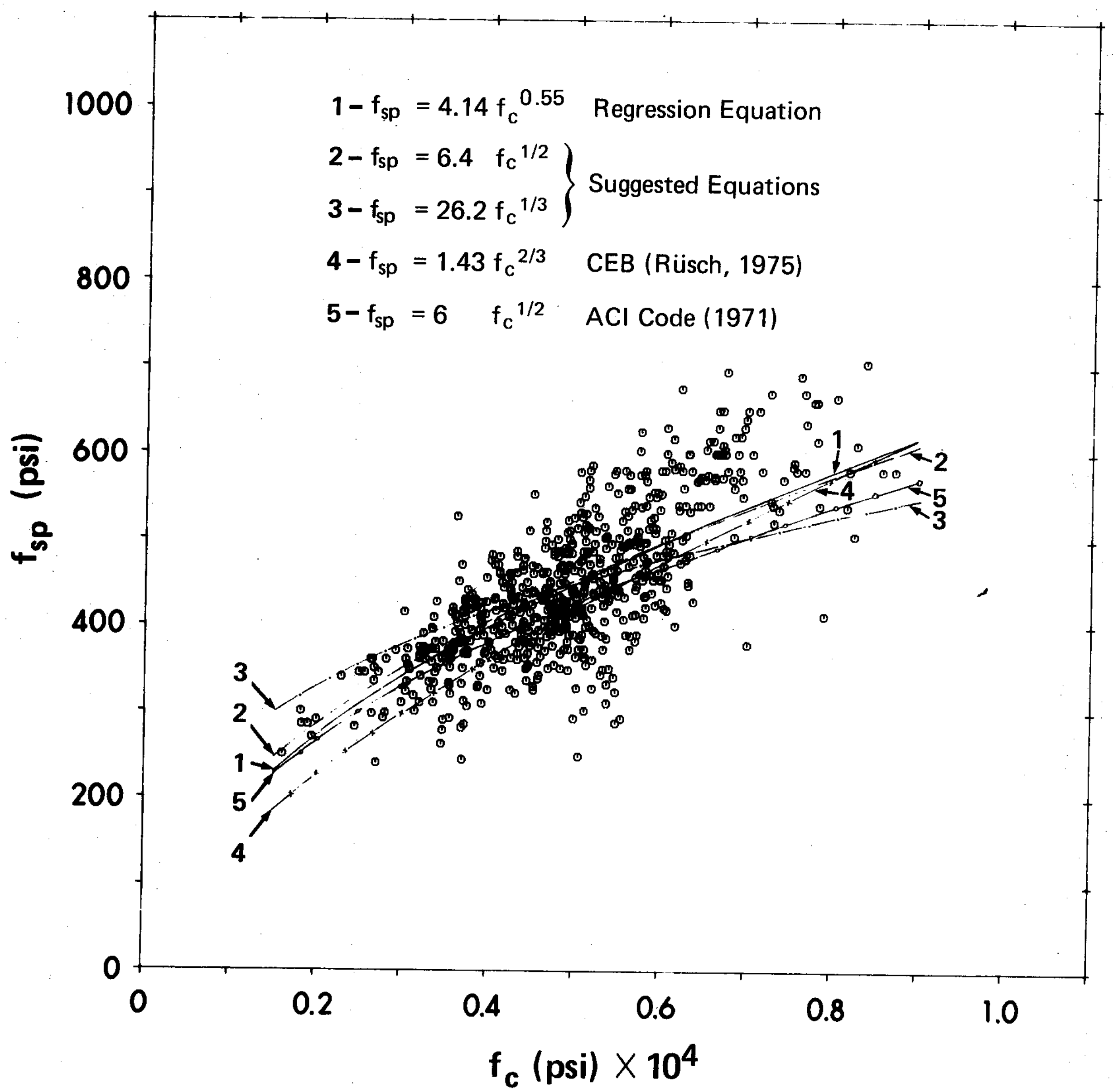


Figure 1.4 Relation Between f_c and f_{sp}

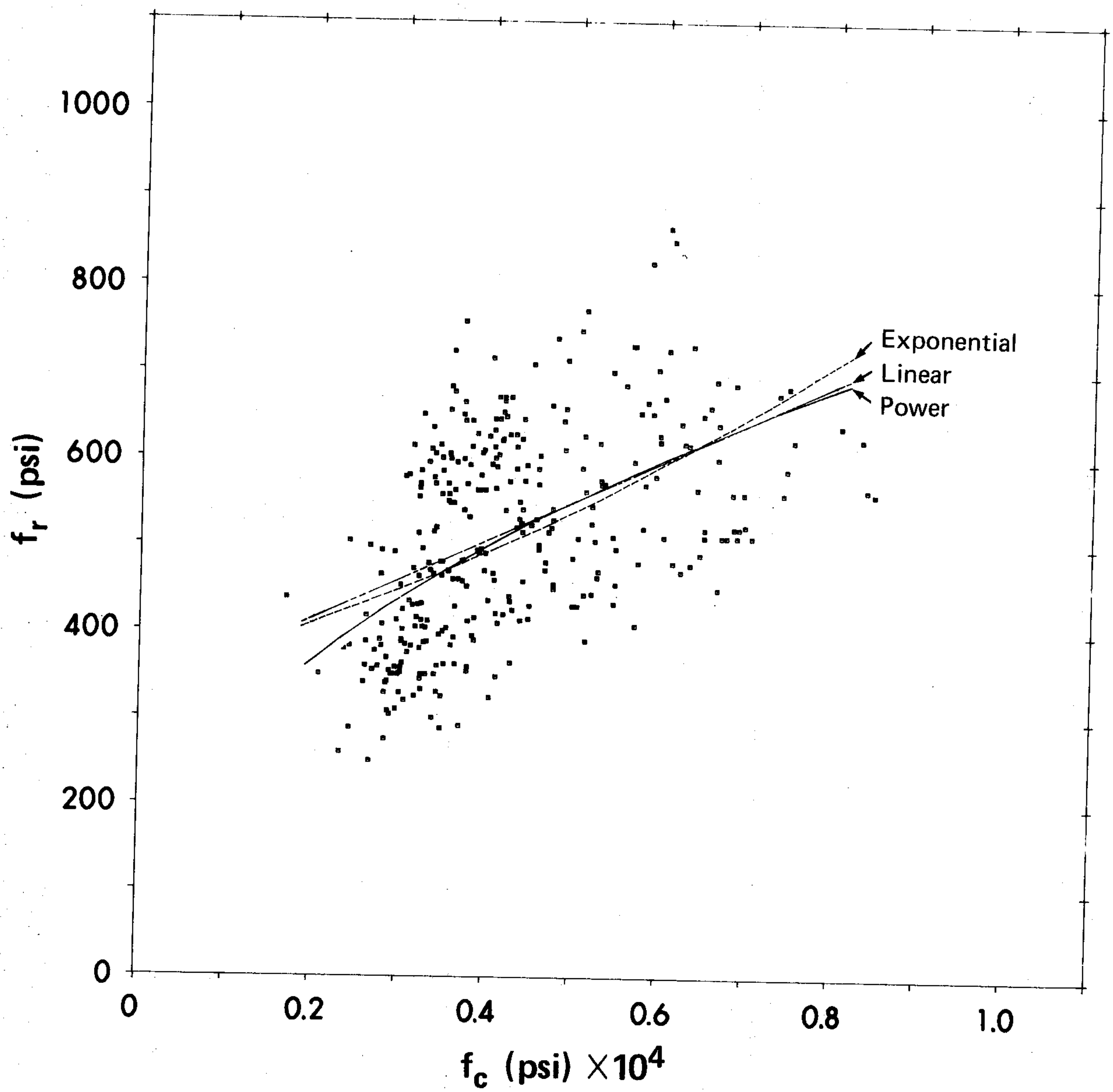


Figure 1.5 Relation Between f_c and f_r

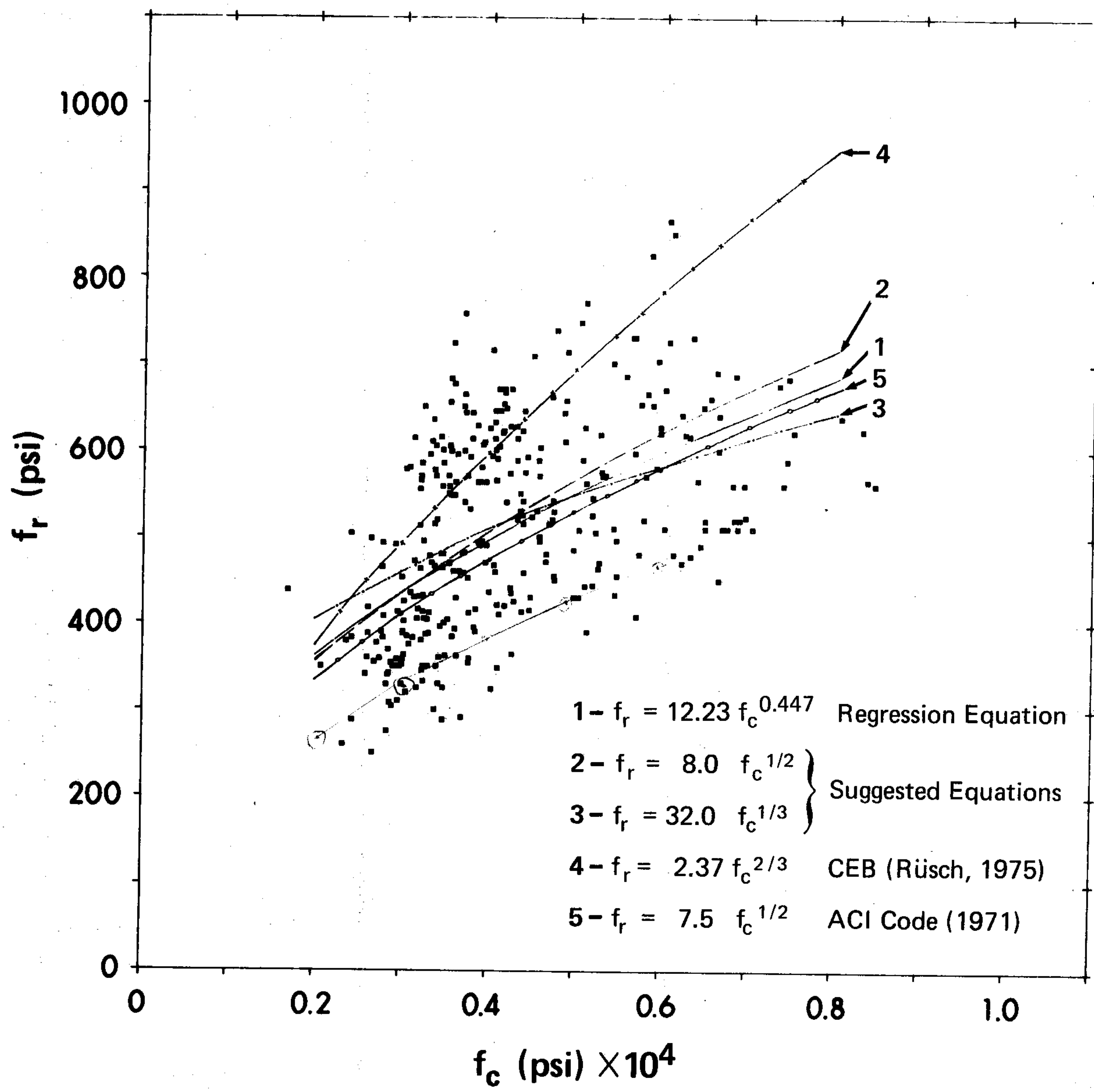


Figure 1.6 Relation Between f_c and f_r

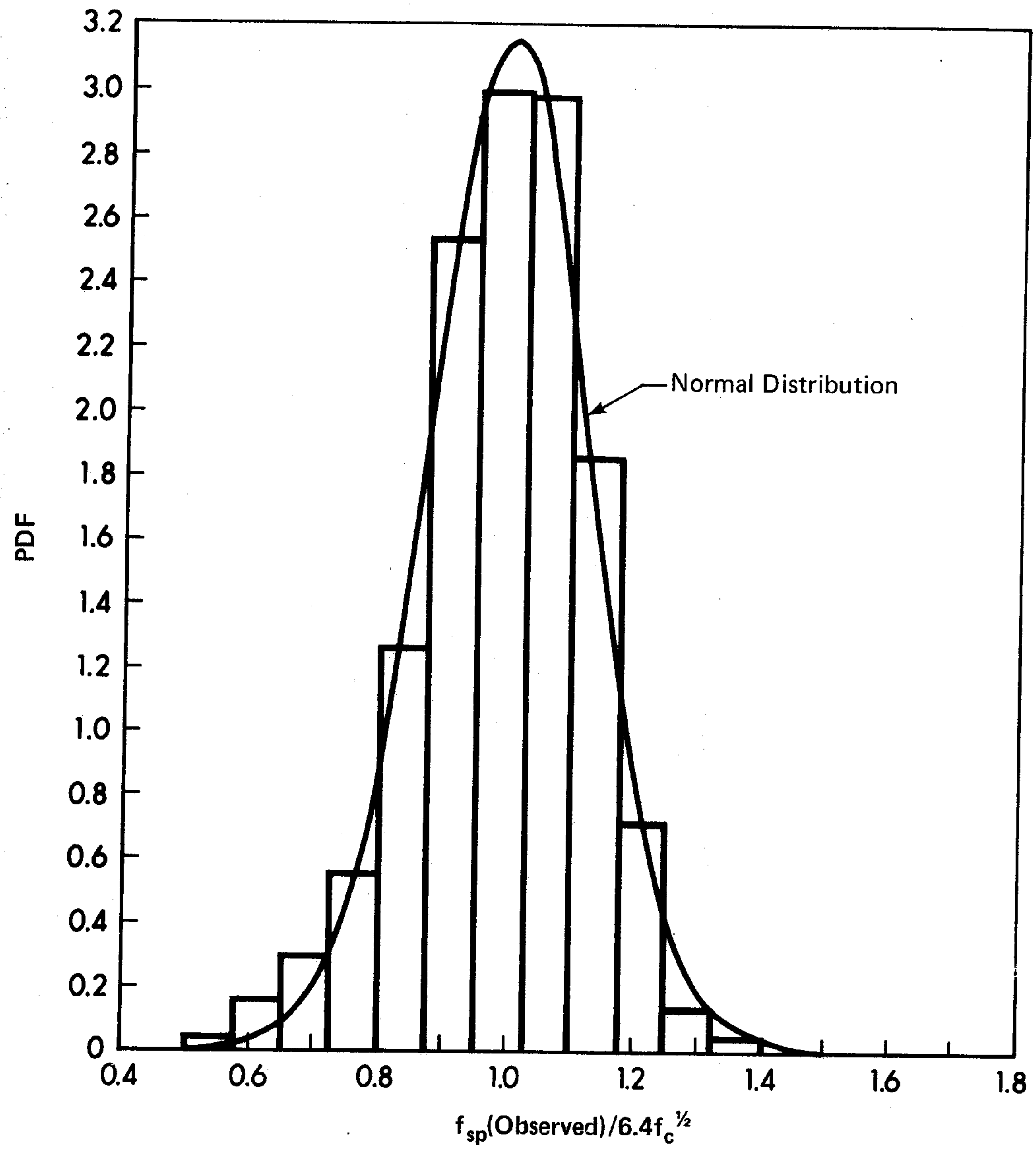


Fig. 1.7 Probability Density Function for the Ratio f_{sp} (Observed)/6.4 $f_c^{1/2}$

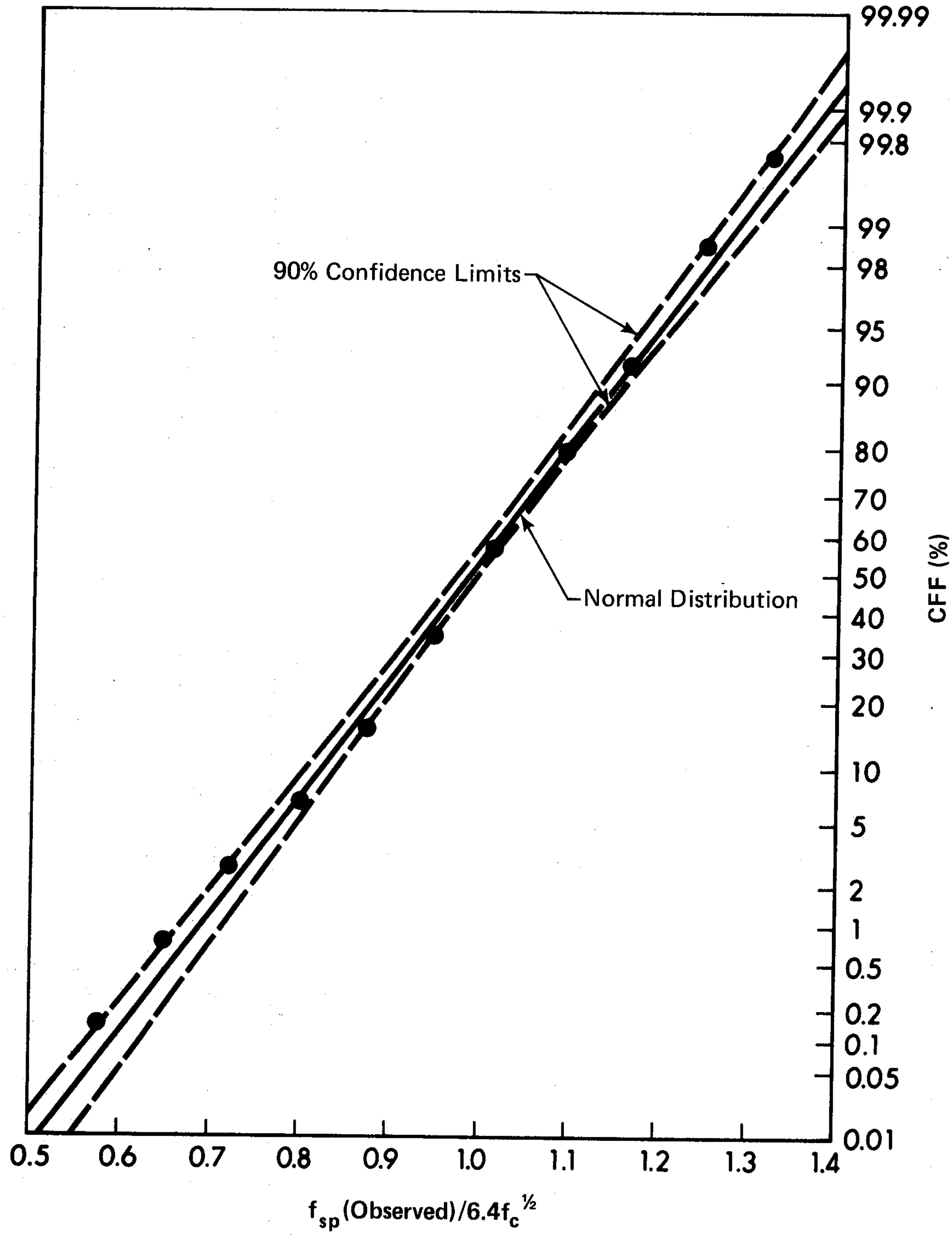


Fig. 1.8 **Cumulative Frequency Function for the Ratio f_{sp} (Observed)/$6.4f_c^{1/2}$**

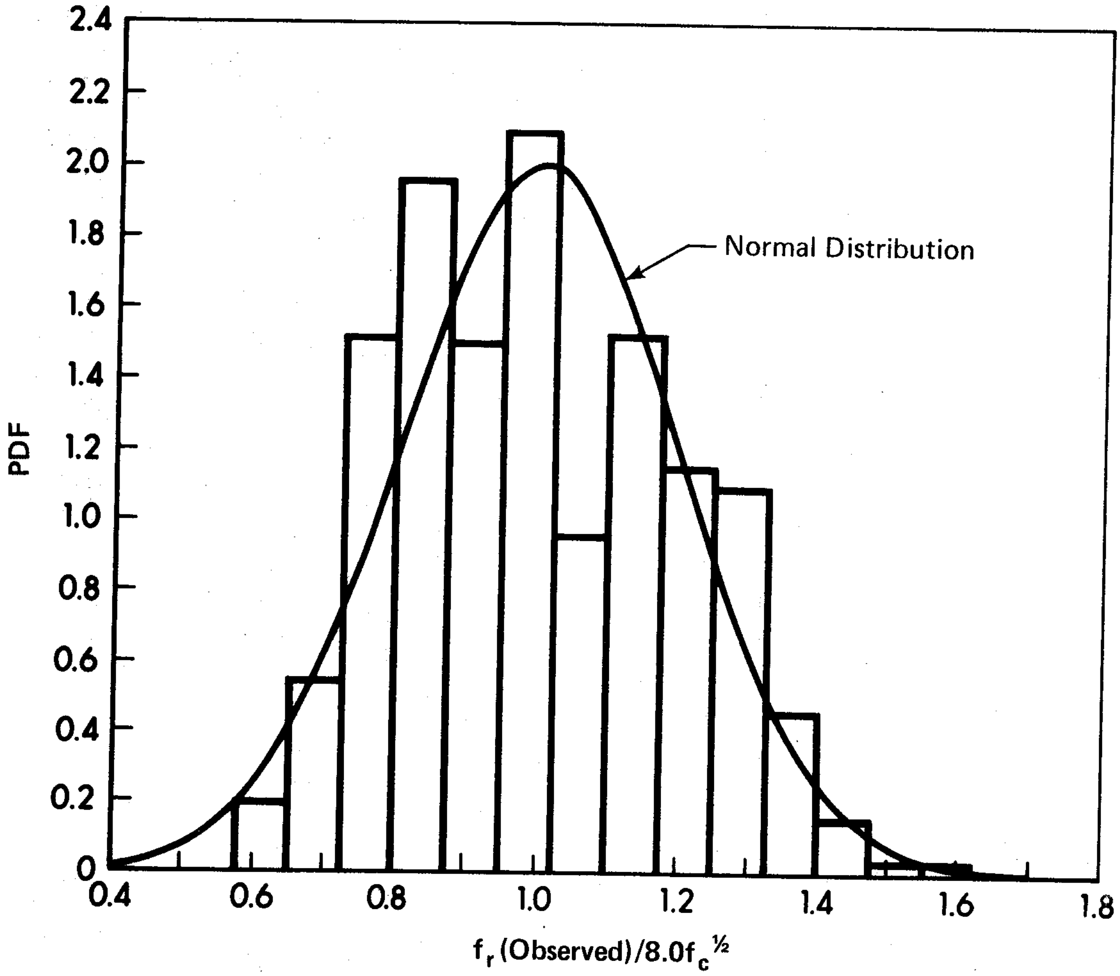


Fig. 1.9 Probability Density Function for the Ratio f_r(Observed)/8.0 $f_c^{1/2}$

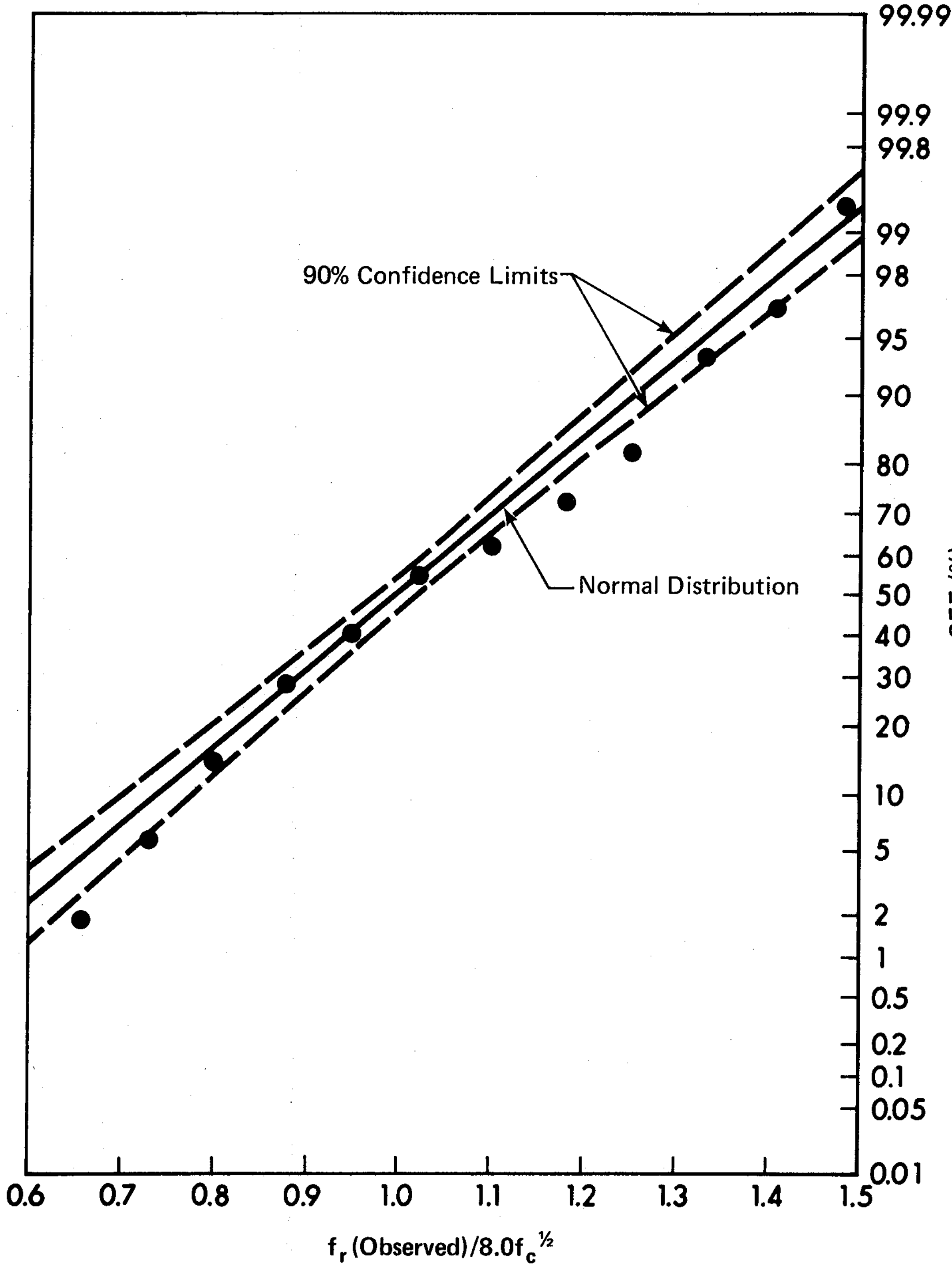


Fig. 1.10 Cumulative Frequency Function for the Ratio f_r (Observed)/$8.0f_c^{1/2}$

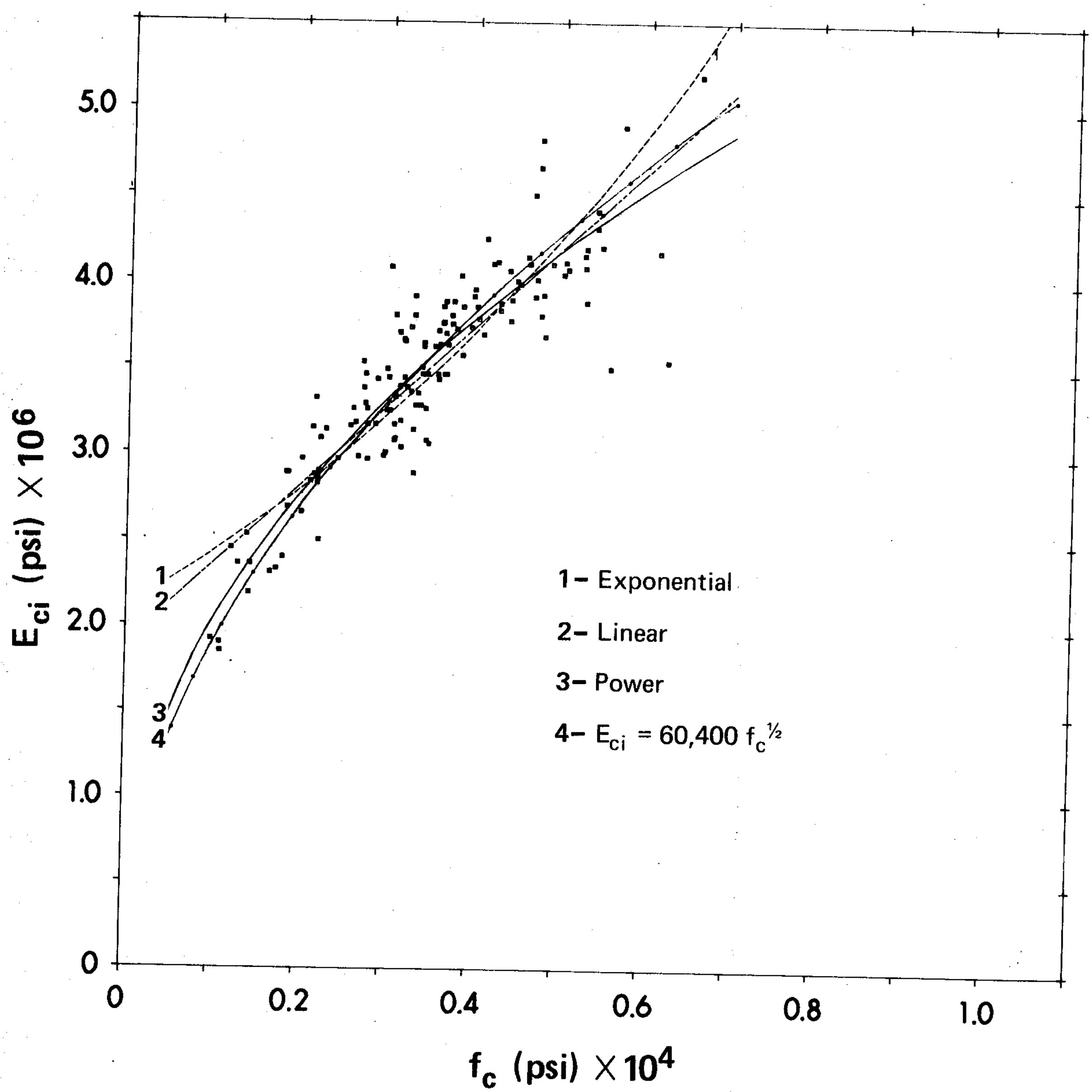


Figure 1.11 Relation Between f_c and E_{ci}

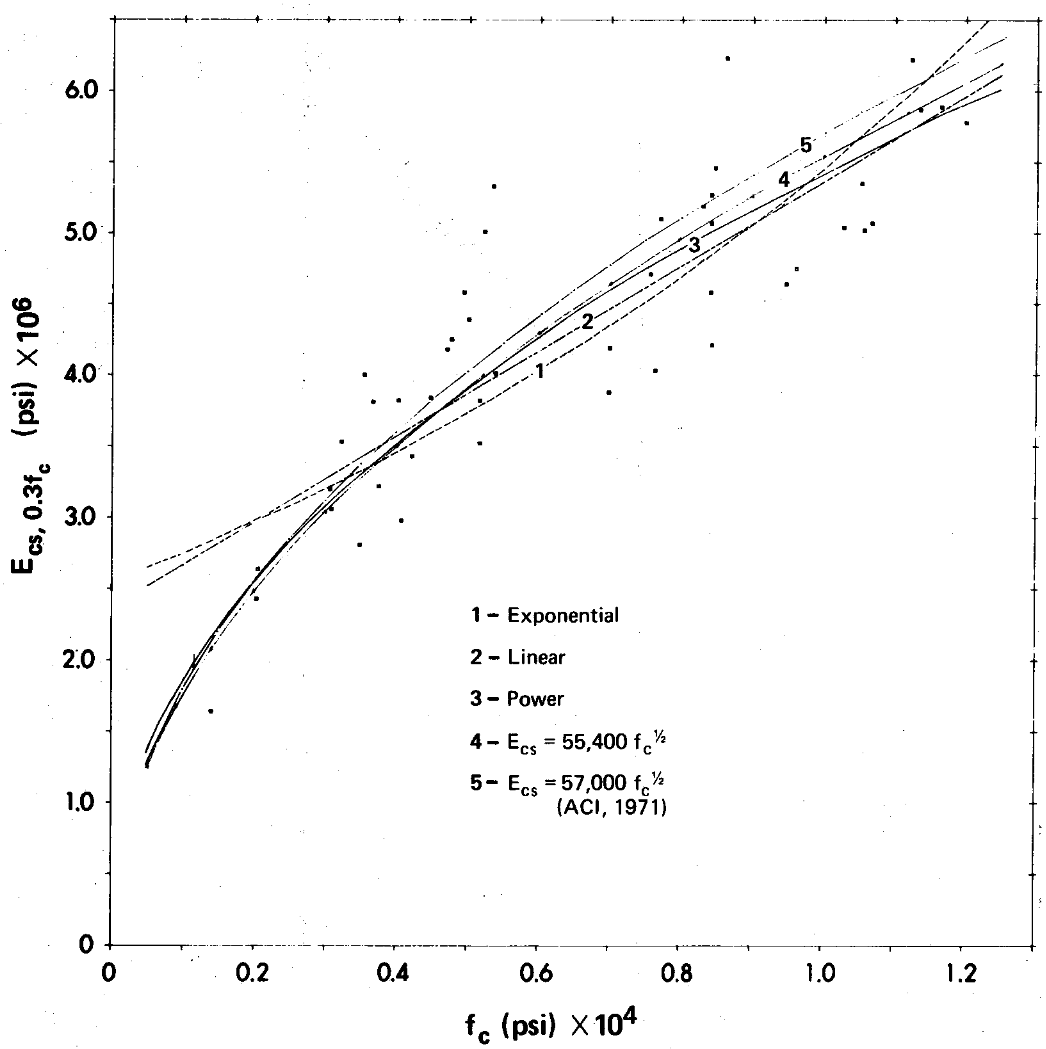


Figure 1.12 Relation Between f_c and $E_{cs,\ 0.3f_c}$

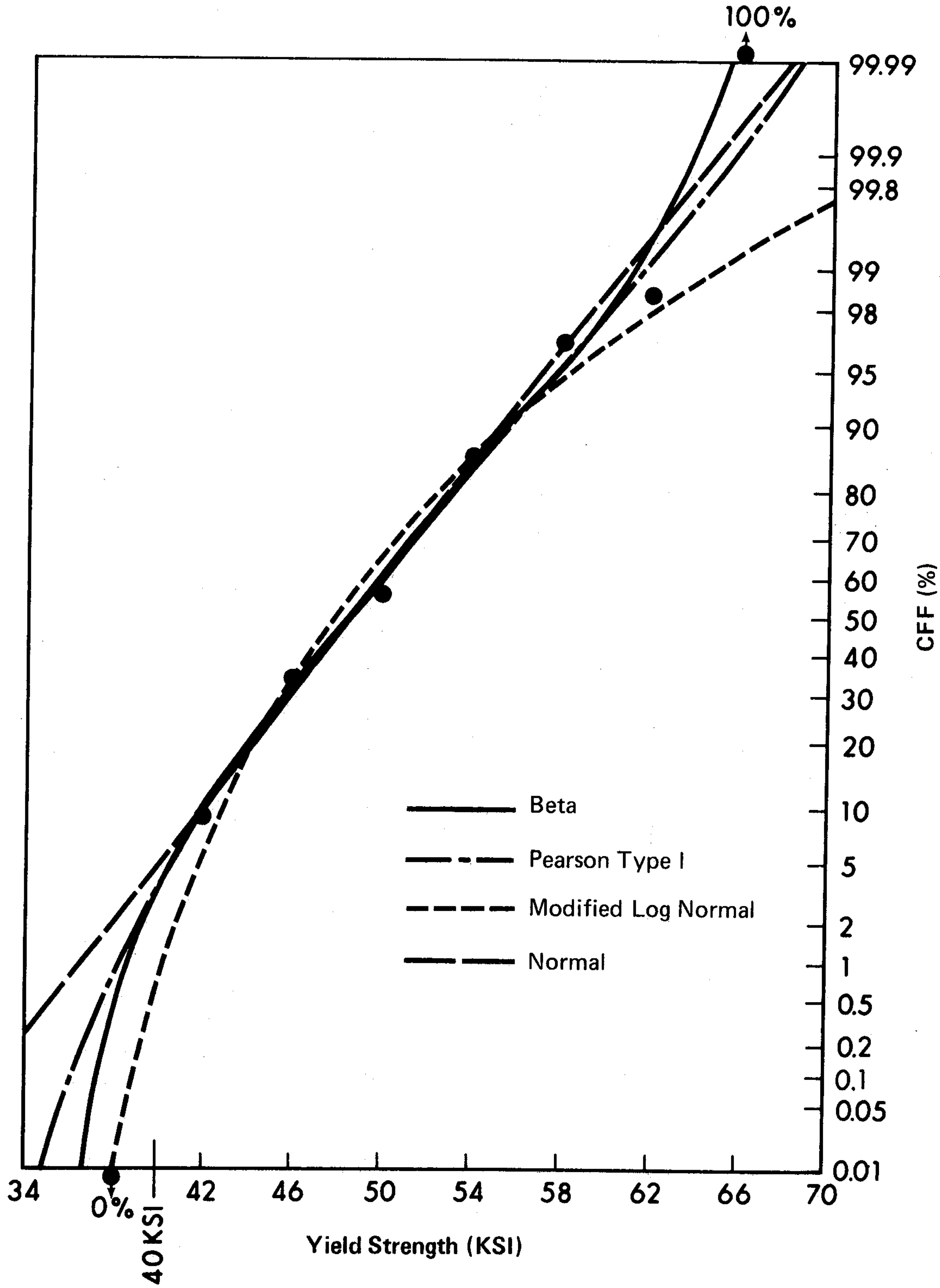


Fig. 2.1 Distribution for Grade 40 Steel

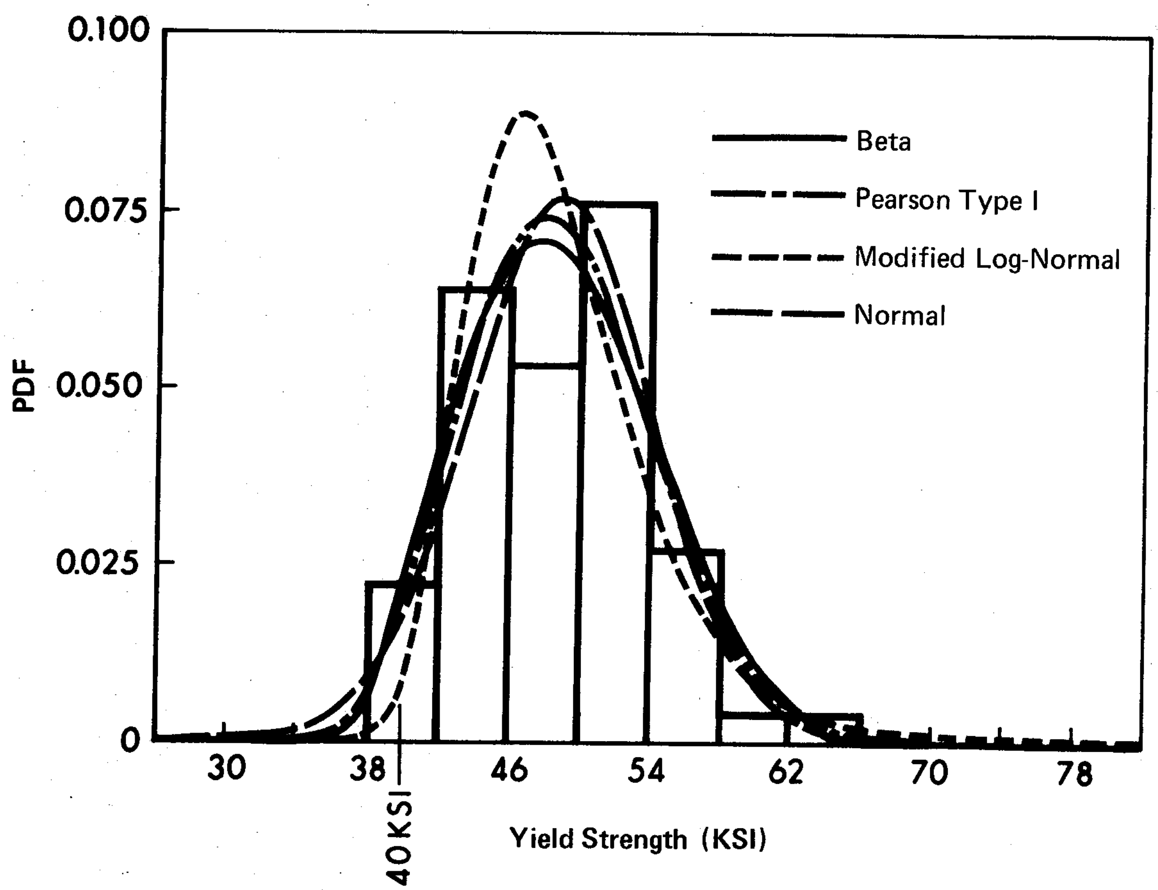


Fig. 2.2 Distribution for Grade 40 Steel

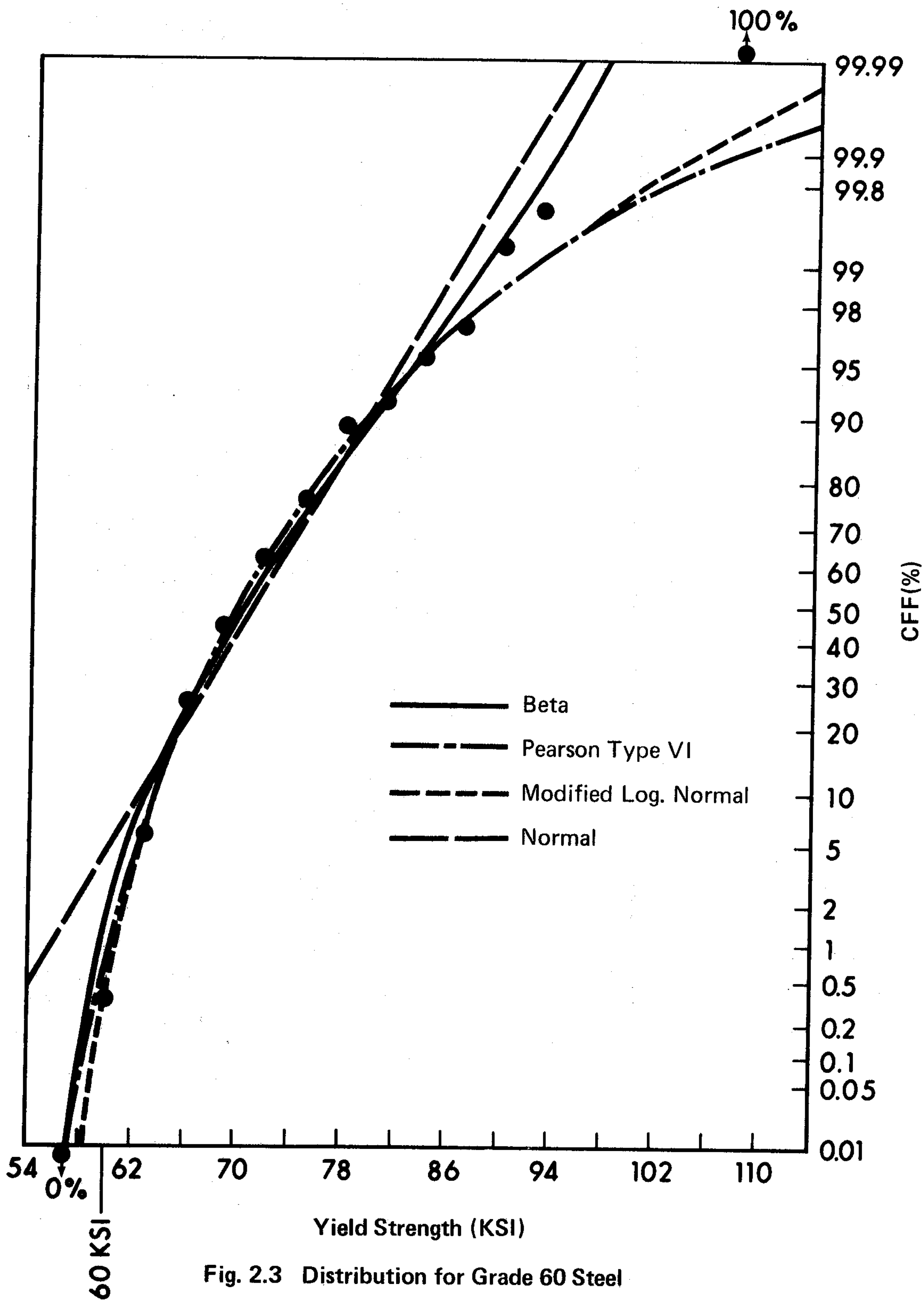


Fig. 2.3 Distribution for Grade 60 Steel

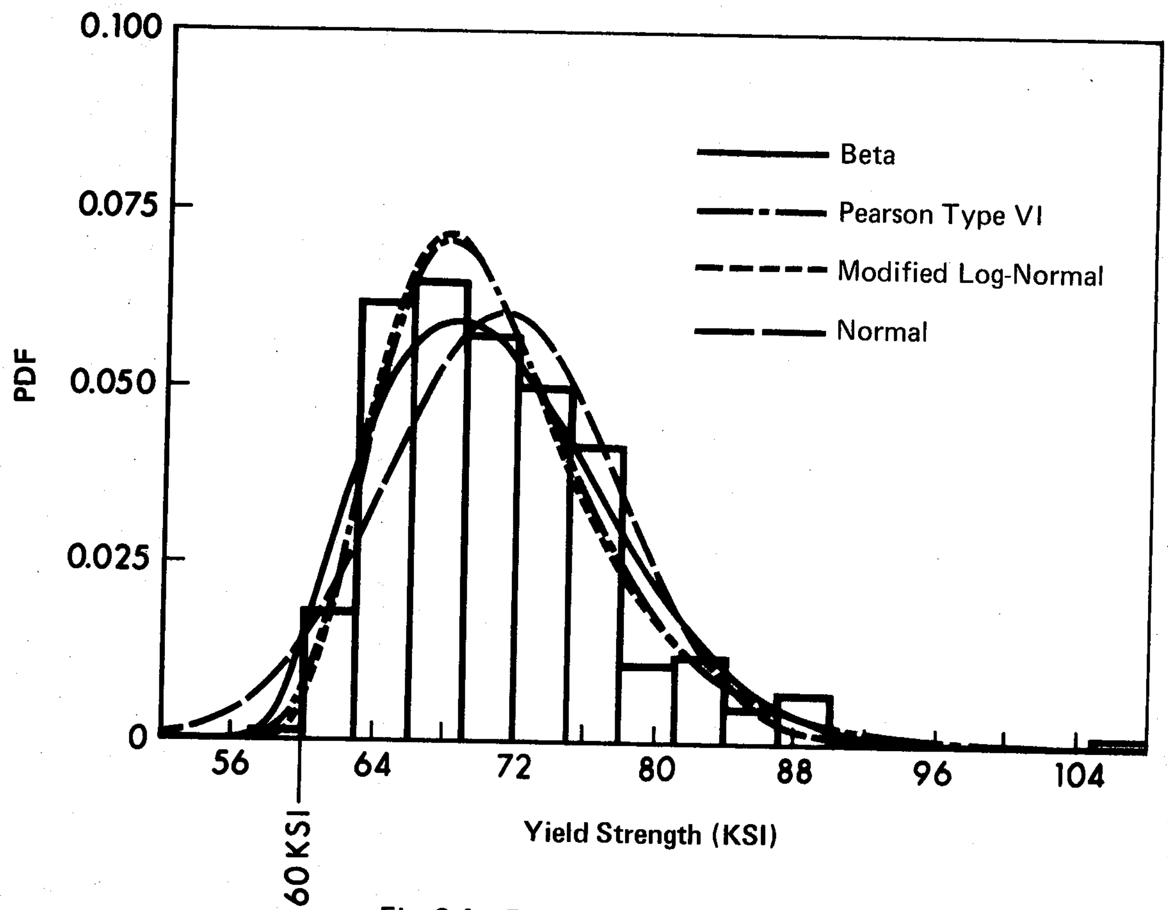


Fig. 2.4 Distribution for Grade 60 Steel

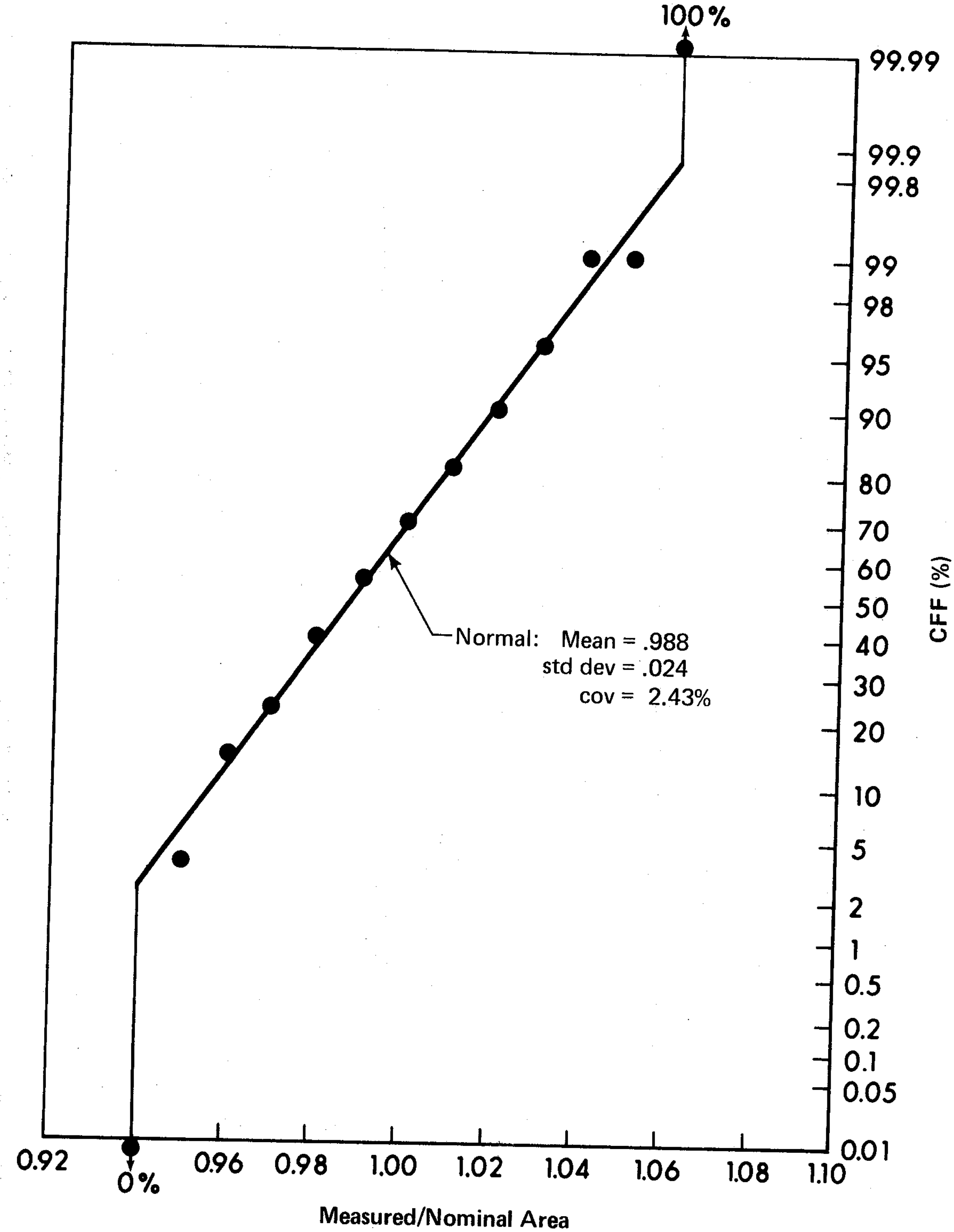


Fig. 2.5 Distribution for A_m/A_n of Reinforcing Bars of Grade 40 and Grade 60 Steel

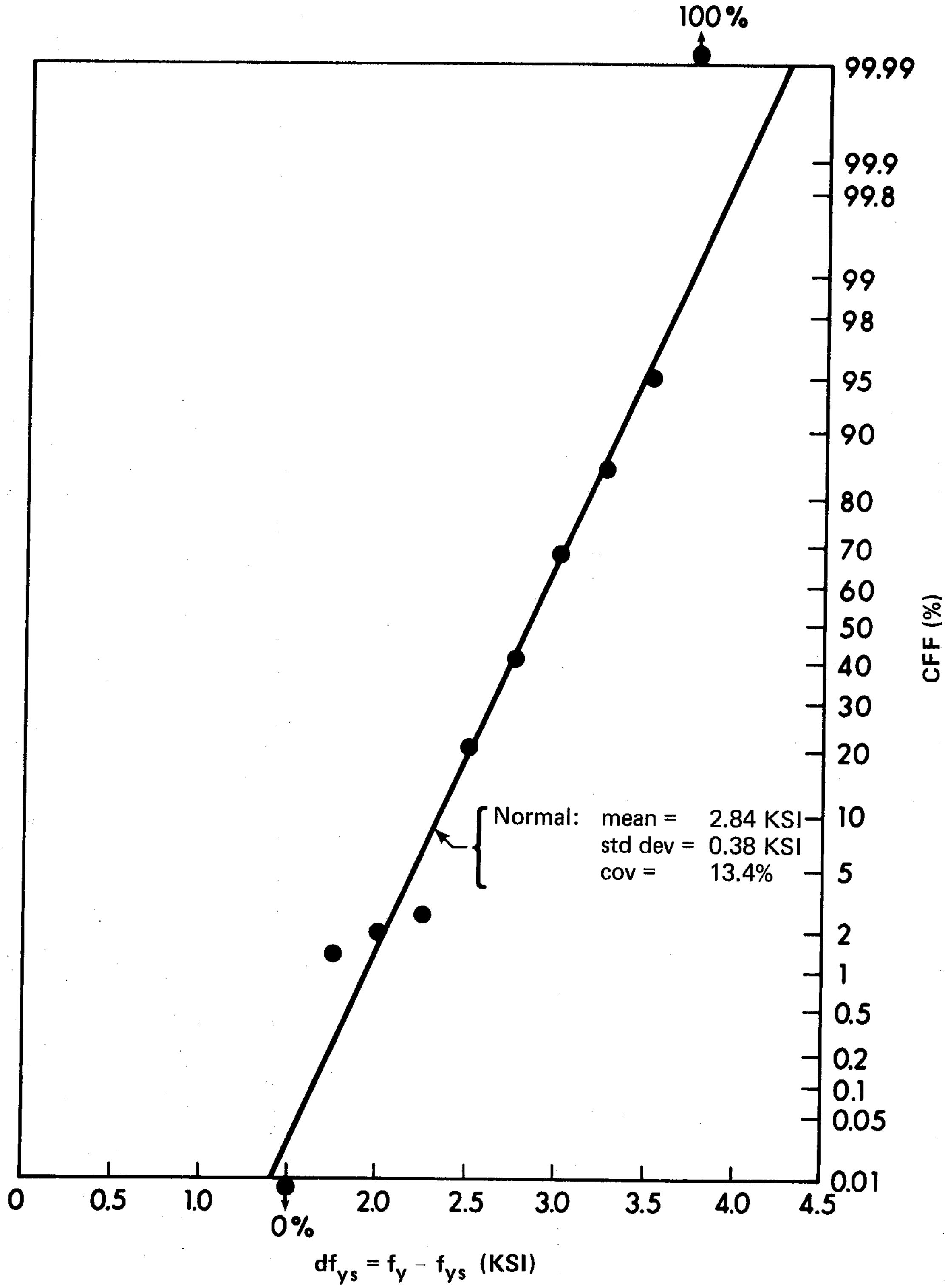


Fig. 2.6 Distribution for df_{ys} of Grade 40 Reinforcing Bars

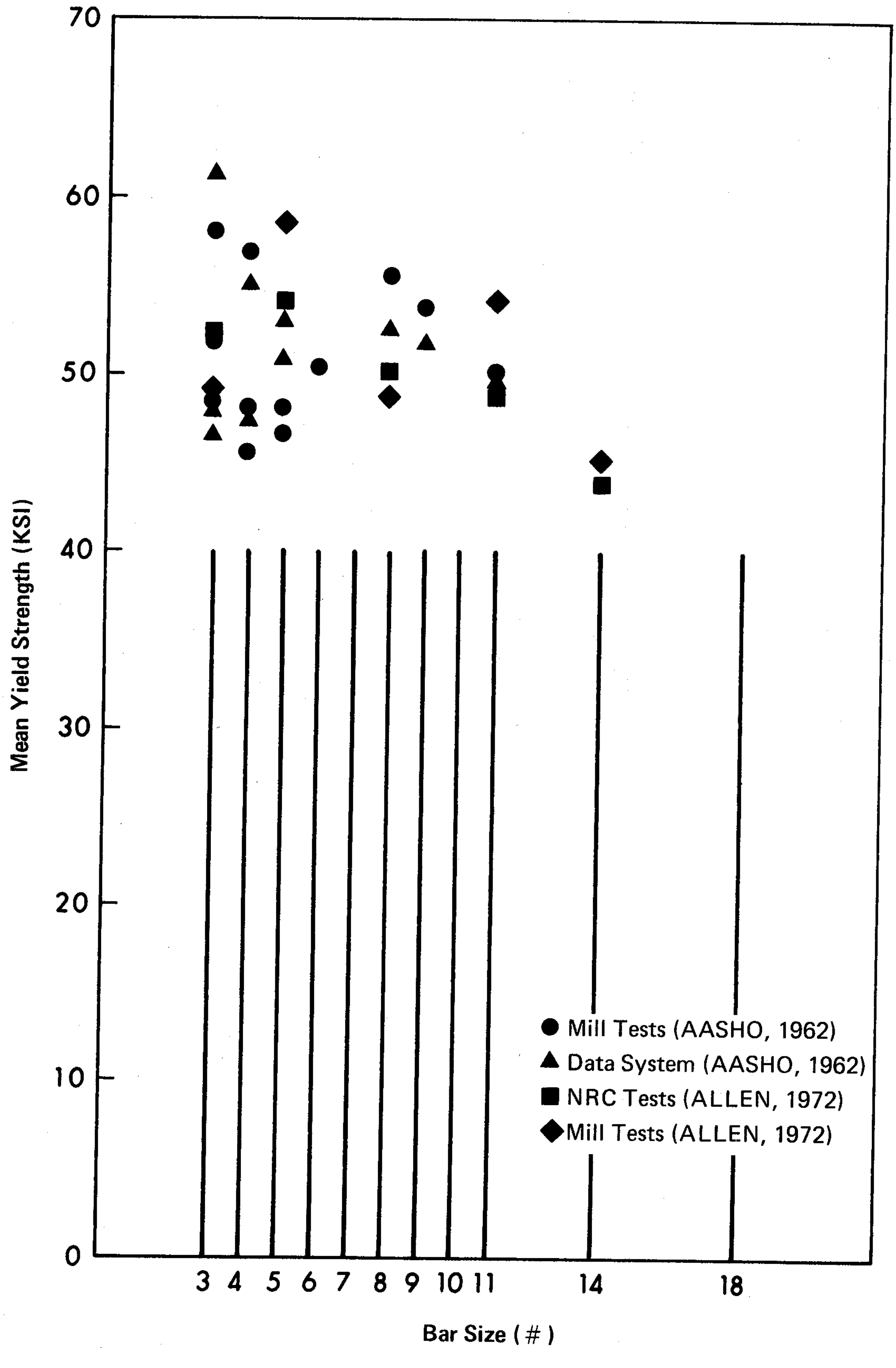


Fig. 2.7 Effect of Bar Diameter on Yield Strength, Grade 40

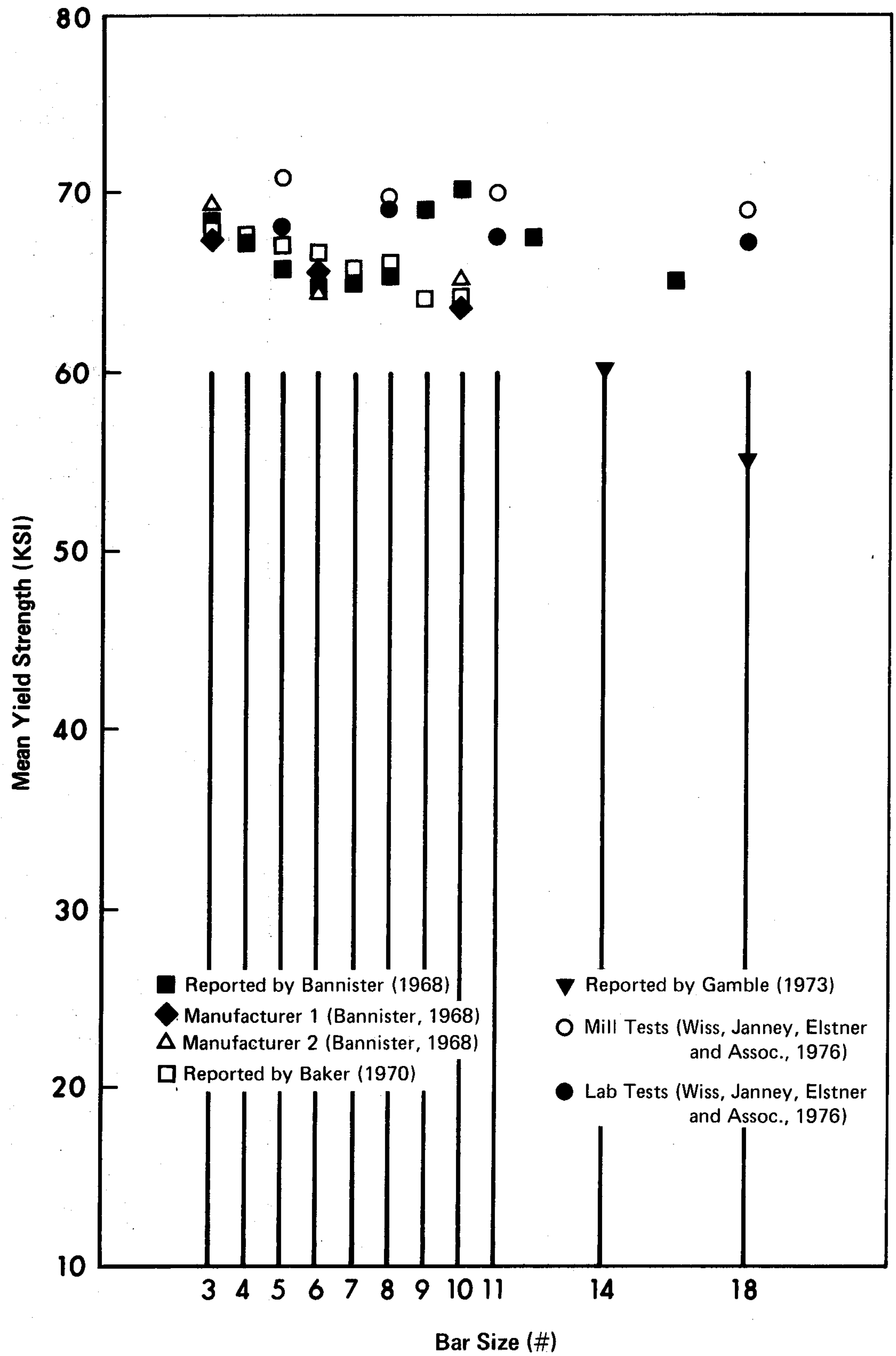


Fig. 2.8 Effect of Bar Diameter on Yield Strength, Grade 60

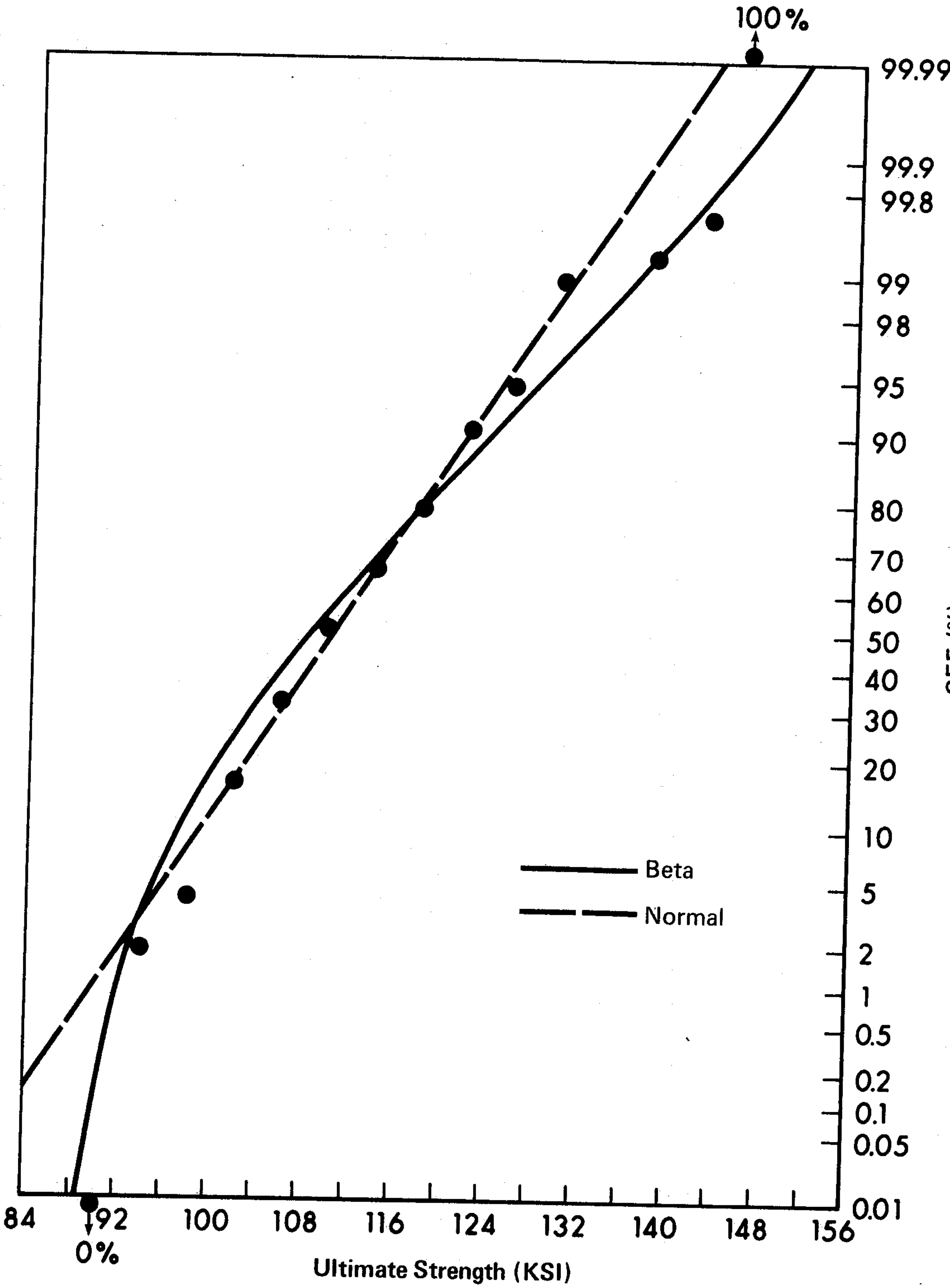


Fig. 2.9 Distribution for Grade 60 Steel

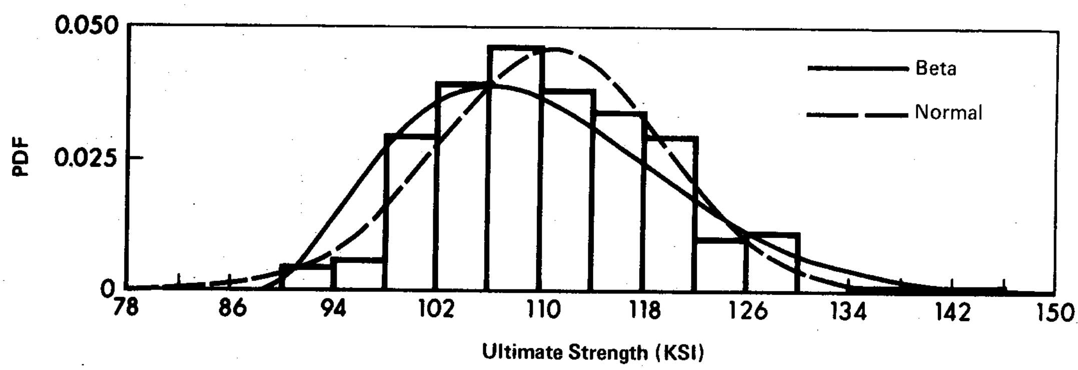
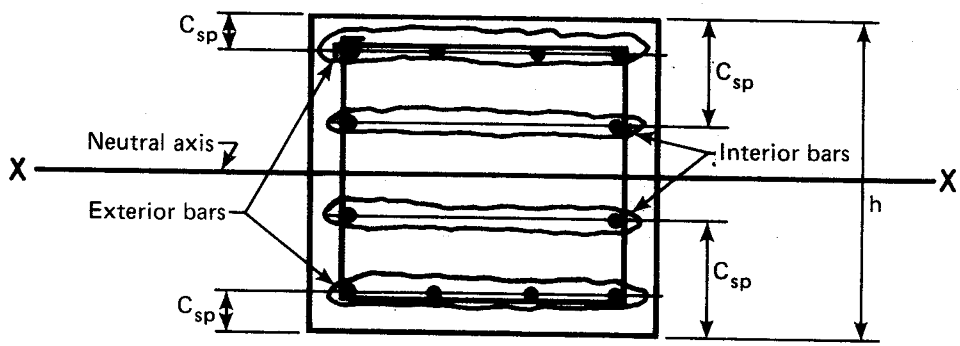
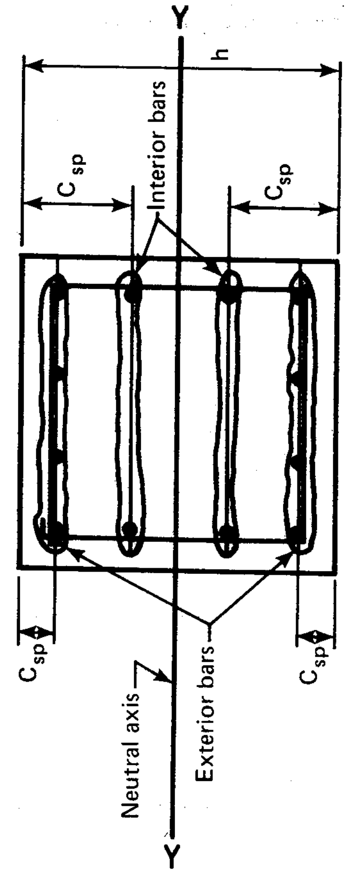


Fig. 2.10 Distribution for Grade 60 Steel



(a) Dimensions perpendicular to X- axis



(b) Dimensions perpendicular to Y-axis

Fig. 3.1 Reinforcing Steel Placement in Columns

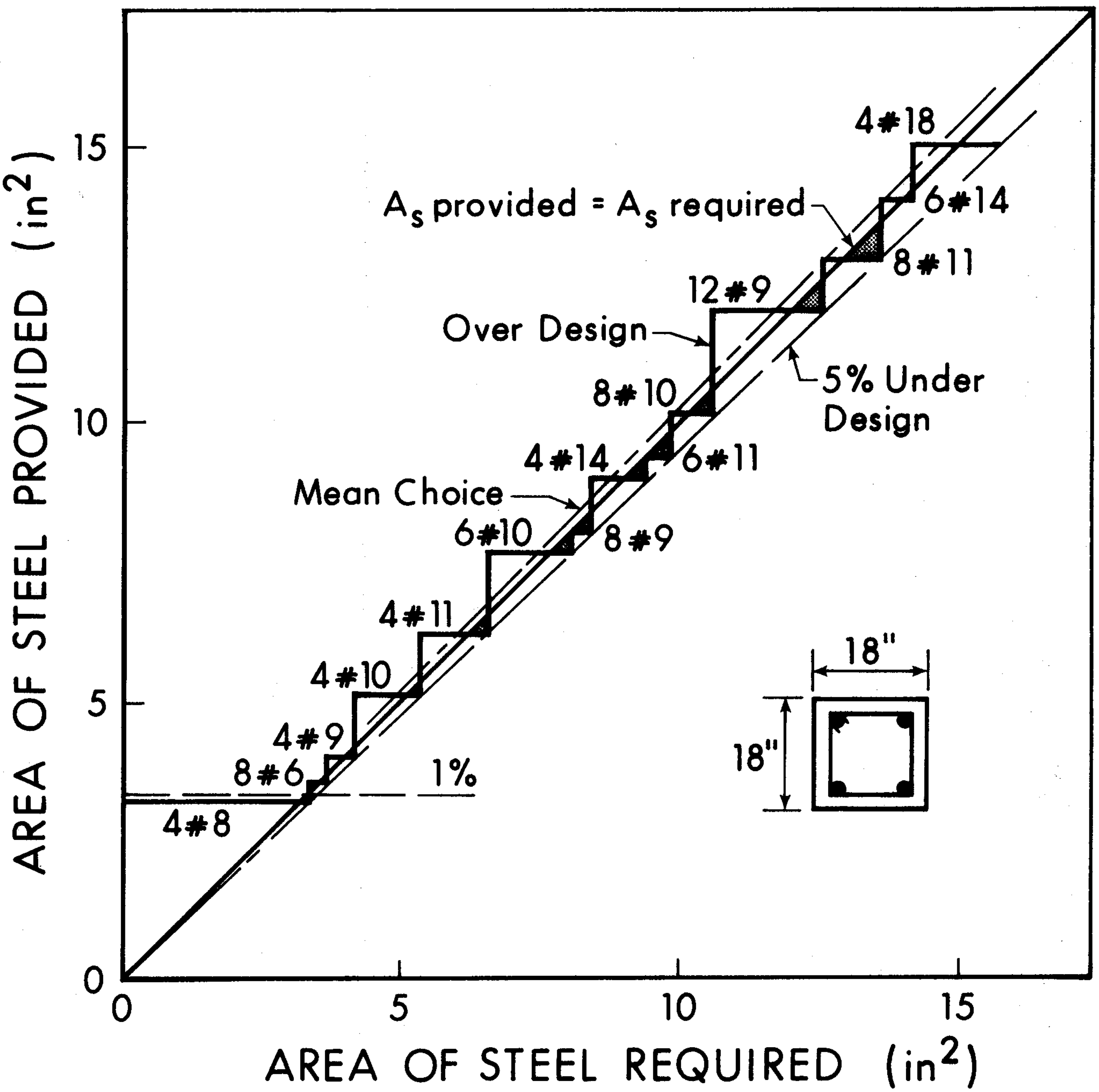


Fig. 3.2 Effect of Selection of Discrete Bar Sizes on Choice of Reinforcement in a Tied Column

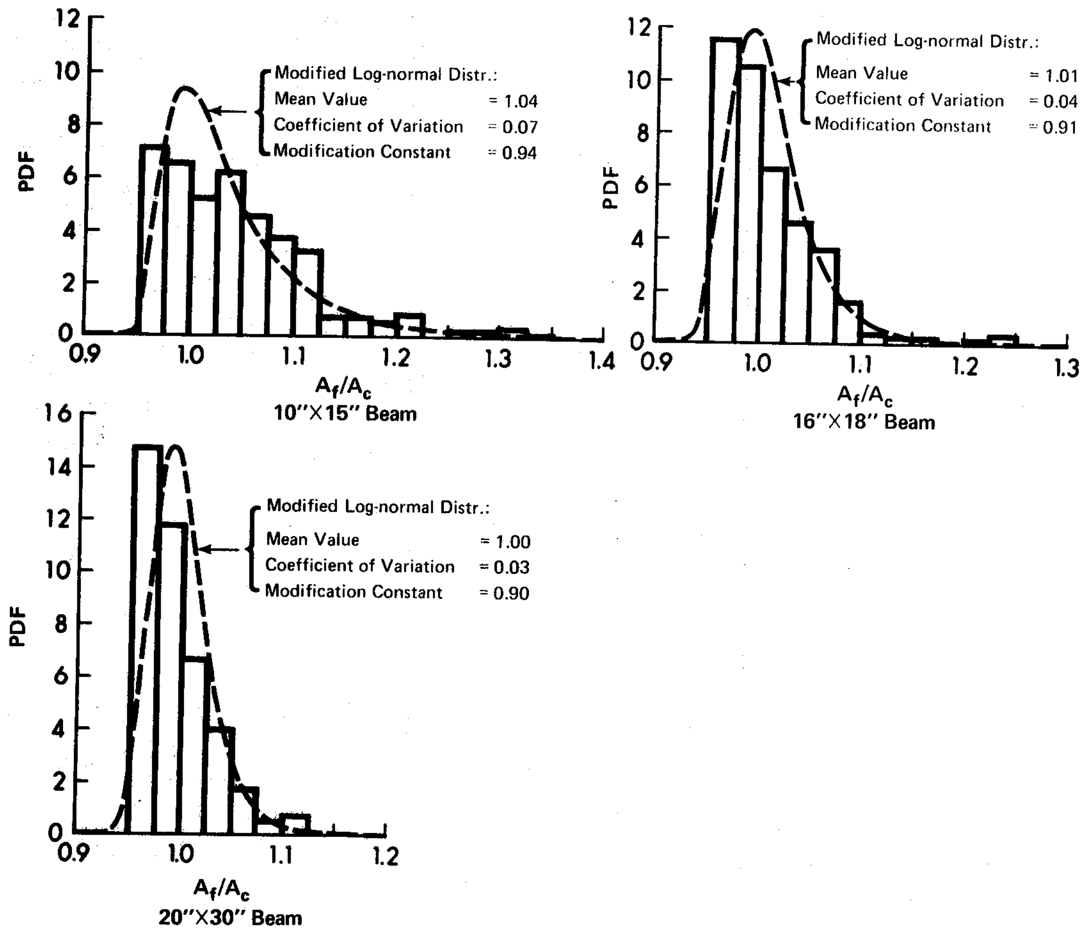


Fig. 3.3 Variation of Ratio A_f/A_c for Flexural Tension Steel in Beams

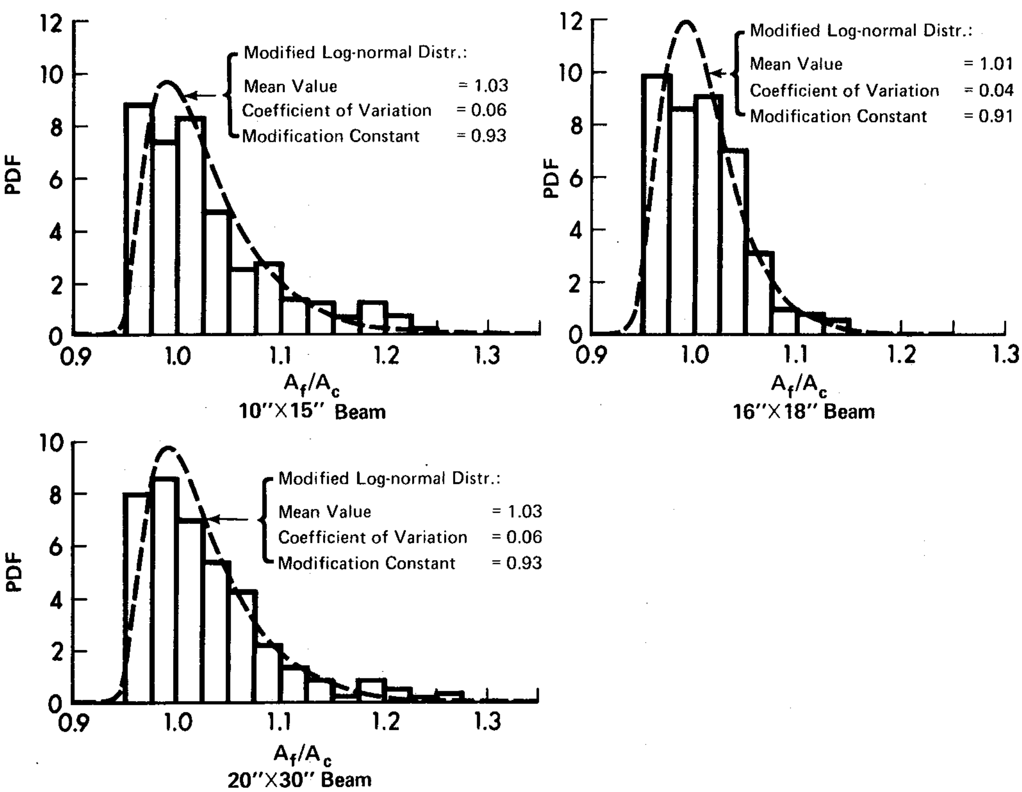


Fig. 3.4 Variation of Ratio A_f/A_c for Transverse Steel (Stirrups) in Beams

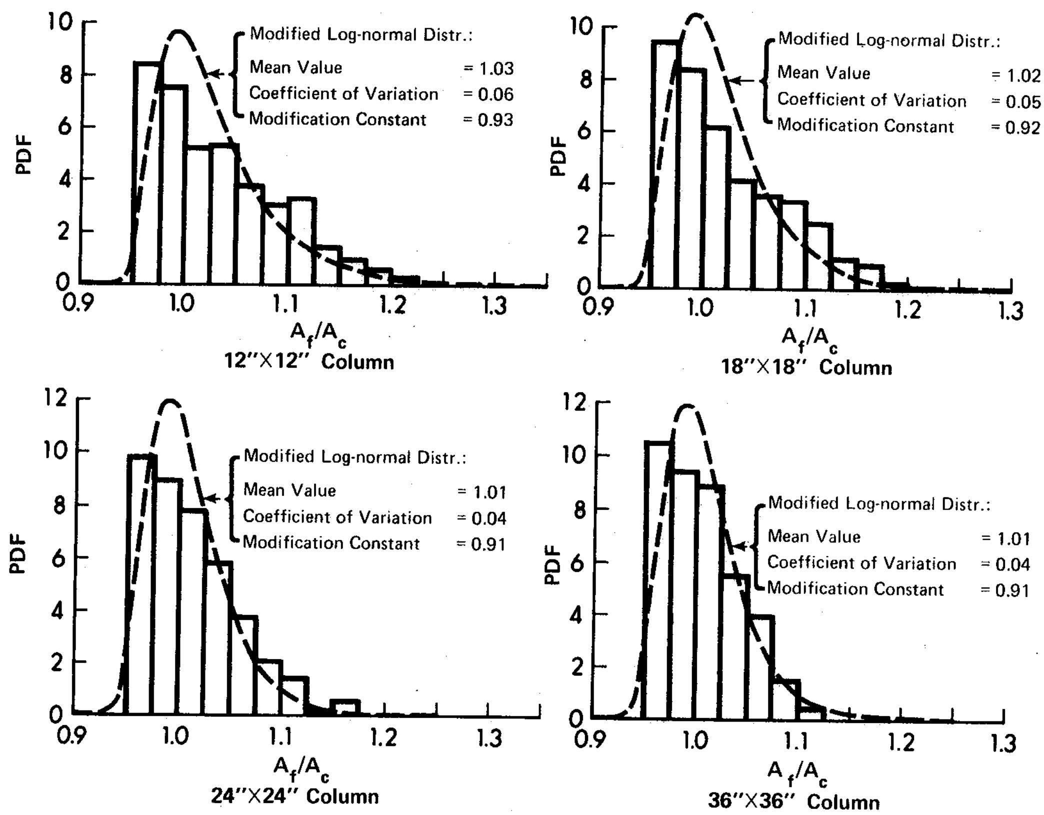


Fig. 3.5 Variation of Ratio A_f/A_c for Vertical Steel in Columns

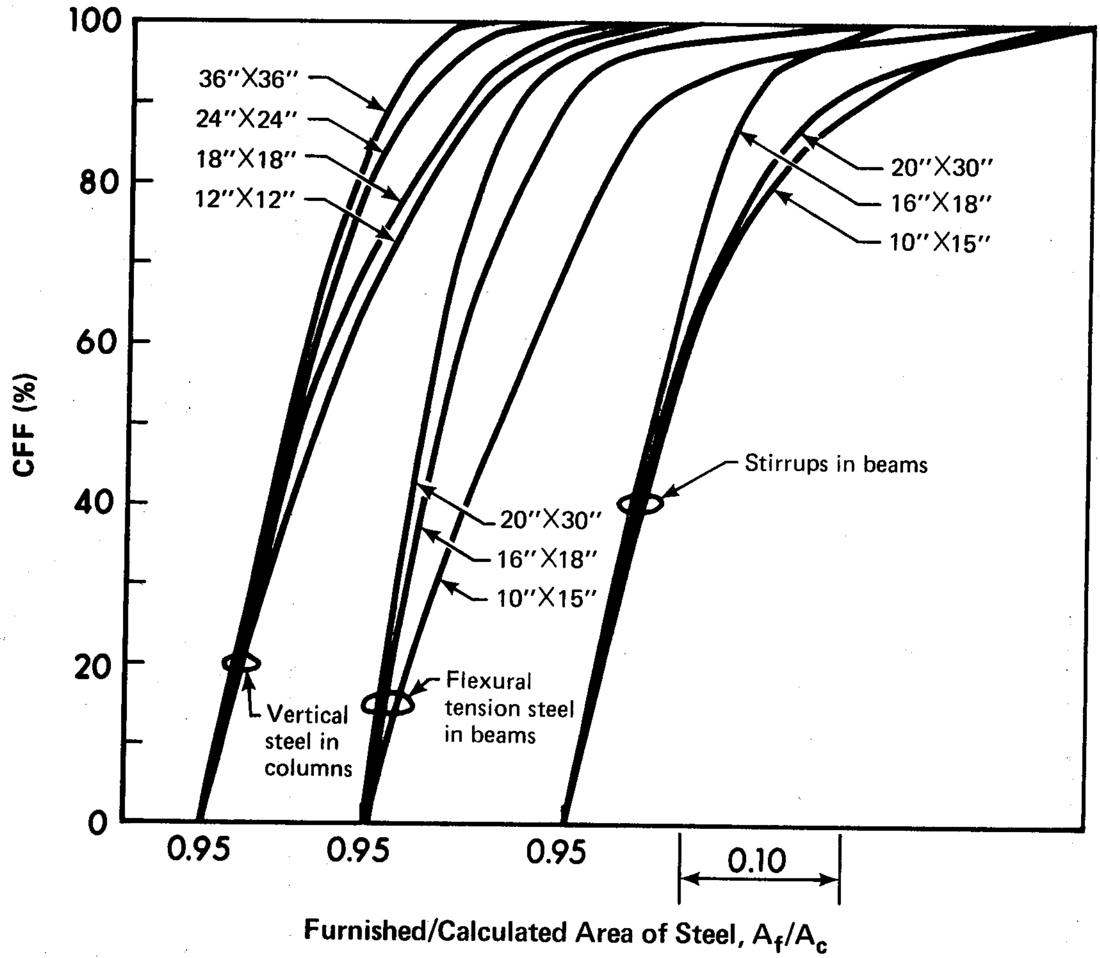


Fig. 3.6 Effect of Size of Cross Section on Distribution of A_f/A_c