University of Alberta

DEVELOPMENT OF PROBABILITY BASED RESISTANCE FACTORS AND COMPANION-ACTION LOAD FACTORS FOR CONCRETE DESIGN IN CANADA

by

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To

Gaya and Kalana

who bring joy and purpose

to my life and work

Abstract

The load and resistance factors currently used for concrete buildings need updating. They do not reflect the current levels of quality control, refined variabilities established in recent studies, load-resistance correlation in the tension failure of columns and so on. The probability methods and design formats can be improved to achieve more consistent reliability. The primary objective of this investigation is to develop an optimum set of load and resistance factors for the design of concrete structures in Canada.

Variables that affect the reliability of a structure are established in probabilistic terms. Present levels of quality control and specifications are reflected in these variables. The companion action load factor format is selected because it models real load combinations more accurately than the probability factor format and thus leads to more consistent reliability. Acceptable levels of reliability are established by evaluating existing successful practice (viz., CSA A23.3-M84) and by reviewing recommendations of other researchers.

The reliability analysis is based on a variable limit state equation developed by combining the 'true' limit state, where the structure is about to fail, and the design limit state that is used in the design office. This method proved to be a versatile analysis tool.

The load and resistance factors and load combinations are developed for use with design methods specified in CSA A23.3-94. Practical limitations and simplicity of load combinations are taken into account. The resistance factor for concrete is increased leading to a general improvement in economy. Adequate and more uniform safety is obtained through revised load combinations and factors.

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a	=	shear span in shear resistance or depth of compressive stress block for flexural resistance
A'_s	=	compression reinforcement area
A_I	=	influence area
A_s	=	tension reinforcement area
A_T	=	tributary area
A_v	=	area of one vertical stirrup
B	=	modelling parameter
b_w	=	web width
c	=	influence coefficient
C_a	=	accumulation factor
C_b	=	basic roof snow load factor
C_d	=	directional effect factor
C_e	=	exposure height factor
C_g	=	dynamic gust response factor
C_{gr}	=	ground to roof conversion factor
C_m	=	modeling uncertainty factor
C_p	=	pressure coefficient or the shape factor
C_s	=	slope factor
C_w	=	wind exposure factor
C_w	=	wind pressure calculation factor
D	=	dead load
d	=	depth
E	=	earthquake load
e	=	eccentricity
E(X)	=	expected value of variable X
E_{cistrR}	=	initial tangent modulus of elasticity of in-situ concrete for a loading rate
F_1	=	the ratio of the average strength of standard cylinder specimens to the specified strength f_c
F_2	=	the ratio of the average *in-situ* strength to the average strength of 28 day old standard cylinder specimens
f_c'	=	specified concrete strength
f_{cstr35}	=	*in-situ* compressive strength of concrete tested at the standard loading rate of 35 psi/sec
f_{cstrR}	=	*in-situ* concrete compressive strength loaded at a rate of R psi/sec
f_{rstrR}	=	*in-situ* concrete flexural tensile strength loaded at a rate of R psi/sec
f_{spstrR}	=	*in-situ* concrete splitting tensile strength loaded at a rate of R psi/sec
$F_X(x)$	=	cumulative distribution function (CDF) of variable X at a value x

$f_X(x)$ = probability density function (PDF) of variable X at a value x
f_{ys} = yield strength of stirrups
h = height
L = live load due to occupancy
L_{max} = maximum total live load
L_o = minimum specified uniform live load
L_s = sustained live load
L_{smax} = maximum sustained live load
M = true variable applied moment
M_{fn} = factored nominal applied moment
M_{nr} = factored nominal moment resistance
$M_{nr\text{-}pure}$ = factored nominal pure moment resistance (Figure 7.20)
M_o = true uncertain moment resistance
M_{pure} = true pure moment resistance in true space (Figure 7.19)
P = true axial load (random variable)
p = external or internal wind pressure
P_{fn} = factored nominal axial load
P_o = true uncertain axial resistance
Q = load
q = reference wind pressure
Q_{lv} = long term variable load
Q_p = permanent load
Q_{sv} = short term variable load
Q_v = variable load
R-square = coefficient of determination
r_{creal} = random variable relating real cylinder strength to design compressive strength
r_D = normalized dead load variable = D/D_n
$r_{in\text{-}situ}$ = random variable relating *in-situ* strength to real cylinder strength
R_{nr} = factored nominal resistance which is a deterministic value
R_o = true resistance (random variable)
r_R = random variable relating strengths at R psi/sec and 35 psi/sec
r_{vi} = normalized ith variable load = Q_{vi}/Q_{vni}
S = snow load
s = stirrup spacing
S_r = rain on snow load
S_s = ground snow load
T = design period in years
t_i = holding time of the principal variable load
t_j = holding time of the companion variable load
V = coefficient of variation (COV) , wind velocity in wind load calculations, base shear in earthquake load calculations

v	=	zonal velocity ratio or Peak Horizontal Ground Velocity (PHV)
Var(x)	=	variance of x
V_c	=	shear resistance due to concrete
V_d	=	daily maximum wind velocity
V_f	=	factored shear force
V_{model}	=	COV of the calculation model itself
V_s	=	shear resistance provided by shear reinforcement
V_{spec}	=	represents errors introduced by the test specimen
$V_{t/c}$	=	COV obtained directly from the comparison of the measured and calculated strengths
V_{test}	=	represents the uncertainties in the loads measured in an experiment
W	=	wind load
w_i	=	weighting for load ratio i
y	=	depth of ground snow

α	=	load factor
α_1	=	ratio of average stress in rectangular compression block to the specified concrete strength
α_p	=	load factor for permanent load
α_v	=	load factor for variable load
β	=	reliability index
β_1	=	ratio of depth of rectangular compression block to depth of the neutral axis
β_T	=	target reliability index
χ	=	a factor which is 1 if (β - β_T) is positive and 2 if it is negative
δ	=	bias factor
ϕ	=	resistance factor
ϕ_c	=	resistance factor for concrete
ϕ_s	=	resistance factor for reinforcement
$\Phi(x)$	=	cumulative dstribution function of the standard normal variable X at value x
γ	=	weight density of snow
μ	=	mean
ν	=	arrival rate of an infrequent event/load
θ	=	slope of the idealized tension failure line in true space (Figure 7.19)
θ_n	=	slope of the idealized tension failure line in factored nominal space(Figure 7.20)
ρ	=	longitudinal reinforcement ratio for columns tension reinforcement ratio for beams
ρ_v	=	stirrup ratio = $A_v/b_w s$

σ	=	standard deviation
ψ	=	load combination probability factor
ψ_{ij}	=	factor applied to the j^{th} characteristic load to convert it to a companion load (when combined with the ith principal load)

<u>subscripts</u>

c	=	compressive strength
D	=	dead load
d	=	daily
E	=	earthquake load
f	=	factored
i	=	related to the i^{th} load
j	=	related to the j^{th} load
L	=	live load due to occupancy
n	=	nominal
o	=	true resistance variable
R	=	loading rate R psi/sec (R x 6.895 x 10^{-3} MPa/sec)
r	=	factored resistance
S	=	snow load
str	=	in-situ strength or strength in a structure
W	=	wind load

LIST OF ABBREVIATIONS

a.p.t.	=	arbitrary-point-in-time
ACI	=	American Concrete Association
ANSI	=	American National Standards Institute
ASCE	=	American Society of Civil Engineers
CEB	=	*Comite Euro-International du Beton*
COV	=	coefficient of variation
CSA	=	Canadian Standards Association
MRI	=	mean recurrence interval (return period)
NBCC	=	National Building Code of Canada
PDF	=	probability density function
EUDL	=	equivalent uniform distributed load
std.dev.	=	standard deviation
CDF	=	cumulative distribution function

CHAPTER 1

INTRODUCTION

1.1 Structural Reliability

The safety of a structure depends on many parameters, such as applied loads, material strengths, member dimensions, modelling and analysis parameters. Recognizing that these parameters are probabilistic variables, safety can be assessed only in probabilistic terms. The reliability of a structure can be defined as the probability that the structure is safe. This probability can be defined at various levels of accuracy which depend on the idealization of the structural reliability. The accuracy of the method chosen in a reliability analysis should be compatible with the accuracy of the parameters and variables used in the study.

The design formats used to idealize and combine design loads and resistances should be chosen to represent the real situation as accurately as possible. They should also be simple and clear enough to be used in design. The load and resistance factors should ensure adequate safety in all possible situations. An optimum set of load and resistance factors and design formats should provide uniform safety for all structural members, structural actions and load ratios.

1.2 Scope of Thesis

The objective of this thesis is to develop an optimum set of load and resistance factors to be used in the design of concrete structures in Canada. These factors are developed for ultimate limit state design considering flexure, combined flexure and axial load, and shear. Special consideration is given to tension failure of short columns where the axial load effect acts to prevent failure. The load and resistance factors and load combinations are developed for use with design methods specified in CSA A23.3 - 94.

It is necessary to establish acceptable measures of reliability upon which to base the factors. The desirable target level of safety, i.e., a target reliability index, β_T, is chosen by observing reliability indices implicit in existing successful practice and by reviewing levels of safety used and recommended by other researchers. The reliability indices implicit in practice are obtained by evaluating the reliability of CSA A23.3-M84.

The reliability analysis is based on a limit state equation. A method of formulating a limit state equation is developed by combining the 'true' random limit state equation where the structure is about to fail and the design limit state equation that is used in the design office. This method is a versatile problem solving tool as it is able to model the real life limit states for complex situations, without difficulty.

A load factor format and a resistance factor format that could provide uniform safety with varying load ratios and design situations are chosen. The loads and resistances are defined and established in probabilistic terms. The basic variables on which the load effect and resistance depend are obtained from environmental data or from the literature. Using these basic variables the loads and member resistances are simulated using Monte Carlo simulation methods. Variable modelling and distribution fitting are used to establish the type of statistical distribution. Current levels of quality control and material variability are reflected in these variables.

1.3 Organization of Thesis

Chapter 2 outlines the reliability and probability theory that is used in this study. In Chapter 3 different safety formats are discussed and a load factor format and resistance factor format are selected. Load and resistance variables are established, statistically modelled, and simulated in Chapters 4 and 5. The desirable target level of safety, i.e., a target reliability index, β_T, is investigated and selected in Chapter 6. Chapter 7 gives details of the reliability analysis and optimization of safety factors. The reliability obtained using the proposed factors is compared with that of the existing factors in CSA A23.3-94. Chapter 8 gives a summary of the work and the conclusions, and suggests areas of future research.

CHAPTER 2

THEORETICAL BASIS

2.1 Introduction

This chapter outlines the types of reliability theories and other theories used in this study. In carrying out this study, simulation methods and distribution fitting methods are developed. These methods are also presented herein.

2.2 Reliability Methods

The reliability of a structure may be defined as the probability that the structure is safe. There are many idealizations and reliability models of structures. Madsen et al.(1986) ciassify these structural reliability methods based on the information used in the analysis.

Reliability methods that employ only one "characteristic" value of each variable are called level I methods. Most design formats used in a design office, including load and resistance factor formats, are examples of level I methods.

Reliability methods that employ two parameters of each variable (commonly the mean and variance) are called level II methods. Reliability index methods are examples of level II methods.

Reliability methods that employ probability of failure as a measure, and therefore require a knowledge of the joint distribution of all variables, are called level III methods.

Finally, a reliability method that compares a structural prospect with a reference prospect according to the principles of engineering economic analysis is called a level IV method. In these methods the costs and benefits of construction, maintenance, repair, consequences of failure, and interest on capital are considered. Such methods are appropriate for structures of major economic importance.

In this study, the load and resistance factor method which is a level I method used in design codes, is calibrated by a level II method. Therefore, level II methods and other related theory necessary to implement such a method are presented here.

2.3 Reliability Indices

The definitions and notation in this section are based on Madsen et al. (1986). Reliability index methods are level II methods. They are also called second moment methods as the uncertainties of the structural reliability are expressed in terms of expected value (first moment) and covariance (second moment) of each relevant parameter. These parameters are called the basic variables and are denoted by random variables Z_i. The basic variables include loading parameters, strength parameters and other variables associated with geometrical variations and model uncertainty. It must be possible for each set of values, z_i, of the basic variables to state whether or not the structure has failed. (The random variables are denoted by upper case letters, Z_i, and the sets of values are denoted by lower case letters, z_i, according to standard probability notation.) This leads to a division of the z-space into a safe set S and a failure set F. The two sets are separated by a failure surface, L_z, defined by the limit state function, $g(z_i)$ as follows:

[2.0]
$$g(z_i) > 0 \qquad z_i \in S$$
$$g(z_i) = 0 \qquad z_i \in L_z$$
$$g(z_i) < 0 \qquad z_i \in F$$

The failure surface for a simple two variable case is shown in Fig. 2.1(a).

The different types of reliability indices are defined based on the shape of the failure surface and the second moment representation of the basic variables (Madsen et al. 1986). These reliability indices are defined below.

2.3.1 Cornell Reliability Index

The random variable obtained by replacing the parameters z_i in the failure function with the corresponding random variables Z_i is called a safety margin, M, given by $M = g(Z_i)$. The safety margin can also be defined as the capacity minus the demand and takes a negative value for failure situations and positive value for safe situations. The Cornell reliability index, β_c, is defined as

[2.1 a] $\beta_c = E(M) / \sigma_M$ or,

[2.1 b] $\beta_c \, \sigma_M = E(M)$

where, $E(M)$ = expected value of M

σ_M = standard deviation of M

Thus β_c is a measure of how far the failure surface is from E(M) in units of σ_M.

2.3.2 Mean Value First Order Second Moment Reliability Index

The safety margin, M, is often a non-linear function and consequently its moments cannot be calculated with the second moment representation of the basic variables. One way to avoid this problem is to linearize the safety margin around some point. If the linear safety margin, M_{FO}, is used in the definition of β then the reliability index is called the first order second moment reliability index. Often the mean value point is used as the linearization point and the reliability index is known as the mean value first order second moment reliability index.

This method has many drawbacks because of the arbitrariness of the linearization point. This causes errors when the $g(z_i)$ function is non-linear and the mean value is some distance from the failure surface. It also causes mechanical variance, i.e., β is dependent on how the limit state is formulated. These problems can be avoided if the linearization point is on the failure surface. On the failure surface $g(z_i)$ and its derivatives are independent of the formulation.

2.3.3 Hasofer and Lind Reliability Index

The reliability index can be interpreted as a measure of the distance to the failure surface. In the one dimensional case the standard deviation of the safety margin is used as the scale. Hasofer and Lind (1974) proposed a similar scale in the case of many basic variables. They used non-homogeneous linear mapping of basic variables to obtain reduced variables that are a set of normalized and uncorrelated variables, X_i. These X_i are standard normal variables. The Hasofer and Lind reliability index is the distance in the reduced space between the origin and the closest point to the origin on the failure surface.

The point on the failure surface which is closest to the origin is called the "design point" where failure is most likely to occur. To find this design point, as shown in Fig. 2.1, the basic variables, Z_i, are transformed to reduced variables, X_i, with zero mean and unit variance,

[2.2] $$X_i = \frac{Z_i - \mu_{Zi}}{\sigma_{Zi}}$$

where μ_{Zi} = mean value of Z_i

σ_{Zi} = standard deviation of Z_i

For a problem involving two variables, X_1 and X_2, the reduced variable space can be plotted as shown in Fig. 2.1(b). Here the circles represent values of X_1 and X_2 which are one, two and three standard deviations from the origin.

Substituting [2.2] in $g(z_i) = 0$

[2.3] $$g_X(x_i) = 0$$

The failure surface defined by this equation is shown in Fig. 2.1(b) for the two variable case.

The design point with coordinates (x_1^*, x_2^*, ...x_n^*) is the point on this surface which is closest to the origin. It is determined by solving the following system of equations (Ellingwood et al. 1980) and searching for the direction cosines α_i that minimize β.

[2.4] $$\alpha_i = \frac{\partial g_X / \partial X_i}{\left[\Sigma(\partial g_X / \partial X_i)^2\right]^{1/2}}$$

[2.5] $$x_i^* = -\alpha_i \beta$$

[2.6] $$g_X(x_1^*, x_2^*, ...x_n^*) = 0$$

The α_i are a measure of the sensitivity of the reliability to the independent variables. If the reliability is insensitive to the independent variable it can be disregarded.

2.4 Reliability Methods Used

The Hasofer and Lind reliability index is used to assess the reliability because it measures the reliability at the design point where failure is most likely to occur. When the limit state function is linear, [2.4] to [2.6] can be solved easily. But when the limit state function is non-linear the design point has to be assumed and an iterative process has to be used to obtain the Hasofer Lind reliability index.

The Hasofer and Lind equations assume that the variables Z_i are distributed normally. Because this is not always true in structural reliability, a normal approximation of the tail is used for each variable. This normal tail approximation is made at the design point so that both the true distribution and the approximated normal distribution have the same ordinate and the same tail area. An algorithm to find the design point and the related β while approximating the true distribution with a normal distribution was developed by Rackwitz and Fiessler (1978). Further details of this algorithm can be found in Madsen et al. (1986). The FORM (First Order Reliability Methods) subroutine (Gollwitzer et al. 1990) that implements this analysis is used in this study.

2.5 Simulation of Variables and Modelling Variables

For problems involving mathematical combinations of random variables with known or assumed probability distributions, Monte Carlo simulation is appropriate to model the process. In each simulation, a particular set of values of the random variables is generated in accordance with the respective distributions. These values are combined according to the mathematical equation to generate one value of the variable being simulated. By repeating the process, a simulated sample of the variable is obtained. This simulated sample is then analyzed and fitted with various statistical distributions until a satisfactory distribution is found. Since we are interested in the rare events, where the value of the variable is some distance from the mean value, the distribution is generally fitted in the appropriate tail region.

The distributions of the basic variables, on which the load or resistance depend, are established in Chapters 4 and 5. Therefore, the member resistance or loads can be simulated using these distributions.

In this process it is necessary to generate values of the variables which have specific probability distributions. For example, to simulate the flexural resistance of a beam it is necessary to generate values of f_y and f_c' from their specific probability distributions. This can be achieved by first, generating a uniformly distributed random number between zero and one, and then, by obtaining the corresponding random number with the specified probability distribution through appropriate transformations. One method of doing so, that of Ang and Tang (1984b), is described here.

Let a random variable X have a cumulative distribution function (CDF) of $F_X(x)$. Then, as shown in Fig. 2.2,

[2.7] $$F_X(x) = x_u$$

[2.8] $$x = F_X^{-1}(x_u)$$

x_u is a value of the standard uniform variate, X_u.

$$F_{Xu}(x_u) = x_u$$

$$P(X_u \leq x_u) = x_u$$

$$\begin{aligned} P(X \leq x) &= P[F_X^{-1}(X_u) \leq x] \\ &= P[X_u \leq F_X(x)] \\ &= F_X(x) \end{aligned}$$

This means that if $(x_{u1}, x_{u2}, \ldots x_{un})$ is a set of values from X_u then the corresponding set of values obtained through $x_i = F_X^{-1}(x_{ui})$ will have the desired CDF $F_X(x)$.

Generation of random numbers in this way is called the inverse transform method. This method has limitations as the $F_X^{-1}(x_u)$ has to be available. There are many probability distributions for which the CDF cannot be inverted analytically. For such cases, other methods are available (Ang and Tang 1984b). Methods of transforming variables are developed in conjunction with the principles of the inverse transform method. The methods developed are explained below.

The simulations are made using the statistical software SAS (SAS Institute Inc. 1993). In this program a random normal distribution is generated using an internal function. Therefore lognormal distributions could also be generated using appropriate transformations. Since the Gumbel distribution is a double exponential function, the transformation can be performed by taking the double $\log_e$ of the cumulative probability.

These methods are used in the SAS programs shown in Appendix A and B. The three most commonly used distributions are the normal, lognormal and Gumbel. The generation of these are explained below.

2.5.1 Generating a Variable That has a Normal Distribution

The SAS program can generate a random standard normal distribution by using the function *rannor(seed)*. The result is a random number that fits a normal distribution with a mean equal to 0, and a standard deviation of 1.0. The 'seed' value is taken as -1 as recommended in the SAS Manual (SAS Institute Inc. 1993). For example, to simulate the resistance of a column in bending and compression it is necessary to generate values of the modulus of elasticity of steel reinforcing bars, E_s. The variable E_s has a normal distribution with a mean of 201,000 MPa and a standard deviation of 6598 MPa (as established in Chapter 5). A random value of E_s is generated by;

[2.9] $$E_s = 201000 + 6598 \times rannor(-1)$$

2.5.2 Generating a Variable That has a Lognormal Distribution

This is done by using the relationship between the normal and lognormal distributions. It is illustrated by generating values of the yield strength, f_y, which has a shifted lognormal distribution with a μ_{fy} equal to 442 MPa, σ_{fy} equal to 26.5 MPa and shift equal to 365 MPa (as established in Chapter 5). Therefore, f_y - 365 has a lognormal distribution with a mean of 442-365 MPa and a standard deviation of 26.5 MPa. Then, based on the definition of a lognormal distribution, $\ln(f_y-365)$ is normally distributed with mean, λ, and standard deviation, ζ. From the equation of the mean and standard deviation of a lognormal distribution

[2.10] $$\mu_{fy} - \text{shift} = 442\text{-}365 = \exp(\lambda + \zeta^2/2)$$

[2.11] $$\sigma_{fy} = 26.5 = \exp(\lambda)\sqrt{\exp(\zeta^2)\left[\exp(\zeta^2) - 1\right]}$$

By solving simultaneous equations [2.10] and [2.11],

$\lambda = 0.334$

$\zeta = 4.288$

Using these $\ln(f_y - 365)$ is generated and consequently f_y as shown below

$\ln(f_y - 365) = 0.334 + 4.288 \times$ *rannor(-1)*

$f_y = \exp\,[0.334 + 4.288 \times$ *rannor(-1)*$] + 365$

2.5.3 Generating a Variable That has a Gumbel Distribution

The following transformation is done for Gumbel distributions. For wind and snow parameters the maximum value in 30 years is generated by using the annual maximum established from climatic data.

Let X be the annual maximum variable which has a Gumbel distribution.
Then X_{30}, maximum value in 30 years also has a Gumbel distribution.

$F_X(x) = \exp\{-\exp[-\alpha\,(x - u)]\}$

From extreme value theory,

[2.12] $$F_{X30}(x_{30}) = [\exp\{-\exp[-\alpha\,(x_{30} - u)]\}]^{30}$$

From [2.7],

$x_u = [\exp\{-\exp[-\alpha\,(x_{30} - u)]\}]^{30}$

[2.13] $$x_{30} = u - \frac{\ln\left[-\ln\left(\sqrt[30]{x_u}\right)\right]}{\alpha}$$

Equation [2.13] is used to generate the required Gumbel distributions.

In the case of wind velocity the annual maximum wind velocity, V, for Vancouver has a Gumbel distribution with parameters u equal to 65.1 km/hr and α equal to 0.151 $(\text{km/hr})^{-1}$ (as established in Chapter 4). Therefore using [2.13], and similar format for 10 year and 100 year maximum, the following equations are used to generate the required wind velocities. The *ranuni(seed)* function in the SAS software generates a variable with a uniform distribution from 0.0 to 1.0.

$x_u = ranuni(-1)$

$V_{10} = 65.1 - \{\ln[-\ln(x_u^{1/10})]\}/0.151$

$V_{30} = 65.1 - \{\ln[-\ln(x_u^{1/30})]\}/0.151$

$V_{100} = 65.1 - \{\ln[-\ln(x_u^{1/100})]\}/0.151$

2.6 Distribution Fitting and Parameter Estimation

Distribution fitting and parameter estimation is done by transforming variables and using regression analysis (least squares estimation) to check the fit and obtain parameters. These methods are used in the SAS programs shown in Appendix A and B. The three most commonly used distributions the normal, lognormal and Gumbel are explained below. General methods of goodness of fit, such as the Kolmogrov-Smirnov test, were not used in this study because most of the distributions that are encountered here are not normal distributions. The data was always examined by viewing scatter plots and percentile plots to check for outliers which might unduly affect the results of the regression. The outlier cutoffs are taken to be $\mu \pm 2.7\,\sigma$ (Jobson 1991). In the climatic observations such outliers were left out of the analysis as they probably were erroneous observations. Because the outliers were removed, the maximum likelihood estimates obtained from the least squares estimation were not affected by a single erroneous data point.

If the simulated variable is called 'simx', by ordering the simulated data, and using a plotting function, a data set of simx and corresponding cumulative probability, F_i, can be obtained. The plotting function for i^{th} ordered value is taken as F_i equal to $i/(n+1)$ where n is the number of simulated values used for fitting.

2.6.1 Checking the Fit of Data to a Normal Distribution

The following method is developed to check the fit of simulated data to the normal distribution.

The SAS program has an inverse function of the standard normal distribution. This is called *probit*(F_i) function.

If 'simx' is normally distributed with mean, μ, and standard deviation, σ, then $\frac{\text{simx} - \mu}{\sigma}$ will have a standard normal distribution.

Therefore, $$probit(F_i) = \frac{\text{simx} - \mu}{\sigma}$$

[2.14] $$probit(F_i) = \frac{1}{\sigma}\text{simx} - \frac{\mu}{\sigma}$$

If the simx is normally distributed, regressing *probit*(Fi) on simx would give a value of R-square (coefficient of determination) close to 1.0 and the intercept and variable coefficients will give the parameters μ and σ of simx. This type of regression is used to check the fit and find the best fit distribution parameters.

2.6.2 Checking the Fit of Data to a Lognormal Distribution

Checking the fit of data to a lognormal distribution is done by using the relationship between the normal and lognormal distributions. The inverse function of the standard normal distribution, *probit*(F_i), is used as in section 2.6.1.

If simx is lognormally distributed then ln(simx) is normally distributed, say with parameters λ and ζ. Then $\frac{\ln(\text{simx}) - \lambda}{\zeta}$ would have a standard normal distribution.

Therefore, $$\frac{\ln(\text{simx}) - \lambda}{\zeta} = probit(F_i)$$

$$\ln(\text{simx}) = \zeta\,[probit(F_i)] + \lambda$$

If the regression of ln(simx) on *probit*(Fi) gave good fitness parameters, it implies that simx is lognormally distributed. Further the parameters λ and ζ can be obtained from the regression coefficients. From these values, using [2.10] and [2.11], the parameters μ and σ of simx can be obtained.

2.6.3 Checking the Fit of Data to a Gumbel Distribution

If simx has a Gumbel distribution then:

$$F_X(\text{simx}) = F_i = \exp\{-\exp[-\alpha\,(\text{simx} - u)]\}$$

$$-\ln\,[-\ln(F_i)] = \alpha\ \text{simx} - \alpha\, u$$

Therefore by regressing $-\ln\,[-\ln(F_i)]$ on simx the fit can be checked and the distribution parameters α and u can be found from the regression coefficients.

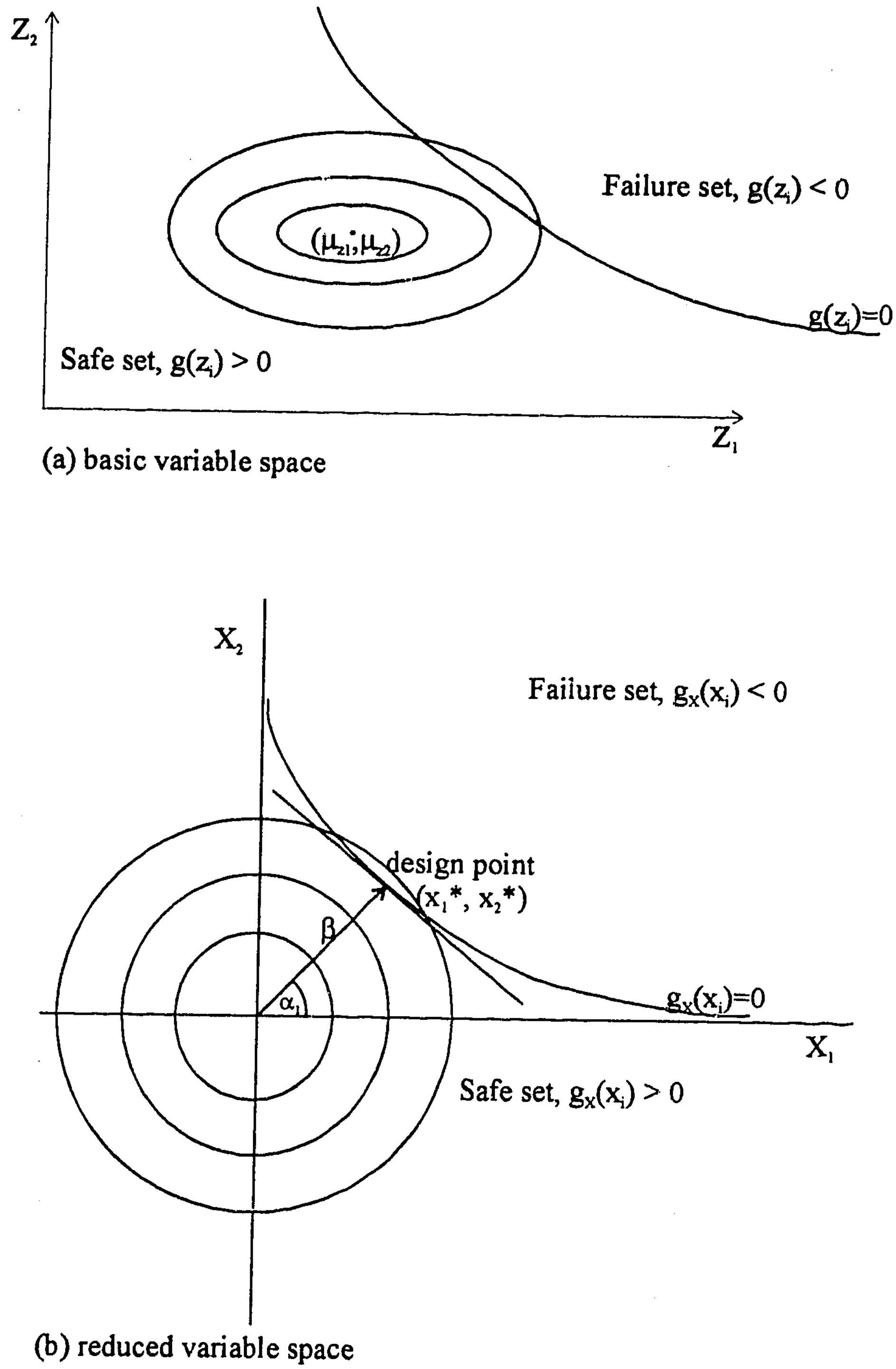


Figure 2.1 - Variable transformation in Hasofer and Lind reliability index

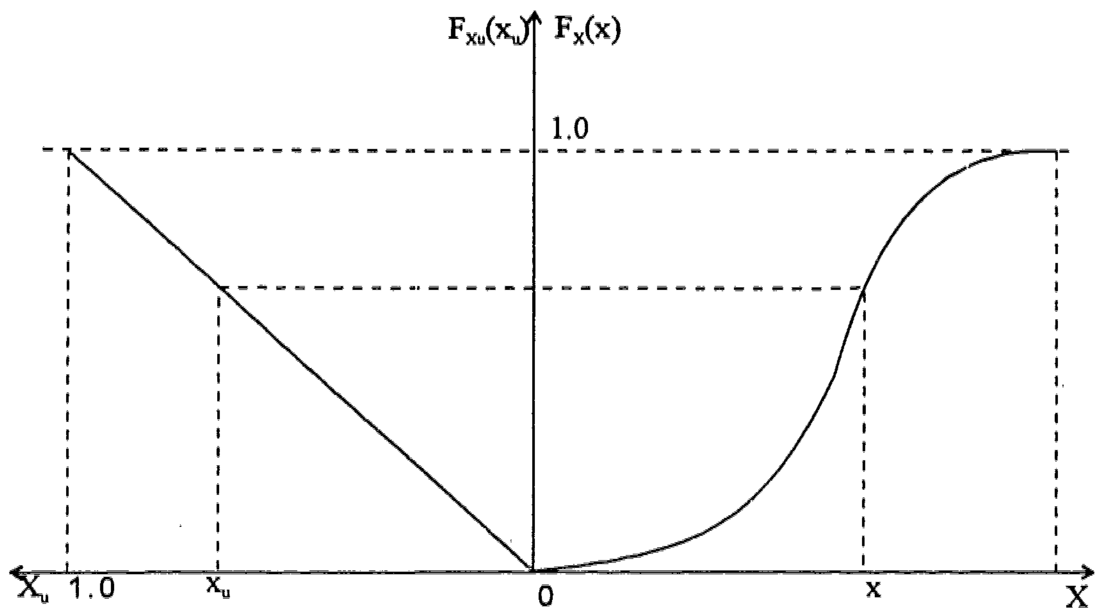


Figure 2.2 - Inverse transform method

CHAPTER 3

SAFETY FORMATS

3.1 Introduction

Fluctuations in loads, variability in material and dimensional properties, and uncertainties in analytical models must be considered in providing safety in a structural design. Different safety formats have been adopted by various structural design codes to address these items. In current limit state design codes, the two main steps in providing safety are: specifying loads and material properties at a defined exceedance probability, and applying partial safety factors to these specified loads and nominal resistances.

In the first step, a margin of safety is provided by defining the specified loads at a low probability of exceedance, generally around 5%, and by defining the specified material strengths at a higher probability of exceedance, generally around 90 to 95%. This is because the loads and material properties are the most critical variables in structural safety. The dimensional and model variabilities can be implicitly taken into account in the partial safety factors and code formulations for sectional resistance, or can be considered in the design calculations.

In the second step, partial safety factors are developed to provide a predetermined margin of safety between the specified loads and resistances. There are several formats in which partial safety factors are applied to loads and resistances. These formats are presented and summarized herein and their relative merits are discussed. Finally, the formats chosen for use in this study are presented.

3.2 Load Combination Formats

3.2.1 Introduction

Loads vary randomly in time and space in a very complex manner. Design loads are based on a number of simplifications including: uncoupling of loads from structural characteristics; reduction of the problem to a limited set of parameters, such as static wind pressure for wind load; and the use of conventional structural analysis to predict load effects. Time variations are normally included in design by a statistical analysis of peak values during relatively long time periods coupled with an analysis of short-time events around such peaks to obtain, for example, wind gust factors (Turkstra and Madsen 1980).

If two time-varying uncorrelated loads were considered, the sum of the two maximum loads will generally be significantly higher than the maximum value of the sum of the two loads. To obtain a realistic estimate of the maximum load one should explicitly consider the time variations of loads in the particular load combination. A structured methodology or format for load combination is essential for structural design. These formats should provide a list of realistic and critical combinations to be considered and a set of corresponding load factors to be applied to specified values of the individual loads. Such a list of combinations and a related set of load factors is developed.

3.2.2 Background

All loads vary in space, that is, if the loads on a family of similar buildings are sampled a variation would be seen even in the dead load. However the variation of loads with time is quite different for different types of loading. To provide for the many design situations which can arise, loads are categorized according to their variation in time, as either permanent or variable loads. Permanent loads (Q_p) are loads such as those due to self weight and prestressing force. Variable loads (Q_v) are quite varied in nature and can be further subdivided. Sometimes variable loads are divided according to the duration of the load, into long term variable loads (Q_{lv}), such as occupancy live loads and snow loads in some regions, and short term variable loads (Q_{sv}), such as transient live loads, wind and earthquake loads. More recently, recognizing that earthquake loads are infrequent or rare, variable loads have been further divided into frequent and infrequent loads. These infrequent or rare loads may need special consideration as explained further in the discussion of earthquake loads and the specified exceedance probability format.

For variable loads, codes specify load values, normally defined as a value that has some small probability of being exceeded in any given year. For environmental loads this probability is taken by convention as 0.033 in the NBCC (1995) and 0.02 in the ASCE 7-95 (formerly ANSI A58.1) standard. Equivalently, the design load is said to have a mean recurrence interval (MRI) equal to the inverse of this probability, i.e.; 30 years = 1/ 0.033 in the NBCC and 50 years = 1/0.02 in the ASCE 7-95 standard. In this study, the specified loads which have a small probability of exceedance are called "characteristic" values to distinguish them from frequent or companion values that are defined later.

The critical combinations of loads applied on structures involve both higher percentile loads such as characteristic loads and lower percentile loads which are encountered more often. To account for these lower percentile loads, companion loads are defined. These are the values of variable loads that are likely to be present along with another variable load that has its characteristic value. The variable load that has a maximum or a characteristic value is termed the "principal load" and the other variable loads present are termed "companion loads". As companion loads are a relatively new concept, the definition of these values are not as well established as for the characteristic loads. A more detailed explanation of companion load values used, in this study, is given in section 3.2.7.

Ferry Borges and Castanheta (1972) proposed a model to represent loads as random variables. Loads are assumed to change after prescribed, deterministic, equal, elementary intervals of time t. The interval t is called the holding time of the load. The magnitudes during different elementary intervals are identically distributed and mutually independent random variables. The holding time can therefore be defined as the shortest interval of time over which the load can be considered to be sensibly constant. They also proposed that these loads be arranged in order of increasing elementary time interval and that the combination of a pair of loads be obtained using the ratio of the two elementary time intervals and binomial theory to account for the zero values of the loads. By progressively adding two sets of loads at a time any number of loads could be combined. Turkstra and Madsen (1980) confirmed that the Borges and Castanheta processes provide a sufficient description of loads for design combination analysis.

This approach to developing load combinations, has a number of difficulties with its use. As Ellingwood et al. (1980) noted, a major short coming is the necessity of making assumptions regarding the number of basic intervals and the probability of a non-zero load value within each one. Information regarding the elementary intervals and the probability of non-zero values generally is not available nor is it easily recoverable from available load data. The safety criteria are quite sensitive to the selection of these parameters.

"Turkstra's rule" is an alternative way of handling load combinations (Turkstra 1972). It states that the maximum lifetime load is most likely to occur when one of the variable loads is at its maximum, and other loads take more frequent values.

The two basic load combination methods used in design codes are the "probability factor design format" and "companion action factor format" which are described in section 3.2.3. and 3.2.4. A third method known as the "specified exceedance probability format" described in section 3.2.5. is sometimes used for combinations including infrequent loads.

3.2.3 Probability Factor Design Format

The probability factor format is given by:

$$[3.1] \qquad \text{Total Factored Load} = \alpha_p Q_p + \psi \left(\sum_{\text{all } i} \alpha_{vi} Q_{vi} \right)$$

where

Q_p is the characteristic value of the permanent load

α_p is the permanent load factor

Q_{vi} is the characteristic value of the i^{th} variable load

α_{vi} is the load factor for the i^{th} variable load

ψ is a load combination probability factor to account for the fact that extreme values of different loads are unlikely to occur simultaneously. When only one variable load acts, ψ is equal to 1.0 . If more than one variable load acts, ψ is less than one.

The load factors α_p and α_{vi} account for variations in the loads themselves, and variations in the load effects due to uncertainties in the load models, and variations due to the structural analysis.

This format is presently used in the NBCC and in ACI 318 to specify the basic loading cases. However, some researchers consider this format to be illogical and without basis (Ferry Borges and Castanheta 1972 , Wen 1977, Turkstra and Madsen 1980) as discussed further in section 3.2.6.

3.2.4 Companion-action Load Factor Format

In some codes, especially European codes, the load combinations are considered to consist of the permanent load with one variable load at the characteristic value while all other variable loads take companion values.

This format, proposed by *Comite Euro-International du Beton* (CEB)(1976) and the Joint Committee on Structural Safety (1974), is a family of design load combinations of the general form:

[3.2] $$\text{Total Factored Load} = \alpha_p Q_p + \alpha_{vi} Q_{vi} + \sum_{\text{all } j} \alpha_{vij}\, \psi_{ij}\, Q_{vj}$$

where,

Q_p = characteristic values of permanent loads (e.g., self weight)

Q_{vi} or Q_{vj}= characteristic values of variable loads

α_p = load factor for permanent load

α_{vi} = load factor for variable load Q_{vi} when it is the principal load

α_{vij} = load factor for variable load Q_{vj} when Q_{vi} is the principal load

ψ_{ij} = factor to reduce the characteristic value of Q_{vj} to a companion value of Q_{vj}

The subscripts i, j, denote particular load types and $\psi_{ij}\, Q_{vj}$ is called the companion value of the load.

Here the variable load Q_{vi}, termed the principal load, takes a maximum value and the other variable loads, Q_{vj}, take companion values. This format is based on Turkstra's rule. For simplicity in usage, companion values of loads are presented as $\psi_{ij}\, Q_{vj}$, i.e., as a fraction of the characteristic values.

3.2.5 Specified Exceedance Probability Design Format

In a study of a calibration of the new CSA code for fixed offshore structures, Maes (1986a) divided loads into three categories; frequent loads, occasional loads and rare or infrequent loads. Infrequent loads are generated by rare events, such as earthquakes, i.e., events with a low probability of occurrence. The probability density function (PDF) for such a load has a spike at zero load when the arrival rate of the infrequent event is small. Figure 3.1(a) shows the probability distribution of an infrequent load given that an event occurs. However, such events are rare. The distribution for all time intervals in which events might occur is as shown in Fig. 3.1(b) which has a concentration of values at zero (Jordaan and Maes 1991). Infrequent loads are characterized by a probability density function that is very flat in the neighborhood of the design point. It is clear from Fig. 3.1 that the extremal PDF cannot be characterized by a COV since the expected value of the extreme load is likely to be zero for low occurrence rates of load events. It is much more important to focus on the arrival rate, ν of the infrequent event, i.e., the expected number of occurrences per year, as well as on the rate of decline of the tail of the extremal distribution.

Maes (1986a) shows that for earthquake distributions, characterized by long tails because of the many contributing mechanisms, more uniform safety is attained with design loads specified directly on the basis of an exceedance criterion, than by the use of partial factors. This is reiterated by Jordaan and Maes (1991), who found that the partial factor design format does not yield consistent safety in the case of rare events. This is chiefly due to the fact that the exceedance probability of large infrequent loads decreases in very different ways than that for frequent loads. The tail of an infrequent load depends more on the type of load, the frequency of occurrence of extreme events, the importance of modelling uncertainties and other factors. As a result, it is virtually impossible to express a single partial load factor to serve every possible application. A better approach is to specify directly the ultimate design load that would have a load factor of 1.0. This is referred to as the "specified exceedance probability design format".

The NBCC (1990) effectively adopted the specified exceedance probability design format in defining earthquake loads and earthquake load factors. The seismic load factor was separated from that of the wind and assigned the value of 1.0, in contrast to 1.5 in the NBCC (1985) which was used with a lesser load. The seismic zoning maps used in the NBCC (1990), are based on the probability of exceedance of the seismic ground motion parameters of 10% in 50 years (which is mathematically equivalent to a specified probability of exceedance of 0.0021 per annum). The previous value was 0.01 per annum used in conjunction with the higher load factor.

In essence the specified exceedance probability design format is used when the principal load is an infrequent load. The characteristic value of the principal infrequent load is specified at a low probability of exceedance such as 0.0021 per annum. A load factor of 1.0 is applied to this load.

3.2.6 Comparison of Load Factor Formats

The analytical study on load processes done by Turkstra and Madsen (1980) shows that the maximum lifetime structural load is most likely to occur when one of the variable loads is a maximum and the others take expected values. Linear combinations of a large class of loads routinely encountered in structural design were considered in their study. To evaluate the design cases, the sensitivity of results to a variety of load models and parameters was examined. It was found that fixed holding time models, as discussed in Section 3.2.2. seemed adequate for analysis of conventional design loads. Numerical analysis for combinations of sustained live load, transient live load, snow load, wind load and earthquake load was based on normalized design values, a spectrum of influence coefficients, the Rackwitz-Fiessler algorithm, and a relatively simple set of Borges-Castenheta load models. The theoretical values obtained from this numerical analysis were compared with results of three combination formats. Typical errors associated with combinations of loads according to the basic formats were examined. The formats examined were; the upper bound format in which design load effects are simply added with no correction for time variations, the probability factor format, and the companion action load factor format.

Turkstra and Madsen (1980) reported that the simple addition of design loads led to results ranging from 5 to 82% conservative compared to theoretical values for the sets of loads studied. The probability factor approach also led to significant variations relative to theoretical values. With a probability factor, ψ, of 0.7 the errors ranged from 23% of the theoretical values (that is 23% conservative compared to the theoretical values) to -23% of the theoretical values (that is 23% unconservative based on the theoretical values). Choice of suitable factors, however, can force the approach to be always conservative, but this would lead to undue conservatism and a large variation in reliability. Use of the companion action approach yielded results with an error almost always less than 10% and normally within 5% of the theoretical values in the cases studied by Turkstra and Madsen.

Ferry Borges and Castanheta (1972) used the vectorial representation of statistical loads to examine the combination of loads. For the combination of two and three variable loads, they concluded that the probability factor method is illogical. Wen (1977) in his study of statistical combination of loads stated "Design based on constant load combination factor recommended by current building codes is not risk-consistent. Depending on the relative magnitude of the individual load effects, it may seriously under or overestimate the combined load effect".

These various studies strongly suggest that the load combination factor format presented in the NBCC (1990) does not represent the actual load combinations as closely as the companion action load factor format, is not as consistent in reliability, and may lead to significant errors. Several modern codes (CEB 1976, ASCE Standard 7-88, AISC Specification 1986) have adopted the companion action load factor format.

3.2.7 Load Combination Format and Load Definitions Used

The companion action load factor format is used for load combinations including frequent loads. Companion action load factors are developed to reflect actual load combinations. The principal loads are taken at their characteristic values and accompanying loads are as defined in this section.

3.2.7.1 Selection of Companion Loads

In the companion action load factor format, the principal load is taken at its characteristic value. The accompanying loads are taken at the maximum value they are likely to have when the principal load is at its characteristic value. These are termed companion values and are defined below.

Loads are idealized by Borges-Castenheta processes with elementary time intervals. The holding times (as defined in section 3.2.2) used for different types of loads are given in Table 3.1. These values were based on the frequency of load changes as indicated by recorded loads, and recommended values given by Turkstra & Madsen (1980) and Ferry Borges and Castanheta (1972).

In view of the different possible combinations of loads, the following methods are developed for the definition of companion loads and are used in this study.

Case 1 - The holding time of the principal variable load, t_i is greater than the holding time of the companion load, t_j, as shown in Fig. 3.2

The principal variable load is taken at its characteristic maximum value. During the holding time of the principal load a number of companion load values will occur. Therefore for the critical combination, the companion load is taken as the maximum likely to occur during the holding period t_i of the principal load.

For example, consider the combination of dead, snow and wind. If the snow load is the principal variable load, the wind load would be the companion variable load. Therefore from Table 3.1, t_i equals 1 day and t_j equals 1 hour. Thus t_i is greater than t_j one could use the characteristic value of the snow, and the companion value of wind load which is the maximum hourly wind during the 24 hour holding time of the principal load. Therefore the companion value of wind load for this combination of loads is the maximum hourly wind likely to occur in any given day.

Case 2 - The holding time of the principal variable load, t_i is smaller than the holding time of the companion load, t_j, as shown in Fig. 3.3

Again the principal load is the characteristic maximum. Because the holding time of the companion load is longer than that of the principal load, only one value of the companion load will occur during the holding time of the principal load. The companion load is taken as the arbitrary-point-in-time (a.p.t.) value. This a.p.t. value is defined as the most likely load that could occur at anytime, e.g., snow has been recorded on a daily basis, therefore, the daily snow load records are used to establish the a.p.t. snow load distribution.

For example, consider the combination of dead, snow and live, with snow as the principal variable load. This gives t_i equal to 1 day and t_j equal to 8 years. Since t_i is smaller than t_j, the characteristic value of the snow load and the companion value of the sustained live load which is the a.p.t. load are used. The characteristic load is the maximum daily snow load with a 30 year return period and the a.p.t. load is the expected value of the sustained live load as obtained from survey data.

For load combinations that include earthquake loads, the specified exceedance probability design format has been used. The earthquake load, which is always the principal load, is taken at a specified probability of exceedance of 0.0021 per annum. A factor of 1.0 is applied on the earthquake load. The companion loads are taken to be the arbitrary point in time (a.p.t.) values which are the expected values at any time. For the dead load, in earthquake load combinations, a factor of 1.0 is applied because this is the likely value and because the calculated equivalent static earthquake load is dependent on the dead load or mass of the structure. With earthquake loads, the other companion loads have been taken as the a.p.t.

values and are represented as some fraction of the characteristic load. Load factors for NBCC are reevaluated based on these formats.

3.3 Resistance Factor Format

3.3.1 Introduction

In deriving resistance factors and related formats many items need to be taken into account. Some of the major items are: individual material strength variations, dimensional variability, uncertainty in the model used to estimate the resistance, type of failure (sudden or gradual), computational ease, accuracy and user acceptance (Mirza and MacGregor 1982). Traditionally the factored resistance has been written in one of two formats. These are the structural action resistance factor format and the material partial safety factor format. The relative merits of these two methods are discussed below.

3.3.2 Structural Action Resistance Factor Format

In this format the resistance factor is applied to the particular structural action such as flexure, shear, or axial compression. That is:

Factored resistance = ϕ Nominal resistance

This format is used in ACI Standard 318-89.

3.3.3 Material Partial Safety Factor Format

This format applies the resistance factors to the material strengths. That is:

Factored resistance = Resistance computed using $\phi_c f_c'$, $\phi_s f_y$ etc.

For example:

$$\text{Factored moment resistance of a beam} = \phi_s A_s f_y \left(d - \frac{\phi_s A_s f_y}{2 \alpha_1 \phi_c f'_c b}\right)$$

This format is used by CEB, CSA A23.3 and CSA S474, where material resistance factors are specified for each type of structural material. Few codes have included dimensional variations explicitly.

3.3.4 Comparison of Resistance Factor Formats

A comparison of the structural action resistance factor format and material partial safety factor format is given in Table 3.2. As seen in the table, the two methods are comparable and for final reliability they can give similar results.

3.3.5 Resistance Factor Format Used

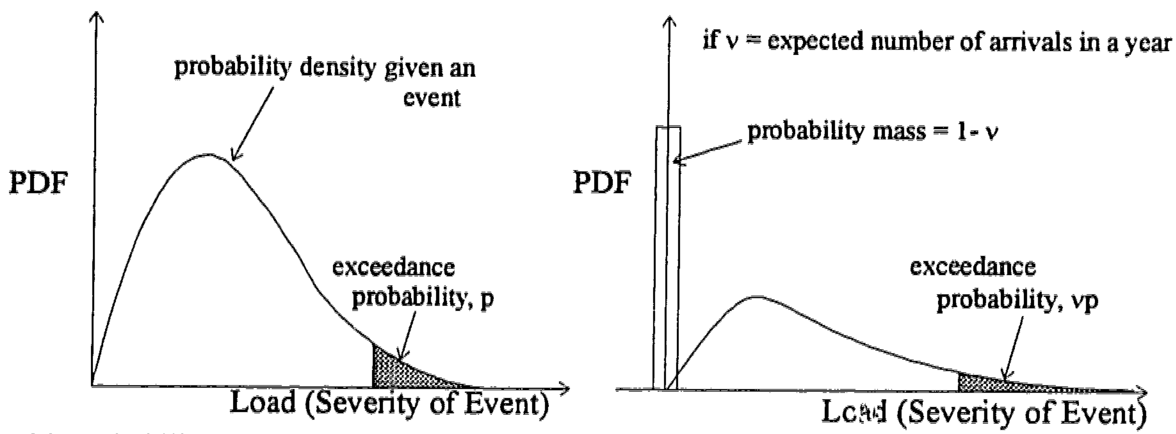
This study considers structures made of reinforced concrete which is a composite material. The material partial safety factor format is appropriate for composite materials as mentioned in Table 3.2. Further, this format is used in CSA A23.3. Hence this format is used herein.

Table 3.1 - Holding Times Used for Different Types of Loading

Load type	Holding time
Sustained live	8 years
Transient live	6 hours
Earthquake	30 seconds
Wind	1 hour
Snow	1 day

Table 3.2 - Comparison of Resistance Factor Formats

Major items to consider	Structural action factor format	Material partial factor format
1. Variability of material properties	Cannot consider individual variabilities of materials if more than one material is involved.	This method can consider individual strength variations.
2. Dimensional variability	Not considered directly.	Not considered directly unless dimensional variability is included.
3. Model error	Handled directly.	This can be handled indirectly.
4. Type of failure	Handled directly.	This can be handled indirectly.
5. Computational ease and accuracy	Two methods are comparable.	Two methods are comparable.
6. User familiarity	Not currently used by CSA A23.3, but used in ACI standards.	Currently used by CSA A23.3 standard.
7. Effect of member classification, i.e., whether considered as a beam, column etc.	Significant changes in resistance factors even in the same equation, according to member classification	Resistance factors do not depend on member classification



(a) Probability density given an event (b) Probability density

Figure 3.1 - Probability distribution of a rare event

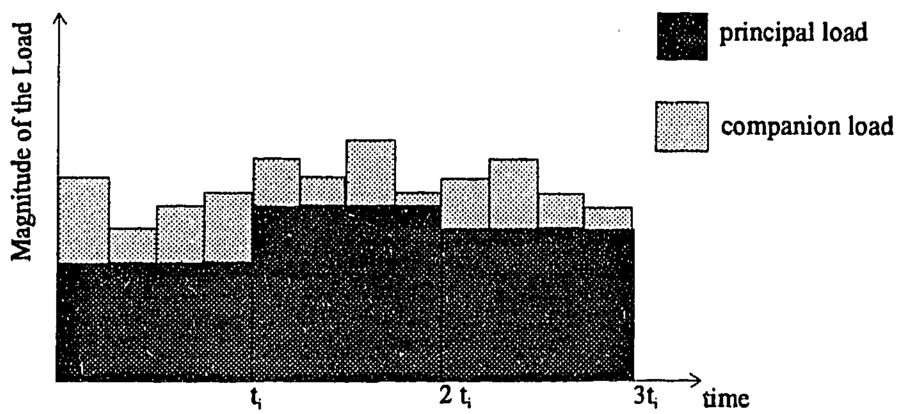


Figure 3.2 - Case 1: Holding time of principal load, t_i is greater than the holding time of companion load, t_j

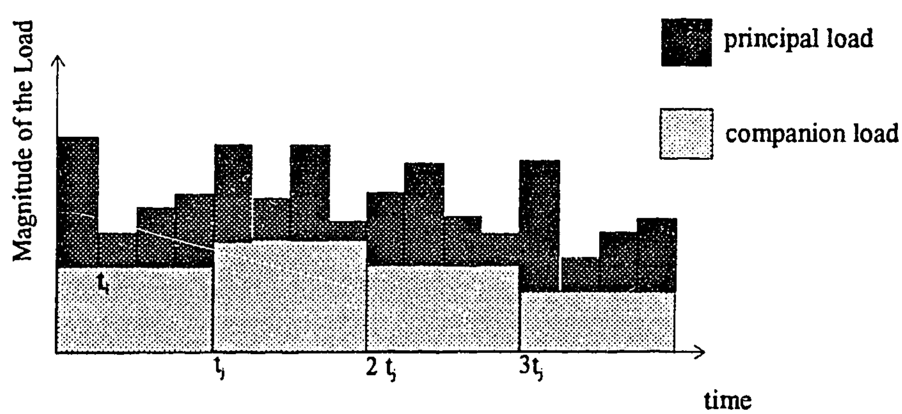


Figure 3.3 - Case 2: Holding time of principal load, t_i is smaller than the holding time of companion load, t_j

CHAPTER 4

LOAD DISTRIBUTIONS AND PARAMETER VALUES

4.1 Introduction

In this chapter, the dead, live, snow, wind and earthquake loads are established. The companion action load factor format as chosen in Chapter 3 is used for frequent loads and the specified exceedance probability factor format for earthquake loads. The required load values, as defined in Chapter 3, are established.

All load parameters are normalized because normalized loads are required for the reliability analyses. The loads are normalized by dividing the load variable by the specified load as given in the NBCC (1990). The mean value of the normalized variable is termed the bias factor.

The terms 'specified', 'nominal', 'true' variable used in referring to the load and resistance are explained in Section 6.1.

The variability of the load effect is due to the variability in the load, the load modelling, and analysis. To account for these uncertainties in the load effect the structural load is multiplied by a modelling parameter, B, and an influence coefficient, c, (Ellingwood et al. 1980). Uncertainty arises from the load model which transforms the actual load into a statically equivalent uniformly distributed load (EUDL). The variability due to this load modelling is reflected in the coefficient of variation of the parameter B. Variability is also introduced by the analysis that transforms the EUDL into the load effect. This variability is accounted for by the coefficient of variation of the influence coefficient, c. The bias factors of B and c are generally taken as 1.0.

4.2 Choosing Representative Geographic Locations

Buildings are subjected to environmental loads, as well as dead and live. Therefore, it is necessary to select various geographic locations that can be considered representative of environmental conditions in Canada.

Initially 15 cities, approximately uniformly distributed across Canada were chosen and used for preliminary studies as given in Sections 4.5 and 4.6. These are Vancouver, Kelowna, Yellowknife, Edmonton, Regina, Saskatoon, Winnipeg, Thunder Bay, Sault Ste Marie, Toronto, Montreal, Fredericton, Halifax, Charlottetown, and St. John's. However for reasons summarized below all the cities are not used in all the analyses.

By observing the snow depth and wind speed data it is seen that the load parameters of some cities are similar, for example, the annual maximum ground snow depth in Edmonton has a mean that is 8 % higher and a standard deviation 3% higher than that in Regina. Therefore the reliability analysis need not be done for both cities as results would be very similar. Accordingly, 7 of the cities, Vancouver, Kelowna, Yellowknife, Regina, Winnipeg, Toronto and St.John's, are chosen so that the whole range of wind speed and snow depth distributions are represented. The snow and wind loads are established for these cities. In the NBCC (1990) the specified earthquake loads are zero in Regina and Winnipeg and sufficient data are not available for Kelowna and Yellowknife. The earthquake load is obtained at Toronto, Vancouver and St.John's.

Once the wind and snow loads were simulated (as explained in Sections 2.5, 4.5.3, and 4.6.3) and used in the reliability analysis it was realized that some cities gave reliability indices, β s, that are significantly different than other cities with the same load ratios. This may be due in part to the fact that the NBCC (1990) specified loads are regionally smoothed to remove anomalous values, while the data used in this study are for the specific city.

The load factors should not be established for the location with the most critical or variable loads. Instead the load factors should be based on load parameters that are moderate or representative for Canada. The specified loads at locations which have parameters significantly different from the moderate values should be adjusted as discussed later in Section 7.5.7. The locations where the bias factor of the load is within 1 ± 0.33 times the overall average bias factor are used in the reliability studies. As explained in Section 4.5.4. the snow loads used in the analyses are for Kelowna, Regina, Winnipeg, Toronto and St.John's and as explained in Section 4.6.5 the wind loads used in the analyses are for Vancouver, Winnipeg, Toronto and St.John's. Load parameters and their influence on reliability is further discussed in Section 7.5.7.

4.3 Dead Load

4.3.1 Introduction

Because the dead load is to be modelled as a variable the bias factor, coefficient of variation and type of distribution must be established. In the NBCC (1990) specified dead load for a structural member is taken to consist of:

(a) self weight of the member,

(b) weight of all permanently supported construction material,

(c) weight of partitions,

(d) weight of permanent equipment.

These loads are relatively constant during the lifetime of the structure. The uncertainty is due to variations in size of members, density of material, the weight of the non-structural members and variability in the weight estimation. Of these the variability in non-structural items such as roofing, partitions, etc. is the main cause of the uncertainty (Ellingwood et al. 1980).

4.3.2 Estimating the Dead Load

As the dead load is often quite predictable and there is relatively little variation with time, some researchers have assumed that the bias factor, i.e., the ratio of mean load to nominal (or specified) load, is 1.0 and that the coefficient of variation, V_D is small as summarized in Table 4.1.

The values in Table 4.1 include the variabilities due to modelling and analyzing the loads. Ellingwood et al.(1980) in estimating the variability of dead load for concrete structures considered the variabilities of the density of concrete, dimensions of members and the superimposed load. When the model and analysis variabilities were included they found the bias factor, δ, equal to 1.03, and the coefficient of variation, V_D equal to 0.093.

Most investigators have assumed that the probability distribution of the dead load is normal (Ellingwood et al. 1980). As the variability of the dead load is low relative to other loads this assumption is more than adequate in modelling the dead load.

4.3.3 Summary of Values Used

As only concrete structures are considered, a bias factor of 1.03 and V_D= 0.093 are used to model the dead load. The probability distribution of the dead load is assumed to be normal.

4.4 Live Load Due to Occupancy

4.4.1 Introduction

The specified live load due to use of floors and roofs depends on the intended use and occupancy. The NBCC (1990) gives minimum values for concentrated loads and uniformly distributed loads. Because concentrated loads influence local areas and would rarely govern any major element in a concrete structure they are not considered further.

When the uniformly distributed load acts on a large tributary area, a live load reduction factor is applied as follows (NBCC 1990):

Where a structural member supports a tributary area greater than 80 m^2 used for assembly occupancies designed for a live load of 4.8 kPa or more, or for storage, manufacturing, retail stores, garages or a foot bridge, the live load, excluding snow, is multiplied by $0.5 + \sqrt{20 / A_T}$.

Where a structural member supports a tributary area greater than 20 m^2 for any use or occupancy, other than assembly occupancies and those mentioned above, the live load, excluding snow, is multiplied by $0.3 + \sqrt{9.8 / A_T}$.

where A_T = tributary area in square meters excluding the area supporting snow.

4.4.2 Probabilistic Model of Live Load

The gravity live load acting on a structural component of an office building can be represented by two distinct parts (McGuire and Cornell 1974) :

The first, a sustained part, acts continually in time, and represents the ordinary office furniture including bookcases and desks and their contents, and normal personnel. This load is assumed to be a spatially varying random function. In any particular area this load is assumed to be constant with time until a load change takes place. These load changes are assumed to occur as Poisson arrivals as shown in Fig. 4.1(a). Such a load change may represent a change in tenancy. Zero sustained live loads are also possible. Load surveys describe this sustained component acting on the structure at any point in time. Consequently the distribution of sustained load is the distribution of arbitrary-point-in-time load and is obtained directly from load survey results.

The second portion of the live load is a transient or extraordinary load which represents short-term loads such as might be caused by an extraordinary grouping of people during an office party or by the stacking of furniture in one area during remodelling of the premises. These extraordinary loads shown in Fig. 4.1(b) occur essentially instantaneously, and they are assumed to arrive as Poisson events. Very few sets of data exist on this type of load because live load surveys have not generally included unusual loading situations (Chalk and Corotis 1980). A model for the extraordinary load was proposed by McGuire and Cornell (1974) and is explained in Section 4.4.3.2.

4.4.3 Estimating the Live Load

Major data surveys in the UK by Mitchell and Woodgate (1971) and in the USA by Culver (1976) have been conducted and several different models have been proposed. The surveys, and consequently the studies, have focused on office buildings. Ellingwood et al.(1980) concluded that the results for several other occupancies (e.g., residences, retail establishments) are similar enough so that these statistics may be applied to them also.

Most surveys and many models have dealt with applied load and not the structural load effect although the latter is of most importance in setting load factors. To reduce the complexity of dealing with loads that vary spatially the loads are generally considered to be uniformly distributed over some area. This uniformly distributed load is called the "equivalent uniformly distributed load" (EUDL) and is the uniform load intensity which would produce the same load effect as the actual spatially varying live load if applied over the appropriate floor area. Two different concepts used in defining these "equivalent uniformly distributed loads" are those of tributary areas and influence areas.

In the case of a member which supports the load directly, such as a slab, the tributary area is the area supported by the member bounded by the lines of support. In the case of a member which does not support the load directly but supports other members, the tributary area is defined as the area bounded by the lines of support of the member and the lines of zero shear in the members supported, assuming a uniformly distributed load is acting on the structure (NBCC 1990).

McGuire and Cornell (1974) define the influence area as the area over which the influence surface for the load effect is significantly different from zero. (For simple framed structures A_I is equal to $2A_T$ for beams and A_I is equal to $4A_T$ for columns.) They emphasize the importance of using the influence area, A_I, rather than the conventional tributary area, A_T, in comparing load effects. They studied the load effects produced by occupancy loads on beams and columns for an office building. They observed that when the upper fractiles of the total EUDL for midspan moment and column load effects are compared, the fractiles for the two load effects differ by less than 6% if calculated based on influence area. However, if based on tributary area the upper fractiles for the two load effects differ by about 12 to 30%. Therefore their study concluded that if common probability level for columns and beams was the goal load effects should be compared on the basis of influence area rather than tributary area. The estimates of the EUDL based on influence area are used herein.

The statistical characteristics of both the maximum total live load, L_{max}, and the arbitrary-point-in-time live load, which is the sustained live load, L_s, need to be established as normalized loads, obtained by dividing the loads by the specified or nominal load (L_n) given in the NBCC (1990).

For beams with A_I greater than 40 m^2 ($A_T > 20$ m^2), the nominal live load, L_n, is

[4.1] $$L_n = L_o\left(0.3 + \sqrt{\frac{9.8 \times 2}{A_I}}\right)$$

$$= L_o\left(0.3 + \sqrt{\frac{19.6}{A_I}}\right) \text{ kN / m}^2$$

and for columns with A_I greater than 80 m^2 ($A_T > 20$ m^2) it is

[4.2] $$L_n = L_o\left(0.3 + \sqrt{\frac{9.8 \times 4}{A_I}}\right)$$

$$= L_o\left(0.3 + \sqrt{\frac{39.2}{A_I}}\right) \text{ kN / m}^2$$

where L_o = Minimum specified uniform load for the particular type of use and occupancy (NBCC 1990)

4.4.3.1 Sustained Live Load

As explained in Section 4.4.2 the sustained live load is the arbitrary-point-in-time load. The following probabilistic load model is given for sustained loads by Peir and Cornell (1973):

[4.3] $$w(x,y) = m + \gamma_{bld} + \gamma_{flr} + \varepsilon(x,y)$$

where, $w(x,y)$ = instantaneous sustained live load intensity

m = mean live load for the type of occupancy considered

γ_{bld} = deviation of the building average of unit load from m

γ_{flr} = deviation of the floor average of unit load from the building average ($m + \gamma_{bld}$)

$\varepsilon(x,y)$ = stochastic process representing the deviation from the floor average ($m + \gamma_{bld} + \gamma_{flr}$)

Based on this model and assuming that $\varepsilon(x,y)$ is a "white noise process" (Chalk and Corotis 1980) the mean and variance of the equivalent uniformly distributed load (EUDL) L are:

[4.4] $$E(L) = m$$

[4.5] $$Var(L) = \sigma^2_{bld} + \sigma^2_{flr} + [\sigma^2_s / A_I]\kappa$$
$$= \sigma^2 + [\sigma^2_s / A_I]\kappa$$

where σ^2_{bld} = variance among building average loads

σ^2_{flr} = variance among floor average loads

σ^2 = $\sigma^2_{bld} + \sigma^2_{flr}$

σ^2_s = an experimental constant

A_I = Influence Area

κ = mean square of the height of the influence surface divided by the average height of the influence surface taken over the area A_I

These parameters have been established from load surveys. The results from the main surveys are summarized in Table 4.2. Ellingwood and Culver (1977) established the value of κ equal to 2.2 for both beam and column load effects.

As the survey data represent the sustained load, using the weighted average values from the surveys the following expressions for the sustained load, L_s, are obtained.

[4.6] $$E(L_s) = 0.566 \quad kN/m^2$$

[4.7] $$Var(L_s) = 0.044 + 3.212 / A_I \quad kN^2/m^4$$

These values account for the variation in the structural load. To obtain the load effect, these values have to be augmented with the variation in influence coefficient, c, and modelling parameter, B (as discussed in Section 4.1). Based on Ellingwood et al. (1980) the coefficients of variation of these are taken to be V_c = 0.05 and V_B = 0.10 with a bias factor of one for each.

Corotis and Doshi (1977) report that Mitchell and Woodgate (1971), investigated the variation of loading with time due to change in occupancy, noting that the average period of occupancy (which is the average holding time of the sustained load) is 8.8 years. Ellingwood and Culver (1977) on analyzing survey data found that the average occupancy duration is 8 years. The average holding time for sustained live loads is taken as 8 years herein.

Chalk and Corotis (1980), Corotis and Doshi (1977), Ellingwood and Culver (1977) concluded that the sustained live load has a Gamma probability distribution. This distribution provides a good estimate of the observed loads in the upper tail region. Therefore the Gamma distribution is used to model L_s.

4.4.3.2 Maximum Total Live Load

The total live load is the sustained load together with an extraordinary or transient live load as shown in Fig. 4.1(c). As the transient load is not measured in load surveys, a theoretical model was proposed by McGuire and Cornell (1974). Each transient load event is modelled by a random number of randomly-sized loaded areas or load cells occurring randomly in space. The average number of load cells per unit area (and therefore the average load per unit area) decreases with increasing area. As there is little data to estimate the parameters involved in this model, parameters are assumed.

For purposes of structural design, the mean lifetime maximum total, EUDL L_{max}, is of interest, as it is this EUDL which is analogous to the design live load specified in building codes and standards. Ellingwood and Culver (1977) show by Monte Carlo simulations that the upper fractiles (0.9-0.99) of L_{max} may be estimated as the corresponding upper fractiles of the maximum sustained load, L_{smax} plus the mean of the extraordinary load for the period over which L_{smax} acts.

Two sets of expressions for the mean and variance of L_{max} are given in Table 4.3. As may be deduced, there is significant difference between the two mean loads for small influence areas. Ellingwood and Culver (1977) attribute this difference to the parameters used in the transient load model. In particular, they suggest the discrepancy is related to the parameters representing the average number of load cells in an area. The expressions developed for the U.S.A. data are used herein.

To obtain the total variability in the maximum live load effect the above load variability has to be augmented with modelling parameter, B, and influence coefficient, c, uncertainties (as discussed in Section 4.1). Based on Ellingwood et al. (1980) a bias factor of 1.0 and V_B= 0.20 and V_c= 0.05 are used herein. Note that the V_B is higher than for the sustained load as modelling the transient load adds more variability.

McGuire and Cornell (1974), Ellingwood and Culver (1977) and Ellingwood et al. (1980) fitted the upper fractiles of L_{max} to a Type I Extreme value distribution. This distribution is used to model L_{max}.

4.4.4 Summary of Values Used

The following values are used:

From [4.1], the nominal load for beams with A_I >40 m^2,

$$L_n = L_o\left(0.3 + \sqrt{\frac{9.8 \times 2}{A_I}}\right)$$

$$= L_o\left(0.3 + \sqrt{\frac{19.6}{A_I}}\right) \quad kN/m^2$$

From [4.2], for columns with A_I >80 m^2,

$$L_n = L_o\left(0.3 + \sqrt{\frac{9.8 \times 4}{A_I}}\right)$$

$$= L_o\left(0.3 + \sqrt{\frac{39.2}{A_I}}\right) \quad kN/m^2$$

where L_0 = Minimum specified uniform load for the particular type of use and occupancy as given in NBCC (1990)

The arbitrary-point-in-time live load or sustained live load (L_s) has a Gamma probability distribution and an average holding time of 8 years with

[4.6] $E(L_s) = 0.566$ kN/m^2

[4.7] $Var(L_s) = 0.044 + 3.212 / A_I$ kN^2/m^4

with $V_C = 0.05$ and $V_B = 0.10$

Maximum Total Live Load (L_{max}) has a Type I Extreme Value distribution with (from Table 4.3)

$$E(L_{max}) = 0.895 + 7.6/\sqrt{A_I} \quad kN/m^2$$

$$Var(L_{max}) = 0.033 + 4.025/A_I \quad kN^2/m^4$$

with $V_C = 0.05$ and $V_B = 0.20$

4.5 Snow Load

4.5.1 Introduction

The roof snow load is modelled as a random variable. The two types of snow load considered are, the characteristic snow load and the arbitrary-point-in-time (a.p.t.) load. Bias factors, coefficients of variation and types of distributions for the ground snow load are established at representative locations in Canada. The roof snow load is dependent on many other basic variables. These variables are modelled and then a simulation is performed to find the roof snow load.

According to the NBCC (1990), the specified loading due to snow accumulation on a roof or any other building surface subject to snow accumulation shall be calculated from the formula

[4.8] $S = S_s (C_b C_w C_s C_a) + S_r$

where

S_s = the ground snow load in kPa

S_r = the associated rain load in kPa

C_b = the basic roof snow load factor of 0.8

C_w = the wind exposure factor

C_s = the slope factor

C_a = the accumulation factor

The factor $C_{gr} = (C_b\, C_w\, C_s\, C_a)$ is called the ground to roof conversion factor.

The ground snow load depends on two basic variables;

[4.9] $$S_s = \gamma\, y$$

where, γ = weight density of snow

y = depth of ground snow

Both γ and y are variables with significant variability at a given site. Therefore [4.8] can be re-written as:

[4.10] $$S = \gamma\ y\, C_{gr} + S_r$$

To model the roof snow load as a probability distribution γ, y, C_{gr} and S_r need to be modelled for different types of roofs.

4.5.2 Statistical Description of Variables

4.5.2.1 Background

Newark et al. (1989) state that since 1961 Canadian estimates of the maximum snow depth (y) are made using the Type I (Gumbel) extreme value distribution,. Ellingwood and Redfield (1983), have researched the appropriate model for the USA. They concluded that the lognormal distribution is preferable to extreme value distributions for describing annual maximum water-equivalent data, subject to the proposition that it is desirable to specify only one probability distribution for describing ground snow loads. The observation that data at a minority of stations appeared to be fit by Type I or Type II was thought to be due to sampling errors caused by limited sample size. It is noted that water-equivalent snow load data take into account variations due to both γ and y.

A comparison of current values and distributions used for defining ground snow load is given in Table 4.4.

The 30 year maximum snow load is taken as the characteristic snow load. The distribution of the 30 year maximum can be obtained from the annual maximum distribution. Therefore the annual maximum snow depth is first established and the 30 year maximum is derived from it. The a.p.t. value is taken as the load that could occur at anytime. As snow depth has been recorded on a daily basis, the daily snow load records are used to establish the a.p.t. snow load distribution.

4.5.2.2 Annual maximum Ground Snow Depth

The raw data for this analysis are obtained from Atmospheric Environment Service, Environment Canada. Stations across the country record rainfall in millimetres, snowfall and ground snow depth in centimetres on a daily basis throughout the year. Snowfall data have been recorded for many years, but the ground snow depth has been documented starting on or after 1955.

As the total accumulated ground snow depth records are relatively short, an attempt was made to find a relationship between snowfall and ground snow depth, with the hope of utilizing snowfall records. It was found that in consistently cold climates (e.g. Edmonton) the snowfall and ground snow depth followed the same probability distribution with the same dispersion coefficient but with different mean values. However, in coastal regions and in areas with highly varying weather this was not so; the ground snow depth varied much more than the snowfall. Therefore, it was decided to use only the ground snow depth data.

The following steps are followed to derive a distribution for the annual maximum ground snow depth:

(1) From the raw data the maximum ground snow depth in each month is obtained.

(2) If the records for more than two winter months are missing that year is rejected (the winter months are taken as October to March). The cities and years of record are listed in Table 4.5.

(3) The maximum depth in a year is obtained and a frequency table is formed by sorting the data in ascending order and using the plotting position $F_i = i / (n+1)$ where, i is the rank of the maximum annual depth and n is the total number of years of record.

(4) The depth data is fitted to a Gumbel distribution. The fitting is done using a transformation of the Gumbel distribution function (Ang and Tang 1984b):

[4.11] $$F(x) = \exp(-\exp(-\alpha(x-u)))$$

where, u = modal value

α = inverse dispersion parameter

This is rearranged as:

[4.12] $$-\ln(-\ln(F(x))) = \alpha x - u\alpha$$

which has the form of $Y = a x + b$

(5) By performing a regression analysis of x on Y, the best estimates of the parameters α and u are obtained. Using the regression analysis, the fit of the Gumbel distribution to the data is assessed. The fit is good as indicated by the coefficients of determination (R-squared values) in column 9 of Table 4.5. The overall goodness of fit of the Gumbel distribution is significant as shown by the fact that the F-statistic (i.e., mean square due to regression/mean square due to error) in column 10 of Table 4.5 is large.

(6) The mean, μ, and standard deviation, σ, of the annual maximum snow depth is obtained from u and α as follows (Ang and Tang 1984b):

[4.13] $$\mu = u + \frac{0.577}{\alpha}$$

[4.14] $$\sigma = \frac{\pi}{\sqrt{6}\,\alpha}$$

4.5.2.3 Arbitrary-point-in-time Ground Snow Depth

The following steps are followed to derive a distribution for the a.p.t. ground snow load:

(1) From the raw data, which are the daily ground snow depths for around 37 years, the average ground snow depth for each day of the winter period is obtained.

(2) The average daily snow depth for the winter period is extracted. The 'winter period' for each city is selected by observing the average daily ground snow depth for that city. The period where there are predominantly non-zero values is taken to be the winter period.

(3) A frequency table is formed by sorting the data in ascending order and using the plotting position $F_i = i / (n+1)$ where, i is the rank of the maximum annual depth and n is the total number of years of record.

(4) The best fit distributions are found by using transformations and regression analyses. Eight types of distributions; i.e., uniform, Gumbel, normal, f, lognormal, gamma, beta, and t distributions, and the log and exponential transformations are tested. The results are summarized in Table 4.6.

Other than for Vancouver, the a.p.t. snow depth values are fitted best by a uniform distribution. In Table 4.6 the parameters 'a' and 'b' for the uniform distribution are the minimum and maximum respectively, i.e., the mean is given by (a+b)/2 and variance by $[(b-a)^2]/12$. For Vancouver, the Gamma distribution is found to be the best fit distribution (with parameters 'a' and 'b'). As the Gamma distribution is positively skewed, the lower values of snow depth have a higher probability of occurrence in Vancouver, as would be expected in a temperate climate.

4.5.2.4 Snow Density

Determining a representative value of snow density for a particular location is a complicated problem. Snow density has been the subject of many investigations using many techniques and according to literature can vary from as little as 10 kg/m^3 in the case of freshly fallen snow to as much as 700 kg/m^3 in the case of thawing firm snow (Newark 1984).

The work of McKay and Findley (1971) classifying average snow density by forest region was adapted by Newark (1984). McKay and Findley reasoned that the type of forest in Canada is governed to a large degree by climatic conditions and therefore that snow density could be classified by forest regions. Newark's values are given in Table 4.7.

Newark (1984) has accounted for the local variation in snow density. His values are used in this study and are given in Table 4.7. As the type of distribution is not specified, a normal distribution is assumed. The forest region and parameters of unit weight are listed in Table 4.8 for each of the cities.

4.5.2.5 Rain-on-Snow

Rain falling on roofs carrying a heavy snow load is an important aspect of roof failures in North America. The weight of water, both in capillary and transient liquid form, should be considered as an additional load which must be supported by the roof (Colbeck 1977).

The maximum weight of liquid depends on properties of snow, size, slope and shape of roof, spacing and type of drains and intensity and duration of rainstorm. The NBCC (1990) takes S_r to be the 24 hour maximum rainfall during the winter months but not exceeding $S_s(C_bC_wC_sC_a)$. The factors influencing the rain load may be indirectly considered; e.g., if the slope of the roof is high, C_s is low and consequently the maximum limit of S_r is lowered.

The annual maximum winter rain load is obtained by analyzing the 24 hour winter rain load data. The procedure outlined in Section 4.5.2.2 is used, with the rain depth used instead of the ground snow depth. The Gumbel distribution is found to fit the data except in the case of Yellowknife. Therefore the rain load parameters are obtained for this distribution. The results are given in Table 4.9. The lack of fit for Yellowknife is probably due to the fact that the winter rain load in Yellowknife is quite small.

4.5.2.6 Ground-to-roof Conversion Factor

Isyumov (1977) reports that the coefficient of variation of the roof snow load is approximately double that of the ground snow load.

O'Rourke (1977) states that the most important parameter affecting the conversion factor is the wind speed during and after the storm. The conversion factor should reflect wind speed, roof geometry (i.e., slope, size and valleys), exposure to wind action, orientation of roof to wind, air temperature, rainfall and thermal characteristics of roof.

It is reported by Taylor (1980) that although the Institute for Research and Construction of the National Research Council has been measuring snow on roofs since 1956, no surveys lasted for more than 12 years (up to 1980) and they are not, therefore, on a sound statistical basis. As a result, roof snow loads in the NBCC (1990) are obtained by multiplying the 30 year return ground load by various coefficients that depend on external geometry and exposure to wind.

There is, however, no statistical connection between roof and ground loads because the coefficients are not statistically derived.

Taylor (1992) re-evaluated the roof snow loads observed between 1967 and 1982. He used depths and specific gravities of snow recorded on 44 single and multi-level flat-roofed buildings between Halifax and Edmonton. Histograms of normalized maximum snow loads on upper and lower roofs given by him are used to obtain an estimate of the ground-to-roof conversion factor, C_{gr}, from

[4.15] $$S = S_s(C_b C_w C_s C_a) + S_r = S_s C_{gr} + S_r$$

The histograms include the class frequency as well as the cumulative frequencies. A distribution for the factor, C_{gr}, is derived as follows:

(1) From Taylor's histograms the mean, standard deviation and frequency tables for the maximum roof to ground ratio (or ground-to-roof conversion factor, C_{gr}) is obtained for both upper and lower roofs away from drifts. Upper roofs are either the roofs of the single level flat roofs or the top level of the multi-level-flat roofs. All other multi-level-roofs are considered to be lower roofs.

(2) The best fit distribution for this data is found to be a normal distribution; with R-square (coefficient of determination) values of 0.90 and 0.96 respectively for upper and lower roofs.

(3) As the characteristics of the lower roofs are not given, the category of roofs according to the NBCC (1990) (C_w = 1.0 or 0.75) is unknown. Therefore, the upper roof values only are used. The NBCC gives values of C_b =0.8, C_w =1.0 or 0.75, C_s = 1.0 and C_a= 1.0 for an upper flat roof. It has been assumed that the roofs are exposed to the wind and thus C_w is taken as 0.75. Using C_w = 0.75 rather than 1.0 tends to conservative load factors.

From the results summarized in Table 4.10, C_{gr} is taken to have a normal distribution with a mean of 0.32 and standard deviation of 0.20.

4.5.3 Simulation of Roof Snow Load

4.5.3.1 Introduction

The roof snow load, S, is taken as:

$$[4.17] \qquad S = C_{gr} S_s + S_r = C_{gr}\, y\, \gamma + S_r$$

The two types of roof snow loads that have to be simulated are the characteristic roof snow load and the arbitrary-point-in-time roof snow load. For both types the component variables C_{gr}, y, γ and S_r are established in the previous sections as distributions with the relevant parameters. To simulate the roof snow load values C_{gr}, y, γ and S_r are generated using the simulation procedures given in Section 2.5 and distribution fitting to S is carried out as given in Section 2.6.

4.5.3.2 Simulation Procedure

The basic steps followed in the simulation of S are:

1. Generate 10,000 sets of randomly selected values each of C_{gr}, y, γ and S_r
2. Calculate the corresponding 10,000 values of S
3. Check the fit of the values of C_{gr}, y, γ and S_r against the assumed distributions
4. Check the generated values of S.

   If the values (mean, quantiles) change significantly from simulation to simulation the number of data points generated may be insufficient. Check the mean against the calculated value of the mean. Check the mean against the values from the NBCC (1990).
5. Fit different distributions to the generated values of S.

For the characteristic values of S the distributions are fitted to the upper tail (above the 95th percentile). For the a.p.t. values of S the distributions are fitted to all 10,000 points of generated data.

6. Find the best fit distribution and the corresponding statistical parameters.

A program to carry out this procedure is developed in the programming language of SAS (SAS Institute Inc. 1993). A similar program developed for wind loads is given in Appendix A.

4.5.3.3 Simulation of Characteristic Roof Snow Load

For characteristic roof snow load, the variables C_{gr} and γ have normal distributions. To obtain the characteristic y the annual maximum ground snow depth is established as a Gumbel distribution. Using these established parameters and extreme value theory the distribution and parameters of the 1 in 30 year maximum ground snow depth y, the characteristic y, can be found. The 1 in 30 maximum of a Gumbel distribution is also a Gumbel distribution according to extreme value theory (Castillo 1988). The 1 in 30 year maximum S_r is also obtained using the same procedure as for the characteristic y. The 30 year maximum snow load is combined with the 30 year maximum rain load as a heavy snow storm can turn into a rain storm.

In calculating the values of S, it is assumed (as in the NBCC 1990, Section 4.1.7.1.) that S_r, which is the rain component cannot be larger than the snow component C_{gr} S_s. As in such a case it is assumed that the snow would be washed away. Therefore, if S_r is less than C_{gr} S_s, S is taken as C_{gr} S_s plus S_r. If S_r is greater than C_{gr} S_s, the maximum rain load S_r that could be added to the snow without washing it away is assumed to be equal to C_{gr} S_s.

The results of the simulations are summarized in Table 4.11. (The reasons for choosing representative cities and the subsequent deletion of some cities from the analysis is explained in Section 4.2.) Columns (2) to (9) give parametric values of the distributions used to generate values of C_{gr}, y, γ and S_r. In columns (10) and (11) the mean and standard deviation of the 10,000 values of simulated S are given. Column (12) gives the number of times when the generated rain component is greater than the generated snow component (out of 10,000 generations). Columns (13) to (15) gives the mean value of S calculated using the mean values of C_{gr}, y, γ and S_r in [4.17]. Mean value of C_{gr} and γ are given in column (2) and (6) respectively. The S is taken equal to the smaller of C_{gr} S_s+S_r and 2C_{gr} S_s. The mean value of y and S_r are obtained using the model parameter, u, and inverse dispersion parameter, α, as follows:

If,

y = annual maximum which has a Gumbel distribution with parameters u and α

y_{30}= 30 year maximum which is the 1 in 30 maximum of y

the mean of y = $\mu_y = u_y + 0.577/\alpha_y$

from Simiu (1979)

the mean of $y_{30} = \mu_y + \ln 30 / \alpha_y = u_y + 0.577/\alpha_y + \ln 30 / \alpha_y$

Therefore, using [4.10] mean value of S, μ_S, is given by

[4.18] $\mu_S = \mu_{Cgr} \times [u_y + (0.577 + \ln 30)/\alpha_y] \times \mu_\gamma + [u_{Sr} + (0.577 + \ln 30) / \alpha_{Sr}]$

Columns (16) to (18) give the values of ground snow load, S_s and winter rain component, S_r, from the NBCC (1990). S is calculated using [4.17] with C_{gr} = 0.6 from Table 4.10. Columns (19) to (22) summarize the results for the distribution fittings. The best fit distribution and the corresponding adjusted R-square, obtained when the data is regressed against the transformed probabilities are given in columns (19) and (20). The best fit distribution parameters are given in columns (21) and (22). 'u' is the mode of the distribution with units of kN/m^2 and α is the inverse dispersion parameter with units of (kN/m^2)$^{-1}$. In columns (23) to (26) the normalized values are given, e.g., the columns (23) and (24) are the values in columns (21) and (22) divided by the nominal value in column (18). The mean and standard deviation values in column (25) and (26) are obtained by using the parameters in columns (23) and (24).

4.5.3.4 Simulation of a.p.t. Roof Snow Load

In simulating the values of the a.p.t. roof snow load, S, C_{gr} and γ have normal distributions and y has a uniform or gamma distribution. The a.p.t. rain component, S_r, is neglected as it is very much smaller than the snow component during the winter period. Therefore, the a.p.t. S is taken as $C_{gr}\ S_s = C_{gr}\ y\ \gamma$. The best fit distribution for S (for all the values generated) is obtained as for the characteristic values.

The results are summarized in Table 4.12. The a.p.t. values are generally below 20% of the characteristic values and the coefficient of variation of the a.p.t. values is high (around 1.0), which is expected as the average value of snow load on any given (winter) day has a high variability.

4.5.4 Summary of Values Used

The normalized a.p.t. and characteristic snow loads each have a Type I Extreme Value distribution. The parameters, are summarized in Table 4.13.

As seen in Table 4.13 there is a significant difference between the parameters for the different cities. This is due in part to the fact that the NBCC (1990) ground snow loads are area weighted averages (Newark et al. 1989). It is not possible to include this area weighting in the analysis of S because only one station is studied from a region. The overall average of the bias factor (mean of the normalized load) for the characteristic snow load is 0.928. The bias factors for Yellowknife and Vancouver are very different from the overall average. Unlike the majority of the cities Vancouver has a highly variable marine snow environment and Yellowknife has an arctic climate. Thus Vancouver and Yellowknife are excluded from the snow load analyses in succeeding chapters. The reasons for choosing representative cities and the subsequent deletion of some cities from the analysis is explained in Section 4.2. The applicability of the results in different locations is discussed in Section 7.5.7.

4.6 Wind Loads

4.6.1 Introduction

The wind load is modelled as a variable. Bias factors, coefficients of variation and types of distributions for the wind load are established at representative locations in Canada. As the wind load (wind pressure or suction) is dependent on any other basic variables, the basic variables are first modelled and then a simulation is performed to model the wind load.

According to the NBCC (1990), the specified wind pressure or suction due to wind on part or all of a surface of a building is calculated from

[4.19] $$p = q\, C_e C_g C_p$$

where,

p = the specified external or internal pressure acting statically and in a direction normal to the surface either as a pressure directed towards the surface or as a suction directed away from the surface,

q = the reference velocity pressure

C_e = the exposure height factor

C_g = the dynamic gust response factor

C_p = the pressure coefficient or the shape factor, i.e., the external or internal pressure coefficient

The variations in the wind pressure are also dependent on two other factors, namely, the directional effect factor, C_d , and the modelling uncertainty factor, C_m. The reference velocity pressure, q, varies with the wind velocity and the density of air. The basic equation for the wind pressures gives the reference velocity pressure, q, in terms of the density of air, r, and the reference wind speed, V.

[4.20] $$q = 1/2\, r\, V^2 = C_w V^2$$

Therefore, the equation of the true wind pressure (when treated as a variable) is:

[4.21] $$p = q\, C_e C_g C_p C_d C_m$$
$$= C_w V^2 C_e C_g C_p C_d C_m$$

Detailed descriptions of each of these variables follow.

4.6.2 Statistical Description of Variables

4.6.2.1 Background

Allen (1975), Ellingwood et al.(1980) and Davenport (1981) have established and used most of the wind related variables. A summary of the factors used in their studies are given in Table 4.14 where some of the values given above were not given directly but were implied or could be derived from the given data.

In Allen (1975) the 'C_w' in $q = C_w V^2$ was taken as a constant value. The maximum annual wind speed, V_{ann}, was plotted on Extreme Value Type I paper, for two Canadian locations. On examination of the 30 year velocity pressure (q), a value of 0.20 for the COV was assumed. Directionality and modelling was accounted for by applying a reduction of 0.85 in each case to the bias factor of the wind load. The COV values of directionality and modelling were neglected.

Ellingwood et al. (1980) have taken a return period of 50 years, as used in the U.S.A. . The directionality effects were accounted for by applying a reduction of 0.85 only to the bias factor. The modelling factor was assumed to be included in the C_e, C_g and C_p factors. The distribution fitting was done in the 90th percentile and above.

Davenport (1981) developed factors for the wind loading of a tall building computed according to the detailed method outlined in the NBCC. Explanations on how these values were obtained are not given in that paper.

As seen in Table 4.14 the values given in the three papers are similar.

4.6.2.2 Estimation of Variation in Wind Velocity (V)

This is determined by extreme value analysis of meteorological observations of hourly mean wind speeds. These observations are taken at sites chosen to be representative of open exposure and adjusted to correspond to an elevation of 10 m above the ground surface. The appropriate value for wind velocity, V, is chosen in accordance with the annual probabilities of exceedance for the different structural components given in Table 4.15.

Canadian Climatic Normals (1951-80) published by Environment Canada give environmental load related variables including wind velocity. These climatic normals are readily available whereas the raw data of all observations have to be retrieved from Environment Canada's mainframe computer in Ottawa and is expensive to obtain and manipulate. The climatic normals give average and maximum hourly wind speeds for the period of time that data is reported. These wind speed values from climatic normals are used to model the variable wind speed by using extreme value theory. To check the accuracy of such an analysis a few sets of direct observations are obtained and analyzed as in Section 4.5.2.2. Then, the two methods of analysis are compared.

4.6.2.3 Method of Analysis Based on Climatic Normals

The data given in the climatic normals include, for wind, the number of years of observation, T_o; the mean hourly wind speed, V_{ave}; and the maximum hourly wind speed in T_o years, V_{max}. Assuming the hourly wind speed, V, is an independent variable, V_n is taken as the annual maximum hourly wind speed. The variable V_n can also be described as the 1 in 'n' hour maximum wind speed where 'n' is the number of hours in a year, i.e., n is equal to 8766.

The units used are as follows:

Period of observation = T_o years

Mean hourly wind speed $= V_{ave}$ km/hr

Maximum hourly wind speed in T_o yr. $= V_{max}$ km/hr

Assuming V is an independent variable

Take V = Hourly wind speed (km/hr)

V_n = Annual maximum hourly wind speed (km/hr)

The probability density functions of V and V_n are shown in Fig. 4.2 and P_A is the probability that V is greater than V_{max}.

From Fig. 4.2 (a),

$$P(V > V_{max}) = P_A$$

[4.22] $$P(V < V_{max}) = 1 - P_A$$

From Fig. 4.2(b),

$$P(V_n > V_{max}) = 1 /(T_o)$$

[4.23] $$P(V_n < V_{max}) = 1 - 1 /(T_o)$$

As there are 8766 hours in a year n is equal to 8766 hours.

Using extreme value theory, as V_n is the 1 in n maximum of V,

$$P(V_n < V_{max}) = P(V < V_{max})^n = P(V < V_{max})^{8766}$$

substituting using [4.22] and [4.23]

$$1 - 1 /(T_o) = (1 - P_A)^{8766}$$

from which (assuming wind velocities in successive hours are uncorrelated)

[4.24] $$P_A = 1 - [1 - 1 /(T_o)]^{1/8766}$$

From this equation, as T_o is given, P_A can be found. Using [4.24] and by assuming the types of distributions for V and V_n the parameters of the two distributions can be determined.

It is assumed that the distribution of V is lognormal and therefore the distribution of V_n can be taken as an extreme value Type I (Gumbel).

If the random variable V has a lognormal distribution (Ang and Tang 1984a) the probability density function of V is

$$f_V(v) = \frac{1}{\sqrt{2\pi}\,\varsigma\, v} \exp\left[-\frac{1}{2}\left(\frac{\ln v - \lambda}{\varsigma}\right)^2\right] \text{ with}$$

[4.25a] $E(V) = \exp(\lambda + \varsigma^2/2)$

[4.25b] $\sigma_V = \exp(\lambda + \varsigma^2/2)\sqrt{e^{\varsigma^2} - 1}$

and the cumulative distribution function of V is

[4.26] $F_V(v) = \Phi[(\ln v - \lambda)/\varsigma]$

where $\Phi[x]$ is used to designate the cumulative distribution function of the standard normal variate X.

From [4.25a],

[4.27] $V_{ave} = \exp(\lambda + \varsigma^2/2)$

and from [4.22] and [4.26],

[4.28] $1 - P_A = \Phi[(\ln V_{max} - \lambda)/\varsigma]$

As V_{ave}, P_A, and V_{max} are known quantities λ and ς can be found from the two simultaneous equations [4.27] and [4.28].

Using extreme value theory, when V_n is the 1 in n maximum of V which has a cumulative distribution function $F_V(v)$, and V_n has a extreme value Type I distribution with a mode of u_n and an inverse dispersion parameter α_n (Ang and Tang 1984b)

$$F_V(u_n) = 1 - 1/n$$

thus from [4.26],

[4.29] $\Phi[(\ln u_n - \lambda)/\varsigma] = 1 - 1/n$ and

[4.30] $\alpha_n = n\, f_V(u_n)$

$$= \frac{n}{\sqrt{2\pi}\,\varsigma\, u_n} \exp\left[-\frac{1}{2}\left(\frac{\ln u_n - \lambda}{\varsigma}\right)^2\right]$$

From [4.29] and [4.30] u_n and α_n for the distribution of V_n can be found and then the mean and standard deviation of V_n can be found using extreme value theory.

[4.31] $\mu_{V_n} = u_n + 0.577 / \alpha_n$

[4.32] $\sigma_{V_n}^2 = \pi^2 / (6\,\alpha_n^2)$

In the analysis it is assumed that the 8766 hours are chosen randomly and that the maximum is chosen as V_n. But this is not so because the 8766 consecutive hours which make up a year are not randomly chosen. The errors introduced by this are believed to be small based on comparison with hourly wind speeds for three cities. In writing [4.23] it is assumed that the maximum hourly wind speed in T_o years, V_{max}, is the modal value of the 1 in T_o year maximum hourly wind speed. This is the best estimate from the available data.

Example - (Summary given in Table 4.16)

From the data given in the climatic normals for Edmonton wind speeds

T_o = 28 years

V_{ave} = 16.7 km/hr

V_{max} = 71 km/ hr

Using [4.24]

$$P_A = 1 - [1 - 1/(28)]^{1/8766}$$
$$= 4.1 \times 10^{-6}$$

Substituting into [4.27] and [4.28]

$$16.7 = \exp(\lambda + \varsigma^2/2)$$
$$1 - 4.1 \times 10^{-6} = \Phi[(\ln 71 - \lambda)/\varsigma]$$

solving these equations give

$$\lambda = 2.758$$
$$\varsigma = 0.340$$

Equation [4.29] gives $\Phi[(\ln u_n - 2.758)/.340] = 1 - 1/8766$

which gives $u_n = 55.150$ km/hr

Equation [4.30] gives

$\alpha_n = [\,8766/(\sqrt{2\pi} \times 0.340 \times 55.15)\,]\, \exp[-1/2\ \{(\ln 55.15 - 2.758)/0.340\}^2]$

$\alpha_n = 0.209$ hr/km

and from [4.31] and [4.32]

$$\mu_{Vn} = u_n + 0.577 / \alpha_n$$
$$= 55.150 + 0.577/0.209$$
$$= 57.9 \text{ km/hr}$$

$$\sigma_{Vn} = \pi / (\sqrt{6}\,\alpha_n)$$
$$= p / (\sqrt{6} \times 0.209)$$
$$= 6.1 \text{ km/hr}$$

This analysis is done for all fifteen cities and is summarized in Table 4.16.

The data modelled are the annual maximum hourly wind speeds (V_n). The 30 year maximum hourly wind speed, V_{30}, is found by using extreme value theory. In this an extreme distribution is estimated and then used to estimate a further extreme value distribution. Because V_n has a Type I extreme value distribution, V_{30} would also have a Type I extreme value distribution. The mean (μ_{V30}) and coefficient of variation (V_{V30}) is obtained from (Simiu 1979):

[4.33] $\mu_{V30} = \mu_{Vn} (1 + \sqrt{6} / \pi\, V_{Vn}\, \ln(30))$

[4.34] $\mu_{V30}\, V_{V30} = \mu_{Vn}\, V_{Vn}$

4.6.2.4 Comparison of Methods of Analysis

To check the accuracy of the analysis given in section 4.6.2.3, three sets of hourly data are obtained, and analyzed as in section 4.5.2.2. The results of the two methods of analysis are compared in Table 4.17.

The location parameters i.e.; the mean and mode, u_n, differ by a maximum of about 9 %. The dispersion parameters (standard deviation and α_n) differ more but when both the location and dispersion parameters are used to estimate 10, 30 and 100 year wind pressures the differences in pressures computed by the two methods become relatively small as shown in Table 4.18 and as discussed subsequently.

Table 4.18 gives, in columns 2 to 4 , values derived using the information given in the climatic normals using extreme value theory and assuming that the maximum annual hourly wind speed (V_{Ann}) has a Gumbel / Extreme Value Type I distribution. 'q' values are derived from V_{10}, V_{30} and V_{100} using $q = C_w V^2$. Columns 5 to 7 give values of q_{10}, q_{30} and q_{100} derived from hourly data and columns 8 to 10 give the same values obtained from the NBCC (1990). In columns 11 to 13, bias factors for values from climatic normals, i.e.; mean/nominal value from the NBCC, are given. Columns 14 to 16 give bias factors for values from hourly data analysis. Columns 17 to 19 give the ratio of bias factors for wind pressures obtained by dividing those obtained from the climatic normals by those from the hourly observations for the 10, 30, and 100 year return period winds. This ratio for the two methods of analysis ranges between 0.87 and 0.98.

Therefore, the somewhat approximate but less expensive analysis of using the climatic normals to obtain the various velocity values, as given in Table 4.16, is used.

4.6.2.5 Estimation of Variation in Pressure Calculation Factor (C_w)

From [4.20], C_w is a factor that depends on the air density which in turn depends on the atmospheric pressure and the air temperature. The atmospheric pressure is a direct function of the elevation above sea level, but also varies about the mean with changes in the weather. Some researchers (Allen 1975, Ellingwood et al. 1980) have considered the variation in C_w to be negligible.

Variations in air density over Canada and the related values of the C_w factor were examined by Boyd (1967) where values of C_w are given for many cities at different elevations. Observations made in January and July were used to estimate the effect of the variation of temperature on density. These observations were used with the equation for C_w in terms of temperature and pressure given by Boyd to estimate the location and dispersion parameters of C_w. The distribution of C_w is assumed to be normal and the results are given in Table 4.19 . The normalized C_w is the variable C_w divided by the nominal value 50×10^{-6} given in the NBCC (1990) when the velocity is in km/hr. The values of C_w given in Table 4.19 are used to model the wind load.

4.6.2.4 Exposure Height Factor (C_e)

The exposure height factor reflects changes in wind speed with height, and also the effects of variations in the surrounding terrain and topography. Hills can significantly amplify the wind speeds near the ground.

The parametric values from the existing literature given in Table 4.14 are similar. This is especially true for the Canadian studies by Allen and Davenport whose values result in similar parameters for the final wind pressure. Each factor considered in these studies does not take the same variations into account, but collectively they do consider the same effects. It is reasonable therefore to take the estimates for all the factors from one study. The parameters given by Allen (1975) are used, mainly because the types of distributions and methods of derivation are explained in detail and more variables are established by him. The effects or variations taken into account and how they are accounted for are known. The subsequent study by Davenport (1981) is for tall buildings only and the factor variabilities are not described.

The values and distribution types for C_e used are given in Table 4.20.

4.6.2.7 Dynamic Gust Response Factor (C_g)

The dynamic gust response factor, C_g, is intended to take into account the superimposed dynamic effect of gusts. This factor is defined as:

C_g = maximum effect of the loading / mean effect of the loading

The dynamic response includes the action of

(a) random wind gusts acting for short durations (such as 3 seconds) over all or part of the structure,

(b) fluctuating pressures induced by the wake of the structure, including "vortex shedding forces", and

(c) fluctuating forces induced by the motion of the structure itself through the wind.

These forces act on the external surfaces of the structure as a whole or on cladding components and may also affect internal surfaces. They may act longitudinally , laterally or torsionally and further they may be amplified by resonance of the structure at one or more of its natural frequencies (NBCC 1990, Davenport 1967).

The statistical estimate for C_g was taken as given by Allen (1975). The values used are given in Table 4.20.

4.6.2.8 Shape Factor or Pressure Coefficient (C_p)

The pressure coefficient is defined as,

C_p = wind induced pressure / velocity pressure at the reference height

Pressures on the surface of the structure vary considerably with the shape, wind direction and profile of the wind velocity. Pressure coefficients are usually determined from wind tunnel experiments on small scale models, although in a few recent instances measurements on full-scale buildings have been used directly.

The statistical estimates for C_p are taken as given by Allen (1975). The values used are given in Table 4.20.

4.6.2.9 Directional Effect Factor (C_d)

To assume that the worst wind speed consistently comes from the worst direction (that is the direction for which the pressure coefficient is highest) is clearly conservative. It is apparent that the alignment of the peak pressure coefficient contour with the direction of the prevailing winds has a large influence on the risk. If the peak pressure coefficient aligns with a direction of relatively weak winds, the reduction can easily reach a factor of 2 (Davenport 1983).

The statistical estimates for C_d are taken as given by Allen (1975). The values used are given in Table 4.20.

4.6.2.10 Modelling Uncertainty Factor (C_m)

The factor C_m, accounts for the overall predictive accuracy of wind tunnel modelling , meteorological estimates, changes in exposure and analytical modelling (Davenport 1983).

The errors in wind tunnel modelling can be directly assessed from full scale to wind tunnel test comparisons. Meteorological estimates carry uncertainties caused by sampling errors (due to short records), instrumental errors, archiving errors and climate change. These are changes in exposure due to increase in such factors as the height or number of neighboring buildings.

The statistical estimates for C_m are taken as given by Allen (1975). The values used are given in Table 4.20.

4.6.2.11 Summary of Statistical Description of Variables

A summary of the statistical descriptions of the factors or variables used in the simulation of the wind load is given in Table 4.21. Columns 2 to 5 give the parameters of C_w and the annual maximum velocity for each city. Using extreme value theory the 10, 30 and 100 year maximum velocity values are calculated. These values are combined to give q and are normalized by dividing by the nominal values from the NBCC (1990) as given in columns 8 to 10. The variable V_d is the daily maximum wind speed modelled by a Gumbel distribution. These values are estimated by finding the ratio of the annual maximum to daily maximum from the hourly data for Edmonton, Vancouver and Toronto. Both the annual maximum and the daily maximum are found to be fitted best by a Gumbel distribution. This ratio is then applied to the annual maximum values given in columns 4 and 5 to obtain the daily maximum values given in columns 6 and 7. Normalized values of the other factors are given in the remainder of the Table. As C_m and C_d are taken as constants (see Table 4.20) they are not shown in Table 4.21.

4.6.3 Simulation of Wind Load Distribution

4.6.3.1 Introduction

From [4.21]

$$p = C_w V^2 C_e C_p C_g C_d C_m$$

when normalized with respect to the NBCC (1990) values, the normalized pressure is $(C_w V^2)(C_e C_p C_g C_d C_m) / q_n$. This expression is used in the simulation of the wind load (pressure), p.

The three characteristic wind loads are the 10, 30 and 100 year maximum wind loads. The three companion loads that have to be simulated are the daily, 2 year and 8 year maximum wind loads, selected because, as explained in Chapter 3, other potential principal loads have these holding time. The characteristic and companion values are normalized with respect to the specified loads from the NBCC (1990).

4.6.3.2 Simulation Procedure

The generation of variables and simulation is done according to the procedure outlined in Sections 2.5 and 4.5.3.2 (Computer program given in Appendix A.). The summary of the values used is given in Table 4.21.

4.6.3.3 Results of Simulation

Results of the simulations for characteristic values are given in Table 4.22. Results for p_{10}, p_{30} and p_{100}, i.e. the simulated values of the mean and standard deviation are given in columns 2 to 7. Mean values of p_{10}, p_{30} and p_{100} are calculated as a check for the simulated values. These are given in columns 8 to 10 and the values are virtually identical to those obtained from the simulations. Next different distributions are fitted to the simulated values and the best fit distributions and their parameters are summarized in columns 11 to 22. These values are used.

Results of the simulations for companion values are given in Table 4.23. The daily, 2 year and 8 year maximums (p_d, p_2 and p_8 respectively) are simulated and the simulated values of the mean and standard deviation are given in columns 2 to 7. Mean values of p_d, p_2 and p_8 are calculated as a check for the simulated values. These are given in columns 8 to 10. These values are almost equal to the values obtained from the simulations as given in columns 2, 4 and 6. Next different distributions are fitted to the simulated values and the best fit distributions and their parameters are summarized in columns 11 to 22. These values are used.

4.6.4 Summary of Values Used

The normalized companion and characteristic values of wind loads have Type I Extreme Value distributions except that the daily maximum, p_d, has a lognormal distribution. The statistical parameters of these loads, used here are summarized in Table 4.24.

Table 4.24 shows a significant difference between the parameters for the different cities. The overall average of the bias factor (mean of the normalized load) for the p_{30} is 0.731. The bias factors for Kelowna, Yellowknife and Regina fell outside of the range of 1 ± 0.33 times the overall average and they are not considered in the calibration work done in the next chapters. This problem is discussed in Section 7.5.7.

4.7 Load due to Earthquakes

4.7.1 Introduction

The philosophy of seismic design does not lend itself well to the development of a material-independent load criterion, and the problem of how earthquake loads should be treated in load combinations is still unresolved (Ellingwood et al. 1980). New approaches to probabilistic seismic design have been studied in Canada by Maes (1986a,b) and Heidebrecht et al. (1983). Maes (1986a) developed a seismic design approach for use in the CSA code for fixed offshore structures (CSA-S471) . The NBCC (1990) has adopted the probabilistic estimates by Heidebrecht et al. (1983) for use in defining seismic forces. As it is not within the scope of this study to change the definitions and formulations of the loads defined in the NBCC, the seismic forces defined in the NBCC (1990) are used.

Lateral load effects due to seismic activity are determined for conventional buildings using the equations given in the NBCC (1990):

[4.35] $$V = (V_e / R)\, U$$

where,

V = minimum design base shear

V_e = equivalent lateral seismic force representing elastic response

R = force modification factor

U = 0.6 is a calibration factor and,

[4.36] $V_e = v\ S\ I\ F\ W$

where,

v = zonal velocity ratio

S = seismic response factor

I = importance factor

F = foundation factor

W = weight of structure

These equations are used to estimate the statistical descriptions of seismic forces. The uncertainty in the base shear, V, is dominated by that of the zonal velocity ratio, v.

In the formulation for the NBCC (1990) seismic provisions, only the zonal velocity ratio, v, is specified explicitly, whereas the zonal acceleration ratio, a, is used implicitly via the seismic response factor, S. The zonal velocity ratio, v, is derived from the probabilistic study of ground motions in the 1 second period range, normalized to a spectral velocity of 1.0 m/s. Similarly, a zonal acceleration ratio, a, is obtained from probabilistic evaluation of the ground motions in the 0.2 second period range, normalized to 1.0 g (Heidebrecht et al. 1983).

4.7.2 Statistical Description of Variables

4.7.2.1 Background

Ellingwood et al. (1980) have estimated the earthquake load by describing the base shear in terms of the 50-year maximum peak ground acceleration, A. This analysis was based on the Algermissen and Perkins estimates of peak ground accelerations associated with a 10% probability of being exceeded in 50 years. The peak ground acceleration was described by using a Type II extreme value distribution. They also established a load factor to be used with the above definition of the earthquake load.

In a calibration study for the CSA code for fixed offshore structures, Maes (1986a) explains the need for the specified exceedance probability design format. In this format no partial factor is used for the earthquake load, but, instead, the design load is directly specified at a given probability of exceedance. This specified exceedance probability design format is used for earthquake load combinations as described in Section 3.2.5.

In 1990, the NBCC effectively adopted the specified exceedance probability design format in defining earthquake loads. The probability of exceedance of the seismic ground motion parameters was changed to 10% in 50 years (an annual probability of exceedance of 0.0021) from the previous value of 0.01 per annum used in earlier codes based on the probability factor design format.

4.7.2.2 Zonal Velocity Ratio, v

When using [4.35] and [4.36] to estimate the minimum base shear, V, the zonal velocity, v, dominates the uncertainty in the base shear. The zonal velocity ratio, v, is derived from the probabilistic study of Peak Horizontal Ground Velocity, PHV, in the 1 second period range, normalized to a spectral velocity of 1.0 m/s. The specified exceedance level for PHV is 10% in 50 years.

Figure 4.3 is used to explain the development of the distribution of the peak horizontal ground velocity that has a 10% chance of exceedance in a 50 year building life. Table J-2 of the Commentary J of the NBCC (1990) gives three points on the tail of the annual maximum horizontal ground velocity. By fitting a probability distribution to this estimate of the tail, the distribution of the annual maximum horizontal ground velocity is obtained. Figure 4.3(a) shows the three points and the resulting distribution for Vancouver. By using extreme value theory, the 50 year maximum horizontal ground velocity, x_{50}, is derived from this annual maximum distribution and is shown in Fig. 4.3 (b). The probability distribution of $\ln(x_{50})$ is given in Fig. 4.3(c) and it is a Type I distribution. Using extreme value theory again the 1 in 10 maximum value of $\ln(x_{50})$ is obtained and is shown as a Type I distribution in Fig. 4.3(d). Finally the Type I distribution of Fig. 4.3 (d) is transformed into the Type II distribution of Fig. 4.3 (e) which gives the peak horizontal ground velocity with a 10% probability of exceedance in a 50 year life.

A sample calculation of this analysis is given below for the city of Vancouver and is also given in Table 4.25.

Table J-2 of the NBCC (1990) Commentary J gives the following values of PHV for Vancouver:

for a probability of exceedance = 0.01 PHV = 0.077 m/s

= 0.005 PHV = 0.12 m/s

= 0.0021 PHV = 0.21 m/s

For ease of representation in the figure let X represent the variable annual PHV as shown in Fig. 4.3.

From the three points on the tail of the distribution in Fig. 4.3(a)

$\Phi((\ln(0.21)-\lambda)/\varsigma) = 1-0.0021 = 0.9979$

[4.37] $(\ln(0.21)-\lambda)/\varsigma = 2.86$

similarly for the other two points

[4.38] $(\ln(0.12)-\lambda)/\varsigma = 2.575$

[4.39] $(\ln(0.077)-\lambda)/\varsigma = 2.33$

Solving [4.37], [4.38] and [4.39] the estimates are

$\lambda = -6.91$

$\varsigma = 1.87$

note: $E(X) \approx \exp(\lambda) = 0.001$

When the variable X has a lognormal distribution the n^{th} largest value of X, X_n, would converge to the Type II asymptotic form with the following parameters (Ang and Tang 1984b, pp 218)

[4.40] $v_n = \exp(\varsigma \sqrt{2\ln n} - \varsigma(\ln \ln n + \ln 4\pi) / (2\sqrt{2\ln n}) + \lambda)$

[4.41] $k = \sqrt{2 \ln n} \ / \varsigma$

Using these equations and n equal to 50 the following parameters for the distribution of Y (or X_{50}) as shown in Fig. 4.3 (b) are obtained

$v_n = 0.050$

$k_n = 1.499$

When the variable Y has a Type II asymptotic distribution then ln Y has a Type I asymptotic distribution with the following parameters (Ang and Tang 1984b, pp 217)

[4.42] $u_n = \ln (v_n)$

[4.43] $\alpha_n = k_n$

Using these equations

$u_n = -2.991$

$\alpha_n = 1.499$

The mean and standard deviation of ln(Y) shown in Fig. 4.3(c) can be found .

[4.44] $\mu_n = u_n + 0.577 / \alpha_n$

[4.45] $\sigma_n^2 = \pi^2 / (6\ \alpha_n^2)$ also,

[4.46] $COV_n = \sigma_n / \mu_n$

Since ln(Y) has a Type I extreme value distribution, the 1 in 10 maximum of ln(Y) would also have a Type I extreme value distribution. The mean (μ_{nn}) and coefficient of variation (COV_{nn}) can be obtained from (Simiu 1979):

[4.47] $\mu_{nn} = \mu_n (1 + \sqrt{6} / \pi \ COV_n \ \ln(10))$

[4.48] $\mu_{nn}\ COV_{nn} = \mu_n\ COV_n$ or $\sigma_n = \sigma_{nn}$

Using [4.44] to [4.48]

$\mu_n = -2.606$

$\sigma_n = 0.856$

$COV_n = -0.328$

$\mu_{nn} = -1.070$

$\sigma_{nn} = 0.856$

Using [4.44] and [4.45] the parameters of ln (Z) depicted in Fig. 4.3(d) are

$u_{nn} = -1.455$

$\alpha_{nn} = 1.499$

Using [4.42] and [4.43] the parameters of Z which is the zonal velocity ratio with a 10% probability of exceedance in 50 years, as shown in Fig. 4.3(e), are

$v_{nn} = 0.233$

$k_{nn} = 1.499$

Table 4.25 gives values for Toronto and St.John's as well for which values are given in Table J-2.

4.7.3 Summary of Values Used

The specified exceedance probability factor format is used with earthquake load combinations therefore the load factor on the earthquake is always 1.0. There are many factors involved in establishing earthquake loads but they do not have to be established to develop the relevant load factors. Because loads are specified differently in this format, v is established as an example in this study. Table 4.26 gives the statistical descriptions of v, which has a Type II extreme value distribution. The other parameters in [4.35] and [4.36] depend on many factors such as the foundation, the soil conditions, the dead weight of the structure, ductility of the structure and so on. The variabilities in these parameters have not been established.

Table 4.1 - Literature review for dead load parameters

Reference	$\delta = E(D)/D_n$	COV_D
Allen (1976)	1.00	0.10
Ellingwood (1978)	1.00	0.10
Ellingwood et.al.(1980)	1.05	0.10
Ellingwood et.al.(1980) (for concrete structures)	1.03	0.093
Ravindra and Galambos (1978)	1.00	0.08
Lind (1976)	1.05	0.09

Table 4.2 - Live load survey results for equations (4.4) and (4.5)

Parameter	U.K. data	U.S.A. data	Weighted average of surveys on areas greater than 18.6 m^2
m (kN/m^2)	0.565 (Peir & Cornell, 1973)	0.555 (Ellingwood & Culver, 1977)	0.566 (Lai, 1981)
σ^2 (kN^2/m^4)	0.0466 (Lai, 1981)	0.0601 (Ellingwood & Culver, 1977)	0.044 (Lai, 1981)
σ^2_s(kN^2/m^2)	1.782 (McGuire & Cornell, 1974)	1.407 (Ellingwood & Culver, 1977)	1.460 (Lai, 1981)

Table 4.3 - Estimated expressions for the parameters of L_{max}

	McGuire & Cornell (1974) using U.K. data	Ellingwood & Culver (1977) using U.S.A. data
$E(L_{max})$ [kN/m^2]	$0.714 + 36.548/\sqrt{A_I}$	$0.895 + 7.588/\sqrt{A_I}$
$\sigma^2(L_{max})$ [kN^2/m^4]	$0.0005 + .719/A_I$	$0.033 + 4.025/A_I$

Table 4.4- Current definitions of ground snow load

	Canada (Newark et al. 1989)	USA (Ellingwood and Redfield 1983)
(1) Distribution of annual extreme ground snow load = $S_s = \gamma y$	Type I extreme value / Gumbel distribution - for both S_s and y, as γ is taken as a constant.	Lognormal distribution
(2) Mean recurrence interval (MRI)	30 yrs	50 yrs
(3) Probability of exceedance in any given year	0.033	0.02
(4) Distribution of lifetime extreme ground snow load	Type I extreme value / Gumbel distribution	Type I extreme value / Gumbel distribution

Table 4.5 - Annual maximum ground snow depth

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Depth (m)					
Location	Station	Number of years	Years considered	Modal value, u	Inverse dispersion parameter, α	μ	σ	R-square	F-statistic
Vancouver, Int'l Airport	1108447	37	1955 - 91	0.10	7.72	0.17	0.17	0.96	898
Kelowna , Airport	1123970	22	1970 - 91	0.22	8.72	0.29	0.15	0.98	1031
Yellowknife, Airport	2204100	37	1955 - 91	0.40	8.23	0.47	0.16	0.98	2128
Edmonton, Municipal Airport	3012208	37	1955 - 91	0.27	7.48	0.35	0.17	0.96	892
Regina, Airport	4016560	37	1955 - 91	0.26	8.64	0.33	0.15	0.98	1787
Saskatoon, Airport	4057120	37	1955 - 91	0.23	9.29	0.29	0.14	0.99	4923
Winnipeg, Int'l Airport	5023222	37	1955 - 91	0.32	5.78	0.42	0.22	0.99	2911
Thunder Bay, Airport	6048261	37	1955 - 91	0.45	3.56	0.61	0.36	0.93	447
Sault Ste Marie, Airport	6057592	30	1962 - 91	0.66	4.40	0.79	0.29	0.97	958
Toronto, University	6158350	37	1955 - 91	0.21	13.88	0.25	0.09	0.95	661
Montreal, McGill Univ.	7025280	37	1955 - 91	0.48	4.60	0.60	0.28	0.93	445
Fredricton, CDA	8101600	33	1959 - 91	0.44	5.31	0.55	0.24	0.97	1069
Halifax, Citadel	8202220	19	1964 - 91	0.22	5.32	0.33	0.24	0.89	148
Charlottetown, CDA	8300406	33	1959 - 91	0.42	5.60	0.53	0.23	0.98	1603
St.John's Airport	8403506	37	1955 - 91	0.47	3.47	0.64	0.37	0.97	1025

Table 4.6 - Arbitrary-point-in-time ground snow depth (m)

	Years of record	Winter period	Best fit distrn. for y	R-square	Distribution Parameters for y	
					a	b
Vancouver	1955 - 91	Nov.16 - Mar.15	Gamma	0.97	2.0**	50.0**
Kelowna	1970 - 91	Nov.1 - Mar.31	Uniform	0.97	0.0	0.161
Yellowknife	1955 - 91	Oct.1 -May.15	Uniform	0.96	0.0	0.479
Edmonton	1955 - 91	Oct.1- Mar.31	Uniform	0.97	0.0	0.235
Regina	1955 - 91	Oct.1- Apr.30	Uniform	0.97	0.0	0.233
Saskatoon	1955 - 91	Oct.1 - Apr.30	Uniform	0.97	0.0	0.212
Winnipeg	1955 - 91	Oct.16 - Apr.30	Uniform	0.98	0.0	0.309
Thunder Bay	1955 - 91	Oct16 - Apr30	Uniform	0.98	0.0	0.460
Sault Ste Marie	1962 - 91	Nov.1 - Apr.30	Uniform	0.97	0.0	0.573
Toronto	1955 - 91	Nov.1 - Mar.31	Uniform	0.99	0.0	0.098
Montreal	1955 - 91	Nov.1 - Apr.15	Uniform	0.97	0.0	0.427
Fredricton	1959 - 91	Nov.16 - Apr.30	Uniform	0.97	0.0	0.350
Halifax	1964 - 91	Nov.16 - Apr.15	Uniform	0.98	0.0	0.114
Charlottetown	1959 - 91	Nov.16 - May.15	Uniform	0.98	0.0	0.310
St.John's	1955 - 91	Nov.1 - May.15	Uniform	0.97	0.0	0.301

** Distribution parameters:

For the gamma distribution, a and b are distribution parameters (a is a dimensionless integer and b has units m^{-1}) where, mean=a/b and variance=a/b^2

For the uniform distribution, a is the minimum and b is the maximum in meters.

Table 4.7 - Average seasonal snow density by region based on type of forest

Region	Regional number	μ (kg/m^3)	σ (kg/m^3)	COV
A = Acadian	1	220	50	0.23
AG = Aspen Grove	2	220	40	0.18
B = Boreal	3	190	60	0.32
C = Coast	4	430	25	0.06
CL = Columbia	5	360	35	0.10
GL = Great Lakes	6	220	60	0.27
M = Montane	7	260	25	0.10
P = Prairie	8	210	40	0.19
SA = Subalpine	9	360	30	0.08
T = Tundra	10	300	80	0.27
TA = Taiga	11	200	80	0.40

Table 4.8 - Snow density

Location	Forest region	Unit weight (kN/m^3) μ	σ
Vancouver	SA	3.53	0.29
Kelowna	M	2.55	0.25
Yellowknife	TA	1.96	0.78
Edmonton	AG	2.16	0.39
Regina	P	2.06	0.39
Saskatoon	AG	2.16	0.39
Winnipeg	AG	2.16	0.39
Thunder Bay	AG	2.16	0.39
Sault Ste Marie	GL	2.16	0.59
Toronto	GL	2.16	0.59
Montreal	GL	2.16	0.59
Fredricton	A	2.16	0.49
Halifax	A	2.16	0.49
Charlottetown	A	2.16	0.49
St.John's	A	2.16	0.49

Table 4.9 - Annual maximum winter rain load

	Rain (mm)				Rain load (kN/m^2)	
	u	α	R-square	F-statistic	u	α
Vancouver	32.88	0.08	0.96	1168	0.323	8.134
Kelowna	6.07	0.34	0.96	523	0.059	34.495
Yellowknife	-0.12	0.81	0.65	86	-0.001	82.571
Edmonton	1.43	0.55	0.98	2179	0.014	55.668
Regina	0.57	0.34	0.85	525	0.006	34.152
Saskatoon	0.11	0.47	0.78	338	0.001	48.287
Winnipeg	1.67	0.15	0.82	240	0.016	15.144
Thunder Bay	6.38	0.12	0.97	1479	0.063	12.297
Sault Ste Marie	14.16	0.13	0.98	1240	0.139	12.896
Toronto	20.64	0.14	0.99	19911	0.202	14.031
Montreal	20.12	0.12	0.99	12259	0.197	12.524
Fredricton	25.79	0.09	0.97	2958	0.253	9.637
Halifax	43.44	0.07	0.95	732	0.426	6.780
Charlottetown	20.94	0.10	0.98	3687	0.205	9.855
St.John's	33.60	0.07	0.99	4615	0.330	7.464

Table 4.10 - Ground to roof conversion factor

	Upper Roof	Lower roof
Distribution of C_{gr} (R-square)	Normal (0.90)	Normal (0.96)
μ for C_{gr}	0.317	0.304
σ for C_{gr}	0.200	0.235
1990 NBCC value for C_{gr}	0.6	cannot be sure

Table 4.11 - Simulations of snow load, S - characteristic values

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	C_{gr} - normal (non-dimensionalized)		y - from annual maximum, Gumbel (m)		γ - normal (kN/m^3)		Sr - from annual maximum Gumbel (kN/m^2)		S - from simulation (kN/m^2)		Number of obs. where $S_r > C_{gr} S_s$	S - from approximate calculation of mean values		
Location	μ	σ	u	α	μ	σ	u	α	μ	σ	(out of 10,000)	$C_{gr} S_s + S_r$	$2 C_{gr} S_s$	S
Vancouver	0.32	0.20	0.10	7.72	3.53	0.29	0.32	8.13	1.24	0.76	6303	1.50	1.38	1.38
Kelowna	0.32	0.20	0.22	8.72	2.55	0.25	0.06	34.50	0.70	0.43	1489	0.72	1.10	0.72
Yellowknife	0.32	0.20	0.40	8.23	1.96	0.78	0.00	82.57	0.58	0.47	911	0.59	1.10	0.59
Regina	0.32	0.20	0.26	8.64	2.06	0.39	0.01	34.15	0.57	0.37	1315	0.59	0.94	0.59
Winnipeg	0.32	0.20	0.32	5.78	2.16	0.39	0.02	15.14	0.93	0.58	1929	0.97	1.38	0.97
Toronto	0.32	0.20	0.21	13.88	2.16	0.59	0.20	14.03	0.63	0.41	7341	0.83	0.68	0.68
St.John's	0.32	0.20	0.47	3.47	2.16	0.49	0.33	7.46	1.75	1.09	4209	1.97	2.21	1.97

(1)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
	Values from NBCC			Best fit distribution for S (kN/m^2) (above 95th percentile)				Normalized S (w.r.t. column (18))			
Location	S_s (kN/m^2)	S_r (kN/m^2)	S (kN/m^2)	Type	Adj.R-sq	u	α	u	α	μ	σ
Vancouver	1.40	0.20	1.04	Gumbel	0.994	1.28	2.77	1.23	2.88	1.43	0.45
Kelowna	1.10	0.10	0.76	Gumbel	0.996	0.65	4.06	0.86	3.09	1.04	0.42
Yellowknife	2.00	0.10	1.30	Gumbel	0.997	0.40	2.95	0.31	3.84	0.46	0.33
Regina	1.30	0.10	0.88	Gumbel*	0.989	0.54	4.59	0.62	4.04	0.76	0.32
Winnipeg	1.70	0.20	1.22	Gumbel	0.984	0.82	2.87	0.67	3.50	0.84	0.37
Toronto	0.80	0.40	0.88	Gumbel	0.995	0.79	5.75	0.90	5.06	1.01	0.25
St.John's	2.60	0.60	2.16	Gumbel	0.994	1.69	1.67	0.78	3.61	0.94	0.36

* Normal distribution function gave the best fit but Gumbel also gave a good fit

Table 4.12 - Simulation of snow loads - a.p.t. values

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
	C - normal (non-dimensionalized)		y **			γ - normal (kN/m³)		S - from simulation (kN/m²)		S ***	Best fit distribution for S (kN/m²)				Normalized S (w.r.t. NBCC values from Table 4.11)			
Location	μ	σ	distrn.	a	b	μ	σ	μ	σ	S	Type	Adj. R-sqr	u	α	u	α	μ	σ
Vancouver	0.32	0.20	Gamma	2.00	50.00	3.53	0.29	0.05	0.05	0.04	Gumbel	0.97	0.02	26.90	0.017	36.99	0.032	0.03
Kelowna	0.32	0.20	Uniform	0.00	0.16	2.55	0.25	0.07	0.06	0.07	Gumbel	0.99	0.04	20.69	0.051	15.00	0.090	0.09
Yellowknife	0.32	0.20	Uniform	0.00	0.48	1.96	0.78	0.15	0.16	0.15	Gumbel	0.98	0.07	7.88	0.126	4.69	0.249	0.27
Regina	0.32	0.20	Uniform	0.00	0.23	2.06	0.39	0.08	0.07	0.08	Gumbel	0.99	0.04	17.26	0.072	10.21	0.129	0.13
Winnipeg	0.32	0.20	Uniform	0.00	0.31	2.16	0.39	0.11	0.10	0.11	Gumbel	0.99	0.06	12.53	0.061	12.17	0.108	0.11
Toronto	0.32	0.20	Uniform	0.00	0.10	2.16	0.59	0.03	0.03	0.03	Gumbel	0.99	0.02	37.45	0.027	25.53	0.050	0.05
St.John's	0.32	0.20	Uniform	0.00	0.30	2.16	0.49	0.10	0.10	0.10	Gumbel	0.99	0.06	12.76	0.029	25.12	0.052	0.05

** Distribution parameters:

For Gamma distribution, a and b are distribution parameters (a is a dimensionless integer and b has units m^{-1})
where, mean=a/b and variance=a/b^2

For Uniform distribution , a is the minimum and b is the maximum in metres;i.e., mean=(a+b)/2 and variance=$((b-a)^2)/12$

***S - from approximate calculation of mean values

Table 4.13 - Summary of values used in this study

	a.p.t.. normalized S (non-dimensionalized)				Characteristic normalized S (non-dimensionalized)			
Location	u	α	μ	σ	u	α	μ	σ
Vancouver	0.017	36.99	0.032	0.03	1.232	2.88	1.43	0.45
Kelowna	0.051	15.00	0.090	0.09	0.856	3.09	1.04	0.42
Yellowknife	0.126	4.69	0.249	0.27	0.310	3.84	0.46	0.33
Regina	0.072	10.21	0.129	0.13	0.619	4.04	0.76	0.32
Winnipeg	0.061	12.17	0.108	0.11	0.674	3.50	0.84	0.37
Toronto	0.027	25.53	0.050	0.05	0.899	5.06	1.01	0.25
St.John's	0.029	25.12	0.052	0.05	0.782	3.61	0.94	0.36

Table 4.14 - Statistical data for wind load factors

Variable	Allen (1975)			Ellingwood et al. (1980)			Davenport (1981)		
	δ	COV	Distn	δ	COV	Distn	δ	COV	Distn
V_{ann}	Table given		TypI	Table given		*	*	*	*
C_w	*	*	*	1.0	0.05	*	*	*	*
q_{30}	1.1	0.20	TypI	*	*	*	1.0	0.25	*
C_e	1.0	0.08	*	1.0	0.16	*	1.0	0.10	*
C_p	1.0	0.10	*	1.0	0.12	*	1.0	0.10	*
C_g	1.0	0.10	*	1.0	0.11	*	1.0	0.05	*
C_d	0.85	0.0	*	0.85	0.0	*	0.85	0.00	*
C_m	0.85	0.0	*	included above		*	*	*	*
p_{30yr}	0.80	0.26	TypI	0.78	0.37	$TypI_{50 yr}$	0.85	0.29	*
p_{daily}	0.08	1.0	lgnm	0.01	0.07	Typ I	*	*	*

where, δ = bias coefficient COV = Coefficient of Variation

Distn = Distribution TypI = Extreme Value Type I

* = Not mentioned lgnm = Lognormal distribution

Table 4.15 - Probability of exceedance of V, for different elements

For the design of :	Probability of exceedance in any one year
Cladding	0.10
Structural members for deflection and vibration	0.10
Structural members for strength	0.033
Structural members for strength for post-disaster buildings	0.010

Table 4.16 - Results of analysis based on climatic normals

Station	T_o	V_{ave}	V_{max}	P_A	λ	ζ	Annual maximum wind velocity			
							u_n	α_n	μ	σ
	years	(km/hr)	(km/hr)	x 10^{-6}			(km/hr)	$(km/hr)^{-1}$	(km/hr)	(km/hr)
Vancouver	28	16.2	89	4.15	2.71	0.399	65.1	0.151	68.9	8.48
Kelowna	22	10.4	56	5.31	2.26	0.401	42.1	0.232	44.6	5.52
Edmonton	28	16.7	72	4.15	2.76	0.340	55.1	0.209	57.9	6.12
Regina	28	23.5	142	4.15	3.07	0.422	101.9	0.091	108.3	14.07
Saskatoon	28	21.9	105	4.15	3.02	0.366	78.8	0.136	83.1	9.41
Winnipeg	28	18.0	89	4.15	2.82	0.373	66.4	0.158	70.1	8.10
Thunder Bay	28	19.8	80	4.15	2.93	0.324	62.0	0.195	65.0	6.57
Sault Ste Marie	20	23.0	89	5.85	3.08	0.321	71.3	0.172	74.7	7.47
Toronto	28	17.6	97	4.15	2.79	0.400	70.9	0.139	75.1	9.26
Montreal	28	18.1	90	4.15	2.83	0.374	67.1	0.156	70.8	8.21
Fredericton	28	15.7	80	4.15	2.68	0.380	59.4	0.174	62.7	7.38
Halifax	28	27.3	97	4.15	3.26	0.293	77.1	0.174	80.4	7.38
Charlottetown	28	22.5	97	4.15	3.06	0.340	74.3	0.155	78.0	8.25
St John's	28	23.9	137	4.15	3.09	0.409	99.4	0.097	105.3	13.29
Yellowknife	28	17.0	72	4.15	2.78	0.335	55.3	0.211	58.1	6.07

Table 4.17 - Comparison of two methods of establishing annual wind speeds (1953-1980)

Estimates of annual max. velocity	Edmonton			Vancouver			Toronto		
	Based on normals	Based on hourly data	Error	Based on normals	Based on hourly data	Error	Based on normals	Based on hourly data	Error
u	55.1	60.2	-8.5%	65.1	60.5	7.6%	70.9	65.2	8.7%
α	0.21	0.22	-4.5%	0.15	0.11	35.6%	0.14	0.10	36.6%
μ	57.9	62.9	-7.9%	68.9	65.6	5.0%	75.1	70.9	5.9%
σ	6.1	5.8	4.7%	8.5	11.5	-26.2%	9.3	12.6	-26.8%

Table 4.18 - Comparison of extreme values of wind pressures obtained by two methods

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Source	Climatic normals			Hourly data			NBCC			Climatic normals			Hourly data			Normals/Hourly		
										bias factors			bias factors			ratio of bias factors		
Station	q_{10} (kPa)	q_{30} (kPa)	q_{100} (kPa)	q_{10} (kPa)	q_{30} (kPa)	q_{100} (kPa)	q_{n10} (kPa)	q_{n30} (kPa)	q_{n100} (kPa)	q_{10}/q_n (kPa)	q_{30}/qn (kPa)	q_{100}/q_n (kPa)	q_{10}/q_n (kPa)	q_{30}/qn (kPa)	q_{100}/q_n (kPa)	q_{10}	q_{30}	q_{100}
Vancouver	0.35	0.42	0.49	0.36	0.45	0.55	0.45	0.55	0.67	0.79	0.76	0.74	0.80	0.81	0.82	0.98	0.93	0.89
Kelowna	0.15	0.18	0.21				0.34	0.43	0.53	0.44	0.41	0.39						
Edmonton	0.24	0.27	0.32	0.26	0.29	0.33	0.32	0.40	0.51	0.74	0.69	0.63	0.80	0.73	0.65	0.92	0.95	0.97
Regina	0.89	1.06	1.26				0.34	0.39	0.46	2.62	2.72	2.74						
Saskatoon	0.50	0.58	0.68				0.36	0.44	0.54	1.39	1.33	1.26						
Winnipeg	0.36	0.42	0.49				0.35	0.42	0.49	1.02	1.00	1.00						
Thunder Bay	0.29	0.34	0.39				0.25	0.29	0.34	1.18	1.17	1.15						
Sault Ste Marie	0.39	0.45	0.51				0.32	0.37	0.43	1.21	1.21	1.20						
Toronto	0.42	0.50	0.59	0.44	0.54	0.67	0.39	0.48	0.59	1.08	1.03	0.99	1.12	1.13	1.14	0.96	0.91	0.87
Montreal	0.37	0.43	0.50				0.31	0.37	0.44	1.18	1.16	1.14						
Fredericton	0.29	0.34	0.40				0.30	0.37	0.46	0.96	0.91	0.86						
Halifax	0.44	0.50	0.57				0.40	0.52	0.67	1.10	0.96	0.85						
Charlottetown	0.43	0.50	0.58				0.46	0.55	0.66	0.94	0.91	0.88						
St John's	0.83	0.99	1.17				0.60	0.73	0.89	1.39	1.35	1.32						
Yellowknife	0.24	0.27	0.32				0.34	0.43	0.53	0.70	0.64	0.60						
Whitehorse	0.24	0.28	0.32				0.28	0.34	0.42	0.86	0.81	0.75						

Table 4.19 - Estimate of variation in C_w

	Elevation		Temperature (°C)		Pressure	C_w *		Normalized C_w	
Station	ft	m	Jan	July	millibars	μ (x 10^{-5})	σ (x 10^{-6})	μ	σ
Vancouver	3	0.9	4	16	1013.2	4.82	0.51	0.96	0.010
Kelowna	430	131.1	-6	22	1008.8	4.84	1.20	0.97	0.024
Yellowknife	205	62.5	-29	15	1011.1	5.15	2.13	1.03	0.043
Regina	577	175.9	-15	20	1007.3	4.94	1.57	0.99	0.031
Winnipeg	239	72.8	-18	20	1010.7	4.98	1.73	1.00	0.035
Toronto	113	34.4	-3	22	1012.0	4.83	1.07	0.97	0.021
St.John's	134	40.8	-3	18	1011.8	4.86	0.91	0.97	0.018

* when V is given in km/hr

Table 4.20 - Estimates of wind load factors common to all cities

	δ	COV	Distribution
C_e	1.0	0.08	Normal
C_p	1.0	0.10	Normal
C_g	1.0	0.10	Normal
C_d	0.85	0.0	Constant
C_m	0.85	0.0	Constant

Table 4.21 - Summary of values used in the simulation

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	C_w - normal distribution (kPa(hr/km)^2)		V - from annual maximum - Gumbel distribution (km/hr)		V_d - daily maximum - Gumbel distribution (km/hr)		q_n - from NBCC (kPa)			Normalized C_e - normal distribution		Normalized Cg - normal distribution		Normalized C_p - normal distribution	
Location	μ (x 10^{-5})	σ (x 10^{-6})	u	α	u	α	q_{10}	q_{30}	q_{100}	μ	σ	μ	σ	μ	σ
Vancouver	4.82	0.51	65.082	0.151	21.48	0.11	0.45	0.55	0.67	1.0	0.08	1.0	0.10	1.0	0.10
Kelowna	4.84	1.20	42.114	0.232	13.90	0.17	0.34	0.43	0.53	1.0	0.08	1.0	0.10	1.0	0.10
Yellowknife	5.15	2.13	55.337	0.211	18.26	0.16	0.34	0.43	0.53	1.0	0.08	1.0	0.10	1.0	0.10
Regina	4.94	1.57	101.946	0.091	33.64	0.07	0.34	0.39	0.46	1.0	0.08	1.0	0.10	1.0	0.10
Winnipeg	4.98	1.73	66.413	0.158	21.92	0.12	0.35	0.42	0.49	1.0	0.08	1.0	0.10	1.0	0.10
Toronto	4.83	1.07	70.889	0.139	23.39	0.10	0.39	0.48	0.59	1.0	0.08	1.0	0.10	1.0	0.10
St.John's	4.86	0.91	99.365	0.097	32.79	0.07	0.60	0.73	0.89	1.0	0.08	1.0	0.10	1.0	0.10

Table 4.22 - Results from simulation of wind loads - characteristic values

1	2	3	4	5	6	7	8	9	10
	Normalized p_{10} - from simulation		Normalized p_{30} - from simulation		Normalized p_{100} - from simulation		p - from calculations (mean values)		
Location	μ	σ	μ	σ	μ	σ	p_{10}	p_{30}	p_{100}
Vancouver	0.55	0.15	0.53	0.14	0.52	0.13	0.55	0.53	0.51
Kelowna	0.31	0.08	0.29	0.07	0.28	0.07	0.31	0.29	0.27
Yellowknife	0.53	0.13	0.48	0.12	0.45	0.11	0.52	0.48	0.45
Regina	1.90	0.52	1.96	0.51	1.98	0.49	1.87	1.94	1.96
Winnipeg	0.75	0.20	0.73	0.19	0.73	0.18	0.74	0.72	0.72
Toronto	0.76	0.20	0.73	0.18	0.70	0.17	0.75	0.72	0.69
St.John's	0.98	0.27	0.96	0.24	0.93	0.23	0.98	0.95	0.92

1	11	12	13	14	15	16	17	18	19	20	21	22
	Best fit distribution for normalized p_{10}				Best fit distribution for normalized p_{30}				Best fit distribution for normalized p_{100}			
Location	Type	R-square	u	α	Type	R-square	u	α	Type	R-square	u	α
Vancouver	Gumbel	1.00	0.41	7.40	Gumbel	0.99	0.42	8.30	Gumbel	0.99	0.43	9.40
Kelowna	Gumbel	0.99	0.23	13.00	Gumbel	0.99	0.22	15.20	Gumbel	0.99	0.22	17.40
Yellowknife	Gumbel	0.97	0.39	8.00	Gumbel	0.97	0.37	9.50	Gumbel	0.97	0.36	10.90
Regina	Gumbel	0.99	1.45	2.20	Gumbel	0.99	1.61	2.30	Gumbel	0.99	1.76	2.50
Winnipeg	Gumbel	0.98	0.49	5.10	Gumbel	0.98	0.51	5.70	Gumbel	0.98	0.54	6.10
Toronto	Gumbel	0.98	0.55	5.30	Gumbel	0.98	0.56	6.10	Gumbel	0.98	0.57	6.90
St.John's	Gumbel	0.97	0.65	3.90	Gumbel	0.96	0.68	4.40	Gumbel	0.97	0.72	4.90

Table 4.23 - Results from simulation of wind loads - companion values

1	2	3	4	5	6	7	8	9	10
	Normalized p_d - from simulation		Normalized p_2 - from simulation		Normalized p_8 - from simulation		p - from calculations (mean values)		
Location	μ	σ	μ	σ	μ	σ	p_d	p_2	p_8
Vancouver	0.054	0.054	0.35	0.10	0.44	0.12	0.045	0.34	0.43
Kelowna	0.029	0.028	0.19	0.06	0.24	0.06	0.024	0.18	0.23
Yellowknife	0.047	0.040	0.33	0.09	0.40	0.10	0.042	0.33	0.40
Regina	0.191	0.186	1.25	0.38	1.59	0.44	0.163	1.23	1.57
Winnipeg	0.072	0.067	0.48	0.14	0.60	0.16	0.062	0.48	0.59
Toronto	0.073	0.073	0.47	0.14	0.59	0.16	0.061	0.47	0.59
St.John's	0.096	0.096	0.62	0.18	0.78	0.21	0.080	0.61	0.77

1	11	12	13	14	15	16	17	18	19	20	21	22
	Best fit distribution for normalized p_d				Best fit distribution for normalized p_2				Best fit distribution for normalized p_8			
Location	Type	R-square	μ	σ	Type	R-square	u	α	Type	R-square	u	α
Vancouver	lognormal	0.99	0.054	0.054	Gumbel	0.95	0.165	8.50	Gumbel	0.96	0.244	7.80
Kelowna	lognormal	0.98	0.029	0.028	Gumbel	0.99	0.124	18.50	Gumbel	0.99	0.174	16.70
Yellowknife	lognormal	0.99	0.047	0.040	Gumbel	0.97	0.221	11.30	Gumbel	0.97	0.293	10.32
Regina	lognormal	0.98	0.191	0.186	Gumbel	0.98	0.893	2.80	Gumbel	0.98	1.240	2.50
Winnipeg	lognormal	0.99	0.072	0.067	Gumbel	0.97	0.287	6.80	Gumbel	0.98	0.396	6.21
Toronto	lognormal	0.99	0.073	0.073	Gumbel	0.98	0.301	7.30	Gumbel	0.98	0.430	6.60
St.John's	lognormal	0.98	0.096	0.096	Gumbel	0.96	0.358	5.20	Gumbel	0.96	0.515	4.76

Table 4.24 - Summary of wind load values used

	Normalized characteristic values											
	p_{10}				p_{30}				p_{100}			
Location	u	α	μ	σ	u	α	μ	σ	u	α	μ	σ
Vancouver	0.41	7.40	0.483	0.173	0.42	8.30	0.491	0.155	0.43	9.40	0.487	0.136
Kelowna	0.23	13.00	0.275	0.099	0.22	15.20	0.262	0.084	0.22	17.40	0.257	0.074
Yellowknife	0.39	8.00	0.460	0.160	0.37	9.50	0.429	0.135	0.36	10.90	0.411	0.118
Regina	1.45	2.20	1.717	0.583	1.61	2.30	1.860	0.558	1.76	2.50	1.987	0.513
Winnipeg	0.49	5.10	0.603	0.251	0.51	5.70	0.610	0.225	0.54	6.10	0.636	0.210
Toronto	0.55	5.30	0.656	0.242	0.56	6.10	0.652	0.210	0.57	6.90	0.649	0.186
St.John's	0.65	3.90	0.802	0.329	0.68	4.40	0.813	0.291	0.72	4.90	0.834	0.262

	Normalized companion values									
	p_d*		p_2				p_8			
Location	μ	σ	u	α	μ	σ	u	α	μ	σ
Vancouver	0.054	0.054	0.16	8.50	0.233	0.151	0.24	7.80	0.318	0.164
Kelowna	0.029	0.028	0.12	18.50	0.156	0.069	0.17	16.70	0.208	0.077
Yellowknife	0.047	0.040	0.22	11.30	0.272	0.113	0.29	10.32	0.349	0.124
Regina	0.191	0.186	0.89	2.80	1.099	0.458	1.24	2.50	1.471	0.513
Winnipeg	0.072	0.067	0.29	6.80	0.372	0.189	0.40	6.21	0.489	0.207
Toronto	0.073	0.073	0.30	7.30	0.380	0.176	0.43	6.60	0.518	0.194
St.John's	0.096	0.096	0.36	5.20	0.469	0.247	0.51	4.76	0.636	0.269

* All variables have Gumbel distributions, except p_d which has a lognormal distribution.

Table 4.25 - Analysis of zonal velocity

	v at annual probability of exceedance, p									
Location	0.01	0.005	0.0021	ζ	λ	exp(λ)	v_n	k	u_n	α_n
Vancouver	0.077	0.12	0.21	1.866	-6.913	0.00100	0.0502	1.50	-2.99	1.50
Toronto	0.014	0.023	0.038	1.867	-8.603	0.00018	0.0093	1.50	-4.68	1.50
St.John's	0.013	0.026	0.052	2.593	-10.361	0.00003	0.0073	1.08	-4.91	1.08

Location	μ_n	σ_n	$(COV)_n$	μ_{nn}	σ_{nn}	u_{nn}	α_{nn}	v_{nn}	k_{nn}
Vancouver	-2.61	0.856	-0.328	-1.07	0.856	-1.45	1.50	0.233	1.50
Toronto	-4.30	0.856	-0.199	-2.76	0.856	-3.14	1.50	0.043	1.50
St.John's	-4.38	1.189	-0.272	-2.24	1.189	-2.78	1.08	0.062	1.08

Table 4.26 - Summary of zonal velocity parameters

Location	v	k	Nominal	Normalized values v	Normalized values k
Vancouver	0.23	1.50	0.20	1.17	0.30
Toronto	0.04	1.50	0.05	0.86	0.07
St.John's	0.06	1.08	0.05	1.24	0.05

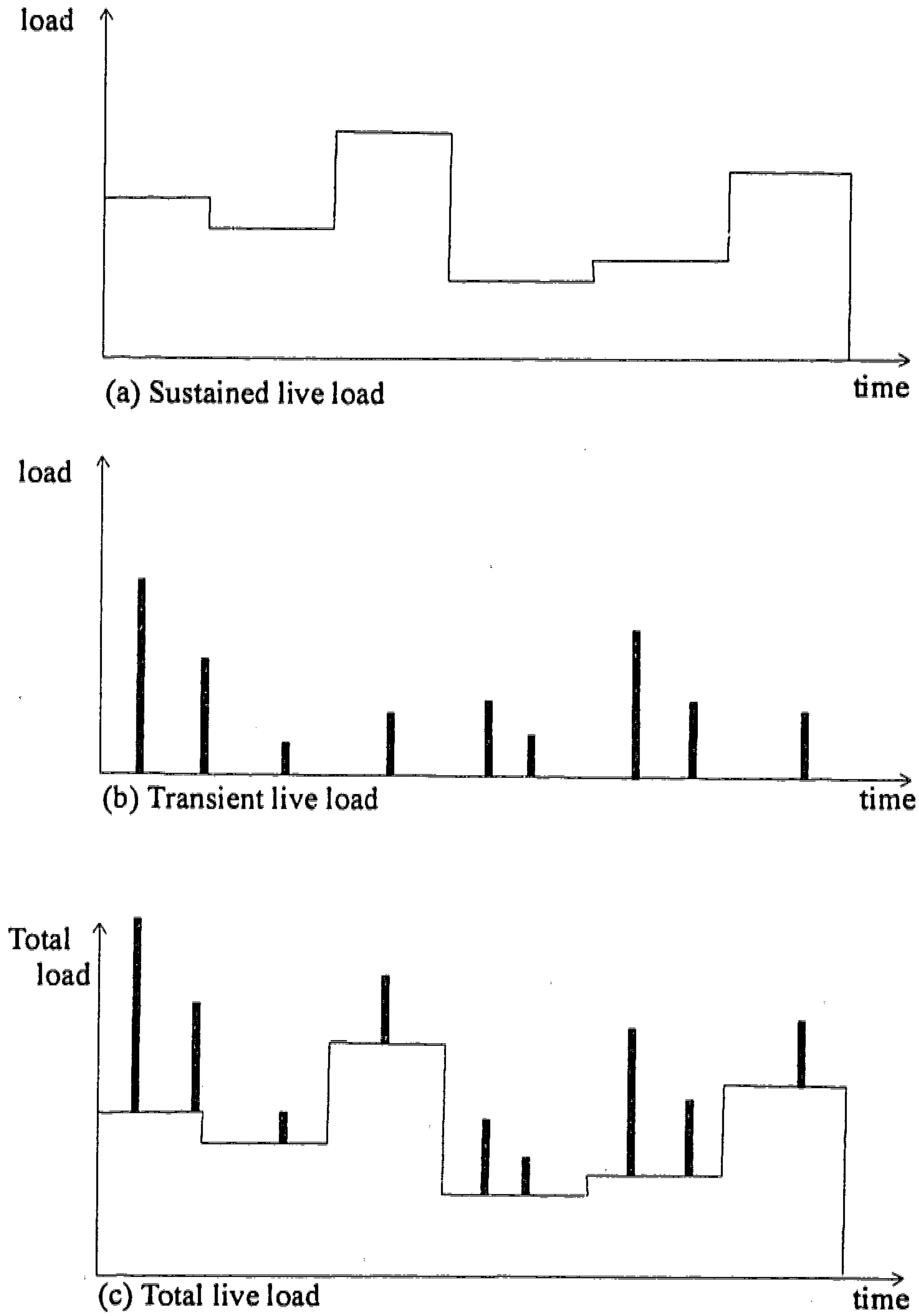
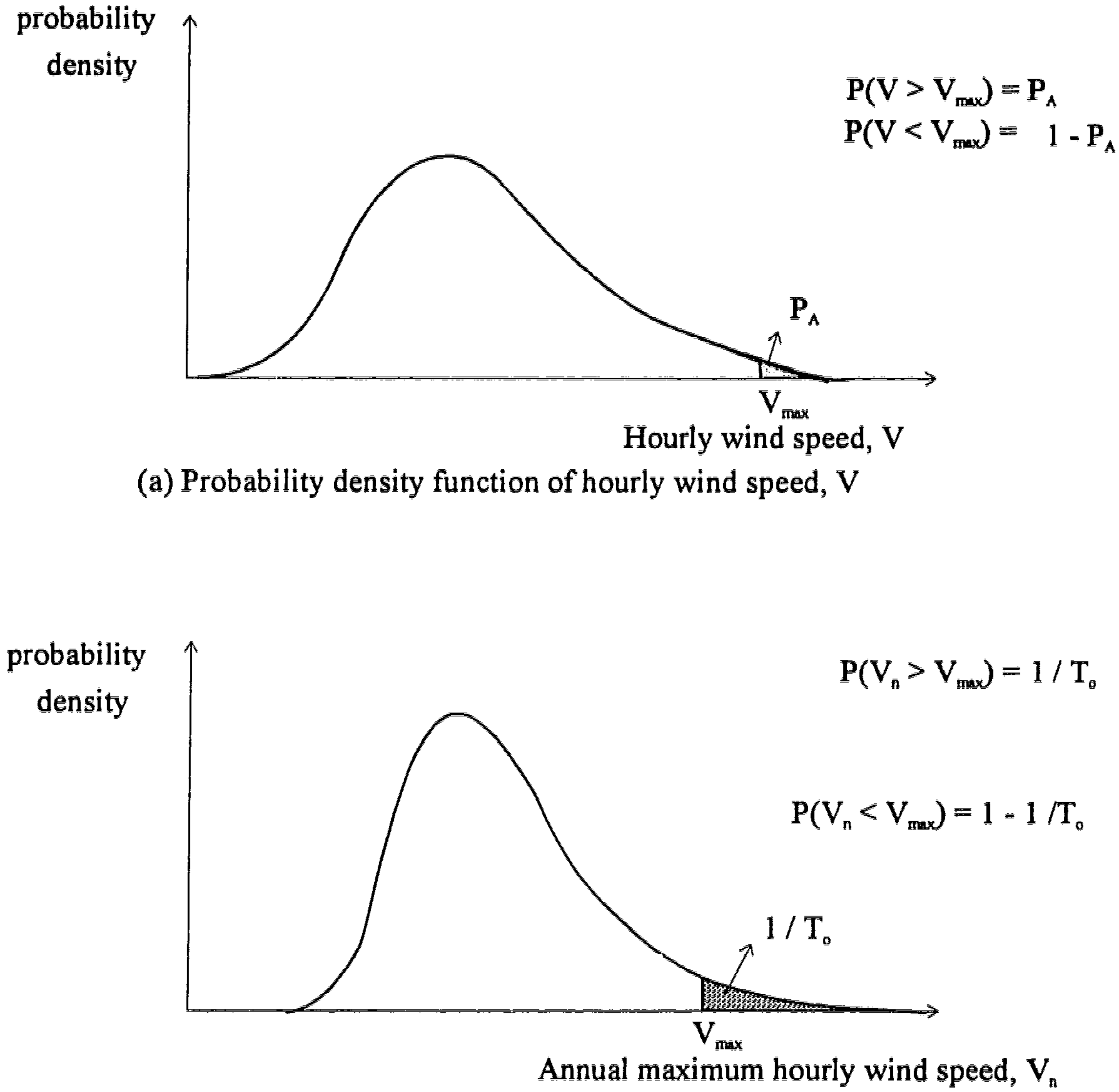


Figure 4.1 - Model of live load



(a) Probability density function of hourly wind speed, V

(b) Probability density function of annual maximum hourly wind speed, V_n

Figure 4.2 - Probability density functions for wind speed

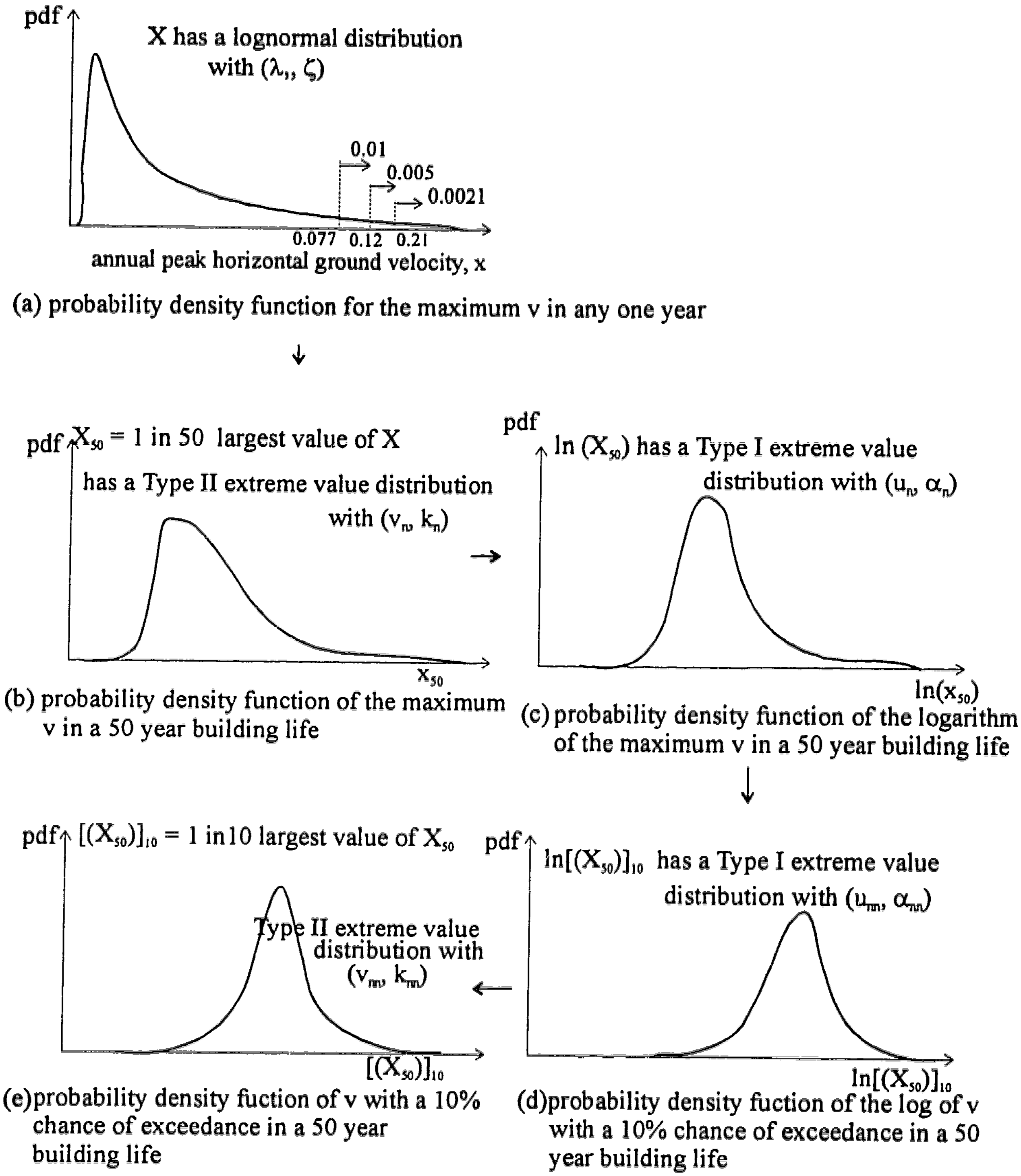


Figure 4.3 - Development of relevant distribution for the earthquake load

CHAPTER 5

RESISTANCE DISTRIBUTIONS AND PARAMETER VALUES

5.1 Introduction

As an initial step in the calculation of resistance factors, it is necessary to study the variability of the strength of reinforced concrete members in flexure, shear, and combined axial force and flexure. The true strength of a reinforced concrete member differs from the calculated or "nominal resistance". This is due to variations in the basic variables such as the material strengths and dimensions of the member, as well as the uncertainties inherent in the equations used to compute the member strength. The variability of these basic variables have been established by other researchers.

Section 5.2 of this chapter documents these basic resistance variables. Simulation methods used to calculate the member resistances are given in Section 5.3. Finally, Section 5.4 summarizes the results of simulations performed using the basic variables.

The terms 'specified', 'nominal' and 'true' variable used in referring to the load and resistance are explained in Section 6.1.

5.2 Basic Variables

The true strength of reinforced concrete members depends on the variability of the basic variables that affect the strength. These basic variables are the properties of concrete, properties of steel reinforcement, dimensions of the member and the model error. These are considered individually.

5.2.1 Properties of Concrete

5.2.1.1 Concrete Strength in Compression

Bartlett and MacGregor (1994) have established a relationship between the compressive strength of concrete in a structure and the cylinder strength, both tested at a standard rate of loading, assumed to be 35 psi/sec (0.241 MPa/sec). In their study, the relationship between the specified strength of concrete, f_c', and the corresponding compressive strength in an element or structure, f_{cstr}, is described by two independent factors, F_1 and F_2.

[5.1] $$\bar{f}_{cstr35} = F_1 F_2 f_c'$$

Factor F_1 represents the ratio of the average strength of standard cylinder specimens to the specified strength, f_c'. To have some assurance that the material provided will meet specifications, the concrete producer typically ensures that F_1 exceeds 1.0. Factor F_2 is the ratio of the average *in-situ* strength to the average strength of 28 day old standard cylinder specimens, and depends on the age and height of the element and on the quality of curing provided.

For conventional concretes the mean value of the product of F_1 F_2 for cast-in-place and non-steam-cured pre-cast construction is described by a lognormal distribution. Cast-in-place concrete has a mean of 1.2 for shallow elements and 1.3 for tall elements (elements greater than 0.45 m high). For non-steam-cured precast construction the mean values are roughly 6% less, i.e., 1.13 and 1.23 for shallow and tall elements respectively. This mean value of the product of F_1 F_2 has a coefficient of variation (COV) of 18.6% for cast-in-place construction and 15% for non-steam-cured precast construction. The COV due to *in-situ* strength variation throughout the structure (i.e., the within structure variation) is 0.130 for cast-in-place and 0.103 for precast construction. Thus the overall COV of the *in-situ* strength of concrete, f_{cstr}, for a cast-in-place structure composed of many members and cast from many batches, is 0.227 as shown in Table 5.1. Similarly the overall COV of the *in-situ* strength of concrete in precast units is 0.182.

These values for the mean and COV of f_{cstr35} derived by Bartlett and MacGregor (1994) are augmented by the r_R factor given by Mirza et al. (1979) to account for a loading rate, R, that is different than the standard 35 psi/sec (0.241 MPa/sec). The *in-situ* concrete strength in compression was given as:

$$[5.2] \qquad f_{cstrR} = f_c' \, r_{creal} \, r_{in-situ} \, r_R$$

where

f_{cstrR} = *in-situ* concrete compressive strength loaded at a rate of R psi/sec

r_{creal} = random variable relating real cylinder strength to design compressive strength

$r_{in-situ}$ = random variable relating *in-situ* strength to real cylinder strength

r_R = random variable relating strengths at R psi/sec and 35 psi/sec

The mean *in-situ* concrete strength in compression is given by:

$$[5.3] \qquad \bar{f}_{cstrR} = \bar{f}_{cstr35} 0.89(1 + 0.08 \log R)$$

where R = loading rate in psi/sec

Mirza et al. (1979) found the variation in f_{cstrR} due to the loading rate in a structure to be negligible therefore it is neglected in this study.

The terms F_1 and F_2 used by Bartlett and MacGregor (1994) are equivalent to the terms r_{creal} and $r_{in-situ}$. Therefore, using [5.3] and substituting for $\bar{f}_{cstr35}$ from [5.1],

$$[5.4] \qquad \bar{f}_{cstrR} = F_1 \, F_2 \, f_c' \; 0.89(1 + 0.08 \log R)$$

This equation is used to find the mean value of *in-situ* concrete strength and the results are given in Table 5.2(a) and 5.2(b) for shallow and tall members respectively. The loading rate R, in the structure, used in this analysis is $f_c'/3600$ MPa/sec corresponding to a one hour loading to failure. The variation due to the loading rate is negligible (Mirza et al. 1979) therefore the coefficient of variation given by Bartlett and MacGregor (1994) remains the same.

It should be mentioned that in reinforced concrete design the equivalent stress in the concrete stress block is taken as 0.85 f_c' before 1994 and as α_1 f_c' in CSA A23.3-94 where α_1 equal to 0.85 - 0.0015 f_c' but not less than 0.67. This, at least in part, takes the loading rate into consideration. This α_1 factor is used in the calculation of nominal resistance to take the loading rate into account. But in the calculation of true random concrete strength the variable values developed herein are used and the loading rate is accounted for by the r_R factor.

5.2.1.2 Concrete Strength in Tension and Modulus of Elasticity

Mirza et al. (1979) concluded that the probability model of the splitting tensile strength of concrete in a structure can be described with a normal distribution with mean value and coefficient of variation calculated from:

[5.5] $$\bar{f}_{spstrR} = 6.4\left[\bar{f}_{cstr35}\right]^{1/2} [0.96\,(1+0.11\ \log R)] \times 6.895 \times 10^{-3} \quad \text{MPa}$$

[5.6] $$V^2_{spstrR} = \frac{V^2_{cstrR}}{4} + 0.13^2 \geq V^2_{cstrR}$$

where V_{cstrR} is the coefficient of variation of f_{cstrR}. As V_R is assumed to be negligible $V^2_{cstrR} = V^2_{cstr35}$.

Similarly, based on Mirza et al. (1979), the probability model of the strength of *in-situ* concrete in flexural tension is assumed to follow a normal distribution with the mean and coefficient of variation calculated from:

$$[5.7] \qquad \bar{f}_{rstrR} = 8.3\left[\bar{f}_{cstr35}\right]^{1/2} [0.96\,(1 + 0.11 \times \log R)] \times 6.895 \times 10^{-3} \quad \text{MPa}$$

$$[5.8] \qquad V^2_{rstrR} = \frac{V^2_{cstrR}}{4} + 0.20^2 \geq V^2_{cstrR}$$

The distribution for initial tangent modulus of elasticity of *in-situ* concrete was assumed to be a normal distribution. The mean and coefficient of variation are given by:

$$[5.9] \qquad \bar{E}_{cistrR} = 60{,}400\left[\bar{f}_{cstr35}\right]^{1/2} \; (1.16 - 0.08 \times \log t) \times 6.895 \times 10^{-3}\,\text{MPa}$$

$$[5.10] \qquad V^2_{cistrR} = \frac{V^2_{cstrR}}{4} + 0.08^2$$

where t = loading duration in seconds

These equations are used with $\bar{f}_{cstr35}$ and V^2_{cstr35} estimates given by Bartlett and MacGregor (1994) to obtain the results given in Table 5.2(a) and 5.2(b) for shallow and tall members respectively.

5.2.2 Properties of Reinforcement

Mirza and MacGregor (1979b) analyzed data for the mechanical properties of reinforcement bars from 273 mill tests. They found that the data was positively skewed and was in good agreement with a shifted lognormal distribution in the lower tail and mid regions. They also found that the beta distribution fitted the whole range of data. They estimated that the difference between the mill test yield strength and static yield strength was approximately 28 MPa. However the distributions and parameters they established were for bars that were manufactured according to imperial units, having yield strengths of 60 ksi (413 MPa).

More recent data from quality control tests by Canadian steel manufacturers was investigated by Nessim et al. (1993) for a steel grade of 400 MPa and bar sizes between 20M and 35M. They concluded that the bar size does not have a significant influence on either the mean or the coefficient of variation of the yield strength. The mill test yield strength for grade 400 steel was found to have a mean value of 470 MPa and a COV of 0.06. The yield strength was defined as the yield force divided by the nominal area, and therefore the variability of the yield force combines variabilities of both the yield strength and the cross-sectional area. As loading rate effects were neglected by Nessim et al. (1993), the static yield strength was not calculated. In this study loading rates are taken into account and therefore the mean of 470 MPa is adjusted by subtracting the difference of 28 MPa between mill test yield strength and static yield strength established by Mirza and MacGregor (1979b). Therefore, the mean of the static yield is estimated by 470 - 28 = 442 MPa and the COV is 0.06 (i.e., a standard deviation of 26.5 MPa) for the current manufacturing practices.

Many types of distributions have been used to describe the yield strength. Nessim et al. (1993) used a lognormal distribution to describe the steel yield strength. MacGregor (1976) gives a fitting of yield strength data produced in Sweden as an example and in this a positively shifted lognormal distribution was used. On looking at yield strength data a positive skewness and a certain minimum limit is evident. This is expected since quality control procedures weed out under-strength heats. Therefore, a beta or shifted lognormal distribution would be most appropriate in describing the yield strength. A beta distribution fit the whole distribution in Mirza and MacGregor's (1979b) analysis. The shifted lognormal distribution gave a good fit in the lower tail region which is the critical region. Consequently, a shifted lognormal is used to describe the static yield strength.

The shift in the static yield strength is equal to the minimum static yield strength that is possible in reinforcement used in structures. In Canada billet steel bars for concrete reinforcement are manufactured and controlled by CSA Standard G30.18 which states that Grade 400 steel has to have a minimum yield strength of 400 MPa. The standard also states that (depending on the size of the heat) at least one tension test shall be made of each bar size rolled from a heat. Therefore bars that are marketed have been tested and found to have a minimum dynamic yield strength of 400 MPa. But there is a strength variation within the bars manufactured from one heat. Therefore the minimum yield strength has a variability of its own. The minimum dynamic yield strength is assumed to have a mean of 400 MPa and the minimum static yield strength a mean of 400 - 28 MPa. The coefficient of variation within a bar size from the same source is estimated to be 1- 4 % (Mirza and MacGregor 1979b). It is assumed that from the same heat the COV is 1%. Therefore the minimum static yield strength has a COV of 1% which corresponds to a standard deviation of $0.01 \times (400 - 28) = 3.72$ MPa. Assuming the minimum static yield has a normal distribution and that the minimum value lies two standard deviations below the mean, the minimum of the minimum static yield strength (or shift) is estimated as, $\Delta ys = (400 - 28) - 2 \times 3.72 = 365$ MPa. These values are summarized in Table 5.3. This is the lowest static yield strength allowed in the distribution.

The probability distribution of modulus of elasticity of reinforcing steel is considered normal with parameters given in Table 5.3 (Mirza and MacGregor 1979b).

Because the reinforcement in a concrete member must be some combination of whole bars, the area of steel actually provided may differ from the calculated. This effect was considered to have a modified (shifted) lognormal distribution having a mean of 1.01, a COV of 0.04 and a modification constant (shift) of 0.91 below which the modified lognormal distribution equals zero (Mirza and MacGregor

1982). It is recognized that their study used imperial bar sizes and the SI bar sizes would give slightly different values.

5.2.3 Dimensions

The differences between the nominal and as-built dimensions of concrete sections are best characterized by the means and standard deviations of the differences as shown in Table 5.3 (Mirza and MacGregor 1979a). Since the dimensional variations in beams are roughly independent of beam size, the COV decreases as the member size increases. This was not strictly the case for column dimensions, which were found to be somewhat size-dependent.

5.2.4 Model Error

The model error is the variability caused by the calculation model used to quantify the strength. Test data from two studies (Hognestad 1951, and Ibrahim 1994) is analyzed to obtain the model error and check calculations. However, due to the few points of applicable data the variability of the model error could not be adequately estimated. Therefore, the model error is calculated according to Mirza and MacGregor (1982). Because the theoretical models used to estimate beam and beam-column action and shear are similar in both their study and this study, their model errors are assumed to be applicable to this study as well. To determine the model error the accurate calculation procedure was compared with tests to get the mean and COV of the ratio of test strength to calculated strength.

The values of model error used are given in Table 5.4.

5.3 Calculation of Member Resistance

A method of obtaining the statistics of the unfactored member resistance, R_o (i.e., the 'true' member resistance) and the nominal factored resistance, R_{nr} (i.e., the factored resistance calculated using nominal or specified values as opposed to using true values) is established. The statistics of the unfactored 'true' member resistance represent the real life values. The nominal factored resistance represents the factored resistance calculated by the design engineer. The results of these analyses are presented in Section 5.4.

The member resistances are obtained as follows:

1. Representative cross sections and loading ratios are selected for each type of structural action, viz., flexure, shear and combined flexure and axial load.

2. For each structural action a relatively accurate method of calculating member resistance is selected. In general these methods are more accurate and comprehensive than design procedures. The model error explained in Section 5.2.4 is used to account for the bias and variability in the theoretical computational method.

3. For each representative cross section the following calculations are performed:

(a) The nominal factored resistance is calculated using the nominal material strengths, nominal dimensions, material resistance factors and the calculation procedure used in design i.e., CSA A23.3-M84 or CSA A23.3-94 as appropriate.

(b) A Monte Carlo simulation is used to generate the probability distribution of the unfactored 'true' resistance. This involved the following :

(i) A set of material strengths and dimensions is generated randomly from the statistical distributions of each basic variable. (Theoretical details of this procedure are given in section 2.5.)

(ii) These values are used with the accurate calculation procedure to estimate the theoretical unfactored capacity of the member.

(iii) This procedure is repeated 5000 times to generate a sample of unfactored resistance values for the particular cross section. The number of simulations used is selected by testing how many simulations are necessary to give repeatable results.

(iv) Different types of distributions are fitted to the sample of unfactored resistance values and the best fit distribution is chosen. The distributions are fitted to the lower tail of the 5000 point sample. The tail is taken to be below the 10th percentile, so that the fitted tail has 500 points (Theoretical details are explained in Section 2.6).

(v) Checks are performed by deterministic direct calculation of the means, maximums and minimums. In some instances theoretical predictions are compared with test results.

5.3.1 Beam and Beam Column Action

The SAS statistical package (SAS Institute Inc., 1993) is used to develop a program which simulates the resistance and fits distributions to the simulated values.

For a given axial load the flexural capacity of a reinforced concrete member is computed by deriving the moment-curvature diagram for the cross section. The maximum moment capacity for the particular axial load is then taken as the highest point on the moment-curvature diagram. This approach allowed either compression or tension failure to be detected in concrete members without a change in calculation procedures or equations. For beams, the axial load is set equal to zero. For columns, a sufficient number of axial load levels are considered to develop an interaction diagram that is used to determine the strengths at various eccentricity ratios.

The calculations are based on the following assumptions:

(1) The strains are assumed to be proportional to the distance from the neutral axis.

(2) The concrete and steel stresses are calculated as functions of the strains.

(3) The concrete stresses in compression are computed using the stress strain curve given by Thorenfeldt et al. (1987). Thorenfeldt et al. found that the stress strain diagrams for concretes of all strength classes (including strength classes with characteristic cube strength up to 105 MPa) can be approximated by curves of the same type, given by the equation:

$$\frac{f_{cx}}{f_{cm}} = \frac{n(\varepsilon_{cx}/\varepsilon_o)}{n-1+(\varepsilon_{cx}/\varepsilon_o)^{nk}} \qquad [5.11]$$

where,

f_{cx} = stress

f_{cm} = maximum stress on the stress-strain curve

ε_{cx} = strain

ε_o = strain at maximum stress f_{cm}

$= \dfrac{f_{cm}\, n}{E_c(n-1)}$

E_c = initial tangent modulus (MPa)

$n = 0.8 + \dfrac{f_{cm}}{17}$ (MPa)

$k = 1.0$ for $\dfrac{\varepsilon_{cx}}{\varepsilon_o} \leq 1.0$

$= 0.67 + \dfrac{f_{cm}}{62}$ but not less than 1.0 for $\dfrac{\varepsilon_{cx}}{\varepsilon_o} > 1.0$

These curves have shapes similar to the curves recorded when testing at constant strain rate, including the descending part.

The maximum stress on the stress-strain curve, f_{cm}, for concrete in the member was found to be 0.85 times the cylinder strength by Hognestad (1951). Grant (1976) also found that using 0.85 to 0.87 times cylinder strength for the maximum stress on the stress-strain diagram gave a good fit to data. When calculations were checked against test data, $f_{cm} = 0.85\ f_{cstrR}$ is found to give a good fit. Therefore this value is used to define the curve.

(4) No allowance is made for shrinkage and creep.

(5) For concrete in tension a linear brittle stress-strain diagram with the tensile strain at rupture equal to f_{rstr}/E_{cistr} is assumed.

(6) An elastic-plastic stress-strain curve is assumed for reinforcing bars. Mirza and MacGregor (1982) found that of all the assumptions made, this had the greatest effect on the accuracy of the solutions for reinforced concrete members. Their calculations suggest that inclusion of strain hardening would increase the ultimate moment by an amount ranging from less than 5% for steel ratios representative of beams to as much as 20% for very lightly reinforced slabs. Other researchers (Grant 1976, Mirza and MacGregor 1982, Ellingwood et al. 1980) have ignored the effect of strain hardening because the deformations required to utilize strain hardening are very large and are accompanied by a risk of failure due to bond or shear before a complete hinge system develops. An exception to this is thin, lightly reinforced slabs in which large redistribution of moments is possible. Ellingwood et al. (1980) showed that the effect of the increase in mean strength due to strain hardening on the resistance factor, ϕ, in such members, is offset by the increased variability resulting from the large variability of the effective depth in shallow members.

(7) According to the CSA A23.3 continuous flexural members have to be designed for pattern loading. Even though some redistribution is allowed this redistribution in beams depends on many variables and is difficult to assess in probabilistic terms. A beam is assumed to fail if one section reaches its moment capacity. This would be the case for statically determinate members. As members are often statically indeterminate this assumption adds conservatism to the factors developed.

5.3.2 Shear in Beams

The shear strength regression equation developed by Zsutty (1971) is used to estimate the true shear resistance in beams.

$$V = b\,d\left[2.17\left(f'_c\,\rho\frac{d}{a}\right)^{1/3} + \frac{A_v}{b\,s}f_{ys}\right] \qquad [5.12]$$

where, b = web width

d = depth of beam

ρ = longitudinal tension reinforcement ratio

a = shear span

A_v = area of one vertical stirrup

s = stirrup spacing

f_{ys} = yield strength of stirrups

Zsutty derived this equation for beams in the literature that have $A_v f_{ys}/bs$ greater than 60psi (0.414 MPa), s/d smaller than 0.5 and a/d greater than 2.5. Previously, he derived an equation having only the first term in the square brackets for beams without stirrups and a/d greater than 2.5.

5.4 Resistance Statistics

5.4.1 Beams

The analysis explained in Section 5.3.1 is performed on the beams given in Table 5.5. The specified material strengths are taken as f_y equal to 400 MPa and f_c' equal to 35 MPa.

The following design rules and material resistance factors obtained from CSA - A23.3-M84 and 94 are used together with the nominal values of f_y and f_c' to calculate the factored moment resistance (M_{nr}). The calibration of β_T is based on the 1984 code since there has been a decade of successful performance of designs based on this code. However, load and resistance factors developed are applicable to the 1994 code.

[5.13] $$M_{nr} = \phi_s A_s f_y (d - a/2)$$

[5.14] $$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b}$$

where

$\alpha_1 = 0.85$ according to A23.3-M84

[5.15] $\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67$ according to A23.3-94

These calculated M_{nr} values, according to A23.3-M84 and A23.3-94, are also given in Table 5.5. The α_1 values for the 1994 code are smaller than in the 1984 code, but because the beam reinforcement is taken proportional to the balanced ρ, which is higher in the 1994 code, the M_{nr} values for the 1994 code are higher.

Using the basic variables (*in-situ* concrete strength, static yield strength and dimensions) established, 5000 possible beam configurations are simulated for each nominal beam. Thus 5000 possible values of the unfactored moment resistance, M_o, are generated for each beam. Several types of distributions are fitted to the 500 points in the lower tail (i.e., below the 10th percentile) and the normal probability distribution curve gave one of the best fits in each case. (Distribution fitting and parameter estimation procedures are explained in section 2.6.) By performing a statistical analysis the statistical parameters are obtained for the normal distribution fitted to the tail. The statistical parameters obtained, viz., the mean and standard deviation, for the beams are given in Table 5.6 . These can be compared to the factored nominal strengths given in Table 5.5. The goodness of fit for the simple linear regression model is usually expressed in terms of the R-square, the coefficient of determination. The R-square value is the proportion of the variation in the dependent variable explained by the fitted linear regression relationship, in terms of the explanatory variable.

These values do not include the model error factor. As it is a directly mulitiplicative value it is included when doing the reliability analysis.

5.4.2 Short Columns

The analysis explained in Section 5.3.1 is performed on the columns defined by the nominal dimensions and material properties given in Table 5.7.

Using the nominal material strengths and dimensions and equilibrium and compatibility conditions with the stress block based on the CSA - A23.3 (M84 and 94) the nominal factored resistances given in Table 5.8 are calculated. These are for eccentricities of e/h=0.1 for the compression failures and e/h equal to 0.7 and 1.2 for tension failure of Column 1 and Column 2, respectively.

For each nominal cross section and a number of constant axial load levels, P_o, 5000 possible columns were simulated using the basic variables established earlier. For each of these 5000 simulated columns a moment curvature diagram is developed and the maximum moment on the curve is taken as the unfactored moment resistance, M_o. These values are analyzed and distributions are fitted to the lower tail (i.e., below the 10th percentile). Distribution fitting is done according to the theory developed in Section 2.6. (The SAS programs developed for the simulation, moment curvature analysis, interaction diagram development, statistical analysis and distribution fitting are given in Appendix B.) The normal probability distribution gave the best fit and the statistical parameters obtained are given in Table 5.9. The value P_x ,given in Table 5.9, is a deterministic arbitrary value which is taken higher than the maximum pure axial load. This allowed the interaction diagram to be estimated at similar intervals.

For values of axial load above P_o equal to 0.6 P_x some simulations gave negative values of moment due to the variability in the location of the steel. Simulations beyond P_o equal to 0.6 P_x are not carried out because they are not necessary to estimate the lower tails of the distributions for compression and tension failure at the chosen e/h ratios. The distribution of the axial load at zero moment is estimated purely to complete the interaction diagram.

The five curves shown in Figs. 5.1 and 5.2 are the 2.3, 15.9, 50, 84.1, and 97.7 percentile curves from the Monte Carlo simulation. These are the mean and one or two standard deviations from the mean. The lines are spaced approximately equal distances apart indicating that the distribution is close to a normal distribution.

Fig. 5.3 compares the percentile values of the simulated unfactored resistance, P_o, with the nominal factored resistance, P_{nr}, and nominal unfactored resistance, P_n, where nominal values are calculated according to CSA A23.3-94. The value of P_n is between the 2.3 and the 50^{th} percentile lines and P_{nr} is below even the 2.3 percentile.

Lines representing eccentricity ratios, e/h, equal to 0.1 and 0.7 are given on Fig. 5.1. The intersection of these lines with the variable interaction diagram gives the distribution for the compression failure and tension failure at e/h equal to 0.1 and 0.7. Similar lines for Column 2 at e/h equal to 0.1 and 1.2 are given on Fig. 5.2.

5.4.2.1 Compression Failure

The distribution for compression failures is taken as the distribution of the axial forces, P, obtained from the intersection of the line for e/h=0.1 and the curves in Figs. 5.1 and 5.2. The lower tail of the unfactored axial load resistance, P_o, (obtained at e/h equal to 0.1) is estimated from the 2.3, 15.9 and 50.0^{th} percentile lines in Figs. 5.1 and 5.2. These values are given in Table 5.10.

The percentile lines in Figs. 5.1 and 5.2 and Table 5.10 are almost equally spaced. This shows that the distribution of P_o is symmetric and normally distributed. Therefore, the distances between the 2.3, 15.9 and 50^{th} percentiles are equal to the standard deviation of the distribution. The mean is given by the 50^{th} percentile. The distribution is fitted to the lower tail values as this is the critical region for the resistance. The resistance statistics are given in Table 5.10.

5.4.2.2 Tension Failure

When evaluating present practice for calibration purposes the moment resistance is used as the resistance for tension failure. For this purpose, the distribution for tension failures is taken as the distribution of the moment resistance, M_o, obtained from the intersection of the line for e/h equal to 0.7 (or e/h equal to 1.2 for Column 2) and the percentile curves in Figs. 5.1 and 5.2. The estimates for the lower tail of this unfactored moment resistance, M_o, are given in Table 5.11.

The values in Figs. 5.1 and 5.2 and Table 5.11 show that the distribution of M_o is not skewed and as the percentile values are equi-distant it is normally distributed. Therefore, the distances between the 2.3, 15.9 and 50^{th} percentiles are equal to the standard deviation of the distribution. The mean is given by the 50^{th} percentile. The distribution is fitted to the lower tail values as this is the critical region for the resistance. The resistance statistics are given in Table 5.11.

The values in Table 5.10 and 5.11 do not include the model error factor. As it is a directly mulitiplicative value it is included when doing the reliability analysis.

The variable interaction diagrams are also used to obtain variables M_{pure} and θ for a more accurate tension failure analysis explained in section 7.5.1. (See Fig. 7.19 for a definition of M_{pure} and θ.) The values of M_{pure} and θ used are obtained from Figs. 5.1 and 5.2. The variability of M_{pure} is evaluated at the e/h values where tension failure is assumed to be most likely. These values are given in Table 5.12. The nominal values $M_{pure-nr}$ and θ_n (defined in Figure 7.20) are also given in Table 5.12

5.4.3 Shear in Beams

The range of properties of the beams chosen are within the range of values for which the theoretical model in Section 5.3.2. is developed. As the factored design strength is obtained according to CSA A23.3-94 the maximum and minimum reinforcement limitations and spacing limitations in this code are also satisfied in choosing the beams. All the beams have the same dimensions but have different longitudinal and shear reinforcement.

The nominal properties of the beams are given in Table 5.13 together with the notations used to identify the beams. In the beam notation s1, s2, and s3 refers to beams with ρ equal to 0.43, 1.08 and 3.59 % while v0, v1, and v2 refers to beams with ρ_v ,i.e., A_v / b s equal to 0.0, 0.2 and 0.6% . The nominal resistances calculated according to CSA A23.3-94 are also given in Table 5.13. As shear resistance is not evaluated according to CSA A23.3-M84 this resistance is not necessary.

Monte Carlo simulations are used, with Zsutty's regression equation and the basic variables established earlier, to estimate the shear resistance of the 9 beams chosen. The results of the simulations with the estimated shear resistance statistics are given in Table 5.14.

Table 5.1 - Values of COV from Bartlett and MacGregor (1994)

Variable	for cast-in-place construction	for non-steam-cured pre-cast construction
Mean of F_1 F_2	0.186	0.150
Within structure variation of F_1 F_2	0.130	0.103
F_1 F_2	$\sqrt{0.186^2 + 0.130^2}$ $= 0.227$	$\sqrt{0.150^2 + 0.103^2}$ $= 0.182$

Table 5.2(a) - Properties of concrete for shallow members*

Property of concrete		Quality of construction							
		Average (cast-in-place)				Excellent (factory precast)			
		Mean	f_{cstrR}			Mean	f_{cstrR}		
	MPa	f_{cstr35}	μ	σ	COV	f_{cstr35}	μ	σ	COV
Compressive strength									
for f_c' =	20.00	24.00	21.33	4.84	0.227	22.56	20.01	3.64	0.182
	30.00	36.00	32.45	7.36	0.227	33.84	30.44	5.54	0.182
	35.00	42.00	38.06	8.64	0.227	39.48	35.70	6.50	0.182
	40.00	48.00	43.70	9.92	0.227	45.12	40.99	7.46	0.182
Modulus of rupture									
for f_c' =	20.00	24.00	2.93	0.67	0.230	22.56	2.84	0.62	0.220
	30.00	36.00	3.63	0.83	0.230	33.84	3.51	0.77	0.220
	35.00	42.00	3.94	0.91	0.230	39.48	3.81	0.84	0.220
	40.00	48.00	4.22	0.97	0.230	45.12	4.09	0.90	0.220
Splitting strength									
for f_c' =	20.00	24.00	2.23	0.51	0.227	22.56	2.16	0.39	0.182
	30.00	36.00	2.76	0.63	0.227	33.84	2.67	0.49	0.182
	35.00	42.00	2.99	0.68	0.227	39.48	2.90	0.53	0.182
	40.00	48.00	3.21	0.73	0.227	45.12	3.11	0.57	0.182
Modulus of elasticity									
for f_c' =	20.00	24.00	21511	2986	0.139	22.56	20856	2527	0.121
	30.00	36.00	26345	3658	0.139	33.84	25543	3095	0.121
	35.00	42.00	28456	3951	0.139	39.48	27589	3342	0.121
	40.00	48.00	30421	4223	0.139	45.12	29494	3573	0.121

* a shallow member is one that is less than 0.45 m in height.

Table 5.2(b) - Properties of concrete for tall members*

Property of concrete		Quality of construction							
		Average (cast-in-place)				Excellent (factory precast)			
		Mean	f_{cstrR}			Mean	f_{cstrR}		
	MPa	f_{cstr35}	μ	σ	COV	f_{cstr35}	μ	σ	COV
Compressive strength									
for f_c' =	20.00	26.20	23.36	5.30	0.227	24.44	21.74	3.96	0.182
	30.00	39.30	35.54	8.06	0.227	36.66	33.07	6.02	0.182
	35.00	45.85	41.68	9.46	0.227	42.77	38.78	7.06	0.182
	40.00	52.40	47.85	10.86	0.227	48.88	44.53	8.10	0.182
Modulus of rupture									
for f_c' =	20.00	26.20	3.07	0.71	0.230	24.44	2.96	0.65	0.220
	30.00	39.30	3.80	0.87	0.230	36.66	3.66	0.81	0.220
	35.00	45.85	4.12	0.95	0.230	42.77	3.97	0.87	0.220
	40.00	52.40	4.42	1.02	0.230	48.88	4.26	0.94	0.220
Splitting strength									
for f_c' =	20.00	26.20	2.34	0.53	0.227	24.44	2.25	0.41	0.182
	30.00	39.30	2.89	0.66	0.227	36.66	2.79	0.51	0.182
	35.00	45.85	3.14	0.71	0.227	42.77	3.02	0.55	0.182
	40.00	52.40	3.36	0.76	0.227	48.88	3.24	0.59	0.182
Modulus of elasticity									
for f_c' =	20.00	26.20	22475	3120	0.139	24.440	21707	2630	0.121
	30.00	39.30	27526	3822	0.139	36.660	26586	3221	0.121
	35.00	45.85	29732	4128	0.139	42.770	28716	3479	0.121
	40.00	52.40	31785	4413	0.139	48.880	30699	3719	0.121

* a tall member is one that is greater than 0.45 m in height.

Table 5.3 - Properties of reinforcement and dimensions

	Average(cast-in-place)			Excellent (pre-cast)			Distribution	Shift
Property	μ	σ	COV	μ	σ	COV		
Reinforcement								
Static yield strength (MPa)								
f_y = 400 MPa	442.0	26.5	0.06	442.0	26.5	0.06	shifted lognorm.	365.0
Modulus of elasticity (MPa)	201000	6599	0.03	201000	6599	0.03	normal	
Effect of discrete bar sizes, A_f/A_c	1.01	0.04	0.04	1.01	0.04	0.04	shifted lognorm.	0.91
Deviation of beam dimensions from nominal values								
Flange width (mm)	--	--		4.1	6.4		normal	
Stem width (mm)	2.3	4.8		0.0	4.8		normal	
Flange depth (mm)	0.8	11.9		0.0	4.8		normal	
Overall depth (mm)	-3.0	6.4		3.0	4.1		normal	
Depth of top steel (mm)	-6.4	17.5		3.0	8.6		normal	
Depth of bottom steel (mm)	-4.8	12.7		3.0	8.6		normal	
Stirrup spacing (mm)	0.0	13.5		0.0	6.9		normal	
Beam spacing and span (mm)	0.0	17.5		0.0	8.6		normal	
Deviation of slab dimensions from nominal values								
effective depth (mm)								
Top reinforcement	-19.1	15.9		0.0	2.4		normal	
Bottom reinforcement	-7.9	15.9		0.0	2.4		normal	
Deviation of column dimensions from nominal values								
Overall width and thickness (mm)	1.5	6.4		--	--	--	normal	
Concrete cover for h x h column								
exterior bars	6.4+ 0.004h	4.2		--	--	--	normal	
interior bars	1.0	5.1+ 0.033h		--	--	--	normal	

From: Nessim,et.al.(1993), Mirza and MacGregor (1979a and b, 1982)

Table 5.4 - Model error

For resistance of	δ	COV
Flexure and Axial load	1.00	0.035
Shear	1.09	0.11

From Mirza and MacGregor (1982)

Table 5.5 - Nominal beam properties

Beam	b (mm)		h (mm)		d (mm)	ρ / ρ_b	M_{nr} A23.3-M84 (Nmm)	M_{nr} A23.3-M94 (Nmm)
	b_t	b_c	h_w	h_f			$\times 10^6$	$\times 10^6$
R-0.14	200	200	330	-	267	0.14	23.3	25.1
R-0.31	200	200	330	-	267	0.31	48.4	51.5
R-0.57	200	200	330	-	267	0.57	80.1	83.5
R-0.71	200	200	330	-	267	0.71	93.8	96.6
T-0.14	300	1200	600	150	525	0.14	541.6	582.6
T-0.71	1200	300	600	150	525	0.57	545.2	561.3

Table 5.6 - Simulation results for the unfactored moment resistance of beams, M_o

Beam designation	Fit to normal distribution Adjusted R^2	μ (Nmm) × 10^6	σ (Nmm) × 10^6	COV %
R-0.14	0.9923	32.1	2.04	6.36
R-0.31	0.9939	68.4	4.65	6.80
R-0.57	0.9982	116.	9.01	7.7
R-0.71	0.9966	144.	17.6	12.2
T-0.14	0.9958	753.	35.6	4.7
T-0.71	0.9966	844.	93.3	11.1

Table 5.7 - Nominal column properties

Property	Column 1	Column 2
f_y (MPa)	400	400
f_c' (MPa)	35	35
E_s (MPa)	200×10^3	200×10^3
b (mm)	450	450
h (mm)	450	450
cover (mm)	40	40
d (mm)	387	378
ρ	0.01	0.04
$A_s = A_s'$	2#25	3#45

Table 5.8 - Nominal factored resistance of columns

	Column 1		Column 2	
	A23.3-M84	A23.3-94	A23.3-M84	A23.3-94
P_{nr} for compression failure, N ($\times 10^6$)	3.41	3.24	4.94	4.76
M_{nr} for tension failure, Nmm ($\times 10^6$)	244	242	611	578

Table 5.9 (a) - Simulated M_o values for Column 1

Axial load level		μ of M_o	σ of M_o	Fit to normal tail
P_o/P_x *	P_o (N) $\times10^6$	(Nmm) $\times10^6$	(Nmm) $\times10^6$	R^2
0.0	0.00	168	8.70	0.9977
0.1	0.847	304	13.0	0.9950
0.2	1.69	420	33.5	0.9853
0.3	2.54	467	60.3	0.9953
0.35	2.97	468	78.0	0.9973
0.4	3.39	458	99.0	0.9872
0.5	4.24	408	126	0.9954
0.6	5.08	297	138	0.8911

* P_x is taken as a deterministic number equal to $0.9\, f_c'\,(h\times b - A_s - A'_s) + f_y(A_s + A'_s)$ calculated using mean values.

Note: At M_o=0.0 the distribution of P_o is normal with mean = 7.53 $\times10^6$ N and standard deviation = 1.12 $\times10^6$ N.

Table 5.9(b) - Simulated M_o values for Column 2

Axial load level		μ of M_o	σ of M_o	Fit to normal tail
P_o/P_x *	P_o (N) $\times 10^6$	(Nmm) $\times 10^6$	(Nmm) $\times 10^6$	R^2
0.0	0.00	577	27.9	0.9982
0.1	1.10	738	38.0	0.9962
0.2	2.19	865	80.1	0.9971
0.3	3.29	789	93.5	0.9965
0.4	4.38	694	109	0.9959
0.5	5.48	580	130	0.9967
0.6	6.57	450	141	0.9929

* P_x is taken as a deterministic number equal to $0.9\, f_c'\,(h\, b - A_s - A'_s) + f_y(A_s+A'_s)$ calculated using the mean values.

Note: At M_o=0.0 the distribution of P_o is Normal with mean = 9.92×10^6 N and standard deviation = 1.01×10^6 N.

Table 5.10 - Compression failure resistance values for P_o (N)

Value	Column 1 at e/h =0.1	Column 2 at e/h =0.1
2.3 percentile	4.1×10^6	5.9×10^6
15.9 percentile	4.8×10^6	6.7×10^6
50.0 percentile	5.5×10^6	7.5×10^6
μ	5.5×10^6	7.5×10^6
σ	0.7×10^6	0.8×10^6

Table 5.11 - Tension failure resistance values for M_o (Nmm)

Value	Column 1 at e/h =0.7	Column 2 at e/h =1.2
2.3 percentile	2.8×10^8	6.7×10^8
15.9 percentile	3.1×10^8	7.3×10^8
50.0 percentile	3.4×10^8	7.9×10^8
μ	3.4×10^8	7.9×10^8
σ	0.3×10^8	0.6×10^8

Table 5.12 - Tension failure properties

	M_{pure} (Nmm)		θ	$M_{pure-nr}$	θ_n
	μ	σ		(Nmm)	
Column 1	1.68×10^8	33.5×10^6	3.0	1.24×10^8	2.97
Column 2	5.77×10^8	57.0×10^6	3.1	4.59×10^8	3.38

Table 5.13 - Nominal shear beam properties

Property	Beam								
	s1v0	s1v1	s1v2	s2v0	s2v1	s2v2	s3v0	s3v1	s3v2
b_w (mm)	305	305	305	305	305	305	305	305	305
d (mm)	457	457	457	457	457	457	457	457	457
a (mm)	1140	1140	1140	1140	1140	1140	1140	1140	1140
f'_c (MPa)	35	35	35	35	35	35	35	35	35
f_y (MPa)	400	400	400	400	400	400	400	400	400
A_s (mm^2)	600	600	600	1500	1500	1500	5000	5000	5000
A_v (mm^2)	0	200	200	0	200	200	0	200	200
s (mm)	-	320	110	-	320	110	-	320	110
V_{cn} (N) x 10^3	88.29	98.95	98.95	88.29	98.95	98.95	88.29	98.95	98.95
V_{sn} (N) x 10^3	0.00	97.11	282.50	0.00	97.11	282.50	0.00	97.11	282.50
V_{nr} (N) x 10^3	88.29	196.06	381.45	88.29	196.06	381.45	88.29	196.06	381.45

Table 5.14 - Simulated shear resistance values

Beam	tail fit to normal			
	R^2	μ (x 10^3) N	σ (x 10^3) N	COV
s1v0	0.992	109	18.0	16.5
s1v1	0.998	251	31.8	12.6
s1v2	0.997	511	59.9	11.7
s2v0	0.995	149	25.5	17.1
s2v1	0.999	291	39.1	13.4
s2v2	0.997	555	68.2	12.3
s3v0	0.994	224	38.2	17.1
s3v1	0.996	363	48.7	13.4
s3v2	0.997	632	78.4	12.4

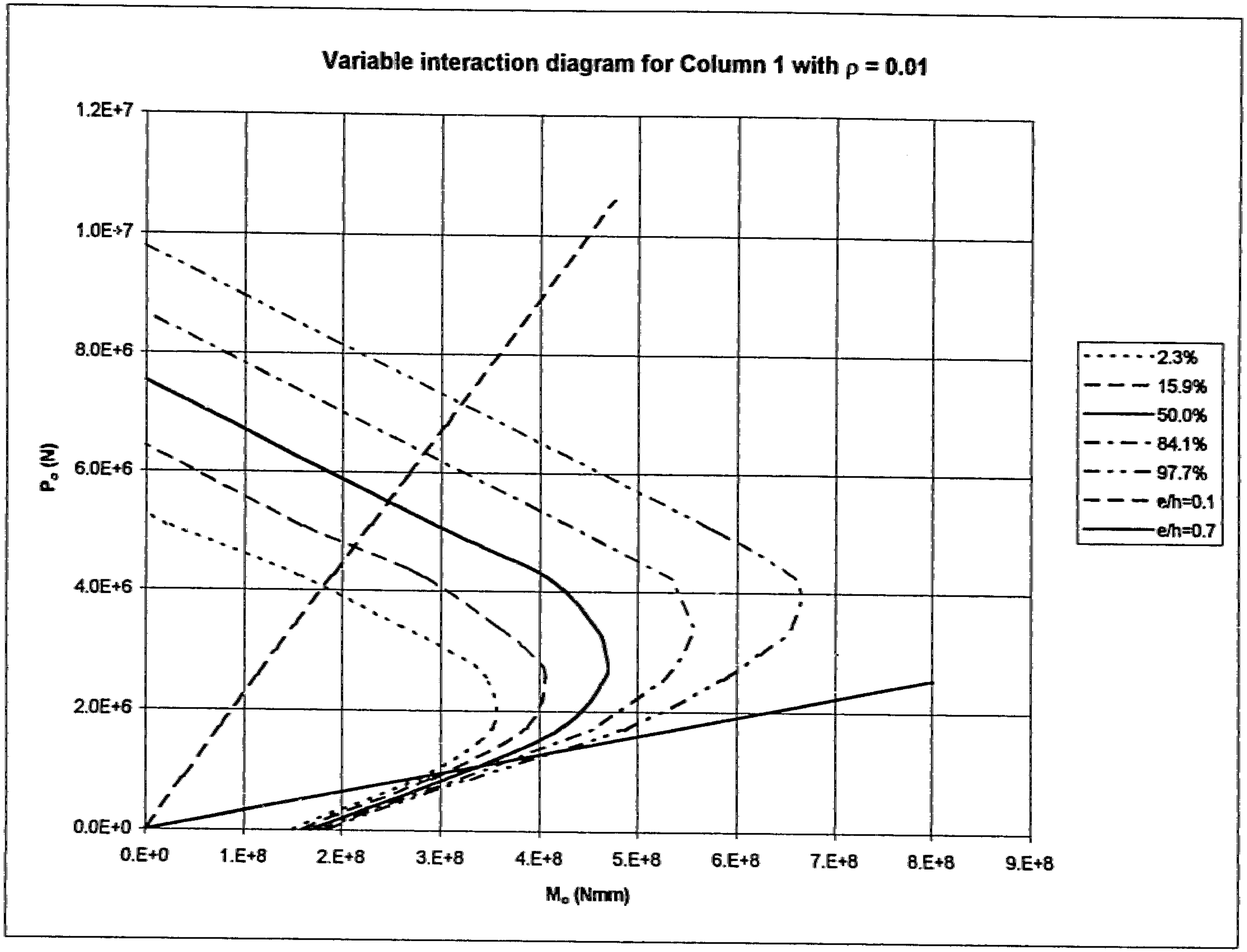


Figure 5.1 - 'True' unfactored simulated resistance for Column 1

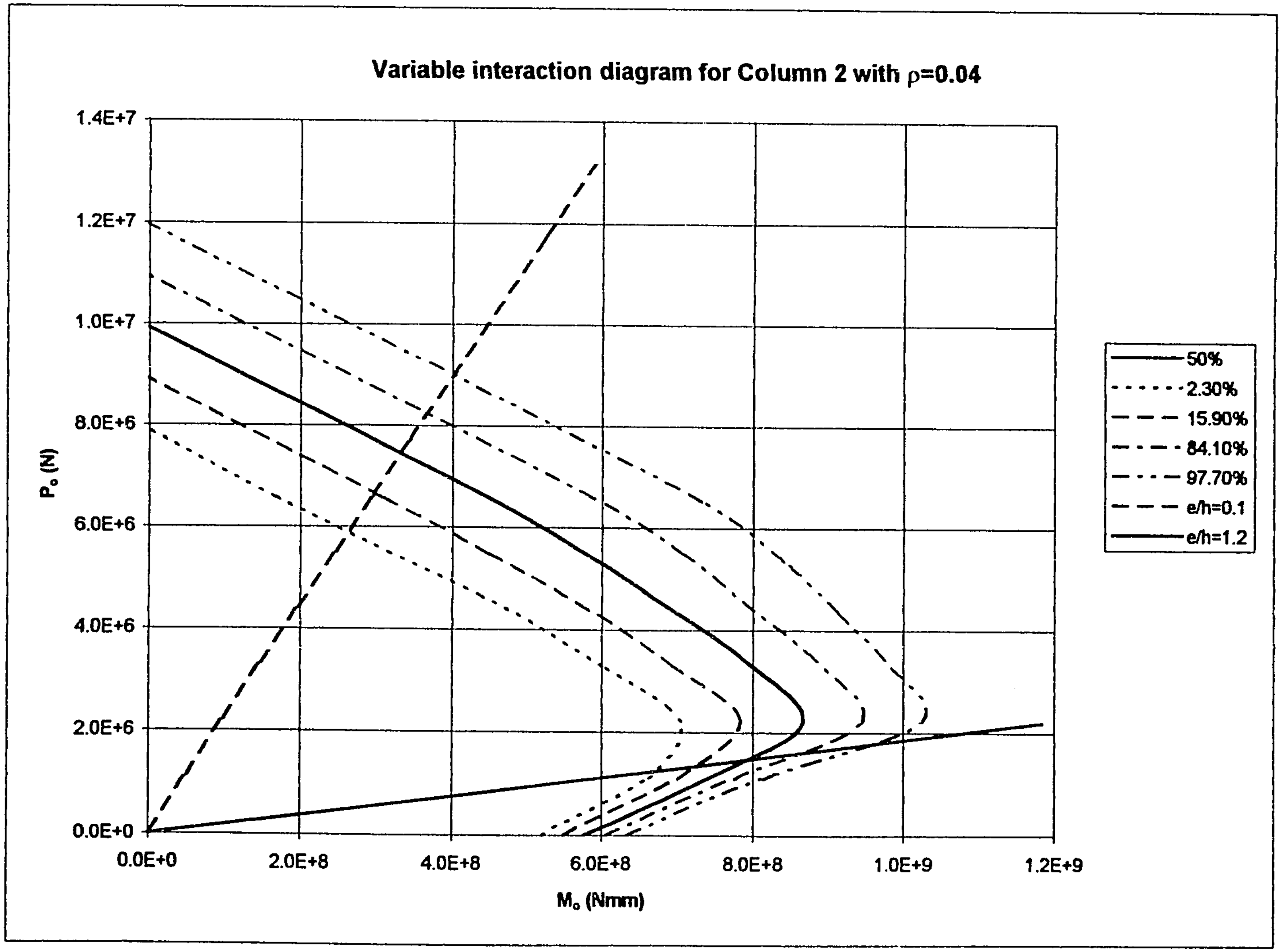


Figure 5.2 - 'True' unfactored simulated resistance for Column 2

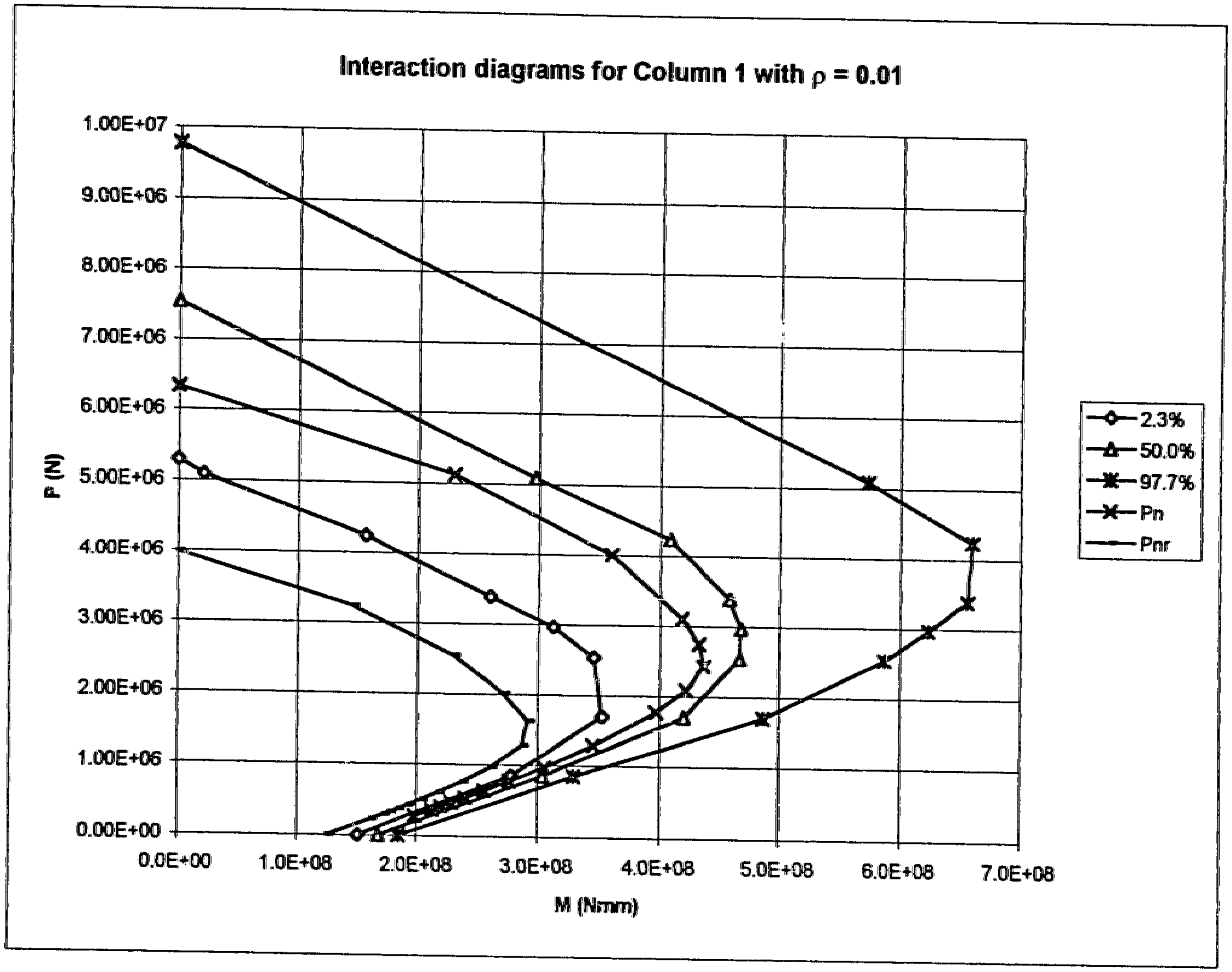


Figure 5.3 - Comparison of simulated 'true' resistance values (P_o) with factored (P_{nr}) and unfactored (P_n) nominal resistance values

CHAPTER 6

TARGET RELIABILITY INDICES

6.1 Introduction

Probability based load and resistance factors are derived for a set of target reliability indices selected to provide a predetermined probability of failure. Before load and resistance factors can be derived it is necessary to establish target reliability indices, β_T.

In Section 6.3 the reliability indices that are implicit in present design practice by CSA A23.3-M84 are established. The evaluation is based on the load and resistance probability distributions and the first order second moment (FOSM) reliability theory detailed in Chapters 2 to 5.

The target beta values are also be based on recommendations given in other studies and the CSA standard S408 - 1981 "Guidelines for the Development of Limit State Design". Section 6.2 reviews these studies and standard. The selection of β_T values for this study is outlined in section 6.4.

For each load and resistance variable there are mainly 2 types of values that are used herein. The 'true' variable value is the best estimate of the load or resistance and is a random variable represented by a probability distribution and relevant parameters. The 'nominal' value is the deterministic value used in design as the load or resistance. The nominal load is called the specified load in design codes and therefore this terminology is also used when considering load values from codes. The nominal load and resistance values are multiplied by load or resistance factors to obtain factored nominal values. In this study two limit states are considered; one in terms of the 'true' variable values and the other in terms of the factored nominal values. These limit state equations are developed in Section 6.3.

6.2 Literature Review

Traditionally, consequences of failure are taken into account in selecting the β_T and the related notional probability of failure. Many types of consequences are considered in reliability studies. Some of these are (MacGregor 1976, Allen 1992): economic cost of failure, potential loss of life, cost to society in lost time and indirect costs, type of failure (warning of failure, post failure strength, alternate load paths), importance of element in structural system, inspection level, and the level of risk accepted by society.

Most of these are subjective considerations. They have been quantified and studied by researchers and β_T values are recommended accordingly. Some of these studies and recommended values are discussed below.

Different definitions or reliability formulations will yield slightly different values of β. Reliability formulations such as the methods used in this study give more accurate β values than the β values obtained by using methods such as the lognormal formulation. The lognormal formulation has been used by many researchers due to its simplicity in calculation. As the method of formulation is significant it is noted in the summary of the references given below.

6.2.1 Allen (1992)

A target reliability index for use in bridge evaluation has been determined on the basis of life safety considerations and is calibrated to experience in the design and evaluation of bridges. For a typical case of an element without alternate paths of support, the β_T is given as 3.0 for gradual failure and 3.5 for sudden failure. These factors were determined using the lognormal reliability model.

6.2.2 Mirza and MacGregor (1982)

Mirza and MacGregor (1982) chose β_T values for the calibration of ϕ factors to be used with the NBCC load factors. The ϕ values were chosen to produce β values close to those computed for practical cases of reinforced concrete beams designed according to CSA A23.3-M78. The β_T values were set to 3.0 and 3.5 respectively for structures exhibiting gradual failure and sudden failure. These values fell near the lower end of the range of β values obtained from evaluation of CSA A23.3, and for this reason ϕ values were chosen conservatively.

6.2.3 Kennedy and Gad Aly (1980)

Performance factors were determined for steel building columns and beams made from rolled W, welded W, and class H hollow structural sections using the lognormal formulation. For all types of failure and members a β_T value of 3.0 was used.

6.2.4 Ellingwood et al. (1980)

In their study the β_T values selected for deriving load factors were representative of those associated with existing designs in the USA. The evaluation using first order second moment methods yielded β values from 1.5 for some metal tension members to over 7.0 for certain masonry walls. It was observed that many flexural and compression members tended to fall within the range $\beta = 2.5$ to 3.0 for the D + L, D + S and D + L + W load combinations. These are among the most common combinations governing designs in large parts of the USA and present designs in these cases were considered satisfactory. Therefore it was considered appropriate for β_T's to be within this range. The following β_T values were used:

$\beta_T = 3.0$ for D + L and D + S

$\beta_T = 2.5$ for D + L + W

$\beta_T = 1.75$ for D + L + E

Ellingwood (1994) mentions that the apparently lower reliability for W and E is a result of inconsistencies propagating into new specifications due to calibration to older practice.

6.2.5 CSA Standard S408 -1981

CSA Standard S408 - 1981 gives the guidelines for the development of limit states design. It is mentioned that life safety, economics, and socio-economic considerations should be considered in determining acceptable levels of reliability. Calculated reliability indices should be compared with failure rates in service to obtain β_T values. It is estimated that the present failure rate in Canada corresponds to a probability of failure of around 10^{-4} in 50 years for engineered structures. The probability of failure of 10^{-4} for a normal probability curve indicates a reliability index of 3.71. The following target beta values are quoted as β_T indices currently in use for steel and concrete buildings:

$\beta_T = 3.5$ for gradual failure

$\beta_T = 4.0$ for sudden failure

6.3 Evaluation of CSA A23.3-M84

The objective of this evaluation is to determine the β values implied in CSA A23.3-M84. This code has been used successfully for more than 10 years. While some areas of the code may be overly conservative, there are no known areas where the code is excessively unconservative. This evaluation helps identify gross errors in the proposed methodology and also identifies opportunities where the specifications in A23.3 can be improved.

The A23.3-M84 code uses the NBCC probability factor format for load combinations and the material partial safety factor format for the resistance. The evaluation is done by performing the reliability analysis using the FORM program (Golliwitzer et al. 1990) explained in Section 2.4.

To use FORM it is necessary to establish a limit state equation that represents the interaction of the loads and resistances at the limit state. A general equation for use with all types of members and ultimate limit states is developed.

Let R_o = unfactored 'true' resistance (variable)

R_{nr} = factored nominal resistance (deterministic)

R_n = nominal resistance (deterministic)

Q_D = dead load effect

Q_{vi} = i^{th} variable load effect

α_D = dead load factor

α_{vi} = load factor for the i^{th} variable load

subscript n = nominal value

If, as in most similar studies, the structural action resistance factor format is used and only one factor is applied to the resistance the factored resistance R_{nr} could be written as ϕR_n. However, the material partial safety factor format is used in this study, therefore a resistance factor for each material has to be applied. The overall resistance is not multiplied by all the factors, i.e., $R_{nr} \neq \phi R_n$. This is illustrated by an example. For a beam the nominal moment resistance is

$$R_n = M_n = A_s f_y \left(d - \frac{A_s f_y}{2 \alpha_1 f_c' b}\right)$$

and the factored nominal resistance is:

$$R_{nr} = M_{nr} = \phi_s A_s f_y \left(d - \frac{\phi_s A_s f_y}{2 \alpha_1 \phi_c f_c' b}\right) \neq \phi R_n$$

This illustrates that ϕ_s and ϕ_c cannot be extracted as an overall multiplicative factor. Therefore the limit state equation was developed to accommodate the material partial safety factor format.

For a specific member at an ultimate limit state, the 'true' unfactored resistance is equal to the unfactored load effects applied

[6.1] $$R_o - D - \Sigma Q_{vi} = 0$$

In design, the design engineer sets the factored nominal resistance, R_{nr} equal to the factored nominal load effects applied

[6.2] $$R_{nr} - \alpha_D D_n - \sum_{\text{all } i} \alpha_{vi} Q_{vni} = 0$$

where,

R_o, D, Q_{vi} are 'true' random variables

R_{nr} is the factored nominal resistance which is a deterministic value

D_n, Q_{vni} are deterministic nominal/specified load values

α_D, α_{vi} are deterministic load factors

By multiplying and dividing each term in [6.1] by nominal values

$$\frac{R_o}{R_{nr}} R_{nr} - \frac{D}{D_n} D_n - \sum_{\text{all } i} \frac{Q_{vi}}{Q_{vni}} Q_{vni} = 0$$

but $\frac{D}{D_n} = r_D$ is the normalized dead load variable and $\frac{Q_{vi}}{Q_{vni}} = r_{vi}$ is the normalized i^{th} variable load. Also R_{nr} can be substituted using [6.2]

$$\frac{R_o}{R_{nr}} [\alpha_D D_n + \sum_{\text{all } i} \alpha_{vi} Q_{vni}] - r_D D_n - \sum_{\text{all } i} r_{vi} Q_{vnii} = 0$$

dividing by D_n

[6.3] $$\frac{R_o}{R_{nr}} [\alpha_D + \sum_{\text{all } i} \alpha_{vi} \frac{Q_{vni}}{D_n}] - [r_D + \sum_{\text{all } i} r_{vi} \frac{Q_{vni}}{D_n}] = 0$$

This equation is the basic ultimate limit state equation. Therefore for this analysis the variables R_o, r_D and r_{vi} are necessary with the appropriate loading ratios $\frac{Q_{vni}}{D_n}$, the nominal value R_{nr} (which has been calculated using ϕ_s and ϕ_c) and the load factors α_D and α_{vi}. The model error term is included in R_o.

For load combinations that involve wind or earthquake loading the loading rate would be higher. The member strength increases for a higher loading rate. This effect of the rate of loading has been included in the calibration by multiplying the resistance variable R_o by 1.05 (Ellingwood et al. 1980).

Typical loading ratios are obtained from the literature (Ellingwood et al. 1980) and also by considering common values in design. The values used are given in Table 6.1.

6.3.1 Beams

The most commonly encountered load combinations for beams are D, D+L, D+S and D+L+W. These combinations are used in the calibrations.

The results for D+L are given in Figs. 6.1 and 6.2 for rectangular beams. The beam with a steel ratio of 0.71 ρ_b (approximately the maximum allowed by A23.3-M84) has a significantly lower reliability for low L/D ratios, as seen in Fig. 6.2, indicating that the maximum steel ratio allowed in beams might need to be revised to a lower value than 0.71 ρ_b. Even though such a beam is nominally under-reinforced, due to the variability in beam parameters some members are over-reinforced, giving a higher variability in member strength. The three beams that have 0.14, 0.31 and 0.53 ρ_b have similar results. A representative value for β for the D+L load case is around 3.75 for the beams with lower ratios of steel and around 3.5 for the beam with $\rho = 0.71\ \rho_b$.

The results for D+S are given in Fig. 6.3. The β values depend heavily on the snow load parameters and especially on the variability of the snow load. The typical values of β taken at middle of the typical range (that is around S/D equal to 1.0) range from 2.6 for Kelowna, which has a highly variable snow load, to 3.25 for Toronto which has a low variability in its snow load.

The results for D+L+W are given in Fig. 6.4. The β for wind is in the range of 2.5 to 3.5 for the practical range of W/D values. When the wind variability is high, as it is in St.John's, the values of β decrease with higher values of W/D.

6.3.2 Short Columns

For short columns compression and tension failures are considered separately. For compression failure the load combinations D, D+L and D+L+W are used. For tension failure the load combination D+W is used with the limit state equation written in terms of moment.

6.3.2.1 Compression Failure

The beta values for the D and D+L combinations are given in Fig. 6.5. The β value for the dead load only case is above 3.5. For D+L the β value is between 3.5 and 4.0 for the typical L/D ratios for compression failure. The case of dead load only seems to be less reliable than others. This suggests that a separate load case with only dead load and a higher dead load factor is needed in the design code.

The W/D load ratio, based on axial effects, for compression failure ranges from 0 to 0.5 . Fig. 6.6 shows that the β values are uniform just above 4.0 for most of the typical load ratios.

6.3.2.2 Tension Failure

The results for the load case of D+W are given in Fig. 6.7. The β values are highly dependent on the wind load parameters. Therefore the calculated reliability index is quite different for different cities.

6.4 Selection of Target Beta Values

Code calibration provides a degree of comparability between structures designed by the new and old criteria. It also permits reduction of the large range in member reliability in the old criteria that resulted from their inconsistent treatment of uncertainties. Unfortunately, calibration also may allow inconsistencies in current practice to propagate into the new specifications as suggested in Section 6.2.4 (Ellingwood 1994).

The evaluation of β values implicit in A23.3-M84 yielded a large range of values. Although the average β values are around 3.5, the whole range is between 2.0 and 5.0 This high range of β is partly due to the probability factor load combination format that is used in CSA A23.3-M84 (as shown in Chapter 7, Figs. 7.29 to 7.36). It is seen that the β is lowest at high load ratios of variable loads. The β values are also lower when only the dead load is present.

The literature review suggests that a desirable range of β_T values is from 3.0 to 3.5. The values used are given below.

6.4.1 Summary of Target Reliability Indices Used

Considering all the β values obtained in the calibration and the values used by other researchers, the following β_T values are used in developing the load and resistance factors.

β_T = 3.00 to 3.25 for gradual failure

β_T = 3.25 to 3.50 for sudden failure

These β_T values apply to loads that are defined at a probability of exceedance of 0.033 and for normal structures with a design life of 30 years.

Table 6.1 - Typical loading ratios

Member	Type of failure	Load combination	Loading ratios investigated[‡]	
			L/D or S/D	W/D or E/D
Reinforced concrete beams	Flexural	D+L	0.0 to 1.5	
		D+S	0.0 to 1.5	
		D+L+W	0.0 to 1.5	0 to 1.0
Short column	Compression	D+L	0.0 to 0.75	
		D+L+W	0.0 to 0.75	0 to 0.5
	Tension	D+L+W	0.0 to 0.75	0.25 to 5.0
		D+L+E	0.0 to 0.75	0.5 to 5.0
		D+W		2.0 to 5.0

[†] used specified values with reduction factors where appropriate.

[‡] Larger ranges of loading ratios were initially used but for the optimization of load factors these ranges were used.

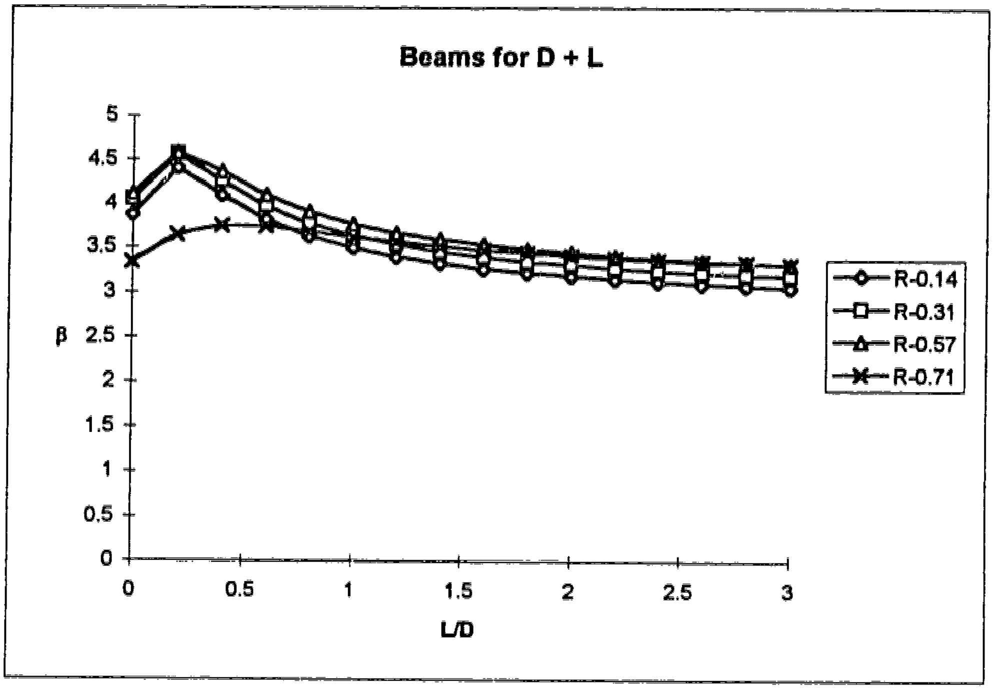


Figure 6.1 - Reliability index of beams for D+L according to A23.3-M84

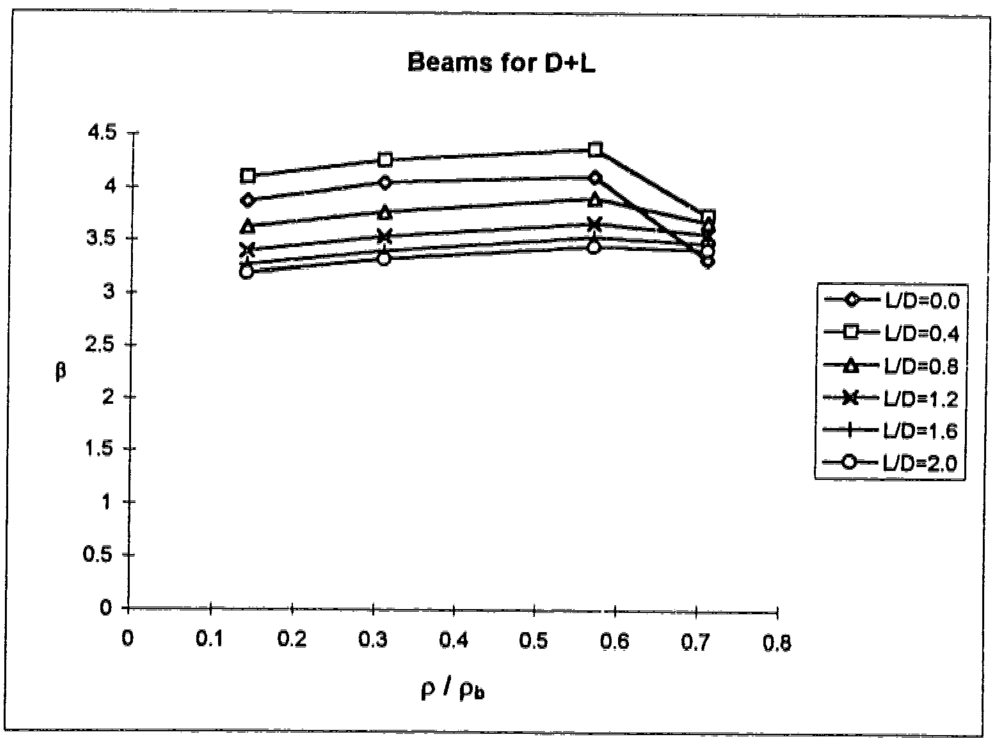


Figure 6.2 - Variation of reliability index with ρ for A23.3-M84

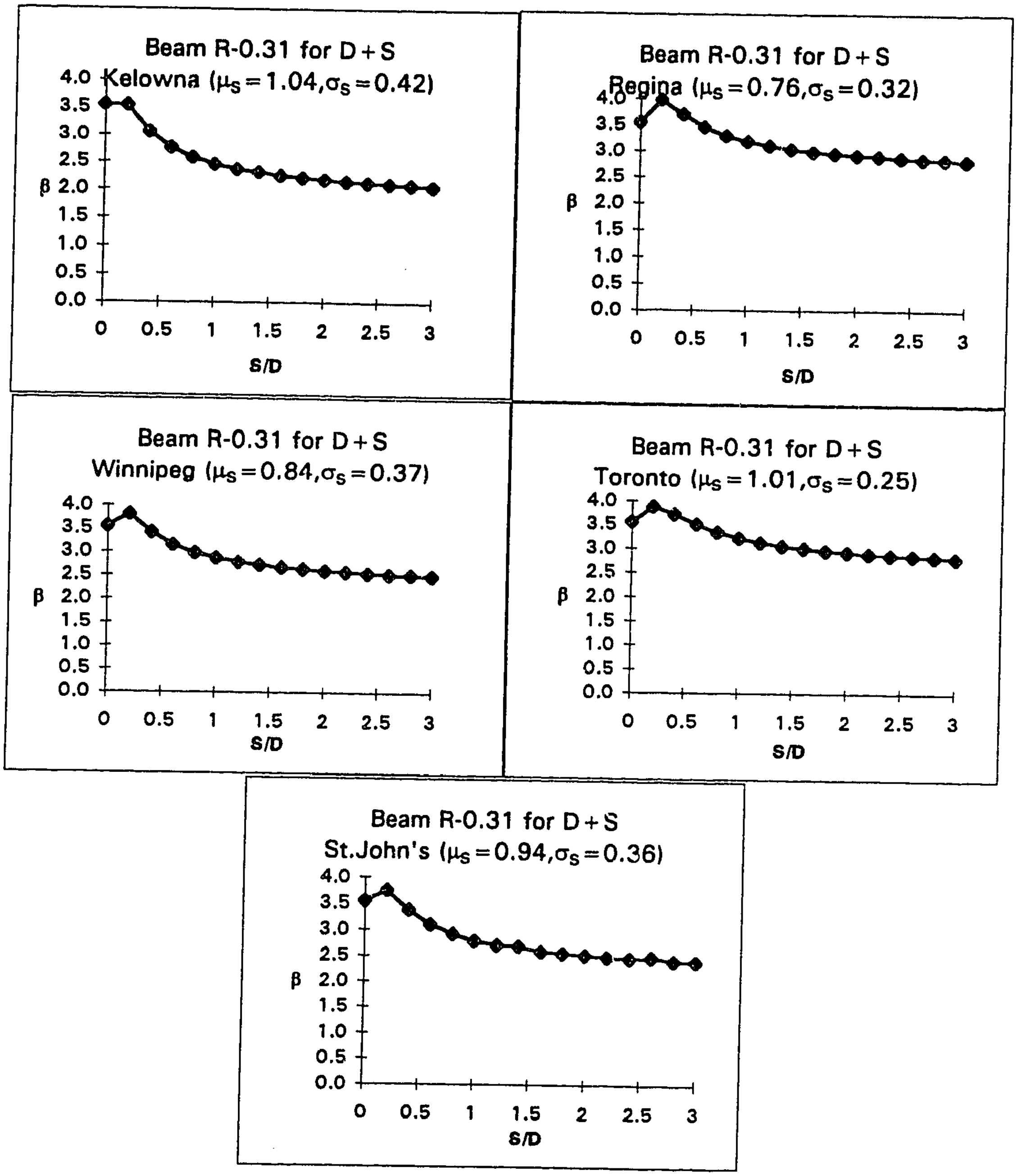


Figure 6.3 - Reliability index of beams for D+S according to A23.3-M84

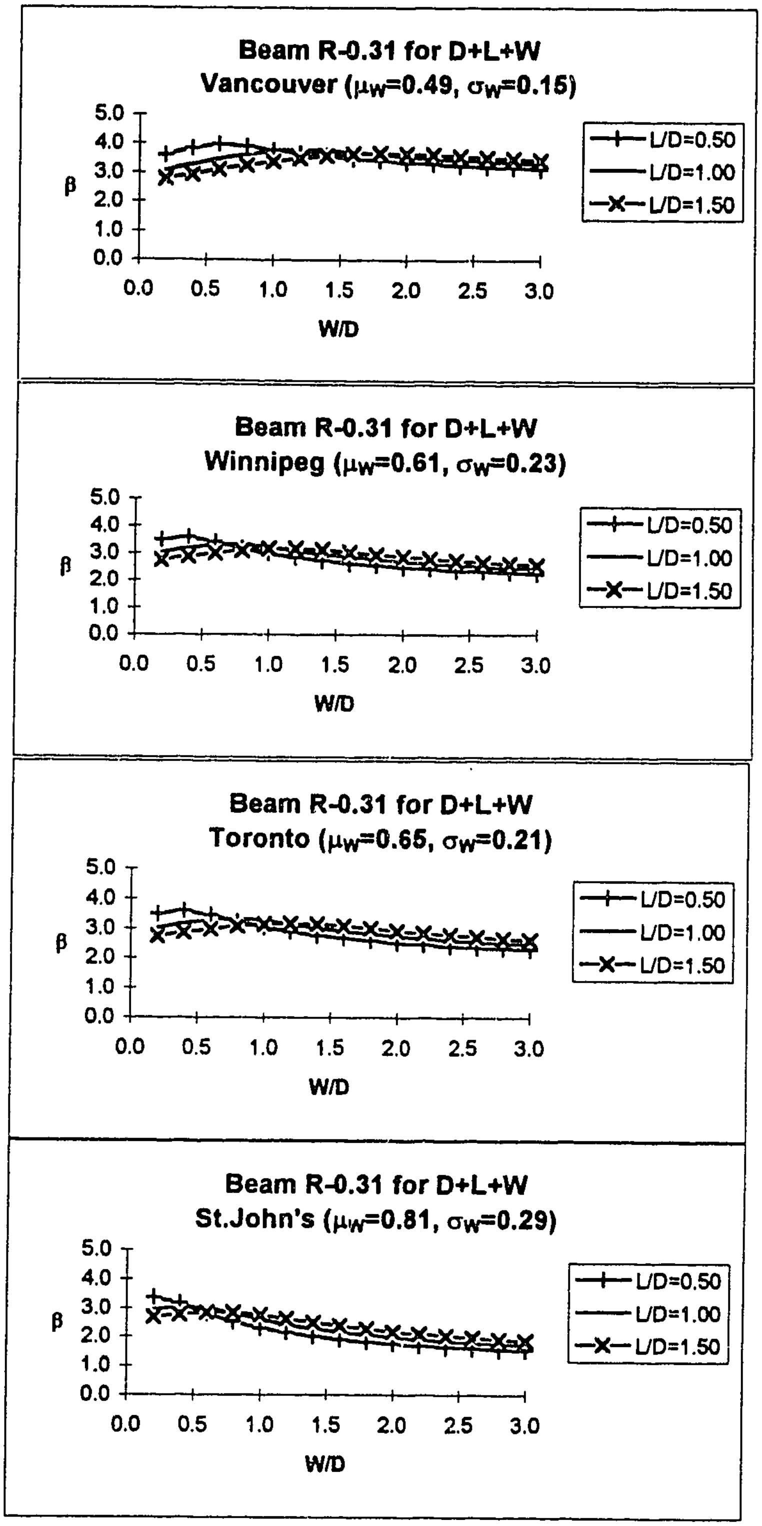


Figure 6.4 - Reliability index of beams for D+L+W according to A23.3-M84

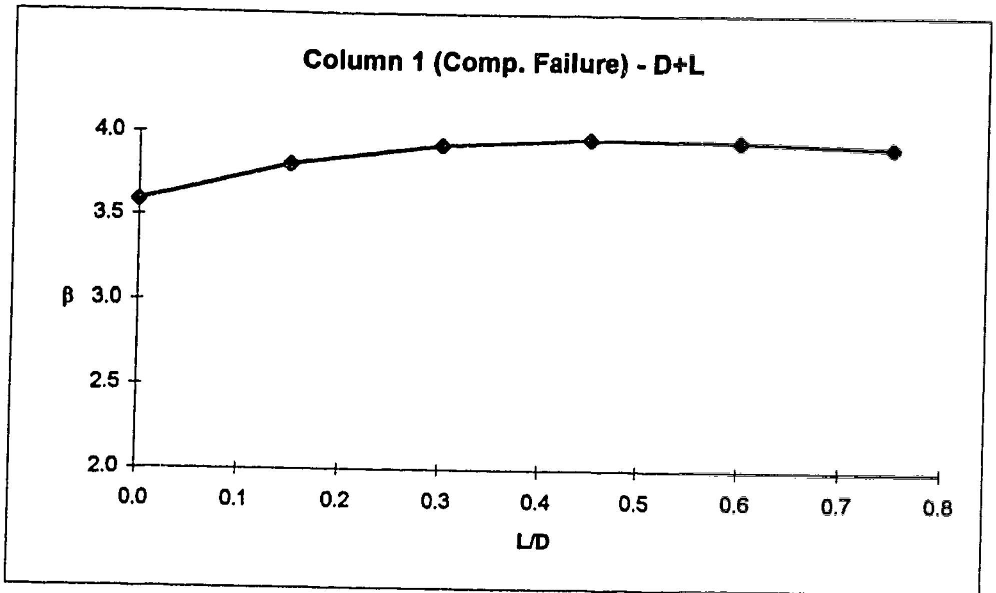


Figure 6.5 - Reliability index of Column 1 for D+L according to A23.3-M84

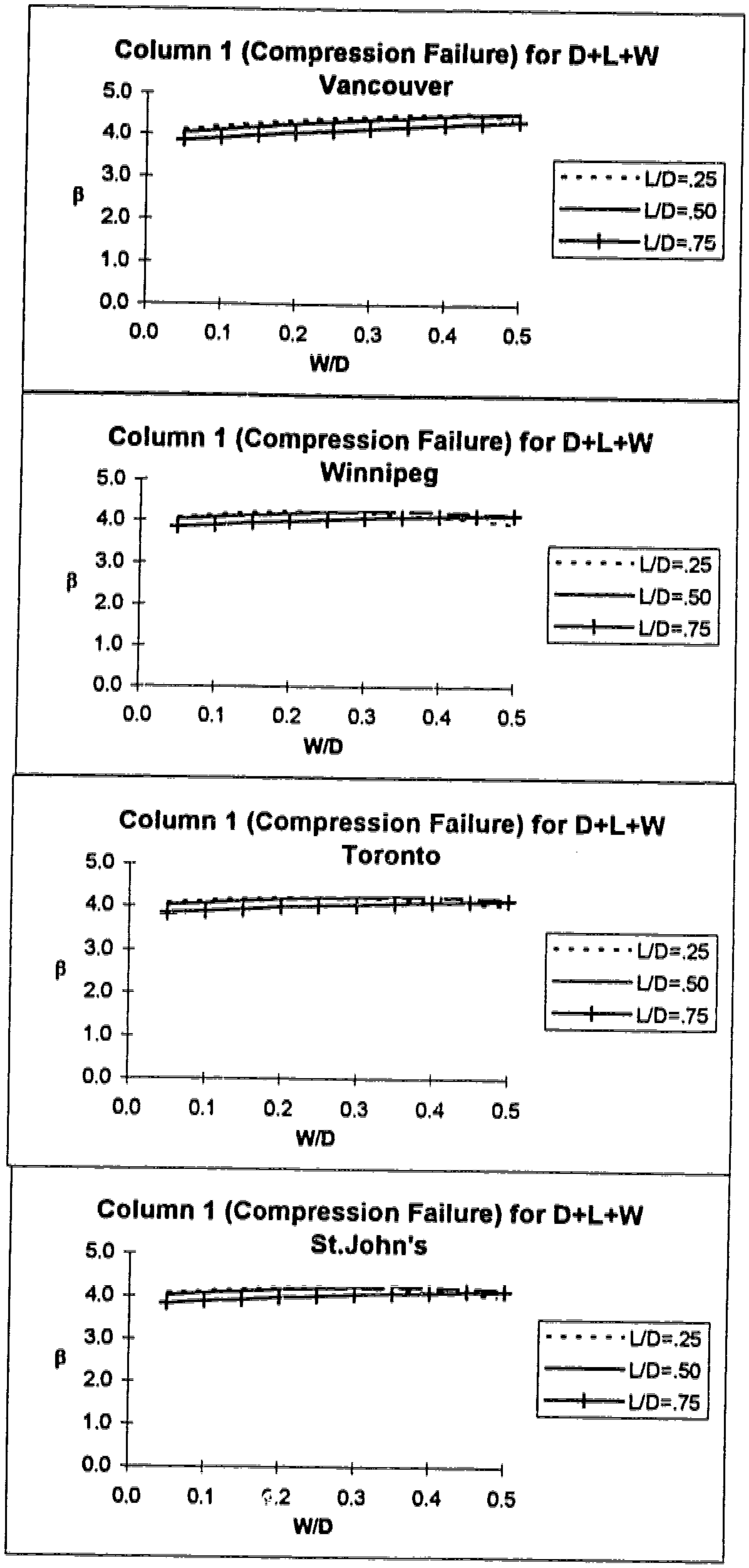


Figure 6.6 - Reliability index of Column 1 for D+L+W according to A23.3-M84

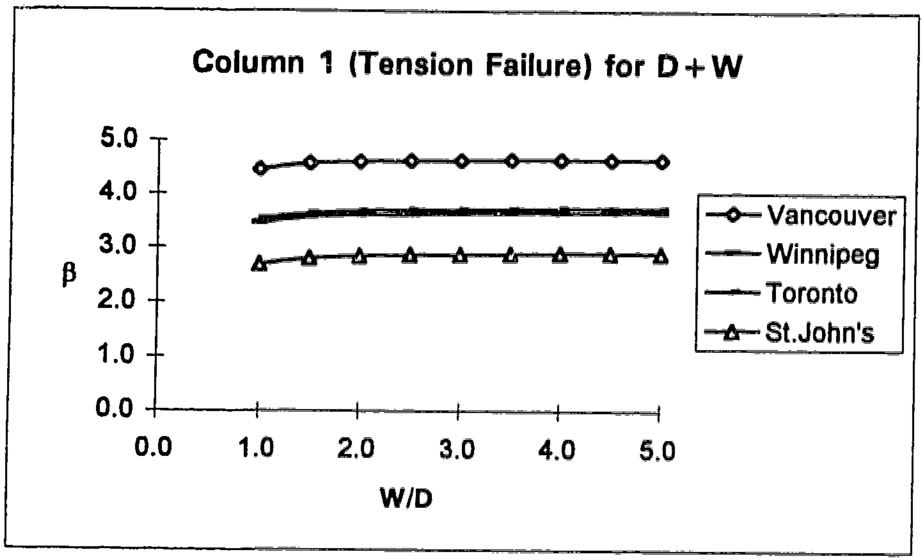


Figure 6.7 - Reliability index of Column 1 for D+W according to A23.3-M84

CHAPTER 7

OPTIMIZATION OF LOAD AND RESISTANCE FACTORS

7.1 Introduction

The previous work in Chapters 2 to 6 is used to optimize load and resistance factors. The load and resistance variables established in Chapters 4 and 5 are used to obtain load and resistance factors for the safety formats chosen in Chapter 3. In doing so, the first order second moment methods and related theory outlined in Chapter 2 are used, and the target reliability index, β_T, as established in Chapter 6 is provided.

The objective here is to optimize the load and resistance factors used in limit state design of reinforced concrete buildings. The load values in NBCC (1990) are used as the specified values (also called nominal values). The NBCC (1995) values are compared at the end of the chapter as they were not available during the time this study was conducted. Ultimate limit states of flexure, shear, and combined flexure and axial load are considered. Special consideration has been given to tension failures of columns and the unique characteristics of such a failure.

The terms 'specified', 'nominal', 'true' variable used in referring to the load and resistance are explained in Section 6.1.

7.2 Background

Probability based load and resistance factors are known to be dependent on many factors. Some of these factors such as the load and resistance variabilities established in this study can be taken into account explicitly in selecting the load and resistance factors. However, other dependencies, even though known, cannot be explicitly taken into account because of practical limitations. One example is the dependency of the load and resistance factors on the load ratios, the ratio of variable loads to dead load. As it is not practical to have load and resistance factors dependent on the load ratios in a design code, the load and resistance factors have to be specified as constant over the whole range of load ratios. To get the optimum constant value that can give reliabilities close to the target it is necessary to employ methods of optimization.

As there are many independent variables involved, one can theoretically conceive of a multitude of possibilities to consider in the optimization process. However the possibilities can be limited, using practical considerations and by observing trends in the behaviour of load and resistance factors. Ellingwood et al. (1980) made the following observations which are helpful in planning the optimization:

(1) Resistance factors are relatively insensitive to the time varying load or loads in the combination.

(2) Load factors do not appear to be especially sensitive to the resistance statistics.

(3) A certain amount of coupling exists between the load and resistance factors but it is relatively weak.

7.3 Method of Analysis and Strategy in Optimization

7.3.1 Formulation of the Limit State Equation

The reliability analysis is based on the limit state equation given in Section 6.3. This equation is developed by considering the two idealizations or versions of the ultimate limit states that impact on the reliability of a member. The first version is the 'true' limit state at which the unfactored 'true' strength of the member equals the unfactored 'true' load effect. These strengths and load effects are uncertain variables and are taken as probability distributions. The other version that impacts on the reliability is the factored limit state considered by the design engineer. The structure is designed such that the factored nominal resistance equals the factored specified load effect. This equation involves deterministic values. By combining the 'true' random limit state equation and the deterministic factored design equation the limit state equation, [6.3], is developed. Later these same principles of formulating the limit state equation are used as a tool to formulate more complex cases encountered. This formulation is a versatile problem solving tool.

Equation 6.3 is the basic ultimate limit state equation used in the reliability considerations. Here R_o, r_D and r_{vi} are random variables and other parameters are deterministic. The algorithm described in Section 2.4 is used with this limit state equation to obtain the reliability index. The deterministic load and resistance factors are adjusted so that the reliability index falls within the desirable target range.

7.3.2 Strategy in Optimization

Based on the observations reported in Section 7.2, the resistance factors are optimized for the dead and live load combinations. Once these resistance factors are selected, load factors for the time varying load combinations are optimized. Because there is interdependency, these two steps are repeated until the reliability index is as close as possible to the target values.

As seen in Figs. 7.1 and 7.2 the resistance factors (ϕ factors) change the reliability over the whole range of load ratios but the load factors (α factors) affect the reliability where that particular load is dominant (e.g. α of the live load affects the reliability most when the live load ratio, L/D, is large). This observation helped in the optimization by indicating which factor should be changed. For example, when the present code factors are used, the reliability index for members is high in the whole range of load ratios indicating that the resistance factors are too conservative. On the other hand, if the reliability had been too high for large L/D ratios this would have indicated an over-conservative α_L .

In the optimization process, a data set of the safety factors with the corresponding β values are generated for each combination of load and resistance variables. This data set contained all the safety factors that would give β values close to the target range. For example for the D+L combination when ϕ_c and ϕ_s are being optimized, a data set of all combinations of (ϕ_c, ϕ_s, L/D) and β values corresponding to each combination are generated.

This data set is utilized in three ways to find the optimum factors.:

(1) Graphs of load ratio vs. ϕ

Graphs of load ratio vs. ϕ values that gave β values that are within the target reliability range are drawn. This displayed the ϕ values necessary to obtain the target reliabilities as shown in Fig. 7.3. From these the combination of ϕ values that ensured the target reliability are chosen.

(2) Using an optimization function

As observed earlier the influence of the load factors on the reliability changes with load ratio. A set of optimum load factors can be obtained by defining a function that measures the closeness of the reliability index, β, obtained with each set of load factors, to the target reliability index, β_T. The set of load factors that gives the minimum dispersion is considered the optimum set for that range of load ratios. Because some load ratios are more frequently encountered in practice than others, the optimization function must be weighted accordingly. The optimization function is based on the square of the difference between the β values. Unconservative differences are weighted twice as heavily as conservative differences:

[7.1] $$\text{Optimization function} = \sum_{i=1}^{n} (\beta - \beta_T)^2 \chi \; w_i$$

where χ = a factor which is 1 if (β - β_T) is positive and 2 if it is negative

w_i = weighting for load ratio i

i = Load ratios ranging 1 to n

The weightings (given in Table 7.1) are taken to be similar to those used by (Ellingwood et al. 1980) based on judgment rather than empirical data. Because the optimization function including the weightings is subjective the results are considered to be only a good estimate of the optimum factors. The optimum factors obtained using this method are termed the 'theoretical optimum'.

(3) Graphs of load ratio vs. β

Once the 'theoretical optimum' is estimated as detailed in (2) above, it is plotted as a load ratio vs. β chart as shown in Fig. 7.4. The theoretical optimums for the different load combinations varied widely. For practical use in a design office there are practical constraints on the value which could be chosen. These are discussed in Section 7.5. The load factors are adjusted according to these constraints while trying to keep the reliabilities close to the target. Load ratio vs. β graphs are used in these adjustments to observe how different the final reliabilities are from those obtained with the 'theoretical optimum'.

7.4 Optimization of Resistance Factors (ϕ factors)

7.4.1 Trends Observed

First, some of the trends in the reliability with the ϕ factors are observed. As mentioned earlier, the resistance factors affect the reliability of the whole range of load ratios almost uniformly as seen in Fig. 7.1. Figures 7.5 and 7.6 show the effect of ϕ_c and ϕ_s on the reliability of an under-reinforced beam in flexure. It is clear that the reliability in flexure depends strongly on ϕ_s and not very much on ϕ_c. Similarly, as seen in Figs. 7.7 and 7.8 reliability of columns that develop compression failures depends more on ϕ_c than ϕ_s.

Figure 7.9 shows the different shapes of β curves obtained for different types of beams (using the A23.3-94 code stress block). The flexural resistances of beams with low steel ratios have lower COV values and lower M_o/M_{nr} values than beams with high steel ratios. The shape of the β vs. load ratio curve is determined by the ratios of 'true' to factored nominal resistance (and load) and their COV values. When the COV of the resistance is low the reliability increases at L/D close to ratios 0.2 where the load variation is low. At L/D close to 0.0 the reliability decreases because one load reaching its maximum is more likely than two loads reaching maximums. At higher load ratios where the COV of the load is high the beams with lower steel ratios and consequently lower M_o/M_{nr} values have a lower reliability. When the COV of the resistance is high (as in R-0.71 and columns) the β is not affected significantly by the load ratio and a flatter curve is obtained. Out of the sample of beams chosen for this study beams R-0.71 (rectangular section with ρ equal to 0.71 ρ_{bal}) and T-0.14 (T beam with ρ equal to 0.14 ρ_{bal}) are the most critical (have the lowest β values in the different L/D ratios). Therefore, these two are chosen as the two critical beams to investigate.

Similarly in columns, as seen in Fig. 7.10, Column 1 (the column with ρ =0.01) gave the lower reliability and is chosen as the critical column to investigate. Cases other than the load case shown in Fig. 7.10 confirmed this decision.

7.4.2 Beam Flexure

Figures 7.11 and 7.12, for beams R-0.71 and T-0.14 respectively, give the ranges of ϕ_s possible, with different ϕ_c and L/D values, to give β values within the target range of 3.00 to 3.25. Only four values of ϕ_c are considered for beams because the reliability of beams in flexure is not strongly affected by ϕ_c. The D+L combination is considered in Figs. 7.11 and 7.12. The low reliability when L/D is close to zero suggests that a separate load case with D only should be considered. The highest ϕ_s values that ensure β is within or above the target range are chosen from Figs. 7.11 and 7.12 and are summarized in Table 7.2. The value of ϕ_s at L/D equal to 0.0 is ignored as this is governed by the D only load case introduced in Section 7.5. From Table 7.2, ϕ_s equal to 0.85 is chosen because this would provide the necessary reliability when ϕ_c is in between 0.6 and 0.7. This is the value used in CSA A23.3-M84 and 94.

7.4.3 Column Compression Failure

Figs. 7.13 and 7.14, for Columns 1 and 2 respectively, give the ranges of ϕ_c possible with different ϕ_s and L/D values. These factors ensure β values within the target range of 3.25 to 3.5, as compression failure is a brittle failure. Only four values of ϕ_s are considered for columns because the reliability of columns failing in compression is not strongly affected by ϕ_s. The highest ϕ_s that ensures the β is within or above the target range is chosen from the Figs. 7.13 and 7.14 and are summarized in Table 7.3. It is clear from Table 7.3 that Column 1 is the critical column. Again, the ϕ_c value at L/D equal to 0.0 is ignored because it is governed by the D only load case introduced in Section 7.5. From Table 7.3 ϕ_c equal to 0.65 is chosen as this would provide the necessary reliability in all cases considered. Column 1 approaches the maximum fraction of the column load which can be carried by concrete in a practical tied column and hence approaches the case most influenced by ϕ_c.

7.4.4 Beam Shear

As observed in Chapter 5 the bias factors of the shear resistance vary widely suggesting that the shear resistance equation for the simplified method given in A23.3-94 does not adequately consider all relevant variables. The reliability according to A23.3-94 for the nine different beams is shown in Fig. 7.15. In the beam notation s1, s2, and s3 refers to beams with ρ equal to 0.43, 1.08 and 3.59 % while v0, v1, and v2 refers to beams with ρ_v, i.e., A_v / b s % equal to 0.0, 0.2 and

0.6% . The amount of flexural reinforcement is not accounted for in the equation specified in A23.3-94. As shear reinforcement decreases, the effect of longitudinal reinforcement is more significant. As developing an adequate equation for shear is beyond the scope of this study, the types of members addressed are limited.

If the limitation specified in A23.3-94 clause 11.2.8.1 is applied, namely, that V_f shall be less than or equal to $V_c/2$ for beams without stirrups, the reliability for beams without stirrups would be governed by this limitation leading to higher values of β for some beams as shown in Fig. 7.16. However this limitation does not apply to footings and slabs where shear reinforcement is usually not used. The shear mechanisms in these cases are more complicated and are not considered.

Note N11.3.5. of the handbook for A23.3-94 (CPCA 1995) says:

> "There is considerable evidence that [A23.3] Equation 11-6 overestimates the shear strength of large lightly reinforced members that do not contain stirrups. Because of this concern [A23.3] Equation 11-6 is restricted to members that contain stirrups or that have effective depths not exceeding 300 mm. The concrete contribution, V_c, for large members with no stirrups is to be determined from [A23.3] Equation 11-7. The shear strength of large members with no stirrups is strongly influenced by the amount of longitudinal reinforcement the member contains. In the simplified method this influence is neglected and hence, [A23.3] Equation 11-7 can give conservative results for heavily reinforced sections."

Zsutty's regression equation, [5.12], which is used to estimate the 'true' shear strength, does not adequately represent the strength of lightly reinforced beams with stirrups. Furthermore, the depth is only considered in the ratio a/d and not by itself.

Beams that have a longitudinal reinforcement percentage greater than 1.0% are considered here after. The members without stirrups that are not governed by V_f

smaller or equal to $V_c/2$ are also not considered. These limitations, eliminate beams s1v0, s1v1 and s1v2. The shear strength of beams s2v0 and s3v0 is governed by the V_f smaller or equal to $V_c/2$ limitation and is not critical. The beams considered in the analysis are shown in Fig. 7.17 which shows that beam s2v1 is the most critical. This beam is further investigated with different combinations of factors, with the results given in Table 7.4. The code equation for the shear carried by the concrete can be written as: V_c equal to $C \lambda \phi_c \sqrt{f'_c}\, b_w d$ where CSA A23.3-94 gives C equal to 0.20 for use with ϕ_c equal to 0.60. As the ϕ_c is increased, it is necessary to decrease the constant C in the V_c equation to produce similar reliability indices. Table 7.4 shows that ϕ_c equal to 0.65, ϕ_s equal to 0.85 and the constant in the V_c equation equal to 0.17 would give β values within the target range taken as 3.25 to 3.50 for members displaying brittle failures.

7.4.5 Optimum Resistance Factors Chosen

As explained in the preceding sections the following resistance factors are chosen: ϕ_c equal to 0.65 and ϕ_s equal to 0.85 for concrete and non-prestressed reinforcement respectively, with V_c equal to $0.17 \lambda \phi_c \sqrt{f_c'}\, b_w d$ for members with flexural reinforcement ratio greater than or equal to 1.0%. This value of V_c is applicable to members with stirrups or beams without stirrups that are governed by clause 11.2.8.1 of A23.3-94.

7.5 Optimization of Load Factors (α factors)

As explained in Chapter 3 the companion action load factor format is used for load combinations with the specified exceedance probability format used in the cases where earthquake loads are involved.

The load types considered are: the permanent load D and the variable loads L, S, W and E. The variable loads take principal values (denoted by subscript i) or companion values (denoted by subscript ij). This gives the following possible combinations to consider in developing load factors:

(1) D

(2) $D + L_i$

(3) $D + W_{ij} + L_i$

(4) $D + S_{ij} + L_i$

(5) $D + S_i$

(6) $D + L_{ij} + S_i$

(7) $D + W_{ij} + S_i$

(8) $D + S_{ij} + W_i$

(9) $D + L_{ij} + W_i$

(10) $D + W_i$ where the load effects oppose each other

(11) $D + E_i$

(12) $D + S_{ij} + E_i$

(13) $D + L_{ij} + E_i$

Load combinations (1) to (4) and especially (1) and (2) are important for floor systems where gravity loads govern. Snow load combinations are important for roof systems and columns. Wind and earthquake loads are important for lateral load carrying systems such as columns and beams in framed structures, for floor and roof diaphragms and for bracing systems.

In Section 6.3 it is explained that the limit state equation is developed by combining the true limit state equation and the factored nominal design equation. As designers use only specified loads and are not aware of the magnitude of the companion loads, the factored nominal design equation is written based on specified loads. Thus the factors developed are applied to the specified characteristic loads. The true variable limit state however has to model the real load combinations on a structure and thus is written in terms of principal values and companion values, both modelled as random variables.

As explained in Section 7.3, the optimization function method and the graphical method using load ratio vs. β charts are used with the thirteen combinations mentioned above. For each combination, the method of choosing the practical optimum is explained in Sections 7.5.2 to 7.5.5. The resistance factors summarized in 7.4.5 are used to define the factored resistances.

Two important observations about the behaviour of load factors are shown in Fig. 7.18. The dead load factor affects the reliability in the lower ranges of the load ratio L/D. The variable load factor affects the reliability in the higher ranges of the load ratio.

Special considerations are necessary for developing load factors for the tension failure of columns. These are explained in Section 7.5.1.

7.5.1 Special Considerations for Tension Failure of Columns

In column failures both axial load and applied moment contribute to the load effect. Historically the compression failure limit state has been formulated in terms of axial force and the tension failure limit state has been formulated in terms of moment.

In the case of a compression failure, a higher axial load and a higher moment would both lead to failure and therefore an axial load formulation is appropriate. However, in the tension failure region a lower axial load and a higher moment would tend to lead to failure because of the shape of the tension failure portion of the interaction diagram (See, for example, Figs. 5.1 and 5.2). Therefore a formulation of the limit state for moment alone does not account for the contributing effect of the axial load to the strength. To overcome this problem the limit state equation is formulated using both axial load and moment terms. The same principles used in the development of the limit state equation, [6.3], are used.

First, consider the interaction diagrams in, Fig. 7.19 which is in terms of 'true' uncertain variables and in Fig. 7.20 which is in terms of factored nominal values. The axial load, P/A_g, and moment, $M/A_g h$, are used so that the dimensions on both axes are the same. In Fig. 7.19 the interaction diagram is a variable diagram like the ones developed in Chapter 5. The load effect, $(P/A_g, M/A_g h)$, is represented by a PDF and is given by the average plotted as a point and the variation plotted as percentiles. In Fig. 7.20 the interaction diagram is a deterministic factored diagram and the load is a point. The proximity of the load point to the interaction diagram is a gauge of the safety. The tension failure region of each diagram can be idealized as a straight line, the equation of this line can be used to formulate the limit state equation.

Let θ = slope of the idealized tension failure line in 'true' space, as shown in Fig. 7.19

M_{pure} = the 'true' pure moment resistance in true space, which is a normal distribution, as shown in Fig. 7.19

also θ_n = slope of the idealized tension failure line in factored nominal space, as shown in Fig. 7.20

$M_{nr\text{-}pure}$ = the factored nominal pure moment resistance (Fig. 7.20)

Both θ and M_{pure} are 'true' resistance characteristics and are obtained from the variable interaction diagrams established in Chapter 5. The true variable load point is given as $(P/A_g , M/A_g h)$ in Fig. 7.19.

The deterministic factored values θ_n and $M_{nr\text{-}pure}$ are factored nominal resistance characteristics. These values include the new resistance factors developed in Section 7.4. The load point in factored nominal space is $(P_{fn}/A_g , M_{fn}/A_g h)$ in Fig. 7.20.

For the limit state in true space (i.e., when the load point is on the tension failure line) ,

$$\frac{P}{A_g} = \theta\left(\frac{M}{A_g\, h}\right) - \theta\left(\frac{M_{pure}}{A_g\, h}\right)$$

The limit state in factored nominal space is

$$\frac{P_{fn}}{A_g} = \theta_n\left(\frac{M_{fn}}{A_g\, h}\right) - \theta_n\left(\frac{M_{nr-pure}}{A_g\, h}\right)$$

These two equations are simplified to

$$\frac{P}{\theta} + \left(\frac{M_{pure}}{h}\right) - \left(\frac{M}{h}\right) = 0 \qquad [7.2]$$

$$\frac{P_{fn}}{\theta_n} + \left(\frac{M_{nr-pure}}{h}\right) - \left(\frac{M_{fn}}{h}\right) = 0 \qquad [7.3]$$

These equations are used to formulate the limit state equation. The D and W load case is considered because the moment is maximum due to the wind and the axial load is minimum, which is a critical condition for tension failure. A similar situation is encountered with D and E, but this case is handled with the other earthquake load combinations.

For the combination of D, L and W, the live load would add a moment and axial load. Whether this combination is more critical depends on the relative values of L/D and W/D. On analyzing this case it was found that when W/D is low and L/D is high the reliability is low. This means that in some cases when W/D is low, adding live load will decrease the reliability. This occurs if the increase in the moment is more significant than the increase in axial load, but this condition is rare and the change in β was never greater than 0.2. Therefore, only the D with W combination was considered for tension failure.

For D+W, assume the axial load due to wind is negligible for columns undergoing tension failures and let

$P = P_D$ and $P_{fn} = P_{Dfn}$

$M = M_D + M_W$ and $M_{fn} = M_{Dfn} + M_{Wfn}$

substituting these in [7.2] and [7.3]

$$\left(\frac{P_D}{\theta}+\frac{M_{pure}}{h}\right)-\frac{1}{h}(M_D+M_W)=0$$

$$\left(\frac{P_{Dfn}}{\theta_n}+\frac{M_{nr-pure}}{h}\right)-\frac{1}{h}(M_{Dfn}+M_{Wfn})=0$$

By combining these equations in the same manner as in Section 6.3

[7.4]

$$\frac{1}{\theta}r_D+\frac{M_{pure}}{M_{nr-pure}}\left(\alpha_D\frac{M_{Dn}}{P_{Dn}\,h}+\alpha_W\frac{M_{Wn}}{P_{Dn}\,h}-\frac{\alpha_D}{\theta_n}\right)-\left(r_D\frac{M_{Dn}}{P_{Dn}\,h}+r_W\frac{M_{Wn}}{P_{Dn}\,h}\right)=0$$

This is the limit state equation used in the reliability analysis of tension failure. As mentioned before, M_{pure}, r_D and r_W are the variables and have the values established in Chapters 4 and 5. The load ratios are not in conventional form, so, they have to be developed as follows:

$M_{Dn} = P_{Dn}\, e_{Dn}$

Assume $e_{Dn}/h = 0.1$ and $M_{Wn}/M_{Dn} = 2.0$ to 5.0

Therefore,

$$\frac{M_{Wn}}{P_{Dn}\,e_{Dn}}=2.0\text{ to }5.0,\qquad \text{i.e.,}\quad \frac{M_{Wn}}{P_{Dn}\,h}=0.2\text{ to }0.5$$

$$\frac{M_{Dn}}{P_{Dn}\,h}=0.1$$

Using these ranges of load ratios and the limit state equation, [7.4], the reliability analysis is performed for tension failure. Values of α_D and α_W that yielded β values closest to the range of target β values are chosen. The results of this analysis are given later when the D+W load case is considered.

7.5.2 Load Combinations with L as the Principal Variable Load

7.5.2.1 Combinations (1): D, and (2): D + L_i

The 'theoretical optimums' obtained with the D + L_i combination for the two critical beams R-0.71 and T-0.14 and the critical Column 1 are given in Table 7.5. But, because it is assumed that a dead load factor less than 1.2 would not be accepted by the design community, a dead load factor of 1.2 is selected. With α_D = 1.2 different values of α_L are compared graphically. It is found that the existing factor in NBCC (1990) of 1.5 would give the best results with α_D equal to 1.2 except at very low L/D ratios where the reliability index would be too low. Therefore α_D equal to 1.2 and α_L equal to 1.5 are chosen for the D + L_i combination and a D only load case which would govern in this range of low L/D values is developed.

For the D only load case to govern over 1.2D + 1.5L the α_D for this case would have to be higher than 1.2. Values above 1.2 in steps of 0.05 are tried and 1.3 is found to give reliabilities closest to the target β range.

The reliability indices obtained with the theoretical optimum and the practical optimum chosen above is shown in Figs. 7.21, 7.22 and 7.23 for beam R-0.71, beam T-0.14 and Column 1, respectively. The lack of consistent reliability for beam T-0.14 is due to the fact that the α_D is set equal to 1.2.

The D only case governs up to L/D equal to 0.07. Therefore if L/D is equal to 0 the member would be designed for dead load only. At L/D equal to 0.07 the factored specified loads for D only and D +L_i are the same; i.e., $1.2D_n + 1.5L_n = 1.3D_n$.

When L/D is less than 0.07, the member would experience both D and L_i but is designed for the factored dead load only. Therefore a different kind of limit state equation is developed to obtain the reliability for $0.0 < L/D < 0.07$. In this case the 'true' limit state equation is written for both $D + L_i$ and the factored design equation is written for the D only case with α_D equal to 1.3.

For $0.0 < L/D < 0.07$

the 'true' random limit state equation

$R_o = D + L$

governing design limit state equation

$R_{nr} = 1.3\ D_n$

by combining these two equations, the limit state equation is

$$\frac{R_o}{R_{nr}}(1.3) - \left(r_D + r_L \frac{L_n}{D_n}\right) = 0 \qquad [7.5]$$

The reliability for the case where D only governs is obtained using this equation and is shown in Figs. 7.21, 7.22 and 7.23. Beyond L/D equal to 0.07 the $D + L_i$ design case governs and the true load combination is also the same.

It is seen from Table 7.5 and Figs. 7.21, 7.22 and 7.23 that the beam T-0.14 is the critical member when load ratios are in the mid to high range. The reliability in the lower range of load ratios is important in the choice of α_D which is already chosen. Therefore beam T-0.14 is taken as the critical member in choosing all other load factors.

7.5.2.2 Combination (3): $D + W_{ij} + L_i$

With α_D equal to 1.2 the theoretical optimum set of load factors obtained are α_W equal to 0.1 and α_L equal to 1.4 for St.John's. (This city is chosen because it has 'moderate' load parameters. Further details of this choice are given in Section 7.5.7) In this combination the factored load $\alpha_W W_n$ equal to $0.1W_n$ is insignificant and can be ignored with little effect on β. The relevant wind load safety check is the load case $D + L_{ij} + W_i$ developed later.

7.5.2.3 Combination (4): $D + S_{ij} + L_i$

With α_D equal to 1.2 the theoretical optimum set of load factors obtained had $\alpha_S \leq 0.1$ and $\alpha_L \leq 1.4$ for all the cities considered. In this combination the factored load $\alpha_S S_n$ equal to $0.1S_n$ is insignificant and can be ignored. The relevant snow load safety check would then be the load case $D + L_{ij} + S_i$ developed later.

7.5.3 Load Combinations with S as the Principal Variable Load

7.5.3.1 Combination (5): $D + S_i$

For this combination the theoretical optimum is $\alpha_D \leq 1.1$ and α_S equal to 1.9. However, due to practical considerations α_D is chosen to be 1.2 as explained in Section 7.5.2.1. With α_D equal to 1.2 the optimum α_S is 1.7.

7.5.3.2 Combinations (6): $D + L_{ij} + S_i$, and (7): $D + W_{ij} + S_i$

For the $D + L_{ij} + S_i$ combination with α_D equal to 1.2, the theoretical optimum load factors are α_L equal to 0.2 and α_S equal to 1.9. For the $D + W_{ij} + S_i$ combination the theoretical optimum load factors are α_W equal to 0.5 and α_S equal to 1.3. Figures 7.24 and 7.25 show that the compromise of 1.2 D + 0.5 L (or 0.4W) + 1.7 S is not unconservative and is also close to the target β values. In the interest of having easier combinations to use in design these factors are adopted.

7.5.4 Load Combinations with W as the Principal Variable Load

7.5.4.1 Combination (8): $D + S_{ij} + W_i$

With α_D equal to 1.2 the theoretical optimum set of load factors obtained had α_S smaller or equal to 0.1 and α_W smaller or equal to 1.2 for all the cities considered. In this combination the factored load $\alpha_S S_n$ equal to $0.1S_n$ is insignificant and can be ignored. The relevant wind load safety check would then be the load case $D + L_{ij} + W_i$ developed in the next paragraph.

7.5.4.2 Combination (9): $D + L_{ij} + W_i$

With α_D equal to 1.2 the theoretical optimum load factors are α_L equal to 0.2 and α_W equal to 1.6. Since α_L is equal to 0.5 is used above with snow load, this value is checked. The optimum α_W is then equal to 1.5. These two sets of values are compared in Fig. 7.26. The α_L equal to 0.5 and α_W equal to 1.5 are chosen.

7.5.4.3 Combination (10): $D + W_i$

The $D + L_{ij} + W_i$ load case will take care of the cases where D and W effects add. Member failures where D and W effects oppose each other are considered herein. As member failure is considered, D opposing W will not be critical for flexure and shear. As overturning of a member is not a member failure it has not been considered. This $D + W_i$ combination is developed for the tension failure of columns.

The tension failure analysis is explained in Section 7.5.1. The optimum factors are found to depend heavily on the wind load parameters. As explained in Section 7.5.7 the final factors are obtained for cities that gave moderate results. The theoretical optimum obtained is α_D equal to 0.85 and α_W equal to 1.5. These values are shown in Fig. 7.27.

7.5.5 Load Combinations with E as the Principal Variable Load

7.5.5.1 Combinations (11): $D + E_i$, (12): $D + S_{ij} + E_i$, and (13): $D + L_{ij} + E_i$

As explained in Chapter 3 the specified exceedance probability format is used with E combinations. The earthquake load distribution is established for a specified exceedance probability of 0.0021 which is equivalent to a one in 475 year load. This is a more extreme distribution than the one in 30 year load. Compared with the 30 year load the annual probability of exceedance of E is approximately 16 times less.

The load combinations that include E are formatted differently because of the nature of the earthquake load (infrequent load). An earthquake occurs rarely with comparatively short holding times. Therefore, when E is the principal load the expected values of the other loads are their a.p.t. value. In all three load combinations, E is the principal load with α_E equal to 1.0 and the other loads take a.p.t. or expected values which are 1.0D, 0.2S and 0.5L (except in the case of storage live load which is not considered).

The target β values of 3.0 to 3.5 were established for a building life of 30 years. While all other loads (for normal buildings) are specified at an annual probability of exceedance of 0.033 the earthquake loads are specified at an annual probability of exceedance of 0.0021. Because the probabilities of exceedance are so different the target β values are not applicable to the earthquake loads.

7.5.6 Optimum Load Factors Chosen

The following 13 load combinations are selected in sections 7.5.2 to 7.5.5. The factors are developed to be used with the specified loads D, L, S, W and E.

(1) 1.3 D
(2) 1.2 D + 1.5 L
(3) 1.2 D + 0.1 W + 1.4 L
(4) 1.2 D + 0.1 S + 1.4 L
(5) 1.2 D + 1.7 S
(6) 1.2 D + 0.5 L + 1.7 S
(7) 1.2 D + 0.4 W + 1.7 S
(8) 1.2 D + 0.1 S + 1.2 W
(9) 1.2 D + 0.5 L + 1.5 W
(10) 0.85 D + 1.5 W
(11) 1.0 D + 1.0 E
(12) 1.0 D + 0.2 S + 1.0 E
(13) 1.0 D + 0.5 L + 1.0 E

As mentioned before, the factors which are equal to 0.1 give insignificant factored loads. Combination (2) governs over cases (3) and (4) in almost all practical cases. Therefore combinations (3) and (4) are neglected.

When combination (5) is compared with (6) and (7) it is seen that (5) will never be the governing load case. Therefore combination (5) is neglected.

Similarly, combination (8) is negligible when compared to combination (9). When load effects add up combination (11) is negligible when compared to combination (12) and (13). However, when load effects oppose as in tension failure (11) will govern over (12) and (13).

When load combinations (3), (4), (5) and (8) are deleted the proposed load combinations and load factors are:

1.3 D

1.2 D + 1.5 L

1.2 D + 0.5 L (or 0.4 W) + 1.7 S

1.2 D + 0.5 L + 1.5 W

0.85 D + 1.5 W

1.0 D + 1.0 E

1.0 D + 0.2 S (or 0.5 L) + 1.0 E

7.5.7 Variable Load Parameters and Their Influence on Reliability

The limit state equation is developed for a limit state that exists at a certain geographical location. Therefore the load variables in the equation should represent the local variation of the environmental loads. In past studies, average parameters of environmental loads have been used for all locations. To represent better the local variation the environmental load distributions for representative cities are established and these values are used to analyze the reliability at each location.

Fifteen major cities distributed uniformly throughout Canada are initially chosen as representative cities. However, the number of cities considered is reduced as explained in Section 4.2 and the cities considered for the final analysis are given in Table 7.6.

The reliability analysis explained in this chapter is done for these cities. It is clear from the results that the optimum safety factors are sensitive to the load parameters. This indicated that considering the individual cities separately is important. If the average of the climatic parameters is used the most critical combinations of load parameters may not be considered. However, it would also not be reasonable to penalize all locations in Canada for the critical load parameters that existed in a few locations. Therefore, the dependency of reliability on the load parameters is further investigated.

The optimum load factors obtained for the different cities are found to be proportional to the mean, μ, and standard deviation, σ, of the normalized load. Table 7.6 gives the values of the normalized loads. Figure 7.28 shows that β depends strongly on the μ and σ of the normalized load. Table 7.7 shows that the load factors required are proportional to the product $\mu \times \sigma$.

Two strategies are available. The first is to derive the load factors for the city having the highest $\mu \times \sigma$ in which case, they would be conservative, sometimes excessively so, for all other cities. The second strategy would be to derive the load factors for a group of cities having a moderate combined value of $\mu \times \sigma$ and to adjust the specified climatic loads for cities with values of $\mu \times \sigma$ that are significantly different from the moderate values. The second strategy is used in this study.

To get consistent safety in different cities the specified load has to be such that μ and σ of the normalized load gives a moderate combined value; i.e., around 0.25 to 0.35 depending on the combination, as shown in Fig. 7.28.

It should be noted that the μ and σ of the normalized load both depend on the nominal load specified for that city; for example,

for wind load, $\mu = \dfrac{\overline{W}}{W_n}$ and $\sigma = \dfrac{\sigma_W}{W_n}$

where, $\overline{W}$ is the 'true' mean of the wind load

σ_W is the 'true' standard deviation of the wind load

W_n is the nominal/specified wind load.

Therefore, by changing the nominal/specified wind load for a city both the μ and σ of the normalized load are changed.

Thus by specifying an appropriate nominal load for each city the $\mu \times \sigma$ values of all cities can be made similar so that the reliability of the factors would be uniform for all locations. Load factors are developed for cities where the normalized load parameters are moderate. For the other cities considered in this study the results indicate the specified snow load should be increased for Kelowna (because α_L and α_S are both higher for Kelowna as seen in Table 7.7). The specified snow load for St.John's, Toronto and Winnipeg seem appropriate. The specified wind load needs to be decreased in Vancouver and perhaps increased in Regina. Further study is necessary in specifying load parameters.

The above recommendations are developed using the snow and wind loads in the NBCC (1990). The NBCC (1995) values were available only very late in 1995. The only changes in 1995 version for the cities used in this study are an increase in snow load parameters for Kelowna and a decrease in wind load parameters for Vancouver. These changes agree with the findings herein.

The sensitivity of the load factors to the load parameters can be considered by looking at the 'theoretically optimum' load factors. As subjective measures, such as graphical comparison, are used to obtain the final load factors their sensitivity to the load factors cannot be measured independently. Table 7.7 shows that 'theoretically optimum' load factors are proportional to $\mu \times \sigma$. By comparing St. John's and Winnipeg it is seen that a 9 % difference in $\mu \times \sigma$ did not have an impact on the 'theoretical optimum' load factors. When comparing Toronto and Winnipeg it is seen that a 24% difference in snow load $\mu \times \sigma$ made a 12% difference in the corresponding load factor.

7.6 Comparison of Existing and Proposed Load and Resistance Factors

The load and resistance factors proposed in this study are compared with the existing factors specified in A23.3-94 and NBCC (1990). Figures 7.29 to 7.3< give graphical representations of β for the different load combinations. The load combinations with E are not given here because the target β value do not apply to them as discussed in Section 7.5.5.

The true failure is modelled using Turkstra's rule (Turkstra 1972) which states that the critical load combination is most likely when one variable load takes a maximum and the other loads take companion values. The governing load cases are handled by using the governing case for the factored design equation and actual load combination for the 'true' variable equation (as detailed in section 7.5.2.1).

When a two load combination and three load combination are compared, the true load idealization gets more complicated. This is because the two load combination involves only characteristic loads while the three load combination involves both characteristic and companion loads. As many situations are possible, the combination giving the lowest reliability of these possibilities are given in the Figs. 7.29 to 7.36. The proposed load and resistance factors give β values very close to the target range.

Table 7.1 - Weightings used for different load ratios

S/D	Weight
0.2	25.5
0.4	20.8
0.6	17.1
0.8	13.0
1.0	9.3
1.2	6.9
1.4	3.7
1.6	2.3
1.8	1.4

L/D	Weight
0.0	0.0
0.2	11.8
0.4	23.5
0.6	23.5
0.8	17.6
1.0	9.4
1.2	7.1
1.4	5.9
1.6	1.2
1.8	0.0

W/D	Weight
0.5	10
1.0	10
1.5	10
2.0	10
2.5	10
3.0	10
3.5	10
4.0	10
4.5	10
5.0	10

Table 7.2 - Highest ϕ_s values that ensure β within or above the target range

	Beam	
ϕ_c	R-0.71	T-0.14
0.60	1.00	0.85
0.65	0.95	0.85
0.70	0.90	0.85
0.75	0.875	0.825

Table 7.3 - Highest ϕ_c values that ensure β within or above the target range

	Column	
ϕ_s	Column 1	Column 2
0.80	0.675	0.750
0.85	0.675	0.725
0.90	0.650	0.700
0.95	0.650	0.675

Table 7.4 - Results of shear analysis

ϕ_c	0.7	0.65	0.65	0.7
ϕ_s	0.85	0.85	0.85	0.85
Constant*	0.2	0.2	0.17	0.17
L/D		β		
0	2.73	2.91	3.26	3.11
0.4	3.12	3.28	3.61	3.46
0.8	3.12	3.27	3.58	3.44
1.2	3.05	3.20	3.49	3.36
1.6	2.99	3.13	3.41	3.29
2	2.95	3.08	3.36	3.24
2.4	2.91	3.05	3.32	3.20
2.8	2.89	3.02	3.28	3.16

*Constant in V_c equation

Table 7.5 - 'Theoretical optimums' for D + L_i combination

Member	α_D	α_L
Beam R-0.71	1.15	1.50
Beam T-0.14	1.00	1.65
Column 1	1.25	1.25

Table 7.6 - Normalized characteristic load parameters used in this study

Load	City	Normalized load μ	σ	COV	μ x σ
D		1.03	0.096	0.093	0.10
L		0.92	0.24	0.26	0.22
S	Kelowna	1.04	0.41	0.39	0.43
	Regina	0.76	0.32	0.42	0.24
	Winnipeg	0.84	0.37	0.44	0.31
	Toronto	1.01	0.25	0.25	0.25
	St.John's	0.94	0.36	0.38	0.34
W	Vancouver	0.49	0.15	0.31	0.07
	Winnipeg	0.61	0.23	0.38	0.14
	Toronto	0.65	0.21	0.32	0.14
	St.John's	0.81	0.29	0.36	0.23

Table 7.7 - Normalized load parameters with required load factors

			Theoretical optimum with $\alpha_D = 1.2$			
			$D+L_{ij}+S_i$		$D+L_{ij}+W_i$	
Load	City	μ x σ	α_L	α_S	α_L	α_W
S	Kelowna	0.43	0.5	1.9		
	Regina	0.24	0.2	1.7		
	Winnipeg	0.31	0.2	1.9		
	Toronto	0.25	0.2	1.7		
	St.John's	0.34	0.2	1.9		
W	Vancouver	0.07			< 0.1	< 1.1
	Winnipeg	0.14			0.2	1.2
	Toronto	0.14			0.2	1.2
	St.John's	0.23			0.2	1.6

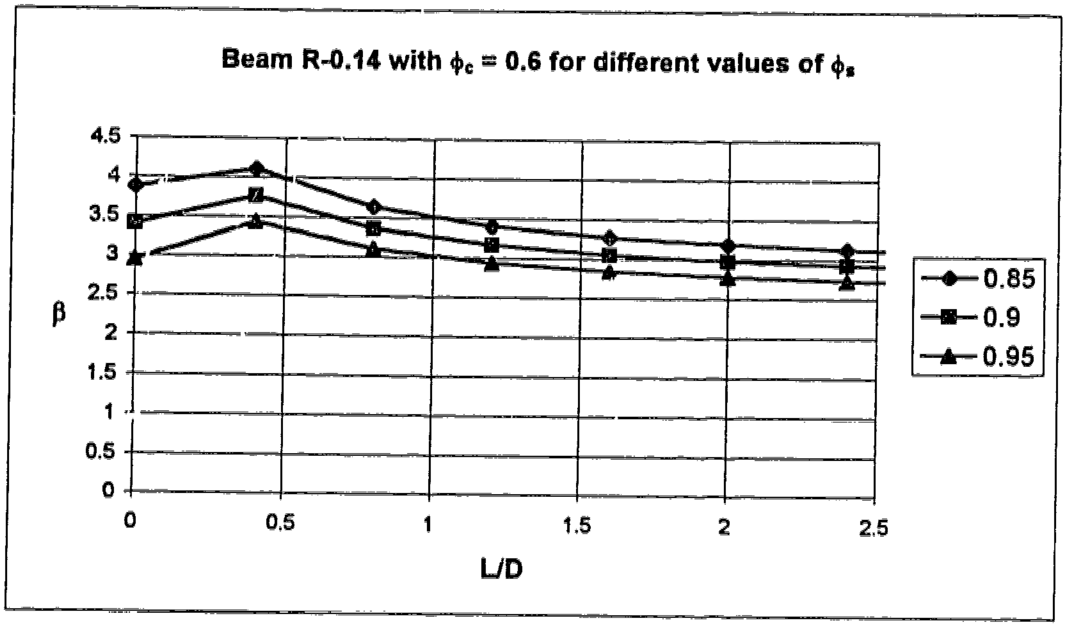


Figure 7.1 - Effects of changing a resistance factor

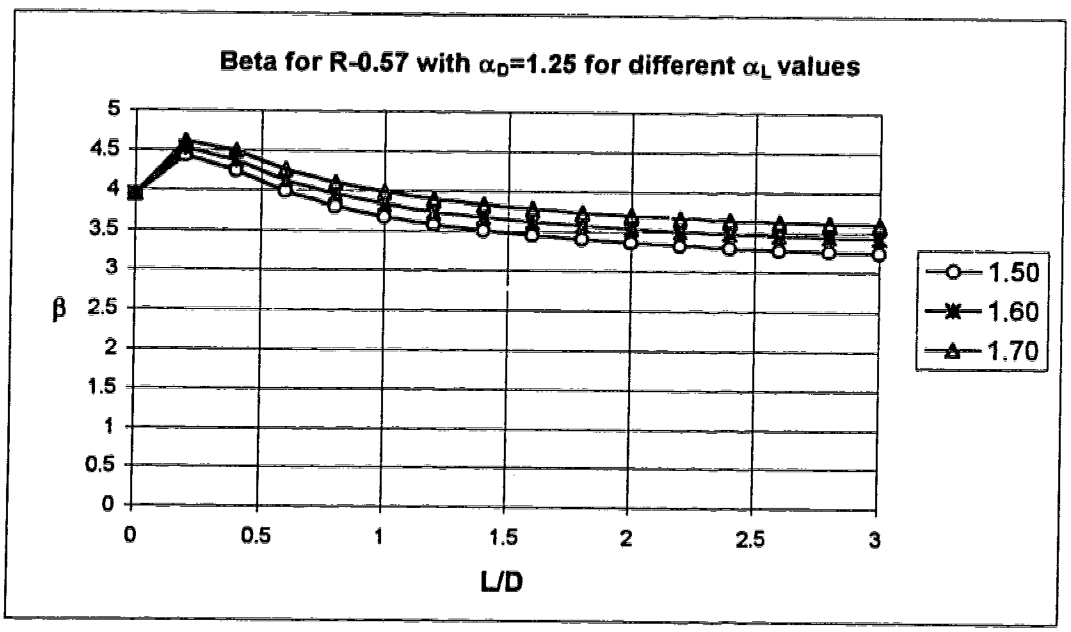


Figure 7.2 - Effects of changing a load factor

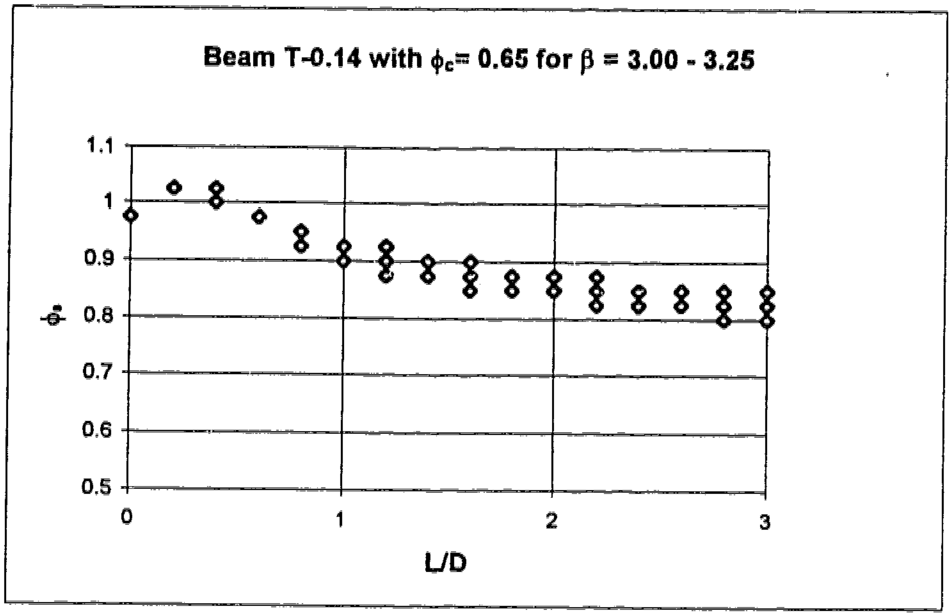


Figure 7.3 - Graph of ϕ_s vs L/D for the target β range

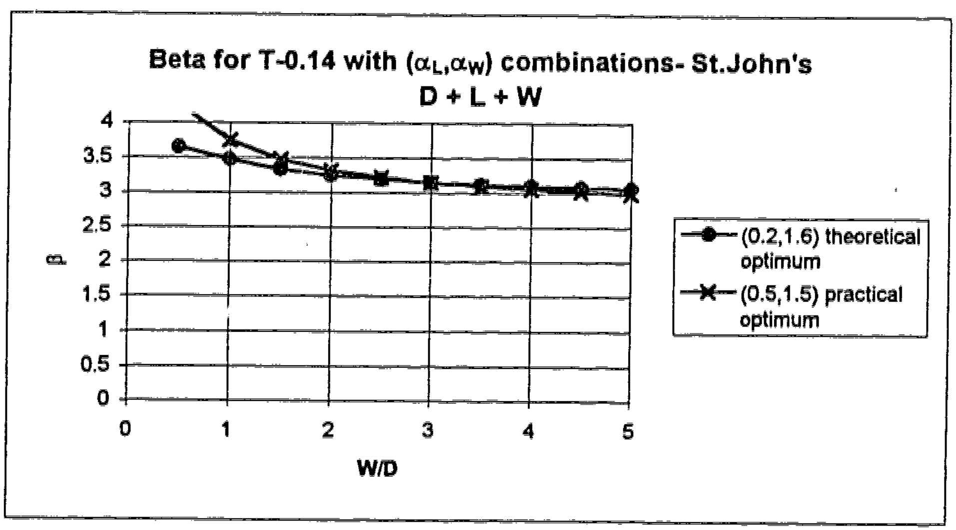


Figure 7.4 - Graphical representation of adjusted optimum factors

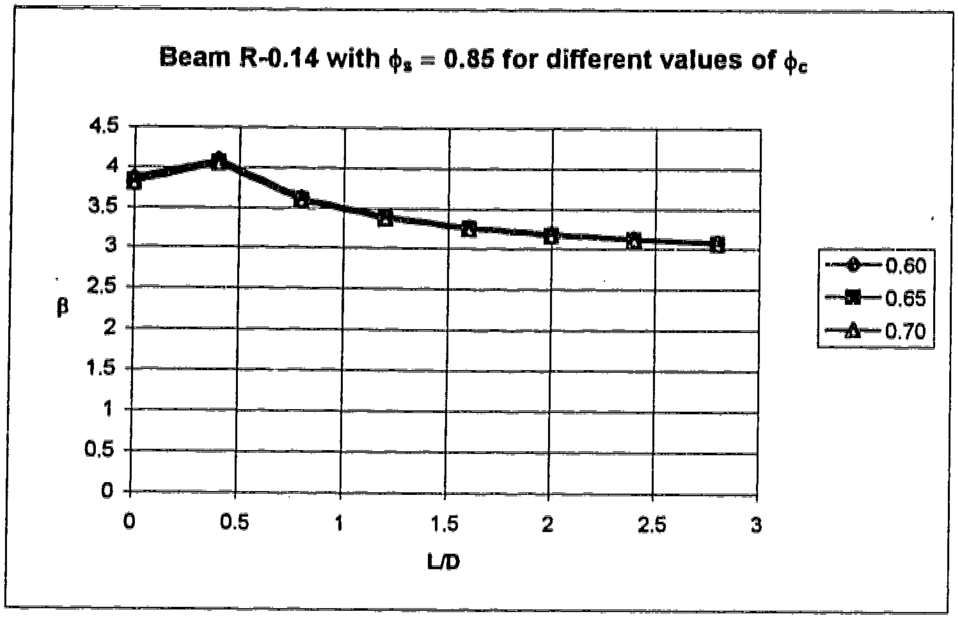


Figure 7.5 - Effect of ϕ_c on the reliability of under-reinforced beams in flexure

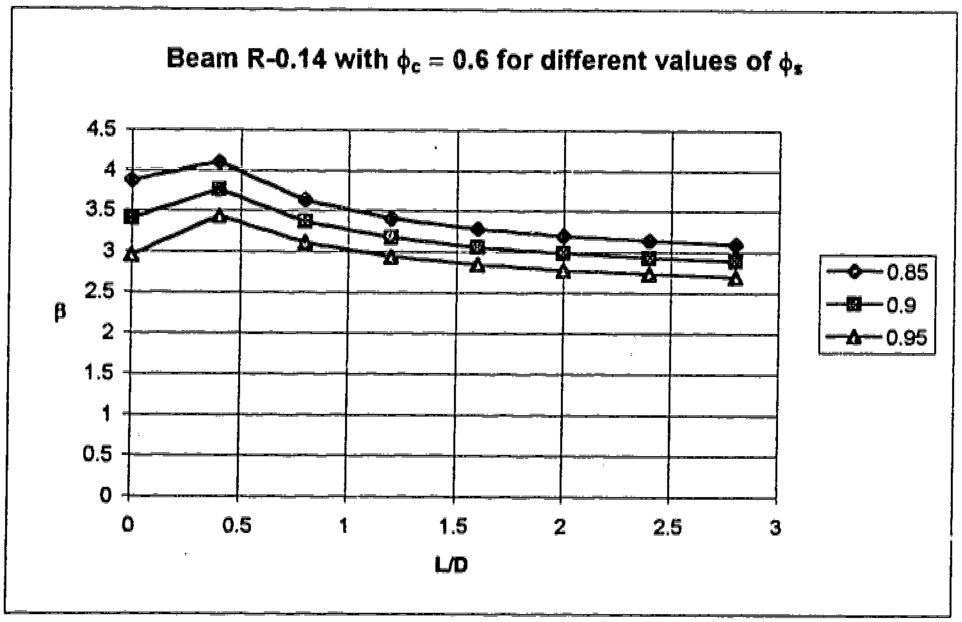


Figure 7.6 - Effect of ϕ_s on the reliability of under-reinforced beams in flexure

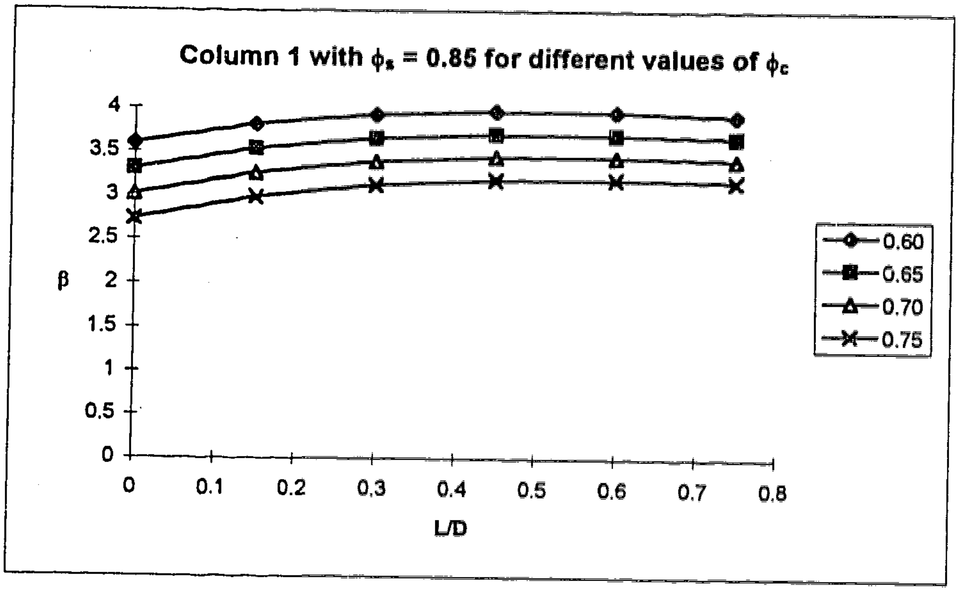


Figure 7.7 - Effect of ϕ_c on the reliability of columns (compression failure)

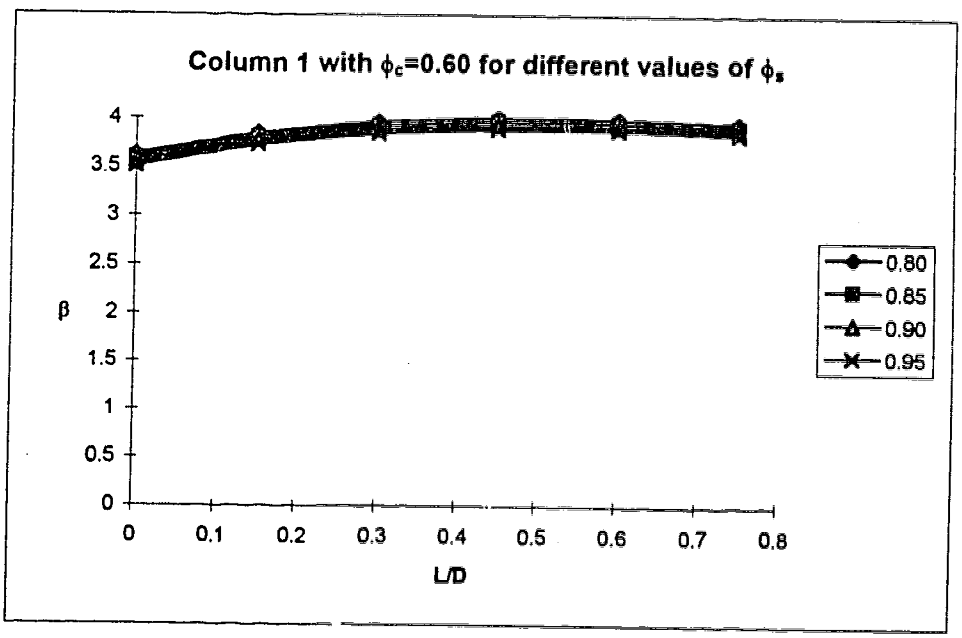


Figure 7.8 - Effect of ϕ_s on the reliability of columns (compression failure)

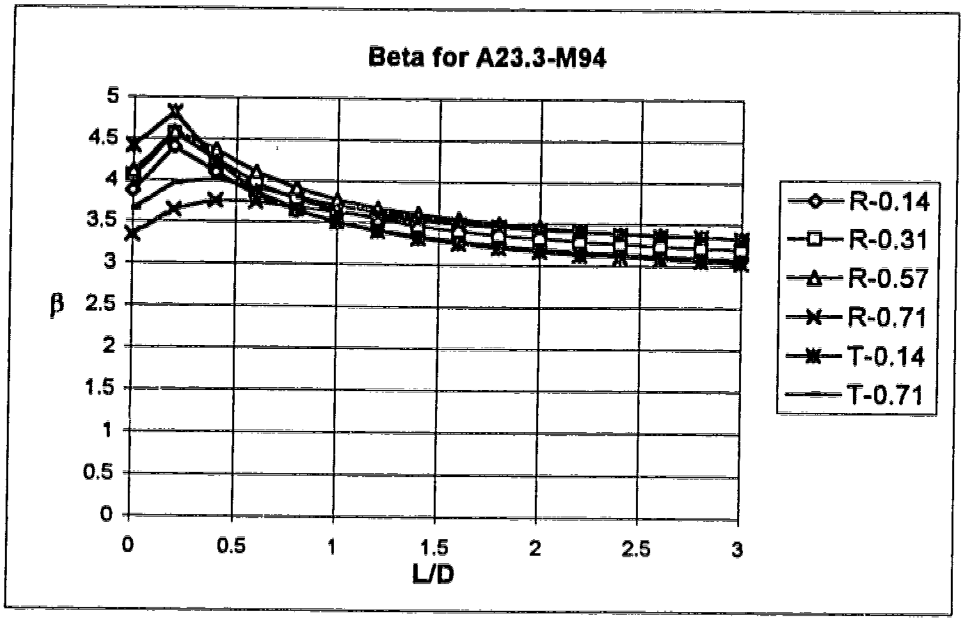


Figure 7.9 - Reliability of different kinds of beams

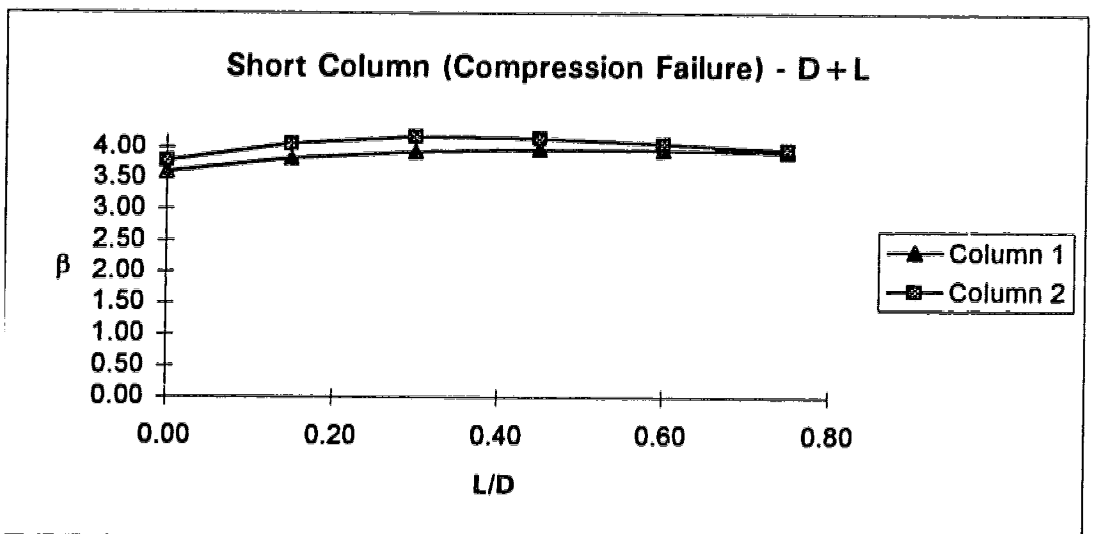


Figure 7.10 - Column reliability for A23.3-94

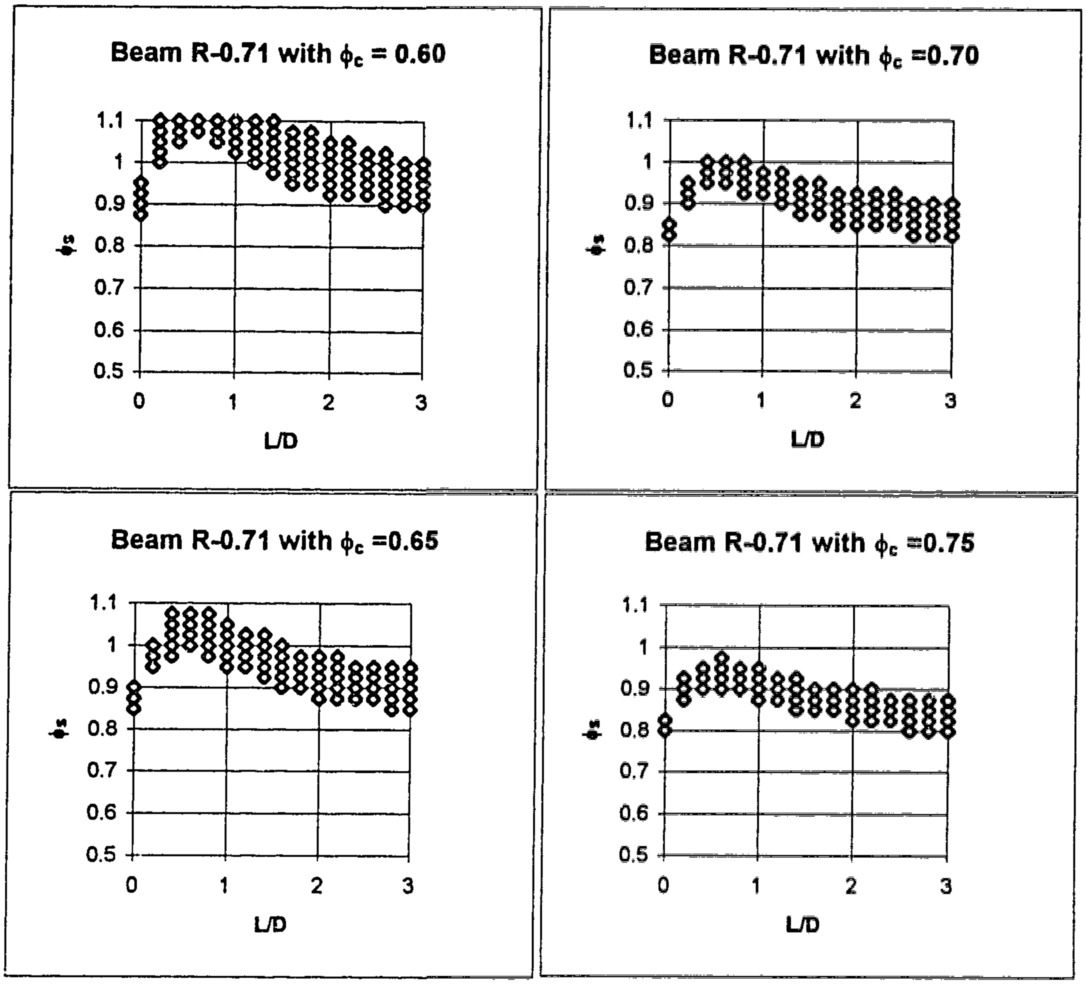


Figure 7.11 - Combinations of ϕ_c and ϕ_s, for R-0.71, that ensure β = 3.00 to 3.25

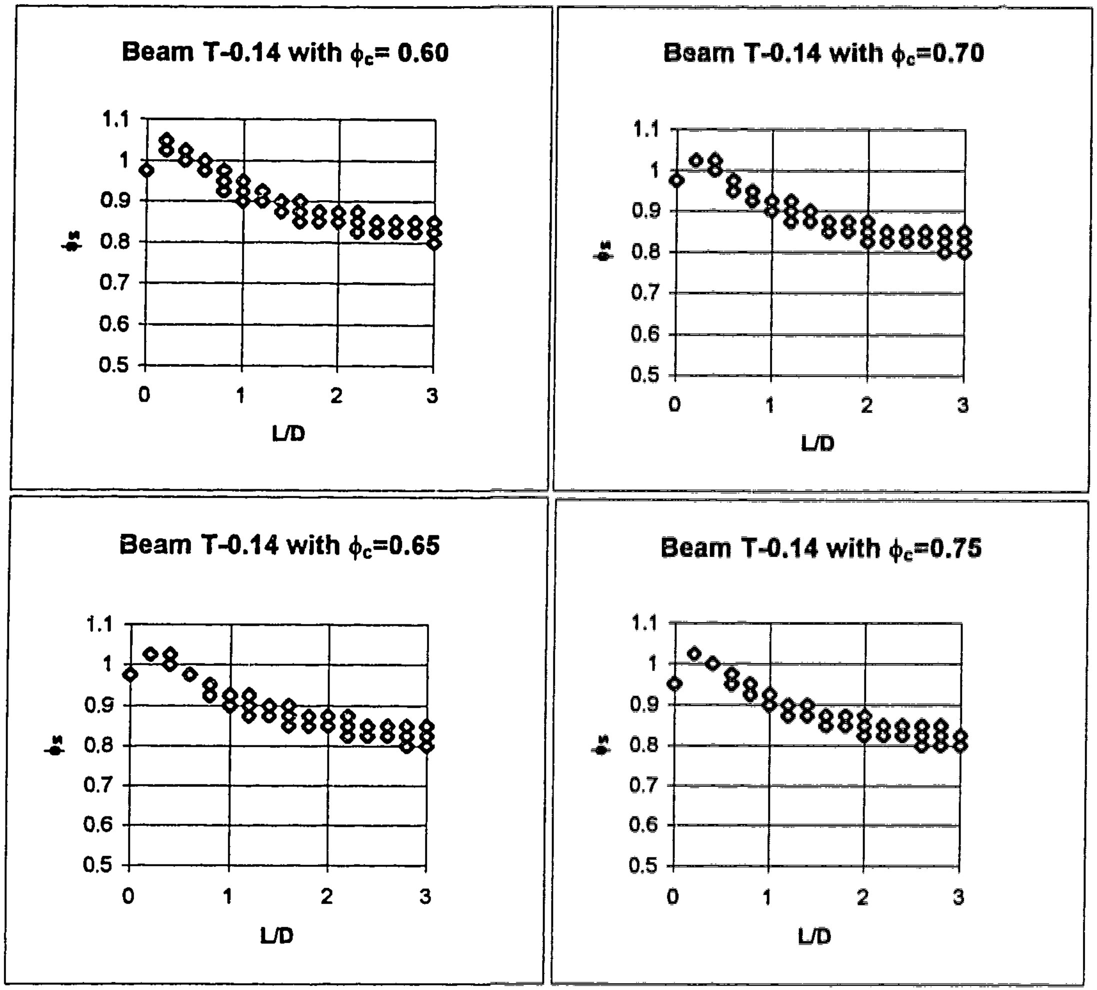


Figure 7.12 - Combinations of ϕ_c and ϕ_s, for T-0.14, that ensure β = 3.00 to 3.25

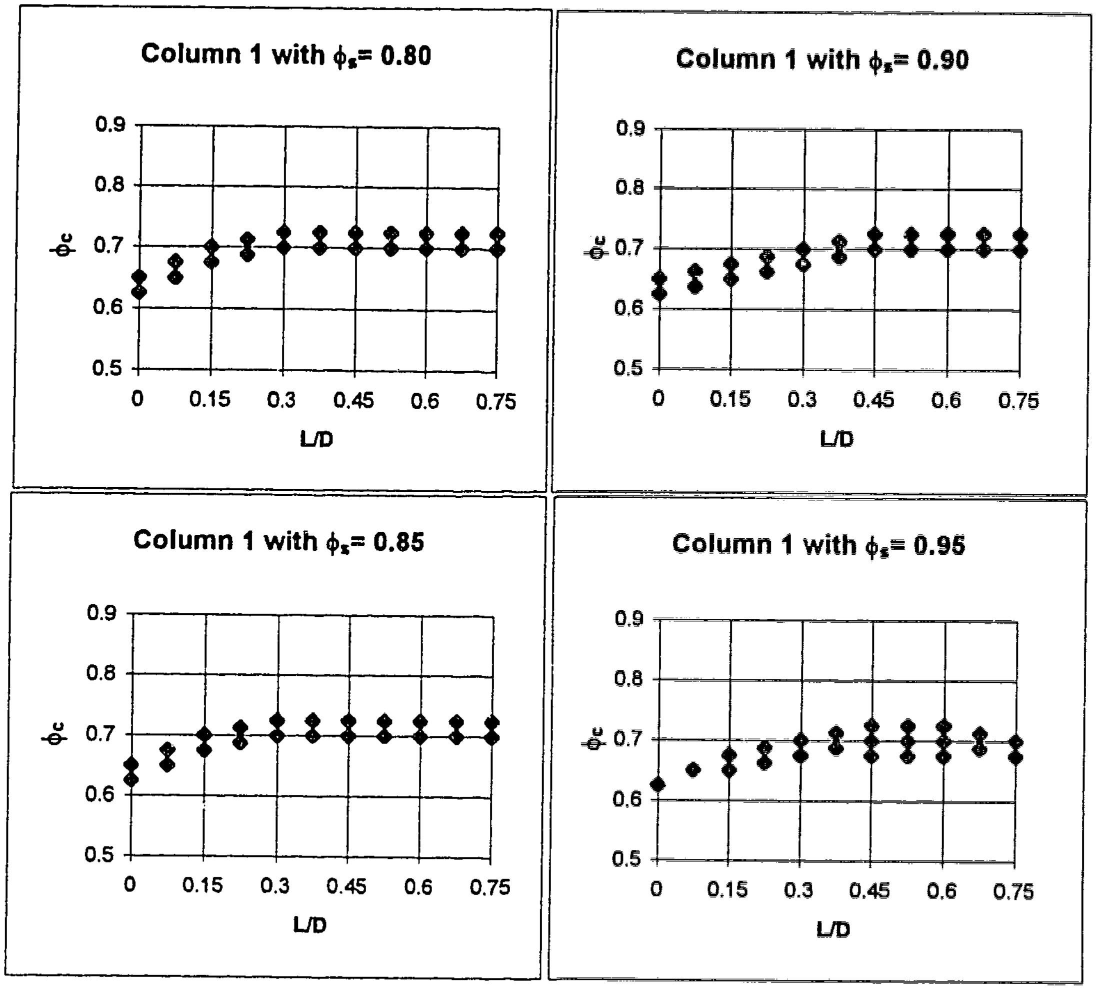


Figure 7.13 - Combinations of ϕ_c and ϕ_s, for Column 1, that ensure β = 3.25 to 3.50

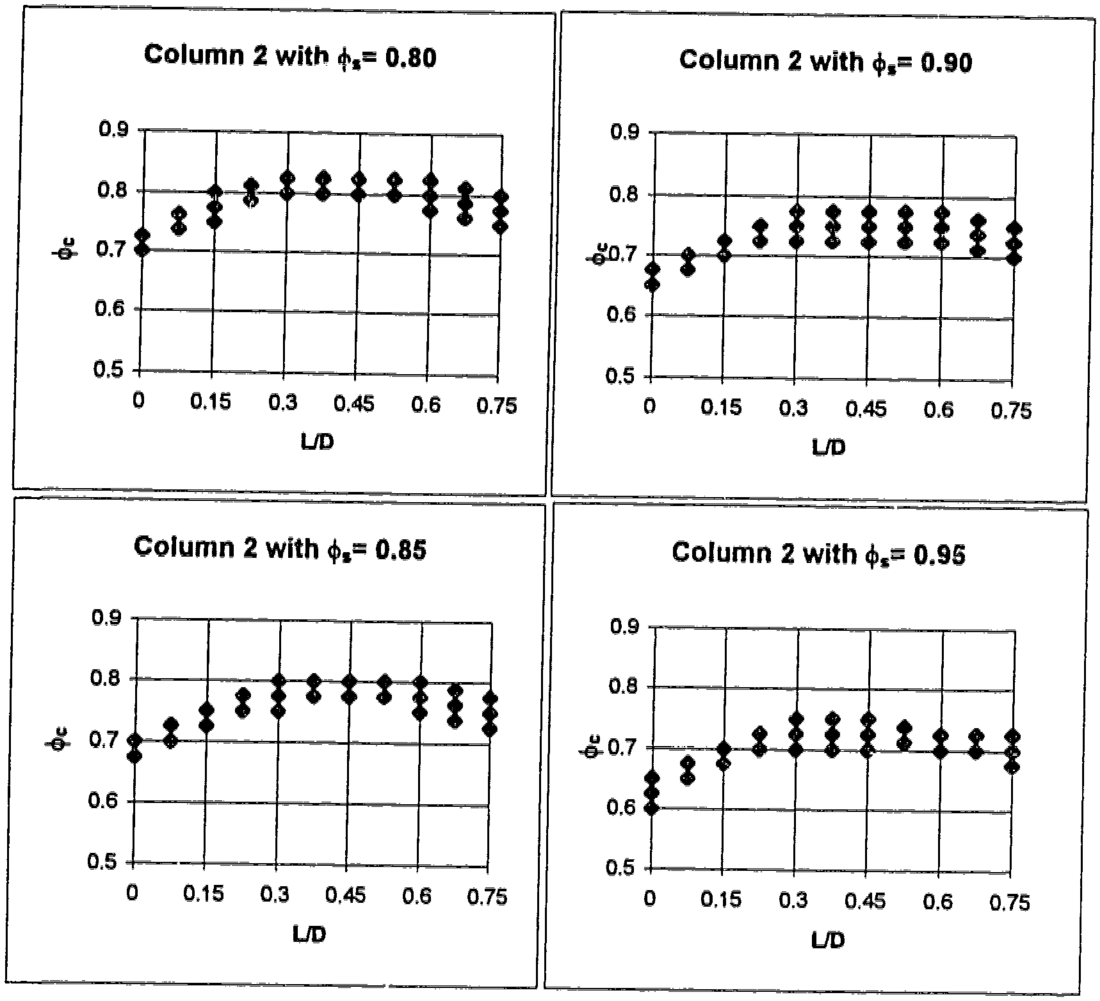


Figure 7.14 - Combinations of ϕ_c and ϕ_s, for Column 2, that ensure $\beta = 3.25$ to 3.50

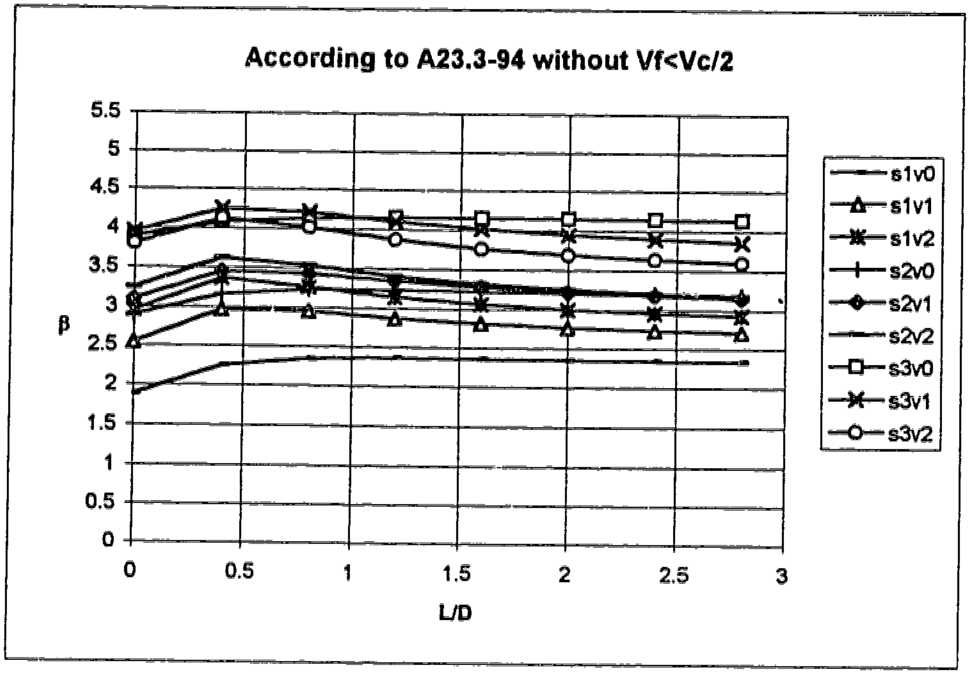


Figure 7.15 - Shear reliability of nine different beams

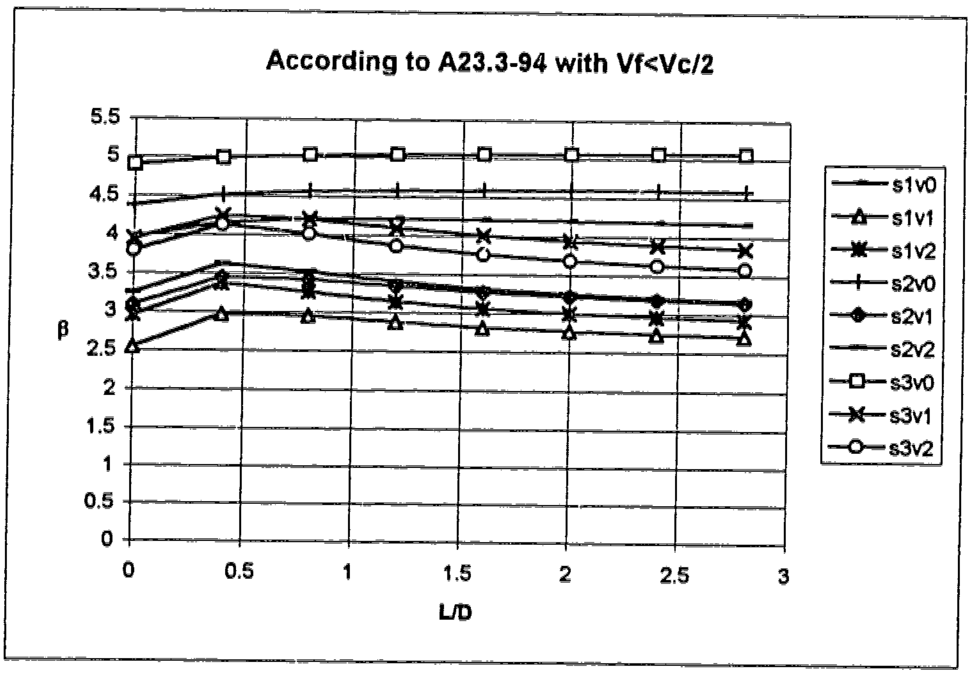


Figure 7.16 - Shear reliability of nine different beams with limitation for beams without stirrups

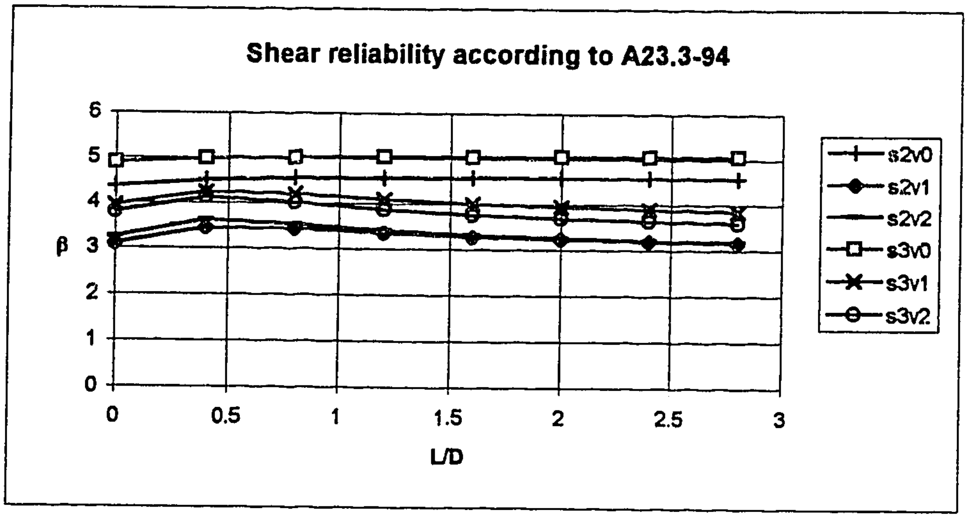


Figure 7.17 - Shear reliability of beams considered in this study

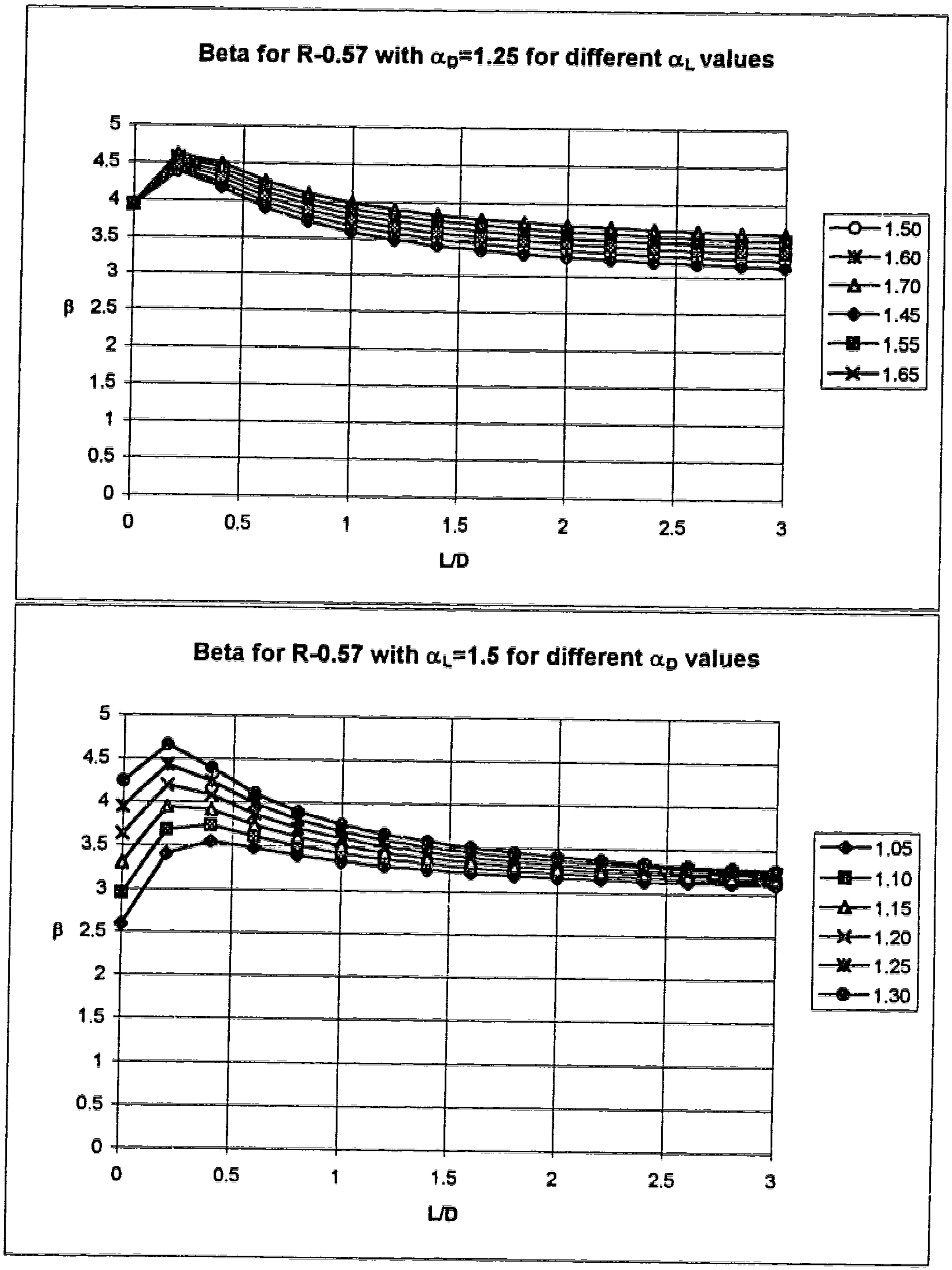


Figure 7.18 - Effect on β when the dead load and live load factors are changed individually

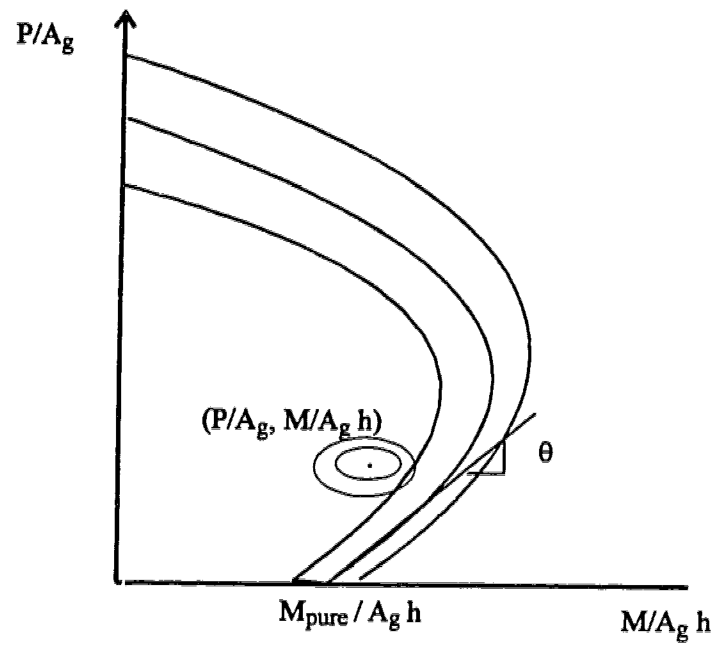


Figure 7.19 - Interaction diagram in 'true' space

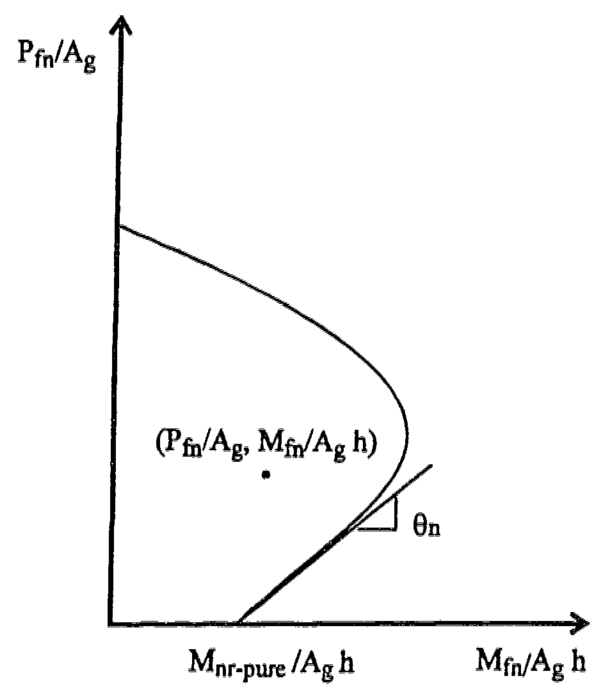


Figure 7.20 - Interaction diagram in factored nominal space

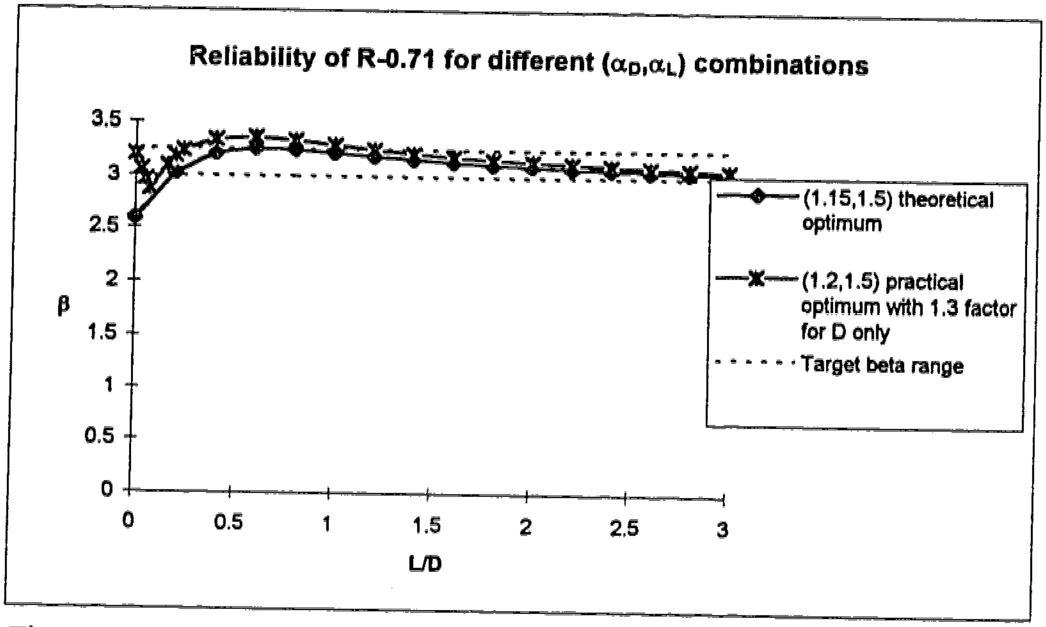


Figure 7.21 - Reliability of optimum load factors for beam R-0.71 for D+L_i combination (with a D only case introduced for the practical optimum)

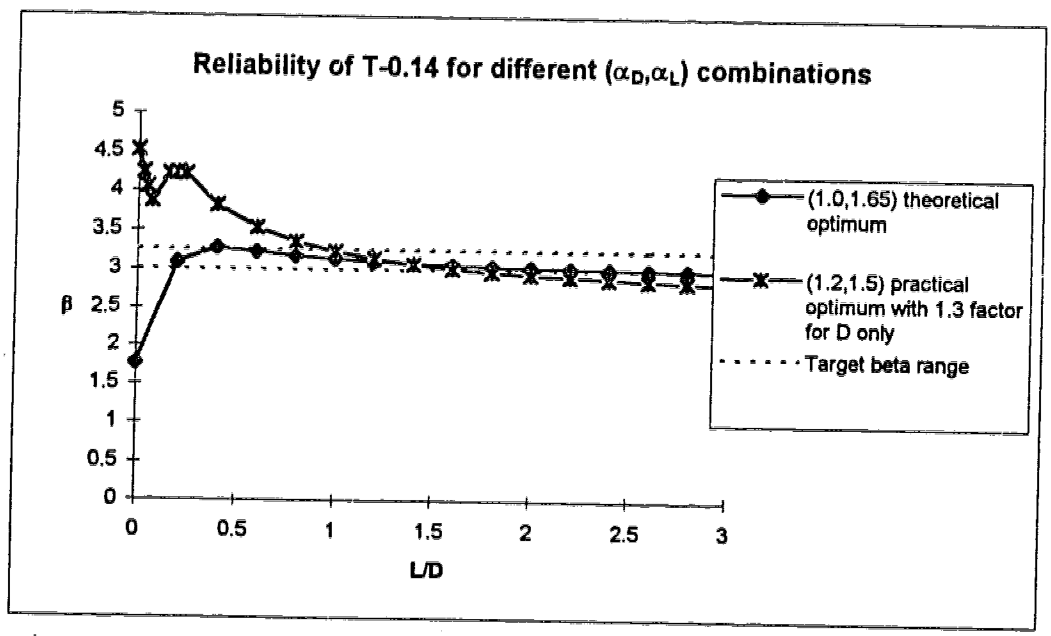


Figure 7.22 - Reliability of optimum load factors for beam T-0.14 for D+L_i combination (with a D only case introduced for the practical optimum)

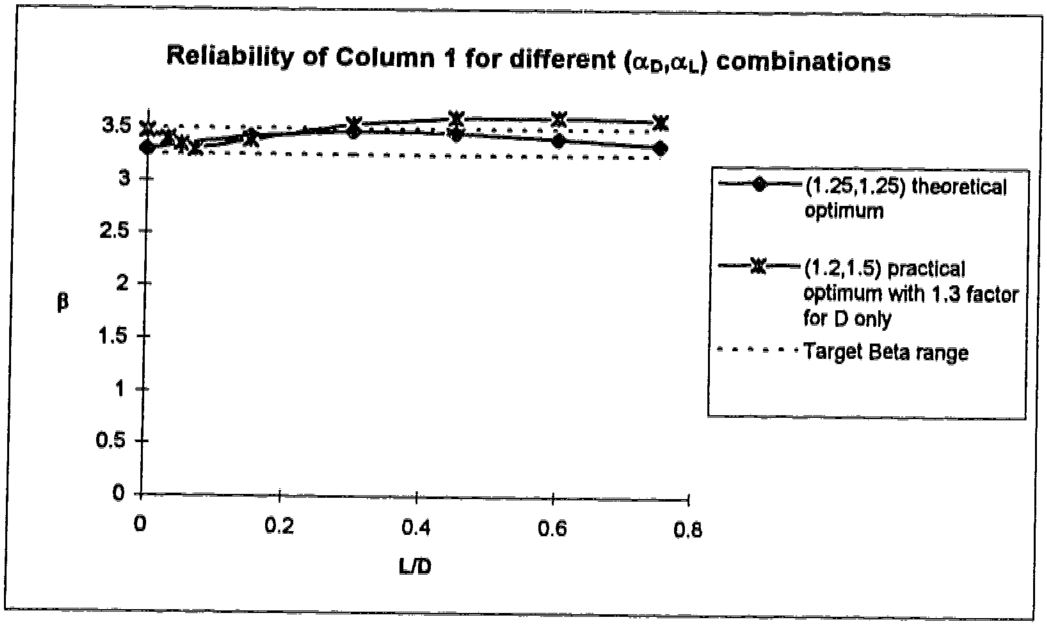


Figure 7.23 - Reliability of optimum load factors for Column 1 for D+L_i combination (with a D only case introduced for the practical optimum)

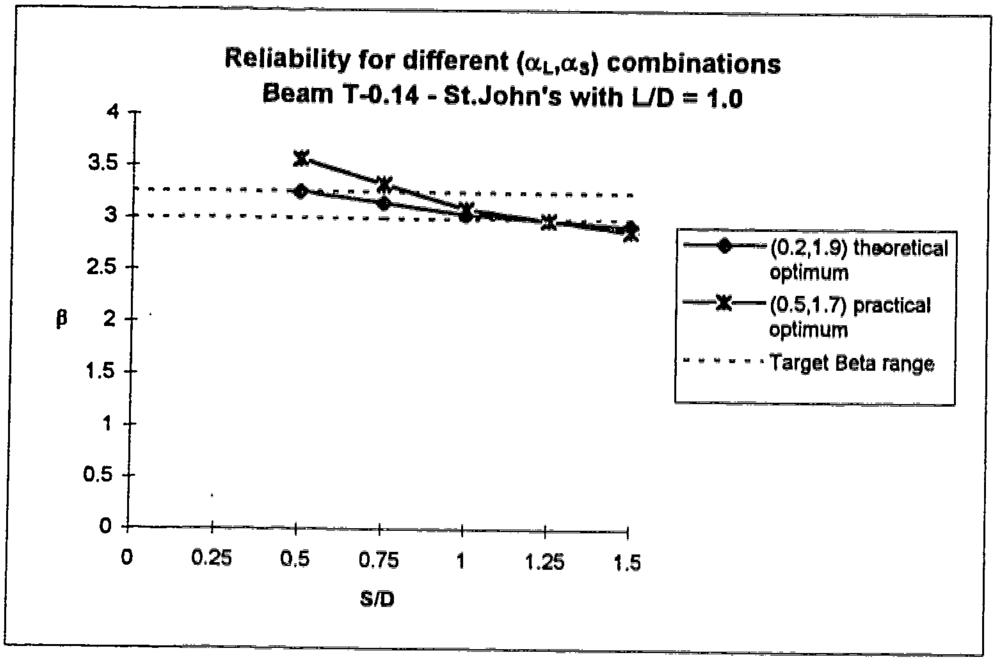


Figure 7.24 - Reliability of optimum load factors for beam T-0.14 for $D+L_{ij}+S_i$ combination

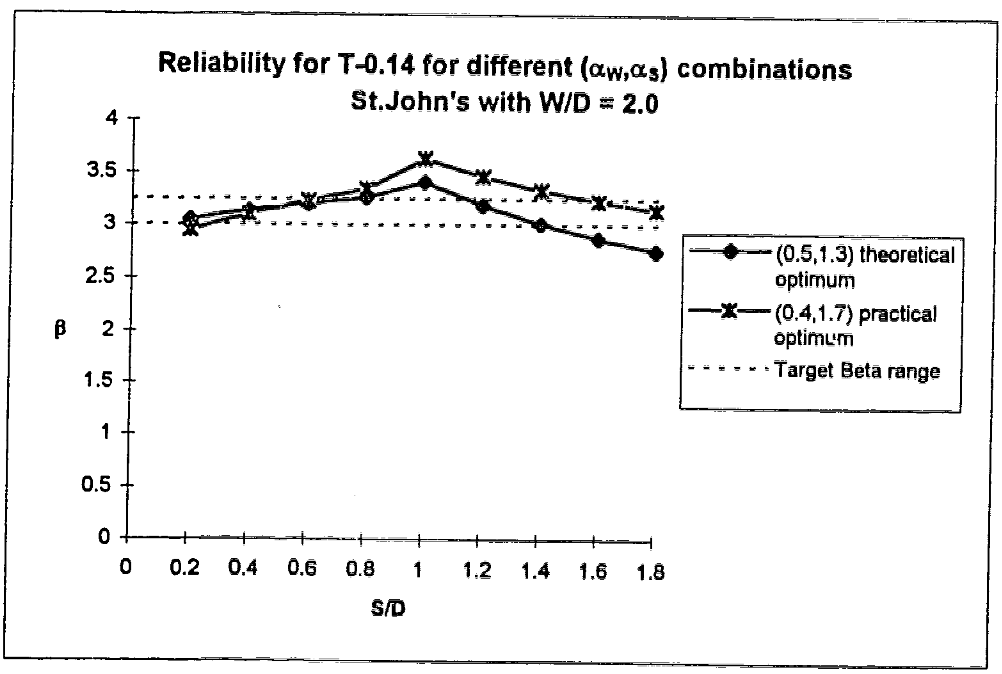


Figure 7.25 - Reliability of optimum load factors for beam T-0.14 for $D+W_{ij}+S_i$ combination

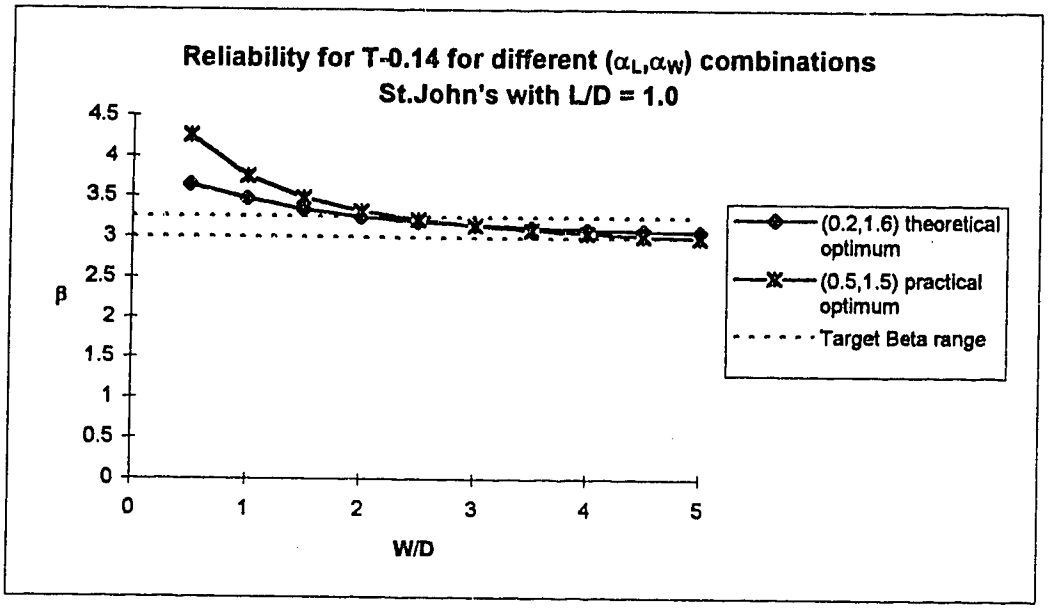


Figure 7.26 - Reliability of optimum load factors for beam T-0.14 for $D+L_{ij}+W_i$ combination

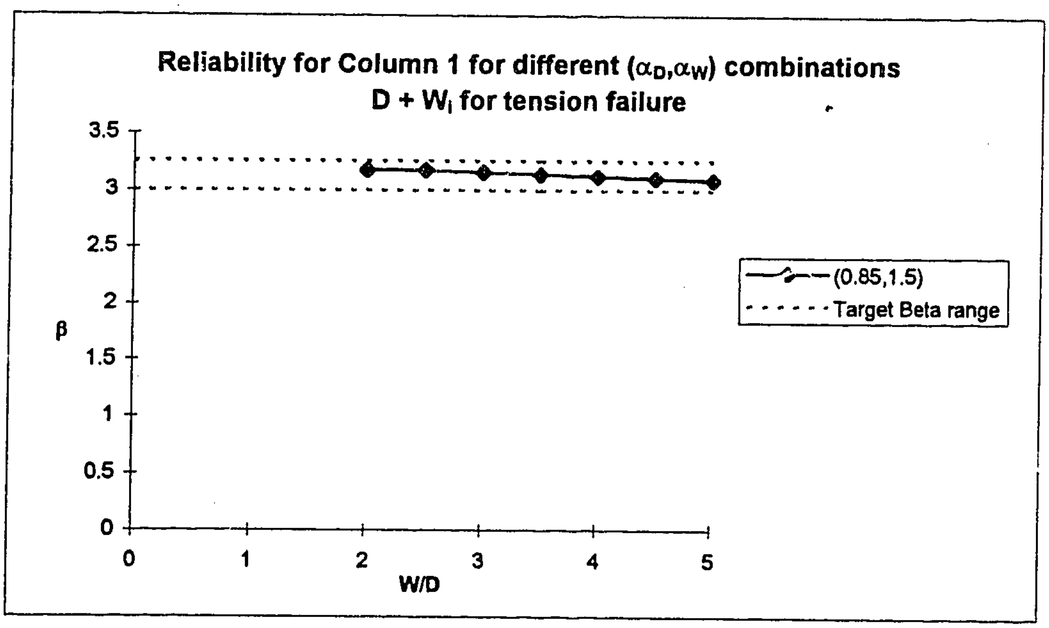
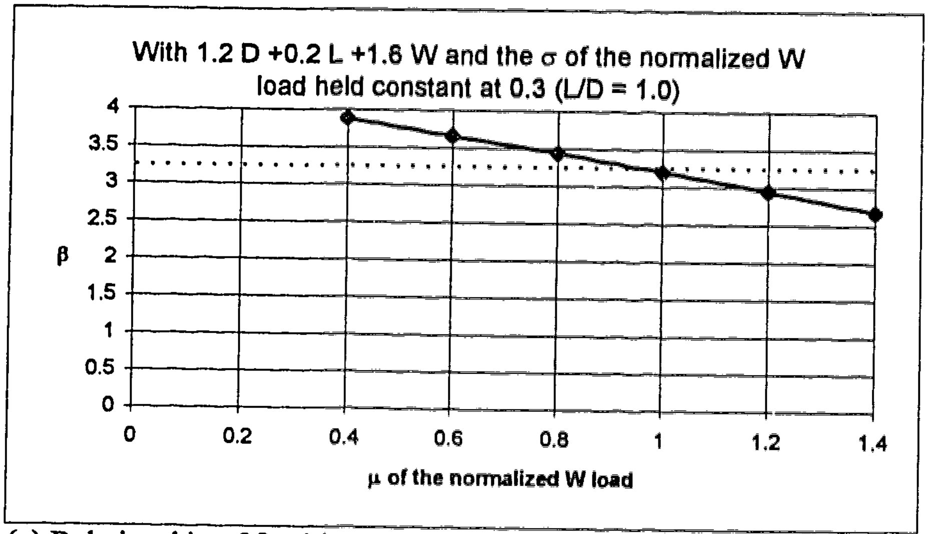
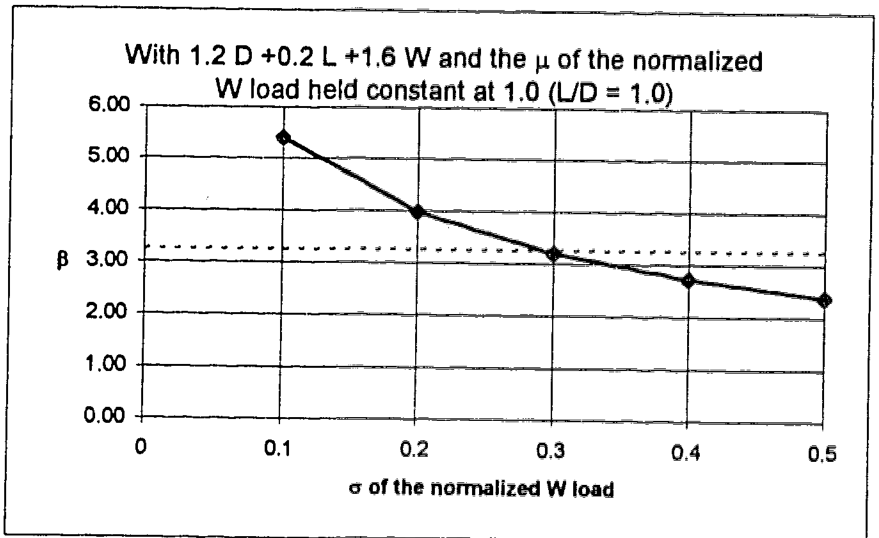


Figure 7.27 - Reliability of optimum load factors for Column 1 for D + W_i combination (Tension failure)



(a) Relationship of β with μ of normalized load



(b) Relationship of β with σ of normalized load

Figure 7.28 -Dependancy of β on the μ and σ of the normalized principal load

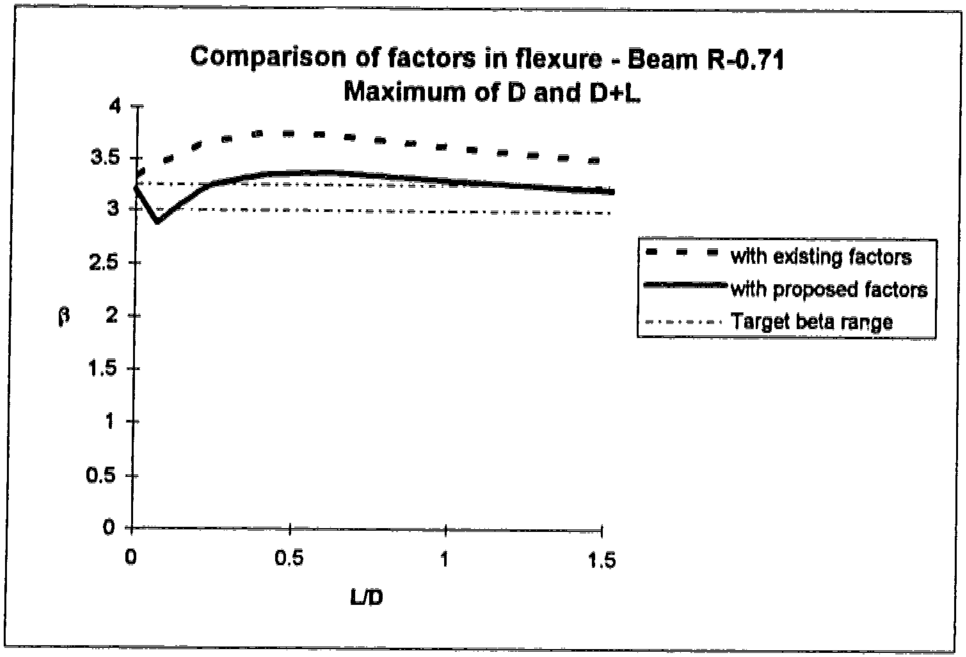


Figure 7.29 - Comparison of proposed load and resistance factors with those from CSA A23.3-94 (Beam R-0.71 for max. of D and D+L)

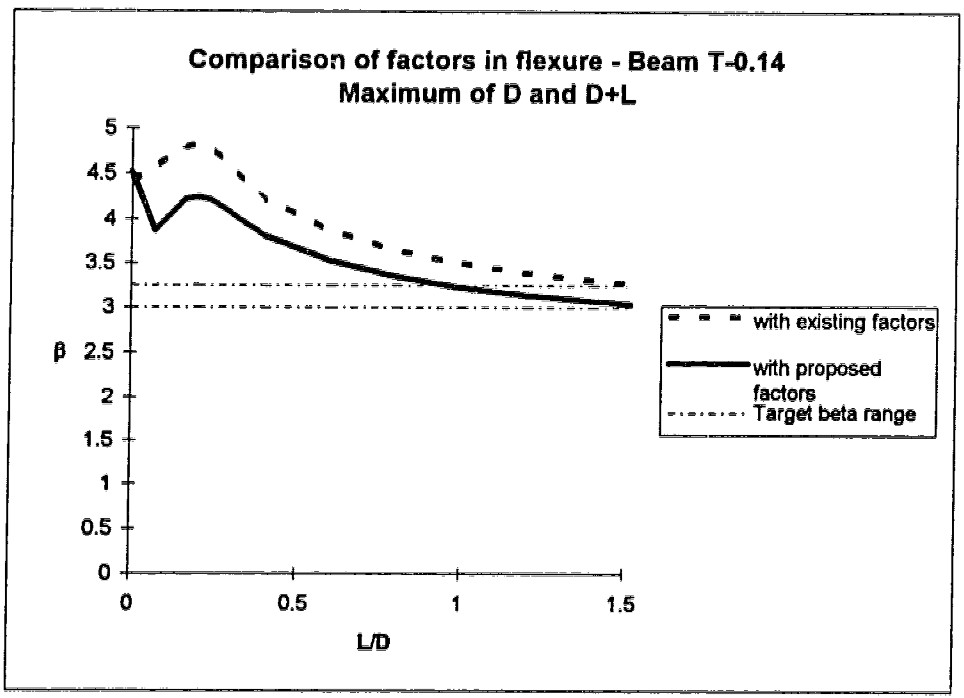


Figure 7.30 - Comparison of proposed load and resistance factors with those from CSA A23.3-94 (Beam T-0.14 for max. of D and D+L)

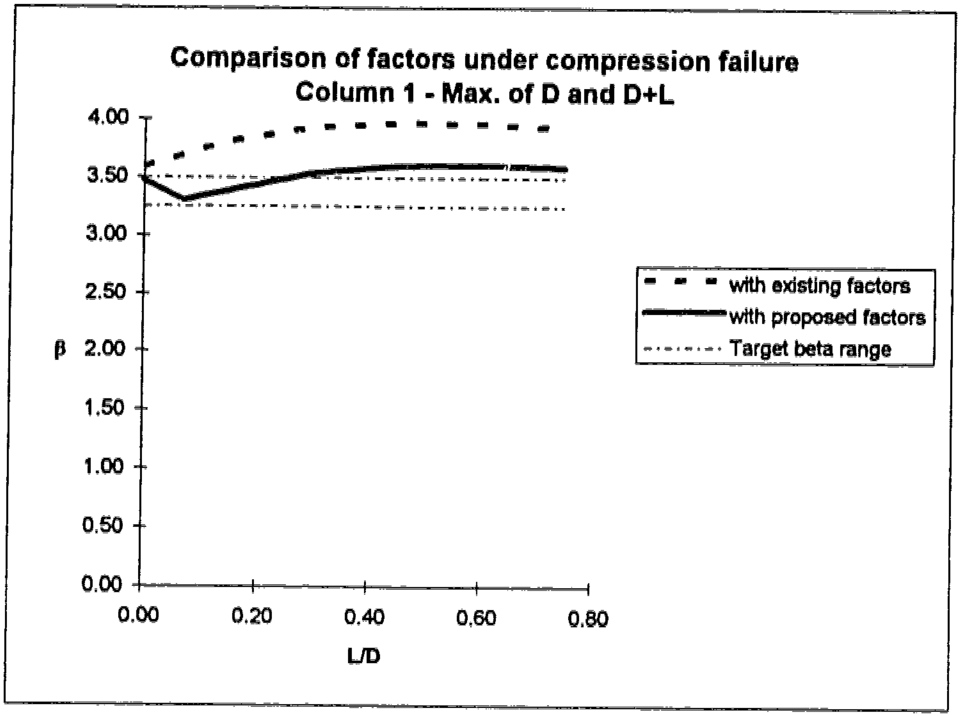


Figure 7.31 - Comparison of proposed load and resistance factors with those from CSA A23.3-94(Column 1 for max. of D and D+L)

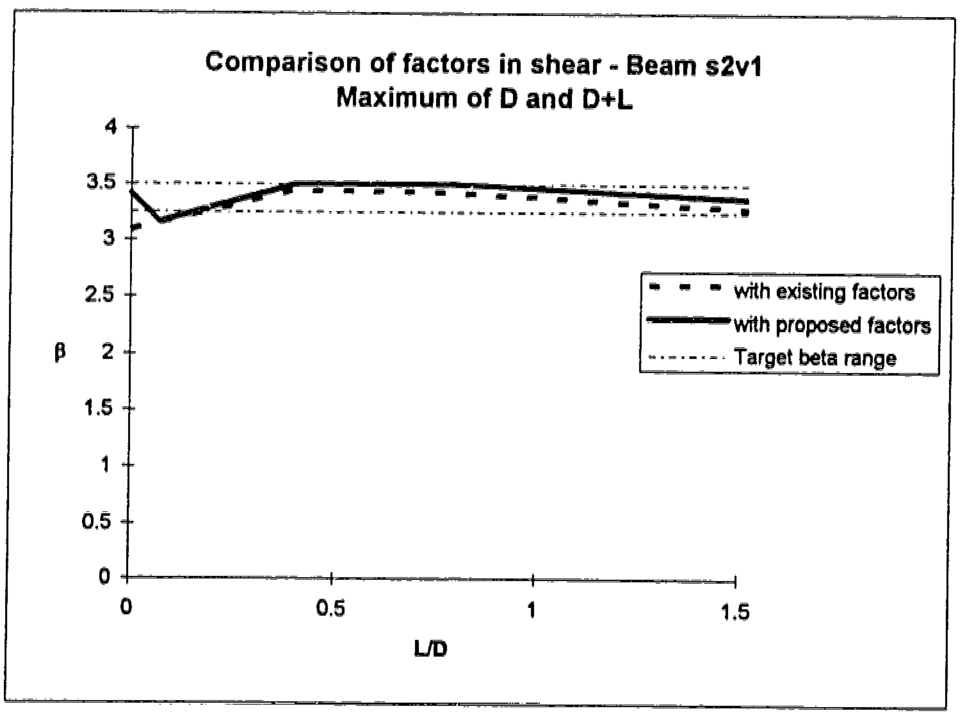


Figure 7.32 - Comparison of proposed load and resistance factors with those from CSA A23.3-94 (Beam s2v1 for max. of D and D+L)

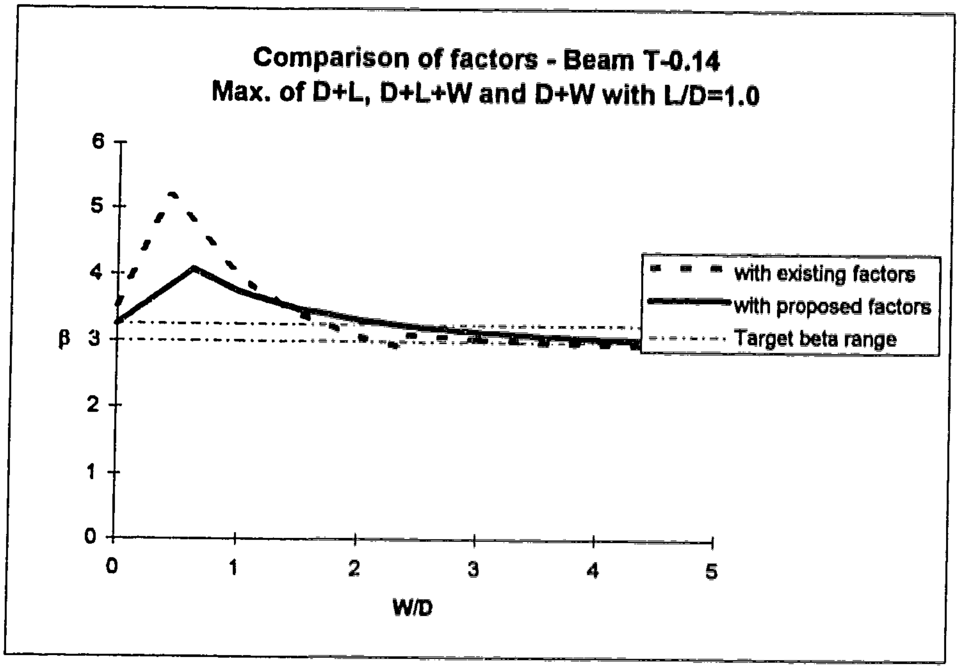


Figure 7.33 - Comparison of proposed load and resistance factors with those from CSA A23.3-94 (Beam T-0.14 for max. of D+L, D+L+W and D+W)

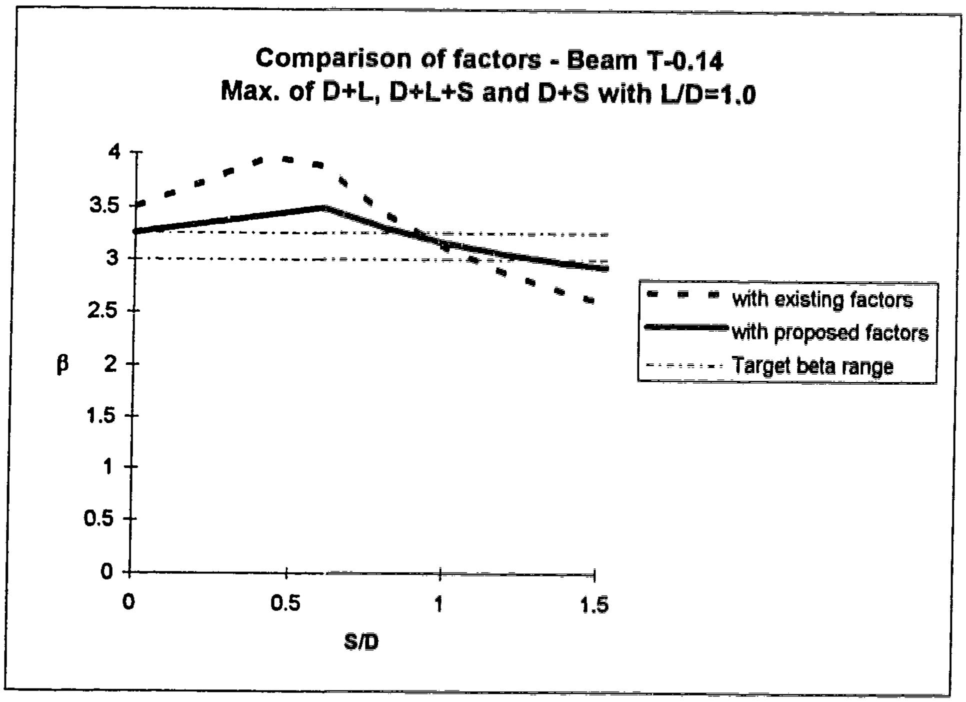


Figure 7.34 - Comparison of proposed load and resistance factors with those from CSA A23.3-94 (Beam T-0.14 for max. of D+L, D+L+S and D+S)

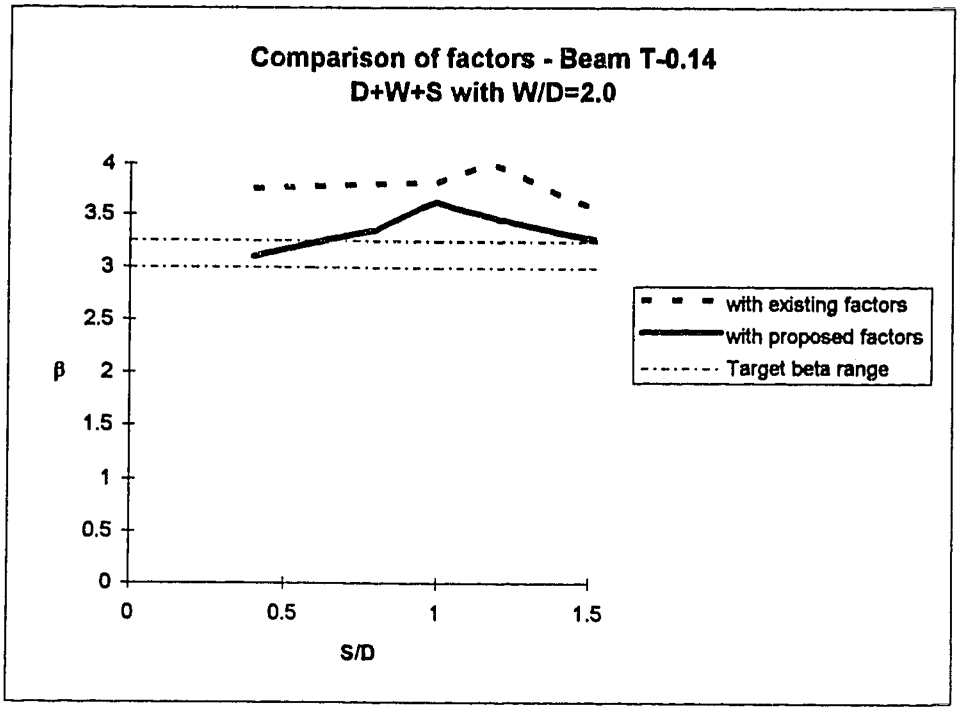


Figure 7.35 - Comparison of proposed load and resistance factors with those from CSA A23.3-94 (Beam T-0.14 for D+W+S)

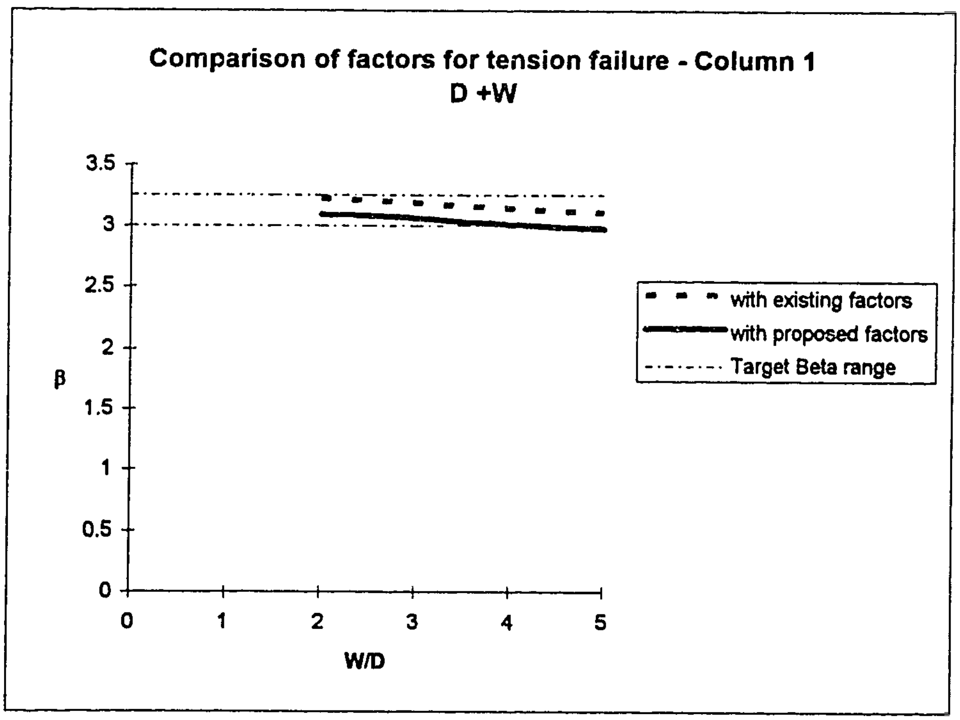


Figure 7.36 - Comparison of proposed load and resistance factors with those from CSA A23.3-94 (Beam T-0.14 for D+W)

CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 Summary

The objective of this thesis is to develop an optimum set of load and resistance factors to be used in the design of concrete structures in Canada. Load and resistance factors are utilized in the design of structures to provide a desired reliability.

A preliminary study of the types of reliability methods is performed. The Hasofer and Lind reliability index is chosen for use because it provides mechanical invariance and is a measure of the reliability at the 'design point' where failure is most likely to occur. The Hasofer and Lind reliability method is essentially a first order second moment method but by using the normal approximation of the critical tail regions of the variables the distribution type can also be incorporated in the analysis. This is done using the Rackwitz Fiessler algorithm. The FORM (First Order Reliability Methods) subroutine (Gollwitzer et al. 1990) that performs this algorithm is adapted to perform the main reliability analysis.

A load factor format and a resistance factor format that could provide uniform safety with varying load ratios and design situations is necessary. The different load and resistance idealizations and the methods of finding the maximum probable load combination are studied. The Borges and Castenheta processes are found to represent variable loads adequately. The companion action load factor method is selected because the load combination format for combinations of frequent loads because it is the best representation of the maximum load combinations. The specified exceedance probability design format is selected for load combinations including earthquake loads because this format better represents combinations of infrequent loads. Similarly, the material resistance factor format is selected as the most appropriate format for concrete structures in Canada.

The loads and resistances that are necessary to derive the load and resistance factors are defined and established in probabilistic terms. The basic variables that affect the load effect and resistance are obtained from environmental data or from the literature.

Data for snow and wind loads, ground snow depth, rain on snow and wind speed data are obtained from Environment Canada. Fifteen cities geographically distributed though out Canada are initially chosen as representative locations. The variables necessary to simulate the snow load and the wind speed are obtained for all these cities. The observation that similar values occurred in several cities allowed a reduction in the number of cities considered. Therefore seven cities are chosen so that the whole range of wind speeds and snow depths are represented. These seven cities are Vancouver, Kelowna, Yellowknife, Regina, Winnipeg, Toronto and St.John's.

Other basic variables that affect the snow and wind load, such as ground to roof snow load conversion factor, snow density, air density, and so on are obtained by analyzing data given in the literature. Where appropriate, these are also obtained for the representative cities.

Using the basic variables, the wind and snow loads are simulated using Monte Carlo simulation methods. This is done for all seven representative cities. A computer program is developed to generate the random variables, model variables, carry out the simulations and fit distributions. This program is written in SAS software programming language and an example is given in Appendix A.

The distributions of the earthquake load are developed using extreme value theory and data from literature. This is done for the representative cities that had significant earthquake loading.

Once the wind and snow loads were simulated and used for the reliability analysis it was realized that the different cities gave significantly different load factors. Because it is not possible to calibrate the load factors to suit this whole range, the locations where the bias factor of the load is within plus or minus one third of the overall average bias factor for the seven cities are used for the reliability studies. As explained in Sections 4.5.4 and 4.6.5 the snow load values used in the calibration are for Kelowna, Regina, Winnipeg, Toronto and St.John's and the wind load values are for Vancouver, Winnipeg, Toronto and St.John's.

The proposed load factors were obtained for locations with moderate load parameters. The implications of the climatic variability are discussed in Section 7.5.7 where it is recommended that the specified load values in NBCC for some locations need reconsideration. The existing load factors were derived for average load parameters, therefore, in effect those factors were also derived for locations with moderate parameters. Thus the proposed factors are no less applicable to the whole of Canada than the existing factors.

Basic variables for member resistances are obtained from the literature. Current levels of quality control and types of variability are reflected in these variables. Member resistances are simulated using Monte Carlo simulation methods. For the simulation of beam and column resistances the moment curvature analysis, variable modeling, simulation and distribution fitting are done by developing a SAS program. These simulation results are utilized to develop flexural strength distributions for beams and variable interaction diagrams for the columns. The shear resistance is simulated using Zsutty's (1971) regression equations.

It is necessary to establish acceptable measures of reliability upon which to base the load and resistance factors. The desirable target level of safety, i.e., a target reliability index, β_T, is chosen by evaluating reliability indices implicit in existing successful practice and by reviewing levels of safety used and recommended by other researchers. The reliability indices implicit in practice are obtained by evaluating the reliability of CSA A23.3-M84.

The reliability analysis is based on a limit state equation. A method of formulating a limit state equation is developed by combining the 'true' limit state, where the structure is about to fail, and the design limit state, that is used in the design office. The 'true' limit state is written in terms of variables which are the best estimates of the true load and resistance. This combined limit state equation is a versatile problem solving tool as it was able to model complex real life limit state situations.

The load and resistance factors are developed for ultimate limit state design considering flexure, combined flexure and axial load, and shear. A method of analyzing the reliability of column tension failure is developed. Tension failure of columns where the axial load effect due to gravity load acts to prevent failure has to be analyzed both in terms of moment and axial force so that the characteristics of the tension failure portion of the interaction diagram can be taken into account.

The load and resistance factors are optimized to give desirable safety indices in all situations, but practical limitations and simplicity of load combinations are also taken into account. Optimization is done using both an optimization function as well as graphical methods. A wide range of possible load combinations are considered. Once the load factors were developed, some load combinations were found to be trivial as they would not govern over the other cases. The load combinations presented in the conclusions are the critical load combinations that need to be considered. The load and resistance factors and load combinations are developed for use with design methods specified in CSA A23.3 - 94 and the loads specified in NBCC 1995.

8.2 Conclusions

8.2.1 Results of Evaluating NBCC 1990 and CSA A23.3-M84

The beam with a steel ratio of 0.71 ρ_b (approximately the maximum allowed by A23.3-M84) has a significantly lower reliability for low live to dead ratios than the beams with lower steel ratios. This suggests that the maximum steel ratio allowed in beams should be revised to a lower value than 0.71 ρ_b. It appears that even though such a beam is nominally under-reinforced, some members will be over-reinforced due to the variability in beam parameters giving a higher variability in member strength. A representative value for β for the dead plus live load case is around 3.75 for the beams with lower ratios of steel and around 3.5 for the beam with $\rho = 0.71\ \rho_b$.

For beams with dead plus snow loads the typical values of β, range from 2.6 for Kelowna, which has a highly variable snow load, to 3.25 for Toronto which has a low variability in its snow load. It is found that β for dead plus live plus wind loads is in the range of 2.5 to 3.5 for the practical range of W/D values. It is also observed that when the wind variability is high, the values of β decrease with higher values of W/D.

For column compression failures, the β values obtained are around 3.5 to 4.0. For tension failures of columns β varied significantly according to the load parameters.

The evaluation of β values implicit in NBCC 1990 and A23.3-M84 yielded a large range of values. Although the average β values are around 3.5, the whole range is between 2.0 and 5.0.

8.2.2 Selection of Target Beta Values

As the existing probability factor load combination format cannot yield uniform reliabilities as a function of load ratios, a conservative average β is necessary to make the reliability adequate for the critical load ratios. Therefore the average value $\beta = 3.5$ obtained from evaluating existing practice is considered to be a conservative value for β_T.

The literature review suggests that 3.0 to 3.5 is a desirable range of β_T values. Considering all the β values obtained in the calibration and the values used by other researchers the following β_T values are selected for use in developing the load and resistance factors in this study.

β_T = 3.00 to 3.25 for gradual failure

β_T = 3.25 to 3.50 for sudden failure

8.2.3 Selection of Load And Resistance Formats

Companion action load factors reflect actual load combinations more closely than the probability factor format. The companion action load factor format is used to develop load factors for frequent loads. The principal loads are taken at characteristic values and accompanying companion load definitions are given in Section 3.2.7.

For load combinations that include earthquake loads the specified exceedance probability design format is used. The earthquake load, which is always the principal load, is taken at a specified probability of exceedance of 0.0021 per annum. A factor of 1.0 is applied on the earthquake load. For the dead load, in earthquake load combinations, a factor of 1.0 is used because this is the likely value and because the earthquake load is dependent on the dead load or mass of the structure. With earthquake loads the other companion loads have been taken as the a.p.t. values and are represented as some fraction of the characteristic load. Load factors for NBCC were reevaluated based on these formats.

The resistance factors are developed using the material resistance factor format. The resistance factors are developed for use with design methods specified in CSA A23.3 - 94.

8.2.4 Recommended Resistance Factors

The optimum set of resistance factors for concrete buildings in Canada, developed in this study, are; $\phi_c = 0.65$ and $\phi_s = 0.85$ for non-prestressed concrete and reinforcement respectively. These resistance factors require a minor revision in the calculation of V_c. It is recommended that $V_c = 0.17\ \lambda\ \phi_c\ \sqrt{f_c'}\ b_w\ d$ be used for members with flexural reinforcement ratios greater than or equal to 1.0%. This value of V_c is applicable to members with stirrups or members without stirrups that are governed by clause 11.2.8.1 of A23.3-94.

8.2.5 Recommended Load Factors and Combinations

A wide range of possible load combinations are analyzed and the appropriate load factors are derived. Out of these combinations the following set of combinations are found to be the critical cases that would govern. Therefore the proposed load combinations and load factors are:

1.3 D

1.2 D + 1.5 L

1.2 D + 0.5 L (or 0.4 W) + 1.7 S

1.2 D + 0.5 L + 1.5 W

0.85 D + 1.5 W

1.0 D + 1.0 E

1.0 D + 0.2 S (or 0.5 L) + 1.0 E

Environmental load parameters are found to vary significantly throughout Canada and locally. Averaging load parameters for reliability studies is found to be inappropriate. A number of cities are identified where the NBCC specified loads appear to need adjustment. These anomalous cities are not used in the load and resistance factor calibration.

The load factors are found to depend heavily on the normalized load parameters. Load factors are developed for load parameters in cities with moderate normalized load parameters. For the cities with moderate load parameters the nominal load parameters specified in NBCC (1990) are appropriate for the load distributions of that city. For the other cities considered in this study the results indicate (as seen in Table 7.5 and 7.6) the specified snow load should be increased for Kelowna and decreased for Regina. The specified wind load should be decreased in Vancouver.

8.3 Future Research Areas

In this study shear in beams is considered. The shear in slabs and footings are complicated mechanisms which have not been considered in the reliability analysis. An appropriate regression model is required for shear in slabs and footings. This should be researched and established in probabilistic terms.

As already noted, the reliability at different locations varied significantly according to the normalized load parameters. The reliability of a member depended heavily on the mean and the standard deviation of the normalized load. By specifying nominal loads according to the load distributions and variabilities uniform reliability could be achieved. Further investigation is necessary to specify loads appropriately.

Loads due to prestressing, imposed deformation (such as temperature, creep, shrinkage and support settlement) and earth pressure need to be investigated in a reliability analysis.

Extension of this study to steel, timber and masonry structures is necessary, so that common load factors for all materials can be established.

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APPENDIX A

SAS PROGRAM TO SIMULATE AND MODEL WIND LOAD

The following program was developed to simulate and model the wind load for Vancouver. Similar programs were developed for snow load etc. The parameters of the basic variables used in this program is established in section 4.6. The background to the procedure for which the program is used is given in sections 4.5.3 and 4.6.3

```
options nocenter;
libname lib 'c:\saslib';

*****Simulating Cw,V,Ce,Cp,Cg and p;
data wlvan1;
 do I = 1 to 10000;
  *Generating Cw;
  Cw = 4.82e-5+5.10e-7*rannor(-1);
  *Generating V10,V30 and V100;
  u3 = ranuni(-1);
  V10 = 65.082 - (log(-log((u3)**(1/10))))/.151;
  V30 = 65.082 - (log(-log((u3)**(1/30))))/.151;
  V100 = 65.082 - (log(-log((u3)**(1/100))))/.151;
  *Generating Ce;
  Ce = 1.0+ .08*rannor(-1);
  *Generating Cp;
  Cp = 1.0+ .1*rannor(-1);
  *Generating Cg;
  Cg = 1.0+ .1*rannor(-1);
  *Calculate p10,p30 and p100;
  p10 = Cw*V10**2/.45*Ce*Cp*Cg*.85*.85;
  p30 = Cw*V30**2/.55*Ce*Cp*Cg*.85*.85;
  p100 = Cw*V100**2/.67*Ce*Cp*Cg*.85*.85;
  output;
 end;run;


*****Check if Cw has a normal distribution;
data checkC;
 set wlvan1(keep=Cw);
proc sort;
 by Cw;
data sorted1; set checkc;
  obs=_n_;
  prob=obs/10001;
  fx=4.82e-5+5.10e-7*probit(prob);
```

```
proc reg; model fx=c;
  title 'Check if Cw has a normal distribution';
run;

*****Check if V's have maxima of Gumbel distribution;
data checkV;
  set wlvan1(keep=V10);
proc sort;
  by V10;
data sorted2; set checkV;
   obs=_n_;
   prob=obs/10001;
   fx=65.082 - (log(-log((prob)**(1/10))))/.151;
proc reg; model fx=V10;
  title 'Check if V10 has a maxima of Gumbel distribution';
run;
data checkV;
  set wlvan1(keep=V30);
proc sort;
  by V30;
data sorted2; set checkV;
   obs=_n_;
   prob=obs/10001;
   fx=65.082 - (log(-log((prob)**(1/30))))/.151;
proc reg; model fx=V30;
  title 'Check if V30 has a maxima of Gumbel distribution';
run;
data checkV;
  set wlvan1(keep=V100);
proc sort;
  by V100;
data sorted2; set checkV;
   obs=_n_;
   prob=obs/10001;
   fx=65.082 - (log(-log((prob)**(1/100))))/.151;
proc reg; model fx=V100;
  title 'Check if V100 has a maxima of Gumbel distribution';
run;

*****Check if Ce has a normal distribution;
data checkd;
  set wlvan1(keep=Ce);
proc sort;
  by Ce;
data sorted3; set checkd;
```

```
    obs=_n_;
    prob=obs/10001;
    fx=1+.08*probit(prob);
 proc reg; model fx=Ce;
   title 'Check if Ce has a normal distribution';
 run;

 *****Check if Cp has a normal distribution;
 data checkd;
   set wlvan1(keep=Cp);
 proc sort;
   by Cp;
 data sorted3; set checkd;
    obs=_n_;
    prob=obs/10001;
    fx=1+.1*probit(prob);
 proc reg; model fx=Cp;
   title 'Check if Cp has a normal distribution';
 run;


 *****Check if Cg has a normal distribution;
 data checkd;
   set wlvan1(keep=Cg);
 proc sort;
   by Cg;
 data sorted3; set checkd;
    obs=_n_;
    prob=obs/10001;
    fx=1+.1*probit(prob);
 proc reg; model fx=Cg;
   title 'Check if Cg has a normal distribution';
 run;


 *****Find the best fit distribution to p (above the 95th percentile);
 data wlvan2;
   set wlvan1(keep=p10);
 proc sort;
   by p10;
 proc univariate;
 data wlvan95; set wlvan2;
    obs=_n_;
    fi=obs/10001;
    if fi>=.95;
```

```
run;

data fits;
  set wlvan95;
 r1=fi ;
 r2=log(fi) ;
 r3=exp(fi) ;
 r4=-log(-log(fi)) ;
 r5=probit(fi) ;
 fratio=1.0; dfi=5.0; df2=10.0;  r6=fnonct(fratio,dfi,df2,fi/2);
 r7=probit(fi);
 r7=exp(r7);
 ag=5.0; r8=gaminv(fi,ag);
 ab=2.0; bb=5.0; r9=betainv(fi,ab,bb);
 t=1.0; df=20.0; r10=tnonct(t,df,fi);
 proc reg ; model r1=p10; run;
 proc reg ; model r2=p10; run;
 proc reg ; model r3=p10; run;
 proc reg ; model r4=p10; run;
 proc reg ; model r5=p10; run;
 proc reg ; model r6=p10; run;
 proc reg ; model r7=p10; run;
 proc reg ; model r8=p10; run;
 proc reg ; model r9=p10; run;
 proc reg ; model r10=p10; run; quit;
```

APPENDIX B

SAS PROGRAM TO SIMULATE AND MODEL COLUMN STRENGTH

The following program was developed to simulate and model column strength. This program simulates the moment resistance of the Column 1 at the axial load level 0.1 P_x. Similar programs were developed for other axial load levels so that the variable interaction diagram could be developed. The parameters of the basic variables used in this program is established in Chapter 5. The background to the procedure for which the program is used is given in section 5.4.2.

```
options nocenter;
data mean;
  input fc1n fyn As1n Asn dn d1n bnn hn;
  cards;
41.68 442 1023 1023 379 72 451.5 451.5
;

data constP;
 set mean;
Px= 0.9*fc1n*(bnn*hn-Asn-As1n)+fyn*(Asn+As1n);
  prop=0.1;
  P = prop*Px;

data basic (keep = fc1 fy ftr Ec As1 As Es d d1 b h P phi Mr);
  set constP (keep=prop P);
*****Simulating basic variables;
 do sec = 1 to 5000;
   *Generating b and h;
   bn=450.0;
   db = 1.5 + 6.4*rannor(-1);
   b = bn + db;
   h = b;
   *Generating d;
   d = 379 + 7.7*rannor(-1);
   *Generating d1;
   d1 = 72 + 4.2*rannor(-1);
    *Generating fy;
   lnfy = 4.288 + 0.3345*rannor(-1);
   fy = exp(lnfy)+365;
```

```
*Generating fc1,ftr & Ec;
lnfc1 = 3.705 + 0.224*rannor(-1);
fc1 = 0.85*exp(lnfc1);
ftr=8.3*((fc1*145.033)**0.5)*0.96*(1+0.11*log10(fc1*145.033/60/60))*6895e-6;
Ec=60400*((fc1*145.033)**0.5)*0.96*(1.16+0.08*log10(60*60))*6895e-6;
*Generating Es;
Es = 201000 + 6598*rannor(-1);
*Generating As;
lnAfAc = -2.383 + 0.4*rannor(-1);
AfAc = exp(lnAfAc)+0.91;
Asn = 1012.5;
As = Asn*AfAc;
As1=As;



*****Find coordinates of M-phi diagram;

stop=100. ;
Mr=0.0e0;
format Mr e10.;
if prop=0.99 then do;
 star=0.5;
 incr=0.5;
 end;
else if prop=0.0 or prop=0.1 then do;
 star=20.;
 incr=2.;
 end;
else do;
 star =3.;
 incr =2.;
 end;
do i = star to stop by incr;
 phi = i*1.e-6;
 format phi e10.;
 ep1=0.001;
 do j = 1 to 70;
  ep4= ep1- phi*h;
  c=ep1*h/(ep1-ep4);
  if c>h then cact =h ;
  else cact=c;
******************Find Cc and Mcc;
  if cact>h/2 then do;
   step=cact/25;
```

```
      mark=25;
      end;
    else do;
      step=cact/15;
      mark=15;
      end;
    epex=ep1+step/2*phi;
    Cc=0.0;
    Mcc=0.0;
    n=0.8+fc1/17;
    Ec2=3320*sqrt(fc1)+6900;
    ep0=n*fc1/(Ec2*(n-1));
    do k=1 to mark;
     epx=epex-k*step*phi;
     x=epx/ep0;
     if x<=1.0 then kk=1.0;
     else kk=0.67+fc1/62;
     if kk<1.0 then kk=1.0;
     fcx=fc1*n*x/(n-1+x**(n*kk));
     Cc=Cc + fcx*step*b;
     Mcc=Mcc+ fcx*step*b*(h/2+step/2-k*step);
     end;
    epct=-ftr/Ec;
    dt=epct/phi;
    if cact-dt<=h then do;
    Cc=Cc + 0.5*ftr*dt*b;
    Mcc=Mcc + 2/3*dt*0.5*ftr*dt*b;
    end;
*********************;
   ep2=ep1-(ep1-ep4)*d1/h;
   if Es*ep2>-fy and Es*ep2<fy then fs2= Es*ep2;
    else if Es*ep2>fy then fs2=fy;
    else fs2=-fy;
   x2=ep2/ep0;
   if x2>0.0 then do;
    if x2<=1.0 then kk2=1.0;
    else kk2=0.67+fc1/62;
    if kk2<1.0 then kk2=1.0;
   fc2=fc1*n*x2/(n-1+x2**(n*kk2));
   Cs=(fs2-fc2)*As1;
   end;
   else do;
    Cs=(fs2)*As1;
    end;
   ep3=ep1-(ep1-ep4)*d/h;
```

```
    if Es*ep3>-fy and Es*ep3<fy then fs3= Es*ep3;
     else if Es*ep3>fy then fs3=fy;
     else fs3=-fy;
    x3=ep3/ep0;
    if x3>0.0 then do;
     if x3<=1.0 then kk3=1.0;
     else kk3=0.67+fc1/62;
     if kk3<1.0 then kk3=1.0;
    fc3=fc1*n*x3/(n-1+x3**(n*kk3));
    end;
    if c<d then T=fs3*As;
    else T=(fs3-fc3)*As;
    sigP=Cc+Cs+T;
    dif=P-sigP;
   * put "ep1 sigP " ep1 sigP;
    if abs(dif)<(P+6000)*1e-3 then go to out;
    if prop=0.99 or prop=0.9 then change=3.e-10;
    else if prop=0.0 or prop=0.1 then change=7.e-10;
    else change=4.e-10;
    ep1=ep1+dif*change;
    if ep1<0 then go to out2;
    end;
  if abs(dif)>(P+6000)*1e-3 then go to out3;
  out:;
  M=Mcc+Cs*(h/2-d1)+T*(h/2-d);
  if M>Mr then Mr=M;
  else i=stop;
 *  put "Mr"  Mr;
  * put "phi" phi;
  end;
 if M>=Mr then go to out4;
 phi=phi-incr*1.e-6;
 output;
 end;
 out2:IF EP1<0 THEN put "ERROR!!! NEGATIVE EP1";
 out3:If abs(dif)>(P+5000)*1e-3 then put "P didnot converge";
 out4:If M>=Mr then put "Mr is not a maximum";

 run;


*****Find the best fit distribution to Mr ;
data fitmo;
 set basic(keep=Mr);
```

```
  title 'Check fit of Mr';
 proc sort;
  by Mr;
 proc univariate;
 data fitmoreg; set fitmo;
   obs=_n_;
   fi=obs/5001;
   if fi<=0.10;
   run;

 data fitsmo;
  set fitmoreg;


r1=fi ;
r2=log(fi) ;
r3=exp(fi) ;
r4=-log(-log(fi)) ;
r5=probit(fi) ;
fratio=1.0; dfi=5.0; df2=10.0;  r6=fnonct(fratio,dfi,df2,fi/2);
r7=probit(fi);
lnmr=log(Mr);
ag=5.0; r8=gaminv(fi,ag);
ab=2.0; bb=5.0; r9=betainv(fi,ab,bb);
t=1.0; df=20.0; r10=tnonct(t,df,fi);
proc reg ; model r1=Mr; run;
proc reg ; model r2=Mr; run;
proc reg ; model r3=Mr; run;
proc reg ; model r4=Mr; run;
proc reg ; model r5=Mr; run;
proc reg ; model r6=Mr; run;
proc reg ; model lnmr=r7; run;
proc reg ; model r8=Mr; run;
proc reg ; model r9=Mr; run;
proc reg ; model r10=Mr; run;
run;quit;
```