University of Alberta

Optimization of Steam Utility Network Operation

by

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Abstract

Energy and utilities costs often represent one of the largest operating costs at manufacturing plants and they are areas where companies can reduce cost if optimal operating strategy is applied for efficient steam distribution and electricity generation. In addition to the financial incentive, environmental benefits can be achieved by reducing inefficient use of resources with optimal operating strategies.

While there has been an extensive research activity for the optimization of utilities network, the interaction of the constraints and the feasibility of the optimal solution when facing uncertainties were rarely questioned. In this work, the utilities network is formulated as a chance constraint problem and the solution of the optimization problem would ensure the feasibility of the solution when facing uncertainties or when the constraints interact with one another. The optimal operating strategies obtained from solving the chance constraint problem are compared with the operating strategies obtained using linear programming, which do not take process uncertainties into account, and the operating conditions currently in place at the manufacturing plant. Several scenarios are also developed to demonstrate how the optimal solution would shift when process and economic conditions vary. In summary, this thesis provides a new approach to solve the utilities optimization problem that explicitly includes the desired feasible probability of the system to handle process uncertainties within the system.

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List of Symbols

$x_{Boiler1}$	Steam flow rate into the 900 psi common header from the Boiler #1
$x_{Boiler2}$	Steam flow rate into the 900 psi common header from the Boiler #2
$x_{AcidPlant1}$	Steam flow rate into the 900 psi common header from the Acid Plant #1
$x_{AcidPlant2}$	Steam flow rate into the 900 psi common header from the Acid Plant #2
$x_{900headerin}$	Total steam flow rate into the 900 psi common header
x_{Tur1in}	Steam flow rate into the Turbine #1 from the 900 psi common header
x_{Gen1in}	Steam flow rate into the Generator #1 from the 900 psi common header
x_{Tur2in}	Steam flow rate into the Turbine #2 from the 900 psi common header
x_{Gen2in}	Steam flow rate into the Generator #2 from the 900 psi common header
$x_{PLS93in}$	Steam flow rate into the 900-psi-to-35-psi Pressure Letdown Station from the 900 psi common header
x_{TCin}	Steam flow rate into the Turbo-Compressor from the 900 psi common header

$x_{Umicore9}$ Steam flow rate to the Umicore plant from the 900 psi common header

x_{Metal9} Steam flow rate to the Metal plant from the 900 psi common header

$x_{Sulzer9}$ Steam flow rate to the Sulzer plant from the 900 psi common header

$x_{HPStill9}$ Steam flow rate to the High Pressure Still unit from the 900 psi common header

$x_{PLS94in}$ Steam flow rate into the 900-psi-to-450-psi Pressure Letdown Station from the 900 psi common header

$x_{PLS911in}$ Steam flow rate into the 900-psi-to-160-psi Pressure Letdown Station #1 from the 900 psi common header

$x_{PLS912in}$ Steam flow rate into the 900-psi-to-160-psi Pressure Letdown Station #2 from the 900 psi common header

x_{TTin} Steam flow rate into the Terry Turbine from the 900 psi common header

$x_{900headerout}$ Total steam flow rate out of the 900 psi common header

$x_{PLS94out}$ Steam flow rate into the 450 psi common header from the outlet of the 900-psi-to-450-psi Pressure Letdown Station

x_{FGWHT} Steam flow rate into the 450 psi common header from the Fuel Gas Waste Heat Boiler

$x_{450headerin}$ Total steam flow rate into the 450 psi common header

$x_{PLS43in}$ Steam flow rate into the 450-psi-to-35-psi Pressure Letdown Station from the 450 psi common header

x_{PTout} Combined steam flow rate out of the Process Turbines to the 160 psi common header

$x_{PLS41in}$ Steam flow rate into the 450-psi-to-160-psi Pressure Letdown Station from the 450 psi common header

$x_{450headerout}$ Total steam flow rate out of the 450 psi common header

$x_{PLS41out}$ Steam flow rate into the 160 psi common header from the outlet of the 450-psi-to-160-psi Pressure Letdown Station

$x_{PLS911out}$ Steam flow rate into the 160 psi common header from the outlet of the 900-psi-to-160-psi Pressure Letdown Station #1

$x_{PLS912out}$ Steam flow rate into the 160 psi common header from the outlet of the 900-psi-to-160-psi Pressure Letdown Station #2

$x_{PKGBoiler}$ Steam flow rate into the 160 psi common header from the Package Boiler

$x_{160headerin}$ Total steam flow rate into the 160 psi common header

$x_{Sulzer1}$ Steam flow rate to the Sulzer plant from the 160 psi common header

$x_{Agrium1}$ Steam flow rate to the Agrium plant from the 160 psi common header

$x_{PLS131in}$ Steam flow rate into the 160-psi-to-35-psi Pressure Letdown Station from the 160 psi common header

$x_{160headerout}$ Total steam flow rate out of the 160 psi common header

$x_{Tur1out}$ Steam flow rate into the 35 psi common header from the outlet of the Turbine #1

$x_{Tur2out}$ Steam flow rate into the 35 psi common header from the outlet of the Turbine #2

$x_{PLS93out}$ Steam flow rate into the 35 psi common header from the outlet of the 900-psi-to-35-psi Pressure Letdown Station

x_{TCout} Steam flow rate into the 35 psi common header from the outlet of the Turbo-Compressor

$x_{PLS43out}$ Steam flow rate into the 35 psi common header from the outlet of the 450-psi-to-35-psi Pressure Letdown Station

$x_{PLS13out}$	Steam flow rate into the 35 psi common header from the outlet of the 160-psi-to-35-psi Pressure Letdown Station
x_{PLSPKG}	Steam flow rate into the 35 psi common header from the outlet of the Packaged-Boiler-to-35-psi Pressure Letdown Station
$x_{35headerin}$	Total steam flow rate into the 35 psi common header
x_{oxy3}	Steam flow rate to the Oxy process from the 35 psi common header
$x_{LeachSW3}$	Steam flow rate to the south west Leach Plant from the 35 psi common header
$x_{LeachS3}$	Steam flow rate to the south Leach Plant from the 35 psi common header
$x_{LeachSE3}$	Steam flow rate to the south east Leach Plant from the 35 psi common header
x_{NH33}	Steam flow rate to the Ammonia Plant from the 35 psi common header
x_{GR35}	Steam flow rate to the Gas Reform Plant from the 35 psi common header
$x_{Sulzer3}$	Steam flow rate to the Sulzer Plant from the 35 psi common header
x_{vent3}	Steam flow rate to the vent from the 35 psi common header
$x_{35headerout}$	Total steam flow rate out of the 35 psi common header
$x_{900excess}$	Excess steam flow rate into the 900 psi common header
$x_{450excess}$	Excess steam flow rate into the 450 psi common header
$x_{160excess}$	Excess steam flow rate into the 160 psi common header
$x_{35excess}$	Excess steam flow rate into the 35 psi common header
$\frac{dE_{System}}{dt}$	Accumulation of energy in the system
$\dot{E_{in}}$	Internal energy at the inlet of the turbine generator

$\dot{E}_{out}$	Internal energy at the outlet of the turbine generator
g	Gravity constant
x_{900stm}	Flow rate of 900 psi steam into the turbine generator
x_{35stm}	Flow rate of 35 psi steam out of the turbine
x_{cond}	Flow rate of condensate out of the generator
v_{900stm}	Velocity of steam into the turbine
v_{35stm}	Velocity of steam out of the turbine
z_{900stm}	Relative height to reference point for 900 psi steam
z_{35tm}	Relative height to reference point for 35 psi steam
$\triangle E_{Actual}$	Electricity generated from the turbine generator
$\eta_{overall}$	Overall efficiency of the turbine generator
H_{900stm}	Enthalpy of the 900 psi at the turbine generator inlet condition
H_{350stm}	Enthalpy of the 35 psi at the turbine generator outlet condition
Y	Output Data for the Empirical Model
X	Input Data for the Empirical Model
ϕ	System Parameter for the Empirical Model
ε	Modelling Error for the Empirical Model
$\hat{y}$	Predicted Output for the Empirical Model
x	Measured Data for the Empirical Model in Chapter 2
$\beta_0 \ldots \beta 5$	System Parameter for the Empirical Model
R^2	Coefficient of Determination
x	Decision Variables
A	Utilities Network Information Matrix
b	Right Hand Side Matrix for Supply and Demand of Steam

c	Economic Information Matrix
$C_{Electricity}$	Unit Price for Electricity
C_{35}	Unit Price for 35 psi Steam
C_{900}	Unit Price for 900 psi Steam
E	Amount of Electricity
α_{Tur1}	Model Parameter for Turbine 1 in Turbine Generator Model 1
α_{Gen1}	Model Parameter for Generator 1 in Turbine Generator Model 1
α_{Tur2}	Model Parameter for Turbine 2 in Turbine Generator Model 2
α_{Gen2}	Model Parameter for Generator 2 in Turbine Generator Model 2
β_{TG1}	Constant Term for Turbine Generator Model 1
β_{TG2}	Constant Term for Turbine Generator Model 2
x_{900_users}	Users of 900 psi Steam
$x_{900\_suppliers}$	Suppliers of 900 psi Steam
x_{450_users}	Users of 450 psi Steam
$x_{450\_suppliers}$	Suppliers of 450 psi Steam
x_{160_users}	Users of 160 psi Steam
$x_{160\_suppliers}$	Suppliers of 160 psi Steam
x_{35_users}	Users of 35 psi Steam
$x_{35\_suppliers}$	Suppliers of 35 psi Steam
E_{TG1_max}	Maximum Electricity Production Allowed on Turbine Generator 1
E_{TG2_max}	Maximum Electricity Production Allowed on Turbine Generator 2

x_{Tur1_min}	Minimum Flow Rate Requirement for Turbine 1
x_{Tur2_min}	Minimum Flow Rate Requirement for Turbine 2
$\alpha_{PLSconvert}$	Conversion Parameter for Pressure Letdown Station Inlet Flow Rate to Outlet Flow Rate
$x_{PLS93in}$	900-to-35 psi Pressure Letdown Station Inlet
$x_{PLS93out}$	900-to-35 psi Pressure Letdown Station Outlet
$\alpha_{Tur1convert}$	Inlet-to-Outlet Conversion Parameter for Turbine 1
x_{Tur1in}	Turbine 1 Inlet
$x_{Tur1out}$	Turbine 1 Outlet
$\alpha_{Tur2convert}$	Inlet-to-Outlet Conversion Parameter for Turbine 2
x_{Tur2in}	Turbine 2 Inlet
$x_{Tur2out}$	Turbine 2 Outlet
P^*_{new}	New Optimal Objective Function Value
P^*_{old}	Old Optimal Objective Function Value
λ^*_a	Shadow Price Corresponding to Active Constraint
$x_{450\_headerin}$	Flow Rate into the 450 psi Common Header
$x_{450\_headerout}$	Flow Rate out of the 450 psi Common Header
$x_{160\_headerin}$	Flow Rate into the 160 psi Common Header
$x_{160\_headerout}$	Flow Rate out of the 160 psi Common Header
α	Probability for Joint Probability Constraints
α_i	Probability for i^{th} term in Individual Probability Constraint Problem
$\alpha_1^{'} \ldots \alpha_8^{'}$	Probability for Each of the Constraints in Individual Probability Constraint Problem
$A_1 \ldots A_n$	Various Probabilities for Constraints 1 to n

$\bar{A}_i$	Average of the Various Probabilities for Constraints 1 to n
$\alpha^{'}$	Probability for Individual Probability Constraint Problem
$\bar{b}$	Average of Matrix b
σ_b	Standard Deviation of Matrix b
ξ	Z-score of Right Hand Side Matrix b
z_{alpha}	Z-score Corresponding to Probability α

Chapter 1

Introduction

Optimization of utility systems is often of interest to operating companies because energy cost is usually the largest part of the operating cost for a process plant [Papalexandri et al., 1998]. With a deregulated electricity market, such as Alberta's, and an increasing cost of fuel, the economic incentive for operating companies to determine the optimal strategy for the process system is obvious. Additional benefits such as saving energy, reducing environmental pollutant and greenhouse gas emissions [Luo et al., 2012] can be achieved by maintaining the utilities system at the optimal operating condition.

To limit the impact of fluctuating electricity cost, industrial users usually lock in power price through fixed price contracts [Li and Flynn, 2006] with associated penalty cost for excess usage. If the price of electricity is allowed to vary based on market condition, [Li and Flynn, 2006] proposed a demand-side management strategy to optimize electricity usage during high and low electricity price periods. Such strategy would work well for small scale consumers such as household usages, however, it is difficult for large scale industrial users to follow because they usually operate 24 hours a day and usually need a long time span to adjust processes to react to different electricity prices. In addition, if the industrial user has the ability to generate electricity on-site, the optimal operating strategy can also include supply-side management to hedge against high electricity price periods.

The purpose of this project is to find an optimal operating strategy for a utility network to allocate steam among different equipment such that the cost of the process is minimized while satisfying the process steam requirement. To do so, existing methods to solve a utility system optimization problem will be explored. It will require modelling of the system equipment and formulation of the optimization problem before applying known algorithms to solve the optimization problem. Various case studies will be developed to determine a general operating strategy for the utilities system to handle variations in unit prices for utilities and different levels of energy demand.

1.1 Project Background and Overview

In a typical utility network at a metal processing plant, there can be multiple routes for steam to travel from high pressure common header to lower pressure common headers. For this project, there are four steam common headers at different pressure. Each of the common headers is connected to a number of equipment. Some of the equipment can be considered solely as either a supplier or user of steam while some equipment can be considered as both user of high pressure steam and the supplier of lower pressure steam. For example, a boiler can be considered as a supplier of steam by producing steam to the common header, whereas a turbine can take a high pressure steam and produce lower pressure steam by converting part of the energy to electricity. With multiple paths available to allocate steam from a higher pressure steam common header to lower pressure steam common header to satisfy all process requirements, the operating cost can be different depending on how the steam is allocated. Therefore, the focus of this project is to find the optimal operating strategy to allocate steam among different equipment based on both process and economic conditions.

This project is advantageous for the company because it can minimize the operating cost by reducing excess steam production and maximize revenue by generating electricity when appropriate while satisfying the steam requirement. In addition, a general optimal operating strategy could provide a guideline to run the operation so the process could be optimal continuously.

1.2 Literature Review/Existing Methods

There have been significant research efforts and progress in the optimization of utilities system in the last three decades. Various optimization methods for electric utility resource planning were reviewed by [Hobbs, 1995]. Most of the studies have formulated the optimization problem using mixed integer programming framework where some of the variables are restricted to be integers, and these models can be developed using linear or nonlinear models. Papoulias and Grossmann [1983] proposed a systematic optimization approach to formulate the optimization problem using mixed-integer linear programming (MILP) and [Kalitventzeff, 1991] provided a solution method for the MILP problem to solve the utility network control strategy problem.

While the mixed integer programming offered a great solution for systems requiring discrete decisions such as an on/off switch, these models only consider operations at current conditions. To solve the optimization problem for a longer period, Hui and Natori [1996] suggested formulating the optimization problem using multi-period mixed integer programming. The multiple periods can be developed based on periodical variation such as daily or seasonal variations. Unfortunately, this study considered steam and electricity profiles as constants and uncertainties associated with future demands were not considered.

Further improvements to address uncertainties in mixed integer programming were discussed by [Papalexandri et al., 1996] and [Papalexandri et al., 1998] where multi-period models would include the range of past variations during normal operation to address uncertainties in the energy demand. Recently, [Velasco-Garcia et al., 2011] took into account for shut-downs and start-ups of utility operating units and optimized the system with successive mixed integer linear programming (SMILP) that resulted in significant cost savings. These methods can be challenging to implement if the range or the duration of the process variations are too great to distinguish in discrete time periods.

Besides the mixed integer programming method, there are other methods available to determine the optimal control or operating strategies for utilities systems. For example, Yi et al. [1998] proposed a heuristic rule-based expert system to minimize the net cost of providing energy to the plant. The expert system was developed based on steady-state modelling and simulation of steam generation and distribution among a number of common headers. Newton's iteration method and linear programming algorithms were used in the simulation to obtain the optimal result. The study done by [Kim and Han, 2001] included switching cost of operation determined using dynamic programming to improve short-term heuristic optimal planning model. Similar heuristic model was presented by [Halasz et al., 2002] using decision-mapping method. These proposed systems offered a good starting point for problem formulation and provided a suggestion to deal with disturbances and uncertainties in the system. Nevertheless, since the model developed did not incorporate disturbances and uncertainties, and any changes to process condition was dealt with heuristic approach based on a series of “If-Then" rules to control the steam allocation, it is likely for the result obtained from a heuristic approach to be sub-optimal compared to the result obtained from a statistical method. In addition, the strategy provided by the expert system failed to address any variation in the economic information, which is a major factor for determining an optimal control strategy for the steam system.

In addition to the investigation of the optimal control strategy for the utilities system, other applications have also used similar framework to minimize the operational cost. Maréchal and Kalitventzeff [1998] used a combination of mixed integer optimization and an expert system to define optimal configurations of the utility system to satisfy the minimum energy requirement at minimum cost. Kim et al. [2002] investigated the preventative optimization framework that considered emergency situations in the optimization models by using quantitative constraints for the utility system to handle unexpected equipment failures. Yi and Han [2001] and [Yi et al., 2003] integrated re-planning and rule-based optimal operation to handle prediction errors for energy demands during multi-period operational planning. The modelling of a nonlinear planning and scheduling problem for refinery operation using large-scale mixed integer programming was discussed by [Pinto et al., 2000]. Pinto had shown how objective function and constraints in optimization models could be formulated for refinery production. Both discrete and continuous time representations approaches were tested by Pinto for optimization results using mixed integer framework and the work has focused on the development of nonlinear models. Zhang et al. [2001] and Micheletto et al. [2008] discussed the overall refinery optimization through the integration of different process units in a mixed integer optimization model. These papers concluded that a better optimization result can be achieved by exploiting network interactions among different process units rather than optimize each unit separately.

Recent development in the utilities system optimization includes the investigation into how to bring the system online and how to include additional cost information such as environmental impact into the scope. Han et al. [2006] developed an online optimization system and applied it to the condensing steam turbine network. Luo et al. [2012] proposed a multi-period mixed integer linear programming problem that takes into account for charges related to environmental costs.

1.3 Issues and Problems

Most of the existing techniques for optimization studies focus on mixed integer problem framework, and uncertainties within the process are generally handled with the formulation of multi-period models. These methods tend not to consider the possible interactions between constraints specified for the process. The constraints are usually formulated for a single purpose such as maximum electricity generation or minimum flow rates; however, it is possible that variations in one of the constraints could have an impact on other constraints in the system. Therefore, the effects on optimal solution due to constraint interactions are generally ignored in existing methods.

Existing methods also generally ignore the possibility of the optimal solution going into the infeasible region due to uncertainties in the optimization model. These uncertainties can cause the optimal solution to shift and if the optimal solution moves outside of the feasible region, the solution will be sub-optimal. Therefore, a way to explicitly state how frequent the optimal solution should remain in the feasible region into the optimization model and maintain the operation at optimum is required.

Some papers have also proposed using heuristic approaches when dealing with process variations. This may have provided acceptable results for operations, but a more rigorous operating strategy should be developed such that a decision support can be provided to operators to run the process at optimum.

Since the study at hand will not deal with turning on or off the equipment during the operation, mixed integer program is not necessary. The problem can be easily formulated using linear or nonlinear programming.

1.4 Thesis Objective and Scope

The objective of this thesis is to develop a robust optimization scheme so that the optimal operating strategy is determined based on process and economic conditions while ensuring that the process remains in the feasible region when facing naturally occurring uncertainties within the process system. The approach to this optimization problem is to solve it as a scheduling and planning problem that incorporates both process models and economic objective function. Process models are to be developed for the steam network and its associated equipment, and the economic objective function is to be formulated based on maximizing the profit. The uncertainties in the process will be incorporated into the optimization model using probability constraints. After the optimization problem is formulated, case studies will be conducted to simulate the effect of varying supply and demand of steam, and changing utility prices.

The scope of the project includes any sources and sinks connected to high pressure steam (900 psi steam) and low pressure steam (35 psi steam) common headers. This means any equipment that produces or takes steam from the common headers would be included in the scope of this thesis. The decision variables for the optimization problem will be determined based on degree of freedom analysis and the remaining variables are treated as constants that can not be changed throughout this thesis. All of the data used for this work is obtained from our industrial partner and the details of this data can be found in Appendix A.

1.5 Thesis Structure

An attempt is made in this thesis to develop models that represent the steam system and integrate various optimization techniques to solve for optimal operating strategies and maintain its optimal result when facing uncertainties.

In Chapter 2, the modelling of the steam network and relevant equipment are discussed. The steam network model is developed using first principles model while the turbine generators models are obtained using both first principles model and data driven methods.

In Chapter 3, the formulation of the optimization model using Linear Programming is explored. The optimal operating strategy for the utilities plant is determined by solving for the optimum point from the optimization model. Finally, case studies are conducted to examine possible scenarios that could be faced by the operating company.

In Chapter 4, a formulation of the optimization model with probability constraints to handle uncertainties in the process is suggested. A solution technique is developed to transform the Joint Probability Constraint problem to a Linear Programming equivalent problem to take advantage of the solver program available. Case studies are also conducted for comparison between deterministic and stochastic optimization models as well as the current operation.

Chapter 2

Model Development for Utilities Plant

In this chapter, process models are developed to represent the steam network and its associated equipment in a typical utilities plant including turbine generators and pressure letdown stations. After these models are developed, they will be used in subsequent chapters for optimization analyses. The chapter begins with a description of the processes in the utilities plant and strategies to develop models required to represent the system. Following that, the steam network model is developed based on steady state mass balance and the nominal values are obtained using the average of the data collected from the plant. The turbine generator models are developed based on both white box modelling through the use of energy balance and black box modelling through the use of data analysis. By comparing the models developed using black box modelling and white box modelling in terms of accuracy of the fit and simplicity of the structure, the resulting grey box model is considered to be the best model to closely represent the actual system because it combines the process knowledge and the structure obtained from the white box modelling. Simulation of the steam network and validation with available data are conducted along with model development to ensure the accuracy of the model. After the model development, possible sources of disturbances and uncertainties, as well as their effects on the models are discussed. Finally, the objective of the operation is explored in order to assist the development of the optimization model in subsequent chapters.

2.1 Process Description

A schematic of a typical utilities plant with four common steam headers and a number of equipment can be found in Figure 2.1.

For this project, the sources, sinks and process equipment of the steam system can be classified based on the direction of the steam flow with respect to the common headers. If the equipment only provides steam to the common headers, then the equipment is defined as the source of steam. For example, the major sources of steam in the system are Boilers #1 and #2. If the equipment only takes steam from the common header, then the equipment is defined as the sink of steam. The major sinks of the system are denoted as “User" in Figure 2.1. If the equipment takes high pressure steam for process usage and provides lower pressure steam to another header, it is considered process equipment. The main process equipment in the utilities plant includes two turbine generators, air compressor turbines, and pressure letdown stations.

Boilers #1 and #2 supply high pressure steam at 900 psi, which is fed into the 900 psi common header near the left end of Figure 2.1. Equipment such as air compressor turbines consume high pressure steam from the header, and the remaining high pressure steam is sent to other plants to satisfy their steam demand. An additional package boiler is available and activated to supply mid pressure steam at 160 psi to cover the shortage of steam when one of the two boilers is shut down for service or when there is an excess demand for 160 psi and/or 35 psi steam.

If the high pressure steam passes through turbine generators, electricity can be produced as a byproduct from the conversion of high pressure steam (900 psi) to low pressure steam (35 psi) and condensate; however, a minimum flow rate requirement of at least 7000 lb/hr of high pressure steam for the generator has to be satisfied in order to keep the generator in continuous operation.

There are two ways to manipulate turbine generators to control the generation of electricity. One of the ways is to adjust the inlet flow rate to the turbine generator from the high pressure steam header while keeping other variables constant. By doing so, the ratio of the condensate to low pressure steam flow rate, which is used to determine the split between the condensate and low pressure steam flow rate from the high pressure flow rate in the turbine generator, remains the same but the adjustment in inlet flow rate for 900 psi steam changes the amount of electricity

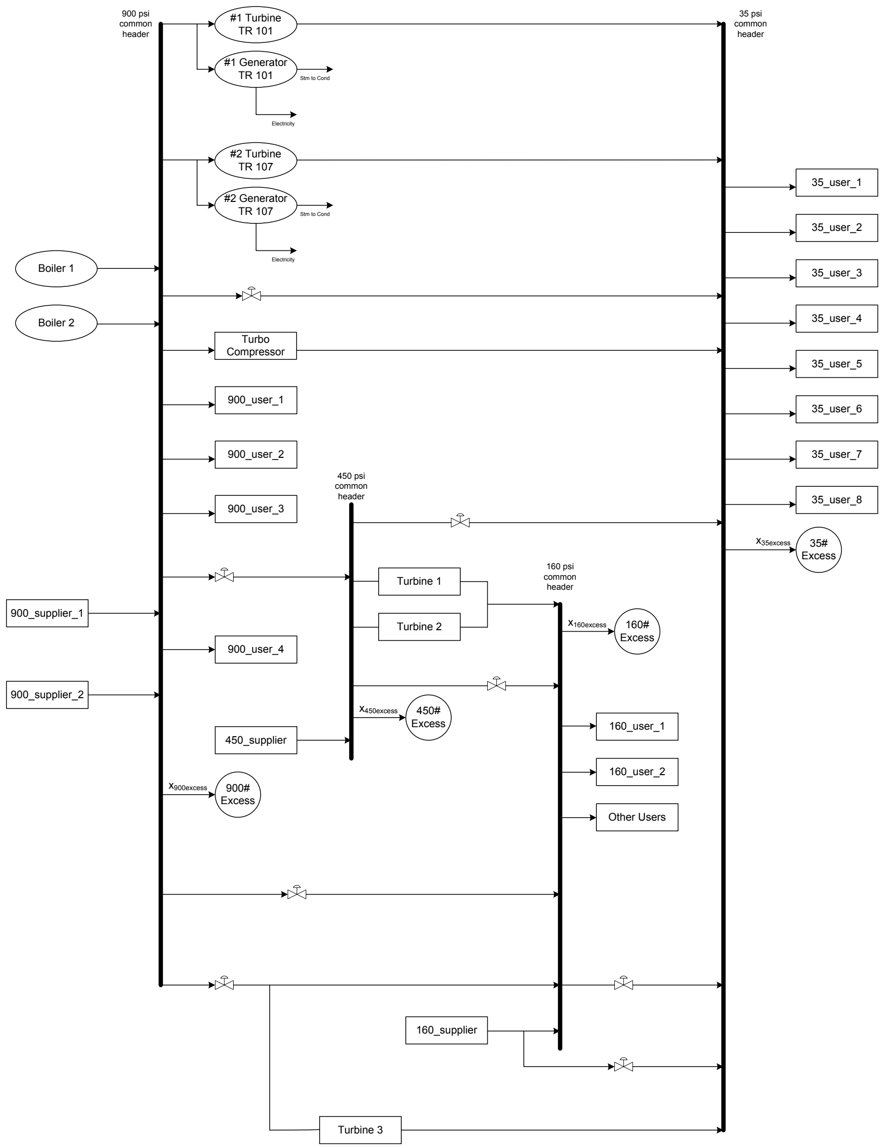


Figure 2.1: Process Flow Diagram of the Utilities Plant

produced. As the inlet flow rate of the turbine generator increases, the generation of electricity also increases. The other method to control the generation of electricity is to change the extraction flow rate of 35 psi steam from the turbine generator while keeping the inlet flow rate of the high pressure steam constant. In this case, the ratio of the condensate to low pressure steam flow rate is adjusted, and the generation of electricity increases as the ratio favours the split to the condensate flow rate.

Since there are two ways to operate the turbine generators, 900 psi steam flow rate and 35 psi steam flow rate are used as decision variables to describe the equipment. An illustration of the Turbine Generator can be found in Figure 2.2

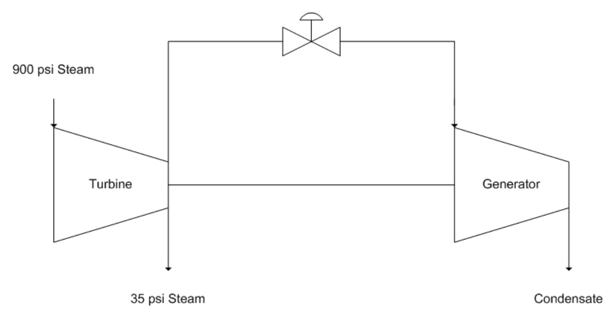


Figure 2.2: Process Flow Diagram of a Turbine Generator

In addition to the turbine generators, the high pressure steam can also pass through pressure letdown stations (PLS) to produce low pressure steam when there is excess high pressure steam or if there is an additional demand for low pressure steam. When high pressure steam passes through a pressure letdown station, the outlet mass flow rate stays the same as the inlet mass flow rate of the pressure letdown station, and high pressure steam is expanded to low pressure steam with heat lost to the atmosphere.

With many potential manipulated variables available for adjustment to meet the various levels of steam demand such as steam flow rates from boilers and steam flow rates to turbine generators and pressure letdown stations, it is important to compare all of the available operating strategies for steam distribution for the optimal result that would minimize the cost of steam generation, while meeting the site's steam requirement. Nevertheless, models are required to describe the system in detail before formulating the optimization problem. In particular, the steam flow model based on mass flow rates is required to determine whether the steam demand from the users is satisfied while turbine generator models are needed for the prediction of electricity generation.

2.2 Steam Network Model

As is the case with most process engineering analysis, the starting point of the analysis typically includes either the mass or energy balances for the system. In order to model the utilities plant thoroughly, a set of steady state mass balance equations are used to simulate the steam flow within the utilities plant based on the direction of steam flow into or out of common headers in Figure 2.1. During the development of the steam network model, it is assumed that there is no error in the measurement readings unless the readings are negative values, in which case the values are adjusted to zero. Loss of steam due to leaks along the pipeline or inside the equipment will be neglected. In addition, the steam network model assumes that all of the steam suppliers and users in the system have been taken into account during the model development.

For the 900 psi common header, the supplier of steam are the boiler #1 and #2 as well as the steam coming in from the acid plant. The users of the high pressure steam include two turbine generators with outlet connecting to the 35 psi common header, four pressure letdown stations to letdown high pressure steam to all three lower common headers, two air compressor turbines that produce 35 psi steam, and demand from various plants. The mass balance equations around the 900 psi common header can be written as the following:

Into the 900 psi Steam Header:

$$x_{Boiler1} + x_{Boiler2} + x_{AcidPlant1} + x_{AcidPlant2} = x_{900headerin} \tag{2.1}$$

Out of the 900 psi Steam Header:

$$\begin{aligned} x_{Tur1in} + x_{Gen1in} + x_{Tur2in} + x_{Gen2in} + x_{PLS93in} + \\ x_{TCin} + x_{Umicore9} + x_{Metal9} + x_{Sulzer9} + x_{HPStill9} + \\ x_{PLS94in} + x_{PLS911in} + x_{PLS912in} + x_{TTin} = x_{900headerout} \end{aligned} \tag{2.2}$$

Similarly, for the flow rates into and out of the 450 psi common header, the main supplier of steam is through the Secondary Reformer, also known as the Fuel Gas Waste Heat Boiler, while the letdown from 900 psi common header can also act as an additional supplier of 450 psi steam to the header. The main users of 450 psi steam are the turbines in Gas Reform plant with the outlet connecting to the 160 psi header. If there is excess 450 psi steam in the 450 psi header, it can be letdown to 160 psi and 35 psi common headers through pressure letdown station if required. With the information provided on the 450 psi common header, the mass balance equations around the 450 psi common header can be written as the following:

Into the 450 psi Steam Header:

$$x_{PLS94out} + x_{FGWHT} = x_{450headerin} \tag{2.3}$$

Out of the 450 psi Steam Header:

$$x_{PLS43in} + x_{PTout} + x_{PLS41in} = x_{450headerout} \tag{2.4}$$

The main consistent supplier of 160 psi steam to the common header is the outlet of the Gas Reformer turbines. There are also two pressure letdown stations available for letdown from 900 psi header and one pressure letdown station for letdown from 450 psi common header. The package boiler is activated when the letdown from high pressure steam (900 psi) does not provide enough steam to satisfy the demands for 160 psi and 35 psi steam. There are several plants that take the 160 psi steam and the excess steam is again available for letdown to 35 psi common header. Therefore, the mass balance around 160 psi common header can be written as the following:

Into the 160 psi Steam Header:

$$x_{PTout} + x_{PLS41out} + x_{PLS911out} + x_{PLS912out} + x_{PKGBoiler} = x_{160headerin} \tag{2.5}$$

Out of the 160 psi Steam Header:

$$x_{Sulzer1} + x_{Agrium1} + x_{PLS131in} = x_{160headerout} \tag{2.6}$$

At the lowest level of the common headers, any outlet that is connected to the 35 psi header from higher pressure steam can be considered as supplier of 35 psi steam and they include two turbines, two compressors and four available pressure letdown stations. The 35 psi steam is sent to various plants for process use and the excess is vented. The mass balance around 35 psi common header can be written as the following:

Into the 35 psi Steam Header:

$$x_{Tur1out} + x_{Tur2out} + x_{PLS93out} + x_{TCout} + x_{PLS43out} + \\ x_{PLS13out} + x_{PLSPKG} = x_{35headerin} \tag{2.7}$$

Out of the 35 psi Steam Header:

$$x_{oxy3} + x_{LeachSW3} + x_{LeachS3} + x_{LeachSE3} + x_{NH33} + x_{GR35out} + x_{Sulzer3} + x_{vent3} + x_{35excess} = x_{35headerout} \quad (2.8)$$

Using Equations (2.1) to (2.8), the steam flow around the four common headers are represented with mass balance equations that address steam heading into and coming out each of the common headers.

Steam Model Validation

With the steam flow model developed for the utilities plant based on steady state mass flow equations, it is imperative to determine whether the models can represent the system accurately enough to conduct further analysis. One of the ways to determine the validity of the model is to check whether there is more steam supply than steam demand in each of the common headers after accounting for all of the suppliers of steam. This is the case because all of the suppliers of steam are known, but there could be leaks in the pipeline or users/sinks there were not accounted for in Equations (2.1) to (2.8). As a result, there could be more steam heading into the common header than coming out of the common header. In order to reflect this fact in the steam model in this thesis and not have steam accumulating in the common headers, "excess steam" is used to denote the difference between steam heading into the common header and steam coming out of the common header and they are defined as the following:

Excess 900 psi Steam:

$$x_{excess900} = x_{900headerin} - x_{900headerout} \quad (2.9)$$

Excess 450 psi Steam:

$$x_{excess450} = x_{450headerin} - x_{450headerout} \quad (2.10)$$

Excess 160 psi Steam:

$$x_{excess160} = x_{160headerin} - x_{160headerout} \quad (2.11)$$

Excess 35 psi Steam:

$$x_{excess35} = x_{35headerin} - x_{35headerout} \quad (2.12)$$

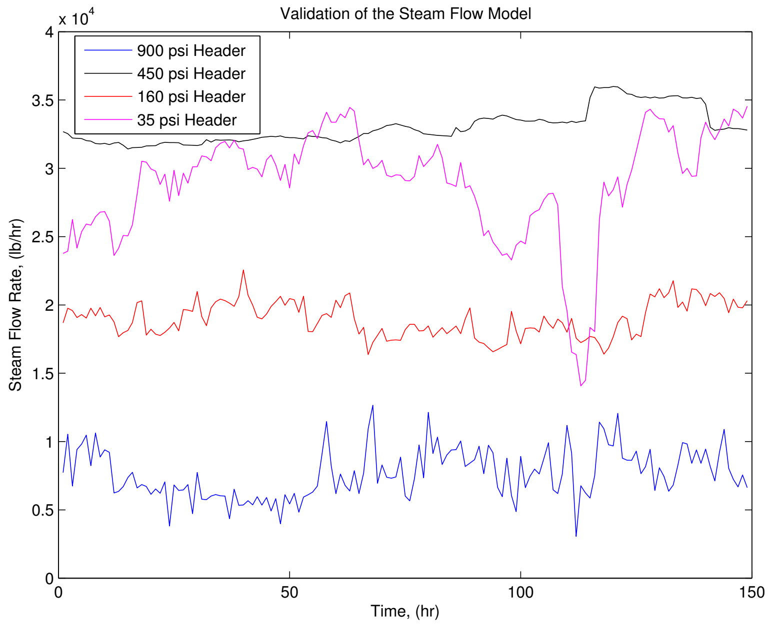


Figure 2.3: Validation of the Steam Flow Model

The data used to determine the excess steam is shown in Appendix A and Figure 2.3 shows the result of the excess steam in each of the common headers.

From the figure, it can be seen that all of the common headers contain positive excess steam. Therefore, the process requirement by the current steam distribution system to meet the user demand is satisfied. The excess steam accounts for about 3% of the 900 psi steam produced, 46% of the steam into 450 psi common header, 26% of the steam into 160 psi common header and 23% of the steam into 35 psi common header. The presence of positive excess steam in the common headers means that there may be leaks along the line or that there could be miscellaneous sinks that were not accounted for.

2.3 Turbine Generator Modelling

Similar to many modelling exercises, one can choose to model using first principles or data based techniques or a combination of both when modelling around a piece of process equipment. The advantage of modelling the process using first principles is that one can have insight into the physical meaning of model parameters; however, it could be difficult to develop the model using first principles and incorporate all of the necessary information into the first principles model as the process gets more complicated. On the other hand, part of the advantage of an empirical modelling is that it does not require an intensive understanding of the physical process and it could make it easier to develop a model. Nevertheless, the model parameters may not always have physical interpretation. Both techniques will be explored and discussed in this section for their pros and cons before a proper format of the model is chosen.

2.3.1 First Principles Model

Modelling using first principles, more specifically, thermodynamic principles will be discussed first. Since the turbine generators involve the conversion of internal energy to electricity energy via mechanical means, the modelling of turbine generator comprises of a set of energy balance equations around the unit.

Using the first law of thermodynamic [Çengel and Boles, 2006], the energy balance around the turbine generator can be written as:

$$\dot{E_{in}} - \dot{E_{out}} = \frac{dE_{System}}{dt} \tag{2.13}$$

This is a steady flow process and there is no accumulation of mass or energy. Therefore, the term $\frac{dE_{System}}{dt}$ is equal to 0 and Equation (2.13) can be expanded to include internal energy, kinetic energy, potential energy and electricity to be generated with the following equation.

$$H_{900stm}x_{900stm} + \frac{1}{2}x_{900stm}v_{900stm}^2 + x_{900stm}gz_{900stm} - \left(H_{35stm}x_{35stm} + \frac{1}{2}x_{35stm}v_{35stm}^2 + x_{35stm}gz_{35stm} + E_{Available}\right) = 0 \tag{2.14}$$

Since there is almost no difference in velocity and height between the inlet and outlet of turbine, the change in potential and kinetic energy can be assumed to be zero and the equation can be simplified to the following equation.

$$E_{Available} = H_{900stm}x_{900stm} - H_{35stm}x_{35stm} \tag{2.15}$$

In reality, the conversion of internal energy to electricity can never achieve 100% conversion due to mechanical limitations such as friction or heat loss [Çengel and Boles, 2006]. Therefore, to incorporate inefficiencies such as friction or heat loss as a physical realization of the energy conversion in the modelling of turbine generators, an efficiency parameter is included in the Equation (2.15) as the following:

$$E_{Actual} = \eta_{overall} \left(H_{900stm} x_{900stm} - H_{35stm} x_{35stm} \right) \tag{2.16}$$

Equation (2.16) is nonlinear because H_{900stm} and H_{35stm} are functions of pressure and temperature [Felder and Rousseau, 2005]. Nevertheless, given pressure and temperature measurements at the inlet and outlet of the turbine generators, the specific enthalpy of the steam at those locations can be obtained from the International Association for the Properties of Water and Steam (IAPWS) steam table. Therefore, the inherently nonlinear equation is linearized with the assumption that the specific enthalpy terms can be obtained to linearize the nonlinear equation. At this point, the only thing remaining before obtaining the predicted electricity generation is the efficiency parameter. After determining the efficiency parameter using process data, the generation of electricity can be predicted from Equation (2.16) given process information such as temperature, pressure and steam flow rates.

Efficiency Parameter

The efficiency of the turbine generator can be obtained using two different methods. One of the methods is to compare the actual electricity generated with the total energy available for electricity generation with 100% conversion rate. The other option to obtain the efficiency parameter is through the use of empirical modelling. Both techniques will be explored and the merits of each technique will be discussed and a decision will be given for the one to use for the model. Nevertheless, only the first option will be discussed in this section while the second option will be discussed in the following section.

Since the electricity generated by turbine generators can be measured, the efficiency parameter can be obtained by comparing the total available energy and the actual measured output of the electricity shown in Equation (2.17) [Çengel and Boles, 2006], which provides an option to obtain the efficiency parameter for the said option.

$$\eta_{overall} = \frac{E_{Actual}}{E_{Available}} \tag{2.17}$$

As mentioned before, E_{Actual} in Equation (2.17) can be obtained from measurement while $E_{Available}$ can be obtained from Equation (2.15).

Using historical data obtained from the plant for electricity generated, the overall efficiency parameter for the Turbine Generator, $\eta_{overall}$, is around 25%. The overall efficiency parameter accounts for the efficiency from the turbine as well as the generator. The detailed calculation of the value of the efficiency parameter can be found in Appendix B.

With an overall efficiency of 25% and specific enthalpy associated to inlet and outlet of the Turbine Generator #1, the first principles model for electricity generation can be obtained using Equation (2.16) and the resulting model is:

$$E_{real} = 0.002605x_{35stm} + 0.0929x_{cond} \tag{2.18}$$

Using the same procedure, the overall efficiency for Turbine Generator 2 is calculated to be around 22% and the electricity generation model is obtained as:

$$E_{real} = 0.002297x_{35stm} + 0.0819x_{cond} \tag{2.19}$$

2.3.2 Empirical Modelling

The other way to model the steady state turbine generator is through the use of an empirical model. Given the input and output data, the parameters for the turbine generator can be obtained using the least square regression analysis. To simplify the process, the noise is assumed to be independent and identically distributed Gaussian noise. The output for the system is the electricity generated and the input can be the flow rate of 900 psi, 35 psi steam or condensate or any combination of the three for the black box model. Assuming there is no previous knowledge of the model structure, a series of tests is required to determine the necessary input(s) for the turbine generator model.

Model Testing

To obtain the model structure for the turbine generator model, various linear regression models will be attempted to determine whether they have good fit with the data. It will start with simple linear regression models and then gradually increase in the complexity of the model structure to multiple linear regression. If the data fitting is not satisfactory, a more complicated model will be proposed and tested until a satisfactory result is obtained. After a satisfactory fitting on the model is obtained, the residual analysis will be conducted to determine if there are outliers in the data set. If this is the case, the outliers will be removed to determine if the model can be improved.

Based on the model determined from first principles, the empirical model is expected to have a model structure similar to

$$\hat{y} = \beta_1 x_1 + \beta_2 x_2 \tag{2.20}$$

Nevertheless, other model structures were investigated by adding or subtracting either parameter(s) or input variable(s) to determine if a better structure can be obtained. By removing one of the regressors, the model structure would resemble that of a simple linear regression model. The simplest model structure investigated has the following format:

$$\hat{y} = \beta_1 x_1 \tag{2.21}$$

The output variable for the model is electricity and the input variable can be any of the flow rates for condensate, 900 psi steam, or 35 psi steam. The best input variable was determined to be the condensate flow rate because it has the best coefficient of determination at 59.4% and has the following form:

$$\hat{y} = 0.0940 x_{cond} \tag{2.22}$$

The t-test is used to test the contribution of a single variable to the model. For this model, T_0 and $T_{0.025,17279}$ are calculated to be 998.3939 and 1.9601 respectively. Since T_0 is greater than $T_{0.025,17279}$, the null hypothesis: $H_0 : \beta = 0$ is rejected and it can be concluded that the condensate flow rate contributes significantly to the model.

Nevertheless, the fitting is considered poor by common convention, hence, simple linear regression models with additional parameters and complexity were investigated. The following three model structures were investigated to determine if a better model structure could be achieved with additional parameters.

$$\hat{y} = \beta_0 + \beta_1 x \tag{2.23}$$

$$\hat{y} = \beta_1 x + \beta_2 x^2 \tag{2.24}$$

$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 \tag{2.25}$$

A total of nine simulations were conducted for the above three model structures with each of the possible input variables. None of the above model structures investigated has a better coefficient of determination than Equation (2.22) and β_2 is effectively zero for the models investigated.

Since the simple linear regression models did not provide an adequate model for the Turbine-Generators, multiple regression models are investigated. The simplest multiple regression model structure would have a structure similar to Equation (2.20) but the structure with an intercept similar to the following equation is investigated.

Model 1:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \tag{2.26}$$

Three other model structures with additional parameters were also investigated to determine if a better model could be obtained with increasing complexity in model structure.

Model 2:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 \tag{2.27}$$

Model 3:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 \tag{2.28}$$

Model 4:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 \tag{2.29}$$

Similar to the simple linear regression models investigated, the input variables can be a combination of condensate flow rate and 900 psi steam flow rate or 35 psi steam flow rate. Model structures without the intercept, β_0, were also investigated to determine if the intercept should be part of the model parameters. In total, 24 models were investigated and the models were judged on their accuracy based on the fitting of the data as well as the simplicity of the model in terms of the order of the model.

Multiple Regression Result and Discussion

The multiple regression method described in [Montgomery and Runger, 2007] is applied to all 24 models similar to the procedures used for simple linear regression models. The coefficients of determination of the simulated results are presented in Table 2.1. The minus sign, "-", in the table signifies the models without intercept.

R^2	Condensate and 35 psi Steam	Condensate and 900 psi Steam	35 psi and 900 psi steam
Model 1	93.31%	93.31%	93.31%
Model 2	93.38%	93.53%	93.76%
Model 3	93.85%	93.63%	93.66%
Model 4	93.87%	93.87%	93.87%
Model 1-	82.83%	82.83%	82.83%
Model 2-	89.57%	93.36%	88.11%
Model 3-	92.57%	92.03%	92.86%
Model 4-	93.52%	93.52%	93.52%

Table 2.1: Coefficient of Determination Result for Multiple Regression Models

According to the results obtained from the multiple linear regression models, several observations and conclusions can be made with respect to the coefficient of determination. First, the fitting for all multiple regression models is better than the fitting for all simple linear regression models. The coefficients of determination for multiple linear regression models are close to each other in value and most of them are above 90%. By adding one extra input variable to the Turbine Generator model as shown in Model 1, the coefficient of determination has increased by 30% or more when compared to simple linear regression models. Beyond that, any further addition in the number of parameters in the model, as shown in Models 2 to 4, does not have any dramatic improvement on the coefficient of determination.

In addition, the coefficient of determination for models with the intercept is better than those without. This is the case for all models tested, however, the difference is larger for Model 1 with the intercept. Since Model 1 with condensate and 35 psi steam flow rates for input variables as shown in Equation (2.30) resembles the model structure obtained from the first principles models, this model is the most likely structure for the turbine generator model.

$$y = -362.7790 + 0.0400x_{35stm} + 0.0905x_{cond} \tag{2.30}$$

Equation 2.31 shows the 95% confidence interval for the coefficient of determination for Model 1 with intercept.

$$93.11\% \leq R^2 \leq 93.50\% \tag{2.31}$$

Since the confidence interval on the coefficient of determination is above 90%, the data fitting with the model structure is considered adequate.

Figure 2.4 shows the comparison between the predicted electricity output from the Turbine Generator 1 model and the actual electricity output. Based on the figure, most of the data points fall in a straight 45 degree line indicating that the predicted electricity output is similar to the actual measurement.

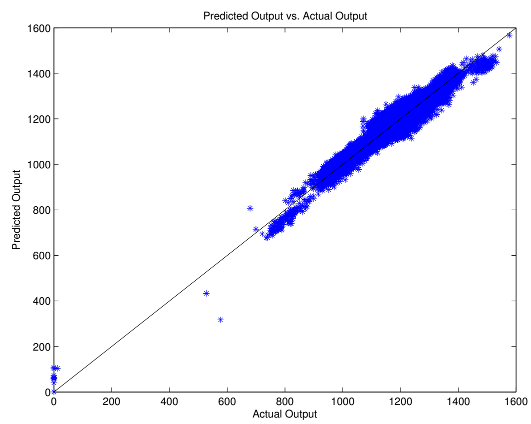


Figure 2.4: Predicted Output vs. Actual Output for Turbine Generator 1 Model

To check whether the model determined is adequate, residual analyses were conducted to validate the model. In particular, time series plot, histogram, standardized residual test, residual versus prediction test and residual versus regressor test were conducted.

Figure 2.5 shows the time series plot of the residuals. From the figure, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Figure 2.6 shows the histogram test of the residuals. The figure shows that the residuals are close to being normally distributed but there could be an outlier in the data set.

To determine whether there are outliers in the data set, standardized residuals were calculated. Figure 2.7 shows the plot of the standardized residuals. From the figure, approximately 95% of the data falls between +2 and -2. The other 5% of the data that falls outside the range can be attributed mostly to natural occurring event. Data point 29 has an unusually large standardized residual of 8.89, which suggests the data point could be an outlier.

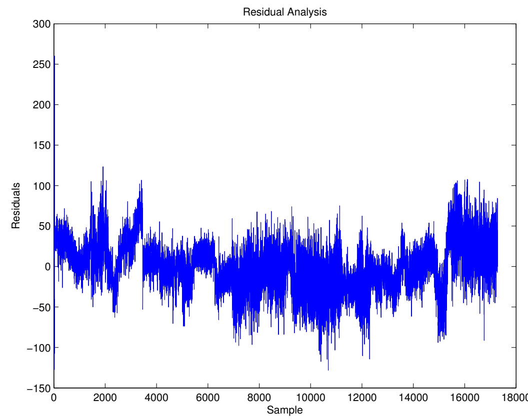


Figure 2.5: Time Series Plot of the Residuals

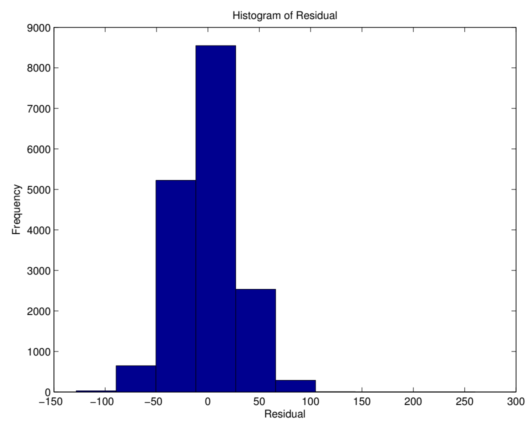


Figure 2.6: Histogram of the Residuals

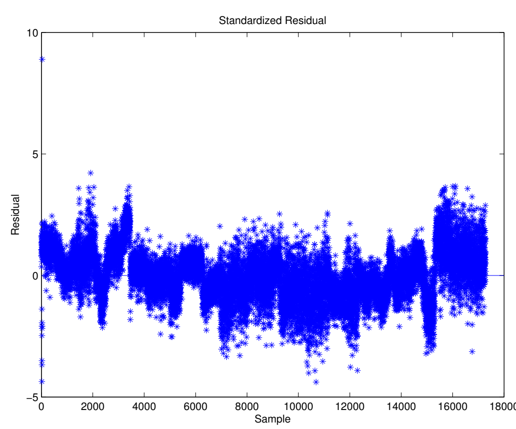


Figure 2.7: Standardized Residual Plot

Figure 2.8 shows the comparison of the residuals against predicted electricity output. Most of the residuals range from +100 to -100 for predictions around 800 to 1400. The data point with residual of 260 seems to be an outlier. Similar to the observation from Figure 2.7, this point corresponds to data point 29.

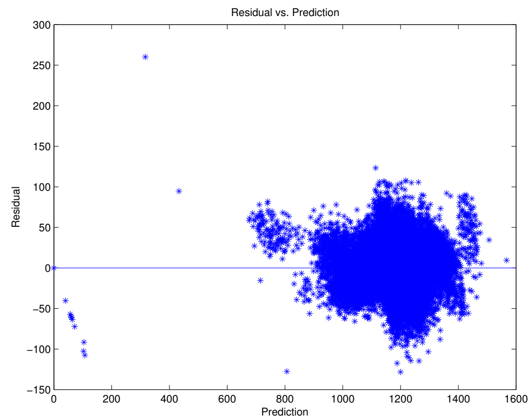


Figure 2.8: Residual vs. Predicated Electricity Output

Figure 2.9 shows the residual versus regressor tests. Similar to previous figures, most of the residuals fall within the range of +100 to -100 except one data point that corresponds to data point 29.

Based on the residual analyses performed, it was determined that the data point 29 is likely an outlier. Therefore, the data point is removed to test if a better model can be obtained.

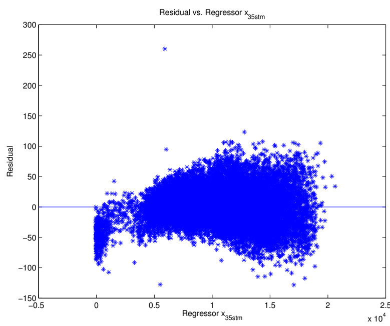
After the outlier is taken out of the original data set, the resulting regression model is obtained in Equation (2.32).

$$y = -364.5379 + 0.0400x_{35stm} + 0.0906x_{cond} \quad (2.32)$$

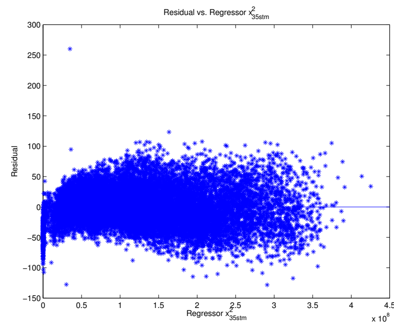
Since the null hypothesis for t-test is rejected for all three parameters, it is determined that all parameters contributed significantly to the model.

The coefficient of determination R^2 and the adjusted R^2 for this model are determined to be 93.33%. This is a slight improvement compared to the 93.31% obtained for the R^2 and the adjusted R^2 with the outlier.

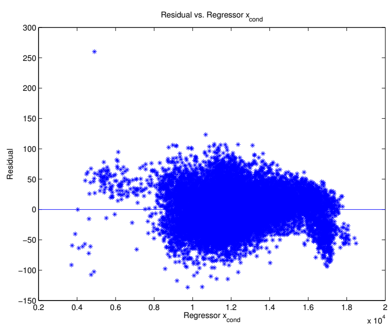
Figures 2.10 to 2.15 show the result of the residual analysis without the outlier. The general trend in these figures is very similar to those obtained with the outlier. The figures that show a clear improvement from removing the outlier are the



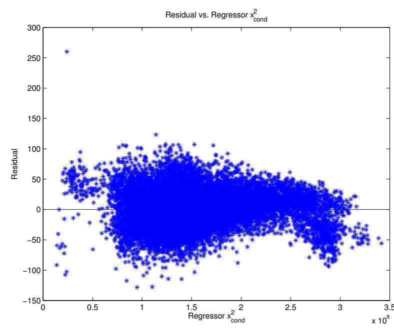
(a) x_{35stm}



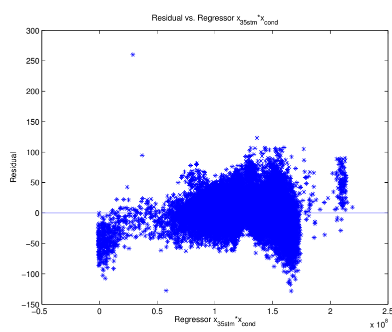
(b) x_{35stm}^2



(c) x_{cond}



(d) x_{cond}^2



(e) $x_{35stm}x_{cond}$

Figure 2.9: Residual vs. Regressors

histogram and the standardized residual plot.

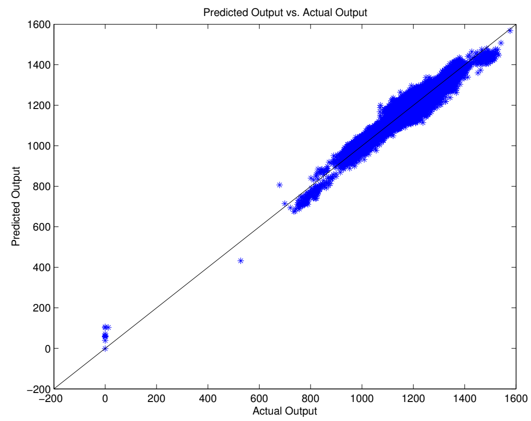


Figure 2.10: Predicted Output vs. Actual Output for Turbine Generator 1 Model

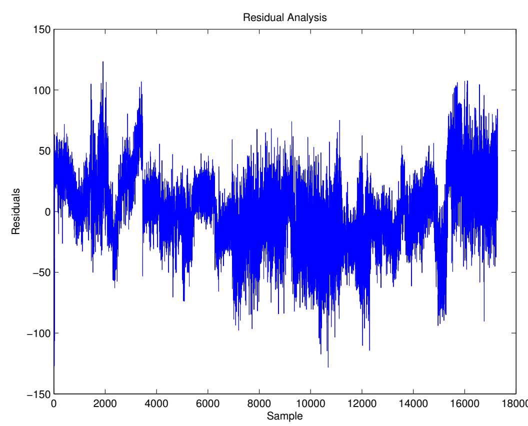


Figure 2.11: Time Series Plot of the Residuals

The histogram in Figure 2.12 mirrors closer to a normal distribution than the histogram in Figure 2.6. The standardized residual plot in Figure 2.13 shows that 95% of the data points fall within plus and minus 2, and no point outside of plus and minus 5. Therefore, based on the improvements observed from these two figures, it is determined that a better regression model is obtained by removing the outlier.

There is 95% confidence that each of the parameters in Turbine Generator 1 model is in the confidence interval shown in Equations (2.33) to (2.35).

$$-370.8313 \leq \beta_0 \leq -358.2446 \tag{2.33}$$

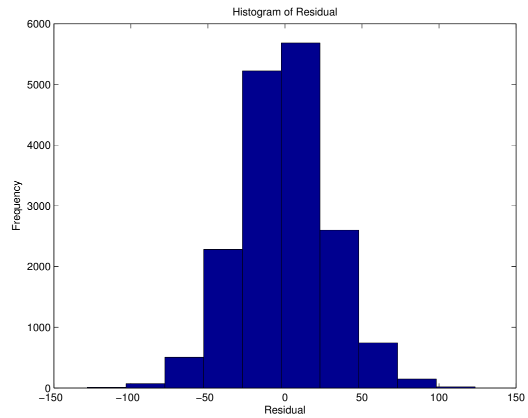


Figure 2.12: Histogram of the Residuals

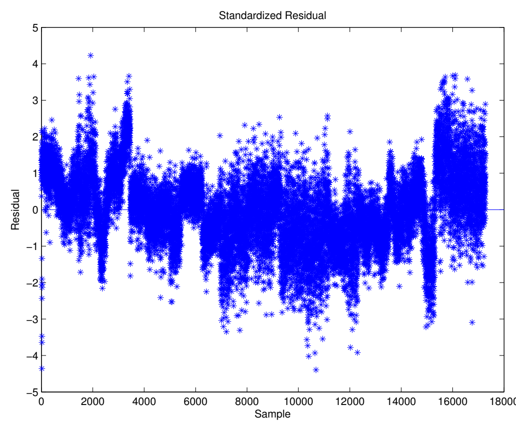


Figure 2.13: Standardized Residual Plot

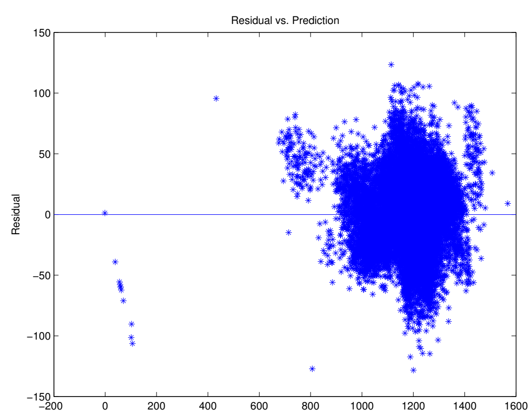
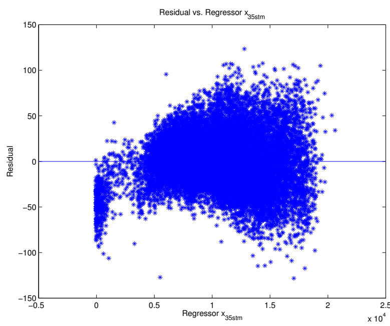
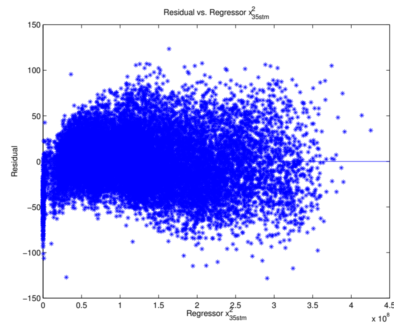


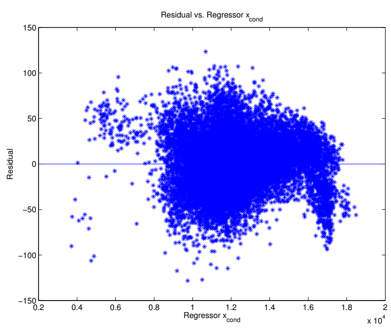
Figure 2.14: Residual vs. Predicated Electricity Output



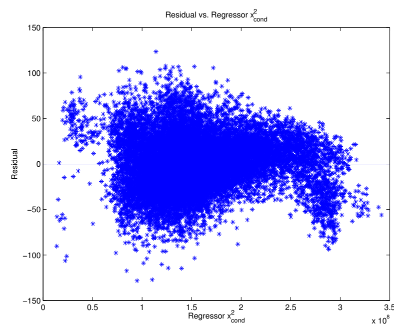
(a) x_{35stm}



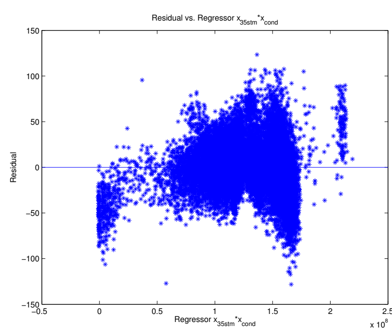
(b) x_{35stm}^2



(c) x_{cond}



(d) x_{cond}^2



(e) $x_{35stm}x_{cond}$

Figure 2.15: Residual vs. Regressors

$$0.0398 \leq \beta_1 \leq 0.0402 \quad (2.34)$$

$$0.0902 \leq \beta_2 \leq 0.0909 \quad (2.35)$$

The Turbine Generator 1 model determined is valid for 0 to 1,577 kW, 0 to 20,626 lb/hr 35 psi steam flow rate, and 3,707 to 18,484 lb/hr condensate flow rate. Extrapolation beyond the range provided may not be accurate.

Similar analysis was conducted for Turbine Generator #2 and the best model obtained has a 94.45% fit with the following form:

$$y = -603.0405 + 0.0435x_{35stm} + 0.1074x_{cond} \quad (2.36)$$

Equations (2.37) to (2.39) show the 95% confidence interval for the parameters in the Turbine Generator 2 model.

$$-609.4267 \leq \beta_0 \leq -596.6544 \quad (2.37)$$

$$0.0433 \leq \beta_1 \leq 0.0436 \quad (2.38)$$

$$0.1070 \leq \beta_2 \leq 0.1078 \quad (2.39)$$

The Turbine Generator 2 model determined is valid for 931 to 1,589 kW, 3,088 to 19,957 lb/hr 35 psi steam flow rate, and 7,653 to 17,591 lb/hr condensate flow rate. Extrapolation beyond the range provided may not be accurate.

2.4 Disturbances and Uncertainties

In this section, the potential sources of disturbances and uncertainties in the steam network will be discussed. The effects of these disturbances and uncertainties on the models developed from previous sections will also be explored.

One of the major disturbances to the steam network is the variation in user demand for steam from the network. Some of the steam users are core process units such as High Pressure Still while others can be external plants such as Metals plants. Since the steam demand from these users depends on the process conditions they face, the steam requirement for these units will change if the process conditions change for their process or business. The steam demand from these users can change as frequently as the variations of their process conditions. These disturbances can potentially exhibit random walk characteristics and the level of steam demand can be inconsistent because there is no control over the demand from some of the users. On the other hand, the supply of steam is also a source of disturbance for the steam network. Some of the steam supplies come from external plants, such as the Acid Plants, and the rest comes from the boilers. Similar to the variation for steam demand, the steam load from external plants and the boiler swings can be significant and the timing of variation is usually unpredictable.

Another major source of disturbances can be caused by leaks in the common headers. When leaks take place in the steam network, it causes a consistent steady loss of steam from the system and the problem lasts until the leak is repaired.

Other sources of disturbances include the variation in the consistency of the temperature and pressure in the common headers. The variation of the temperature in the common headers can be caused by line loss due to temperature difference with the ambient temperature. The variation of the pressure in the common headers can be caused by the amount of steam being drawn or supplied to the steam network. These types of disturbances can vary from time to time and they may not be consistent. Leaks in the steam header could also affect the variation in temperature and pressure. Since it is likely that there are leaks somewhere along the steam header, it is highly likely that disturbances in temperature and pressure could cause the difference between the model and the actual process as well as affecting the variation in the efficiency of the turbine generators.

Even though the optimization model is not yet formulated, any change in the economics is going to have an effect on the optimization solution. In particular, the unit prices for products such as electricity and 35 psi steam as well as the unit price for material such as the 900 psi steam are going to be used in the optimization model. Depending on how market prices vary for steam and electricity, the optimal solution would shift as well. More details on the effects of varying market values on steam and electricity will be discussed in later chapters.

Typically any of the measured data would have uncertainties due to measurement errors. Therefore, any of the flow rate measurements used in the development of flow rate models would contain some form of uncertainties. In addition, since the temperature and pressure measurements were used in the development of the overall efficiency parameters, the efficiency parameters obtained would also have uncertainties attached to their values. Furthermore, since the parameters calculated for turbine generator models were based on model fitting the data collected, the parameters obtained would also contain uncertainties.

2.5 Operating Objectives

With many potential manipulated variables available for adjustment to meet the various level of steam demand such as boilers output, turbine generator flow rates and the usage of pressure letdown stations, it is important to compare the financial return on each of the paths for the optimal result. Currently, the Energy Management System (EMS) is available to provide steam allocation among boilers, turbines and pressure letdown stations. The objective of the EMS is to meet the steam and electrical requirements of the plant at minimum cost subject to the operating constraints imposed on the process and generation equipment. The system provides an excellent tool in controlling the pressure in common headers by exchanging electricity production for process steam, should there be an increase in steam demand and the other way around when the steam demand drops. However, since the system does not take variation in market value into account for steam and electricity generation and any increases in energy demand by the plant are met by exchanging electrical generation for process steam, this means that the control arrangement does not produce the most cost effective electrical and steam balance. Therefore, an optimization scheme based on financial and process information is to be proposed to find the optimal distribution path for steam in the utilities plant. Also, the effects of market variation will be taken into account for the optimal operating strategy by solving the optimization model for different economic conditions.

Although boilers outputs are available as manipulated variables, due to the complexity of the process involved in modelling the boilers, they are left out of the scope as possible manipulated variables for this project. It is determined that the main purpose of the boilers is to maintain sufficient amount of steam in the system, and the EMS is more than capable of making sure that this is the case so there is enough steam in the utilities plant for all of the users.

Nevertheless, with the presence of turbine generators and pressure letdown station, multiple paths exist in which high pressure steam can go through to produce low pressure steam. Even though the mass flow is preserved with the use of pressure letdown stations, the loss of energy in the form of heat loss is irrecoverable. On the other hand, although part of the energy is converted to electricity through turbine generator, the poor efficiency of the equipment makes the decision of distributing steam through turbine generator less obvious as a better alternative. The dilemma of choosing the best path to distribute steam with necessary process and economic information presents the problem as a classic planning and scheduling problem that can be solved using optimization problem approach.

The solution from the planning and scheduling optimization problem will yield set points for operators to run the process at the given financial and process conditions. As illustrated in Figure 2.16, the output from the optimizer will be used as set points by the controller to control valves that manipulate flow rates to Turbines, Generators and Pressure Reducing Valve.

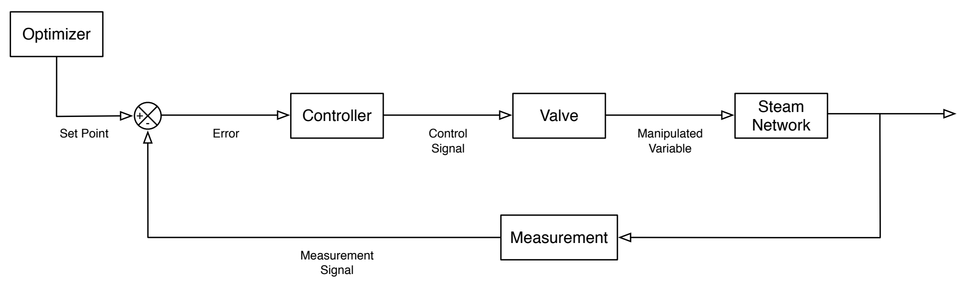


Figure 2.16: Block Diagram of the Optimizer and the Control System

The primary objective of the optimizer is to optimize the cost (or revenue) of running the utilities plant according to the market conditions while meeting the process requirements. This will involve finding the deterministic solution of the optimization problem, which will be discussed in Chapter 3. The secondary objective of the operation is to find the optimal point to operate the process so the utilities plant would minimize the frequency of the plant operating outside the feasible regions. This will involve the use of chance constraint and stochastic linear programming techniques, which will be discussed in Chapter 4.

Chapter 3

Steam Network Optimization Using Deterministic Linear Programming

In this chapter, optimization problems for the steam network will be formulated and analyzed to provide operating strategies. The chapter begins with the formulation of a linear optimization problem. In particular, the information used to formulate the objective function as well as the system constraints will be discussed. After the formulation of the deterministic optimization problem, solution techniques will be explored. Upon obtaining the optimal solution, the nominal optimization result will be compared with the current operation to assess the potential benefit that can be achieved from the optimization scheme. A post-optimality analysis will be given and the sensitivity of the optimization result will be discussed. Following the analysis of the nominal case, several case studies will be developed to study the effect of varying process and economic conditions on the optimization result. The various cases developed will reflect variations in utility prices and the level of steam supply and demand. Finally, results from optimization problems will be summarized as operating strategies for the steam network so the steam distribution can be manipulated based on process and economic condition.

3.1 Linear Programming Formulation

A Linear Programming problem has the general form:

$$\begin{array}{cc} \max & c^T x \\ \text{Subject to:} & Ax \leq b \end{array} \tag{3.1}$$

The problem consists of an objective function to be maximized or minimized and a set of inequality/equality constraints that describe the process model, process operation, and safety constraints.

3.1.1 Objective Function Formulation

The objective function for scheduling and planning problems typically uses economic functions such as cost, revenue or profit. For this project, the objective function considers the net economic benefit of the utility operation with a profit function, which consists of unit prices of steam and electricity, and flows of high and low pressure steam. The profit function is defined as the difference between the revenue and the cost and it has the following form:

$$\text{Profit} = \text{Revenue - Cost} \tag{3.2}$$

To define items that generate revenue and cost, it is necessary to classify raw materials and products of the process. Since the utility system can be considered as taking high pressure steam to generate electricity or low pressure steam, the 900 psi steam is categorized as raw material while electricity and 35 psi steam are categorized as products of the process. By considering the flow of 900 psi steam and its associated unit price as the cost of the process, and electricity, flow of 35 psi steam and their associated unit prices as the revenue of the process, the profit function in Equation (3.2) can be rewritten as:

$$\text{Profit} = C_{electricity}E + C_{35}x_{35stm} - C_{900}x_{900stm} \tag{3.3}$$

The coefficient, $C_{electricity}$ (\$/kWh), denotes the unit price for electricity while the coefficients, C_{35} (\$/lb) and C_{900} (\$/lb), denote unit prices for the 35 psi steam, 900 psi steam respectively. Since variables x_{900stm} (lb/hr), x_{35stm} (lb/hr), and E (kW) in Equation (3.3) do not show how steam distribution can be manipulated, a model to express the relationship between the variables given in the equation with the operation of the steam network is needed. This can be achieved by material and energy balance equations across the steam network.

The mass balance equation for 900 psi steam available for distribution has the following form:

$$x_{900stm} = x_{Tur1in} + x_{Gen1} + x_{Tur2in} + x_{Gen2} + x_{PLSin} \tag{3.4}$$

The mass balance equation for 35 psi steam generated from the 900 psi steam has the following form:

$$x_{35stm} = x_{Tur1out} + x_{Tur2out} + x_{PLSout} \tag{3.5}$$

All of the variables in Equations (3.4) and (3.5) denote steam flow rates with the unit of (lb/hr).

The following equation shows the total electricity generated from the two turbine generators.

$$E = \alpha_{Tur1}x_{Tur1in} + \alpha_{Gen1}x_{Gen1} + \beta_{TG1} + \alpha_{Tur2}x_{Tur2in} + \alpha_{Gen2}x_{Gen2} + \beta_{TG2} \tag{3.6}$$

The four efficiency variables denoted by α (kWh/lb) in Equation (3.6) are determined in Chapter 2.

By combining Equations (3.4) to (3.6) into Equation (3.3), the resulting equation for profit is obtained as:

$$\begin{aligned} \text{Profit} = {} & (C_{35} + C_{electricity}\alpha_{Tur1} - C_{900})x_{Tur1in} + (C_{electricity}\alpha_{Gen1} - C_{900})x_{Gen1} \\ & + (C_{35} + C_{electricity}\alpha_{Tur2} - C_{900})x_{Tur2in} + (C_{electricity}\alpha_{Gen2} - C_{900})x_{Gen2} \\ & + (C_{35} - C_{900})x_{PLSin} + C_{electricity}(\beta_{TG1} + \beta_{TG2}) \end{aligned} \tag{3.7}$$

All other overhead cost information associated with equipment maintenance and amortization fees are not included in the objective function because the goal of this thesis is to find the optimal steam distribution strategy for operational purpose based on steam and electricity unit prices. Also, the inclusion of these overhead cost would not affect how the optimal strategies are determined because the overhead cost is a lumped sum deduction to the profit calculated and as a result, it has no effect on the decision variables except shifting the objective function value.

3.1.2 Formulation of Constraints

The constraints used in optimization problems typically include mass and energy balances as well as other process information such as minimum flow rates required for the turbine generators and physical limitations such as maximum electricity that can be generated from the turbine generators.

A system of mass balance equations is used to derive the steam flow model and the energy balance analysis is used to determine the turbine generator models. The details for the modelling of steam flow system and the turbine generators were given in the previous chapter.

Since the Energy Management System currently in place supplies enough steam in each of the common headers to satisfy the steam demand from various users, there is always a net positive flow of steam in each of the common headers. Therefore, four constraints, one for each of the common headers, can be formulated based on the process requirement that a positive steam flow is necessary to meet the steam demand.

900 psi Header Constraint:

$$x_{Tur1in} + x_{Gen1} + x_{Tur2in} + x_{Gen2} + x_{PLSin} + x_{900\_users} \leq x_{900\_suppliers} \tag{3.8}$$

450 psi Header Constraint:

$$x_{450\_suppliers} \geq x_{450\_users} \tag{3.9}$$

160 psi Header Constraint:

$$x_{160\_suppliers} \geq x_{160\_users} \tag{3.10}$$

35 psi Header Constraint:

$$x_{Tur1out} + x_{Tur2out} + x_{PLSout} + x_{35\_suppliers} \geq x_{35\_users} \tag{3.11}$$

Equations (3.8) to (3.11) are developed based on Equations (2.9) to (2.12) to ensure that there is a net positive flow in each of the common headers. Therefore, they can be very similar to each other except that some of the supplier and user units are combined together as a variable.

Furthermore, three additional equations are developed to relate the inlet flow rates and outlet flow rates of Pressure Letdown Station and Turbine Generators 1 and 2 in Equations (3.12) to (3.14) respectively.

$$\alpha_{PLSconvert} x_{PLS93in} = x_{PLS93out} \tag{3.12}$$

$$\alpha_{Tur1convert} x_{Tur1in} = x_{Tur1out} \tag{3.13}$$

$$\alpha_{Tur2convert} x_{Tur2in} = x_{Tur2out} \tag{3.14}$$

The $\alpha_{PLSconvert}$, $\alpha_{Tur1convert}$, and $\alpha_{Tur2convert}$ are factors converting inlet flow rates to outlet flow rates.

With regards to the generation of electricity, there is also an inherent physical limitation on how much electricity can be produced from each of the common headers. The constraints on how much electricity can be produced from each of the Turbine-Generators can be expressed as the following two functions.

Electricity Generation Constraint on Turbine Generator 1:

$$\alpha_{Tur1}x_{Tur1in} + \alpha_{Gen1}x_{Gen1} + \beta_{TG1} \leq E_{TG1\_max} \tag{3.15}$$

Electricity Generation Constraint on Turbine Generator 2:

$$\alpha_{Tur2}x_{Tur2in} + \alpha_{Gen2}x_{Gen2} + \beta_{TG2} \leq E_{TG2\_max} \tag{3.16}$$

The Equations (3.15) and (3.16) show that the generation of electricity would reach maximum after a certain flow rate into the turbine generators. As a result, any increase in steam flow to the turbine generator beyond the physical limit would not generate additional electricity.

In addition, in order to maintain a smooth continuous process, there needs to be a minimum flow rate into the turbines. These bounds can be expressed as the following:

Turbine 1 Minimum Flow

$$x_{Tur1in} \geq x_{Tur1\_min} \tag{3.17}$$

Turbine 2 Minimum Flow

$$x_{Tur2in} \geq x_{Tur2\_min} \tag{3.18}$$

Combining all of the above constraints and the objective function into linear programming framework, the optimization problem can be formulated as:

$$
\begin{aligned}
\max_{x} \quad & (C_{35} + C_{electricity}\alpha_{Tur1} - C_{900})x_{Tur1in} \\
& +(C_{electricity}\alpha_{Gen1} - C_{900})x_{Gen1} \\
& +(C_{35} + C_{electricity}\alpha_{Tur2} - C_{900})x_{Tur2in} \\
& +(C_{electricity}\alpha_{Gen2} - C_{900})x_{Gen2} \\
& +(C_{35} - C_{900})x_{PLSin} + C_{electricity}(\beta_{TG1} + \beta_{TG2}) \\
\text{Subject to:} \quad & \left\{ \begin{array}{c} x_{Tur1in} + x_{Gen1} + x_{Tur2in} + \\ x_{Gen2} + x_{PLSin} + x_{900\_users} \end{array} \right\} \leq x_{900\_suppliers} \\
& x_{450\_suppliers} \geq x_{450\_users} \\
& x_{160\_suppliers} \geq x_{160\_users} \\
& x_{Tur1out} + x_{Tur2out} + x_{PLSout} + x_{35\_suppliers} \geq x_{35\_users} \\
& \alpha_{Tur1}x_{Tur1in} + \alpha_{Gen1}x_{Gen1} + \beta_{TG1} \leq E_{TG1\_max} \\
& \alpha_{Tur2}x_{Tur2in} + \alpha_{Gen2}x_{Gen2} + \beta_{TG2} \leq E_{TG2\_max} \\
& x_{Tur1in} \geq x_{Tur1\_min} \\
& x_{Tur2in} \geq x_{Tur2\_min}
\end{aligned}
\tag{3.19}
$$

3.2 Degrees of Freedom Analysis

In order to fully describe the processes in the system, eight mass balance equations from Equations (2.1) to (2.8) are used to represent steam flows into and out of each of the common headers. Two equations from Equations (2.18) and (2.19) for electricity generation are developed based on energy balances around Turbine Generators. In addition, three equations from Equations (3.12) to (3.14) are developed to relate the inlet flow rates and outlet flow rates of Pressure Letdown Station and Turbine Generators 1 and 2 respectively.

In total, there are 18 variables and 13 equations. The reduced row echelon form of these variables and equations show that these 13 equations are linearly independent to each other. Therefore, there are 5 degrees of freedom available for the optimization model. To obtain an unique solution, additional information in the form of inequality functions is required. For example, the steam supply to each of the common headers has to be greater than or equal to the steam demand from the common headers, the electricity generated from the turbine generator has to be less than or equal to its maximum capacity, and the flow rate to turbine generators has to be greater than or equal to their minimum flow rates to keep the equipment running continuously. By combining all of the additional information together, the constraints developed in optimization problem (3.19) represent a reduced space with 5 variables and eight inequality functions and a feasible solution can be found using linear programming.

3.3 Solution Techniques

Once the optimization problem is formulated, there are several solution techniques available to solve linear programming problems. A collection of available solution techniques can be categorized into simplex or non-simplex methods that are summarized in [Chong and Zak, 2008].

The simplex method, which finds the optimal feasible solution by testing adjacent vertices from a feasible starting origin to improve the objective function value, was invented by George Dantzig in 1947 [Dantzig and Thapa, 1997]. When the starting origin is not in the feasible set, an extension of the original simplex method covered by the Big M method in [Murty, 1983] is used.

The simplex methods provide practical solutions to linear programming problems, however, the amount of time required to find the optimal solution increases rapidly as the size of the system grows. As demonstrated by [Klee and Minty, 1972], the computation time increases exponentially with the size of the system. Therefore, interior point methods were proposed as a non simplex alternative to solve the computation time issue.

The major differences between simplex methods and interior point methods come from the starting point, and the stopping criteria. The optimal solution for simplex methods is obtained by comparing the objective function value at the adjacent vertex until the optimal solution is reached. For interior point methods, the search process starts from inside of the feasible region and then moves closer to the vertex of the optimal solution until a predefined stopping criterion is met.

An idea of the interior point method was initially developed from Khachiyan's ellipsoid algorithm [Khachiyan and Todd, 1993], which had the polynomial running time in its algorithm and utilized the concept of duality to solve linear programming problems [Chong and Zak, 2008], but the algorithm was impractical to solve linear programming problems because it was considered too slow. It was not until the breakthrough in 1984 by [Karmarkar, 1984] that revitalized the interest in interior point methods to solve linear programming problems. Among many proposed improvements to the interior point method, the predictor corrector method proposed by [Mehrotra, 1992] is commonly considered to be the most efficient and successful interior point method to date to solve large scale problems.

The CPLEX, optimizer which uses the interior point method, will be used to solve the optimization problems.

3.4 Optimization Result and Comparison

In this section, the optimal operating strategy is to be determined by solving the optimization Problem (3.19) using nominal operating condition and the sensitivity of the solution is to be investigated. After that, the result of the optimization Problem (3.19) will be compared with the current operation.

3.4.1 Nominal Operation

After the formulation of the optimization problem in Equation (3.19), all of the variables except inlet flows to Turbine 1, Generator 1, Turbine 2, Generator 2, and Pressure Letdown Station can be obtained from either the process or market data or they can be obtained using system identification techniques. For example, the amount of steam being supplied by the boilers or steam used by various plants can be obtained from the historian system. On the other hand, the parameters used for turbine generators are obtained from system identification techniques discussed in Chapter 2. The details of the process data used in the optimization problem are described in the Appendix A.

With all of the available information, the optimal nominal operation can be solved using GAMS and the result is obtained in Table 3.1.

Table 3.1 shows the optimal solution to decision variable, x^*, objective function value, p^*, and shadow price, λ^* from first to third row respectively. The decision variables are determined to be flow rates to Turbine 1, Generator 1, Turbine 2, Generator 2, and Pressure Letdown Station and their respective flow rates can be found in the third column of first row in Table 3.1.

The second row in Table 3.1 shows the optimal profit that can be achieved is \$-83.210 per hour if the steam is distributed based on the optimal operating strategy determined in the table. An optimal profit of \$-83.210 per hour means that it is costing the company to operate the utilities system.

The third row in Table 3.1 shows the shadow price obtained from the optimization

Flow Rate (lb/hr), x^*	Turbine 1	0
	Generator 1	7,000
	Turbine 2	37,500
	Generator 2	7,000
	Pressure Letdown Station	0
Profit ($/kW), p^*	-83.210	
Shadow Price, λ^*	900 psi Header (lb/hr)	-0.0006
	450 psi Header (lb/hr)	0
	160 psi Header (lb/hr)	0
	35 psi Header (lb/hr)	0
	Electricity Generation 1 (kW)	0
	Electricity Generation 2 (kW)	0
	Generator 1 Extraction Flow Rate (lb/hr)	-0.0043
	Generator 2 Extraction Flow Rate (lb/hr)	-0.0033

Table 3.1: Nominal Case Optimization Result

problem. These shadow prices can be used in the post-optimality analysis to study the sensitivity of the optimal solution.

Shadow Price and Constraint Conditions

Shadow prices, also known as Lagrange multipliers or Kuhn-Tucker multipliers, are often used in post-optimality analysis to determine how the optimal objective function value would change with a unit change in the constraints. The conventional optimality conditions require the Lagrange multipliers to be zero for inactive inequality constraints and greater than or equal to zero for active inequality constraints. The status of the inequality constraints can be identified as active or inactive depending on whether the inequality constraint becomes an equality constraint at the optimal solution. Based on the optimality conditions, the shadow prices shown in Table 3.1 can be divided into two parts. The first part contains values other than 0 and these constraints correspond to the active constraints. The second part contains values equal to 0, and these constraints can correspond to active constraints or inactive constraints and further analysis is needed to determine whether the constraint is active or not. To determine if those constraints are active, it is necessary to check if the inequality constraints become equality constraints at optimum. If the constraints are not equality constraints at the optimum, then the difference for the constraint to become equality constraint is considered "slack" for the constraint.

Table 3.2 shows the status of the constraints at the optimum bounded by their

900 psi Header (lb/hr)	-51,500 $\leq$ -51,500 $\leq \infty$
450 psi Header (lb/hr)	$0 \leq 0 \leq 0$
160 psi Header (lb/hr)	$0 \leq 0 \leq 0$
35 psi Header (lb/hr)	-11,700 $\leq$ 37,500 $\leq \infty$
Electricity Generation 1 (kW)	-$\infty \leq$ 633.5000 $\leq$ 3,487.7790
Electricity Generation 2 (kW)	-$\infty \leq$ 2,383.0500 $\leq$ 3,728.0405
Generator 1 Extraction Flow Rate (lb/hr)	7,000 $\leq$ 7,000 $\leq \infty$
Generator 2 Extraction Flow Rate (lb/hr)	7,000 $\leq$ 7,000 $\leq \infty$

Table 3.2: Constraint Status for Nominal Case

upper and lower limits. Based on the information, one can quickly determine whether a constraint is active or inactive depending on if the constraint is equal to either the lower or upper boundaries. If the constraint is equal to one of the boundaries, then the constraint is active. If the constraint is not equal to one of the boundaries, then the constraint is inactive, and one can quickly find how much slack there is for the constraint. For example, one can quickly determine that constraints for 900 psi header, 450 psi header, 160 psi header, Generator 1 Extraction Flow Rate and Generator 2 Extraction Flow Rate are active and constraints for 35 psi header, Electricity Generation 1, and Electricity Generation 2 are inactive.

For the inactive constraints, the 35 psi Header, Electricity Generation 1 and Electricity Generation 2 are respectively 25,800 lb/hr, 2,854.279 kW and 1,344.9905 kW away from reaching the constraint boundary to become active constraints.

Based on the Equation (D.7) in Appendix D, it can be seen that the shadow price can be used to determine how variations in the vector b would affect the objective function value. Each of the non-zero shadow prices represents how the objective function value would vary if the corresponding constraints were to be relaxed by a unit of the constraint. Since the shadow prices represents the rate of change in objective function value with respect to vector b, for a unit change in vector b without changing the active constraint set, the new objective function value would be:

$$P^*_{new} = P^*_{old} + (\lambda^*_a)^T. \tag{3.20}$$

For example, if the constraint for Extraction Flow Rate from Generator 1 was to be relaxed from 7,000 lb/hr to 7,001 lb/hr, the original objective function value of \$-83.2097/hr would change by -0.0043 to \$-83.2140/hr. Similarly, a relaxation of the constraint for Extraction Flow Rate from Generator 2 from 7,000 lb/hr to 7,001 would make a \$-0.0033/hr change to the objective function value and bring it to \$-83.149486/hr.

Although the variation in the objective function values given in the examples is quite small, the shadow prices can be used as one of the tools to predict the variability of the objective function values given a small change in the constraints that does not result in a change of active constraint set. Based on the shadow prices obtained in Table 3.1, it can be inferred that the worst way to improve plant profitability is to increase the Extraction Flow Rate from Generator 1, because it has the most negative impact on the objective function in terms of profit.

3.4.2 Comparison with the Current Operation

The current operating conditions for the utilities system are shown in Table 3.3.

Flow Rate (lb/hr), x^*	Turbine 1	9,000
	Generator 1	14,000
	Turbine 2	5,000
	Generator 2	14,000
	Pressure Letdown Station	2,000
Profit (\$/kW), p^*	-147.903	

Table 3.3: Current Operation

Based on the current operating strategy, the operation would lose \$147.903 per hour.

The first obvious comparison between the current operating condition and the optimal operating condition determined from nominal optimization problem is the benefit or additional profit that can be achieved. By comparing the profit sections in Table 3.1 and Table 3.3, the optimization scheme would decrease the cost of operation by roughly 45% or roughly \$65 an hour, which would translate to a saving of \$570,000 a year.

Another comparison can be made for the total flow rate through the five decision variables. Even though the total amount of 900 psi steam going into the common header is roughly the same for both cases, the total flow rate through the five decision variables is 44,000 lb/hr for the current operating condition and 51,500 lb/hr for the optimal operating strategy. The difference between the two cases is 7,500 lb/hr of steam, which roughly corresponds to the amount of excess 900 psi steam calculated in the previous chapter. Therefore, it is likely that the difference between the two cases was caused by leaks in the 900 psi common header as suspected in the previous chapter.

Another difference between the two cases is the operating strategy. The optimal operating strategy minimizes the use of Turbine 1 and 2, Generator 1 and the pressure letdown station. The current operating strategy has steam flow to all five decision variable greater than their minimum flow rates. With lower cost of operation associated with the optimal operating strategy, process operation improvements can be made by switching from the current operating strategy to the new optimal operating strategy.

3.5 Case Studies for Different Operating Scenarios

With the set up of linear programming problem in Problem (3.1), uncertainties and variations can occur in any of the A matrix or b and c vectors and result in a different operating strategy. This thesis will mainly focus on the effects of changing b and c vectors as demonstrated in a detailed derivation in Appendix D since the scope of the project is to investigate the effect of changing utility unit prices and level of steam supply and demand. In this section, various scenarios will be developed to study the effects of varying matrices in linear programming on the optimization system.

3.5.1 Changes in Economics

The average price is used for electricity and steam, however, it is possible that the actual price would deviate from the average prices used in the nominal operating case. With the deregulated electricity market in Alberta, it is possible to have a wide fluctuation in electricity price.

Any change in the economic condition is reflected in the changing c vector. To simulate such effect on the optimization problem, the optimization problem can be formulated as the following:

$$\begin{array}{cc} \max & (c^T \pm \varepsilon)x \\ \text{Subject to:} & Ax \leq b \end{array} \tag{3.21}$$

The only difference in Problem (3.21) compared to Problem (3.1) is that the objective function parameters are allowed to vary.

The results of the optimization problem when only the electricity price is allowed to vary are shown in Table 3.4.

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	0	7,000	0
4.8 ≤ E < 11.2	0	7,000	37,500	7,000	0
11.2 ≤ E < 12.2	0	7,000	16,452	28,048	0
12.2 ≤ E < 14.2	9,788	7,000	0	34,712	0
E ≥ 14.2	0	16,788	0	34,712	0

Table 3.4: Optimization Result for Varying Electricity Price

Table 3.4 shows the strategy to operate flow rates for the five decision variables from the second to the sixth column according to different levels of electricity prices, which are indicated in the left hand side column. It can be seen from Table 3.4 that the operating strategies would remain the same for a small range of variation in the electricity price before a new operating policy is implemented. This is the case because small changes in the electricity price may not be enough to affect the variations in the active constraints until a certain point where the direction of optimality would point to a different set of active constraints resulting in a different operating strategy. Therefore, the electricity price is given as a range on the left hand column in Table 3.4.

For each of the cases studied in Table 3.4, the objective function value is given as the following:

Electricity Price (¢/kW)	Objective Function Value, f, (\$/hr)
E < 4.8	f < -107.754
4.8 ≤ E < 11.2	-107.754 ≤ f < 23.635
11.2 ≤ E < 12.2	23.635 ≤ f < 57.716
12.2 ≤ E < 14.2	57.716 ≤ f < 133.666
E ≥ 14.2	f ≥ 133.666

Table 3.5: Objective Function Value for Varying Electricity Price

The result obtained in Table 3.5 shows the objective function value increases as electricity price increases and the break even price is at 10.1 ¢/kW. This is the case because the unit price for electricity is used as the price for a product and as a result, when the unit price for electricity goes up, the revenue value also goes up.

Similar to the case with varying electricity price, the optimization result from varying steam unit price would also give various ranges of operating strategies depending on the steam unit price used in the optimization problem. The main difference is that the unit price for 900 psi steam is considered to be the raw material cost and as the 900 psi steam price goes up, the objective function value would go down. On the other hand, the 35 psi steam is considered to be one of the products of the steam system, and increasing the 35 psi steam unit price would have the same effect as the increasing electricity unit price. The overall effect on the optimization result would depend on both changing unit price for 900 psi and 35 psi steam and the overall amount of 900 psi steam and 35 psi steam flow through the system.

3.5.2 Changes in Steam Supply and Demand

Since the nominal case uses the average flow rate for supply and demand of steam in the system, the actual level of steam being supplied or used is often deviated from the average flow rate.

Any change in the supply and demand of steam is reflected in the adjustment of the right hand side vector b. In order to simulate the deviation of the supply and demand level of steam, the average flow rate is allowed to vary one standard deviation above or below the average flow rate.

If only the supply or the demand of steam is allowed to vary compared to the nominal case, the optimization problem would vary according to the following case:

$$\begin{array}{cc} \max & c^T x \\ \text{Subject to:} & Ax \leq (b \pm \varepsilon) \end{array} \tag{3.22}$$

According to the formulation above, the suppliers and users of steam can vary at any common headers. Since the decision variables are not directly involved in supplying or using steam from 450 psi or 160 psi common headers, the following two inequality functions have to hold true at all times for the optimization system to proceed.

$$x_{450headerin} \geq x_{450headerout} \tag{3.23}$$

$$x_{160headerin} \geq x_{160headerout} \tag{3.24}$$

Similarly, since the objective for the boilers supplying 900 psi steam is to balance the steam requirement in the system and are currently controlled by the Energy Management System, the level of steam supply is maintained at the average level.

With that in mind, the only process flow rates allowed to deviate from the average flow rates are the users attached to 900 psi and 35 psi common headers.

If these users are allowed to vary one standard deviation above and below the average flow rates used in the nominal operation, there can be nine combinations as described in the following table.

	900 psi User Level	35 psi User Level
Case 1	Low	Low
Case 2	Low	Medium
Case 3	Low	High
Case 4	Medium	Low
Case 5	Medium	Medium
Case 6	Medium	High
Case 7	High	Low
Case 8	High	Medium
Case 9	High	High

Table 3.6: Objective Function Value for Varying Electricity Price

In Table 3.6, Low, Medium, and High represent Low, Medium, and High levels of steam demand from users attached to their respective common headers. The Medium flow rate indicates that the flow rates used in the optimization problem is average for each of the process users, where as the Low flow rate indicates that the flow rates are one deviation below the medium flow rates and the High flow rate indicates that the flow rates are one deviation above the medium flow rates.

Table 3.7 shows the optimization result for varying user level at current electricity price of 6 ¢/kW.

Cases	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
Case 1	0	7,000	43,000	7,000	0
Case 2	0	7,000	43,000	7,000	0
Case 3	0	7,000	43,000	7,000	0
Case 4	0	7,000	37,500	7,000	0
Case 5	0	7,000	37,500	7,000	0
Case 6	0	7,000	37,500	7,000	0
Case 7	0	7,000	32,700	7,000	0
Case 8	0	7,000	32,700	7,000	0
Case 9	0	7,000	32,700	7,000	0

Table 3.7: Optimization Result for Varying Steam User Level

From Table 3.7, it can be seen that the operating strategy is virtually the same for each of the cases except the level of 35 psi steam extracted from the Turbine 2 is different to accommodate the level 900 steam available in the system. This is the case because there is enough 35 psi steam in the system to satisfy high user demand level for 35 psi steam and adjusting the user demand for 900 psi steam affects the amount of 900 psi steam available to be distributed among the decision variables. Therefore, the higher the user demand for 900 psi steam, the fewer the amount of steam available to be distributed through Turbine 2.

If the electricity price were also allowed to vary for the optimization problem, the optimal operating strategies for the various cases described in Table 3.6 would be the following:

Case 1:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	43,000	7,000	0
$11.2 \leq E < 12.2$	0	7,000	25,696	24,304	0
$12.2 \leq E < 14.2$	15,288	7,000	0	34,712	0
$E \geq 14.2$	0	22,288	0	34,712	0

Table 3.8: Case 1 Result

Table 3.8 shows the result for Case 1 with varying electricity price. The break even price for electricity is 9.5 ¢/kW. After accounting for user requirement, there is an excess of 57,000 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 13,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Low and Low levels for 900 psi and 35 psi steam users respectively.

Case 2:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	43,000	7,000	0
$11.2 \leq E < 12.2$	0	7,000	25,696	24,304	0
$12.2 \leq E < 14.2$	15,288	7,000	0	34,712	0
$E \geq 14.2$	0	22,288	0	34,712	0

Table 3.9: Case 2 Result

Table 3.9 shows the result for Case 2 with varying electricity price. The break even price for electricity is 9.5 ¢/kW. After accounting for user requirement, there is an excess of 57,000 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 11,700 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Low and Medium levels for 900 psi and 35 psi steam users respectively.

Case 3:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	43,000	7,000	0
$11.2 \leq E < 12.2$	0	7,000	25,696	24,304	0
$12.2 \leq E < 14.2$	15,288	7,000	0	34,712	0
$E \geq 14.2$	0	22,288	0	34,712	0

Table 3.10: Case 3 Result

Table 3.10 shows the result for Case 3 with varying electricity price. The break even price for electricity is 9.5 ¢/kW. After accounting for user requirement, there is an excess of 57,000 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 10,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Low and High levels for 900 psi and 35 psi steam users respectively.

Case 4:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	37,500	7,000	0
$11.2 \leq E < 12.2$	0	7,000	16,452	28,048	0
$12.2 \leq E < 14.2$	9,788	7,000	0	34,712	0
$E \geq 14.2$	0	16,788	0	34,712	0

Table 3.11: Case 4 Result

Table 3.11 shows the result for Case 4 with varying electricity price. The break even price for electricity is 10.1 ¢/kW. After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 13,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Medium and Low levels for 900 psi and 35 psi steam users respectively.

Case 5:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	0	7,000	0
4.8 ≤ E < 11.2	0	7,000	37,500	7,000	0
11.2 ≤ E < 12.2	0	7,000	16,452	28,048	0
12.2 ≤ E < 14.2	9,788	7,000	0	34,712	0
E ≥ 14.2	0	16,788	0	34,712	0

Table 3.12: Case 5 Result

Table 3.12 shows the result for Case 5 with varying electricity price. The break even price for electricity is 10.1 ¢/kW. After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 11,700 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Medium and Medium levels for 900 psi and 35 psi steam users respectively.

Case 6:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	0	7,000	0
4.8 ≤ E < 11.2	0	7,000	37,500	7,000	0
11.2 ≤ E < 12.2	0	7,000	16,452	28,048	0
12.2 ≤ E < 14.2	9,788	7,000	0	34,712	0
E ≥ 14.2	0	16,788	0	34,712	0

Table 3.13: Case 6 Result

Table 3.13 shows the result for Case 6 with varying electricity price. The break even price for electricity is 10.1 ¢/kW. After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 10,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Medium and High levels for 900 psi and 35 psi steam users respectively.

Case 7:

Table 3.14 shows the result for Case 7 with varying electricity price. The break even price for electricity is 10.7 ¢/kW. After accounting for user requirement, there is an excess of 46,700 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 13,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in High and Low levels for 900 psi and 35 psi steam users

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	0	7,000	0
4.8 ≤ E < 11.2	0	7,000	32,700	7,000	0
11.2 ≤ E < 12.2	0	7,000	8,384	31,316	0
12.2 ≤ E < 14.2	4,988	7,000	0	34,712	0
E ≥ 14.2	0	11,988	0	34,712	0

Table 3.14: Case 7 Result

respectively.

Case 8:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	0	7,000	0
4.8 ≤ E < 11.2	0	7,000	32,700	7,000	0
11.2 ≤ E < 12.2	0	7,000	8,384	31,316	0
12.2 ≤ E < 14.2	4,988	7,000	0	34,712	0
E ≥ 14.2	0	11,988	0	34,712	0

Table 3.15: Case 8 Result

Table 3.15 shows the result for Case 8 with varying electricity price. The break even price for electricity is 10.7 ¢/kW. After accounting for user requirement, there is an excess of 46,700 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 11,700 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in High and Medium levels for 900 psi and 35 psi steam users respectively.

Case 9:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	0	7,000	0
4.8 ≤ E < 11.2	0	7,000	32,700	7,000	0
11.2 ≤ E < 12.2	0	7,000	8,384	31,316	0
12.2 ≤ E < 14.2	4,988	7,000	0	34,712	0
E ≥ 14.2	0	11,988	0	34,712	0

Table 3.16: Case 9 Result

Table 3.16 shows the result for Case 9 with varying electricity price. The break even price for electricity is 10.7 ¢/kW. After accounting for user requirement, there

Electricity Price	Decision Variable Strategies
E < 4.8	Only run Generator 1 and 2 at minimum flow rate
4.8 ≤ E < 11.2	Run Generator 1 and 2 at minimum flow rate and the rest of the steam pass through Turbine 2
11.2 ≤ E < 12.2	Run Generator 1 at minimum flow rate and pass the rest of the steam through Turbine Generator 2 to maximize electricity generation
12.2 ≤ E < 14.2	Run Generator 2 at maximum flow rate for maximum electricity generation and Generator 1 at minimum flow rate and pass the rest of the steam through Turbine 1
E ≥ 14.2	Run Generator 2 at maximum flow rate and pass the rest of the steam through Generator 2

Table 3.17: Summary of Operating Strategies with Changing b and c Vectors

is an excess of 46,700 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 10,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in High and High levels for 900 psi and 35 psi steam users respectively.

Summary

From the 9 cases simulated for varying electricity price and varying user demands from 900 psi and 35 psi steam users, a general trend among these cases can be observed. An overall strategy observed based on the results of nine tables described in this section could be found in Table 3.17.

First, when electricity price goes below 4.8 ¢/kW, generators are running at minimum required flow rates to keep the continuous process. This is the case because the 900 psi steam is more valuable compared to 35 psi steam or electricity. Therefore, the optimal operating strategy for the system is to run the generators at minimum and not produce any 35 psi steam. In this case, the excess 900 psi steam can be sold to other plants if possible, however, when selling 900 psi steam is not possible, the production of the 900 psi steam should be lowered to reduce the amount of 900 psi steam in the system. Nevertheless, since this is not within the scope specified for the project, the detailed solution technique will not be discussed here.

When electricity price is greater than or equal to 4.8 ¢/kW but less than 11.2 ¢/kW, the economic value obtained from the combination of 35 psi steam and electricity is greater than that of 900 psi steam. Since passing excess 900 psi steam through Turbines would generate electricity, and 35 psi steam can be extracted from the process, whereas passing the steam through Generators would only generate electricity, the optimal economic strategy when the electricity price is between 4.8 ¢/kW and 11.2 ¢/kW is through the generation of both 35 psi steam and electricity. Also, since Turbine Generator 2 is more efficient than Turbine Generator 1, the load is increased on Turbine Generator 2 before Turbine Generator 1. Therefore, the excess 900 psi steam in the system is used to generate 35 psi steam by passing through Turbine 2.

The economic value obtained from the combination of 35 psi steam and electricity generated is still greater than that of 900 psi steam for the ranges between 11.2 ¢/kW and 12.2 ¢/kW as well as between 12.2 ¢/kW and 14.2 ¢/kW.

For the electricity price range between 11.2 ¢/kW and 12.2 ¢/kW, the electricity becomes even more valuable, and hence the Generator 2 is used to produce more electricity, however, optimal solution still dictates that a portion of 35 psi steam to be generated because the optimal solution is to maximize the electricity generated from Turbine Generator 2.

When the electricity prices goes between 12.2 ¢/kW and 14.2 ¢/kW, Generator 2 is used to produce maximum electricity allowed; hence, Turbine 2 is no longer extracting any more 35 psi steam. The remaining excess 900 psi steam is therefore routed through Turbine 1.

When electricity price is greater than or equal to 14.2 ¢/kW, the electricity is the most valuable product and electricity generation should be prioritized. Therefore, all of the excess 900 psi steam is directed through Generators 1 and 2 to generate electricity. Since Generator 2 is more efficient compared to Generator 1, the electricity generation is at the maximum for Generator 2 while Generator 1 generates electricity with the remaining 900 psi steam.

If 900 psi and 35 psi steam suppliers are allowed to vary to simulate the changing levels of steam supply and demand, then the optimal strategy would be dictated by the amount of excess steam in the common headers. In general, Turbine Generator 2 would be fully utilized before increasing the load on Turbine Generator 1 and the price ranges for the operation would remain the same.

Special Case

All of the cases studied, when the demand side is allowed to change, resulted in excess 35 psi steam in the common header before taking decision variables into account. Therefore, the optimal decision variables are not required to satisfy the 35 psi steam user demand. This is the case because some of the equipment such as the Package Boilers is considered to be constant and supply constant steam to the system. Nevertheless, this may not be the case since the package boiler can be incorporated as one of the decision variables for future projects or there could be other instances when there would be a need to use the decision variables to satisfy the 35 psi steam user demand.

One of the ways to simulate the case where decision variables need to satisfy the 35 psi steam user demand is by removing the package boilers for Case 6. By doing so, 1,000 lb/hr of steam is required from one of the decision variables.

Case 10:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	1,000	7,000	0
$4.8 \leq E < 11.2$	0	7,000	37,500	7,000	0
$11.2 \leq E < 12.2$	0	7,000	16,452	28,048	0
$12.2 \leq E < 14.2$	9,788	7,000	0	34,712	0
$E \geq 14.2$	1,000	15,788	0	34,712	0

Table 3.18: Case 10 Result

Table 3.18 shows the result for Case 10 with varying electricity price. After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and a need of 1,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Medium and High levels for 900 psi and 35 psi steam users respectively.

Table 3.18 is very similar to Table 3.13 with the operating strategies across various ranges of electricity price. The differences between the two cases are for the ranges when electricity price is below 4.8 ¢/kW and when it is above 14.2 ¢/kW. Turbine 2 is used to extract 35 psi steam to satisfy the user demand when the electricity price is below 4.8 ¢/kW and Turbine 1 is used to extract 35 psi steam when the electricity price is above 14.2 ¢/kW. This is the case because Turbine 2 is more efficient than Turbine 1, so Turbine 2 is used when electricity price is low. When electricity price is high, Turbine Generator 2 is already running at capacity, hence Turbine 1 is used to satisfy the 35 psi steam demand when the electricity price is above 14.2 ¢/kW.

3.5.3 Process Equipment Failure

The last parameter that could change in Linear Programming framework is the "A" matrix in the constraint functions. Typically, the A matrix contains process information such as parameter values for Turbine Generators for electricity generation, as well as the steam conversion from high pressure steam to low pressure steam. If the A matrix in linear programming optimization framework is allowed to vary, the optimization problem could be formulated as the following:

$$\begin{array}{cc} \max & c^T x \\ \text{Subject to:} & (A \pm \varepsilon)x \leq b \end{array} \tag{3.25}$$

One of the ways that causes the A matrix in linear programming framework to change is through process failure scenarios. Since there are two turbine generators available to make electricity from 900 psi steam and have 35 psi steam extraction available, it is important to see how the optimization problem would react when facing process failure of one of these two turbine generators.

To simulate the process failure scenarios, one of the two turbine generators is taken offline while leaving the other in the process. By using average flow rates for steam user demand and supply levels like the nominal case or Case 5 shown in Table 3.12, the optimal operating strategy when Turbine Generator 1 fails is given in Table 3.19 for varying electricity price.

Turbine Generator 1 Failure:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	0	0	7,000	0
4.8 ≤ E < 11.2	0	0	44,500	7,000	0
E ≥ 11.2	0	0	28,217	23,283	0

Table 3.19: Turbine Generator 1 Failure

Similar to Table 3.12, there are different operating strategies for various electricity price ranges. Unlike Table 3.12 with 5 different operating strategies at 5 different electricity price ranges, Table 3.19 only has 3 operating strategies at 3 different electricity price ranges. This is due to the fact that Turbine Generator 1 is no longer functioning, and thus the extra load has to be given to Turbine Generator 2.

Recall that after accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of

11,700 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Medium levels for 900 psi and 35 psi steam users. Therefore, when the electricity price goes below 4.8 ¢/kW, Turbine Generator 2 is operating at minimum level required to keep the continuous process because the value of 900 psi steam is greater than the combined economic value from 35 psi steam and electricity. The combined economic value from 35 psi steam and electricity is greater than that of 900 psi steam when the electricity price is between 4.8 ¢/kW and 11.2 ¢/kW so the excess 900 psi steam is completely used by passing through Turbine Generator 2 and extracted as 35 psi steam. When the electricity price reaches 11.2 ¢/kW and above, the economic value for electricity is greater than that of 900 psi steam and 35 psi steam combined. Therefore, Turbine Generator 2 is running at the peak capacity with maximum electricity generated from the excess 900 psi steam available. In order to generate the maximum amount of electricity with the amount of 900 psi steam available, the operating strategy shown in Table 3.19 sets the ratio around the turbine and the generator so the maximum electricity generation can be achieved.

Similarly, when Turbine Generator 2 fails, the optimal operating strategy is given in Table 3.20 for varying electricity price.

Turbine Generator 2 Failure:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 5.2$	0	7,000	0	0	0
$5.2 \leq E < 14.2$	44,500	7,000	0	0	0
$E \geq 14.2$	23,227	28,273	0	0	0

Table 3.20: Turbine Generator 2 Failure

The result obtained in Table 3.20 for the optimal operating strategy is similar to the result obtained in Table 3.19 for failure of Turbine Generator 1. The reason why there are 3 different operating strategies for 3 different electricity price ranges is because of the economic value of 900 psi steam, 35 psi steam and electricity. At the first level, the 900 psi steam is more valuable than the other two products, whereas the middle price ranges is where the combination of 35 psi steam and electricity become more valuable, and the last prices range shows that the electricity is the most valuable product of the three.

A major difference between Table 3.19 and Table 3.20 is the cut off point for electricity price. This is the case because the efficiency for Turbine Generator 1 is different from that of Turbine Generator 2. Since Turbine Generator 2 is more efficient compared to Turbine Generator 1, when Turbine Generator 2 fails, Turbine Generator 1 would require a higher electricity price before changing the operating conditions to extract 35 psi steam or to generate electricity.

Since there is already an excess 35 psi steam in the common header, two special cases were developed so that the simulations for process failure scenarios can also be taken into account when excess 35 psi steam is needed from one of the decision variables. In this case, the 35 user steam flow rates were set to the high level and the package boiler is taken offline.

Turbine Generator 1 Failure and No Package Boiler:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	0	1,000	7,000	0
$4.8 \leq E < 11.2$	0	0	44,500	7,000	0
$E \geq 11.2$	0	0	28,217	23,283	0

Table 3.21: Turbine Generator 1 Failure and No Package Boiler

Turbine Generator 2 Failure and No Package Boiler:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 5.2$	1,000	7,000	0	0	0
$5.2 \leq E < 14.2$	44,500	7,000	0	0	0
$E \geq 14.2$	23,227	28,273	0	0	0

Table 3.22: Turbine Generator 2 Failure and No Package Boiler

After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and a need of 1,000 lb/hr of 35 psi steam available in the 35 psi common header when the user demands are in Medium and High levels for 900 psi and 35 psi steam users respectively.

The results obtained from Table 3.21 and Table 3.22 are similar to Table 3.19 and Table 3.20 except that a minimum amount of steam is passed through Turbine 2 and Turbine 1, respectively, to meet the steam requirement at the lowest electricity price range. There are no other differences because the 35 psi steam requirement is already met from the 35 psi steam extracted from the Turbines in these cases.

Chapter 4

Optimal Operating Strategy with Process Uncertainties

In this chapter, stochastic optimization problems will be formulated and the results will be analyzed to provide operating strategies for the operation of utilities system. The chapter begins with a discussion on why process uncertainties can cause an issue for the feasibility of the solution determined from the deterministic optimization problems. Following that, various ways to formulate the optimization problem to resolve issues surrounding the feasibility of the optimal solution are proposed. In particular, the formulation of the optimization problem using joint probability constraints and individual probability constraints are discussed and used to handle process uncertainties within the process. After the optimization problem is formulated with the probability constraints, a solution technique to solve the optimization problem is explored. Using the solution technique presented as the back-off solution, the stochastic optimization problem can be reformulated into a linear programming equivalent problem and existing algorithms for linear programming problems can be used to obtain the optimal solution. A comparison is done for the optimal solutions obtained from the deterministic optimization problem and the stochastic optimization problem. Finally, case studies were developed in a similar way to the ones done in Chapter 3 to show the effects of varying supply and demand of steam (which can be reflected by the changing b vector), economics (reflected by the changing c vector) and equipment failure scenarios (reflected by the changing A matrix) on optimal solution.

4.1 Uncertainties in Steam Network Model

According to Problem (3.1), matrix A and vectors b and c in linear programming framework represent process information, supply and demand level of steam, and economic information respectively. The parameters for these matrices are obtained based on the data collected from the plant, and the optimal operating condition determined from linear programming framework is dependent on the information used in these matrices. Nevertheless, the actual process and economic conditions in the plant may be different from the conditions specified in the linear programming framework due to process uncertainties or variations in process and economic information. Therefore, the actual optimal solution in the plant can be different from the one determined from the optimization model.

Variations in process and economic information can be identified as the mismatch between the data observed at the plant and the data used in the optimization model that can be explained by adjusting the process parameters in matrix A and vectors b and c in linear programming framework. An example of a process variation would be changing the set point for the steam intake of a turbine generator, which would affect the parameter used in the b vector in linear programming framework. On the other hand, the uncertainties in process and economic information are normally the differences that could not be explained by changing the parameters in the optimization model. For example, the optimization model would not be able to capture any leaks along the header; however, such leaks could affect how the optimal solution would behave when the system is in operation. Another source of uncertainties can come from measurement readings and it can cause the operating conditions to be suboptimal. If the difference between process condition observed at the plant and the condition specified in the optimization model is caused by variations in the process/economic conditions, then a new optimal operating condition can be obtained by running the deterministic optimization model again with the new observed conditions. On the other hand, if the difference is caused by uncertainties within the system, then the system would have to run more conservatively than the solution obtained from the deterministic model in order to ensure the system remains in the feasible region. Since the new operating solution, due to process and economic variation, can be easily obtained from re-running the deterministic model, this chapter will focus on the methods available to maintain the optimal operation when facing process and economic uncertainties.

4.1.1 Effects of Uncertainties on Optimal Solutions

To understand how to maintain an optimal solution when facing uncertainties, one has to understand how optimal solution shifts when uncertainties are introduced to optimization models. Case studies made in Chapter 3 explored how variations in the optimization problem would affect the optimal solution. In general, the optimal solution is determined at the vertices of optimization system, and depending on which of the A matrix or b and c vectors varies, a generalized trend of the effect on the optimal solution can be given as the following:

- Changing b vector would cause the constraint boundaries to shift either inward or outward and the new constraint intersection would be the new optimal solution
- Changing c vector changes the profit contour line that searches for the optimal solution and the optimal solution shifts from its original location when the active constraints are changed
- Changing the A matrix of an equality constraint affects the slope of the constraints and the objective function
- Changing the A matrix of an inequality constraint affects the slope of the constraints

With the movements of constraints and profit contour lines, the location of the optimal solutions could change as a result of the uncertainties. A detailed discussion regarding the variation of the matrices and its effect on the optimal solution can be found in [Monder, 2001].

To illustrate how uncertainties can cause the optimal solution to go outside the "feasible region", various regions have to be identified. Consider a system with two decision variables as illustrated in Figure 4.1 where the solid lines are constraints for Variables 1 and 2. Assuming the profit contour line points to the intersection of the two constraints, the optimal solution for the system is found at the intersection.

The feasible region for the optimal solution of the system is the shaded area bounded by the constraints and the two axes (assuming that the decision variables are bounded to be greater than or equal to zero). The area outside of the shaded area would be considered as the infeasible region.

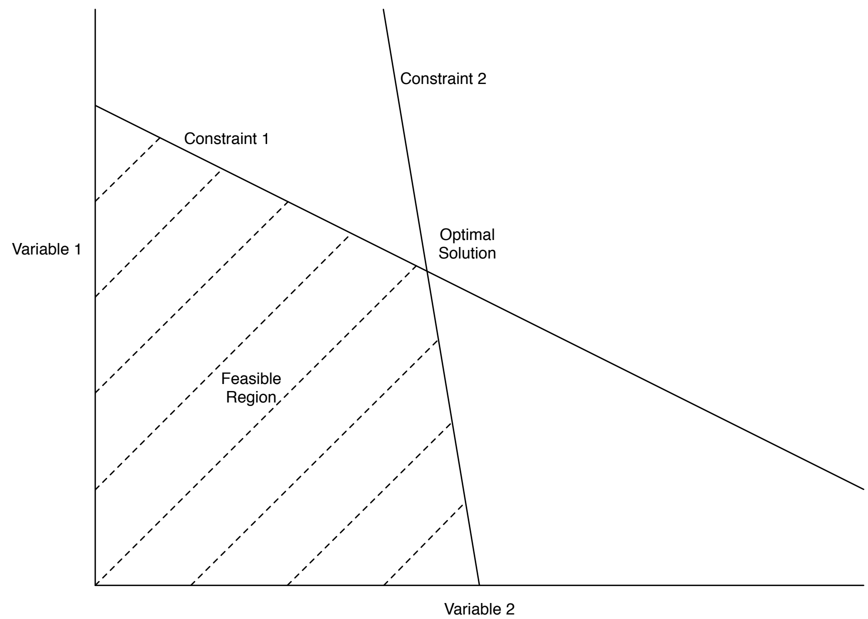


Figure 4.1: Optimal Solution Point

If the constraints of the system are allowed to move due to uncertainties in vector b, the optimality of the stochastic system can be illustrated by Figure 4.2. In the figure, the dotted lines indicate the confidence interval on how the b vector can change. By holding the profit contour line (c vector) constant, the thick dashed lines show where the new constraint intersections are in response to changing b vector. In other words, the box shows where the new optimal solution would be when b vector is allowed to change while holding c vector constant. Clearly, if one of the constraints moves in the opposite direction of the feasible region, the new optimal solution would go into the infeasible region.

4.1.2 Proposed Solution

To prevent optimal solutions from going into infeasible regions, the constraint box centred on the original optimal solution can move inside the feasible region as illustrated in Figure 4.3. By doing so, the optimal solution would remain feasible when the b vector varies within the confidence interval due to uncertainties. Therefore, the topic of discussion for this chapter is to find the location to back-off into the feasible region to ensure that changing b vector does not result in an infeasible operation.

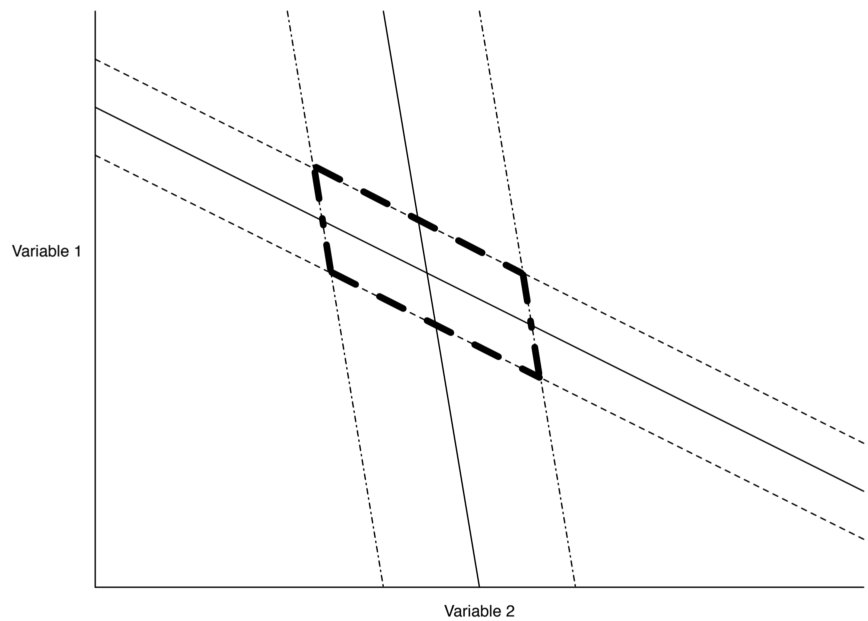


Figure 4.2: Constraint Box which Encompasses Infeasible Region

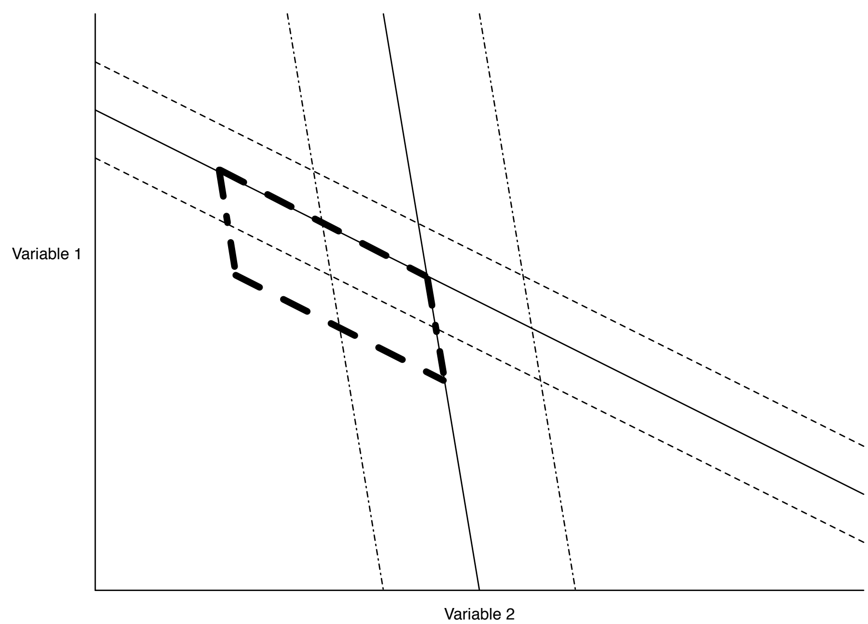


Figure 4.3: Constraint Box in the Feasible Region

The location where the optimal solution should back-off to depends on the desired confidence interval, which determines how the parameters of the constraints vary and how big the constraint box is. The confidence interval can be explicitly incorporated into the optimization problem as a pre-determined probability for the chance constraints to ensure the system remains in the feasible region.

After the chance constraint problem is formulated, the stochastic optimization problem is reformulated into a linear programming equivalent problem so the solutions techniques for linear programming problems can be used to solve for the optimal solution.

4.1.3 Pros and Cons of Post-Optimality Approach and Back-Off Solution Approach

With deterministic optimization approach discussed in Chapter 3 and stochastic optimization approach discussed in this Chapter, there are two methods available to deal with variations or uncertainties in the system. The deterministic optimization uses post-optimality approach to analyze how variations in the problem parameters affect the optimal solution by using the analysis of active constraint set and shadow prices. Stochastic optimization can result in a back-off solution approach to incorporate uncertainties into the optimization problem and delivers an optimal solution that guarantees the process to remain feasible for the desired frequency. Since there are two methods available to deal with variations or uncertainties in the system, it is important to discuss the advantages and disadvantages of the two methods.

The advantage of the post-optimality approach is that it provides an effective way to check how the objective function would vary using shadow prices when the constraint is relaxed. It should be noted that the solution would remain valid only if the active constraint set is not changed from relaxing the constraint. If the active constraint set is changed during the process, the information obtained from the shadow price no longer applies to the new objective function value and the optimal solution has to be re-evaluated. On the other hand, the post-optimality approach is also unable to maintain the feasibility of the optimal solution with changing parameters in the optimization problem. Since the optimization model uses the average of the collected data, the parameters used in the optimization problem could change and result in the variation of the optimal solution towards the infeasible region.

The back-off solution is a more proactive approach with regards to dealing with uncertainties in the optimization problem compared to the post-optimality approach. The back-off solution takes the desired feasibility of the system into account for the optimization problem and determines an optimal solution that can withstand variations in the parameters within the limits specified and ensure that the process remains feasible. The major problem with the back-off solution, however, is to determine how uncertain and how accurate the problem parameters are. As a result, the optimal solution determined from the back-off approach is often more conservative than the optimal solution obtained using deterministic linear programming.

Depending on the application, both methods can offer insights on how the system would react when the process faces uncertainties. For a normal continuous industrial application, it is often desirable to actively maintain the optimal solution in the feasible region so that adverse effects resulting from the process operating in the infeasible region can be avoided.

4.2 Formulation Using Probability Constraint

There are two possible ways to formulate optimization problems using probability constraints. One of the ways is through the use of joint probability constraints where one probability is given for all of the constraints in the optimization problem as shown in the following system.

$$\begin{array}{cc} \max & c^T x \\ \text{Subject to:} & \Pr\{Ax \leq b\} \geq \alpha \end{array} \tag{4.1}$$

The Problem (4.1) shows the optimization problem using joint probability constraint, which is similar to Problem (3.1) except all of the constraints have to satisfy the desired feasibility given by α, which is the probability for the inequality to hold.

Rather than setting one probability for the whole system, the desired probability can also be stated for each of the constraints in the system. The individual probability constraint assigns a probability, α_i, to each of the system constraints and it has the following form.

$$\begin{array}{cc} \max & c^T x \\ \text{Subject to:} & \Pr\{A_i x \leq b_i\} \geq \alpha_i \end{array} \tag{4.2}$$

Depending on the application, both joint probability constraint and individual probability constraint can be used for the optimization problem formulation to incorporate uncertainties or feasibilities in the model.

4.2.1 Advantages and Disadvantages for Joint Probability Constraint and Individual Probability Constraint

The advantage of the joint probability constraint is the ease of maintainability for large optimization problems. This is the case because only one desired feasible probability is required to be explicitly stated for the overall system rather than specifying the feasible probability for each of the constraints. This can be especially useful for systems with a large number of constraints, for which it can be challenging to maintain all of the feasible probabilities in the system. Therefore, joint probability constraint is better suited for large scale processes and for cases where the constraints are correlated between the elements of vector b.

The individual probability constraint on the other hand requires every constraint to be explicitly specified with a feasible probability and it can be challenging to fulfill especially for large scale processes. Furthermore, the interaction between individual probability constraints can also make it difficult to maintain the overall feasibility of the process. Therefore, the individual probability constraints are better suited for optimization problems where the elements of vector b are independent and identically distributed from each other. Nevertheless, an individual probability constraint problem can be easily transformed into a linear programming equivalent problem, so the optimal solution can be determined using available solution techniques for linear programming even if the feasible solution can turn out to be overly conservative.

4.2.2 Formulation of the Stochastic Optimization Model

A deterministic optimization problem is required before formulating the stochastic optimization model using chance constraints. The deterministic optimization problem is formulated in Equation (3.19). To formulate the stochastic optimization model using joint probability constraint, the only thing that needs to be added to the Equation (3.19) is the desired probability for the feasible region.

$$
\begin{aligned}
\max_x \quad & (C_{35} + C_{electricity}\alpha_{Tur1} - C_{900})x_{Tur1} \\
& +(C_{electricity}\alpha_{Gen1} - C_{900})x_{Gen1} \\
& +(C_{35} + C_{electricity}\alpha_{Tur2} - C_{900})x_{Tur2} \\
& +(C_{electricity}\alpha_{Gen2} - C_{900})x_{Gen2} \\
& +(C_{35} - C_{900})x_{PLS} + C_{electricity}(\beta_{TG1} + \beta_{TG2})
\end{aligned}
$$

Subject to:

$$
\Pr\left\{\begin{array}{c}
x_{Tur1} + x_{Gen1} + x_{Tur2} + x_{Gen2} \\
+x_{PLS} + x_{900\_users} \leq x_{900\_suppliers} \\
x_{450\_suppliers} \geq x_{450\_users} \\
x_{160\_suppliers} \geq x_{160\_users} \\
x_{Tur1} + x_{Tur2} + x_{PLS} + x_{35\_suppliers} \geq x_{35\_users} \\
\alpha_{Tur1}x_{Tur1} + \alpha_{Gen1}x_{Gen1} + \beta_{TG1} \leq E_{TG1\_max} \\
\alpha_{Tur2}x_{Tur2} + \alpha_{Gen2}x_{Gen2} + \beta_{TG2} \leq E_{TG2\_max} \\
x_{Tur1} \geq x_{Tur1\_min} \\
x_{Tur2} \geq x_{Tur2\_min}
\end{array}\right\} \geq \alpha \quad (4.3)
$$

Even though optimization problems with probability constraint have forms that are similar to linear programming framework, with the way the stochastic optimization problem is set up currently in Problem (4.3), it could not take advantage of the solver program available. Since linear programming has many solution techniques available to solve for the optimal solution and its post-optimality analysis can be utilized to analyze the sensitivity of the optimal solution, the idea is to transform the chance constraint problem into a linear programming equivalent problem.

Since it is easier to transform individual probability constraint optimization problems to linear programming problems, the joint probability constraint problem is first transformed into an individual probability constraint problem before being transformed into linear programming equivalent problem.

4.3 Solution Technique

To transform a joint probability constraint (JPC) optimization problem into an individual probability constraint (IPC) optimization problem, statistical manipulation of the constraints is used. The resulting individual probability constraint optimization problem can be transformed into linear programming equivalent problems using the back-off solution technique. After the linear programming equivalent problem is obtained, the existing solution techniques and solver programs can be used to solve the linear programming problem that incorporates the uncertainties in the optimization problem.

4.3.1 Approximating JPC with IPC

The format for the optimization problem with individual probability constraints is shown in Problem (4.4).

$$
\begin{aligned}
\max_{x} \quad & (C_{35} + C_{electricity}\alpha_{Tur1} - C_{900})x_{Tur1} \\
& +(C_{electricity}\alpha_{Gen1} - C_{900})x_{Gen1} \\
& +(C_{35} + C_{electricity}\alpha_{Tur2} - C_{900})x_{Tur2} \\
& +(C_{electricity}\alpha_{Gen2} - C_{900})x_{Gen2} \\
& +(C_{35} - C_{900})x_{PLS} + C_{electricity}(\beta_{TG1} + \beta_{TG2}) \\
\text{Subject to:} \quad & \Pr\left\{ \begin{array}{c} x_{Tur1} + x_{Gen1} + x_{Tur2} + \\ x_{Gen2} + x_{PLS} + x_{900\_users} \leq x_{900\_suppliers} \end{array} \right\} \geq \alpha_1^{'} \\
& \Pr\left\{x_{450\_suppliers} \geq x_{450\_users}\right\} \geq \alpha_2^{'} \\
& \Pr\left\{x_{160\_suppliers} \geq x_{160\_users}\right\} \geq \alpha_3^{'} \\
& \Pr\left\{x_{Tur1} + x_{Tur2} + x_{PLS} + x_{35\_suppliers} \geq x_{35\_users}\right\} \geq \alpha_4^{'} \\
& \Pr\left\{\alpha_{Tur1}x_{Tur1} + \alpha_{Gen1}x_{Gen1} + \beta_{TG1} \leq E_{TG1\_max}\right\} \geq \alpha_5^{'} \\
& \Pr\left\{\alpha_{Tur2}x_{Tur2} + \alpha_{Gen2}x_{Gen2} + \beta_{TG2} \leq E_{TG2\_max}\right\} \geq \alpha_6^{'} \\
& \Pr\left\{x_{Tur1} \geq x_{Tur1\_min}\right\} \geq \alpha_7^{'} \\
& \Pr\left\{x_{Tur2} \geq x_{Tur2\_min}\right\} \geq \alpha_8^{'}
\end{aligned} \tag{4.4}
$$

An approximation method is needed to transform the joint probability constraint problem shown in Problem (4.3) to the individual probability constraint problem shown in Problem (4.4).

As shown in Problem (4.4), the probability for each of the constraints can be different. To simplify the approximation process, the probabilities for the constraints in the individual probability constraint optimization problem are assumed to be the same.

There are several ways where a joint probability constraint problem can be approximated into an individual probability constraint equivalent problem, namely through the use of Boole's inequality and independent events functions.

Boole's inequality states:

$$\Pr\left(A_1 \cup A_2 \cup \ldots \cup A_n\right) \leq \sum_{i=1}^{n} \Pr\left(A_i\right) \tag{4.5}$$

As shown by [Monder, 2001], the inequality can easily be shown to have an equivalent form:

$$\Pr\left(A_1 \cap A_2 \cap \ldots \cap A_n\right) \geq 1 - \sum_{i=1}^{n} \Pr\left(\bar{A}_i\right) \tag{4.6}$$

On the other hand, the independent events function states:

$$\Pr\left(A_1 \cap A_2 \cap \ldots \cap A_n\right) \geq \prod_{i=1}^{n} \Pr\left(A_i\right) \tag{4.7}$$

The difference between the two methods gives an upper and a lower bound on the probabilities for the constraints and Boole's inequality would give a more conservative approximation.

Both methods can be used to calculate the equivalent probability for individual probability constraints from joint probability constraints. By using either one of the methods, the resulting probability for individual probability constraint problem is the same for all of the constraints. After the probability is calculated, the individual probability constraint problem would have the following form:

$$\begin{aligned}
\max_{x} \quad & (C_{35} + C_{electricity}\alpha_{Tur1} - C_{900})x_{Tur1} \\
& +(C_{electricity}\alpha_{Gen1} - C_{900})x_{Gen1} \\
& +(C_{35} + C_{electricity}\alpha_{Tur2} - C_{900})x_{Tur2} \\
& +(C_{electricity}\alpha_{Gen2} - C_{900})x_{Gen2} \\
& +(C_{35} - C_{900})x_{PLS} + C_{electricity}(\beta_{TG1} + \beta_{TG2}) \\
\text{Subject to:} \quad & \Pr\left\{ \begin{array}{c} x_{Tur1} + x_{Gen1} + x_{Tur2} + \\ x_{Gen2} + x_{PLS} + x_{900_users} \leq x_{900_suppliers} \end{array} \right\} \geq \alpha' \\
& \Pr\left\{x_{450_suppliers} \geq x_{450_users}\right\} \geq \alpha' \\
& \Pr\left\{x_{160_suppliers} \geq x_{160_users}\right\} \geq \alpha' \\
& \Pr\left\{x_{Tur1} + x_{Tur2} + x_{PLS} + x_{35_suppliers} \geq x_{35_users}\right\} \geq \alpha' \\
& \Pr\left\{\alpha_{Tur1}x_{Tur1} + \alpha_{Gen1}x_{Gen1} + \beta_{TG1} \leq E_{TG1_max}\right\} \geq \alpha' \\
& \Pr\left\{\alpha_{Tur2}x_{Tur2} + \alpha_{Gen2}x_{Gen2} + \beta_{TG2} \leq E_{TG2_max}\right\} \geq \alpha' \\
& \Pr\left\{x_{Tur1} \geq x_{Tur1_min}\right\} \geq \alpha' \\
& \Pr\left\{x_{Tur2} \geq x_{Tur2_min}\right\} \geq \alpha'
\end{aligned} \tag{4.8}$$

The danger of using this approximation method is that the resulting probability for each of the individual probability constraint can be very conservative and it becomes more conservative as the number of inequality constraints increases. Nevertheless, it is easier to transform individual probability constraint into a linear programming equivalent problem such that the solution techniques available for linear programming can be used to solve for chance constraint problems.

4.3.2 Transforming IPC to LP: Back-Off Solution

After the individual probability constraint optimization problem is obtained, the problem still could not be solved using solution techniques available to linear programming problems in its present form. Nevertheless, it is possible to transform the problem from individual probability constraint problem to a linear programming problem through the back-off approach.

As shown by [Zhao et al., 2009], consider a probability constraint with the following form:

$$\Pr(ax \leq b) \geq \alpha \tag{4.9}$$

After normalizing the constraint with mean and standard deviation of vector b, Equation (4.9) becomes:

$$\Pr(\frac{ax - \bar{b}}{\sigma_b} \leq \frac{b - \bar{b}}{\sigma_b}) \geq \alpha \tag{4.10}$$

The Equation can be simplified to:

$$\Pr(\frac{ax - \bar{b}}{\sigma_b} \leq \xi) \geq \alpha \tag{4.11}$$

where ξ is the Z-score of the right hand side vector and

$$\frac{b - \bar{b}}{\sigma_b} = \xi \tag{4.12}$$

By definition from a normal distribution, the probability of $Z \geq z_{\alpha/2}$ is given as:

$$\Pr(Z \geq z_{\alpha/2}) = \frac{\alpha}{2} \tag{4.13}$$

By extension, in order to obtain a probability of α, the normalized b has to be either greater than $-Z_\alpha$ or less than Z_α

$$\Pr(\xi \geq -z_\alpha) = \alpha \tag{4.14}$$

or

$$\Pr(\xi \leq z_\alpha) = \alpha \tag{4.15}$$

The original probability constraint in Equation (4.9) states that the probability of $ax \leq b$ has to be greater than or equal to α. In other words, the cumulative probability of the bell distribution curve has to be greater than or equal to α when the tail end reaches point b. To achieve that, the normal distribution curve on b has to move towards $-z_\alpha$ rather than z_α. Solving the system simultaneously with Equation (4.11) and Equation (4.14) results in

$$ax \leq \bar{b} - z_\alpha \sigma_b \tag{4.16}$$

The result of the derivation in Equation (4.16) shows that the constraint is backed off from its original constraint location by moving into the feasible region, which makes physical sense since the idea of incorporating uncertainties into the optimization problem requires the optimal solution to be backed off from its original edge to withstand the possible uncertainties in the system. Figure 4.4 shows the locations of the new constraint box, indicated by the thick dotted box, and the original constraints box, indicated by the thick solid box.

Recall that the constraint box shows how the optimal solution would change when uncertainties are introduced. In the case of a new constraint box, new optimal solutions caused by uncertainties still remain in the feasible region bounded by the original constraints. Therefore, the new optimal solution obtained is able to remain feasible when variations caused by uncertainties are within the feasible probability provided.

Similarly, a back off solution for three other possible chance constraints with different direction of inequalities compared to Equation (4.9) can be obtained using a similar method.

If the original probability constraint has the following form:

$$\Pr(ax \geq b) \leq \alpha \tag{4.17}$$

The back-off solution is:

$$ax \leq \bar{b} - z_\alpha \sigma_b \tag{4.18}$$

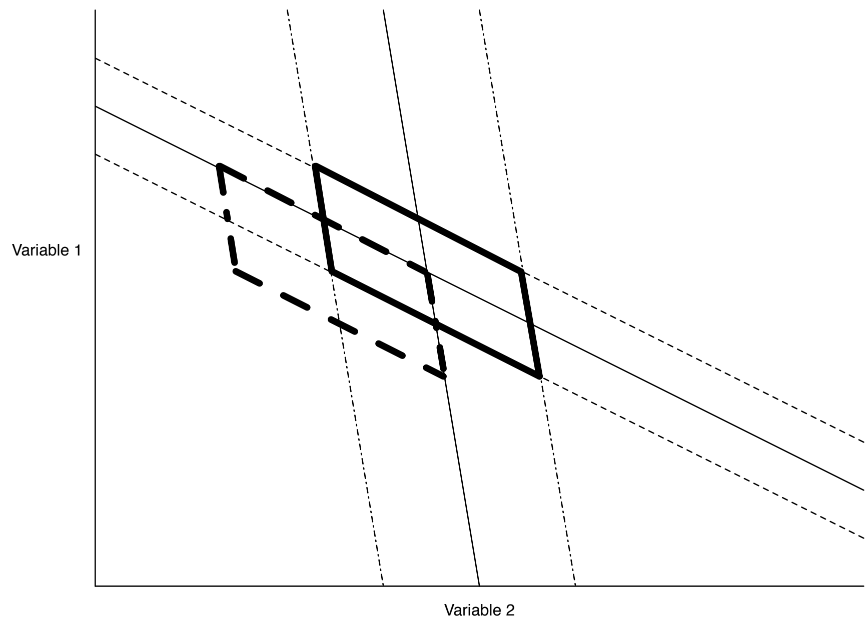


Figure 4.4: Back-Off Solution for $\Pr(ax \leq b) \geq \alpha$

Since the back off solution in Equation (4.18) is the same as Equation (4.16), Figure 4.4 also shows how the constraint box is moved when Equation (4.17) is the original probability constraints.

If the original probability constraint is of the following two forms:

$$\Pr(ax \geq b) \geq \alpha \tag{4.19}$$

$$\Pr(ax \leq b) \leq \alpha \tag{4.20}$$

The back-off solution for Equation (4.19) and Equation (4.20) can be obtained as:

$$ax \geq \bar{b} + z_{\alpha}\sigma_b \tag{4.21}$$

Figure 4.5 shows how the constraint box is moved due to back off solution compared to its original constraint box location.

4.4 Case Study

For this thesis, the feasible probability is set at 95% for the joint probability constraint optimization problem. Nevertheless, the feasible probability is an arbitrary

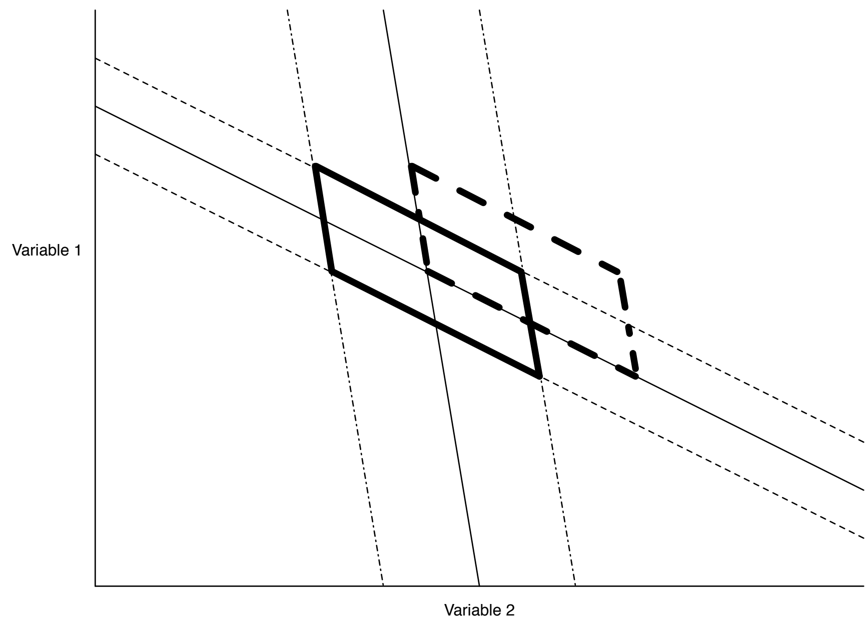


Figure 4.5: Back-Off Solution for $\Pr(ax \geq b) \geq \alpha$ and $\Pr(ax \leq b) \leq \alpha$

value and it can be changed to suit different needs.

The probability for each of the constraints in individual probability constraint optimization problem is calculated to be 98.33% using Boole's inequality and 98.30% if the constraints are independent events. The results from Boole's inequality and independent events provide the upper and lower bounds respectively for the individual constraint probability.

Since the probability for each of the individual constraints calculated using Boole's inequality is higher than the probability calculated using independent events, the result from Boole's inequality is the probability that the process has to back off to. This is the case because the optimization problem would be able to remain feasible from both Boole's inequality and independent events' perspectives if the process is backed off using 98.33%.

The corresponding Z-scores for probabilities obtained using Boole's inequality is 2.13. After obtaining the Z-score of the probabilities, the deterministic optimization problem is backed off with appropriate equations.

After incorporating all of the information into the optimization problem, the stochastic optimization result for the decision variables under the current electricity price of 6 ¢/kW is shown in Table 4.1

Flow Rate (lb/hr), x^*	Turbine 1	0
	Generator 1	7,000
	Turbine 2	26,900
	Generator 2	7,000
	Pressure Letdown Station	0
Profit (\$/kW), p^*	-89.040	
Shadow Price, λ^*	900 psi Header (lb/hr)	-0.0006
	450 psi Header (lb/hr)	0
	160 psi Header (lb/hr)	0
	35 psi Header (lb/hr)	0
	Electricity Generation 1 (kW)	0
	Electricity Generation 2 (kW)	0
	Generator 1 Extraction Flow Rate (lb/hr)	-0.0043
	Generator 2 Extraction Flow Rate (lb/hr)	-0.0033

Table 4.1: Nominal Case Optimization Result

Table 4.1 shows the optimal solution for decision variables, objective function value and the shadow prices from the first to third row respectively. The optimal operating strategy for decision variables is given in the first row. Under this stochastic optimization scheme, the optimal objective function value obtained is \$-89.040 per hour as evident from the second row. This result is slightly more costly compared to the result of \$-83.210 per hour obtained from deterministic optimization problem, however, it is still significantly better than the current operation of \$-147.903 per hour.

Besides the difference in objective function value, the operating strategy is very similar to that determined from deterministic optimization problem. There is less steam passing through the Turbine 2 because the steam is conserved to handle uncertainties in the process. In this case, since all of the steam demand is satisfied from sources other than the steam flow rates from decision variables; the 900 psi steam is conserved to handle potential process uncertainties. As a result, the stochastic optimization problem would conserve some amount of the 900 psi steam to deal with any difference in the demand of 900 psi steam compared to the demand expected by the optimization model. Under certain circumstances when the steam demand does not vary much, the excess steam may be vented through the strategies developed in Chapter 3 to gain a better profit by reducing the operating cost.

Similar to the sensitivity analysis performed in Chapter 3, the shadow price, λ, represents the rate of change in objective function value with respect to unit change in vector b without changing the active constraint set. For a unit change in vector b without changing the active constraint set, the new objective function can be determined using Equation (3.20).

Table 4.2 shows whether a constraint is active or inactive at the optimal solution. A constraint is considered "active" if the inequality constraint becomes an equality constraint at the optimal solution. It can be seen that the constraints for 900 psi Header, Extraction Flow Rates from Generator 1 and Generator 2 are active. On the other hand, the constraints for 35 psi Header, and Electricity Generation 1 and 2 are inactive and they are respectively 34,000 lb/hr, 2854.279 kW, and 1806.091 kW away from reaching the boundary to become active constraints.

	Lower	Level	Upper
900 psi Header (lb/hr)	-40,900	-40,900	+ Inf
450 psi Header (lb/hr)	0	0	0
160 psi Header (lb/hr)	0	0	0
35 psi Header (lb/hr)	-7,100	26,900	+ Inf
Electricity Generation 1 (kW)	- Inf	633.5000	3,487.7790
Electricity Generation 2 (kW)	- Inf	1,921.950	3,728.0405
Generator 1 Extraction Flow Rate (lb/hr)	7,000	7,000	+ Inf
Generator 2 Extraction Flow Rate (lb/hr)	7,000	7,000	+ Inf

Table 4.2: Constraint Status for Optimal Operation Using Stochastic Programming

According to Equation (4.16), the back-off location of the constraint box depends on the α. Therefore, as α gets larger, the constraint box would move further into the feasible region to result in a more conservative result.

4.5 Various Scenarios for Changing Process and Economic Conditions

Since the back-off solution is a linear programming equivalent problem, any of the matrix A and vectors b and c could vary similarly to the various cases discussed in Chapter 3. In this section, the scenarios will be visited again for a comparison against the results obtained using deterministic linear programming.

4.5.1 Changes in Economic Condition

The Table 4.3 shows the operating strategy for steam distribution at different electricity price levels.

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	0	7,000	0
4.8 ≤ E < 11.2	0	7,000	26,900	7,000	0
E ≥ 11.2	0	7,000	0	33,900	0

Table 4.3: Optimization Result for Varying Electricity Price

Similar to the case studied using deterministic optimization problem, the result shows multiple distinct operating strategies corresponding to different electricity price levels. The operating strategy seems to follow that of the deterministic optimization problems with the lowest three levels of electricity prices. When electricity price is below 4.8 ¢/kW, the 900 psi steam is only used to satisfy the minimum steam requirement to keep the generators in operation because the 900 psi steam is more valuable than electricity and 35 psi steam. When the electricity price is between 4.8 ¢/kW and less than 11.2 ¢/kW, all of the 900 psi steam available in the common header is passed down to the 35 psi steam common header through Turbine 2 because the combination of 35 psi steam and electricity is more valuable than the value from 900 psi steam. When the electricity price is greater than 11.2 ¢/kW, all of the available 900 psi steam is turned into electricity via the use of Generator 2 because electricity is the most valuable product at this given market condition.

The associate objective function shows a similar increasing trend when electricity price increases as the objective function values obtained using deterministic optimization problem shown in Table 3.5. The break even price for electricity is 11.4 ¢/kW.

4.5.2 Changes in Steam Supply and Demand

Parameters in vector b are changed to study the effects of uncertainties in steam supply and demand on the optimal solution. In Chapter 3, the vector b was allowed to change by one standard deviation for the case study. To illustrate that the back off technique would guarantee feasibility for a pre-determined percentage, the parameters in vector b are allowed to change by a Z-score of 2.13.

Nine cases were developed to simulate the effect of varying vector b similar to the Table 3.6 in Chapter 3. The difference here is rather than having variations in the “actual" user demand, it is actually the difference between the expected process conditions caused by potential uncertainties and the actual process conditions.

For example, the actual user demand in Case 1 is lower than the expected extra user demand used in the stochastic optimization problems in anticipation for the uncertainties.

The results for the nine cases are obtained as:

Case 1:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	37,500	7,000	0
$11.2 \leq E < 12.2$	0	7,000	16,452	28,048	0
$12.2 \leq E < 14.2$	9,788	7,000	0	34,712	0
$E \geq 14.2$	0	16,788	0	34,712	0

Table 4.4: Case 1 Result

Table 4.4 shows the result for Case 1 with varying electricity price. The break even price for electricity is 10.1 ¢/kW. After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 11,700 lb/hr of 35 steam available in the 35 psi common header when the user demands are lower than the expected extra demand for both of 900 psi and 35 psi steam users.

Case 2:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	37,500	7,000	0
$11.2 \leq E < 12.2$	0	7,000	16,452	28,048	0
$12.2 \leq E < 14.2$	9,788	7,000	0	34,712	0
$E \geq 14.2$	0	16,788	0	34,712	0

Table 4.5: Case 2 Result

Table 4.5 shows the result for Case 2 with varying electricity price. The break even price for electricity is 10.1 ¢/kW. After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 7,100 lb/hr of 35 steam available in the 35 psi common header when the user demands are lower than the expected extra demand for 900 psi users and meeting the expected extra demand from 35 psi steam users.

Case 3:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	37,500	7,000	0
$11.2 \leq E < 12.2$	0	7,000	16,452	28,048	0
$12.2 \leq E < 14.2$	9,788	7,000	0	34,712	0
$E \geq 14.2$	0	16,788	0	34,712	0

Table 4.6: Case 3 Result

Table 4.6 shows the result for Case 3 with varying electricity price. The break even price for electricity is 10.1 ¢/kW. After accounting for user requirement, there is an excess of 51,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 3,700 lb/hr of 35 steam available in the 35 psi common header when the user demands are lower than the expected extra demand for 900 psi users and higher than the expected extra demand for 35 psi steam users.

Case 4:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	26,900	7,000	0
$E \geq 11.2$	0	7,000	0	33,900	0

Table 4.7: Case 4 Result

Table 4.7 shows the result for Case 4 with varying electricity price. The break even price for electricity is 11.4 ¢/kW. After accounting for user requirement, there is an excess of 40,900 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 11,700 lb/hr of 35 steam available in the 35 psi common header when the user demands are meeting the expected extra demand for 900 psi users and lower than the expected extra demand for 35 psi steam users.

Case 5:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	26,900	7,000	0
$E \geq 11.2$	0	7,000	0	33,900	0

Table 4.8: Case 5 Result

Table 4.8 shows the result for Case 5 with varying electricity price. The break even price for electricity is 11.4 ¢/kW. After accounting for user requirement, there is an excess of 40,900 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 7,100 lb/hr of 35 steam available in the 35 psi common header when the user demands are meeting the expected extra demand for both 900 psi users and 35 psi steam users.

Case 6:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	26,900	7,000	0
$E \geq 11.2$	0	7,000	0	33,900	0

Table 4.9: Case 6 Result

Table 4.9 shows the result for Case 6 with varying electricity price. The break even price for electricity is 11.4 ¢/kW. After accounting for user requirement, there is an excess of 40,900 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 3,700 lb/hr of 35 steam available in the 35 psi common header when the user demands are meeting the expected extra demand for 900 psi users and higher than the expected extra demand for 35 psi steam users.

Case 7:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	16,500	7,000	0
$E \geq 11.2$	0	7,000	0	23,500	0

Table 4.10: Case 7 Result

Table 4.10 shows the result for Case 7 with varying electricity price. The break even price for electricity is 12.8 ¢/kW. After accounting for user requirement, there is an excess of 30,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 11,700 lb/hr of 35 steam available in the 35 psi common header when the user demands are higher than the expected extra demand for 900 psi users and lower than the expected extra demand for 35 psi steam users.

Case 8:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	16,500	7,000	0
$E \geq 11.2$	0	7,000	0	23,500	0

Table 4.11: Case 8 Result

Table 4.11 shows the result for Case 8 with varying electricity price. The break even price for electricity is 12.8 ¢/kW. After accounting for user requirement, there is an excess of 30,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 7,100 lb/hr of 35 steam available in the 35 psi common header when the user demands are higher than the expected extra demand for 900 psi users and meeting the expected extra demand for 35 psi steam users.

Case 9:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 4.8$	0	7,000	0	7,000	0
$4.8 \leq E < 11.2$	0	7,000	16,500	7,000	0
$E \geq 11.2$	0	7,000	0	23,500	0

Table 4.12: Case 9 Result

Table 4.12 shows the result for Case 9 with varying electricity price. The break even price for electricity is 12.8 ¢/kW. After accounting for user requirement, there is an excess of 30,500 lb/hr of 900 psi steam available in the 900 psi common header and an excess of 3,700 lb/hr of 35 steam available in the 35 psi common header when the user demands are higher than the expected extra demand for both 900 psi users and 35 psi steam users.

There is one major difference between the first three cases and the last six cases. The difference between the two groups is the number of price levels available for the operating strategy at different electricity prices. This is due to the increased steam demand from the 900 psi steam header that reduces the amount of steam available in the common header, hence reducing the need to operate Turbine 1 at high electricity prices for the last six cases.

Electricity Price	Decision Variable Strategies
E < 4.8	Only run Generator 1 and 2 at minimum flow rate
4.8 ≤ E < 11.2	Run Generator 1 and 2 at minimum flow rate and the rest of the steam pass through Turbine 2
11.2 ≤ E < 12.2	Run Generator 1 at minimum flow rate and pass the rest of the steam through Turbine Generator 2 to maximize electricity generation
12.2 ≤ E < 14.2	Run Generator 2 at maximum flow rate for maximum electricity generation and Generator 1 at minimum flow rate and pass the rest of the steam through Turbine 1. If not enough 900 psi steam is available, continue the strategy from the last price level.
E ≥ 14.2	Run Generator 2 at maximum flow rate and pass the rest of the steam through Generator 2. If not enough 900 psi steam is available, continue the strategy from the last price level.

Table 4.13: Summary of Operating Strategies with Changing b and c Vectors

Summary of the Operating Strategies for Changing Right Hand Side Vector

Since there are 9 cases simulated for varying electricity prices and varying user demands from 900 psi and 35 psi steam users, a general trend among these cases was observed. An overall strategy observed from these results can be found in Table 4.13.

The operating strategy obtained using the probability constraint is similar to the strategy obtained from deterministic optimization problem except there is less 900 psi steam available to distribute among decision variables. When there is not enough 900 psi steam available for distribution between the five decision variables, then the operating strategy at that electricity price level would take on the operating strategy of the previous lower electricity price level.

Special Cases

Similar to Chapter 3, all of the nine cases studied have excess steam in the 35 psi common header, which means that it is not necessary to use any of the decision variables to satisfy the additional demand of 35 psi steam users. Two special cases were developed to simulate cases where 35 psi steam is required from the distribution of the 900 psi steam. The special cases were developed using both Case 2 and Case 9 as basis with the Package Boiler, which supplies 35 psi steam, turned off. The results were obtained as the following:

Special Case 2:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	3,900	7,000	0
4.8 ≤ E < 11.2	0	7,000	37,500	7,000	0
11.2 ≤ E < 12.2	0	7,000	16,452	28,048	0
12.2 ≤ E < 14.2	9,788	7,000	0	34,712	0
E ≥ 14.2	3,900	12,888	0	34,712	0

Table 4.14: Special Case 2 Result

Special Case 9:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	7,000	7,300	7,000	0
4.8 ≤ E < 11.2	0	7,000	16,500	7,000	0
E ≥ 11.2	0	7,000	7,300	16,200	0

Table 4.15: Special Case 9 Result

Similar to the special cases developed in Chapter 3, the operating strategies obtained in Table 4.14 and Table 4.15 are similar to the regular cases shown in Case 2 and Case 9 except a portion of the 900 psi steam is conserved to satisfy the demand of 35 psi steam.

4.5.3 Changing Process Information

After variations in b and c vectors were simulated for the stochastic optimization problem, the scenario where process equipment fails is simulated for the effects of changing A matrix in the stochastic optimization problem. The scenarios were developed similar to the ones used in Chapter 3 where Turbine Generator 1 and Turbine Generator 2 were turned off one at a time to see the effects on operating strategy. The results for the various cases are shown below.

No Turbine Generator 1:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	0	0	7,000	0
4.8 ≤ E < 11.2	0	0	33,900	7,000	0
E ≥ 11.2	0	0	10,410	30,499	0

Table 4.16: No Turbine Generator 1

Similar to the result obtained when the process equipment fails under deterministic optimization problem, there are only three different operating strategies required at three different ranges of electricity unit price. The price at which the operating strategy changes for the stochastic model in Table 4.16 is the same as the price for the operating strategy under deterministic optimization problem in Table 3.19. The result shown in Table 4.16 is in fact the same as the result shown in Table 3.19 except for the different levels of steam being distributed. This is obvious since the stochastic optimization result reserves part of the steam in the headers in case of a swing in the steam usage.

No Turbine Generator 2:

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
$E < 5.2$	0	7,000	0	0	0
$5.2 \leq E < 14.2$	33,900	7,000	0	0	0
$E \geq 14.2$	4,231	36,669	0	0	0

Table 4.17: No Turbine Generator 2

Similarly, the result shown in Table 4.17 is the same as the result shown in Table 3.20 obtained from deterministic optimization problem. The only difference is again the amount of steam distributed through certain decision variables.

Both scenarios developed for cases when turbine generators fail using stochastic optimization problems show how optimal solutions would vary and how the process should be operated at various electricity price levels, however, neither of these scenarios requires additional steam from the 900 psi common header for distribution to satisfy users of all levels of steam. This is the case because there is enough 35 psi steam from suppliers that satisfy the user demand. As a result, before the optimization problem determines how to allocate the excess steam in the system, there is already an excess of 7,100 lb/hr of 35 lb steam in the system. Therefore, two special cases were developed to study how the system would react when that excess steam is no longer present in the system. To do so, the flow rate from the package boiler is turned off, reducing the amount of 35 psi steam supplied by 11, 000 lb/hr.

The results for the above two cases without 35 psi steam supplied from the package boiler are shown below.

Special Case with No Turbine Generator 1 and No Package Boiler

Special Case with No Turbine Generator 2 and No Package Boiler

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 4.8	0	0	3,900	7,000	0
4.8 ≤ E < 11.2	0	0	33,900	7,000	0
E ≥ 11.2	0	0	10,410	30,499	0

Table 4.18: No Turbine Generator 1 and No Package Boiler

Electricity Price (¢/kW)	Tur1 (lb/hr)	Gen1 (lb/hr)	Tur2 (lb/hr)	Gen2 (lb/hr)	PLS (lb/hr)
E < 5.2	3,900	7,000	0	0	0
5.2 ≤ E < 14.2	33,900	7,000	0	0	0
E ≥ 14.2	4,231	36,669	0	0	0

Table 4.19: No Turbine Generator 2 and No Package Boiler

The results from the two special cases were very similar to the cases with failures in turbine generators shown in Chapter 3, except that the amount of steam that passes through the functional turbine when electricity price is at the lowest level. This is the case because additional 35 psi steam is required to satisfy user demand, and the strategy is modified to reflect that.

4.5.4 Summary

The operating strategy obtained from the stochastic optimization problem would result in a cost operation of $-89.040 per hour. Compared to the result obtained from deterministic optimization problem, which has the cost operation of $-83.210 per hour, the result obtained from the stochastic optimization problem is more conservative and is $5.83 per hour more expensive. Nevertheless, the solution obtained from the stochastic optimization problem is capable of dealing with uncertain events in the process for a desired percentage of the time and the “desired percentage" for the feasibility in this study was set to be 95%. By having a more conservative result, the solution from the stochastic optimization problem holds on to more steam in the common headers to deal with potential increase of steam usage from the users. If the users do indeed increase their demand, the extra steam available in the common header can be used to satisfy the increase in demand. If the users do not increase their demand, the extra steam can still go down to lower pressure common header with the strategies described in Chapter 3.

Since the chance constraint optimization problem is reformulated into linear programming equivalent problems, the matrix A and vectors b and c in the stochastic optimization problem can vary in similar ways to those in linear programming. Scenarios were simulated when the c vector is allowed to vary and the result obtained is very similar to the result obtained for deterministic optimization problems, except there are only three levels of electricity price needed for different operating strategies compared to the five needed for deterministic optimization problems. In fact, only three levels of electricity prices are needed for six of the nine cases developed for a combination of variations in vector b in stochastic optimization problem. The only three cases that require five levels of electricity price are the cases when the variation in b vector is based on lower than expected extra demand for 900 psi steam users. This is the case because as the amount of available 900 psi steam decreases, it reduces the need to pass the excess steam through additional equipment. Similarly, the results obtained from variations in A matrix are also similar to the ones obtained from deterministic optimization problem.

Overall, the operating strategy obtained from the stochastic optimization problems is very similar to the operating strategy obtained from the deterministic optimization problem. The main difference is that solutions from the stochastic optimization problems are more conservative in terms of the cost associated with the operation. Nevertheless, the operating strategies between the five decision variables remain the same.

Chapter 5

Conclusion and Future Work

One of the general philosophies for operating companies in today's economy is to maximize profitability through the efficient allocation of resources while reducing their environmental impact. Even though various optimization schemes have been developed to achieve these goals separately, the ability to meet these objectives together while facing uncertainties has gained importance. These ideas have led to the study of stochastic linear programming to optimize operating strategies.

In this thesis, an optimization study was performed for a steam network at a utility plant. The objective of this study was to find an optimal operating strategy at a given process and economic condition while taking into account for uncertainties.

Before formulating the optimization problem, the steam network and turbine generator models were required to describe the process. The steam network was modelled using steady state material balances around each of the common headers and the turbine generator models were modelled using empirical model for its form and parameters.

After the models were developed for the steam network and the Turbine Generators, the optimization model was formulated using linear programming. The objective function was formulated to maximize the profit where 35 psi steam and electricity were considered as revenue generating products, and 900 psi steam was considered as raw material with cost. The constraints were formulated using process information such as positive flow in each of the common headers, maximum electricity generation limit from turbine generators, and minimum flow rates to generators. Since nominal values were used to determine some of the parameters in the optimization model without any consideration for uncertainties, the optimization model formulated was considered deterministic.

The optimal solution to the deterministic optimization model was compared with the current operating condition and the optimal operating strategy was able to save roughly \$80 an hour or \$700,000 every year when electricity price is at 6 ¢/kW. Various scenarios were tested to imitate the effects on the optimal solution when the process and/or the economic condition changes.

Since there could be uncertainties associated with economic condition (vector c), supply and demand levels of steam (vector b), and process information (matrix A), the operating condition determined by the deterministic optimization problem could have become infeasible. To rectify the issue, chance constraints were used to explicitly incorporate the feasible probability of the optimal solution remaining in the feasible region into the optimization problem.

To take advantage of the solving techniques available, the chance constraint optimization problem was reformulated into linear programming equivalent problem using back-off technique. The resulting optimization problem produced a more conservative result than the deterministic optimization problem, but the result was still better than the current operation.

Based on the case studies investigated, the summary of the operating strategies from deterministic optimization model depends largely on the electricity price range. When the electricity is below a certain price level, the 900 psi steam is conserved because of its value. As the electricity price increases, that operating strategy shifts in favour of the combined production of electricity and 35 psi steam. As the electricity price increases further, the operating strategy favours more towards the generation of electricity as expected. The study also found that Turbine Generator 2 is more efficient than Turbine Generator 1. Therefore, the steam distribution is increased for Turbine Generator 2 for maximum electricity generation before increasing the steam flow to Turbine Generator 1 as electricity price increases from one level to the next. Last but not least, the pressure letdown stations were never used because Turbine Generator 1 and Turbine Generator 2 were capable of handling the available steam in the system economically. In this case, pressure letdown stations are only needed for safety reasons to relieve excess steam in the system. When the availability of steam in the common headers differs due to varying matrices A or b, the operating strategies developed above remain true for these scenarios.

5.1 Future Work

Although work was done to determine the optimal operating strategies under normal operating conditions as well as special conditions by handling uncertainties using probability constraints, there are many issues remaining to be examined. Potential areas to be explored include the expansion of the decision variables list, different formulation of the objective function and constraints, and the effects of varying parameters in matrix A.

The optimization models used for the thesis included five decision variables, which were Turbine 1, Generator 1, Turbine 2, Generator 2, and Pressure Letdown Station for steam from 900 psi header to 35 psi header while assuming other variables to be constant and could not be manipulated. The list of decision variables can be expanded to include other decision variables such as natural gas inputs to boilers for 900 psi steam output, package boiler output and other flow rates to pressure letdown stations in the system. Expanding the list of decision variables in the optimization models will provide an opportunity for operators to manipulate the availability of steam in the system, which could potentially enhance the optimization result by ensuring steam is only generated when it is required.

Currently, the objective function was formulated using one stage objective function, which means that at a given process and economic condition, an optimal operating strategy is given for that particular condition and there is no regard for what would happen afterward until the optimization model is compiled again. A different formulation of the objective function could have included a penalty cost to the objective function so that the objective function value would reflect the cost of having excess steam in the steam headers. Similar result could probably be achieved through the formulation of Multi-Stage Objective Function.

One of the assumptions used in formulating the optimization model was that the constraints used in the model represent the absolute process limit that shall not be violated. Since the constraints were formulated based on the average of the data, the true limit or the boundary of the constraint for the process may not be the one used in the optimization model. Therefore, a true limit or boundary of the process constraints should be found and used in the optimization model. Another way of dealing with the issue is through the use of the conditional constraints, which allows the constraints to be violated but less than a given absolute value.

Lastly, this thesis treated process equipment failure scenarios as simplified scenarios for the impact of changing matrix A on the optimal solution. No derivation work was provided in this thesis to study how changing parameters in matrix A would impact the optimal solution. Therefore, a rigorous derivation to relate changing parameter in matrix A and the optimal solution could be part of the future work.

Bibliography

Yunus A. Çengel and Michael A. Boles. *Thermodynamics: An Engineering Approach*. McGraw-Hill, fifth edition, 2006.

Edwin K. P. Chong and Stanislaw H. Zak. *An Introduction to Optimization*. John Wiley & Sons, third edition, 2008.

Gerorge B. Dantzig and Mukund N. Thapa. *Linear Programming : Introduction*, volume 1. Springer, 1997.

Richard M. Felder and Ronald W. Rousseau. *Elementary Principles of Chemical Processes*. John Wiley & Sons, third edition, 2005.

L. Halasz, A.B. Nagy, T. Ivicz, F. Friedler, and L.T. Fan. Optimal retrofit design and operation of the steam-supply system of a chemical complex. *Applied Thermal Engineering*, 22:939 – 947, 2002. ISSN 1359-4311.

In-Su Han, Young-Hak Lee, and Chonghun Han. Modeling and optimization of the condensing steam turbine network of a chemical plant. *Industrial & Engineeering Chemistry Research*, 45(2):670 – 680, 2006. ISSN 08885885.

Benjamin F. Hobbs. Optimization methods for electric utility resource planning. *European Journal of Operational Research*, 83(1):1 – 20, 1995. ISSN 03772217.

Chi-Wai Hui and Yukikazu Natori. An industrial application using mixed-integer programming technique: A multi-period utility system model. *Computers & Chemical Engineering*, 20:S1577 – S1582, 1996. ISSN 00981354.

IAPWS. Knovel steam tables [electronic resource] : an implementation of equations published by the international association for the properties ofwater and steam (iapws)., 2006.

B. Kalitventzeff. Mixed integer non-linear programming and its application to the management of utility networks. *Engineering Optimization*, 18(1-3):183, 1991. ISSN 0305215X.

N. Karmarkar. A new polynomial-time algorithm for linear programming. *Combinatorica*, 4(4):373, 1984. ISSN 02099683.

Leonid Khachiyan and Michael Todd. On the complexity of approximating the maximal inscribed ellipsoid for a polytope. *Mathematical Programming*, 61(1-3): 137, 1993. ISSN 00255610.

Jeong Hwan Kim and Chonghun Han. Short-term multiperiod optimal planning of utility systems using heuristics and dynamic programming. *Industrial & Engineeering Chemistry Research*, 40(8):1928 – 1938, 2001. ISSN 08885885.

Jeong Hwan Kim, Sangjun Ju, Heui-Seok Yi, In-Su Han, and Chonghun Han. Preventive optimization framework for unexpected equipment failures in the utility system with quantitative emergency handling constraints. *Industrial & Engineeering Chemistry Research*, 41(24):6070 – 6081, 2002. ISSN 08885885.

V. Klee and G. L. Minty. *How Good is the Simplex Algorithm?* Inequalities III. Academic Press, New York, 1972.

Ying Li and Peter Flynn. Electricity deregulation, sport price patterns and demand-side management. *ENERGY*, 31:908–922, 2006.

Lennart Ljung. *System identification: Theory for the User*. Prentice Hall, 1999.

Xianglong Luo, Bingjian Zhang, Ying Chen, and Songping Mo. Operational planning optimization of multiple interconnected steam power plants considering environmental costs. *Energy*, 2012. ISSN 0360-5442.

François Maréchal and Boris Kalitventzeff. Process integration: Selection of the optimal utility system. *Computers & Chemical Engineering*, 1998.

Sanjay Mehrotra. On the implementation of a primal-dual interior point method. *SIAM Journal on Optimization*, 2:575–601, 1992.

Sandra R. Micheletto, Maria C.A. Carvalho, and José M. Pinto. Operational optimization of the utility system of an oil refinery. *Computers & Chemical Engineering*, 32(Process Systems Engineering: Contributions on the State-of-the-Art Selected extended Papers from ESCAPE '16/PSE 2006.):170 – 185, 2008. ISSN 0098-1354.

Dayadeep S. Monder. Real-time optimization of gasoline blending with uncertain parameters. Master's thesis, University of Alberta, 2001.

Douglas C. Montgomery and George C. Runger. *Applied Statistics and Probability for Engineers*. John Wiley & Sons, fourth edition, 2007.

Katta G. Murty. *Linear Programming*. Wiley, first edition, 1983.

K. P. Papalexandri, E.N. Pistikopoulos, B. Kalitventzeff, M.N. Dumont, K. Urmann, and J. Gorschluter. Operation of a steam production network with variable demands modelling and optimization under uncertainty. *Computers & Chemical Engineering*, 20(European Symposium on Computer Aided Process Engineering-6):S763 – S768, 1996. ISSN 0098-1354.

Katerina P. Papalexandri, Efstratios N. Pistikopoulos, and B. Kalitventzeff. Modelling and optimization aspects in energy management and plant operation with variable energy demands-application to industrial problems. *Computers & Chemical Engineering*, 22(9):1319 – 1333, 1998. ISSN 00981354.

Soterios A. Papoulias and Ignacio E. Grossmann. A structural optimization approach in process synthesis—i. utility systems. *Computers and Chemical Engineering*, 7: 695 – 706, 1983. ISSN 0098-1354.

J.M. Pinto, M. Joly, and L.F.L. Moro. Planning and scheduling models for refinery operations. *Computers & Chemical Engineering*, 24:2259 – 2276, 2000. ISSN 0098-1354.

Patricia Velasco-Garcia, Petar Sabev Varbanov, Harvey Arellano-Garcia, and Günter Wozny. Utility systems operation: Optimisation-based decision making. *Applied Thermal Engineering*, 2011. ISSN 1359-4311.

Heui-Seok Yi and Chonghun Han. The integration of complete replanning and rule-based repairing for optimal operation of utility plants. *Korean Journal of Chemical Engineering*, 18(4):442 – 450, 2001. ISSN 02561115.

Heui-Seok Yi, Yeong-Koo Yeo, and Jin-Kuk Kim. A rule-based steam distribution system for petrochemical plant operation. *Industrial & Engineeering Chemistry Research*, 37(3):1051 – 1062, 1998. ISSN 08885885.

Heui-Seok Yi, Jeong Hwan Kim, Chonghun Han, Jae Hak Jung, Moon Yong Lee, and Jie Tae Lee. Periodical replanning with hierarchical repairing for the optimal operation of a utility plant. *Control Engineering Practice*, 11(Process Dynamics and Control):881 – 894, 2003. ISSN 0967-0661.

J. Zhang, X. X. Zhu, and G. P. Towler. A simultaneous optimization strategy for overall integration in refinery planning. *Industrial & Engineeering Chemistry Research*, 40(12):2640 – 2653, 2001. ISSN 08885885.

Chao Zhao, Yu Zhao, Hongye Su, and Biao Huang. Economic performance assessment of advanced process control with lqg benchmarking. *Journal of Process Control*, 19:557 – 569, 2009. ISSN 0959-1524.

Appendix A

Data Used

The process data for each of the equipment tags is obtained every minute from midnight of October 9, 2010 to midnight of October 21 2010. However, only the first 75 hours of data is used for the optimization problem because of the steady state properties.

The utilities cost information is only available as monthly data from October 2009 to September 2010.

From the data obtained, statistical properties such as mean and standard deviation can be obtained for each of the tags.

For nominal operation, the average flow rates are used for each of the suppliers and users of steam and the average monthly cost is used for both steam and electricity prices.

The average prices of $0.00921/lb and $0.00715/lb are used for 900 psi and 35 psi steam and the price of of $0.06/kW is used for the average electricity price.

Appendix B

Efficiency Calculation for Turbine Generators

From the available measurement, the average temperature and pressure at the inlet of the Turbine Generators #1 and #2 are 900 psi and 585°F. Based on the temperature and pressure measure, the specific enthalpy of the steam at the inlet of the Turbine Generators #1 and #2 are found to be 1248 Btu/lb from the IAPWS steam table [IAPWS, 2006]. Similarly, the the average temperature and pressure at the outlet of the Turbine Generators #1 and #2 are 35 psi and 350°F, and the specific enthalpy is 1213 Btu/lb. The average flow rates for Turbine Generator #1 inlet (900 psi steam), and outlet (35 psi steam) are 22,946 lb/hr, and 10,486 lb/hr respectively.

The total energy available to be converted to electricity is:

$$\triangle E_{Available} = (H_{900stm}x_{900stm} - H_{35stm}x_{35stm}) \tag{B.1}$$

Substituting the variables with numbers available

$$\triangle E_{Available} = 22,946 \text{ (lb/hr)} * 1,248 \text{ (Btu/lb)} - 10,485 \text{ (lb/hr)} * 1,213 \text{ (Btu/lb)}$$

$$\triangle E_{Available} = 15,918,303 \text{ (Btu/hr)} = 4,665 (kW)$$

Recall Equation (2.17) that the overall efficiency parameter can be obtained as:

$$\eta_{overall} = \frac{\triangle E_{Actual}}{\triangle E_{Available}} \tag{B.2}$$

With the average of the electricity measured to be 1,183 kW, the overall efficiency parameter can be calculated as:

$$\eta_{overall} = \frac{1,183(kW)}{4,665(kW)}$$

$$\eta_{overall} = 25.4\%$$

Therefore, the overall efficiency for Turbine Generator #1 is calculated to be 25.4%.

For Turbine Generator #2, the average 900 psi steam flow rate, and 35 psi steam flow rate and electricity generated are 19,620 lb/hr, 6,504 lb/hr, and 1,090 kW respectively. Using Equation (B.1), the total available energy is calculated to be 4,864 kW, which means the overall efficiency for Turbine Generator #2 is calculated to be about 22.4% using Equation (B.2).

Appendix C

Simple Linear Regression and Multiple Regression Results

C.1 Simple Linear Regression Result for Turbine Generator 2

For the cases when condensate flow rate is used as the input variable:

Model 1:

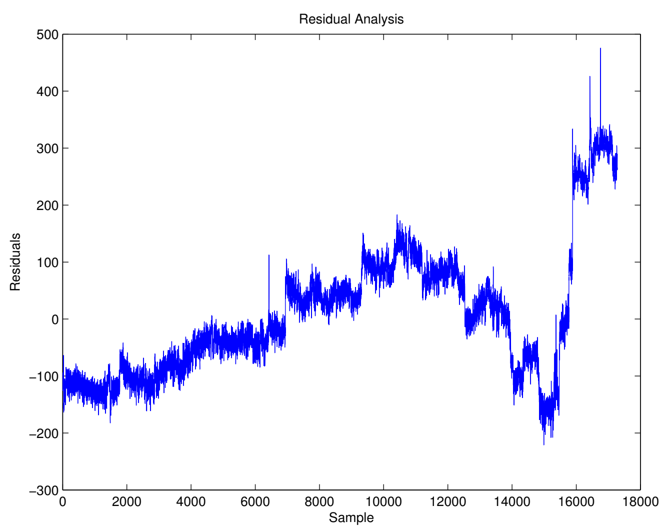


Figure C.1: Model 1 Using Condensate as the Input

$\beta_1 = 0.0861$. $R^2 = 47.44\%$

From the simulation, Model 1 with condensate flow rate is obtained as:

$$\hat{y} = 0.0861x \tag{C.1}$$

A coefficient of determination, R^2, of 47.44% is relatively poor for the model. From Figure C.1, there maybe nonlinearity or lack of fit in the model since there is an apparent trend in the residual plot. Overall, this model seems good for a model with one parameter, but further improvement seems possible.

Model 2:

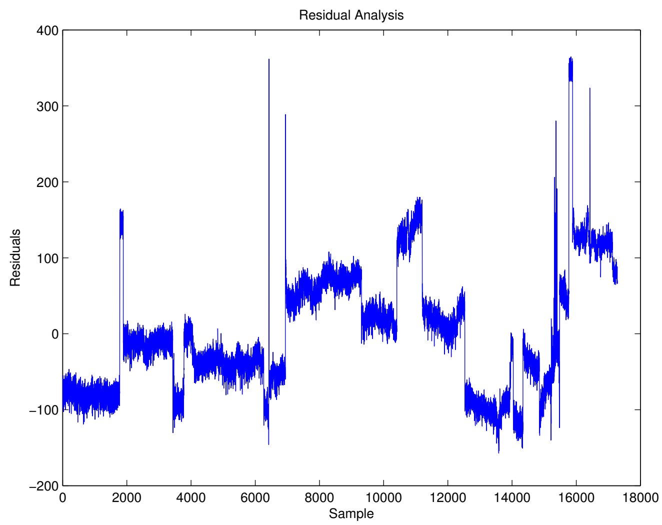


Figure C.2: Model 2 Using Condensate as the Input

$\beta_0 = 853.3855$ $\beta_1 = 0.0215$ $R^2 = 10.28\%$

From the simulation, Model 2 with condensate flow rate is obtained as:

$$\hat{y} = 853.3855 + 0.0215x \tag{C.2}$$

A coefficient of determination, R^2, of 10.28% is poor for the model especially considering that the model has one more parameter compared to Model 1 and exhibits a worse fitting result. Figure C.2 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is a poor model.

Model 3:

$\beta_1 = 0.1489$ $\beta_2 = -4.711 * 10^{-6}$ $R^2 = 14.90\%$

From the simulation, Model 3 with condensate flow rate is obtained as:

$$\hat{y} = 0.1489x - 4.711 * 10^{-6}x^2 \tag{C.3}$$

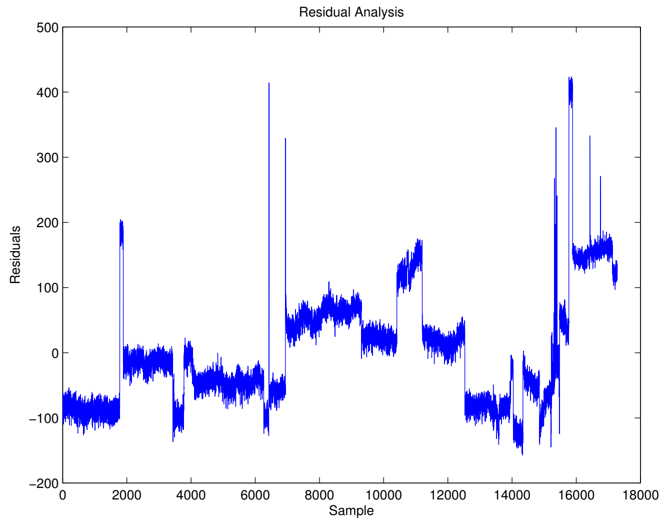


Figure C.3: Model 3 Using Condensate as the Input

A coefficient of determination, R^2, of 14.90% is poor for the model especially considering that the model has one more parameter compared to Model 1 and exhibits a worse fitting result. Figure C.3 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is a poor model.

Model 4:

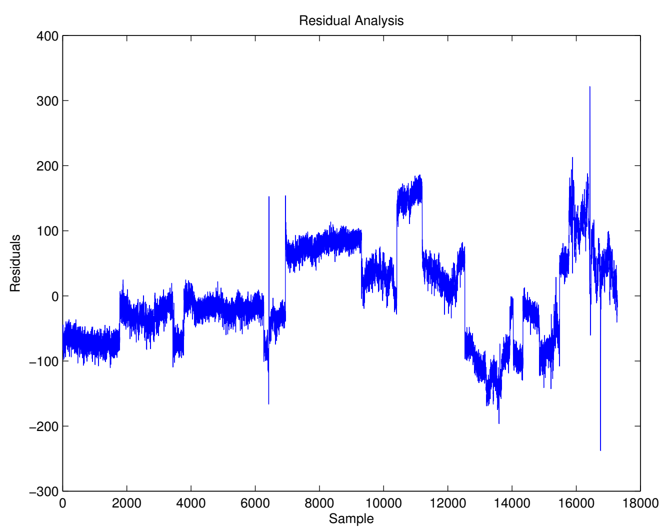


Figure C.4: Model 4 Using Condensate as the Input

$\beta_0 = 3081.4$ $\beta_1 = -0.300$ $\beta_2 = 1.343 * 10^{-8}$ $R^2 = 25.48\%$

From the simulation, Model 4 with condensate flow rate is obtained as:

$$\hat{y} = 3081.4 - 0.300x + 1.343 * 10^{-8}x^2 \tag{C.4}$$

A coefficient of determination, R^2, of 25.48% is poor for the model especially considering that the model has one more parameter compared to Model 2 and 3 and exhibits a worse fitting result. Figure C.4 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is a poor model.

After comparing the four models, it is determined that Model 1 is the best model among the four models proposed when condensate is used as the input variable. However, due to the its low fitting score, further improvement on the model with different model structure may be achieved.

For the cases when 35 psi steam flow rate flow rate is used as the input variable:

Model 1:

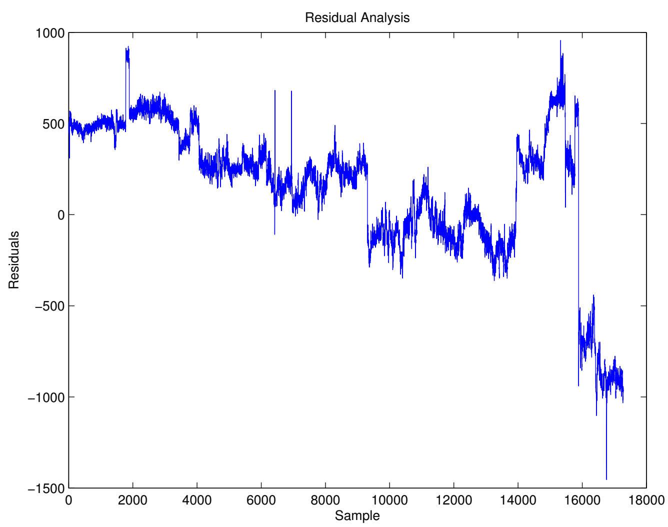


Figure C.5: Model 1 Using 35 psi Steam Flow Rate as the Input

$\beta_1 = 0.1297$. $R^2 = 51.62\%$

From the simulation, Model 1 with 35 psi steam flow rate flow rate is obtained as:

$$\hat{y} = 0.1297x \tag{C.5}$$

A coefficient of determination, R^2, of 51.62% is relatively poor for the model. From Figure C.5, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot. Overall, this model seems good for a model with one parameter, but further improvement seems possible.

Model 2:

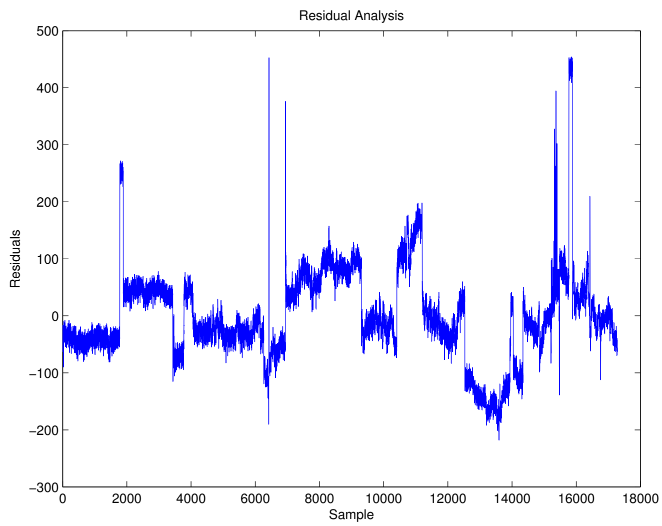


Figure C.6: Model 2 Using 35 psi Steam Flow Rate as the Input

$\beta_0 = 1077.8$ $\beta_1 = 7.581 * 10^{-6}$ $R^2 = 7.53\%$

From the simulation, Model 2 with 35 psi steam flow rate flow rate is obtained as:

$$\hat{y} = 1077.8 + 7.581 * 10^{-6}x \tag{C.6}$$

A coefficient of determination, R^2, of 7.53% is very poor for the model. Figure C.6 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is unacceptable.

Model 3:

$\beta_1 = 0.2480$ $\beta_2 = -1.159 * 10^{-5}$ $R^2 = 47.26\%$

From the simulation, Model 3 with 35 psi steam flow rate flow rate is obtained as:

$$\hat{y} = 0.2480x - 1.159 * 10^{-5}x^2 \tag{C.7}$$

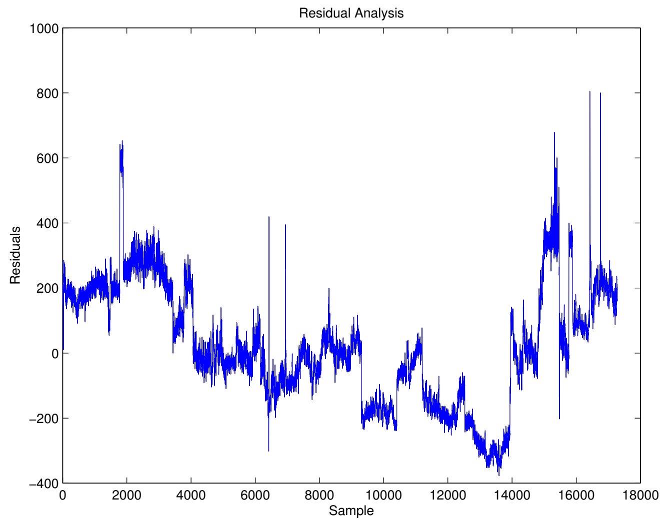


Figure C.7: Model 3 Using 35 psi Steam Flow Rate as the Input

A coefficient of determination, R^2, of 47.26% is poor for the model especially considering that the model has one more parameter compared to Model 1 and exhibits a worse fitting result. Figure C.7 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is a poor model.

Model 4:

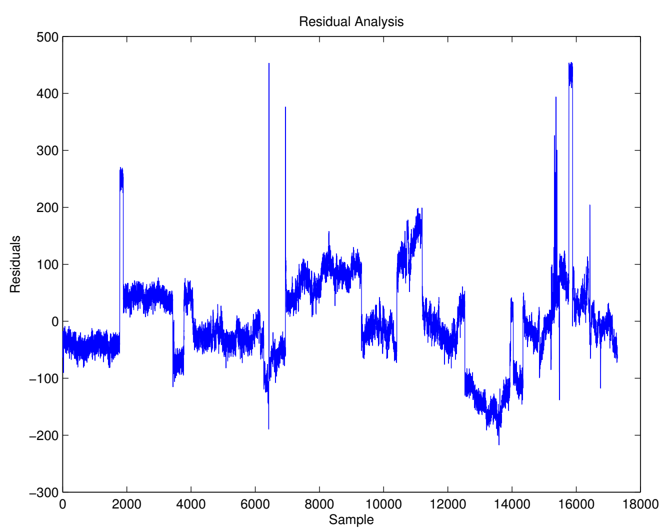


Figure C.8: Model 4 Using 35 psi Steam Flow Rate as the Input

$\beta_0 = 1083.7$ $\beta_1 = 6.143 * 10^{-6}$ $\beta_2 = 7.568 * 10^{-11}$ $R^2 = 7.55\%$

Note that this model failed the null hypothesis for β_2, which means that the parameter is statistically insignificant.

From the simulation, Model 4 with 35 psi steam flow rate flow rate is obtained as:

$$\hat{y} = 1083.7 + 6.143 * 10^{-6}x + 7.568 * 10^{-11}x^2 \quad \text{(C.8)}$$

A coefficient of determination, R^2, of 7.55% is poor for the model especially considering that the model has one more parameter compared to Model 2 and 3 and exhibits a worse fitting result. Figure C.8 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is unacceptable.

After comparing the four models, it is determined that Model 1 is the best model among the four models proposed when 35 psi steam flow rate is used as the input variable. However, due to the its low fitting score, further improvement on the model with different model structure may be achieved.

For the cases when 900 psi steam flow rate flow rate is used as the input variable:

Model 1:

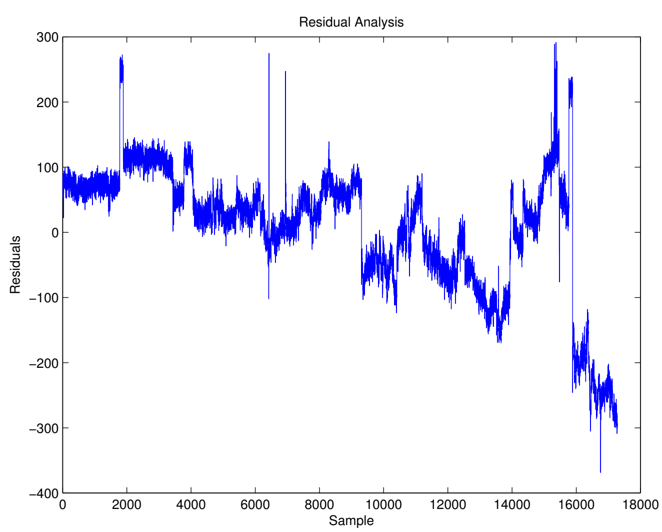


Figure C.9: Model 1 Using 900 psi Steam Flow Rate as the Input

$\beta_1 = 0.0545$. $R^2 = 59.10\%$

From the simulation, Model 1 with 900 psi steam flow rate flow rate is obtained as:

$$\hat{y} = 0.0545x \tag{C.9}$$

A coefficient of determination, R^2, of 59.10% is relatively poor for the model. From Figure C.9, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot. Overall, this model seems good for a model with one parameter, but further improvement seems possible.

Model 2:

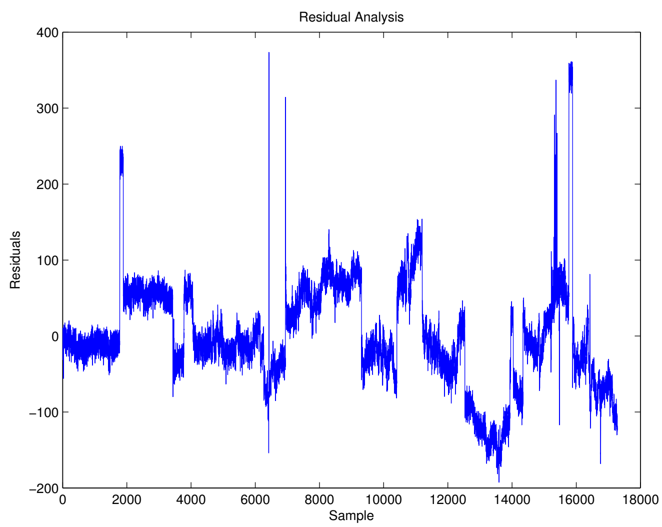


Figure C.10: Model 2 Using 900 psi Steam Flow Rate as the Input

$\beta_0 = 671.4752$ $\beta_1 = 0.0224$ $R^2 = 32.94\%$

From the simulation, Model 2 with 900 psi steam flow rate flow rate is obtained as:

$$\hat{y} = 671.4752 + 0.0224x \tag{C.10}$$

A coefficient of determination, R^2, of 32.94% is poor for the model especially considering that the model has one more parameter compared to Model 1 and exhibits a worse fitting result. Figure C.10 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is a poor model.

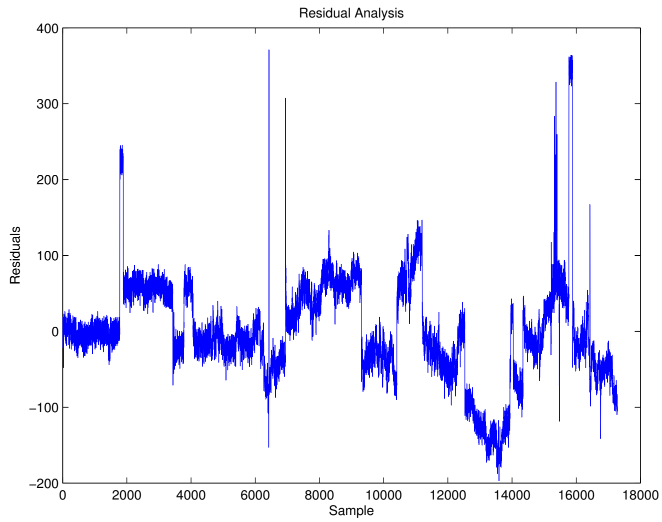


Figure C.11: Model 3 Using 900 psi Steam Flow Rate as the Input

Model 3:

$\beta_1 = 0.0844$ $\beta_2 = -1.412 * 10^{-6}$ $R^2 = 33.73\%$

From the simulation, Model 3 with 900 psi steam flow rate flow rate is obtained as:

$$\hat{y} = 0.0844x - 1.412 * 10^{-6}x^2 \tag{C.11}$$

A coefficient of determination, R^2, of 33.73% is poor for the model especially considering that the model has one more parameter compared to Model 1 and exhibits a worse fitting result. Figure C.11 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is a poor model.

Model 4:

$\beta_0 = -558.9279$ $\beta_1 = 0.1353$ $\beta_2 = -2.558 * 10^{-8}$ $R^2 = 36.12\%$

From the simulation, Model 4 with 900 psi steam flow rate flow rate is obtained as:

$$\hat{y} = -558.9279 + 0.1353x - 2.558 * 10^{-8}x^2 \tag{C.12}$$

A coefficient of determination, R^2, of 36.12% is poor for the model especially considering that the model has one more parameter compared to Model 2 and 3 and

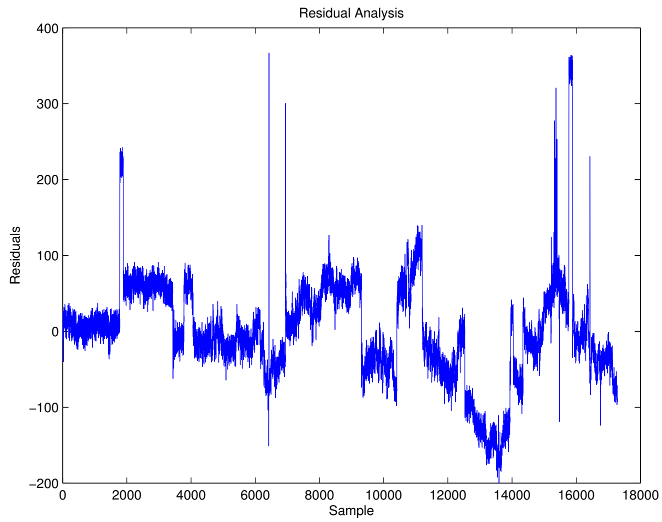


Figure C.12: Model 4 Using 900 psi Steam Flow Rate as the Input

exhibits a worse fitting result. Figure C.12 shows there is nonlinearity or lack of fit in the model affecting the data since there is an apparent shifting of the trend in the residual plot. Overall, this model is unacceptable.

C.2 Multiple Regression Result for Turbine Generator 1

For the cases when 35 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and with intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = -362.7790 + 0.0400x_1 + 0.0905x_2 \quad (C.13)$$

A coefficient of determination, R^2, of 93.31% is obtained as a result. From Figure C.13, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = -322.8908 + 0.0359x_1 + 0.0874x_2 + 3.337 * 10^{-9}x_1x_2 \quad (C.14)$$

A coefficient of determination, R^2, of 93.38% is obtained as a result. From Figure C.14, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

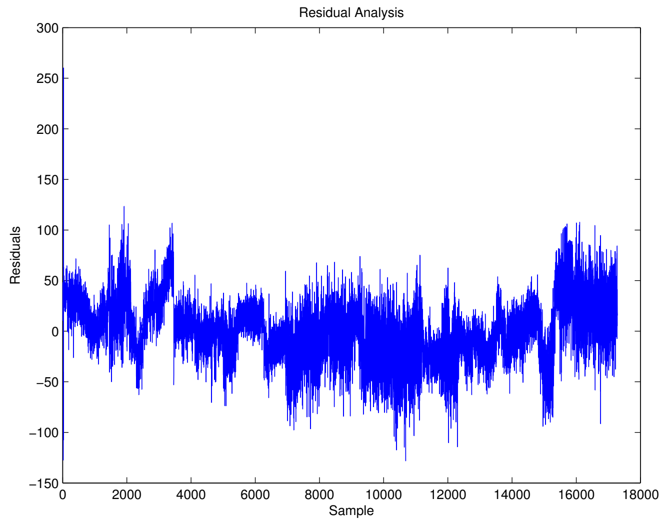


Figure C.13: Model 1 Using 35 psi Steam and Condensate Flow Rates as Input Variables

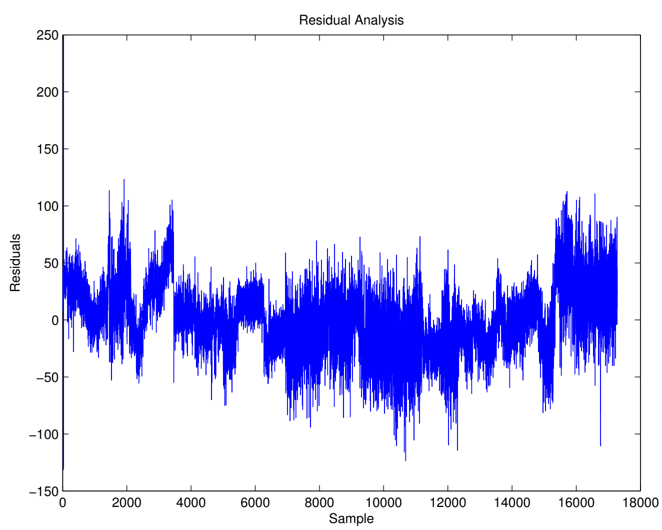


Figure C.14: Model 2 Using 35 psi Steam and Condensate Flow Rates as Input Variables

Model 3: The model is obtained has the following structure:

$$\hat{y} = -276.1182 + 0.0518x_1 + 0.0654x_2 - 5.351 * 10^{-9}x_1^2 + 1.056 * 10^{-8}x_2^2 \quad \text{(C.15)}$$

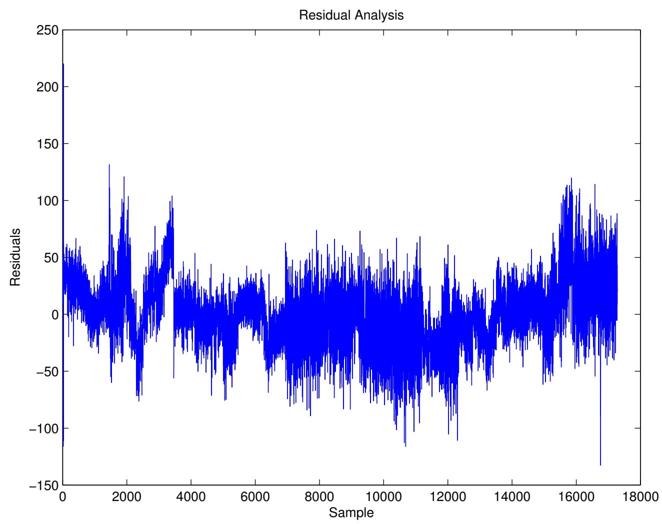


Figure C.15: Model 3 Using 35 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 93.85% is obtained as a result. From Figure C.15, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = -367.5381 + 0.0589x_1 + 0.0748x_2 - 6.541 * 10^{-9}x_1^2 + 8.211 * 10^{-9}x_2^2 - 3.961 * 10^{-9}x_1x_2 \quad \text{(C.16)}$$

A coefficient of determination, R^2, of 93.87% is obtained as a result. From Figure C.16, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 35 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and without intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = 0.0297x_1 + 0.0701x_2 \quad \text{(C.17)}$$

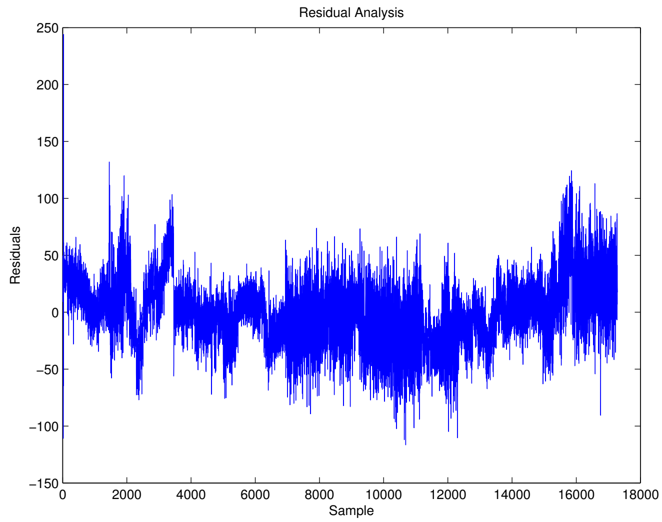


Figure C.16: Model 4 Using 35 psi Steam and Condensate Flow Rates as Input Variables

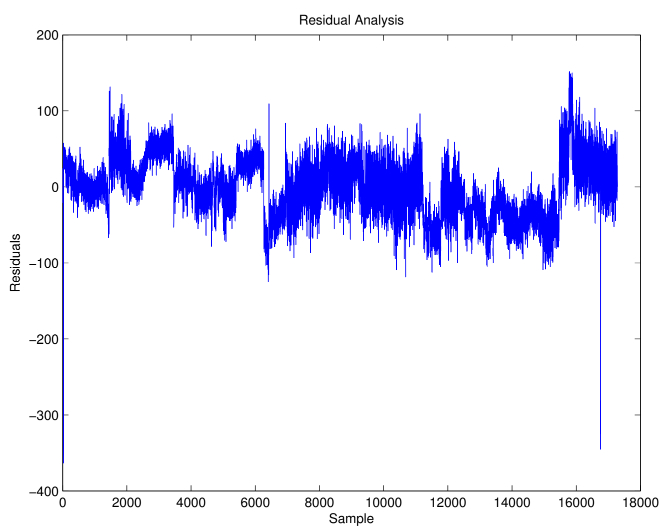


Figure C.17: Model 1 Using 35 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 82.83% is obtained as a result. From Figure C.17, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = 0.0160x_1 + 0.0661x_2 + 1.544 * 10^{-6}x_1x_2 \tag{C.18}$$

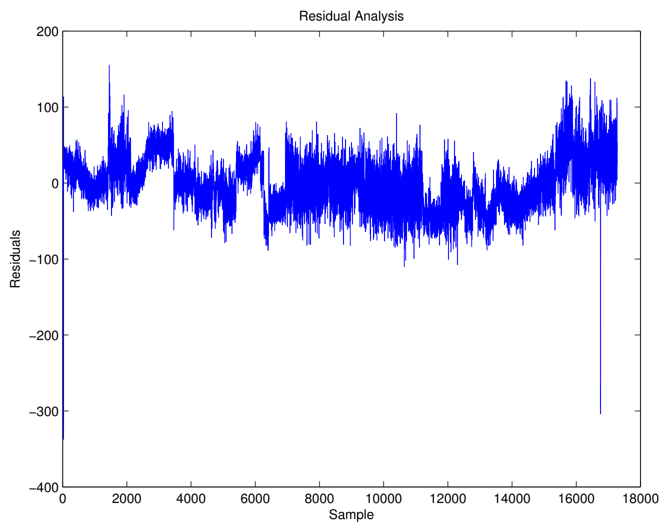


Figure C.18: Model 2 Using 35 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 89.57% is obtained as a result. From Figure C.18, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = 0.0549x_1 + 0.0215x_2 - 7.397 * 10^{-7}x_1^2 + 2.704 * 10^{-6}x_2^2 \tag{C.19}$$

A coefficient of determination, R^2, of 92.57% is obtained as a result. From Figure C.19, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = 0.0358x_1 + 0.0333x_2 - 3.070 * 10^{-7}x_1^2 + 1.984 * 10^{-6}x_2^2 + 7.719 * 10^{-7}x_1x_2 \tag{C.20}$$

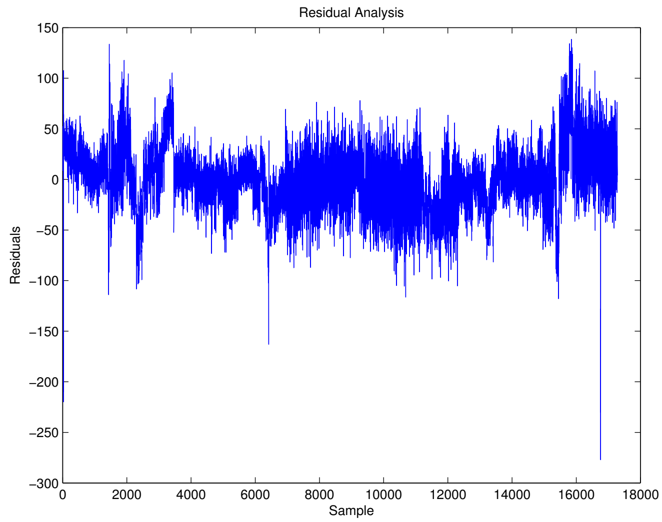


Figure C.19: Model 3 Using 35 psi Steam and Condensate Flow Rates as Input Variables

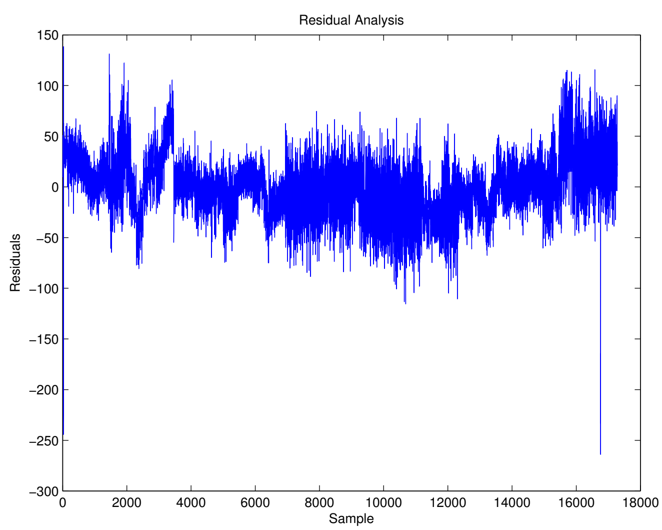


Figure C.20: Model 4 Using 35 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 93.52% is obtained as a result. From Figure C.20, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and with intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = -362.7790 + 0.0400x_1 + 0.0505x_2 \quad (C.21)$$

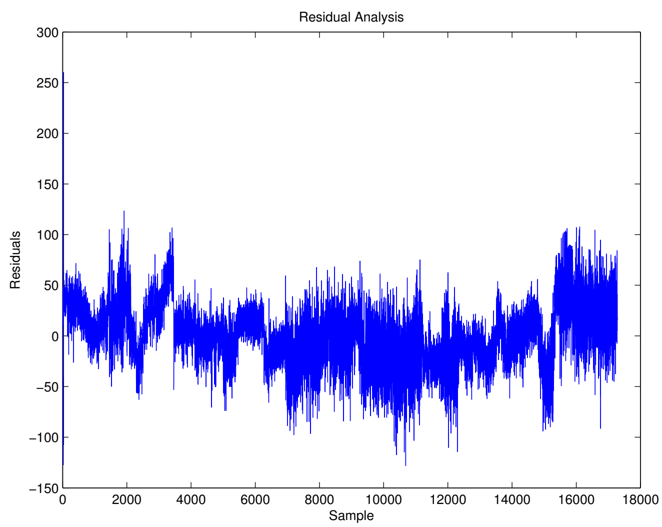


Figure C.21: Model 1 Using 900 psi steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 93.31% is obtained as a result. From Figure C.21, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = -141.5912 + 0.0298x_1 + 0.0316x_2 + 8.695 * 10^{-9}x_1x_2 \quad (C.22)$$

A coefficient of determination, R^2, of 93.53% is obtained as a result. From Figure C.22, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = -612.2737 + 0.0691x_1 + 0.0388x_2 - 6.468 * 10^{-9}x_1^2 + 4.429 * 10^{-9}x_2^2 \quad (C.23)$$

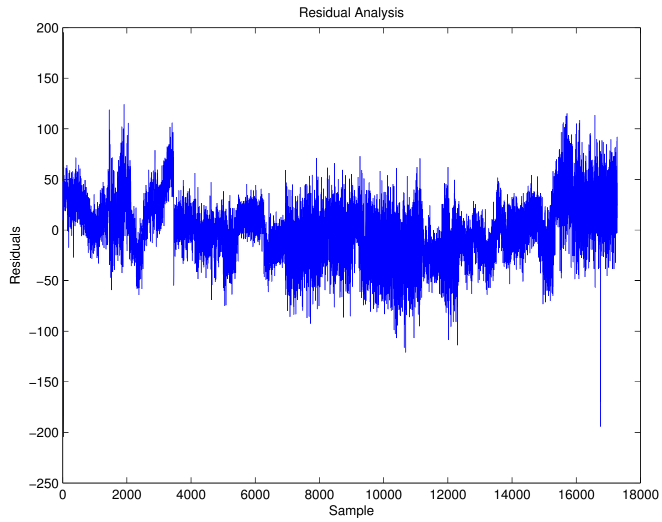


Figure C.22: Model 2 Using 900 psi steam and Condensate Flow Rates as Input Variables

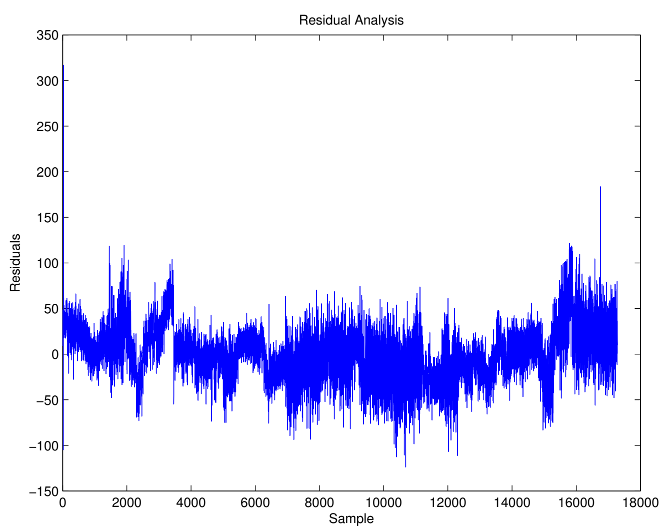


Figure C.23: Model 3 Using 900 psi steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 93.63% is obtained as a result. From Figure C.23, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = -367.5381 + 0.0589x_1 + 0.0159x_2 - 6.541 * 10^{-9}x_1^2 + 5.631 * 10^{-9}x_2^2 + 9.122 * 10^{-9}x_1x_2 \quad (C.24)$$

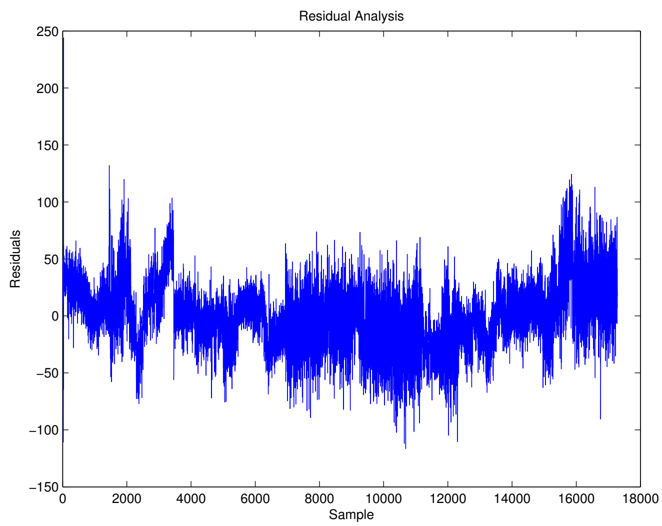


Figure C.24: Model 4 Using 900 psi steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 93.87% is obtained as a result. From Figure C.24, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and without intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = 0.0297x_1 + 0.0404x_2 \quad (C.25)$$

A coefficient of determination, R^2, of 82.83% is obtained as a result. From Figure C.25, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = 0.0236x_1 + 0.0203x_2 + 1.367 * 10^{-6}x_1x_2 \quad (C.26)$$

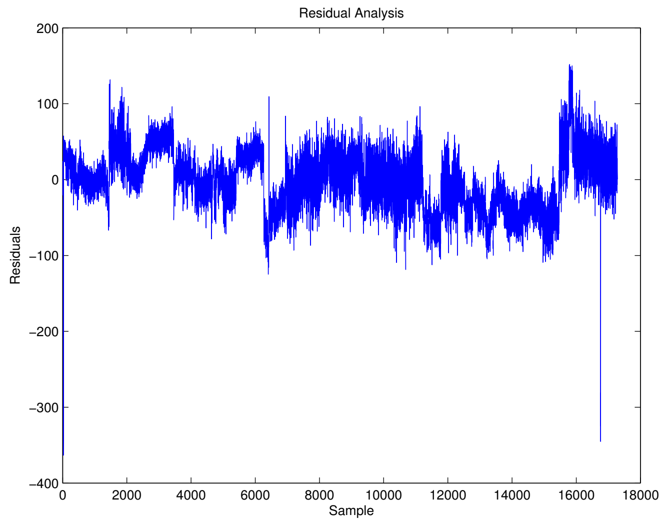


Figure C.25: Model 1 Using 900 psi steam and Condensate Flow Rates as Input Variables

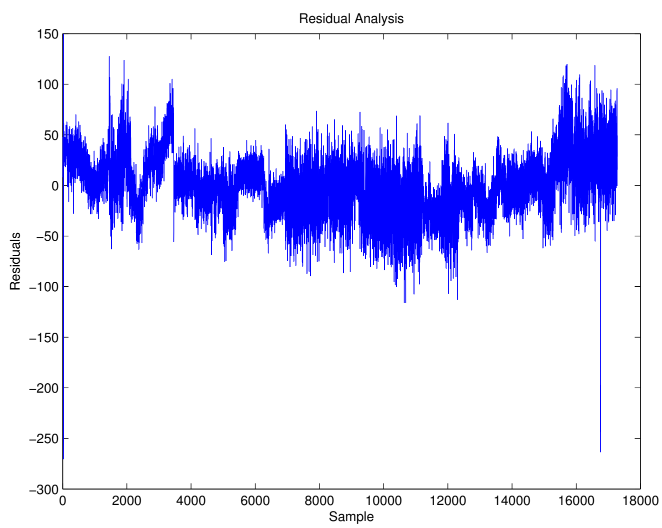


Figure C.26: Model 2 Using 900 psi steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 93.36% is obtained as a result. From Figure C.26, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = 0.0200x_1 + 0.0299x_2 + 4.245 * 10^{-7}x_1^2 + 7.909 * 10^{-7}x_2^2 \quad \text{(C.27)}$$

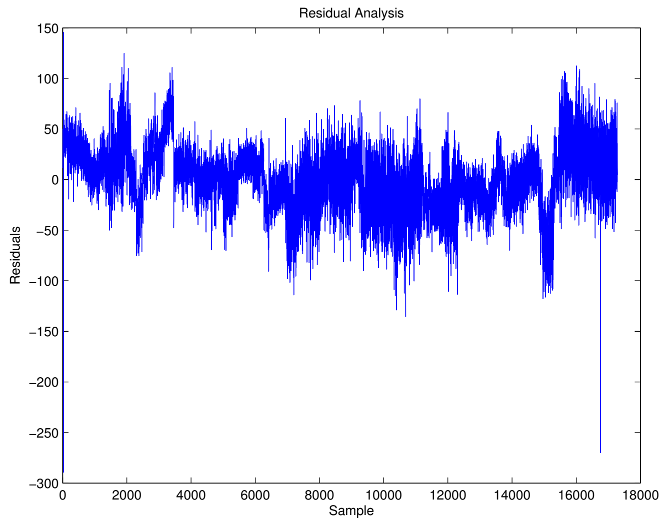


Figure C.27: Model 3 Using 900 psi steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 92.03% is obtained as a result. From Figure C.27, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = 0.0358x_1 - 0.0025x_2 - 3.070 * 10^{-7}x_1^2 + 7.591 * 10^{-7}x_2^2 + 1.532 * 10^{-6}x_1x_2 \quad (C.28)$$

A coefficient of determination, R^2, of 93.52% is obtained as a result. From Figure C.28, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and 35 psi steam flow rate are used as the input variable for x_1 and x_2 respectively and with intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = -362.7790 + 0.0905x_1 + 0.0505x_2 \quad (C.29)$$

A coefficient of determination, R^2, of 93.31% is obtained as a result. From Figure C.29, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = -503.2125 + 0.0968x_1 - 0.0316x_2 - 6.277 * 10^{-9}x_1x_2 \quad (C.30)$$

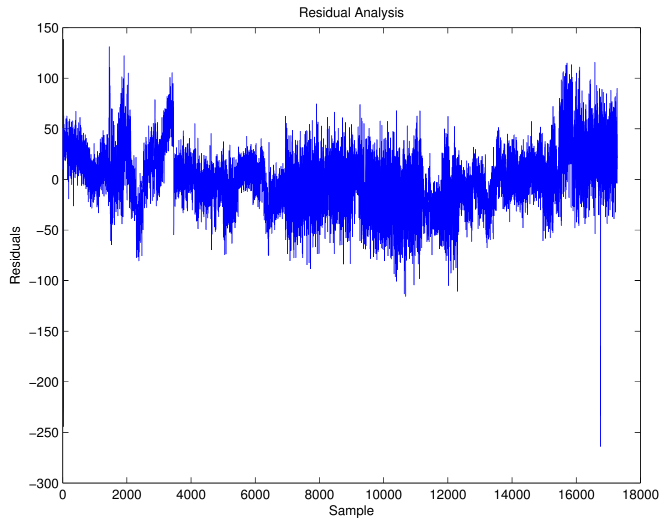


Figure C.28: Model 4 Using 900 psi steam and Condensate Flow Rates as Input Variables

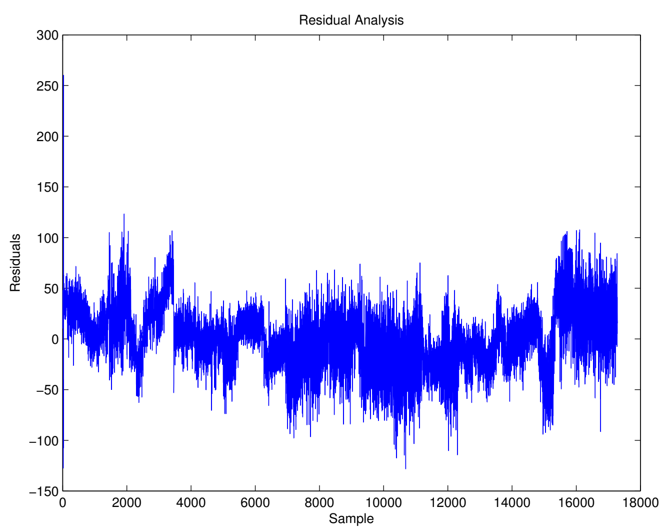


Figure C.29: Model 1 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

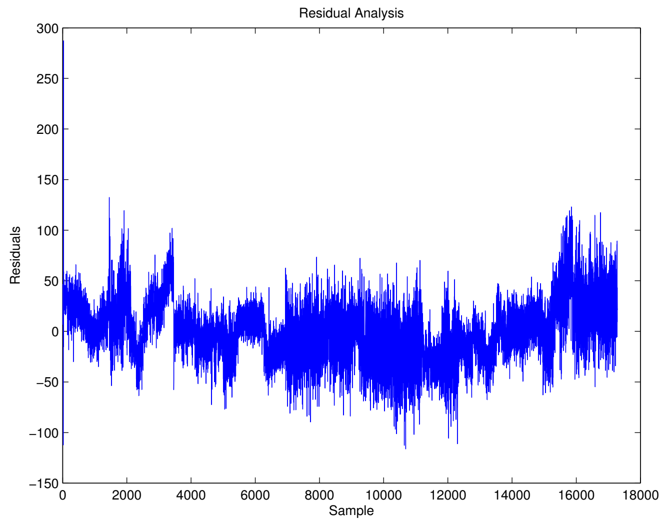


Figure C.30: Model 2 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

A coefficient of determination, R^2, of 93.76% is obtained as a result. From Figure C.30, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = -520.2978 + 0.1027x_1 - 0.0455x_2 - 2.781 * 10^{-9}x_1^2 - 2.358 * 10^{-9}x_2^2 \quad (C.31)$$

A coefficient of determination, R^2, of 93.66% is obtained as a result. From Figure C.31, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = -367.5381 + 0.0748x_1 - 0.0159x_2 + 8.211 * 10^{-9}x_1^2 + 5.631 * 10^{-9}x_2^2 - 2.038 * 10^{-8}x_1x_2 \quad (C.32)$$

A coefficient of determination, R^2, of 93.87% is obtained as a result. From Figure C.32, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and 35 psi steam flow rate are used as the input variable for x_1 and x_2 respectively and without intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = 0.0701x_1 - 0.0404x_2 \quad (C.33)$$

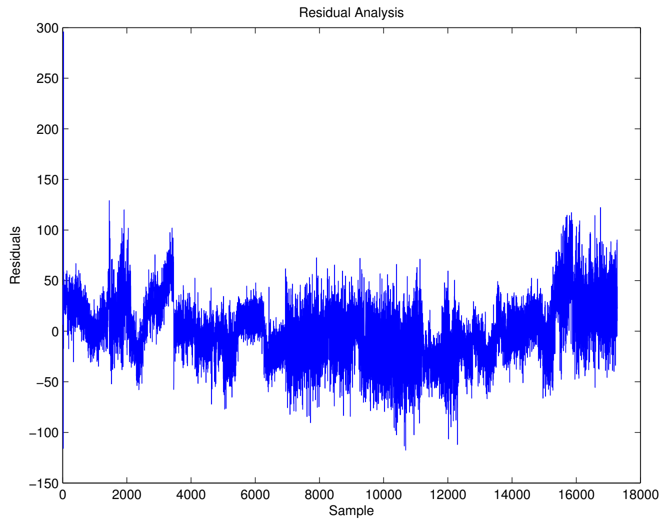


Figure C.31: Model 3 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

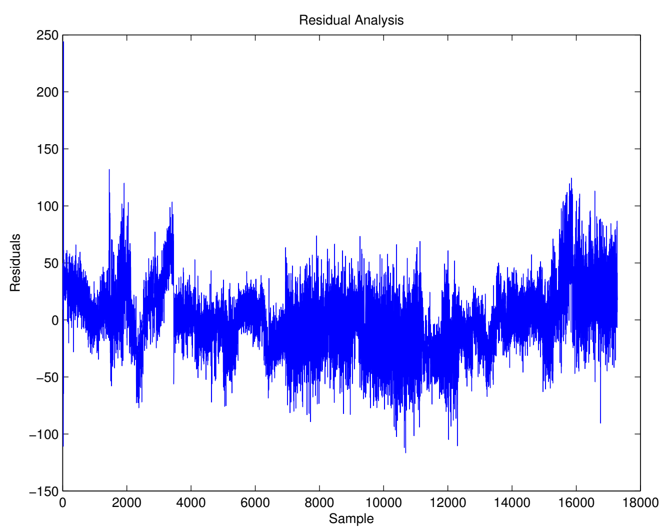


Figure C.32: Model 4 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

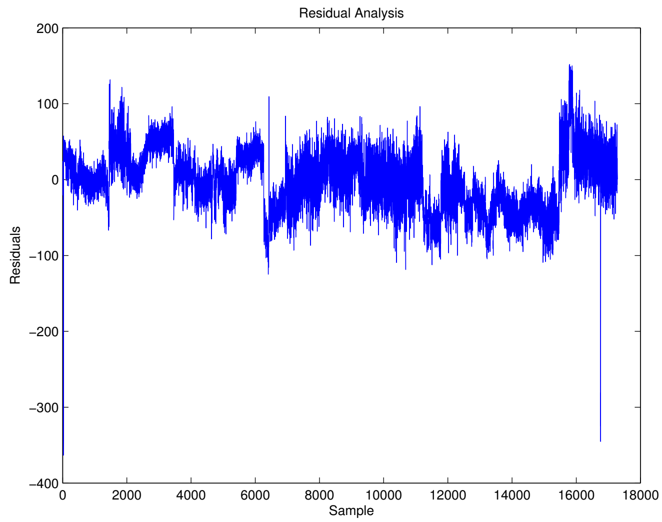


Figure C.33: Model 1 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

A coefficient of determination, R^2, of 82.83% is obtained as a result. From Figure C.33, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = 0.0720x_1 - 0.0627x_2 + 7.643 * 10^{-7}x_1x_2 \tag{C.34}$$

A coefficient of determination, R^2, of 88.11% is obtained as a result. From Figure C.34, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = 0.0534x_1 - 0.0387x_2 + 8,229 * 10^{-7}x_1^2 - 5.934 * 10^{-7}x_2^2 \tag{C.35}$$

A coefficient of determination, R^2, of 92.86% is obtained as a result. From Figure C.35, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = 0.0333x_1 + 0.0025x_2 + 1.984 * 10^{-6}x_1^2 + 7.591 * 10^{-7}x_2^2 - 3.050 * 10^{-6}x_1x_2 \tag{C.36}$$

A coefficient of determination, R^2, of 93.52% is obtained as a result. From Figure C.36, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

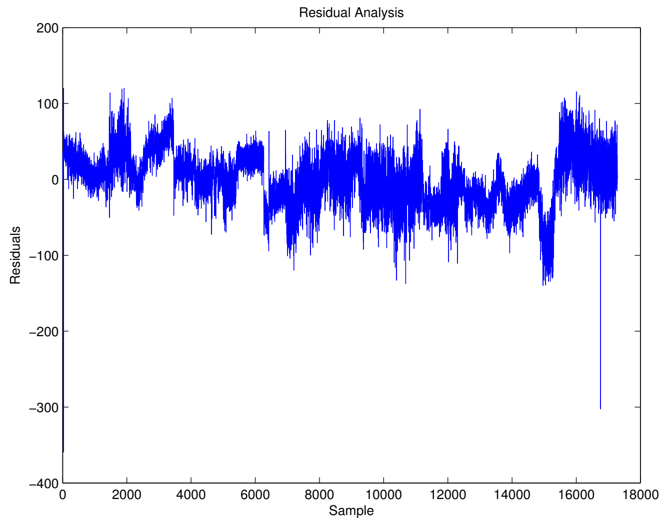


Figure C.34: Model 2 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

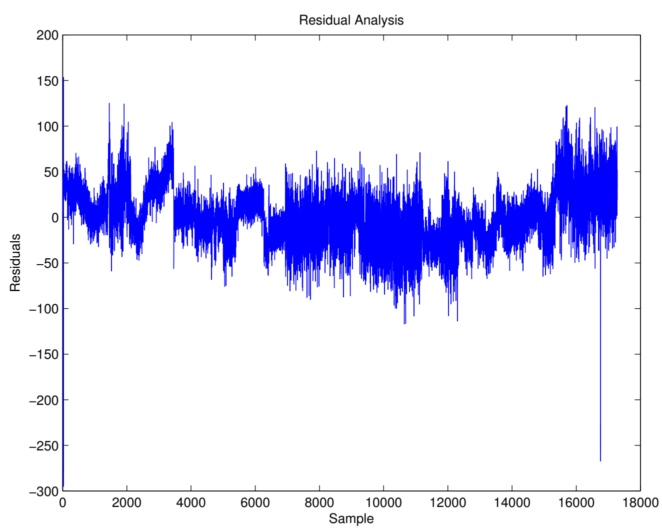


Figure C.35: Model 3 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

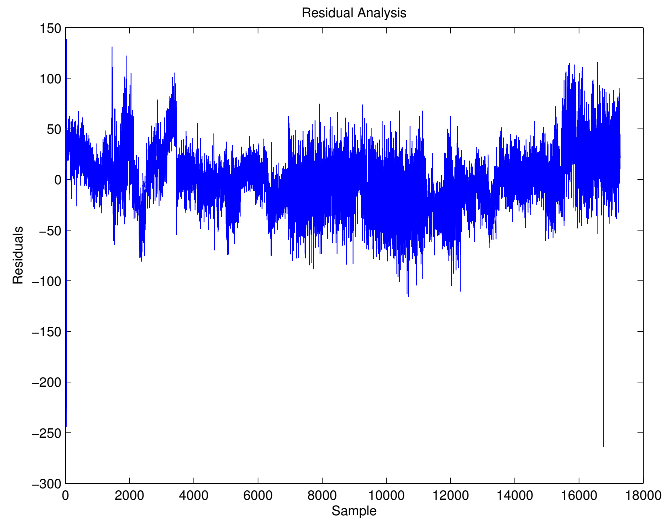


Figure C.36: Model 4 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

C.3 Multiple Regression Result for Turbine Generator 2

For the cases when 35 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and with intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = -603.0405 + 0.0435x_1 + 0.1074x_2 \quad (C.37)$$

A coefficient of determination, R^2, of 94.45% is obtained as a result. From Figure C.37, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = -434.2122 + 0.0199x_1 + 0.0935x_2 + 2.002 * 10^{-8}x_1x_2 \quad (C.38)$$

A coefficient of determination, R^2, of 95.57% is obtained as a result. From Figure C.38, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = -759.1146 + 0.0584x_1 + 0.1194x_2 - 7.419 * 10^{-9}x_1^2 - 3.757 * 10^{-9}x_2^2 \quad (C.39)$$

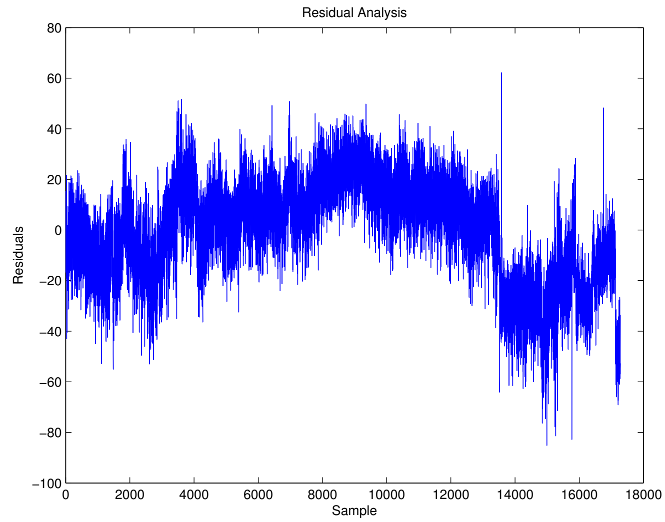


Figure C.37: Model 1 Using 35 psi Steam and Condensate Flow Rates as Input Variables

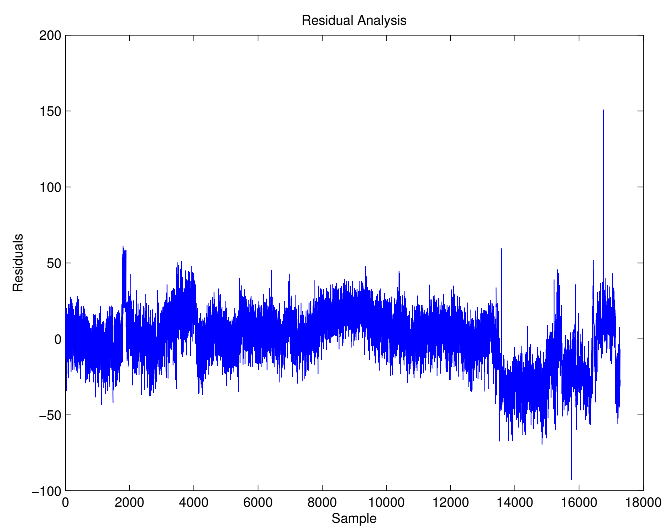


Figure C.38: Model 2 Using 35 psi Steam and Condensate Flow Rates as Input Variables

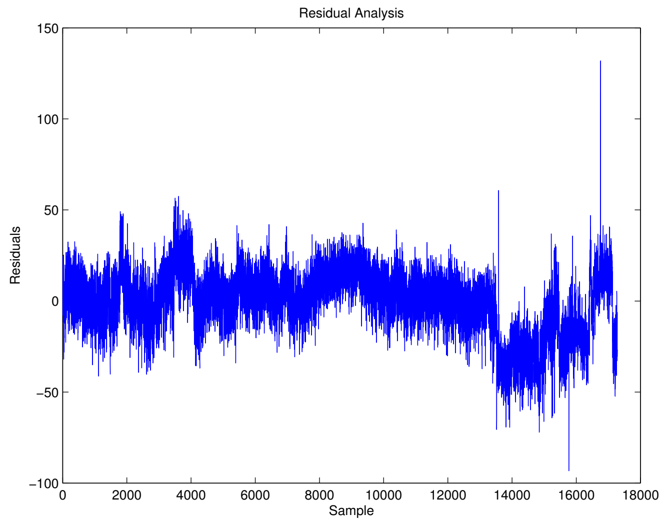


Figure C.39: Model 3 Using 35 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 95.81% is obtained as a result. From Figure C.39, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = -1153.2 + 0.1000x_1 + 0.2000x_2 - 1.222 * 10^{-9}x_1^2 - 1.403 * 10^{-9}x_2^2 - 1.725 * 10^{-9}x_1x_2 \quad \text{(C.40)}$$

A coefficient of determination, R^2, of 95.86% is obtained as a result. From Figure C.40, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 35 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and without intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = 0.0297x_1 + 0.0695x_2 \quad \text{(C.41)}$$

A coefficient of determination, R^2, of 71.69% is obtained as a result. From Figure C.41, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = -0.0120x_1 + 0.0624x_2 + 4.241 * 10^{-6}x_1x_2 \quad \text{(C.42)}$$

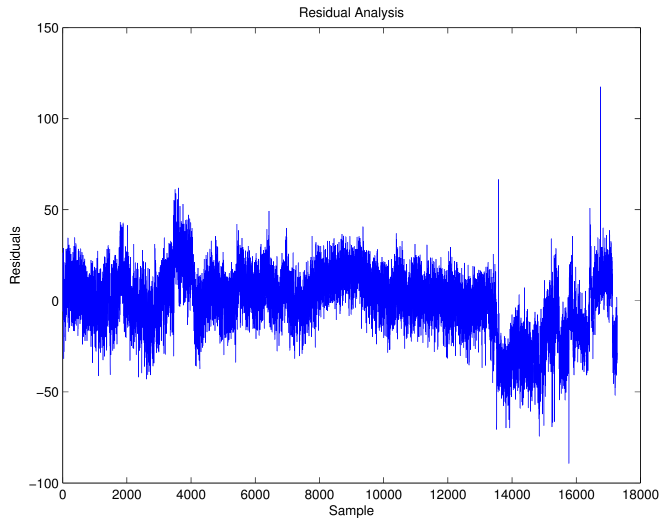


Figure C.40: Model 4 Using 35 psi Steam and Condensate Flow Rates as Input Variables

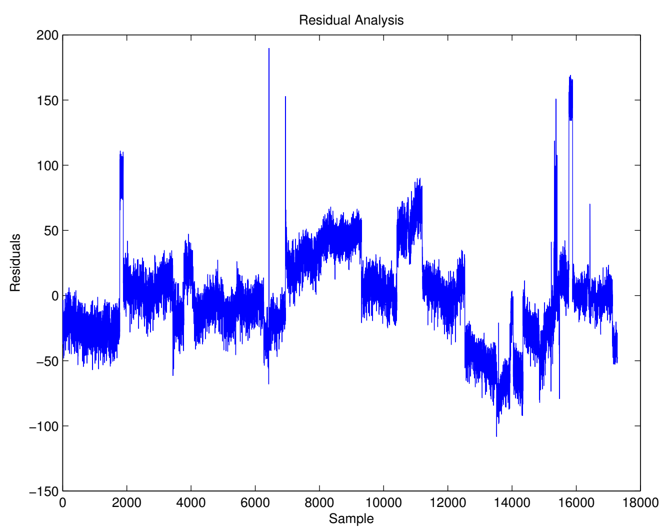


Figure C.41: Model 1 Using 35 psi Steam and Condensate Flow Rates as Input Variables

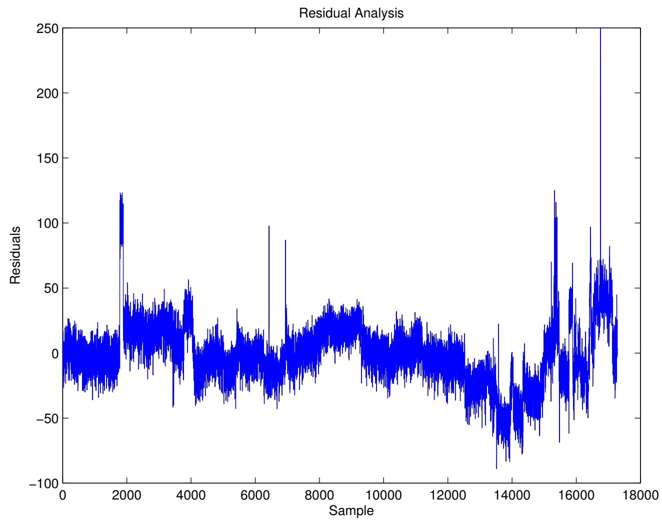


Figure C.42: Model 2 Using 35 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 90.79% is obtained as a result. From Figure C.42, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = 0.0606x_1 + 0.0085x_2 - 1.032 * 10^{-6}x_1^2 + 3.644 * 10^{-6}x_2^2 \quad \text{(C.43)}$$

A coefficient of determination, R^2, of 94.88% is obtained as a result. From Figure C.43, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = 0.0169x_1 + 0.0313x_2 - 1.602 * 10^{-7}x_1^2 + 2.180 * 10^{-6}x_2^2 + 2.384 * 10^{-6}x_1x_2 \quad \text{(C.44)}$$

A coefficient of determination, R^2, of 95.48% is obtained as a result. From Figure C.44, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and with intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = -603.0405 + 0.0435x_1 + 0.0640x_2 \quad \text{(C.45)}$$

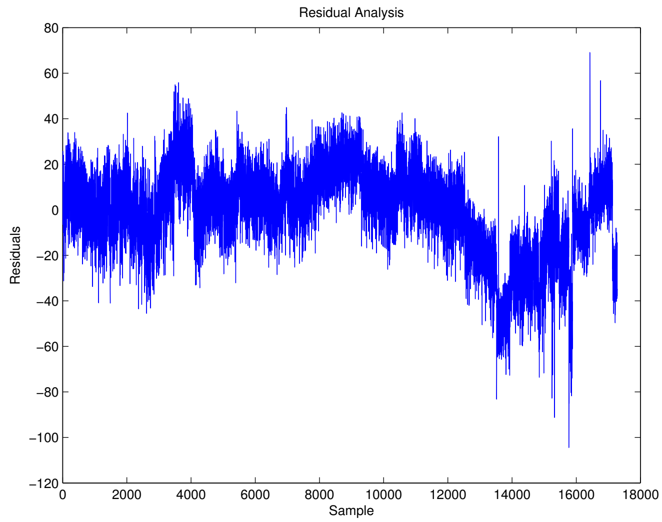


Figure C.43: Model 3 Using 35 psi Steam and Condensate Flow Rates as Input Variables

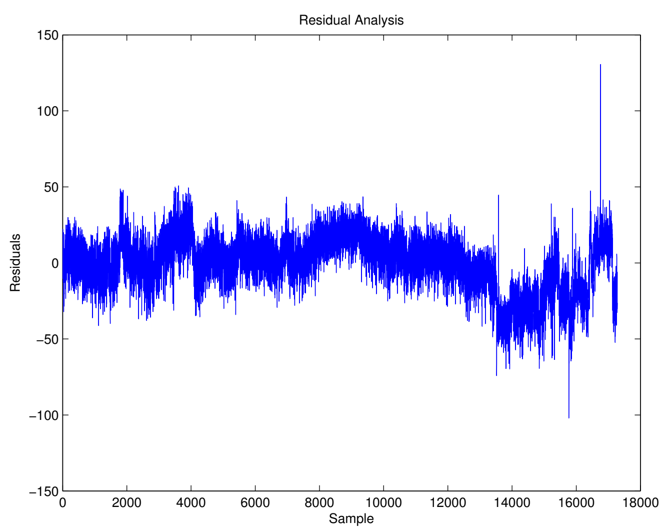


Figure C.44: Model 4 Using 35 psi Steam and Condensate Flow Rates as Input Variables

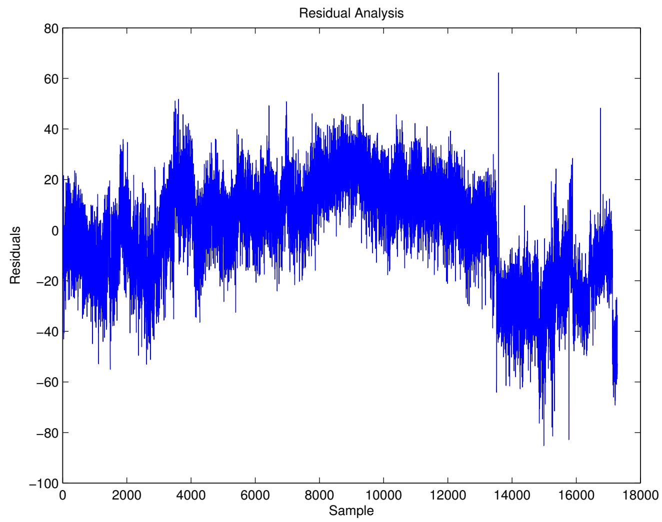


Figure C.45: Model 1 Using 900 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 94.45% is obtained as a result. From Figure C.45, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = 67.4908 + 0.0129x_1 + 0.0105x_2 + 2.459 * 10^{-6}x_1x_2 \quad \text{(C.46)}$$

A coefficient of determination, R^2, of 95.41% is obtained as a result. From Figure C.46, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = -1488.9 + 0.1000x_1 + 0.1000x_2 - 1.536 * 10^{-9}x_1^2 - 8.541 * 10^{-10}x_2^2 \quad \text{(C.47)}$$

A coefficient of determination, R^2, of 95.83% is obtained as a result. From Figure C.47, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = -1153.2 + 0.1000x_1 + 0.1000x_2 - 1.222 * 10^{-9}x_1^2 - 9.005 * 10^{-10}x_2^2 + 7.191 * 10^{-10}x_1x_2 \quad \text{(C.48)}$$

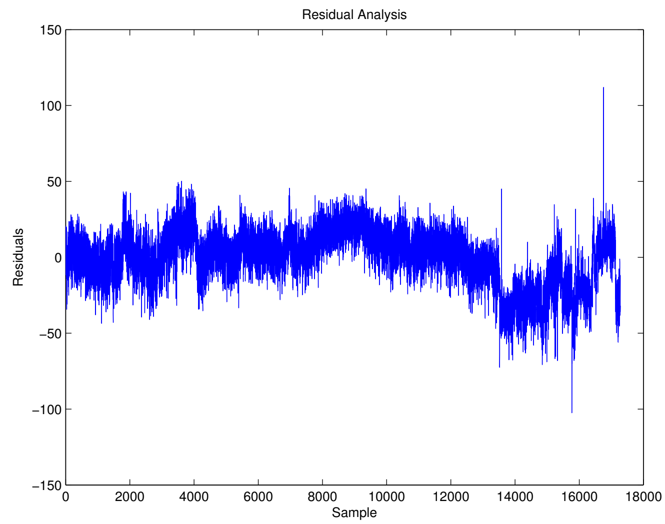


Figure C.46: Model 2 Using 900 psi Steam and Condensate Flow Rates as Input Variables

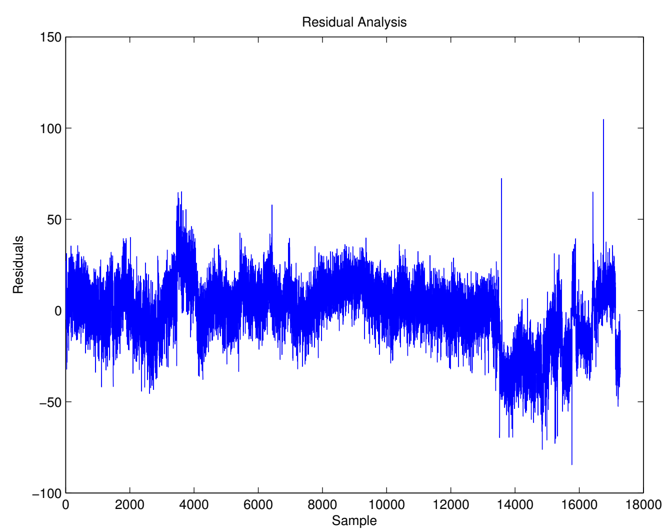


Figure C.47: Model 3 Using 900 psi Steam and Condensate Flow Rates as Input Variables

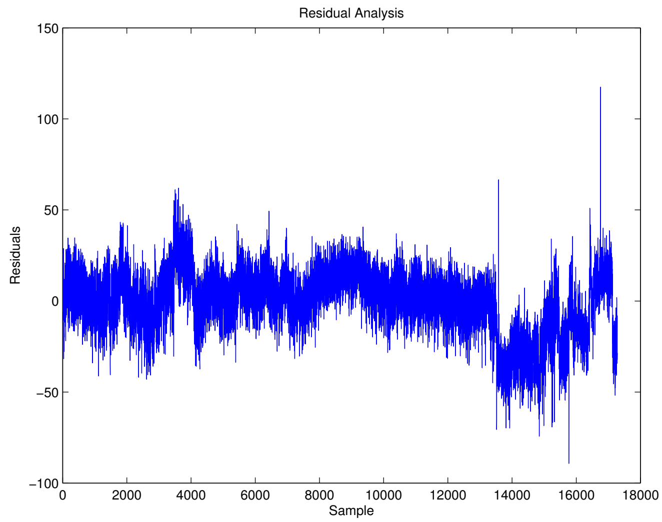


Figure C.48: Model 4 Using 900 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 95.86% is obtained as a result. From Figure C.48, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and condensate flow rate are used as the input variable for x_1 and x_2 respectively and without intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = 0.0297x_1 + 0.0398x_2 \tag{C.49}$$

A coefficient of determination, R^2, of 71.69% is obtained as a result. From Figure C.49, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = 0.0158x_1 + 0.0157x_2 + 2.228 * 10^{-6}x_1x_2 \tag{C.50}$$

A coefficient of determination, R^2, of 95.42% is obtained as a result. From Figure C.50, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = 0.0512x_1 - 0.0320x_2 - 2.627 * 10^{-7}x_1^2 + 3.512 * 10^{-7}x_2^2 \tag{C.51}$$

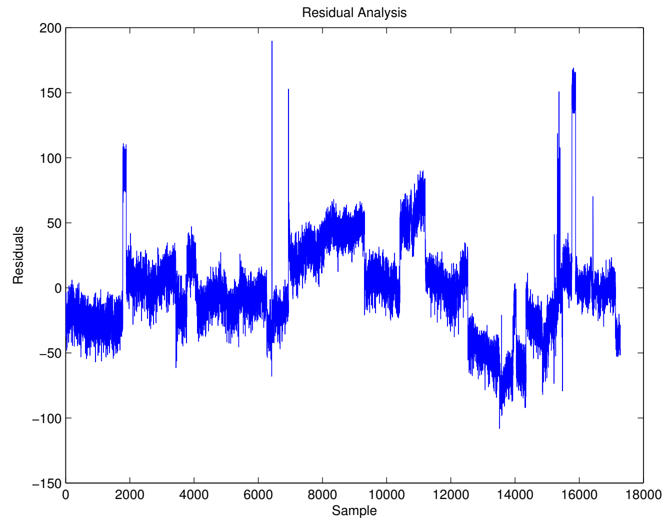


Figure C.49: Model 1 Using 900 psi Steam and Condensate Flow Rates as Input Variables

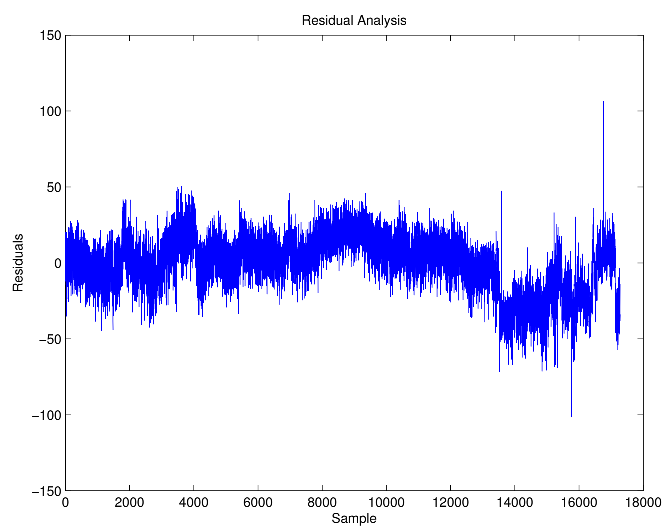


Figure C.50: Model 2 Using 900 psi Steam and Condensate Flow Rates as Input Variables

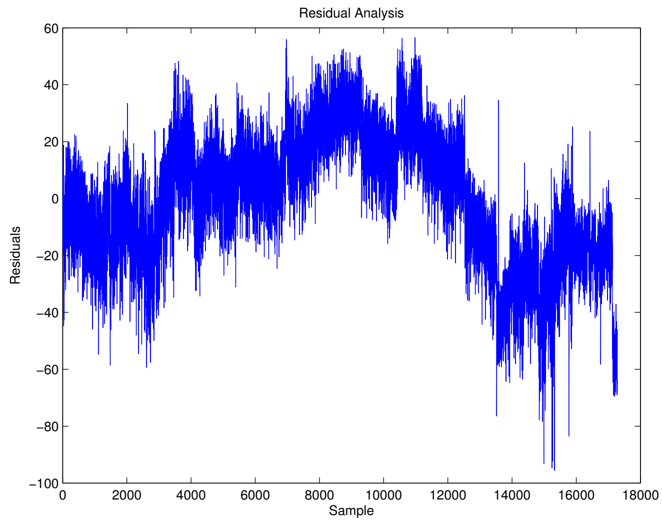


Figure C.51: Model 3 Using 900 psi Steam and Condensate Flow Rates as Input Variables

A coefficient of determination, R^2, of 92.53% is obtained as a result. From Figure C.51, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = 0.0169x_1 + 0.0144x_2 - 1.602 * 10^{-7}x_1^2 - 3.645 * 10^{-7}x_2^2 + 2.704 * 10^{-6}x_1x_2 \quad (C.52)$$

A coefficient of determination, R^2, of 95.48% is obtained as a result. From Figure C.52, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and 35 psi steam flow rate are used as the input variable for x_1 and x_2 respectively and with intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = -603.0405 + 0.1074x_1 - 0.0640x_2 \quad (C.53)$$

A coefficient of determination, R^2, of 94.45% is obtained as a result. From Figure C.53, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = -829.1762 + 0.1181x_1 - 0.0388x_2 - 1.135 * 10^{-8}x_1x_2 \quad (C.54)$$

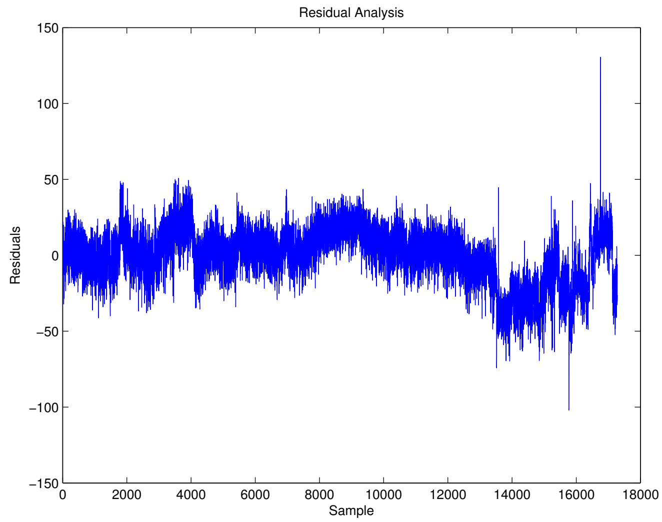


Figure C.52: Model 4 Using 900 psi Steam and Condensate Flow Rates as Input Variables

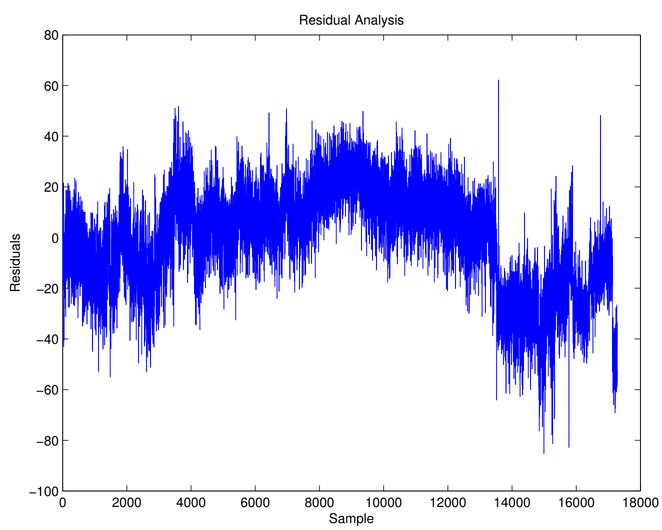


Figure C.53: Model 1 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

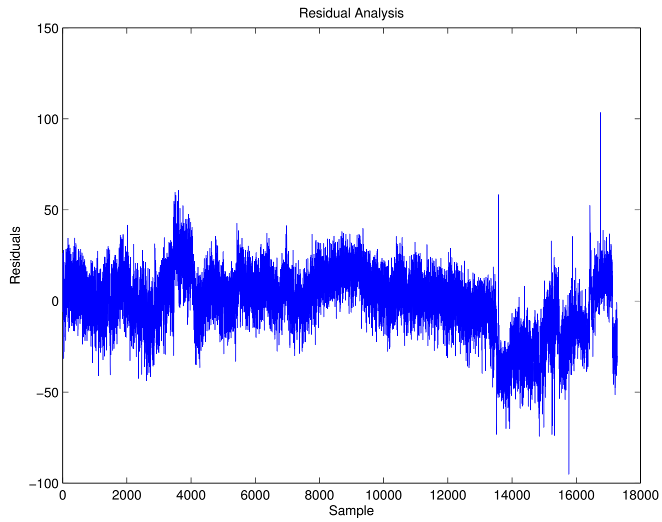


Figure C.54: Model 2 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

A coefficient of determination, R^2, of 95.80% is obtained as a result. From Figure C.54, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = -1026.5 + 0.1000x_1 - 0.1000x_2 - 7.909 * 10^{-10}x_1^2 - 4.363 * 10^{-10}x_2^2 \quad \text{(C.55)}$$

A coefficient of determination, R^2, of 95.85% is obtained as a result. From Figure C.55, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = -1153.2 + 0.2000x_1 - 0.1000x_2 - 1.403 * 10^{-9}x_1^2 - 9.005 * 10^{-10}x_2^2 + 1.082 * 10^{-9}x_1x_2 \quad \text{(C.56)}$$

A coefficient of determination, R^2, of 95.86% is obtained as a result. From Figure C.56, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

For the cases when 900 psi steam and 35 psi steam flow rate are used as the input variable for x_1 and x_2 respectively and without intercept:

Model 1: The model is obtained has the following structure:

$$\hat{y} = 0.0695x_1 - 0.0398x_2 \quad \text{(C.57)}$$

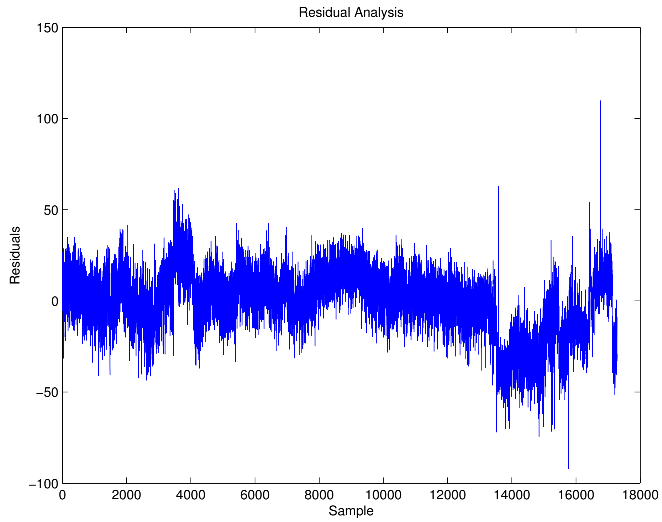


Figure C.55: Model 3 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

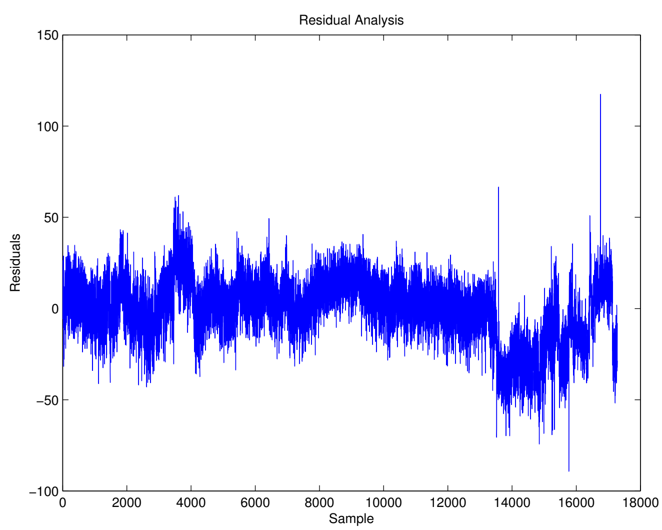


Figure C.56: Model 4 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

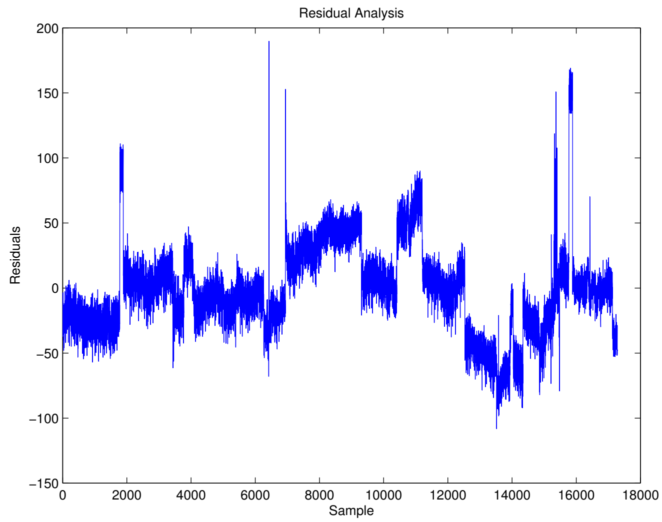


Figure C.57: Model 1 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

A coefficient of determination, R^2, of 71.69% is obtained as a result. From Figure C.57, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 2: The model is obtained has the following structure:

$$\hat{y} = 0.0729x_1 - 0.0724x_2 + 1.082 * 10^{-6}x_1x_2 \quad (C.58)$$

A coefficient of determination, R^2, of 81.01% is obtained as a result. From Figure C.58, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 3: The model is obtained has the following structure:

$$\hat{y} = 0.0403x_1 - 0.0392x_2 + 1.601 * 10^{-6}x_1^2 - 1.438 * 10^{-6}x_2^2 \quad (C.59)$$

A coefficient of determination, R^2, of 95.41% is obtained as a result. From Figure C.59, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Model 4: The model is obtained has the following structure:

$$\hat{y} = 0.0313x_1 - 0.0144x_2 + 2.180 * 10^{-6}x_1^2 - 3.645 * 10^{-7}x_2^2 - 1.975 * 10^{-6}x_1x_2 \quad (C.60)$$

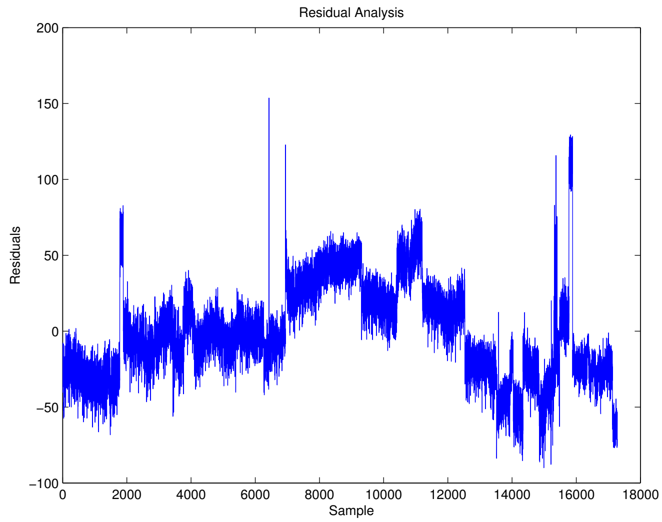


Figure C.58: Model 2 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

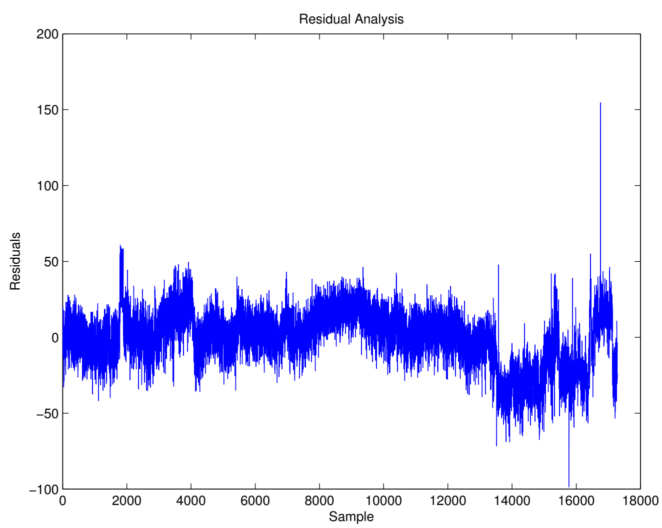


Figure C.59: Model 3 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

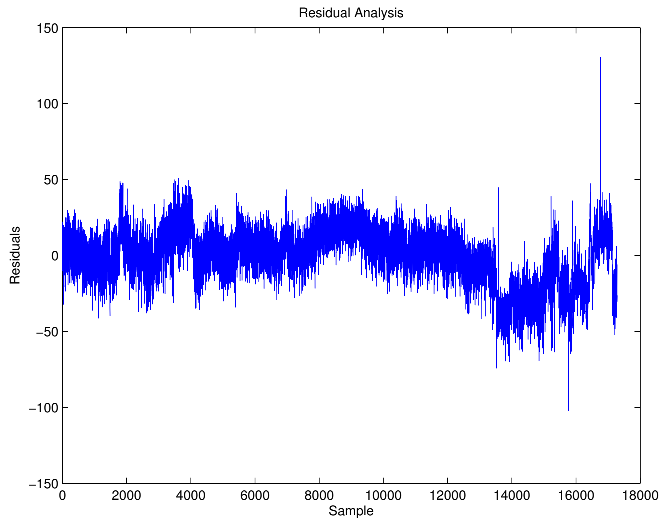


Figure C.60: Model 4 Using 900 psi Steam and 35 psi Steam Flow Rates as Input Variables

A coefficient of determination, R^2, of 95.48% is obtained as a result. From Figure C.60, it is difficult to determine if there is nonlinearity or lack of fit in the model since there is no apparent trend in the residual plot.

Appendix D

Post Optimality Analysis

In this appendix, the relevant derivations required for post optimality analysis is demonstrated.

For linear programming problem, the optimal objective function value is given as:

$$P^* = c^T x^* \tag{D.1}$$

And the optimal decision variable is obtained as:

$$x^* = A_a^{-1} b_a \tag{D.2}$$

The subscript “a" represent the active constraints in those matrices.

By substituting Equation (D.2) into Equation (D.1), the resulting equation is:

$$P^* = c^T A_a^{-1} b_a \tag{D.3}$$

With the expression in Equation (D.3), the effects on the objective function value from changing matrices without changing the active constraints can be derived.

The effects on objective function value from changing c matrix is given as:

$$\frac{dP}{dc} = (x^*)^T \tag{D.4}$$

The effects on objective function value from changing b matrix is given as:

$$\frac{dP}{db} = c^T A_a^{-1} \tag{D.5}$$

At the optimum, the Lagrangian multiplier can be obtained as:

$$\lambda_a^* = (A_a^T)^{-1} c \tag{D.6}$$

Therefore, the Equation (D.5) can be rewritten as:

$$\frac{dP}{db} = (\lambda_a^*)^T \tag{D.7}$$

Based on Equation (D.6), the effects on the Lagrangian multiplier from changing matrices, without changing the active constraints can be derived.

The effects on the Lagrangian multiplier from changing c matrix is given as:

$$\frac{d\lambda_a^*}{dc} = (A_a^T)^{-1} \tag{D.8}$$

The effects on the Lagrangian multiplier from changing b matrix is given as:

$$\frac{d\lambda_a^*}{db} = 0 \tag{D.9}$$

From Equation (D.2), the effects on the optimal solution from changing matrix without changing the active constraint can be derived.

The effects on the optimal solution from changing c matrix is given as:

$$\frac{dx^*}{dc} = 0 \tag{D.10}$$

The effects on the optimal solution from changing b matrix is given as:

$$\frac{dx^*}{db} = A_a^{-1} \tag{D.11}$$