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TOPICS IN SUPERSYMMETRY

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled TOPICS IN SUPERSYMMETRY submitted by Kenneth Andrew Peterson in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE in PHYSICS.

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Supervisor

[signature]

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John Cameron

Date 27 October 1986

I would like to dedicate this thesis to my parents

K.L. and M.E. Peterson

who have always kept an interest in whatever I've done.

Abstract

Supersymmetry has been developed to solve many theoretical problems of the Standard Model, specifically the Gauge Hierarchy Principle. Supersymmetry predicts partner particles for each particle. None of these partners have yet been observed.

These new particles, however, may induce new radiative corrections. We have done a calculation that uses radiative corrections due to supersymmetry in the decay of the muon, $\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$, in order to get lower bounds on the masses of supersymmetric particles.

Recently there has been a renewed interest in composite models. For supersymmetric composite models the effective low energy interactions induced by compositeness may appear suppressed by fewer powers of the composite mass scale than in the nonsupersymmetric case. Doing calculations involving two decays, $\pi \to e^+ \nu_e$, $\mu N \to eN$, we set limits on a composite energy scale in SUSY/composite models by again using their effects on radiative corrections.

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1. INTRODUCTION TO SUPERSYMMETRY

1.1 Introduction

Ever since the discovery of supersymmetry[1-3] in the early seventies, through to its modern manifestations,[4-5] and onward to its natural appearance in superstring theory,[6-8] supersymmetry has been of ever increasing interest to particle physicists. This interest comes in spite of no direct experimental evidence of any of supersymmetry's predictions. The real reason behind the interest in supersymmetry is purely theoretical, as it resolves many of the problems of the standard model.

Gauge theories have become the backbone of particle physics. They are totally dominant as a description of the fundamental interactions. The first successful gauge theory that was proposed was QED as a description of electromagnetism. It is based on a local abelian gauge theory which leaves the Lagrangian invariant under the gauge group U(1). The theory was apparently plagued with numerous infinities, but means were devised whereby these infinities could be disposed of through renormalization and regularization. Agreement with experiment was spectacular and QED remains the most successful theory to date. With the extension of the gauge principle to non-Abelian gauge theories by Yang and Mills[9], the modern era of gauge theories was born.

The next gauge theory to be developed was the electroweak theory unifying the electromagnetic and weak forces. This theory was proposed independently in 1967 and 1968 by Weinberg[10] and Salam[11]. It unifies the weak and electromagnetic forces in a Yang-Mills gauge theory with the $W^{\pm}$, Z^0 and photon acting as gauge bosons. The important breakthrough, however, was the principle of spontaneous symmetry breaking. Previous to the Weinberg-Salam model Yang-Mills theories invariant under SU(2) had been developed with gauge bosons of charges ±1,0. However, the bosons had to be massless, contradicting experiment. For a continous parameter $\mu^2 > 0$ in the Lagrangian the ground state of the system possesses the full symmetry of the Lagrangian and the quanta that appear in the theory are four real scalar bosons of mass μ. However, for $\mu^2 < 0$ the symmetry of the ground state is broken and one obtains one scalar quantum with positive mass, the Higgs boson, and three massless scalar Goldstone bosons.[12] Now Goldstone bosons are not seen experimentally, but this is taken care of in a nice fashion. When one adds the four vector gauge quanta (2 degrees of freedom each) of a Lagrangian invariant under SU(2)×U(1) (3 quanta from SU(2) and 1 from U(1)) to the three scalar quanta from symmetry breaking (where the massless scalar bosons have 1 degree of freedom) one obtains three massive vector gauge bosons (3 degrees of freedom), leaving one vector gauge boson massless.[13-15] These correspond to the $W^{\pm}$, Z^0 and photon respectively. A description of this can be found in Commins and Bucksbaum.[17] Again problems with renormalization of these theories was suspected until 1971 when t'Hooft[18] proved that this symmetry breaking mechanism produced renormalizable formulations of massive Yang-Mills theories.

Non-Abelian gauge theories provided the basis for the description of the strong force as well. This theory, dubbed Quantum Chromodynamics (QCD), is based on a non-Abelian SU(3) of colour charge. This time the $SU(3)_c$ remains exact and the eight vector bosons called gluons are massless. The direct product of these gauge groups, $SU(3)_c \times SU(2)_L \times U(1)$, with the Weinberg and Salam model as generalized by GIM for the incorporation of hadrons[19] forms a not quite unified description of the strong, electromagnetic and weak interactions known as the Standard Model. The one thing the theory lacks is a description of the relative strengths of the strong, electromagnetic and weak forces. However, it is quite consistent with experimental findings.

With the direct observation of the intermediate vector bosons, $W^{\pm}$, Z^0, of the Weinberg-Salam theory,[20-24] gauge theories are well entrenched in explaining the fundamental interactions. What lies beyond the Standard Model has been the subject of much interest and speculation. The most natural way of extending the standard model is to inbed it in a "unifying" gauge group. The first such attempt was proposed by Georgi and Glashow in 1974.[25] They imbedded the standard model in the simplest possible Lie group, SU(5), with the resulting theory having one coupling constant at the unifying mass scale. It also produces 24 vector bosons, 12 being the standard photon, $W^{\pm}$, Z^0, and gluons, the other 12 being much heavier. However, the nicest part, which is why these models were created, was that it fixed many parameters and relationships that previously had to be determined by experiment. One of the major predictions of the theory was its prediction of proton decay, which is mediated by one of the 12 new vector bosons. Unfortunately experimental limits on proton decay seem to indicate that this model's prediction for the proton lifetime is too short.[26] Other than this, the SU(5) model holds up remarkably to experiment.

As it turns out, though, it is theoretical considerations that are the greatest challenge to Grand Unified Theories (GUTS). The best known such challenge is the "Gauge Hierarchy Problem." Perhaps this could better be known as the "Particle Physicist Employment Program" (PPEP), as in its simplest form it states no useful physics appears between $O(10^2)$ GeV and $O(10^{15})$ GeV.[27] More elegantly stated, to split the grand unified and weak mass scales by $O(10^{12})$ one must fine tune Higgs sector parameters to O(25) digit accuracy. This is hardly the work of a naturally predictive theory.

One has (For reviews see the papers by Ellis[5,28] and Nilles.[29]) quadratically divergent contributions to the Higgs propagator.

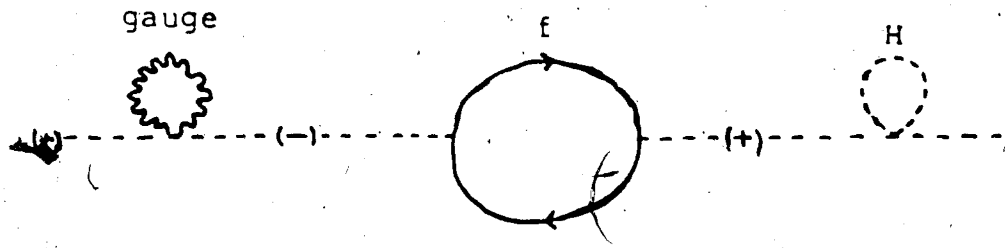

Fig. 1.1

The contribution from Fig 1.1 gives a mass shift.

$$\delta m_H^2 = O(\alpha^n)(\Lambda = O(m_X \text{ or } m_P))^2. \quad \text{Eq. 1.1}$$

Other mechanisms also tend to pull the Higgs mass to $O(M_X^2)$ or $O(M_P^2)$

As a final blow to the old ideas, we have the inability to incorporate gravitational interactions (the final frontier) into a Yang-Mills type gauge theory. Clearly some new ideas were needed to make Grand Unification a much more complete idea. One of these new ideas happens to be supersymmetry.

The basic approach to extending our unified gauge theories based on Lie groups is to increase the symmetry, which restricts the theory, and thus increases the predictive power of the theory. A theorem of Coleman and Mandula[30] tells us that in more than 1+1 dimensions to have a non-trivial S-matrix, the only possible conserved quantities that transform as tensors under the Lorentz group are the following:

The usual space time symmetries	P_μ	Energy-Momentum
	$M_{\mu\nu}$	Lorentz-Invariance
Lorentz Invariant Quantum Numbers	Q_i	electric charge etc.

Thus it would appear that for an interacting field theory no new symmetries could be added as the theory would become trivial and lose all its predictive power. It would seem we are at the end of the rope. What else can one do? Well you can always change ropes. If one extends[31] the theories based on Lie algebras to ones based on graded Lie algebras one can still construct a conserved current even in the presence of interactions and hence have a non-trivial theory. Theories based on these graded Lie algebras have come to be known as Supersymmetric Gauge Theories. They represent the next, but not the last step, in unified theories.

One of the recent developments (or resurrections -- depending how you see it) of particle physics that gives further support to supersymmetry is superstrings. These are theories of one dimensional extended quanta in higher dimensions which have a gauged (local) supersymmetry implying supergravitational interactions, as well as local internal gauge interactions. The most popular gauge group for string theories at present is $E_8 \times E_8$. Now E_8 has maximal subgroup $SU(3) \times E_6$, and E_6 represents a natural grand unified gauge group with $N=1$ SUSY.[32-34] One such possible breakdown of E_6 at low energies would be $SU(3) \times SU(2) \times U(1) \times U(1)$.[33]

It would now appear that at least theoretically one has incentive for supersymmetry from above and below. For this reason many particle physicists look forward to experimental evidence supporting or contradicting SUSY. On this note I will now proceed to some of the basic precepts and algebra of supersymmetry.

1.2 The Algebra of Supersymmetry

In 1974, Wess and Zumino[3] invented a Lagrangian with a remarkable new symmetry. This symmetry transformed bosons into fermions and vice-versa. The Lagrangian is in terms of a (dim 1) complex scalar field ϕ^i, a (dim 3/2) chiral fermion ψ_L^i and a (dim 2) auxiliary complex scalar field F^i. It also contains a "superpotential" $W(\phi^i)$ which for a renormalizable theory must be:

$$W = c_i\phi^i + m_{ij}\phi^i\phi^j + g_{ijk}\phi^i\phi^j\phi^k. \qquad \text{Eq. 1.2}$$

The Lagrangian is then:

$$L_{SUSY} = L_{KE} + L_{PE} + L_{YUK}$$

$$L_{KE} = \sum_i \partial_\mu\phi_i^*\partial^\mu\phi^i + \bar{\psi}_{Li}(i\not{\partial})\psi_L^i + F_i^*F^i$$

$$L_{PE} = \sum_i \left[\left[\frac{\partial W}{\partial\phi^i}\right]F^i + \left[\frac{\partial W}{\partial\phi^i}\right]^* F_i^* \right] \qquad \text{Eq. 1.3}$$

$$L_{YUK} = -\sum_{ij} \left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}(\bar{\psi}_R^{ci})\psi_L^j + \left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right]^*(\bar{\psi}_L^j)\psi_R^{ci} \right]$$

$$= \sum_{ij}\left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}(\psi_L^i)^T C^{-1}\psi_L^j - \left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right]^*(\bar{\psi}_L^j)C(\bar{\psi}_L^i)\right].$$

The Lagrangian is invariant, up to a total divergence, under the following transformation characterized by a constant anticommuting Majorana spinor ϵ:

$$\delta\phi^i = \bar{\epsilon}\psi_L^i$$

$$\delta\psi_L^i = \left[\frac{1-\gamma_5}{2}\right](F^i - i\not{\partial}\phi^i)\epsilon \qquad \text{Eq. 1.4}$$

$$\delta F^i = -i\bar{\epsilon}\not{\partial}\psi_L^i$$

One can use this to show (Appendix B) that $(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\phi$

gives

$[\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] = \bar{\epsilon}_1 \gamma^\mu \epsilon_2 P_\mu$ Eq. 1.5

or

$\{Q_\alpha, Q_\beta\} = (-\gamma^\mu C)_{\alpha\beta} P_\mu,$ Eq. 1.6

the algebra of the group, where Q is the generator of the group.

However, for a much more concise method of showing the algebra, I will use the method employed by S.D. Joglekar.[35] Firstly, the algebra of SUSY is an extension of the familiar Poincaré algebra which obeys the laws:

$[P_\mu, P_\nu] = 0$

$[M_{\mu\nu}, P_\rho] = -ig_{\mu\rho}P_\nu + ig_{\nu\rho}P_\mu$ Eq. 1.7

$[M_{\mu\nu}, M_{\rho\sigma}] = ig_{\nu\rho}M_{\mu\sigma} - ig_{\mu\rho}M_{\nu\sigma} - ig_{\nu\sigma}M_{\mu\rho} + ig_{\mu\sigma}M_{\nu\rho}.$

Now we must find the place of our generator Q. We shall characterize SUSY tranformations as $e^{\bar{\epsilon}Q}$. Now since $\delta A = \bar{\epsilon}\psi_L^i$, dim $\epsilon = -½$ (dim A = 1, dim ψ = 3/2), which implies dim Q = ½, since $\bar{\epsilon}Q$ must be dimensionless. Furthermore $\bar{\epsilon}Q$ must be spinless and commuting. Hence:

1. Dim Q = ½
2. Q must carry spin ½
3. Q is an anticommuting object.

Now Q must transform as a spinor under Lorentz transformations, so

$e^{i\omega^{\mu\nu}M_{\mu\nu}} Q e^{-i\omega^{\mu\nu}M_{\mu\nu}} = e^{½ i\sigma_{\mu\nu}\omega^{\mu\nu}} Q.$ Eq. 1.8

Taking an infinitesimal transformation, we have

$[Q_\alpha, M_{\mu\nu}] = ½(\sigma_{\mu\nu}Q)_\alpha = ½(\sigma_{\mu\nu})_{\alpha\beta}Q_\beta$ Eq. 1.9

Next we consider $[Q_\alpha, P_\mu]$. This has dimension 3/2. However, there are no operators of dimension 3/2 under consideration (Dim P=1, dim $M_{\mu\nu}$= dim r×p = 0). Thus

$$[Q_\alpha, P_\mu] = 0 \qquad \text{Eq. 1.10}$$

This says the Q_α are unaffected by space-time translations. This comes from the fact P_μ, Q_α are supertranslations in superspace.

Lastly we have $\{Q_\alpha, Q_\beta\}$ which has dimension 1. Furthermore, $\{Q_\alpha, Q_\beta\}$ is an (α, β) element of a 4×4 matrix that is symmetric in α, β. Recalling $\gamma^\mu C$ and $\sigma_{\mu\nu} C$ provide the basis for 4×4 symmetric matrices we write

$$\{Q_\alpha, Q_\beta\} = A(\gamma^\mu C)_{\alpha\beta} P_\mu + B(\sigma^{\mu\nu} C)_{\alpha\beta} M_{\mu\nu} \qquad \text{Eq. 1.11}$$

where Dim B = 1, Dim A = 0. Now

$$[\{Q_\alpha, Q_\beta\}, P_\mu] = \{Q_\alpha, [Q_\beta, P_\mu]\} + \{[Q_\alpha, P_\mu], Q_\beta\} = 0. \qquad \text{Eq. 1.12}$$

Thus

$$B(\sigma^{\lambda\kappa} C)_{\alpha\beta} [M_{\lambda\kappa}, P_\mu] = 0 \qquad \text{Eq. 1.13}$$

which implies B = 0. Fixing the (as yet arbitrary) normalization of Q one gets

$$\{Q_\alpha, Q_\beta\} = -(\gamma^\mu C)_{\alpha\beta} P_\mu . \qquad \text{Eq. 1.14}$$

The negative sign is there since it is related to the norm of the states, which must be positive.

So our extension to Poincaré-algebra is characterized by

$$[Q_\alpha, P_\mu] = 0$$

$$[Q_\alpha, M_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu})_{\alpha\beta} Q_\beta \qquad \text{Eq. 1.15}$$

$$\{Q_\alpha, Q_\beta\} = -(\gamma^\mu C)_{\alpha\beta} P_\mu .$$

One last characteristic of Q_α is that since ϵ is a Majorana spinor, Q_α must be one as well, Q_α satisfies

$$Q_\alpha = C_{\alpha\beta}\bar{Q}_\beta \qquad \text{Eq. 1.16}$$

This implies

$$\{Q_\alpha,\bar{Q}_\beta\} = \{Q_\alpha,-Q_\gamma C^{-1}_{\gamma\beta}\} = -\{Q_\alpha,Q_\gamma\}C^{-1}_{\gamma\beta} \qquad \text{Eq. 1.17}$$

$$= (\gamma^\mu C)_{\alpha\gamma}C^{-1}_{\gamma\beta}P_\mu = (\gamma^\mu)_{\alpha\beta}P_\mu$$

The above rules apply only for N=1 SUSY. For extensions of SUSY generated by sets of Majorana spinors $Q_{\alpha j}$ j=1,2...N. The third rule of Eqn 1.15 is replaced by,

$$\{Q_{\alpha i},Q_{\beta j}\} = -\delta_{ij}(\gamma^\mu C)_{\alpha\beta}P_\mu + C_{\alpha\beta}Z_{ij} + (\gamma_5 C)_{\alpha\beta}Z'_{ij} \qquad \text{Eq. 1.18}$$

where the generators Z_{ij} and Z'_{ij} are antisymmetric in i,j. They are called central charges and commute with all the generators of the system. Only N≤4 is possible for gauge theories since $Q_{\alpha j}$ changes helicity by ½ a unit. For theories including gravity N≤8 is possible. We will only deal with N=1.

1.3 Auxiliary Field Equations

We consider the "F" term in the Lagrangian

$$L_F = \sum_i F_i^* F^i + \left[\frac{\partial W}{\partial \phi^i}\right] F^i + \left[\frac{\partial W}{\partial \phi^i}\right]^* F_i^*, \qquad \text{Eq. 1.19}$$

and use the Euler-Lagrange equation

$$\partial_\mu \frac{\delta L}{\delta(\partial_\mu \phi)} - \frac{\delta L}{\delta \phi} = 0, \qquad \text{Eq. 1.20}$$

to give

$$F_i^* + \frac{\partial W}{\partial \phi^i} = 0, \qquad \text{Eq. 1.21a}$$

$$F^i + \left[\frac{\partial W}{\partial \phi^i}\right]^* = 0. \qquad \text{Eq. 1.21b}$$

This leaves the F term as

$$L_F = -\sum_i F_i^* F^i = -\sum_i \left|\frac{\partial W}{\partial \phi^i}\right|^2 \qquad \text{Eq. 1.22}$$

This is the potential term of our Lagrangian. The full Lagrangian rewritten now is:

$$L_{SUSY} = L_{KE} + L_{POT} + L_{YUK}$$

$$L_{KE} = \sum_i \left[\partial^\mu \phi_i^* \partial_\mu \phi^i + i\bar{\Psi}_{Li} \not{\partial} \psi_L^i \right]$$

$$L_{POT} = -\sum_i \left|\frac{\partial W}{\partial \phi^i}\right|^2 \qquad \text{Eq. 1.23}$$

$$L_{YUK} = -\sum_{ij} \left[\frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \bar{\Psi}_R^{ci} \psi_L^j + \left[\frac{\partial^2 W}{\partial \phi^i \partial \phi^j}\right]^* \bar{\Psi}_L^j \psi_R^{ci}\right].$$

For a gauge invariant SUSY theory the Lagrangian that appears is slightly different. It also has a second auxiliary field "D" which can be eliminated in the same fashion as "F". The complete gauge invariant SUSY Lagrangian may be found in Appendix C.

For gauge invariant theories that do not require renormalizability one will find higher order terms. I will not discuss these here, but the relevant Lagrangian may be found in Chapter 3.

1.4 Sparticles

For N=1 SUSY one has the following supermultiplets of particles;

Gauge (1,½)

Chiral (½,0).

Here the charges Q_a carry no quantum numbers, so the partners must share the same quantum numbers with their sparticles. This means no known particle may be the spartner of another. As a result we have a rich spectroscopy of particles given in Table 1.1 [From ref(5)].

Table 1.1 Supersymmetric Sspectroscopy

Particle	Spin	Sparticle	Spin	Mass
quark q	½	squark $\tilde{q}$	0	≥ O(15)GeV
lepton l	½	slepton $\tilde{l}$	0	≥ O(15)GeV
photon γ	1	photino $\tilde{\gamma}$	½	
gluon g	1	gluino $\tilde{g}$	½	≥ O(2)GeV
$W^{\pm}$	1	wino $\tilde{W}$	½	≥ O(15)GeV
Z^0	1	zino $\tilde{Z}$	½	
Higgs $H^{\pm}$	0	Higgsino $\tilde{H}^{\pm}$	½	≥ O(15)GeV
Higgs H^0	0	Higgsino $\tilde{H}^0$	½	

The two Higgs doublets $H_{1,2}$ of opposite hypercharges are needed to give masses to leptons and charge +2/3 and charge -1/3 quarks, as well as to cancel triangle anomalies. The lower limits on masses come from non-observation of direct production from e^+e^- annihilation (mainly $e^+e^- \rightarrow \tilde{X}^+\tilde{X}^-$). All results are model dependent.

Due to non-observation of the spartners, one is led to conclude that most or all of the spartner masses must be large. This is in direct contradiction to unbroken SUSY which requires that all of partners have identical masses to their sparticles. Thus SUSY must be broken enough to allow the sparticles to obtain a large mass. This breaking may be spontaneous, dynamical, or by explicit "soft" operators, without losing the desired features of the theory. There is a wealth of different schemes for symmetry breaking much too numerous to mention here. For an excellent review see the paper by Ellis.[28] Despite the wealth of different scenarios many of the models have similiar features. The models generally predict the masses of some of the spartners around O(100 GeV).

1.5 Couplings

We can now look at the general form of couplings of chiral supermultiplets. [Here we will follow the notations of ref. (5)]

Φ_i = (ψ_i of spin ½, ϕ_i of spin 0) Eq. 1.24

It is conventional to work with identical helicities for all the chiral fermions ψ_i; generally they all will be left-handed. Thus instead of using some species of right-handed fermions ψ_R (ie. q_R, e_R), we will use its conjugate antiparticle field:

$$\psi_R \rightarrow (\psi^c)_L = C(\bar{\psi}_R)^T \qquad \text{Eq. 1.25}$$

$$\psi_L \rightarrow \psi_L$$

Standardizing helicities leads to more convenient forms for the couplings. However, since the Lagrangian is hermitian it contains both particles and antiparticles of opposite helicities (ie. the Lagrangian would contain both left and right handed parts).

The non-gauge couplings of chiral smultiplets Φ_i are fixed by choosing a polynomial $W(\phi)$ called the super-potential. In order for the theory to be renormalizable it must be cubic:

$$W = a_{ij}\phi^i\phi^j + b_{ijk}\phi^i\phi^j\phi^k. \qquad \text{Eq. 1.26}$$

Yukawa interactions involving two fermions are obtained by differentiating out two of the spin zero ϕ_i's in W and replacing them by their fermionic partners, ψ_L^i, in Φ_i. Thus:

$$L_Y = \frac{\partial^2 W}{\partial\phi^i\partial\phi^j}(\bar{\psi}_R^{ci}\psi_L^j) + \text{H.C.} \qquad \text{Eq. 1.27}$$

$$= a_{ij}(\bar{\psi}_R^{ci}\psi_L^j) + b_{ijk}\phi_k(\bar{\psi}_R^{ci}\psi_L^j) + \text{H.C.}$$

The first term on the R.H.S. of Eqn 1.27 is the fermion mass matrix, while the second term is a true Yukawa interaction.

Multiple scalar boson interactions are obtained by;

$$V_F = \sum_i \left|\frac{\partial W}{\partial\phi^i}\right|^2 = \sum_i |\ a_{ij}\phi_j + b_{ijk}\phi_j\phi_k\ |^2. \qquad \text{Eq. 1.28}$$

The first term of the R.H.S. gives the scalar boson mass squared matrix.

$$(m_B)^2_{ij} = a_{ik}a^*_{kj} = (m_F m_F^*)_{ij}. \qquad \text{Eq. 1.29}$$

It is immediately obvious that this is equal to the fermion mass squared matrix. The second term of the equation gives the quartic scalar interactions like one gets from a Higgs Potential.

The fermion gauge interactions come directly from the standard gauge interactions as N=1 SUSY adds no internal quantum numbers. Thus for left-handed fermions, ψ_L, in different representations, T_L^a, of the gauge group with generators a, one has the standard gauge interaction;

$$g(\bar{\psi}_L \gamma^\mu T_L^a \psi_L) A_\mu^a. \qquad \text{Eq. 1.30}$$

As a direct result of no internal quantum numbers, one can write down couplings for the scalar partners, ϕ_i, of the fermions, ψ_i, in the chiral multiplet, Φ_i;

$$g(\phi^* \overleftrightarrow{\partial}^\mu T_L^a \phi_L) A_\mu^a. \qquad \text{Eq. 1.31}$$

There are also gaugino interactions obtained by substituting one of the ψ (or ϕ) and the gauge boson A_μ^a by their supersymmetric partners: (For A_μ^a this means $A_\mu^a \to \chi^a$.)

$$L_{A-} = \sqrt{2} g(\phi_L^* T_L^a)(\bar{\chi}^a \psi_L) + \text{h.c.} \qquad \text{Eq. 1.32}$$

where $(\bar{\chi}^a \psi)$ is the Lorentz scalar combination of the two fermions. Eqns. 1.30 thru 1.32 are valid for both left-handed fields, both the normal ψ_L and the conjugate field ψ_L^c. As a result eqns. 1.31 and 1.32 draw a distinction between ϕ_L and ϕ_L^c despite the fact ϕ is a spin zero field and has no intrinsic spin. Thus if a conjugate e_L^c does not couple to a conjugate neutrino, ν_L^c via a gauge boson W, neither will a conjugate (s)electron $e_L^c(\tilde{e}_L{}^c)$ couple to a conjugate sneutrino, $\tilde{\nu}_L^c$, via a gaugino, $\tilde{W}$, (gauge boson, W). In addition to the expected interactions there are also additional scalar quartic interactions

$$V_D = \tfrac{1}{2} g^2 \sum_{a,\phi} |\phi_L^* T_L^a \phi_L|^2 \qquad \text{Eq. 1.33}$$

which contribute to the full potential,

$$V = V_F + V_D. \qquad \text{Eq. 1.34}$$

The full interaction Lagrangian and the resulting Feynmann rules are given in Appendix C.

1.6 Masses

In general, for mass matrices of the squarks and sleptons, we can have mixing in both flavour and chirality (L-R) space. Before SUSY is broken, one has conventional Higgs-fermion Yukawa interactions derived from a superpotential term;

$$W(\phi) = b_{H\bar{q}q} q_L q_L^c H \qquad \text{Eq. 1.35}$$

which with the aid of Eqn 1.28 gives

$$V_F = |b_{H\bar{q}q}|^2 [\ |\tilde{q}_L \tilde{H}|^2 + |\tilde{q}_L^c \tilde{H}|^2 + |\tilde{q}_L \tilde{q}_L^c|^2\ ] \qquad \text{Eq. 1.36}$$

Now using a Higg's vacuum expectation value $\langle 0|H|0\rangle = v$ such that $m_q = b_{H\bar{q}q} v$, one obtains the following contributions for the $(\tilde{q}_L, \tilde{q}_R)$ mass matrix;

$$[\tilde{q}_L^* \quad \tilde{q}_R^*] \begin{bmatrix} m_q^2 & 0 \\ 0 & m_q^2 \end{bmatrix} \begin{bmatrix} \tilde{q}_L \\ \tilde{q}_R \end{bmatrix}. \qquad \text{Eq. 1.37}$$

Clearly this mass matrix can be diagonalized in flavour space simultaneously with the conventional quark matrix, and one finds as expected from eqn 1.29, that the squark (slepton) mass is equal to the quark (lepton) mass. This is clearly not so experimentally, so to rescue the situation one must break SUSY.

Phenomenologically acceptable models exist with spontaneously broken supergravity and manifest themselves in the low energy theory as softly explicitly broken global supersymmetry. Since we are only interested in the low energy sector [O(100 GeV)], we will concern ourselves with global supersymmetry which looks like an ordinary gauge theory with particular couplings, seen earlier, and masses, where the mass scales come from "explicit soft breaking terms" that originate in the aforementioned supergravity theories. Thus we can now go on to some general mass matrices.

The mass degeneracy is removed by introducing SUSY breaking mass squared terms of the general form:

$$\tilde{m}_q^2|\tilde{q}|^2 + \tilde{m}_\ell^2|\tilde{\ell}|^2 + \tilde{m}_\nu^2|\tilde{\nu}|^2. \qquad \text{Eq. 1.38}$$

Each of the mass matrices $\tilde{m}_q^2$ and $\tilde{m}_\ell^2$ can a priori be a general matrix in flavour and "helicity" space. However, careful analysis[5] has shown that this leads to unacceptable flavour changing neutral current interactions and the gaugino interactions would no longer be flavour diagonal giving catastrophically large contributions to the K_1-K_2 mass matrix. Phenomenological considerations severely constrain the form of the mass matrices, and as a result of more detailed analysis[5] one gets the following flavour diagonalized form;

$$[\tilde{q}_L^* \quad \tilde{q}_R^*] \begin{bmatrix} L^2\tilde{m}^2 + m_q^2 & A\tilde{m}m_q \\ A\tilde{m}m_q & R^2\tilde{m}^2 + m_q^2 \end{bmatrix} \begin{bmatrix} \tilde{q}_L \\ \tilde{q}_R \end{bmatrix} \qquad \text{Eq. 1.39}$$

where $\tilde{m}_L^2 = L^2\tilde{m}^2$, $\tilde{m}_R^2 = R^2\tilde{m}^2$, and $L^2 \neq R^2$ in general, $A \approx O(1)$ and $\tilde{m} \approx O(20 \text{ to } 1000)$ GeV.

1.7 Implications and Intentions

Supersymmetry, by its introduction, has at least doubled the number of fundamental particles and introduced many new couplings. To the chagrin of many, none of these particles have been found. However, much work has gone into mapping out the phenomenological consequences and physical implications of SUSY. An "R-parity" (essentially a charge carried only by superpartners) survives, at least as a discrete symmetry, in most versions of SUSY[29] and dictates that superpartners are produced only in association. The constraints on the masses given in Table 1.1 come from non-observation of pair production at existing e^+e^- machines, and represents one of many ways that direct evidence for SUSY has been searched for. As well, in many theories the lightest supersymmetric particle (LSP) is quasi-stable and thus may be detected in cosmological searches.[36]

In the second chapter of this thesis we perform a calculation that searches for possible indirect effects of SUSY via radiative corrections. Specifically we have calculated supersymmetric radiative corrections to muon decay. For a similiar treatment of $\pi_{\ell 2}$ decay and the supersymmetric lamb shift, one can see the work of Campbell and Scott.[37-38]

In Chapter 3 we perform some calculations where we have assumed a composite nature for leptons and hadrons. Here we have assumed the existence of supersymmetry and used a supersymmetric composite model to calculate radiative corrections that appear suppressed by fewer orders of the composite mass scale than in non-supersymmetric composite models. This is due to the fact that if we calculated a pure composite decay our results would be of order $\frac{1}{M^2}$, where M is a binding energy for the compositeness. For our combined SUSY/composite model decay our results are of order $\frac{1}{MM_S}$ where M_S is a supersymmetric mass presumably much smaller than M. Although we make use of compositeness, we make use of no particular composite model, but rather treat compositeness as a black box. This will make our discussion quite general.

Chapter 4 will end this thesis with a summary of results and conclusions.

2. SUPERSYMMETRIC RADIATIVE CORRECTIONS TO MUON DECAY

2.1 Introduction

Although the energy to observe supersymmetry directly may not be available as yet, there are other ways to observe supersymmetry. One such way is to calculate the effects of radiative corrections at the one loop level that are introduced because of the exchange of supersymmetric particles and then compare these with experimental parameters.

Effects with this origin have already been studied in the literature. One loop corrections to the lepton anomalous magnetic moment have been examined,[39-42] and put constraints on the slepton and electroweak gauge fermion masses. Radiatively induced strong interaction parity violation has been evaluated,[43-44] and puts severe constraints on the mass splitting between left and right chiral squarks. Radiatively induced flavour changing neutral interactions have been calculated in these theories,[45-47] and constrain the masses, mass splittings, and mixing angles for scalars and their coupling to gauge fermions. Finally one loop corrections to pion decay and the supersymmetric lamb shift have been calculated,[37] giving limits on the masses of scalar particles and gauge fermions.

We calculate one loop corrections to muon decay. Comparisons of the scalar/psuedoscalar portions of the decay with experiment give limits on the masses of scalar particles and gauge fermions.

2.2 Mass Mixing Revisited

In chapter one we learned that the mass squared matrix for squarks and sleptons will in general be a 2n×2n matrix, where n is the number of quarks or leptons of a given cha In the work discussed here we have no need for flavour (mixing) indices, which would always appear summed over at intermediate stages in our process, giving a result of order unity (GIM-unsupressed). Thus we have the mass squared matrix we wrote down in chapter one

$$[\tilde{f}_L^* \quad \tilde{f}_R^*] \begin{bmatrix} L^2\tilde{m}^2 + m_f^2 & A\tilde{m}m_f \\ A\tilde{m}m_f & R^2\tilde{m}^2 + m_f^2 \end{bmatrix} \begin{bmatrix} \tilde{f}_L \\ \tilde{f}_R \end{bmatrix} \qquad \text{Eq. 1.39}$$

which for $L^2 \approx R^2$ and $\tilde{m} >> m_f$ reduces to

$$[\tilde{f}_L^* \quad \tilde{f}_R^*] \begin{bmatrix} \tilde{m}^2 & \Delta^2 \\ \Delta^2 & \tilde{m}^2 \end{bmatrix} \begin{bmatrix} \tilde{f}_L \\ \tilde{f}_R \end{bmatrix} \qquad \text{Eq. 2.1}$$

where $\tilde{m}$ is the mass of the scalar particle and $\Delta^2 \approx \tilde{m}m_f$, m_f being the mass of the associated fermion. In some models, spontaneously broken supergravity gives a major contribution to $\tilde{m}^2$ that is independent of the particle. In particular, for a specific class of models this is the gravitino mass squared. Thus in general one can set $\tilde{m}^2$ equal to some large mass squared, M_S^2, which is independent of the particle. It is the Δ^2 term that interests us though. It provides the mixing parameter that allows us to flip chiralities and calculate normally supressed rates.

For the low energy processes we will consider, the momentum transfers are assumed to be small. The resulting amplitude can then be described by a local operator of the light external particles, describing an effective contact interaction.[48,45] By bounding the matrix elements for these operators from comparison with experiment we can then get constaints on the supersymmetric content of the theory.

2.3 Muon Decay

We will now look at the decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ or $\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$. This will be of some interest since we will be able to calculate possible deviations from V-A. This can be achieved by flipping the helicities of the sleptons, in particular the selectron, so as to achieve a right-handed electron (left-handed anti-electron).

First let us consider Fig. 2.1, which leads to the largest contribution.

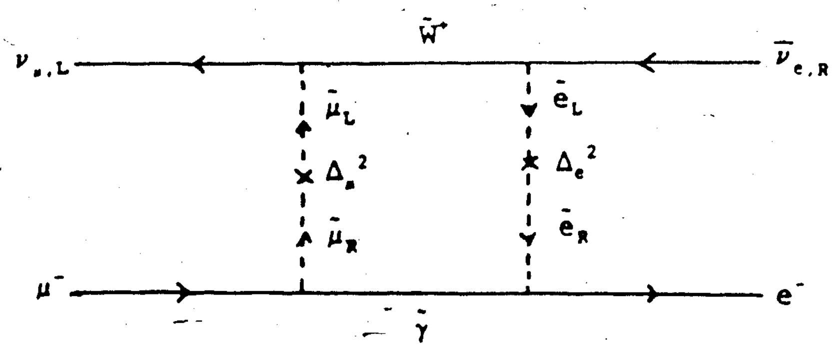

Fig. 2.1

Using

$\psi_1 = \psi_{\bar{\nu}_e}$,

$\psi_2 = \psi_{\nu_\mu}$

as our neutrino and anti-neutrino wavefunction, and ψ_e, ψ_μ as our electron and muon wavefunctions respectively, we write down the resulting amplitude:

$$\text{Amp} \sim \frac{(-\sqrt{2})^2(\sqrt{2})^2}{(\sqrt{2})^2} e^2 g^2 \int \frac{d^4k}{(2\pi)^4} i^6 \left[\frac{\bar{\psi}_2 R(\not{k}+\tilde{M}_w) L\psi_1}{(k^2-\tilde{M}_w^2)}\right]$$

$$\times \left[\frac{\bar{\psi}_e L(\not{k}+\tilde{M}_\gamma) R\psi_\mu}{(k^2-\tilde{M}_\gamma^2)}\right] \frac{1}{(k^2-\tilde{M}_{e_R}^2)} \Delta_e^2 \frac{1}{(k^2-\tilde{M}_{e_L}^2)} \qquad \text{Eq. 2.2}$$

$$\times \frac{1}{(k^2-\tilde{M}_{\mu_R}^2)} \Delta_\mu^2 \frac{1}{(k^2-\tilde{M}_{\mu_L}^2)}$$

In determining this amplitude we have used the Feynmann rules from Appendix C, and have ignored all external momenta. Putting $\tilde{M}_\gamma = 0$ and using symmetric integration we get:

$$\text{Amp} \sim -2e^2g^2[\bar{\psi}_2\gamma^\mu L\psi_1][\bar{\psi}_e\gamma_\mu R\psi_\mu]\times I \qquad \text{Eq. 2.3}$$

where

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2(k^2-\tilde{M}_w^2)} \frac{\Delta_e^2}{(k^2-\tilde{M}_{e_R}^2)(k^2-\tilde{M}_{e_L}^2)} \frac{\Delta_\mu^2}{(k^2-\tilde{M}_{\mu_R}^2)(k^2-\tilde{M}_{\mu_L}^2)}, \qquad \text{Eq. 2.4}$$

which by assuming a common mass for the remaining sparticles simplifies to

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{\Delta_e^2\Delta_\mu^2}{(k^2-M_S^2)^5} = \frac{-i}{192\pi^2} \frac{\Delta_e^2\Delta_\mu^2}{M_S^6}. \qquad \text{Eq. 2.5}$$

The calculation of the integral is performed in Appendix D.

Therefore, one gets

$$\text{Amp} \sim \frac{1}{96\pi^2} e^2 g^2 [\bar{\Psi}_2 \gamma^\mu L \psi_1][\bar{\Psi}_e \gamma_\mu R \psi_\mu] \frac{\Delta_e^{\,2} \Delta_\mu^{\,2}}{M_S^{\,6}}. \qquad \text{Eq. 2.6}$$

(I have dropped a factor of i here)

We now Fierz reorder this using the Fierz transform found in Appendix E:

$$[\bar{\Psi}_2 \gamma^\mu L \psi_1][\bar{\Psi}_e \gamma_\mu R \psi_\mu] = -2[\bar{\Psi}_2 R \psi_\mu][\bar{\Psi}_e L \psi_1] \qquad \text{Eq. 2.7}$$

which gives;

$$\text{Amp} \sim \frac{-e^2 g^2}{48\pi^2} [\bar{\Psi}_2 R \psi_\mu][\bar{\Psi}_e L \psi_1] \frac{\Delta_e^{\,2} \Delta_\mu^{\,2}}{M_S^{\,6}}. \qquad \text{Eq. 2.8}$$

Lastly making the identification that

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^{\,2}} \qquad \text{Eq. 2.9}$$

and $\Delta_e^{\,2}$, $\Delta_\mu^{\,2}$ are of the order of $m_e M_S$, $m_\mu M_S$ respectively, one obtains

$$\text{Amp} \sim \frac{G_F}{\sqrt{2}} \frac{-e^2}{6\pi^2} [\bar{\Psi}_2 R \psi_\mu][\bar{\Psi}_e L \psi_1] \frac{m_e m_\mu M_W^{\,2}}{M_S^{\,4}}. \qquad \text{Eq. 2.10}$$

We now move onto the second possible diagram and consider Fig. 2.2.

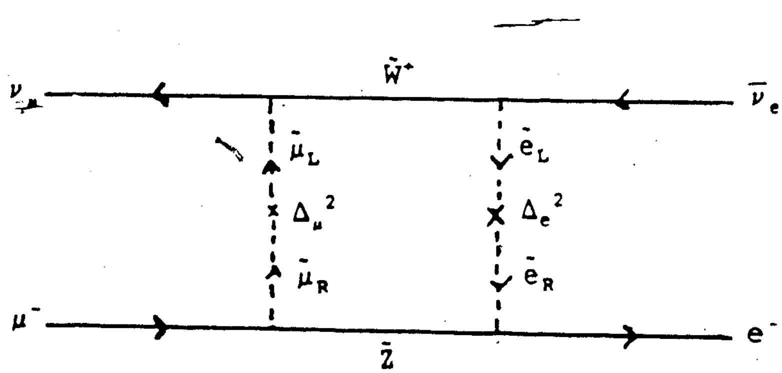

Fig. 2.2

It has

$$\text{Amp} \sim \frac{(-\sqrt{2})^2(\sqrt{2})^2}{(\sqrt{2})^2} g^4 \frac{\sin^2\theta_W}{\cos^2\theta_W} \int \frac{d^4k}{(2\pi)^4} i^6$$

$$\times \left[\frac{\bar{\Psi}_2 R(\not{k}+\tilde{M}_W)L\psi_1}{(k^2-\tilde{M}_W^2)}\right] \left[\frac{\bar{\Psi}_e L(\not{k}+\tilde{M}_Z)R\psi_\mu}{(k^2-\tilde{M}_Z^2)}\right] \qquad \text{Eq. 2.11}$$

$$\times \frac{\Delta_e{}^2}{(k^2-\tilde{M}_{e_R}^2)(k^2-\tilde{M}_{e_L}^2)} \frac{\Delta_\mu{}^2}{(k^2-\tilde{M}_{\mu_R}^2)(k^2-\tilde{M}_{\mu_L}^2)}.$$

One could consider leaving out the chiral mass mixing term on the $\tilde{\mu}$ line, but then one would have $L\not{k}L=0$ leaving the integral odd in k. Simplifying we have

$$\text{Amp} \sim -2g^4 \frac{\sin^2\theta_W}{\cos^2\theta_W}[\bar{\Psi}_2\gamma^\mu L\psi_1][\bar{\Psi}_e\gamma_\mu R\psi_\mu]\times I, \qquad \text{Eq. 2.12}$$

where

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{k^2\Delta_e{}^2\Delta_\mu{}^2}{(k^2-M_S{}^2)^6} = \frac{-i}{480\pi^2} \frac{\Delta_e{}^2\Delta_\mu{}^2}{M_S{}^6}, \qquad \text{Eq. 2.13}$$

which leaves after Fierz reordering and the use of Eqn 2.9 with

$$\frac{M_W{}^2}{\cos^2\theta_W} = M_Z{}^2 \qquad \text{Eq. 2.14}$$

$$\text{Amp} \sim -\frac{G_F}{\sqrt{2}}\frac{G_F}{\sqrt{2}}\frac{8\sin^2\theta_W}{15\pi^2}[\bar{\Psi}_2 R\psi_\mu][\bar{\Psi}_e L\psi_1]\frac{m_e m_\mu M_W{}^2}{M_S{}^4}M_Z{}^2. \qquad \text{Eq. 2.15}$$

There could only be one other contribution (Fig 2.3),

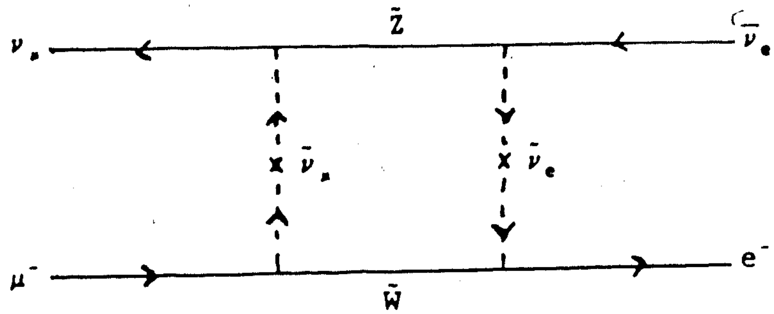

Fig 2.3

but this involves chiral mass mixing of a sneutrino which is expected to be negligible.

Adding the two amplitudes (Eqns. 2.10 and 2.15) we obtain the total amplitude due to SUSY one loop diagrams:

$$\text{Amp} \sim \frac{-1}{\pi^2}\frac{G_F}{\sqrt{2}}\left[\frac{e^2}{6} + \frac{8\sin^2\theta_W}{15}\frac{G_F}{\sqrt{2}}M_Z^2\right]\frac{m_e m_\mu M_W^2}{M_S^4}$$
$$\times\left[\bar{\psi}_2\left[\frac{1+\gamma_5}{2}\right]\psi_\mu\right]\left[\bar{\psi}_e\left[\frac{1-\gamma_5}{2}\right]\psi_1\right]. \qquad \text{Eq. 2.16}$$

We can now compare this to the general four-fermion interaction for muon decay.[49]

$$L_{int} = -\frac{G_F}{\sqrt{2}}\sum_i[\bar{\psi}_e\Gamma_i\psi_1][\bar{\psi}_2\Gamma_i(G_i+G_i'\gamma_5)\psi_\mu]. \qquad \text{Eq. 2.17}$$

This gives us

$$G_S=G_S'=-G_P=-G_P' = \frac{1}{4\pi^2}\left[\frac{e^2}{6} + \frac{8\sin^2\theta_W}{15}\frac{G_F}{\sqrt{2}}M_Z^2\right]\frac{m_e m_\mu M_W^2}{M_S^4}, \qquad \text{Eq. 2.18}$$

which upon substitution of known quantities gives us

$$G_S = \frac{2.0\times10^8}{M_S^4} \qquad \text{Eq. 2.19}$$

where M_S is given in MeV. Now also from Stoker[49] we have

$$(G_S-G_P')^2 + (G_S'-G_P)^2 < 0.066 \qquad \text{Eq. 2.20}$$

which gives

$$|G_S| < 0.091. \qquad \text{Eq. 2.21}$$

This gives us a lower bound on M_S of

$$M_S{}^4 > \frac{2.0\times10^8}{0.091} = 2.2\times10^9 \text{ (Mev)}^4$$

$$M_S > 200 \text{ MeV}. \qquad \text{Eq. 2.22}$$

Obviously this lower bound is much lower than lower bounds established by other methods. What we find is that radiative corrections to muon decay give us no useful information in terms of supersymmetric masses.

3. COMBINED SUSY/COMPOSITE MODEL DECAYS

3.1 Introduction

Recently there has been a renewed interest in composite models for quarks and leptons. In this thesis we make claims to no particular model, or even the existence of compositeness itself. Rather we devise a means by which to calculate a lower bound to any possible mass scale for a composite system. In this method we shall treat compositeness as a black box and will perform our calculation simply by considering all gauge invariant possibilties.

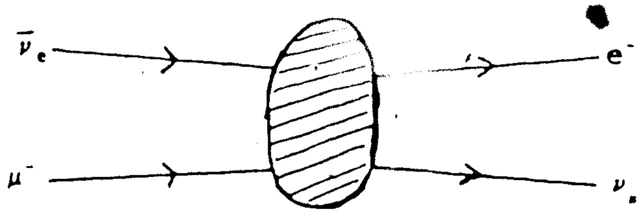

Fig. 3.1

When one calculates a straight composite decay, as exemplified by fig. 3.1, one finds that the amplitude is proportional to $\frac{1}{M^2}$, where M is an energy scale for the compositeness. This comes from simple dimensional analysis. Since the action, $S = \int d^4x\, L$, must be dimensionless, one has from the matter Lagrangian for fermions, $L_M = \psi \partial\!\!\!/ \psi$, giving $S = \int d^4x (\psi \partial\!\!\!/ \psi)$, that $\dim[\psi]=3/2$, as $\dim[d^4x]=-4$ and $\dim[\partial]=+1$. Now our amplitude for the decay will go like $L = A(\bar{\psi}\psi)(\bar{\psi}\psi)$, which implies from $S = \int d^4x\, A(\bar{\psi}\psi)(\bar{\psi}\psi)$, that $\dim[A]=-2$, or in other words $A \propto \frac{1}{M^2}$, since M is the only available mass scale. Having our amplitude proportional to $\frac{1}{M^2}$ means that its contribution is going to be small. One may now ask; is there any way to reduce this to a $\frac{1}{M}$ proportionality? The answer is yes, and the solution is to introduce supersymmetry.

The matter Lagrangian for a scalar, $L_M = (\partial_\mu\phi)^*(\partial^\mu\phi)$ or $L_M = \phi^*\partial^2\phi$, implies that $\dim[\phi]=1$ in order to make the action dimensionless.

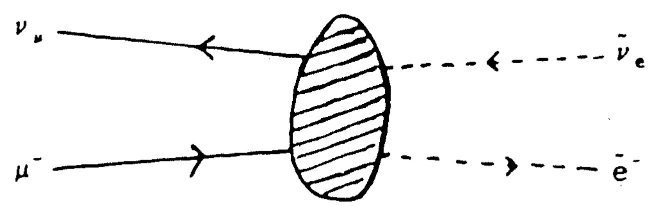

Fig. 3.2

Thus a decay like figure 3.2, which goes like $L = B(\bar{\psi}\psi)(\phi^*\phi)$ implies from $S = \int d^4x\, B(\phi^*\phi)(\bar{\psi}\psi)$, that $\dim[B]=-1$ so that $B \propto \frac{1}{M}$.

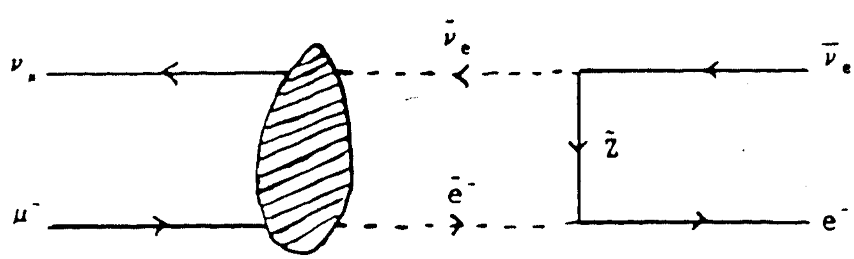

Fig. 3.3

Our complete decay, diagrammed in fig. 3.3, would then go as $\frac{1}{MM_S}$, since the mass of the superpartner scalars and spinors appearing in the loop are characterized by the supersymmetry breaking scale M_S.

We now have a decay that goes as $\frac{1}{MM_s}$. For models where the supersymmetry breaking scale is much less than the scale of compositeness (ie. $M_s \ll M$), our chance of observing contributions from compositeness is enhanced. In this chapter we will make just such an attempt for muon decay, $\pi_{\ell 2}$ decay, and $\mu N \rightarrow eN$ conversion.

3.2 General Supersymmetric Lagrangian

In order to use our black box approach we must know all the possible couplings that can take place. We can discover these possible couplings from the Lagrangian. However, since we no longer require renomalizability we are not restricted to the Lagrangian of Appendix C. Rather the Lagrangian is superseded by a more general Lagrangian given by Weinberg.[50] It is:

$$
\begin{aligned}
L = {} & J_{ij}\partial_\mu\phi_i\partial^\mu\phi_j^* + J^{-1}_{ij}\frac{\partial f}{\partial\phi_i}\left[\frac{\partial f}{\partial\phi_j}\right]^* \\
& + J_{ij}\bar{\psi}_i\not{\partial}\psi_j \qquad\qquad \text{Eq. 3.1} \\
& + (\psi_L^i)^T C^{-1}\psi_L^j\left[\frac{\partial^2 f}{\partial\phi_i\partial\phi_j} - J^{-1}_{km}\left[\frac{\partial f}{\partial\phi_k}\right]\frac{\partial^3 d}{\partial\phi_m^*\partial\phi_i\partial\phi_j}\right] \\
& - \bar{\psi}_L^i C(\bar{\psi}_L^j)^T\left[\left[\frac{\partial^2 f}{\partial\phi_i\partial\phi_j}\right]^* - J^{-1}_{mk}\left[\frac{\partial f}{\partial\phi_k}\right]^*\frac{\partial^3 d}{\partial\phi_m\partial\phi_i^*\partial\phi_j^*}\right] \\
& - \bar{\psi}_L^i\gamma^\mu\psi_L^j\left[\frac{\partial^3 d}{\partial\phi_i\partial\phi_j^*\partial\phi_m}\partial_\mu\phi_m - \frac{\partial^3 d}{\partial\phi_i\partial\phi_j^*\partial\phi_m^*}\partial_\mu\phi_m^*\right] \\
& - [(\psi_L^i)^T C^{-1}\psi_L^j][\bar{\psi}_L^k C\bar{\psi}_L^m]\left[\frac{\partial^4 d}{\partial\phi_i\partial\phi_j\partial\phi_k^*\partial\phi_m^*}\right. \\
& \left. + J^{-1}_{np}\frac{\partial^3 d}{\partial\phi_n^*\partial\phi_i\partial\phi_j}\frac{\partial^3 d}{\partial\phi_p\partial\phi_k^*\partial\phi_m^*}\right]
\end{aligned}
$$

where

$$J_{ij} = \frac{\partial^2 d}{\partial\phi_i \partial\phi_j^*}. \qquad \text{Eq. 3.2}$$

The f term is an exact analog of the superpotential (ie. $f=f(\phi)$), whereas $d=d(\phi,\phi^*)$. Now we are specifically interested in the part of the Lagrangian that will generate two scalars and two fermions with the added consideration that since we want to have a dimension five effective interaction we will have no derivative terms (ie. terms with $\partial_\mu\phi_i$), which require a dimension six d term. This leaves us with the third and fourth lines of eqn. 3.1:

$$L = (\psi_L^i)^T C^{-1}\psi_L^j\left[\frac{\partial^2 f}{\partial\phi_i\partial\phi_j} - J^{-1}_{km}\left[\frac{\partial f}{\partial\phi_k}\right]\frac{\partial^3 d}{\partial\phi_m^*\partial\phi_i\partial\phi_j}\right] \qquad \text{Eq. 3.3}$$

$$-\bar{\Psi}_L^i C(\bar{\Psi}_L^j)^T\left[\left[\frac{\partial^2 f}{\partial\phi_i\partial\phi_j}\right]^* - J^{-1}_{mk}\left[\frac{\partial f}{\partial\phi_k}\right]^*\frac{\partial^3 d}{\partial\phi_m\partial\phi_i^*\partial\phi_j^*}\right]$$

Furthermore, in order to produce 2 scalars and 2 fermions we will need dim=5 f and d terms. This means f is of the form $\phi\phi\phi\phi$ and d is of the form $\phi^*\phi\phi$ or $\phi\phi^*\phi^*$.

One last and extremely important constraint on the f and d terms is that they must be gauge invariant. The only possible dim=5 terms that are $SU(3)\times SU(2)\times U(1)$ invariant and which are also invariant under "R-parity" are:[50,51]

$(L_L E_L^c H_L'^*)_d$ $\quad$ $(Q_L D_L^c H_L'^*)_d$ $\quad$ $(Q_L U_L^c H_L^*)_d$

$(L_L L_L H_L' H_L')_f$ $\quad$ $(Q_L Q_L U_L^c D_L^c)_f$ $\quad$ $(Q_L U_L^c L_L E_L^c)_f$

$(Q_L Q_L Q_L L_L)_f$ $\quad$ $(U_L^c U_L^c D_L^c E_L^c)_f$.

Here the supermultiplets are denoted by capital letters: the spin ½ component of the left chiral scalar superfield $Q_L=(U_L,D_L)$ is the quark doublet $q_L=(u_L,d_L)$; the spin ½ components of the left chiral scalar superfield U_L^c, D_L^c are the conjugate particles u_L^c, d_L^c of the singlet quarks. H_L and H_L' are colour-singlet electroweak-doublet left-chiral Higgs superfields. Also of value to us will be dim=3 and 4 f terms, the SU(3)×SU(2)×U(1) and "R-parity" invariant ones are:[50,51]

$$(H_L'H_L)_f$$

$$(H_L L_L E_L^c)_f \qquad (H_L Q_L D_L^c)_f \qquad (H_L' Q_L U_L^c)_f .$$

The dim=4 f and d terms represent the proper dimensionality for a dimensionless action. Any higher or lower dimension terms must carry the proper proportionality constant to correct for the dimensions. Thus a dim=5 term requires a $(\text{mass})^{-1}$ term to correct for the extra dimension. This is where our $\frac{1}{M}$ proportionality comes in.

Now for the second term in each line of eqn. 3.3, that is the terms involving d, we will want a dim=5 d term as this is the minimum nonvanishing d term for $\frac{\partial^3 d}{\partial \phi_m^* \partial \phi_i \partial \phi_j}$. Now because we require two scalars; we need a dim=4 f term for $\frac{\partial f}{\partial \phi_k}$ to go with the dim=5 d term. This, then, requires a dim=4 d term for J_{km}^{-1} to complete the whole term. Now dim=4 d terms are just kinetic terms. They must be in the form $\phi^*\phi$. Since all dim=5 d terms are of the form H_L _ _ or H_L' _ _ where the blanks are the two fermions we require, the dim=4 d term for J_{km}^{-1} must be $H_L H_L^*$ or $H_L' H_L'^*$.

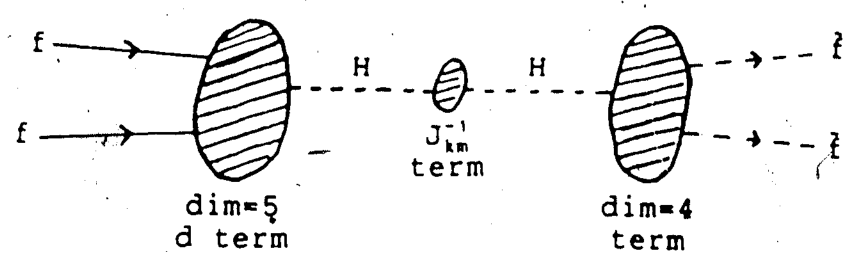

Fig. 3.4

These will look something like fig. 3.4, which represents a contribution to Higgs exchange diagrams which, since we know little about Higg's exchange, we are not particularily interested in any corrections to it.

This leaves us solely with the first term in each line of eqn. 3.3, which deals only with one f term. These are quite simple to handle. All we need is a dim=5 f term which has the four chiral smultiplets we need. All such possible gauge invariant f terms were given above.

3.3 SUSY/Composite Model for μ Decay

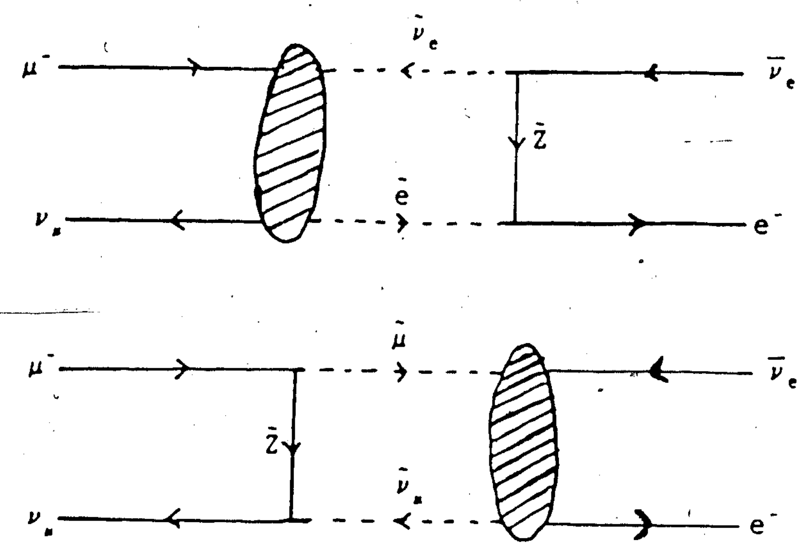

Fig. 3.5

To investigate muon decay we are interested in diagrams such as fig. 3.5 and all other permutations of this diagram. The dim=5 f term that is required for this would be one that involves four leptons. Quite simply there is no such gauge invariant f term.

Although I mentioned earlier that any possible d terms will be Higgs exchange contributions I will proceed to investigate possible d terms in order to give an example of the choosing process. One can choose a dimensionality 5 d term like $d = \frac{1}{M} L_L E^c_L H'^*_L$, but then you also require a dimensionality 4 f term like $f = H_L L_L E^c_L$ in order to get the right external states. This in turn requires the d term for J^{-1}_{km} to be $d = H_L H'^*_L$, which is not gauge invariant. Therefore one cannot construct a term like this.

One can also investigate possible dim=6 (quartic) d terms. Although these will have a $\frac{1}{M^2}$ dependence, they will be coupled to a dim=3 f term (quadratic) that has M dependence. The only possible dim=3 term we can choose is

$f_1 = M' L_L H_L'$

which means the d term for J_{km}^{-1} has to be

$d_2 = H_L' H_L'^{*}$.

So the d term must be

$d_3 = \frac{1}{M^2} H_L'^{*}$ _ _ _.

Now the final three positions must be leptons, either L_L or E_L^c. We also require two incoming and two outgoing states. The L_L in the f_1 term is one incoming term, so we require one more incoming and two outgoing terms. This combined with the need for one or three isodoublets means the remaining positions must be $L_L L_L^* L_L^*$ or $L_L^* E_L^c E_L^{c*}$ (E_L^c is an outgoing state. So

$d_3 = \frac{1}{M^2} H_L'^{*} L_L^* L_L^* L_L$ or $\frac{1}{M^2} H_L'^{*} L_L^* E_L^c E_L^{c*}$.

These are both gauge invariant terms, but once again they represent contributions to Higg's exchange diagrams, so we are not particularily interested in them.

This exhausts all the possibilities for using a combined SUSY/composite model for $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. It also exhausts any possibilities for doing $\mu^- \to e^- e^+ e^-$ since the fact that two of the leptons were neutrinos was never used.

Thus this method gives no limits on either of these cases.

3.4 Application of SUSY/Composite Model to $\pi_{\ell 2}$ Decay

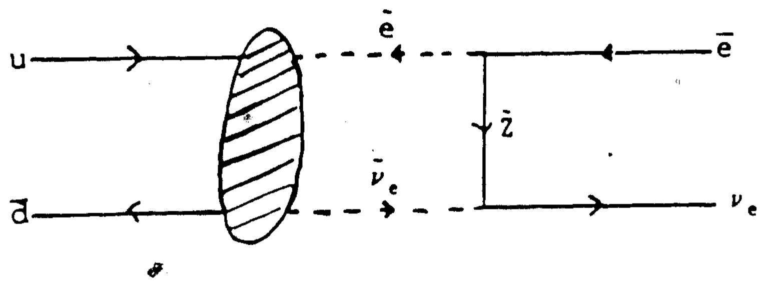

Fig. 3.6

Figure 3.6 shows just one of six possible contributions to $\pi_{\ell 2}$ decay from a SUSY/composite model. The remaining 5 contributions are just the 5 remaining permutations of the 4 external lines. The compositeness is shown in Fig 3.7.

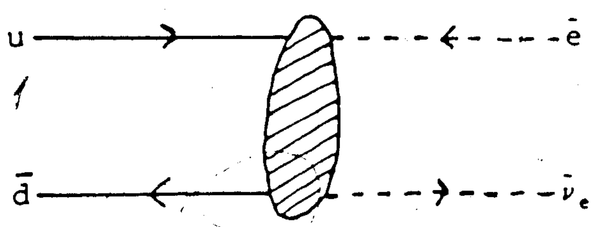

Fig. 3.7

In all these calculations we will use

$$f = \frac{1}{M} Q_L U^c_L L_L E^c_L .$$

This gives us the proper incoming and outgoing states. Using this f one gets the contribution from fig. 3.7 by:

$$-\bar{\psi}_{Li} C(\bar{\psi}^j_L)^T \left[\frac{\partial^2 f}{\partial\phi_i \partial\phi_j}\right]^*$$

$$= -\bar{d}_L C(\bar{u}^c_L)^T \left[\frac{\partial^2 f}{\partial\phi_i \partial\phi_j}\right]^* \qquad \text{Eq. 3.4}$$

$$= -\frac{1}{M} (\bar{d}_L u_R)(\phi^*_{\tilde{\nu}_L} \phi^*_{\tilde{e}^c_L}).$$

Now the scalar terms couple like:

$$\left[c_L^\nu\sqrt{2}\frac{g}{\cos\theta_w}\phi_{\nu_L}\bar{\nu}_L\psi_{\tilde{z}}\right]\left[-c_R^e\sqrt{2}\frac{g}{\cos\theta_w}\phi_{e_L^c}\bar{e}_L^c\psi_{\tilde{z}}\right]$$

$$= -2\frac{g^2}{\cos^2\theta_w}c_L^\nu c_R^e\ (\bar{\nu}_L\psi_{\tilde{z}})(-(e_R)^T C^{-1}C(\bar{\psi}_{\tilde{z}})^T)\ \phi_{\nu_L}\phi_{e_L^c}$$

$$= -2\frac{g^2}{\cos^2\theta_w}c_L^\nu c_R^e\ [\bar{\nu}_L(\psi_{\tilde{z}}\bar{\psi}_{\tilde{z}})e_R]\ \phi_{\nu_L}\phi_{e_L^c} \qquad \text{Eq. 3.5}$$

$$= -2\frac{g^2}{\cos^2\theta_w}c_L^\nu c_R^e\ [\bar{\nu}_L\frac{i}{\not{k}-\tilde{M}_z}e_R]\ \phi_{\nu_L}\phi_{e_L^c}.$$

We used $-c_R^e$ since the coupling constant for the left-handed conjugate is equal to minus the right-handed coupling constant (ie. $c_L^c = -c_R$). c^f is the coupling constant of f to Z^0.

We now proceed on with the amplitude:

$$\text{Amp} \sim 2ic_L^\nu c_R^e\ \frac{g^2}{\cos^2\theta_w}\ \frac{1}{M}\int\frac{d^4k}{(2\pi)^4}\ (\bar{d}Ru)\ \left[\frac{\bar{\nu}R(\not{k}+\tilde{M}_z)Re}{k^2-\tilde{M}_z^2}\right]$$

$$\times\ \frac{1}{(k^2-\tilde{M}_e^2)}\ \frac{1}{(k^2-\tilde{M}_\nu^2)} \qquad \text{Eq. 3.6}$$

$$\sim 2ic_L^\nu c_R^e\ \frac{g^2}{\cos^2\theta_w}\ \frac{M_S}{M}[\bar{d}Ru][\bar{\nu}Re]\ \times I,$$

where

$$I = \int\frac{d^4k}{(2\pi)^4}\ \frac{1}{(k^2-M_S{}^2)^3} = \frac{-i}{32\pi^2 M_S{}^2}. \qquad \text{Eq. 3.7}$$

Symmetric integration and a common supersymmetric mass have been used in the simplication of the integral. The integral is evaluated in Appendix D. Thus

$$\text{Amp} \sim c_L^\nu c_R^e\ \frac{1}{16\pi^2}\ \frac{g^2}{\cos^2\theta_w}\ \frac{1}{MM_S}\ (\bar{d}Ru)(\bar{\nu}Re). \qquad \text{Eq. 3.8}$$

When we take all 6 possible diagrams into account one gets

$$\text{Amp} \sim \frac{C}{16\pi^2}\,\frac{g^2}{\cos^2\theta_w}\,\frac{1}{MM_S}\,(\bar{d}Ru)(\bar{\nu}Re), \qquad \text{Eq. 3.9}$$

where

$$C = c_L^\nu c_R^e + c_L^d c_R^u - \tfrac{1}{2}c_L^d c_R^e - \tfrac{1}{2}c_L^\nu c_R^u + \tfrac{1}{2}c_L^\nu c_L^d + \tfrac{1}{2}c_R^e c_R^u$$

$$= -\frac{1}{8} + \frac{4}{3}\sin^2\theta_w - \frac{13}{18}\sin^4\theta_w \qquad \text{Eq. 3.10}$$

$$= 0.143$$

The other diagrams were calculated in much the same manner as the first one, however, the last four must be Fierz reordered using

$$(\bar{d}Re)(\bar{\nu}Ru) = -\tfrac{1}{2}(\bar{d}Ru)(\bar{\nu}Re) - \frac{1}{8}(\bar{d}\sigma_{\mu\nu}u)(\bar{\nu}R\sigma^{\mu\nu}e)$$

$$= -\tfrac{1}{2}(\bar{d}Ru)(\bar{\nu}Re) \qquad \text{Eq. 3.11}$$

since

$$\bar{d}\sigma_{\mu\nu}u = 0$$

as $\sigma_{\mu\nu}$ is a spin 2 operator and the pion is spin zero.

Lastly, one should also consider fig. 3.8.

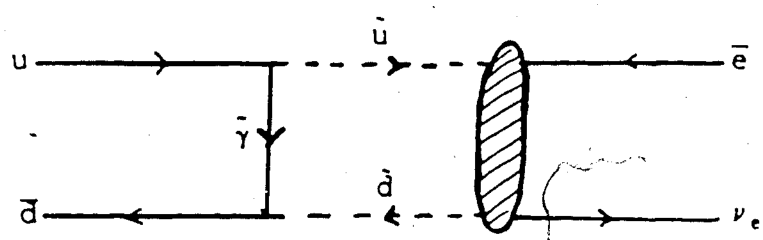

Fig. 3.8

One has for this diagram:

$$\text{Amp} \sim -\frac{2}{9}i(\sqrt{2})^2 \frac{e^2}{M} \int \frac{d^4k}{(2\pi)^4} \left[\frac{\bar{d}R(\not{k}+\tilde{M}_\gamma)Ru}{k^2-\tilde{M}_\gamma^2} \right] (\bar{\nu}Re)$$

$$\times \frac{1}{(k^2-\tilde{M}_u^2)} \frac{1}{(k^2-\tilde{M}_d^2)} \qquad \text{Eq. 3.12}$$

$$= -\frac{4}{9}ie^2 \frac{\tilde{M}_\gamma}{M} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{(\bar{d}Ru)(\bar{\nu}Re)}{(k^2-\tilde{M}_\gamma^2)(k^2-\tilde{M}_u^2)(k^2-\tilde{M}_d^2)}$$

which as $\tilde{M}_u \to \tilde{M}_d \to M_S$, $\tilde{M}_\gamma << M_S$

$$\to \frac{-4}{9} \frac{e^2}{16\pi^2} \frac{\tilde{M}_\gamma}{M_S^2 M} (\bar{d}Ru)(\bar{\nu}Re). \qquad \text{Eq. 3.13}$$

This amplitude would be negligible for

$$\frac{e^2}{g^2} \frac{\tilde{M}_\gamma}{M_S} = \frac{1}{\sin\theta_w} \frac{\tilde{M}_\gamma}{M_S} << 1, \qquad \text{Eq. 3.14}$$

which we assume to be true. We thus will ignore this contribution.

The total decay amplitude is thus represented by eqn. 3.9

$$\text{Amp} \sim \frac{C}{16\pi^2} \frac{g^2}{\cos^2\theta_w} \frac{1}{MM_S} (\bar{d}Ru)(\bar{\nu}Re) \qquad \text{Eqn. 3.9}$$

Using eqns. 2.9 and 2.14 this simpifies to

$$\text{Amp} \sim \frac{C}{2\pi^2} \frac{G_F}{\sqrt{2}} \frac{M_Z{}^2}{MM_S} (\bar{d}Ru)(\bar{\nu}Re) \qquad \text{Eq. 3.15}$$

$$\sim \frac{C}{8\pi^2} \frac{G_F}{\sqrt{2}} \frac{M_Z{}^2}{MM_S} [\bar{d}\gamma_5 u][\bar{\nu}(1+\gamma_5)e]$$

since $\bar{d}u=0$ as the pion is a pseudoscalar particle.

Now one can parametrize the four fermion local interaction by:

$$L \sim \frac{G_F}{\sqrt{2}} [(\bar{d}\gamma_\mu\gamma_5 u)(\bar{\nu}_e\gamma^\mu(1-\gamma_5)e)f_{AL}^e \qquad \text{Eq. 3.16}$$

$$+ (\bar{d}\gamma_5 u)(\bar{\nu}_e(1+\gamma_5)e)f_{PL}^e] + \text{R.H. terms}.$$

The ratio

$$R = \frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu \nu_\mu)}$$

can be written (Shankar).[52]

$$R = R_0 \left[1 + 2 \frac{m_\pi}{m_e} \frac{a_P}{a_A} f^e_{PL} \right] / \left[1 + 2 \frac{m_\pi}{m_\mu} \frac{a_P}{a_A} f^\mu_{PL} \right] \qquad \text{Eq. 3.17}$$

Where R_0 is the ratio from standard weak interactions alone (with radiative corrections included),

$R_0 = 1.233 \times 10^{-4}$.

Also defining

$$\langle 0 | \bar{d} \gamma_\mu \gamma_5 u | \pi^+(k) \rangle = \frac{k_\mu m_\pi a_A}{\sqrt{2m_\pi}} \qquad \text{Eq. 3.18}$$

$$\langle 0 | \bar{d} \gamma_5 u | \pi^+(k) \rangle = \frac{m_\pi^2 a_P}{\sqrt{2m_\pi}}$$

and from current algebra

$$\frac{a_P}{a_A} = \frac{m_\pi}{m_u + m_d} \simeq 14. \qquad \text{Eq. 3.19}$$

So now

$$R = R_0 \left[1 + 28 \frac{m_\pi}{m_e} f^e_{PL} \right] \Big/ \left[1 + 28 \frac{m_\pi}{m_\mu} f^\mu_{PL} \right]. \qquad \text{Eq. 3.20}$$

Comparisons of eqns. 3.14 and 3.15 gives

$$f^e_{PL} = \frac{C}{8\pi^2} \frac{M_Z^2}{MM_S}. \qquad \text{Eq. 3.21}$$

Using experimental values, and ignoring the contribution from $28 \frac{m_\pi}{m_\mu} f^\mu_{PL}$, since $\frac{m_e}{m_\mu} << 1$, Shankar gets[52]

$f^e_{PL} < 1.4 \times 10^{-6}$.

This implies

$$\frac{C}{8\pi^2} \frac{M_Z^2}{MM_S} < 1.4 \times 10^{-6} \qquad \text{Eq. 3.22}$$

which for $M_S \simeq M_Z$ gives

$M > M_Z(1.3\times10^3) = (90\text{GeV})(1.3\times10^3)$

$M > 1\times10^5$ GeV. Eq. 3.23

3.5 Composite Model Limits for $\mu N \rightarrow eN$

In our black box composite model approach there is nothing that says we must conserve lepton number. All we must do is make sure our coupling is gauge invariant. This will be useful in that we can now calculate possible decays where $\mu^- \rightarrow e^-$, in particular we will investigate $\mu N \rightarrow eN$. To do this we will have to investigate possible u quark and d quark contributions. We will start with u quark contributions.

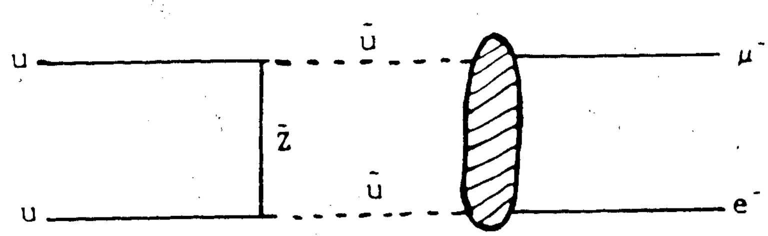

Fig. 3.9

Fig. 3.9 represents 1 of 6 possible diagrams to represent this decay. The remaining 5 are the remaining permutations of the 4 external lines. Taking

$f = \frac{1}{M} Q_L U_L^c L_L E_L^c$,

for the above diagram one gets

$$(\psi_L^i)^T C^{-1}\psi_L^j \frac{\partial^2 f}{\partial\phi_i\partial\phi_j} = \frac{1}{M}(e_L^c)^T C^{-1}\mu_L(\phi_{u_L^c}\phi_{u_L})$$

$$= -\frac{1}{M}(\bar{e}_R\mu_L)\phi_{u_L^c}\phi_{u_L}$$

and Eq. 3.24

$$-\bar{\Psi}_L^i C(\bar{\Psi}_L^j)^T\left[\frac{\partial^2 f}{\partial\phi_i\partial\phi_j}\right]^* = -\frac{1}{M}(\bar{e}_L\mu_R)\phi^*_{u_L^c}\phi^*_{u_L}.$$

This is then coupled to

$$(-\sqrt{2})^2 c_L^u(-c_R^u)\ \frac{g^2}{\cos^2\theta_w}\ (\bar{\Psi}_{\tilde{z}}u_L^c)(\bar{\Psi}_{\tilde{z}}u_L)\phi^*_{u_L^c}\phi^*_{u_L}$$

$$= -2c_L^u c_R^u\ \frac{g^2}{\cos^2\theta_w}\ [-(\psi_{\tilde{z}})^T C^{-1}C(\bar{u}_R)^T][\bar{\Psi}_{\tilde{z}}u_L]\phi^*_{u_L^c}\phi^*_{u_L} \qquad \text{Eq. 3.25}$$

$$= -2c_L^u c_R^u\ \frac{g^2}{\cos^2\theta_w}\ [\bar{u}_R(\psi_{\tilde{z}}\bar{\Psi}_{\tilde{z}})u_L]\phi^*_{u_L^c}\phi^*_{u_L}.$$

Similiarly, the second term couples like

$$(\sqrt{2})^2 c_L^u(-c_R^u)\ \frac{g^2}{\cos^2\theta_w}\ (\bar{u}_L^c\psi_{\tilde{z}})(\bar{u}_L\psi_{\tilde{z}})\phi_{u_L^c}\phi_{u_L}$$

$$= -2c_L^u c_R^u\ \frac{g^2}{\cos^2\theta_w}\ [\bar{u}_L(\psi_{\tilde{z}}\bar{\Psi}_{\tilde{z}})u_R]\phi_{u_L^c}\phi_{u_L}. \qquad \text{Eq. 3.26}$$

The amplitude is then

$$\text{Amp} \sim 2ic_L^u c_R^u\ \frac{g^2}{\cos^2\theta_w}\ \frac{1}{M}\int \frac{d^4k}{(2\pi)^4}\left[\left[\frac{\bar{u}L(\not{k}+\tilde{M}_z)Lu}{k^2-\tilde{M}_z^2}\right](\bar{e}L\mu) + \left[\frac{\bar{u}R(\not{k}+\tilde{M}_z)Ru}{k^2-\tilde{M}_z^2}\right](\bar{e}R\mu)\right]\frac{1}{(k^2-\tilde{M}_u^2)^2}$$

$$= 2ic_L^u c_R^u\ \frac{g^2}{\cos^2\theta_w}\ \frac{M_S}{M}\int\frac{d^4k}{(2\pi)^4}\ \frac{1}{(k^2-M_S{}^2)^3} \qquad \text{Eq. 3.27}$$

$$\times\ [(\bar{u}Lu)(\bar{e}L\mu)+(\bar{u}Ru)(\bar{e}R\mu)]$$

$$= c_L^u c_R^u\frac{1}{16\pi^2}\ \frac{g^2}{\cos^2\theta_w}\frac{1}{MM_S}\ [(\bar{u}Lu)(\bar{e}L\mu)+(\bar{u}Ru)(\bar{e}R\mu)],$$

where once again the integral is calculated in Appendix D.

When all 6 possible u quark contributions are considered one gets

$$\text{Amp} \sim -\frac{C}{16\pi^2}\frac{g^2}{\cos^2\theta_W}\frac{1}{MM_S}\left[(\bar{u}Lu)(\bar{e}L\mu)+(\bar{u}Ru)(\bar{e}R\mu)\right], \qquad \text{Eq. 3.28}$$

where

$$C = -c_L^u c_R^u - c_L^e c_R^e + \tfrac{1}{2}c_L^u c_R^e + \tfrac{1}{2}c_L^e c_R^u - \tfrac{1}{2}c_R^u c_R^e - \tfrac{1}{2}c_L^u c_L^e$$

$$= \frac{1}{8} + \frac{5}{6}\sin^2\theta_W - \frac{13}{9}\sin^4\theta_W \qquad \text{Eq. 3.29}$$

$$= 0.240.$$

Once again the final four are Fierz reordered using

$$R\times R = \tfrac{1}{2}R\times R + \frac{1}{8}R\sigma^{\mu\nu}\times\sigma_{\mu\nu},$$

$$\text{Eq. 3.30}$$

$$L\times L = \tfrac{1}{2}L\times L + \frac{1}{8}L\sigma^{\mu\nu}\times\sigma_{\mu\nu}.$$

The $\sigma^{\mu\nu}$ term is then neglected since it will not induce a coherent contribution to $\mu\to e$ conversion in the ground state nucleus. Using eqns. 2.9 and 2.14 the amplitude becomes,

$$\text{Amp} \sim -\frac{C}{2\pi^2}\frac{G_F}{\sqrt{2}}\frac{M_Z^2}{MM_S}\left[(\bar{u}Lu)(\bar{e}L\mu)+(\bar{u}Ru)(\bar{e}R\mu)\right], \qquad \text{Eq. 3.31}$$

Again the pseudo-scalar term will not induce a coherent contribution to $\mu\to e$ conversion in the ground state nucleus, so

$$\text{Amp} \sim -\frac{C}{4\pi^2}\frac{G_F}{\sqrt{2}}\frac{M_Z^2}{MM_S}\left[(\bar{u}u)(\bar{e}L\mu) + (\bar{u}u)(\bar{e}R\mu)\right]$$

$$\text{Eq. 3.32}$$

$$\sim -\frac{C}{4\pi^2}\frac{G_F}{\sqrt{2}}\frac{M_Z^2}{MM_S}(\bar{u}u)(\bar{e}\mu).$$

We can now move onto contributions from d quarks.

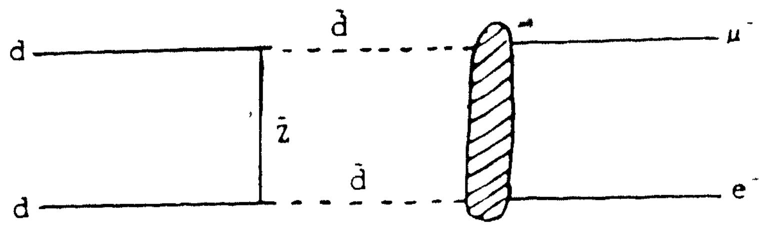

Fig. 3.10a

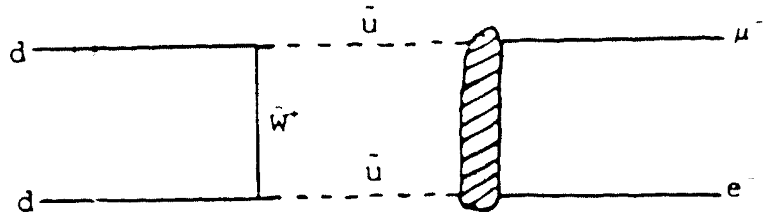

Fig. 3.10b

The first possibility represented by fig 3.10a can be thrown out immediately since one can not find a suitable f term that involves incoming and outgoing d quarks. The second possibility represented by fig 3.10b requires a little more investigation. Since the composite part of this diagram is exactly the same as in the u quark contribution (fig. 3.9), we again get

$\frac{1}{M}(\bar{e}_R\mu_L)(\phi_{u_L^c}\phi_{u_L})$

and

$\frac{1}{M}\ (\bar{e}_L\mu_R)(\phi^*_{u_L^c}\phi^*_{u_L})$.

However, the $\tilde{W}$ does not couple to $\phi_{u_L^c}$, therefore, no contribution.

Our amplitude is just the u quark contribution given by eqn. 3.32

$$\text{Amp} \sim -\frac{C}{4\pi^2}\frac{G_F}{\sqrt{2}}\frac{M_Z^{\,2}}{MM_S}\,(\bar{e}\mu)(\bar{u}u). \qquad \text{Eq. 3.32}$$

For $\mu N \rightarrow eN$, the effective four-fermi interaction is

$$L_{eff} = -\frac{G_F}{\sqrt{2}}\sum_{i=0,1} g_s^i\bar{e}(1+\alpha_s^i\gamma_5)\mu S^i + g_p^i\bar{e}(1+\alpha_p^i\gamma_5)\mu P^i, \qquad \text{Eq. 3.33}$$

where

$$S^0 = \tfrac{1}{2}(\bar{u}u + \bar{d}d)$$

$$S^1 = \tfrac{1}{2}(\bar{u}u - \bar{d}d)$$

and $\qquad$ Eq. 3.34

$$P^0 = \tfrac{1}{2}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$$

$$P^1 = \tfrac{1}{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d).$$

Once again, the pseudo-scalar term will not induce a coherent contribution to $\mu \rightarrow e$ conversion in the ground state nucleus. Thus the interaction reduces to

$$L_{eff} = -\frac{G_F}{\sqrt{2}}\sum_{i=0,1} g_s^i\bar{e}(1+\alpha_s^i\gamma_5)\mu S^i. \qquad \text{Eq. 3.35}$$

Now for our particular process, we have $g_s = g_s^0 = g_s^1$, and $\alpha = 0$, so that

$$L_{eff} = -\frac{G_F}{\sqrt{2}}\,g_s(\bar{e}\mu)(\bar{u}u), \qquad \text{Eq. 3.36}$$

which implies, by comparison with eqn 3.32, that

$$g_s = \frac{C}{4\pi^2}\frac{M_Z^{\,2}}{MM_S}. \qquad \text{Eq. 3.37}$$

From experiment we have (Bryman[53])

$$R = \frac{\Gamma(\mu^- Ti \rightarrow e^- Ti)}{\Gamma(\mu^- Ti \rightarrow \text{all})} \leq 6\times 10^{-12}, \qquad \text{Eq. 3.38}$$

where the process $\Gamma(\mu^- Ti \rightarrow \text{all})$ is for our purpose $\Gamma(\mu N \rightarrow \nu N')$.

Now theoretically, for $\alpha=0$, $g_s=g_s^0=g_s^1$, we have:[54]

$$R = \frac{1}{2}\frac{\omega_H(Z)}{\Gamma(\mu N\to\nu N)}(g_s)^2\left[1+\frac{(Z-N)}{3A}\right]^2, \qquad \text{Eq. 3.39}$$

which for Ti^- has the following values:

$Z=22$

$N=26$

$A=48$

$$\frac{\omega_H(22)}{\Gamma(\mu N\to\nu N)} = 228.^{54}$$

This means

$$(g_s)^2 = 2R\frac{\Gamma(\mu N\to\nu N')}{\omega_H(Z)}\left[1+\frac{(Z-N)}{3A}\right]^{-2}$$

$$\le 2(6\times10^{-12})\frac{1}{228}\left[1-\frac{4}{144}\right]^{-2},$$

or Eq. 3.40

$$g_s \le 2\times10^{-7}.$$

Now using our value for g_s,

$$g_s = \frac{C}{4\pi^2}\frac{M_Z{}^2}{MM_S}, \qquad \text{Eq. 3.37}$$

we have:

$$M = \frac{C}{4\pi^2}\frac{M_Z{}^2}{M_S}\frac{1}{g_s}. \qquad \text{Eq. 3.41}$$

Using $M_Z \simeq M_S$ this reduces to

$$M = \frac{C}{4\pi^2}\frac{M_Z}{g_s} \ge 2\times10^6\text{GeV}. \qquad \text{Eq. 3.42}$$

Both our decays, $\pi_{\ell 2}$ decay and $\mu N \to eN$, appear to give fairly strong constaints on a composite model energy scale, both giving binding energies of $O(10^{5-6}$ GeV). The constraints from π_{l2} decay are the most important. It is a flavour diagonal decay and thus one cannot insert some arbitrary flavour conserving mechanism to suppress the rate. It would appear that any supersymmetric composite model would have an energy scale well above presently available energies.

4. SUMMARY AND CONCLUSIONS

In this thesis we calculated contributions to one loop radiative corrections from supersymmetry in order to get a bound on supersymmetric masses. Specifically we calculated one loop corrections to muon decay. However, this only gave us a lower bound of 300 MeV, a somewhat useless lower bound in that non-observation through direct methods gives us a lower bound well above this. The reason why this method of investigating radiative corrections failed us here is that its contribution was proportional to $\frac{m_e m_\mu}{M_S^2}$. This leads to a very small contribution that even better experimental limits will not be able to detect. The calculation does, however, serve a useful purpose. For one; it allows me to demonstrate a indirect method of observing supersymmetry by its effect on low energy processes, a method that has had better success[37,39-47] in calculating limits on supersymmetric masses in the past. Secondly, it warns us about the downfalls of using muon decay for such calculations, since you invariably get the small $\frac{m_e m_\mu}{M_S^2}$ contribution.

Our second investigation was an investigation of possible contributions of compositeness to radiative corrections in low energy processes. Since just a basic composite model calculation goes as $\frac{1}{M^2}$, in order to get a larger contribution we make it a combined SUSY/composite model decay. With the introduction of scalar fermions, one achieves a contribution that goes as $\frac{1}{MM_S}$, and since the specific proportionality we get is $\frac{M_Z^2}{MM_S} \simeq \frac{M_Z}{M}$, this works extremely well. The result we obtain from this is a lower bound of around 10^5 GeV for a composite energy scale. This is a fairly high limit on the mass scale, and precludes possible direct observation of super-compositeness for quite some while.

In terms of the over all picture, particle physics still awaits the direct observation of supersymmetric particles in the range just above 50 GeV. Without this supersymmetry still lacks credibility. However, there is great hope that these particles will be found in this range.

Meanwhile, compositeness has been replaced by string theory in the hearts of most theorists. If we consider strings as a form of "compositeness", it has an energy scale to observe it at the Planck energy, well above any experimental means of detection. So the search for an experimentally verifiable grand unified theory continues. After all, you can't do physics if you don't have GUTS.

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APPENDIX A -- SPINORS AND C-CONJUGATION

Conventions in this thesis will follow those of Bjorken and Drell.[55] Specifically, the Dirac equation is:

$$(p_\mu \gamma^\mu - m)\psi = 0. \qquad \text{Eq. A-1}$$

The γ^μ's obey the commutation relation

$$\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu}, \qquad \text{Eq. A-2}$$

$g^{\mu\nu}$ being the metric +---.

The most common representation for the γ^μ's being

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \gamma^1 = \begin{bmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{bmatrix} \qquad \text{Eq. A-3}$$

where

$$1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{Eq. A-4}$$

is a 2×2 matrix, and the σ's are the fami[illegible] Pauli matrices.

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

$$\sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \qquad \text{Eq. A-5}$$

Other frequently appearing 4×4 matrices are:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu], \qquad \text{Eq. A-5}$$

giving

$$\sigma^{ij} = \begin{bmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{bmatrix}, \qquad \text{ijk cyclic;}$$

$$\sigma^{0i} = i\begin{bmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{bmatrix}.$$

A free Dirac particle boosted from the rest frame spinor χ is,

$$\psi = \begin{bmatrix} \chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi \end{bmatrix}. \qquad \text{Eq. A-6}$$

The Pauli adjoint is defined by

$$\bar{\psi} = \psi^\dagger\gamma^0. \qquad \text{Eq. A-7}$$

One can find other useful relations in Appendix A of Bjorken and Drell's Relativistic Quantum Mechanics.[55]

The projection operators are

$$P_L = L = \frac{1 - \gamma_5}{2}; \qquad P_R = R = \frac{1 + \gamma_5}{2}. \qquad \text{Eq. A-8}$$

So

$$\psi_L = L\psi = \left[\frac{1-\gamma_5}{2}\right]\psi \qquad \text{Eq. A-9}$$

$$\psi_R = R\psi = \left[\frac{1+\gamma_5}{2}\right]\psi$$

Furthermore,

$$\bar{\psi}_L = \bar{\psi}R \qquad \text{Eq. A-10}$$

$$\bar{\psi}_R = \bar{\psi}L$$

giving

$$\bar{\psi}\psi = \bar{\psi}L\psi + \bar{\psi}R\psi \qquad \text{Eq. A-11}$$

$$= \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$$

and

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}R\gamma^\mu L\psi + \bar{\psi}L\gamma^\mu R\psi \qquad \text{Eq. A-12}$$

$$= \bar{\psi}_L\gamma^\mu\psi_R + \bar{\psi}_R\gamma^\mu\psi_L$$

as

$$R\gamma^\mu = \gamma^\mu L$$

Eq. A-13

$$L\gamma^\mu = \gamma^\mu R$$

and

$$L^2 = L$$

Eq. A-14

$$R^2 = R.$$

Useful relations are

$$RO_1 = O_1 L \qquad \text{Eq. A-15}$$

$$LO_1 = O_1 R$$

for $O_1 = \gamma^\mu,\ \gamma_5\gamma^\mu$

and

$$RO_1 = O_1 R \qquad \text{Eq. A-16}$$

$$LO_1 = O_1 L$$

for $O_1 = 1,\ \gamma_5,\ \sigma^{\mu\nu}$

Another important concept is charge conjugation. The conjugate field is defined by:

$$\psi^c = C(\bar{\psi})^T \qquad \text{Eq. A-17}$$

$$\bar{\psi}^c = -\psi^T C^{-1}$$

where C is defined by

$$C\gamma^\mu C^{-1} = -(\gamma^\mu)^T. \qquad \text{Eq. A-18}$$

In our representation

$$C = C^{-1} = -C^T = -C^\dagger = i\gamma^2\gamma^0, \qquad \text{Eq. A-19}$$

so

$$\psi^c = i\gamma^2\gamma^0\gamma^0\psi^* = i\gamma^2\psi^*. \qquad \text{Eq. A-20}$$

For chiral fermions;

$$\psi^c_{L,R} = C(\overline{\psi}_{R,L})^T \qquad \text{Eq. A-21}$$

$$\overline{\psi}^c_{L,R} = -(\psi_{R,L})^T C^{-1}.$$

Notice the inverse relations are also

$$\psi_{L,R} = C(\overline{\psi}^c_{R,L})^T \qquad \text{Eq. A-22}$$

$$\overline{\psi}_{L,R} = -(\psi^c_{R,L})^T C^{-1}.$$

One other useful relationship is

$$CO_iC^{-1} = -(O_i)^T \qquad \text{Eq. A-23}$$

for $O_i = \gamma^\mu,\ \sigma^{\mu\nu}$

and

$$CO_iC^{-1} = (O_i)^T \qquad \text{Eq. A-24}$$

for $O_i = 1,\ \gamma_5,\ \gamma^\mu\gamma_5$.

We will also encounter objects known as Majorana Spinors. They are defined by:

$$\eta^c = \eta \qquad \text{Eq. A-25}$$

ie) $\eta = C(\overline{\eta})^T$

$$\overline{\eta} = -(\eta)^T C^{-1},$$

and can be constructed from chiral spinors as follows:

$$\eta = \psi_R + \psi_L{}^c \qquad \text{Eq. A-26}$$

$$\zeta = \psi_L + \psi_R{}^c.$$

APPENDIX B -- Q ANTI-COMMUTATION LAW

In this appendix we set out to calculate $(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\phi$, giving $[\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] = \bar{\epsilon}_1\gamma^\mu\epsilon_2 P_\mu$. We will need several relations first. We will derive these using the fact that ϵ is a Majorana spinor so

$$\epsilon = \epsilon^c = C(\bar{\epsilon})^T \qquad \text{Eq. B-1}$$

The needed relations are:

i)
$$\begin{aligned} \bar{\epsilon}_2\epsilon_1 &= -\epsilon_2{}^T C^{-1} C \bar{\epsilon}_1{}^T \\ &= -\epsilon_2{}^T \bar{\epsilon}_1{}^T \qquad \text{Eq. B-2} \\ &= \bar{\epsilon}_1\epsilon_2 \end{aligned}$$

since ϵ_1, ϵ_2 anticommute.

ii)
$$\bar{\epsilon}_2\gamma_5\epsilon_1 = \bar{\epsilon}_1\gamma_5\epsilon_2. \qquad \text{Eq. B-3}$$

since $\gamma_5{}^T = \gamma_5$, and it commutes with C.

iii)
$$\begin{aligned} \bar{\epsilon}_2\gamma^\mu\epsilon_1 &= -\epsilon_2{}^T C^{-1}\gamma^\mu C \bar{\epsilon}_1{}^T \\ &= \epsilon_2{}^T(\gamma^\mu)^T \bar{\epsilon}_1{}^T \qquad \text{Eq. B-4} \\ &= -\bar{\epsilon}_1\gamma^\mu\epsilon_2 \end{aligned}$$

iv)
$$\begin{aligned} \bar{\epsilon}_2\gamma^\mu\gamma_5\epsilon_1 &= -\epsilon_2{}^T C^{-1}\gamma^\mu\gamma_5 C \bar{\epsilon}_1{}^T \\ &= \epsilon_2{}^T(\gamma^\mu)^T\gamma_5\bar{\epsilon}_1{}^T \qquad \text{Eq. B-5} \\ &= -\bar{\epsilon}_1\gamma_5\gamma^\mu\epsilon_2 \\ &= \bar{\epsilon}_1\gamma^\mu\gamma_5\epsilon_2 \end{aligned}$$

and finally

v)
$$\begin{aligned} \bar{\epsilon}_2\sigma^{\mu\nu}\epsilon_1 &= -\epsilon_2{}^T C^{-1}\sigma^{\mu\nu} C \bar{\epsilon}_1{}^T \\ &= \epsilon_2{}^T(\sigma^{\mu\nu})^T \bar{\epsilon}_1{}^T \qquad \text{Eq. B-6} \\ &= -\epsilon_1\sigma^{\mu\nu}\epsilon_2. \end{aligned}$$

Recalling our infinitesimal transformation that the

Lagrangian is invariant under

$$\delta\phi^i = \bar{\epsilon}\psi_L^i$$

$$\delta\psi_L^i = \left[\frac{1-\gamma_5}{2}\right](F^i - i\partial\!\!\!/\phi^i)\epsilon \qquad \text{Eq. B-7}$$

$$\delta F^i = -i\bar{\epsilon}\partial\!\!\!/\psi_L^i .$$

Now:

$$\delta_{\epsilon_1}\delta_{\epsilon_2}\phi^i = \delta_{\epsilon_1}(\bar{\epsilon}_2\psi_L^i)$$

$$= \bar{\epsilon}_2\left[\frac{1-\gamma_5}{2}\right](F^i - i\partial\!\!\!/\phi^i)\epsilon_1$$

so

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\phi^i \qquad \text{Eq. B-8}$$

$$= \bar{\epsilon}_2\left[\frac{1-\gamma_5}{2}\right](F^i - i\partial\!\!\!/\phi^i)\epsilon_1 - \bar{\epsilon}_1\left[\frac{1-\gamma_5}{2}\right](F^i - i\partial\!\!\!/\phi^i)\epsilon_2$$

$$= \bar{\epsilon}_1\left[\left[\frac{1-\gamma_5}{2}\right]F^i + i\left[\frac{1+\gamma_5}{2}\right]\partial\!\!\!/\phi^i\right]\epsilon_2 - \bar{\epsilon}_1\left[\frac{1-\gamma_5}{2}\right](F^i - i\partial\!\!\!/\phi^i)\epsilon_2$$

$$= \bar{\epsilon}_1 i\partial\!\!\!/\phi^i\epsilon_2$$

$$= \bar{\epsilon}_1\gamma^\mu\epsilon_2 P_\mu\phi^i .$$

Doing the same with F^i

$$\delta_{\epsilon_1}\delta_{\epsilon_2}F^i = \delta_{\epsilon_1}(-i\bar{\epsilon}_2\partial\!\!\!/\psi_L^i)$$

$$= -i\bar{\epsilon}_2\partial\!\!\!/\left[\left[\frac{1-\gamma_5}{2}\right](F^i - i\partial\!\!\!/\phi^i)\epsilon_1\right]$$

$$= -i\bar{\epsilon}_2\left[\frac{1+\gamma_5}{2}\right]\left[\partial\!\!\!/F^i - i\Box\phi^i\right]\epsilon_1$$

so

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})F^i \qquad \text{Eq. B-9}$$

$$= -i\bar{\epsilon}_2\left[\frac{1+\gamma_5}{2}\right](\partial\!\!\!/F^i - i\Box\phi^i)\epsilon_1 + i\bar{\epsilon}_1\left[\frac{1+\gamma_5}{2}\right](\partial\!\!\!/F^i - i\Box\phi^i)\epsilon_2$$

$$= \bar{\epsilon}_1\left[\left[\frac{1-\gamma_5}{2}\right]i\partial\!\!\!/F^i - \left[\frac{1+\gamma_5}{2}\right]\Box\phi^i\right]\epsilon_2 + \bar{\epsilon}_1\left[\frac{1+\gamma_5}{2}\right](i\partial\!\!\!/F^i + \Box\phi^i)\epsilon_2$$

$$= \bar{\epsilon}_1 i\partial\!\!\!/F^i\epsilon_2$$

$$= \bar{\epsilon}_1\gamma^\mu\epsilon_2 P_\mu F^i .$$

Lastly we do

$$\delta\epsilon_1 \delta\epsilon_2 \psi_L^i$$

$$= \delta\epsilon_1\left[\left[\frac{1-\gamma_5}{2}\right](F^i - i\gamma^\mu\partial_\mu\phi^i) \right]\epsilon_2 \qquad \text{Eq. B-10}$$

$$= \left[\frac{1-\gamma_5}{2}\right][-i\bar{\epsilon}_1\gamma^\mu\partial_\mu\psi_L^i - i\gamma^\mu\partial_\mu(\bar{\epsilon}_1\psi_L^i)]\epsilon_2$$

$$= \left[\frac{1-\gamma_5}{2}\right]\left[-\bar{\epsilon}_1\gamma^\mu P_\mu\left[\frac{1-\gamma_5}{2}\right]\psi^i - \gamma^\mu\bar{\epsilon}_1 P_\mu\left[\frac{1-\gamma_5}{2}\right]\psi^i \right]\epsilon_2.$$

Must use Fierz transform. From Appendix E.

$$L\times L = \tfrac{1}{2}L\times L + \frac{1}{8}L\sigma_{\mu\nu}\times\sigma^{\mu\nu}.$$

Thus

$$[(\bar{\epsilon}_1\gamma^\mu)L(P_\mu\psi^i)]L\epsilon_2 \qquad \text{Eq. B-11}$$

$$= -\tfrac{1}{2}[\bar{\epsilon}_1\gamma^\mu L\epsilon_2]LP_\mu\psi^i - \frac{1}{8}[\bar{\epsilon}_1\gamma^\mu\sigma^{\alpha\beta}\epsilon_2]L\sigma_{\alpha\beta}P_\mu\psi_i$$

where the extra minus sign comes from the anti-commuting spinors. Also

$$(\bar{\epsilon}_1 LP_\mu\psi^i)(L\gamma^\mu\epsilon_2) \qquad \text{Eq. B-12}$$

$$= -\tfrac{1}{2}(\bar{\epsilon}_1 L\gamma^\mu\epsilon_2)LP_\mu\psi^i - \frac{1}{8}(\bar{\epsilon}_1\sigma^{\alpha\beta}\gamma^\mu\epsilon_2)L\sigma_{\alpha\beta}P_\mu\psi^i.$$

Thus

$$\delta\epsilon_1\delta\epsilon_2\psi_L^i = \tfrac{1}{2}[\bar{\epsilon}_1\gamma^\mu L\epsilon_2 + \bar{\epsilon}_1 L\gamma^\mu\epsilon_2]P_\mu\psi_L^i \qquad \text{Eq. B-13}$$

$$+ \frac{1}{8}[\bar{\epsilon}_1\gamma^\mu\sigma^{\alpha\beta}\epsilon_2 + \bar{\epsilon}_1\sigma^{\alpha\beta}\gamma^\mu\epsilon_2]\sigma_{\alpha\beta}P_\mu\psi_L^i$$

$$= \tfrac{1}{2}(\bar{\epsilon}_1\gamma^\mu\epsilon_2)P_\mu\psi_L^i$$

$$+ \frac{1}{8}\bar{\epsilon}_1[\gamma^\mu\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\gamma^\mu]\epsilon_2(\sigma_{\alpha\beta}P_\mu\psi_L^i).$$

We will now need

$$\bar{\epsilon}_2\gamma^\mu\sigma^{\alpha\beta}\epsilon_1 = -\epsilon_2^T C^{-1}\gamma^\mu CC^{-1}\sigma^{\alpha\beta}C\bar{\epsilon}_1^T$$

$$= -\epsilon_2^T(\gamma^\mu)^T(\sigma^{\alpha\beta})^T\bar{\epsilon}_1^T \qquad \text{Eq. B-14}$$

$$= \bar{\epsilon}_1\sigma^{\alpha\beta}\gamma^\mu\epsilon_2.$$

Finally, one gets

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\psi_L^i$$

$$= (\bar{\epsilon}_1\gamma^\mu\epsilon_2)P_\mu\psi_L^i + \frac{1}{8}\bar{\epsilon}_1[\gamma^\mu\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\gamma^\mu]\epsilon_2\sigma_{\alpha\beta}P_\mu\psi_L^i$$

$$-\frac{1}{8}\bar{\epsilon}_1[\sigma^{\alpha\beta}\gamma^\mu + \gamma^\mu\sigma^{\alpha\beta}]\epsilon_2\sigma_{\alpha\beta}P_\mu\psi_L^i$$

$$= (\bar{\epsilon}_1\gamma^\mu\epsilon_2)P_\mu\psi_L^i. \qquad \text{Eq. B-15}$$

We have finally shown that

$$[\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] = \bar{\epsilon}_1\gamma^\mu\epsilon_2 P_\mu \qquad \text{Eq. B-16}$$

To get the anti-commutation law use

$$[AB,CD] = ABCD - CDAB \qquad \text{Eq. B-17}$$

$$= -AC\{B,D\} + \{A,C\}DB + A\{B,C\}D$$

$$- C\{A,D\}B.$$

Thus

$$[\bar{\epsilon}_{1\alpha}Q_\alpha, \bar{\epsilon}_{2\beta}Q_\beta] = -\bar{\epsilon}_{1\alpha}\bar{\epsilon}_{2\beta}\{Q_\alpha, Q_\beta\} \qquad \text{Eq. B-18}$$

since

$$\{\bar{\epsilon}_{1\alpha}, \bar{\epsilon}_{2\beta}\} = 0$$

$$\{\bar{\epsilon}_{1\alpha}, Q_\beta\} = \{\bar{\epsilon}_{2\beta}, Q_\alpha\} = 0.$$

Furthermore,

$$\bar{\epsilon}_1\gamma^\mu\epsilon_2 = \bar{\epsilon}_1\gamma^\mu C\bar{\epsilon}_2^T = \bar{\epsilon}_{1\alpha}(\gamma^\mu C)_{\alpha\beta}\bar{\epsilon}_{2\beta}, \qquad \text{Eq. B-19}$$

giving

$$-\bar{\epsilon}_{1\alpha}\bar{\epsilon}_{2\beta}\{Q_\alpha, Q_\beta\} = \bar{\epsilon}_{1\alpha}\bar{\epsilon}_{2\beta}(\gamma^\mu C)_{\alpha\beta}P_\mu$$

which implies

$$\{Q_\alpha, Q_\beta\} = -(\gamma^\mu C)_{\alpha\beta}P_\mu. \qquad \text{Eq. B-20}$$

APPENDIX C – FULL LAGRANGIAN AND FEYNMANN RULES

The general gauge invariant SUSY Lagrangian[56,57] is;

$$L = L_{GAUGE} + L_{MATTER} + L_{SCALAR} + L_{YUKAWA}, \qquad \text{Eq. C-1}$$

where

$$L_G = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \tfrac{1}{2}\bar{\chi}^a\gamma^\mu(D^{ab}_\mu\chi^b) \qquad \text{Eq. C-2}$$

$$L_M = (D_\mu\phi)^*_i\,(D^\mu\phi)^i + i\bar{\Psi}_{Li}\gamma^\mu D_\mu{}^i_j\psi^j_L \qquad \text{Eq. C-3}$$

$$-\sqrt{2}g\bar{\chi}^a\phi^*_i T^{ai}{}_j\psi^j_L - \sqrt{2}g\bar{\Psi}_{Li}T^{ai}{}_j\phi^j\chi^a$$

$$L_Y = -\sum_{ij}\left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right]\bar{\Psi}^{ci}_R\psi^j_L + \left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right]^*\bar{\Psi}^j_L\psi^{ci}_R \qquad \text{Eq. C-4}$$

$$= \sum_{ij}\left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right](\psi^i_L)^T C^{-1}\psi^j_L - \left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right]^*\bar{\Psi}^j_L C(\bar{\Psi}_{Li})^T \qquad \text{Eq. C-5}$$

$$L_S = -\sum_i\left[\frac{\partial W}{\partial\phi^i}\right]^*\left[\frac{\partial W}{\partial\phi^i}\right] - \tfrac{1}{2}\sum_a(g\phi^*_i T^{ai}{}_j\phi^j - \zeta^a)^2 \qquad \text{Eq. C-6}$$

where ζ^a appears only for invariant abelian subalgebras,

$$W = c_i\phi^i + m_{ij}\phi^i\phi^j + g_{ijk}\phi^i\phi^j\phi^k \qquad \text{Eq. C-7}$$

is the superpotential, a and b are adjoint indices, i and j are complex representation indices (ie. $(\phi^i)^* = (\phi^*)_i$), and;

$$D_\mu\phi = (\partial_\mu - ig\vec{T}\cdot\vec{A}_\mu)\phi,$$

$$D_\mu\psi_L = (\partial_\mu - ig\vec{T}\cdot\vec{A}_\mu)\psi_L$$

so $\qquad$ Eq. C-8

$$D_\mu\psi^c_R = (\partial_\mu + ig\vec{T}^T\cdot\vec{A}_\mu)\psi^c_R,$$

$$(D_\mu\chi)^a = \partial_\mu\chi^a + gf^a_{bc}A^b_\mu\chi^c,$$

and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^a_{bc}A^b_\mu A^c_\nu \qquad \text{Eq. C-9}$$

and this theory is invariant under gauge invariant SUSY transformations.

Now from L_M we get the "Gauge-Yukawa" coupling to gauge fermions χ^a. We consider a Dirac fermion $\psi = \psi_L + \psi_R$ whose chiral pieces transform under a gauge group

$$\delta\psi_L^i = i\theta^a T_{Lj}^{ai}\psi_L^j \qquad \text{Eq. C-10}$$

$$\delta\psi_R^i = i\theta^a T_{Rj}^{ai}\psi_R^j,$$

where $T_{L,R}$ are the generators of the representation of $\psi_{L,R}$. Furthermore, in a supersymmetric theory the chiral components of ψ appear in left/right chiral multiplets $(\phi_{L,R};\psi_{L,R})$. The Gauge-Yukawa couplings are then

$$\begin{aligned} L_{g-y} &= i\sqrt{2}g\,[\phi^*_{Ri}\overline{\chi}^a_L T^{ai}_{Rj}\psi^j_R - \overline{\psi}_{Ri}\chi^a_L T^{ai}_{Rj}\phi^j_R] \\ &+ i\sqrt{2}g\,[\phi^*_{Li}\overline{\chi}^a_R T^{ai}_{Lj}\psi^j_L - \overline{\psi}_{Li}\chi^a_R T^{ai}_{Lj}\phi^j_L] \qquad \text{Eq. C-11} \\ &= i\sqrt{2}g\,[\phi^*_{Ri}T^{ai}_{Rj}\overline{\chi}^a R\psi^j - \overline{\psi}_i L\chi^a T^{ai}_{Rj}\phi^j_R] \\ &+ i\sqrt{2}g\,[\phi^*_{Li}T^{ai}_{Lj}\overline{\chi}^a L\psi^j - \overline{\psi}_i R\chi^a T^{ai}_{Lj}\phi^j_L], \end{aligned}$$

where g is the gauge coupling constant. We have also slightly redefined the scalar wavefunction in going from L_M to L_{g-y}, putting $\phi\to i\phi$. We now obtain the following vertices by simply multiplying L_{g-y} by i.

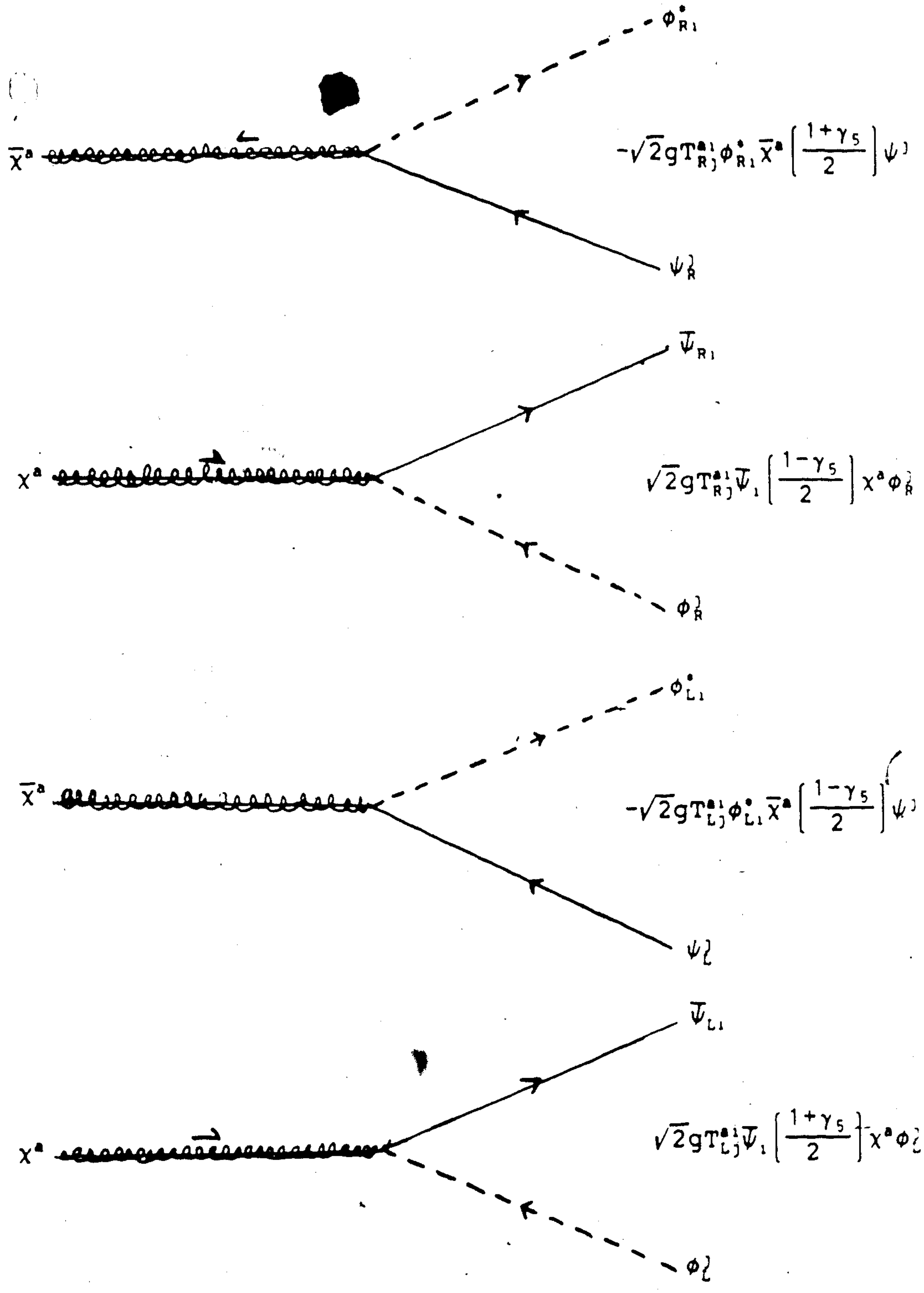
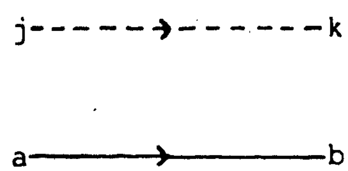

$\bar{\chi}^a$
ϕ^*_{Ri}
ψ^j_R
$-\sqrt{2} g T^{ai}_{Rj} \phi^*_{Ri} \bar{\chi}^a \left[\frac{1+\gamma_5}{2}\right] \psi^j$
χ^a
$\bar{\psi}_{Ri}$
ϕ^j_R
$\sqrt{2} g T^{ai}_{Rj} \bar{\psi}_i \left[\frac{1-\gamma_5}{2}\right] \chi^a \phi^j_R$
$\bar{\chi}^a$
ϕ^*_{Li}
ψ^j_L
$-\sqrt{2} g T^{ai}_{Lj} \phi^*_{Li} \bar{\chi}^a \left[\frac{1-\gamma_5}{2}\right] \psi^j$
χ^a
$\bar{\psi}_{Li}$
ϕ^j_L
$\sqrt{2} g T^{ai}_{Lj} \bar{\psi}_i \left[\frac{1+\gamma_5}{2}\right] \chi^a \phi^j_L$

Fig. C-1

The rules shown in fig. C-1 are the basic Feynmann rules that were adapted for each particular calculation.

The propagators for the scalars and fermions are;



$$\frac{-i\delta^j{}_k}{p^2-M^2}$$

$$\frac{-i\delta^a{}_b}{\not{p}-m}.$$

Fig. C-2

APPENDIX D -- THE INTEGRALS

In this thesis we make use of several integral of the form

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-M^2)^n} \quad \text{and} \quad \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2-M^2)^n}. \qquad \text{Eq. D-1}$$

In this appendix I demonstrate a method to evaluate these.

For demonstration I will use the integral found in Eqn. 2.1.

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-M^2)^5} \qquad \text{Eq. D-2}$$

$$I = \int \frac{d^3k}{(2\pi)^4} \frac{dk_0}{[k_0{}^2-\bar{k}^2-M^2]^5}.$$

We can evaluate the integral over k_0 by investigating

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2-a^2)^5} = \int_{-\infty}^{\infty} \frac{dx}{(x-a+i\epsilon)^5(x+a-i\epsilon)^5}. \qquad \text{Eq. D-3}$$

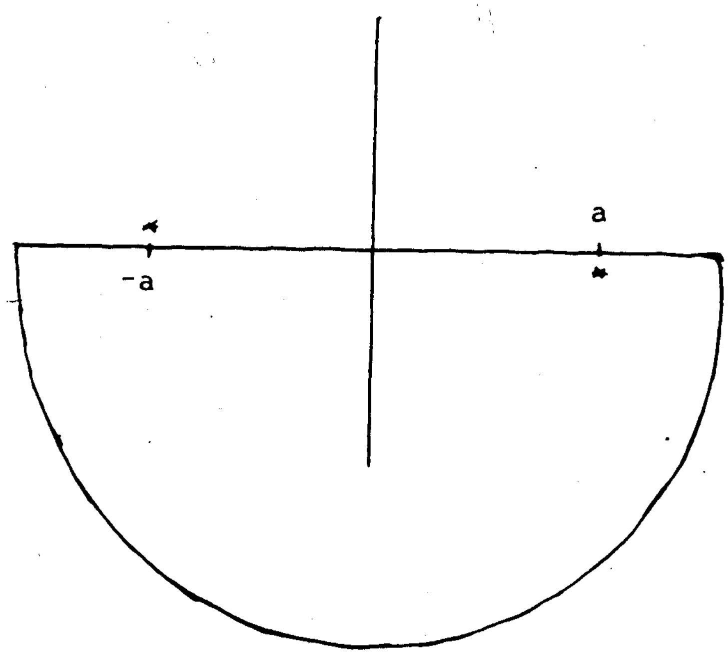


Fig. D-1

Using Cauchy's Theorem one gets:

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2-a^2)^5} = \frac{-2\pi i}{4!} \frac{d^4}{dx^4} \left[\frac{1}{(x-a)^5} \right]_{x=a}$$

$$= \frac{-2\pi i}{4!} \frac{5\cdot 6\cdot 7\cdot 8}{(x-a)^9} \Bigg|_{x=a} \qquad \text{Eq. D-4}$$

$$= \frac{-35}{128} \frac{\pi i}{a^9}.$$

Thus we now need only calculate

$$\int \frac{d^3k}{(k^2+M^2)^{9/2}} = 4\pi \int_0^{\infty} \frac{k^2 dk}{(k^2+M^2)^{9/2}} \qquad \text{Eq. D-5}$$

which we can evaluate using a formula from Gradshteyn and Ryzhik.[58]

$$\int_0^{\infty} \frac{x^{\mu-1}}{(p+qx^{\nu})^{n+1}} = \frac{1}{\nu p^{n+1}} \left[\frac{p}{q}\right]^{\mu/\nu} \frac{\Gamma(\mu/\nu)\Gamma(n+1-\mu/\nu)}{\Gamma(n+1)} \qquad \text{Eq. D-6}$$

So

$$4\pi \int_0^{\infty} \frac{k^2}{(k^2+M^2)^{9/2}} = \frac{4\pi}{2} \frac{(M^2)^{3/2}}{(M^2)^{9/2}} \frac{\Gamma(3/2)\Gamma(3)}{\Gamma(9/2)}$$

$$= \frac{32\pi}{105} \frac{1}{M^6}. \qquad \text{Eq. D-7}$$

Finally

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-M^2)^5} = \frac{-1}{(2\pi)^4} \frac{35\pi i}{128} \frac{32}{105} \frac{\pi}{M^6}$$

$$= \frac{-i}{192\pi^2} \frac{1}{M^2}. \qquad \text{Eq. D-8}$$

Similiarly, we find that

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-M^2)^3} = \frac{-i}{32\pi^2 M^2}. \qquad \text{Eq. D-9}$$

For

$$\int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2-M^2)^6} = \int d^3k \int dk_0 \frac{k_0^2-k^2}{(k_0^2-k^2-M^2)^6} \qquad \text{Eq. D-10}$$

we split it into two integrals, namely

$$\int dx \frac{x^2}{(x^2-a^2)^6} \quad \text{and} \int \frac{dx}{(x^2-a^2)^6} \qquad \text{Eq. D-11}$$

which we evaluate in the same manner as before, i.e. one has

$$\int \frac{x^2 dx}{(x-a+i\epsilon)^6(x+a-i\epsilon)^6} = \frac{-2\pi i}{5!}\frac{d^5}{dx^5}\left[\frac{x^2}{(x+a)^6}\right]_{x=a}$$

$$= \frac{-7}{2^8}\frac{\pi i}{a^9}. \qquad \text{Eq. D-12}$$

In the end we get a final result

$$\int \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-M^2)^6} = \frac{-1}{480}\frac{i}{\pi^2}\frac{1}{M^6}. \qquad \text{Eq. D-13}$$

One last integral we wish to evaluate is

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\tilde{M}_\gamma^2)}\frac{1}{(k^2-\tilde{M}_u^2)}\frac{1}{(k^2-\tilde{M}_d^2)} \qquad \text{Eq. D-14}$$

We can evaluate this with the use of a formula given by Shankar[52]:

$$\int \frac{d^4k}{(2\pi)^4}\frac{(k^2)^n}{\prod_{i=1}^m (k^2-m_i^2)} = \frac{i(-1)^{m+n+2}}{16\pi^2}\frac{\sum_{i=1}^m m_i^{2n+2}\ln(m_i^2)}{\prod_{(j\neq i)(j=1)}^m (m_j^2-m_i^2)} \qquad \text{Eq. D-15}$$

This gives us

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\tilde{M}_\gamma^2)}\frac{1}{(k^2-\tilde{M}_u^2)}\frac{1}{(k^2-\tilde{M}_d^2)} \qquad \text{Eq. D-16}$$

$$= \frac{-i}{16\pi^2}\left[\frac{\tilde{M}_\gamma^2\ln(\tilde{M}_\gamma^2)}{(\tilde{M}_u^2-\tilde{M}_\gamma^2)(\tilde{M}_d^2-\tilde{M}_\gamma^2)} + \frac{\tilde{M}_u^2\ln(\tilde{M}_u^2)}{(\tilde{M}_\gamma^2-\tilde{M}_u^2)(\tilde{M}_d^2-\tilde{M}_u^2)} + \frac{\tilde{M}_d^2\ln(\tilde{M}_d^2)}{(\tilde{M}_\gamma^2-\tilde{M}_d^2)(\tilde{M}_u^2-\tilde{M}_d^2)}\right]$$

which for $\tilde{M}_\gamma << \tilde{M}_u, \tilde{M}_d$

$$\rightarrow \frac{i}{16\pi^2}\frac{\ln(\tilde{M}_u^2)-\ln(\tilde{M}_d^2)}{\tilde{M}_d^2-\tilde{M}_u^2}, \qquad \text{Eq. D-17}$$

which, using L'Hopitals Rule, further reduces, for

$\tilde{M}_u \simeq \tilde{M}_d \simeq M_S$, to

$$\rightarrow \frac{-i}{16\pi^2}\,\frac{1}{M_S{}^2}. \qquad \text{Eq. D-18}$$

APPENDIX E -- FIERZ IDENTITIES

In this thesis we make use of Fierz reordering. The Fierz identities derive from the observation that the direct product of any two matrices, A_{ij}, B_{km}, may be decomposed into a sum of other direct products[59]

$$A_{ij}B_{km} = \sum_r C^{(r)}_{im} \; D^{(r)}_{kj} \; . \qquad \text{Eq. E-1}$$

When this is applied to particular combinations of γ-matrices, and various conditions are imposed, like their behavior under Lorentz transformations, one finds

$$I_{ij}I_{km} = \tfrac{1}{4}\{ \; I_{im}I_{kj} + (\gamma_5)_{im}(\gamma_5)_{kj} + (\gamma^\mu)_{im}(\gamma_\mu)_{kj} - (\gamma^\mu\gamma_5)_{im}(\gamma_\mu\gamma_5)_{kj} + \tfrac{1}{2}(\sigma^{\mu\nu})_{im}(\sigma_{\mu\nu})_{kj} \; \} \qquad \text{Eq. E-2}$$

which we write in the following shorthand

$$I\times I = \tfrac{1}{4}\{I\times I + \gamma_5\times\gamma_5 + \gamma^\mu\times\gamma_\mu - \gamma^\mu\gamma_5\times\gamma_\mu\gamma_5 + \tfrac{1}{2}\sigma^{\mu\nu}\times\sigma_{\mu\nu}\} \qquad \text{Eq. E-3}$$

If we now want something like R×L one multiplies eqn E-2 by $R_{pi}L_{mq}$, to obtain

$$L\times R = \tfrac{1}{4}\{RL\times I + R\gamma_5 L\times\gamma_5 + R\gamma^\mu L\times\gamma_\mu - R\gamma^\mu\gamma_5 L\times\gamma_\mu\gamma_5 + \tfrac{1}{2}R\sigma^{\mu\nu}L\times\sigma_{\mu\nu}\} \qquad \text{Eq. E-4}$$

which using the properties of L,R reduces to

$$\begin{aligned} L\times R &= \tfrac{1}{4}[\gamma^\mu L\times\gamma_\mu - \gamma^\mu\gamma_5 L\times\gamma_\mu\gamma_5] \\ &= \tfrac{1}{4}[\gamma^\mu L\times\gamma_\mu + \gamma^\mu L\times\gamma_\mu\gamma_5] \qquad \text{Eq. E-5} \\ &= \tfrac{1}{4}[\gamma^\mu L\times\gamma_\mu(1+\gamma_5)] \\ &= \tfrac{1}{2}\gamma^\mu L \times \gamma_\mu R. \end{aligned}$$

Of course, the converse is also true

$$\gamma^\mu L \times \gamma_\mu R = 2L\times R. \qquad \text{Eq. E-6}$$

Similiarly, one has

$$L\times L = \tfrac{1}{2}L\times L + \frac{1}{8}L\sigma^{\mu\nu}\times\sigma_{\mu\nu},$$

$$R\times R = \tfrac{1}{2}R\times R + \frac{1}{8}R\sigma^{\mu\nu}\times\sigma_{\mu\nu}.$$

Eq. E-7