THE UNIVERSITY OF ALBERTA

STABILITY REGIONS OF A REGULATED SYNCHRONOUS MACHINE

BY

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A THESIS

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ABSTRACT

The small signal dynamic stability of power systems has been a subject of interest for some twenty years. It continues to grow in importance as the control requirements of the generating stations become more sophisticated and demanding. In this thesis, the small signal performance of a regulated synchronous machine connected to an infinite bus is described by a set of differential equations of the form $(\dot{x}) = (A)(x)$. The transformation of the system into Schwarz form is used and the stability conditions for the system under study are established. The closed stability regions are obtained as the result of this study and the effects of various system parameters on stability regions are studied.

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CHAPTER I

INTRODUCTION

1.1 Background

Electrical power systems, considered from the point of view of their electro-mechanical operation, are very complicated. Power system engineers have devoted much thought and effort to stability studies since about 1925. The tendency of a power system or its component parts to develop forces to maintain synchronism and equilibrium is known as stability. In general, stability studies are classified by whether they involve steady-state or transient conditions. Dynamic stability is the term associated with the small signal performance and is applied to operations above the ordinary steady-state limits. It can be realized by the use of automatic control devices such as voltage and speed regulators. The small signal dynamic stability of electric power systems has been a subject of major theoretical and practical interest for some twenty years. It continues to grow in importance as the control requirements of the power plants become more sophisticated and demanding.

Power system stability studies as usually conducted, with all of their assumptions and approximations, are far from an exact science. Possibly some of the approximations can be justified because of the power system conditions analyzed. However, they will almost never correspond to the actual conditions existing on the system when the actual disturbance occurs. Also, it is virtually impossible to know how the actual power system loads vary with frequency and voltage magnitude. It is certain that all the loads do not act as fixed impedances. It is equally certain that they do not remain as constant current loads. Possibly, sufficient information will never be available on loads to represent them, and even if the information were available, it might be too complex to represent them accurately in a practical digital study. Usually it will not be possible to represent all of the generating plants connected to an interconnected system. This results in further approximation since there seems to be no exact simple equivalent that represents the generating plants. Further, the representation of individual generating plants also introduces errors because the machine impedances are not really constants. It appears that this can result in a paradoxical situation where the system will be calculated to be transiently stable and also calculated to be unstable under steady-state. Most analytical treatments [9, 10] of stability problems have been restricted to one or two generating sets because of the computational difficulties involved in applying the older methods. However, the techniques of modern control theory have partly removed this difficulty, subject to the requirement that the system be described by a set of differential equations in the state-space form

$$[\dot{x}] = [A][x] \qquad (1)$$

Laughton [8] has proposed a method of obtaining the [A] matrix of a multi-machine power system by using matrix elimination techniques to extract [A] from the complete algebraic and differential equations of the whole system. The paper by Undrill [7] describes a method of building up the [A] matrix of the multi-machine power system from the submatrices describing individual elements of the synchronous system. The matrix building approach offers significant savings in computer storage in comparison with the matrix elimination approach of Laughton [8]. The transient response of a dynamical system for small perturbations about an equilibrium (operating point) is completely described by a set of such equations. In addition, using the digital computer it is possible to decrease the time necessary for computation of stability analyses in this form.

In this thesis, the dynamic system which consists of a regulated synchronous machine connected to an infinite bus is taken for the purpose of the study. The system is expressed in the vector-matrix form (1) and the characteristic polynomial is obtained. The transformation into Schwarz form is performed and the necessary and sufficient conditions for stability established. Using these conditions, the stability regions are obtained and the change of these regions with variation of system parameters is observed.

1.2 Objective of Thesis

The stability study of the individual unit can be carried out and the necessary and sufficient conditions for stability established, assuming the rest of the system as an infinite bus. These can be used to find the stable regions for the system under study. Liapunov's criteria for stability and eigenvalue methods presented in the literature [13, 14] give satisfactory results for smaller systems but become too cumbersome for large systems. It seems reasonable therefore to look for conditions which could simplify the matrix representation of a system, i.e. minimize the computational effort. Consequently, the following objectives can be formulated.

1. To find the necessary and sufficient conditions for stability
2. To find the stability regions for the given system
3. To look for the possibility of extending the procedures employed in 1. and 2. to multi-machine systems.

Generally, the necessary and sufficient conditions for stability should yield the closed stability regions which should be of particular interest since no paper has been published so far on this subject.

CHAPTER II

MATHEMATICAL MODEL

2.1 Description of the System

The dynamic system taken for the purpose of this study consists of a regulated synchronous machine connected to an infinite bus. The system is generally considered to be non-linear and can be expressed in the vector-matrix form as follows:

$$\dot{x} = f(x) \tag{2}$$

The individual generating unit connected to an infinite bus has been investigated by Yu and Vongsuriya in [3] and by Kasturi and Doraraju in [4] in the configuration as shown in Fig. 1. The system under study consists of the synchronous machine, the voltage regulator, the speed governor and the tie-line. In addition, the machine with its control equipment is connected to an infinite bus. The steady-state phasor diagram for the system under study is shown in Fig. 2, where

e_d - armature voltage in d axis

e_q - armature voltage in q axis

e_t - armature terminal voltage

e - infinite bus-bar voltage

e_{fd} - field winding applied voltage

δ - load angle

r - tie-line resistance between generator and infinite bus

X - tie-line reactance between generator and infinite bus

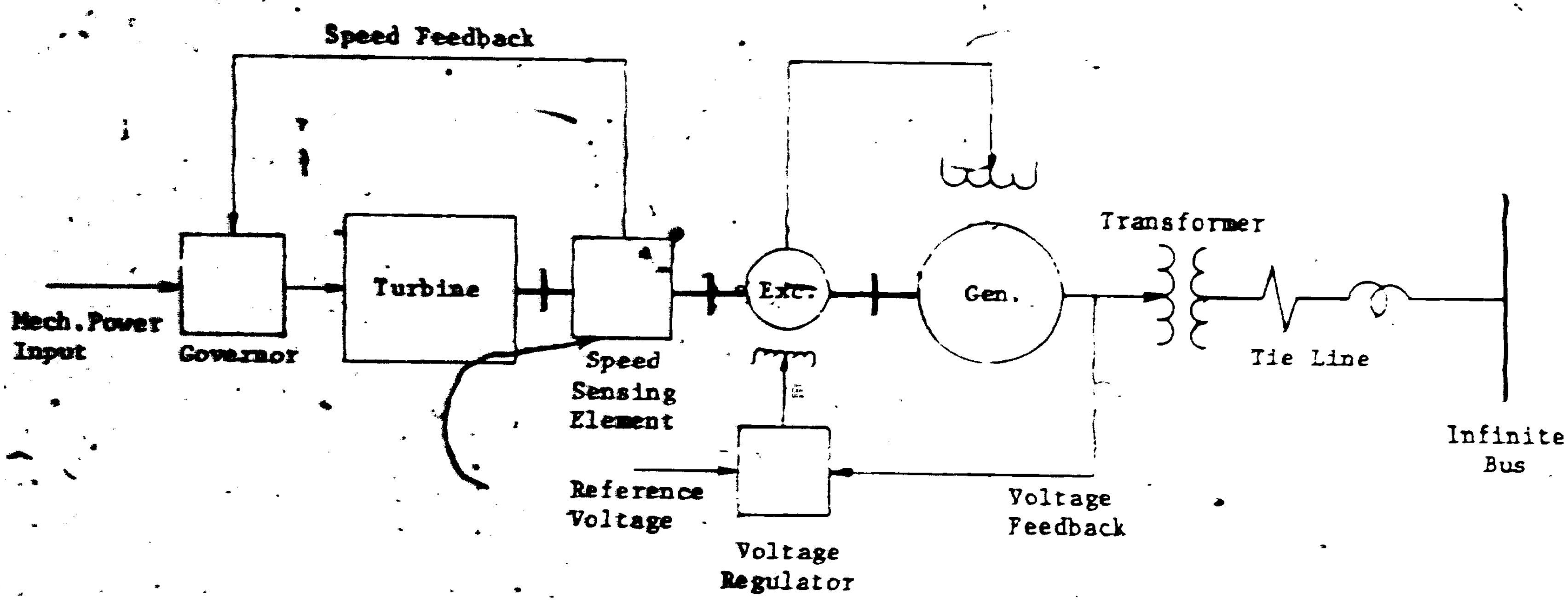

Speed Feedback
Mech. Power Input
Governor
Turbine
Speed Sensing Element
Exc.
Gen.
Transformer
Tie Line
Infinite Bus
Reference Voltage
Voltage Regulator
Voltage Feedback

Figure 1

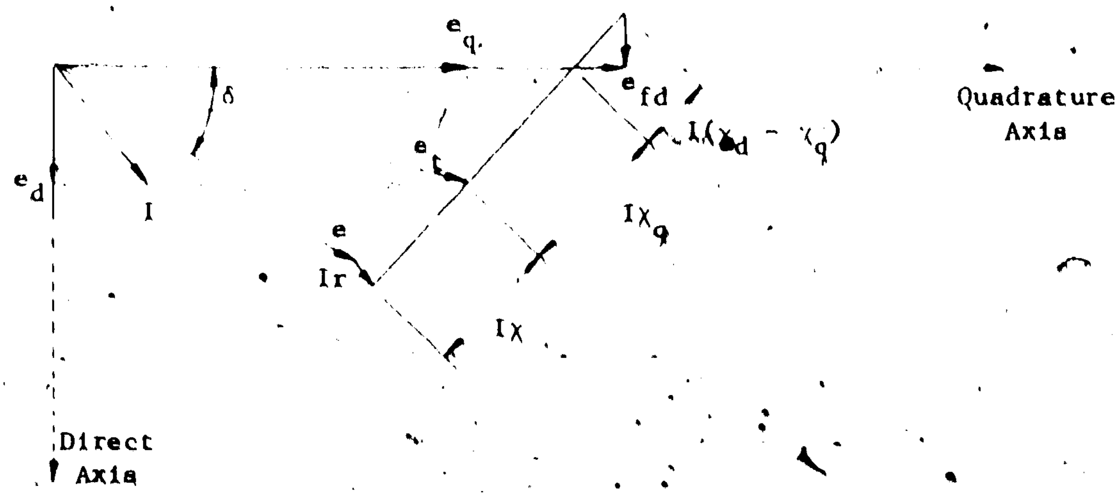


Figure 2

X_d - synchronous reactance in d axis

X_q - synchronous reactance in q axis

I - the phase current

The mathematical equations describing the state of the model at any instant consists of

A. The control system equations

B. Power transfer equations relating the mechanical input and electrical output power

C. Machine equations

In writing the equations governing the system under consideration, the following assumptions are considered:

1. Each stator winding is distributed to produce a sinusoidal magnetomotive force wave along the air gap
2. Stator slots produce negligible variations in the rotor inductances
3. The armature resistance of the synchronous machine is neglected
4. Damping due to damper bars has been neglected
5. Transformer voltages in Park's equations have been neglected compared to speed voltages
6. Saturation in the machine is neglected
7. The regulator is assumed to have no dead zone or limits

The above assumptions are usual in power system studies and they are the same as those in [3] and [4].

2.2 A. The Control System Equations

Assuming a conventional voltage regulator and speed governor, the control system can be written in a form which is commonly used in similar analyses [3, 4]:

$$g(s) = \frac{\Delta e_{fd}}{\Delta e_t} = \frac{-\mu'_e (1 + T_s s)}{1 + (T_e + T_s \mu'_e)s + T_e T_s s^2} \qquad (3)$$

$$f'(s) = \frac{\Delta T_i}{\Delta(s\theta)} = \frac{-\mu_m}{(1 + T_1 s)(1 + T_2 s)} \qquad (4)$$

where

Δ - small change around initial operating point

s - Laplace transform variable

θ - instantaneous angular position of rotor

$s\theta$ - angular velocity

T_i - mechanical power input to rotor

$\mu_e' = (X_{afd}/r_f)\mu_e$ - overall regulator gain

T_s - stabilizer time constant

T_e - exciter time constant

$\mu_s' = 1+\mu_{ex}\mu_{st}$ - overall stabilizer gain

μ_m - governor gain

T_1, T_2 - governor time constants

μ_{ex} - exciter gain

μ_{st} - stabilizer gain

$\mu_e = \mu_{ex}\mu_{st}\mu_r$ - regulator gain

μ_r - convertor gain

r_f - field winding resistance

$X_{afd}, X_{akd}, X_{akq}$ - mutual reactances between stator and rotor

2.3 B. Tie-Line Voltage Equations

The tie-line voltage equations derived from the steady-state phasor diagram shown in Fig. 2 of the system shown in Fig. 1 are as follows:

$$e_d = e\sin\delta + r\,i_d - X\,i_q$$

$$e_q = e\cos\delta + r\,i_q + X\,i_q$$

$$e_t^2 = e_d^2 + e_q^2$$

$$I^2 = i_d^2 + i_q^2 \qquad (5)$$

Considering a constant frequency for the infinite bus, the instantaneous angular displacement, velocity and acceleration of the machine respectively are

$$\theta = \theta_o + \delta$$

$$s\theta = s\theta_o + s\delta = \omega_o + s\delta \qquad \text{(velocity)}$$

$$s^2\theta = s^2\omega \qquad \text{(acceleration)}$$

In these equations $s\theta_o = \omega_o$ is the synchronous speed and ω is the undamped natural frequency. The system of equations (5) and the following system of machine equations can be obtained from [11], where also more detailed information can be found.

2.4 C. Machine Equations

Park's synchronous machine equations [11, 12] are used for the purpose of this study. The synchronous machine equations are as follows:

$$\psi_{fd} = \frac{X_{afd}}{X_{ffd}} \psi_f$$

$$e_{fd} = \frac{X_{afd}}{r_f} e_f$$

$$\psi_d = \frac{G(s)}{\omega_o} e_{fd} - \frac{X_d(s)}{\omega_o} i_d$$

$$X_d(s) = \frac{1 + T_d' s}{1 + T_{do}' s} X_d$$

$$e_q = G(s)\, e_{fd} - X_d(s)\, i_d$$

$$G(s) = \frac{X_{afd}}{r_f(1+T_{do}'s)}$$

$$e_d = X_q\, i_q \qquad (6)$$

where

$\psi_d, \psi_q, \psi_{kd}, \psi_{kq}$ – armature flux linkages in d and q axes, and damper flux linkages in d and q axes, respectively

$X_{ffd}, X_{kkd}, X_{kkq}$ – rotor self reactances

i_d, i_q, i_{kd}, i_{kq} – armature and damper currents in d and q axes

T_{do}' – direct axis transient open-circuit time constant

T_d' – direct axis transient short-circuit time constant

Further notation will be introduced here to complete the nomenclature of symbols used in the following text:

r_a – armature winding resistance in d or q axis

T_{ele} – electrical torque

P – real power output of machine

Q - reactive power output of machine

H - inertia constant

M - moment of inertia

D - damping coefficient of machine

ω - undamped natural frequency

In this study the stability of the power system due to small disturbancies will be examined. The following nine equations are the usual equations for the similar stability studies obtained by linearizing the operational equations (3), (4), (5) and (6) about an operating point.

$$\Delta e_{to} = \frac{e_{do}}{e_{to}} \Delta e_{do} + \frac{e_{qo}}{e_{to}} \Delta e_{qo}$$

$$\Delta e_{do} = - \psi_q s\Delta\delta + \chi_q \Delta i_{qo}$$

$$\Delta e_{qo} = \psi_d s\Delta\delta - \psi_d(s)\, \Delta i_{do} + G(s)\, \Delta e_{fdo}$$

$$\Delta e_{fdo} = g(s) \Delta e_{to}$$

$$\Delta T_i = f'(s)\, s\Delta\delta$$

$$\Delta e_{do} = e_o \cos\delta \Delta\delta + r\Delta i_{do} - \chi \Delta i_{qo}$$

$$\Delta e_{qo} = e_o \sin\delta \Delta\delta + r\Delta i_{qo} + \chi \Delta i_{do}$$

$$\omega_o \Delta T_{ele} = i_{do} \Delta e_{do} + i_{qo} \Delta e_{qo} + e_{do} \Delta i_{do} + e_{qo} \Delta i_{do} - T_{ele}\, s\Delta\delta$$

$$\Delta T_i = Ms^2 \Delta\delta + Ds \Delta\delta + \Delta T_{ele} \qquad (7)$$

The quantities e_{do}, e_{qo}, e_{to}, δ_o etc. represent the steady-state values at the particular operating point. Eliminating T_i, T_{ele} and e_t in the preceding equations, five homogeneous equations are obtained, which can be put into the matrix form as follows:

$$\begin{bmatrix} 1 & 0 & -r & \chi & -v_o\cos\delta_o \\ 0 & 1 & -\chi & -r & v_o\sin\delta_o \\ 1 & 0 & 0 & -\chi_2 & \psi_{qo}s \\ -h(s)v'_{do} & -h(s)v'_{qo}+1 & \chi_{d(s)} & 0 & -\psi_{do}s \\ i_{do} & i_{qo} & v_{do} & v_{qo} & \omega J(s) \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta i_g \\ \Delta i_d \\ \Delta i_2 \\ \Delta \delta \end{bmatrix} = \underline{0} \quad (8)$$

where

$$h(s) \doteq g(s)\,G(s)$$

$$J(s) \doteq Ms^2 + s\left(D - \frac{T_{ele}}{\omega_o}\right) - f(s) \quad (9)$$

$$f(s) \doteq sf'(s)$$

A characteristic equation of the form

$$a_o p^7 + a_1 p^6 + a_2 p^5 + a_3 p^4 + a_4 p^3 + a_5 p^2 + a_6 p + a_7 = 0 \quad (10)$$

can be obtained from the characteristic determinant of (8). Linearization of the system equations and the derivation of the characteristic equation is shown in more detail in [3] and [4].

CHAPTER III

CONDITIONS FOR STABILITY

3.1 Derivation of necessary and sufficient Conditions for Stability

The theorems given by Ogata [1] will be stated. This theorem gives the answers to the problem of deriving necessary and sufficient conditions for stability of a linear dynamic system and will be employed later to derive these conditions for the system under investigation.

Theorem 8 - 9; [1] p. 466:

Consider the linear time-invariant system

$$\dot{y} = Wy \qquad (11)$$

where

$$W = \begin{vmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -b_n & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -b_{n-1} & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -b_3 & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -b_2 & -b_1 \end{vmatrix} \qquad (12)$$

and $b_1, b_2, b_3, \ldots, b_n$ are real quantities. The origin of the system is asymptotically stable if and only if

$$b_1 > 0,\ b_2 > 0,\ b_3 > 0 \ \ldots\ldots\ldots\ b_n > 0.$$

(W is called the Schwarz matrix)

The above theorem gives directly the necessary and sufficient conditions for stability for the linear time-invariant system (11). The dynamic system under study is described by (1) and is of the form $\dot{x} = Ax$. If we can find the similarity transformation and transform the system under study into the form given by (11), the whole problem of stability can be solved and necessary and sufficient conditions for stability established. In [2] we can find the confirmation of this assumption, since on the page 100 is stated as follows: "once the matrix A has been transformed into Schwarz form, the stability problem is solved immediately; the necessary and sufficient conditions for asymptotic stability being that all b_i's are positive ($i = 1, 2, 3, \ldots, n$)." Another theorem from [1] will be introduced and the similarity transformation given by this theorem will be employed later.

Theorem 8 - 11, [1] p. 467:

If a real constant matrix C

$$C = \begin{bmatrix} 0 & 1 & \cdots\cdots & 0 \\ 0 & 0 & \cdots\cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots\cdots & 1 \\ -a_n & -a_{n-1} & \cdots\cdots & a_1 \end{bmatrix}$$

is similar to the Schwarz matrix W,

that is

$$C = T^{-1}WT \tag{13}$$

(T:non-singular), then the number of eigenvalues of C which have negative real parts is equal to the number of positive terms in the sequence

$$b_1, b_1b_2, b_1b_2b_3, \ldots\ldots, b_1b_2b_3 \ldots b_n;$$

provided that none of the b_is is zero.

Using this theorem, the following relationship will be used in order to transform the C matrix into the W matrix:

$$W = TCT^{-1} \tag{14}$$

where

- W - is Schwarz matrix
- C - Companion matrix associated with characteristic polynomial
- T - Transformation matrix
- T^{-1} - Inverse of the Transformation matrix

Generally speaking, every non-derogatory matrix A can be converted to the companion form and to the Schwarz form if the particular transformation matrix and its inverse can be found.

The characteristic equation of the system under study (10) can be rewritten in the following form:

$$p^7 + \frac{a_1}{a_o} p^6 + \frac{a_2}{a_o} p^5 + \frac{a_3}{a_o} p^4 + \frac{a_4}{a_o} p^3 + \frac{a_5}{a_o} p^2 + \frac{a_6}{a_o} p + \frac{a_7}{a_o} = 0 \qquad (15)$$

To outline further investigation at this point the following must be done in order to establish the necessary and sufficient conditions for the dynamic system:

1. Find the companion matrix C of the system
2. Find the transformation matrix T and its inverse
3. Perform the matrix multiplication and obtain the Schwarz form
4. Establish the necessary and sufficient conditions for stability for the system under study

Using [1] and [2], the companion matrix associated with the characteristic polynomial given by (15) was found to be as follows:

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{a_7}{a_o} & -\frac{a_6}{a_o} & -\frac{a_5}{a_o} & -\frac{a_4}{a_o} & -\frac{a_3}{a_o} & -\frac{a_2}{a_o} & -\frac{a_1}{a_o} \end{bmatrix} \qquad (16)$$

To obtain the transformation matrix T, two different methods can be followed. One is due to S. G. Loo and is shown in detail in [2]. The other method which seems to be more practical for the purpose of this study is that given by C. F. Chen and H. Chu and shown in [5]. These two authors also give the method for constructing the inverse of the transformation matrix [6] and this will be employed later in the text.

Using the material [5], the transformation matrix T was found to be as follows:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{C_{62}}{C_{61}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{C_{52}}{C_{51}} & 0 & 1 & 0 & 0 & 0 \\ \frac{C_{43}}{C_{41}} & 0 & \frac{C_{42}}{C_{41}} & 0 & 1 & 0 & 0 \\ 0 & \frac{C_{33}}{C_{31}} & 0 & \frac{C_{32}}{C_{31}} & 0 & 1 & 0 \\ \frac{C_{24}}{C_{21}} & 0 & \frac{C_{23}}{C_{21}} & 0 & \frac{C_{22}}{C_{21}} & 0 & 1 \end{bmatrix} \qquad (17)$$

where C_{ij} /i,j = 1, 2, 3 . . . . . .n/ are the elements of the Routh array for the given characteristic polynomial. The positions of the Routh array elements are indicated as matrix double subscript notations.

As mentioned in the preceding text, 6 gives the material for obtaining the inverse of the transformation matrix T; and it was found to be of the following form:

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{C_{62}}{C_{61}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{C_{52}}{C_{51}} & 0 & 1 & 0 & 0 & 0 \\ \begin{vmatrix} \frac{C_{62}}{C_{61}} & 1 \\ \frac{C_{43}}{C_{41}} & \frac{C_{42}}{C_{41}} \end{vmatrix} & 0 & -\frac{C_{42}}{C_{41}} & 0 & 1 & 0 & 0 \\ 0 & \begin{vmatrix} \frac{C_{52}}{C_{51}} & 1 \\ \frac{C_{33}}{C_{31}} & \frac{C_{32}}{C_{31}} \end{vmatrix} & 0 & \frac{C_{32}}{C_{31}} & 0 & 1 & 0 \\ \begin{vmatrix} \frac{C_{62}}{C_{61}} & 1 & 0 \\ \frac{C_{43}}{C_{41}} & \frac{C_{42}}{C_{41}} & 1 \\ \frac{C_{24}}{C_{21}} & \frac{C_{23}}{C_{21}} & \frac{C_{22}}{C_{21}} \end{vmatrix} & 0 & \begin{vmatrix} \frac{C_{42}}{C_{41}} & 1 \\ \frac{C_{23}}{C_{21}} & \frac{C_{22}}{C_{21}} \end{vmatrix} & \text{[illegible]} & \frac{C_{22}}{C_{21}} & 0 & 1 \end{bmatrix} \quad (18)$$

where C_{ij} $(i,j = 1, 2, 3 \ldots\ldots n)$ are the same elements of the Routh array for a given characteristic polynomial as before. Further we have to construct the Routh array in order to evaluate the transformation matrix and its inverse. In [1] we can find the method for constructing such an array and it was found to be as follows:

λ^7	1	a_2/a_0	a_4/a_0	a_6/a_0
λ^6	a_1/a_0	a_3/a_0	a_5/a_0	a_7/a_0
λ^5	b_1	b_2	b_3	0
λ^4	c_1	c_2	c_3	0
λ^3	d_1	d_2	0	0
λ^2	e_1	e_2	0	0
λ	f_1	0	0	0
λ^0	g_1	0	0	0

The coefficients for our seven-degree polynomial $a_0, a_1, a_2 \ldots\ldots a_7$ are known and we have to express all the elements of this array in terms of these coefficients. In other words, we want to know b_1, b_2, b_3, c_1, c_2, c_3, d_1, d_2, e_1, e_2, f_1 and g_1 in terms of a_i's $(i = 1, 2, 3 \ldots\ldots n)$. The following relationships hold.

$$b_1 = a_1a_2 \text{ [illegible]}$$

$$b_2 = a_3a_4 - a_2a_5/a_0a_3$$

$$b_3 = a_5a_6 - a_4a_7/a_0a_5$$

$$c_1 = (a_1a_2 - a_0a_3)\ a_3^2 - (a_3a_4 - a_2a_5)\ a_1^2/(a_1a_2 - a_0a_3)\ a_0a_3$$

$$c_2 = (a_3a_4 - a_2a_5)\ a_5^2 - (a_5a_6 - a_4a_7)\ a_3^2/(a_3a_4 - a_2a_5)\ a_0a_5$$

$$c_3 = a_7/a_0$$

$$d_1 = c_1b_2 - b_1c_2/c_1$$

$$d_2 = c_2b_3 - b_2c_3/c_2$$

$$e_1 = d_1c_2 - c_1d_2/d_1$$

$$e_2 = a_7/a_0$$

$$f_1 = e_1d_2 - d_1e_2/e_1$$

$$g_1 = a_7/a_0 \qquad (19)$$

These relationships allow the expression of all the elements of the Routh array in terms of coefficients of the characteristic polynomial. Now substituting the following values from the Routh array into T and T^{-1} matrices in order to get them to the suitable form for matrix multiplication (14):

$$C_{22}/C_{21} = a_3/a_1 \qquad C_{23}/C_{21} = a_5/a_1$$

$$C_{24}/C_{21} = a_7/a_1 \qquad C_{32}/C_{31} = b_2/b_1$$

$$C_{33}/C_{31} = b_3/b_1 \qquad C_{42}/C_{41} = c_2/c_1$$

$$C_{43}/C_{41} = c_3/c_1 \qquad C_{52}/C_{51} = d_2/d_1$$

$$C_{62}/C_{61} = e_2/e_1 \tag{20}$$

Substituting equations given by (20) into (17), the matrix T is

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{e_2}{e_1} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{d_2}{d_1} & 0 & 1 & 0 & 0 & 0 \\ \frac{c_3}{c_1} & 0 & \frac{c_2}{c_1} & 0 & 1 & 0 & 0 \\ 0 & \frac{b_3}{b_1} & 0 & \frac{b_2}{b_1} & 0 & 1 & 0 \\ \frac{a_7}{a_0} & 0 & \frac{a_5}{a_1} & 0 & \frac{a_3}{a_1} & 0 & 1 \end{bmatrix} \tag{21}$$

Similarly, substitute the elements of the Routh array (20) into (18). To make the notation simpler, let

$$K = \begin{bmatrix} \dfrac{C_{62}}{C_{61}} & 1 \\ \dfrac{C_{43}}{C_{41}} & \dfrac{C_{42}}{C_{41}} \end{bmatrix} = e_2c_2 - c_3e_1/e_1c_1$$

$$L = \begin{bmatrix} \dfrac{C_{52}}{C_{51}} & 1 \\ \dfrac{C_{33}}{C_{31}} & \dfrac{C_{32}}{C_{31}} \end{bmatrix} = d_2b_2 - b_3d_1/d_1b_1$$

$$M = \begin{bmatrix} \dfrac{C_{42}}{C_{41}} & 1 \\ \dfrac{C_{23}}{C_{21}} & \dfrac{C_{22}}{C_{21}} \end{bmatrix} = c_2a_3 - a_5c_1/c_1a_1$$

$$N = \begin{bmatrix} \dfrac{C_{62}}{C_{61}} & 1 & 0 \\ \dfrac{C_{43}}{C_{41}} & \dfrac{C_{42}}{C_{41}} & 1 \\ \dfrac{C_{24}}{C_{21}} & \dfrac{C_{23}}{C_{21}} & \dfrac{C_{22}}{C_{21}} \end{bmatrix} = a_3(c_2e_2 - c_3e_1) + c_1(a_7e_1 - a_5e_2)/a_1c_1e_1$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{e_2}{e_1} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{d_2}{d_1} & 0 & 1 & 0 & 0 & 0 \\ K & 0 & -\frac{c_2}{c_1} & 0 & 1 & 0 & 0 \\ 0 & L & 0 & -\frac{b_2}{b_1} & 0 & 1 & 0 \\ -N & 0 & M & 0 & -\frac{a_3}{a_1} & 0 & 1 \end{bmatrix} \quad (22)$$

Now matrices T and T^{-1} are a suitable form for matrix multiplication and obtaining the Schwarz form (14) gives the relationship for transformation into Schwarz form and in this equation T is given by (21), T^{-1} by (22) and C by (16). After multiplication, the Schwarz matrix is

$$W = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{e_2}{e_1} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{e_2}{e_1} - \frac{d_2}{d_1} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{d_2}{c_1} - \frac{c_2}{c_1} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{c_2}{c_1} - \frac{b_2}{b_1} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{b_2}{b_1} - \frac{a_3}{a_1} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -\frac{a_3}{a_1} - \frac{a_2}{a_0} & -\frac{a_1}{a_0} \end{bmatrix} \quad (23)$$

When the system is transformed into the form given by (11), the necessary and sufficient conditions for asymptotic stability can be established. Theorem 8-9 introduced earlier in the text and given in [1] can now be applied in order to obtain the stability conditions for the dynamic system under study. It is stated in the above mentioned theorem that the necessary and sufficient conditions for asymptotic stability are that all b_i's are positive ($i = 1, 2, 3 \ldots\ldots n$). If this is applied to the Schwarz matrix given by (23), the following seven conditions for stability can be found.

1. $a_1/a_o > 0$
2. $a_2/a_o > a_3/a_1$
3. $a_3/a_1 > b_2/b_1$
4. $b_2/b_1 > c_2/c_1$
5. $c_2/c_1 > d_2/d_1$
6. $d_2/d_1 > e_2/e_1$
7. $e_2/e_1 > 0$ (24)

The set of inequalities given by (24) forms the necessary and sufficient conditions for asymptotic stability for the dynamic system given by (15). It was mentioned previously that all constants involved in these conditions can be expressed in terms of coefficients of the characteristic polynomial, namely in terms of $a_o, a_1, a_2 \ldots a_7$.

For the time being they may be left in the form given by (24) because this seems to be the simplest form in which to express the stability conditions for our case. Speaking generally, the stability conditions given by (24) must be satisfied simultaneously for some operating point and only then is it possible to assume that the dynamic system is stable at that particular operating point. Choosing some systems of coordinates, it will be possible to find stability regions in that plane if it is checked for stability at every point in that particular plane. In other words, for some point in the chosen plane to be stable the conditions given by (24) must be satisfied simultaneously.

3.2 Multimachine Systems

In the preceding text a single-machine system connected to an infinite bus has been taken for analysis. However, with the state variable approach given by [7] and [8], the study can be extended to multimachine systems. Speaking generally, the A matrix must be obtained for the considered system and the characteristic equation of the system established. Then the companion matrix of the investigated system must be found and using the transformation (14) the Schwarz form obtained. Theorem 8-9 can be applied again and the necessary and sufficient conditions for asymptotic stability of a dynamic system established.

The method of analysis used in this study is quite general. To obtain the stability regions, the same technique can be used in multimachine systems as was used in the single-machine case.

Dynamic stability analysis of a large interconnected power system is extremely time-consuming and laborious and may even exceed the storage capacity of modern fast computers because of high order of the A matrix. Hence methods have been developed to obtain simplified models of the systems based on the speed of response of the variables or the nodes. One of such methods is given by Kappurajulu and Elangovan [15] and gives the reduced model of dynamic systems. It seems reasonable, therefore, to consider the possibility of reducing the system with more than three machines to some simplified form in order to simplify the stability calculations.

3.3 Outline of the Method by Yu and Vongsuriya

The steady-state stability limits of a regulated synchronous machine connected to an infinite bus were investigated by Y. N. Yu and K. Vongsuriya in [3]. The open stability regions were obtained as a result of this study and the method of the approach used in [3] will be outlined briefly in the following text. The system under investigation in this paper is that shown in Fig. 1 and the stability of the system due to small load disturbance is studied. Generally, the Routh-Hurwitz criterion and D-partition method are used in this steady-state stability study. The characteristic equation of the investigated system (10) is obtained and it can be written in such a form that the voltage regulator gain μ'_e and the stabilizer gain μ'_s can be separated from the remainder of the equation.

$$\mu'_s P(p) + \mu'_e Q(p) + R(p) = 0 \qquad \text{(a)}$$

To find the stability boundary on a $\mu_s' - \mu_e'$ plane, let

$$p = j\omega \qquad \text{(b)}$$

which corresponds to the imaginary axis on the complex frequency plane. Separating P, Q and R in (a) into real and imaginary parts,

$$\begin{aligned} P(j\omega) &= P_1(\omega) + jP_2(\omega) \\ Q(j\omega) &= Q_1(\omega) + jQ_2(\omega) \\ R(j\omega) &= R_1(\omega) + jR_2(\omega) \end{aligned} \qquad \text{(c)}$$

the following can be obtained:

$$\begin{aligned} \mu_s' P_1(\omega) - \mu_e' Q_1(\omega) + R_1(\omega) &= 0 \\ \mu_s' P_2(\omega) - \mu_e' Q_2(\omega) + R_2(\omega) &= 0 \end{aligned} \qquad \text{(d)}$$

Hence

$$\mu_s' = \frac{1}{\Delta} \begin{bmatrix} -R_1(\omega) & Q_1(\omega) \\ -R_2(\omega) & Q_2(\omega) \end{bmatrix}$$

$$-\mu_e' = \frac{1}{\Delta} \begin{bmatrix} P_1(\omega) & -R_1(\omega) \\ P_2(\omega) & -R_2(\omega) \end{bmatrix} \qquad \text{(e)}$$

where

$$\Delta = \begin{bmatrix} P_1(\omega) & Q_1(\omega) \\ P_2(\omega) & Q_2(\omega) \end{bmatrix}$$

For a non-trivial solution to exist, Δ must be non-zero. Theoretically, the complete stability boundary on the $\mu'_s - \mu'_e$ plane can be determined by varying ω from zero to infinity. Practically, only a variation of ω from zero to 0.02 is necessary to obtain all the useful information. A point test on stability by the Routh-Hurwitz criterion is necessary to determine which side of the boundary is stable.

The effects of the saliency and the short-circuit ratio of the synchronous machine and that of the tie-line resistance and reactance on the stability of the system were investigated in this paper.

CHAPTER IV

MODELLING OF THE SYSTEM

4.1 System Parameters

For the purpose of the study, the system will be introduced here with all the necessary machine data, regulator data and governor data. The system and its constants are the same as those used in [4]. The following constants are expressed in p.u. values and time in seconds.

Machine data:

$P = 1.0 \qquad M = .0337$

with

$H = 5.3 \qquad T'_{do} = 6.0 \qquad T'_d = 1.13$

$X_d = 2.0 \qquad X_q = 1.7 \qquad r = .0223$

$X = 0.18 \qquad e_t = 1.0$

Regulator settings:

$T_e = 1.12 \qquad T_s = 2.5$

Governor settings:

$T_1 = T_2 = 1.0 \qquad \mu_m = 10.0$

As stated earlier, a graphical display of the stable regions of the system under investigation is desired. For this purpose a Cartesian system, such that its X and Y coordinates correspond to μ'_e and μ'_s which are the overall stabiliser and overall regulator gain respectively, is used. These two parameters are controllable and were used in the similar study given by [3] to display the stability limits of a regulated synchronous machine.

4.2 Initial Conditions of Synchronous Machine

The initial values i_{do}, i_{qo}, v_{do}, v_{qo}, ψ_{do}, ψ_{qo}, v_{fdo}, v_o and δ_o of a salient pole machine are found by the following expressions. In the steady-state, Park's equations become

$$v_{do} = \chi_q i_{qo}$$

$$v_{qo} = \frac{\chi ad}{R_F} v_{fdo} - \chi_d i_{do}$$

$$P = v_{do} i_{do} + v_{qo} i_{qo}$$

$$Q = v_{qo} i_{do} - v_{do} i_{qo}$$

$$v_{do} = v_o \sin\delta + r\, i_{do} - \chi\, i_{do}$$

$$v_{qo} = v_o \cos\delta + \chi\, i_{do} + r\, i_{qo}$$

$$v_{to}^2 = v_{do}^2 + v_{qo}^2$$

In these equations v_{do}, v_{qo}, i_{do}, i_{qo}, v_o, δ_o and v_{fdo} are unknown, while v_{to}, P and Q are given. During the analyses in this thesis, P and Q will change but v_{to} will be kept constant (v_{to} = 1.0 = constant). The unknowns will be calculated from the following relationships derived from the Park's equations for the steady-state.

$$i_{qo} = \frac{P\, v_{to}}{\sqrt{(P\, \chi_q)^2 + (v_{to}^{\,2} + \chi_q\, Q)^2}}$$

$$v_{do} = \chi_q\, i_{qo}$$

$$v_{qo} = \sqrt{v_{to}^{\,2} - v_{do}^{\,2}}$$

$$i_{do} = \frac{Q + \chi_q\, i_{do}^{\,2}}{v_{qo}}$$

$$v_{fdo} = (v_{qo} - \chi_d\, i_{do})\, \frac{R_F}{\chi_{ad}}$$

$$\psi_{do} = -\, v_{do}$$

$$\psi_{qo} = v_{qo}$$

$$v_o = \sqrt{(v_{do} - r\, i_{do} + \chi\, i_{qo})^2 + (v_{qo} - \chi\, i_{do} - r\, i_{qo})^2}$$

$$\delta_o = \operatorname{arctg} \frac{v_{do} - r\, i_{do} + \chi\, i_{qo}}{v_{qo} - \chi\, i_{do} - r\, i_{qo}}$$

Further and more detailed information on deriving the initial conditions of a synchronous machine can be found in [3] and [4]. The initial conditions of the synchronous machine are calculated for all different power factors used in this study. These values are used later in the process of establishing the coefficients of the characteristic polynomial, that is, they are used for the computation of the following constants: $A_1, A_2, \ldots\ldots\ldots A_8, A_9$. Results are tabulated in Appendix I.

4.3 Coefficients for the Characteristic Polynomial

The stability conditions were established in terms of the coefficients of the characteristic polynomial (24) of the dynamic system. The set of equations for calculating these coefficients is given here. From the characteristic equation (10) the coefficients of the polynomial are

$$a_o = T_a M B_1$$

$$a_1 = T_a B_4 + T_d M B_1$$

$$a_2 = T_a B_5 + T_c M B_1 + T_d B_4 - \mu_a' A_1 M T_1 T_2 T_s$$

$$a_3 = T_d B_3 + T_b M B_1 + T_c B_1 + T_d B_5 + \mu_m B_1 T_E T_s - \\ - \mu_a' (M A_1 T_E + B_6 T_1 T_2 T_s)$$

$$a_4 = T_b B_4 + T_c B_5 + T_d B_3 + \mu_m (B_1 (T_E + T_s \mu_a') + B_2 T_E T_s) - \\ - \mu_a' (M A_1 T_F + A_5 T_1 T_2 T_s + B_6 T_E) + M B_1$$

$$a_5 = T_b B_5 + T_c B_3 + \mu_m (B_1 + B_2 (T_E + T_s \mu'_s) - \mu'_e A_1 T_s) -$$

$$- \mu'_e (M A_1 + A_5 T_E + B_6 T_F) + B_4$$

$$a_6 = T_b B_3 + \mu_m (B_2 - \mu'_e A_1) - \mu'_e (A_5 T_F + B_6) + B_5$$

$$a_7 = B_3 - \mu'_e A_5 \qquad (28)$$

In the above set of equations, constants $B_1, B_2 \ldots\ldots B_6, T_1, T_s, T_E, T_F, T_a, T_b, T_c, T_d$ and $A_1, A_2 \ldots\ldots A_8, A_9$ are introduced. The equations specifying the above constants and the set of equations (28) are in agreement with [3] and [4].

To calculate the constants $B_1, B_2 \ldots\ldots B_6$ the following relationships can be used:

$$B_1 = A_2 x_d T'_d + A_8 T'_{do}$$

$$B_2 = A_2 x_d + A_3$$

$$B_3 = A_7 x_d + A_9$$

$$B_4 = A_6 x_d T'_d + A_8 T'_{do} + B_1 \left(D - \frac{T_{eo}}{\omega}\right) + B_2 M$$

$$B_5 = (A_6 + A_7 T'_d) x_d + A_8 + A_9 T'_{do} + B_2 \left(D - \frac{T_{eo}}{\omega}\right)$$

$$B_6 = A_1 \left(D - \frac{T_{eo}}{\omega}\right) + A_4 \qquad (29)$$

where

$$T_{eo} = i_{qo}\,\psi_d - i_{do}\,\psi_q$$

and for calculation of T_a, T_b, T_c, T_d, T_E and T_F the following set of relationships can be used:

$$T_a = T_1\,T_2\,T_s\;\;T_e$$

$$T_b = T_1 + T_2\,T_e + T_s\,\mu_s'$$

$$T_c = T_1\,T_2 + T_e\,T_s + (T_1 + T_2)\;(T_e + T_s\,\mu_s')$$

$$T_d = T_1\,T_2(T_e + T_s\,\mu_s') + (T_1 + T_2)\;T_e\,T_s$$

$$T_E = T_1\,T_2 + T_2\,T_s + T_1\,T_s$$

$$T_F = T_1 + T_2 + T_s$$

The last set of equations is for calculation of A_1, A_2 ...., A_8, A_9. The following equations are used to calculate these constants.

$$A_1 = -\,\omega\,[v_{do}'\,r\chi_q - v_{qo}'\,(r^2 + \chi^2 + \chi\chi_q)]$$

$$A_2 = \omega\,[\chi + \chi_q]$$

$$A_3 = \omega\,[r^2 + \chi^2 + \chi\chi_q]$$

$$A_4 = \psi_{qo}\,[-\,v'_{do}\,\{v_{do}\,\chi + v_{qo}\,r + i_{qo}\,(r^2 + \chi^2)\} +$$

$$+\, v'_{qo}\,\{v_{do}\,r - v_{qo}\,\chi + i_{do}\,(r^2 + \chi^2)\}]$$

$$A_5 = v_o\,\sin\delta_o[v'_{do}\,i_{qo}r\chi_q - v'_{qo}\,\{v_{do}(\chi + \chi_q) + r\,(v_{qo} + i_{do}\chi_q)\}] +$$

$$+\, v_o\cos\delta_o\ \ v'_{do}\chi_q\,(v_{do} + i_{qo}\chi) - v'_{qo}\,\{-\,v_{do}r + \chi(v_{qo} + i_{do})\}]$$

$$A_6 = \psi_{qo}\,(v_{qo} - i_{do}\,\chi + i_{qo}\,r)$$

$$A_7 = v_o\,\sin\delta_o\,i_{qo}\,(\chi + \chi_q) + v'_o\cos\delta_o\,(v_{qo} - i_{do}\chi q - i_{qo}r)$$

$$A_8 = \psi_{do}\ \ v_{do}\,(\chi + \chi_q) + v_{qo}r + i_{qo}\,\{(r^2 + \chi^2) + \chi\chi_q\} +$$

$$+\, i_{do}r\,\chi_q] + \psi_{qo}\,[-\,v_{do}r + v_{qo}\,\chi - i_{do}(r^2 + \chi^2)]$$

$$A_9 = v_o\sin\delta_o\,[v_{do}\,(\chi + \chi_q) + v_{qo}r + i_{do}r\,\chi_q] +$$

$$+\, v_o\cos\delta_o\,[-\,v_{do}r + v_{qo}\,\chi + i_{do}\,\chi\chi_q] \qquad (31)$$

The equations for the computation of initial conditions for synchronous machine were introduced earlier (27) and similarly the equations for the computation of all constants (31, 30, 29) necessary to evaluate the coefficients of the characteristic polynomial $a_1, a_2, a_3 \ldots\ldots\ldots a_n, a_o$.

As was mentioned before, it is convenient to leave these coefficients in terms of μ'_s and μ'_a (overall stabilizer and regulator gain respectively) and then to draw the stability regions for the particular study on this plane. A digital computer program (Appendix II) is used to check as many points as required to establish stability regions. This program checks individual points on a grid.

In summary, the following steps must be taken in order to obtain the stability regions for the dynamic system under investigation.

1. Calculate the initial conditions of synchronous machine
2. Calculate constants T_a, T_b, T_c, T_d, T_E and T_F
3. Calculate constants A_1, A_2 .... A_8, A_9
4. Calculate constants B_1, B_2, B_3, B_4, B_5 and B_6
5. Establish the coefficients for the characteristic polynomial of the system, namely a_o, a_1 .... a_n
6. Check for stability on $\mu'_a \mu'_e$ plane for as many points as is required.

CHAPTER V

RESULTS

5.1 Stability Regions

In this study the effects of the following parameters on the stability region of the system under study will be examined.

1. Changes in power factor
2. Changes in damping coefficient
3. Stabilizer time constant effect
4. Exciter time constant effect
5. Effect of a one-time constant governor
6. Effect of a two-time constant governer

These studies are introduced in the following text and all the conclusions drawn later in the text are based on these studies.

5.2 Effect of Different Power Factor on Stability Region

In this study, the real power output of the machine is kept constant ($P = 1.0$ p.u.) in all investigated cases. The reactive power output of the machine is varied and the following distinct values of Q are used.

$$Q = .6;\ .3;\ 0;\ -.3$$

The system data used here are those introduced earlier and given by (25). All constants are in p.u. values and time is in seconds. Using the set of equations (27), the initial conditions of the synchronous machine were calculated for all different values of Q and are shown in Appendix I in tabulated form.

For each different value of Q different values of constants T_a, T_b, T_c, T_d, T_E, T_F, A_1, A_2 .... A_8; A_9 and B_1, B_2 .... B_6 and also the coefficients of the characteristic polynomial a_o, a_1 .... a_n have to be calculated, since the initial conditions of the synchronous machine vary with different values of Q.

Example for $P_o = 1.0$ and $Q = .6$

To calculate T_a, T_b, T_c, T_d, T_E, T_F the set of equations (30) is used and the values are found to be

$$T_a = 2.8$$

$$T_b = 3.12 + 2.5\ \mu'_a$$

$$T_c = 6.04 + 5\ \mu'_a$$

$$T_d = 6.72 + 2.5\,\mu_s'$$

$$T_E = 6$$

$$T_F = 4.5$$

The next step is to calculate constants $A_1, A_2 \ldots\ldots\ldots A_8, A_9$. The set of relationships given by (31) will serve this purpose and the constants are

$$A_1 = .239 \qquad A_6 = .4755$$

$$A_2 = 1.88 \qquad A_7 = -.9244$$

$$A_3 = .339 \qquad A_8 = -.783$$

$$A_4 = -.126 \qquad A_9 = 1.058$$

$$A_5 = -.263$$

To calculate the constants $B_1, B_2 \ldots\ldots\ldots B_6$ the set of equations given by (29) is used and the constants are

$$B_1 = 6.33 \qquad B_4 = 15.667$$

$$B_2 = 4.1 \qquad B_5 = 21.397$$

$$B_3 = -.79 \qquad B_6 = .59$$

With these constants the coefficients of the characteristic polynomial for the dynamic system can be calculated. The set of equations (28) will serve this purpose, giving coefficients of the characteristic polynomial as follows:

$$a_0 = .597$$

$$a_1 = 45.299 + .532\ \mu_s'$$

$$a_2 = 166.479 + 40.232\ \mu_s' - .0201\ \mu_e'$$

$$a_3 = 414.11 + 132.359\ \mu_s' - 1.523\ \mu_e'$$

$$a_4 = 358.61 + 302.427\ \mu_s' - 2.918\ \mu_e'$$

$$a_5 = 186.874 + 152.042\ \mu_s' - 7.055\ \mu_e'$$

$$a_6 = 59.932 - 1.975\ \mu_s' - 1.797\ \mu_e'$$

$$a_7 = .263\ \mu_e' - .79$$

As was mentioned earlier, it is convenient to leave these coefficients in terms of μ_s' and μ_e' (the overall stabilizer and regulator gain, respectively) in order to draw the stability regions on this particular plane. When the coefficients are in this form, the program for digital computer shown in Appendix II can be used. This program employs the conditions for stability directly in the form given by (24) and derived in the text previously. This program checks for stability for as many points on $\mu_s'\ \mu_e'$ plane as required and specified in the program. All points on $\mu_s'\ \mu_e'$ plane which satisfy the stability conditions constitute the stable region on this plane. Using the information given by digital computer, results can be displayed graphically as in Fig. 3a. In this figure the stable region of the regulated synchronous machine at $P = 1.0$ and $Q = .6$ p.u. connected to an infinite bus is shown. All points which lie inside the triangular region satisfy the stability conditions (24) and the points outside this region are unstable, i.e. they do not satisfy the conditions for asymptotic stability.

Using this figure for a chosen value of the overall stabilizer gain, it is possible to find the maximum allowable voltage regulator gain; the minimum allowable voltage regulator gain shows fairly constant value for all possible settings of μ_s'.

Using the same procedure, the stability regions for all other power factors can be established (Q = .3; 0; -.3). All necessary constants and the coefficients of the characteristic polynomial were calculated and the stability regions established. The results are graphically shown in Fig. 3b and 3c, where the effect of different power factor on stability region of dynamic power system can be seen.

It can be seen in the above mentioned figures that the minimum allowable voltage regulator gain is constant in all investigated cases. Both the maximum voltage regulator and stabilizer gains decrease their values and reach certain minimum and then again increase their values as the system goes from lagging power factor (positive Q) towards leading power factors.

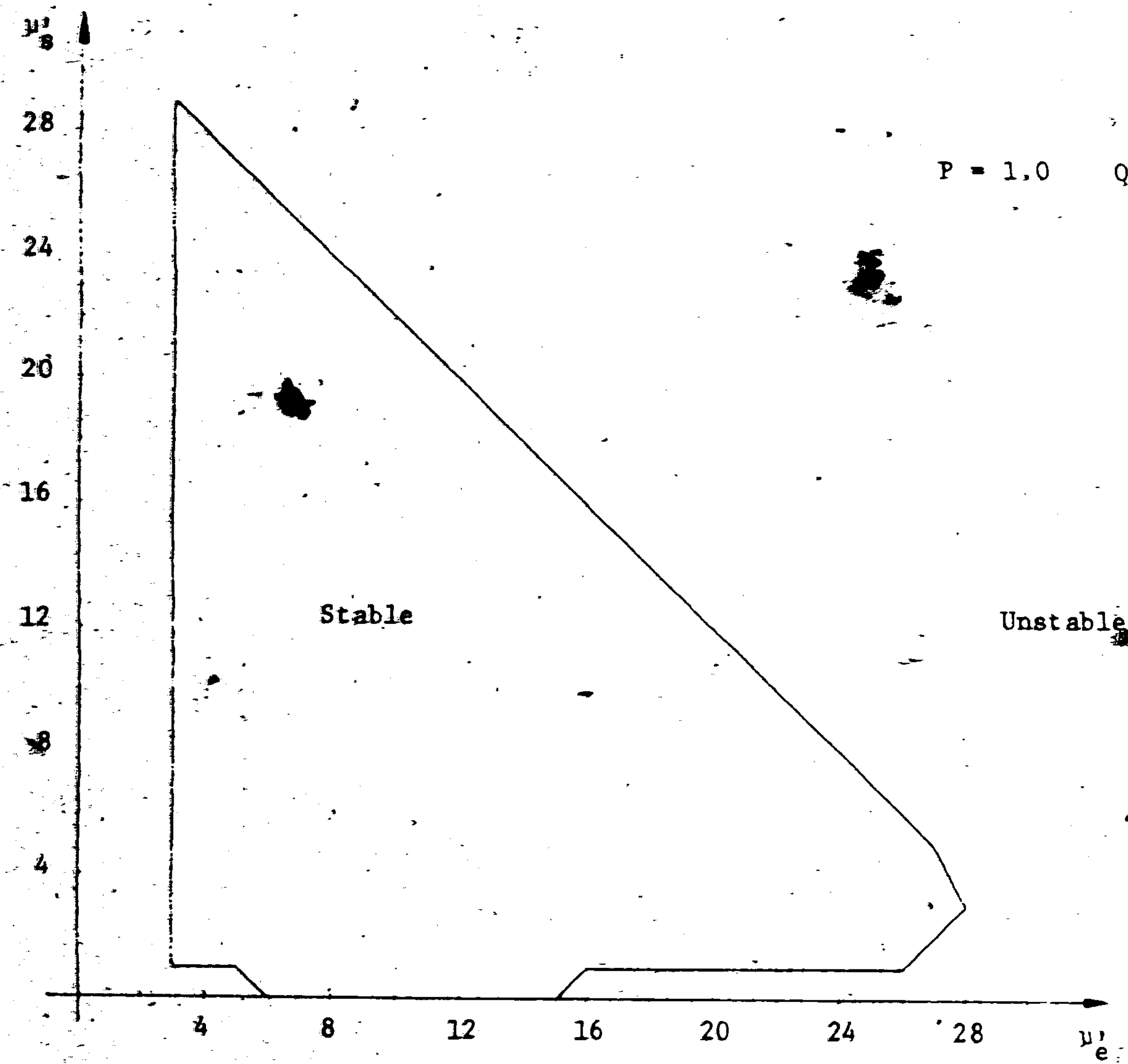


Figure 3a

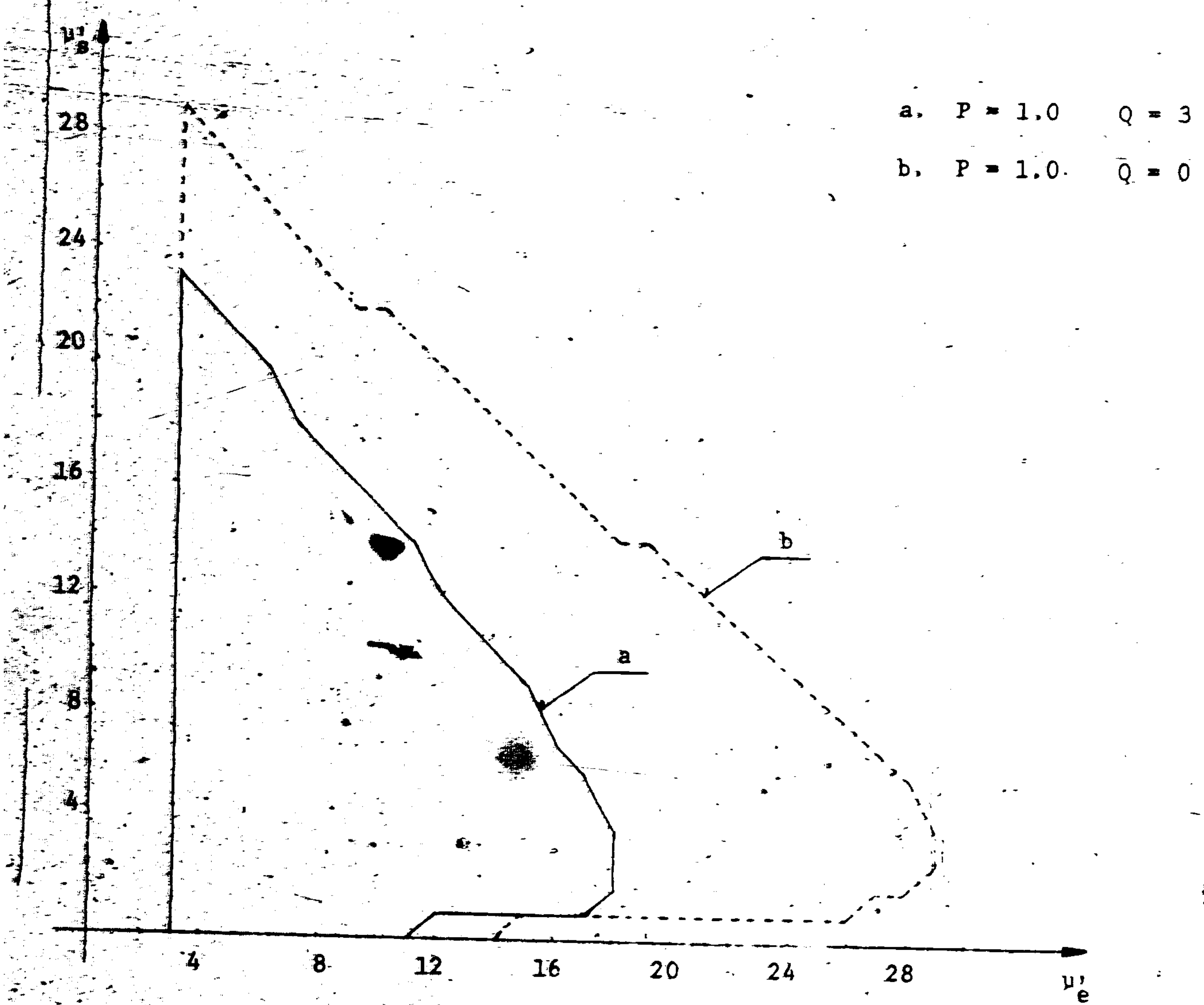
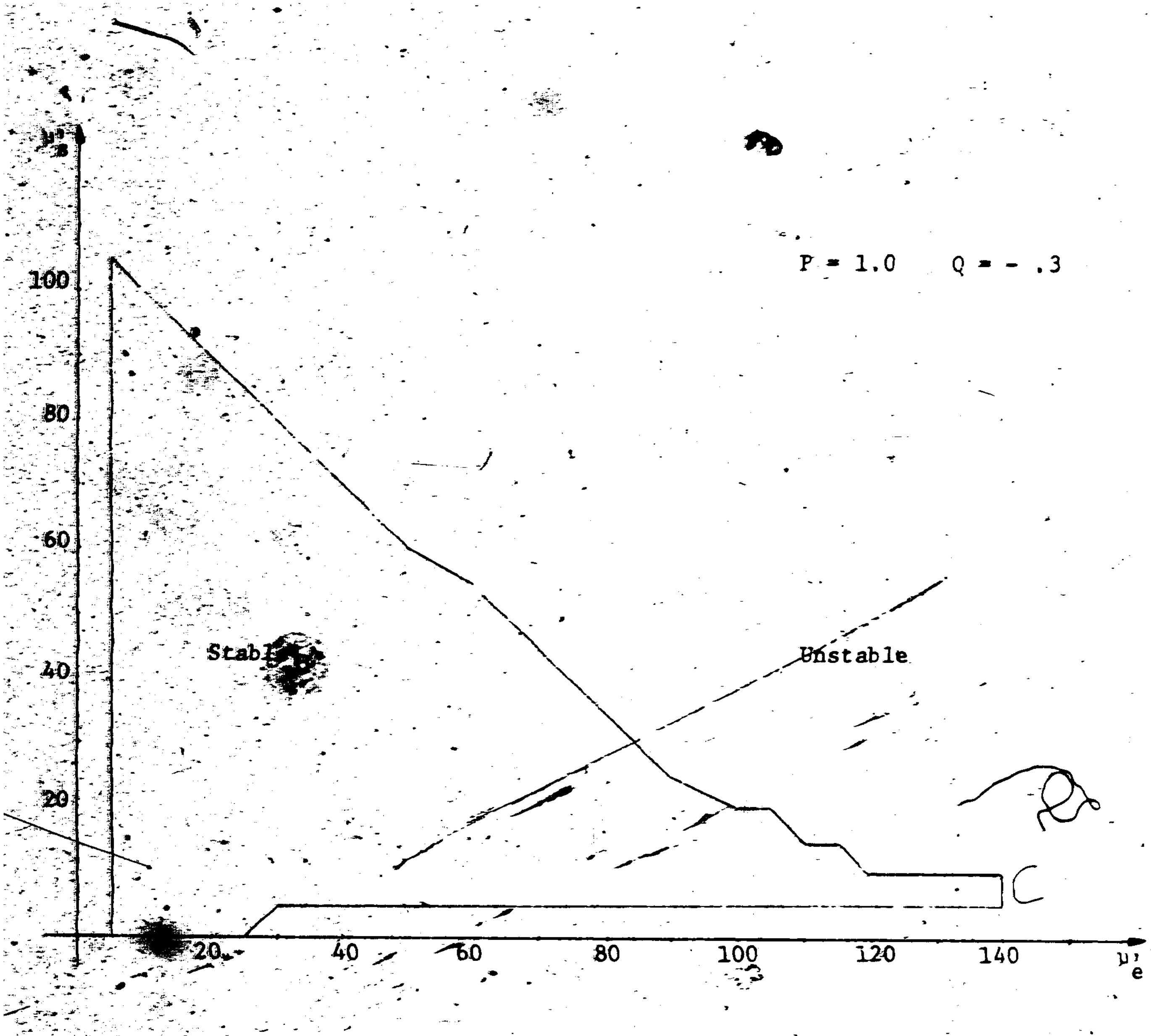


Figure 3b



μ'_s
100
80
60
40
20
20
40
60
80
100
120
140
μ'_e
P = 1.0 Q = - .3
Stable
Unstable

Figure 3c

5.3 Effect of Different Damping Coefficients on Stability Region

In this study the real and reactive power output of the synchronous machine is constant during the investigation at

$$P = 1.0 \qquad Q = .6$$

The damping coefficient D is varied and the effect of different damping coefficients on stability region studied. During the study these three distinct values for damping coefficient are assumed:

$$D = 1;\ 1.5;\ 2 \text{ p.u.}$$

All other data introduced by (25) are kept constant during the investigation.

The results of this study are graphically shown in Fig. 4, where the effect of different damping coefficients on stability region of the dynamic system under investigation can be seen.

From Fig. 4 it can be seen that the larger damping coefficient has a favourable effect on voltage regulator and stabilizer gain settings. As it is expected, the stability region is larger for larger damping coefficient and it allows larger maximum allowable gains. The study shows that there is a very distinctive change in both maximum allowable (voltage regulator and stabilizer) gain settings if the damping coefficient is changed.

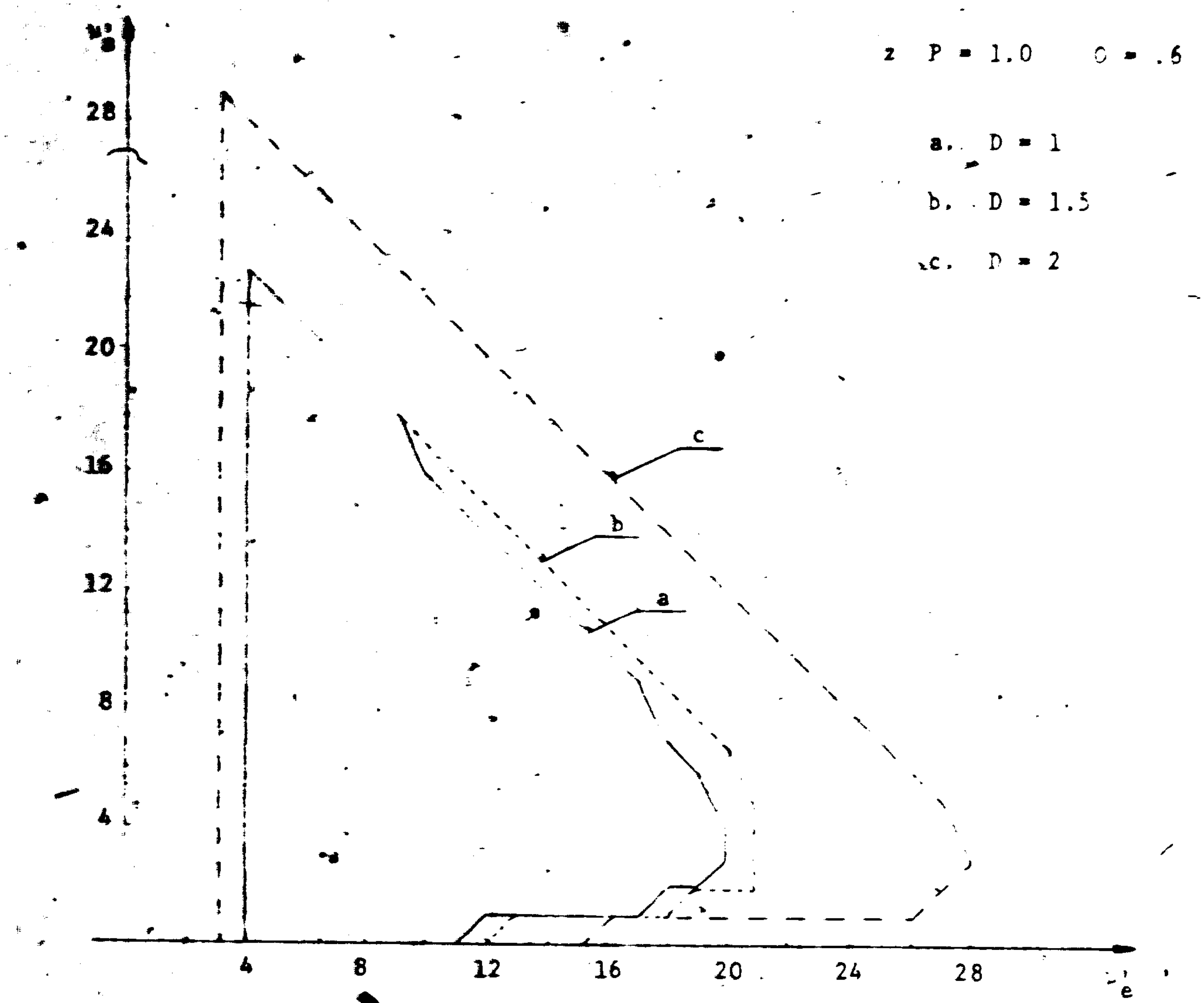


Figure 4

5.4 Stabilizer Time Constant Effects on the Stability Region

The real and reactive power output of the machine is constant at the following values:

$$P = 1.0 \qquad Q = 0$$

The stabilizer time constant is varied during the investigation and the effect of this variation on stability region of the dynamic system is studied. During this study three distinct values of the stabilizer time constant are used:

$$T_s = 1.5;\ 2.5;\ 3.5 \text{ (seconds)}$$

All other data used in this study are those introduced by (25) and they are kept constant during this investigation.

The results were processed graphically and they are shown in Fig. 5. In this figure the stabilizer time constant effect on the stability region of the dynamic system under investigation can be seen.

From Fig. 5 it can be seen that the minimum allowable voltage regulator gain is constant in all investigated cases. In the high μ_e' region, a larger stabilizer time constant allows a smaller voltage regulator gain at the low μ_s' portion. The maximum allowable stabiliser gain has the same value in all investigated cases.

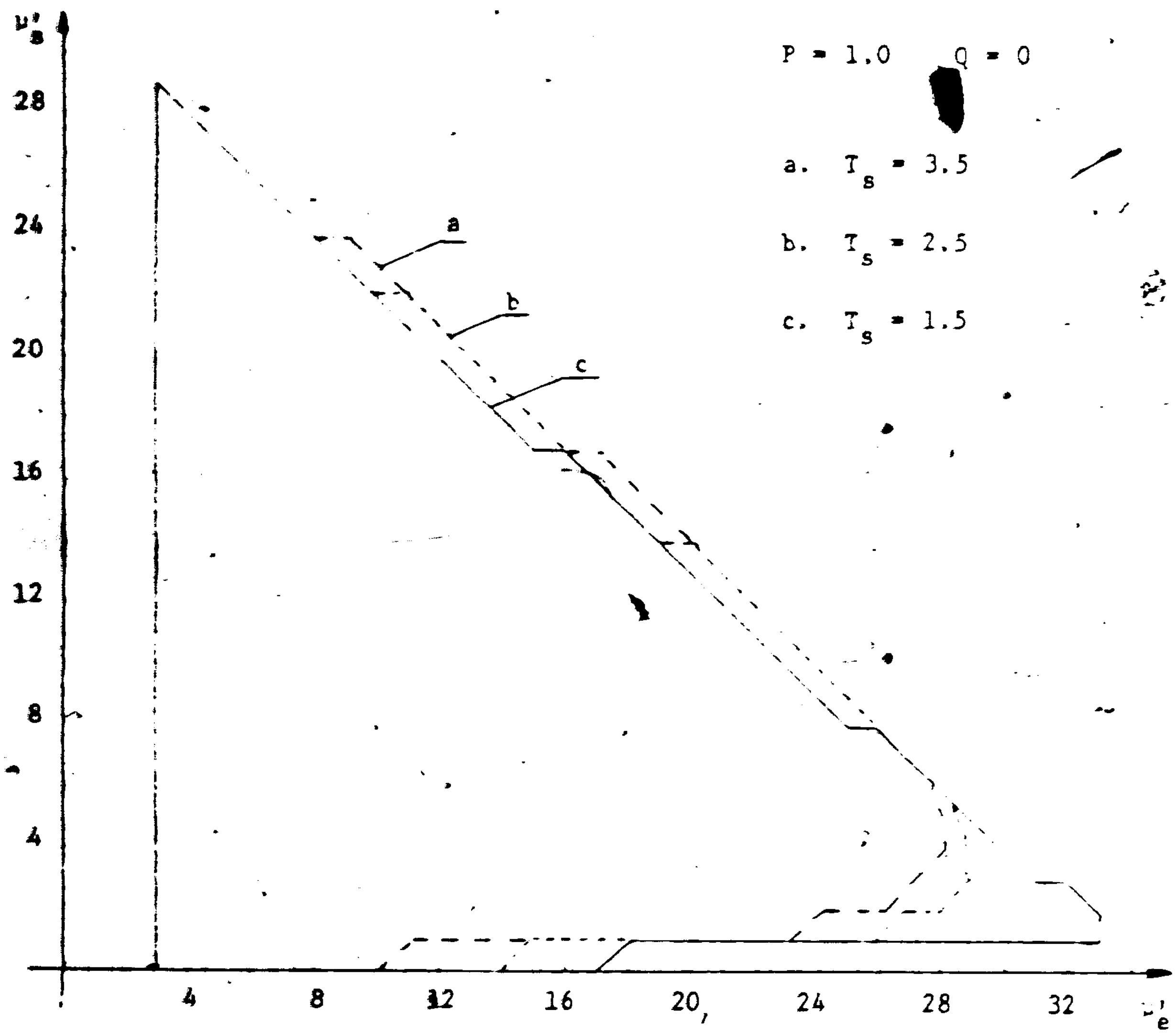

P = 1.0 Q = 0
a. T_s = 3.5
b. T_s = 2.5
c. T_s = 1.5
a
b
c
μ'_s
28
24
20
16
12
8
4
4
8
12
16
20
24
28
32
μ'_e

Figure 5

5.5 Exciter Time Constant Effect on the Stability Region

In this study the real and reactive power output of the machine is kept constant at

$$P = 1.0 \qquad Q = 0$$

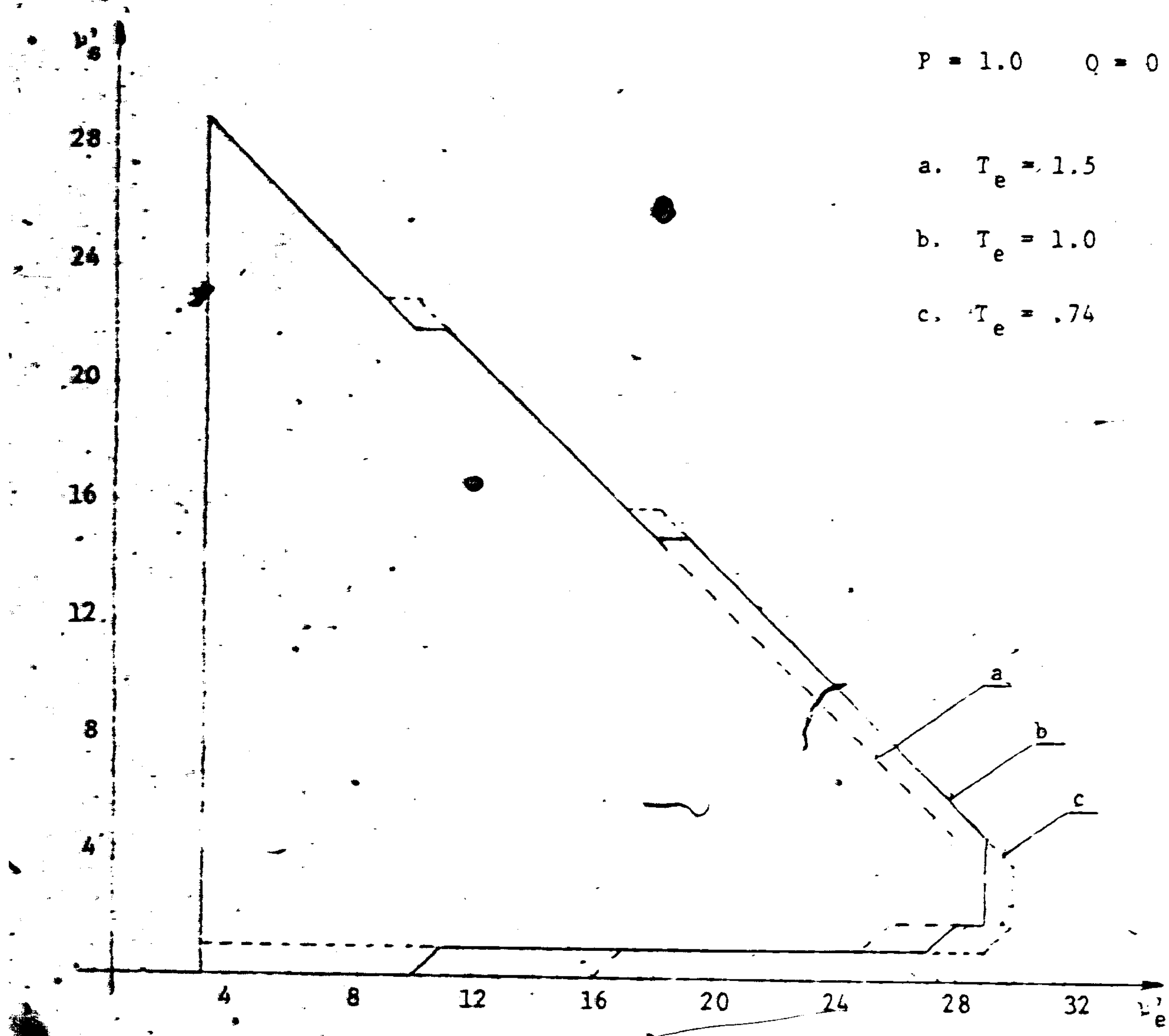
The exciter time constant is varied during this investigation and the effect of this variation on stability region of the investigated dynamic system is studied. The exciter time constant is given the following values:

$$T_e = .74;\ 1.0;\ 1.5 \quad \text{(seconds)}$$

During this investigation all additional data are supplied by (25) and they are kept constant.

The results are shown graphically in Fig. 6. In this figure the exciter time constant effect on the stability region of the dynamic system under study can be seen.

In general, the larger the exciter time constant the smaller the maximum allowable voltage regulator gain. The minimum voltage regulator gain shows constant value in all investigated cases and so does the maximum allowable stabilizer gain.



P = 1.0 Q = 0
a. T_e = 1.5
b. T_e = 1.0
c. T_e = .74
a
b
c
ν_s'
28
24
20
16
12
8
4
4
8
12
16
20
24
28
32
ν_e'

Figure 6

5.6 One-Time Constant Governor Effect on the Stability Region

As in the previous study, the real and reactive power output of the machine is kept constant at

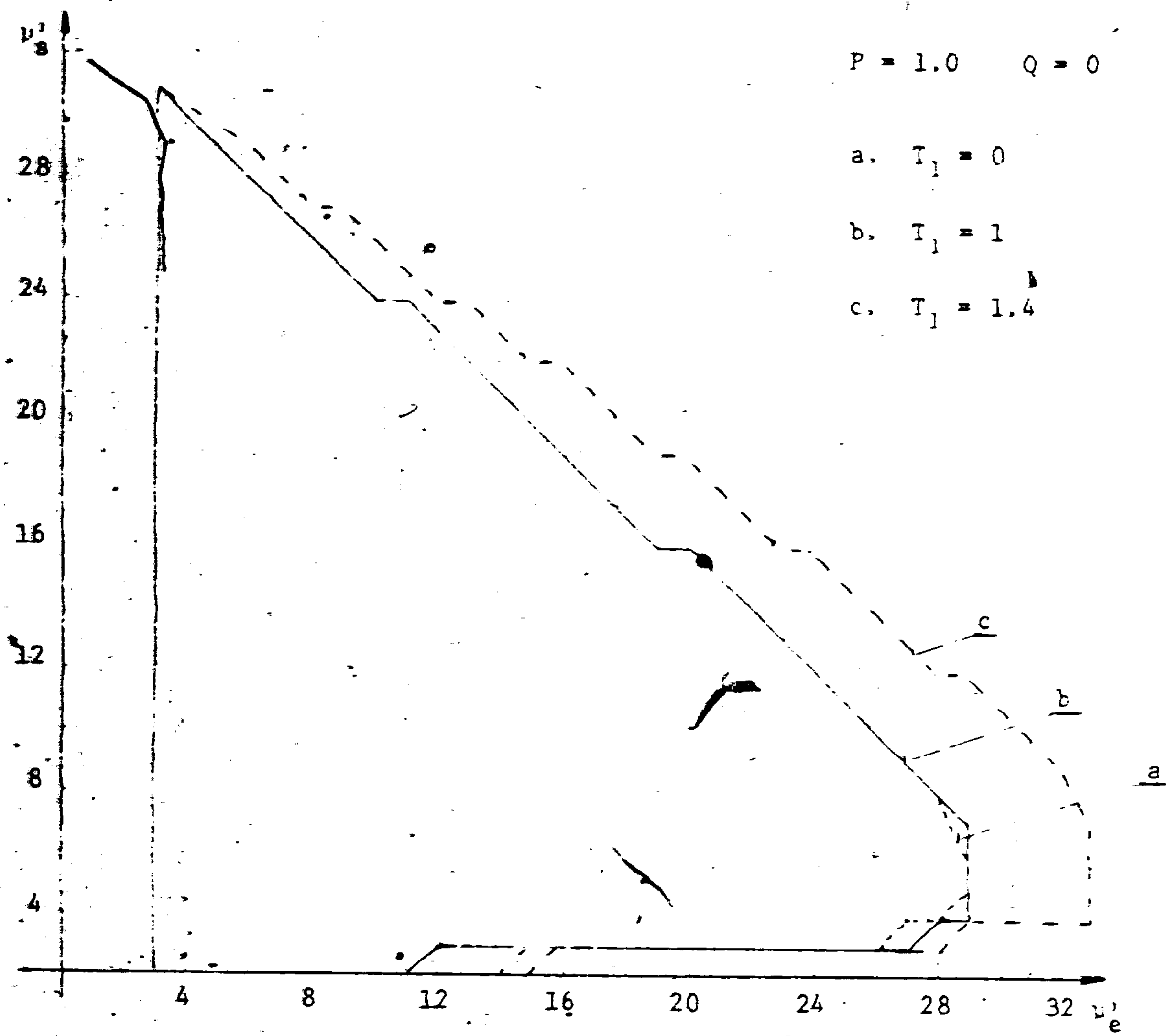
$$P = 1.0 \qquad Q = 0$$

The effect of a one-time constant governor is studied here and therefore this constant is varied in this study. The following three values are considered:

$$T_1 = 0;\ 1;\ 1.4 \quad \text{(seconds)}$$

Fig. 7 shows the one-time constant governor effect on the stability region of the dynamic system under study.

From Fig. 7 it can be seen that at low μ_s' portion of the high μ_e' region, a fast-acting governor allows only a smaller voltage regulator gain than a slow-acting governor. There is no difference between cases in the low region and the maximum allowable stabilizer gain shows constant value.



μ'_s
28
24
20
16
12
8
4
4
8
12
16
20
24
28
32
μ'_e
P = 1.0 Q = 0
a. T_1 = 0
b. T_1 = 1
c. T_1 = 1.4
a
b
c

Figure 7

5.7 Two-Time Constant Governor Effect on the Stability Region

The real and reactive power output of the machine is kept constant at these values:

$$P = 1.0 \qquad Q = 0$$

Two-time constant governor effect is studied here and therefore this constant is varied during this study. The following three distinct values were chosen for the purpose of the study:

$$T_2 = 0;\ 1;\ 2 \quad \text{(seconds)}$$

The results are shown in Fig. 8. This figure shows the two-time constant governor effect on the stability region for dynamic system under study. From Fig. 8 it can be seen that for a system with a two-time constant governor the effect of the values of the time constant on the voltage regulator gain settings is smaller than that of a system with a one-time constant governor. The minimum voltage regulator gain and the maximum allowable stabilizer gain is constant and there are only very small changes in the low μ_s' region.

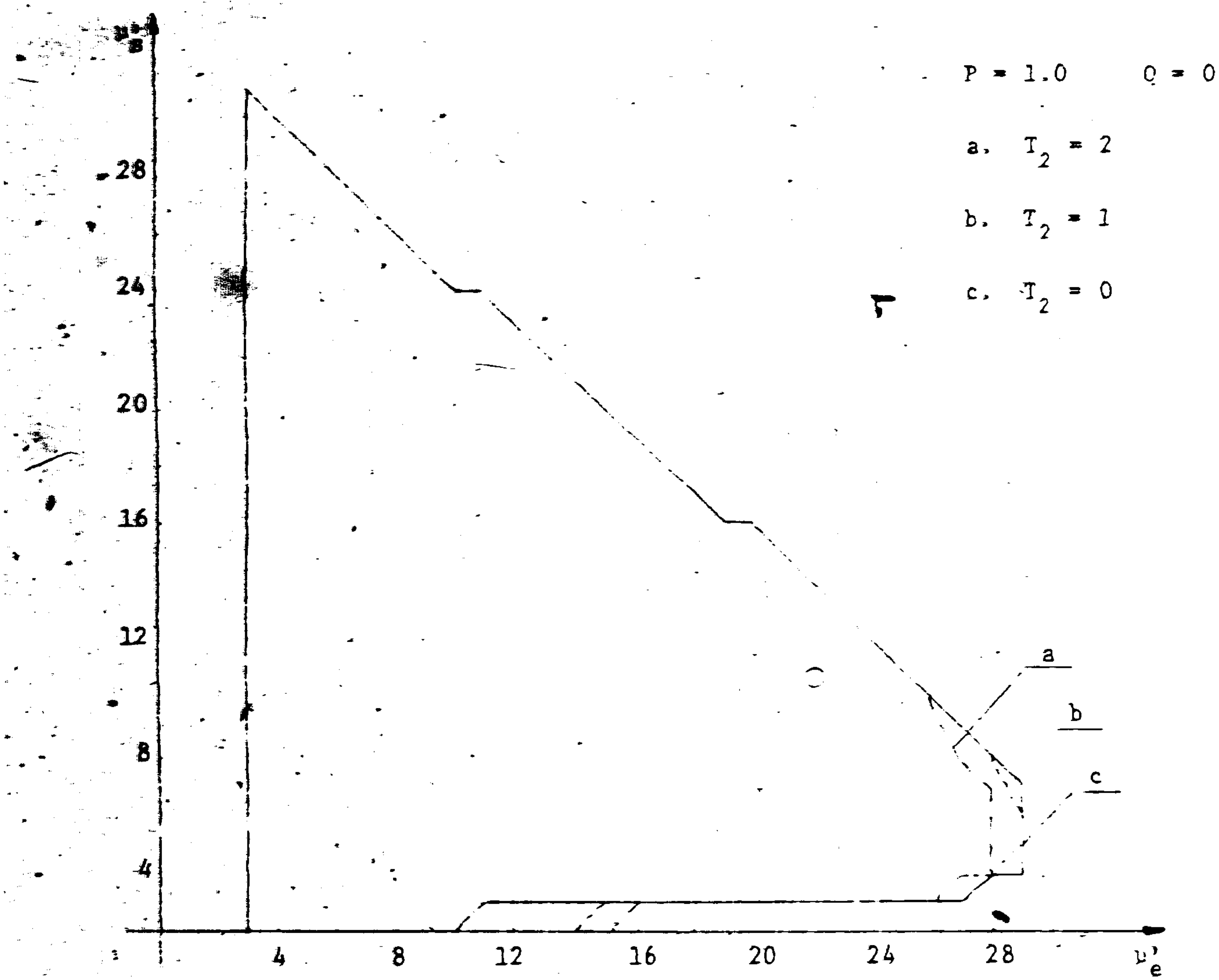


Figure 8

CHAPTER VI

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

6.1 Summary and Conclusions

The dynamic system studied in this thesis consists of a regulated synchronous machine connected to an infinite bus. Using the transformation of the system into Schwarz form (14) and the necessary and sufficient conditions of a dynamic system established in this thesis (24) the closed stability regions on μ_s' μ_e' plane were obtained. The method presented here is fairly simple and can be used for both designing and power system operational practice. The closed stability regions obtained here should be of some interest as no paper has been published so far on this subject.

The method of approach used in this thesis is quite general and gives the opportunity to study the effect of many parameters on the stability region of a dynamic system. Six parameters were chosen for this purpose in this thesis and the studies were conducted and introduced earlier in the text. The following general results are observed in these studies.

It can be observed that the general shape of the stability region for the system under study is triangular. This is an agreement with the operational power system practice, where for practical purposes smaller values of the overall stabilizer gain and higher values of the overall regulator gain are chosen. There seems to be a definite trend to stay in the lower part of the stability region in order to ensure good controllability of a power system. It can be verified by [3] or [4] where the voltage regulator data are introduced for the systems studied in those papers.

It can also be seen that in the studies presented in this thesis most of the changes in the stability regions are in the low μ'_s portion of the high μ'_e region. The minimum allowable voltage regulator gain shows fairly constant value in all presented studies.

Once the stability conditions for the system under study are obtained, the rest of the calculations (constants and coefficient of the characteristic polynomial) can be computerized. The study as presented here does not require special output or computer storage facilities.

Generally, all results and conclusions presented earlier are in agreement with [3]. In that study, however, open stability regions were obtained and the system under investigation in that paper had different parameters than the dynamic system in this study. Therefore, only a general comparison of these two studies is possible.

6.2 Suggestions for Further Research

The stability study of the individual unit is presented above and necessary and sufficient conditions are established, assuming the rest of the system as an infinite bus. These conditions can be compared with conditions for stability of the system with more than one generating unit in order to find any possible variations in both stability conditions and stable regions of the system.

For practical studies more information on the variation of system parameters would be useful and more effective results can be obtained. If the limits within which a parameter varies were known, some domain of attraction could be established for the investigated system. This would be the common area for all stability regions obtained by varying the system parameters within the assumed limits. This information can be of vital importance in operational practice as well as in design of power systems.

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APPENDIX I

The real power output of the machine is kept constant in all cases ($P = 1.0$ p.u.) and the reactive power output of the machine (Q) is varied, showing the following initial conditions:

	Q = .6	Q = [illegible]	Q = [illegible]	Q = - .3
i_{qo}	.38	[illegible]	.5154	.565
v_{do}	.628	.75	.8762	.96
v_{qo}	.777	.659	.9367	.20
i_{do}	.96	.955	.482	1.16
v_{fdo}	$.017\times10^{-4}$	1.25×10^{-4}	1.39×10^{-4}	2.12×10^{-4}
ψ_{do}	- .628	- .75	- .8762	- .96
ψ_{qo}	.777	.659	.482	.20
v_o	.85	.936	.995	1.04
δ_o	31^{o}	$59^{o}30'$	$72^{o}20'$	$91^{o}10'$

APPENDIX II

FORTRAN IV G PROGRAM

```
C PROGRAM TO FIND THE STABILITY REGIONS
C OVERALL STABILIZER GAIN IS X
C OVERALL EXCITATION GAIN IS Y
      DO 6 J=1,32
      DO 7 I=1,32
      X=(-3+I)
      Y=J
      WRITE(6,10)X,Y
C COEFFITIENTS OF CHARACTERISTIC POLYNOMIAL WILL FOLLOW
      A=.594
      B=(35.361+(.53*X))
      C=(123.648+(31.36*X)-(.597*Y))
      D=(345.607+(97.667*X)-(1.218*Y))
      E=(306.133+(101.4*X)-(2.187*Y))
      F=(163.277+(135.087*X)-(6.511*Y))
      G=(53.15-(1.975*X)-(1.675*Y))
      H=(.263*Y)-.79
C CONDITIONS FOR STABILITY WILL FOLLOW
      COND1=(B/A)
      WRITE(6,11)COND1
      COND2=(B*C)-(A*D)
      WRITE(6,11)COND2
      B1=((B*C-A*D)/(A*B))
      B2=((D*E-C*F)/(A*D))
      B3=((F*G-E*H)/(A*F))
      C1=((B1*(D/A))-(B2*(B/A)))/B1
      C2=((B2*(F/A))-(B3*(D/A)))/B2
      C3=(H/A)
      D1=((C1*B2)-(B1*C2))/C1
      D2=((C2*B3)-(B2*C3))/C2
      E1=((D1*C2)-(C1*D2))/D1
      E2=(H/A)
      COND3=((D/B)-(B2/B1))
      WRITE(6,11)COND3
      COND4=((B2/B1)-(C2/C1))
      WRITE(6,11)COND4
      COND5=((C2/C1)-(D2/D1))
      WRITE(6,11)COND5
      COND6=((D2/D1)-(E2/E1))
      WRITE(6,11)COND6
      COND7=(E2/E1)
      WRITE(6,11)COND7
   10 FORMAT(1H ,2E13.4)
   11 FORMAT(1H ,E14.7)
    7 CONTINUE
    6 CONTINUE
      STOP
      END
```