Articles

RUSSELL'S MARGINALIA IN HIS COPIES OF FREGE'S WORKS

Bernard Linsky
Philosophy / U. of Alberta
Edmonton, ab t6g 2e5
bernard.linsky@ualberta.ca

A transcription of Russell's marginal comments in his copies of Frege's works, from his readings of Frege in 1902. The greatest number are in the early sections of *Grundgesetze der Arithmetik*, Vol. 1, but there are also marginal comments in *Begriffsschrift*, *Grundlagen der Arithmetik*, "Über Formale Theorien der Arithmetik", "Über Begriff und Gegenstand", "Function und Begriff", "Kritische Beleuchtung einiger Punkte in E. Schroeders …" and two corrections of typographical errors in "Über Sinn und Bedeutung".

1. ORIGIN AND DESCRIPTION OF THE MARGINALIA

On 16 June 1902 Bertrand Russell wrote his famous letter to Gottlob Frege, announcing the paradox of the class of all classes that are not members of themselves. Russell reports in that letter: "For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work."[1] Russell had turned to a study of Frege's work after having

[1] "Letter to Frege", in Jean van Heijenoort, ed., *From Frege to Gödel* (Cambridge, Mass.: Harvard U. P., 1967), pp. 124–5; *SLBR*, 1: 245–6 (without the postscript).

russell: the Journal of Bertrand Russell Studies
The Bertrand Russell Research Centre, McMaster U.
n.s. 24 (summer 2004): 5–36
ISSN 0036-01631

completed the body of *The Principles of Mathematics*.[2] There he discovered not only that Frege had anticipated many of his ideas, but also that Frege's work was subject to the paradox. In the second paragraph of the letter, Russell writes, "I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library." Frege did indeed send offprints as requested, and Russell continued to study Frege's works carefully during 1902 before sending the last of the *Principles* to the publisher in December.[3] The final published version includes discussions of the paradox in Chapter X, "The Contradiction" and an Appendix A, "The Logical and Arithmetical Doctrines of Frege". This article presents evidence of that study of Frege by transcribing Russell's marginalia in his copies of Frege's works.

The Bertrand Russell Archives at McMaster University house Russell's library, including two bound collections of Frege's works.[4] One volume contains Frege's *Grundgesetze der Arithmetik*, originally published in two volumes, the first in 1893, the second, famously in press when Russell's letter arrived, in 1903. It contains an appendix discussing the paradox. Russell had the two volumes bound together, dating the first one "Oct. 1900".[5] An accompanying volume, titled "Pamphlets",

[2] Cambridge: Cambridge U. P., 1903; with new Introduction, London: Allen and Unwin, 1937. Russell's study of Frege and the chronology of events around the letter and the writing of *The Principles of Mathematics* are recounted by Gregory H. Moore in the introductory material to *Papers* 3: xxxvii–xxxix.

[3] Frege lists five papers that he has sent to Russell in his response of 22 June 1902; "A Critical Elucidation …", "On the Notation of Mr. Peano, etc.", "On Concept and Object", "On Sense and Meaning" and "On the Formal Theories of Arithmetic" (Gottlob Frege, *Philosophical and Mathematical Correspondence*, ed. G. Gabriel *et al.* and Brian McGuinness [Chicago: U. of Chicago P., 1980], p. 131). Russell inscribed the offprint on Peano's notation "B. Russell | übbereicht vom Verfasser".

[4] Russell's library also contains copies of *Die Grundlagen der Arithmetik* and *Begriffsschrift* that belonged to Ludwig Wittgenstein. In 1919 Russell purchased books and even some furniture that Wittgenstein left behind in Cambridge. *The Autobiography of Bertrand Russell* describes this transaction (2: 100). Wittgenstein's copies of Frege are beautifully bound in two matching volumes by the "Wiener Werk Stätte", a workshop in Vienna that made furniture as well as artistic bindings. There are no marginalia in either volume.

[5] He dated his copies of *Die Grundlagen der Arithmetik* "June 1902" and *Function and Begriff* "July 1902", respectively.

contains various other works of Frege, including *Begriffsschrift*, *Die Grundlagen der Arithmetik*, *Function und Begriff*, and offprints of several articles including "Über Begriff und Gegenstand" and "Über Sinn und Bedeutung". It seems reasonable to assume that the copy of the *Grundgesetze* is the one that Russell studied before writing his letter, and that several of the articles were those sent by Frege in reply to the letter.

What follows is a report of the comments that Russell wrote in the margins of those copies of Frege's works. In addition to comments, there are scores of passages that are highlighted with a vertical line along the edge of the text. Even in the papers, such as "Über Sinn und Bedeutung", in which the marginal comments are only to correct two typographical errors, there are numerous, now famous passages, which are marked with marginal lines. They occur on almost every page.

The marginal comments are the smallest part of Russell's notes on the works of Frege. The Russell Archives also possess over 80 pages of notes by Russell on all of these works. Extensive notes on the *Grundgesetze*, almost entirely consisting of reformulations of Frege's theorems and proofs in Russell's Peano-inspired notation, begin at page 74, exactly where the marginal comments give out. It appears that after annotating the early parts of the *Grundgesetze* Russell decided to make notes for the rest of his reading. Other comments from the marginalia appear in a series of numbered pages beginning with an outlined "Appendix on Frege". The notes summarize Frege's position in the *Grundgesetze* and other works in the "Pamphlets" volume. These are the notes on which Appendix A of the *Principles* was based. Many of the ideas indicated in the marginalia reappear in those notes and, in turn, in Appendix A.[6] The marginal comments thus seem to record the moment when Russell first encountered these points in Frege's thought that in turn figure in the *Principles* and through so much of Russell's later philosophy.

[6] The Russell Archives contain over 85 pages of various "Notes on Frege" in its document 230.030420, which have been identified and described by Moore, *Papers* 3: 692–3. Two pages of these are printed as Plate VIII, at page 39. One group of has been published as Appendix 1, "Frege on the Contradiction", in *Papers* 4: 607–19. Another group of 25 consecutively numbered pages are notes on the *Grundgesetze der Arithmetik*. The notes for the "Appendix on Frege" occupy eleven consecutively numbered pages of notes organized by topic and a further 23 pages organized by source.

II. SIGNIFICANCE OF THE MARGINALIA

Together the marginalia and notes show that Russell was a careful reader of Frege, primarily interested in understanding Frege's views and technical accomplishments, but also occasionally critical. They do not reveal the moment when Russell observed that the paradox would apply to Frege's system, nor indeed do they include any reference to the paradox. The marginalia in the *Grundgesetze* most likely precede the letter to Frege, as Russell notes that he has already made a study of Frege's writings. They may therefore precede Russell's conclusion that the paradox would affect Frege. Russell notes the important passages, and often comments on some which have been especially significant in the history of Frege scholarship. One gets the impression of Russell working out Frege's views as he read, for example, initially thinking that there is something wrong with Frege's notorious Axiom V, which identifies the course of values of coextensive concepts, then deleting the objection and citing the passage that resolves the difficulty. At other points he voices an objection that we appreciate all too well, having struggled with the very different views of Frege and Russell on some issues. (Thus the comment, "He does not realize that everything is a *Gegenstand* <object>", after Frege has painstakingly explained that a concept word cannot appear in the place where the name of an object can.)

The comments on the *Grundgesetze* begin in the introduction (p. xiii) where Russell approves ("Hear Hear!") of a statement of what we would call realism. At a footnote on the same page, he responds to a reported view that numbers are just physical signs with "Good God!" At page xviii an extended discussion of how Frege views the objective, abstract (or "*Nichtwirklich*") and subjective realms is met with "Splendid". Clearly in agreement with Frege's metaphysical and epistemological preliminaries, Russell sets to working out the technical views on logic and mathematics.

At *Grundgesetze*, §2 we have Russell asking "What is the *Sinn* of $\xi^2 = 4$? This is a most puzzling question." That the *Bedeutung*, or denotation, of a predicate is not its extension but rather an "unsaturated" function, took much work by Frege scholars before later textual finds resolved the issue. Russell wonders about the *Sinn* or sense of such expressions.

At *Gg*, §3 Russell begins a series of objections to Frege's exposition of the "*Werthverlauf*", or "course-of-values", of a function. Russell's prob-

lems arise because of Frege's view of concepts as functions from objects to truth values. For Frege, the extension of a concept is its course of values. Other functions, such as those expressed by "*sin x*" and "$x + 4$", also have a course of values. Russell seemed to think that Frege's Axiom V, which identifies the course of values of coextensive functions, in fact only made sense for the special case of concepts, not for arbitrary functions. Another comment, in which Russell suggests that *sin x* and *cos x* are a problem for the principle, suggests the view that a course of values is much like a representation or drawing of the graph of a function, which correlates arguments with values. Since the sine and cosine functions have graphs displaced by only 90 degrees along the *x* axis, on this view they are in some sense the same. Perhaps he thought briefly that the course of values of a function is its range, and *cos* and *sin* do have the same range. In any case the confusion seems to have been cleared up as he read further.[7]

Students of Frege will appreciate Russell's comment on the footnote to *Grundlagen*, §68 where Frege suggests that he could have defined numbers as concepts rather than extensions. At *Gl*, §79 where Frege presents his definition of the ancestral of a relation, Russell first objects, but then changes his mind. Is this his first encounter with the definition, or is he just puzzled by the somewhat unusual wording? Finally, at page 21 of "Function und Begriff" we find Russell remarking that Frege's notion of a supposition as opposed to an assertion ("$—a$" versus "$\vdash a$") reminds him of Meinong.[8]

Other comments have Russell correcting minor errors in Frege, in each case appropriately, and beginning the transition to the technical notes on the *Grundgesetze* by comparing Frege's definitions with his and rewriting definitions and proofs in his own notation. *Gg*, §31 is a much discussed section where Frege offers a proof that every sign in the language has a unique denotation. Both Frege and Russell's initial response to the paradox was to see it as proving that not every concept has a course of values. At the paragraph where the case of terms for a course of

[7] The ultimate conclusion of these worries is expressed in Appendix A, §484, and in the correspondence with Frege, in a letter dated 24 July 1902. See Frege, *Philosophical and Mathematical Correspondence*, pp. 138–9.

[8] This point appears in *PoM*, Appendix A, §477.

values is considered, Russell confesses: "I don't understand this paragraph".[9] How nice it is to think that Russell sensed a problem here.

III. TEXT OF THE MARGINALIA

Marginalia are reported opposite the passage by which they were written. Standard English translations are used, with some slight modifications of font, and the notation of German words added. Page numbers before the English translation passages report the original pagination. Some trivial changes of punctuation have been made to accord more with the German versions. Square brackets and question marks in the marginalia column are Russell's own. German words are italicized, with Russell's underlining retained. Editorial commentary is in angle brackets, including indications of lines that were struck through as deleted. Russell employed his standard manuscript abbreviations in composing his marginalia. Since their meaning is not doubtful, they have been expanded silently here. Page references are to the original editions, but they are included in the translated editions, and most passages are identified by a section number. Russell's notation, in particular the use of ∪ for disjunction, rather than his more standard ∨, helps to date the marginalia from after May of 1902.[10] Michael Byrd, however, suggests that some of the marginalia on the *Grundgesetze* may precede that date.[11] He suggests that Russell read initial portions, perhaps even up to where the marginalia give out, some time earlier, during the "year and a half" that Russell told Frege he had been acquainted with the *Grundgesetze*.

Some of the marginalia are in ink, with the editorial remark "<ink:>". This is further evidence of distinct readings by Russell.

Rather than attempt to recreate Frege's *Begriffsschrift* notation, I have provided formulas from Frege in a contemporary notation that is meant to suggest the original. Thus rather than the concavity to express universal quantifiers, ∀ is used, and with the Gothic letters that Frege used for bound variables. Negation is expressed with ¬ rather than Frege's nega-

[9] The objection is explained in *PoM*, Appendix A, §484.

[10] See Moore, *Papers* 3: xlv, for a discussion of changes of notation in this period.

[11] Michael Byrd, "Part II of *The Principles of Mathematics*", *Russell*, n.s. 7 (1987): 60–70. Byrd remarks on several of these marginal comments, as well as on Wittgenstein's copies of Frege's books.

tion stroke, and the complex arrangement of strokes indicating a conditional is replaced by $\rightarrow$ with complex consequents placed in square brackets. Russell's zigzag arrangement of strokes that Peano printed as a dagger on its right side in "The Logic of Relations" (1901) is also replaced by $\rightarrow$. Special signs in Frege's theory of one–one relations and number are replaced by new symbols, meant to suggest the original. Thus a superscript -1 indicates the converse of a relation, $\#u$ the number of u. The symbol $\llcorner$ should be longer. Of Russell's marginal lines, only the double ones are represented here, by an initial ||.[12]

Passages from Frege	**Marginalia**
Grundgesetze der Arithmetik[13] ("*Gg*")	
Gg, p. [v]	<in pencil, in top right-hand corner:> 1δ1
Gg, p. xiii Just as the geographer does not create a sea when he draws boundary lines and says: the part of the ocean's surface bounded by these lines I am going to call the Yellow Sea, so too the mathematician cannot really create anything by his defining.	Hear Hear!

[12] This transcription was drafted on the basis of photocopies from the books in Russell's library. This article was checked against the originals by the editor, who inserted the indication "<ink:>" where appropriate and corrected several transcription errors.

[13] The original is G. Frege, *Grundgesetze der Arithmetik. Begriffsschriftlich abgeleitet*, Vol. 1 (Jena: Hermann Pohle, 1893); Vol. 2 (Jena: Hermann Pohle, 1903); reprinted with the same pagination by Georg Olms (Hildesheim, 1962). The English translation is from Frege, *Basic Laws of Arithmetic: Exposition of the System*, trans. and ed. Montgomery Furth (Berkeley and Los Angeles: U. of California P., 1967).

Passages from Frege	Marginalia
Gg, p. xiii, fn. 1 Cf. E. Heinem, *Die Elemente der Functionslehre*, in Crelle's *Journal*, vol. 74, p. 173: "As for definition, I adopt the purely formalistic standpoint; what I call numbers are certain tangible signs, so that the existence of these numbers is thus unquestionable."	<ink:> Good God!
Gg, p. xiv But that an oval figure produced on paper with ink should by a definition acquire the property of yielding one when added to one, I can only regard as a scientific superstition. One could just as well by a pure definition make a lazy pupil diligent.	Excellent!
Gg, p. xv <In one sense a law asserts what is; in the other it prescribes what ought to be.> Only in the latter sense can the laws of logic be called "laws of thought": so far as they stipulate the way in which one ought to think. Any law asserting what is, can be conceived as prescribing that one ought to think in conformity with it, and is thus in that sense a law of thought.	Good
Gg, p. xviii We can generalize this still further: for me there is a domain of what is objective, which is distinct from that of what is actual, whereas the psychological logicians without ado take what is not actual <*Nichtwirklichen*> to be subjective. <…> Whatever ideas there may be of the number one in individual souls, they are still to be as carefully distinguished from the number one, as ideas of the moon are to be distinguished from the moon itself.	Splendid

Passages from Frege	Marginalia
Gg, p. xx "In general it is either the ideated or the idea, for the two are one and the same: what is ideated is the idea, and the idea is what is ideated."	!!
Gg, p. xxiv–xxv Now let us see how for the psychological logicians, more delicate distinctions within the subject matter are blotted out. For the case of characteristic mark and property this has already been mentioned; a related case is the distinction between object and concept, which I stress, and also that between concepts of first and second level. Naturally these distinctions are indiscernible to psychological logicians; for them everything is just idea. With this goes their wrong conception of those judgments that in everyday language we express by using "there is". This existence Herr Erdmann jumbles up with actuality (Vol. 1, p. 311), which, as we saw, also is not clearly distinguished from objectivity. Of what thing are we really asserting that it is actual if we say that there are square roots of four? Is it 2 or −2? But neither the one nor the other is named here in any way at all. And if I wished to say that the number 2 acts or is active or actual, this would be false and wholly different from what I mean by the sentence, "There are square roots of four." The confusion before us is just about the grossest possible; for it is not between concepts of the same level, but rather between a concept of first level and a concept of second level.	<ink:> Important

Passages from Frege	Marginalia
Gg, "*Einleitung*", p. 2 In fact what Dedekind really means when he calls a system part of a system (p. 2) is the subordination of a concept under a concept or an object's falling under a concept: cases that he distinguishes no better than Schröder, owing to an error of conception shared by them both; for Schröder too at bottom regards the elements as what constitute his *class*.	Peano's distinction of $\supset$ and ϵ
Gg, p. 2 Dedekind continues the above passage: "On the other hand, for certain reasons we will here wholly exclude the empty system, which contains no element, although for other investigations it can prove convenient to invent such a system."	<ink:> No null-class in extension
Gg, p. 3 In this way, then, it will finally be acknowledged that a statement of number contains an assertion about a concept.	Number essentially Num *a*
Gg, §2 I further say a name *expresses* <*drücke*> its sense and *denotes* <*bedeute*> its denotation. I *designate* <*bezeichne*> with the name that which it denotes <*bedeutet*>. Thus the function $\xi^2 = 4$ can have only two values, namely the True for the arguments 2 and —2, and the False for all other arguments.	What is the <u>*Sinn*</u> of $\xi^2 = 4$? This is a most puzzling question.

Passages from Frege	Marginalia
Gg, §3 I use the words "the function $\Phi(\xi)$ has the same *course-of-values* <*Werthverlauf*> as the function $\Psi(\xi)$" generally to denote the same as <*gleichbedeutend mit*> the words "the functions $\Phi(\xi)$ and $\Psi(\xi)$ have always the same value for the same argument".	<ink:> Df [bad, if other than propositional functions included]
Gg, §3 With such functions, whose value is always a truth-value, one may accordingly say, instead of "course-of-values of the function", rather "extension of the concept" <*Umfang des Begriffes*>, and it seems appropriate to call directly a *concept* <*Begriff*> a function whose value is always a truth-value.	? *Begriff* = $a \ni (x \in a = \text{true} \,.\vee.\, x \in a = \text{false})$ Df This seems a bad Df, if this is his meaning.
Gg, §5 We have already said that in a mere equation there is as yet no assertion; "2 + 3 = 5" only designates a truth-value, without its being said which of the two it is. Again, if I wrote "(2 + 3 = 5) = (2 = 2)" and presupposed that we knew 2 = 2 to be the True, I still should not have asserted thereby that the sum of 2 and 3 is 5; rather I should only have designated the truth-value of *"2 + 3 = 5"'s denoting the same as "2 = 2"*. We therefore require another special sign to be able to assert something as true. For this purpose I let the sign "⊢" precede the name of the truth-value, so that for example in "$\vdash 2^2 = 4$", it is asserted that the square of 2 is 4. I distinguish the *judgment* from the *thought* in this way: by a *judgment* I understand the acknowedgement of the truth of a *thought*.	In grammar, actual assertion is distinguished by the indicative verb, the mere proposition, apart from its assertion, being best expressed by a verbal noun.

Passages from Frege	Marginalia
Gg, §5 "—$2^2 = 5$" denotes the False, thus the same thing as does "$2^2 = 5$"; as against, this, "—2" denotes the False, thus something different from the number 2.	Thus negation should mean the falsehood of identity.
Gg, §5 Of the two signs of which "⊢" is composed, only the judgment-stroke contains the act of assertion.	Assertion is thus something new, over and above all the concepts in the asserted proposition and over and above truth and falsehood. This is obviously correct: if *P* is a proposition, "the truth of *P*" is not the same as "*P* is true".
Gg, §7 "$\Gamma = \Delta$" shall denote the True if Γ is the same as Δ; in all other cases it shall denote the False.	|| All equality is identity
Gg, §8 <…> asserts: *there is* at least one solution of the equation "$2 + 3x = 5x$". In the same way, $\neg \forall \mathfrak{a} \neg \mathfrak{a}^2 = 1$; in words: *there is* at least one square root of 1.	<inked out over pencil:> ~~It says more: it says there are concepts not satisfying the equation. "Some, not all" is its meaning.~~

Passages from Frege	Marginalia
Gg, §8 Accordingly we might incline toward these functions as corresponding: $(\xi + \xi = 2 \cdot \xi) = (\forall \mathfrak{a}\ \xi + \mathfrak{a})$, $(\xi + \xi = 2 \cdot \xi) = (\forall \mathfrak{a}\ \mathfrak{a} + \xi)$, $(\xi + \xi = 2 \cdot \xi) = (\forall \mathfrak{a}\ \mathfrak{a} + \mathfrak{a})$; but against the first two notions is the fact that the denotation of "$\forall \mathfrak{a}\ \mathfrak{a} = \mathfrak{a}$" occurring in "$\forall \mathfrak{a}\ [(\mathfrak{a} + \mathfrak{a} = 2 \cdot \mathfrak{a}) = (\forall \mathfrak{a}\ \mathfrak{a} + \mathfrak{a})]$" is already established, and may not be called back into question.	<ink:> = <second "+" deleted and replaced by "="[14]> = < second "+" deleted and replaced by "="> = <second "+" deleted and replaced by "="> = <second "+" deleted and replaced by "=">
Gg, §9 If $\forall \mathfrak{a}\ \Phi(\mathfrak{a}) = \Psi(\mathfrak{a})$ is the True, then by our earlier stipulation (§3) we can also say that the function $\Phi(\xi)$ has the same course-of-values as the function $\Psi(\xi)$;....	He seems to overlook that not only are the classes of values the same, but equal values are correlated. <ink:> [But cf. p. 7][15]
Gg, §9 .... and indicated then that negative, irrational, in short all numbers were to be defined as extensions of concepts.	how?
Gg, §9 Similarly, $\grave{\epsilon}(\epsilon^2 = 4)$ is the course-of-values of the function $\xi^2 = 4$....	$\grave{\epsilon}(\epsilon^2 = 4) = x\ \ni\ (x^2 = 4)$

[14] Furth makes these changes as well.

[15] The reference to p. 7 is to the definition at the beginning of §3 where Russell remarks, also in ink, "Df [bad, if other than propositional functions included]".

Passages from Frege	Marginalia
Gg, §10 Although we have laid it down that the combination of signs "$\dot{\epsilon}\,\Phi(\epsilon) = \dot{\alpha}\,\Psi(\alpha)$" has the same denotation as "$\forall\mathfrak{a}\ \Phi(\mathfrak{a}) = \Psi(\mathfrak{a})$", this by no means fixes completely the denotation of a name like "$\dot{\epsilon}\,\Phi(\epsilon)$".	<ink:> mistake: e.g. sin *x* and cos *x* But cf. Df p. 7, which justifies F. <Frege>[16] <ink:> This whole § is very difficult. Observe that *Begriff* = propositional function: thus (p. 19) $\xi + 3$ is not a *Begriff*.
Gg, §11 1. If to the argument there corresponds an object Δ such that the argument is $\dot{\epsilon}\,(\Delta = \epsilon)$, then let the value of the function $\backslash\xi$ be Δ itself; 2. if to the argument there does not correspond an object Δ such that the argument is $\dot{\epsilon}\,(\Delta = \epsilon)$, then let the value of the function be the argument itself.	<ink:> Df
Gg, §11 We have here a substitute for the definite article of ordinary language, which serves to form proper names out of concept words.	corresponding to ι <inverted iota, R's description symbol>

[16] Again the definition at the beginning of §3.

Passages from Frege	**Marginalia**
Gg, §12 … I introduce the function of two arguments $\zeta \rightarrow \xi$ by stipulating that its value shall be the False if the True be taken as the ζ-argument and any object other than the True be taken as ξ-argument, and that in all other cases the value of the function shall be the True.	Odd Df It amounts to this, that the proposition is true when ξ is a necessary condition of ζ, whether sufficient or not. [See e.g. top of p. 24 <§13>] In short, $\zeta \supset \xi$ approximately<.> Strictly, it means $\xi \cup \sim\zeta$ [which $\supset \zeta \supseteq \xi$] <BR wrote "$\supseteq$" above a deleted $\supset$.>
Gg, §12 … if the function $\neg\zeta \rightarrow \xi$ has as value always the True if the function $\zeta \rightarrow \xi$ has as value the <u>True</u>, and conversely.	*Falsche* <False, with "?" sign before it deleted>[17] <Russell's underlining>
Gg, §12 … in words: 2 is not greater than 3 *and* the sum of 2 and 3 is 5.	$\sim P \cap Q$
Gg, §12 … in words: 3 is greater than 2 *and* the sum of 2 and 3 is 5.	$P \cap Q$
Gg, §12 … in words: *neither* is the third power of 2 the second power of 3, *nor* is the second power of 1 the first power of 2.	$\sim P \cap \sim Q$
Gg, §12 … of the following two at least one is true: either that the square of 1 is greater than 3, *or* that 1 is smaller than 3.	$P \cup Q$

[17] This error is corrected by Frege in the "Berichtigungen" on p. [xvi] of the second volume of *Grundgesetze*.

Passages from Frege	Marginalia
Gg, §12 … $\Delta \rightarrow [\Lambda \rightarrow \Theta]]$ is the same truth-value as $\Lambda \rightarrow [\Delta \rightarrow \Theta]]$	$\Delta \,.\, \supset \,.\, \Lambda \supset \Theta$ $= \Delta\, \Lambda \supset \Theta$
Gg, §12 … $\Xi \rightarrow [\Delta \rightarrow [\Lambda \rightarrow \Theta]]$ is the False if and only if both Λ and Δ and Ξ are the True while Θ is not the True;….	<deleted in ink over pencil:> $\Theta \,.\, \supset : \Lambda \,.\, \supset \,.\, \Delta \supset \Xi :.$ $= . \, \Theta\, \Lambda\, \Delta \supset \Xi$
Gg, §13 *if* the square of something is 1, *then* its fourth power also is 1. But one can also say: *every* square root of 1 is also fourth root of 1; or: *all* square roots of 1 are fourth roots of 1. Here we have the *subordination* <*Unterordnung*> of a concept under a concept, a *universal* <*allgemein*> affirmative proposition.	$a \supset b \,. = : x \in a \,. \supset_x . \, x \in b$
Gg, §14 From the propositions "$\vdash \Delta \rightarrow \Gamma$" and "$\vdash \Delta$" we may infer "$\vdash \Gamma$"; ….	This is approximately $a, b \in P \,.\, a \,.\, a \supset b \,.\, \supset \,.\, b$
Gg, §15 From the two propositions "$\vdash \Delta \rightarrow \Gamma$" and "$\vdash \Theta \rightarrow \Delta$" we may infer the proposition "$\vdash \Theta \rightarrow \Gamma$".	Transitiveness of his relation.
Gg, §15 A subcomponent may be inter-changed with the main component if the truth-value of each is simultaneously *reversed*.	$\xi \cup \sim \xi =$ $\sim \xi \cup \xi$ $= \sim (\zeta \sim \xi)$ $= \sim \zeta \cup \sim \sim (\xi)$ $= (\sim \zeta) \cup \sim (\sim \xi)$

Passages from Frege	Marginalia
Gg, §17 Thus it remains permissible to extend such a scope over several propositions, and this renders the Roman letters suitable to do duty in inferences, which the Gothic letters, with the strict closure of their scopes, cannot. If we have the premisses "$\vdash x^2 = 1 \rightarrow x^8 = 1$" and "$\vdash x^2 = 1 \rightarrow x^4 = 1$" and infer the proposition "$\vdash x^4 = 1 \rightarrow x^8 = 1$", in making the transition we extend the scope of the "x" over both of the premisses and the conclusion, in order to perform the inference, although each of these propositions still holds good apart from this extension.	Important <Russell corrects the exponents of x in the antecedents of the first and last conditionals to 4 and 2, respectively.[18]>
Gg, §18 … in "$\vdash a \rightarrow a$" we have a particular case of (I), which will be understood together with (I) without explicit notice.	$a + \overline{a} = \dot{2}$
Gg, §21 We commonly speak here of a function of a function, but inaccurately; for if we recall that functions are fundamentally different from objects, and further that the value of a function for an argument is to be distinguished from the function itself, then we see that a function-name can never occupy the place of a proper name, because it carries with it empty places that answer to the unsaturatedness of the function.	He does not realize that everything is a *Gegenstand* <object>.

[18] Furth corrects the exponents differently, from $x^2 = 1 \rightarrow x^4 = 1$ and $x^4 = 1 \rightarrow x^8 = 1$ and concluding $x^2 = 1 \rightarrow x^8 = 1$.

Passages from Frege	Marginalia
Gg, §21 Thus another function can never occur as argument of the function $X(\xi)$, though indeed the value of function for an argument can do so: e.g., $\Phi(2)$, in which case the value is $X(\Phi(2))$.	He is right in this instance but functions of relations are logically possible.
Gg, §21 If we say, "the function $\sim\forall\mathfrak{a} \sim \phi(\mathfrak{a})$", then "$\phi$" is a proxy <*vertritt*> for the sign of an argument, just as "ξ" in the expression "the function $\xi^2 = 4$" is a proxy for a proper name that could appear as sign of an argument. "ϕ" in our present case is not to be assigned to the function any more than "ξ" in the previous case.	Important
Gg, §21 We now call those functions whose arguments are objects *first-level functions* <*Functionen erster Stufe*>; on the other hand, those functions whose arguments are first-level functions may be called *second-level functions* <*Functionen zweiter Stufe*>. The value of our function $\sim\forall\mathfrak{a} \sim \phi(\mathfrak{a})$ is always a truth-value, whatever first-level function we may take as argument. To conform with earlier nomenclature, we shall accordingly call it a concept: namely a *second-level concept*, to distinguish it from *first-level concepts* which are first-level functions.	Df
Gg, §22 In $\neg[\forall\mathfrak{a}\,[\phi(\mathfrak{a}) \rightarrow \mathfrak{a} = 2] \rightarrow \neg\phi(2)]$ we also have a second-level concept, which we could call *property of the number 2 that belongs to it exclusively*.	$\exists\phi\ni\{\phi a \,.\, \supset_a \,.\, a = 2 : \phi 2\}$

Passages from Frege	Marginalia
Gg, §26 If from a proper name we remove a proper name that forms a part of it or coincides with it, at some or all of the places where the constituent name occurs—but in such a way that these places remain recognizable as capable of being filled by one and the same arbitrary proper name (i.e., as being *argument places of type 1* <*Argumentstellen erster Art*>), then I call that which we obtain by this means a *name* <*Namen*> of a first-level function of one argument. Such a name, combined with a proper name filling the argument-place, forms a proper name.	Df of function (or rather explanation)
Gg, §31 The matter is less simple with "$\grave{\epsilon}\phi(\epsilon)$"; for with this we are introducing not merely a new function-name, but simultaneously answering to every name of a first-level function of one argument, a new proper name (course-of-values-name) <*Werthverlaufsnamen*>; in fact not just for those known already, but in advance for all such that may be introduced in the future.	I don't understand this paragraph
Gg, §31 By our stipulations, that "$\grave{\epsilon}\,\Psi(\epsilon) = \grave{\epsilon}\,\Phi(\epsilon)$" is always to have the same denotation as "$\forall\mathfrak{a}\,[\Psi(\mathfrak{a}) = \Phi(\mathfrak{a})]$", that "$\grave{\epsilon}\,(—\epsilon)$" is to denote the True, and that "$\grave{\epsilon}\,[\epsilon = \neg\forall\mathfrak{a}\,(\mathfrak{a} = \mathfrak{a})]$" is to denote the False, a denotation is assured in every case for a proper name of the form "$\Gamma = \Delta$" if "Γ" and "Δ" are fair <*rechte*> course-of-values-names or names of truth values.	Yes, but not when one is one and the other the other: if $\xi = x \ni \phi x$, we have only determined —ξ for the case $\phi x = \Lambda . \cup . \sim \phi x = \Lambda$

Passages from Frege	Marginalia
Gg, §33 3. The name defined must be simple; that is, it may not be composed of any familiar names or names that are yet to be defined; for otherwise it would remain in doubt whether the definitions of the names were consistent with one another.	~~?~~
Gg, §37 $\vdash (\forall \mathfrak{e}\, \forall \mathfrak{d}\, [\mathfrak{e} \cap (\mathfrak{d} \cap p) \rightarrow$ $[\forall \mathfrak{a}\, \mathfrak{e} \cap (\mathfrak{a} \cap p) \rightarrow \mathfrak{d} = \mathfrak{a}]]) = \mathrm{I}p$[19]	Df of Nc→1
Gg, §38 The definition of the latter offered in §72 (p. 85) of my *Grundlagen* is: The expression "The concept *F* is equinumerate with the concept *G*" is to mean the same as [*gleich-bedeutend mit*] the expression "there exists a relation ϕ that correlates one to one the objects falling under the concept *F* with the objects falling under the concept *G*".	Num
Gg, §45 … then we say "Θ follows in the Y-series after Δ". Accordingly $\forall \mathfrak{F}\, \forall \mathfrak{d}\, \mathfrak{F}(\mathfrak{d}) \rightarrow [\forall \mathfrak{a}\, \mathfrak{d} \cap (\mathfrak{a} \cap Y) \rightarrow \mathfrak{F}(\mathfrak{a})]$ $\rightarrow [[\forall \mathfrak{a}\, \Delta \cap (\mathfrak{a} \cap Y) \rightarrow \mathfrak{F}(\mathfrak{a})] \rightarrow \mathfrak{F}(\Theta)]$ is the truth value of Θ's following after Δ in the Y-series.[20]	$b \in \mathrm{seq}_R\, a$. $= :: \phi x$. xRy . $\supset_{x,y} \phi y$: aRy . $\supset_y$. $\phi y : \supset_\phi$. ϕb Df <bottom of page:> $Y_\Delta \supset F_\xi$:. $x \in F_\xi$. $\supset_x$. $Y_x \supset F_\xi : \supset_\xi$. $\Theta \in F_\xi$:. $\supset_\Theta \Theta = \mathrm{seq}\, \Delta_Y$ [Bad Df]

[19] "$\mathfrak{e} \cap (\mathfrak{d} \cup p)$" means $\mathfrak{e}$ bears relation p to $\mathfrak{d}$; "Ip" means p is a many-one relation, or function.

[20] I do not follow Furth in using a "T" in place of Frege and Russell's Greek capital upsilon.

Passages from Frege	**Marginalia**
Gg, §47 Summary of the Basic Laws <*Grundgesetze*> $\vdash a \rightarrow a$ (I (§18) <propositions not reproduced here>	 Excluded Middle Pp's <Primitive propositions>
Gg, §48 2. *Interchange of subcomponents.* The subcomponents of the same proposition may be interchanged with one another as desired.	~~This is the associative law of addition~~
Gg, §48 3. *Contraposition* <*Wendung*>. A subcomponent may be interchanged in a proposition with the main component if the truth-value of each is simultaneously reversed.	contraposition
Gg, §48 4. *Amalgamation of identical subcomponents.* A subcomponent occurring more than once in the same proposition need be written only once.	Law of tautology
Gg, §48 6. *Inference (a).* If a subcomponent of a proposition differs from another proposition only in lacking the judgment-stroke, then a proposition may be inferred that results from the first proposition by suppressing that subcomponent.	$x, y \in P$. ~~y . $x \cup \breve{y}$. $\supset$. x~~ x . $x \supset y$. $\supset$. y [Ass]

Passages from Frege	Marginalia
Gg, §48 7. *Inference (b)* If the same combination of signs (proper name <*Eigenname*> or Roman object-mark <*Gegenstandsmarke*>) appears in one proposition as main component and in another as subcomponent, a proposition may be inferred in which the main component of the second is main component, and all subcomponents of either, save the one mentioned, are subcomponents.	$a \supset b \,.\, b \supset c \,.\supset.\, a \supset c$
Gg, §48 8. *Inference (c)* If two propositions agree in their main components, while a subcomponent of one differs from a subcomponent of the other only in a negation-stroke's being prefixed, then a proposition may be inferred in which the common main component is main component, and all subcomponents of either, save the two mentioned, are subcomponents.	$a \cup b\bar{\alpha} \,.\, a \cup b \cup \alpha \supset a \cup b$ ——— $p \supset q \,.\, {\sim}p \supset q \,.\supset.\, q$
Gg, §53 A) Proof of the proposition $v \cap (u \cap \rangle\, {}^{-1}q) \to [\, u \cap (v \cap \rangle\, q) \to \#u = \#v]$ <${}^{-1}q$ is the converse of the relation q. "$u \cap (v \cap \rangle\, q)$" means that q maps u into v.>	$u, v \in \mathrm{Cls}$ ~~.⊃ : Nc'u = Nc'v.~~ <A ⊃ . deleted in the beginning of the next line:> $\exists\, 1{\to}1 \cap R \ni (\rho = u \,.\, \breve{\rho} = \sigma) \,.\supset.\, \mathrm{Nc}\, u = \mathrm{Nc}\, v$
Gg, §54 (δ $v \cap (u \cap \rangle\, {}^{-1}q) \to$ $[\, u \cap (v \cap \rangle\, q) \to$ $[\neg[\forall \mathfrak{q}\ v \cap (w \cap \rangle\, {}^{-1}\mathfrak{q}) \to \neg\, w \cap (v \cap \rangle\, \mathfrak{q})]$ $\to [\neg[\forall \mathfrak{q}\ v \cap (w \cap \rangle\, {}^{-1}\mathfrak{q}) \to \neg\, w \cap (u \cap \rangle\, \mathfrak{q})]]]]$	$v = u \cup .$ $w \sim = v \,.\cup.$ $u \sim = v$

Passages from Frege	Marginalia
Gg, §54 (ϵ $v \cap (u \cap \backepsilon {}^{-1}q) \rightarrow$ $[u \cap (v \cap \backepsilon q) \rightarrow$ $[\neg[(\forall \mathfrak{q}\ \ u \cap (w \cap \backepsilon {}^{-1}\mathfrak{q}) \rightarrow \neg w \cap (u \cap \backepsilon \mathfrak{q})] \rightarrow$ $[\neg[\forall \mathfrak{q}\ v \cap (w \cap \backepsilon {}^{-1}\mathfrak{q}) \rightarrow \neg w \cap (v \cap \backepsilon \mathfrak{q})]]]]$	$w = v \,.\, \cup \,.$ $w \sim = u \,.\, \cup \,.$ $u \sim = v$
Gg, §54 $u \cap (v \cap \backepsilon q) \rightarrow u \cap (v \cap \backepsilon {}^{-1-1}q)$ (η	$R \in 1 \rightarrow 1 \,.\, \supset \,.\, \breve{\breve{\rho}} = \rho$
Gg, §54 (χ $v \cap (u \cap \backepsilon {}^{-1}q) \rightarrow$ $[u \cap (v \cap \backepsilon q) \rightarrow$ $[w \cap (u \cap \backepsilon p) \rightarrow$ $[u \cap (w \cap \backepsilon {}^{-1}p) \rightarrow$ $[\neg[\forall \mathfrak{q}\ \ v \cap (w \cap \backepsilon {}^{-1}\mathfrak{q}) \rightarrow \neg w \cap (v \cap \backepsilon \mathfrak{q})]]]]]$	$w = v \,.\, \cup \,.\, u \sim = w \,.$ $\cup \,.\, u \sim = v$ $u = v \,.\, u = w \,.\, \supset \,.\, v = w$[21]
Gg, §54 $\vdash \dot{\alpha}\, \dot{\epsilon} (\neg [\forall \mathfrak{r}\ \mathfrak{r} \cap (\alpha \cap q) \rightarrow \neg \epsilon \cap (\mathfrak{r} \cap p)])$ $= p \backepsilon q$ (B <Def of $p \backepsilon q$>	Relative X^n <relative product>
Gg, §54 $\vdash {}^{-1}(p \backepsilon q) = {}^{-1}p \backepsilon {}^{-1}q$ (ν	$(\breve{P}\breve{Q}) = \breve{P}\breve{Q}$
Gg, §54 $(\neg [\forall \mathfrak{r}(\mathfrak{r} \cap (m \cap q) \rightarrow \neg d \cap (\mathfrak{r} \cap p))]) =$ $d \cap (m \cap (p \backepsilon q))$ (τ	This is my Df

[21] As Frege's conditionals are read from bottom to top, so should these marginalia; $u = v \,.\, u = w \supset \,.\, v = w$ is the first antecedent, $u \,.\, \sim = v$ the second, and $w = v \,.\, \cup \,.\, u \sim = w$ the consequent.

Passages from Frege	Marginalia
Gg, §87 <top of p. 113 *bis*>	<beneath a slanted line:> Already occurred <Russell refers to the binding fault of pp. 113–20 being duplicated in his copy.>
Gg, §108, ll. 13–19 <W>hen we assign a number to the concept $\Phi(\xi)$, or as one usually says, when we count the objects falling under the concept $\Phi(\xi)$, we arrange the numerals beginning with one until we reach a numeral '*N*' in order, so that it is thereby determined that the coordinating relation maps the concept $\Phi(\xi)$ onto the concept "member of the series of numerals from 'one' to '*N*'" and that the converse relation maps the latter onto the former.	Counting
Gg, Vol. 2, §150, l. 3 The first <mistake> comes from confusing the number with its bearer, like confusing colour or <*oder*> thing coloured <*pigmentum*>.	*mit* <replacing "*oder*">
Gg, Vol. 2, §164, p. 161, ll. 18–19 Set aside for a moment our knowledge of the irrational numbers!	x
Gg, Vol. 2, §217	Df of $\downarrow$'*s*, p. 171. 7th line is $\breve{P}\,\vert Q \in \eth$'*s* <Russell refers to *Gg*, 2: 171, the 7th line of the definition '$\downarrow s$ at the bottom of the page.>

Passages from Frege	Marginalia
<"Pamphlets" volume:> *Begriffsschrift* [22]	
§5 It is not less easy to see that $[B \to A] \to \Gamma$ denies the case in which B is affirmed but A and Γ are denied.	<ink:> There is an inconsistency here. The proposition should mean $B \supset A . \supset . \Gamma$, but not $\sim B \cup A \cup \Gamma$.[23]
§9 *The Function* Let us suppose that there is expressed in our formalized language the circumstance of hydrogen's being lighter than carbon dioxide. <...>	<ink:> Important §.
§9 Conversely, the argument may be determinate and the function indeterminate.	<ink:> ?
§10 *we may thus regard* $\Phi(A)$ *as a function of the argument* Φ.	<ink:> ?

[22] The original is *Begriffsschrift, Eine der Arithmetischen Nachgebildete Formelsprache des Reinen Denkens* (Halle: Louis Nebert, 1879). The English translation by Stephan Bauer-Mengelberg is in van Heijenoort, pp. 1–82.

[23] Van Heijenoort reports in *From Frege to Gödel* that "There is an oversight here, already pointed out by Schröder (1880, p. 88)". Russell reports in his letter to Frege of 24 June 1902, a response to Frege's letter of 22 June, that he had "already corrected" the mistake (Frege, *Correspondence*, p. 133). This marginal note, then, precedes 24 June 1902.

Passages from Frege	Marginalia
§18 ¬ ¬ *a* means the denial of the denial, hence the affirmation of *a*. Thus *a* cannot be denied and (at the same time) ¬ ¬ *a* affirmed. *Duplex negatio affirmat*. The denial of the denial is affirmation.	(31) & (41) [preface] together give ⊢ (¬ ¬ *a* ≡ *a*) <¬ ¬ *p* follows *Begriffsschrift* notation.>
Grundlagen der Arithmetik[24] ("*Gl*") *Gl*, p. vii Often it is only after immense intellectual effort, which may have continued over centuries<....>	x
Gl, §26 It is in this way that I understand objective to mean what is independent of our sensation, intuition and imagination, and of all construction of mental pictures out of memories of earlier sensations, but not what is independent of reason,—for what are things independent of the reason? To answer that would be as much to judge without judging, or to wash the fur without wetting it.	Mistaken
Gl, §44 Abstract number, then, is *the empty form of difference.*	Jevons

[24] The original is *Die Grundlagen der Arithmetik, Eine logisch mathematische Untersuchung über den Begriff der Zahl* (Breslau: Verlag von Wilhelm Koebner, 1884). The English translation is from *The Foundations of Arithmetic: a Logico-Mathematical Enquiry into the Concept of Number*, trans. J. L. Austin (Oxford: Basil Blackwell, 1974).

Passages from Frege	Marginalia
Gl, §55 It is tempting to define 0 by saying that the number 0 belongs to a concept if no object falls under it. But this seems to amount to replacing 0 by "no", which means the same. The following formulation is therefore preferable: the number 0 belongs to a concept, if the proposition that *a* does not fall under the concept is true universally, whatever *a* may be. Similarly we could say: the number 1 belongs to a concept *F*, if the proposition that *a* does not fall under *F* is not true universally, whatever *a* may be, and if from the propositions "*a* falls under *F*" and "*b* falls under *F*" it follows universally that *a* and *b* are the same.	Dfs of 0 and 1. not adequate.
Gl, §62 Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs.	?
Gl, §68 the direction of the line *a* is the extension of the concept "parallel to line *a*"; the shape of triangle *t* is the extension of the concept "similar to triangle *t*".	|| Principle of abstraction
Gl, §68 My definition is therefore as follows: the number which belongs to the concept *F* is the extension <footnote> of the concept "equal to the concept *F*"	Df of Nc'*u*

Passages from Frege	Marginalia
Gl, §68 <footnote to the passage above> I believe that for "extension of the concept": we could write simply "concept". But this would be open to the two objections: 1. that this contradicts my earlier statement that the individual numbers are objects, as is indicated by the use of the definite article in expressions like "the number two" 2. that concepts can have identical extensions without themselves coinciding. I am, as it happens, convinced that both these objections can be met<….>	2nd objection is fatal to my mind.
Gl, §70 For example, it would scarcely be possible to put the proposition "the Earth is bigger than the Moon" into other words so as to make "the Earth and the Moon" appear as a composite subject; the "and" must always indicate that the two things are being put in some way on a level. However, this does not affect the issue.	You overlook sense
Gl, §72 <…> this correlation has to be one-one. By this I understand the two following propositions both hold good: 1. If d stands in the relation ϕ to a, and if d stands in the relation ϕ to e, then generally, whatever d, a and e may be, a is the same as e. 2. If d stands in the relation ϕ to a, and if b stands in the relation ϕ to a, then generally, whatever d, b and a may be, d is the same as b.	Df of 1→1

Passages from Frege	Marginalia
Gl, §79 The proposition "If every object to which x stands in the relation ϕ falls under the concept F, and if from the proposition that d falls under the concept F it follows universally, whatever d may be, that every object to which d stands in the relation ϕ falls under the concept F, then y falls under the concept F, whatever concept F may be" is to mean the same as "y follows in the ϕ-series after x"<....>	$xR^{N}y \,.=:\breve{\rho}x \subset s \,.\, \breve{\rho}(s) \supset_s . \, y \in s$ Df [~~There is something wrong with this Df~~] <The "$\breve{\rho}(s)$" replaces a deleted "$\breve{\rho}$".>
Gl, §89 In calling the truths of geometry synthetic and a priori, he revealed their true nature.	?
"Über formale Theorien der Arithmetik"[25] p. 4, l. 30 Kunsterzeugniss bestehend,	<Russell inserted a comma after the first word and deleted one after the second.>
"Über Begriff und Gegenstand"[26] p. 200 I do not want to say it is false to say concerning an object what is said here concerning a concept; I want to say it is impossible, senseless, to say so.	Important

[25] *Sitzungsberichte der Jenaischen Gesellschaft für Medizin und Naturwissenschaft*, 19 (1885): Suppl. 2, pp. 94–104. Translated as "On Formal Theories of Arithmetic" by E.-H. W. Kluge, in Frege, *Collected Papers on Mathematics, Logic, and Philosophy*, pp. 112–21.

[26] The original is "Über Begriff und Gegenstand", *Vierteljahrsschrift für wissenschaftliche Philosophie*, 16 (1892): 192–205. English translation by P. Geach as "On Concept and Object", in Frege, *Collected Papers on Mathematics, Logic, and Philosophy*, ed. Brian McGuinness (Oxford: Basil Blackwell, 1984), pp. 182–94.

Passages from Frege	Marginalia
p. 203, l. 23	d <correcting "*hie*">
Function und Begriff[27] p. 6 From this we may discern that it is the common element of these expressions that contains the essential peculiarity of a function, i.e. what is present in "$2.x^3 + x$" over and above the letter "x". We could write this somewhat as follows: "$2.(\ )^3 + (\ )$"	This might stand for $2x^3 + y$
p. 16 A statement contains (or at least purports to contain) a thought as its sense; and this thought is in general true or false; i.e. it has in general a truth value, which must be <*ebenso*> regarded as what the sentence means, just as (say) the number 4 is what the expression "2 + 2" means or London is what the expression "the capital of England" means.	? <Russell's underlining>
p. 18 Here I can only say briefly: An object is anything that is not a function, so that an expression for it does not contain any empty place.	|| This is not correct, for predicates etc. seem to be neither.

[27] The original is *Function und Begriff* (Jena: Hermann Pohle, 1891). English translation by P. Geach as "Function and Concept" in Frege, *Collected Papers on Mathematics, Logic, and Philosophy*, pp. 137–56.

Passages from Frege	**Marginalia**
p. 21 According to the view I am here presenting, "5 > 4" and "1 + 3 = 5" just give us expressions for truth-values, without making any assertion. This separation of the act from the subject matter of judgment seems to be indispensible; for otherwise we could not express a mere <u>supposition</u> <*Annahme*> <....>	Meinong! <Russell's underlining>
"Über Sinn und Bedeutung" [28] p. 27 Comprehensive knowledge of the thing meant <*der Bedeutung*> would require us to be able to say immediately whether any given sense attaches to it. To such knowledge we never attain <"*gelanges*">.	<"*gelanges*" emended to "*gelangen*">
p. 37 Let us compare, for instance, the two sentences "Copernicus believed that the planetary orbits are circles" and Copernicus believed that the apparent motion of the Sun is produced by the real motion of the Earth"<....>	<Russell inserts " before the second occurrence of "Copernicus".>
"Kritische Beleuchtung einiger Punkte" [29] p. 444, l. 15	<The doublet "*eine*" is deleted.>

[28] The original is "Über Sinn und Bedeutung", *Zeitschrift für Philosophie und philosophische Kritik*, 100 (1892): 25–50. English translation by Max Black as "On Sense and Meaning", in Frege, *Collected Papers on Mathematics, Logic, and Philosophy*, pp. 157–77.

[29] The original is "Kritische Beleuchtung einiger Punkte in E. Schröders Vorlesungen über die Algebra der Logik", *Archiv für Systematische Philosophie*, 1 (1895) 433–56. English translation by Max Black as "A Critical Elucidation of Some Points in E. Schröder, *Vorlesungen über die Algebra der Logik* [Lectures on the Algebra of Logic]", in Frege, *Collected Papers on Mathematics, Logic, and Philosophy*, pp. 210–28.

Passages from Frege	Marginalia
p. 444 In the discussion set forth above we may take *P* to be itself likewise a class comprising a number of individuals; for, as the author says (p. 148), such a class can be presented as an object of thought and consequently as an individual. Now if *Q*, as before, is the class of objects that coincide with *P*, then *Q* is a singular class containing only *P* as an individual. Now if it were right to hold that a singular class coincides with the only individual it contains, then *Q* would coincide with *P*. Let us now suppose that *a* and *b* are different objects, contained within *P* as individuals; then they would also be contained within *Q*; i.e. both *a* and *b* would coincide with *P*. Consequently *a* would also coincide with *b*, contrary to our permissible supposition that they are different.	This is quite precise. But it only proves that the class as one differs from the class as many. Cf my pp. 75–6.
p. 451, l. 4 *v subter b*,	<o is substituted for *b*.[30]>
p. 451, l. 27 es gibt keine *a*, oder:	<*b* is substituted for *a*.>

[30] The objects of this comment and that at p. 444, l. 15, are identified in the translation as misprints.