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Fostering a Conceptual Understanding Approach to Teaching the Arithmetical Operations

by

Matthew Clements

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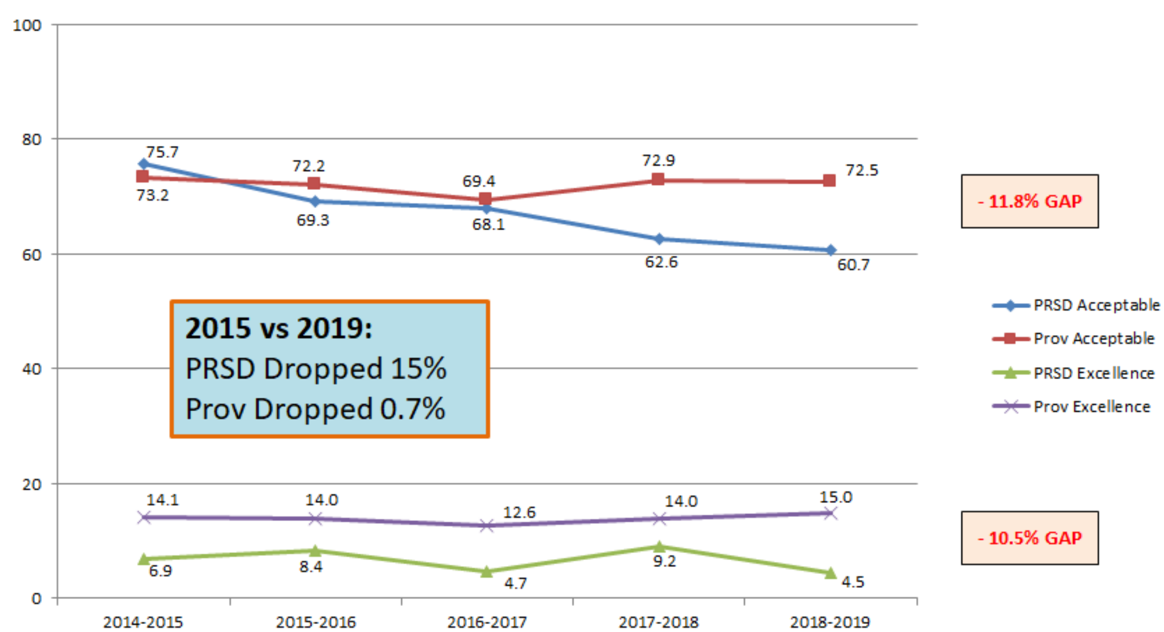
The disappointing Grade 6 Provincial Achievement Test (PAT) scores for Peace River School Division (PRSD) for the 2018-2019 year (the most recent scores available due to COVID-19 interruptions) marked the fourth consecutive year the divisional scores had declined. As shown in Figure 1, only 60.7% of PRSD Gr. 6 students achieved the acceptable standard on the PAT in 2019, compared to 75.7% in the spring of 2015. While the portion of students achieving the excellence standard fluctuated during the same timeframe, the trend remained low and approximately 10% below than the provincial average.

Figure 1

Grade 6 Provincial Achievement Test Scores For Mathematics



Note. Peace River School Division versus provincial Gr. 6 mathematics Provincial Achievement Test scores, 2014-2019. Data from Alberta Education (2019).

Part A of the Grade 6 PAT measures students' ability to perform the four arithmetic operations (addition, subtraction, multiplication, and division) on whole numbers and decimal numbers without a calculator. As seen in Figure 2, PRSD scores in this section — as with the overall scores in Figure 1 — were concerningly low. While the division does not have data tracking student performance with the arithmetic operations in grades previous to Grade 6, conversations with teachers, report card data, classroom visits, and observing students indicate the deficits seen in Table 1 are part of a trend that begins in the primary classrooms.

Table 1

Grade 6 Provincial Achievement Scores on Arithmetical Operations, 2019

			Below provincial average	**Above provincial average**	**At provincial average**
		Part A: Operations			
School	Number of Students	**Addition**	**Subtraction**	**Multiplication**	**Division**
PROVINCE		80%	63%	73%	68%
School 1	4	77%	70%	83%	75%
School 2	53	60%	45%	50%	48%
School 3	31	70%	30%	58%	38%
School 4	16	73%	33%	43%	55%
School 5	3	57%	25%	50%	43%
School 6	14	93%	75%	90%	83%
School 7	7	97%	75%	68%	53%
School 8	2	83%	88%	88%	75%
School 9	6	40%	25%	40%	40%
School 10	32	67%	43%	60%	54%
School 11	8	80%	58%	60%	58%

Note. Peace River School Division Gr. 6 mathematics Provincial Achievement Test Part A scores from spring of 2019. Data from Alberta Education (2019).

From my own experience of some 16 years teaching mathematics to primary, intermediate, and junior high school students, there is a strong correlation between students' familiarity with the arithmetic operations and their success across the mathematics curriculum — students with a poor grasp of the foundational operations tend to struggle throughout the rest of the mathematics curriculum as increasingly higher-level and complex concepts assume and require fluency with the operations. A crucial step in raising PRSD students' scores, then, is improving the instruction (and thus student learning) of the four operations.

Principals can play a key influential role in the improvement of operation instruction in their schools. As detailed by Cranston (2018), recent trends in education have "emphasized the development of instructional leadership skills to promote good teaching and high levels of student achievement" (p. 3). The leading-the-learning role of a principal is referenced throughout the Alberta Government's (2020) Leadership Quality Standard (LQS). The second leadership quality standard of modeling commitment to professional development can be achieved, in part, by: modeling professional learning; collaborating with teachers to build school-wide capacities and expertise; using educational research to augment pedagogical practice; and "seeking, critically reviewing and applying educational research" (Alberta Government, 2020, p. 4). The fourth competency of leading a learning community can be fulfilled, in part, by fostering a school-wide responsibility for student success, and "creating meaningful, collaborative learning opportunities for teachers and support staff" (Alberta Government, 2018, p. 4).

Perhaps most important in the context of boosting student mathematics scores is the leadership quality standard competency of providing instructional leadership. This competency involves establishing instructional excellence through: enhancing teachers' ability to respond to students' needs; ensuring instruction addresses the appropriate curricular outcomes; and “demonstrating a strong understanding of effective pedagogy and curriculum” (Alberta Government, 2018, p. 4). The importance of instructional leadership was further emphasized by the Peace River School Division (2019) in its Three-Year Education Plan for 2019-2022, which listed analysis of PAT results and formation of divisional and school-based PLCs focused on the development of best instructional practices as division-wide strategies to ensure its students are academically successful. Principals experiencing difficulty in finding the time or expertise to deliver this professional development may consider the research of Sprott (2019), who found allowing teachers to visit “different educational spaces [i.e. classrooms] not only helped them gain new teaching strategies, but moving to different physical contexts also provided new lenses through which to view their own practice” (p. 324). Establishing instructional excellence in one or a few classes initially, then, may serve as a key strategy to spreading the best practices described hereafter throughout a school.

In summary, in order to empower teachers with the knowledge and expertise they require to deliver effective mathematics instruction, school leaders will need to provide quality professional development opportunities in mathematics (Bender & Crane, 2011). Instructional leadership theory will inform and aid efforts to align mathematics teachers’ instruction of the arithmetic operations with strategies that promote conceptual understanding rather than rote memorization of abstract procedures.

Rationale for a Conceptual Understanding Approach

Conceptual understanding (CU) can be established through a concrete-representative-abstract (CRA) instructional sequence that builds deeper understanding beyond the superficial, and far too arbitrary for young learners, standard algorithms most veteran teachers would have learned when they attended elementary mathematics classes as the sole and superlative methodology for approaching operations. As stated by Hattie (2003), expert teachers employ a deeper representation of concepts, allowing them to “spontaneously relate what is happening to these deeper sets of principles ... detect and concentrate more on information that has instructional significance ... able to predict and determine what types of errors students might make … [and] can be much more responsive to students" (p. 5).

The CRA approach to teaching arithmetic operations attempts to address the lack of CU resulting from a purely procedural, or abstract, approach. “This lack of understanding goes beyond operations and written algorithms and causes difficulties related to concepts such as fractions, ratios, and proportions,” according to Milton, Flores, Moore, Taylor, & Burton (2019, p. 32). Rather than simply learning to follow, by rote, a standard algorithm, students “must be actively involved in reasoning and discussion ... with problems that are accessible to students through manipulation of objects, pictures, and other representations” (Milton et al., 2019, p. 33).

In CRA sequence, then, students first learn operations through manipulation of concrete objects, before using pictures or drawings to represent the concrete manipulations and their own understandings (Milton et al., 2019). Only when students are fluent with the concrete and representative approaches should teachers approach the operation with an abstract, or symbolic, algorithm. In this sequence, students build on a conceptual understanding of the operation itself, rather than relying solely on a disconnected abstract procedure (Milton et al., 2019). In short, it is important that students establish a conceptual understanding before the teacher introduces an abstract-symbolic procedure (Miller, Harris, Strawser, Jones, & Mercer, 1998).

Relatively recent research into the pedagogy of establishing fact fluency — an essential skill if students are to use any arithmetical operation efficiently — further highlights the shortcomings of the traditional methodology. Researchers are finding the popular methods of memorizing facts and timed tests – such as traditional flashcards (which include simply the fact and product in numerical form, i.e. "4 x 3" and "12") and "Mad Minutes" – are actually detrimental to students of all achievement levels. Boaler argues rote memorization without understanding and timed drills emphasize speed rather than understanding (2015).

According to Kling and Bay-Williams (2015):

> Decades of drill and timed testing have failed our students, often leading to a lack of fluency and a negative disposition toward mathematics. Even in cases where students are able to successfully complete tasks, such as timed tests, one might question the value of such assessments. Does a perfect score on a timed test really tell us anything about that student's *understanding*? (p. 558)

Additionally, Baroody (2006) argued it can be difficult to motivate students – particularly those with learning difficulties or no interest in mathematics – to memorize a long list of facts purely for memorization's sake. "Teachers must overcome this reluctance either by profusely rewarding progress … or, if necessary, by resorting to punishment … or the threat of it" (p. 24).

Challenges to Implementing a Conceptual Understanding Approach

While educational leaders will, of course, face a myriad of challenges in fostering a division-wide CU approach to teaching the arithmetical operations, there are two aspects I will focus on here: educational reform and the politicization of mathematics; and teacher experience and familiarity with the CU approach itself as well as teaching strategies for its incorporation in day-to-day classroom activities.

Educational Reform

Championing a CU approach in mathematics instruction is, unfortunately and perhaps surprisingly, not without political and societal complications and pushback. In 2013, a parent from the small town of Calmar, Alberta, began publicly voicing her concerns with mathematics instruction in the province, blaming declining student performance on the inadequacy of so-called “discovery math.” Dr. Nhung Tran-Davies was concerned her then eight-year-old was struggling with the arithmetic strategies used in her mathematics class to the point of hating the subject, despite successful pre-tutoring at home in previous years. Confessing she was “neither a teacher nor a mathematician,” the mother of three began an online petition declaring the trend of discovery mathematics had “clearly failed the first wave of children subjected to [the] grand experiment,” and demanded the provincial government “overhaul” the curriculum to return to emphasizing “the importance of acquiring and mastering the basic mathematics skills” (Tran-Davies, 2013). The blame for student performance was not to be placed on teachers, Tran-Davies clarified at the time, but on Alberta Education steering the curriculum into so-called “new math” waters:

> In theory, the "new math" curriculum is alluring with the promise to help students develop a deeper understanding of how calculations work by allowing students to learn different "strategies" to get to a "reasonable" answer, to perhaps help students problem-solve. In practice, the "new math" glares of absurdities in that students are led through multiple convoluted "strategies" to get to a solution, with no emphasis on mastering any one method. As a result, the importance of knowing basic mathematics facts (e.g., algorithms, time tables, automatic recalls, vertical additions) is diluted down to a weak understanding and poor grasp of basic mathematical concepts in addition/subtractions and

> multiplication/divisions, which in effect ill-equip our children to reconfigure equations in their own minds, problem-solve, and think critically. (Tran-Davies, 2013)

Tran-Davies' concerns struck a chord, and within months her petition had garnered thousands of signatures from parents and educators alike demanding a "back to basics" reform of Alberta's mathematics curriculum. Tran-Davies was named one of the province's most influential people of 2014 ("Dr. Nhung Tran-Davies," 2014). College and university professors joined the list of accusers, complaining their students were arriving in post-secondary courses without the prerequisite mathematics skills. Upon signing Tran-Davies' petition, retired University of Alberta engineering professor Ken Porteous wrote in an open letter to then Alberta Education Minister Jeff Johnson:

> The discovery approach has no place in arithmetic at the junior elementary level. There is nothing to discover. The tried and true methods … work just fine as they have for centuries. There is no benefit and in fact a huge downside to students being asked to discover other methods of performing these operations and picking the one which they like. This just leads to confusion which ultimately translates into frustration, a strong dislike for mathematics and a desire to drop out of any form of mathematics course at the earliest opportunity. If the so called "new math" works so well, how is it that clerks in stores are frequently unable to make change without a calculator? (Staples, 2014)

It may surprise many contemporary critics of "new math" (or its interchangeable moniker among detractors, "discovery math") to know they are repeating a call for a return to mathematics instruction "basics" over a half-century old. Nor is the demand to reform the mathematics curriculum such that it will adequately prepare students for the future a fresh one. The term "new math" originated in the 1960s to describe North American efforts to shift the emphasis of mathematics instruction from rote memorization of facts and techniques to critical thinking, flexibility, and creativity (Phillips, 2016). The reform itself began in the United States in the 1950s and quickly became a system-wide change as the School Mathematics Study Group (SMSG), the predominant voice for change comprised of mathematicians and mathematics teachers, transformed instruction by producing new textbooks and teacher guides; by 1965, it is estimated at least half of America's students were using the new mathematics textbooks (Phillips, 2016). While the SMSG steered clear of modifying the curriculum itself, it fulfilled two key conditions for successful reform described by Cohen and Mehta (2017): it successfully raised awareness of a perceived instructional shortcoming and offered a solution; the government responded favourably to pressure from educational organizations (e.g., the SMSG); and it "offered the educational tools, materials, and practical guidance" teachers needed to implement the reform (p. 2). The reform was rooted in liberalism, with the goal of giving as many students possible a background in mathematics that would keep as many doors open as possible (Phillips, 2016).

Notably missing from Cohen and Mehta's criteria for successful reform was buy-in from parents and other parties outside education academia. Criticism arose from parents and even mathematicians who complained that proven procedures through direct instruction were more important than fostering a conceptual understanding in students (Sherwood, 2015). Predominant among parental concerns was the complaint they did not recognize their children's mathematics work. The cynical term "New Math" was coined as the name for parents' frustration, as shown in satirist Tom Lehrer's song of the same name:

Hooray for New Math

New-hoo-hoo Math

It won't do you a bit of good to review Math

It's so simple

So very simple

That only a child can do it! (Lehrer, 1965)

Remarkably, the "new" arithmetic specifically highlighted and maligned in Lehrer's song is the very Base-10-rooted standard algorithm now coveted by modern traditionalists.

Changes were made to Alberta's K-9 mathematics curriculum in 2016 in response to "back to basics" demands. Language in the curriculum front matter was modified to stipulate students would, in fact, explicitly learn the standard, traditional algorithms, albeit while they "investigate a variety of strategies" (Alberta Education, 2016). Notes were added to Grades 1 to 6 clarifying which collections of basic facts students are expected to recall. Lists of strategies for certain outcomes were culled, also. For example, the Grade 3 outcome of understanding and recalling addition and subtraction basic facts no longer listed the possible strategies of using doubles, making 10, using the commutative property, using the property of zero, or thinking addition for subtraction. In its new mathematics curriculum mandated for instruction starting in the fall of 2022, the Alberta Government eradicated nearly all references to CU methodology, outright mandating the instruction of, and student use of, standard algorithms (Alberta Education, 2022).

It behooves mathematics teachers and educational leaders, then, to be aware of the local and regional politicization of their subject matter — and legalities, even, given potential curricular requirements — before utilizing instructional strategies having the potential to ignite passions and opinions not necessarily informed by research. Rather than facing an angry parental pushback after the fact, one might choose to proactively promote the benefits of the CU approach. In either case, educators would do well to have their justification laid out in advance.

Teacher Efficacy and Experience

With the impetus for change identified by contemporary research, I conducted a qualitative study in 2021 to identify and describe some of the challenges preventing or hindering PRSD's elementary mathematics teachers from approaching fact fluency instruction via research-based pedagogy focused on establishing conceptual understanding rather than rote memorization. Despite running numerous professional development sessions with our division's intermediate teachers in my role as Numeracy Program Coordinator, many were still using the traditional approaches of rote memorization and timed drills no longer aligned with contemporary research on effective mathematics pedagogy. Teachers also questioned how they were supposed to assess students' fact fluency if the traditional method of timed drills was removed. Without this quantitative tool, numerous teachers asked how the curricular outcome of fact fluency could be measured and reported upon. It was hoped the study would identify some of the specific concerns around this question. By identifying some of the previously unidentified challenges (as perceived by teachers in the PRSD), it was hoped mitigation strategies could then be developed to assist and encourage teachers in making the pedagogical change.

Three teachers — "James" (Grade 5), "Jeff" (Grade 5), and "Anne" (Grades 4 to 6) — were interviewed with the goal of identifying the overall nature of the challenges as well as narrowing down the specifics of their concerns to aid in developing mitigation strategies. These grades were selected because they contain a heavy curricular emphasis on both establishing and applying fact fluency. Also, the success of teachers of these grades in establishing fact fluency has a major and long-lasting impact on students' mathematical self-efficacy and skill sets in future years; this is exacerbated by later grades' curricular emphasis on other concepts assuming and depending upon students developing fact fluency in earlier years. James and Jeff taught in the same middle school, while Anne taught at a different K-6 school. These teachers were among those in the division who had demonstrated an openness to learning and applying new pedagogy and had, at the least, received professional development on the conceptual understanding approach to fact fluency. If they had not followed through on consistently implementing the newer approach, their reasons had the potential to be highly informative for my research.

The respondents were interviewed with the goals of identifying the overall nature of the challenges to implementing a conceptual understanding approach to fact fluency and narrowing down the specifics of their concerns to aid in developing potential strategies to mitigate those challenges. Respondents were asked the questions listed in Appendix A (with follow up questions inserted where necessary to elicit more details or explore ideas arising from initial responses) to describe their feelings of self-efficacy, knowledge, and motivations around implementing the newer pedagogy.

All three respondents cited difficulties related to the conceptual understanding approach being a change from traditional practice. Some of these difficulties were related to the teachers themselves; for instance, the respondents each spoke to the challenge of needing to break from how they were taught and thus came to understand the pedagogical approach. Respondents also stated the change in approach was difficult from students' perspectives as they were typically taught previously with the rote memorization approach. One respondent added some parents have outright refused the new approach, demanding their children be taught with the same approach used when they were students.

Respondents also cited challenges related to their own lack of knowledge of the new approach, as well as a general lack of time to do their own research into it. Another common challenge cited by the respondents was a lack of instructional time. The respondents unanimously felt the new method required more time than the drills-based rote memorization approach, which they found especially challenging in the last two years of pandemic teaching and the related frequent absences of both themselves and their students. Anne also felt larger class sizes particularly made the new approach more laborious. James added the extra time he felt the new approach required made it particularly difficult to attempt to adopt mid-year, suggesting it needed to be budgeted into long-term plans at the launch of the year to be implemented successfully and with fidelity.

Other challenges falling into this theme included: how to support students in making it a habit to use the new methodology, especially when the teacher isn't supervising their work; monitoring student progress in developing strategies related to conceptual understanding is more problematic and time consuming than the easy-to-assess quantitative data provided by timed drills related to rote memorization; lack of experience and confidence in the newer pedagogy; vague curricular expectations for fact fluency interpreted as the need for memorization; recent politicization of mathematics pedagogy in Alberta that disparages research-based approaches as ineffective ("new math"); and lack of resources in contrast to the plethora of easily accessed rote memorization tools such as software, games, and flashcards. Finally, Anne and James referenced a lack of peers championing the new approach who could be called upon for advice and support, as well as a lack of locally generated evidence to support the research claims in favour of the conceptual understanding strategies.

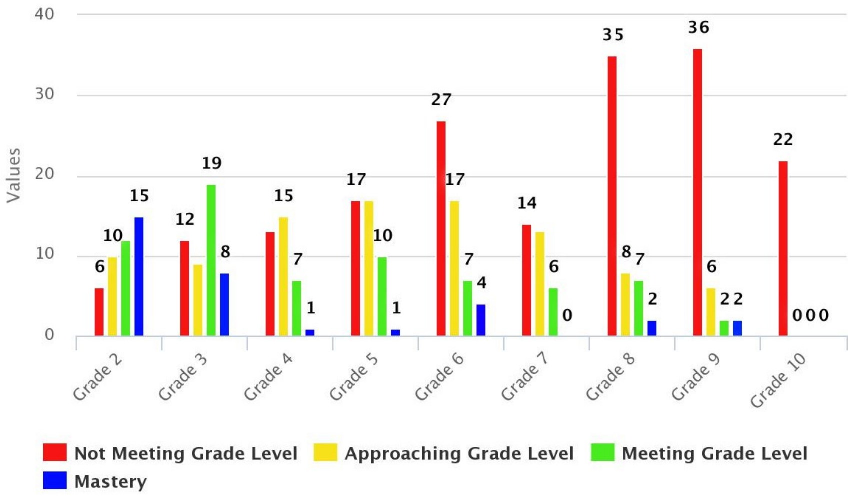
Implications for Indigenous Learners

It is a fortunate convenience research on effective instructional strategies for Indigenous learners aligns well with the CU approach championed here. It is no longer excusable for educators to lean on the tired, superficial, and ineffective approach to designing culturally relevant mathematics lessons by finding artifacts on which to apply, or find within, elements of specific concepts. Pity the Grade 8 teacher attempting to make the Pythagorean Theorem culturally relevant, let alone the senior high school calculus teacher. It is an understandable approach, given the opinion mathematics is resistant, at best, to cultural relevancy. The many professional development sessions promising to imbed traditional Indigenous games or learnings in mathematics lessons I have attended have always seemed to make ambitious claims only leading to mild disappoint at best; I am unable to cite a single example where either the cultural or curricular element (or both) was watered down beyond usefulness. The connection always seemed forced or contrived — an attempt to show how the square peg and round hole could get along.

While no doubt well-intentioned, PRSD data shows this weak attempt at serving Indigenous learners in the mathematics class as an afterthought simply does not work. A new focus on data-driven decision making in recent years has produced PRSD's first grade-by-grade breakdown of performance on a start-of-the-year benchmark assessment allowing comparison between Indigenous (see Figure 2) and non-Indigenous (see Figure 3) student achievement. While the performance of both groups in the junior high grades warrants concern and intervention, the percentage of Indigenous students experiencing difficulties (i.e., not meeting or approaching grade-level expectations) is noticeably higher in most grades and begins to dominate the picture earlier, beginning in Grade 4 compared to the same phenomenon beginning in Grade 7 for non-Indigenous students.

Figure 2

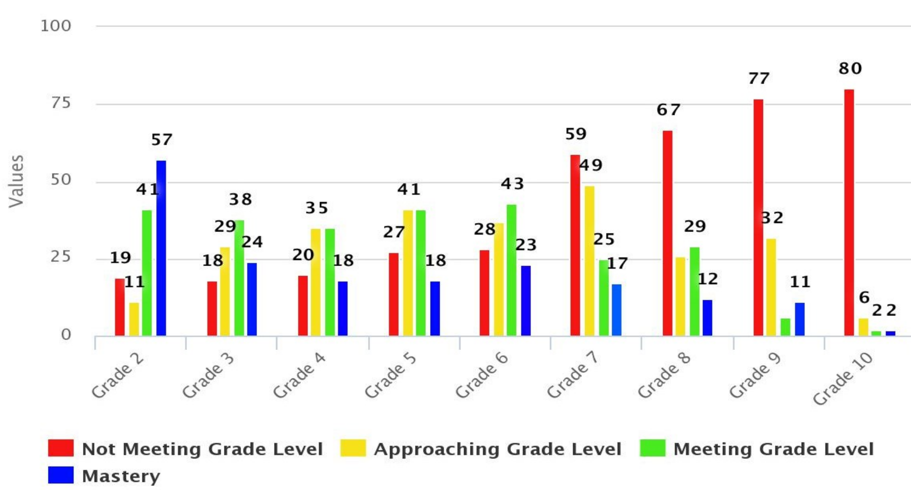
Grade 2-10 Indigenous Students' Mathematics Performance, Fall 2021



Note. Data showing the portion of Indigenous students in each grade not meeting, approaching, meeting, and mastering grade-level expectations, based on results from a universal benchmark assessment administered in the fall.

Figure 3

Grade 2-10 Non-Indigenous Students' Mathematics Performance, Fall 2021



Note. Data showing the portion of non-Indigenous students in each grade not meeting, approaching, meeting, and mastering grade-level expectations, based on results from a universal benchmark assessment administered in the fall.

The district's numeracy achievement gap between Indigenous and non-Indigenous students is typical in Canada, according to research by Barber and Jones (2021). Our results also fall in line with their finding this gap tends to begin in intermediate grades (Grades 4 to 6) and widens in junior high. Indigenous students are a minority in our division, which also tends to worsen the achievement gap, according to Barber and Jones (2021) who cite possible "discrimination, bullying, or racism" contributing, at least in part, to this gap (p. 2). Barber and Jones point to studies showing socioeconomic factors contribute considerably to this achievement gap, and, certainly, Hattie's (n.d.) work demonstrates socioeconomic status has an influence on student achievement. However, Hattie's studies attribute a relatively low influence factor to socioeconomic status, compared with the very high effectiveness of instructional factors such as collective teacher efficacy and response to intervention (n.d.). The historical abuse and racism systemic in Canada's institutions, politics, and education have clearly had a negative effect on Indigenous academic achievement (Stavrou & Miller, 2017). While appreciating the considerable and tragic effect socioeconomic factors and colonialism have had on Canada's Indigenous people, the focus here will be on instructional approaches which may begin to address the achievement gap.

My former impression that mathematics is a standalone topic out of reach — aloof, even — of cultural nuances and interpretations has been shattered. Indeed, believing mathematics to be culture-free is part of the problem. While mathematics is widely considered a neutral, culture-free, and universal endeavour, in reality, it has the "inherent agency to exclude, create barriers, regulate access to further learning, and prompt discomfort and anxiety in teachers and students" (Stavrou & Miller, 2017, p. 100). Barton (1996) argued widely-held beliefs disparaging traditional Indigenous mathematics as not as valuable or valid as Western mathematics are partly to blame and even rooted in colonialism: "These acts of colonialism work to destroy Indigenous knowledge through the imposition and internalization of the colonisers' way of knowing, by the colonised" (p. 135). Stavrou and Miller (2017) point to the mindset of Western mathematics as "uncontestable, objective, and disassociated from experience, history, and cultures" (p. 95) as cause, in part, for the Western worldview of Indigenous ways of knowing as inferior. This perspective is grounded in the idea of mathematics as largely immune to and incompatible with cultural learning styles, as I once believed.

The importance of embedding cultural relevancy in mathematics is supported by some North American educational researchers who believe it may be an effective mechanism for closing the learning gap while providing a more inclusive learning environment (Stavrou & Miller, 2017). The drive for culturally relevant mathematics, however, is often superficial, as I've experienced, and can even unintentionally perpetuate harmful stereotypes when taught by teachers who are not authorities on the culture or language (Stavrou & Miller, 2017). Researchers such as Sleeter (1997) saw little to no merit in efforts to embed culture in mathematics, and Stavrou and Miller (2017) believed "superficial pedagogical examples, which are intended to acknowledge multiculturalism, may serve to affirm instructor goodness but do nothing to address the fundamental problem of inequality" (p. 108).

A potentially very effective approach to instructing Indigenous mathematics students can be found in the so-called "mathematics wars" between traditionalists and reformists. Further encouraging, it is the approach the majority of current educational research endorses for all students. Russell and Chernoff (2012) categorized the traditional side of teaching mathematics as using methods such as rote practice and direct instruction where the teacher is the sole authority on knowledge. Whereas reformists believe mathematics should be learned through investigation and inquiry, with the teacher in a role of guidance and support. The reform perspective holds conceptual understanding as the ultimate goal of mathematics education, achieved through exploration and constructed by the individual; alternate answers and solutions are encouraged (Russell & Chernoff, 2012). Perhaps surprisingly, the traditionalist approach has some similarities with Indigenous ways of knowing, with Russell and Chernoff (2012) seeing alignment with the value given to the knowledge of Elders, family, and the community: “even the traditionalists’ focus on a single answer, a single way of achieving that answer, and the symbolic representation of such knowledge is acceptable within an Indigenous worldview, which accepts and values different ways of knowing (p. 121). It should not be surprising, though, the alignment ends there. Traditionalists’ preference of direct instruction of a single, correct method runs counter to the Indigenous preference for interaction, diversity, and subjectivity (Russell & Chernoff, 2012). Stavrou and Miller (2012) agreed, arguing the traditionalist approach to mathematics is detrimental to decolonization efforts as the upholding of Western standards as absolute and objective inherently implies Indigenous knowledge and ways of knowing are “archaic and inferior” (p. 108).

Thus, the reformist perspective is far better aligned with an Indigenous worldview valuing generational sharing and building of knowledge where students are encouraged to explore and interact with their environment and community members:

> These learning strategies, along with the teacher’s role as a facilitator of learning (through engaging students in discussions of mathematical ideas and discoveries), would

> also bring about recognition and valuing of diverse ways of knowing and knowledge, which is also foundational to an Indigenous worldview. (Russell & Chernoff, 2012, p. 122)

Munroe, Borden, Murray, Toney, and Meader (2013) agreed the reformist approach to mathematics aligned with Indigenous ways of thinking, and even championed its constructivist methods as a key to decolonizing education as it fit with the ideals and goals of modern education:

> We hope that the burgeoning worldwide concern for an education that prepares children for the uncertainties and complexities of the 21st century will renew respect for Indigenous knowledges and serve to decolonize education from the tyranny of belief that wisdom and knowledge come through separation and classification as is the case in European thought. (p. 332)

It is a happy, and unintended, coincidence, then, the emphasis on conceptual understanding over rote memorization of symbolic procedures (e.g., standard algorithms beloved by traditionalists) in the Numeracy Framework is in direct alignment with the reformist perspective. Given the arguments of researchers above in favour of reformist approaches to mathematics, I am hopeful this approach will suit our Indigenous learners while minimizing the differentiation load on our teachers.

Implementation

When the division created its first numeracy program coordinator position several years ago, it was an easy match; no one else in the division had been so publicly fervent about improving mathematics instruction, and I had an arsenal of evidence-based practices I could readily speak to. I was eager to set about the work of sharing my approaches to mathematics instruction to other schools, supporting teachers through professional development, and create common programming and assessments to, hopefully, improve universal instruction and learning of mathematics and improve the division's flagging achievement scores. Faced with a myriad of potential issues to address — wide variances in assessment practices, instructional strategies not aligned with research or evidence, teachers openly negative and unconfident about teaching the subject, and undifferentiated lessons, among others — one of my first goals was to establish some research-based instructional strategies, starting with effective strategies for teaching the arithmetic operations. Adams, Mombourquette, and Townsend (2019) suggest reflecting upon colleague's efficacy when contemplating how to grow leadership from within, and I had a firmly held belief — I still do — most of our elementary mathematics teachers tackle the subject with reluctance and recycled instructional strategies; through no fault of their own, they simply haven't had the time, opportunity, or interest to familiarize themselves with contemporary research-based practices and resort to the ineffective teacher-centred approaches they experienced in elementary mathematics. I set about creating a research-based how-to manual on teaching each of the operations and within the first two months on the job I was in schools presenting to teachers.

Upon reflection, I suppose I assumed most teachers would adopt the new practices when faced with what I felt was incontrovertible reasoning for doing so. I did not yet appreciate, however, "people will not venture into uncertainty unless there is an appreciation that difficulties encountered are a natural part of the process" (Fullan & Miles, 1992, p. 749). I had my share of trailblazers, motivated by novelty and excited by risks, and pioneers conscious of the need for change and eager to explore new techniques, but I did not account for the number of unenthusiastic stay-at-homes needing an inexhaustible supply of prodding and the saboteurs doing their absolute best to stop the change (Schlechty, 2018).

When presenting on the need to change instructional practices, I often cited the division's flagging performance on the Alberta Provincial Achievement Tests (PATs) for Grade 6 and 9 as evidence. The PATs, though, are an example of solely externally mandated goals and simply were insufficient motivation for some educators to change (Adams et al., 2019). Wanting student achievement data hitting closer to home for our teachers, I brought in a universal screening tool for the beginning of the year to help the division track student progress year-to-year and target professional development efforts, schools to target supports where most needed, and inform teachers of the needs in their classrooms. The results of the first year's screening brought into the light many troubling areas in achievement from Grades 2 to 10, including very troubling results when it came to student fluency with the operations in all grades. My presentations on the need for instruction building conceptual understanding — rather than rote memorization of abstract procedures and algorithms — became rooted in data derived from within our organization. As supported by Adams et al., it was the groundwork of a vision backed by data: "[o]f most importance is that effective leaders interpret and create opportunities to align data with that which has been collaboratively and collectively defined as the broadly understood mission and vision" (2019, p. 7).

My initial efforts lacked, upon reflection, a cohesive and well-communicated vision that would link the initiatives directly to the needs of students in the eyes of our teachers (Adams et al., 2019). This began to be addressed in the second year when I was tasked with authoring the division's first Numeracy Framework (NF) that would clarify and bring commonality to the hodgepodge of instruction and assessment approaches from classroom to classroom, school to school. A crucial part of this plan was a division-wide assessment framework which included the aforementioned screener and a progress monitoring tool I developed to provide data on grade-level foundational numeracy skills, the Numeracy Common Assessment Tool (NCAT). This tool tracked student achievement on the operations, place value, number sense, and the relationship between fractions and decimals from Grade 1 to 9 (as relevant to each grade) and provides values to all levels of our education community. At the classroom level, the NCAT provides teachers across our geographically expansive division with common achievement expectations and data to target instruction and interventions. At the school level, school leaders can use the data to better target classroom supports. Divisionally, the data informs of particular schools, grades, and concepts requiring supports and teachers in need of professional development. There is vast amounts of research showing the importance of educational leaders using formative assessments — such as the NCAT — to support teachers and students. Hollingworth points to the importance of teachers using formative assessment data to fuel professional learning communities (PLCs) that result in high student achievement and teacher efficacy (2012). School leaders have an important role in participating in these PLCs to help teachers understand and apply formative assessment principles (Hollingworth, 2012). Davies, Busick, Herbst, and Sherman attested formative assessments are not just an important aspect of student-centred instruction but a powerful leadership tool for helping staff themselves develop (2014).

The Numeracy Framework (NF) was released in the spring of 2021 for official implementation in the 2021-2022 school year with the intention of organizing programs and resources to maximize the potential for student success in numeracy. It simultaneously represents my newfound appreciation for the importance of elements of the Leadership Development Program, in that it aims to establish pedagogical and assessment norms across the division, is intended to work in concert with a fresh focus on numeracy achievement in the PRSD's 2021-2024 Education Plan, supports site-based leadership's efforts to monitor and support student achievement, and draws on research-based instructional practices emphasizing a CU approach.

Implementing Change Via Structural, Political, and Human Frameworks

How can PRSD restructure itself to ensure the elements of its new NF — particularly the CU element for my purposes here — are implemented by all elementary mathematics teachers? While there is a strong acceptance, and even enthusiasm in many cases, for the NF from most principals and teachers, there remains a contingent of approximately a dozen teachers from various schools who have outright indicated reluctance to adhere to tenets of the plan. In addition to the remaining staff who still need to "come on board" or actively engage with the NF, there is the potential for the plan to be shelved and join the graveyard of well-intentioned but unfulfilled and forgotten initiatives and resources lining principals' and teachers' shelves. For the NF to be successful, the implementation plan will need to engage affected parties at multiple levels (i.e., district, administration, and classrooms), include regular, if not frequent, revisiting throughout the year, and function as one of the strategies that support the fulfillment of a larger vision - in this case, the division's three-year education plan and its goals.

Structural Frame. The structural frame emphasizes that "the right combination of goals, roles, relationships, and coordination is essential to organizational performance" (Bolman & Deal, 2017, p. 46). Central office staff should support vertical coordination efforts via professional development planning and control systems. They will create an overarching PD plan. This is a shift from the recent practice of leaving site-based PD decision-making up to individual principals. The centralized plan aligns with the three divisional goals for educational leaders to use as a frame for site-based PD plans. Training and resources relevant to numeracy topics will be disseminated at the monthly educational leaders meetings. A “teach-the-teacher” format will enable educational leaders to adapt and coordinate PD towards their own staff within the context of their needs. During these times, central staff can perform a needs assessment to determine what further supports should be afforded to schools. The central PD plan will establish a component of each PD day to be designated to interschool collaboration efforts. This will enable teacher teams to collaborate while partially solving the issue of time, a frequently cited barrier to implementing district-wide mathematics improvement initiatives (Mason et al., 2019).

Principals will play an important vertical coordinating role as authorities in their schools. They must establish expectations for staff in relation to adopting the NF and nudge them in the direction of relevant PD in support of the aim. Further, principals must actively participate in PD themselves as this has been positively linked to student achievement (Dempster, 2012). A leadership for learning framework such as the one outlined by Dempster (2012) could be deployed to support principals navigating this process; at its core is the need to anchor the school team to a shared moral purpose. Staff will inevitably challenge this new initiative; some, indeed, already have. Principals should embrace feedback while continually directing staff back to the rationale for the NF and its connection to that moral purpose, which, in turn, should be grounded in a strong evidence base and a strategy Dempster dubbed *disciplined dialogue*. Both align with the existing Collaborative Response process (Hewson & Adrian, 2013) driving PRSD decision-making around student instruction and support. A process that will allow principals to check that data is being collected as appropriate and to create dialogue about what is shown, why it is shown, and what should be done to respond effectively

Human Resources Frame. Providing instructional leadership as a foundation for the implementation of the NF will allow educational leaders to guide educators to resources and tools to improve their results as an overarching goal of the framework. To empower educational leaders, they will be given training at the outset and throughout the school year on the NF and be supported by central office personnel to implement its components. They will also be guided on how to read and discuss implications of the student achievement data with their staff. Providing time for teachers to attend professional learning communities (PLC) is a crucial investment in PRSD employees. They will need the time to take ownership of the outcomes and discuss the results of the assessment. Grade-level PLCs will foster the collective accountability of teachers, providing them with the opportunity to be part of the decision-making process (Fullan, 2014).

Political Frame. It is vital to the success of the Numeracy Framework (NF) that its implementation be analyzed through the lens of the political framework detailed by Bolman and Deal (2017) given the array of competing staff interests at the outset. The common approach to some aspects of mathematics instruction promoted and prescribed by the NF is a stark change from the division's history of leaving most instructional choices up to individual teachers, and there is already known to be a spectrum of perspectives from eager champions to saboteurs resentful of the change. Political skills and awareness of staff perspectives will aid leaders in choosing an effective balance between collaborative and more adversarial approaches (Bolman & Deal, 2017).

Symbolic Frame. Parents have indicated the importance and their support of children developing proficiency in numeracy skills (Peace River School Division, 2021). How those skills are established with students in PRSD schools and how parents can further assist their children with numeracy development is much less clearly defined. Further, how has PRSD demonstrated their commitment toward numeracy skill development with respect to the majority of its stakeholders? The NF within the context of the symbolic frame serves as a pathway forward while seeking to overcome these challenges.

If parents have an awareness of the NF, they can engage teachers with mutually understood language and adopt clearer expectations for their child's skill development. To this end, PRSD's central office should deploy an ongoing, vibrant, and engaging communication campaign that outlines the NF in an approachable manner. This campaign will act as a symbol illustrating a renewed commitment to improving numeracy within the organization. It will also serve as an advertisement for PRSD with the capacity to shape community perceptions — a showcase of the best educational institution for parents looking to invest in their child's future.

Potential Research

While mathematics instruction researchers such as Bray (2014), Boaler (2015), and King and Bay-Williams (2015) cite the importance of employing visual flashcards to build students' fact fluency, there is a dearth of research demonstrating the implied improvement in students' fact fluency and application over the use of traditional flashcards. Existing research on the efficacy of procedures for building fact fluency tend to be quantitative in nature, focusing on the comparative number of facts students can recall in a timed test, in isolation of application in problem solving contexts, after using different rote memorization methods: Burns, Aguilar, Young, Preast, Taylor, and Walsh compare the rote memorization methods of traditional drill and incremental rehearsal (2019); Rave and Golightly examine the effectiveness of the rote Rocket Math program (2014); and Nelson, Burns, Kanive, and Ysseldyke compare traditional flashcards with mnemonic strategies (2013). Pantages approaches the purpose of the research proposed here by comparing fact fluencies developed by rote memorization versus conceptual strategies but falls short of analyzing students' ability to apply said fluency (2008).

Furthermore, quantitative studies require testing involving a time limit so as to compare with a control group (e.g., using traditional flashcards). As pointed out by Boaler (2015), timed fact recall is highly discouraged and considered outright detrimental to students of all abilities, so I propose a potential study would avoid this type of measurement and opt for a qualitative study comparing the effects of traditional and visual flashcards on students' efficacy in applying their fact fluency in problem solving via interviews with students on their mental strategies for solving given problems.

Should the study confirm the declarations of Boaler (2015), Bray (2014), and King and Bay-Williams (2015), the results could encourage more teachers to employ visual flashcards (thus CU) as an instructional strategy for achieving fact fluency. The benefits of visual flashcards would likely extend beyond student's fluency with and application of fact fluency; given the positive effect fact fluency has on student performance throughout mathematics curricula (Baroody, 2006), there would also be the potential for increased student engagement and motivation in mathematics in general.

Provided promising results, further research would be warranted with varying sample types, such as students struggling in mathematics in general and junior high students needing remediation with fact fluency. More research could also be done comparing longer-term retention of fact fluency with traditional versus visual flashcards. Furthermore, should the results of this proposed study provide evidence using visual flashcards provides a learning advantage, there would be ample cause to share the method with a broader group of teachers and thus positively influence the learning of a greater number of students.

Concluding Remarks

There is a clear need to ground and address the challenges and solutions in implementing the NF and its CU component in the frameworks presented by Bolman and Deal (2017). Each lens brings to light a different (but related) set of issues needing to be addressed if the components of the NF are to be realized in each of our elementary mathematics classrooms and thus across the division as a whole. Additionally, these analyses emphasize a common aspect of a potentially successful plan — multiple levels of personnel within the division must be purposely involved. This is further supported by Fullan (2018), whose research attested teacher PD and PLCs lack impact without direct principal involvement with the explicit purpose of improving student learning, and Leithwood, Louis, and Anderson (2012), who stated "leadership affects student learning when it is targeted at working relationships, improving instruction and, indirectly, student achievement" (p. 234). Dufour and Eaker (2009) similarly argued student achievement depends on education teams working collaboratively and collectively. Finally, Shlechty (2009) supported a multi-tiered approach to implementing the NF with his research showing leaders successfully transform learning organizations when they (a) establish a shared understanding of the need to change and a vision of how to do so, (b) maintain direction by developing structures for continuity, and (c) providing ongoing support and fostering collaboration.

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Appendix A

Interview Schedule

1. Describe your experience and context (i.e., grades taught) in teaching fact fluency so far. Describe one or two of your go-to strategies for teaching fact fluency? What makes those strategies some of your favourites? (What are the benefits?)
2. Do you feel the newer approach to fact fluency focusing on conceptual understanding is an improvement over the traditional approach of rote memorization? How important is it that teachers adopt the newer approach?
3. What are some of the research-driven criticisms around the traditional approach to fact fluency? (i.e. Rote memorization and timed drills.) Are these criticisms valid? Why do you think that?
4. What does it look like to establish fact fluency via conceptual understanding? What are some of the teaching strategies potentially involved?
5. In your experience or observations, what are some of the barriers or challenges in adopting the newer approach to building fact fluency? What do you feel is the most significant/difficult/common challenge? Why?
6. What would make it easier for you to adopt — or convince you to adopt — a conceptual understanding approach to building fact fluency?