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Spatial Reasoning Rules in Multimedia Management Systems

by

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Abstract

In this paper we consider various spatial relationships that are of general interest in multimedia databases for data retrieval. We present a unified representation of spatial objects for both topological and directional relations. Such a representation is based on Allen's temporal interval algebra. We extend the frequently used four directional relations *south, north, west*, and *east* into twelve directional relations by adding *southwest, southeast, northwest, northeast, left, right, above*, and *below*. Furthermore, we define a set of topological relations which are equivalent to the widely used eight topological relations. One of major contributions of this paper is to present a complete and formal definitions of both directional and topological relations. However, we also present a set of spatial inference rules, which allow us to make heterogeneous spatial relation deductions from existing directional and topological relations. For example, if we know A *north* of B, B *overlap* with C, and C *north* of D, then we derive A *above* D. This paper contains proofs for all the inference rules. Since all the rules are

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propositional Horn clauses, they can be easily integrated into any multimedia database by either using a simple inference engine or using a lookup table.

Keywords: spatial object, spatial relation, topology, direction, interval, inference rule.

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1 Introduction

Representation and processing of spatial relationships are important characteristics of multimedia information systems. A major focus of recent research has been on the design of sound and complete reasoning systems [RCC92, SYH94, PS94, Ege94, PTSE95, SF95, GPP95] because spatial reasoning forms a vital part of spatial query languages. A spatial inference engine within a multimedia database management system (MMDBMS) can support spatial analysis without transforming any spatial knowledge into the domain of underlying coordinates and point-region representations. Reasoning with imprecise and incomplete information may be achieved in a purely qualitative matter or, when necessary and available, augmented by quantitative information. Two major kinds of spatial relations have been extensively studied: directional relations such as *left, above, north, south*, and topological relations such as *inside, overlap, disjoint*.

Qualitative spatial reasoning is necessary because the common-sense reasoning which humans apply to spatial issues is generally qualitative. For example, in most cases we are only interested in *whether object A is north of object B*, in stead of *whether object A has the same longitude, but smaller latitude than object B*. High-precision quantitative measurements are of limited use in these cases. Once we have many objects in our database, it is very expensive, if not impossible, to store all the spatial relations among them. In fact some relations may rarely be used because of lack of interest from users. A straightforward approach to attacking this problem is to explicitly store the most frequently used spatial relations and generate rarely used relations on demand. This strategy needs generalized spatial representations to convey the existing spatial knowledge and such representations should be support easy computation by a spatial reasoning (inferencing) engine.

Logic-based representations are used for qualitative spatial reasoning since they provide a natural and flexible way to represent spatial knowledge [PS94]. Such representations usually have well defined semantics and easily understood inference rules which can be integrated into any deductive system. Several researchers [MJ90, Fre91] have suggested that Allen's temporal interval algebra [All83] could be used to represent spatial relationships among objects. However, to the best of our knowledge, very few attempts [SF95, NSN95] have been made to reason about heterogeneous (both directional and topological) relationships using this temporal interval algebra. In those very few attempts, the directional relations are restricted to: *north, south, west* and *east*[1]. This is certainly not sufficient since other directional relations such as *southwest, northeast,* etc. are frequently used in daily life. However, the complexity of reasoning rules increases dramatically even when only a few new directional relations are added.

In this paper we introduce a unified representation of spatial relationships for multimedia objects. The unified representation is based on Allen's temporal interval algebra [All83] and defines both topological and directional relations. We extend the above-mentioned four directional relations to twelve directional relations (adding *southwest, southeast, northwest, northeast, left, right, above, below*). We introduce a set of rules to deduce other heterogeneous relations from the existing directional and topological relations. For example, if we know A *north* of B, B *overlap* with C, and C *north* of D, then we can deduce A *above* D. We also prove the correctness of all the inference rules. Since all the rules are propositional Horn clauses, they can be easily integrated into any multimedia database by either using an inferencing engine or using a lookup table.

The remainder of this paper is organized as follows. In Section 2 we discuss related spatial representation and reasoning work. In Section 3 we introduce the temporal interval algebra and give complete definitions of the directional and topological relations. All the inference rules are discussions in Section 4. We prove the correctness of these rules in Section 5. We discuss how our inference rules may be used in multimedia applications in Section 6 and give our conclusions in Section 7.

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[1]Some systems use *above, below, left* and *right* to represent those four directional relations respectively.

2 Related Work in Spatial Reasoning

Egenhofer [EF91] points out that there are eight fundamental topological relations that can hold between two planar regions. These relations are computed by four intersections over the concepts of *boundary* and *interior* of pointsets between two regions embedded in a two-dimensional space. For example, let A^0 and B^0 be the interiors of objects A and B respectively and ∂A and ∂B be the boundaries of A and B respectively, then the combinations of intersections ($A^0 \cap B^0, A^0 \cap \partial B, \partial A \cap B^0, \partial A \cap \partial B$) between interiors and boundaries form a set of topological relations. These four intersections result in eight topological relations: *disjoint, contains, inside, meet, equal, covers, covered_by*, an *overlap*, depending on whether or not each intersection is empty.

Papadias et al. [PS94, PTSE95, GPP95] assume a construction process that detects a set of special points, called *representative points*, in an image. Every spatial relation in the modeling space can be defined using only the representative points. Two kinds of representative points are considered: *directional representative points*, which are used to define directional relations, and *topological representative points*, which are used to define topological relations. For example, some possible directional representative points can be the centroid (center of mass) of an object, the lower-left and upper-right corners of an object's minimum bounding rectangle, or a reference to a known object. Therefore, in the case of using two representative points, the directional relations between objects can be defined as intervals which may facilitate the retrieval of spatial objects from a database using an R-tree based indexing mechanism [PTSE95]. Their topological reasoning work is based on Egenhofer's eight topological relations in two dimensional space. The topological relations are divided into three levels of resolution (high, medium, and low) according to the applications. The objective is to reduce the computational complexity whenever possible by using lower resolution.

A sound and complete spatial reasoning system is presented in [SYH94]. The soundness and completeness require that each object be *connected*, which means that the object does not have disjoint parts. A set of generalized inference rules are defined without relying on particular internal data representations. Therefore, a user can choose any spatial representation and still use the rules. The spatial inference engine can be easily integrated into a spatial DBMS. However, a serious drawback of this inference system is its low expressive power. There are only four directional relations (left, right, above and below) and three topological relations (inside, outside and overlap) in two dimensional space. In three dimensional space, two additional relations (front and behind) are considered. Therefore, the application domain of such a system is very restrictive.

Sharma and Flewelling [SF95] propose a heterogeneous (including both topological and directional relations) reasoning system. A canonical model incorporating Allen's interval relations are used, which results in a powerful heterogeneous reasoning mechanism. The spatial objects are approximated by their minimum bounding rectangles and the topological and directional relations are mapped onto interval relations. They defined a *composition* of spatial relations as an inference mechanism that permits the derivation of a spatial relation between two objects based on their relation with a common object. Compositions are performed using the composition table for interval relations. The results of the compositions are then reverse mapped onto directional relations. The system has similar capabilities to the system in [SYH94]. A major problem of this model is the simple (only four) directional relations that it considers. Another problem is that the inference rules are restricted to one pattern: *directional* $\wedge$ *topological* $\wedge$ *directional*. For example, A *west* of $B \wedge B$ *meets* $C \wedge C$ *west* of D implies A *west* of D.

Nabil et al. [NSN95] propose a two dimensional projection interval relationship (2D-PIR) to represent spatial relationships based on Allen's interval algebra and Egenhofer's 4-intersection formalism. Then a graph representation for pictures based on 2D-PIR can be constructed. This work concentrates on defining an efficient algorithm for picture matching. Abdelmoty et al. [AEG94] extend Egenhofer's 4-intersection formalism to represent *orientational* relations. The orientational relations always require a reference object called an *origin* to establish a spatial relation. Each object's bounding rectangle together with four lines extending from the corners of the rectangle are used to divide the space external to the object into four semi-infinite areas. The directional relations between two objects are defined using the intersections of the components of these areas. This research reveals that the closer the objects are, the stronger the dependency between the different spatial types of relations. Hernández [Her94] defines the composition of topological and directional relations taken together with the result being pairs of topological/directional relations. Composition is accomplished using *relative topological orientation nodes* as a store for the intermediate results and allows inferences such as if A *disjoint/right* B, B *disjoint/right-back* C then A *disjoint/right* or *disjoint/right-back* C. This work is extended in [CSE94] to handle composition of distance and directional relations.

3 Mapping from Intervals into Heterogeneous Spatial Relations

It is common to use object approximations to index the data space for efficient querying and retrieval in multimedia databases [PTSE95]. Depending on the application domain, there are several options in choosing object approximations. Minimum Bounding Rectangles (MBRs) have been used extensively to approximate objects because they need only two points for their representation. While MBRs demonstrate some disadvantages when approximating non-convex or diagonal objects, they are the most commonly used approximations in spatial applications. Hence, we use MBRs to represent objects in our system.

In this section we first discuss Allen's temporal interval algebra and how it can be used to represent an object's spatial properties. Then we introduce 12 directional relations and 6 topological relations which subsume Egenhofer's 8-topological relations. Our discussion is restricted to a two dimensional (2D) space.

3.1 The Temporal Interval Algebra

Allen [All83] gives a temporal interval algebra (Table 1) for representing and reasoning about temporal relations between events represented as intervals. These temporal relations have often been cited [Bee89, SF95, NSN95] for their simplicity and ease of implementation with constraint propagation algorithms. The elements of the algebra are sets of the seven basic relations and their inverses that can hold between two intervals.

Relation	Symbol	Inverse	Meaning
A before B	`b`	`bi`	AAA BBB
A meets B	`m`	`mi`	AAABBB
A overlaps B	`o`	`oi`	AAA BBB
A during B	`d`	`di`	AAA BBBBB
A starts B	`s`	`si`	AAA BBBBB
A finishes B	`f`	`fi`	AAA BBBBB
A equal B	`e`	`e`	AAA BBB

Table 1: 13 Temporal Interval Relations

The temporal interval algebra is essentially topological relations in one dimensional space enhanced by the distinction of the order of the space. A 2D space is usually represented by two orthogonal axes, x and y. An object approximated by an MBR can be represented by two points (such as lower left corner and top right corner). These two points can be projected onto the x and y axes and each projection can be seen as an interval. It is obvious that the MBRs approximation is the ideal technique to capture topological relations if the interval algebra is used.

Using the interval algebra to capture both directional and topological relations of spatial objects can offer more information about spatial relations between objects as compared to traditional methods [NSN95]. In other words, it has greater expressive power than traditional methods.

3.2 Definitions of Spatial Relations

In this subsection we introduce a unified representation, based on interval relations, to capture both directional and topological relations between spatial objects. We consider 12 directional relations and 5 topological relations in our system. The 12 directional relations are classified into the following three categories:

- *strict directional relations*: north, south, west, and east;
- *mixed directional relations*: northeast, southeast, northwest, and southwest;
- *positional directional relations*: above, below, left, and right.

The definitions of these relations in terms of Allen's temporal algebra are given in Table 2. We use A, B, C, etc. to represent arbitrary spatial objects and their projected intervals on x and y axes are denoted as A_x and A_y respectively. $\wedge$ and $\vee$ are the standard logical *AND* and *OR* operators, respectively. The notation $\{\}$ is used to substitute the $\vee$ operator over interval relations. For example $A_x\,\{\mathtt{b},\mathtt{m},\mathtt{o}\}\,B_x$ is equivalent to $A_x\,\mathtt{b}\,B_x \vee A_x\,\mathtt{m}\,B_x \vee A_x\,\mathtt{o}\,B_x$.

In the context of using the MBR approximation, Egenhofer's eight topological relations can be reduced. Among the eight topological relations there are two inverse relations: *covers* vs *covered_by* and *inside* vs *contains*. This brings the number of our topological relations down to six without reducing the expressiveness. Fewer relations certainly result in a simpler system and less computation costs for reasoning. We use `EC` to denote that two objects are externally connected. This relation is usually denoted by the *meet* topological relation. The reason for avoiding *meet* is to distinguish this relation from the temporal algebra's *meet* relation. All six topological relations are defined in the last part of Table 2.

Figure 1 shows all the cases of A *north* of B ($A\,\mathtt{NT}\,B$). Since $A\,\mathtt{NT}\,B \equiv A_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y$, A's y interval must be above B's y interval ($A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y$). At the same time the intervals of A_x and B_x must satisfy one of the following conditions:

Relation	Meaning	Definition
$A\,\mathtt{ST}\,B$	South	$A_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{b},\mathtt{m}\}\,B_y$
$A\,\mathtt{NT}\,B$	North	$A_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y$
$A\,\mathtt{WT}\,B$	West	$A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge A_y\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B_y$
$A\,\mathtt{ET}\,B$	East	$A_x\,\{\mathtt{bi},\mathtt{mi}\}\,B_x \wedge A_y\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B_y$
$A\,\mathtt{NW}\,B$	Northwest	$(A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y) \vee (A_x\,\{\mathtt{o}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y)$
$A\,\mathtt{NE}\,B$	Northeast	$(A_x\,\{\mathtt{bi},\mathtt{mi}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y) \vee (A_x\,\{\mathtt{oi}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y)$
$A\,\mathtt{SW}\,B$	Southwest	$(A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge A_y\,\{\mathtt{b},\mathtt{m},\mathtt{o}\}\,B_y) \vee (A_x\,\{\mathtt{o}\}\,B_x \wedge A_y\,\{\mathtt{b},\mathtt{m}\}\,B_y)$
$A\,\mathtt{SE}\,B$	Southeast	$(A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge A_y\,\{\mathtt{b},\mathtt{m},\mathtt{o}\}\,B_y) \vee (A_x\,\{\mathtt{oi}\}\,B_x \wedge A_y\,\{\mathtt{b},\mathtt{m}\}\,B_y)$
$A\,\mathtt{LT}\,B$	Left	$A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x$
$A\,\mathtt{RT}\,B$	Right	$A_x\,\{\mathtt{bi},\mathtt{mi}\}\,B_x$
$A\,\mathtt{BL}\,B$	Below	$A_y\,\{\mathtt{b},\mathtt{m}\}\,B_y$
$A\,\mathtt{AB}\,B$	Above	$A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y$
$A\,\mathtt{EQ}\,B$	Equal	$A_x\,\{\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{e}\}\,B_y$
$A\,\mathtt{IS}\,B$	Inside	$A_x\,\{\mathtt{d}\}\,B_x \wedge A_y\,\{\mathtt{d}\}\,B_y$
$A\,\mathtt{CV}\,B$	Cover	$(A_x\,\{\mathtt{di}\}\,B_x \wedge A_y\,\{\mathtt{fi},\mathtt{si},\mathtt{e}\}\,B_y) \vee (A_x\,\{\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{di},\mathtt{fi},\mathtt{si}\}\,B_y) \vee$ $(A_x\,\{\mathtt{fi},\mathtt{si}\}\,B_x \wedge A_y\,\{\mathtt{di},\mathtt{fi},\mathtt{si},\mathtt{e}\}\,A_y)$
$A\,\mathtt{OL}\,B$	Overlap	$A_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,B_y$
$A\,\mathtt{EC}\,B$	Externally Connected	$(A_x\,\{\mathtt{m},\mathtt{mi}\}\,B_x \wedge A_y\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{m},\mathtt{mi},\mathtt{e}\}\,B_y) \vee$ $(A_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{m},\mathtt{mi},\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{m},\mathtt{mi}\}\,B_y)$
$A\,\mathtt{DJ}\,B$	Disjoint	$A_x\,\{\mathtt{b},\mathtt{bi}\}\,B_x \vee A_y\,\{\mathtt{b},\mathtt{bi}\}\,B_y$

Table 2: Directional and Topological Relation Definitions

- A_x and B_x starts together but B_x lasts longer $(A_x\,\{\mathtt{s}\}\,B_x)$ or A_x and B_x starts together and A_x lasts longer $(A_x\,\{\mathtt{si}\}\,B_x)$;
- A_x and B_x finish at the same time with B_x starting first $(A_x\,\{\mathtt{f}\}\,B_x)$ or A_x and B_x finish at the same time with A_x starting first $(A_x\,\{\mathtt{fi}\}\,B_x)$;
- A_x is a subinterval of B_x $(A_x\,\{\mathtt{d}\}\,B_x)$ or B_x is a subinterval of A_x $(A_x\,\{\mathtt{di}\}\,B_x)$;
- A_x and B_x are equal $(A_x\,\{\mathtt{e}\}\,B_x)$.

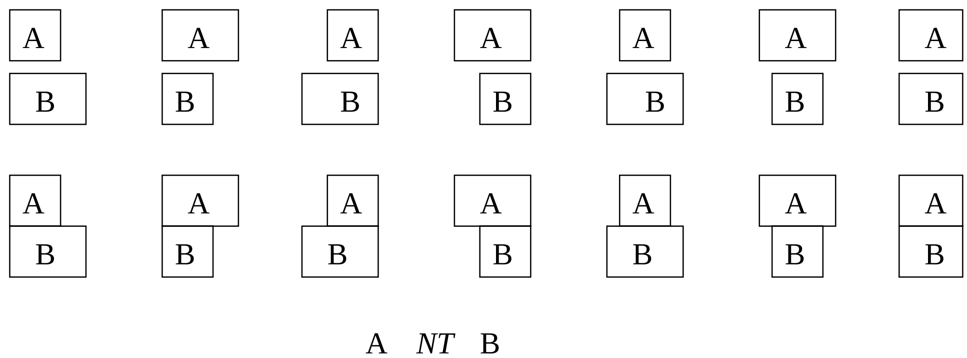


Figure 1: All the Cases of `NT`

Figure 2 shows all the cases of A *northwest* of B ($A\,\mathtt{NW}\,B$). A *northwest* of B ($A\,\mathtt{NW}\,B$). Since the definition of northwest is $A\,\mathtt{NW}\,B \equiv (A_x\ \{\mathtt{b},\mathtt{m}\}\ B_x \wedge A_y\ \{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\ B_y) \vee (A_x\ \{\mathtt{o}\}\ B_x \wedge A_y\ \{\mathtt{bi},\mathtt{mi}\}\ B_y)$, we may have following three cases:

- If A_x is before B_x ($A_x\ \{\mathtt{b}\}\ B_x$), A_y can be after, met by, or overlapped by B_y ($A_y\ \{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\ B_y$). These cases correspond (a), (b), and (c) of Figure 2 respectively.

- If A_x meets B_x ($A_x\ \{\mathtt{m}\}\ B_x$), A_y can be after, met by, or overlapped by B_y ($A_y\ \{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\ B_y$). These cases correspond (d), (e), and (f) of Figure 2 respectively.

- If A_x overlaps with B_x ($A_x\ \{\mathtt{o}\}\ B_x$), A_y can only be either after or met by B_y ($A_y\ \{\mathtt{bi},\mathtt{mi}\}\ B_y$). These cases correspond (g), and (h) of Figure 2 respectively.

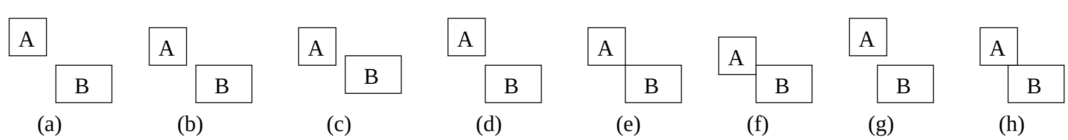


Figure 2: All the Cases of `NW`

Figure 3 shows all the topological relations. While any two spatial objects always have a topological relation, they may not have any directional relation. For instance, consider objects A and

B in the case of $A\ \texttt{OL}\ B$ in Figure 3. A and B have no any directional relation. This coincides with our intuition about spatial objects.

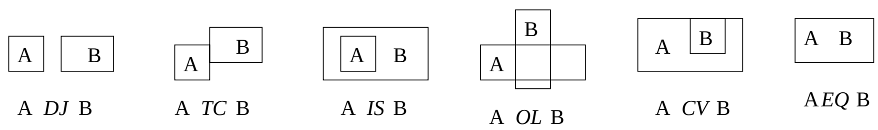


Figure 3: Definitions of Topological Relations

The definition of A *above* B ($A\ \texttt{AB}\ B \equiv A_y\ \{\texttt{bi},\texttt{mi}\}\ B_y$) requires that A's projection on the y-axis is greater than or equal to B's projection on the y-axis. The *above* relation includes A *north* of B ($A\ \texttt{NT}\ B$) because A *north* of B requires A's projection on the y-axis to be greater than or equal to B's projection on the y-axis and some restrictions on the x-axis projections. Furthermore, the *above* relation includes part of A *northwest* of B ($A\ \texttt{NW}\ B$) because the requirement of A's projection on the y-axis greater than or equal to B's projection on the y-axis is implied in some cases of relation *northwest*. Similarly, the *above* relation includes part of A *northeast* of B ($A\ \texttt{NE}\ B$) for the same reason. Our positional relations are more general than those defined in [SYH94] because only the top half (A and B are not externally connected) satisfy the relation *above* among all the cases of *north* shown in Figure 1.

The definition of A *overlap* B ($A\ \texttt{OL}\ B$) indicates that object A shares some region with object B. If this shared region reduces to either a line or a point, then we say that object A touches object B ($A\ \texttt{TC}\ B$). A *disjoint* B ($A\ \texttt{DJ}\ B$) means that object A shares no region with object B.

In our definition, if two objects overlap, they do not have any directional relation. This is certainly an arguable definition. Let us look at Figure 4. It is natural to say A *northwest* of B in (a) and A *west* of B in (c). However, it may not be reasonable to claim that these relations are still hold in cases (b) and (d) respectively. The problem comes from the representation of the temporal interval algebra which does not distinguish the degree of the overlap regions in these cases. All overlaps are treated the same. Even worse, in Figure 4(e) A and B do not have a clear directional relation. This may not be satisfactory in some fine-grain multimedia applications.

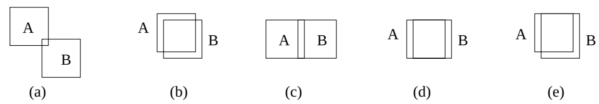


Figure 4: Some Non-directional Spatial Cases

Nevertheless, using the interval relations (algebra) to capture both directional and topological relations of spatial objects can offer more information about spatial relations than traditional methods [NSN95]. In other words, it has greater expressive power than traditional methods. Adopting such an interval algebra is especially attractive in multimedia objectbase systems, compared to GIS and image systems, because many multimedia systems already support Allen's temporal algebra in their temporal models. Hence, no special treatment is required for spatial intervals from an implementation point of view.

4 Rules

Rules have been extensively used in knowledge representation and reasoning within computer science area, especially in artificial intelligence. This is because rules are easy to understand and they can be efficiently implemented. We use rules to infer new spatial knowledge from existing knowledge. Before introducing the spatial inference rules, we define two more notations. If relation x implies relation y, we denote it by $x \Rightarrow y$. Also if $x \Rightarrow y$ and $y \Rightarrow x$, we denote it by $x \Leftrightarrow y$.

Rule 1 (Reflexivity) The following topological relations are reflexive:

$A \,\texttt{EQ}\, A \qquad\qquad A \,\texttt{OL}\, A.$

Rule 2 (Symmetry) The following topological relations are symmetric:

$A \,\texttt{EQ}\, B \Leftrightarrow B \,\texttt{EQ}\, A \qquad A \,\texttt{OL}\, B \Leftrightarrow B \,\texttt{OL}\, A \qquad A \,\texttt{EC}\, B \Leftrightarrow B \,\texttt{EC}\, A \qquad A \,\texttt{DJ}\, B \Leftrightarrow B \,\texttt{DJ}\, A.$

Rule 3 (Inverse Property) The following directional relations are inverses of each other:

$A \,\texttt{NT}\, B \Leftrightarrow B \,\texttt{ST}\, A \qquad A \,\texttt{NE}\, B \Leftrightarrow B \,\texttt{SW}\, A \qquad A \,\texttt{NW}\, B \Leftrightarrow B \,\texttt{SE}\, A$

$A \,\texttt{ET}\, B \Leftrightarrow B \,\texttt{WT}\, A \qquad A \,\texttt{LT}\, B \Leftrightarrow B \,\texttt{RT}\, A \qquad A \,\texttt{AB}\, B \Leftrightarrow B \,\texttt{BL}\, A.$

Rule 4 (Transitivity) Let $\theta \in \{\texttt{NW}, \texttt{NE}, \texttt{SW}, \texttt{SE}, \texttt{LT}, \texttt{RT}, \texttt{AB}, \texttt{BL}, \texttt{IS}, \texttt{EQ}\}$ then

$A\theta B \wedge B\theta C \Rightarrow A\theta C.$

Rule 5 (Implication) Some relations imply other relations:

$A\,\texttt{IS}\,B \Rightarrow A\,\texttt{OL}\,B \qquad A\,\texttt{CV}\,B \Rightarrow A\,\texttt{OL}\,B \qquad A\,\texttt{EQ}\,B \Rightarrow A\,\texttt{OL}\,B$

$A\,\texttt{NT}\,B \Rightarrow A\,\texttt{AB}\,B \qquad A\,\texttt{ST}\,B \Rightarrow A\,\texttt{BL}\,B$

$A\,\texttt{WT}\,B \Rightarrow A\,\texttt{LT}\,B \qquad A\,\texttt{ET}\,B \Rightarrow A\,\texttt{RT}\,B.$

Rule 6 The relationships between $\{\texttt{IS}, \texttt{CV}\}$ and $\{\texttt{OL}, \texttt{DJ}\}$ are:

$A\,\texttt{IS}\,B \wedge A\,\texttt{OL}\,C \Rightarrow B\,\texttt{OL}\,C \qquad A\,\texttt{IS}\,B \wedge B\,\texttt{DJ}\,C \Rightarrow A\,\texttt{DJ}\,C$

$A\,\texttt{CV}\,B \wedge B\,\texttt{OL}\,C \Rightarrow A\,\texttt{OL}\,C \qquad A\,\texttt{CV}\,B \wedge A\,\texttt{DJ}\,C \Rightarrow B\,\texttt{DJ}\,C.$

The first formula indicates that if A is *inside* of B and A *overlaps* C, then B *overlaps* C, while the second formula indicates that if A is *inside* of B and A is *disjoint* from C, then A is *disjoint* from C. We have similar cases for the relation *cover* ($\texttt{CV}$).

Rule 7 This rule indicates relationships between topological relation $\{\texttt{IS}, \texttt{CV}\}$ and the positional directional relations $\{\texttt{LT}, \texttt{RT}, \texttt{AB}, \texttt{BL}\}$. Suppose $\theta \in \{\texttt{LT}, \texttt{RT}, \texttt{AB}, \texttt{BL}\}$ then

$A\,\texttt{IS}\,B \wedge B\theta C \Rightarrow A\theta C \qquad A\,\texttt{CV}\,B \wedge A\theta C \Rightarrow B\theta C.$

Rule 8 This rule indicates relationships between topological relations $\{\texttt{IS}, \texttt{CV}\}$ and the strict directional relations $\{\texttt{ST}, \texttt{WT}, \texttt{NT}, \texttt{ET}\}$:

$A\,\texttt{IS}\,B \wedge B\,\texttt{NT}\,C \Rightarrow A\,\texttt{AB}\,C \qquad A\,\texttt{IS}\,B \wedge B\,\texttt{ST}\,C \Rightarrow A\,\texttt{BL}\,C$

$A\,\texttt{IS}\,B \wedge B\,\texttt{WT}\,C \Rightarrow A\,\texttt{LT}\,C \qquad A\,\texttt{IS}\,B \wedge B\,\texttt{ET}\,C \Rightarrow A\,\texttt{RT}\,C$

$A\,\texttt{CV}\,B \wedge A\,\texttt{NT}\,C \Rightarrow B\,\texttt{AB}\,C \qquad A\,\texttt{CV}\,B \wedge A\,\texttt{ST}\,C \Rightarrow B\,\texttt{BL}\,C$

$A\,\texttt{CV}\,B \wedge A\,\texttt{WT}\,C \Rightarrow B\,\texttt{LT}\,C \qquad A\,\texttt{CV}\,B \wedge A\,\texttt{ET}\,C \Rightarrow B\,\texttt{RT}\,C.$

Rule 9 Suppose $\theta \in \{\texttt{LT}, \texttt{RT}, \texttt{AB}, \texttt{BL}\}$. The relationships between $\texttt{OL}$ and $\{\texttt{LT}, \texttt{RT}, \texttt{AB}, \texttt{BL}\}$ are

$A\theta B \wedge B\,\texttt{OL}\,C \wedge C\theta D \Rightarrow A\theta D.$

This rule captures the interaction between the *overlap* ($\texttt{OL}$) relation and the positional directional relations. For example, from A *left* of B, B *overlap* C, and C *left* of D we can deduce A *left* of D.

Rule 10 The relationships between $\texttt{OL}$ and $\{\texttt{ST}, \texttt{WT}, \texttt{NT}, \texttt{ET}\}$ are

$A\,\texttt{NT}\,B \wedge B\,\texttt{OL}\,C \wedge C\,\texttt{NT}\,D \Rightarrow A\,\texttt{AB}\,D \qquad A\,\texttt{WT}\,B \wedge B\,\texttt{OL}\,C \wedge C\,\texttt{WT}\,D \Rightarrow A\,\texttt{LT}\,C$

$A\,\texttt{ST}\,B \wedge B\,\texttt{OL}\,C \wedge C\,\texttt{ST}\,D \Rightarrow A\,\texttt{BL}\,C \qquad A\,\texttt{ET}\,B \wedge B\,\texttt{OL}\,C \wedge C\,\texttt{ET}\,D \Rightarrow A\,\texttt{RT}\,C.$

This rule captures the interaction between the *overlap* ($\texttt{OL}$) relation and the strict directional relations. If we have A *north* of B, B *overlap* C, and C *north* of D, then we can deduce A *above* of D. Note that we cannot deduce A *north* of D, which appears to hold intuitively. Consider Figure 5, both (a) and (b) satisfy formula one. In Figure 5(a) we do have A *north* of D. However, in Figure 5(b) we have A *northeast* of D. Therefore, A *north* of D does not hold generally. This is because the strict and mixed directional relations are exclusive. That is, at most, one can hold between any two objects.

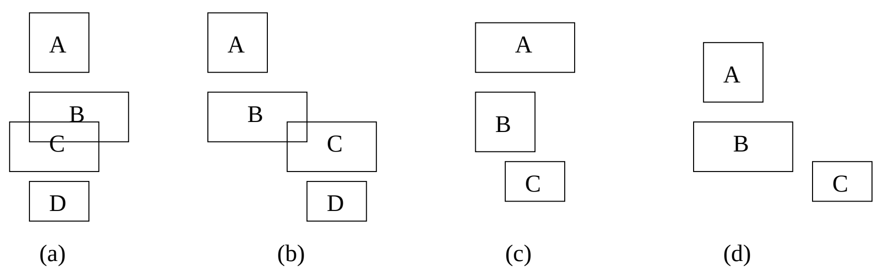


Figure 5: Relations *north* and *overlap* only imply relation *above*

Rule 11 The relationships between $\{\texttt{ST}, \texttt{WT}, \texttt{NT}, \texttt{ET}\}$ and $\{\texttt{NW}, \texttt{NE}, \texttt{SW}, \texttt{SE}\}$:

$$A \,\texttt{NT}\, B \wedge B \,\texttt{NW}\, C \Rightarrow A \,\texttt{AB}\, C \qquad A \,\texttt{NT}\, B \wedge B \,\texttt{NE}\, C \Rightarrow A \,\texttt{AB}\, C$$
$$A \,\texttt{ST}\, B \wedge B \,\texttt{SW}\, C \Rightarrow A \,\texttt{BL}\, C \qquad A \,\texttt{ST}\, B \wedge B \,\texttt{SE}\, C \Rightarrow A \,\texttt{BL}\, C$$
$$A \,\texttt{WT}\, B \wedge B \,\texttt{NW}\, C \Rightarrow A \,\texttt{LT}\, C \qquad A \,\texttt{WT}\, B \wedge B \,\texttt{SW}\, C \Rightarrow A \,\texttt{LT}\, C$$
$$A \,\texttt{ET}\, B \wedge B \,\texttt{NE}\, C \Rightarrow A \,\texttt{RT}\, C \qquad A \,\texttt{ET}\, B \wedge B \,\texttt{SE}\, C \Rightarrow A \,\texttt{RT}\, C.$$

This rule captures the interactions between strict directional relations and mixed directional relations and indicates that the resulting relation is a positional directional relation. For example, if A is *north* of B and B is *northwest* of C, then A is *above* B. Note we cannot be certain whether A *north* of B or A *northwest* of B, although we know one of them is true. This is demonstrated in (c) and (d) of Figure 5.

Rule 12 The relationships between $\{\texttt{NW}, \texttt{NE}, \texttt{SW}, \texttt{SE}\}$ and $\{\texttt{ST}, \texttt{WT}, \texttt{NT}, \texttt{ET}\}$ are

$$A \,\texttt{NW}\, B \wedge B \,\texttt{NT}\, C \Rightarrow A \,\texttt{AB}\, C \qquad A \,\texttt{NE}\, B \wedge B \,\texttt{NT}\, C \Rightarrow A \,\texttt{AB}\, C$$
$$A \,\texttt{SW}\, B \wedge B \,\texttt{ST}\, C \Rightarrow A \,\texttt{BL}\, C \qquad A \,\texttt{SE}\, B \wedge B \,\texttt{ST}\, C \Rightarrow A \,\texttt{BL}\, C$$

$$A\,\mathtt{NW}\,B \wedge B\,\mathtt{WT}\,C \Rightarrow A\,\mathtt{LT}\,C \qquad A\,\mathtt{SW}\,B \wedge B\,\mathtt{WT}\,C \Rightarrow A\,\mathtt{LT}\,C$$

$$A\,\mathtt{NE}\,B \wedge B\,\mathtt{ET}\,C \Rightarrow A\,\mathtt{RT}\,C \qquad A\,\mathtt{SE}\,B \wedge B\,\mathtt{ET}\,C \Rightarrow A\,\mathtt{RT}\,C.$$

These are the same relationships as **Rule 11** but with different orders of the directional relations.

Rule 13 The relationships between $\{\mathtt{LT},\mathtt{RT},\mathtt{AB},\mathtt{BL}\}$ and $\{\mathtt{ST},\mathtt{WT},\mathtt{NT},\mathtt{ET},\mathtt{NW},\mathtt{NE},\mathtt{SW},\mathtt{SE}\}$ are

$$A\,\mathtt{LT}\,B \wedge B\,\{\mathtt{WT},\mathtt{NW},\mathtt{SW}\}\,C \Rightarrow A\,\mathtt{LT}\,C \qquad A\,\mathtt{RT}\,B \wedge B\,\{\mathtt{NE},\mathtt{ET},\mathtt{SE}\}\,C \Rightarrow A\,\mathtt{RT}\,C$$

$$A\,\mathtt{AB}\,B \wedge B\,\{\mathtt{NE},\mathtt{NT},\mathtt{NW}\}\,C \Rightarrow A\,\mathtt{AB}\,C \qquad A\,\mathtt{BL}\,B \wedge B\,\{\mathtt{SE},\mathtt{ST},\mathtt{SW}\}\,C \Rightarrow A\,\mathtt{BL}\,C.$$

This rule captures the interactions between positional directional relations and other directional relations.

Rule 14 Suppose $\theta \in \{\mathtt{LT},\mathtt{RT},\mathtt{AB},\mathtt{BL}\}$. The relationships between $\mathtt{EC}$ and $\{\mathtt{LT},\mathtt{RT},\mathtt{AB},\mathtt{BL}\}$ are

$$A\theta B \wedge B\,\mathtt{EC}\,C \wedge C\theta D \Rightarrow A\theta D.$$

This rule captures the interactions between the *externally connected* relation and the positional directional relations. Suppose A *left* of B, B *externally connected* to C, and C *left* of D, this rule allows us to infer that A *left* of D.

Rule 15 The relationships between $\mathtt{EC}$ and $\{\mathtt{ST},\mathtt{WT},\mathtt{NT},\mathtt{ET}\}$ are

$$A\,\mathtt{NT}\,B \wedge B\,\mathtt{EC}\,C \wedge C\,\mathtt{NT}\,D \Rightarrow A\,\mathtt{AB}\,D \qquad A\,\mathtt{ST}\,B \wedge B\,\mathtt{EC}\,C \wedge C\,\mathtt{ST}\,D \Rightarrow A\,\mathtt{BL}\,D$$

$$A\,\mathtt{WT}\,B \wedge B\,\mathtt{EC}\,C \wedge C\,\mathtt{WT}\,D \Rightarrow A\,\mathtt{LT}\,D \qquad A\,\mathtt{ET}\,B \wedge B\,\mathtt{EC}\,C \wedge C\,\mathtt{ET}\,D \Rightarrow A\,\mathtt{RT}\,D.$$

This rule shows the interactions between the *externally connected* relation and the strict directional relations.

5 Correctness of the rules

In this section we prove the correctness (soundness) of all the inference rules given in the previous section. Since in our proofs we frequently use the results of the transitivity table described in [All83] as Figure 4, we include the table here as Table 3 and cite it it as *Trans. Table*. Before we give the proofs some lemmas are necessary.

	b	bi	d	di	o	oi	m	mi	s	si	f	fi
b	b		b o m d s	b	b	b o m d s	b	b o m d s	b	b	b o m d s	b
bi		bi	bi oi mi d f	bi	bi oi mi d f	bi	bi oi mi d f	bi	bi oi mi d f	bi	bi	bi
d	b	bi	d		b o m d s	bi oi mi d f	b	bi	d	bi oi mi d f	d	b o m d s
di	b o m di fi	bi oi di mi si	o oi d di e	di	o di fi	oi di si	o di fi	oi di si	di fi o	di	di si oi	di
o	b	bi oi di mi si	o d s	b o m di fi	b o m	o oi d di e	b	oi di si	o	di fi o	d s o	b o m
oi	b o m di fi	bi	oi d f	bi oi mi di si	o oi d di e	bi oi mi	o di fi	bi	oi d f	oi bi mi	oi	oi di si
m	b	bi oi di mi si	o d s	b	b	o d s	b	f fi e	m	m	d s o	b
mi	b o m di fi	bi	oi d f	bi	oi d f	bi	s si e	bi	d f oi	bi	mi	mi
s	b	bi	d	b o m di fi	b o m	oi d f	b	mi	s	s si e	d	b m o
si	b o m di fi	bi	oi d f	di	o di fi	oi	o di fi	mi	s si e	si	oi	di
f	b	bi	d	bi oi mi di si	o d s	bi oi mi	m	bi	d	bi oi mi	f	f fi e
fi	b	bi oi mi di si	o d s	di	o	oi di si	m	si oi di	o	di	f fi e	fi

Table 3: The Transitivity Table From FIGURE 4 in [All83].

Lemma 1 $\{\mathtt{d}, \mathtt{di}, \mathtt{s}, \mathtt{si}, \mathtt{f}, \mathtt{fi}, \mathtt{e}\}$ and $\{\mathtt{d}, \mathtt{di}, \mathtt{s}, \mathtt{si}, \mathtt{f}, \mathtt{fi}, \mathtt{o}, \mathtt{oi}, \mathtt{e}\}$ are symmetric:

$A\,\{\mathtt{d}, \mathtt{di}, \mathtt{s}, \mathtt{si}, \mathtt{f}, \mathtt{fi}, \mathtt{e}\}\,B \Leftrightarrow B\,\{\mathtt{d}, \mathtt{di}, \mathtt{s}, \mathtt{si}, \mathtt{f}, \mathtt{fi}, \mathtt{e}\}\,A$

$A\,\{\mathtt{d}, \mathtt{di}, \mathtt{s}, \mathtt{si}, \mathtt{f}, \mathtt{fi}, \mathtt{o}, \mathtt{oi}, \mathtt{e}\}\,B \Leftrightarrow B\,\{\mathtt{d}, \mathtt{di}, \mathtt{s}, \mathtt{si}, \mathtt{f}, \mathtt{fi}, \mathtt{o}, \mathtt{oi}, \mathtt{e}\}\,A$.

Proof: The proof is trivial if we notice that all the relations and their inverses are in the set. For example, from $A\,\{\mathtt{d}\}\,B \Leftrightarrow B\,\{\mathtt{di}\}\,A$, we can have $A\,\{\mathtt{d}, \mathtt{di}\}\,B \Leftrightarrow B\,\{\mathtt{d}, \mathtt{di}\}\,A$. ■

Lemma 2 $A\,\{\mathtt{o}\}\,B \wedge B\,\{\mathtt{o}\}\,C \Rightarrow A\,\{\mathtt{b}, \mathtt{m}, \mathtt{o}\}\,C$. (a direct result from Trans. Table)

Lemma 3 $A\,\{\mathtt{b}, \mathtt{m}\}\,B \wedge B\,\{\mathtt{o}\}\,C \Rightarrow A\,\{\mathtt{b}\}\,C$. (a direct result from Trans. Table)

Lemma 4 $A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,B \wedge B\,\{\mathtt{bi}, \mathtt{mi}\}\,C \Rightarrow A\,\{\mathtt{bi}\}\,C$.

Proof: $A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,B$ indicates that A is always to the right of B and $B\,\{\mathtt{bi}, \mathtt{mi}\}\,C$ indicates that B is always to the right of C. Therefore, in a one dimensional space, A is always to the right of C, i.e., $A\,\{\mathtt{bi}\}\,C$. Furthermore $A\,\{\mathtt{mi}\}\,C$ cannot be true because A and B have neither the relation *started_by* ($\mathtt{si}$) nor the relation *equal* ($\mathtt{e}$). ■

Lemma 5 $A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,B \wedge B\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,C \Rightarrow A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,C$.

Proof: $A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,B \wedge B\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,C$

$\Leftrightarrow (A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,B \wedge B\,\{\mathtt{bi}, \mathtt{mi}\}\,C) \vee (A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,B \wedge B\,\{\mathtt{oi}\}\,C)$

$\Rightarrow A\,\{\mathtt{bi}\}\,C \vee (A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,B \wedge B\,\{\mathtt{oi}\}\,C)$ (Lemma 4)

$\Rightarrow A\,\{\mathtt{bi}\}\,C \vee A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,C$ (Trans. Table)

$\Rightarrow A\,\{\mathtt{bi}, \mathtt{mi}, \mathtt{oi}\}\,C$. ■

Lemma 6 $A\,\{\mathtt{b}, \mathtt{m}, \mathtt{o}, \mathtt{d}, \mathtt{s}\}\,B \wedge B\,\{\mathtt{b}, \mathtt{m}\}\,C \Rightarrow A\,\{\mathtt{b}\}\,C$.

Proof: $A\,\{\mathtt{b}, \mathtt{m}, \mathtt{o}, \mathtt{d}, \mathtt{s}\}\,B \wedge B\,\{\mathtt{b}, \mathtt{m}\}\,C$

$\Leftrightarrow (A\,\{\mathtt{b}, \mathtt{m}, \mathtt{o}\}\,B \wedge B\,\{\mathtt{b}, \mathtt{m}\}\,C) \vee (A\,\{\mathtt{d}, \mathtt{s}\}\,B \wedge B\,\{\mathtt{b}, \mathtt{m}\}\,C)$

$\Rightarrow A\,\{\mathtt{b}\}\,C \vee (A\,\{\mathtt{d},\mathtt{s}\}\,B \wedge B\,\{\mathtt{b},\mathtt{m}\}\,C)$ (Inverse Property and Lemma 4)

$\Rightarrow A\,\{\mathtt{b}\}\,C \vee (A\,\{\mathtt{b}\}\,C)$ (Trans. Table)

$\Rightarrow A\,\{\mathtt{b}\}\,C.$ ■

Lemma 7 $A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B \wedge B\,\{\mathtt{b},\mathtt{m}\}\,C \Rightarrow A\,\{\mathtt{b},\mathtt{m}\}\,C$

Proof: $A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B \wedge B\,\{\mathtt{b},\mathtt{m}\}\,C$

$\Leftrightarrow (A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,B \wedge B\,\{\mathtt{b},\mathtt{m}\}\,C) \vee (A\,\{\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B \wedge B\,\{\mathtt{b},\mathtt{m}\}\,C)$

$\Rightarrow A\,\{\mathtt{b}\}\,C \vee (A\,\{\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B \wedge B\,\{\mathtt{b},\mathtt{m}\}\,C)$ (Lemma 6)

$\Rightarrow A\,\{\mathtt{b}\}\,C \vee (A\,\{\mathtt{b},\mathtt{m}\}\,C)$ (Trans. Table)

$\Rightarrow A\,\{\mathtt{b},\mathtt{m}\}\,C.$ ■

Lemma 8 $A\,\{\mathtt{b},\mathtt{m}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C \Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C.$

Proof: $A\,\{\mathtt{b},\mathtt{m}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C$

$\Leftrightarrow (A\,\{\mathtt{b}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C) \vee (A\,\{\mathtt{m}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C)$

$\Rightarrow (A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C) \vee (A\,\{\mathtt{m}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C)$ (Trans. Table)

$\Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C \vee (A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C)$ (Trans. Table)

$\Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C$ ■

Lemma 9 $A\,\{\mathtt{b},\mathtt{m}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{m},\mathtt{mi},\mathtt{e}\}\,C \Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,C.$

Proof: $A\,\{\mathtt{b},\mathtt{m}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{m},\mathtt{mi},\mathtt{e}\}\,C$

$\Leftrightarrow (A\,\{\mathtt{b},\mathtt{m}\}\,B \wedge B\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C) \vee (A\,\{\mathtt{b},\mathtt{m}\}\,B \wedge B\,\{\mathtt{m},\mathtt{mi}\}\,C)$

$\Rightarrow (A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C) \vee (A\,\{\mathtt{b},\mathtt{m}\}\,B \wedge B\,\{\mathtt{m},\mathtt{mi}\}\,C)$ (Lemma 8)

$\Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C \vee (A\,\{\mathtt{b}\}\,B \wedge B\,\{\mathtt{m},\mathtt{mi}\}\,C) \vee (A\,\{\mathtt{m}\}\,B \wedge B\,\{\mathtt{m},\mathtt{mi}\}\,C)$

$\Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C \vee (A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C) \vee (A\,\{\mathtt{f},\mathtt{fi},\mathtt{e}\}\,C)$ (Trans. Table)

$\Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C \vee A\,\{\mathtt{f},\mathtt{fi},\mathtt{e}\}\,C$

$\Rightarrow A\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,C.$ ■

Theorem **Rules 1-15** are correct.

Proof:

Rule 1 Since $A_x\,\{\mathtt{e}\}\,A_x$ and $A_y\,\{\mathtt{e}\}\,A_y$, from the definition of EQ we have $A\,\mathtt{EQ}\,A$. The same is for the OL.

Rule 3 $A\,\mathtt{NT}\,B \Leftrightarrow A_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y$

$\Leftrightarrow B_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,A_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y$ (Lemma 1)

$\Leftrightarrow B_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,A_x \wedge B_y\,\{\mathtt{b},\mathtt{m}\}\,A_y$ (Inverse property)

$\Leftrightarrow B\,\mathtt{ST}\,A.$

It is similar to other strict directional relations. As for the positional directional relations, we have

$A\,\mathtt{LT}\,B \Leftrightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \Leftrightarrow B_x\,\{\mathtt{bi},\mathtt{mi}\}\,A_x \Leftrightarrow B\,\mathtt{RT}\,A$

$A\,\mathtt{AB}\,B \Leftrightarrow A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y \Leftrightarrow B_y\,\{\mathtt{b},\mathtt{m}\}\,A_y \Leftrightarrow B\,\mathtt{BL}\,A.$

Rule 4 $A\,\mathtt{NW}\,B \wedge B\,\mathtt{NW}\,C \Leftrightarrow [(A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y) \vee (A_x\,\{\mathtt{o}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y)]\wedge$
$[(B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y) \vee (B_x\,\{\mathtt{o}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi}\}\,C_y)]$

There are four cases to consider:

1. $A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y \wedge B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y$

   $\Leftrightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y \wedge B_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y$

   $\Rightarrow A_x\,\{\mathtt{b}\}\,C_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y \wedge B_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y$ (Lemma 6)

   $\Rightarrow A_x\,\{\mathtt{b}\}\,C_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y$ (Lemma 5)

   $\Rightarrow A\,\mathtt{NW}\,C$

2. $A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y \wedge B_x\,\{\mathtt{o}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi}\}\,C_y$

   $\Leftrightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge B_x\,\{\mathtt{o}\}\,C_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y \wedge B_y\,\{\mathtt{bi},\mathtt{mi}\}\,C_y$

   $\Rightarrow A_x\,\{\mathtt{b}\}\,C_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,B_y \wedge B_y\,\{\mathtt{bi},\mathtt{mi}\}\,C_y$ (Lemma 3)

   $\Rightarrow A_x\,\{\mathtt{b}\}\,C_x \wedge A_y\,\{\mathtt{bi}\}\,C_y$ (Lemma 4)

   $\Rightarrow A\,\mathtt{NW}\,C$

3. $A_x\,\{\mathtt{o}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y \wedge B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y$

   $\Leftrightarrow A_x\,\{\mathtt{o}\}\,B_x \wedge B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y \wedge B_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y$

$\Rightarrow A_x\,\{\mathtt{o}\}\,B_x \wedge B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge A_y\,\{\mathtt{bi}\}\,C_y$ (Lemma 4)

$\Rightarrow A_x\,\{\mathtt{b}\}\,C_x \wedge A_y\,\{\mathtt{bi}\}\,C_y$ (Lemma 3)

$\Rightarrow A\,\mathtt{NW}\,C$

4. $A_x\,\{\mathtt{o}\}\,B_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y \wedge B_x\,\{\mathtt{o}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi}\}\,C_y$

   $\Leftrightarrow A_x\,\{\mathtt{o}\}\,B_x \wedge B_x\,\{\mathtt{o}\}\,C_x \wedge A_y\,\{\mathtt{bi},\mathtt{mi}\}\,B_y \wedge B_y\,\{\mathtt{bi},\mathtt{mi}\}\,C_y$

   $\Rightarrow A_x\,\{\mathtt{o}\}\,B_x \wedge B_x\,\{\mathtt{o}\}\,C_x \wedge A_y\,\{\mathtt{bi}\}\,C_y$ (Lemma 4)

   $\Rightarrow A_x\,\{\mathtt{b},\mathtt{m},\mathtt{o}\}\,C_x \wedge A_y\,\{\mathtt{bi}\}\,C_y$ (Lemma 2)

   $\Rightarrow A\,\mathtt{NW}\,C$

Similar proof can be constructed for other mixed directional relations.

Rule 9 $A\,\mathtt{LT}\,B \wedge B\,\mathtt{OL}\,C \wedge C\,\mathtt{LT}\,D$

$\Leftrightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge B_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C_x \wedge B_y\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C_y \wedge C_x\,\{\mathtt{b},\mathtt{m}\}\,D_x$

$\Rightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge B_x\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{o},\mathtt{oi},\mathtt{e}\}\,C_x \wedge C_x\,\{\mathtt{b},\mathtt{m}\}\,D_x$ (drop y-interval)

$\Rightarrow A_x\,\{\mathtt{b},\mathtt{m},\mathtt{o},\mathtt{d},\mathtt{s}\}\,C_x \wedge C_x\,\{\mathtt{b},\mathtt{m}\}\,D_x$ (Lemma 8)

$\Rightarrow A_x\,\{\mathtt{b}\}\,D_x$ (Lemma 6)

$\Rightarrow A\,\mathtt{LT}\,D$

Others are similar.

Rule 13 $A\,\mathtt{LT}\,B \wedge B\,\{\mathtt{WT},\mathtt{NW},\mathtt{SW}\}\,C \Rightarrow A\,\mathtt{LT}\,C$. There are three cases to consider:

1. $A\,\mathtt{LT}\,B \wedge B\,\mathtt{WT}\,C$

   $\Leftrightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge B_y\,\{\mathtt{d},\mathtt{di},\mathtt{s},\mathtt{si},\mathtt{f},\mathtt{fi},\mathtt{e}\}\,C_y$

   $\Rightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x$ (drop y-interval)

   $\Rightarrow A_x\,\{\mathtt{b}\}\,C_x$ (Lemma 6)

   $\Rightarrow A\,\mathtt{LT}\,C$

2. $A\,\mathtt{LT}\,B \wedge B\,\mathtt{NW}\,C$

   $\Leftrightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge [(B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi},\mathtt{oi}\}\,C_y) \vee (B_x\,\{\mathtt{o}\}\,C_x \wedge B_y\,\{\mathtt{bi},\mathtt{mi}\}\,C_y]$

   $\Rightarrow A_x\,\{\mathtt{b},\mathtt{m}\}\,B_x \wedge (B_x\,\{\mathtt{b},\mathtt{m}\}\,C_x \vee B_x\,\{\mathtt{o}\}\,C_x)$ (drop y-interval)

$\Rightarrow (A_x\,\{\mathtt{b,m}\}\,B_x \wedge B_x\,\{\mathtt{b,m}\}\,C_x) \vee (A_x\,\{\mathtt{b,m}\}\,B_x \wedge B_x\,\{\mathtt{o}\}\,C_x)$

$\Rightarrow (A_x\,\{\mathtt{b}\}\,C_x) \vee (A_x\,\{\mathtt{b,m}\}\,B_x \wedge B_x\,\{\mathtt{o}\}\,C_x)$ (Lemma 6)

$\Rightarrow A_x\,\{\mathtt{b}\}\,C_x \vee A_x\,\{\mathtt{b}\}\,C_x$ (Lemma 3)

$\Rightarrow A_x\,\{\mathtt{b}\}\,C_x$

$\Rightarrow A\,\mathtt{LT}\,C$

3. $A\,\mathtt{LT}\,B \wedge B\,\mathtt{SW}\,C$

$\Leftrightarrow A_x\,\{\mathtt{b,m}\}\,B_x \wedge [(B_x\,\{\mathtt{b,m}\}\,C_x \wedge B_y\,\{\mathtt{b,m,o}\}\,C_y) \vee (B_x\,\{\mathtt{o}\}\,C_x \wedge B_y\,\{\mathtt{b,m}\}\,C_y]$

$\Rightarrow A_x\,\{\mathtt{b,m}\}\,B_x \wedge (B_x\,\{\mathtt{b,m}\}\,C_x \vee B_x\,\{\mathtt{o}\}\,C_x)$ (drop y-interval)

$\Rightarrow A_x\,\{\mathtt{b}\}\,C_x$ (proof of case 2 of Rule 13)

$\Rightarrow A\,\mathtt{LT}\,C$

Rule 14 $A\,\mathtt{LT}\,B \wedge B\,\mathtt{EC}\,C \wedge C\,\mathtt{LT}\,D$

$\Leftrightarrow A_x\,\{\mathtt{b,m}\}\,B_x \wedge [(B_x\,\{\mathtt{m,mi}\}\,C_x \wedge B_y\,\{\mathtt{d,di,s,si,f,fi,o,oi,m,mi,e}\}\,C_y)\vee$
$(B_x\,\{\mathtt{d,di,s,si,f,fi,o,oi,m,mi,e}\}\,C_x \wedge B_y\,\{\mathtt{m,mi}\}\,C_y)] \wedge C_x\,\{\mathtt{b,m}\}\,D_x$

$\Rightarrow A_x\,\{\mathtt{b,m}\}\,B_x \wedge (B_x\,\{\mathtt{m,mi}\}\,C_x \vee B_x\,\{\mathtt{d,di,s,si,f,fi,o,oi,m,mi,e}\}\,C_x)\wedge$
$C_x\,\{\mathtt{b,m}\}\,D_x$ (Drop y-interval)

$\Leftrightarrow (A_x\,\{\mathtt{b,m}\}\,B_x \wedge B_x\,\{\mathtt{m,mi}\}\,C_x \wedge C_x\,\{\mathtt{b,m}\}\,D_x)\vee$
$(A_x\,\{\mathtt{b,m}\}\,B_x \wedge B_x\,\{\mathtt{d,di,s,si,f,fi,o,oi,m,mi,e}\}\,C_x \wedge C_x\,\{\mathtt{b,m}\}\,D_x)$

$\Rightarrow (A_x\,\{\mathtt{b,m}\}\,B_x \wedge B_x\,\{\mathtt{m,mi}\}\,C_x \wedge C_x\,\{\mathtt{b,m}\}\,D_x)\vee$
$(A_x\,\{\mathtt{b,m,o,d,s,f,fi,e}\}\,C_x \wedge C_x\,\{\mathtt{b,m}\}\,D_x)$ (Lemma 9)

$\Rightarrow (A_x\,\{\mathtt{b,m}\}\,B_x \wedge B_x\,\{\mathtt{m,mi}\}\,C_x \wedge C_x\,\{\mathtt{b,m}\}\,D_x) \vee (A_x\,\{\mathtt{b,m}\}\,D_x)$ (Lemma 7)

$\Rightarrow (A_x\,\{\mathtt{b,m,o,d,s,f,fi,e}\}\,C_x \wedge C_x\,\{\mathtt{b,m}\}\,D_x) \vee (A_x\,\{\mathtt{b,m}\}\,D_x)$ (Trans. Table)

$\Rightarrow (A_x\,\{\mathtt{b,m}\}\,D_x) \vee A_x\,\{\mathtt{b,m}\}\,D_x$ (Lemma 7)

$\Rightarrow A_x\,\{\mathtt{b,m}\}\,D_x$

$\Rightarrow A\,\mathtt{LT}\,C$

The proofs of Rules 2, 5-8 are trivial, the proof of Rule 10 is similar to the proof of Rule 9, and the proofs of Rules 11 and 12 are similar to the proof of Rule 13. They all are omitted. ∎

6 Applications

We have witnessed an increasing interest in multimedia technology in recent years. In particular, image and video databases have received enormous attention. One important aspect of image and video databases is the spatial relations between objects. A query subsystem is usually provided to support efficient image or video retrievals based on user queries. In reality, user queries usually contain spatial constraints or relations which must be satisfied when the results are returned to users. How can our spatial inference rules be used to retrieve data from an MMDBMS? Since all the rules are propositional Horn clauses, we claim that they can be easily integrated into any MMDBMS by either using a logic language with a simple inference engine (like DATALOG and LDL [Ull85, TZ86]) or using a lookup table. In terms of spatial properties, there is not much difference between a video and an image database. Therefore, we restrict our discussion to image databases only. We assume that there exists an image database with an image processing subsystem which is able to extract image features, such as salient objects, events, spatial relations between objects etc. Intuitively, a salient object is a semantic entity contained in the image which is meaningful in the application domain [GRV96]. For example, at the physical representation level (e.g., bitmap), a salient object is defined as a subset of the image pixels. This subsystem is doing image preprocessing and generates all the information for building image indexes within the database.

Let us consider a locomotive image example as shown in Figure 6. Suppose the salient objects are the cab (cab), the window (window), the left big wheel (lbw), the right big wheel (rbw), the left small wheel (lsw), the right small wheel (rsw), the smokestack (smokestack), and the body (body). From the image, we have the spatial relations: {window IS cab, cab DJ lbw, cab LT body, cab AB lbw, body EC rbw, body NW rsw, body NT rbw, smokestack NE body, lbw WT rbw, rbw WT lsw}.

The following interesting spatial relations can be derived:

- Since the cab is to the left of the body and the body is to the northwest of the right small wheel (cab LT body $\wedge$ body NW rsw), we can derive that the cab is to the left of the right wheel (cab LT rsw) by **Rule 13**.

- Since the window is inside the cab and the cab is disjoint from the left big wheel (window IS cab$\wedge$ cab DJ lbw), we can derive that the window is disjoint from the left big wheel (window DJ lbw) by **Rule 6**.

- Given the window is inside the cab and the cab is above the left big wheel (window IS cab $\wedge$ cab AB lbw), we can derive that the window is above the left big wheel (window AB lbw) by **Rule 7**.

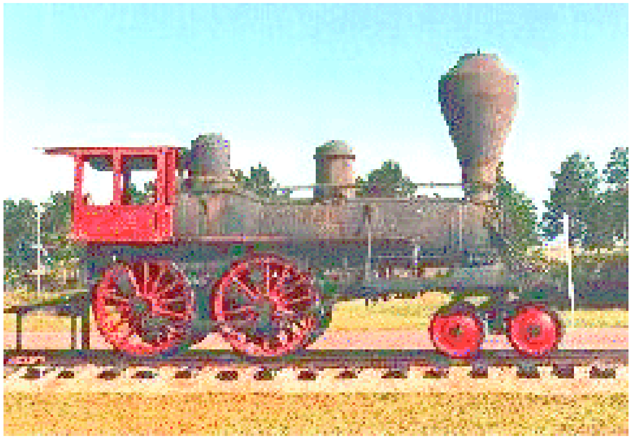
- Since the smokestack is to the northeast of the body and the body is to the north of the left big wheel (smokestack NE body $\wedge$ body NT rbw), we can derive that the smokestack is above the right big wheel (smokestack AB rbw) by **Rule 12**.

- Given the left big wheel is to the west of the right big wheel and the right big wheel is to the west of the left small wheel (lbw WT rbw $\wedge$ rbw WT lsw), we can derive that the left big wheel is to the west of the left small wheel (lbw WT lsw) by **Rule 4**. Furthermore, we can infer that the left big wheel is to the left of the left small wheel (lbw LT lsw) by **Rule 5**.

- Since the cab is to the left of the body and the body is externally connected to the right big wheel, and the right big wheel is to the left of the left small wheel (cab LT body $\wedge$ body EC rbw $\wedge$ rbw LT lsw, rbw LT lsw is derived from rbw WT lsw from **Rule 5**), we can derive that the left big wheel is to the west of the left small wheel (lbw WT lsw) by **Rule 4**. Furthermore, we can deduce that the cab is to the left of the left small wheel (cab LT lsw) by **Rule 14**.

The above derived relations are not complete, i.e., there are many other relations which are derivable from the given relations.

Depending on the data models we can either use *metadata* (data about data) or *attributes* (associated with objects in object-oriented DBMS) to capture the semantics of images and such semantics includes object spatial properties. As discussed before, we assume that some basic spatial relations are generated a priori either by image processing algorithms or manually, or by a hybrid mechanism which is both. These relations are usually stored as metadata or object attributes and will be used



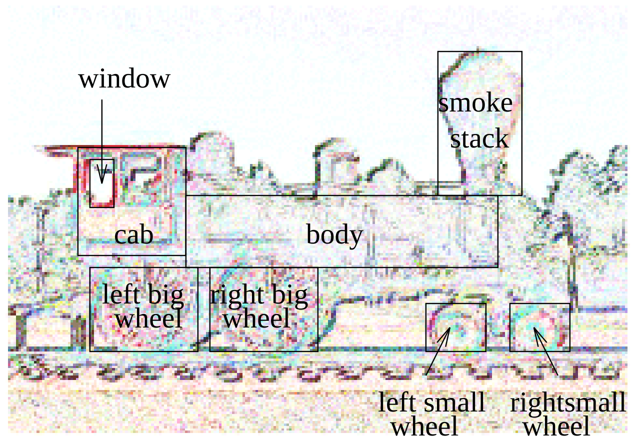


Figure 6: A Locomotive Image and Salient Objects

by the query subsystem for efficient query processing. Together with content-based indexing, the availability of metadata or object attributes avoids the invocation of the expensive image processing algorithms each time a query is processed. Metadata or object attributes may not be *definite* in the sense that it is possible to have more than one relation between two objects. Furthermore they may not be *complete* or may not be necessary to be complete in the sense that not all the spatial relations are explicitly captured by the metadata or object attributes. There are two major reasons that cause such incompleteness [SYH94] . First, it could be impossible for existing image processing algorithms to recognize all the objects and their relations. Second, those implied relations may not be stored explicitly in order to save space. Saving space is particularly attractive [YGBC89] in a distributed environment where the metadata or object attributes are stored at the user sites with limited storage facilities, while the actual images may be stored in remote image archives. The query subsystem executed at the user sites uses the metadata or object attributes to determine the images that need to be retrieved from the remote sites.

7 Conclusions

In this paper we have introduced a unified representation of spatial objects for both topological and directional relations. Such a representation is based on Allen's temporal interval algebra. We have extended the most frequently used four directional relations into twelve directional relations, i.e, adding *southwest, southeast, northwest, northeast, left, right, left, above, below* directional relations. Six topological relations which are adapted from Egenhofer's eight topological relations within our context are discussed in our system. One major contribution of this paper is to have a complete and formal definition of these heterogeneous relations. Another major contribution is to introduce a set of rules to deduce other heterogeneous relations from existing directional and topological relations and prove the correctness of these inference rules. For example, if there are A *north* of B, and B *overlap* C, and C *north* of D, then we have A *above* D. Possible applications are also discussed.

It is straightforward to extend our work into three dimensional space if we only consider two relations *in_front_of* and *behind* as in [SYH94]. For instance, we could have the following:

$A\ \texttt{in\_front\_of}\ B \Leftrightarrow A_z\ \{\texttt{b},\texttt{m}\}\ B_z$ and $A\ \texttt{behind}\ B \Leftrightarrow A_z\ \{\texttt{bi},\texttt{mi}\}\ B_z$

where A_z and B_z are the interval projections over the Z-axis three dimensional space. Then, the set of inference rules can be extended. In order to gain some insightful experience we will integrate these inference rules into our prototype of a video database, based on the Common Video Object Tree model, using a locally developed object-oriented DBMS.

References

[AEG94] A. I. Abdelmoty and B. A. El-Geresy. An intersection-based formalism for representing orientation relations in a geographic database. In *Proceedings of the 2nd ACM Conference on Advances in GIS Theory*, Gaithersburg, MD, 1994.

[All83] J. F. Allen. Maintaining knowledge about temporal intervals. *Communications of ACM*, 26(11):832—843, 1983.

[Bee89] P. V. Beek. Approximation algorithms for temporal reasoning. In *Proceedings of the 11th International Joint Conference on Artificial Intelligence*, pages 1291—1296, Detroit Michigan, August 1989.

[CSE94] E. Clementini, J. Sharma, and M. J. Egenhofer. Modelling topological spatial relations: Strategies for query processing. *Computers and Graphics*, 18(6):815—822, 1994.

[EF91] M. Egenhofer and R. Franzosa. Point-set topological spatial relations. *International Journal of Geographical Information Systems*, 5(2):161—174, 1991.

[Ege94] M. Egenhofer. Spatial SQL: A query and presentation language. *IEEE Transactions on Knowledge and Data Engineering*, 6(1):86—95, January 1994.

[Fre91] C. Freksa. Conceptual neighborhood and its role in temporal and spatial reasoning. In *Proceedings of the IMACS Workshop on Decision Support Systems and Qualitative Reasoning*, pages 181—187, Amsterdam, North-Holland, 1991.

[GPP95] M. Grigni, D. Papadias, and C. Papadimitriou. Topological inference. In *Proceedings of the 14th International Joint Conference on Artificial Intelligence*, pages 901–906, 1995.

[GRV96] V. N. Gudivada, V. V. Raghavan, and K. Vanapipat. A unified approach to data modeling and retrieval for a class of image database applications. In V. S. Subrahmanian and S. Jajodia, editors, *Multimedia Database Systems: Issues and Research Directions*, pages 37—78. Springer Verlag, 1996.

[Her94] D. Hernández. *Qualitative Representation of Spatial Knowledge*. Springer-Verlag, New York, 1994.

[MJ90] A. Mukerjee and G. Joe. A qualitative model for space. In *Proceedings of the 8th AAAI*, pages 721—727, Boston, MA, July 1990.

[NSN95] M. Nabil, J. Shepherd, and H. H. Ngu. 2D projection interval relationships: A symbolic representation of spatial relationships. In *Proceedings of 4th International Symposium on Large Spatial Databases*, pages 292—309, Portland, Maine, USA, August 1995.

[PS94] D. Papadias and T. Sellis. The qualitative representation of spatial knowledge in two-dimensional space. *The VLDB Journal (Special Issue on Spatial Databases)*, 4:100—138, 1994.

[PTSE95] D. Papadias, Y. Theodoridis, T. Sellis, and M. J. Egenhofer. Topological relations in the world of minimum bounding rectangles: A study with R-trees. In *Proceedings of ACM SIGMOD International Conference on Management of Data*, pages 92—103, San Jose, CA, May 1995.

[RCC92] D. A. Randell, Z. Cui, and A. Cohn. A spatial logic based on regions and connection. In *Proceedings of the 3rd International Conference on Principles of Knowledge Representation and Reasoning*, pages 165—176, Cambridge, MA, 1992. Morgan Kaufmann.

[SF95] J. Sharma and D. M. Flewelling. Inferences from combined knowledge about topology and directions. In *Proceedings of 4th International Symposium on Large Spatial Databases*, pages 279—291, Portland, Maine, USA, August 1995.

[SYH94] P. Sistla, C. Yu, and R. Haddack. Reasoning about spatial relationships in picture retrieval systems. In *Proceedings of the 20th International Conference on Very Large Data Bases*, pages 570—581, 1994.

[TZ86] S. Tsur and C. Zaniolo. LDL: A logic based data language. In *Proceedings of the 12th International Conference on Very Large Data Bases*, pages 33—41, Koyoto, Japan, 1986.

[Ull85] J. D. Ullman. Implementation of logical query language for databases. In *Proceedings of ACM SIGMOD International Conference on Management of Data*, pages 444—444, Austin, Texas, May 1985.

[YGBC89] C. T. Yu, K. Guh, D. Brill, and A. Chen. Partition strategy for distributed query processing in fast local networks. *IEEE Transactions on Software Engineering*, pages 780—793, 1989.