National Library of Canada

Acquisitions and Bibliographic Services Branch

395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale du Canada

Direction des acquisitions et des services bibliographiques

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Your file Votre référence

Our file Notre référence

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

Canada

UNIVERSITY OF ALBERTA

DYNAMIC ANALYSIS AND SIMULATION OF MOTION OF RIGID LINKED BODIES

BY ©

JOHN PIERRE BAUDIN

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the degree of DOCTOR OF PHILOSOPHY.

DEPARTMENT OF PHYSICAL EDUCATION AND SPORT STUDIES

Edmonton, Alberta

SPRING, 1993

National Library
of Canada

Acquisitions and
Bibliographic Services Branch

395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale
du Canada

Direction des acquisitions et
des services bibliographiques

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Your file Votre référence

Our file Notre référence

The author has granted an irrevocable non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of his/her thesis by any means and in any form or format, making this thesis available to interested persons.

L'auteur a accordé une licence irrévocable et non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de sa thèse de quelque manière et sous quelque forme que ce soit pour mettre des exemplaires de cette thèse à la disposition des personnes intéressées.

The author retains ownership of the copyright in his/her thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without his/her permission.

L'auteur conserve la propriété du droit d'auteur qui protège sa thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

ISBN 0-315-82040-3

Canada

UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: JOHN PIERRE BAUDIN

TITLE OF THESIS: DYNAMIC ANALYSIS AND SIMULATION OF MOTION OF RIGID LINKED BODIES

DEGREE: DOCTOR OF PHILOSOPHY

YEAR THIS DEGREE WAS GRANTED: 1993

Permission is hereby granted to the University of Alberta Library to reproduce single copies of this thesis and to lend or sell such copies for private, scholarly or scientific research purposes only.

The author reserves all other publication and other rights in association with the copyright in the thesis, and except as hereinbefore provided neither the thesis nor any substantial portion thereof may be printed or otherwise reproduced in any material form whatever without the author's prior written permission.

11511-37 A Avenue
Edmonton, Alberta
T6J 0J6

April 23/93

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled DYNAMIC ANALYSIS AND SIMULATION OF MOTION OF RIGID LINKED BODIES submitted by JOHN PIERRE BAUDIN in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY.

P. Gervais Ph.D. (Co-Supervisor)
R.B. Wilberg Ph.D. (Co-Supervisor)
W.W. Armstrong Ph.D.
Jacques Bobet
J. Bobet Ph.D.
M. W. Green Ph.D.
E. Sprigings Ph.D. (External Examiner)

February 18, 1993

DEDICATION

This thesis is dedicated to all of those people who inspired and supported me in the pursuit of my life long educational goals. Especially it is dedicated to my wife Joan, daughter Jessica, son Daniel, parents John and Corinne, and my uncle Dennis.

ABSTRACT

The purpose of this research was to determine the ability of the recursive, forward dynamics method of Armstrong and Green(1985) to simulate three dimensional human motion in a non-support state using as input the kinetic and kinematic parameters calculated from real motion.

Two rigid link models were used; a three link wooden model with 12 degrees of freedom and a 14 link, 45 degrees of freedom model of the human body. Using three dimensional position and orientation data, taken from cinematographical records, for each link of each model, a computer program was written to calculate dynamic values. Linear and angular kinematic data was determined for all segments and a Newtonian approach was used to solve the inverse equations of motion and produce kinetic values of net reaction forces and net torques acting at each joint.

The quality of the results of the dynamic calculations was found to be dependent on the quality of the data used to determine the link positions and orientations at each timestep. The data from the dynamic calculations was used as input into a computer simulation program that solved the forward dynamics equations using the recursive methods.

The simulation results demonstrated that the recursive technique was easy to implement and was quite user friendly. The methods proved to be very fast with computer to simulated time ratios as low as 4:1, on a personal computer, when simulating motion for the 14 link human model in a state of non-support. The accuracy of the simulations when compared to the original motion was found to be, in large part, dependent on the quality of the dynamic data input into the simulations. As a demonstration of its' flexibility, the simulation technique was also applied to two different types of problems.

It was concluded that the recursive, forward dynamics method of Armstrong and Green (1985) provides a fast and relatively accurate technique for simulating the motion of rigid linked bodies in a non-support state, over short periods of time, using net joint torques as the primary input functions. These results show that it has the potential to be a flexible and efficient tool for the biomechanics researcher.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my supervisory committee members Dr. Pierre Gervais, Dr. Robert Wilberg and Dr. William Armstrong. Their unfailing support, assistance, guidance and prodding allowed me to fulfill this lifelong goal.

I would like to extend my appreciation to the other members of the examining committee; Dr. Jacque Bobet, Dr. Mark Green, Dr. Eric Sprigings, and Dr. Len Wankel. Their questions and input helped to make this thesis something I am proud of.

To my brother Brett, a huge thank you. Without his guidance and assistance in most of the computer aspects of this research, its' completion may never have happened.

I would also like to thank Dr. Juris Terauds for opening my eyes to the wonderful discipline of Biomechanics and Dr. Wendy Bedingfield for providing me with the opportunity to enter the doctoral program.

Finally, to all my family, friends and colleagues, thank you for all the support and encouragement you gave me as I worked over the many years to finish this thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The accurate representation of human motion has long been one of the goals of the biomechanist; a formidable task due to the extremely complex makeup of the human body. One popular method is the use of mathematical equations, borrowed from the study of rigid body mechanics, to model the human body while in motion. These equations are often complex and extremely difficult to solve by hand. However, with all of the advances in computer technology, the biomechanics researcher has ready access to a tool that can greatly reduce time required for their solution. Computer simulation of human movement involves the incorporation of these mathematical equations into a program that will produce an output that represents human motion.

"Whereas the usefulness of analysis of human motion lies in determining what is occurring in the actual movement, simulation allows the freedom to investigate the consequences of altering a movement in any desired manner." (Mann and Sorensen, 1979b, p 86). Miller (1974), Vaughan (1984) and Yeadon et al. (1990d) also feel that the biggest advantage of computer simulation lies in its potential ability to answer "what if" questions such as: "What would happen if an athlete's angular momentum was increased at takeoff in the long jump?" or "Would an adjustment in the moment of inertia of an artificial limb improve the gait of a handicapped person?" Other advantages include:

1. Safety - the athlete is freed from attempting potentially dangerous maneuvers before their feasibility is tested through a simulation.

2. Savings in Time - it is often possible to answer questions about movement patterns much faster with simulation than with traditional methods of biomechanical analysis such as cinematography.
3. Optimization - it may be possible to predict optimal performance by manipulating the variables critical to the motion.
4. Savings in Expenses - using computer simulation to solve human motion problems could prove to be less expensive than the methods usually used by the biomechanist. (Vaughan, 1984).
5. Perfect Repeatability - experimentation with a computer model has the advantage that results are perfectly repeatable, and cannot be obscured by uncontrollable biological variations.

With the body modelled as a series of rigid links, the simulation of human motion is a dynamics problem which can be solved using one of two approaches. A kinematic method uses equations which describe the motions of the model segments without regard for the forces that are required to produce them. The positions of the segments are determined from equations which describe their velocities and accelerations. The alternative is to use kinetic equations of motion which are based on the forces and torques that determine the movement of the body links.

Of the two, the kinetic approach has the potential to better answer the 'what if' questions that motivate the biomechanics researcher to use computer simulation. A kinematic simulation may produce a desired movement in the model segments but as noted by Dapena *"... it does not take into account whether the perturbed angle patterns will require joint torques beyond the capabilities of the subject"*(Dapena, 1981, p. 86). In contrast, kinetic simulation uses a direct dynamics approach and is driven by the forcing functions of the model. When a satisfactory output is achieved the biomechanist can examine the input to determine the feasibility of its reproduction in a real life situation.

Recently, in discussion of direct dynamics computer simulation methods, van Soest and van den Bogert (1992) stated that *"development of new methods should only be considered when existing methods can definitely not be used."* (p. 10) In recent years, three dimensional simulation methods of a general multi linked full human model using a kinetic approach have been developed but in some cases they have not been completely evaluated relative to their complete potential for use in biomechanics. Wilhelms (1985), Isaacs and Cohen (1987), Casolo and Legnani (1989), Lake (1990) and Meglan (1991) are examples of some of the researchers using kinetic (sometimes termed dynamic) methods for simulating human motion. In most cases the equations of motion for the human body are developed in matrix format and then solved using techniques, such as Gaussian elimination combined with an integration method such as the Runge-Kutta. These matrix methods, while accurate and flexible, tend to be computationally intensive and very demanding on computer hardware, especially if the body is modeled with a large number of links.

More efficient alternatives to using large matrix methods for solving dynamics equations of motion have been developed by Armstrong (1979), Luh, Walker and Paul (1980) and Featherstone (1983). In all instances a recursive solution is used to solve the equations and has a computational requirement that varies linearly with the number of links in the system. All methods were originally developed for dynamics calculations in mechanical manipulators but the methods of Armstrong have been applied to simulation of human movement (Armstrong and Green, 1985).

In their model Armstrong and Green have the human body represented as a tree-like, fourteen link structure that is connected by three degree of freedom spherical joints. Based on the original work of Armstrong (1979) on general n-link manipulators, it was shown that a linear relationship exists between the linear acceleration of the proximal joint of a link and the amount of angular acceleration it has. As well, a linear relationship was established between the linear acceleration of a link and the reaction force it produces on the adjoining proximal link. It was demonstrated that this allows a recursive approach to solve the equations of motion and with internal and external forces and torques as input into the model, simulated human motion could be produced.

This simulation method was originally developed to be used in the animation of human motion with primary concern being the production of realistic 'looking' movement. Subsequent work using this method was done by Forsey and Wilhelms(1988) and Lake(1990). In these instances, the method proved to be very fast but recurring problems, related to the control and development of forcing functions that can produce realistic looking human motion, have slowed progress. To date, it appears that this recursive solution to the forward dynamics equations of motion has not been evaluated from a biomechanical perspective. However, its computational efficiency suggests that it may be an excellent method with which to simulate human motion.

A fast, flexible and accurate forward dynamics simulation technique is important for the field of biomechanics because it would better permit practical evaluation of the 'what if' questions about human movement. Having to wait hours or even days for the output of a simulation run after modifying some input value will test the patience of even the most dedicated researcher. A technique limited to work only in two dimensions or capable of handling only a small number of body segments would also have limited applications. The recursive solution, proposed by Armstrong and Green (1985), seems to offer potential in these respects, however it must be evaluated before any conclusions can be stated as to its' biomechanical applications.

1.1 Purpose of the Research.

The purpose of this research was to determine the ability of the recursive method of Armstrong and Green(1985) to simulate three dimensional human motion in a non support state using as input the kinetic and kinematic parameters calculated from real motion. The method was examined with respect to accuracy, calculation speed, flexibility and user friendliness. This first required the development of a computer program for the dynamic analysis of three dimensional motion of rigid linked bodies acted upon by internal torques and only the external force of gravity.

1.2 Limitations of the Research.

This research will be limited by:

1. The ability of the researcher to locate and digitize, from a projection of 16mm film or video image, anatomical locations (joint centers and segment end points) of the human body, and other reference

points required in the kinematic and kinetic analysis of the movement.

2. The error introduced in cinematographical procedures such as use of only two cameras, perspective error, lens distortion and vibration of the cameras.
3. The error introduced when determining the body segment parameters for the subjects. This error will be introduced in all inverse and forward dynamic solutions.
4. The round off error introduced by the computer hardware.

1.3 Delimitations of the Research.

The study will be delimited to:

1. An examination of an open loop, non-support movement problem as defined in the work of Vaughan et al. (1982)
2. Study of a basic three link mechanical model as preliminary work in both the analysis and simulation phases of the research.
3. Two human subjects performing selected basic movement patterns in a state of free fall.

1.4 Definition of Terms.

The following definitions will be adopted throughout the research:

Calibration Tree - a calibrated structure which provides a set of reference points (control points) that are required by the Direct Linear Transformation method of calculating three-dimensional coordinates from two synchronized film or video records.

Computer Simulation - a procedure which uses computer hardware and software to create a representation of a system of interest.

CST Ratio - the ratio of computer processing time to simulated motion time. (Meglan, 1991).

Determinate Problem - a system of equations in which the number of equations is equal to the number of unknowns, therefore usually uniquely solvable.

Digital Filter - a data smoothing technique that accepts as input a sequence of equally spaced numbers y(t), and operates on them to produce as output another number sequence, y'(t) of limited frequency. (Wood, 1982)

Digitized Coordinates - a set of Cartesian coordinates taken from a projected image as determined from a digitizing board or video display.

Direct or Forward Dynamics Problem - a problem in which the forcing functions applied to a mechanical system are known and the objective is to determine the resulting motion of the system. (Vaughan et al. , 1982 p.197)

Direct Linear Transformation (DLT) - a two-pass linear regression technique with which three-dimensional coordinates of a point in space can be determined by using films from different observations. (Kollias, 1984 p.11)

Forcing Functions - a general term for the forces and torques that act on a mechanical system.

Inverse Dynamics Problem - a problem in which the motion of a mechanical system is completely specified and the objective is to find the forcing functions causing that motion. (Vaughan et al. , 1982 p.197)

Mathematical Model - a set of mathematical equations that represent a system of interest.

Newtonian Mechanics - an approach used for solving problems in rigid body mechanics based on Newton's Second Law of Motion and its angular analogue.

Recursive Method - a method by which coefficients for equations of motion for each link of a treelike structure are first calculated on an inward pass from the most distal link of each branch to the root link of the structure. The equations are then solved during a pass back out each branch from the root to the most distal link.

Rigid Body Model - a representation of the human body with the following assumptions:

1. The body segments are considered to be rigid, of uniform density and simple geometric shape.
2. The rigid links rotate about fixed axes.
3. Tissue deformation and the asymmetrical location of the internal organs are considered negligible. (Miller,1979 p. 118)

Spline - a data smoothing technique in which a continuous function of polynomials of degree *m*, pieced together at knots, and having *m-1* continuous derivatives, is passed through noisy data.

Inertial Coordinate System (ICS) - a non-rotating, uniformly moving, right handed Cartesian coordinate system attached to the earth.

Local Coordinate System (LCS) - a right handed Cartesian coordinate system which is attached to each link of the body being studied. It moves with the segment and is aligned so that one of the axes is along the principal, longitudinal axis of the link.

CHAPTER 2

REVIEW OF LITERATURE

Many researchers in different disciplines have developed mathematical models to represent human movement. These models have been used for both the dynamic analysis and simulation of motion. The following review of literature will focus on those simulation procedures which use the rigid link model to create a representation of the human body in a dynamic state. The first section will briefly outline the model and simulation procedures for works from the discipline of biomechanics while the second section will examine computer animation within the field of computer science. This is not intended as a comprehensive review of all the related literature but rather is a selective cross section. The material is presented in chronological order to provide an understanding of the development of the topic with the focus being directed at research done in the last two decades.

2.1 Related Biomechanics Literature

A nine segment, hinge-connected system of rigid bodies was used by **Ramey (1973)** to represent the human body in simulations of the long jump. The equations of motion were based on the principle of conservation of angular momentum while the body segment parameters were chosen from the data of the United States Air Force mean man. The flight phase of three different styles of the jump was examined (sail, hang and hitch-kick), using this two dimensional model, in an effort to determine the amount of angular momentum required in their successful execution.

For each of the three styles Ramey used the same takeoff velocities, takeoff height and landing height. The flight of the center of mass of the system was based on the solution of a simple ballistics equation, neglecting air resistance. Time histories of the angular displacement for all of the body segments for the model were calculated from film data and used as input into the simulation. With these as the initial parameters for the model, the angular momentum required at takeoff was adjusted until an optimal landing position was achieved for each style of jump. In all cases the distance jumped was the same but the angular momentum at takeoff was different. Ramey then used the model to predict the type of landing that would be required if the athlete did not have the correct angular momentum at takeoff. This was done by adjusting the angular displacement histories of the body segments until a good landing was achieved.

A study of free-fall motion was conducted by one of the pioneers of computer simulation of human movement; Doris Miller in a study of diving **(Miller, 1975)**. A three dimensional, four segment model of the body, a modification of Hanovan's (1964) model, was used to create non-twisting dives in either the pike or layout positions. She, like Ramey (1973, 1981), based her equations of motion on the principle of conservation of angular momentum. The work was patterned after the research that had been done on astronauts in weightless situations.

The simulation was used to examine the effect that resulted when altering one or more of: takeoff velocity, trunk angle at takeoff, angular velocity of the arms and legs throughout the dive, total angular momentum of the diver, height of the diving surface, and physical characteristics of the diver. Inputs into the model include the angular displacements and velocities of the arms and legs as well as the total angular momentum of the system. The program produced quantitative results of the angular velocity of the head-trunk segment as well as a three dimensional perspective view of the diver which was produced by a CalComp Plotter. When the simulated results were compared with experimental data for adult male divers they compared favorably.

The kip-up maneuver was the topic for the research of **Gosh and Boykin (1975)**. The human body for this model was represented as three segments; arms, legs, and head and trunk. The links were joined by frictionless hinges and the control variables for the simulation were torques at the hip and shoulder joints. The equations of motion of the three-link system controlled by these torques were solved with the use of Lagrange equations and control theory.

A skilled gymnast was filmed as he performed kip-ups; in the minimum time and with the minimum expenditure of energy. These experimental values were compared to those produced by the simulation; with reasonable results. The differences were attributed to the difficulty in determining the torque values from the film data because of the large amount of deformation of the torso during the execution of the skill.

The development of a more interactive computer program, that could be used by students of biomechanics and coaches, was the incentive for the research performed by **Boysen, Francis and Thomas (1977)**. They used the PLATO system that combines the interactive and graphical capabilities of the computer to study planar motion of the human body under free-fall conditions. A

Lagrangian approach was used to derive the equations of motion for a two dimensional, five segment model. The model was validated by comparing the trunk angles predicted by the simulation with those obtained at corresponding time intervals from cine film of a diver performing two dives: a forward dive in the pike position and; a forward one and one-half somersault in the pike position. The results were good for the simple forward dive but not for the somersault.

The attractiveness of this simulation was the ease with which it could be used. The user simply entered the inter-segment angles at selected time intervals. The program first checked to see if the body formation is anatomically feasible and then displayed the figure on a raster display. As well the torques that are required to achieve the position were output to allow the user to determine whether or not the movement is possible. As **Vaughan (1984)** states: "The possibilities of such a system are far reaching, not only for the teacher or coach but also for the biomechanics researcher who wants to gain a deeper insight into the complex mechanisms governing human motion."(p. 394)

The most comprehensive model for the simulation of human movement has been developed by **Hatze (1976, 1977, 1979, 1980, 1981a, 1981b, 1984)**. Based on his belief that optimization of performance and prevention of injuries in sport are the prime objectives of biomechanics research **Hatze (1979)** states:

> "These objectives necessitate a systems approach to the biomechanics of sport whereby the total human neuro-musculoskeletal control system is considered in its entirety. The model system will obviously be only a greatly simplified version of the real biosystem, the simplifications introduced being consistent with the purpose of the investigation (the jaw muscles, for instance,

> being practically irrelevant for a model designed to investigate the long jump). Such a systems model contains all the segmental parameters (masses, locations of centers of mass, principal moments of inertia) and neuromuscular constants for a given individual, and its dynamics is described by a system of nonlinear, non-autonomous, ordinary first order differential equations. The control parameters in this model are the actual neural controls, motor unit recruitment rate and average stimulation rate, for each of the muscles involved."(p. 237)

Obviously this systems approach is beyond the skills of the biomechanist therefore necessitating the use of a research team. The group assembled by Hatze included; bioengineers, mathematicians, physiologists, physical educators and computing scientists.

Hatze (1976) began some of his earlier work by looking closely at biological motion including that of humans. In this paper he develops a quantitative method of expressing what he calls biomotion. This motion is different from the motion of a non-living system because it is characterized by its non-repeatability and is best represented by a Fourier series.

The work was continued with the development of a complete set of differential equations which describe the dynamic behavior of the total human musculo-skeletal system in **Hatze (1977)**. The 'link-mechanical' part of the system is represented by a seventeen segment model with joints of six degrees of freedom. The equations of motion for this system are solved using a Lagrangian approach. Differential equations for the excitation dynamics and the contraction dynamics of the human muscle represent the 'musculomechanical' part of the system. This is the point where the knowledge of exercise physiology must be brought into the development of the model as these equations incorporate: "such well-known functions as the force-velocity relation for the shortening as well as lengthening of the contractile element, the length-tension relation, the dynamics of the active state, the non-linear dependence of the force production on the stimulation rate, and all the complicated interrelationships which exist between the excitation and contraction variables." **(Hatze, 1977, p. 803)**

The input into this model was the same as that actually used by the human body to produce movement: motor unit recruitment and stimulation rate. Hatze used optimal control theory with his model to produce an optimal performance for a kick. The validation involved the comparison of the results of the simulation with that produced from film data. Specifically the time histories of the angular displacement for the thigh and leg are examined.

The details of the mathematical model for the seventeen link representation of the human body are discussed in **Hatze (1980)**. The segments of this model, which includes the shoulders as separate links, were not represented as simple geometric shapes as in other models, but rather as combinations of these simple shapes. The result was a figure which more closely approximates the general features of the human body. Each geometrical element of each segment was of the same density, based on the work of Dempster (1955).

All of the body segment parameters for the model are derived using sophisticated mathematics which requires the use of a computer. Using a battery of 242 anthropometric measurements from a subject, it is possible to individualize the model, an important feature when attempting to simulate the motion for a particular individual

Based on much of this previous work **Hatze (1981b)** presented a comprehensive model for human motion simulation. Entry of the anthropometric measurements and the control inputs of 46 muscle groups (as taken from EMG readings) were required to begin the simulation. In this particular study the model was used to solve an optimization problem for the takeoff phase of the long jump.

The output from the computer program included: the kinematics of the body segments; the histories of the constraint forces; trajectory of the center of mass; velocity of the center of mass; total angular momentum; muscle and joint reaction forces; and the mechanical energy and power for each of the 17 segments. The validation of this simulation was done by comparing the ground reaction force histories produced by a force plate during the takeoff phase of the long jump with those determined from the simulation results. 'Good agreement' was found between the two sets of data.

Hemami and Farnsworth (1977) developed a five link, five degree of freedom model to simulate human walking. Newtonian equations of motion were used. Input into the simulation was a reference position of the joint angles. The model was limited to single support contact with the ground and movement in the sagittal plane. The simulation produced results which were similar to that of a measured gait cycle. In a follow-up study **Hemami, Tomovic et al. (1978)** examined the swing phase of walking using a similar model. This research used joint torques as driving functions and the simulation did run but motion was not similar to normal human gait.

Walton and Kane (1978) were concerned with developing an interactive computer simulation technique for human motion that could be used as a coaching aid. They used a three link, two dimensional model with three DOF to recreate the free-fall phase of diving and gymnastics maneuvers. They do not explain exactly how the equations of motion are solved except to say the internal angles of the body are generated by using three cubic polynomials. These polynomials appear to have been generated by the user inputting values for the internal angles at the beginning and end of the flight phase as well as at two other times during the action. There was the additional constraint that at these times the angular velocity of these internal joints was zero.

The program calculated the trunk angle with respect to the vertical as well as the location of the center of mass. Finally a graphical display was output on a CRT screen at selected time intervals to be observed by the user. The system was tested by both a gymnastic and diving coach, both who felt that the system would be an aid to the coach.

Normal walking, sport walking, and sprinting were the subjects of analysis for **Aleshinsky and Zatsiorsky (1978)**. A fifteen link chain model is used to represent the human body in three dimensions. The equations of motion were solved using a Newtonian approach. Input into the model included: coordinates of 'reference points' of the subject taken from bilateral stereophotogrammetric films; body segment parameters of the subject as determined by a radioisotope method; and external forces as taken from two force plates. The output from the program was joint moments in either digital form or graphical form (as a function of time). A 'stick figure' recreation of the body which they called a 'kinetogram' could simulate the motion from the frontal, sagittal or transverse views. There was also the option of outputting these kinetograms with: the location of the center of mass of the segments and the entire body, or with the vectors of the net joint forces, a useful feature for the researcher.

The, now outlawed, somersault long jump was simulated by **Mann and Sorensen (1979b)** using a fourteen segment model . Their biomechanical model was based on displacement data, generated either theoretically or from actual performances, and allowed for comprehensive planar movement analysis and simulation. The simulation was divided into two parts: first the limbs were simulated; then based on these results, a complimentary trunk action was added to develop a complete simulation.

For the simulation of the limb segments, boundary conditions of muscle moments and joint forces at the joints were used as input into the model. These conditions were determined from previous research and values published by Plagenhoef (1971) . The results chosen to be used were those with the most potential of generating total body rotation, a necessity of the somersault long jump. These simulations were combined with an 'appropriate' trunk action to produce the total body simulation. Although, the model simulated the entire action, from final foot plant to landing, the takeoff phase was discussed in most detail, a definite departure from most other simulations which tend to focus on the free-fall phase.

Hubbard (1980) simulated the pole vault as a system which included a three segment model of the vaulter and a pole modeled as an elastica. In this study the motion of the vaulter-pole system was assumed to take place entirely in the sagittal plane of the vaulter and perpendicular to the bar. As well, the twisting motion of the athlete during the flyaway phase was ignored. The inputs into the model were the linear and angular displacements and velocities of the vaulter at takeoff and the joint torques at the hips, shoulders and wrists until pole release. In this model the torque at the wrists was considered a good approximation of the effect of two arms applying forces to the pole. The bond graph technique was used to derive the equations of motion which were solved with the aid of a computer. The input of torque values into the model was done on a trial and error basis until an adequate vaulter trajectory was simulated. The authors suggest that this is "not unlike the process through which a real vaulter learns to vault." The torque time histories that resulted were deemed to be 'within reason'. The angles that were reported throughout the jump were reasonable except those of the arms; since upon release of the pole they had not attained the handstand-like position found with good vaults. No attempt was made to compare the torque time histories of the simulation with those from real life vaulting situations.

Gait analysis was the topic for research of **Onyshko and Winter (1980)** who used a seven segment model of the human body. The feet, legs, thighs and HAT (head, arms and trunk) are connected by joints at the hips, knees and ankles and are limited to movement in the sagittal plane. A Direct Dynamic Problem approach is used with the initial conditions being the limb angles and velocities, and input for the model being the joint moments.

Using initial conditions obtained from data produced at the Gait Laboratory of the University of Waterloo the equations of motion were solved basically using a Lagrangian mechanics approach. The output was examined after each time interval and the input joint moments were adjusted until a gait within a few percent of the original data was attained. Minor perturbations of the input data were also introduced in the model to produce atypical gait patterns, an option which the authors feel could help predict the results of therapy and surgery.

A fifteen segment, three dimensional model of the human body was used by **Dapena (1981)** to analyze and simulate the airborne phase of the high jump. All segments except the trunk were treated as thin rods with the moment of inertia about the longitudinal axis equal to zero. The segment masses and center of mass locations were taken from Dempster's (1955) data while the segment moments of inertia were taken from Whitsett (1963). As with many other simulations of free-fall motion, these methods were based on the conservation of angular momentum.

The research was divided into two stages. Stage one included analyzing a film of the jump and calculating the three dimensional coordinates of the segment end points. Using this information and body segment parameter data of Dempster (1955), Clauser et al. (1969) and, Whitsett (1963), Dapena's output from this phase was: the initial orientation of the trunk; the successive orientations of the limbs relative to the trunk; several anthropometric lengths; the path of the center of mass of the subject and the angular momentum of the body about the center of mass. The output of phase one of the research was used as input into the simulation phase of the research. The output of this second phase was three dimensional coordinates of the segment end points. By comparing this output to the original coordinates taken from the film record of phase one, a check on the validity of the model demonstrated that the simulation was 'reasonably' accurate for about 0.6 - 0.8 seconds.

Having verified that the simulation procedure was sufficiently accurate the author then adjusted the input values. While maintaining the initial velocity and angular momentum values, the segment motions relative to the trunk were modified to produce a successful jump. However, the author does warn the reader that a major limitation of the model is ". .. that it does not take into account whether the perturbed angle patterns will require joint torques beyond the capabilities of the subject."

Unsatisfied with the limitations of the two dimensional model, **Ramey and Yang (1981)** developed a three dimensional, nine segment model of the human body. As with the study of Ramey (1973), only the free-fall portion of the long jump was examined, again using angular momentum and angular displacement histories of the limbs as the variables which could be modified. In this study the model was used to simulate the hitch-kick and somersault long jumps. The three dimensional nature of the model resulted in more complicated equations of motion but the work produced similar results to that of the previous study done by Ramey (1973).

A kinematic simulation system developed at Simon Fraser University is described by **Calvert, Chapman and Patla (1982).** The model of the body consists of 20 segments (five for the torso, two for the neck, head, hands, arms, forearms, thighs, legs and feet) with 23 joints that were simulated in three dimensions. The interesting feature of this simulation procedure is that its input consists of an alphanumeric representation of Labanotation, a dance notation. This leads the authors to suggest that a possible application of the simulation is as a tool to assist in the notation of dance and also for the visualization of dance notation.

The output from the model was produced using a number of different computer systems which produced figures ranging from stick men to 'bubble or sausage persons'. The results of the simulation were compared to those produced directly from film data with satisfactory results.

Marshall, Jensen and Wood (1985) developed a general method of simulating the motion of a body represented by n open chain links connected by pin joints. This model was limited to two dimensional motion in the saggital plane. The equations of motion were solved using a Newtonian approach, with the torque histories of the segments, the initial angular displacements and velocities, and the acceleration histories for the proximal end of the nth segment serving as input.

The data for the input were obtained by performing a kinetic analysis of film for: an underhand throw, a rugby punt, and a standing vertical jump. For the underhand throw only the arm, forearm and wrist were included in the model while for the punt the foot, leg and thigh were included. The jump was considered to be a symmetrical whole body movement so the body was modelled as five segments (feet, legs, thighs, trunk and arms). The output from the simulation was the angular acceleration, velocity and displacement of each of the segments.

When compared to data recovered from the original film the results of the simulations were good for periods up to about 0.45 segment-seconds (ie. 3 segments times 0.16 seconds or five segments times 0.09 seconds) after which time the model became unstable. This problem was attributed to round-off error caused by the precision of the computer used in the integration. As a result the simulation of the vertical jump had to be done in a piecewise manner. The authors also used their technique to produce an optimal performance (ie. maximum foot velocity) in the kicking action by modifying the input torques.

A two dimensional, nine segment model was used by **Bourassa and Morel (1985)** to simulate the running motion of a human. Each lower limb was divided into four segments (forefoot, foot, leg and thigh) with the entire upper body modelled as a single HAT segment. Data from real running were used to determine initial conditions and joint torques as the forcing functions. Procedures similar to Onyshko and Winter were used to derive the Lagrangian dynamic equations of motion and the simulation was restricted to a maximum of one point contacting the ground. After making manual changes to the joint torques being used as input they were able to produce a simulated running sequence. They extended their work to use the techniques with a five segment model and examine the single stance, double support, and swing phases of human locomotion using an optimal control approach(**Morel, Bourassa and Marcos, 1985**). **Bourassa (1991)** continued this work with simulation of running over an elastic ground surface and was able to produce realistic motion with a PC system using an INTEL 80386 processor with a math coprocessor. Sixty images were produced in about 1 minute of computing time.

Gervais (1986) used Lagrangian methods to derive the equations of motion. An optimization approach was used to predict then simulate an optimal performance of a human motion. The prediction of optimal performance of a handspring one and one half front salto vault was made by first developing a deterministic model of the task's performance objective of maximizing the points awarded by judges for execution of the skill. The variables of postflight height and distance were identified as those which, if maximized, would result the best score. The simulation of the postflight phase of the vault was achieved using an optimization scheme that used a cost function based on angular momentum and parabolic path of the center of mass. The results of the simulation produced motion that was judged to have received a higher score than the real vault data upon which it was based.

Pandy and Berme (1988) used joint torques as input into a simulation model of the lower extremity in human walking. Solutions to the equations of motion were done using a recursive Newton-Euler formulation. Two algorithms were presented; the first dealt with the open chain problem of single support while the second looked at the closed chain problem of double support. Simulations of planar motion for the two different phases were undertaken using initial conditions from experimental gait data and joint moments chosen on a trial and error basis by the researchers. The resulting simulation produced motion that compared favorably in the single support phase with experimental data recorded for a normal adult male. In the case of double support the simulation was not as successful, as the step length was less than that of real human motion.

Fujii (1989) used Newtonian equations of motion to simulate human motion in two dimensions. The input into this simulation system was the joint torque histories, initial link coordinates and velocities and external forces and torques. A seven segment model of a vertical jump and the running stride were simulated with results being compared to the original data. In both simulations a reasonable result was achieved.

A three dimensional model of seven segments (two lower limbs of three segments plus HAT) connected by spherical joints was developed by **Matthijsse**

and Breedveld (1989). It was used to simulate both the single and double support phases of walking. Multibond graph methods were used to develop the dynamic equations of the motions. The simulation was driven with joint moments and the location of the center of pressure of the foot while contact with the floor was simulated as a moving revolute joint. No results of the simulation were presented.

Marshall et al. (1989) continued their earlier work and used five and six segments (two segments per leg and HAT) to simulate the single stance phase of normal gait. The sixth segment, the foot, was added to examine the heel rising. The simulation methods of their earlier work were used again but the joint torques were calculated using optimum control methods with the state variables being time histories of the joint angles and angular velocities taken from experimentally gathered gait data. The results of the simulation were used to evaluate seven different optimality functions for the entire gait cycle. They concluded that no single criterion seemed to be operative during the motion.

A two dimensional twenty segment model, connected by nineteen hinge joints (total of twenty DOF) was used to simulate the locomotion of a horse by **van den Bogert, Schamhardt, and Crowe (1989).** Eleven of the joints were controlled using torque functions with a linear feedback system while the remaining eight joints were kinematically controlled using joint angle histories. The ground reaction forces were generated using a visco-elastic model of ground-hoof interaction. A mixed forward/inverse dynamics approach was used in the simulation which produced movement of the horse center of gravity which agreed with *in vivo* measurements. In follow-up research (**van den Bogert, Schamhardt, Sauren and Hartman, 1989**) the *DADS* software package was used to make the numerical calculations. *DADS* is a general purpose package for the design and analysis of multibody systems developed by Computer Aided Design Software Inc. Again in two dimensions but now with twenty five segments, pony locomotion was simulated using joint kinematics and torques as well as some musculotendon actuators. Six seconds of motion similar to a normal pony walking was produced using 68.07 hours of CPU time on an Apollo DN4000 computer.

Casolo and Legnani (1988,1989,1990) in a series of papers developed a system for the three dimensional analysis and simulation of rigid link bodies. Using a formulation which combines screw calculus with matrix methods, solutions to both the forward and inverse dynamics problems of general systems of rigid bodies are presented. A software library, *SPACE_LIB*, of programs written in C using these methods was developed and used to simulate the motion of the human body in horse vaulting. The program uses as input: the inertial characteristics of the body; initial state conditions of segment position, orientation, and velocity; the relative motion of the segments and the linear velocity and acceleration of the joints throughout the simulation. With the body modeled as fourteen rigid segments connected by revolute joints (nineteen DOF total) results of the simulation compared favorably with the display of the original data (from a film record of gymnastics competition) from which the input parameters were taken.

Yeadon (1990 a, b, c, d). in a series of papers developed a comprehensive model for the simulation of human airborne movement. With the body modelled as eleven rigid links and having a total of seventeen DOF the twisting and somersaulting of freefall motion was simulated. As with other simulations of human motion experiencing no external forces or torques, the equations of motion are developed around the conservation of angular momentum. Input into the simulation included: the total angular momentum of the system; segmental inertia parameters; initial orientation angles of the whole body and; the time histories of fourteen internal orientation angles which described the body configuration. Output from the simulation was the time histories of the orientation angles of the whole body. Input data were obtained from film of gymnasts performing twisting somersaults executed from a trampoline. The simulation output orientation angles for the whole body were compared to that of the original data. The maximum deviations from the original data were found to be 0.04 revolutions for somersault, seven degrees for tilt and 0.12 revolutions for twist.

Airborne movements were also the focus of a three dimensional simulation model of **Hong and Bruggemann (1991)**. Newton's laws and **Wittenburg's (1977)** theory of the dynamics of rigid body systems were used to develop the equations of motion for high bar dismounts or flights with regrasp and airborne phase of diving. The body was modelled as eleven segments with twenty one DOF, with input into the simulation being the time histories of location of the joint centers or the joint angles. The output was the location and orientation of the body. An interactive procedure allowed for the modification of joint motions within constraints that kept the joint angular velocities realistic. Data derived from gymnastic and diving competitions was used for input into the model. The simulation produced results that varied from the measured results by 2% for the somersault angle and 6% for the twist angle.

Meglan (1991) used a generalization of the Newton-Euler method to develop a numerical technique to solve both the inverse and forward dynamic problems of rigid linked motion. The human body was modelled as thirteen rigid links with 34 DOF and contact with the ground was with a passive, nonlinear mechanical model of the foot. Several different numerical integration methods were used to solve the forward dynamics equations of motion of the 34 x 34 matrix. An analysis module was capable of calculating all of the kinetics of the body based on the input of inertial body parameters and the time history of three dimensional joint locations. Forcing functions for the simulation module could be any combination of joint torques, muscle forces, external forces and torques.

Using only joint torques as drivers, and the initial kinematic state and segment parameters of the body as input, the simulation of a number of different human motions was attempted. Reasonable results were obtained for the motion in a state of free fall and when the figure was allowed to collapse under only the influence of gravity. The ratio of CPU time to simulated motion time (CST ratio) was reported in the range of 306:1 to 14391:1 for these basic motions depending on the integration technique used. Simulation of an ankle plantar flex jump and walking simulation had poor results. Meglan concluded that the model was very sensitive to the quality of the input joint torque patterns and initial conditions therefore any error present in this data had a negative effect on the results of the simulation. Another major problem was encountered with the integration techniques as the equations tended to become stiff very quickly.

2.2 Related Computer Simulation Literature.

Much of the early work with computer simulated models of the human body was motivated by the study of ergonomics (the study of the interaction of human beings with their environment). The systems that have been developed are outlined below.

Fetter (1982) developed several models beginning in 1959 with the simulation of a landing officer on the deck of an aircraft carrier. Later, beginning in 1970, he developed models of the human body which he called 'First Man' up to 'Fourth Man and Woman'. 'First Man' was modelled with seven segments representing the upper body, the sizes of which were based on the 50th percentile American male. This model was used to study the instrument panel of the Boeing 747 aircraft. 'Second Man' modelled the entire body with nineteen segments and was used to simulate motion in ergonomic studies plus running and high jumping. The anthropometric data for the 'Third Man and now Woman' were based on the data found in Humanscaie 1/2/3(Diffrient et al. ,1974). Finally in 1977 the fourth generation of models was developed with an emphasis on complete body contours and better visual displays of the models.

The Chrysler Corporation developed a fifteen link model of the human body called *'Cyberman'* **(Blakely 1980)** for use in the automobile industry. The motion of this model was done by the operator repositioning the body segments. The body segment paramaters were developed based on 1960 census data.

Ergonomics, in this case reach analysis, was again the motivation for the development of the Computerized biomechanical man or *'Combiman'* by the Aerospace Medical Research Laboratory and the University of Dayton Research Institute **(Bapu,1980)**. This model had three links and angular motion was constrained to realistic values. Here the operator designated the moves of the various limbs and could then examine the resulting body configuration in the surrounding environment.

Kingsley, Schofield and Case (1981) developed another general ergonomic tool called *'Sammie'* which had twenty one segments. The human model was based on data gathered from the general population. The operator specified the motion and the program could return the success of reaching movements but did not indicate possible interaction with the environment on the way to the goal. As with *Combiman* the motions of the body segments were constrained to realistic ranges.

Dooley(1982) describes other ergonomic systems that were developed. *'Boeman'* used by the Boeing Corporation in 1969 was a human model based on the data of anthropometric studies. Rockwell International developed *'Buford'* to use in reach and clearance studies of a work space around the model.

As well as ergonomics the more general study of human motion has also motivated computer scientists. Although concerned primarily with how pleasing the simulation looks to the human eye and control methods, the study of human 'animation' does provide the biomechanist with a body of knowledge to draw upon. Following are described some of the key works that have recently been done in this area.

A general method for the animation of articulated figures was presented by **Zeltzer (1982 a,b)**. The simulation was based entirely on kinematics of the motion controlled by 'local motor programs' driven by higher level 'goal directed' routines. No attempt was made to examine the forcing functions which might be required by a requested movement. The simulation technique was successfully used to generate the motion of a human model walking.

The *PODA* animation system was developed by **Girard and Maciejewski (1985)** to simulate the motion of multi-legged animals. In this work the simulation is based on kinematics, some dynamics and spline interpolations. Vertical control of the animation was done primarily with dynamics by specifying ground reaction forces and the force of gravity on the legs during a gait cycle which was divided into the three phases termed the 'push duration' , 'fall duration' and 'restore duration'. Angular motion in *PODA* was restricted to rotations about the yaw and roll axes. The horizontal motion of the model was specified by the animator with a cubic spline along which the model moved at a designated velocity. By inputting both the leg kinematics and forces, simulation of a fourteen legged insect was carried out. In another paper, **Girard (1987)** describes the use of *PODA* to simulate the motion of a nineteen link human model.

Wilhelms (1985) developed an animation system called *Deva* which allowed both the kinematic and dynamic specification of motion of multiple rigid link bodies. The Gibbs-Appell dynamics formulation was used for the equations of motion and solved using a fourth order Runge-Kutta integration technique. The segments were joined by multiple DOF joints (usually one to three rotational) and an initial segment was attached to the inertial frame by a six DOF joint. The simulation used either functions of segment position and orientation or functions of forces and torques as the drivers. Constraints were placed on the segment motions using a 'freeze' function based on springs and dampers. Ground forces were calculated using impulse-momentum formulas. The controlling functions for the movement were developed with the aid of a motion control editor called *'Virya'*. A number of basic simulations were done using the system: raising and dropping a four link model of the arm; a six segment, thirteen DOF man falling to the ground (with and without friction) and; 'Joe' an eleven segment twenty four DOF man raising his knees to his chest while lying on the floor. The author found the results very encouraging but found the computational time excessive which "often meant waiting overnight for several seconds of animation"**(Wilhelms, 1990 p. 272).**

Due to the large computational demands the Gibbs-Appell method was abandoned and follow-up work done by **Forsey and Wilhelms (1988)** used the recursive method of Armstrong and Green (1985). The result was *'Manikin'* which produced simulations that were done at interactive speeds. The main problem with this simulation was the design of torque and force functions which could produce goal directed movement. To provide this control, Wilhelms decided to implement an Euler formulation for articulated bodies in a return to matrix methods. (Wilhelms, 1990) The equations were expressed in terms of inertial motion which resulted in a sparse matrix. Then using sparse matrix techniques to solve the equations, simulation speeds approaching those of Armstrong and Green were attained.

D'Alembert's principal of virtual work is the basis for the equations of motion used by **Isaacs and Cohen (1987)** to simulate motion in multi-linked systems of rigid bodies. The system *'DYNAMO'* uses inverse dynamics to compute the forcing functions in the forward dynamics implementation of the simulation. Input into the system are the physical characteristics of the linked figure, initial state of the system, and behavioral functions which defined the desired motion. At each time step in the simulation a four step procedure was followed. First the behavior functions were executed. Next the joint forces and torques were calculated based on the behavior functions. The torques and forces are calculated using springs and dampers. Finally, the forward dynamic equations of motion were built then solved using a Gaussian elimination scheme. Kinematic constraints could be applied to the solution to provide the animator with a degree of control over the simulation. *DYNAMO* successfully simulated the motion of; a tree blowing in the wind, a swinging chain, a person on a swing, a whip controlled by a two link arm, a three link arm catching and throwing a ball, and a man kicking his leg out on both a floor with friction and without. It is reported that computational time for the simulations was inversely dependent on the time step needed to maintain accuracy and exponentially dependent on the number of degrees of freedom. Using a DEC VAX 8700 the reported times for one second of simulation ranged from 12.6 seconds for the four DOF swing to 1800 seconds for the 39 DOF kicker. The authors also report the values of the torque at the shoulder and wrist calculated by the inverse dynamics routines. Unfortunately, these values, which seem quite large, were not compared to torque values reported in biomechanics literature.

The *KLAW* (Keyframe-Less Animation of Walking) system was developed by **Bruderlin and Calvert (1989)** to animate walking. Goal directed control was combined with dynamic equations of motion based on Lagrangian mechanics and solved using numerical integration techniques. The human was modelled with four links; a telescopic support leg, two segments in the swing leg, and the single segment for the HAT. During the stance phase the leg is represented first as an inverted telescopic pendulum and then by a rigid inverted pendulum. The swing phase was divided into three phases that are controlled both kinematically and kinetically. The model had five DOF in the stance phase and seven DOF during the swing phase. Input into the model was body height and mass plus locomotion parameters of forward velocity, step frequency and, step length. The *KLAW* system was used to produce a wide variety of realistic human walking motions.

The animation of ballroom dancing was the motivation for the work of **Lake (1990)** in his development of *BDAS* (Ballroom Dance Animation System). The recursive dynamics methods of Armstrong and Green (1985) were used to solve the dynamics equations of motion for the sixteen link model of the human body joined by three DOF revolute joints. This forward dynamics solution uses as input forces and torques to control the motion. Lake derived these forcing functions by applying springs and dampers or by using a function representing the force acting across parallel elastic muscle elements which was taken from the biomechanics literature. Contact of the figure with the floor is modelled with springs and dampers and a horizontal friction component.

In a departure from most simulation programs for rigid link models Lake used quaternions to express the orientations and rotations of the model segments rather than Euler angles. He feels that there are some significant advantages to working with quaternions. They uniquely specify all orientations and are continuous for all orientations. They are free from 'gimbal lock' which is the loss of one rotational degree of freedom when two rotation axes are superimposed on each other. Quaternions can also uniquely determine rotation axes unlike Euler angles.

To control the driving functions, low level motor programs were developed such as 'move limb' which attempted to move a limb as smoothly as possible from a current position to the goal position within a specified time limit. A second class of motor programs was used to help the limb motor programs move the human figure to a specific dance position. A final class of motor programs acted again on the low level motor programs to simulate the functions that control automatic functions such as balance. Direction to these lower motor levels is provided by upper level editors which are used to create and modify the motion patterns to simulate that of ballroom dancing.

The *BDAS* system was used on an IRIS 3130 computer with a floating point accelerator to simulate the basic dance moves of raising arms into position and then forward, back and side steps. The results produced realistic but not completely natural looking motion. Problems arose when determining stable values for the constants of the springs and dampers governing the motion of the body segments. Most of the values resulted in limb oscillations which caused numerical instability in the integration routines and then a failure of the simulation. The problem was partially solved by increasing the moments of inertia of the links by a factor of 300.

2.3 Summary

The literature reveals a number of different approaches to the computer simulation of human movement using rigid link models. Early works were probably limited by the hardware available at the time and tended to be two dimensional with degrees of freedom usually less than ten. Although some two dimensional work is still done research is generally directed at three dimensions but the problems of simulating full body motion still have researchers limiting the DOF in their studies through various constraint methods. Much of the time this limitation is necessary because of the excessively large demands made on the computer hardware by the methods chosen to solve the equations of motion.

The methods of solving the equations of motion generally fall into three categories. Kinematic approaches range from the 'key frame' methods found in computer animation to the use of the principle of conservation of angular momentum found in airborne studies in biomechanics. Kinetic or dynamic solutions primarily use Newtonian or Lagrangian methods with most of the recent works following the formulations of the former. The final category of simulation works combine the kinematics and kinetics. In the field of biomechanics this tends to be done so that kinematic constraints are applied to primarily kinetic simulations. In computer animation the kinematics allows the motion to be controlled in part by the traditional keyframing techniques but the in between frames use kinetic solutions to produce the motion. The result is a more realistic looking animation.

The usefulness of simulation methods depends to a large extent on the validity of the results. As **Zeigler (1984)** states "validity is measured by the extent of agreement between real system data and model-generated data" (p. 5). In the case of computer animation, the validation process is relatively simple as a qualitative approach is usually taken with the human eye being the measurement tool. In biomechanics this method is also used but many studies also take a quantitative look at the results, usually by comparing the orientations and/or positions of the model segments.

The usefulness of a simulation procedure is also related to its flexibility and efficiency. While some of the works reviewod present methods with many possible applications, most have constraints that limit their use. This should not be viewed as a criticism of the researchers because in most cases these limitations were implemented to make the problem manageable. But a concern does come to light in that, in many cases, little or no follow-up research was carried out in an attempt to remove some of these constraints and pursue the 'potential' many simulation procedures demonstrated. It might be speculated that some of these solutions were abandoned simply because they were too inefficient to use.

Efficiency is usually affected in two ways. It is first influenced by 'user friendliness' which is determined by the design of the software and the required and supporting hardware. Many of the early studies suffered from this problem but to a large extent it has disappeared today because of advances in both hardware and software. The other major factor affecting efficiency is the method used to solve the equations of motion. This is particularly a problem with three dimensional studies where the number of DOF can be very large. Most researchers have chosen to put the equations into large matrix formats and then solve them using numerical methods. While very flexible they tend to be computationally demanding even for relatively powerful computers. The exceptions to this are studies based on 'recursive' solutions to the equations of motion. This component of the efficiency should be easy to determine but in fact most papers do not report the computational demands of the simulations. The power of computers available to the earlier researchers would suggest that many of the more complex simulations would have been prohibitively slow. In recent studies the time values that are reported still suggest that work with these simulations could be very trying on the patience of the scientist. Those works using a recursive solution seem to be the only exception.

Finally, it should be noted that the primary difference between the biomechanics literature and that of computer science is reflected in the differing goals of the research. This is summarized nicely by Girard and Maciejewski (1985) "In contrast to industrial robots and biomechanical simulations, animation does not necessarily require the computation of actual forces. The application of dynamics to animation is simplified by the fact that we are interested only in what can be seen."(p. 267) This may help to explain why to a large extent the two disciplines have almost ignored each other's works. However, it is obvious that the two fields of study have a great deal to learn from each other.

Many problems and concerns still remain to be solved if simulation is to prove useful as a tool for the biomechanist in the study of movement. It is clear that new research should be primarily focused on work in three dimensions and that unrealistic constraints on the simulation must be avoided. It also seems that to advance the field it is not necessary to develop a totally new solution. Rather, extension of existing procedures may prove to be more fruitful. Thus, it is within this framework that the present study was undertaken.

CHAPTER 3

THREE DIMENSIONAL ANALYSIS OF RIGID LINK MOTION

Dynamics is "the part of mechanics dealing with analysis of bodies in motion." (Beer and Johnson 1976, p. 399). This chapter is devoted to the dynamic analysis of multi-linked rigid bodies in motion in three dimensional space. The programs for the dynamic analysis and simulation phases of this research were developed on personal computers operating under the Microsoft DOS operating system in the C programing language using the Microsoft 'Quick C' compiler.

3.1 The Models.

Two rigid link models were used during this research:

3.1.1 The three link model

A three link wooden model as illustrated in Figure 3.1 was developed for use during the early stages of each phase of the research. The small number of links made it easier to isolate problems as they arose. All three links of the model were solid wooden cylinders joined by two simple one DOF hinge joints oriented at right angles to each other. The forcing functions of this model were provided by rubber bands attached to two adjoining links in such a manner as to cause flexion at their joints when the band was in a state of stretch. An example of the motion produced by the model can be seen in figure 3.2.

The use of this model helped to minimize some of the problems associated with conducting research on only a model of the human body. The physical properties of the links for this model can be determined with greater accuracy than with human models. The lengths of each segment can be directly measured; the mass of each segment is determined by simply weighing each one; and the mass moments of inertia for each can be calculated by using formulas developed for common geometric shapes.

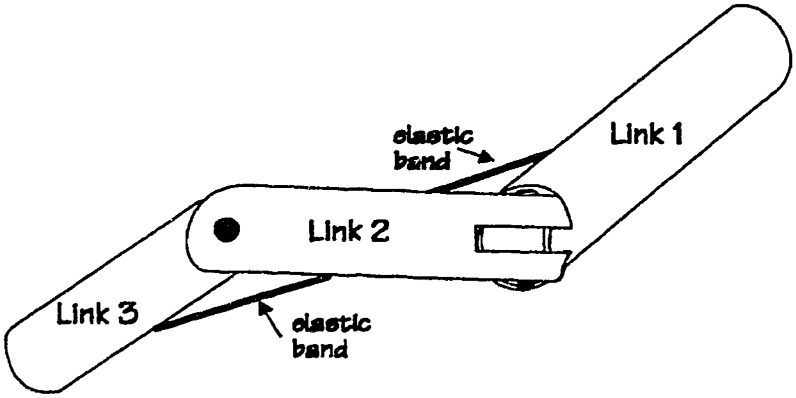


Figure 3.1 The three link model.

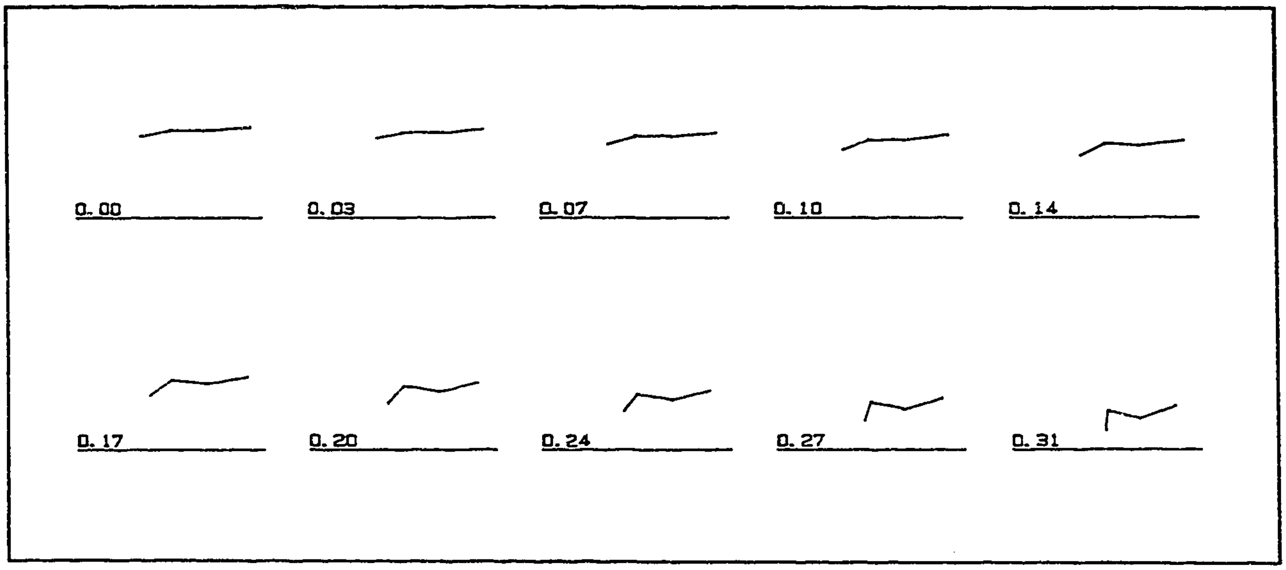


Figure 3.2 An example of the motion produced by the 3 - link wooden Model with 1 DOF joints and moved by elastic bands.

For a cylinder the mass moments of inertia are given by:

$$I_x = I_y = {}^1\!/_{12}\, m(3a^2 + L^2) \quad \text{and} \quad I_z = {}^1\!/_2\, ma^2 \tag{3.1}$$

where m is the mass of the segment and a is its radius and L is its length (Beer and Johnston, 1977). These physical characteristics for this model can be found in Table 3.1.

Table 3.1 Physical characteristics of three link Model

Link	Mass (kg)	Length (m)	I_x (kg*m^2)	I_y (kg*m^2)	I_z (kg*m^2)
1	0.45	0.555	0.008834	0.008834	0.000293
2	0.33	0.365	0.004557	0.004557	0.000211
3	0.22	0.270	0.001963	0.001963	0.000171

The joints and link endpoints were marked with high contrast tape which allowed for easy identification during the digitizing process. The small number of links made for easier debugging of all computer software during early stages of development and simplified the evaluation of output from the analysis and simulation programs.

3.1.2 The Human Model

A 14 segment, rigid link model was used to represent the human body for all of the research. The 14 segments used in the model and illustrated in Figure 3.3 were:

1. Trunk.
2. Head and Neck.
3. Right Arm.
4. Right Forearm.

5. Right Hand.
6. Left Arm.
7. Left Forearm.
8. Left Hand.
9. Right Thigh.
10. Right Leg.
11. Right Fcot.
12. Left Thigh.
13. Left Leg.
14. Left Foot.

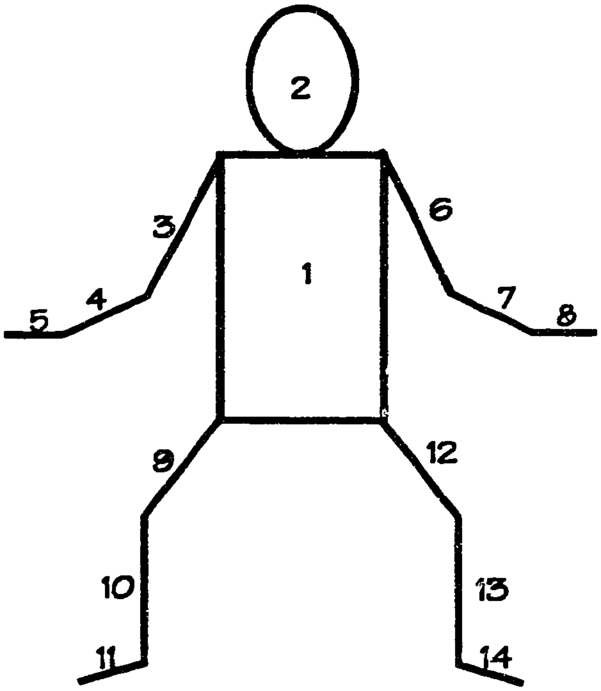


Figure 3.3 Configuration of the 14 link human model

The MIT Human Scale 1/2/3 data of Diffrient et al. (1974) found in Table 3.2 was used to calculate the mass and the location of the center of mass for each segment. The principal mass moments of inertia for each link were calculated by normalizing the data, from Whitsett (1963) in Table 3.3, according to the mass and height of the human subject.

Table 3.2

Mass and Location of Center of Mass for Body Segments,

from Diffrient et al. (1974)

Segment	Location of Center of Mass from Proximal Joint (% of segment length)	Mass (% of total body mass)
Trunk	50.0	45.80
Head and Neck	50.0	9.60
Arm	43.6	3.30
Forearm	43.0	1.90
Hand	28.0	0.65
Thigh	43.3	10.10
Leg	43.3	4.50
Foot	45.0	1.45

Table 3.3

Principal Moments of Inertia for Human Body Segments

from Whitsett (1963)

Segment	I_x (kg*m^2)	I_y(kg*m^2)	I_z(kg*m^2)
Head and Neck	0.0249	0.0249	0.0169
Trunk	1.2606	1.3555	0.3218
Arm	0.0214	0.0214	0.0025
Forearm	0.0076	0.0076	0.0011
Hand	0.0005	0.0005	0.0005
Thigh	0.1055	0.1055	0.0209
Leg	0.0506	0.0506	0.0051
Foot	0.0038	0.0038	0.0008

I_x principal moment about the frontal axis.

I_y principal moment about the transverse axis.

I_z principal moment about the longitudinal axis.

The equations used for this normalization, taken from Dapena (1978) are:

For the transverse and frontal axes

$$\mathrm{I} = (I * \mathrm{M} * \mathrm{H}^2) / (M * H^2) \qquad (3.2)$$

and for the longitudinal axis

$$\mathrm{I} = (I * \mathrm{M}^2 * H) / (M^2 * \mathrm{H}) \qquad (3.3)$$

where:

I is the principal moment as given by Whitsett;

M = 74.2 kg., the average mass of Whitsett's subjects;

H = 1.7555 m., the average height of Whitsett's subjects;

M = the mass of the subject in this study;

H = the height of the subject in this study.

A computer routine was written to calculate the principal moments of inertia for all links of the human model based on Table 3.3, equations 3.2 and 3.3, and the height and weight of the subjects for this study given in Table 3.4. The resulting principal mass moments of inertia for the two subjects of this study are shown in table 3.5

Table 3.4
<u>Height and Weight of Subjects as Used to Calculate Principal Moments of Inertia for All Model Segments.</u>

SUBJECT	HEIGHT (cm)	WEIGHT (kg)
One	165	56.8
Two	170	65.0

Table 3.5

Principal Moments of Inertia for Body Segments For Study Subjects

Segment	Subject	I_x (kg*m^2)	I_y (kg*m^2)	I_z (kg*m^2)
Head and Neck	One	.015052	.015052	.011144
	Two	.020455	.020455	.013392
Trunk	One	.764685	.822112	.212203
	Two	1.039193	1.117235	.255010
Arm	One	.012936	.012936	.001649
	Two	.017580	.017580	.001981
Forearm	One	.004594	.004594	.000725
	Two	.006243	.006243	.000872
Hand	One	.000302	.000302	.000302
	Two	.000411	.000411	.000396
Thigh	One	.063774	.063774	.013782
	Two	.086668	.086668	.016562
Leg	One	.030587	.030587	.003363
	Two	.041568	.041568	.004041
Foot	One	.002297	.002297	.000528
	Two	.003122	.003122	.000634

The motions studied for the human model were the tuck jump, split jump and straddle jump. In all jumps the subjects were facing forward in the x direction of the ICS. During the tuck jump the subject flexes (primarily around the y axis of the ICS) the thighs at the hips and the legs at the knees on the ascending phase followed by extension of the thighs and legs during descent in preparation for landing. A stick figure example of this is seen in figure 3.4. The split jump has the subject abduct the thighs and legs at the hip joint (primarily around the x axis of the ICS) during the ascent and then adduct them during the descent as seen in figure 3.5. The straddle jump, as shown in figure 3.6, involves both abduction and flexion of the thighs and legs at the hip on ascent and then adduction and extension when descending. The resulting rotation is around all three axes of the ICS for this jump.

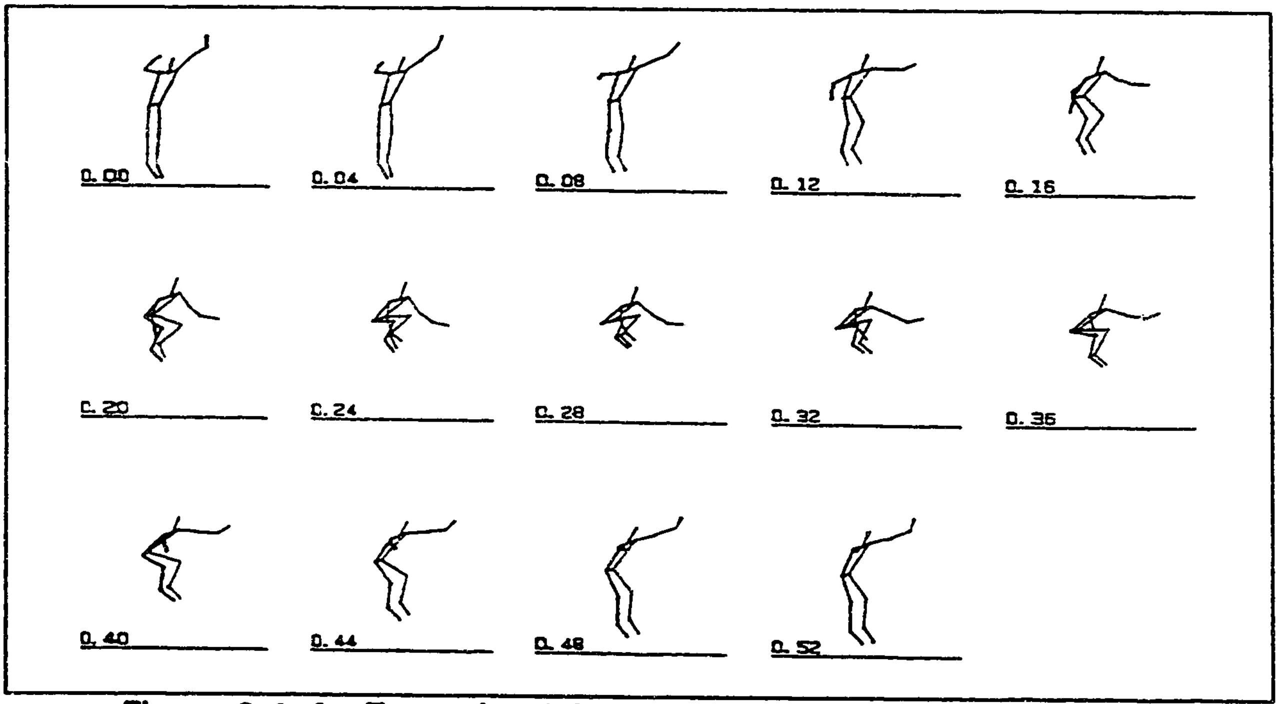


Figure 3.4 An Example of the tuck jump studied in this thesis

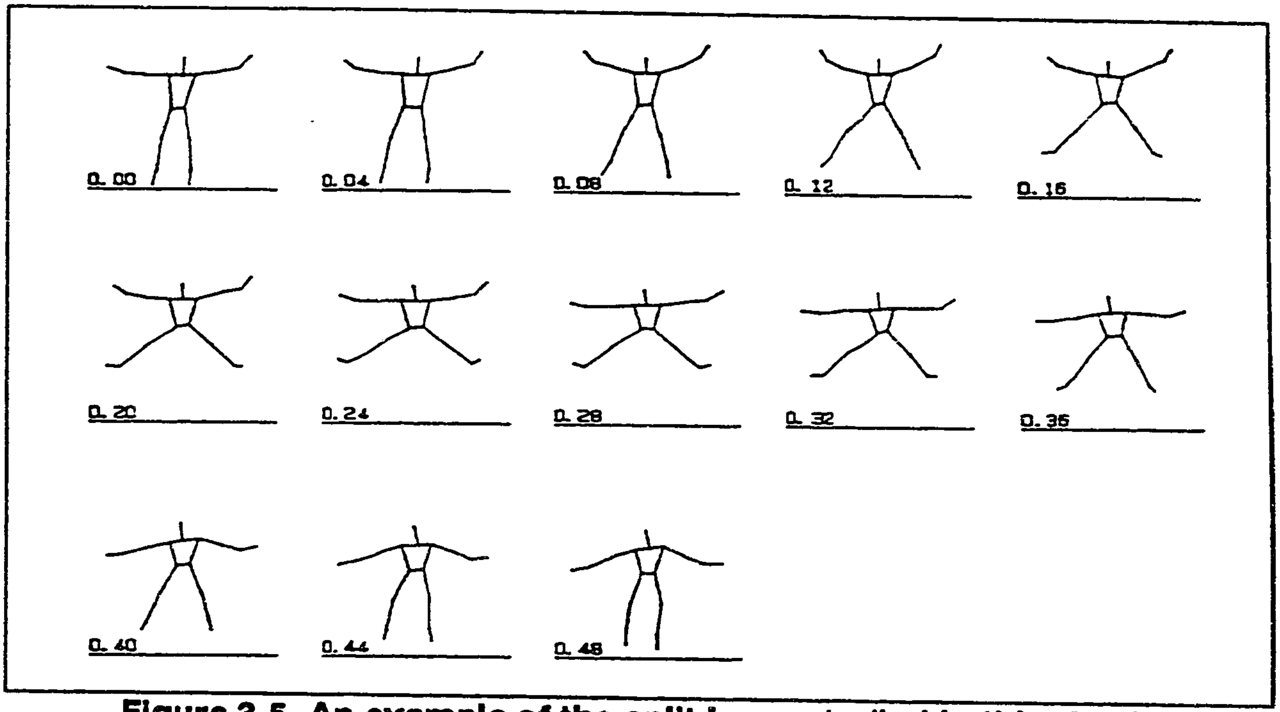


Figure 3.5 An example of the split jump studied in this thesis

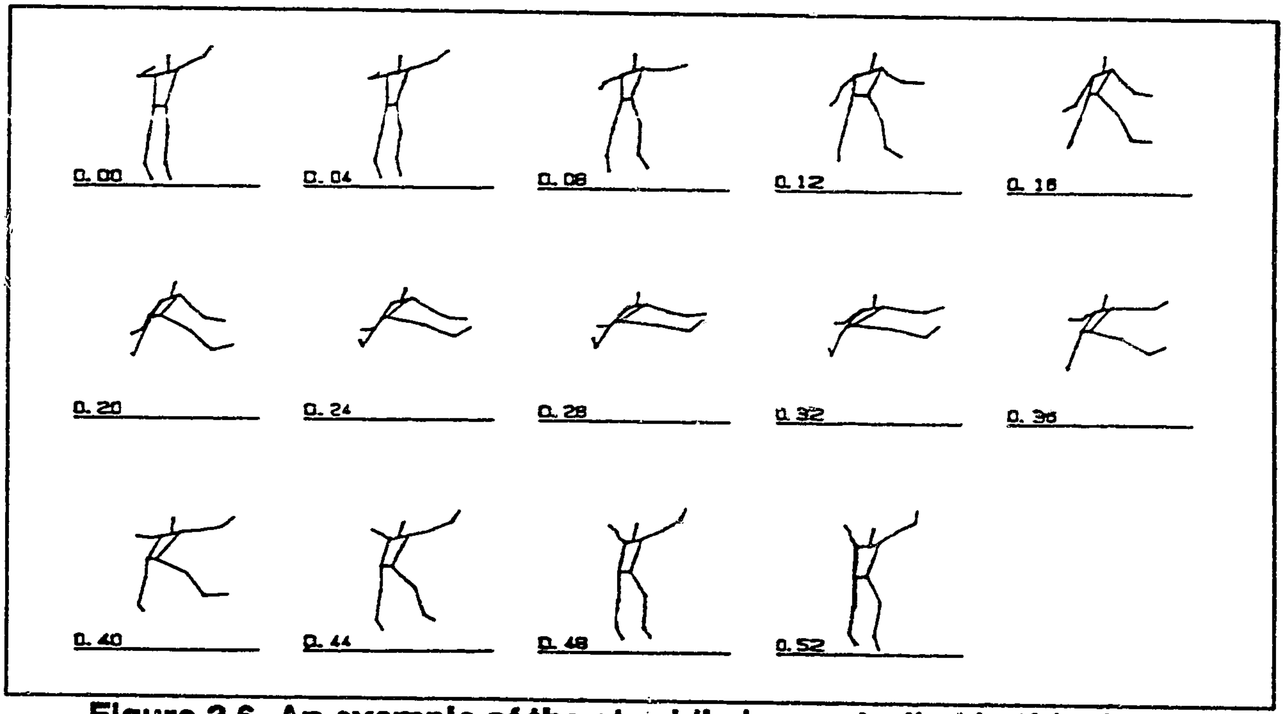


Figure 3.6 An example of the straddle jump studied in this thesis

3.2 Data Collection and Production of Spatial Coordinates.

Two different systems were used to determine the spatial coordinates of segmental endpoints for a three link mechanical model and a 14 link model of the human body. In both instances a Direct Linear Transformation (DLT) technique was used to determine the three dimensional spatial coordinates of segment endpoints, for both the three link model and the human subjects, from film and video records of the free fall motion. The DLT technique developed by Abdel-Aziz and Karara, (1971) has been used by many biomechanics researchers (Shapiro, 1978; Walton, 1981; Miller et al. , 1980; Kollias, 1984a; Dapena and Chung, 1988) with good success and today it is an accepted method in the discipline.

The first system used to calculate three dimensional coordinates for link end points was a 16 mm film system. A calibration tree, consisting of 27 control points, was constructed to cover the entire area where the motion of the subject and three link model was to take place. The location of all control points was measured with respect to a visible origin and; it was filmed using two Photosonics 1 PL phase-locked 16mm cameras that were perpendicular to each other. Each camera was equipped with an Angenieux 12 - 120 zoom lens. After removing the reference tree the motion of the subject and three link model was filmed during a free-fall state. The cameras were operated at 50 frames per second with an exposure time of .00125 seconds, using Ectachrome 7250 film (400 ASA). Their speed was checked with the use of a timing light generator operating at a frequency of 10 Hz. Once developed, the data film was projected onto a GTCO Corporation digitizing board by a M-16c Vanguard projector. The image size to real life size ratio was 1:4.9. The Cartesian coordinates, for all segment endpoints, were returned by the digitizing system and stored on the hard drive of an IBM PC-XT computer.

Seventeen points were digitized for the human subject:

Top of the head.
Center of shoulder joints.
Center of elbow joints.
Center of wrist joints.
End of hands.
Center of hip joints.
Center of knee joints.
Center of ankle joints.
End of Toes.

To assist the researcher in identifying these points during the digitizing process, they were marked with black spots on a backing of white tape located at joint centers and segment endpoints.

For the three link model the 4 points digitized were :

Each end of the model.
The two hinge joints.

The two dimensional coordinates for each view for each trial were passed to an HP 9825B computer. This data was then put through a program developed by Kollias (1984b) for the DLT technique that was used to calculate the three dimensional coordinates for all points. This data was then smoothed using a second order, recursive, Butterworth digital filter with a cutoff frequency set at 8.25 Hz.. This cutoff frequency was decided upon after visual observation of the data with different frequencies.

The second set of data was collected using a commercially available video based system available from Ariel Life Systems Inc. called the Ariel Performance Analysis System (*APAS*). This is a comprehensive package of hardware and software capable of producing three dimensional coordinates of points in space based on the input of video images produced by two SVHS video cameras operating at 60 fields per second. As with the film based system, the space where the motion took place was first calibrated by filming fourteen control points of known location. The image size to real life size ratio was 1:13.6. The motion of a second subject and the same three link model were again filmed in a state of free-fall. The same joint centers and segment endpoints were marked with white tape against a black background for both the subject and the model before filming. The video images from both cameras were stored on the computer hard disc with a 'frame grabbing' program. The points of interest on the images were then digitized primarily by the automatic system with point tracking problems solved by intervention of the researcher and subsequent manual digitizing. The data was then passed on to another program which uses the DLT technique to transform the two views of two dimensional coordinates into three dimensional coordinates. This data was then smoothed using a quintic spline with the required fit based upon the estimate of error calculated automatically during the digitizing process when 25% of the frames were randomly redigitized.

3.3 Three Dimensional Kinematic Analysis.

Kinematics is "the study of the geometry of motion; Kinematics is used to relate displacement, velocity, acceleration and time, without reference to the cause of motion" Beer and Johnston (1976, p. 399).

The first step, in the analysis of the three dimensional data produced from the video and film records, was to develop a computer program to perform a kinematic analysis of the motion of both the three link wooden model and the fourteen link human model. Input into the program came from an ASCII file containing: the number of frames of motion to be analyzed; the time step between each frame; the segmental parameters of mass, location of center of mass, and the principal mass moments of inertia; and the three dimensional coordinates of the joints and segment endpoints.

The first module of the program produced a three dimensional display of the models on a computer monitor. The program routines are based on the three dimensional graphics work of Adams(1989). The program produces a stick-figure display with the capability of rotating and translating the models in space. A basic animation technique is included to allow for the examination of movement of all segments.

3.3.1 Coordinate Systems, Orientation Angles and Transformations

The movement of the rigid links in a system in space can be defined in relation to a non-rotating frame or coordinate system common to all of the body segments. For this thesis, this stationary coordinate system will be termed the inertial coordinate system (ICS). It will be defined by a frame consisting of three mutually orthogonal axes x, y, z following the 'right hand' convention. The unit coordinate vectors for this system will be given by i, j, k. The orientation of any link *i* in the ICS is defined as the orientation of a frame of reference which is fixed to the link. For this thesis such a frame or system will be termed a local coordinate system (LCS). For each link *i* there will be a right handed frame defined by three mutually orthogonal axes x_i, y_i, z_i. With the origin of the LCS located at the proximal end of the link, the unit coordinate vectors for this system will be given by i_l, j_l, k_l and as shown by Alexander and Colbourne (1980) and Yeadon(1990a) can be defined as:

i_l is parallel to **a**.

j_l is parallel to i_l x **b**. (3.4)

k_l= i_lx j_l.

where **a** is a vector pointing from the proximal end to the distal end of link *i* and **b** is a vector pointing from the proximal end to the distal end of a link attached at the proximal joint of link *i*. If **a** and **b** are parallel a singularity would exist when calculating j_l, using this method, but at no time for any of the data did this situation occur. The orientation of the local frame with respect to the inertial frame will be given by the xyz-convention Euler angles which are defined by Goldstein(1980) as successive rotations through the yaw angle ϕ about the z axis, the pitch angle θ about the y' axis and the roll angle ψ about the x'' axis and shown in figure 3.7.

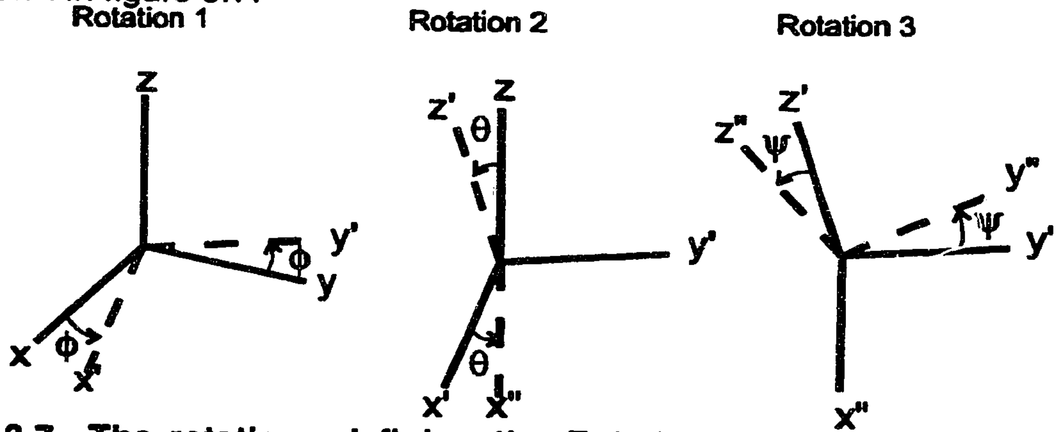


Figure 3.7 The rotations defining the Eulerian angles which define the orientation of the local frame of a link in relation to the inertial frame.

Knowing the directions of i_l, j_l, k_l in the ICS Yeadon(1990a) shows that a transformation matrix $[T]_{I\text{-}L}$ which can transform a vector from the coordinates in the ICS to coordinates in the LCS is given by:

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \qquad (3.5)$$

where:

$$i_l = a_1 i + a_2 j + a_3 k$$

$$j_l = b_1 i + b_2 j + b_3 k$$

$$k_l = c_1 i + c_2 j + c_3 k$$

Goldstein(1980) also shows that this matrix is given by:

$$\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ \sin\psi\sin\theta\cos\phi-\cos\psi\sin\phi & \sin\psi\sin\theta\sin\phi+\cos\psi\cos\phi & \cos\theta\sin\psi \\ \cos\psi\sin\theta\cos\phi+\sin\psi\sin\phi & \cos\psi\sin\theta\sin\phi-\sin\psi\cos\phi & \cos\theta\cos\psi \end{bmatrix} \qquad (3.6)$$

Therefore:

$$\sin\theta = -a_3$$

$$\cos\theta = (1 - \sin^2\theta)^{1/2}$$

$$\sin\phi = a_2/\cos\theta \qquad (3.7)$$

$$\cos\phi = a_1/\cos\theta$$

$$\sin\psi = b_3/\cos\theta$$

$$\cos\psi = c_3/\cos\theta$$

and any angle β is given at time t by:

$$\beta_t = \arctan(\sin\beta_t/\cos\beta_t) \text{ for } \cos\beta_t > 0.$$

$$\beta_t = \pi + \arctan(\sin\beta_t/\cos\beta_t) \text{ for } \cos\beta_t <= 0. \qquad (3.8)$$

The orientation of any link at any time t can now be defined in relation to the ICS using the Euler angles calculated as above. Yeadon(1990a) shows that a time history of these orientation angles β'_t without the discontinuities is given by:

$$\beta'_t = \beta'_{t-1} + X \tag{3.9}$$

where X is in the range $-\pi < X <= \pi$ and defined by:

$$\cos X = \cos(\beta_t - \beta'_{t-1}) \tag{3.10}$$

$$\sin X = \sin(\beta_t - \beta'_{t-1})$$

The location of the center of mass and the orientation angles of each segment in the ICS at each time step were now calculated. The output curves from these calculations could be displayed for all three dimensions for all links. The curves could be stepped through while at the same time displaying the model in a separate window on the display screen. As a diagnostic aid the unit coordinate vectors representing the axes of the LCS could be displayed for each link. A layout of the analysis screen can be seen in figure 3.8.

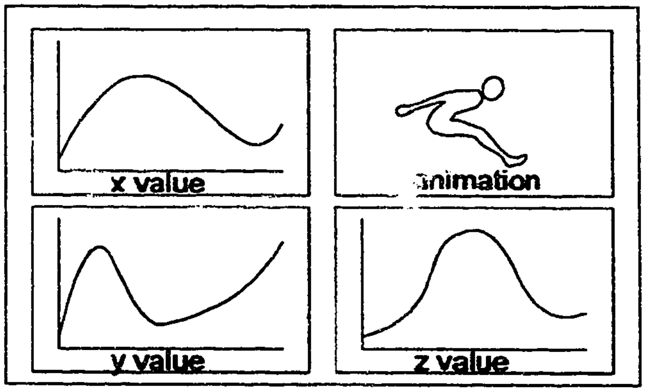


Figure 3.8 **An example of an analysis screen layout. The animation window is in the upper right corner with the remaining windows used for plotting the data relative to each axis as indicated.**

3.3.2 Velocities and Accelerations

The linear velocity and acceleration of each segment's center of mass in the ICS were then calculated using a central difference technique (Miller and Nelson, 1973; Wood 1982). The velocity and acceleration at any time *t* are given by:

$$x'_i = (x_{t+1} - x_{t-1})/(2\Delta t) \qquad (3.11)$$

$$x''_i = (x_{t+1} - 2x_t + x_{t-1})/(\Delta t)^2$$

where x indicates the position of the center of mass.

The angular velocity ω and rate of change of angular velocity $\dot{\omega}$ in the LCS of any link *i* at any time *t* are calculated using a finite difference technique :

$$\omega^i = (\Delta B^i_{t-1,t} + \Delta B^i_{t,t+1})/(2\Delta t) \qquad (3.12)$$

$$\dot{\omega}^i = (\Delta B^i_{t,t+1} - \Delta B^i_{t-1,t})/(\Delta t)^2$$

where the components of ΔB^i representing angular displacements are taken from the transformation matrix $[A^i_{t,t+1}]$ (using the same methods as used to calculate the Euler orientation angles above) where:

$$[A^i_{t,t+1}] = [T^i_{t+1}]_{I-L}[T^i_t]^T_{I-L} \qquad (3.13)$$

with $[T^i_t]^T_{I-L} = [T^i_t]^{-1}_{I-L}$. $[T^i_t]^T_{I-L}$ can also be represented by $[T^i_t]_{L-I}$, a transformation matrix which when applied to any vector in LCS transforms it to the ICS.

As with the position and orientation data this kinematic data could also be displayed, for evaluation purposes, in the same graphical format as in figure 3.7.

3.4 Three Dimensional Kinetic Analysis of Rigid Link Motion

"Kinetics is the study of the forces associated with motion, including forces causing motion and forces resulting from motion" (Hall, 1991). In this research an inverse dynamics approach was used to perform a kinetic analysis of the motion of the two rigid link models. This study examines an open loop, non-support problem therefore, knowing the motion produced from the kinematic analysis, it is possible to calculate the forcing functions producing the motion.

3.4.1 Newton-Euler equations of motion.

An approach, based on Newton's Second Law of Motion, is used to solve the equations of motion for the rigid body system. In the simplest form, the basic equations in vector format, for each segment of the system, are given by:

For linear motion

$$\Sigma \mathbf{F} = m\mathbf{a} \tag{3.14}$$

For angular motion

$$\Sigma \mathbf{M} = \mathbf{I}\alpha \tag{3.15}$$

For analysis of a rigid body in space if a coordinate system is attached to the links these formula can be expanded as:

For linear motion equation 3.14 becomes:

$$\Sigma F_x = m\ a_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z \tag{3.16}$$

For angular motion, I from 3.15 represents an inertia tensor and gives:

$$\begin{aligned}
\Sigma M_x &= I_{xx}\alpha_x - (I_{yy}-I_{zz})\omega_y\omega_z - I_{xy}(\alpha_y - \omega_x\omega_z) - I_{yz}(\omega_y^2 - \omega_z^2) - I_{zx}(\alpha_z - \omega_x\omega_y) \\
\Sigma M_y &= I_{yy}\alpha_y - (I_{zz}-I_{xx})\omega_z\omega_x - I_{yz}(\alpha_z - \omega_y\omega_x) - I_{zx}(\omega_z^2 - \omega_x^2) - I_{xy}(\alpha_x - \omega_y\omega_z) \qquad (3.17) \\
\Sigma M_z &= I_{zz}\alpha_z - (I_{xx}-I_{yy})\omega_x\omega_y - I_{zx}(\alpha_x - \omega_z\omega_y) - I_{xy}(\omega_x2 - \omega_y^2) - I_{yz}(\alpha_y - \omega_z\omega_x)
\end{aligned}$$

If the coordinate system attached to the rigid link is oriented to coincide with the principal axes of the body, the above angular equations in 3.17 can be greatly simplified to:

$$\begin{aligned} \Sigma M_x &= I_{xx}\alpha_x - (I_{yy} - I_{zz})\omega_y\omega_z \\ \Sigma M_y &= I_{yy}\alpha_y - (I_{zz} - I_{xx})\omega_z\omega_x \\ \Sigma M_z &= I_{zz}\alpha_z - (I_{xx} - I_{yy})\omega_x\omega_y \end{aligned} \qquad (3.18)$$

which are called Euler's equations.

3.4.2 Solution to the equations of motion.

Given the kinematics, mass and moments of inertia, the equations of motion for link *i* in a multi-segment model, at any moment in time, as shown by Huang, et al. (1982), are given by:

$$\begin{aligned} F^i_{xp} &= -F^i_{xd} - m^i g^i_x + m^i a^i_x \\ F^i_{yp} &= -F^i_{yd} - m^i g^i_y + m^i a^i_y \\ F^i_{zp} &= -F^i_{zd} - m^i g^i_z + m^i a^i_z \end{aligned} \qquad (3.19)$$

where g represents the acceleration due to gravity and:

$$\begin{aligned} M^i_{xp} &= -M^i_{xd} + I^i_x\alpha^i_x - (I^i_y - I^i_z)\omega^i_y\omega^i_z \\ M^i_{yp} &= -M^i_{yd} + I^i_y\alpha^i_y - (I^i_z - I^i_x)\omega^i_z\omega^i_x - F^i_{zp}\ell^i_p + F^i_{zd}\ell^i_d \\ M^i_{zp} &= -M^i_{zd} + I^i_z\alpha^i_z - (I^i_x - I^i_y)\omega^i_x\omega^i_y + F^i_{yp}\ell^i_p - F^i_{yd}\ell^i_d \end{aligned} \qquad (3.20)$$

where:

p indicates values at the proximal end of the link.

d indicate values at the distal end of the link.

ℓ indicates the distance to the segment center of mass from the *p* or *d* end of the link.

To satisfy conditions of continuity at each joint the forces and torques exerted by one link on the next are of equal magnitude but in the opposite direction to that applied by the second on the first. This results in the following at each joint:

$$\begin{bmatrix} F^{i+1}{}_{xd} \\ F^{i+1}{}_{yd} \\ F^{i+1}{}_{zd} \end{bmatrix} = -\,[T^{i+1}]_{I-L}[T^{i}]_{L-I} \begin{bmatrix} F^{i}{}_{xp} \\ F^{i}{}_{yp} \\ F^{i}{}_{zp} \end{bmatrix} \tag{3.21}$$

and

$$\begin{bmatrix} M^{i+1}{}_{xd} \\ M^{i+1}{}_{yd} \\ M^{i+1}{}_{zd} \end{bmatrix} = -\,[T^{i+1}]_{I-L}[T^{i}]_{L-I} \begin{bmatrix} M^{i}{}_{xp} \\ M^{i}{}_{yp} \\ M^{i}{}_{zp} \end{bmatrix} \tag{3.22}$$

Based on the above solutions to the equations of motion and continuity conditions, and using the calculated kinematic values, the kinetic routines were developed for the dynamic analysis module of the program. For each branch in the model (1 for the three link model and 5 for the human model) the equations were solved on an inward pass from the most distal link to a base link (link 1 for the three link model and the trunk link for the human model). The net internal torques and net reaction forces occurring at each joint were calculated in both of the ICS and LCS. These values for all 3 dimensions could be graphed then tracked while observing the animated movement of the models.

3.4.3 Check of kinetic values.

A final routine was created to check the correctness of the kinetic solutions. The net forces and torques that would be required to produce the motion of the base link were first calculated as if it had no adjoining segments.

This was then compared to the sums of the values calculated at the proximal joints of all limbs attached to the base link. Aleshinsky and Zatsiorsky(1978) and Meglan(1991) suggest that the agreement between these values can give an indication of the consistency of either the kinetic calculations or the estimated physical parameters of the links. The graphical display of the two sets of data were plotted against each other for comparison.

3.5 Dynamic Analysis Results

3.5.1 Error analysis

The error involved in the data collection can be classified as either systematic or random.

Systematic error introduces consistent biases into the data (Wood, 1982) and is found in the three dimensional coordinates used in the dynamic analysis procedures explained above. In the case of the data collected using the film based method the sources of this error are: a) the least square method used to derive the calibration coefficients; b) the limited precision of measurement of the control points on the calibration tree and; c) the digitizing error of these points (Kollias, 1984a). The criterion for the estimation of this error was the root mean square (RMS) difference of the measured vs the calculated coordinates of the control points. This error was found to be RMS (X) = 0.47 cm., RMS (Y) = 0.30 cm. and RMS(Z) = 0.29 cm.. This is only slightly larger than that found by Kollias (1984a) using the same techniques. The *APAS* system does not provide a method of calculating this error but it can be assumed that it was present.

Another source of systematic error in the analysis methods was the use of the Whitsett (1963) model data and the normalization equations of Dapena to calculate the principal mass moments of inertia for the body segments. The use of the data of Diffrient et al. (1974) provided similar errors for the estimation of segment mass and location of the center of mass.

The random error is introduced primarily by the digitizing efforts of the researcher when using the methods of Kollias (1984b). The selection of the exact joint and limb endpoints is not possible and some random error is introduced in this process. The vibrations of the cameras and digitizing equipment would also introduce some random error. This error was estimated by redigitizing 10 randomly selected frames. The RMS difference between all the points was: X = 1.56 cm., Y = 0.75 cm., Z = 0.89 cm.. The data containing this random error was smoothed using the digital filter set at a cutoff frequency of 8.25 Hz.. The automatic digitizing methods of the *APAS* system would not introduce a random error like this but the target tracking and selection algorithms would introduce some systematic error.

Using methods similar to those of Walton (1981), a final analysis of the error introduced by the digitizing process was undertaken. The use of the rigid link model in dynamic analysis assumes a constant length of the links. Therefore, it seemed appropriate to examine these values as calculated for each link at each time step of the analysis. Tables 3.6 and 3.7 illustrate the difference between the measured values of the segments and the mean of the values for a complete trial calculated from the 3 dimension coordinates produced by the data

Table 3.6

Percentage deviations of the computed mean segmental lengths for one trial from values of direct measurement for the three link model. Values are from the trials with the largest errors.

LINK	Manual Process (Film)	Automatic Process (Video)
Link 1	0.22%	1.29%
Link 2	4.93%	2.19%
Link 3	6.29%	1.48%

Table 3.7

Percentage deviations of the computed mean segmental lengths for one trial from values of direct measurement for the human subjects. Values are from the trials with the largest errors.

LINK	Manual Process (Film)	Automatic Process (Video)	Values Reported By Walton (1981)
Right Arm	24.3%	3.39%	22.7%
Left Arm	9.6%	4.15%	16.0%
Right Forearm	2.9%	8.26%	9.80%
Left Forearm	5.4%	16.5%	8.80%
Right Hand	17.8%	0.91%	Not reported
Left Hand	5.0%	4.09%	Not reported
Right Thigh	11.2%	5.26%	19.0%
Left Thigh	9.2%	2.63%	19.0%
Right Leg	5.5%	1.50%	6.30%
Left Leg	4.2%	8.00%	6.30%
Right Foot	12.7%	10.0%	Not reported
Left Foot	12.3%	10.0%	Not reported

collection techniques outlined. The values reported here are from trials producing the most error. The percentages reported for Walton's work are the best values he found.

These tables reveal that there is a difference between the measured and calculated values. What is also obvious is that the automatic digitizing process generally produces more accurate segment length values than the manual method. The differences found in the three link model are generally lower than those for the human model. This probably resulted because the segment end points where more easily observed. The values for the human model, though higher, compare favorably with the work of Walton(1981) especially when it is remembered that the worst values of this study are compared to the best of the earlier research.

As noted in equation 3.20 the lengths of the segments are used in the calculation of the net moments at the joints. Further examination of the deviation in the segment lengths revealed that this deviation changed throughout a single trial. An example of this can be clearly seen in Figure 3.9 which shows the change in the length of the right thigh during one of the trials. Based on these observations it was decided to use the mean values of segment length for each trial in the calculations of net moments. This approach to the varying lengths problem was previously used by Marshall et al. (1985).

3.5.2 Kinematic Results.

The kinematic modules of the analysis program were capable of producing both linear and angular values for all links in each model. Rather than reporting all the results for all of the segments in all of the trials (as this would produce a huge amount of data) only examples of typical findings will be reported and any problems that were encountered.

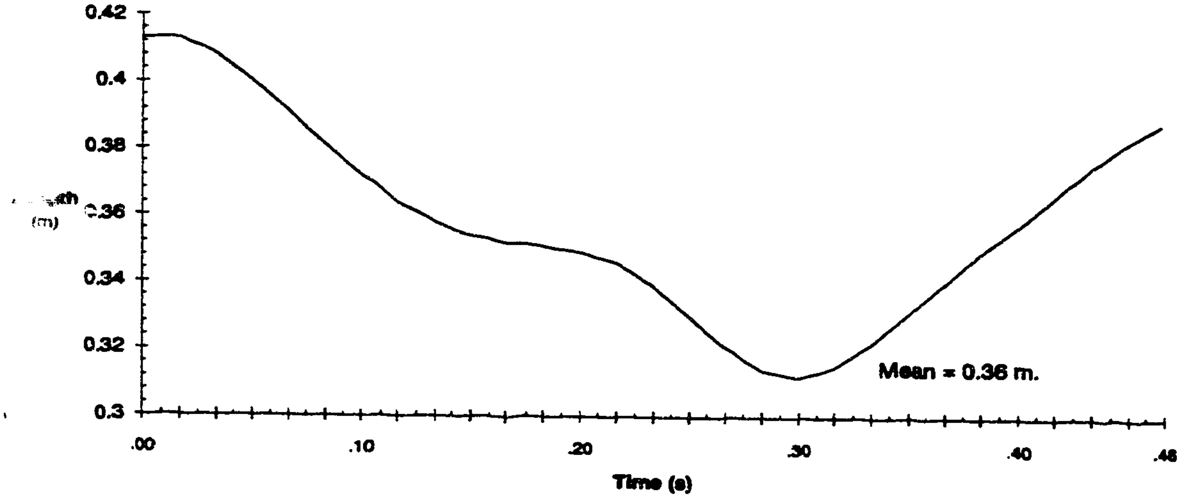


Figure 3.9 An example of the change of segment length for the right thigh during a single trial (video) as a result of digitizing error.

The linear kinematics for the model segments produced results that would be expected. Figure 3.10 is an example of typical values found for Link 2 of the three link model while those for the right forearm of a human performing a split jump can be seen in figure 3.11.

Initial examination of the angular kinematics for the three link model as exemplified in figure 3.12 revealed some unexpected results. The angular velocity and accelerations around the X axis in the LCS (the long axis of the link) are very high for the first 0.10 seconds. This was typical for all of the links in all trials during this time period. After a lengthy evaluation of the links' motion, it was determined that this was being caused when the joint angles were near π radians. As the orientation of the links was determined in part by using the cross product between vectors pointing in the direction of the two adjoining segments and the angular velocity and acceleration were based on these orientation angles this problem can be explained. When the joint angle was near π radians

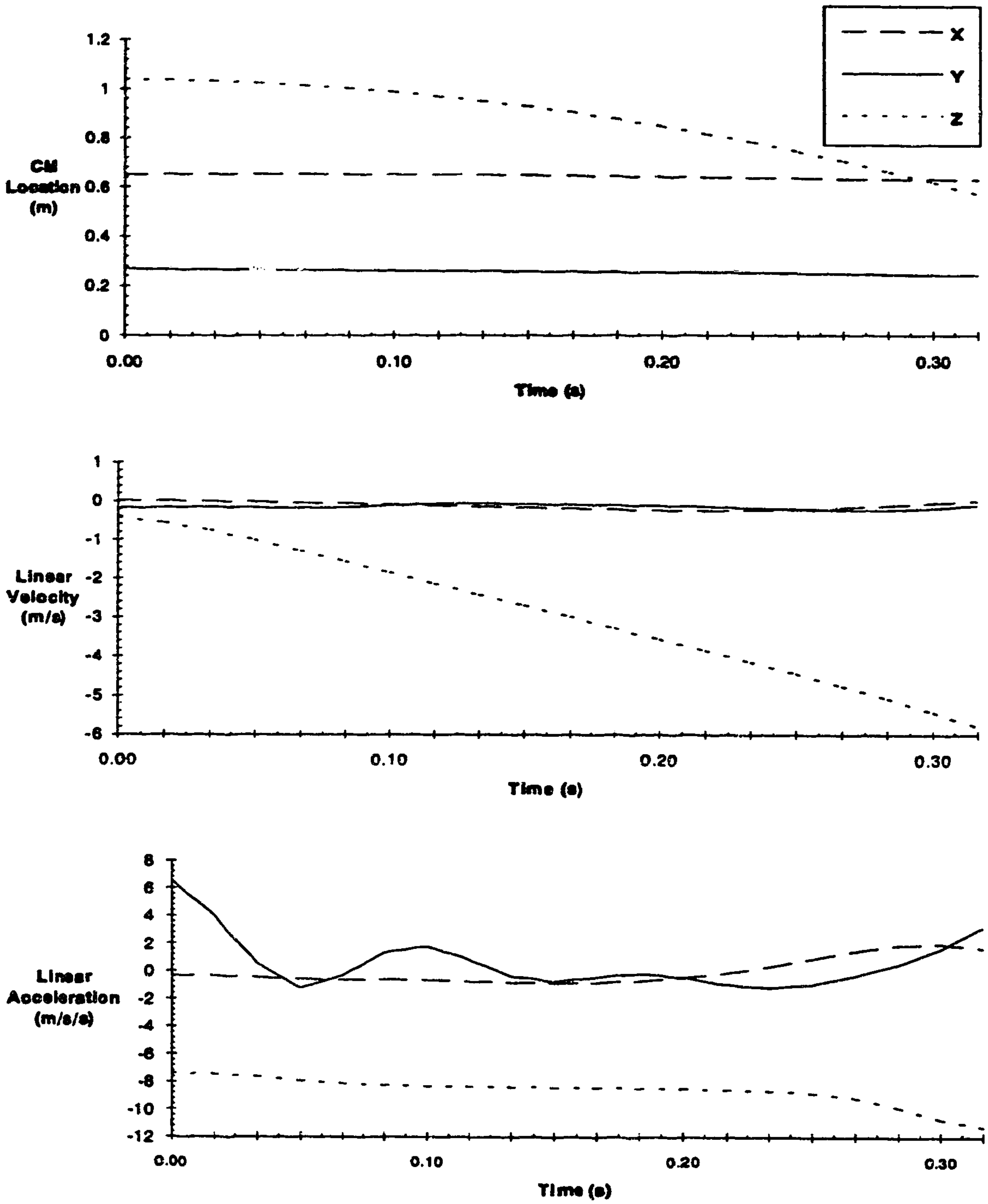


Figure 3.10 An example of the linear kinematics of the CM of link 2 for the three link model during a single trial in the ICS.

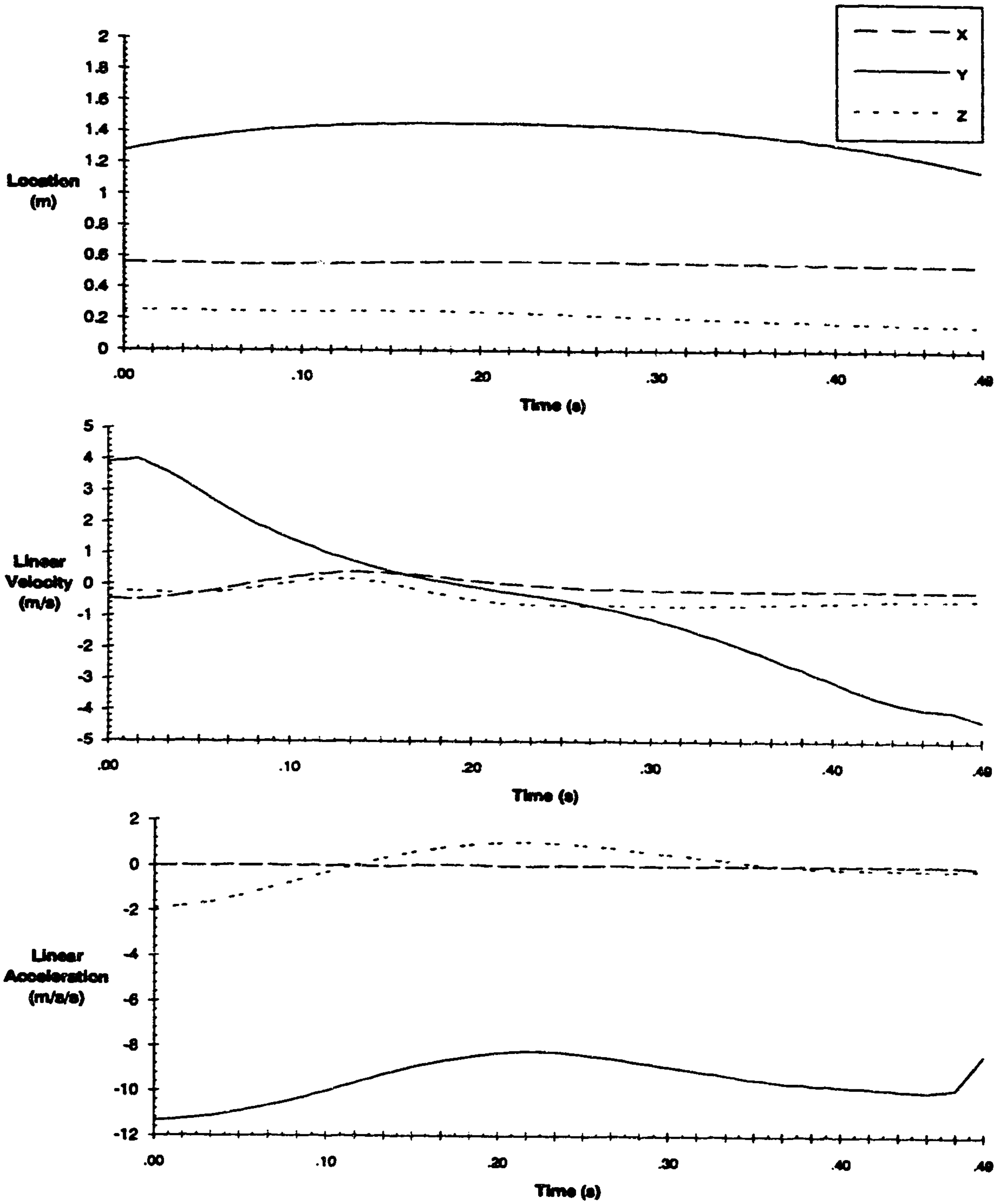


Figure 3.11 An example of the linear kinematics in the ICS for the CM of the forearm for the human model during a split jump

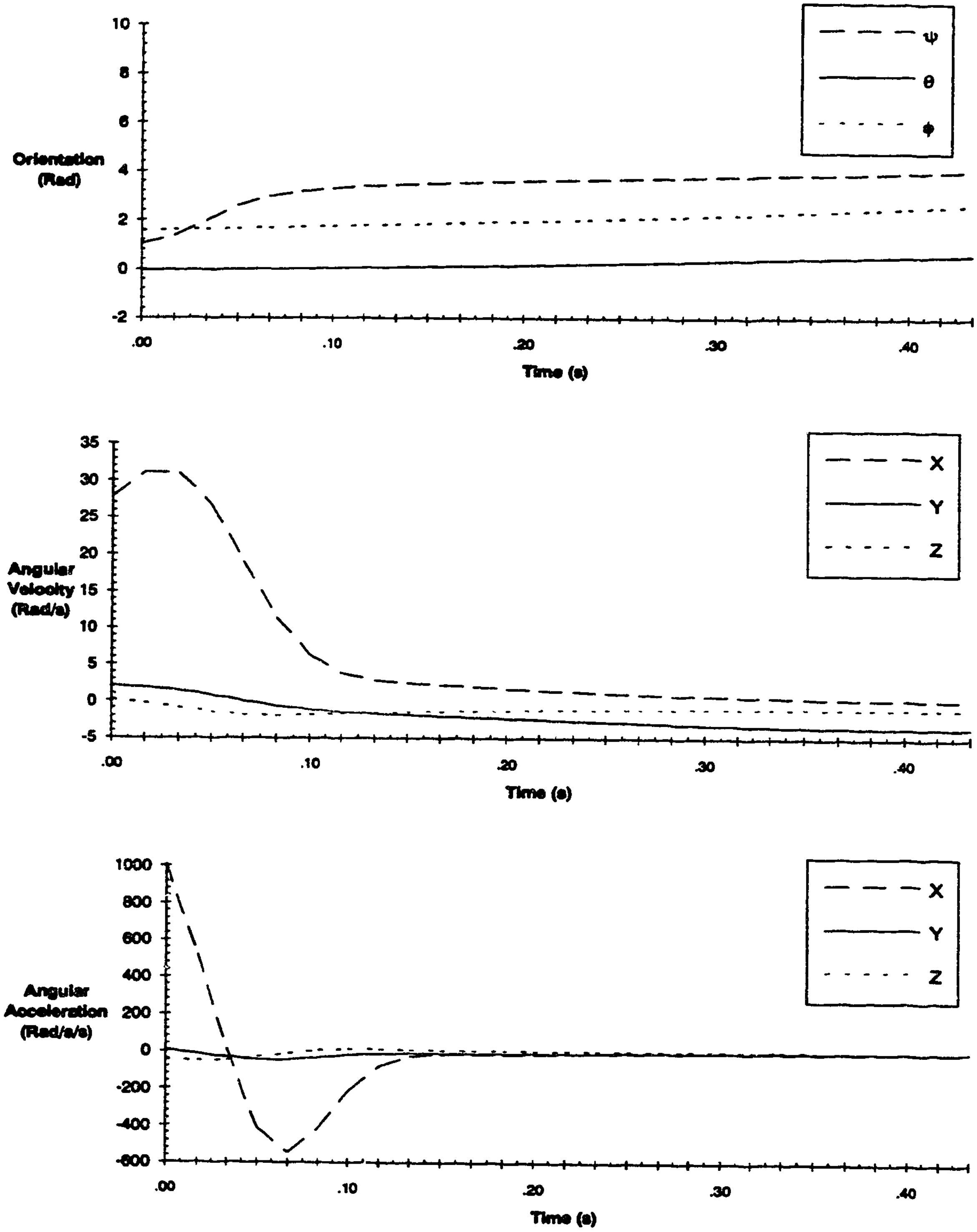


Figure 3.12 An example of the angular kinematics in LCS for link one of the three link model during a single trial

the digitizing error at the segment endpoints was large enough to cause the cross product vector, j_i from equations 3.4, to change direction rapidly from one time step to the next. Once the joint angle becomes sufficiently less than π radians the digitizing error does not have the same effect on the direction of the cross product vector. Only when this position is reached by adjoining segments will the angular kinematics not be as susceptible to the digitizing error. In the case of the three link model this meant waiting until the elastic bands had sufficiently flexed the joint. Therefore, for all trials of this wooden model the dynamic analysis and subsequent simulation began when this time had been reached.

The problem for the human model was much more difficult to deal with as the body joints did not simply continue to flex in the chosen motions. Rather, they went through flexion and extension reaching the problem position around π radians at various times. This required an examination of each segment to determine the best way of calculating reasonable orientation angles. There were no real problems for the foot and leg as the ankle joint seldom approached the problem angle. The thigh proved to be more difficult as in all of the motions the knee was often in a position at or near full extension at various times depending on the motion. This resulted in very large angular velocities and accelerations around the X axis (long axis) of the thigh that were obviously incorrect. Much better results were obtained when the cross product of the leg and thigh vectors was replaced with the vector from the cross product between the foot and leg. That is for each lower extremity in this study:

$$j_{thigh} = j_{leg} = j_{foot} \tag{3.23}$$

The knee is generally modelled as a one DOF joint and the ankle, although actually having two DOF, primarily moved through flexion and extension with little inversion and eversion for the movement patterns in this study and therefore could be considered to have only one DOF. If modelled this way the vectors representing the cross products at each joint will point in the same direction.

For the upper extremities the problem could not be solved in a similar manner because the hand does not behave the same way as the foot. The cross product vector at the elbow joint was the only reasonable alternative as the arms were bent at the elbow for the majority of time during the trials. The orientation of the hand was calculated using this vector as well. The two DOF nature of the wrist joint suggests that this would be an inappropriate constraint but observation of the trials indicated that for this study it was reasonable. Thus for each upper extremity in this study:

$$\mathbf{j}_{hand} = \mathbf{j}_{forearm} = \mathbf{j}_{arm} \qquad (3.24)$$

The calculation of orientation angles for the trunk and the head segments also presented a problem as these two segments are joined by a three DOF joint. This was solved by creating a segment that had end points at the shoulder joints. The cross product between the vector representing this temporary segment and either the head or trunk segments produced better orientation angles for these links. Thus for the head and trunk in this study **b** in equations 3.4 is a vector pointing from the left shoulder joint to the right shoulder joint. In the case of the trunk this is reasonable as the rotation of the trunk around its long axis is often reflected in the motion of this shoulder segment and was certainly the case in this study. For the head this could not be generally considered true but again observation of the motions in this study indicated it too was reasonable.

With these adjustments made in the methods for calculating orientation angles the angular kinematics for the human trials were recalculated. Figure 3.13 is an example of the angular kinematics for the right thigh during a tuck jump. As this motion primarily involves rotation around the Y axis the velocity and acceleration curves generally seem appropriate for the motion. The maximum angular velocity for the thigh in all of the jumps was 11.5 radians/second and the maximum angular acceleration was 208 radians/second^2 . These values are similar to the values reported by Huang et al. (1983) for simulated kicking, Putnam (1981) for punting a football and van Soest et al. (1985) for vertical jumping, all which exhibit similar types of movement of the thigh around the hip joint. The high rates of angular velocity and acceleration around the X axis should also be noted in figure 3.13. This was not expected but examination of the data for all jumps revealed that this was common and caused because the foot at this time is in the most plantar flexed position putting the ankle joint much nearer the problem position of π radians than at any other time.

An examination of the data for all of the jump trials revealed similar types of results but with the different patterns of motion reflected in the respective angular kinematics.

Despite obtaining what generally seems to be reasonable angular kinematic results, it is clear that better methods must be used to calculate three dimensional segment orientation angles in future research studies using a linked

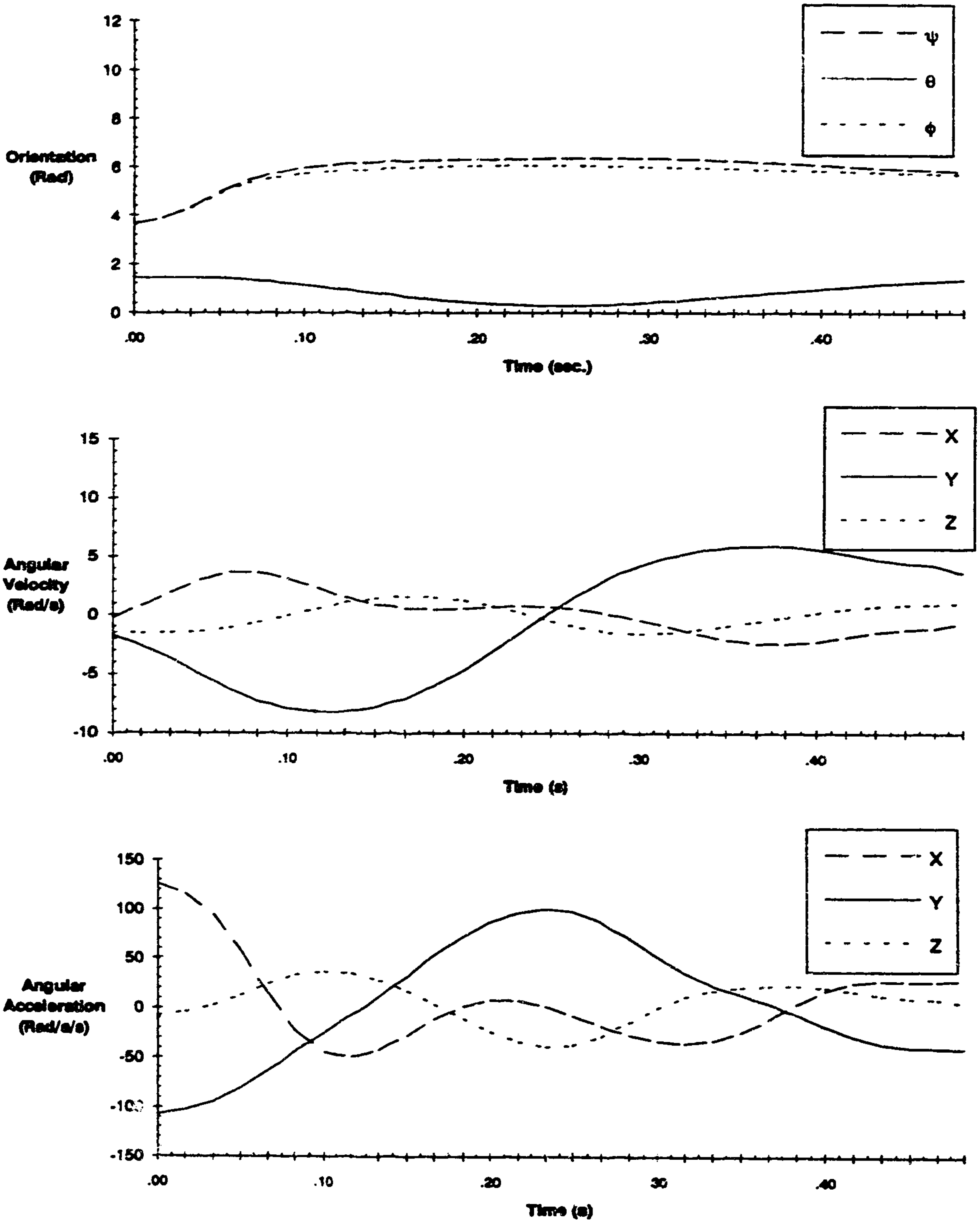


Figure 3.13 An example of the angular kinematics in LCS for the right thigh of the human model during a tuck jump.

rigid body model. It is clear that another point not located on the long axis for each link must be included . The selection of another point that is visible in the view of both cameras is quite difficult but Vaughan et al. (1992) describes points that can be used in human gait analysis and may work in more complex motions as examined in this study. The use of additional cameras during the data collection would also help with this problem. A second alternative is the rigid attachment of wands or frame-like structures, containing the necessary third point, to each link as suggested by Davis et al. (1990).

3.5.3 Kinetic Results.

As with the kinematic analysis it was possible to provide large quantities of data with the kinetics but discussion will be based on typical findings. Figure 3.14 provides examples of the net joint reaction forces calculated for link 3 of the wooden model and the left leg during a tuck jump. In this trial for the wooden model the primary motion of link 3 is a rotation in the XY plane about a Z axis in the ICS. As seen in figure 3.14 the joint reaction forces are generally negative in the Y direction until the link slows, stops and then changes direction resulting in a change of the joint reaction force to positive. The final change of direction i Y is as the link begins to be affected by the elastic band as it rebounds at the end of the range of motion. Evaluation of other links and other trials for the wooden model produced results which also were appropriate for their patterns of motion.

As the principal movement for the leg in the tuck jump is primarily in a vertical direction it is appropriate to examine those force values for the Z axis.

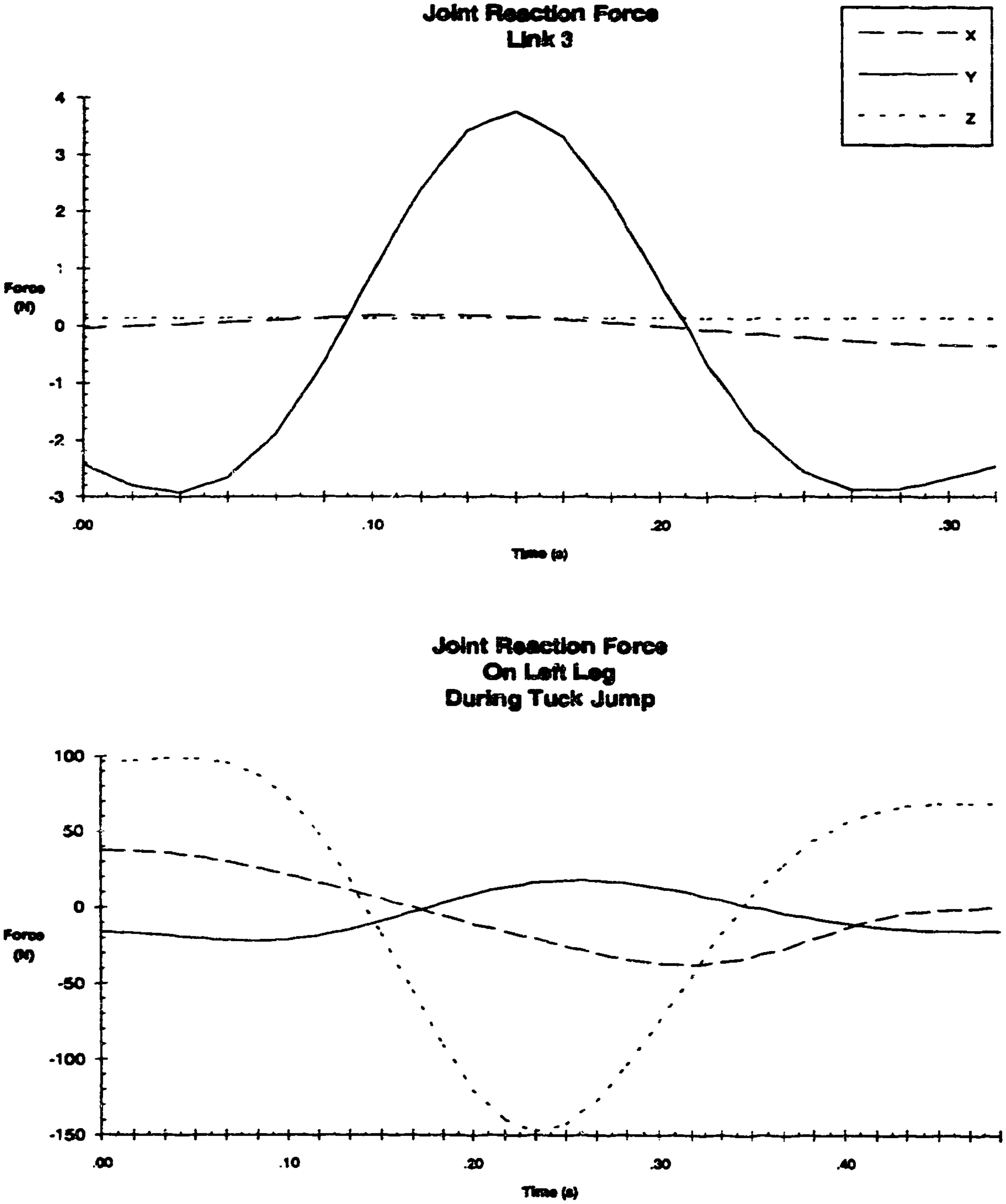


Figure 3.14 Examples of joint reaction forces for the three link model and the left leg during a tuck jump for the human model.

During the first third of the movement, the reaction force in the leg (at the knee) is positive as movement begins upward. This is followed by a negative value as the leg decelerates when the top of the tuck is approached and the direction of the movement changes to downward. The force value changes to positive again as the leg is again accelerated upward in preparation for landing. These values seem appropriate for this action and likewise, an evaluation of the values for the other two directions produces similar conclusions. The magnitudes of the forces are slightly less than those found by Zernicke and Roberts (1978) and Huang et al. (1983) in their 2 dimensional studies of kicking. An examination of other links and other trials produced joint reaction forces with similar results.

The rotation of link three in the wooden model is reflected in the torque patterns of figure 3.15. The motion of link three in the ICS for this trial was

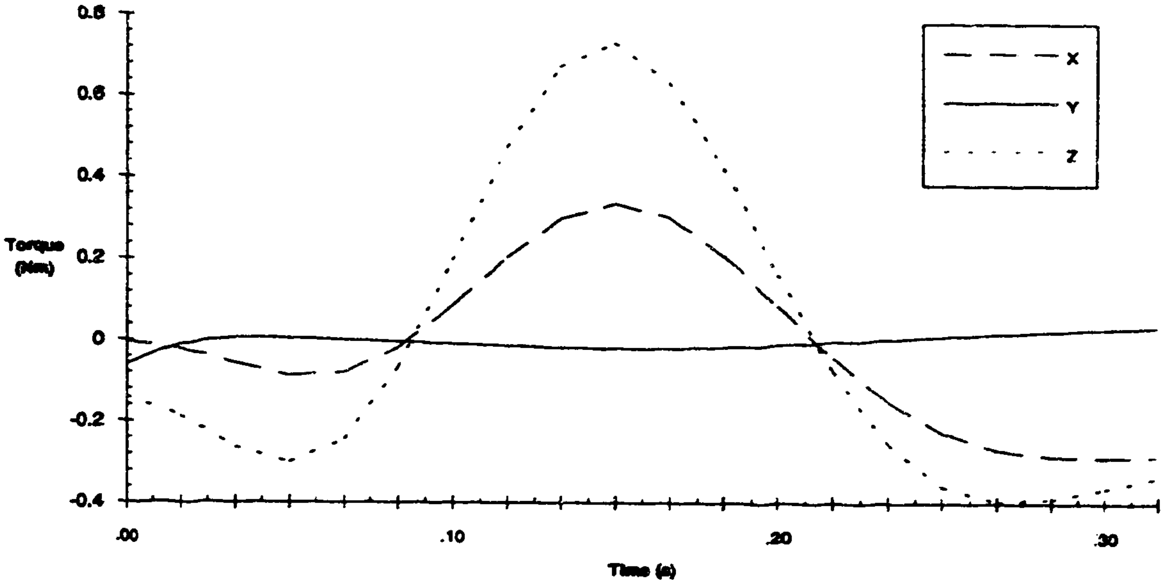
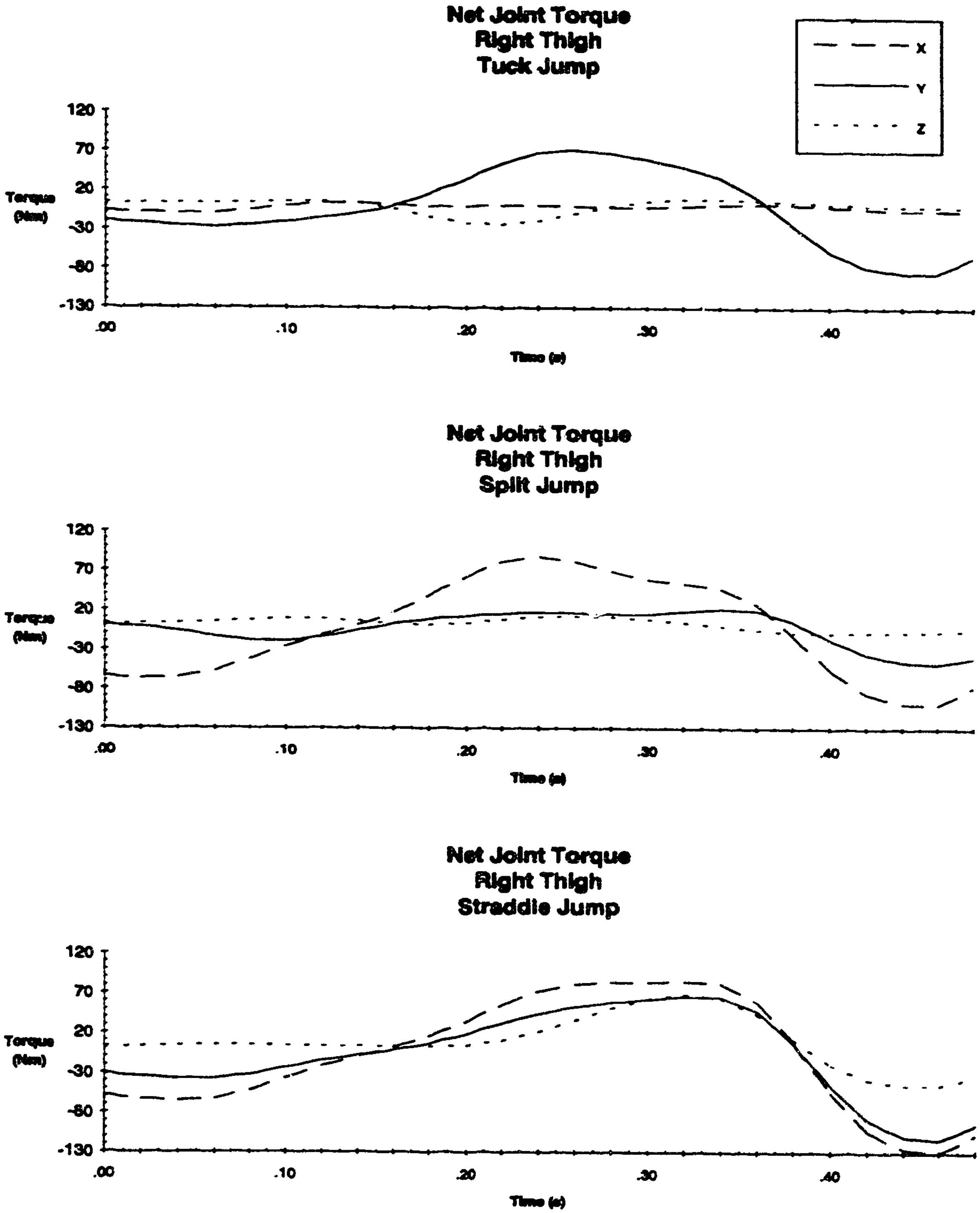


Figure 3.15 An example of net joint torques at the proximal joint of link three of the wooden model

primarily clockwise around the Z axis and slightly clockwise around the X axis. The pattern of the torques around the X and Z axes were appropriate as the values are in the negative direction during the period when the elastic band is acting. This is followed by a period in the positive direction, as the joint friction only is acting, after the elastic is completely contracted. The positive torque increases as the link reaches the end of the range of motion and changes its direction of rotation about the joint. The final change of direction of the torque represents that caused by the friction at the joint acting in a direction opposite the final motion. The other trials for the wooden model produced similar results.

The primary motion for the tuck jump involves rotation of the thigh about the Y axis running laterally through the hip joint. As can be seen in figure 3.16 for the tuck jump the net torque about this axis shows the highest values. The pattern of progression from negative to positive and back to negative again also agree with what would be expected for this jump. In contrast the motion around the hip, in the split jump, involves primarily rotation about the posterior-anterior oriented X axis. In the graph of the torque patterns for this jump the largest values are around the X axis.

One of the purposes of this study was to develop a dynamic analysis system for three dimensional motion. While the torque values for the first two types of motion could have been found, with a reasonable degree of accuracy, using 2 dimensional techniques, the motion of the straddle jump would greatly limit the accuracy of any such data produced for the hip. The final graph of figure 3.16 illustrates the torque values for the right thigh at the hip joint during a straddle jump. This jump requires the rotation of the thigh about all three inertial


Net Joint Torque
Right Thigh
Tuck Jump

Net Joint Torque
Right Thigh
Split Jump

Net Joint Torque
Right Thigh
Straddle Jump

Figure 3.16 An example of net joint torques of the right thigh during three different types of jump.

axes and this is clearly demonstrated by the net torque values calculated for this trial. The lower values for the Z direction indicate less rotation for this orientation although considerably more than found for either the tuck or split jumps. The magnitudes and patterns seen for the other axes are what would be expected. No results were found for similar types of motion in the literature and again only values for 2 dimensional analysis of kicking can be compared. The maximum torque values are very similar to those found by Putnam (1981) for punting but about one half of the maximum values found by Huang et al. (1983) for a simulated kick and [illegible] legged vertical jump examined by van Soest et al. (1985). Although [illegible] not reasonable to present all of the torque values found in this study, an examination of them generally revealed data similar in nature to that discussed in depth.

Using the methods described in section 3.4.3 the kinetic data were also evaluated. Figures 3.17 and 3.18 illustrate the difference between the forces found through direct analysis of the linear motion of the center of mass of the trunk (measured) and the sum of the forces acting at the proximal end of each limb (calculated). Two examples are shown; the first from a trial of the split jump done with the automatic digitizing represents an example of the best results, and the second from a trial of a straddle jump done with the manual digitizing represents an example of the worst. The difference between the measured and calculated data are quite large especially in the case of the manually digitized straddle jump. A similar analysis of the torque values for the same two trials can be seen in figures 3.19 and 3.20. Here again the differences are quite large with the better results coming for the split jump with the data collected using the automatic digitizing feature of the *APAS*. These are the values calculated after

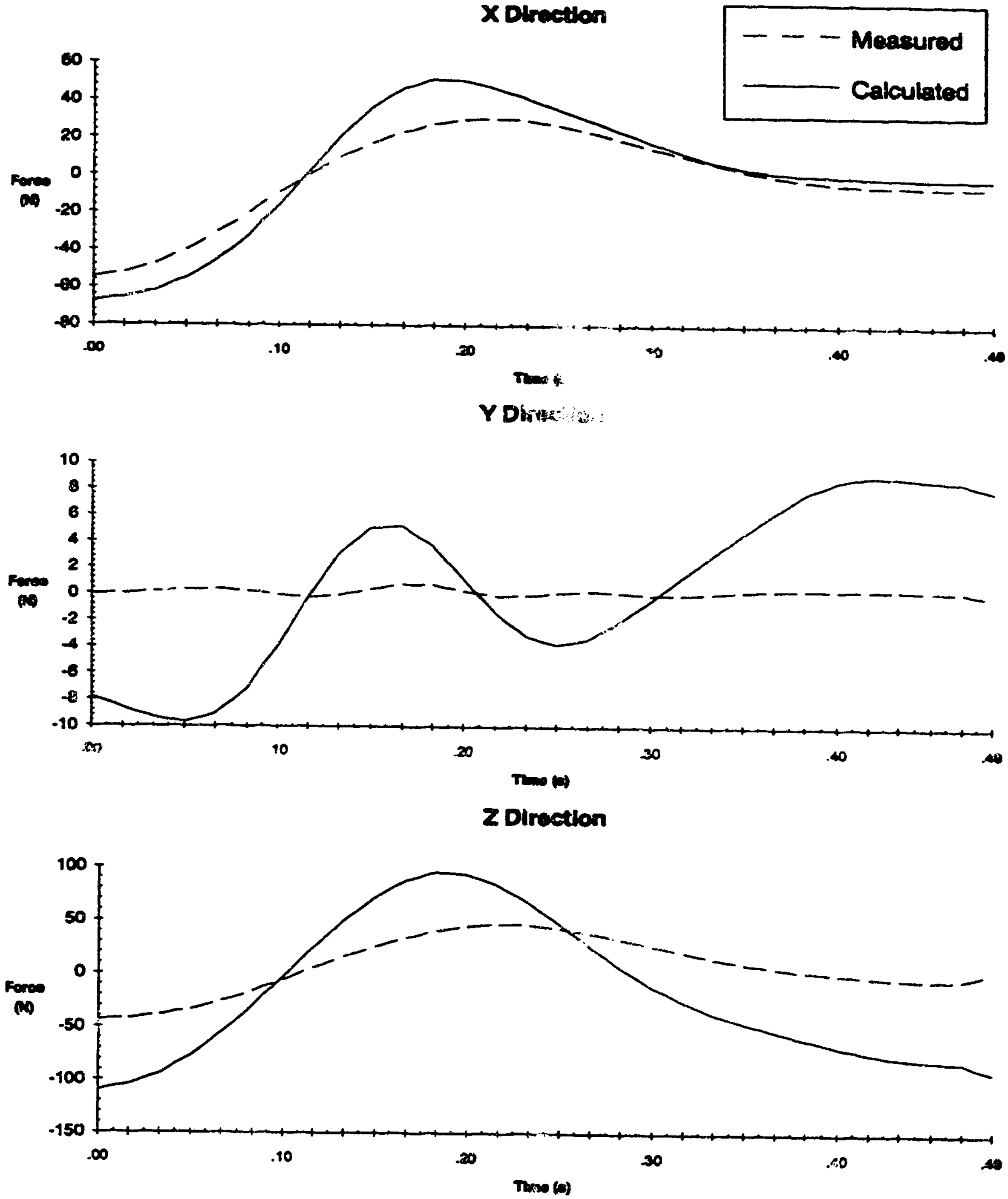


Figure 3.17 Force values from measurements of the trunk vs sum of calculated forces at hip and shoulder joints for a split jump. Data for this trial is from an automatically digitized video record.

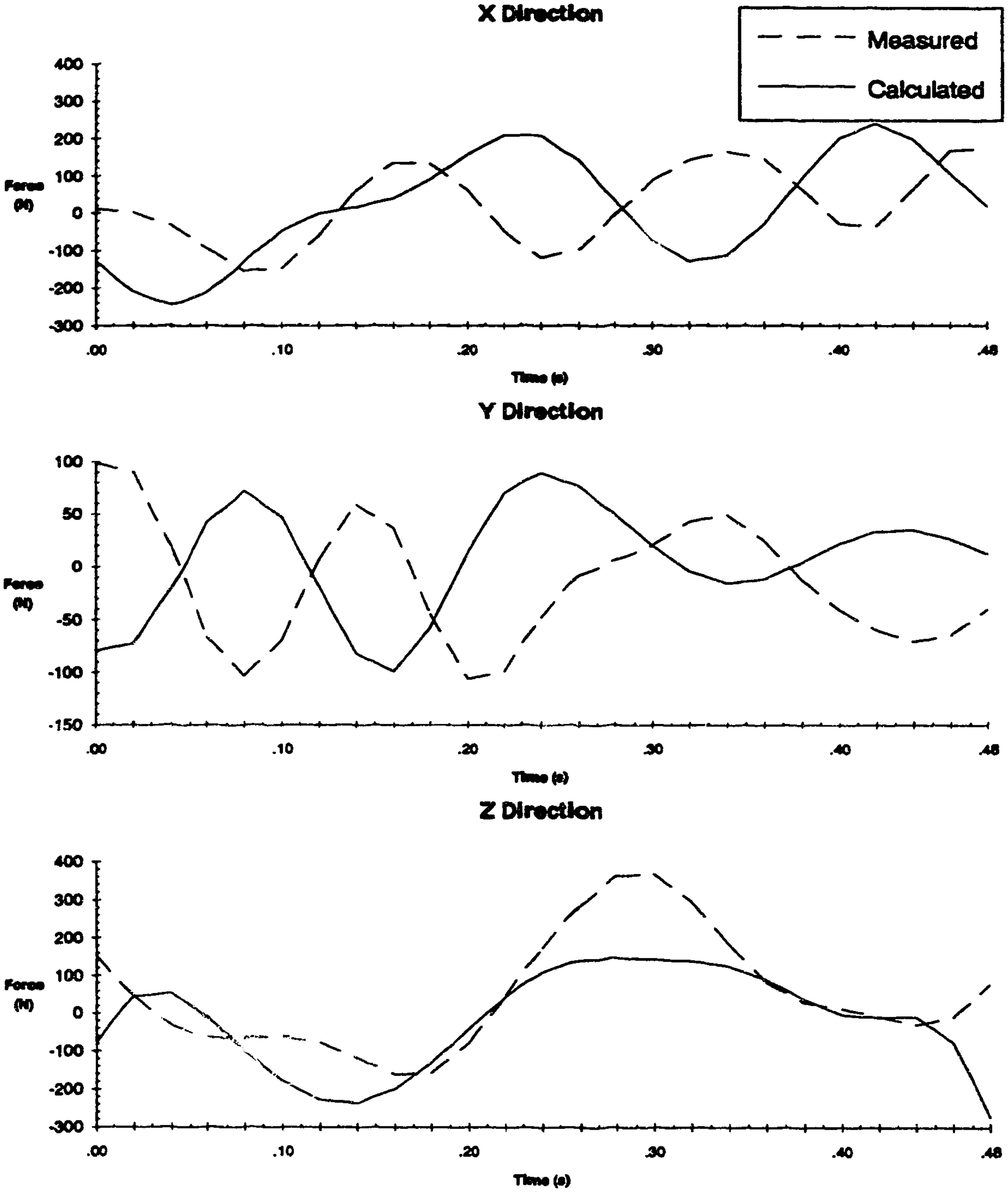


Figure 3.18 Force values from measurements of the trunk vs sum of calculated forces at hip and shoulder joints for a straddle jump. Data for this trial is from a manually digitized film record.

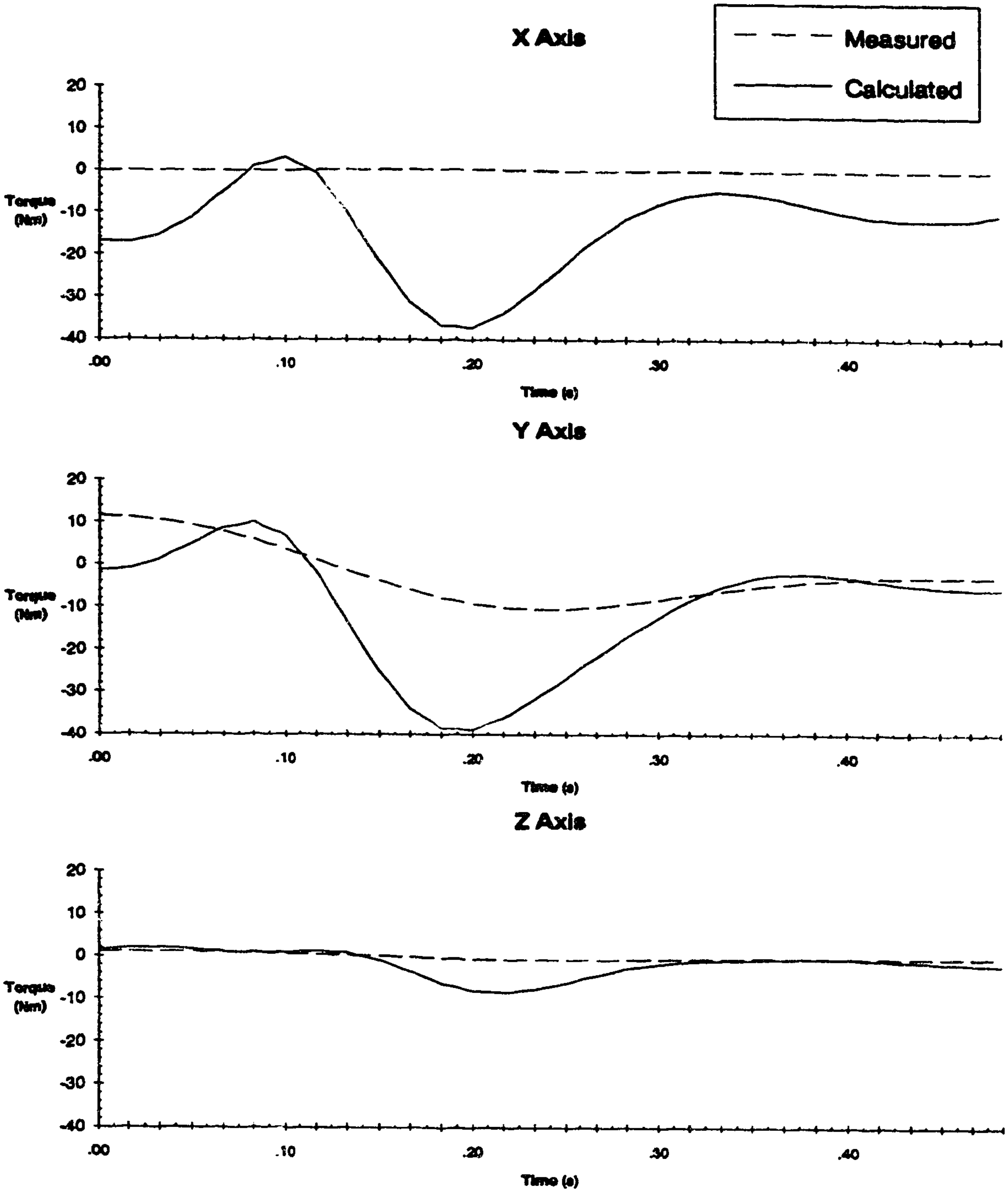


Figure 3.19 Torque values from measurements of the trunk vs sum of calculated torques at hip and shoulder joints for a split jump. Data for this trial is from an automatically digitized video record.

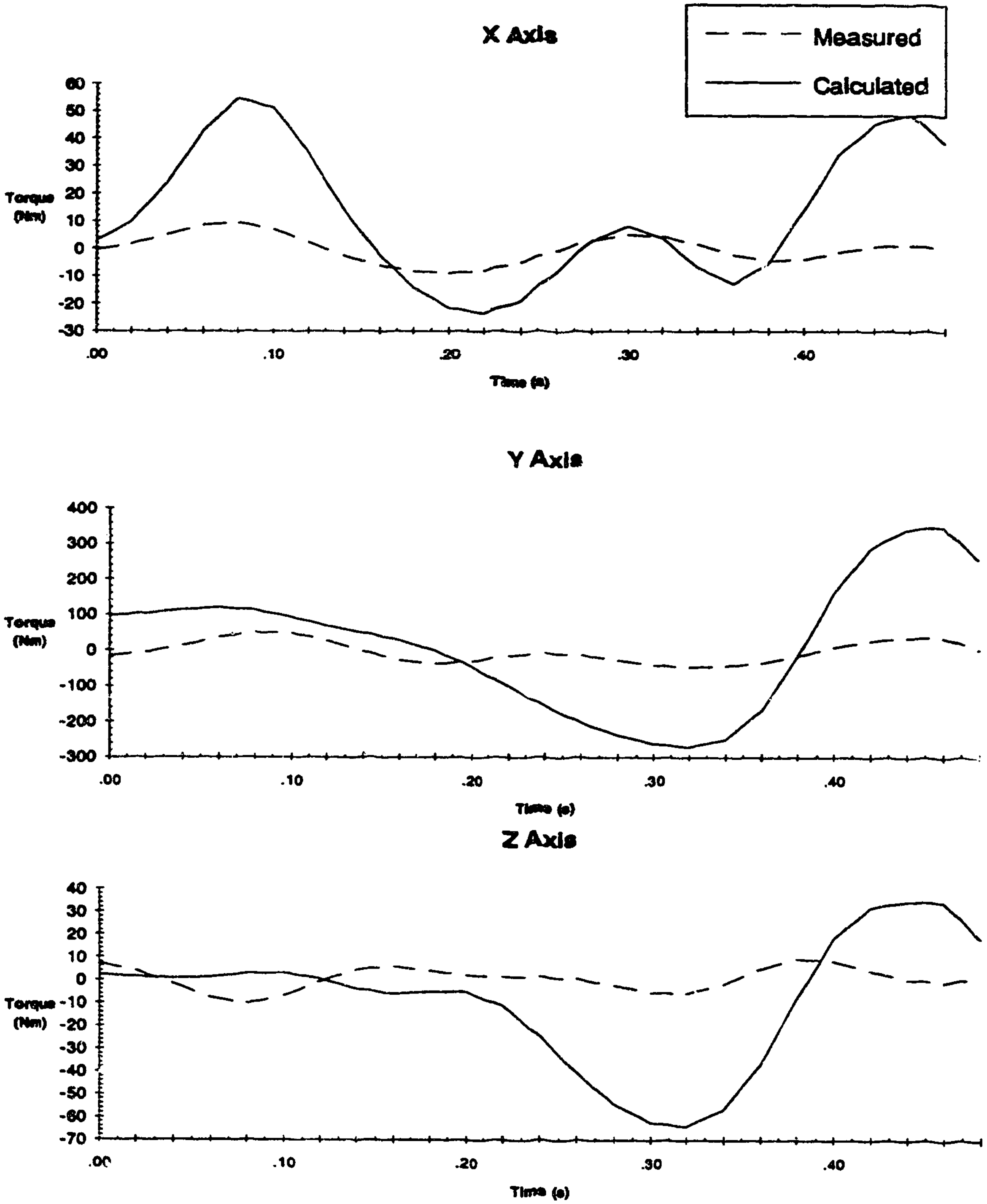


Figure 3.20 Torque values from measurements of the trunk vs sum of calculated torques at hip and shoulder joints for a straddle jump. Data for this trial is from a manually digitized film record.

making adjustments to the angular kinematic data of the model links using the methods described earlier. Ideally these difference values would have been much closer to zero but, with all of the possible sources of error that can be introduced into the kinetic solutions by the kinematic errors and the calculations of body segment parameters, it is not surprising. Meglan (1991) reported differences in the same range as found in this study for both force and torque. The exception was that the torque values around the Y axis for the straddle jump (figure 3.20) were almost three times as great at some points in time as those reported by Meglan (1991). As this trial was from the manually digitized data this larger difference may be attributed to the greater error for this collection method as previously discussed in this chapter. It should also be noted that the data for Meglan's study were collected with the aid of the *VICON* automatic digitizing system. No other literature was found which did a similar quantitative analysis.

As is demonstrated by the difference between the data collected with manual digitizing methods and that collected with the automatic digitizing, these differences can be made more acceptable with careful data collection. The use of better techniques for calculating body segment parameters may also help to minimize the difference. However, an evaluation of the differences for both force and torque values in the motion of the wooden three link model, for which it was possible to be much more accurate when calculating its physical properties, does not seem to support this hypothesis. The difference found was about the same relative magnitude.

When using the techniques of rigid linked body analysis, total removal of these differences is probably not possible. Output from the dynamic analysis routine developed for this study would seem to indicate that automatic data collection systems such as *APAS* do produce better results than the systems based on manual digitization of film records of human motion.

3.6 Summary of Dynamic Analysis Results.

The purpose of this phase of the research was the development of a computer program for the dynamic, three dimensional, analysis of rigid linked models in a state of free fall. The program produced was capable of taking an ASCII file of three dimensional coordinates of link end points and physical parameters of the model, and producing kinematic and kinetic results. When implemented on a personal computer it proved to be flexible and very easy to use.

The results for the linear kinematics seemed to be good but the angular kinematics analysis originally produced values that appeared to be incorrect at certain times during each trial When the joint between adjacent segments approached an angle of π radians orientation angles fluctuated wildly, apparently as a result of digitizing error. This forced the use of some constraints on the orientation angles for the links to remove some of this artifact.

On initial evaluation, the results of the kinetic analysis appeared to be reasonable and, where comparison was possible, generally agreed with the literature. However, a comparison of the kinetic values produced by the dynamic analysis with that expected due to the motion of the model's base links revealed a relatively large difference. This seems to indicate that error was still present in the dynamic results. This must be taken into consideration in the next chapter which will evaluate a computer simulation technique that uses much of this dynamic information as input.

CHAPTER 4

EVALUATION OF A SIMULATION TECHNIQUE FOR LINKED BODIES

A direct or forward dynamics approach to problems of rigid body motion enables the researcher to calculate the motion of a mechanical system given all of the forcing functions that will act on it (Vaughan, Hay and Andrews 1982). *"There are a number of ways to formulate the dynamic equations, such as the Gibbs-Appell, the Lagrangian, the D'Alembert, and the Euler formulations"* (Wilhelms, 1990, p. 267). The computational complexity of most forward dynamics algorithms is O(n^3) or worse (Featherstone, 1983) as a result of having to work with large matrices.

Armstrong and Green (1985) first used a "recursive" solution from Armstrong (1979), a very efficient method O(n), to solve the equations of motion for tree linkages in human form. Additional work has been done using this solution (Armstrong, Green and Lake 1986, 1987; Forsey and Wilhelms,1988; Lake 1990) with most of it directed at producing "realistic looking" and controlled animation of human motion. Lake (1990) did use functions obtained from biomechanical studies of muscle contraction and reported by Hatze (1981) but no direct effort has been made to calculate forcing functions from real human motion and input them into the simulation technique. The dynamic analysis system developed in Chapter 3 is capable of producing the kinematic and kinetic data necessary as input for the simulation. The purpose of Chapter 4 is to further examine the capabilities of this simulation method, especially as a research tool for the biomechanist.

4.1 Direct Dynamics - The Equations of Motion.

Armstrong and Green (1985) represent the human body as a multiple rigid link, tree-like structure connected at three degree of freedom spherical joints. A root link is then attached to the real world inertial frame by a joint with three rotational degrees of freedom and three translational degrees of freedom. For this research, models similar to that used in the dynamic analysis of Chapter 3 have been used. In the case of the three link, mechanical model the root link was considered to be the long link (link 1) of the structure while the entire trunk served the same role for the 14 link human model. Each link, except the root link, is considered to be attached at its proximal end to another link termed its "parent" and may be attached at its distal end to 1 or more links called "sons". Thus, the entire structure of the models in this research can be seen as illustrated in figures 4.1 and 4.2 and described in tables 4.1 and 4.2.

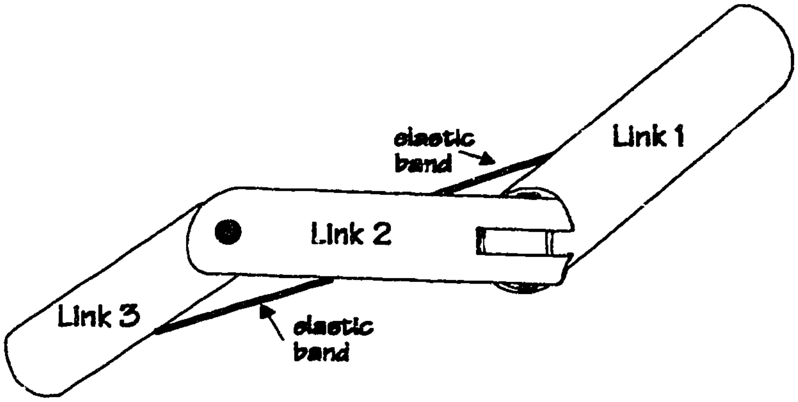


Figure 4.1 Three link model

Table 4.1 Structural Relationships of Segments in the three link Model

Segment Name	Segment Number	Parent of Segment	Number of Sons
Long	1	None	1
Middle	2	1	1
Short	3	2	0

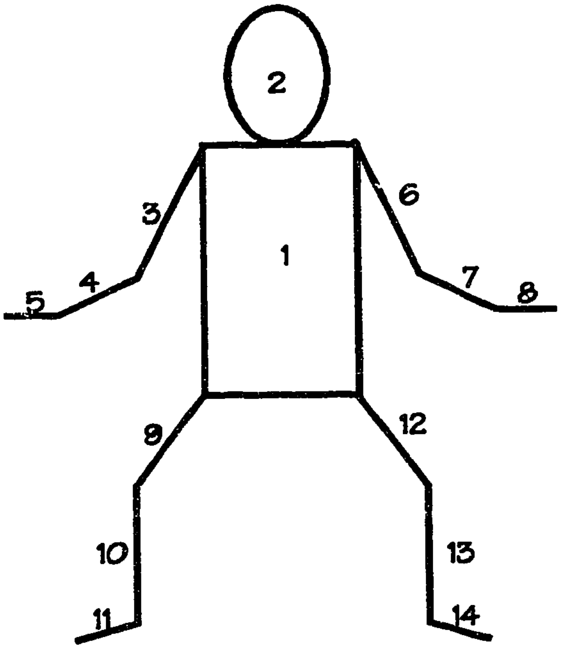


Figure 4.2 14 Link Human Model

As in solving the inverse dynamics problem presented in Chapter 3 a stationary inertial coordinate system (ICS) and moving link coordinate system (LCS) attached to each segment are used in solving the equations of motion for the forward dynamics problem of simulation. The axes of the LCS are again aligned to the calculated principal axes of each segment. All of the terms used in the equations of motion for this method are defined in Table 4.3.

Table 4.2
Structural Relationships of Segments in 14 Link Human Model.

Segment Name	Segment Number	Parent of Segment	Number of Sons
Trunk	1	None	5
Head	2	1	0
RtArm	3	1	1
RtForearm	4	3	1
RtHand	5	4	0
LtArm	6	1	1
LtForearm	7	6	1
LtHand	8	7	0
RtThigh	9	1	1
RtLeg	10	9	1
RtFoot	11	10	0
LtThigh	12	1	1
LtLeg	13	12	1
LtFoot	14	13	0

Table 4.3

Terms used in Equations of Motion for Simulation

From Armstrong and Green (1985)

Term	Definition
m^i	mass of link i (scalar quantity).
$\mathbf{a}_G$	acceleration of gravity in ICS.
$\mathbf{p}^i$	position vector of proximal joint of link *i* in ICS
$\mathbf{v}^i$	velocity of proximal joint of link *i* in ICS.
$\mathbf{f}^i_E$	external force vector in ICS acting on link *i* at the point $\mathbf{p}^i_E$.
$\mathbf{g}^i_E$	external torque acting on link *i* in ICS.
$\mathbf{a}^i$	acceleration of proximal hinge of link *i* in LCS.
ω^i	angular velocity of link *i* in LCS.
$\dot{\omega}^i$	rate of change of angular velocity of link *i* in LCS.
$\mathbf{c}^i$	vector from the proximal hinge to the center of mass of link *i* in LCS.
$\mathbf{f}^i$	force at proximal joint of link *i* exerted on its parent in LCS.
$\mathbf{g}^i$	torque that link *i* exerts on its parent in LCS.
$\mathbf{p}^i_E$	vector from proximal joint of link *i* to location of the application of the external force $\mathbf{f}^i_E$ in LCS.
ℓ^i	vector from proximal joint of parent of link *i* to the proximal joint of link *i* in the LCS of the parent.
$\mathbf{R}^i$	rotation matrix used to convert vectors in the frame of link *i* to the frame of the parent.
$\mathbf{R}^{iT}$	the inverse of $\mathbf{R}^i$ used to convert vectors in the LCS of the parent of link *i* to the LCS of link *i*.
$\mathbf{R}^i_I$	rotation matrix used to convert vectors in the LCS of link *i* to the ICS.
$\mathbf{R}^{iT}_I$	the inverse of $\mathbf{R}^i_I$ used to convert vectors in ICS to the LCS of link *i*.
$\mathbf{J}^i$	matrix for the principal moments of inertia of link *i*.

The first equation of motion shows the relationship of the angular momentum of link i to all of the torques applied to it:

$$\mathbf{J}^i\dot{\omega}^i = \mathbf{g}^i_\Sigma - m^i\mathbf{c}^i \times \mathbf{a}^i + \sum_{s \in S_i} \mathbf{l}^s \times \mathbf{R}^s\mathbf{f}^s \qquad (4.1)$$

where S_i represents the set of all sons of link i and where:

$$\mathbf{g}^i_\Sigma = -\omega^i \times (\mathbf{J}^i\omega^i) - \mathbf{g}^i + \sum_{s \in S_i} \mathbf{R}^s\mathbf{g}^s + \mathbf{R}_I^{iT}\mathbf{g}^i_E + m^i\mathbf{c}^i \times \mathbf{R}_I^{iT}\mathbf{a}_G + \mathbf{p}^i_E \times \mathbf{R}_I^{iT}\mathbf{f}^i_E \qquad (4.2)$$

In equation 4.1 the second term ($-m^i\mathbf{c}^i \times \mathbf{a}^i$) is the 'fictitious' torque caused by the acceleration of the link mass in the moving LCS while the second term ($\sum \mathbf{l}^s \times \mathbf{R}^s\mathbf{f}^s$) is the torque coming from all of the sons of link i as a result of the forces applied to the sons. In equation 4.2 the term ($-\omega^i \times (\mathbf{J}^i\omega^i)$) is another 'fictitious' torque caused by the rotation of the segment in the LCS. The term $-\mathbf{g}^i$ is the reaction torque (as per Newton's 3rd Law) coming from the parent as a result of the torque applied by link i on its parent. Next is added the sum of the torques coming from the sons ($\sum \mathbf{R}^s\mathbf{g}^s$) and the external torques($\mathbf{R}_I^{iT}\mathbf{g}^i_E$). The final two terms are causing torques on the proximal joint of link i due to gravity ($m^i\mathbf{c}^i \times \mathbf{R}_I^{iT}\mathbf{a}_G$) and any external forces ($\mathbf{p}^i_E \times \mathbf{R}_I^{iT}\mathbf{f}^i_E$).

The next two equations give the forces acting on the proximal hinge of link i:

$$\mathbf{f}^i = \mathbf{f}^i_\Sigma - m^i\mathbf{a}^i + m^i\mathbf{c}^i \times \dot{\omega}^i + \sum_{s \in S_i} \mathbf{R}^s\mathbf{f}^s \qquad (4.3)$$

where:

$$\mathbf{f}^i_\Sigma = -m^i\omega^i \times (\omega^i \times \mathbf{c}^i) + \mathbf{R}_I^{iT}(\mathbf{f}^i_E + m^i\mathbf{a}_G) \qquad (4.4)$$

In equation 4.3 the term ($-m^i\mathbf{a}^i$) is the force resulting from the acceleration of the LCS of link i while the next term ($m^i\mathbf{c}^i \times \dot{\omega}^i$) is the force caused by the angular acceleration of the LCS. The final term ($\sum \mathbf{R}^s \mathbf{f}^s$) is the sum of the forces coming from the sons of link i.

In equation 4.4 the centrifugal force caused by the rotating LCS of link i is given by the term ($-m^i \omega^i \times (\omega^i \times \mathbf{c}^i)$) while the last term ($\mathbf{R}_i^{iT} (\mathbf{f}_E^i + m^i \mathbf{a}_G)$) accounts for the external forces and the force due to gravity.

The final equation of motion relates the acceleration of the proximal hinge of a son of link i to the linear and angular acceleration of link i:

$$\mathrm{R}^s \mathrm{a}^s = \omega^i \times (\omega^i \times \ell^s) + \mathbf{a}^i - \ell^s \times \dot{\omega}^i \tag{4.5}$$

The term ($\omega^i \times (\omega^i \times \ell^s)$) is the centripetal acceleration of link *i* in the LCS while the last term ($-\ell^s \times \dot{\omega}^i$) is the linear acceleration caused by the rate of change of angular velocity of link *i* in the LCS.

The equations of motion described above can be used in a forward dynamics manner to calculate the kinematics of motion of all the segments in a rigid linked model given the torques at the joints and any external torques and forces. These equations can be solved using matrix methods such as Gaussian Elimination but they tend to be numerically intensive when the number of links is large as with the 14 link human model being used in this research. The method chosen by Armstrong and Green (1985) to solve these equations is of a recursive nature and is much faster when there is a large number of links in the system being modeled.

The following two linear equations form the basis of the solution:

$$\dot{\omega}^i = \boldsymbol{K}^i \mathbf{a}^i + \boldsymbol{d}^i \tag{4.6}$$

$$\mathbf{f}^i = \boldsymbol{M}^i \mathbf{a}^i + \boldsymbol{f}'^i \tag{4.7}$$

With $\boldsymbol{K}^i$, $\boldsymbol{d}^i$, $\boldsymbol{M}^i$ and $\boldsymbol{f}'^i$ being recursive coefficients, equation 4.6 shows a linear relationship between the rate of change of angular velocity of link *i* and its linear acceleration while equation 4.7 shows a linear relationship between the acceleration of link i and the reactive force it applies to its parent link.

In reorganizing equations 4.1 and 4.3 by replacing selected values with equations 4.5, 4.6, and 4.7, Armstrong and Green demonstrated that the recursive coefficients can be calculated for each link by an inward pass from the most distal links of the model to the root link. Upon reaching the root link there is no reaction force applied to its parent (because it has no parent), equation 4.7 can then be solved for $\mathbf{a}^i$ and 4.6 solved for $\dot{\omega}^i$. Now an outward pass from the root link to the most distal links allows calculation of the ω^i for each link *i*. Using a basic Euler integration technique, it is then possible to calculate for each link:

$$\omega^i = \omega^i + (\delta t \dot{\omega}^i)/2 \qquad (4.8)$$

$$\delta \mathbf{u}^i = \delta \mathbf{u}^i + \delta t \omega^i \qquad (4.9)$$

where δt is the time step value. In equation 4.9, $\delta \mathbf{u}^i$ is an incremental rotation vector used to determine the new position of each link.

This solution to the equations of motion is available in the public domain software "*DynaTree*" (written in C) by W.W. Armstrong.

A program was developed, using the forward dynamics approach, to simulate airborne motion of multiple segment, rigid link models in an open loop situation. The "*DynaTree*" software was used to carry out the dynamics computations.

Input into this module were:

1. Number of links in the model.
2. Structural relationships of links in the model (See tables 4.1 and 4.2)
2. Link masses.
3. Principal moments of inertia for links (See Tables 3.1 and 3.3)

4. Vector from proximal joint of parent link to proximal joint of son in LCS.
5. Vector to center of mass from the proximal joint for each link in LCS.
6. Vector to proximal joint of root link in ICS.
7. Initial angular velocity for each link in LCS.
8. Initial linear velocity of root link in ICS.
9. Internal joint torques for each time step in ICS. At each time step these values were converted into values in the LCS by applying the transformation matrix $\mathbf{R}_i'^{T}$ from Table 4.3

4.2 Results of the Simulation.

Van Soest and van den Bogert (1991) suggest criteria to use when evaluating general purpose direct dynamics software packages. The software in this study cannot be considered as general purpose but much of the offered criteria can still be applied. The four areas of evaluation, as taken from their paper, for the simulation software developed in this study were:

1. User friendliness.
2. Calculation speed and numerical efficiency.
3. Accuracy.
4. Flexibility.

A series of simulations was run using the data produced by the dynamics module outlined in the previous chapter. The following discussion of the results will address typical findings, problems and concerns.

4.2.1 User Friendliness.

The definition of the model structure used in the simulation module was easily done and modifications were a simple matter of making adjustments in the input ASCII file. This permitted, for example, studying the effect of decreasing the number of links in the human model or of using a different link as the root link in the three link wooden model. Modifications to the dynamic input data and segment moments of inertia could be made immediately prior to each simulation run.

The output from the dynamics simulation routines was the Cartesian coordinates, in the ICS, for both the proximal and distal endpoints of each link in the system being studied. Using this data, the same three dimensional display and animation routines used in the dynamic analysis program were used to output a visual display of the simulation results. In the animation window a simultaneous display of the same model, using the original data used for the dynamics analysis, was also possible. The angular orientation and position of each link could be plotted for each time step and compared with that of the original motion in three windows on the same screen as the animation, in a screen layout similar to that used in the dynamics module (See figure 3.8). The ability to examine both the real and simulated graphs and animation proved to be extremely valuable when evaluating the simulation runs.

In general the software proved to be quite user friendly but it was only used by the author and to date has not been tested by other researchers. However, it would seem to have good potential as a more general purpose program that could be used in biomechanics laboratories.

4.2.2 Calculation Speed and Numerical Efficiency.

The simulation program performed exceptionally well in the personal computer environment it was developed for. It was tested on three different computers all operating under Microsoft DOS 5.0, each with a different CPU and all using a floating point math co-processor chip. Table 4.4 reports the ratio of computer processing time to the simulated motion time (CST ratio) for the 3 different systems. These values are based on an integration step size of 17 ms with the results sampled every 17 ms for output to the display routines. These were the values most often used when the simulations were run.

Table 4.4
Computer Processing Time to Simulated Time Ratio (CST ratio) for Two Different Models on Different Computers

Computer CPU (Clock Speed)	Three link Model (12 DOF) CST ratio	Human Model (45 DOF) CST ratio
Intel 286 (10 Mhz)	7.8:1	40:1
Intel 386 SX (20 Mhz)	2.5:1	13:1
Intel 486 (33 Mhz)	0.79:1	4:1

An examination of this table reveals the power of the recursive solution for simulation of rigid linked models. These values are much better than CST ratios reported by Isaacs and Cohen (1987) and Meglan (1991). Isaacs and Cohen using a DEC VAX 8700 report values of 46.2:1 for a 9 DOF whip with kinematic constraints on 2 arm-like links and values of 1800:1 for a 39 DOF human model executing a kick, again with some kinematic constraints. Meglan reports values ranging from 306:1 to 14391:1, depending on the integration method used, for simulation of a 34 DOF human model tumbling in space.

4.2.3 Accuracy.

In all figures representing the motion of the two different models the view orientation and perspective are always the same for the simulated and real data.

The simulation module was first used to recreate the motion of the three link wooden model. The first attempts used: the same physical properties as used in the dynamics module; the initial orientation and state as determined by the kinematics routines; and the net joint torque values calculated in the kinetics routines. Figure 4.3 illustrates the real motion and the simulated motion in a typical trial. As can be seen the simulated motion only closely resembles the real motion for the first 0.07 seconds and then rapidly deteriorates until the simulation becomes unstable after 0.27 seconds and fails. In an attempt to quantify the difference, the angular orientation of the different links was examined. Figure 4.4 is a graph illustrating the values of real versus simulated angular orientations around all three axes for link one, which is the rightmost link in figure 4.3. The RMS of the difference between the two orientation angles and the maximum difference are also shown for each axis.

Initially it was felt that reducing the integration step size would minimize this problem and allow the Euler integration technique to perform better. In fact, as the step size was reduced from 0.017 seconds to 0.001 seconds there was some very slight improvement but results became worse as it was further reduced. Even with the improvements the results were still poor.

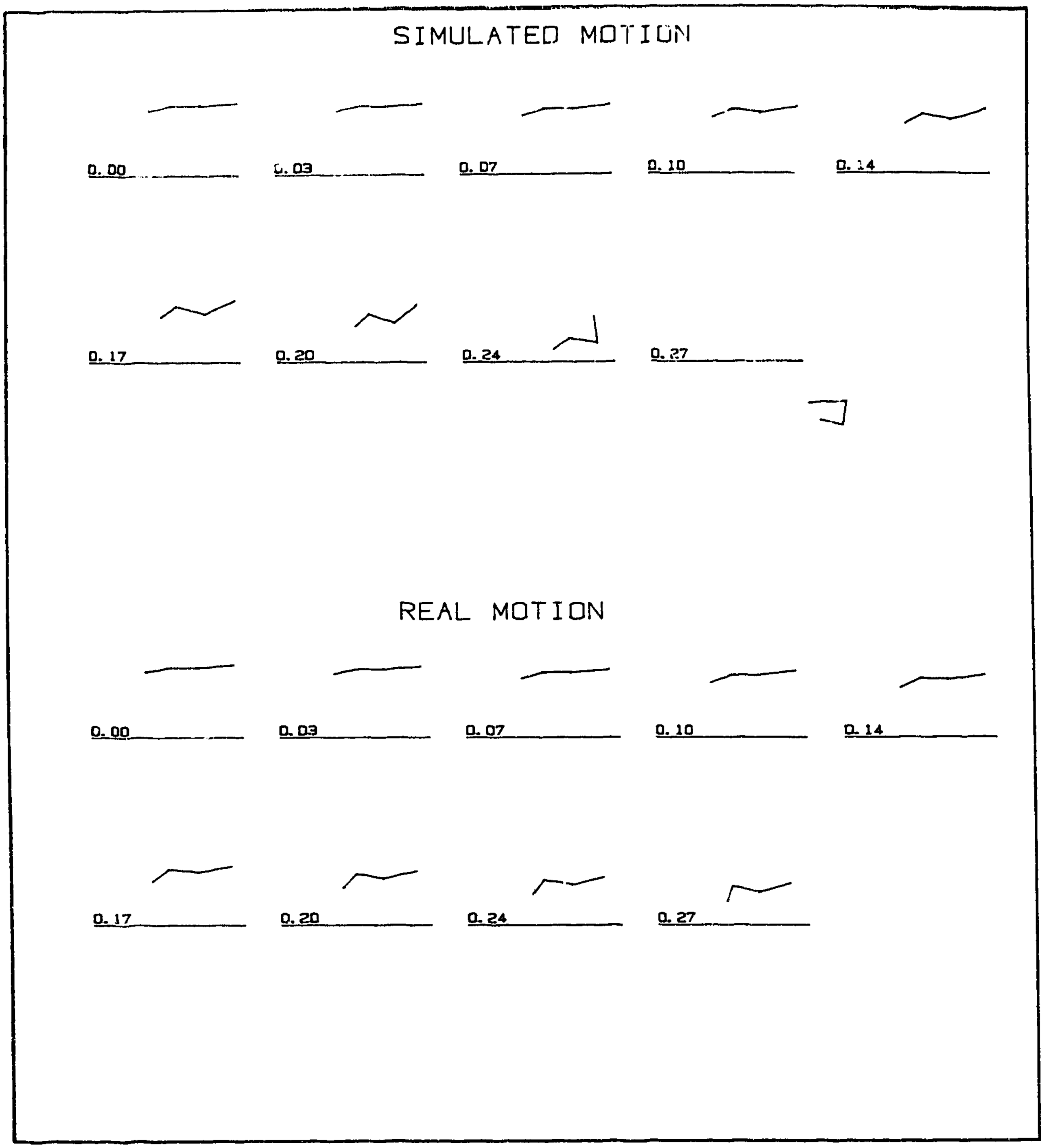


Figure 4.3 Example of real vs simulated motion of a three link wooden model with no constraints on the direction of the applied torques.

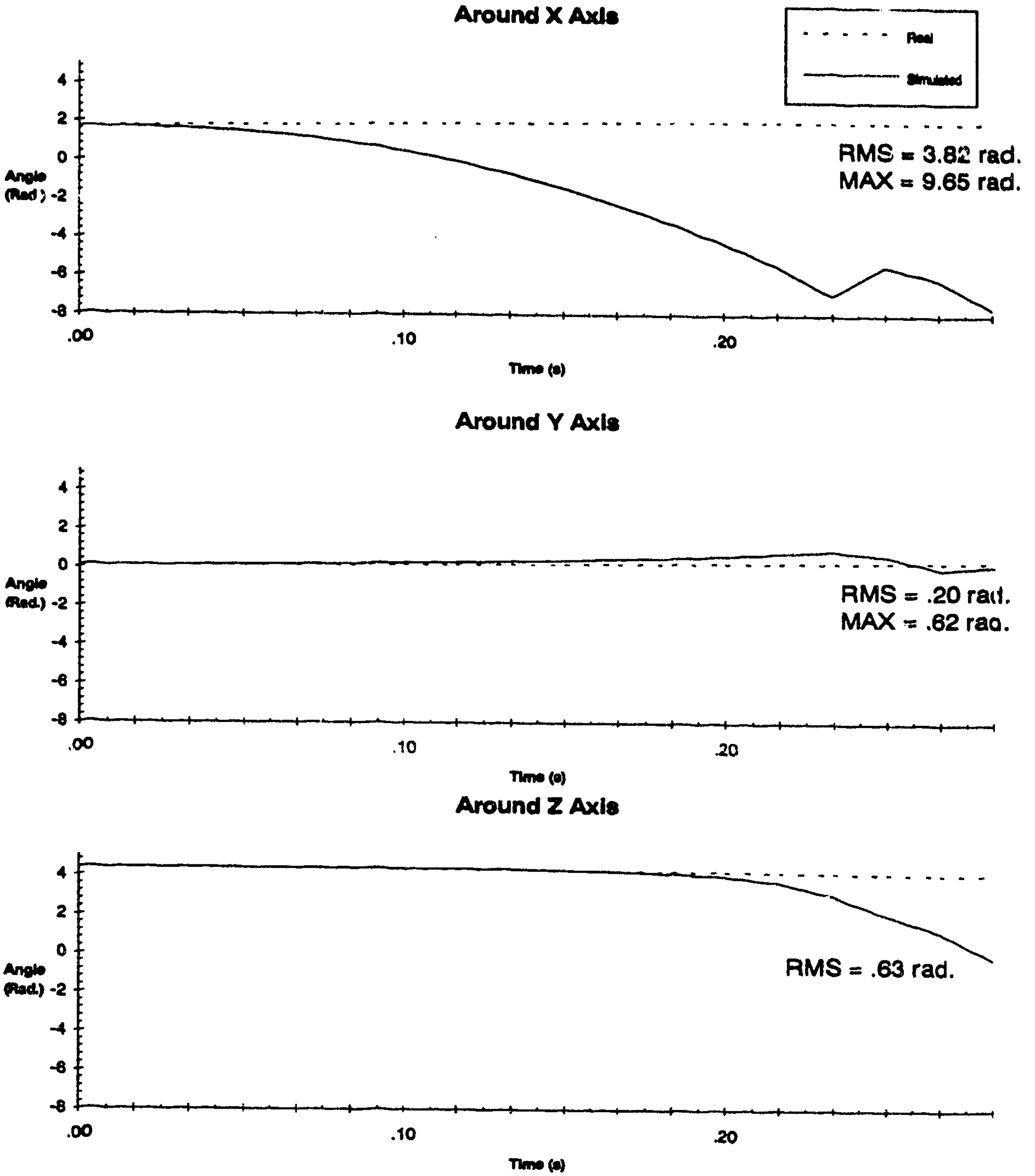


Figure 4.4 Angular orientation of link 1 in the wooden model for the real and simulated motions of figure 4.3 with no constraints on the direction of the applied torques.

What was obvious when observing the animations of the simulation was the rapid angular acceleration around the X axis (longitudinal axis) as illustrated by the rapid change of angular orientation around this axis in figure 4.4. It seemed that this acceleration was causing the simulation to become unstable and ultimately the simulation was failing. The hinge joints in the wooden model should have meant there would be no angular acceleration around the X axis and that the torque values being applied around this axis should have been zero. To prevent this angular acceleration, the torque vector acting at the joint was projected onto a vector which was the cross product of the two link vectors connected at the joint. This in effect constrained the torque to act around the joint in only a flexion/extension action, a reasonable constraint for this model. The simulations for the wooden model were then rerun with much better results as exemplified in figure 4.5. which is the same trial as in figure 4.3. A quantitative analysis of the angular orientations of the links confirmed the improvement as can be seen when comparing figures 4.4 and 4.6 which look at link one in the same trial. The values in figure 4.6 are much closer to the real values and the RMS and maximum difference values for the difference have also dropped. Although not always providing such a big improvement, a similar evaluation of the other trials revealed a similar results. Table 4.5 provides average values for the nine simulation trials of the wooden model used in the study. The simulations of all nine trials using this method of constraining the torques can be found in Appendix A. Based on the visual observation of stick figure outputs, these seem to be more accurate than those produced by Marshall et al. (1985) in the two dimensional forward dynamics simulation of the three link underhand throw.

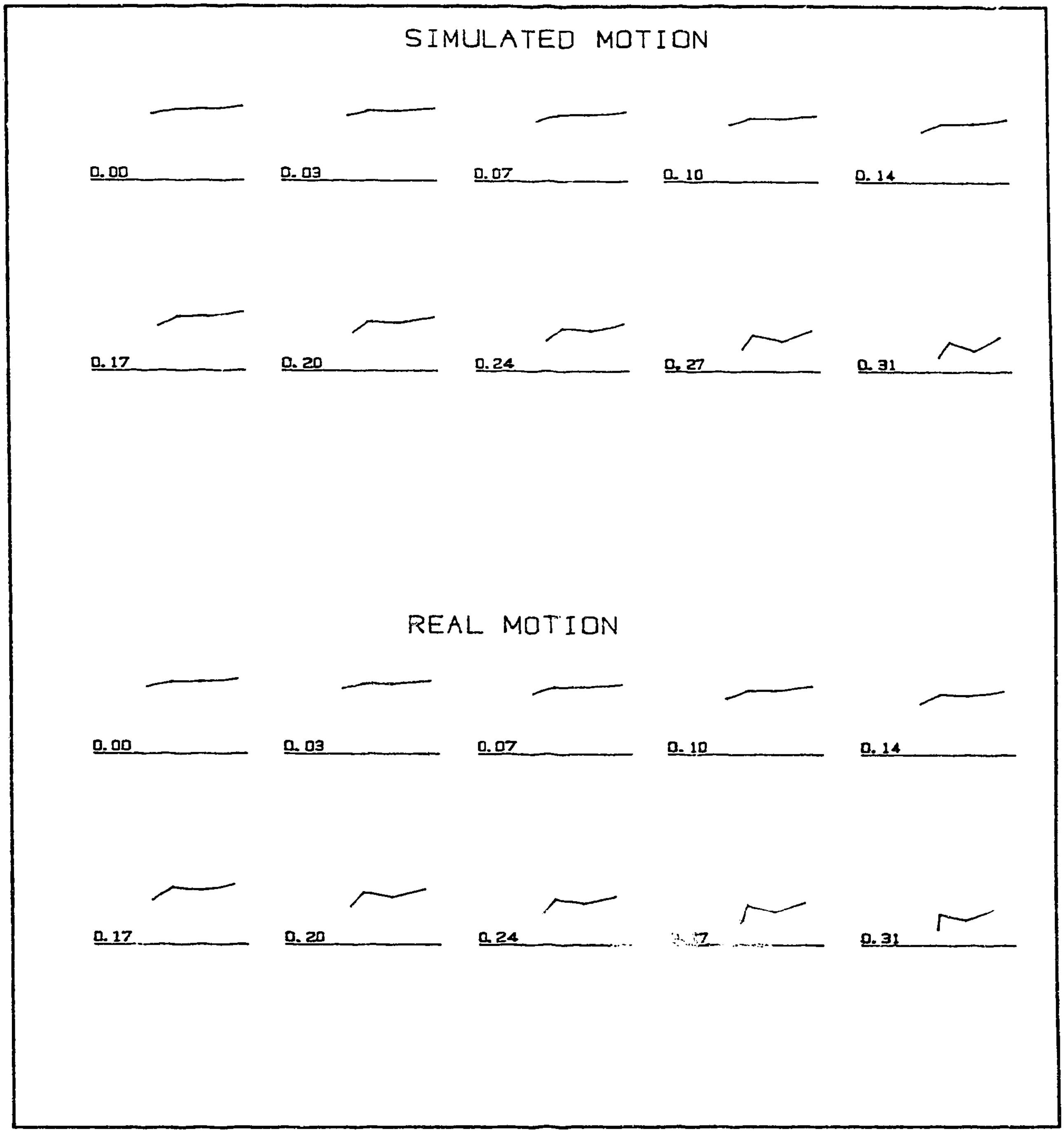


Figure 4.5 Example of real vs simulated motion of a three link wooden model with torques constrained to act only in flexion/extension.

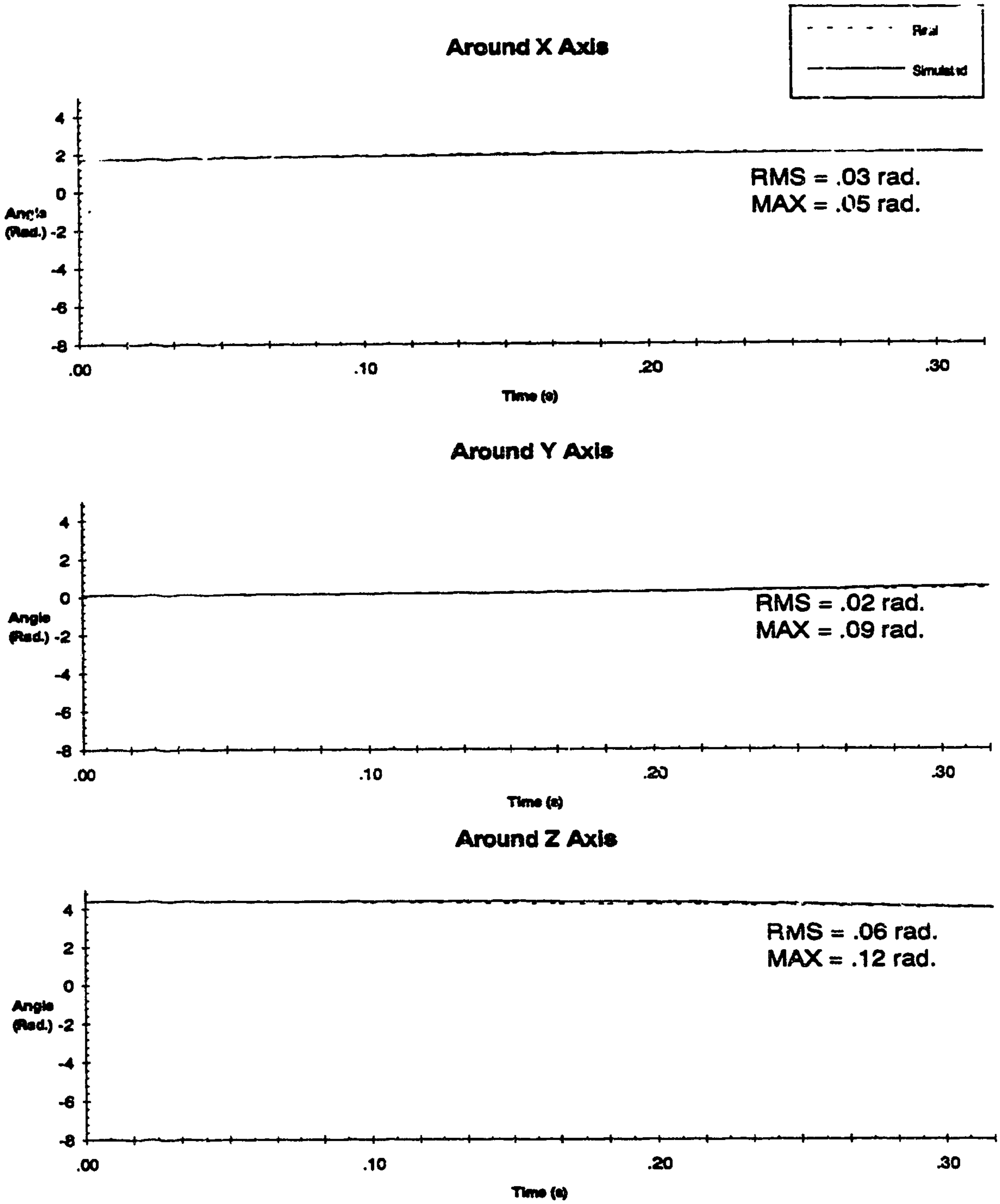


Figure 4.6 Angular orientation of link 1 in the wooden model for the real and simulated motions of figure 4.5 with torques constrained to act only in flexion/extension.

Table 4.5
Mean, RMS and maximum, of difference between real and simulated angular orientation for nine trials of a three link wooden model when torques were constrained to act in flexion/extension only.

Link	Axis	Mean RMS of difference between real and simulated angular orientation (rad.)	Mean maximum difference between real and simulated angular orientation (rad.)
One	X	.24	.57
	Y	.06	.16
	Z	.11	.29
Two	X	.21	.50
	Y	.07	.16
	Z	.10	.24
Three	X	.37	.97
	Y	.15	.43
	Z	.43	1.24

Simulations of the human motion were next run, again with the first attempts inputting the same body segment parameters as used in the dynamics calculations. The initial kinematic state and net joint torque values calculated in this module were also used as inputs for the human model simulations. The first runs did not use any constraints at all and as with the wooden model the simulations failed prior to completion. It appeared that the quality of the data driving the simulations affected both the run duration and quality of the output. As demonstrated in the previous chapter the data produced from the automatic digitizing system was more accurate than that produced by manual methods. This same data also produced better simulations and failed later in each trial. The simulations based on the manually collected data failed after about 0.16

seconds while those using the automatically collected data ran for about 0.30 seconds. Figure 4.7 is an example of a simulation of a tuck jump based on data from the automatic digitizing system. As can be seen, the simulation completely fails after 0.31 seconds of a 0.48 second run. Illustrations of the other simulation trials can be found in Appendix B.

An examination of the simulations revealed that, as with the wooden model, there were gradual increases in the angular velocity around the long axes of the links, as the run progressed, until the procedure became numerically unstable and failed. The same constraint technique as used with the wooden model was used next in an attempt to stabilize the human simulations. The subsequent tests ran longer but the results were still poor in the latter half of the trials, as is illustrated in figure 4.8 which used the same data as was used to produce the simulation of figure 4.7. The other trials, produced similar results, indicating that the application of constraints in simulating human motion is not trivial and requires further study.

Further evaluation of all the trials, when no constraints were applied during the simulations, generally indicated a gradual increase in the deviation between the real and simulated motions until the simulation became unstable. In an attempt to verify that it was this accumulation of error that was, at least in part, causing the simulations to fail, a 'piecewise' method was next implemented. The program was modified so that the operator could choose times during the motion when the simulation could be stopped and then restarted at the next time step. Before restarting, all segments of the model were given their 'correct' positions, orientations and velocities as determined by the original dynamics calculations.

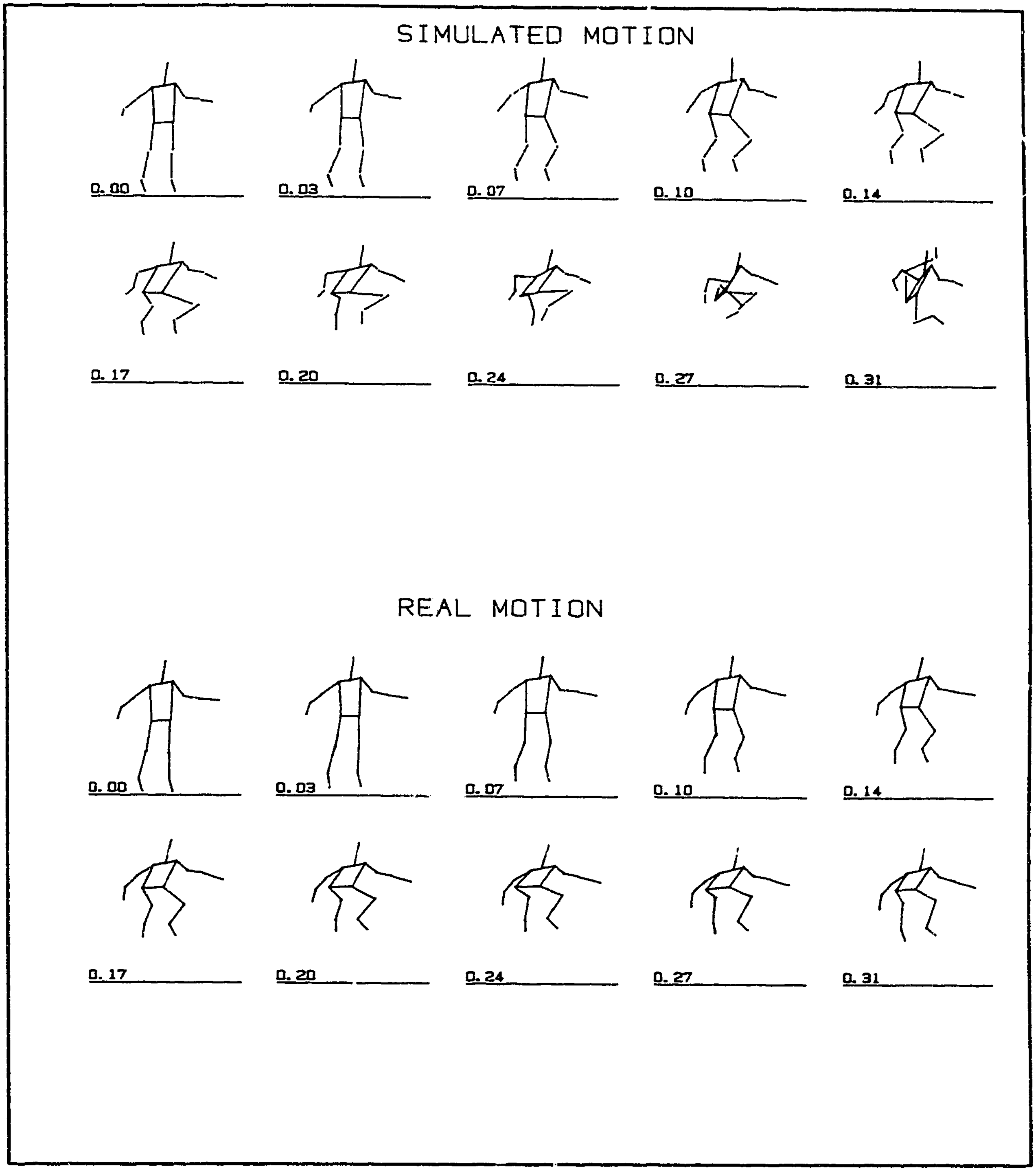


Figure 4.7 Example of real vs simulated motion of a tuck jump with no constraints.

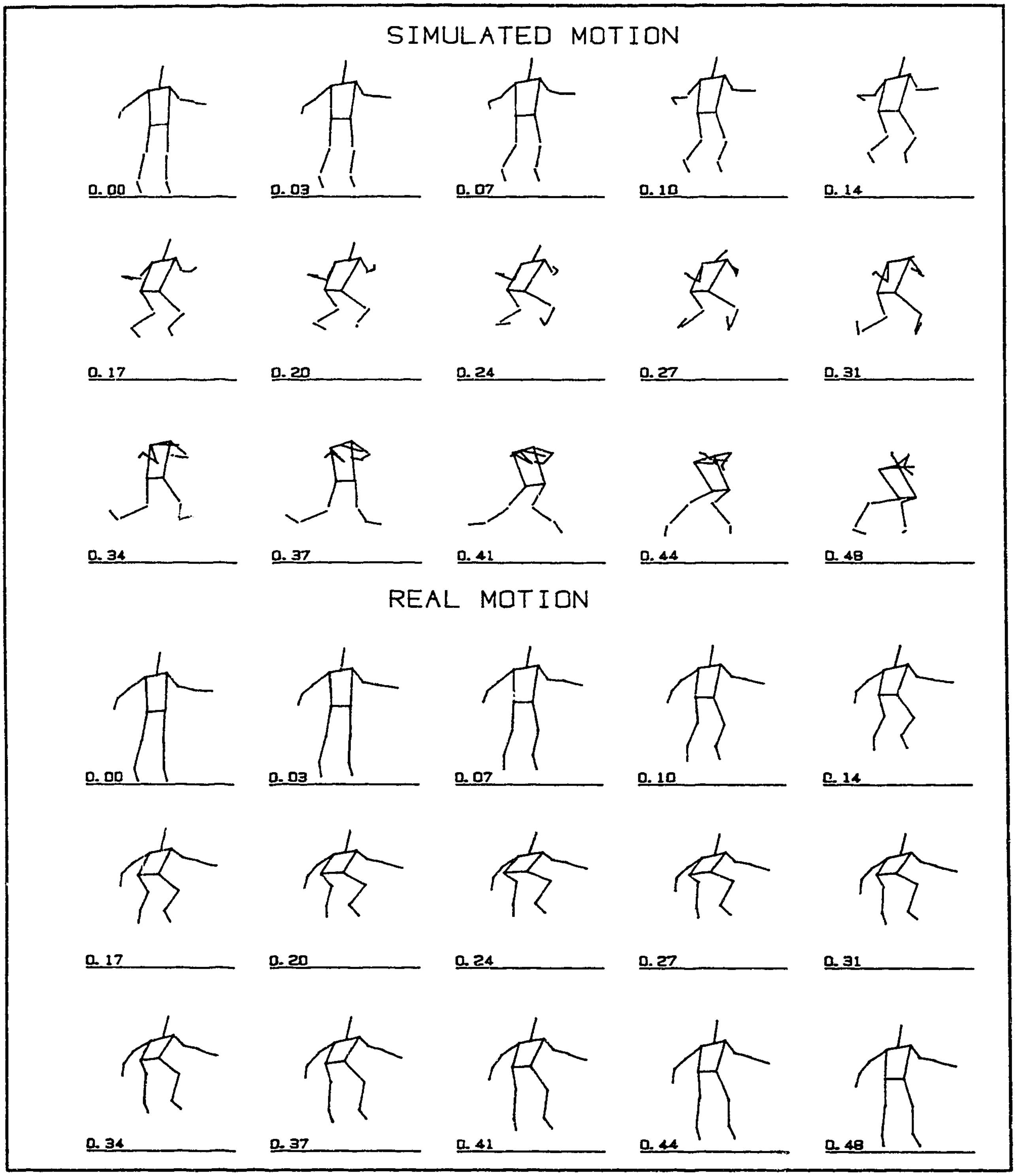


Figure 4.8 **Example of real vs simulated motion of a tuck jump with constraints forcing the torques to act only in flexion/extension.**

Figure 4.9 is an example of the piecewise simulation of the same tuck jump as in figures 4.7 and 4.8 with the location of the simulation restart indicated. Clearly, the results of the second half are better than either of the other runs. Further tests were done by increasing the number of 'knots' in the simulation. As expected the results improved with each additional knot. Figures 4.10 and 4.11 are examples of simulations of a split jump and straddle jump using this 'piecewise' technique. The split jump used data from the automatic digitizing methods and required the use of only one knot to complete the simulation for the whole trial. The straddle jump was simulated with data produced from manual digitizing methods and required the use of two knots to complete the trial. Similar results were produced for the other trials and can be seen in Appendix C.

The outputs from the piecewise simulations seemed to show that the gradual deterioration of the simulation was, at least in part, the result of an accumulation of error. This type of deterioration was found not only in this study as a review of the sample data shown by Dapena (1981), Marshall et al. (1985), Yeadon et al. (1990d) and Meglan (1991) reveals similar results although not always to such a degree. The two studies using a forward dynamics approach to the simulation problem (Marshall et al., 1985, and Meglan, 1991) both also attributed, in part, the accumulation of error to the failure of their methods to accurately reproduce human motion. Clearly, if the torque applied to a link is incorrect, even if only slightly, it will move that link into the wrong orientation for the next time step. Then, even if the next torque applied to the same link is correct, its subsequent orientation will still be wrong. Possibly this second deviation will be even larger since only another incorrect torque could put the segment into the correct orientation. Therefore, if the simulation method is

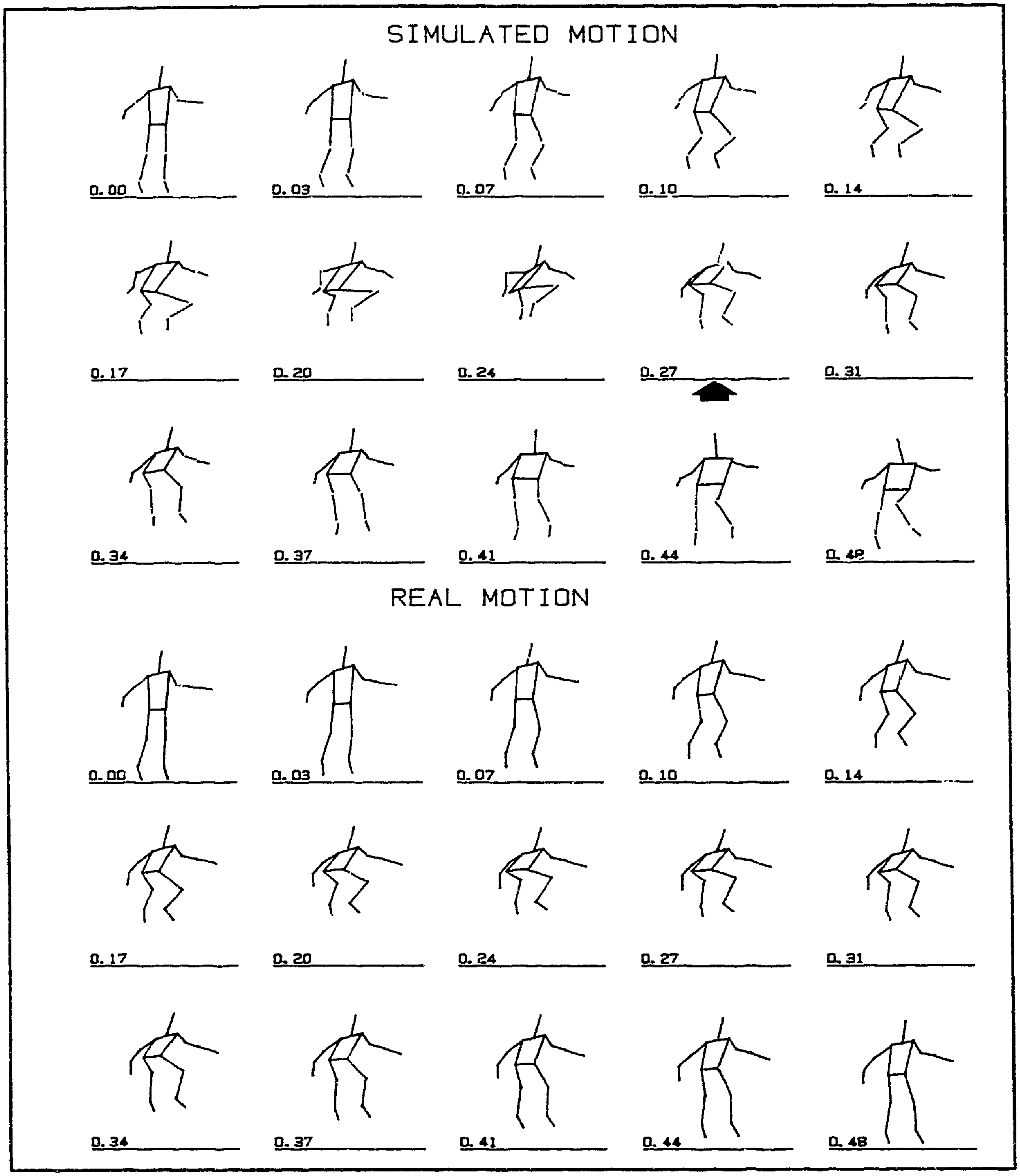


Figure 4.9 Example of real vs piecewise simulated motion of a tuck jump. The arrow indicates the time of the simulation restart.

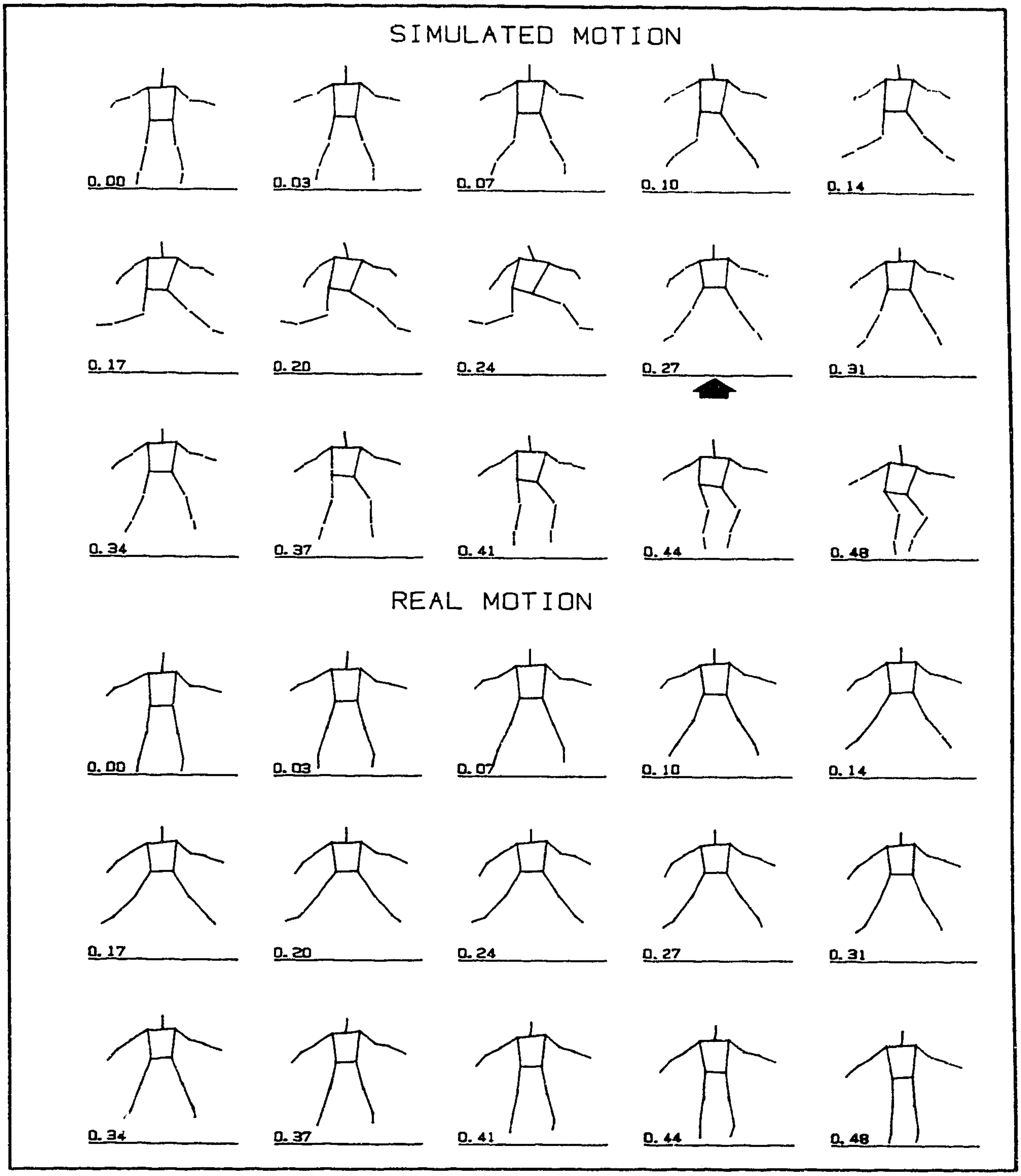


Figure 4.10 Example of real vs piecewise simulated motion of a split jump. The arrow indicates the time of the simulation restart.

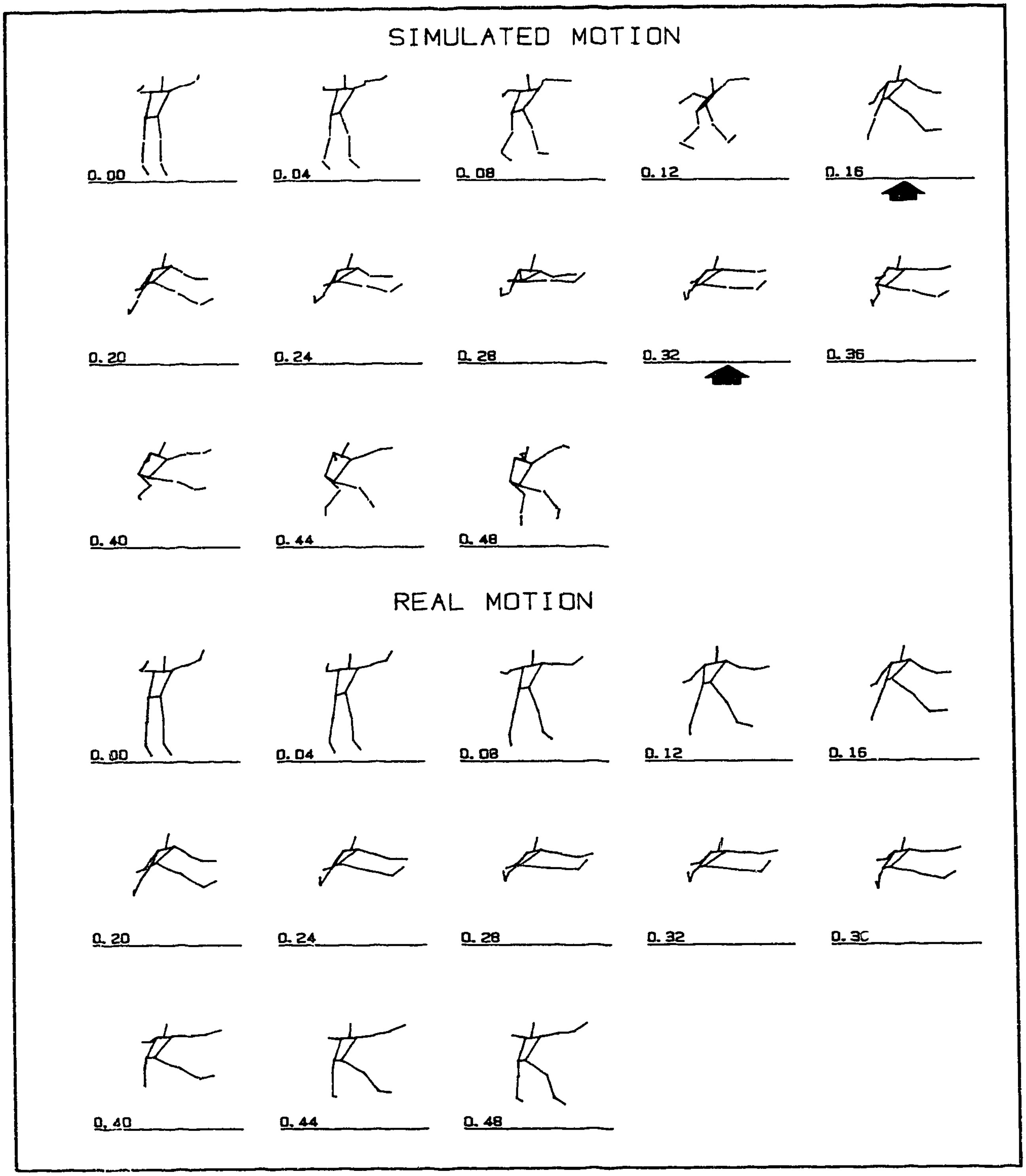


Figure 4.11 **Example of real vs piecewise simulated motion of a straddle jump. The arrows indicate the times of the simulation restarts.**

responding properly to the input data it cannot produce results that exactly resemble the original motion.

In addition to the poor input data, there are other conceivable explanations for the increase in error, as a function of time, during the simulations: the recursive solution in *DynaTree* was being solved incorrectly; the Euler integration technique was inadequate for the type of problem; or the mathematical model defining the movement was inadequate.

The *DynaTree* software has a routine which checks to make sure the equations are being solved correctly. This is done by:

1. Having solved equation 4.7 for $\mathbf{a}^i$ this value is then used to solve equation 4.1 for $\dot{\omega}^i$. This value is compared to the value of $\dot{\omega}^i$ solved for in equation 4.6.
2. The value of $\mathbf{f}^i$ found in solving equation 4.7 is compared to that found when equation 4.3 is solved using $\mathbf{a}^i$ and $\dot{\omega}^i$ of equation 4.6.

This routine was run using data from the study with results showing identical values, in double precision, for each comparison. Thus, it seemed that the forward dynamics equations were being solved correctly and not contributing to the decline in the quality of the simulations.

The Euler integration technique, used in *DynaTree,* was originally implemented because of its efficiency and ease of programming with the recursive method. This technique works well if the function varies slowly over a time step (Green, 1990) and initially seemed to be adequate for human motion problems not involving impact. Euler's method was used previously by Onyshko et al. (1980) and they did not suggest any problems were caused by its application in a gait simulation. However, Marshall et al. (1985) also used this technique but did suggest that the poor stability of their simulation method could be improved by using Gear's integration method. Meglan (1991) also found that the Adams-Basforth-Moulton predictor-corrector method produced the best results for simulation of man in space. It would seem that the problem of simulating human motion can not be represented by a slow varying function and that another integration techniques should be applied. Green (1990) suggests that predictor-corrector methods as used by Meglan (1991) should be the methods of choice. It would therefore be reasonable to implement another integration technique into *DynaTree* to determine if the simulation results improved. After personal communication with the software's originators it was determined that this was no trivial problem and it would require extensive programming to implement different integration schemes into the DynaTree software. It was felt that these changes were not congruent with the stated purposes of this study. However, it is clear that future work with the recursive solution must examine the effect of the integration technique on the error in the simulation output.

The results from the simulations of the three link wooden model were found to be better than that for the fourteen link human model. The data from the dynamic analysis did not seem to be better for the wooden model and cannot totally account for the difference in the quality of the simulation outputs. This suggests that the assumptions associated with a rigid linked model are not reasonable for accurately representing the human body in motion.

The potential sources of error discussed above make it extremely difficult to prove that the recursive solution can accurately recreate three dimensional human motion.

4.2.4 Flexibility of the Simulation Technique.

Though no perfect validation of the recursive solution was possible in this research the results were of sufficient quality to suggest its' potential use as a tool with which to study biomechanical problems of rigid linked motion. The final phase in the evaluation of the recursive simulation technique was to use it in two different problems that could be presented to the biomechanics researcher. This would provide some indication of the flexibility of the technique when applied to practical problems.

4.2.4 .1 Simulation of a New Motion.

Discussion in chapter one suggested that an advantage of using simulation in research was that it allowed the study of the consequences of making adjustments in movement patterns or answering 'what if' questions. This would be the next test of Armstrong and Green's recursive simulation model as a tool for the biomechanics researcher in this thesis. An attempt was made to reproduce a different, but recognizable, motion using one of the existing data bases of dynamic variables produced in the dynamics module.

The gymnastics movement of a back somersault has some similarities with a tuck jump. In both instances the athlete jumps into the air and tucks the legs up and close to the trunk by simultaneously flexing the thighs around the hips and, the legs around the knees. The primary difference between the two skills is the requirement that the trunk has a backward rotation at takeoff for the back somersault but a slight forward rotation for the tuck jump. The similarity between the two skills suggest that it might be possible to use the basic torque patterns of the tuck jump to build, with the aid of simulation, a back somersault.

As the primary difference between the two skills appears to be the kinematics of the trunk upon leaving the ground it would seem that by simply adjusting the initial state of the trunk at takeoff for the tuck jump it should be possible to create a back somersault. Bruggeman (1983) reports an angular velocity backward of 4 rad./s and linear takeoff velocities of 4.3 m/s vertically and 2.69 m/s horizontally backward for single back somersaults. Hwang et al. (1990) reports linear takeoff velocities of 4.3 m/s vertically and 2.1 m/s horizontally backward for double back somersaults. Therefore, the first attempts at creating a back somersault involved only setting the initial state of the trunk with a backward angular velocity of 4 rad./s, a vertical velocity upward of 4.3 m/s and a horizontal velocity of 2.0 m/s backward. All other linear and angular velocities of the trunk were set to zero. All remaining initial state values for all body segments were left the same as for a basic vertical jump which was used as the motion upon which the somersault was built.

The simulation was now run using the torque functions that had been calculated in the dynamics module for a tuck jump. The first problem that became obvious was the failure of the simulation as a result of build up of high angular velocities around the long axes of the links. In this part of the study it was not possible to use a piecewise approach as no 'real' values for the back somersault were available to allow restarts as the simulation progressed. Although the constraint method used in the three link model did not produce good results for comparison with real trials of human motion it was felt that the constraint would be appropriate for the back somersault. The limiting of the motion of the joints to one DOF seemed reasonable in this instance and provided the necessary stability for the model to run for reasonable periods of time. All subsequent trials run to simulate the back somersault used this constraint on the joint torques. Figure 4.12 is an illustration of the first attempt and it is obvious that this is not a back somersault. The simulation starts out reasonably well but at about 0.27 seconds the angle between the thighs and the trunk begins to increase and as a result a tight tuck is never reached. This angle keeps on increasing until by about .37 seconds the thighs are in an anatomically impossible orientation in relation to the trunk. The last frame also shows that the model is still very high in the air and certainly in no position to land. Initially the results were discouraging and it was obvious that the similarity between the tuck jump and the back somersault was not as great as originally thought.

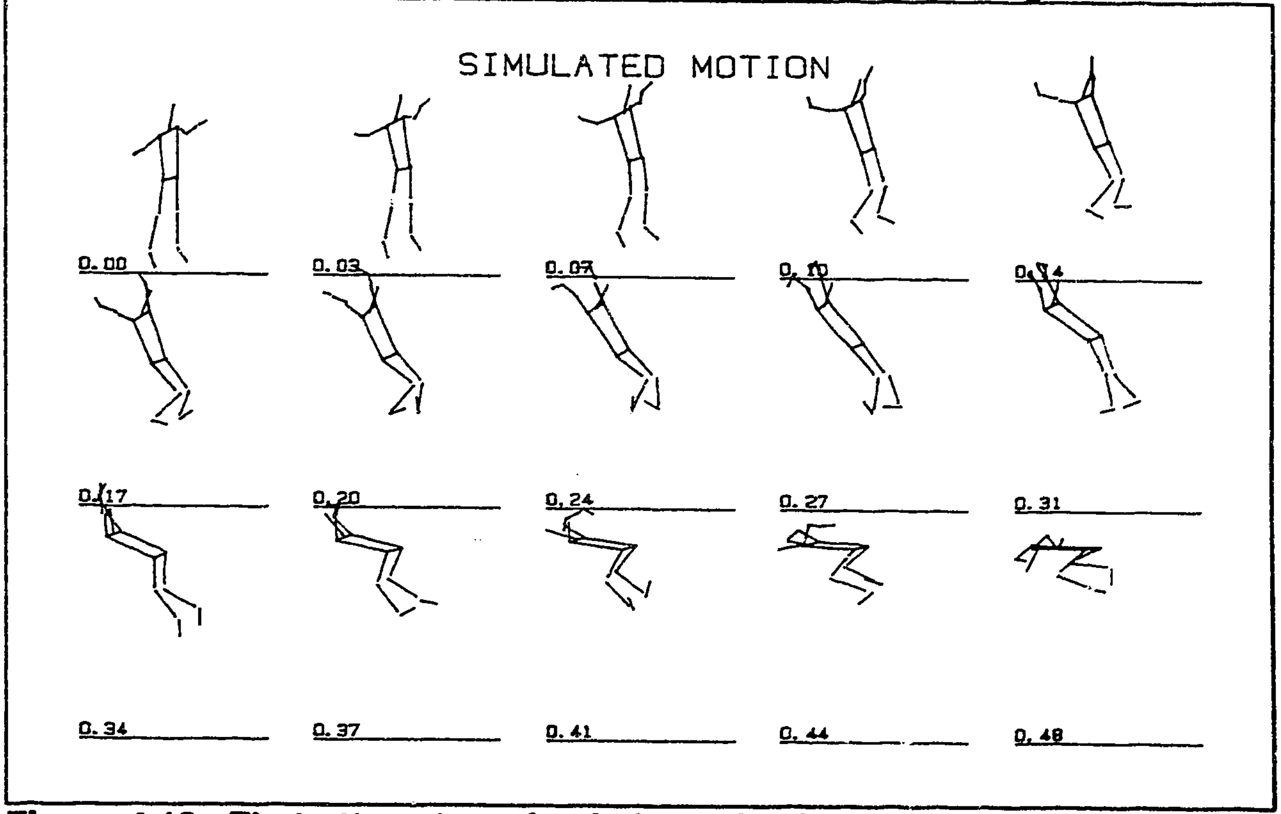


Figure 4.12 First attempt at simulating a back somersault using original torque values of a tuck jump and a modified initial state for the trunk in a simple vertical jump.

The first necessary adjustment was to deal with the problem of the athlete remaining too high in the air, at the end of the simulation, to actually permit a landing. It is obvious that the greater vertical takeoff velocity used for the somersault (3.5 m/s vs 2.5 m/s) dictated that the model would be in the air longer than for the tuck. To achieve this the simulation was allowed to run for gradually longer periods of time until the human model was near enough to the ground to land. Also the torque data controlling the simulation had to be extended to act for the duration of the simulation.

A closer examination of the movement pattern of the thighs was next undertaken. It was then obvious that they continue to rotate in the same direction throughout the somersault unlike the tuck jump where there is a direction change. The original torque around the Y axis causing this direction change for the right thigh can be seen in figure 4.13. and is the same for the left thigh. For the simulation to allow the thighs to get into a tight tuck and hold it longer this pattern had to be changed. This was done by manually editing the individual torque values at each time step and then passing them through a digital filter to smooth out the function. This approach to adjusting the net joint moments that were driving the simulation was previously used by both Onyshko et al. (1980) and Marshall et al. (1985).

First, because of the trunk rotating away from the thighs in the back somersault, a greater initial torque had to be applied. In an analogy to a practical coaching situation this is equivalent to the coach directing the athletes to "pull harder with the thighs when you leave the ground." The second change was a decrease in the magnitude of the torque causing the braking of the thigh rotation. Unlike the tuck jump, the thighs must continue to rotate in the same direction until landing, therefore not all of the angular velocity in this direction needed to be stopped. Without such a decrease in the braking torque the thighs would stop their rotation backwards and begin to rotate in the opposite direction. The modified torque pattern for the thighs, which seemed to achieve the goal of a good tuck, is illustrated in figure 4.13. The torques around the other axes were considered of less importance and in an effort to simplify the problem these torque values were set to zero. Although not discussed here, a similar trial and error process was used to adjust the torques at the knee to achieve a reasonable motion for the legs. The result for this torque around the Y axis is seen in figure 4.13. In an effort to simplify the problem torque values in the other segments were set to zero. Finally, the initial orientation of the trunk was also adjusted so that the model was leaning backward at takeoff as would be expected of an athlete performing a back somersault. The simulation was then run based on these adjustments.

The model was still unable to produce a reasonable landing position. The head had a tendency to begin rotating very rapidly in the direction of the somersault. The angular analogue of Newton's first law of motion would dictate that this would cause the remainder of the body to rotate slower. Consequently torques were adjusted for the head and neck segment to prevent most of this unwanted rotation. The result of this final change allowed the simulation to be relatively successful as can be seen in figure 4.14. The movement of the arms were ignored as attempts at adjusting them proved to be extremely difficult. No effort was made to create a perfect landing position but only to put the body into an orientation that would prevent landing on its head. Although the result of the simulation would not receive a score of 10.0 from a gymnastics judge the general motion for a back somersault was achieved.

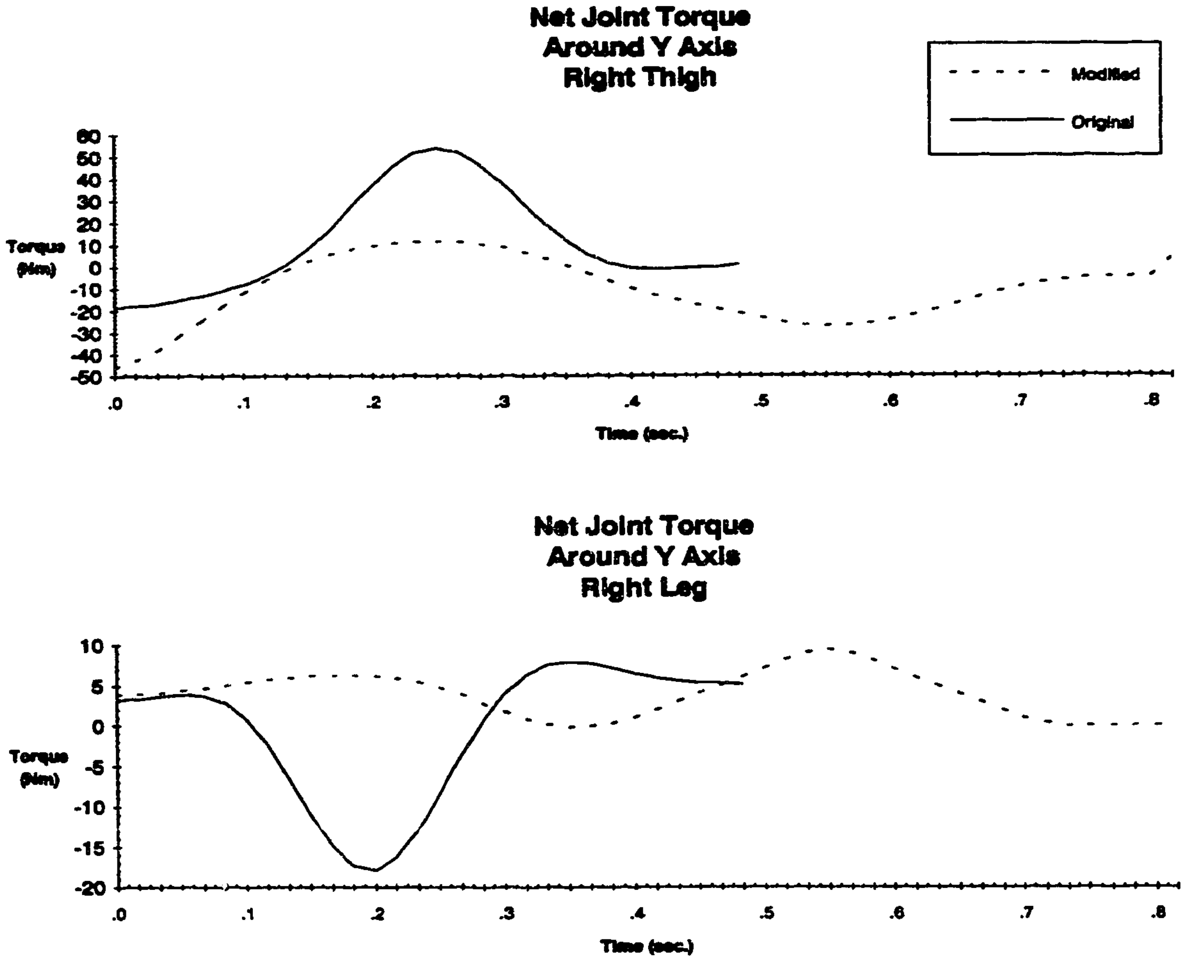


Figure 4.13 Torque values for original tuck jump and adjusted values used to simulate the back somersault.

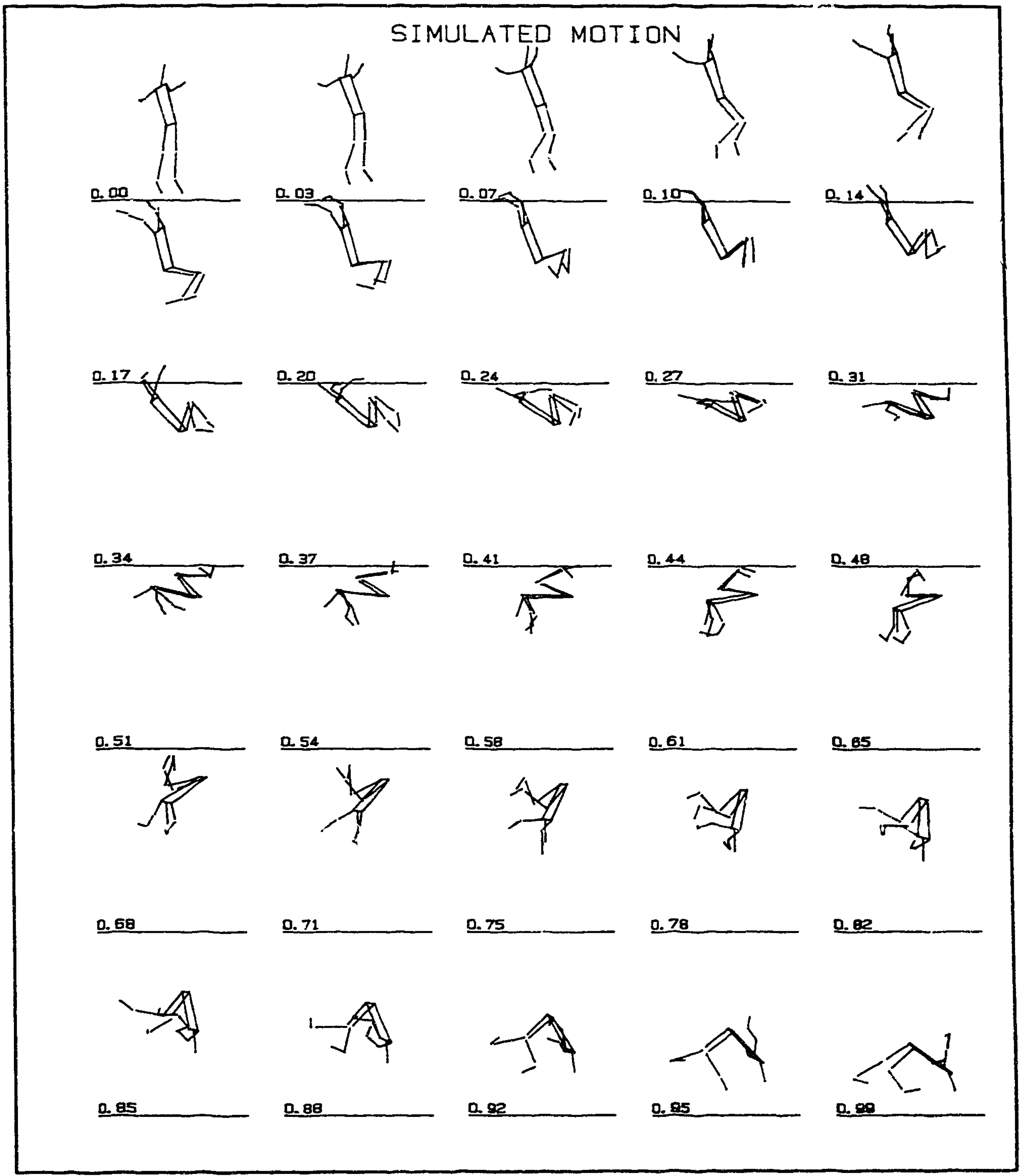


Figure 4.14 Simulation of a back somersault driven by modified torques and initial states from a tuck Jump.

After the desired motion was produced the adjusted torque values were compared to maximum values determined earlier in the previous chapter. As can be seen in figure 4.13 the maximum values for the right thigh are within the range of values seen in figure 3.18. This is, of course, the advantage of such a kinetic simulation technique, as it allows a knowledgeable operator to determine the possibility of a human actually being physically capable of performing the simulated movement. However, even though these torque patterns may be individually possible, it cannot be concluded that the torque functions that finally produced the back somersault are similar to those that would be found when a human is actually performing such a movement. The constraints placed on some of the forcing functions in this study clearly do not represent those found in reality.

This process of a creating a 'new' movement proved to be more difficult than was originally thought. Manually entering each torque at each time step was very laborious and the use of a graphically based editor such as *Virya* used by Wilhelms (1985) would certainly speed up this process. Also, adjusting torques around one axis often caused unwanted movements around other axes for adjoining segments. Clearly, the complexity of human movement and the 45 DOF nature of this model would allow only persons with an extremely good understanding of such motion to make effective use of this tool in its' present format.

Despite the noted difficulties the recursive technique generally performed well and made the simulations possible on the personal computers used in this study. The very short periods of time required to complete each run, after adjustments were made to the input data, helped the user friendliness of this simulation method, and demonstrated its practical potential for use in the sporting and biomechanics research communities. However, before it can be truly applied as a tool for simulating human motion, a better method must be found for controlling the accumulation of angular momentum around the long axes of the segments that usually causes the simulations to fail. It could not be expected that all human motion can be simulated with this constraint being placed on the joints which have more than one DOF.

4.2.4.2 A Torque Adjustor.

During the evaluation of the simulation model it was not possible to provide a perfect validation of the recursive methods. It was demonstrated that error in the input data caused, in part, an accumulation of error in the output. A review of many abstracts from the recent International Congress on Biomechanics (Marshall et al., 1991) reveals that this is an ongoing problem in this discipline. It may be that a different approach to finding accurate torque values for human motion is required. The results of the present study seem to indicate that the recursive solution is reasonably accurate in its response to input data over short periods of time. If it were assumed that it was perfectly accurate and that the failure of the simulations in the study were only a result of poor input data, then it might be possible to use this extremely efficient simulation technique to calculate joint torques which could recreate the original motion. The torque functions, originally produced in the dynamics module, could be iteratively adjusted until the simulation output matched that of the goal motion from the original film and video records. This is similar to the approach used by Chao and Rim (1973) on two dimensional data. This potential application of computer simulations of rigid linked models was the final test of the flexibility of the recursive technique. A computer routine was developed to try and adjust the input torque functions with the goal of producing simulated motion that more closely matched the original motion of the film and video records.

Figure 4.15 is a flow chart of the 'best fit' routine that was developed to adjust the net joint torques causing motion in the rigid linked models used in this research. At each time step for each DOF of each link the torque values were iteratively adjusted in small increments until the resulting orientation of the link in the next time step matched that of the original data within an error tolerance of 0.02 radians. For each iteration the simulation was run for 1 time step. The short time periods of the simulation runs helped to minimize the problems created by the integration technique. This was done in an outward pass along each branch of the model. As the deviation was reduced the size of the increment was decreased. Three stopping criteria were applied:

1. Convergence took place.
2. The routine stalled, that is there were no changes in the deviation after 5 successive iterations.
3. A maximum number of iterations was reached before either 1. or 2.

A series of tests was run with both the three link model and the human model using the same net joint torque values as originally employed in earlier evaluation of the simulation technique. In general it was found that at each time step convergence was either achieved quite quickly or not at all. Study of the data and the progress of the routine failed to indicate what caused the failure of convergence. Different starting points and methods of adjusting the increment at each iteration were tried with varying degrees of success but none was able to produce convergence all the time. Finally, in a last attempt at producing convergence in all instances, a second pass inward from the most distal link to the most proximal for each branch was added. Interestingly, it had almost no effect on the results.

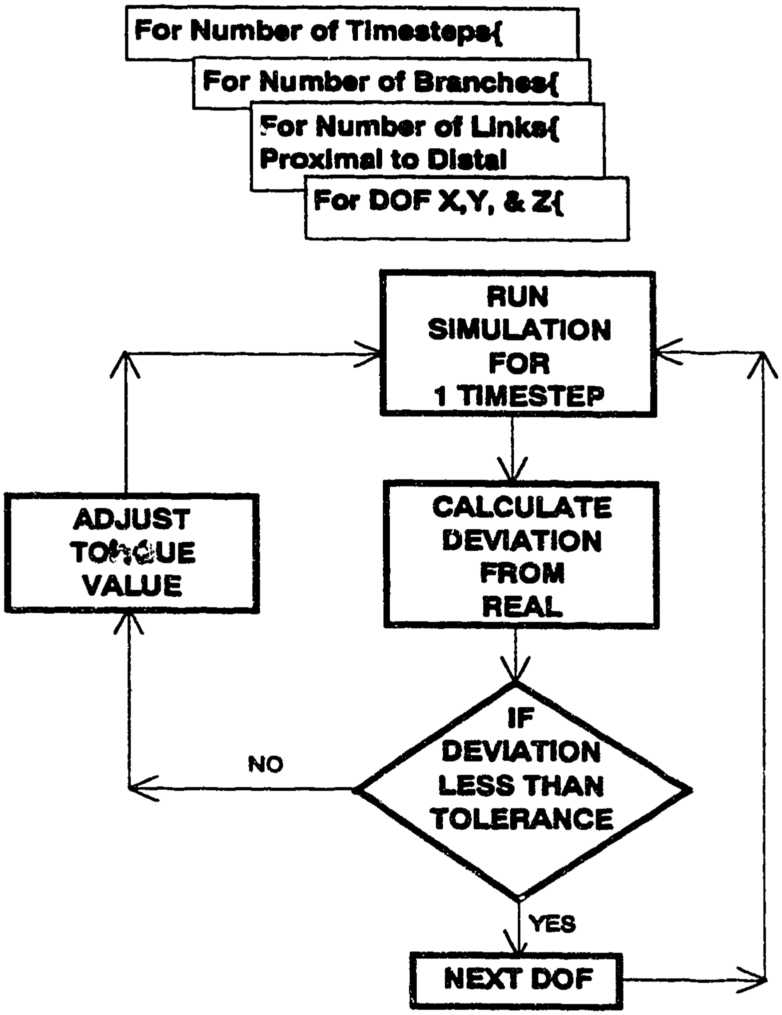


Figure 4.15 Flow chart of the torque adjustor routine.

Reasonable results were achieved when the routine was applied to the three link model as convergence was achieved in most instances. Although no trial produced convergence at every time step, the new adjusted torque data was used to run complete simulations for the three link model. The results were different but no better than with the original data even when the tolerance was reduced to 0.01 radians. For the human model the adjusted data produced much worse results, usually causing the simulation to fail early. Clearly, the individual torque functions for each DOF are not independent and the approach of adjusting them one at a time is completely inadequate.

Despite the discouraging results of the tests, the recursive method did permit efficient testing of the different approaches. For the three link model it was possible to perform iterations at a rate of 1200 per minute and for the human model at 260 iterations per minute. This permitted the adjustment of torques for 30 time steps in one to two minutes for the three link model and about five to ten minutes for the human model. While the exact stopping criteria and the rate of convergence had a large effect on these times, generally this performance allowed the convenient testing of other possible solutions to the problem.

Despite the failure of the methods applied in this study, the concept of using simulation feedback as a way of adjusting torque data to produce a goal motion remains interesting. However, it should also be noted that even if the routine had been successful, it could not have been concluded that the adjusted torque functions were in fact equal to those producing the real motion in the two models.

4.3 Summary of the Simulation Results.

The purpose of the final phase of this thesis was the implementation and evaluation of Armstrong and Green's recursive solution to forward dynamics equations of motion as a simulation procedure that would be driven by dynamic data derived from real motion. The result was a very fast simulation method that functioned well on personal computers and proved to be very user friendly.

In testing the accuracy of the recursive methods, a gradual deterioration in the quality of the results was seen in all trials. The problem was largely attributed to the accumulation of errors introduced into the solutions by the dynamic data driving the simulations. In the case of the three link model, it was shown that when the torques were constrained to act appropriately only in flexion/extension better simulations were produced. A piecewise approach to the simulation of the human model also provided better results by partially controlling the effect of this error accumulation. Instability problems were also attributed, in part, to the use of the Euler integration technique suggesting the implementation of a more sophisticated method such as a predictor-corrector in future work with the recursive solution.

In a practical application of the technique and a test of its' flexibility an attempt was made to create a 'new' motion, that of a back somersault. This was successfully done by using kinematic data from the literature and then applying net joint torques that were modifications of data originally calculated for the tuck jump in the dynamics module. In a second application, the recursive method was iteratively applied, in a feedback based best fit approach, to try and produce torque data that could more accurately simulate the original motion of the two models. Although not producing the desired results the recursive method was found to be very efficient, suggesting that it could be similarly applied in future research using more sophisticated best fit routines.

CHAPTER 5

SUMMARY AND CONCLUSIONS

The purpose of this research was to determine the ability of the recursive method of Armstrong and Green(1985) to simulate three dimensional human motion in a non support state using as input the kinetic and kinematic parameters calculated from real motion. This was accomplished by first developing a computer program to do dynamic analysis of three dimensional motion of rigid linked bodies. The data from this module was then used as input into a computer simulation module based on the Armstrong and Green (1985) recursive solution to the forward dynamics equations of motion. The research was limited to the study of two models: a three link wooden mechanical model with 12 DOF falling through space; and the airborne phase of a human model with 45 DOF performing three different types of jump.

5.1 Dynamic Analysis

The dynamic analysis module calculated kinematic and kinetic data for all links of the models. The kinematic results were based on three dimensional coordinates of the model segment endpoints calculated by applying the DLT transformation technique to digitized points taken from film and video records of the original motion. An inverse dynamics approach, based on Newtonian methods, was used to solve the equations of motion and calculate the net forces and torques at all joints. The results of the dynamics module were displayed in a graphical format that allowed for their analysis while simultaneously observing a three dimensional animation of the model movement in a separate window.

The linear kinematics results were good but problems were found with the angular kinematics which had a detrimental effect on the angular kinetics. The problems were caused by the difficulty of correctly determining the angular orientation of each segment in three dimensional space. Consequently, constraints were applied to the angular kinematics to produce more reasonable values which were then used in the calculations of segment kinetics. The kinematic and kinetic data were generally similar to that found in the literature. A final check was made on the accuracy of the overall kinetic values using a method similar to Meglan (1991). The sum of the net joint torques and reaction forces coming in from the model branches was compared to the values which would produce the motion of the root link. The difference was relatively large at times for some data, indicating that there was error in the kinetic values that were being calculated or, in the values used for the body segment parameters of the links. The error found to be present in the dynamic data had to be considered in the evaluation of the simulation model.

5.2 The Recursive Simulation Model.

The recursive solutions for linked rigid body motion of Armstrong and Green (1985) were applied in a computer program to be used in simulating the movement of the two models. The kinematic and kinetic input into the simulation technique was data calculated in the dynamics module.

A series of tests were run on the three link, 12 DOF wooden model and the 14 link, 45 DOF human model to evaluate the performance of the simulation module. The recursive methods used by the simulation proved to be very efficient, with computer to simulated time ratios ranging from 40:1 seconds to 4:1

seconds on three different personal computers, when simulating the motion of a 45 DOF human model. Initially the simulations showed a gradual decline in quality as the run progressed until it became numerically unstable and failed. The cause of this decline appeared to be an accumulation of error resulting from the poor quality of the dynamic data used as inputs. However when reasonable constraints were applied to the driving torques the simulations for the three link wooden model were good. The accumulation of error in human motion trials was controlled using a piecewise approach to the simulations. A series of tuck jumps, split jumps and straddle jumps were simulated in this manner with varying degrees of success that appeared to be dependent on the quality of the data input into the simulations and the limitations of the Euler integration technique.

In an application of the simulation module the kinematic and kinetic input variables of a tuck jump were modified to produce a back somersault movement. This was where the potential advantage of a kinetic driven technique was demonstrated. After creating the simulated motion by adjusting the torque values it was a simple matter to compare these values to previously computed values based on real movements. In the case of the back somersault created with the simulation, the torque values used were within the range of values found for the straddle jump in the dynamics analysis of this research.

An attempt was also made at improving the net joint torques produced in the dynamics module with the aid of the recursive technique. The simulation was used as a method for checking the effect of iteratively adjusting the joint torques in the search for values which would produce motion correct within a tolerance of 0.02 radians. While the approach generally appeared to work, convergence to the goal orientation was not achieved for all links.

5.3 Conclusions

The recursive solution to the forward dynamics equations of motion of Armstrong and Green (1985) has for the first time been evaluated, if only in part, from a biomechanics perspective. Based on the findings of this research and within its limitations and delimitations the following conclusions can be made:

1. The dynamic analysis computer routines developed in this study were capable of producing three dimensional kinematic and kinetic data for the airborne motion of rigid linked bodies. The accuracy of the results was dependant on the quality of the three dimensional positional and orientation data upon which the calculations are based.
2. The recursive solution of Armstrong and Green (1985) provides a very fast method for simulating the motion of rigid linked bodies in a non support state, using net joint torques calculated from cinematographic records of real motion as the primary forcing functions. Computer to simulated time ratios as low as 4:1 for a 45 DOF, 14 link human model were found when this method was run on a personal computer.
3. The recursive method is relatively accurate over short periods of time but errors tend to accumulate over longer spans. These errors would appear to be dependent upon the quality of the input data and the integration technique applied.

4. The *DynaTree* software which used the recursive methods was relatively easy to implement, found to be flexible and quite user friendly.

These conclusions suggest that this recursive method may have the potential to be a very flexible and powerful tool for the biomechanics researcher. Clearly, this research has really only introduced the technique to the discipline of biomechanics, and its future applications in this field will be determined only after further research addresses the remaining problems, some of which have been identified here.

5.4 Recommendations and Directions for the Future

The findings of this study would support further work into the areas of dynamic analysis and simulation of human motion in three dimensions. Recommendations for future work in these areas include:

1. More sophisticated integration methods should be tested with the recursive method. Instability problems in the simulations may be avoided.
2. Expansion of the kinetics routines to produce data for three dimensional human motion when acted upon by external forces. Problems of single support could be studied first.
3. Examination of the application of justified constraints which may improve the simulation technique's performance. For example, poor and unrealistic motions of the body limbs could be improved by applying kinematic or kinetic constraints based on anatomical research about segment ranges of motion. The single DOF joints

of the knees and elbows could also be constrained using the methods outlined by Armstrong (1979).

4. The use of quaternions, for determining link orientations in space, should be studied in the context of dynamic analysis of rigid linked models. As they uniquely specify all orientations in space and are continuous for all orientations they may help solve some of the problems encountered in the kinematic and kinetic routines used in this research.
5. The recursive simulation methods of Armstrong and Green (1985) should continue to be evaluated when the forcing functions are based on data produced by biomechanical research. The first steps in this direction should include its performance with more complicated airborne human motion and in problems of single support.
6. A study should be conducted to fully evaluate the possibility of using kinetically based simulation techniques to improve the quality of the torques and forces produced by traditional inverse dynamics methods.
7. The power of the recursive model suggests that practical applications for its use should be developed for both the lab and field. The capability of smaller personal computer systems to drive these simulations puts this tool within the reach of labs with small hardware budgets, undergraduate students, and possibly even coaches of sport technique.

8. A method for easily determining the torque values used by the kinetic simulation method should be developed. A possible solution may be the combined use of a kinematic 'keyframing' type technique combined with a kinetic analysis method as used in this research. A similar approach was used by Issacs and Cohen (1987). A graphically based torque editor should also be developed such as that used by Wilhelms (1985).
9. The simulation of human movement has been studied quite extensively in the fields of computer science and biomechanics. Unfortunately, with only a few exceptions, the two disciplines have worked almost in isolation from each other. Although the goals are generally different, collaborative work between the two would probably accelerate progress.

BIBLIOGRAPHY

Abdel-Aziz, Y.I. and Kara, H.M. (1971) Direct linear transformation from comparator coordinates into object-space coordinates. Symposium of Close-Range Photography.

Adams, L. (1989) High performance CAD graphics in C, Windcrest Books, Blue Ridge Summit, PA.

Aleshinsky, S.Y. and Zatsiorsky V.M. (1978) Human locomotion in space analyzed biomechanically through a multi-link chain model. Journal of Biomechanics 11, 101-108.

Alexander, M.J.L. and Colbourne, J. (1980) A method of determination of the angular velocity vector of a limb segment. Journal of Biomechanics 13, 1089-1093.

Andrews, G.C. & Kesavan, H.K. (1977) Simulation of multibody systems using the vector-network model. Dynamics of multibody systems symposium, Munich Germany, 1-13.

Armstrong, W. W. (1979) Recursive solution to the equations of motion of an n-link manipulator. Proceedings of the fifth world congress on theory of machines and mechanisms 2, 1343-1346.

Armstrong , W.W. and Green, M. W. (1985) The dynamics of articulated rigid bodies for purposes of animation. The Visual Computer 1, 231-240.

Badler, N.I. and Smoliar, S.W. (1979) Digital representations of human movement. Computing Surveys 11, 19-38.

Bapu, P.,Evans,S., Kitka, P., Korna, M. and McDaniel, J. (1980) User's Guide to Combiman Programs. University of Dayton Research Institute, Dayton, Ohio, February.

Beer, F. and Johnston, E. (1976) Mechanics for Engineers (3rd ed.) McGraw-Hill, USA.

Blakely, F.M. (1980) Cyberman. Chrysler Corporation, Detroit, Michigan, June.

Bogert, A.J. van den, Schamhardt, H.C., Sauren, A.A.H.J. and Hartman, W. (1989) Computer simulation of equine locomotion and its application to the study of force distribution in the limbs. In Computer simulation of locomotion in the horse. Ph. D. thesis of Bogert, A.J. van den , Utrecht University.

Bogert, A.J. van den, Schamhardt, H.C. and Crowe, A. (1989) Simulation of quadrupedal locomotion using a dynamic rigid body model. Journal of Biomechanics 22, 33-41.

Bourassa, P and Morel, Y. (1985) A mathematical model for computer simulation of human running. Biomechanics IX-B Human Kinetics Publishers, Champaign, Illinois, 325-330.

Bourassa, P. (1991) A software program for running motion dynamics analysis and animation. XII International Congress of Biomechanics, Perth, Australia, 412-413.

Boysen, J.P., Francis, P.R. and Thomas,R. (1977) Interactive computer graphics in the study of human body planar motion under free fall conditions. Journal of Biomechanics 10, 783-787.

Bruggeman, P. (1983) Kinematics and kinetics of the backward somersault takeoff from the floor. In H. Matsui & K. Kobayashi (Eds.), Biomechanics VIII-B, Human Kinetics, Champaign, IL, 793-800.

Brudelin, A. and Calvert, T.W. (1989) Goal directed dynamic animation of human walking. Computer Graphics, SIGGRAPH '89 23, 233-244.

Casolo, F. and Legnani, G. (1988) A consistent matrix method for the kinematics and dynamics of chains of rigid bodies. Atti del X Congresso Nazionale AIMETA, Bari, Italy.

Casolo, F. and Legnani, G. (1989) A new approach to the dynamics of human motion. 2nd International Symposium on Computer Simulation in Biomechanics, Davis , California, USA.

Casolo, F. and Legnani, G. (1990) Dynamic simulation of whole body motion; An application to horse vaulting. Atti del X Congresso Nazionale AIMETA, Pisa, Italy, 681- 686.

Calvert, T.W., Chapman, J. and Patla, A. (1982) Aspects of kinematic simulation of human movement IEEE Computer Graphics and Applications 2, 41-50.

Chaffin, D.B. (1969) A computerized biomechanical model - Development of and use in studying gross body actions. Journal of Biomechanics 2, 429-441.

Chao, E.Y. and Rim, K. (1973) Application of optimization principles in determining the applied moments in human leg joints during gait. Journal of Biomechanics 6, 487-510.

Dainis, A. (1980) Whole body and segment center of mass determination from kinematic data. Journal of Biomechanics 13, 647-651.

Dapena, J. (1981) Simulation of modified human airborne movements Journal of Biomechanics 14, 81-89.

Dapena, J. and Chung, S.C. (1988) Vertical and radial motions of the body during the take-off phase of high jumping. Medicine and Science in Sports and Exercise. 20, 290-302.

Davis, R.B., Ounpuu, S, Tyburski, D.J. and Deluca, P.A. (1991) A comparison of two dimensional and three dimensional techniques for the determination of joint rotation angles. Proceedings of the International Symposium on 3-D Analysis of Human Movement. Montreal, 67-70.

Diffrient, N., Tilley, A.R. and Bardagjy, J.C. (1974) Humanscale 1/2/3, The MIT Press. USA.

Dooley, M. (1982) Anthropometric modeling programs - A survey. IEEE Computer Graphics and Applications 2, 17-26.

Featherstone, R. (1983) The calculation of robot dynamics using articulated-body inertias. International Journal of Robotics Research 2, 13-30.

Fetter, W.A. (1982) A progression of human figures simulated by computer graphics. IEEE Computer Graphics and Applications 2, 9-16.

Forsey, D. and Wilhelms, J. (1988) Techniques for interactive manipulation of articulated bodies using dynamic analysis. Proceedings Graphics Interface '88 June, 8-15.

Fujii, N. (1989) A general simulation system of two-dimensional multi-link model of the human body. XII International Congress of Biomechanics, Los Angeles, California, USA, 296.

Gervais, P. (1986) An individualized approach to the optimization of a human motion. Unpublished Ph.D. Thesis, University of Alberta, Alberta, Canada.

Ghosh, T.K. and Boykin, W.H. (1975) Dynamics of the human kip-up maneuver. Journal of Dynamic Systems, Measurement and Control 97, 196-201.

Girard, M. and MacieJewski, A.A. (1985) Computational modeling for the computer animation of legged figures Siggraph '85 Proceedings, 263-270.

Girard, M. (1987) Interactive design of 3-D computer-animated legged animal locomotion. IEEE Computer Graphics Applications June, 39-51.

Green, M. (1990) Using dynamics in computer animation: Control and solution issues. Making them Move: Mechanics, Control, and Animation of Articulated Figures. Morgan Kaufman Publishers, Inc. San Mateo, California, USA, 281-314.

Hardt, D.E. and Mann, R.W. (1980) A five body - three dimensional dynamic analysis of walking. Journal of Biomechanics 13, 455-457.

Hatze, H. (1981) A comprehensive model for human motion simulation and its application to the take-off phase of the long Jump. Journal of Biomechanics 14, 135-142.

Hatze, H. (1980) A mathematical model for the computational determination of parameter values of anthropomorphic segments. Journal of Biomechanics 13, 833-843.

Hatze, H. (1981) The use of optimally regularized fourier series for estimating higher-order derivatives of noisy biomechanical data. Journal of Biomechanics 14, 13-18.

Hatze, H. (1983) Computerized optimization of sport motions: an overview of possibilities, methods and recent developments. Journal of Sport Sciences 1, 3-12.

Hatze, H. (1979) Biomechanics of sport - What should the future really hold. Journal of Biomechanics 12, 237.

Hatze, H. (1977) A complete set of control equations for the human musculo- skeletal system. Journal of Biomechanics 10, 799-805.

Hatze, H. (1976) A method for describing the motion of biological systems. Journal of Biomechanics 9, 101-104.

Hatze, H. (1980) A mathematical model for the computational determination of parameter values of anthropomorphic segments. Journal of Biomechanics 13, 833-843.

Hatze, H. (1984) Quantitative analysis, synthesis and optimization of human motion. Human Movement Science 3, 5-25.

Hemami, H. and Farnsworth, R.L. (1977) Postural and gait stability of a planar five link biped by simulation. IEEE Transactions on Automatic Control AC-22, 452-458.

Hemami, H, Tomovic, et al. (1978) Finite state control of planar bipeds with application to walking and sitting. Journal of Biomedical Engineering 2, 477-494.

Hong, Y. and Bruggemann, G.P (1991) A three dimensional simulation model for the development of new airborne movements in gymnastics, diving and acrobatics. XII International Congress of Biomechanics, Perth, Australia, 414-415.

Huang, T.C., Roberts, E.M. and Youm, Y. (1982) Biomechanics of kicking. Human Body Dynamics: Impact, Occupational and Athletic Aspects. Clarendon Press, Oxford. 409-443.

Hubbard, M. (1980) Dynamics of the polevault. Journal of Biomechanics 13, 965-976.

Hwang, I, Seo, G. and Cheng Lui, Z. (1990) Takeoff mechanics of the double backward somersault. International Journal of Sport Biomechanics 6, 177-186.

Isaacs, P and Cohen, R. (1987) Controlling dynamic simulation with kinematic constraints, behavior functions, and inverse dynamics. Computer Graphics 21, 215-224.

King, A.I. (1984) A review of biomechanical models. Journal of Biomechanical Engineering 106, 97-104.

King, A.I. and Chou, C.C. (1976) Mathematical modelling, simulation and experimental testing of biomechanical system crash response. Journal of Biomechanics 9, 301-317.

Kingsley, N. A., Schofield, N.A. and Case, K. (1981) SAMMIE - A computer aid for man-machine modeling. Computer Graphics 15,163-169.

Kollias, I. (1984a) Three dimensional analysis of hammer throwing. Unpublished Ph.D. Thesis, University of Alberta, Edmonton, Alberta, Canada.

Kollias, I. (1984b) DLT for calibration of large areas (Using Kararas's model), Unpublished Manuscript, University of Alberta, Edmonton, Alberta, Canada.

Lake, R. (1990) Dynamic motion contol of an articulated figure. Masters Thesis, University of Alberta, Edmonton, Alberta, Canada.

Luh, J., Walker, M., and Paul, P. (1980) On-line computational scheme for mechanical manipulators, Journal of Dynamic Systems, Measurement and Control 102, 69-76.

Mann, R.V. and Sorenson H.C. (1979) A comprehensive computer model to simulate human motion. Human motion Motor skills: Theory into practice 3, 85-93.

Mann, R.V. and Sorensen, H.C. (1979) Investigating complex movements by computer simulation - application to the flip long jump. Research Quarterly. 50, 36-54.

Marshall, R.N., Jensen, R.K. and Wood, G.A (1985) A general Newtonian simulation of an N-segment open chain model. Journal of Biomechanics. 18, 359-367.

Marshall, R.N., Wood, G.A. et al. (1989) Performance objectives in human movement: A review and application to the stance phase of normal walking. Human Movement Science 8, 571-594.

Marshall, R.N., Wood, G.A., Elliott, B.C., Ackland, T.R., and M^{c}Nair, P.J. (1991) Book of Abstracts - XIIIth International Congress on Biomechanics. The Department of Movement Studies, The University of Western Australia, Perth, Australia.

Matthijsse, P.C. and Breedveld, P.C. (1989) Modeling and simulation of human gait in three dimensions with multibond graphs. XII International Congress of Biomechanics, Los Angeles, California, USA, 208.

Meglan, D.A. (1991) Enhanced analysis of human Locomotion. Ph.D. Thesis, Ohio State University.

Miller, D.I. (1974) Simulation of sports techniques by digital computer and programmable calculator. Journal of Health, Physical Education and Recreation 45, 65-67.

Miller, D.I. (1979) Modelling in biomechanics: an overview. Medicine and Science in Sports 11, 115-122.

Miller, D.I. (1975) A computer simulation of the airborne phase of diving. In T.p. Martin(ed.) Biomechanics of Sports: Selected Readings, 230-234.

Miller, D.I. and Nelson, R.C. (1973) Biomechanics of Sports (1st ed.), Lea and Febiger, Philadelphia, U.S.A.

Miller, N.R., Shapiro, R. and Mclaughlin, T.M. (1980) A technique for obtaining spatial kinematic parameters of segments of biomechanical systems from cinematographic data. Journal of Biomechanics 13, 535-547.

Mochon, S. and McMahon, T.A. (1980) Ballistic walking. Journal of Biomechanics 13, 49-57.

Onyshko, S. and Winter, D.A. (1980) A mathematical model for the dynamics of human locomotion. Journal of Biomechanics 13, 2, 1980 361-368

Pandy, M.G. and Berme, N. (1988) A numerical method for simulating the dynamics of human walking. Journal of Biomechanics 21, 1043-1052.

Panjabi, M. (1979) Validation of mathematical models. Journal of Biomechanics 12, 238.

Putnam, C.A. (1981) Interaction between segments during a kicking motion. Biomechanics VIII - B. Nagoya, Japan, 688-694.

Ramey, M.R. (1973) Significance of angular momentum in long jumping. Research Quarterly 44, 488-497.

Ramey, M.R. & Yang, A.T. (1981) A simulation procedure for human motion studies. Journal of Biomechanics, 14, 203-213.

Seigler, S., Seliktar, R. and Hyman, W. (1982) Simulation of human gait with the aid of a simple mechanical model. Journal of Biomechanics 15, 415-425.

Shapiro, R. (1976) Direct linear transformation method for three-dimensional cinematography. Research Quarterly 49, 199-205.

Soest, A.J van and Bogert, A.J. van den (1991) Criteria for the comparison of direct dynamics software systems to be used in the field of biomechanics. Abstracts for the Third International Symposium on Computer Simulation in Biomechanics. Perth, Australia, 10-11.

Vaughan, C.L., Davis, B.L. and O'Connor, J. C. (1992) Dynamics of Human Gait, Human Kinetics Publishers, Champaign , IL.

Vaughan, C.L. (1984) Computer simulation of human motion in sports biomechanics. American College of Sports Medicine Series The Collamore Press, D.C. Heath & Company, 373-416.

Vaughan, C.L., Hay, J.G. and Andrews, J. (1982) Closed loop problems in biomechanics. part I - A classification system. Journal of Biomechanics 15, 197-200.

Vaughan, C.L., Hay, J.G., and Andrews, J. (1982) Closed loop problems in biomechanics. part II - An optimization approach. Journal of Biomechanics 15, 201-210.

Walton, S. (1981) A generalized technique for quantifying gross human motion. Ph. D. Thesis, Pennsylvania State University.

Walton, S. & Kane, R. (1978) Interactive computer graphics - a new coaching aid. Biomechanics VI-A , 439-444.

Whitsett, C.E. (1963) Some dynamic response characteristics of weightless man. AMRL Technical Report 63-18, Wright-Patterson Airforce Base, Ohio.

Wilhelms, J. (1985) Graphical simulation of the motion of articulated bodies such as humans and robots, with particular emphasis on the use of dynamic analysis. Ph. D. Thesis, University of California, Berkley.

Wilhelms, J. (1990) Dynamic experiences. Making them Move: Mechanics, Control, and Animation of Articulated Figures. Morgan Kaufman Publishers, Inc. San Mateo, California, USA, 265-279

Wittenberg, J. (1977) Dynamics of Systems of Rigid Bodies, B.G. Tuebner, Stuttgart, Germany.

Wood, G.A. (1976) Data smoothing and differentiation procedures in biomechanics. Exercise and Sport Science Reviews 10, 308-362.

Woltring, H.J. (1980) Planar control in multi-camera calibration for 3-D gait studies. Journal of Biomechanics 13, 39-48.

Yeadon, M. R. (1990a) The simulation of aerial movement I. The determination of orientation angles from film data. Journal of Biomechanics 23, 59-66.

Yeadon, M. R. (1990b) The simulation of aerial movement II. A mathematical inertia model of the human body. Journal of Biomechanics 23, 67-74.

Yeadon, M. R. (1990c) The simulation of aerial movement III. The determination of the angular momentum of the human body. Journal of Biomechanics 23, 75-83.

Yeadon, M. R. , Atha, J., and Hales, F.D. (1990 d) The simulation of aerial movement IV. A computer simulation model. Journal of Biomechanics 23, 85-89.

Zeigler, Bernard, P. (1984) Theory of Modelling and Simulation, Robert E. Krieger Publishing Company, Malabar, Florida.

Zeltzer, David (1982) Representation of complex animated figures. Graphics Interface '82 Conference Proceedings. 205-211.

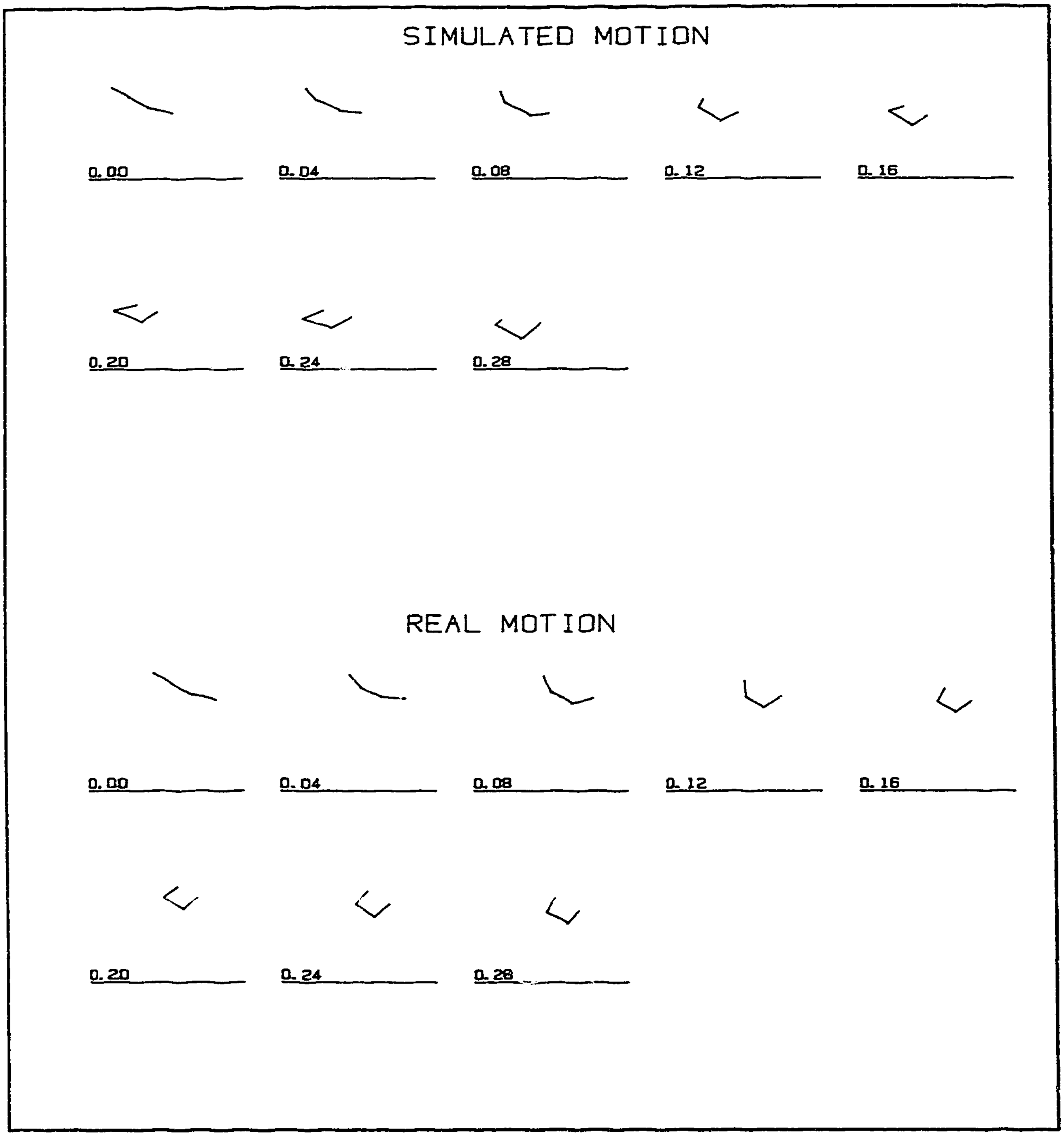
Zeltzer, David (1982) Motor control techniques for figure animation. IEEE Computer Graphics and Applications 2, 53-68.

Zernicke, R.F. and Roberts, E.M. (1978) Lower extremity forces and torques during systematic variation of non-weight bearing motion. Medicine and Science in Sports 10, 21-26.

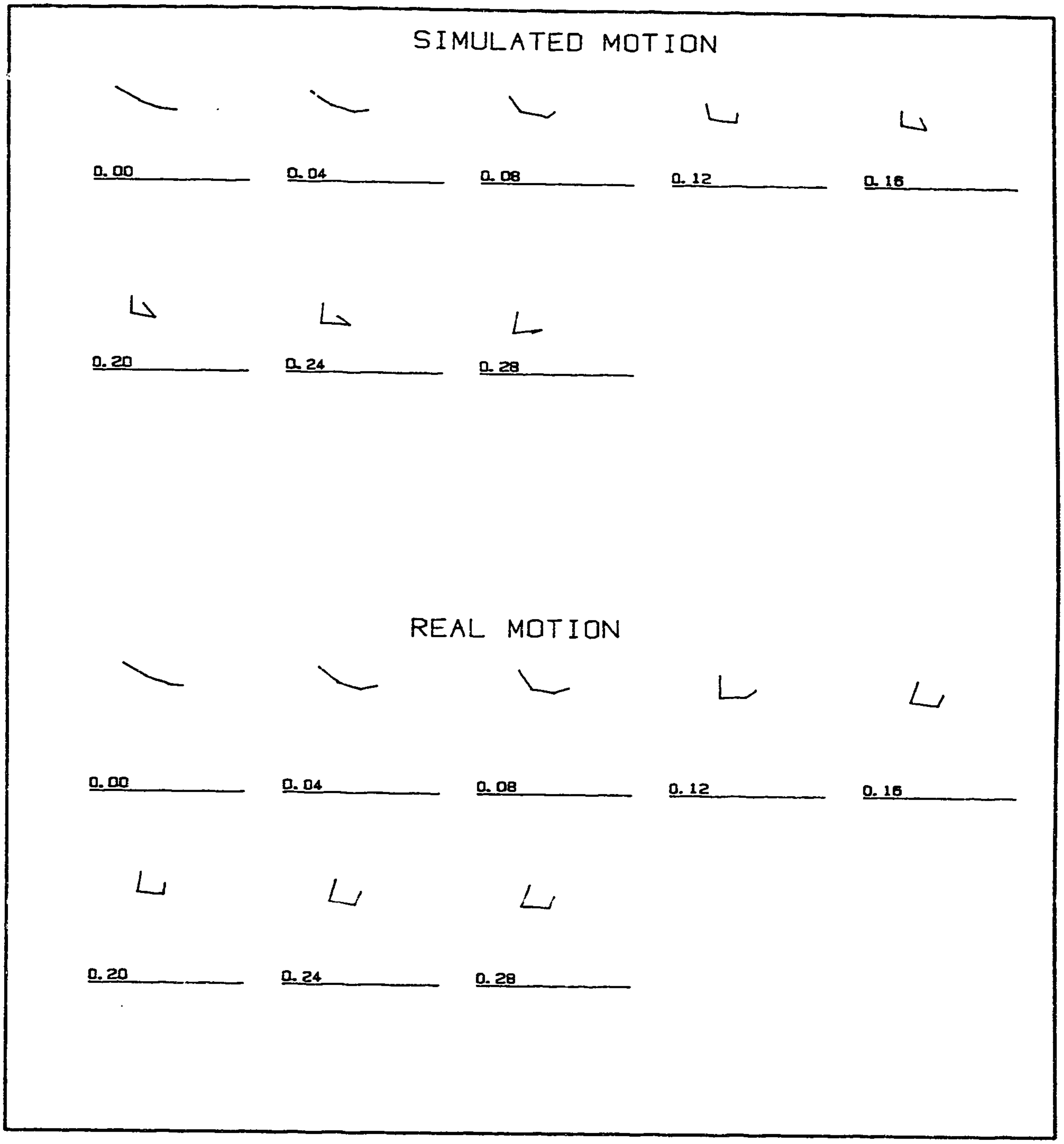
APPENDIX A

SIMULATIONS OF A THREE LINK WOODEN MODEL

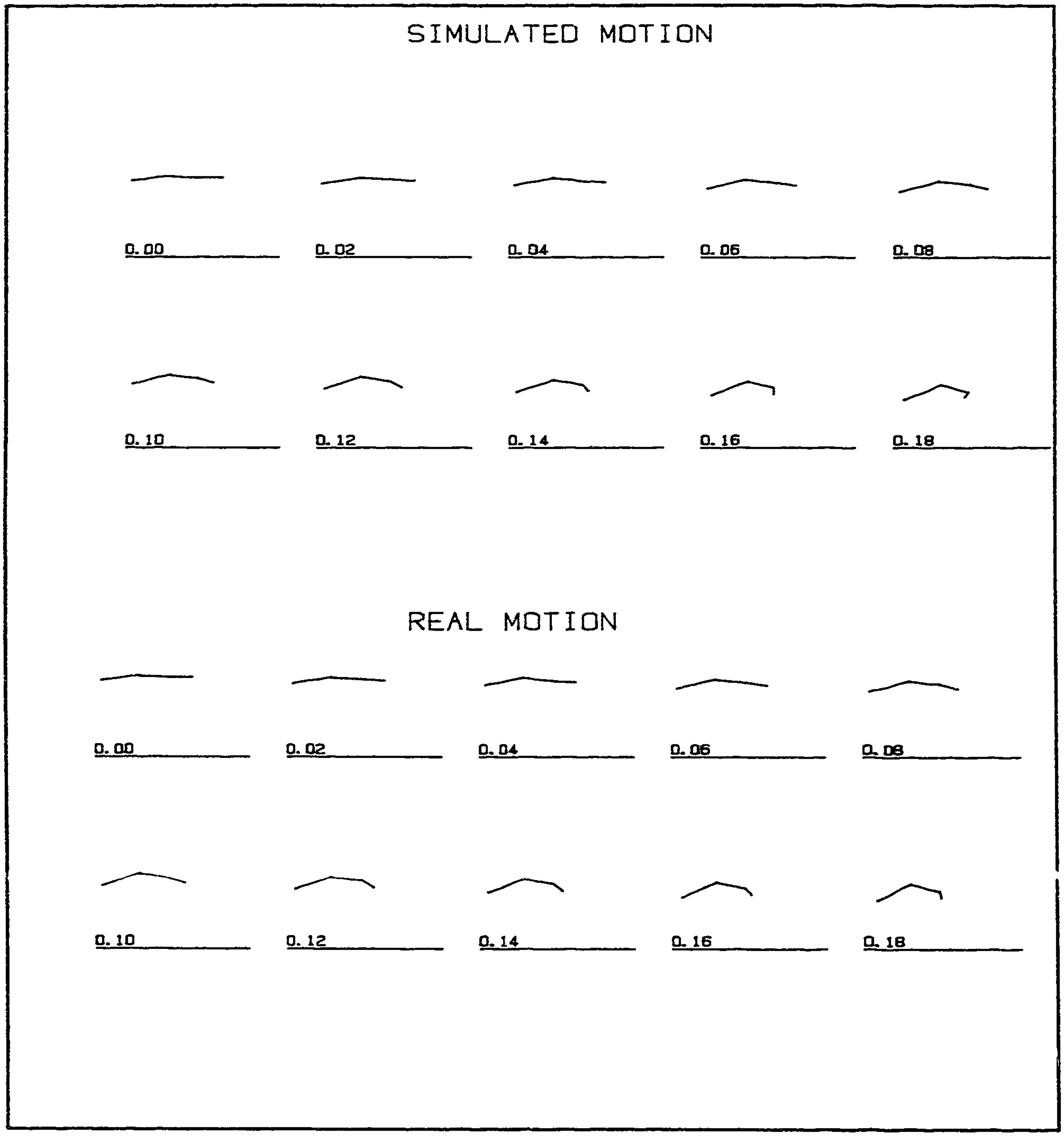
Real vs simulated motion of a three link, 12 degree of freedom wooden model with torques constrained to act only in flexion/extension.



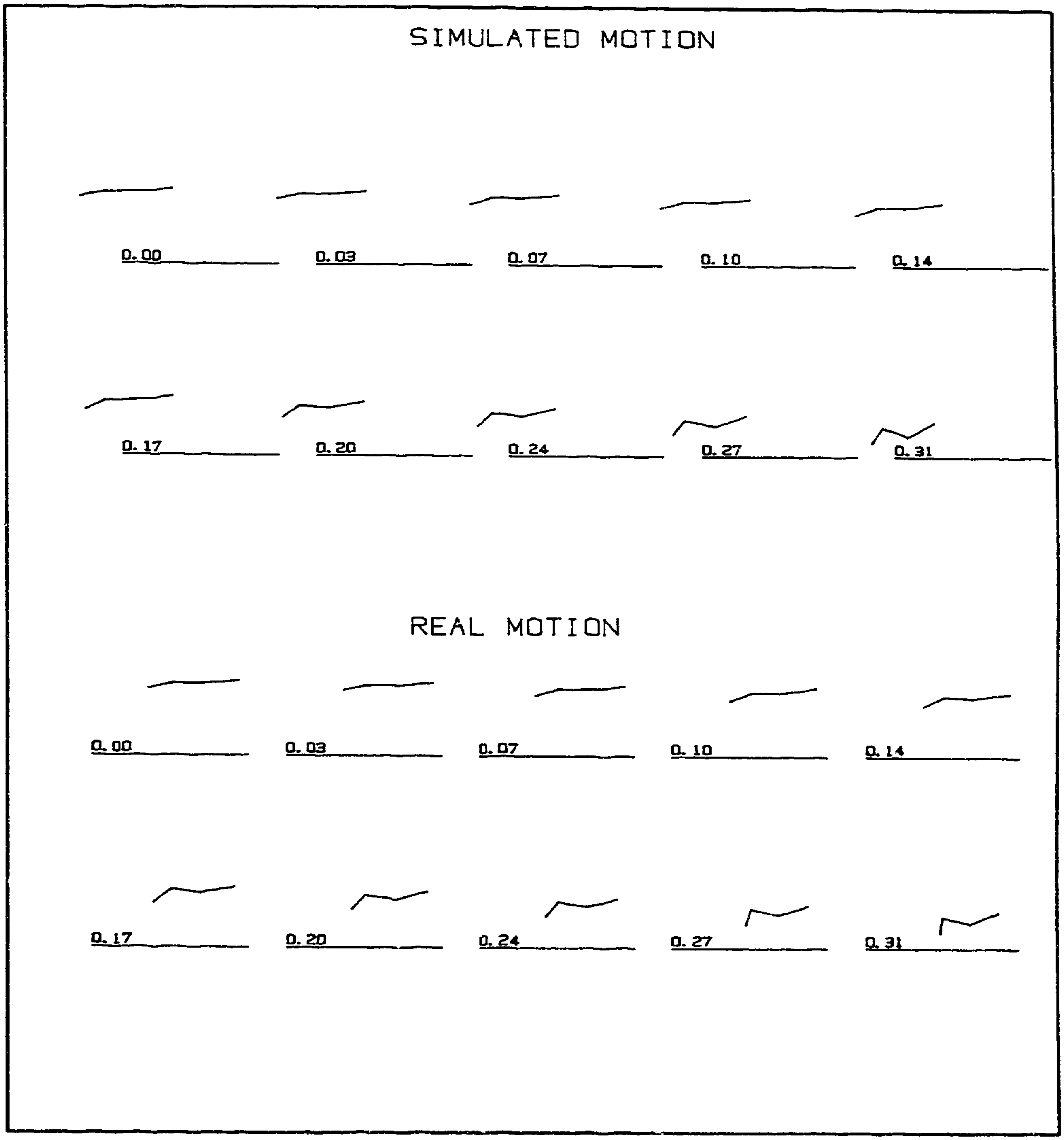
Trial 1- Manually digitized data with torques constrained to act in flexion/extension at the joints.



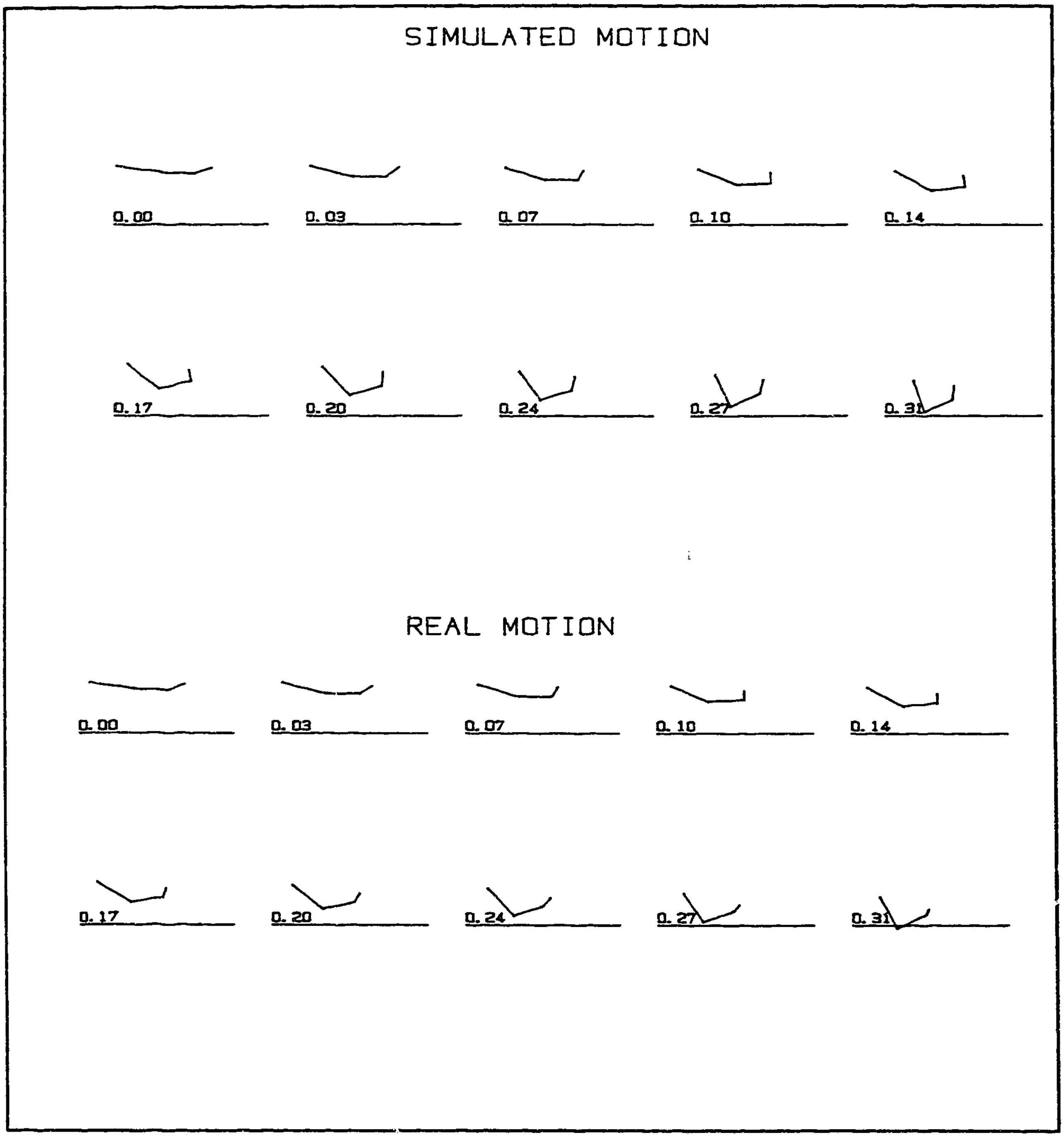
Trial 2- Manually digitized data with torques constrained to act in flexion/extension at the joints.



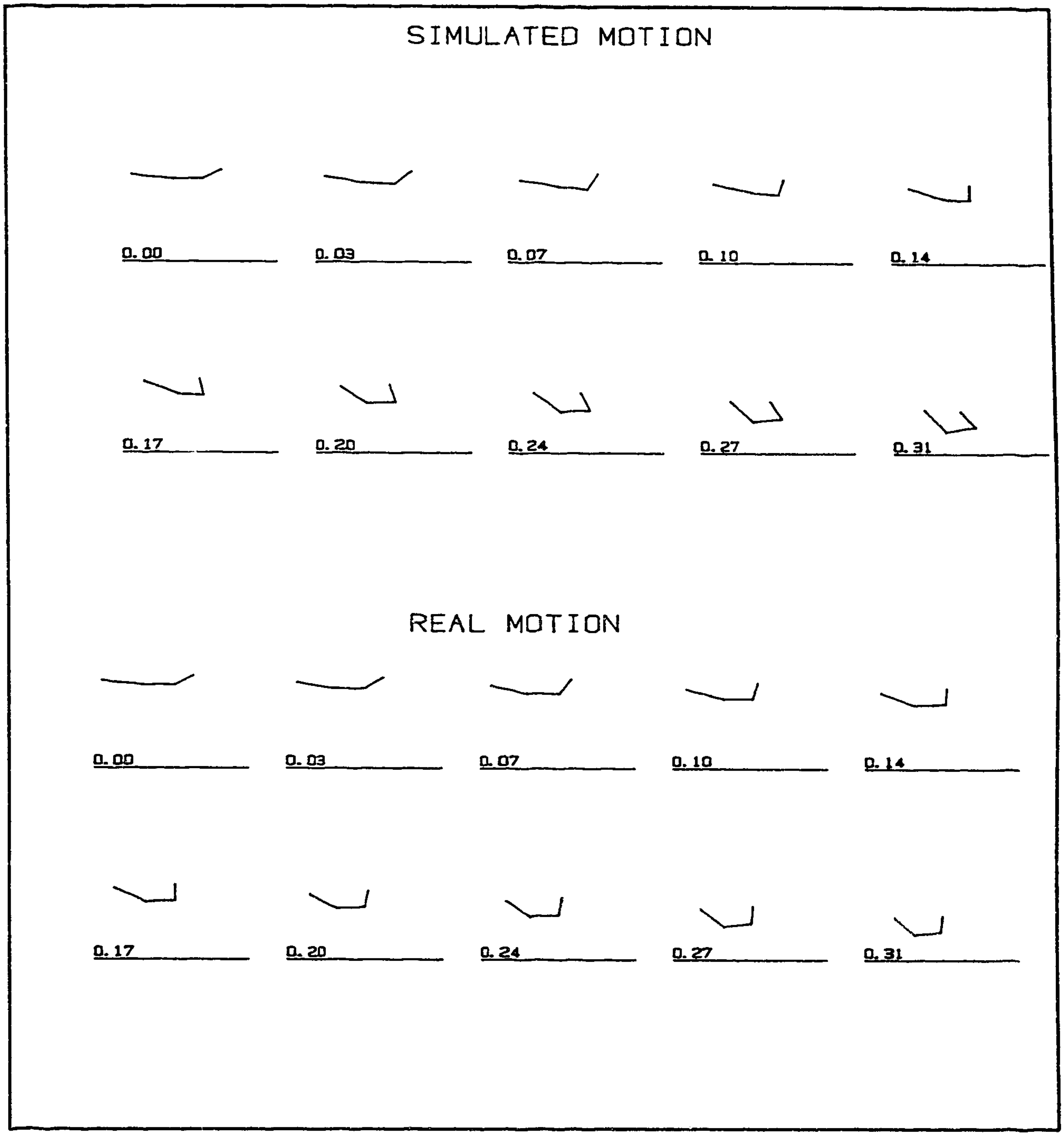
Trial 3 - Automatically digitized data with torques constrained to act in flexion/extension at the joints.



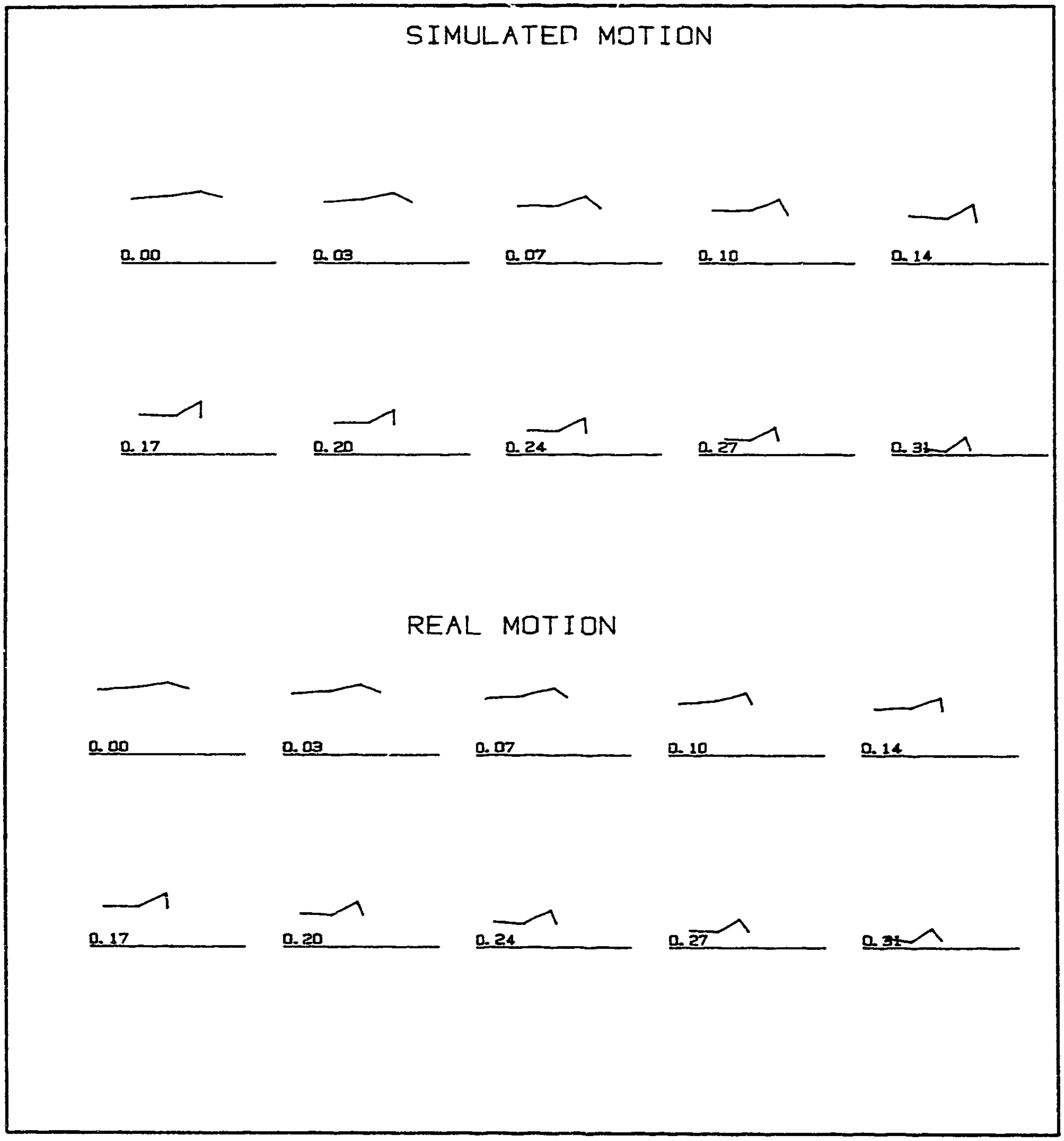
Trial 4 - Automatically digitized data with torques constrained to act in flexion/extension at the joints.



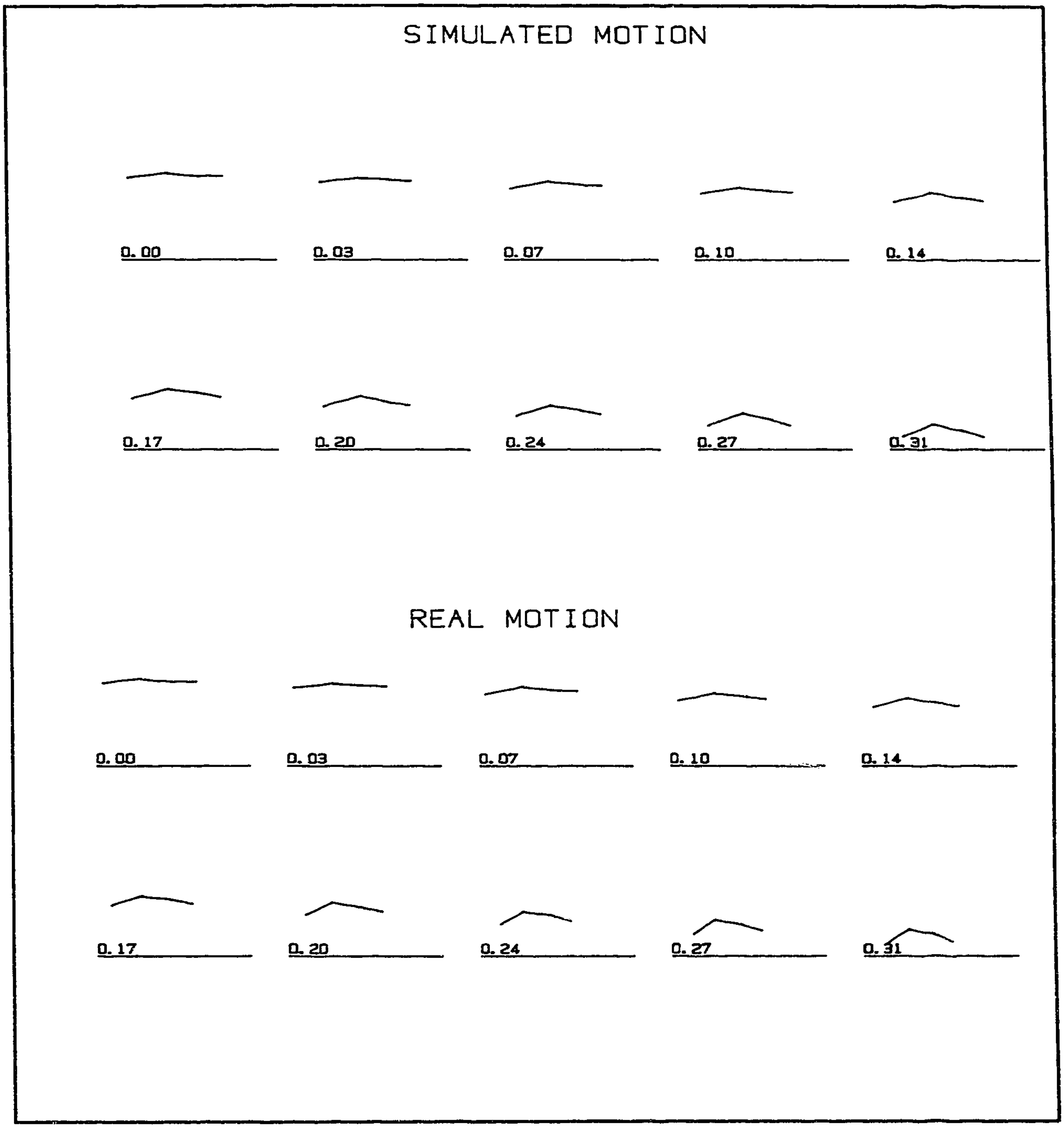
Trial 5 - Automatically digitized data with torques constrained to act in flexion/extension at the joints.



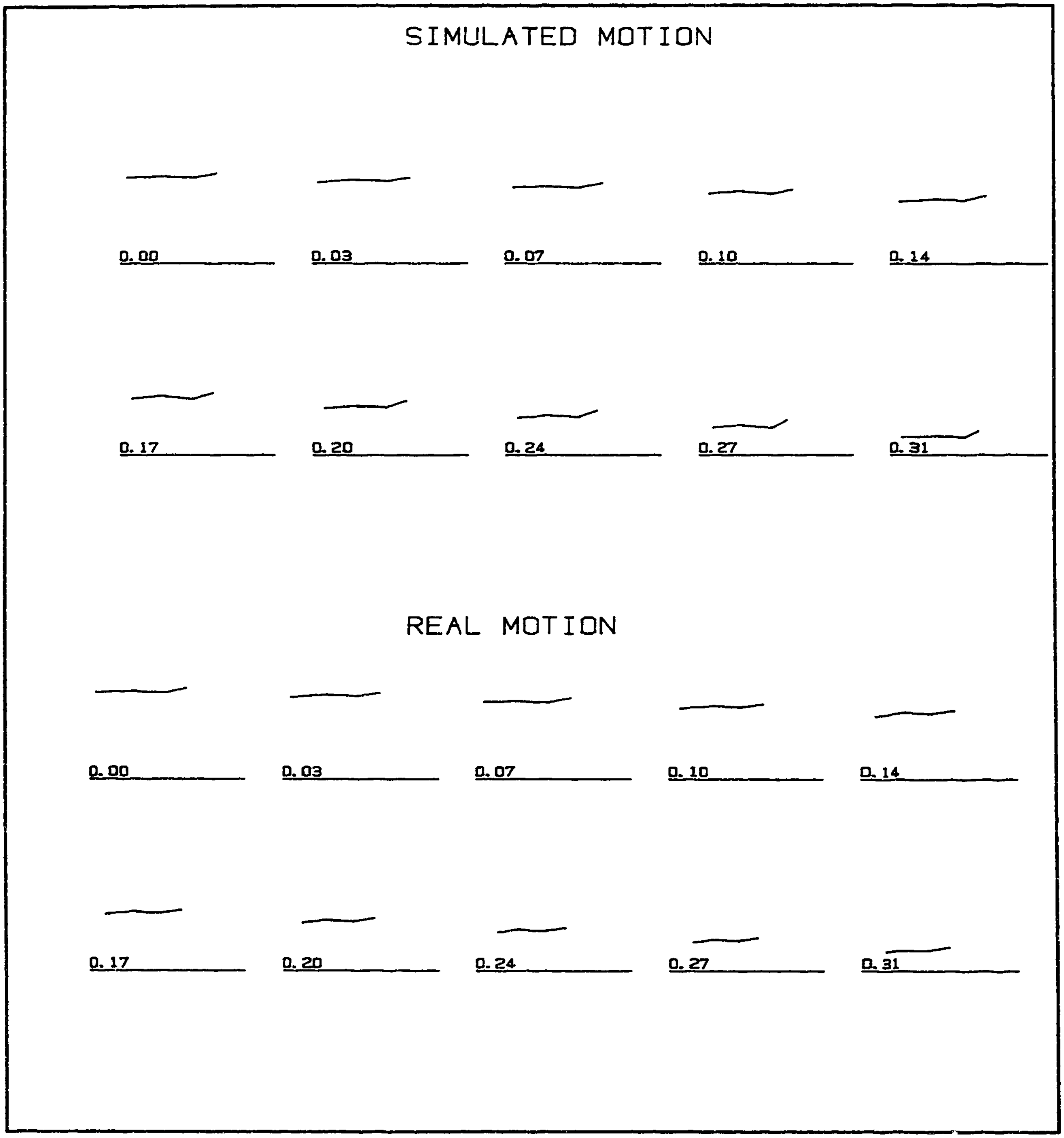
Trial 6 - Automatically digitized data with torques constrained to act in flexion/extension at the joints.



Trial 7 - Automatically digitized data with torques constrained to act in flexion/extension at the joints.



Trial 8 - Automatically digitized data with torques constrained to act in flexion/extension at the joints.

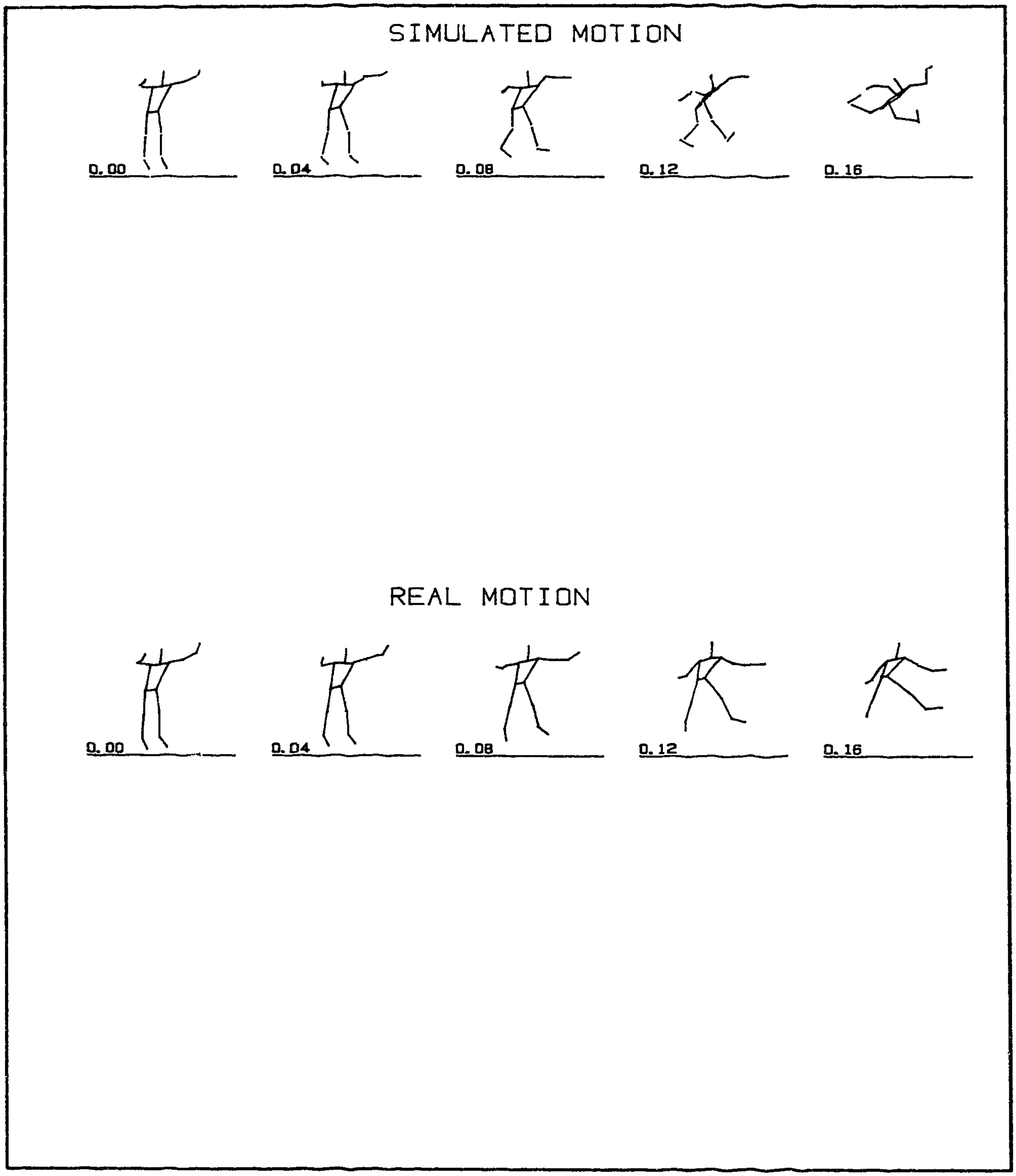


Trial 9 - Automatically digitized data with torques constrained to act in flexion/extension at the joints.

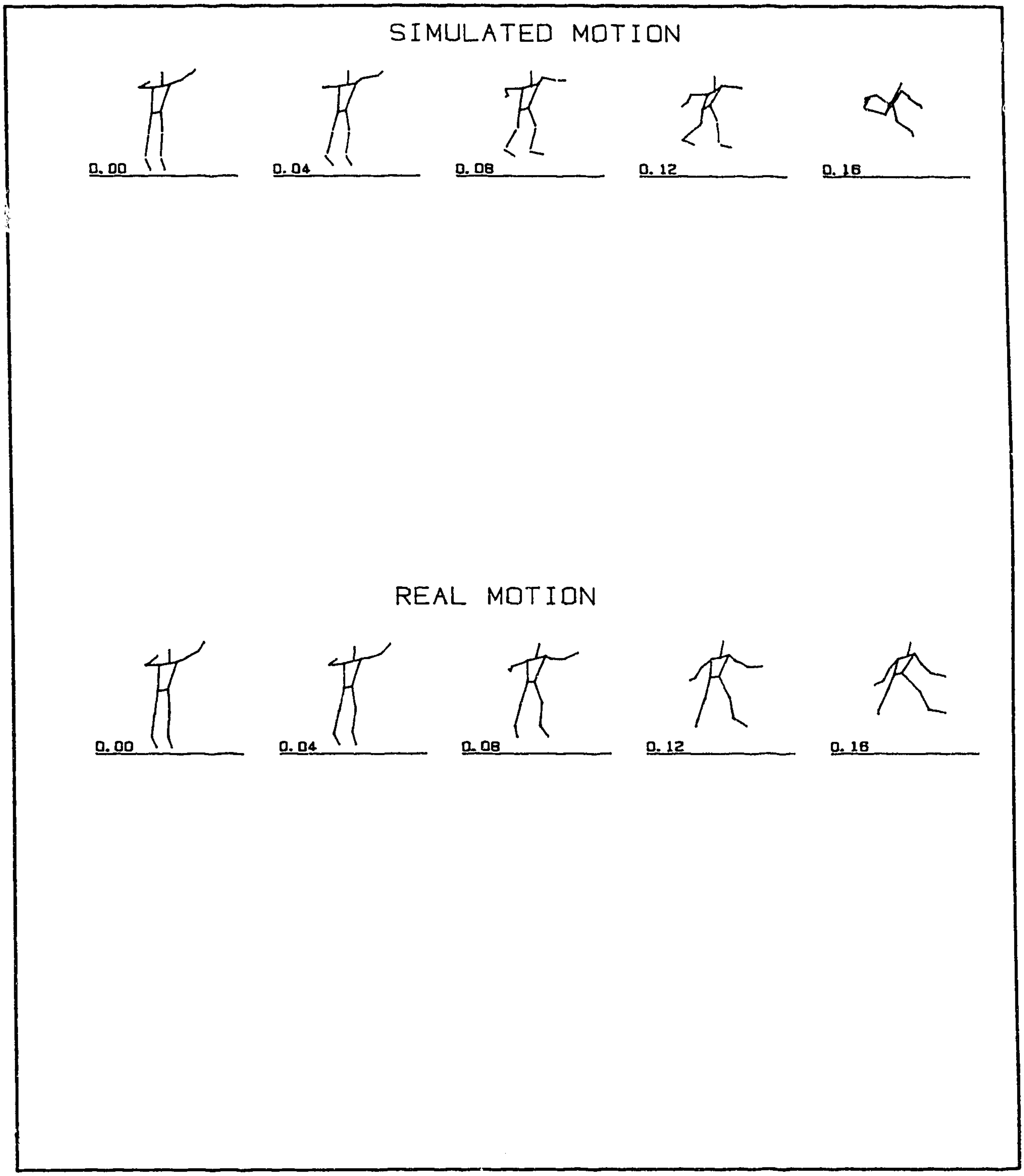
APPENDIX B

HUMAN MODEL SIMULATIONS WITH NO CONSTRAINTS

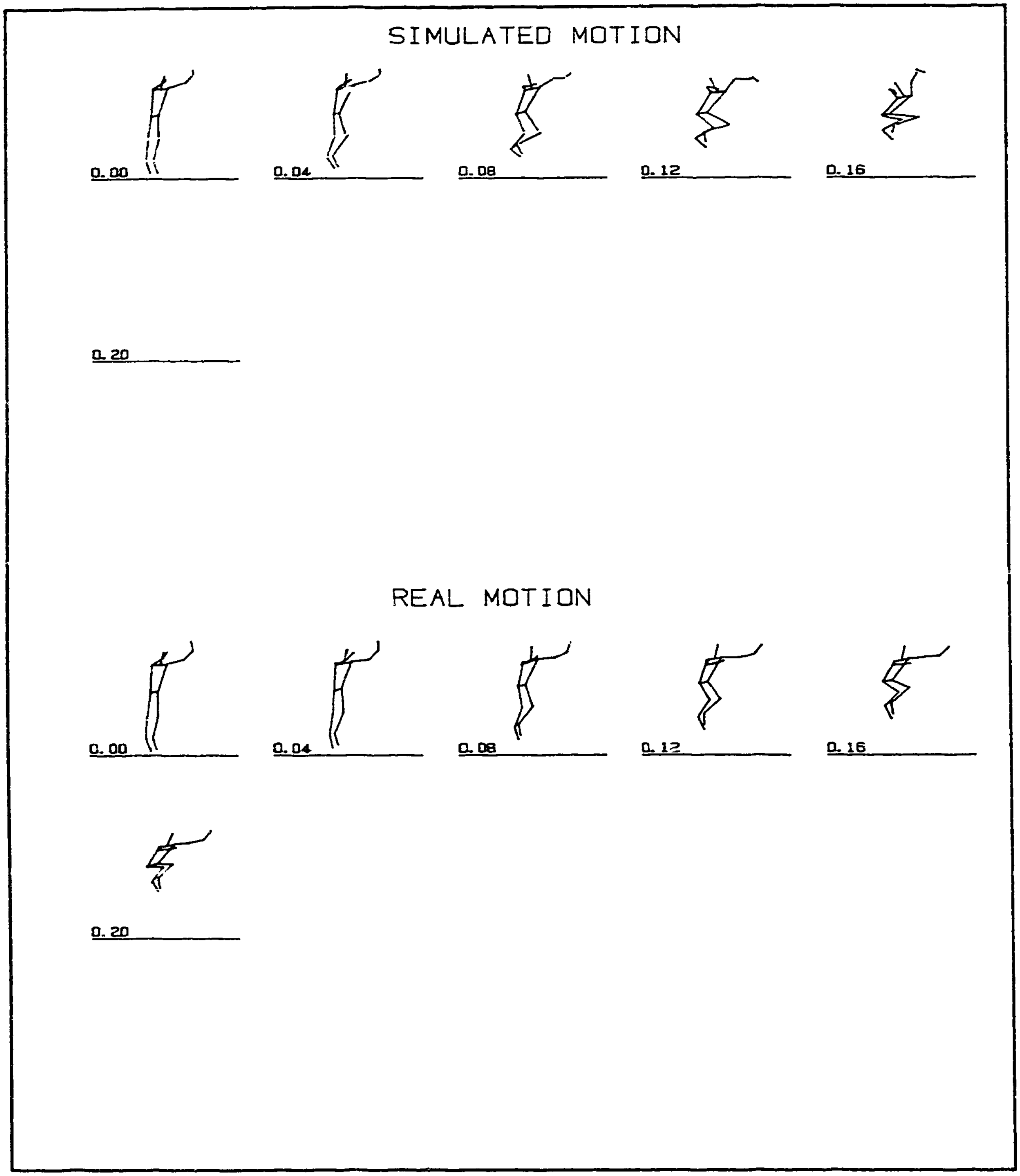
Real vs simulated motion of a 14 link, 45 degree of freedom human model with no constraints being applied.



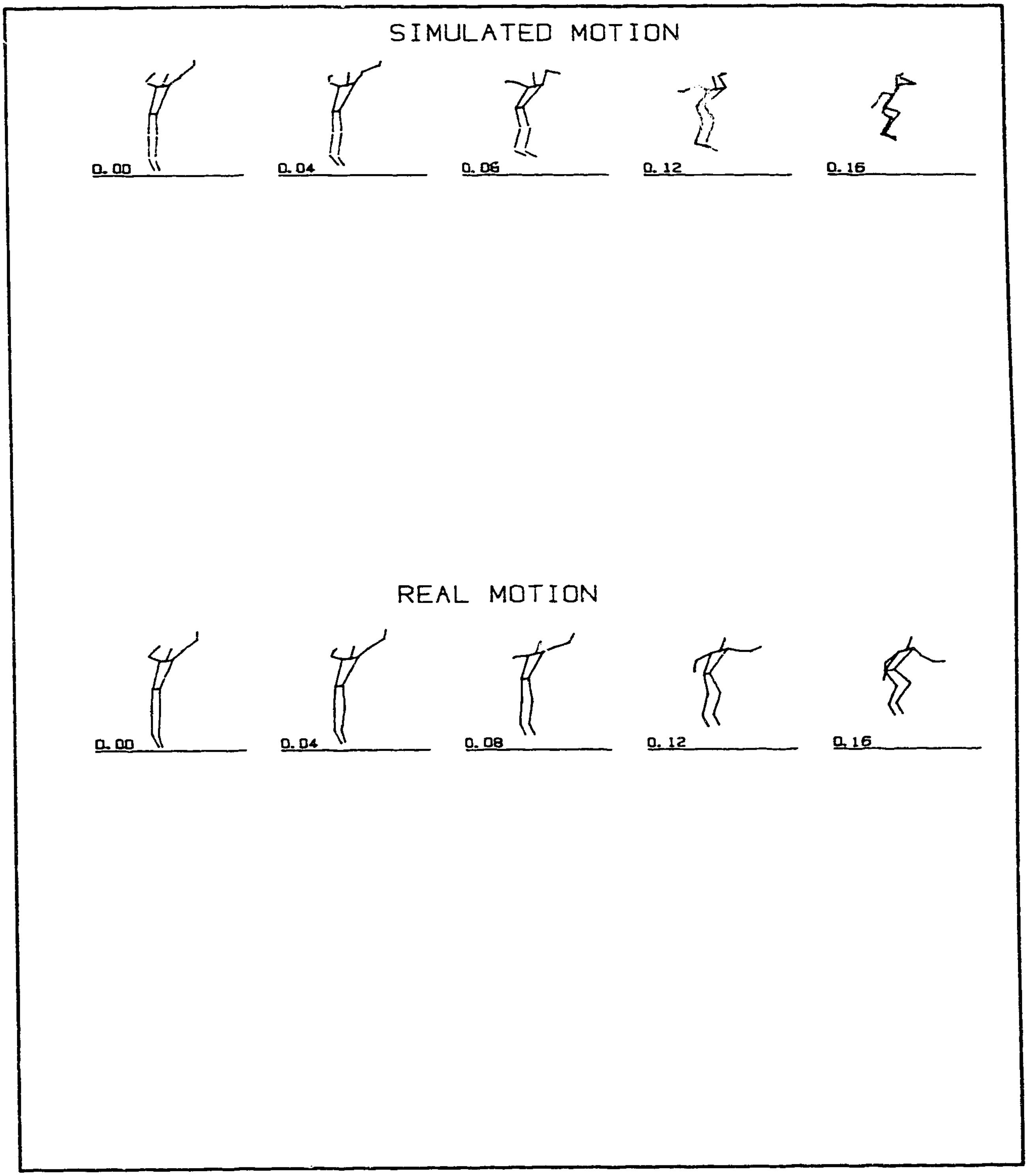
Straddle Jump Trial 1 - Manually digitized data and no constraints.



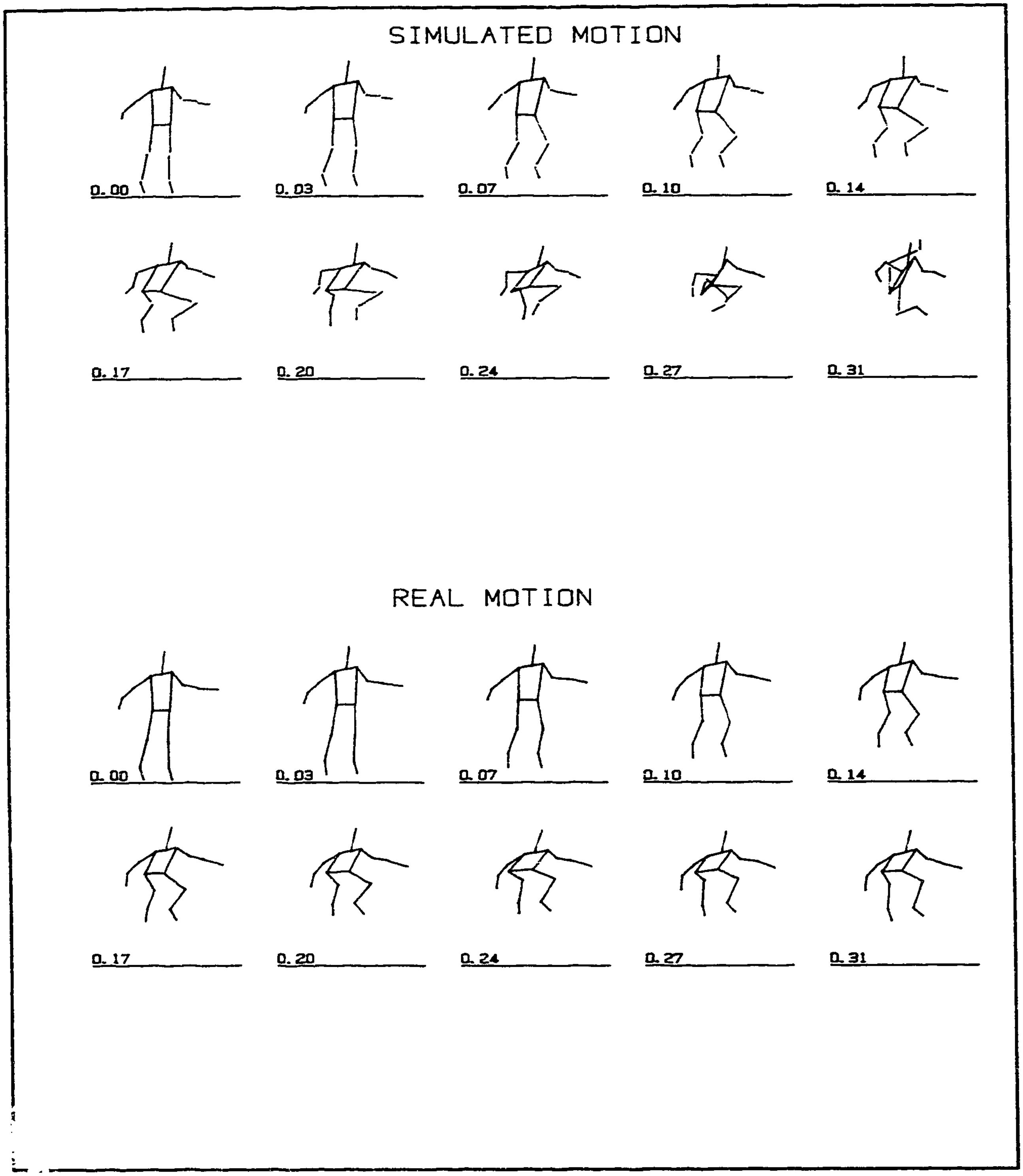
Straddle Jump Trial 2 - Manually digitized data and no constraints.



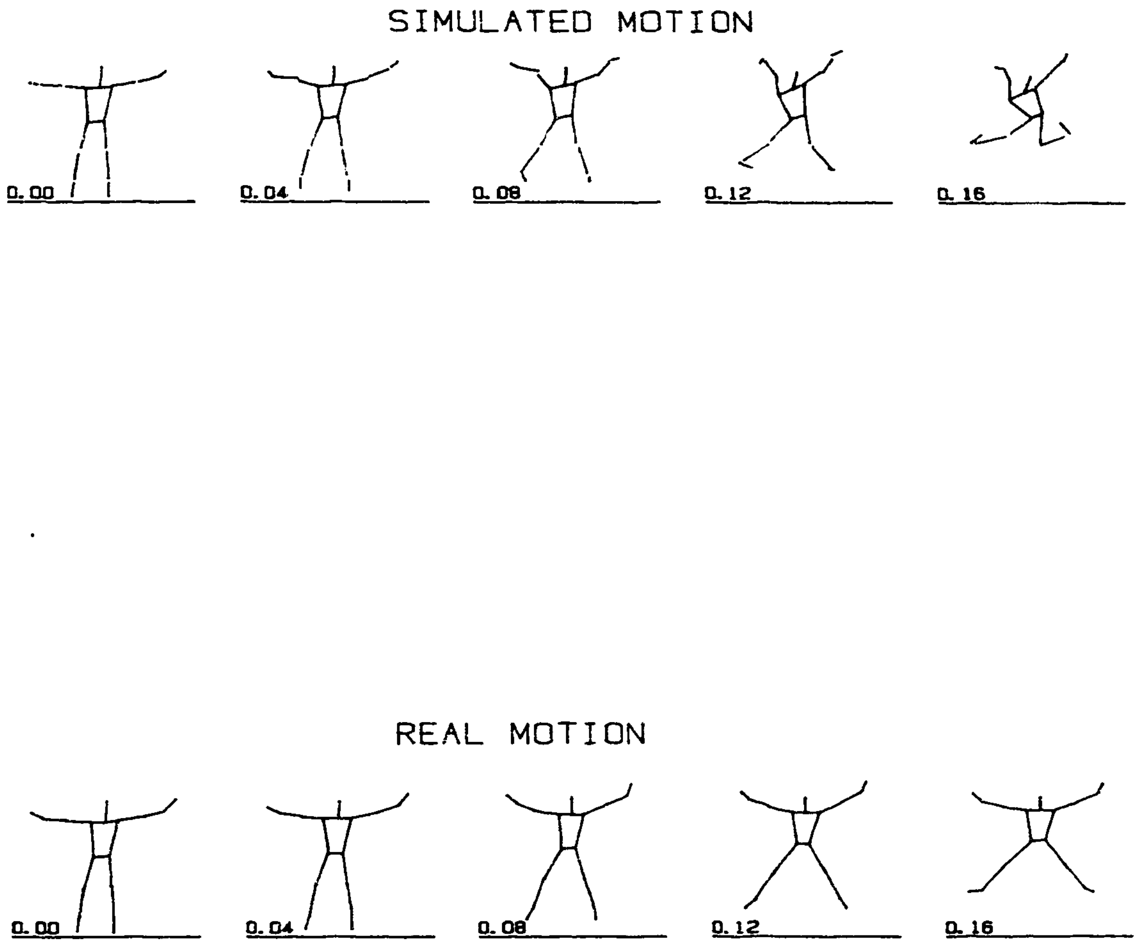
Tuck Jump Trial 1 - Manually digitized data and no constraints.



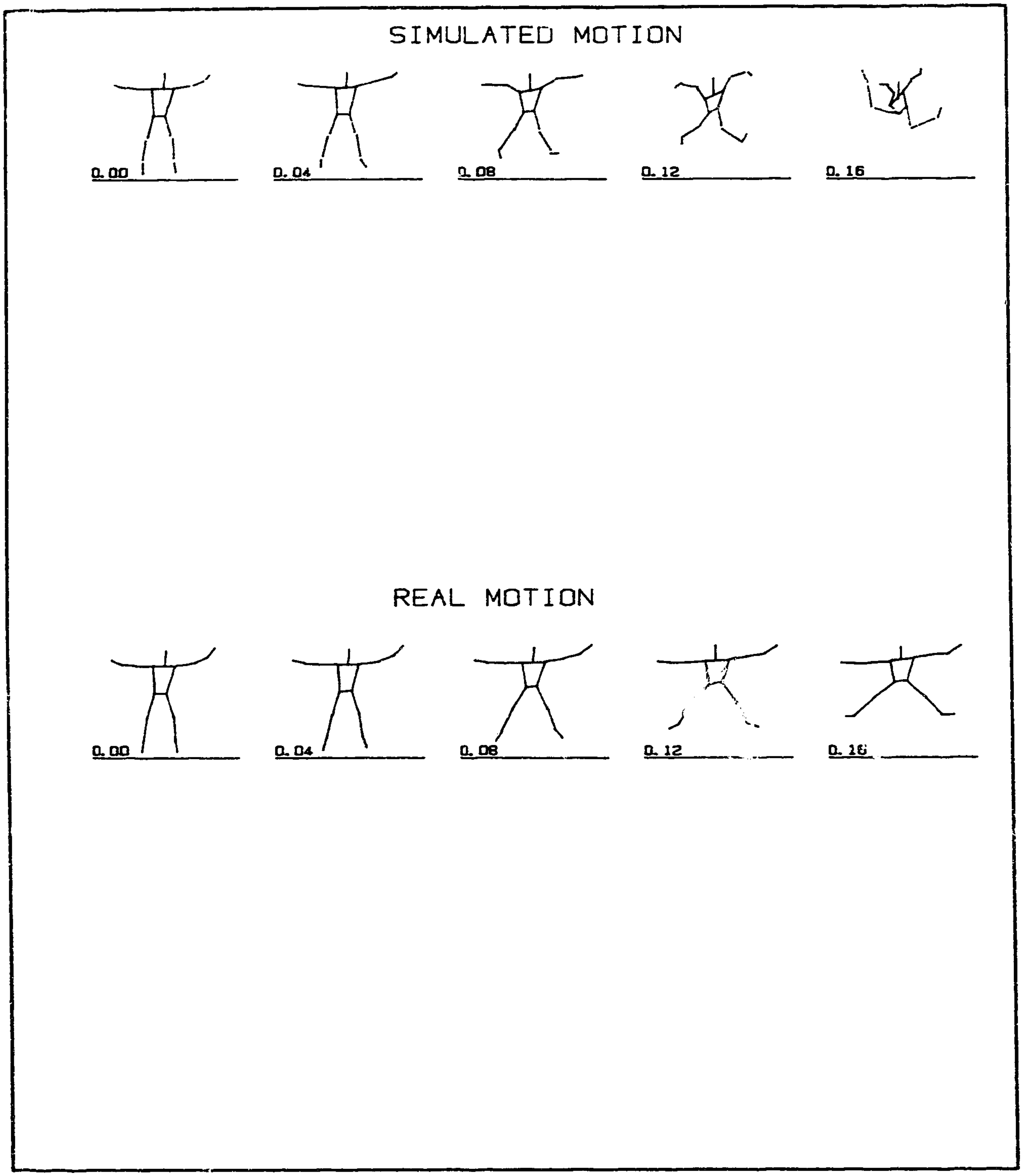
Tuck Jump Trial 2 - Manually digitized data and no constraints.



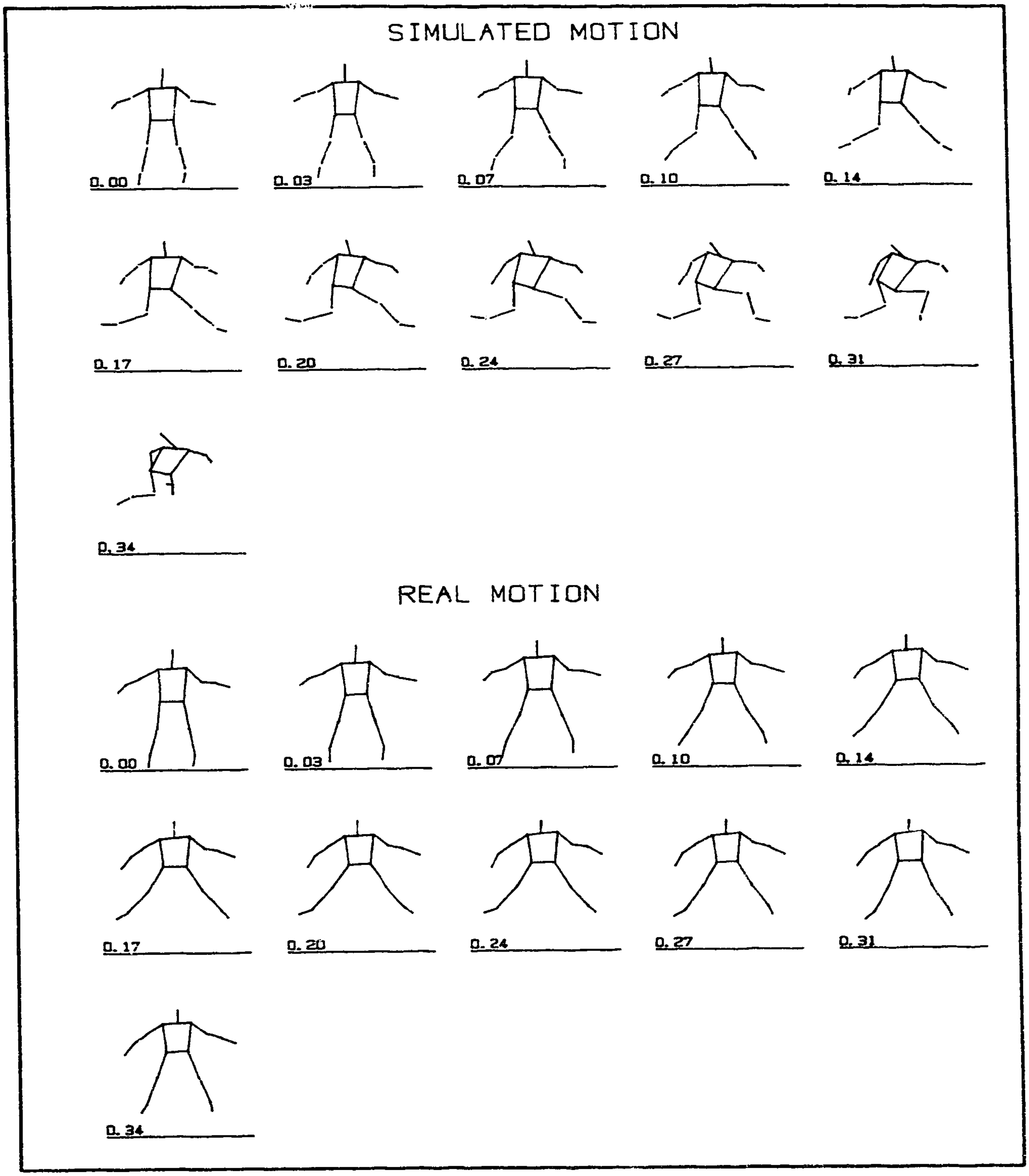
Tuck Jump Trial 3 - Automatically digitized data and no constraints.



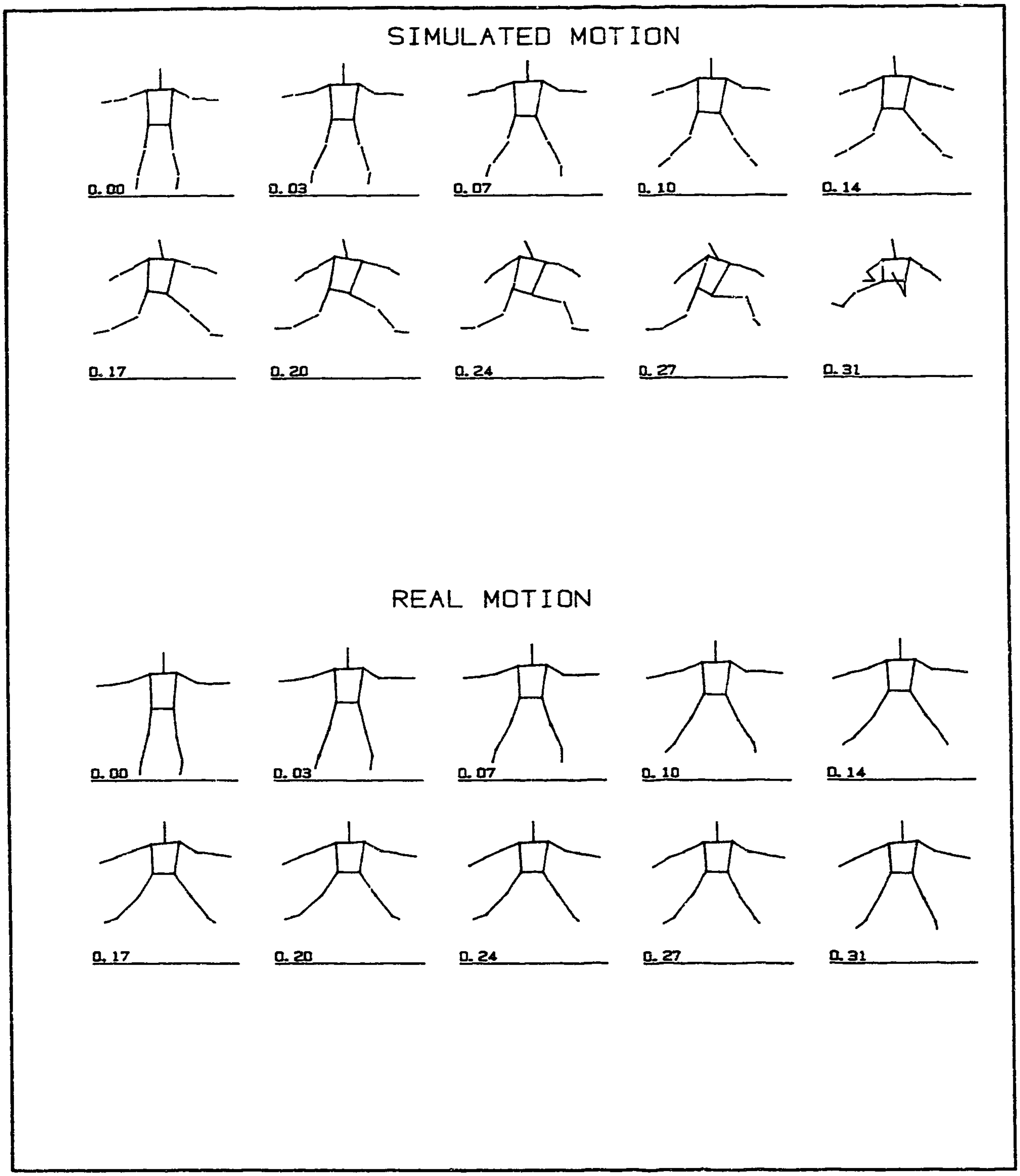
Split Jump Trial 1 - Manually digitized data and no constraints.



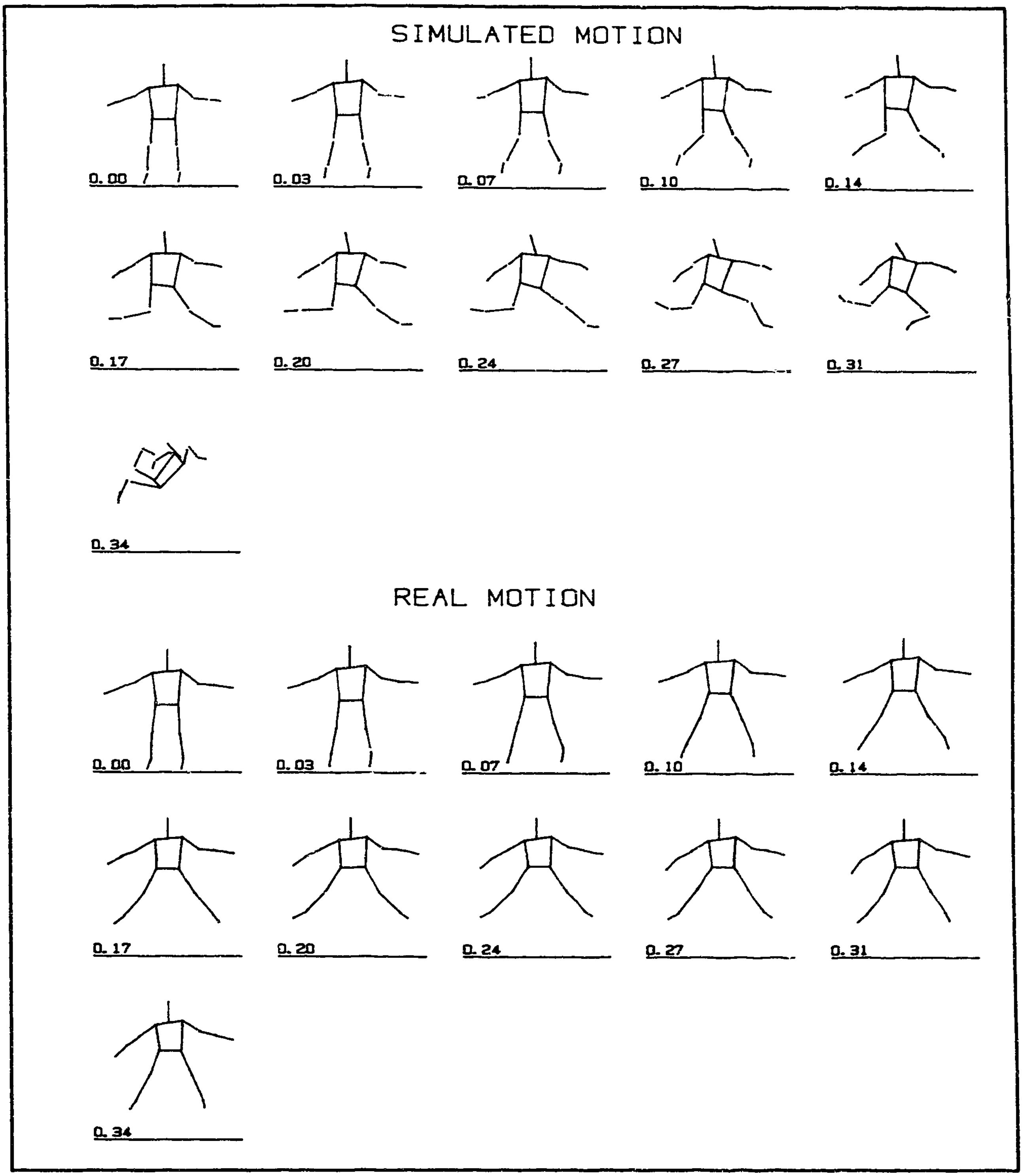
Split Jump Trial 2 - Manually digitized data and no constraints.



Split Jump Trial 3 - Automatically digitized data and no constraints.



Split Jump Trial 4 - Automatically digitized data and no constraints.

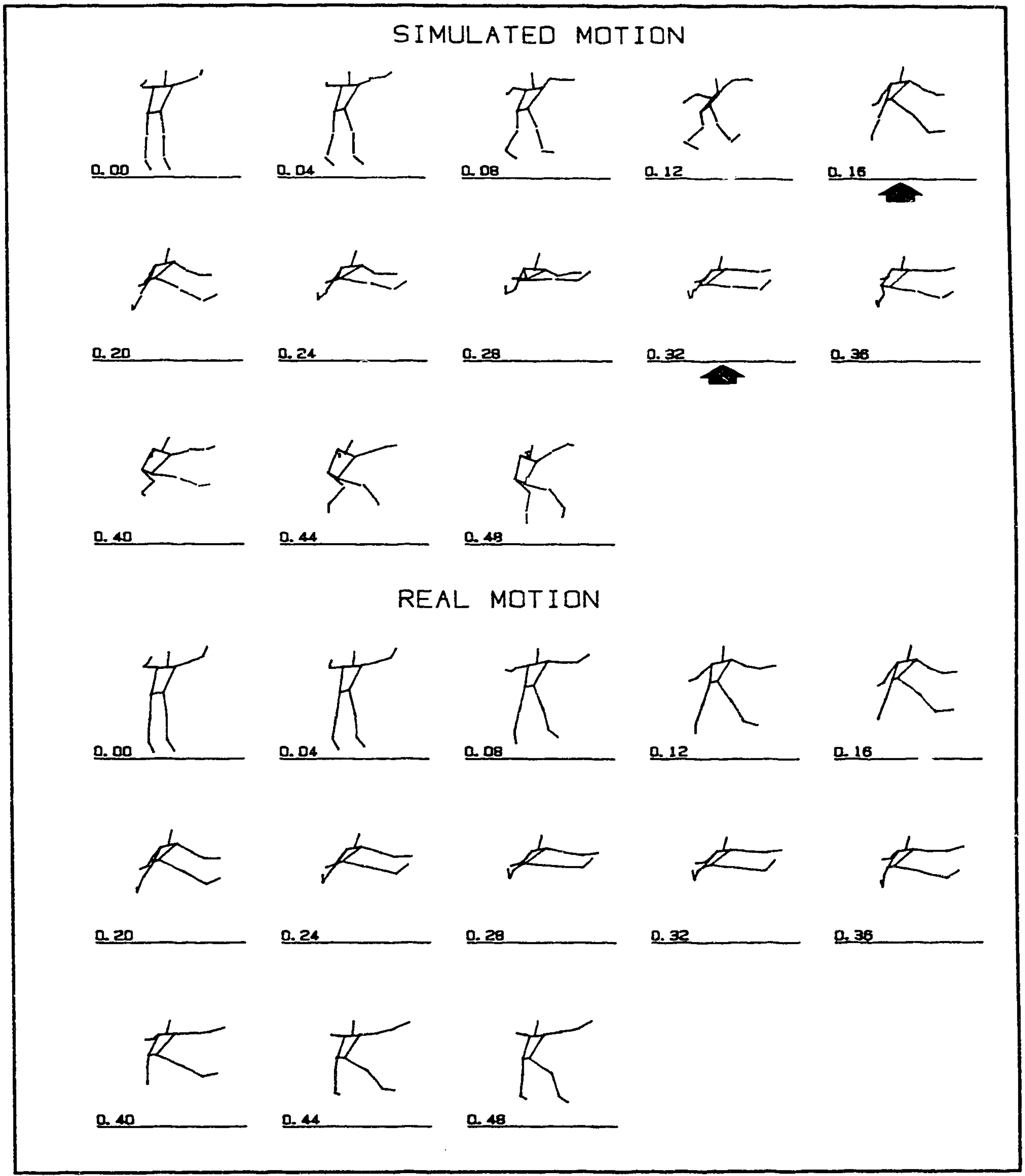


Split Jump Trial 5 - Automatically digitized data and no constraints.

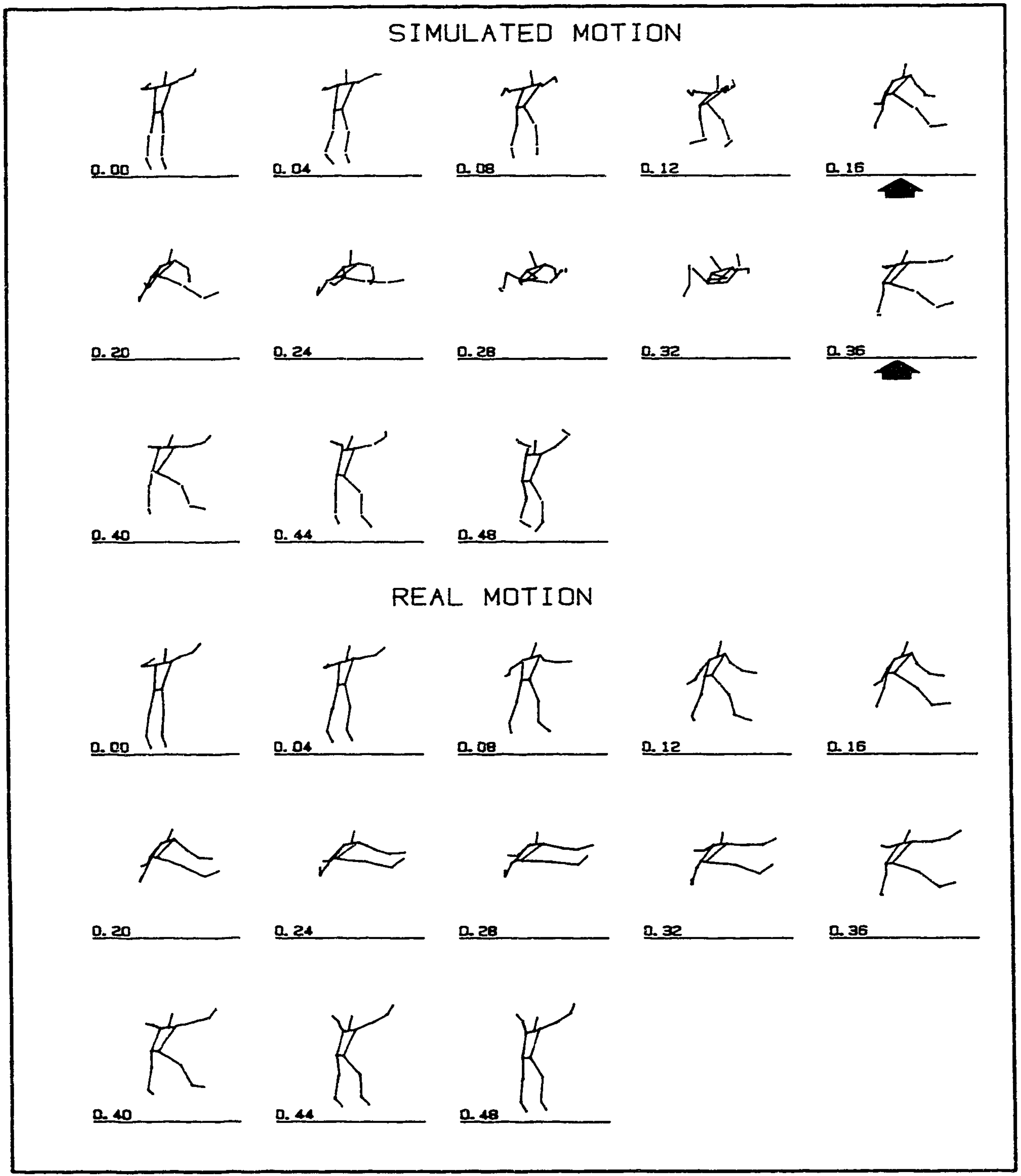
APPENDIX C

HUMAN MODEL PIECEWISE SIMULATIONS

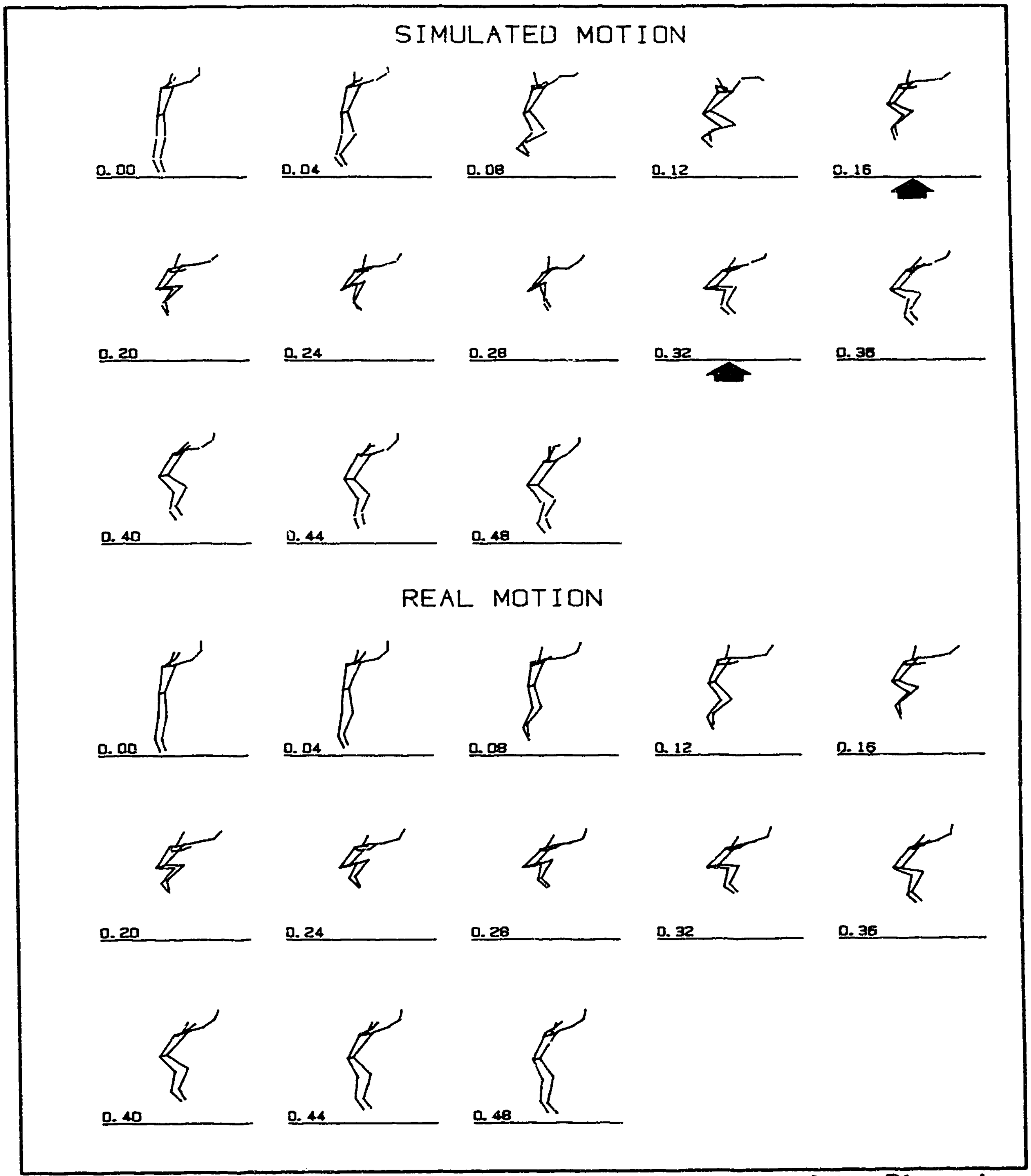
Real vs simulated motion of a 14 link, 45 degree of freedom human model with simulation restarts indicated by the arrows.



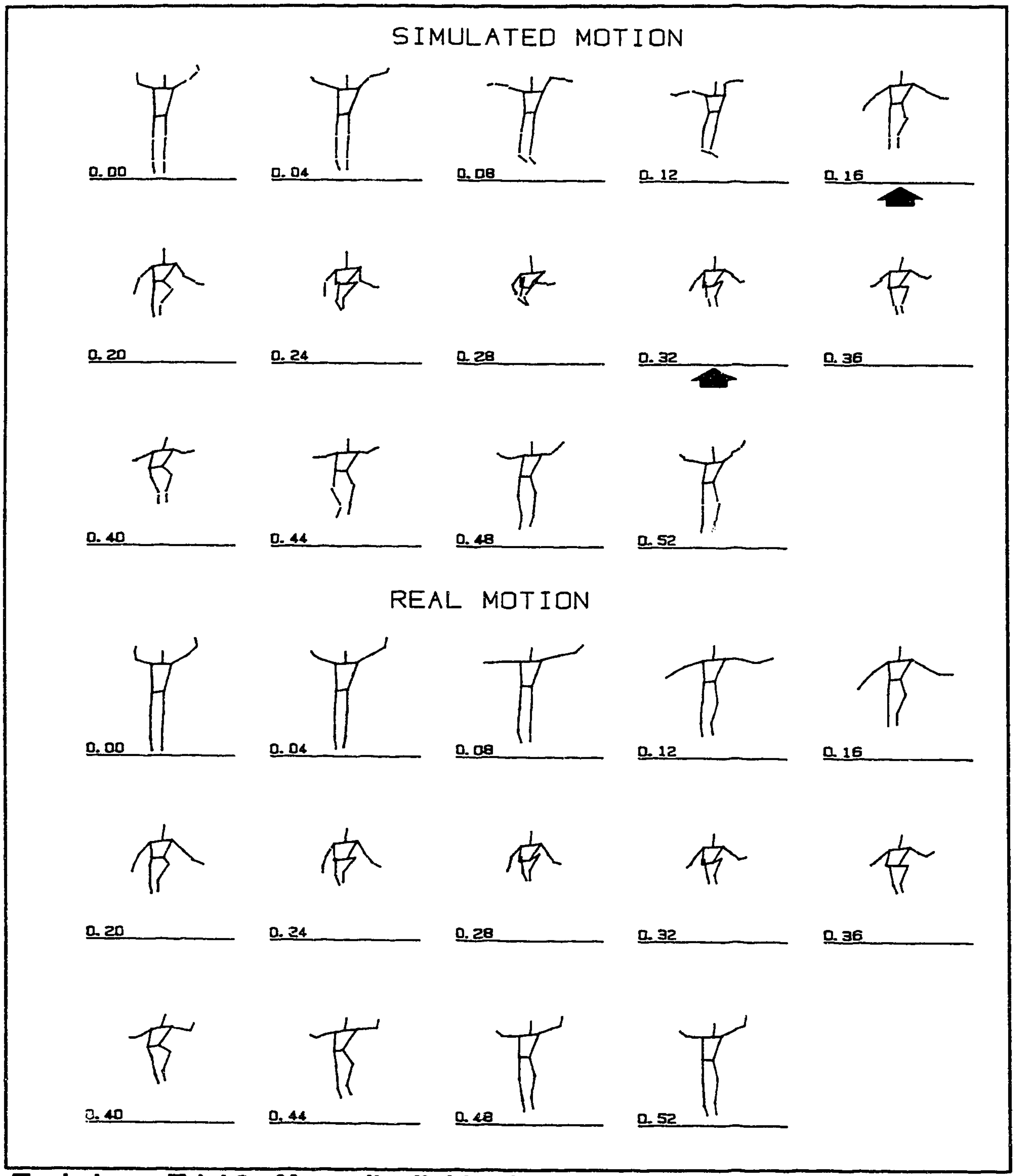
Straddle Jump Trial 1 - Manually digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



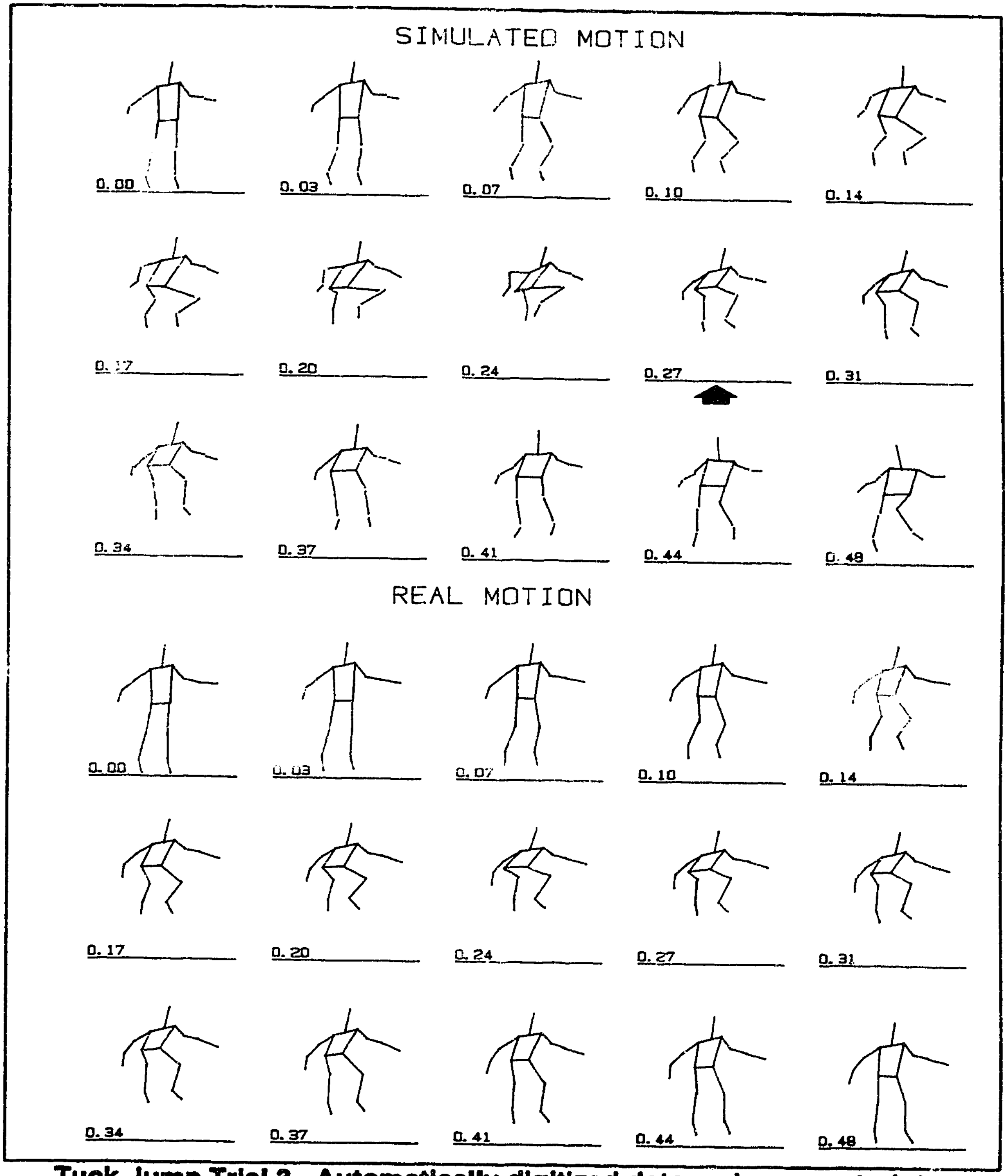
Straddle Jump Trial 2 - Manually digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



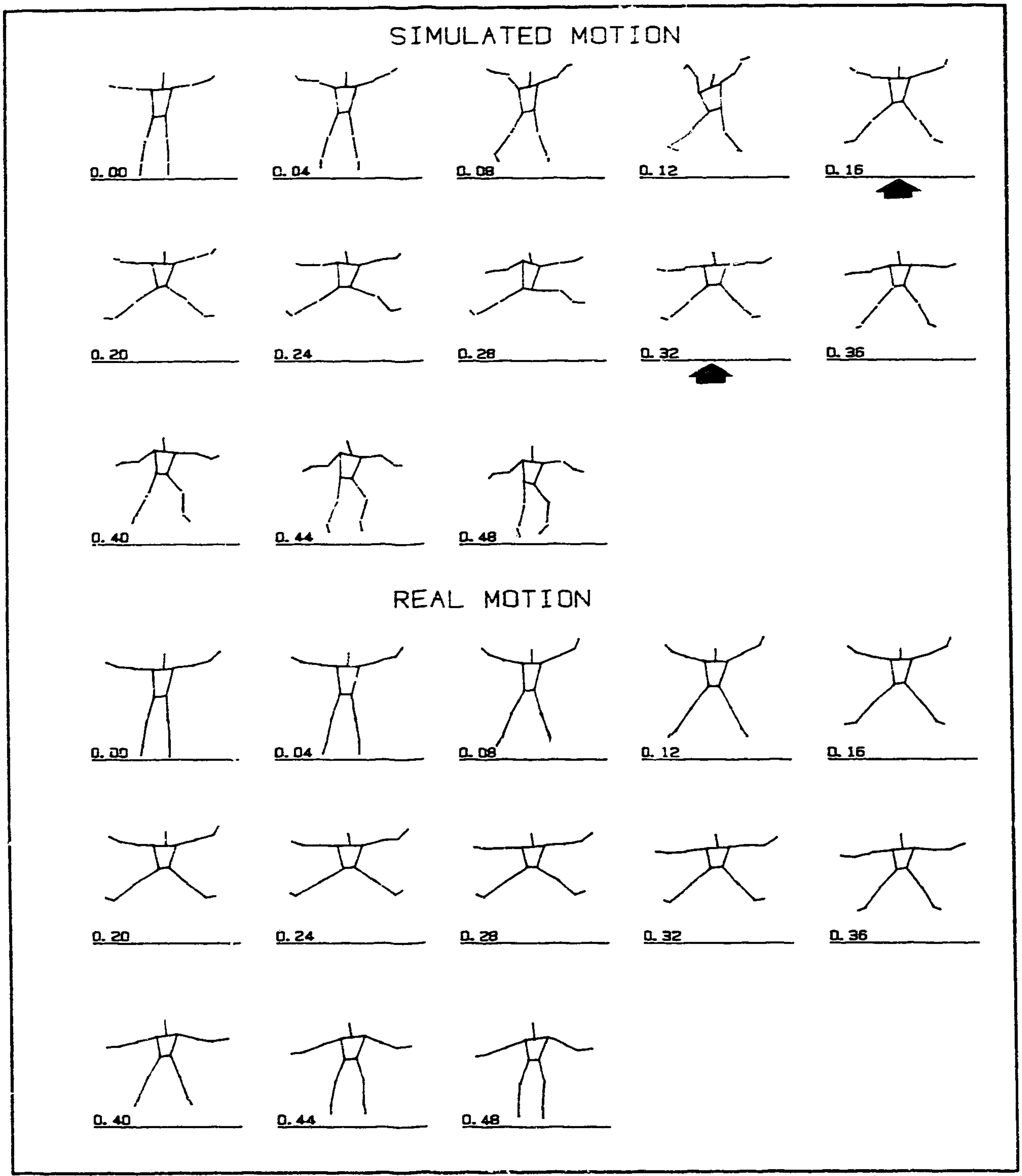
Tuck Jump Trial 1 - Manually digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



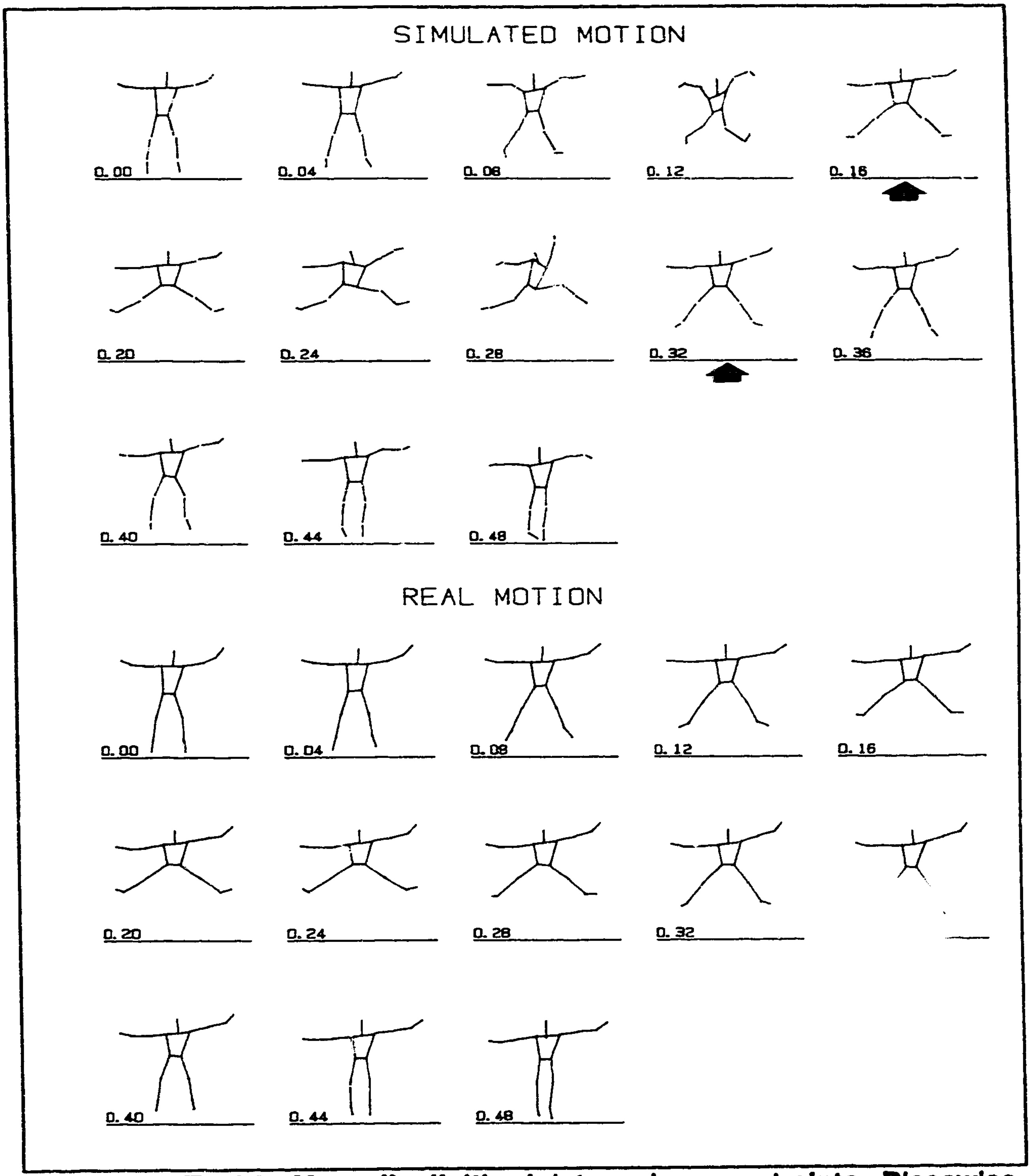
Tuck Jump Trial 2 - Manually digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



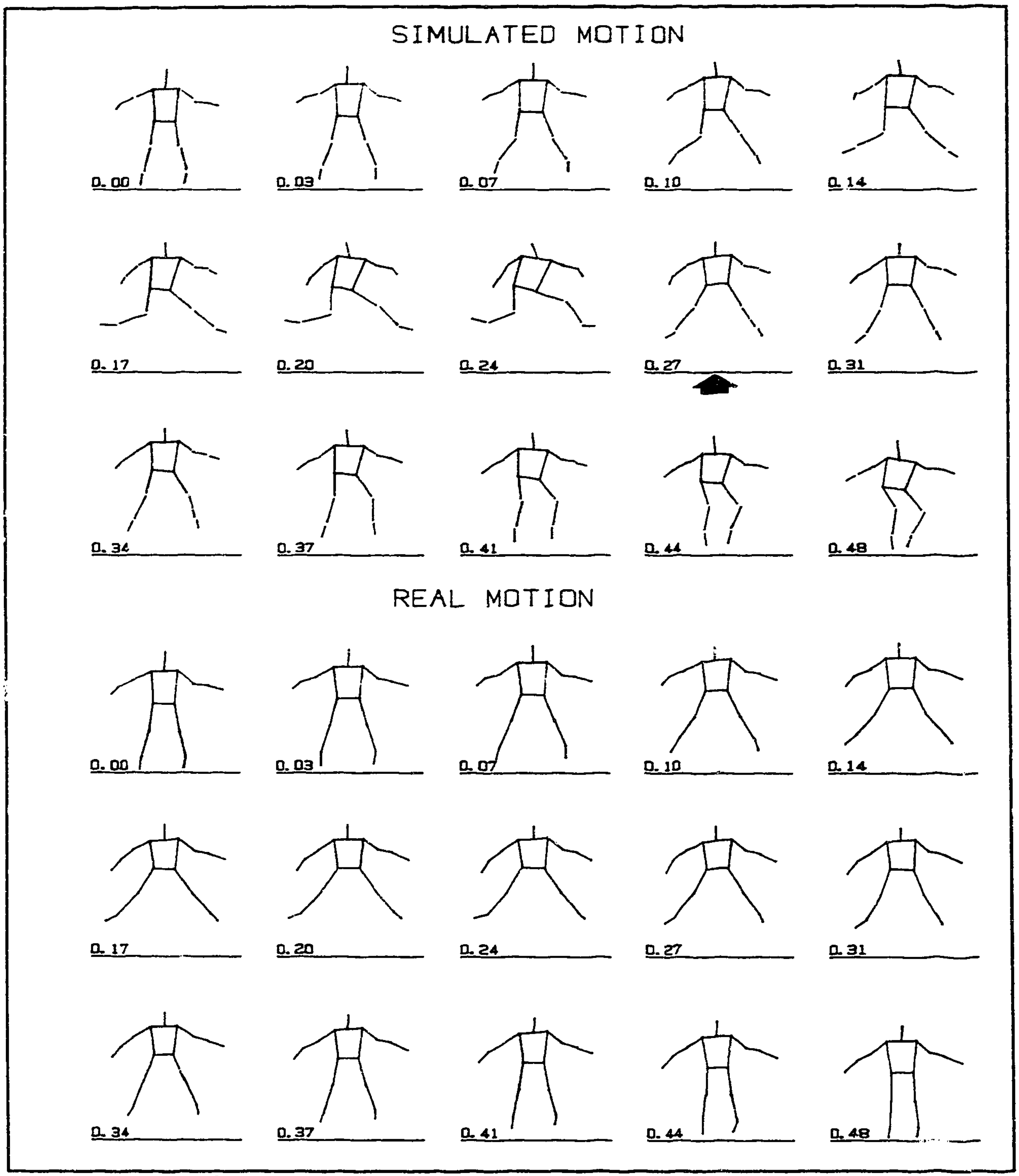
Tuck Jump Trial 3 - Automatically digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



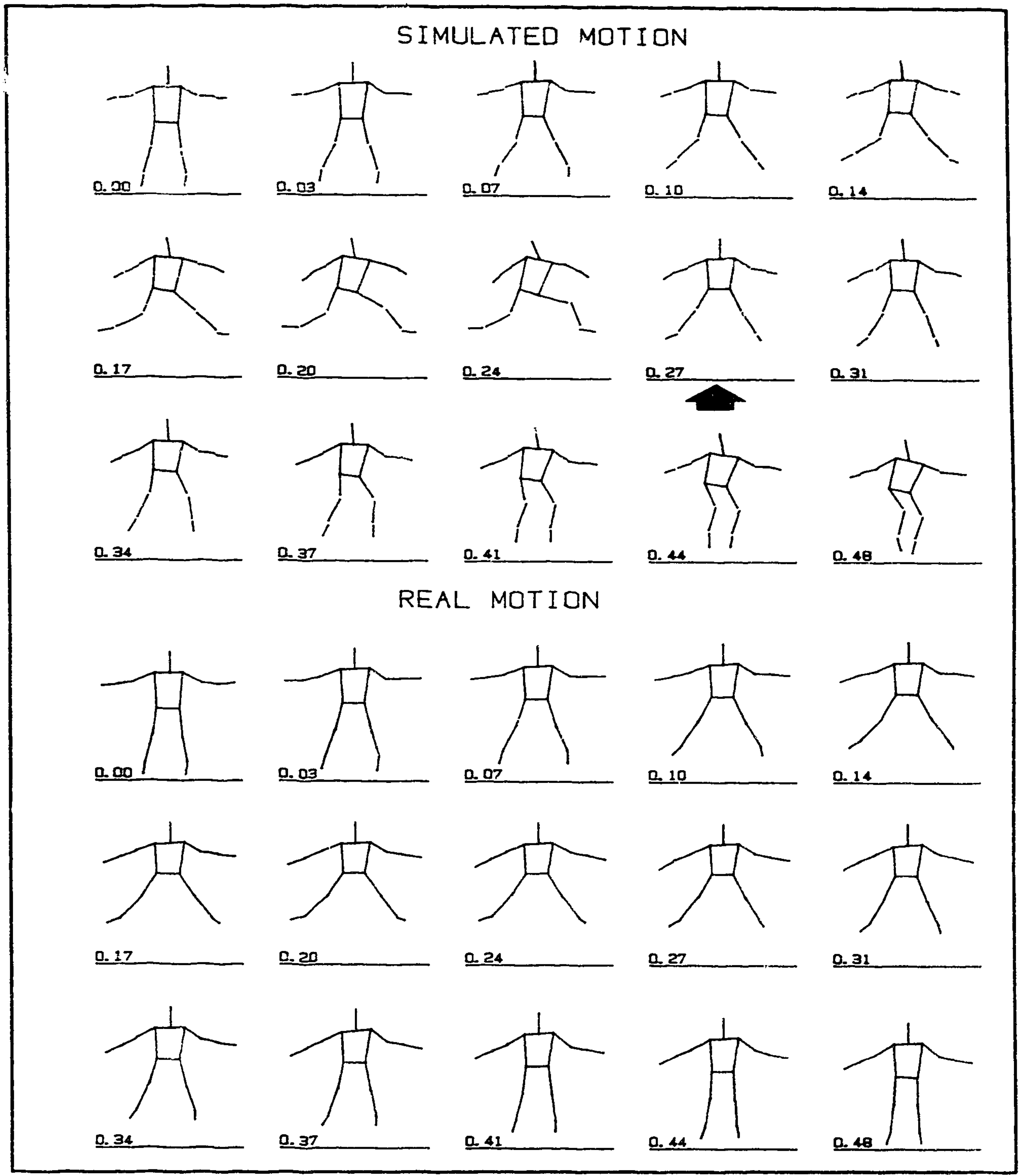
Split Jump Trial 1 - Manually digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



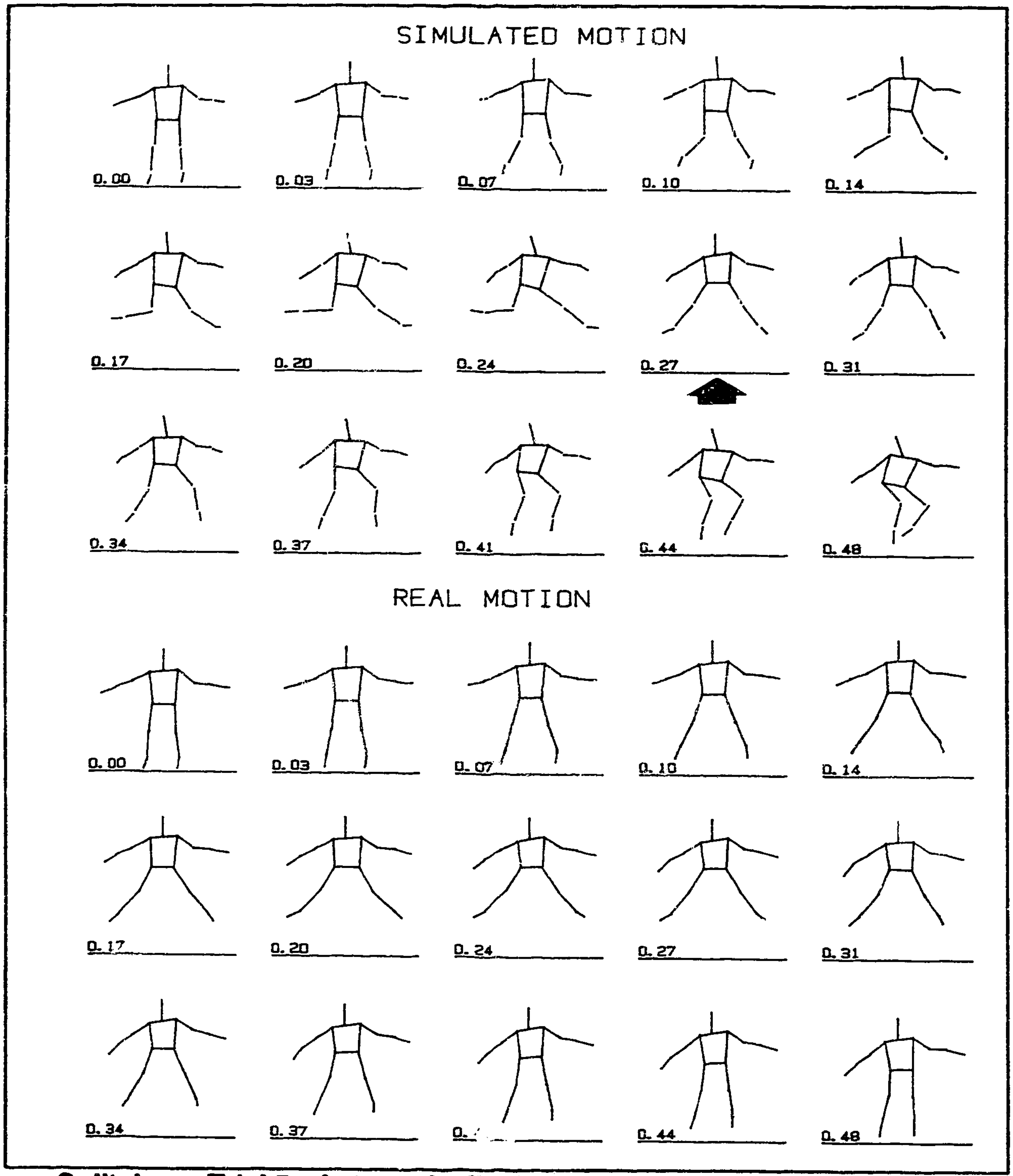
Split Jump Trial 2 - Manually digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



Split Jump Trial 3 - Automatically digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



Split Jump Trial 4 - Automatically digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.



Split Jump Trial 5 - Automatically digitized data and no constraints. Piecewise simulation with restarts indicated by the arrows.