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THE UNIVERSITY OF ALBERTA

IRRIGATION AND DRAINAGE

AS

INFLUENCED BY WEATHER:

A SIMULATED MODEL

by

DAVID WILLIAM SCOTT

C

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH  
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FACULTY OF GRADUATE STUDIES AND RESEARCH

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## Abstract

Due to the unpredictable nature of weather, crop growth, crop water requirements and drainage are variables of nature, over which man has no control. It is therefore desirable to know how these variables react to different weather patterns over a period of time sufficient to include most different combinations of weather. Average trends in irrigation and drainage can then be studied.

The primary objective of this investigation was to develop an accurate model of seasonal crop growth for the Lethbridge area by including weather and crop growing conditions. A digital computer was used to generate weather via the Monte Carlo sampling technique and to simulate crop growth and soil moisture during the growing season. The distribution of drainage and irrigation was then evaluated. The average rate of drainage occurrence per day and the average yield per drainage period were the parameters upon which this study was based.

The results indicated that the rate of increase in daily consumptive use greatly affected the occurrence of drainage while the daily rate of consumptive use did not show any significant effect upon drainage occurrence. Furthermore, the amount of drainage occurring on a particular day is determined mostly by the consumptive use rate. High water use results in low drainage while low water use produces high drainage rates. A set of probability tables is presented as a guide to the probable

dates of irrigation.

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## 1. Introduction

Irrigation has been practised primarily in arid and semi-arid regions of the world where natural rainfall is insufficient for good crop growth. In semi-arid regions, such as southern Alberta, irrigation water has been used mainly as a supplement to natural rainfall. Rainfall in this region is sufficient to support crop growth throughout the growing season. However, the summer months in which crop consumptive use is maximum are relatively dry. The main purpose, therefore, of irrigation is to provide a means of controlling the moisture level of the soil in order that optimum conditions for crop production are maintained. Both the quality and the quantity of the crop will increase, thereby decreasing the risk of crop damage or loss.

Drainage problems are sometimes a result of improper irrigation practices. Water is often applied at the irrigators' convenience or according to a fixed schedule which has little concern for the needs of the crop or the interrelationship between the soil and the crop. Soils, which have an impermeable layer close to the surface, often experience a rise in the water table following an excessive irrigation. Small temporary sloughs, either in the irrigated field itself or in neighbouring fields, and salt accumulation on the surface are the end results.

Drainage problems, however, are not exclusively attributable to improper irrigation practices. Often, as is the case in southern Alberta, an irrigation during the early

growth stages of the crop is followed by an untimely rainfall and then by a prolonged period of drought. Excess soil water during the early growth stages will damage the crop making it more susceptible to drought later on. Proper irrigation scheduling is therefore essential.

The two major factors, therefore, which limit crop production in southern Alberta, are: 1) the lack of sufficient rainfall during the months of peak consumptive use and, 2) an excess of irrigation water during the early crop growth stages when rainfall is maximum.

The purpose of this research is to evaluate which has the greater influence on irrigation and drainage; crop consumptive use or weather. Information regarding the occurrence and the amount of irrigation was available from the Irrigation Guide records. However, information regarding drainage and flooding were non-existent. Hence, it was decided to construct a model which would simulate the weather distribution and daily soil moisture content from April 1st to October 31st for a period of 200 years. Lethbridge was chosen as the area for this study because it represents the area of highest concentration of irrigation in southern Alberta and because daily weather data were readily available.

The objectives of this research are fourfold.

1. To obtain probability distributions of rainfall and potential evapotranspiration and to derive the conditional probabilities for rainy and non-rainy days for

the Lethbridge area. Weather records dating from 1922 to 1966 are available for use.

2. To simulate the soil-crop-water system throughout the entire growing season with the weather probabilities as the inputs to the model. Four major irrigated crops are used: Soft Wheat, Potatoes, Sugar Beets and Alfalfa.

3. To obtain from the simulation model probability distribution curves of irrigation lapse times for each irrigation and each crop.

4. To qualitatively analyse both irrigation and drainage as intermittent stochastic processes in terms of the average number of occurrences per day and the average yield per occurrence.

## 2. Review of Literature.

Many attempts to simulate the soil-plant-water system have been made in order to aid in the farm decision process. Some researchers (10,35,48,49,50) have developed models which aid in the selection of machinery for harvesting operations or for scheduling farm operations based on weather probabilities. Other models have been developed to aid in the decision of irrigation scheduling (1,9,14,20,30,39,40,41,44,47,59,60), and to simulate the plant response to environmental conditions (11,13). Still other models have been built to simulate the movement of water through the soil (6,34), or the response of a watershed to precipitation (45).

### 2.1 The Moisture Budget.

The relationship between the essential components of the plant-soil-water system can best be expressed mathematically by the following differential equation.

$$\frac{dQ}{dt} = I - O = (R_n + IRR) - (CU + Dr + Ro)$$

where: Q = amount of stored water in the soil at time t  
I = inflow into the soil medium  
O = outflow from the soil medium  
R<sub>n</sub> = precipitation  
IRR = irrigation water  
CU = crop consumptive use  
Dr = drainage from the root zone  
Ro = surface runoff  
t = time

The above soil moisture budget represents a simple accounting procedure which continually updates the soil moisture content in discrete intervals of time (dt might

represent minutes, hours, days, etc.). The method can be applied to the entire root zone or to distinct soil zones within the root zone. Robertson et al (46) applied this budgeting technique to predict the timing of irrigation on two plots of land. A black Bellani plate was used to determine the daily potential evapotranspiration rates. The amount of irrigation water required by the budgeting technique and that specified by the electrical resistance block was within one inch. The soil moisture budgeting technique has since been used in the majority of mathematical soil moisture models.

## 2.2 Evapotranspiration.

Various methods have been developed throughout the years to estimate, either theoretically or empirically, each of the individual terms of the moisture budget. Early researchers realized that one of the most important and most difficult variables to estimate was that of potential evapotranspiration. They realized that the evaporation of water from both the soil and the plant required energy and that this energy was a function of the immediate climatic parameters such as temperature and radiation. The methods of estimating evapotranspiration are classified as 1) mass transfer methods, 2) energy balance methods, 3) combination methods, and 4) empirical methods based on meteorological data. The first three methods involve a complicated theoretical approach to the energy balance between the heat transfer to and from the plant and its environment. Many of



the variables are extremely difficult to measure; however, the results are fairly accurate. The last method estimates evapotranspiration from readily available climatic data via empirically or experimentally derived mathematical expressions. Meteorological data such as radiation, temperature, humidity and wind speed are usually available for most areas and are the main parameters upon which the expressions are based. However, satisfactory results under all conditions necessarily may not be achieved. A few of the empirical methods are described in the following text.

In 1950, Blaney and Criddle, as cited by Gray (19), developed a simplified formula for estimating consumptive use in the arid western regions of the United States. It relates mean monthly temperature (T), monthly percent of annual daytime hours (p) and a monthly crop coefficient (k) to consumptive use (CU). Stated mathematically:

$$CU = \frac{kT_{mp}}{100} = kf$$

This method gives reliable monthly and seasonal estimates.

Penman, as cited by Hardee (20), combined the energy balance equation and experimentally derived aerodynamic equations to obtain an expression which included such weather variables as short wave and long wave radiation, wet and dry bulb psychrometric constants, mean wind speed, and saturation vapor pressure at both the mean air temperature, and at the dew point temperature. Jensen et al (30) proposed a formula for estimating potential

evapotranspiration by an approximate energy balance-aerodynamic equation which employed mean daily temperature and solar radiation. Actual evapotranspiration was obtained by multiplying potential evapotranspiration with a crop coefficient which reflected the effects of sensible and latent heat flux and net radiation. Linacre (36), in 1967, related evapotranspiration to radiation and temperature. Such variables as latent heat of vaporization, short and long wave radiation, water vapor pressure, specific heat of air, net flux of heat into the atmosphere, air density, saturation deficit and two crop resistant parameters were employed. The net flow of heat took into consideration the percentage of bright sunshine, and the temperatures for both cloudy and non-cloudy days. An attempt was made by Linacre to incorporate two crop resistant parameters which measured the ability of the plant to release water into the atmosphere. These parameters had to be experimentally determined and were unique to a specific crop and location.

Christiansen and Hargreaves, as cited by Hardee (20), produced a formula which involves several dimensionless coefficients, each of which expresses the effect of mean temperature, mean wind velocity, mean relative humidity, and elevation, respectively. Radiation and a crop coefficient were also included. The result, when all the coefficients were multiplied together, yielded potential evapotranspiration. If a weather variable was not available, the respective coefficient could be set to unity.

Eagleman, in 1971, (16) developed a third degree regression model which related the soil moisture ratio to the ratio of actual to potential evapotranspiration. The soil moisture ratio was defined as the ratio of the current soil moisture content to the total water capacity of the soil. Eagleman found the relationship to be curvalinear.

In 1965, Baier and Robertson (2) proposed a linear regression model which would estimate daily latent evaporation from simple meteorological observations and astronomical data readily available from tables. The versatility of this method was enhanced by the fact that any combination of up to six variables could be used. Estimates of potential evapotranspiration were obtained directly from the model by multiplying latent evaporation by a coefficient of 0.0034. This model will be discussed in more detail in a later section.

Holmes and Robertson (26,27) recognized that as the plant roots expanded and the soil moisture decreased, the rate of plant water use also decreased. Soil moisture drying curves, which adjusted the evapotranspiration rate in relation to the season and the soil moisture content, were derived experimentally from laboratory and field observations for various soils and crops. Holmes also recognized the fact that as the plant roots reached a certain soil depth, the actual evapotranspiration rate decreased sharply from the potential rate. From these two important concepts, the Modulated Soil Moisture Budget was

developed. The soil was divided into five zones, each with equal water holding capacities. The actual evapotranspiration was determined by the above mentioned soil moisture curves and the amount of water extracted from each zone was determined by a set of arbitrary coefficients. Kerr (32,33) had used the basic principles of the Modulated Budget in the development of a moisture budget which considered the effects of the crop height, soil and plant rooting characteristics on the rate of moisture use by crops.

Baier and Robertson (3) later developed a model called the Versatile Soil Moisture Budget which made use of the basic concepts of the modulated budget. In addition, the concept of atmospheric demand rates as a function of the AE/PE ratio and a matrix of crop coefficients which reflected the amount of water the root system could extract from each soil zone were instituted. The coefficients were varied for each soil zone and for each stage of growth of the crop throughout the season in order to attempt to simulate the probable water extraction pattern of the root system.

Other soil moisture models have attempted to simulate consumptive use in various ways. Weaver (56), in 1967, described the algorithm which Pierce had developed in 1966 to estimate soil moisture deficit under corn, meadow and wheat. Consumptive use was calculated by multiplying potential evapotranspiration together with several

correction factors which included day length, soil moisture dryness, rainfall and crop stage. Each correction factor in turn was determined by a non-linear regression equation.

Windsor and Chow (59,60) incorporated the relationship between crop potential evapotranspiration and turgor loss point in order to determine moisture stress days. Crop potential evapotranspiration was estimated from a Weather Bureau Class A evaporation pan and a dimensionless crop coefficient which accounted for the type of crop and stage of crop development. Soil dryness curves, similar to those used by Holmes, were used to convert potential crop evapotranspiration to actual crop evapotranspiration.

David (14) and Rasheed et al (44) developed regression models which related the day of the growing season to the rate of actual evapotranspiration. Rochester and Busch (47), in 1972, developed a scheduling model to improve the management of irrigation systems. Pan evaporation measurements were multiplied by a coefficient, which varied according to the day of the growing season, to determine daily actual evapotranspiration estimates. Richardson and Ritchie (45) developed empirical relationships to predict separately soil and plant evaporation from a watershed.

The problem with any soil moisture budgeting technique is, to properly estimate potential evapotranspiration and thus crop consumptive use. To date, only the Versatile Soil Moisture Budget contains crop, soil and water parameters to estimate crop water use. For this reason, the Versatile

Soil Moisture Budget was chosen as the model to simulate soil moisture conditions under several irrigated crops for this study.

Literature which deals with the relationship between weather and irrigation is scarce. Many models have been built to produce probability distributions of seasonal irrigation water requirements. Coligado et al (12) presented a risk analysis of irrigation requirements for each week of the growing season for numerous stations across Canada. The risks were computed for different combinations of total available soil moisture capacities and consumptive use factors. No analyses have been found by the author which attempts to depict the behaviour of drainage water in relation to irrigation and rainfall. Data concerning the amount and the time of occurrence of deep percolation under natural conditions over a period of several years is virtually non-existent.

Soils which have a moisture content in excess of field capacity have been reported by many researchers to take two to three days to reach equilibrium. It is generally accepted that deep percolation rates level off when field capacity has been reached. However, Wilcox (57) reported that drainage never ceases and that there is no leveling off point. Wilcox concluded that evapotranspiration, measured by common soil moisture depletion methods, contains some unknown quantity of deep drainage. Willardson and Pope (58) explained that unsaturated drainage is usually accounted for

in most moisture models by the evapotranspiration parameter.

Since very little is known about unsaturated drainage and the fact that any unsaturated drainage is probably accounted for by the consumptive use term, the use of the Versatile Soil Moisture Budget was further justified. The Budget assumes that no unsaturated drainage occurs between soil layers and that deep percolation is that amount of water in excess of field capacity.

### 2.3 Description of the Area.

Daily weather data for 45 years for six Alberta stations were available on magnetic tape at the Department of Agricultural Engineering, University of Alberta. Of these six stations, only two, Lethbridge and Medicine Hat, were located in the southern regions of the province. Since Lethbridge has the largest concentration of irrigation, it was chosen as the study area for this thesis. A general description of the area follows. The climatic information and soil description were taken from Hobbs (21) and Nielson (40) respectively.

#### 2.3.1 Location.

Lethbridge is located at north latitude  $49^{\circ}42'$  and west longitude  $112^{\circ}47'$ . It is situated 2,961 feet above sea level.

#### 2.3.2 Climate.

The climate of the Lethbridge area is extremely variable from month to month. Short, warm summers followed by long, cold winters are typical. Lethbridge lies within

the influence of the Chinook winds which tend to reduce the severity of the cold winter months and to alleviate the extreme summer heat. These winds, being relatively warm and dry, originate on the eastern slopes of the Rocky Mountains. During the winter months, the winds may displace cold air masses while during the summer months, they may effect cooler temperatures but cause high moisture stress and drought injury to crops.

Lethbridge has an average annual precipitation of 16.18 inches (1902-1969). Approximately 75 percent (12.43 inches) of the total occurs during the months of April to October and 32 percent occurs during the critical growing months of May and June when the crops are young and shallow rooted. June has the highest rainfall amount, averaging about 3.21 inches as shown in figure 1. These average values were calculated from the 45 years of daily weather data available on computer tape.

During the winter months, it is not unusual to have one foot or less of snow cover or no snow cover at all. Warm Chinook winds often raise the temperature sufficiently to remove any snow cover within several days. A midwinter rainfall is not uncommon.

### 2.3.3 Soils Description.

Most of the soils in the immediate Lethbridge area fall into the order of Chernozemic soils. They are characterized by a thick dark brown "A" horizon.

Chernozemic dark brown soils were formed under slightly more



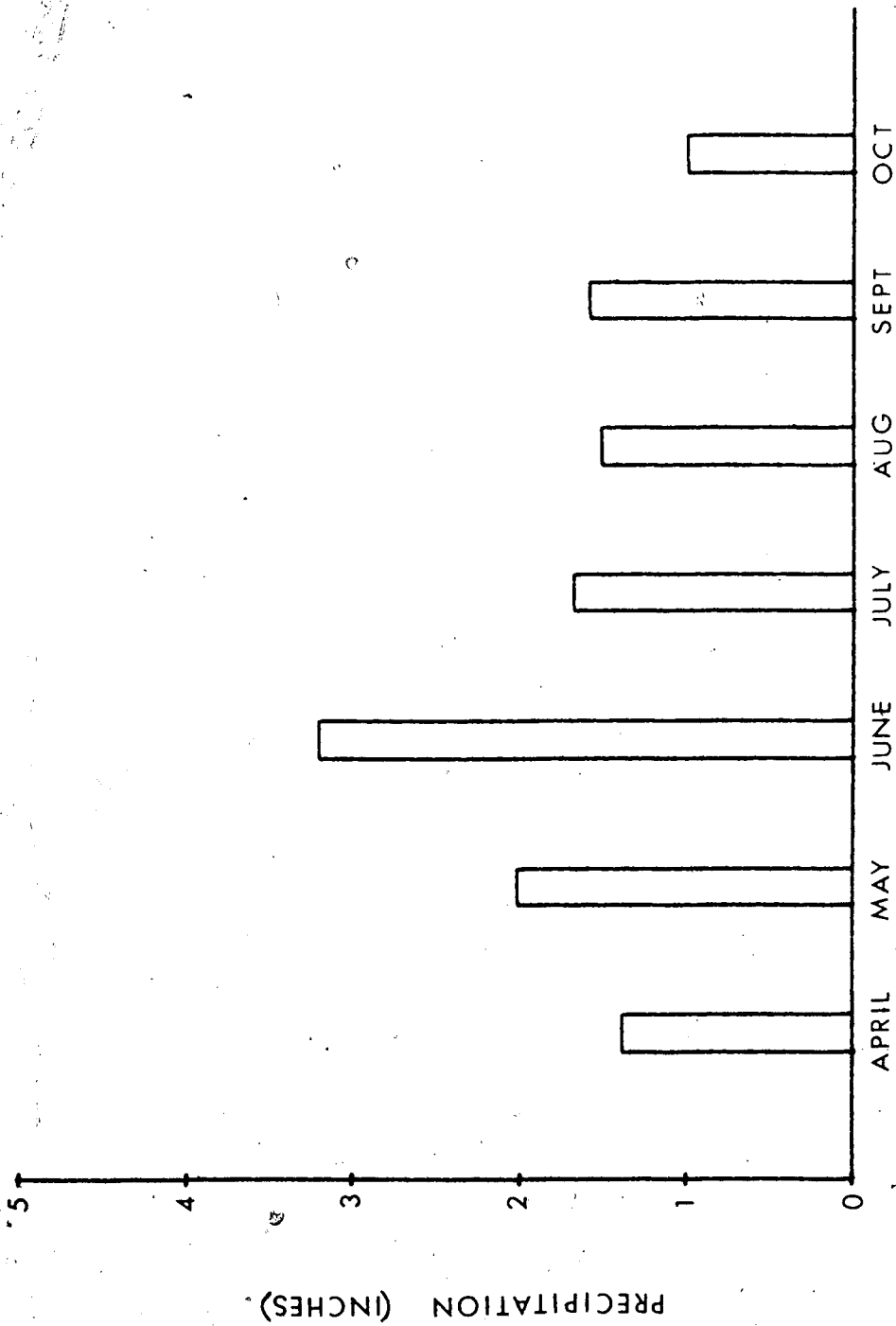


Figure 1. Average total monthly precipitation for Lethbridge.

humid semiarid conditions than the brown soils of the more eastern parts of southern Alberta. The upper layer is of a clay, silt and sand mixture called Glacio-Lacustrine deposits. The permeability of this layer varies considerably, but is generally moderately to rapidly permeable, affording good to very good irrigating conditions.

The lower layer is a glacial deposit called Till. It is massive and largely structureless. The thickness varies between 60 to 130 feet. Sand and gravel are present, but relatively rare. In some areas, the till forms the present land surface while in other areas it underlies the Lacustrine deposits. The depth at which the till is situated, where overburden is present, ranges between 2 feet and 40 feet with the average depth being 5 feet. Since the permeability of this layer is very low (0.2 iph or less), drainage problems are often a result of the existence of the till on irrigated lands. Table 1 presents a brief description of some of the more common soil types of the Lethbridge area.

#### 2.3.4 Drainage Studies.

Experiments by Rapp and van Schaik (43) in shallow glacial till soils, indicated that the irrigation amount and irrigation frequencies influenced the position of the water table considerably more so than did natural rainfall. The water table was observed to rise close to the surface after an irrigation, and the amount of rise was found to be

TABLE 1: A DESCRIPTION OF SOME SOUTHERN ALBERTA SOILS.  
(Bowser et al, 8)

|                    | Horizon | Depth<br>(ins) | H.C.<br>(iph) | Description                          |
|--------------------|---------|----------------|---------------|--------------------------------------|
| Chin Light<br>Loam | Ah      | 0 - 4          | 1.5           | brown loam                           |
|                    | Bj      | 4 -15 -        | 1.0           | brown-dark brown loam                |
|                    | Cca     | 15-26          | 0.7           | light brownish grey loam             |
|                    | Csk     | 26-48          | 0.7           | yellowish brown loam to<br>silt loam |
|                    | Till    | 48-            | 0.2           | glacial till                         |

Irrigability- good to very good. Glacial till averages  
4 feet from the surface.

Shallow Chin - horizon characteristics same as above  
- glacial till averages 2 feet from surface  
causing high water tables well within the  
root zone.  
- irrigability fairly good to good.

|                         |      |       |     |   |
|-------------------------|------|-------|-----|---|
| Cavendish<br>Loamy Sand | A    | 0 - 7 | 2.5 | brown fine sandy loam                       |
|                         | B    | 7 -24 | 1.5 | brown sandy loam                            |
|                         | Cc   | 24-40 | 2.5 | light yellowish brown sand<br>sand to sand  |
|                         | Ck   | 40-60 | 3.0 | light yellowish brown loamy<br>sand to sand |
|                         | Till | 60-   |     | glacial till                                |

glacial till averages 5 feet below the surface  
irrigability - good to very good

|            |     |       |     |  |
|------------|-----|-------|-----|--|
| Maleb Loam | Ah  | 0 - 4 | 1.0 | brown loam - loose                             |
|            | Bc  | 4 -12 | 0.3 | brown to dark brown heavy<br>loam to clay loam |
|            | Cea | 12-18 | 0.5 |  |
|            | Csk | 18-24 |     | clay loam till - blocky                        |
|            | C   | at 36 | 0.2 | granite, ironstone, coal                       |

irrigability good to very good if good topography  
exists.

dependant upon the amount of irrigation. The subsequent recession of the water table took three to four days and was considered to be primarily due to crop consumptive use. A duration of 3 to 4 days of high water table was found not to be injurious to shallow rooted crops; however, a considerable amount of dead roots were found on deep rooted crops.

Excessive irrigation was also observed to be a problem. It was estimated by Rapp that some fields were irrigated by as much as 2 to 3 inches of water in excess of field capacity. Because of the low hydraulic conductivity of the till, temporary potholes or sloughs could form causing eventual crop root damage and salinity problems. Sloughs reduce the productive acreage of the farm and increase the cost of operation.

Drainage problems, although not entirely due to irrigation mal-practice, can be alleviated by developing efficient irrigation methods.

### 3. The Consumptive Use Model.

Any soil moisture model which simulates soil moisture on a daily basis must employ a fairly sophisticated means of determining daily crop consumptive use. As stated previously, the method developed by Baier and Robertson (3) is the most refined mathematical model of consumptive use devised to date. A detailed description of the model follows.

#### 3.1 The New Versatile Soil Moisture Budget.

The Versatile Soil Moisture Budget is a method by which climatic, plant and plant-soil interrelationships are implemented to estimate crop consumptive use. The expression is as follows:

$$AE_i = \sum_{j=1}^n \left[ K_j \frac{S'_j(i-1)}{S_j} Z_j PE_i e^{-w(PE_i - \overline{PE})} \right] \quad (1)$$

where:  $AE_i$  = actual evapotranspiration on day i  
 $K_j$  = coefficient matrix accounting for the amount of water in percent of PE extracted by plant roots from different zones j during the growing season  
 $S'_j(i-1)$  = available soil moisture in the jth zone at the end of day i-1  
 $S_j$  = total available water capacity in the jth zone  
 $Z_j$  = adjustment factor for different types of soil dryness curves  
 $J$  = soil zone number  
 $PE_i$  = potential evapotranspiration for day i  
 $w$  = adjustment function accounting for the effects of varying PE rates on the AE:PE ratio  
 $\overline{PE}$  = long term average daily PE value for the month or season

The crop coefficients,  $K_j$ , describe the percent of PE which is removed from each soil zone. In essence,  $K_j$  is a

matrix of consumptive use factors: the columns represent the various stages of growth on a time scale and the rows represent the individual soil moisture zones. Hence, in this manner, a particular  $K_j$  coefficient may only apply to one soil moisture zone over a period of time defined by the length of the current stage of growth. The  $K_j$  coefficients must be determined by iterative comparisons between computed and observed soil moistures. Alternatively, they may be estimated so as to represent the most probable soil moisture pattern under prevailing environmental conditions. A third alternative, provided experimentally determined average consumptive use curves are available for different crops, is to compute on a short term basis (i.e. 5 to 10 day intervals), daily consumptive use values averaged over a period of several years of simulated crop growth. Iterative comparisons between the experimental and simulated curves may then be performed. Although more expensive, the latter method will provide accurate results on a long term basis. The  $K$  coefficients for this study were determined using both the first and the latter techniques.

The term  $S_j^{(i-1)}/S_j$  describes the ratio of the current available soil moisture to the total available soil moisture capacity in zone  $j$ . This ratio is used in conjunction with the  $Z$  term which is a vector of 100 coefficients corresponding to the value of the moisture ratio. The product  $S_j^{(i-1)}/S_j * Z_j$  represents the amount of water, expressed as a percentage of PE, extracted from

zone j according to the current moisture content of that zone. Various proposals for the relationship between the AE/PE ratio and the soil moisture content are presented in figure 2. Each curve (A through H) has associated with it a Z-vector similar to the A and H vectors presented in table 2. Baier (4) concluded from a comparison of observed soil moisture with estimates obtained from the Versatile Budget using five types of relationships that the type G curve would yield the best results for grass grown in Matilda loam soil. He further recommended that this curve be used as a "first approximation in most medium textured, non-irrigated soil" (5, pp 10). Baier also encouraged the use of the type A curve for sandy soils as well as "for soils under irrigation when a moisture content close to field capacity is maintained throughout the growing season" (5, pp 9). The type H curve, which is a compromise between the A and G curves, was chosen for use in the model. The Z-vectors for the A and the H curves are presented in table 2.

The exponential term of the Versatile Budget accounts for the varying daily atmospheric demand rates. The W term is a regression equation developed by Baier et al (3) and is described below.

$$W = 7.91 - 0.11 \frac{S'_j(1-1)}{S_j} 100 \quad (2)$$

This value is dependent on the soil moisture ratio of each soil zone.

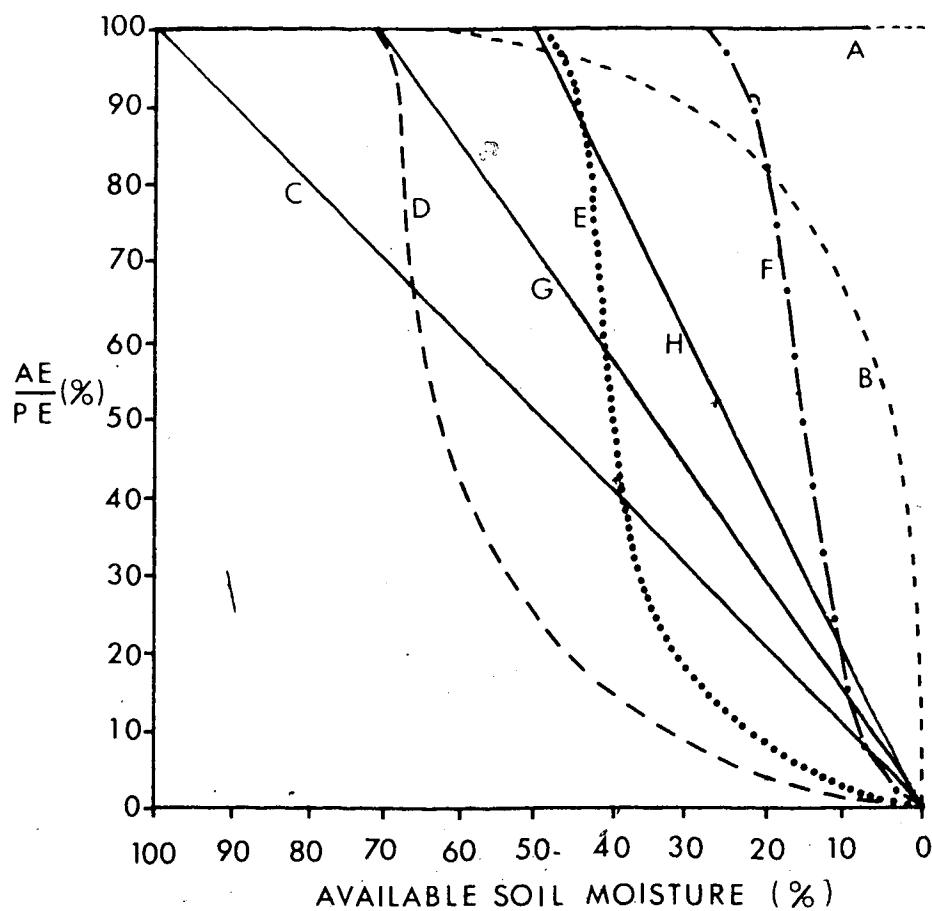


Figure 2. Various proposals for the relationship between the AE:PE ratio and the current available soil moisture. (Baier et al, 5)



TABLE 2. Z - TABLES SOIL DRYNESS CURVES A AND H.

A TABLE

| 99.99 | 50.00 | 33.00 | 25.00 | 20.00 | 16.66 | 14.28 | 12.50 | 11.11 | 10.00 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 9.09  | 8.33  | 7.69  | 7.14  | 6.67  | 6.25  | 5.88  | 5.56  | 5.26  | 5.00  |
| 4.76  | 4.55  | 4.35  | 4.17  | 4.00  | 3.85  | 3.70  | 3.57  | 3.45  | 3.33  |
| 3.23  | 3.13  | 3.30  | 2.94  | 2.86  | 2.78  | 2.70  | 2.63  | 2.56  | 2.50  |
| 2.44  | 2.38  | 2.33  | 2.27  | 2.22  | 2.17  | 2.13  | 2.08  | 2.04  | 2.00  |
| 1.96  | 1.92  | 1.89  | 1.82  | 1.85  | 1.82  | 1.79  | 1.75  | 1.72  | 1.69  |
| 1.96  | 1.92  | 1.89  | 1.85  | 1.82  | 1.79  | 1.75  | 1.72  | 1.69  | 1.67  |
| 1.64  | 1.61  | 1.59  | 1.56  | 1.54  | 1.52  | 1.49  | 1.47  | 1.45  | 1.43  |
| 1.41  | 1.39  | 1.37  | 1.35  | 1.33  | 1.32  | 1.30  | 1.28  | 1.27  | 1.25  |
| 1.23  | 1.22  | 1.20  | 1.19  | 1.18  | 1.16  | 1.15  | 1.14  | 1.12  | 1.11  |
| 1.10  | 1.09  | 1.08  | 1.06  | 1.05  | 1.04  | 1.03  | 1.02  | 1.01  | 1.00  |

H TABLE

| 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |
|------|------|------|------|------|------|------|------|------|------|
| 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| 1.96 | 1.92 | 1.88 | 1.85 | 1.81 | 1.78 | 1.75 | 1.72 | 1.69 | 1.67 |
| 1.64 | 1.61 | 1.59 | 1.56 | 1.53 | 1.52 | 1.49 | 1.47 | 1.45 | 1.43 |
| 1.40 | 1.38 | 1.35 | 1.34 | 1.33 | 1.31 | 1.29 | 1.28 | 1.26 | 1.25 |
| 1.23 | 1.21 | 1.19 | 1.18 | 1.17 | 1.15 | 1.14 | 1.13 | 1.12 | 1.11 |
| 1.10 | 1.08 | 1.07 | 1.06 | 1.05 | 1.04 | 1.03 | 1.02 | 1.01 | 1.00 |

### 3.2 Potential Evapotranspiration.

The value of PE in equation 1 may be determined by either the Bellani Plate Atmometer, Penman's equation, or by a regression equation developed by Baier and Robertson (2). The latter method involves the estimation of daily latent evaporation from a combination of simple meteorological observations and astronomical data readily available from tables. Three to six terms were employed in a series of eight equations. As the number of terms included in the equation increased from three to six the multiple correlation coefficients increased from 0.68 to 0.84. The expression using all six terms is described below.

$$\begin{aligned} EL = & -53.39 + 0.337 \text{ MAX} + 0.531 (\text{MAX}-\text{MIN}) + 0.017 Q_0 \\ & + 0.0512 Q_s + 0.977 \text{ WIND} + 1.77 (\text{E}_w-\text{E}_s) \end{aligned} \quad (3)$$

where: EL = latent evaporation  
 MAX = maximum daily temperature  
 MIN = minimum daily temperature  
 Q<sub>0</sub> = solar radiation received at the top of the atmosphere  
 Q<sub>s</sub> = solar radiation received on a horizontal surface  
 WIND = total daily wind mileage  
 E<sub>w</sub> = saturation vapor pressure at mean air temperature  
 E<sub>s</sub> = saturation vapor pressure at mean dew point

The value of Q<sub>s</sub> may be determined from the expression:

$$Q_s = Q_0 \left\{ 0.251 + 0.616 \frac{n}{N} \right\} \quad (4)$$

where: n = daily hours of bright sunshine  
 N = total hours between sunrise and sunset  
 Q<sub>0</sub> and Q<sub>s</sub> are as above.

Because 33 of the 45 years of weather records available for the Lethbridge area contained measurements of

only daily temperatures and precipitation, it was decided to use the equation containing only four terms as described below.

$$EL = -108.8 + 1.13 \text{ MAX} + 0.920 (\text{MAX-MIN}) + 0.359 Q_0 + 0.131 \text{ WIND} \quad (5)$$

Potential evapotranspiration is obtained by multiplying EL by 0.0034.

Because the regression equations were developed from daily weather data recorded across Canada over several years, reasonable estimates of latent evaporation for most parts of Canada can be expected with the use of this equation.

### 3.3 The Soil Moisture Zones.

Baier et al (5) adopted six standard soil moisture zones which contained respectively 5.0, 7.5, 12.5, 25.0, 25.0, 25.0 percent of the total available moisture in the root zone. The adoption of the six zones made it possible to describe the plant water extraction characteristics in any soil type regardless of the depth at which the moisture was located. Several assumptions were made with the use of these soil moisture zones.

1. The soil zone receives water in successive order from top to bottom in a step-wise fashion. If the amount of water entering the first zone is greater than the capacity of that zone, the remaining water enters the next zone. If it is less than the capacity of the zone, the water will remain in that zone and no drainage will occur into the next

zone.

2. Because of the above assumption, water is assumed to infiltrate into the soil zones instantaneously.

3. Drainage is assumed to be that amount of water above the total soil moisture deficit of all six zones. This amount is assumed to leave the soil zone as deep percolation on the same day that the water was applied.

### 3.4 Runoff

In order to incorporate runoff into the Versatile Budget, Baier and Robertson implemented a simple relationship between soil moisture in the top zone and daily precipitation.

$$\text{RUNOFF} = \text{RR}_i - I \quad (6)$$

$$I = 0.9177 + 1.811 \ln \text{RR}_i - 0.00973 \ln \text{RR}_i \frac{S'_1(1-1)}{S_1} 100 \quad (7)$$

where:  $\text{RR}_i$  = the rainfall for a 24 hour period ending in the morning of day  $(i+1)$ .

$I$  = amount of infiltration into the soil

$$\frac{S'_1(1-1)}{S_1} = \text{the available soil moisture in percent of capacity of } (S_1) \text{ in the top zone at the end of day } (i-1).$$

Runoff is assumed to occur if the total daily rainfall exceeds 1.00 inch. The topography is assumed to be level.

In general, irrigation sprinkler nozzles used in southern Alberta discharge water at a rate of 0.5 inches per hour. The majority of soils in the Lethbridge area possess hydraulic conductivities above that of the nozzle discharge. A list of the various types of soils and their respective

hydraulic conductivities are presented in table 1. It was therefore assumed that runoff from sprinkler irrigation was negligible and any runoff that did occur was due solely to precipitation exceeding 1.00 inch per day.

#### 4. Selection of the Proper K-Coefficients.

In order for the Versatile Budget to effectively simulate the moisture withdrawal from each soil zone, the K-coefficients had to be selected so as to represent the most probable soil moisture extraction pattern for the four crops under study. The K-coefficients were obtained by iterative comparisons between actual and estimated soil moisture. The procedure followed is described below.

##### 4.1 Experimental Soil Moisture Data.

Before iterative comparisons could be made, experimental field measurements of soil moisture had to be obtained. Field data was necessary in order that comparisons between the daily soil moisture contents of different crops, as simulated by the Versatile Budget, could be made against actual values as measured in the field.

Hobbs and Krogman (24) had carried out experiments at Vauxhall on the consumptive use rates of 12 irrigated crops, each grown in 15 foot square plots of land. Vauxhall lies approximately 30 miles east of Lethbridge. When the soil moisture content of each plot had depleted to approximately 50 percent of the total soil moisture capacity, the plots were irrigated. The soil moisture content was determined prior to an irrigation and the amount of water applied was just sufficient to bring the soil moisture to field capacity. It was assumed that deep percolation was negligible. From the soil moisture content readings and the total irrigation and rainfall water applied to each plot, a

reasonable estimate of the rate of consumptive use between irrigations was obtained.

The soil moisture readings, the total available soil moisture, and the irrigation dates and amounts for the years 1960 to 1963 were obtained from Hobbs (22) for Soft Wheat, Potatoes, Sugar Beets and Alfalfa. This data was then used to estimate the K-coefficients.

#### 4.2 Weather Data.

The Versatile Budget requires that potential evapotranspiration be estimated from daily maximum and minimum temperatures, solar radiation and wind velocity. The daily temperatures and precipitation for the Vauxhall area were obtained from the "Monthly Records of Meteorological Observations in Canada" (38). Solar radiation received at the top of the atmosphere was obtained from Smithsonian Tables (37) and the monthly average wind velocities were gathered from table 7 of Rutledge (48). Ten years of daily wind velocities (1956 - 1966) were taken from the computer tape containing the daily weather data and averaged on a monthly basis. Equation 5 was then used to calculate daily potential evapotranspiration from April 1st to October 31st for the years 1960 to 1963.

The long term average PE value in the exponential term of the Versatile Budget was taken from the monthly averages for Lethbridge as determined by Rutledge in table 4 (48). Equation 3 was used by Rutledge to determine daily PE values. According to the values, Medicine Hat and

Lethbridge showed very little difference in their monthly PE values. Hence, since Vauxhall lies approximately between the two stations, it was felt that the conditions at Lethbridge would be sufficiently close to conditions at Vauxhall. This procedure of selecting long term averages of PE values had to be done since daily weather data for the Vauxhall station was not readily available on computer tape. Furthermore, the purpose of performing the iterative comparison between actual and simulated data was to obtain only approximate K-coefficients for each crop. Later, the K-coefficients would be readjusted, using accurate average PE values for Lethbridge, to fit average consumptive use curves for all of southern Alberta. Hence, the accuracy of the PE term in the Versatile Budget is only minor at this point.

#### 4.3 The Z-Table.

The data obtained from Hobbs indicated that the daily rate of consumptive use was quite high. This suggested that either the type H or type A curves of figure 2 would be suitable for simulating the soil-water relationships. Both curves stipulate that AE equals PE for soil moisture contents above 50 percent. Having no other basis for selection, the type H curve was chosen. This curve is represented by the H table in table 2.

#### 4.4 Method.

The K-coefficients for each crop were determined by iterative comparisons between actual soil moisture contents



and the Versatile Budget estimated soil moisture contents prior to each irrigation. Figure 3 shows an example of the output from the simulation and the corresponding experimental values as obtained from Hobbs (22).

The ending dates of the stages of growth, as represented by each row of the K-coefficient matrix, were determined in accordance with the consumptive use curves derived by Hobbs et al (24). The coefficients used for the periods prior to planting were those suggested by Baier et al (5) for fallow. They are 0.60, 0.15, 0.05, 0.00, 0.00, 0.00. The coefficients used for the period subsequent to harvest for Wheat and Alfalfa were those recommended for sod (0.50, 0.20, 0.15, 0.10, 0.03, 0.02). The coefficients recommended for fallow were employed for Potatoes and Sugar Beets.

| Year | Month and Day | Crop Growth Stage | Daily Rainfall | Daily PE | Daily Consumptive Use | Soil Moisture Zones |      |      |      |      |      | Total Soil Moisture Content (simulated) | Drainage | Total Soil Moisture Content (experimental) |
|------|---------------|-------------------|----------------|----------|-----------------------|---------------------|------|------|------|------|------|---|----------|--|
|      |               |                   |                |          |                       | 1                   | 2    | 3    | 4    | 5    | 6    |   |          |  |
| 60   | 505           | 1                 | 1.00           | 0.05     | 0.02                  | 0.21                | 0.31 | 0.53 | 1.05 | 1.05 | 1.05 | 4.20                                    | 0.98     | 4.20                                       |
| 60   | 704           | 3                 | 0.0            | 0.17     | 0.09                  | 0.01                | 0.01 | 0.03 | 0.27 | 0.41 | 0.56 | 1.39                                    | 0.0      | 1.70                                       |
| 60   | 712           | 4                 | 0.0            | 0.23     | 0.22                  | 0.00                | 0.02 | 0.15 | 0.73 | 0.37 | 0.45 | 1.70                                    | 0.0      | 2.60                                       |
| 60   | 718           | 4                 | 0.0            | 0.52     | 0.10                  | 0.00                | 0.01 | 0.04 | 0.51 | 0.21 | 0.21 | 0.76                                    | 0.0      | 1.00                                       |
| 60   | 727           | 4                 | 0.0            | 0.28     | 0.27                  | 0.00                | 0.02 | 0.12 | 0.62 | 0.74 | 0.73 | 2.23                                    | 0.0      | 2.70                                       |
| 60   | 808           | 5                 | 0.0            | 0.18     | 0.15                  | 0.03                | 0.10 | 0.03 | 0.20 | 0.32 | 0.37 | 1.05                                    | 0.0      | 1.20                                       |
| 60   | 817           | 5                 | 0.0            | 0.13     | 0.16                  | 0.00                | 0.0  | 0.10 | 0.59 | 0.41 | 0.88 | 1.55                                    | 0.0      | 2.20                                       |
| 60   | 831           | 5                 | 0.0            | 0.21     | 0.22                  | 0.03                | 0.01 | 0.27 | 0.84 | 0.91 | 0.98 | 3.02                                    | 0.0      | 3.10                                       |
| 60   | 906           | 6                 | 0.0            | 0.12     | 0.17                  | 0.01                | 0.01 | 0.09 | 0.41 | 0.46 | 0.60 | 1.57                                    | 0.0      | 1.20                                       |
| 60   | 911           | 6                 | 0.0            | 0.23     | 0.35                  | 0.03                | 0.03 | 0.24 | 0.82 | 0.87 | 0.93 | 2.99                                    | 0.0      | 3.40                                       |
| 60   | 921           | 7                 | 0.0            | 0.10     | 0.08                  | 0.04                | 0.02 | 0.07 | 0.08 | 0.35 | 0.49 | 1.05                                    | 0.0      | 1.60                                       |
| 60   | 1005          | 1                 | 0.0            | 0.19     | 0.01                  | 0.02                | 0.06 | 0.17 | 0.61 | 0.62 | 0.54 | 1.84                                    | 0.0      | 2.70                                       |

Figure 3. A sample output, of the Versatile Budget simulation for Sugar Beets during 1960. ( Note: all units are in inches. )

## 5. The Weather Model.

### 5.1 Monte Carlo Sampling.

The Monte Carlo sampling technique is a method by which a sample of an independent variable can be synthetically generated, in a sequential fashion, with a given frequency distribution. This involves transforming a random independent number from a uniform probability distribution and, by use of the graphical method, producing a sample from the desired frequency distribution. A number between, but not including, 0.0 and 1.0 is generated by a random number generator and is applied to the cumulative distribution to obtain a sample of the independent random variable.

The major advantage of sequential generation is the ability to create a synthetic record longer than existing historical records. In this way, most of the possible combinations of the variable sequences will be included in the synthetic sample depending on the length of generation. In the present study, the behavior of the plant-soil-water relationships under most weather conditions will be simulated. The amount and frequency of occurrence of both irrigation and drainage will reflect the soil-crop-water behavior under varying weather conditions.

### 5.2 Weather Distributions

Weather includes such variables as rainfall, temperature, wind, etc. It is common knowledge that such variables fluctuate randomly from day to day or from hour to

hour and also that these variables are a function of the time of day, month or year. For instance, temperature is maximum during the summer months and minimum during the winter months, but the maximum and minimum temperatures, on a daily basis, are random. Such a phenomena is known as a Stochastic process and the values it assumes over time are known as a time series. Daily monthly and annual values of rainfall, for example, form a discrete time series. Each random variable of a time series has associated with it a certain probability distribution at any particular point in time. If the distribution remains constant throughout the process, the variable is said to be stationary. Otherwise, it is non-stationary. Most hydrologic processes are non-stationary over long time periods. They are treated, therefore, as stationary processes over short time periods.

Three variables are necessary to generate weather on a daily basis. They are wet and dry day sequences, daily rainfall and daily potential evapotranspiration. A computer program was written in FORTRAN to read in daily precipitation amounts and maximum and minimum temperatures for the Lethbridge station from the computer tape containing the daily weather data. The temperatures were used to calculate potential evapotranspiration (PE) according to equation 5. The date, precipitation and PE values were then printed onto a second tape from which subsequent work was to be performed.

### 5.3 Wet and Dry Day Probabilities.

Weather is composed of a series of wet days followed by a series of dry days. Hopkins and Robillard (28) performed a statistical analysis of daily rainfall occurrence for three areas in the Prairie Provinces. They found events on successive days to be statistically dependant and that a first-order transitional probability model would serve to approximate the occurrence of dry days. However, the model did underestimate slightly the total number of rainy days in the month. Feyerherm and Dean Bark (18) stated that where interest lies in computing probabilities for relatively short sequences of wet and dry days, the first-order Markov chain appeared to be quite adequate. In an earlier paper, Feyerherm and Dean Bark (17) had presented the first order Markov chain for wet and dry sequences in mathematical form as described below.

$$P(x_t, x_{t+1}, x_{t+2}, \dots, x_{t+n}) = P(x_t) P(x_{t+1}|x_t) P(x_{t+2}|x_{t+1}) \\ P(x_{t+3}|x_{t+2}) \dots P(x_{t+n}|x_{t+n-1}) \quad (8)$$

where:  $x_t$  = the event that day  $t$  is wet (W) or dry (D)

and

$$P(D_t) = \frac{\text{No. of years the (t) day is dry}}{\text{No. of years of records}}$$

$$P(D_{t+n}|W_{t+n-1}) = \frac{\text{No. of years (t+n) day is dry and (t+n-1) day is wet}}{\text{No. of years t+n-1 day is wet}}$$

Each probability in the expression is dependant on the events of the previous day. Because simulation by the first

order Markov chain is on a daily basis, the conditional probabilities of a wet day preceded by a dry day and a wet day preceded by a wet day need only to be determined. Jones et al (31) used the Markov chain principle to calculate a series of conditional probabilities for each week of the year. They assumed that the probabilities remained constant over a seven day period. A polynomial equation was then fitted to the probabilities and a reasonably good fit was obtained. The two polynomial curves showed that the conditional probabilities followed definite seasonal trends. Hence, the method used by Jones was applied to the Lethbridge data to determine if a similar seasonal trend existed in the data.

Daily rainfall records spanning a period of 45 years (1922 to 1966) were used to calculate the rainfall model parameters. The data for Lethbridge and five other Alberta stations were available on magnetic tape. The conditional probabilities for rainfall were calculated as follows:

$$P(W|D)_i = \frac{\sum \text{wet day following a dry day (i)}}{\text{total days following a dry day (i)}} \quad (9)$$

$$P(W|W)_i = \frac{\sum \text{wet days following a wet day (i)}}{\text{total days following a wet day (i)}} \quad (10)$$

$P(W|D)_i$  represents the probability that any day during the  $i$ th period was wet given that the preceding day was dry.

$P(W|W)_i$  is the probability that any day during the  $i$ th period was wet given that the preceding day was wet. Both

$P(W|D)_i$  and  $P(W|W)_i$  were calculated for each 5-day period

from April 1st to October 31st making a total of 43 time periods in all. It was assumed for the purposes of this study that the probabilities did not change considerably over any 5-day period.

A further assumption was made regarding the definition of a wet day. If the amount of rainfall received was equal to or greater than 0.01 inch, the day was considered to be wet. A base level of 0.01 inch was used because of the fact that the top soil zone of the Versatile Budget has the capacity of holding only 5% of the total soil moisture. This value can be small. Hence, a rainfall of 0.01 inch will influence the moisture content of the top soil zone sufficient to warrant the use of this amount as the basis for a wet day. Furthermore, it could not be assumed that daily consumptive use never reached values of zero inches during the spring and fall months. Therefore, 0.01 inches could affect the top soil zone on days experiencing zero inches of consumptive use. As well, days on which "traces" were recorded were designated as dry days.

In order to determine if the probabilities followed a seasonal trend, the probabilities were plotted against their corresponding period number and a 6th degree polynomial equation was fitted to both the  $P(W|D)$  and  $P(W|W)$  data. An F-test was performed on both plots to test the equations for significance. It was found that both polynomials were significantly different at the 95% level of probability. Figure 4 shows the actual values plotted against the

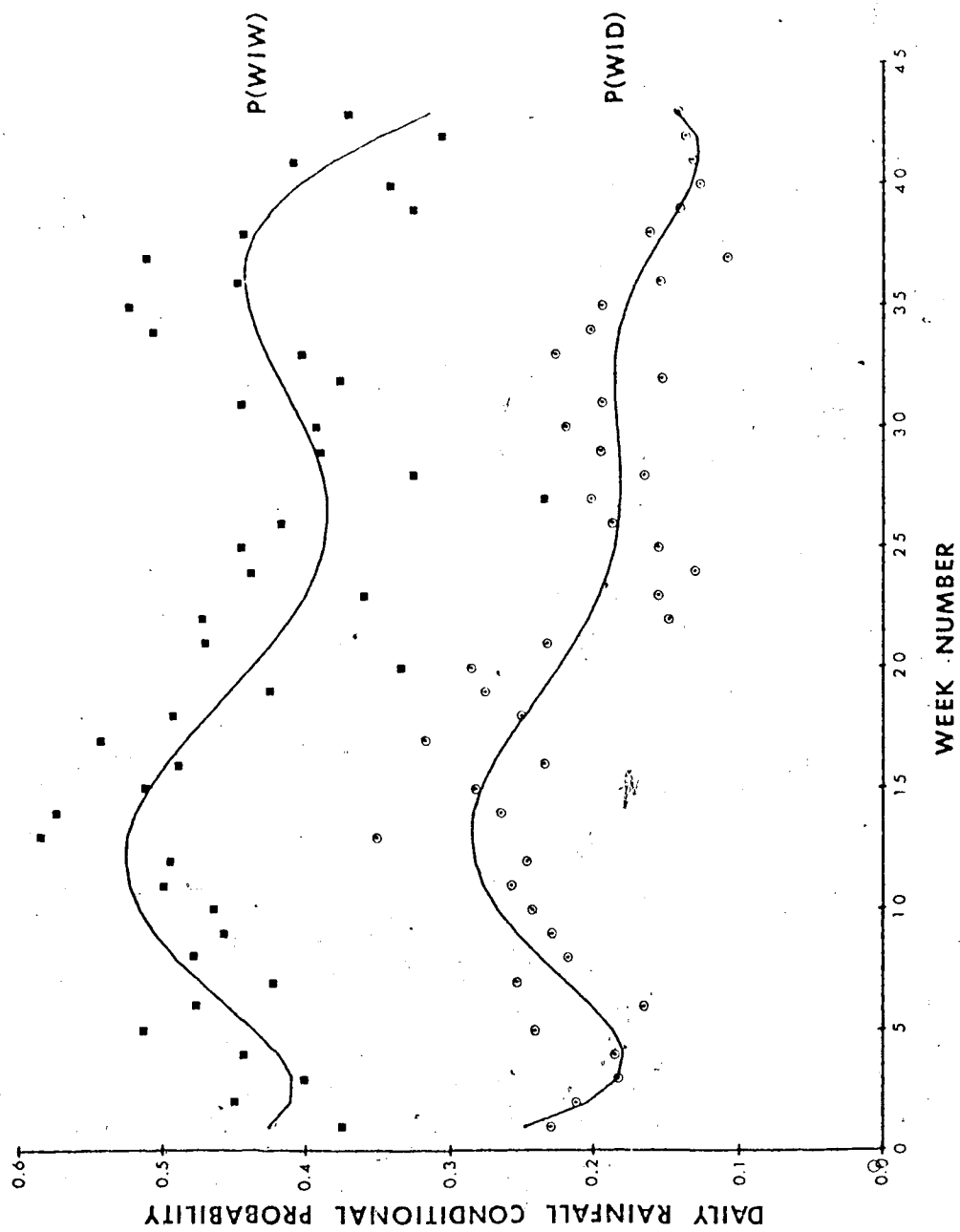


Figure 4. Comparison of actual and predicted values of daily rainfall conditional probabilities for days following a dry day and days following a wet day.



predicted values using the 6th order polynomial equations.

The equations are:

$$P(W|D) = 0.32542 - (9.6446 \times 10^{-2})X + (2.1051 \times 10^{-2})X^2 \\ - (1.77 \times 10^{-3})X^3 + (7.0055 \times 10^{-5})X^4 - (1.3067 \times 10^{-6})X^5 \\ + (9.3216 \times 10^{-9})X^6 \quad (11)$$

$$P(W|W) = 0.46017 - (4.8552 \times 10^{-2})X + (1.3869 \times 10^{-2})X^2 \\ - (1.2516 \times 10^{-3})X^3 + (4.878 \times 10^{-5})X^4 - (8.5935 \times 10^{-7})X^5 \\ + (5.5955 \times 10^{-9})X^6 \quad (12)$$

where X represents the 5-day period number.

The coefficients of determination were 0.67 and 0.45 for equations 11 and 12 respectively. Figure 4 indicates that both  $P(W|D)$  and  $P(W|W)$  have definite seasonal trends. Also indicated is the fact that there is a strong tendency, especially in the latter half of the growing season, for a dry day to follow a dry day as suggested by the relatively low values of  $P(W|D)$ . Furthermore, the values of  $P(W|W)$ , as the season progresses, decrease thereby increasing the probability of dry days to occur. This partly shows why the average monthly precipitation from July to September, as illustrated in figure 1, is less than May and June. The sixth order polynomial equations were used to determine wet and dry day sequences in the Monte Carlo model.

#### 5.4 The Rainfall Model.

The next step involved in the simulation of daily rainfall is to select an appropriate distribution function which will characterize precipitation on a daily basis. Some investigators (7,14,15,20,52,53,61) have suggested that rainfall can be characterized by the gamma function. The

cumulative gamma distribution function is given by the following expression.

$$F(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^x e^{-x/\beta} x^{\alpha-1} dx \quad (13)$$

where:  $F(x)$  = cumulative distribution function  
 $x$  = precipitation amount in inches  
 $\beta$  = shape parameter dependant on the variability of rainfall amounts  
 $\alpha$  = scale parameter dependant on the magnitude of the rainfall amounts  
 $\Gamma(\alpha)$  = complete gamma function

Thom (53) used the concept of mixed distributions to illustrate the use of the inverse gamma distribution tables. It was realized by Thom that the nonoccurrence of precipitation was caused by a set of meteorological variables different from those causing a measurable amount of precipitation. Therefore, the distribution must be broken up into two parts as described below.

$$G(x) = (1 - p) + pF(x) \quad (14)$$

where:  $G(x)$  = the precipitation distribution  
 $F(x)$  = the precipitation distribution of measurable amounts (as described above)  
 $p$  = the probability of occurrence of a measurable amount of precipitation

Equation 14 considers both the probability of a day being wet or dry as well as the probability of receiving  $x$  inches should it be a wet day. The parameters,  $\alpha$  and  $\beta$ , were determined by the maximum likelihood method, equations 15 and 16, which follow.

$$\alpha = \frac{1 + \sqrt{1 + 4/3A}}{4A} - \Delta e \quad (15)$$

$$\beta = \frac{\bar{x}}{\alpha} \quad (16)$$

where:  $\alpha$  and  $\beta$  are the gamma parameters

$$A = \ln \bar{x} - \frac{1}{N} \sum_{i=1}^n \ln x_i$$

$\Delta e$  = correction factors given in table 82 of Yevjevich (62).

$\bar{x}$  = average rainfall within a given time interval

$N$  = number of days of rainfall

$x_i$  = amount of rainfall for day  $i$

From the weather records available on magnetic tape, a computer program was written in FORTRAN to calculate the  $\alpha$  and  $\beta$  parameters for days following a wet day and for days following a dry day. Since the cumulative distribution can not be easily calculated from equation 13, an expansion equation, as given by Thom (53), was used. The equation is as follows.

$$F(t; \alpha) = \frac{t^\alpha}{\Gamma(\alpha + 1)e^t} \left[ 1 + \frac{t}{\alpha+1} + \frac{t^2}{(\alpha+1)(\alpha+2)} + \dots \right] \quad (17)$$

where:  $F(t; \alpha)$  = gamma distribution function

$t$  =  $X/\alpha$

$X$  = precipitation (inches)

$\alpha$  = scale parameter

The parameters were calculated over 15 and 16 day intervals, depending on whether the month had 30 or 31 days. This made a total of 14 intervals in the season starting from April 1st. It was assumed that seasonal variation in

precipitation amounts would vary little over 15 day periods. A second program was written to construct the cumulative frequency distribution of precipitation following both wet and dry days using the actual data. The actual distributions were plotted on log probability paper against the theoretical function for each of the 28 time intervals. Figure 5 represents a sample plot of actual versus theoretical cumulative rainfall distribution following a dry day. The Chi-squared test was used on a random sample of ten plots in order to determine if the actual distribution followed the gamma function. Table 3a lists the Chi-squared values and their respective degrees of freedom for each distribution chosen. Nine of the ten samples chosen were found not to be significantly different from the theoretical distribution at the 90 percent level of probability. Therefore, the incomplete gamma function was used to describe the daily rainfall occurrences for the entire growing season. The  $\alpha$  and  $\beta$  parameters are listed in table 4.

#### 5.5 The Potential Evapotranspiration Model.

A computer program was written to calculate daily potential evapotranspiration via equation 5 between the dates of April 1st to October 31st for each of the 45 years of records available on magnetic tape. The term  $Q_0$  (solar radiation recieved at the top of the atmosphere) was obtained from Smithsonian tables (37), while WIND (monthly average wind velocities) were taken from table 7 of

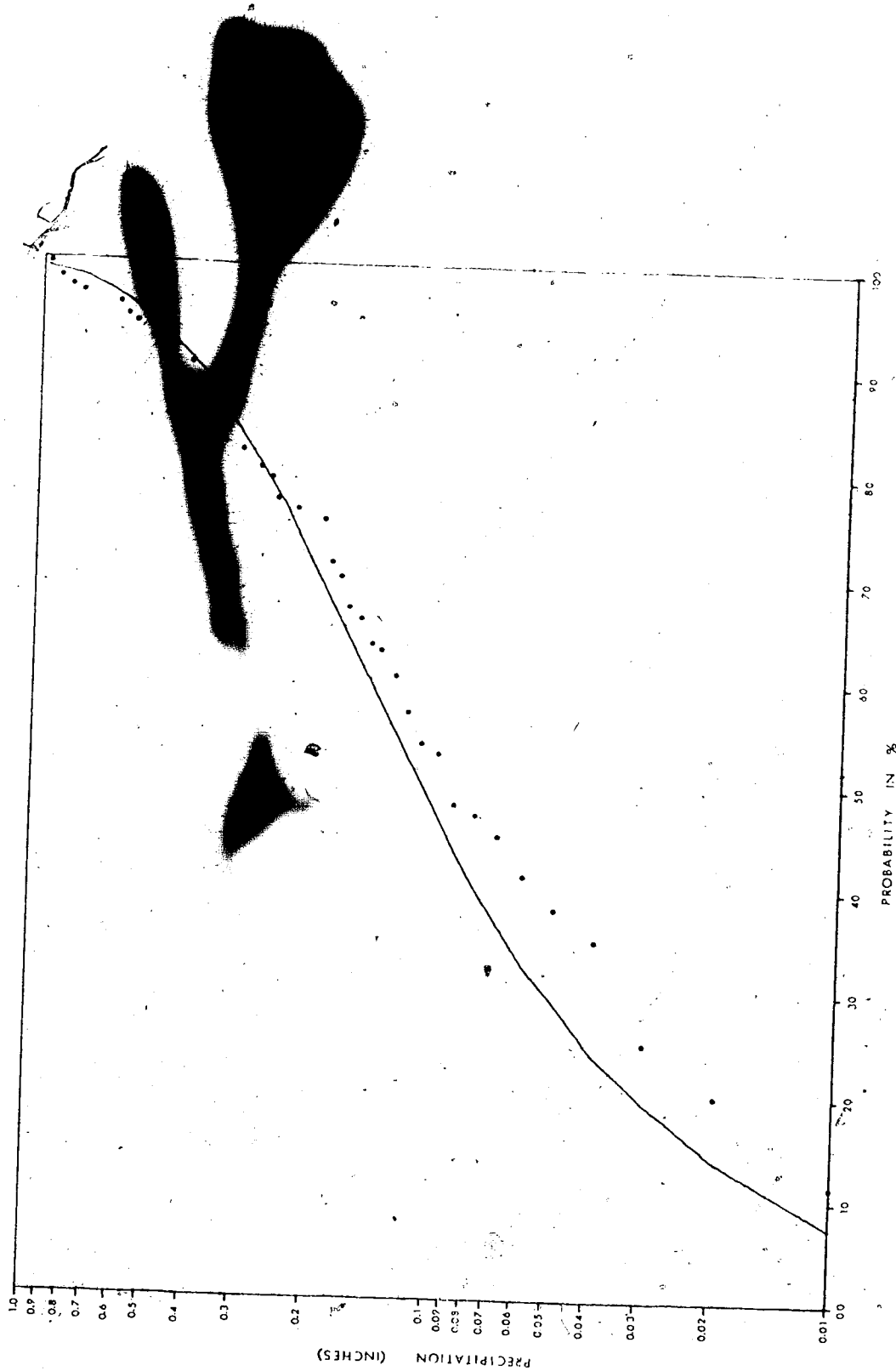


Figure 5. Comparison of actual and theoretical cumulative distribution of precipitation following a non-rainy day: May 15 - 30.

TABLE 3. CHI-SQUARED TEST - PRECIPITATION AND POTENTIAL EVAPOTRANSPIRATION.

a) PRECIPITATION

| Interval  | Type of Day | Degrees of Freedom | Chi-Squared Values |
|-----------|-------------|--------------------|--------------------|
| Apr 1-15  | Dry         | 3                  | 7.365 *            |
| Apr 16-30 | Dry         | 4                  | 7.531 n.s.         |
| Jul 16-31 | Dry         | 3                  | 1.209 n.s.         |
| Aug 16-31 | Dry         | 4                  | 2.384 n.s.         |
| Oct 1-15  | Dry         | 2                  | 2.984 n.s.         |
| Apr 1-15  | Wet         | 3                  | 3.562 n.s.         |
| May 16-31 | Wet         | 5                  | 6.036 n.s.         |
| Jul 1-15  | Wet         | 4                  | 4.868 n.s.         |
| Sep 1-15  | Wet         | 4                  | 6.583 n.s.         |
| Oct 15-31 | Wet         | 3                  | 3.215 n.s.         |

b) POTENTIAL EVAPOTRANSPIRATION

| Interval  | Type of Day | Degrees of Freedom | Chi-Squared Values |
|-----------|-------------|--------------------|--------------------|
| Apr 1-15  | Wet         | 2                  | 9.703 ***          |
| Jun 1-15  | Wet         | 4                  | 4.267 n.s.         |
| Jul 1-15  | Wet         | 5                  | 4.797 n.s.         |
| Aug 16-31 | Wet         | 4                  | 3.329 n.s.         |
| Oct 1-15  | Wet         | 2                  | 6.676 **           |
| Apr 16-30 | Dry         | 4                  | 11.211 **          |
| May 16-31 | Dry         | 5                  | 7.889 n.s.         |
| Jun 1-15  | Dry         | 3                  | 7.005 *            |
| Jul 16-31 | Dry         | 4                  | 13.176 **          |
| Sep 1-15  | Dry         | 4                  | 12.847 **          |

\* significant at the 0.10 level.

\*\* significant at the 0.05 level.

\*\*\* significant at the 0.01 level.

n.s. not significant.

TABLE 4. A LIST OF THE  $\alpha$  AND  $\beta$  PARAMETERS OF THE INCOMPLETE GAMMA FUNCTION FOR PRECIPITATION.

| Time Interval | Dry Day Preceding<br>$\alpha$ | Dry Day Preceding<br>$\beta$ | Wet Day Preceding<br>$\alpha$ | Wet Day Preceding<br>$\beta$ |
|---------------|-------------------------------|------------------------------|-------------------------------|------------------------------|
| April 1-15    | 1.042437                      | 0.134301                     | 1.034741                      | 0.130065                     |
| April 16-30   | 0.835857                      | 0.225257                     | 0.759484                      | 0.291298                     |
| May 1-15      | 0.814344                      | 0.207961                     | 0.740754                      | 0.309259                     |
| May 16-31     | 0.860329                      | 0.199111                     | 0.693046                      | 0.372689                     |
| June 1-15     | 0.803625                      | 0.261213                     | 0.804689                      | 0.381127                     |
| June 16-30    | 0.765032                      | 0.285950                     | 0.659652                      | 0.636569                     |
| July 1-15     | 0.686373                      | 0.295029                     | 0.720244                      | 0.346013                     |
| July 16-31    | 0.939405                      | 0.186009                     | 0.893318                      | 0.284008                     |
| Aug 1-15      | 0.856398                      | 0.199735                     | 0.654811                      | 0.344156                     |
| Aug 16-31     | 0.790676                      | 0.290288                     | 0.778251                      | 0.301041                     |
| Sept 1-15     | 0.863120                      | 0.260740                     | 0.828852                      | 0.258705                     |
| Sept 16-30    | 0.791574                      | 0.243743                     | 0.899845                      | 0.250745                     |
| Oct 1-15      | 0.893684                      | 0.168571                     | 0.730132                      | 0.253239                     |
| Oct 16-31     | 0.764728                      | 0.208680                     | 0.890063                      | 0.276635                     |

o Rutledge (48).

Because of the increase in relative humidity during rainfall, potential evapotranspiration, on the average, will be lower on wet days than on dry days. Hence, it was decided to create two sets of distributions, one to describe daily PE on wet days and one to describe daily PE on dry days. Each set of PE distributions would therefore characterize the daily temperature, solar radiation and cloud cover. A program was written in FORTRAN to read in the daily PE values from magnetic tape and to construct cumulative distributions on a bimonthly basis for PE on dry days and wet days. A total of 28 sets of data were then plotted on normal probability paper. The concept of mixed distributions, as discussed earlier, was again employed in the construction of the PE distributions. Only those PE values greater than zero were used to create the distribution while those values equal to zero were used to determine the probability of the occurrence of a measurable amount of PE. These probabilities are presented in table 5.

Because most of the data plotted as straight lines on normal probability paper, the normal distribution was assumed to apply. The straight lines were fitted to the data according to the mean and standard deviation of their respective distribution. A Chi-squared test was performed on a random sample of ten plots to determine if the normal distribution applied. A list of the Chi-squared values and their respective degrees of freedom are given in table 3.



TABLE 5. BIMONTHLY PROBABILITIES OF POTENTIAL  
EVAPOTRANSPIRATION ON WET AND DRY DAYS.

| Interval  | P(PE D) | P(PE W) |
|-----------|---------|---------|
| Apr 1-15  | 0.8180  | 0.4520  |
| Apr 16-30 | 0.9267  | 0.6022  |
| May 1-15  | 0.9810  | 0.8079  |
| May 16-31 | 1.0000  | 0.9336  |
| Jun 1-15  | 1.0000  | 0.9665  |
| Jun 16-30 | 1.0000  | 0.9957  |
| Jul 1-15  | 1.0000  | 1.0000  |
| Jul 16-31 | 1.0000  | 0.9932  |
| Aug 1-15  | 1.0000  | 0.9935  |
| Aug 16-31 | 1.0000  | 0.9268  |
| Sep 1-15  | 0.9882  | 0.7821  |
| Sep 16-30 | 0.9059  | 0.5269  |
| Oct 1-15  | 0.8569  | 0.5455  |
| Oct 16-31 | 0.7221  | 0.2810  |

TABLE 6. SUMMARY OF THE SMIRNOV-KOLMOGOROV STATISTIC FOR  
DAILY PE VALUES OCCURRING ON DRY DAYS.

| Interval  | Size | Statistic  |
|-----------|------|------------|
| Apr 1-15  | 408  | 0.10 **    |
| Apr 16-30 | 454  | 0.065 *    |
| May 1-15  | 464  | 0.05 n.s.  |
| May 16-31 | 480  | 0.040 n.s. |
| Jun 1-15  | 407  | 0.05 n.s.  |
| Jun 16-30 | 444  | 0.06 n.s.  |
| Jul 1-15  | 466  | 0.04 n.s.  |
| Jul 16-31 | 573  | 0.03 n.s.  |
| Aug 1-15  | 521  | 0.025 n.s. |
| Aug 16-31 | 553  | 0.04 n.s.  |
| Sep 1-15  | 500  | 0.04 n.s.  |
| Sep 16-30 | 461  | 0.06 n.s.  |
| Oct 1-15  | 466  | 0.08 *     |
| Oct 16-31 | 433  | 0.10 **    |

\* significant at the 0.05 level

\*\* significant at the 0.01 level

n.s. not significant.

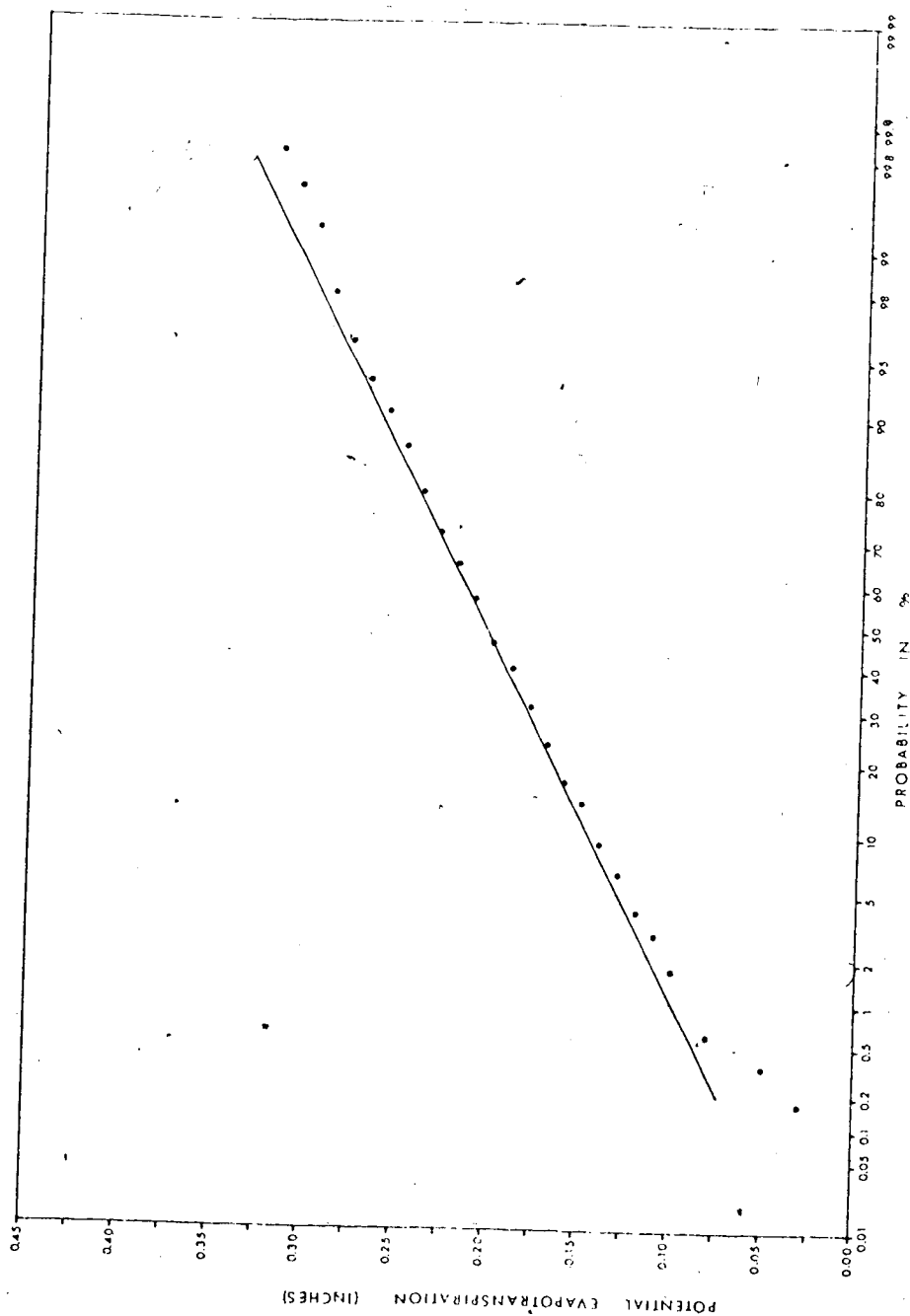


Figure 6. Comparison of actual and theoretical cumulative distribution of daily PE occurring on a non-rainy day: July 16 - 31.

TABLE 7. A LIST OF THE MEAN AND STANDARD DEVIATION - POTENTIAL  
EVAPOTRANSPIRATION.

| Interval  | Dry      |                  | Wet      |                  |
|-----------|----------|------------------|----------|------------------|
|           | Mean     | Days<br>St. Dev. | Mean     | Days<br>St. Dev. |
| Apr 1-15  | 0.089853 | 0.049389         | 0.063544 | 0.045377         |
| Apr 16-30 | 0.117159 | 0.056946         | 0.067477 | 0.041745         |
| May 1-15  | 0.138815 | 0.055099         | 0.086932 | 0.058175         |
| May 16-31 | 0.159042 | 0.053060         | 0.112098 | 0.057719         |
| Jun 1-15  | 0.166708 | 0.051293         | 0.117413 | 0.059707         |
| Jun 16-30 | 0.176374 | 0.049125         | 0.129652 | 0.061352         |
| Jul 1-15  | 0.200021 | 0.043207         | 0.157081 | 0.057409         |
| Jul 16-31 | 0.206195 | 0.045414         | 0.149383 | 0.056705         |
| Aug 1-15  | 0.192687 | 0.048151         | 0.147451 | 0.053931         |
| Aug 16-31 | 0.168047 | 0.053644         | 0.113613 | 0.061664         |
| Sep 1-15  | 0.137580 | 0.058546         | 0.085985 | 0.053772         |
| Sep 16-30 | 0.112798 | 0.055075         | 0.079310 | 0.055548         |
| Oct 1-15  | 0.089764 | 0.054423         | 0.069577 | 0.050918         |
| Oct 16-31 | 0.075751 | 0.044581         | 0.057273 | 0.039392         |

For PE occurring on wet days, only two of the five distributions were significantly different. These distributions occurred during the spring and fall months when weather conditions are unstable and in a state of change. During the summer months, the distributions were not significantly different from the theoretical distributions. In the case of PE occurring on dry days, the situation was quite different. Only the distribution representing the latter half of May was non-significant. The distribution representing the first half of June was significant at the 0.01 percent level and all other distributions were significantly different at the 0.05 percent level. Therefore, it was assumed that the PE values occurring on dry days did not follow the normal distribution. However, because the straight lines, as depicted by the mean and standard deviation of the data, in most cases, fitted the plotted points extremely well, it was decided to perform a non-parametric test with the use of the Smirnov-Kolmogorov statistic. This test assumes that the distribution is continuous and that the fitted straight line to the data is distribution free. Potential evapotranspiration, because it is measured to the nearest 0.01 inch, can be considered to be a continuous event. The Smirnov-Kolmogorov test indicated that ten of the 14 distributions were not significantly different at the 95 percent level. A list of the Smirnov-Kolmogorov statistic is presented in table 6. The normal distribution was

accepted as characteristic of daily potential evapotranspiration amounts. A sample distribution for the period July 16-31 is given in figure 6 . The means and standard deviations are listed in table 7 and were used to simulate daily PE events.

#### 5.6 The Overwinter Precipitation Model.

The last parameter of the weather model which remains to be discussed is that of precipitation during the winter months. There are essentially two directives which can be taken in the matter. One is to develop the rainfall and the PE models for the entire year thereby providing a means of simulating weather for all twelve months of the year. The main objective, however, in developing a weather model is to simulate actual soil moisture conditions on a daily basis. This can be done satisfactorily and with sufficient ease during the summer months, but it is extremely difficult to simulate water movement in frozen soil.

VanSchaik and Rapp (55) performed lysimeter experiments in which soil moisture contents and water tables were monitored during two winters for both bare and grass covered soils with a shallow water table. Two major points were concluded from their research. The water table showed a general downward movement during the winter, but this sometimes was nullified by warm Chinook periods. As well, the soil moisture content of a soil with a shallow water table increased substantially due to upward capillary movement of water. However, the moisture content of the

upper 10 inches of soil could only be increased by snowmelt or fall irrigation.

Further research by Hobbs and Krogman (25) indicated that the fall soil moisture was linearly related to overwinter precipitation storage. Experiments were performed on four crops with four irrigation treatments. Overwinter changes in the root zone soil moisture were recorded for eight seasons from the harvesting date to the planting date of each crop. It was found that the crop species did not significantly affect the soil moisture content at the harvest date nor did the amount of precipitation stored in the root zone during the winter months. The storage of overwinter precipitation was found to be inversely proportional to the fall soil moisture and was expressed by a linear regression model as follows.

$$\Delta M = 6.6 - 0.46M_f$$

where:  $M_f$  = fall soil moisture

$\Delta M$  = overwinter increase in soil moisture

The correlation between storage and precipitation showed that the storage was more dependent upon spring precipitation than on fall or winter precipitation.

Rutledge (48) had assumed that the amount of overwinter precipitation which was stored in the soil was 35 percent of the total overwinter precipitation for the Lethbridge area. This estimate was based on experimental work performed at Swift Current by Staple and Lehane. Since

this method was based upon actual values of overwinter precipitation, the method, as used by Rutledge, was adopted into the model. A program was written to construct a frequency distribution of the overwinter total precipitation. The mean precipitation was found to be 4.35 inches with a standard deviation of 1.24 inches. A Chi-squared test yielded a value of 2.1559 with 5 degrees of freedom. This value was not significantly different from the normal function at the 90% level of probability. The Monte Carlo sampling technique was used to select at the end of each season a value of overwinter precipitation, 35 percent of which was added to the soil to arrive at a soil moisture content for April 1st of the next season. The first year of the simulation run was assumed to be 75 percent of the total available capacity.

## 6. Programming.

Several points of interest in the construction of the cropping model should be indicated before proceeding any further. It was the initial intent of the author to write the program in GPSS (General Purpose Simulation System). This language has the ability to perform Monte Carlo sampling of distributions with the least amount of experience required on the part of the programmer. Only two statements are required to simulate a day of rainfall and likewise only two statements are required to construct a cumulative frequency distribution from the output variables. Hence, a cropping model was built using GPSS in which daily rainfall and PE amounts were determined by the Monte Carlo sampling technique. The daily soil moisture contents for the four crops were calculated using the Versatile Budget. The model was built and a dry run was performed. It was found that 4 seconds of computing time were required to simulate one day of crop growth. This was far too slow if a total of 200 years of 214 days each (April 1st to October 31st) were to be simulated. This would have amounted to approximately 171,200 seconds or 47 hours of computing time. The cost would have been astronomical. Hence, it was decided to rewrite the program in FORTRAN - G. Rewriting the Monte Carlo model in FORTRAN proved to be much more difficult and time consuming than in GPSS. One subroutine each had to be devoted to the rainfall and PE models while construction of the desired frequency distributions of the



output variables required three subroutines.

The program, when completed, was run for a period of one year. The model, this time, required only 4 seconds of computing time to simulate one season of crop growth. Hence, to complete 200 seasons of simulation, a maximum of 13 minutes computing time would be required. This was a considerable reduction in time and more in keeping with the current financial situation. After considerable editing, the efficiency of the program was increased and the model actually took 10 minutes to execute.

The model was divided into eleven parts: a main program and ten subroutines. A listing of the program and flow charts of the major subroutines is presented in Appendix A. Some of the minor things which had to be considered in the construction of the model will now be discussed at this point.

#### 6.1 Random Number Generator.

During the course of each day of simulation, two variables, rainfall and potential evapotranspiration, had to be simulated. Therefore, two random numbers per day were required, making a total of 428 numbers per season. Also, a random number was required to determine whether or not March 31st, at the beginning of each season, was to be a wet or a dry day. This information was then used to determine the precipitation functions to be used in calculating daily rainfall on April 1st. Furthermore, a random number was required to determine the amount of overwinter precipitation

so that the soil moisture condition at the start of each season could be calculated. Hence a total of 430 uniformly distributed random numbers were required for one year of simulation. This made a total of 86,000 numbers for the entire 200 years. A random number generator had to be selected so that it could produce up to 100,000 numbers without exhibiting circularity. Also, it had to have the capability of producing the same sequence of random numbers during different runs in order that comparisons of drainage distributions could be made with and without irrigation. A pseudo-random number generator called GGU1 from the IMSL package (International Mathematical Statistical Language, 29) was found to be suitable for the task. Statistical Chi-squared tests had shown that 126,000 numbers could be generated without circularity occurring. The random numbers were stored in a two dimensional array, RND(2,214), where the columns represented the day number of the season and the rows represented the random numbers used to calculate precipitation and potential evapotranspiration, respectively.

## 6.2 Monte Carlo Sampling.

The application of the random numbers described above to the precipitation and the PE distributions were carried out in two different manners worthy of a brief discussion.

### 6.2.1 Precipitation.

Because calculating the precipitation with the use of equation 11 involves a great deal of iteration, computer

time would have been increased substantially. Instead, the values for the gamma distribution for  $\alpha = 0.5, 1.0, \text{ and } 1.5$ , as given in table II, p 29, of Thom (53) and in the tables of Pearson (42), were stored in the array, GAM(29,4). The Lagrange interpolating polynomial, as described by Stark (51), was used to perform a two-way interpolation of the tables. The basic equation is of the form

$$P_1(x) = f(x_0) \frac{(x - x_1)}{(x_0 - x_1)} + f(x_1) \frac{(x - x_0)}{(x_1 - x_0)}$$

such that  $P_1(x) = f(x_0)$  and  $P_1(x_1) = f(x_1)$  at the two tabulated points  $x_0$  and  $x_1$ . Tests performed by hand calculation showed that interpolated values were in close agreement with the theoretical distribution of both the low and high probability ranges.

#### 6.2.2 Potential Evapotranspiration.

A subroutine, MDNRIS, from the IMSL statistical computer package (29), was used to determine daily PE values. A random number was selected from the array RND and it was then transformed into a standard normal deviate  $z = (x-u)/s$  using the above mentioned subroutine. For each bimonthly period, a regression equation of the type

$$y = az + b$$

was used to calculate daily PE amounts. The  $z$  term refers to the standard normal deviate corresponding to the cumulative probability,  $y$  stands for the associated daily PE value,  $s$  stand for the standard deviation and the mean,  $u$  the PE distribution (table 7).

### 6.3 Decision to Irrigate.

Irrigation was performed when the total soil moisture content had been depleted to 50% of its total capacity to hold moisture. The decision to irrigate Wheat and Alfalfa was based upon the total moisture within all six zones. The decision to irrigate Potatoes and Sugar Beets, on the other hand, was based upon the total moisture only within those soil zones from which the roots were actively extracting water. In other words, if the  $K$  - coefficient for a particular zone during a particular crop stage was zero, the moisture within that zone was not included in the total sum of soil moisture. In this way, excessive irrigation during the early crop growth stages could be avoided. Wheat and Alfalfa, however, do not require careful irrigation practices as do Potatoes and Sugar Beets. The generally recommended practice for Wheat is to give the crop one thorough irrigation prior to the time of peak consumptive use during the middle of July. For Alfalfa, 3 - six inch irrigations are recommended during the season. Hence, it was decided that all six zones would be used to determine total soil moisture for Wheat and Alfalfa.

Hobbs et al (23) had reported on the response of various crops to several minimum allowable soil moisture levels. Yield data, for like crops irrigated by three different treatments, were compared. Irrigation was performed when the soil moisture content became 1) 25%, 2) 50%, 3) 75% of the total available soil moisture. The

results are tabulated in table 8 for the four crops under study.

TABLE 8. SUMMARY OF THE MINIMUM IRRIGATION LEVELS FOR FOUR DIFFERENT CROPS (Hobbs et al, 23).

| Crop                     | Irrigation Level (%) |
|--------------------------|----------------------|
| Soft Wheat               | 50                   |
| Potatoes                 | 75                   |
| Sugar Beets              | 25                   |
| Alfalfa (1st year stand) | 75                   |
| Alfalfa (2nd year stand) | 50                   |

Ten years of crop growth was simulated with the above criteria used to determine the irrigation day. The results indicated that Wheat averaged about 4 irrigations per season, Potatoes and Alfalfa averaged 14 , and Sugar Beets, 3 irrigations per season. An examination of the Irrigation Gauge data for the years 1969 to 1973 indicated that many farmers were irrigating approximately when the soil moisture content was 50 percent of the total moisture capacity for all crops. Furthermore, the Irrigation Gauge recommended from 3 to 4 irrigations per season for Wheat, 3 to 4 irrigations for Potatoes, 3 to 5 irrigations for Sugar Beets and from 5 to 6 irrigations for Alfalfa. Hence, the irrigation levels for all crops were adjusted to the 50 percent level and the model was run again. This time the average number of irrigations corresponded to the recommended number.

## 7. Results And Conclusions.

### 7.1 Actual vs Simulated Data.

Before any meaningful data could be gathered from the model, it was necessary to perform a check on the program to verify the accuracy of both the rainfall and the potential evapotranspiration models. Such a check is necessary if the soil moisture content, and thus irrigation and drainage, is to be simulated with reasonable accuracy under weather conditions typical of the Lethbridge area. Both the simulated and the actual sets of data were compared by examining averages, lengths of dry day sequences and their respective  $\lambda_1$  and  $\lambda_2$  parameters.  $\lambda_1$  refers to the rate occurrence of an event while  $\lambda_2$  signifies the yield density of the event. These two parameters will be explained in a later section.

The average total simulated rainfall of 45 years for the period from April 1st to October 31st was 11.96 inches compared to the actual average of 12.43 inches computed from 1922 to 1966 for Lethbridge. Table 9 lists the bimonthly averages of rainfall and potential evapotranspiration.

The author attempted to find a statistical test which could be applied to the data to show that the actual average monthly values did not differ significantly from the simulated monthly values. However, because the actual values were not derived from a theoretical formula, no statistical test could be found. Instead, the correlation coefficient ( $r$ ) and the standard error of estimate ( $S_{xy}$ ) of

TABLE 9. SUMMARY OF SIMULATED AND ACTUAL WEATHER DATA - 45 YEARS.

## Precipitation

| Interval  | Actual           |                      | Simulated        |                      |
|-----------|------------------|----------------------|------------------|----------------------|
|           | Mean<br>(inches) | St. Dev.<br>(inches) | Mean<br>(inches) | St. Dev.<br>(inches) |
| Apr 1-15  | 0.54             | 0.4235               | 0.48             | 0.3086               |
| Apr 16-30 | 0.85             | 0.7665               | 0.64             | 0.5523               |
| May 1-15  | 0.88             | 0.8837               | 1.04             | 0.7378               |
| May 16-31 | 1.14             | 1.2348               | 1.19             | 0.8495               |
| Jun 1-15  | 1.57             | 1.2158               | 1.45             | 0.7771               |
| Jun 16-30 | 1.65             | 1.3676               | 1.43             | 1.0948               |
| Jul 1-15  | 1.03             | 1.0205               | 0.76             | 0.6166               |
| Jul 16-31 | 0.66             | 0.7900               | 0.82             | 0.6231               |
| Aug 1-15  | 0.66             | 0.6534               | 0.66             | 0.4913               |
| Aug 16-31 | 0.86             | 0.8165               | 0.93             | 0.8943               |
| Sep 1-15  | 0.83             | 0.8009               | 0.70             | 0.6006               |
| Sep 16-30 | 0.77             | 0.7535               | 0.69             | 0.5434               |
| Oct 1-15  | 0.48             | 0.4791               | 0.56             | 0.5364               |
| Oct 16-31 | 0.52             | 0.6976               | 0.63             | 0.5630               |

## Potential Evapotranspiration

| Interval  | Actual           |                      | Simulated        |                      |
|-----------|------------------|----------------------|------------------|----------------------|
|           | Mean<br>(inches) | St. Dev.<br>(inches) | Mean<br>(inches) | St. Dev.<br>(inches) |
| Apr 1-15  | 0.93             | 0.4057               | 0.94             | 0.2552               |
| Apr 16-30 | 1.35             | 0.5287               | 1.43             | 0.3024               |
| May 1-15  | 1.75             | 0.4801               | 1.73             | 0.2461               |
| May 16-31 | 2.25             | 0.4589               | 2.13             | 0.2795               |
| Jun 1-15  | 2.18             | 0.4105               | 2.20             | 0.2128               |
| Jun 16-30 | 2.40             | 0.4009               | 2.40             | 0.2293               |
| Jul 1-15  | 2.80             | 0.3692               | 2.84             | 0.1824               |
| Jul 16-31 | 3.11             | 0.3869               | 3.07             | 0.2633               |
| Aug 1-15  | 2.73             | 0.3163               | 2.74             | 0.2396               |
| Aug 16-31 | 2.46             | 0.4430               | 2.49             | 0.2451               |
| Sep 1-15  | 1.78             | 0.4267               | 1.82             | 0.2585               |
| Sep 16-30 | 1.31             | 0.5173               | 1.31             | 0.2809               |
| Oct 1-15  | 1.14             | 0.4468               | 1.22             | 0.2552               |
| Oct 16-31 | 0.77             | 0.4224               | 0.79             | 0.1805               |

the data were used to describe the disparity between the two sets of data. The correlation coefficient is a measure of the degree to which the variables vary together or a measure of the intensity of association. The standard error of estimate is measure of the variability of the estimated data about the actual data. In essence, it is the standard deviation of Y holding X constant.

Agreement between actual and simulated rainfall was found to be quite good. The correlation coefficient was 0.9177 and the standard error of estimate was 0.1192. The standard deviations of the simulated data, in general, were slightly lower than those of the actual data. This probably can be attributed to the fact that the continuous functions estimating the conditional probabilities of rainy and non-rainy days (figure 4) were used in lieu of the actual probabilities. The actual probabilities have more variation than do the functions and therefore would effect higher standard deviations in the average binmonthly rainfall of the simulated data.

In conjunction with the total amount of binmonthly rainfall is the distribution of consecutive periods of dry days throughout the entire season. Figure 7 represents the actual versus the simulated relative frequencies of the number of consecutive days separating wet days for the entire season. The total number of simulated dry days for 45 years was 1,448 compared to the actual number of dry days of 1,442. The longest simulated dry run was 34 days while



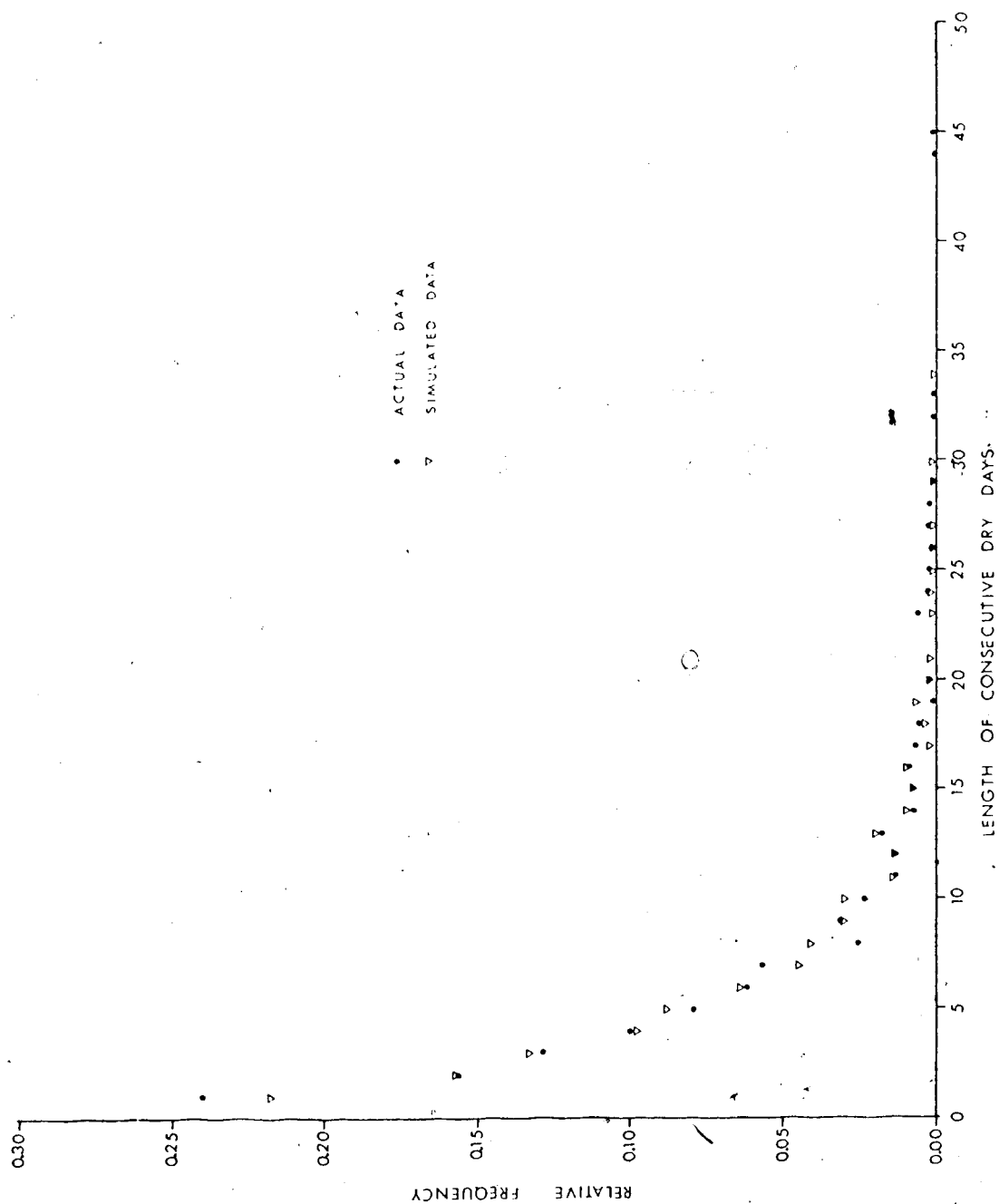


Figure 7. Relative frequencies of dry day runs for actual and simulated data: April 1 - Oct. 31.

the longest actual dry run was 45 days. When the model was run for 200 years, the longest simulated dry run was found to be 40 days. The actual data showed that dry day runs of 44 and 45 days occurred once. It was thought that had the actual daily rainfall conditional probabilities (figure 4) been employed instead of the probabilities depicted by the polynomial equations 11 and 12, more actual values of dry day runs and therefore average rainfall amounts would have been obtained from the simulation model. However, this possibility was not tested.

An alternative method of describing the rainfall pattern was employed to compare actual and simulated data. The season from April 1st to October 31st was divided into 43 - five day intervals. Within each time interval the number of wet days and the total amount of precipitation were summed over the 45 years of both the simulated and the actual data. Figure 8a and 8b show plots of the average number of wet days per day and the average amount of precipitation yield per wet day for the actual and simulated data. Good agreement exists between the actual and the generated number of wet days per day except for the month of May in which the simulated number of wet days slightly overestimates the actual data. The correlation coefficient and the standard error of estimate for figure 8a were found to be 0.7191 and 0.0460 respectively. This indicates that the distribution of wet days follows the actual distribution reasonably close. The amount of simulated precipitation

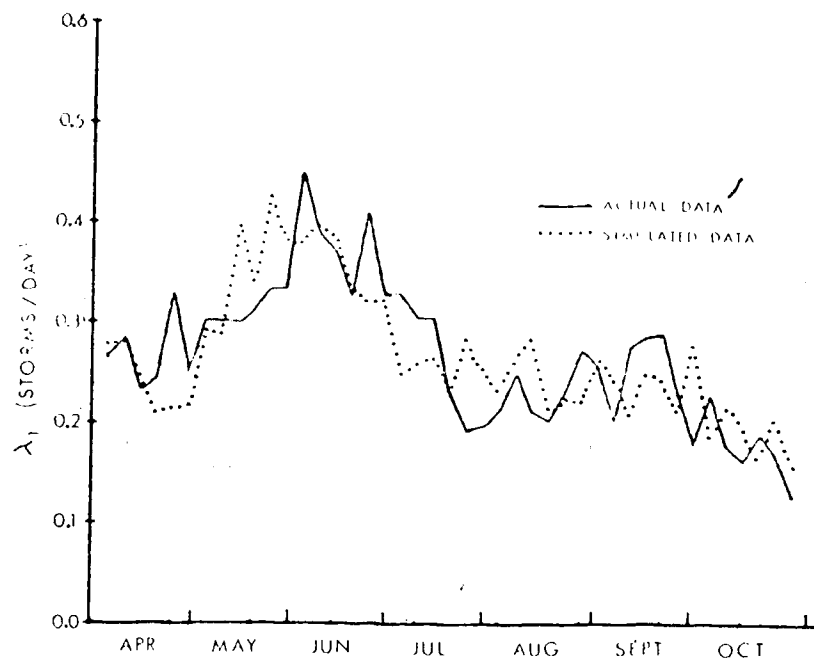


Figure 8a. Actual and simulated  $\lambda_1$  values: - 45 years.

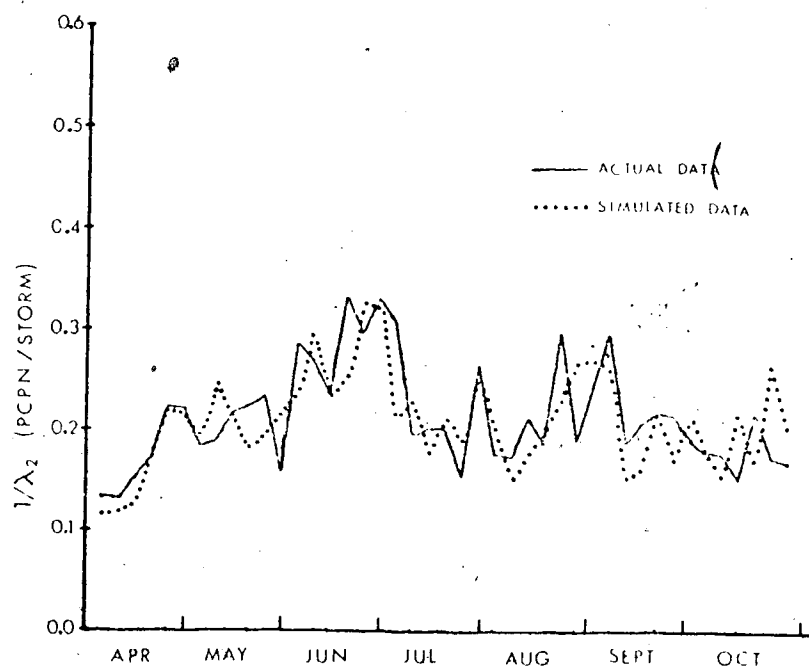


Figure 8b. Actual and simulated  $1/\lambda_2$  values: - 45 years.

which each storm yields, according to figure 8b, also estimates fairly well the actual data for the entire season. The  $r$  and the  $S_{xy}$  values for this case were calculated to be 0.6533 and 0.0367 respectively. Although the simulated and the actual data do not correlate very well, the dispersion is very small.

Based on these comparisons it can be concluded that the Markov Chain model combined with the incomplete gamma function can be effectively used to simulate daily rainfall data by way of the Monte Carlo sampling technique for the Lethbridge area.

The bimonthly average values of potential evapotranspiration from the simulation compares very favorably with the actual values in Table 9. The average total simulated PE for the entire season was 27.11 inches compared to the actual value of 26.97: a difference of 0.14 inch. The  $r$  value and the  $S_{xy}$  value were found to be 0.9359 and 0.2703 respectively. The maximum discrepancy which occurs during the periods of April 16-30 and Sept 1-15, is 0.08 inch. Since the actual PE bimonthly averages were computed from the daily values estimated by equation 5, the actual PE values are only estimates. Because the theoretical distributions of PE are closer to the actual data than the theoretical distributions of rainfall, the discrepancies of the mean PE values are much less. However, the variation of PE in the actual data is substantially greater than the variation of PE in the simulated data as

noted by their respective standard deviations. Since the conditional probability functions, as employed in the incomplete gamma distributions of rainfall, were continuous, the discrepancy between the standard deviations of the simulated data and the actual data were small. The conditional probabilities for the PE distributions (table 5) were calculated on a 15 day interval basis and therefore were discreet. This might have caused much lower dispersion in the simulated values and therefore much lower values of standard deviations were realized. However, this did not seem to affect the mean values of PE.

The outputs from the weather model have shown to compare very favorably with the actual weather data for the Lethbridge area.

A further refinement of the K-coefficients was carried out at this point. Ten years of simulated crop growth was performed for each crop. The simulation season was divided into 43 time intervals of 5 days each. Daily consumptive use values were summed for each time interval. Over the 10 years of simulation. Average daily consumptive use values for each time interval were then plotted against the experimental curves. The K-coefficients were adjusted until the curves showed a good fit. Figures 9 to 12 represent the simulated versus actual consumptive use curves and Table 10 lists the coefficient matrix for each crop.

The years 1960 to 1963 were in general warmer and dryer than usual. Hence, the crop consumptive use values

TABLE 10. K - COEFFICIENTS FOR FOUR CROPS.

## A) Wheat

| Dates<br>Ending | Soil Zones |     |     |     |     |     |
|-----------------|------------|-----|-----|-----|-----|-----|
|                 | 1          | 2   | 3   | 4   | 5   | 6   |
| May 4           | .60        | .15 | .05 |     |     |     |
| May 24          | .55        | .30 | .10 |     |     |     |
| June 12         | .50        | .40 | .20 | .10 |     |     |
| July 5          | .40        | .35 | .20 | .20 | .10 |     |
| July 12         | .40        | .30 | .25 | .20 | .10 | .05 |
| July 20         | .40        | .30 | .25 | .20 | .10 | .10 |
| Aug 1           | .40        | .30 | .25 | .15 | .10 | .10 |
| Aug 10          | .45        | .30 | .20 | .10 | .05 | .05 |
| Aug 20          | .45        | .30 | .20 | .10 | .05 | .05 |
| Oct 31          | .50        | .20 | .15 | .10 | .03 | .02 |

## B) Potatoes

| Dates<br>Ending | Soil Zones |     |     |     |     |     |
|-----------------|------------|-----|-----|-----|-----|-----|
|                 | 1          | 2   | 3   | 4   | 5   | 6   |
| May 10          | .60        | .15 | .05 |     |     |     |
| June 4          | .15        | .10 | .03 | .02 |     |     |
| June 25         | .30        | .20 | .10 | .03 | .02 |     |
| July 10         | .45        | .30 | .20 | .10 | .03 | .02 |
| Aug 1           | .40        | .35 | .25 | .15 | .10 | .05 |
| Aug 12          | .45        | .35 | .25 | .15 | .05 | .05 |
| Sept 18         | .40        | .30 | .20 | .10 | .05 | .03 |
| Oct 31          | .60        | .15 | .05 |     |     |     |

## C) Sugar Beets

| Dates<br>Ending | Soil Zones |     |     |     |     |     |
|-----------------|------------|-----|-----|-----|-----|-----|
|                 | 1          | 2   | 3   | 4   | 5   | 6   |
| Apr 25          | .60        | .10 | .05 |     |     |     |
| June 5          | .15        | .10 | .05 | .03 | .02 |     |
| June 26         | .20        | .15 | .10 | .10 | .05 | .02 |
| July 10         | .25        | .20 | .15 | .10 | .10 | .05 |
| Aug 1           | .35        | .25 | .20 | .15 | .10 | .05 |
| Sept 1          | .35        | .25 | .25 | .20 | .10 | .10 |
| Sept 15         | .45        | .25 | .20 | .20 | .15 | .10 |
| Oct 10          | .30        | .25 | .25 | .20 | .20 | .10 |
| Oct 31          | .60        | .15 | .05 |     |     |     |

TABLE 10. cont'd.

## D) Alfalfa

| Dates<br>Ending | Soil Zones |     |     |     |     |     |
|-----------------|------------|-----|-----|-----|-----|-----|
|                 | 1          | 2   | 3   | 4   | 5   | 6   |
| Apr 17          | .60        | .15 | .05 |     |     |     |
| May 24          | .50        | .20 | .15 | .12 | .08 | .05 |
| June 18         | .50        | .25 | .23 | .22 | .15 | .10 |
| July 3          | .50        | .25 | .15 | .15 | .10 | .10 |
| July 26         | .50        | .25 | .15 | .15 | .10 | .10 |
| Aug 25          | .40        | .20 | .18 | .15 | .12 | .05 |
| Sept 17         | .35        | .25 | .20 | .15 | .15 | .10 |
| Oct 31          | .50        | .20 | .15 | .10 | .03 | .02 |

were greater than the average values as presented by Hobbs (24). An attempt to bring the average consumptive use values down to a more general level was made. However, because the values were greatly unaffected by any large change in the K-coefficients, it was extremely difficult to force the simulated and actual consumptive use curves to coincide perfectly without drastically changing the entire coefficient matrices. Thus, discrepancies exist in figures 9 to 12. However, it is felt that the simulated curves assume values between the average values and those of the dryer years of 1960 to 1963. Inevitably, the power of the Versatile Budget to simulate daily consumptive use could greatly be enhanced if better coefficients had been selected both during the growing season and during the spring and fall seasons and had there been more accurate consumptive use curves available for each crop.

#### 7.2 Intermittent Processes.

A few researchers (54,63) have regarded daily rainfall as an intermittent stochastic process. A stochastic process is a random variable, defined in a probability space, and dependent on time. If the random variable assumes zero values for some positions along the time scale and greater than zero values for all other positions, the process is said to be intermittent. Rainfall, evaporation, runoff, and floods are intermittent processes. Similarly, irrigation dates and drainage can be considered as intermittent stochastic processes. They are both dependent on the soil



moisture level which in turn is a derived variable influenced by the two stochastic variables of precipitation and consumptive use. The amount and occurrence of drainage are stochastic whereas only the irrigation frequencies are stochastic. The amount of irrigation water applied to the field is that amount required to replenish the soil moisture deficit to field capacity at the 50 percent level. It is therefore a fixed quantity and has no need to be considered in this study. Because irrigation water replenishes the soil to exactly field capacity in the model, any drainage which does occur will be due to the combined effect of the amount and the occurrence of rainfall. The definition of drainage, therefore, as employed in this study, is that amount of water which is in excess of field capacity on day (i).

Yevjevich (63) describes two basic parameters of an intermittent process. They are:

- $\lambda_1$  = average number of bursts per unit time interval
- $\lambda_2$  = average number of bursts per unit yield

The  $\lambda_1$  and  $\lambda_2$  parameters are periodic functions of time with the year as the period. The term  $\lambda_2$  is best described by its inverse, the average water yield per burst. Because of daily and seasonal variations,  $\lambda_1$  and  $\lambda_2$  will vary with time. However, if the time interval is very small, they can be considered as constants within that time interval.

The two parameters were calculated according to the following formulae.

$$\lambda_1 = \frac{\sum_{y=1}^N e_y(i)}{5N}$$

$$\lambda_2 = \frac{\sum_{y=1}^N e_y(i)}{\sum_{y=1}^N x_y(i)}$$

where:  $e_y(i)$  = the number of bursts within the  $i$ th time interval and the  $y$ th year  
 $x_y(i)$  = the total water yield during the  $i$ th time interval and the  $y$ th year  
 $N$  = total number of years  
 $y$  = the  $y$ th year  
 $i$  = the  $i$ th time interval in the  $y$ th year

The interval of time over which the parameters were calculated was chosen as 5 days as it was felt that the parameters would vary little over this time span. The parameters were calculated for both irrigation and drainage as well as the actual and simulated rainfall.

#### 7.2.1 Drainage: $\lambda_1$ Parameters.

Figures 13 through to 16 present the  $\lambda_1$  and the  $1/\lambda_2$  curves for three variables, two of which are drainage and one irrigation. Drainage a, represented by the solid line, depicts the seasonal trend of drainage when irrigation water has been applied to the soil for the entire simulation run.

Drainage b, represented by the dotted line, depicts the behaviour of drainage when no irrigation water at all has been applied to the soil for the 200 years of simulation.

The dashed line represents the behavior of the  $\lambda_1$  parameter

for irrigation. The  $1/\lambda_2$  irrigation parameters maintained a constant value of 3.5 inches for the entire season for each of the four crops. Therefore, they were not presented in the figures and will not be discussed to any great length. Figures 13 to 16 also show the seasonal behavior of the average densities of the standard deviations for the  $\lambda_1$  and  $1/\lambda_2$  curves for each crop. The average densities are simply the standard deviations for each interval divided by the number of days within the interval. This value, then, represents the average standard deviation on a daily basis.

Figures 13a to 13d represent the  $\lambda_1$  curves of drainage for Soft Wheat, Potatoes, Sugar Beets and Alfalfa respectively. An examination of the  $\lambda_1$  curves for all four crops indicate that there are two general trends, one for Wheat and Alfalfa and one for Potatoes and Sugar Beets. The trends are as follows.

**Wheat and Alfalfa:**

1. The maximum value of  $\lambda_1$  occurs during the month of June.
2. A secondary maximum occurs during September.
3. Minimum values extend through July and August.
4. There is a sharp decline at the beginning of July.

**Potatoes and Sugar Beets:**

1. The peak  $\lambda_1$  values occur at the beginning of June and the end of May.
2. High values prevail during May and June.

3. Minimum values occur during July and August.

4. There is a gradual decrease in  $\lambda_1$  during June.

Two trends mentioned above are common to all four crops. The maximum value of the  $\lambda_1$  curves occur during June, and the value of  $\lambda_1$  during April 1-15 and from July onwards are approximately equal.

The average densities of the standard deviations of the  $\lambda_1$  curves (figures 14a and 14b) follow the same seasonal trends as do their respective  $\lambda_1$  curves. In other words, on a long term basis, as the average rate of occurrence of drainage increases, the range of the rate of occurrence increases. It is also noted that the  $\lambda_1$  curves and their respective standard deviations are almost identical throughout the entire season for Wheat and Alfalfa as well as for Potatoes and Sugar Beets. Yet, during May and June, figures 9 and 12 show that the average consumptive use rate of Alfalfa is much higher than for Wheat. A similar situation exists for Potatoes and Sugar Beets during August and September (figure 10 and 11). The  $\lambda_1$  curve and their standard deviations are almost identical, yet the consumptive use curve for Sugar Beets shows that its average consumptive use is higher than Potatoes. However, in both cases, it is noted that the slopes of the curves or the rate of increase of CU from one day to the next is approximately equal. This suggests that the drainage frequency is influenced by the daily rate of increase of CU rather than the absolute daily amount of CU. This fact is further

exemplified by the differences which exist between the shallow rooted crops and the other crops. The slope of the CU curves are much shallower for Potatoes and Sugar Beets (figures 10 and 11) than for Wheat and Alfalfa (figures 9 and 12) during the months of May and June. Drainage, therefore, has a much greater rate of occurrence for the crops showing the lower rate of daily increase of CU.

The conclusions drawn from the above analyses are listed below.

1. The daily amounts of consumptive use affect the average rates of drainage slightly. Crops which have higher daily consumptive use values but equal rates of increase, will not experience any appreciable difference in their average drainage rates.
2. It follows from the above that drainage rates are not influenced by the cumulative amount of consumptive use over a period of time.
3. The slope or the rate of increase of daily consumptive use affects the drainage rates greatly. Low rates of increase cause high rates of drainage while high rates of increase cause low drainage rates. Therefore, a crop will not experience very many drainage problems if its rate of daily increase in water use is high during the early crop growth stages.

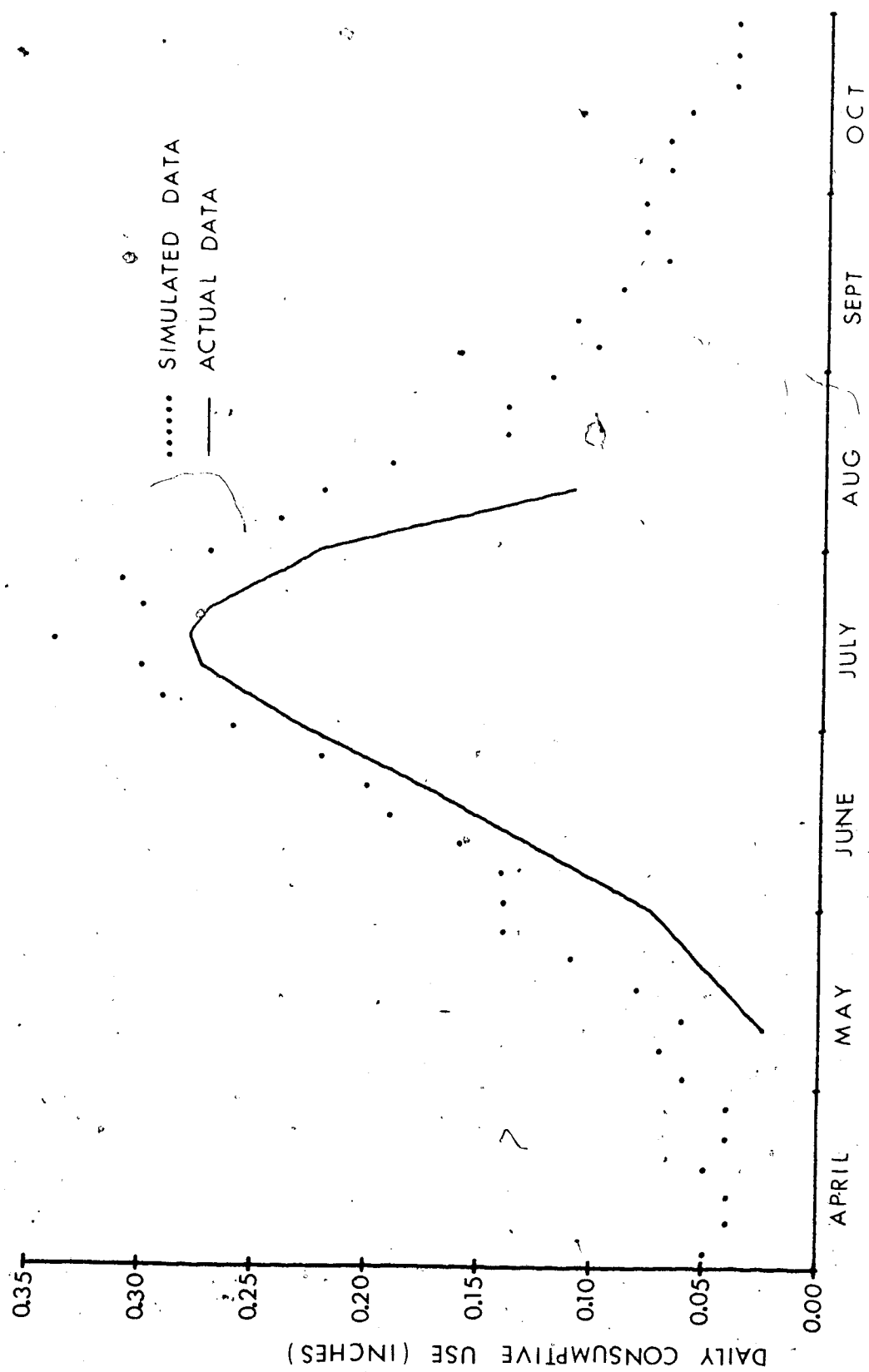


Figure 9. Comparison of actual and simulated daily consumptive use averages for wheat.

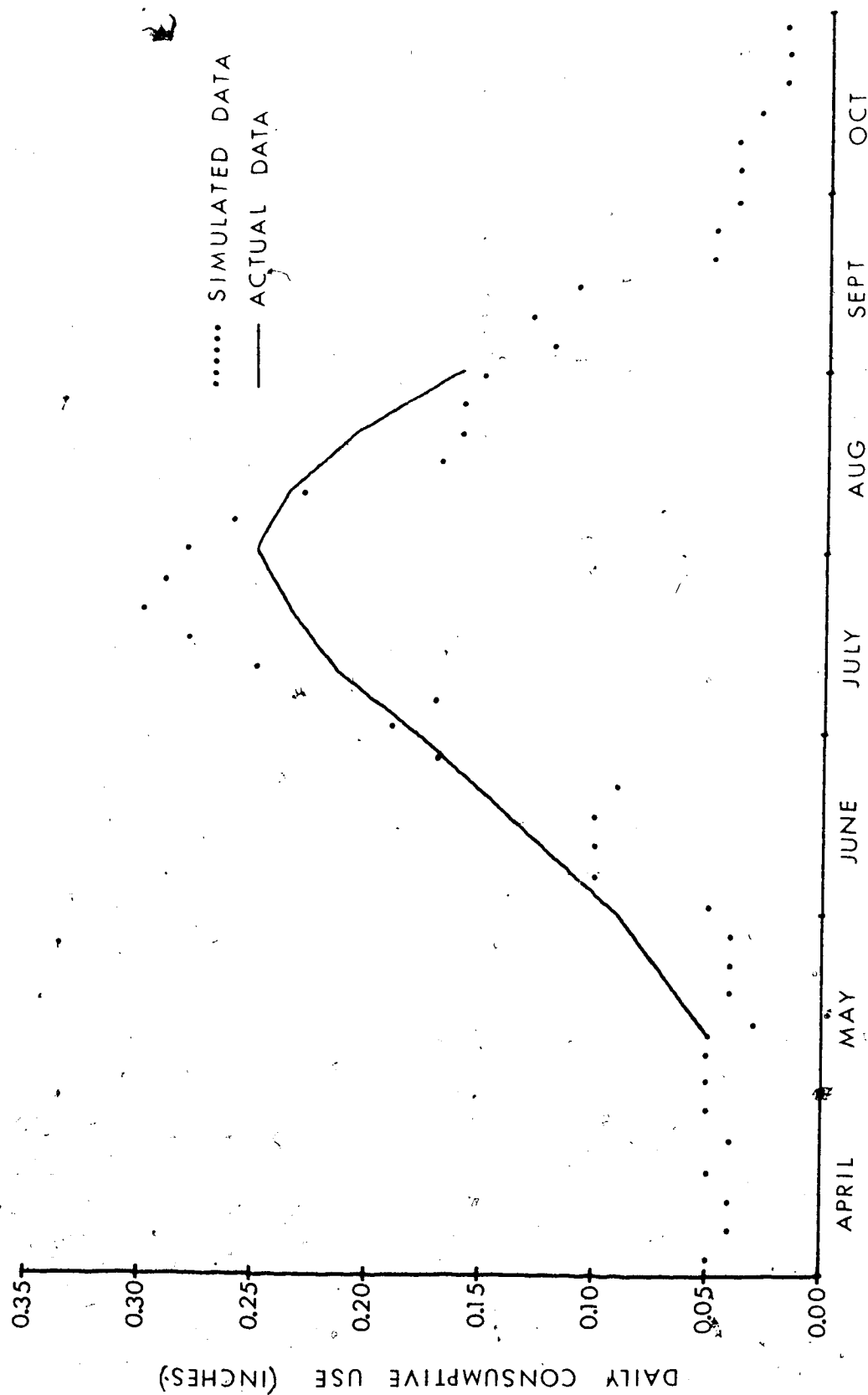


Figure 10. Comparison of actual and simulated daily consumptive use averages for Potatoes.

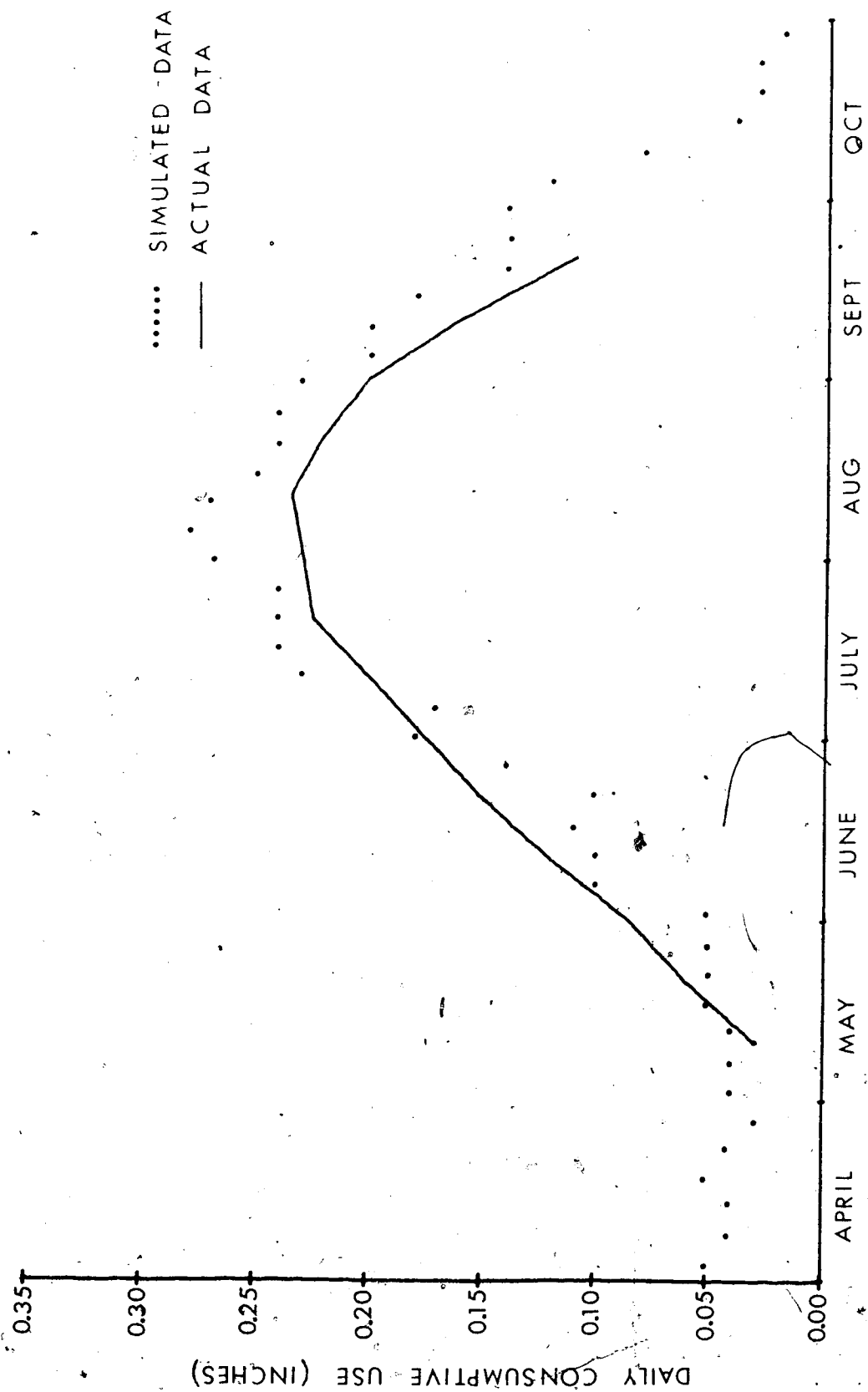
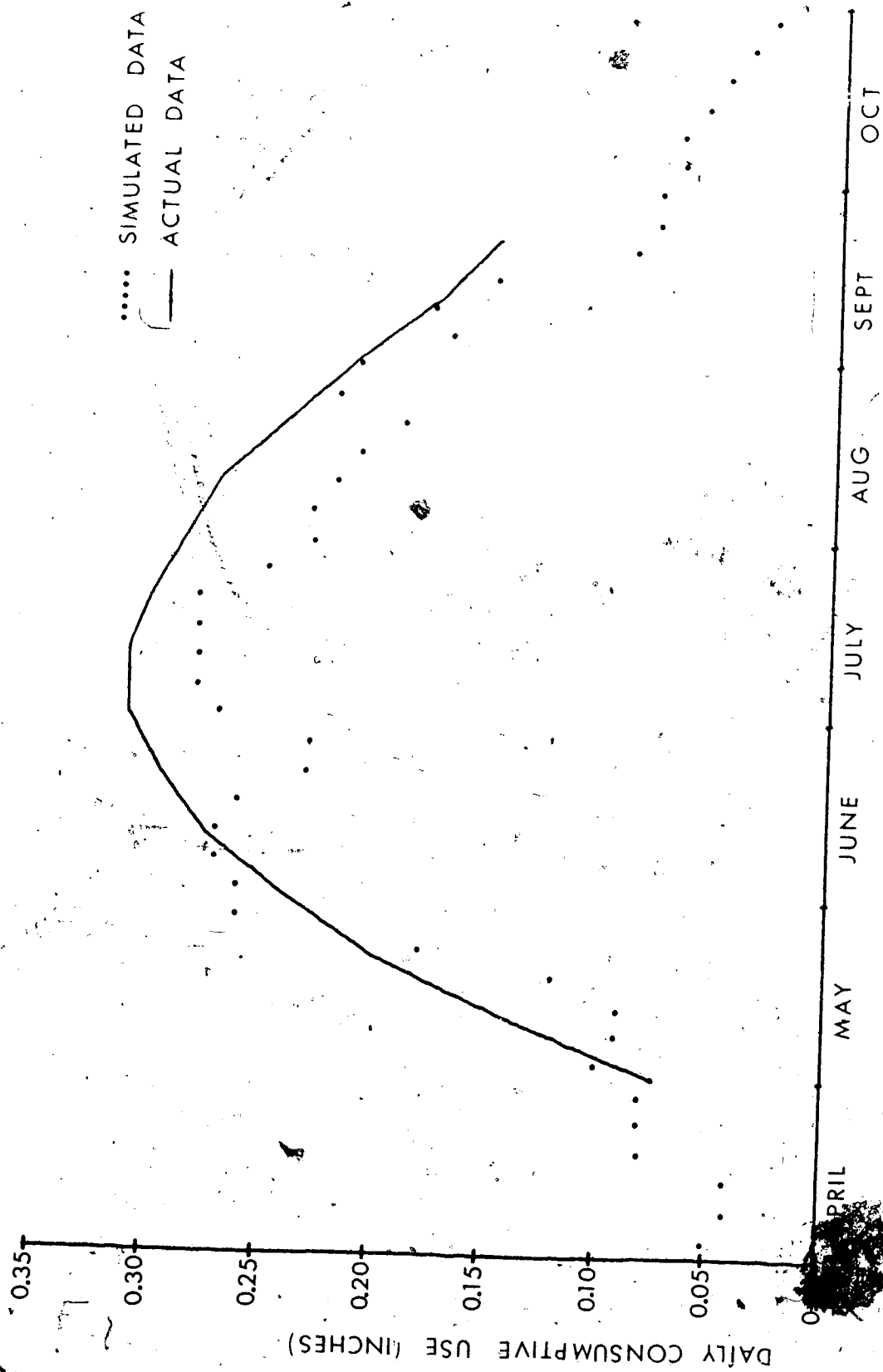


Figure 11. Comparison of actual and simulated daily consumptive use averages for Sugar Beets.





12. Comparison of actual and simulated daily consumptive averages for Alfalfa.

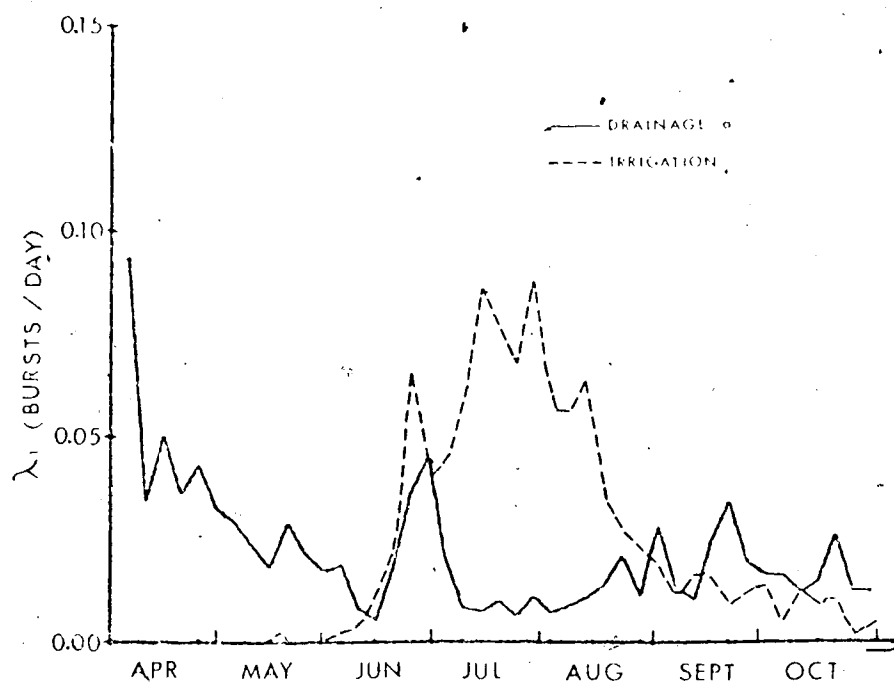


Figure 13a.  $\lambda_1$  curves for Wheat.

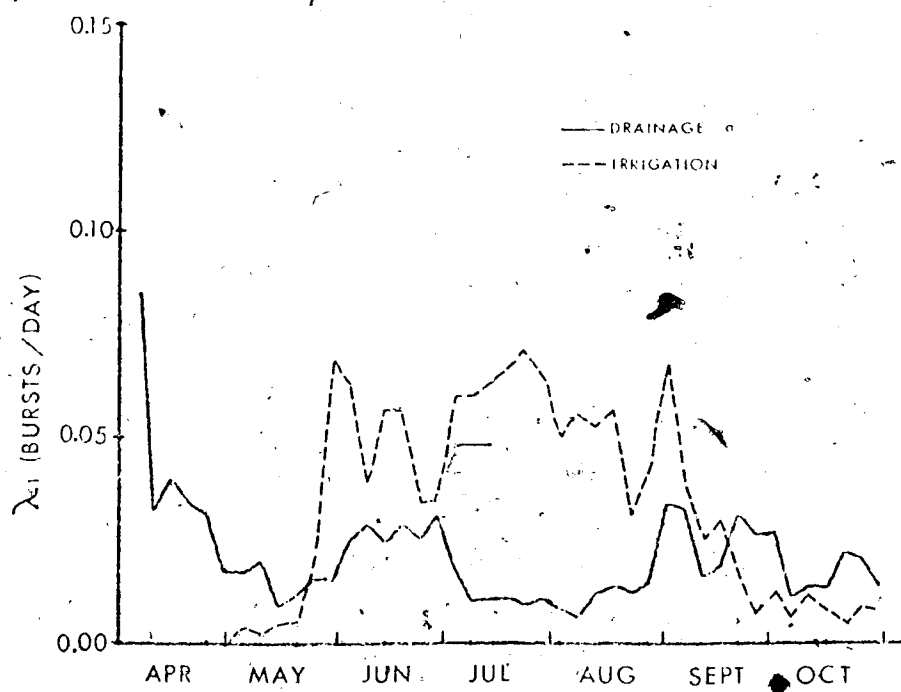


Figure 13b.  $\lambda_1$  curves for Alfalfa.

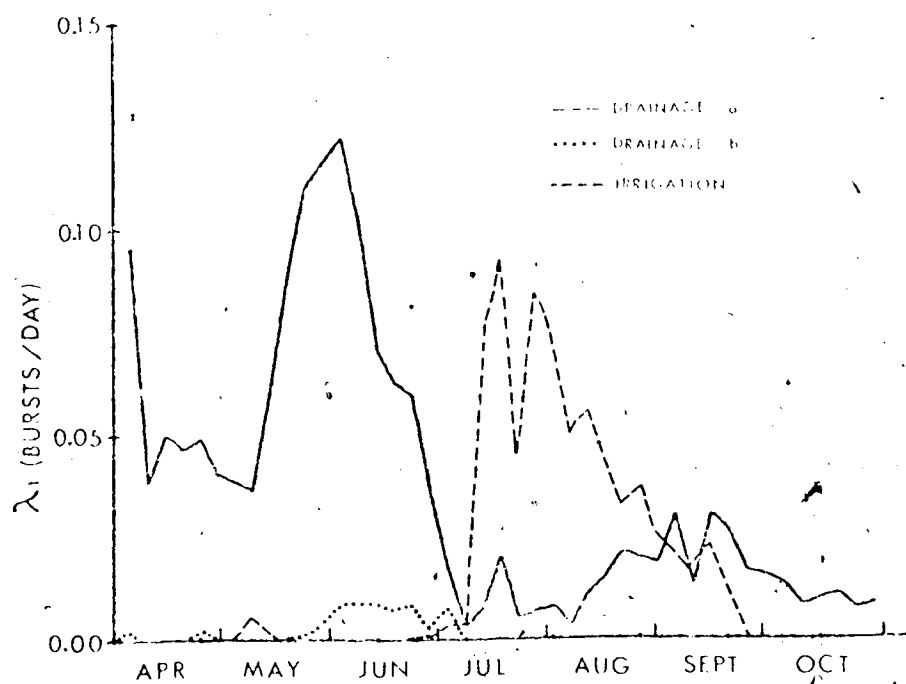


Figure 13c.  $\lambda_1$  curves for Potatoes.

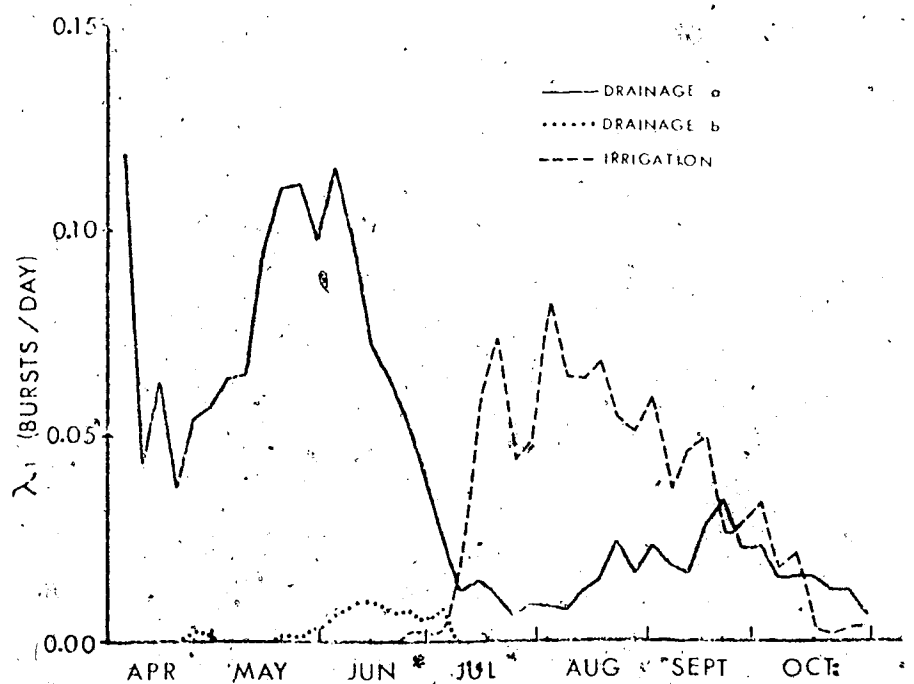


Figure 13d.  $\lambda_1$  curves for Sugar Beets.

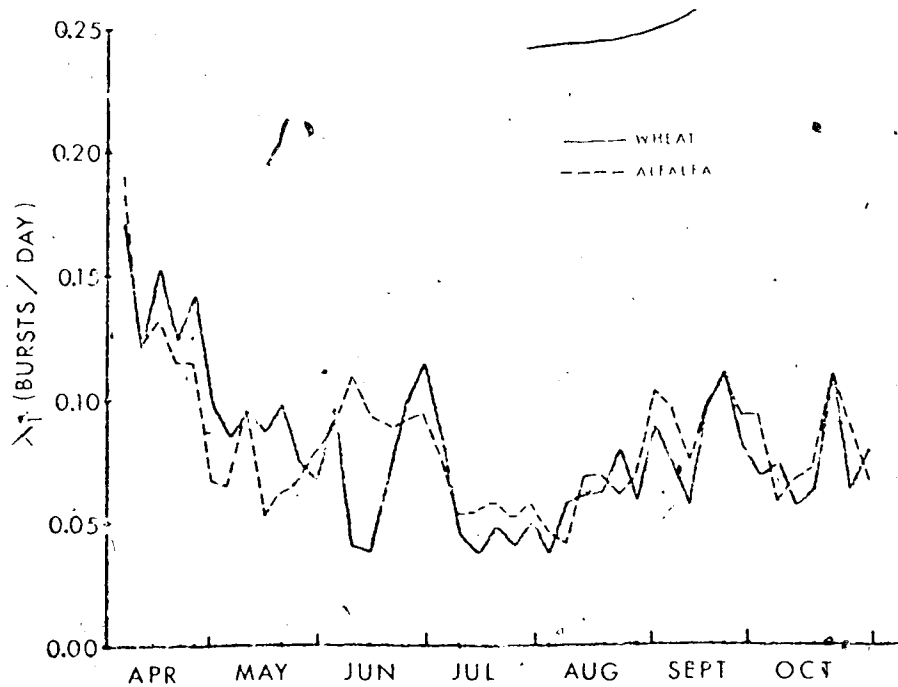


Figure 14a. Standard deviation of the  $\lambda_1$  curves for Wheat and Alfalfa.

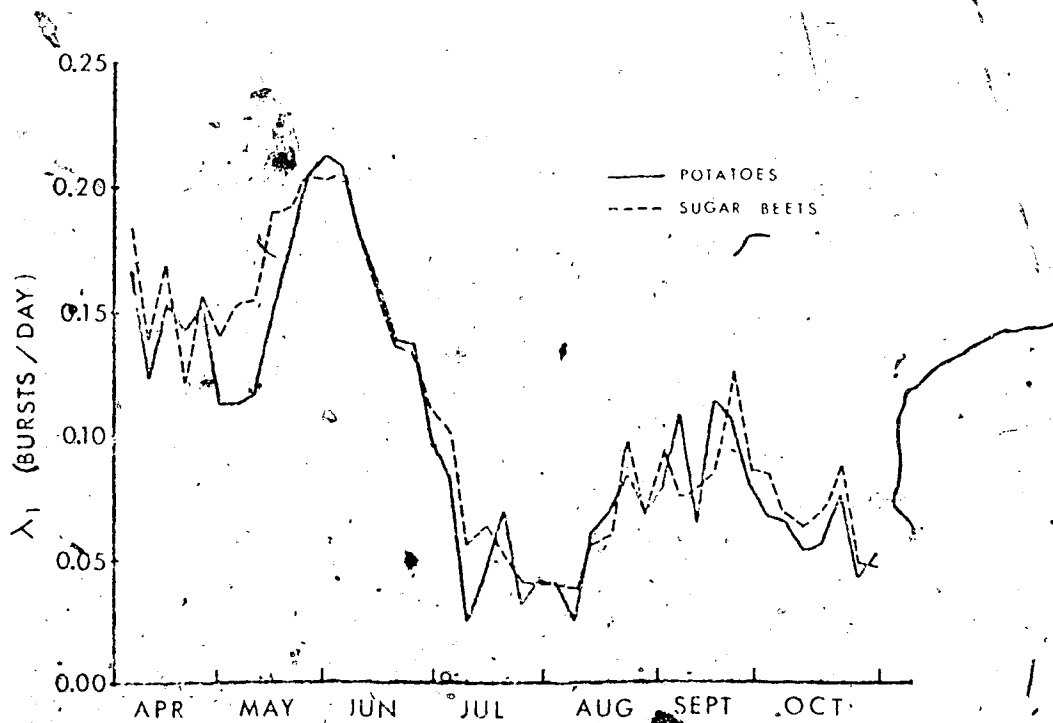


Figure 14b. Standard deviation of the  $\lambda_1$  curves for Potatoes and Sugar Beets.

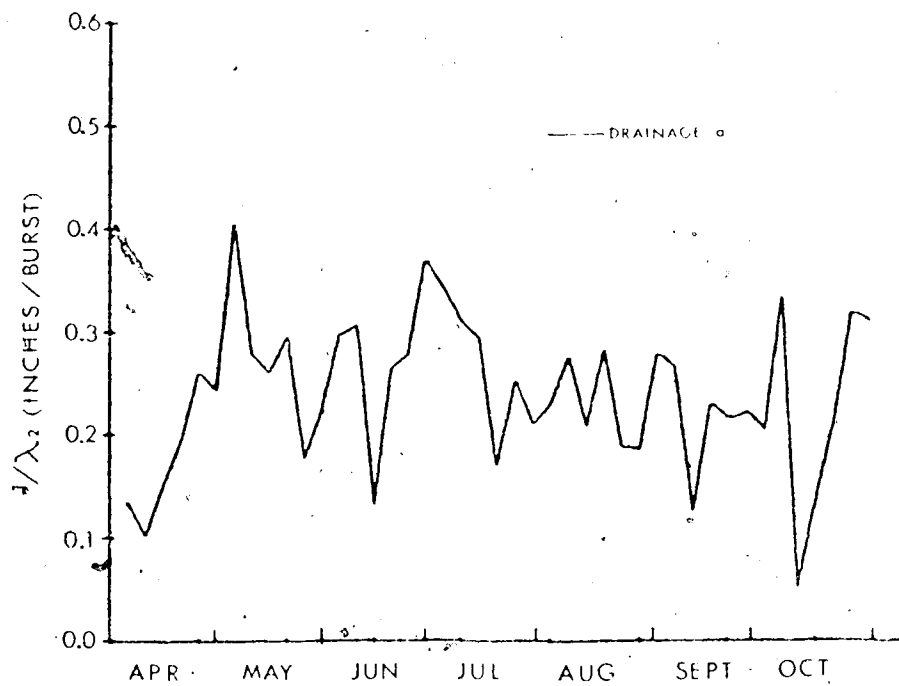


Figure 15a.  $1/\lambda_2$  curve for Wheat.

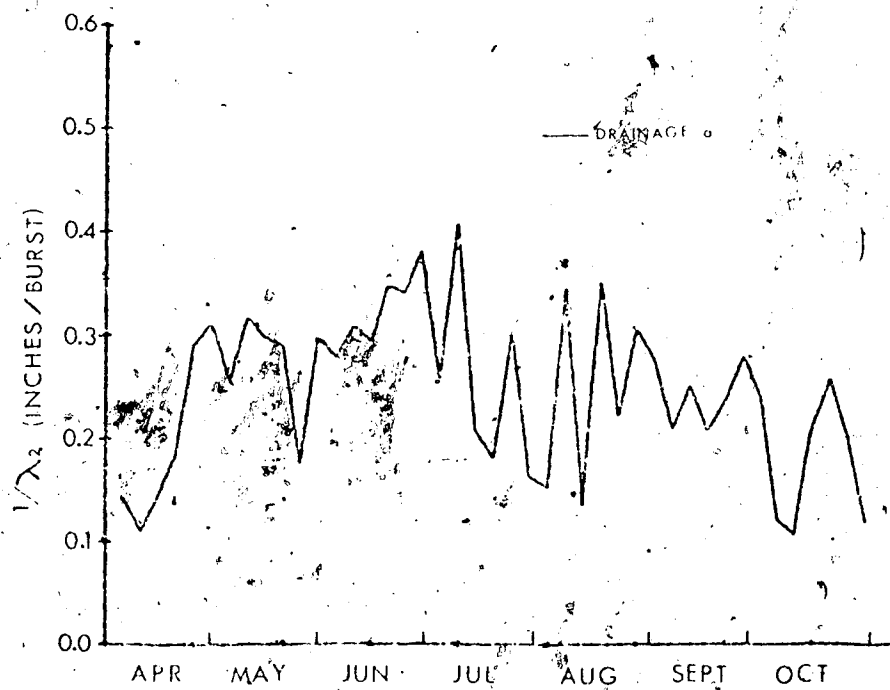


Figure 15b.  $1/\lambda_2$  curve for Alfalfa.

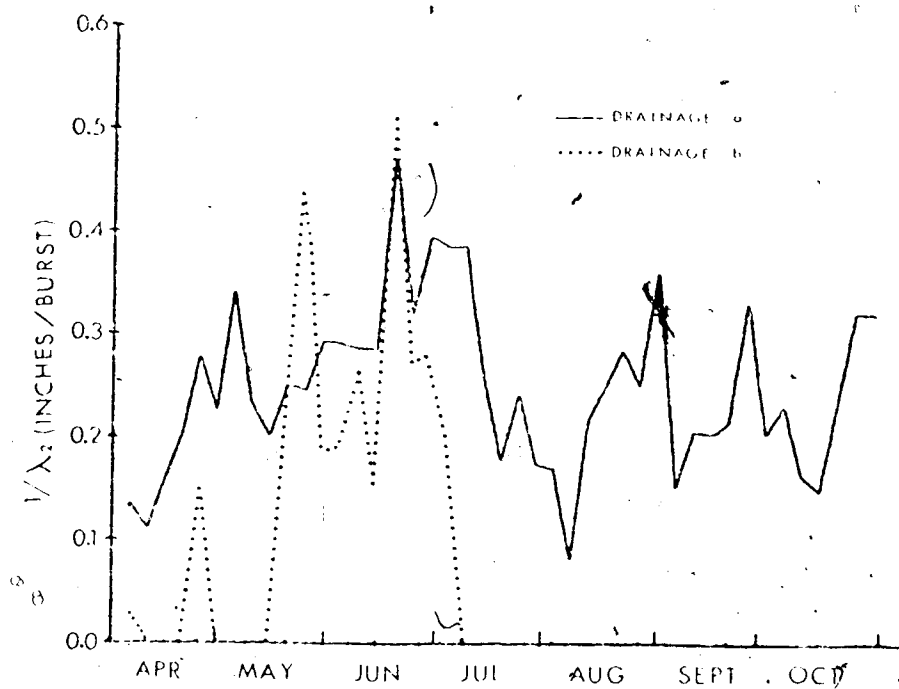


Figure 15c.  $1/\lambda_2$  curves for Potatoes.

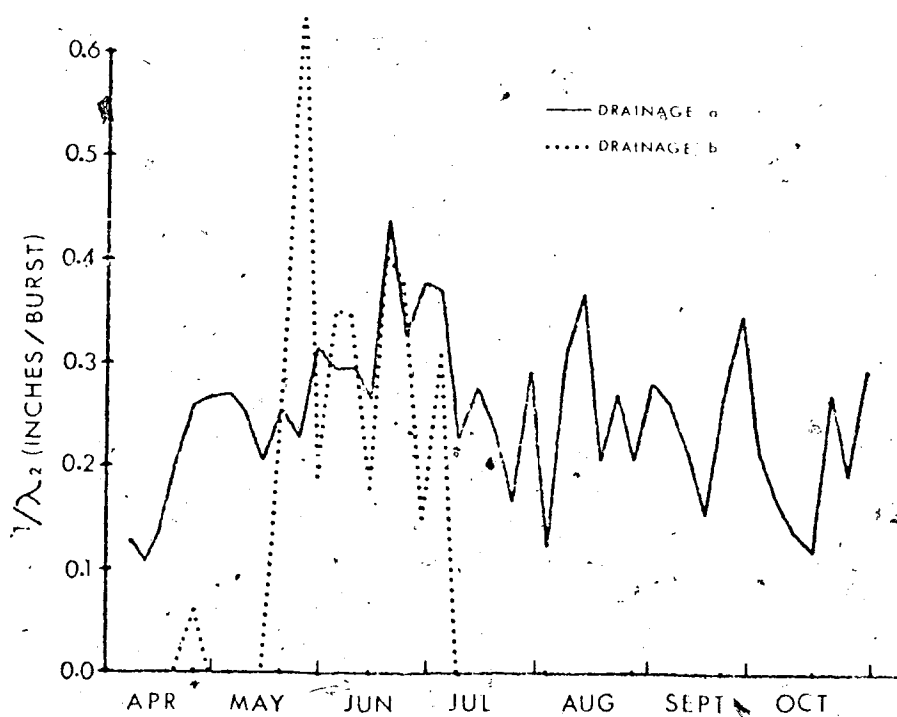


Figure 15d.  $1/\lambda_2$  curves for Sugar Beets.

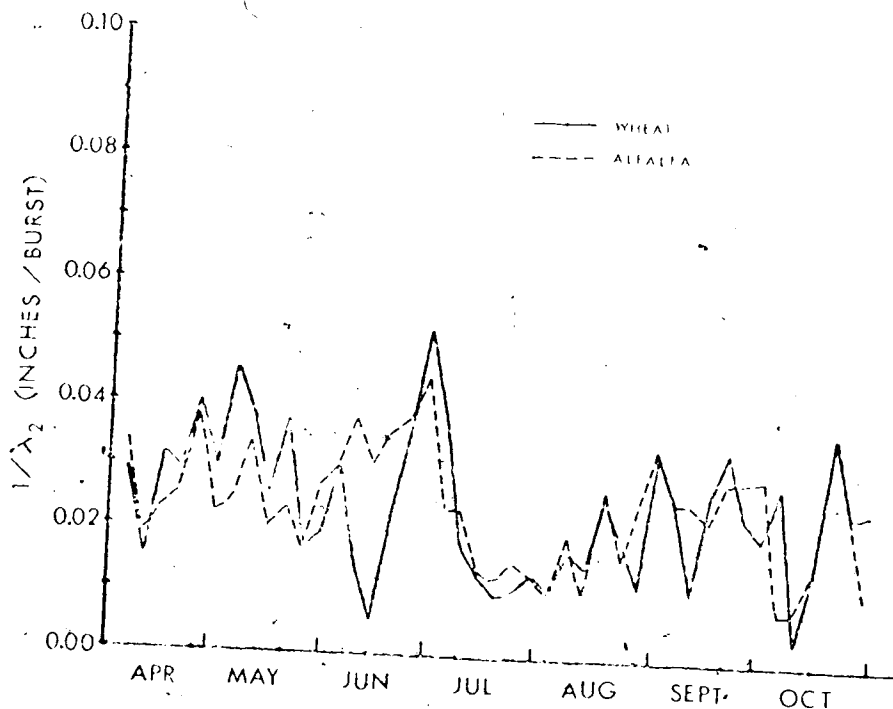


Figure 16a. Standard deviation of the  $1/\lambda_2$  curves for Wheat and Alfalfa.

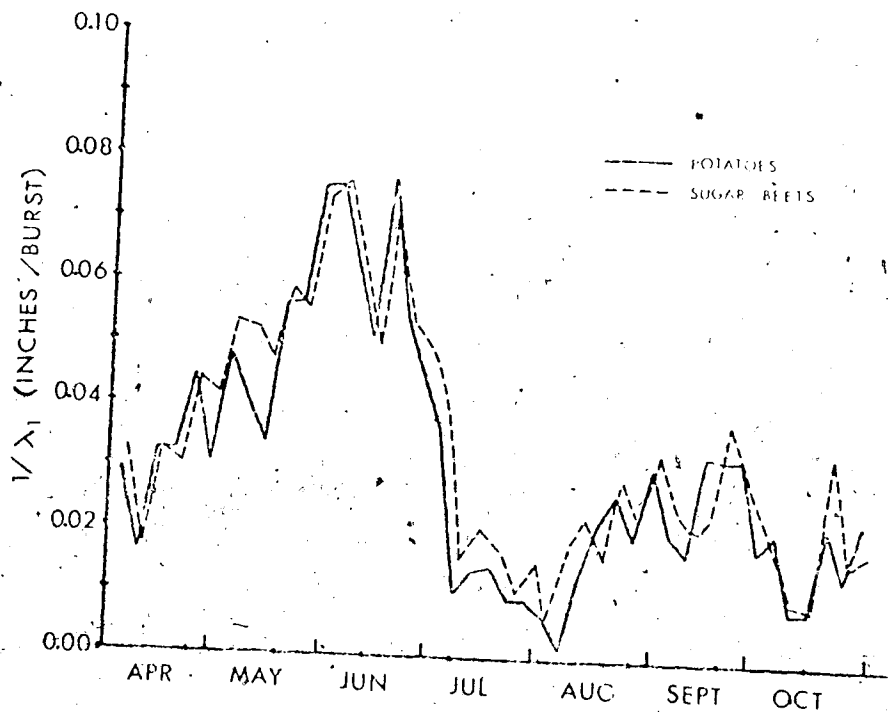


Figure 16b. Standard deviation of the  $1/\lambda_1$  curves for Potatoes and Sugar Beets.

### 7.2.2 Drainage: $\lambda_2$ Parameters.

An examination of the  $1/\lambda_2$  curves (figures 15a to 15d) indicate that the amount of drainage was much more variable than the occurrence of drainage. No distinct seasonal trends prevailed, however.

The  $1/\lambda_2$  curves maintained constant average values of approximately 0.25 inches per burst throughout the months of May and June and then gradually decreased to 0.20 inches from July to October. During the month of June, however, the yield per burst appears to reach average values of between 0.30 and 0.35 inches for most of the crops except Wheat. This apparently is the result of the fact that the  $1/\lambda_2$  curve for rainfall peaks during the same month and therefore effects a small increase in the amount of drainage.

The variability of the drainage yields between the values of 0.20 and 0.30 inches for all of the four crops corresponds to the average values of rainfall yield as illustrated in figure 8b. In other words, since the amount of drainage apparently is unaffected by consumptive use rates, it may be assumed, therefore, that it is affected by the amount of rainfall the soil receives. An examination of all the  $1/\lambda_2$  curves yields the speculation that the drainage curves follow the same general trend as do the precipitation curves.

Figures 16a and 16b show the seasonal behavior of the standard deviation for the  $1/\lambda_2$  curves for all four crops.



Except for the months of May and June, the standard deviations approximate each other fairly closely. A comparison of the average daily consumptive use curves for Potatoes and Sugar Beets (figures 10 and 11) shows that the values are approximately identical from April to June. Consequently, it can be expected that the mean and the standard deviations of the amount of drainage to be approximately identical. A similar comparison for Wheat and Alfalfa (figures 9 and 12) shows that although there is a large discrepancy in the consumptive use curves during May and June, there is relatively little discrepancy in their respective  $1/\lambda_2$  curves. The discrepancy, however, does show up in the standard deviations curves. The difference between the consumptive use curves for Wheat and Alfalfa and Potatoes and Sugar Beets is quite marked during May and June. However, this difference is not reflected to any great degree in the  $1/\lambda_2$  curves but is very pronounced in the standard deviation curves.

From the above comparisons, it can be concluded that the daily consumptive use rates have much more influence in determining the daily variability rather than the mean drainage yields. The daily consumptive use rates determine the variability of the drainage amounts whereas the daily rainfall amounts will determine the upper limit of the amount of daily drainage. Therefore, a shallow rooted crop, because it exhibits lower consumptive use rates during May and June, will not exhibit higher average drainage yields.

but will exhibit a higher range over which the drainage yields can vary. In general, the long term drainage yield will correspond to the average rainfall amount whereas the variability of individual drainage bursts will be determined by the daily consumptive use rates of the crop in question.

### 7.2.3 Irrigation Parameters

The  $\lambda_1$  curves for irrigation are plotted as dashed lines in figures 13a to 13d so that comparisons between drainage and irrigation can be made. Examination of the irrigation  $\lambda_1$  curves indicate that the maximum concentration of irrigation occurs during July and August for most of the crops. Alfalfa, however, shows that irrigation is more or less constant from June to September. This is probably due to the fact that Alfalfa has the highest total consumptive use over the entire growing season. Wheat, Potatoes, and Sugar Beets are irrigated mainly during July and August when the amount and the occurrence of precipitation is low, the consumptive use rates are maximum and the chance of drainage is minimal.

### 7.2.4 Drainage on Unirrigated Soil

Figures 13 and 15 also show the behaviour of the  $\lambda_1$  and the  $1/\lambda_2$  parameters of drainage for crops which have not been irrigated. No drainage problems for both Wheat and Alfalfa existed whereas Potatoes and Sugar Beets did show slight problems during June and part of July. The amount of drainage water tended to average about the same with or without irrigation. This is shown by the variation in the

1/2 curves. Hence, it can be concluded that irrigation water, even though it is applied at the exact instance the soil deficit reaches the 50 percent level, contributes substantially to the drainage problems of irrigated soils.

### 7.3 Irrigation Lapse Times.

The probability curves presented in figures 17 to 20 represent the cumulative probability distribution of the irrigation lapse times for each individual irrigation and crop. An irrigation lapse time is defined as that interval of time, in days, between the beginning of an interval to an irrigation day. The beginning of the interval, in this case, was selected as April 1st. The difference between the nth irrigation and April 1st is called the lapse time.

The curves were derived in the usual manner of constructing frequency distributions. The dates for each individual irrigation and for each crop were stored in a frequency table from which cumulative probabilities were calculated according to the following plotting position.

$$P_k = \frac{\sum_{i=1}^k n_i}{N + 1}$$

where:

$P_k$  = cumulative probability of the kth item  
 $n_i$  = absolute frequency of the ith item  
 $N$  = total sum of all absolute frequencies

The cumulative probabilities for irrigation dates were calculated and tabulated during the simulation run and then plotted on normal probability paper as shown in figures 17 to 20.

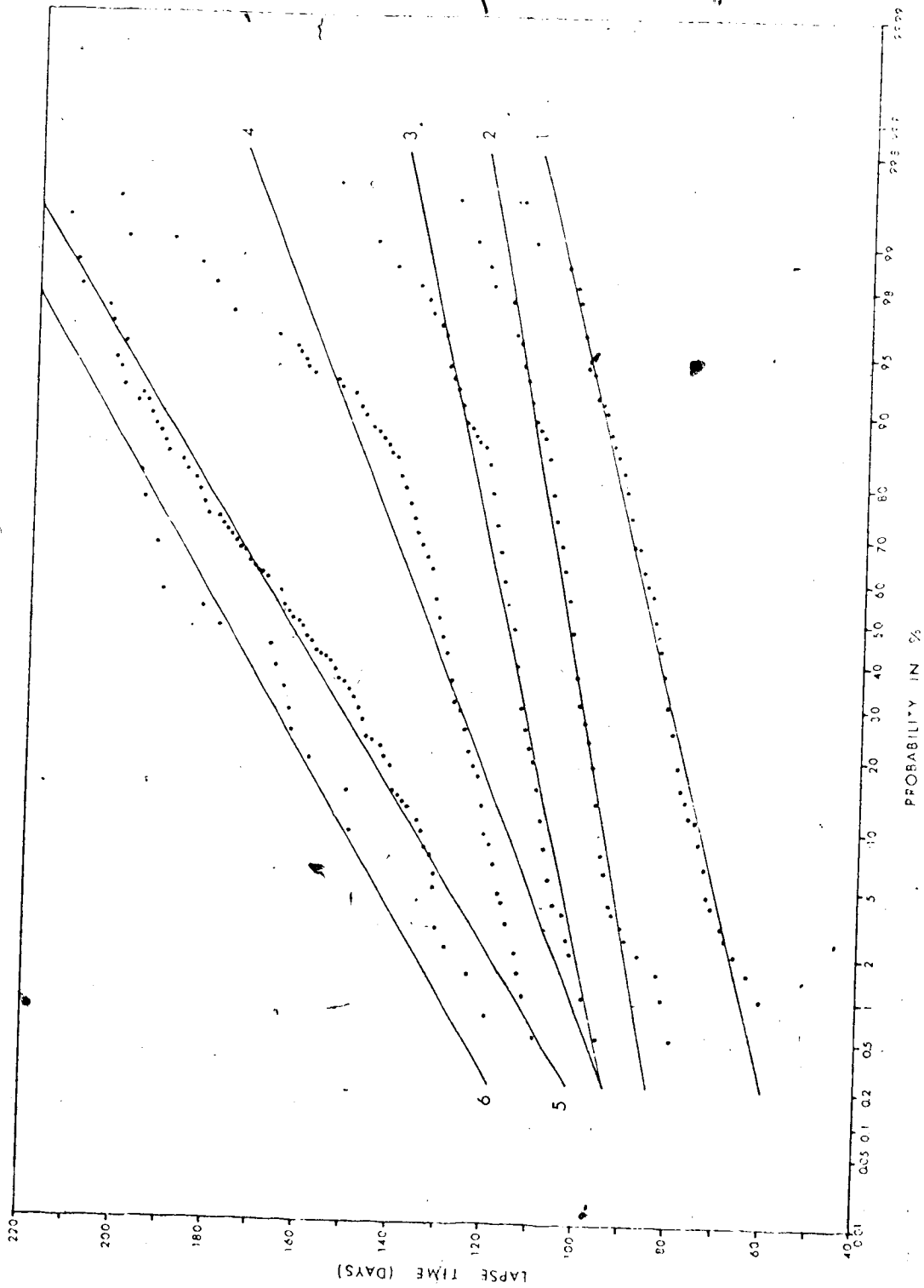


Figure 17. Cumulative distribution of irrigation lapse dates for Wheat.

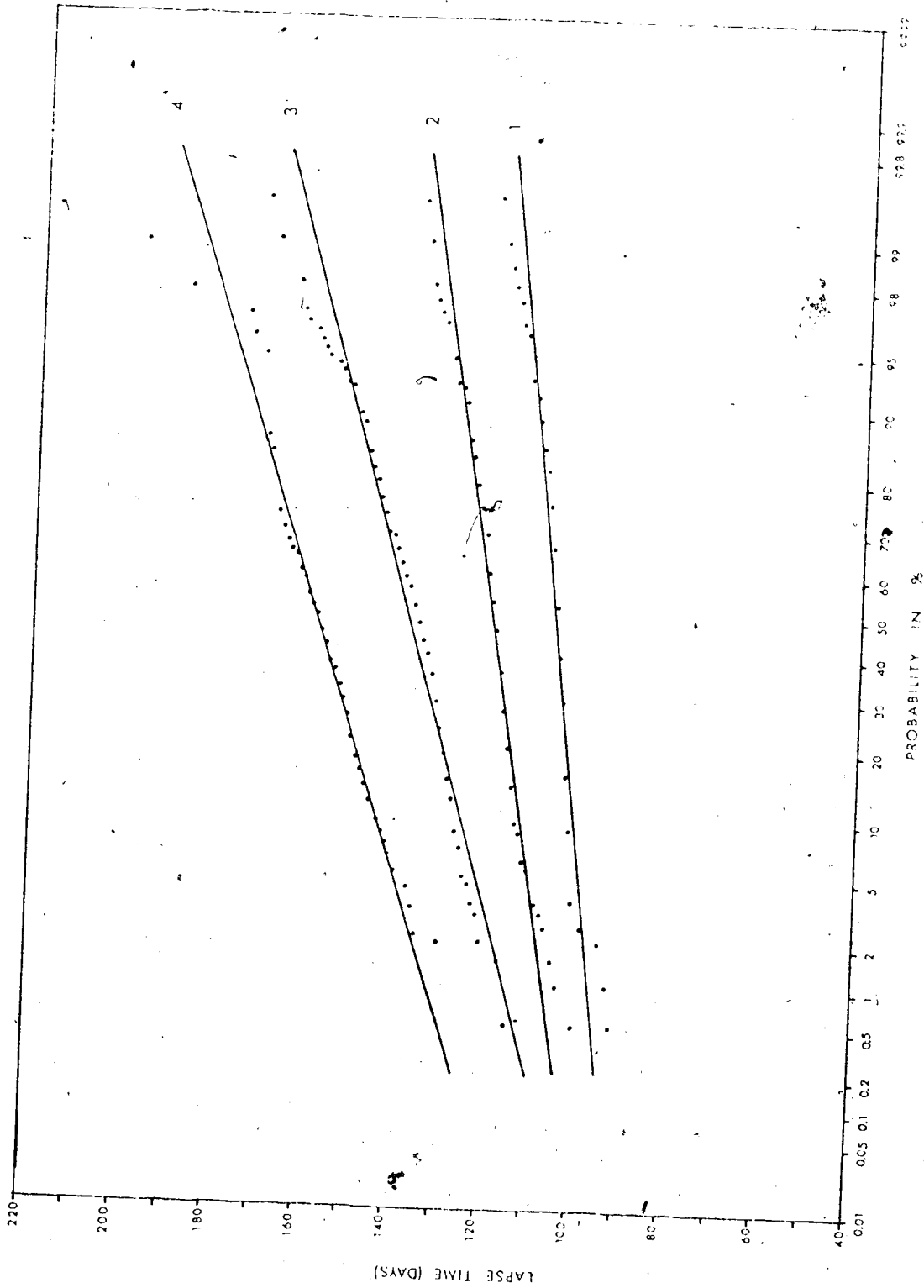


Figure 18: Cumulative distribution of irrigation lapse dates for Potatoes.

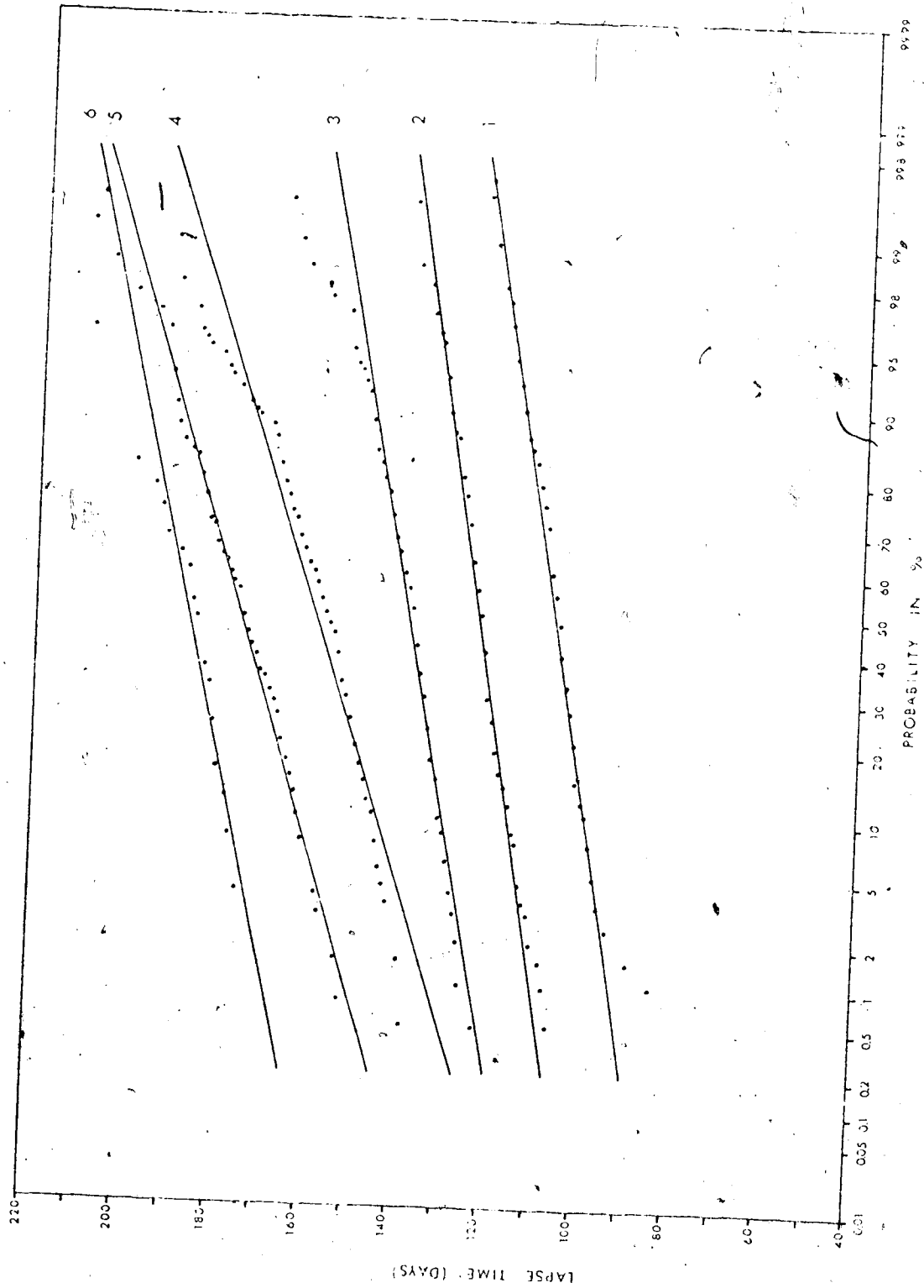


Figure 19. Cumulative distribution of irrigation lapse dates for Sugar Beets.

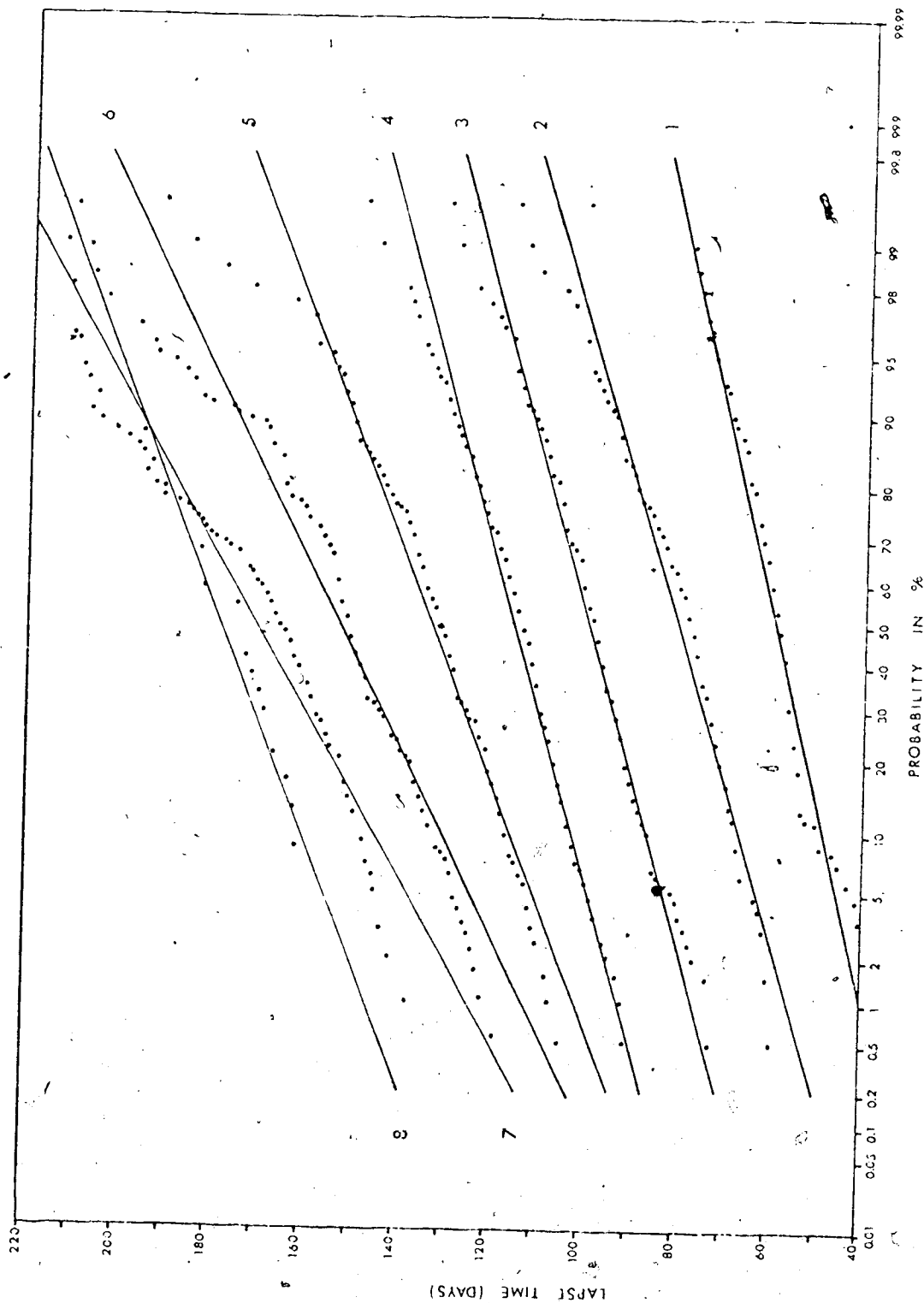


Figure 20. Cumulative distribution of irrigation lapse dates for Alfalfa.

TABLE 11. DESCRIPTION OF THE IRRIGATION PROBABILITY CURVES.

| Crop        | Irrigation Number | N   | Prob. | Mean Date | St. Dev. of Date |
|-------------|-------------------|-----|-------|-----------|------------------|
| Wheat       | 1                 | 200 | 100.0 | June 25   | 8.9              |
|             | 2                 | 200 | 100.0 | July 13   | 6.8              |
|             | 3                 | 200 | 100.0 | July 26   | 8.2              |
|             | 4                 | 194 | 97.0  | Aug 13    | 14.3             |
|             | 5                 | 138 | 69.0  | Sept 12   | 22.1             |
|             | 6                 | 21  | 10.5  | Sept 26   | 21.0             |
|             | 7                 | 1   | 0.5   | Oct 8     | 0.0              |
| Potatoes    | -1                | 5   | 2.5   | May 10    | 1.2              |
|             | -2                | 1   | 0.5   | May 13    | 0.0              |
|             | -3                | 1   | 0.5   | June 23   | 0.0              |
|             | 1                 | 193 | 96.5  | July 15   | 4.1              |
|             | 2                 | 200 | 100.0 | July 29   | 5.8              |
|             | 3                 | 195 | 97.5  | Aug 17    | 10.0             |
|             | 4                 | 92  | 46.0  | Sept 6    | 11.2             |
|             | 5                 | 3   | 1.5   | Sept 15   | 0.6              |
| Sugar Beets | -1                | 1   | 0.5   | Apr 25    | 0.0              |
|             | -2                | 1   | 0.5   | May 28    | 0.0              |
|             | 1                 | 198 | 99.0  | July 16   | 6.3              |
|             | 2                 | 200 | 100.0 | Aug 2     | 5.9              |
|             | 3                 | 200 | 100.0 | Aug 18    | 7.4              |
|             | 4                 | 196 | 98.0  | Sept 6    | 11.6             |
|             | 5                 | 129 | 64.5  | Sept 23   | 11.0             |
|             | 6                 | 24  | 12.0  | Oct 4     | 7.6              |
| Alfalfa     | 1                 | 200 | 100.0 | May 30    | 8.6              |
|             | 2                 | 200 | 100.0 | June 20   | 10.6             |
|             | 3                 | 200 | 100.0 | July 9    | 10.0             |
|             | 4                 | 200 | 100.0 | July 25   | 10.4             |
|             | 5                 | 199 | 99.5  | Aug 12    | 13.9             |
|             | 6                 | 178 | 89.0  | Sept 1    | 17.4             |
|             | 7                 | 98  | 49.0  | Sept 19   | 19.9             |
|             | 8                 | 23  | 11.5  | Sept 26   | 14.0             |

- 1 preseason irrigation  
 2 irrigation during emergence  
 3 irrigation between emergence and flowering  
 - N too small for a distribution (curve not shown)



TABLE 12. SUMMARY OF THE SMIRNOV-KOLMOGOROV STATISTIC FOR THE IRRIGATION DISTRIBUTIONS.

| Crop        | Irrigation Number | N   | Statistic |      |
|-------------|-------------------|-----|-----------|------|
| Wheat       | 1                 | 200 | 0.065     | n.s. |
|             | 2                 | 200 | 0.080     | n.s. |
|             | 3                 | 200 | 0.130     | *    |
|             | 4                 | 194 | 0.140     | *    |
|             | 5                 | 138 | 0.070     | n.s. |
|             | 6                 | 21  | 0.155     | n.s. |
|             | -                 | 1   | -         | -    |
| Potatoes    | -1                | 5   | -         | -    |
|             | -2                | 1   | -         | -    |
|             | -3                | 1   | -         | -    |
|             | 1                 | 193 | 0.120     | **   |
|             | 2                 | 200 | 0.080     | n.s. |
|             | 3                 | 195 | 0.075     | n.s. |
|             | 4                 | 92  | 0.090     | n.s. |
|             | -                 | 3   | -         | -    |
| Sugar Beets | -1                | 1   | -         | -    |
|             | -2                | 1   | -         | -    |
|             | 1                 | 198 | 0.100     | **   |
|             | 2                 | 200 | 0.045     | n.s. |
|             | 3                 | 200 | 0.070     | n.s. |
|             | 4                 | 196 | 0.100     | **   |
|             | 5                 | 129 | 0.050     | n.s. |
|             | 6                 | 24  | 0.115     | n.s. |
| Alfalfa     | 1                 | 200 | 0.115     | **   |
|             | 2                 | 200 | 0.100     | **   |
|             | 3                 | 200 | 0.075     | n.s. |
|             | 4                 | 200 | 0.070     | n.s. |
|             | 5                 | 189 | 0.085     | n.s. |
|             | 6                 | 178 | 0.125     | *    |
|             | 7                 | 98  | 0.115     | n.s. |
|             | 8                 | 23  | 0.150     | n.s. |

- 1 preseason irrigation  
 2 irrigation during emergence  
 3 irrigation between emergence and flowering  
 - N too small for a distribution (curve not shown)

With each distribution curve there is associated a probability. For instance, for 200 of the 200 simulated years, Wheat received at least one irrigation each year, whereas, a total of five irrigations were performed for only 28 years. Therefore, the probability associated with the first and the fifth irrigation are 1.0 and 0.14 respectively. Table 11 lists the curve numbers with their respective probabilities. The table indicates that Wheat had at least three irrigations per season, Potatoes had two irrigations, Sugar Beets had three, and Alfalfa had four irrigations. In the case of Potatoes and Sugar Beets, the probabilities associated with the first irrigations are not 1.0 because of the fact that the conditions (i.e. the number of soil zones) upon which the irrigation dates were based were different during the early stages of growth than in the later stages of growth. In the drier years the first irrigation might have occurred when the roots occupied only the first four soil zones, whereas, in the wetter seasons, sufficient rainfall had permitted the roots to extend into the sixth zone prior to the first irrigation. Table 11 lists the total number of irrigations,  $N$ , the irrigation probability and the mean and standard deviation of the irrigation dates.

According to the probabilities, most of the first irrigations had occurred after the roots had entered the sixth zone. This corresponds to the approximate dates of June 25 and June 5 for Potatoes and Sugar Beets

TABLE 13. IRRIGATION DATES WITH PROBABILITY EQUAL OR LESS THAN . - WHEAT.

| Irrigation<br>Number | 0.05    | 0.10    | 0.20    | 0.30    | 0.40    | 0.50    | 0.60    | 0.70    | 0.80    | 0.90    | 0.95    |
|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1                    | June 10 | June 14 | June 18 | June 20 | June 23 | June 25 | June 27 | June 30 | July 3  | July 7  | July 10 |
| 2                    | July 2  | July 4  | July 8  | July 10 | July 12 | July 13 | July 15 | July 17 | July 19 | July 21 | July 23 |
| 3                    | July 13 | July 16 | July 20 | July 22 | July 25 | July 27 | July 29 | July 31 | Aug 3   | Aug 6   | Aug 11  |
| 4                    | July 20 | July 26 | Aug 1   | Aug 5   | Aug 9   | Aug 13  | Aug 16  | Aug 20  | Aug 25  | Aug 31  | Sept 5  |
| 5                    | Aug 7   | Aug 15  | Aug 25  | Aug 31  | Sept 6  | Sept 12 | Sept 18 | Sept 23 | Oct 1   | Oct 10  | Oct 18  |
| 6                    | Aug 22  | Aug 30  | Sept 8  | Sept 15 | Sept 21 | Sept 26 | Oct 1   | Oct 7   | Oct 13  | Oct 23  | Oct 30  |

TABLE 14. IRRIGATION DATES WITH PROBABILITY EQUAL OR LESS THAN - POTATOES.

| Irrigation<br>Number | 0.05    | 0.10    | 0.20    | 0.30    | 0.40    | 0.50    | 0.60    | 0.70    | 0.80    | 0.90    | 0.95    |
|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1                    | July 9  | July 10 | July 12 | July 13 | July 15 | July 16 | July 17 | July 18 | July 19 | July 21 | July 22 |
| 2                    | July 20 | July 22 | July 24 | July 26 | July 28 | July 29 | July 31 | Aug 1   | Aug 3   | Aug 6   | Aug 8   |
| 3                    | July 31 | Aug 4   | Aug 8   | Aug 11  | Aug 14  | Aug 16  | Aug 19  | Aug 22  | Aug 25  | Aug 29  | Sept 2  |
| 4                    | Aug 18  | Aug 22  | Aug 27  | Aug 31  | Sept 3  | Sept 6  | Sept 9  | Sept 13 | Sept 15 | Sept 20 | Sept 24 |

TABLE 15. IRRIGATION DATES WITH PROBABILITY EQUAL OR LESS THAN - SUGAR BEETS.

| Irrigation Number | 0.05    | 0.10    | 0.20    | 0.30    | 0.40    | 0.50    | 0.60    | 0.70    | 0.80    | 0.90    | 0.95    |
|-------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1                 | July 6  | July 8  | July 11 | July 13 | July 15 | July 16 | July 18 | July 19 | July 21 | July 24 | July 26 |
| 2                 | July 23 | July 25 | July 27 | July 29 | July 31 | Aug 2   | Aug 3   | Aug 5   | Aug 7   | Aug 9   | Aug 11  |
| 3                 | Aug 6   | Aug 8   | Aug 11  | Aug 13  | Aug 15  | Aug 17  | Aug 19  | Aug 21  | Aug 23  | Aug 26  | Aug 29  |
| 4                 | Aug 19  | Aug 23  | Aug 28  | Sept 1  | Sept 4  | Sept 7  | Sept 10 | Sept 13 | Sept 16 | Sept 22 | Sept 26 |
| 5                 | Sept 5  | Sept 9  | Sept 14 | Sept 17 | Sept 20 | Sept 23 | Sept 26 | Sept 29 | Oct 2   | Oct 7   | Oct 11  |
| 6                 | Sept 21 | Sept 24 | Sept 28 | Sept 30 | Oct 2   | Oct 4   | Oct 6   | Oct 8   | Oct 11  | Oct 14  | Oct 17  |

TABLE 16. IRRIGATION DATES WITH PROBABILITY EQUAL OR LESS THAN - ALFALFA.

| Irrigation Number | 0.05    | 0.10    | 0.20    | 0.30    | 0.40    | 0.50    | 0.60    | 0.70    | 0.80    | 0.90    | 0.95    |
|-------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1                 | May 16  | May 19  | May 23  | May 25  | May 28  | May 31  | June 2  | June 4  | June 6  | June 10 | June 18 |
| 2                 | June 3  | June 6  | June 11 | June 15 | June 18 | June 21 | June 23 | June 26 | June 29 | July 4  | July 8  |
| 3                 | June 23 | June 26 | July 1  | July 4  | July 7  | July 9  | July 11 | July 14 | July 17 | July 22 | July 25 |
| 4                 | July 8  | July 12 | July 17 | July 20 | July 22 | July 25 | July 28 | July 30 | Aug 2   | Aug 7   | Aug 10  |
| 5                 | July 20 | July 25 | July 31 | Aug 5   | Aug 8   | Aug 12  | Aug 15  | Aug 19  | Aug 23  | Aug 29  | Sept 3  |
| 6                 | Aug 3   | Aug 9   | Aug 17  | Aug 23  | Aug 27  | Aug 31  | Sept 5  | Sept 10 | Sept 15 | Sept 23 | Sept 30 |
| 7                 | Aug 17  | Aug 24  | Sept 2  | Sept 8  | Sept 13 | Sept 18 | Sept 23 | Sept 29 | Oct 5   | Oct 14  | Oct 21  |
| 8                 | Sept 3  | Sept 8  | Sept 14 | Sept 19 | Sept 22 | Sept 26 | Sept 29 | Oct 3   | Oct 8   | Oct 14  | Oct 18  |

respectively. These dates are taken from table 10. Because there were so few irrigations prior to these dates (Potatoes - 7 and Sugar Beets - 2) these irrigations were not plotted.

As can be seen from Figures 17 to 20, the plotted points followed fairly straight lines on normal probability paper. Thus, a Chi-squared test was performed to test the assumption that the irrigation dates followed a normal function. All were found to be highly significant. Therefore, it was decided to perform a Smirnov-Kolmogorov distribution free test on the data. Only seven of the 24 distributions were found to be significantly different. Table 12 lists the Smirnov-Kolmogorov statistic.

Because of the fact that an irrigator considers the type of theoretical distribution to be irrelevant, it was felt that the lines, as depicted by the means and standard deviation, would serve the purpose of characterizing the irrigation distributions. Tables 13 to 16 list the cumulative probabilities and their respective irrigation dates in tabular form. A broad spectrum of probability levels was used in an attempt to consider as many different types of weather patterns to which these computations might be relevant. For instance, the low levels of irrigation probabilities may be relevant during years in which the season is exceptionally dry, whereas, the high levels may be of greater interest during excessively wet seasons.

#### 7.4 Summary of results.

A summary of the results are listed below.

1. Irrigation contributes significantly to drainage problems. Wheat and Alfalfa experienced peak drainage rates of 0.05 and 0.03 bursts per day with irrigation and zero drainage rates without irrigation. Similarly, Potatoes and Sugar Beets exhibited peak drainage rates of 0.125 and 0.12 bursts per day with irrigation compared to only 0.01 bursts per day without irrigation.
2. Irrigation water is mainly applied during July and August. Dry seasons will require post-season irrigations. Irrigation should not be performed during May and June for the shallow rooted crops.
3. Drainage problems are more critical for shallow rooted crops during the early growth stages than during later stages. May and June have the highest drainage rates of approximately 0.125 bursts per day with a standard deviation of 0.20 bursts per day. In other words, drainage problems can occur every 3 to 13 days with an average of an 8 day return period. The variability of rainfall plus low consumptive use rates during these months are the major causes of drainage problems.

4. The amount of daily rainfall determines the upper limit of the daily drainage amounts.
5. The daily consumptive use rates determine the actual daily amounts of drainage. High consumptive use rates will decrease drainage yields whereas low consumptive use rates will increase drainage yields.
6. The daily rate of increase of consumptive use has a profound influence on the rate of occurrence of drainage. Wheat and Alfalfa averaged a daily rate of increase of 0.004 inches and had a peak drainage rate of 0.05 bursts per day while Potatoes and Sugar Beets averaged 0.003 inches but had a peak drainage rate of 0.125 bursts per day during May and June.
7. The average rate of drainage is affected only slightly by the individual daily rates of consumptive use.
8. The rate of occurrence of drainage is highest during May and June for shallow rooted crops.
9. All crops experienced the least drainage problems during the latter half of July. The occurrence of drainage averaged 0.01 burst per day (100 days per burst) with an average deviation of 0.05 bursts per day (20 days per burst). The yield per drainage was about 0.20

inches per burst plus or minus 0.01 inches per  
burst.



## 8. Conclusions.

The main objective of this study was to develop an irrigation and a crop growth simulation model which could be used as a tool to obtain information regarding the behaviour of soil drainage to weather and to different crops. Incorporated into the model were theoretical distributions of rainfall and potential evapotranspiration and conditional probabilities of rainy and non-rainy days. A model of consumptive use was employed to determine crop water use according to the water extraction patterns of the roots and the dryness curves of the soil. Soil moisture conditions under four crops were thus simulated over a period of 200 years.

Actual weather records for Lethbridge, Alberta, were used to develop the weather model for the simulation. It was found that both the rainfall amounts and the rainfall probabilities were dependent upon the time of the year. Furthermore, rainfall amounts of less than 0.10 inch constituted a significant portion of each rainfall distribution during the season. The rainfall probabilities showed definite seasonal trends and were considered to be important in simulating weather.

The weather model was run on the computer and 45 years of simulated data were shown to compare favorably with actual data for Lethbridge. It was concluded that the best method of comparing actual and simulated rainfall was to compare their  $\lambda_1$  and  $1/\lambda_2$  parameters. Although the

correlation between the actual and simulated was not substantially high, the standard error of estimate was very small indicating that the average fluctuation between the actual and the simulated values was insignificant.

The Versatile Soil Moisture Budget was used to calculate daily consumptive use. The accuracy of this model was found to be mainly dependant upon the selection of the K-coefficients. Manipulation of the K-coefficients in order that the proper average consumptive use curves might be assumed proved to be extremely difficult and time consuming. On the other hand, to adjust the coefficients so that the simulated soil moisture content coincided with actual field data proved to be rather easy. However, it was felt that this latter method would not be sufficiently accurate in a Monte Carlo model which requires long term average values. Therefore, it was concluded that the Versatile Soil Moisture Budget can be used in a Monte Carlo model to provide the basic crop variables provided that the K-coefficients are selected so that local long term average consumptive use curves are simulated.

Probability distributions of irrigation lapse dates were obtained from the model for each irrigation and each crop. From the slopes of the distributions, it was concluded that at least the first two irrigation dates for each crop were relatively uninfluenced by wet and dry years. This is illustrated by the shallow slopes of the distribution lines. The dates of the latter most

irrigations were substantially influenced by wet and dry years. In these cases, steeper slopes indicating larger variability are prevalent. Due to the high consumptive use rates, the variability of irrigations and thus the slopes of the distribution lines are minimum during June and July. In September and October, when consumptive use is low, rainfall contributes more to the soil moisture thereby increasing the variability of irrigation dates and increasing the slopes of the distributions. An irrigator, through the use of such probability curves, could decide the approximate date of irrigation provided he knows the cumulative amount of rainfall from April 1st to the present date.

The  $\lambda_1$  and  $1/\lambda_2$  curves and their respective standard deviations provided a means of investigating the behavior of soil drainage under the influence of irrigation, consumptive use and rainfall. Moreover, it was shown that drainage was a direct result of irrigation practices and not rainfall. Little or no drainage was observed when irrigation practices were not simulated. These curves also suggested that the shallow rooted crops are more susceptible to over-irrigation than deep rooted crops during the early growth stages. As the crop matures the risk of damaging a crop decreases. Furthermore, the standard deviation of the  $1/\lambda_2$  curves suggest that the amount of water which drains from the soil is dependant on crop consumptive use during the early growth stages. It therefore was concluded that the  $\lambda_1$  and  $1/\lambda_2$  curves are a valuable method of viewing the trend of both

drainage and rainfall.

## 9. Recommendations.

1. The accuracy of the daily consumptive use model could undoubtedly be improved with the use of K-coefficients which could better approximate the average consumptive use curves for each crop. Selection of the K-coefficients should be based upon more up to date experimentally determined consumptive use curves. Hence, research regarding water use for various crops is needed.
2. A better method of determining planting dates based on rainfall, temperature, and soil moisture conditions should be developed in order to make the length of the growing season a variable in accordance with the weather.
3. The length of each crop growth stage is, in reality, affected by the soil moisture conditions and the weather. A method of varying each stage of growth according to the amount of rainfall received and the potential evapotranspiration should be developed. This ability would enhance the effectiveness of the K-coefficients to simulate daily consumptive use.
4. The possibility of obtaining probabilities of the number of rainy days and the number of drainage periods within a given time interval should be investigated. As well, the probability of the total amount of rainfall and drainage within a given time period should also be obtained.
5. The simulation model should be extended to include other major crops, different soil moisture capacities, different soil types and different localities.

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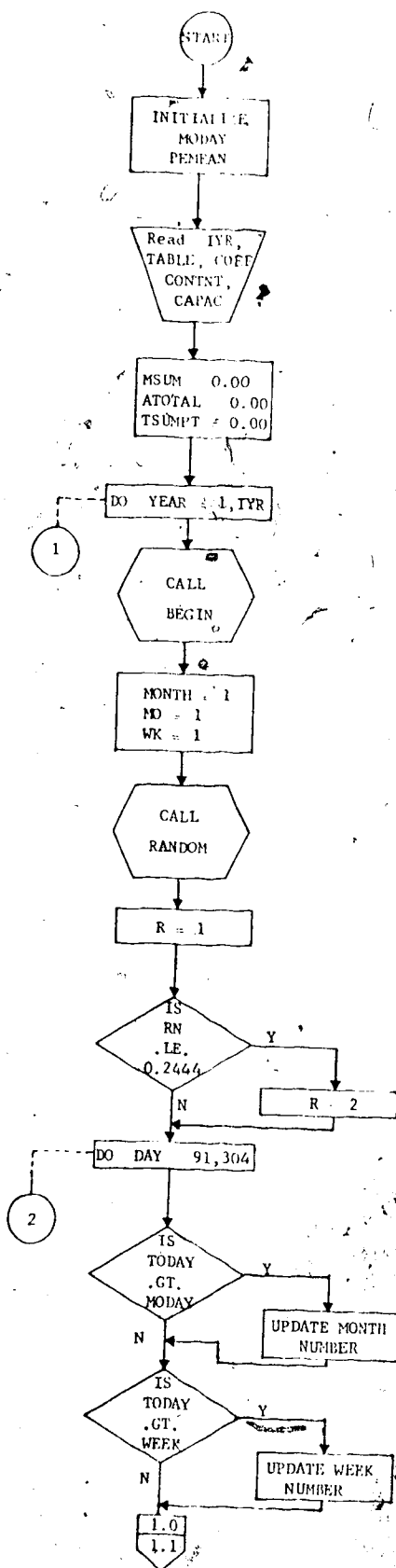
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## Appendix A

The cropping model was written in FORTRAN - G language. It consists of a main program and ten subroutines. One subroutine each is devoted to the rainfall and the P.E. models, one to the overwinter precipitation model, and one to the cropping model. Two subroutines are devoted to frequency tabulations while two other subroutines initialize the constants for the entire model and set several variables to their initial values at the start of each year.

A listing of the program is given on the following pages. Flow charts of the more important subroutines are also presented.

## MAIN PROGRAM



Initialize the ending dates of each month and each total monthly PE value.

Input total number of years to be simulated and crop specifications.

Initialize summers to zero.

Do for each year to be simulated.

Initialize summers and counters to zero.

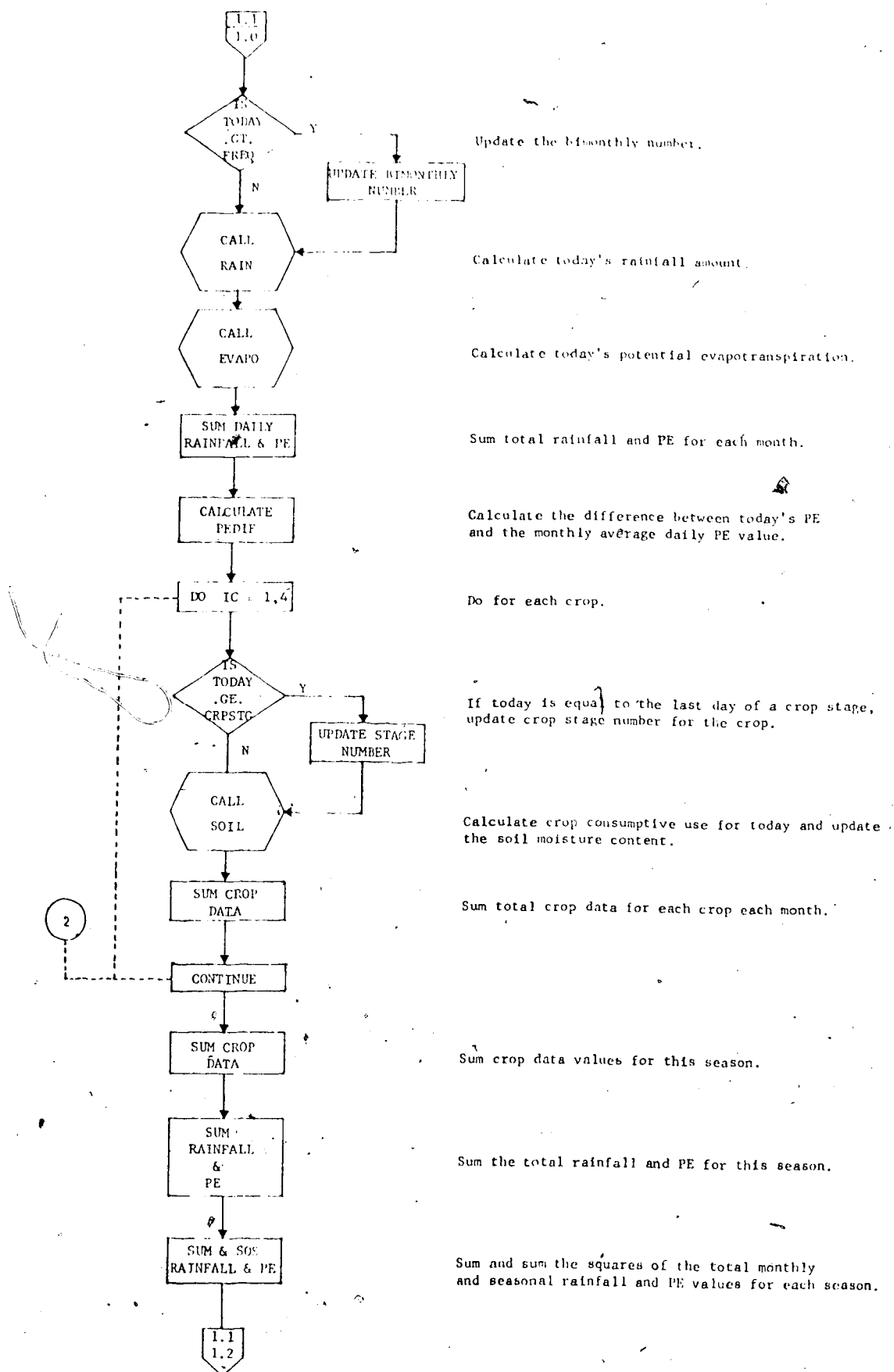
Initialize month, bimonth and week numbers to 1.

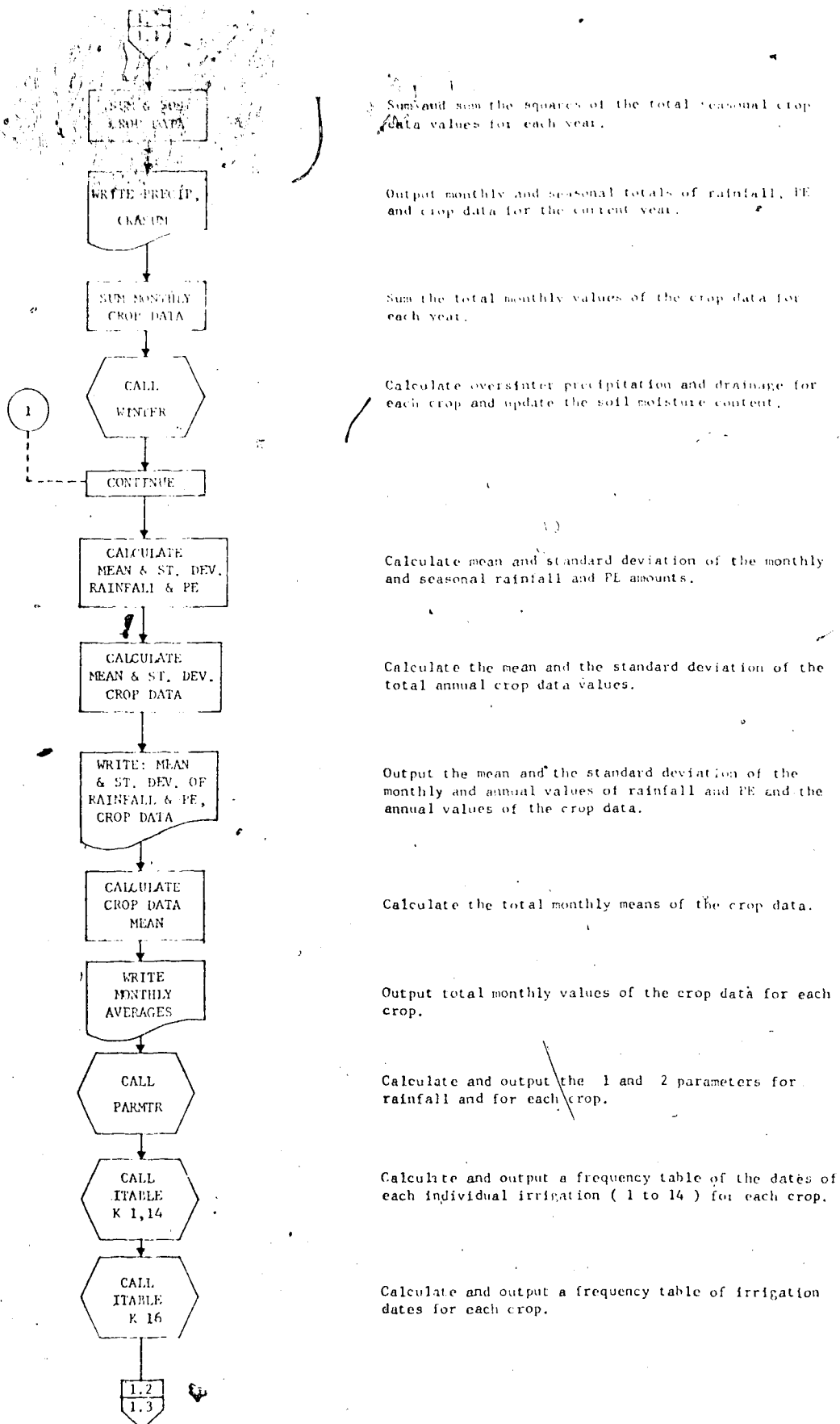
Generate 430 pseudo-random numbers for the entire season.

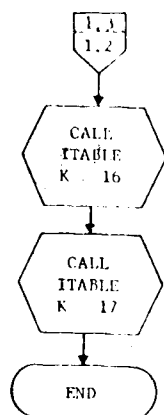
If first random number is less than the probability of rainfall for March 31st,  $R = 2$  otherwise  $R = 1$ .

Update the number of the month.

Update the number of the current week.







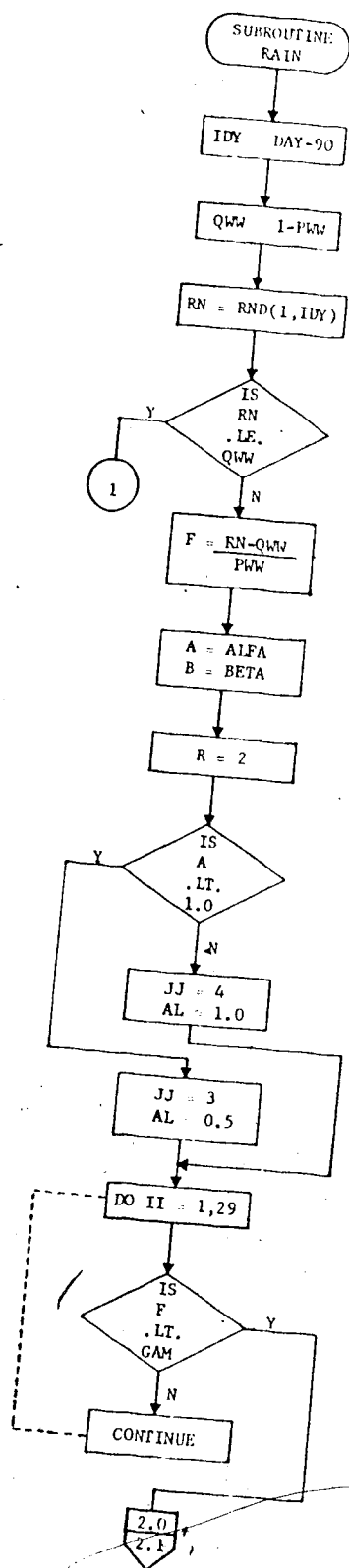
Calculate and output a frequency table of drainage dates for each crop.

Calculate and output a frequency table of runoff dates for each crop.



# SUBROUTINE RAIN

Subroutine to determine daily rainfall values.



Reset day number of year in relation to April 1st.

Calculate probability of a non-rainy day occurring today.

Select the next sequential random number.

Is today dry?

Adjust RN for a mixed distribution.

Select  $\alpha$  and  $\beta$  values of the theoretical gamma distribution for rainfall.

Set  $R = 2$  indicating rain today.

If  $\alpha$  is less than 1.0.

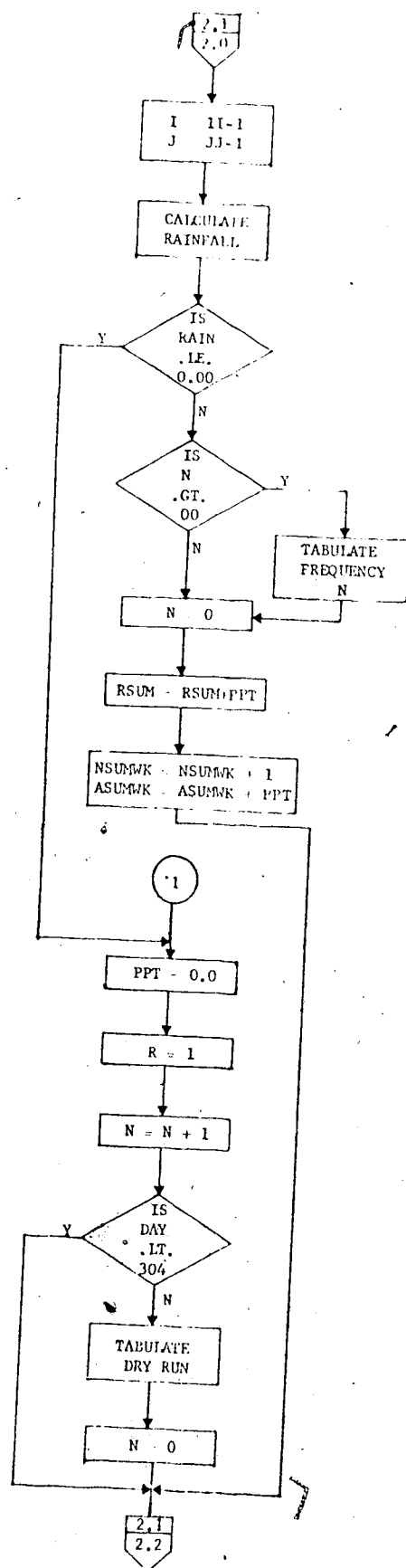
Select maximum column number 4 and maximum  $\alpha$  value of 1.0

Select column number 3 and  $\alpha$  value 0.5

Do for each row of the gamma table.

Select row number of the gamma table by comparing  $F$  to the probability in the gamma table.

If  $F$  is less than  $GAM$ , exit the do loop.



Select the row and column which lie on the opposite side of the F probability and the alpha value respectively.

Calculate rainfall by a 2-way interpolation of the rows and columns selected above. (Lagrange method, Stark, 51)

Is rainfall less than or equal to zero?

If length of consecutive dry days is greater than zero, tabulate the frequency of N.

Set length of dry runs to zero.

Sum rainfall amounts on a bimonthly basis.

Sum the total number of rainy days and the total amount of rain on a weekly basis.

Set rainfall to zero.

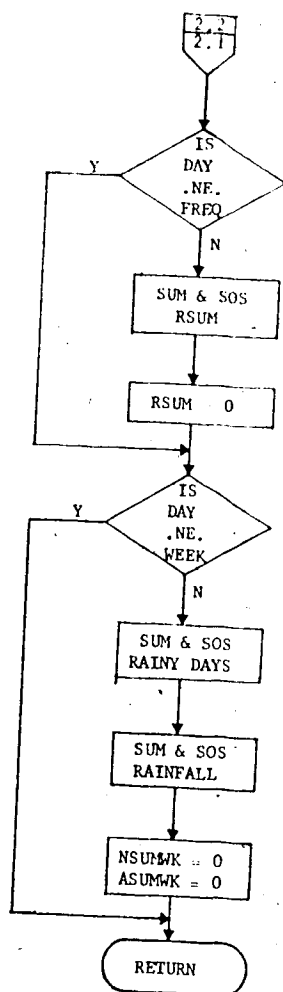
Set R to 1 indicating no rain today.

Sum the number of consecutive non-rainy days.

If today is not October 31st.

Tabulate frequency of last dry run

Reset dry run to zero.



If today is not the last day of the current bimonthly period.

Sum and sum the squares of the total monthly rainfall amounts.

Reset summation to zero.

If today is not the last day of the current 5-day period.

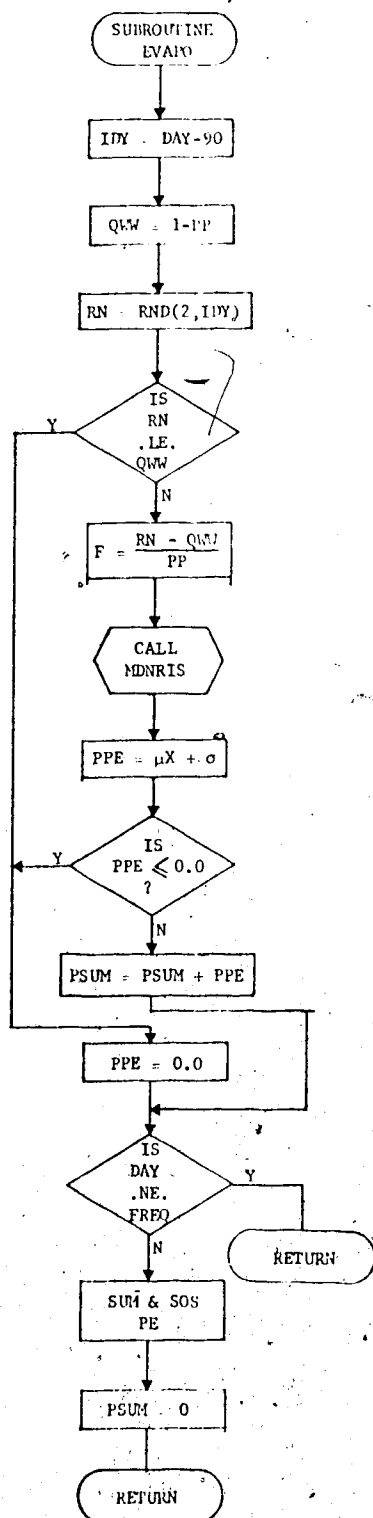
Sum and sum the squares of the number of rainy days in the last 5-day period.

Sum and sum the squares of the total amount of rainfall in the last 5-day period.

Reset summers to zero.

SUBROUTINE  
EVAPO

Subroutine to determine daily potential evapotranspiration.



Reset day number of year with respect to April 1st.

Probability of zero inches of PE occurring today.

Select next sequential random number.

Does today experience zero inches of PE?

Adjust RN for a mixed distribution.

Calculate standard deviate (X) of probability P  
MSL statistical package (29).

Calculate today's PE value given the mean and the standard deviation of the frequency distribution of the current bimonthly period.

If today's PE is zero or less.

Sum daily PE amounts on a bimonthly basis.

Set today's PE to zero.

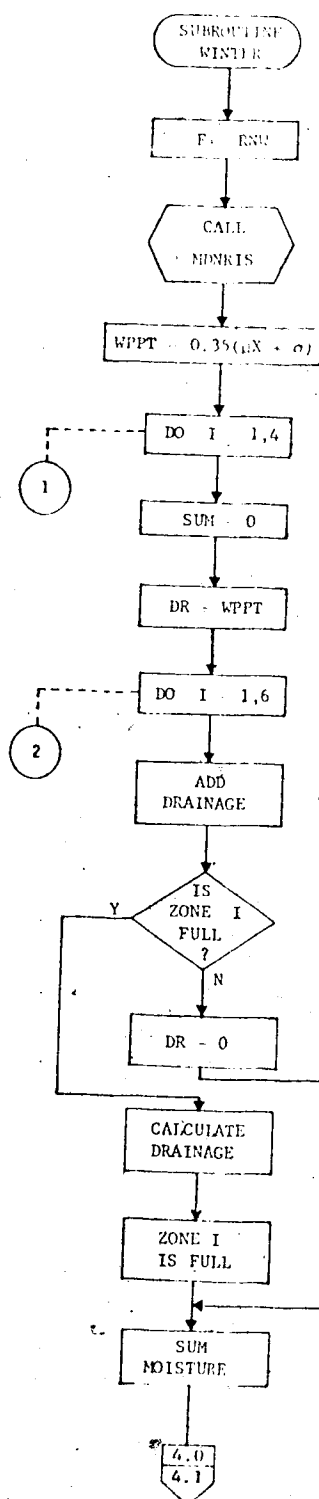
If today is not the last day of the current bimonthly period.

Sum and sum the squares of the total PE amount in the last bimonthly period.

Reset summer to zero.

SUBROUTINE  
WINTER

Subroutine to calculate overwinter precipitation, overwinter drainage and to update the soil moisture content for April 1st of the next year. The subroutine also outputs statistics for overwinter drainage.



Select last random number generated for this year.

Calculate standard deviate X of P.  
IMSL statistical package (29).

Calculate overwinter precipitation.

Do for crops 1 to 4.

Set summer to zero.

Set drainage equal to precipitation.

Do for each soil zone.

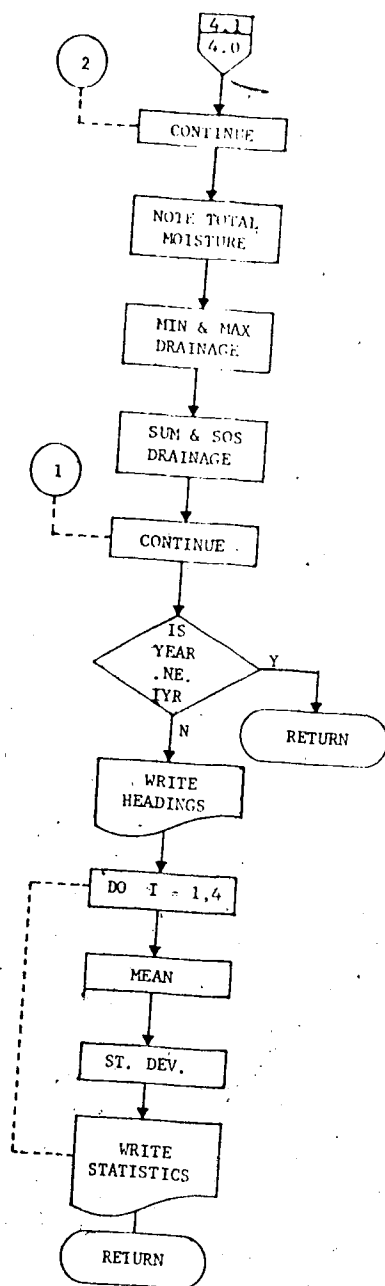
Add drainage from zone I-1 to zone I.

No drainage into zone I + 1.

Calculate drainage into zone I + 1.

Set zone I to capacity.

Sum water content in all 6 zones.



Note total water content in all 6 zones.

Select minimum and maximum values of drainage.

Sum and sum the squares of overwinter drainage.

If the current year is not the last year to be simulated - return.

Output table headings.

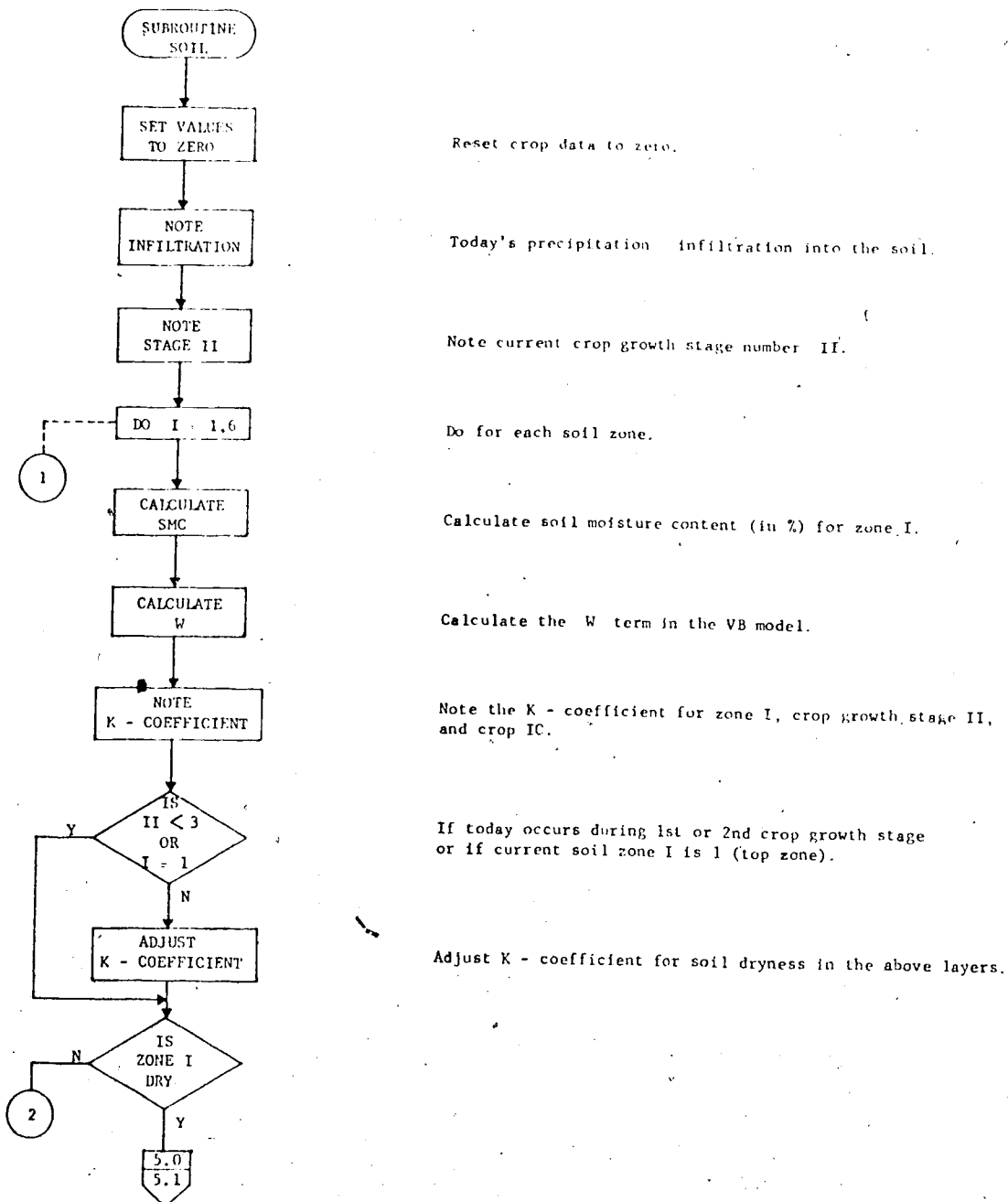
Do for each crop.

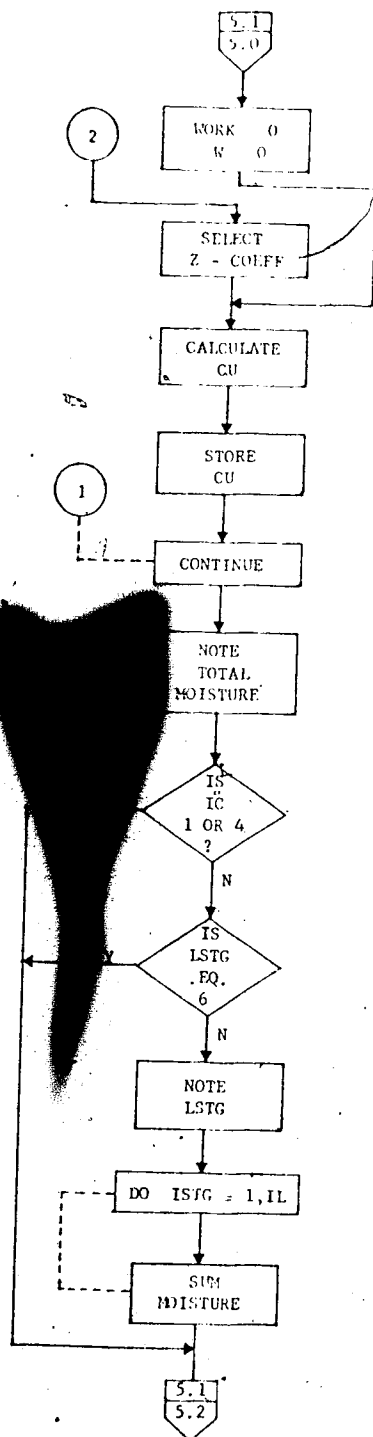
Calculate mean value of overwinter drainage.

Calculate standard deviation of overwinter drainage.

SUBROUTINE  
SOIL

Subroutine which utilizes the Versatile Soil Moisture Budget to:  
 1) calculate daily consumptive use values  
 2) update the soil moisture status for each soil zone  
 3) make irrigation decisions





Set values to zero.

Select coefficient from Z - table according to the soil moisture content (in %) in zone I.

Calculate consumptive use from zone I.

Store consumptive use values.

Note total moisture in all 6 soil zones.

If crop is Wheat or Alfalfa.

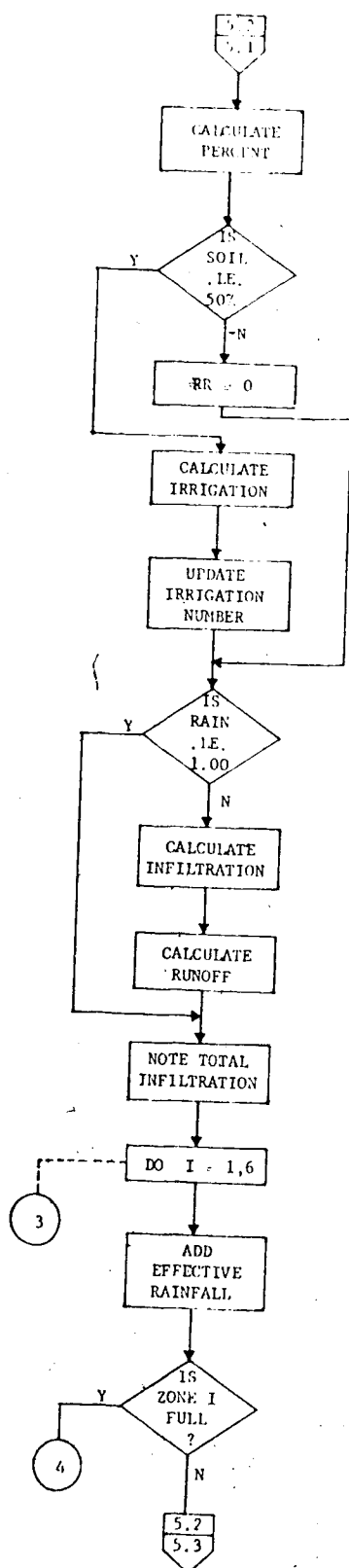
If Potatoes and Sugar Beet roots have penetrated into the 6th soil zone.

Note deepest zone into which roots have penetrated.

Do for zones no. 1 to LSTG.

Sum moisture in zones 1 to IL.





Calculate soil moisture percent of only those zones where roots exist.

If soil moisture content is less than 50%.

No irrigation water today.

Calculate amount of irrigation water to be applied.

Update current irrigation count.

If rainfall is less than 1.0 inch.

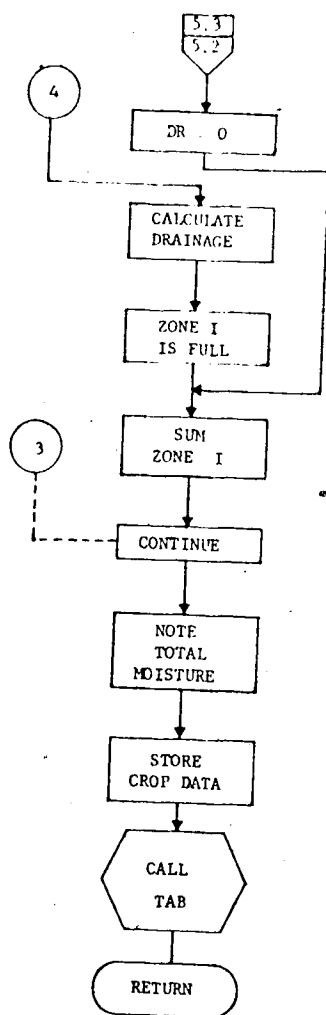
Calculate water infiltration into the soil.

$\text{Runoff} = \text{rainfall} - \text{infiltration}$

$\text{Total infiltration} = \text{irrigation} + \text{infiltration}$

Do for each soil zone.

Add drainage from zone I-1 and subtract consumptive use from zone I.



No drainage from zone I.

Calculate drainage into zone I+1

Set zone I to capacity.

Sum water content in all zones.

Note total water content in all 6 zones.

Store all crop data in array AMOUNT.

Update frequencies of occurrences of the dates of each crop data.

```

BLOCK DATA
C SUBROUTINE INITIALIZING ARRAYS AND VECTORS
  INTEGER CRPSTG, WEEK, FREQ, STAGE, DAY, WK, R, YEAR
  COMMON /BUDG/COEF(6,10,4), TABLE(100), CRMSUM(5,4,7), WEEK(43), CRASUM
  1(5,4), CONTNT(7,4), CAPAC(7), CRPSTG(10,4), PRECIP(8,2), FREQ(14), STAGE
  2(4), AMCUNT(5), IRRNO(4), PPT, PPE, DAY, WK, NO, PEDIF, R, YEAR, IC
  COMMON /PKOB/PWW(43,2), PE(14,2,2), GAM(29,4), ALFA(14,2), BETA(14,2),
  1PP(14,2)
  DATA CRPSTG/124,144,163,186,193,201,213,222,232,304,
  1      130,155,176,191,213,224,261,304,000,000,
  1      115,156,177,191,213,244,258,283,304,000,
  1      107,144,169,184,207,237,260,304,000,000/
  DATA FREQ/105,120,135,151,166,181,196,212,227,243,258,273,288,304/
  DATA ALFA/1.042437,0.835857,0.814344,0.860329,0.803625,0.765032,
  1      0.686373,0.939405,0.856398,0.790676,0.863120,0.791574,
  2      0.893684,0.764728,1.034741,0.759484,0.749754,0.693046,
  3      0.804689,0.659652,0.720244,0.893318,0.654811,0.778251,
  4      0.828852,0.899845,0.730132,0.890063/
  DATA BETA/0.134301,0.225257,0.207961,0.199111,0.261213,0.285950,
  1      0.295029,0.180009,0.199735,0.290288,0.260740,0.243743,
  2      0.168571,0.208680,0.130065,0.291298,0.309259,0.372689,
  3      0.381127,0.636569,0.346013,0.284008,0.344156,0.301041,
  4      0.258705,0.250745,0.252239,0.276635/
  DATA PP/0.8180,0.9265,0.9810,7*1.00,0.9882,0.9059,0.8569,0.7221,
  1      0.4520,0.6022,0.8079,0.9336,0.9665,0.9957,1.0000,0.9932,
  2      0.9935,0.9286,0.7824,0.5269,0.5455,0.2810/
  DATA PE/0.089853,0.117159,0.138815,0.159042,0.166708,0.176374,
  1      0.200021,0.206195,0.192687,0.168047,0.137580,0.112798,
  2      0.099764,0.075751,0.049389,0.056946,0.055059,0.053060,
  3      0.051293,0.049125,0.043207,0.045414,0.048151,0.053644,
  4      0.058546,0.055075,0.054423,0.044581,0.063544,0.067477,
  5      0.086932,0.112098,0.117413,0.129652,0.157081,0.149383,
  6      0.147451,0.113613,0.085985,0.079310,0.069577,0.057273,
  7      0.045377,0.041745,0.058175,0.057719,0.059707,0.061352,
  8      0.057409,0.056705,0.053931,0.061664,0.053772,0.055548,
  9      0.050918,0.039392/
  DATA GAM/0.0000,0.0100,0.0500,0.1000,0.1500,0.2000,0.2500,0.3000,
  1      0.3500,0.4000,0.4500,0.5000,0.5500,0.6000,0.6500,0.7000,
  2      0.7500,0.8000,0.8500,0.9000,0.9500,0.9900,0.9925,0.9950,
  1      0.9980,0.9990,0.9995,0.9999,1.0000,0.0000,0.0001,0.0020,
  1      0.0079,0.0179,0.0321,0.0508,0.0742,0.1029,0.1375,0.1787,
  1      0.2275,0.2853,0.3542,0.4367,0.5371,0.6617,0.8212,1.0361,
  1      1.3528,1.9207,3.3174,3.5753,3.9403,4.7754,5.4144,6.0585,
  1      7.5680,14.9907,0.000,0.0101,0.0513,0.1054,0.1625,0.2231,
  1      0.2877,0.3567,0.4308,0.5108,0.5978,0.6931,0.7985,0.9163,
  1      1.0498,1.2040,1.3863,1.6094,1.8971,2.3026,2.9957,4.6052,
  1      4.8932,5.2984,6.2152,6.9081,7.6010,9.2104,16.900,0.0000,
  1      0.0574,0.1759,0.2922,0.3989,0.5026,0.6063,0.7118,0.8208,
  1      0.9346,1.0547,1.1830,1.3215,1.4731,1.6416,1.8324,2.0542,
  1      2.3208,2.6585,3.1257,3.9074,5.6724,5.8839,6.4207,7.3994,
  1      8.1348,8.8667,10.5545,18.3712/
  DATA PWW/0.2477,0.2029,0.1822,0.1786,0.1865,0.2011,0.2190,0.2372,
  1      0.2539,0.2676,0.2776,0.2834,0.2851,0.2830,0.2775,0.2692,
  2      0.2590,0.2475,0.2355,0.2236,0.2126,0.2029,0.1947,0.1885,
  3      0.1842,0.1817,0.1808,0.1812,0.1824,0.1838,0.1850,0.1853,
  4      0.1842,0.1812,0.1762,0.1691,0.1601,0.1489,0.1397,0.1312,
  5      0.1267,0.1295,0.1436,0.4244,0.4091,0.4089,0.4189,0.4350,
  6      0.4540,0.4734,0.4911,0.5059,0.5169,0.5236,0.5258,0.5238,
  7      0.5178,0.5085,0.4965,0.4825,0.4673,0.4517,0.4365,0.4224,
  8      0.4069,0.3995,0.3917,0.3867,0.3845,0.3851,0.3884,0.3939,
  9      0.4013,0.4098,0.4188,0.4274,0.4348,0.4401,0.4424,0.4408,
  1      0.4345,0.4227,0.4051,0.3811,0.3508,0.3146/
  DATA WEEK/95,100,105,110,115,120,125,130,135,140,145,150,155,160,
  1      165,170,175,180,185,190,195,200,205,210,215,220,225,230,
  1      235,240,245,250,255,260,265,270,275,280,285,290,295,300,
  1      304/

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END

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C MAIN PROGRAM: 200 YEARS SIMULATION OF WEATHER AND CROP GROWTH
C
C VARIABLE DESCRIPTION
C CRPSTG ARRAY CONTAINING ENDING DATES FOR EACH CROPSTAGE
C WEEK VECTOR OF ENDING DATES OF CONSECUTIVE 5-DAY PERIODS
C FREQ VECTOR OF ENDING DATES OF EACH BIMONTHLY PERIOD
C STAGE CROP STAGE NUMBER
C WK WEEK NUMBER
C MO BIMONTHLY NUMBER
C MONTH MONTH NUMBER
C R PREVIOUS DAY INDICATOR (1 - DRY, 2 - WET)
C MODAY VECTOR OF ENDING DATES OF EACH MONTH
C COEF ARRAY CONTAINING K-COEFFICIENT MATRIX FOR EACH CROP
C CONTNT CURRENT SOIL MOISTURE CONTENT FOR EACH SOIL ZONE
C CAPAC SOIL MOISTURE CAPACITY OF EACH ZONE
C YEAR YEAR NUMBER
C DAY DAY NUMBER IN THE YEAR (91 TO 304)
C PEMEAN VECTOR CONTAINING AVERAGE DAILY PE FOR EACH MONTH
C IYR TOTAL NUMBER OF YEARS TO BE SIMULATED
C PRECIP MONTHLY AND ANNUAL TOTALS OF RAINFALL AND PE
C CRMSUM SUMMATION OF MONTHLY CROP DATA
C CRASUM SUMMATION OF ANNUAL CROP DATA
C AMOUNT VECTOR CONTAINING CROP DATA VALUES
C PPT DAILY RAINFALL VALUE (IN.)
C PPE DAILY PE VALUE (IN)
C IC CROP NUMBER
C 1. WHEAT
C 2. POTATOES
C 3. SUGAR BEETS
C 4. ALFALFA
C IT CROP DATA ITEM NUMBER
C 1. IRRIGATION QUANTITY
C 2. DRAINAGE
C 3. DEFICIT
C 4. CU
C 5. RUNOFF
C TSUMPT MEAN AND ST. DEV. OF MONTHLY AND ANNUAL RAINFALL AND PE TOTALS
C ATOTAL MEAN AND ST. DEV. OF ANNUAL CROP DATA VALUES
C MSUM TOTAL SUM OF CROP DATA VALUES FOR EACH MONTH
C
REAL MSUM(5,4,7),TSUMPT(8,2,2),ATOTAL(5,4,2),PEMEAN(7),AVG(5),CROP
1*8(4)
INTEGER CRPSTG,WEEK,FREQ,STAGE,DAY,WK,R,YEAR,MODAY(7)
COMMON /BUDG/COEF(6,10,4),TABLE(100),CRMSUM(5,4,7),WEEK(43),CRASUM
1(5,4),CONTNT(7,4),CAPAC(7),CRPSTG(10,4),PRECIP(8,2),FREQ(14),STAGE
2(4),AMOUNT(5),IERN(4),PPT,PPE,DAY,WK,MO,PEDIF,R,YEAR,IC
DATA CROP/'WHEAT','POTATOES','SUG BEET','ALFALFA'/
DATA MODAY/120,151,181,212,243,273,304/,ASTRIK/'****'/
DATA PEMEAN/0.076,0.129,0.153,0.191,0.167,0.103,0.062/
C INPUT NUMBER OF YEARS TO BE SIMULATED
READ(4,1) IYR
1 FORMAT(I3)
C INPUT CROP SPECIFICATIONS
READ(5,2) TABLE,COEF,CONTNT,CAPAC
2 FORMAT(1Q(10F5.2/),40(6F4.2/),(7F5.2))
C INITIALIZE SYSTEM COUNTERS
CALL INTIAL
C SET ANNUAL SUMMATIONS TO ZERO
DO 100 K=1,7
DO 100 J=1,4
DO 100 I=1,5
100 MSUM(I,J,K)=0.00
DO 101 I=1,2
DO 101 J=1,4
DO 101 K=1,5
101 ATOTAL(K,J,I)=0.00
DO 102 I=1,2
DO 102 J=1,2
DO 102 K=1,8
102 TSUMPT(K,J,I)=0.00

```

```

C
C BEGIN SIMULATION OF SEASON
C
  DO 3000 YEAR=1,IYR
C RESET ANNUAL COUNTERS AND SUMMATIONS
  CALL BEGIN
  MONTH=1
  MO=1
  WK=1
C OBTAIN PSEUDO-RANDOM NUMBERS FOR ENTIRE YEAR
  CALL RANDOM(RN)
  R=1
C IF 1ST RANDOM NUMBER LESS THAN THE PROBABILITY OF
C RAINFALL ON MARCH 31ST
  IF(RN.LE.0.2444)R=2
C
C BEGIN DAILY SIMULATION
C
  DO 2000 DAY=91,304
C UPDATE MONTHLY, WEEKLY AND BIMONTHLY COUNTERS
  IF(DAY.GT.MODAY(MONTH))MONTH=MONTH+1
  IF(DAY.GT.WEEK(WK))WK=WK+1
  IF(DAY.GT.FREQ(MO))MO=MO+1
C CALCULATE RAINFALL AND PE FOR TODAY
  CALL RAIN
  CALL EVAPO
C SUM DAILY RAINFALL AND PE FOR EACH MONTH
  IF(PPT.GT.0.00)PRECIP(MONTH,1)=PRECIP(MONTH,1)+PPT
  PRECIP(MONTH,2)=PRECIP(MONTH,2)+PPE
  PEDIF=PPE-PEMEAN(MONTH)
C
C CALCULATE CU AND SOIL MOISTURE FOR EACH CROP
C
  DO 2000 IC=1,4
C UPDATE CROP STAGE NUMBER
  IF(DAY.GE.CRPSIG(STAGE(IC),IC))STAGE(IC)=STAGE(IC)+1
C CALCULATE CU AND UPDATE SOIL M.CM FOR TODAY
  CALL SOIL
C SUM DAILY CROP DATA FOR EACH MONTH
  DO 1200 IT=1,5
1200 CRMSUM(IT,IC,MONTH)=CRMSUM(IT,IC,MONTH)+AMOUNT(IT)
2000 CONTINUE
C SUM MONTHLY CROP DATA FOR EACH SEASON
  DO 200 I=1,7
  DO 200 IC=1,4
  DO 200 IT=1,5
200 CRASUM(IT,IC)=CRASUM(IT,IC)+CRMSUM(IT,IC,I)
C SUM DAILY RAINFALL AND PE OVER ENTIRE SEASON
  DO 201 II=1,2
  DO 201 I=1,7
201 PRECIP(8,II)=PRECIP(8,II)+PRECIP(I,II)
C SUM TOTAL MONTHLY RAINFALL AND PE FOR EACH SEASON
  DO 205 J=1,2
  DO 205 I=1,8
  TSUMPT(I,J,1)=TSUMPT(I,J,1)+PRECIP(I,J)
205 TSUMPT(I,J,2)=TSUMPT(I,J,2)+PRECIP(I,J)*PRECIP(I,J)
C SUM ANNUAL CROP DATA FOR EACH SEASON
  DO 206 IC=1,4
  DO 206 IT=1,5
  ATOTAL(IT,IC,1)=ATOTAL(IT,IC,1)+CRASUM(IT,IC)
206 ATOTAL(IT,IC,2)=ATOTAL(IT,IC,2)+CRASUM(IT,IC)*CRASUM(IT,IC)
C OUTPUT TOTAL MONTHLY RAINFALL AND PE
  WRITE(1,3)((PRECIP(I,J),I=1,8),J=1,2)
3 FORMAT(7F6.2,F8.2,' -',7F6.2,F8.2)
C OUTPUT TOTAL ANNUAL CROP DATA
  WRITE(2,4) CRASUM
4 FORMAT(20F7.2)
C SUM MONTHLY CROP DATA FOR EACH SEASON
  DO 260 MO=1,7
  DO 260 IC=1,4
  DO 260 IT=1,5
260 NSUM(IT,IC,MO)=NSUM(IT,IC,MO)+CRMSUM(IT,IC,MO)

```

```

C  CALCULATE OVERWINTER PRECIPITATION
    CALL WINTER(IYR)
3000  CONTINUE
    Y=FLOAT(YEAR)
C  CALCULATE MEAN AND ST. DEV. FOR RAINFALL AND PE
    DO 310 IT=1,2
        DO 310 M=1,8
            SS=TSUMPT(M,IT,1)*TSUMPT(M,IT,1)
            TSUMPT(M,IT,2)=SQRT((TSUMPT(M,IT,2)-SS/Y)/(Y-1.00))
310    TSUMPT(M,IT,1)=TSUMPT(M,IT,1)/Y
C  CALCULATE MEAN AND ST. DEV. FOR CROP DATA
    DO 320 IC=1,4
        DO 320 IT=1,5
            SS=ATOTAL(IT,IC,1)*ATOTAL(IT,IC,1)
            ATOTAL(IT,IC,2)=SQRT((ATOTAL(IT,IC,2)-SS/Y)/(Y-1.00))
320    ATOTAL(IT,IC,1)=ATOTAL(IT,IC,1)/Y
C  OUTPUT MEANS AND ST. DEV.
        WRITE(1,6) (ASIRIK,K=1,103),TSUMPT
6        FORMAT(103A1/(7F6.2,F8.2,' -',7F6.2,F8.2))
        WRITE(2,7) (ASTRIK,K=1,140),ATOTAL
7        FORMAT(140A1/(20F7.2))
C  OUTPUT MONTHLY AVERAGES FOR CROP DATA
        WRITE(6,9)
9        FORMAT('1',30X,'MONTHLY AVERAGES FOR:-')
        DO 360 IC=1,4
            WRITE(6,10) CROP(IC)
10       FORMAT('-',12X,'CROP.....',A8,5X,'MO',10X,'IRR',6X,'DR',5X,'DEF'
            1,4X,'C.U.  RUNOFF')
            DO 360 MO=1,7
                DO 350 IT=1,5
350         AVG(IT)=MSUM(IT,IC,MO)/Y
360       WRITE(6,11) MO,AVG
11       FORMAT(' ',36X,I2,5X,5F8.2)
C  CALCULATE Y1 AND Y2 PARAMETERS
    CALL PARMTR(YEAR)
C  CALCULATE FREQUENCY DISTRIBUTIONS
C      1. DATES OF EACH IRRIGATION (1ST, 2ND, 3RD, ETC.)
C      2. IRRIGATION DATES COLLECTIVELY
C      3. DRAINAGE DATES
C      4. RUNOFF DATES
    CALL ITABLE(1,14,'DATES ',YEAR)
    CALL ITABLE(15,15,'IR DATES',YEAR)
    CALL ITABLE(16,16,'DR DATES',YEAR)
    CALL ITABLE(17,17,'RUNOFF ',YEAR)
    STOP
    END

```

## SUBROUTINE INTIAL

SUBROUTINE TO INITIALIZE SUMMERS AND COUNTERS TO ZERO

## VARIABLE DESCRIPTION

C F(I,J,K) FREQUENCY TABULATION OF IRRIGATION DATES, DRAINAGE  
 C DATES AND RUNOFF DATES FOR EACH CROP  
 C I = 1-14 IRRIGATION NUMBER DURING A SEASON  
 C = 15 IRRIGATION DATES TAKEN COLLECTIVELY  
 C = 16 DRAINAGE DATES  
 C = 17 RUNOFF DATES  
 C J = 1-4 CROP NUMBER  
 C I = 1 - 43 WEEK NUMBER  
 C ANT(I,J,K) WEEKLY SUMMATION OF IRRIGATION AND DRAINAGE  
 C K = 1 RAINFALL  
 C = 2-5 DRAINAGE FOR EACH WEEK AND CROP  
 C = 6-10 IRRIGATION FOR EACH WEEK AND CROP  
 C = 10-13 CU FOR EACH WEEK AND CROP  
 C J = 1 SUM  
 C = 2 SUM OF SQUARES  
 C I = 1-43 WEEK NUMBER  
 C NUMBER TOTAL NUMBER OF OCCURRENCES OF IRRIGATION AND DRAINAGE  
 C FOR EACH WEEKLY PERIOD (SUBSCRIPTS SAME AS ABOVE)  
 C IRRNO IRRIGATION NUMBER  
 C STAGE NUMBER OF CURRENT CROP GROWTH STAGE  
 C CRASUM SUMMATION OF ANNUAL CROP DATA  
 C CRMSUM SUMMATION OF MONTHLY CROP DATA  
 C PRECIP MONTHLY AND ANNUAL TOTALS OF RAINFALL AND PE

INTEGER CRPSTG, WEEK, FREQ, STAGE, DAY, WK, R, YEAR, F\*2, SEQ  
 COMMON /BUDG/COEF(6,10,4), TABLE(100), CRMSUM(5,4,7), WEEK(43), CRASUM  
 1(5,4), CONTNT(7,4), CAPAC(7), CRPSTG(10,4), PRECIP(8,2), FREQ(14), STAGE  
 2(4), AMCUNT(5), IRRNC(4), PPT, PPE, DAY, WK, MO, PEDIF, R, YEAR, IC  
 COMMON /PARM/ANT(43,2,13), NUMBER(43,2,9), PT(14,2,2), SEQ(100)  
 COMMON F(214,4,17)

## C RESET SIMULATION COUNTERS

DO 1 I=1,17  
 DO 1 J=1,4  
 DO 1 K=1,214  
 1 F(K,J,I)=00  
 DO 7 I=1,13  
 DO 7 J=1,2  
 DO 7 K=1,43  
 7 ANT(K,J,I)=0.00  
 DO 8 I=1,9  
 DO 8 J=1,2  
 DO 8 K=1,43  
 8 NUMBER(K,J,I)=000  
 DO 9 I=1,2  
 DO 9 J=1,2  
 DO 9 K=1,14  
 9 PT(K,J,I)=0.00  
 DO 10 I=1,100  
 10 SEQ(I)=00  
 RETURN

## C RESET SEASONAL COUNTERS

ENTRY BEGIN  
 DO 5 I=1,4  
 IRRNO(I)=00  
 STAGE(I)=1  
 DO 5 J=1,5  
 CRASUM(J,I)=0.00  
 DO 5 K=1,7  
 5 CRMSUM(J,I,K)=0.00  
 DO 6 J=1,2  
 DO 6 I=1,8  
 6 PRECIP(I,J)=0.00  
 RETURN  
 END

```

      SUBROUTINE RAIN
C
C  SUBROUTINE TO DETERMINE DAILY RAINFALL
C
C  VARIABLE DESCRIPTION
C      PWW      CONDITIONAL PROBABILITY OF RAINFALL FOR EACH WEEK
C                GIVEN THAT THE PREVIOUS DAY WAS DRY(R=1) OR WET(R=2)
C      QWW      PROBABILITY OF A NON-RAINY DAY
C      GAM      INVERSE GAMMA VALUES AS PER TABLE II, THOM (53)
C      ALFA     ALFA VALUES OF THE ESTIMATED GAMMA FUNCTION FOR RAINFALL
C      BETA     BETA VALUES OF THE ESTIMATED GAMMA FUNCTION FOR RAINFALL
C      PT       BIMONTHLY SUM AND SUM OF SQUARES FOR PRECIPITATION AND PE
C      RSUM     BIMONTHLY SUMMATION OF RAINFALL
C      NSUMWK   WEEKLY SUMMATION OF THE NUMBER OF RAINY DAYS
C      ASUMWK   WEEKLY SUMMATION OF RAINFALL AMOUNTS
C      SEQ      TABULATION OF CONSECUTIVE NON-RAINY DAY RUNS
C      RN       PSEUDO-RANDOM NUMBER
C
      INTEGER CRPSTG, WEEK, FREQ, STAGE, DAY, WK, R, YEAR, SEQ
      COMMON /BUDG/ COEF( 6, 10, 4), TABLE(100), CRMSUM(5, 4, 7), WEEK(43), CRASUM
      1(5, 4), CONTNT( 7, 4), CAPAC( 7), CRPSTG( 10, 4), PRECIP( 8, 2), FREQ(14), STAGE
      2( 4), AMOUNT( 5), IKRNO( 4), PPT, PPE, DAY, WK, MO, PEDIF, R, YEAR, IC
      COMMON /PROB/ PWW( 43, 2), PE( 14, 2, 2), GAM( 29, 4), ALFA( 14, 2), BETA( 14, 2),
      IPP( 14, 2)
      COMMON /PARM/ AMT( 43, 2, 13), NUMBER( 43, 2, 9), PT( 14, 2, 2), SEQ( 100)
      COMMON /RNDM/ RDUM, RND( 2, 214), RNW
      DATA RSUM, ASUMWK, NSUMWK/2*0.00, 00/, N/00/
      IDY=DAY-90
C  PROB. OF NON-RAINY DAY OCCURRING TODAY
      QWW=1.00000-PWW(WK, R)
C  SELECT RANDOM NUMBER
      RN=RND( 1, IDY)
C  IF TODAY IS DRY
      IF(RN.LE.QWW)GO TO 1
C  ADJUST RN FOR MIXED DISTRIBUTION
      F=(RN-QWW)/PWW(WK, R)
C  SELECT ALFA AND BETA VALUES
      A=ALFA(MO, R)
      B=BETA(MO, R)
      R=2
C  SELECT COLUMNS TO BE INTERPOLATED
      IF(A.LT.1.0)GO TO 2
      JJ=4
      AL=1.0
      GO TO 3
2      JJ=3
      AL=0.5
C  CALCULATE TODAYS RAINFALL - LEGRANGE INTERPOLATION, STARK (51)
3      DO 4 II=1, 29
          IF(F.LT.GAM(II, 1))GO TO 5
          CONTINUE
4          I=II-1
          J=JJ-1
          Y2=(A-AL)*2.00
          Y1=1.0-Y2
          X2=(F-GAM(I, 1))/(GAM(II, 1)-GAM(I, 1))
          X1=1.0-X2
          PPT=((GAM(I, J)*X1+GAM(II, J)*X2)*Y1+(GAM(I, JJ)*X1+GAM(II, JJ)*X2)*Y2
          1)*B
          IF(PPT.LE.0.00)GO TO 1
C  TABULATE LENGTH OF CONSECUTIVE DRY DAY RUNS
      IF(N.GT.00)SEQ(N)=SEQ(N)+1
      N=00
      RSUM=RSUM+PPT
      NSUMWK=NSUMWK+1
      ASUMWK=ASUMWK+PPT
      GO TO 6
C  IF NO RAINFALL
1      PPT=0.0
      R=1
      N=N+1
      IF(DAY.LT.304)GO TO 6

```



```
SEQ(N)=SEQ(N)+1
N=00
C SUM BIMONTHLY RAINFALL
6 IF(DAY.NE.FREQ(MO))GO TO 10
  PT(MO,1,1)=PT(MO,1,1)+RSUM
  PT(MO,2,1)=PT(MO,2,1)+RSUM*RSUM
  RSUM=0.00
C SUM WEEKLY RAINFALL AMOUNTS AND OCCURRENCES
10 IF(DAY.NE.WEEK(WK))RETURN
  NUMBER(WK,1,1)=NUMBER(WK,1,1)+NSUMWK
  NUMBER(WK,2,1)=NUMBER(WK,2,1)+NSUMWK*NSUMWK
  AMT(WK,1,1)=AMT(WK,1,1)+ASUMWK
  AMT(WK,2,1)=AMT(WK,2,1)+ASUMWK*ASUMWK
  NSUMWK=0
  ASUMWK=0.00
  RETURN
  END
```

## SUBROUTINE EVAPO

```

C
C SUBROUTINE TO DETERMINE DAILY POTENTIAL EVAPOTRANSPIRATION
C
C     PSUM    SUMMATION OF DAILY PE
C     PP      CONDITIONAL PROBABILITIES OF PE OCCURRING
C     QWW     PROBABILITY OF NO PE OCCURRING
C     RN      RANDOM NUMBER
C     PE      MEAN AND STANDARD DEVIATION FOR EACH PE DISTRIBUTION
C
C     INTEGER CRPSTG, WEEK, FREQ, STAGE, DAY, WK, R, YEAR, SEQ
C     COMMON /BUDG/COEF(6,10,4),TABLE(100),CRMSUM(5,4,7),WEEK(43),CRASUM
C     1(5,4),CCNTNT(7,4),CAPAC(7),CRPSTG(10,4),PRECIP(8,2),FREQ(14),STAGE
C     2(4),AMOUNT(5),IKRNO(4),PPT,PPE,DAY,WK,MO,PEDIF,R,YEAR,IC
C     COMMON /PROB/PWW(43,2),PE(14,2,2),GAM(29,4),ALFA(14,2),BETA(14,2),
C     1PP(14,2)
C     COMMON /PARM/AMT(43,2,13),NUMBER(43,2,9),PT(14,2,2),SEQ(100)
C     COMMON /RNDM/RDUM,KND(2,214),RNW
C     DATA PSUM/0.00/
C     IDY=DAY-90
C 1  PROBABILITY OF NO PE OCCURRING TODAY
C     QWW=1.0000-PP(MO,R)
C     RN=RND(2,IDY)
C 2  IF NO PE OCCURS TODAY
C     IF(RN.LE.QWW)GO TO 7
C 3  ADJUST RN FOR MIXED DISTRIBUTION
C     F=(RN-QWW)/PP(MO,R)
C 4  CALCULATE STANDARD VARIATE AND PE FOR TODAY
C     CALL MDNRIS(F,X,IER)
C     PPE=PE(MO,2,R)*X+PE(MO,1,R)
C 5  SUM DAILY PE
C     IF(PPE.LE.0.00)GO TO 7
C     PSUM=PSUM+PPE
C     GO TO 8
C 6  PPE=0.00
C 7  SUM DAILY PE FOR EACH WEEK
C 8  IF(DAY.NE.FREQ(MO))RETURN
C     PT(MO,1,2)=PT(MO,1,2)+PSUM
C     PT(MO,2,2)=PT(MO,2,2)+PSUM*PSUM
C     PSUM=0.00
C     RETURN
C     END

```

```

SUBROUTINE WINTER(IYR)
C
C SUBROUTINE TO CALCULATE TOTAL OVERWINTER PRECIPITATION
C
C VARIABLE DESCRIPTION
C   RNW   RANDOM NUMBER FOR OVERWINTER PRECIPITATION
C   WPPT  OVERWINTER PRECIPITATION
C   MIN   MINIMUM DRAINAGE OVER 200 YEARS
C   MAX   MAXIMUM DRAINAGE OVER 200 YEARS
C   DR    OVERWINTER DRAINAGE DUE TO WPPT
C   MEAN  SUM AND SUM OF SQUARES OF OVERWINTER PRECIPITATION
C
C   INTEGER CRPSTG, WEEK, FREQ, STAGE, DAY, WK, R, YEAR
C   REAL MEAN(4,2), MAX(4), MIN(4)
C   COMMON /BUDG/COEF(6,10,4), TABLE(100), CRMSUM(5,4,7), WEEK(43), CRASUM
C   1(5,4), CONTNT(7,4), CAPAC(7), CRPSTG(10,4), PRECIP(8,2), FREQ(14), STAGE
C   2(4), AMOUNT(5), IRRNO(4), PPT, PPE, DAY, WK, MO, PEDIF, R, YEAR, IC
C   COMMON /RNDM/RDUM, RND(2,214), RNW
C   DATA MEAN, MAX, MIN/12*0.00, 4*1000.0/
C CALCULATE OVERWINTER PRECIPITATION (MONTE CARLO SAMPLING)
F=RNW
CALL MDNRIS(F,X,IER)
WPPT=(1.242474*X+4.350465)*0.350000
IF(WPPT.LE.0.00)WPPT=0.00
C CALCULATE SOIL MOISTURE CONTENT FOR EACH CROP NEXT SPRING
DO 32 ICP=1,4
SUM=0.00
DR=WPPT
DO 30 I=1,6
CONTNT(I,ICP)=CONTNT(I,ICP)+DR
IF(CONTNT(I,ICP).GT.CAPAC(I))GO TO 31
DR=0.00
GO TO 30
31 DR=CONTNT(I,ICP)-CAPAC(I)
CONTNT(I,ICP)=CAPAC(I)
30 SUM=SUM+CONTNT(I,ICP)
CONTNT(7,ICP)=SUM
IF(DR.LT.MIN(ICP))MIN(ICP)=DR
IF(DR.GT.MAX(ICP))MAX(ICP)=DR
MEAN(ICP,1)=MEAN(ICP,1)+DR
MEAN(ICP,2)=MEAN(ICP,2)+DR*DR
32 CONTINUE
C OUTPUT MEAN AND ST. DEV. OF OVERWINTER DRAINAGE
IF(YEAR.NE.IYR)RETURN
WRITE(6,1)
WRITE(6,2)
FORMAT('1 OVERWINTER DRAINAGE FOR EACH CROP')
2 FORMAT('-. ,30X, ' CROP MAXIMUM MINIMUM MEAN ST DEV')
Y=FLOAT(YEAR)
DO 40 I=1,4
XM=MEAN(I,1)/Y
VAR=(MEAN(I,2)-MEAN(I,1)*MEAN(I,1)/Y)/(Y-1.0)
SD=0.00
IF(VAR.GT.0.00)SD=SQRT(VAR)
40 WRITE(6,3) I, MAX(I), MIN(I), XM, SD
3 FORMAT('0',30X,15,3F10.2,F10.6)
RETURN
END

```

```

SUBROUTINE RANDOM(RN)
C
C SUBROUTINE TO OBTAIN PSEUDO-RANDOM NUMBERS
C
C VARIABLE DESCRIPTION
C   RR      VECTOR CONTAINING 430 PSEUDO-RANDOM NUMBERS FOR ONE SEASON
C   RDUM    RANDOM NUMBER FOR MARCH 31ST. OF EACH SEASON
C   RND     ARRAY OF RANDOM NUMBERS FOR PRECIPITATION (1) AND PE (2)
C   RNW     RANDOM NUMBER FOR OVERWINTER PRECIPITATION
C
C   REAL SEED*8,RR(430)
C   COMMON /RNDM/RDUM,RND(2,214),RNW
C   EQUIVALENCE (RND(1),RR(2))
C THE SEED NUMBER IS THAT VALUE RECOMMENDED BY IMSL PACKAGE (29)
C   DATA SEED/0.123457D0/
C   CALL GGU1(SEED,430,RR)
C   RN=RR(1)
C   RETURN
C   END

```

## SUBROUTINE SOIL

```

C
C SUBROUTINE TO CALCULATE DAILY CU AND SOIL MOISTURE CONTENT
C FOR EACH CROP (BASED ON THE VERSATILE SOIL MOISTURE BUDGET)
C
C VARIABLE DESCRIPTION
C   CONTNT    CURRENT SOIL MOISTURE IN EACH SOIL ZONE (IN)
C   CAPAC     POTENTIAL SOIL MOISTURE IN EACH SOIL ZONE (IN)
C   SMC       SOIL MOISTURE RATIO
C   W         AS PER VERSATILE BUDGET
C   COEF      K-COEFFICIENT, ZONES 1-6, CROP STAGES 1-10, CROP 1-4
C   TABLE    Z-TABLE OF 100 COEFFICIENTS DEPICTING SOIL DRYNESS CURVES
C   COP       K-COEFFICIENT ADJUSTED FOR DRYNESS IN LOWER ZONES
C   AE        ACTUAL EVAPOTRANSPIRATION FOR EACH SOIL ZONE
C   PEDIF     DIFFERENCE BETWEEN DAILY PE AND MONTHLY AVERAGE PE
C   CU        DAILY CONSUMPTIVE USE
C   LSTG      SOIL ZONE NUMBER INTO WHICH ROOTS HAVE PENETRATED
C   RR        IRRIGATION AMOUNT
C   DR        DRAINAGE
C   RUN       RUNOFF
C   SUMCON    TOTAL MOISTURE IN ZONES INTO WHICH ROOTS HAVE PENETRATED
C   SMR       SOIL MOISTURE RATIO OF SOIL ZONES INTO WHICH ROOTS HAVE PENETRATED
C   SUMCAP    TOTAL WATER CAPACITY FROM TOP ZONE TO ZONE I
C   OGER      NATURAL LOGRITHM OF DAILY RAINFALL
C   AIN       WATER INFILTRATION INTO SOIL
C   AMOUNT    VECTOR STORING CROP DATA VALUES
C   DEL       VECTOR STORING AE FOR EACH SOIL ZONE

```

```

C   REAL COF(6),DEL(6),SUMCAP(6)
C   INTEGER CRPSTG, WEEK, FREQ, STAGE, DAY, WK, R, YEAR, SMR, LSTG(10,4)
C   COMMON /BUDGET/ COEF(6,10,4), TABLE(100), CKMSUM(5,4,7), WEEK(43), CRASUM
C   1(5,4), CONTNT(7,4), CAPAC(7), CAPSTG(10,4), PRECIP(8,2), FREQ(14), STAGE
C   2(4), AMOUNT(5), IRRNO(4), PPT, PPE, DAY, WK, MO, PEDIF, R, YEAR, IC
C   DATA LSTG/6,3,4,5,7*6,4,5,8*6,5,18*6/
C   DATA SUMCAP/0.35,0.87,1.75,3.50,5.25,7.00/
C RESET CROP DATA TO ZERO
C   RR=0.0
C   DR=0.0
C   CU=0.0
C   RUN=0.00
C   AIN=PPT

```

```

C
C CALCULATION OF A.E. FOR EACH SOIL ZONE
C
C SELECT CROP STAGE
C   II=STAGE(IC)
C DO FOR EACH SOIL ZONE
C   DO 100 I=1,6
C   CALCULATE SOIL MOISTURE RATIO
C     SMC=CONTNT(I,IC)/CAPAC(I)
C   CALCULATE W TERM
C     W=7.91-0.11*SMC*100.0
C     IF(W.LT.0.0)W=0.
C   SELECT K-COEFFICIENTS
C     COF(I)=COEF(I,II,IC)
C   IF II LESS THAN 3RD CROP GROWTH STAGE OR I EQUALS 1ST SOIL ZONE
C     IF(II.LT.3.OR.I.EQ.1)GO TO 2
C   ADJUST K-COEFFICIENT FOR DRYNESS IN ABOVE LAYERS
C     DO 1 J=2,I
C       K=J-1
C       COF(I)=COF(I)+COF(I)*COF(K)*(1.-CONTNT(K,IC)/CAPAC(K))
C     IT=SMC*100.
C     IF(IT.GT.0)GO TO 3
C   IF SOIL MOISTURE RATIO IS ZERO
C     WORK=0.
C     W=0.
C     GO TO 4
C   SELECT SOIL DRYNESS COEFFICIENT FROM Z-TABLE
C     WORK=TABLE(IT)
C   CALCULATE AE FOR ZONE I
C     AE=COF(I)*WORK*PPE*SMC*EXP(-W*PEDIF)

```

```

      IF(AE.C...IC))AE=CONTNT(I,IC)
C  STORE AE ... CH ZONE
      DEL(I)=...
C  CALCULATE
      CU=CU+AE
100  CONTINUE
C
C  DEC... TO
C
      IF...=CON... (7,IC)
      IF...EQ...IC.EQ.4)GO TO 10
      IF...G(I...).EQ.6)GO TO 10
      IL=...G(I...C)
      SUMC...
      DO 11 I=1,IL
11  SUMC...ON+CONTNT(ISTG,IC)
10  SMR=SUMC...SUMCAP(IL)*100.0
      IF(SMR...50)GO TO 20
      RR=0.
      GO TO 20
20  RR=SUMC...IL)/2.00
      IRRNO(IC)=IRRNO(IC)+1
C
C  APPLYING PRECIPITATION TO EACH ZONE
C
28  IF(PPT.LE...00)GO TO 29
C  CALCULATE AMOUNT OF RUNOFF
      OGER=ALOG(PPT)
      AIN=0.91770+1.81100*OGER-0.97300*OGER*CONTNT(1,IC)/CAPAC(1)
      IF(AIN.GT.PPT)AIN=PPT
      RUN=PPT-AIN
29  DR=RR+AIN
      SUM=0.
C  UPDATE TODAY'S MOISTURE CONTENT
      DO 30 I=1,6
      CONTNT(I,IC)=CONTNT(I,IC)+DR-DEL(I)
      IF(CONTNT(I,IC).LT.0.)CONTNT(I,IC)=0.
      IF(CONTNT(I,IC).GT.CAPAC(I))GO TO 31
      DR=0.00
      GO TO 32
31  DR=CONTNT(I,IC)-CAPAC(I)
      CONTNT(I,IC)=CAPAC(I)
32  SUM=SUM+CONTNT(I,IC)
30  CONTINUE
      CONTNT(7,IC)=SUM
C  STORE CROP DATA
      AMOUNT(1)=RR
      AMOUNT(2)=DR
      AMOUNT(4)=CU
      AMOUNT(3)=CAPAC(7)-CONTNT(7,IC)
      AMOUNT(5)=RUN
C  TABULATE FREQUENCIES OF IRRIGATION
      CALL TAB
      RETURN
      END

```

## SUBROUTINE TAB

C SUBROUTINE TO TABULATE IRRIGATION FREQUENCIES AND TO SUM IRRIGATION  
C AND DRAINAGE WEEKLY

## C VARIABLE DESCRIPTION

C IRSUM WEEKLY SUMMATION OF IRRIGATION  
C DRSUM WEEKLY SUMMATION OF DRAINAGE  
C NIRSUM WEEKLY SUMMATION OF IRRIGATION OCCURRENCES  
C NDRSUM WEEKLY SUMMATION OF DRAINAGE OCCURRENCES  
C AMT SUMMATION AND SUM OF SQUARES OF IRRIGATION AND DRAINAGE AMOUNTS  
C NUMBER SUMMATION AND SUM OF SQUARES OF IRRIGATION AND DRAINAGE OCCURRENCES  
C AMOUNT VECTOR CONTAINING CROP DATA  
C F(I,J,K) ARRAY CONTAINING FREQUENCIES FOR IRRIGATION, DRAINAGE AND RUNOFF DATES  
C I = DAY OF YEAR (1-214)  
C J = CROP (1-4)  
C K = 1-14 (NUMBER OF IRRIGATIONS IN THE SEASON)  
C = 15 (COMBINED IRRIGATION DATES IN SEASON)  
C = 16 (DRAINAGE DATES)  
C = 17 (RUNOFF DATES)

REAL IRSUM(4),DRSUM(4)

INTEGER NIRSUM(4),NDRSUM(4)

INTEGER CRPSTG, WEEK, FREQ, STAGE, DAY, WK, R, YEAR, F\*2, SEQ

COMMON /BUDG/ COEF(6,10,4), TABLE(100), CRMSUM(5,4,7), WEEK(43), CRASUM  
1(5,4), CONTNT(7,4), CAPAC(7), CRPSTG(10,4), PRECIP(8,2), FREQ(14), STAGE  
2(4), AMOUNT(5), IRRNO(4), PPT, PPE, DAY, WK, MO, PEDIF, R, YEAK, IC  
COMMON /PARM/ ANT(43,2,13), NUMBER(43,2,9), PT(14,2,2), SEQ(100)

COMMON F(214,4,17)

DATA IRSUM,DRSUM,NIRSUM,NDRSUM/8\*0.00,8\*00/  
ID=DAY-90

IF(AMOUNT(1).LE.0.00)GO TO 6

UPDATE FREQUENCY OF IRRIGATION DATES

F(ID,IC,IRRNO(IC))=F(ID,IC,IRRNO(IC))+1

F(ID,IC,15)=F(ID,IC,15)+1

SUM IRRIGATION AMOUNT AND OCCURRENCES

IRSUM(IC)=IRSUM(IC)+AMOUNT(1)

NIRSUM(IC)=NIRSUM(IC)+1

IF(AMOUNT(2).LE.0.00)GO TO 7

UPDATE FREQUENCY OF DRAINAGE DATES

F(ID,IC,16)=F(ID,IC,16)+1

SUM DRAINAGE AMOUNT AND OCCURRENCES

DRSUM(IC)=DRSUM(IC)+AMOUNT(2)

NDRSUM(IC)=NDRSUM(IC)+1

IF(AMOUNT(4).LE.0.00)GO TO 8

ITC=IC+9

SUM AND SUM OF SQUARES OF CU

AMT(WK,1,ITC)=AMT(WK,1,ITC)+AMOUNT(4)

AMT(WK,2,ITC)=AMT(WK,2,ITC)+AMOUNT(4)\*AMOUNT(4)

IF(AMOUNT(5).LE.0.00)GO TO 9

UPDATE FREQUENCY OF RUNOFF DATES

F(ID,IC,17)=F(ID,IC,17)+1

IF(IC.LT.4)RETURN

IF IC REPRESENTS LAST OF THE 4 CROPS

IF(DAY.NE.WEEK(WK))RETURN

IF DAY IS LAST DAY IN WEEK (WK)

DO 5 I=1,4

J=I+1

SUM AND SUM OF SQUARES OF DRAINAGE AMOUNT AND OCCURRENCES

AMT(WK,1,J)=AMT(WK,1,J)+DRSUM(I)

AMT(WK,2,J)=AMT(WK,2,J)+DRSUM(I)\*DRSUM(I)

NUMBER(WK,1,J)=NUMBER(WK,1,J)+NDRSUM(I)

NUMBER(WK,2,J)=NUMBER(WK,2,J)+NDRSUM(I)\*NDRSUM(I)

J=I+5

SUM AND SUM OF SQUARES OF IRRIGATION AMOUNT AND OCCURRENCES

AMT(WK,1,J)=AMT(WK,1,J)+IRSUM(I)

AMT(WK,2,J)=AMT(WK,2,J)+IRSUM(I)\*IRSUM(I)

NUMBER(WK,1,J)=NUMBER(WK,1,J)+NIRSUM(I)

NUMBER(WK,2,J)=NUMBER(WK,2,J)+NIRSUM(I)\*NIRSUM(I)

C RESET SUMMATIONS TO ZERO  
 DRSUM(I)=0.00  
 NDRSUM(I)=00  
 IRSUM(I)=0.00  
S NIRSUM(I)=00  
 RETURN  
 END



```

SUBROUTINE PARMTR(YEAR)
C
C SUBROUTINE TO CALCULATE AND OUTPUT LAMDA1 AND LAMDA2 PARAMETERS
C
C VARIABLE DESCRIPTION
C   LAM1 OCCURRENCE PER DAY
C   LAM2 YIELD PER OCCURRENCE
C   VAR1 DENSITY OF VARIANCE OF LAM1
C   V2 VARIANCE OF LAM2
C   RATIO VAR1/LAM1
C   PROD PRODUCT OF LAM1 AND LAM2
C   SD1 DENSITY OF STANDARD DEVIATION OF LAM1
C   SD2 DENSITY OF STANDARD DEVIATION OF LAM2
C   MEAN MEAN OF WEEKLY VALUES OF CU, PRECIPITATION AND PE
C   VAR WEEKLY STANDARD DEVIATION OF CU, PRECIPITATION AND PE
C   SEQ FREQUENCY OF CONSEQUITIVE DRY DAY RUNS
C
C INTEGER WK, YEAR, SEQ
C REAL LAM1, LAM2, MEAN(4), SD(4), CROP*8(4)
C COMMON /PARM/AMT(43,2,13), NUMBER(43,2,9), PT(14,2,2), SEQ(100)
C DATA CROP/'WHEAT','POTATOES','SUG BEET','ALFALFA'/
C
C Y=FLOAT(YEAR)
C DO 50 IC=1,9
C   OUTPUT Y1 AND Y2 STATISTICS FOR RAINFALL, IRRIGATION AND DRAINAGE
C   IF(IC.EQ.1)WRITE(6,1)
C   IF(IC.GE.2.AND.IC.LE.5)WRITE(6,2) CROP(IC-1)
C   IF(IC.GE.6.AND.IC.LE.9)WRITE(6,3) CROP(IC-5)
C   WRITE(6,4)
C   YY=YEAR*5.0
C   DAYS=5.0
C   DO 50 WK=1,43
C   IF(WK.LT.43)GO TO 10
C   YY=4.0*YEAR
C   DAYS=4.0
C   CALCULATE Y1 STATISTICS
10  X=NUMBER(WK,1,IC)
C   LAM1=X/YY
C   V1=(NUMBER(WK,2,IC)-X*X/Y)/(Y-1.0)
C   SD1=0.00
C   IF(V1.GT.0.00)SD1=SQRT(V1)/DAYS
C   VAR1=V1/DAYS
C   IF(X.EQ.0.00)X=1.0
C   RATIO=(V1*Y)/X
C   X1=AMT(WK,1,IC)
C   CALCULATE Y2 STATISTICS
C   LAM2=X1/X
C   V2=(AMT(WK,2,IC)-X1*X1/Y)/(Y-1.0)
C   SD2=0.00
C   IF(V2.GT.0.00)SD2=SQRT(V2)/DAYS
C   PROD=LAM1*LAM2
50  WRITE(6,55) WK, LAM1, VAR1, SD1, RATIO, LAM2, SD2, PROD
55  FORMAT(' ', I3, 7F10.4)
1  FORMAT('1 RAINFALL PARAMETERS')
2  FORMAT('1 DRAINAGE PARAMETERS.....', A8)
3  FORMAT('1 IRRIGATION PARAMETERS.....', A8)
4  FORMAT('1 9X, 'LAM1', 6X, 'VAR1', 3X, 'ST DEV1', 5X, 'RATIO', 6X, 'LAM2', 3
1X, 'ST DEV2', 3X, 'PRODUCT')
5  FORMAT('1 CONSUMPTIVE USE STATISTICS: MEAN AND STANDARD
1DEVIATION')
6  FORMAT('1 18, A8, T36, A8, T56, A8, T77, A8)
7  FORMAT('1 0 WEEK', 4(6X, 'MEAN ST DEV'))
8  FORMAT('1 PRECIPITATION AND POTENTIAL EVAPOTRANSPIRATION'//
1 MEANS AND STANDARD DEVIATIONS'//////
2 ST. DEV. POT.EVAPO. MONTH RAINFALL
ST. DEV.')
C   CALCULATE AND OUTPUT CU STATISTICS
C   WRITE(6,5)
C   WRITE(6,6) (CROP(I), I=1,4)
C   WRITE(6,7)
C   YY=YEAR*5.0

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DO 105 WK=1,43
IF(WK.EQ.43)YY=YEAR*4.0
DO 100 IC=10,13
ITC=IC-9
X=AMT(WK,1,IC)
MEAN(ITC)=X/YY
VAR=(AMT(WK,2,IC)-X*X/YY)/(YY-1.0)
SD(ITC)=0.00
IF(VAR.GT.0.00)SD(ITC)=SQRT(VAR)
100 CONTINUE
105 WRITE(6,9) WK,(MEAN(I),SD(I),I=1,4)
9 FORMAT(' ',15,4(F10.2,F10.6))
C CALCULATE AND OUTPUT RAINFALL AND PE STATISTICS
WRITE(6,8)
DO 150 I=1,14
DO 140 J=1,2
MEAN(J)=PT(I,1,J)/Y
VAR=(PT(I,2,J)-PT(I,1,J)*PT(I,1,J)/Y)/(Y-1.0)
SD(J)=0.00
IF(VAR.GT.0.00)SD(J)=SQRT(VAR)
140 CONTINUE
150 WRITE(6,151) I,(MEAN(J),SD(J),J=1,2)
151 FORMAT('0',6X,14,9X,F5.2,10X,F7.4,16X,F5.2,10X,F7.4)
C CALCULATE AND OUTPUT CONSEQUITIVE DRY DAY RUN STATISTICS
WRITE(6,160)
160 FORMAT('1 RELATIVE FREQUENCIES OF DRY DAY RUNS.'////)
ISUM=00
DO 70 I=1,100
ISUM=ISUM+SEQ(I)
SUM=ISUM
WRITE(6,161) ISUM
161 FORMAT('-',30X,'RUN LENGTH FREQUENCY PERCENT TOTAL FREQUE
INCY',18)
DO 80 I=1,100
IF(SEQ(I).EQ.00)GO TO 80
PER=SEQ(I)*100.0/SUM
WRITE(6,102) I,SEQ(I),PER
102 FORMAT(' ',30X,16,8X,16,7X,F6.2)
80 CONTINUE
RETURN
END

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SUBROUTINE ITABLE(X1,K2,X,IY)
C
C SUBROUTINE TO CALCULATE AND OUTPUT CUMULATIVE FREQUENCY DISTRIBUTIONS
C
C VARIABLE DESCRIPTION
C
C K COUNTER SPECIFYING IRRIGATION NUMBERS K1 TO K2
C NZ NUMBER OF CROPS HAVING NO KTH IRRIGATION
C SUM TOTAL SUM OF DAY NUMBER OF THE YEAR FOR KTH IRRIGATION
C SUM2 SUM OF SQUARES OF SUM
C IN DAY NUMBER OF THE YEAR
C N TOTAL NUMBER OF KTH IRRIGATIONS
C MAX LATEST DAY ON WHICH KTH IRRIGATION WAS PERFORMED
C AVG AVERAGE DAY OF THE KTH IRRIGATION
C SD STANDARD DEVIATION OF DAY NUMBER OF THE KTH IRRIGATION
C XI UPPER LIMIT OF DAY IN FREQUENCY DISTRIBUTION
C F OBSERVED FREQUENCY OF IRRIGATION FOR EACH DAY, IRRIGATION AND CROP
C DF PERCENT OF TOTAL OBSERVED FREQUENCY
C AF CUMULATIVE PERCENTAGE OF TOTAL OF EACH OBSERVED FREQUENCY
C R CUMULATIVE REMAINDER OF TOTAL OF EACH OBSERVED FREQUENCY
C XM MULTIPLE OF MEAN
C DEV PERCENT OF 200 YEARS OF EACH FREQUENCY
C THE ABOVE CODES ALSO APPLY FOR DRAINAGE AND RUNOFF
C
C DIMENSION X(2)
C REAL*8 SUM,SUM2,CROP(4)
C INTEGER*2 F
C COMMON F(214,4,17)
C DATA CROP/'WHEAT','POTATOES','SUG BEET','ALFALFA'/
C NZ=0
C DO 50 K=K1,K2
C IF NZ=4, NO MORE IRRIGATIONS TO CONSIDER
C IF(NZ.EQ.4)RETURN
C NZ=0
C DO FOR EACH CROP
C DO 50 J=1,4
C SUM=0.00
C SUM2=0.00
C N=0
C SUM AND SUM OF SQUARES OF VARIATE
C DO 2 I=1,214
C IF(F(I,J,K).EQ.00)GO TO 2
C IN=I+90
C SUM=SUM+F(I,J,K)*IN
C SUM2=SUM2+F(I,J,K)*IN*IN
C N=N+F(I,J,K)
2 CONTINUE
C MAX=IN-90
C IF(N.GT.00)GO TO 3
C IF TOTAL FREQUENCY OF KTH IRRIGATION FOR CROP J IS ZERO
C NZ=NZ+1
C GO TO 50
3 Y=FLOAT(N)
C MEAN AND ST. DEV. OF VARIATE
C AVG=SUM/Y
C IF(N.NE.1)SD2=(SUM2-Y*AVG*AVG)/FLOAT(N-1)
C SD=0.00
C IF(SD2.GT.0.00)SD=SQRT(SD2)
C OUTPUT HEADINGS
C WRITE(6,100) CROP(J),K,X
100 FORMAT('ENTRIES IN TABLE',10X,'MEAN ARGUMENT',10X,'STANDARD DEVIATION',10X,'CROP NO...',A8,10X,'ITEM NO...',I3,5X,2A4)
C WRITE(6,101) N,AVG,SD
101 FORMAT(' ',12X,I4,13X,F10.4,18X,F10.4)
C WRITE(6,150)
150 FORMAT(11X,'UPPER',7X,'OBSERVED',6X,'PER CENT',2(6X,'CUMULATIVE'),16X,'MULTIPLE',6X,'PER CENT')
C WRITE(6,151)
151 FORMAT(11X,'LIMIT',6X,'FREQUENCY',6X,'OF TOTAL',6X,'PERCENTAGE',7X,1,'REMAINDER',7X,'OF MEAN',6X,'OF 200')
C CALCULATE FREQUENCY STATISTICS
C Y=FLOAT(IY)

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D=FLOAT(N+1)
IF(N.LT.30)D=FLOAT(N)
AF=0.
DO 51 I=1,MAX
IF(F(I,J,K).EQ.0)GO TO 51
DF=F(I,J,K)*100./D
AF=AF+DF
R=100.-AF
XI=FLOAT(I+90)
XM=0.00
IF(AVG.GT.0.00)XM=XI/AVG
DEV=F(I,J,K)*100.0/Y
WRITE(6,152) XI,F(I,J,K),DF,AF,R,XM,DEV
152 FORMAT(' ',9X,F6.2,9X,I6,8X,F6.2,2(10X,F6.2),8X,F6.3,8X,F7.3)
51 CONTINUE
WRITE(6,153)
153 FORMAT('REMAINING FREQUENCIES ARE ALL ZERO')
50 CONTINUE
RETURN
END

```