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Revisiting Llewellyn's Absolute Stability Criterion for Bilateral Teleoperation Systems under Non-passive Operator or Environment

Ali Jazayeri and Mahdi Tavakoli

***Abstract*— Stability of a haptic teleoperation system is influenced by the typically uncertain, time-varying and/or unknown dynamics of the operator and the environment. For a stability analysis that is independent of the operator and the environment dynamics, Llewellyn's absolute stability criterion proposes certain conditions on the two-port network representing the teleoperator (comprising the master, the controller and communication channel, and the slave) *assuming that the terminations (i.e., the operator and the environment) are passive*. These are less-than-accurate assumptions. It is desirable to extend Llewellyn's result to the cases where the operator or the environment is non-passive. This paper revisits Llewellyn's criterion and relaxes the assumption of passivity for one of the terminations. The possibly non-passive termination is realistically assumed to have a complex impedance with an upper or lower bound on its amplitude or real part, respectively. Although the proposed stability criteria are useful for any application of two-port network systems, we specifically apply them on bilateral teleoperation systems and find the stability conditions when the operator or the environment is not passive; this is a result that Llewellyn's absolute stability criterion cannot afford.**

I. INTRODUCTION

Absolute stability of a two-port network guarantees stability of the network when the two ports are terminated to passive but otherwise arbitrary one-port networks. An equivalent definition of absolute stability of a two-port network requires seeing a passive input impedance (i.e., driving-point impedance) from one of the ports when the other port is terminated to a passive one-port network (i.e., termination); see Fig. 1 [1]. The notion of absolute stability has been applied to two-port network systems with limited information about the impedance of the terminations. For instance, in a bilateral teleoperation system, the models of the human operator and the environment are typically unknown. If the models of the human operator and the environment are assumed passive but otherwise arbitrary, absolute stability criteria will impose conditions on the teleoperator model parameters for stability of the overall teleoperation system – the teleoperator consists of the master, the communication channel and the slave (Fig. 2).

A well-known absolute stability criterion for two-port networks has been proposed by Llewellyn [1], [2], [3]. Llewellyn's absolute stability criterion gives closed-form conditions on the immittance parameters (impedance, admittance, hybrid [4]) of the two-port network for it to

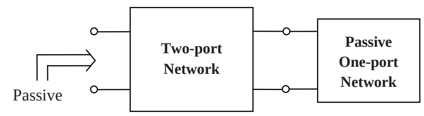

Fig. 1. Absolute stability of a two-port network is equivalent to passivity of the input impedance seen from a port when the other port is terminated to a passive one-port network.

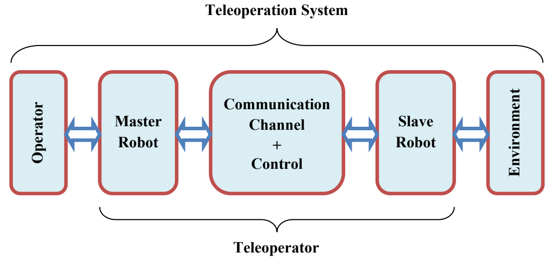

Fig. 2. The teleoperation system versus the teleoperator.

be absolutely stable. Llewellyn's absolute stability conditions require the assumption of passivity of both of the terminations of the two-port network. Yet, they allow the terminations' impedances to take on any amplitude and phase. On the other hand, in practical systems, a two-port network's termination may be non-passive (i.e., active) but with an upper bound on the amplitude of its impedance or a lower bound on the real part of its impedance; later in the paper, we will discuss specific examples of such non-passive terminations for bilateral teleoperation systems. Therefore, in this paper, stability analysis of a two-port network coupled to a possibly non-passive termination with certain constraints on its impedance is considered.

Past work has modeled the bounded impedance of a termination as a shunt impedance in parallel to a very large impedance in order to utilize the existing absolute stability criteria [3]. In another work, the minimum and maximum impedances of a termination are modelled as series and shunt impedances and the remaining part of the termination is assumed to be passive [5]. In an interesting recent work, the range of stabilizing terminations for a two-port network has been derived using scattering parameters and reflection coefficient with the aid of a 3-dimensional graphical approach [6]. This analysis determines bounds on the reflection coefficient of the termination to guarantee the stability of the coupled system.

Passivity of an LTI one-port network is equivalent to the positive-realness of its driving-point impedance [7]. Deviating from the assumption of passivity, the one-port network's complex impedance may be allowed to have a real part that stays greater than a negative number (i.e., the right-half plane (RHP) is shifted by a finite amount to the left in the complex impedance plane). This shift accounts for the

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[1]Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada

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difference between a passive and a non-passive (i.e., positive-real and non-positive-real) one-port network.

Interestingly, to have a stable coupled system, it suffices if, after terminating the two-port network to a passive or non-passive one-port network, the driving-point impedance at the remaining port (which is currently open) is passive. Connecting a passive termination at the currently open port of this two-port network will inevitably result in a passive and thus stable system even though the opposite port might have been connected to a non-passive termination.

In this paper, using Mobius transformation on regions of the complex impedance plane, a powerful stability analysis tool has been developed that is appropriate for control system analysis and design. Similar to Llewellyn's criterion, the developed absolute stability condition has a closed form, and consequently can be explicitly used for theoretical as well as graphical controller design. In this paper, the analysis is framed in two categories in order to answer two specific questions:

- Find the largest region in the complex impedance plane such that any termination with an impedance in that region – regardless of being passive or non-passive – when coupled to a two-port network will result in a passive driving-point impedance at the other port of the two-port network. We will show that this region is a disc in the complex plane.
- Find a strip in the complex impedance plane such that any termination with an impedance in that strip – regardless of being passive or non-passive – when coupled to a two-port network will result in a passive driving-point impedance at the other port of the two-port network.

Needless to say, in both of the above cases, the impedance parameters of the two-port network will appear in the stability conditions. It is interesting to note that, as described later, lines and circles are mapped to one another via the transformation that describes the input versus load impedance of a two-port network; that is why we have considered discs and strips in the above two general cases.

The paper is organized as follows. First, Llewellyn's absolute stability criterion has been discussed and a proof has been provided in Section II. This is followed, in Sections III and IV, by our two main theorems to ensure the passivity of the driving-point impedance of the open port of a two-port network that has been terminated to a potentially non-passive (i.e., passive or non-passive) one-port network at its opposite port. Concluding remarks and future work have been presented in Section VI.

II. Llewellyn's Absolute Stability Criterion for Passive Terminations

Before looking into our new absolute stability condition, since Llewellyn's absolute stability criterion has an explicit relationship with both the disc-like and the crescent-like absolute stability in Sections III and IV, let us consider Llewellyn's absolute stability criterion.

Assuming an LTI model, a two-port network (shown in 3-a) is expressed by the impedance (Z) parameters as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{1}$$

Theorem 1: [1] A two-port network is absolutely stable if and only if

(i) Z_{11} and Z_{22} have no poles in the RHP,

(ii) Pure imaginary poles of Z_{11} and Z_{22} are simple and have positive residues, and

(iii) For all real positive frequencies ω,

$$\begin{gathered} \operatorname{Re} Z_{11}(j\omega) \geq 0, \operatorname{Re} Z_{22}(j\omega) \geq 0 \\ 2 \operatorname{Re} Z_{11}(j\omega) \operatorname{Re} Z_{22}(j\omega), - \operatorname{Re} Z_{12}(j\omega) Z_{21}(j\omega) \\ -|Z_{12}(j\omega) Z_{21}(j\omega)| \geq 0 \end{gathered} \tag{2}$$

where Z_{ij} are the two-port network impedance parameters and may be replaced by any immitance parameters P_{ij}.

Proof: [8] Conditions (i) and (ii) of Theorem 1 are necessary conditions for ensuring positive realness of Z_{11} and Z_{22} in zero-impedance conditions for ports 2 and 1, respectively [1]. Let us consider the third condition of Theorem 1. As shown in Fig. 3-b, the two-port network is connected to a passive impedance z_2 and the input impedance seen from the other port is assumed to be Z_{a1}. The two-port network will be absolutely stable if Z_{a1} is passive as well. It is easy to show that Z_{a1} can be expressed as

$$Z_{a1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + z_2} \tag{3}$$

or as a Mobius transformation as

$$Z_{a1} = \frac{z_2(Z_{11}) + (Z_{11}Z_{22} - Z_{12}Z_{21})}{z_2 + (Z_{22})} \tag{4}$$

The Mobius transformation maps circles and lines from one complex plane to lines and circles in another complex plane [9]. The borderline of passivity in the z_2 complex plane is a vertical line coincident with the $j\omega$-axis; any impedance to the right of this line (i.e., with a positive real value) is passive. As shown in Lemma 1 in Appendix, if $\operatorname{Re} Z_{22} \geq 0$ (and for a similar reason $\operatorname{Re} Z_{11} \geq 0$), then the Mobius transformation of the line can be shown to be a circle with a radius r_o and a centre at ω_o where

$$r_o = \frac{|Z_{12}Z_{21}|}{2R_{22}}, \omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2R_{22}} \tag{5}$$

where $R_{22} = \operatorname{Re} Z_{22}$. Consequently, the right half plane in the z_2-plane (i.e., class of positive-real impedances) is mapped to a disc as depicted in Fig. 4. Now, the condition for passivity of the resulting mapped impedance, i.e. Z_{a1}, is that it entirely lies in the right half plane. In other words,

$$\operatorname{Re} \omega_o - r_o \geq 0 \tag{6}$$

Substituting (5) in the above leads to

$$\frac{2 \operatorname{Re} Z_{11} \operatorname{Re} Z_{22} - \operatorname{Re}(Z_{12}Z_{21}) - |Z_{12}Z_{21}|}{2R_{22}} \geq 0 \tag{7}$$

This completes the proof. ■

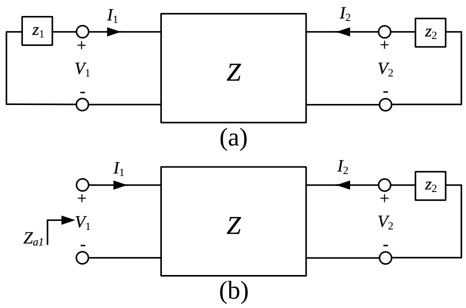


Fig. 3. (a)Two-port network and (b) driving point impedance $Z_{a1} = V_1/I_1$ when port 2 is terminated to a passive impedance z_2.

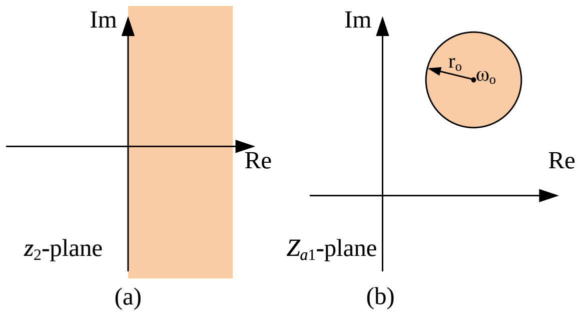


Fig. 4. Mobius transformation maps the right half of the impedance plane of z_2-plane (a) to a disc in the Z_{a1}-plane (b).

III. Stability of Systems Comprising a Non-passive "Disc-Like" Impedance

Llewellyn's absolute stability criterion is valid only for the case where the connecting termination z_2 is passive, which is not a practical assumption in some applications. For instance, in a bilateral teleoperation system, the human operator is not necessarily passive but will have a limited impedance determined by physical properties of his/her arm. While human operator behaves passive in a grasping task [10], he/she is non-passive during manipulation of the master robot. Non-passivity of the human operator is happening for example when the teleoperator is designed to be passive and the environment is also energy-distilling, thus the human operator must be the source of energy. Also, instead of commonly used second order model for the human operator, it is assumed that the impedance is arbitrary with a limited amplitude in its impedance. The following analysis finds all passive and non-passive impedances z_2 such that the resulting input impedance Z_{a1} is passive.

Theorem 2: Consider a two-port network system modelled in (1). Assume that port 2 of the two-port network is terminated to an impedance z_2 and the driving-point impedance seen from port 1 is Z_{a1}. Assume that z_2 has a maximum impedance of $Z_{2,max}$, and that port 1 of the two-port network is terminated to another passive impedance. Then, the necessary and sufficient condition for stability of the overall system (comprising the two-port network and the terminations at ports 1 and 2) is

(i) Z_{11} and Z_{22} have no poles in the right half of the complex plane,

(ii) Pure imaginary poles of Z_{11} and Z_{22} are simple and have positive residues, and

(iii) For all real positive frequencies ω,

$$R_{11} \geq 0, R_{22} \geq 0$$
$$Z_{2,max} \leq \frac{|Z_{12}Z_{21}| - 2|Z_{22}R_{11} + Z_{12}Z_{21}|}{2R_{11}} \quad (8)$$

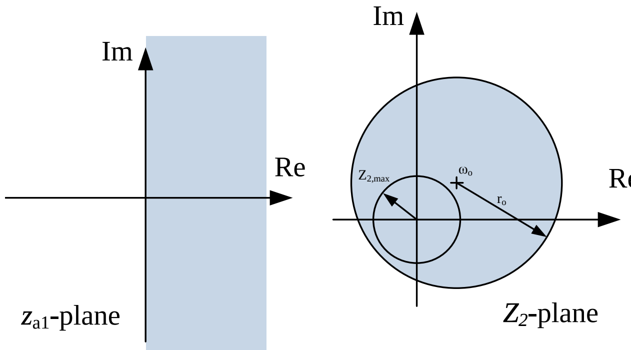
where R_{11} and R_{22} are the real part of Z_{11} and Z_{22}, respectively. The argument $j\omega$ has been omitted for brevity.

Proof: Similar to Theorem 1, conditions (i) and (ii) are necessary for stability of the two-port network. Condition (iii) has the following proof.

- Step 1: To find the mapping from Z_{a1}-plane to z_2-plane, the Mobius transformation (3) is rearranged to find the inverse transformation as
$$z_2 = -Z_{22} - \frac{Z_{12}Z_{21}}{-Z_{11} + Z_{a1}} \quad (9)$$
The inverse transformation is another Mobius transformation, which can be seen as the transformation for a two-port network with the following impedance matrix:
$$\begin{bmatrix} -Z_{22} & Z_{12} \\ Z_{21} & -Z_{11} \end{bmatrix} \quad (10)$$
- Step 2: Using Lemma 1 of Appendix, the Mobius transformation (9) maps the right half plane $\text{Re}\, Z_{a1} \geq 0$ onto a disc z_2 in the z_2-plane, with a centre at ω_o and a radius of r_o given by
$$r_o = \frac{|Z_{12}Z_{21}|}{2R_{11}}, \omega_o = -Z_{22} - \frac{Z_{12}Z_{21}}{2R_{11}} \quad (11)$$
The impedance that terminates port 2 should be entirely placed in the disc defined by z_2 to guarantee passivity of the Z_{a1}.
- Step 3: Although the result in Step 2 determines bounds for all passive and non-passive terminations, this general result can be customized for applications where the termination's impedance has known shape and amplitude. For instance, consider the human operator in a bilateral teleoperation system. When the master robot is released by the operator, this is equivalent to zero impedance for the operator. At the other extreme, the operator's impedance reaches a maximum amplitude $Z_{2,max}$ if the operator pushes as hard as possible against the master robot. As shown in Fig. 5-b, the disc with radius of $Z_{2,max}$ centered at the origin should be inside the stability disc given by the mapping found in Step 3. Therefore, the condition for passivity of Z_{a1} becomes
$$r_o > |\omega_o| + Z_{2,max} \quad (12)$$
substituting r_o and ω_o from (11), the condition becomes
$$Z_{2,max} \leq \frac{Z_{12}Z_{21}}{2R_{11}} - |Z_{22} + \frac{Z_{12}Z_{21}}{R_{11}}| \quad (13)$$
which can be rearranged as (8) and completes the proof. ■

IV. Stability of Systems Comprising a Non-passive "Strip-Like" Impedance

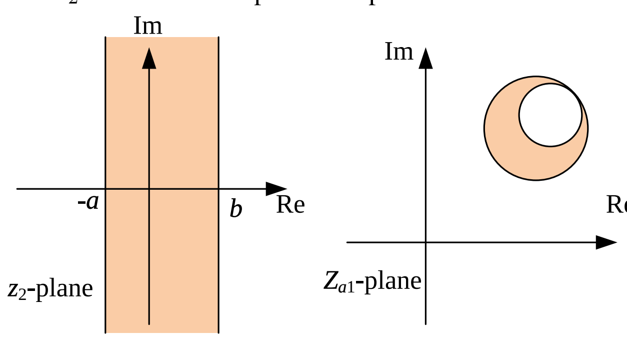
In this section, it is assumed that the impedance z_2 that terminates port 2 of the two-port network has a potentially negative real part (i.e., z_2 may be non-passive). Specifically, it is assumed that z_2 covers a strip in the complex plane as depicted in Fig. 6-a. Using this strip is a generalization of termination impedance considered in Llewellyn's absolute



(a) (b)

Fig. 5. In disc-like impedance analysis (Theorem 2) determines the region in the termination impedance complex plane (b) such that the driving point impedance (a) remains passive. The impedance of port 2's termination with maximum amplitude of $Z_{2,max}$ should be entirely in the stability disc.

stability criterion. In fact, when the upper limit b and the lower limits $-a$ are set to be infinity and zero, respectively, then z_2 is restricted to passive impedances.



(a) (b)

Fig. 6. In analysis of the strip-like impedances the strip in the z_2-plane (a) is mapped to a rotated crescent in the Z_{a1}-plane (b).

Theorem 3: Consider a two-port network system modelled in (1). As shown in Fig. 3-b, assume that port 2 of the two-port network is terminated to an impedance z_2 and the driving-point impedance seen from port 1 is Z_{a1}. Assume that z_2 satisfies $-a \le \operatorname{Re} z_2 \le b$, where a and b are positive and real numbers, and that port 1 of the two-port network is terminated to another passive impedance. Then, the necessary and sufficient condition for stability of the overall system (comprising the two-port network and two terminations) is

(i) Z_{11} and Z_{22} have no poles in the RHP,
(ii) Pure imaginary poles of Z_{11} and Z_{22} are simple and have positive residues, and
(iii) For all real positive frequencies ω,

$$2R_{11}R_{22} - \operatorname{Re}\{Z_{12}Z_{21}\} - |Z_{12}Z_{21}| - 2R_{11}a \ge 0$$
$$R_{11} \ge 0, R_{22} \ge a \quad (14)$$

Proof: Similar to the proof of Theorem 1, conditions (i) and (ii) are necessary condition for stability of the two port network. Condition (iii) is derived in the following steps.

- Step 1: The transformation from z_2 into Z_{a1} is expressed based on the two-port network's impedance

$$Z_{a1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + z_2} \quad (15)$$

  which can be expressed as a Mobius transformation from z_2 into Z_{a1} consistent with (4).

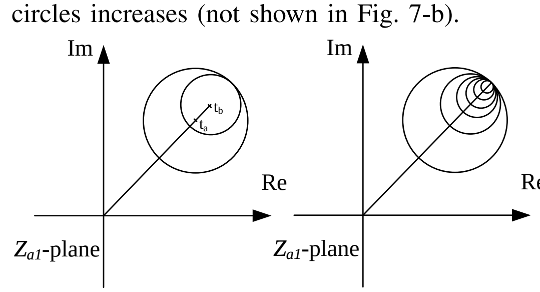
- Step 2: The strip $-a \le \operatorname{Re} z_2 \le b$ is mapped to a crescent in Z_{a1}-plane as explained in the following. The borderlines of the strip are the vertical lines at $\operatorname{Re} z_2 = -a$ and $\operatorname{Re} z_2 = b$ for the lower and the upper bounds, respectively. These two lines are transformed to two circles in the Z_{a1}-plane. Using a similar transformation as in Lemma 1 in Appendix, it is easy to show that the radii and centers of the two circles are

$$^{a}r_o = \frac{|Z_{12}Z_{21}|}{2(R_{22}-a)}, {}^{a}\omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2(R_{22}-a)} \quad (16)$$

  for the line defined by $\operatorname{Re} z_2 = -a$ (provided that $R_{22} \ge a$), and

$$^{b}r_o = \frac{|Z_{12}Z_{21}|}{2(R_{22}+b)}, {}^{b}\omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2(R_{22}+b)} \quad (17)$$

  for the line defined by $\operatorname{Re} z_2 = b$.
  In the following, we will prove that the circle corresponding to $\operatorname{Re} z_2 = b$ is entirely inside the circle corresponding to $\operatorname{Re} z_2 = -a$. As it is described in the third step of the proof of Lemma 1 in Appendix, the two circles result from an expansion/contraction term $Z_{12}Z_{21}$ followed by a translation by the amount Z_{11} in the Z_{a1}-plane. Consequently, as depicted in Fig. 7-a, the line connecting the centres of these two circles ($\overline{t_a t_b}$) will go through the origin. Another conclusion from Lemma 1 in Appendix is that the length of the line segment between the centres of the two circles (i.e., $|\overline{t_a t_b}|$) is identical to the differences between to the radii of the two circles (i.e., $|{}^{a}r_o - {}^{b}r_o|$). Therefore, as shown in Fig. 7-a, the two circles must be tangent at their farthest points from the origin. Additionally, changing the bounds on the real part of z_2 will result in the circles shown in Fig. 7-b. As the real value of z_2 is allowed to increase, the radius of the smaller circles decreases. Also, as the real value of z_2 is allowed to decrease further into the negative values, the radius of the larger circles increases (not shown in Fig. 7-b).



(a) (b)

Fig. 7. The vertical lines in the z_2-plane are mapped to circles in the Z_{a1}-plane. The vertical line at $\operatorname{Re} z_2 = -a$ is mapped to the larger circle in the Z_{a1}-plane while the vertical line at $\operatorname{Re} z_2 = b$ is mapped to the smaller circle (a). As the real value of z_2 is allowed to increase (i.e., larger b), the radius of the smaller circle decreases while the circles still share the same tangent point.

- Step 3: The crescent found in Step 2 needs to be in the right half the complex Z_{a1}-plane for passivity of the driving-point impedance at port 1. Because the outer circle corresponding to $\operatorname{Re} z_2 = -a$ contains the inner circle corresponding to $\operatorname{Re} z_2 = b$ and two circles overlap each other in a region farthest possible from the origin, the necessary and sufficient condition for passivity of the driving-point impedance at port 1 is

$$\mathrm{Re}\,{}^{a}\omega_o - {}^{a}r_o \geq 0 \tag{18}$$

Substituting ${}^{a}r_o$ and ${}^{a}\omega_o$ from (16) and (18) yields

$$\mathrm{Re}\{Z_{11} - \frac{Z_{12}Z_{21}}{2(R_{22}-a)}\} - \frac{|Z_{12}Z_{21}|}{2(R_{22}-a)} \geq 0 \tag{19}$$

which can be rearranged as (14). This completes the proof.

■

Remarks:

- The value of the upper limit of the impedance b does not appear in the stability condition (14) due to the fact that the inner circle is not the source of any constraint when ensuring the passivity of the driving-point impedance Z_{a1}. In other words, stability depends on the lower limit of the real part of the impedance connected to port 2 of the two-port network.
- The difference between the new stability condition (14) for strip-like termination impedances and Llewellyn's absolute stability criterion is in the last term of (14).
- Unlike Llewellyn's absolute stability criterion, the new stability conditions in Theorems 2 and 3 are not symmetric with respect to the network parameters. In other words, swapping the terminations at ports 1 and 2 of a two-port network does not change Llewellyn's absolute stability conditions but it may affect the new conditions in Theorems 2 and 3.
- The regions in z_2-plane and Z_{a1}-plane found for strip-like impedances (Fig. 6) and the same regions for disc-like impedances (Fig. 4) might demonstrate an overlap but one is not a subset of the other. Whether to use the stability condition for a strip-like impedance or a disc-like impedance depends on the application at hand; this will be discussed further in the next section.

V. Application to Bilateral Teleoperation

The stability conditions derived in Sections III and IV can be used to relax the assumption of passivity for one of the (one-port network) terminations for a two-port network while preserving the stability of the overall system. An important application of these new criteria is in bilateral teleoperation systems if one of the terminations (human operator and environment) is non-passive. For a non-passive human operator, the stability criterion presented in Section III is most appropriate because the impedance of the human operator's hand has a limited amplitude and can be modeled as a disc with finite radius in the complex impedance plane. On the other hand, for a non-passive environment, the stability criterion in Section IV is useful because stiff environment (very large impedance amplitude) and non-passive environments (negative real impedance) can be covered. In practice, the operator is in most cases non-passive (at least during manipulation tasks if not during sensing and grasp tasks [10]), and the environment might demonstrate non-passive behaviour, e.g., when gravity forces are acting on it. Another example of non-passive environment is in beating-heart telesurgical systems where the slave is interacting with an environment that emits energy. In the following, our new stability conditions will be applied to the position error based teleoperation control architecture for either of non-passive human operator and non-passive environments.

A position error based (PEB) architecture for a bilateral teleoperation system is discussed in [11].

The hybrid matrix representing the system is [11]

$$H = \begin{bmatrix} Z_m + C_m \frac{Z_s}{Z_{ts}} & \frac{C_m}{Z_{ts}} \\ -\frac{C_s}{Z_{ts}} & \frac{1}{Z_{ts}} \end{bmatrix} \tag{20}$$

where $Z_{ts} = Z_s + C_s$ and $Z_{tm} = Z_m + C_m$, where the master and the slave robots are modelled as $Z_m = M_m s$ and $Z_s = M_s s$ and the local position controllers are $C_m = K_{v_m} + K_{p_m}/s$ and $C_s = K_{v_s} + K_{p_s}/s$. The impedance matrix (1) can be found from the hybrid matrix as

$$Z = \begin{bmatrix} Z_{tm} & C_m \\ C_s & Z_{ts} \end{bmatrix} \tag{21}$$

In the following two subsections, the new stability conditions found in Theorem 2 and Theorem 3 will be applied to the bilateral teleoperation system described by (21). Note that in these stability conditions, port 2's termination (z_2) is the non-passive termination. Thus, in the case where the human operator is non-passive, z_2 is the human operator's impedance and port 1 is terminated by the passive environment. On the other hand, when the environment is non-passive, z_2 is the environment impedance and port 1 is terminated by the passive operator.

A. Non-passive human operator with known bounds on the amplitude of the impedance

A non-passive human operator's impedance has a disc shape in the complex plane. With this assumption, the results of Section III are applied to the teleoperation system described above. The stability conditions (8) become

- $K_{v_m} \geq 0$
- $K_{v_s} \geq a$
- $Z_{2,max} \leq \frac{|M_1 - jM_2|}{2K_{v_m}} - |M_3 - \frac{M_1}{K_{v_m}}|$
  where $M_1 = K_{v_m}K_{v_s} - K_{p_m}K_{p_s}/\omega^2$, $M_2 = (K_{v_m}K_{p_s} + K_{v_s}K_{p_m})/\omega$ and $M_3 = K_{v_s} - jK_{p_s}/\omega + j\omega M_s$.

B. An environment with a non-passive strip-like impedance

The non-passive impedance of the environment is assumed to have a strip-like shape with a lower bound of $-a$ and an upper bound of b for its real part in the complex plane. The stability condition in Theorem 3 is applied to the system and the results are as follows. The conditions in (14) yield

- $K_{v_m} \geq 0$
- $K_{v_s} \geq a$
- $a^2 - a(\frac{K_{v_s}^2 + K_{p_s}^2}{\omega^2 K_{v_s}}) \geq 0$ and $a \leq \frac{K_{v_m}K_{v_s} + K_{p_m}K_{p_s}/\omega^2}{2K_{v_m}}$
  where the solution of the above inequalities becomes $K_{p_s}/K_{v_s} = K_{p_m}/K_{v_m}$ and $a = 0$, meaning that the termination has to be passive to satisfy the stability of the closed-loop system.

VI. Conclusions and Future Work

Llewellyn's absolute stability analysis for a two-port network assumes the passivity of both of its terminations. This is not a realistic assumption in the context of bilateral teleoperation systems. To extend the result of Llewellyn, in this paper a powerful stability analysis tool has been developed based on Mobius transformations between the impedance of the termination coupled to a port of the two-port network and the driving-point impedance seen at the opposite port. The new stability criteria have been applied to a position-error-based bilateral teleoperation system for non-passive operator and environment models.

Appendix

In this appendix, the lemma related to the Mobius transformation is presented and the proof is given.

Lemma 1: Let us consider the following conformal mapping:

$$Z_{a1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + z_2} \tag{22}$$

The right half plane in the z_2-plane is transformed to a disc in the Z_{a1}-plane with radius of

$$r_o = \frac{|Z_{12}Z_{21}|}{2R_{22}} \tag{23}$$

and centre at

$$\omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2R_{22}} \tag{24}$$

Proof: A Mobius transformation can be split to a set of three transformation, namely a linear transformation ($\zeta_1 = z_2 + Z_{22}$), an inversion ($\zeta_2 = 1/\zeta_1$) and another linear transformation ($\zeta_3 = Z_{11} - Z_{12}Z_{21}\zeta_2$) [12]. As a result of these three transformation the vertical line passing the origin in the z_2-plane is transformed to the Z_{a1}-plane.

It should be noted that a line or a circle can be expressed as the following unified equation in the complex plane:

$$Az\bar{z} + \bar{B}z + B\bar{z} + C = 0 \tag{25}$$

where A, B and C are the parameters of the circle or line in the plane. Also, $A = 0$ reduces (25) to a line equation.

The three transformations are as follow:

1) The first transformation is a linear transformation as $\zeta_1 = z_2 + Z_{22}$ that translates the left half plane to the right side by the real part of Z_{22}, i.e. R_{22} (Fig. 8-b). The resulting line is expressed as $\mathrm{Re}\{\zeta_1\} = R_{22}$, which can be converted to the general circle and line equation (25) as $\zeta_1 + \bar{\zeta_1} = 2R_{22}$ (i.e. $A = 0$, $B = 1$ and $C = -2R_{22}$).
2) The second transformation is an inversion $\zeta_2 = 1/\zeta_1$. Substitution of the definition of the new transformation into result of step 1 reads as $1/\zeta_2 + 1/\bar{\zeta_2} = 2R_{22}$, which can be expressed in the general form of $-2R_{22}\zeta_2\bar{\zeta_2} + \zeta_2 + \bar{\zeta_2} = 0$(i.e. $A = -2R_{22}$, $B = 1$ and $C = 0$). This is an equation for a circle centered at $\omega_2 = -B/A$ and with radius of $r_2 = \frac{\sqrt{|B|^2 - AC}}{|A|}$ [12]. The centre and radius will be found as $\omega_2 = 1/2R_{22}$ and $r_2 = 1/2R_{22}$, respectively (Fig. 8-c). It should be noted that R_{22} has to be positive.
3) The third tansformation is $\zeta_3 = Z_{11} - Z_{12}Z_{21}\zeta_2$. Similar to the first transformation, the third transformation is a linear transformation (Fig. 8-d). For this transformation the magnifying factor is $Z_{12}Z_{21}$ and translation is Z_{11}. Therefore, the circle will be expanded or contracted by factor of and $Z_{12}Z_{21}$ and the radius becomes $r_o = \frac{|Z_{12}Z_{21}|}{2R_{22}}$ and the centre of the circle will be translated to $\omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2R_{22}}$. This completes the proof of the lemma.

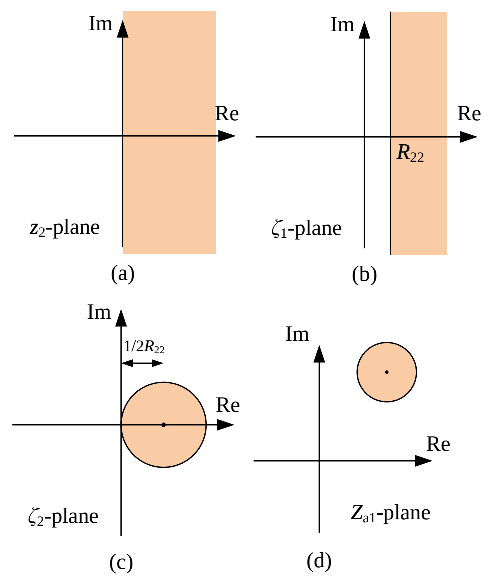


Fig. 8. The Mobius transformation has been split into three transformations: from (a) to (b) is a linear transformation (horizontal translation), from (b) to (c) is an inversion, and from (c) to (d) is another linear transformation with expansion/contraction in addition to a translation.

■

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