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6 **Exploiting the Full Potential of Bayesian Networks in Predictive**
7 **Ecology**

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Summary

1. Although ecological models used to make predictions from underlying covariates have a record of success, they also suffer from limitations. They are typically unable to make predictions when the value of one or more covariates is missing during the testing. Missing values can be estimated but methods are often unreliable and can result in poor accuracy. Similarly, missing values during the training can hinder parameter estimation of many ecological models. Bayesian networks can handle these and other limiting issues, such as having highly correlated covariates. However, they are rarely used to their full potential.
2. Indeed, Bayesian networks are commonly used to evaluate the knowledge of experts by constructing the network manually and often (incorrectly) interpreting the resulting network causally. We provide an approach to learn a Bayesian network fully from observed data, without relying on experts and show how to appropriately interpret the resulting network, both to identify how the variables (covariates and target) are interrelated and to answer probabilistic queries.
3. We apply this method to the case-study of a mountain pine beetle infestation and find that the trained Bayesian network has a predictive accuracy of 0.88 AUC. We classify the covariates as primary and secondary in terms of contributing to the prediction and show that the predictive accuracy does not deteriorate when the secondary covariates are missing and degrades to only 0.76 when one of the primary covariates is missing.
4. As a complement to the previous work on constructing Bayesian networks by hand, we show that if instead, both the structure and parameters are learned only from data, we can achieve more accurate predictions as well as generate new insights about the underlying processes.

Keywords: Bayesian network, structure learning, automatic learning, pest, mountain pine beetle, invasive species, machine learning, risk modeling

41 1 Introduction

42 Predictions are essential in aquatic and terrestrial ecology, whether the focus lies in changes
43 in ecosystem composition, structure, and richness to preserve the biodiversity and ecosystem
44 function, or in the spatial distribution of individuals and species to inform conservation and
45 invasive species policies. The field of predictive ecology focuses on how to make such predictions,
46 particularly in the context of climate change, and has grown exponentially since the 1990s,
47 given the quality and quantity of available ecological data (Purves *et al.*, 2013; Mouquet *et al.*,
48 2015). Simple and advanced statistical and machine-learning approaches have been used to
49 this end, and some have reported great success. Commonly applied models include mechanistic
50 equations, individual-based models, generalized linear models (Aukema *et al.*, 2008; Preisler
51 *et al.*, 2012), generalized additive models, MaxEnt (Merow *et al.*, 2013), decision trees, support
52 vector machines, and artificial neural networks, (Marmion *et al.*, 2009; Youssef *et al.*, 2016).

53 These standard models, however, lack some practical features, which questions their use as
54 predictors. They are unable to make predictions when the value of a covariate is missing, a typical
55 issue because some covariates are expensive or logistically impossible to collect. To impute the
56 missing values can be unreliable as modelling assumptions are needed so as to “guess” them.
57 The assumptions may even conflict with those posed by the original model using the imputed
58 values. Another approach is to produce a model that does not involve any covariate that is ever
59 missing. This can be problematic as well, because (i) those covariates are not fixed in the area
60 of interest: the value of a covariate may be missing at location A, but present at location B,
61 and the opposite may hold for another covariate; and (ii) even if a covariate is only measured
62 in the lab and never on the field, incorporating it in the model can still reveal its effect on the
63 response variable. Most models also cannot reveal the co-effect of more than one covariate on
64 the response variable, and some do not allow for statistical inference. Moreover, those that are
65 used for statistical inference cannot handle correlated covariates.

66 *Bayesian networks (BNs)* can deal with these issues. They are directed acyclic graphs, whose

67 nodes are the response variable and covariates, and the links between the nodes show how these
68 nodes are related to each other. Both links from covariate to response and from covariate to
69 covariate are allowed in the network. BNs are graphical, and hence often simpler to understand
70 than complex systems of equations (*e.g.*, Bode *et al.*, 2017; Eklöf *et al.*, 2013; Troyanskaya *et al.*,
71 2003; Rish *et al.*, 2009), deepening our understanding of natural phenomena as well as allowing
72 for accurate predictions. However, there are two main issues with how BNs are typically applied
73 in practice: *(i)* they are rarely used to their full potential, and *(ii)* they are misinterpreted as
74 *causal* networks. The common practice of applying BNs is to manually construct the structure
75 (network), based on the knowledge of experts, then either set the parameters manually or learn
76 them from data, and finally, read the links as causal relationships in the resulting BN. Although
77 useful in assessing the qualitative descriptions of an ecological process, this approach relies heavily
78 on our prior understanding of the process, and hence, is only as good as our understanding. If,
79 instead, both the structure and parameters of the BN are learned only from the data, there will
80 be room for more accurate predictions as well as new insights about the process. Moreover, BNs
81 are not causal networks, but essentially a set of conditional (in)dependencies that factorize the
82 joint probability distribution of all of the variables. Causal deductions, hence, may not be made,
83 although some hypotheses may be tested.

84 We complement previous studies on BNs that used the knowledge of experts (Marcot *et al.*,
85 2006; Chen & Pollino, 2012) by focusing on learning the structure, and proper model interpret-
86 ation in the form of conditional probabilistic inferences rather than causal deductions. The goal
87 of this paper is *(i)* to discuss the advantages of different ecological modeling approaches, and
88 highlight what BNs can offer in this context; *(ii)* to provide a systematic approach for training
89 a BN completely from data, without incorporating the prior knowledge of experts, and then
90 evaluating and interpreting the resulting BN; and *(iii)* to apply this method to the case study
91 of a mountain pine beetle (MPB) outbreak.

92 **2 Materials and Methods**

93 2.1 ADVANTAGES OF BAYESIAN NETWORKS

94 In what follows, we first briefly introduce BNs and then compare them with other modeling
 95 approaches in predictive ecology (Table 1). Here, we focus on the “typical” situation with each
 96 model; for example, the prediction accuracy of a properly trained BN being typically high does
 97 not imply that it is always higher or even as high as other highly accurate models.

Table 1 Comparison of models in predictive ecology. See Sections 2.1.2 to 2.1.9 for explanations of the model characteristics.

Model	Model characteristic										
	<i>Capable of generative learning</i>	<i>Handles missing values at training</i>	<i>Handles missing values at testing</i>	<i>Level of non-linearity that can be handled</i>	<i>Allows for statistical inference</i>	<i>Tolerates correlated inference and model selection</i>	<i>Provides insights on variables in statistical inference</i>	<i>Ease of incorporation of variables co-effects</i>	<i>Does not need prior knowledge</i>	<i>Predictive accuracy</i>	<i>Marginalizable</i>
Mechanistic equation	•		•			•	•				•
Individual-based model			•			•	•				
Generalized linear model				•			•	•	•		
Generalized additive model			•	•			•	•			
MaxEnt		•					•		•		
Decision tree			•				•	•	•		
Support vector machine (linear)				•				•	•		
Neural network				•				•	•		
Bayesian network	•	•	•	•	•	•	•	•	•	•	•

(empty) typically low • typically medium • typically high

98 2.1.1 Introduction to Bayesian networks

Given a set of n random variables $\mathcal{Z} = \{Z_i\}_{i=1}^n$ (consisting of the response variable and $n - 1$ covariates), a BN factorizes the joint probability $P(\mathcal{Z})$ according to a specified directed acyclic

graph whose nodes are the variables \mathcal{Z} , following the equation

$$P(\mathcal{Z}) = \prod_{i=1}^n P(Z_i | \text{Pa}_{Z_i}), \quad (1)$$

99 where Pa_{Z_i} denotes the *parents* of Z_i in the graph, *i.e.*, those nodes that have an outgoing edge
 100 that leads to Z_i (Fig. 1 in SI). The individual factors $P(Z_i | \text{Pa}_{Z_i})$ are known as *conditional*
 101 *probability distributions (CPDs)* (Koller & Friedman, 2009). A BN encodes the claim that given
 102 the *Markov blanket* $\text{MB}(Z_i)$ of a node Z_i – which is the set of its parents, children and the other
 103 parents of its children – the node becomes independent from the remaining of the nodes, written
 104 $Z_i \perp (\mathcal{Z} - Z_i - \text{MB}(Z_i)) | \text{MB}(Z_i)$. This provides the essentials for understanding how the variables
 105 relate to each other. We, therefore, refer to the nodes in the Markov blanket of the target node
 106 Z_i as *primary* covariates and to others as *secondary*. The estimation of the target node based
 107 on the values of the primary covariates does not change if the values of the secondary covariates
 108 are additionally known. The conditional independencies also reduce the number of parameters
 109 needed to represent the joint distribution $P(\mathcal{Z})$. It is possible to learn from data, both the graph
 110 and the CPDs, known as the *structure* and *parameters* of the BN (Section 2.2).

The factorization in Eq. (1) is sufficient to define BNs and draws a clear line between BNs and causal networks. To explain, assume that we are modeling the co-occurrence of two competitive species, with densities denoted by X_1 and X_2 , each corresponding to a node in a BN. We could link these two distributions using either a directed edge from X_1 to X_2 , decomposing the joint density distribution of the two species as $P(X_1, X_2) = P(X_1)P(X_2 | X_1)$, or a directed edge from X_2 to X_1 , resulting in $P(X_1, X_2) = P(X_2)P(X_1 | X_2)$. The first relies on the distribution of X_1 and the conditional distribution of X_2 given X_1 , and the reverse holds for the second. Both of these models can be used to make acceptable predictions if one can estimate parameters $P(X_2 | X_1)$ and $P(X_1 | X_2)$ effectively. However, none of the models are causal: neither of X_1 or X_2 is causing the other. The edge simply means probabilistic dependence and dictates the

factorization of the joint distribution. Now, assume that the species distributions are each partly “caused” by a third variable *vegetation*, denoted by V . Should we construct a BN based on this “causal understanding”, we would add the node V and link it to both X_1 and X_2 without connecting the two. This results in the joint probability distribution of the two species and vegetation

$$P(X_1, X_2, V) = P(V)P(X_1 | V)P(X_2 | V).$$

However, this is not the only way to model the joint distribution. Depending on the training data, one may obtain a more accurate model in terms of data fitting by also linking X_1 to X_2 (or *vice versa*), resulting in

$$P(X_1, X_2, V) = P(V)P(X_1 | V)P(X_2 | X_1, V).$$

This might be because vegetation is not the only cause of the two, and another factor, say temperature, also plays a role, which is not included in our variable list but is highly correlated with X_1 , and hence, provides a better estimation of X_2 by linking the two. One may yet use a different model, where X_1 and X_2 are not linked to each other but both linked to V , resulting in

$$P(X_1, X_2, V) = P(V | X_1, X_2)P(X_1)P(X_2).$$

111 This is particularly useful if we know the distributions of the species densities, *i.e.*, $P(X_1)$ and
 112 $P(X_2)$, but not that of vegetation $P(V)$, and we know how vegetation can be estimated based
 113 on the distribution of the two species, *i.e.*, $P(V | X_1, X_2)$. None of the links in this model are
 114 causal.

115 2.1.2 *Generative vs discriminative learning*

116 Consider the response variable Y and set of covariates (features) \mathcal{X} that are used to estimate
117 Y . One may pursue either of the two learning tasks with respect to these variables: *generative*,
118 that is to learn the joint probability distribution $P(Y, \mathcal{X})$, or *discriminative*, that is to learn
119 the conditional probability $P(Y | \mathcal{X})$. The joint probability distribution $P(Y, \mathcal{X})$ represents the
120 probability of any given assignment to all of the variables Y and \mathcal{X} in the data, or loosely
121 speaking, how all the variables are related to each other. On the other hand, the conditional
122 probability $P(Y | \mathcal{X})$ represents the probability of Y happening given \mathcal{X} , or in other words, in
123 which cases does Y happen. So discriminative learning focuses only on the probability of the
124 response variable whereas generative learning also reveals the probability of the covariates. For
125 example, an ecologist may be interested in two species' co-occurrence, which is a generative
126 question given by the distribution $P(X_1, X_2)$, were X_1 and X_2 are the density of the species.
127 On the other hand, the same ecologist may be interested in whether the density of species X_1
128 (as a response variable) can be estimated using that of species X_2 (as the covariate), which is a
129 discriminative question, given by $P(X_1 | X_2)$.

130 Note that knowing the “true” joint distribution $P(Y, \mathcal{X})$ allows knowing the conditional distri-
131 bution $P(Y | \mathcal{X})$. However, because small errors in estimating $P(Y, \mathcal{X})$, which typically happen
132 in practice, might lead to large errors in the associated values of $P(Y | \mathcal{X})$ (Ng & Jordan, 2002),
133 each learning task deserves its own treatment. Although potentially capable of modelling the
134 joint probability distribution, mechanistic models are not commonly used for this purpose as it
135 would require a great deal of prior knowledge of the process. Roughly speaking, none of the
136 models in Table 1, except for BNs, are effective at generative learning.

137 2.1.3 *Missing data*

138 Datasets often have many instances (observations) where the value of one or more of the covariates
139 and/or response variables are missing. Missing values can occur both at the time of training and

140 testing of a model.

141 Should the training dataset contain missing values, most traditional statistical methods such
142 as regressions would use *casewise deletion*, that is, to remove the entire instance (observation)
143 from the dataset if the value of one or more variable is missing (Harrell, 2015). Casewise deletions
144 can lead to bias in the estimated parameters if the degree to which the variable’s value is likely
145 to be missing is correlated with the actual range of values, *e.g.*, when a temperature sensor fails
146 to record values below -10°C . Casewise deletions also result in losing the information provided
147 by the remaining variables in the instance with missing values. Therefore, *imputation* is often
148 used to estimate the missing values, which can be as simple as using the variable’s mean or
149 the variable’s value from a similar instance, or can be more complex, such as using the *chained*
150 *equation method* (Harrell, 2015). However, in essence, imputation is presuming a model for the
151 variables with missing values, which may conflict the actual model that is going to be trained on
152 the imputed dataset, resulting in a poor predictor. As with BNs, methods such as *expectation*
153 *maximization (EM)* and *structural EM* can be used to learn the parameters and structure,
154 without imputation or casewise deletion (Koller & Friedman, 2009).

155 Should the testing dataset contain missing values, almost all models fail to make predictions
156 as each covariate has to take some value, *i.e.*, they cannot be left with “NA”s (not available).
157 Imputation comes with the above mentioned shortcomings. Another alternative is to use expert
158 knowledge to obtain probable limits for the covariates with missing values, and run the model on
159 those limits to get a probable range for the prediction. For example, in climate change models,
160 the exact concentration of the pathway of a covariate such as greenhouse gas emission that will be
161 followed in the future is unknown. Therefore, models use a series of scenarios ranging from best
162 to worst case scenario in order to predict changes in CO_2 emissions and temperatures (Pachauri
163 *et al.*, 2015). There is, however, no need of these rough approximations when applying BNs. By
164 marginalizing over the unobserved covariates, BNs can predict the target variable based on any
165 observed subset of the covariates.

166 2.1.4 *Nonlinearity of the relationship between the covariates and response variable*

167 In many real-world situations, the response variable may be related to the covariates in a highly
168 nonlinear manner. Simple models such as linear regressions, however, assume a linear relation-
169 ship. To capture some levels of non-linearity, generalized linear models extend the regressions by
170 applying functions such as $\log(\cdot)$ and $\text{logit}(\cdot)$ to the covariates. Other extensions, such as general-
171 ized additive models, fit a smooth curve to the data for each covariate, thereby allowing complex
172 nonlinear relationships (Guisan *et al.*, 2002). Another extension is the machine-learning method
173 MaxEnt (Phillips *et al.*, 2006) that is able to link highly non-linear response curves and estimate
174 the probability distribution of the response variable using maximum entropy. Likewise, support
175 vector machines classify the covariate space using hyper-planes, and hence, are linear, yet can
176 allow for some nonlinearity by first transforming the space using nonlinear kernels (Scholkopf &
177 Smola, 2001). Process-based models can also build in highly complex nonlinear relationships. In
178 all of these cases, the relationships between the response variable and covariates must be entirely
179 described, based on a *a priori* model, a constraint that is relaxed in some other machine learn-
180 ing models. For example, classification trees can represent any function over the set of discrete
181 covariates, but does not need to be defined beforehand. Note, this may require a very deep
182 classification tree. Moreover, the fact that a classification tree can represent a complex function
183 does not mean it can be learned effectively. Likewise, BNs are flexible in dealing with nonlinear
184 relationships. Over a set of discrete variables, BNs can represent an arbitrary joint probability
185 distribution $P(Y, \mathcal{X})$, which can represent any arbitrary conditional distribution $P(Y|\mathcal{X})$.

186 2.1.5 *Hypothesis testing, statistical inference and model selection*

187 The objective of hypothesis testing is to make inference through deduction. It consists of devising
188 one or more working hypotheses and challenging them with data for corroboration (Hilborn &
189 Mangel, 1997; Stephens *et al.*, 2005). The hypothesis to test is translated into a mathematical
190 equation and is verified using methods such as least squares and maximum likelihood. So to test

191 a hypothesis, one needs (i) a mathematical equation representing a biological hypothesis and
192 (ii) a test statistic with a distribution that can be determined, representing the model accuracy
193 when confronted to data. The complexity of machine-learning models usually prevents us from
194 obtaining a simple equation representing the hypothesis, but this is not the case for BNs.

195 For example, consider a process with the response variable Y and covariates X_1 and X_2 .
196 One may hypothesize that the response variable Y depends on both X_1 and X_2 but becomes
197 independent of X_2 , given X_1 . Namely, the response variable depends directly only on covariate
198 X_1 , and that X_1 itself depends only on X_2 . This can be modelled by a BN with three nodes for
199 the variables and two links: one from X_1 to Y and another from X_2 to X_1 . The BN assigns the
200 following likelihood to each observation of the above process:

$$P(Y, X_1, X_2) = P(Y | X_1)P(X_1 | X_2)P(X_2). \quad (2)$$

201 The *null hypothesis* in this case is that there is no dependence among the variables: they are
202 mutually independent. This results in a BN without any links between the nodes, yielding the
203 following likelihood:

$$P(Y, X_1, X_2) = P(Y)P(X_1)P(X_2). \quad (3)$$

204 Given an observation, each of the probabilistic terms on the right hand-side of the above equations
205 is simply a parameter provided that the BNs are discrete. Hence, the likelihood of a specified
206 dataset for each of the BNs will be a polynomial in the parameters, the maximum of which is
207 straightforward to derive. This allows for classical hypothesis testing, *e.g.*, by employing the
208 *likelihood ratio test*, to reject the null hypothesis. Alternatively, among all BNs with the nodes
209 Y , X_1 and X_2 , one may find ‘the best’ using *multiple working hypotheses*, based on the *Akaike*
210 *information criterion* (AIC; Akaike, 1974) or *Bayesian information criterion* (BIC; Schwarz,
211 1978). Therefore, with BNs, we are able to make inferences and obtain insights on the ecologically
212 relevant covariates (*e.g.* Cooper & Herskovits, 1992; Pollino *et al.*, 2007a; Milns *et al.*, 2010).

213 2.1.6 *Prior knowledge of the processes*

214 Unlike mechanistic models that typically need a comprehensive knowledge of the involved pro-
215 cesses to make accurate predictions, phenomenological methods such as traditional statistics and
216 especially machine learning have more leeway. One does not need to have any knowledge about
217 the ecological process to train and test a support vector machine, or neural network, for example.
218 Although one may argue that the functions used in a neural network or the number of nodes
219 and layers are parameters to be determined beforehand, yet these too can be selected automat-
220 ically based on the training data or general rules of thumb. The level of autonomous learning is
221 even higher with BNs. The whole structure and parameters of a discrete BN can be completely
222 learned from data (McCann *et al.*, 2006a). The same goes for decision trees.

223 Although they can be trained autonomously, BNs allow experts to incorporate their know-
224 ledge into the network by forcing or preventing links between the nodes and additionally adding
225 *latent variables* that are unobservable and often abstract variables, such as habitat quality. In-
226 deed, the spectrum of autonomous learning for BNs ranges from neither to both structure and
227 parameters learned based on experts' knowledge.

228 2.1.7 *Correlations*

229 Often two or more of the covariates in a process are highly correlated. This hinders statistical
230 inference as the effects of the correlated covariates on the response variable are difficult to separate
231 (Stewart, 1987; Dormann *et al.*, 2013). This would happen if we were building a model, say a
232 logistic regression, with two covariates that are both relevant to the response variable, and also are
233 highly correlated with each other. Thus, typically one of the variables is eliminated beforehand,
234 either randomly, based on ecological relevance, measurement feasibility, and proximity to the
235 mechanisms (Harrell, 2015; Dormann *et al.*, 2013), or by using some autonomous technique
236 such as *minimum-redundancy maximum-relevance* (Peng *et al.*, 2005). However, this prevents
237 understanding the impact of both of the correlated covariates together on the response variable.

238 Process-based models do not suffer from correlation (except for parameter estimability), yet they
239 require the mechanisms to be *a priori* known (Dormann *et al.*, 2013). Nevertheless, a BN whose
240 structure is learned from data, does not require any prior knowledge, and reveals the differences
241 of the correlated covariates in terms of their probabilistic dependence to other covariates as well
242 as the response variable.

243 2.1.8 Predictive accuracy

244 Despite the complexity of ecological systems (Levin, 1992; Anand *et al.*, 2010), some machine-
245 learning models are reported to make accurate predictions. In contrast, process-based and tra-
246 ditional statistical models are rarely able to reach the same level of accuracy (*e.g.* Elith *et al.*,
247 2006). Particularly, process-based models are known for their inability to make good predictions,
248 although this has been challenged by, for example, (Håkanson, 2004), who presented an accur-
249 ate mechanistic model for aquatic systems. Within machine-learning models, neural networks
250 are acknowledged for accurate performance in highly complex tasks such as image recognition
251 (Egmont-Petersen *et al.*, 2002). However, this does not mean that neural networks necessarily
252 outperform simpler models in practice. Firstly, finding the optimal number of layers and nodes is
253 not always practical due to limited computational resources. Secondly, proper estimation of the
254 many parameters of a neural network often requires massive data. Hence, while asymptotically
255 effective, neural networks may not be as successful as simple models when the available data is
256 insufficient. Finally, if the system in question is actually simple, then neural networks, especially
257 deep ones, can easily overfit the training data. Simpler models may be a better choice also in
258 this case.

259 2.1.9 Marginalizability

260 The notion of marginalizability addresses the possibility of separately studying how a particular
261 covariate or subset of the covariates informs us about the response variable. We call a model

262 *marginalizable* if it allows us to compute the probability of the response variable Y given any
263 subset \mathcal{Z} of the covariates \mathcal{X} ; that is, $P(Y|\mathcal{Z})$. Most predictive models allow us to compute
264 $P(Y|\mathcal{X})$, that is, the likelihood of the response given all of the covariates. However, only those
265 that perform a generative task, *i.e.*, learning $P(Y,\mathcal{X})$, allow us to marginalize the likelihood
266 over the variables $\mathcal{X} - \mathcal{Z}$, to obtain the likelihood conditioned on only those variables that we
267 are interested in: $P(Y|\mathcal{Z})$. Therefore, only BNs and those mechanistic models developed to
268 formulate the joint probability $P(Y,\mathcal{X})$, are marginalizable.

269 2.2 LEARNING BAYESIAN NETWORKS FROM DATA

270 We explain, step by step, how to learn and then use a BN to make predictions and acquire
271 biological insights. Most steps are general enough to be applied by any statistical/machine-
272 learning method in the context of model selection or prediction making.

273 2.2.1 Setup

274 Ecological processes are typically modelled by a response variable Y and a set of covariates \mathcal{X} . If
275 the process is spatial and temporal, then each instance (observation) of the process has a unique
276 pair of identities: (*i*) the time t of the instance, the unit of which indicates the frequency of
277 the observations, *e.g.*, a year, month, or day, and (*ii*) a general index g , roughly to distinguish
278 the instances location-wise. For example, if the process of interest is Cyanobacteria bloom in
279 lakes, then g indicates the label of the lakes. If the interest is in the spread of an infestation
280 over a given area, then we may divide the area into $r \times r$ squares for say $r = 1\text{km}$, and label
281 them by $g = 1, 2, \dots$. We may exclude time when modelling a stationary quantity, *e.g.*, the
282 joint distribution of several species in a specific area. Similarly, we may exclude the index g , if
283 all instances are taken from the same location, *e.g.*, from the same lake. Also, note that time
284 and especially the index g are not necessarily two covariates of the process. Indeed, time must
285 be excluded from the set of covariates if the goal is to obtain a model that can be applied to

286 times different from those in the available data, *e.g.*, to predict the future (see SI). Similarly,
 287 the index g may be excluded; however, one must acknowledge the possible performance loss
 288 when applying the model to areas far-away from the training area, with dramatically different
 289 geographic features.

290 For illustration purposes, in what follows, we consider a spatial and temporal process. For
 291 each index g and time t , let $\mathcal{X}_{g,t}$ denote the set of covariates and $Y_{g,t}$ denote the response
 292 variable (Table 2). Although the response variable can be continuous or integer, in order to use
 293 acknowledged performance measures such as AUC (Section 2.2.5), we restrict it to be binary. For
 294 example, given an index g and time t , the response variable $Y_{g,t}$ may represent the presence, $Y_{g,t} =$
 295 1, or absence, $Y_{g,t} = 0$, of infestation or a species of interest. The covariates can be correlated
 296 with each other and may include variables that are not known *a priori* to contribute to the
 297 response variable. Our goal is to estimate (learn) the joint probability distribution $P(Y_{g,t}, \mathcal{X}_{g,t})$
 using available data.

Table 2 Variable notation.

Notation	Variable
t	time
g	general index
$Y_{g,t}$	response variable
$\mathcal{X}_{g,t}$	set of covariates
$P(\cdot)$	probability function

298

299 2.2.2 Step 1: Data discretization

300 The random variables in a BN can be either continuous or categorical. However, if they are
 301 continuous, we must predetermine their distributional forms, *e.g.*, a Gaussian distribution. To
 302 avoid making such assumptions, we use discrete BNs where every variable is categorical. We
 303 discretize all continuous variables by considering various number of intervals or discretization
 304 levels (say 2, 3, \dots , 10) and using data to determine which number leads to a higher performance
 305 score. If a continuous variable's range does not have evident thresholds in terms of the biological

306 context, we use *Hartemink’s information-preserving algorithm* (Hartemink, 2001) to quantify
307 the values in a way that maximizes the mutual information shared by the variables (Cover &
308 Thomas, 2012).

309 2.2.3 Step 2: Partitioning the dataset into train and test

310 The typical machine-learning approach to learn, then evaluate a model, is to randomly partition
311 the dataset in two subsets, *train* and *test*, where the greater portion (train) is used to estimate
312 the model, and the remaining portion (test) to evaluate the trained model. However, evaluation
313 concerns are raised if the instances of the original data are randomly partitioned into train and
314 test. Indeed, using this method, the train and the test datasets are extremely similar (see SI).
315 For each instance of the test dataset, it is highly likely to have a matching instance in the train
316 dataset due to correlations in time and space. The purpose of a test dataset is to simulate how
317 the model performs when applied in practice to a new dataset. If the goal is to make predictions
318 in the future, say next month, we set the train dataset to be the data from the final observations
319 (instances) and let the train dataset be the remaining instances. Namely, we make the train and
320 test datasets time-wise disjoint.

321 2.2.4 Step 3: Learning

322 **Step 3.1. Learning the BN structure.** For each of the k -level quantified training datasets,
323 we find the structure that results in the lowest BIC or the lowest AIC. Although this can be done
324 by performing an exhaustive search on all possible BN structures – *i.e.*, directed acyclic graphs,
325 with the response variable and covariates as the node-set – we instead use efficient algorithms,
326 *e.g.*, (Silander & Myllymaki, 2012), which is implemented in the R package `bnstruct` (Franzin
327 *et al.*, 2017). Both BIC and AIC criteria penalize having more parameters, which reduces the
328 chance of over-fitting to the training dataset. The choice of BIC or AIC depends on the main
329 goal of the study, the model complexity, and the number of instances relative to the number of

330 parameters (Aho *et al.*, 2014).

331 Note this approach is computationally infeasible if there are too many variables, *e.g.*, more
332 than 25, or too many discretization levels. Then one may, instead, either use a fixed (*a priori*
333 known) BN structure, *e.g.*, naive Bayes, or learn a ‘close-to-optimal’ (*a priori* unknown) BN
334 on the training dataset using acknowledged searching-algorithms (Table 3, Fig. 1). We learn
335 the structure of the *a priori* unknown networks by the `bnlearn` package in R (Scutari, 2009).
336 The input to each algorithm is the variables and the corresponding training dataset, and the
337 output is a BN structure whose nodes are the variables. In case the learned structure contains
338 undirected links, we randomly assign directions as long as directed cycles and v-structures do
339 not appear. This is because BNs must not contain cycles by definition, and the introduction of
340 v-structures can change the performance of the resulting BN (Koller & Friedman, 2009). So for
341 each discretization level k , we obtain a BN structure according to one of the algorithms or fixed
342 structures in Table 3.

343 **Step 3.2. Learning the BN parameters.** After finding the highest-scoring BN structure
344 for each of the k -level quantified training datasets, we learn the associated CPD parameters on
345 the same training dataset and denote the resulting BN by \mathcal{B}_k^* . We use the *Bayesian parameter*
346 *estimation* approach (Koller & Friedman, 2009), implemented in `bnlearn`. To this end, for each
347 quantization level k , we obtain a BN \mathcal{B}_k^* that best fits the training data in terms of BIC, AIC,
348 or other constraints listed in Table 3.

349 2.2.5 Step 4: Evaluation

350 How to choose among the different \mathcal{B}_k^* s from the previous step? Namely, what number of discret-
351 ization levels results in ‘the best’ BN? We cannot compare them directly using a performance
352 measure that involves the likelihood of the data, *e.g.*, log-likelihood, AIC and BIC, because the
353 \mathcal{B}_k^* s do not use the same data but different discretized versions of it.

354 However, all BNs use the same number of discretization levels for the response variable. So

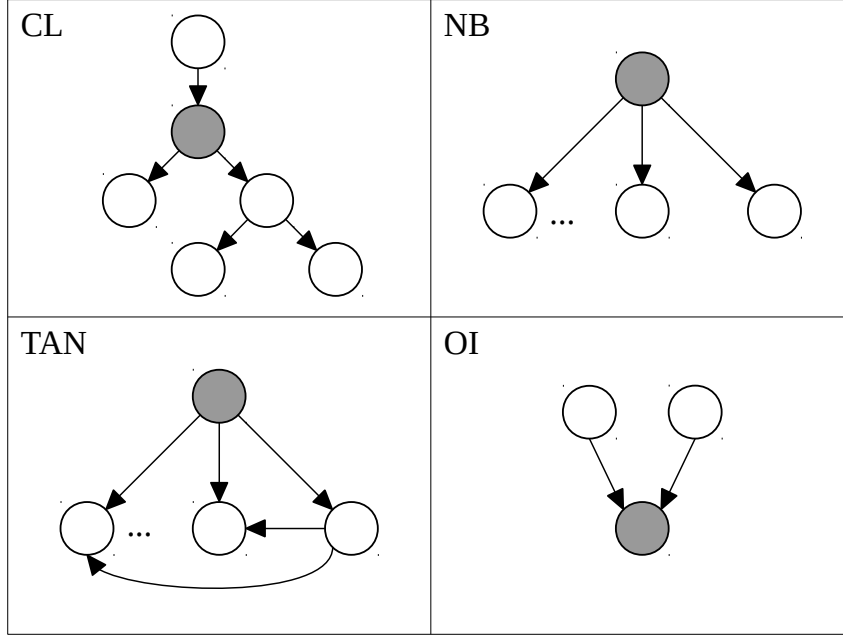


Fig. 1 Structure of different Bayesian networks CL: Chow-Liu, NB: naive Bayes, TAN: tree-augmented naive Bayes, and OI: one-memory infestation (Section 3). Grey and white circles represent the target $I_{g,t}$ and its covariates. In the OI case, the covariates are $I_{g,t-1}^{\text{Missed}}$ and $I_{g,t-1}^{\text{Managed}}$.

355 we can compare them based on how well they predict the response variable on the test dataset.

356 Each network allows us to compute $P(Y_{g,t} | \mathcal{X}_{g,t})$, that is, the chances of the observed response

357 variable given the covariates, for every instance in the dataset. Correspondingly, we compare

358 the *area under receiver operating characteristic curve (AUROC or simply AUC)* (Metz, 1978;

359 Bradley, 1997) score of the BNs on the test dataset (see SI). The choice of AUC is to make our

360 results comparable with the huge body of literature using this performance score as the final

361 performance of a classifier. For each discretization level k , we calculate the AUC score of \mathcal{B}_k^*

362 and pick the highest-scoring one as our final BN. If there is a tie between the top BNs, we break

363 it by looking at the *area under precision-recall curve (AUPR)* (Raghavan *et al.*, 1989; Saito &

364 Rehmsmeier, 2015) scores; that is, among the top BNs with a deficit of at most, say 0.01, from the

365 top AUC, we pick the one with the highest AUPR. The AUPR score better handles unbalanced

366 data by looking at precision rather than the false positive rate (Davis & Goadrich, 2006; Saito

367 & Rehmsmeier, 2015).

368 Given the temporal nature of our task, we evaluate the final model on a single test dataset,

Table 3 Bayesian networks to compare with the learned one.

Abbreviated name	Based on the algorithm/structure	Type of the algorithm	Description
GS	Grow shrink	Constraint based	Uses conditional independence tests on the training dataset to detect the Markov blankets of the variables (Margaritis & Thrun, 1999)
IAMB	Incremental association Markov blanket	Constraint based	Detects Markov blankets with an attempt to avoid <i>false positives</i> , <i>i.e.</i> , fault infestation predictions (Tsamardinos <i>et al.</i> , 2003)
IIAMB	Interleaved incremental association Markov blanket	Constraint based	A variant of IAMB to maintain the size of the Markov blanket as small as possible (Tsamardinos <i>et al.</i> , 2003)
HC	Hill climbing	Local search	Starts from a random directed graph and adds or removes an edge only if it results in a higher score (BIC in our case) on the train dataset (Margaritis, 2003)
CL	Chow-Liu	Global search	Finds the undirected spanning tree of the variables to minimize the <i>Kullback-Leibler distance</i> from the actual distribution (Chow & Liu, 1968) (Fig. 1)
NB	Naive Bayes	-	The most basic yet often successful BN formed by the response variable ($Y_{g,t}$ in our case), linking to all of the covariates (Koller & Friedman, 2009) (Fig. 1)
TAN	Tree-augmented naive Bayes	-	A NB network with a spanning tree among the covariates that can be learned from the train dataset (Friedman <i>et al.</i> , 1997) (Fig. 1)

369 as explained in Section 2.2.3. If instead, one divides the original dataset into several yearly-
370 separated folds and uses cross-validation to obtain the AUC and AUPR values for each fold,
371 then one could also provide confidence intervals for the reported AUC and AUPR values.

372 2.2.6 Step 5: Interpretation

373 Given an index g and year t , the final BN obtained from the above steps, determines the joint
374 probability distribution of the response variable $Y_{g,t}$ and covariates $\mathcal{X}_{g,t}$. Perhaps the most
375 important implication of the obtained BN is the primary ($\mathcal{X}_{g,t}^1$) and secondary ($\mathcal{X}_{g,t}^2$) division of
376 the covariates $\mathcal{X}_{g,t}$ with respect to the response variable. Namely, if we just know the primary

377 covariates, there is no need to know the secondary covariates, *i.e.*,

$$P(Y_{g,t} | \mathcal{X}_{g,t}^1, \mathcal{X}_{g,t}^2) = P(Y_{g,t} | \mathcal{X}_{g,t}^1). \quad (4)$$

378 Moreover, other conditional independencies between the covariates themselves can be identified
379 based on the *d-separations* of the BN (Koller & Friedman, 2009).

380 Also, based on the CPDs, we can investigate knowing which covariates increases the prob-
381 ability of the response variable most. For example, consider the covariate temperature $T_{g,t}$,
382 discretized into the two ranges $[20^\circ\text{C}, 30^\circ\text{C})$ and $[30^\circ\text{C}, 40^\circ\text{C})$. We can see how the response
383 variable $Y_{g,t}$ depends on this covariate by sweeping through these quantified levels, *e.g.*,

$$P(Y_{g,t} = 1 | T_{g,t} \in [20^\circ\text{C}, 30^\circ\text{C})) = 0.3 \quad \text{and} \quad P(Y_{g,t} = 1 | T_{g,t} \in [30^\circ\text{C}, 40^\circ\text{C})) = 0.4. \quad (5)$$

384 Hence, the response variable being equal to 1 is most likely when temperature is in the range
385 $[30^\circ\text{C}, 40^\circ\text{C})$. Note that this is only if other covariates are unknown. Now, comparing this with
386 the similar probability conditioned on a different covariate, clarifies which is more informative
387 to the response variable.

388 2.2.7 Step 6 (optional): Sensitivity analysis

389 We examine the prediction accuracy (AUC and AUPR) of the best model when a primary co-
390 variate becomes unobservable. This roughly shows the contribution of each covariate to the
391 prediction, although it is, indeed, the co-effect of all the covariates that leads to accurate predic-
392 tions.

393 2.2.8 Step 7 (optional): Comparison with simple Bayesian networks

394 To further assess the prediction performance of the final BN, we may compare its AUC (or
395 AUPR) with that of simple BNs consisting of a single or two covariates linked to the response

396 variable. These BNs might be considered as the ‘null model’.

397 Recall that our final BN is designed to perform a generative task, that is to reveal the
398 relationships between the variables, not a discriminative task, that is to predict the response
399 variable. However, if the BN performs well on the first, it is likely to also do well on the second.
400 Yet, the opposite does not hold (Ng & Jordan, 2002). So even if any of these simple BNs
401 predicts the response variable better than our final BN, it does not question the capability of
402 our BN in explaining the probabilistic relationships between the variables. The same may hold
403 in the previous optional step: the AUC score of the BN may increase after removing some of the
404 covariates. This can also be explained by the fact that our final BN is the best fit to the data
405 under the performance score that we used, which is BIC (or AIC) not AUC.

406 Nevertheless, in such cases, we may train a BN with a different set of covariates for prediction
407 purposes. For example, we may find that subset of the covariates that results in a BN scoring
408 the highest AUC on the training dataset.

409 **3 The mountain pine beetle case study**

410 We illustrate the learning and interpretation of BNs via the data on the MPB infestation in the
411 Cypress Hills park—an interprovincial park located in Alberta and Saskatchewan (Fig. 2 in SI).
412 Endemic-level populations of MPB have existed in Cypress Hills since the 1980’s. However, a
413 MPB outbreak started in 2006 and propagated in the park, where it continues until now.

414 **3.1 BIOLOGY AND MANAGEMENT**

415 MPB presents two main population phases: an *endemic phase* with small population size where
416 beetles attack weak and stressed pines with the help of other bark beetles, and an *epidemic*
417 *phase* where the number of individuals is large enough to overcome the defences of large and
418 healthy pines (Safranyik & Carroll, 2006). In summer, beetles will emerge from a tree, mate,
419 and attack new pines to lay eggs in galleries under the bark. New MPB infestations are reported

420 to frequently appear in south and west-facing slopes (Safranyik, 2004). During the tree growing
421 season, water-stress negatively impacts the pine's ability to build its defense against bark beetles
422 (Safranyik, 1978; Lusebrink *et al.*, 2016). Indeed, pines use water to make a toxic resin that
423 is exuded during a beetle attack to prevent beetles from attracting conspecifics and inhibit the
424 formation of galleries and oviposition (Raffa & Berryman, 1983; Erbilgin *et al.*, 2017). MPB
425 emergence and flights are reduced with high temperatures during the dispersal season (Safranyik
426 & Carroll, 2006). MPB can disperse at short distances within a stand or, more rarely, fly above
427 the canopy to use the wind to travel long distances of the order of tens to thousands of kilometers
428 (Safranyik & Carroll, 2006; Robertson *et al.*, 2007). Once the eggs are laid, the adults die. Over
429 the fall, winter, and spring, eggs become larvae then pupae before finishing their transition to
430 adult and emerging in the summer. Individuals need a minimum of 833 degree days to complete
431 their transition to adult (Safranyik *et al.*, 1975, 2010).

432 The Forest Service Branch of the Saskatchewan Ministry of Environment follows a strict
433 direct control approach. At the start of every fall, the park is surveyed aerially to collect geo-
434 referenced data on red-top trees – *i.e.*, trees that are dead or dying from a MPB infestation at
435 the previous year. Then, on the ground, managers survey 50 meter-radius circular plots around
436 each red-top tree to find recently infested trees during the summer. The newly-found infestations
437 are later controlled in late fall/winter using a fell and burn method.

438 Our goal is to provide a set of covariates that potentially impact the MBP infestation in
439 Cypress Hills area, understand how they are related to each other and to the infestation, and
440 find which covariates are sufficient for an accurate prediction. We also are interested to test
441 some of the claims in the literature, *e.g.*, lower humidity increases the chances of infestation
442 (Lusebrink *et al.*, 2016), and to find what values of the highly correlated covariates degree-days
443 and maximum temperature, that are typically not included together in a model, makes infestation
444 most likely. These objectives are well suited to BNs.

445 3.2 METHODS

446 We divide the studying area into $100\text{m} \times 100\text{m}$ squares and label them by $g = 1, 2, \dots$. We
447 choose one year as our time unite and define the response variable $I_{g,t}$ as the presence or absence
448 of infestation in pixel g at the fall of year t . We use the covariates listed in Table 4 and quantify
449 them into $k = 1, 2, \dots, 7$ levels. Our data includes the values of $\mathcal{X}_{g,t}$ and $I_{g,t}$ over the years
450 $t = 2006, 2007, \dots, 2018$ and for 18 317 different pixels g in Cypress Hills, resulting in a total of
451 238 121 instances (see SI for an instance of the data).

452 We compare the AUC and AUPR scores of our final model with those of what we call the
453 *one-memory infestation (OI)* Bayesian network, consisting of $I_{g,t-1}^{\text{Managed}}$ and $I_{g,t-1}^{\text{Missed}}$, being linked
454 to the target $I_{g,t}$, considered as the null model (Fig. 1).

455 3.3 RESULTING BAYESIAN NETWORK

456 We find the BN with the best BIC score on the train dataset with 6 discrete levels, *i.e.*, \mathcal{B}_6^*
457 (Fig. 3), as our “best model” to explain the MPB infestation, with AUC = 0.88 and AUPR
458 = 0.28. The OI model scores 0.75 for AUC and 0.19 for AUPR – both lower than our selected
459 model. According to the structure of \mathcal{B}_6^* , the infestation $I_{g,t}$ in location g at year t is directly
460 connected to $I_{g,t-1}^{\text{Missed}}$, $I_{g,t-1}^{\text{Managed}}$, $I_{\mathcal{N}_g,t-1}^{\text{Missed}}$, $I_{\mathcal{N}_g,t-1}^{\text{Managed}}$, B_g , and $T_{g,t-1}^{\text{max}}$. These together with $C_{g,t-1}$, form
461 the Markov blanket of the infestation node, and hence, are the primary covariates and sufficient
462 for estimating infestation with 0.88 AUC score. Other covariates are all indirectly linked to
463 infestation and are secondary covariates. Given \mathcal{B}_6^* , one can obtain conditional independencies
464 of the covariates to infestation using d-separations and plot the CPDs (see SI).

465 3.4 SENSITIVITY TO MISSING COVARIATES

466 The prediction accuracy of \mathcal{B}_6^* does not deteriorate when the values of any of the secondary
467 covariates are missing. Upon missing values for the primaries, the model can still accurately
468 predict infestation as it can use some of the secondary covariates (Table 5).

Table 4 Description of the covariates $\mathcal{X}_{g,t}$.

Name	Symbol	Description	Unit
Aspect	A_g	Compass direction that the slope at pixel g faces	◦
Distance to infested border	B_g	Distance of the centre of pixel g to the border of the whole area of interest that was initially infested (Fig. 2 in SI)	km
Degree days	$D_{g,t-1}$	Sum of daily temperatures above 5.5°C from fall of year $t - 1$ to summer of year t	Celsius degree-day
Maximum temperature	$T_{g,t-1}^{\max}$	Highest maximum daily temperature in July and August of year t	°C
Wind speed	$W_{g,t-1}$	Average daily wind speed in July and August of year t	km/h
Relative humidity	$R_{g,t-1}$	Average daily relative humidity in spring of year t	%
Cold tolerance	$C_{g,t-1}$	An index in $[0, 1]$ representing the ability of the larvae to survive the cold season of year $t - 1$, as defined in (Régnière & Bentz, 2007)	
Pine cover	$P_{g,t-1}$	Pine density in summer of year t	%
Managed last year infestation	$I_{g,t-1}^{\text{Managed}}$	Defined to be 1 if pixel g includes at least one tree that was infested and managed (controlled) at year $t - 1$, and 0 otherwise (Fig. 2)	–
Missed last year infestation	$I_{g,t-1}^{\text{Missed}}$	Defined to be 1 if pixel g includes at least one tree that was infested and missed (not controlled) at year $t - 1$, and 0 otherwise	–
Missed neighbors' last year infestation	$I_{\mathcal{N}_g,t-1}^{\text{Missed}}$	MPB's ability to disperse at short distances within a stand, defined as $I_{\mathcal{N}_g,t-1}^{\text{Missed}} = \sum_{i=1}^3 \frac{1}{2^i} \sum_{g' \in \mathcal{N}_g^i} I_{g',t-1}^{\text{Missed}}, \quad I_{\mathcal{N}_g,t-1}^{\text{Missed}} \in [0, 6],$	–
Managed neighbors' last year infestation	$I_{\mathcal{N}_g,t-1}^{\text{Managed}}$	where \mathcal{N}_g^i are those pixels that are essentially at a distance of $i \times 100\text{m}$ from g (Fig. 3 in SI). Defined similarly to $I_{\mathcal{N}_g,t-1}^{\text{Missed}}$, with the difference that $I_{g',t-1}^{\text{Missed}}$ is replaced by $I_{g',t-1}^{\text{Managed}}$	–

469 3.5 DISCUSSION

470 The final model we have chosen to explain the MPB infestation in the Cypress Hills area is the
471 BN \mathcal{B}_6^* with 6 discretization levels, scoring 0.88 AUC on the test dataset. For a managed MPB
472 outbreak in the Cypress Hills area, the model postulates the following covariates as primary (and
473 hence sufficient for an 0.88 AUC prediction) at each location, at each time: (1,2) presence of
474 infestation in last year, both managed and missed, (3,4) neighbors' degree of infestation in last

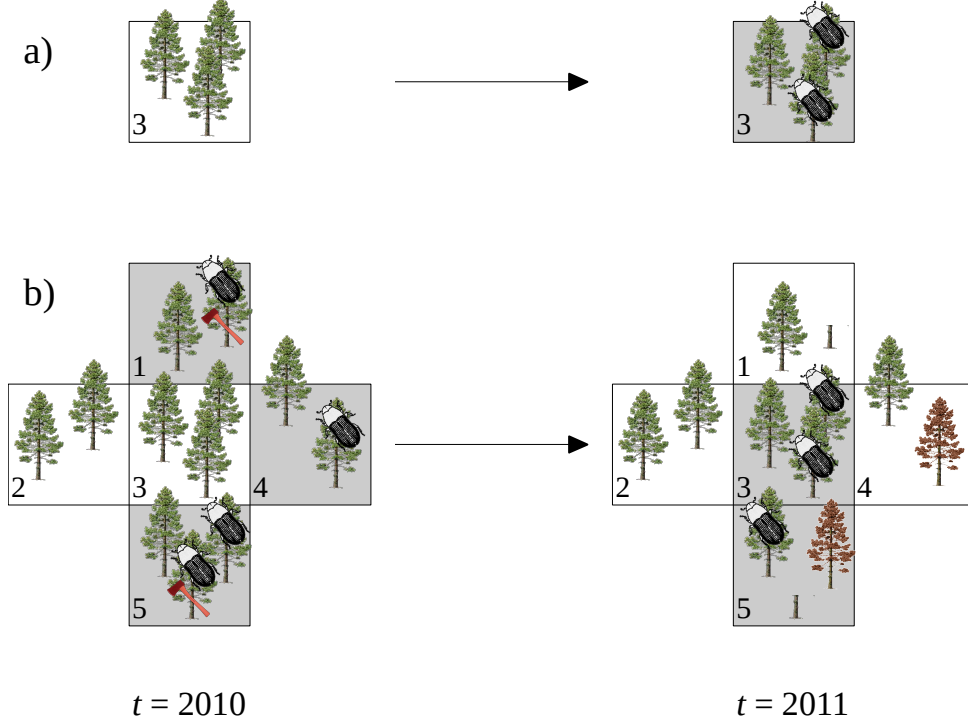


Fig. 2 Infestation status. Gray and white are used to indicate the presence and absence of infestation in a pixel. (a) None of the trees in pixel 3 were infested at year 2010 ($I_{3,2010} = 0$); however, at least one tree was infested at year 2011 ($I_{3,2011} = 1$). (b) All infested trees in pixel 1 that were infested at year 2010 were managed at the same year ($I_{1,2010}^{\text{Managed}} = 1$, $I_{1,2010}^{\text{Missed}} = 0$), there were no infested trees in pixel 2 at year 2010 ($I_{2,2010}^{\text{Managed}} = 0$, $I_{2,2010}^{\text{Missed}} = 0$), all infested trees in pixel 4 that were infested at year 2010, were missed at the same year, and hence, turned red in the following year ($I_{4,2010}^{\text{Managed}} = 0$, $I_{4,2010}^{\text{Missed}} = 1$), some infested trees were missed and some were managed in pixel 5 ($I_{5,2010}^{\text{Managed}} = 1$, $I_{5,2010}^{\text{Missed}} = 1$). Missed and managed neighbors' last year infestation for pixel 3 at year 2010 are, thus, $I_{\mathcal{N}_3,2010}^{\text{Missed}} = I_{\mathcal{N}_3,2010}^{\text{Managed}} = \frac{1}{2} + \frac{1}{2}$, presuming that $\mathcal{N}_3^2 = \mathcal{N}_3^3 = \emptyset$.

Table 5 AUC and AUPR scores of ‘the best’ BN \mathcal{B}_6^* , when one of the primary covariates is missing.

Missing covariate	AUC	AUPR
Nothing missing	0.882	0.277
Maximum temperature	0.889	0.350
Cold tolerance	0.881	0.290
Distance to infested border	0.890	0.309
Missed neighbors' past infestation	0.760	0.220
Managed neighbors' Past infestation	0.879	0.284
Missed last year infestation	0.811	0.103
Managed last year infestation	0.869	0.206
Last year infestation (both missed and managed)	0.784	0.068

475 year, both managed and missed, (5) distance to the border where the infestation was initiated, (6)

476 maximum temperature in July and August of that year, and (7) cold tolerance in the cold season

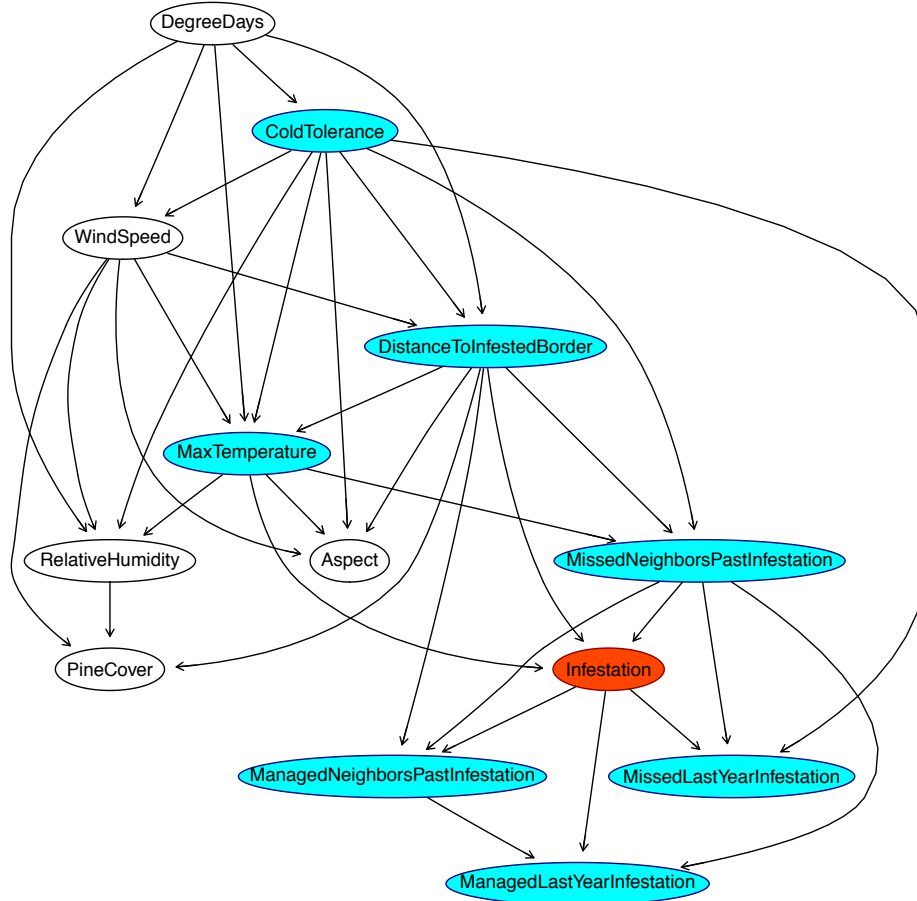


Fig. 3 The structure of ‘the Best’ Bayesian network (\mathcal{B}_6^*). We choose this structure as the one to explain and predict MPB infestation. The response variable and its Markov blanket are in red and Cyan.

477 of that year; *n.b.*, the remaining covariates are secondary and are used to predict infestation if
 478 one or more of the primary covariates are missing.

479 Given this BN, we can provide a wide range of *ceteris paribus* claims revealing the co-effects
 480 of the covariates on the presence of infestation (see SI). For example, if we know maximum
 481 daily temperature is high (above 31.2°C), the interval of relative humidity that results in the
 482 highest infestation risk sharply changes from medium to low. This is in line with the claim in
 483 (Safranyik, 1978; Lusebrink *et al.*, 2016) that lower humidity increases the infestation probability.
 484 However, for maximum daily temperatures lower than 31.2°C, the infestation likelihood is high
 485 for both low and high relative humidity. This inconsistency can be solved by looking at maximum
 486 temperature and relative humidity together. We find that humid areas require low maximum

487 daily temperature, while dry areas require high maximum daily temperature for a considerable
488 risk of infestation (above 20%).

489 As another example, a MPB needs 833 degree days to complete its transition to adults and
490 the minimum number of degree-days in the data is 1054 (Safranyik *et al.*, 1975, 2010). Therefore,
491 degree-day never prevents infestation in our data and just reflects the negative impact of high
492 summer temperatures. This, however, does not mean that degree day is useless in our model.
493 First of all, as mentioned earlier, in the absence of some of the primary covariates, the model
494 effectively estimates infestation via the information on degree day and other present covariates.
495 Secondly, although highly correlated, degree day and maximum temperature are different, and
496 the model reveals their coexistence effect on the infestation: for low (resp. high) degree-days,
497 infestation becomes more likely as maximum temperature increases (resp. decreases; see SI).

498 We emphasize that one may not make causal conclusions based on the structure of the
499 model. Clearly, the edge from infestation to managed-last-year-infestation does not imply that
500 this year’s infestation has caused last year’s (managed) infestation. It only means that the
501 two are probabilistically dependent. The same holds for all other links, such as the one from
502 maximum temperature to infestation: although temperature may be “causing” infestation, one
503 may not conclude so just based on the BN. One may refer to the literature on causality and
504 the corresponding tests in order to verify the causality of a link in a BN (Pearl, 2009; Pearl &
505 Mackenzie, 2018). On the other hand, the absence of an edge between, for example, degree day
506 and infestation does not necessarily mean that the two are independent. They may be dependent
507 but become conditionally independent if some other covariates are known here.

508 In summary, the learned BN contributes to the prediction and understanding of MPB infesta-
509 tions by (1) accurately predicting MPB infestations, (2) identifying the primary set of covariates
510 that are sufficient for making these predictions, (3) making acceptable predictions when data on
511 some of the primary covariates are unavailable, (4) revealing the previously unknown co-effects
512 of the covariates on infestation likelihood, (5) identifying the most informative covariate(s) to

513 infestation likelihood, and (6) proposing a BN structure that can serve as the basis for future
514 causality tests between the variables. Points 1, 2, 3, and 5 are particularly useful to forest man-
515 agers to plan ahead of time and know what data to collect. See SI for a more elaborate discussion
516 on the MPB case study.

517 Nevertheless, as with almost all other machine-learning models, BNs are generally constructed
518 under the stationary assumption, implying fixed structure and parameters over time. This may
519 result in poor performance when the model is used to make predictions at a time different from
520 those in the training dataset, provided that the “true ecological process” is non-stationary. For
521 example, a BN trained on data collected during the beginning of an outbreak may not accurately
522 predict the declining phases of the outbreak. Similar concerns are raised when using the learned
523 BN in environmental situations, where the ranges of the covariates are very different from those
524 in the training dataset. We refer the reader to (Zhu & Wang, 2015; Zhou *et al.*, 2008; Robinson
525 *et al.*, 2010) for relaxing the stationary assumption.

526 4 Discussion

527 Although traditional models used to make ecological predictions from underlying covariates have
528 a record of success, they also suffer from limitations. They cannot make predictions when one or
529 more covariates are missing; unless the missing values are imputed using other methods which can
530 be unreliable and result in low prediction accuracy. They also do not allow for statistical inference
531 when some of the covariates are highly correlated. BNs can handle these issues. Specifically,
532 they provide a primary and secondary ordering of the covariates, where primary covariates are
533 essential to predicting the target variable and secondary covariates, while not always essential,
534 can be helpful in making predictions when the values of some covariates are missing.

535 However, BNs are not used to their full potential in the literature as their structure is typically
536 constructed based on the knowledge of experts. Moreover, the obtained BN is often read causally,
537 a questionable practice as BNs are different from causal networks.

538 We have complemented previous work by providing a systematic approach to obtain a BN
539 fully from data. We have demonstrated the approach via a MPB case study, where no knowledge
540 of experts was involved in finding either the structure or CPDs. The resulting BN predicts
541 infestations fairly accurately, even in the absence of any of the selected covariates that are
542 involved in the model.

543 Researchers have utilized BNs to visualize their understanding of the causal relationships
544 between the variables involved in ecological processes (Borsuk *et al.*, 2004; Amstrup *et al.*, 2008;
545 Johnson *et al.*, 2010; Newton, 2010; Aps *et al.*, 2009; Pollino *et al.*, 2007b). The resulting
546 networks have been often used as predictors and sometimes reported to be fairly successful on
547 a test dataset. This is an acceptable approach to assess the *a priori* knowledge of the experts
548 or when there is no data available to learn the BN structure. However, by means of the results
549 for our MPB case study, we challenge claims that put forward this approach as “the (only) right
550 one” for constructing a BN. Examples include synthesizing existing knowledge into the model is
551 necessary and structural learning is only for modeling poorly understood systems or those difficult
552 to characterize (Chen & Pollino, 2012), modellers must demonstrate causal relations (McCann
553 *et al.*, 2006b), models based on theories about causal relations are generally better (Uusitalo,
554 2007), and network structure is a matter of judgement and should reflect expert knowledge and
555 stakeholder needs (Gutierrez *et al.*, 2011). Some researchers have looked into fixed (naive Bayes)
556 and partially learnable (Tree-augmented naive Bayes) structures (Aguilera *et al.*, 2010), yet this
557 is different from learning fully based on data.

558 In general, for modelling the joint probability distribution of the variables involved in an
559 ecological process, *i.e.*, a generative task, BNs seem to be the first and often best candidate,
560 especially if the governing dynamics are yet unknown to be mechanistically modeled. However,
561 if the sole purpose is to predict the response variable, *i.e.*, a discriminative task, other models
562 may show a higher prediction accuracy, although unlike BNs, they typically cannot deal with
563 missing values in the covariates. We are currently exploring ways to use BNs as well as other

564 models, to predict infestation many years in the future (Ramazi *et al.*, accepted).

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578 **6 Authors' contributions**

579 All authors conceived the ideas, interpreted the results and drafted the manuscript. P.R. de-
580 veloped the methods and under-took the analysis. All authors gave final approval for publication.

581 **7 Data Accessibility**

582 The dataset analyzed in the current study is described in (Kunegel-Lion *et al.*, 2020a) and avail-
583 able from Dryad repository (<https://doi.org/10.5061/dryad.70rxwdbt9>) (Kunegel-Lion *et al.*,
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