Structural Engineering Report No. 132

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DESIGN STRENGTHS OF STEEL BEAM-COLUMNS

by

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DECEMBER 1985

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Design Strengths of Steel Beam-Columns

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December 1985

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DESIGN STRENGTHS OF STEEL BEAM-COLUMNS

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SUMMARY

Many of the deficiencies of the present Canadian rules for designing steel beam-columns can be removed if different formulations are used for designing against in-plane failure and out-of-plane buckling.

An extension of the present use of non-linear elastic analysis methods allows a common formulation for the in-plane strengths of braced and unbraced beam-columns, and generally leads to more accurate predictions than the present forms.

Two alternative methods are developed for improving the design rules for estimating the out-of-plane strengths of beam-columns. The first of these retains the present familiar form, while the second uses a new form which will lead to significant economies in some cases.

A method is also proposed for combining the separate formulations for the in-plane and out-of-plane strengths so as to estimate the biaxial bending resistances of beam-columns.

KEYWORDS: Beams, buckling, columns, flexure, steel, structural design, structural engineering, torsion.

1. INTRODUCTION

Recent research (1,2,3,5) into the inelastic flexural-torsional buckling of steel beam-columns has shown that the present design interaction equation used in the Canadian Standard CAN-S16.1 (4) is unnecessarily conservative in many cases. This conservatism arises from compromises made to ensure that a single equation will safely model both in-plane failure by bending and buckling, and out-of-plane flexural-torsional buckling. In addition, difficulties arise when interpreting the individual terms of the interaction equation, which may have different meanings in each failure mode. If separate design criteria were provided, then each could be improved and clarified independently so as to model its failure mode more accurately.

The purpose of this paper is to review the methods given in the present Canadian Standard for designing steel beam-columns against in-plane failure and out-of-plane buckling, and to suggest improvements and clarifications. For this purpose, the discussion will be confined to compact I-section beam-columns which are bent only in the planes of their webs. For simplicity, the resistance factor will be omitted, that is $\phi \equiv 1$.

2. REVIEW OF PRESENT DESIGN RULES

2.1 Structural Analysis for Member Actions

The Canadian Standard (4) generally permits the use of elastic analysis to determine the member actions in frames. It is desirable that the elastic analysis should incorporate any non-linear stability effects which significantly modify the member actions from those predicted by a linear elastic analysis.

For frames which may sway (Fig. 1(a)) as a result of horizontal forces, initial out-of-plumb, or asymmetry in the structure or its gravity loading, the Standard permits two types of elastic analysis. In the first, the non-linear stability effects on the member end moments M_1*, M_2* and forces $C*$ (Fig. 2) are included, either as a result of an exact analysis or by means of an approximation. In the second, the Standard implies that the linear elastic end moments M_1, M_2 and forces C may be used in conjunction with an estimate C_{exs} of the member force at elastic frame sway buckling to approximate the non-linear actions by

[1] $$C* \approx C$$

[2] $$M_1* \approx M_1/(1 - C/C_{exs})$$

These methods are generally satisfactory for predicting the elastic member end actions, although Eq. [2] may be unnecessarily conservative when the linear elastic deflected shape of the frame is markedly dissimilar from its elastic buckled shape. However, they do not always predict the maximum moment in the member length.

A similar result occurs for the members of frames which are effectively rigidly braced against sway (Fig. 1(b)), for which the Standard permits a linear elastic analysis to be used to determine the member actions M_1, M_2, and C. The non-linear effects of stability on the moment distributions of the members of sway and braced frames are allowed for approximately in the member resistance rules discussed in the following sub-sections.

Finally, it should be pointed out that elastic analyses can at best only give close approximations to the member behaviour at failure, since yielding may change the stiffnesses of the members, and alter the moment and force distributions (16,17,18). However the use of an inelastic non-linear method of structural analysis is not yet a practical design procedure.

2.2 Cross-Section Resistance

The cross-section resistance of a compact beam-column is reached when the cross-section becomes fully plastic (7,10,12,14). For an I-section beam-column bent in the plane yz of the web, this condition is approximated in the Standard by

[3] $$\frac{C}{C_y} + 0.85 \frac{M_x^*}{M_{Px}} \leqslant 1, \text{ and}$$

[4] $$\frac{M_x^*}{M_{Px}} \leqslant 1, \text{ in which}$$

[5] $$C_y = F_y A$$

is the "squash" load,

[6] $$M_{Px} = F_y Z_x$$

is the plastic moment of the section, A and Z_x are the area and the plastic modulus of the cross-section, and F_y is the yield stress. These approximations are of reasonable accuracy (7,10,12,14). However, the Standard allows M_x to be used instead of M_x* when the linear elastic method is used to analyse the structure. This is unconservative.

2.3 In-Plane Stability

2.3.1 Beam-Columns Prevented From Swaying

Beam-columns which are prevented from swaying may fail in-plane by a combination of in-plane buckling and bending effects (7,10,12,14). The Standard approximates this condition by

[7] $$\frac{C}{C_{rb}} + \frac{\omega_{xb} M_x}{M_{Px}\left(1 - C/C_{exb}\right)} \leq 1 \text{ , and}$$

[8] $$\omega_{xb} = 0.6 - 0.4\beta \not< 0.4$$

in which C_{rb} is the column resistance of the braced member, C_{exb} is the axial force at elastic buckling of the braced member, and β is the ratio of the lesser and greater end moments (β = −1 for uniform bending, and β = +1 for double curvature bending).

The term $\omega_x M_x/(1 - C/C_{exb})$ provides an approximation for the elastic effects of in-plane stability on beam-columns with unequal end moments (9). It is known to lead to reasonably accurate strength estimates for single curvature bending ($\beta < 0$). However, its predictions for double curvature bending ($\beta > 0.5$) are somewhat erratic (7,10,12,14), being rather conservative for moderate moments, and optimistic for high moments, although this latter problem is avoided when the cross-section resistance limit given by Eq. [4] and [5] is satisfied. The conservatism is due in part to the cut-off value of $\omega_{xb} = 0.4$, which is based on the out-of-plane buckling behaviour of beams.

Although it is clear from the in-plane nature of the derivation (7,10,12,14) of Eq. [7] and [8] that the value used for C_{rb} should be the in-plane value C_{rxb}, this is not made clear in the Standard, and a common practice is to use the lower of C_{rxb} and the out-of plane value C_{ry}. When C_{ry} is the lower, then this practice is unnecessarily conservative.

2.3.2 Sway Beam-Columns

2.3.2.1 Linear Elastic Analysis

When the end moments on a sway beam-column are determined by linear elastic analysis, then the Standard requires its resistance to in-plane stability failure to be approximated by modifying Eq. [7] and [8] to

[9] $$\frac{C}{C_{rs}} + \frac{\omega_{xs} M_x}{M_{Px}(1 - C/C_{exs})} = 1 \text{ , and}$$

[10a] $\omega_{xs} = 1.0$ for $-1 \leqslant \beta < 0$, or

[10b] $\omega_{xs} = 0.85$ for $0 \leqslant \beta \leqslant 1$

Comparisons with more precise analyses (16,17,19) indicate that these equations are generally satisfactory, except for some conservatism for double curvature bending under moderate moments.

2.3.2.2 <u>Non-Linear Elastic Analysis</u>

When the end moments M_x* in a sway beam-column are obtained by non-linear analysis, then the Standard permits the use of Eq. [7,8] for a braced beam-column with M_x* substituted for M_x, instead of Eq. [9,10] for a sway beam-column, so that

[11] $$\frac{C}{C_{rb}} + \frac{\omega_{xb} M_x^*}{M_{Px} (1 - C/C_{exb})} \leqslant 1$$

An attempt might be made to justify this by the argument that since the effects of sway buckling are included in the non-linear analysis for M_x*, only the effects of braced mode buckling need be incorporated into the design interaction equation.

However, there are a number of difficulties created by the use of Eq. [11]. First of all, the allowance for sway buckling is indirect, and appears to disappear when $M_x = 0$. It is often not recognised that the second term of Eq. [11] does not disappear when $M_x = 0$, but approaches a non-zero value so that the apparent limiting solution of

Eq. [11] for $M_x = 0$ is $C = C_{exs}$, and not $C = C_{rb}$.

Because the apparent limit of Eq. [11] of $C = C_{exs}$ for $M_x = 0$ is greater than the design strength C_{rxs} of a sway column, it appears that the effects of residual stresses and geometrical imperfections which reduce C_{exs} to C_{rxs} are omitted from Eq. [11]. Thus the beam-column rule at its limit appears to be inconsistent with the column rule, and there appears to be an overestimate of strength which is significant for intermediate length members with small bending actions.

It is not obvious that these difficulties are avoided by the incorporation of an initial out-of-plumb into the calculation of the non-linear M_x^*. It is the initial out-of-plumb which leaves a non-zero M_x^* effect even when $M_x = 0$, and which is amplified to very high values as the sway buckling load C_{exs} is approached. Even so, the beam-column strength for $M_x = 0$ is not generally equal to the column strength C_{rxs}, because the out-of-plumb value given in the Standard is inconsistent with the reduction from C_{exs} to C_{rxs}.

Finally, Eq. [11] appears to suggest that the braced axial load capacity C_{rxb} can be approached for beam-columns at low values of M_x^*. This is misleading, because amplification of the out-of-plumb effects induces substantial moments M_x^*, so that the strength values indicated at low values of M_x^* are fictitious.

2.4 Out-of-Plane Buckling

The out-of-plane strength of a beam-column bent in the plane of its web is closely related to its resistance to flexural-torsional buckling (13,14). The Standard provides a method of designing against out-of-plane failure by modifying the in-plane stability equations to

$$[12] \qquad \frac{C}{C_r} + \frac{\omega_x M_x}{M_{rx}(1 - C/C_{ex})} \leq 1$$

in which M_{rx} is the beam resistance of the member, which may be reduced below the full plastic moment M_{px} by beam flexural-torsional buckling effects.

This equation is unsatisfactory for a number of reasons. First of all, it is clear from its derivation (14) that the out-of-plane value C_{ry} should be used for C_r. However, when C_{rx} is less than C_{ry}, as it often is for unbraced frames which have out-of-plane end restraints, then the use of the lower value is unnecessarily conservative.

Secondly, the value used for ω_x is a compromise between the effects of in-plane beam-column stability and out-of-plane beam buckling. Recent research (6) has shown that the value of ω_x for the flexural-torsional buckling of beam-columns should vary with the axial load C, and that the general form of Eq. [12] is unsatisfactory. Further, there is no evidence to support the use of the in-plane sway values of ω_x given by Eq. [10] for the out-of-plane buckling of beam-columns which have out-of-plane end restraints.

Finally, the value of ω_x is used twice in Eq. [12], once by its explicit use, and once by its implicit use in the determination of M_{rx}. Thus Eq. [12] does not reduce to the correct end point $M_x = M_{rx}$ when C = 0. In this respect it is of interest to note that a proposal has been made (15) to determine the inelastic beam strength (C = 0) from

[13] $$M_{rx} = \frac{M_{rxu}}{\omega_x} \ngtr M_{Px}$$

in which M_{rxu} is the strength of a beam in uniform bending ($\beta = -1$).

3. RECENT RESEARCH ON OUT-OF-PLANE BUCKLING

The out-of-plane beam-column design rule of the Canadian Standard is based on the commonly used "linear" approximation (14) for elastic buckling

[14] $$\frac{C}{C_{ey}} + \frac{\omega_b M_x}{M_{eu}\left(1 - C/C_{ex}\right)} = 1 \text{ , and}$$

[15] $$1/\omega_b = 1.75 + 1.05\beta + 0.3\beta^2 \ngtr 2.3$$

in which M_{eu} is the elastic buckling resistance of a beam in uniform bending

[16] $$M_{eu} = r_o\sqrt{\left(C_{ey}C_{ez}\right)}$$

and C_{ez} is the elastic torsional buckling load of a column

[17] $$C_{ez} = (GJ + \pi^2 EI_w/L^2)/r_o^2$$

A recent study (6) has indicated that Eq. [14,15] are sometimes unnecessarily conservative, especially for high moment gradients ($\beta > 0.5$). Instead, a return to the "parabolic" approximation

[18] $$\left(\frac{\omega_{bc} M_x}{M_{eu}}\right)^2 = (1 - C/C_{ey})(1 - C/C_{ez})$$

has been suggested, with the moment gradient factor changed to

[19] $$\omega_{bc} = (1-\beta)/2 + \{(1+\beta)^3/8\}\{0.22(C/C_{ey})^3 - 0.103(C/C_{ey})^2 - 0.138(C/C_{ey}) + 0.378\}$$

or more simply to

[20] $$\omega_{bc} \approx \frac{1-\beta}{2} + \left(\frac{1+\beta}{2}\right)^3 \{0.4 - 0.23(C/C_{ey})\}$$

A series of studies of the inelastic buckling of beam-columns (1,2,3,5) has led to an adaptation of this recommendation to

[21] $$\left(\frac{\omega_{bc} M_x}{M_I}\right)^2 = (1 - C/C_I)(1 - C/C_{ez}) \quad \text{and}$$

[22] $$\omega_{bc} = \frac{1-\beta}{2} + \left(\frac{1+\beta}{2}\right)^3 \{0.4 - 0.23\,(C/C_I)\}$$

in which M_I and C_I are inelastic buckling resistances given by

[23] $$\frac{M_I}{M_{Px}} = 1.008 - 0.245 \frac{M_{Px}}{M_{eu}} \ngtr 1.0, \text{ and}$$

[24] $$\frac{C_I}{C_y} = 1.035 - 0.181 \sqrt{\left(\frac{C_y}{C_{ey}}\right)} - 0.128 \frac{C_y}{C_{ey}} \ngtr 1.0$$

These recommendations ignore any non-linear in-plane effects, since these were small for the beam-columns studied. However, when there are likely to be significant in-plane prebuckling deflections, then it would be appropriate to replace M_x by $M_x/(1 - C/C_{ex})$ so that

[25] $$\left\{\frac{\omega_{bc} M_x}{M_{eu}(1 - C/C_{ex})}\right\}^2 = \left(1 - \frac{C}{C_{ey}}\right)\left(1 - \frac{C}{C_{ez}}\right), \text{ or}$$

[26] $$\left\{\frac{\omega_{bc} M_x}{M_I(1 - C/C_{ex})}\right\}^2 = \left(1 - \frac{C}{C_I}\right)\left(1 - \frac{C}{C_{ez}}\right)$$

for elastic or inelastic buckling, respectively.

For design purposes, these equations need to be reduced for the effects of initial imperfections. It is suggested that this can be accomplished by using

[27] $$\left\{\frac{\omega_{bc} M_x^*}{M_{rxu}}\right\}^2 = \left(1 - \frac{C}{C_{ry}}\right)\left(1 - \frac{C}{C_{ez}}\right) \text{ , and}$$

[28] $$\omega_{bc} = \frac{1-\beta}{2} + \left(\frac{1+\beta}{2}\right)^3 \left\{0.4 - 0.23 \frac{C}{C_{ry}}\right\}$$

This equation is significantly different from the single equation (Eq. [12]) often used for both in-plane stability and out-of-plane buckling. It is therefore suggested that in-plane stability should be checked separately.

4. PROPOSALS FOR NEW DESIGN RULES

4.1 General

The reviews in the preceding sections of the Canadian design rules for the non-linear structural analysis, cross-section resistance, in-plane stability, and out-of-plane buckling of beam-columns have indicated a number of areas where improvements can be made. The following sub-sections present and discuss proposals for these improvements.

These proposals include a general application of the methods of non-linear elastic frame analysis to estimate the maximum elastic moment M* in a beam-column (Fig. 2). The use of this maximum moment then allows the in-plane strength rules for braced and unbraced members to be unified in a new form. Separate rules based on recent research are proposed for out-of-plane buckling, and a method is suggested for combining the in-plane and out-of-plane rules to provide rules for biaxial bending.

4.2 Non-Linear Structural Analysis

4.2.1 Sway Frames

The present design rules for sway frames include an approximate cyclic method of non-linear elastic analysis in which artificial storey shears are computed from the axial forces and the out-of-plumb and sway deflections caused by transverse loads. These are then used to approximate the non-linear elastic end moments, $M_1{}^*$ and $M_2{}^*$ (Fig. 2). A more precise method of non-linear elastic analysis is also permitted which formulates equilibrium for the deformed structure.

It is suggested that it should also be permissible to estimate the end moments $M_1{}^*$, $M_2{}^*$ by using Eq. [2], since this method is well known, widely used, and already implied in the present strength design rules. This method is accurate when the linear elastic deflected shape is the same as the elastic buckled shape, and is usually conservative.

It is also proposed that the maximum non-linear elastic moment M^* in the member (Fig. 2) should be estimated. For many sway frames, this will be the larger of the end moments $M_1{}^*$, $M_2{}^*$, but not always. For a member with end moments $M_1{}^*$, $\beta M_1{}^*$ and no transverse loads, the maximum moment is given by

[29] $$M^* = M_1{}^* \quad \text{while } \beta \geq -\cos\alpha \text{ , and}$$

[30] $$M^* = M_1{}^* \sqrt{\{1 + (\beta \operatorname{cosec}\alpha + \cot\alpha)^2\}} \quad \text{otherwise, where}$$

[31] $$\alpha = \sqrt{\{CL^2/EI_x\}}$$

These exact solutions are often approximated by

[32] $$M^* = \omega_x M_1{}^*/(1 - CL^2/\pi^2 EI_x)$$

where ω_x is given by Eq. [8]. Note that these formulations are consistent with using an effective length factor of unity in the elastic buckling load C_{ex}. This is appropriate because the use of the non-linear end moments M_1*, M_2* allows the member to be analysed for M* independently of the rest of the structure.

4.2.2 Frames Prevented From Swaying

No specific provision is made in the present design rules for using a non-linear elastic analysis to estimate the maximum end moments M_1*, M_2* in a frame prevented from swaying. This is because the application of the cyclic non-linear analysis method would lead in this case to the same result as a linear elastic analysis, and so it is appropriate to use the linear end moments, M_1 and M_2.

It is proposed that the maximum non-linear elastic moment M* in a non-sway member should be determined in the same way as for sway members, using Eq. [24-32] for members with no transverse loads.

4.3 In-Plane Strength

The in-plane behaviour of compact beam-columns with end moments has been investigated theoretically and experimentally (8,9,17,19). Their predicted ultimate strengths are usually presented as interaction curves which vary from the column strength C_{rx} when there are no bending actions to the beam strength M_{px} when there is no axial force, as shown in Fig. 3. These interaction curves are often (8,9) but not always

(16,17) presented using the greater linear elastic end moment M_1. However, a more meaningful comparison can be made when the maximum non-linear elastic moment M* is used, as shown in Fig. 3. The interaction curves fall within the bounds of nominal first yield and full plasticity, as shown in Fig. 3, and are terminated by the column strength C_{rx} and the beam strength M_{px}. The shapes of the predicted curves typically vary between extremes which depend on the ratio β of the non-linear end moments M_1* and M_2*, but which are practically independent of the bracing of the frame, except insofar as this influences the column strength C_{rx}. For uniform bending (β = -1) the curves are almost linear, but for double curvature bending (β = +1) the curves are approximately parabolic, and approach or reach the full plasticity limit.

The reasons for this dependence on the end moment ratio β are illustrated in Fig. 4. For beam-columns in uniform bending (β = -1), the maximum moment results from the combined effects of the end moments and the moments caused by the axial force C and the initial crookedness or out-of-plumb v_o and deflection v. The maximum strength is reached after first yield because of the plastic reserve of strength, but before the full plasticity condition can be reached.

On the other hand, the maximum non-linear moment M* in a beam-column in double curvature bending (β = +1) is virtually unaffected by moments caused by axial force and deflection. Thus the maximum strength of a member with moderate axial force is reached when M* approaches the full plasticity condition. However, when the member has a high axial force, the moments caused by initial crookedness or out-of-plumb v_o become important, especially as the axial force approaches the column strength C_{rx}, which therefore forms an upper bound to its strength.

It is proposed herein that the member strength should be approximated by a cubic interpolation between linear and parabolic limits, and given by

$$[33] \qquad \frac{M_x^*}{M_{Px}} \leq \{1 - (\frac{1+\beta}{2})^3\} (1 - \frac{C}{C_{rx}}) + (\frac{1+\beta}{2})^3 (1 - \frac{C}{C_{rx}})^{1/2} /0.85$$

but should not exceed the plasticity limits defined by

$$[34] \qquad M^*/M_{Px} \leq 1 \text{ , and}$$

$$[35] \qquad M^*/M_{Px} \leq (1 - C/C_y)/0.85$$

This leads to the set of curves shown in Fig. 5, which agree more closely with the research predictions than do the present design rules. Further, these curves may be used for all beam-columns with end moments, because the use of M_1^* and βM_1^*, to calculate M^* allows the member to be considered independently of the frame action.

It should be pointed out that some research predictions (16,17) are significantly higher than those given by Eq. [33], especially for members with low moments. These result principally from inelastic member stiffness changes, which lead to higher end restraints in indeterminate frames than those which can be predicted by non-linear elastic analysis (18). Thus the proposed rules may be improved further, provided a simple method can be found for non-linear inelastic analysis. However, this is beyond the scope of the present paper, and should be the subject of future research.

4.4 Out-of-Plane Buckling

The proposed separation of the design rules for in-plane strength and out-of-plane buckling allows the out-of-plane rule to be improved to

$$[36] \qquad M_x^* \leqslant \left(1 - \frac{C}{C_{ry}}\right) \frac{M_{rxu}}{\omega_x} \ngtr M_{Px}$$

where ω_x is given by Eq. [8], and M_{rxu} is the beam strength ($C=0$) for uniform bending ($\beta=-1$), which includes any lateral buckling reductions below M_{Px}. This proposal satisfies the criticisms of the present rule made in Section 2.4.

Alternatively, the results of recent research (see Section 3) may be used to further improve the out-of-plane strength formulation to that of Eq. [27,28].

4.5 Biaxial Bending

For beam-columns which are bent biaxially about both principal axes, it is expected that the strength may be approximated by the linear interaction equation

[37] $$\frac{M_x^*}{M_{rxo}} + \frac{M_y^*}{M_{ryo}} \leqslant 1$$

in which M_{rxo} is the lower of the design strengths of the beam-column bent about the x axis ($M_y^* = 0$) determined from Sections 4.3 and 4.4 for in-plane and out-of-plane behaviour, and M_{ryo} is correspondingly defined for the beam-column when bent about the y axis ($M_x^* = 0$).

Linear interaction equations of this type are generally conservative, and it may be advantageous to develop non-linear equations of the form

[38] $$\left(\frac{M_x^*}{M_{rxo}}\right)^{\eta_x} + \left(\frac{M_y^*}{M_{ryo}}\right)^{\eta_y} \leqslant 1$$

The powers η_x and η_y in this equation will need to be determined in the light of available research findings on the inelastic biaxial bending strength of beam-columns.

5. CONCLUSIONS

A review of the design rules of the present Canadian Standard (4) for steel beam-columns bent about the x axis reveals a number of deficiencies. Many of these arise from compromises made to allow a single equation to represent the two different failure modes of in-plane stability and out-of-plane buckling. Recent research has indicated that a significantly different formulation is desirable for members which fail by out-of-plane buckling. Accordingly, it is proposed that different methods should be used to represent the resistances to these two failure modes.

An examination of research findings demonstrates that the in-plane strengths of braced and unbraced beam-columns may be expressed in a common form, provided that any differences in their column strengths are accounted for. This common form requires an estimate to be made of the maximum non-linear elastic moment in the beam-column, and it is proposed that this could be made by a simple extension of the present explicit and implicit methods of the standard. A simple computational procedure is then developed for estimating the in-plane strength, which generally leads to more accurate predictions than the present methods.

Two alternative methods are developed for improving the design rules for estimating the out-of-plane strengths of beam-columns. The first of these retains the present familiar form, while the second uses a new form which will lead to significant economies for beam-columns with high moment gradients and moderate axial loads. Finally, a method is proposed for combining the separate formulations for the in-plane and out-of-plane strengths so as to estimate the resistance of a beam-column to biaxial bending.

ACKNOWLEDGEMENTS

The work described in this paper has been supported by research grants by the Natural Sciences and Engineering Research Council of Canada, and by the University of Alberta. The author gratefully acknowledges the stimulating and helpful discussions with Dr. G.L. Kulak, Dr. D.J.L. Kennedy, and Dr. J.J. Cheng of the Department of Civil Engineering and Dr. P.F. Adams, Director of the Centre for Frontier Engineering Research of the University of Alberta on the subject matter of this paper.

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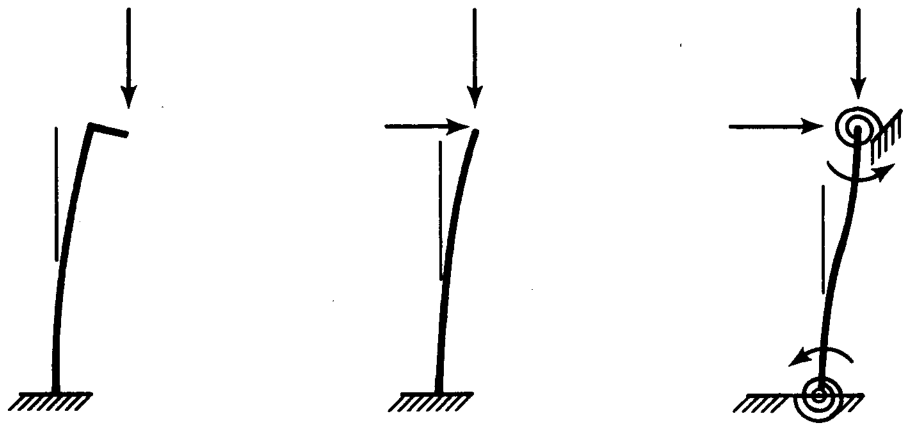
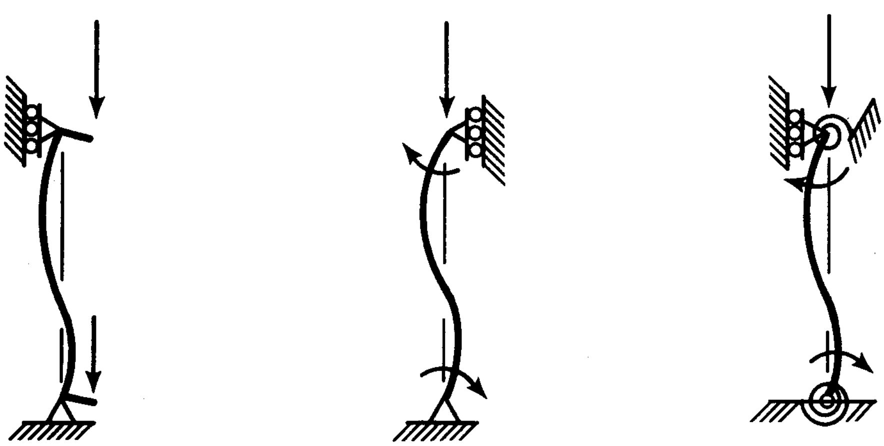
APPENDIX II - NOTATION

A	Cross-sectional area
C	Axial compression force
C^*	Value determined for C by non-linear elastic analysis
C_{ex}, C_{ey}	Elastic buckling loads for flexure about x,y axes
C_{exb}, C_{exs}	Values of C_{ex} for braced and sway columns
C_{ez}	Elastic torsional buckling load
C_I	Inelastic value of C_{ey}
C_r	Resistance to axial compression
C_{rb}, C_{rs}	Values of C_r for braced and sway columns
C_{rx}, C_{ry}	Values of C_r for failure about x,y axes
C_{rxb}, C_{rxs}	Values of C_{rx} for braced and sway columns
C_y	Squash load
C_{yr}	Reduced full plastic load
E	Young's modulus of elasticity
F_y	Yield stress
G	Shear modulus of elasticity
I_w	Warping section constant
I_x, I_y	Second moments of area about x,y axes
J	Torsion section constant
L	Member length
M_1, M_2	End moments determined by linear elastic analysis
M_1^*, M_2^*	End moments determined by non-linear elastic analysis
M_{eu}	Elastic buckling moment for uniform bending
M_I	Inelastic value of M_{eu}

M_{Px}	Full plastic moment for bending about x axis
M_{rx}	Resistance to bending about x axis
M_{rxo}	Beam-column moment resistance when $M_y^* = 0$
M_{rxu}	Resistance to uniform bending about x axis
M_{ryo}	Beam-column moment resistance when $M_x^* = 0$
M_x, M_y	Moments about x,y axes determined by linear elastic analysis
M_x^*, M_y^*	Maximum non-linear elastic moments about x,y axes
M_{yx}	Moment at nominal first yield for bending about x axis
r_o	Polar radius of gyration = $\sqrt{\{(I_x + I_y)/A\}}$
v	Deflection
v_o	Initial crookedness or out-of-plumb
x,y	Principal axes perpendicular to and in plane of web
z	Distance along centroidal axis
Z_x	Plastic section modulus about x axis
α	$= \sqrt{\{CL^2/EI_x\}}$
β	Ratio of non-linear elastic end moments
η_x, η_y	Powers for biaxial bending interaction equation
ω_b	Moment gradient factor for beams
ω_x	Moment gradient factor for bending about x axis
ω_{xb}, ω_{xs}	Values of ω_x for braced and sway beam-columns
ω_{bc}	Moment gradient factor for flexural-torsional buckling of beam-columns

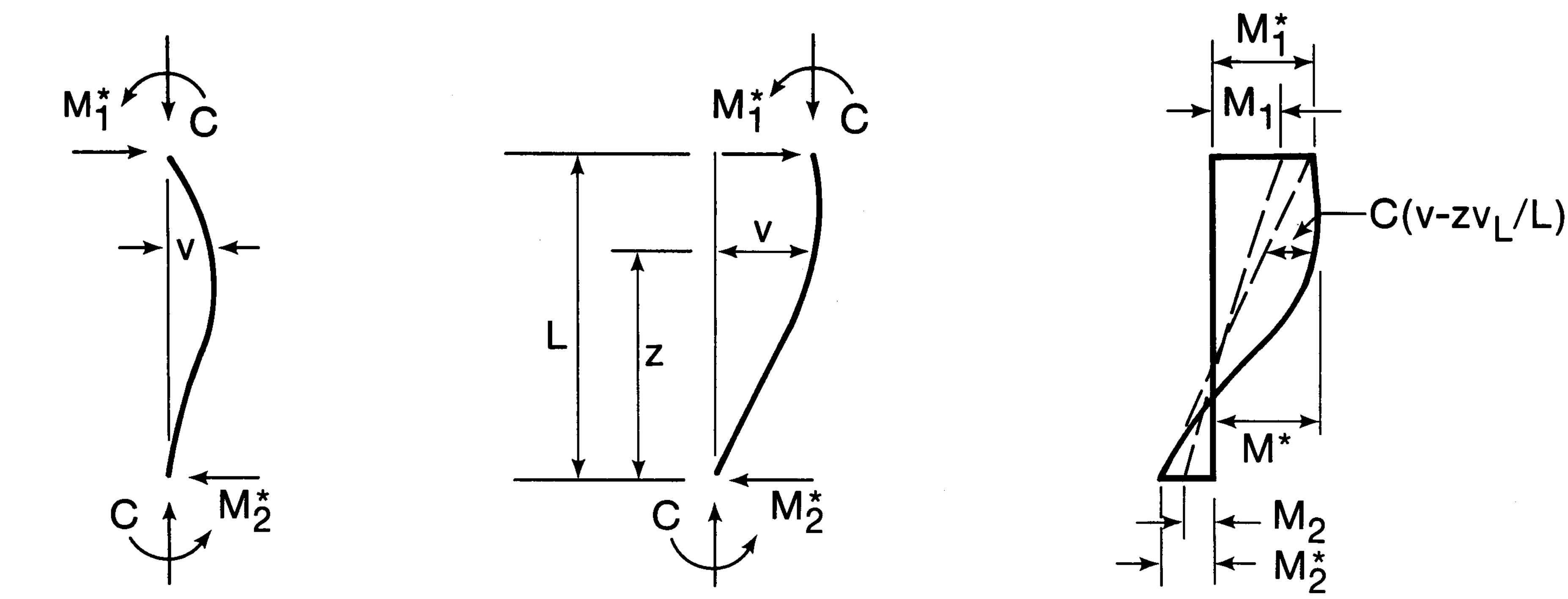


(a) Unbraced



(b) Rigidly Braced

Fig. 1. Beam-Columns



(a) Braced Member (b) Unbraced Member (c) Bending Moment

Fig. 2. Bending of Beam—Columns

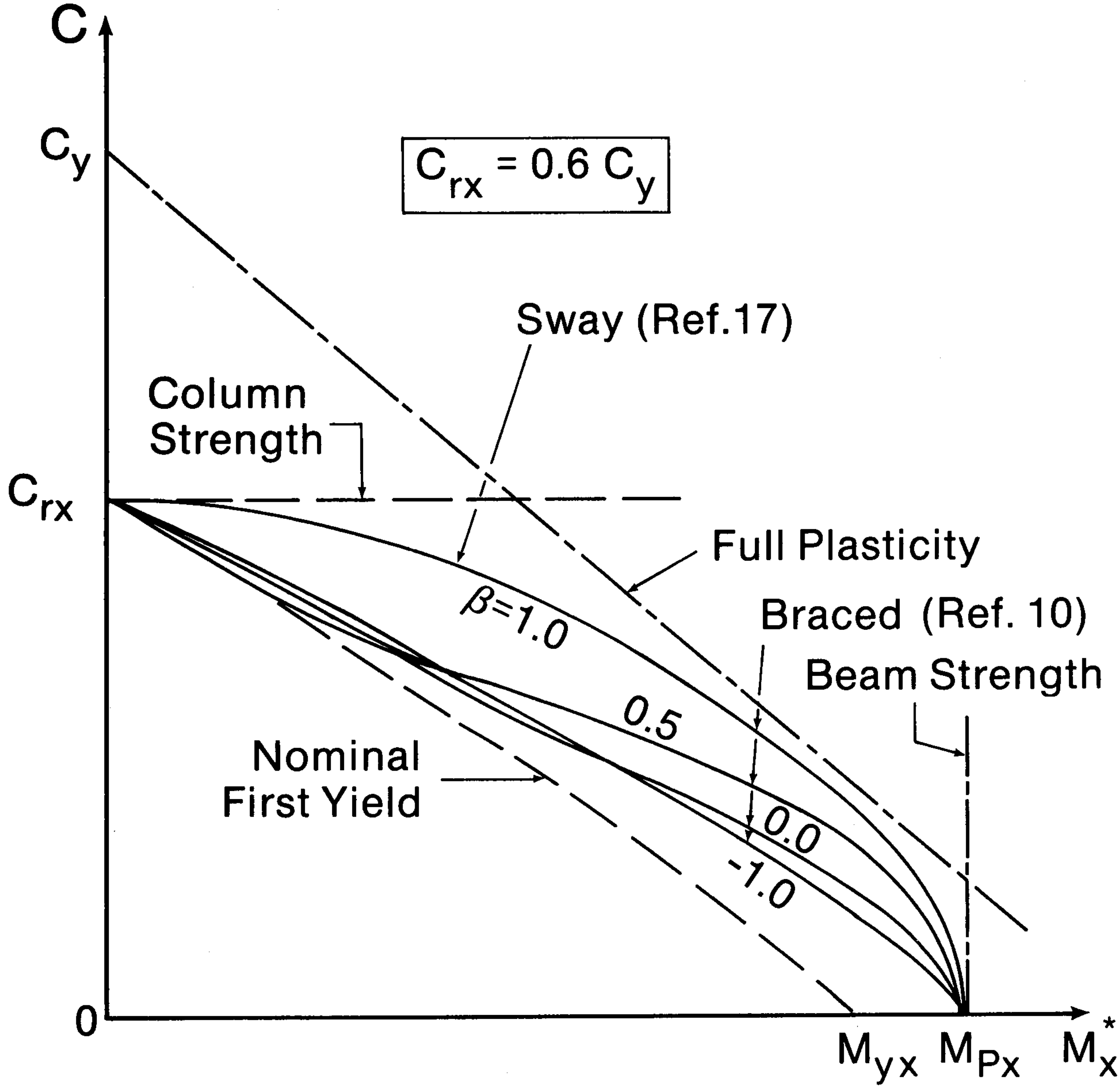


Fig. 3. Analytical Predictions of Beam-Column Strength

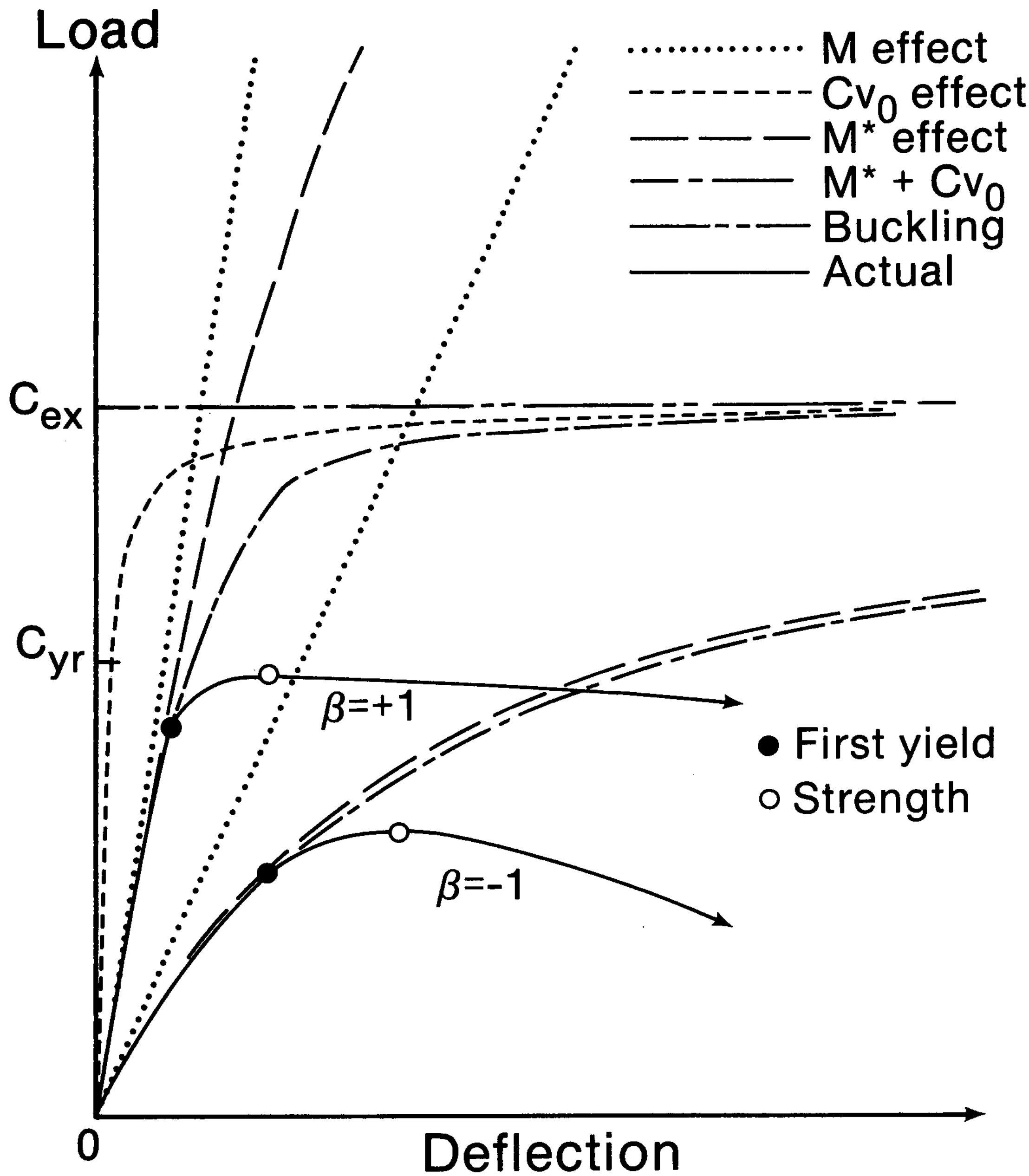


Fig. 4. Load-Deflection Behaviour

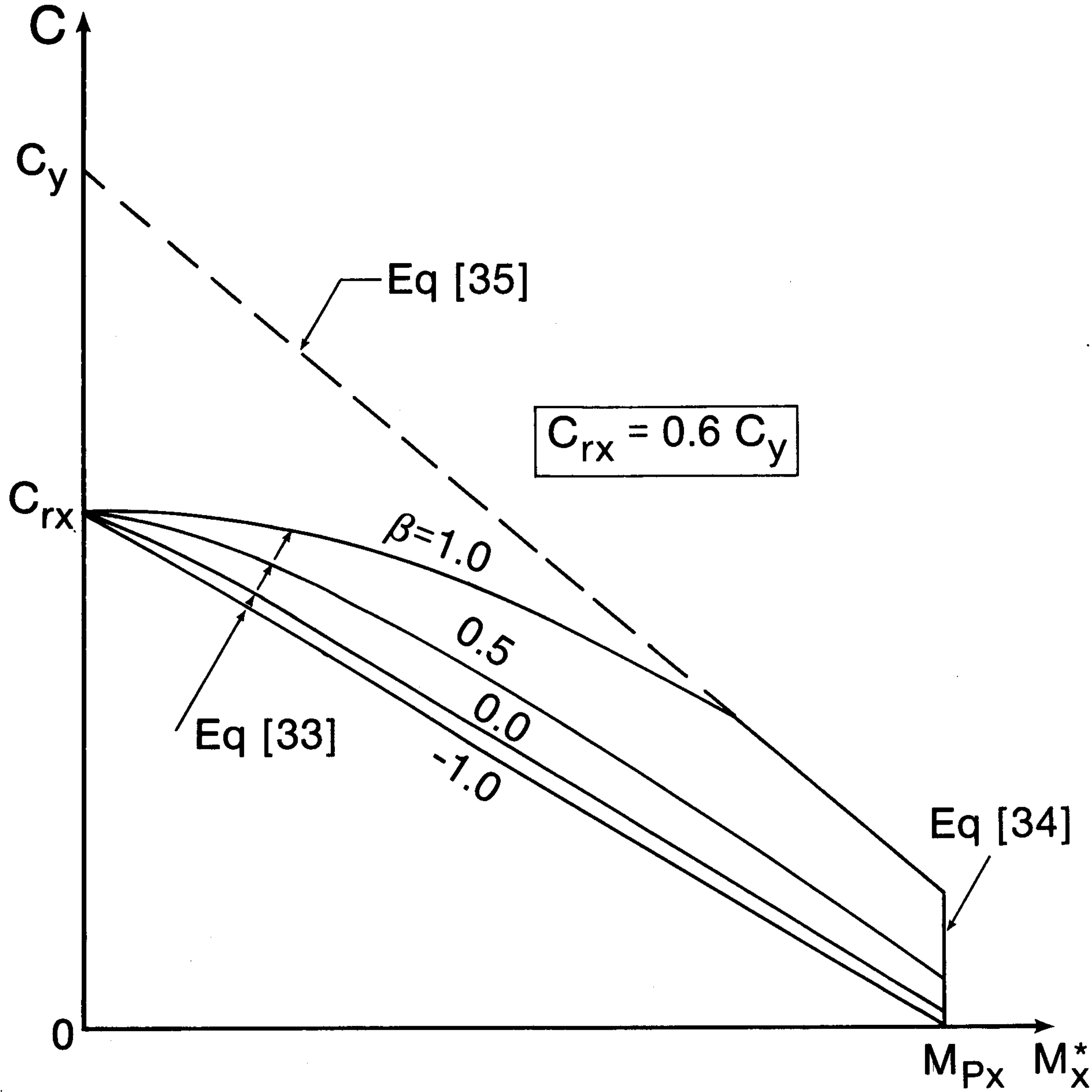


Fig. 5. Proposed In-Plane Strengths