UNIVERSITY OF ALBERTA

COMPUTER GENERATED GEOLOGICAL MAPS FROM CROSS-SECTIONS:

an example from the Rocky Mountain Foothills near Grande Cache, Alberta

by

Kevin Wayne Yakiwchuk ©

A thesis submitted to the Faculty of Graduate Studies and Research

in the partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

DEPARTMENT OF GEOLOGY

EDMONTON, ALBERTA

FALL, 1994

ISBN 0-315-95136-2

Name Kevin Yakiwchuk

Dissertation Abstracts International is arranged by broad, general subject categories. Please select the one subject which most nearly describes the content of your dissertation. Enter the corresponding four-digit code in the spaces provided.

Earth Sciences - Geology

SUBJECT TERM

0372

SUBJECT CODE

U·M·I

Subject Categories

THE HUMANITIES AND SOCIAL SCIENCES

COMMUNICATIONS AND THE ARTS
Architecture 0729
Art History 0377
Cinema 0900
Dance 0378
Fine Arts 0357
Information Science 0723
Journalism 0391
Library Science 0399
Mass Communications 0708
Music 0413
Speech Communication 0459
Theater 0465

EDUCATION
General 0515
Administration 0514
Adult and Continuing 0516
Agricultural 0517
Art 0273
Bilingual and Multicultural 0282
Business 0688
Community College 0275
Curriculum and Instruction 0727
Early Childhood 0518
Elementary 0524
Finance 0277
Guidance and Counseling 0519
Health 0680
Higher 0745
History of 0520
Home Economics 0278
Industrial 0521
Language and Literature 0279
Mathematics 0280
Music 0522
Philosophy of 0998
Physical 0523
Psychology 0525
Reading 0535
Religious 0527
Sciences 0714
Secondary 0533
Social Sciences 0534
Sociology of 0340
Special 0529
Teacher Training 0530
Technology 0710
Tests and Measurements 0288
Vocational 0747

LANGUAGE, LITERATURE AND LINGUISTICS
Language
General 0679
Ancient 0289
Linguistics 0290
Modern 0291
Literature
General 0401
Classical 0294
Comparative 0295
Medieval 0297
Modern 0298
African 0316
American 0591
Asian 0305
Canadian (English) 0352
Canadian (French) 0355
English 0593
Germanic 0311
Latin American 0312
Middle Eastern 0315
Romance 0313
Slavic and East European 0314

PHILOSOPHY, RELIGION AND THEOLOGY
Philosophy 0422
Religion
General 0318
Biblical Studies 0321
Clergy 0319
History of 0320
Philosophy of 0322
Theology 0469

SOCIAL SCIENCES
American Studies 0323
Anthropology
Archaeology 0324
Cultural 0326
Physical 0327
Business Administration
General 0310
Accounting 0272
Banking 0770
Management 0454
Marketing 0338
Canadian Studies 0385
Economics
General 0501
Agricultural 0503
Commerce-Business 0505
Finance 0508
History 0509
Labor 0510
Theory 0511
Folklore 0358
Geography 0366
Gerontology 0351
History
General 0578
Ancient 0579
Medieval 0581
Modern 0582
Black 0328
African 0331
Asia, Australia and Oceania 0332
Canadian 0334
European 0335
Latin American 0336
Middle Eastern 0333
United States 0337
History of Science 0585
Law 0398
Political Science
General 0615
International Law and Relations 0616
Public Administration 0617
Recreation 0814
Social Work 0452
Sociology
General 0626
Criminology and Penology 0627
Demography 0938
Ethnic and Racial Studies 0631
Individual and Family Studies 0628
Industrial and Labor Relations 0629
Public and Social Welfare 0630
Social Structure and Development 0700
Theory and Methods 0344
Transportation 0709
Urban and Regional Planning 0999
Women's Studies 0453

THE SCIENCES AND ENGINEERING

BIOLOGICAL SCIENCES
Agriculture
General 0473
Agronomy 0285
Animal Culture and Nutrition 0475
Animal Pathology 0476
Food Science and Technology 0359
Forestry and Wildlife 0478
Plant Culture 0479
Plant Pathology 0480
Plant Physiology 0817
Range Management 0777
Wood Technology 0746
Biology
General 0306
Anatomy 0287
Biostatistics 0308
Botany 0309
Cell 0379
Ecology 0329
Entomology 0353
Genetics 0369
Limnology 0793
Microbiology 0410
Molecular 0307
Neuroscience 0317
Oceanography 0416
Physiology 0433
Radiation 0821
Veterinary Science 0778
Zoology 0472
Biophysics
General 0786
Medical 0760

EARTH SCIENCES
Biogeochemistry 0425
Geochemistry 0996
Geodesy 0370
Geology 0372
Geophysics 0373
Hydrology 0388
Mineralogy 0411
Paleobotany 0345
Paleoecology 0426
Paleontology 0418
Paleozoology 0985
Palynology 0427
Physical Geography 0368
Physical Oceanography 0415

HEALTH AND ENVIRONMENTAL SCIENCES
Environmental Sciences 0768
Health Sciences
General 0566
Audiology 0300
Chemotherapy 0992
Dentistry 0567
Education 0350
Hospital Management 0769
Human Development 0758
Immunology 0982
Medicine and Surgery 0564
Mental Health 0347
Nursing 0569
Nutrition 0570
Obstetrics and Gynecology 0380
Occupational Health and Therapy 0354
Ophthalmology 0381
Pathology 0571
Pharmacology 0419
Pharmacy 0572
Physical Therapy 0382
Public Health 0573
Radiology 0574
Recreation 0575
Speech Pathology 0460
Toxicology 0383
Home Economics 0386

PHYSICAL SCIENCES
Pure Sciences
Chemistry
General 0485
Agricultural 0749
Analytical 0486
Biochemistry 0487
Inorganic 0488
Nuclear 0738
Organic 0490
Pharmaceutical 0491
Physical 0494
Polymer 0495
Radiation 0754
Mathematics 0405
Physics
General 0605
Acoustics 0986
Astronomy and Astrophysics 0606
Atmospheric Science 0608
Atomic 0748
Electronics and Electricity 0607
Elementary Particles and High Energy 0798
Fluid and Plasma 0759
Molecular 0609
Nuclear 0610
Optics 0752
Radiation 0756
Solid State 0611
Statistics 0463

Applied Sciences
Applied Mechanics 0346
Computer Science 0984

Engineering
General 0537
Aerospace 0538
Agricultural 0539
Automotive 0540
Biomedical 0541
Chemical 0542
Civil 0543
Electronics and Electrical 0544
Heat and Thermodynamics 0348
Hydraulic 0545
Industrial 0546
Marine 0547
Materials Science 0794
Mechanical 0548
Metallurgy 0743
Mining 0551
Nuclear 0552
Packaging 0549
Petroleum 0765
Sanitary and Municipal 0554
System Science 0790
Geotechnology 0428
Operations Research 0796
Plastics Technology 0795
Textile Technology 0994

PSYCHOLOGY
General 0621
Behavioral 0384
Clinical 0622
Developmental 0620
Experimental 0623
Industrial 0624
Personality 0625
Physiological 0989
Psychobiology 0349
Psychometrics 0632
Social 0451

UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: Kevin Wayne Yakiwchuk

TITLE OF THESIS: COMPUTER GENERATED GEOLOGICAL MAPS FROM CROSS-SECTIONS: an example from the Rocky Mountain Foothills near Grande Cache, Alberta.

DEGREE: Master of Science

YEAR THIS DEGREE GRANTED: 1994

Kevin Yakiwchuk

31 Ranchero Rise, N.W.

Calgary, Alberta

T3G - 1C8

DATED: October 7, 1994

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled COMPUTER GENERATED GEOLOGICAL MAPS FROM CROSS-SECTIONS: an example from the Rocky Mountain Foothills near Grande Cache, Alberta submitted by Kevin Wayne Yakiwchuk in partial fulfillment of the requirements for the degree of Master of Science in Geology.

Supervisor : Dr. P. Erdmer

Dr. H. Charlesworth

Dr. B. Nesbitt

Dr. D. Schmitt

DATED: OCT 04 1994

ABSTRACT

A computer program was written to construct a geological map of an area underlain by cylindrically folded strata. The program uses as input 1) a digital elevation model of the area's topographic surface, 2) a digitized cross-section, 3) the position and orientation of the cross-section, and 4) the orientation of the fold-axis. The program was used to complete a 1:15000 scale geological map of a 120 km² area west of Sheep Creek in the Rocky Mountain Foothills of west-central Alberta. The program is particularly useful in poorly exposed areas where the structure is known largely from drillhole intersection data.

ACKNOWLEDGMENTS

This project began while the author was employed by Smoky River Coal Limited during the summer of 1992. The author wishes to thank everyone in the geology department at Smoky River Coal for their help during that summer and over the duration of the project. I am especially indebted to Brian Klappstein for his assistance and invaluable advice. Thanks also to Smoky River Coal Limited for making available the extensive database of outcrop and drillhole data.

During this study financial assistance was provided in the form of numerous scholarships and Teaching Assistantships as well as a Graduate Research Assistantship from the University of Alberta. I was also the recipient of the Province of Alberta Graduate Scholarship over a portion of this study.

Dr. Henry Charlesworth is thanked for his guidance and willingness to review and assist throughout the project. I appreciate the fact that he agreed to supervise one last graduate student after his retirement. Dr. Philippe Erdmer is acknowledged for his support and donation of computing facilities. Peter Jahans' assistance and suggestions were much appreciated during the development of the computer program. Last, but not least, I am grateful to my wife Kristin who showed enormous patience during my countless hours working at the computer and during summers working away from home.

TABLE OF CONTENTS

APPENDICES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Geological maps are used for engineering, agricultural, forestry and environmental purposes. They play an essential role in the mineral, petroleum and coal industries and are an integral part of most geological studies. Geological maps of areas underlain by deformed strata are generally constructed from outcrop data. Where outcrops are scarce, map construction has to be based on geological cross-sections put together from drillhole or seismic data. Constructing a geological map of an area from cross-section data by hand is extremely tedious, especially where the topographic relief is high and the orientation of the fold-axis is not known with any degree of certainty.

The objectives of this thesis are two-fold: 1) to develop a computer-based procedure for constructing a geological map of an area adjacent to a cross-section using data from the cross-section, and 2) to use this procedure in order to complete a 1:15000 scale geological map of the 120 km^2 Beaverdam Creek map-area in west-central Alberta (Figure 1). Chapter 2 of the thesis describes the computer-based mapping procedure. Equations used in the procedure are given in appendices. Chapter 3 describes the geology of the Beaverdam Creek area. Chapter 4 discusses how the computer-based mapping procedure was used in a poorly exposed 8.5 km^2 part of the Beaverdam creek area. Chapter 5 summarizes the work completed during the project and draws conclusions concerning the usefulness of the mapping procedure.

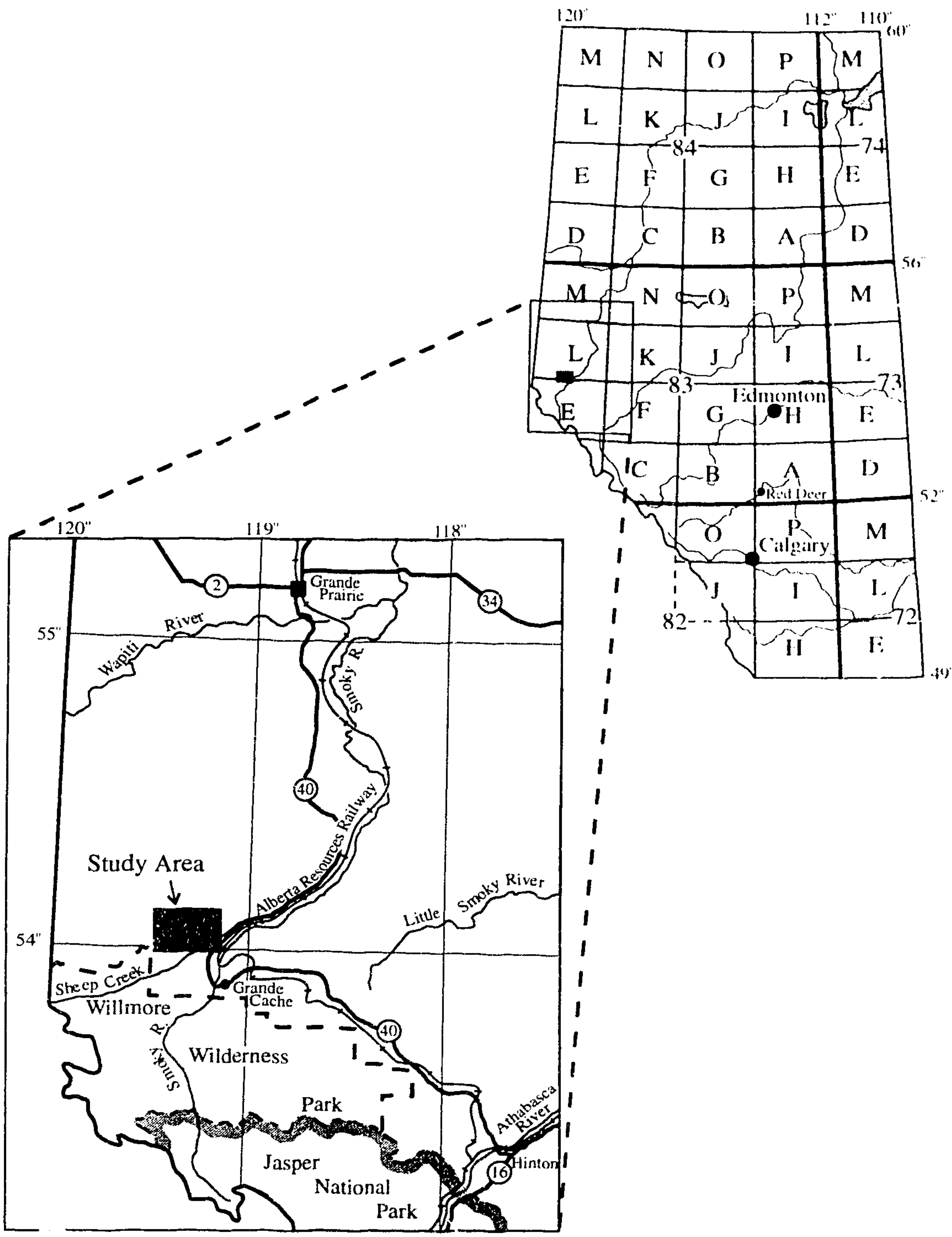


Figure 1. Map showing the location of the study area.

CHAPTER 2

PROGRAM DESCRIPTION

Introduction

In the past, computer programs such as TRIPOD (Charlesworth *et al.* 1987) have applied the principle of axial projection to aid in the production of geological cross-sections by projecting outcrop and drillhole data onto a chosen plane of section. A computer-based procedure was developed to perform the reverse of this process, that is, to construct geological maps of cylindrically folded terrains from cross-sectional data. The procedure uses the principle of axial projection (see for example Marshak and Mitra, 1988, p. 269) to project the cross-sectional data parallel to a given fold-axis orientation onto a computer-modelled topographic surface.

Before the program can be executed four items are required: 1) a Digital Elevation Model (DEM) consisting of the three-dimensional coordinates of points on the topographic surface adjacent to the cross-section, 2) a digitized geologic cross-section, 3) the location and orientation of the cross-section, and 4) the average fold-axis orientation for the area adjacent to the cross-section. Items 1, 2 and 3 must be supplied to the program in the format specified in Appendix A. Item 4 is entered during the execution of the program.

Outline of Algorithm

In its most basic form the algorithm consists of two main steps: (1) modelling the topographic surface of the area adjacent to the cross-section, and (2) locating the topographic traces of stratigraphic horizons and structures such as faults present on the cross-section.

Modelling the topographic surface

A three-dimensional model of the topographic surface must be constructed before the subsurface data can be projected onto it. To model the surface a Delaunay triangulation is performed whereby the planar coordinates of the DEM points (elevations are not used in this process) are joined to form triangles which are as equilateral as possible. The topographic model is such that the circumcircle of no triangle contains another data point, i.e. each triangle is a Delaunay Triangle (Figure 2). When the elevations of the vertices of the triangles are taken into account, Delaunay triangulation produces a good three-dimensional model, each triangle representing a planar portion of the topographic surface.

The triangulation algorithm was modified from the FORTRAN program ACORD (Watson, 1982). Constructing the topographic model begins with an initial arbitrary triangle which surrounds all points in the DEM (Figure 3a). The DEM data points are introduced into the model one at a time (Figure 3b). As each new point is added triangles in the previous model whose circumcircles contain the new point are identified and deleted (Figure 3c). This is accomplished by comparing, for each triangle, the radius of the circumcircle and the distance between its center and the new point. Finding the center and radius of a circumcircle from the coordinates of a triangle's apices is discussed in Appendix B. The deleted triangles are replaced with triangles which use the new point as one of their apices (Figure 3d). The introduction of each new point increases the number of triangles in the topographic model by two. When all points have been added, triangulation is complete (Figures 3e and 3f). Triangles involving points from the initial surrounding triangle are deleted from the model leaving only those involving DEM data.

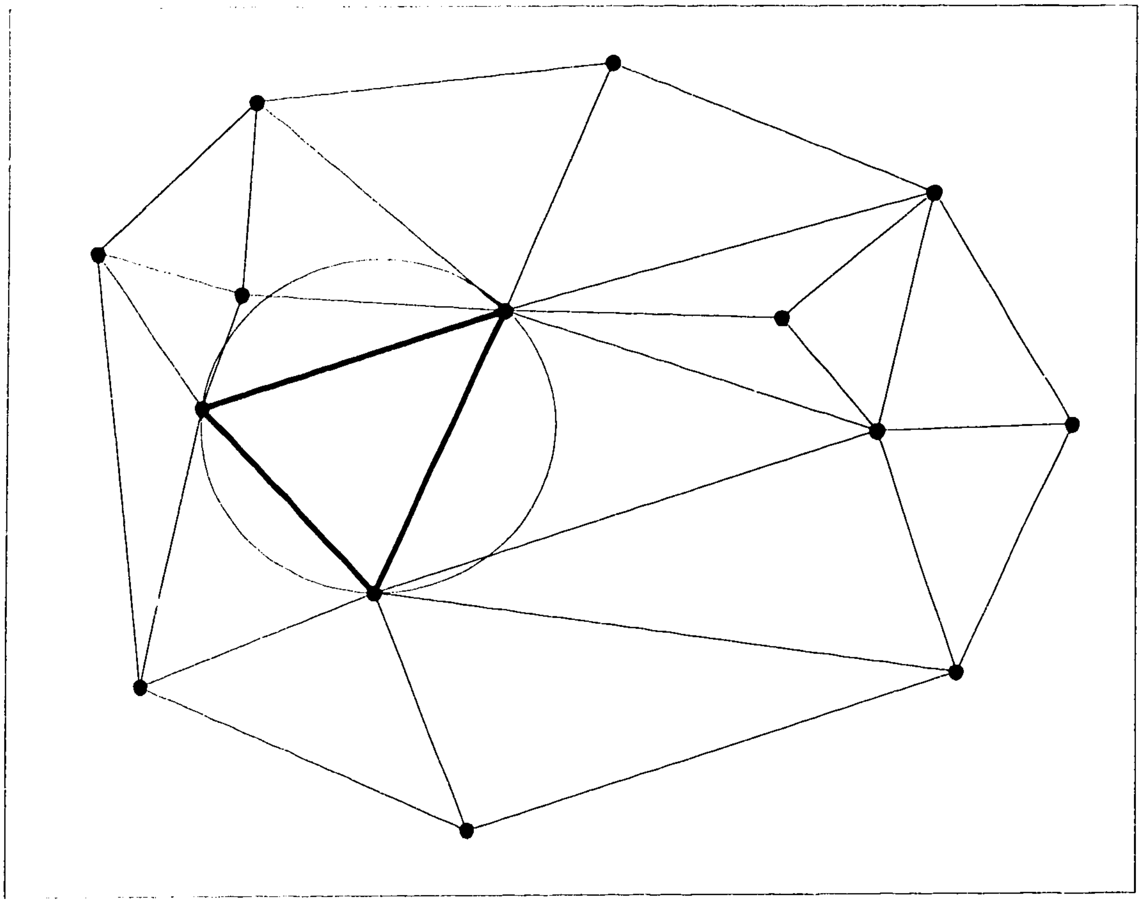


Figure 2. Delaunay triangulation of a set of points on a surface. Triangulation is such that the Delaunay triangles are as equilateral as possible and the circumcircle of each triangle contains no other point. The constructed Delaunay Triangles are shown along with one circumcircle.

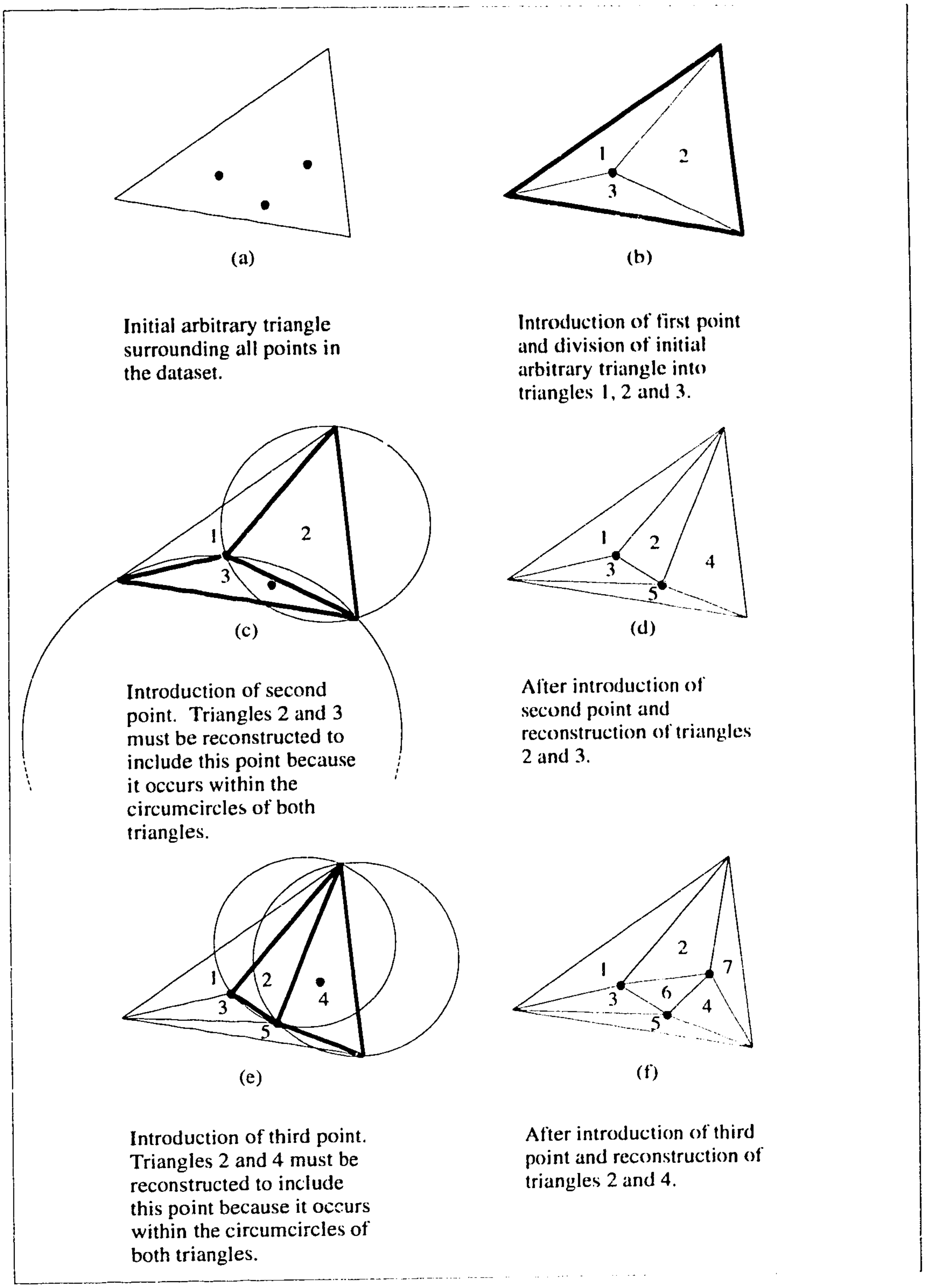


Figure 3. Process of Delaunay triangulation showing reconstruction of triangles after introduction of each point.

Map construction

The principal idea behind the program is to project traces on the cross-section parallel to a certain direction, the fold-axis, until they intersect the topographic surface. A number of terms are defined here so that they may be used in the following discussion (also see Figure 4).

A modelled topographic surface consists of a number of Delaunay Triangles where each triangle lies on a distinct plane referred to as a *Triangle Plane* and the area within the triangle represents a portion of the topographic surface. A trace on a cross-section is approximated by a number of linear segments referred to as *Section Segments*. The plane common to a Section Segment and the Projection Direction is referred to as a *Segment Plane*. The complete trace of a Section Segment projected onto the topographic surface is referred to as a *Segment Trace*.

There are two fundamental parts to the map construction algorithm: 1) locating the trace of a Section Segment projected onto a Triangle Plane, and 2) determining if the trace passes through the Triangle Plane's Delaunay Triangle and, if it does, identifying that part that does so that it may be stored. The steps in the procedure used to project cross-sectional data onto a modelled topographic surface are identified in Figure 5.

1) Projection of the Section Segment onto the Triangle Plane

The Section Segment is projected onto the Triangle Plane by projecting its end points (Figure 4). The line joining the two projected end points defines the trace of the Section Segment projected onto the Triangle Plane. Each end point is projected onto the Triangle Plane by calculating the point of intersection of the Triangle Plane and the line parallel to the Projection Direction through the end point. The calculations used to find these intersections are described in Appendix C.

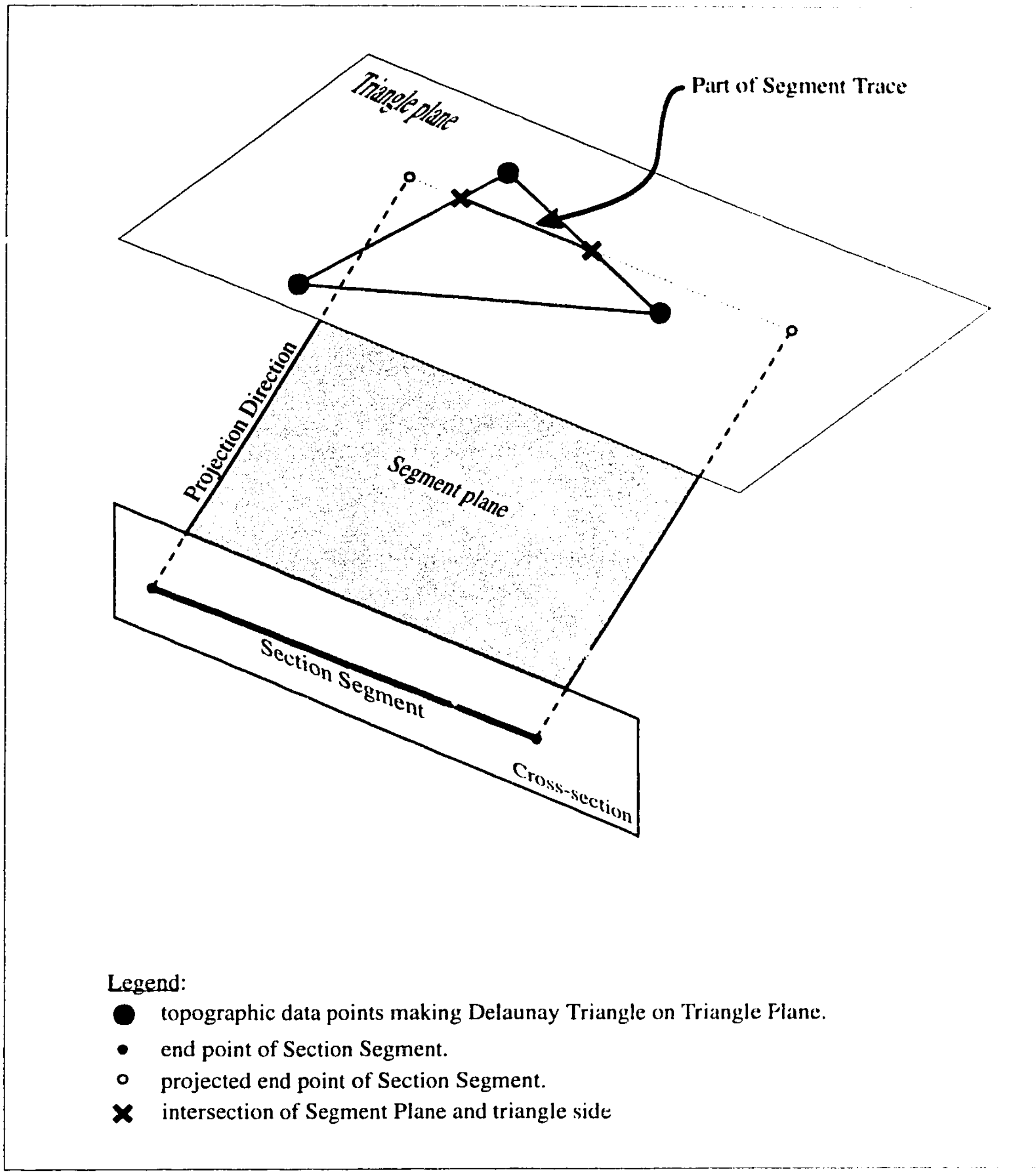


Figure 4. Projection of a Section Segment onto a Triangle Plane. The part of the projected Section Segment that is inside the Delaunay Triangle is part of the Segment Trace.

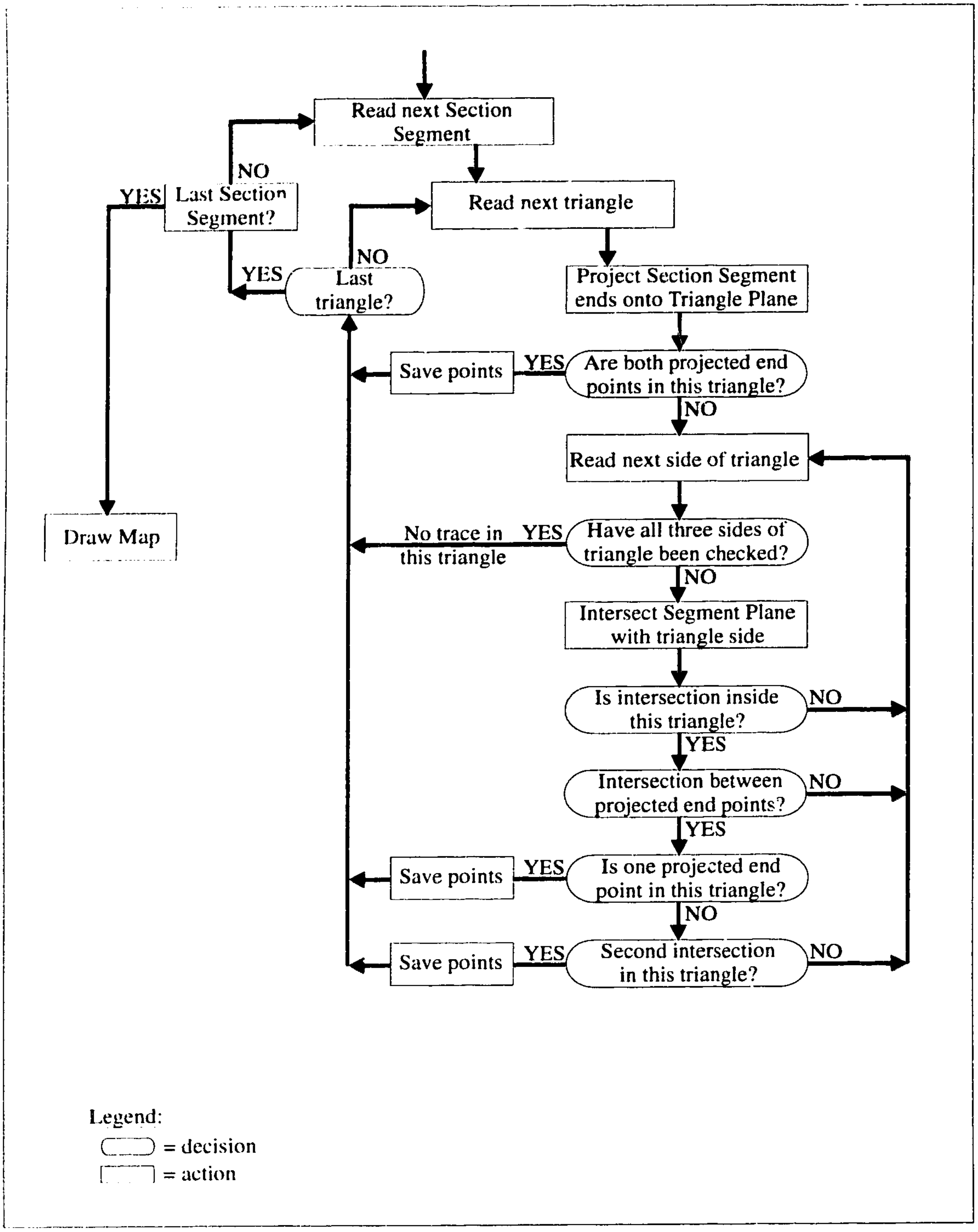


Figure 5. Flow chart for projection of cross-sectional data onto a modelled topographic surface. For definitions of Section Segment, Triangle Plane and Segment Plane see text and Figure 4.

2) Locating the part of the trace within the Delaunay Triangle

Since the area of the Triangle Plane outside the Delaunay Triangle does not represent a portion of the topographic surface only that part of the projected Section Segment within the Delaunay Triangle belongs to the Segment Trace. The goal of the algorithm is to cycle through each Delaunay Triangle in the model and, if the triangle contains part of the Segment Trace, to locate and store the two ends of the trace in that triangle (Figure 4). For the situation portrayed in Figure 4, the Segment Trace passes in through one side of the triangle and out through another. The part of the Segment Trace within the triangle is defined by the points where the Segment Plane intersects these two sides of the triangle. The calculations used to locate the points of intersection of a Segment Plane with the sides of a Delaunay Triangle are described in Appendix C.

There are four possible relationships between the projected end points of a Section Segment and a Delaunay Triangle:

a) Both projected end points lie within the Delaunay Triangle, in which case the entire Segment Trace is defined by these points (Figure 6a).

b) One projected end point lies within the Delaunay Triangle, in which case the Segment Plane intersects one of the sides of the triangle and that part of the Section Segment in the triangle connects the intersection point and the projected end point (Figure 6b).

c) Neither projected end point lies within the Delaunay Triangle and the Segment Plane intersects two sides of the triangle, in which case that part of the Segment Trace in the triangle connects the two intersection points (Figure 6c),

d) Neither projected end point lies within the Delaunay Triangle and the Segment Plane does not intersect the triangle, in which case no part of the Segment Trace passes through the triangle (Figure 6d).

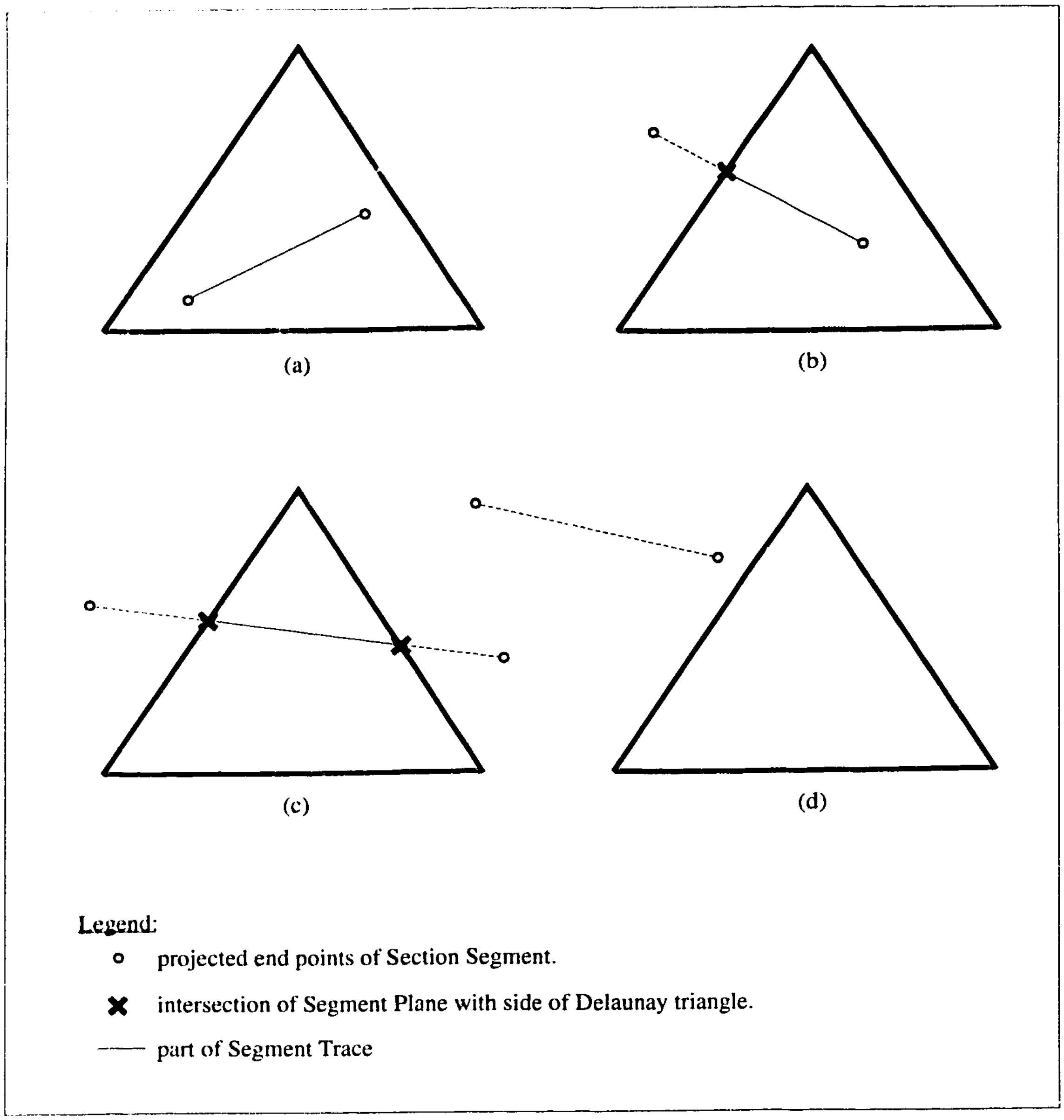


Figure 6. Possible relationships between a Delaunay Triangle and the projected end points of a Section Segment.

The calculations used to determine the relationship between the projected end points of a Section Segment and a Delaunay Triangle are given in Appendix D.

Planes and lines represented by mathematical equations are infinite in three-dimensions so every Segment Plane actually intersects all three sides of a triangle (Figure 7). For an intersection to be on the Segment Trace, it must be 1) inside the Delaunay Triangle's circumcircle, and 2) between the projected end points of the Section Segment (Figure 7 - Insert). The process of determining if the intersection point lies between the projected end points is carried out by comparing the distances between points (Appendix E). The intersection is not between the points when the distance between it and either of the projected end points is greater than the distance between the projected end points.

Summary of algorithm flow

The two steps which must be performed to produce a map of a cross-section are: 1) Delaunay triangulation of the DEM, and 2) projection of the cross-sectional data parallel to the fold-axis onto the modelled topography.

The triangulation procedure produces a three-dimensional model of the topographic surface by dividing it into a number of planar triangular portions. The resulting triangles are such that they are as equilateral as possible and no triangle's circumcircle contains another point, that is they are Delaunay Triangles.

Traces on a cross-section are divided into Section Segments which are projected onto the Delaunay Triangles to locate the Segment Trace. The projection algorithm begins by reading one segment from the cross-section and one triangle from the topography model (Figure 5). Next, the ends of the segment are projected onto the Triangle Plane. If both projected end points lie within the triangle the complete Segment Trace is defined by these points. If one or none of the projected end points lies within the triangle the intersections

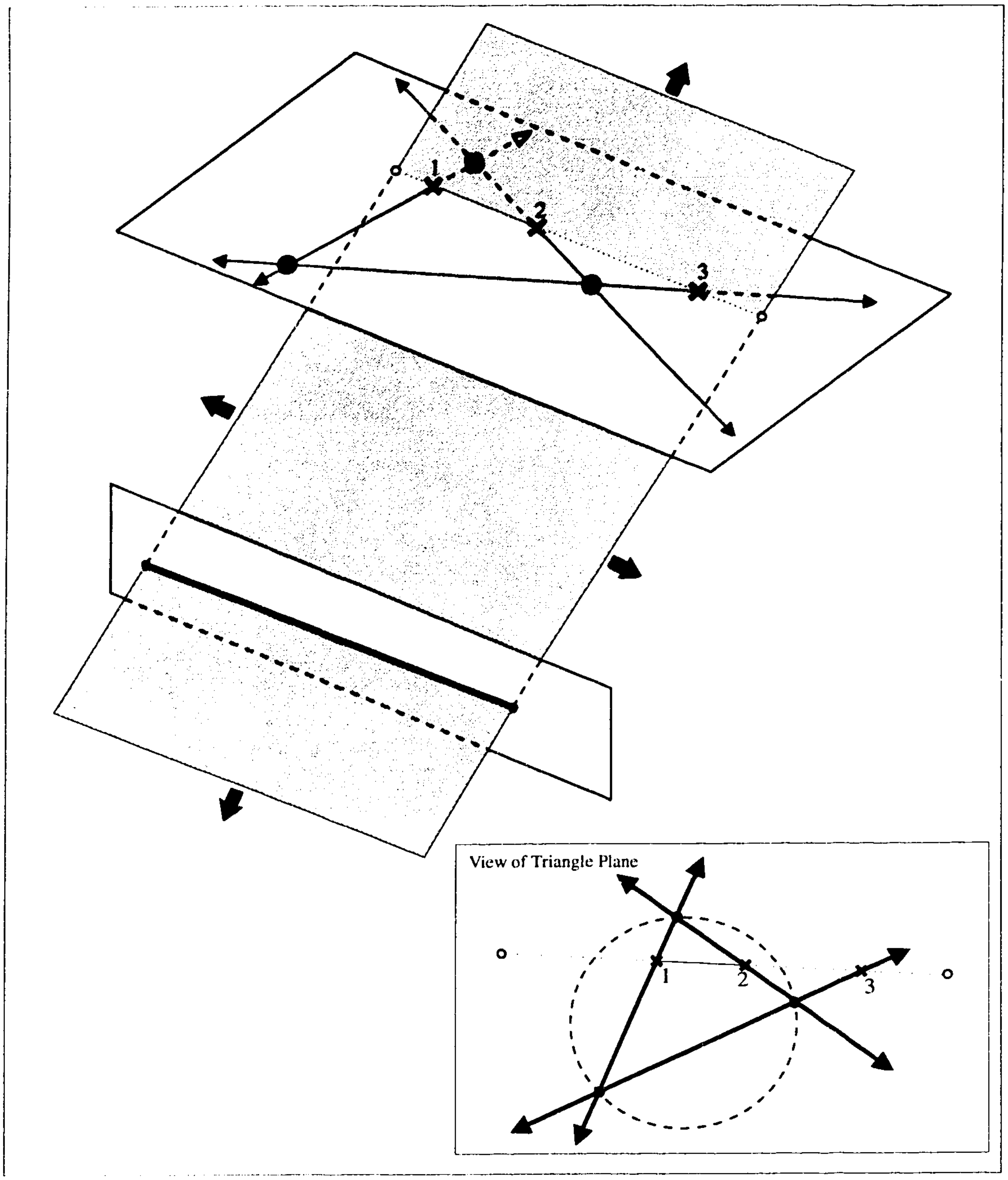


Figure 7. Realistic depiction of the process of projecting a Section Segment onto a Triangle Plane. Arrows indicate that the Segment Plane and the lines along the sides of a Delaunay Triangle are infinite in all directions. Note that three intersections exist but only those which are inside the triangle 's circumcircle and between the two projected end points belong to the Segment Trace.

of the Segment Plane with the sides of the triangle are calculated. Only intersections which are within the triangle's circumcircle and between the projected end points belong to the Segment Trace. Where one projected end point lies within the triangle, the Segment Plane intersects one side of the triangle and the part of the Segment Trace in the triangle joins the intersection and the projected end point. Where neither projected end point lies within the triangle, the Segment Plane intersects two or no sides of the triangle. If it intersects two sides, the part of the Segment Trace in the triangle joins the two intersections. If it intersects no sides, the Segment Trace does not pass through the triangle. Each time that a triangle containing a portion of the Segment Trace is found, the ends of that portion of the trace are saved.

When the algorithm has checked a triangle, the next triangle is read from the topography model and processed. When the algorithm has checked every triangle in the topography model for part of the Segment Trace the next Section Segment is read from the cross-section and the entire process is repeated. Note that because a Segment Plane may intersect the topographic surface in more than one place, for each Section Segment every triangle in the topography model has to be checked. When the last Section Segment has been projected, the calculations are complete. The resulting map is displayed by plotting the portions of the Segment Traces that were located and stored during the calculations.

Mapping Demonstration

A simple example is outlined here in order to demonstrate the computer-based mapping procedure. The topography of the sample study area is displayed in Figure 8a. A cross-section involving three folded horizons (A, B and C) cut by a southwest dipping thrust fault (F) was constructed through the area (Figure 8b). The map of the cross-section

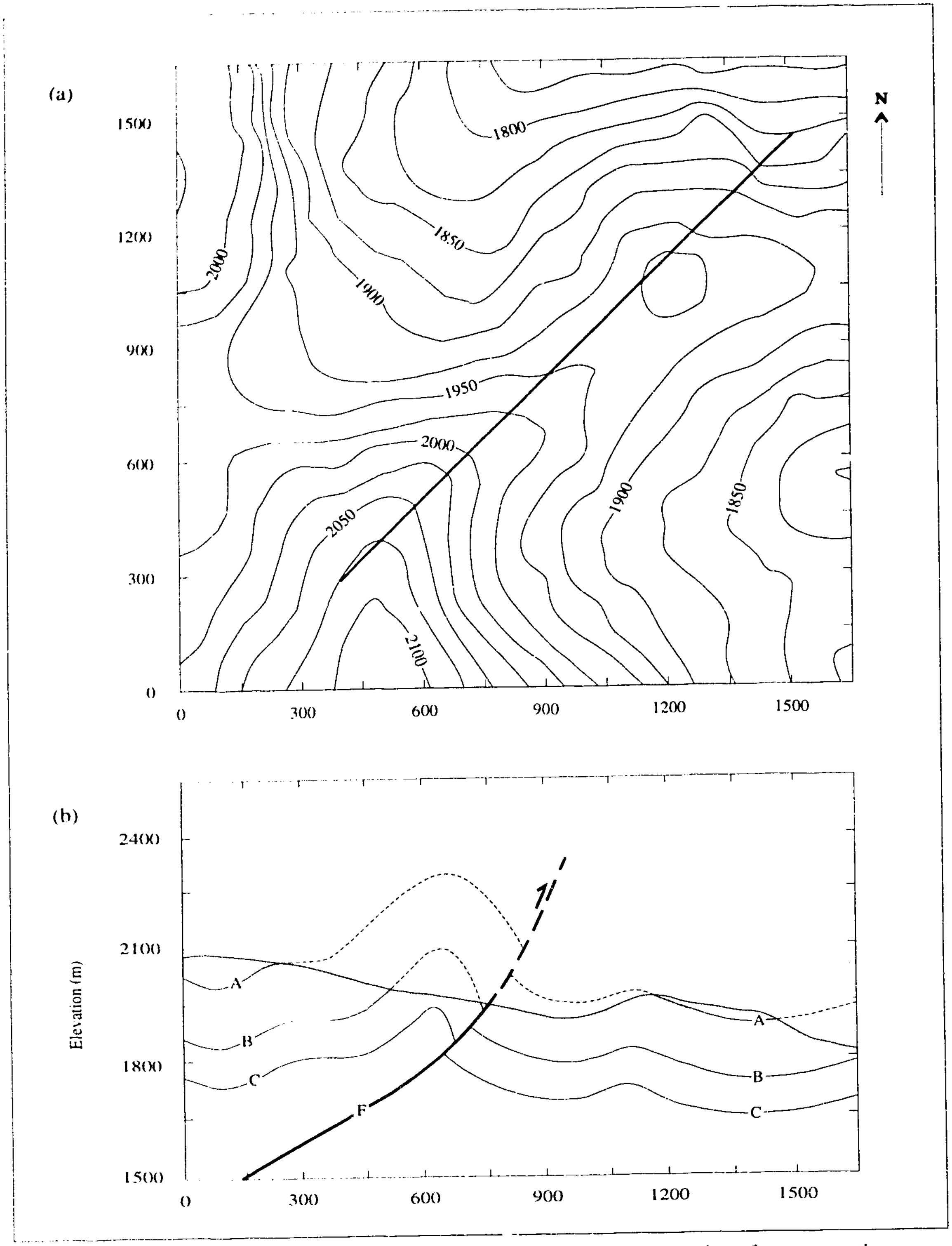


Figure 8. Topographic contours on a sample study area (a) showing the location of a cross-section (b) to be projected onto its surface. Scale of both drawings is 1:15000.

(Figure 9) was produced by projecting the digitized cross-sectional data at 134° 5° onto the modelled topographic surface.

Software and Hardware

The mapping program is called MapXS. It must be run on an IBM compatible PC under the Microsoft Windows operating environment. The program was written in Borland's Turbo Pascal for Windows Version 1.5. A copy of the program may be obtained by contacting Dr. Philippe Erdmer, Department of Geology, University of Alberta, Edmonton, T6G - 2E3.

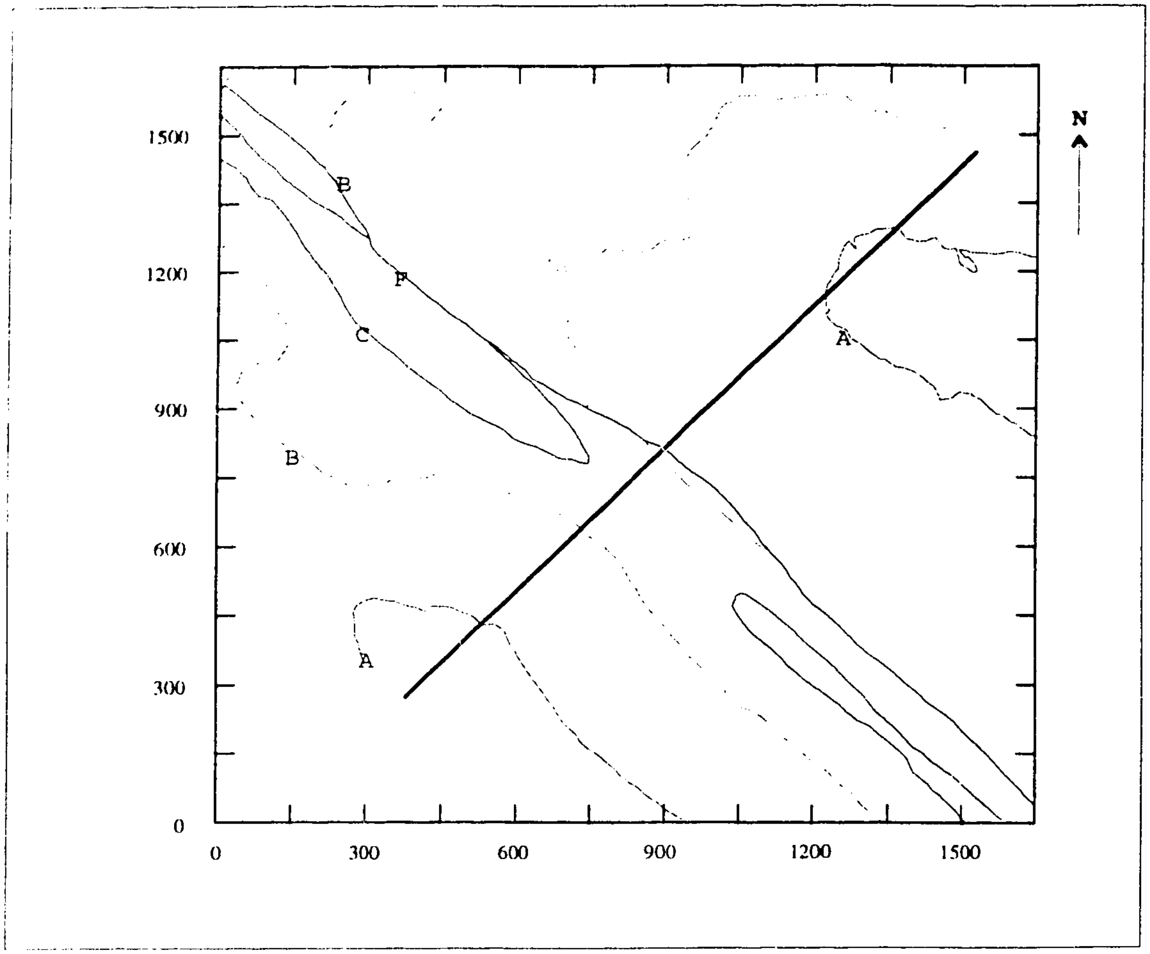


Figure 9. Map produced from projection of cross-section in Figure 8-b at 134° 5" onto topographic surface in Figure 8-a. Scale of drawing is 1:15000.

CHAPTER 3

GEOLOGY OF THE BEAVERDAM CREEK MAP-AREA

Introduction

The Beaverdam Creek map-area is in the Foothills of the Rocky Mountains near the town of Grande Cache, Alberta. It lies between latitudes 53°59′ and 54°06′ N, and longitudes 119°07′ and 119°25′ W and forms part of the Grande Cache (NTS 83E/14) and Copton Creek (NTS 83L/3) mapsheets (Figure 1). The Beaverdam Creek area is bound to the west by Sheep Creek, and to the northwest and southeast by the Muskeg and Cowlick thrusts respectively. The area is about 350 km west of Edmonton and can be accessed by Highway 40, which connects Hinton with Grande Cache. This highway extends north from Grande Cache to the offices and plant facilities of Smoky river Coal Limited. From there, several exploration roads and a forestry road allow access to most of the 120 km² of the map area itself. Trail bikes or four-wheel drive vehicles are required to reach some areas.

Previously published geological maps of the area include the Grande Cache (Irish and Thorsteinsson 1957) and Copton Creek (Irish 1955) mapsheets both at a scale of 1:63360. McMechan (1989) produced a 1:50000 scale geologic map which includes the study area. The area is bordered to the southwest by a 1:15000 scale map (Langenberg *et al.*, 1987).

General Geology

The Beaverdam Creek area is located in the Inner Foothills of the Rocky Mountain fold and thrust belt. In this area the Inner Foothills are bound to the southwest by the Rocky Pass thrust and to the northeast by the Muskeg thrust (Mountjoy 1978). The Inner Foothills generally expose Lower Cretaceous and the Outer Foothills Upper Cretaceous and Paleocene strata. In the study area a thick succession of marine and non-marine clastic rocks ranging in age from Late Jurassic to Late Cretaceous are exposed. Thick economic coal seams are present in some non-marine Cretaceous strata.

Deformation caused by the Laramide Orogeny took place between early Campanian and late Eocene and proceeded from southwest to northeast (Bally *et al.* 1966; Price 1981). The main episode of deformation in the Grande Cache area is estimated to have occurred at the end of the Paleocene (Kalkreuth and McMechan 1984).

Shortening of the stratigraphic succession is accomplished by folding and thrusting and averages 30-33% in the Grande Cache area (Langenberg 1985; Langenberg *et al.* 1987). Folds are of the chevron variety. The majority of the faults are southwest-dipping thrusts with ramps that cut up section to the northeast and flats that parallel bedding. The major thrusts are rooted in a basal detachment at the top of the Precambrian crystalline basement, which is at a depth of approximately 5500 m below sea level (Mountjoy 1978).

Data

The data used to construct the geological map of the area were provided by Smoky River Coal Limited in the form of computer files. The format of these files is compatible with the software package TRIPOD (Charlesworth *et al.* 1987) that was used to store, analyze and display the data. Some of the data were collected by the author during the summer of 1992 while employed by Smoky River Coal Limited.

Outcrops

Structural and stratigraphic data were collected in the field by plotting the locations of 2778 outcrops on aerial photographs and recording lithological observations and orientational measurements. Outcrop positions were later transferred to 1:6000 scale topographic base maps with a 1000 foot grid spacing and 10 foot contour interval. The eastings and northings of outcrops were recorded from the base maps using an engineer's scale and the elevation was estimated from the nearest contour.

The grid system used on these maps is the 3° Universal Transverse Mercator grid (3TM). This map projection is one of the provincial standard projections and is similar to the standard 6° Universal Transverse Mercator grid (UTM). The origin of the 3TM grid is at the intersection of the 120° W meridian and the equator.

Drillholes

Data from 1311 drillholes placed in computer files include location, orientation, stratigraphic horizons intersected and depths to these horizons. Stratigraphic horizons were picked by Smoky River Coal geologists based on geophysical logs and drillers' reports. The mineable coal is within the Gates Formation so this is where the majority of the horizon picks were made.

The orientations of intersected horizons were available for 129 of the drillholes. The orientation measurements were obtained from dipmeter data and recorded as the dip-direction (the azimuth of the direction of dip) and dip as well as the depth of these measurements.

Deviation measurements were recorded at approximately 15 m intervals for 1149 drillholes which deviated substantially from straight. A deviation measurement consists of the azimuth of the drillhole, its deviation from the vertical, and the depth of the measurement.

Stratigraphy

Over 1200 m of marine and non-marine clastics are exposed in the map area. The strata range from the Jurassic and Lower Cretaceous Nikanassin Formation to the Upper

Cretaceous Kaskapau Formation (Figure 10). The lower part of the Nikanassin as well as the Moosebar, Shaftesbury and Kaskapau Formations represent marine episodes of sedimentation. The upper part of the Nikanassin Formation as well as the Cadomin, Gladstone, Gates and Dunvegan Formations are non-marine. In general, the succession is characterized by competent sandstone units separated by incompetent shales.

A number of coal seams are present within the Gates Formation. A few of the thicker seams within the Grande Cache Member are being mined by Smoky River Coal Limited. The Grande Cache Member and all contained coal seams show a trend of decreasing thickness towards the north and west.

Structural Geology

The structural style of the area is characterized by chevron folds and thrust faults. The major thrust faults are, from southwest to northeast, the Cowlick, Syncline Hills and Muskeg thrusts (Irish and Thorsteinsson 1957; see also Figure 11, in pocket). The area can be divided into the Syncline Hills and Muskeg thrust sheets, with each being named after the thrust over which it moved (Figure 12).

Using statistical tests and other criteria (Charlesworth *et al.* 1976), the study area was divided into 7 domains within which folding can be considered cylindrical (Figure 12). Domain boundaries, chosen by trial-and-error, are generally parallel and perpendicular to the regional strike. Originally, domain boundaries were chosen where a change in geometry was apparent from the outcrop data. Later, they were altered or refined as a better understanding of the geology was gained. Some domain boundaries are the traces of

Stage	Group	Formation (Thickness[1])	Member (Thickness[1])	Description[1,2]
Cenomanian		Kaskapau (160 m)		dark gray shale and siltstone; lower part only present in study area.
		Dunvegan (50 m)		brown to reddish weathering sandstone interbedded with shale, siltstone and thin coal seams.
		Shaftesbury (150 m)		dark gray fissile shale; interbedded siltstone and sandstone; ironstone concretions common; contact with Gates Formation marked by pebble conglomerate bed.
Albian	Luscar (525 - 595 m)	Gates (335- 380 m)	Mountain Park (175 - 190 m)	gray-orange weathering very fine- to medium-grained cross-bedded sandstone; thin sandstone and shale units with minor coal occur between thick sandstones; base placed at first prominent sandstone above highest major coal seam in Grande Cache Member.
			Grande Cache (130 - 160 m)	gray-orange weathering shale and fine-grained sandstone; upper part contains fine- to medium-grained sandstone; contains coal seams Nos. 3 to 11 with No. 3 at base and No. 11 at top; includes thick economic coal seams.
			Torrens (30 m)	gray-orange weathering fine-grained sandstone; minor pebble conglomerate beds; contact with Moosebar Formation placed at base of first massive sandstone.
		Moosebar (60 m)		dark gray shale; interbedded hummocky cross-stratified fine-grained sandstone and shale present in middle and upper parts.
		Gladstone (100-110 m)		brownish weathering, fining upward sequences of sandstone, shale and minor coal; contains coal seam No. 1, coal seam No. 2 at top.
		Cadomin (30-45 m)		upper and lower parts: grayish weathering chert and quartzite pebble conglomerate; middle part: fine- to medium-grained, brownish-gray weathering, cross-bedded sandstone; minor coal above lower conglomerate.
Aptian		Nikanassin (>400 m)		grayish-orange weathering sandstone with interbedded shale, silty shale and shaley sandstone; upper part contains minor thin coal; contact with underlying Fernie Formation not seen in study area.

Figure 10. Table of formations. Contact with underlying strata: ——— sharp, ······· gradational, ～～～ disconformable. Stratigraphic nomenclature from Langenberg and McMechan (1985). Coal seams numbered according to system introduced by Landes (1963). Data Sources: [1] Langenberg et al. (1987), [2] Wrightson (1979).

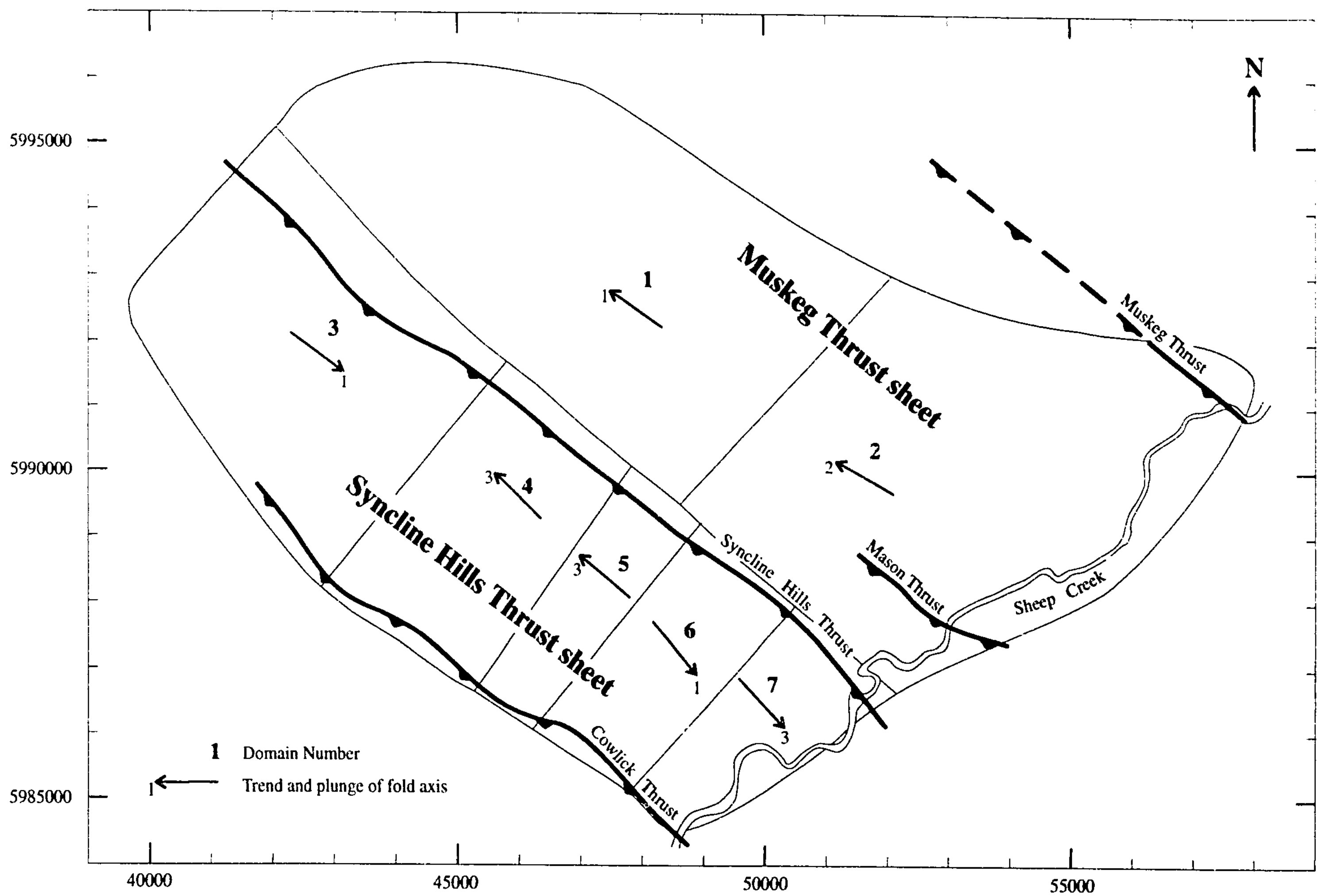


Figure 12. Map of the study area showing major thrust faults and boundaries of established cylindrical domains. The trend and plunge of the best-fit cylindrical fold-axis within each domain is shown. Coordinates are in 3TM grid.

major geological features such as thrusts; others are generally drawn through areas with few outcrops and drillholes.

The trend and plunge of the fold-axis for each domain are shown in Figure 12. The orientation of the fold-axis in each domain was determined by carrying out a moment of inertia analysis of outcrop orientations within that domain (Charlesworth *et al.* 1976 ; Appendix F). In domains 2, 4 and 5 a sufficient number of dipmeter orientations were also available to produce stereoplots. The drillholes that contain dipmeter data in domains 2 and 5 are located on single limbs of folds and so the poles to bedding plot as a cluster and a fold-axis orientation can not be determined from them (Figures 17 and 22). In domain 4 the dipmeter orientations plot as a girdle with a fold-axis orientation of 316° 5° (Figure 20). This orientation is comparable to the orientation of 313° 3° determined from outcrop data (Figure 19). Outcrop data tend to be more randomly dispersed than the drillholes and so the fold-axis determined from the outcrop data is assumed to supply a statistically more accurate orientation.

In the Muskeg thrust sheet, which encompasses domains 1 and 2, plunges are gentle towards the northwest. Plunge variations in the Syncline Hills thrust sheet are mainly due to changes in the geometry of Two Camp Creek Anticline. Plunges in domains 3, 6 and 7 are towards the southeast, in domains 4 and 5 towards the northwest.

In folded terrains, the axial projection method can be used to construct structural cross-sections. The software package TRIPOD does this by projecting the data in each domain parallel to its fold-axis onto a chosen plane of section (Charlesworth *et al.* 1976; Charlesworth *et al.* 1987). Faults and traces of stratigraphic contacts are then drawn on the resulting plots. Six structural cross-sections (Figure 13, in pocket), whose locations are shown on Figure 11, were constructed.

The cross-sections reveal that the majority of the folds are chevron in style. Folds of this type are expected in shortened sequences of relatively thin, alternating, competent and incompetent layers such as is present in the study area (Ramsay 1974). The chevron nature of folding is also evident from the pattern displayed on some of the stereoplots (see for example Figure 23).

Syncline Hills thrust sheet

The Cowlick thrust can be followed for over 100 km along strike. It forms the southwestern boundary of the study area and limits the Syncline Hills thrust sheet to the southwest. This fault dips to the southwest at between 50° and 60° and carries Nikanassin over Gates strata.

The Syncline Hills thrust can be traced across the entire Beaverdam Creek map-area more or less following the valley of Beaverdam Creek. This fault dips to the southwest at between 55° and 65° and thrusts Gates and Shaftesbury Formations over Shaftesbury and Dunvegan Formations. To the southeast, the Syncline Hills thrust is estimated to have a dip-slip displacement in the order of 1300 m (Langenberg *et al.* 1987).

Two Camp Creek Anticline, a major regional structure which dominates the Syncline Hills thrust sheet, is a chevron structure with a maximum amplitude of nearly 2500 m at the top of the Cadomin Formation near section EE'. Southwest of here the fold plunges at between 1° and 3° southeast; to the northwest it plunges 3° northwest. The amplitude of the fold at the top of the Cadomin Formation reaches a low of nearly 1800 m near section BB' before increasing again to about 2000 m near section AA'. Strata on the southwest limb have steep to vertical dips along sections CC', DD' and EE'. On section DD' this limb has been overturned and cut by a northeast-dipping backthrust which places

Nikanassin Formation over Torrens Formation.

Two Camp Creek Anticline is asymmetric along sections DD', EE' and FF'. Its northeast limb dips steeply at between 45° and 60°. The southwest limb, excluding the nearly vertical strata near the crest of the anticline, dips shallowly at 10° to 30°. The asymmetry of the structure is verified by the pattern displayed on the stereoplots of outcrop bedding orientations in domains 5, 6 and 7 which show distinct clusters formed by a shallow southwest-dipping panel and a steep northeast-dipping panel (Figures 21, 23 and 25). Cross-section FF' displays a nearly horizontal panel of strata on the southwest limb of the fold. The stereoplot of bedding orientations in this area displays a pattern which resembles that of a box fold (Figure 25). The nearly horizontal panel of strata shows up as a separate cluster near the center of the stereoplot.

On sections AA', BB' and CC' the region between the crest of Two Camp Creek Anticline and Cowlick thrust includes a number of smaller folds and faults. The folds here are, from southwest to northeast, Dipslope Anticline, Marmot Basin Syncline, Cadomin Crest Anticline and Chevron Syncline. A number of unnamed folds exist between Dipslope Anticline and Marmot Basin Syncline. Cross-sections CC' and DD' show some of the minor thrust faults that are present in this region. Drillhole data suggest that these faults are of limited extent and die out within the Mountain Park or Grande Cache Members. These folds and faults are not present on cross-sections DD', EE' and FF' where the southwest limb of Two Camp Creek Anticline is a continuously southwest-dipping panel up to Cowlick thrust.

Muskeg thrust sheet

The Muskeg thrust marks the northeastern boundary of the Beaverdam Creek map-area. Its position is known only in the extreme eastern portion of the map area. Sufficient data were not available to extend, with any degree of confidence, the trace of the fault along strike to the northwest. In the footwall of this fault are shales of the Kaskapau Formation (Langenberg *et al.* 1987). No marker horizon can be matched across the fault so its displacement is not known.

The Mason thrust, a major thrust on the east side of Sheep Creek, dies out approximately 2.5 km west of Sheep Creek. Displacement along this fault is in the order of 100 m along section FF'. It can not be detected along section EE'.

The Muskeg thrust sheet is dominated by a broad, open anticlinorial structure. Near Sheep Creek this structure includes, from southwest to northeast, McEvoy Anticline, Winder Syncline and Barrett Anticline as well as a number of unnamed folds. On the east side of Sheep Creek these structures plunge towards the southeast (Langenberg *et al.* 1987). The structures reach their maximum amplitudes east of section FF' before plunging towards the northwest. A very small area exists adjacent to, and west of, Sheep Creek in which plunges are towards the southeast. Due to its very limited extent, this area was included in domain 2.

McEvoy Anticline can be seen on sections CC', EE' and FF'. Along section FF', McEvoy Anticline, along with an unnamed syncline and anticline to the northeast, forms a box fold. The presence of this box fold is verified by the pattern displayed on the stereoplot of outcrop orientations in domain 2 (Figure 16). McEvoy anticline has a maximum amplitude of about 1300 m at the top of the Cadomin Formation near section FF' but on section CC' it exists as only a slight flexure on the southwest limb of the anticlinorium.

Winder Syncline is a prominent feature on the eastern side of Sheep Creek where it extends along strike for over 5 km (Langenberg *et al.* 1987). West of Sheep Creek it splits into two synclines with an intermediate anticline. The northern syncline and the anticline are visible on section FF' just south of Barrett Anticline. The southern syncline terminates after a short distance and can not be detected on section FF'.

Barrett Anticline reaches a maximum amplitude of 1100 m at the top of the Cadomin Formation near section FF'. The fold can not be detected along section EE'. To the southeast Barrett Anticline is a prominent chevron fold that continues along strike for over 6 km (Langenberg *et al.* 1987).

In domain 1 the anticlinorium includes a number of folds on its surface, the majority of which are unnamed. Most of these folds exist as only slight flexures along the surface of the anticlinorium. Along section FF' the southwest limb of the anticlinorium dips down into, and is cut off by, Mason thrust. Further northwest this limb is cut by Syncline Hills thrust.

The geology of the region in between McEvoy Anticline and Syncline Hills thrust is poorly constrained. Very few outcrop data are available for this region and there are no drillholes. It is also very difficult to interpret geology on airphotos of the area likely due to the fact that this region is underlain by recessive shales of the Shaftesbury Formation.

CHAPTER 4

APPLICATION OF MAPPING PROCEDURE

The computer-based mapping procedure developed as part of this project uses the principle of axial-projection to produce geological maps from cross-sectional data. The procedure has been applied in a part of the Beaverdam Creek map-area where the scarcity of outcrops is made up for by numerous drillholes.

The 8.5 km² area is bound to the southwest and northeast by the Cowlick and Syncline Hills thrusts, respectively, and to the northwest and southeast by cross-sections AA' and CC' respectively (Figure 11). The area is located entirely within domain 3 which has an average fold-axis orientation of 124° 1°. Cross-section BB' cuts approximately through the area's center and so was chosen to be projected throughout the selected sample study area.

A Digital Elevation Model of the area was produced by extracting the coordinates of points from a computer database of topography. The points were selected on a grid with a 25 meter spacing to produced a model which includes 13765 data points. The Digital Elevation Model was then triangulated to produce a topographic model which could be used by the computer-based mapping procedure.

The tops of the 4 Formations and 2 Members on the cross-section were digitized to produce an ASCII file containing 330 points. The coordinates of these points, relative to the origin chosen on the cross-section during digitizing, were converted to 3TM coordinates using the procedure discussed in Appendix A.

The cross-sectional data were projected onto the modelled topographic surface at 124° 1° to produce the map shown in Figure 25 (in pocket). This map shows Two Camp Creek Anticline with the Cadomin and Nikanassin Formations cropping out in its core where the anticline's surface trace has been widened where it is cut by creeks. The cross-section shows the Torrens Formation folded into a tight anticline-syncline pair on the southwest limb of Two Camp Creek Anticline. This anticline-syncline pair projects onto the map between Two Camp Creek Anticline and Chevron Syncline just east of the section line. The Torrens and Moosebar Formations crop out in the Cadomin Crest Anticline southwest of Two Camp Creek Anticline where the creek valleys have cut across these structures and also along strike from here to the east of the cross-section. At the southwest end of the cross-section, Torrens, Moosebar and Gladstone Formations crop out in Dipslope Anticline. Between Dipslope Anticline and Cadomin Crest Anticline are a number of anticlines and synclines exposing mainly Grande Cache and Mountain Park Members.

The surface geology in this area, as displayed in Figure 11, was interpreted using the computer-generated map as a guideline. Slight modifications to the traces were made so that they conformed with known surface outcrops. The computer-generated map is generally consistent with the available outcrop data everywhere except at its southeasternmost edge where it shows the Torrens and Moosebar Formations at surface. Available outcrop data do not support this interpretation. The discrepancy here between observed outcrops and the computer-generated map is likely due to the fact that this is near the boundary between domain 3 and domain 4 where the orientation of the fold axis changes from 124° 1° to 313° 3° respectively.

CHAPTER 5

SUMMARY AND CONCLUSIONS

One of the two main objectives of this thesis, to develop a computer-based procedure for constructing geological maps from cross-sectional data, has been fully realized. The procedure can be applied in areas where the strata are cylindrically folded and where a Digital Elevation Model and a digitized cross-section are available. The procedure enables the surface geology to be displayed in areas where structure is known largely from drillhole intersection data.

The main advantages of this procedure are as follows.

1) It is quick, taking up only minutes of the geologist's time whereas the graphical procedure could take up to several days.

2) The constructed geological map is extremely accurate and is reproducible.

3) It allows the geologist easily and quickly to modify the cross-section and/or fold-axis orientation to generate new maps where discrepancies exist between the computer-generated map and the few available outcrops.

4) It can be used to locate accurately the surface position of a target zone such as a coal seam whose position is known only from drillhole data.

The second objective, to produce a 1:15000 scale map of the120 km^2 Beaverdam Creek map-area, has also been accomplished. The area is characterized by chevron folding and northeast verging thrust faults. The map was completed by projecting a cross-section based on drillhole data throughout an 8.5 km^2 part of this area where outcrop data are particularly sparse.

REFERENCES

Anton, H., 1984. Elementary Linear Algebra, 4th. Ed., John Wiley and Sons, New York, 403 pages.

Bally, A.W., Gordy, P.L., and Stewart, G.A., 1966. Structure, seismic data, and orogenic evolution of southern Canadian Rocky Mountains. Bulletin of Canadian Petroleum Geology, v. 14, pp. 337-381.

Charlesworth, H.A.K., Langenberg, C.W., and Ramsden, J., 1976. Determining axes, axial planes, and sections of macroscopic folds using computer-based methods. Canadian Journal of Earth Sciences, Vol. 13, pp. 54-65.

Charlesworth, H.A.K., Gold, C., Wynne, D., and Guidos, J., 1987. A microcomputer-based system for collecting, storing, retrieving and processing structural, stratigraphic and positional data from outcrops and drillholes; Computer Manual. University of Alberta, 65 pages.

Irish, E.J.W., 1955. Copton Creek, Alberta. Geological Survey of Canada, Map 1041A.

Irish, E.J.W. and Thorsteinsson, R., 1957. Grande Cache, Alberta. Geological Survey of Canada, Map 1049A.

Kalkreuth,W. and McMechan, M.E., 1984. Regional pattern of thermal maturation as determined from coal-rank studies, Rocky Mountain Foothills and Front Ranges north of Grande Cache, Alberta - implications for petroleum generation. Bulletin of Canadian Petroleum Geology, v. 32, p. 249 - 271.

Landes, K., 1963. Exploration report, Smoky River area. McIntyre Mines, internal report.

Langenberg, C.W., 1985. The geometry of folded and thrusted rocks in the rocky Mountain Foothills near Grande Cache, Alberta. Canadian journal of Earth Sciences, v. 22, pp. 1711 - 1719.

Langenberg, C.W., Kalkreuth, W., and Wrightson, C. B., 1987. Deformed Lower Cretaceous coal-bearing strata of the Grande Cache area, Alberta. Alberta Research Council, Bulletin No. 56, 54 pages.

Langenberg, C.W. and McMechan, M.E., 1985. Lower Cretaceous Luscar Group (revised) of the northern and north central Foothills of Alberta. Bulletin of Canadian Petroleum Geology, v. 33, pp. 7-77.

Marshak, S. and Mitra, G., 1988. Basic Methods of Structural Geology. Prentice Hall, New Jersey, 446 pages.

McMechan, M.E., 1989. Copton Creek, Alberta. Energy, Mines and resources, Open File Map 2185.

Mountjoy, E.W., 1978. Mount Robson. Geological Survey of Canada, Map 1499A.

Price, R.A., 1981. The Cordilleran foreland thrust and fold belt in the southern Canadian Rocky Mountains; *in* Thrust and nappe tectonics, edited by K.R. McClay and N.J. Price. Geological Survey of London, Special Publication 9. pp. 427-448.

Ramsay, J.G., 1974. Development of chevron folds. Bulletin Geological Society of America, v. 85, pp. 1741-1754.

Rorres, C. and Anton, H., 1984. Applications of Linear Algebra, 3rd Ed., John Wiley and Sons, New York, 364 pages.

Watson, D.F., 1982. ACORD: Automatic Contouring of Raw Data. Computers & Geosciences, 8(1), pp. 97-101.

Wrightson, C. B., 1979. Structure and stratigraphy of Campbell Flats Anticlinorium near Grande Cache, Alberta. Unpublished University of Alberta M.Sc. thesis, 116 pages.

APPENDIX A

DATA FILE FORMATS AND CROSS-SECTION DIGITIZING

Topographic Data Files

The topographic data must be placed in an ASCII file such that each line of the file contains the x (easting), y (northing) and z (elevation) coordinates of a single point on the topographic surface. The coordinates must be space delimited and there may not be any spaces between individual lines. At present the algorithm accepts up to 32760 data points but this number could be increased if necessary by editing the source code.

Example of part of a topography data file:

```
45643.0 87492.1 1784.7
45650.6 87485.6 1783.7
45658.2 87479.2 1783.4
45665.9 87472.7 1782.3
45673.5 87466.2 1781.5
45681.1 87459.7 1781.3
45688.7 87453.3 1780.9
45696.4 87446.8 1780.3
45704.0 87440.3 1780.0
45711.6 87433.8 1779.4
45719.2 87427.4 1779.1
```

Cross-sectional Data Files

The cross-sectional data must be placed in an ASCII file such that each line of the file contains the x (easting), y (northing) and z (elevation) coordinates of a single point followed by a code labeling the horizon that the point lies on:

Example of part of a cross-section data file:

```
46301.84 87347.70 1613.00 S4
46357.33 87413.84 1636.00 S4
46408.90 87475.29 1663.00 S4
```

46417.69 87485.78 1667.00 S4
46420.14 87488.69 1668.00 S4
46423.64 87492.87 1669.00 S4
46478.36 87558.07 1661.00 S4
999 999 999 999
46135.88 87149.92 1510.00 4
46184.20 87207.52 1535.00 4
46248.36 87283.97 1557.00 4
46305.49 87352.05 1578.00 4
46359.84 87416.83 1601.00 4
46409.69 87476.24 1627.00 4

Horizon codes should identify the contacts of stratigraphic units. Data on a line must be space delimited and there may not be any spaces between individual lines. Distinct horizon traces must be separated by a line containing four space delimited repetitions of the number 999.

Digitizing Cross-sections

Cross-sections are digitized in two dimensions. These two dimensions are referred to here as the X and Y dimensions where the X dimension is the distance of a point from the origin horizontally on the cross-section, and the Y dimension is the distance of the point from the origin parallel to the sides of the cross-section. A computer procedure was written to transform the digitized points into their true three-dimensional (X, Y, Z) coordinates so that they could be used by the computer-based mapping program.

When calibrating a digitizer the software requests that up to four points be selected and their coordinates entered. A point for which the true coordinates are known should be selected as the origin of the cross-section and this point should be calibrated as (0, 0). The coordinates of the remaining calibration points must be input with respect to this selected origin.

The spacing of digitized points on a cross-section trace should fit the geology being traced. For example, digitizing a straight line requires that only its two end points be stored, whereas a curved trace needs more points to represent it accurately.

The digitizing software must be capable of storing the coordinates of the points along with a code for each labeling the horizon that it lies on. Separate traces of the same horizon must not be digitized one after the other. For example, if a horizon named A has been cut by a fault then two separate traces of this horizon exist on the cross-section. If these two traces are digitized one after the other then the conversion procedure used to transform the data will not be able to distinguish between the end of the first trace and the beginning of the second. Either of two solutions may be used to solve this problem:

1) digitize the first trace of horizon A, digitize another horizon, and then digitize the second trace of horizon A,

2) digitize the first trace of horizon A and then digitize the second trace of horizon A giving it a distinct horizon code such as A2. The conversion routine will be able to distinguish between the two traces and, once the conversion is complete, the code can be changed back to A using any ASCII text editor.

The conversion program is called DigConv and must be run under the Microsoft Windows operating environment. The procedure requests two pieces of information: 1) the orientation of the cross-section, and 2) the position of the cross-section given by the true three-dimensional coordinates of the point chosen as the origin during digitizing.

APPENDIX B

CENTER AND RADIUS OF A CIRCLE GIVEN THREE POINTS ON ITS PERIMETER

The center and radius of a circle can be obtained from three points on its perimeter. The center and radius are determined by applying these three points to the general form of the equation of a circle which is:

$$(X - h)^2 + (Y - k)^2 = r^2 \qquad (1)$$

where the center is (h, k) and the radius is r.

Given three points:

$Pa = (Xa, Ya)$
$Pb = (Xb, Yb)$
$Pc = (Xc, Yc)$

there is a unique circle whose perimeter lies on these points given by the equation (see for example Rorres and Anton, 1984, p. 3):

$$C_1 (X^2 + Y^2) + C_2X + C_3Y + C_4 = 0 \qquad (2)$$

Substituting the coordinates of the three points into equation 2 yields:

$$C_1 (Xa^2 + Ya^2) + C_2Xa + C_3Ya + C_4 = 0 \qquad (3)$$
$$C_1 (Xb^2 + Yb^2) + C_2Xb + C_3Yb + C_4 = 0 \qquad (4)$$
$$C_1 (Xc^2 + Yc^2) + C_2Xc + C_3Yc + C_4 = 0 \qquad (5)$$

Equations 2 through 5 may be used to write the equation of the circle in its determinant form:

$$\begin{vmatrix} X^2+Y^2 & X & Y & 1 \\ Xa^2+Ya^2 & Xa & Ya & 1 \\ Xb^2+Yb^2 & Xb & Yb & 1 \\ Xc^2+Yc^2 & Xc & Yc & 1 \end{vmatrix} = 0$$

Cofactor expansion of this determinant along the first row yields (Anton, 1984, p. 75):

$$C_1 = Xa\,(Yb - Yc) - Ya\,(Xb - Xc) + (XbYc - YbXc)$$

$$C_2 = -\,[\,(Xa^2 + Ya^2)\,(Yb - Yc) - Ya\,((Xb^2 + Yb^2) - (Xc^2 + Yc^2)) + ((Xb^2 + Yb^2)\,Yc - Yb\,(Xc^2 + Yc^2))\,]$$

$$C_3 = (Xa^2 + Ya^2)\,(Xb - Xc) - Xa\,((Xb^2 + Yb^2) - (Xc^2 + Yc^2)) + ((Xb^2 + Yb^2)\,Xc - Xb\,(Xc^2 + Yc^2))$$

$$C_4 = -\,[\,(Xa^2 + Ya^2)\,(XbYc - YbXc) - Xa\,((Xb^2 + Yb^2)\,Yc - Yb\,(Xc^2 + Yc^2) + Ya\,((Xb^2 + Yb^2)\,Xc - Xb\,(Xc^2 + Yc^2))\,]$$

All constants in equation 2 have been solved for at this point. Multiplying through and collecting like terms in this equation yields:

$$C_1\,X^2 + C_2\,X + C_1\,Y^2 + C_3\,Y + C_4 = 0$$

Complete the squares to get the general form of the equation of a circle (equation 1):

$$(X + C_2 / 2C_1)^2 + (Y + C_3 / 2C_1)^2 = -\,(C_4 / C_1) + (C_2 / 2C_1)^2 + (C_3 / 2C_1)^2$$

where:

$$\text{center} = (-\,C_2 / 2C_1\,,\ -\,C_3 / 2C_1),$$

$$\text{radius} = [\,-\,(C_4 / C_1) + (C_2 / 2C_1)^2 + (C_3 / 2C_1)^2\,]^{1/2}$$

APPENDIX C

POINT OF INTERSECTION OF A PLANE AND A LINE

Using the coordinates of three points on a plane and two points on a line the point of intersection of the plane and line can be calculated. If all five points are not known the point of intersection may be calculated using some additional data: 1) if only two points on the plane are known then the azimuth and inclination of a line on the plane can be used to locate a third point, 2) if only one point on the line is known then the trend and plunge of the line can be used to locate a second point. The coordinates of an additional point on the plane or line are calculated as follows:

$$X = Xk + l \times D$$
$$Y = Yk + m \times D$$
$$Z = Zk + n \times D$$

where:

X, Y, Z = the coordinates of the required additional point
Xk, Yk, Zk = the coordinates of a known point on the plane or line
l = sin(azimuth) x cos(inclination)
m = cos(azimuth) x cos(inclination)
n = -sin(inclination)
D = arbitrary distance along the line

I. Equation of a plane

The general form of the equation of a plane is:

$$AX + BY + CZ + D = 0 \quad (1)$$

Given the coordinates of three points on the plane,

Pa = (Xa, Ya, Za)
Pb = (Xb, Yb, Zb)
Pc = (Xc, Yc, Zc)

and substituting these coordinates into equation (1) yields,

$$AXa + BYa + CZa + D = 0 \quad (2)$$

$$AXb + BYb + CZb + D = 0 \quad (3)$$

$$AXc + BYc + CZc + D = 0 \quad (4)$$

Equations 1 through 4 may be used to write the equation of the plane in its determinant form (see for example Rorres and Anton 1984, p. 6):

$$\begin{vmatrix} X & Y & Z & 1 \\ Xa & Ya & Za & 1 \\ Xb & Yb & Zb & 1 \\ Xc & Yc & Zc & 1 \end{vmatrix} = 0$$

Cofactor expansion of this determinant along the first row yields,

$$A = Ya\,(Zb - Zc) + Yb\,(Zc - Za) + Yc\,(Za - Zb)$$

$$B = -\,[\, Xa\,(Zb - Zc) + Xb\,(Zc - Za) + Xc\,(Za - Zb)\,]$$

$$C = Xa\,(Yb - Yc) + Xb\,(Yc - Ya) + Xc\,(Ya - Yb)$$

$$D = -\,[\, Xa\,(YbZc - ZbYc) + Xb\,(ZaYc - YaZc) + Xc\,(YaZb - ZaYb)\,]$$

II. <u>Equations of a line:</u>

The parametric equations of a line can be determined from the coordinates of two points on that line. For example, given:

$$Pd = (Xd, Yd, Zd)$$

$$Pe = (Xe, Ye, Ze)$$

the equations of the line are (see for example Anton, 1984, p. 123):

$$X = Xd + S\,(Xe - Xd) \quad (5)$$

$$Y = Yd + S\,(Ye - Yd) \quad (6)$$

$$Z = Zd + S\,(Ze - Zd) \quad (7)$$

where: S = a scalar,

III. Point of Intersection

If equations 5, 6 and 7 are substituted for *X*, *Y*, and *Z* respectively in equation 1 the point of intersection of the plane and the line can be calculated. This substitutution yields,

$$A[Xd + S (Xe - Xd)] + B [Yd + S (Ye - Yd)] + C [Zd + S (Ze - Zd)] + D = 0$$

and solving for *S*:

$$S = - [A (Xd) - B (Yd) - C (Zd) - D] \ / \ [A (Xe - Xd) + B (Ye - Yd) + C (Ze - Zd)]$$

The X, Y and Z coordinates of the point of intersection are determined by substituting the value of *S* in equations 5, 6 and 7.

APPENDIX D

DETERMINING IF A POINT LIES WITHIN A POLYGON

The process described here can be used to determine if a point lies within a polygon that does not have re-entrants.

The polygon is made up of vectors whereby the head of one vector is at the tail of the next (Figure 14). If a perimeter vector is multiplied with a vector from its tail to the test point the direction to that vector is obtained. When two vectors are multiplied such that the second vector is clockwise to the first vector, where the angle in that direction is less than 180°, then the result will be a positive number. If the second vector is counter-clockwise to the first then the result is a negative number. If the direction to the vector which includes the test point is the same for each perimeter vector then that point is within the polygon.

For example, given the coordinates of the corners of a three-sided polygon (Figure 14):

$Pa = (Xa, Ya)$

$Pb = (Xb, Yb)$

$Pc = (Xc, Yc)$

and the coordinates of a point:

$Q = (Xq, Yq)$

the direction of the vector containing Q with respect to:

vector PaPb is:

$$(Xq - Xa)(Yb - Ya) - (Yq - Ya)(Xb - Xa) = \text{RESULT } 1$$

vector PbPc is:

$$(Xq - Xb)(Yc - Yb) - (Yq - Yb)(Xc - Xb) = \text{RESULT } 2$$

vector PcPa is:

$$(Xq - Xc)(Ya - Yc) - (Yq - Yc)(Xa - Xc) = \text{RESULT } 3$$

If the results for all three perimeter vectors are either all positive or all negative then the point lies within the polygon, otherwise the point is outside the polygon.

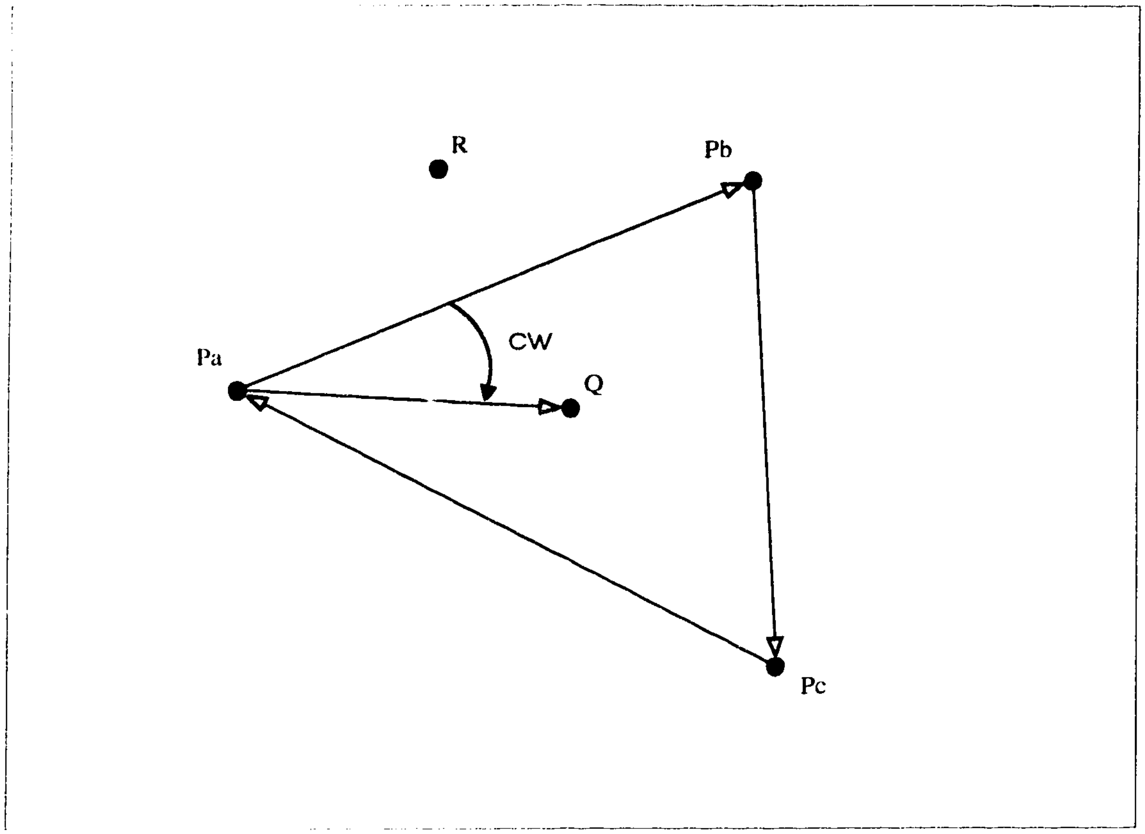


Figure 14. Determining if a point is within a polygon. If the direction of the vector from the tail of a perimeter vector to the test point is the same for each perimeter vector then the point is inside the polygon. It can be seen that point Q is clockwise to all perimeter vectors while point R is counter-clockwise to one and clockwise to the other two perimeter vectors.

APPENDIX E

DISTANCE BETWEEN TWO POINTS

The distance between two point is found using the coordinates of the points.

Given:

$Pa = (Xa, Ya, Za)$

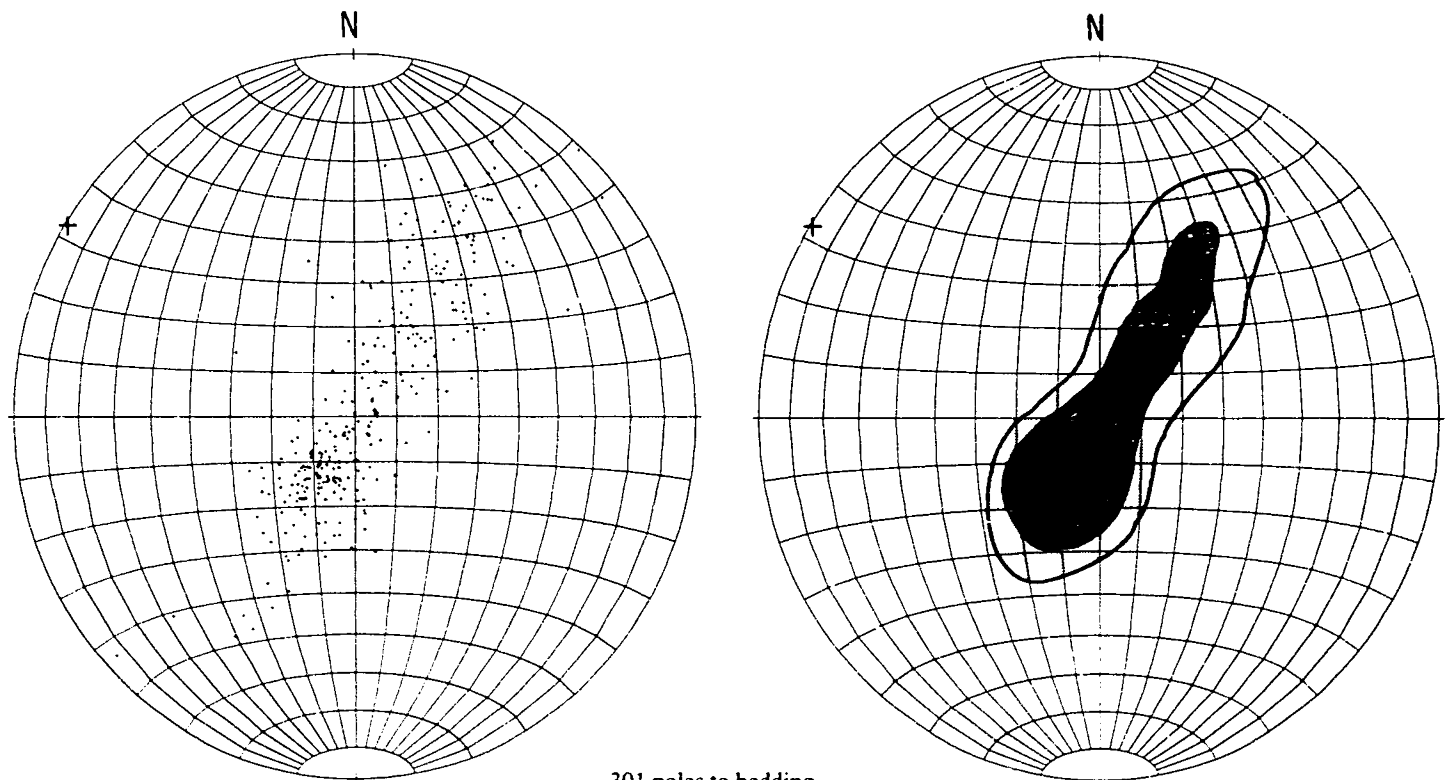
$Pb = (Xb, Yb, Zb)$

the distance between the two points is given as:

$$\text{Distance} = [\,(Xb - Xa)^2 + (Yb - Ya)^2 + (Zb - Za)^2\,]^{1/2}$$

APPENDIX F

SCATTER PLOTS AND DENSITY DIAGRAMS OF OUTCROP AND DIPMETER BEDDING ORIENTATIONS IN DOMAINS 1 - 7



301 poles to bedding
+ = Fold axis = 302° 2°

Figure 15. Equal area scatter plot and probability density diagram for bedding orientations measured at outcrops in Domain 1. Density diagram contoured at 2, 5 and 10 times a uniform density using a concentration parameter of 100.

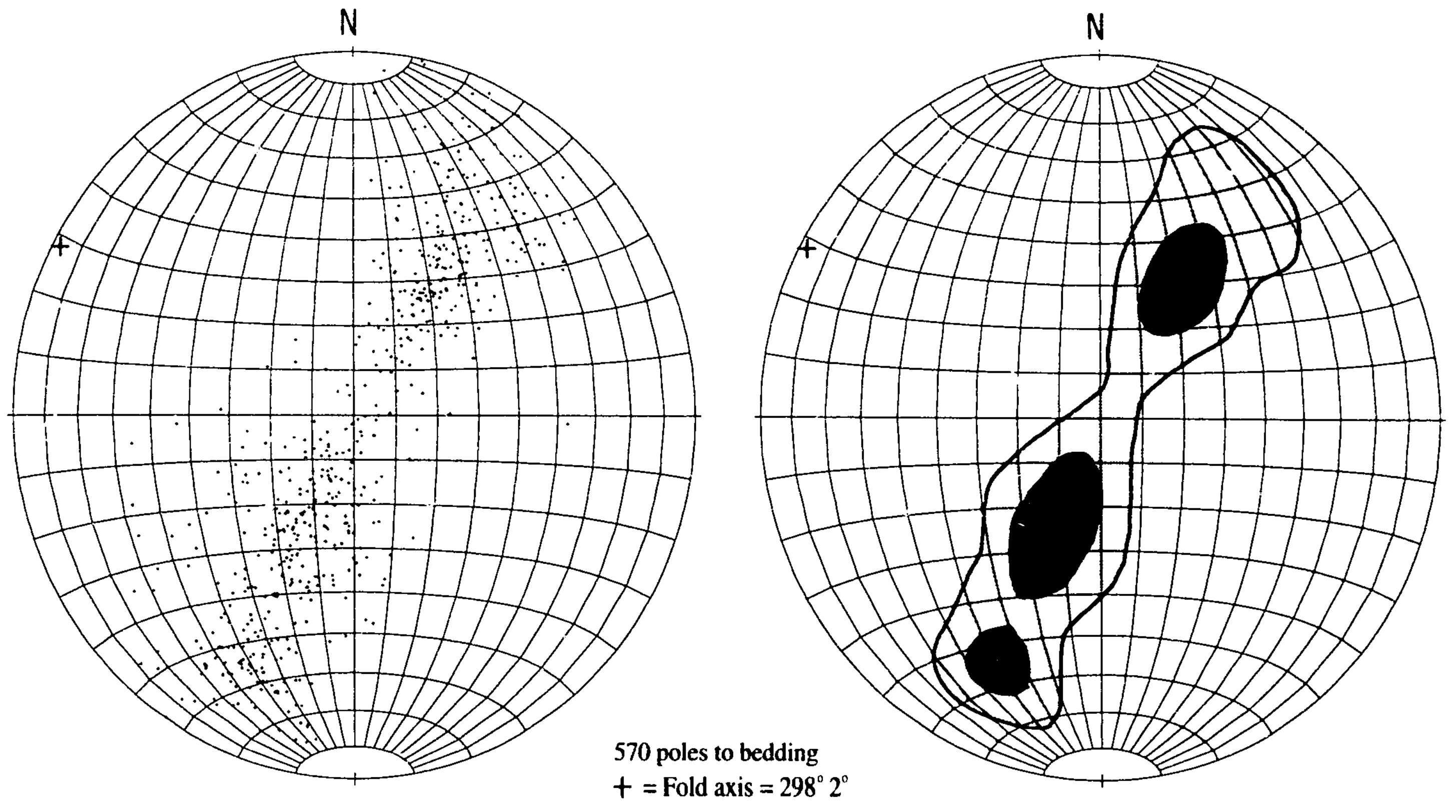


Figure 16. Equal area scatter plot and probability density diagram for bedding orientations measured at outcrops in Domain 2. Density diagram contoured at 2 and 5 times a uniform density using a concentration parameter of 80.

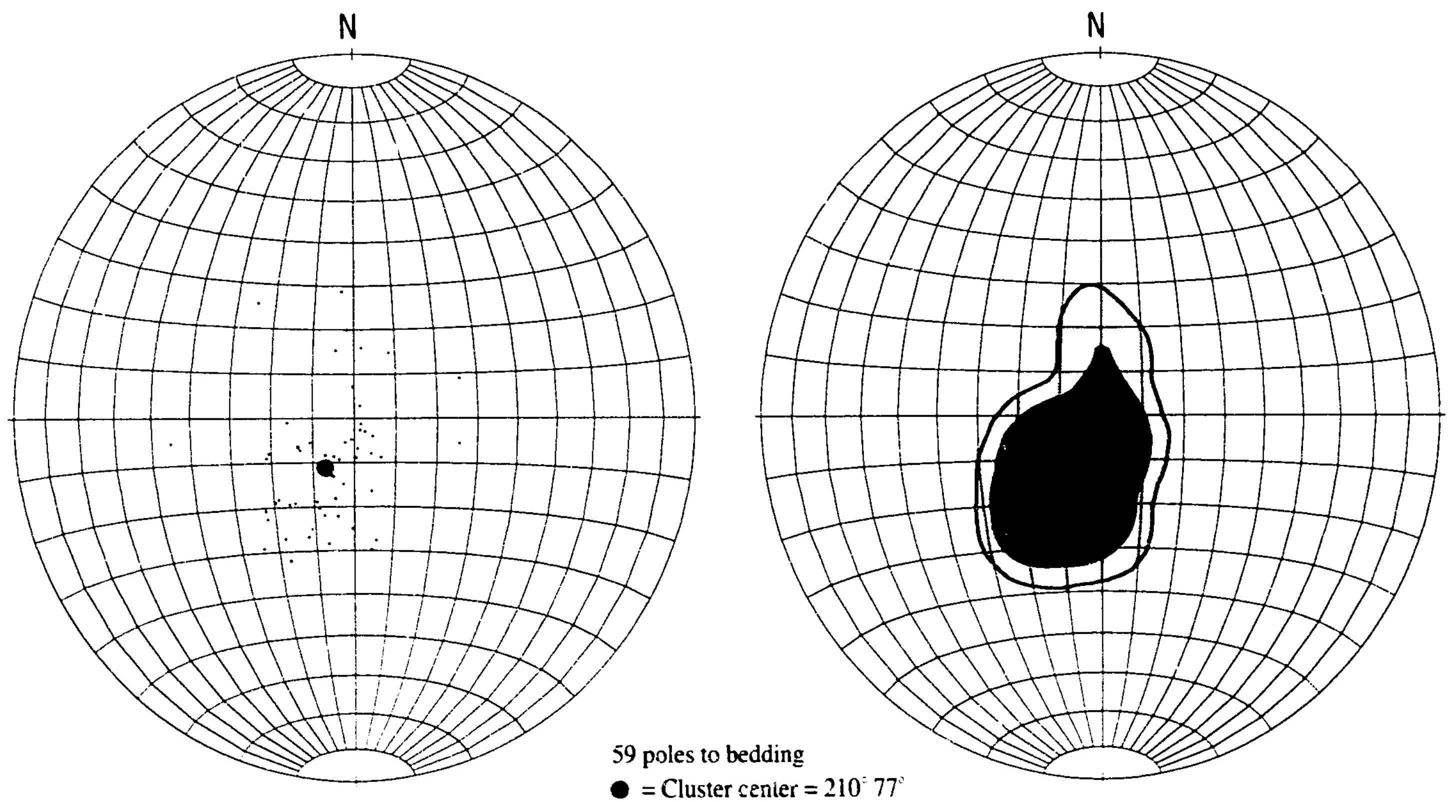


Figure 17. Equal area scatter plot and probability density diagram for dipmeter bedding orientations in Domain 2. Density diagram contoured at 2, 5 and 10 times a uniform density using a concentration parameter of 80.

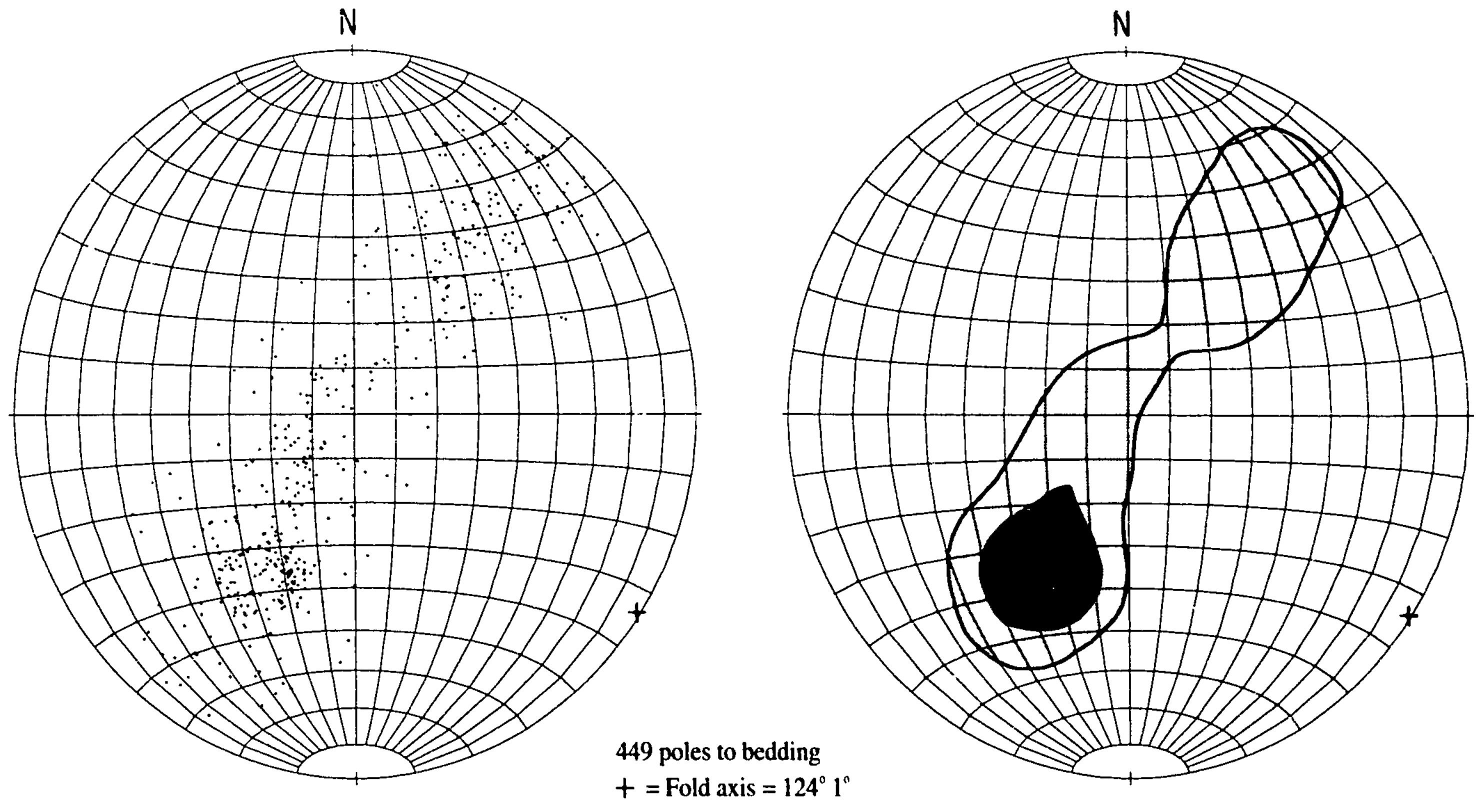


Figure 18. Equal area scatter plot and probability density diagram for bedding orientations measured at outcrops in Domain 3. Density diagram contoured at 2, 5 and 10 times a uniform density using a concentration parameter of 57.

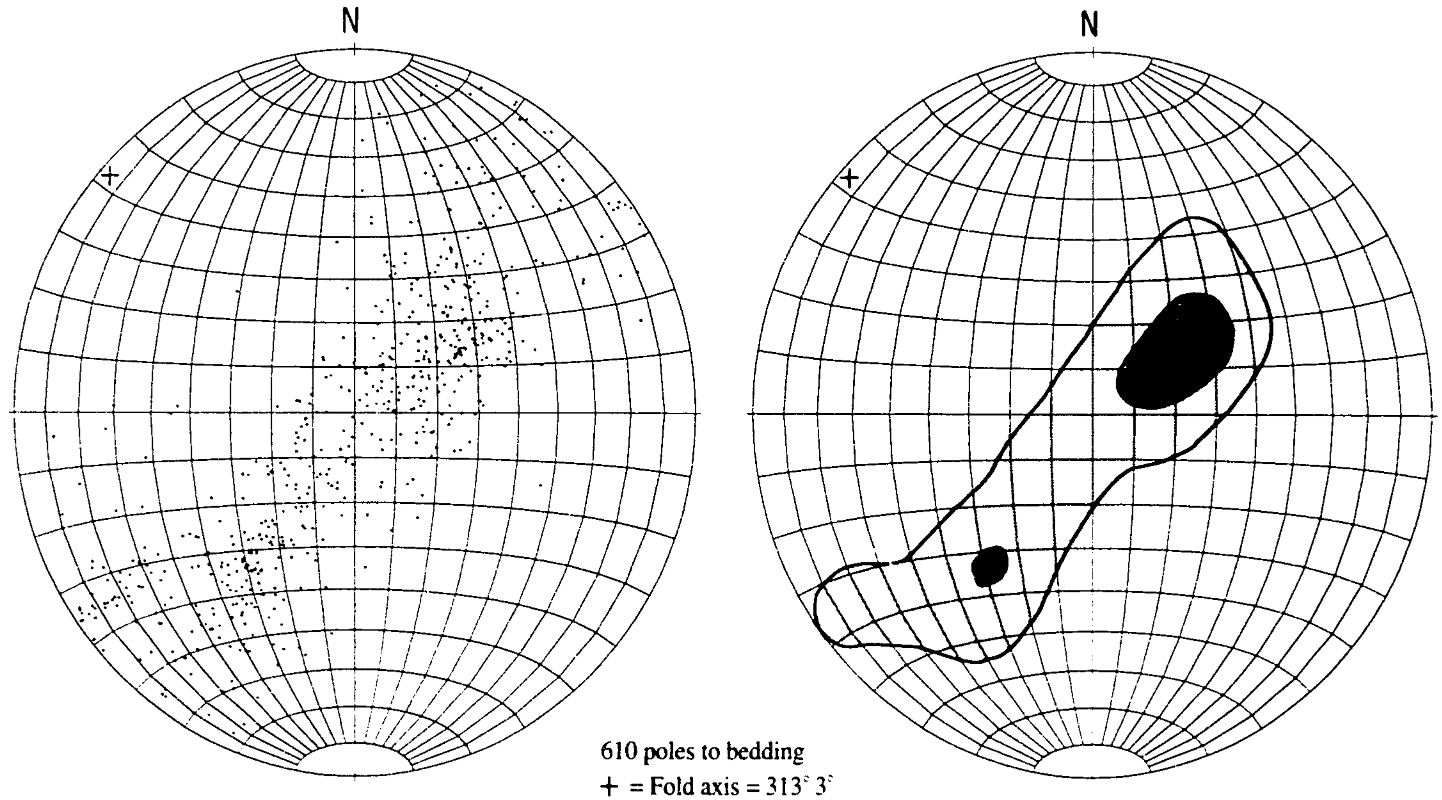


Figure 19. Equal area scatter plot and probability density diagram for bedding orientations measured at outcrops in Domain 4. Density diagram contoured at 2 and 5 times a uniform density using a concentration parameter of 56.

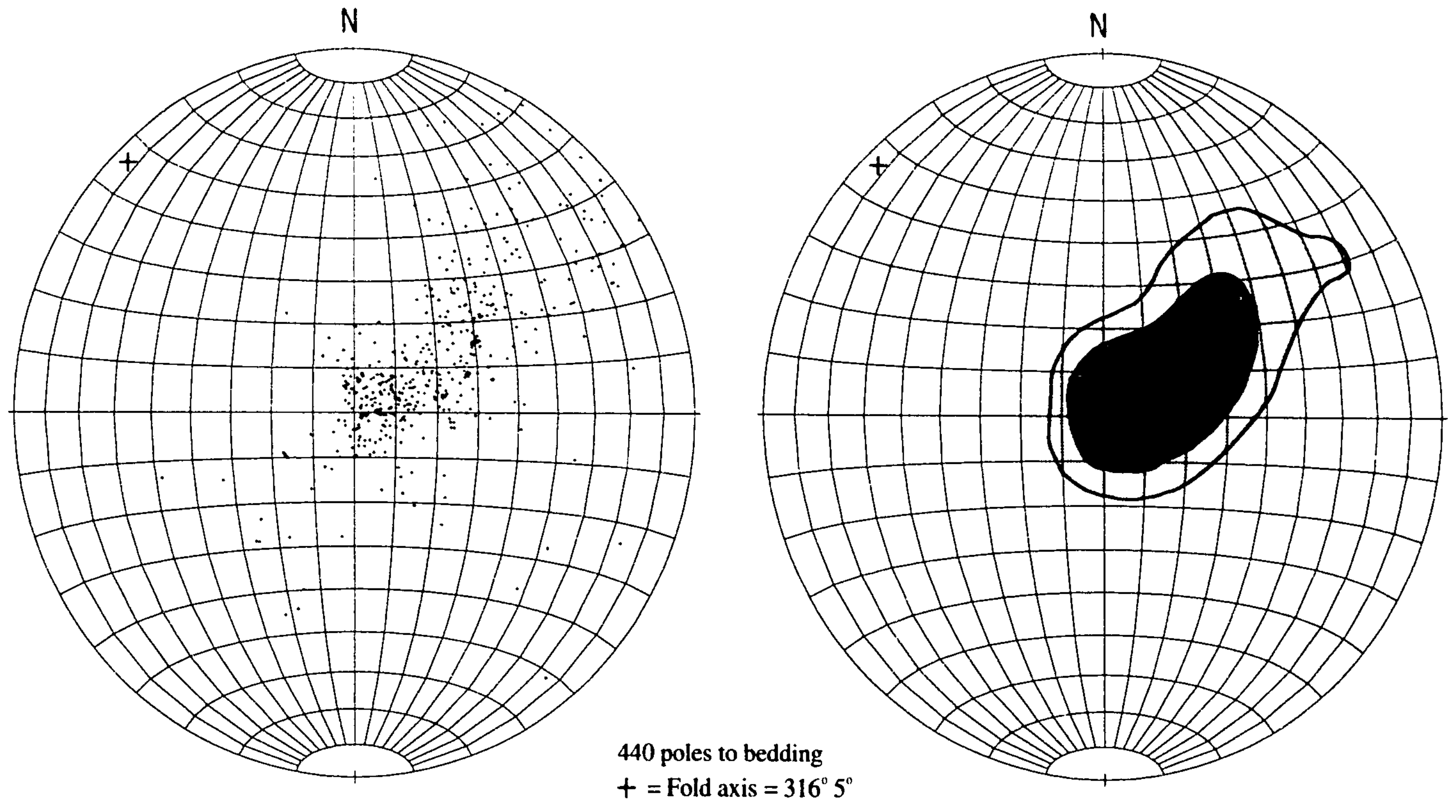


Figure 20. Equal area scatter plot and probability density diagram for dipmeter bedding orientations in Domain 4. Density diagram contoured at 2, 5 and 10 times a uniform density using a concentration parameter of 56.

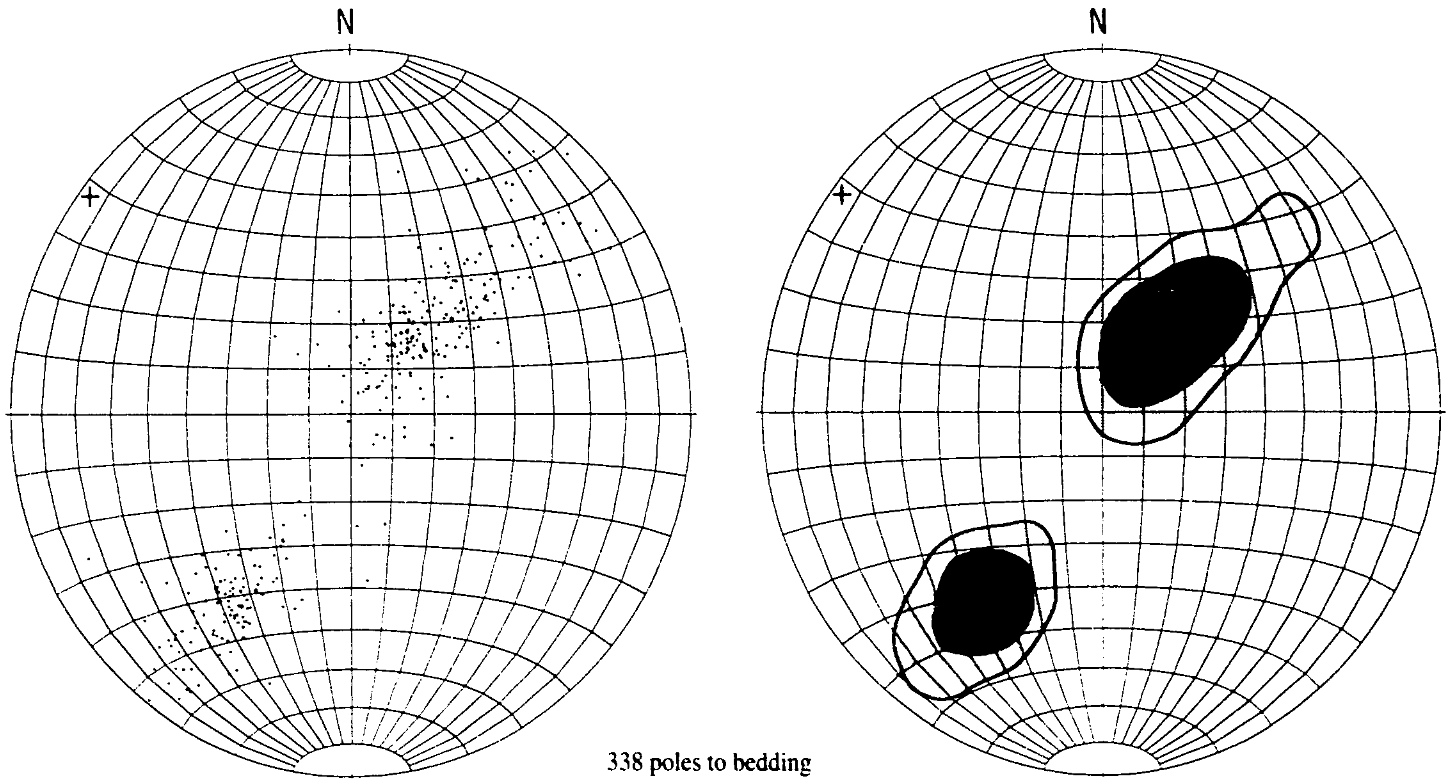


Figure 21. Equal area scatter plot and probability density diagram for bedding orientations measured at outcrops in Domain 5. Density diagram contoured at 2, 5, and 10 times a uniform density using a concentration parameter of 74.

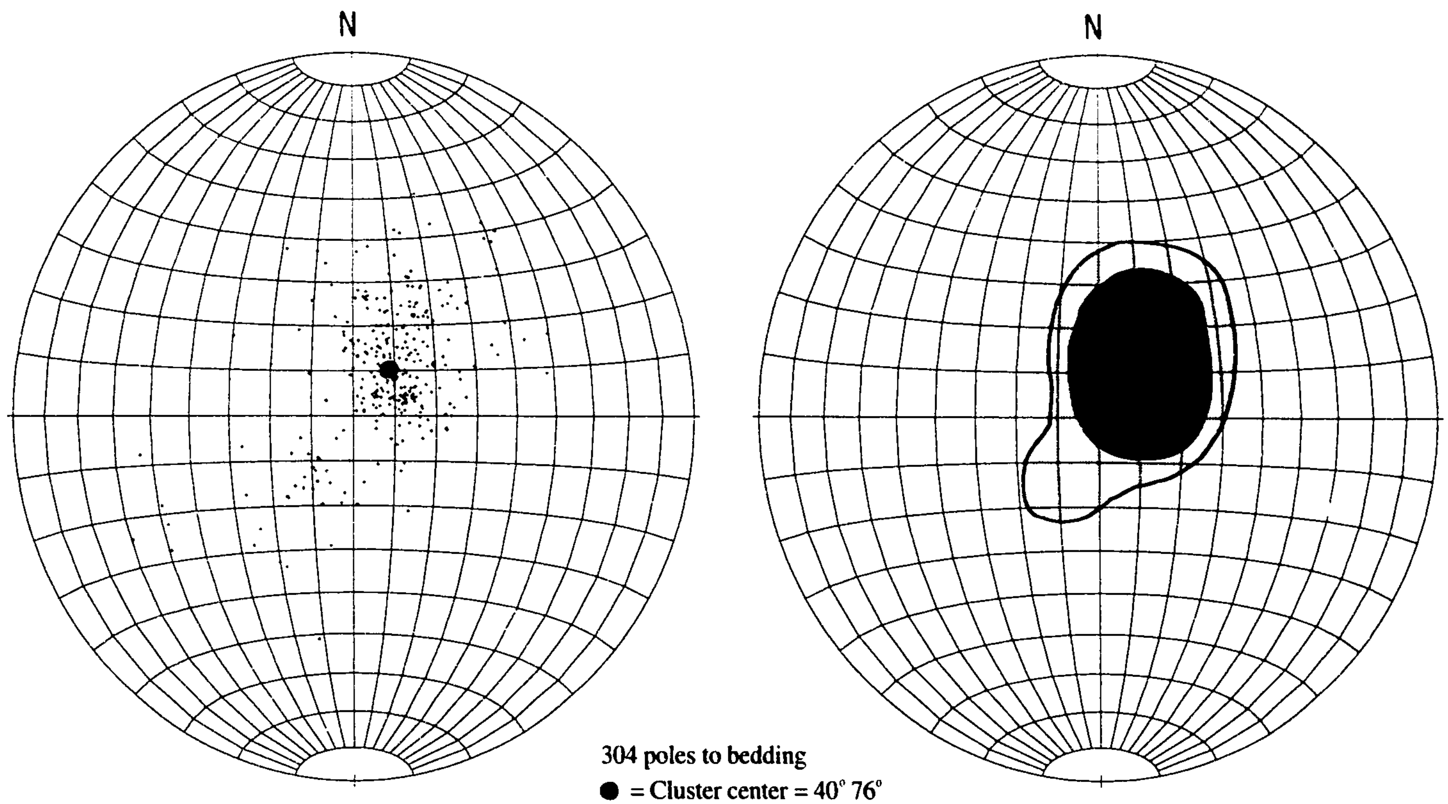


Figure 22. Equal area scatter plot and probability density diag:am for dipmeter bedding orientations in Domain 5. Density diagram contoured at 2, 5 and 10 times a uniform density using a concentration parameter of 74.

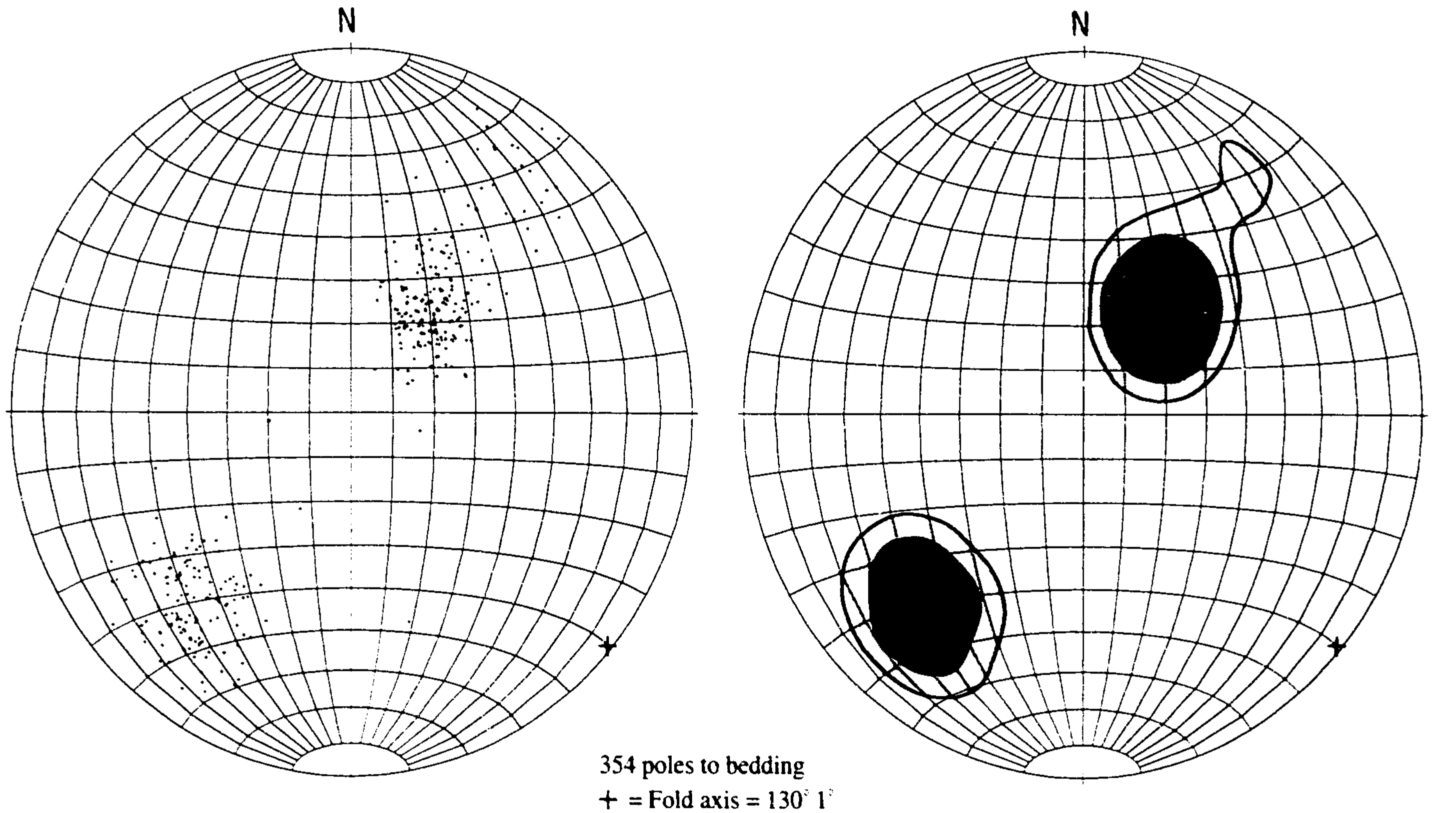


Figure 23. Equal area scatter plot and probability density diagram for bedding orientations measured at outcrops in Domain 6. Density diagram contoured at 2, 5, and 10 times a uniform density using a concentration parameter of 1 15.

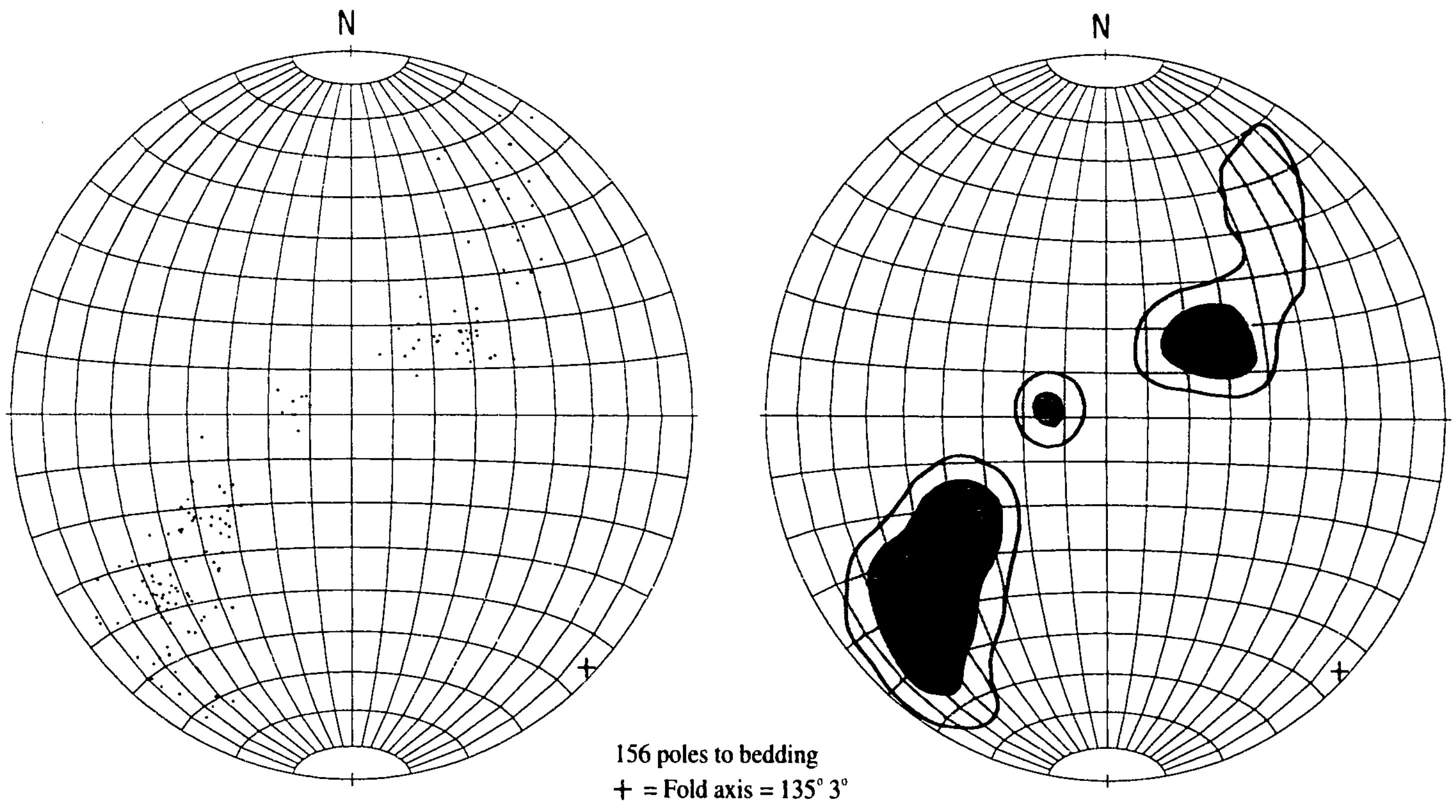


Figure 24. Equal area scatter plot and probability density diagram for bedding orientations measured at outcrops in Domain 7. Density diagram contoured at 2, 5, and 10 times a uniform density using a concentration parameter of 132.