University of Alberta

Qin Jiushao and His *Mathematical Treatise in Nine Sections* in Thirteenth-Century China

by

Ke-Xin Au Yong

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Arts
in
History

History and Classics

Fall 2011
Edmonton, Alberta

Your file Votre référence
ISBN: 978-0-494-81281-5
Our file Notre référence
ISBN: 978-0-494-81281-5

Abstract

Researching Qin Jiushao (ca. 1202 – 1261), a mathematician in Southern Song China, presents several difficulties for the historian. Qin Jiushao now suffers from a very controversial image in historiography due to the nature of our sources. This study presents a re-evaluation of existing sources and proposes that Qin's image should be rehabilitated. Qin's treatise, the *Shushu jiuzhang*, marks the beginning of the maturation of Chinese algebra, and as such, it has great mathematical and historical significance. Its influence was also apparent in Korea. Although there can be no question as to the importance of the *Shushu jiuzhang* in the history of Chinese mathematics and of mathematical development worldwide, it is difficult to explain why its influence remained almost nil for six centuries after it was completed. I will argue that one of the primary reasons was the lack of state support for mathematical education since the Southern Song dynasty.

Acknowledgement

This project could not have been completed without the guidance and support provided by my excellent professors at the Department of History and Classics who initiated me into the world of historical research. I would especially like to thank Dr. Jennifer Jay for her encouragement and for tirelessly overseeing this project from its conception to its completion. In addition, I have to thank Dr. Ryan Dunch and Dr. Jean DeBernardi for their invaluable time and advice. My thanks also go to Dr. Gregory Blue of the University of Victoria for his encouragement and many helpful suggestions, and to my sister, Ke Xian Au Yong, M.Sc., for sharing her expertise in algebra when my own knowledge proved insufficient.

I would also like to thank the Social Sciences and Humanities Research Council for its generous support of my research.

Last but not least, I need to thank my family for their patience, understanding, and acceptance of my decision to go back to being a student instead of getting a "real" job.

Table of Contents

Introduction 1

Chapter 1: Qin Jiushao: A Question of (Im)morality and Talent 8

Qin Jiushao's Family, Childhood, and Education 8

Adulthood – Pursuing Mathematics in the Midst of a Troubled Bureaucratic Career 14

The Qin Jiushao to be Remembered in History 18

Chapter 2: The *Shushu jiuzhang* and its Significance 24

A Selective Discussion of the *Shushu jiuzhang* 24

Comparison of the *Shushu jiuzhang* with the *Ten Mathematical Classics* 43

Comparison of the *Shushu jiuzhang* with the Works of the other Song-Yuan Masters 46

Qin Jiushao's Motivation for Writing the *Shushu jiuzhang* 48

Chapter 3: The Influence of Qin Jiushao and the *Shushu jiuzhang* 50

The *Shushu jiuzhang* in China – From the Thirteenth to the Nineteenth Century 51

Why Song-Yuan Algebra Had Such Limited Influence in China 56

The *Shushu jiuzhang* in East Asia 62

The *Shushu jiuzhang* in Europe 65

Qin Jiushao and the *Shushu jiuzhang* Today 65

Conclusion 67

Bibliography of Works Consulted 71

Appendix A: Qin Jiushao's Preface to the *Shushu jiuzhang*, with Translation into English 79

Appendix B: Zhou Mi's Entry about Qin Jiushao in the *Guixin zazhi xuji* (Miscellaneous Notes From the Guixin Quarter, First Addendum), with Translation into English 84

List of Figures

Figure 1 – Biographical map of Qin Jiushao 9

Figure 2 – Example of container used to collect precipitation 32

Figure 3 – Original illustration from the *Shushu jiuzhang* for Problem 5 of Section 4 35

Introduction

The pinnacle of mathematics in pre-modern China is embodied in a series of treatises written by four men: Qin Jiushao 秦九韶 (ca. 1202 – 1261), Li Ye[1] 李冶 (ca. 1192 – 1265), Yang Hui 楊輝 (fl. 1261 – 1275), and Zhu Shijie 朱世傑 (fl. 1280 – 1303). These treatises remained unmatched and unsurpassed in their level of sophistication until modern mathematics from the West began to flourish in China during the Qing 清 dynasty (1644 – 1911). They were all completed in the thirteenth and early fourteenth centuries during the Southern Song 南宋 (1127 – 1279) and Yuan 元 (1279 – 1368) dynasties, marking this period as the golden age of Chinese mathematics. Therefore, historians today refer to Qin, Li, Yang, and Zhu collectively as the four Song-Yuan masters.

The focus of this study is Qin Jiushao and his treatise, the *Shushu jiuzhang* 數書九章 (Mathematical Treatise in Nine Sections) which was completed in 1247. This treatise is the only piece of Qin's writings that remains today, and we have no evidence that he ever wrote another mathematical treatise. The *Shushu jiuzhang* enjoys the eminent position in Chinese history as the mathematical text that contains the earliest use of the zero symbol in China, the first systematic description of what is known today as the Chinese Remainder Theorem, and the first occurrence of solutions to algebraic equations which are of degrees higher than the third. It predates the treatises written by the other three Song-Yuan masters, so it could be said that Qin Jiushao was the first to usher in the golden age of Chinese mathematics and the one who set the

[1] Some historical texts refer to Li Ye as Li Zhi 李治. It is believed that Li Zhi was his original name, but he changed it because it violated the name taboo for the Tang emperor, Tang Gaozong 唐高宗 (r. 649 – 683), with whom he shared the same name.

standards by which historians henceforward measure sophistication in Chinese algebra. In this view, Qin's work really is the milestone that marks the beginning of the maturation of Chinese algebra.

Qin Jiushao's outstanding contribution to Chinese mathematics has been highlighted in all the general histories written about the development of Chinese mathematics published since the early twentieth century. Among the most notable of these general histories are the ones written by Mikami Yoshio (1875 – 1950), Li Yan 李儼 (1892 – 1963), Joseph Needham (1900 – 1995), and Jean-Claude Martzloff. In addition to giving accounts of Qin Jiushao and his work, these scholars have also raised several points that will be explored further in this study.

In his 1913 monograph, *The Development of Mathematics in China and Japan*, Japanese mathematician Mikami pointed out that Qin Jiushao's treatise includes all the arithmetic rules that had been laid out in two earlier mathematical classics: the *Jiuzhang suanshu* 九章算術 (Nine Chapters on the Mathematical Art)[2] from the Han 漢 dynasty (206 BCE – 220 CE) and the *Haidao suanjing* 海島算經 (Sea-Island Mathematical Classic) from the Three Kingdoms period (220 – 280).[3] However, it would be incorrect to assume that Qin Jiushao had merely made use of previous scholarship to compose his own treatise without contributing anything new to the field of mathematics. This study will show that, even when Qin Jiushao did not invent any new mathematical method, his innovative use of existing methods did, in fact, create new possibilities for later mathematicians.

---

[2] To avoid confusion between this treatise and Qin Jiushao's *Shushu jiuzhang*, the *Jiuzhang suanshu* will henceforward be referred to as the *Nine Chapters*.

[3] Mikami Yoshio, *The Development of Mathematics in China and Japan* (New York: Chelsea Publishing Company, 1913), 65.

In volume 3 of *Science and Civilization in China*, Needham observed that it was odd that we have not been able to find any connection between Qin Jiushao and the other three Song-Yuan masters.[4] There is no evidence that these four mathematicians knew about the work of one another. Needham believed this was due to the fact that there remains a great deal of Chinese mathematical literature yet to be uncovered.[5] However, the lack of connection between Qin Jiushao and his contemporary mathematicians also makes one wonder just how widely circulated their treatises actually were. We do know that China's publishing industry was already very well developed and widely available during the Southern Song dynasty. Yet, we have not been able to find any evidence that Qin Jiushao ever printed his treatise. Why Qin would write a mathematical treatise and never seek to have it printed is a fascinating historical question that will be investigated in my study.

When examining the life and work of Qin Jiushao, one natural question is to ask the extent of his influence on later mathematicians. So far, we have not been able to trace any direct inheritance of his methods by the mathematicians after him. The *Shushu jiuzhang* had passed into near-total obscurity by the fourteenth century. The influence of the other three Song-Yuan masters was also extremely limited in China, to the point where the mathematicians of the Ming 明 dynasty (1368 – 1644) could no longer understand the methods of Qin Jiushao and his contemporaries.[6] In other words, Chinese mathematics experienced a break in development immediately after the golden age of the

---

[4] Joseph Needham, *Mathematics and the Sciences of the Heavens and the Earth*, vol. 3 of *Science and Civilization in China* (London: Cambridge University Press, 1959), 41.

[5] Ibid, 42.

[6] Jean-Claude Martzloff, *A History of Chinese Mathematics*, trans. Stephen S. Wilson (Berlin: Springer-Verlag Berlin Heidelberg, 1997), 20.

Song-Yuan masters and went into decline. The sophisticated algebra of the four Song-Yuan masters became forgotten and was only rediscovered during the Qing dynasty. Both historians Li Yan and Jean-Claude Martzloff have presented possible reasons for this break and decline, but their opinions seem to be directly opposite to each other. Li Yan attributed this sudden decline to the impracticality of Song-Yuan mathematics.[7] In other words, he believed that the type of mathematics that Qin Jiushao and his contemporaries did, though brilliant, did not fulfil the practical demands of the common people's daily lives, so it did not become popularized or lead to even greater achievements. On the other hand, Martzloff observed that treatises like the *Shushu jiuzhang* were actually "too oriented towards real problems," thus making the problems too specific to achieve wider application or influence.[8] Here we seem to have a contradiction: how could Song-Yuan mathematics be practical and impractical at the same time? This contradiction will be resolved in my study. I will argue that there is, in fact, no contradiction at all. Both Li's and Martzloff's analyses are valid, but they were looking at the same question from two different angles. Li Yan formulated his conclusion based on the methods used by the Song-Yuan masters, while Martzloff was looking at the nature of the problems they chose to solve. I will present a synthesis of these two opinions to explain why Qin Jiushao had such limited influence in the Yuan-Ming period and why mathematics went into decline after the four masters. However, this is only half of the answer. I believe a crucial part of the answer has been neglected in previous scholarship. In my study, I will also argue that the lack of state support in promoting mathematical education was a major factor in causing the decline of Chinese mathematics.

---

[7] Li Yan, *Zhongguo gudai shuxue jianshi* (Simplified History of Ancient Chinese Mathematics), 2nd ed. (Taipei: Jiuzhang chubanshe, 1981), 205.

[8] Martzloff, 55.

Besides the general histories mentioned above, there are also studies that concentrate specifically on Qin Jiushao and his work. The only major one in English is Ulrich Libbrecht's *Chinese Mathematics in the Thirteenth Century: the Shu-shu chiu-chang of Ch'in Chiu-shao*. In it, Libbrecht described the traditional role of mathematics in China as being the servant of astronomy.[9] In other words, Libbrecht believed that, traditionally, the Chinese valued mathematics not for its own sake, but because it allowed them to make advancements in astronomical observations and calendrical science. However, I will demonstrate in my study that Qin Jiushao's pursuit of mathematics was not to serve the purposes of astronomy. Instead, the Chinese Remainder Theorem that Qin presented in his treatise is actually a classic case of using astronomy to serve the purposes of mathematics.

In recent decades, Chinese historians of mathematics have been very active in researching Qin Jiushao and his work. Xu Pinfang 徐品方 and Kong Guoping 孔國平 have recently published a joint monograph exploring Qin Jiushao's biographical information and historical image. One major study on the *Shushu jiuzhang* has been done collectively by Chen Xinchuan 陳信傳, Zhang Wencai 張文材, and Zhou Guanwen 周冠文. In their work, they provide a complete translation of the *Shushu jiuzhang* into modern vernacular Chinese and mathematical symbols. The version of the *Shushu jiuzhang* that they used is the 1842 edition produced by Qing mathematician Song Jingchang 宋景昌 (fl. ca. 1842). My study is based on the version reproduced in Chen, Zhang, and Zhou's monograph.

---

[9] Ulrich Libbrecht, *Chinese Mathematics in the Thirteenth Century: the Shu-shu chiu-chang of Ch'in Chiu-shao* (Cambridge, Massachusetts: MIT Press, 1973), 4.

My thesis holds that Qin Jiushao and his work have special significance for the fields of history, mathematics, and education and that they provide important clues to help explain why Chinese mathematics went into decline after the early fourteenth century. I proceed to demonstrate this by addressing the subject of Qin Jiushao and his work on three levels: what we can uncover about them, what they reveal about their times, and how they fit into the global context. The first chapter will focus on the life and times of Qin Jiushao. He is a very controversial figure in history because the nature of our available sources makes it difficult to come to many definite conclusions about his life and his character. I will attempt to resolve the controversy while constructing a sketch of his life and the environment in which he lived.

Chapter two will address the technical part of this study. The contents of the *Shushu jiuzhang* will be summarized and compared with several of the other major mathematical works of pre-modern China. It will be compared with the *Ten Mathematical Classics* (*Suanjing shishu* 算經十書) in terms of both scope and level of difficulty. The *Ten Mathematical Classics* were the state-prescribed mathematical textbooks used in the imperial academy during the Tang 唐 (618 – 906) and Northern Song 北宋 (960 – 1127) dynasties. Even though the Southern Song government permanently removed mathematics from the imperial academy's curriculum, it is known that these textbooks continued to survive and be used by private tutors during Qin Jiushao's time. In other words, the *Ten Mathematical Classics* were still the foundation of mathematical education during the thirteenth century. By comparing the *Shushu jiuzhang* with the *Ten Mathematical Classics*, we will be better able to envision just how much Qin Jiushao enriched the mathematics of his time. The *Shushu jiuzhang* will also be compared with the works of the other three Song-Yuan masters. This will enable us to assess more accurately what methods in the *Shushu jiuzhang* are uniquely the creation of Qin rather than what thirteenth-century mathematicians knew in general. In addition, a comparison of the topics covered by all four of these mathematicians will allow us to draw conclusions about the type of mathematics prevalent in the thirteenth century and disprove Libbrecht's remark that mathematics was the servant of astronomy. This chapter will also explore the significance of Qin Jiushao's work in the history of mathematics worldwide and end with a discussion of the possible motivations for Qin to write the *Shushu jiuzhang*.

Finally, the last chapter will explore the extent of Qin Jiushao's influence on later mathematicians. The practical impact of his treatise will be examined by detailing the extent of its circulation across East Asia. Even though Chinese mathematics went into decline after the Song-Yuan masters and though Qin had minuscule impact on the Yuan-Ming mathematicians, the sophisticated algebra of the Song-Yuan masters was revived during the nineteenth century and has remained an active field of research ever since. Therefore, it was only after a delay of almost six hundred years that Qin Jiushao's contribution to mathematics finally became acknowledged and that his influence finally made itself felt.

Chapter 1: Qin Jiushao: A Question of (Im)morality and Talent

A complete biography of Qin Jiushao is difficult to construct due to the lack of historical sources. The limited amount of information we can gather come mainly from local gazetteers, Qin's own preface to his treatise, and the writings of his contemporaries, Liu Kezhuang 劉克莊 (1187 – 1269) and Zhou Mi 周密 (1232 – 1298), both of whom were officials and well-known literary figures. From these sources, we know that Qin Jiushao was a minor official who lived during the later half of the Southern Song dynasty. However, not only is the information provided by these sources extremely sketchy, but historians have also begun to question the reliability of Liu Kezhuang's and Zhou Mi's accounts, which paint Qin Jiushao's character in a very unfavourable light. The result is a controversy over Qin's historical image. In this chapter, I will examine our current state of knowledge about Qin Jiushao and attempt to present an objective and fair evaluation of how he should be remembered in history.

Qin Jiushao's Family, Childhood, and Education

We have no official records of Qin Jiushao's birth and death years, but historians generally believe he was born around 1202 and died around 1261.[10] Even though Qin Jiushao himself named Lujun 魯郡 of Shandong 山東 province as his hometown, he was most likely born in Sichuan 四川. This conclusion is based on our knowledge that his father was from Anyue 安岳 in Sichuan and had

[10] In their 2007 book, Chinese historians Xu Pinfang and Kong Guoping wrote that evidence had been found which suggested that Qin Jiushao was born in 1208. However, that evidence had not yet been verified at the time of publication of their monograph. See Xu Pinfang and Kong Guoping, *Zhongshiji shuxue taidou – Qin Jiushao* (Beijing: Kexue chubanshe, 2007), 1.

spent most of his bureaucratic career in that province.[11] This means that the Qin family was under the jurisdiction of the Southern Song, but the province of Shandong was already in the hands of the Jurchen Jin 金 dynasty (1115 – 1234) by 1127. Therefore, Lujun must have been the Qin family's ancestral home rather than where Qin Jiushao was actually born.[12]

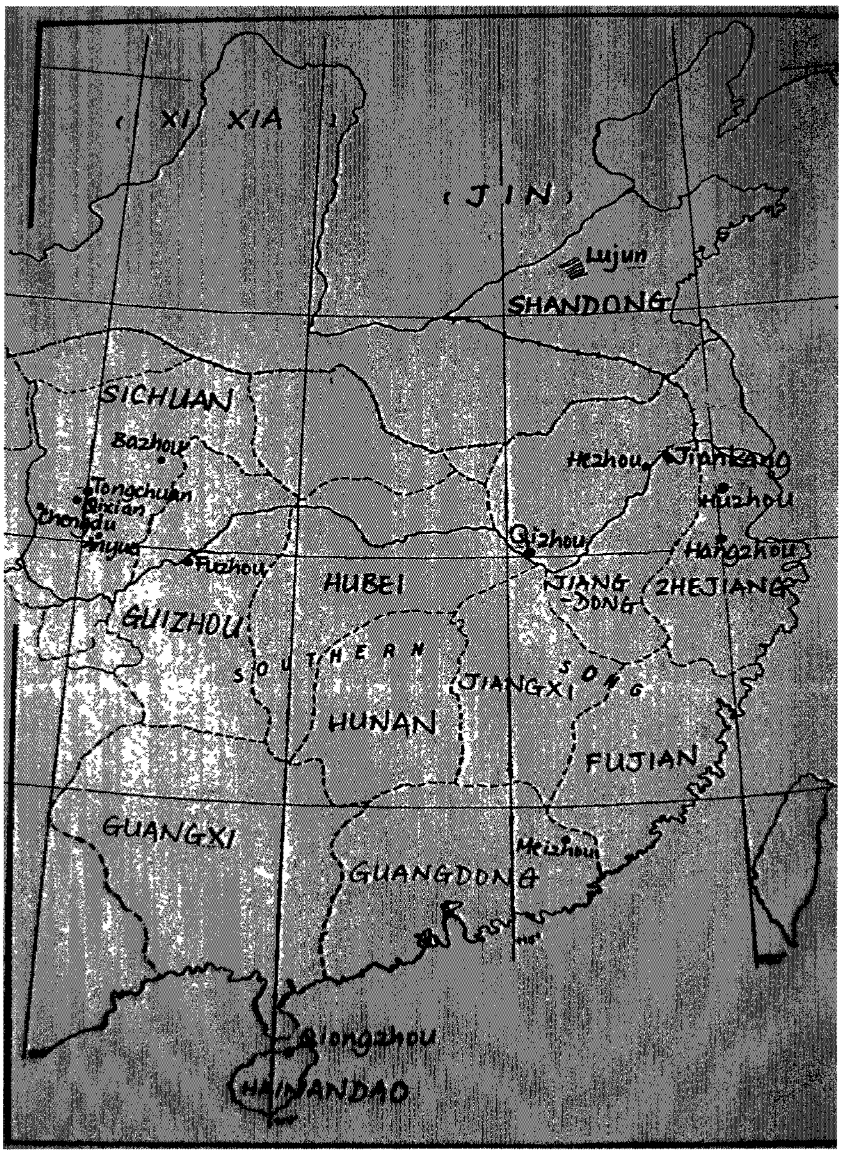

Figure 1 – Biographical map of Qin Jiushao (based on Libbrecht, 23)

---

[11] Libbrecht, 24-25.
[12] Ibid, 24.

From an inscription in Fuzhou 涪州,[13] we learn that Qin Jiushao's courtesy name is Daogu 道古, and his father's name is Qin Jiyou 秦季槱 with the courtesy name of Hongfu 宏父.[14] We have no information about other members of the Qin family, but we learn from Zhou Mi that Qin Jiushao also had an older brother, and that each had at least one son.[15] According to research done by Chinese scholars, Qin Jiushao's brother's name was Qin Jiuliang 秦九良.[16]

Qin Jiushao was from a literati family. According to official records, Qin Jiushao's father, Qin Jiyou, attained his *jinshi* 進士 degree in 1193.[17] He then served in various positions within the Southern Song government. He was the prefect of Bazhou 巴州[18] of Sichuan in 1219 when a military rebellion broke out in the area. He subsequently left Bazhou with his family, but it is not certain where they went. It is likely that they moved directly to the Hangzhou 杭州 area at this time.[19] In 1224, Qin Jiyou became an assistant librarian (*bishu shaojian* 秘書少監) in Hangzhou, the Southern Song capital. However, the very next year, he became appointed as the prefect of Tongchuan 潼川[20] and had to return again to Sichuan, where it is believed he passed away within a few years.[21] From the above-mentioned inscription in Fuzhou which is dated 1226, it is apparent that Qin Jiushao accompanied his father back to Sichuan at this time to take up the

[13] Present-day Fuling 涪陵 in the province of Sichuan

[14] Libbrecht, 22.

[15] Zhou Mi, *Guixin zazhi xuji* (Miscellaneous Notes from the Guixin Quarter, First Addendum) (Beijing: Zhonghua shuju, 1991), 325.

[16] Xu and Kong, 5.

[17] Libbrecht, 24. The *jinshi* degree was awarded to men who had passed the highest level of the civil service examinations.

[18] Present-day Bazhong 巴中 in Sichuan

[19] Han Xianglin, "Shi xi Qin Jiushao yi jia yu ju Huzhou de yuanyin" (Why did the Family of Qin Jiushao Live in Huzhou?), *Journal of Huzhou Teachers' College* 23, no. 2 (April 2001): 96-97.

[20] Present-day Santai 三臺 in Sichuan

[21] Libbrecht, 24; Xu and Kong, 95.

appointment in Tongchuan. The inscription was made to commemorate their joint excursion to the Stone Fish, which was a popular sightseeing attraction.[22] The conspicuous absence of Qin Jiushao's brother's name in the inscription has led historians to speculate that Qin Jiyou might have favoured his younger son over the older since he only took Jiushao with him to the Stone Fish.[23] Qin Jiushao would have been around twenty-four years old at the time of the excursion.

Being the son of an official, Qin Jiushao's early life was certainly filled with opportunities that not everyone could enjoy. He was able to travel around Sichuan and the Hangzhou area as he accompanied his father to take up various appointments. While the family was living in Hangzhou, Qin Jiushao made use of this opportunity to enrich himself with an education on more technical subjects. Qin Jiushao's writings do not indicate the names of his teachers, so it is unclear who they were. He first studied astronomy under a *taishi* 太史, which was the title of an official of the imperial Board of Astronomy (*taishiju* 太史局).[24] This might well have been made possible by his father's official connections. He then learnt mathematics from a scholar who was living as a recluse.[25] By Qin Jiushao's time, the only means to learn mathematics was through private education because it was not taught in the schools instituted by the government during the Southern Song dynasty.

---

[22] Libbrecht, 22.

[23] Xu and Kong, 103.

[24] Chen Xinchuan, Zhang Wencai, and Zhou Guanwen. *Shushu jiuzhang jinyi ji yanjiu* (Studies of the Mathematical Treatise in Nine Sections and its Translation into Modern Vernacular) (Guiyang, Guizhou: Guizhou jiaoyu chubanshe, 1993), 6. The imperial Board of Astronomy was responsible for observing and predicting celestial phenomena. One of its most important tasks was to maintain an accurate calendar.

[25] Ibid, 6.

This neglect of mathematics by the government had not always been the case. Mathematics had been part of the curriculum of the imperial academy (*guozijian* 國子監) since the Sui 隋 dynasty (581 – 618). This continued during the Tang dynasty even though mathematics was cancelled from the curriculum at times. Still, the Tang government made a very important contribution to mathematical education by standardizing for the first time the textbooks that were to be used throughout the country. Mathematics even became one of the civil service examination subjects, and successful candidates started their careers as the lowest-ranked officials.[26]

The Northern Song dynasty continued the Tang example: mathematics was included as part of the curriculum of the imperial academy except for the times when it was cancelled. However, the Northern Song government had a more far-reaching impact on public education and mathematical development than the Tang. In the period immediately following the founding of the Song, the government began converting private academies into public ones in order to open up educational opportunities. In 1043, it took a further step by forming state schools (*zhouxue* 州學) in all the prefectures. Wherever the number of students exceeded two hundred, county schools (*xianxue* 縣學) were formed in addition to the prefectural schools. These schools were very well-supported by the government. An indication of this and of the Song government's financial prowess is the state endowment of one hundred and fifty acres of land to each prefectural school in 1071.[27]

---

[26] Li Yan and Du Shiran, *Chinese Mathematics: A Concise History*, trans. John N. Crossley and Anthony W.C. Lun (Oxford: Clarendon Press, 1987), 104-106.

[27] Richard L. Davis, "Custodians of Education and Endowment at the State Schools of Southern Sung," *Journal of Song-Yuan Studies* 25 (1995): 97-98.

In 1084, the government gave a boost to mathematical education and development by printing the *Ten Mathematical Classics* and distributing them to the prefectural and county schools for use as textbooks.[28] This helped ensure the widespread proliferation of these basic mathematical textbooks that laid the foundation for the advanced mathematics done by the four Song-Yuan masters. However, already during the Northern Song dynasty, there was a strong sentiment at court that mathematics was a superfluous subject that did not help prepare students for an official career.[29] Concurrently, the civil service examinations became geared more and more towards the memorization and interpretation of Confucian texts.[30] Since the principal purpose of the public education system was to train students for taking the civil service examinations, mathematics became an unnecessary subject. After the Northern Song fell to the Jurchens, mathematics was completely dropped from all state-implemented education until the Qing dynasty. In the meantime, no Chinese government funded another large-scale printing of mathematical texts.

Unable to learn mathematics through public schools, Qin Jiushao had to turn to a private tutor. The unstable political climate and economic prosperity during both the Northern and Southern Song dynasties contributed to the flourishing of private education.[31] The frequent military conflicts with other states and the rivalry between the war and peace factions at court might have

---

[28] Li and Du, 109.

[29] Ibid, 110.

[30] Qiao Weiping, *Zhongguo Song Liao Jin Xia jiaoyu shi* (History of Education during the Song, Liao, Jin, and Xia Dynasties), in *Baijuanben Zhongguo quanshi* (Complete History of China), ed. Shi Zhongwen and Hu Xiaolin (Beijing: Renmin chubanshe, 1994), 10-12.

[31] Ren Shixian, *Zhongguo jiaoyu sixiang shi* (History of Chinese Educational Thought) (Shanghai: Shanghai shudian, 1936), 166; Qiao, 2.

discouraged some scholars from pursuing a bureaucratic career.[32] Teaching then became an alternative as the economic prosperity allowed more people to hire private tutors. Many scholars who wanted to join the bureaucracy but were not recruited might also have turned to teaching as the means to support themselves. The demand for and the availability of private tutors ensured a strong private education system which provided Qin Jiushao the means to learn mathematics.

Adulthood – Pursuing Mathematics in the Midst of a Troubled Bureaucratic Career

As an adult, Qin Jiushao had the reputation of being a multi-talented individual. He was said to have been brilliant in everything he did, including astronomy, music, mathematics, architecture, polo, archery, and swordplay.[33] He followed his father's example and entered into a life-long career as a government official. We have no records of him first taking the civil service examinations before joining the bureaucratic ranks. However, being the son of a prefect, it is likely that he did. His actual performance on the examinations probably had little impact on his career because it seems that he got most of his positions through his or his father's connections. In 1229, Qin Jiushao became a *xianwei* 縣尉, which was a ninth-ranked military position, in Qixian 郪縣 of Sichuan.[34] In other words, his career as an official began when he was around twenty-seven years old. Since this occurred in his home province where his father was still serving as a prefect, it is likely that his father had something to do with his appointment. It seems that Qin Jiushao served in this position for several years. A note

---

[32] Ren, 166.
[33] Zhou, 325.
[34] Xu and Kong, 85.

addressed to "Sheriff Qin Jiushao" and dated 1233 has been found among the writings of Li Liu 李劉 (literary name Meiting 梅亭, fl. 1230's), who was a fiscal intendant in Chengdu 成都, the provincial capital of Sichuan.[35] It must have been around this time that Qin learnt parallel prose (*pianliwen* 駢儷文) from Li.[36] Qin would later demonstrate his proficiency in this style of writing by utilizing it in the last part of his preface to his mathematical treatise.

In 1234, the situation between Southern Song China and her northern neighbours underwent a decisive change: the Mongols successfully conquered the Jurchen Jin empire and became the Southern Song's new enemy to the north. In 1235, the Mongols began invading Sichuan, where Qin Jiushao was still serving as a military official. In his preface to the *Shushu jiuzhang*, Qin wrote that he spent ten years at the frontier amid the flying arrows and stone missiles, where he devoted his spare time to mathematical learning and research in spite of being troubled by ongoing warfare.[37] However, from 1235 until Qin's preface was written in 1247, there is evidence that he did not spend the entire period in Sichuan. At some point between 1235 and 1239, Qin Jiushao left Sichuan and received a promotion to become the sub-prefect of Qizhou 蘄州.[38] According to a memorial to the emperor written by Liu Kezhuang, a contemporary official and poet, Qin Jiushao's treacherous behaviour in Qizhou was responsible for causing a military revolt there.[39] This memorial did not seem to have any effect since Qin shortly received another promotion to become the prefect of Hezhou 和州.[40] Again, Liu Kezhuang wrote a memorial to the emperor attacking Qin Jiushao.

---

[35] Libbrecht, 26-27.
[36] Zhou, 325.
[37] Chen, Zhang, and Zhou, 10.
[38] Libbrecht, 28. Qizhou is now Qichun 蘄春 in the province of Hubei 湖北.
[39] Ibid.
[40] Present-day Hexian 和縣 in Anhui 安徽

This time, he alleged that Qin had smuggled salt and sold it illegally to the commoners.[41] The truth of these allegations remains controversial, but according to Zhou Mi, a contemporary official and social critic, Qin became very rich around this time, thereby suggesting that Qin was, in fact, a corrupt official.[42]

In 1239, Qin Jiushao was recommended by long-time friend Wu Qian 吳潛 (1196 – 1262) to a position in Huzhou 湖州.[43] They had met while Qin was still the *xianwei* in Qixian.[44] Wu Qian was certainly one of Qin Jiushao's most influential connections because Wu occupied many high positions and would go on to become a minister of state in the 1250's. Qin Jiushao apparently also received a piece of land in Huzhou from Wu, on which he built a luxurious mansion.[45] It is believed that this mansion then became the permanent residence of Qin Jiushao's family, including his mother.[46]

In 1244, Qin Jiushao had to leave for Jiankang 建康[47] to take up the post of vice-administrator there.[48] His term of office ended prematurely when his mother passed away in the same year. At this time, he returned to Huzhou to carry out the customary mourning for three years, where it is believed he wrote the *Shushu jiuzhang*. The date of Qin's preface corresponds to the end of the mandated mourning period, so it is probable that he spent much of these three years on more mathematical research and writing his treatise. When the *Shushu jiuzhang* was completed, Qin would have been around forty-five years old. After

[41] Libbrecht, 28.
[42] Zhou, 325.
[43] Xu and Kong, 105.
[44] Ibid, 104.
[45] Zhou, 325.
[46] Han, 97.
[47] Present-day Nanjing 南京 in the province of Jiangsu 江蘇
[48] Libbrecht, 28.

that, there is no evidence that Qin ever became involved in mathematical work again.

After he came out of mourning, Qin Jiushao was recommended to the imperial government on account of his proficiency in calendrical science.[49] This occurred some time between 1247 and 1254 and was apparently done by Chen Zhensun 陳振孫 (? – 1261) who was in charge of the imperial academy at the time.[50] It is unclear whether or not this recommendation led to anything, but official records show that Qin was soon back in Jiankang holding a military position.[51] In 1255, he was relieved of his position, whereupon he returned to Huzhou.[52] In 1257, he became the prefect of Qiongzhou 瓊州.[53] This appointment apparently came about based on the recommendation of a chief minister with a notorious reputation, Jia Sidao 賈似道 (1213 – 1275), who had considerable influence in court at the time, his sister being one of the emperor's concubines.[54] This recommendation could have been the result of a bribe by Qin Jiushao because there is no evidence that he had connections to Jia previously. Qin Jiushao only stayed in Qiongzhou for a few months. He was dismissed due to allegations of bad behaviour and corruption.[55]

In 1259, Qin Jiushao's official career received a new boost from Wu Qian, who had just been appointed as a minister of state. Qin followed Wu into the capital and subsequently received two new appointments, first to Huzhou and then to Jiangxi 江西. Both appointments were extremely short-lived as

---

[49] Zhou, 325.
[50] Libbrecht, 29; Xu and Kong, 54.
[51] Libbrecht, 29.
[52] Xu and Kong, 247.
[53] Present-day Qiongxian 瓊縣 in Hainandao 海南島
[54] Zhou, 325.
[55] Xu and Kong, 250-251.

allegations of inappropriate conduct continued to plague him. His most outspoken critic was again Liu Kezhuang whose memorials to the emperor proved much more effective this time in ousting him.[56]

In 1260, the rivalry between the two ministers of state, Jia Sidao and Wu Qian, reached its conclusion with Wu's defeat when he was charged with deceiving the emperor and was banished. In 1262, an imperial decree was passed to dismiss all of Wu Qian's associates and to forbid them to ever again hold official positions. In the meantime, Qin Jiushao had been sent to Meizhou 梅州[57] to be the prefect. Zhou Mi wrote that Qin died there while still in office.[58] Therefore, most historians infer that Qin must have died in 1261, before the imperial decree was passed. However, others like Xu and Kong think Qin's connections to Jia Sidao and his location in out-of-the-way Meizhou were enough to prevent Qin from being affected by the decree.[59] In any case, historians have not been able to find any evidence that Qin Jiushao lived beyond the 1260's. Therefore, he most probably did not live to see the completion of the Mongol conquest of China, and his entire life was spent under the Southern Song.

The Qin Jiushao to be Remembered in History

From the above, we can see that Qin Jiushao was by no means a professional mathematician. His main occupation was an official who had held both military and civil positions at various times. It is interesting to note the contrast between Qin Jiushao and Shen Gua 沈括 (1031 – 1095), the much better known eleventh-century Song official who enjoys a very positive image in

---

[56] Ibid, 251.
[57] Present-day Meixian 梅縣 in the province of Guangdong 廣東
[58] Zhou, 325.
[59] Xu and Kong, 256.

historiography. Shen really was a polymath. His *Dream Pool Essays* (Mengxi bitan 夢溪筆談) delved into mathematics, astronomy, zoology, botany, and the history of technology among other subjects. Qin certainly could not match Shen in either versatility or reputation, but in terms of mathematical abilities, Qin could be said to surpass Shen. However, for Qin Jiushao, mathematics was only a part-time pursuit to be indulged in either as an escape or as a hobby. Therefore, though a keen learner, he most likely was never involved in teaching mathematics to others. It is then especially noteworthy that his mathematical treatise is of such high intellectual worth.

The historical evidence we have about Qin Jiushao indicates that he was a corrupt official and an individual of questionable morality in spite of his intellectual brilliance. This evidence is currently limited to the writings of Liu Kezhuang and Zhou Mi. According to Liu, who never commented on Qin Jiushao's mathematical talent, Qin was a terrible and corrupt official who was involved in illegal salt trade and whose behaviour had caused soldiers to rebel.[60] Zhou Mi's account paints Qin in an even worse light. He described Qin Jiushao as someone who was multi-talented, but extremely indiscreet in his relations with women, extravagant, boastful, and dangerous. According to Zhou, Qin had no qualms about poisoning people to whom he objected, including his own nephew.[61]

With such accounts provided by Qin Jiushao's contemporaries, compounded with the lack of testimonials to the contrary, it is no wonder that Qin's character has a very negative reputation in much of the historiography about him. In all the biographies Libbrecht has written about Qin, the former

---

[60] Libbrecht, 28.
[61] Zhou, 324-328.

does not question Liu Kezhuang's and Zhou Mi's statements. Instead, Libbrecht used them to reconstruct Qin's life, thereby confirming that Qin had indeed caused a military revolt and mistreated the common people. In the *Dictionary of Scientific Biographies*, Ho Peng Yoke, one of the leading historians in the field of the history of Chinese science and technology, takes the same attitude.

Another point against Qin Jiushao is his connection to Jia Sidao. According to Zhou Mi, Qin became the prefect of Qiongzhou on Jia's recommendation. Jia Sidao himself has a very negative reputation in historiography as a corrupt and incompetent minister who was willing to betray his country for his own interests.[62] By associating with Jia and benefitting from his favours, Qin Jiushao became tainted by association and he seemed like one who would resort to any means to secure his own profit. Therefore, some historians protest very vehemently against Qin's character even while acknowledging his intellectual brilliance.[63]

Even though intellectual brilliance and an immoral character are not mutually exclusive qualities in a person, Qin Jiushao's mathematical abilities seem to have been a major factor in sparking a movement to redeem the image of his character. When Chinese scholars began doing extensive research into Qin Jiushao's work during the Qing dynasty, several of them questioned the credibility of Zhou Mi's account of Qin Jiushao's character. For example, Qing mathematician Jiao Xun 焦循 (1763 – 1820) pointed out that Zhou Mi's account was based on information provided by a certain Chen Shengguan 陳聖觀, so Zhou Mi most likely did not personally know Qin Jiushao. Therefore, Zhou's

---

[62] Xu and Kong, 249.

[63] See, for example, Li Zhaohua, *Zhongguo shuxue shi* (History of Chinese Mathematics) (Taipei: Wenjin chubanshe, 1995), 136.

information might very well have been inaccurate. Another Qing mathematician, Lu Xinyuan 陸心源 (1834 – 94), pointed out that Qin Jiushao had close ties to Wu Qian, who is remembered as a respectable official in direct contrast to Jia Sidao. Therefore, Lu argued that Qin could not have been the despicable man that Zhou Mi would like people to believe he was.[64]

The movement to redeem Qin Jiushao's historical image has continued into recent times. Many historians today side with the Qing scholars mentioned above and dismiss Zhou Mi's account as inaccurate in terms of its description of Qin's character and behaviour.[65] There are also historians who argue that Liu Kezhuang and Zhou Mi intentionally distorted Qin's character because they wanted to please Jia Sidao, so they did everything they could to attack Jia's political opponent Wu Qian and others close to Wu, including Qin Jiushao.[66]

Therefore, there currently exist two schools of thought regarding Qin Jiushao. One school takes the information written by Liu Kezhuang and Zhou Mi as solid evidence of Qin's questionable character. The other school calls Liu's and Zhou's credibility into question and argues instead that Qin Jiushao was admirable in terms of both moral character and intellectual brilliance. With such limited historical evidence available and in the absence of any new evidence, it is indeed difficult to decide which school is correct. However, an objective evaluation of existing evidence is possible and shall be attempted here.

Liu Kezhuang's memorials to the emperor are currently our most solid source of evidence that Qin Jiushao behaved problematically in his various official positions in Qizhou, Hezhou, Huzhou, and Jiangxi. Liu Kezhuang himself was from Fujian 福建 and served in various official positions in both Hangzhou

---

[64] Xu and Kong, 263-264.
[65] See, for example, Han, 98.
[66] See, for example, Xu and Kong, 263.

and Fujian during his career.[67] In other words, Liu and Qin never seemed to have been in the same administrative district at the same time. Therefore, it is questionable that Liu could have been so intimately aware of Qin's behaviour as to be able to accuse him justly of inappropriate conduct, so it is quite possible that Liu's testimonial was indeed tainted by the factional rivalry between Jia Sidao and Wu Qian.

However, the same cannot be said about Zhou Mi's account. After all, Zhou's account was written after Wu Qian had already been driven from power and after Qin Jiushao had passed away. Therefore, by that time, there could have been no further reason to attack Qin in the interest of factional rivalry. Furthermore, Zhou Mi was not an associate or supporter of Jia Sidao, so he could have had no reason to aim his writings at pleasing Jia. However, Zhou's account cannot be taken as the absolute truth either. It is doubtful that Zhou ever met Qin Jiushao himself, and there is no means of verifying the accuracy of his source of information. Furthermore, his *Guixin zazhi* 癸辛雜識 (Miscellaneous Notes from the Guixin Quarter), in which we find the entry about Qin Jiushao, was not written as a historical text, but a record of the conversations and gossip shared between him and his friends.[68] Therefore, Zhou Mi's evaluation of Qin Jiushao's character is probably not entirely reliable.

How, then, should we construct the image of Qin Jiushao in history? We are certain that he was an official who happened to have been brilliant in mathematics. Because there is no biography of him in the *Songshi* 宋史 (Standard History of the Song Dynasty), we can safely say that he never attained

[67] Xu and Kong, 251.

[68] Jennifer W. Jay, *A Change in Dynasties* (Bellingham, Washington: Centre of East Asian Studies, Western Washington University, 1991), 233.

great fame or influence on the national political scene. Though his reputation as an official and as an individual has been considerably damaged by his contemporaries, the absence of more objective corroborating evidence suggests that Qin Jiushao's character might have been unjustly maligned. However, there can be no doubt as to his brilliance as a mathematician, and proof of this will be discussed in the next chapter.

Chapter 2: The *Shushu jiuzhang* and its Significance

There can be no doubt as to the authenticity of Qin Jiushao's treatise, the *Shushu jiuzhang*.[69] It stands as proof of Qin Jiushao's outstanding mathematical abilities and as a valuable source of information about Southern Song China. This chapter will explore the qualities that mark the *Shushu jiuzhang* as a milestone work in the history of mathematics and the reasons why it was written. My goal here is to contextualize Qin Jiushao's work in the midst of both earlier and later mathematical texts and to disprove Libbrecht's remark that "mathematics was the servant of the more important sciences of the heaven."[70]

A Selective Discussion of the Shushu jiuzhang

True to its title, the *Shushu jiuzhang* (Mathematical Treatise in Nine Sections) is divided into nine sections, with each section covering a different category of problems. Each section contains nine problems, totalling eighty-one problems along with their solutions. The nine categories are:

1. *Dayan lei* 大衍類 (indeterminate analysis)
2. *Tianshi lei* 天時類 (heavenly phenomena)
3. *Tianyu lei* 田域類 (boundaries of fields)
4. *Cewang lei* 測望類 (measuring at a distance/telemetry)
5. *Fuyi lei* 賦役類 (taxes and levies of service)
6. *Qiangu lei* 錢穀類 (money and grain)
7. *Yingjian lei* 營建類 (fortifications and buildings)

[69] Libbrecht, 35.
[70] Ibid, 4.

8. *Junlü lei* 軍旅類 (military affairs)

9. *Shiwu lei* 市物類 (commercial affairs)

The above list shows that Qin Jiushao delved into a wide variety of topics in his treatise. In order to fully demonstrate Qin's methods and the type of mathematics that he did, I will be using examples from each of the nine sections. The section that has drawn the most attention among historians and mathematicians alike is the one on the *dayan* rule or indeterminate analysis. This is because it contains the first systematic description of what is known today as the Chinese Remainder Theorem. It is used to solve problems like the famous "Sunzi 孫子 problem" which dates from around the fourth century CE.[71] The "Sunzi problem" is as follows:

> The number of objects we have is unknown. If we count the objects by threes, there is a remainder of two. If we count them by fives, there is a remainder of three. If we count them by sevens, the remainder is two. How many objects do we have? Answer: 23

Although the "Sunzi problem" had already been solved in the fourth century, there remained a strong interest among the Chinese literati in brain-teasers like this, and this was why the "Sunzi problem" was widely circulated and widely known in pre-modern China.[72] The "Sunzi problem" is only one special case of indeterminate problems. Qin Jiushao made an extremely important contribution to the history of mathematics by laying out a general method that could be used to solve all similar types of problems. The *dayan* method involves

---

[71] This problem was originally found in the *Sunzi suanjing* 孫子算經 (Master Sun's Mathematical Classic), whose author remains unknown. Because this text contains references to Buddhism, it must have been written after Buddhism had spread to China during the first centuries CE. Therefore, its author could not have been Sun Wu 孫武, author of *Sunzi bingfa* 孫子兵法 (Master Sun's Art of War), who lived during the sixth century BCE.

[72] Ho Peng Yoke, and He Guanbiao, *Zhongguo keji shi gai lun* (An Overview of the History of Technology in China) (Hong Kong: Zhonghua shuju, 1988), 82.

reducing certain given numbers until they are relatively prime, followed by a series of multiplication and division operations to obtain the answer. An English translation of Qin Jiushao's step-by-step instructions for the *dayan* method has been provided by both Libbrecht and Dauben.[73] Here, a demonstration of how it works can be seen in the following example taken from Qin Jiushao's *dayan* section.

> **Section 1 (indeterminate analysis), Problem 5:** Three farmers divided their harvested rice equally among themselves to be sold. Farmer A sold his rice to the local government and had 3 *dou* 斗 2 *sheng* 升 of rice remaining. Farmer B sold his rice to the commoners of Anji[74] 安吉 and had 7 *dou* of rice remaining. Farmer C sold his to people in Pingjiang[75] 平江 and had 3 *dou* remaining. How much rice did they have originally?

Solution:

It is known that the *hu* 斛 (measuring vessel) used by the government has a capacity of 8 *dou* 3 *sheng*; the one used in Anji is 1 *shi* 石 1 *dou*; and the one used in Pingjiang is 1 *shi* 3 *dou* 5 *sheng*.

Converting these capacities to the same unit (1 *shi* = 10 *dou* = 100 *sheng*):

Government *hu* = 83 *sheng*

Anji *hu* = 110 *sheng*

Pingjiang *hu* = 135 *sheng*,

thus giving us the three numbers that have to be reduced until they are relatively prime.

---

[73] See Libbrecht, 328-332; Joseph W. Dauben, "Chinese Mathematics," in *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook*, ed. Victor Katz (Princeton: Princeton University Press, 2007), 314-315.

[74] Present-day Huzhou 湖州 in the province of Zhejiang 浙江

[75] Present-day Suzhou 蘇州 in the province of Jiangsu 江蘇

In modern notation, this problem can be represented in the following ways:

$N \equiv 32 \pmod{83}$
$N \equiv 7 \pmod{110}$
$N \equiv 3 \pmod{135}$,[76] where N is the amount of rice each farmer started out with, or

$N = 83x + 32$, where x is the number of government *hu* of rice sold by Farmer A
$N = 110y + 7$, where y is the number of Anji *hu* of rice sold by Farmer B
$N = 135z + 3$, where z is the number of Pingjiang *hu* of rice sold by Farmer C

Going back to the numbers 83, 110, and 135, 110 and 135 have a common factor of 5. According to Qin's *dayan* rule, only the odd number, 135, needs to be reduced here by dividing it by 5.

83, 110 135 $\rightarrow$ 83, 110, 27

These numbers are now relatively prime. These are the *dingshu* 定數:

$a_1 = 83,\ a_2 = 110,\ a_3 = 27$

Multiplying the *dingshu* together, we get the *yanmu* 衍母:

$83 \times 110 \times 27 = 246\ 510$

To get the *yanshu* 衍數 (m_i, $1 \le i \le 3$), two of the three *dingshu* (a_i) have to be multiplied together each time such that:

$m_1 = a_2 \times a_3 = 110 \times 27 = 2970$
$m_2 = a_1 \times a_3 = 83 \times 27 = 2241$
$m_3 = a_1 \times a_2 = 83 \times 110 = 9130$

Then, get the remainder (*jishu* 奇數 g_i) of $m_i \div a_i$:

$m_1 \div a_1 = 2970 \div 83 = 35 \text{ r } 65$
$m_2 \div a_2 = 2241 \div 110 = 20 \text{ r } 41$
$m_3 \div a_3 = 9130 \div 27 = 338 \text{ r } 4$

---

[76] $s \equiv t \pmod{u}$ means that s and t have the same remainder when divided by u.

Therefore, g_1 = 65, g_2 = 41, g_3 = 4.

Then, apply the *dayan qiuyi shu* 大衍求一術 (ie. solve the congruence $k_i \times g_i \equiv 1(\mathrm{mod}\, a_i)$). Here, we need to find three numbers (k_i) such that their products with the *jishu*, when divided by the *dingshu*, will yield a remainder of one.

$$k_1 \times 65 \equiv 1(\mathrm{mod}\, 83)$$
$$k_1 = 23$$

$$k_2 \times 41 \equiv 1(\mathrm{mod}\, 110)$$
$$k_2 = 51$$

$$k_3 \times 4 \equiv 1(\mathrm{mod}\, 27)$$
$$k_3 = 7$$

Perform the operation $k_i * m_i$ to get the *yongshu* 用數 (S_i):

$$S_1 = k_1 \times m_1 = 23 \times 2970 = 68\ 310$$

$$S_2 = k_2 \times m_2 = 51 \times 2241 = 114\ 291$$

$$S_3 = k_3 \times m_3 = 7 \times 9130 = 63\ 910$$

Using the remainders given in the problem (R_1 = 32, R_2 = 70, and R_3 = 30), multiply these by the *yongshu* (S_i):

$$N_1 = R_1 \times S_1 = 32 \times 68\ 310 = 2\ 185\ 920$$
$$N_2 = R_2 \times S_2 = 70 \times 114\ 291 = 8\ 000\ 370$$
$$N_3 = R_3 \times S_3 = 30 \times 63\ 910 = 1\ 917\ 300$$

Adding these together, we get a *zongshu* 總數:

$$N_1 + N_2 + N_3 = 12\ 103\ 590$$

Divide this by the *yanmu*. The remainder will be N, the answer to this problem.

12 103 590 ÷ 246 510 = 49 r 24 600

Therefore, N = 24 600. Each farmer had 24 600 *sheng* or 246 *shi* of rice originally.

Altogether, they had 246 × 3 = 738 *shi* of rice.

If we use the individual measurement standards of each different place to calculate:

Farmer A (government *hu*):

$$p_1 = \frac{246 \times 100}{83} \approx 296 \text{ } \textit{shi}$$

Farmer B (Anji *hu*):

$$p_2 = \frac{246 \times 100}{110} \approx 224 \text{ } \textit{shi}$$

Farmer C (Pingjiang *hu*):

$$p_3 = \frac{246 \times 100}{135} \approx 182 \text{ } \textit{shi}$$

Qin Jiushao did not invent the *dayan* rule himself. In his preface, Qin stated that the *dayan* rule had already been in use among astronomers for calendrical calculations.[77] However, it was not well known among mathematicians. Being a mathematician himself who had studied under astronomers, Qin was in the perfect position to help spread the *dayan* rule to mathematical circles and push it into more widespread use. Here we see a method in astronomy being applied to solve problems in mathematics. Therefore, at least in Qin Jiushao's perspective, mathematics did not simply exist

[77] Chen, Zhang, and Zhou, 5.

to serve astronomy, but was an independent field that could also absorb methods from astronomy.

In the *dayan* section, Qin Jiushao demonstrated that the application of the *dayan* rule was not simply limited to calendrical calculations. He also used it to solve a variety of problems that reflect a concern with government administrative tasks, finances, construction, and volume problems. Therefore, besides one question on calendrical calculations and one on oracle readings, Qin also had one question about four counties dividing the labour of building a dyke, one on monetary exchange, two on calculating volume based on different-sized containers, two on the distance travelled by government messengers, and one about laying the foundation of a building with four different types of tiles. In other words, the situations described in Qin Jiushao's problems reflect an emphasis on practical day-to-day affairs. This emphasis is apparent throughout his treatise. Because of this emphasis, Qin Jiushao's treatise is very significant historically as a reflection of small details within Southern Song society. From this first section on the *dayan* rule, we learn about different measurement standards used during the Southern Song dynasty. In the question about building a dyke, Qin specified that one *li* [78] 里 was equal to three hundred and sixty *bu* 步, while one *bu* was equal to five *chi* 尺 and eight *cun* 寸. The fact that Qin Jiushao had to define how long the *li* and the *bu* were as units of length suggests that these were not commonly known or that their definitions varied across time or space. Indeed, we know that one *bu* was equal to eight *chi* during the Zhou 周 dynasty (1066 – 221 BCE), and six *chi* during the Qin 秦 dynasty (221 – 206 BCE).[79] Even within the confines of Qin Jiushao's text, we find that Qin himself was not consistent in

[78] One *li* is equivalent to about one third of a mile.
[79] Chen, Zhang, and Zhou, 65.

his definitions of the measuring units. For example, in the second question of section 3, he defined one *li* as being equal to three hundred *bu* instead of three hundred and sixty.[80]

In the question about monetary exchange, Qin mentioned using coins to exchange for *jiuhui* 舊會. *Jiuhui* refers to old *huizi* 會子, which was a type of paper currency issued by the Southern Song government from the year 1160 onwards.[81] This type of currency was re-issued every three years, and the value of each new generation of *huizi* was defined at the time of issuance. By 1240, the newest *huizi* in use was the eighteenth generation. Qin's problem reflects the fact that this type of currency became devalued over the years because the government had issued too much of it to cover rising military costs.[82]

In the problem about three farmers dividing their harvested rice to be sold, whose solution is given above, we learn that volume measures were not standardized throughout the empire during the Southern Song. Instead, different locales had different measures even though the name of the measurement unit used remained identical.[83] In the problem, Qin Jiushao implied that the standard of measurement for rice set by the government was not, in fact, adopted everywhere. Villages and cities could use their own local measuring standards, so conversion calculations were necessary for transactions between different locales.

The questions in Section 2 (heavenly phenomena) deal with the calendar, astronomical phenomena, and precipitation. More specifically, the first three questions deal with calendrical calculations. The fourth question is about

---

[80] Ibid, 165.
[81] Ibid, 326.
[82] Ibid, 72.
[83] Ibid, 82.

calculating the speed of the planet Jupiter based on data collected from astronomical observations. The fifth question is about calculating the length of shadows on summer and winter solstices under different calendars. The last four questions are about calculating the depth of precipitation on level ground based on the amount collected on a slope or in certain containers.

From these questions, we gain several insights into Chinese astronomy. First of all, the winter solstice served as an important marker on the calendar for Chinese astronomers for the purpose of calculations. Second, Qin Jiushao made reference to a calculation method invented by astronomer He Chengtian 何承天 (370 – 447) in one of the questions. This is clear evidence that He's method had survived into the thirteenth century and was part of the long tradition of Chinese astronomy. From the fourth question in this section, we learn that observing the position of Jupiter was also part of that tradition.

From the questions dealing with calculating the depth of precipitation, we learn that containers such as the following were being used to collect precipitation during the Southern Song dynasty:

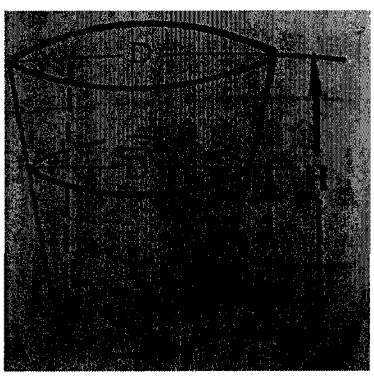

Figure 2 – Example of container used to collect precipitation
(illustration taken from Chen, Zhang, and Zhou, 140)

Qin Jiushao stated that some people assumed that the depth of precipitation collected in such containers was the same as the depth on level ground.[84] Qin rightly pointed out that this type of assumption was incorrect. Therefore, in this case, calculations had to be done in order to find the precise depth of precipitation accumulated on level ground. Although Qin did not explicitly give a general formula for this type of calculations, his method for finding the depth of precipitation on level ground based on the depth of precipitation collected in containers like the one above can be reduced to the following formula:

$$h_1 = \frac{h}{3(DH)^2}\{[dH+(D-d)h]^2+(dH)^2+[dH+(D-d)h]dH\}.$$ [85]

The third section (boundaries of fields) of Qin Jiushao's treatise deals with calculating the area of different-shaped agricultural fields. The first question is about calculating the area of a kite-shaped field. Here, Qin Jiushao solved it with a fourth-degree equation:

$$-x^4 + 763\,200\,x^2 - 40\,642\,560\,000 = 0$$

The method used by Qin to extract the root of this equation is similar to the one developed by British mathematician William George Horner (1786 – 1837) in 1819. Qin's method is known as the *zhengfu kaifang fa* 正負開方法 (positive and negative root extraction method). He was not the creator of this method, but his treatise is the earliest we have today that clearly explains it.[86] Although it has been pointed out by scholars that there is a more direct way of

---

[84] Ibid, 138.
[85] Ibid, 139-140.
[86] Libbrecht, 180.

solving this problem without resorting to a fourth-degree equation, Qin Jiushao's actual intention might have been to demonstrate the workings of the *zhengfu kaifang fa* rather than insisting that it was the only way to solve it.[87]

Another method worthy of note in this section is Qin Jiushao's formula for finding the area of a triangle based on the length of its three sides:

$$S = \sqrt{\frac{1}{4}[c^2a^2 - (\frac{c^2 + a^2 - b^2}{2})^2]}$$

where a,b,c are the lengths of the sides of the triangle. It is similar to the formula developed by Heron (ca. 10 – 70) of Alexandria, but Qin Jiushao seemed to have developed it independently.[88]

In section 4 (measuring at a distance/telemetry) of his treatise, Qin Jiushao dealt with problems involving calculating distances or heights based on observations from far away. For example, his first question is about calculating the height of a distant mountain and the distance between the mountain and the town of the observer using the known height of a tree in-between. For questions like this, Qin Jiushao made use of the principle of similar triangles to solve them. However, in the fifth question of this section, Qin actually made use of a tenth-degree equation to find the solution. This is the first known instance of a tenth-degree equation being solved in Chinese mathematics. The question deals with finding the circumference and diameter of a circular town wall from afar:

> **Section 4 (measuring at a distance/telemetry), Problem 5:**
> There is a round town of which we do not know the circumference and the diameter. There are four gates [in the wall]. Three *li* outside the northern [gate] there is a high tree. When we go outside the southern gate and turn east, we must cover 9 *li* before

[87] Chen, Zhang, and Zhou, 163.
[88] Ibid, 168.

we see the tree. Find the circumference and the diameter [x^2] of the town (π= 3).[89]

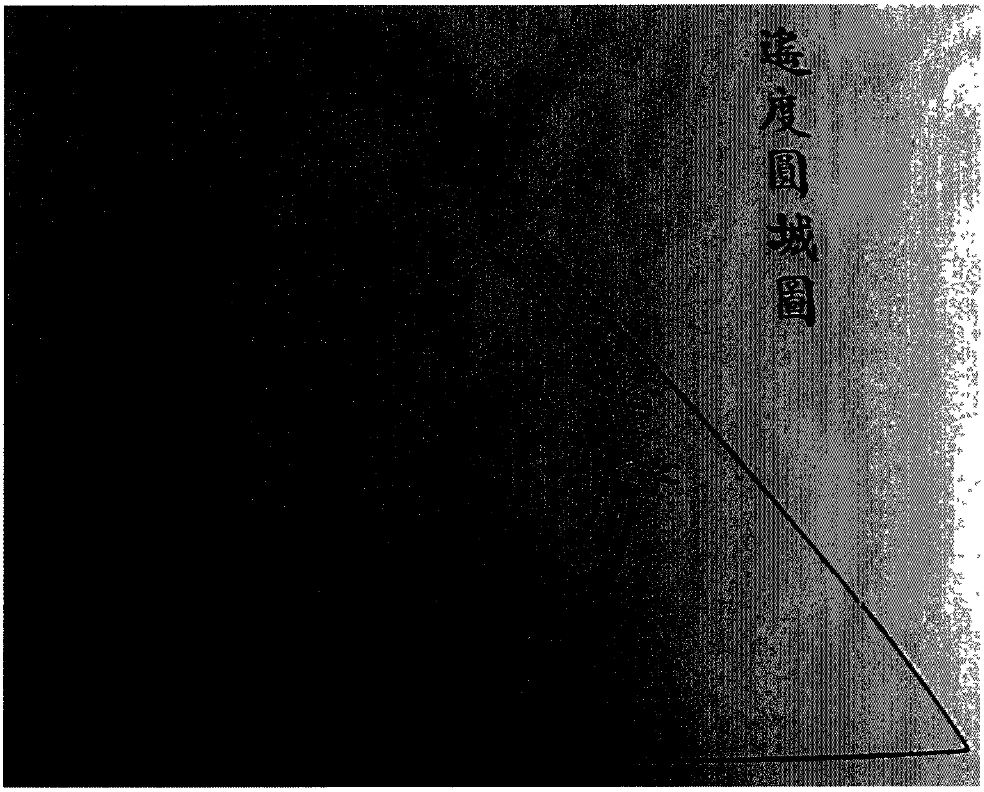

Figure 3 – Original illustration from the *Shushu jiuzhang* for Problem 5 of Section 4 (illustration taken from Libbrecht, 136)

The equation Qin used to solve it is

$$x^{10} + 15x^8 + 72x^6 - 864x^4 - 11664x^2 - 34992 = 0,$$

where x^2 was equal to the diameter of the town wall. Again here, scholars have questioned Qin's usage of such a complex method to solve a simple question. If the diameter of the town wall was not specified as x^2, but a simple y instead, then the equation could be reduced to a fifth-degree equation:

$$y^5 + 15y^4 + 72y^3 - 864y^2 - 11664y - 34992 = 0.$$ [90]

[89] Libbrecht, 134-135.

Furthermore, by making use of simple geometrical principles, this problem could actually be solved with a third-degree equation instead.[91] Therefore, although historians generally acknowledge that this problem really showcases the sophistication of Qin Jiushao's mathematical abilities, they are divided as to why Qin chose such an overly complex way to solve it. For example, historian Qian Baocong 錢寶琮 believed that Qin Jiushao's intention was to show off his own skills, while Chen, Zhang, and Zhou convincingly argue that Qin Jiushao's motive was purely the scholarly desire to demonstrate that the *zhengfu kaifang fa* could be used to solve equations of any degree.[92] Therefore, Qin Jiushao's choice of method should be acknowledged for its potential to further mathematical research and studies. Indeed, engaging in mathematical work should not simply be about finding the simplest solution to a problem. Instead, it should also encompass the notion of pushing the limits of what could be done.

One of the most striking themes in this section is about military and warfare. Qin Jiushao included three questions that dealt with calculating the size of the enemy's camp or fortress or how far away it was. Later on in the treatise, Qin would devote an entire section to military affairs. Given that Qin lived in times of regular warfare, it is certainly not surprising that the military theme is so visible throughout his work.

Another point worthy of note in this section is that Qin Jiushao used different values for *pi* (ratio of a circle's circumference to its diameter) in different questions. In the problem quoted above, he used $\pi = 3$. In the sixth problem of this section, he used $\pi = \frac{22}{7}$. In a previous section, he also used the value of

[90] Ibid, 136.
[91] Chen, Zhang, and Zhou, 240.
[92] Ibid, 242-243.

$\sqrt{10}$ for *pi*. Chinese mathematicians and astronomers have a long history of working with *pi*, which dates at least to the Han dynasty. The actual values used varied among individual mathematicians.[93] For example, Zhang Heng 張衡 (78 – 139 CE) used $\pi=\sqrt{10}$ (≈ 3.16228); Wang Fan 王蕃 (219 – 257 CE) used $\pi=\frac{142}{45}(\approx 3.15556)$; and Liu Hui 劉徽 (ca. 3rd century CE) came up with $\pi=3.141024$ or $\pi=\frac{157}{50}(=3.14)$ as an approximate in fraction form.[94] By the fifth century, astronomer Zu Chongzhi 祖衝之 (429 – 500 CE) was able to come up with a much more accurate value of pi. He determined it to be approximately equal to $\frac{355}{113}(\approx 3.14159)$, but also used $\frac{22}{7}$ as an approximate value.[95] The fact that Qin Jiushao used such a variety of values for *pi* suggests that he was very likely aware of the works written by most, if not all, of the mathematicians and astronomers listed above. The actual choice of the value of *pi* to be used for each question indicates that Qin was more concerned with ease of calculation than with the accuracy of the result.

In section 5 (taxes and levies of service) of his treatise, Qin Jiushao made extensive use of ratios to deal with the very practical matters of calculating taxes, rent, and labour. For example, the first question is about six villages that had land that varied in quality. The question is to calculate the amount of taxes levied on each type of land based on the principle that owners of better-quality land were expected to pay more taxes. However, this was not the reality during Qin Jiushao's time. During the Southern Song dynasty, taxes levied were based

---

[93] Today, the value of pi has been determined to be 3.14159265…
[94] Li and Du, 66-68.
[95] Ibid, 83.

solely on the size of the land owned, and did not take into account the quality of the land and how much it could yield. It is obvious that Qin Jiushao devoted a lot of time to creating this question because he had to specify how much of each grade of land could be found in each of the six villages. This is by far the longest question in the entire treatise, taking up thirty-two pages in the original manuscript.[96] Historians have, of course, speculated why Qin devoted so much time to a question that was based on a make-believe tax system. One likely explanation is that Qin had been influenced by the Neo-Confucian philosopher Zhu Xi 朱熹 (1130 – 1200) who advocated for a fairer tax system that took land quality into account.[97]

Section 6 (money and grain) of the treatise is perhaps the most interesting section because the problems here cover a great variety of practical situations involving money and grain, and this shows Qin Jiushao's willingness to engage his mathematics in regular day-to-day affairs. The first question is about four towns having to transport their collected taxes to the capital. It involves calculating the value of silver and cloth, and converting old *huizi* currency to new *huizi*. The second question is about calculating the amount of prepaid transportation costs that should be refunded after a shipment of rice failed to reach its final destination. The third question is about comparing the price of rice from five different districts while taking transportation costs into account, and involves converting various volume measuring standards to the government standard. The fourth question is about calculating the dimensions of the measuring containers needed for given volumes. The fifth describes a situation in which the government has to borrow warehouses to temporarily store grain

---

96 Chen, Zhang, and Zhou, 282.
97 Ibid, 283.

while awaiting transport. It is a volume question that asks for the amount of grain that could be stored in the warehouses given the dimensions. In the sixth question, the government is described as having a set amount of money to buy grain and pay for transportation costs and middlemen fees, so the question is to calculate the actual amount of grain that could be bought. The seventh question involves dividing tax revenues among three government departments according to a given ratio, and the eighth is about the government lending out money with a monthly interest of 6.5%. The last question in this section is the most notable out of the nine. The situation described in the question is about a farmer paying 1534 *shi* of rice as tax. However, upon examination, it was discovered that the 1534 *shi* included both rice and another grain, probably millet. In order to calculate the actual amount of pure rice paid by the farmer, Qin Jiushao proposed first taking a random sample from what the farmer paid, and then calculating based on the ratio of rice to millet in the sample. This use of a random sample is considered a very innovative approach in the history of mathematics. Western mathematicians did not use it until the seventeenth century.[98]

The seventh section (fortifications and buildings) of the treatise deals with construction problems. Here, Qin Jiushao looked at calculating the amount of material needed to build various structures, the number of workmen needed, and the amount of money needed to pay the wages of the labourers. The detail in which Qin delved into these problems suggests that he really was very knowledgeable in the field of construction. In fact, it is believed that Qin was personally involved in the designing and building of his mansion in Huzhou.[99]

---

[98] Ibid, 367.
[99] Han, 97.

The eighth section (military affairs) of the treatise deals with military matters, including problems on calculating the dimensions of the camp needed to house a given number of soldiers, the number of soldiers required for certain battle formations, and the amount of time needed to makes weapons for the soldiers. The methods employed in this section are generally not complex. The most involved solution is perhaps the one for the final problem of this section, which requires the use of the Rule of Double False Positions taken from the *Nine Chapters*. The problem can be translated as follows:

> **Section 8 (military affairs), Problem 9**: There are three types of material in the warehouse for making clothing for the soldiers: hemp cloth, floss silk, and inferior silk. If we use 8 bolts of hemp cloth for every 6 soldiers, then we are short by 160 bolts. If we use 9 bolts for every 7 soldiers, then we have 560 bolts extra. As for floss silk, if we use 150 *liang* 兩 for every 8 soldiers, then we have 16 500 *liang* extra. If we use 170 *liang* for every 9 soldiers, then we have 14 400 *liang* extra. As for the inferior silk, if we use 13 *jin* 斤 for every 4 soldiers, then we are short by 6804 *jin*. If we use 14 *jin* for every 5 soldiers, then we have just the right amount. How many soldiers are there? How much hemp cloth, floss silk, and inferior silk are there? Answer: 15 120 soldiers, 30 000 bolts of hemp cloth, 300 000 *liang* of floss silk, and 42 336 *jin* of inferior silk[100]

This type of problem is generally classified as *ying buzu* 盈不足 (excess and deficit). In the problem above, three cases are presented. The case with the hemp cloth is considered as being one excess and one deficit. The case with the floss silk is considered as being two excesses. The case with the inferior silk is considered as being one deficit and one exact fit. The methods required to deal with all three cases are already dealt with in the *Nine Chapters*. Qin Jiushao did not create anything new here, so his intention here was probably to demonstrate the workings of old methods in new situations.

[100] One *jin* was approximately equal to 600 grams, while 1 *jin* = 16 *liang*.

As mentioned before, it is not surprising that Qin Jiushao has so many questions devoted to the military theme in his treatise. After all, he lived in an era when warfare was common. Furthermore, it must be recalled that he had actually held several military positions throughout his career. Therefore, it is a natural choice for him to deal with questions that involve battle formations and military supplies because those must have been familiar subjects to him.

The final section (commercial affairs) of the *Shushu jiuzhang* deals with commercial matters. The first problem uses a system of linear equations to solve for the value of three different types of goods. The problem reads as follows:

> **Section 9 (commercial affairs), Problem 1**: "Three times we close a bargain; each time we pay for the goods an amount of precisely 1 470 000 *guan*.[[101]] The first time they send 3500 bundles of garu wood, 2200 *jin* of tortoiseshell, and 375 cases of frankincense. The next time they send 2970 bundles of garu wood, 2130 *jin* of tortoiseshell, and $3056\frac{1}{4}$ cases of frankincense. The last time they send 3200 bundles of garu wood, 1500 *jin* of tortoiseshell, and 3750 cases of frankincense. We wish to know the value of each bundle [of garu wood, each *jin* of tortoiseshell, and each case of frankincense]."[102] Answer: 300 *guan*, 180 *guan*, and 64 *guan*

In his solution, Qin Jiushao was essentially working with the following system of equations:

$$\begin{cases} 3500x + 2200y + 375z = 1470000 \\ 2970x + 2130y + 3056\frac{1}{4}z = 1470000, \\ 3200x + 1500y + 3750z = 1470000 \end{cases}$$

where x = the price of one bundle of garu wood, y= the price of one *jin* of tortoiseshell, and z= the price of one case of frankincense.

[101] *Guan* 贯 is a monetary unit. 1 *guan* = 1000 coins.
[102] Libbrecht, 153.

The method he used to solve this system is the same as the modern method of elimination used today.[103] Qin used this method again in the very next question which dealt with calculating the amount of products that should be paid back to each of four investors who had invested varying amounts of goods and money into a joint maritime trade venture. The original matrix-like diagrams that Qin included in this solution have been preserved.[104] The diagrams are actually pictures of the counting board, and they illustrate the entire process of elimination that Qin went through to find the answer. Therefore, it is certain that Qin used the counting board to solve simultaneous equations.[105] The remaining problems in this section deal mainly with the exchange of products and the calculation of interest.

From the above summary of the contents of the *Shushu jiuzhang*, we come to the following conclusions. First of all, it is obvious that Qin Jiushao was very consistent in situating his problems in the context of a wide variety of real-life situations. This is characteristic of all Chinese mathematical treatises in general.[106] Therefore, in this case, Qin Jiushao was a true follower of the Chinese mathematical tradition. Even though he presented very abstract and sophisticated methods such as the *dayan* rule and the *zhengfu kaifang fa* (positive and negative root extraction method), he still tied them to practical problems. In other words, he never pushed beyond the boundaries of tradition, and he never delved into theoretical mathematical research. He provided methods to solve specific types of problems, but he never came up with a general theory for them.

---

[103] Chen, Zhang, and Zhou, 480.
[104] Ibid, 473.
[105] Libbrecht, 153.
[106] Ibid, 9.

The difficulty level of Qin Jiushao's problems is by no means uniform throughout his treatise. Some questions, such as the one involving the tenth-degree equation, are extremely complicated. Even though Qin provided an answer and solution instructions for each of his questions, he did not explain why his methods worked. Furthermore, he assumed that the reader was already familiar with all the methods laid out in the *Nine Chapters.* This is apparent from the fact that he simply referred to these methods by name and did not elaborate on what they were or where they came from. Therefore, we can conclude that Qin Jiushao's intended audience was not beginners in mathematics, but people who were already quite competent in it. To achieve that competency during Qin Jiushao's time, one would first have to be familiar with the more elementary mathematical texts such as the *Nine Chapters*. In this sense, Qin Jiushao's treatise is a clear sign of the progress of mathematical development in China. This treatise contained ideas that went beyond those presented in previous treatises, and raised the cumulative body of mathematical thought in China to a higher level of sophistication. In writing the *Shushu jiuzhang*, Qin Jiushao was effectively creating a new building block of mathematical knowledge that could be laid on top of the foundation made by the earlier texts and marked a new step forward in the history of Chinese mathematics.

Comparison of the **Shushu jiuzhang** *with the* **Ten Mathematical Classics**

In order to understand how the *Shushu jiuzhang* differs from earlier Chinese mathematical texts, it is necessary to compare them in terms of both scope and level of difficulty. Unfortunately, most of the mathematical texts that were compiled before the *Shushu jiuzhang* have been lost. The most prominent texts among those still remaining today are the *Ten Mathematical Classics*, which had served as official textbooks in the imperial academy during the Tang and the Northern Song dynasties. This collection of books were printed by the government in 1084 and distributed to public schools throughout Song China, so they were the mathematical texts that enjoyed the widest circulation in pre-modern China. Therefore, a comparison of the *Shushu jiuzhang* with the *Ten Mathematical Classics* will be presented here.

The *Shushu jiuzhang* actually covers many of the same topics and methods as the *Ten Mathematical Classics*. Like the *Zhoubi suanjing* 周髀算經 (Arithmetic Classic of the Gnomon and the Circular Paths of Heaven), which is one of the classics, the *Shushu jiuzhang* has questions dealing with right triangles and similar triangles. Like the *Haidao suanjing* (Sea-Island Mathematical Classic), another of the *Ten Mathematical Classics*, the *Shushu jiuzhang* also deals with calculating heights and distances based on observations and measurements from a distant point. Problems on calculating area of figures are covered in the *Shushu jiuzhang* as well as the classics *Sunzi suanjing* (Master Sun's Mathematical Classic) and *Wucao suanjing* 五曹算經 (Mathematical Manual of the Five Government Departments). As mentioned before, the *Shushu jiuzhang* contains a general method that could be used solve remainder problems like the "Sunzi problem" from the *Sunzi suanjing*, thereby marking an increased understanding of such problems. Some of the classics also dealt with systems of equations and root extractions, but these never went beyond the third degree. Therefore, Qin Jiushao's ability to handle a tenth-degree equation marks a significant advancement in the algebraic abilities of Chinese mathematicians.

Historians have often remarked that the *Shushu jiuzhang* is very similar to the *Nine Chapters*, which is certainly the most influential out of the *Ten Mathematical Classics*. As historian of Chinese mathematics Lam Lay Yong remarked,

> The predominant influence of the [Nine Chapters] on the mathematical thought in China for over a millennium can be compared with the influence of Euclid's "Elements" on geometry in the West. Most Chinese mathematical texts borrow ideas from the [Nine Chapters] in one form or the other...[107]

Like the author of the *Nine Chapters*, Qin Jiushao also divided his treatise into nine separate categories, and he made extensive use of the methods laid out in the *Nine Chapters*. Besides the Rule of Double False Position discussed above, the methods for calculating area and volume, extracting square and cube roots, and solving systems of linear equations used by Qin Jiushao are all covered in the *Nine Chapters*. Qin Jiushao himself was also aware of this. In his preface, he stated that, out of all the methods he used in his treatise, only the *dayan* rule had not been covered in the *Nine Chapters*.[108] Therefore, it seems as though he intended his treatise to be an extension of or complement to the *Nine Chapters*. This is why the *Shushu jiuzhang* is sometimes described as the most representative successor to the *Nine Chapters* among all Chinese mathematical texts.[109]

However, the full significance of the *Shushu jiuzhang* in the history of Chinese mathematics goes beyond its ability to complement the *Nine Chapters*. It has the special status of being one of the handful of works that represent the climax of the history of mathematics in pre-modern China. The sophistication of

---

[107] Lam Lay Yong, "Chu Shih-chieh's *Suan-hsüeh ch'i-meng* (Introduction to Mathematical Studies)," *Archive for History of Exact Sciences* 21 (1979): 12.

[108] Chen, Zhang, and Zhou, 9.

[109] Ibid, 1.

its methods should not be viewed as a logical and inevitable end result of the tremendous influence of the *Nine Chapters*. Rather, the mathematics that Qin Jiushao did was a far and unexpected leap above the foundations laid by the *Nine Chapters*. By suddenly pushing the method of root extraction from the third to the tenth degree, Qin really was doing the unexpected and opened up possibilities of mastering even greater challenges in algebra. Furthermore, by bringing the *dayan* rule from the field of astronomy into mathematics, Qin Jiushao was essentially opening up a new area for investigation in Chinese mathematics. Therefore, rather than viewing the *Shushu jiuzhang* as a mere legacy of earlier works such as the *Nine Chapters*, its own unique legacy for posterity must also be acknowledged, and this will be discussed in the next chapter.

Comparison of the* Shushu jiuzhang *with the Works of the other Song-Yuan Masters

The uniqueness of the *Shushu jiuzhang* can only be fully understood by comparing this treatise with those written by the other three Song-Yuan masters: Li Ye, Yang Hui, and Zhu Shijie. Their works were all completed within the time span from 1247 to 1303, but historians have not been able to find any evidence that they ever met one another. Since they did not make reference to one another's work in their treatises, it is not possible to trace any sign of direct influence among the four. As mentioned before, the treatises written by the four Song-Yuan masters are considered the embodiment of the golden age of Chinese mathematics because the sophistication of their work was unmatched and unsurpassed in China until modern mathematics took hold. A comparison of all these treatises allows us to uncover their common characteristics and to discover how differently Qin Jiushao approached mathematics in contrast to his contemporaries.

Although Qin Jiushao's *Shushu jiuzhang* has the earliest known record of the use of the zero symbol in China, it is unlikely that he was the actual inventor of it. Li Ye also used the symbol in his treatise *Ceyuan haijing* 測圓海鏡 (Sea Mirror of Circle Measurements) which was completed in 1248, just one year after the *Shushu jiuzhang*.[110] He remained in the Hebei 河北 and Shanxi 山西 area throughout his lifetime, which was controlled by the Jurchens followed by the Mongols.[111] In other words, it is highly unlikely that Li Ye could have come into contact with Qin Jiushao's work and learnt the zero symbol from him. Therefore, the use of the zero symbol is probably an influence of Indian mathematics and was probably quite widespread in both northern and southern China by the middle of the thirteenth century.[112] The use of the counting board to solve equations must also have been a widespread phenomenon because it is also evident in the treatises written by Li Ye and Zhu Shijie.

Qin Jiushao engaged in many similar topics as the other three Song-Yuan masters, such as root extraction, solving systems of equations, and geometry with circles and triangles. Although Qin included a few questions on astronomy and the calendar in his treatise, the vast majority of his problems deal with topics that are purely mathematical. This preoccupation with purely mathematical topics is also evident in the works of the other three Song-Yuan masters who are known for their work on solving equations, magic squares, the Pascal triangle, and sum of series. This shows that all four Song-Yuan masters were not

---

[110] Dauben, 323.

[111] Ibid, 323.

[112] Indian mathematical texts have spread to China and been translated into Chinese since at least the Sui dynasty. See Li and Du, 107.

interested in finding new ways in which mathematics could serve astronomy. Instead, they were all pursuing mathematics for its own sake. However, the direction in which Qin Jiushao pursued mathematical work is quite different from the others. The *dayan* rule, which is one of the main highlights of Qin's work, is not covered in any of the treatises by the other three masters probably because the latter did not have a background in astronomy. Therefore, giving the *dayan* rule a place in Chinese mathematics is Qin Jiushao's unique contribution.

Qin Jiushao's Motivation for Writing the Shushu jiuzhang

The other three Song-Yuan masters seemed to have been more prolific than Qin Jiushao: they each wrote more than one mathematical treatise while Qin seems to have written only one. It is known that Li Ye, Yang Hui, and Zhu Shijie each wrote at least one treatise that covered elementary mathematics: Li wrote *Yigu yanduan* 益古演段 (New Steps in Computation) in 1259; Yang completed Yang Hui suanfa 楊輝算法 (Yang Hui's Mathematical Treatise) in 1275; Zhu wrote Suanxue qimeng 算學啓蒙 (Introduction to Mathematical Studies) in 1299. The problems and methods covered in these texts tend to be basic, so it is very likely that they were actually meant to be textbooks or useful reference books for beginning students in mathematics. Qin Jiushao is the only one out of the four who did not write a text for beginners. In the preface to the *Shushu jiuzhang*, Qin stated explicitly that his treatise was meant to be read by learned scholars at their leisure, almost as a recreational hobby.[113] For Qin, mathematics was a very important part of the education of gentlemen. In his preface, he referred to the fact that mathematics had been considered one of the six gentlemanly arts that

[113] Chen, Zhang, and Zhou, 10.

must be mastered by the children of the nobility during the Zhou dynasty.[114] He believed that this was because mathematics had a wide application and was useful in helping humans understand the Way (*dao* 道). He lamented the fact that, after the era of the great mathematicians of the Han dynasty, people began looking down on mathematics, and mathematicians became mere tools to be used, not people to become. This had led to the utilization of old methods by rote rather than with real understanding, thus resulting in the slow development of mathematics. Qin explained that he wrote his treatise after achieving some results from investigating the work of previous scholars, and he hoped that his treatise would be helpful in both practical usage and scholarly research.[115]

Therefore, we can infer from the above that, for Qin Jiushao, writing a mathematical text was not about creating a teaching tool, but about sharing new findings with others who were also interested in the same field. In this sense, we see in Qin Jiushao a true mathematician. However, this attitude of Qin would have negative repercussions on the extent of his influence, and this will be discussed in the next chapter.

---

114 Ibid, 8; Li and Du, 22.

115 Chen, Zhang, and Zhou, 8-10.

Chapter 3: The Influence of Qin Jiushao and the *Shushu jiuzhang*

The full significance of the *Shushu jiuzhang* was not recognized during Qin Jiushao's lifetime. It was an obscure text that failed to spark the interest of Chinese mathematicians until after they began studying European mathematics during the Qing dynasty. Therefore, an interim of almost six hundred years passed before Qin Jiushao's work was finally able to exert its influence. This chapter will discuss both the circumstances that led to this delay and those that led to its far-reaching impact on both mathematics and historical research since then. I will argue that the lack of state support for mathematical education was a major factor in limiting the influence of Qin Jiushao during the Southern Song, Yuan, and Ming dynasties. Another factor is the fact that the problems which he solved were not really necessary for the everyday needs of the people. These problems, while practical and involving agricultural fields, measurements, etc., were not immediate concerns of the ordinary people, especially those living in the cities. Officials did not take an interest in Qin's work either because all the arithmetic that they needed to know were already covered in the *Ten Mathematical Classics*. However, once mathematics finally became recognized as an important subject worth studying, scholars in both the East and the West were quick to recognize the significance of the *Shushu jiuzhang*, thus resulting in the celebrity status Qin Jiushao enjoys in the history of Chinese mathematics and in Sichuan today.

The **Shushu jiuzhang** *in China – From the Thirteenth to the Nineteenth Century*

The means by which the *Shushu jiuzhang* survived until the nineteenth century was through several handwritten copies. There was no printed copy until 1842. The fact that it was not printed really handicapped its ability to circulate during the six centuries immediately after its completion in 1247. Why Qin Jiushao never printed his work remains a mystery, but possible reasons will be explored here. It certainly could not have been due to any technical difficulty or problem in accessing publishing technology. After all, it is known that the printing industry during the Southern Song was extremely well developed and widely available.[116] Although movable type printing technology was already invented in China in the mid-eleventh century, it did not become popularized, and woodblock printing remained the mainstream form of publishing technology.[117] The Song government already had experience printing mathematical texts with woodblock printing when it published the *Ten Mathematical Classics* for distribution throughout the country in 1084. This shows that, in terms of the technical aspects, it was possible to print the *Shushu jiuzhang* when it was completed in 1247. Of course, it would not have been done by the government this time because, as mentioned earlier, the Southern Song government did not consider mathematics a necessary subject for study or a worthwhile area for investment. However, there were two other means by which Qin Jiushao could have had his own treatise printed for wider circulation. One was through a private printer; the other was through a commercial printer.

---

[116] Yang Weisheng, et al. *Liang Song wenhua shi* (History of Song Culture) (Hangzhou: Zhejiang daxue chubanshe, 2008), 407.
[117] Ibid, 424.

Private printing is known to have been undertaken by literati families who were extremely wealthy during the Southern Song dynasty. There were at least forty such families who printed works with their own funds. These works were usually meant to be preserved for posterity rather than sold for profit, so the number of copies printed tended to be small. Therefore, these works were generally limited to the writings of family members, their teachers, and local celebrities.[118]

Rather than investing in private printing, it was also possible to submit manuscripts to commercial publishers to be printed during the Southern Song dynasty. There were over one hundred commercial publishers spread throughout Southern Song territory, with at least twenty concentrated in the Hangzhou area, so Qin Jiushao definitely had access to them when he finished the *Shushu jiuzhang* in Huzhou.[119]

It seems that Qin Jiushao never ventured to print his work through either private or commercial channels. During the Song dynasty, manuscripts for publication had to be approved by the imperial academy beforehand. The works that ended up being denied publication were usually those that might threaten the security of the state or those that were written by people who were considered political renegades. Works dealing with astrology and astronomy were banned from publication in order to prevent the formation of religious cults and the creation of new calendars among the common people.[120] Being a mathematical treatise with no political connotations, astrological pretensions, or astronomical secrets, the *Shushu jiuzhang* itself was not subversive, though the questions

---

[118] Ibid, 422.
[119] Ibid, 423.
[120] Ibid, 430-434; Benjamin A. Elman, *A Cultural History of Civil Examinations in Late Imperial China* (Berkeley: University of California Press, 2000), 465.

dealing with the calendar might have been found problematic by the censor. It is also possible that Qin Jiushao might have had political enemies like Liu Kezhuang who prevented the publication of his work. Another possibility is that commercial publishers did not accept his manuscript for publication because they did not think printing it for sale would be profitable, and that Qin Jiushao was not quite rich enough to have it printed privately. However, based on my analysis in the previous chapter of Qin's reasons for writing the treatise, a more likely reason was that he probably only meant to have it circulate among his own friends and acquaintances whom he knew would be interested in it. This conclusion is in line with recent research that asserts the continued precedence of manuscript culture during the Song dynasty despite the widespread availability of printing technology. Indeed, it was quite common for Song scholars *not* to print their own works.[121] Among the literati, be they either writers or collectors, the manuscript form was deemed to be preferable.[122]

Qin Jiushao's original manuscript is no longer extant. However, in 1249, he allowed Chen Zhensun to copy the *Shushu jiuzhang* into the latter's collection, *Zhizhai shulu jieti* 直齋書錄解題 (Catalogue of the Books of Zhizhai and Explanation of their Contents), which ranks among the highest-quality book collections of the Southern Song.[123] The two of them likely had a close friendship because Chen also received copies of two rare calendars from Qin to be included

[121] Joseph P. McDermott, *A Social History of the Chinese Book: Books and Literati Culture in Late Imperial China* (Hong Kong: Hong Kong University Press, 2006), 47.

[122] Ronald Egan, "To Count Grains of Sand on the Ocean Floor: Changing Perceptions of Books and Learning in the Song Dynasty," in *Knowledge and Text Production in an Age of Print: China, 900 – 1400*, ed. Lucille Chia and Hilde De Weerdt (Leiden: Brill, 2011), 41. Books like the Confucian classics were the exception to this rule because the literati counted on such texts being widely available to help them prepare for the civil service examinations. See McDermott, 43.

[123] Yang et al., 448.

in his collection.[124] In addition, as mentioned earlier, Chen is believed to have recommended Qin to the central government some time between 1247 and 1254. In Chen's collection, the *Shushu jiuzhang* is called the *Shushu dalue* 數術大略 (Outline of Mathematical Methods), which might have been Qin's original title for his treatise. In Zhou Mi's *Guixin zazhi*, the treatise is known as *Shuxue dalue* 數學大略 (Outline of Mathematics).

More copies of the *Shushu jiuzhang* were probably made by various people during the course of the thirteenth and fourteenth centuries because by the time it was compiled into the *Yongle dadian* 永樂大典 (imperial catalogue compiled for Emperor Yongle (r. 1403 – 1425) of the Ming dynasty) between 1403 and 1408, its name had changed again. In the *Yongle dadian*, the *Shushu jiuzhang* is called *Shuxue jiuzhang* 數學九章 (Nine Chapters on Mathematics). A copy of the *Shushu jiuzhang* was also preserved in the *Wenyuange shumu* 文淵閣書目, a catalogue of the Ming imperial library compiled in 1441.

After the turn of the seventeenth century, we have evidence of the *Shushu jiuzhang* circulating more widely in society. The best-known copy made at this time was by Zhao Qimei 趙琦美 (1563 – 1624). In 1616, Zhao added a copy of the *Shushu jiuzhang* to his own book collection and wrote a table of contents for it. His version was copied from the one made by Wang Yinglin 王應遴 (1545 – 1620) based on the *Wenyuange* version. In both versions, Qin Jiushao's treatise is known by its modern name, the *Shushu jiuzhang*.

The compilers of the *Siku quanshu* 四庫全書 (imperial catalogue compiled for Emperor Qianlong 乾隆 (r. 1736 – 1795) of the Qing dynasty), however, went

[124] Xu and Kong, 245.

back to the *Yongle dadian* for their copy of the *Shushu jiuzhang*. Therefore, they used the name "Shuxue jiuzhang." This *Siku quanshu* version, compiled in 1781, is annotated with comments trying to explain Qin Jiushao's methods, but most of these comments only serve to demonstrate the commentator's inability to understand Qin's work.[125]

The *Siku quanshu* was compiled during the time when a renewed understanding of Song-Yuan algebra was only beginning to take shape. Much of this "mathematical renaissance" was due to the influence of Mei Wending 梅文鼎 (1633 – 1721) and his grandson Mei Gucheng 梅瑴成 (? – 1763) who encouraged Qing mathematicians to study and recover old Chinese mathematical texts.[126] Until then, Qing mathematicians had mainly been preoccupied with translating and studying mathematical works imported from Europe by the Jesuits. This interest in mathematics was not simply an influence of the Jesuits, but reflected a growing preoccupation with concrete studies (*shixue* 實學) and with the desire to rid "true" Confucianism of the distorting effects of Neo-Confucianism.[127] When Song-Yuan algebra began to be understood by the Qing mathematicians, they were surprised to discover that China had actually already achieved mathematical results that were comparable to the Western works they were studying. Therefore, Qin Jiushao became of particular interest to them. Biographical and bibliographical information about Qin and his treatise was uncovered by Qian Daxin 錢大昕 (1728 – 1804). In

---

[125] Libbrecht, 43.

[126] Ibid, 44.

[127] Catherine Jami, "Learning Mathematical Sciences during the Early and Mid-Ch'ing," in *Education and Society in Late Imperial China, 1600 – 1900*, ed. Benjamin A. Elman and Alexander Woodside (Berkeley: University of California Press, 1994), 226-227.

1796, Qian's student, Li Rui 李銳 (1768 – 1817), undertook the first mathematical analysis of the *Shushu jiuzhang*. However, Li's work was never published.[128]

In the early nineteenth century, Shen Qinpei 沈欽裴 (fl. early nineteenth century) began working on correcting the mistakes in Zhao Qimei's copy of the *Shushu jiuzhang*. Illness prevented him from finishing it, so his student, Song Jingchang, continued his work and produced a copy of the *Shushu jiuzhang* complete with reading notes. Song Jingchang's work, the *Shushu jiuzhang zaji*, corrects the textual errors that were the result of centuries of corruption due to copyists' errors and the Ming mathematicians' inability to understand the text.[129] In addition to using Shen Qinpei's notes, Song also made use of Li Rui's and Mao Yuesheng's 毛岳生 (1791 – 1841) work when writing the *Shushu jiuzhang zaji*.

Song Jingchang's work and Zhao Qimei's copy that he used were the basis of the first printed version of the *Shushu jiuzhang*. In 1842, Yu Songnian 郁松年 (fl. ca. 1842) had both the *Shushu jiuzhang* and the *Shushu jiuzhang zaji* printed as part of the *Yijia tang congshu* 宜稼堂叢書 (Yijia tang Collectanea). He also wrote an introduction for the *zaji*. This 1842 version became the basis of all subsequent reproductions of Qin Jiushao's treatise.

Why Song-Yuan Algebra Had Such Limited Influence in China

From the above description of the fate of the *Shushu jiuzhang* after it was written, it is evident that Qin Jiushao's work remained very obscure in China during the Yuan, Ming, and early Qing dynasties. The situation of the other three

[128] Libbrecht, 45-46.
[129] Ibid, 37.

Song-Yuan masters was also very similar. Their treatises also did not circulate widely and their methods were very little understood after their death until the mathematical renaissance of seventeenth and eighteenth centuries. Out of the four masters, only Li Ye's treatises were printed shortly after completion. Yang Hui's work was only partially preserved by being compiled into the *Yongle dadian*, and Zhu Shijie's treatises were only rediscovered in China through Korean reprints.[130] Why this collection of outstanding mathematical treatises became the endpoint of mathematical development in China and never led to even more breakthroughs is a puzzling historical question. As mentioned earlier, historians Li Yan and Jean-Claude Martzloff have come up with different explanations for this phenomenon. Li Yan argued that the main reason was that the sophisticated algebra of the four Song-Yuan masters was incompatible with the real-life demands of society at the time. It was very difficult to find real-life situations that required using methods like high-degree root extraction. When compounded with the fact that these Song-Yuan treatises could not be easily understood, this led to a general reluctance to learn the mathematics of the Song-Yuan masters, resulting in it soon becoming forgotten.[131]

Martzloff agreed with Li Yan that the Song-Yuan masters' treatises were too difficult to understand without oral instruction and that the problems solved by them were not useful economically. However, for Martzloff, a major reason for the discontinuity in the development of mathematics in China was the insistence on solving real problems. This made the problems presented in treatises like the

[130] Ibid, 6.
[131] Li Yan, *Zhongguo gudai shuxue jianshi*, 206.

Shushu jiuzhang too specific for later mathematicians to generalize the methods for wider application.[132]

I think both Li's and Martzloff's analyses are valid. There was certainly a discrepancy between the research interests of the Song-Yuan masters and the everyday demands of society at the time. Even though the situations described in their problems are tied to real life, solving these problems was not a necessity in real life. For example, if we consider problem 5 from Qin Jiushao's *dayan* section discussed in the previous chapter, we can see that the situation described here of farmers taking their harvested rice to various places to be sold is something that could easily happen in real life. However, in real life, the farmers would already have known how much rice they were starting out with before dividing it amongst themselves to be sold. In other words, this question would never have come up in reality, and Qin Jiushao was actually creating an artificial problem that he could solve with his *dayan* rule. Given the artificial nature of these "real-life" problems, it is easy to see the logic of Martzloff's observation that it was very difficult to apply Qin Jiushao's groundbreaking methods like the *dayan* rule to other problems. The same can be said of the other three Song-Yuan masters. Their sophisticated methods for solving systems of equations and finding the sum of series, and their interesting work on magic squares and the Pascal Triangle simply did not have an application in everyday life, but belonged to the realm of pure mathematical research.

Therefore, what we see in China during that time was that basic mathematical texts circulated more widely than the sophisticated ones. Texts like the *Nine Chapters* continued to be used as mathematical textbooks and be influential on new generations of mathematicians. Also, we see a new type of

---

[132] Martzloff, 20, 55.

mathematical text gaining popularity. Due to the growing economic prosperity and the increase in trade, people looked for faster calculation methods, and the abacus came into widespread use. Therefore, new texts dealing with calculating with the abacus emerged during the fourteenth century and became popular.[133] In addition, the simpler texts written by Yang Hui and Zhu Shijie, *Yang Hui suanfa* and *Suanxue qimeng*, also proved to be more popular than their more sophisticated works. These texts made their way into Korea very early and were used in the civil service examinations there.[134]

However, the above analysis only presents part of the reason why Chinese mathematics went into decline after the four Song-Yuan masters. The inherent difficulty of the Song-Yuan texts, the incompatibility between Song-Yuan algebra and real-life requirements, and the difficulty of applying Song-Yuan algebra to a wider class of problems only serve to demonstrate that the Chinese had little internal motivation to continue the work of Qin Jiushao and the other masters. I believe another crucial part of the reason for this decline was the lack of external structure to promote mathematical development. In other words, I argue that the lack of state support for mathematical education during this time was an important factor that led to the decline of Chinese mathematics.

The government had traditionally played the central role in the collection, preservation, and teaching of mathematical texts. Mathematics began to be taught at the imperial academy during the Sui dynasty. During the Tang, an official set of mathematical textbooks was prescribed for the first time in Chinese history by Emperor Gaozong to be used in the imperial academy and across the whole country. The *Ten Mathematical Classics*, which include treatises dating

---

[133] Li Yan, *Zhongguo gudai shuxue jianshi*, 206-207.
[134] Martzloff, 106.

from as early as the Han dynasty, were selected and annotated by court mathematician Li Chunfeng 李淳風 (602 – 670) and others, then approved for use as textbooks. The Tang government also provided the motivation for the sons of lower officials and commoners to learn mathematics by making it one of the subjects of the civil service examinations. Candidates who passed the *mingsuan* 明算 (understanding mathematics) examinations were allowed to enter the bureaucracy as the lowest-ranked officials.[135]

In 1084, the Northern Song government demonstrated its concern for mathematical education by printing the *Ten Mathematical Classics* and distributing them to schools throughout the country. This is undoubtedly the main reason why these texts circulated so widely and survived so well into the modern era. However, the government did not start another movement to collect and preserve more mathematical texts to be used as textbooks. Therefore, most of the mathematical texts written since the *Ten Mathematical Classics* became rare items that easily slipped into oblivion.

It is not a coincidence that Chinese mathematics experienced its golden age and started its decline within two centuries of the government ending its support of mathematical education in 1127. The teaching of mathematics had been sponsored by the government during the Sui, Tang, and Northern Song dynasties. With such a long tradition of promoting mathematical education and cultivating mathematical students, it is not surprising that mathematics continued to thrive in limited circles up to the early fourteenth century. From Yang Hui's and Zhu Shijie's treatises, we find references to many mathematical texts that were probably produced in the twelfth and early thirteenth centuries. Although

[135] Li and Du, 105-106.

these texts are no longer extant, we know that their difficulty level was somewhere between the *Ten Mathematical Classics* and the Song-Yuan masters' treatises, thus forming a bridge between the foundational texts and the most sophisticated ones.[136] This shows that the mathematical breakthroughs of the Song-Yuan masters did not emerge out of the blue, but were the result of a large body of accumulated mathematical research. Although Qin Jiushao did not make references to older texts except for the *Nine Chapters*, he did state in his preface that he had consulted various scholars on mathematics before writing his treatise.[137] Therefore, it is evident that Qin had also relied on scholarship that had accumulated up to 1247. However, because the government did not endeavour to collect, preserve, and teach these new texts, these important stepping stones that led up to the golden age of Chinese mathematics were soon lost, thus denying people the means required to understand the texts of the Song-Yuan masters. With this gap in knowledge, it becomes clear why few people during the Yuan and Ming dynasties were able to make the intellectual leap to understand Song-Yuan algebra. This is why the works of the Song-Yuan masters were not able to push Chinese mathematics to even greater heights of sophistication and why they became known as the terminal point in the development of Chinese mathematics.

If, on the other hand, the government had continued to maintain mathematical education and to collect newly written mathematical texts to be used as textbooks, then the history of mathematical development in China would definitely have been very different. If the government had utilized its immense

[136] See Ho and He, 118-122, and Yun Feng, *Zhongguo Yuandai keji shi* (History of Technology in Yuan China), in vol. 14 of *Baijuanben Zhongguo quanshi* (Complete History of China), ed. Shi Zhongwen and Hu Xiaolin (Beijing: Renmin chubanshe, 1994), 45.

[137] Chen, Zhang, and Zhou, 10.

publishing resources at the state-run schools and local government centres to preserve new mathematical texts, then far fewer texts would have become lost. If the government had continued to collect new mathematical texts and have them taught at the state-run schools, then practicality and economic usefulness would not have been the sole factors that determined whether or not a mathematical text would be passed onto posterity. After all, mathematics should not just be valued for its ability to solve real problems encountered in everyday life. Chinese mathematicians of the late Ming and early Qing period realized this as they poured more and more energy into studying the abstract mathematics imported from Europe. After the works of the Song-Yuan masters were rediscovered and re-understood, they came to be highly valued, not because the Qing mathematicians had suddenly found practical usage for them, but because the mathematics done by the Song-Yuan masters was in no way inferior to the modern mathematics from the West. Therefore, if the government had continued to promote mathematical development by preserving texts and having them taught, then Qin Jiushao's *Shushu jiuzhang* would have been able to fulfil its potential much earlier as one of China's proudest mathematical achievement in her intellectual exchange with other countries.

The* Shushu jiuzhang *in East Asia

Although Korea, Japan, and Vietnam all have a long history of importing books from China either through official diplomatic exchange or through trade, the spread and influence of the *Shushu jiuzhang* were most evident in Korea. In 1858 and 1867, Korean mathematician Nam Bying-gil[138] 南秉吉 (1820 – 1869)

[138] Also written as Nam Pyong-Gil

wrote treatises that included discussions of Qin Jiushao's work on telemetry and the *dayan* rule. The 1858 treatise focuses on questions about telemetry or measuring at a distance, which correspond to section 4 of the *Shushu jiuzhang*. Nam also included questions of this type taken from the *Nine Chapters* and the *Haidao suanjing*, thus indicating that both of these treatises had spread to Korea and that Nam believed Qin Jiushao's work had something valuable to add to the two classics. In his treatise, Nam quoted section 4 of the *Shushu jiuzhang* in its entirety except for the counting board diagrams, corrected the mistakes in the text he had, and added his own pictorial explanations. An analysis of his work shows that the original copy of the *Shushu jiuzhang* that he used was different from both the *Siku quanshu* and the 1842 *Yijiatang* copies. This indicates that another version of the *Shushu jiuzhang* produced by another source must have been available to him in Korea.[139]

How Nam came by his copy of the *Shushu jiuzhang* is not certain. There are several possible channels that could have spread the *Shushu jiuzhang* to Korea from the time it was completed in 1247 to the time Nam wrote his explanation in 1858. When Qin Jiushao wrote his treatise, Korea was under the rule of the Koryo 高麗 dynasty (918 – 1392). Official relations between Southern Song China and Koryo had been cut off by that time because Koryo was under the suzerainty of the Jurchen Jin.[140] However, maritime trade carried out by private merchants, especially by the Chinese, had flourished since the eleventh

---

[139] Guo Shirong, "Qin Jiushao Shushu jiuzhang zai Chaoxian bandao de liuchuan yu yingxiang" (Qin Jiushao's Mathematical Treatise in Nine Chapters in Korea), *Nei Menggu shifan daxue xuebao, ziran kexue ban* (Journal of Inner Mongolia Normal University, Natural Sciences edition) 34 (2005): 315.

[140] Gari Ledyard, "Yin and Yang in the China-Manchuria-Korea Triangle," in *China Among Equals: The Middle Kingdom and Its Neighbours, 10th – 14th Centuries*, ed. Morris Rossabi (Berkeley: University of California Press, 1983), 324-325.

century.[141] Regular trade continued between China and Korea throughout the Yuan, Ming, and Qing dynasties. Because the Koreans really valued Chinese books at this time,[142] it is possible that a handwritten copy of Qin Jiushao's *Shushu jiuzhang* might have been brought over by the merchants.

Another possibility is that the *Shushu jiuzhang* could have spread to Korea during the Mongol Yuan dynasty via the Koreans themselves. The Mongols completed their conquest of the Jin in 1234, of Koryo in 1259, and of Southern Song in 1279. The resulting *pax mongolica* across China and Korea relaxed previous travel restrictions and led to even more cultural exchange between the two countries. The Mongols intermarried with the Koryo imperial clan, and all the Koryo kings who ruled after Wonjong 元宗 (r. 1259 – 1274), except the very last, grew up in Beijing.[143] Officials, scholars, monks, and traders from Korea also came over to China, and it is possible that some of them might have come into contact with a copy of the *Shushu jiuzhang* and brought it back to Korea.

Whether or not the *Shushu jiuzhang* gained a significant presence in Japan and Vietnam is less certain. However, it is possible that it might have spread to these two countries through traders. In the case of Japan, it is also possible that the *Shushu jiuzhang* might have spread there via Korea during the Japanese military expeditions of 1592 – 1598.

---

[141] Shiba Yoshinobu, "Sung Foreign Trade: Its Scope and Organization," in *China Among Equals: The Middle Kingdom and Its Neighbours, 10th – 14th Centuries*, ed. Morris Rossabi (Berkeley: University of California Press, 1983), 107.

[142] Yang et al., 807.

[143] Ledyard, 325.

The Shushu jiuzhang *in Europe*

The first European to write about the *Shushu jiuzhang* was probably British missionary Alexander Wylie (1815 – 1887). In 1852, he published a paper in the *North China Herald*, which included a discussion of Qin Jiushao's *dayan* rule. This paper was translated by K.L. Biernatzki (fl. ca. 1856) into German in 1856, and it was this translation that first made Qin Jiushao's work known in Europe.[144] After that, German mathematicians such as L. Mathiessen (1830 – 1906) and M.B. Cantor (1829 – 1920) started doing research on Qin's *dayan* rule. In 1875, Mathiessen pointed out that reasoning behind Qin's *dayan* rule was the same as that behind the method developed in 1801 by C.F. Gauss (1777 – 1855) to solve indeterminate problems. Mathiessen later also worked out a mathematical proof of the *dayan* rule. In 1880, Cantor wrote a very positive evaluation of Qin Jiushao's method. Qin's *dayan* rule was the basis of what would later become known in the West as the Chinese Remainder Theorem.

Qin Jiushao and the Shushu jiuzhang *Today*

Since the nineteenth century, Qin Jiushao and his treatise have continued to generate scholarly interest. Besides the numerous studies referred to above, Chinese scholars have remained very active in promoting research in this particular area. In May of 1987, an international conference was held at the Beijing Normal University to commemorate the 740th anniversary of the completion of the *Shushu jiuzhang*. In April of 2004, a national conference that focused specifically on Qin Jiushao was held in Huzhou.

In Qin Jiushao's father's hometown of Anyue, Sichuan, Qin has gained somewhat of a celebrity status and become a local attraction. His name has

---

[144] Libbrecht, 311.

become a brand name for locally produced food items. In 1993, the government of Anyue approved plans to build a commemorative hall for Qin Jiushao. This hall was completed in 2000.[145]

The educational value of the *Shushu jiuzhang* in the teaching of mathematics today has only recently begun to be explored. In 2006, the results of a pilot study were published in the *International Journal of Science and Mathematical Education*.[146] This study involved giving a group of secondary-school students in Singapore problems taken out of the *Shushu jiuzhang* for them to translate into English and then solve. Although the results of this study could not conclusively prove that including thirteenth-century Chinese mathematics into the curriculum would improve students' cognitive skills and academic achievement, this is certainly an area that should be explored further. Although I argued in the previous chapter that Qin Jiushao did not create the *Shushu jiuzhang* as a teaching tool, using his problems and his methods to teach in schools today will definitely be a valuable supplement to our modern system of mathematical education which still focuses predominantly on the achievements of Western mathematicians.

---

145 Xu and Kong, 266-273.

146 See Ng Wee Leng, "Effects of an Ancient Chinese Mathematics Enrichment Programme on Secondary School Students' Achievement in Mathematics," *International Journal of Science and Mathematical Education* 4 (2006): 485-511.

Conclusion

> Mathematics has a particularly important role in the history of science because the great Scientific Revolution, which brought all modern science, ecumenical and universal, into existence, depended essentially upon the mathematization of hypotheses about Nature, combined with relentless experimentation.
>
> ~ Joseph Needham[147]

My study has explored one piece of the history of Chinese mathematics through a broad investigation of one man and his treatise: Qin Jiushao and the *Shushu jiuzhang*. This topic has proved particularly suitable for tying together history, mathematics, and education because Qin and his treatise have relevance for all three fields. Because available information about Qin Jiushao remains very limited, any biography written about him at this time can only provide a cursory sketch of who he was and how he lived. In chapter 1, I discussed the details of his life that are described in official records and the writings of himself and his contemporaries. His image in historiography remains very controversial due to the lack of indisputable evidence that can prove either his innocence or his villainy. However, I called into question the reliability of Liu Kezhuang's and Zhou Mi's accounts because the circumstances under which they were written are highly suspect. Without evidence to prove them wrong either, I can only suggest that Qin Jiushao's character should be treated with a greater degree of leniency in historiography.

Qin Jiushao was not a professional mathematician even though his principal contribution to Chinese history was in the field of mathematics. In his own perspective, however, he was, first and foremost, a bureaucrat. His entire adult life was dedicated to an official career. For him, mathematics was only a

[147] In "Foreword" of Li and Du

part-time, though important, pursuit. However, in his treatise, we find in him qualities of a true mathematician. He did not believe that mathematics should only serve a utilitarian purpose, but should be pursued for its own sake. This was why he sometimes presented methods and solutions that were much more complicated than necessary to solve his problems. He also did not believe that mathematics could only play a servile role to astronomy. Instead, he demonstrated that mathematics and astronomy were related fields that could benefit from each other by using mathematics to solve calendrical and astronomical problems and by using astronomy's *dayan* rule to solve mathematical problems.

Although Qin Jiushao lamented the low social status of mathematics and mathematicians during his time, that was precisely when Chinese mathematics attained its most glorious achievements, and these are embodied in the treatises written by him and the other three Song-Yuan masters. I argued that the break in mathematical development that occurred after them was partly the result of the earlier break in state-sponsored mathematical education. Because the government stopped providing the structure necessary for promoting mathematical learning and an uninterrupted accumulation of mathematical knowledge, Chinese mathematics went into decline.

The *Shushu jiuzhang*'s value to us today is much more than being a historical text and one of the most representative works of Chinese mathematics. Qin Jiushao's problems and methods can still serve as valuable educational tools for students of mathematics today. It has already been pointed out that some of the methods presented in it predate similar achievements by Western mathematicians. This suggests that, by incorporating some history into mathematical education, students will be able to understand the history of mathematical development worldwide.

Even though my study focuses only on Qin Jiushao and his treatise, my thesis has bearing on our understanding of the development of Chinese mathematics from the eleventh to the nineteenth century. The glory of Song-Yuan mathematics was indirectly the end result of centuries of government investment in mathematical education. Without the government's active involvement in supporting mathematical education through funding classes and printing textbooks, there would certainly have been even fewer people involved in mathematical research as Confucian teachings quickly dominated the civil service examinations. After that involvement ended permanently in the twelfth century, the development of Chinese mathematics became endangered as no one in the private sector had the means of sponsoring widespread mathematical education. Mathematics then became reduced to an area of recreational study that was only pursued by very few interested individuals because it was no longer the means to a career. Even as the circle of mathematicians diminished, the fate of their treatises became further endangered as fewer and fewer people became involved in the preservation and circulation of their work. As a result, the vast majority of Chinese mathematical texts became lost, and people could no longer trace the path that led up to the sophisticated algebraic methods of the Song-Yuan masters. As I argued earlier, Song-Yuan algebra was not simply the result of the personal genius of the four masters, but was also the result of centuries of small mathematical breakthroughs accumulated up to their time. Because we no longer have access to records of these breakthroughs in the mathematical texts written after the *Ten Mathematical Classics*, it becomes clear why people after the early fourteenth century found Song-Yuan algebra so difficult to understand

and yet so particularly ingenious. With the influx of Western mathematics came a renewed ability to understand the algebra of the Song-Yuan masters. Therefore, it was thanks to European algebra that Chinese algebra was finally revitalized and brought back into the discourse of mathematical development worldwide.

Any historical phenomenon as widespread as the decline of Chinese mathematics, however, is usually not due to one single reason. There must have been a variety of factors involved. Some of these have already been explained in previous studies such as the ones by Li Yan and Jean-Claude Martzloff. These studies tend to attribute the reason to the mathematics itself, stating that Song-Yuan algebra was simply too difficult to understand or was not useful enough to be studied. My thesis presents another factor to be considered. Instead of focusing only on the mathematics, I propose that external factors must also be considered in future studies in this area because mathematics, though an objective science, does not exist in a vacuum. Its developmental potential is limited by the boundaries set by the society in which it exists, so its developmental history must always be contextualized within the history of that society.

Bibliography of Works Consulted

Au Yong, Ke-Xin. "The Mathematics and Mathematicians of Thirteenth-Century China." Unpublished paper presented at the World History Association Conference, Beijing, July 7-10, 2011.

Borthwick, Sally. Education and Social Change in China: The Beginnings of the Modern Era. Stanford: Hoover Institution Press, Stanford University, 1983.

Chen, Lifu, et al. Zhongguo kexue zhi fazhan 中国科学之发展 (The Development of Science in China). Taipei: Zhongyang wenwu gongyingshe, 1978.

Chen, Lifu, and Zhao Liangwu. Zhongxi shuxue shi de bijiao 中西数学史的比较 (Comparison of the History of Mathematics Between China and the West). Taipei: Taiwan shangwu yinshuguan, 1991.

Chen, Xinchuan, Zhang Wencai, and Zhou Guanwen. Shushu jiuzhang jinyi ji yanjiu 数书九章今译及研究 (Studies of the Mathematical Treatise in Nine Sections and its Translation into Modern Vernacular). Guiyang, Guizhou, China: Guizhou jiaoyu chubanshe, 1993.

Cheng, Yu, and Zhang Hesheng. Liao Xi Xia Jin Yuan – jin ge tie ma 辽西夏金元一金戈铁马 (History of the Liao, Xi Xia, Jin, and Yuan Dynasties). Vol. 12 of Huashuo Zhongguo 话说中国 (The Story of China). Shanghai: Shanghai wenyi chubanshe, 2003.

Cheng, Yu, and Zhang Hesheng. Song – wencai yu beichuang de jiaoxiang 宋一文采与悲怆的交响 (Song Dynasty – the Symphony of Brilliance and Sadness). Vol. 11 of Huashuo Zhongguo 话说中国 (The Story of China). Shanghai: Shanghai wenyi chubanshe, 2003.

Chia, Lucille. "The Development of the Jianyang Book Trade, Song-Yuan." Late Imperial China 17, no. 1 (June 1996): 10-48.

Dauben, Joseph W. "Chinese Mathematics." In The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook. Princeton: Princeton University Press, 2007.

Davis, Richard L. "Custodians of Education and Endowment at the State Schools of Southern Sung." Journal of Song-Yuan Studies 25 (1995): 95-119.

Deng, Zhicheng. Song Liao Jin Xia Yuan 宋辽金夏元 (History of the Song, Liao, Jin, Xia, and Yuan Dynasties). Vol. 4 of Zhongguo er qian nian shi 中国二千年史 (The Two-thousand Year History of China). Taipei: Zhonghua shuju, 1983.

Ebrey, Patricia Buckley, and Maggie Bickford, ed. Emperor Huizong and Late Northern Song China: The Politics of Culture and the Culture of Politics. Cambridge, Massachusetts: Harvard University Asia Centre, 2006.

Egan, Ronald. "To Count Grains of Sand on the Ocean Floor: Changing Perceptions of Books and Learning in the Song Dynasty." In Knowledge and Text Production in an Age of Print: China, 900 – 1400, edited by Lucille Chia and Hilde De Weerdt. Leiden: Brill, 2011.

Elman, Benjamin A. A Cultural History of Civil Examinations in Late Imperial China. Berkeley: University of California Press, 2000.

Franke, Herbert, ed. Sung Biographies. Wiesbaden: Franz Steiner Verlag GmbH, 1976.

Franke, Herbert, and Denis Twitchett, ed. Alien Regimes and Border States 907 – 1368. Vol. 6 of The Cambridge History of China. New York: Cambridge University Press, 1994.

Fu, Pu. Zhongguo shuxue fazhan shi 中国数学发展史 (History of the Development of Chinese Mathematics). Taipei: Yutai gongsi zhonghua yinshuachang, 1982.

Gernet, Jacques. Daily Life in China on the Eve of the Mongol Invasion 1250 – 1276. Translated by H.M. Wright. London: George Allen & Unwin Ltd., 1962.

Guo, Shirong. "Qin Jiushao Shushu jiuzhang zai Chaoxian bandao de liuchuan yu yingxiang" 秦九韶《数书九章》在朝鲜半岛的流传与影响 (Qin Jiushao's Mathematical Treatise in Nine Chapters in Korea). Nei Menggu shifan daxue xuebao, ziran kexue ban 内蒙古师范大学学报，自然科学版 (Journal of Inner Mongolia Normal University, Natural Sciences edition) 34 (2005): 310-319.

Guo, Zhimeng. "Zhongguo Song Liao Jin Xia keji shi" 中国宋辽金夏科技史 (History of the Technology of the Song, Liao, Jin, and Xia Dynasties). In vol. 12 of Baijuanben Zhongguo quanshi 百卷本中国全史 (Complete History of China), edited by Shi Zhongwen and Hu Xiaolin. Beijing: Renmin chubanshe, 1994.

Han, Xianglin. "Shi xi Qin Jiushao yi jia yu ju Huzhou de yuanyin" 试析秦九韶一家寓居湖洲的原因 (Why did the Family of Qin Jiushao Live in Huzhou?). Journal of Huzhou Teachers' College 23, no. 2 (April 2001): 96-98.

Hang, Kan. Liang Song: zai fanhua zhong chenmo 两宋：在繁华中沉没 (The Song Dynasty: Drowning in the Midst of Extravagance). Vol. 7 of Chinese Civilization in a New Light. Edited by Liu Wei. Hong Kong: Shangwu yinshuguan, 2001.

Hang, Kan. Liao Xia Jin Yuan: caoyuan diguo de rongyao 辽夏金元：草原帝国的荣耀 (The Liao, Xiao, Jin, and Yuan Dynasties: the Glory of the nomadic dynasties). Vol. 8 of Chinese Civilization in a New Light. Edited by Liu Wei. Hong Kong: Shangwu yinshuguan, 2001.

Henthorn, William E. A Guide to Reference and Research Materials on Korean History: An Annotated Bibliography. Honolulu: Institute of Advanced Projects, East-West Centre, 1968.

Ho, Peng Yoke. Li, Qi and Shu: An Introduction to Science and Civilization in China. Seattle: University of Washington Press, 1985.

Ho, Peng Yoke, and He Guanbiao. Zhongguo keji shi gai lun 中国科技史概论 (An Overview of the History of Technology in China). Hong Kong: Zhonghua shuju, 1988.

Hsu, Zhuo-yun. Zhongguo wenhua yu shijie wenhua 中国文化与世界文化 (Chinese Culture and World Culture). Guiyang: Guizhou renmin chubanshe, 1991.

Huang, Chunyan. Gaoli shi shiji gai yao 高丽史史籍概要 (An Annotated Bibliography of Historical Works on Koryo). Lanzhou: Gansu renmin chubanshe, 2007.

Huang, K'uan-ch'ung. Shishi, wenxian yu renwu – Song shi yanjiu lunwen ji 史事，文献与人物－宋史研究论文集 (Dissertations on Song History). Taipei: Dongda tushu gongsi, 2003.

Huang, K'uan-ch'ung. Song shi cong lun 宋史丛论 (Dissertations on Song History). Taipei: Xinwenfeng chuban gongsi, 1993.

Huang, K'uan-ch'ung. "Trends in Historical Research in Taiwan and Mainland China." Translated by David C. Wright. Journal of Sung-Yuan Studies 25 (1995): 265-302.

Jami, Catherine. "Learning Mathematical Sciences during the Early and Mid-Ch'ing." In Education and Society in Late Imperial China, 1600 – 1900, edited by Benjamin A. Elman and Alexander Woodside. Berkeley: University of California Press, 1994.

Jay, Jennifer. A Change in Dynasties. Bellingham, Washington: Centre of East Asian Studies, Western Washington University, 1991.

Jin, Yufu. Song Liao Jin shi 宋辽金史 (History of the Song, Liao, and Jin Dynasties). 3rd ed. Hong Kong: Longmen shudian, 1966.

Katz, Victor, ed. Using History to Teach Mathematics: An International Perspective. Washington, DC: The Mathematical Association of America, 2000.

Ke, Dake. "Zhongguo Song Liao Jin Xia xisu shi" 中国宋辽金夏习俗史 (History of the Customs of the Song, Liao, Jin, and Xia Dynasties). In vol. 12 of

Baijuanben Zhongguo quanshi 百卷本中国全史 (Complete History of China), edited by Shi Zhongwen and Hu Xiaolin. Beijing: Renmin chubanshe, 1994.

Kim, Tai-jin, ed. A Bibliographical Guide to Traditional Korean Sources. Seoul: Asiatic Research Centre, Korea University, 1976.

Korea: Its Land, People and Culture of All Ages. Seoul: Hakwon-Sa Ltd, 1960.

Lam, Lay Yong. "Chu Shih-chieh's Suan-hsüeh ch'i-meng (Introduction to Mathematical Studies)." Archive for History of Exact Sciences 21 (1979): 1-31.

Langlois Jr., John D., ed. China Under Mongol Rule. Princeton: Princeton University Press, 1981.

Ledyard, Gari. "Yin and Yang in the China-Manchuria-Korea Triangle." In China Among Equals: The Middle Kingdom and Its Neighbours, 10th – 14th Centuries, edited by Morris Rossabi. Berkeley: University of California Press, 1983.

Lee, Thomas H.C. "Books and Bookworms in Song China: Book Collection and the Appreciation of Books." Journal of Sung-Yuan Studies 25 (1995): 193-218.

Li, Guohao, Zhang Mengwen, and Cao Tianqin, ed. Explorations in the History of Science and Technology in China. Shanghai: Shanghai Chinese Classics Publishing House, 1982.

Li, Wenlin. Shuxue shi gailun 数学史概论 (Overview of the History of Mathematics). Taipei: Jiuzhang chubanshe, 2003.

Li, Yan, Zhongguo gudai shuxue jianshi 中国古代数学简史 (Simplified History of Ancient Chinese Mathematics). 2nd ed. Taipei: Jiuzhang chubanshe, 1981.

Li, Yan. Zhongguo suanxue shi 中国算学史 (History of Chinese Mathematics). Shanghai: Shanghai shudian, 1938.

Li, Yan, and Du Shiran. Chinese Mathematics: A Concise History. Translated by John N. Crossley and Anthony W.C. Lun. Oxford: Clarendon Press, 1987.

Li, Zhaohua. Zhongguo shuxue shi 中国数学史 (History of Chinese Mathematics). Taipei: Wenjin chubanshe, 1995.

Li, Zefen. Song Liao Jin Yuan lishi lunwen ji 宋辽金元历史论文集 (Dissertations on the History of the Song, Liao, Jin, and Yuan Dynasties). Taipei: Liming wenhua shiye gongsi, 1991.

Libbrecht, Ulrich. Chinese Mathematics in the Thirteenth Century: the Shu-shu chiu-chang of Ch'in Chiu-shao. Cambridge, Massachusetts: MIT Press, 1973.

Lin, Congyuan. Shuxue shi – gudian pian 数学史－古典篇 (History of Mathematics – Classical Period). Hsinchu: Fanyi chubanshe, 1995.

Lin, Mingcan. Zhongguo wenhua shi 中国文化史 (History of Chinese Culture). Taipei: New Wun Ching Developmental Publishing Co., Ltd., 2005.

Liu, Xueyao. Zhongguo wenhua shi jiang gao 中国文化史讲稿 (A Script on Chinese Cultural History). Taipei: Zhishufang chubanshe, 2005.

Liu, Yizhi. Zhongguo wenhua shi 中国文化史 (Chinese Cultural History). Shanghai: Shanghai guji chubanshe, 2001.

Liu, Yingsheng. Yuan shi 元史 (History of the Yuan Dynasty). Vol. 12 of Ershiwu shi xin bian 二十五史新编 (The New Twenty-five Standard Histories). Shanghai: Shanghai guji chubanshe, 2002.

Liu, Zhiqin, and Tingjia Wu. Zhongguo wenhua shi gai lun 中国文化史概论 (Overview of Chinese Cultural History). Taipei: Wenjin chubanshe, 1994.

Lorge, Peter. War, Politics and Society in Early Modern China 900 – 1795. London: Routledge, 2005.

Martzloff, Jean-Claude. A History of Chinese Mathematics. Translated by Stephen S. Wilson. Berlin: Springer-Verlag Berlin Heidelberg, 1997.

McDermott, Joseph P. A Social History of the Chinese Book: Books and Literati Culture in Late Imperial China. Hong Kong: Hong Kong University Press, 2006.

McKnight, Brian E. Village and Bureaucracy in Southern Sung China. Chicago: University of Chicago Press, 1971.

Mikami, Yoshio. The Development of Mathematics in China and Japan. New York: Chelsea Publishing Company, 1913.

Mote, F.W. Imperial China 900 – 1800. Cambridge, Massachusetts: Harvard University Press, 1999.

Needham, Joseph. Mathematics and the Sciences of the Heavens and the Earth. Vol. 3 of Science and Civilization in China. London: Cambridge University Press, 1959.

Needham, Joseph. The Grand Titration: Science and Society in East and West. London: George Allen & Unwin Ltd., 1969.

Ng, Wee Leng. "Effects of an Ancient Chinese Mathematics Enrichment Programme on Secondary School Students' Achievement in Mathematics." International Journal of Science and Mathematical Education (2006) 4: 485-511.

Pratt, Keith. "Politics and Culture within the Sinic Zone: Chinese Influences on Medieval Korea." In Korean History: Discovery of Its Characteristics and Developments, edited by the Korean National Commission for UNESCO. Seoul: Hollym Corporation, 2004.

Qiao, Weiping. "Zhongguo Song Liao Jin Xia jiaoyu shi" 中国宋辽金夏教育史 (History of the Education of the Song, Liao, Jin, and Xiao Dynasties). In vol. 12 of Baijuanben Zhongguo quanshi 百卷本中国全史 (Complete History of China), edited by Shi Zhongwen and Hu Xiaolin. Beijing: Renmin chubanshe, 1994.

Qiu, Xieyou, Zhou He, and Tian Boyuan. Guoxue dao du 国学导读 (Guide to Sinology). Taipei: Sanmin shuju, 1993.

Ren, Shixian. Zhongguo jiaoyu sixiang shi 中国教育思想史 (History of Chinese Educational Thought). Shanghai: Shanghai shudian, 1937.

Rowe, William T. "Approaches to Modern Chinese Social History." In Reliving the Past: the Worlds of Social History, edited by Olivier Zunz. Chapel Hill: University of North Carolina Press, 1985.

Shen, Kangshen, John N. Crossley, and Anthony W.C. Lun. The Nine Chapters on the Mathematical Art: Companion and Commentary. New York: Oxford University Press, 1999.

Shi, Xun, and Zhu Baoshu, ed. Zhongguo Song dai wenhua 中国宋代文化 (Culture of the Song Dynasty). Zhengzhou: Henan renmin chubanshe, 2000.

Shiba, Yoshinobu. "Sung Foreign Trade: Its Scope and Organization." In China Among Equals: The Middle Kingdom and Its Neighbours, 10th – 14th Centuries, edited by Morris Rossabi. Berkeley: University of California Press, 1983.

Song, Lian. Yuan shi 元史 (Standard History of the Yuan Dynasty). Beijing: Zhonghua shuju, 1976.

Tuotuo. Song shi 宋史 (Standard History of the Song Dynasty). Beijing: Zhonghua shuju, 1985.

Wang, Xiangrong. Zhong Ri guanxi shi wenxian lun kao 中日关系史文献论考 (Discussion of the Sources on Sino-Japanese Relations). Changsha: Yuelu shushe, 1985.

Wang, Yi. Menggu Yuan yu Wang shi Gaoli ji Riben de guanxi 蒙古元与王氏高丽及日本的关系 (Relations among the Mongol Yuan, the Koryo Dynasty, and Japan). Taipei: Taiwan shangwu yinshuguan, 1973.

Wilkinson, Endymion. Chinese History: A Manual. Cambridge, Massachusetts: Harvard University Asia Centre, 2000.

Woolf, D.R., ed. A Global Encyclopedia of Historical Writing. New York: Garland Publishing, Inc., 1998.

Wu, Wenjun, ed. Qin Jiushao yu Shushu jiuzhang 秦九韶与数书九章 (Qin Jiushao and the Shushu jiuzhang). Beijing: Beijing shifan daxue chubanshe, 1987.

Xiao, Qiqing. Yuan dai de zuqun wenhua yu keju 元代的族群文化与科举 (The Group Culture and the Civil Service Examinations of the Yuan Dynasty). Taipei: Lianjing chuban shiye gongsi, 2008.

Xu, Pinfang, and Kong Guoping. Zhong shiji shuxue taidou – Qin Jiushao 中世纪数学泰斗—秦九韶. Beijing: Kexue chubanshe, 2007.

Xu, Yiming, Chen Jiangfeng, and Liu Taiheng, ed. Zhongguo wenhua lungang 中国文化论纲 (Discussions of Chinese Culture). Kaifeng: Henan University Press, 1992.

Yabuuchi, Kiyoshi. Zhongguo, kexue, wenming 中国，科学，文明 (China, Science, Civilization). Translated by Liang Ce and Zhao Weihong. Beijing: Zhongguo shehui kexue chubanshe, 1987.

Yang, Weisheng, et al. Liang song wenhua shi 两宋文化史 (History of Song Culture). Hangzhou: Zhejiang daxue chubanshe, 2008.

Yao, Yingting, ed. Song dai wenhua shi 宋代文化史 (History of Song Culture). Kaifeng: Henan daxue chubanshe, 1992.

Ye, Tan, and Jiang Songyan. Song Liao Xia Jin Yuan wenhua shi 宋辽夏金元文化史 (History of the Culture of the Song, Liao, Xia, Jin, and Yuan Dynasties). Shanghai: Dongfang chuban zhongxin, 2007.

Zen, H.C. "Science: Its Introduction and Development in China." In Symposium on Chinese Culture, edited by Sophia H. Chen Zen. Shanghai: China Institute of Pacific Relations, 1931.

Zeng, Jinyi, ed. Zhongxi kexue jishu sixiang bijiao 中西科学技术思想比较 (Comparison of Eastern and Western Scientific Thoughts). Guangzhou: Guangdong gao deng jiaoyu chubanshe, 1993.

Zha, Youliang. "Qin Jiushao zai shuxue shang de gongxian (yi)" 秦九韶在数学上的贡献（一）(The Contribution of Qin Jiushao to Mathematics – part 1).

Journal of Guangxi University for Nationalities (Natural Science Edition) 12, no. 2 (May 2006): 28-36.

Zha, Youliang. "Qin Jiushao zai shuxue shang de gongxian (er)" 秦九韶在数学上的贡献（二）(The Contribution of Qin Jiushao to Mathematics – part 2). Journal of Guangxi University for Nationalities (Natural Science Edition) 12, no. 3 (August 2006): 42-51.

Zhang, Jiaju. Liang Song jingji zhongxin de nan yi 两宋经济重心的南移 (The Southward Move of Song China's Economic Centre). Wuhan: Hubei renmin chubanshe, 1957.

Zhang, Qizhi, ed. Zhongguo sixiang xueshuo shi – Song Yuan juan 中国思想学说史一宋元卷 (History of Chinese Thought – Song and Yuan Dynasties). Guilin: Guangxi shifan daxue chubanshe, 2008.

Zhang, Shengzhen, and Guo Hongmao. Zhong Ri guanxi shi 中日关系史 (History of Sino-Japanese Relations). Beijing: Social Sciences Academic Press, 2008.

Zhou, Baozhu, Yang Qianmiao, and Wang Zengyu. Bei Song shi, nan Song shi 北宋史，南宋史 (History of the Northern Song and the Southern Song). Vol. 10 of Ershiwu shi xin bian 二十五史新编 (The New Twenty-five Standard Histories). Shanghai: Shanghai guji chubanshe, 2002.

Zhou, Liangxiao, and Gu Juying. Yuan dai shi 元代史 (History of the Yuan Dynasty). Shanghai: Shanghai renmin chubanshe, 1993.

Zhou, Mi. Guixin zazhi xuji 癸辛杂识续集 (Miscellaneous Notes from the Guixin Quarter, First Addendum). Beijing: Zhonghua shuju, 1991.

Appendix A: Qin Jiushao's Preface to the *Shushu jiuzhang*, with Translation into English[ϒ]

周教六藝，數實成之。學士大夫，所從來尚矣。其用本太虛生一，而周流無窮，大則可以通神明，順性命；小則可以經世務，類萬物，詎容以淺近窺哉？若昔推策以迎日，定律而知氣。髀矩浚川，土圭度晷。天地之大，囿焉而不能外，況其間總總者乎。爰自河圖、洛書，開發秘奧，八卦、九疇、錯綜精微，極而至於大衍、皇極之用。而人事之變無不該，鬼神之情莫能隱矣。聖人神之，言而遺其粗；常人昧之，由而莫之覺。要其歸，則數與道非二本也。漢去古未遠，有張蒼、許商、乘馬延年、耿壽昌、鄭［元］、張衡、劉洪之論，或明天道，而法傳於後，或計功策，而效驗於時。後世學者自高，鄙不之講，此學殆絕，惟治歷疇人，能為乘除，而弗通於開方衍變。若官府會事，則府史一二紊。算家位置，素所不識，上之人亦委而聽焉。持算者惟若人，則鄙之也宜矣。嗚呼！樂有制氏，僅記鏗鏘，而謂與天地同和者止於是，可乎。今數術之書，尚三十餘家。天象歷度，謂之綴術；太乙、壬、甲，謂之三式，皆曰內算；言其秘也。九章所載，即周官九數，系於方圓者為叀術，皆曰外算，對內而言也。其用相通，不可岐二。獨大衍法不載九章，未有能推之者，歷家演法頗用之，以為方程者誤也。且天下之事多矣，古之人先事而計，計定而行。仰觀俯察，人謀鬼謀，無所不用其謹，是以不愆於成，載籍章章可覆也。後世興事造始，鮮能考度，浸浸乎天紀人事淆缺矣。可不求其故哉？九韶愚陋，不閒於藝。然早歲侍親中都，因得訪習於太史，又嘗從隱君子受數學。際時狄患，歷歲遙塞，不自意全於矢石間。嘗險罹憂，荏苒十祀，心槁氣落，信知夫物莫不有數也。乃肆意其間，旁諏方能，探索杳渺，粗若有得焉。所謂通神明，順性命，固膚末於見，若其小者，竊嘗設為問答，以擬於用。積多而惜其棄，因取八十一題，釐為九類，立術具草，間以圖發之。恐或可備博學多識君子之余觀，曲藝可遂也。原進之於道，倘曰，藝成而下是惟疇人府史流也，烏足盡天下之用，亦無譽焉。時淳祐七年九月魯郡秦九韶敘。

且系之曰：
昆侖磅礴，道本虛一，聖有大衍，微寓於易，奇余取策，
群數皆捐，衍而究之，探隱知原，數術之傳，以實為體，
其書九章，惟茲弗紀，歷家雖用，用而不知，小試經世，
姑推所為，述大衍第一。

七精四穹，人事之紀，追綴而求，宵星晝晷，歷久則疏，
性智能革，不尋天道，模襲何益，三農務穡，厥施自天，
以滋以生，雨膏雪零，司牧閔焉，尺寸驗之，積以器移，
憂喜皆非，述天時第二。

[ϒ] The English translation provided here is a free, rather than a literal, translation.

魁隗粒民，甄度四海，蒼姬井之，仁政攸在，代遠庶蕃，
墾萵日廣，步度庀賦，版圖是掌，方圓異狀，袤窳殊形，
重術精微，孰究厥真，差之毫釐，謬乃千百，公私共弊，
蓋謹其籍，述田域第三。

莫高匪山，莫浚匪川，神禹奠之，積矩攸傳，智創巧述，
重差夕桀，求之既詳，揆之罔越，崇深廣遠，度則靡容，
形格勢禁，寇壘仇墉，欲知其數，先望以表，因差施術，
坐悉微渺，述測望第四。

邦國之賦，以待百事，畡田經入，取之有度，未免力役，
先商厥功，以衰以率，勞逸乃同，漢猶近古，稅租以算，
調均錢谷，河萵之扞，惟仁隱民，猶已溺飢，賦役不均，
寧得勿思，述賦役第五。

物等斂賦，式時府庾，粒粟寸絲，褐夫紅女，商徵邊糴，
後世多端，吏緣為欺，上下俱殫，我聞理財，如智治水，
澄源浚流，維其深矣，彼昧弗察，慘急煩刑，去理益遠，
吁嗟不仁，述錢谷第六。

斯城斯池，乃棟乃宇，宅生寄命，以保以聚，鴻功雉制，
竹個木章，匪究匪度，財蠹力傷，圍蔡而栽，如予西素，
匠計靈台，俾漢文懼，惟武圖功，惟儉昭德，有國有家，
茲焉取則，述營建第七。

天生五材，兵去未可，不教而戰，維上之過，堂堂之陣，
鵝鸛為行，營應規矩，其將莫當，師中之吉，惟智仁勇，
夜算軍書，先計攸重，我聞在昔，輕則寡謀，殄民以幸，
亦孔之憂，述軍旅第八。

日中而市，萬民所資，賈貿墆鬻，利析錙銖，蹛財役貧，
封君低首，逐末兼並，非國之厚，述市易第九。*

The six gentlemanly arts were taught during the Zhou dynasty, and mathematics was one of them. Scholars and officials think mathematics is very important because it has a variety of uses, including discovering the wishes of

* As reproduced in Chen Xinchuan, Zhang Wencai, and Zhou Guanwen, *Shushu jiuzhang jinyi ji yanjiu* (Guiyang, Guizhou: Guizhou jiaoyu chubanshe, 1993), 1-8.

Heaven and planning daily activities. How could mathematics be regarded as a shallow field of knowledge?

In the past, people calculated the length of days, predicted the pattern in weather changes, measured the height of mountains and the width of rivers, and observed the length of shadows to fix the solstices. Everything in the universe is related to numbers. Developing from the obscure and mysterious ways of the *Hetu* and *Luoshu* squares, the eight trigrams and nine domains continued with profound and subtle interchanges, arriving at the *dayan* rule of indeterminate analysis and Shao Yong's 邵雍 Supreme World-Ordering Principles. And thus, we know that no transformation in the world of humans occurs without cause and no situation in the world of ghosts and spirits defies explanation. Although the sages of the past were very wise and understood many things, the words they have left to us are too simplistic, making it difficult for the average person to fully grasp their meaning. The reason why mathematics has such widespread effect is because numbers originate from the same source as the Way.

The Han dynasty is not too far from antiquity. It had men like Zhang Cang, Xu Shang, Chengma Yannian, Geng Shouchang, Zheng Yuan, Zhang Heng, and Liu Hong who understood the Way. They have either left writings for posterity or had great impact during their own times. After them, some scholars became too arrogant and looked down on mathematics as a subject not worth studying or teaching. Only some astronomers and mathematicians still have some knowledge of calculation with numbers, but they do not fully understand the more sophisticated methods like root extraction and *dayan.* People think that only a very small minority of people within the government needs to understand mathematics. The status of mathematicians has always been low because they are seen as mere tools. If a person who makes musical instruments only records the sounds made by the instruments, can we say that he has made beautiful music in harmony with the rest of the world?

There are currently over thirty schools of mathematics. Those that deal with observing heavenly phenomena, calendar-making, and fortune-telling are called *neisuan* (internal calculations) because their methods are not revealed to outsiders. In the *Jiuzhang suanshu* (Nine Chapters on the Mathematical Art), there are topics that deal with surveying and measuring. These are called *waisuan* (external calculations) in contrast to the *neisuan*. However, in practical usage, *neisuan* and *waisuan* are mutually related and should not be treated as two separate types of calculations. Only the *dayan* rule is not recorded in the *Nine Chapters*, and no one has yet discussed its basic method, but it is commonly used in calendar-making. It would be wrong to consider the *dayan* rule the same as the methods to solve determinate equations.

There are many matters in this world. When people of the past wished to resolve some practical questions, they always made a detailed plan first, and then carried it out accordingly. They observed heavenly phenomena and surveyed the land with many effective methods. In order to leave their findings to posterity, they recorded them in writing. However, over time, it becomes more and more difficult to investigate into their writings.

I am only an unintelligent person. However, in my youth, I stayed in the capital with my parents and I had the chance to study under a member of the Board of Astronomy. I also learnt mathematics from a recluse scholar. During the time of troubles with the barbarians, I was at the distant frontier. After experiencing danger for ten years, I grew despondent. However, I believed that everything in the world is related to numbers. Therefore, I visited knowledgeable scholars, investigated their findings, and came to a few of my own results. I did not find any of the big relationships that would allow us to understand the will of Heaven, but I did discover some little ones related to daily life. I have designed them into question-and-answer format for the sake of convenience of usage. Out of the many problems I have accumulated over the years, I have selected eighty-one and grouped them into nine categories. I have also listed the methods needed to solve these problems and included my own detailed workings in the solutions. For some questions, I have also added diagrams. If these problems can become worthy reading material for knowledgeable scholars at their leisure, then my wish is fulfilled. If the world finds my writings useless, that is fully understandable as well. Dated the ninth month of the seventh year of the Chunyou period (1247 CE), by Qin Jiushao of Lujun.

[poetic description of the nine sections of the *Shushu jiuzhang* in parallel prose not translated]

Appendix B: Zhou Mi's Entry about Qin Jiushao in the *Guixin zazhi xuji* (Miscellaneous Notes From the Guixin Quarter, First Addendum), with Translation into English[∂]

秦九韶字道古，秦鳳間人。年十八，在鄉裡為義兵首，豪宕不羈，嘗隨其父守郡。父方宴客，忽有彈丸出父後，眾賓駭愕，莫知其由。頃加物色，乃九韶與一妓狎，時亦抵筵，此彈之所以來也。既出東南，多交豪富。性極機巧，星象、音律、算術，以致營造等事，無不精究。遡嘗從李梅亭學駢儷詩詞，遊戲、毬馬、弓、劍，莫不能知。性喜奢好大，嗜進謀身。或以歷學薦於朝，得對，有奏稿及所述數學大略。與吳履齋交尤稔，吳有地在湖州西門外，地名曾上，正當苕水所經，入城面勢浩蕩。乃以術攫取之，遂建堂其上，極其宏敞：堂中一間，橫亙七丈，求海筏之奇材為前楣，位置皆自出心匠。凡屋脊雨翠搏風，皆以磚為之。堂成七間，後為列屋，以處秀姬、管弦，制樂度曲，皆極精妙，用度無算。將持<u>盍</u>於諸大閫，會其所養兄之子與其所生親子妾通，事泄，即幽其妾，絕其飲食而死。又使一隸偕此子以行，授以毒藥及一劍曰：導之無人之境，先使仰藥；不可，則令自裁；又不可，則擠之於水中。其隸偽許，而送之所生兄之寓鄂渚者，歸告事畢。已而浸聞其實，隸懼而逃，秦並購之，於是罄其所蓄，自行，且求其子及隸，將甘心焉。語人曰：我且賫十萬錢如揚，惟秋壑所以處我。既至，遍謁台幕。洪恕齋勳為憲起而賀曰：比傳令嗣不得其死，今君訪求之，是傳者妄也，可不賀乎。秦不為久之，賈為宛轉得瓊州行未至，怒<u>迓</u>者之不如期，取馭卒戮之。至郡數月，罷歸，所攜獲富。已未透渡，秦喜色洋洋然。既未有省者，則又曰生活皆為人攪了也。時吳履齋在鄞，亟往投之。吳時將入相，使之先行，曰當思所處。秦復追隨之。吳旋得謫，賈當國，徐摭秦事，竄之梅州，在梅治政不輟，竟殂於梅。其始謫梅離家之日，大堂前大楣中斷，人謂不祥。秦亡後，其養子復歸，與其弟共處焉。余嘗聞楊守齋雲，往守霅川日，秦方居家，暑夕與其姬好和於月下，適有僕汲水庭下，意謂其窺己也。翌日遂加以盜名，解之郡中，且自至白郡，就欲黥之。楊公頗知其事，以其罪不至此，遂從杖罪斷遣。秦大不平，然匿怨相交如故，楊知其怨己，每闞其亡而往謁焉，直至替滿而往別之，遂延入曲室，堅欲苛留，楊力辭之，遂薦湯一杯，皆如墨色，楊恐甚，不飲而歸。蓋秦向在廣中，多蓄毒藥，如所不喜者，必遭其毒手，其險可知也。陳聖觀雲。

Qin Jiushao, whose courtesy name is Daogu, was from Qinfeng. When he was eighteen *sui* (sixteen or seventeen years old), he was the head of the

[∂] The English translation provided here is a free, rather than a literal, translation.

volunteer militia in his hometown. He helped his father to defend the town, but his character was unrestrained. Once when his father was hosting a feast, a pellet suddenly flew from behind his back, surprising everyone. Upon investigation, it turned out that Qin Jiushao had been playing with a performing girl, and the pellet had flown from their direction.

When Qin Jiushao was in the southeast, he made friends with the rich. He was exceptionally intelligent, and excelled in astronomy, music, mathematics, and construction among other things. He learnt *pianli* prose from Li Meiting. He was also very good at games, polo, archery, and swordplay.

Qin Jiushao was extravagant and boastful by nature. He did everything he could to advance his own career. He was recommended to the imperial court on account of his proficiency in calendrical science. His memorials to the emperor and his mathematical treatise are extant.

He was very close to Wu Lüzhai (Wu Qian). Wu had a piece of land outside the western gate of Huzhou, named Zengshang. The Tiao River flowed through it before entering the city. Qin Jiushao got the land from Wu through trickery and built a great hall on it. One of the rooms in the hall was seventy feet wide. For the front lintels, he used materials from overseas. For the ridges, he used tiles. The hall had seven rooms. The back of the hall had buildings for housing female musicians. Qin wrote marvelous music. His expenses were incalculable.

A monk came begging at the women's quarters, and word leaked out that Qin Jiushao's older brother's son, whom Qin had adopted, was having an affair with Qin's own son's concubine. Qin locked up that concubine and had her starved to death. He then ordered a servant to accompany the nephew on a journey. He gave the servant some poison and a sword, and instructed him to take the nephew to a secluded spot and feed him the poison. If that could not be done, then the servant was to make him commit suicide. If that could not be done either, then the servant had to push him into the water. The servant pretended to obey Qin, but took the nephew to his brother's house instead. After Qin Jiushao learnt the truth, the servant fled out of fear. Qin took out all his savings and went after his nephew and servant himself. He told people, "I have brought 100 000 cash with me, but will only lodge in a ditch." He visited people who were in high positions. Hong Shuzhai congratulated him, "There was a rumour that your [adopted] son was already dead. But since you are looking for him now, that means the rumour was false. Is this not worth congratulating?"

Jia Sidao had Qin Jiushao sent to Qiongzhou to take up a position. Before Qin arrived, he became angered when the people who were supposed to receive him did not come promptly, so he had the government messenger killed. After spending several months at Qiongzhou, he was dismissed. He took tremendous wealth with him on his return home.

When the Mongols crossed the Yangzi River, Qin Jiushao looked pleased. But he soon complained that his means of livelihood had been stolen by others. He then sought out Wu Qian who was in the Yin district. Wu had just been promoted to become a minister, and he promised Qin that he would give him a position. After Wu became demoted, Jia Sidao became in charge of national affairs. He put Qin Jiushao away in Meizhou. While at Meizhou, Qin performed his official duties rigorously and died there. The day he left for Meizhou, the front lintel of his home broke. People said it was an inauspicious sign. After Qin Jiushao died, his adopted son returned and lived with his brother.

Yang Shouzhai said that, one evening, Qin Jiushao was being intimate with one of his women when he saw a servant nearby. Qin thought that the servant had been spying on him, so he accused him of theft the very next day, turned him in, and demanded that the servant be tattooed as punishment. Yang thought that tattooing would be too heavy a punishment, so he sentenced the servant to a beating instead. Qin became very upset and Yang knew that Qin began holding a grudge against him. When Yang's term in office was about to be completed, he visited Qin Jiushao to bid him farewell. Qin refused to let him leave his home, and offered Yang a cup of black liquid. Yang became very afraid and left. Qin Jiushao had collected many poisons while in Guangdong. Those whom he did not like would definitely be harmed by him. This was how dangerous he was. Account related by Chen Shengguan.