Improving AI in Skat through Human Imitation and Policy Based Inference

by

Douglas Rebstock

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Department of Computing Science

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Abstract

Creating strong AI systems for trick-taking card games is challenging. This is mostly due to the long action sequences and extremely large information sets common in this type of game. Thus far, search-based methods have shown to be most effective in this domain.

In this thesis, I explore learning model-free policies for Skat, a popular three player German trick-taking card game. The policies are parametrized using deep neural networks (DNNs) trained from human game data. I produce a new state-of-the-art system for bidding and game declaration by introducing methods to a) directly vary the aggressiveness of the bidder and b) declare games based on expected value while mitigating issues with rarely observed state-action pairs. While bidding and declaration were improved, the cardplay policy performs marginally worse than the search-based method, but runs orders of magnitude faster.

I also introduce the Policy Based Inference (PI) algorithm that uses the resultant model-free policies to estimate the reach probability of a given state. I show that this method vastly improves the inference as compared to previous work, and improves the performance of the current state-of-the-art search based method for cardplay.

Preface

This thesis is based on joint work of which I was the first author. Chapters 3 and 4 are based on joint work with Chris Solinas and Michael Buro [RSB19]. Chapters 5 and 6 are based on joint work with Chris Solinas, Michael Buro and Nathan R. Sturtevant [Reb+19]. The remaining chapters include work from these papers. Both papers will appear in the proceedings of IEEE Conference on Games (CoG) 2019.

Education without values, as useful as it is, seems rather to make man a more clever devil.

– C.S. Lewis

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Chapter 1

Introduction

Since the early days of computing, there has been a strong fascination with games. While settling for a Tic-Tac-Toe solver, Charles Babbage had aims of creating a Chess player back in the 1840s, while Alan Turing hand simulated the first game of computer Chess in the early 1950s [CHH02]. The fascination with applying computers to games has only grown since these early days of computing, and it is an important factor in the field of Artificial Intelligence (AI). Games are the perfect testbed for intelligent systems, since they can be arbitrarily complex and have the power to capture the imagination.

Perfect information games are ones in which all the information is public. Chess is an example of a perfect information game, since the player knows the exact state they are in simply by observing the board. Over the years, there has been great success in creating super-human AI systems for games of this type. Super-human AI systems have been created in games that once were seen as quintessentially human: Logistello for Othello [Bur97], Deep Blue for Chess [CHH02], Chinook for Checkers [Sch+07], and most recently AlphaGo for Go [Sil+16]. Imperfect information games feature private information, so the players do not necessarily know which state they are in. The techniques used for perfect information games are very well understood, but creating strong AI systems becomes much more difficult in the imperfect information domain.

In this thesis, I demonstrate novel methods to improve upon the state-of-the-art AI system for Skat, a popular 3 player trick-taking card game. Like most card games, it is an imperfect information game since the players keep their hands private. Like Contract Bridge, Skat features a bidding and declaration phase, followed by card play. The winner of the bidding becomes the soloist and plays against the other two players who form a team. Skat is a large game in terms of both the size of the information sets, and the number of histories. Informally, an information set is a set of game states that a player cannot tell apart given his observations and histories are the sequence of states and actions. At the beginning of cardplay, the size of an information set can be as large as $\approx 2.8 \cdot 10^9$. Overall, there are $\approx 4.4 \cdot 10^{19}$ terminal histories in the pre-cardplay portion alone and many more when taking cardplay into account.

Recent advances in Poker [BS18; Mor+17] demonstrate that techniques based on counterfactual regret minimization (CFR) [Zin+08] can be effective in large imperfect information games. For these large games, the general approach for using CFR methods is to first abstract the game into a smaller version of itself, solve that, and then map those strategies back to the original game. This process implies a game-specific tradeoff between abstraction size and how well the strategies computed on the abstraction translate to the real game. In Skat, however, the size of the game makes this prohibitively difficult. For example, in Poker where CFR has shown to be effective, before the flop (when the first three cards are revealed) there are 1326 possible combinations of public and private cards from the point of view of one of the players [Joh13]. In Skat, there can be 225,792,840 such combinations from the point of view of the soloist, when only looking at games in which the player picked up the skat. The sheer size difference makes it difficult to construct abstractions that are small enough to use with CFR methods, but expressive enough to capture the per-card dependencies that are vital to success in the full game.

The characteristics in Skat that prevent the application of CFR techniques are common to other popular trick-taking-card games, like Spades, Hearts, and Contract Bridge. The most common approach for these games is to use determinized search algorithms. Determinized search algorithms allow for the application of perfect information algorithms to imperfect information games.

These algorithms are composed of two steps: sampling and evaluation. First, a state is sampled from the player's current information set. The state is then evaluated using a perfect information algorithm such as minimax. While this has been quite successful, open-handed simulations have been criticized across the literature [FB98; RN16] because they assume that a strategy can take different actions in different states that are part of the same information set. Also, with the large size of the games, determinized search algorithms can be very computationally expensive.

Inspired by the recent success in AlphaGo, I explored the use of supervised learning on human data to create neural network parametrized policies for Skat. With these networks, I present a method for varying the aggressiveness of the bidder by viewing the output of the imitation network as a distribution of the aggressiveness of the humans and selecting the action that maps to the desired percentile of bidder aggression. I improve upon the declaration policy performance by accounting for rarely-seen state-action pairs without generating new experience. These contributions lead to a new state-of-the-art bidding system for Skat, and a reasonably strong card player that performs orders of magnitude faster than search based methods.

While these policies can be used directly, they can also be used for inference. Inference is a central concept in imperfect information games. It involves using a model of the opponent's play to determine their private information based on the actions taken in the game so far. Because the states that constitute the player's information set are not always equally likely, inference plays a key role in the performance of determinized search algorithms.

In this thesis, I present an algorithm for performing inference in trick-taking card games called Policy Inference (PI). It works by using models of the opponents policy to estimate the reach probability of a given state by taking the product of all transition probabilities. I use the previously discussed learned policies as my models for the other players, and apply the algorithm to Skat. This leads to improvements over the previous state-of-the-art techniques for inference in Skat, and improved performance of the determinized search based cardplay method.

The rest of this thesis is organized as follows. Firstly, I provide relevant background information and related work. In the next chapter, I present my work on learned pre-cardplay policies in Skat, followed by a chapter on learning cardplay policies. I then present the PI algorithm, and directly demonstrate its effectiveness in inference and how it improves the current-best search based method for cardplay in Skat. I finish the thesis with conclusions and possible future work.

Chapter 2

Background and Related Work

2.1 Background Material

2.1.1 Imperfect Information Games

Perfect information games feature only public information. In these games, all players know the exact state, s, they are in, so they can directly evaluate their position. Chess is an example of a perfect information game, since the position of all pieces are always visible. It is not necessary, however, for all game information to be visible at all times. As long as all decisions and states are visible and the players move sequentially, it is perfect information. For instance, a card game in which the initial deal is known and all actions are public is a perfect information game, since with perfect recall, all information is public. Recall is the ability to retain information from previous states.

Tree search is commonly used in solving perfect information games. In tree search, the player evaluates different sequences of moves that follow from the current state, and selects an action based on the result of this search. Figure 2.1 shows a very simple game that consists of two actions, one taken by Player 1 (P_1) and then the second by Player 2 (P_2). The value of the outcomes are shown at the leaf nodes. For instance, the leftmost leaf node has a value of (10,0) which means P_1 would get a reward of 10 while P_2 would get nothing.

Before deciding to choose A or B, P_1 can reason what would P_2 do if they were to choose either action. This game is trivial, since P_2 would choose $Right$ if P_1 chose A and $Left$ if P_1 chose B. Thus, P_1 can look ahead and see the

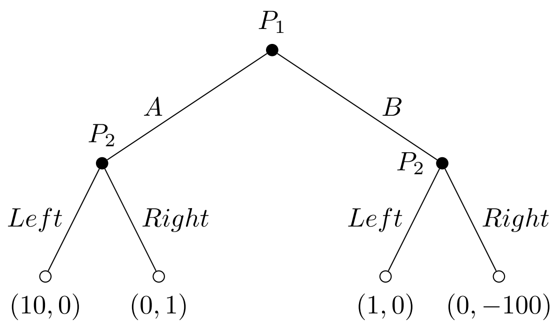


Figure 2.1: Simple Game tree for perfect information game.

only positive outcome happens if they choose B, resulting in a reward of 1. This is a trivial example of lookahead search.

Imperfect information games feature private information. Due to this private information, a player does not always know which state they are in, but only which set of states they could be in which is termed the information set, I. This causes problems with traditional search techniques, since the player cannot directly perform search on the current state if they do not know what the state is. Within this context, a player must infer the probability that they are in a given state. When viewed from the start of the game, this is called the reach probability, η.

Figure 2.2 shows an example of an imperfect information game tree. The only difference in this game is that P_2 does not observe the action of P_1, so P_2 does not know which of the two states they are in. These two states make up an information set for P_2 and it is demarcated with a dashed box in the tree. Suppose you were P_2, and you wanted to do a look ahead search. They only problem is that you do not know what the next state will be, since you don't know where you are. While this problem can be solved using game theory, it would suffice to say that it would be trivial to solve if P_2 knew the probability that they are in any given state. With this knowledge, they could simply find the expected value of moving left or right and select the one with the greater value.

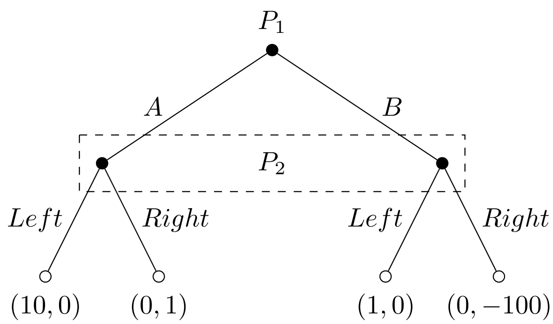


Figure 2.2: An example of an imperfect information game. Tree is similar to Figure 2.1, but now P_2 cannot distinguish between states inside the information set (dotted rectangle) since they could not observe the private move made by P_1.

2.1.2 Trick-taking Card Games

Card games are a prominent example of imperfect information games. Typically, players hold private information in the form of their hands, the hidden cards they hold. Trick-taking card games like Contract Bridge, Skat, and Hearts are card games in which players sequentially play cards to win tricks. A trick leader plays a card, and the remaining players play a card in turn. As the game progresses, information set sizes shrink rapidly due to hidden information being revealed by player actions.

2.1.3 Skat

In this section I provide the reader with the necessary background related to the three player trick-taking card game of Skat. Originating in Germany in the 1800s, Skat is played competitively in clubs around the world. The following is a shortened explanation that includes the necessary information to understand the work presented here. For more in-depth explanation about the rules of Skat, interested readers should refer to `https://www.pagat.com/schafk/skat.html`.

Skat is played using a 32-card deck which is built from a standard 52-card by removing 2,3,4,5,6 in each suit. A hand consists of each of the three players being dealt 10 cards with the remaining two kept face down in the so-called skat.

Games start with the bidding phase. In the cardplay phase, the winner of the bidding phase plays as the soloist against the team formed by the other two players. of the game. Upon winning the bidding, the soloist decides whether or not to pickup the skat followed by discarding two cards face down, and then declares what type of game will be played during cardplay. The game type declaration determines both the rules of the cardplay phase and also the score for each player depending on the outcome of the cardplay phase. Players typically play a sequence of 36 of such hands and keep a tally of the score over all hands to determine the overall winner.

The game value, which is the number of points the soloist can win, is the product of a base value (determined by the game type, see Table 2.1) and a multiplier. The multiplier is determined by the soloist having certain configurations of Jacks and other high-valued trumps in their hand and possibly many game type modifiers explained in Table 2.2. An additional multiplier is applied to the game value for every modifier.

After dealing cards, the player to the right of the dealer starts bidding by declaring a value that must be less than or equal to the value of the game he intends to play — or simply passing. If the soloist declares a game whose value ends up lower than the highest bid, the game is lost automatically. Next, the player to the dealer's left decides whether to accept the bid or pass. If the player accepts the bid, the initial bidder must proceed by either passing or bidding a higher value than before. This continues until one of the player's decides to pass. Finally, the dealer repeats this process by bidding to the player who has not passed. Once two players have passed, the remaining player has won the bidding phase and becomes the soloist. At this point, the soloist decides whether or not to pick up the skat and replace up to two of the cards in his hand and finally declares a game type.

Cardplay consists of 10 tricks in which the trick leader (either the player who won the previous trick or the player to the left of the dealer in the first trick) plays the first card. Play continues clockwise around the table until each player has played. Passing is not permitted and players must play a card of the same suit as the leader if they have one — otherwise any card can be

Table 2.1: Game Type Description

Type	Base Value	Trumps	Soloist Win Condition
Diamonds	9	Jacks and Diamonds	$\geq$ 61 card points
Hearts	10	Jacks and Hearts	$\geq$ 61 card points
Spades	11	Jacks and Spades	$\geq$ 61 card points
Clubs	12	Jacks and Clubs	$\geq$ 61 card points
Grand	24	Jacks	$\geq$ 61 card points
Null	23	No trump	losing all tricks

Table 2.2: Game Type Modifiers

Modifier	Description
Schneider	$\geq$90 card points for soloist
Schwarz	soloist wins all tricks
Schneider Announced	soloist loses if card points < 90
Schwarz Announced	soloist loses if opponents win a trick
Hand	soloist does not pick up the skat
Ouvert	soloist plays with hand exposed

played. The winner of the trick is the player who played the highest card in the led suit or the highest trump card.

In suit and grand games, both parties collect tricks which contain point cards (Jack:2, Queen:3,King:4,Ten:10,Ace:11) and non-point cards (7,8,9). Unless certain modifiers apply, the soloist must get 61 points or more out of the possible 120 card points in the cardplay phase to win the game. In null games the soloist wins if he loses all tricks.

Comparison to Contract Bridge

While Skat is a popular trick-taking card game, more research has been done in Contract Bridge, a game with many similarities to Skat. For this reason, I present a lot of related work in the domain of Contract Bridge. Both are trick-based games featuring bidding and cardplay phases. The numerical Skat bidding seems simpler than Bridge bidding because no bidding conventions have to be followed and there is not much room (and incentive) in Skat to convey hand information to other players. However, Skat adds uncertainty in form of unknown Skat cards in the bidding phase. Other notable differences include cardplay for points in Skat rather than achieving a certain number of tricks, Skat's hidden discard move, and changing coalitions versus fixed 2 vs. 2 team play in Contract Bridge.

2.2 Related Work

In this section, I will present prior work on trick-taking card games, use of supervised learning from human data, inference in search, and finally the specifics of the current state-of-the art computer Skat program.

One WBridge5 [WBr19] and Jack [Sof19] have had recent success in the World Computer Bridge Championship [Fed19], but due to the commercial natures of these AI systems, their implementation details are not readily available.

2.2.1 Determinized Search Techniques

Determinized search techniques seek to apply perfect information algorithms to imperfect information games. In a perfect information game, a player always know which state they are in, so they can perform a tree search rooted at the current state. For an imperfect information game, the player only knows which set of states they could possibly be in, the information set. As seen earlier, this makes the naive application of the search impossible. Determinized search techniques sidestep this issue by assuming a given s within I is the current state (determinizing), and performing search rooted at this state. These search techniques are split into two main parts, sampling and evaluation. Sampling is the repeated determinization of the state, and evaluation is the determination of the values of the possible actions through the use of search.

Long et al. [Lon+10] explain why trick-taking card games are an appropriate setting for determinized search algorithms. These algorithms are considered state-of-the-art in several trick-taking card games.

Perfect Information Monte Carlo Tree Search (PIMC)

PIMC [Lev89] is a form of determinized search in which all possible actions are evaluated by sampling states from the current information state, and averaging

```
PIMC(InfoSet I, int n)
  for a ∈ A(I) do
  | v[a] = 0
  end
  for i ∈ {1..n} do
    s ← Sample(I, p)
    for a ∈ A(I) do
    | v[a] ← v[a] + PerfectInfoVal(s, a)
    end
  end
  return argmax_a v[a]
```

Algorithm 1: vanilla PIMC. In the algorithm, a is action, p is probability, and v is value.

the values over the states which were determined using perfect information algorithms. Algorithm 1 is a basic version of PIMC.

The first successful application of determinized search in a trick-taking card game was GIB [Gin01] in Contract Bridge with the of PIMC. PIMC has shown great success in other trick-taking card games including Skat [Bur+09], and Hearts [Stu08]. The current state-of-the-art for cardplay in Skat uses PIMC [SRB19].

Monte Carlo Tree Search (MCTS)

MCTS [Cou06; KS06] has shown to be a very strong search algorithm for perfect information games, as demonstrated by the strength of the AlphaGo architecture in Go [Sil+16], Chess and Shogi [Sil+17]. Broadly speaking, it works by maintaining search statistics for each node of the tree based upon rollouts that passed through that node. UCT [KS06], a form of MCTS, chooses actions in the rollouts by treating each decision as a bandit problem.

Determinized UCT applies UCT in the imperfect information domain by determinizing the state for each rollout. Determinized UCT was applied to Skat [SBH08] with little success. ISMCTS [CPW12] is the application of MCTS to imperfect information games by maintaining nodes for information sets as opposed to states as is normally seen in MCTS. ISMCTS still relies on determinization of the state for each rollout. Singe Observer(SO)-ISMCTS is the single observer variant, and works under the assumption that all moves are fully observable. Multiple Observer(MO)-ISMCTS takes into account partially observable moves, and maintains separate tree for each distinct observer.

Imperfect Information Monte Carlo Tree Search (IIMC)

Later, Imperfect Information Monte-Carlo Search [FB13] was introduced. This replaces the perfect information evaluation, with an imperfect information evaluation. For example, PIMC could be used for the the evaluation. This was shown to be very effective in Skat [FB13], and is still the strongest card player for Skat. The biggest issue with this method though is that its extremely computationally expensive. Currently, it is not a feasible solution when time is of importance, which is typically the case in an online decision making setting.

Issues with Determinized Search

In perfect information game trees, the values of nodes depend only on the values of their children, but in imperfect information games, a node's value can depend on other parts of the tree. This issue, called non-locality, is one of the main reasons why determinized search has been heavily criticized in prior work [FB98; RN16]. The other main issue identified is strategy fusion. The strategy for an information set should be the same for any state within that set since they are indistinguishable by definition. However, during the determinized rollouts the strategy is now specific to the state. Fusing these strategies together does not necessarily provide a good overall strategy, and clearly does not take into account the roll of hidden information.

2.2.2 Inference in Search

In the context of imperfect information games, inference is the reasoning that leads to the beliefs held on other players' private information. Inference helps with non-locality by biasing state samples so that they are more probable with respect to the actions that the opponent has made. This seems to improve the overall performance of determinized algorithms. However, the gains provided by inference come at the cost of increasing the player's exploitability. If the inference model is incorrect or has been deceived by a clever opponent, using it can result in low-quality play against specific opponents.

Previous applications of determinized search in trick-taking card games acknowledge the relationship between inference and playing performance. In Skat, Kermit [Bur+09; FB13] used a table-based technique to bias state sampling based on opponent bids and declarations. This approach only accounts for a limited amount of the available state information and neglects important inference opportunities that occur when opponents play specific cards. Solinas et al. [SRB19] extend this process by using a neural network to make predictions about individual card locations. By assuming independence between these predictions, the probability of a given configuration was calculated by multiplying the probabilities corresponding to card locations in the configuration. This enables information from the cardplay phase to bias state sampling. While this method is shown to be effective, the independence assumption does not align with the fact that for a given configuration, the probability that a given card is present is highly dependent on the presence of other cards. For instance, their approach cannot capture situations in which a player's actions indicate that their hand likely contains either the clubs Jack or the spades Jack, but not both.

While not providing the likelihood of states, rule based policies can be used to bias the sampling of states in determinized algorithms. Ginsberg [Gin01] uses rule based bidding in GIB to sample states that are consistent with the assumed bidding policy of the other players within the PIMC algorithm. Likewise, Amit and Markovitch [AM06] use a rule based policy for bidding in Bridge to limit the number of inconsistencies in sampled worlds in their proposed determinized search algorithm.

In other domains, Richards and Amir [RA07] model the opponent's policy using a static evaluation technique and then perform inference on the opponent's remaining tiles given their most recent move in Scrabble.

Sturtevant and Bowling [SB06] build a generalized model of the opponent from a set of candidate player strategies. My use of aggregated human data could be viewed as a general model that captures common action preferences from a large, diverse player base.

2.2.3 Counter Factual Regret (CFR)

CFR techniques [Zin+08] work by minimizing counterfactual regret. The counterfactual value, $v^{\sigma}(I)$ is the expected payoff for a player if they took actions to reach I multiplied by the reach probability η, given the strategy profiles σ of all players. $v^{\sigma}(I, a)$ is the same, except that the player takes a at I. The instantaneous regret is the difference of these two values, basically how much the player regrets making the decision to choose a. The counterfactual regret is the sum over these regrets. Regret matching [HM00] is typically used for minimizing regret. CFR is shown to able to find an ϵ-Nash equilibrium for two player zero sum games [Zin+08].

These techniques have proven to be very successful in Poker [BS18; Mor+17]. For larger games, the general approach for using CFR methods is to first abstract the game into a smaller version of itself, solve that, and then map those strategies back to the original game. This was the case for the Poker applications. This is problematic for trick-taking card games, such as Skat. With the vast amount of possible holdings and the highly interactive nature of these hands, creating good abstractions is very difficult. For example, for a given game type, Skat has 225,792,840 possible holdings from the point of view of the soloist after picking up the skat, much larger than the 1326 possible holdings that a player has in Poker. Recent advances in Regression CFR [Wau+15] may provide a means of using functional representation within the CFR framework for Skat, but it is not clear how this would be effectively applied to a game of this size.

2.2.4 Supervised and Reinforcement Learning

With the recent success in machine learning, directly applying supervised and reinforcement learning to games has shown great promise. Within the domain of perfect information games, the foundation of AlphaGo [Sil+16] was built upon policy and value networks trained on expert human data. One prominent use of learning in the context of trick-taking card games is for bidding, with great success demonstrated in Contract Bridge.

Yeh et al. [YHL18] directly use reinforcement learning algorithms to learn a value network for bidding in Bridge, however, this was limited to the case where the opposing team always passes. While this greatly simplifies the problem, it ignores key elements of the game. Rong et al. [RQA19] train a network to predict the individual card locations and use this as an input into the policy network. Both the inference network and the policy network were trained using expert human data, but like [Sil+16], both the networks are refined using reinforcement learning. While showing positive results, both these contributions are evaluated on the double dummy results, a heuristic for actual cardplay strength. This is done to avoid using search based algorithms that are too costly to produce game play.

In the cardplay setting, Baier et al. [Bai+18] leverage policies trained from supervised human data to bias MCTS results in the trick-taking card game Factory Spades. This is similar to our approach in that it uses human data to train an opponent model, but different because their model is not used to infer opponent hidden information.

In the domain of Hanabi, Foerster et al. present Bayesian Auto Decoder (BAD) a method of inference for cooperative multi-agent reinforcement learning problems [Foe+18]. Hanabi is a cooperative card game that features private information, although interestingly enough a player's hand is only kept from themselves. By maintaining a public belief and shared policies, players are able to better infer their holdings and the beliefs of others. While this line of research seems promising, it is not obvious how to apply it to the adversarial setting in Skat and Bridge.

2.2.5 State-of-the-Art in Skat

Previous work on Skat AI has applied separate solutions for decision-making in the pre-cardplay and cardplay phases. The cardplay phase has received the most attention — probably due to its similarity to cardplay in other trick-taking card games.

The current state-of-the-art for the pre-cardplay phase [Bur+09] uses PIMC and evaluates the leaf nodes after the discard phase using the GLEM framework [Bur98]. The evaluation function is based on a generalized linear model over table-based features indexed by abstracted state properties. These tables are computed using human game play data. Evaluations take the player's hand, the game type, the skat, and the player's choice of discard into account to predict the player's winning probability. The maximum over all player choices of discard and game type is taken and then averaged over all possible skats. Finally, the program bids if the such estimated winning probability is higher than some constant threshold.

Despite its shortcomings, PIMC Search [Lev89] continues be the state-of-the-art cardplay method for Skat, with the determinized states solved using minimax search. Inference is done using the card location method described earlier [SRB19] and is used to bias the sampling of the states in the search algorithm, so that the likelihood of sampling the state is proportional to the estimated probability the player is in that state. When time for decision making is not a factor,the current strongest computer Skat player uses IIMC with PIMC used for the evaluation [FB13], however, I do not examine this implementation in my thesis since I find it to be too computationally expensive for an online player.

Together, the described solutions make up Kermit, the current strongest Skat AI that has shown to play at expert human level strength [Bur+09].

Chapter 3

Imitation Based Policies for Pre-Cardplay in Skat

It was the goal of the work in this chapter to produce strong and fast policies for Skat using supervised learning on data from Skat games played between humans. First, I describe the training of imitation networks using the data, and how I use those networks to create pre-cardplay policies for Skat. Then, I present a simple policy that selects the maximum output from the imitation network that corresponds to a legal action. Next, I study the issue of overly conservative bidding that comes as a result of the simple aforementioned policy, and how I can remedy the issue. Finally, I explore using a value network in conjunction with the imitation network for creating the declaration/pickup phases' policies, in order to perform better than using either separately.

3.1 Pre-cardplay Method

The pre-cardplay phase has 5 decision points: max bid for the Bid/Answer Phase, max bid for the Continue/Answer Phase, the decision whether to pickup the skat or declare a hand game, the game declaration and the discard. The bidding phases feature sequential bids between two players, but further bids can only be made if the other player has not already passed. This allows a player to effectively pre-determine what their max bid will be. This applies to both the Bid/Answer and Continue/Answer phases. However, in the Continue/Answer phase remaining players must consider which bid caused a player to pass in the first bidding phase. The Declaration and Discard phases happen simultaneously and could be modelled as a single decision point, but for simplicity's sake I separate them.

For each decision point, a separate DNN was trained using human data from a popular Skat server, DOSKV [DOS18]. The server is open to the public, thus the spread of skill levels is quite large. For discard, separate networks were trained for each game type except for Null and Null Ouvert. These were combined because of their similarity and the low frequency of Ouvert games in the dataset.

The features for each network are one-hot encoded. The features and the number of bits for each are listed in Table 3.1. The Bid/Answer network uses the Player Hand, and Player Position features. The Continue/Answer network uses the same features as Bid/Answer, plus the Bid/Answer Pass Bid, which is the pass bid from the Bid/Answer phase. The Hand/Pickup network uses the same features as Continue/Answer, plus the Winning Bid. The Declare network uses the Player Hand + Skat feature in place of the Player Hand feature, as the skat is part of their hand at this point. The Discard networks use the same features as the Declare network, with the addition of the Ouvert feature, which indicates whether the game is played with the soloist's hand revealed.

Note that the game type is not included in the feature set of the Discard networks because they are split into different networks based on that context. Assuming no skip bids (not normally seen in Skat) these features represent the raw information needed to reconstruct the game state as observed from the player. Thus, abstraction and feature engineering in this approach is limited.

The outputs for each network correspond to any of the possible actions in the game at that phase. The legality of the actions depend on the state. Table 3.2 lists the actions that correspond to the outputs of each network, accompanied by the number of possible actions.

The networks all have identical structure, except for the input and output layers. Each network has 5 fully connected hidden layers and use rectified linear units (ReLU) [NH10]. The network structure can be seen in Figure 3.1. The

Table 3.1: Network input features

Features	Width
Player Hand	32
Player Position	3
Bid/Answer Pass Bid	67
Winning Bid	67
Player Hand + Skat	32
Ouvert	1

Table 3.2: Corresponding actions to Network Outputs

Phase	Action	Width
Bid/Answer	MaxBid	67
Continue/Answer	MaxBid	67
Hand/Pickup	Game Type or Pickup	13
Declare	Game Type	7
Discard	Pair of Cards	496

Table 3.3: Pre-cardplay training set sizes and imitation accuracies

Phase	Train Size (millions)	Train Acc.%	Test Acc.%
Bid/Answer	23.2	83.5	83.2
Continue/Answer	23.2	79.7	80.1
Pickup/Hand	23.2	97.3	97.3
Declare	21.8	85.6	85.1
Discard Diamonds	2.47	76.3	75.7
Discard Hearts	3.13	76.5	75.0
Discard Spades	3.89	76.5	75.5
Discard Clubs	5.07	76.6	75.8
Discard Grand	6.21	72.1	70.3
Discard Null	1.46	84.5	83.2

largest of the pre-cardplay networks consists of just over 2.3 million weights. Tensorflow [Aba+16] was used for the entire training pipeline. Networks are trained using the ADAM optimizer [KB14] to optimize cross-entropy loss with a constant learning rate set to 10^{-4}. The middle 3 hidden layers incorporate Dropout [Sri+14], with keep probabilities set to 0.6. Dropout is only used for learning on the training set. Each network was trained with early stopping [Pre98] for at most 20 epochs. The size of the training sets, and accuracies after the final epoch are listed for each network in Table 3.3. These accuracies appear to be reasonable, given the number of options available at each decision point. The test dataset sizes were set to 10,000. Training was done using a

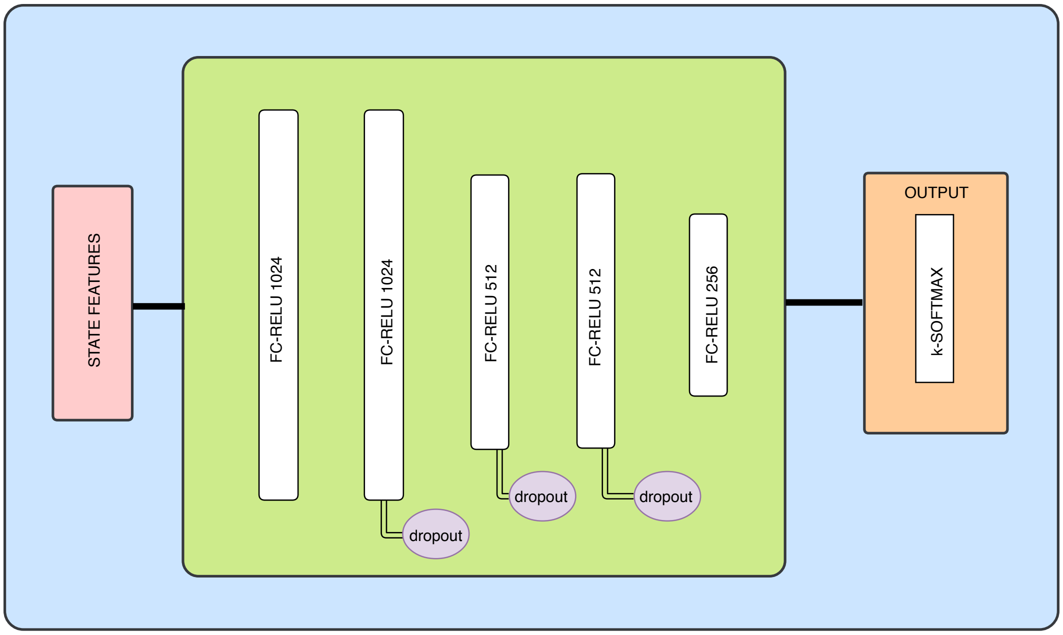


Figure 3.1: Network architecture used across all game types for both soloist and defenders.

single GPU (Nvidia GTX 1080 Ti) and took around 8 hours for the training of all pre-cardplay finalized networks.

These networks are optimized to predict what action a player from this dataset would choose given a representation of the game context as an input, which is done with the goal of using the networks to produce a policy that is representative of the human players. For this reason, I refer to them as imitation networks. One issue is that while the exact actions during the bidding phase are captured, the intent of how high the player would have bid is not. The intent is based largely on the strength of the hand, but how high the player bids is dependent on the what point the other player passed. For example, if a player decided their maximum bid was 48 but both other players passed at 18, the maximum bid reached is 18. For this reason, the max bid data was limited to the two players who passed in the bidding phase. The max bid for these players is known since they either passed at that bid (if they are the player to bid) or at the next bid (if they are the player to answer).

In the approach I chose, the output of the bidding networks corresponds to the maxbid a player reached. The data is only representative of players who passed in the bidding phase, and does not use the data of the bidding winner since the maxbid they intended to reach is not available to train on. The argmax on the outputs provides the most likely maxbid as predicted by the network, but it would not utilize the sequential structure of the bids. What I propose is to take the bid that corresponds to a given percentile, termed A. In this way, the distribution of players aggressiveness in the human population can be utilized directly to alter the aggressiveness of the player. Let B be the ordered set of possible bids, with b_i being the i^{th} bid, with b_0 corresponding to passing without bidding. The maxbid is determined using

$$\text{maxbid}(s, A) = \min(b_i) \text{ s.t. } \Sigma_{j=0}^{i} p(b_j; \theta | I) \geq A \tag{3.1}$$

where $p(b_j)$ is the output of the trained network, which is interpreted as the probability of selecting b_j as the maxbid for the given Information Set I and parameters θ in the trained network. Given the returned maxbid b_i and the current highest bid in the game b_{curr}, the policy for the bidding player is

$$\pi_{bid}(b_i, b_{curr}) = \begin{cases} b_{current+1} & \text{if } b_i > b_{current} \\ \text{pass} & \text{otherwise} \end{cases} \tag{3.2}$$

while the policy for the answer player is

$$\pi_{ans}(b_i, b_{curr}) = \begin{cases} \text{yes} & \text{if } b_i \geq b_{current} \\ \text{pass} & \text{otherwise} \end{cases} \tag{3.3}$$

Another limiting factor in the strength of direct imitation is that the policy is trained to best copy humans, regardless of the strength of the move. While the rational player would always play on expectation, it appears there is a tendency for risk aversion in the human data set. For example, the average human seems to play far fewer Grands than Kermit. Since Grands are high risk / high reward, they are a good indication of how aggressive a player is. To improve the pre-cardplay policy in the Hand/Pickup and Declare phases, one can instead select actions based on learned values for each possible game type. Formally, this policy is

$$\pi_{MV}(I, a; \theta) = argmax(v(I, a; \theta)) \tag{3.4}$$

where v is the output of the trained network for the given Information Set I, action a, and parameters θ, and is interpreted as the predicted value of the game.

Two additional networks were trained, one for the Hand/Pickup phase and one for the Declare phase. These networks were identical to the previous ones, except linear activation units are used for the outputs. The network was trained to approximate the value of the actions. The value labels are simply the endgame value of the game to the soloist, using the TP system. For winning, the value is simply the TP awarded, but for losing the value is the TP lost plus an additional negative 40 points in order to take into account the defenders' bonus for winning.

The loss was the mean squared error of prediction on the actual action taken. For the Hand/Pickup network, the train and test loss were 607 and 623 respectively. For the Declare network, the values were 855 and 898. These values seem quite large, but with the high variance and large scores in Skat, they are in the reasonable range. For instance, the lowest base value for a game is 18. If a player were to win this game, they would be awarded 68 TP. If they were to lose, they would lose 108 TP and both their opponents would gain 40 TP. Altogether, the smallest point swing in terms of relative standings from a win to a loss is 208 points, which creates a potential for high values for the squared error loss.

The π_{MV} seen in Equation 3.4 is problematic. The reason for this is that many of the actions, while legal, are never seen in the training data within a given context. This leads to action values that are meaningless, which can be higher than the other meaningful values. For example, Null Ouvert is rarely played, has high game value and is most often won. Thus the network will predict a high value for the Null Ouvert action in unfamiliar situations which in turn are not appropriate situations to play Null Ouvert. This results in an overly optimistic player in the face of uncertainty, which can be catastrophic. This is demonstrated in the results section.

To remedy this issue, I decided to use the supervised policy network in tandem with the value network. The probability of an action from the policy network is indicative of how often the move is expected to be played given the observation. The higher this probability is, the more likely the network has seen a sufficient number of "relevant" situations in which the action was taken. With a large enough dataset, I assume that probabilities above a threshold indicate that there is enough representative data to be confident in the predicted action value. 0.1 was chosen as the threshold. There is no theoretical reason for this exact threshold, other than it is low enough that it guarantees that there will always be a value we are confident in. Furthermore, the probability used is normalized after excluding all illegal actions.

The policy for the Hand/Pickup and Declare phases using the method described above is

$$\pi_{MLV}(I, a; \theta) = argmax(v_L(I, a; \theta)) \tag{3.5}$$

where

$$v_L(I, a; \theta) = \begin{cases} v(I, a; \theta) & \text{if } p_{legal}(a; \theta | I) \geq \lambda \\ -\infty & \text{otherwise} \end{cases} \tag{3.6}$$

in which p_{legal} is the probability normalized over all legal actions and λ is a constant set to 0.1 in our case.

3.2 Bidding Experiments

Since Kermit is the current strongest Skat AI system [Bur+09] [SRB19], it is used as the baseline for the rest of this paper. Since the network-based player learned off of human data, it is assumed that defeating Kermit is indicative of the overall strength of the method, and not based on exploiting it's specific policy.

Because Skat is a 3-player game, each match in the tournament is broken into six games. In a match, all player configurations are considered, with the exception of all three being the same bot, resulting in six games (see Table 3.4). In each game, once the pre-cardplay phase is finished, the rest of the game is played out using Kermit cardplay, an expert level player based on PIMC search which samples 160 worlds — a typical setting. The results for each bot is the resultant average over all the games played. Each tournament was ran

Table 3.4: Player configurations in a single match consisting of six hands (K=Kermit, NW=Network Player)

Game Number	Seat1	Seat2	Seat3
1	K	K	NW
2	K	NW	K
3	K	NW	NW
4	NW	NW	K
5	NW	K	NW
6	NW	K	K

for 5,000 matches. All tournaments featured the same identical deals in order to decrease variance.

Different variations of the pre-cardplay policies were tested against the Kermit baseline. Unless otherwise stated, the policies use the aggressiveness transformation discussed in the previous section, with the A value following the policies prefix. The variations are:

- Direct Imitation Max **(DI.M)**: selects the action deemed most probable from the imitation networks
- Direct Imitation Sample **(DI.S)**: like DI.M, but samples instead of taking the argmax
- Aggressive Bidding **(AB)**: like DI.M, but uses the aggressiveness transformation in bidding
- Maximum Value **(MV)**: like AB, but selects the maximum value action in the Hand/Pickup and Declare phases
- Maximum Likely Value **(MLV)**: like AB but uses the maximum likely value policy, π_{MLV}, in the Hand/Pickup and Declare phases

While the intuition behind the aggressiveness transformation is rooted in increasing the aggressiveness of the bidder, the choice for A is not obvious. MLV and AB were investigated with A values of 0.85, 0.89, 0.925. Through limited trial and error, these values were chosen to approximately result in the player being slightly less aggressive, similarly aggressive, and more aggressive than

Kermit's bidding, as measured by share of soloist games played in the tournament setting. MV was only tested with A of 0.89 since it was clear that the issue of overoptimism was catastrophic.

An overview of the game type selection breakdown is presented in Table 3.5, while an overview on the performance is presented in Table 3.6. To measure game playing performance I use the Fabian-Seeger tournament point (TP) scoring system which awards the soloist (50 + game value) points if they win. In the case of a loss, the soloist loses (50 + 2· game value) points and the defenders are awarded 40 points. All tournament points per game **(TP/G)** difference values reported were found to be significant, unless otherwise stated. These tests were done using Wilcon signed-rank test [WKW70], with a significance level set to p=0.01. This test is valid for paired data that is on a ratio scale and it does not assume a normal distribution. The data is paired since the deal is the same for all games within a match and a ratio scale is used for the scoring, thus the test is appropriate.

Clearly, naively selecting the max value (MV) in the Hand/Pickup and Declare phases causes the bot to perform very poorly as demonstrated by it performing -75.6 TP/G worse than the Kermit baseline. It plays 96% of its suit games as hand games, which is extremely high to the point of absurdity. In the previous section, I explained how actions that are not representative of normal play in the dataset provide values that are overly optimistic, and these results align with this explanation.

Direct Imitation Argmax (DI.M) performed much better, but still performed slightly worse than the baseline by 2.1 TP/G. Direct Imitation Sample (DI.S) performed 4.2 TP/G worse than baseline and slightly worse than DI.M. The issue with being overly conservative is borne out for both these players with the player being soloist approximately half as often as Kermit.

The direct imitation with the aggressiveness transformation (AB) performed better than Kermit for the lower values, but slightly worse for AB.925. None of these values were statistically significant. The best value for A was 0.85 (AB.85) which leads to +0.4 TP/G against Kermit. The advantage decreases with increasing A values. At the 0.85 A value, the player is soloist a fewer of 1.81 times per 100 games played, indicating it is a less aggressive bidder than Kermit.

The players selecting the max value declarations within a confidence threshold (MLV) performed the best overall, outperforming the AB players at each A value level. The best overall player against Kermit is the MLV.85 player. It outperforms Kermit by 1.2 TP/G, 0.8 TP/G more than the best AB player.

The actual breakdown of games is quite interesting, as it shows that the AB and MLV players are markedly different in their declarations. Across the board, AB is more conservative as it plays more Suit games and less Grand games (worth more and typically more risky) than the corresponding MLV player.

These results indicate that the MLV method that utilizes the networks trained on human data provides the new state-of-the-art for Skat bots in pre-cardplay.

Table 3.5: Game type breakdown by percentage for each player, over their 5,000 match tournament. Soloist games are broken down into types. Defense games (Def) and games that were skipped due to all players passing (Pass) are also included. The K vs X entries list breakdowns of Kermit playing against player(s) X with identical bidding behavior.

Match	Grand	Suit	Null	NO	Def	Pass
DI.S	6.8	17.1	1.2	0.8	68.9	5.2
DI.M	6.6	16.0	0.9	0.9	69.3	6.3
MV.89	8.2	25.4	0.0	0.3	64.8	1.4
MLV.85	10.6	19.0	1.6	1.3	65.6	1.9
MLV.89	11.0	19.8	1.7	1.4	64.8	1.4
MLV.925	11.6	20.5	1.8	1.5	63.6	1.1
AB.85	8.7	21.5	1.1	1.2	65.6	1.9
AB.89	9.2	22.3	1.1	1.2	64.8	1.4
AB.925	9.7	23.1	1.2	1.3	63.6	1.1
K vs DI.S	10.7	21.0	3.8	2.0	57.1	5.5
K vs DI.M	10.7	21.6	3.8	2.0	55.5	6.4
K vs *.85	11.0	17.5	2.5	1.8	64.9	2.4
K vs *.89	10.9	16.8	2.2	1.8	66.4	1.9
K vs *.925	11.0	15.8	2.0	1.7	68.0	1.5

Table 3.6: Tournament results over 5,000 matches between learned pre-cardplay polices and the baseline player (Kermit). All players use Kermit's cardplay. Rows are sorted by score difference (TP/G=tournament points per game, S=soloist percentage) Starred TP/G diff. values were not found to be significant at p=0.01.

Player (P)	TP/G(P)	TP/G(K)	diff.	S(P)	S(K)	diff.
MV.89	-41.3	34.3	-75.6	33.8	31.7	2.1
DI.S	19.2	23.3	-4.1	25.9	37.2	-11.3
DI.M	20.6	22.7	-2.1	24.3	38.1	-13.8
AB.925	22.4	22.6	-0.2*	35.3	30.5	4.9
AB.89	22.9	22.5	0.4*	33.8	31.7	2.1
AB.85	23.0	22.6	0.4*	32.5	32.8	-0.3
MLV.925	23.0	22.3	0.8*	35.3	30.5	4.9
MLV.89	23.3	22.3	1.0*	33.8	31.7	2.1
MLV.85	23.6	22.4	**1.2**	32.5	32.8	-0.3

Chapter 4

Learning Policies from Human Data For Cardplay in Skat

4.1 Cardplay Method

With improved pre-cardplay policies, the next step was to create a strong and fast cardplay policy using the human data. To do this, a collection of networks were trained to predict the human play in the dataset using the same network architecture used for the pre-cardplay imitation networks. Six networks were trained in all; defender and soloist versions of Grand, Suit, and Null. The networks take the game context encoded into features as the input, and predicts the action the player would take as the output.

To capture the intricacies of the cardplay phase, handcrafted features were used — they are listed in Table 4.1. Player Hand represents all the cards in the players hand. Hand Value is the sum of the point values of all cards in a hand (scaled to the maximum possible value). Lead cards represents all the cards the player led (first card in the trick). Sloughed cards indicate all the non-Trump cards that the player played that did not follow the suit. Void suits indicate the suits which a player cannot have based on past moves. Trick Value provides the point value of all cards in the trick (scaled to the max possible value). Max Bid Type indicates the suit bid multipliers that match the maximum bid of the opponents. For example, a maximum bid of 36 matches with both the Diamond multiplier, 9, and the Clubs multiplier, 12. The special soloist declarations are encoded in Hand Game, Ouvert Game, Schneider Announced,

Table 4.1: Network input features

Common Features	Width
Player Hand	32
Hand Value	1
Played Cards (Player, Opponent 1&2)	32*3
Lead Cards (Opponent 1&2)	32*2
Sloughed Cards (Opponent 1&2)	32*2
Void Suits (Opponent 1&2)	5*2
Current Trick	32
Trick Value	1
Max Bid Type (Opponent 1&2)	6*2
Soloist Points	1
Defender Points	1
Hand Game	1
Ouvert Game	1
Schneider Announced	1
Schwarz Announced	1

Soloist Only Features	Width
Skat	32
Needs Schneider	1

Defender Only Features	Width
Winning Current Trick	1
Declarer Position	2
Declarer Ouvert	32

Suit/Grand Features	Width
Trump Remaining	32

Suit Only Features	Width
Suit Declaration	4

and Schwartz announced. Skat encodes the cards placed in the skat by the soloist, and Needs Schneider indicates whether the extra multiplier is needed to win the game. Both of these are specific to the soloist networks. Specific to the defenders are the Winning Current Trick and the Declarer Ouvert features. Winning Current Trick encodes whether the current highest card in the trick was played by the defense partner. Declarer Ouvert represents the soloist's hand if the soloist declared Ouvert. Trump remaining encodes all the trump cards that the soloist does not possess and have not been played, and is used in the Suit and Grand networks. Suit Declaration indicates which suit is trump

Table 4.2: Cardplay train/test set sizes

Phase	Train Size (millions)	Train Acc.%	Test Acc.%
Grand Soloist	53.7	80.3	79.7
Grand Defender	105.4	83.4	83.1
Suit Soloist	145.8	77.5	77.2
Suit Defender	289.1	82.3	82.2
Null Soloist	5.4	87.3	86.3
Null Defender	10.7	72.9	71.6

based on the soloist's declaration, and is only used in the Suit networks. These features are one-hot encoded, except for Trick Value, Hand Value, Soloist and Defender points, which are floats scaled between 0 and 1. The network has 32 outputs — each corresponding to a given card.

The resultant data set sizes, and accuracies after the final epoch are listed in Table 4.2. The test set had a size of 100,000 for all networks. The accuracies are quite high, however, this doesn't mean much in isolation as actions can be forced, and the number of reasonable actions is often low in the later tricks. Training was done using a single GPU (Nvidia GTX 1080 Ti) and took around 16 hours for the training of the finalized cardplay networks. The largest of the cardplay networks consists of just under 2.4 million weights.

4.2 Cardplay Experiments

Bidding policies from the previous chapter, as well as original Kermit bidding, were used in conjunction with the learned cardplay networks. The cardplay policy (C) takes the argmax over the legal moves of the game specific network's output, and plays the corresponding card. AB and MLV bidding policies were tested at all three bidding aggressiveness levels. I also tried sampling of the policy network (CS), to see how it affected the play. The sampling was only done for Kermit bidding as it performed worse than the argmax policy.

Each variant played against Kermit in the same tournament setup from the previous section. Again, all TP/G difference reported were found to be significant. Results are reported in Table 4.3. Again, the Wilcoxon signed-

Table 4.3: Cardplay tournament results over 5,000 matches between bots using pre-cardplay policies from the previous section and the learned cardplay policies. All variants were played against the baseline, Kermit (TP/G=tournament points per game, S=soloist percentage). All difference values Δ were found to be statistically significant.

Player (P)	TP/G(P)	TP/G(K)	Δ	S(P)	S(K)	Δ
K+C	21.9	24.6	-2.6	31.9	31.9	0.0
K+CS	21.1	26.0	-4.9	31.9	31.9	0.0
AB.89+C	22.4	24.5	-2.1	33.8	31.7	2.1
AB.925+C	22.3	24.2	-1.9	35.3	30.5	4.9
AB.85+C	22.6	24.4	-1.8	32.5	32.8	-0.3
MLV.925+C	22.8	24.1	-1.3	35.3	30.5	4.9
MLV.89+C	23.1	24.4	-1.3	33.8	31.7	2.1
MLV.85+C	23.4	22.4	**1.0**	32.5	32.8	-0.3

rank test is used for all TP/G difference values reported with a significance level of 0.01.

The strongest full network player was MLV.85+C, and it outperformed Kermit by 1.0 TP/G. All the rest of the cardplay network players performed worse than Kermit. Like the bidding results, all MLV players performed better than all AB players. Kermit's search based cardplay is quite strong, and it appears to be stronger than the imitation cardplay, as demonstrated by it outperforming K+C by 2.6 TP/G. The sampling variant K+CS performed 4.9 TP/G worse than the search based method. I propose two main explanations for why argmax performs better than sampling; keeping the trajectory within a space closer to the training set and avoiding mistakes. The trajectory explanation comes from the idea that by choosing a more likely action given a world, we have a greater chance of transitioning to a state that resembles worlds that we have seen. The mistake avoidance argument rests on the idea that if human players randomly make mistakes, these mistakes would appear as a noisy training signal. By taking the most likely action, we are effectively filtering out this noise. The overall poorer performance of the cardplay policy is probably due to the effectiveness of search to play near perfectly in the later half of the game when a lot is known about the hands of the players. While these results are compelling, it should be noted that further investigation into the interplay between the bidding and cardplay policies is required to get a better understanding of their strengths. One advantage the imitation network cardplay has is that its much faster, taking turns at a constant rate of around 2.5 ms, as compared to Kermit which takes multiple seconds on the first trick, and an average time of around 650 ms (both using a single thread on consumer-level CPU).

To investigate the assumption that the search based method is stronger in the later stages of the game, I performed experiments using hybridized players. These players used the learned cardplay policy up to a given trick, and the Kermit cardplay after that. Unlike the previous experiments, I used the mirrored adversarial tournament setup. Two games are played per match; player A plays as soloist against two clones of player B and the reverse of that. All matches start at cardplay, and feature the same starting positions that have been created using Kermit pre-cardplay policies. 9 tournaments were ran, with the Kermit policy taking over after varying trick numbers. This ranged from the starting at the first trick, to not starting at all. The results are shown in Table 4.4. Hybrid-1 performs the strongest, with a +0.7 TP/G over Kermit. It is not significant at p = 0.01, however the p value is still quite small at 0.028. As the cardplay policy is used further along into the game past the first trick, the performance decreases. This fits with the theory that search based methods are stronger in later stages of the game.

Since I used the mirrored setup, I could separate the defense and soloist play. In this way, I could see how the combination of different hybridizations for soloist and defense would perform. The difference in TP/G for each combination is shown in Table 4.5, with positive values indicating that the hybridized combination outperforms Kermit. Def0-Sol0 is actually pure Kermit since Kermit takes over prior to the first trick for both soloist and defense. The reason why it performs worse than it's identical counterpart can be attributed to the randomness in the sampling of worlds. The strongest cardplayer combination is Def1-Sol3 which outperforms Kermit by 0.9 TP/G.

Table 4.4: Cardplay tournament results over 5,000 matches between bots using pre-cardplay policies from the previous section and the learned cardplay policies. All variants were played against the baseline, Kermit (TP/G=tournament points per game, S=soloist percentage). Difference values Δ not found to be statistically significant at p=0.01 have been starred.

Player (P)	TP/G (P)	TP/G (K)	Δ TP/G
Hybrid-0	22.1	22.1	-0.1*
Hybrid-1	22.3	21.5	**0.7***
Hybrid-2	22.1	21.7	0.3*
Hybrid-3	22.2	21.9	0.3*
Hybrid-4	21.8	22.4	-0.5*
Hybrid-5	21.5	23.0	-1.5
Hybrid-6	20.6	23.8	-3.1
Hybrid-7	20.0	24.8	-4.7
Hybrid-8	20.0	25.1	-5.2
Hybrid-9	19.8	25.3	-5.5

Table 4.5: Mirrored cardplay tournament results over 5,000 matches between Kermit and hybridized bots. DefX indicates that the hybridized bot used the Kermit card player starting at trick X for defense and the learned policy prior. SolX indicates the same thing, except in the soloist context. All values are how many TP/G the hybridized bot outperformed Kermit. All pre-cardplay done using Kermit.

	Def0	Def1	Def2	Def3	Def4	Def5	Def6	Def7	Def8	Def9
Sol0	-0.1	0.7	0.5	0.2	-0.4	-1.2	-2.1	-3.3	-3.7	-4.0
Sol1	-0.1	0.7	0.5	0.2	-0.4	-1.2	-2.1	-3.3	-3.7	-4.0
Sol2	-0.3	0.5	0.3	-0.1	-0.6	-1.4	-2.3	-3.5	-3.9	-4.2
Sol3	0.1	0.9	0.6	0.3	-0.3	-1.1	-1.9	-3.2	-3.6	-3.9
Sol4	-0.2	0.6	0.4	0.0	-0.5	-1.3	-2.2	-3.4	-3.8	-4.1
Sol5	-0.4	0.4	0.2	-0.2	-0.8	-1.5	-2.4	-3.6	-4.0	-4.3
Sol6	-1.1	-0.3	-0.5	-0.9	-1.5	-2.3	-3.1	-4.4	-4.8	-5.1
Sol7	-1.5	-0.7	-0.9	-1.3	-1.9	-2.7	-3.5	-4.7	-5.2	-5.4
Sol8	-1.5	-0.7	-0.9	-1.3	-1.9	-2.6	-3.5	-4.7	-5.2	-5.4
Sol9	-1.6	-0.8	-1.0	-1.4	-2.0	-2.8	-3.6	-4.8	-5.3	-5.5

Chapter 5

Policy Based Inference in Skat

In the previous two chapters I have demonstrated that neural networks trained off of human data can be used to create strong policies for the game of Skat. While these policies can be used directly, the search based method proves to be stronger in the cardplay portion of Skat. It is the goal of the work in the chapter to improve the strength of the inference used in the search based method. In this section, I present an algorithm that uses these policies to improve inference in these games. As explained earlier, inference is a key part of determinized search and specifically, in the search based cardplay in Kermit.

5.1 Inference Method

To determine the probability of a given state s in an information set I, we need to calculate its reach probability η. If we can perfectly determine the probability of each action that leads to this state, we can simply multiply all the probabilities together and get η. Each s in I has a unique history h, the sequence of all previous s and a that lead to it. $h \cdot a$ represents the history appended with the action action taken at that state. Thus, there is a subset of h, containing all the $h \cdot a$ for a given s. Formally:

$$\eta(s|I) = \prod_{h \cdot a \sqsubseteq s} \pi(h, a) \tag{5.1}$$

For trick-taking card games, the actions are either taken by the world (chance nodes in dealing), other players' actions, and our actions. Transition

```
EstimateDist(InfoSet I, int k, OppModel π)
    S ← SampleSubset(I, k)
    for s ∈ S do
        η(s) ← 1
        for h, a ∈ StateActionHistory(s) do
            η(s) ← η(s) * π(h, a)
        end
    end
    return Normalize(η)
```

Algorithm 2: Estimate the state distribution of an information set given an opponent model and the actions taken so far.

probabilities of chance nodes can be directly computed since these are only related to dealing, and the probability of our actions can be taken as 1 since we chose actions that lead to the given state with full knowledge of our own policy. This leaves us with determining the move probability of the other players. If we have access to the other players' policies, we can use Equation (5.1) to perfectly determine the probability we are in a given state within the information set. If we repeat this process for all states within the information set, we can calculate the probability distribution across states. If we can perfectly evaluate the value of all the state-action pairs, we can select the action that maximizes this expected value which provides an optimal solution.

There are two main issues with this approach. The first is that we either do not have access to the other players' policies, or they are expensive to compute. This makes opponent/partner modelling necessary, in which we assume a computationally inexpensive model of the other players, and use them to estimate the reach probability of the state. The second problem is that the number of states in the information set can be quite large. To get around this, one can sample the worlds and normalize the distribution over the subset of states. Because in Skat the information set size for a player prior to card-play can consist of up to 281 billion states, we employed sampling. I use Algorithm 2 to estimate a state's relative reach probability. When sampling is not needed, this becomes an estimate of the states true reach probability.

While I have access to the policy of the current strongest Skat bot, using its policy directly would be computationally intractable because it uses an expensive search based method that also performs inference. Also, it is the goal of this research to develop robust inference that is not based upon the play of a single player. Thus, I decided to use policies learned directly from a large pool of human players. These policies are parameterized by the deep neural networks trained on human games seen in the previous chapters. The features for the pre-cardplay networks are a lossless one-hot encoding of the game state while considerable feature engineering was necessitated for the cardplay networks. Separate networks were trained for each distinct decision point in the pre-cardplay section, and for each game type for the cardplay networks. More details on the training and the dataset can be found in the previous chapters.

The decision points in pre-cardplay are bidding, picking up or declaring a hand game, choosing the discard, and declaring the game. The decision points in the cardplay section are every time a player chooses what card to play. While inference would be useful for decision-making in the pre-cardplay section, we are only applying it to cardplay in this paper. As such, one can abstract the bidding decisions into the maximum bids of the bidder and answerer in the bid/answer phase, and the maximum bids of bidder and answerer in the continue/answer phase. For these maximum bid decision points, the maximum bid is only observable if the player passes. For the cases in which the intent of maximum bid is hidden, the probability attached to that decision point is the sum of all actions that would have resulted in the same observation, namely the probability of all maximum bids greater than the pass bid. The remaining player decision points are pickup or declare a hand game, discard and declare, and which card to play. As these are not abstracted actions, the probability of the move given the state can be determined directly from the appropriate imitation network detailed in the previous section.

The current state-of-the-art Skat bot, Kermit, uses search-based evaluation that samples card configurations. A card configuration is the exact location of all cards, and thus doesn't take into account which cards where originally present in the soloist's hand prior to picking up the skat. Depending on the game context, there are either 1 or 66 (12 choose 2) states that correspond to a card configuration during the cardplay phase. Two variants of inference were explored. The first variant samples card configurations. Decision points are ignored for inference if there are multiple states with the same card configuration but different features (input to the network). The second variant samples states directly, thus avoiding this issue. For this implementation, the need to distinguish states that share a configuration only occurs when a player does inference on the soloists actions prior to picking up the two hidden cards in the skat. Sampling card configurations will be treated as the default approach for PI. When states are sampled instead, the inference will be labelled PIF, for Policy Inference Full.

5.2 Direct Inference Evaluation

In this section, I test the quality of the inference directly. These inference modules are the original Kermit Inference (KI) [Bur+09], card-location inference (CLI) [SRB19], and no inference (NI).

To measure the inference quality directly, I measured the True State Sampling Ratio (TSSR) [SRB19] for each main game type, separately for defender and soloist. TSSR measures how many times more likely the true state will be selected than uniform random.

$$TSSR = \eta(s^*|I) \ / \ (1/|I|) = \eta(s^*|I) \cdot |I| \tag{5.2}$$

$\eta(s^*|I)$ is the probability that the true state is selected given the information set I, and $|I|$ is the number of possible states. Since the state evaluator of Kermit does not distinguish between states within the same card configuration, I will slightly change the definition to measure how many times more likely the card configuration (world) will be sampled than uniform random.

Since the players tested use a sampling procedure when the number of worlds is too large, the $TSSR$ value cannot easily be directly computed as this would require all the world probabilities to be determined. I therefore estimate it empirically. Since sampling was performed without replacement, I use the given inference method to evaluate η given that the true world was sampled k times. The resulting values can combined to get the combined probability that the true world is selected:

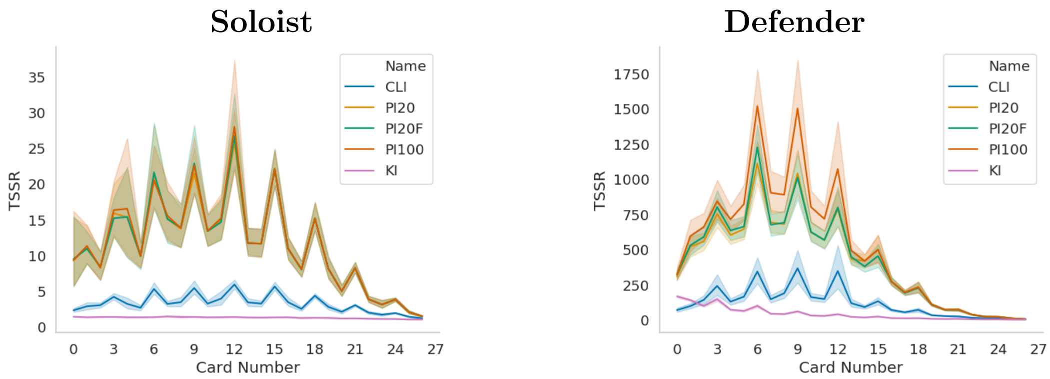
$$TSSR = |I| \cdot \Sigma_k BinDist(k, p) \cdot k \cdot \eta(s^*|I, k) \tag{5.3}$$

where $BinDist$ is the probability mass of the binomial distribution with k successes and probability of sampling the true world p which is $1/|I|$. Terms of the summation were only evaluated if the $BinDist(k, p)$ value were significant, which I cautiously thresholded at 10^{-7}.

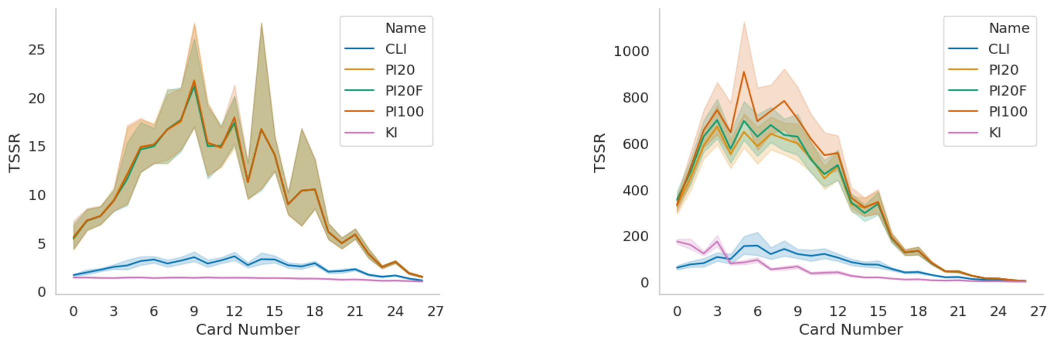
When the number of worlds is less than the set threshold parameter specific to the player, all the worlds are sampled directly to compute the value. For the sake of the TSSR experiments, null games were further subdivided into the two main variants, null and null ouvert. Null ouvert is played with an open hand for the soloist, thus making the inference quite different from that of regular null games from the perspective of the defenders. For each game in the respective test set, the TSSR value was calculated for each move for the soloist, and one of the defenders. The test set was taken from the human data, and was not used in training. The number of games in the test sets were 4,000 for grand and suit, 3,800 for null, and 13,000 for null ouvert.

Figure 5.1 shows the average TSSR metric after varying number of cards have been revealed. The inference variants tested are PI20, PIF20, PI100, CLI, and KI. NI was not tested because it will always have a value of 1. PI20 and PI100 sample 20,000 and 100,000 card configurations respectively, while PIF20 samples 20,000 states. CLI samples 500,000 card configurations, and KI samples 3200 card configurations in soloist, and a varying number in defense. CLI inference was not implemented for null ouvert.

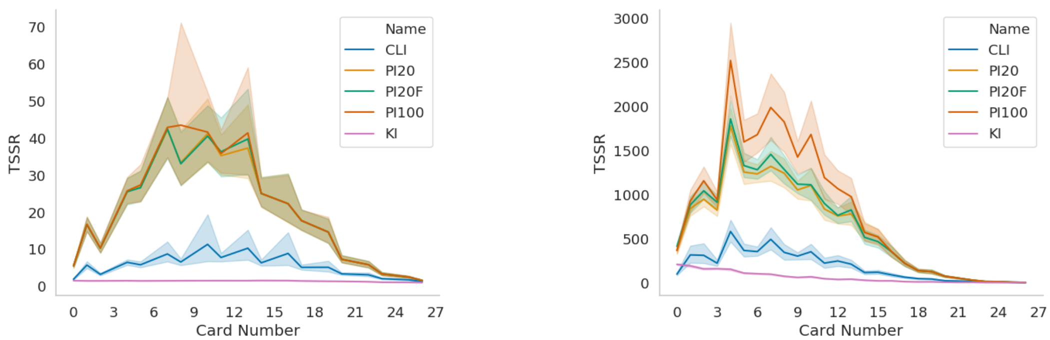
TSSR is higher on defense, with the exception of null ouvert. This is likely due to there being many more possible worlds in the defenders information set because of the hidden cards in the skat. Also, the defender can use the declaration of the soloist for inference, which is a powerful indicator of the soloist's hidden cards. Null ouvert does not follow this trend because there are only 66 possible worlds at most in defense while there are 184,756 for the



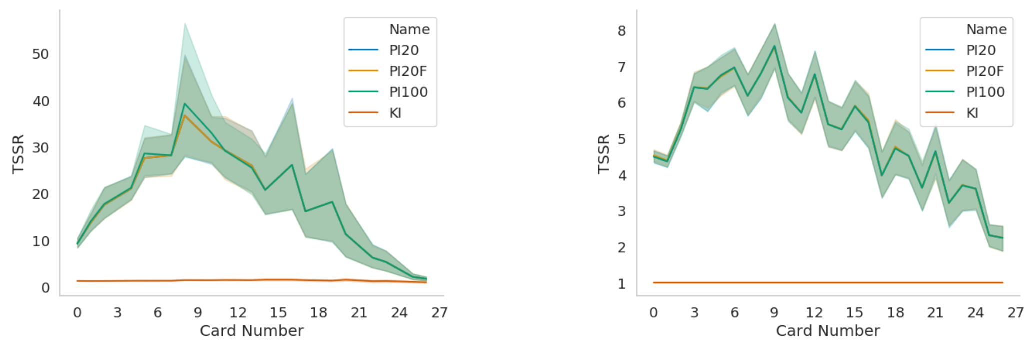
(a) Grand



(b) Suit



(c) Null



(d) Null Ouvert

Figure 5.1: Average *TSSR* after *Card Number* cards have been played. Data is separated by game type and whether the player to move is the soloist (left) or a defender (right).

soloist. This allows for higher TSSR values for the soloist.

PI20, PIF20, and PI100 all achieve significantly higher TSSR values than the other methods, across all game-types and roles. KI performs better than CLI at the beginning of games, but surpasses KI once more cards are played.

PI100 appears to consistently perform better in defense than the other Policy Inference variants, while PIF20 appears to perform slightly better than PI20 in the first half of defender games, but not significantly so. All TSSR values trend down to 1 at the endgame, as the number of possible worlds approaches 1.

One common feature across all games is the spiking of TSSR values, which is best exemplified in suit games. The spiking is consistent between players within the same game type and role graph. However, between graphs it is not consistently occurring at the same number of cards played. I do not see an obvious reason for this. However, these tests were done on human games and thus I am not controlling for inherent biases in the distribution. Further investigation is needed to determine why these spikes occur.

It is clear from these results that the Policy Inference approach provides larger TSSR values, since the error envelopes are completely separated in the figure. It also should be noted that perfect inference would not result in the upper bound TSSR value which is equal to the number of worlds. Even with perfect knowledge of the opponents' policies, uncertainty is inherent and thus a player with perfect TSSR value is not possible.

Chapter 6

Applying Policy Based Inference in Determinized Search

With the vastly improved inference from the previous chapter, I investigate the effect of applying the PI algorithm in the PIMC search that had already been implemented in the Kermit AI system. It is the goal of this work to improve the state-of-the art cardplay system by improving the inference in that system. As explained earlier, the inference can be used to bias the sampling of the states in the determinizing step of PIMC.

6.1 Cardplay Tournament

To test the performance of PI in cardplay, we played 5,000 matches for each of suit, grand, and null games between the baseline players and the PI based player in a pairwise setup. Only the cardplay phase of the game is played, while the bidding and declaration is taken directly from the human data-set. These games were held out from the policy training set. In a match, each player will play as soloist against two copies of the opponent, as well as against two copies of itself. The baseline players are all versions of Kermit, with the only difference being the inference module used. These inference modules are KI, CLI, with the addition of no inference (NI). This experiment is designed to see if (a) the performance of the player improves as measured by its play against opponents and (b) to determine the extent to which the defender and soloist performance is responsible for this difference.

For each match-up I report the average tournament points per game (TP/G) for the games in which the players played against each-other. The games in which the player played against a copy of itself were used to determine the difference in the effectiveness of the defenders and soloists.

$AvBB$ denotes a match-up in which the soloist is of type A while the defenders are both of type B. The value of the game $AvBB$ is in terms of the soloist score, therefore it is the sum of the soloist's score and the negation of the defenders' score. In this notation, the performance of player A relative to player B is given as

$$\Delta TP/G = [AvBB - BvAA]/3 \tag{6.1}$$

The value is divided by 3 since it is enforced that a player is soloist 1/3 the time in the tournament setup. To directly compare the performance of the defenders, one can measure the performance difference between scenarios where the only difference is the change in defenders.

$$\Delta Def/G = [(AvBB + BvBB) - (AvAA + BvAA)]/6 \tag{6.2}$$

A negative value for $\Delta Def/G$ indicates A performs better than B in defense. The same concept can also be applied to directly compare the efficacy of the soloist.

$$\Delta Sol/G = [(AvAA - BvAA) + (AvBB - BvBB)]/6 \tag{6.3}$$

A positive value for $\Delta Sol/G$ indicates A performs better than B as the soloist.

The results for the tournament match-ups are shown in Table 6.1. All Δ values reported have a * attached if they are not found to be significant at a p value of 0.01 when a Wilcoxon signed-rank [WKW70] test was performed. This test is valid for paired data that is on a ratio scale and does not assume a normal distribution. The data is paired since the pre-cardplay is the same for all games within a match and a ratio scale is used for the scoring, thus the test is appropriate.

The general trend is that PI performs the best, followed by CLI, then KI, then NI. This fits with the expectation that better TSSR values seen in

Table 6.1: Tournament results for each game type. Shown are average tournament scores per game for players NI (No Inference), CLI (Card-Location Inference), PI (Policy Inference), and KI (Kermit's Inference) which were obtained by playing 5,000 matches against each other, each consisting of two games with soloist/defender roles reversed. The component of ΔTP attributed to Def and Sol is also indicated

Matchup	Game Type	TP	Δ TP	Δ Def	Δ Sol
	Suit	17.6 : 20.8	-3.2	-2.8	-0.4*
KI : CLI	Grand	37.0 : 39.0	-2.0	-1.8	-0.1*
	Null	17.2 : 19.8	-2.6	-2.7	0.1*
	Suit	16.5 : 21.6	-5.1	-4.2	-0.9
KI : PI	Grand	36.4 : 39.6	-3.1	-2.3	-0.8*
	Null	17.3 : 19.7	-2.4	-2.9	0.6*
	Suit	16.1 : 24.1	-8.0	-7.4	-0.6*
NI : CLI	Grand	36.6 : 40.5	-3.9	-3.4	-0.5*
	Null	16.0 : 22.5	-6.5	-6.6	0.1*
	Suit	19.5 : 17.2	2.3	1.6*	0.7*
PI : CLI	Grand	37.9 : 37.2	0.6*	0.2*	0.5*
	Null	18.8 : 17.3	1.6	1.1	0.4*
	Suit	23.3 : 18.6	4.6	4.5	0.1*
KI : NI	Grand	39.7 : 38.4	1.3	1.3	0.1*
	Null	21.6 : 17.8	3.8	3.9	-0.0*
	Suit	14.6 : 25.0	-10.4	-9.1	-1.4
NI : PI	Grand	36.5 : 40.0	-3.6	-3.1	-0.4*
	Null	15.8 : 22.1	-6.3	-6.8	0.5

Figure 5.1 would translate into stronger game performance. Another interesting result is that the majority of the performance gain seems to be from the defenders, as demonstrated by the ΔDef values being consistently larger than the ΔSol values. The most interesting match-up is PI : CLI since it roots the previous state-of-the-art skat inference against the new policy-based method. PI outperforms CLI by 2.3, 0.6, and 1.6 TP/G in suit, grand, and null, respectively. The grand result did not provide statistical significance.

The major drawback of the PI inference is the runtime. When combined with evaluation, PI20 takes roughly 5 times longer to make a move than CLI.

Table 6.2: Tournament results for each game type. Shown are average tournament scores per game for players CLI (Card-Location Inference), PI20 (Policy Inference with 20,000 card configurations sampled), PIF20 (Policy Inference with 20,000 states sampled), PI20 (Policy Inference with 100,000 card configurations sampled), and C (Cheating Inference) which were obtained by playing 5,000 matches against each other, each consisting of two games with soloist/defender roles reversed.

Game Type	Suit		Grand		Null	
Matchup	TP	Δ TP	TP	Δ TP	TP	Δ TP
PI : CLI	19.5 : 17.2	2.3	37.9 : 37.2	0.6*	18.8 : 17.3	1.6
PIF20 : CLI	19.3 : 17.7	1.6	37.7 : 37.2	0.5*	18.0 : 17.7	0.3*
PI100 : CLI	19.5 : 16.7	2.9	38.2 : 36.1	2.1	18.1 : 17.1	1.0*
C : CLI	14.7 : 18.0	-3.3	30.0 : 38.5	-8.5	20.8 : 11.0	9.8
PI : CLI (6way)	19.1 : 17.8	1.2	37.6 : 37.3	0.4*	18.3 : 17.7	0.6

Further experiments were conducted to test the effect of increasing the number of sampled worlds to 100,000 (PI100) and sampling states instead of card configurations (PIF20). In addition, a cheating version of Kermit was introduced (C) in which it places all probability on the true state. All programs were tested against CLI with only the mirrored adversarial setup used. The rest of the experimental setup was identical.

The results in Table 6.2 indicate that PI100 performs stronger than the other PI variants in suit and grand, when playing against CLI, however, only the grand result is significantly better. The opposite is true for null, in which PI20 performs strongest out of the PI variants. This result contradicts the idea that a higher TSSR value necessarily corresponds to better cardplay performance. Cheating inference performs worse than CLI in all but null games. This is interesting because it puts all the probability mass on the true world, but still plays worse than a player that is not cheating. This result in conjunction with the worse null game score for PI100 indicates that further investigation into the exact role inference quality has within the context of PIMC is required. Also, PI20 outperforms PIF20 over all game types, showing that there can be benefits to sampling card configurations instead of states when

Table 6.3: Tournament results for each game type in the 6-way match between CLI and PI20. 5,000 matches were played for each game type.

Game Type	ΔSol	ΔDef_{CLI}	ΔDef_{PI}
Suit	1.0	-1.1	0.6*
Grand	0.4*	0.0*	0.2*
Null	0.2*	-0.6*	0.5*

there is a limited sampling budget.

One further experiment was performed to determine whether performance gains would be present with mixed defenders. This is interesting since it is possible the gain would only be present if the partner's inference was compatible with their own. For the sake of time, this was only done for the CLI and PI20 matchup. The added arrangements are $AvAB$, $AvBA$, $BvAB$, and $BvBA$. With these added, we now have six games for each tournament match. The results for this match-up are included in Table 6.2. PI is consistently stronger than CLI (the grand result is not significant), but the effect size is smaller. This is expected because PI is now defending against PI in the mixed setup games. To further analyze the the relative effectiveness of the players as soloist against only the mixed team defenders, we can calculate:

$$\Delta Sol = [(AvAB - BvAB) + (AvBA - BvBA)]/6 \tag{6.4}$$

A positive value for ΔSol means that PI is more effective than CLI as soloist in the mixed setting. ΔDef_B measures the difference in effectiveness of a mixed defense (A and B) and a pure defense of A's. It is calculated by averaging the effect of swapping in player B into defense for all match-ups that included two A's on defense. The reverse can be done to find the effect of swapping in A to form a mixed defense. A positive value for ΔDef_{PI} means defense improved when it was added, and same for ΔDef_{CLI}.

While all the values in Table 6.3 show the same trends of PI performing better on defense and soloist across all game types, the effect is only statistically significant for ΔSol and ΔDef_{CLI} in the suit games.

Chapter 7

Conclusion

In this thesis I have demonstrated that pre-cardplay policies for Skat can be learned from human game data and that it performs better than Kermit's pre-cardplay — the prior state-of-the-art. Naively imitating all aspects of the pre-cardplay by taking the argmax over the legal actions (DI-M) resulted in a bidding policy that performed an average of 2.1 TP/G worse than the Kermit baseline. The same procedure but with sampling (DI-S) resulted in the player performing 4.1 TP/G worse than baseline. Using the novel method to increase the aggressiveness of the bidder led to it performing 0.4 TP/G better than the baseline, with A set to 0.85 (AB.85). Using this in conjunction with game declaration based on the predicted values and probabilities of actions (MLV.85), resulted in the best overall pre-cardplay policy, beating the baseline by 1.2 TP/G. Also, the time for pre-cardplay decisions are much faster, as it does not rely on search.

The direct imitation cardplay policy decreases the strength of the overall player, performing 2.8 TP/G worse than the Kermit player when utilizing the Kermit bidder. The best overall full network based player was MLV.925+C, which outperformed Kermit by 1.0 TP/G. This full network player is order of magnitudes faster than the search based player, and in this tournament setup, performs better. One drawback is that while more computation can be done to improve the search (improving the number of worlds sampled for example), the same cannot be done for the network player. While the learned cardplay policy was shown to be weaker overall than the search based method, the hybrid player that uses the policy then switches over to search after a few moves is shown to be stronger than the search based policy alone.

Policy Inference (PI) appears to provide much stronger inference than its predecessors, namely Kermit Inference (KI) and Card Location Inference (CLI) as demonstrated by the TSSR value figures. Across the board, the higher TSSR values translate into stronger game-play as demonstrated in card-play tournament settings. PI20 outperforms CLI by 2.3, 0.6, and 1.6 TP/G in suit, grand, and null games, respectively. Also, it seems that increasing the number of states sampled increases the performance of PI, however, this did not translate into the null game type. Further investigation into this null game result is needed. I would expect that when substantially increasing the sampling threshold, the state sampling employed by PIF20 would be more effective. But under this limited sampling regimen, sampling card configurations is more effective than sampling states.

7.1 Future Work

Now that success has been established for training model-free policies from human data in Skat, the next logical step is to improve these policies directly through experience similar to the process shown in the original AlphaGo [Sil+16]. In [Sri+18] regret minimization techniques often used to solve imperfect information games are related to model-free multi-agent reinforcement learning. The resulting actor-critic style agent showed fast convergence to approximate Nash equilibria during self-play in small variants of Poker. Applying the same approach may be difficult because of Skat's size and the fact that it is not zero-sum, but starting with learning a best response to the full imitation player discussed in this work should be feasible and may yield a new state-of-the-art player for all phases of the game. The next step would be to create a fully-fledged self-play regime for Skat. Learning policies through self-play has shown to yield strategies that are "qualitatively different to human play" in other games [Sil+17]. This could be problematic because Skat involves cooperation on defense during the card play phase. Human players use conventions and signals to coordinate and give themselves the best chance of defeating the soloist. In order to play well with humans, policies need to account for these signals from their partner and send their own. Continually incorporating labelled data from human games may help alleviate this problem.

Also, with a strong and fast network based policy, the IIMC Skat player [FB13] can be sped up by using the network player for the lower level imperfect information algorithm. The policy can also be used for move ordering and pruning in the existing search based methods.

Future work related to inference in trick-taking card games should focus on the relationship between opponent modelling and exploitability. In particular, measuring the exploitability of our current policy trained from human data to see how our algorithm would fare against the worst-case opponent could lead to insight regarding the robustness of our approach. Likewise, adjusting player models online could enable us to better exploit our opponents and cooperate with team-mates. Another direction is to experiment with heuristics that allow our algorithm to prune states that are highly unlikely and stop considering them altogether. This could help us sample more of the states that are shown to be realistic given our set of human games and possibly improve the performance of the search.

And finally, testing the current iteration of Kermit against human players would be illuminating, as the strength has dramatically improved since the last time it was measured against human experts [Bur+09].

References

[Aba+16] Martn Abadi, Paul Barham, Jianmin Chen, Zhifeng Chen, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Geoffrey Irving, Michael Isard, et al. "Tensorflow: A System for Large-scale Machine Learning." In: *OSDI*. Vol. 16. 2016, pp. 265–283. 19

[AM06] Asaf Amit and Shaul Markovitch. "Learning to Bid in Bridge." In: *Machine Learning* 63.3 (2006), pp. 287–327. 13

[Bai+18] Hendrik Baier, Adam Sattaur, Edward Powley, Sam Devlin, Jeff Rollason, and Peter Cowling. "Emulating Human Play in a Leading Mobile Card Game." In: *IEEE Transactions on Games* (2018). 15

[BS18] Noam Brown and Tuomas Sandholm. "Superhuman AI for Heads-up No-limit Poker: Libratus Beats Top Professionals." In: *Science* 359.6374 (2018), pp. 418–424. 2, 14

[Bur97] Michael Buro. "The Othello Match of the Year: Takeshi Murakami vs. Logistello." In: *ICGA Journal* 20.3 (1997), pp. 189–193. 1

[Bur98] Michael Buro. "From Simple Features to Sophisticated Evaluation Functions." In: *International Conference on Computers and Games*. Springer. 1998, pp. 126–145. 16

[Bur+09] Michael Buro, Jeffrey Richard Long, Timothy Furtak, and Nathan R Sturtevant. "Improving State Evaluation, Inference, and Search in Trick-Based Card Games." In: *IJCAI*. 2009, pp. 1407–1413. 11, 13, 16, 23, 37, 48

[CHH02] Murray Campbell, A Joseph Hoane Jr, and Feng-hsiung Hsu. "Deep Blue." In: *Artificial intelligence* 134.1-2 (2002), pp. 57–83. 1

[Cou06] Rémi Coulom. "Efficient Selectivity and Backup Operators in Monte-Carlo Tree Search." In: *International conference on computers and games*. Springer. 2006, pp. 72–83. 11

[CPW12] Peter I Cowling, Edward J Powley, and Daniel Whitehouse. "Information Set Monte Carlo Tree Search." In: *IEEE Transactions on Computational Intelligence and AI in Games* 4.2 (2012), pp. 120–143. 11

[DOS18] DOSKV. *DOSKV: Deutcher Online Skatverband*. `https://www.doskv.de/skat-spielen.htm`. 2018. URL: `https://www.doskv.de/skat-spielen.htm`. 18

[Fed19] World Bridge Federation. *World Computer Bridge Championship*. `https://bridgebotchampionship.com/`. 2019. URL: `https://bridgebotchampionship.com/`. 10

[Foe+18] Jakob N Foerster, Francis Song, Edward Hughes, Neil Burch, Iain Dunning, Shimon Whiteson, Matthew Botvinick, and Michael Bowling. "Bayesian Action Decoder for Deep Multi-agent Reinforcement Learning." In: *arXiv preprint arXiv:1811.01458* (2018). 15

[FB98] Ian Frank and David Basin. "Search in Games with Incomplete Information: A Case Study Using Bridge Card Play." In: *Artificial Intelligence* 100.1-2 (1998), pp. 87–123. 3, 12

[FB13] Timothy Furtak and Michael Buro. "Recursive Monte Carlo Search for Imperfect Information Games." In: *Computational Intelligence in Games (CIG), 2013 IEEE Conference on*. IEEE. 2013, pp. 1–8. 12, 13, 16, 48

[Gin01] Matthew L Ginsberg. "GIB: Imperfect Information in a Computationally Challenging Game." In: *Journal of Artificial Intelligence Research* 14 (2001), pp. 303–358. 11, 13

[HM00] Sergiu Hart and Andreu Mas-Colell. "A Simple Adaptive Procedure Leading to Correlated Equilibrium." In: *Econometrica* 68.5 (2000), pp. 1127–1150. 14

[Joh13] Michael Johanson. "Measuring the size of large no-limit poker games." In: *arXiv preprint arXiv:1302.7008* (2013). 2

[KB14] Diederik P Kingma and Jimmy Ba. "Adam: A Method for Stochastic Optimization." In: *arXiv preprint arXiv:1412.6980* (2014). 19

[KS06] Levente Kocsis and Csaba Szepesvári. "Bandit Based Monte-Carlo Planning." In: *European conference on machine learning*. Springer. 2006, pp. 282–293. 11

[Lev89] David NL Levy. "The Million Pound Bridge program." In: *Heuristic Programming in Artificial Intelligence* (1989). 10, 16

[Lon+10] Jeffrey Richard Long, Nathan R Sturtevant, Michael Buro, and Timothy Furtak. "Understanding the Success of Perfect Information Monte Carlo Sampling in Game Tree Search." In: *Twenty-Fourth AAAI Conference on Artificial Intelligence*. 2010. 10

[Mor+17] Matej Moravk, Martin Schmid, Neil Burch, Viliam Lis, Dustin Morrill, Nolan Bard, Trevor Davis, Kevin Waugh, Michael Johanson, and Michael Bowling. "Deepstack: Expert-level Artificial Intelligence in Heads-up No-limit Poker." In: *Science* 356.6337 (2017), pp. 508–513. 2, 14

[NH10] Vinod Nair and Geoffrey E Hinton. "Rectified Linear Units Improve Restricted Boltzmann Machines." In: *Proceedings of the 27th international conference on machine learning (ICML-10)*. 2010, pp. 807–814. 18

[Pre98] Lutz Prechelt. "Automatic Early Stopping Using Cross Validation: Quantifying the Criteria." In: *Neural Networks* 11.4 (1998), pp. 761–767. 19

[RSB19] Douglas Rebstock, Christopher Solinas, and Michael Buro. "Learning Policies from Human Data for Skat." In: *arXiv preprint arXiv:1905.10907* (2019). iii

[Reb+19] Douglas Rebstock, Christopher Solinas, Michael Buro, and Nathan R Sturtevant. "Policy Based Inference in Trick-Taking Card Games." In: *arXiv preprint arXiv:1905.10911* (2019). iii

[RA07] Mark Richards and Eyal Amir. "Opponent Modeling in Scrabble." In: *IJCAI*. 2007, pp. 1482–1487. 13

[RQA19] Jiang Rong, Tao Qin, and Bo An. "Competitive Bridge Bidding with Deep Neural Networks." In: *arXiv preprint arXiv:1903.00900* (2019). 15

[RN16] Stuart J Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach*. Malaysia; Pearson Education Limited, 2016. 3, 12

[Sch+07] Jonathan Schaeffer, Neil Burch, Yngvi Björnsson, Akihiro Kishimoto, Martin Müller, Robert Lake, Paul Lu, and Steve Sutphen. "Checkers is Solved." In: *science* 317.5844 (2007), pp. 1518–1522. 1

[SBH08] Jan Schäfer, Michael Buro, and Knut Hartmann. "The UCT Algorithm Applied to Games with Imperfect Information." In: *Diploma, Otto-Von-Guericke Univ. Magdeburg, Magdeburg, Germany* (2008). 11

[Sil+16] David Silver, Aja Huang, Chris J Maddison, Arthur Guez, Laurent Sifre, George Van Den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, et al. "Mastering the Game of Go with Deep Neural Networks and Tree Search." In: *nature* 529.7587 (2016), p. 484. 1, 11, 14, 15, 47

[Sil+17] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. "Mastering the Game of Go Without Human Knowledge." In: *Nature* 550.7676 (2017), p. 354. 11, 47

[Sof19] Kuijf & Kuijf Software. *Jack*. `http://www.jackbridge.com/eindex.htm`. 2019. URL: `http://www.jackbridge.com/eindex.htm`. 10

[SRB19] Christopher Solinas, Douglas Rebstock, and Michael Buro. *Improving Search with Supervised Learning in Trick-Based Card Games*. 2019. eprint: `arXiv:1903.09604`. 11, 13, 16, 23, 37

[Sri+18] Sriram Srinivasan, Marc Lanctot, Vinicius Zambaldi, Julien Pérolat, Karl Tuyls, Rémi Munos, and Michael Bowling. "Actor-critic Policy Optimization in Partially Observable Multiagent Environments." In: *Advances in Neural Information Processing Systems*. 2018, pp. 3426–3439. 47

[Sri+14] Nitish Srivastava, Geoffrey Hinton, Alex Krizhevsky, Ilya Sutskever, and Ruslan Salakhutdinov. "Dropout: a Simple Way to Prevent Neural Networks from Overfitting." In: *The Journal of Machine Learning Research* 15.1 (2014), pp. 1929–1958. 19

[SB06] N. Sturtevant and M. Bowling. "Robust Game Play Against Unknown Opponents." In: *Autonomous Agents and Multiagent Systems (AAMAS)*. ACM. 2006, pp. 713–719. URL: `http://www.cs.du.edu/~sturtevant/papers/softmaxn.pdf`. 13

[Stu08] Nathan Sturtevant. "An Analysis of UCT in Multi-player Games." In: *ICGA Journal* 31.4 (2008), pp. 195–208. 11

[Wau+15] Kevin Waugh, Dustin Morrill, James Andrew Bagnell, and Michael Bowling. "Solving Games with Functional Regret Estimation." In: *AAAI*. Vol. 15. 2015, pp. 2138–2144. 14

[WBr19] WBridge5. *WBridge5*. `http://www.wbridge5.com/`. 2019. URL: `http://www.wbridge5.com/`. 10

[WKW70] Frank Wilcoxon, SK Katti, and Roberta A Wilcox. "Critical Values and Probability Levels for the Wilcoxon Rank Sum Test and the Wilcoxon Signed Rank Test." In: *Selected tables in mathematical statistics* 1 (1970), pp. 171–259. 25, 42

[YHL18] Chih-Kuan Yeh, Cheng-Yu Hsieh, and Hsuan-Tien Lin. "Automatic Bridge Bidding Using Deep Reinforcement Learning." In: *IEEE Transactions on Games* 10.4 (2018), pp. 365–377. 15

[Zin+08] Martin Zinkevich, Michael Johanson, Michael Bowling, and Carmelo Piccione. “Regret Minimization in Games with Incomplete Information.” In: *Advances in neural information processing systems*. 2008, pp. 1729–1736. 2, 14