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(Signed)....K. Edugyan....

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GHANA

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SPECTRAL ANALYSIS OF COCOA PRICES

by

KWEKU EDUGYAN

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS

DEPARTMENT OF ECONOMICS

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Spectral Analysis of Cocoa Prices" submitted by Kweku Edugyan in partial fulfilment of the requirements for the degree of Master of Arts.

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Supervisor

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ABSTRACT

The cocoa price mechanism is examined by spectral analysis and cross-spectral analysis using spot prices of cocoa. First, spectral analysis is carried out for prices from the New York Cocoa Exchange and then for prices from the London Cocoa Terminal. Secondly, cross-spectral analysis is carried out to determine any lead-lag relationships which may exist between the two series. It emerges from the spectral analytic study that there are significant cyclical and seasonal fluctuations with definite time periods in the price series and from the cross-spectral analysis that the two series are related.

ACKNOWLEDGEMENTS

My special thanks go to Dr. Ben Korda for supervising this thesis, and especially for rescuing the thesis from the oblivion into which it would otherwise have fallen, and to Drs. A. Buse and J. D. Jobson for their advice and extremely helpful suggestions.

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CHAPTER 1

INTRODUCTION

This thesis deals with the analysis of economic time series, where a time series may be considered as a set of observations arranged sequentially in time. The series we analyse is spot prices of cocoa. Our strategy will be to look at the series, not in the domain of time, but in the frequency domain, that is, by spectral analysis. The terms will become clearer as we progress.

The study begins in Chapter 2, where we take a close look at the behaviour of the time series of cocoa spot prices by examining its trace. Graphical presentations of two realizations from two different markets, the New York Cocoa Exchange and the London Cocoa Terminal Association, are given.

In Chapter 3, a description of the forerunner of spectral analysis, periodogram analysis, and of spectral and cross-spectral analysis are presented together with the works of earlier economists in the field. This chapter is made up of history, theory and applications.

Chapter 4 presents the practical estimation aspects of spectral and cross-spectral analysis. It was found necessary to include the material presented here in order to promote a better understanding of the theory.

The results of our analysis are presented in Chapter 5. Here we not only look at prices in the two markets separately but also the interrelationships between the two series.

Chapter 6 is a summary of results and is the closing chapter.

Whilst a number of studies have been done on the cocoa industry and on cocoa prices, notably by Labys and Granger [28] and Weymar [37], none of these looks at prices in a way that has been done in this study. Also, since the method of analysis employed here is mathematical, an occasional equation here and there has been unavoidable and in fact has been necessary throughout the presentation.

The power spectral estimates have been obtained by using a computer program developed by Karreman [27] for the IBM 7090, and modified here for the IBM 360/67.

CHAPTER 2

COCOA PRICES

2-1 The Spot and Futures Prices for Cocoa

Observers on the cocoa scene believe that cocoa price movements fall into three groups: a long term, a medium term and a short term. It is also believed that each movement has its own generating mechanism. In this study an attempt will be made to discover any periodic movements alleged to exist in cocoa prices.

The dynamics of the long term movements is assumed to stem from that simple model of cyclical price behaviour, the cobweb mechanism. Low prices which come about as a result of over-supply, no doubt, lead to decline in new planting, which after a time lag, leads to price rise, resulting in more planting, more cocoa yield after a lag, and, finally, a price fall. Things may not be that simple in the real world, but this is the basic picture. We note that the minimum time span between planting and harvesting is four to six years. Expectations generate the medium term movements; expectations, that is, of consumers and inventory holders, and the short term month-to-month price fluctuations are accounted for by speculators in the world cocoa markets.

The short term fluctuations arise because a very high percentage of transactions in cocoa are forward sales. That is, the cocoa may be sold through intermediary middlemen and brokers long before it is shipped to the consuming countries, and hence some elements of speculation and hedging enter into cocoa transactions, with the hedgers taking a position in the futures market opposite to that which they would hold in the spot or cash market and the speculators buying and hoping that a favourable change in futures prices would yield them profit. Basically, the speculator aims to make uncertain profit from his transactions in the market by virtue of his expectations and forecast, his price expectations being based sometimes on spot prices which are prices for delivery now, sometimes on futures prices which are prices for delivery at a future date, and sometimes on both.

Futures trading in cocoa occurs on the New York Cocoa Exchange, the London Cocoa Market Association and in Amsterdam, the first two being more important than the third, but with the New York Cocoa Exchange the largest of the three. It is alleged that the three markets are related and that any active buying of futures on the New York market, say, may reduce activity in London and Amsterdam. The relatedness of two of these markets will be tested in a later chapter. It is also alleged that London prices give an indication of opening prices in New York since by the time the New York market opens at 10:00 a.m., trading will have started long before in London.

Prices gyrate a lot on the futures market for cocoa, the causes being demand and supply conditions on the market. On the supply side are the vagaries of the weather and random disturbances such as strikes, wars and the threat of nationalization. Demand shifts come from variables like shifts in tastes, prices of substitutable commodities, and speculative buying which most often stems from change of governments and currency devaluation. Because prices are sensitive to random disturbances, cocoa prices are controlled to some extent on the New York Cocoa Exchange by setting price limits. Trades in futures during any day are not allowed to be made at prices varying by more than one cent per pound above or below the previous closing price. Prices are generally quoted in cents and hundredth of a cent per pound, and the minimum fluctuation is referred to as a point. On the London market, trades in futures are not allowed to be made at prices which vary by more than 20 shillings per hundred weight above or below the previous closing price. Then there is a thirty minutes recess, after which trading may go on without limits. The same factors which determine the prices of the futures also strongly influence the formation of spot prices on the three markets. In fact, spot and futures prices tend to move in the same direction, especially if an unexpected event causes a violent swing in prices. Spots and futures market prices moving in opposite directions is unusual. There is also a tendency for the two prices to be about the same during a delivery month for a given grade of the commodity. In this study, spot prices of Accra beans on the New York and London markets are used for analysis. This is quite in keeping with the fact that Ghana, the world's largest producer, accounts for nearly 40 per cent of the world production of cocoa, whilst the United States and the United Kingdom are the largest importers and consumers. However, New York prices tend to set world cocoa prices.

Prices paid to producers in the big three producing countries of Ghana, Nigeria and Brazil do not reflect world prices. The reason is that the governments of these countries have all instituted Marketing Boards which purchase the cocoa beans from the producers at prices fixed by the Boards. The Boards then sell and export the cocoa.

2-2 Price Behaviour

Table 2-1 shows the monthly average spot prices of main crop Accra beans in New York in cents per pound, and Table 2-2 the monthly average spot prices on the London market in shillings per hundredweight. The plot of the

TABLE 2-1

MONTHLY-AVERAGE SPOT PRICES OF MAIN CROP ACCRA COCOA BEANS IN NEW YORK (CENTS/LB)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1946	8.9	8.9	8.9	8.9	8.9	8.9	8.9	8.9	8.9	14.0	19.5	24.5
1947	25.9	26.6	28.0	28.8	28.2	30.1	32.7	34.5	40.4	49.5	51.0	43.0
1948	43.6	43.6	39.4	35.4	33.2	41.6	44.6	44.2	40.4	40.2	38.0	32.0
1949	26.6	20.3	18.5	19.9	19.1	18.9	21.3	23.0	20.1	20.8	24.8	25.9
1950	27.2	25.1	22.8	24.0	28.6	30.8	35.6	40.5	42.0	37.2	36.2	34.6
1951	36.8	37.7	38.3	38.4	38.2	38.3	35.0	35.3	34.0	32.2	29.8	32.4
1952	33.1	35.8	38.4	38.1	38.4	37.8	38.1	35.4	33.3	34.0	31.8	30.8
1953	31.8	30.0	32.8	33.9	33.4	34.6	38.5	38.6	40.3	40.0	44.9	46.8
1954	54.2	53.5	57.8	61.9	63.9	64.8	68.9	67.8	53.7	47.1	51.7	48.3
1955	49.5	47.7	40.3	37.7	36.4	37.2	36.1	32.6	33.2	33.9	32.8	32.6
1956	29.5	27.7	26.6	25.8	26.2	27.2	28.6	28.3	28.0	26.3	27.2	26.6
1957	23.7	22.7	22.5	25.5	26.5	30.5	30.5	32.1	34.8	35.8	41.6	40.7
1958	42.0	44.9	43.4	43.5	46.9	48.9	47.8	47.4	42.5	38.8	44.3	41.4
1959	37.6	36.5	38.7	37.3	37.4	37.4	36.4	37.7	37.8	36.6	34.1	31.4
1960	30.0	28.7	27.3	28.1	28.6	28.4	28.9	28.0	29.0	29.6	28.1	25.6
1961	23.3	21.8	20.6	22.9	23.0	21.9	22.0	21.0	20.8	22.5	25.1	25.2
1962	23.0	20.1	21.3	21.0	21.6	20.6	22.8	20.2	20.1	20.4	21.1	21.8
1963	22.9	24.6	24.4	25.7	28.1	25.4	24.1	24.0	25.3	26.9	25.9	26.5
1964	25.7	23.6	23.8	22.2	22.3	22.0	23.5	23.0	23.7	23.6	23.1	23.8
1965	23.0	20.6	17.0	16.4	15.5	13.8	22.2	15.0	16.7	17.1	18.5	21.5
1966	22.6	22.4	23.2	25.2	24.4	24.9	27.2	26.6	23.4	23.9	23.3	26.1
1967	27.5	29.7	29.1	27.8	28.0	28.3	27.6	28.3	30.3	29.5	31.3	31.5
1968	31.6	29.8	30.2	30.6	29.9	29.1	29.6	31.0	36.8	39.4	46.0	48.7
1969	44.7	44.4	44.4	44.6	44.7	46.1	47.8	45.8	45.3	47.0	48.6	44.9
1970	39.0	34.8	33.7	32.3	29.4	29.7	32.5	38.0	38.0	36.2	34.0	32.4
1971	29.7	27.9	27.1	27.2	25.1	26.6	28.4	29.0	27.0	25.1	24.4	23.9
1972	25.8	26.7	28.2	28.6	30.3	31.1	32.3	34.5	36.7	38.0	37.3	

Sources: Cocoa Statistics (Quarterly), Food and Agricultural Organization, Rome;
Cocoa Market Reports (Monthly), Gill & Duffus Ltd., London.

TABLE 2-2

MONTHLY-AVERAGE SPOT PRICES OF MAIN CROP ACCRA COCOA BEANS IN LONDON (SHILLINGS/CWT)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1956	244.00	227.50	213.75	208.50	214.75	220.00	228.75	233.33	228.17	211.17	216.42	212.92
1957	198.17	185.00	182.50	202.58	209.67	241.92	243.58	265.50	284.00	291.50	342.42	328.42
1958	333.58	357.17	342.92	343.42	376.25	369.75	375.83	357.92	335.25	325.50	354.75	329.08
1959	299.50	281.67	296.92	292.17	298.50	297.92	283.08	281.25	285.67	284.08	277.00	248.58
1960	237.33	229.17	221.50	229.67	229.50	230.00	232.58	228.92	225.75	229.33	219.17	197.42
1961	183.75	170.83	160.42	180.00	180.33	175.75	176.83	169.92	167.58	181.08	204.58	208.00
1962	183.33	166.08	170.17	171.00	172.08	171.00	171.50	167.33	162.08	164.67	170.25	171.58
1963	185.17	202.42	200.92	213.75	233.58	217.00	203.67	196.08	203.33	216.83	213.67	211.25
1964	207.25	188.75	190.25	181.92	181.58	185.95	192.08	190.17	192.42	190.42	190.00	195.67
1965	192.08	171.92	138.33	132.92	127.42	111.42	97.92	123.17	153.25	137.58	147.25	169.67
1966	179.50	177.33	185.08	201.33	198.58	202.92	217.10	213.00	189.17	195.33	188.33	206.83
1967	223.17	243.33	241.17	234.75	226.08	227.42	223.83	229.00	244.58	246.00	273.08	290.00
1968	293.50	281.75	279.83	284.17	281.83	281.50	284.92	294.92	330.83	374.00	440.33	468.17
1969	436.42	424.67	423.83	421.17	420.33	418.33	431.92	418.08	410.75	427.00	434.58	398.67
1970	342.50	309.25	307.75	305.67	277.25	268.83	290.83	331.33	341.90	332.33	318.00	300.08
1971	273.40	252.00	243.40	241.30	227.60	237.17	249.25	258.00	236.70	211.30	205.90	198.60
1972	219.70	223.50	232.40	236.70	249.50	260.50	281.30	294.90	318.20	319.00	323.20	

Source: Cocoa Statistics (Quarterly, Food and Agricultural Organization, Rome.

data in Table 2-1 is shown in Figure 2-1 and that of Table 2-2 in Figure 2-2. The trace shown in Figure 2-1 is one realization of the cocoa price series. In Figure 2-1, the point zero corresponds to January 1946. The plot terminates with November 1972 price. The plot reveals some interesting aspects of cocoa prices not immediately discernible from Table 2-1: fluctuations in spot prices. These fluctuations are shown in Figure 2-1. The price of Ghana cocoa was 8.9 cents per pound just after World War II in September 1946, 51.0 cents per pound in November 1949, 18.9 cents per pound in June 1949 and 68.9 cents per pound in July 1954, the highest price during the 323 months or the 27-year period examined here. In March 1957, the price dropped to 22.5 cents per pound, rose to 48.9 cents per pound in June 1958 and hit rock bottom in July 1965 at 12.2 cents per pound. In between the high and low points were numerous erratic fluctuations.

Figure 2-2 shows the plot of the London series. The zero point corresponds to January 1956 and again the plot terminates with November 1972 price. Again the fluctuations in prices are apparent. The overall shape of the plot bears close resemblance to the shape of the plot of the New York series for the same period. The resemblance is so close that a plot of the New York data for the period January 1956 - November 1972, which is the period covered

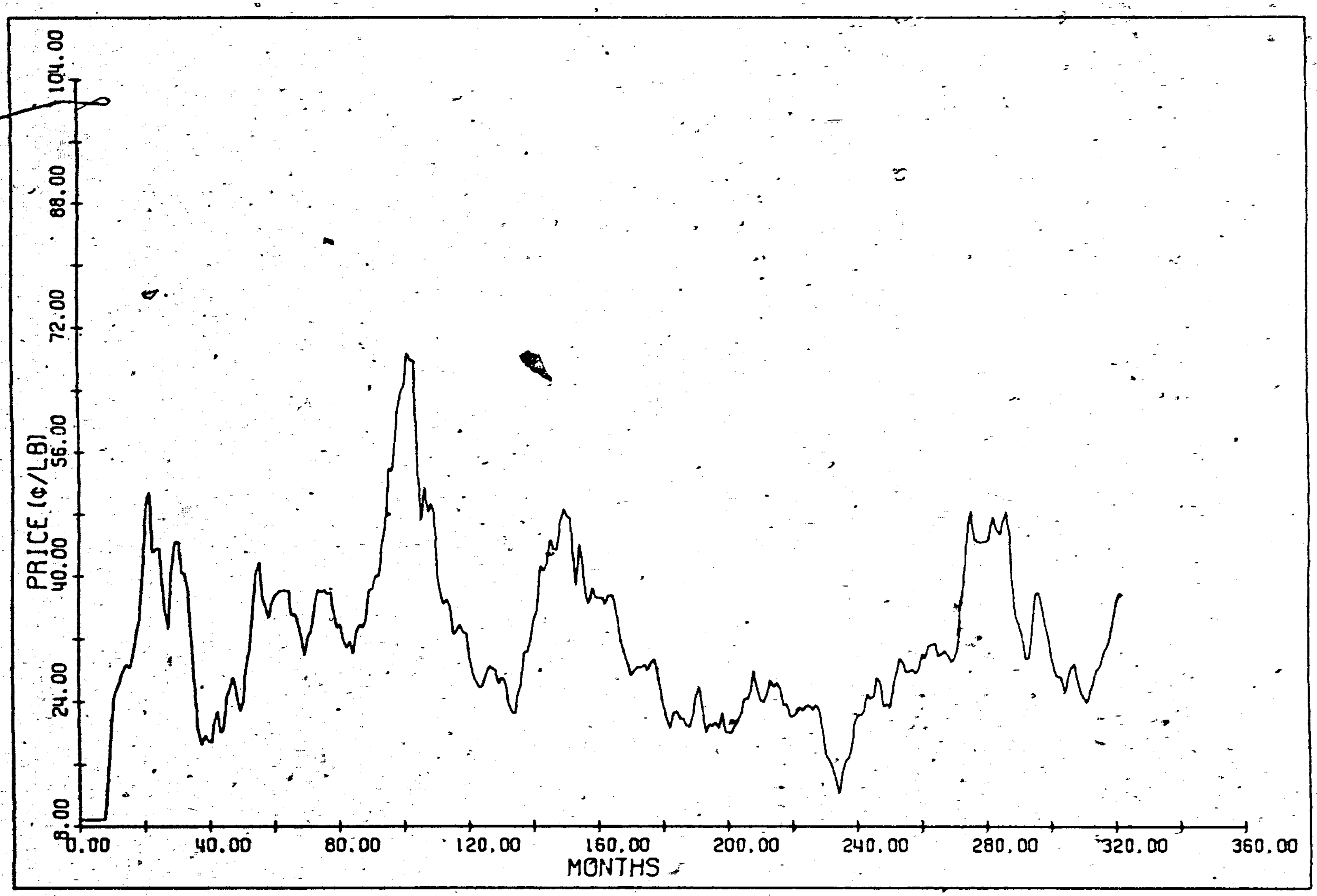

Figure 2-1 Plot of Cocoa Price Series, New York Spot Accra, Monthly, January 1946 - November 1972.

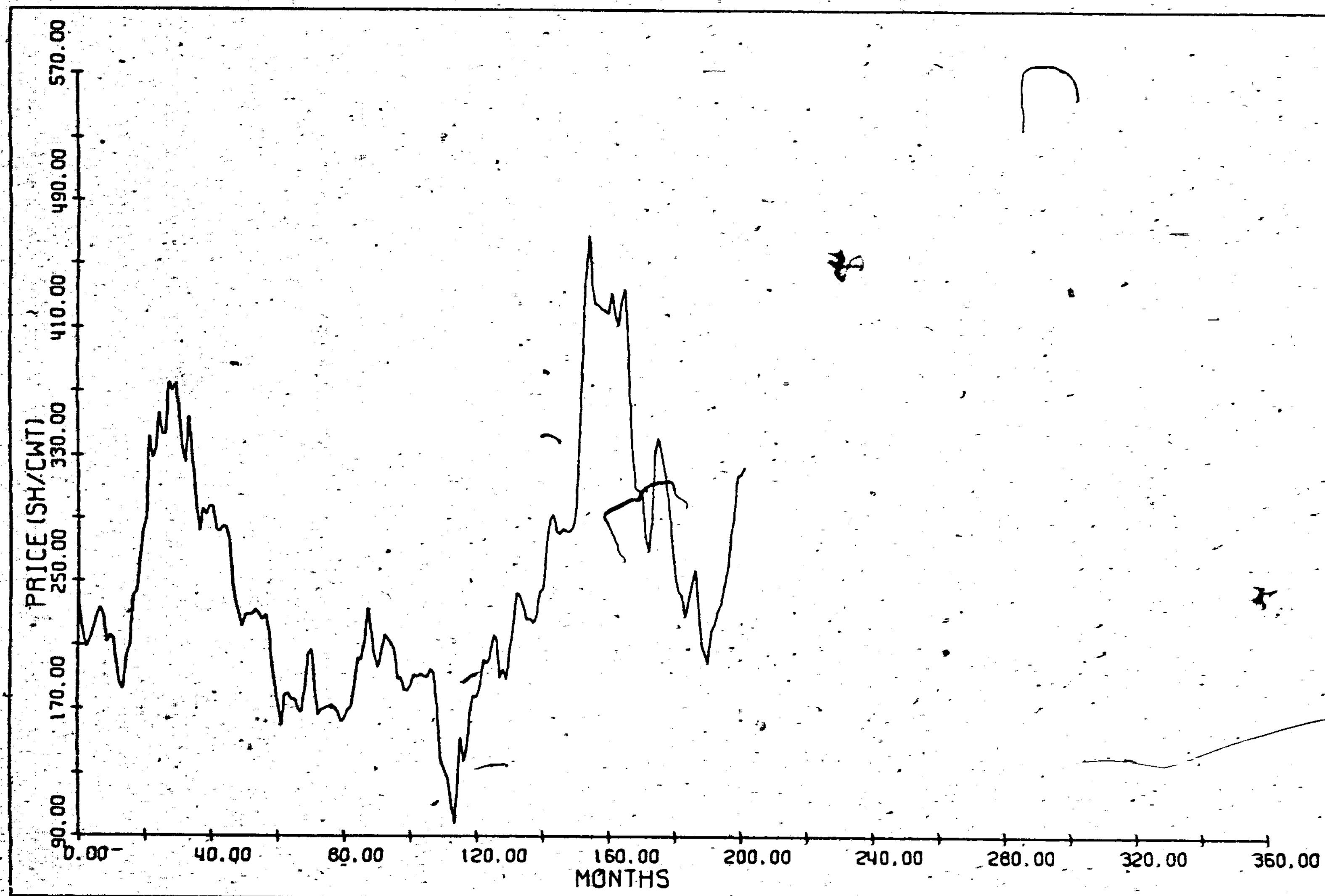

Figure 2-2 Plot of Cocoa Price Series, London Spot Accra, Monthly, January 1956 - November 1972.

by the London series, is shown in Figure 2-3, for careful comparison. An examination of Figures 2-2 and 2-3 immediately shows similar patterns in price movements in the two markets. The prices are not the same however, as a closer look at the fine ends of the two graphs reveals. This is to be expected because of different movements of futures prices on the two markets due to different environment disturbances, one such disturbance being the threat of currency devaluation.

To find out if there is a trend in the series, the New York price series for the period January 1946 to November 1972 was regressed on time t, t = 1, 2,.......,323, to obtain the equation

$$P_t = \underset{(1.1389)}{32.5496} - \underset{(0.006)}{0.0074t}$$

The t-statistic for the independent variable, time, was found to be -1.2197 and the t-statistic for the coefficient was found to be 28.5817. The trend is thus negligible and hence no attempt has been made to remove it from the data for spectral analysis on the basis of this fact.

The fact that prices of cocoa from December 1972 upward were unavailable from the source of data collection at the time of this study makes it impossible to comment on the behaviour of prices after November 1972.

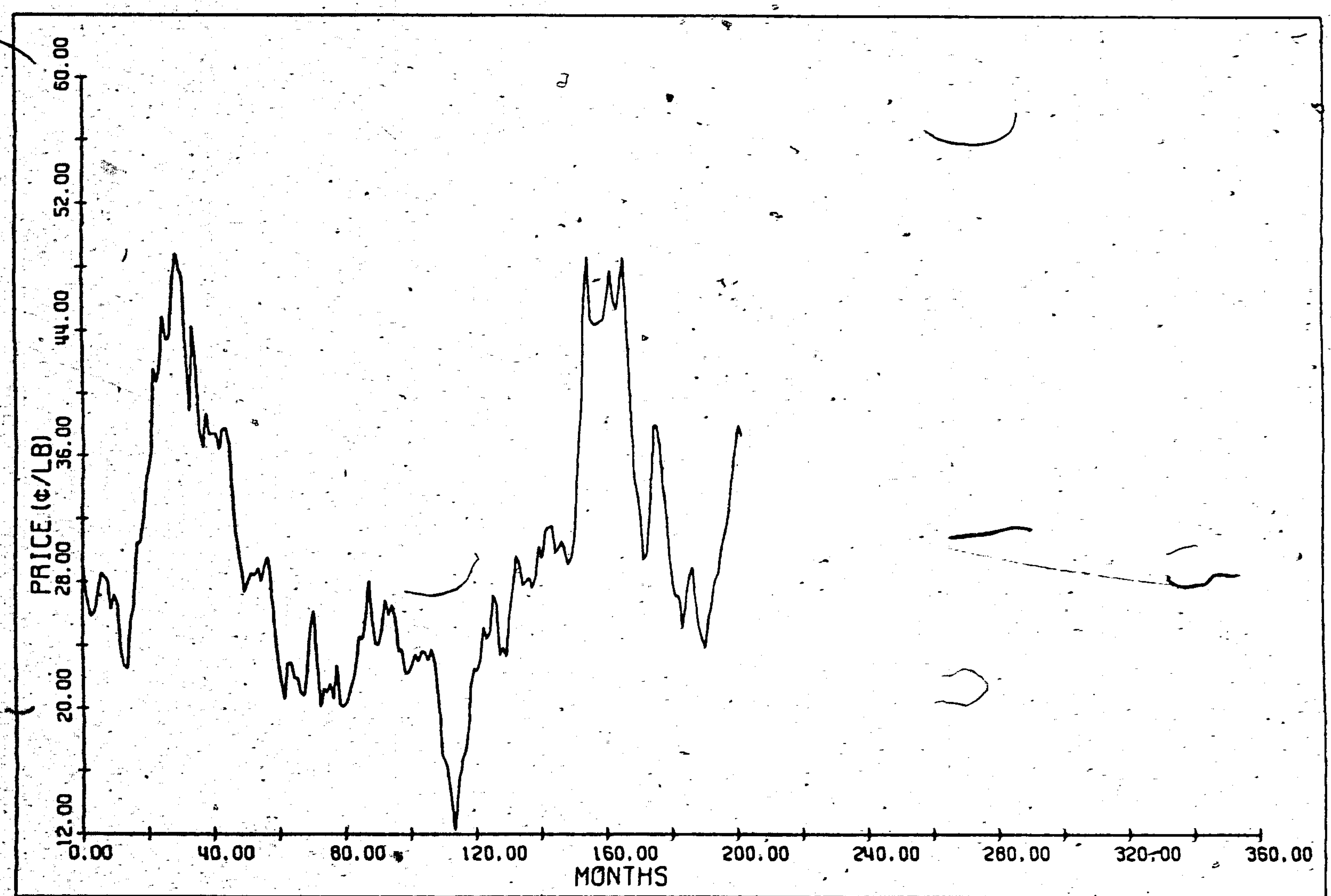

Figure 2-3 Plot of Cocoa Price Series, New York Spot Accra, Monthly, January 1956 - November 1972.

In the next chapter we take a look at the method which will be used to analyse the time series of spot cocoa prices depicted in the graphs of this chapter.

CHAPTER 3

FREQUENCY DOMAIN METHODS IN ECONOMICS: HISTORY, THEORY, APPLICATIONS

3-1 Motivation

Economic time series may be described as being made up of four components: trend, cyclical, seasonal and irregular. The trend is a long term movement which shows the overall tendency of the series and hence changes as the process changes through time. It may be considered to be of infinite period, and consequently, of very short frequency. It thus shows up in the near zero frequency range when one applies the technique adopted in this study. It shows up as a peak in the spectrum and may thus mask the emergence of any long cycle in the series. We shall not be very concerned with the trend here. This is not because the trend is unimportant, but our method of analysis is not suited to its study. There exist other simpler and effective ways of dealing with the trend.

The cyclical component of the time series will concern us here. This component may be regular or irregular and describes the fluctuating nature of most economic variables. It shows up in spectrum as a peak, and the narrower the peak, the more distinguishable and regular it is.

However, the absence of a peak does not imply the absence of cycles in the series. If the cycles are sufficiently irregular, the peaks may not show up even though cycles are present.

The seasonal component is a variation which occurs within the year and is more regular than the cycle. The cause of the seasonal pattern is mainly climatic but other factors, institutional and traditional, may contribute to its presence. This component may also be detected in the spectrum as a peak. Since we shall be pre-occupied with the time series of an agricultural commodity our interest in the seasonal component will not be minimal.

The irregular component is a purely random phenomena arising out of the activities of speculators and hedgers and random decisions of producers and consumers in a nation's economy. The component does not show up in the spectrum as a peak but as a flat curve because a purely random phenomenon has a uniform spectrum over all frequencies. This component will also occupy our attention in this study.

As seen above, a time series, or equivalently, a stochastic process, may be purely random or not purely random. Economic time series are never purely random. Whatever the case may be, one way of analysing the process is to look at its autocovariance function, defined by

$$R(\tau) = E[(P_t - \bar{p})(P_{t+\tau} - \bar{p})]$$

for the discrete parameter time series

$$\left\{ P_t, \ t = 0, \pm1, \pm2, \ldots\ldots\ldots \right\}$$

In practice, $R(\tau)$ is estimated by

$$\hat{R}(\tau) = \frac{1}{n} \sum_{t=1}^{n-\tau} (P_t - \bar{p})(P_{t+\tau} - \bar{p})$$

where $\bar{p} = \frac{1}{n} \sum_{t=1}^{n} P_t$

$\bar{p}$ is the mean and n is the length of the process. τ is referred to as the time lag of the points in the series. A plot of $R(\tau)$ against τ provides information for determining the autoregressive scheme of a process or for testing its randomness. This is a time-domain approach. For two time series with different scales of measurement, it is useful to look at the normalized autocovariance function called the autocorrelation function and defined by

$$\gamma(\tau) = \frac{R(\tau)}{R(0)}$$

where $R(0)$, is, by definition, the variance of the series. $\gamma(\tau)$ has the property that it lies between -1 and +1. Again, a plot of $\gamma(\tau)$ against τ, called the CORRELOGRAM, gives a picture of the way in which the series damps out with time lag τ between points in the series.

Yet another method, available to us for analysing time series, is to look at its spectrum. This approach is rather involved but quite rewarding, as we shall see anon.

There are two basic principles underlying such an approach. The first is that a time series or a stochastic process, can be decomposed into a superposition of sine and cosine waves of different frequencies and random amplitudes or variations. The second is that the process is stationary. The process can then be described by the lower moments of its probability distribution, these moments being the mean, the variance the covariance and the Fourier transform of the covariance function, the power spectrum. It is this third approach which is adopted here.

The question immediately arises as to why we want to examine the power spectrum and not the autocovariance or the autocorrelation function. After all, the observations are functions of time. The answer is that we are looking for cyclical patterns in the time series. Time domain methods stress the dependence of observations along the time axis, and for neighbouring observations which influence one another, the autocovariance or the autocorrelation is difficult to interpret. The "frequency" or "periodic" approach is best suited to our purpose. But, why look for cycles at all? It is contended that if one is aware that cyclical or seasonal patterns exist with definite time periods one may be more capable of analysing the patterns further and devising methods for the purposes of control.

3-2 The Periodogram

The beginnings of spectral analysis go far back to the use of the periodogram of Sir Arthur Schuster [35] to estimate the spectrum. The procedure, periodogram analysis, utilizes the principle that any collection of equally spaced values of a variable may be represented by a finite Fourier series, or that a time series may be represented as a superposition of sinusiodal waveforms with independent amplitudes. The periodogram is the plot of a function $f_n(\lambda_j)$ against the frequency λ_j where one version of $f_n(\lambda_j)$ is

$$f_n(\lambda_j) = \frac{1}{\pi n}\left\{\left(\sum_{t=1}^{n} P_t \mathrm{Cos}\lambda_j t\right)^2 + \left(\sum_{t=1}^{n} P_t \mathrm{Sin}\lambda_j t\right)^2\right\}$$

$$\lambda_j = \frac{2\pi j}{n},$$

for a discrete parameter time series

$$\left\{P_t,\ t = 0,\ \pm 1,\ \pm 2, \ldots\ldots\ldots \right\}$$

of length n.

The periodogram was supposed to be able to discover correct cycles in a series, but this only held true if the series under investigation were strictly and genuinely periodic. If the series were not strictly periodic, erroneous conclusions could be drawn. The first reason behind the periodogram presentation here is that earlier frequency analyses in economics were based on this method of analysis which was later found to yield an inconsistent estimate of the spectrum. Its use in economics dates back to Moore [30] who used the periodogram to study rainfall in the Ohio Valley and to Beveridge [3] who used it to discover the existence of cycles in Western and Central European wheat prices. An extensive account of the use of the periodogram to study stock prices using monthly closing quotations of the Dow-Jones industrial stock price averages is given by Davis [6]. Davis's book also covers other uses of the periodogram such as the analysis of annual data of commercial paper rates of the Cleveland Trust Company for the period 1831-1930, annual data of Rail Bond prices of the Cleveland Trust Company for the period 1831-1930 and of business failures in the United States during the period 1867-1932. The last study was actually done by Greenstein [19]. The second reason is that even though the original periodogram was found to be an inconsistent estimate of the power spectrum, periodogram averaging in the frequency domain through the use of the Fast Fourier Transform now presents one fast and efficient method of estimating the spectrum.

The original periodogram may be said to have lived a short life. Its discontinued use stemmed from the fact that, firstly, it was able to detect "hidden periodicities" even in series so constructed as to contain none. Secondly, it was found to be an inconsistent estimate of the power spectrum. And as if the above two were not enough, its computation was quite time consuming. Generally, too, the methods of the physical sciences were thought to be inappropriate for the study of social science phenomena. But with the advancement of mathematical methods in statistics, one of the results of which was the search for consistent estimates of the spectrum, and with the advent of the computer which wrought drastic changes in computational speeds, the study and application of frequency domain methods came to the fore, especially in the engineering sciences.

The economists, whether it was in the spirit of keeping up with the Joneses, or to render their analysis more rigorous, or both, were not to be outdone. Works on frequency domain methods of analysis in economics were not lacking in the 1960's. Not by periodogram analysis, though, but by spectral analysis since the latter method yields a consistent estimate of the power spectrum.

3-3 What is Spectral Analysis?

Spectral analysis is a statistical method of estimation. The estimate we seek is the power spectrum or the spectral density function. These two are almost the same, but not quite. A fine distinction will be stated later on in the discussion. Like its predecessor the periodogram analysis, spectral analysis is based on the principle of a time series being decomposable into sinusoidal waveforms under the assumption of stationarity. Wold [38] in 1938, showed that for a stationary discrete parameter time series

$$\left\{ P_t,\ t = 0,\ \pm 1,\ \pm 2,\ .\ .\ .\ .\ . \right\}$$

there exists a non-decreasing bounded function $F_p(\lambda)$ defined on $-\pi \leq \lambda \leq \pi$ such that

$$R(\tau) = \int_{-\pi}^{\pi} e^{i\lambda\tau}\, dF_p(\lambda)$$

$$t = 0,\ \pm 1,\ \pm 2,\ \ldots\ldots$$

The function $F_p(\lambda)$ is called the <u>spectral distribution</u> of the time series and can be decomposed into an absolutely continuous function $F_{ac}(\lambda)$, a singular continuous function $F_{sc}(\lambda)$ and a purely discontinuous function or step function $F_d(\lambda)$. Since $F_{sc}(\lambda)$ has no practical significance from our point of view and economic time series are not strictly period so that $F_d(\lambda)$ also holds no meaning for us, these two will be dropped out of the discussion, leaving $F_{ac}(\lambda)$ as the function of interest. $F_{ac}(\lambda)$ is assumed to have a derivative $f(\lambda)$ which is a non-negative function. $f(\lambda)$ is the spectral density function or the power spectrum.

It is perhaps useful to be aware that the Wold representation in terms of the covariance sequence of the series is not the only existing one for arriving at the spectrum of the series. In the literature, one may also encounter what is known as the Cramer representation which is given in terms of the series itself as

$$P_t = \int_{-\pi}^{\pi} e^{i\lambda\tau} \, dz_p(\lambda)$$

No occasion arises for the use of this representation and is mentioned here as a point of interest.

3-4 Stationarity

As we mentioned in the two previous sections, the assumption of stationarity is crucial to spectral estimation. A time series or stochastic process may be stationary or non-stationary. Formally, a stochastic process is a sequence

$$\left\{ P_t, \ t \in T \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4\text{-}1)$$

of random variables, the set T, called the index set, being arbitrary and infinite. When T is countable, the stochastic process is said to be a DISCRETE-TIME process. If T is an open or closed interval on the real line, the stochastic process is said to be a CONTINUOUS-TIME process. In (3.4 - 1), the index set T is time. In Chapter 5, we deal with a sample of the process called a REALIZATION or SAMPLE FUNCTION of the stochastic process. This realization extends indefinitely into the past and the future, and constitutes a single time series. A stochastic process is a universe of such series.

A stochastic process

$$\left\{ P_t, \ t \in T \right\}$$

is strictly stationary if for any finite set of integers $\left\{ t_1, t_2, \ldots\ldots, t_n \in T \right\}$ and for every integer t, the distribution of $P_{t_1}, P_{t_2}, \ldots\ldots, P_{t_n}$ is the same as the distribution of $P_{t_1+t}, P_{t_2+t}, \ldots\ldots, P_{t_n+t}$. If the first order moments exist, then stationarity implies that

$$E(P_t) = E(P_{t+s}), \quad \ldots\ldots \quad (3.4\text{-}2)$$

$$s, t = -2, -1, 0, 1, 2, \ldots\ldots$$

Most economic time series do not fulfill the assumption of strict stationarity. A weaker form of stationarity must therefore be considered. Fortunately, this weaker form of stationarity, known variously as WIDE-SENSE STATIONARITY, COVARIANCE STATIONARITY, SECOND-ORDER STATIONARITY or MEAN-SQUARE STATIONARITY is adequate for spectral analysis. If a stochastic process is covariance stationary with mean zero, then the following conditions hold

$$E[P_t^2] < \infty, \quad \ldots\ldots \quad (3.4\text{-}3)$$

$$E[P_t P_{t-\tau}] = R(\tau) < \infty \quad \ldots\ldots \quad (3.4\text{-}4)$$

The conditions are stated with mean zero because for such a process no generality is lost by assuming that

$$E(P_t) = 0$$

Generally, $R(\tau)$ is a function time t and the time lag τ, but for covariance stationary processes, $R(\tau)$ is a function of τ only. A strictly stationary process need not have finite second moments, and hence need not be covariance stationary, but if it does have second moments, then the strictly stationary process is also wide-sense stationary. The converse is, however, not true. But if a covariance stationary process is also Gaussian, then it is necessarily strictly stationary. These statements can be easily proved.

3-5 The Spectrum as a Decomposition of Variance

The discussion in this section is carried out for the continuous case. Consider a time series

$$p_t, \; -\infty < t < \infty,$$

The autocovariance of the series is

$$E[p_t p_{t-\tau}] = R_p(\tau)$$

By the Wold representation, the time series possesses a spectral density function or power spectrum defined by

$$f_p(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\lambda\tau} R_p(\tau) d\tau$$

obtained by taking the Fourier transform of $R_p(\tau)$. It is assumed that $f_p(\lambda)$ has a continuous second derivative.

Now (a)[1] $R_p(\tau) = R_p(-\tau)$

(b) $\sin\lambda\tau R_p(\tau)$ is an odd function of τ, so that

$$\int_{-\infty}^{\infty} \sin\lambda\tau R_p(\tau)d\tau = 0$$

and (c) $e^{i\lambda\tau} = \cos\lambda\tau - i \sin\lambda\tau$

Hence $$f_p(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \cos\lambda\tau R_p(\tau)d\tau, \quad -\infty<\lambda<\infty$$

$f_p(\lambda)$ is a real-valued function. It is also symmetric about the origin.[2]

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[1]This follows from the definition of $R_p(\tau)$.

In $R_p(\tau) = E[p_t p_{t+\tau}]$

put $t + \tau = x$, then $t = x - \tau$

and
$$R_p(\tau) = E[p_t p_{t+\tau}]$$
$$= E[p_{x-\tau} p_x]$$
$= R_p(-\tau)$, and we are done

[2]This is seen as follows:

$$f_p(\lambda) = \int_{-\infty}^{\infty} R_p(\tau)\, e^{-i\lambda\tau} d\tau$$

and $$f_p(-\lambda) = \int_{-\infty}^{\infty} R_p(\tau)\, e^{i\lambda\tau} d\tau$$

Putting $x = -\tau$, and noting that $R(-x) = R(x)$,

we have $f_p(-\lambda) = \int_{-\infty}^{\infty} R_p(x)\, e^{-i\lambda x}\, dx = f_p(\lambda)$.

Thus

$$f_p(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \cos\lambda\tau R_p(\tau)d\tau, \quad -\infty<\lambda<\infty$$

$$= \frac{2}{2\pi} \int_{0}^{\infty} \cos\lambda\tau R_p(\tau)d\tau,$$

$$= \frac{1}{\pi} \int_{0}^{\infty} \cos\lambda\tau R_p(\tau)d\tau,$$

By taking the inverse Fourier transform of $f_p(\lambda)$, one obtains

$$R_p(\tau) = \int_{-\infty}^{\infty} e^{i\lambda\tau} f_p(\lambda)d\lambda, \quad -\infty<\tau<\infty$$

which reduces to

$$R_p(\tau) = \int_{-\infty}^{\infty} \cos\lambda\tau f_p(\lambda)d\lambda \quad \ldots\ldots (3.5 - 1)$$

In particular, if $\tau = 0$,

$$\sigma_p^2 = \mathrm{Var}(p_t) = R_p(0)$$

$$= \int_{-\infty}^{\infty} (\cos 0) f_p(\lambda)d\lambda$$

That is,

$$\sigma_p^2 = \int_{-\infty}^{\infty} f_p(\lambda)d\lambda \quad \ldots\ldots (3.5 - 2)$$

Equation (3.5 - 2) states that the spectral density or power spectrum of a series may be considered as the decomposition of the variance at different frequencies. That is, by looking at the spectrum we are actually looking at the contribution to variance of the series made by each frequency component. The spectrum is thus the frequency analogue of the variance in the time domain, and hence spectral analysis may be likened to the classical analysis of variance. For a finite sample record of size n, with finite sample autocovariance, the sample spectral density function is defined as

$$f_{p_n}(\lambda) = \frac{1}{2\pi} \int_{-n}^{n} R_{p_n}(\tau) e^{-i\lambda\tau}\, d\tau$$

$$= \frac{1}{\pi} \int_{o}^{n} \mathrm{Cos}\lambda\tau R_{p_n}(\tau)\, d\tau \quad \ldots\ldots\ldots\ldots (3.5\text{-}3)$$

For the discrete parameter process the equivalent of equation (3.5-3) is

$$f_{p_n}(\lambda) = \frac{1}{2\pi} \sum_{\tau=-n}^{n} R_{p_n}(\lambda) e^{-i\lambda\tau}$$

$$= \frac{1}{2\pi} R_{p_n}(0) + \frac{1}{\pi} \sum_{\tau=1}^{n} R_{p_n}(\tau)\cos\lambda\tau \ \ldots (3.5\text{-}4)$$

$f_p(\lambda)$ is known as the population spectral density and $f_{p_n}(\lambda)$ as the sample spectral density. It is shown by Anderson [2, pp. 438-496] that

$$E[f_{p_n}(\lambda)] = f_p(\lambda)$$

as

$$n \to \infty$$

but that the variance of $f_{p_n}(\lambda)$ does not approach zero. $f_{p_n}(\lambda)$ is not a consistent estimator of $f_p(\lambda)$. Some writers distinguish between spectral density and power spectrum.

The term spectral density is then given to the spectral estimates computed from autocorrelations and the term power spectrum is reserved for the estimates when autocovariances are used in the computations (See Jenkins [25]). In this sense, the term power spectrum will henceforth be used since our estimates are computed with autocovariances.

This section gives three facts:

A. The power spectrum of a time series is the Fourier cosine transform of the autocovariance function. For practical estimation purposes, the sample power spectrum is defined as the Fourier cosine transform of the <u>estimate</u> of the autocovariance function.

B. The sample autocovariance function and the sample power spectrum are a Fourier transform pair.

C. From equation (3.5 - 2), it is noted that the spectrum of a time series may be considered as the decomposition of the variance at different frequencies.

To close this section, it may be mentioned that a very long series of data is needed to obtain a good estimate of the spectrum. Granger and Hatanaka [15, p. 61] consider n = 200 a desirable minimum, although they note that crude estimates have been obtained with series as short as 80 observations.

Spectral analysis does not require the specification of a model. Explanation of causality is not its objective unless one employs cross-spectral analysis. It is nonparametric, and only seeks to confirm or deny the existence of cycles in a time series. In the next section we take a brief look at cross-spectral analysis.

3-6 A Look at Cross-Spectral Analysis

For ease of exposition, let us denote the New York spot price series by

$$\{x_t, t = 1, 2, \ldots, 203\}$$

and the London series by

$$\{y_t, t = 1, 2, \ldots, 203\}$$

with spectra $f_x(\lambda)$ and $f_y(\lambda)$ respectively. Assume both series to be stationary for the moment. As under auto-spectrum, one could examine the relationship between the two series by looking at cross-covariance functions:

$$R_{xy}(\tau) = E[x_t y_{t+\tau}]$$

$$R_{yx}(\tau) = E[y_t x_{t+\tau}]$$

or by looking at the cross-correlation function

$$\gamma_{xy}(\tau) = \frac{R_{xy}(\tau)}{\sqrt{R_{xx}(0) \cdot R_{yy}(0)}}$$

and plotting either $R_{xy}(\tau)$ or $\gamma_{xy}(\tau)$ against τ. We note in passing that

$$R_{xy}(\tau) = R_{yx}(-\tau)$$

and

$$R_{yx}(\tau) = R_{xy}(-\tau)$$

However, as before, we prefer to look at the relationship between the two series in the frequency domain and once again the Wold representation comes to the rescue to yield

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} f_{xy}(\lambda) e^{i\lambda\tau} d\lambda$$

where

$$f_{xy}(\lambda) = p_{xy}(\lambda) + iq_{xy}(\lambda)$$

is called the POWER CROSS-SPECTRUM. The real functions $p_{xy}(\lambda)$ and $q_{xy}(\lambda)$ are called respectively the power CO-SPECTRUM and power QUADRATURE spectrum and are defined by

$$p_{xy}(\lambda) = 2\int_{o}^{\infty} [R_{xy}(\tau) + R_{xy}(-\tau)] \cos 2\pi\lambda\tau d\tau, \; \lambda>o$$

and

$$q_{xy}(\lambda) = 2\int_{o}^{\infty} [R_{xy}(\tau) - R_{xy}(-\tau)] \sin 2\pi\lambda\tau d\tau \quad \lambda>o$$

Interest will not be focused on $f_{xy}(\lambda)$ itself but on two functions derived from it. These are the COHERENCE which is defined by

$$C(\lambda) = \frac{p_{xy}^2(\lambda) + q_{xy}^2(\lambda)}{f_x(\lambda) . f_y(\lambda)}$$

and the PHASE defined by

$$\Psi(\lambda) = \arctan \left[\frac{q_{xy}(\lambda)}{p_{xy}(\lambda)}\right]$$

The coherence measures the degree of association between the two series at different frequencies. It is the frequency domain analogue of the correlation coefficient, thus a value of $C(\lambda)$ near 1 indicates that the components are closely related at the frequencies in question while a low value near zero indicates no relation. The plot of $C(\lambda)$ against λ, $0 \leq \lambda \leq \pi$, is known as the COHERENCE DIAGRAM. The phase measures the lead-lag relationship between the two series at different frequencies, that is, it tells us whether or not one series is leading another. It goes without saying that the coherence diagram must be examined first since there is no gain in examining the lead-lag relationships of two series which are not related. A plot of $\Psi(\lambda)$ against λ, $0 \leq \lambda \leq \pi$, is known as the PHASE DIAGRAM. The estimation of these functions will be among the subjects discussed in the next two chapters.

3-7 Some Earlier Works in Economics Using Spectral and Cross-Spectral Analysis

In this section works done by economists using spectral and cross-spectral analysis will be reviewed. We first give a bird's eye view of the field, and then concentrate on two topics on which detailed work has been done.

3-7-1 Overview

The works of economists on spectral analysis have been as varied as their interests. Applications to econometric models have been provided by Naylor, Wertz and Wannacott [33], Chow [4], Hatanaka and Suzuki [22], Granger [11, 12] and Howrey [24]. The long swing hypothesis has been examined by Adelman [1], Hatanaka and Howrey [21], Howrey [23] and Harkness [20]. Purely theoretical presentation has been given by Godfrey [8]. The textile industry has not escaped the attention of Naylor, Wallace and Sasser [32], nor the automobile industry of Sweden the discerning eyes of Brandes, Farley, Hinich and Zackrisson [4]. Using hypothetical yields on British Government Securities, Granger and Rees [18] have used cross-spectral analysis to examine the term structure of interest rates. General works on spectra have been done by Granger [14] and Morgenstern [31], Granger on the spectral shape of an economic variable and Morgenstern on the justification for employing spectral analysis in economics. Spectral methods of dealing with seasonal adjustment of data have been the occupation of Nerlove [34] and Godfrey and Karreman [10]. Lastly, stock market prices have been analysed by Granger and Morgenstern [16, 17] Godfrey, Granger and Morgenstern [9], and Granger [13].

Of the works mentioned above, interest will be focussed mainly on the long swing hypothesis and the spectral analysis of stock market prices, because it is on these two that extensive work has been done.

3-7-2 The Long Swing Hypothesis

Adelman [1] was the first to use spectral analysis to study business cycles in the economy of the United States. She sought to confirm or reject the long swing hypothesis which postulates the existence of fluctuations of duration ranging between fifteen and twenty years in the United States economy. Her approach was to filter the series for output, capital stock, employment, investment, productivity of labour and other variables, compute their spectra, and look for the long cycle. Her conclusion was that there existed no cycles ranging from fifteen to twenty years. The cycles given by the spectral estimates, Adelman observed, were spurious ones introduced by the filtering process and spectrum averaging.

Howrey [23] has determined the spectral estimates of various macroeconomic variables with regard to the long swing hypothesis. He examined the spectra of series of gross national product, consumer durables, industrial production and many others, and came to the conclusion that the long swing frequency band which should be centered around 1/20th cycle per year was very weak indeed. So weak, in fact, as to render them insignificant. He also pointed out that the use of filters introduced biases into the spectral estimates, and consequently, long swings were noticed where none existed.

A year after Howrey's paper mentioned above had been published, Hatanaka and Howrey [21] questioned Adelman's methodology and her conclusions. To them Adelman had not chosen an appropriate truncation point for the proper resolution of the spectral estimates. Nor had she successfully surmounted the problem of trend removal. The filtering process had, therefore, lessened the significance of some important peaks in the spectrum, and because of improper resolution the spectral estimates were not independent enough to warrant any good statistical testing. Therefore, her conclusion that the estimates of the spectrum obtained offered no evidence of long swings was premature; further investigation was called for.

It may be surmised from the above that the use of filters has marred the attempt to discover the long swing. Filtering is, in fact, the most difficult aspect of spectral estimation. A filter designed to remove power at low frequencies should remove power at the desired frequencies. However, this is not always the case. For example, Slutsky [36] has shown that if sum and difference filter is applied to white noise whose spectrum is the same over all frequencies, the output is a sine wave. It is apparent, then, that the presence of some cycles in economic time series may be explained by the smoothing procedures used on the data.

The problem of trend elimination, when testing the long swing hypothesis for the Canadian economy, fills a greater part of Harkness's paper. Harkness [20] chose harmonic regression as the trend elimination procedure because of its predictable effects on low frequency variations. Using forty-eight series of wholesale prices, output, imports, exports, population, immigrant arrivals, total sales of Canadian bonds and total sales of corporation bonds, with some of the series in money terms and some in real terms, all data Canadian and all 60 to 100 years long, he found an average period of 10 to 14 years long for all the series. The long swing hypothesis is a fact for the Canadian economy, and this makes Harkness's results very interesting. It also confirms Hatanaka and Howrey's rejection of the Adelman conclusion. It is here conjectured that long swings may exist for the United States economy also.

One further result which makes Harkness's study interesting is that the long swings which appeared in the series in real terms were longer than those which appeared in the series in money terms.

3-7-3 The Random Walk Hypothesis

The stock market is a speculator's market; its analysis is of importance to investors, financial analysts and, of course, to speculators. The question that has been asked with regard to stock market prices is this: do stock market prices follow a trend, knowledge of which will make future prices predictable or do the prices follow a random walk? The random walk hypothesis states that prices of stock in pteriod t denoted by p_t equal prices in period t - 1, denoted by p_{t-1}, plus white noise u_t. Symbolically,

$$p_t = p_{t-1} + u_t$$

Three cases may be distinguished. The series of prices p_t is a strict random walk if u_t and u_{t-s}, $s \neq 0$, are independent. If the condition for strict random walk holds, and in addition the u_t are all identically normally distributed, then p_t is a Wiener process. If u_t and u_{t-s}, $s \neq 0$, are just uncorrelated, we have a second order martingale. A colourful interpretation of random walk is that it represents the steps of a drunken man as he moves along a straight line. A similar interpretation for martingale is that the process represents the winnings of a gambler who on each play of the game either loses or wins. That is, if p_t represents the capital of a gambler after time t and if the best are 'fair' in the sense that they result in zero expected gain to the gambler, then p_t, $t > 0$, forms a martingale. A Wiener process is a stochastic process which may be used to describe the highly irregular motion of microscopic particles suspended in a liquid.

The implications of the random walk hypothesis run counter to the beliefs of practical financial analysts.

For, the model implies that past values of absolute stock market prices contain no pertinent information as a guide for predicting future prices. The method used by Granger and Morgenstern [17] in their analysis was to estimate the spectrum of first differences of price series of stocks without transforming the data. This provided useful information only for the short run. To discover the long run properties of the series, the trend was removed by a moving average procedure before calculating the spectrum. The results from both analysis were that the random walk model provided a good fit for very short term movements of stock prices, but not for the long run. In the short run, therefore, stock market prices cannot be predicted. Godfrey, Granger and Morgenstern [9] reached the same conclusion by taking the logarithm of price series from the London and New York markets, removing the trend by regression and estimating the spectrum of first differences of the logarithmic price series. The work of Godfrey, Granger and Morgenstern differed from the work of Granger and Morgenstern in that the three authors not only utilized data from the London market but also used cross-spectral analysis to show that the random walk mechanism operated even when the market was closed.

The random walk mechanism basically operates on two assumptions. The first is that price changes are random variables, independent and identically distributed. The second is that the changes conform to some probability distribution, usually taken to be Gaussian, with variance assumed finite. Mandelbrot [29], however, has suggested that the differences of the logarithm of the prices, say γ_t, is a ramdom sequence, independent and identically distributed from a Pareto-Levy distribution with infinite variance, where

$$\log p_t - \log p_{t-1} = \gamma_t$$

This is the Stable Paretian hypothesis. Further discussion of the Stable Paretian hypothesis may be found in Fama [7].

It has been alleged by some practical financial analysts that stock market prices are predictable and that one can make money by "playing the market." But if statistical tests with real world data indicate that the behaviour of prices on the stock market, at least for the short run, is not predictable but random, and if in particular the random process is a random walk or a martingale, then one may be cautious in speculating since one stands as much chance of winning as of losing.

Recently, Labys and Granger [28] have attempted to find out if commodity prices follow a random walk. They examined the first differences of futures prices for thirteen commodities and first differences of cash prices for six of the thirteen commodities whose futures were also studied. Some of the commodities were potatoes, eggs, corn, oats, cocoa, copper, flax, soybeans and rye. The observations, mostly monthly but some weekly and some daily, covered mostly the period January 1950 to July 1965. The commodities were taken from the American market. Groups of one to four were assigned to the commodities on the basis of how near random walk the price changes were found to be. Cocoa spot and futures prices fell into group one, the group whose price changes closely resembled a random walk, since the spectra of the members of this group were well within the 95% confidence band constructed about the horizontal average of the spectral estimates. Examination of daily price changes for cocoa futures covering the period August 1964 to July 1965 revealed negative autocorrelation for cocoa prices. Since prices showed a steady downward movement in that period an explanation for the upward sloping spectrum was hard to come by. Lastly, using monthly data for the period 1950 to 1965, no significant lead-lag relationship was found for spot and futures prices of cocoa.

The main conclusion drawn from the exercise was that whereas the random walk model provides a very good fit for stock market price changes, the "model is a cruder approximation to the truth for commodity prices" [28, p. 67].

CHAPTER 4

PRACTICAL CONSIDERATIONS

4-1 Autospectrum Estimation

For the practical estimation of the autospectrum (to distinguish it from the cross-spectrum) use is made of the estimate of the mean and the estimate of the autocovariance function of the price series. These estimates are given by (Jenkins and Watts [26], pp. 171-189)

$$\bar{p} = \frac{1}{n} \sum_{t=1}^{n} p_t ,$$

$$\hat{C}_{p_n}(\tau) = \frac{1}{n-\tau} \sum_{t=1}^{n-\tau} (p_t - \bar{p})(p_{t+\tau} - \bar{p}) ,$$

However, knowledge of these two estimates and the Fourier transform of the autocovariance function would not necessarily yield a consistent estimate of the power spectrum. For, that simple procedure will be appropriate only when the time series under investigation is strictly periodic. For time series not strictly periodic, it is necessary to estimate the average band centered around a frequency rather than the power at a precise frequency. However, when this is done, the bias in the estimate is taken along with the true estimate of the spectrum. If the bias is small, the spectrum is said to be reproduced with high

FIDELITY or RESOLUTION. When the fidelity is small, a large variance or low STABILITY is most likely to result. If the estimate has a small variance, it is said to have high STABILITY. High stability may result in low fidelity and low stability in high fidelity. In practice a compromise must be made between the two.

To achieve this compromise, the usual principle is to apply a weighting function $w(\tau/m)$ known in the time domain as the LAG WINDOW, to the estimated autocovariance function before taking its Fourier transform. The Fourier transform of the lag window, that is, the representation of the lag window in the frequency domain, is known as the AVERAGING KERNEL or SPECTRAL WINDOW. The typical shape of a lag window is one large lobe centered at some frequency λ_o, flanked by smaller lobes or side lobes. An acceptable lag window would possess small side lobes to reduce or eliminate LEAKAGE, and permit a reasonable compromise to be made between fidelity and stability. The effect of leakage, a distortion of the spectrum caused by the manner of observation, is to blur the power spectrum, consequently spreading power to nieghbouring frequencies instead of being concentrated at a single point. It thus introduces bias into the spectral estimates. The source of the trouble lies in dealing with finite instead of infinite samples. The suppression of the side lobes, or equivalently, the reduction of leakage, is one reason for the use of lag windows.

There are several lag windows in the literature: Bartlett, Hamming, Hann, Dirichlet, Fejer, Lanczos, Blacman-Tukey and Parzen. The Parzen window has been chosen for our estimation. It is of the form

$$w\left(\frac{\tau}{m}\right) = \begin{cases} 1-6\left(\frac{\tau}{m}\right)^2 \left(1-\frac{\tau}{m}\right), & \text{for } 0 \le \tau \le \frac{m}{2} \\ 2\left(1-\frac{\tau}{m}\right)^3, & \text{for } \frac{m}{2} \le \tau \le m \\ 0, & \text{for } \tau > m \end{cases}$$

where m is the maximum number of lags or number of frequency bands for which the spectrum is estimated. For this window, the corresponding estimate of the autocovariance function is taken to be

$$\hat{C}_{p_n}(\tau) = \frac{1}{n}\left[\sum_{t=1}^{n-(\tau-1)} p_t p_{t-(\tau-1)} - \frac{1}{n}\left(\sum_{t=1}^{n} p_t\right)\left(\sum_{t=1}^{n} p_t\right)\right]$$

The desirable properties of the Parzen window are that statistical variability is reduced when using this window and also the power spectral estimates are nonnegative, a fact which is consistent with theory. The Blackman-Tukey lag window, on the other hand, can give negative estimates. This is not necessarily undesirable since negative estimates may indicate low power, leakage or possibly oscillating power in the neighbouring frequency bands of the negative estimate. When the Parzen weighting function is applied, equation (3.5-4) becomes

$$f_{p_n}(\lambda_k) = \frac{1}{2\pi} R_{p_n}(0) + \frac{1}{\pi}\sum_{\tau=1}^{m} \cos\lambda_k \tau w(\frac{\tau}{m}) R_{p_n}(\tau) \quad \ldots\ldots\ldots\ldots (4.1\text{-}1)$$

and the equation used for the actual estimation is

$$f_{p_n}(k) = \frac{1}{2\pi}\left\{\hat{C}_{p_n}(0) + 2\sum_{\tau=1}^{m} \hat{C}_{p_n}(\tau)\cos\frac{\pi k \tau}{m}\, w(\frac{\tau}{m})\right\} \ldots\ldots (4.1\text{-}2)$$

where $k = 0, 1, \ldots, m$ is a given time lag. The value of k which gives the largest value of $f_{p_n}(k)$ is the lag most significant in explaining the variance of prices.

From equations (4.1-1) and (4.1-2), it is seen that

$$\lambda_k = \frac{\pi k}{m}$$

$$k = 0, 1, \ldots\ldots\ldots m$$

Now $\lambda_k = 2\pi f$

where f is the frequency in cycles per unit of time and λ_k is the angular frequency in radians per unit of time.

Hence $f = \frac{k}{2m}$

and $\delta = \frac{2m}{k} = \frac{1}{f}$

where δ is the period.

Varying the number of lags or the truncation point m affects the smoothness of the spectral estimates. Several truncation points, 45, 60, 75 and 90 were tried before choosing 60 lags. There is no known method of choosing an optimal value of m. The number of lags has been chosen by examining the spectra obtained using the various lags of 45, 60, 70 and 90 and also with the advice of Granger and Hatanaka [15, p. 61] in mind that for small n, m taken as n/5 or n/6 is a reasonable choice. Figures 5-1 and 5-2 show plots of the spectra of the cocoa price series estimated with 60 and 75 lags. It is noted that if m is too large, spurious peaks appear in the spectrum and may be mistaken for genuine peaks. In this case, as stated earlier, the bias is small but the variance is large, and the spectrum is said to be well resolved, or has high fidelity, but low stability. If m is small, the variance is small but the bias is large and the spectrum has high stability but low fidelity. In the latter case, a genuine peak may be smoothed out of existence if m is too small.

4-2 Filtering

It was observed in Section 3-1 that economic time series are traditionally decomposed into trend, cycle, seasonal and irregular components. The presence of trends in mean and variance of the series renders the series non-stationary, so that application of spectral analysis would yield incorrect results.

There are two approaches to the problem. One is to estimate the spectrum of the series without any data transformation and then disregard the peaks in the low frequency range of the spectrum since it is in the low frequency range that trends manifest themselves in the spectrum. However, the problem is not solved that way, for it is also in this range that cycles of long periods make their appearance. If the raw series is therefore estimated, a peak in the low frequency range of the spectrum would most likely be the result of cycles with long periods, and trends. This leads to the second approach: transform the series in a way to remove the trend and make it as near stationary as possible before spectral estimation. However, the problem of filtering is not a simple one. One must consider the effect of the filtering process not only on the trend but also on other components of the series. Filtering is, in fact, based on the assumption that the four components can be linearly added and that they are independent. But this assumption is much open to doubt. A filter designed to remove a trend may also remove a cycle. Any adjustment of data to make it stationary is, therefore, at best, an approximation. In view of this, and the fact that the estimated trend using the New York series was negligible, the data used here have not been adjusted in any way.

4-3 <u>Aliasing</u>

Aliasing is a problem which arises from data sampling. If data are sampled at very close intervals such as weekly and monthly, the data tend to be correlated and if at long intervals such as yearly, aliasing results. If data are taken at intervals of Δt years per sample, or in terms of frequency, $\frac{1}{\Delta t}$ samples per year, then the information content of data taken at $\frac{1}{2}(\Delta t)$ years per sample or $\frac{2}{\Delta t}$ samples per year is lost. In such a situation, the highest frequency about which there is information is $\frac{1}{\Delta t}$ and is called the FOLDING or ALIASING or NYQUIST frequency. Frequencies in the original data above the Nyquist frequency will be folded back into the range 0 to $\frac{1}{\Delta t}$ and are said to be aliased with the frequencies in the range 0 to $f = \frac{1}{\Delta t}$. It must be remembered that shorter periods of time imply higher frequencies and vice versa. The frequencies which are aliased add power to the spectral estimates in the 0 to $f = \frac{1}{\Delta t}$ frequency range. The result is biased estimates. One method of dealing with the aliasing problem is to sample at such small intervals that no data can be ignored at the cut-off frequency point. The second method is to filter the data before sampling to eliminate information above the desired maximum frequency. Since the data analysed here were collected monthly, no information can be obtained regarding periods of less than 2 months or frequencies greater than that corresponding to the 2 month period. Also due to the symmetry in the definition of power spectrum the only frequency range worthy of practical consideration is the range 0 to π.

4-4 Cross-Spectrum Estimation

The estimation of cross-spectrum follows almost the same steps as the estimation of auto-spectrum. Using the notation of Section 3-6, we compute the estimate of

$$R_{xy}(\tau) = R_{xy}(-\tau) = E[x_t y_{t-\tau}]$$

by

$$\hat{C}_{xy}(\tau) = \frac{1}{n}\left[\sum_{t=1}^{n-(\tau-1)} x_t y_{t-(\tau-1)} - \frac{1}{n}\left(\sum_{t=1}^{n} y_t\right)\left(\sum_{t=1}^{n} x_t\right)\right]$$

and of

$$R_{yx}(\tau) = R_{xy}(-\tau) = E[y_t x_{t-\tau}]$$

by

$$\hat{C}_{xy}(\tau) = \frac{1}{n}\left[\sum_{t=1}^{n-(\tau-1)} y_t x_{t-(\tau-1)} - \frac{1}{n}\left(\sum_{t=1}^{n} x_t\right)\left(\sum_{t=1}^{n} y_t\right)\right]$$

where τ is the maximum number of lags. Using these estimates and the Parzen weighting function, the COSPECTRUM and the QUADRATURE SPECTRUM are computed respectively as

$$p_{xy}(k) = \frac{1}{2\pi}\ \hat{C}_{xy}(0) + \sum_{\tau=1}^{m} [\hat{C}_{xy}(\tau) + \hat{C}_{yx}(\tau)]\cos\frac{\pi k\tau}{m} w(\tau/m)$$

and

$$q_{xy}(k) = \frac{1}{2\pi} \sum_{\tau=1}^{m} [\hat{C}_{xy}(\tau) - \hat{C}_{yx}(\tau)]\ \sin\frac{\pi k\tau}{m} w(\tau/m)$$

where k and $w(\tau/m)$ have the same meaning as they had under autospectrum. From the co-spectrum and the quadrature spectrum emerge

(1) the cross-spectrum:

$$f_{xy}(k) = p_{xy}(k) + iq_{xy}(k)$$

(2) the coherence

$$c(k) = \frac{p^2_{xy}(k) + q^2_{xy}(k)}{f_x(k) \cdot f_y(k)}$$

and

(3) the phase:

$$\Psi(k) = \arctan \left[\frac{q_{xy}(k)}{p_{xy}(k)}\right]$$

The phase angle is measured in radians. The value of k which gives the highest value of coherence indicates that the two prices series are highly correlated in that particular frequency and the period of the related cycles can be computed as before as

$$\delta = \frac{2m}{k}$$

The difference in phase between the two price series can be computed by

$$\frac{2\pi}{\delta} = \frac{\Psi(k)}{\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.4\text{-}1)$$

where β is the phase difference;

From (4.4 - 1), we have that

$$\beta = \frac{\delta\Psi(k)}{2\pi}$$

$$= \Psi(k) \frac{m}{\pi k} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.4\text{-}2)$$

It is important to note the fact that if no lag exists between the two series the phase $\Psi(k)$ will be zero for all k. We also note that the phase difference β could be fractional as well as integer since fractional lags are possible to observe. Also, as mentioned earlier, it is not useful examining the phase $\Psi(k)$ if the coherences do not indicate that any relationship exists between the two series, and lastly if one series is not simply lagged to the other but there is a feedback between the two series, the complicated theoretical shape of the phase diagram permits no meaningful interpretation of the lag structure. To be specific, if spot prices of cocoa on the London market influence significantly, in some way, the spot prices on the New York market, and vice versa, then the phase diagram will look complicated and we cannot tell the lag structure of one series on the other by examining the phase diagram. The phase diagram of the two series shown in Tables 2-1 and 2-2 will be examined in the next chapter. There is as yet no satisfactory method for investigating feedback of economic variables in the frequency domain.

CHAPTER 5

RESULTS

5-1 Introduction

In this chapter, the results obtained from the analysis using methods described in Chapter 4 and the data of Tables 2-1 and 2-2 are presented.

5-2 Spectrum of Data from the New York Market

The spectra of raw cocoa price series, New York spot Accra, are shown in Figures 5-1 and 5-2. For the estimation of these spectra use was made of all the 323 monthly observations. Throughout the presentation the data used has been post-war data since wars affect the low frequency range of the spectrum (Granger and Hatanaka [15, p. 199]).

The first observation to be made concerns the shape of the spectra. The spectra decrease with increasing frequency and display the "typical" shape made famous by Granger [14]. This shape is typical of most economic time series even when there is no trend in mean. The typical shape suggests that the series can be represented by a first or second order auto-regressive model. The shape also tells us that the series are positively correlated. One advantage of spectral analysis is that the frequency

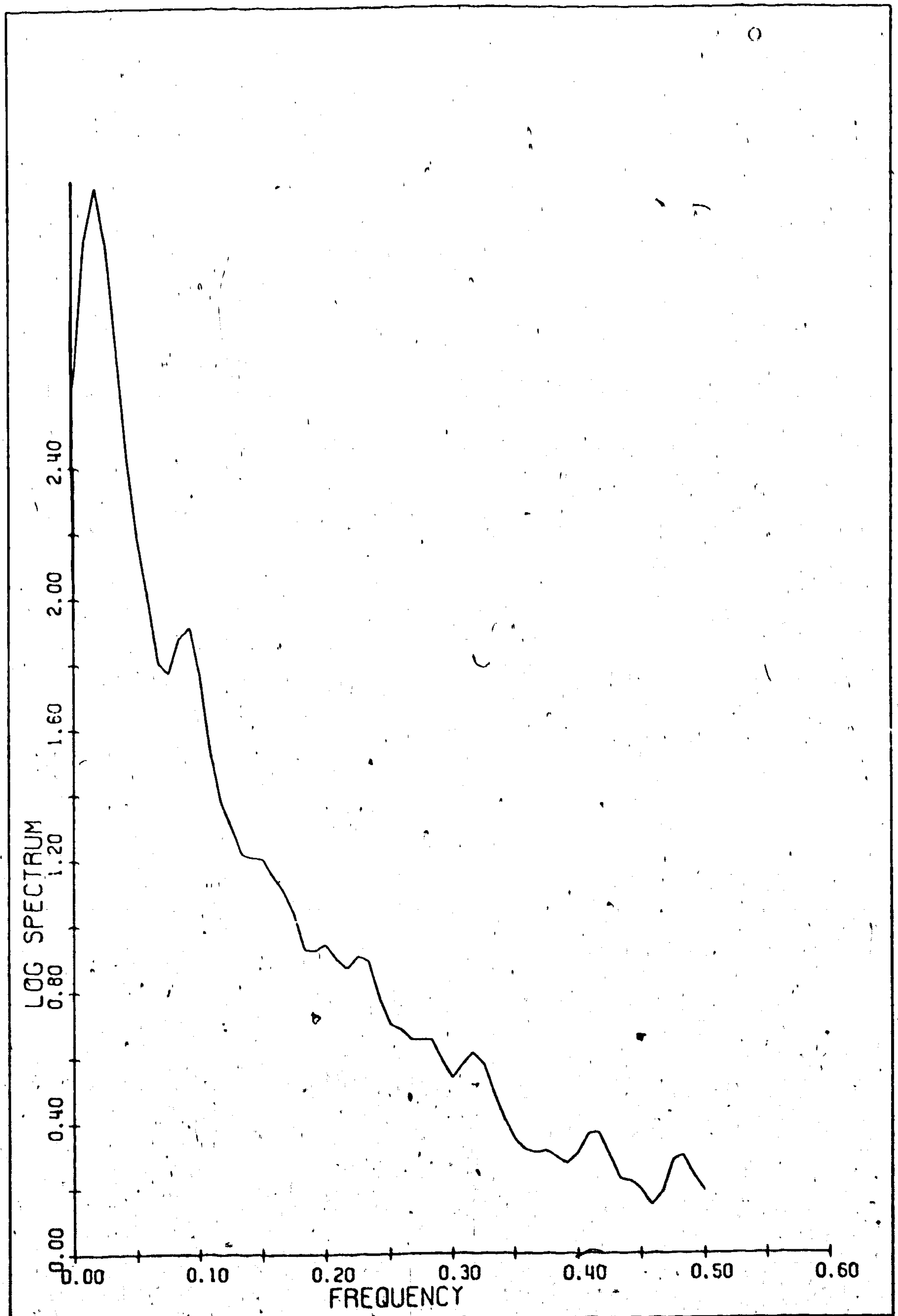

Figure 5-1 Spectrum of Series Shown in Figure 2-1, 60 lags.

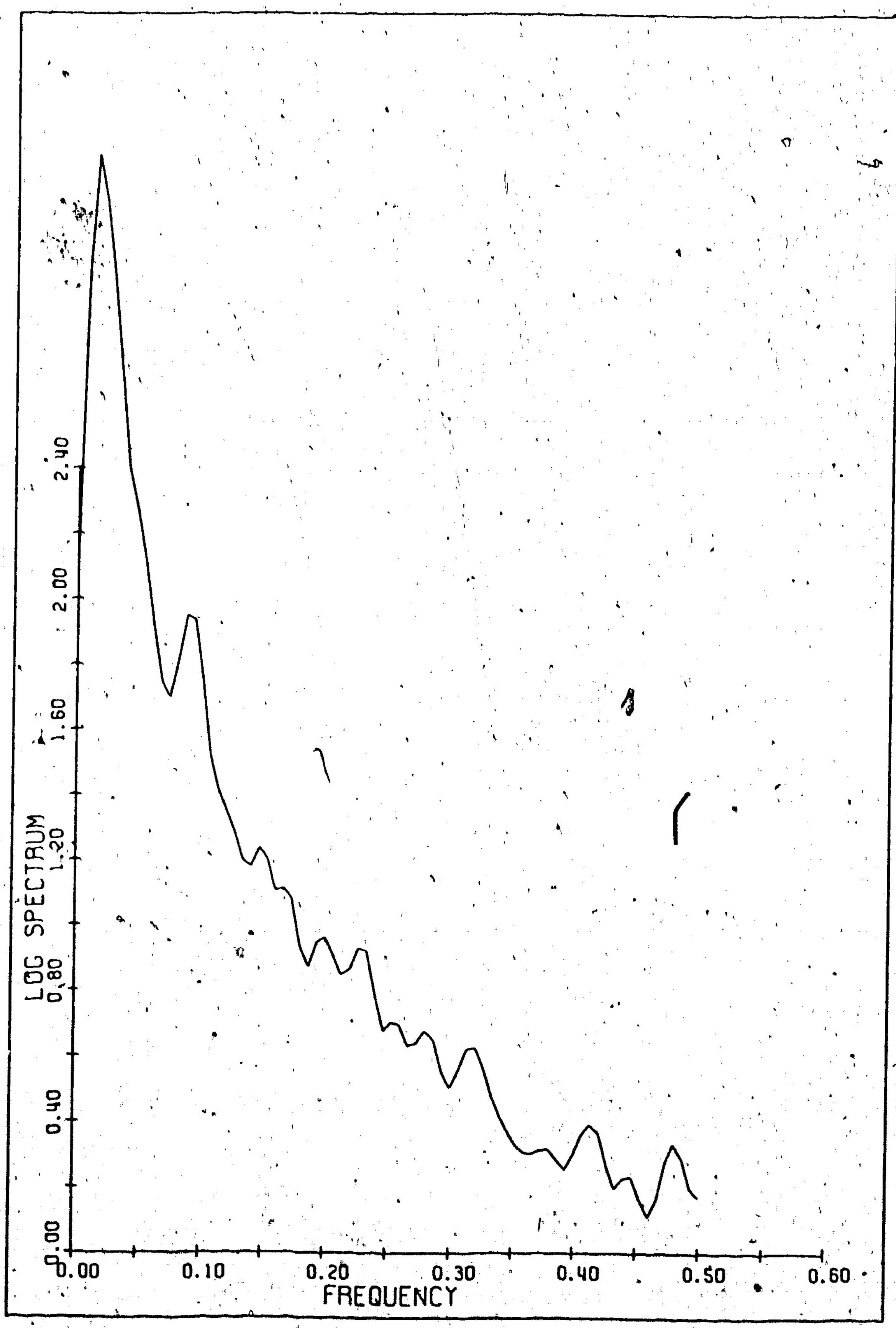

Figure 5-2 Spectrum of Series Shown in Figure 2-1, 75 lags.

analysis allows the concepts of long run and short run to be specifically stated. An examination of the spectra in Figures 5-1 and 5-2 reveals different periodicities in the price series. Before examining the peaks in the spectrum, we hasten to comment that we are estimating a function and not a point, so that consideration is given to bands of frequencies as well as points centered on these bands. We also interpret the spectra bearing in mind that the trend in the price series is negligible. Another point to note is that Granger and Morgenstern [16, p. 138] state that any trend in the data shows up in the frequency range 0 to π/n, and Granger [14] states that the trend shows up in the range 0 to $2\pi/n$. For 323 observations, $\pi/n = 0.0097$ which is very small indeed, whilst $2\pi/n = 0.019$. We now proceed to examine the peaks in the spectra. Using 60 lags the peaks appeared in frequency bands 0.0083 to 0.0250 centered on 0.0167; 0.0833 to 0.1000 centered on 0.0917; 0.1917 to 0.2083 centered on the frequency 0.2000; 0.3038 to 0.3250 centered on the frequency 0.3167; 0.4000 to 0.4250 centered on 0.4167 and finally in the band 0.475 to 4.4917 with centre at 0.4833. The corresponding periods using the points in the centers of the bands are 59.88 months or 4.99 years, 10.91 months, 5 months, 3.16 months, 2.40 and 2.03 months. For the estimation using 75 lags, the corresponding centered frequencies are 0.0133, 0.0867, 0.2000, 0.3200, 0.4133 and

0.4800 corresponding to periods of 75.19 months or 6.27 years, 11.53 months, 5 months, 3.13 months, 2.42 months and 2.08 months. From the estimation using 60 lags the band 0.0083 to 0.0250 gives an interval of 40 to 120 months, and from the band 0.0067 to 0.0200 obtained by using 75 lags we have a time interval of 50 to 149 months. A close examination of Figure 5-2 shows that the spectrum shows slight instability. For instance, a peak shows at the frequency of 0.2800 with period of 3.57 months. This peak is not so pronounced in the estimation using 60 lags. There the peak is overshadowed by the one at frequency 0.2000. The peak at 0.2800 in Figure 5-2 is therefore spurious. In fact, for 60 lags, we have 20 degrees of freedom where ν, the number of degrees of freedom per estimate, is calculated as

$$\nu = 2\left(\frac{n}{m}\right) b_1$$

where b_1 is the standardized bandwidth of the Parzen window and is equal to 1.86 (Jenkins and Watts, [26, p. 252]), whereas with 75 lags we are left with approximately 16 degrees of freedom. Now, if the number of degrees of freedom is large, the estimate is more reliable in the sense that it has smaller variance. What we have done by increasing m is to reduce the bias and increase the variance per estimate, thus introducing peaks which were not present in the estimation using 60 lags.

5-3. Spectrum of Data From the London Market

Here, we are dealing with 203 observations covering the period January 1956 to November 1972, and for the purpose of comparison the New York data was divided into two parts and spectral estimation was carried with data covering the same period as the data from the London Market. Figure 5-3 shows the plot of the spectrum of the cocoa series, London spot Accra, estimated with 40 lags. Figure 5-4 shows the same series estimated with 50 lags. The spectrum estimated with 40 lags shows a peak in the band centered on the frequency 0.0250 which corresponds to a 40-month cycle, and a peak in the band centered on the frequency 0.4125 corresponding to a period of 2.42 months. The spectrum is very smooth even with the estimation using m equal to 1/5 of the data and a degree of freedom per estimate of approximately 19. Examination of the spectrum estimated with 50 lags and hence with 15 degrees of freedom, shown in Figure 5-4, indicates a sharp peak in the frequency band 0.0100 to 0.0300 centered on 0.0200, and weak peaks in the bands centered on frequencies 0.1100 and 0.4100, corresponding to periods of 50 months or 4.17 years, 9 months and 2.44 months. The corresponding New York data estimated with 40 lags, shown in Figure 5-5 yielded periods also of 40 months and 2.42 months, the frequencies being 0.025 and 0.4125 respectively. The presence of 40-month cycles and 2.42 month cycles argues strongly for the presence of the

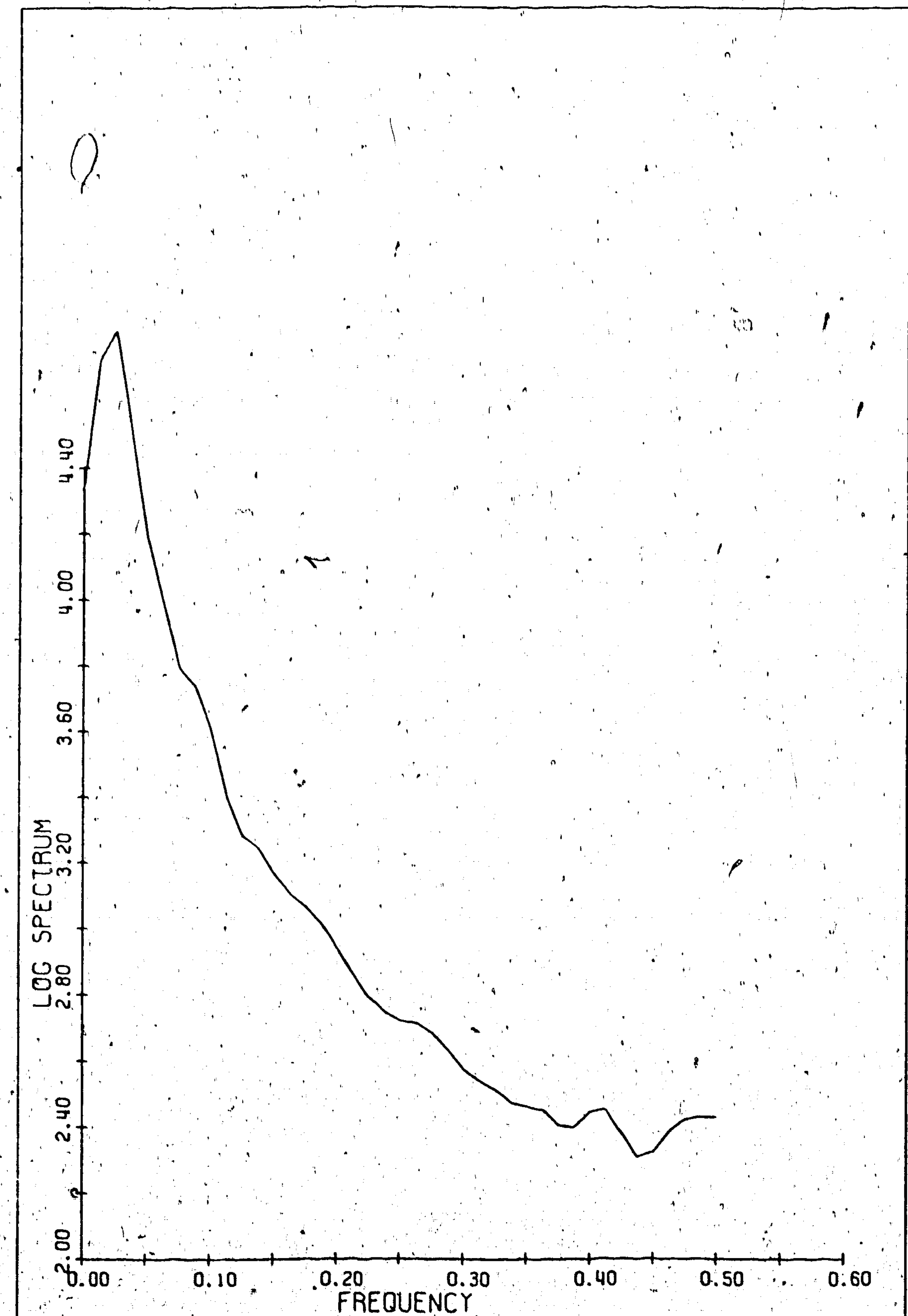

Figure 5-3 Spectrum of London Data, 40 lags.

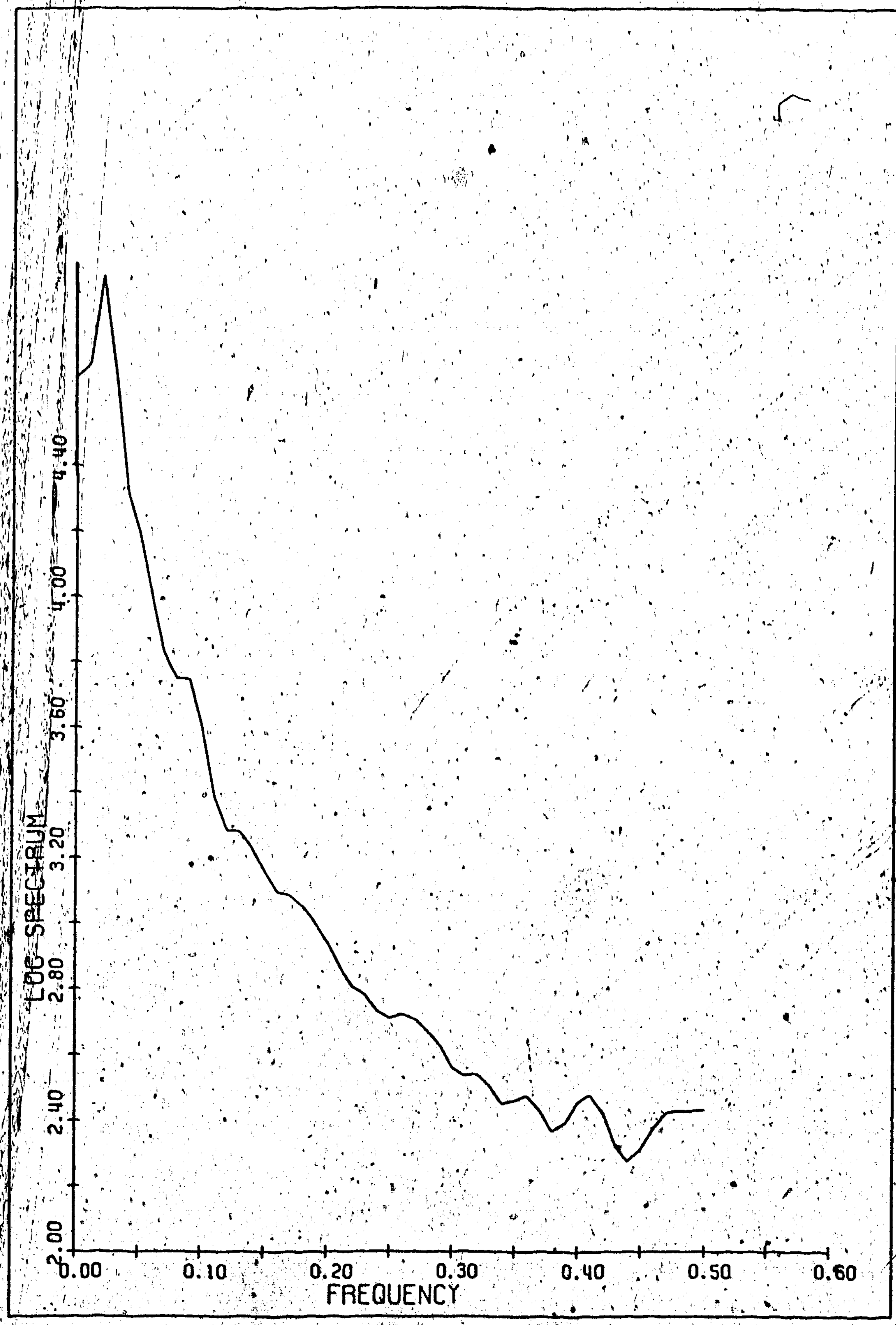

Figure 5-4 Spectrum of London Data, 50 lags.

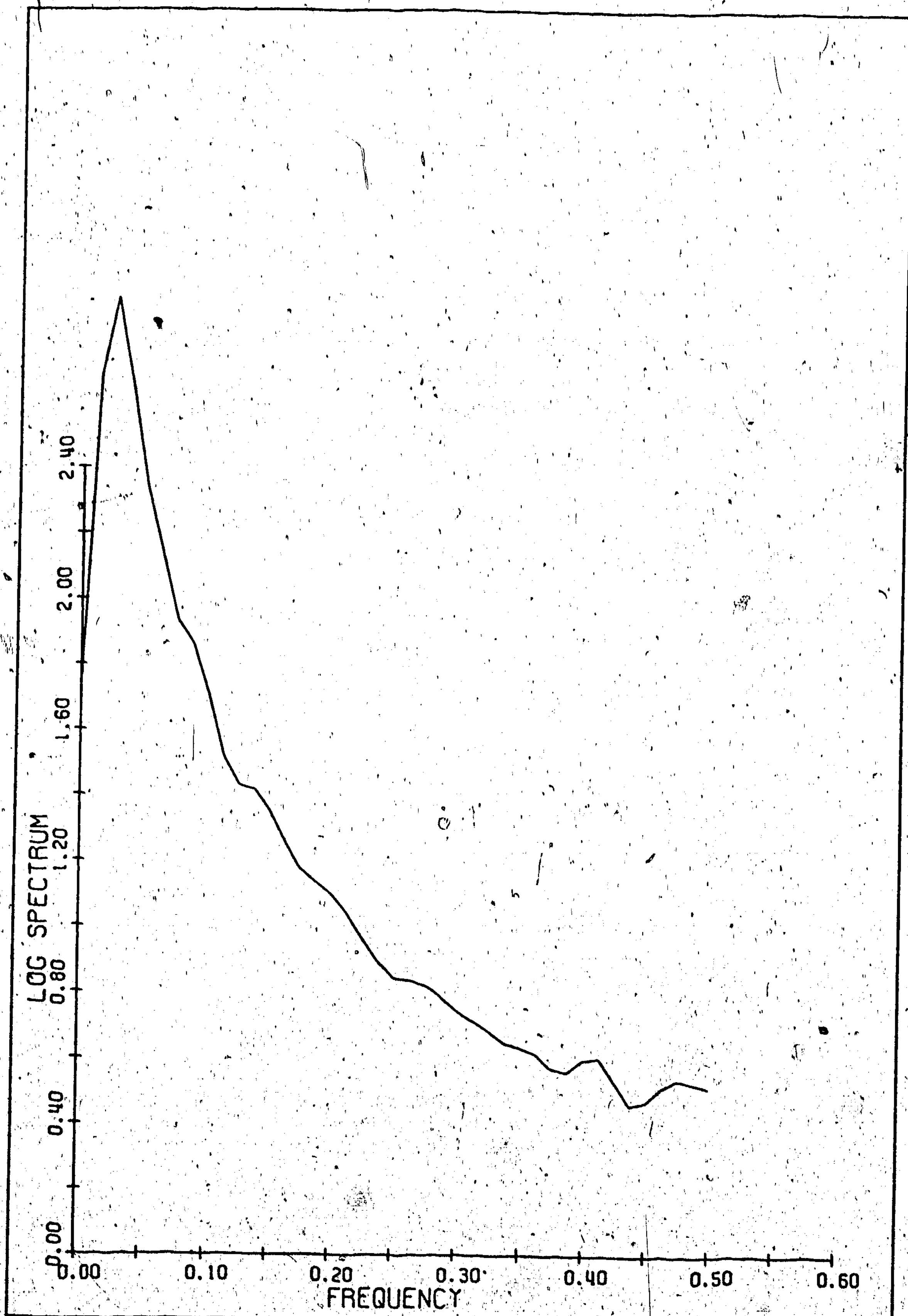

Figure 5-5 Spectrum of New York Series, 40 lags (1956 - 1972 data).

annual component in the two short series.

5-4 Cross-Spectra of the Two Series

As stated earlier, the objective of cross-spectral analysis is to discover the relationships between the two series and to find out if one series is leading the other. In Figure 5-6 is shown the coherence diagram for the two short series and the corresponding phase diagram. The coherence and the phase diagrams are shown on larger scales in Figures 5-7 and 5-8 respectively. All coherences lie between 0 and 1. The coherence diagram indicates a number of significant correlations at various frequencies. At these frequencies the two series are subjected to the same influences.

Taking the periods at which the two series are strongly related as the starting point, we next examine the phase diagram for any lead-lag relationships. That one series may be leading another is indicated by the fact that the phase diagram is not flat. However, for there to be any meaningful relationship between the two series, the individual cyclical components corresponding to the frequencies at which the coherences are high must also be significant for each of the series considered separately. This happens to be the case for the 40-month component only. Since there is more activity on the New York Exchange than there is on the London Terminal and since the United States

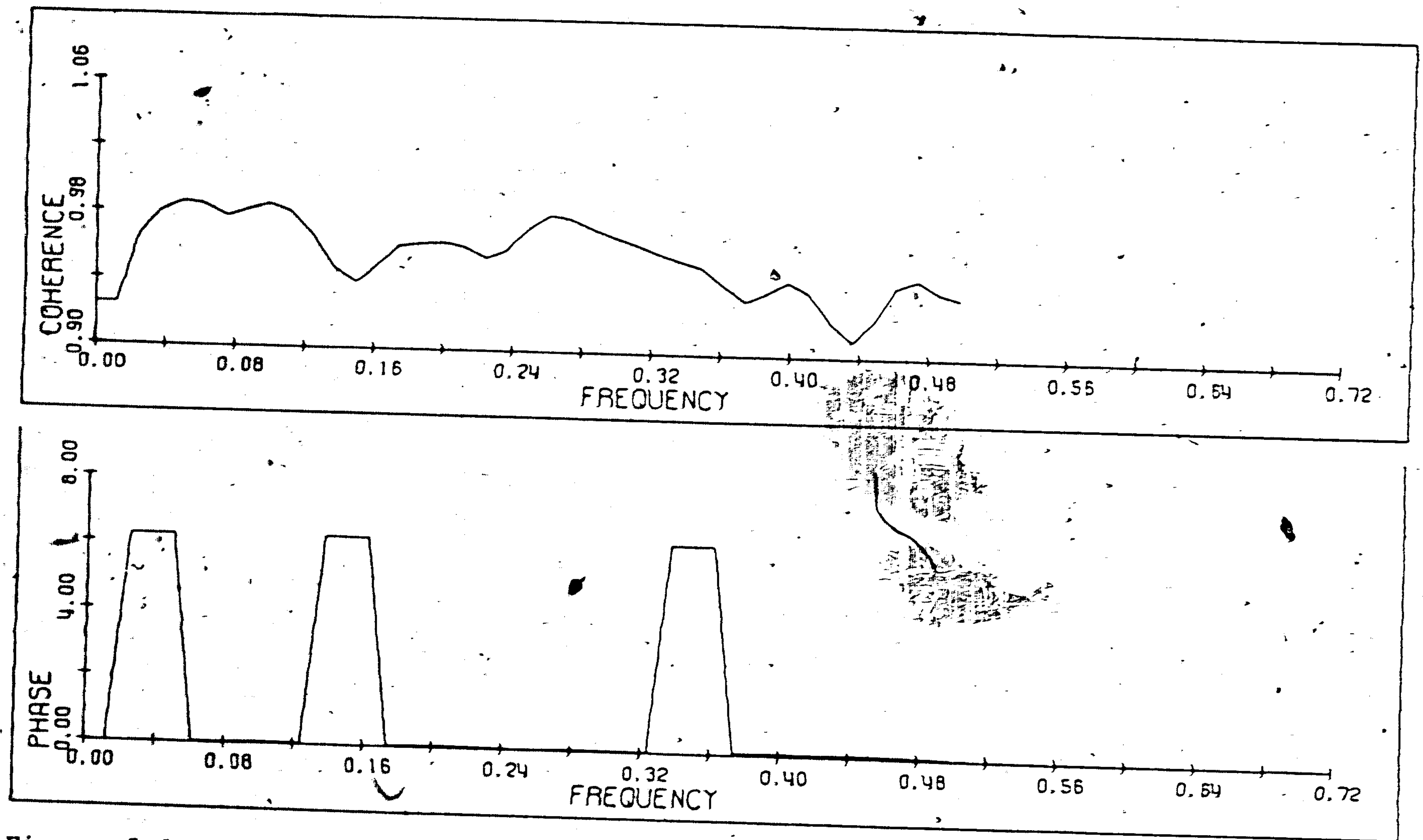

Figure 5-6. Coherence and Phase Diagrams of New York and London Series.

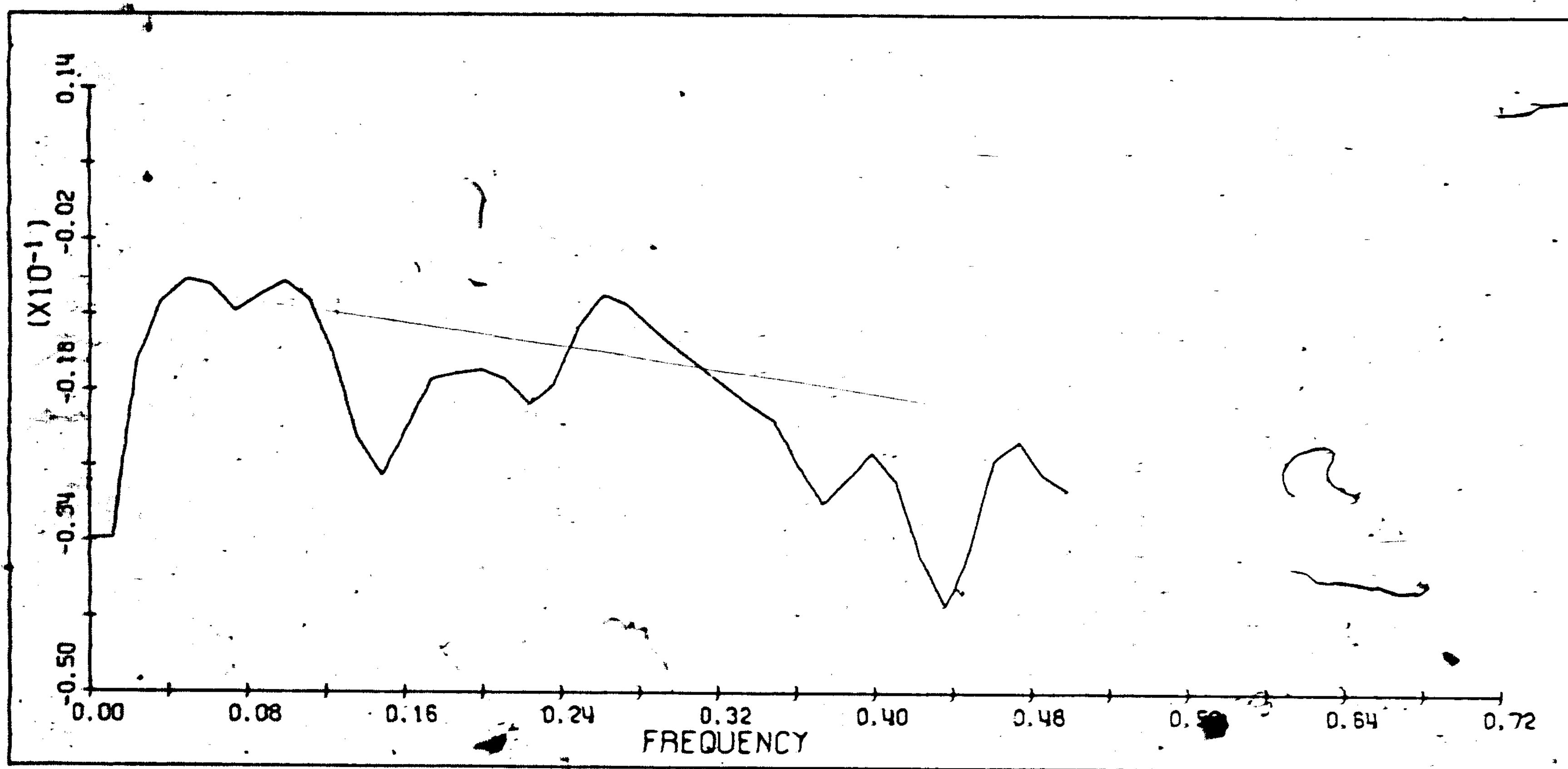

Figure 5-7 Coherence, Larger Scale.

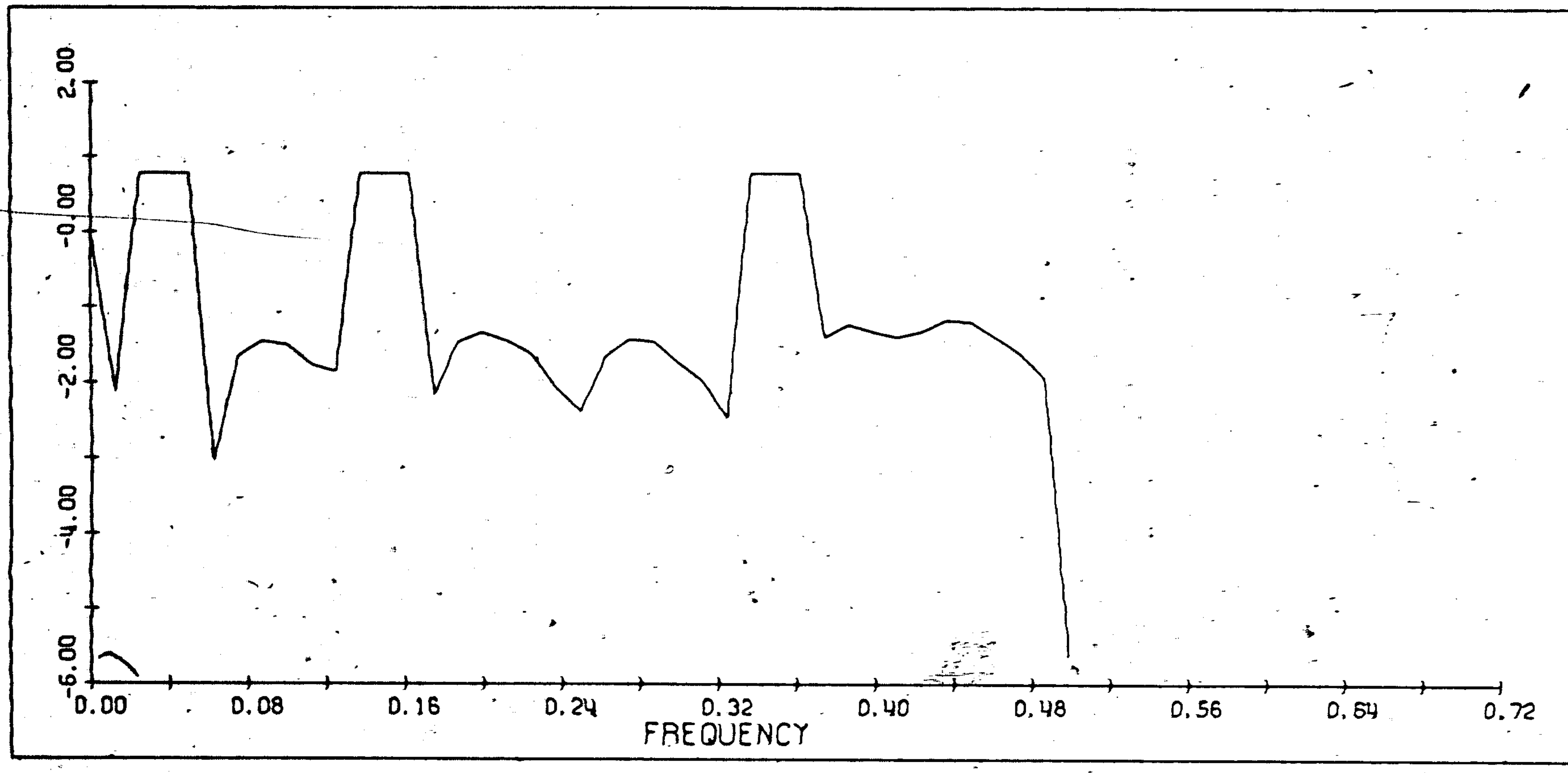

Figure 5-8 Phase, Larger Scale.

is the largest importer and consumer, it may be asserted that the New York series leads the London series. Table 5-1 gives some values of the coherence and phase where the coherence values are high. Table 5-1 also shows some values of phase differences calculated according to equation (4.4-2). As mentioned above only the 40-month component is of interest, and for this 40 months per cycle fluctuation, it is the London series which leads the New York series by 2 days. No cogent reason can be advanced for the peculiar shape of the phase diagram except to say that in such highly organized markets as the London Cocoa Terminal and the New York Cocoa Exchange, there is a strong likelihood of feedback of information regarding prices between the two markets. The plots of the raw series in Figures 2-3 and 2-4, showing similar overall shapes may confirm this.

The graph in Figure 5-7 shows that the coherences are not uniformly high. This points to the fact that there are factors appreciably affecting one series but not the other, at various frequencies. To find out if this is the case we have plotted the SPECTRAL ESTIMATES OF THE RESIDUALS given by

$$S(k) = [1 - c(k)]\,[f_y(k)] \quad ,$$

$c(k)$ being the squared coherency and $f_y(k)$ the spectrum of the London Series.

TABLE 5-1

COHERENCE, PHASE AND PHASE DIFFERENCES FOR LONDON AND NEW YORK SERIES AT SOME POINTS OF HIGH COHERENCE VALUES

FREQUENCY	PERIOD (Months)	COHERENCE	PHASE (Radians)	PHASE DIFFERENCE (Months)
0.0250	40	0.966	6.272	-0.07
0.05	20	0.986	6.269	-0.05
0.10	10	0.985	0.031	+0.05
0.2625	3.8	0.982	0.021	+0.01

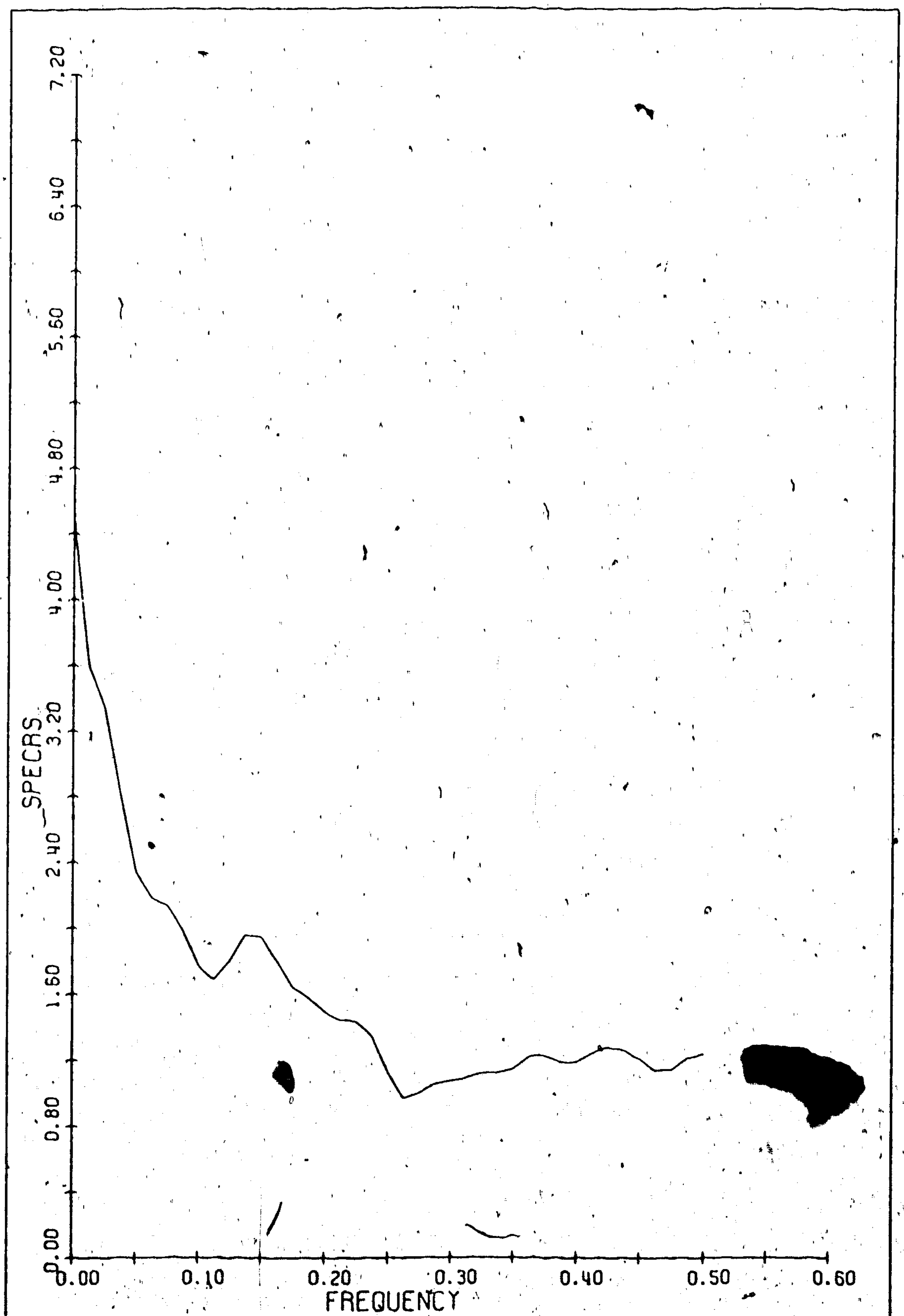

Figure 5-9 Spectral Estimates of the Residuals.

$$\{ y_t, \ t = 1, 2, \ldots\ldots\ldots 203 \}$$

The residuals are similar to the residuals in ordinary regression analysis and the estimates give one an idea of the possible periodicities in the London series which are not present in the New York series. The graph of S(k) is shown in Figure 5-9. The only significant peak occurs in the frequency band 0.1250 - 0.1625 and is centered on the frequency 0.1375 which corresponds to a period of 7.27 months. This indicates that some factor affects the London series causing a fluctuation with a period of 7.27 months, but that this factor is not significant for the New York series. It would be interesting, from the point of view of speculation, to know what this factor is.

CHAPTER 6

CONCLUSIONS

In this thesis an attempt has been made to study the fluctuations in spot prices of cocoa in the frequency domain. The overall shapes of the spectra obtained using the two series from the London Cocoa Terminal and the New York Cocoa Exchange indicate that diverse mechanisms are at work in generating the series of spot cocoa prices. The typical shape implies that a long run component exists, and indeed long cycles of 5 years using 60 lags and of 6 years using 75 lags were found. If the intervals are considered, we have periods of 3.33 to 10 years using 60 lags and 4.17 to 12.4 years using 75 lags. A 40-month cycle showed up even in the short series. However Weymar [37], using methods other than spectral analysis states a cycle of about 11 years. The facts about the cocoa tree are the following. In Grenada, a life time of 80 years has been recorded; in West Africa, 50 years; and in West Indies 100-year old trees have been known to yield fruit. The young tree begins to yield in its fifth year with a variation of 4 to 6 years. Optimum yield is supposed to occur at about the 30th year and then a decline sets in. Once, however, the yield begins, it bears fruit twice a year; the major one around December and the minor one in June.

The fact that the peak with period of 10-12 months is well defined indicates its permanence. This, of course, is a seasonal fluctuation, with harmonics of 5, 3, and 2.4 months. The 10-12 months period and the 5 month harmonic may be explained on the grounds that there are two harvesting periods for cocoa, a "main crop" which is harvested from October to March and a "mid crop" which is harvested from May to June. It is possible that the annual fluctuation is caused by the major harvest and the 5 month fluctuation by the minor harvest. Demand and supply conditions in the market and the level of inventories may thus explain these price shifts. The 2 to 3 months harmonics may be explained of the basis of hedging and speculative behaviour on the market since the parts of the spectra in the frequency ranges 0.30 to 0.5 are relatively flat.

Cross-spectral analysis indicated that the different price fluctuations are governed by the same factors. What affects spot prices in New York affects prices in London. An impending change of government in Ghana, say, may cause the same shifts in prices on the two Exchanges. The shape of the phase diagram rendered it quite difficult to interpretation. However, on the assumption of absence of feedbacks, quite an unlikely one, a lead of 2 days was found. The fact that the phase values were not zero indicated some form of relationship, though. Also, since the phase diagram was found to be made up of straight lines with different slopes, there was strong indication that one series was leading the other and that the lead-lag relationship does change in the time domain as we move from one band of frequency to another. It suggests that different models may be fitted to different ranges of frequency. The two markets are apt, however, to possess feedback of information which significantly affects prices. Further work needs to be done in this area. Examination of the spectral estimates of the residuals showed that there is a seasonal fluctuation of period 7.27 months which though not very noticeable on the New York market is quite significant on the London market. No explanation can be advanced here for such a fluctuation.

What are the implications of the above findings for decision making? What has been done here is to establish the EXISTENCE of phenomena. It is the first step towards prediction and regulation. Before an attempt is made to predict and control one must first establish the existence of the phenomenon. Having established existence one can then undertake further study of the process to know its properties. Only then can prediction and control proceed.

The spectra of prices could help in model building. Knowledge of the cycles of long periods and the seasonal fluctuations and of the shape of the spectra could lead to the construction of a realistic predictive model for cocoa prices which may be used to damp out some of the price fluctuations since these fluctuations affect the incomes of both producers and consumers.

The cross-spectral method may be used to study not only two price series as done here, but also the price-production relationship, price-consumption relationship, crop-failure and price relationship and many other variables. Our study showed that the two price series from the different markets are governed by the same factors at several definite time periods, with the 40 month periodic fluctuation being the most significant here. The price fluctuation of period 7.27 months which was found significant for the London market but not for the New York market could be put to use by speculators on the World Cocoa market if its nature could be ascertained, for example as to when such a fluctuation occurs, and what causes it, by further analysis.

The findings here in no way contradict the results obtained by Labys and Granger [28] that cocoa price changes follow a random walk. As stated earlier, the shape of the phase diagram indicates that different models may be fitted to the different frequency ranges of the spectrum. The high frequency range may be fitted with a model which is quite different from the model required to fit the low frequency range. This just says that different generating mechanisms combine to produce the series. It could be a combination of an autoregressive process and white noise, or even a combination of more than two or three generating mechanisms. We note that the random walk model is not put forward to explain long term changes in either commodity or stock prices. It is a "short-run" model. In fact, an examination of Table 3-1 in Labys and Granger [28, p. 69] reveals the fact that after filtering the monthly cocoa futures prices for the period January 1950 to July 1965 and estimating the spectrum a business cycle of 48 months was found.

We conclude by stating that spectral analysis is a fruitful approach to time series analysis in that it allows examination of the series in terms of time periods. Spectral analysis is also useful as an aid to model building. Finally, it is affirmed that fluctuations with definite time periods do exist in cocoa spot price series. These fluctuations may last years or months. Existence is a fact: it is on the question of how long these fluctuations last that observations differ.

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