Multi-Objective Optimization for Reinforcement Detailing Design and Work Planning on a Reinforced Concrete Slab Case

by

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ABSTRACT

Reinforced steel rebar is fabricated in the form of one-dimensional stocks, designed according to structural engineering code, and installed in various structural components. Cutting one-dimensional stocks to fit to project-specific requirements results in cutting losses, which is the major contributor in the generation of construction waste. Previous research efforts developed mathematical models aimed to analytically minimize cutting losses in preliminary engineering designs, but few have offered insight on the integration of engineering design, workface plan, detailed estimating, plus environmental factors for optimization, let alone considering minimizing total reinforcing installation cost as the ultimate objective. This study introduces an optimization model that contains three optimization stages. Integer programming (IP) technique is applied at the first stage to generate optimal rebar stock procurement plan and cutting plan for each rebar layout arrangement alternative. Next, a discrete event simulation (DES) tool is used to aid in estimating crew installation cost and field productivity in rebar cutting, handling and installation. The final stage is to apply Pareto optimization techniques so as to simultaneously optimize total cost and material waste, resulting in the optimal trade-off solution for decision making. A reinforcing concrete (RC) slab-on-grade case is adopted as test-bed case to demonstrate that the proposed methodology is capable of producing trade-off solutions in terms of reducing wastage and lowering total cost by identifying the optimal slab steel layout arrangement plan. Based the proposed methodology, “What if” scenario analysis is also provided to further investigate the potentials of the proposed method to guide the practitioners in making the most informed decisions.

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NOMENCLATURE

a = long side length of rectangular slab;

b = short side length of rectangular slab;

n_x = the cutting of the quantity of rebar lengthwise in one row;

n_y = the cutting of the quantity of rebar crosswise in one row;

m_x = number of rows of horizontal rebar;

m_y = number of rows of vertical rebar;

x = horizontal rebar cutting length;

y = vertical rebar cutting length;

c = concrete cover depth in reinforced concrete;

h = distance of edge (i, j);

L_i = stock length;

i = To denote different stock lengths from 1 to n;

j = To denote different cutting patterns from 1 to n;

r_{ij} = the quantity of x long cutting rebar associated with cutting pattern j;

s_{ij} = the quantity of y long cutting rebar associated with cutting pattern j;

z_{ij} = is the total quantity required for cutting pattern j of i stock length rebar;

w_{ij} = the cutting losses associated with cutting pattern j and rebar stock i;

$\sum z_{ij} s_{ij}$ = the total quantity of y long rebar;

$\sum z_{ij} r_{ij}$ = the total quantity of x long rebar;

$\sum z_{ij} w_{ij}$ = the total cutting losses (in ton);

α = material cost in \$/ft;

γ = waste processing cost in \$/ft;

$n_{x-\max}$ = max allowable cutting of the quantity of rebar lengthwise in one row;

$n_{y-\max}$ = max allowable cutting of the quantity of rebar crosswise in one row;

N_{layout} = total numbers of feasible rebar layout arrangement plan;

d = diameter of reinforcing steel rebar;

$r_{ij-\max}$ = maximum number of x-long rebar that can be supplied by the stock i;

$s_{ij-\max}$ = maximum number of y-long rebar that can be supplied by the stock i;

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Chapter 1. INTRODUCTION

1.1 Introduction

Reinforcing steel cost accounts for a significant portion of total project budget. For regular reinforced concrete structure, reinforcement cost accounts for about 16% of the entire project cost (Kim et al. 2004); as for some steel dominated structure, the cost of the reinforcing steel can even reach up to 60% of the entire project's cost (Porwal and Hewage, 2012). Hence, making improvement on engineering and management practices in this regard is vitally important for enhancing cost efficiency in construction.

In regards to rebar detailing design, two major factors are commonly taken into consideration: rebar material cost and onsite installation cost in connection with particular rebar layout arrangements in the structural component under construction (Golfeto et al. 2009). On one hand, different rebar layout arrangement plans (i.e. rebar engineering design) in the structural component –together with the available rebar stock types in terms of length and size in the market gives rise to different material cost. A particular rebar layout arrangement plan would result in specific stock procurement and cutting plans, producing a certain amount of waste resulting from cutting rebar stocks. Hence, the rebar layout arrangement plan along with rebar stocks available in the market determines stock procurement and cutting plans and rebar material cost. On the other hand, rebar installation cost differs markedly among different rebar layout arrangement plans. Specifically, labour hours consumed in stock processing (i.e. cutting, bending, and etc.), delivering, placing, and tying are all related to rebar procurement and cutting plans. Field productivity, defined as "a measure of the overall effectiveness of an operating system in utilizing labour, equipment, and capital to convert labour efforts into useful output" (Hendrickson and Au 1989), is defined in labour-hour per ton in this thesis, and would vary due to different rebar layout arrangement plans. In short, the layout arrangement plan determines the material cost and the field productivity of reinforcing installation.

Other than material cost and field productivity, environment concerns and sustainability issues are also not negligible in planning the rebar layout arrangement. In the process of rebar processing, the generation of waste is inevitable when rebar is procured in straight market-supplied lengths for on-site fabrication (Salem et al. 2007). Rebar length needs required in a project vary according to the sizes of the construction component, whereas the standard lengths for rebar available in the Canadian market are limited to 20 ft (6 m), 30 ft (9 m), 40 ft (12 m), and 60 ft (18 m) (Porwal and Hewage, 2012). When the market stock is cut into required lengths, the losses generated from cutting are unavoidable. The disposal of the cutting losses consumes prohibitively expensive resources; in reality, it is not uncommon that rebar waste mixed with other construction waste ends up in landfill instead of recycling, potentially contaminating the living environment. It is worth mentioning that lean construction research is in pursuit of cost-effectiveness by minimizing waste of materials, time, and effort (Nikakhtar et al. 2015). However, in some cases, reducing material waste does not necessarily lead to total cost saving without adequately factoring in engineering design, procurement plans, and workface plans in a holistic system. Therefore, the following have been identified as significant problems which need to be addressed in planning rebar installation: (1) how to optimize the procurement of rebar stocks to result in the minimum material cost, (2) how to design the rebar layout arrangement to improve labour productivity (labour hour/ton) under stock availability and engineering design constraints, and (3) how to reduce the generated material waste in order to address environment protection and sustainable development.

In this thesis, an integrated approach is proposed by simultaneously considering rebar design details, rebar procurement, waste reduction, and workface planning so that it would lead to reduction in waste material together with generation of a cost-efficient workface plan. This, however, is in contrast with the traditional project delivery method that assigns rebar detailing design, procurement planning, cost estimating, and installation planning to separate responsibilities at the planning stage, thereby causing poor communication, uneconomical design, unfeasible planning and higher material waste (Chen et al. 1996). The basic idea underlying the research closely aligns with the Integrated Project Delivery (IPD) concept, which implies construction performance can be considerably improved through developing a project team that focuses on work processes and decisions benefitting the entire project rather than individual team members (Tatum 2012). The integration allows evaluation of many alternatives for design and construction, adding new insight in project management. By involving experienced engineering designers and field superintendents and foremen, it is anticipated that both total cost and material waste can be further reduced, while simultaneously improving productivity and sustainability performances at the workface level on a construction site. The IPD concept is essentially a holistic approach to planning and executing construction, by which all project participants work in highly collaborative relationships through all phases of design, fabrication, and construction in order to achieve efficiency and effectiveness (Tatum 2012). In this thesis, the IPD concept is loosely borrowed to enhance installation productivity and facilitate cost estimating by integrating the perspectives of cost estimator, structural design engineer, and field foremen/superintendent, while allowing for evaluation of many alternatives for design and construction and adding new insight to project management.

The proposed concept is presented by placing the focus on steel bar used on concrete slabs which are commonly seen reinforced concrete structural elements. In particular, slab-on-grade is used as case study in this thesis to illustrate the proposed concept. Slab-on-grade has relatively straightforward rebar layout arrangement design with standard crew installation processes irrespective of the locale of the construction site. Note the slab on grade is different from suspended slab which may have more complicated rebar layout arrangement (e.g. two-layered reinforcement) due to more critical loading conditions. The case study for this thesis research is based on a classic textbook problem in Peurifoy and Oblender (2002). This problem was originally designed to give an overview of reinforcing quantity takeoff, construction operations, and construction cost estimating based on a slab on grade. Nonetheless, a complete case on rebar procurement, rebar processing, workface installation plan, and waste management was not provided in the original source. Inspired by the IPD concept and its applications, the current research further extends the textbook problem to shed light on the rebar layout arrangement (i.e. rebar detailing design), rebar procurement planning, rebar cutting planning, and workface installation planning in an integrated fashion. As such, engineers, procurement specialists, and field supervisors work collaboratively and focus on the whole process of rebar engineering design, procurement, processing, and installation. To materialize such integration, mathematical formulations and modelling based on applications of Integer Programming (IP) technique, Discrete-Event Simulation technique, and Pareto Optimization technique are implemented in a systematic way. Detailed research objective and methodologies are elaborated in subsequent sections.

1.2 Research Scope and Objective

The integration of rebar layout arrangement planning, rebar stock procurement planning, rebar stock cutting planning, and workface operation planning is challenging and complex in consideration of the following:

(1) The conflicting factors associated with the complicated interplay among engineering design, procurement plan, rebar processing plan, and workface crew operation plan in determination of reinforcement cost and material waste.

(2) Various practical constraints on engineering design, rebar stock availability, and crew activities in the field.

(3) Scientific modelling, formulating, and solving the one-dimensional rebar stock cutting problem to simultaneously minimize total rebar-related project cost and total material waste.

Having identified research problems and challenges, the main purpose of the thesis is to develop an integrated optimization approach for planning reinforced steel installation in the preliminary planning stage. The proposed integrated planning methodology aims at providing new insight and effective decision support in reinforcement detailing design and work planning in construction based on generally available information. Particular objectives on current research are described as below:

- Investigating the integrated reinforcement planning problem and clarifying inputs and outputs for optimization analysis;

- Formulating mathematical models by considering engineering design constraints, procurement constraints, and crew installation operations constraints;
- Devising optimization algorithms to simultaneously optimize rebar layout arrangement plan, rebar stock procurement plan, rebar stock processing plan and crew operation workface plan considering total rebar-related project cost and total material waste;
- Simulating rebar installation operations according to the optimized rebar layout arrangement plan, rebar stock procurement plan, and rebar stock processing plan alternatives; determining installation duration, cost, and productivity in a quantitatively reliable way;
- Applying the proposed integrated optimization methodology in a classic reinforced concrete slab case (slab on grade) to demonstrate that the proposed methodology is capable of producing the optimum trade-off solution in terms of reducing wastage and lowering total cost simultaneously.

1.3 Thesis Organization

The present thesis consists of five chapters. Chapter 1 gives an overview of the thesis and identifies research objectives. Chapter 2 presents a comprehensive literature review of the state of art in studying the one-dimensional rebar cutting problem. Chapter 3 introduces the core multi-objective optimization methodology proposed in this thesis. Chapter 4 focuses on the implementation of the proposed methodology in a reinforced concrete slab project. Research conclusion and recommended future work are presented in Chapter 5.

Table 1: Thesis organization

Chapter	Title	Sections
Chapter 1.	Introduction	• Introduction • Research Scope and Objective • Thesis Organization
Chapter 2.	Literature Review	• One-dimensional Cutting Stock Problem • Operation Simulation
Chapter 3.	Methodology	• Methodology Overview • Rebar Layout Arrangement Plan • Rebar Stock Procurement and Cutting Plan • Reinforcing Total Cost • Mathematical Formulation and Optimization
Chapter 4.	Case Study	• Background • Scenario One (Base Scenario) • Scenario Two (With Maximum Carrying Length Limit) • Scenario Three (Lap Length at 30d) • Scenario Four (Waste Disposal Cost at $600/ton) • Scenario Five (Rebar Purchasing Cost at $1.5/feet) • Discussion of Results • Cross Validation
Chapter 5.	Conclusion	

Chapter 2. LITERATURE REVIEW

2.1 One-dimensional cutting stock problem

Reducing rebar consumption has attracted the interest of researchers for decades. Previous researchers have made substantial efforts on finding ways to cut rebar usage in an attempt to minimize cutting losses. The rebar cutting problem is a typical one-dimensional material cutting optimization problem (Salim and Bernold, 1994; Lin 1994; Karelahti 2002; Salem et al., 2007; Kasimbeyli et al. 2011; Debrah 2011; Feifei et al. 2012; Moon et al 2017). The one-dimensional cutting stock problem (CSP) is known for achieving the best cutting pattern (i.e. how to cut stock rebar) so as to meet particular construction project requirements, with rebar cutting losses being the major cause of the construction material waste (Salem et al. 2008). The main objective for the classical CSP is minimizing the material waste, when the order quantity (i.e. required rebar type and length as per design drawings) has been predetermined (Gilmore and Gomory 1961; Arbelt 2001; Kim et al., 2004; Mishra et al. 2014; Salem et al., 2007; Shahin and Salem, 2004; Kasimbeyli et al. 2011; Debrah 2011), resulting in the optimized cutting pattern and corresponding waste. In general, researchers have focused on two kinds of methodology on achieving the best cutting pattern, namely: (1) analytical methods; and (2) heuristic methods.

For the analytical approach, around the early 1960's, linear programming (LP) was introduced to find the solution of the cutting stock problem (CSP). Gilmore & Gomory (1961) formulated the CSP into the LP approach, they used a constraint matrix to show the various ways of cutting rebar. The proposed special column generation technique conquered the problem of listing too many cutting patterns. This method is relatively efficient, but may not arrive at an optimal solution, because of rounding off the fractional part to an integer (Salem et al. 2008). Beyond the basics of the Gilmore and Gomory' method, Dyckhoff (1981) raised a new model which was characterized by a dynamic use of simply structured cutting patterns. This method was superior when dealing with a large number of stock lengths and order lengths.

For the heuristic approach, Haessler (1975) developed a heuristic procedure that could potentially control both the trim losses and the excessive cutting patterns. Roodman (1986) introduced a set of heuristic procedures for efficiently generating good solutions to one-dimensional cutting stock problems in which there were multiple stock lengths available. Next, Gradišar et al. (1999) proposed another Sequential Heuristic Procedure (SHP) to solve the CSP. They raised an item-oriented solution, through a combination of approximation and heuristics that minimized the influence of ending conditions leading to a near optimal solution. Afterwards, the Genetic Algorithms (GA) based solution was developed by Salem et al. (2007). They proposed a GA model, which combined Linear Programming (LP) and Integer Programing (IP) to solve the one-dimension CSP. The generated cutting schedules, using the proposed GA model, showed a high potential in cost saving when compared to a real world workshop's cutting schedules.

Though significant contributions have been made to reduce cutting losses and control material cost, few previous research endeavors have addressed the need in minimizing cost from the project perspective and integrated the engineering processes at the workface level (i.e. rebar cutting/bending to segment ready to install, rebar segment delivery, and rebar installation) into the optimization. While making a profit is one of the primary objectives for a construction contractor, to set a single objective of minimizing material waste or minimizing material cost would be insufficient. A more comprehensive optimization consideration is needed to assist construction contractors in making key decisions by integrating design, procurement and installation in a practical way. In the following section, the CSP problem will be redefined and re-formulated as a multi-objective optimization problem by considering minimizing material waste and minimizing total project cost simultaneously.

2.2 Operation Simulation

Simulation modelling of complicated, dynamic, and interactive processes in construction is essentially a computer-supported implementation of a systematic approach. Here, a system is an integrated combination of the components and activities designed to follow a common purpose; a system exists to achieve a better understanding of the problem and hence help to create a ‘tool’ to resolve the problem (Riley and Towill 2001). Simulation modelling builds a logical model of a system for experimenting with the system on a computer (Prisker 1986). Valid simulation models provide practical tools to assist construction managers in (1) facilitating productivity level estimation for complicated processes, (2) improving repetitive process scheduling, and (3) planning adequate resource assignment that minimizes time and cost (Gonzales-Quevedo et al. 1993). Discrete event simulation tools can play a major role in detailed cost estimating and work planning. A special purpose simulation environment called SIMPHONY was introduced to facilitate construction crew estimating and operations planning for achieving cost efficiency and productivity in building simulation models (Hajjar and AbouRizk 1999). Meanwhile, the simplified discrete-event simulation approach (SDESA) was proposed with the goal of streamlining simulation modeling into a process of designing an enhanced version of activity-on-node (AON+) diagram model. Specifically, the simulation modeling process of SDESA is to create a network diagram model which is relatively stable in representing dynamic resource allocation and resource transit between various locations in a construction system. To some extent, the whole process of simulation modeling resembles preparing the AON network model for the critical path method (CPM) (Lu 2003; Lu et al. 2008).

Owing to the fast development of simulation tools, tremendous endeavors have been made over the past decades, in regards to workflow modelling, simulation methods and practical applications, aimed to simplify simulation and promote implementation in the practice of construction engineering and management. Nonetheless, the application of simulation modeling has been widely confined to the workface planning such as resource allocation and activity scheduling. There is a lack of modelling frameworks and practical applications in the literature which link simulation modeling with engineering design, temporary facility design, and material quantity takeoff in an integrative, seamless approach (Lu et al. 2017).

Chapter 3. METHODOLOGY

3.1 Methodology Overview

This research aims at optimizing rebar detailing design, rebar stock procurement plan, rebar stock cutting plan, and rebar material waste under practical work planning constraints by applying advanced optimization techniques. In this thesis, rebar detailing design refers to the rebar layout arrangement plan that includes detailed lap design, rebar type and quantity, spacing, and location. Note the design in current research excludes possible rebar bending at the two ends of the structure which is less important to current problem setting and hence ignored in the proposed problem solution. To achieve the objectives, three methodological stages are introduced.

Stage I: In the first stage, minimizing cutting losses as the main objective is executed to: (1) generate all the possible rebar layout arrangement plans under current work planning constraints; (2) generate all the possible rebar cutting plans for each rebar layout arrangement plan under rebar stock availability constraints; (3) generate the optimal cutting plans with the least material waste for each rebar layout arrangement plan, and (4) discard rebar layout arrangement plans that fail to satisfy the preset material waste limit.

Stage II: In the second stage, Discrete Event Simulation (DES) is applied by (1) modelling the rebar installation operation; (2) inputting for each component in the operation simulation model based on Stage I outputs, and a result, deciding crew production rates; and (3) evaluating installation duration, cost, and productivity for each layout arrangement alternative from Stage I.

Stage III: In the third stage, Pareto Optimization is introduced to conduct trade-off analysis by (1) calculate non-domination value for each rebar layout arrangement plan according Pareto optimization criteria; and (2) output optimal trade-off solution with smallest non-domination value. An overview of the methodology is shown in Figure 1.

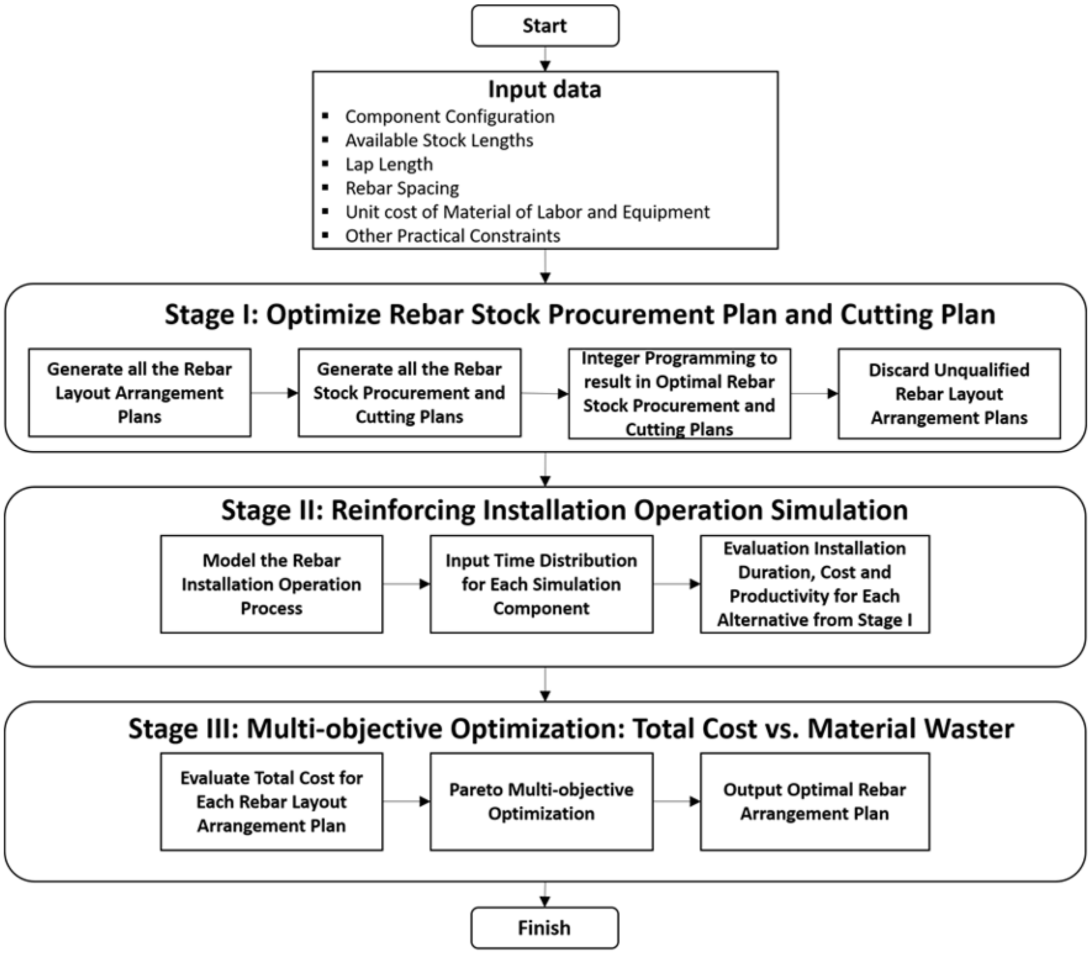


Figure 1: Methodology Overview

In order to solve the proposed rebar engineering and management problem on the slab case, the following constraints are imposed and assumptions are made:

(1) The lengths of both the long side and the short side of the slab are longer than the longest stock size available in the market; rebar lapping exists along both sides of the slab by a distance range from 28d to 40d (d is the diameter of reinforcing steel rebar) due to bar size and concrete strength (Concrete Design Handbook 2006, Explanatory Notes on CSA

A23.3-04, Page 279);

(2) The material waste limit is defined as a percentage applied on the total material required as per engineering design, for which the maximum material waste shall not exceed. As the waste limit may differ from contractor to contractor, the material waste limit is assumed as 15% in current thesis.

(3) The rebar stocks are cut into order length (according to cutting plan) at steel processing area by ironmen. For on-site management requirements, the procured rebar stocks need to cut into identical lengths either along the long direction or short direction;

(4) Rebar being arranged in both directions is of identical engineering specifications.

3.2 Rebar Layout Arrangement Plan

The slab in the current case study is of the typical rectangular design with the long side length a, and the short side length b, (where a>b). To satisfy the engineering design specifications, the procured rebar stocks need to be processed to fit into the slab. The procured rebar stocks need to cut and lapped according to rebar layout arrangement plan and rebar cutting plan. Lapped splices are used for joining two individual rebar segments.

A sample rebar layout arrangement plan is given in Figure 2. The rebar layout arrangement is determined by two integer parameters n_x and n_y, where n_x is the rebar counts lengthwise in one row, or in the long direction, while n_y is the rebar counts crosswise in one row, or in the short direction. The horizontal rebar cutting length is denoted as x, and vertical rebar cutting length is denoted as y. Then the lap length along the long direction can be formulated as $(x * n_x - a)/(n_x - 1)$, while the lap length along the short direction can be formulated as $(y * n_y - b)/(n_y - 1)$. The rebar is placed along both vertical and horizontal directions, spacing at a length denoted by c. Concrete cover, which is specifically defined in the current thesis as the least distance between the center line of rebar cross section and the outer edge surface of the concrete, is denoted as h. The concrete cover between the center line of rebar cross section and the outer facade surface of the slab is not relevant to the proposed problem thus not defined. Figure 2 is given to illustrate the concrete cover defined in the current thesis.

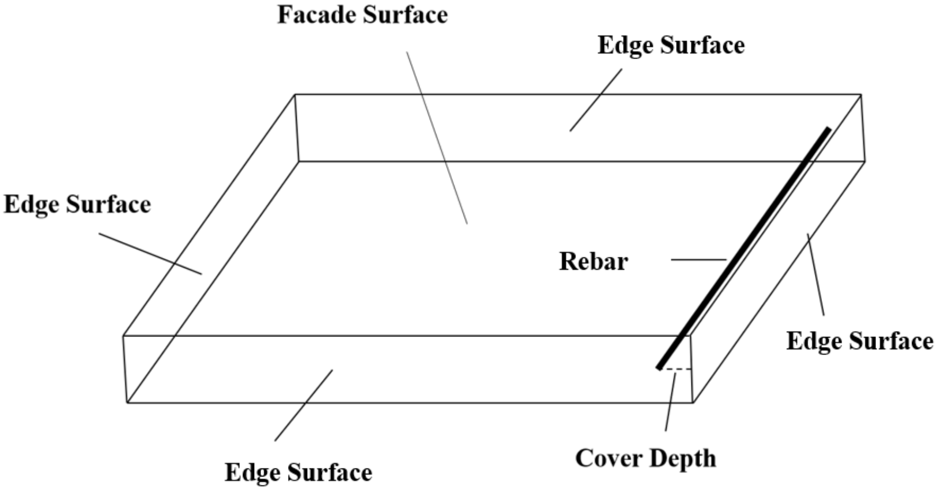


Figure 2: Concrete Cover

The rows of rebar can be decided by slab configuration, rebar spacing, and concrete cover depth. Then, the rows of rebar in short direction of the slab is formulated as $m_x = 1 + roundup[(b - 2h)/c]$; while the rows of rebar in long direction of the slab is formulated as $m_y = 1 + roundup[(a - 2h)/c]$. So the total pieces of rebar required in a slab is $m_x * n_x + m_y * n_y$.

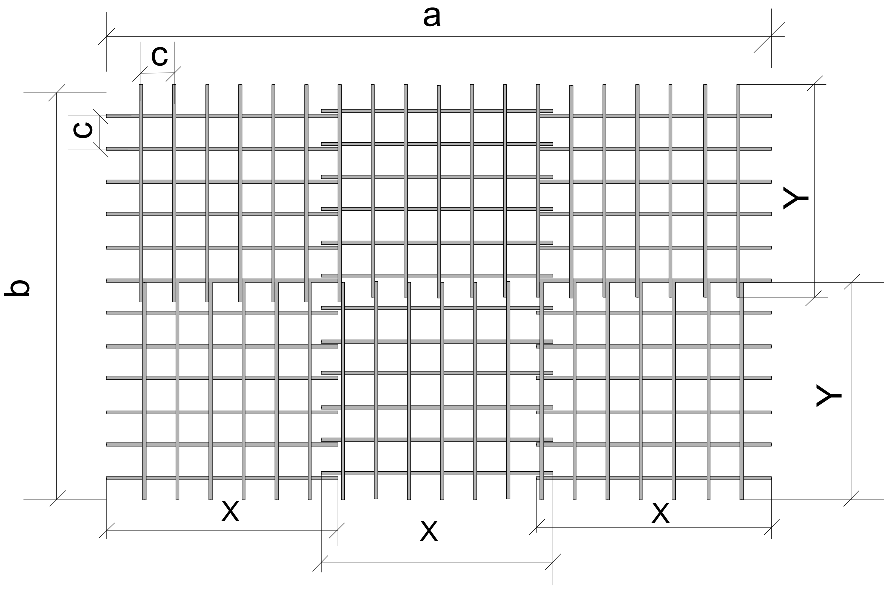


Figure 3: Rebar Layout Arrangement Plan Sample ($n_x = 3, n_y = 2$)

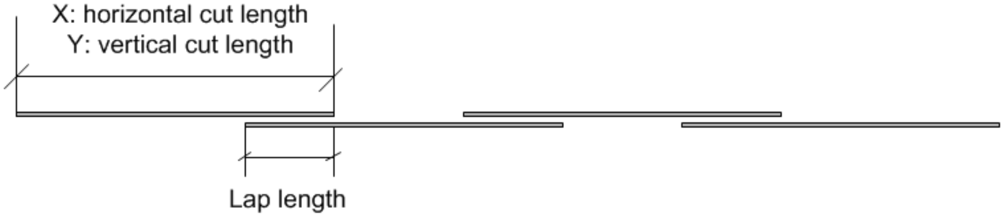


Figure 4: Rebar Lap

3.3 Rebar Stock Procurement Plan and Cutting Plan

When the rebar layout arrangement plan is set, the next step is to generate rebar stock procurement plan and cutting plan subject to available rebar stocks in the market and identify the optimal cutting pattern with minimal material waste.

Suppose the stock length supplied by the market is L_i , then they need to cut into demand length x and y according to rebar layout arrangement plan. For stock length i, suppose it could potentially provide a total of j cutting patterns; the quantity of x long rebar pieces (or segments) associated with cutting pattern j, is denoted as r_{ij}; and the quantity of y long rebar pieces associated with cutting pattern j, is denoted as s_{ij}; z_{ij} is the total number of required stock length i for cutting pattern j; w_{ij} is the cutting losses (i.e. waste) of cutting pattern j for stock length i. So $\sum z_{ij}r_{ij}$ is the total quantity of x long rebar pieces, $\sum z_{ij}s_{ij}$ is the total quantity of y long rebar pieces. $\sum z_{ij}w_{ij}$ is denoted as the total cutting losses. Figure 5 below gives an example of rebar stock cutting plan.

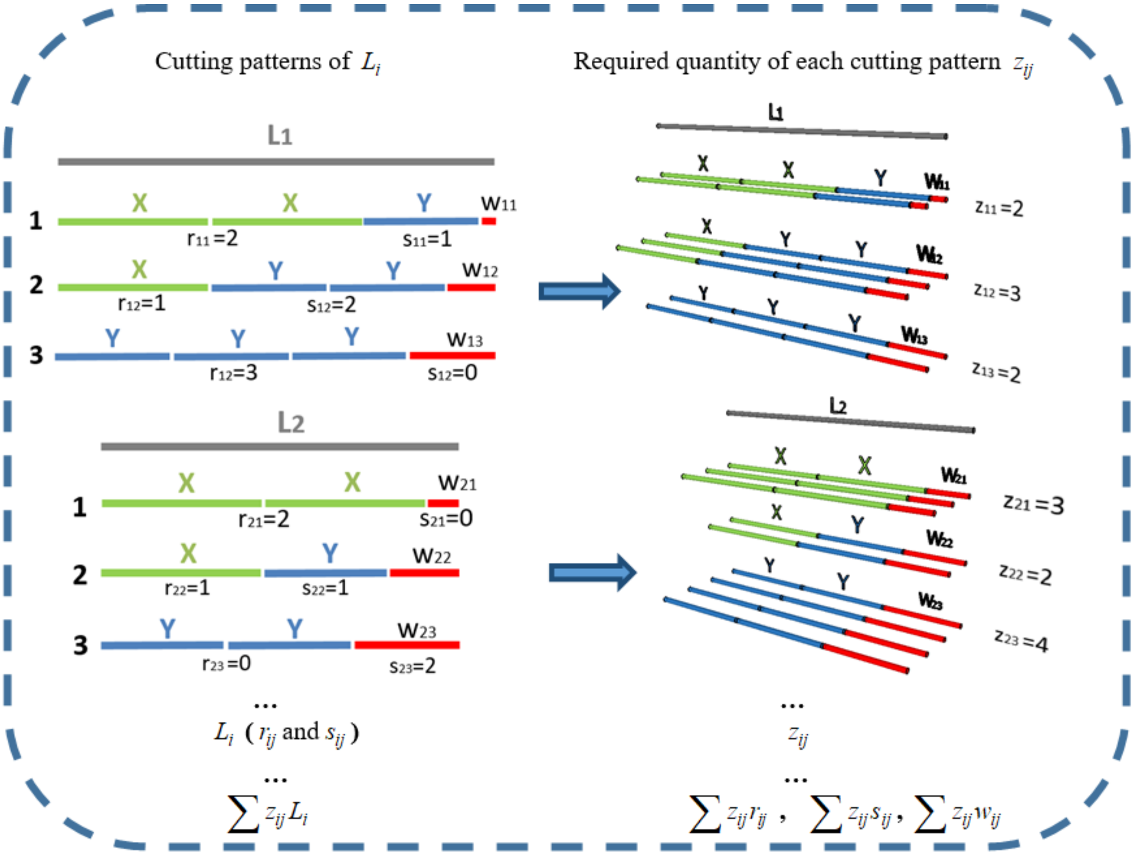


Figure 5: Rebar stock cutting plan sample

3.4 Reinforcing Total Cost

The total cost in this thesis is defined to be composed of (1) material cost; (2) installation cost, and (3) waste processing cost. In particular, material cost comes directly from the procured rebar stocks, which is related to the per foot cost, denoted as α ($/ft). Similar to material cost, waste processing cost relates directly with the total amount of waste (in feet) and the per foot waste processing cost, denoted as γ ($/ft). Installation cost, which refers to the labour and equipment cost for rebar installation, is calculated by analyzing detailed rebar installation operation simulation which is to be elaborated in the subsequent section. Cost formulations are shown in Eq.(3-1), Eq.(3-2) and Eq.(3-3),

$$Total = C_{material} + C_{install} + C_{waste} \tag{3-1}$$

$$C_{material} = \alpha \sum z_{ij} L_i \tag{3-2}$$

$$C_{waste} = \gamma \sum w_{ij} \tag{3-3}$$

Where $\sum z_{ij} L_i$ is the total length of rebar stocks in feet; $\sum w_{ij}$ is the total material waste in feet.

3.5 Mathematical Formulation and Optimization

A three-staged methodology was applied in sequential order to generate optimal rebar layout arrangement plan along with corresponding optimal rebar stock procurement plan and cutting plan. Stage I is to optimize rebar stock procurement plan and cutting plan for each rebar layout arrangement plan. Stage II is to simulation rebar installation operations to results in installation duration and field productivity. Stage III is to optimize optimal rebar layout arrangement plan by identifying the best tradeoff between minimum total cost and minimum total material waste through multi-objective optimization.

3.5.1 Stage I: Optimize Rebar Stock Procurement and Cutting Plan

The methodology of Stage I optimization can be summarized in Figure 6. Further, Stage I optimization process is decomposed to three consecutive steps.

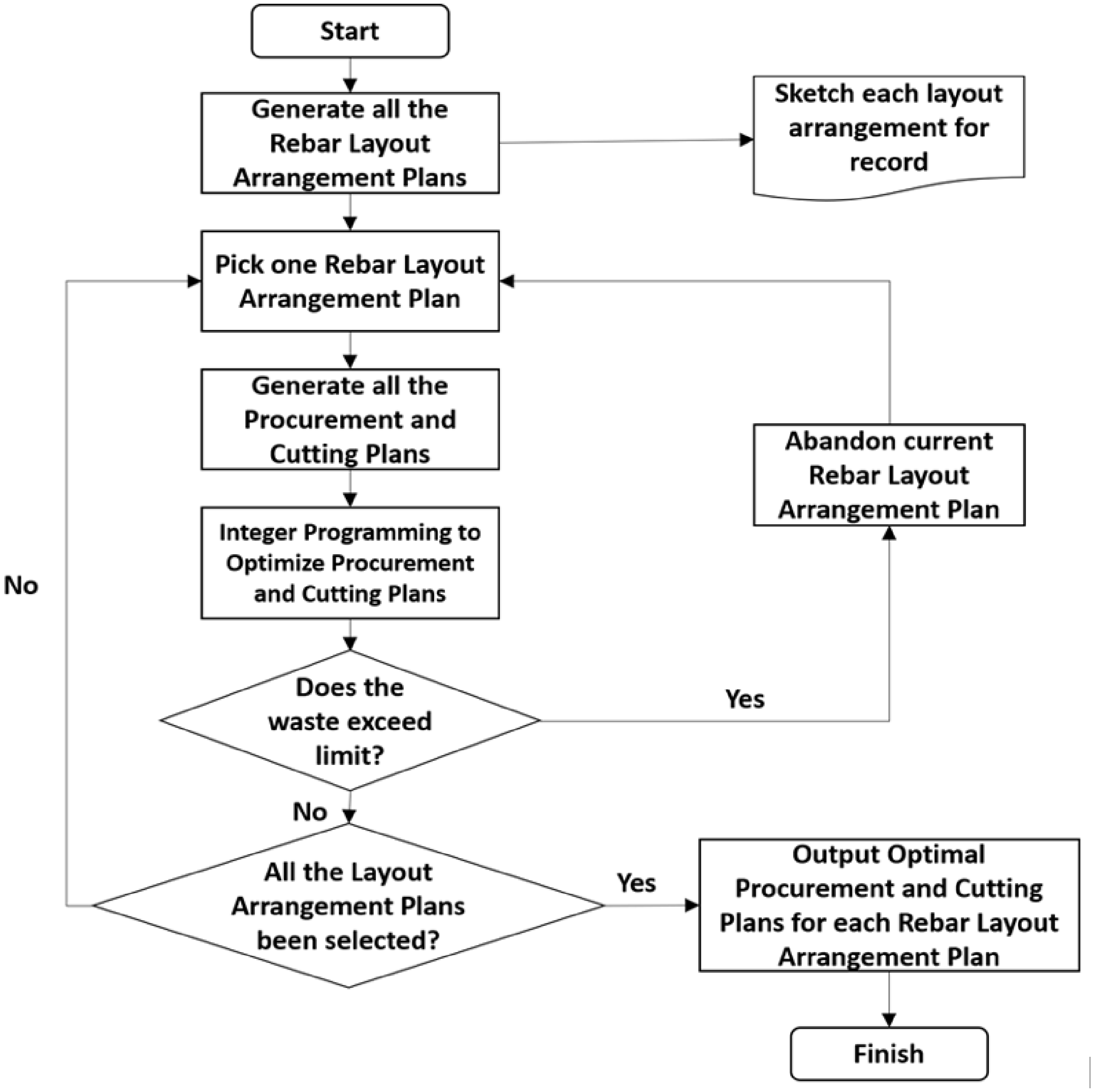


Figure 6: Stage I optimization process

Step 1: Generating rebar layout arrangement plans

The first step of Stage I optimization is to generate all the feasible rebar layout arrangement plans under design and work planning constraints.

The rebar detailing design is determined by two integer parameters n_x and n_y. Note that n_x and n_y are finite integer with lower and upper bounds denoted as $roundup\left(\frac{a}{L}\right) \leq n_x \leq n_{x-max}$ and $roundup\left(\frac{b}{L}\right) \leq n_y \leq n_{y-max}$ respectively. n_{x-max} and n_{y-max} belong to positive integers set N^* and are determined by experienced operations personnel based on practical feasibility. Any combination of n_x and n_y results in a rebar layout arrangement plan. Hence, the total numbers of rebar layout arrangement plan N_{layout} can be denoted by Eq.(3-4),

$$N_{layout} = \left[n_{x-max} - roundup\left(\frac{a}{L}\right) + 1\right]\left[n_{y-max} - roundup\left(\frac{b}{L}\right) + 1\right] \tag{3-4}$$

Rebar lapping exists with a distance of l_d, thus horizontal rebar cutting length x and vertical rebar cutting length y for each rebar layout arrangement plan can be denoted by Eq.(3-5) and Eq. (3-6),

$$x = [l_d * d * (n_x - 1) + a]/n_x \tag{3-5}$$

$$y = \left[l_d * d * \left(n_y - 1\right) + b\right]\Big]/n_y \tag{3-6}$$

Step 2: Generate rebar stock procurement plan and cutting plans

This step is to generate all the possible rebar stock procurement plans and rebar stock cutting plans, by (1) calculating the maximum number of x-long rebar that can cut from the rebar stock i, denoted as r_{ij-max}; (2) letting r_{ij} = 0, 1, 2...r_{ij-max}, and calculating s_{ij} respectively; hence each combination of r_{ij} and s_{ij} defines a cutting pattern for stock i; and (3) iterating the above process for each type of rebar stock and recording all the cutting patterns. r_{ij-max}, s_{ij} and w_{ij} are calculated by Eq. (3-7), Eq.(3-8), and Eq.(3-9),

$$r_{ij-max} = rounddown(L_i/x) \tag{3-7}$$

$$s_{ij} = rounddown(L_i - r_{ij} * x)/y \tag{3-8}$$

$$w_{ij} = L_i - r_{ij} * x - s_{ij} * y \tag{3-9}$$

By following the above steps, all the possible rebar stock procurement plans and rebar stock cutting plans can be obtained.

Step 3: Integer Programming

For each rebar layout arrangement plan, Integer Programming technique is applied to obtain the optimal rebar stock procurement plan and cutting plan in terms of minimal material waste. The solutions are further used as input for operation simulation in the subsequent step.

1. Decision variables:

z_{ij}: integer variable denoting the quantity of cutting pattern j from rebar stock i.

2. Minimizing total cutting losses:

The objective is to minimize total cutting losses, which can be written as:

$$Min \quad Waste = \sum z_{ij}w_{ij} + \left(\sum z_{ij}r_{ij} - m_x * n_x\right) * x + \left(\sum z_{ij}s_{ij} - m_y * n_y\right) * y \quad (3\text{-}10)$$

In the the objective function, cutting losses $\sum z_{ij}w_{ij}$ and surplus cut $\left(\sum z_{ij}r_{ij} - m_x * n_x\right) * x + \left(\sum z_{ij}s_{ij} - m_y * n_y\right) * y$ in both directions are considered as waste. Note surplus cut are redundant materials cut from rebar stock by default once cutting patterns for rebar stocks are fixed.

3. Constraints:

The constraints are imposed to ensure enough rebar pieces cut from stocks to fit in rebar layout arrangement plan, which can be given as Eq. (3-11) and Eq. (3-12):

$$\sum z_{ij}r_{ij} \geq m_x * n_x \quad (3\text{-}11)$$

$$\sum z_{ij}s_{ij} \geq m_y * n_y \quad (3\text{-}12)$$

3.5.2 Stage II: Rebar Installation Workface Planning and Simulation

In this thesis, the rebar installation operation is decomposed into four activities: rebar cutting, rebar carrying/delivery, rebar placing, and rebar tying, as demonstrated in Figure 7.

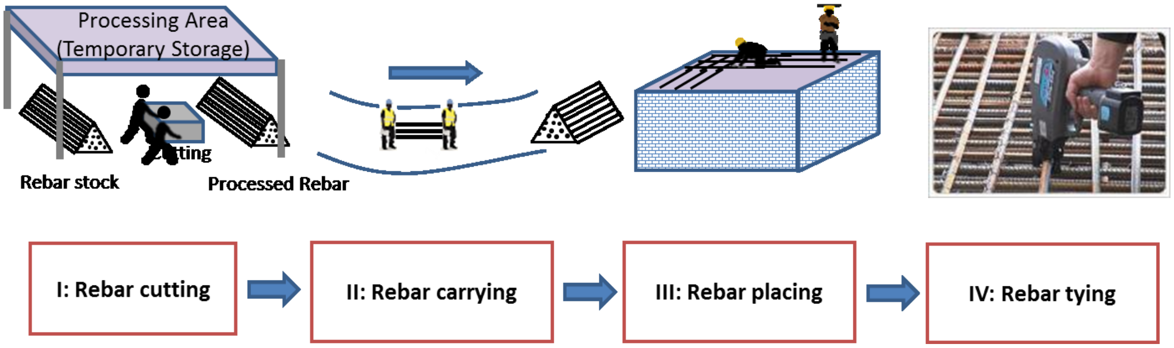


Figure 7: Installation operation process

Activity I: Cutting rebar stocks into rebar pieces according to the stock cutting plan: two steel setters work as a team to cut the rebar, the production rate is triangularly distributed at [2,3,4] pieces/min;

Activity II: Carrying rebar pieces to the slab site: two ground labourers work as a team to carry the completed rebar pieces to the slab precast site at a speed of [80, 100, 120] feet /min, then return to stockpile at the same speed; it takes [0.5,1,2] min to load and unload rebar pieces. The two ground labourers can carry 6 rebar pieces in each trip.

Activity III: Placing and spacing the rebar pieces: two rodmen work as a team with production rate at [1, 2, 3] pieces/min;

Activity IV: Tying the rebar pieces: the two rodmen work as a team with production rate at [6,10,12] ties/min;

Note all the production rates are derived by averaging the data collected from three experienced field supervisors who specialized in building construction for more than five years. Note, the production rates are triangularly distributed as [minimum, regular, maximum], whereas the minimum denotes the ideal condition; regular denotes the averaged condition; maximum denotes the worst condition considering field productivity factors such as weather, field coordination, labour experience, and etc.

In this thesis, the SDESA simulation modelling tool is used to construct the computerized simulation model (Lu 2003). The SDESA software is straightforwardly designed as if the users were analyzing the process with simple Activity-on-Node (AON) method (Chan and Lu, 2008). By simply adding resources and time distributions to each activity, an “AON Plus” model can be established in SDESA for operation simulation analysis.

For current rebar installation process, the “AON Plus” is presented in Figure 3-7. The “AON Plus” model comprises of three operation flows corresponding with the three work packages. In the first one, two rebar setters cut the rebar pieces from procured rebar stocks according to the cutting plan at the rebar processing area. In the next one, two ground labourors grab already-processed rebar pieces and deliver them to the construction site, then return. This operation starts at the on-site rebar processing area and ends at the construction site. Lastly, two rodmen place the delivered rebar pieces and tie them up. This operation takes place at the construction site.

The “AON Plus” model starts with rebar cutting work package. Ground labourers can start to work as soon as 6 rebar pieces are generated. In the “AON Plus” model, the "rebar delivery" work package starts with activity "load 6 pieces". When the two ground labourers load rebar pieces, activity “Deliver” in the model is activated to represent rebar delivery activity from the rebar processing area to the construction site. A disposable resource “Rebar Ready To Install” is generated once 6 rebar pieces are unloaded (end of “Unload 6 Pieces” activity of Rebar delivery work package) to the construction site. At the beginning of the third work package, “Rebar Ready To Install” is required as a resource before initiating the installation process, meaning the third work package can only be initiated when one cycle of rebar delivery is done at the construction site, ready for immediate installation. Two activities, namely "Rebar Placing" and "Rebar Tying" are carried out consecutively. Once all the cycles of rebar cutting, rebar delivery and rebar placing/tying are completed, the simulation terminates, and the total operation duration is obtained. With total operation duration known, installation cost and field productivity can be calculated. The base “AON Plus” model which is established for SDESA simulation analysis is shown in Figure 8.

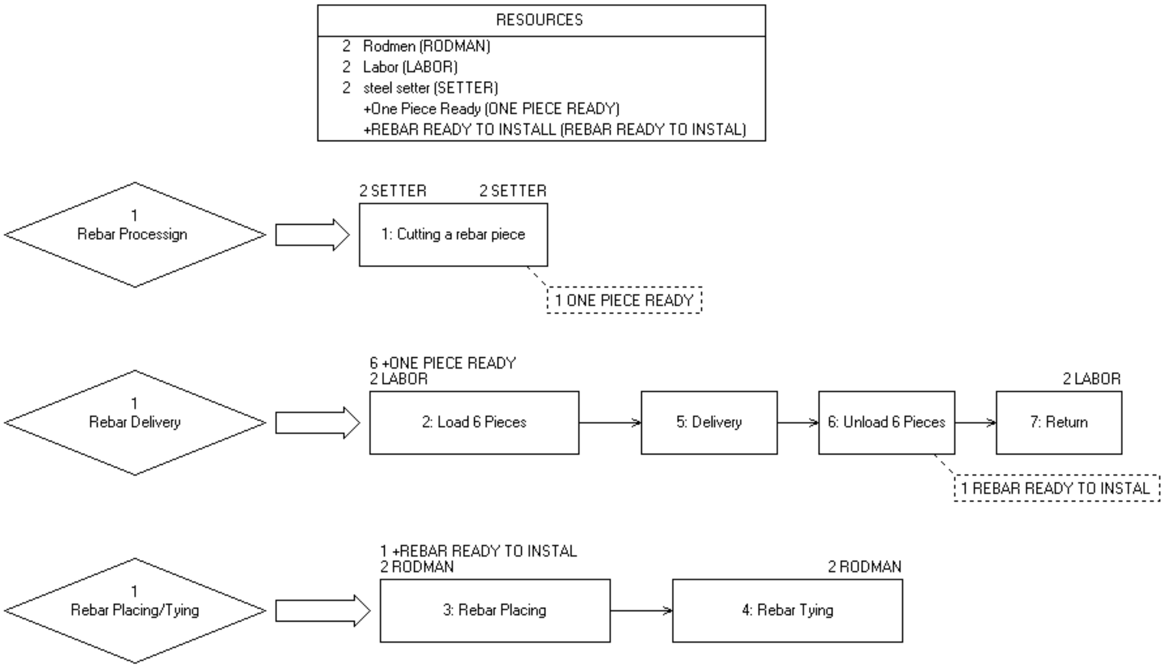
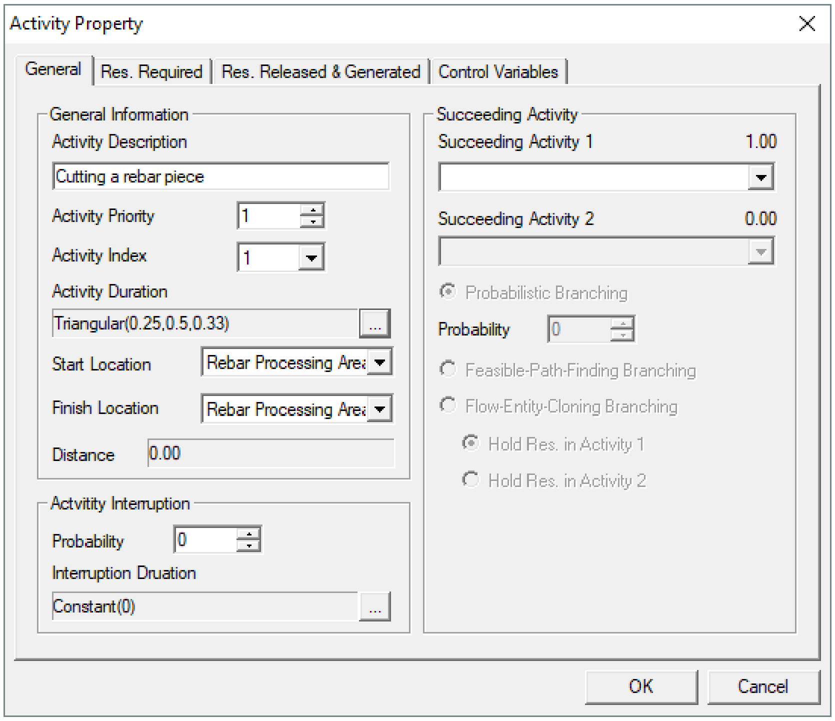


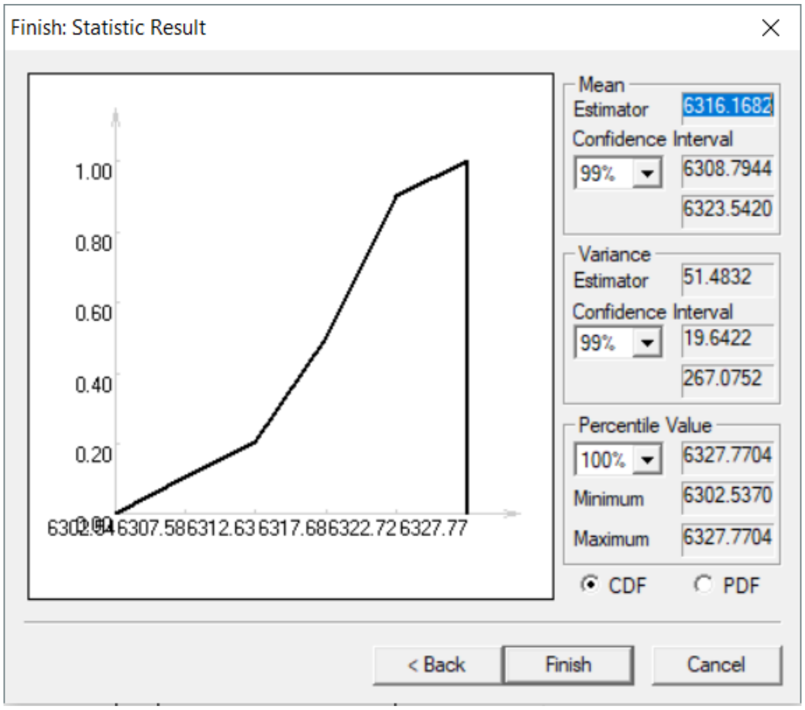
Figure 8: SDESA modelling for reinforcing operation

For each activity in the SDESA model, the triangular distributed time is defined as shown in Figure 9. After inputting every activity, users specify the simulation run count and click “Run” in the software. In this thesis, simulation run count is set to be 100. A sample simulation output indicating total installation time is shown in Figure 10.



Activity Description: Cutting a rebar piece
Activity Priority: 1
Activity Index: 1
Activity Duration: Triangular(0.25,0.5,0.33)
Start Location: Rebar Processing Are
Finish Location: Rebar Processing Are
Distance: 0.00
Succeeding Activity 1: 1.00
Succeeding Activity 2: 0.00
● Probabilistic Branching
Probability: 0
○ Feasible-Path-Finding Branching
○ Flow-Entity-Cloning Branching
● Hold Res. in Activity 1
○ Hold Res. in Activity 2
Actvitity Interruption Probability: 0
Interruption Druation: Constant(0)

Figure 9: Activity time input in SDESA



Mean
Estimator: 6316.1682
Confidence Interval: 99%: 6308.7944 – 6323.5420
Variance
Estimator: 51.4832
Confidence Interval: 99%: 19.6422 – 267.0752
Percentile Value: 100%: 6327.7704
Minimum: 6302.5370
Maximum: 6327.7704
● CDF ○ PDF

Figure 10: Average duration of total reinforcing installation time

3.5.3 Stage III: Multi-objective optimization: total cost vs. material waste

From Stage I, optimized rebar stock procurement plan and cutting plan for each feasible rebar layout arrangement plan are obtained; and from stage II, crew installation cost for each feasible rebar layout arrangement plan is obtained. Hence, with information obtained in Stage I and Stage II, total reinfocing cost can be evaluated for each feasible rebar layout arrangement plan as per Eq.(3-1). Next, with material waste and total cost known for each rebar layout arrangment plan, Pareto multi-objective optimization is applied to generate the optimal solution achieving an equilibrium between the two objectives, namely minimizing total cost and minimizing material waste. For the multi-objective optimization problem, a Pareto optimal solution (i.e. Pareto front) is commonly applied to simultaneously optimize each objective (Orabi et al. 2009).

After material waste and total cost of all the possible rebar layout arrangement plans are calculated, Pareto multi-objective optimization algorithm sorts all the alternatives and assigns a rank that represents the non-domination of each solution in comparison with the other solutions. Note the non-domination is the number of times that the particular objective value derived of a certain solution is smaller than all other possible solutions. Then, the Pareto optimal solution(s) are selected by comparing the ranks of all the input alternative rebar layout arrangement plans. Then Pareto multi-objective optimizatioprocess is illustrated in Figure 11.

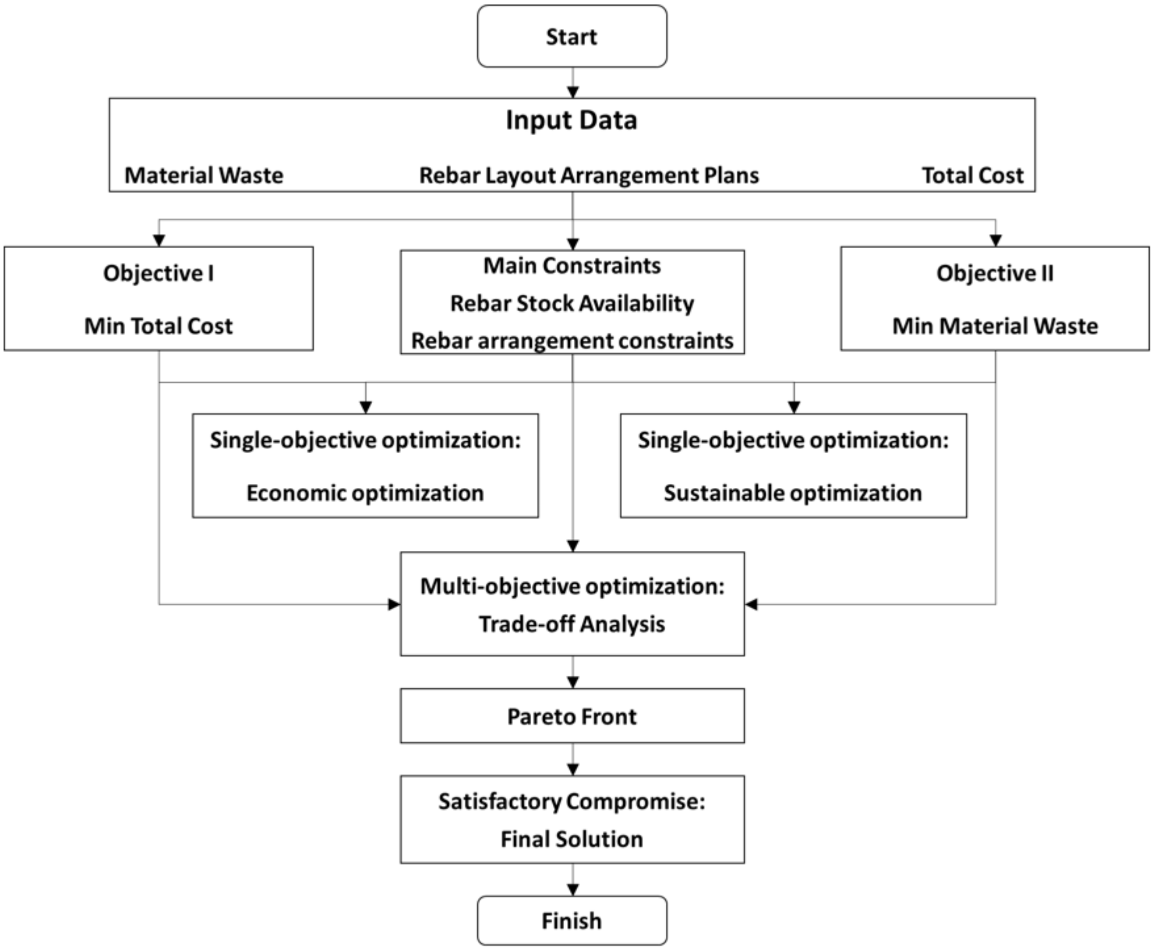


Figure 11: Flowchart of Multi-objective Optimization Process

Chapter 4. CASE STUDY

4.1 Background

In order to illustrate and verify the proposed approaches, a reinforced concrete (RC) slab design and installation case – which was originally used for detailed estimating in Peurifoy and Oberlender (2002) is adapted. The detailed slab configuration and rebar layout arrangement solution for the original case in Peurifoy and Oberlender (2002) is presented in Appendix A.

The RC slab is the work package of a one-story garage building construction. The RC slab which is 70 feet (21.34m) long and 57 feet (17.37m) wide is reinforced with 15M steel rebar along both long and short directions. The steel rebar is placed on the slab spaced at 1 foot (0.304m) along both directions, concrete cover depth in reinforced concrete is 0.5 feet (0.15m). Confirmed by experienced field engineers, a maximum of eight pieces of rebars of identical length is allowed to be placed along both long and short directions. Rebar pieces are overlapped and tied for connection with a lap length at 40d. Material waste limit is 1222 feet (372.47 m) (15% of total rebar length as per engineering design); in the current case, total rebar length is calculated as $m_x * b + m_y * a$ = 57*57 + 70*70 = 8149 feet. According to rebar vendors, the available rebar stock size for 15M rebar are 20, 30, 40 feet (6, 9, 12m) long, respectively. The material manager and the field superintendent would both benefit from identifying the optimized solution for the rebar layout arrangement plan on the slab if cutting loss and total cost are reduced simultaneously.

In the case study, material cost and waste material disposal cost are directly provided by industry partners specialized in steel structures. Material cost for 15M rebar is $2100/ton;

waste material disposal cost $380/ton. Derived from RS Means (2017), steel setter hourly rate is $60/hr; cutting machine hourly rate is $115/hr; ground labour hourly rate is $45/hr; rodmen hourly rate is $60/hr. Two steel setters (for rebar processing), two ground labouors (for rebar handing/delivery), plus two rodmen (for rebar placing and tying onsite) make up the crew.

4.2 Scenario One (Base Case Scenario)

In the first base-case scenario, model inputs are as per background information. All the feasible rebar layout plans are generated to fit slab design configurations. A maximum of 8 pieces along each side is constrained.

4.2.1 Rebar Detailing Design Results

In the first optimization stage, optimized rebar layout arrangement plans with the corresponding rebar stock procurement and cutting plans were obtained by applying Integer Programming. In this scenario, a total of 24 feasible alternative rebar layout arrangement plans were obtained from optimization analysis, as shown in Table 2. Detailed calculations of the 24 rebar layout arrangement plans are given in Appendix C. The optimization process is programmed in the Python language (Python 3.5) by calling optimization functional boxes in CPLEX (CPLEX 12.61)

Table 2: Cutting plan for selected detailing design (scenario one)

Layout #	n_x	n_y	$\boldsymbol{x}$	$\boldsymbol{y}$	z_{ij}	L_i	r_{ij}	s_{ij}	$\sum w_{ij}$ **(ft)**
1	3	2	24.67	29.5	171	30	1	0	982
					140	30	0	1	
2	4	2	19	29.5	228	20	1	0	298
					140	30	0	1	
3	8	2	10.5	29.5	106	40	3	0	901
					140	40	1	1	
4	**4**	**3**	**19**	**20.33**	**18**	**20**	**1**	**0**	**158.07**

Layout #	n_x	n_y	$\boldsymbol{x}$	$\boldsymbol{y}$	z_{ij}	L_i	r_{ij}	s_{ij}	$\sum w_{ij}$ **(ft)**
					210	**40**	**1**	**1**	
5	5	3	15.6	20.33	75	20	1	0	1184.07
					210	40	1	1	
					280	20	0	1	
6	6	4	13.33	15.75	280	30	1	1	670.114
					62	20	1	0	
7	7	4	11.71	15.75	200	40	2	1	506
					80	20	0	1	
8	8	4	10.5	`15.75	228	40	2	1	962
					52	20	0	1	
9	3	5	24.67	13	171	40	1	1	759
					90	30	0	2	
10	4	5	19	13	114	40	2	0	928
					175	30	0	2	
11	5	5	15.6	13	285	30	1	1	854
					65	20	0	1	
12	6	5	13.33	13	119	40	1	2	116.78
					112	40	2	1	
13	7	5	11.71	13	150	40	2	1	766
					100	40	1	2	
14	4	6	19	11.17	228	20	1	0	1137.86
					140	40	0	3	
15	5	6	15.6	11.17	285	30	1	1	1213.86
					45	40	0	3	
16	6	6	13.33	11.17	119	40	2	1	743.169
					112	40	1	2	
17	3	7	24.67	9.86	171	40	1	1	982.213
					80	40	0	4	
18	4	7	19	9.86	228	20	1	0	298.356
					123	40	0	4	
19	6	7	13.33	9.86	114	40	3	0	69.898
					122	40	0	4	
					1	20	0	2	
20	7	7	11.71	9.86	133	40	3	0	717.494
					122	40	0	4	
					1	20	0	2	
21	4	8	19	8.88	228	20	1	0	858
					140	40	0	4	
22	6	8	13.33	8.88	52	40	3	0	7.906
					187	40	1	3	
23	7	8	11.71	8.88	280	30	1	2	346
					40	40	3	0	
24	8	8	10.5	8.88	202	40	2	2	402
					52	40	1	3	

4.2.2 Operation Planning Results

SDESA "AON Plus" simulation models were built for each rebar layout arrangement plan. Take layout 8 (which is the same as solution in Peurifoy and Oberlender 2012) as an example, a total of 438 rebar pieces needs to be installed (as per Table A-1 in Appendix A). Hence, a number 438 is initialized for rebar cutting process. For rebar delivery process, a total of roundup (438/6) = 73 trips were initialized, as shown in Figure 12. Correspondingly, a total of 73 rebar placing/tying packages (6 pieces of rebar per package) are initialized as per SDESA modelling strategy.

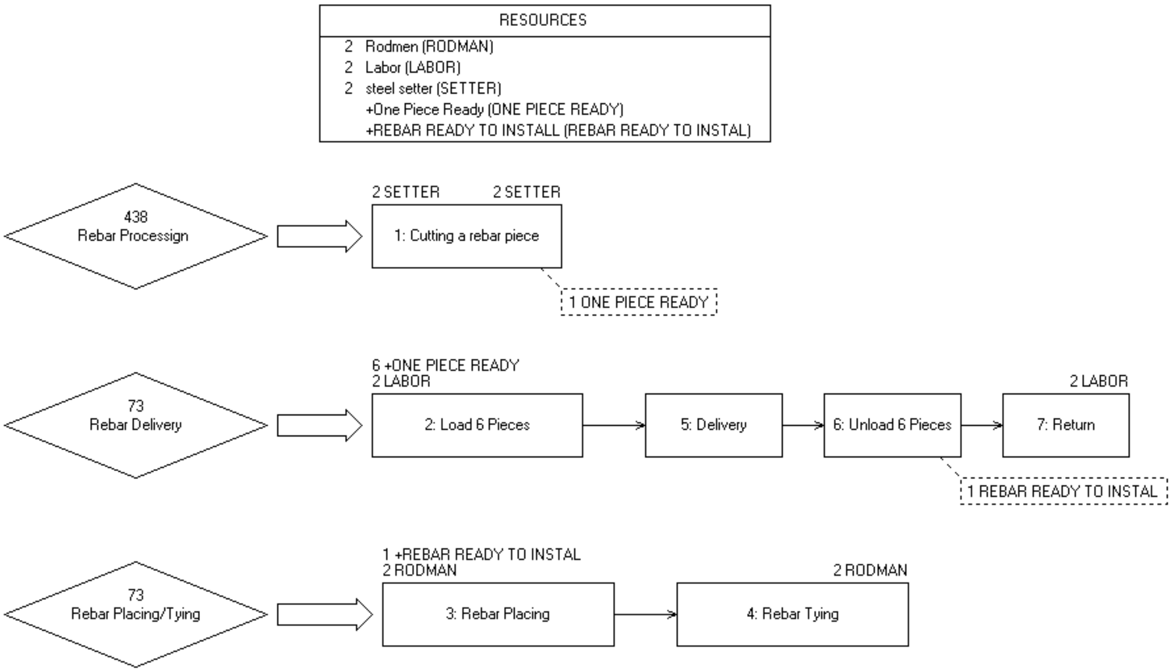


Figure 12: "AON Plus" simulation model for layout 1

Following the same modelling methodology, a total of 24 distinct models were established each corresponding with a particular cutting plan. By running the models for 100 times, the average duration of the field operations was obtained for each layout arrangement plan. With the job duration obtained from simulation, the installation cost can be calculated. In this thesis, field productivity is defined in terms of labour input over the work installed (labour hour/ton). The total weight is fixed as per slab configuration and rebar spacing. The total length of rebar in the slab is $m_x * b + m_y * a$ = 57*57 + 70*70 = 8149 feet (2484 m), so the total weight equals to total length * rebar weight density = 8149 feet *0.00479 ton/ft = 3.903 ton. There are in total 6 workers onsite involved in reinforcing installation operations. Therefore, total installation labour hours can be calculated as 6 times total working hours. Simulated operation duration and calculated installation cost, labour hours, and productivity for each cutting plan are summarized in Table 3.

Table 3: Simulation results for 24 rebar layout arrangement plans (scenario one)

Layout #	**n_x**	**n_y**	**Duration (hour)**	**Installation Cost ($)**	**Labour hours (hours)**	**Productivity (labour hour/ per ton)**
1	3	2	6.73	2994.85	40.38	10.35
2	4	2	6.98	3106.10	41.88	10.73
3	8	2	10.23	4552.35	61.38	15.73
4	4	3	8.04	3577.80	48.24	12.36
5	5	3	8.45	3760.25	50.7	12.99
6	6	4	10.87	4837.15	65.22	16.71
7	7	4	11.23	4997.35	67.38	17.26
8	8	4	11.87	5282.15	71.22	18.25
9	3	5	9.14	4067.30	54.84	14.05
10	4	5	10.11	4498.95	60.66	15.54
11	5	5	10.92	4859.40	65.52	16.79
12	6	5	11.45	5095.25	68.7	17.60
13	7	5	12.8	5696.00	76.8	19.68
14	4	6	10.98	4886.10	65.88	16.88
15	5	6	11.54	5135.30	69.24	17.74
16	6	6	12.95	5762.75	77.7	19.91
17	3	7	11.12	4948.40	66.72	17.09
18	4	7	11.61	5166.45	69.66	17.85
19	6	7	13.65	6074.25	81.9	20.98
20	7	7	13.96	6212.20	83.76	21.46
21	4	8	13.02	5793.90	78.12	20.02
22	6	8	14.12	6283.40	84.72	21.71
23	7	8	14.98	6666.10	89.88	23.03
24	8	8	15.43	6866.35	92.58	23.72

4.2.3 Pareto Trade-off Analysis

Next, Pareto multi-objective optimization was applied to achieve a balance between total cost and material waste. Firstly, the total reinforcing cost consisting of material cost, installation and waste disposal cost was evaluated for each rebar layout arrangement plan, as shown in Table 4.

Table 4: Total reinforcing cost evaluations (scenario one)

Layout #	n_x	n_y	**Material Cost ($)**	**Installation Cost ($)**	**Waste Disposal Cost ($)**	**Total Reinforcing Cost ($)**
1	3	2	9330.00	2994.85	178.34	12503.19
2	4	2	8760.00	3106.10	54.12	11920.22
3	8	2	9840.00	4552.35	163.63	14555.98
4	4	3	8760.00	3577.80	28.71	12366.51
5	5	3	9900.00	3760.25	215.04	13875.29
6	6	4	9640.00	4837.15	121.70	14598.85
7	7	4	9600.00	4997.35	91.90	14689.25
8	8	4	10160.00	5282.15	174.71	15616.86
9	3	5	9540.00	4067.30	137.84	13745.14
10	4	5	9810.00	4498.95	168.54	14477.49
11	5	5	9850.00	4859.40	155.10	14864.50
12	6	5	9240.00	5095.25	21.21	14356.46
13	7	5	10000.00	5696.00	139.12	15835.12
14	4	6	10160.00	4886.10	206.65	15252.75
15	5	6	10350.00	5135.30	220.45	15705.75
16	6	6	9240.00	5762.75	134.97	15137.72
17	3	7	10040.00	4948.40	178.38	15166.78
18	4	7	9480.00	5166.45	54.19	14700.64
19	6	7	9440.00	6074.25	12.69	15526.94
20	7	7	10200.00	6212.20	130.31	16542.51
21	4	8	10160.00	5793.90	155.82	16109.72
22	6	8	9560.00	6283.40	1.44	15844.84
23	7	8	10000.00	6666.10	62.84	16728.94
24	8	8	10160.00	6866.35	73.01	17099.36

The reinforcing total cost and material waste percentage for each rebar layout arrangement plan are listed in Table 5. Next, none-domination values were calculated for each key performance indicator. By summing none-domination values of the two key performance indicators of each layout arrangement plan, a rank was assigned in order to identify the optimal trade-off solution.

Table 5: Total Cost vs. Material Waste (scenario one)

Plan No.	n_x	n_y	**Waste (%)**	**Non-Dominance (1)**	**Total Cost ($)**	**Non-Dominance (2)**	**Non-Dominance (1)+(2)**	**Rank**
1	3	2	10.53%	4	12503.19	22	26	10
2	4	2	3.40%	19	11920.22	24	43	2
3	8	2	9.16%	8	14555.98	17	25	11
4	**4**	**3**	**1.80%**	**21**	**12366.51**	**23**	**44**	**1**
5	5	3	11.96%	1	13875.29	20	21	15
6	6	4	6.95%	15	14598.85	16	31	7
7	7	4	5.27%	16	14689.25	15	31	7
8	8	4	9.47%	6	15616.86	8	14	21
9	3	5	7.96%	12	13745.14	21	33	5
10	4	5	9.46%	7	14477.49	18	25	11
11	5	5	8.67%	9	14864.50	13	22	14
12	6	5	1.26%	22	14356.46	19	41	3
13	7	5	7.66%	13	15835.12	6	19	17
14	4	6	11.20%	3	15252.75	10	13	23
15	5	6	11.73%	2	15705.75	7	9	24
16	6	6	8.04%	11	15137.72	12	23	13
17	3	7	9.78%	5	15166.78	11	16	20
18	4	7	3.15%	20	14700.64	14	34	4
19	6	7	0.74%	23	15526.94	9	32	6
20	7	7	7.03%	14	16542.51	3	17	19
21	4	8	8.44%	10	16109.72	4	14	21
22	6	8	0.08%	24	15844.84	5	29	9
23	7	8	3.46%	18	16728.94	2	20	16
24	8	8	3.96%	17	17099.36	1	18	18

A Pareto tradeoff graph is provided to further illustrate the results in Table 5, as demonstrated in Figure 13.

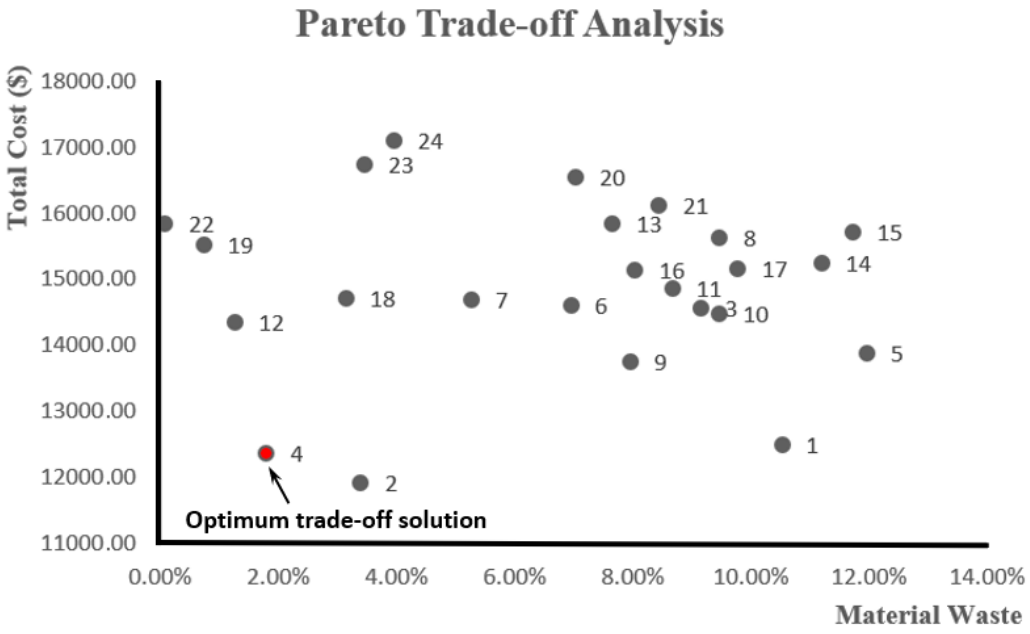


Figure 13: Pareto tradeoff analysis (scenario one)

According to the results from Pareto multi-objective optimization, the rebar layout arrangement plan No. 4 in Table 5, which is the same as the solution in Peurifoy and Oberlender (2002), has the highest rank, thus providing the best trade-off plan with regards to the two objectives. Specifically, the rebar layout arrangement plan No. 4 has the fourth minimum material waste of 1.80%, and second lowest total cost at $12366.51, among all the 24 alternative layout plans.

The resulting work plans for the optimum solution in terms rebar stock procurement plan, rebar stock cutting plan, field productivity, and waste disposal are summarized in Table 6. The results in Table 6 are intended to provide practical work plans for all the parties in the IPD framework and guide project execution in the field.

Table 6: Work plans of optimum solution (scenario one)

Procurement Plan	**Cutting Plan**	**Field Productivity (labour hour/ton)**	**Waste Disposal (feet)**
18 pieces of 20 ft stock 210 pieces of 40 ft stock	20 feet stocks cut into one 19 feet piece; 40 feet stocks cut into one 19 feet piece and one 20.33feet piece	12.36	158.07

4.3 Scenario Two (With Maximum Carrying Length Limit)

In the second scenario, a maximum rebar piece length limit is imposed. It is practically assumed that the maximum carrying length of a rebar segment by the ground labourers is 16 feet (5m). Otherwise, material handling equipment (such a small crane or skid steer) may be used for rebar delivery.

4.3.1 Rebar Detailing Design Results

In this scenario, a total of 13 feasible alternative rebar layout arrangement plans were obtained from optimization analysis, as shown in Table 7.

Table 7: Cutting plan for selected detailing design (scenario two)

Layout #	n_x	n_y	***x***	***y***	z_{ij}	L_i	r_{ij}	s_{ij}	$\sum w_{ij}$ **(ft)**
1	6	4	13.33	15.75	280	30	1	1	670.114
					62	20	1	0	
2	7	4	11.71	15.75	200	40	2	1	506
					80	20	0	1	
3	8	4	10.5	`15.75	228	40	2	1	962
					52	20	0	1	
4	5	5	15.6	13	285	30	1	1	854
					65	20	0	1	
5	6	5	13.33	13	119	40	1	2	116.78
					112	40	2	1	
6	7	5	11.71	13	150	40	2	1	766
					100	40	1	2	
					83	40	0	3	
7	5	6	15.6	11.17	285	30	1	1	1213.86

Layout #	n_x	n_y	x	y	z_{ij}	L_i	r_{ij}	s_{ij}	$\sum w_{ij}$ (ft)
					45	40	0	3	
8	6	6	13.33	11.17	119	40	2	1	743.169
					112	40	1	2	
					4	40	0	3	
9	6	7	13.33	9.86	114	40	3	0	69.898
					122	40	0	4	
					1	20	0	2	
10	7	7	11.71	9.86	133	40	3	0	717.494
					122	40	0	4	
					1	20	0	2	
					1	20	0	2	
11	6	8	13.33	8.88	52	40	3	0	7.906
					187	40	1	3	
12	7	8	11.71	8.88	280	30	1	2	346
					40	40	3	0	
13	8	8	10.5	8.88	202	40	2	2	402
					52	40	1	3	

4.3.2 Operation Planning Results

Since no rebar cutting plans are altered in this scenario, simulation results for the 13 rebar layout arrangement plans are directly truncated from Table 3 by eliminating the infeasible layout designs as per setting in current scenario. The results are summarized in Table 8.

Table 8: Simulation results for 13 rebar layout arrangement plans (scenario two)

Layout #	n_x	n_y	**Duration (hour)**	**Installation Cost ($)**	**Labour hours (hours)**	**Productivity (labour hour/ per ton)**
1	6	4	10.87	4837.15	65.22	16.71
2	7	4	11.23	4997.35	67.38	17.26
3	8	4	11.87	5282.15	71.22	18.25
4	5	5	10.92	4859.40	65.52	16.79
5	6	5	11.45	5095.25	68.7	17.60
6	7	5	12.8	5696.00	76.8	19.68
7	5	6	11.54	5135.30	69.24	17.74
8	6	6	12.95	5762.75	77.7	19.91
9	6	7	13.65	6074.25	81.9	20.98
10	7	7	13.96	6212.20	83.76	21.46
11	6	8	14.12	6283.40	84.72	21.71
12	7	8	14.98	6666.10	89.88	23.03
13	8	8	15.43	6866.35	92.58	23.72

4.3.3 Pareto Trade-off Analysis

Similar to scenario one, the total cost and material waste results for each rebar layout arrangement plan are listed in Table 9. Non-dominance values were calculated, based on which a rank was assigned to each layout plan alternative.

Table 9: Total Cost vs. Material Waste (scenario two)

Plan No.	n_x	n_y	**Waste (%)**	**Non-Dominance (1)**	**Total cost ($)**	**Non-Dominance (2)**	**Non-Dominance (1)+(2)**	**Rank**
1	6	4	6.95%	7	14598.85	8	15	7
2	7	4	5.27%	8	14689.25	9	17	4
3	8	4	9.47%	2	15616.86	2	4	12
4	5	5	8.67%	3	14864.50	7	10	9
5	**6**	**5**	**1.26%**	**11**	**14356.46**	**13**	**24**	**1**
6	7	5	7.66%	5	15835.12	5	10	9
7	5	6	11.73%	1	15705.75	1	2	13
8	6	6	8.04%	4	15137.72	12	16	5
9	6	7	0.74%	12	15526.94	11	23	2
10	7	7	7.03%	6	16542.51	3	9	11
11	6	8	8.44%	10	16109.72	10	23	2
12	7	8	0.08%	13	15844.84	6	19	5
13	8	8	3.46%	9	16728.94	4	13	8

A Pareto tradeoff graph is provided to further illustrate the results in Table 9, as demonstrated in Figure 14.

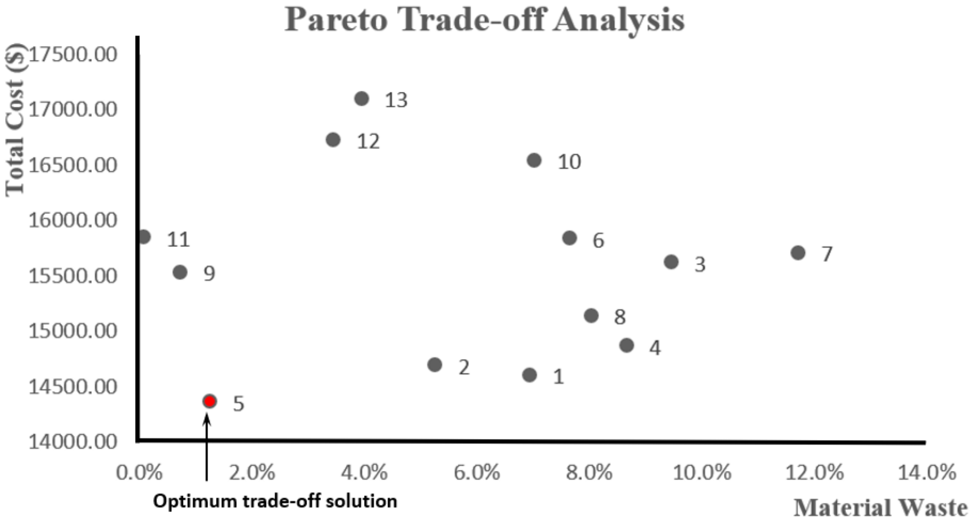


Figure 14: Pareto tradeoff analysis (scenario two)

According to the results from Pareto multi-objective optimization, the rebar layout arrangement plan No. 5 in Table 9 has the highest rank, thus representing the best trade-off plan between the two objectives. Specifically, rebar layout arrangement plan No. 5 has the third minimum material waste of 1.26%, and lowest total cost at $13794.06, among all the 13 alternative layout plans. The specific work plans for the optimum solution is summarized in Table 10.

Table 10: Work plans of optimum solution (scenario two)

Procurement Plan	Cutting Plan	Field Productivity (labour hour/ton)	Waste Disposal (feet)
231 pieces of 40 ft stock	119 pieces of 40 feet stocks cut into one 13.33 feet piece, and two 13 feet pieces 112 pieces of 40 feet stocks cut into two 13.33 feet pieces, and one 13 feet piece	17.60	116.78

4.4 Scenario Three (Lap Length at 30d)

In the third scenario, contractor decides to use high capacity concrete due to a structural consideration, which justifies the use of shorter lap length (Concrete Design Handbook 2006, Explanatory Notes on CSA A23.3-04, Page 279). A lap length at 30d is selected in this scenario and all the feasible rebar layout plans are generated to fit slab design configurations. A maximum of 8 pieces along each side is imposed as previous scenarios.

4.4.1 Rebar Detailing Design Results

By changing the lap length variable in the program (Appendix B), optimized rebar layout arrangement plans with the corresponding rebar stock procurement and cutting plans were obtained by applying Integer Programming. In this scenario, a total of 22 feasible alternative rebar layout arrangement plans were obtained from optimization analysis, as shown in Table 11.

Table 11: Cutting plan for selected detailing design (scenario three)

Layout #	n_x	n_y	x	y	z_{ij}	L_i	r_{ij}	s_{ij}	$\sum w_{ij}$ **(ft)**
1	3	2	24.37	29.28	171	30	1	0	1062.96
					140	30	0	1	
2	4	2	18.67	29.28	228	20	1	0	404.04
					140	30	0	1	
3	8	2	10.12	29.28	106	40	3	0	1108.13
					140	40	1	1	
4	**4**	**3**	**18.67**	**20.04**	**18**	**20**	**1**	**0**	**294.84**
					210	**40**	**1**	**1**	
5	3	4	24.37	15.42	109	20	0	1	534.56
					171	40	1	1	
6	6	4	12.97	15.42	280	30	1	1	887.8
					62	20	1	0	
7	7	4	11.34	15.42	200	40	2	1	747.54
					80	20	0	1	
8	3	5	24.37	12.65	171	40	1	1	932.01
					90	30	0	2	
9	4	5	18.67	12.65	114	40	2	0	1126.44
					175	30	0	2	
10	5	5	15.25	12.65	285	30	1	1	1077.52
					65	20	0	1	
11	6	5	12.97	12.65	119	40	1	2	365.63
					112	40	2	1	
12	7	5	11.34	12.65	150	40	2	1	1038.34
					100	40	1	2	
13	4	6	18.67	10.8	228	30	1	1	607.24
					64	40	0	3	
14	6	6	12.97	10.8	285	30	1	1	997.27
					45	40	0	3	
15	4	7	18.67	9.48	228	20	1	0	559.08
					123	40	0	4	
16	6	7	12.97	9.48	114	40	3	0	379.16
					122	40	0	4	
					1	20	0	2	
17	7	7	11.34	9.48	133	40	3	0	1050.24
					122	40	0	4	
					1	20	0	2	
18	8	7	10.12	9.48	456	20	1	1	203.4
					9	40	0	4	
19	4	8	18.67	8.49	228	20	1	0	1148.84
					140	40	0	4	

Layout #	n_x	n_y	$\boldsymbol{x}$	$\boldsymbol{y}$	z_{ij}	L_i	r_{ij}	s_{ij}	$\sum w_{ij}$ **(ft)**
20	6	8	12.97	8.49	52	40	3	0	349.54
					187	40	1	3	
21	7	8	11.34	8.49	280	30	1	2	315.1
					40	40	3	0	
22	8	8	10.12	8.49	202	40	2	2	793.16
					52	40	1	3	

4.4.2 Operation Planning Results

SDESA “AON Plus” simulation models were built for each rebar layout arrangement plan in Table 11. A total of 22 distinct models were established each corresponding with a particular cutting plan. Simulated operation duration and calculated installation cost, labour hours, and productivity for each cutting plan are summarized in Table 12.

Table 12: Simulation results for 22 rebar layout arrangement plans (scenario three)

Layout #	$\mathbf{n_x}$	$\mathbf{n_y}$	**Duration (hour)**	**Installation Cost ($)**	**Labour hours (hours)**	**Productivity (labour hour/ per ton)**
1	3	2	6.73	2994.85	40.38	10.35
2	4	2	6.98	3106.10	41.88	10.73
3	8	2	10.23	4552.35	61.38	15.73
4	4	3	8.04	3577.80	48.24	12.36
5	3	4	7.89	3511.05	47.34	12.13
6	6	4	10.87	4837.15	65.22	16.71
7	7	4	11.23	4997.35	67.38	17.26
8	3	5	9.14	4067.30	54.84	14.05
9	4	5	10.11	4498.95	60.66	15.54
10	5	5	10.92	4859.40	65.52	16.79
11	6	5	11.45	5095.25	68.7	17.60
12	7	5	12.8	5696.00	76.8	19.68
13	4	6	10.45	4650.25	62.7	16.06
14	6	6	12.95	5762.75	77.7	19.91
15	4	7	11.61	5166.45	69.66	17.85
16	6	7	13.65	6074.25	81.9	20.98
17	7	7	13.96	6212.20	83.76	21.46
18	8	7	14.25	6341.25	85.5	21.91
19	4	8	13.02	5793.90	78.12	20.02
20	6	8	14.12	6283.40	84.72	21.71
21	7	8	14.78	6577.10	88.68	22.72
22	8	8	15.43	6866.35	92.58	23.72

4.4.3 Pareto Trade-off Analysis

Next, Pareto multi-objective optimization was applied to strike a balance between total cost, and material waste. Firstly, the total reinforcing cost consisting of material cost, installation and waste disposal cost was evaluated for each rebar layout arrangement plan, as shown in Table 13.

Table 13: Total reinforcing cost evaluations (scenario three)

Layout #	n_x	n_y	**Material Cost ($)**	**Installation Cost ($)**	**Waste Disposal Cost ($)**	**Total Reinforcing Cost ($)**
1	3	2	9330.00	2994.85	193.05	12324.85
2	4	2	8760.00	3106.10	73.38	11866.10
3	5	2	9840.00	4552.35	201.25	14392.35
4	6	2	8760.00	3577.80	53.55	12337.80
5	7	2	9020.00	3511.05	97.08	12531.05
6	8	2	9640.00	4837.15	161.24	14477.15
7	3	3	9600.00	4997.35	135.76	14597.35
8	4	3	9540.00	4067.30	169.26	13607.30
9	5	3	9810.00	4498.95	204.58	14308.95
10	6	3	9850.00	4859.40	195.69	14709.40
11	7	3	9240.00	5095.25	66.40	14335.25
12	8	3	10000.00	5696.00	188.58	15696.00
13	3	4	9400.00	4650.25	110.28	14050.25
14	4	4	9240.00	5762.75	181.12	15002.75
15	5	4	9480.00	5166.45	101.54	14646.45
16	6	4	9440.00	6074.25	68.86	15514.25
17	7	4	10200.00	6212.20	190.74	16412.20
18	8	4	9480.00	6341.25	36.94	15821.25
19	3	5	10160.00	5793.90	208.64	15953.90
20	4	5	9560.00	6283.40	63.48	15843.40
21	5	5	9610.00	6577.10	57.23	16187.10
22	6	5	10160.00	6866.35	144.05	17026.35

The reinforcing total cost and material waste percentage for each rebar layout arrangement plan are listed in Table 14. Next, none-domination values were calculated for each key performance indicator. By summing none-domination values of the two key performance indicators of each layout arrangement plan, a rank was assigned in order to identify the optimal trade-off solution.

Table 14: Total Cost vs. Material Waste (scenario three)

Plan No.	n_x	n_y	Waste (%)	Non-Dominance (1)	Total Cost ($)	Non-Dominance (2)	Non-Dominance (1)+(2)	Rank
1	3	2	11.39%	2	12324.85	21	23	13
2	4	2	4.61%	16	11866.10	22	38	2
3	8	2	11.26%	4	14392.35	14	18	15
4	**4**	**3**	**3.37%**	**20**	**12337.80**	**20**	**40**	**1**
5	3	4	5.93%	14	12531.05	19	33	3
6	6	4	9.21%	10	14477.15	13	23	13
7	7	4	7.79%	12	14597.35	12	24	10
8	3	5	9.77%	9	13607.30	18	27	7
9	4	5	11.48%	1	14308.95	16	17	16
10	5	5	10.94%	5	14709.40	10	15	17
11	6	5	3.96%	18	14335.25	15	33	3
12	7	5	10.38%	7	15696.00	7	14	19
13	4	6	6.46%	13	14050.25	17	30	5
14	6	6	10.79%	6	15002.75	9	15	17
15	4	7	5.90%	15	14646.45	11	26	8
16	6	7	4.02%	17	15514.25	8	25	9
17	7	7	10.30%	8	16412.20	2	10	21
18	8	7	2.15%	22	15821.25	6	28	6
19	4	8	11.31%	3	15953.90	4	7	22
20	6	8	3.66%	19	15843.40	5	24	10
21	7	8	3.28%	21	16187.10	3	24	10
22	8	8	7.81%	11	17026.35	1	12	20

A Pareto tradeoff graph is provided to further illustrate the results in Table 14, as demonstrated in Figure 15.

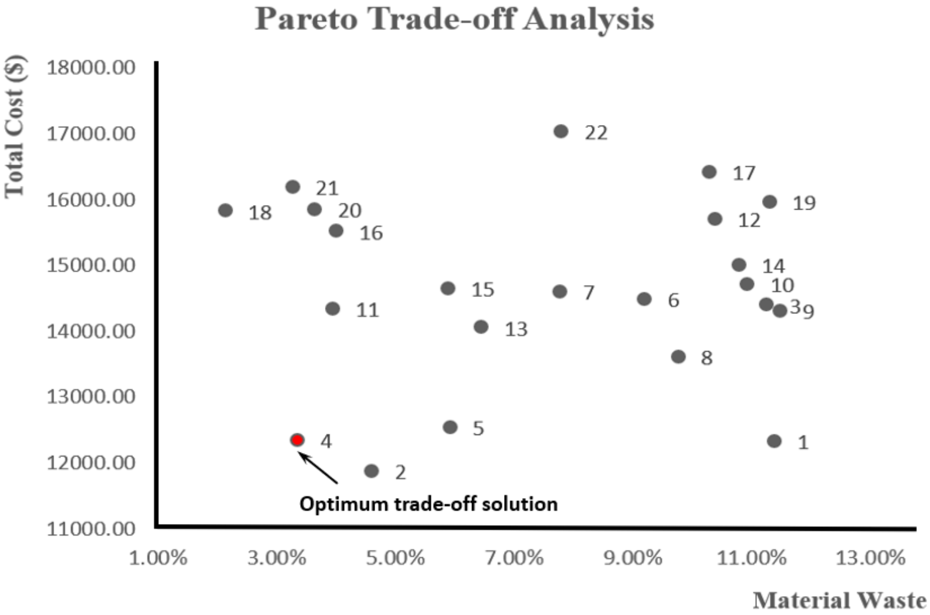


Figure 15: Pareto tradeoff analysis (scenario three)

According to the results from Pareto multi-objective optimization, the rebar layout arrangement plan No. 4 in Table 14, which is the same as the solution in Peurifoy and Oberlender (2002), remains to be the best trade-off plan regardless of the change of lap length. Specifically, the rebar layout arrangement plan No. 4 has the third minimum material waste of 3.37%, and third lowest total cost at $12337.80, among all the 22 alternative layout plans.

The specific work plans for the optimum solution in terms rebar stock procurement plan, rebar stock cutting plan, field productivity, and waste disposal are summarized in Table 15 to provide practical work plans for all the parties in the IPD framework and guide project execution in the field.

Table 15: Work plans of optimum solution (scenario three)

Procurement Plan	Cutting Plan	Field Productivity (labour hour/ton)	Waste Disposal (feet)
18 pieces of 20 ft stock 210 pieces of 40 ft stock	20 feet stocks cut into one 18.67 feet piece; 40 feet stocks cut into one 18.67 feet piece and one 20.04 feet piece	12.36	294.84

4.5 Scenario Four (Waste Disposal Cost at $600/ton)

In the fourth scenario, waste disposal cost is assumed to increase to $600/ton due to a new environmental by-law issued by the local government which imposes an environmental protection tax to material waste. The rest settings remain identical to base case scenario. Without changing any design specifications in this scenario, rebar detailing designs and operation plans remain the same as the base case scenario. The waste disposal cost was reevaluated for each rebar layout arrangement plan and total reinforcing cost was updated accordingly, as shown in Table 16.

Table 16: Total reinforcing cost evaluations (scenario four)

Layout #	n_x	n_y	Material Cost ($)	Installation Cost ($)	Waste Disposal Cost ($)	Total Reinforcing Cost ($)
1	3	2	9330.00	2994.85	282.23	12607.08
2	4	2	8760.00	3106.1	85.65	11951.75
3	8	2	9840.00	4552.35	258.95	14651.30
4	4	3	8760.00	3577.8	45.43	12383.23
5	5	3	9900.00	3760.25	340.30	14000.55
6	6	4	9640.00	4837.15	192.59	14669.74
7	7	4	9600.00	4997.35	145.42	14742.77
8	8	4	10160.00	5282.15	276.48	15718.63
9	3	5	9540.00	4067.3	218.14	13825.44
10	4	5	9810.00	4498.95	266.71	14575.66
11	5	5	9850.00	4859.4	245.44	14954.84
12	6	5	9240.00	5095.25	33.56	14368.81

Layout #	n_x	n_y	Material Cost ($)	Installation Cost ($)	Waste Disposal Cost ($)	Total Reinforcing Cost ($)
13	7	5	10000.00	5696	220.15	15916.15
14	4	6	10160.00	4886.1	327.02	15373.12
15	5	6	10350.00	5135.3	348.86	15834.16
16	6	6	9240.00	5762.75	213.59	15216.34
17	3	7	10040.00	4948.4	282.29	15270.69
18	4	7	9480.00	5166.45	85.75	14732.20
19	6	7	9440.00	6074.25	20.09	15534.34
20	7	7	10200.00	6212.2	206.21	16618.41
21	4	8	10160.00	5793.9	246.59	16200.49
22	6	8	9560.00	6283.4	2.27	15845.67
23	7	8	10000.00	6666.1	99.44	16765.54
24	8	8	10160.00	6866.35	115.53	17141.88

Next, none-domination values were calculated for material waste and total reinforcing cost for all the 24 rebar layout arrangement designs, by which a rank was assigned to identify the optimal trade-off solution. The results are shown in Table 17.

Table 17: Total Cost vs. Material Waste (scenario four)

Plan No.	n_x	n_y	Waste (%)	Non-Dominance (1)	Total Cost ($)	Non-Dominance (2)	Non-Dominance (1)+(2)	Rank
1	3	2	10.53%	4	12607.08	22	26	10
2	4	2	3.40%	19	11951.75	24	43	2
3	8	2	9.16%	8	14651.30	17	25	11
4	**4**	**3**	**1.80%**	**21**	**12383.23**	**23**	**44**	**1**
5	5	3	11.96%	1	14000.55	20	21	15
6	6	4	6.95%	15	14669.74	16	31	7
7	7	4	5.27%	16	14742.77	14	30	8
8	8	4	9.47%	6	15718.63	8	14	21
9	3	5	7.96%	12	13825.44	21	33	5
10	4	5	9.46%	7	14575.66	18	25	11
11	5	5	8.67%	9	14954.84	13	22	14
12	6	5	1.26%	22	14368.81	19	41	3
13	7	5	7.66%	13	15916.15	5	18	17
14	4	6	11.20%	3	15373.12	10	13	23
15	5	6	11.73%	2	15834.16	7	9	24
16	6	6	8.04%	11	15216.34	12	23	13
17	3	7	9.78%	5	15270.69	11	16	20
18	4	7	3.15%	20	14732.20	15	35	4
19	6	7	0.74%	23	15534.34	9	32	6

20	7	7	7.03%	14	16618.41	3	17	19
21	4	8	8.44%	10	16200.49	4	14	21
22	6	8	0.08%	24	15845.67	6	30	8
23	7	8	3.46%	18	16765.54	2	20	16
24	8	8	3.96%	17	17141.88	1	18	17

A Pareto tradeoff graph is provided to further illustrate the results in Table 17, as demonstrated in Figure 16.

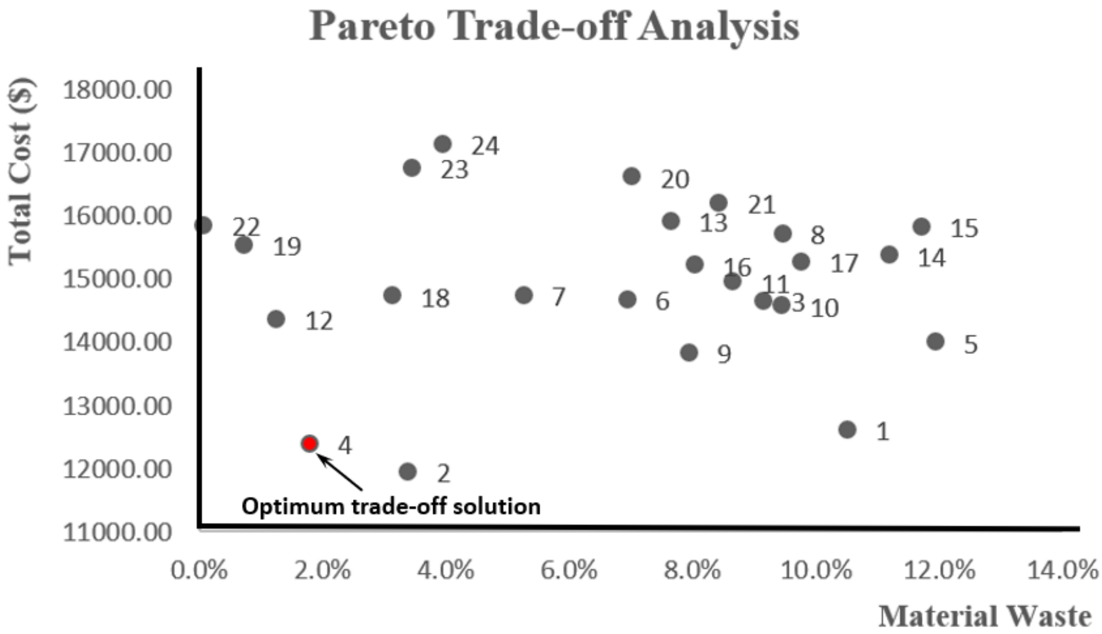


Figure 16: Pareto tradeoff analysis (scenario four)

According to the results from Pareto multi-objective optimization, the rebar layout arrangement plan No. 4 in Table 17, which is the same as the solution in Peurifoy and Oberlender (2002), remains to be the best trade-off plan irrespective of the rise of waste disposal cost. The specific work plans for the optimum solution in terms rebar stock procurement plan, rebar stock cutting plan, field productivity, and waste disposal thus remains the same as scenario 1.

4.6 Scenario Five (Rebar Purchasing Cost at $3150/ton)

In the fifth scenario, it is practically assumed that the price of steel rises from $2100/ton to

$3150/ton (1.5 times original price) due to the impact from external environment (e.g. tariff on steel). The rest settings remain identical to base case scenario. Without changing any design specifications in this scenario, rebar detailing designs and operation plans remain the same as the base case scenario. The material cost was reevaluated for each rebar layout arrangement plan and total reinforcing cost was updated accordingly, as shown in Table 18.

Table 18: Total reinforcing cost evaluations (scenario five)

Layout #	**n_x**	**n_y**	**Material Cost ($)**	**Installation Cost ($)**	**Waste Disposal Cost ($)**	**Total Reinforcing Cost ($)**
1	3	2	13995.00	2994.85	178.34	17168.19
2	4	2	13140.00	3106.10	54.12	16300.22
3	8	2	14760.00	4552.35	163.63	19475.98
4	**4**	**3**	13140.00	3577.80	28.71	16746.51
5	5	3	14850.00	3760.25	215.04	18825.29
6	6	4	14460.00	4837.15	121.70	19418.85
7	7	4	14400.00	4997.35	91.90	19489.25
8	8	4	15240.00	5282.15	174.71	20696.86
9	3	5	14310.00	4067.30	137.84	18515.14
10	4	5	14715.00	4498.95	168.54	19382.49
11	5	5	14775.00	4859.40	155.10	19789.50
12	6	5	13860.00	5095.25	21.21	18976.46
13	7	5	15000.00	5696.00	139.12	20835.12
14	4	6	15240.00	4886.10	206.65	20332.75
15	5	6	15525.00	5135.30	220.45	20880.75
16	6	6	13860.00	5762.75	134.97	19757.72
17	3	7	15060.00	4948.40	178.38	20186.78
18	4	7	14220.00	5166.45	54.19	19440.64
19	6	7	14160.00	6074.25	12.69	20246.94
20	7	7	15300.00	6212.20	130.31	21642.51
21	4	8	15240.00	5793.90	155.82	21189.72
22	6	8	14340.00	6283.40	1.44	20624.84
23	7	8	15000.00	6666.10	62.84	21728.94
24	8	8	15240.00	6866.35	73.01	22179.36

Next, none-domination values were calculated for material waste and total reinforcing cost for all the 24 rebar layout arrangement designs, by which a rank was assigned to identify the optimal trade-off solution. The results are shown in Table 19.

Table 19: Total Cost vs. Material Waste (scenario five)

Plan No.	n_x	n_y	**Waste (%)**	**Non-Dominance (1)**	**Total Cost ($)**	**Non-Dominance (2)**	**Non-Dominance (1)+(2)**	**Rank**
1	3	2	10.53%	4	17168.19	22	26	10
2	4	2	3.40%	19	16300.22	24	43	2
3	8	2	9.16%	8	19475.98	15	23	13
4	**4**	**3**	**1.80%**	**21**	**16746.51**	**23**	**44**	**1**
5	5	3	11.96%	1	18825.29	20	21	14
6	6	4	6.95%	15	19418.85	17	32	7
7	7	4	5.27%	16	19489.25	14	30	9
8	8	4	9.47%	6	20696.86	7	13	22
9	3	5	7.96%	12	18515.14	21	33	5
10	4	5	9.46%	7	19382.49	18	25	11
11	5	5	8.67%	9	19789.50	12	21	14
12	6	5	1.26%	22	18976.46	19	41	3
13	7	5	7.66%	13	20835.12	6	19	17
14	4	6	11.20%	3	20332.75	9	12	23
15	5	6	11.73%	2	20880.75	5	7	24
16	6	6	8.04%	11	19757.72	13	24	12
17	3	7	9.78%	5	20186.78	11	16	20
18	4	7	3.15%	20	19440.64	16	36	4
19	6	7	0.74%	23	20246.94	10	33	5
20	7	7	7.03%	14	21642.51	3	17	19
21	4	8	8.44%	10	21189.72	4	14	21
22	6	8	0.08%	24	20624.84	8	32	7
23	7	8	3.46%	18	21728.94	2	20	16
24	8	8	3.96%	17	22179.36	1	18	18

A Pareto tradeoff graph is provided to further illustrate the results in Table 19, as demonstrated in Figure 17.

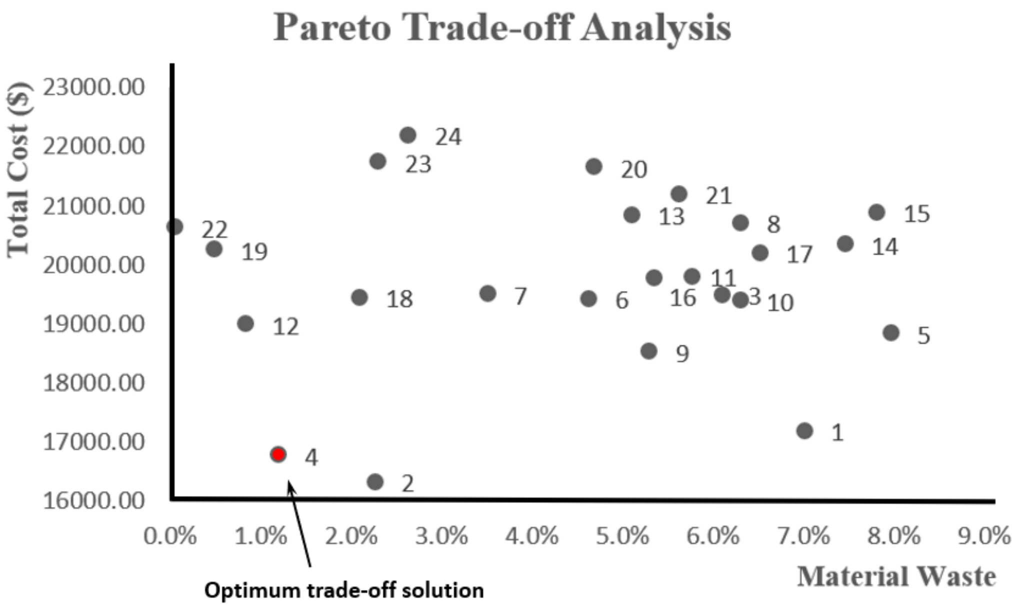


Figure 17: Pareto tradeoff analysis (scenario five)

According to the results from Pareto multi-objective optimization, the rebar layout arrangement plan No. 4 in Table 19, which is the same as the solution in Peurifoy and Oberlender (2002), remains to be the best trade-off plan in spite of the rise in steel cost. The specific work plans for the optimum solution in terms rebar stock procurement plan, rebar stock cutting plan, field productivity, and waste disposal thus remains the same as scenario 1.

4.7 Discussion of Results

It is observed from above "what if" analysis that solution in Peurifoy and Oberlender (2002) (n_x=4, n_y=3) remains to be the optimum trade-off solution for four practically assumed scenarios, except scenario two where solution in Peurifoy and Oberlender (2002) is opted out by the imposed constraints on rebar carrying length. The reasons of the robustness of solution in Peurifoy and Oberlender (2002) is meticulously investigated and listed below:

(1) Installation cost is correlated to the engineering design. The more rebar segments and rebar laps to be installed, the higher the installation cost is; the solution in Peurifoy and Oberlender (2002) has the third minimum requirements for rebar segment and rebar lap (in terms of quantity to be installed), thus having the third minimum installation cost among all the feasible alternatives;

(2) Material waste disposal cost is correlated to the generated material waste. The solution in Peurifoy and Oberlender (2002) has the third minimum material waste, making its material waste disposal cost the third minimum among all the feasible plans;

(3) Material cost is correlated to procurement plan; the more rebar stocks are procured, the higher the material cost is. The solution in Peurifoy and Oberlender (2002) has the lowest material costs among all the feasible layout designs due to its "perfect fit" for the slab configuration in current thesis.

In scenario 3, the change of lap length from 40d to 30d does not affect the optimum solution. Though the change in lap length nearly double material waste (158.07 feet for 40d scenario vs. 294.84 feet for 30d scenario), the installation cost, material procurement cost, and material waste of the solution in Peurifoy and Oberlender (2002) remains to be superior to most of the alternative designs. In particular, installation cost is the third minimum, material procurement cost is the minimum, material waste is the third minimum. Thus, in combination, the solution in Peurifoy and Oberlender (2002) remains to be the optimum trade-off solution.

In scenario 4, the rise of material cost from $2100/ton to $3150/ton does not change the optimum solution in current case. As stated previously, the material cost is correlated to procurement plan, where the solution in Peurifoy and Oberlender (2002) has the least amount of rebar to be procured. Hence, the optimum solution remains the same.

In scenario five, the rise of material waste disposal cost does not change the optimum solution in current case. On one hand, the material waste disposal cost contributes only a small portion to the total reinforcing cost (0.5% to 2%), the rise of the material waste disposal cost has limited impact to total reinforcing cost. On the other hand, the amount of waste of the solution in Peurifoy and Oberlender (2002) is relatively small, despite the increase on material waste disposal unit cost, the total material waste disposal cost still remains to be the third minimum among all the alternative designs.

A comparison table between the optimum solutions of all the scenarios is given in Table 20 (in total three optimum solutions; optimum solutions of scenario four and scenario five is identical to scenario three). Among the three optimum solutions, optimum solution in scenario two has the least material waste generated, but consumes the most labor-hours because more rebar segments and rebar laps are designed to be installed.

Table 20: Optimum solution comparison

Scenario	**Procurement Plan**	**Cutting Plan**	**Field Productivity (labour hour/ton)**	**Waste Disposal (feet)**
1	18 pieces of 20 ft stock 210 pieces of 40 ft stock	20 feet stocks cut into one 19 feet piece; 40 feet stocks cut into one 19 feet piece and one 20.33feet piece	12.36	158.07
2	231 pieces of 40 ft stock	119 pieces of 40 feet stocks cut into one 13.33 feet piece, and two 13 feet pieces 112 pieces of 40 feet stocks cut into two 13.33 feet pieces, and one 13 feet piece	17.60	116.78

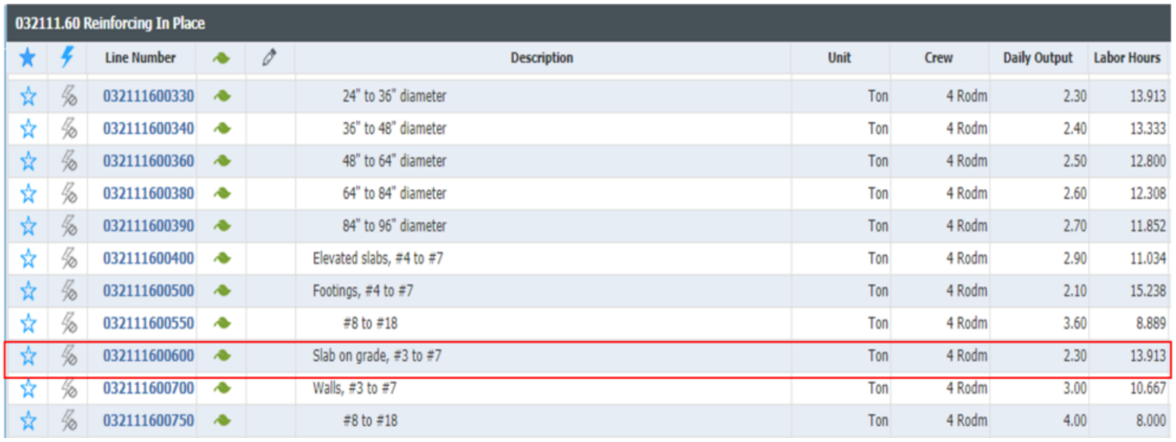
Scenario	Procurement Plan	Cutting Plan	Field Productivity (labour hour/ton)	Waste Disposal (feet)
3	18 pieces of 20 ft stock 210 pieces of 40 ft stock	20 feet stocks cut into one 18.67 feet piece; 40 feet stocks cut into one 18.67 feet piece and one 20.04 feet piece	12.36	294.84

Having analyzed above, the change of lapping length, the rise of material cost, and the rise of material waste disposal cost will not change the current optimum solution in this case study. However, the results do not necessarily conclude the change of above parameters will not affect optimum solutions in other cases (e.g. the material cost of the optimum solution in base case scenario is not the minimum among all the solutions, then the rise of material cost may change the optimum result). Due to the existence of the complicated interplay among engineering design, procurement plan, rebar processing plan, and workface crew operation plan in terms of reinforcement cost and material waste, it is suggested that practitioners apply the proposed method to analyze case by case, scenario by scenario in order to guide decision makings in practice.

In short, this case study is conducive to illustrating how the proposed approach can be effectively used to search for and shed light on the optimum trade-off in rebar layout arrangement plan among a wide range of alternatives subject to practical work planning constraints. Decision makers can evaluate optimized solutions through Pareto trade-off analysis and select an optimal slab rebar layout arrangement plan under different practical constraints.

4.8 Cross Validation against Productivity Benchmark

The productivity results generated from current research is loosely benchmarked with RSMeans data. Note, RSMeans (2017) is a comprehensive benchmarking database that provides average cost and production performance information for labour and equipment that are typically applied in practice organized as per the industry-wide work breakdown structure (i.e. MasterFormat); whereas specific job conditions are not considered. This “rough” benchmark does not intend to find the exact match in RSMeans; rather it aims to demonstrate the results generated in current thesis fall in the right order and range. A screenshot from RSMeans is shown in Figure 18.



032111.60 Reinforcing In Place

★	⚡	Line Number		✎	Description	Unit	Crew	Daily Output	Labor Hours
☆		032111600330			24" to 36" diameter	Ton	4 Rodm	2.30	13.913
☆		032111600340			36" to 48" diameter	Ton	4 Rodm	2.40	13.333
☆		032111600360			48" to 64" diameter	Ton	4 Rodm	2.50	12.800
☆		032111600380			64" to 84" diameter	Ton	4 Rodm	2.60	12.308
☆		032111600390			84" to 96" diameter	Ton	4 Rodm	2.70	11.852
☆		032111600400			Elevated slabs, #4 to #7	Ton	4 Rodm	2.90	11.034
☆		032111600500			Footings, #4 to #7	Ton	4 Rodm	2.10	15.238
☆		032111600550			#8 to #18	Ton	4 Rodm	3.60	8.889
☆		032111600600			Slab on grade, #3 to #7	Ton	4 Rodm	2.30	13.913
☆		032111600700			Walls, #3 to #7	Ton	4 Rodm	3.00	10.667
☆		032111600750			#8 to #18	Ton	4 Rodm	4.00	8.000

Figure 18: Productivity Benchmark from RSMeans

The national-wide reinforcing productivity benchmark including rebar cutting, delivering and reinforcing in place is 13.913 LH/Ton. Productivity results generated in current thesis range from 10 to 25 LH/Ton in different rebar layout arrangement plans. Regardless of the wide variation derived from job specific factors such as rebar layout design, rebar cutting plan, rebar delivery method and distance, and other site specific constraints, the validity of results generated in current thesis is proven to a certain extent by showing the derived productivity falls in line with the productivity benchmark. Ideally, the results generated in current thesis need to be benchmarked with estimates from professional industry practitioners through a bottom-up estimating process considering all the specific job conditions. Due to limited resource, this “precise” benchmark is not available and hence not presented in current thesis.

Chapter 5 CONCLUSIONS

The steel cost incurred on a building or infrastructure construction project accounts for a large proportion of the entire project cost. Therefore, efficient planning on reinforcing steel in terms of procurement, design and installation is critical to achieve a more cost-efficient project delivery. Other than the profitability goal, sustainability-related goals (such as material waste) in connection with rebar stock processing, engineering design and material procurement are equally important. Having found that previously related research has not simultaneously considered profitability and sustainability, this research has introduced an optimization method for rebar layout arrangement design illustrated with a slab case. The proposed optimization methodology has given rise to a sustainable construction plan featuring the optimal trade-off between reducing cutting losses and lowering the total cost. In this research, the concepts of sustainability, integrated project delivery, and workface planning have been brought into practical effect through mathematical programming formulations, resulting in analytically optimal solutions ready for workface execution. Specifically, the problem has been formulated in the form of Integer programming, and Multi Objective Optimization; the optimal trade-off plan in terms of optimal wastage and total cost for selecting slab rebar layout arrangement is thus achievable. To some extent, the proposed methodology has converted an empirical rebar layout arrangement problem in construction engineering into an analytical problem for optimization by taking on an integrated project delivery vision.

Based on a classic slab-on-grade case for illustrating rebar design and the detailed takeoff and estimating found in the textbook, the effectiveness of the proposed methodology is demonstrated in terms of generating optimum trade-off work plans. The outputted slab rebar layout arrangement plans provide decision makers alternatives for cost estimating and work planning based on specific requirements and strategic objectives. Hence, the research deliverables cater to the pressing needs of the construction industry for enhancing the constructability, sustainability and cost efficiency in construction engineering. The decision-makers can take advantage of the optimized rebar designs for achieving cost saving and sustainable design. Engineers with limited experience in slab rebar detailing design will be assisted in delivering a valid, feasible and sustainable solution by applying the proposed methodology.

Despite advances made and new insight obtained through the current research, some further improvements on the research reported in this thesis will be worthy to be pursued, in the following three directions:

(1) In the current research, a slab-on-grade case is used for demonstrating the proposed methodology. It is necessary to extend current research applications to more slab types (e.g. flat slab, hollow core slab, and etc.) as well as other building components such as columns and beams in order to realize the full potential of the postulated research;

(2) As the optimization approach relies on cost data as input, it is foreseen that there is a need to improve the reliability of cost data in order to achieve more reliable optimized solutions;

(3) The whole optimization methodology needs to be automated by seamlessly linking optimization with simulation analyses at different stages for the benefits of end users; coupling the automated methodology onto building information modeling (BIM) platform for rebar design-construction integrated analysis and visualization will further enhance the

impact of the proposed research.

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Appendix A. Slab Configuration and Rebar Layout Arrangement Solution by Peurifoy and Oberlender (2002)

In Peurifoy and Oberlender (2002), the steel rebar is placed on the slab spaced at 1 foot (0.304m) along both directions, concrete cover depth in reinforced concrete is considered as 0.5 feet (0.15m). Rebar pieces are overlapped and tied for connection with a lap length at 40d (d is the diameter of rebar). 4 rebar pieces are designed to be placed horizontally and 3 rebar pieces are design to be placed vertically. Specifically, the 4 rebar pieces in a horizontal row is 19.00ft (5.79m) for each while the 3 rebar pieces in vertical row is 19.67 ft (6.00m) for each. Detailed configurations are shown in Figure A-1, Figure A-2 and Figure A-3.

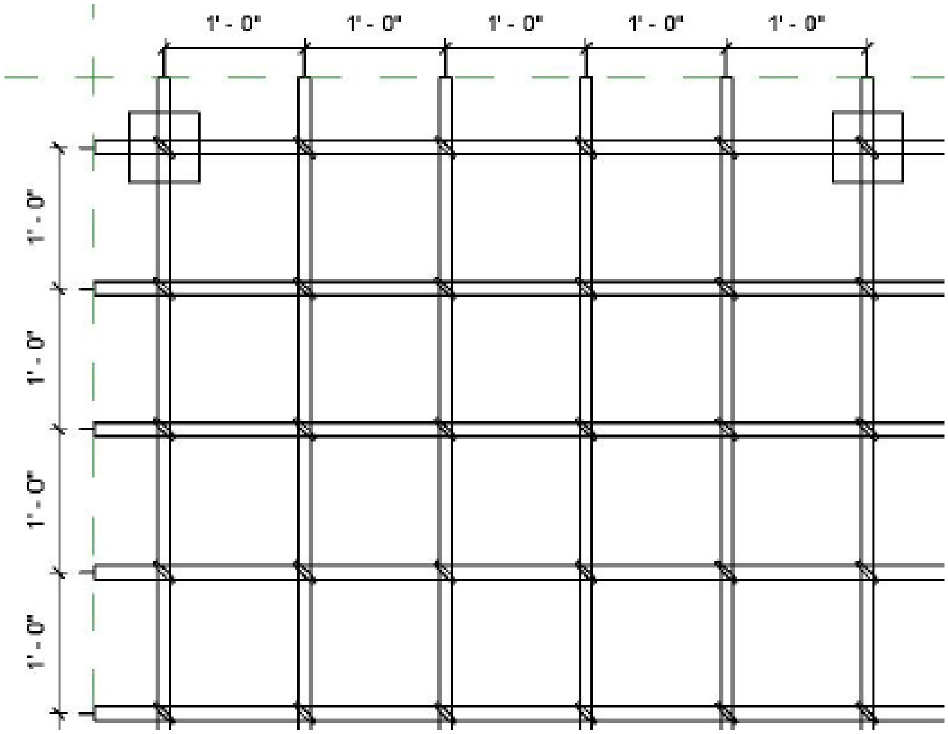

Figure A-1: Rebar Spacing (Lu 2017)

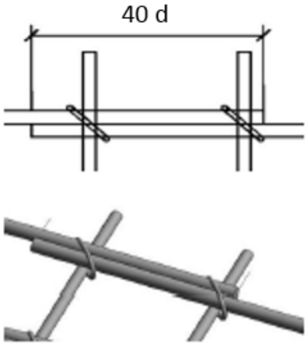

Figure A-2: Rebar Lapping (Lu 2017)

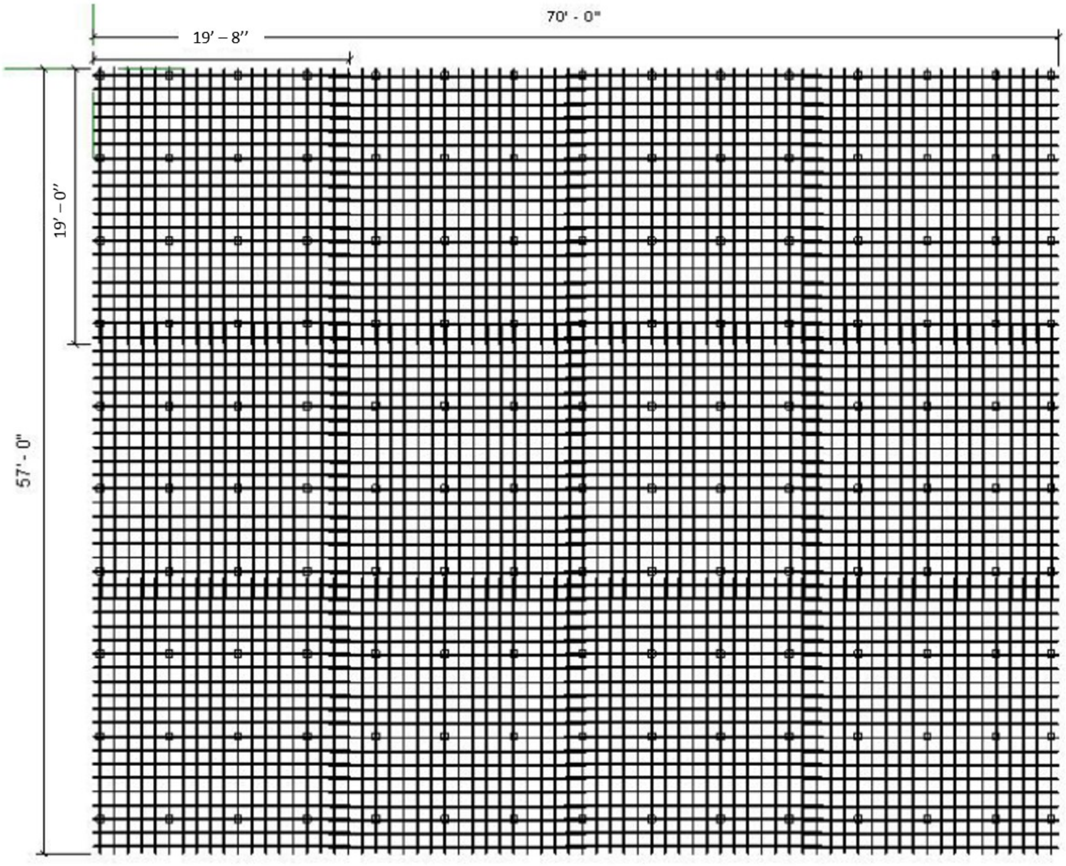

Figure A-3: Rebar Layout Arrangement Plan (Lu 2017)

A total of 18 pieces of 20 feet long rebar stock and 210 pieces of 40 feet long rebar stock are needed to fit in the rebar layout arrangement plan. As for rebar stock cutting plan, two cutting patterns are optimized: the 18 pieces 20 feet long stocks are optimized to be cut into one 19 feet piece with 1ft cutting loss; and 210 pieces of 40 feet long stocks are optimized to be cut into one 19 feet pieces and one 19.67 feet piece with 1.33 feet cutting loss. Rebar stock cutting plan is further illustrated in Figure A-4. Consequently, the cutting plan in Peurifoy and Oberlender (2002) will generate a total of 158.07 feet of material waste. The detailed calculations for rebar layout arrangement plan in Peurifoy and Oberlender (2002) are elaborated in Table A-1.

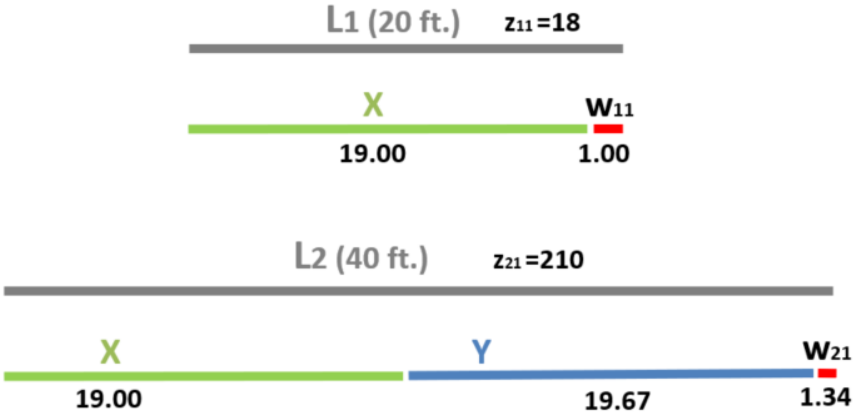

Figure A-4: Cutting plan for layout in Peurifoy and Oberlender (2002)

Table A-1: Detailed calculations for layout in Peurifoy and Oberlender (2002)

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	4
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(4-1)+70]/4=19.00
Number of row	m_x	$m_x=roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	4×57=228

Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	3
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(3-1)+57]/3=19.67
Number of column	m_y	$m_y=roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_x m_x$	3×70=210

Appendix B. Program for Optimization of Rebar Stock Procurement and Cutting Plan

The optimization process is programmed in the Python language (Python 3.5) by calling optimization functional boxes in CPLEX (CPLEX 12.61). CPLEX is an optimization software package developed by IBM for solving integer programming problems, convex and non-convex quadratic programming problems, and convex quadratically constrained problems. Codes were written in Python 3.5 environment as below:

Variables	Description
L	rebar stock alternatives
a	length of short side of slab
b	length of long side of slab
c	rebar spacing
d	rebar diameter
e	Lapping length
wt	waste tolerance

Import Functional Box

```
import networkx as Nx
import numpy as np
import matplotlib.pylab as plt
import cplex
import math
```

Initializing

```
L = [20, 30, 40]
a = 55
b = 70
c = 1
d = 0.05
```

```
e = 40d
wt = 0.15(a^2+b^2)
m1 = math.ceil(b / c) + 1             # number of row along short side
m2 = math.ceil(a / c) + 1             # number of row along long side
optimal_value = []
```

Integer Programming

```
for nx in range(1, 9):
        x = ((nx - 1) * e * d + a) / nx
        if x > max(L) or x < 2:
              continue
        for ny in range(1, 9):
              y = ((ny - 1) * e * d + b) / ny
              if y > max(L) or y < 2:
                     continue
                r = []
                s = []
               w = []
               c = []
                t = []
               ll = []
           current_type = 0
           for l in L:
                         nx_max = math.floor(l / x)
                    for n_x in range(nx_max):
                                r.append(n_x)
                                n_y = math.floor((l - x * n_x) / y)
                                s.append(n_y)
```

```
                  w.append(l - n_x * x - n_y * y)
                  t.append(current_type)
                  c.append(n_x + n_y)
                  ll.append(l)
            current_type += 1
           ind = np.argsort(w)
       ub = [1000] * len(r)
          demand_x = nx * m1
          demand_y = ny * m2
          number_of_patterns = len(r)
          model = cplex.Cplex()
    model.variables.add(lb=[0] * len(r), ub=ub, types=[model.variables.type.integer] *
number_of_patterns)
    model.variables.add(types=[model.variables.type.binary] * number_of_patterns
    obj = [0 for i in range(number_of_patterns * 2)]
    for i in range(number_of_patterns):
    model.objective.set_linear([(i, obj[i]) for i in range(number_of_patterns * 2)])
    model.linear_constraints.add(lin_expr=[
      cplex.SparsePair(ind=[i for i in range(number_of_patterns)],
              val=[r[i] for i in range(number_of_patterns)])],
      senses="G", rhs=[demand_x])
    model.linear_constraints.add(lin_expr=[
      cplex.SparsePair(ind=[i for i in range(number_of_patterns)],
              val=[s[i] for i in range(number_of_patterns)])],
      senses="G", rhs=[demand_y])
    for i in range(number_of_patterns):
      model.linear_constraints.add(lin_expr=[cplex.SparsePair(ind=[i, i +
number_of_patterns], val=[-1, M])],
                    senses="G",
```

```
                        rhs=[0])
model.linear_constraints.add(lin_expr=[
    cplex.SparsePair(ind=[i + number_of_patterns for i in range(number_of_patterns)],
                    val=[1] * number_of_patterns)],
    senses="L", rhs=[U])
model.parameters.mip.tolerances.mipgap.set(4e-6)
model.parameters.mip.display.set(0)
model.solve()
if model.solution.get_status() == 101 or model.solution.get_status() == 102:
    print ("nx ",nx,"ny ",ny)
    ww = 0
    z = model.solution.get_values(0, number_of_patterns - 1)
    for i in range(len(z)):
        if z[i] > 0.5:
            ww += w[i] * z[i]
            print ("l", ll[i], "r", r[i], "s", s[i], "z", z[i])
    print ("waste", ww)
```

Appendix C. Rebar layout arrangement plan detailed calculations for base case scenario (24 plans)

Plan 1

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	3
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(3-1)+70]/3=24.67
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	3×57=171
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	2
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(2-1)+57]/2=29.5
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	2×70=140

Plan 2

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	4
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(4-1)+70]/4=19
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	4×57=228
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	2
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(2-1)+57]/2=29.5
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	2×70=140

Plan 3

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	8
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(8-1)+70]/8=10.50
Number of row	m_x	$m_x=roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	8×57=456

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	2
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(2-1)+57]/2=29.5
Number of column	m_y	$m_y=roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	2×70=140

Plan 4

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	4
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(4-1)+70]/4=19
Number of row	m_x	$m_x=roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	4×57=228

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	3
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(3-1)+57]/3=20.33
Number of column	m_y	$m_y=roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	3×70=210

Plan 5

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	5
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(5-1)+70]/5=15.6
Number of row	m_x	$m_x=\mathit{roundup}[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	5×57=285
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	3
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(3-1)+57]/3=20.33
Number of column	m_y	$m_y=\mathit{roundup}[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	3×70=210

Plan 6

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	6
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(6-1)+70]/6=13.33
Number of row	m_x	$m_x=\mathit{roundup}[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	6×57=342
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	4
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(4-1)+57]/4=15.75
Number of column	m_y	$m_y=\mathit{roundup}[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	4×70= 280

Plan 7

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	7
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(7-1)+70]/7=11.71
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	7×57=399

Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	4
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(4-1)+57]/4=15.75
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	4×70= 280

Plan 8

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	8
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(8-1)+70]/8=10.50
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	8×57=456

Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	4
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(4-1)+57]/4=15.75
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	4×70= 280

Plan 9

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	3
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(3-1)+70]/3=24.67
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	3×57=171

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	5
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(5-1)+57]/5=13
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	5×70= 350

Plan 10

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	4
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(4-1)+70]/4=19
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	4×57=228

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	5
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(5-1)+57]/45=13
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	5×70= 350

Plan 11

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	5
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(5-1)+70]/5=15.6
Number of row	m_x	$m_x=roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	5×57=285

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	5
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(5-1)+57]/5=13
Number of column	m_y	$m_y=roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	5×70= 350

Plan 12

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	6
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(6-1)+70]/6=13.33
Number of row	m_x	$m_x=roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	6×57=342

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	5
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(5-1)+57]/5=13
Number of column	m_y	$m_y=roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	5×70= 350

Plan 13

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	7
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(7-1)+70]/7=11.71
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	7×57=399
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	5
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(5-1)+57]/5=13
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	5×70= 350

Plan 14

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	4
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(4-1)+70]/4=19
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	4×57=228
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	6
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(6-1)+57]/6=11.17
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	6×70= 420

Plan 15

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	5
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(5-1)+70]/5=15.6
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	5×57=285

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	6
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(6-1)+57]/6=11.17
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	6×70= 420

Plan 16

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	6
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(6-1)+70]/6=13.33
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	6×57=342

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	6
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(6-1)+57]/6=11.17
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	6×70= 420

Plan 17

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	3
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(3-1)+70]/3=24.67
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	3×57=171

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	7
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(7-1)+57]/7=9.86
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	7×70= 490

Plan 18

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	4
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(4-1)+70]/4=19
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	4×57=228

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	7
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(7-1)+57]/7=9.86
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70

Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	7×70= 490

Plan 19

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	6
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(6-1)+70]/6=13.33
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	6×57=342

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	7
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(7-1)+57]/7=9.86
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	7×70= 490

Plan 20

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	7
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(7-1)+70]/7=11.71
Number of row	m_x	$m_x = roundup[(b - 2h) / c] + 1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	7×57=399

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	7
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(7-1)+57]/7=9.86

Number of column	m_y	$m_y = roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	7×70= 490

Plan 21

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	4
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(4-1)+70]/4=19
Number of row	m_x	$m_x = roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	4×57=228

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	8
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(8-1)+57]/8=8.875
Number of column	m_y	$m_y = roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	8×70= 560

Plan 22

Horizontal Layout

Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	6
rebar length	x	$x=[40d\times(n_x-1)+a]/n_x$	[40×0.05×(6-1)+70]/6=13.33
Number of row	m_x	$m_x = roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	6×57=342

Vertical Layout

Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	8
rebar length	y	$y=[40d\times(n_y-1)+b]/n_y$	[40×0.05×(8-1)+57]/8=8.875

Number of column	m_y	$m_y = roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	8×70= 560

Plan 23

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	7
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(7-1)+70]/7=11.71
Number of row	m_x	$m_x = roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	7×57=399
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	8
rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(8-1)+57]/8=8.875
Number of column	m_y	$m_y = roundup[(a-2h)/c]+1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	8×70= 560

Plan 24

Horizontal Layout			
Description	Notation	Equation	Calculation Results
Rebar quantity in each row	n_x	-	8
rebar length	x	$x = [40d \times (n_x - 1) + a] / n_x$	[40×0.05×(8-1)+70]/8=10.50
Number of row	m_x	$m_x = roundup[(b-2h)/c]+1$	Roundup[(57-2×0.5)/1]+1=57
Qty of rebar piece needed	$n_x m_x$	$n_x m_x$	8×57=456
Vertical Layout			
Description	Notation	Equation	Calculation Results
Rebar number in each row	n_y	-	8

rebar length	y	$y = [40d \times (n_y - 1) + b] / n_y$	[40×0.05×(8-1)+57]/8=8.875
Number of column	m_y	$m_y = roundup[(a - 2h) / c] + 1$	Roundup[(70-2×0.5)/1]+1=70
Qty of rebar piece needed	$n_y m_y$	$n_y m_y$	8×70= 560