ANALYSIS OF
STEEL PLATE SHEAR WALLS

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Abstract

In the last 15 years a few highrise structures have been successfully completed which employ steel walls to resist lateral loads. These shear walls are formed from thin steel plates which in turn are framed by the beams and columns of the structural system. However, uncertainty still exists as to how the walls should be designed, as very little technical information has been published dealing with suitable methods of analysis. A study was therefore undertaken to review the existing steel shear wall systems and to develop an analytical technique suitable for studying the force transfer in a storey-size steel panel subject to shear.

The existing steel shear cores generally employ a series of heavily-reinforced steel panels proportioned to ensure that buckling does not occur in the working load range. This approach is overly conservative for panels not subject to cyclic loads, as it completely neglects the post-buckling strength of a steel panel. An analytical theory which recognizes the contribution of the post-buckling strength of a web to the overall shear resistance has been developed in this report.

A model was developed to represent analytically the resistance provided by the tension zone which arises in a buckled web. The tension zone in the model was represented as a series of inclined truss members, orientated at the same inclination as the diagonal tension stress. Using this

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model, a series of parametric computer studies were done to examine the influence of various factors on the strength and stiffness characteristics of a shear wall panel. The panel stiffness was found to be uniquely related to panel height, panel length, web thickness, and column stiffness. The stress distribution is not uniform throughout the web and is primarily a function of the column stiffness. The study also showed that the stiffness limits, and not the strength, will usually govern the design of a shear wall panel.

The conclusions drawn from the analytical study should be compared with experimental studies of shear wall webs. Further research is needed to develop a procedure to predict the stress distribution in a given web.

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A . . . . Area of diagonal of Pratt truss having
lateral stiffness equivalent to that of
a shear wall web

A_b . . . . Beam area

A_c . . . . Column area

A_w . . . . Web area

E . . . . Modulus of elasticity

h . . . . Panel height

H . . . . Total height of structure

I_c . . . . Column moment of inertia

L . . . . Panel length

s . . . . Width of Basler's tension field

V . . . . Applied shearing force

w . . . . Plate thickness of shear wall web

W . . . . Total work done

α . . . . Angle of inclination of the diagonal
tension

Δ . . . . Storey displacement

ϵ . . . . Strain

ϕ . . . . $\tan^{-1} L / h$

ρ . . . . Member extension due to an applied load

σ . . . . Stress

σ_b . . . . Stress in beam

σ_c . . . . Stress in column

σ_v . . . . Shearing stress

σ_w . . . . Stress in web

θ.................Angle of inclination of Basler's tension

field for plate girders

$\int_v$ ....Integrate over the volume

$\sum$ ....Summation

1. INTRODUCTION

1.1 Introduction

Shear wall systems in highrise buildings have conventionally been built almost exclusively of reinforced concrete. In the last ten years several structures have been successfully completed which break this tradition. These innovative structures employ a series of steel plates, framed by columns and girders, to transfer the lateral loads.

The investigation presented herein deals with the use of steel plates framed by structural beams and columns to form a shear wall system. The main drawback deterring Canadian engineers from using this framing scheme is uncertainty as to how the walls should be analyzed. Very little technical information has been published describing suitable analysis techniques and such design requirements as connection and stiffening details. There have been descriptive articles written about the structures that have been built using steel shear walls but these contain only vague references to methods of analysis. The purpose of the following study is to briefly review the steel shear wall systems currently in use and to develop an analysis technique suitable for studying the transfer of forces in a steel wall subjected to shear.

1.2 Lateral Load Resisting Systems

A building's structural system must be designed to resist two types of loads, gravity loads and lateral loads. Gravity loads include the self-weight of the structure and its contents, while lateral loads result from the action of wind, seismic, and blast forces on the structure. The latter type become of prime importance in the design of structures having a large ratio of height to width, in which case the wind loads are significant, and also for those structures built in areas of high seismic risk.

Each of these load types require different considerations in design. Gravity loads normally dictate the size of structural framing components such as beams and columns. To absorb the lateral loads to which a structure is subjected, resistive elements must be built into the structure in conjunction with the framing system designed to withstand the vertical forces.

Traditionally, one of four lateral load resisting systems has been used in the structural design of buildings. These commonly used systems are:

a. moment-resisting frames

b. braced frames

c. so-called "tubular" structures

d. shear walls

Moment resisting frames have full continuity provided at the beam-to-column connections to resist the lateral loads. These frames usually require very large member sizes

at the lower levels of the building to limit horizontal
deflections. The economy of the rigid framing system
decreases gradually after the structure reaches about 20
storeys.

Braced frames commonly consist of a system of diagonal
members between the columns and beams or a K-brace system,
in which the horizontal member is supported at midspan.
These two types of bracing systems are illustrated in
Fig 1.1. Each forms a vertical truss to carry the lateral
loads. They have proven to be efficient for structures up to
about 40 storeys high. One disadvantage of this system is
the difficulty of penetrating the braced walls, as required
for mechanical services and doorways. Control of drift may
also be a problem.

For buildings over about 40 storeys in height, the
optimum bracing system consists of a series of closely
spaced perimeter columns that act together as a cantilevered
box.

Shear wall systems are the most common form of lateral
bracing used for structures in Canada having about 15 to 40
storeys. A shear wall system consists of a series of plane
walls, often surrounding an interior service area, to form a
central core. Lateral forces acting on the outside walls of
a building are normally transferred through the floor,
acting as a horizontal diaphragm, to the shear core. The
loads are then delivered to the ground through the shear
core.

The shear core of a highrise building subjected to wind loads can be idealized as a vertical cantilever beam loaded uniformly, as shown in Fig 1.2. Structural engineers are familiar with the concept that a member which carries load perpendicular to its longitudinal axis will do so most efficiently when the largest proportion of the cross-section is concentrated at the extreme fibre of the section. Thus, it would be reasonable to expect that the shear core in a building would act most efficiently when a large proportion of its mass was placed at the extremities. Based on this conclusion, shear cores built up from thin steel plates, with the building's columns and beams acting as flanges and stiffners, respectively, seems to be an attractive system to investigate for use in highrise buildings.

A shear core constructed of steel plates has certain advantages over more conventional lateral load resisting systems. The following is a brief look at some of the factors that are important to a structural designer considering the use of a steel shear wall system.

Comparing the steel shear walls first to a moment resisting frame, the total steel cost can be substantially reduced using the former system. Steel savings have been cut by as much as 50% in structures employing a steel shear wall system rather than a comparable moment-resisting frame (1).

When the alternative is a reinforced concrete shear core, the steel system offers reduced foundation costs, as it is a much lighter system. This feature makes steel plate

particularly adaptable for upgrading the lateral load resistance of an existing building while not overstressing its foundation. Reinforced concrete shear walls for use in seismic zones must be heavily reinforced and, even then, in the event of strong cyclic loading, their ductility and capacity to absorb energy is limited. In addition, due to their high stiffness, reinforced concrete walls designed by a static analysis can occasionally result in inadequate sizes when compared to a dynamic analysis (2). The strength and ductility of steel plates make them a suitable alternative for resisting cyclic loads. For this application, the steel plates should be stiffened to prevent buckling.

Increased speed of erection is another advantage of steel plate shear walls. Connection angles or fish plates can be shop-welded to the columns and beams framing the walls and then bolted or welded to the steel plates in the field. Elimination of trade interdependence is another bonus of the all-steel system. No longer does the rate of construction of a concrete core control the rate of construction of the steel frame. Greater flexibility of the core arrangement can also be achieved through use of a steel core.

A feature that is of prime importance to the building's owner is the increase in usable floor space that results from the use of steel shear walls. Concrete walls or steel bracing would often result in much thicker walls to provide

the required strength.

Steel shear cores also have some disadvantages which preclude their use under certain circumstances. These factors must be considered when judging the system's suitability for a particular project and include:

a. possible vibration problems, particularly if the wall encloses an elevator system.

b. fireproofing requirements.

1.3 Steel Shear Wall Systems Currently in Use

In the last ten years a significant number of structures have been designed employing steel plate shear walls as the lateral load resisting system. The design for the first structure of this type was started in the Japan in the late 1960's. By the close of the seventies the method had been used in the design of a significant number of structures.

North American reviews have been published documenting four structures in the U.S. and one in Japan that have been built using steel shear walls. There is evidence that many more steel plate shear wall structures have been built in Japan in the last decade, but little information about their design has been published in English. The purpose of the following section is to briefly examine some of the existing buildings which have steel plate shear walls as part of their structural system. These structures cover a wide

variety of structural needs, ranging from a hospital in a very high seismic zone to a 53 storey office building. They will be reviewed paying particular attention to the reasons the designers gave for choosing this particular system to resist lateral loads. In addition, some of the unique features of the analysis, design, and assembly procedure will be discussed, where available, for each structure.

It is believed that the first structure built using a system of steel plates to resist lateral loads was the Shin Nittetsu Building, commonly referred to as the Nippon Steel Building, after the company who owns and occupies it. It was completed in Tokyo, Japan in 1970 (3). A typical floor plan of this 20 storey office tower is shown in Fig. 1.3 (4).

The main factor controlling the choice of steel shear walls for this 20 storey office tower was the owner's desire to create a prototype highrise building which would promote the use of Nippon Steel Co. products in new and promising ways.

Built in an area of high seismic risk, earthquake motions produced the controlling lateral forces for which the structure had to be designed. There were two major structural systems used simultaneously to resist these lateral loads. For a typical floor, as shown in Fig. 1.3, there were 42 perimeter H-columns. These columns were designed to act, in conjunction with the girders, as rigid frames to resist wind in the longitudinal direction. Seismic forces in the transverse direction were to be absorbed by

the five continuous H-shaped steel plate shear walls.

The plate thickness of the steel walls ranged from 4.5 mm, in the longitudinal walls in the upper 10 storeys, to 12 mm, in the lower transverse walls. The 2.75 m x 3.70 m steel plates were stiffened horizontally and vertically with steel channels to prevent shear buckling at loads less than the design load. A typical stiffened panel is detailed in Fig. 1.4. To give the steel panels the required fire rating, the plates were encased with drywall material, which had a maximum thickness of 50 mm.

To calculate the bending strength in the longitudinal direction, the structure was considered to be an earthquake resistant wall composed of five H-shaped beams running the full height of the structure (5). To simplify the analysis, it was assumed that the shear walls did not carry any vertical gravity load. Steel beams were designed to carry all the gravity loads on each floor, in addition to laterally induced axial forces. The model used to analytically describe the shear core can be described as follows. The plates which were orientated longitudinally, (AB, BC, DE, and EF in Fig. 1.5), were replaced with a set of diagonal braces resistant to simultaneous tension and compression. The areas of these braces were calculated to give the same stiffness characteristics as an unbuckled steel plate when subjected to a shearing force. This was done by assuming that the shear deflection of a plate is equivalent to the extension of a single diagonal brace in a

truss having the same dimensions as the plate. The resulting brace model for one of the "flanges" of the H-shaped wall is shown in Fig. 1.6.

A computer program, using the stiffness method, was used to analyze this model for the member forces and deformations resulting from the applied quasi-static design wind and earthquake loads. In addition, a dynamic analysis of the structure's elastic and elasto-plastic response to four earthquake time histories was carried out (6). The structure was designed on the basis of these analyses, ensuring that its deformations were within the given tolerances and that the material behaviour remained in the elastic range. Experiments were done on scaled models of steel shear walls at the University of Tokyo to test the assumptions on which the design was based (4).

Japan's second structure employing steel shear walls in the core was the Shinjuku Nomura Tower. Work on this 53 storey office tower began in 1975, five years after completion of the Nippon Steel Building. The designers decided to use steel walls because a patent conflict arose over the right to use reinforced concrete walls in Tokyo highrises and because the cost of a steel braced core was found to be prohibitive (7).

The structural steel frame of this highrise has three major components; a main frame resisting longitudinal lateral forces, a sub-frame to support vertical loads, and steel shear walls in the core area to resist horizontal

loads in the short direction. A typical floor plan is given in Fig. 1.7 (7). The 32 square box columns are rigidly framed to large girders to form the main frame. Sixty-four wide flange columns, joined to smaller girders with simple shear connections, resist the gravity floor loads. the eight T-shaped continuous steel shear walls in the core area, surrounding elevators, stairways, and the mechanical shaft, provide transverse lateral resistance.

Steel plates, 3 m by 5 m, were bolted to the steel framing members to form the continuous shear wall. Plate thicknesses ranged from 6 to 12 mm. In Fig. 1.8, a plate-to-column connection detail is shown. The number of high-strength bolts per panel varied from 200 to 500 depending on the load in the panel. The connection plates were shop-welded to the columns and field-bolted to the wall panel. This erection process required very precise drilling and alignment techniques and created many delays in the field. Both the contractor and the designer agreed that a completely welded connection would have been a better alternative. Despite this shortcoming, frame erection was rapid, requiring less than 3.5 days per floor, on average. To achieve the recommended fire rating, sprayed-on fire protection was applied to the plates. A wire mesh was first attached to the panels to provide an anchor for the sprayed on material. The result was a shear wall made of steel plate, as thin as 6mm, with sprayed-on fire protection up to 50 mm thick.

The analysis of this structure was similar to that for the Nippon Steel Building, i.e., the steel shear walls were modelled with an equivalent strut system having the same force-deformation characteristics.

In about 1976, the steel shear wall concept was used in the design of three structures in the United States. Two of these were hospitals in California, the first to be designed under the very strict building codes that resulted from the destructive San Fernando Earthquake of 1971.

The Olive View Hospital, just outside Los Angeles, was one of these first hospitals to use steel shear walls. It was built on the same site as a former hospital that was destroyed by seismic forces in 1971. New structures at this site had to be designed for ground accelerations up to 0.69g (8). To resist such large forces, reinforced concrete shear walls would have to be extremely thick, requiring elaborate reinforcing layouts around windows and other openings. Such thicknesses would have taken valuable floor space. As well, concrete walls would have resulted in a greater mass than steel walls having equivalent strength. Since the seismic force to which a structure is subjected, for a given ground acceleration, is proportional to the structure's mass, a heavier structure must be designed to withstand a larger seismic force than a comparable but lighter structure. Based on these considerations, the designers opted to use steel plate shear walls in the upper four storeys and reinforced concrete walls in the first two stories. The use of the

steel walls accounted for substantial savings in steel weight over the other alternative of using a moment-resisting frame. Although more fabrication was required for the former system, the net result was a more economical design.

The structural system consists of a steel space frame to carry the vertical loads and a combination of steel and reinforced concrete walls to carry the horizontal seismic loads. The roof and floors act as horizontal diaphragms to transfer the siesmic forces to the walls.

The steel walls used in this structure consisted of steel panels one storey high and one bay wide. The top two storeys contained 16 mm steel plate and the lower two had 19 mm plate. The panels were stiffened horizontally and vertically with steel channel sections (9). These channels were joined to the plate with toe welds and were located around the window openings. Channels were also shop-welded to each side of the plate at the floor line for reinforcing. The inside channel also carried the metal deck for each floor.

On site, the panels were connected to the framing columns and to each other with high-strength bolts. At their bases, the walls were bolted to a 760 mm deep steel girder which rested on the 360 mm reinforced concrete walls used to stiffen the lower two storeys of the building.

The same structural engineering firm that was responsible for the design of the Olive View Hospital was,

at the same time, involved in another project employing steel shear walls. This was the 30 storey Dallas Hyatt Regency Hotel, completed in Dallas, Texas in 1978 (1).

The lateral load resisting system in this highrise consisted of steel braced frames in the long direction and steel plate shear walls, which were also bearing walls, in the short direction. Since these walls took some of the vertical loads, as well as the lateral loads, steel was saved by reducing the column and beam dimensions. The diagonal bracing system was not used in the short direction because it would have resulted in the loss of considerable interior space. The cross-braced frame system was feasible, however, for wind in the opposite direction where it could be encased by corridor walls (10). Diagonal bracing was also used in the lower two storeys to allow for convenience in providing architectural openings. Use of a moment resisting frame was rejected by the designer as it would have used more steel and very large member sizes. Concrete shear walls would have added to the construction time of this predominately steel structure and, because of the structure's complex shape, it would have been difficult to use a concrete frame.

The walls were built up from ASTM A36 steel plates having thicknesses from 13 to 29 mm. Depending on the stress in the wall, two or three T-sections were welded to the wall to act as stiffeners to resist buckling. The panels were further strengthened against torsion by welding steel

channel sections to each plate below the floor line (1).

The engineering consultant responsible for the structural design described the shear wall system as a "vertical plate girder" and compared it to the use of steel plates in the ship building industry to form steel bulkheads (11). The same stiffened plate concepts as are used in ship design were used to design the plates subject to shearing forces in the Olive View Hospital and the Dallas Hyatt Regency Hotel. References on plate stability derive methods of calculating the buckling stress and the ultimate stress of plates subjected to shear (12,13). Plates having longitudinal and transverse stiffeners are also discussed. Although the designers do not state it directly, it is assumed that the steel plates are proportioned so that under service loads the stresses in the plate will be less than the buckling stress. Most stress and deflection calculations for the walls were done by hand using these plate buckling theories, eliminating the need for a finite element analysis. The computer program SAPIV provided a computer check of the calculations (14). The shear wall system was modelled for this computer analysis using plane stress elements for the steel plates and beam elements for the columns, beams, and stiffeners. Under the top design wind loads, the 82 m high tower deflected only 200 mm.

The first highrise hospital to be built under California's strict 1973 building code was the Moffitt Hospital (15). Work on this 16 storey structure commenced in

1977 in San Francisco, at a site just four miles away from the epicentre of the disastrous 1971 earthquake.

The designer decided to use steel plate walls rather than reinforced concrete in this instance because the latter would have had to be up to 1.2 m thick to resist the strong seismic forces, pre-empting too much floor space. Acting in conjunction with the main framing system of five steel shear walls, arranged as shown in Fig. 1.9, there is a reinforced concrete shear wall. This wall was placed around the elevator core where a steel wall could possibly have led to vibration problems. These two components were designed to resist 100% of the lateral forces. The structure had an additional safety feature built in; a ductile steel moment-resisting frame. This frame was able to resist up to 25% of the applied lateral load.

The contractor erected the steel panels, which ranged in thickness from 10 mm to 32 mm, in two-storey tiers (16). In the field, the panels were first connected to the framing members with high-strength bolts. After final alignment, the panels were fillet welded to the steel columns and plate girders.

To give the walls the required rigidity, and to meet fireproofing requirements, the steel panels were covered on both sides with 250 mm of reinforced concrete. Steel reinforcing ties were spaced 610 mm centre-to-centre and were continuous through both the steel and concrete. Due to difficulties that would have arisen trying to construct

forms for these concrete additions, shotcrete was used.

Although no details are given concerning the analysis procedure, the walls were most likely designed on the basis that the actual shear stress must be kept less than the shear buckling stress, in a similar manner to that used for the plates stiffened with steel sections.

The most recent use of steel shear walls in the United States was in upgrading an existing seven storey hospital in Fresno, California to meet the most recent seismic codes (17). The designers used a combination of steel bracing, steel plate and reinforced concrete shear walls to increase the structure's resistance to lateral loads by five times. The controlling factor in the decision to include a system of steel walls in this instance was the considerable savings in weight that could be realized over using only reinforced concrete walls. Choosing the lightest alternative was extremely important for this structure because the designers wanted to avoid overloading the existing spread footings or overstressing the floor slabs.

A random pattern of stiffening elements was developed, by a trial-and-error procedure, through a series of three-dimensional computer analyses. An important consideration in the placing of the bracing elements was the need to keep critical hospital functions in operation. Steel K-bracing was the most common form of bracing used, but where the position of openings eliminated its use, steel plates were used. These plates were typically 10 mm thick

and stiffened by attaching 4-gage, 115 mm deep steel decking. Over doorway openings 20 mm thick plates replaced the thinner plate used elsewhere. Reinforced concrete walls, 280 mm thick, were used only to brace the elevator core.

1.4 Current Design Philosophy

It appears that most of the research that has been done to date on steel shear wall systems, both analytically and experimentally, can be credited to the Japanese. The results of these studies are primarily available only in Japanese, limiting their use as a resource to North American engineers. One of the few exceptions is a paper describing an experimental program conducted on steel shear walls by the research division of a large Japanese contracting firm in the early 1970's (2). This research established the basic principles for the method of analysis that has been used, almost exclusively, for the stiffened plates used in existing steel shear wall systems. A brief look at the program, and the conclusions drawn from it, will provide insight into the current design approach.

The aim of this research was to test the response of a stiffened plate to alternating horizontal loads, a major concern in Japanese structures, since Japan is located in an active seismic zone. The testing was carried out in two phases. First, twelve panels, 2100 mm by 900 mm, having various plate thicknesses and different stiffener sizes and

arrangements, were tested. All used boundaries (the columns and beams in a real structure) that could be characterized as infinitely stiff. From the results of these tests, the strength and stiffness characteristics of steel shear walls subjected to several load reversals were studied and the following conclusions drawn:

a. Steel plates having stiffeners on both sides exhibit more stable behaviour than those reinforced on one side only.

b. The stiffener arrangement did not affect the shape of the hysteresis loop.

c. The plate thicknesses and stiffener sizes that were tested all proved to provide sufficient ductility.

The test results also provided the basis for establishing design criteria for the stiffeners.

Utilizing the experience gained from the first series of tests, stiffened steel shear walls for a 32 storey building were designed. Two full-scale models of one-bay, two-storey sections of this shear core were tested in the laboratory as the second phase of the experimental program. One of these specimens had openings in the panel and the other one did not.

The test results from phase 1 provided the basis for designing the stiffeners in these panels. They were proportioned so that the shear walls would not buckle until the yield stress of the material had been exceeded. This was achieved by ensuring that the calculated plate buckling

stress was above the proportional limit of the material. The
second desired conditon was that the panel would buckle
between the ribs, with the latter remaining unbent, rather
than overall buckling of the panel and stiffeners together.
To achieve this buckling mode the ribs were designed to be
adequately stiff, based on the theory of plate stability.

The results of phase 2 were compared with an
elasto-plastic finite element analysis, based on the
assumption that the wall does not buckle in the elastic
range. There was good agreement between the experimentally
measured and the theoretically obtained load-deflection
curves. From this, it was concluded that the stiffeners
could be suitably designed on the basis of the conclusions
drawn from the results of the first series of tests, and,
for these stiffened panels, the strength and rigidity could
be calculated by shear theory.

All of the panels, both in the test program and in the
structures discussed in the previous section, use stiffened
steel plates. In each case, it appears that these panels
were designed to resist the applied shearing forces without
buckling under working stress conditions. The critical
buckling stresses were determined from the theory of plate
stability, accounting for the additional rigidity
contributed by the stiffening elements. This approach
neglects completely the benefit of any strength contributed
by the panel after buckling has occurred and bases strength
calculations on shear theory only.

For many years, the post-buckling strength has been recognized as a significant contribution to the overall resistance of a panel to a shearing force. In fact, it has been deemed unduly conservative to overlook this post-buckling strength both in the design of steel plate girders for buildings or bridges and in the design of aluminum shear webs for the aircraft industry. These observations lead to the design approach proposed in the following study; i.e. to use a thin, unstiffened steel panel, for which it is assumed that buckling can take place at working stress levels, and make use of the strength inherent in the subsequent buckled configuration. One fact that should be recognized at the onset of the study is that this method is not intended for use in circumstances where load reversals are significant, such as would occur under earthquake loading. The load-carrying capacity of a buckled plate may not be satisfactory under such load reversal.

As an introduction to this design philosophy for a steel shear wall system, the next chapter briefly examines the origins of this post-buckling strength. Also to be discussed are the theories that have been developed to predict the influence of post-buckling strength on the overall strength of a member subjected to shear.
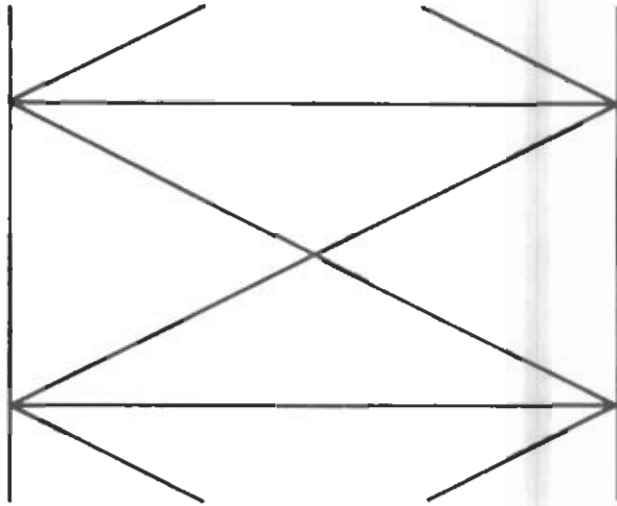
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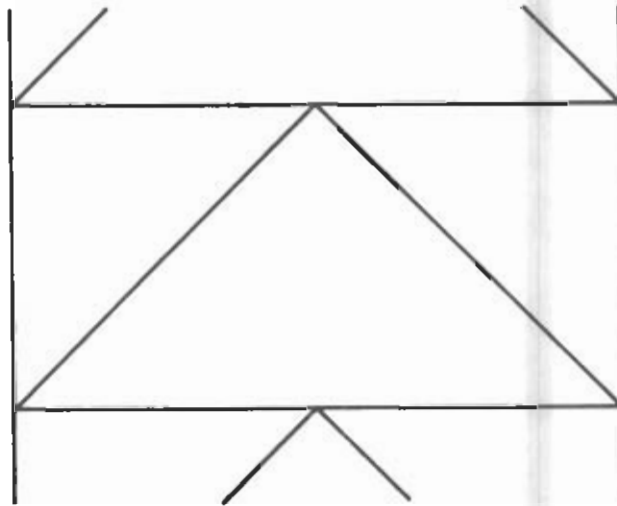
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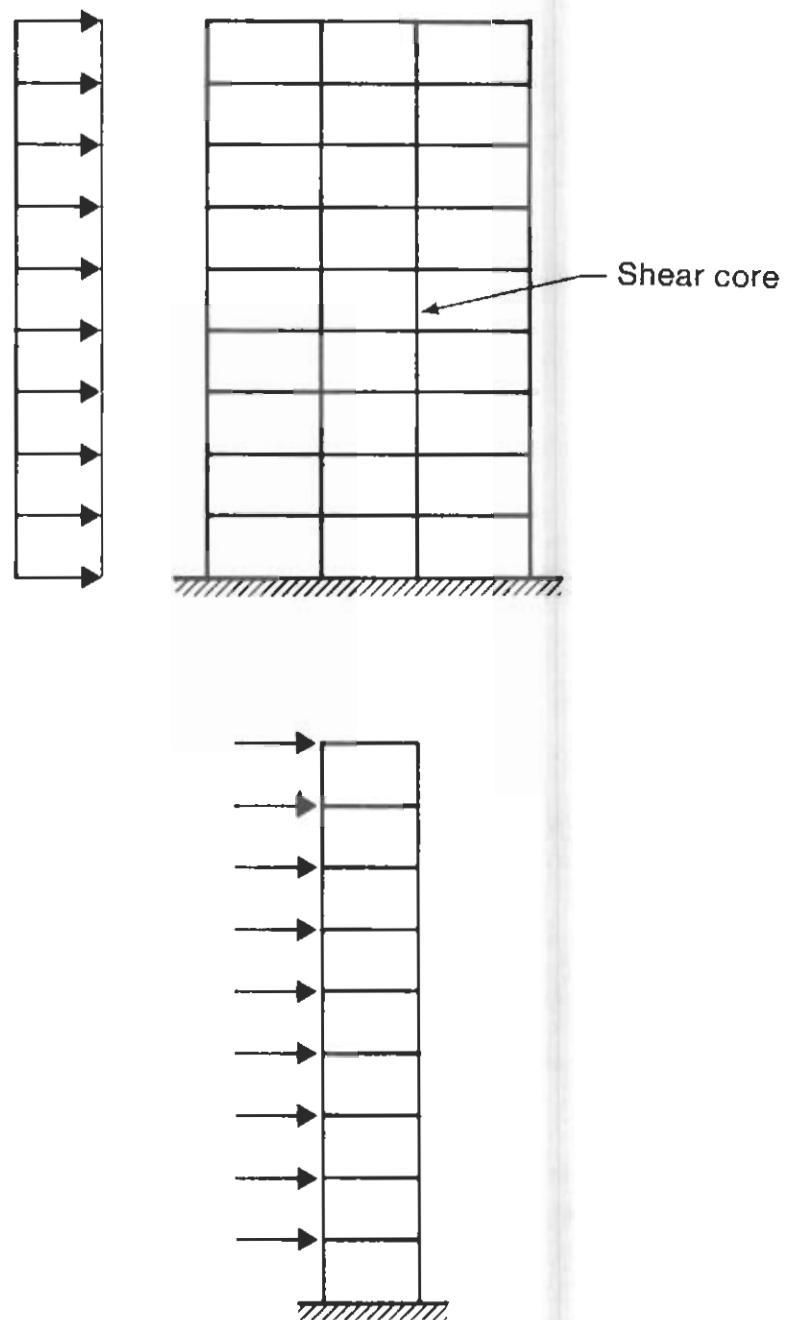
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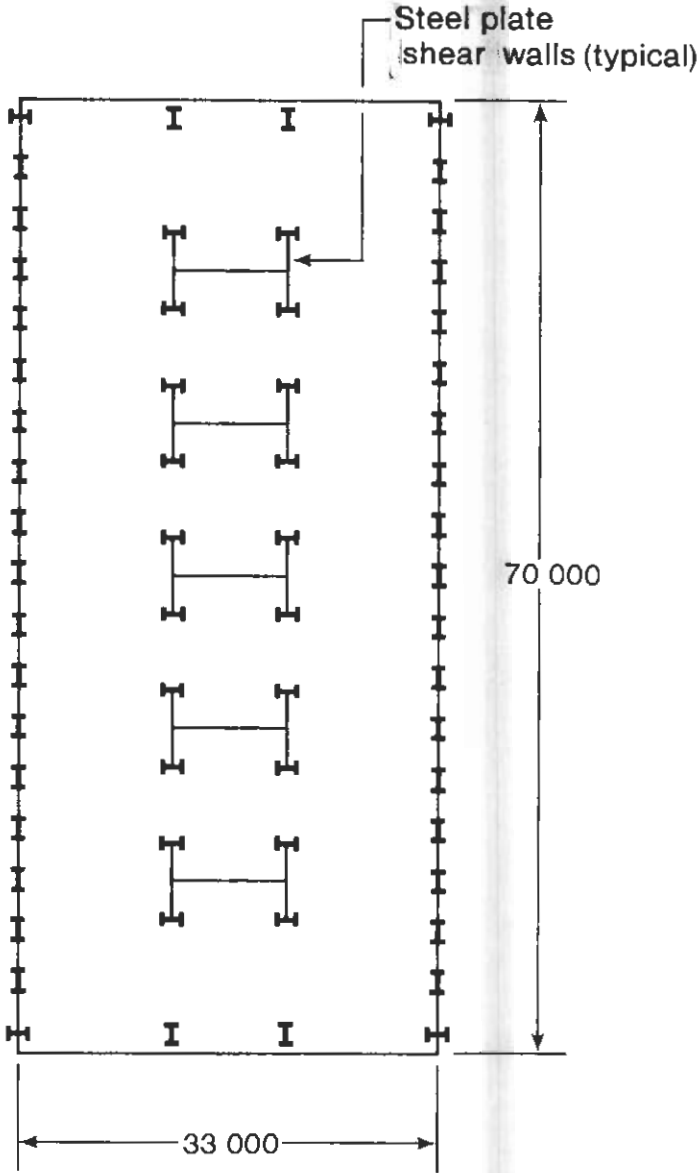
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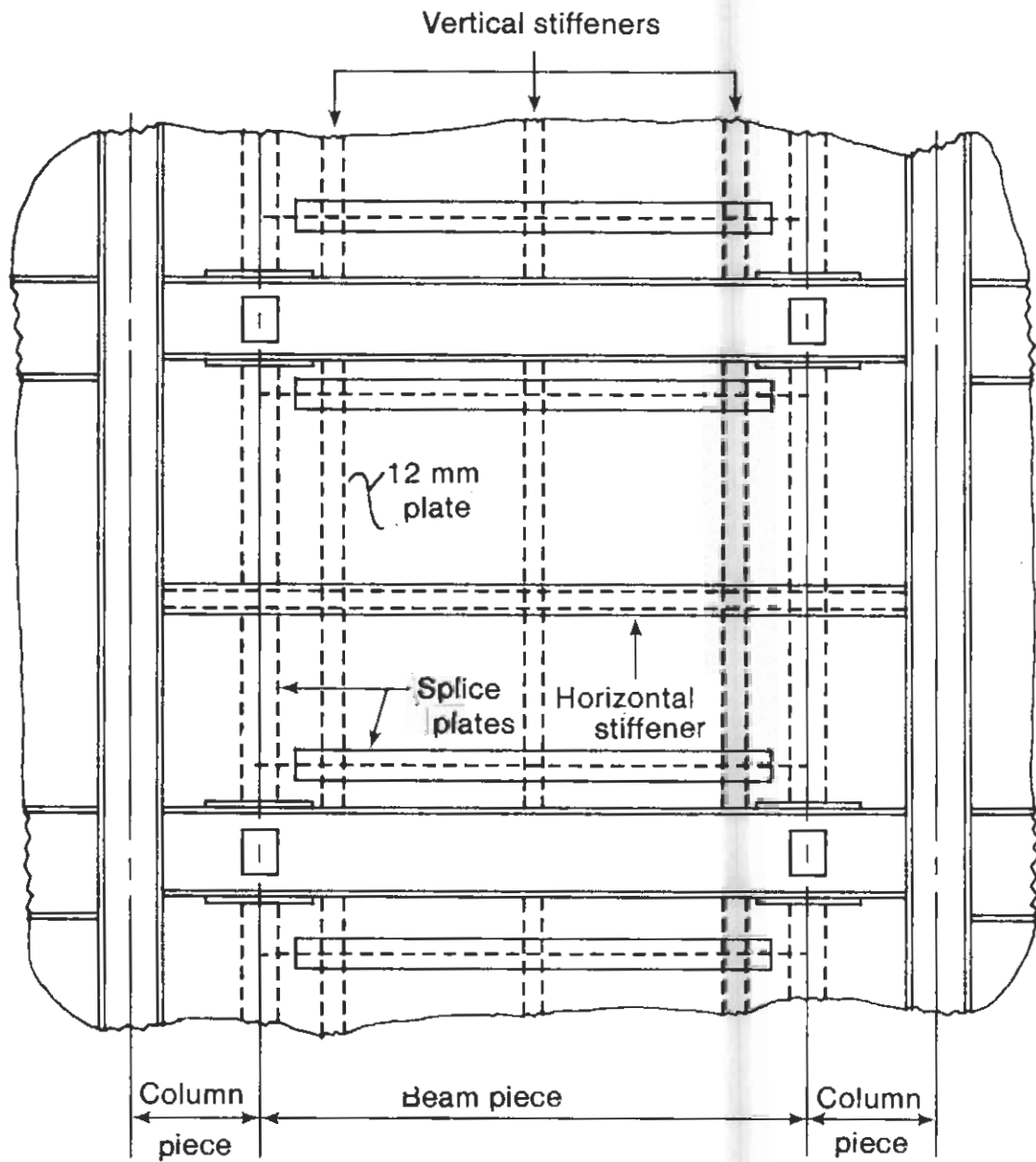
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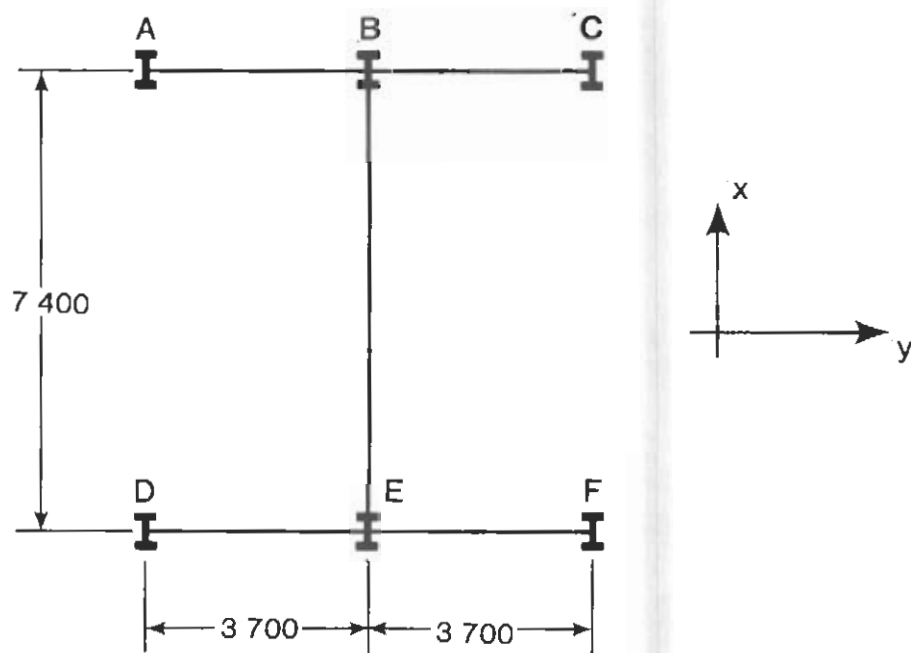
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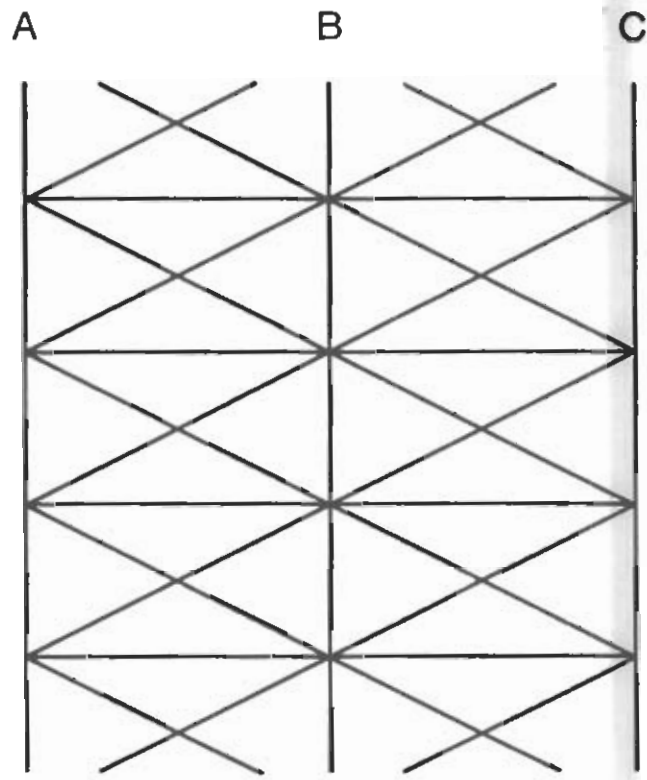
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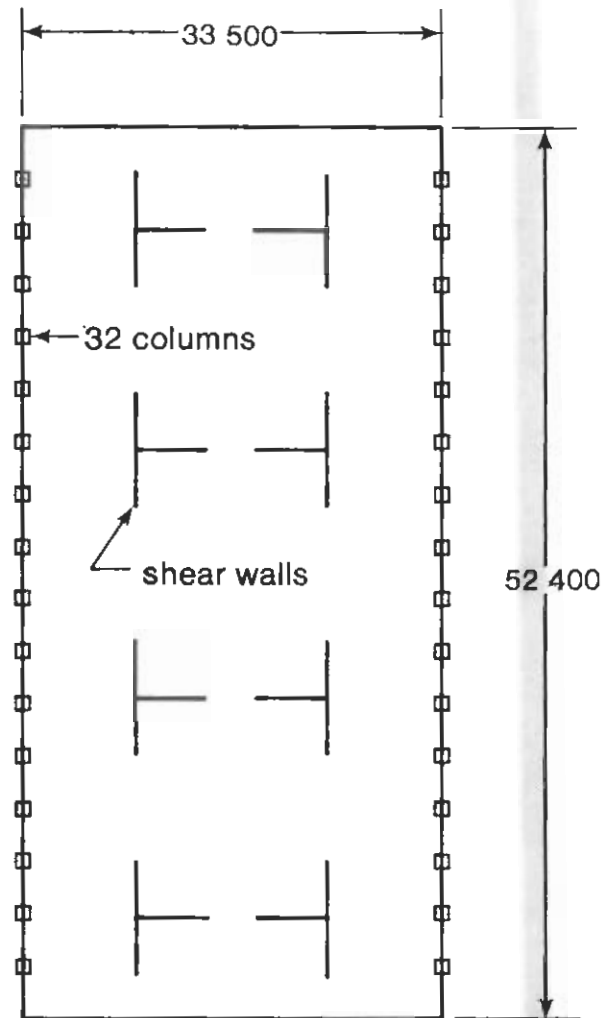
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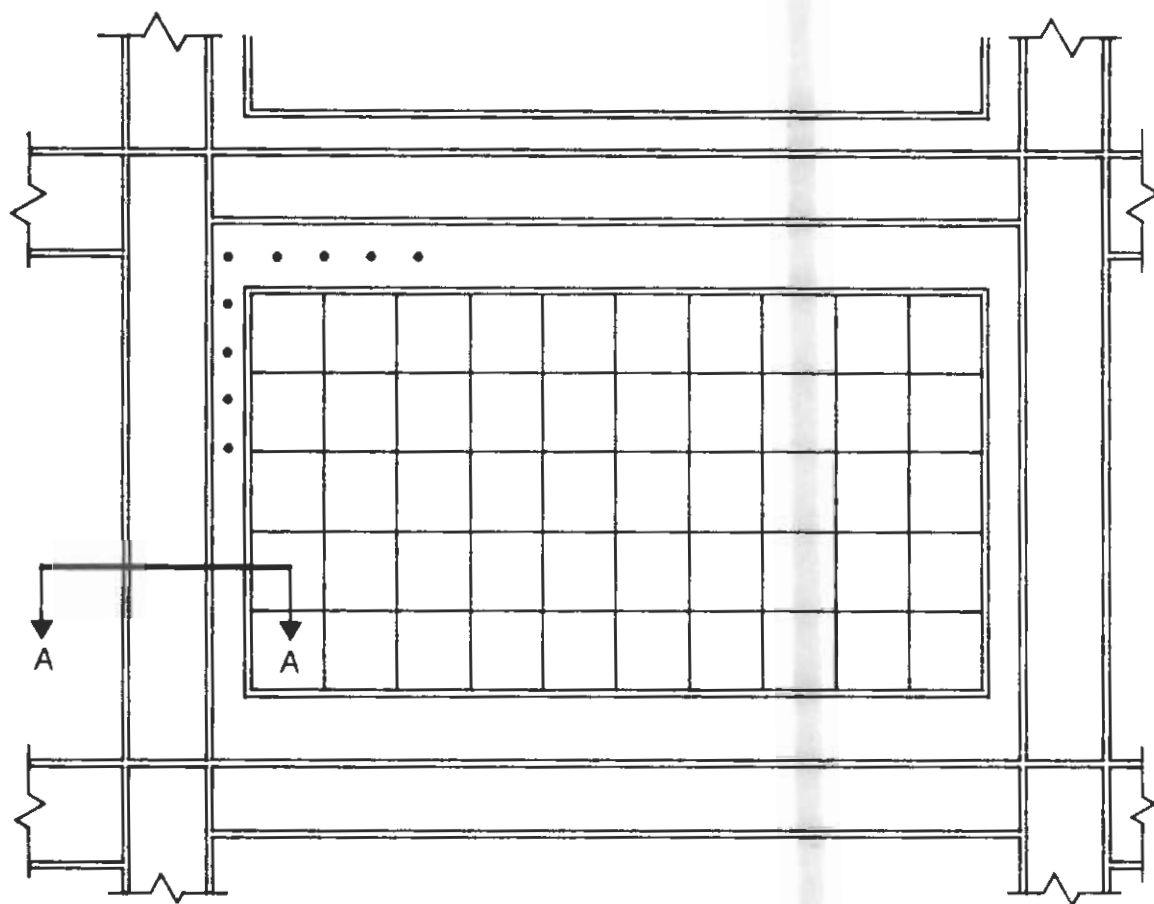
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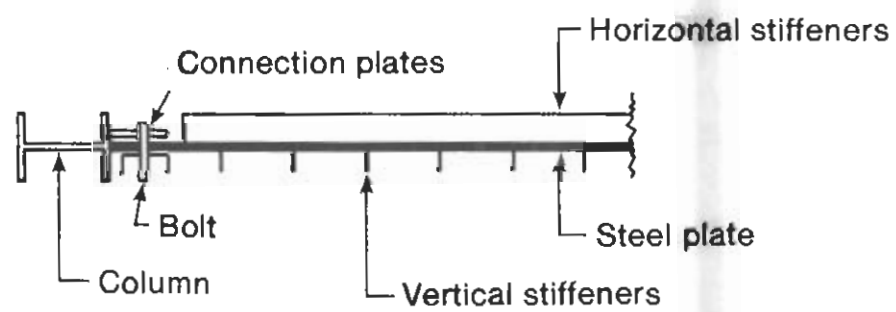
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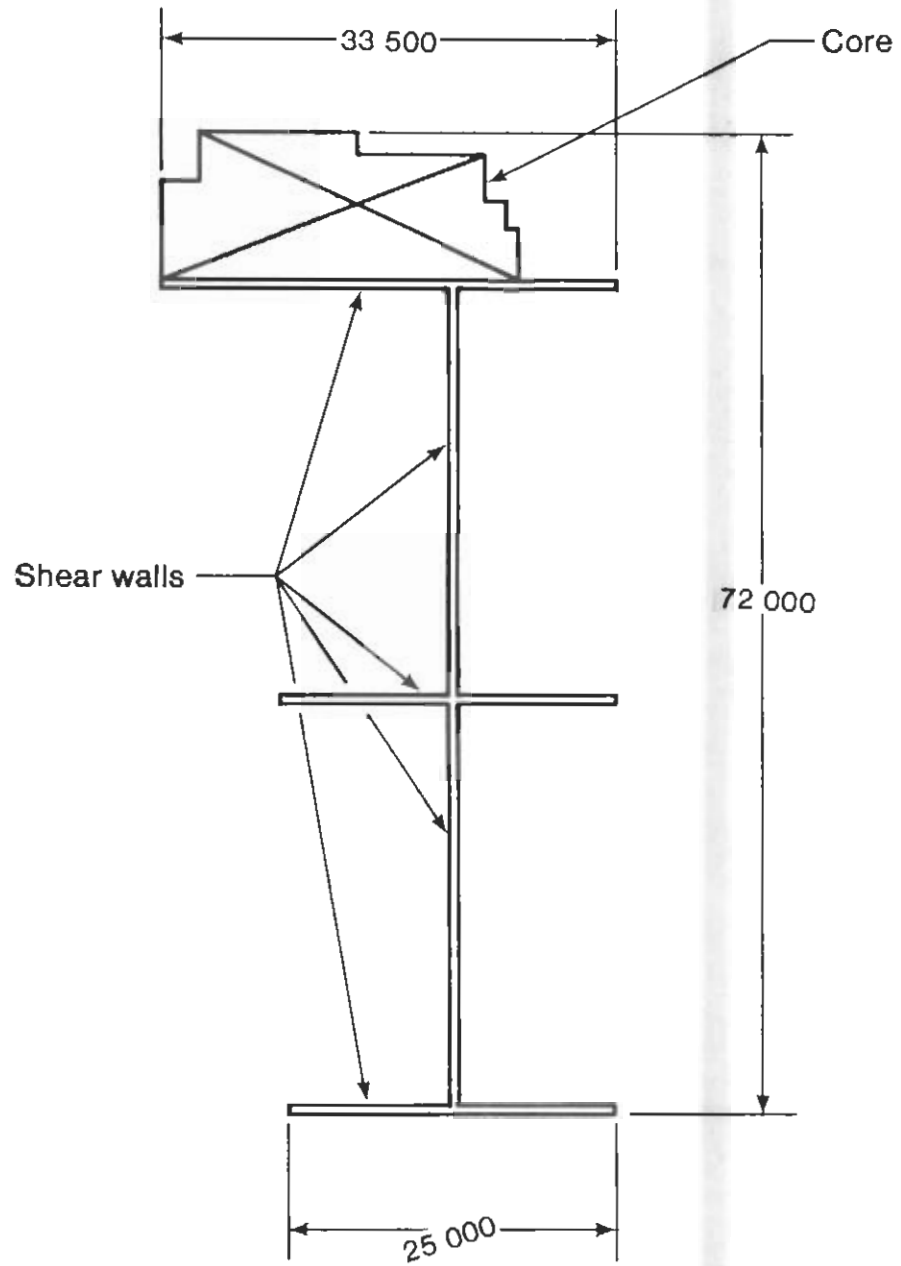
Figure 1.9 Shear Wall Layout in Moffitt Hospital

2. Post-Buckling of Thin Plates Subject to Shear

2.1 Basis of Post-Buckling Shear Strength

For a member subjected to compressive load, there are two distinct criteria normally used to define the failure load; the first is the load which causes the local stresses somewhere on the cross-section to exceed the elastic limit of the material, and the second is that load which causes the member to become unstable, resulting in a premature failure of the member. For very thin members subjected to edge compression, elastic instability commonly occurs before the stresses reach the yield stress. Therefore, in the design of a web plate to resist shear, stability becomes of prime concern.

Elastic instability of plates subjected to shear is marked by out-of-plane movement followed by the formation of inclined waves or buckles. In this case, the instability does not mark the end of the member's usefulness; for although the load-carrying mechanism changes, significant post-buckling strength is available.

Prior to buckling, a web plate subjected to pure shear stresses resists load by so-called "beam-action", defined by the small deformation theory established by Navier and St. Venant (18). In regions where shear predominates, the bending stresses can be neglected and the shear transfer mechanism idealized as shown in Fig. 2.1. When the panel is

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acted upon by a shearing force, V, the resulting shear stresses on any element of the web are σ_v. Because this is a state of pure shear, the shear stresses are equivalent to two principal stresses of equal magnitude, one in tension and one in compression, inclined at 45° to the shear stresses. As the load, V, is increased, the shearing stresses, σ_v, continue to increase correspondingly until the shear buckling stress of the panel has been exceeded. At this point the panel buckles, as evidenced by out-of-plane movement of the plate. The stress at which buckling occurs in a flat plate can be predicted by an equation relating the edge forces to the displacement of the plate (19). This equation takes the form of a fourth-order differential equation, dependent on the support conditions at the plate boundary, the loading conditions, and the plate geometry.

The resisting mechanism developed in the plate is changed by the buckling action. After buckling, the compressive principal stress cannot increase any further. The tensile principal stress, however, is limited only by the yield stress of the material, and it will continue to increase in response to the load after the web has buckled, up to the yield stress level. After the buckling stress has been exceeded, buckles will form in the plate in the direction of the diagonal tension. These folds function as a series of tension diagonals which tend to stabilize the panel, giving it significant post-buckling strength. This shear transfer mechanism, composed of inclined tensile

membrane stresses, is referred to by Wagner as "*tension field action*" (20).

In addition to the primary stress in the web, researchers have identified secondary stresses that occur as a result of post-buckling action (21). Upon development of a tension field in the panel, equilibrium is maintained by a transfer of stress from the tension field to the boundary members. Because the tension field forces create additional reactions in the members bounding the panel, a uniform tension field will develop only if the boundary members are infinitely stiff. The horizontal and vertical components of the inclined web stresses must be completely resolved by the boundary members if the force carrying capability of the tension field is to be fully utilized.

The vertical components of the web stresses act on the horizontal members bounding the panel. If these members are not sufficiently rigid to resist the additional forces, they will bend inward between the uprights that support the plate, as shown in Fig 2.2.a. The tension field forces will be redistributed as a result. If the bending stiffness of the horizontal boundary members is very small, the deflections will be sufficient to relieve the diagonal tension stress in the web that is attached to the horizontals near the middle of the panel. Figure 2.2.b shows this redistribution of web stress by showing tension diagonals only near the uprights.

Similarly, the horizontal components of the tension field cause secondary bending moments in the uprights located at the panel's extremities. Again, if these members are not adequate to resist these additional forces, they will bend into the plate and cause a redistribution of the inclined membrane forces.

As the load increases, the angle of the tension field alters to allow the maximum shear carrying capacity to develop in the panel. The optimum angle of 45°, based on the inclination of the principal stresses in a pure stress field, is seldom achieved, due to the difficulty of providing infinitely rigid supports at the panel boundary. Thus, the post-buckled shear capacity of an actual panel is usually somewhat less than the theoretical maximum.

2.2 Shear Capacity of Aluminum Webs

With the increasing importance of aeronautical engineering in the early 1930's, the need for light membrane shear-supporting structures became apparent. Wagner, in 1931, representing the National Advisory Committee for Aeronautics, was the first to present a theory utilizing the post-buckling strength which develops in thin webs subjected to a shearing force (20). Wagner utilized the fact that a thin web does not "fail" when it buckles; it merely forms inclined folds which serve as a series of tension diagonals, with the stiffeners acting as compression posts. The

web-stiffener system thus behaves as a truss and is capable of carrying loads that may be many times greater than those which cause buckling of the web.

Aluminum alloys, used almost exclusively for aircraft membranes, have a very low modulus of elasticity, and, hence, a low buckling stress. Aluminum alloy webs often buckle at loads considerably less than the design load. This led Wagner to completely neglect the shear resistance of the web before it buckled, assuming the limiting case of a web that forms buckles when subjected to an infinitesimally small load. He called this a *"pure diagonal tension"* web.

Wagner considered a web in pure diagonal tension physically as a series of strips of unit width, inclined at an angle α, and under a uniform stress, σ_v, as shown in Fig. 2.3. A consideration of the internal work done by a panel subjected to a shearing force led to expressions for α, the angle of diagonal tension, the web stress, σ_v, and the stresses in the uprights and the flanges, as a result of the diagonal tension. The derivation and the resulting expressions are detailed in Ref. 21.

As a result of Wagner's work, designers placed a shear web in one of two categories and calculated its ultimate strength accordingly. Webs were either considered as "shear resistant", in which case no buckling was allowed, or they were classified as "pure diagonal tension" webs, completely neglecting the shear capacity of the web prior to buckling. Steel plate girders at that time were in the former category

and aluminum webs were commonly in the latter. Kuhn, et al,
expanded upon Wagner's study of aluminum webs and developed
a theory of "*incomplete diagonal tension*" (21). They studied
the intermediate case of webs falling between the two
extremes of pure diagonal tension and shear resistant webs.
Their work involved empiricism, and was based on the results
of a series of aluminum test webs. Their solution reverted
to a trial-and-error procedure if the flanges were not
infinitely stiff. This limited application of their theory
to specific cases.

2.3 Plate Girder Design

Prior to 1961, the shear buckling stress was taken as
the upper limit of the allowable shear stress when designing
a plate girder web. It was required that intermediate
stiffeners be spaced sufficiently close together so that
shear buckling of the web could not occur before the bending
strength of the section was reached. The shear stresses were
assumed to be resisted completely by beam action, with the
post-buckling shear strength completely neglected. It was
recognized that shear buckling did not really represent the
limit of the girder to carry load. Intuition and the
experience of those in the aircraft industry led engineers
to the conclusion that, after shear buckling had taken
place, a plate girder will also be able to carry additional
load. This will occur in a manner similar to a Pratt truss,

with the web acting analogously to the tension diagonals and the stiffeners acting as the compression posts. This qualitative theory of post-buckling strength was reflected in the use of less conservative factors of safety against web buckling than those required to prevent other stability failures, but the stiffeners were still designed to prevent instability due to shear.

The work done in evaluating the shear-carrying capacity of thin plates for the aeronautics industry created interest in the civil engineering profession to evaluate quantitatively the beneficial effect of membrane forces in the web of plate girders. Certain conditions inherent in the design of a steel plate girder make its design quite different than the design of a thin aluminum web for use in aircraft design. Wagner studied the case of a membrane which buckled almost immediately upon being loaded (20). Thus, he neglected any shear that was carried by beam action and considered the web as a membrane which resisted only tension forces. This hypothesis is more apt for the case of aluminum webs than for steel webs; the former having a much lower modulus of elasticity and, hence, a lower buckling stress than the latter. For steel webs, it is unduly conservative to neglect the shear resistance developed in the web prior to buckling. Also, as mentioned previously, the development of the membrane forces in the web is dependent on the stiffness of the boundary members, the flanges and intermediate stiffeners in the case of a plate girder web.

If the bending stiffness of the flanges is small, the
deflection of the flanges into the web between two
stiffeners is sufficient to relieve the diagonal tension
stress in the areas of the web that are attached to the
flanges, thus preventing complete tension-field action. In
addition, aluminum girders do not generally fail in the same
manner as steel plate girders. Failure of aluminum girder
webs usually occurs when cracks form in the web as a result
of diagonal tension. This type of failure does not usually
occur in steel plate girders because of their greater
ductility.

Basler, in 1961, was the first person to consider the
post-buckling strength of the steel plates normally used in
plate girders (22). Flanges of plate girders, conventionally
designed to carry the girder's bending moment, would be
subjected to a load resulting directly from the tension
field if the post-buckling action was allowed to develop.
The horizontal component of the inclined tension field would
have to be resisted by transverse forces in the flanges.
Basler reasoned that the flanges would have to be
specifically designed to resist this force. To avoid this,
he hypothesized that if no resistance to the membrane action
was offered by a girder flange, that a tension band would
still form, but would be of the form shown in Fig. 2.4.

In Basler's model, the flanges do not anchor the
tension field stresses, but the stiffeners must resolve the
axial forces that are a result of the vertical components of

the inclined tension stresses.

The inclination, θ, and the width, s, of the tension band were chosen to maximize the contribution of the tension field to the web's total shear strength.

From this post-buckling model, Basler developed a theory to evaluate the ultimate shear capacity of a steel plate girder web by superimposing the state of stress up to the point of buckling and the post-buckling benefit. The former stress being transferred by beam action, and the latter contributed by the tension field action.

Basler's work was incorporated into the American and Canadian steel codes for buildings in 1961 and it still forms the basis of stiffened plate girder design in the current codes (23,24). Since 1961, the total shear strength has been considered to consist of both buckling and post-buckling contributions. Stiffeners are not required when the flexural strength of the section can be achieved without diagonal buckling due to shear. Tension field action is expected when stiffeners are used and buckling occurs before shear yielding.

Many variations of the post-buckling tension field theory for plate girders have been developed since Basler's solution was published. The most important contributions are summarized in Ref. 25. The theories all have at least one thing in common; in each, the web's contribution to the panel shear strength was found by superimposing the shear buckling strength and the vertical component of the tension

field. The theories differ basically in the assumptions made
as to the configuration of assumed tension field and the
type of failure mechanism used. In addition, some
researchers attempted to include the influence of flange
flexibility in their models. But, for various reasons,
including limited agreement with test results and the use of
complicated formulas requiring trial-and-error solutions,
the most widely accepted approach to evaluating the ultimate
shear strength of a plate girder has remained that proposed
by Basler in 1961.
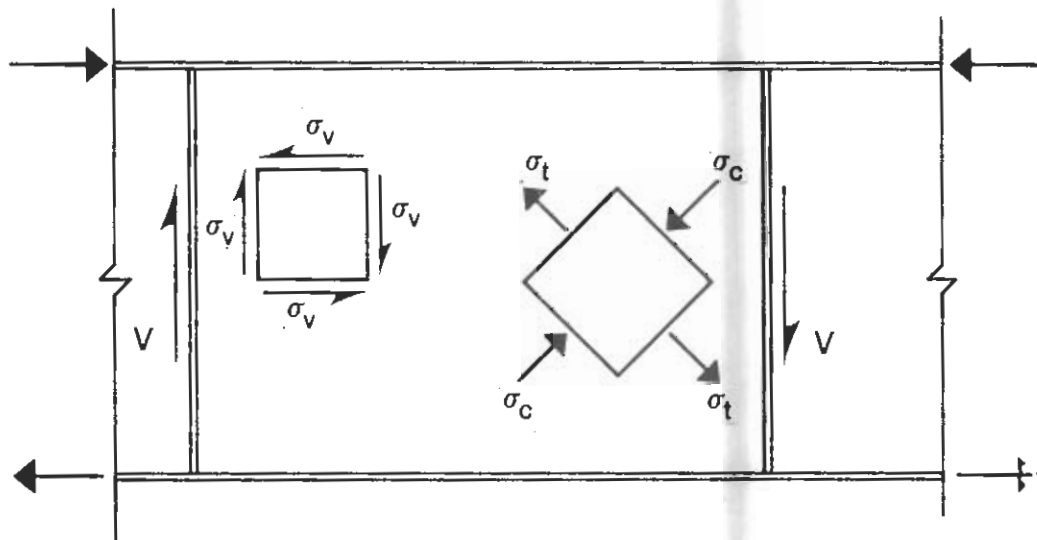
σ_v = shearing stress

$\sigma_c = \sigma_t$ = principal stresses

$$\sigma_v = \sigma_c = \sigma_t$$

Figure 2.1 Shear Transfer Mechanism Before Shear Buckling
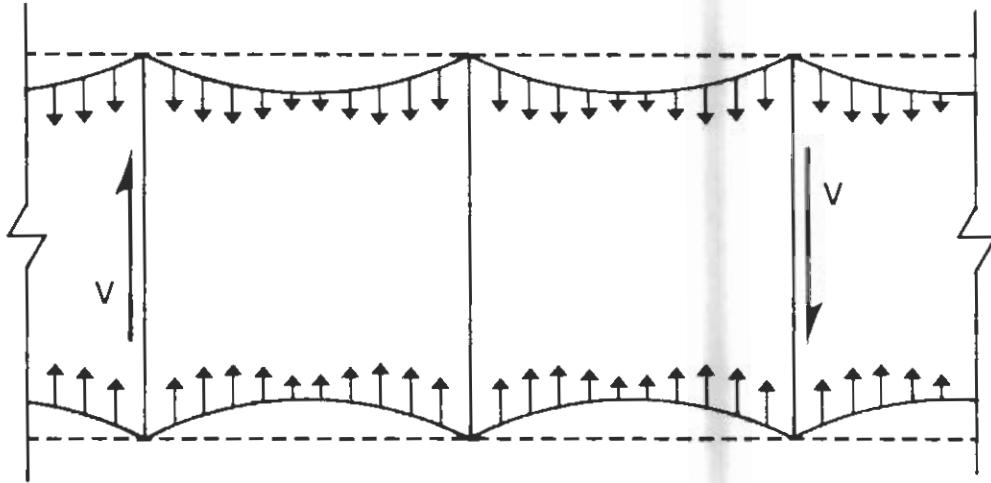
Figure 2.2 a. Vertical Components of Diagonal Tension Acting
on the Horizontals
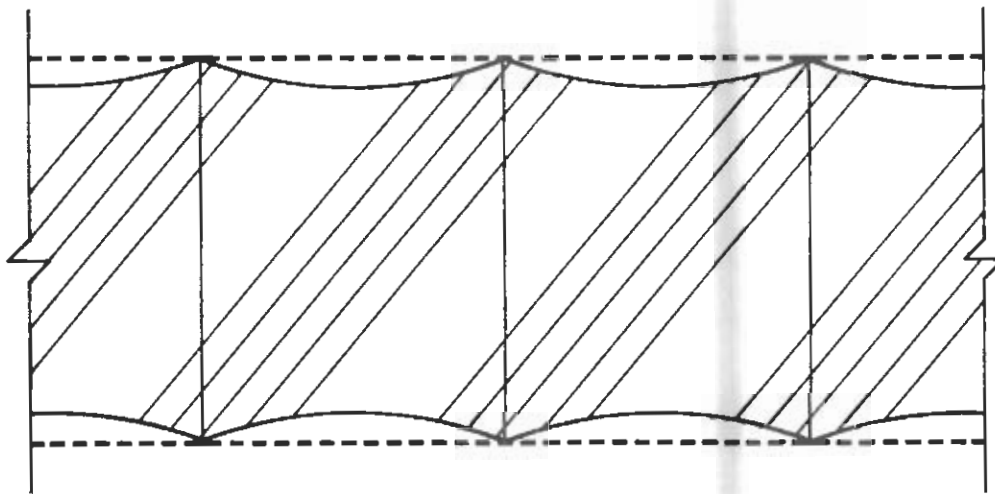
Figure 2.2 b. Redistribution of Tension Field Forces Due to
Deflection of the Horizontals
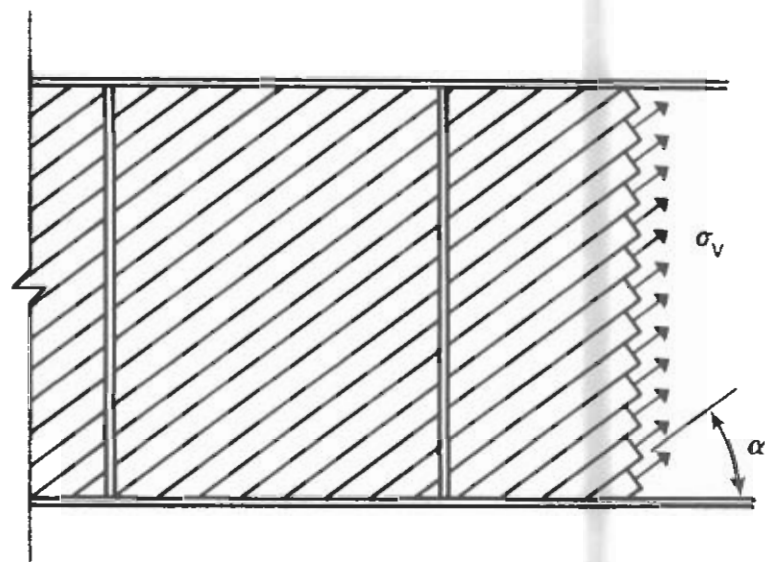
σ_v = inclined tensile stress

Figure 2.3 Idealization of a Diagonal Tension Web
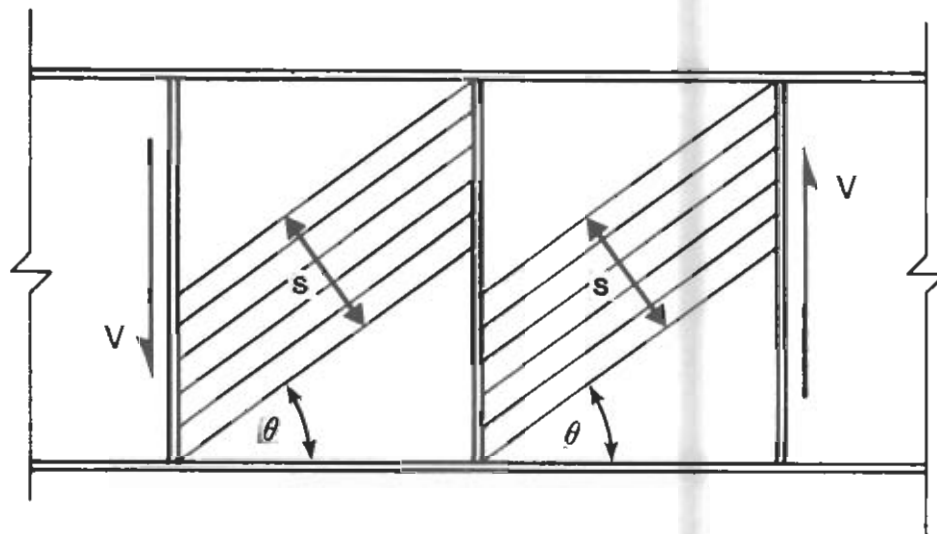
s = width of assumed tension field

θ = inclination of tensile stress

Figure 2.4 Assumed Tension Field for Plate Girders

3. Analytical Model to Study a Steel Shear Wall System

3.1 Post-Buckling Strength of Steel Wall Panel

The presence of the thin web in the steel shear wall system immediately implies stability problems, and buckling can be expected to occur at relatively low loads in the very thin plates present in a steel shear wall system. As was recognized very early by structural designers, a plate does not lose its load-carrying capacity when it buckles. When subjected to a shearing force in excess of that which will cause buckling, a tension zone will develop in the panel which is capable of resisting additional shear up until the tension yield stress of the material is attained.

This shear-resisting process is similar to that initiated subsequent to buckling in the web of a plate girder, as described in the preceding chapter. However, unlike plate girders, for which the total shear strength has two components, the shear carried by beam action prior to buckling and that carried by the tension zone after the critical buckling stress has been exceeded, the panel's shear strength is assumed to derive only from the tension field. It is expected that the thin, storey-size panels will buckle almost immediately upon being loaded. More likely, the fabricated web will have initial out-of-straightness that puts it very close to incipient buckling. Thus, the shear resistance that is attributed to the unbuckled panel

will be neglected.

As pointed out in Chap. 2, there is another important dissimilarity between the tension field that is assumed to develop in a plate girder web and that assumed for the cantilevered shear wall. Basler's partial tension field, as hypothesized for plate girders, neglects completely the portion of the tension field in the area of the flanges, considering the tension zone to develop only between adjacent transverse stiffeners (22). This assumption is based on the fact that the the girder flanges are not considered to be sufficiently rigid to be effective in resisting the additional forces that would be imposed on them by the vertical components of the tension field. For a shear wall panel that has structural members as its "flanges" and "stiffeners", this assumption will be overly conservative. The "flanges" in the shear wall case are the building columns, and the "stiffeners" are the floor beams. Together they provide a frame for the panel that is rigid enough to allow a more complete tension field to develop, similar to that observed in aeronautic structures, and first investigated by Wagner (20). If the beams and columns are sufficiently rigid, the entire panel will carry the inclined tensile forces.

3.2 Strip Model Representation of Shear Wall Panel

Once the critical buckling load is attained, the web buckles. These buckles develop in a regular pattern of inclined folds which follow the lines of the diagonal tension stress. The angle between the columns and the inclined tensile stress in a steel wall panel, α, can be derived in a manner analagous to that used by Wagner in his investigation of aluminum webs for aeronautical applications (20). The derivation, based on the principle of least work, evolved as follows. First, an expression is written for the total internal work done in one panel of the wall, (i.e. a section framed by columns on each side and by beams top and bottom), when acted upon by a shearing force. The expression for the total work is then minimized by differentiating with respect to α. The result is an expression for the angle of inclination of the tension field as a function of the column and beam areas, the panel dimensions, and the thickness of the plate used in the shear web. This derivation is presented in Ref. 21, and is adapted to the case of a shear wall panel in Appendix A of this report. The resulting expression for the inclination of the tension stresses that will develop in a buckled shear wall panel is:

$$\tan^4 \alpha = [1 + (L \times w) / 2 Ac] / [1 + (h \times w) / Ab].....(3.1)$$

where:

α = angle between the column and the diagonal tension
field, degrees

L = length of panel, mm

w = plate thickness, mm

Ac = column area, mm²

h = height of panel, mm

Ab = beam area, mm²

To investigate the distribution of tensile forces in
the buckled plate, it was found to be advantageous to
represent physically the action of the tension zone as a
series of inclined tensile bars. To do this, the area of the
plate is divided into a series of strips of equal width,
each having the same inclination as the tension field that
would arise in the buckled plate. Each strip is modelled as
an inclined truss member, capable of transmitting axial
forces only, and having an area equal to the product of the
strip width and the plate thickness. These bars are joined
with pinned connections to the surrounding frame. The beams
and columns are modelled as beam elements. As developed in
this report, the beam-to-column connection consists of a
hinge at the end of the beam, joining it to a continuous
column. This detail ensures that no moment is transferred
from the rigid joint to the beam. However, if a
moment-resisting connection exists at this location in the
actual structure, the model can be made to conform by
imposing a fixed end condition.

To determine how finely a panel need be subdivided to obtain accurate deflection values, the number of strips per panel was varied for a range of panel dimensions. The deflections that resulted are shown in Table 3.1. Based on this comparison, it was concluded that 10 strips per panel would adequately represent the tension field action for all subsequent shear wall webs to be investigated.

A plane frame computer program was employed to analyze the response of a panel to an applied shearing force using the strip model. By means of this strip representation of the plate, the distribution of tension forces in a given shear panel can be established. The force distribution is valuable in two respects; first, to establish a strength limit for the plate, and second, to design the connection between the plate and the frame.

Initially a one-story panel was examined, considering it to be a typical panel of a highrise core. Certain conditions had to be imposed on the one-storey panel so that it could be considered as a representative "slice" from the multi-storey case.

One of these conditions concerns the intermediate beams which are loaded by the vertical components of the tension field. Since the tension field forces for any two adjacent storeys will differ very little (for the column sizes normally required in a multi-storey shear core) and will oppose one another, the net vertical deflection of the beam was considered to be negligible. To represent this aspect of

a multi-layer system in the one-storey computer model, the interior beams were considered to be infinitely stiff. It should be noted that if an incomplete tension field did arise in the web the vertical components from the fields above and below a particular beam would be concentrated at opposite ends of the member. A small amount of bending would then take place near the ends of the member as a result. Also, special precautions would have to be made in an actual shear wall stack at the extreme top and bottom panels and in a panel adjacent to openings to ensure that a tension field would develop. Provision of a rigid element at these locations would be necessary to anchor the vertical components of the inclined tensile stresses in the adjacent panels in order for the tension zone to be developed.

Actual column sizes had to be used in modelling the one-storey panel to account for the influence of column bending in response to the horizontal components of the tension field stresses. The effect of axial forces in the columns was not included in the one-storey investigation, as it was the response of the panel to the lateral load which was being investigated. In an actual shear core, there would be axial forces and moments in the columns, causing additional bending. This bending would cause additional stresses in the panel but these were considered to be negligible and did not form a part of the present study. However, the contribution of this column bending to overall lateral deflection was recognized in the multi-storey truss

system, as will be discussed below.

In a multi-storey structure, having simple beam-to-column connections, the joints at both ends of the beams at each storey will rotate as the structure is loaded. For the hypothetical case of infinitely stiff beams and columns, this rotation would be equal to the total lateral deflection divided by the height of the structure. Consider a beam-to-column connection in a steel shear wall stack for which the adjacent steel plates above and below the joint have buckled and are functioning as a series of tensile bars. Because the variation in shearing forces acting on any two adjacent storeys will be small, the forces acting on the columns above and below the beam at a particular joint will be approximately equal. In addition, as mentioned previously, the beams act as though infinitely stiff, also as a result of the tension field action. These two influences lead to joint rotations in a multi-storey steel shear wall system that approximate Δ/H, as occur in the hypothetical case of infinitely stiff beams and columns. To represent this phenomena in the single storey model, the joint rotations at the beam-to-column connections at the top and bottom corners of the panel were specified to conform to the ideal slope expected in a highrise structure, i.e., equal to the panel deflection divided by its height.

The lateral load applied to the one-storey model at the top beam-to-column joints was the storey shear that resulted from the applied service load, for the storey under

consideration.

Fig. 3.1 illustrates the model used to represent any typical storey.

This single storey case was expanded to model a multi-storey stack of panels, as would exist in a highrise steel shear core, with only minor modifications. Each panel is assumed to form an independent tension zone, inclined at an angle, α, as it was for the single storey. For the multi-storey case, the joint loads are the actual lateral loads imposed on the structure, not the storey shears, as were used in modelling a specific storey. Unlike the one-storey case, the joint rotations were not specified, but rather, the joints are free to rotate under the applied loads and the influence of the tension zone.

3.3 Equivalent Truss Model

3.3.1 Introduction

Modelling each panel as a series of tensile strips, as previously described, although necessary from a researcher's point of view, would be a prohibitively time-consuming procedure for use in a design office. Simplifications are desirable which will enable a designer to more easily determine the stiffness characteristics of a given wall. The approach adopted is to replace the tension zone of the steel sheet with an equivalent truss element having the same

storey stiffness. A similar approximation was used previously by the Japanese to calculate the bending stiffness of a reinforced steel plate wall (5). The storey displacements can be readily calculated for the steel plate shear wall idealized as an equivalent truss system, as shown in Fig. 3.2. The size of the equivalent truss member is a function of the extent to which a tension field forms and the inclination of this tension zone, both of which depend on the plate geometry and the stiffness characteristics of the perimeter members.

The method of determining the area of a truss element, such that its storey stiffness is the same as that for the tension zone in a shear panel, is as follows. First, consider a Pratt truss with infinitely stiff beams and columns on the boundaries and with one interior diagonal member, as shown in Fig 3.2. An expression for the equivalent storey stiffness of the diagonal brace element in terms of its area, modulus of elasticity, and the truss geometry is found by considering the force in the diagonal that results from an applied shearing force. For a steel shear wall panel having the same geometry as the truss, the work done by the web plate when subjected to the same shearing force is determined. (Only the work done by the web was considered, as the deformations caused by the flexibility of the columns and beams will be accounted for later in the multi-storey truss model used to calculate the lateral deflections.) By equating the external work done by

the shearing force to the resulting internal work in the web, a relationship between the shearing force and the characteristics of the tension field that is expected to develop can be established. The storey stiffness of the web panel is then equated to the storey stiffness of the truss diagonal, and the expression rearranged to give an equation to find the equivalent brace area that would deform identically to a web plate.

There are two limiting tension zones that can be identified in relation to a shear wall panel. The first of these is the ultimate case of a fully developed tension zone occurring uniformly throughout the entire web. Such a complete tension zone will form in panels that are bounded by infinitely stiff columns. The second extreme case is typical of panels bounded by columns that are considered to be completely flexible. For this case, no anchorage is provided to the inclined tensile stresses by the columns. As a result, the tension zone forms only in that section of the panel bounded by the beams. The characteristics of these two tension zones correspond to those hypothesized by Wagner (20) and Basler (22), respectively, for plate girders. The derivations of the areas of the equivalent truss members which are necessary to represent the post-buckling action of each tension field are developed in Appendix B.

Although the actual column stiffness will fall between the limits imposed by these two cases, they were investigated first in an attempt to determine whether or not

one of these models would constitute an acceptable solution.
In the event that neither extreme proved adequate to
describe the actual tension zone, a solution employing the
actual column stiffnesses would have to be adopted.

3.3.2 Brace Area for Panels with Rigid Boundaries

A shear web bounded by columns having a bending
stiffness very much greater than that of the web will be
considered in this section. The equation developed in
Appendix B to determine the area of an equivalent truss
member for this case is:

$$A = (w \times L / 2) \sin^2 2\alpha / \sin \phi \sin 2\phi \ldots\ldots\ldots\ldots(3.2)$$

where:

A = area of equivalent truss member, mm^2

$\tan \phi = L / h$

$\tan^4\alpha = [1 + (L \times w)/2Ac] / [1 + (h \times w)/Ab]$

A series of analytical tests was carried out to examine
the validity of the relationship expressed in Eq. 3.2. These
tests consist of a comparison of the deflection obtained
from the strip model of a panel with that of a
geometrically-similar truss having a single diagonal member.
The area of this diagonal member was calculated from Eq.
3.2.

The hypothetical limiting case is that in which the beams and columns bounding the panel are infinitely rigid. In this case, the panel develops a complete tension zone, inclined at 45°. Under these conditions, the deflections obtained from the two models are essentially equal. Table 3.2 presents the resulting deflections for each model for various panel geometries. Based on these results, two conclusions can be drawn. The deflections for the one-storey shear panel with stiff members on its perimeter can be determined using the truss representation. Also, the close agreement of the two sets of deflections provided evidence that the strip model could adequately depict the behaviour of a shear panel. However, only a comparison with actual test results can positively substantiate the analytical model.

Having ensured that an equivalent truss model could represent the hypothetical case, attention was turned to modelling a panel having more realistic column proportions. It is important to note that the rigidity of the beams employed in this case does not change. They are considered to remain infinitely stiff to account for the opposing, and essentially equal, tension field forces to which the beams in an actual shear wall would be subjected. The following analysis was carried out for a range of column sizes similar to that which could reasonably be expected in a highrise structure. The area of the tension diagonal was calculated using the relation developed for infinitely stiff

boundaries, to investigate its effectiveness with the actual column sizes. The change of inclination of the tension field stresses, which results from altering the column stiffnesses, is included in the equation for the equivalent brace area. However, the reduction in the extent to which the tension field develops, as a result of the greater column flexibility, is not accounted for. The equation was developed on the basis that the web forms a tension zone over its entire area, which may not be the the case in an actual shear wall panel if adequate anchorage for the tension field forces is not provided at the panel extremities.

The deflections obtained from the analysis of the two models can be seen in Table 3.3. It is evident from a comparison of these results that the reduced column stiffness does significantly affect the usefulness of the equation for the equivalent brace area derived on the basis of infinitely rigid columns. Only in the lower levels of a shear core, where the columns are massive, are the truss model deflections comparable to those obtained from the strip representation. For the more flexible columns, found in the panels located in the upper storeys, the truss model predicts a deflection that is several times smaller than that obtained from the strip model. This trend can be explained as follows. The more rigid are the columns, the more efficient is the tension field. Therefore, when the columns become more flexible, the shear resistance of the

web decreases and larger lateral deflections result. As a result, it is concluded that the development of an equivalent brace area based on rigid boundaries is not valid for most panels in an actual shear wall in a highrise building.

3.3.3 Brace Area Derived for Panel with Completely Flexible Columns

The second case examined is the other extreme; a panel framed by completely flexible columns. Assuming no bending resistance is provided by the columns, a partial tension field will develop in the web area anchored by the beams, as shown in Fig. 3.3. This configuration is analogous to the tension zone assumed to form in a plate girder, as first postulated by Basler (22). The formula to determine the diagonal brace area that is required in a truss to provide the same lateral stiffness as this partially buckled plate, as derived in Appendix B, is:

$$A = (w \times L/2) \tan\theta / \sin 4\theta \dots\dots\dots\dots\dots\dots\dots(3.3)$$

where:

$$\tan 2\theta = L / h$$

In contrast to the equation that defines the equivalent brace area for a web with infinitely stiff columns, (Eq. 3.2), this relation for the equivalent brace area depends

only on the panel geometry and the web thickness. The sizes
of the bounding beams and columns do not influence the brace
area. Therefore, for a given panel size, only one brace area
is obtained using this equation.

The area of the diagonal truss member was calculated
using Eqn 3.3, and the truss deflection was compared with
the deflection of a panel for which the assumed tension zone
was modelled as a series of tension bars. For the column
sizes normally used in a highrise, the truss model
deflection was greater than that obtained from the strip
representation.

By considering the columns as completely flexible, the
shear resistance of a large portion of the web is ignored.
Accordingly, the brace area formulated on this basis is
smaller than it would be if the whole web were taken into
account. Without this adjustment, the truss model is
expected to be more flexible than the actual panel. This
conclusion sustantiates the analytical results.

3.3.4 Truss Model Based on Strip Model Deflections

From the foregoing analyses, it is evident that the web
panels for an actual shear wall system fall somewhere
between the two limiting cases that were developed for the
design of plate girders. The Wagner theory, with infinitely
stiff "flanges", (the columns in the steel plate shear wall
system), overestimates the shear strength. Conversely, by
completely neglecting any bending resistance provided by the

columns, as Basler suggests, the deflection predictions are overly conservative. Based on these conclusions, the method of determining the size of a diagonal member was altered, so that the truss representation could adequately predict the deflection characteristics of the shear wall.

The excellent agreement between the strip model and the truss model deflections for a hypothetical panel bounded by infinitely stiff columns and beams implies that the strip model does accurately describe the response of a panel to shear. In addition, when the column flexibility significantly reduces the effectiveness of the tension field, the trend is reflected in the redistribution of forces in the inclined bars. Justification for this last statement is provided by a comparison of the force distribution that results in the strips of a panel bounded by infinitely stiff columns and beams with that which results in a panel with more flexible columns. For the infinitely stiff frame, the forces that arise in the inclined members are uniform across the panel and are all tensile. But, when column flexibility is introduced, the forces in the strips decrease symmetrically, from a maximum value in the centre of the panel. In the extreme case, panels having very flexible supporting columns, the corner strips may be subjected to compressive stresses. This trend is illustrated in Fig 3.4, in which the stresses that arise in the strips are plotted for three panels with a range of column stiffnesses. The same shearing force is applied in

each case. This reflection of the extent to which a tension zone develops in a given panel is another advantage of the strip representation.

Based on the above arguments, the concept of representing the tension zone as a series of strips is retained to facilitate the analysis of the plate shear wall system, at least at the research stage.

In order to use an equivalent truss to predict the lateral deflection of a shear wall stack bordered by realistically proportioned columns, the strip model deflection was employed. Assuming the strip model deflection to be the amount the equivalent truss should deflect, back-calculation was used to determine the necessary diagonal brace area. Knowing the strip model deflection, a virtual work analysis of the truss was used to compute the brace area required in the truss to give it the same lateral stiffness as the strip model.

This indirect method results in a truss system which can be used to predict the stiffness characteristics of a steel shear core. However, in order for this truss model to be an efficient tool for the designer, it is advantageous to replace the preliminary stage of working with the strip model with a more direct method for arriving at the correct diagonal area. A detailed investigation of the factors which influence the panel strength and stiffness characteristics is presented in Chapter 4. The aim of this investigation is to correlate these factors with an equivalent brace area,

thereby developing a systematic approach for the designer to use when proportioning the diagonal member for the truss representation.

3.4 Application of the Strip Model to a Multi-Storey System

3.4.1 Reason for Studying a Multi-Storey Wall

Up to this point, the discussion has been limited to a single panel, framed by the storey floor beams and columns. However, the practical case of a shear core in a multi-storey structure consists of several of these one storey systems joined vertically to form one unit. To examine the validity of considering a one-storey strip model as a representative "slice" of a highrise building, it was necessary to expand the model from a single storey and to analyze a multi-storey stack of panels. In the event that the forces and deflections obtained from an analysis of a series of single storey panels proved sufficient to predict those occurring in a multi-storey structure, each panel in a shear core could then be analyzed individually. The total core deflection could then be calculated as merely the cumulative effect of all the individual panels. The main advantage of this simplification applies to cases where the column sizes and plate thicknesses are common to several adjacent storeys, affording a means of mass production.

3.4.2 Computer Program to Generate Member And Nodal Data

For a multi-storey strip model, the time involved in the manual preparation of the nodal and member data necessary to describe the structural geometry and load characteristics becomes prohibitive. Therefore, to systematize this preliminary step, a computer program was written which is capable of generating the nodal and member data for a specific panel, or series of panels. The input required in order for the program to generate this information is:

a.   the panel geometry; length, height, and thickness,

b.   the angle of inclination of the tensile bars, α,

c.   the number of storeys,

d.   the member properties; the area, moment of inertia and modulus of elasticity of the beams and columns.

Details of the loading and boundary conditions are added to the output from this program. This then constitutes the input data required to describe the structure for the plane frame program used in the analysis. Thus, making use of the program to generate the input data, multi-storey stacks of panels can be investigated readily, without undue time spent in preparation. The program also facilitated the parametric studies that will be discussed in the next section.

3.4.3 Analysis of a Ten Storey Shear Core

The aim of this investigation was to compare the storey deflections that result from an analysis of a multi-storey unit of strip models with the sum of the lateral deflections obtained when each of the panels was analyzed individually as a one storey strip model. The deflections calculated herein are only those due to the presence of the shear wall web itself. Of course, a complete description of deflections would include the effect of column shortening. The ten storey shear wall system which forms the basis for the comparison is shown in Fig 3.5. The applied load was an unfactored wind load, typical of the Canadian climate. This structure is a somewhat hypothetical case, as the column moment of inertia was chosen to be constant throughout the height of the structure in order to simplify the analysis. Although the model was not intended to represent an actual highrise structure, in which the column size would vary throughout the height, the findings of this analysis are nevertheless equally applicable to the more realistic case.

To begin, each storey panel was taken as a single unit, and modelled according to the requirements given previously for a single storey strip model. Each storey was loaded with the shear force that would result at that storey due to the applied wind load and the structure was analyzed storey by storey. The deflections obtained from this analysis are the storey drifts between the storey under consideration and the one immediately below it. Adding the storey drifts for each

storey consecutively produces the sway deflections for the shear wall web of the total structure. The resulting deflections for the ten storey shear core are shown in Table 3.4.

Next, the ten one-storey strip models were combined and analyzed as a multi-storey unit subjected to the actual wind loads. The resulting storey deflections are also presented in Table 3.4.

A comparison of the two sets of deflections shows that by analyzing the structure one storey at a time, the predicted deflections are, at most, 2% less than the deflections obtained from the actual multi-storey strip model. The primary difference between the two cases is in the joint rotations at the beam-to-column connections. Both models assume a hinge at the end of the beam at this location. However, the joints in the one-storey case are set equal to the rotation which would occur in a multi-storey structure if the beams and columns were infinitely stiff, (Δ/h, clockwise). For the multi-storey case, the joints connecting the beams to the columns are permitted to rotate freely under the influence of the applied loads. As was explained earlier, if the tension field forces acting on a column above and below a particular joint were exactly equal, the joint would rotate as if the beams and columns were infinitely stiff, i.e. Δ/h. In the ten storey structure analyzed, the variation of the tension field forces from storey to storey was less than 15%, causing the joint

rotations to deviate only slightly from the assumed value of Δ/h.

With this small discrepancy explained, it is concluded that the approach to modelling a structure one panel at a time will adequately predict the storey deflections of a highrise shear core. Working with a one-story model can significantly reduce the analysis when beam and column sizes are common to several storeys.

Table 3.1 Strip Model Deflections as a Function of the
Number of Strips per Panel

PANEL ASPECT RATO, L/h	PANEL DEFLECTIONS (mm)			
	NUMBER OF STRIPS			
	5	10	20	100
0.25	10.06	10.06	10.05	——
0.50	4.92	4.97	5.01	——
0.75	3.35	3.31	3.34	——
1.00	2.41	2.50	2.51	——
2.50	2.43	2.46	2.47	2.47
3.50	0.72	0.72	0.72	——
4.00	0.63	0.63	0.63	——

Table 3.2 Deflections for Strip Model and Truss Model,
One-storey Panel with Rigid Boundaries

PANEL ASPECT RATIO, L/h	STRIP MODEL DEFLECTION, mm	TRUSS MODEL DEFLECTION, mm
0.25	9.96	10.06
0.50	4.97	5.02
1.00	2.50	2.51
1.50	1.67	1.67
2.00	1.24	1.25
2.50	1.00	1.01
3.00	0.83	0.84
3.50	0.72	0.72
4.00	0.63	0.63

Table 3.3 Deflections for Strip Model and Truss Model,

One-storey Panel with Actual-size Columns

COLUMN MOMENT OF INERTIA, $\times 10^6 mm^4$	STRIP MODEL DEFLECTION, mm	TRUSS MODEL DEFLECTION, mm
17.2	0.41	0.19
70.6	0.31	0.15
502.0	0.15	0.10
1100.0	0.11	0.09
3070.0	0.10	0.08
4390.0	0.11	0.10

Table 3.4 Deflections of Ten Storey Steel Shear Core

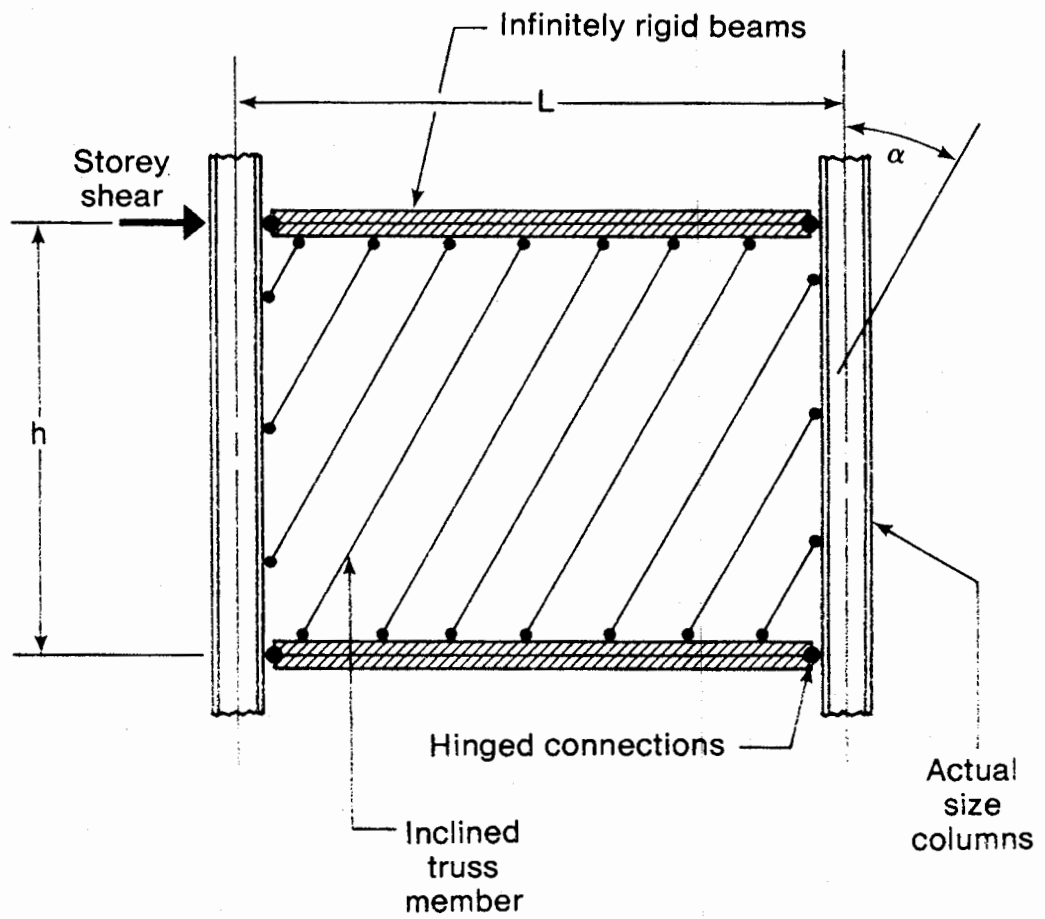| STOREY	DEFLECTIONS, mm			
	One-storey strip model		Multi-storey strip model	Equivalent Pratt truss
	storey drift,	total sway	storey deflections	storey deflections
10	0.11	10.68	10.74	10.70
9	0.33	10.57	10.62	10.59
8	0.55	10.24	10.29	10.26
7	0.76	9.69	9.75	9.72
6	0.93	8.93	8.99	8.96
5	1.18	8.00	8.03	7.99
4	1.39	6.82	6.85	6.81
3	1.60	5.43	5.47	5.43
2	1.81	3.83	3.87	3.83
1	2.02	2.02	2.07	2.02

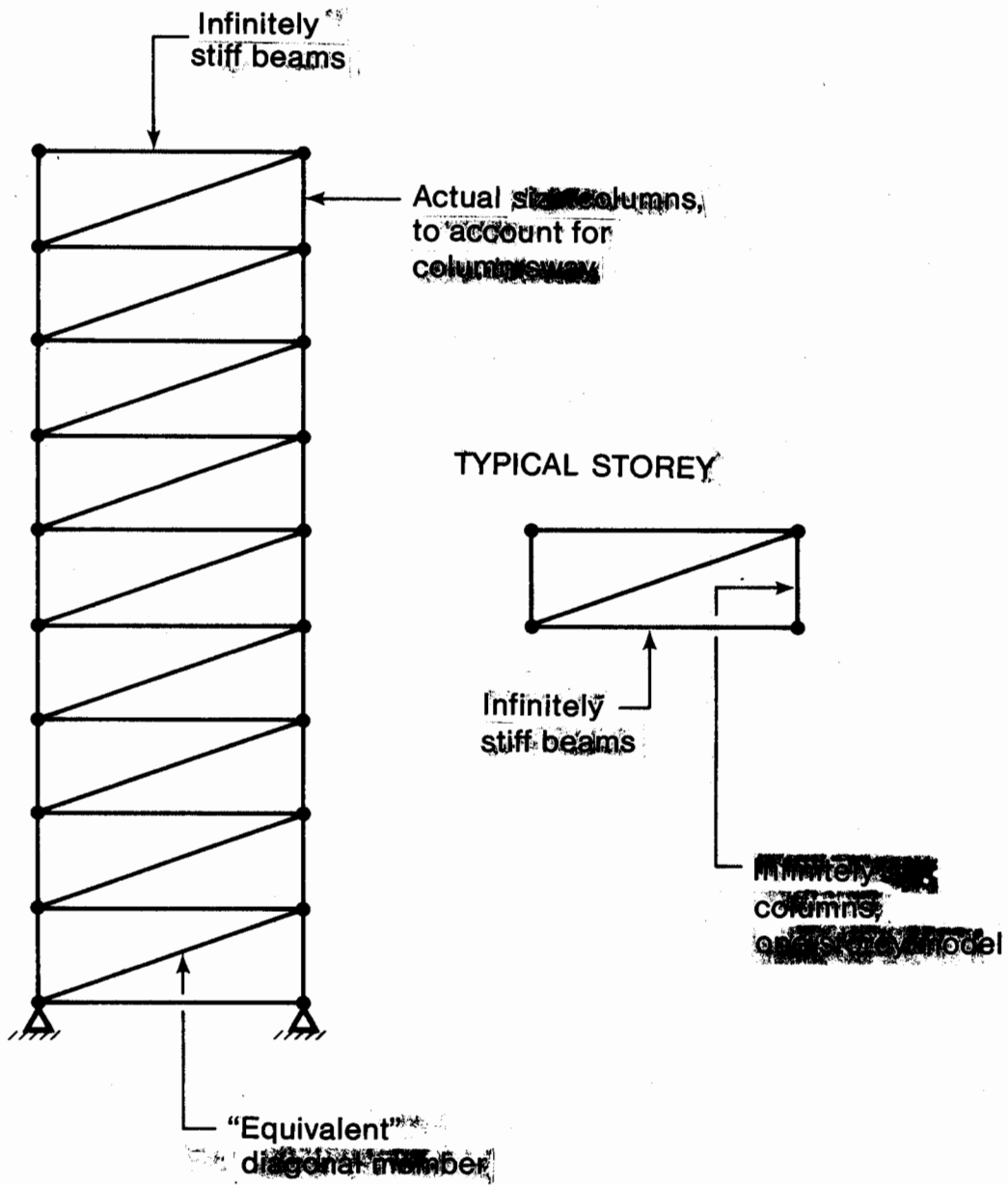Figure 3.1 Strip Model Representation of a Typical Storey

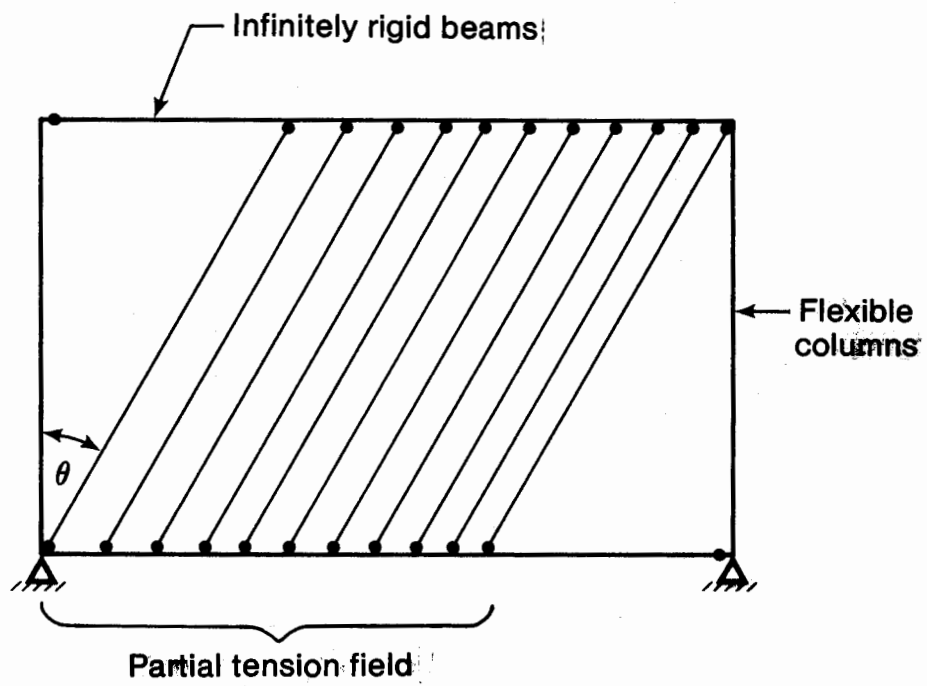Figure 3.2 Shear Core Modelled as an Equivalent Truss System

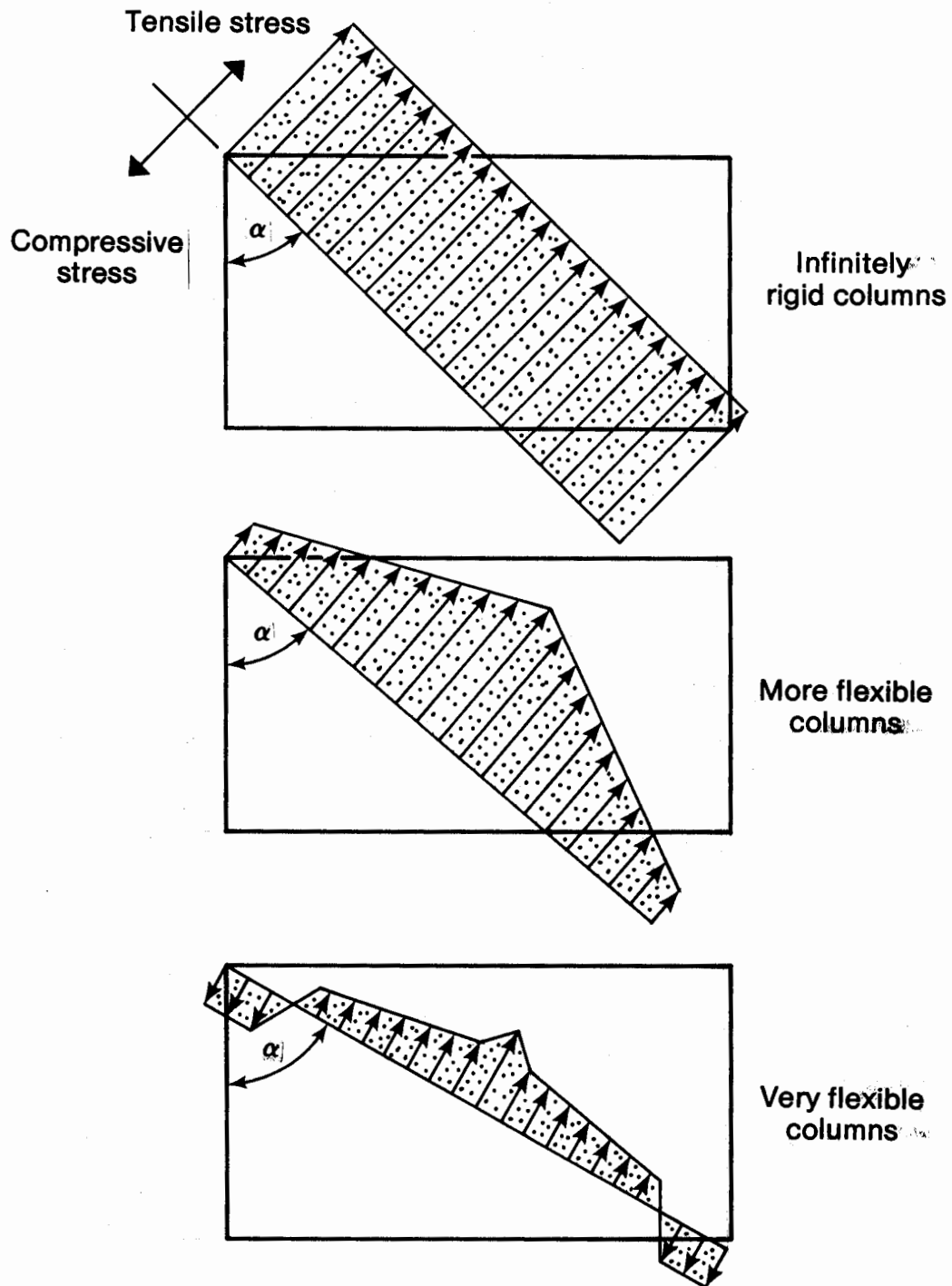Figure 3.3 Tension Zone for Panel with Flexible Columns

Figure 3.4 Distribution of Tension Zone Stresses as Obtained
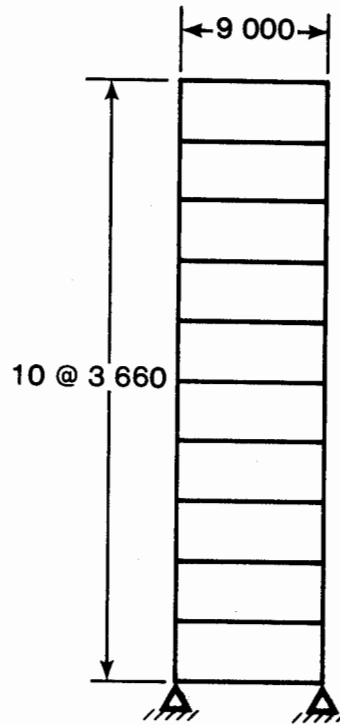from Strip Model
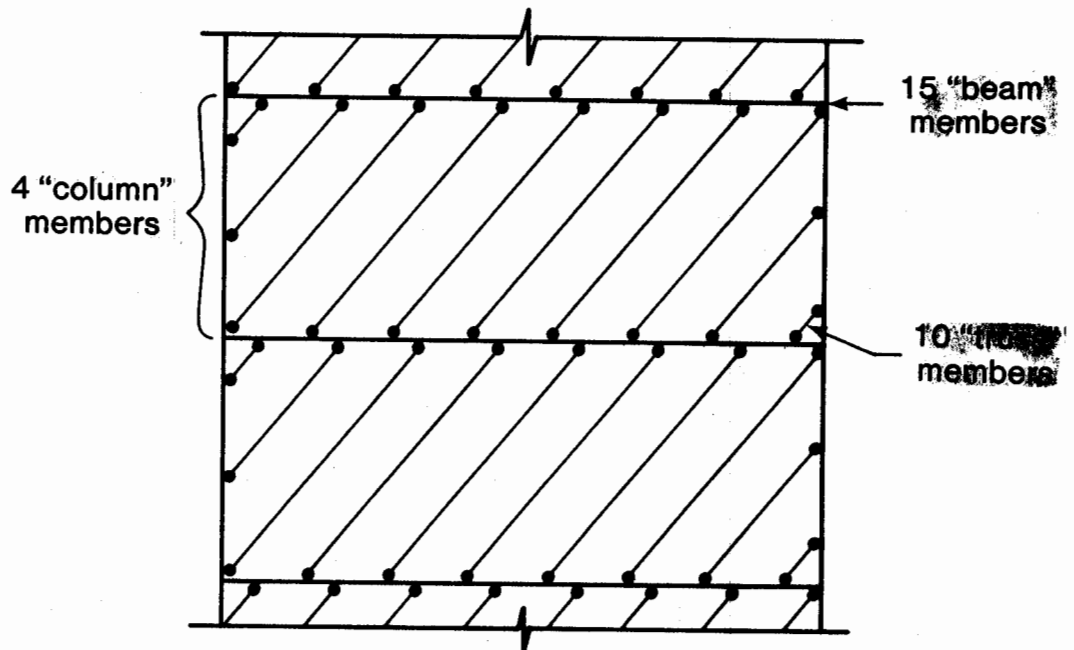
Figure 3.5 a. Multi-storey Shear Wall System

Figure 3.5 b. Detail of the 'Strip' Representation of Two of
the Ten Storeys

4. STRENGTH AND STIFFNESS CHARACTERISTICS OF A STEEL SHEAR WALL PANEL

4.1 Introduction

The extent to which a tension field forms in a given panel and the distribution of the forces in this zone has been shown in Chap. 3 to be influenced by several factors. The most important of these are: panel geometry (length and height), column stiffness, web thickness, and the angle of inclination of the tension field. (This last factor, defined in Eqn. 3.1, is dependent upon the panel length, height, plate thickness, and the column and beam areas.) In this Chapter, the strength and stiffness characteristics of a shear wall web subsequent to buckling will be evaluated in terms of these parameters. The strip model representation will be used as the basis of the examination.

The basic procedure followed was to look at the influence of each of the above-mentioned parameters individually. The parameter to be considered was altered successively for several trials, keeping all other characteristics of the panel constant. This provides a direct evaluation of each parameter's contribution to the panel characteristics. For each trial, a one-storey panel was modelled and analyzed as a series of strips, according to the method described in Chap. 3.

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From this analysis, the area of a diagonal truss element was calculated such that a Pratt truss containing this element would have the same stiffness characteristics as a geometrically-similar strip model. The truss model was important in the calculation of storey drifts, as explained in Chap. 3. The area of this "equivalent" truss element provides a direct indication of the contribution of the buckled web to the lateral rigidity of the panel. As the tension field becomes more effective in resisting shear, the area of the truss element which is required to exhibit the same stiffness characteristics increases accordingly. In the following analysis, "panel stiffness" and "equivalent member area" will be used interchangeably.

The following two sections present the results of the parametric studies as they relate to the strength and stiffness of a steel shear wall panel.

4.2 Parameters that Influence the Web Stiffness

4.2.1 Plate Thickness

The first parameter which was isolated was the thickness of the steel plate which constitutes the web of the panel. The characteristics of the six series of panels that were analyzed are shown in Table 4.1.

For a given panel geometry and column size, a series of plate thicknesses in the range of 6 to 20 mm were chosen,

and the panel was first analyzed to determine its stiffness characteristics for each thickness. Fig 4.1 presents the results of the study for Series T1 in graphical form. The results show that the panel stiffness is a linear function of the thickness of the plate used for the web for this series.

Next, further test cases were examined to see whether or not this relationship remained valid if either the panel geometry or the column moment of inertia was varied.

Keeping the panel geometry fixed, (i.e. a 9.00 m by 3.66 m plate), the influence of a change in column moment of inertia was investigated. For this constant geometry, two additional column sizes were examined, (Series T2 and T3). For each case, the plate thickness was repeatedly varied and the necessary equivalent member area found. The graph given in Fig. 4.2 shows the relationship between plate thickness and panel stiffness for the three column sizes. All three cases can be described as exhibiting linear relationships between plate thickness and equivalent member area, indicating that as plate thickness is increased, panel stiffness increases proportionally. For the two series of panels bounded by relatively rigid columns, (Series T1, I_c = 2800 x 10^6 mm^4 and, Series T2, I_c = 4390 x 10^6 mm^4), the relationship between the plate thickness and the panel stiffness is essentially the same. However, comparing this relationship to that for the panel with very flexible columns, (Series T3, I_c = 177 x 10^6mm^4), a difference in

slope is observed. For the latter, the same increase in plate thickness results in a smaller increase in panel stiffness than it does for the same panel bounded by the more rigid columns.

Column rigidity influences the lateral rigidity of a panel in two distinct ways. First, the lateral deflection of the column itself, which is a component of the total panel deflection, is a function of the column stiffness. Second, and a more complex effect to evaluate, is the influence of the column bending inwards as a result of the horizontal components of the tension field stresses. This column bending, also a function of the column stiffness, causes a redistribution of the forces in the tension field. This dual influence of the column stiffness on the deflection characteristics of a panel, alters the relationship between plate thickness and panel stiffness for any change in column flexibility.

As a second part of the study to determine the influence of plate thickness on the resulting panel stiffness, the panel geometry was varied. For a fourth series, (Series T4 in Table 4.1), the panel length and height, and the column moment of inertia were changed. These changes were made such that the panel aspect ratio, L/h, and the column stiffness parameter, Ic/h, remained the same as they had been for Series T1. The relationship obtained for plate thickness versus equivalent member area in this case remained linear. However, the equation of the line differed

from that obtained previously for the same L/h and Ic/h ratios. The plots for Series T1 and Series T4 are compared in Fig. 4.3.

An examination of these results shows that the panel geometry is a controlling feature of the panel stiffness. For the same aspect ratio, but different physical dimensions, the panel stiffness is not the same. This is illustrated in Fig 4.3. Therefore, the influence of panel length and height must apparently be considered on an individual basis.

To further test this hypothesis, the relationship between the steel plate thickness and the resulting equivalent member area was investigated for a fifth and a sixth series of plates, (Series T5 and T6 in Table 4.1). These last two series consisted of two additional plate geometries. The panels had dimensions of 3.66 m by 3.66 m, (T5), and 9.0 m by 9.0 m, (T6). In addition to the aspect ratio for each panel being the same, the column stiffnesses were the same for each panel. The results of these two series are compared in Fig. 4.4. The slopes of the lines are different, substantiating the observation that the linear relationship that exists between plate thickness and panel stiffness is different for different panel geometries.

4.2.2 Panel Height

Panel height was the next parameter investigated. Five groups of panels were examined, each with a unique combination of panel length, plate thickness, and column moment of inertia. The characteristics of the panels are shown in Table 4.2. The procedure followed was similar to that presented in the preceding section, ie. for each set of panel characteristics the panel height was changed successively, while the other variables were kept constant.

Three of the five series of panels studied had the same length; 9.0 m. For the first series of 9.0 m panels, (Series H1 in Table 4.2), the plate thickness was 10 mm. and the columns supporting the panel were very rigid, (Ic = 4390 x 10^6 mm^4). The panel height was varied several times and the panel reanalyzed after each iteration. A linear relationship was shown to exist between these variables when a semi-log plot of height versus equivalent member area was used. The results are presented graphically in Fig 4.5. This figure shows that as the panel height increases, the panel loses lateral stiffness, i.e. the required "equivalent" member area decreases.

Increasing the column height, with no corresponding increase in its moment of inertia, decreases the column stiffness, (a function of Ic/h). Therefore, the loss of panel rigidity with an increase in panel height can be explained by the fact that the extent to which a tension field forms is influenced by the rigidity of the bounding

members. A fully developed field will result only if the beams and columns bounding the panel have an extremely large bending stiffness. As the columns take on greater flexibility, they will bend into the panel as a result of the horizontal components of the inclined tensile forces. This results in a redistribution of the web forces, and a decrease in the lateral stiffness of the panel.

For the second series of panels, (Series H2 in Table 4.2), the 9.0 m length and the 10 mm steel plate web used in Series H1 were retained, but the the column size was changed. The stiff columns used previously were replaced with more flexible columns, ($Ic = 308 \times 10^6$ mm^4). The purpose of studying this second series of panels was to determine the effect that a change in column size would have on the relationship between the panel height and the overall web stiffness.

The relationship between the panel height and the equivalent member area remained linear on the semi-log plot, (see Fig. 4.5). However, the slopes of the lines for the two series were not the same. For the more flexible column, (Series H2), the overall panel stiffness decreased more rapidly as the height was increased than it did for the stiffer column size, (Series H1).

In order to explain this difference in the relationship between panel height and equivalent member size for the two column sizes, the dual nature of the column's influence on the overall stiffness of the plate is again examined. The

system's lateral stiffness derives from two principal sources; namely the lateral column resistance and the resistance provided by the tension field. A change in column stiffness affects both of these components. Reducing the column stiffness not only increases the lateral deflection due to the column sway, but also induces a less efficient distribution of forces in the tension zone. A change in column size alters the relative importance of both of these contributions. As column flexibility is increased, the loss of efficiency of the tension zone as a result of column bending becomes more significant. Therfore, the relationship between panel height and equivalent member area is changed due to a variation in the column size.

Next, the panel thickness was increased and a third series of panels were examined. These panels have the same length, (9.0 m), and column size, ($I_c = 4390 \times 10^6$ mm^4), as Series H1. Thus, the effect of changing the plate thickness can be determined.

The trend of panel height versus the area of an equivalent member area was again found to be linear on a semi-log plot. Referring to Fig 4.5 and comparing the line for a 14 mm plate thickness, (Series H3), with that for a 10 mm plate, (Series H1), the slopes of the two lines are seen to be essentially the same. Therefore, it can be concluded that by increasing the plate thickness a stiffer panel results, and the direct dependency of equivalent member area on the logarithm of panel height remains the

same.

The next two series of analytical tests were done for a panel 6.0 m in length, having a 10 mm thick plate. Series H4 had flexible columns, ($Ic = 308 \times 10^6$ mm^4), and Series H5 had stiff columns, ($Ic = 4390 \times 10^6$ mm^4). Both of these investigations again show a linear trend between the log of the panel height and the equivalent member area. The results are shown in Fig. 4.5.

The result of changing the column size for the 6.0 m panel was similar to the trend observed for the 9.0 m panels. The slope of the line of log height versus equivalent member area is less for a smaller column size, with all other parameters being equal.

The shorter panels are found generally to be more flexible than the longer panels. However, due to the complex interrelationship of all the parameters, the relationship between the two panel lengths can not be uniquely defined. The panel geometry was found to contribute significantly to the panel stiffness characteristics, as was observed earlier in the investigation of different plate thicknesses.

There are two trends that remain constant for the five series of panels used to test the influence of a change of panel height on the resulting panel stiffness:

1. As the height of a panel is increased, the lateral stiffness of the panel is reduced.
2. The relationships between panel height and panel stiffness are all linear on a semi-log scale.

4.2.3 Panel Length

The relationship between the panel length and the stiffness was examined for four different panel heights, keeping plate thickness and column size constant. The panel characteristics for each of the four series are shown in Table 4.3.

A graph of panel length versus equivalent member area is shown in Fig 4.6 on a log-log scale. For a given column stiffness, a similar trend is observed in all four series. Up to a certain limiting value, the panel stiffness decreases as the panel is lengthened. After a minimum value of panel stiffness has been reached, an increase in the panel length results in an increase in panel stiffness. The extreme left point on each curve represents the least efficient panel configuration.

For the cases examined in Section 4.2.2, i.e. varying the panel height while keeping all other parameters constant, a change in column stiffness was the dominant factor which controlled the panel stiffness. This change in column stiffness was a result of altering the height of the panel. However, for the current series of panels, for which the panel length is the variable, the column stiffness remains constant. Therefore, the factors which influence the change in panel stiffness are:

a. the change in panel geometry,

b. a change in the angle, α, the angle of inclination of the tension field.

(It must be noted that α is a variable which is dependent upon the values chosen for panel length and height, web thickness, and the beam and column areas.) The panel configurations for which the minimum panel stiffness occurs must result from an unfavourable combination of panel geometry and inclination of diagonal tension stresses.

For all of the panels tested in each of the four series of Table 4.4 the angle α is less than 45° and, for a given height, its value increases as the panel is lengthened. The most efficient tension zone configuration occurs for a completely developed tension field and $\alpha = 45°$. Consider an instance in which the panel geometry is kept constant and α is varied by altering the other panel characteristics, (i.e. web thickness and beam and column areas). Neglecting the influence of panel geometry and dealing only with the effect of α on the panel stiffness, the panel stiffness would increase due to an increase in the panel length as the angle α approaches 45°. However, it can be seen from Fig. 4.6 that, for a certain range of panels, the result of an increase in panel length is a decrease in panel stiffness. This effect must, therefore, be due to the influence of the panel geometry.

Considering the angle of the panel diagonal, ϕ, to be representative of the panel geometry, (length or height, per se, also have an effect), combinations of ϕ and α were examined. Panel stiffness was found to decrease due to an increase in length for the following combinations of panel

geometry and the tension field inclination:

$$(\phi - \alpha) + (45° - \alpha) < 0 \ldots\ldots\ldots\ldots\ldots\ldots (4.1)$$

Panel stiffness begins to increase as panel length increases for the following values:

$$(\phi - \alpha) + (45° - \alpha) > 0 \ldots\ldots\ldots\ldots\ldots\ldots (4.2)$$

For $\phi > 45°$, the panel length is linearly related to the resulting panel stiffness for each series. The influence of the inclination of the tension field diminishes for panels in this range, i.e. with $L/h > 1.0$. Above this limit, as the panel lengthens, its rigidity increases as height remains constant.

4.2.4 Column Stiffness

For five sets of panel geometries the column moment of inertia was varied to investigate the relationship between the column stiffness and the panel stiffness. The characteristics of the five series of panels analyzed are given in Table 4.4.

The results show that a linear relationship exists between the log of the column stiffness (as represented by I_c/h) and equivalent member area (Fig 4.7). Fig. 4.7 shows that as the logarithm of the column stiffness parameter increases, the panel stiffness increases proportionately for

each of the five geometries examined. The slopes of the three lines which result for the 9.0 m panels, (C1, C2, and C3), differ by only 8%. Similarly, the two lines for the 6.0 m panels, (C4 and C5), have approximately equal slopes.

The influence of the column stiffness is important to the panel stiffness overall for the reasons that were discussed earlier in the section pertaining to panel height, that is, column sway increases as a result of an increase in column flexibility, and a more flexible column will experience more inward bending as a result of the horizontal components of the tension field forces. The latter effect results in a less efficient distribution of the stresses in the tension field, thus contributing to panel flexibility.

4.3 Strength Considerations

A strength requirement must also be considered in order that the system be safe, (i.e. the maximum stress must be below a certain permissible stress), as well as serviceable, (i.e. deflections less than the allowable value). To define a strength limit for a steel shear wall, the state of stress which exists within the buckled web needs first to be examined.

The approach taken by Basler to establish the failure condition for a buckled plate girder web will first be examined (22). For the web proportions commonly used in plate girders, the shear carried by the web prior to

buckling is significant. Therefore, to define the failure criterion for a plate girder web, two stress components must be recognized. These two components are the critical shear buckling stress and the inclined stress which arises subsequent to buckling. The stresses are combined according to the "energy of distortion" failure condition for plane stress in order to establish a strength limit for the plate girder web. In contrast, an analogous definition for the ultimate stress in a steel shear wall web is not considered to be necessary. For plate thicknesses in the range proposed for use in steel shear walls, the thin plates can be expected to buckle almost immediately upon being loaded. Thus, the stresses which exist prior to buckling are considered to be negligible in comparison to the diagonal tension stresses which arise in the post-buckled web. This, therefore, precludes the need to define the ultimate stress in terms of an interaction between the critical shear buckling stress and the tension field stress. Rather, the stress field in a shear wall web at the ultimate load will be assumed to be one of uniaxial tension. Thus, the tension yield stress of the steel plate can be used to define the failure condition of the buckled shear wall panel.

The strength effectiveness of the steel web has been examined, considering both the magnitude and the distribution of stresses in the web. The distribution of tension forces in a shear wall panel can be obtained from the member forces provided by a strip model analysis of the

panel. From these forces, the corresponding stresses are
calculated by dividing by the strip width and the plate
thickness. The strength limit for the web can be stated by
defining the tensile yield stress of the steel to be the
maximum usable strength of the web. Finally, to ensure a
safe design the factored web stresses must be less than the
factored tensile yield stress of the steel.

The influence of the column stiffness on the stress
distribution within the shear wall web was studied. The web
stresses were calculated from the strip model forces for the
inclined truss members. The panels used in this study were
Series C1, C2, and C3, described in Table 4.4. The stress in
each strip was calculated for each panel and column size.

An indication of the effectiveness of the tension field
that arises in a shear web was obtained by calculating the
average tensile stress in the web. To find the average web
stress, the total force resisted by the inclined tensile
members in the web was found, and this value was divided by
the cumulative strip area for the portion of the web
resisting tension. Fig 4.8 shows the average stress resisted
by the tension zone of the buckled web as a function of the
column stiffness. The figure shows that the total stress
taken by the tension field increases as the column stiffness
parameter becomes larger. The stresses shown are those that
would result at the bottom storey of a ten storey building,
(45 m by 27 m and having a storey height of 3.66 m), in
Edmonton, Alberta, when the structure was subjected to wind

in the longitudinal direction. No indication of the magnitude of the individual stresses that occur in each strip has been provided at this point.

Another way of judging the strength effectiveness of the web is to examine the effectiveness of the various strips in the web model. Figure 4.9 shows the stress distribution in three panels with the same geometry as the column moment of inertia is reduced. It is evident from the figure that the stiffer the column, the more uniform is the stress distribution in the web.

For the most flexible column, (Fig 4.9(c)), the column bending causes compressive stress in the corner strips where column curvature is the greatest. The occurrence of compression within the assumed "tension zone" reduces the overall effectiveness of the post-buckled web. In order to be consistent with the assumption that the stress resisted by the web before buckling occurs is negligible, the forces in the compression areas are neglected. Thus, the force resisted by the web is calculated as the sum of the tensile strip forces.

A graph showing the variation in the tension field stresses as a function of the column stiffness is given in Fig. 4.10. In this figure, the variation in tension field forces is plotted as a function of the column stiffness parameter, I_c/h. The negative values which result for the most flexible columns again indicate that compressive stresses are occurring in the web. In the negative region of

the graph, as the column stiffness increases, the
compressive stresses decrease rapidly, while the tension
stresses increase more slowly. The result of this is an
increase in the ratio of σ (max)/ σ(min), due to an increase
in column stiffness. This trend continues only as long as
there are compressive stresses occurring in the web.

The principal importance of Fig 4.10 relates to the
positive region of the graph. This region depicts webs that
have tensile stresses only. For each of the three geometries
that were investigated, an increase in the column stiffness
results in less variation in the tension field stresses. As
the columns bounding the web become stiffer, they are able
to offer more resistance to the horizontal components of the
inclined stresses. The maximum stresses at the centre of the
panel decrease and the stresses in the strips anchored by
the columns increase. This trend is gradual in the lower
range of column stiffnesses, but, there is a certain
"critical" column stiffness for each of the three panel
geometries. Once the column stiffness exceeds this value,
the variation in tension field forces remains virtually
constant, that is, once a certain column stiffness has been
reached, the stress distribution in the panel remains
unaffected by any further increase in column rigidity. The
members are sufficiently rigid to anchor the tension field
forces.

4.4 Panel Design to Satisfy Strength and Stiffness Tolerances

In order to produce an acceptable design, both the strength and stiffness requirements must be met. (It must be noted that the former relates to factored loads, and the latter to specified loads). To achieve this, one approach is to satisfy the stiffness requirements, as outlined in the previous sections, and to then simply accept the resulting stress field in the web. The stress in the most highly loaded strip would be compared to the factored yield strength of the material to ensure that the stresses are within the acceptable range. Alternatively, the designer's main objective could be to make optimum use of the web material, that is, stress as much of the web as possible to the permissible limit. As a final step in this approach, the resulting structure would have to be examined to see if it satisfies the drift limitations.

Although there is no general way of deciding which of these two sequences in preferable, experience does indicate that the design of cores in highrise buildings is usually controlled by drift. In addition, defining a strength limit for the plate is complicated by the fact that both the magnitude and the distribution of the web stresses are influenced by a change in any of the panel characteristics. A re-examination of the panels investigated herein when considering the panel stiffness, revealed no satisfactory way of predicting the maximum web stress for a given wall

panel. The drift versus strength examination is further complicated by the fact that drift is composed of at least two components, that due to column shortening and that due to the action of the steel plate shear wall web. As a severe test, all of the total drift limitation of h/500 as prescribed by CSA S16.1 (24) can be assigned to the web. Applying this criterion to the panels in this study it was found that the stresses were well below the permissible value. The most critical stress arising in a panel which met the given serviceability requirement was 25% below the permissible tension stress. Specific examples for some of the most flexible panels that were examined in the previous sections are presented in Table 4.5.

4.5 Summary

The aim of the parametric studies was to investigate the factors which influence the strength and stiffness characteristics of a buckled steel wall panel. The results of the study show that the four parameters which influence the effectiveness of the resulting tension zone are inter-related and complex. The following section will briefly summarize the findings of the foregoing parametric studies. It will be shown how the information gained for each parameter can be employed in the design of a specific shear wall panel in order to comply with the established strength and stiffness requirements.

Based on the results obtained from the wall panels exaimned in the preceding sections, (a total of 250 individual panels), it is concluded that the design of any specific shear wall panel will likely be governed by the serviceability condition: if, for a given panel, the drift is within the allowable tolerances, then the web stresses for that panel will be below the permissible values.

It was shown that the trend between each parameter and the resulting panel stiffness can be defined for a given set of panel characteristics. Thus, it should be possible to extend the parametric studies and develop a set of graphs or other design aids to show the equivalent brace area as a function of each parameter. These studies could be done for a variety of panel characteristics and encompass the entire range of panel dimensions, plate thicknesses, and column stiffnesses that would be expected for use in a steel shear wall core. A set of such design curves would enable the designer, having a pre-determined bay size, to choose the required column stiffness and plate thickness so that the panel will meet the drift requirement. An equivalent truss system, for which the area of the diagonal member is obtained from the design curves, would facilitate the drift calculations, as explained in Chap. 3.

To develop a complete set of design curves for the range of all possible panel characteristics is beyond the scope of this report and will be left for future research. However, an example of the use of a set of such curves to

examine the stiffness characteristics of a sample shear wall
will be provided. This example is based on the information
provided by the curves referred to earlier in Chap 4. To
conclude the example, a comparison will be made of the
estimated deflections with an actual strip model analysis of
the panel.

The example will deal with the design of the shear
walls at the first storey level of a 25 storey high building
located in Edmonton, Alberta. The width of the structure
perpendicular to the wind direction is 45 m and the storey
height is 3.66 m. For the first storey, the storey shear due
to the applied wind load is 5760 kN and the factored shear
load is 8640 kN. The former value will be used for the
deflection calculations and the latter for strength
calculations. The first storey has two identical shear
walls, each resisting one-half of the total applied load.
The panel size to be investigated is 9.0 m by 3.66 m. (It is
common for the bay size to be dictated by architectural
considerations and therefore is not considered as a part of
the panel design in this example.) The aim is to choose a
suitable column size and plate thickness for the web, such
that the panels will meet the given drift requirement of
h/500 per storey. According to the work done previously for
the strength requirements, if the deflection tolerances are
met, it is probable that the stress values will fall below
the permissible levels. The actual stresses obtained from a
strip model analysis will be compared with the permissible

strength limit to show that this assumption is valid.

The influence of column size on the effectiveness of the tension zone can be examined with the aid of Fig. 4.8, Series C1. For a panel 3.66 m x 9.0 m, this curve shows that for a stiffness parameter greater than about 500, the average stress taken by the tension field remains approximately constant. The corresponding minimum column moment of inertia for this panel height is 1850×10^6 mm^4. Based on axial loads only, columns in the first storey of a 25 storey building would be in this range. The column size chosen for the panel design was a WWF 500 x 381, ($I_c = 2250 \times 10^6$mm^4 and $A_c = 48\ 600$ mm^2).

For this estimated column size, a virtual work analysis was carried out for the Pratt truss that is pictured in Fig. 4.11 in order to calculate the area of the equivalent diagonal member. The actual column area was used in the Pratt truss model in order to include the contribution of column lengthening and shortening to the panel deflection. The applied load was one-half the unfactored wind shear of 5760 kN and the deflection was the maximum allowable to satisfy the drift requirements. The required member area obtained from this analysis is 22 850 mm^2.

From the graph of the plate thickness versus the member area for this geometry and a stiff column, (Fig. 4.3, Series T1), the predicted thickness is 3.1 mm. It should be noted at this point that although these plates are predicted to have the adequate rigidity and strength, handling

considerations would preclude the use of any plate thinner than about 4.5 mm.

To check the preliminary design, a panel with a 3.5 mm thick web plate was modelled as a series of strips and analyzed. The resulting drift was 6.80 mm and the maximum stress in the tension field due to the factored shear force, was 220 MPa. Since the permissible stiffness and strength limits are 7.32 mm and 270 Mpa, respectively, the panel meets the given tolerances and the predicted panel design based on the design curves is satisfactory.

Table 4.1 Panels Analyzed to Examine the Relationship
Between Plate Thickness and Panel Stiffness

Series	Length ($\times 10^3$mm)	Height ($\times 10^3$mm)	L/h	Column Moment of Inertia ($\times 10^6$mm^4)
T1	9.00	3.66	2.46	4390
T2	9.00	3.66	2.46	2800
T3	9.00	3.66	2.46	177
T4	6.00	2.40	2.50	2927
T5	3.66	3.66	1.00	1138
T6	9.00	9.00	1.00	2800

Table 4.2 Panels Analyzed to Examine the Relationship
Between Panel Height and Panel Stiffness

Series	Plate Thickness (mm)	Panel Length ($\times 10^3$ mm)	Column Moment of Inertia ($\times 10^6$ mm^4)
H1	10	9.0	4390
H2	10	9.0	308
H3	14	9.0	4390
H4	10	6.0	308
H5	10	6.0	4390

Table 4.3 Panels Analyzed to Examine the Influence of Panel

Length on Panel Stiffness

Series	Plate Thickness (mm)	Panel Height ($\times 10^3$mm)	Column Moment of Inertia ($\times 10^6$mm^4)
L1	10	6.00	4390
L2	10	4.50	3293
L3	10	3.66	2678
L4	10	2.50	1829

Table 4.4 Panels Analyzed to Examine the Effect of Column

Stiffness on Panel Rigidity

Series	Panel Length ($\times 10^3$ mm)	Panel Height ($\times 10^3$ mm)	L/h	Plate Thickness (mm)
C1	9.00	3.66	2.46	10
C2	9.00	6.00	1.50	10
C3	9.00	9.00	1.00	10
C4	6.00	4.00	1.50	10
C5	6.00	6.00	1.00	10

Table 4.5 Deflections and Stresses for Various Panels

Ic $(\times 10^6 mm^4)$	L (m)	h (m)	w (mm)	σ_{max} (Mpa)	$\Delta_{calc.}$ (mm)	$\Delta_{allow.}$ (mm)
177	9.0	3.66	4.5	205	8.6	7.3
177	9.0	3.66	6.0	172	6.4	7.3
177	9.0	3.66	10.0	110	4.1	7.3
308	9.0	10.00	10.0	207	34.9	20.0
308	9.0	9.00	10.0	191	21.7	20.0
308	9.0	7.50	10.0	167	12.6	20.0
308	9.0	5.00	10.0	115	5.8	20.0
4390	2.0	3.66	10.0	370	14.9	7.3
4390	3.0	3.66	10.0	257	9.6	7.3
4390	4.0	3.66	10.0	191	7.1	7.3
4390	6.0	3.66	10.0	126	4.6	7.3
177	9.0	9.00	10.0	217	37.8	18.0
308	9.0	9.00	10.0	192	21.7	18.0
1100	9.0	9.00	10.0	148	13.6	18.0
2660	9.0	9.00	10.0	125	11.5	18.0

where:

$$(\sigma) \text{ allowable} = \phi \, Fy$$
$$= 270 \text{ MPa}$$
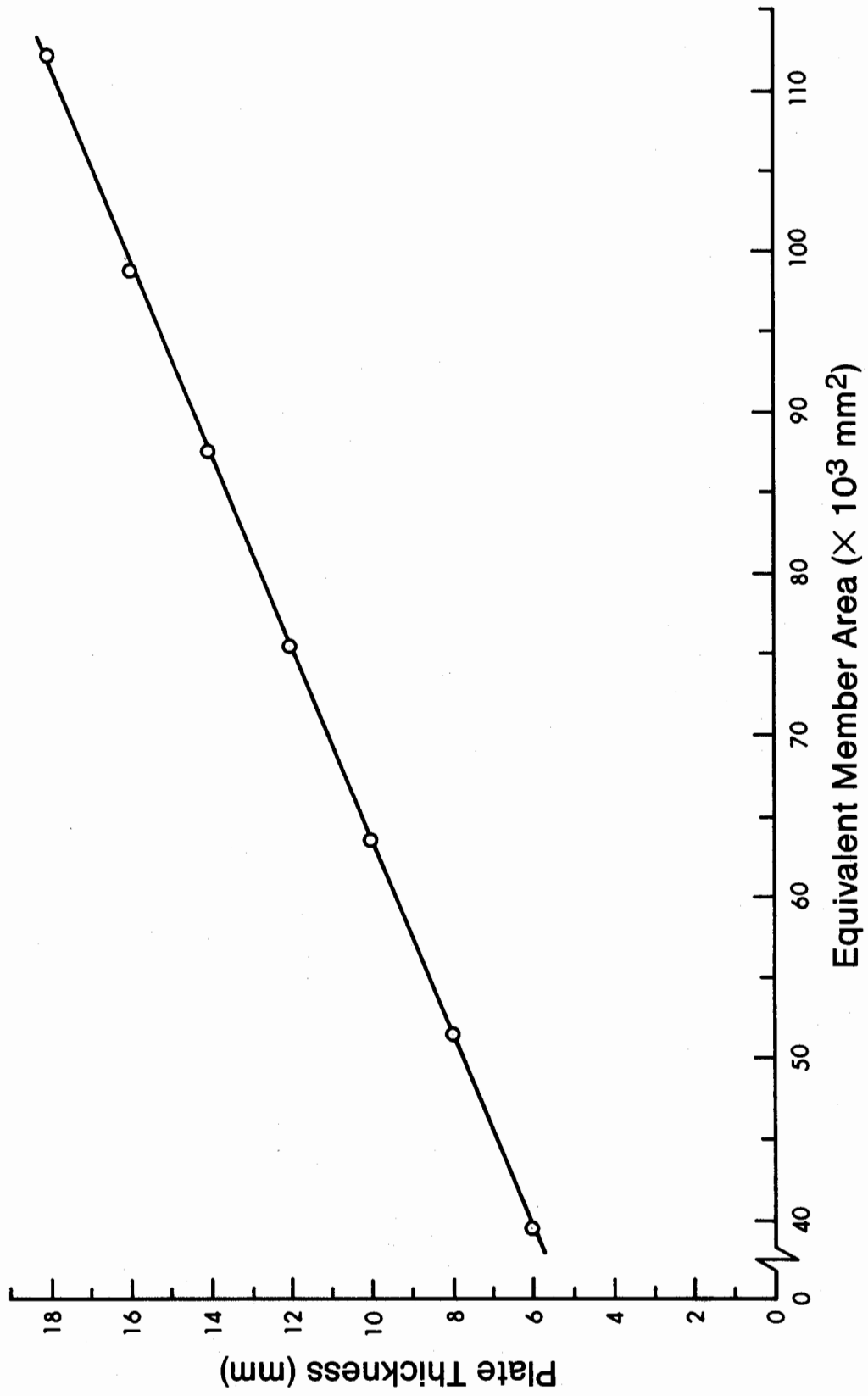
Figure 4.1 Panel Stiffness versus Web Thickness, (Series T1)
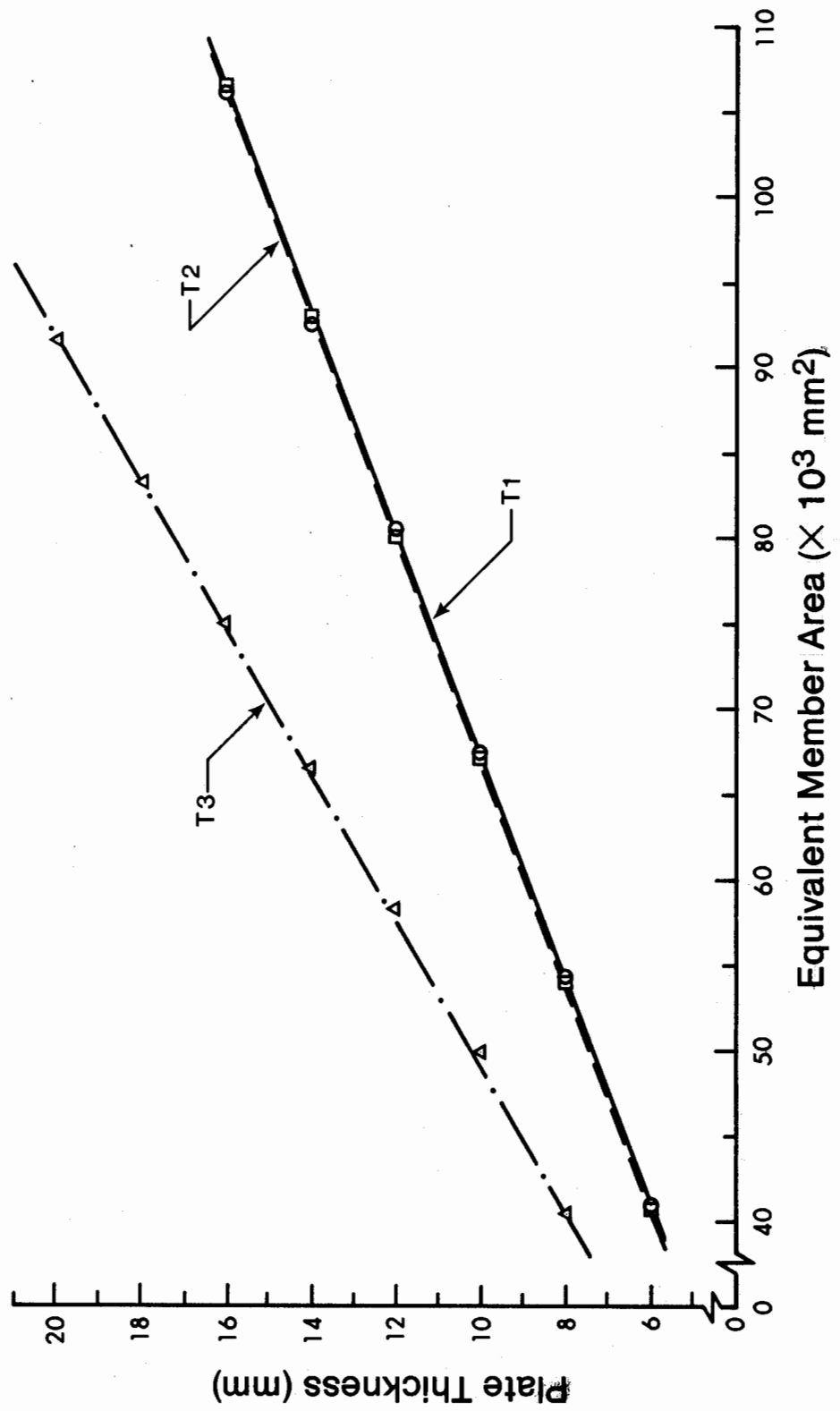
Figure 4.2 Relationship Between Plate Thickness and Panel

Stiffness for Three Column Sizes
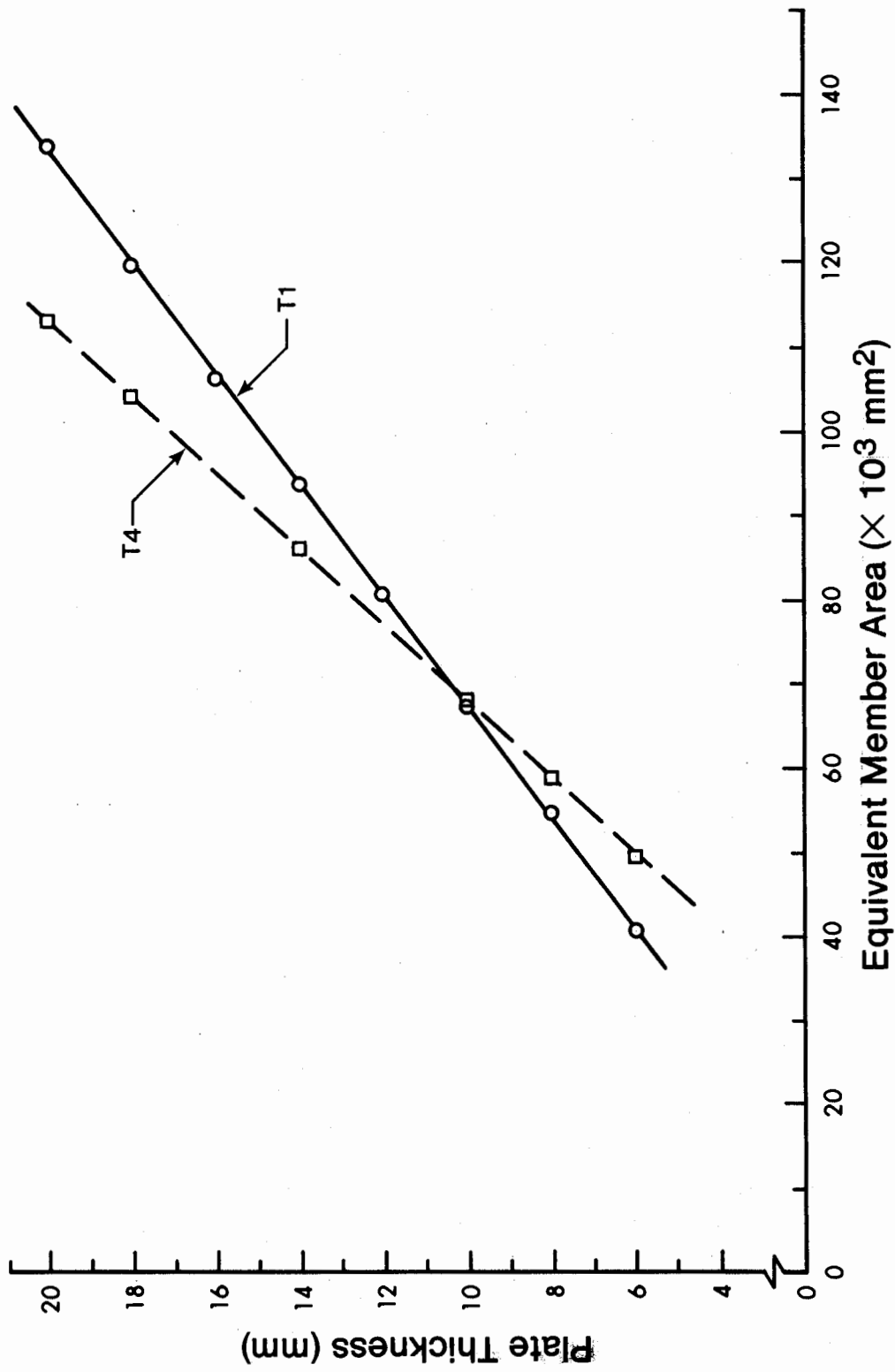
Figure 4.3 Panel Stiffness as a Function of Web Thickness,

Series T1 and Series T4
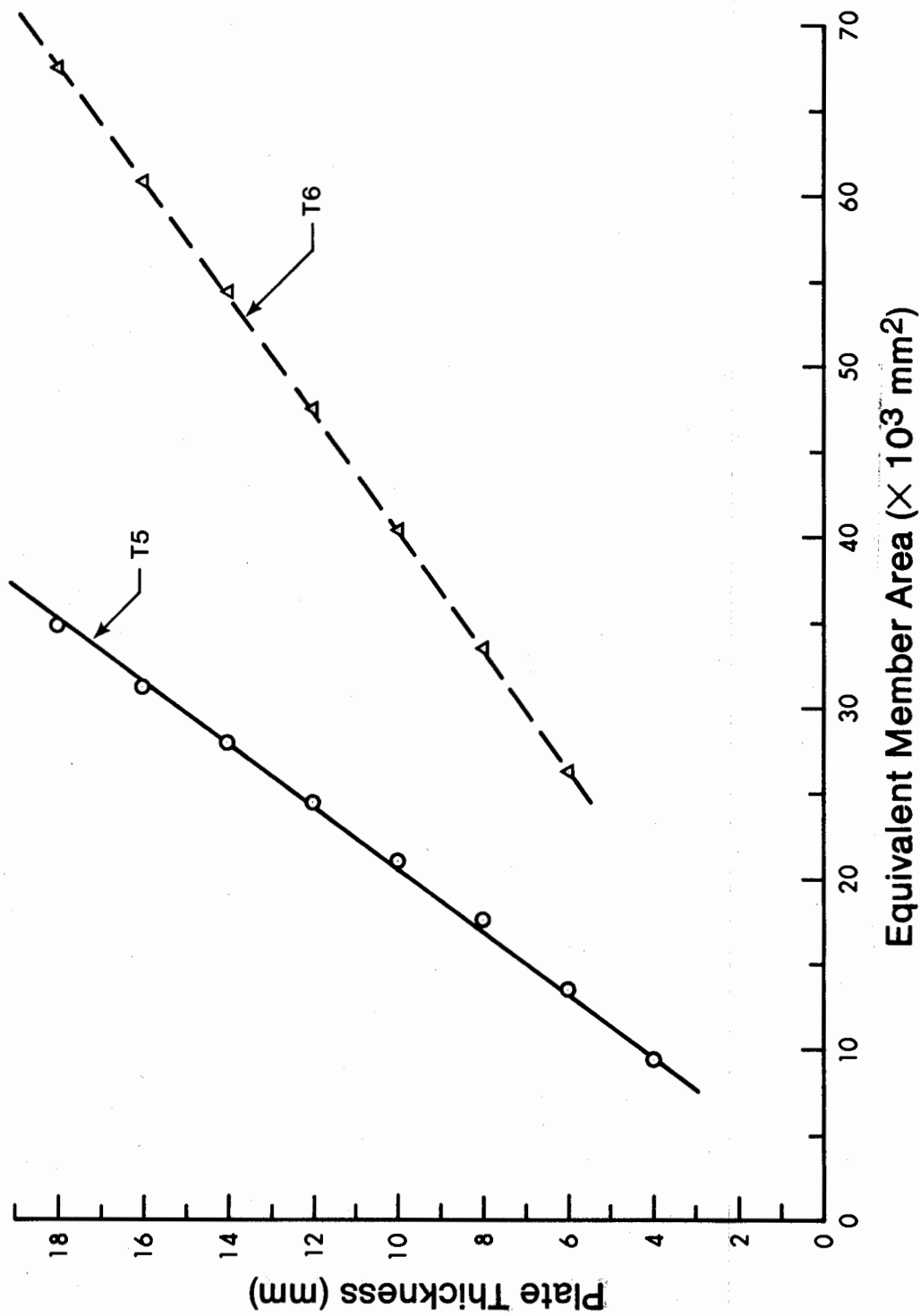
Figure 4.4 Panel Stiffness versus Web Thickness, Series T5
and Series T6
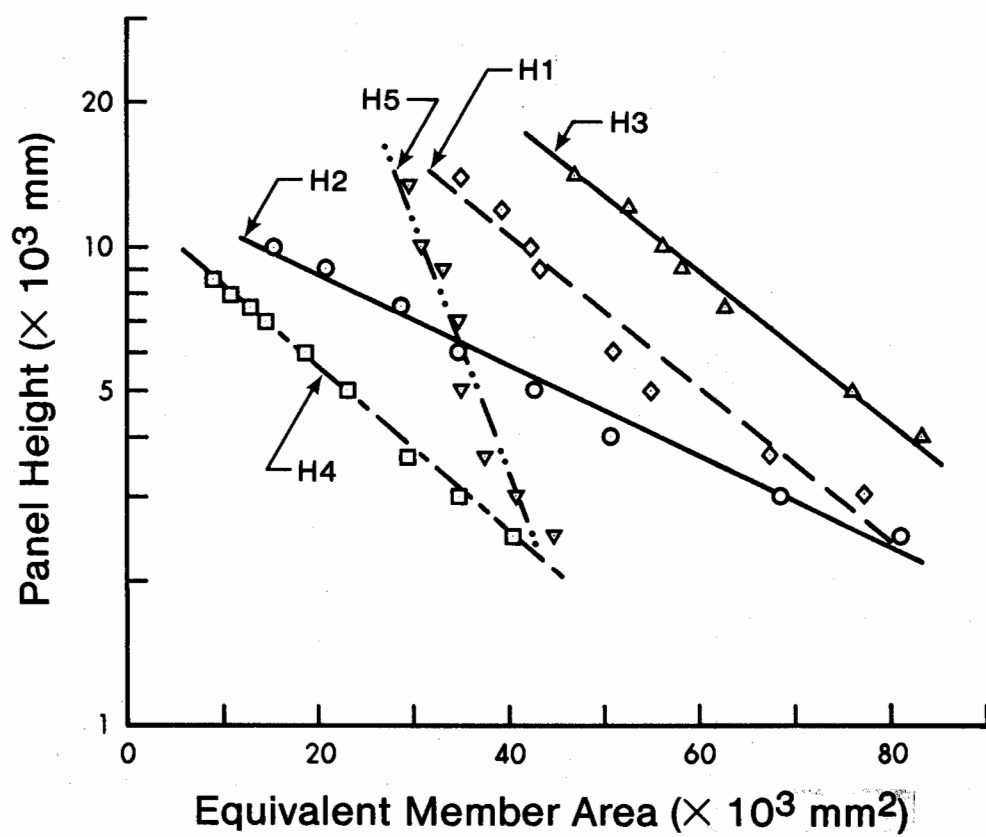
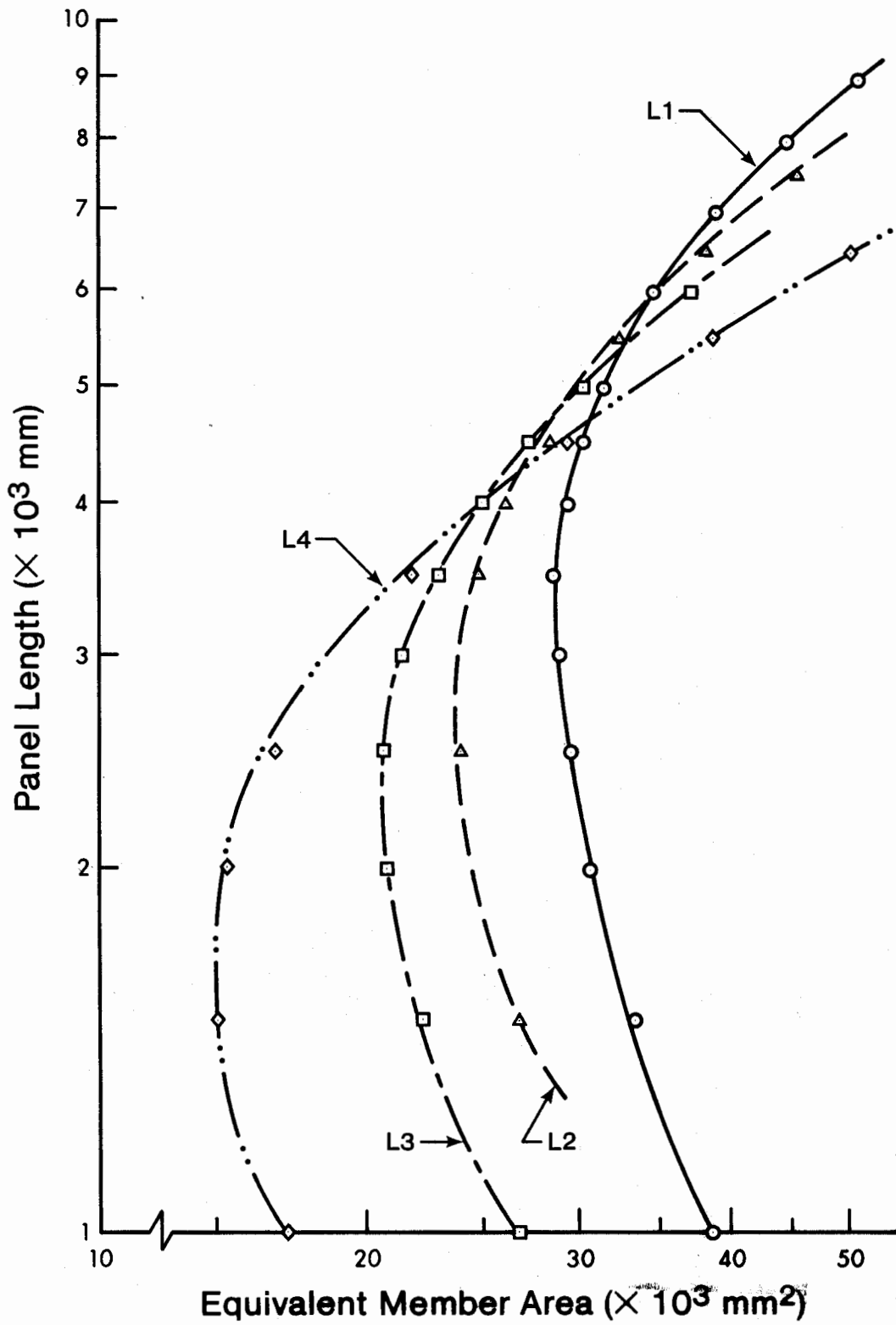Figure 4.5 Panel Stiffness as a Function of Panel Height

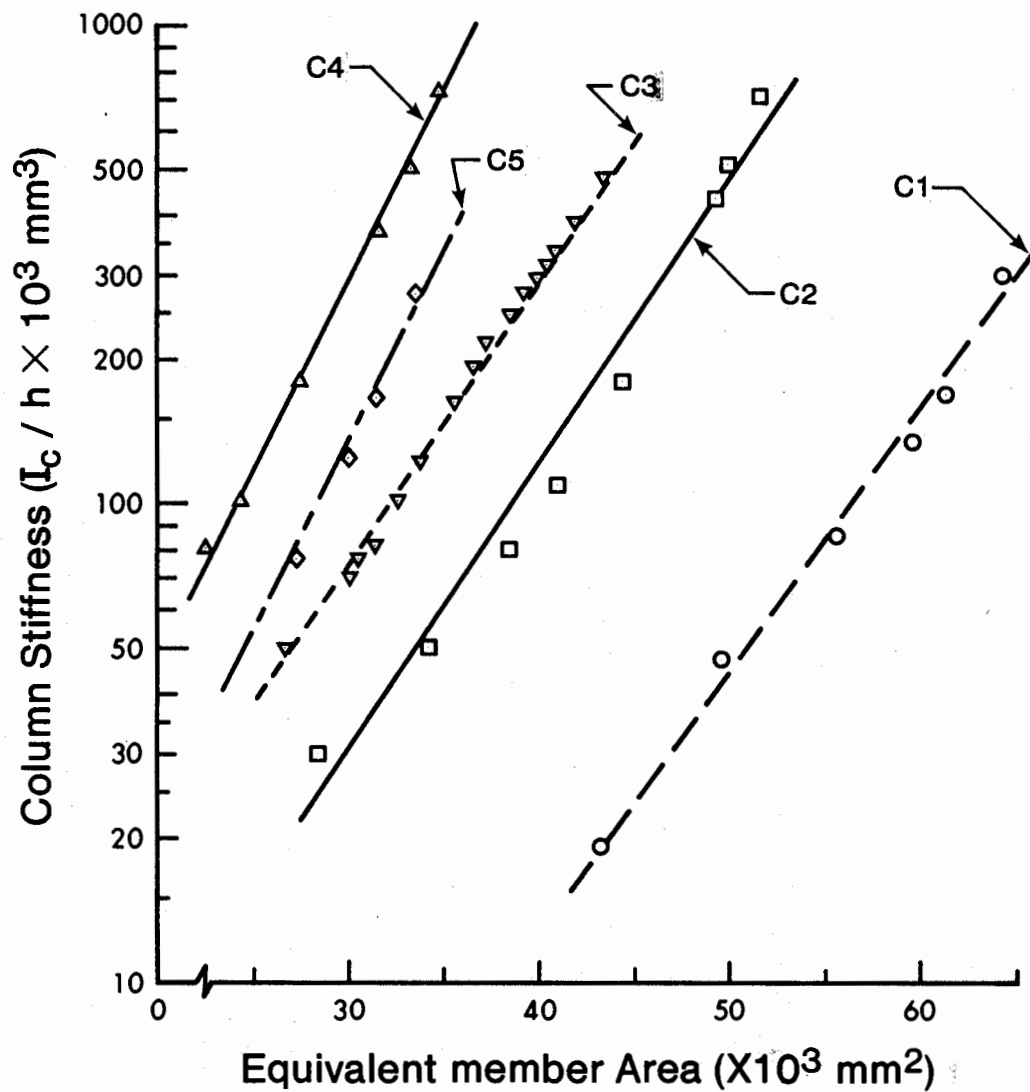Figure 4.6 Influence of Panel Length on Equivalent Member Area

Figure 4.7 Equivalent Member Area as a Function of Column Stiffness
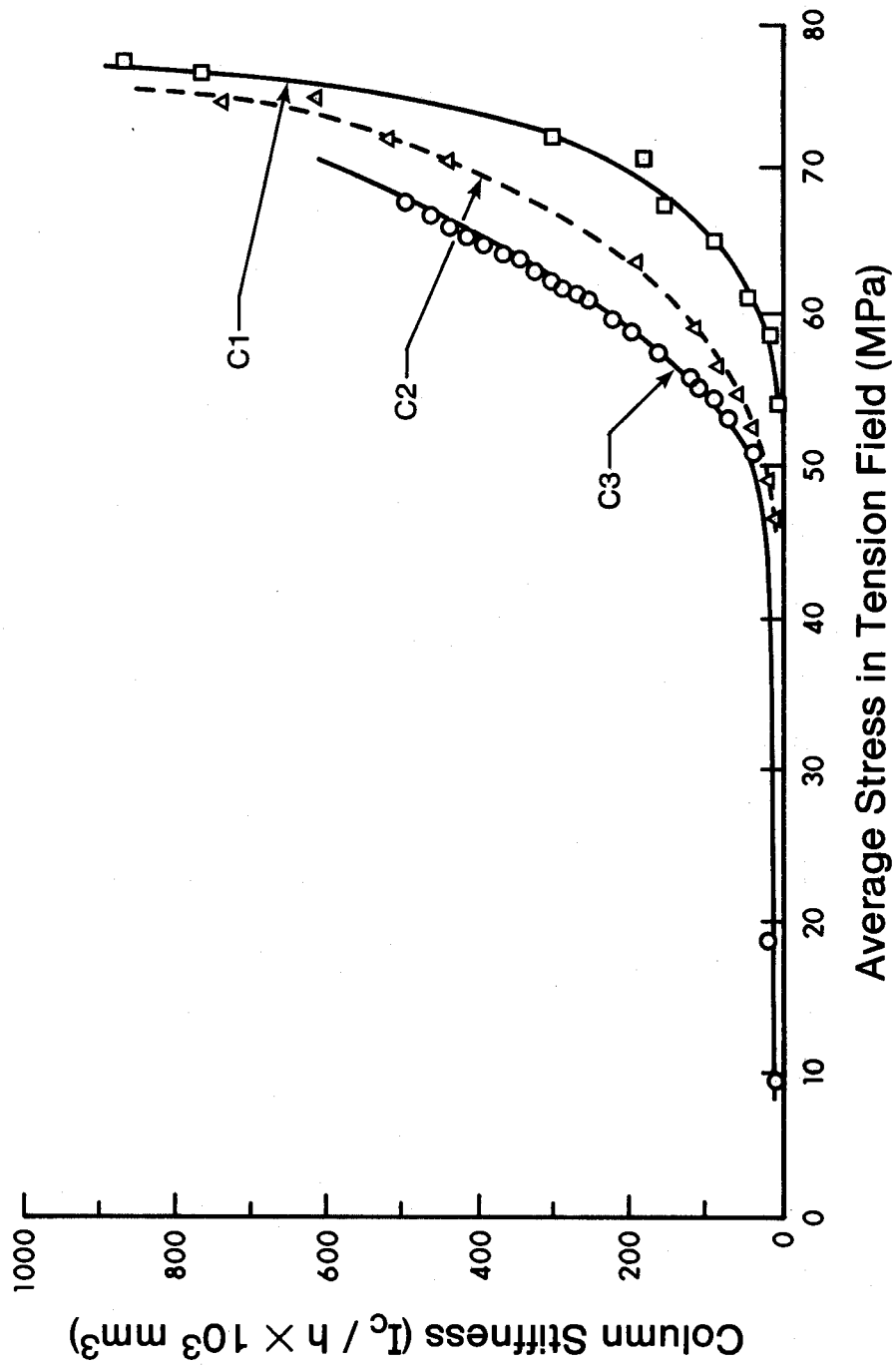
Figure 4.8 Average Stress Resisted by the Web as a Function
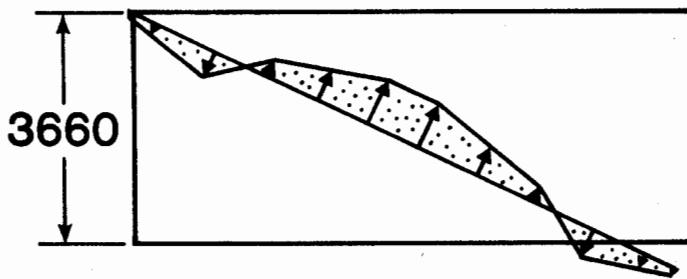of Column Stiffness
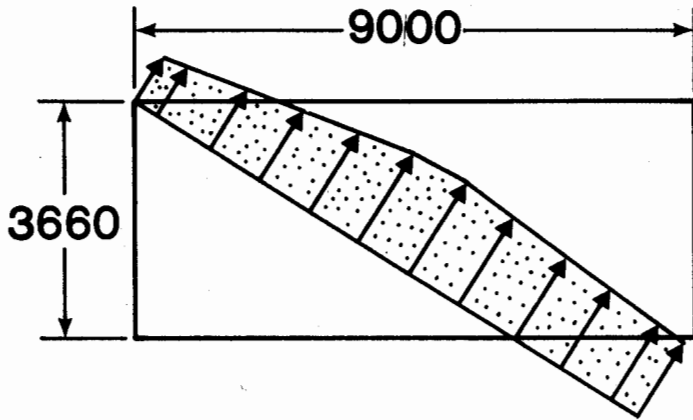
(a)
$I_c = 4390 \times 10^6 \ mm^4$
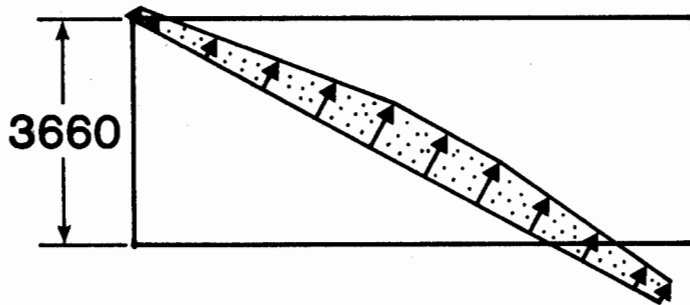
(b)
$I_c = 1100 \times 10^6 \ mm^4$

(c)
$I_c = 308 \times 10^6 \ mm^4$
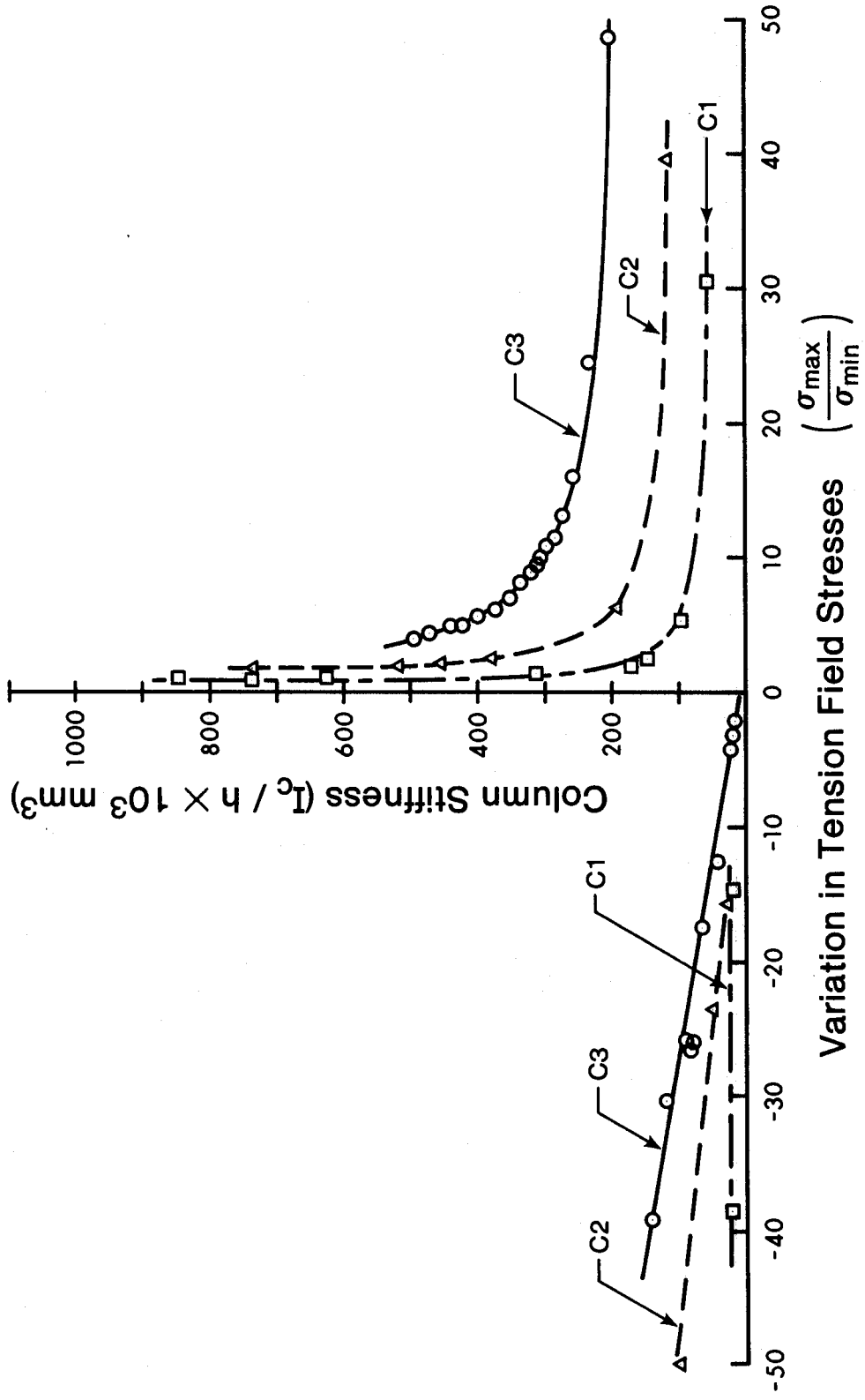
Figure 4.9 Distribution of Forces in the Tension Zone

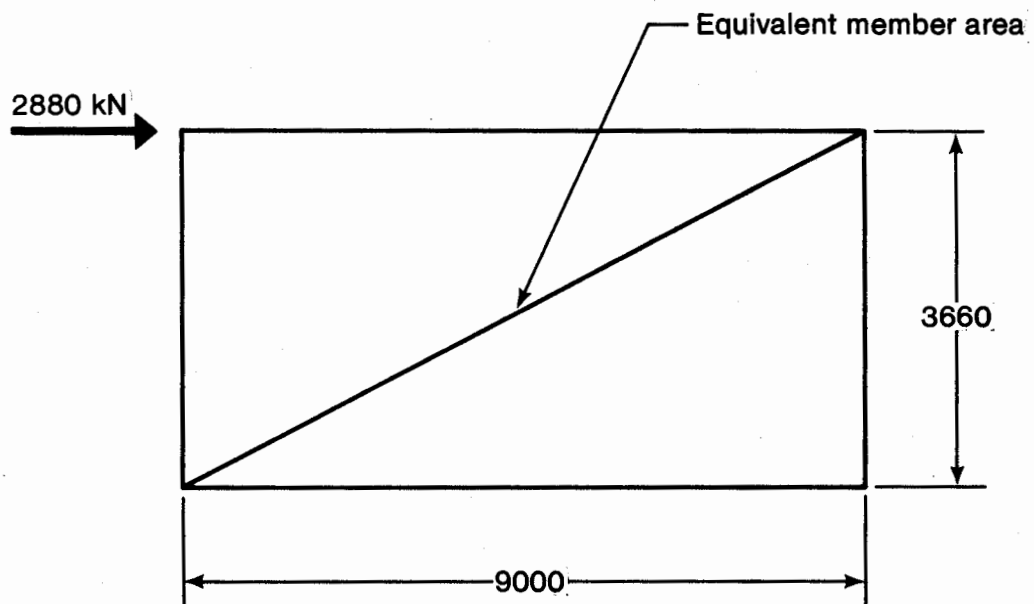Figure 4.10 Column Stiffness versus Variation in Tension

Field Stresses

Figure 4.11 Equivalent Truss Model of Panel

5. Related Design Considerations

5.1 Introduction

Thus far, the report has dealt with only two design considerations; the strength and stiffness characteristics of the panel. Inherent in the design of a steel shear wall core are certain other features which must also be recognized and qualified. Chapter 5 will outline the most important of these characteristics and discuss briefly the considerations relating to their design. The focus in each case will be on providing general information as to the importance of the particular feature, its cause, and the related design provisions. No attempt will be made to quantify a design procedure for each of the specific elements.

5.2 Influence of the Tension Field Action on the Bounding Members

In conjunction with the inclined tensile forces that arise in the web subsequent to buckling, which have been the main emphasis of the previous chapters, additional forces are induced in the beams and columns which surround the panel. These added forces are necessary in order that equilibrium with the tension field forces is maintained and are over and above the forces that result from the applied gravity loads for which the columns and beams are normally

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designed.

The horizontal and vertical components of the inclined web forces are shown in Fig. 5.1. The columns must resist the horizontal and vertical components of the tensile forces in the "strips" which are attached to the columns. Similarly, the beams are loaded with the horizontal and vertical components of the tensile forces that arise in the web adjacent to the beams.

As discussed earlier, the vertical components of the tension fields from the storeys above and below a given beam are nearly equal and act in opposite directions. Hence, their net effect on the beams is negligible. However, the beams must be designed for an axial compressive force that is a result of the lateral load that is applied to the structure. The compressive force can be obtained directly from a strip model analysis. The beam must be designed as a beam-column as this force acts in addition to the usual gravity load effects.

The addition of axial compressive force in the beam could lead to stability problems if sufficient lateral support is not provided. For the most common structural arrangements, this lateral support would be provided by a floor diaphragm attached to the beams. If this is not the case, Kuhn et al (21) hypothesized that the tension field action will provide the necessary restraint. The following explanation provides the basis for this theory as it applies to a steel shear wall panel. If a beam began to buckle out

of the plane of the web, the tension diagonals adjacent to the beam would also move laterally where they were joined to the beam, thus causing the tensile forces to develop components normal to the web. These transverse components would tend to force the beam back into the plane of the web. This creates a distributed load on the beam that is proportional to the out-of-plane displacement of the beam. The solution for the critical buckling load of such a member is a problem in elastic stability (26).

The columns must resist the components of the tension field forces, as shown in Fig. 5.1. The magnitude and distribution of these forces can be obtained from the results of a strip model analysis. The components cause additional axial forces, shears, and moments in the columns which can be calculated directly using the information provided by the strip model. In lieu of this detailed analysis, a general semi-empirical method could be developed analogous to that described by Kuhn et al (21) which accounts for the secondary bending effects due to tension field action on a girder flange.

In summary, the design of the beams and columns adjoining a shear wall web must recognize the additional forces imposed upon them by the tension field action. The column must be designed for additional axial compression and bending, which result from the vertical and the horizontal components of the web forces, respectively, including any secondary effects. The beam design must be altered to

accommodate an additional axial compression.

5.3 Anchor Panels

At the extreme top and bottom panels in a multi-storey shear wall system, the vertical components of the tension field forces which act on the beams are not balanced by equal and opposite forces in an adjacent panel. Thus, the vertical components of the tension field forces must be taken into consideration when designing the exterior beams for these panels.

The vertical components of the tension field at the end of a shear wall stack must either be taken out of the core or resisted internally. The former can be accomplished by providing a rigid element at the extreme top and bottom of the core in order to anchor the inclined stresses in the adjacent panels. At the bottom level of a steel shear core this rigid element could be a large girder, such as is commonly used to connect the superstructure to the foundation. For the top panel, the resisting element could be provided in the form of a vertical truss or a deep girder.

If it is not feasible to include such a rigid element, large distortions will occur in the end panel and the tension field may not develop adequately. Thus, the resistance of the panel would be limited to its buckling strength. This leads to an alternate solution for the design

of these end panels, and that is to proportion the web in
such a way that the shear stresses will not exceed the
critical buckling stress for the panel. To ensure that
buckling will not occur in these "anchor panels" a
restriction must be imposed on the panel dimensions. This is
the approach most commonly taken in the design of a plate
girder (22).

5.4 Connection Details

No matter what the connection detail, the function of
the connection remains the same; to transfer the web forces
from the plate to the surrounding beams and columns. The web
forces can be obtained directly from the member forces in
the inclined bars given in a strip model analysis. The
member forces obtained from the analytical model act over a
finite length of the plate. This length of plate is equal to
the "strip width", which is defined when the plate is
modelled for the analysis. In order to design the
connection, the inclined forces can be resolved into their
horizontal and vertical components. The connection must then
be designed to resist a component of force parallel to the
beam or column and a force component normal to the member.
The conventional methods for designing a bolt or weld in
shear and in tension can then be applied directly.

A variety of combinations of welded and bolted
connections can be used to join a shear wall panel to the

adjoining beams and columns. The following section will describe two possible connection details.

The first connection detail to be described consists of connection (or fish) plates, or angles, shop-welded to the columns and field-bolted to the wall panels. The bolts used in such a system must be designed to provide a friction-type joint, as the formation of the tension field requires that no slip takes place at the connection.

A connection of the type described above was used in one of the Japanese highrises employing a steel shear core that was described earlier (3). A typical connection detail for this structure was shown in Fig. 1.8. Fabrication and construction experience gained during this project showed that there are many problems with the method. The great number of bolts, (200 to 500 per panel, depending on the stress in the panel) required very precise drilling and alignment techniques which resulted in undue time and expense.

A more practical approach is to use all-welded connection, such as is depicted in Fig 5.2. In this system, fish plates are shop-welded to the framing members. In the field, the panel is first connected to the fish plates with a few erection bolts. After final alignment, the panels are field-welded to the fish plates.

5.5 Stiffening Requirements

The method of analysis presented in this report is based on the use of an unstiffened panel, and recognizes the post-buckled strength of the panel. At the present time, it is not recommended for applications where significant load reversals are expected, such as for use in seismic zones. However, the strength and ductility inherent in the steel plates make them suitable for resisting cyclic loads. Under such a circumstance, the plate should be designed to prevent buckling. There are two approaches that can be used to restrict the shear stresses in a steel plate to values less than the critical shear buckling stress. The foundations of these approaches will be outlined in this section.

All of the existing steel shear wall systems that are discussed in Chap. 1 consist of steel plates reinforced with stiffeners. The stiffeners consist of structural members, (channels, T-sections, or angles), welded or bolted to the plates. The stiffeners may be either longitudinal or transverse but their function is the same; to increase the rigidity of the plate. The stiffeners are used to increase the moment of inertia of the section to ensure that buckling does not occur in the service load range. The critical shear buckling stress of the stiffened plate is calculated according to plate stability theory, including the contributions of the stiffeners. The details of such stability problems for horizontally and vertically stiffened plates are discussed by Timoshenko and Wang, respectively

(26,13).

Two considerations that are of prime importance in producing an economically designed steel shear wall web are the efficient use of materials and labour. The method discussed above, advocating the use of a thin plate stiffened with welded ribs, minimizes the amount of material used in the web to carry a given shear stress. However, the design is very labour intensive, as the connection of stiffeners to the plate adds significantly to the time required to fabricate the panel. This aspect of the design becomes of paramount importance in the North American market where labour costs are high. Under such circumstances, the use of an thicker unstiffened plate may be a more desirable alternative.

For applications where plate buckling is deemed undesirable, the two methods described above to increase the shear resistance for a given panel geometry should be evaluated in order to determine the most economical design.
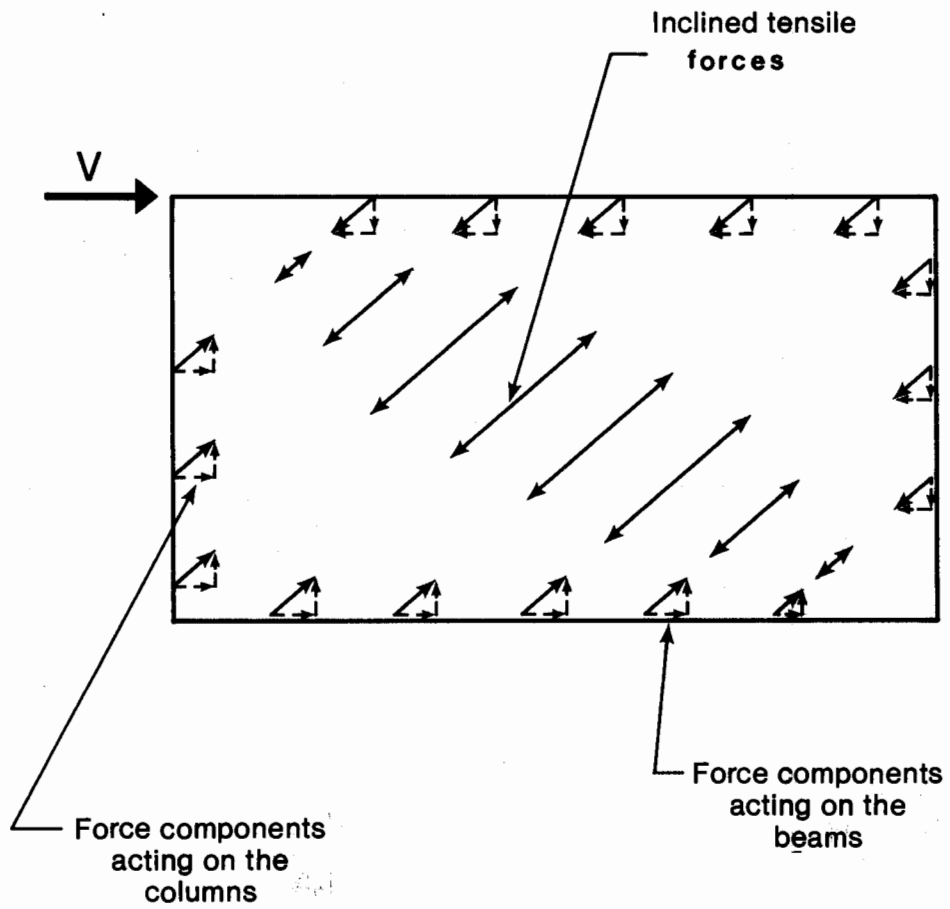
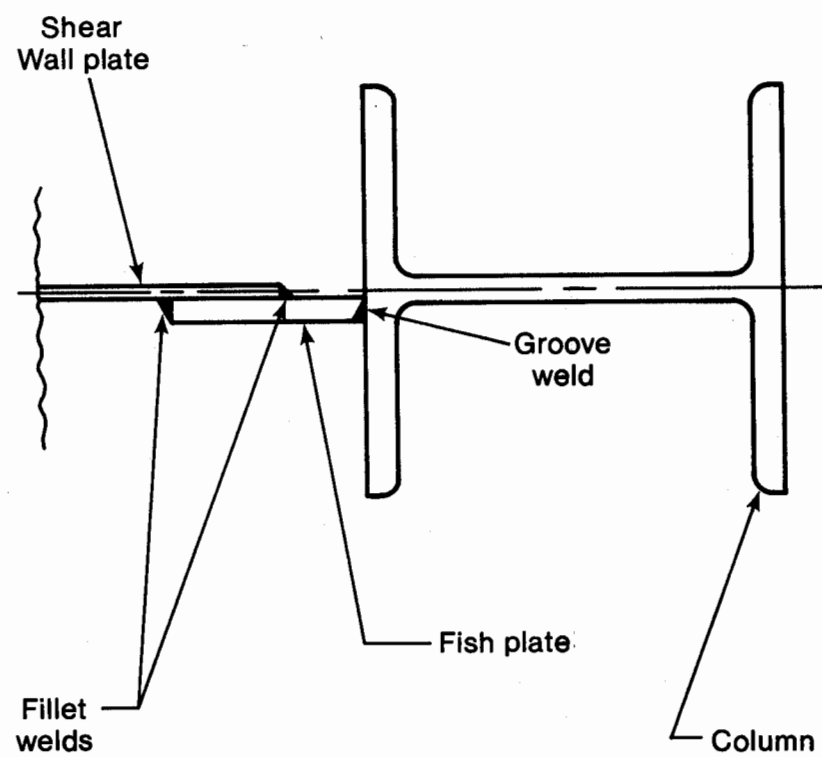Figure 5.1 Components of Inclined Web Forces Acting on Beams and Columns

Figure 5.2 Connection Detail Used to Join Steel Shear Wall
Web to Surrounding Beams and Columns

6. Summary, Conclusions and Recommendations

6.1 Summary

The steel shear wall panels that are currently in use in highrise structures have all been designed as stiffened plates, ensuring that the shear stresses developed in the plate are less than the critical shear buckling stress. This approach completely neglects the strength which is known to develop subsequent to buckling. The study reported herein presents a theoretical approach to this problem which recognizes the post-buckled strength inherent in a steel plate and develops a complementary method of analysis for steel shear walls on this basis.

An analytical model was developed to study the shear resistance provided by a buckled web functioning as a tension field. The tension zone was represented as a series of inclined tensile bars, orientated at the same inclination as the diagonal tension stress in the web. This model provided a means to study the transfer of forces and the resulting stress distribution in a thin steel web. It does not account for any shear carried by the web prior to buckling. In addition, because of the stiff boundaries surrounding the web it is also assumed that a more complete tension field will develop in a shear wall panel than the partial tension field that is assumed to occur only between the stiffeners in plate girder design.

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Using the analytical model, parametric studies were undertaken to determine the influence of the following factors on the strength and stiffness characteristics of a shear wall panel:

a.   column stiffness

b.   web thickness

c.   panel dimensions

The results of the investigation show that column stiffness influences the strength and stiffness characteristics of a shear wall panel in two ways. An increase in column flexibility causes additional lateral column deflection as well as more pronounced bending of the column into the web. It was also found, for given panel dimensions, that the load-carrying capacity of the panel could either be increased by using a thicker web plate or by increasing the column stiffness. In addition, panel height and length were found to have individual influences on the panel characteristics which could not be expressed in terms of a panel aspect ratio.

General information pertaining to certain other features which relate specifically to the design of steel shear wall panels, such as stiffening requirements and connection details, is provided in Chap.5.

6.2 Conclusions

1. It is overly conservative to neglect the post-buckling strength of a shear wall web bounded by columns and beams. An unstiffened steel panel of usual dimensions for building construction will buckle almost immediately upon being loaded. The study showed that strength of the post-buckled tension field is the primary stress-resisting mechanism, and that the shear resistance prior to buckling can be neglected.

2. An investigation of the post-buckling capacity of a panel intended for use in circumstances where load reversals are significant, such as would occur under earthquake loading, did not constitute part of this study. Under such cyclic loads, the performance of a buckled plate may not be satisfactory.

3. The angle of inclination of the diagonal tension forces in a shear wall web is a function of the column and beam areas, the panel dimensions, and the web thickness.

4. The tension field that develops in a buckled shear wall can be modelled analytically by dividing the web into a series of strips of equal width. Each strip is then treated as a planar truss member, capable of transmitting only axial loads.

5. The overall stiffness of a shear wall panel bounded by columns in the size range that is normal for a multi-storey structure falls between two limiting theories, namely that for plates bounded by flexible

columns and that for plates bounded by infinitely stiff columns. These theories are based on studies of post-buckled webs for aircraft structures and steel plate girders, respectively. To effectively model the shear wall panel, the effect of the column flexibility must be included.

6. A Pratt truss model can be developed in order to simplify the stiffness calculations. The diagonal member in this equivalent truss model represents the stiffness characteristics of the tension field in the web and its area is a function of the panel geometry and the stiffness of the perimeter members.

7. A one-storey model is representative of a typical storey in a multi-storey core if certain constraints are imposed. The beams in the one-storey model must be considered as infinitely rigid, joint rotations must be set at the beam-to-column connections, and the applied lateral load is the storey shear.

8. For a given panel, panel stiffness is uniquely related to certain parameters. The relationships are:

   a. An increase in plate thickness results in a proportionate increase in panel stiffness.

   b. As panel height is increased, lateral stiffness of the panel is reduced. There is a linear relationship between the log of the panel height and panel stiffness.

   c. Panel stiffness increases linearly as panel length

increases for values of tan $L/h > 45°$.

d. An increase in the logarithm of column rigidity results in a linear increase in panel stiffness.

9. The stress distribution within the tension field is uniform only for the hypothetical case of infinitely stiff columns. As column flexibility increases, the variation in tensile forces across the web increases. Maximum stress occurs at mid-panel and the stress decreases toward the panel edges. For very flexible columns, compressive stresses will arise in the region of the panel corners, further reducing the effectiveness of the web.

10. For the assumed uniaxial state of stress in the web, the usable strength of the web is a function of the tensile strength of the steel. For a panel that meets the drift requirements, the maximum stress in the panel will most likely be less than the permissible stress.

6.3 Recommendations

1. The results obtained from the analytical study should be compared with deflection and stress values obtained experimentally. A testing program should also be employed to investigate the validity of neglecting the shear resistance that is provided prior to buckling and examine the effect of load reversals on the tension field action.

2. Additional verification of the stress field that exists in the plate should be obtained by a comparison with a finite element analysis of the web and the surrounding boundaries.

3. The parametric studies should be extended in order to prepare a set of design curves which would enable a designer to proportion a shear wall panel and the bounding columns to meet the drift requirements.

4. More study is required to develop a method for predicting the magnitude and distribution of the web stresses for a given set of panel characteristics. This would eliminate the need for using a "strip" model analysis when designing the connection between the plate and the surrounding members.

5. As discussed in Chap. 5, the tension field stresses cause additional forces and moments in the surrounding members. In addition, column flexibility causes secondary bending moments. This effect should be quantified.

6. More complex three-dimensional core arrangements, such as channels, T-sections, and I-sections, should be investigated, as should coupled steel shear cores and panels with holes.

REFERENCES

1. Troy,R.G. and Richard,R.M., *Steel Plate Shear Walls Resist Loads, Cut Costs*, Civil Engineering-ASCE, Vol.49, Feb.1979.

2. Takahashi,Y., Takeda,T., Takemoto,Y., Takagi,M., *Experimental Study on Thin Steel Shear Walls and Particular Steel Bracings Under Alternative Horizontal Load*, Preliminary Report of I.A.B.S.E. Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well-Defined Repeated Loads, Lisbon,Portugal,1973.

3. *Nippon Steel Building at Urban Renewal of Tokiwabashi District*, Private Correspondence with CISC, Jan.1980.

4. Chien, E.,*Steel Plate Shear Wall Structures in Tokyo,Japan*, Private Correspondence with CISC, Jan.1980.

5. *A Guide to the Structure of B-Class Japanese Buildings*, Private Correspondence with CISC, Jan.1980.

6. *Structural Design of Shinjuku Nomura Tower*, Notes from a meeting of ASCE-IABSE Joint Committee for Planning and Design of Tall Buildings, Tokyo,Japan, Sept.1976, Private Correspondence with CISC, Jan.1980.

7. *Patent Problems, Challenge Spawn Steel Seismic Walls*, Engineering News Record, Jan. 26,1978.

8. *Hospital Steel Plate Shear Walls were Designed for a 0.69g Earthquake*, Architectural Record, Mid-August 1978.

9. *Quake-Proof Hospital has Battle-Ship Like Walls*, Engineering News Record, Sept. 21,1978.

10. *Steel Plate Shear Walls Blunt the Wind's Force and Carry Gravity Load in a Towered Hotel*, Architectural Record, Mid-August 1978.

11. *Shear Walls and Slip-Forming Speed Dallas' Reunion Project*, Engineering News Record, July 28, 1977.

12. Bleich,F.,<u>Buckling Strength of Metal Structures</u>, McGraw-Hill Book Company, Inc., New York, N.Y.,1952.

13. Wang,T.K.,*Buckling of Transversely Stiffened Plates Under Shear*, Journal of Applied Mechanics, Vol. 14, 1947.

14. Bathe,K.J., Wilson,E.L., and Peterson,F., <u>SAPIV, A Structural Analysis Program for Static and Dynamic Response of Linear Systems</u>, Earthquake Engineering Research Centre, Report No. EERC 73-11, College of Engineering, University of California, Berkeley, California, 1974.

15. *California Hospital is Quake-Proof, High-Rise*, Engineering News Record, August 10,1978.

16. *Hospital Design Meets Tough Seismic Code*, Engineering News Record, April 21,1977.

17. *Hospital's Bracing is Eclectic Mix*, Engineering News Record, Dec.24,1981.

18. Popov,E.P., <u>Mechanics of Materials</u>, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1978.

19. Salmon,C.G., and Johnson,J.E., <u>Steel Structures: Design and Behaviour</u> Second Edition, Harper and Rowe, Publishers, New York, N.Y., 1980. Chapter 6, Part II.

20. Wagner,H., *Flat Sheet Metal Girders with Very Thin Metal Webs. Part I--General Theories and Assumptions* , National Advisory Committee for Aeronautics, Technical Memo., No. 604, 1931.

21. Kuhn,P., Peterson,J.P., and Levin,L.R., *A Summary of Diagonal Tension, Part I--Methods of Analysis*, National Advisiory Committee for Aeronautics, Technical Note 2662, 1952.

22. Basler,K., *Strength of Plate Girders in Shear* , Journal of the Structural Division, ASCE, Vol. 87, No. ST7, Proc. Paper 2967, Oct. 1961.

23. American Institute of Steel Construction, <u>Specification for the Design, Fabrication, and Erection of Steel Structures for Buildings</u>, New York, N.Y., 1978.

24. CSA CAN3-S16.1-M78, <u>Steel Structures for Buildings--Limit States Design</u>, Canadian Standards Association, Rexdale, Ontario, 1978.

25. Johnston,B.G., editor, <u>The Structural Stability Research Council Guide to Stability Design Criteria for Metal Structures</u>, Third edition, John Wiley & Sons, Inc., New York, N.Y., 1976.

26. Timoshenko,S., <u>Theory of Elasticity</u>, McGraw-Hill Book Co. Inc., New York, N.Y., 1936.

APPENDIX A--Derivation of Angle of Inclination of Diagonal Tension Stress

The aim of the following investigation is to develop a relationship for the angle of inclination of the diagonal tension stress in a buckled steel plate shear wall. The angle, α, is shown in the following diagram as the angle between the columns and the line of action of the inclined tensile forces that result from an applied shearing force, V.
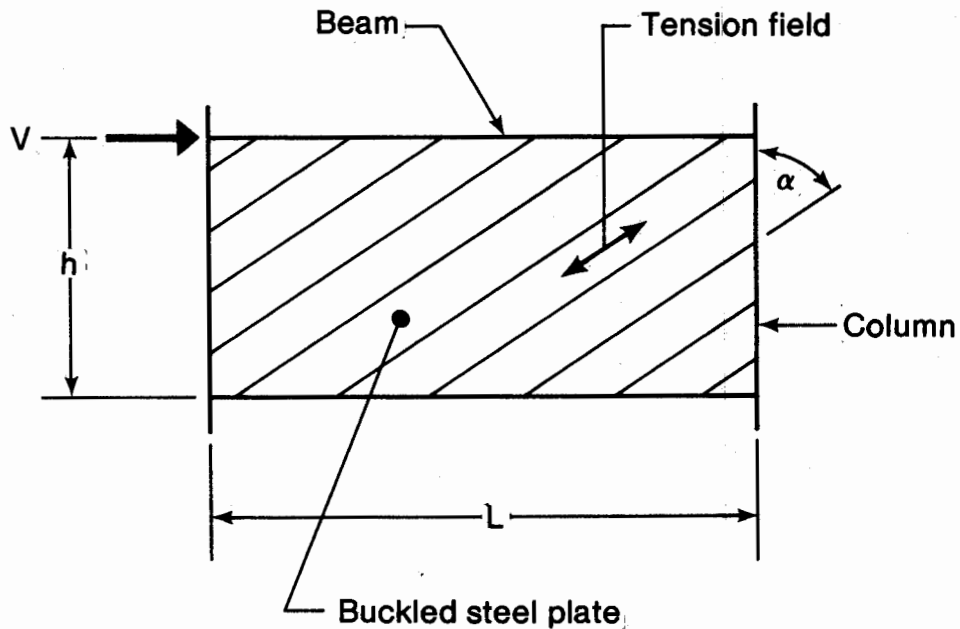
Figure A.1 Analytical Model of a Buckled Steel Shear Wall Web

A one-storey steel wall panel, one-bay wide, framed by beams and columns was considered. It is assumed that the steel plate buckles immediately upon being loaded. Therefore, any shear resisted prior to web buckling is ignored. The axial flexibility of the beams and columns is taken into account. However, as was the case in the original derivation (21), the development presented herein does not account for the bending stiffness of the bounding members. Thus, it is not strictly valid for any web that is not bounded by completely rigid beams and columns.

First, an expression for the total internal work done in the system when acted upon by an external shearing force, V, is written. Using the principle of least work, the expression for the internal work is then differentiated with respect to α and the result set equal to zero to find the critical value of α. The internal work done on one storey is the total work done by the web, the beams, and the columns. Each of these three work components will be evaluated separately, and then the results will be summed to find the total work done.

Consider the work done by the web. For analysis purposes, the web is divided into a series of strips, inclined at the same angle as the diagonal tension, and the stress in each strip is found as follows.
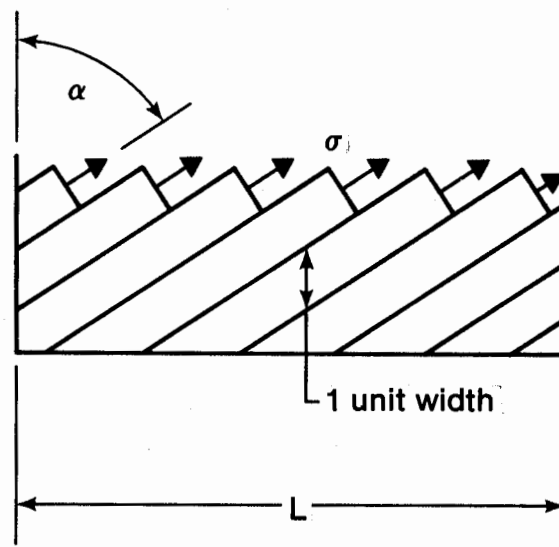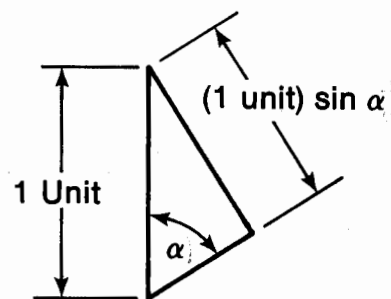
Figure A.2 Strip Representation of a Shear Web

Figure A.3 Geometry of a Unit Strip

Strip area = w x (1 unit)(sin α)

Force in each strip = stress in strip x area of strip

$$= \sigma \times w \ (1 \ unit)(sin \ \alpha)$$

Horizontal length of one strip = (1 unit) x (sin α / cos α)

Number of strips in length L is:

    = (L units) / (1 unit x sin α / cos α)

    = L x (cos α / sin α)

Resultant force for all the strips in length L is:

    R = σ x w x sin α (L cos α / sin α)

    R = σ x w x L x cos α

Horizontal component of R:
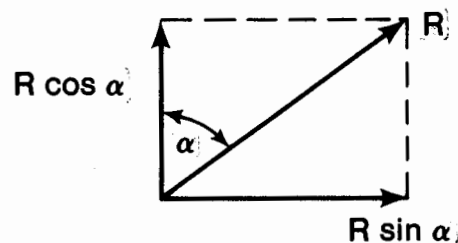
Figure A.4 Components of Total Web Force

The horizontal component of the inclined web force is in equilibrium with the applied horizontal shearing force, V. Equating these two forces;

    V = R sin α = (σ x w x L x cos α) x sin α........(A.1)

Rearranging the equation gives an expression for the stress in each strip:

$$\sigma = V / (L \times w \times \cos \alpha \times \sin \alpha)$$

The general expression for the internal work can be written as:

$$W = 1/2 \quad \sigma \times \epsilon \, dV$$

$$= \int_V 1/2 \times \sigma \times \text{area} \times \epsilon \times \text{length}$$

$$= \sum (\sigma^2 / 2E) \times \text{area} \times \text{length} \dots \dots \dots \dots (A.2)$$

Consider an unit area of the web, having a thickness w, as shown below:
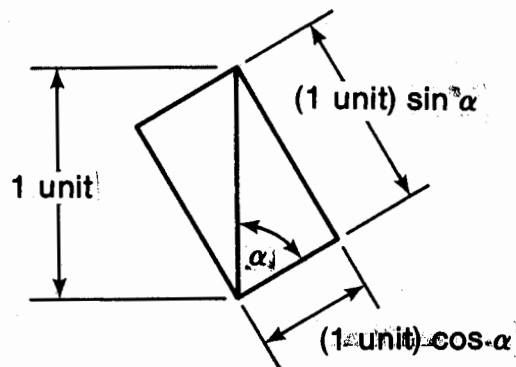
Figure A.5 Geometry of Unit Area

$$\text{unit area} = (1 \text{ unit}) \sin \alpha \times (1 \text{ unit}) \cos \alpha$$

Work for unit area of the web is:

$$W = \sigma^2 / 2E \times (1 \text{ unit}) \sin \alpha \times (1 \text{ unit}) \cos \alpha \times w$$

$$\text{Work for unit area} = (\sigma^2 / 2E) \times w$$

Work for the entire web is:

$$W = (\sigma_w^2 \times w \times L \times h) / 2E \dots\dots\dots\dots\dots\dots\dots (A.3)$$

Next consider the work done by the axial forces in the two columns that form the vertical boundaries to the panel on each side. Assuming that the vertical component of the tension field resultant force (Fig. A.4) will be resisted equally by the two columns, the force in each column is:

$$F = 1/2 \, R \cos \alpha$$

Using $V = R \sin \alpha$, from Eq. A.1, this becomes:

$$F = (V \cos \alpha) / (2 \sin \alpha)$$

(It should be noted that these column forces are approximate values only as the vertical component of the web force which acts along the columns has been neglected. If these forces were taken into account the axial forces in the columns would not be constant but would vary from Vh/L to $Vh/L + V/2 \tan\alpha$.)

The corresponding stress in the columns is:

$$\sigma_c = (V \cos \alpha) / (2 \times A_c \times \sin \alpha)$$

(where Ac is the area of one column)

Substituting this value for stress into the general equation for internal work, (Eq. A.2), results in the the following expression for the work done by the columns:

$$W = (\sigma_c^2 \ / \ 2E) \times Ac \times h \times 2 \text{ columns}$$
$$= (\sigma_c^2 \ / \ E) \ Ac \times h \dots\dots\dots\dots\dots\dots\dots\dots\dots (A.4)$$

Lastly, the contribution of the beams to the total work is evaluated. Consider the free body diagram of the web shown in the following diagram:
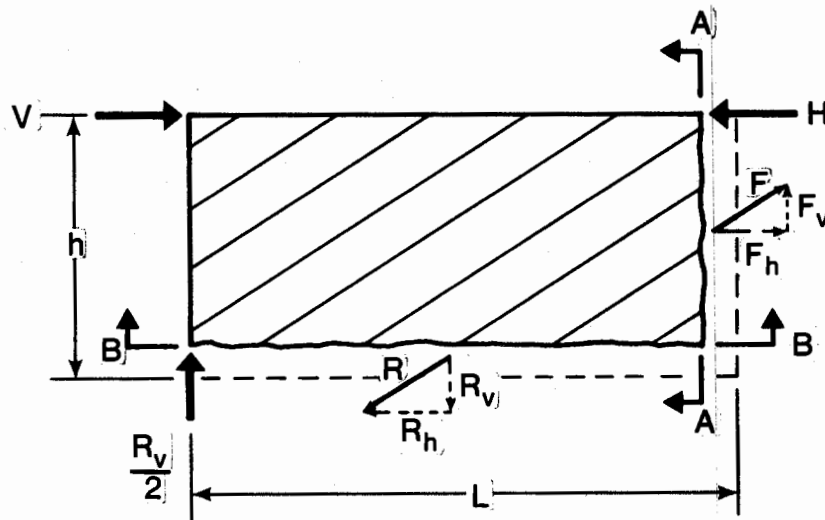
Figure A.6 Free Body Diagram of Portion of Web

The resultant of the inclined strip forces acting on the vertical cut A-A is computed as follows:

F = stress in a strip x strip area x number of strips

$\quad$ = [V/(L x w x cosα x sinα)]x w(1 unit)sinα x h/(1 unit)

$\quad$ = (V x h) / (L x cosα)

The horizontal component of the force is:

$\quad$ Fh = (V x h x sinα) / (L cosα)

The horizontal component of the strip forces acting on the horizontal cut B-B is the same as that developed in Eq. A.1, i.e.,

$\quad$ Rh = R sin α = V

Writing an equation for equilibrium of the horizontal forces the force in the beam, H, can be obtained:

$\quad$ H = V - Rh + Fh

$\quad\quad$ = V - V + (V x h x sin α) / (L cos α)

$\quad\quad$ = (V x h x sin α) / (L cos α)

(It should be noted that if the variation in the axial forces in the columns had been included then the axial force in the top beam would also vary. In this case the axial force in the beam would vary linearly from V + Vhtanα/2L at the left end to Vhtanα/2L at the right end.)

Axial stress in beam:

σ_b = (V h sin α) / (L cos α) Ab

(where Ab is the cross-sectional area of one beam)

Work done by the beam = (σ_b^2 / 2E) x Ab x L . . . .(A.5)

Adding the three work components, (Eqs. A.3, A.4, and A.5), gives the total work done by the panel:

Total Work = [(σ_w^2 / 2E) x w x L x h]

+[(σ_c^2 / E) x Ac x h]

+[(σ_b^2 / 2E) x Ab x L]

In terms of forces, the work equation can be written as:

W = 1/2E [(V) / (L x w x cosα sinα)]²x L x h x w

+1/2E [(V x cosα) / (2 x Ac x sinα)]²x Ac x h x 2

+1/2E [(V x h x sinα) / (Ab x L x cosα)]²x Ab x L..(A.6)

Further simplification of Eq. A.6 results in:

W = [(V² x h) / (2E x L x w x cos²α sin²α)]

+[(V² x h x cos²α) / (4E x Ac x sin²α)]

+[(V² x h² x sin²2α) / (2E x L x Ab x cos²α]

Setting the derivative of the work expression with respect to α equal to zero and solving, results in the following relationship for α:

$$\tan^4 \alpha = [1 + (L \times w) / 2\, Ac] / [1 + (h \times w) / Ab] \ldots\ldots(A.7)$$

APPENDIX B-- Derivation of Equivalent Storey Stiffness

I. Equivalent Storey Stiffness of a Diagonal Brace Element

The purpose of the following section is to develop the relationship between an applied shearing force, V, and the stiffness of a panel of a Pratt truss. Consider a panel, as shown in Fig B.1, having horizontal and vertical members that are infinitely stiff to ensure that the total lateral deflection of the panel accrues from the extension of the diagonal brace.
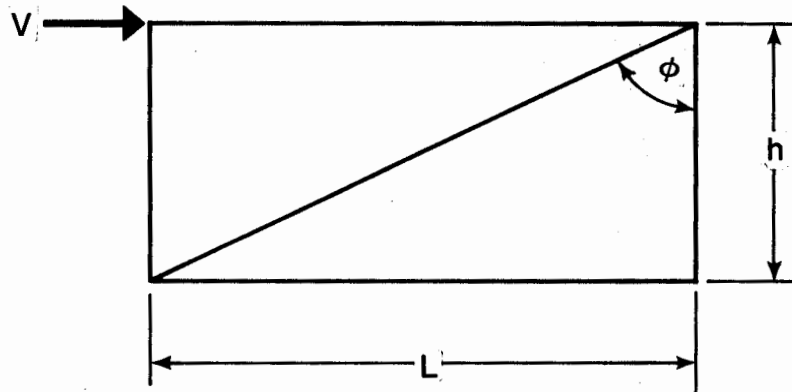

Figure B.1 Equivalent Truss Model of Wall Panel

When a horizontal shearing force, V, is applied to the structure, the resulting force in the inclined member is $V / \sin \phi$. Therefore, the stress in the brace is:

$$\sigma = V / A \sin \phi \dots\dots\dots\dots\dots\dots\dots\dots(B.1)$$

where A represents the brace area.

Rearranging the expression:

$$V = \sigma A \sin \phi \dots\dots\dots\dots\dots\dots\dots\dots(B.2)$$

For a storey displacement, Δ, the bar extension can be expressed as:

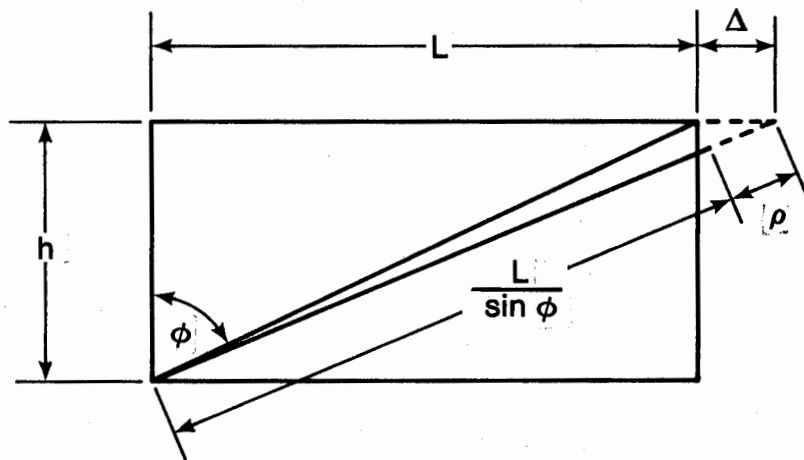$$\rho = \Delta \sin \phi \dots\dots\dots\dots\dots\dots\dots\dots(B.3)$$

Figure B.2 Bar Extension due to Storey Displacement

Noting that $\epsilon = \rho / \text{length} = \rho \sin \phi / L$ and substituting for ρ from Eq. B.3:

$$\epsilon = (\sin^2 \phi \times \Delta) / (L) \dots\dots\dots\dots\dots\dots(B.4)$$

Substituting this result into the relationship between stress and strain ($\sigma = E \times \epsilon$);

$$\sigma = [(E \times \sin^2 \phi \times \Delta) / (L)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B.5)$$

Substituting this relationship for stress into Eq. B.2:

$$V = (A E / L) \sin^3 \phi \times \Delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B.6 \text{ a})$$

Since $\sin \phi = L / (L^2 + h^2)^{1/2}$, this can be written as:

$$V = (A \underbrace{E / L)[L /(L^2 + h^2)^{1/2}]^3}_{} \times \Delta \ldots\ldots\ldots\ldots\ldots (B.6 \text{ b})$$

Equivalent storey stiffness for the brace

II. Equivalent Storey Stiffness of a Fully Developed Tension
    Field

By developing an expression for the equivalent storey
stiffness of a fully developed tension field and equating it
to the storey stiffness of a diagonal brace element, as was
derived in Part I, (refer to Eq. B.6), the area of a truss
element having the same storey stiffness as a truss element
can be found.

For a fully developed tension zone to develop, as shown
in the following diagram, the beams and columns bounding the
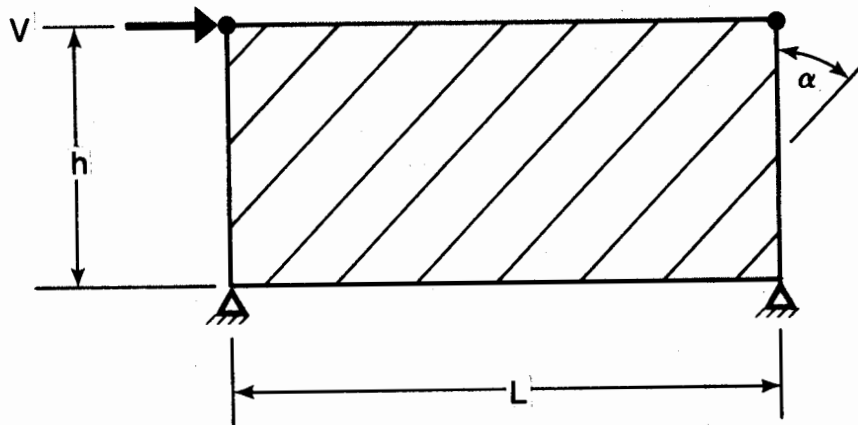web plate must be infinitely rigid.

Figure B.3 Analytical Model of Shear Wall Web

Only the storey stiffness of the tension zone in the web
will be considered.

Work done by the force, V, = (1 / 2) x V x Δ......(B.7)

Work done by the steel plate in the web is given by the first term of Eq. A.6:

$$W = (V^2 \times h) / (2E \times L \times w \times \sin^2 \alpha \cos^2 \alpha).......(B.8)$$

Equating Eqs. B.7 and B.8 and solving for V results in:

$$V = [(L \times w \times \underbrace{E \times \sin^2 \alpha \cos^2 \alpha)} / (h)] \times Δ......(B.9)$$
$$\text{Equivalent storey stiffness of the tension field}$$

Equating Eqs. B.6(a) and B.9 gives:

$$(A \times E / L) \sin^3 \phi = (L \times w \times E \times \sin^2 \alpha \cos^2 \alpha / h)$$

Solving for the area of the equivalent truss element and using the following relationships:

$$\tan \phi = \sin \phi / \cos \phi = L/h$$
$$\sin \phi \cos \phi = 1/2 \sin 2\phi$$
$$\sin^2 \alpha \cos^2 \alpha = 1/4 \sin^2 2\alpha$$

$$A = (w \times L \times \sin^2 2\alpha) / (2 \sin \phi \sin 2\phi).........(B.10)$$

(In using Eq. B.10, the angle α can be obtained from Eq. A.6.)

III. Equivalent Storey Stiffness of a Partially Developed
     Tension Field

For a web panel bounded by flexible columns and
infinitely stif beams, the tension zone will assume the
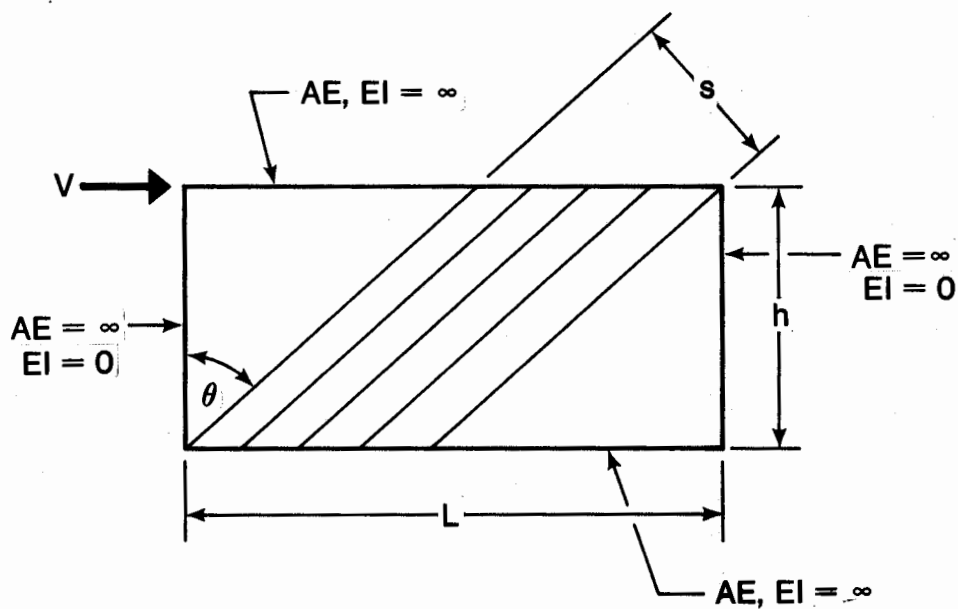configuration shown in Fig. B.4.

Figure B.4 Tension Zone for Web with Flexible Columns

The tension zone develops only in the region of the web
bounded by the beams. This distance, s, is obtained by
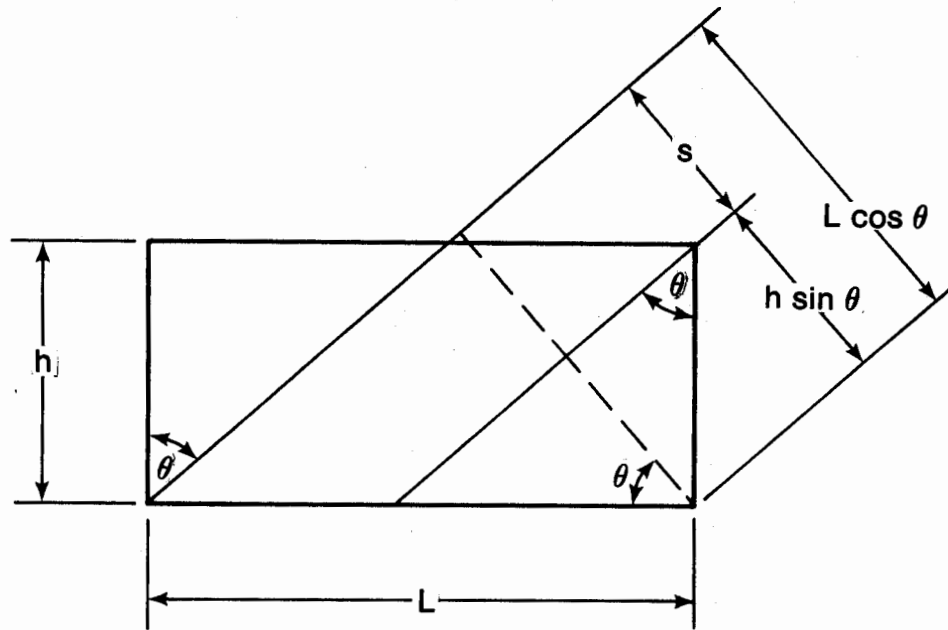considering the geometry:

Figure B.5 Geometry of Partial Tension Zone

From this:

$$s = L \cos \theta - h \sin \theta$$

The resultant force in the direction of the inclined stress is:

$$F = \sigma \times w \times s$$
$$= \sigma \times w \times (L \cos \theta - h \sin \theta)\ldots\ldots\ldots\ldots\ldots\ldots(B.11)$$

The horizontal component of this inclined force is equated to the applied horizontal force, V.

$$V = \sigma \times w (L \cos \theta - h \sin \theta) \sin \theta\ldots\ldots\ldots\ldots\ldots(B.12)$$

By the principle of least work, the most efficient orientation of θ occurs when $dV/d\theta = 0$.

Performing this calculation gives:

$$\tan 2\theta = L / h \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (B.13)$$

Substituting this value of θ into Eq. B.12 above, using various trigonometric identities, and simplifying the result, gives the relationship between V and stress:

$$V = [(w \times L \times \tan \theta \times \sigma) / (2)] \dots\dots\dots\dots\dots (B.14)$$

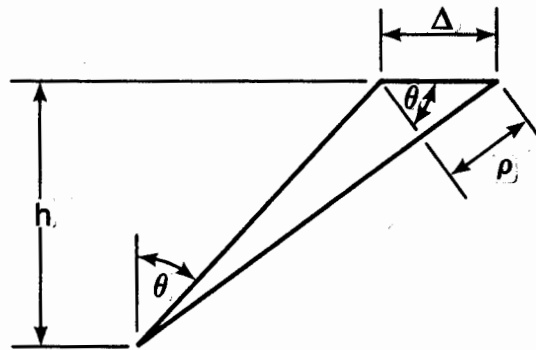The relationship between strain and displacement can be obtained with reference to Fig. B.6.

Figure B.6 Geometry of a Unit Strip

$$\rho = \epsilon \times h / \cos \theta$$

Therefore:

$$\epsilon = \cos \theta \times \rho / h \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (B.15)$$
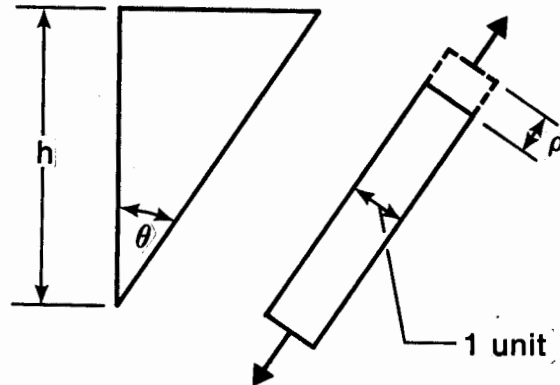
Figure B.7 Resulting Bar Extension

But:

$\rho = \Delta \sin \theta$

Substituting this value of ρ into Eq. B.15 results in the following expression for ϵ:

$\epsilon = \sin \theta \cos \theta \times \Delta / h$

$= \sin 2\theta \times \Delta / 2h$

Substituting this relationship for strain into the usual stress-strain relationship gives:

$\sigma = (E \times \sin 2\theta \times \Delta) / (2h)$

Substituting this expression into Eq. B.14 gives:

$V = (w \times L \times E \times \Delta \times \tan \theta \times \sin 2\theta) / (4h) \ldots \ldots (B.16)$

Since $\phi = \tan^{-1} L/h$, and $\tan 2\theta = L/h$ (Eq. B.16), therefore $2\theta = \phi$ and Eq. B.15 can be further simplified to:

$$V = [(w \times E \times \tan \theta \sin^2 \phi) / (4\cos \phi)] \times \Delta$$

$\underbrace{\qquad\qquad}$ equivalent storey stiffness for the tension zone

Equating the storey stiffness for the tension zone with the stiffness for the diagonal brace (Eq. B.6 a) results in:

$$A E / L \sin^3 \phi = (w E \times \tan \theta \sin^2 \phi) / (4 \cos \phi)$$

which simplifies to:

$$A = (w \times L \times \tan\theta) / 2 \sin 4\theta \dots\dots\dots\dots\dots (B.17)$$

where:

$$\tan 2\theta = L / h$$

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