UNIVERSITY OF ALBERTA

ESSAYS ON APT AND PORTFOLIO RETURNS NORMALITY, AMERICAN FOREIGN CURRENCY OPTION PRICING MODELS, AND INITIAL PUBLIC OFFERINGS AND UNDERWRITING

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled ESSAYS ON APT AND PORTFOLIO RETURNS NORMALITY, AMERICAN FOREIGN CURRENCY OPTION PRICING MODELS, AND INITIAL PUBLIC OFFERINGS AND UNDERWRITING submitted by JASON W. LEE in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY IN FINANCE.

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ABSTRACT

In chapter I, entitled "Arbitrage Pricing Theory and Portfolio Returns Normality", graph-related statistics are applied to give the convergence of portfolio returns to normality. The portfolio, under consideration, can be either a standard portfolio or an arbitrage portfolio. The key to the convergence result is an assumption that the number of dependencies grows slowly compared to the number of assets in the sequence of economies. The normality results of the chapter are applicable to both the "traditional APT" and the "equilibrium forms of the APT", given that they are subject to the same return generating model.

When asset returns are known to be non-normal, a linear mean-risk form of asset pricing models can be obtained by imposing either a restriction on the asset mean returns or the normality restriction on optimal portfolio returns. The empirical analysis of the chapter indicates that the former restriction is generally preferred by a rational agent.

The second chapter, entitled "Tests of American Currency Spot and Futures Option Pricing Models", deals with some empirical issues in the currency option area. The evidence presented here indicates that the American currency spot and futures option pricing models systematically underprice options which are near to expiration, which are 'out of the money', and which are low in volatility. Furthermore, empirical results of the paper generally support the theoretical relative valuation conditions of currency

spot and futures options except the case of Deutsche mark call options. The temporarily higher German interest rate than the U.S. rate during the sampling period may explain this discrepancy.

Chapter III, entitled "Initial Public Offerings and Underwriting", presents a model in which the owner of an IPO firm hires an underwriter in order to increase the firm's market perceived value and/or to reduce the after-market price uncertainty. The unique feature of the model is that the entrepreneur's wealth is affected by the probability that the new issue is fully subscribed by the market. The model allows the possibility that all entrepreneurs, regardless of their firm values, purchase underwriting services as long as the benefits exceed the costs.

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CHAPTER I

D : maximal degree of a dependency graph

E : expectation operator

G : dependency graph

$N(\cdot)$: cumulative normal distribution function

S : sum of a family of random variables

$U(\cdot)$: utility function

V : set of the vertices

Var : variance operator

X : bounded random variable

$f(\cdot)$: probability density function

n : size of an economy

$\tilde{r}$: rate of return

w : portfolio weight

β : factor risk

$\tilde{\delta}$: factor return

$\tilde{\varepsilon}$: residual return

γ : risk premium

μ : mean value

σ : standard deviation

CHAPTER II

$C(\cdot)$: value of an American call option

F : futures exchange rate

$P(\cdot)$: value of an American put option

S : spot exchange rate

T : maturity date of an option

X : exercise price of an option

$V(\cdot)$: value of an American option

dz : standard Wiener process

r : interest rate

σ : standard deviation

CHAPTER III

C : amount of capital to be raised in the IPO market

EW : expected wealth

I : total amount of investment

K : initial investment made by an entrepreneur

$N(\cdot)$: cumulative normal distribution function

P : offer price of an unseasoned equity issue

PTG : proportion of the total investment to be raised in the IPO market

$V(\tilde{\mu})$: market perceived firm value

i : quality of an underwriter

r : revealed fraction of a firm's value

$n(\cdot)$: density function of a standard normal distribution

Π : probability of successful issuance

α : fraction of the ownership retained by an entrepreneur

$\tilde{\mu}$: value which the market will place on an IPO firm

μ_0 : prior mean of $\tilde{\mu}$

μ_f : true or fundamental value of an IPO firm

$\rho(i)$: known fraction of an IPO firm when the firm hires an underwriter with quality i

σ_0 : prior standard deviation of $\tilde{\mu}$

CHAPTER I

ARBITRAGE PRICING THEORY AND PORTFOLIO RETURNS NORMALITY

1. INTRODUCTION

The arbitrage pricing theory (APT) of Ross [1976 and 1977] is an asset pricing model in which cross-sectional variations in assets' expected returns are linearly related to their factor covariance risks. The theory has been viewed as a testable alternative to the capital asset pricing model (CAPM) by many financial researchers and has been the subject of extensive theoretical and empirical research.[1] One of the important advantages of the APT is that it gives a simple linear pricing equation under very weak assumptions. For example, the assumption of normality and/or quadratic utility function is not required in its formulation. In fact, the theory provides, as a very close approximation, the linear functional form of mean returns when asset returns are known to be non-normal.[2]

There are two versions of the APT in the finance literature: a) the "traditional APT" version and b) the "equilibrium forms of the APT".[3] The former version, originally formulated by Ross and extended by Huberman [1982], is based on zero arbitrage arguments. The zero arbitrage condition is defined as the nonexistence of a costless investment that yields a positive and riskless return. In the "traditional APT", the total risk of zero position portfolio returns consists of weighted factor covariance risks and the variance of residual returns. Therefore, convergence of arbitrage portfolio returns to normality, which involves the central limit theorem (CLT) for dependent random variables, is necessary in order to justify two-moment arbitrage arguments (without utility based arguments).

In this paper, graph-related statistics are applied to give the convergence of standard portfolio (i.e., one in which weights add to one) returns to normality in a sequence of economies. The resulting normality is quite general and its derivation requires only two simple conditions. Specifically, uniform bounds on asset returns and slowly growing dependency relative to the number of assets in a sequence of economies are sufficient to give rise to asymptotic normality of portfolio returns. In the portfolio context, the degree of dependency is defined as the number of assets on which an individual asset is dependent. Furthermore, the existence of sufficiently many cross-sections in the economy guarantees that portfolio returns converge at least to approximate normality.[4]

The convergence result of this paper has important implications for the K-factor return generating model and APT. In an economy with a strict factor structure, the portfolio residual return converges in distribution to a normal distribution by the Lindberg-Feller version of the CLT as the number of assets in the economy approaches infinity. This result and the normality of portfolio returns imply the normality of weighted factor returns because a linear combination of two normal variables must be also normal. Note that the asymptotic arguments of the paper are applicable to both a standard portfolio and an arbitrage portfolio.

The "equilibrium forms of the APT" are utility based models. They utilize the first order conditions of expected utility maximization and generally assert a well diversified market portfolio together with specified restrictions on factor model error and an agent's utility. The resulting pricing equation is in the form of a linear mean-risk relation. In this version of the APT, which is known to provide a very close approximation to asset prices, the zero arbitrage condition is the nonexistence of greater expected utility portfolios than the optimum (market portfolio), or put another way, the zero arbitrage condition is the nonexistence of positive marginal utility resulting from a zero marginal position portfolio relative to the optimal portfolio.[5]

When factor returns and asset returns are known to be non-normal, a linear mean-risk form of asset pricing equation can be obtained by imposing either a restriction on the asset mean returns, adopted by the "equilibrium APT", or the normality restriction on optimal portfolio returns, motivated by the asymptotic arguments of this paper. Consequently, these two restrictions represent competing assumptions which require evaluation. An agent incurs a loss in utility from not achieving the expected utility maximizing values of portfolio return moments, permitted by the assets' exact return generating structure. It is postulated here that if the agent's loss in utility, from imposing the mean restrictions of APT, is less than the corresponding utility loss from assuming portfolio returns normality, then it may be argued that the linear pricing equation is better achieved by assuming the APT mean restrictions or vice-versa. Given asset returns following a multivariate non-normal distribution and agents with power utility, we find the APT restriction on means to be preferable to the normality assumption a) when a riskless asset is present and b) when a riskless asset does not exist and economies are not very small. However, this result is subject to error which may cause some irregularities because the empirical investigation is carried out using the historical moments of assets' returns. Note that the asymptotic arguments of this paper require that a portfolio contains a small fraction of each asset in the economy. On the other hand, a portfolio optimization problem utilizing historical data results in portfolio weights with magnitudes substantially exceeding one, in absolute value.

The paper begins with the argument for the portfolio returns normality in a sequence of economies model. In section 3, the form of the losses in expected utility, from APT assumptions versus normality assumptions, are derived in an equilibrium economy. In section 4, numerical analyses in parameterized economies are performed in order to evaluate the comparative utility loss. The fifth section concludes the paper.

2. ASYMPTOTIC NORMALITY OF PORTFOLIO RETURNS IN A SEQUENCE OF

2. ASYMPTOTIC NORMALITY OF PORTFOLIO RETURNS IN A SEQUENCE OF ECONOMIES MODEL

In this section, a theoretical treatment of portfolio returns normality is developed in many asset economies.[6] The analysis begins with a brief review of graph theory and its application to portfolio returns. Portfolio and return restrictions are those found in the sequence of economy derivations of the "traditional" APT.

2.1 Dependency Graph and Portfolio Returns

In order to study dependency among individual asset returns in economy (n), a convenient terminology is introduced subsequently. The superscript (n), which indicates the number of assets in the economy, is omitted for the sake of simplicity.

DEFINITION.[7] *A graph G is a dependency graph for the economy (n) should the following conditions be satisfied.*

(i) There exists a one-to-one correspondence between the assets and the vertices of the graph, where the set of the vertices is represented by V.

(ii) If V_1 *and* V_2 *are two disjoint subsets of* V, *such that no edge in* G *has one end point in* V_1 *and the other in* V_2, *then the corresponding subsets of asset returns are independent.*

In graph theory, the number of vertices in G is called the order of G, represented by $|V|$, while the maximal degree of a graph is the maximal number of edges incident to a single vertex. Therefore, in the portfolio context, the set V is equivalent to the economy (n), disjoint subsets of V are cross-sections in the economy, and the order of the dependency graph is the number of assets. Finally, the maximal degree of dependency, D, is the largest number of assets whose returns are dependent relative to any individual asset return in the economy. In other words, D is the number of assets in the n^{th} economy's largest cross-section.

A portfolio is a set of assets with investment proportions defined over the set. The assets may be real or financial claims to real assets. Then, it is possible that the economy consists of several cross-sections and each of them has its own factors, given that the number of assets is very large (perhaps infinite).

One must note that the existence of several cross-sections does not contradict the validity or the testability of the APT. As Dybvig and Ross [1985] point out, 'it simply means that a subset, under consideration, should include many assets drawn from each of several cross-sections in order to make testing the APT unbiased. Nevertheless, the existence of several cross-sections within the economy will play a crucial role in developing total portfolio returns normality in the next subsection. More specifically, it allows the maximal degree of dependency to be significantly smaller than the number of assets in the economy as $n \longrightarrow \infty$.

2.2. Asymptotic Normality of Portfolio Returns

Applying the concepts of dependency graphs, Janson [1988] establishes a new criterion for the asymptotic normality of sums of dependent random variables. This version of the CLT is more plausible, when applied to portfolio returns, than those based on very strong mixing (VSM) conditions, which include α-mixing, φ-mixing, and m-dependence.[8] The new criterion for asymptotic normality is stated below without proof.[9]

JANSON'S THEOREM. Suppose that $\{X_{ni}, i \in V_n\}$ *is a family of random variables having a dependency graph* G_n. *Suppose further that* $|X_{ni}| \leq A_n$ *almost surely (a.s.) and define* D_n *as the maximal degree of* G_n. *Let* $S_n = \sum_{i \in V_n} X_{ni}$, $\sigma_n^2 = \mathrm{Var} S_n$, *and* $|V_n| \longrightarrow \infty$. *If there exists an integer* $m \geq 3$ *such that*

$$Q_n = \frac{|V_n| D_n^{m-1} A_n^m}{\sigma_n^m} \longrightarrow 0 \text{ as } n \longrightarrow \infty, \tag{1}$$

then

$$\frac{S_n - ES_n}{\sigma_n} \longrightarrow N(0,1).$$

In order for a sequence of random variables to converge in distribution to a normal distribution, it is necessary that all semi-invariants of order greater than two tend to zero.[10] This result, originally suggested by Marcinkiewicz [1939], is the motivation behind Janson's Theorem. When the number of dependencies grows slowly relative to the number of variables, Janson's.[11] Their proof is based on moments rather than semi-invariants. The remainder of this subsection investigates portfolio returns normality in the sequence of economies version of the APT.

Obtaining the asymptotic normality of portfolio returns requires that high order semi-invariants indeed vanish and Q_n, in equation (1), converges to zero as the number of assets increases.[12] Two assumptions will combine to imply this result: 1) returns to individual assets are uniformly bounded and 2) the number of dependencies among asset returns grows slowly relative to the number of assets in the economy. The formal condition of the portfolio returns normality is given in Theorem 1.

THEOREM 1. Consider a portfolio, $p^{(n)}$, *in the* n^{th} *economy where* $w^{(n)}$ *is the vector of portfolio proportions with elements* $w_i^{(n)}$. *Let* G_n *denote the dependency graph of asset returns in the economy. Assume*

(a) $|\tilde{r}_i^{(n)}| \le B_n$ *a.s.,* $i = 1,2,\dots,n$.

(b) The maximal degree of G_n *is given by*

$$D_n = n^a,\ 0 < a < 1.$$

Then,

$$\frac{\tilde{r}_p^{(n)} - \mu_p^{(n)}}{\sigma_p^{(n)}} \longrightarrow N(0,1) \text{ as } n \rightarrow \infty \tag{2}$$

where

$$\tilde{r}_p^{(n)} = \sum_{i=1}^{n} w_i^{(n)} \tilde{r}_i^{(n)},$$

$$\mu_p^{(n)} = E(\tilde{r}_p^{(n)}), \textit{ and}$$
$$\sigma_p^{(n)2} = \mathrm{Var}(\tilde{r}_p^{(n)}).$$

Proof. Let $X_i = w_i^{(n)}\tilde{r}_i^{(n)}$. Then, for all i,

$$|X_i| \le |w_i^{(n)}|B_n \tag{3}$$

because $\tilde{r}_i^{(n)}$ are uniformly bounded by assumption (a). It is also clear that the portfolio return, $\tilde{r}_p^{(n)}$, is equivalent to S_n in Janson's theorem. As in the original APT proof of Ross [1976], let $w_i^{(n)}$ be of order 1/n for simplicity.[13] Replace $|V_n|$ by n, D_n by n^a, and A_n by B_n/n in equation (1). Then it follows that, for any $m \ge 3$,

$$Q_n = \frac{B_n^m \, n^{(m-1)(a-1)}}{\sigma_p^{(n)m}} \longrightarrow 0 \text{ as } n \longrightarrow \infty \tag{4}$$

because B_n is finite and $0 < a < 1$.[14] Therefore,

$$\frac{\tilde{r}_p^{(n)} - \mu_p^{(n)}}{\sigma_p^{(n)}} \longrightarrow N(0,1) \text{ as } n \longrightarrow \infty$$

by Janson's theorem. Q.E.D.

Assumption (b) in the above theorem ensures that the maximal degree of dependency, D_n, grows slowly compared to the number of assets in a sequence of economies. If this assumption does not hold but (D_n/n) is sufficiently small for a large n, then Q_n will be small, i.e., $Q_n < \delta$ for some $\delta > 0$. This will result in an approximation version of the portfolio returns normality instead of the asymptotic normality given by the theorem.[15]

2.3. Portfolio Returns in a Sequence of Economies Model

Consider a sequence of economies where there exist n risky assets in the n^{th} economy. Assume that the returns in a single period, on n assets, are generated by the K-factor return model

$$\tilde{r}^{(n)} = \mu^{(n)} + \beta^{(n)}\tilde{\delta}^{(n)} + \tilde{\varepsilon}^{(n)}, \qquad (5)$$

where

$E(\tilde{\delta}_k^{(n)}) = 0, \ k = 1,2,\ldots,K,$

$E(\tilde{\varepsilon}_i^{(n)}) = 0, \ i = 1,2,\ldots,n,$ and

$V(\tilde{\varepsilon}_i^{(n)}) = \sigma_{\varepsilon_i}^{(n)2} < \infty, \ i = 1,2,\ldots,n.$

$\mu^{(n)}$ is the $(n\times1)$ vector of expected returns in economy (n), conditional on market information,

$\tilde{\delta}^{(n)}$ is the $(K\times1)$ vector of factor returns in economy (n),

$\beta^{(n)}$ is the $(n\times K)$ matrix of factor risks, and

$\tilde{\varepsilon}^{(n)}$ is the $(n\times1)$ vector of residual (idiosyncratic) returns.

In the APT framework, $\mu_i^{(n)}$ depends approximately linearly on its associated factor loadings (i.e.,systematic risks). A portfolio, $p^{(n)}$, in economy (n) has return

$$\tilde{r}_p^{(n)} = w^{(n)}\tilde{r}^{(n)} = \mu_p^{(n)} + \beta_p^{(n)}\tilde{\delta}^{(n)} + \tilde{\varepsilon}_p^{(n)}, \qquad (6)$$

where

$w^{(n)}$ is the $(1\times n)$ vector of portfolio proportions with elements $w_i^{(n)}$, $i = 1,2,\ldots,n,$

$\mu_p^{(n)} = w^{(n)}\mu^{(n)}$ is the portfolio expected return,

$\beta_p^{(n)} = w^{(n)}\beta^{(n)}$ is the $(1\times K)$ vector of portfolio factor risks, and

$\tilde{\varepsilon}_p^{(n)} = w^{(n)}\varepsilon^{(n)}$ is the portfolio residual return.

The portfolio mean and variance are given by

$$E(\tilde{r}_p^{(n)}) = \mu_p^{(n)}$$

and

$$V(\tilde{r}_p^{(n)}) = V(\beta_p^{(n)}\tilde{\delta}^{(n)}) + V(\tilde{\varepsilon}_p^{(n)}), \tag{7}$$

if factor returns are uncorrelated with residual returns, i.e., $E(\tilde{\varepsilon}_i^{(n)}\tilde{\delta}_k^{(n)}) = 0$, $i = 1,2,\ldots,n$, $k = 1,2,\ldots,K$.

If one is willing to assume a strict factor structure, such that $\tilde{\varepsilon}_i^{(n)}$'s are independent across assets, the portfolio's residual standard deviation will be

$$\sigma_{\varepsilon_p}^{(n)} = \{V(w^{(n)}\tilde{\varepsilon}^{(n)})\}^{1/2} = \{\sum_{i=1}^{n} w_i^{(n)2}\sigma_{\varepsilon_i}^{(n)2}\}^{1/2}. \tag{8}$$

Therefore, the portfolio residual return converges in distribution to a normal distribution by the Lindberg-Feller version of the central limit theorem (CLT), provided that the contribution, $w_i^{(n)2}\sigma_{\varepsilon_i}^{(n)2}$, to a portfolio's residual variance by a single asset is negligible as $n \to \infty$.[16] When the number of assets is finite and the portfolio variance is bounded, normality will hold approximately.

In this section, the asymptotic normality of portfolio returns has been developed. The portfolio, under consideration, can be either a standard portfolio or an arbitrage portfolio. No restrictions on the distributional form of each factor return, $\tilde{\delta}_k$, or an asset's residual return, $\tilde{\varepsilon}_i$, have been imposed to obtain Theorem 1. Nevertheless, the portfolio returns normality has an implication for the K-factor return generating model and APT.

The portfolio return, given by equation (6), has two random components: a) the weighted factor returns, $\beta_p^{(n)}\tilde{\delta}^{(n)}$, and b) the portfolio residual return $\tilde{\varepsilon}_p$. When the economy has a strict factor structure and consists of a sufficiently large number of assets, $\tilde{\varepsilon}_p^{(n)}$ converges in distribution to a normal distribution because Lindberg's condition is satisfied. This result and the resulting normality of $\tilde{r}_p^{(n)}$ from Theorem 1 combine to imply the normality of $\beta_p^{(n)}\tilde{\delta}^{(n)}$ because a linear combination of two normally distributed variables must be also normal. Furthermore, the return generating model, given by equation (5), is common to both the "traditional" and the "equilibrium" versions of the APT. Consequently, the normality results of the portfolio returns and the weighted factor returns are applicable to the both versions of the APT.

Observe that Theorem 1 does not require any assumptions with respect to the type of portfolio as long as it has a small proportion of each asset such that no single asset dominates the portfolio. Consequently, the theorem applies to both a standard portfolio ($\sum w_i^{(n)}=1$) and an arbitrage portfolio ($\sum w_i^{(n)}=0$). In particular, the portfolio in Huberman [1982] is a zero beta portfolio in which the effect of the factor returns is completely eliminated from the portfolio. Therefore, the Lindberg-Feller version of the CLT alone is sufficient to show † e normality of zero beta portfolio.

In summary, it has been shown that, in a sequence of economies version of the APT, the distribution of standard and arbitrage portfolio returns converges to a normal distribution. The key to the convergence result is an assumption that the number of dependencies grows slowly compared to the number of assets in the sequence of economies. However, this assumption is not restrictive because the existence of sufficiently many cross-sections in the economy ensures that normality holds at least approximately. In the economy with a strict factor structure, the portfolio residual return converges in distribution to a normal distribution. This result and the normality of portfolio returns give rise to the normality of weighted factor returns.

3. THE UTILITY LOSS FROM EQUILIBRIUM APT ASSUMPTIONS VERSUS A LOCAL NORMALITY ASSUMPTION

If one assumes that the factor returns and asset returns are non-normal, then there is no equilibrium requirement that the optimal portfolio (or a linear combination of factor portfolios) be efficient in mean-variance space conditional on any given information set, or that the means and factor risks are linearly related.[17] However, a linear mean-risk relation may be a good approximation in this environment. There are at least two ways of obtaining the approximation: one approach is to make some assumptions about the properties of the return generating model and the second is to simply assume sufficient normality at finite size economies. The former approach, adopted by the "equilibrium

APT", amounts to a restriction on the asset means, whereas the second approach is the normality restriction on the return moments of the resulting mean-variance optimum portfolio, that is motivated by the asymptotic arguments of section 2.

In this section, an "equilibrium APT" is examined with respect to the loss in utility that is experienced by a rational agent who incorrectly assumes that asset mean returns are given by the APT with zero pricing deviations. Secondly, the utility loss is determined for a rational agent who incorrectly assumes that portfolio returns are normal in the region of the optimal portfolio. By concentrating on the utility loss, the interpretation of a "well diversified" optimal portfolio is avoided because the utility optimization will implicitly define it. Of course, the existence of a utility loss implies that there is an arbitrage return. The analysis begins with the notation and a summary of the literature's treatment of pricing deviation bounds.

3.1 Returns Generation and APT Deviations in a Finite Economy

In a finite universe of n assets in positive supply, the APT assumes that the return from individual asset, i, is generated by

$$\tilde{r}_i = \mu_i + \sum_{k=1}^{K} \beta_{ik}\tilde{\delta}_k + \tilde{\varepsilon}_i, \ \forall \ i = 1,2,\ldots,n, \tag{9}$$

where

μ_i is the mean return for asset i conditional on market wide information,

$\tilde{\delta}_k$ is the value of the k^{th} factor,

$E(\tilde{\delta}_k) = 0,\ k = 1,2,\ldots,K,$

$E(\tilde{\varepsilon}_i) = 0,\ i = 1,2,\ldots,n,$

$V(\tilde{\varepsilon}_i) = \sigma_i^2 < \infty,\ i = 1,2,\ldots,n,$ and

the $\tilde{\varepsilon}_i$'s are independent of each other and of the $\tilde{\delta}_k$'s.[18] The first order utility maximizing conditions of Connor [1984] provide the mean return (pricing) relation

$$\mu_i = \alpha + \sum_{k=1}^{K} \beta_{ik}\gamma_k + x_i,\ \forall\ i = 1,2,\ldots,n, \tag{10}$$

where

α is the implicit riskless interest rate,

γ_k is the risk premium on factor k, and

x_i is the deviation from exact APT.

Dybvig [1983] and Grinblatt and Titman [1983] have provided upper bounds on the value of x_i assuming uncorrelated factor errors, bounds on the agent's absolute risk aversion, bounds on the size of the factor error, and/or other restrictions on the agent's utility function. Both papers take the factors and the risk premia as given and conclude that the upper bound on the pricing error is small and the exact APT is a good approximation in financial markets.

Under exact APT (see Connor [1984] and Chen and Ingersoll [1983]), $x_i = 0$ and therefore the mean pricing relation is

$$\mu_i^* = \alpha + \sum_{k=1}^{K} \beta_{ik}\gamma_k,\ \forall\ i = 1,2,\ldots,n. \tag{11}$$

Substituting (11) in (10) gives the original mean in terms of the new mean and the deviation from APT.

$$\mu_i = \mu_i^* + x_i, \ \forall\ i = 1,2\ldots,n$$

where upon substitution in (9) provides the return generating model in terms of μ_i^* and x_i

$$\tilde{r}_i = \mu_i^* + x_i + \sum_{k=1}^{K} \beta_{ik}\tilde{\delta}_k + \tilde{\varepsilon}_i.$$

Setting $x_i = 0$, as in exact APT, yields the K-factor return generating model

$$\tilde{r}_i^* = \mu_i^* + \sum_{k=1}^{K} \beta_{ik}\tilde{\delta}_k + \tilde{\varepsilon}_i. \tag{12}$$

Equation (12) may be thought of as the return generating model perceived by an uninformed agent who believes that the APT holds exactly. However, equation (12) is an approximation with respect to the more informed generating model given by (9).

3.2 Utility Maximization Under Assumed Zero Pricing Deviations

The agent's problem, under zero deviations from APT pricing, is

$$\max_{\mathbf{w}} U = E[u(\mathbf{w}\tilde{\mathbf{r}}^*)]$$

subject to

$$\mathbf{we} = 1,$$

where u is a von Neumann-Morgenstern utility function of portfolio return, $\tilde{\mathbf{r}}^*$ is given by the vector form of (12), $\mathbf{w}$ is the $(1\times n)$ vector of portfolio proportions with elements w_i, $i = 1,2,\ldots,n$, and $\mathbf{e}$ is the $(n\times 1)$ unit vector. Define the optimal solution to the problem as $\mathbf{w}^*$.

This solution may be contrasted with the optimal solution to the problem when the deviations from APT are not assumed to be zero. The problem in this latter case is

$$\max_{\mathbf{w}} U = E[u(\mathbf{w}\tilde{\mathbf{r}})]$$

subject to

$$\mathbf{we} = 1,$$

where $\tilde{\mathbf{r}}$ is given by the vector form of (9). Let $\mathbf{w}^0$ represent the optimal solution to this problem.

The loss in utility from assuming exact APT, in an economy where there exist deviations from APT, is equivalent to the loss in expected utility from using $\mathbf{w}^*$ in place of $\mathbf{w}^0$. Therefore, the loss in expected utility is $\Delta U^* = U^0 - U^*$, where $U^0 = E[u(\mathbf{w}^0\tilde{\mathbf{r}})]$ and $U^* = E[u(\mathbf{w}^*\tilde{\mathbf{r}})]$. That is, the utility loss is given by; the expected utility obtainable from the portfolio $\mathbf{w}^0$ true return generating model, $\mathbf{w}^0\tilde{\mathbf{r}}$, less the expected utility obtainable from the portfolio $\mathbf{w}^*$ true return generating model, $\mathbf{w}^*\tilde{\mathbf{r}}$. Representative values of this utility loss are derived in section 4.

3.3 Utility Maximization Under Assumed Local Normality

In section 3.2, utility loss has been presented under the "equilibrium APT" derived assumption of zero pricing deviations. Alternatively, consider the maximization problem when the agent incorrectly assumes that the return on the n-asset portfolio is normally distributed. This is a much less stringent assumption than assuming that the n-assets' returns follow a multivariate normal distribution. However, the subsequent analysis will go through with either assumption.

The assumption of "local normality" at the optimum portfolio allows the decision-maker to consider only the mean and variance, of portfolios' returns near the optimum, in defining expected utility. Therefore, the assumed normally distributed portfolio return in this case is $\mathbf{w}^{**}\tilde{\mathbf{r}}^{**}$, where $\tilde{\mathbf{r}}^{**}$ is given by the vector form of (9) with restriction that only the first two moments of $\tilde{\mathbf{r}}$ are used to obtain all the moments of $\mathbf{w}^{**}\tilde{\mathbf{r}}^{**}$ and where μ is given by (10). The maximization problem is then

$$\max_{\mathbf{w}} U = E[u(\mathbf{w}\tilde{\mathbf{r}}^{**})]$$

subject to

$$\mathbf{we} = 1,$$

where $\mathbf{w}^{**}$ represents the optimal solution to this problem. The loss in expected utility from assuming local normality, in an economy where the true optimum portfolio's returns are non-normal, is given by $\Delta U^{**} = U^0 - U^{**}$, where U^0 is as defined in section 2.2 and $U^{**} = E[u(\mathbf{w}^{**}\tilde{\mathbf{r}})]$ is the expected utility obtained from the portfolio $\mathbf{w}^{**}$ true return generating model $\mathbf{w}^{**}\tilde{\mathbf{r}}$.

The utility loss definitions in section 3.2 and 3.3 lead to the following proposition. If the agent's loss in expected utility from the APT zero pricing deviations assumption exceeds the loss in expected utility from assuming the local normality of optimal portfolio returns, i.e. $\Delta U^{*} > \Delta U^{**}$, then by definition the local normality assumption is preferred to the zero pricing deviations assumption and vice-versa.

In investigating the proposition, it is useful to recognize that an agent's problem can be expressed in terms of certainty equivalents instead of expected utilities. That is, an equivalent specification of the agent's problem is

$$\max_{\mathbf{w}} U = u(\theta)$$

subject to

$$\mathbf{we} = 1,$$

where θ is the certainty equivalent rate of portfolio return. In the no assumption case, the optimal θ^0 is a function of all of the moments of the distribution of the optimal portfolio's return, $\mathbf{w}^0\tilde{\mathbf{r}}$. In the case of the APT zero deviations assumption, the assumed optimal certainty equivalent is a function of $\mathbf{w}^*\mu^*$ and all higher moments of $\mathbf{w}^*\tilde{\mathbf{r}}^*$. In the case of assumed local normality, the assumed optimal certainty equivalent is a function of $\mathbf{w}^{**}\mu$ and $V(\mathbf{w}^{**}\tilde{\mathbf{r}})$, but no higher moments of $\mathbf{w}^{**}\tilde{\mathbf{r}}$. However, the true certainty equivalents, θ^* and θ^{**} resulting from $\mathbf{w}^*$ and $\mathbf{w}^{**}$ respectively, are functions of all the moments of $\mathbf{w}^*\tilde{\mathbf{r}}$ and $\mathbf{w}^{**}\tilde{\mathbf{r}}$, respectively. It is these true certainty equivalents which are employed to evaluate the utility losses in the following section.

The remainder of the paper is devoted to determining the preferred assumption, local normality or zero APT mean pricing deviations.

4. AN EVALUATION OF THE UTILITY LOSSES

The analysis of section 3 identified the form of the expected utility losses ΔU^* or ΔU^{**}, from the respective assumptions of zero pricing deviations on means or normality of the optimum portfolio. Because the utility losses are determined relative to the same utility maximizing base, U^0, the relative desirability of the two assumptions can be determined by examining the difference, $U^{**} - U^{*}$. If $U^{**} - U^{*} > 0$ or, equivalently, $\theta^{**} - \theta^{*} > 0$, then the local normality assumption is preferred to the APT mean restrictions assumption and vice versa. For comparison purposes, the certainty equivalent rate of return is employed rather than the expected utility. This approach is useful because it lets us not only determine the preferred assumption but also measure the degree of the relative desirability of the two assumptions.

In general, the loss in expected utility is not available, to our knowledge, in closed form. Therefore, the subsequent analysis investigates numerical solutions to the utility loss across various economies and multivariate return distributions as well as a range of risk preferences in the power utility function.

4.1 Utility Function Parameters and Multivariate Return Parameters

The agent's utility function is a power function given by

$$u(\mathbf{w}\tilde{\mathbf{r}}) = C(1+\mathbf{w}\tilde{\mathbf{r}})^{1-b} \tag{13}$$

where

$$C = \frac{b^b I^{1-b}}{1-b},$$

I is the initial portfolio wealth and b is the constant coefficient of relative risk aversion for wealth. In this paper, three risk aversion values, b = 2.0, 4.0, and 6.0, are employed so that three very different asset portfolios are chosen.[19] Given that the investor's utility function is specified by equation (13), the certainty equivalent rate of return is independent of the size of initial wealth, I. Consequently, unit wealth levels are assumed throughout.

To parameterize the simulation, a sample of monthly returns was obtained on n = 48 randomly selected stocks from the NYSE CRSP data base, over the interval January 1969 to December 1988. Based on the observed data, a multivariate non-normal sample, of T = 500, is generated using the algorithm described by Taylor and Thompson (1986). This method allows us to generate multivariate random vectors from the underlying, but unknown, distribution that gave rise to the random sample. The advantage of this procedure is that it does not require the exact form of the return distribution, which is typically unknown and will never be known to investors. The multivariate density function of the generated random sample is denoted by $f(\tilde{r})$. The mean and covariance parameters of $f(\tilde{r})$ are represented by μ and Σ, respectively. It should be noted that $f(\tilde{r})$ also depends on higher order moments of returns.

With an assumption of multivariate normality, the mean vector and the covariance matrix completely describe the distribution of asset returns. Therefore, a multivariate normal density function, $f^{**}(\tilde{r}^{**})$, was defined by the moments, $\mu^{**} = \mu$ and $\Sigma^{**} = \Sigma$ and T = 500 returns were generated from $f^{**}(\tilde{r}^{**})$ using the IMSL multivariate normal generator.

Eight subsets of 4, 8, 12, 16, 24, 32, 40, and 48 assets were derived from the set of n = 48 stocks. These are a sequence of proper subsets which guarantee that a smaller subset lies inside a larger subset. The return to the riskless asset was parameterized by the mean value of the returns to U.S. Government 1-month treasury bills during the same period. This rate was estimated to be 0.00590 or 0.590 percent per month. Including a riskless gave eight subsets of 5, 9, 13, 17, 25, 33, 41, and 49 assets, respectively.

To this point the multivariate return distributions, f and f^{**}, respectively representing the true return generating process and the process under assumed normality, have been specified. Next the return generating process under assumed APT was specified by first calculating, from f and $u(\mathbf{w}\tilde{\mathbf{r}})$, the true optimal portfolio, $\mathbf{w}^0$, as defined in section 3.2. The beta, β_i, for each asset, i=1,2,...,n, was computed with respect to the risky portion of the optimal portfolio, $\mathbf{w}^0$. When the factor portfolios are contained in the return space of the optimal portfolio, it is well known that,

$$\sum_{k=1}^{K} \beta_{ik}\gamma_k = \beta_i\gamma,$$

where γ is the risk premium on the risky portion of the optimal portfolio. Thus, the mean return on any asset under exact APT is, from (11),

$$\mu_i^* = \alpha + \beta_i\gamma, \quad i = 1,2,\ldots,n,$$

where α is the riskless rate. Therefore, the distribution under assumed APT, $f^{*}(\tilde{r}^{*})$, is the same as $f(\tilde{r})$ except that μ is replaced by μ^{*}.

The multivariate return distributions, for the three cases described thus far, were determined in a market with a riskless asset. This means that the agent's optimal portfolios, $\mathbf{w}^{0}$, $\mathbf{w}^{*}$, and $\mathbf{w}^{**}$ respectively, contain the riskless asset. In the absence of a riskless asset, the same multivariate return distributions were employed except that the riskless asset was not part of the agent's optimal portfolio. The α value in (11) was set numerically equal to 0.00590, which is now interpretable as the zero beta rate rather than the riskless rate.

4.2 Utility Maximization and the Newton-Raphson Algorithm

Under the first assumption, the agent knows the true return process and as discussed previously in section 3.2, $\mathbf{w}$ is chosen to maximize the expected utility

$$\max_{\mathbf{w}} E[u(\mathbf{w}\tilde{\mathbf{r}})] \tag{14}$$

subject to

$$\sum_{i=1}^{n} w_i = 1.$$

Substituting for w_n, by $1 - \sum_{i=1}^{n-1} w_i$ in (14) , gives an unconstrained version of the maximization problem

$$\max U = E[u(\sum_{i=1}^{n-1} w_i \tilde{r}_i + \tilde{r}_n(1 - \sum_{i=1}^{n-1} w_i))] = E[u(\mathbf{w}\tilde{\mathbf{r}})], \tag{15}$$

with first order conditions

$$\frac{\partial U}{\partial w_i} = U_{wi} = E[(\tilde{r}_i - \tilde{r}_n)u_{wi}] = 0, \; i = 1,2,\ldots n-1,$$

and where U denotes the expected utility which is to be maximized, $\mathbf{w}$ is an (n-1) dimensional weight vector, and u_{w_i} is the marginal utility with respect to w_i. This problem can be numerically solved by using the Newton-Raphson algorithm. The advantages of this algorithm are that it converges quickly and that it can be used when the returns follow a non-normal distribution.

The application of the Newton-Raphson algorithm to the utility problem proceeded as follows:[20]

1. Select the starting weight vector, $\mathbf{w}_0$, to be equal set the desired accuracy, $\psi = 1.0 \times 10^{-5}$, and the initial step size, $k_0 = 1$.

2. Evaluate the expected utility

$$U_j = E[u(\mathbf{w}_j\tilde{\mathbf{r}})],$$

where the subscript j refers to the j^{th} iteration and $j = 0$ on startup.[21]

3. Compute a new weight vector

$$\mathbf{w}_{j+1} = \mathbf{w}_j - k_j\mathbf{U}_{\mathbf{w}}\mathbf{U}_{\mathbf{ww}}^{-1}$$

where the subscript $\mathbf{w}$ indicates the partial derivative with respect to $\mathbf{w}$. The Hessian matrix $\mathbf{U}_{\mathbf{ww}}$ must be negative definite.

4. Evaluate

$$U_{j+1} = E[u(\mathbf{w}_{j+1}\tilde{\mathbf{r}})].$$

5a. If $U_{j+1} > U_j$, set $j = j + 1$ and go to step 3.

5b. If $U_{j+1} < U_j$, set $k_{j+1} = k_j/2$ and go to step 3.

6. Repeat steps 3, 4, and 5 until $|U_{j+1} - U_j| \leq \psi$.

This algorithm maximizes the expected utility as long as the Hessian matrix, U_{ww}, is negative definite and implementation usually requires fewer than ten iterations to attain the accuracy level of $\psi = 1.0 \times 10^{-5}$.

Thus, equation (15) was numerically solved, using the Newton-Raphson algorithm, for the optimal weight vectors, w^0, w^*, and w^{**}, obtained under our three different assumptions respecting the return generating model. Given a weight vector, the utility was evaluated in the true return space, with the following outcomes.

(a) If the returns are generated by $\tilde{r}$ in (9), the optimal weight vector, w^0, gives an expected utility of U^0.

(b) If the returns are assumed to be generated under the APT mean restrictions, as $\tilde{r}^*$ in (12), the optimal weight vector, w^*, givės an expected utility of U^* when returns are actually generated by $\tilde{r}$.

(c) If the returns, $\tilde{r}^{**}$, are assumed to follow a multivariate normal distribution with the same first two moments as (9), the optimal portfolio weight vector, w^{**}, provides an expected utility of U^{**} when returns are actually generated by $\tilde{r}$.

Each expected utility was translated to its corresponding certainty equivalent rate of return. Therefore, $\Delta\theta^*$ and $\Delta\theta^{**}$ were equivalent to the utility losses, ΔU^* and ΔU^{**}, from incorrectly assuming APT and normality, respectively.

Because the results are generated by a simulation of the multivariate return distributions and numerical analysis, they are subject to error which may cause irregularities. In order to reduce the degree of this undesirable effect, the procedures in sections 4.1 and 4.2 were repeated four times. The average values of certainty equivalent rates of return were then calculated and tabulated.

4.3 Results of the Numerical Analysis

Tables 1 and 2 show the losses in the certainty equivalent rate of return due to the APT assumption and the normality assumption, with and without a riskless asset, in increasing economy sizes, and for various risk aversions. The tables also give the differences in certainty equivalents, $\theta^{*} - \theta^{**}$, under the two competing assumptions. A positive value of the difference indicates that the APT assumption is preferred to the normality assumption and vice-versa.

When a riskless asset is present in the economy, investors, regardless of their risk aversions, prefer the APT assumption to the normality because it always gives a greater certainty equivalent rate. This result is exhibited in table 1 in which the difference in the certainty equivalents, $\theta^{*} - \theta^{**}$, always assumes a positive value. This value is an increasing function of the size of the economy while it decreases with the risk aversion.

When a riskless asset is not present, the normality assumption is preferable if the economy has few assets and the agent has a low risk aversion. For example, in table 2 an agent with b = 2.0 prefers the normality if $n \le 12$, whereas the agent with b = 6.0 is always better off under the APT assumption. The table also shows that the desirability of the APT assumption measured by the difference in certainty equivalents strictly increases with the size of the economy. However, the difference does not have any apparent relationship with the risk aversion.

Tables 1 and 2 also exhibit the losses in the certainty equivalent from respective assumptions of the APT and the local normality. These values are represented by $\Delta\theta^{*}$ and $\Delta\theta^{**}$, respectively. It is clear from the tables that the loss due to the APT assumption is smaller in the economy with a riskless asset than the one without it. This suggests that the APT pricing is a good approximation and the deviation from it should be small, given that a riskless asset is present in the economy. The loss in the certainty equivalent due to the normality assumption tends to be large when the size of the economy is large. This result may appear to be surprising because portfolio returns normality, developed in section 2, implies that the optimal portfolio returns converge to normality and hence $\Delta\theta^{**}$ should decrease with the size of the economy. Recall that Theorem 1 requires that an asset's proportion of aggregate wealth is small while the only restriction on portfolio weights, imposed in this section, requires the sum of them equal one. On the other hand, a portfolio optimization problem, carried out using historical data, frequently results in unrealistic portfolio weights in the absolute value sense. More specifically, individual assets may have positive or negative portfolio weights, depending on their historical moments, whose magnitudes exceed the total portfolio weight of one. Therefore, the resulting optimal portfolio weights do not satisfy the conditions described in the theorem. However, in a real economy, where the theorem conditions hold, it is likely that these conclusions will change and the normality and APT results will converge as the economy size grows.

The analysis in this study has found that the APT restriction is preferable to the normality restriction in small finite economies and the two are likely to converge as $n \longrightarrow \infty$. However, as evidenced by the literature, it is far more difficult to identify and estimate the APT factors than it is to assume normality; on the other hand, knowledge of the return factors is much more informative from a scientific perspective. Therefore, the relative desirability of the two assumptions will also depend on the ease of identifying and estimating the unobservable parameters and the ultimate use of the model.[22]

5. CONCLUSIONS

Linear risk-return models are desirable and tractable descriptions of equilibrium asset relative prices. The linear relation has been achieved in financial theory by assuming normality or the APT. The literature's development of the APT model has proceeded from the "sequence of economies" version to the "equilibrium" version. This paper has examined some relations between the two APT paradigms and the normality of asset returns. These relations are important, from a statistical perspective, to know whether assuming normality to test APT models is a compatible assumption.

The "traditional APT" utilizes two-moment arbitrage arguments in its formulation. Therefore, arbitrage portfolio returns normality is a result of and is necessary for zero arbitrage analysis. Applying graph-related statistics, we have obtained a sequence of economies proof of portfolio returns normality as $n \longrightarrow \infty$. Asymptotic arguments of this paper are quite general and can be applied to both a standard portfolio and an arbitrage portfolio. Slowly growing dependency of asset returns and their uniform bounds are sufficient to prove the normality. Note that this result will imply a linear asset pricing model in a single period model framework. When the economy has a strict factor structure, portfolio residual returns converge to normality. This result and the asymptotic normality result of this paper combine to imply the normality of weighted factor returns. This implication and the normality result of portfolio returns are applicable to both the "traditional APT" and the "equilibrium APT", provided that they are subject to the same return generating model.

This paper has also addressed the issue of whether the APT restriction on asset mean returns is preferable to restricting an agent's optimal portfolio return to be normal. The analysis in this paper indicates that a rational agent's loss in utility from assuming local normality generally exceeds the loss from APT zero mean return deviations. However, this result must be interpreted with caution. The asymptotic arguments of section 1 require that an asset's proportion of aggregate wealth is small while expected utility maximization, performed using historical data, frequently results in an asset's weight substantially exceeding one in magnitude. Therefore, the empirical results may be distorted due to this potential pitfall. In a real economy, the value of an individual asset is very small relative to the economy's aggregate wealth and hence the conditions of Theorem 1 will be satisfied. Then, it is likely that the normality and APT results will converge.

FOOTNOTES

[1] Theoretical treatments of APT can be found in Huberman [1982], Jobson [1982], Chamberlain [1983], Chamberlain and Rothschild [1983], Chen and Ingersoll [1983], Jarrow and Rudd [1983], Stambaugh [1983], Connor [1984], Ingersoll [1984], Admati and Pfleiderer [1985], Huberman and Kandel [1987], Huberman, Kandel and Stambaugh [1987], Reisman [1988] and Jarrow [1988].

Empirical treatments of APT are in Gehr [1978], Roll and Ross [1980], Gibbons [1980], Reinganum [1981], Chen [1983], Kryzanowski and To [1983], Cho, Elton and Gruber [1984], Chan, Chen and Hsieh [1985], Chen, Roll and Ross [1986], Connor and Korajczyk [1988], Lehmann and Modest [1988], and McElroy and Burmeister [1988].

See also the debates contained in Shanken [1982, 1985], Dhrymes, Friend and Gultekin [1984], Roll and Ross [1984], Dhrymes, Friend, Gultekin and Gultekin [1985], and Dybvig and Ross [1985]. Related work in the interchangeability of the APT and CAPM can be found in Stapleton and Subramanyan [1983], Sharpe [1984], and Wei [1988].

[2] For example, the normality assumption is not contained in the summary list of assumptions described in Dybvig and Ross [1985]. Additionally, Dybvig [1983, abstract] states that, "Ross's Arbitrage Pricing Theory is a tractable and reasonable alternative to the mean-variance model".

[3] See Shanken [1985] and Huberman [1987] for a discussion of the two forms of the APT.

[4] Cross-sections are defined as disjoint subsets of assets in

the economy.

5 This condition is to be contrasted with the condition that the utility of the return from the zero position arbitrage portfolio converges to infinity in an infinite sequence of economies. Clearly, what is most important to a utility maximizing agent is the marginal utility, arising from the arbitrage, and not the utility of the marginal portfolio's return. As Huberman [1982, p190] has demonstrated, the evaluation of the latter utility can easily produce nonsensical results. Ross [1976, appendix 2] provides the utility function restrictions under which the utility of the marginal portfolio's return converges to infinity.

6 In this paper, portfolio returns normality is proved in a single period framework. The number of subperiods in each holding period is fixed and finite. Therefore, the log-normality of holding period returns, which requires continuous compounding, is not applicable in the paper.

7 The definition of a dependency graph is initially introduced by Janson [1988]. The version used here is modified in the portfolio context.

8 α-mixing and φ-mixing conditions can be expressed by their corresponding mixing coefficients:

$$\alpha_n(s) = \sup_{1 \le t \le n-s} \; \sup_{A \in \mathbb{U}_1^t,\, B \in \mathbb{U}_{t+s}^n} |P(A \cap B) - P(A)P(B)|, \text{ and}$$

$$\varphi_n(s) = \sup_{1 \le t \le n-s} \; \sup_{A \in \mathbb{U}_1^t,\, B \in \mathbb{U}_{t+s}^n \;\; P(A)>0} |P(B|A) - P(B)|$$

where $\mathbb{U}_s^t$ denotes the σ-algebra generated by a sequence of dependent random variables. m-dependence is a special case of φ-mixing in which $\varphi_n(s) = 0$ for $s > m$.

See Heinrich [1987] for details.

9 See Janson [1988] for the proof.

10 Consider a random variable X_{ni} which satisfies the conditions in Janson's Theorem such that $X_{ni} \in L^j$. Then, its characteristic function $\phi(t)$ is a continuous and j times differentiable function in the neighborhood of zero. The j^{th} semi-invariant of X_{ni} is now defined as follows:

$$\kappa_j(X_{ni}) = (-i)^j \frac{d^j}{dt^j}\log\phi(0)$$

If X_{ni} is normally distributed, the first two semi-invariants will be the same as its mean and variance while higher order semi-invariants are zeros.

Applying a dependency graph, Janson derives the upper bound on the j^{th} semi-invariant of S_n which is given as follows:

$$|\kappa_j(S_n)| \le C_j n(D_n + 1)^{j-1}A^j, \quad j \ge 1$$

where C_j are some universal constants and S_n is the sum of random variables defined in Janson's Theorem. Observe that S_n follows a normal distribution when this upper bound vanishes for $j \ge 3$.

11 If $m = 3$ in Janson's Theorem, then the criterion for the convergency, given by equation (1), becomes the one derived by Baldi and Rinott [1989a].

12 Semi-invariants are polynomial functions of moments. Therefore, the asymptotic normality can be also obtained when high

order moments vanish.

See Rao [1973, p100-101] for the relationships between moments and semi-invariants.

13 This assumption is not necessary to prove the theorem. It it made only for the sake of simplicity. Otherwise, one can always express $w_i^{(n)} = c_i/n$ where c_i is defined as

$$c_i = \frac{MV_i}{\frac{1}{n}\sum_{i=1}^{n} MV_i}$$

and MV_i is the total market value of asset i. Then, the bound in (3) becomes

$$|X_i| \le c_{max} B_n/n$$

where

$$c_{max} = \max[c_1, c_2, \ldots, c_n].$$

14 Observe that no restriction is made on the value of $\sigma_p^{(n)}$. If the standard deviation of the portfolio return is non-zero, a standard normality result will be obtained. However, when $\sigma_p^{(n)}$ converges to zero as $n \longrightarrow \infty$, then the distribution of $\tilde{r}_p^{(n)}$ will be degenerate. A degenerate case is regarded as a normal distribution throughout the paper.

15 Janson [1988] and Baldi and Rinott [1989a] discuss the approximation version of the CLT for sum of dependent random variables.

16 The portfolio residual can be standardized as

$$\tilde{Y}_n = \frac{\sum_{i=1}^{n} w_i^{(n)} \tilde{\varepsilon}_i^{(n)}}{\sigma_{\varepsilon_p}^{(n)}} = \frac{\tilde{\varepsilon}_p^{(n)}}{\sigma_{\varepsilon_p}^{(n)}}$$

where the portfolio's residual standard deviation, $\sigma_{\varepsilon_p}^{(n)}$, is given by equation (8). Then, the limiting form of the portfolio's standardized residual return is given by the Lindberg-Feller version of the central limit theorem.

17 See Dybvig and Ross [1985a and 1985b] for a treatment of market line deviations.

18 Strictly speaking, any set of assumptions which allows one to write equation (10) from equation (9) are all that is necessary for our analysis contained in sections 3 and 4. For example, correlated errors are permitted. However, for exposition the general format of Dybvig's [1983] analysis of APT bounds is followed here.

19 Previous research has measured the coefficient of relative risk aversion, b, using either wealth or consumption. In particular, Friend and Blume [1975] and Brown and Gibbons [1985] estimate risk aversion to wealth. The ranges of the coefficient obtained in these two studies are 2.5 to 4.0 and 0.09 to 7.09, respectively.

20 See Dyer and McReynolds [1970] for further discussion.

21 The expected utility is calculated by

$$U = \frac{1}{T}\sum_{t=1}^{T} u(1 + \tilde{r}_{pt}),$$

where

$\tilde{r}_{pt} = \sum_{i=1}^{n} w_i \tilde{r}_{it}$ and T = 500.

The first and second order partial derivatives of the expected utility, with respect to the portfolio weights, are obtained by

$$U_i = \frac{1}{T}\sum_{t=1}^{T} (\tilde{r}_{it}-\tilde{r}_{nt})u'(1+\tilde{r}_{pt}), \quad i = 1,2,\ldots,n-1,$$

and

$$U_{ij} = \frac{1}{T}\sum_{t=1}^{T} (\tilde{r}_{it}-\tilde{r}_{nt})(\tilde{r}_{jt}-\tilde{r}_{nt})u''(1+\tilde{r}_{pt}), \quad i,j = 1,2,\ldots,n-1,$$

where $U_{\mathbf{w}}=\{U_i\}$ is the (n×1) vector of first order partials and $U_{\mathbf{ww}}=\{U_{ij}\}$ is the (n×n) matrix of second order partials.

[22] See Gennotte [1986] for a recent discussion of the unobservability of model parameters.

Table 1

Losses in the certainty equivalent rate of return from assuming exact APT versus local normality when a riskless asset is present in economies of size n. The maximum value of the certainty equivalent rate of return, θ^0, is calculated at three values of relative risk aversion which are denoted by b. $\Delta\theta^* = \theta^0 - \theta^*$ and $\Delta\theta^{**} = \theta^0 - \theta^{**}$ represent the decreases in the certainty equivalent rate where θ^* and θ^{**} are the certainty equivalent rates from respective assumptions of exact APT and local normality. (Values in parentheses indicate the certainty equivalent rates expressed as a percentage of θ^0.) The risk-free rate is 0.00590.

number of assets, n	θ^0	$\Delta\theta^*$	$\Delta\theta^{**}$	$\theta^* - \theta^{**}$
b=2.0				
5	0.00968	0.00000 (0.0000)	0.00020 (2.0661)	0.00020 (2.0661)
9	0.01158	0.00000 (0.0000)	0.00040 (3.4542)	0.00040 (3.4542)
13	0.01395	0.00003 (0.2151)	0.00065 (4.6595)	0.00062 (4.4444)
17	0.01723	0.00005 (0.2900)	0.00153 (8.8799)	0.00148 (8.5897)
25	0.02403	0.00015 (0.6242)	0.00355 (14.7732)	0.00340 (14.1490)
33	0.02965	0.00023 (0.7757)	0.00663 (22.3609)	0.00640 (21.5852)
41	0.03390	0.00053 (1.5634)	0.00948 (27.9646)	0.00895 (26.4012)
49	0.04513	0.00100 (2.2158)	0.01830 (40.5495)	0.01730 (38.3337)
b=4.0				
5	0.00780	0.00000 (0.0000)	0.00008 (0.9615)	0.00008 (0.9615)
9	0.00873	0.00000 (0.0000)	0.00018 (2.0057)	0.00018 (2.0057)
13	0.00935	0.00003 (0.2519)	0.00035 (3.5265)	0.00032 (3.2242)
17	0.01158	0.00003 (0.2160)	0.00070 (6.0475)	0.00067 (5.8315)
25	0.01498	0.00008 (0.5342)	0.00175 (11.6861)	0.00168 (11.1519)
33	0.01775	0.00008 (0.4225)	0.00315 (17.7465)	0.00307 (17.3239)
41	0.01988	0.00015 (0.7547)	0.00440 (22.1384)	0.00425 (21.3837)
49	0.02548	0.00038 (1.4720)	0.00808 (30.2257)	0.00770 (30.2257)

b=6.0

5	0.00715	0.00000	0.00008	0.00008
		(0.0000)	(1.0490)	(1.0490)
9	0.00778	0.00000	0.00013	0.00013
		(0.0000)	(1.6077)	(1.6077)
13	0.00860	0.00003	0.00025	0.00022
		(0.3488)	(2.9070)	(2.5581)
17	0.00968	0.00000	0.00053	0.00053
		(0.0000)	(5.4264)	(5.4264)
25	0.01195	0.00008	0.00115	0.00107
		(0.6695)	(9.6234)	(8.9540)
33	0.01380	0.00005	0.00208	0.00203
		(0.3623)	(15.0725)	(14.7101)
41	0.01520	0.00008	0.00293	0.00285
		(0.5263)	(19.2763)	(18.7500)
49	0.01893	0.00023	0.00528	0.00505
		(1.2153)	(27.8996)	(26.6843)

Table 2

Losses in the certainty equivalent rate of return from assuming exact APT versus local normality when a riskless asset is not present in economies of size n. The maximum value of the certainty equivalent rate of return, θ^0, is calculated at three values of relative risk aversion which are denoted by b. $\Delta\theta^{*}=\theta^0-\theta^{*}$ and $\Delta\theta^{**}=\theta^0-\theta^{**}$ represent the decreases in the certainty equivalent rate where θ^{*} and θ^{**} are the certainty equivalent rates from respective assumptions of exact APT and local normality. (Values in parentheses indicate the certainty equivalent rates expressed as a percentage of θ^0.)

number of assets, n	θ^0	$\Delta\theta^{*}$	$\Delta\theta^{**}$	$\theta^{*}-\theta^{**}$
b=2.0				
4	0.00890	0.00065	0.00005	-0.00060
		(7.3034)	(0.6742)	(-6.7416)
8	0.01130	0.00113	0.00035	-0.00078
		(10.0000)	(3.0973)	(-6.9027)
12	0.01380	0.00103	0.00063	-0.00040
		(7.4638)	(4.5652)	(-2.8986)
16	0.01675	0.00110	0.00138	0.00028
		(6.5672)	(8.2388)	(1.6716)
24	0.02330	0.00093	0.00330	0.00237
		(3.9914)	(14.1631)	(10.1717)
32	0.02810	0.00103	0.00600	0.00497
		(3.6655)	(21.3523)	(17.6868)
40	0.03218	0.00125	0.00873	0.00748
		(3.8844)	(27.1287)	(23.2443)
48	0.04310	0.00165	0.01575	0.01410
		(3.8283)	(36.5429)	(32.7146)
b=4.0				
4	0.00533	0.00013	0.00008	-0.00005
		(2.4390)	(1.5009)	(-0.9381)
8	0.00743	0.00015	0.00025	0.00010
		(2.0188)	(3.3647)	(1.3459)
12	0.00898	0.00020	0.00050	0.00030
		(2.2272)	(5.5679)	(3.3408)
16	0.01093	0.00033	0.00100	0.00068
		(3.0192)	(9.1491)	(6.2214)
24	0.01458	0.00053	0.00188	0.00135
		(3.6351)	(12.9387)	(9.2593)
32	0.01750	0.00075	0.00328	0.00253
		(4.2857)	(18.7429)	(14.4571)
40	0.01955	0.00098	0.00445	0.00348
		(5.0128)	(22.7621)	(17.8005)
48	0.02495	0.00113	0.00763	0.00650
		(4.5291)	(30.5812)	(26.0521)

b=6.0

4	0.00215	0.00010	0.00013	0.00003
		(4.6512)	(6.0465)	(1.3953)
8	0.00460	0.00005	0.00025	0.00020
		(1.0870)	(5.4348)	(4.3478)
12	0.00598	0.00010	0.00053	0.00043
		(1.6722)	(8.8629)	(7.1906)
16	0.00780	0.00013	0.00100	0.00088
		(1.6667)	(12.8205)	(11.2821)
24	0.01085	0.00023	0.00168	0.00145
		(2.1198)	(15.4839)	(13.3641)
32	0.01325	0.00040	0.00273	0.00233
		(3.0189)	(20.6038)	(17.5849)
40	0.01478	0.00065	0.00343	0.00278
		(4.3978)	(23.2070)	(18.3092)
48	0.01840	0.00083	0.00553	0.00470
		(4.5109)	(30.0543)	(25.5435)

CHAPTER II

TESTS OF AMERICAN CURRENCY SPOT AND FUTURES OPTION PRICING MODELS

1. INTRODUCTION

Two of the latest innovations in the financial markets that have provided investors alternative mechanisms to hedge foreign currency exposures were options on foreign currencies and options on foreign currency futures contracts. 'he Philadelphia Stock Exchange began trading options on foreign currencies in 1982 and the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME) started trading options on foreign currency futures contracts in 1984.[1] Even though these markets still account for only a small fraction of the total of foreign currency transactions, they have grown very rapidly. Currently, investors can trade foreign currency spot and futures options linked to most of the major currencies, including the British pound, Canadian dollar, Deutsche mark, Japanese yen, and Swiss franc.

Black and Scholes [1973] and Black [1976] were the first authors to study the valuation problems of options and options on futures contracts, respectively. Making certain assumptions regarding the market environments, the riskless rate of interest, and the price changes of the underlying assets, they derive simple closed form solutions to the European-style option valuation equations. Unfortunately, these models are only applicable to European-style contracts, in spite of the fact that most of the options and options on futures contracts traded in the financial markets are American-style. Roll [1977], Geske and Johnson [1983], Johnson [1983], MacMillan [1986], Whaley [1986], and Barone-Adesi and Whaley [1987] study approximation methods of American-style options because there are no known closed form solutions to American-style option valuation equations. These models involve some common assumptions regarding the short-term interest rate and the variance of the underlying asset. More specifically, they all assume that the short-term interest rate and the variance of the underlying asset price change are constant over time. The number of empirical studies, which have been conducted in the foreign currency spot and futures option area, is very limited. This is especially true for the currency futures options. Examples of empirical studies in this area include Bodurtha and Courtadon [1987], Shastri and Tandon [1986, 1986a, 1987], Ogden and Tucker [1987, 1988], and Adams and Wyatt [1987]. A general conclusion of the studies performed on the currency spot option market is that market prices deviate substantially from their corresponding model prices.[2] On the other hand, the tests on the currency futures options market are inconclusive.[3] Regarding the existence of any systematic bias in the currency spot option model, Bodurtha and Courtadon report that the degree of mispricing is related to the time to maturity of the option and the ratio of the spot rate to the exercise price of the option. However, this result is disputed by Adams and Wyatt.

Ogden and Tucker [1988] represent the first paper to study the relative valuation problem of American currency spot and futures options. Their empirical results are consistent with the relative valuation conditions implied by contingent claims pricing models. Unfortunately, their conclusion is not general in the sense that their sample includes only call options on a discount currency and put options on two premium currencies.

The purpose of this paper is to clarify some of the questions still to be answered in the currency spot and futures option areas. More specifically, it investigates whether the American spot and futures option pricing models result in some systematic biases. This paper also attempts to generalize the relative valuation results of Ogden and Tucker [1988] by including both call and put options on premium and discount currencies in the sample.

The paper is organized as follows. Section 2 presents valuation models for American foreign currency spot and futures options. In section 3, the data and the estimates for the volatility of the spot and the futures exchange rates are described. The structure and results of empirical tests are presented in section 4. The final section contains the conclusions.

2. VALUATION EQUATIONS

2.1. Valuation Model for American Currency Options

Bodurtha and Courtadon [1987] develop the partial differential equation describing the dynamics of an American foreign currency option under the following assumptions:

A1] There are no transaction costs and no limits to short sales in financial markets.

A2] There exist no costless arbitrage opportunities. If two assets or portfolios of assets have identical terminal values, they have the same price.

A3] The domestic interest rate, r, and the foreign interest rate, r_f, are constant through time.

A4] The underlying spot rate movements follow the stochastic differential equation

$$dS/S = \mu dt + \sigma dz \tag{1}$$

where S is the spot exchange rate, μ is the expected instantaneous price change relative, σ is the instantaneous standard deviation, and dz is a standard Wiener process. Assumption [A4] implies that the spot rate follows a log-normal distribution. Throughout this paper, the cost-of-carry rate is defined as the domestic interest rate less the foreign interest rate, i.e., $b = r - r_f$.

Let V(S,t,X) represent the value of an American spot option at time t, provided that the exercise price and the expiration date are X and T, respectively. Under the preceding assumptions, V(S,t,X) satisfies the following partial differential equation:

$$\frac{1}{2}\sigma^2 S^2(\partial^2 V/\partial S^2) + (r - r_f)S(\partial V/\partial S) - rV + \partial V/\partial t = 0. \tag{2}$$

The early exercise feature attached to American options allows the holder to exercise the options at any time before the expiration date. Consequently, if C(S,t,X) represents the price of an American currency call option with the exercise price of X and the expiration date of T, the partial differential equation, given by equation (2), must satisfy the following two boundary conditions:

$$C(S,T,X) = \max[0,\ S - X] \text{ and} \tag{3}$$

$$C(S,t,X) = \max[C(S,t^*,X),\ S - X],\ t \le t^* < T, \tag{4}$$

where $C(S,t^*,X)$ is the value of the option at time t^*, given that the option is not exercised. Similarly, $P(S,t,X)$, the value of an American put option at time t, must satisfy the boundary conditions,

$$P(S,T,X) = \max[0,\ X - S] \text{ and} \tag{5}$$

$$P(S,t,X) = \max[P(S,t^*,X),\ X - S],\ t \le t^* < T, \tag{6}$$

where $P(S,t^*,X)$ is the value of the unexercised option at time t^*.

2.2. Valuation Model for American Options on Currency Futures

In order to obtain a valuation model for an American currency futures option, the following assumption is typically made in addition to [A1], [A2], and [A3]:

A5] The price of the underlying futures contract follows the stochastic differential equation

$$dF/F = \alpha dt + \sigma dz \tag{7}$$

where F is the futures rate, α is the expected relative futures rate change, and σ is the instantaneous standard deviation. Whaley [1986] shows that assumption [A5] is consistent with assumption [A4] as long as interest rates are constant over time. In this case, the expected futures rate change relative is equal to the expected spot rate change relative less the cost-of-carry rate, i.e.,

$$\alpha = \mu - (r - r_f) \tag{8}$$

and the standard deviation, σ, is the same for both the underlying spot and futures rate changes.[4] Note that this result is true if and only if the cost-of-carry rate is nonstochastic.

The partial differential equation describing the movements of an American option on currency futures is now given by

$$\frac{1}{2}\sigma^2 F^2(\partial^2 V/\partial F^2) - rV + \partial V/\partial t = 0. \tag{9}$$

Let C(F,t,X) represent the value of an American call option on currency futures, then the necessary boundary conditions are

$$C(F,T,X) = \max[0,\ F - X] \text{ and} \tag{10}$$

$$C(F,t,X) = \max[C(F,t^*,X),\ F - X],\ t \le t^* < T, \tag{11}$$

where $C(F,t^*,X)$ is the value of the unexercised option at time t^*. Similarly, the boundary conditions for an American put option on currency futures are

$$P(F,T,X) = \max[0,\ X - F] \text{ and} \tag{12}$$

$$P(F,t,X) = \max[P(F,t^*,X),\ X - F],\ t \le t^* < T, \tag{13}$$

where $P(F,t^*,X)$ is the value of the unexercised option at time t^*.

It should be noted that there are no known closed form solutions to the partial differential equations (2) and (9) subject to their respective boundary conditions. MacMillan [1986] and Barone-Adesi and Whaley [1987] develop an approximation method to obtain an analytic solution to equation (2) with the proper boundary conditions while Whaley [1986] gives a similar method to solve equation (9).

2.3. Relative Valuation of Currency Spot and Futures Options

The interest rate parity (IRP) theorem states the relationship between the spot and futures exchange rates as follows:

$$F = S \exp[(r - r_f)t] \tag{14}$$

where S (F) represents the spot (futures) rate and r (r_f) is the domestic (foreign) interest rate. When the domestic interest rate is higher (lower) than the foreign interest rate, equation (14) predicts that F > S (F < S) and the currency is called a premium (discount) currency. Note that the underlying commodity for a spot option is the foreign currency itself while it is a futures contract for a futures option.

The difference in the values of American currency spot and futures options is defined as follows:

$$V(S,t,X) - V(F,t,X)$$

where V(S,t,X) and V(F,t,X) are the values of spot and futures options implied by equations (2) and (9), respectively. Because of the nonexistence of closed form solutions to American option models, this difference cannot be analytically shown. However, Ogden and Tucker [1988], using a numerical method, show that the value of a spot call option on a discount (premium) currency is greater (smaller) than that of the corresponding futures option. The opposite is true for the case of put options. Consequently, these two conditions represent the hypotheses to tested in this paper.

3. DATA AND VOLATILITY ESTIMATES

3.1. Data

The data on daily settlement prices for the foreign currency futures contracts, daily closing spot exchange rates, closing option prices, and closing futures option prices was obtained from *The Wall Street Journal*. The selected currency spot and futures option contracts are written on the British pound, Deutsche mark, Japanese yen, and Swiss franc and mature in March, June, September, and December 1989. Canadian dollar contracts are not included in the sample because of infrequent trading in the spot options market. As a proxy for the domestic interest rate, the yield on U.S. treasury bills is utilized. This interest rate is matched to the maturity of the option. The cost-of-carry rate is approximated by taking the yield of U.S. Government 1-month Treasury bills less the 1-month foreign interest rate with similar risk. The foreign interest rates are collected from the *London Financial Times*.

3.2. Estimates for Volatilities

The volatility parameter cannot be directly observed from market prices. Nevertheless, it is a crucial input to the option valuation. In this paper, two alternative estimates for the volatilities of each spot exchange rate and futures rate are presented: a historical standard deviation (HSD) and an implied standard deviation (ISD).

The historical standard deviation is calculated by using the latest 50 observations on the spot exchange rate or the futures rate. On the other hand, the implied volatility is estimated by utilizing the previous date option (futures option) price. The options used to derive this estimate are the ones that are closest to being 'at the money'.[5] In order to reduce potential biases, the volatility estimates for the spot and the futures exchange rates are carried out separately.

Table 3 presents the summary statistics for HSDs and ISDs. As can be seen in the table, the mean of HSDs for futures rates tends to be greater but less stable than that of the ISD estimates during the period of January 1989 to December 1989. However, the estimates of volatilities for spot exchange rates do not exhibit any obvious relationships.

4. RESULTS OF EMPIRICAL TESTS

4.1. Comparison of Market Prices and Model Prices

Solving equations (2) and (9) numerically, subject to their respective boundary conditions, one can value currency spot and futures options given that assumptions [A1] through [A5] are valid. In this paper, the model prices of these instruments are obtained by using the quadratic approximation method suggested by Barone-Adesi and Whaley. This method is computationally tractable and efficient when a large number of options have to be valued. Four different measures are used to compare market and model prices of spot and futures currency options: the mean of the differences in market and model prices in dollars per standard option contract (ME),[6] the mean of the absolute differences in market and model prices (MAE), the mean of the absolute differences as a fraction of the mean of the market prices (PMAE), and the mean of the absolute percent differences in market and model prices (MAPE). These measures are given as follows:

$$ME = \frac{1}{N}\sum_{i=1}^{N} (\Delta P_i), \tag{15}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N} (|\Delta P_i|), \tag{16}$$

$$PMAE = MAE/\{\frac{1}{N}\sum_{i=1}^{N} (P_{market\ i})\}, \text{ and} \tag{17}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N} (|\Delta P_i/P_{market\ i}|) \tag{18}$$

where ΔP is the market price minus its corresponding model price and N is the number of observations.

Tables 4 and 5 present the results of the comparison of actual spot and futures option prices with the corresponding model prices. For the futures option valuation problem, the ISD measure is clearly a better estimate of the futures rate volatility than the HSD because the degree of mispricing is smaller (in all four measures of the model error) when the ISD is used as a model input instead of the HSD. In addition, the tables suggest that the model prices of futures options are good estimators of the corresponding market prices, provided that the ISD is chosen as a model input.[7] However, both the ISD and the HSD estimates of the spot exchange rate result in model prices of spot options that deviate substantially from their market price counterparts even though the ISD estimate gives a smaller degree of mispricing on average.

Potentially, there are two empirical issues which may explain why model prices of spot options deviate substantially from their market prices. The first issue deals with the infrequent trading of spot options. Recall that the ISD is computed from the market price of the most 'at the money' option. However, options with longer time to maturity tend to be not actively traded in the spot option market. Therefore, the option used may not be very close to [illegible] 'at the money', i.e., S/X is not close to 1. In a[illegible]e cost of carry rate, measured by the domestic in[illegible]te minus the foreign interest rate, is an essential inp[illegible]o the spot option valuation. Technically speaking, both the domestic and the foreign interest rates should be matched to the maturity of the option. However, the proper proxy for the foreign interest rate is typically unobservable on a daily basis. Therefore, the differential between the yield of U.S. Government 1-month Treasury bills and the 1-month foreign interest rate with similar risk is used as a measure of the cost-of-carry rate. Unobservability of appropriate foreign interest rates, coupled with the infrequent trading problem, may result in the apparent mispricing of spot options.

One interesting result indicated by Tables 4 and 5 is that the PMAE measure is always much smaller than the MAPE measure. Recall that the PMAE is defined as the mean of the absolute differences divided by the mean of the market prices while the

MAPE is the mean of the absolute percent differences. For a given value of an option's mispricing, the option's contribution to the PMAE is smaller than that to the MAPE when the option's price is lower than the mean of the market prices. Furthermore, the value of a call (put) option is an increasing (decreasing) function of the price of the underlying commodity. Therefore, this result implies that the relative degree of mispricing is substantially greater in the absolute value sense when an option is 'out of the money' than when it is 'in the money'.[8]

4.2. Tests of Biases

Recall that the results discussed in the previous section indicate that the model prices of currency spot and futures options, obtained under the assumptions of section 2, deviate from their corresponding market prices. This is especially true when the underlying commodity is the spot foreign currency and/or when the historical standard deviation is used as a model input. In this section, tests are performed to uncover the existence of any systematic relationship between the model error and the potential sources of bias.

Equation (2) indicates that the model price of a spot option is affected by the spot exchange rate, the exercise price, the volatility, the time to maturity of the option, and the cost-of-carry rate (or the nominal interest rate differential). Note that the model price of a futures option is also influenced by the same variables except the cost-of-carry-rate because this rate for a futures contract must be zero by definition. Consequently, the following regressions are used in this section to test the existence of any systematic bias:

$$MISP = \alpha_0 + \alpha_1 SX + \alpha_2 \hat{\sigma} + \alpha_3 B + \alpha_4 \tau + \varepsilon \quad (19)$$

for spot options and

$$MISP = \alpha_0 + \alpha_1 SX + \alpha_2 \hat{\sigma} + \alpha_3 \tau + \varepsilon \quad (20)$$

for futures options where:

MISP = the model error measured by the log difference of the market and the model prices;

SX = the log difference of the spot exchange rate (or futures rate) and the option's exercise price;

$\hat{\sigma}$ = the ISD or HSD estimate depending on which volatility measure is used as a model input;

τ = the time to maturity measured as a fraction of one year;

B = the domestic interest rate minus the foreign interest rate;

ε = an error term. The reason for the use of logs on variables MISP and SX is that there exists a linear relationship between these two variables while the remaining variables are exponentially related with MISP. Therefore, the use of logs on these two variables is simpler than the use of exponentials on all remaining variables.

Note that the current spot (futures) rate divided by the exercise price is one when the option is 'at the money'. Therefore, a positive (negative) value of SX indicates that a call option is 'in the money' ('out of the money') while the opposite is true for a put option. Tables 6 through 9 contain the results of the regressions. These tables generally exhibit that the R squared value is greater when the historical standard deviation is used as a model input instead of the implied standard deviation.

Variable SX in equations (19) and (20) is used to examine whether the model error is related to the degree of an option being 'in' or 'out of the money'. The results in Tables 6 through 9 indicate that the models tend to underprice 'out of the money' options and overprice 'in the money' options, provided that the ISD estimates are used as model inputs or the underlying commodity is the spot foreign currency. However, when the HSD estimate is used to evaluate futures options, the relationship between the model error and SX is not consistent, or the slope coefficient is not statistically significant.

The volatility estimate, HSD or ISD, is included in the regressions in order to uncover the existence of a systematic variance bias. The results in Tables 6 through 9 show that there exists a negative relationship between the model error and the volatility estimate. This means that the models systematically underprice (overprice) the options when the volatility is low (high). Observe that the slope coefficient is not significant at the 5% level for the Swiss franc futures options when the ISD estimate is used as a model input.

Equations (2) and (9) suggest that the spot option prices are influenced by the cost-of-carry rate. When the domestic interest rate exceeds its foreign counterpart, i.e., the cost-of-carry rate is positive, the American currency option is analogous to the

American option on a dividend paying stock. Thus, it may be optimal to exercise not only an American put option but also an American call option on a premium currency prior to the maturity of the option.[9] If the American spot option model, given by equation (2), is adequate and hence the model price includes the early exercise premium, there should not be any systematic relationship between the model pricing error and the cost-of-carry rate. However, the regression results shown in Tables 6 and 7 generally indicate that the model error is negatively related with the nominal interest rate differential. However, this systematic bias is always consistent and statistically significant only when the underlying currency is the Swiss franc.

Variable τ is included in the regressions to test for the relationship between the model error and the time to maturity of an option. The regression results in Tables 6 through 9 indicate that the model pricing error is negatively related to the time to maturity for all options on all currencies under consideration. This means that the American spot and futures models systematically underprice options close to their expiration dates.

4.3. Test of Relative Valuation Conditions

To obtain the desired sample, each spot option price is paired with the corresponding futures option price traded on the same day. Therefore, each pair of options in the sample have the same exercise price. One problem arises when the test of relative valuation conditions is performed. The futures options expire one week prior to the maturity dates of corresponding spot options. Consequently, the actual values of futures options are adjusted by multiplying by a correction factor given below:

$$\text{maturity correction factor} = V^A_{FT}/V_{FT}$$

where V_{FT} is the theoretical price of a futures option and V^A_{FT} is the corresponding value of the option whose maturity is extended by one week. The value of an option is an increasing function of the time to maturity. Therefore, the adjusted value of a futures option must be greater than its actual price.

The results of the test examining the mean of the differences in corresponding currency spot and futures options are shown in Table 10. When the underlying currency is the pound sterling, the mean of differences in call (put) option prices is positive (negative). Therefore, the spot call (put) option tends to have a greater (smaller) value than its futures option counterpart. This result is consistent with the theory because during the sample period the British interest rate was higher than the U.S. rate and hence its currency is a discount currency. On the other hand, spot call (put) options on premium currencies, such as the Deutsche mark, Japanese yen, Swiss franc, generally have lower (higher) values than futures options. The means of price differences reported in the table are statistically significant except in the case of Deutsche mark call options. This exception can be explained by the behavior of the interest rate differential between Germany and the United States. The German interest rate was very close to the U.S. rate during 1989. In fact, it exceeded the U.S. rate in October and November 1989. Consequently, the Deutsche mark played the role of a discount currency in these two months. This unusual behavior apparently had a greater impact on relative values of call options than those of put options, as indicated by Table 10.

In summ: the results of the empirical tests generally support the relative valuation conditions implic. contingent claims pricing models and are consistent with the findings of Ogden and Tucker [1988]. However, the results in this paper are more general than theirs because the sample in this study include call options on premium currencies and put options on a discount currency.

5. CONCLUSIONS

This study examines whether the American spot and futures option models can predict the market prices of foreign currency spot and futures options. Two alternative estimates for the volatilities of the spot and futures exchange rates are used. The results indicate that the implied standard deviation is preferable to the historical standard deviation as an estimate of the futures exchange rate volatility. In general, the model prices of futures options closely approximate their corresponding market prices. This result is consistent with others' findings. For spot options, both the historical and implied volatilities result in a substantial degree of mispricing. The large degree of mispricing may be attributable to two empirical issues: 1) the actual spot option prices used to estimate the implied volatility often deviate substantially from being 'at the money' due to the infrequent trading of spot options and 2) the foreign risk free rates of interest are not observable on a daily basis.

It is demonstrated that there exist some systematic relationships between the model error and some potential sources of bias. More specifically, the American spot and futures option models systematically underprice options which are near to expiration , which are 'out of the money', especially when the implied standard deviation is used as a model input, and which are low in volatility. Using several different measures of the model error, this paper also provides additional evidence that the relative degree of mispricing is substantially greater for an 'out of the money' option than an 'in the money' option. However, the bias caused by the cost-of-carry rate appears to be less consistent than that caused by the other variables because the regression coefficient is sometimes not significant at the 5% level.

This study also empirically tests relative valuation conditions of American currency spot and futures options. Theoretically, the values of spot call (put) options must be greater (smaller) than those of corresponding futures call (put) options when the foreign interest rate is greater than the U.S. rate. The opposite is true for a premium currency. Empirical results generally support these conditions except in the case of Deutsche mark call options. The Deutsche mark temporarily behaved as a discount currency relative to the U.S. dollar in the sense that the German interest rate was higher than the U.S. rate during a two month period in 1989. The statistically insignificant mean of the differences in its spot and futures call option prices might well be a reflection of this movement of the German interest rate.

FOOTNOTES

1 Even though the Philadelphia Stock Exchange and the IMM are the largest currency spot and futures option markets, respectively, these instruments are also traded at several other exchanges. See Hull [1989] for detail.

2 See, for example, Shastri and Tandon [1986a] and Bodurtha and Courtadon [1987].

3 Shastri and Tandon [1986] conclude that market prices for Deutsche mark and Standard and Poor 500 futures options deviate substantially from the model prices. However, Ogden and Tucker [1987] state that the currency futures option market is efficient.

4 To see this result, apply Ito's lemma to the Interest Rate Parity theorem,

$$F = Se^{b(T-t)} \tag{F1}$$

where $b = r - r_f$.

5 The implied standard deviations are calculated from the market prices of the most sensitive options which are the closest to S/X = 1, provided that S is the spot (futures) exchange rate and X is the exercise price. This approach follows Beckers' empirical study [1981]. He shows that a single ISD computed from the market price of the most sensitive option gives a better estimate of expected volatility than any other method based on weighted implied standard deviations from several options.

6 One standard contract of a currency spot option allows the holder to buy or sell 12,500 British pounds, 62,500 Deutsche marks, 6,250,000 Japanese yens, or 62,500 Swiss francs. The size

of a standard contract for a currency futures option is two times that of a currency spot option.

7 The largest pricing error on average occurs with call options written on Deutsche mark futures contracts, giver that the implied volatility is used as a model input. In this case, the respective PMAE and MAPE measures are 3.47 percent of the mean of the market prices and 0.14458. For all other options, these values are always below 3 percent and 0.10, respectively.

8 When an option is 'out of the money', its value is always smaller than that of an 'in' or 'at the money' option. The reason is that the further an option is out of money, the less likely it will be profitably exercised.

9 Investors may find a situation under which the underlying asset price is low enough to exercise an American put option early, despite the fact that the asset has a zero or negative cost-of-carry rate. On the other hand, it is never optimal to exercise an American call option before its maturity unless the cost-of-carry rate is positive. See Hull [1989] for further details.

TABLE 3
Summary Statistics for Estimates of Volatilities
(Sample Period: January 1989 - December 1989)

Panel A Spot Calls	HSD[a]		ISD[b]	
Currency[c]	Mean	SD[d]	Mean	SD[d]
BP	0.1140	0.0273	0.1285	0.0235
DM	0.1056	0.0220	0.1253	0.0273
JY	0.1128	0.0259	0.1147	0.0254
SW	0.1281	0.0289	0.1299	0.0228
Panel B Spot Puts				
BP	0.1148	0.0270	0.1195	0.0307
DM	0.1057	0.0221	0.1144	0.0280
JY	0.1128	0.0260	0.1034	0.0257
SF	0.1281	0.0289	0.1206	0.0273
Panel C Futures Calls				
BP	0.1250	0.0333	0.1174	0.0169
DM	0.1201	0.0256	0.1174	0.0326
JY	0.1167	0.0325	0.1049	0.0212
SF	0.1302	0.0290	0.1210	0.0211
Panel D Futures Puts				
BP	0.1250	0.0333	0.1173	0.0185
DM	0.1201	0.0256	0.1158	0.0259
JY	0.1167	0.0325	0.1048	0.0187
SF	0.1302	0.0290	0.1199	0.0179

[a] HSD = Historical Volatility
[b] ISD = Implied Volatility
[c] BP = British pound; DM = Deutsche mark; JY = Japanese yen; SF = Swiss franc
[d] SD = Standard Deviation

TABLE 4

Comparison of Market and Model Prices for Spot Options

Panel A Calls

Currency[a]	$\hat{\sigma}$[b]	ME[c]	MAE[d]	PMAE[e]	MAPE[f]
BP	HSD	15.500	49.000	0.17530	0.26795
	ISD	-14.250	36.750	0.13139	0.24787
DM	HSD	-48.710	98.125	0.15859	0.30496
	ISD	6.875	63.125	0.10232	0.19385
JY	HSD	73.125	105.625	0.18565	0.35333
	ISD	6.875	73.750	0.12962	0.27079
SF	HSD	-35.000	103.750	0.17191	0.29051
	ISD	6.250	82.550	0.13142	0.22215

Panel B Puts

Currency	$\hat{\sigma}$	ME	MAE	PMAE	MAPE
BP	HSD	3.125	50.250	0.11464	0.19979
	ISD	0.875	47.750	0.10881	0.19420
DM	HSD	-27.130	66.875	0.12732	0.26259
	ISD	-3.755	55.000	0.10471	0.21232
JY	HSD	27.500	120.625	0.17455	0.36639
	ISD	8.750	73.750	0.10698	0.22465
SF	HSD	-35.625	122.500	0.18318	0.29935
	ISD	5.625	83.750	0.12522	0.22215

[a] BP = British pound; DM = Deutsche mark; JY = Japanese yen; SF = Swiss franc

[b] HSD (ISD) indicates that the historical (implied) volatility is used as a model input.

[c] ME = mean of the differences in market and model prices (in dollars per standard contract

[d] MAE = mean of the absolute differences in market and model prices

[e] PMAE = mean of the absolute differences divided by the mean of the market prices

[f] MAPE = mean of the absolute percent differences in market and model prices

TABLE 5
Comparison of Market and Model Prices for Futures Options

Currency[a]	$\hat{\sigma}$[b]	ME[c]	MAE[d]	PMAE[e]	MAPE[f]
Panel A Calls					
BP	HSD	-4.250	116.750	0.16958	0.24998
	ISD	-0.750	16.000	0.02330	0.05944
DM	HSD	-196.250	132.500	0.10048	0.20635
	ISD	10.000	42.500	0.03473	0.14458
JY	HSD	-48.750	257.500	0.15794	0.24107
	ISD	6.250	45.000	0.02789	0.07234
SF	HSD	-152.500	198.750	0.13595	0.20965
	ISD	13.750	38.750	0.02603	0.08914
Panel B Puts					
BP	HSD	24.250	115.250	0.10287	0.15466
	ISD	1.500	0.00077	0.01722	0.04367
DM	HSD	-182.500	0.00107	0.07360	0.13900
	ISD	12.500	0.00034	0.02316	0.08190
JY	HSD	-120.000	0.00200	0.12493	0.18514
	ISD	7.500	0.00037	0.02326	0.05247
SF	HSD	-146.250	0.00153	0.09764	0.15147
	ISD	13.750	0.00031	0.01982	0.04916

[a] BP = British pound; DM = Deutsche mark; JY = Japanese yen; SF = Swiss franc

[b] HSD (ISD) indicates that the historical (implied) volatility is used as a model input.

[c] ME = mean of the differences in market and model prices (in dollars per standard contract)

[d] MAE = mean of the absolute differences in market and model prices

[e] PMAE = mean of the absolute differences divided by the mean of the market prices

[f] MAPE = mean of the absolute percent differences in market and model prices

TABLE 6

Te[illegible] Biases: Spot Calls

The regression results [illegible] panels A and B are obtained by using the HSD and the ISD as a model input, respectively. Values in parentheses indicate t-statistics.

$$MISP_j = \alpha_0 + \alpha_1 SX_j + \alpha_2 \hat{\sigma}_j + \alpha_3 B_j + \alpha_4 \tau_j + \varepsilon_j$$

Currency[a]	α_0	α_1	α_2	α_3	α_4	R^2
Panel A HSD						
BP	1.120	-5.388	-8.297	-0.142	-0.259	0.344
	(13.94)	(-13.36)	(-14.53)	(-0.11)	(-2.08)	
DM	1.845	-6.459	-13.534	-7.242	-0.473	0.505
	(33.29)	(-29.20)	(-28.77)	(-8.01)	(-5.52)	
JY	1.665	-3.329	-10.882	-9.844	-0.664	0.324
	(22.41)	(-9.68)	(-20.58)	(-7.88)	(-5.61)	
SF	1.373	-2.053	-8.401	-9.155	-0.664	0.385
	(23.99)	(-6.74)	(-18.91)	(-9.61)	(-5.65)	
Panel B ISD						
BP	0.700	-1.577	-6.310	-3.126	-0.567	0.155
	(7.66)	(-3.84)	(-10.65)	(-2.58)	(-4.46)	
DM	0.721	-1.679	-4.196	-4.476	-0.618	0.167
	(16.95)	(-9.26)	(-14.55)	(-5.87)	(-8.49)	
JY	0.953	-2.949	-5.717	-6.[illegible]1	-0.684	0.190
	(14.89)	(-9.60)	(-14.03)	(-5.[illegible])	(-6.66)	
SF	1.087	-1.991	-6.725	-4.847	-0.728	0.211
	(16.02)	(-7.09)	(-14.86)	(-5.25)	(-6.90)	

[a] BP = British pound DM = Deutsche mark
JY = Japanese yen SF = Swiss franc

TABLE 7

Tests of Biases: Spot Puts

The regression results in panels A and B are obtained by using the HSD and the ISD as a model input, respectively. Values in parentheses indicate t-statistics.

$$MISP_j = \alpha_0 + \alpha_1 SX_j + \alpha_2 \hat{\sigma}_j + \alpha_3 B_j + \alpha_4 \tau_j + \varepsilon_j$$

Panel A HSD

Currency[a]	α_0	α_1	α_2	α_3	α_4	R^2
BP	0.815	2.496	-5.881	-1.389	-0.863	0.235
	(12.83)	(8.14)	(-11.44)	(-1.23)	(-8.60)	
DM	1.551	6.257	-11.108	-4.932	-0.793	0.419
	(27.11)	(25.62)	(-23.67)	(-5.16)	(-9.20)	
JY	1.309	1.097	-9.396	-4.904	-1.002	0.286
	(20.27)	(4.08)	(-21.00)	(-4.42)	(-10.50)	
SF	1.330	1.773	-7.688	-9.708	-1.232	0.383
	(23.87)	(6.14)	(-18.28)	(-10.34)	(-12.71)	

Panel B ISD

Currency[a]	α_0	α_1	α_2	α_3	α_4	R^2
BP	0.500	1.309	-4.303	-2.792	-0.548	0.205
	(8.54)	(4.71)	(-13.58)	(-3.04)	(-5.78)	
DM	0.723	4.161	[illegible].080	-1.613	-0.859	0.247
	(15.50)	(18.06)	(-13.08)	(-1.78)	(-10.20)	
JY	0.572	5.024	-3.480	0.680	-0.801	0.264
	(11.17)	(21.66)	(-10.78)	(0.72)	(-9.74)	
SF	0.978	3.762	-5.915	-5.193	-0.777	0.260
	(16.30)	(14.55)	(-15.32)	(-5.74)	(-8.39)	

[a] BP = British pound, DM = Deutsche mark, JY = Japanese yen, SF = Swiss franc

TABLE 8

Tests of Biases: Futures Calls

The regression results in panels A and B are obtained by using the HSD and the ISD as a model input, respectively. Values in parentheses indicate t-statistics.

$$MISP_j = \alpha_0 + \alpha_1 SX_j + \alpha_2 \hat{\sigma}_j + \alpha_3 \tau_j + \varepsilon_j$$

Panel A HSD

Currency[a]	α_0	α_1	α_2	α_3	R^2
BP	0.964	-0.567	-7.157	-0.524	0.435
	(37.51)	(-2.54)	(-38.42)	(-10.45)	
DM	0.962	-0.875	-6.730	-0.791	0.303
	(27.52)	(-4.41)	(-23.56)	(-14.38)	
JY	0.692	0.873	-5.706	-0.515	0.389
	(31.36)	(3.83)	(-33.25)	(-11.35)	
SF	0.810	-0.071	-5.359	-0.798	0.368
	(29.54)	(-0.67)	(-26.32)	(-17.62)	

Panel B ISD

Currency[a]	α_0	α_1	α_2	α_3	R^2
BP	0.186	-1.921	-0.708	-0.441	0.133
	(6.66)	(-13.78)	(-3.20)	(-13.80)	
DM	0.697	-2.141	-4.530	-0.618	0.282
	(26.61)	(-12.33)	(-24.57)	(-12.63)	
JY	0.308	-2.157	-1.941	-0.357	0.172
	(14.26)	(-13.64)	(-10.51)	(-11.17)	
SF	0.487	-2.022	-2.957	-0.484	0.205
	(17.52)	(-13.90)	(-14.66)	(-13.48)	

[a] BP = British pound; DM = Deutsche mark; JY = Japanese yen; SF = Swiss franc

TABLE 9

Tests of Biases: Futures Puts

The regression results in panels A and B are obtained by using the HSD and the ISD as a model input, respectively. Values in parentheses indicate t-statistics.

$$MISP_j = \alpha_0 + \alpha_1 SX_j + \alpha_2 \hat{\sigma}_j + \alpha_3 \tau_j + \varepsilon_j$$

Currency[a]	α_0	α_1	α_2	α_3	R^2
Panel A HSD					
BP	0.593	0.268	-4.359	-0.3[illegible]	0.319
	(29.76)	(1.59)	(-30.05)	(-9.09)	
DM	0.707	0.975	-4.731	-0.674	0.266
	(24.67)	(6.08)	(-20.12)	(-15.09)	
JY	0.541	-0.430	-4.426	-0.494	0.353
	(29.17)	(-2.30)	(-30.34)	(-13.01)	
SF	0.587	-0.050	-3.955	-0.625	0.329
	(27.08)	(-0.34)	(-24.55)	(-17.84)	
Panel B ISD					
BP	0.176	1.187	-0.813	-0.242	0.084
	(7.66)	(9.62)	(-4.54)	(-8.55)	
DM	0.572	1.771	-3.629	-0.493	0.225
	(23.58)	(12.92)	(-19.83)	(-12.77)	
JY	0.153	1.843	-0.568	-0.286	0.160
	(9.81)	(17.44)	(-4.18)	(-13.16)	
SF	0.126	1.397	-0.277	-0.278	0.130
	(6.04)	(14.55)	(-1.76)	(-11.65)	

[a] BP = British pound; DM = Deutsche mark; JY = Japanese yen; SF = Swiss franc

TABLE 10

Price Differences in Currency Spot and Futures Options

Panel A: Calls			
Currency[a]	$\hat{\sigma}$[b]	Mean[c]	s.e.Mean
BP	HSD	0.08048*	0.01312
	ISD	0.07901*	0.01312
DM	HSD	-0.00088	0.01598
	ISD	-0.00462	0.01625
JY	HSD	-0.01760*	0.00485
	ISD	-0.01893*	0.00469
SF	HSD	-0.02667*	0.00600
	ISD	-0.02884*	0.00594

Panel B Puts			
BP	HSD	-0.02457+	0.01653
	ISD	[illegible]02904*	0.01658
DM	HSD	0.01569*	0.00619
	ISD	0.01[illegible]51*	0.00617
JY	HSD	0.06[illegible]9*	0.00570
	ISD	0.06535*	0.00565
SF	HSD	0.01572*	0.00533
	ISD	0.01341*	0.00527

* Indicates significance at the 5% level.

\+ Indicates significance at the 10% level.

a BP = British pound DM = Deutsche mark
JY = Japanese yen SF = Swiss franc

b HSD (ISD) indicates that the historical (implied) volatility is used as a model input.

c The mean differences reported are maturity adjusted. Values are in cents per unit of the underlying foreign currency.

CHAPTER III

INITIAL PUBLIC OFFERINGS AND UNDERWRITING

1. INTRODUCTION

When a firm decides to raise equity capital by issuing unseasoned shares, there is little publicly available information about the firm. The entrepreneur who owns the firm typically holds the best information about the firm's prospects. If he cannot credibly convey his superior knowledge to the market, there will exist a potential market failure of the type presented by Akerlof [1970]. Signalling in the securities market deals with the problem facing corporate insiders who would like to convey their private information about the true value of the firm to the market by employing financial variables or other means, if they have a proper incentive to do so.

A number of studies have suggested various financial and other decision variables which can be used as potential signals in the initial public offering (IPO) market. Leland and Pyle [1977] examine a model in which the proportion of ownership retained by the entrepreneur serves as a signalling device. Gale and Stiglitz [1989] also argue that the size of the initial offering can be informative. Hughes [1986] and Grinblatt and Hwang [1989] extend the Leland and Pyle signalling model by specifically dealing with an additional signalling variable under the entrepreneur's control. These four models generally predict that the greater fraction of entrepreneurial ownership retention signals the greater value of the firm. Allen and Faulhaber [1989], Grinblatt and Hwang, and Welch [1989] present models in which underpricing is a signal that the firm is good. In general, they are motivated by Ibbotson's [1975] conjecture that issuers may want to "'leave a good taste in investors' mouths' so that future underwritings from the issuer could be sold at attractive prices." Therefore, these models may fall apart if the founder of the firm has no plan to reissue equity shares in the near future.[1] There are also several papers which use the quality of the underwriter and/or the auditor as a signal. They include models presented by Boothe and Smith [1986] and Titman and Trueman [1986]. These models generally predict that the higher the quality of the underwriter and/or the auditor, the higher is the market perception of firm value. In addition to the previously mentioned potential signalling devices, some authors employ some types of direct signals to resolve the information asymmetry problem in the IPO market. There is a direct cost for bad firms to mimic good firms in Welch while Hughes studies direct disclosure about the firm as a credible way to convey information. Finally, Milgrom and Roberts [1986] suggest the possibility of using some dissipative signals in the case of repeated sales.

Entrepreneurs in the above studies have a common objective of diversifying their portfolios. Therefore, these models are not applicable when a firm's motivation to go public is to raise capital so as to finance its investment project. Furthermore, they assume that new issues are always fully subscribed. This assumption is not consistent with what happens in the IPO market. The possibility of unsuccessful issuance has always been IPO firm owners' concern and there have been many cases in which new issues are forced to be withdrawn from the market due to investors' lack of interest.

This paper develops a model in which an entrepreneur, who owns the entire firm prior to going public, hires an underwriter in order to obtain a higher market perceived value of the firm and/or to reduce the after-market price uncertainty. There are two types of underwriting contracts; 1) firm commitment and 2) best effort offering. Only the second type is considered in this paper. Rock [1986] argues that there are two principal reasons why a firm enters the IPO market. First, the founder of the firm wants to diversify his portfolio. The entrepreneurs in Leland and Pyle, Hughes, and Grinblatt and Hwang have this motive. The second reason is that the firm has no alternative source of funds to finance its investment project. This motive is consistent with Myers' [1984] pecking order theory which suggests that firms, in deciding on the source of funds for investment, consider using retained earnings first, then debt financing as a second choice, and equity financing only as a last resort. The model developed in this paper exclusively deals with the second motive for issuing unseasoned equity shares. The unique feature of the model is that an entrepreneur's expected wealth is affected by the probability that the new issue is fully subscribed. Thus, an entrepreneur with a good firm may improve his expected wealth through purchasing underwriting services in two ways; 1) the probability of successful issuance increases and 2) the optimal offer price rises and hence his fractional holding of the firm's equity shares goes up. In addition, an entrepreneur with a low value firm may find some situation in which he optimally purchases underwriting services. Under this circumstance, the entrepreneur's motive is to reduce the after-market price uncertainty of the firm and hence to increase the probability that the issue is fully subscribed. Consequently, the model in this paper allows the possibility that all entrepreneurs, regardless of their firm values, purchase underwriting services as long as the benefits of underwriting are greater than the costs. The model also predicts that shares of the IPO firm on average are underpriced in the after-market. This result is consistent with the empirical findings of Ibbotson [1975], Ibbotson and Jaffe [1975], and Ritter [1984]. Finally, the model shows that the presence of underwriters in the economy can improve social welfare because it increases the probability of successful issuance. An increase in this probability implies that more firms in the IPO market can finance their investment projects and hence the economy as a whole invests more in the productive activities. Undoubtedly, this is socially efficient. Therefore, the presence of underwriters can make both individuals and the economy as a whole better off because it can increase the expected wealth of entrepreneurs as well as social welfare.

The next section presents an entrepreneur's decision problem in an economy without underwriting services. The only control variable for him in this economy is the offer price of the new issue. In section 3, an extended model, in which the quality of an underwriter is included as an additional choice variable for the entrepreneur, is developed. In section 4, the results of a numerical analysis and comparative statics are presented. The fifth section concludes the paper.

2. AN ENTREPRENEUR'S PROBLEM IN AN ECONOMY WITHOUT UNDERWRITING

Consider a cohort of firms that come to the market for initial public offerings (IPOs) at time 1. In the absence of any credible information, the market does not know the value of each individual firm. However, it is presumed that the owner of each IPO firm knows the true value of his own firm. For simplicity, the total number of shares outstanding of each firm is normalized to one and the risk-free rate of interest is assumed to be zero.

The sequence of events in this paper is as follows. At time 1, a firm offers unseasoned equity shares to the market. Time 2 represents the deadline by which outside investors make their investment decision regarding the firm's new equity issue. If the issue is fully subscribed by the market, the first trading will take place at time 3. Otherwise, the offer will be withdrawn. No new information will be released between time 2 and time 3. Therefore, the firm value and the price uncertainty, perceived by the market, will remain unchanged during this time interval. At time 4, the firm will have a cashflow, which is to be distributed among shareholders, and will be dissolved subsequently. Figure 1 shows the sequence of events.

Prior to going public, a firm selects an investment project at the cost of K. This amount comes from the entrepreneur's own wealth. To implement the project, the firm has to raise an additional amount of capital C from outside investors in exchange for a fraction $1 - \alpha$ of the firm's ownership.[2] Therefore, the investment project requires the total investment of I, which is the sum of K and C, i.e., $I = K + C$, and the entrepreneur will retain a fraction α of the firm's equity share, given that the issue is fully subscribed.

The process by which the entrepreneur estimates, at time 1, the probability that the new equity issue is fully subscribed by outside investors at time 2 is as follows. The true value of the firm is assumed to be the same as the expected value of the firm's terminal cashflow which occurs at time 4. This is equivalent to the assumption that the firm's cashflow is uncorrelated with the market so that the firm has no systematic component of risk. Note that this assumption has no effect on the conclusions except that it simplifies the model. Prior to going public, the entrepreneur who owns the firm has private information which leads him to assign a specific value to the magnitude of the firm's expected cashflow, μ_f. At time 1, however, the owner does not know the value that will be placed on his firm by the market (or representative agent). His uncertainty is described by the prior distribution

$$\tilde{\mu} \sim N(\mu_0, \sigma_0^2) \tag{1}$$

where μ_0 and σ_0^2 represent the prior mean and variance of the firm's value. Consequently, from the entrepreneur's standpoint at time 1, μ_0 represents the expected value that he believes the market will be willing to pay for his firm at time 2.

Each firm going public files a registration statement with the Securities and Exchange Commission (SEC) and issues a prospectus disclosing material relevant to the prospective buyers. Throughout this paper, this process is assumed to reveal a known fraction r of the firm's true value to the market. This fraction will sometimes be referred as the *revealed fraction*. Let $V(\tilde{\mu})$ denote the value that the market will place on the firm at time 2 where the firm's true or fundamental value is μ_f and the fraction r of the firm value is revealed. The random value, $V(\tilde{\mu})$, is assumed to have the following form:

$$V(\tilde{\mu}) = r \cdot \mu_f + (1 - r) \cdot \tilde{\mu}. \tag{2}$$

Note that if $\mu_f \neq \mu_0$, the entrepreneur perceives an asymmetric information problem: if $\mu_f \neq V(\tilde{\mu})$, there is an ex post asymmetric information problem. Now, the market price and the price uncertainty at time 3 when the first trading takes place, estimated by the owner at time 1, are given by

$$\bar{V} = r \cdot \mu_f + (1 - r) \cdot \mu_0 \tag{3a}$$

and

$$\sigma_V^2 = (1 - r)^2 \cdot \sigma_0^2, \tag{3b}$$

respectively. Recall that no new information is released between time 2 and time 3 and hence these two values do not change during this time interval.

A firm going public has to establish the basic characteristics of its new issue offer when it files a registration with the SEC at time 1. More specifically, it has to select the offer price, P, and the fraction of the ownership, α, to be retained. At this time the firm owner knows that the prospective buyers will subscribe to the new issue if and only if they are reasonably certain that subscribing to the issue will result in a financial gain to them. Specifically, we will assume that investors will subscribe to the issue only if the offer price, P, is less than the value of the firm. Observe that this implies that the outside investors are risk neutral.[3] From the standpoint of the entrepreneur at time 1, the value of $\tilde{\mu}$ is not known. Therefore, he treats $V(\tilde{\mu})$ as a random variable, given by equation (2). Then, the prior probability, assessed by the firm owner at time 1 when the price is set, that investors will subscribe to the new issue at time 2 is given by

$$\Pi = \text{Prob}\{P < V(\tilde{\mu})\} \tag{4}$$

where P is the offer price of the issue. This probability will sometimes be referred as the *probability of successful issuance*. Using the prior distribution of $\tilde{\mu}$, given by equation (1), one can also write this probability as

$$\Pi = 1 - N(S) \tag{5}$$

where $N(\cdot)$ is the cumulative distribution function of a standardized normal random variable and S is defined by

$$S = \frac{P - r\cdot\mu_f}{(1 - r)\cdot\sigma_0} - \frac{\mu_0}{\sigma_0} . \tag{6}$$

It can be seen that the probability of successful issuance is an increasing function of μ_f and μ_0 while it is a decreasing function of P.

If the entrepreneur knows the outside investors' behavior, i.e., they subscribe to the firm's new equity issue if and only if the offer price is lower than the market perceived value, then shares of the IPO firm on average are underpriced, provided that the issue is successful. This may result in an excess demand for the new issue if the market thinks that the firm's offer price is sufficiently low. The "Rules of Fair Practice" of the National Association of Security Dealers require that once the offer price and quantity are filed with the SEC, they cannot be changed. Any excess demand for the issue creates a situation of quantity rationing rather than further adjustment of the offering. Consequently, if new issues are generally underpriced, then the market will experience quantity rationing.

Throughout this paper, it is assumed that the entrepreneur's wealth consists of solely his investment in the firm which he owns. This assumption effectively implies that the entrepreneur does not have any alternative source of funds to finance the firm's investment project when the firm goes public. To emphasize the importance of the successful issuance of the firm's new equity share, it is further assumed that:

A.1. The firm's initial investment, K, is entirely used on non-physical assets such as research and development.

A.2. The entrepreneur's wealth after the public offering is given by $\alpha \cdot \mu_f$ if the issuance is successful. Otherwise, his wealth is zero.

Typically, a young firm does not own a substantial amount of physical assets. This can justify the first assumption. The first part of the second assumption is true by definition as long as the entrepreneur does not own any assets other than the firm. The second part of the assumption A.2. implies that the firm must completely give up its investment project and the entrepreneur's initial investment K dissipates if the firm cannot raise the additional capital requirement, C, to implement the project. The second part of the assumption A.2. is not as strong as it may appear. Upon deciding the basic characteristics of the offer, the issuing firm typically selects a minimum and maximum quantity of the ownership to be sold. If the minimum quantity is not sold at the offer price within a specified period of time, usually 90 days, the offer will be withdrawn. Typically, firms whose offers were withdrawn do not enter the IPO market again.[4] This means that the issuing firm, without alternative sources of funds, cannot implement the investment project and completely loses its investment opportunity in this model. Furthermore, the profitability of the outcome of research and development is not usually known to outsiders and hence the entrepreneur may not be able to recover the entire amount of his initial investment. This observation and the first assumption jointly imply that the entrepreneur must bear some degree of a financial loss if he cannot implement his investment project. Consequently, the second part of the assumption A.2. simplifies the model, it does not alter the conclusions.

The entrepreneur's time 1 decision problem is to choose the offer price of the new issue so as to maximize his time 2 expected wealth. Then, he faces the following optimization problem at time 1:

$$\max_{P} EW = \Pi\cdot\alpha\cdot\mu_f + (1 - \Pi)\cdot 0 \qquad (7)$$

where α = 1 - C/P, i.e. the market pays P per share for $1 - \alpha$ share generating C units of capital,

C/P = the amount of the total IPO proceeds proportional to the offer price or the fraction of the firm's ownership to be sold to outside investors, and

Π = the probability of successful issuance, given by equation (5).

The first order condition to the optimization problem is equivalent to

$$\partial\Pi/\partial P|_{P^*} = - \Pi^*\cdot(1 - \alpha^*) / P^*\cdot\alpha^* < 0 \qquad (8)$$

where α^* is the fraction of the firm's ownership held by the entrepreneur and Π^* is the probability of successful issuance, both evaluated at the optimal offer price P^*. The optimal offer price must satisfy the following second order condition:

$$\partial^2 EW/\partial P^2|_{P^*} \le 0. \qquad (9)$$

A closed form solution for equation (7) does not exist. Consequently, the optimization problem is solved numerically at various values of the parameters. Results are discussed in section 4.

3. AN ENTREPRENEUR'S PROBLEM IN AN ECONOMY WITH UNDERWRITING

In this section, an extended version of the model presented in the preceding section is developed by permitting the presence of underwriting services in the economy. The basic setup is the same as the one in section 2. Preliminary comparative statics are obtained on the basis of the model.

3.1 The Model

Consider now a situation in which an entrepreneur has private information about his firm's prospects, which he would like to convey to the market if the true value of the firm exceeds public expectations. Because this information is usually difficult to verify, the entrepreneur has to use one or more credible ways to communicate with the prospective buyers. The mechanism which is examined in this section is the quality of an underwriter. The role of an underwriter in this model is to lend its ability to reduce the firm's unknown fraction. The quality of an underwriter is measured by the ability that it can correctly certify the firm's true value. Note that a long-run loss for the underwriter from behaving opportunistically can easily exceed its short-run gain because its most important asset is its stock of reputation capital ('goodwill').[5] Therefore, the only issue here is the quality of an underwriter not the probability that it may behave strategically.

Let F denote the fees charged by an underwriter with quality i, $0 \leq i \leq 1$. It is reasonable to assume that a higher quality underwriter exerts greater effort to certify the firm's value correctly and hence demands higher fees. Then, F is assumed to have the following form:

$$F = i \cdot g \cdot C \tag{10}$$

where g is a positive constant, referred to as the *underwriting fee constant*, and C is the amount of IPO proceeds. Thus, the underwriter's fees are proportional to its quality and the amount of capital raised through the issue. It is further assumed that the underwriter receives its fees if and only if the issue is fully subscribed. Recall that the type of underwriting contracts employed in this paper is the best effort offering in which the underwriter receives a fraction of the total IPO proceeds as its fees. This means that the underwriter may not receive any compensation if the issue is withdrawn. Consequently, the expected cost of purchasing quality i underwriting services is given by

$$EF = \Pi \cdot i \cdot g \cdot C. \tag{11}$$

The entrepreneur is assumed to bear the entire amount of the underwriting fees and hence his time 2 expected wealth is reduced by this amount.

The known fraction of the firm value, provided that it selects a quality i underwriter, is now given by

$$\rho(i) = i + (1 - i) \cdot r, \tag{12}$$

where i is non-negative but cannot exceed 1. Note that selecting an underwriter with quality 0 yields the known fraction of r, i.e., $\rho(0) = r$, which is exactly the same as the one in an economy without underwriting services. On the other hand, if the firm hires an underwriter with quality 1, the known fraction will become 1, i.e., $\rho(1) = 1$, which is equivalent to the perfect information case.

From the entrepreneur's viewpoint at time 1, the value that the market will place on his firm at time 2 is now given by

$$V(\tilde{\mu}, i) = \rho(i)\cdot\mu_f + (1 - \rho(i))\cdot\tilde{\mu}, \tag{13}$$

provided that an underwriter with quality i is selected. Then, the market price at time 3 is

$$\bar{V}(i) = \rho(i)\cdot\mu_f + (1 - \rho(i))\cdot\mu_0. \tag{14}$$

Equation (14) reveals the following characteristic of the after-market price:

$$\partial\bar{V}(i)/\partial i = (1 - r)\cdot(\mu_f - \mu_0). \tag{15}$$

This result implies that the after-market price of the firm is an increasing function of i provided that the firm's true value is greater than its prior mean value. The opposite is true for a firm with $\mu_f < \mu_0$. The price uncertainty is now defined by

$$\sigma_V^2 = (1 - \rho(i))^2\cdot\sigma_0^2 = (1 - i)^2\cdot(1 - r)^2\cdot\sigma_0^2 \tag{16a}$$

which is a decreasing function of an underwriter's quality,

$$\partial\sigma_V^2/\partial i = -2(1 - i)\cdot(1 - r)^2\cdot\sigma_0^2 < 0, \tag{16b}$$

regardless of the firm's true value relative to the prior market average. When the firm hires a quality i underwriter, the probability of successful issuance will become

$$\Pi = 1 - N(S) \tag{17}$$

where S is now defined by

$$S = \frac{P - \rho(i)\cdot\mu_f}{(1 - \rho(i))\cdot\sigma_0} - \frac{\mu_0}{\sigma_0}\ . \qquad (18)$$

The effect of a change in the quality of underwriting services on this probability is not very clear at this point but it is discussed in section 4.

An entrepreneur now has two [illegible]rol variables, the offer price of the new issue and the quality of underwriting services. His decision process involves two steps; selecting an underwriter and then deciding the offer price of the new issue.[6] Preselecting the quality of underwriting services, the firm owner faces the following optimization problem:

$$\max_{P}\ EW = \Pi\cdot\alpha\cdot\mu_f - \Pi\cdot i\cdot g\cdot C + (1 - \Pi)\cdot 0 \qquad (19)$$

with the first order condition

$$\partial EW/\partial P = (\partial\Pi/\partial P)(\alpha\cdot\mu_f - i\cdot g\cdot C) + \Pi\cdot C\cdot\mu_f/P^2 = 0. \qquad (20)$$

The first term in the maximization problem, given by equation (19), represents the entrepreneur's time 2 expected wealth without underwriting services while the second term is the expected value of the fees charged by the quality i underwriter. The optimal offer price, P^*, must also satisfy the second order condition

$$\partial^2 EW/\partial P^2 |_{P^*} \leq 0. \qquad (21)$$

Even though the optimization problem does not have a closed form solution, it is clear that the optimal offer price is a function of the underwriter's quality. The effect of a change in i on P^* is discussed in section 4.

Note that the fraction of the firm's ownership retained by the entrepreneur does not have any signalling effect in this model. Once he decides on the optimal offer price, the fraction of the firm's equity share to be sold is readily calculated from the firm's additional capital requirement. This result contrasts with the findings of Leland and Pyle, Hughes, and Grinblatt and Hwang. The reason for this difference is as follows. The entrepreneur in this model does not have any alternative source of funds to finance the firm's investment project. He selects the optimal offer price of the new issue so as to maximize his expected wealth. This price and his budget constraint determine the fraction of the firm's ownership to be sold. On the other hand, the entrepreneur in other models sells the firm's equity shares in order to diversify his portfolio. His willingness to hold a greater fraction of the firm's ownership is perceived by outside investors that the firm is good. Therefore, the entrepreneurial ownership can signal the firm's prospects.

3.2 Preliminary Comparative Statics

The probability of successful issuance, given by equation (17), is a function of several parameters and control variables, i.e.,

$$\Pi = \Pi(\mu_f, \mu_0, \sigma_0, r, P, i). \tag{22}$$

The first order partial derivatives of the probability with respect to the relevant variables are

$$\partial\Pi/\partial\mu_f \quad = -n(S)\cdot(\partial S/\partial\mu_f) \ > 0,$$

$$\partial \Pi / \partial \mu_0 \quad = -n(S) \cdot (\partial S / \partial \mu_0) \quad > 0,$$

$$\partial \Pi / \partial P \quad = -n(S) \cdot (\partial S / \partial P) \quad < 0,$$

$$\partial \Pi / \partial \sigma_0 \quad = -n(S) \cdot (\partial S / \partial \sigma_0),$$

$$\partial \Pi / \partial r \quad = -n(S) \cdot (\partial S / \partial r),$$

and $\partial \Pi / \partial i \quad = -n(S) \cdot (\partial S / \partial i)$

where $n(\cdot)$ is the density function of a standardized normal variable. The first three results state that the probability of successful issuance increases with the firm's true value and its prior mean value but decreases with the firm's offer price. The first two results are explained by the fact that higher values of μ_f and μ_0 imply an increase in the after-market price of the firm. Consequently, outside investors are more willing to subscribe to the firm's new issue, given that all other variables remain unchanged. On the other hand, if the offer price rises without changes in other variables, prospective buyers will realize that it becomes less likely for them to make abnormal returns from the issue and hence they will be less willing to purchase the firm's share. The signs of the last three results cannot be determined at this point and are discussed in Section 4. The after-market price of the firm, written $V(\mu_f, \mu_0, r, i)$, has the following characteristics:

$$\partial V / \partial \mu_f \quad = i + r - i \cdot r \quad > 0,$$

$$\partial V / \partial \mu_0 \quad = (1 - i) \cdot (1 - r) \quad > 0,$$

$$\partial V / \partial r \quad = (1 - i) \cdot (\mu_f - \mu_0), \text{ and}$$

$$\partial V / \partial i \quad = (1 - r) \cdot (\mu_f - \mu_0).$$

The first two results show that the higher the true firm value and its prior mean value are, the higher the price that the firm's share will be traded at once public trading is allowed. The third and fourth characteristics state that the after-market price will be greater, given that the firm's true value is greater than the prior mean value, if the known fraction increases or if the firm purchases underwriting services with higher quality. The opposite is true for the firm whose true value is lower than its prior mean value. The fourth result also implies that an entrepreneur with a high value firm may hire an underwriter in order to convey his private information to the market and hence increase the firm's after-market price, if he has an incentive to do so.

The price uncertainty of the IPO firm, σ_v, is given by $(1 - i)\cdot(1 - r)\cdot\sigma_0$. It is clear that σ_v is a decreasing function of not only the revealed fraction but also the quality of an underwriter. Consequently, an entrepreneur can reduce his firm's price uncertainty when he purchases high quality underwriting services. A decrease in σ_v results in an increase in the probability of successful issuance.

The entrepreneur's expected wealth at time 2, written $EW(\mu_f, \mu_0, \sigma_0, r, g, C, P, i)$, has the following features:

$$\partial EW/\partial \mu_f = (\partial \Pi/\partial \mu_f)\cdot(\alpha\cdot\mu_f - i\cdot g\cdot C) + \Pi\cdot\alpha > 0,$$

$$\partial EW/\partial \mu_0 = (\partial \Pi/\partial \mu_0)\cdot(\alpha\cdot\mu_f - i\cdot g\cdot C) > 0,$$

$$\partial EW/\partial g = -\Pi\cdot i\cdot C < 0,$$

$$\partial EW/\partial C = -\Pi\cdot(\mu_f/P + i\cdot g) < 0,$$

$$\partial EW/\partial P = (\partial \Pi/\partial P)\cdot(\alpha\cdot\mu_f - i\cdot g\cdot C) + \Pi\cdot C\cdot\mu_f/P^2,$$

$$\partial EW/\partial \sigma_0 = (\partial \Pi/\partial \sigma_0)\cdot(\alpha\cdot\mu_f - i\cdot g\cdot C),$$

$\partial EW/\partial r \quad = (\partial \Pi/\partial r)\cdot(\alpha\cdot\mu_f - i\cdot g\cdot C)$, and

$\partial EW/\partial i \quad = (\partial \Pi/\partial i)\cdot(\alpha\cdot\mu_f - i\cdot g\cdot C) - \Pi\cdot g\cdot C$.

Note that $\alpha\cdot\mu_f - i\cdot g\cdot C$ is the entrepreneur's time 2 wealth, provided that the issuance is successful, and hence it must always take a positive value. The first four results state that the entrepreneur's expected wealth increases with the firm's true value and its prior mean value but decreases with the underwriting fee constant and the amount of IPO proceeds. The intuition for the first two results is the same as the intuition for the probability of successful issuance. The third result is explained by the fact that the entrepreneur bears the entire underwriting costs in this model. The reason for the fourth result is that the larger amount the entrepreneur must raise in the IPO market, the smaller the fraction of the firm's ownership that is retained. The sign of the fifth result is unclear. However, realizing the fact that the optimal offer price must satisfy the first and second order conditions of the expected wealth maximization problem, one can be certain that $\partial EW/\partial P$ must be negative (positive) at any value of P greater (lower) than the optimal offer price. It is not possible to find the signs of the last three results at this point because a closed form solution to the optimization problem, given by equation (19), does not exist. The signs of these three results are discussed in section 4.

4. RESULTS OF THE NUMERICAL ANALYSIS[7]

The preceding section develops an IPO model in an economy with underwriting services. Unfortunately, a closed form solution to the entrepreneur's objective function does not exist and the signs of the results of some comparative statics are ambiguous. The purpose here is to reveal the optimal offer price at the preselected level of underwriting services and the signs of some comparative statics, through numerical methods. The benefits of underwriting from both the private and the social welfare viewpoints are also examined.

Consider three types of firms; some with a low value (L), an average value (M), and a high value (H). More specifically, the average value firm is defined as the firm whose true value is exactly the same as its prior mean value, μ_0. The low (high) value firm is the one whose true value is 1/3 lower (higher) than μ_0. These three types of firms have the same prior mean value of $21.3 million. Each type of the firm is further classified into two groups in the sense that the amount of IPO proceeds consists of 50 percent of the total investment for one group while it is 75 percent for another group, i.e., PTG = 0.5 and 0.75, where PTG is the proportion of the IPO proceeds relative to the total investment. There are nine discrete types of underwriters with quality i = 0.1, 0.2,...., and 0.9. Therefore, the perfect information case is not included in this analysis. The market environment is subdivided into four cases with r = 0.2, 0.8 and g = 0.2, 0.4. The environment with r = 0.2 (0.8) represents the case where a small (large) fraction of the firm value becomes known in the absence of underwriters in the market. When g = 0.2

(0.4), the average underwriting costs are low (high) and the average underwriting fees in the market are 10 (20) percent of the total IPO proceeds.[8] It is assumed that all firms in the IPO market require the same amount of total investment at $14.2 million and the prior standard deviation of the firm values in the IPO market is the same as the total investment, i.e., $\sigma_0 = I$.

4.1 Comparative Statics

Tables 11 through 18 reveal that the probability of successful issuance is strictly increasing with the quality of underwriting services i, for firms with $\mu_f \geq \mu_0$, i.e., type M and H firms. However, the effect of changes in i on the probability for a low value firm (L) is not clear. The reason for the contrasting results among the firms is as follows. Section 3 gives the first order partial derivative of the probability of successful issuance with respect to the quality of underwriting services as

$$\partial\Pi/\partial i = -n(S)(\partial S/\partial i).$$

For type M and H firms, tables 11 through 18 show that the probability is always greater than 0.5. This implies that S is smaller than zero and hence $\partial S/\partial i$ must be negative for these firms. Consequently, the probability of successful issuance increases with the quality of underwriting services, for type M and H firms. On the other hand, when the firm value is lower than its prior mean value, $\partial S/\partial i$ can be negative, zero, or positive. This means that the $\partial\Pi/\partial i$ depends on the size of μ_f relative to

μ_0, for a low value firm. Note that the relationship between the probability of successful issuance and the revealed fraction is the same as the one between Π and i. A similar logic shows that there exists a negative relationship between σ_0 and Π, given that each entrepreneur selects the optimal level of underwriting services.[9] In summary, the signs of the partial derivatives of the probability of successful issuance for type M and H firms are

$$\Pi_{M,H} = \Pi_{M,H}(\overset{+}{\mu_f},\ \overset{+}{\mu_0},\ \overset{-}{\sigma_0},\ \overset{+}{r},\ \overset{-}{P},\ \overset{+}{i}) \tag{23}$$

and for a type L firm

$$\Pi_{L} = \Pi_{L}(\overset{+}{\mu_f},\ \overset{+}{\mu_0},\ \overset{-}{\sigma_0},\ \overset{?}{r},\ \overset{-}{P},\ \overset{?}{i}) \tag{24}$$

where the subscript indicates the firm type. It should be noted that the signs of $\partial\Pi/\partial\sigma_0$ for all firms are obtained in the neighborhood of the entrepreneur's optimal choice of underwriting services.

The partial derivatives of the entrepreneur's expected wealth with respect to σ_0 and r in the proceeding section indicate that the expected wealth must respond the same direction to changes in these two parameters as the probability of successful issuance does. Tables 11 through 18 reveal that the relationship between the entrepreneur's expected wealth and the quality of underwriting services depends on the size of the IPO proceeds relative to the total investment, provided that he owns a low value firm. When PTG = 0.75, the expected wealth strictly decreases with i. However, when PTG = 0.5, it tends to be a convex function of i. If he owns any other type of a firm, the relationship between the expected wealth and the quality of an underwriter varies from case to case. For example, when the cost of underwriting is low, i.e., g = 0.2, the entrepreneur with a high value firm finds that his expected wealth always increases with the underwriter's quality. However, if the underwriting cost is high, i.e., g = 0.4, then his expected wealth does not always respond positively to the underwriter's quality. For example, in table 14, PTG = 0.5 and r = 0.8, the expected wealth is a concave function of i with the maximum value occurring at i = 0.3. Furthermore, in table 18, PTG = 0.75 and r = 0.8, the entrepreneur should not purchase any underwriting services because the cost of underwriting exceeds its benefit. For an entrepreneur with an average value firm, the relationship between his expected wealth and the quality of underwriting services does not have any clear tendency either. Figures 2 through 9 show the relationship between the entrepreneur's expected wealth and the quality of underwriting services.

Combining the results in section 3 with those in this section, one can readily observe the signs of the partial derivatives of the expected wealth as follows:

$$EW_{H,H} = EW_{H,H}\,(\overset{+}{\mu_f}, \overset{+}{\mu_0}, \overset{-}{\sigma_0}, \overset{+}{r}, \overset{-}{g}, \overset{-}{C}, \overset{+/-}{P}, \overset{?}{i}), \qquad (25)$$

and

$$EW_L = EW_L\,(\overset{+}{\mu_f}, \overset{+}{\mu_0}, \overset{-}{\sigma_0}, \overset{?}{r}, \overset{-}{g}, \overset{-}{C}, \overset{+/-}{P}, \overset{?}{i}). \qquad (26)$$

Note that the signs of $\partial EW_{H,M,L}/\partial\sigma_0$ are obtained in the neighborhood of the respective entrepreneur's maximum expected wealth.

In tables 11 through 18, for a high value firm, the optimal offer price tends to increase with the quality of underwriting services. An exception occurs when the entrepreneur attempts to raise 75 percent of the total investment from the IPO market in an environment with low underwriting costs and a low revealed fraction, $g = 0.2$ and $r = 0.2$. In this environment, the optimal offer price is a convex function of an underwriter's quality. For the other two types of firms, the offer price shows no apparent relation. The tables also show that the offer prices for all firms increase with the cost of underwriting because the entrepreneurs attempt to at least partially be compensated for the increased cost through higher offer prices. Even though the optimal offer price may depend on several other variables in addition to the preceding two, the relationship is generally ambiguous.

4.2 The Benefits of Underwriting

It has been previously demonstrated that an entrepreneur who owns a firm may purchase underwriting services in order to maximize his expected wealth. Let $EW(i^*, P^*)$ denote the entrepreneur's optimal expected wealth where i^* and P^* represent his optimal choices of the quality of underwriting services and the offer price of the new issue, respectively. Then, the net benefits of underwriting are defined by

$$\Delta EW = EW(i^*, P^*) - EW(0, P^{**}) \tag{26}$$

where P^{**} is the optimal offer price without underwriting and $EW(0, P^{**})$ is the corresponding maximum expected wealth.

Tables 19 and 20 show the benefits of underwriting for various cases. Notice that all entrepreneurs, regardless of their firm values, may benefit from the presence of underwriting services if the circumstance is right. For example, in table 19 an entrepreneur with a low value firm improves his expected wealth by purchasing underwriting services with $i^{*} = 0.9$ when $r = 0.8$ and $g = 0.2$. The tables also reveal that all entrepreneurs may not hire underwriters if the costs and the revealed fraction are high enough, i.e., $g = 0.4$ and $r = 0.8$. These results contrast with the existing IPO models in which only entrepreneurs with high quality firms may signal. Observe that entrepreneurs with low value firms may purchase underwriting services not because they attempt to mimic their counterparts with high value firms but because they can reduce the after-market price uncertainty of the firms and hence increase the probability of successful issuance. Clearly, the results in this model are more general and more realistic than those predicted by other existing models.

It is clear from tables 19 and 20 that an entrepreneur with a higher value firm benefits more than the one with a lower value firm. This result is consistent with the findings of Titman and Trueman and Krinsky and Moore [1989]. The tables also show that the relative benefits of underwriting are greater for an entrepreneur with a high value firm if the proportion of the funds raised from the IPO market increases. Krinsky and Moore predict a similar result.

Tables 11 through 18 state that the marginal costs of increasing the quality of underwriting services to the optimal level are lower for an entrepreneur with a higher value firm when his counterpart w th a lower value firm also hires an underwriter. This result is similar to the one argued by Hughes. Finally, the tables also reveal that the costs of underwriting as a fraction of expected wealth declines with the firm value.

Tables 19 and 20 reveal that the probability of successful issuance increases whenever an entrepreneur optimally purchases underwriting services. This result implies that the presence of underwriters in the economy can increase the social welfare in the sense that the economy as a whole invests more in the productive activities. This is socially efficient behavior. Hiring an underwriter in this paper, of course, involves the costs because of underwriting fees. Nevertheless, the benefits of underwriting exceed the costs as long as an entrepreneur optimally selects the quality of an underwriter. Furthermore, the presence of underwriting services can make some entrepreneurs better off without making anyone worse off, provided that all entrepreneurs maximize their expected wealth. Therefore, both individuals and the economy as a whole can benefit from the presence of underwriters.

5. CONCLUSIONS

The paper has presented a simple model in which an underwriter's quality serves as a means to convey an entrepreneur's private information about his firm's prospects to the market. Underwriting services in this model reduce the unknown fraction of the firm value and hence the difference between the true firm value and its after-market price. Furthermore, underwriting can serve as a mechanism for reducing the price uncertainty of a new firm. For this reason, all entrepreneurs may purchase underwriting services, regardless of their firm values, if they can improve their expected wealth. This result is a major improvement from the current signalling literature in finance which generally predicts that entrepreneurs with high quality firms are the only candidates, who may signal.

It is demonstrated that an entrepreneur should not have any incentive to deviate from his optimal choice of the quality of underwriting services because this behavior lowers his expected wealth. It is also shown that an·entrepreneur, who owns a higher value firm benefits more from the presence of underwriters in the economy. Consequently, he is more likely to hire a more prestigious and more expensive underwriter, thereby reducing the unknown fraction of his firm value. Finally, it is demonstrated that the presence of underwriting services can increase the social welfare because it increases the probability of successful issuance and hence the economy as a whole has more opportunities to invest in the productive activities.

FOOTNOTES

[1] Welch [1989] reports that 288 firms out of 1028 firms, which went public in the 1977-1982 period, reissued by the end of 1987. This number means that fewer than 30 percent of all IPO firms reissue equity shares in the near future, after an initial public offering.

[2] Even though corporate debt is not included in this paper, it does not alter the outcome of the model substantially.

[3] Several IPO models assume that outside investors are risk neutral. For example, see Allen and Faulhaber [1989], Gale and Stiglitz [1989], Hughes [1986], Titman and Trueman [1986], and Welch [1989].

[4] Ritter [1987] has found very few instances of a best effort offer being withdrawn in which the firm subsequently went public.

[5] This argument is initially appeared in Beatty and Ritter [1986]. Tinic [1988] also points out that an underwriter is subject to legal liabilities. This gives another reason why an underwriter does not want to behave opportunistically.

[6] In 95 percent of public securities sales, the issuing firm negotiates the offering terms with the underwriter while the underwriter is selected through a competitive bid in the other five percent. This figure, which is reported by Smith [1986], gives convincing evidence that the issuing firm is able to select its own choice of an underwriter. Once the underwriter is selected, the issuing firm holds a joint meeting with its underwriter to establish the basic characteristics of the offer

including the offer price.

7 The optimization problem in this paper is numerically solved using the Newton-Raphson algorithm. See Dyer and McReynolds [1970] for further discussion.

8 Ritter [1986] reports that the underwriting expense in the best effort contract is 10.26 percent of the gross proceeds during the 1977-1982 period while the total expenses are 17.74 percent. See Ritter for the details.

9 The first order partial derivative of the probability of successful issuance with respect to σ is given by

$$\partial \Pi / \partial \sigma_0 = -n(S)(\partial S / \partial \sigma_0) \tag{F1}$$

where $n(\cdot)$ is the density function of a standard normal variable and S is given by equation (17). The partial derivative of S with respect to σ_0 is $-S/\sigma_0$. As shown in tables 11 through 18, the probability of successful issuance is typically greater than 0.5 when the entrepreneur maximizes his expected wealth. This means that S is smaller than zero and hence S/σ_0 is also negative. Consequently, the partial derivative given by equation (F1) is negative. This implies that the probability of successful issuance decreases with σ_0.

Table 11
Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 50 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.2 and a quality i underwriter charges i times 20 percent of the total IPO proceeds as fees. The total IPO proceeds is $7.1 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i[a]	P[b]	α[c]	Π[d]	EF[e]	EW[f]
Firm Type = Low: Firm Value = $14.2 million					
0.0	15.817	0.551	0.640	0.000	5.006
0.1	15.446	0.540	0.647	0.092	4.875
0.2	15.063	0.529	0.657	0.187	4.747
0.3	14.669	0.516	0.670	0.286	4.626
0.4	14.266	0.502	0.688	0.391	4.517
0.5	13.863	0.488	0.712	0.506	4.427
0.6	13.472	0.473	0.745	0.635	4.372
0.7	13.125	0.459	0.793	0.788	4.379
0.8	12.903	0.450	0.858	0.975	4.504
0.9	13.048	0.456	0.935	1.195	4.857
Firm Type = Average: Firm Value = $21.3 million					
0.0	16.282	0.564	0.671	0.000	8.056
0.1	16.116	0.559	0.694	0.099	8.171
0.2	15.973	0.556	0.721	0.205	8.328
0.3	15.878	0.553	0.752	0.320	8.539
0.4	15.841	0.552	0.788	0.448	8.819
0.5	15.904	0.554	0.829	0.589	9.186
0.6	16.123	0.560	0.873	0.744	9.659
0.7	16.589	0.572	0.917	0.911	10.256
0.8	17.436	0.593	0.955	1.085	10.979
0.9	18.883	0.624	0.983	1.257	11.813
Firm Type = High: Firm Value = $28.4 million					
0.0	16.773	0.577	0.700	0.000	11.460
0.1	16.857	0.579	0.735	0.104	11.983
0.2	17.024	0.583	0.774	0.220	12.953
0.3	17.300	0.590	0.815	0.347	13.297
0.4	17.748	0.600	0.856	0.486	14.100
0.5	18.413	0.614	0.896	0.636	14.996
0.6	19.381	0.634	0.931	0.793	15.965
0.7	20.741	0.658	0.960	0.954	16.972
0.8	22.588	0.686	0.980	1.114	17.974
0.9	24.976	0.716	0.994	1.270	18.935

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 12
Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 50 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.2 and a quality i underwriter charges i times 40 percent of the total IPO proceeds as fees. The total IPO proceeds is $7.1 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i[a]	P[b]	α[c]	Π[d]	EF[e]	EW[f]
Firm Type = Low: Firm Value = $14.2 million					
0.0	15.817	0.551	0.640	0.000	5.006
0.1	15.514	0.542	0.645	0.183	4.783
0.2	15.196	0.533	0.652	0.370	4.561
0.3	14.862	0.522	0.662	0.564	4.343
0.4	14.513	0.511	0.675	0.767	4.130
0.5	14.155	0.498	0.694	0.986	3.928
0.6	13.793	0.485	0.722	1.231	3.746
0.7	13.450	0.472	0.764	1.519	3.604
0.8	13.185	0.462	0.828	1.881	3.546
0.9	13.201	0.462	0.916	2.342	3.671
Firm Type = Average: Firm Value = $21.3 million					
0.0	16.282	0.564	0.671	0.000	8.056
0.1	16.161	0.561	0.692	0.197	8.072
0.2	16.058	0.558	0.718	0.408	8.123
0.3	15.998	0.556	0.748	0.637	8.219
0.4	15.989	0.556	0.782	0.888	8.373
0.5	16.069	0.558	0.821	1.166	8.600
0.6	16.289	0.564	0.865	1.474	8.919
0.7	16.738	0.576	0.910	1.808	9.348
0.8	17.547	0.595	0.951	2.160	9.896
0.9	18.935	0.625	0.981	2.508	10.556
Firm Type = High: Firm Value = $28.4 million					
0.0	16.773	0.577	0.700	0.000	11.460
0.1	16.890	0.580	0.734	0.209	11.879
0.2	17.085	0.584	0.772	0.438	12.373
0.3	17.384	0.592	0.812	0.692	12.951
0.4	17.849	0.602	0.853	0.969	13.615
0.5	18.521	0.617	0.892	1.267	14.361
0.6	19.485	0.636	0.928	1.582	15.172
0.7	20.829	0.659	0.957	1.903	16.019
0.8	22.650	0.687	0.979	2.224	16.861
0.9	25.012	0.716	0.993	2.539	17.666

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 13

Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 50 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.8 and a quality i underwriter charges i times 20 percent of the total IPO proceeds as fees. The total IPO proceeds is $7.1 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i[a]	P[b]	α[c]	Π[d]	EF[e]	EW[f]
Firm Type = Low: Firm Value = $14.2 million					
0.0	12.732	0.442	0.845	0.000	5.310
0.1	12.722	0.442	0.860	0.122	5.272
0.2	12.727	0.442	0.875	0.248	5.242
0.3	12.749	0.443	0.891	0.379	5.224
0.4	12.793	0.445	0.908	0.515	5.219
0.5	12.858	0.448	0.926	0.657	5.230
0.6	12.992	0.454	0.941	0.802	5.258
0.7	13.149	0.460	0.959	0.953	5.309
0.8	13.351	0.468	0.977	1.110	5.386
0.9	13.649	0.480	0.993	1.269	5.495
Firm Type = Average: Firm Value = $21.3 million					
0.0	16.835	0.578	0.942	0.000	11.603
0.1	17.083	0.584	0.951	0.135	11.696
0.2	17.361	0.591	0.958	0.272	11.795
0.3	17.647	0.598	0.967	0.412	11.897
0.4	18.041	0.606	0.972	0.552	12.005
0.5	18.430	0.615	0.978	0.695	12.116
0.6	18.868	0.624	0.984	0.838	12.232
0.7	19.342	0.633	0.989	0.983	12.353
0.8	19.891	0.643	0.993	1.129	12.479
0.9	20.520	0.654	0.997	1.274	12.614
Firm Type = High: Firm Value = $28.4 million					
0.0	21.527	0.670	0.973	0.000	18.511
0.1	22.019	0.678	0.977	0.139	18.662
0.2	22.545	0.685	0.981	0.279	18.810
0.3	23.135	0.693	0.984	0.419	18.953
0.4	23.637	0.700	0.989	0.562	19.092
0.5	24.286	0.708	0.992	0.704	19.227
0.6	24.964	0.716	0.994	0.847	19.358
0.7	25.768	0.724	0.995	0.989	19.487
0.8	26.549	0.733	0.997	1.133	19.612
0.9	27.453	0.741	0.998	1.275	19.732

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 14
Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 50 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.8 and a quality i underwriter charges i times 40 percent of the total IPO proceeds as fees. The total IPO proceeds is $7.1 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i[a]	P[b]	α[c]	Π[d]	EF[e]	EW[f]
Firm Type = Low: Firm Value = $14.2 million					
0.0	12.732	0.442	0.845	0.000	5.310
0.1	12.752	0.443	0.857	0.243	5.150
0.2	12.783	0.445	0.869	0.494	4.995
0.3	12.825	0.446	0.883	0.753	4.846
0.4	12.884	0.449	0.898	1.021	4.706
0.5	12.937	0.451	0.918	1.303	4.576
0.6	13.091	0.458	0.930	1.585	4.459
0.7	13.231	0.463	0.949	1.887	4.359
0.8	13.413	0.471	0.970	2.205	4.280
0.9	13.686	0.481	0.990	2.529	4.233
Firm Type = Average: Firm Value = $21.3 million					
0.0	16.835	0.578	0.942	0.000	11.603
0.1	17.097	0.585	0.950	0.270	11.561
0.2	17.386	0.592	0.958	0.544	11.523
0.3	17.680	0.598	0.966	0.823	11.486
0.4	18.078	0.607	0.971	1.103	11.453
0.5	18.465	0.615	0.977	1.387	11.422
0.6	18.900	0.624	0.983	1.675	11.394
0.7	19.370	0.633	0.988	1.965	11.369
0.8	19.908	0.643	0.993	2.256	11.350
0.9	20.527	0.654	0.997	2.548	11.340
Firm Type = High: Firm Value = $28.4 million					
0.0	21.527	0.670	0.973	0.000	18.511
0.1	22.027	0.678	0.977	0.277	18.524
0.2	22.559	0.685	0.981	0.557	18.531
0.3	23.154	0.693	0.984	0.838	18.534
0.4	23.656	0.700	0.989	1.123	18.531
0.5	24.305	0.708	0.991	1.408	18.524
0.6	24.987	0.716	0.994	1.694	18.512
0.7	25.783	0.725	0.995	1.978	18.497
0.8	26.559	0.733	0.997	2.265	18.479
0.9	27.453	0.741	0.998	2.550	18.457

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 15
Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 75 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.2 and a quality i underwriter charges i times 20 percent of the total IPO proceeds as fees. The total IPO proceeds is $10.65 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i[a]	P[b]	α[c]	Π[d]	EF[e]	EW[f]
Firm Type = Low: Firm Value = $14.2 million					
0.0	19.069	0.442	0.528	0.000	3.313
0.1	18.644	0.429	0.526	0.112	3.091
0.2	18.159	0.415	0.524	0.223	2.863
0.3	17.720	0.399	0.523	0.334	2.628
0.4	17.217	0.381	0.523	0.445	2.387
0.5	16.683	0.362	0.525	0.559	2.137
0.6	16.113	0.339	0.531	0.679	1.880
0.7	15.506	0.313	0.546	0.815	1.615
0.8	14.863	0.283	0.583	0.993	1.352
0.9	14.216	0.251	0.686	1.316	1.129
Firm Type = Average: Firm Value = $21.3 million					
0.0	19.505	0.454	0.563	0.000	5.442
0.1	19.237	0.446	0.580	0.124	5.391
0.2	18.972	0.439	0.601	0.256	5.360
0.3	18.715	0.431	0.627	0.401	5.358
0.4	18.477	0.424	0.661	0.563	5.398
0.5	18.278	0.417	0.703	0.748	5.498
0.6	18.155	0.413	0.756	0.966	5.687
0.7	18.175	0.414	0.820	1.223	6.012
0.8	18.476	0.424	0.893	1.522	6.536
0.9	19.332	0.449	0.958	1.837	7.331
Firm Type = High: Firm Value = $28.4 million					
0.0	19.967	0.467	0.596	0.000	7.895
0.1	19.911	0.465	0.629	0.134	8.180
0.2	19.903	0.465	0.668	0.285	8.538
0.3	19.965	0.467	0.713	0.455	8.986
0.4	20.133	0.471	0.762	0.649	9.544
0.5	20.460	0.479	0.815	0.868	10.235
0.6	21.023	0.493	0.869	1.111	11.071
0.7	21.932	0.514	0.919	1.370	12.055
0.8	23.295	0.543	0.960	1.635	13.159
0.9	25.361	0.580	0.985	1.889	14.341

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 16

Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 75 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.2 and a quality i underwriter charges i times 40 percent of the total IPO proceeds as fees. The total IPO proceeds is $10.65 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i[a]	P[b]	α[c]	Π[d]	EF[e]	EW[f]
Firm Type = Low: Firm Value = $14.2 million					
0.0	19.069	0.442	0.528	0.000	3.313
0.1	18.784	0.433	0.521	0.222	2.979
0.2	18.478	0.424	0.512	0.436	2.642
0.3	18.147	0.413	0.501	0.641	2.301
0.4	17.791	0.401	0.489	0.834	1.955
0.5	17.403	0.388	0.475	1.011	1.604
0.6	16.980	0.373	0.455	1.164	1.247
0.7	16.518	0.355	0.429	1.278	0.884
0.8	16.010	0.335	0.383	1.306	0.516
0.9	15.482	0.312	0.265	1.016	0.158
Firm Type = Average: Firm Value = $21.3 million					
0.0	19.505	0.454	0.563	0.000	5.442
0.1	19.328	0.449	0.576	0.246	5.267
0.2	19.149	0.444	0.594	0.506	5.106
0.3	18.972	0.439	0.615	0.786	4.961
0.4	18.805	0.434	0.643	1.095	4.842
0.5	18.661	0.429	0.679	1.446	4.762
0.6	18.565	0.426	0.726	1.857	4.740
0.7	18.571	0.427	0.788	2.351	4.811
0.8	18.792	0.433	0.865	2.949	5.036
0.9	19.500	0.454	0.943	3.617	5.503
Firm Type = High: Firm Value = $28.4 million					
0.0	19.967	0.467	0.596	0.000	7.895
0.1	19.977	0.467	0.627	0.267	8.046
0.2	20.029	0.468	0.663	0.565	8.254
0.3	20.142	0.471	0.705	0.901	8.533
0.4	20.350	0.477	0.752	1.282	8.899
0.5	20.698	0.485	0.804	1.712	9.372
0.6	21.258	0.499	0.858	2.193	9.967
0.7	22.135	0.519	0.910	2.713	10.691
0.8	23.461	0.546	0.953	3.248	11.530
0.9	25.427	0.581	0.983	3.768	12.454

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 17

Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 75 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.8 and a quality i underwriter charges i times 20 percent of the total IPO proceeds as fees. The total IPO proceeds is $10.65 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i^a	P^b	α^c	Π^d	EF^e	EW^f
Firm Type = Low: Firm Value = $14.2 million					
0.0	14.438	0.262	0.661	0.000	2.464
0.1	14.361	0.258	0.669	0.142	2.312
0.2	14.284	0.254	0.678	0.289	2.162
0.3	14.207	0.250	0.690	0.441	2.013
0.4	14.129	0.246	0.706	0.601	1.867
0.5	14.054	0.242	0.727	0.774	1.725
0.6	13.985	0.238	0.755	0.965	1.591
0.7	13.927	0.235	0.794	1.184	1.469
0.8	13.897	0.234	0.849	1.447	1.371
0.9	13.935	0.236	0.924	1.771	1.322
Firm Type = Average: Firm Value = $21.3 million					
0.0	17.982	0.408	0.879	0.000	7.631
0.1	18.121	0.412	0.893	0.190	7.654
0.2	18.284	0.418	0.908	0.387	7.687
0.3	18.481	0.424	0.922	0.589	7.732
0.4	18.720	0.431	0.935	0.797	7.789
0.5	18.983	0.439	0.949	1.010	7.859
0.6	19.290	0.448	0.962	1.229	7.945
0.7	19.652	0.458	0.973	1.451	8.047
0.8	20.059	0.469	0.986	1.679	8.167
0.9	20.562	0.482	0.995	1.908	8.312
Firm Type = High: Firm Value = $28.4 million					
0.0	22.401	0.525	0.947	0.000	14.102
0.1	22.795	0.533	0.955	0.203	14.243
0.2	23.204	0.541	0.963	0.410	14.387
0.3	23.721	0.559	0.968	0.619	14.531
0.4	24.133	0.569	0.977	0.833	14.677
0.5	24.689	0.580	0.983	1.047	14.823
0.6	25.341	0.590	0.986	1.260	14.971
0.7	25.982	0.601	0.990	1.477	15.120
0.8	26.660	0.612	0.995	1.695	15.271
0.9	27.456	0.741	0.998	1.912	15.430

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 18

Expected Wealth, Optimal Offer Prices, and Underwriting when firms raise 75 percent of the total investment from the IPO market in an environment in which the revealed fraction of the firm value is 0.8 and a quality i underwriter charges i times 40 percent of the total IPO proceeds as fees. The total IPO proceeds is $10.65 million, the total investment is $14.2 million, the prior mean of an IPO firm value is $21.3 million, and the prior standard deviation is $14.2 million.

i[a]	P[b]	α[c]	Π[d]	EF[e]	EW[f]
Firm Type = Low: Firm Value = $14.2 million					
0.0	14.438	0.262	0.661	0.000	2.464
0.1	14.447	0.263	0.657	0.280	2.171
0.2	14.458	0.263	0.650	0.554	1.878
0.3	14.470	0.264	0.642	0.821	1.587
0.4	14.483	0.265	0.631	1.075	1.296
0.5	14.499	0.265	0.614	1.308	1.007
0.6	14.519	0.266	0.587	1.500	0.721
0.7	14.548	0.268	0.537	1.600	0.441
0.8	14.599	0.270	0.420	1.431	0.182
0.9	14.695	0.275	0.107	0.410	0.008
Firm Type = Average: Firm Value = $21.3 million					
0.0	17.982	0.408	0.879	0.000	7 631
0.1	18.154	0.413	0.891	0.379	7.464
0.2	18.345	0.419	0.903	0.770	7.301
0.3	18.564	0.426	0.916	1.170	7.144
0.4	18.813	0.434	0.928	1.581	6.994
0.5	19.078	0.442	0.941	2.005	6.851
0.6	19.379	0.450	0.955	2.440	6.719
0.7	19.737	0.460	0.967	2.883	6.579
0.8	20.114	0.471	0.982	3.345	6.492
0.9	20.571	0.482	0.995	3.814	6.406
Firm Type = High: Firm Value = $28.4 million					
0.0	22.401	0.525	0.947	0.000	14.102
0.1	22.814	0.533	0.954	0.406	14.040
0.2	23.220	0.541	0.962	0.820	13.977
0.3	23.744	0.551	0.967	1.236	13.913
0.4	24.176	0.559	0.976	1.663	13.846
0.5	24.733	0.569	0.981	2.090	13.779
0.6	25.382	0.580	0.984	2.516	13.712
0.7	26.016	0.591	0.989	2.950	13.643
0.8	26.682	0.601	0.994	3.388	13.577
0.9	27.457	0.612	0.998	3.825	13.518

[a] i = underwriter's quality [b] P = offer price ($ million)
[c] α = fraction of entrepreneurial ownership retention
[d] Π = probability that the issue is fully subscribed
[e] EF = expected value of underwriting fees ($ million)
[f] EW = entrepreneur's expected wealth ($ million)

Table 19

Benefits of Underwriting when 50 percent of the total investment is to be raised from the IPO market. The total investment is \$14.2 million, the prior mean of an IPO firm value is \$21.3 million, and the prior standard deviation is \$14.2 million. EW^{*} and EW^{**} are the maximum values of the entrepreneur's expected wealth in the respective economies with and without underwriting services. i^{*} is the quality of underwriting services which the entrepreneur should optimally purchase. ΔEW measures measures the benefits of underwriting as a percentage of EW^{**}. Values in parentheses indicate the probability of successful issuance.

r[a]	g[b]	Firm Type[c]	i^{*}	EW^{*}	EW^{**}	ΔEW
0.2	0.2	L	0.0	5.006 (0.640)	5.006 (0.640)	0.00
		M	0.9	11.813 (0.983)	8.056 (0.671)	46.64
		H	0.9	18.935 (0.994)	11.460 (0.700)	65.23
	0.4	L	0.0	5.006 (0.640)	5.006 (0.640)	0.00
		M	0.9	10.556 (0.981)	8.056 (0.671)	31.03
		H	0.9	17.666 (0.993)	11.460 (0.700)	54.15
0.8	0.2	L	0.9	5.419 (0.993)	5.310 (0.845)	2.05
		M	0.9	12.614 (0.997)	11.603 (0.942)	8.71
		H	0.9	19.732 (0.998)	18.511 (0.973)	6.60
	0.4	L	0.0	5.310 (0.845)	5.310 (0.845)	0.00
		M	0.0	11.603 (0.942)	11.603 (0.942)	0.00
		H	0.3	18.534 (0.984)	18.511 (0.973)	0.12

[a] r is the revealed fraction of the firm.

[b] g is the underwriting fee constant. An underwriter with quality i charges igC as underwriting fees where C is the total IPO proceeds.

[c] Firm type L represents a firm whose true value is 1/3 lower than its prior mean value.

Firm type M represents a firm whose true value is the same as its prior mean value.

Firm type H represents a firm whose true value is 1/3 higher than its prior mean value.

Table 20
Benefits of Underwriting when 75 percent of the total investment is to be raised from the IPO market. The total investment is \$14.2 million, the prior mean of an IPO firm value is \$21.3 million, and the prior standard deviation is \$14.2 million. EW^{*} and EW^{**} are the maximum values of the entrepreneur's expected wealth in the respective economies with and without underwriting services. i^{*} is the quality of underwriting services which the entrepreneur should optimally purchase. ΔEW measures measures the benefits of underwriting as a percentage of EW^{**}. Values in parentheses indicate the probability of successful issuance.

r[a]	g[b]	Firm Type[c]	i^{*}	EW^{*}	EW^{**}	ΔEW
0.2	0.2	L	0.0	3.313 (0.528)	3.313 (0.528)	0.00
		M	0.9	7.331 (0.958)	5.442 (0.563)	34.71
		H	0.9	14.341 (0.985)	7.895 (0.596)	81.65
	0.4	L	0.0	3.313 (0.528)	3.313 (0.528)	0.00
		M	0.9	5.503 (0.943)	5.442 (0.563)	1.12
		H	0.9	12.454 (0.983)	7.895 (0.596)	57.75
0.8	0.2	L	0.0	2.464 (0.661)	2.464 (0.661)	0.00
		M	0.9	8.312 (0.995)	7.631 (0.879)	8.92
		H	0.9	15.430 (0.998)	14.102 (0.947)	9.42
	0.4	L	0.0	2.464 (0.661)	2.464 (0.661)	0.00
		M	0.0	7.631 (0.879)	7.631 (0.879)	0.00
		H	0.0	14.102 (0.947)	14.102 (0.947)	0.00

[a] r is the revealed fraction of the firm.
[b] g is the underwriting fee constant. An underwriter with quality i charges igC as underwriting fees where C is the total IPO proceeds.
[c] Firm type L represents a firm whose true value is 1/3 lower than its prior mean value.
Firm type M represents a firm whose true value is the same as its prior mean value.
Firm type H represents a firm whose true value is 1/3 higher than its prior mean value.

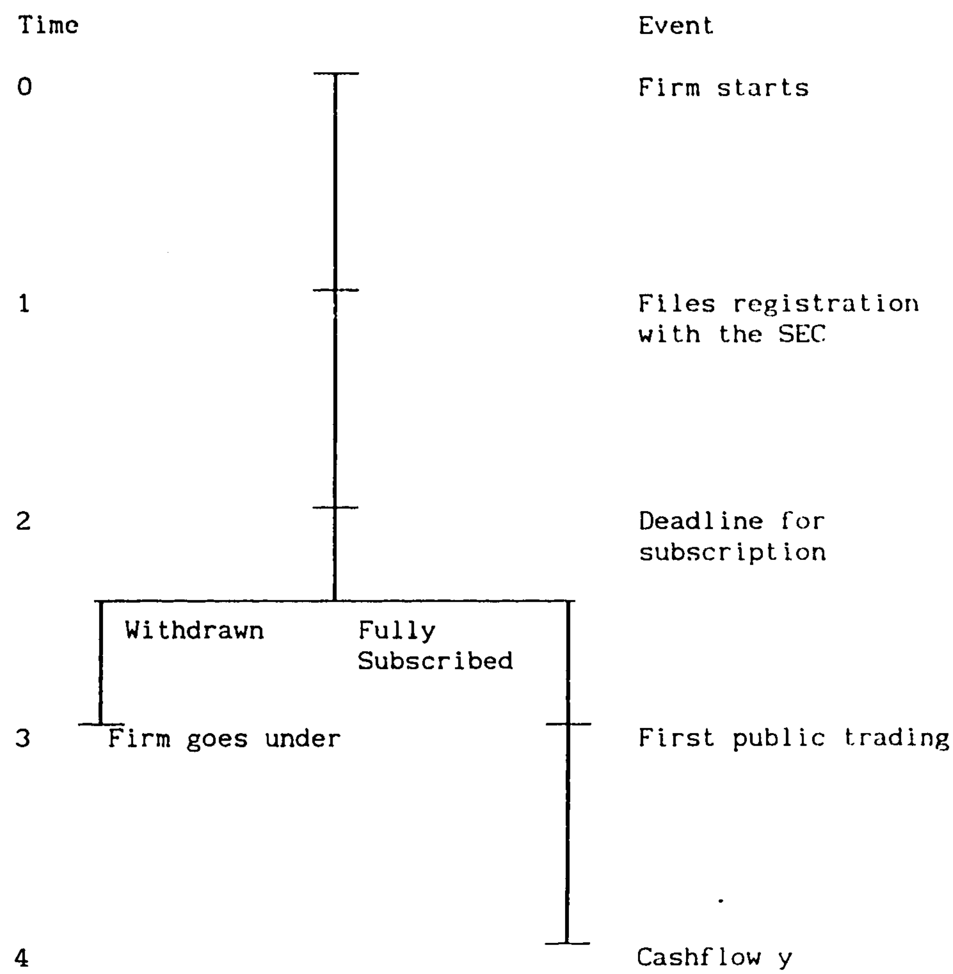


Figure 1. The sequence of events.

In the economy with underwriting services, the firm selects an underwriter between time 0 and 1. If the firm's new issue is not fully subscribed by time 2, then the issue will be withdrawn. Consequently, the firm cannot implement its investment project and will go under.

FIG 2. EXPECTED WEALTH AND UNDERWRITING
(PTG = 50%, r = 0.20, and g = 0.20)

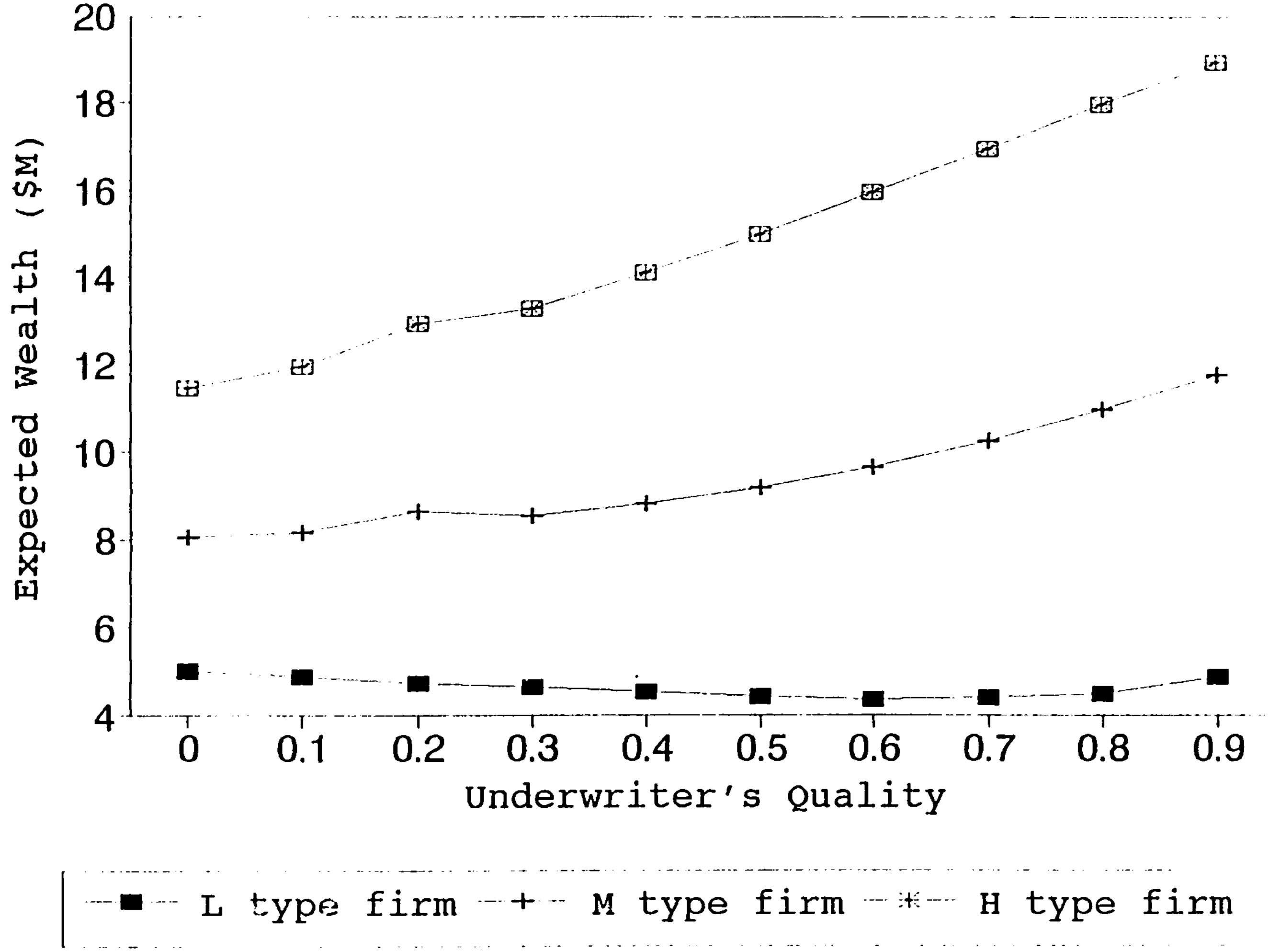


FIG 3. EXPECTED WEALTH AND UNDERWRITING
(PTG = 50%, r = 0.20, and g = 0.40)

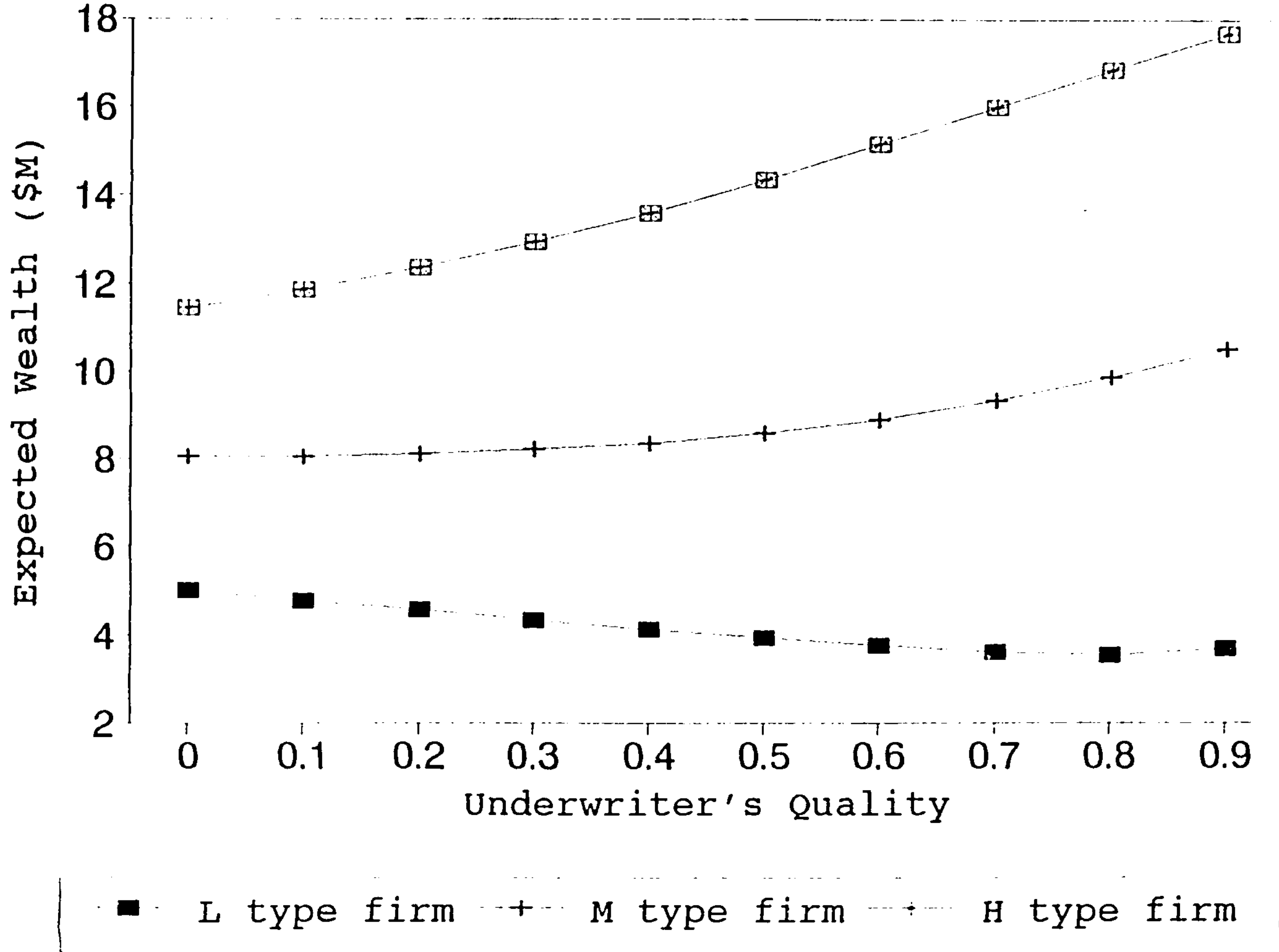


FIG 4. EXPECTED WEALTH AND UNDERWRITING
(PTG = 50%, r = 0.80, and g = 0.20)

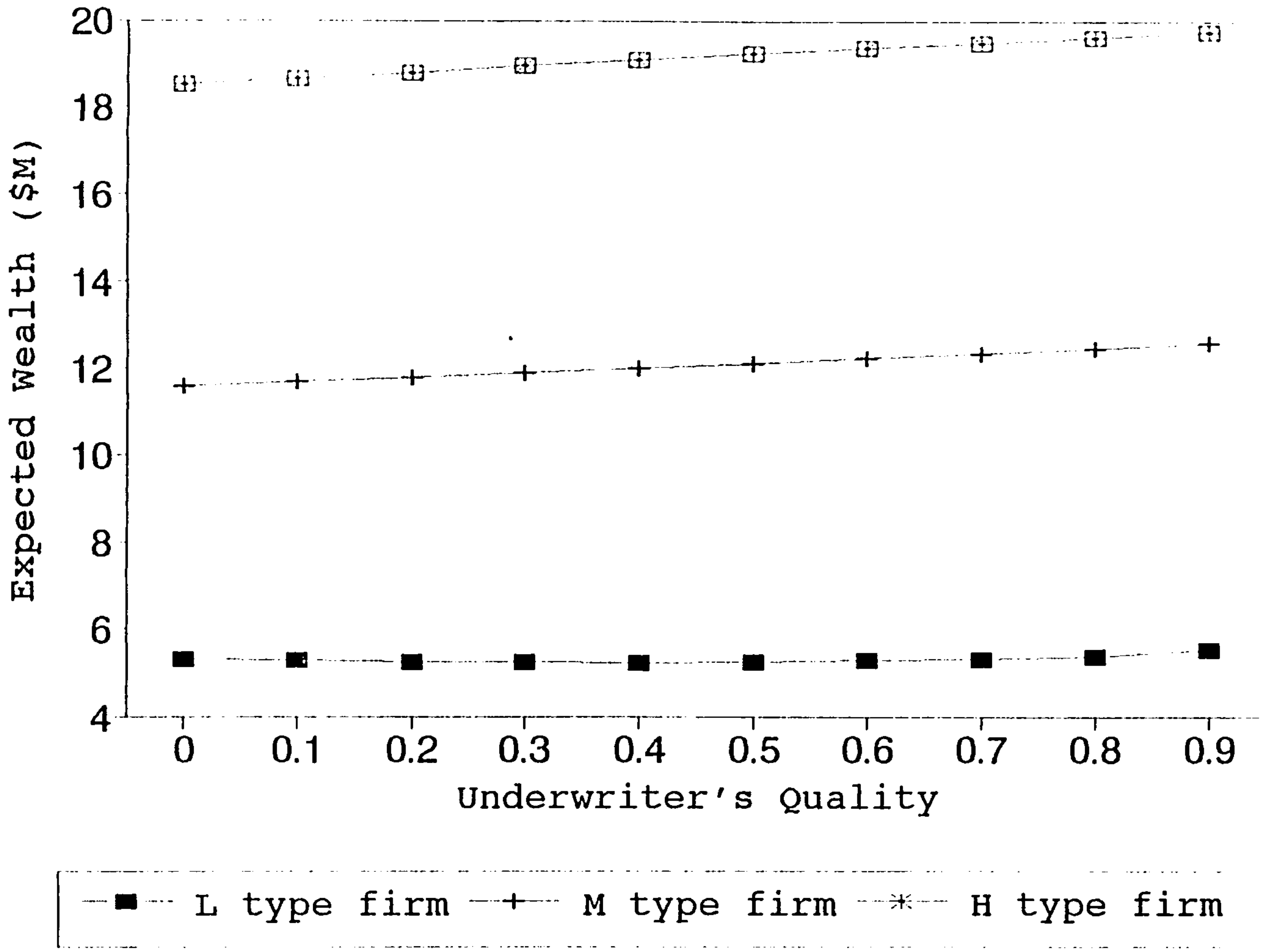


FIG 5. EXPECTED WEALTH AND UNDERWRITING
(PTG = 50%, r = 0.80, and g = 0.40)

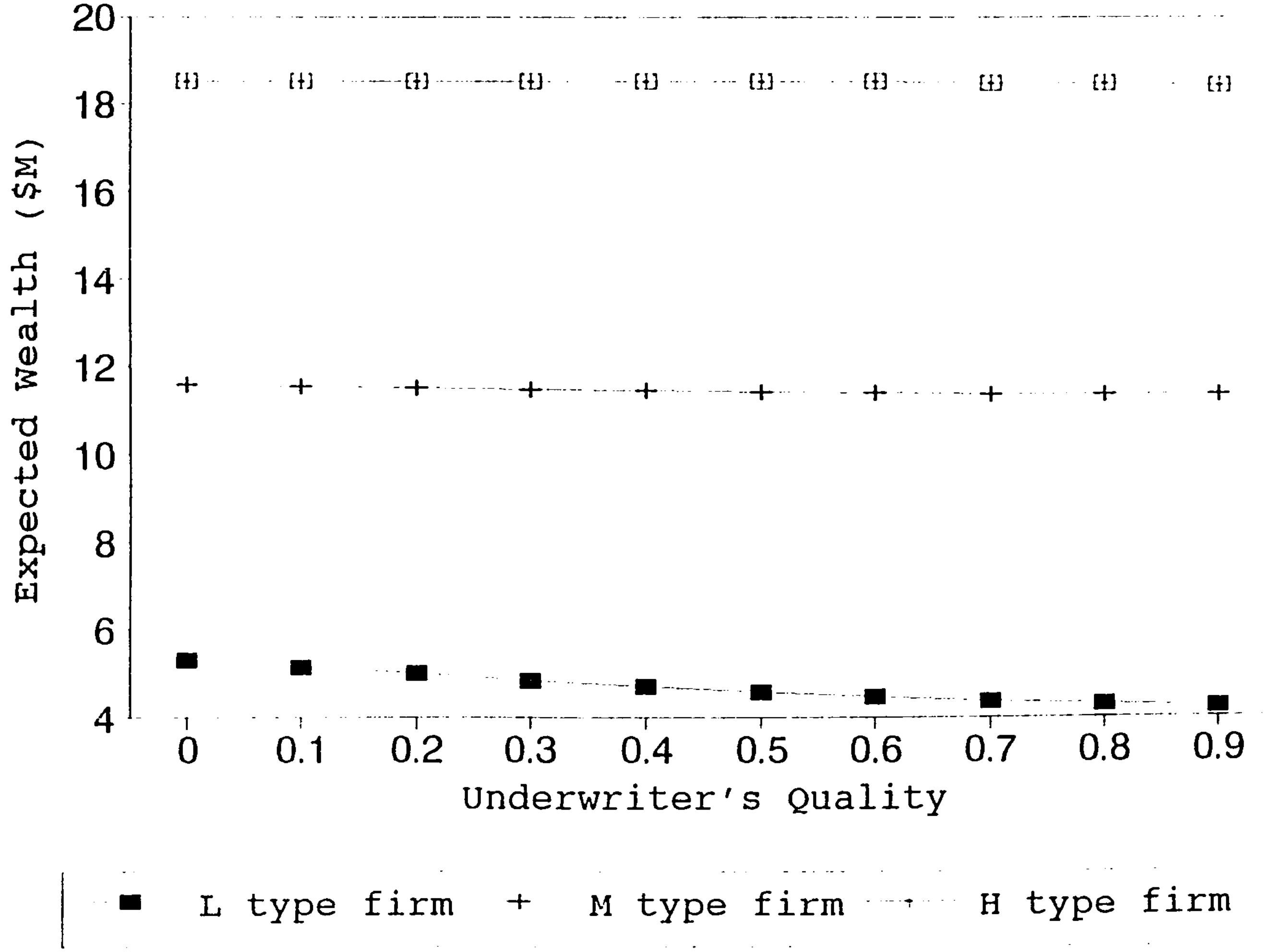


FIG 6. EXPECTED WEALTH AND UNDERWRITING
(PTG = 75%, r = 0.20, and g = 0.20)

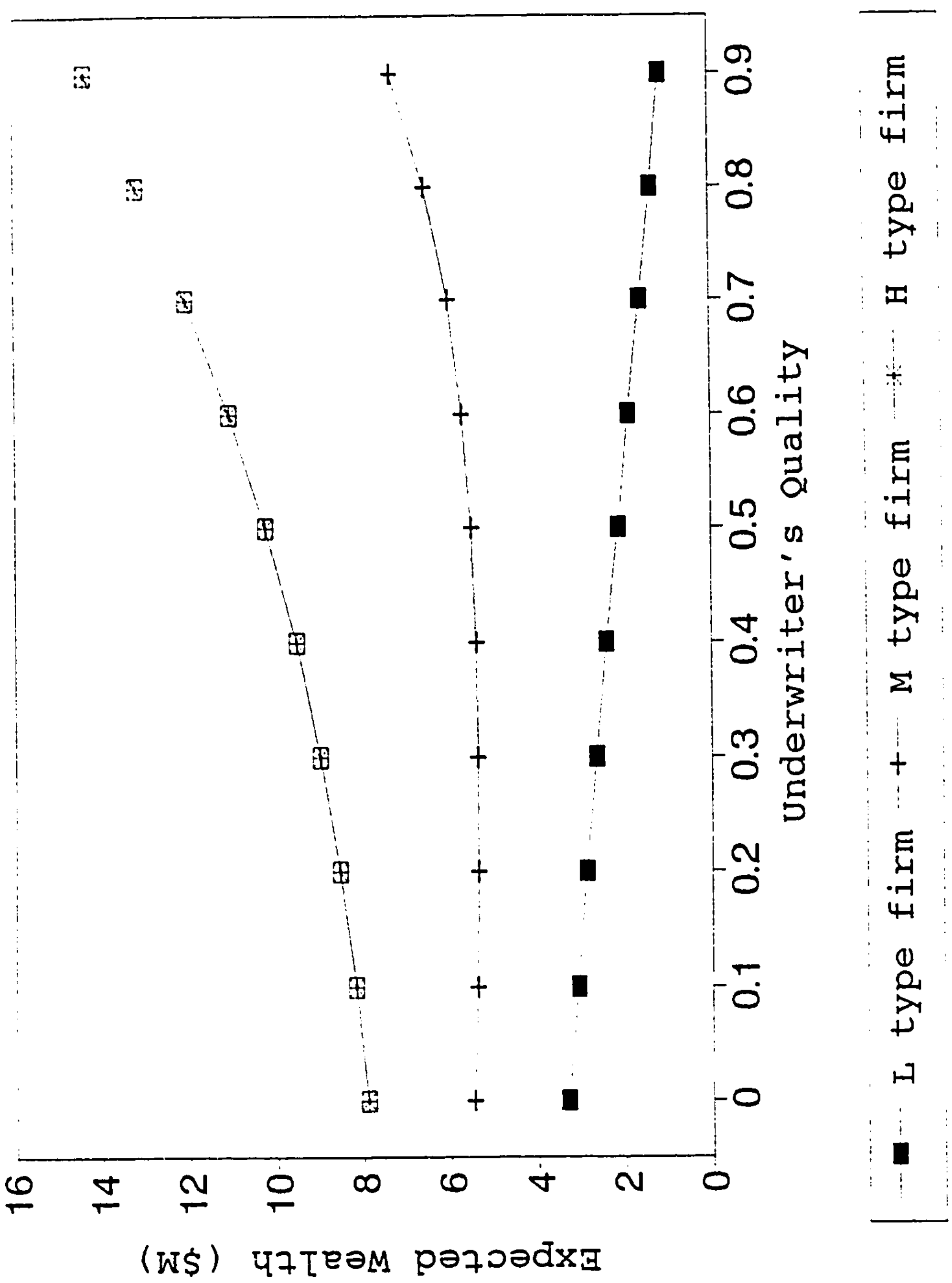


FIG 7. EXPECTED WEALTH AND UNDERWRITING
(PTG = 75%, r = 0.20, and g = 0.40)

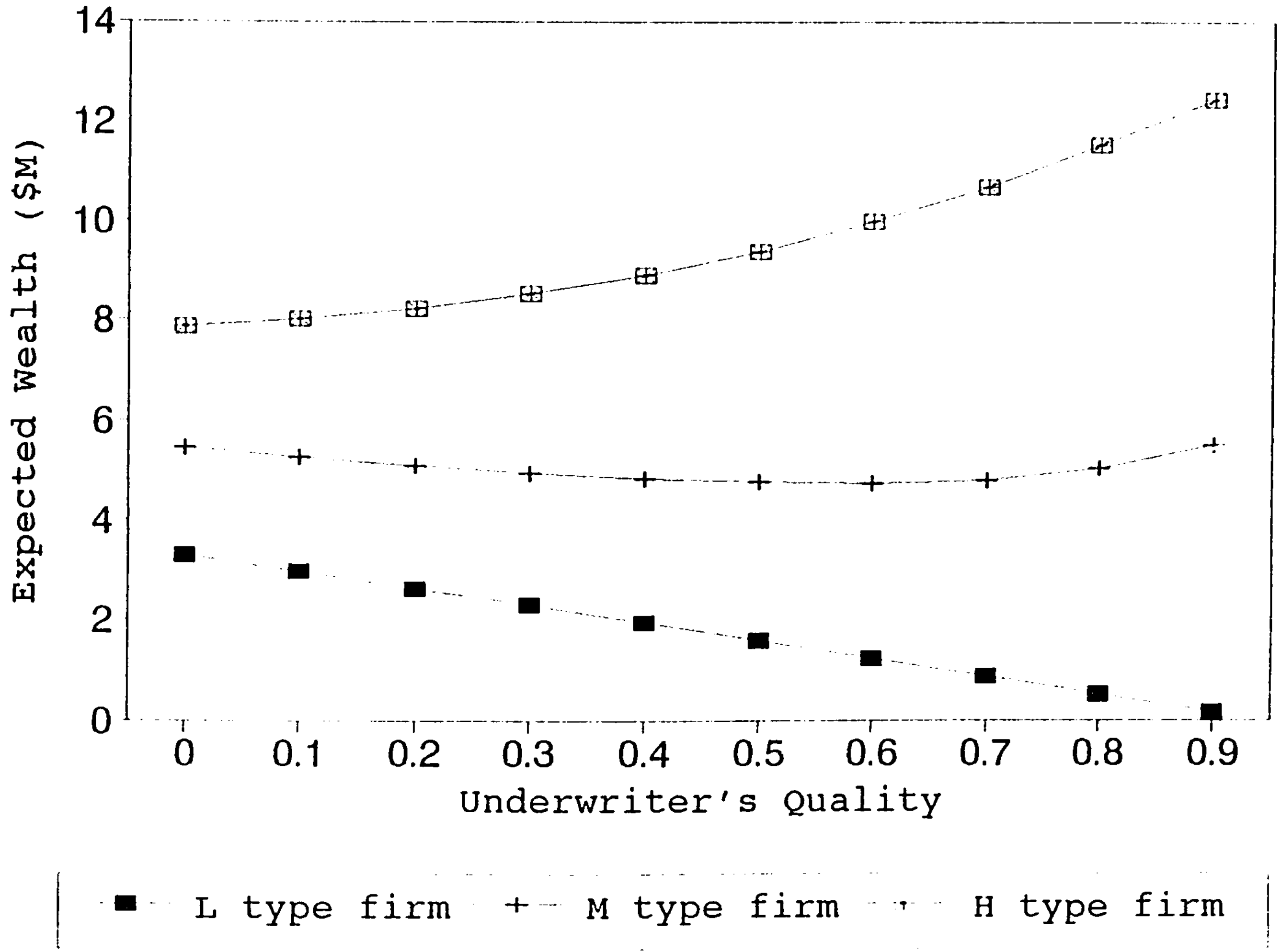


FIG 8. EXPECTED WEALTH AND UNDERWRITING
(PTG = 75%, r = 0.80, and g = 0.20)

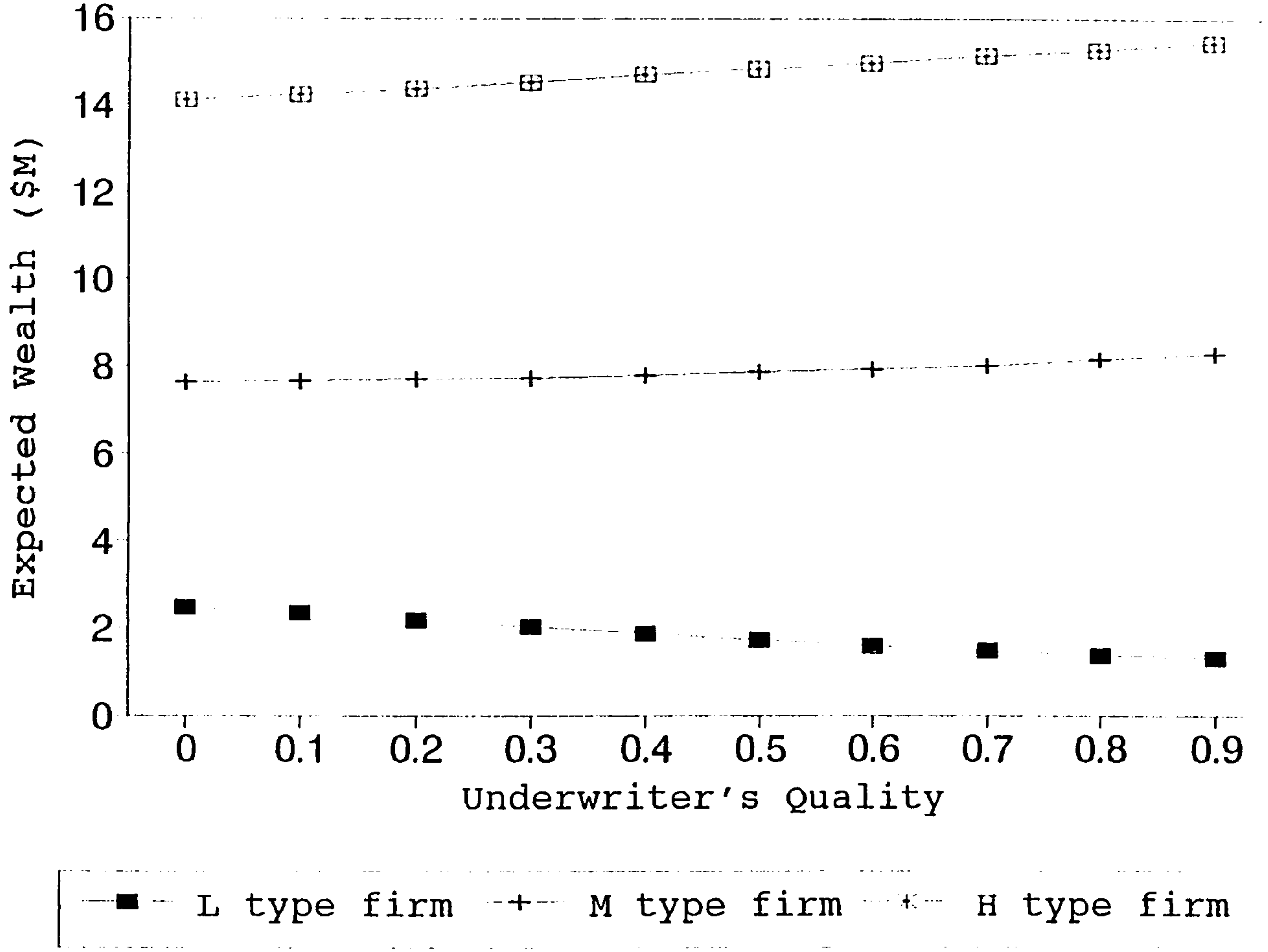
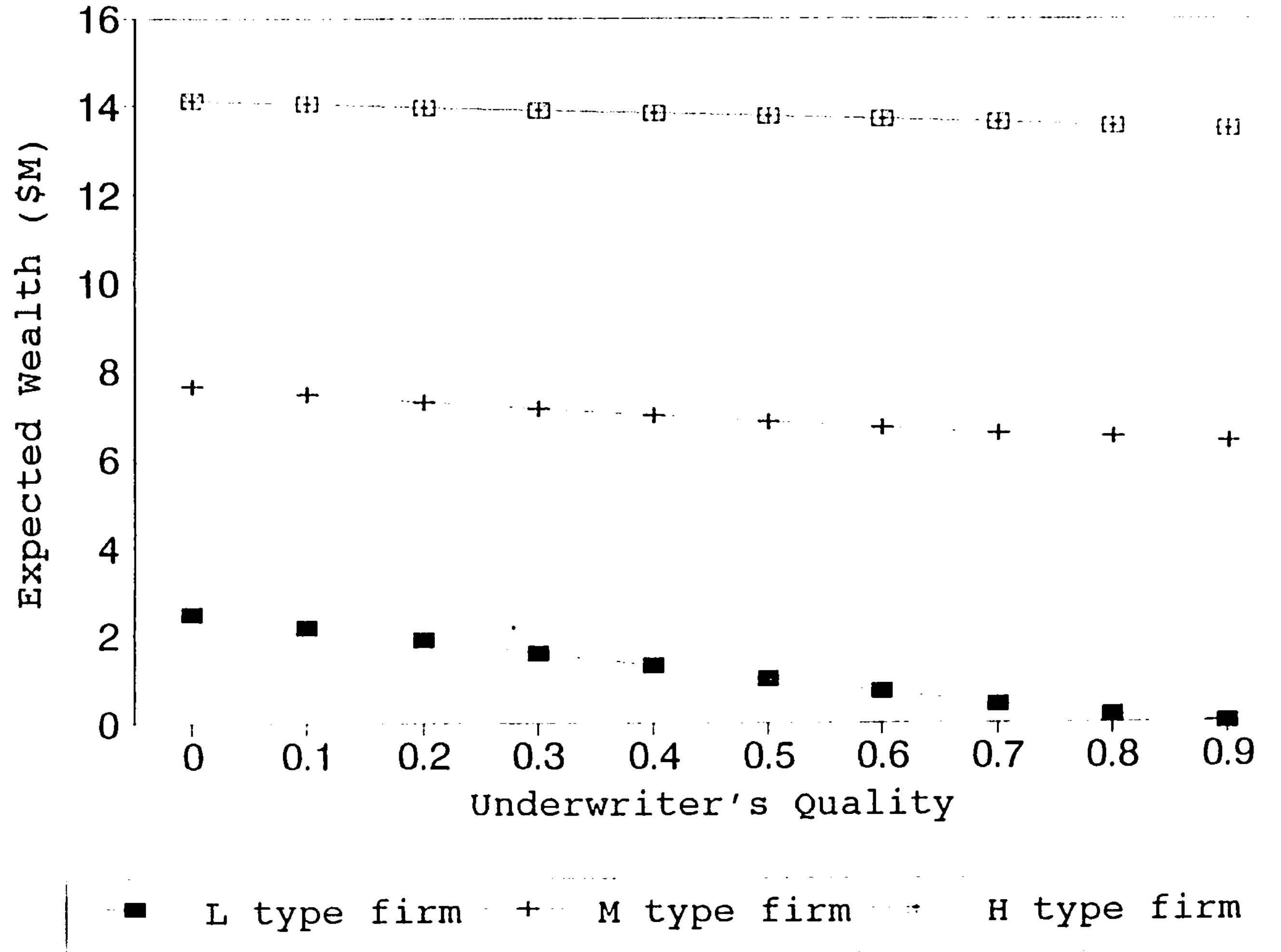


FIG 9. EXPECTED WEALTH AND UNDERWRITING
(PTG = 75%, r = 0.80, and g = 0.40)



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