University of Alberta

Reliability Based Design of Series-Parallel Systems for Minimal Life Cycle Cost

by

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A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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To my parents,

Amarjit and Balwant,

who taught me the way of strategy.

The Way of Strategy

Do not think dishonestly.

The way is in training.

Become acquainted with every art.

Know the ways of all professions.

Distinguish between gain and loss in worldly matters.

Develop intuitive judgment and understanding for everything.

Perceive those things which cannot be seen.

Pay attention even to trifles.

Do nothing which is of no use.

Miyamoto Musashi (1584 - 1645)

Abstract

As modern systems are becoming more complex and automated, an important measure of their effectiveness is their reliability. For a system to perform below a maximum allowed failure rate, a reliability based design (RBD) of the system is imperative.

In this thesis, RBD formulations of series-parallel systems which consist of components whose reliability is a function of time are developed. It is required that these systems perform below a maximum allowed failure rate. Whenever the system reaches the maximum allowed failure rate, preventive maintenance (PM) is performed on the system. If the system fails between these PM intervals then minimal repairs are performed.

Based on two different PM models proposed by Nakagawa (1988), two RBD formulations are developed for systems with monotonically increasing failure rates. In formulation 1, the system's effective age becomes smaller after each PM action but its failure rate function does not change. In formulation 2, the system failure rate is reduced to zero after each PM action but the system deteriorates faster after each PM action. The total cost modeled in both formulations consist of cost of acquisition, installation, preventive maintenance and minimal repair. The average annual cost is minimized over system's useful life to give optimal system design, optimal PM intervals, and optimal system replacement time. Formulation 2 is further modified to incorporate non-zero failure rate at time equal to zero. A salvage value function of a deteriorating system is also proposed which incorporates the economic effects of increasing failure rate and preventive maintenance on the system performance.

Finally, a RBD formulation is developed for systems with components which

follow a bathtub shaped failure rate curve. The total cost consists of costs related to burn-in, warranty, installation, preventive maintenance and minimal repair. The average annual cost is minimized over the system's useful life to give optimal system design, optimal burn-in period, optimal PM intervals, and optimal replacement time for a system.

For all the formulations, the cost minimization is a mixed non-linear integer programming problem. Genetic algorithms are used as an optimization tool due to their flexibility and ease to solve complex non-linear mixed integer programming problems like the one in this thesis. The results have important applications in the area of economic evaluation of automated manufacturing systems where high investment costs are involved to acquire a system which must perform below a given failure rate. The research performed in this thesis is useful for reliability engineers, engineering economists and strategic planners.

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Chapter 1

Introduction

Design of a system is a progression from an abstract notion to something which has form and function. In a concurrent engineering approach, the design process starts with a requirement, and from this point onwards, it evolves through a simultaneous execution of various tasks performed by various design specialists. Each major task of the design process involves evaluation of a design criterion during different phases of a product life cycle (Blanchard, 1992). The important tasks of the design process can be identified as follows:

Task 1	Establishment of market baseline and identification of customer needs.
Task 2	A detailed functional definition of the system, and development of conceptual design to meet the system's functional requirements.
Task 3	Reconfiguration to meet the system's projected performances over its life cycle.
Task 4	Development of the prototype and testing for the prototype's viability.
Task 5	Approval of system design for production.

Task 1 involves the establishment of market baseline and identification of customer needs. Market baseline is established by conducting a market survey of similar products in the market, customer interviews, expert opinions and competitive benchmarking studies. It provides valuable information such as cost, physical parameters, etc., which is necessary during the later part of the design process. Once the market baseline is established, customer needs are identified to be incorporated into design.

In task 2, a detailed functional specification of the system is performed. Utilization of a functional approach as a basis for the identification of design requirements for each hierarchical level of the system is an essential element of the preliminary design. This is accomplished through the development of functional flow block diagrams. These diagrams are developed for the primary purpose of structuring system requirements into functional terms. $IDEF_0$ is a structured analysis methodology capable of representing complex functional relationships of a system. A concise introduction to $IDEF_0$ with an example is presented by Monga, Zuo and Jaisingh (1993) and a state-of-the-art review of $IDEF_0$ is done by Colquohohn, Baines and Crossley (1993). Once the functional specifications of the system are completed, resources are allocated to perform these functions.

In task 3, a system designer is faced with the problem of how the designed system matches its projected performances over the intended period of use. This leads to setting of objectives and requirements based on certain system parameters and checking whether or not these objectives will be met when the system is put in operation. As modern systems become more complex and automated, an important measure of their effectiveness is their reliability. For a system to perform below a maximum allowable failure rate, a reliability based design (RBD) of the system is imperative. A traditional RBD problem involves evaluating various design configurations for minimal system cost or maximum reliability while satisfying other constraints, namely, weight, volume, cost, and/or reliability.

Once the system is reconfigured for reliability and other measures, a prototype is developed and tested for various environmental, technical and safety requirements. If the prototype meets all the user's requirements and standards then the design is finalized for production.

The research performed in this thesis fits in task 3 of the design process, and contributes to the area of RBD of systems with deteriorating components. The research is presented in eight chapters.

In Chapter 2, we review the literature related to optimal redundancy allocation in RBD problems with constant component reliability. Formulations related to the optimization of system design for minimal cost and maximum reliability are presented. Various optimization techniques to solve such problems are discussed. The shortcomings and limitations of RBD formulations with constant component reliability are also discussed.

Chapter 3 provides a brief overview of cost and failure characteristics of the system during its life cycle. Two important maintenance actions, namely, preventive maintenance and minimal repair commonly used in modeling maintenance on deteriorating systems are presented. The characteristics of these maintenance actions are discussed.

In Chapter 4, we discuss in detail the optimization tool used in this thesis, Genetic Algorithms (GAs). This chapter describes step by step, how a simple genetic algorithm works using a simple example. The chapter discusses various GA specific implementation issues and also reviews the application of GAs to various reliability problems.

In Chapter 5, we develop two formulations for the reliability based design of series-parallel systems considering maintenance. All the components have an increasing failure rate. Two types of maintenance actions are modeled, namely, preventive maintenance (PM) and minimal repair. PM is performed when the systems reaches the maximum allowed failure rate. If the system fails between those PM intervals, minimal repair is performed. Based on Nakagawa's age reduction and hazard rate PM models, two models are developed for minimizing the average annual cost of the system. Genetic algorithms are used to find optimal system designs (i.e. number of redundant components at each stage), optimal PM intervals, and optimal system replacement time.

In Chapter 6, the salvage value of the system is taken into consideration for reliability based design of a series-parallel system with an increasing failure rate. Both preventive maintenance and minimal repair are considered. Nakagawa's hazard rate concept for modeling PM is modified to include non-zero failure rates at time zero. The system life cycle cost includes the salvage value of the system apart from acquisition, installation and maintenance cost. A salvage value function of a deteriorating system is proposed. The proposed function incorporates the economic effects of failure rate deterioration and preventive maintenance. The research analyzes the effect of salvage value on system design, economic life of the system and its average annual cost. It has been found that sometimes the economic incentive of capturing the decreasing salvage value of the system results in early replacement of the system, hence making the economic life of the system shorter than the case when the salvage value is ignored. The research also discusses the implications of product life cycles and characteristics of the systems to justify whether such a salvage value function should be included during a design evaluation.

Chapter 7 proposes a RBD model for a series-parallel system considering burn-in, warranty, and maintenance. This model includes all three stages of a system's life cycle, namely, infant mortality, useful period, and the wear-out period. The system life cycle cost includes burn-in, warranty, installation, preventive maintenance and minimal repair. The research provides a brief overview of various warranty policies and their relevance to reliability based design of systems with deteriorating components. In this research we use a warranty policy under which if a failure occurs during a given warranty period then all the related costs are borne by the manufacturer (Murthy and Blischke, 1992). The formulation gives an optimal system design, optimal burn-in period, optimal preventive maintenance intervals and optimal replacement time. The average annual cost of the system is minimized over the system's useful life using Genetic Algorithms. Three distinct types of failure rate bathtub curves are studied to investigate the changes in system design and burn-in period for different lengths of warranty.

Finally Chapter 8 provides a summary of this research and discusses future directions in the area of reliability based design of systems with deteriorating components.

Chapter 2

Review: Reliability Based Design with Constant Component Reliability

2.1 Introduction

The reliable performance of a system is of utmost importance in many industrial, military and everyday life situations. Although the qualitative concept of reliability is not new, its quantitative aspects have only been developed over the past two decades. Such development has resulted from increasing need for systems and components with higher reliability and lower cost. There exist several methods for designing such systems. These methods include using large safety factors, reducing the complexity of the system, increasing the reliability of constituent components through a product improvement program, using structural redundancy and practicing a planned maintenance and repair schedule. A good deal of effort has been focused in the field of optimal redundancy allocation (Tillman, Hwang and Kuo, 1985). In this chapter we briefly review the literature related to optimal redundancy allocation in reliability based design (RBD) problems. Section 2.2 discusses commonly analyzed system configurations in RBD problems. Section 2.3

relates to the formulation of RBD optimization problems with constant component reliability. Various optimization techniques for solving such problems are also discussed. Section 2.4 briefly discusses the shortcomings and limitations of RBD problems with constant component reliability and discusses the reasons to consider RBD problems with components whose reliability is a function of time.

2.2 Reliability Block Diagrams

Assessment of the reliability of a system from its basic elements is one of the main aspects of reliability analysis. Throughout this thesis, a system is referred to as a collection of subsystems, while each subsystem is a collection of components. In reliability analysis of a system it is important to model the relationships between various subsystems and components to reflect the functional configuration of the system. There are various modeling schemes for reliability analysis (Modarres, 1993):

1. Reliability block diagrams
2. Fault tree and success tree methods
3. Event tree method
4. Failure mode and effect analysis
5. Master logic diagram

In this thesis we use reliability block diagrams to model the effect of item failures (or functioning) on system performance. Using reliability block diagrams, we model various configurations which correspond to the functional arrangement of subsystem and components in the system. Next we briefly describe some common system configurations using reliability block diagrams.

Series configuration

In a series configuration, the functional operation of the system depends upon successful operation of all the n subsystems or components (Figure 2.1). If any of the subsystem fails, the system fails. The reliability of the system in Figure 2.1 is the probability that all n subsystems succeed during the intended mission time t. Thus the system reliability R_s for statistically independent subsystems is given by:

$$R_s = R_1 R_2 ... R_n = \prod_{j=1}^{n} R_j \tag{2.1}$$

where R_j represents the reliability of the jth subsystem.

Parallel Configuration

A parallel configuration represents a case when the failure of all the subsystems results in system failure (Figure 2.2). The success of only one subsystem would be sufficient to guarantee the success of the system. Reliability of a parallel configuration is given as:

$$R_s = 1 - \prod_{j=1}^{n} (1 - R_j) \tag{2.2}$$

Parallel-Series and Series-Parallel Configurations

Figure 2.3 shows a parallel-series configuration, in which n subsystems are connected in parallel with subsystem j consisting of m_j components in series. A series-parallel configuration is shown in Figure 2.4. In this configuration, n subsystems are connected in series with the jth subsystem consisting of m_j components connected in parallel. Expressions for system reliability for both series-parallel and parallel-series systems can be derived using the system reliability expressions for series and parallel systems (Kececioglu, 1991b; Misra, 1992).

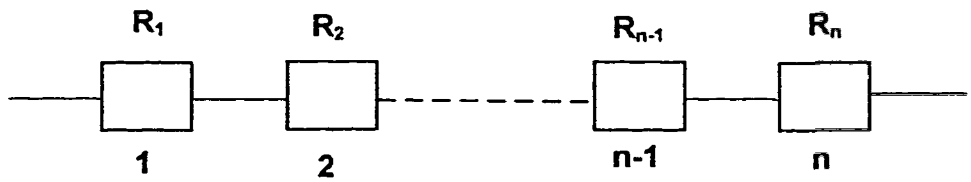


Figure 2.1: A Series Configuration

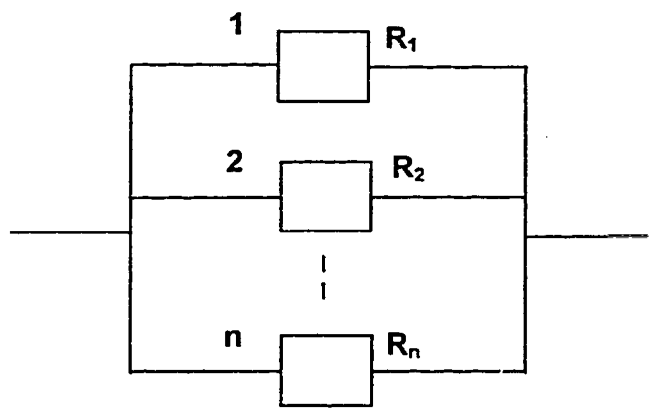


Figure 2.2: A Parallel Configuration

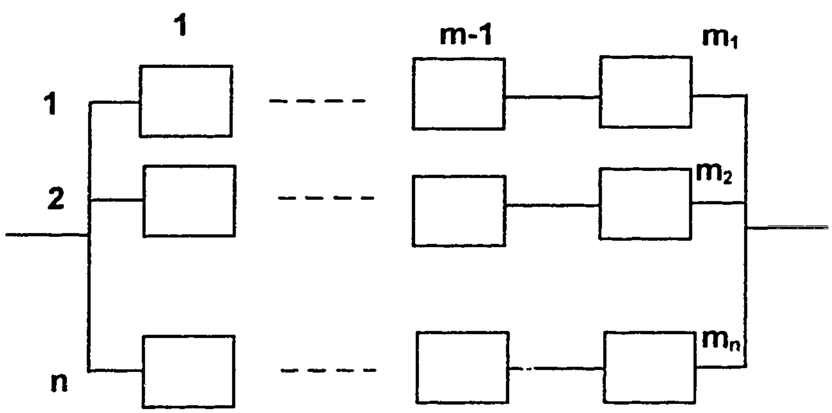


Figure 2.3: A Parallel-Series Configuration

Standby Configurations

A component standby configuration is shown in Figure 2.5. The configuration has a form similar to a series-parallel system. However, in this configuration only one component is active at any given time in a subsystem. When the component fails, one of the parallel components resumes the function of the subsystem until it fails. The process is repeated until all units fail, at which time the system fails. A sub-system standby configuration (Figure 2.6), has a form similar to a parallel-series system. However, the next subsystem becomes active only when the period subsystem fails. Expressions for system reliability and failure rate of the standby system can be obtained through the definitions of individual component reliability (Kececioglu, 1991b; Misra, 1992).

The configurations discussed can be easily generalized into the form where a given number of components or subsystems will be required for a system to be operational. k-out-of-m:G configuration represents the system where the system is operational if at least k out of m components are functional. 1-out-n:G represents the parallel configuration, while n-out-n:G represents a series configuration. There also exist other configurations like bridge network and non-series-non-parallel configuration. For discussions of these configurations, the reader is referred to Kececioglu (1991), Misra (1992) and Modarres (1993).

In this thesis, we limit our discussion to a *series-parallel* configuration with n subsystems in series with each subsystem having $(1 + m_j)$ components in parallel. All the components are statistically independent and all the subsystems act as 1-out-of-m:G configuration. In the next section we review the formulations of an optimal reliability based design problem for series-parallel systems and the optimization techniques used to solve such problems.

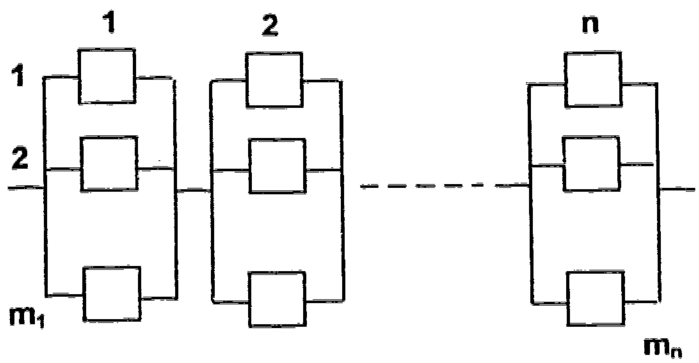


Figure 2.4: A Series-Parallel System

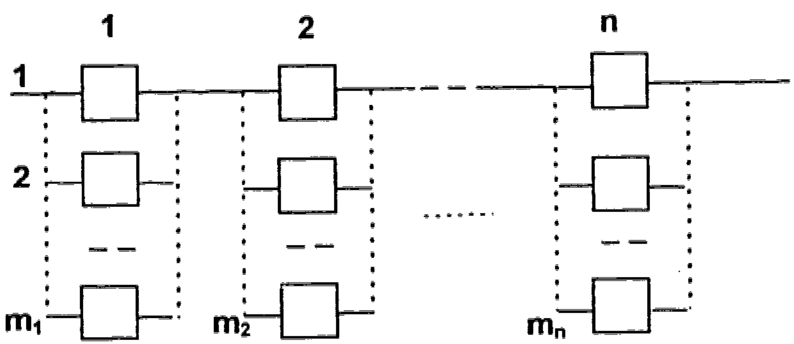


Figure 2.5: A Component Standby Configuration

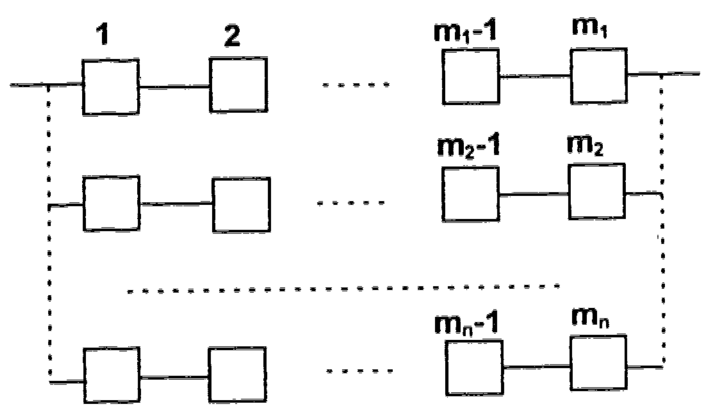


Figure 2.6: A Standby Configuration

2.3 Reliability Based Design: Formulations and Optimization Techniques

For a *series-parallel* system, a typical optimization problem involves finding the optimal number of parallel components in each subsystem that:

1. Minimize the system cost C_s

2. Maximize the system reliability $R_s(m_j)$

The constraints for such problems are either resource or reliability constraints. Resource constraints usually represent constraints of cost, weight, volume or some combinations of these factors. Reliability constraint imposes a minimum requirement of subsystem/system reliability. In general, the optimization problem can be stated as:

Cost Minimization

Minimize

$$C_s = \sum_{j=1}^{n} C_j(m_j) \tag{2.3}$$

Subject to:

$$\sum_{j=1}^{n} g_{zj}(1 + m_j) \leq G_z \quad , z = 1, 2, 3, ... \tag{2.4}$$

$$R_s \geq R \tag{2.5}$$

Reliability Maximization

Maximize

$$R_s(m_j) \tag{2.6}$$

Subject to:

$$\sum_{j=1}^{n} g_{zj}(1 + m_j) \leq G_z \quad , z = 1, 2, 3, ... \tag{2.7}$$

where,

C_s	is the total system cost.
$C_j(m_j)$	is the cost of subsystem j and is a function of number of parallel components m_j at the subsystem j
g_{zj}	is the resource requirement of type z associated with subsystem j
G_z	is the maximum resource of type z available
z	is the number of resource constraints
R	is the minimum required reliability for a system
$R_s(m_j)$	is the system reliability which is a function of m_j for $j = 1, 2, ..., n$

Both of the above problems are nonlinear integer programming problems. They are difficult to solve than general nonlinear programming problems because their solutions must be integers. Many algorithms have been proposed but none has proven to be superior over the others so that it could be classified as a general algorithm for solving nonlinear programming problems (Himmelblau, 1972).

Tillman, Hwang and Kuo (1985) surveyed and classified optimization techniques employed in 77 papers related to nonlinear programming problems. Their conclusions are:

1. *Integer programming* yields integer solutions. The transformation of nonlinear objective functions and constraints into linear form so that integer programming can be applied is a difficult task.

2. *Dynamic programming* has the dimensionality difficulties which increase with the number of constraints and is difficult to solve problems with more than

three constraints.

3. The *sequential unconstrained minimization technique* (SUMT), the *generalized reduced gradient* method (GRG), the *modified sequential simplex pattern search*, and *the generalized Lagrangian function* method are probably few effective techniques when applied to large-scale nonlinear programming problems. However, the solutions are non integers and hence the optimal solution, which must be an integer, is not guaranteed.

Other examples of integer programming solutions to the redundancy allocation problem are presented by Misra and Sharma (1991) and Gen, Ida, Tsujimura and Kim (1990) and Gen, Ida and Lee (1993). In recent years, genetic algorithms have been used by various researchers to solve reliability based design problems (Painton and Campbell (1994); Ida, Gen and Yokota (1994) and Coit and Smith (1994, 1996)). The application of GAs to redundancy allocation in reliability based design problems is discussed in detail in Chapter 4.

2.4 Concluding Remarks

This chapter discusses reliability based design formulations for systems with fixed value of reliabilities. The constant component reliability is the probability that the given component will survive for a given mission period t, hence making the analysis valid only for a given mission period. This limits the application of such formulations to non-repairable systems since the effects of repairs or maintenance cannot be incorporated.

In recent years, the use of preventive maintenance and minimal repair to enhance the system reliability has become prevalent. These actions are economically beneficial because they can extend the system's useful life without actually replacing major components or subsystems. However, to formulate a RBD problem for a repairable system, it is imperative to model component reliability as a function of time. In such a model, the reliability of the system will change with time and will be an important indicator of the system's condition at any given point of time. For such systems it is important that the system's failure rate be below an allowed failure rate or its reliability be above a minimum reliability level.

In Chapter 3, we review failure characteristics, costs and maintenance actions performed on deteriorating system. A brief discussion of various maintenance policies is also presented.

Chapter 3

Review: Reliability Based Design with Deteriorating Components

3.1 System Life Cycle: Cost and Failure Characteristics

A system life cycle consists of four major phases: a) system design, b) system production, c) system operation and maintenance, and d) system retirement and phaseout (Blanchard, 1992). The system design phase encompasses all the activities that develop and define a system which will meet customer's requirement. The system production phase covers the activities related to manufacturing, assembly and testing. The system operation and maintenance phase is normally the longest and includes activities related to installation, operation, maintenance, support and modification of a system throughout its operational life. The system retirement phase consolidates the activities required to remove the system and its supporting facilities.

Typically a system exhibits a failure rate function which has a bathtub shape from the time it becomes a finished product. Initially it has a decreasing failure rate from 0 to some time t_1, a nearly constant failure rate over a range from t_1 to t_2 and an increasing failure rate beyond t_2 (Figure 3.1).

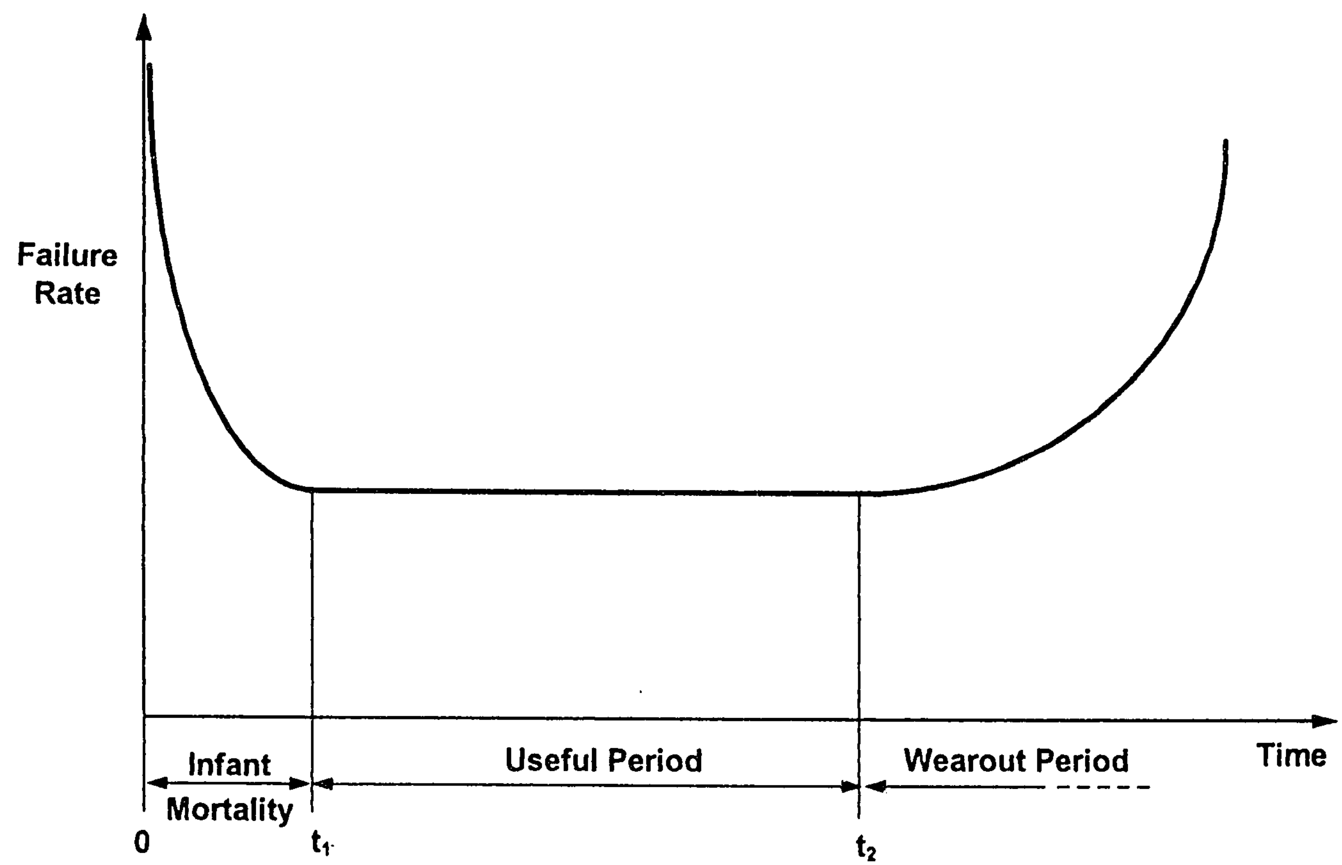


Figure 3.1: A Typical Failure Rate Bathtub Curve

The failures during the initial period are mainly due to defective material, poor manufacturing techniques and assembly processes. In case of repairable systems, such failures are called *teething problems* and may often be fixed through some form of a testing program. This period is called the *infant mortality period*. Failures between t_1 and t_2 are due to chance and are not influenced by system's age. Some of the causes of failure during this period are human errors, misapplication and occurrence of higher than expected random loads. This period is known as *useful life* of the system. Finally, failures over the *wear out phase* reflect an aging process which results in increasing the failure rate with the system's age. Wear out failure causes include corrosion, aging, wear and system deterioration in general. Depending on the type of system, t_1 can be zero or equal to t_2 (O'Connor, 1991).

Life cycle cost is a very important parameter which is evaluated in the life cycle analysis to find an effective design and operating strategy for the system. The life cycle cost is the sum of direct, indirect, recurring, non-recurring and other related cost incurred in design, research, development, operation, maintenance, support and disposal of a system over its life cycle. Since the system operation and maintenance phase is the longest, the system life cycle cost depends heavily on the types and frequency of maintenance actions performed during the operation and maintenance phase. The common strategy employed by the user during this phase is to define various types of maintenance actions for the economical operation of the designed system. In the next section we briefly describe *minimal repair* and *preventive maintenance* as two of the most commonly used maintenance actions to maintain aging systems. The characteristics of these maintenance actions will be later used to develop a maintenance policy for deteriorating systems which must perform under a maximum allowed failure rate.

3.2 Maintenance Actions

3.2.1 Minimal repair

The performance of a system depends on individual components. When a component fails, failure may be reflected on the entire system. If a repair or replacement of the failed component restores the function of the entire system but the failure rate of the system remains as it was just before failure, then the repair is called minimal repair (Valdez-Flores and Feldman, 1989). In such a repair, the majority of components are not replaced, thus the remaining life distribution and failure rate of the system are essentially undisturbed. The concept of minimal repair was introduced by Barlow and Hunter (1960). Barlow and Hunter (1960) also obtained the mean number of failures over an interval when the system is subjected to minimal repair after each failure. They define the occurrence of such failures to follow a non-homogeneous Poisson process with a mean value $H(t)$, where, $H(t) = \int_0^t h(t)\,dt$, is called the cumulative failure rate function and $h(t)$ is the failure rate function.

The number of failures $\{N(t), 0 \leq t < \infty\}$ is a non-homogeneous Poisson process with mean $H(t)$ when it has independent increments that assume only non-negative integer values, and if for all $0 \leq t_1 < t_2$,

$$P[N(t_2) - N(t_1) = k] = \frac{[H(t_2) - H(t_1)]^k}{k!} e^{-(H(t_2)-H(t_1))}. \tag{3.1}$$

Then, the expected number of failures during $[t_1, t_2]$ can be written as:

$$E[N(t_2) - N(t_1)] = H(t_2) - H(t_1) \tag{3.2}$$

This result has been extensively used in the analysis of a variety of maintenance policies involving minimal repair, e.g. Tilquin and Cleroux (1975), Nakagawa

(1980,1981,1986), Boland and Proschan (1982), Phelps (1981, 1983) and Nakagawa and Kowada (1983).

The costs associated with minimal repairs depend on the frequency of failures in a system. The frequency of these failures is related to the individual subsystem's failure rate function. Consider subsystem j in time interval [0,T]. According to Theorem 1 by Boland (1982), the expected minimal repair cost of subsystem j in interval [0,T] is C_{Mj}:

$$C_{Mj} = c_{mj} \int_0^T h_j(t)dt \tag{3.3}$$

where $h_j(t)$ is the failure rate function and c_{mj} is the cost of minimal repair of subsystem j. In the proof of Theorem 1, Boland also defines the number of minimal repairs to the system on $[0, T]$ as a non-homogeneous Poisson process with the mean $H(t)$. This leads to interpret $c_{mj}h_j(t)$ in a naive sort of way as the rate of spending dollar on minimal repair at age t. With this interpretation $c_{mj} \int_0^T h_j(t)dt$ represents the mean number of dollars spent on minimal repair in $[0, T]$. A proof of this result is in Boland (1982).

Under periodic replacement policy with minimal repair at failure the system is replaced at multiple of some period T while performing minimal repair at any intervening system failures. The basic minimal repair model developed by Barlow and Hunter (1960) has been generalized and modified by many authors to fit more realistic situations. Assuming that the cost of minimal repair C_m is less than the cost of replacing the entire system C_r, the long-run expected cost per unit time $C(t)$ using a replacement age t for the basic model is given by:

$$C(t) = \frac{C_m N(t) + C_r}{t} \tag{3.4}$$

where $N(t)$ represents the expected number of failures (and hence the minimal repairs) during interval $(0, t)$. The basic minimal repair model, equation (3.4), was further modified by Barlow and Proschan (1965), Tilquin and Cleroux (1975), Nakagawa (1981), Boland and Proschan (1982). An extensive survey of minimal repair models is available in Valdez-Flores and Feldman (1989).

3.2.2 Preventive Maintenance or Imperfect Repair

Preventive maintenance (PM) categorizes the actions which improve the condition of the system before the system fails. A PM action for a mechanical system may include cleaning, lubrication, adjustment, replacement of small components, etc. Malik (1985) defines PM as an action which improves the condition of the system without replacement. Conventional PM policies assume that the system after each PM intervention is restored like new. This assumption does not hold true in many real situations since any PM performed on system will improve the system condition, however the state/condition of the system is somewhere between as good as new and as bad as old. If a PM does not return the condition of the system to its original state, then it is known as *imperfect repair*. Throughout this thesis we will use the term PM interchangeably with imperfect repair.

Before proceeding with the explanation of various PM models it is important to note that PM is applied to the system before it reaches a failed state and requires two necessary conditions (Jardine, 1973):

1. The total cost of system replacement must be greater after failure than before.

2. The failure rate of the system must be monotonically increasing.

Authors have used different assumptions to develop PM models:

1. The state of the system after PM is *bad as old* with probability p and *good as new* with probability $1-p$ (Murthy and Nguyen, 1981; Brown and Proschan, 1983).

2. PM reduces failure rate, but does not restore the system's effective age to zero (Nakagawa, 1986; Nakagawa, 1988; Jayabalan and Chaudhuri, 1991a).

3. The age of the system is reduced at each PM intervention introducing the concept of an improvement factor (Malik, 1979; Nakagawa, 1980, 1988; Lie and Chun, 1986; Jayabalan and Chaudhuri, 1991; 1992).

4. PM action lowers the rate of system degradation but does not effect the age of the system (Canfield, 1986).

In many areas of engineering, it is extremely desirable to have highly reliable systems (Wood, 1988). Many researchers suggest to perform a PM whenever the system reaches the maximum acceptable level of failure rate or the minimum acceptable level of reliability (Malik, 1979; Lie and Chun, 1986; Jayabalan and Chaudhuri, 1991; 1992). In reality, as the system ages, the post maintenance state of system lies between *bad as old* and *good as new.* Also operation of such systems causes stress which results in system degradation and hence an increase in the level of failure rate with time. Though PM improves the condition of the system, there is a gradual deterioration over time requiring replacement of the system after sometime (Yeh, 1990).

Malik (1986) provides an explanation for system replacement based on the rising cost of maintenance per unit time. He proposes an expression for *average annual cost* of the system at the end of ith interval (AAC_i) given by equation (3.5):

$$AAC_i = \frac{IC + (i-1)MC}{T_i} \tag{3.5}$$

where IC is the installation and acquisition cost, MC is the maintenance cost, i is the number of PM intervals and T_i is the age of system at the end of the ith interval. The system considered by Malik is one with increasing failure rate and subject to a reliability constraint. PM is performed on the system when it reaches minimum allowed reliability and each PM action makes the system younger but the shape of the reliability function remains the same after each PM action. The system's average annual cost is calculated after each interval and the system is replaced when the system's average annual cost is higher than the one at the end of the preceding interval. An extensive survey of mathematical models for optimal replacement, inspections and repair decisions has also been provided by Jardine (1973).

Jayabalan and Chaudhuri (1992c) present a case study of a sequential imperfect preventive maintenance policy performed on bus engines. They present two formulations based on assumptions similar to Nakagawa (1988). According to this maintenance policy, a PM action is performed on the system as soon as it reaches a maximum allowed failure rate. If a failure occurs between these PM actions then minimal repairs are performed. The expected mean cost rate is defined as the total cost per unit time and is calculated after each interval and compared with the last interval. The system is replaced when the expected mean cost rate is larger than the last interval. This maintenance policy described by Jayabalan and Chaudhuri (1992) will be used in our reliability based design formulations for deteriorating systems in the later chapters.

3.3 Concluding Remarks

In this chapter we presented the concepts of the life cycle cost and failure characteristics of deteriorating systems. Two commonly modeled maintenance actions for deteriorating systems, namely, minimal repair and preventive maintenance are also described. The literature review in the area of maintenance management for deteriorating systems shows that much research has been done in the area of optimal maintenance policies involving minimal repair and/or PM. However, for all these policies it assumed that the system design is fixed and hence the effect of system design on system operation and maintenance phase is ignored.

To incorporate the effects of minimal repair and PM in system design, it is imperative to express the system component reliability as a function of time. Thus for an RBD formulation with components whose reliability is a function of time, the objective will be to obtain an optimal system design, which for a given maintenance policy will minimize its costs over its expected useful life. For such problems one can express the objective function as system cost per unit time, which should be minimized subject to resource and failure rate constraints. Such a problem can be modeled as a nonlinear mixed integer programming problem with nonlinear objective function and constraints. In this research, we develop RBD formulations with components whose reliability is a function of time. These formulations are presented in Chapters 5, 6, and 7. Genetic algorithms are used to solve nonlinear mixed integer programming formulation. In the next chapter we discuss, in detail, genetic algorithms and their advantages and limitations.

Chapter 4

Review: Genetic Algorithms

4.1 An Overview of Genetic Algorithms

Genetic Algorithms (GAs) are general purpose search and optimization techniques based on the mechanics of natural selection and genetics (Holland, 1975). The terminology used by GAs is quite similar to that used in natural genetics and close analogy is maintained between the elements of two. In this chapter we will describe step by step, with an example, how a GA works and discuss its applications to reliability based design problems.

GAs work on the principle of *survival of the fittest* strategy among competing sets of coded parameters, where each set represents a point in the search space. This set of coded parameters is referred to as *string* (chromosome). A string is a concatenation of a number of codes (often binary codes) of a given length. The string bits (0 or 1 in binary string) are the equivalent of natural genes. A segment of the string represents a variable and each specific instance of the segment represents, directly or indirectly, a specific value of that variable. There are as many segments in a string as the number of variables, hence a string basically represents a possible solution.

To explain how a simple GA works, we consider the following example with three variables (x_1, x_2, x_3):

Minimize

$$y = x_1\, x_2\, x_3 \tag{4.1}$$

Subject to:

$$x_1x_2 + x_2x_3 + x_1x_3 = 432 \tag{4.2}$$

where x_1, x_2 and x_3 are integers such that $0 \leq x_i \leq 31$ for $i = 1, 2, 3$.

Using binary code to codify different values of each variable, we use a segment of five binary bits. Thus 00000 represents 0, 00001 represents 1, while 11111 represents 31. Thus a string of three segments or fifteen bits will represent a possible solution. For example string 100100101011001 represents $x_1 = 18$, $x_2 = 10$ and $x_2 = 25$.

GAs start by generating an initial population of strings through random selection of string-bit values. The number of strings (chromosomes) in the population is called *population size*. The population size is initially specified by the user, and is kept constant throughout the search.

In our example, in order to randomly generate one string we need to run a binary generator 15 times (equal to number of bits in a string). If the population size is 5, then we need to do that 75 times (15 $\times$ 5). Table 4.1 represents a population of generation zero (the starting generation). The second column in the table represents five randomly generated strings which satisfy equation (4.2). The decoded values of these strings are presented in the next three columns. The last column represents the string's fitness value.

The fitness of a string of the population is evaluated using an objective function. Since GAs seek to maximize the fitness of the solutions, in a maximization problem this fitness is simply expressed as the value of the objective function for that specific

Table 4.1: Population of Generation 0 ($y = x_1x_2x_3$)

String No.	String	x_1	x_2	x_3	Fitness
1	010101110000100	10	28	4	1120
2	000111001010010	3	18	18	972
3	010000100010111	8	8	23	1472
4	001000110011000	4	12	24	1152
5	001100011111110	6	7	30	1260

string. In a minimization problem, the fitness is defined as:

$$\text{Fitness function} = \text{K} - \text{objective function value} \tag{4.3}$$

where, K is a constant, large enough to exclude negative fitnesses. A value commonly used for K is the sum of the minimum and maximum values of the objective function in each generation.

Two operators, *selection* and *crossover*, are used to transform members of the initial population to form a new population. These processes are collectively known as *reproduction*.

The most commonly used selection scheme is stochastic sampling with replacement (Goldberg, 1989). The scheme is based on a biased roulette wheel where each string in a population in a generation has a roulette wheel slot sized in proportion to its relative fitness. If $\sum f'$ represents the sum of raw fitnesses of all strings in the same population, then the relative fitness (or the probability of selection) of the ith string would be $\frac{f'_i}{\sum f'}$. Table 4.2 shows the relative fitness of the population of generation zero in percentage form.

To reproduce we simply spin the roulette wheel as many times as the population size, in our example five times. Each spin specifies the string whose copy will make it to the next stage, i.e. the mating pool. It is evident that the larger the wheel slot

Table 4.2: Raw and relative fitness of population zero

String No.	Raw Fitness	Relative Fitness
1	1120	18.7%
2	972	16.3%
3	1472	24.6%
4	1152	19.3%
5	1260	21.1%

(relative fitness) of a string the higher its probability of having copies in the mating pool thus participating in the creation of the next generation. Let us assume that in a typical sequence of spins of the weighted roulette wheel, strings 1, 4 and 5 are selected once, while string 3 is selected twice and string 2 is not selected at all. The resulting mating pool is shown in Table 4.3. The parent (string) in the mating

Table 4.3: Mating pool for generation 0

String No. Mating Pool	String No. Generation Zero	Fitness of Selected String
1	1	1120
2	3	1472
3	3	1472
4	4	1152
5	5	1260

pool randomly pick their own partner. Two offsprings (children) are produced from each pair of mating partners using genetic operators. A sequence of actions on a typical population is shown in Figure 4.1.

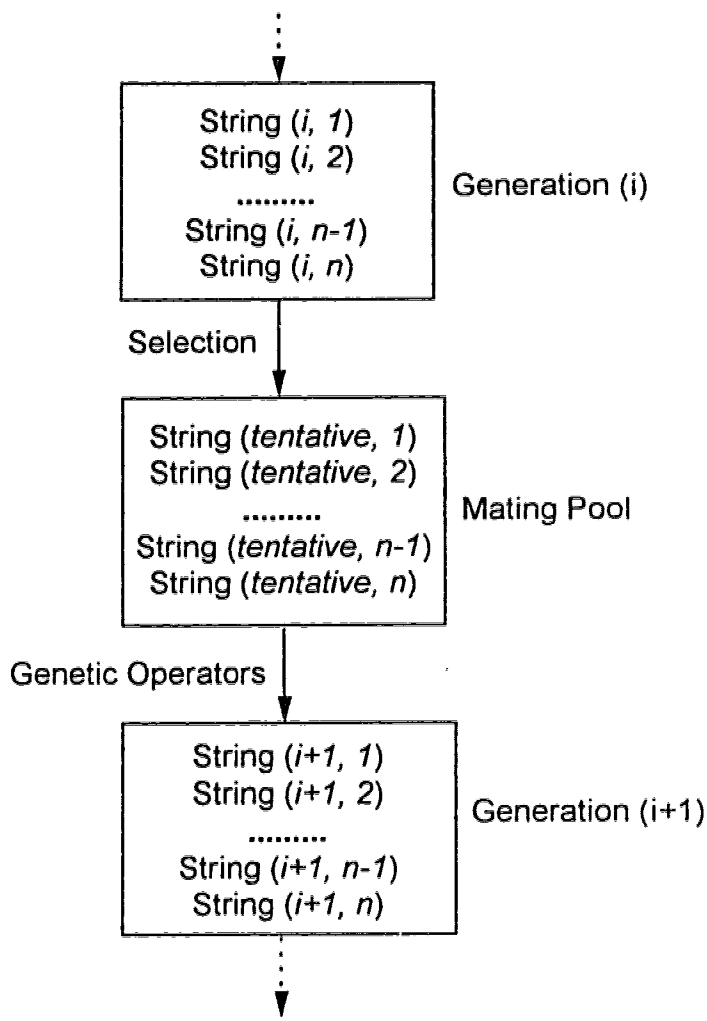


Figure 4.1: Evolution of populations in successive generations

The two commonly applied genetic operators are *crossover* and *mutation*. Crossover is the most important operator of a genetic-based technique. A simple GA uses a one-point crossover scheme. In a one-point crossover scheme a pair of strings is selected at random from the mating pool, followed by a random selection of an integer position k (called the crossover site) along the string where, $1 \leq k \leq l$ and l is the length of the string in bits. Two new strings are then created by swapping all bits from positions $k+1$ to l inclusive between two parents. Crossover of string 1 and 4 from mating pool is shown in Figure 4.2.

The mating process is repeated with other string pairs until all string pairs have produced two offsprings. This number is the same as the population size. Since child strings are generated in pairs, the process would not be able to generate an odd number of children. In case of an odd population size one may generate one more (less) string and then eliminate (add) a single string to the population. One way to do so is to remove the least fit string from the population (add a duplicate of the most fit string of the population).

Crossover results in a randomized, yet structured information exchange. Each child string combines the characteristics of its parent strings. Considering the fact that in every search procedure there is a trade-off between creating new knowledge and exploiting the existing knowledge, one can regard crossover as the means of exploiting the existing knowledge in the GAs. By combining chromosomes to form string patterns that may not have been previously existed in the population, crossover provides a mechanism for exploring new regions of search space (Shariat-Panahi, 1995). In a nutshell, some of the resulting offspring will have a higher fitness than either parent. Offspring with reduced fitness will have a lower chance to reproduce in the subsequent generations. This is what ultimately drives the GA to reach the maximum possible fitness, i.e. the strong gets stronger, and the weak dies out.

Another genetic operator used is *mutation*. Mutation involves the alteration of a randomly selected bit (0 to 1 or 1 to 0) in a randomly chosen string. It is normally applied to post-crossover strings in the mating pool. Mating site is randomly selected along the string (between bits 1 and l inclusive) and the respective bit is altered. Mutation introduces a type of random walk in the search space and prevents the solution from being trapped in local optima (genetic drift). Mutation also allows for the formation of string patterns that may not have been present in the initial, randomly generated finite sized population. In our example, we assume that the random generation gives 4 as a site of mutation (shown in Figure 4.2).

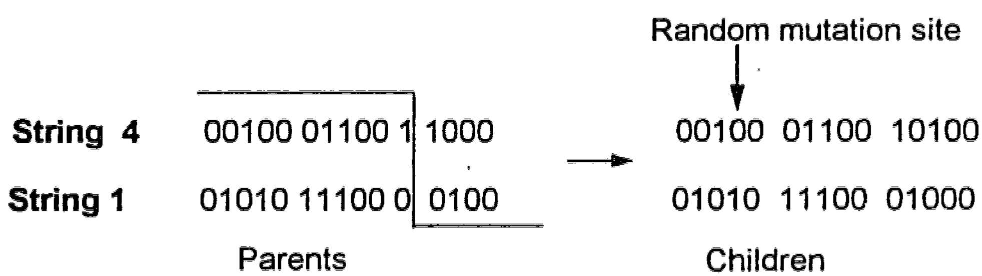


Figure 4.2: Crossover of strings 1 and 4 from the mating pool

The results of genetic operators, crossover and mutation, on the parent strings are shown in Table 4.4. It can be seen that *Child String 2* is invalid as it does not satisfy area constraint (equation (4.2)), however, *Child String 1* exhibits a higher fitness than both its parents. Table 4.5 shows a generation 1 of strings created

Table 4.4: Fitness of parent and child strings

Parent String 1	001000110011000	Fitness	1152
Parent String 2	010101110000100	Fitness	1120
Child String 1	001100110010100	Fitness	1440
Child String 2	010101110001000	Fitness	Invalid

through application of stochastic sampling with replacement selection, crossover and mutation to the population of generation zero. Comparisons between the average fitness of generation zero and one shows that the average fitness of the generations has increased from 1192.2 in generation zero to 1382.4 in generation one.

Table 4.5: Population of Generation 1 ($y = x_1 x_2 x_3$)

String No.	String	x_1	x_2	x_3	Fitness
1	001100110010100	6	12	20	1440
2	111100011100110	30	7	6	1260
3	001001110001010	4	28	10	1120
4	010001011101111	8	23	8	1472
5	010010101010010	9	10	18	1620

The processes of reproduction and mutation are carried out repeatedly until either some convergence criterion is met or a maximum number of generations is reached. Convergence in the context of GAs is measured by the uniformity of the fitnesses of the strings in a population. A common termination criterion is that 95% of these strings share the same fitness or the average fitness of the population falls within 95% of the maximum fitness in the same population (Dejong, 1975).

For this example, the global optimum ($x_1 = x_2 = x_3 = 12$) is found in 9 generations. The basic operation of a genetic algorithm used in this thesis is shown in Figure 4.3.

The stochastic nature of GAs makes theoretical assertions about their convergence properties very difficult. Theoretically, there are no *sufficient condition* for the convergence of GAs, neither is there a way to exactly predict when a GA with a given set of control parameters will converge. Holland (1975) provides a fundamental theorem of genetic algorithms also known as *schema theorem* which explains how GAs work and where their powers come from.

Despite the lack of a solid theoretical proof for the convergence of GAs, results of empirical studies including an abundance of successful applications have established GAs as robust general purpose search technique (see Beasley, Bull and Martin 1993 for a list of related literature).

The following characteristics of GAs make them a preferred alternative optimization tool to solve problems which otherwise are not easily solved by traditional methods:

- A GA searches from a population of solutions, rather than from a single solution. This explicit parallelism helps the GA in determining close to global optimum solutions without the danger of being trapped in a local optimum. Typically, a GA will be able to find a close-to-optimal solution by evaluating fewer than 1% of the points in the search space.

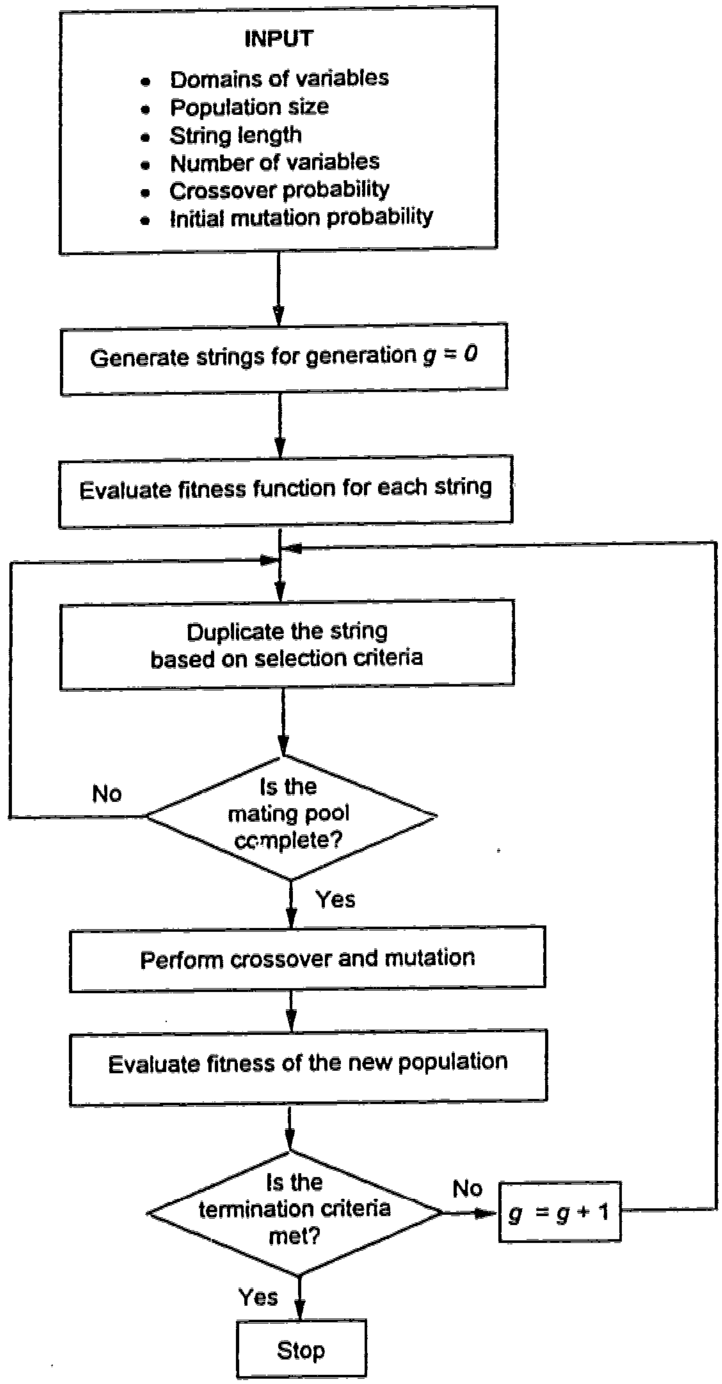


Figure 4.3: Genetic Algorithm operation

- A GA uses objective function information directly, with no need for derivatives or other information. The objective function can involve any type of numeric or non-numeric variables, or other data structures, as long as a coding scheme can be devised to represent the parameter set.
- It is particularly suited to problems with a very large search space.
- It can treat with ease different types of variables (integer, real, non-numeric).

4.2 Problem Specific GA Implementation Issues

GA's ability to find the global optimum and the rate of convergence depend on population size, crossover rate and mutation probability. A too small population size will converge too quickly and often to suboptimal solutions. The larger the population, the more points in the solution space being examined at each generation and hence higher the chance of finding the global optimum in fewer iterations. However a too large population results in long CPU time for any significant improvement and smaller chance of good strings to mate due to crowded populations.

Crossover and mutation play crucial roles in the GA search, especially when they are applied at the same time. When only mutation is applied, the search resembles a random search. On the other hand, when only crossover is applied, the system may quickly find a suboptimal solution (local optima) that exists within the initial population, thus resulting in premature convergence. The trade-off between the two operators allows GAs to successfully converge in most cases to the global optimum. Operation of the two operators is controlled by their prescribed rates or probabilities (expressed as a percentage). These probabilities are either specified by the user or determined by the system according to a given criteria. In a programming sense, they tell the system at each point how many strings must be crossed over and how many must be mutated. A crossover probability of 80% for a population size of 50 means that $40(= 0.8 \times 50)$ of the strings need to be crossed over and ten will directly go to the next generation. A mutation probability of 1% for the same case means that on average, five bits will be switched somewhere in the population.

Various researchers have recommended a value of 0.6 to 1.0 for the crossover probability and a value less than 0.05 for the mutation probability (Grefenstette, 1986). In this thesis, we conducted empirical investigation to obtain appropriate population size, crossover rate and mutation rate for each specific formulation. The results of these investigations are discussed in Chapter 5, 6 and 7 under the section entitled numerical illustration.

4.3 Applications of GAs to Reliability Related Problems

In recent years, GAs have been used by various researchers to solve reliability based design problems. Works by Painton and Campbell (1994); Ida, Gen and Yokota (1994) and Coit and Smith (1994, 1996) are recent examples of GA application to reliability problems.

Painton and Campbell (1994) analyze a fixed design configuration for which incremental decreases in component failure rates and their associated costs are known. They use a GA to find maximum reliability solutions to satisfy specific cost constraints. There are several unique features to their research. Their algorithm is flexible and can be formulated to optimize reliability, mean time between failure (MTBF), the 5th percentile of MTBF distribution or availability. Additionally, they assume that components fail in accordance with the exponential distribution but that the underlying failure rate is a random variable subject to a specified distribution.

Ida, Gen and Yokota (1994) use a GA to find solutions to a redundancy allocation problem where there are several failure modes. This problem had previously been solved by both nonlinear programming and integer programming. They show that the GA quickly converges to the optimal solution after searching only a small percentage of the search space.

Coit and Smith (1994) analyze a series-parallel system design problem with constant component reliability. The system consists of eight subsystems and ten unique component choices for each subsystem. The design problem is to minimize system cost for a given reliability requirement. It is shown that GA readily converges to the optimum solution in under 1000 generations with a population size of 40. In another work by Coit and Smith (1996) it has been shown that GA works exceptionally well for complex redundancy allocation problems. They show that the GA approach is very robust with very few restrictions on the formulation and consistently yields optimal solutions for problems where no feasible solutions could be located using previously published methods.

4.4 Concluding Remarks

It has been recognized that GAs show their power and dominance over other search techniques in problems with large solution spaces and/or complex objective functions. GAs prove to be a robust technique and is definitely useful to be marketed as a universal solver which can handle complex functions. GAs work on a population of solutions at the same time. This, plus the application of mutation, helps the technique to find multiple feasible solutions across the solution space. In the next three chapters we formulate three reliability based design problems and use GAs as an optimization tool to find the optimal design, optimal PM intervals and optimal replacement time of deteriorating system.

Chapter 5

Reliability Based Design Considering Maintenance

5.1 Introduction

A reliability based design problem involves evaluating various design configurations for minimal system costs or maximum reliability while satisfying other constraints, namely, weight, volume, cost, etc. As discussed in Chapter 2, much research has been done in the area of reliability based design (RBD) with constant component reliability. In this chapter we present RBD formulations for systems with components that deteriorate with time. Preventive maintenance (PM) and minimal repairs are considered on the system. Based on PM models proposed by Nagakawa (1988), two formulations are developed. Using the principles of engineering economics, a five step methodology is also proposed to calculate the optimal system design, optimal PM intervals and optimal system replacement time. In each formulation we explain the calculation of PM intervals and minimal repair costs. Each formulation is completed by obtaining an expression for average annual cost of the system. The average annual cost is minimized for maximum allowed failure rate using Genetic Algorithms.

In section 5.2 we discuss system characteristics and costs. Section 5.3 and 5.4

present two formulations based on Nagakawa's sequential PM models. Section 5.5 presents the five step methodology used to calculate the optimal system design, PM intervals and system replacement time. Section 5.6 presents the numerical illustration for both formulations followed by concluding remarks in section 5.7.

5.2 System Characteristics and Costs

The system being considered comprises of n subsystems in series. Subsystem j ($j = 1, ..., n$) consists of $(1 + m_j)$ identical components in active redundancy. All the components are statistically independent. Each subsystem acts as a 1-out-of-m:G configuration. All the components have continuous and strictly increasing failure rates. It is required that the system should operate below a maximum allowed failure rate (In this thesis we use the term failure rate and hazard rate interchangeably).

There are four main components of system costs. They are: (a) acquisition cost of the system, (b) installation cost of the system, (c) cost of minimal repairs, and, (d) cost of performing preventive maintenance. The acquisition cost of the system includes the costs for design, development and production of the system. In both formulations, the acquisition cost of the system is a sum of the products of acquisition costs of the individual components and their respective assembly coefficients (ϕ_j). The assembly coefficients account for subsystem assembly costs. The installation cost of the system is one time cost during system life cycle and is independent of system design.

In the next section we present two formulations using sequential PM models proposed by Nakagawa (1988). These models were developed by Nakagawa by introducing improvement factors (Chun and Lie, 1986) in failure rate and age for a sequential PM policy (Nguyen and Murthy, 1981; Nakagawa, 1986).

5.3 Formulation 1: PM modeled using age reduction concept

In this formulation we model the PM action on the system using an age reduction concept, each PM action on the system makes the system younger by a given improvement factor. However, the failure function of the system does not change after PM (Nakagawa, 1988).

All the subsystems are either maintained preventively or replaced when the system failure rate reaches ξ. The system failure rate reaches ξ for the first time at calendar age T_1. At this moment the system's effective age is also T_1. If the PM action is performed at this moment, the age reduction concept assumes that system's effective age T_1 is reduced to T_1/α, where α is an improvement factor due to PM, such that, $1 \leq \alpha \leq \infty$. During the second interval $T_1 < T \leq T_2$ the system failure rate reaches ξ at T_2 (Figure 5.1). Now when the maintenance is done, the portion of the effective age consumed during the second interval is $\frac{T_2-T_1}{\alpha}$. The improvement by the second maintenance does not affect $\frac{T_1}{\alpha}$ which is the portion of the effective age permanently consumed as a result of wear and environmental effects during the first interval. Thus after maintenance at calendar age T_2, the effective age of the system is $\frac{T_1}{\alpha} + \frac{T_2-T_1}{\alpha} = \frac{T_2}{\alpha}$. Since the failure rate function form of the system does not change after PM and assuming that the improvement factor is the same for all the components in all the subsystems, we can derive the following recursive equation from Figure 5.1.

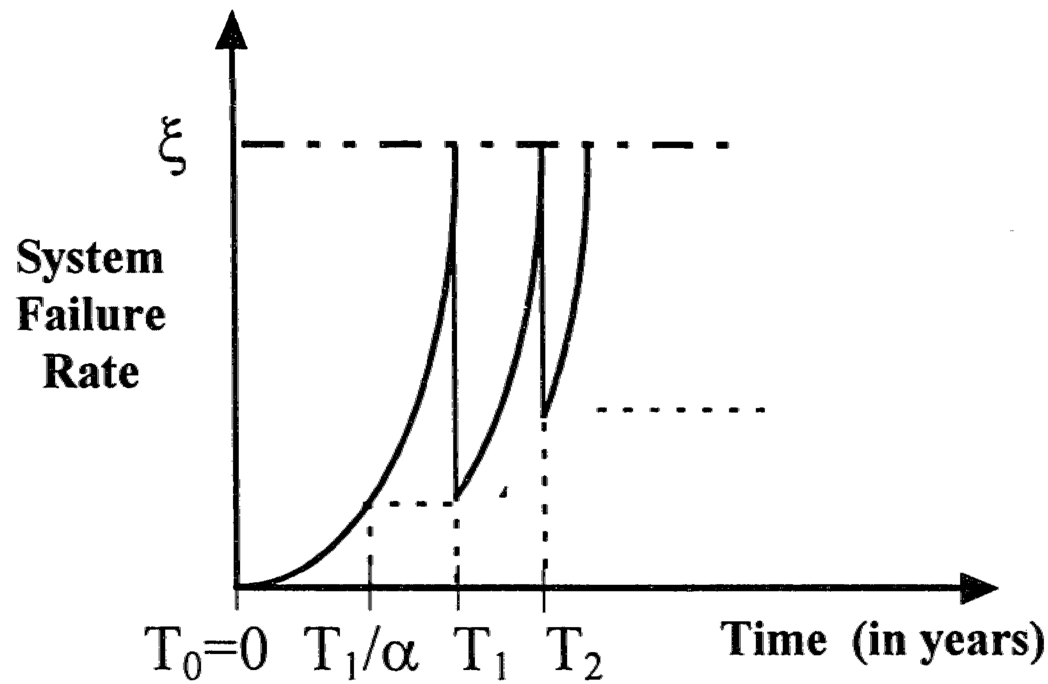


Figure 5.1: System Failure Rate and Preventive Maintenance Intervals (Age reduction concept, Nakagawa 1988)

$$T_i - T_{i-1} = (T_{i-1} - T_{i-2}) - \frac{T_{i-1} - T_{i-2}}{\alpha} \tag{5.1}$$

where $i \geq 2$ and $T_0 = 0$. Equation (5.1) can be simplified to give a closed form of T_i (for $i \geq 2$) as a function of T_i:

$$T_i = T_1 \sum_{k=0}^{i-1} \left(\frac{\alpha - 1}{\alpha} \right)^k \tag{5.2}$$

where, $i = 2, 3, 4, ...$ Figure 5.1 shows the PM scheduling and the system failure rate behavior after each PM action. The system should always operate below the maximum allowable failure rate. Since the system has an increasing failure rate function, we only need to have the following constraint on system failure rate function $h_s(t)$:

$$h_s(T_i) \leq \xi \tag{5.3}$$

Minimal repairs are performed if the system fails within the scheduled PM intervals. Assuming $R_j(0) = 1$, equation (3.3) can be rewritten in terms of the reliability of the jth subsystem $R_j(t)$ as follows:

$$\begin{aligned} C_{Mj} &= -c_{mj} \int_0^T \frac{R'_j(t)}{R_j(t)} dt \\ &= -c_{mj} \ln[R_j(T)] \end{aligned} \tag{5.4}$$

Cost of minimal repairs at the first PM point, T_1, can be written as:

$$C_{Mj1} = -c_{mj} \ln[R_j(T_1)] \tag{5.5}$$

At T_1 PM is performed and the effective age of the system becomes T_1/α instead of T_1. If $x_i = T_i - T_{i-1}$ and $T_0 = 0$, then at the second maintenance point we can find the total expected minimal repair costs during the period $(x_1 + x_2)$, C_{Mj2}:

$$C_{Mj2} = -c_{mj} \left(\ln[R_j(T_1)] + \ln[R_j(A_1 + x_2)] - \ln[R_j(A_1)]\right) \tag{5.6}$$

where A_1 is the effective age of subsystem j after the first PM action and is equal to T_1/α. Simplifying equation (5.6) further we get:

$$C_{Mj2} = c_{mj} \ln \left[\frac{R_j(A_1)}{R_j(T_1)\, R_j(A_1 + x_2)} \right] \tag{5.7}$$

Hence, the expected minimal repair costs till T_i for subsystem j is given by equation (5.8)

$$C_{Mji} = c_{mj} \ln \left[\frac{\prod_{l=1}^{i-1} R_j(A_l)}{\prod_{l=0}^{i-1} R_j(A_l + x_{l+1})} \right] \tag{5.8}$$

where $A_l = T_l/\alpha$ and $A_0 = 0$.

When the system reaches a given maximum allowable failure rate, one of the following actions is selected, (a) keep the system and perform PM, or (b) replace the system with an identical system. This decision is made by comparing the average annual cost (AAC) of the system and checking whether the system has reached its economic life or not. Ignoring the time value of money, AAC of the system is defined as total cost incurred on the system divided by its useful life in years. The economic life of a system is defined as the time interval that minimizes the system's total annual costs. The economic life is also referred to as the minimum cost life or optimal replacement interval.

If PM is performed on the system then all the components undergo PM. Thus the cost of a PM action for subsystem j with (m_j+1) components will be $MC_j(m_j+1)$, where MC_j is the cost of performing PM on a component in subsystem j. The cost of PM on subsystem j at the end of the ith interval is given by:

$$PM_{ji} = (i-1)MC_j(m_j+1) \tag{5.9}$$

Ignoring the time values of cost, the AAC of the system at the end of the ith interval is given by equation (5.10).

$$AAC_i = \frac{IC + \sum_{j=1}^{n} [(1+m_j)AC_j\phi_j + PM_{ji} + C_{Mji}]}{T_i} \tag{5.10}$$

where,

AC_j	Acquisition cost of a component in jth subsystem
C_{Mji}	Minimal repair cost of jth subsystem at the end of the ith interval (Equation (5.8))
IC	Installation cost of the system
PM_{ji}	PM cost on subsystem j at the end of ith interval (Equation (5.9))
T_i	System calendar age at the end of ith interval
ϕ_j	Assembly coefficient of a component in jth subsystem

If the values of m_j are known, then using equations (5.2) and (5.3), one can calculate the values of T_1, T_2,..., T_i, T_{i+1} and subsequently calculate the values of AAC_1, AAC_2,..., AAC_i, AAC_{i+1}. We stop when $AAC_{i+1} > AAC_i$ and $AAC_i < AAC_{i-1}$ and T_i is the economic life of the system. The system is replaced at T_i.

To calculate the optimal system design, optimal PM intervals and economic life of the system we follow a *five step methodology* outlined in section 5.5.

5.4 Formulation 2: PM modeled using hazard rate concept

This formulation is developed using the hazard rate concept for modeling preventive maintenance (Nakagawa, 1988). We assume that the initial failure rate and the reliability of a new system is zero and one, respectively. With the hazard rate concept, a PM restores the system to a failure rate of zero. However, after each additional PM the slope of the failure rate function increases. During the first interval, the component has undergone no PM and the component failure rate is $h_{j1}(t)$. This failure rate corresponds to the original failure rate of the component, that is, $h_{j1}(t) = h_j(t)$. At the end of the first interval the component undergoes PM and its failure rate is reduced instantly to zero but the slope of the new failure rate function is increased (Figure 5.2). In general, the hazard rate of a component in the jth subsystem during the ith interval is $h_{ji}(t)$:

$$h_{ji}(t) = \theta_{ji} h_j(t) \tag{5.11}$$

where θ_{ji} is defined as *failure rate deterioration factor.* Depending on the effect of a PM action on the component, one can mathematically define θ_{ji} to satisfy the following two conditions:

1. $\theta_{j1} = 1$

2. $\theta_{j(i+1)} \geq \theta_{ji}$, where i=1, 2, 3,...

Nakagawa (1988) provides a mathematical expression for hazard rate deterioration factor (θ_{ji}) as:

$$\theta_{ji} = \prod_{k=0}^{i-1} \left(1 + \frac{k}{k+1}\right) \tag{5.12}$$

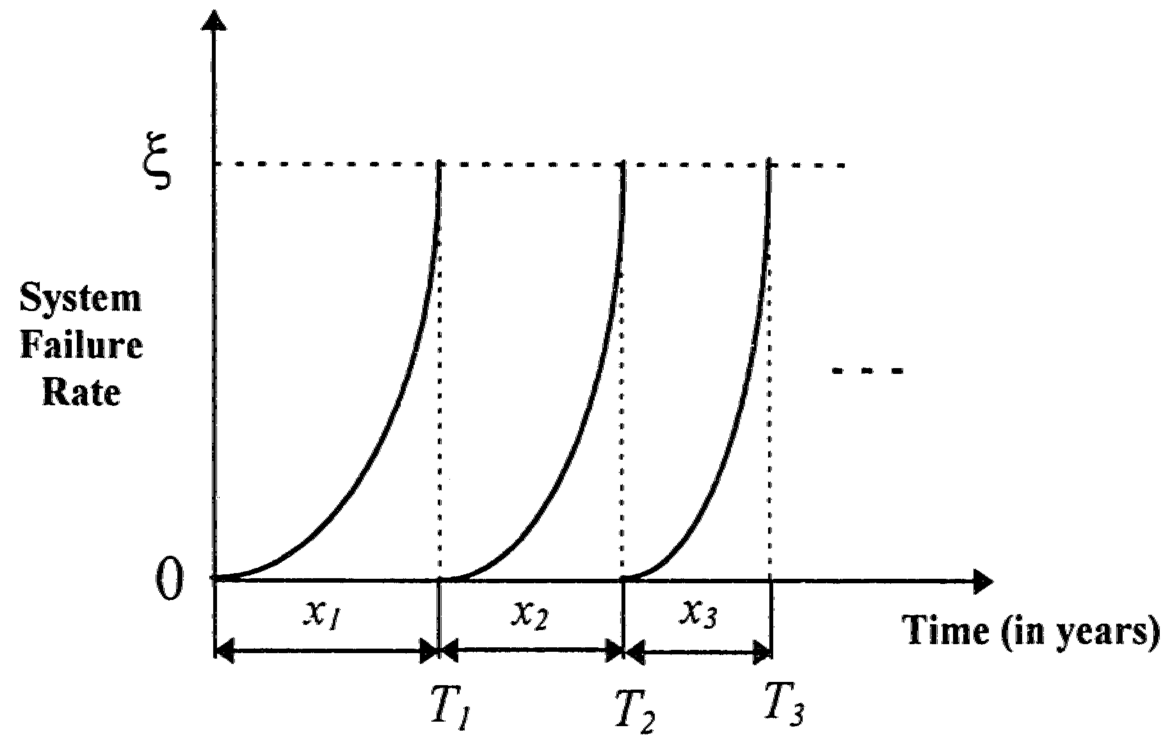


Figure 5.2: System Failure Rate and Preventive Maintenance Intervals (Hazard rate concept, Nakagawa 1988)

According to his proposed formula, the system deteriorates very drastically as the number of PM actions increase. This expression is suitable for systems for which PM actions do not improve the condition of the system considerably. However, for many mechanical systems, PM actions like cleaning, lubrication, adjustment and alignment can improve the condition of the system quite significantly. To model such an action the deterioration factor should increase gradually as the number of PM actions increases. Equation (5.13) gives a general equation for calculating (θ_{ji}).

$$\theta_{ji} = 1 + \sum_{k=0}^{i-1} \left(\frac{Q_j k}{S_j k + P_j} \right) \tag{5.13}$$

where Q_j, S_j and P_j are user defined constants and all are greater than zero. These constants can be defined by the user based on component deterioration characteristics before and after the PM action. A large value of Q_j and relatively smaller values of S_j and P_j will be appropriate for very fast deteriorating system where PM action would make the system operational but would not be able to prevent it from deteriorating drastically after each PM action. On the other hand to model the effect of PM actions on many mechanical systems, the user is suggested to use a lower value of Q_j, and a relatively larger value of S_j and P_j.

Similar to equation (5.11), one can write the following equation for the cumulative hazard function:

$$H_{ji}(t) = \theta_{ji} H_j(t) \tag{5.14}$$

Next we rewrite the system hazard rate in terms of component hazard rate. This will allow us to incorporate different PM factors for different subsystems at the component level. The reliability of a component in subsystem j during the ith interval is:

$$r_{ji}(x_i) = e^{-\int_0^{x_i} h_{ji}(t)dt} = e^{-H_{ji}(x_i)} \tag{5.15}$$

where x_i is the length of the interval i. The system reliability can be written as

$$R_{si}(x_i) = \prod_{j=1}^{n}[1 - [1 - r_{ji}(x_i)]^{(1+m_j)}] \tag{5.16}$$

while the corresponding system hazard rate is

$$h_{si}(x_i) = \frac{-R'_{si}(x_i)}{R_{si}(x_i)} \tag{5.17}$$

Figure 5.2 shows the PM scheduling and the system failure rate behavior after each PM action. The system should always operate below a maximum allowable failure rate, which implies Equation (5.18) should always be satisfied:

$$h_{si}(x_i) \leq \xi \tag{5.18}$$

Minimal repairs are performed if the system fails within the scheduled PM intervals. Using equation (3.3) one can obtain the expected minimal repair cost of subsystem j during the first interval. Since in this formulation the PM actions affect the failure rate function of the component, it is more convenient for us to define minimal repair costs in terms of failure rate and cumulative failure rate function.

Equations (5.19) and (5.20) represent the minimal repair costs related to the system expressed in cumulative failure function rates at the end of first and second intervals, respectively.

$$C_{Mj1} = c_{mj}H_{j1}(x_1) \tag{5.19}$$

$$C_{Mj2} = c_{mj}H_{j1}(x_1) + c_{mj}H_{j2}(x_2) \tag{5.20}$$

Using equation (6.3) one can rewrite equation (5.20) as:

$$C_{Mj2} = c_{mj}\sum_{k=1}^{2}\theta_{jk}H_j(x_k) \tag{5.21}$$

In general, the minimal repair cost of subsystem j at the end of i intervals can be given by equation (5.22)

$$C_{Mji} = c_{mj}\sum_{k=1}^{i}\theta_{jk}H_j(x_k) \tag{5.22}$$

Similar to formulation 1, when the system reaches maximum allowed failure rate, one of the following action is taken. Either keep the system and perform PM or replace the system with an identical system. This decision is made by comparing the average annual cost (AAC) of the system and checking whether the system has reached its economic life or not. The cost due to PM is given by equation (5.9) and the AAC of the system at the end of the ith interval is given by equation (5.23).

$$AAC_i = \frac{IC + \sum_{j=1}^{n} [(1 + m_j)AC_j\phi_j + PM_{ji} + C_{Mji}]}{T_i} \tag{5.23}$$

where

$$T_i = \sum_{k=1}^{i} x_k \tag{5.24}$$

AC_j	Acquisition cost of a component in jth subsystem
C_{Mji}	Minimal repair cost of jth subsystem at the end of the ith interval (Given by equation (5.22))
IC	Installation cost of the system
PM_{ji}	PM cost on subsystem j at the end of ith interval (Given by equation(5.9))
T_i	System calendar age at the end of ith interval
ϕ_j	Assembly coefficient of a component in jth subsystem

If the values of m_j are known, then using equations (5.18), (5.23) and (5.24), one can calculate the values of T_1, T_2,..., T_i, T_{i+1} and subsequently calculate the values of AAC_1, AAC_2,..., AAC_i, AAC_{i+1}. We stop when $AAC_{i+1} > AAC_i$ and $AAC_{i-1} > AAC_i$ and T_i is the economic life of the system and the system is replaced at T_i. However, since m_j is unknown, we cannot calculate the PM intervals, AAC_i or replacement time of the system directly. A *five step methodology* proposed in the next section is used to calculate the optimal system design, PM intervals and economic life of the system.

5.5 Five Step Methodology

To calculate optimal values of m_j we minimize the expected average annual costs AAC subject to failure rate constraint for each value of i to obtain AAC_i^*. Once the optimal system configuration is obtained for the first i intervals, we calculate AAC_{i+1} using the optimal configuration obtained for i. If this AAC_{i+1} is less than AAC_i^* then it implies that for the given optimal configuration for intervals 1 to i, the corresponding value of T_i is not system's economic life. The system design is then optimized for i+1 intervals. This process is repeated and we stop when AAC_{i+1} is greater than AAC_i^*. The basis behind this methodology is that if the system is replaced after the ith interval, the system cost should be minimum for T_i. The five step methodology is listed below:

Step 1 Set $i = 1$

Step 2 Minimize the expected average annual costs AAC subject to failure rate. The optimization problem is formulated as,
Minimize: AAC_i, subject to:
$h_{sk}(x_k) \leq \xi$, where $k = 1$ to i
We then obtain AAC_i^* and $m_j^{(i)}$, where $m_j^{(i)}$ is the optimal values of m_j for the first i intervals.

Step 3 Calculate $AAC_{i+1}(m_j^{(i)})$, expected average annual cost at the end of $(i+1)$ intervals for the system configuration $m_j^{(i)}$.

Step 4 Is $AAC_{i+1}(m_j^{(i)})$ greater than AAC_i^*?
If yes, then stop. $m_j^{(i)}$ is the optimal system configuration with T_i

as the economic life of the system. If no, then go to Step 5.

Step 5 Set $i = i + 1$ and go to Step 2.

The optimization problem at Step 2 has non-linear objective function with a non-linear failure rate constraint. In the next section we present the numerical illustrations of these two formulations and discuss the application of genetic algorithms to obtain optimal designs of the system for minimal average annual costs.

5.6 Numerical Illustration

Consider a system with four subsystems connected in series. We need to determine the number of parallel components in each subsystem. Assume all the components follow a Weibull distribution with shape parameter greater than one. The reliability functions of the components for subsystems 1, 2, 3 and 4 are given by Equations (5.25), (5.26), (5.27) and (5.28), respectively.

$$r_1(t) = e^{-0.5\,t^2} \tag{5.25}$$

$$r_2(t) = e^{-0.15\,t^2} \tag{5.26}$$

$$r_3(t) = e^{-0.055\,t^{1.5}} \tag{5.27}$$

$$r_4(t) = e^{-0.095\,t^2} \tag{5.28}$$

Various costs associated with the system components are presented in Table 5.1. It can be seen that the component that deteriorates faster costs less than the one which deteriorates slower. The installation cost of the system is 400 dollars and is independent of system design. The maximum allowable failure rate of the system is 0.2 failures per year. The maximum number of components allowed in

Table 5.1: Costs related to system components for Formulation 1 and 2

j	AC_j	ϕ_j	MC_j	c_{mj}
1	90	1.11	10	1
2	125	1.2	15	1.5
3	150	1.33	20	2
4	225	1.11	25	2.5

each subsystem is equal to 15. For our research we use Goldberg's simple genetic algorithm (SGA) (Goldberg, 1989). GAs are programmed in Fortran 77 and run on Unix based IBM RS-6000 machine. In the implementation of GA's, we use linear scaling to regulate the number of copies of extraordinary individuals within a population (Goldberg, 1989). Goldberg's scaling routines: procedure prescale, function scale, and procedure scalepop are used. Stochastic remainder selection without replacement is used as the selection method due to its superiority over other selection schemes (Booker, 1982). A linearly decreasing mutation rate is used over the first 40 generations with initial mutation probability of 0.05 and decreasing to 0 at the end of the 40th generation. Mutation in early generations helps the algorithm to fully explore the search space. However, later on, during the run, the creation of a solution via mutation generally will be the one with a lower fitness of that population. Hence, it is not recommended to have mutation after 40 generations as it might be counter productive for obtaining a global optimum solution.

A convergence criteria was set to stop a run when the average fitness of the population is within 0.5% of the maximum fitness. Preliminary runs were carried out to examine the effect of the population size on the performance of the GA. This is very important as too small a population will result in premature convergence of the GA to a suboptimal solution while too large a population results in excessive CPU time. Primarily, we are concerned with a fact that a GA does not absolutely guarantee convergence to a global optimum solution. Because of this, we monitor the best solution that has been encountered for all the populations generated during a single run. One hundred runs are performed for each population size in order to assess the probability of success in determining the globally optimal solution. The results of the preliminary runs indicate that, for this problem:

- The probability that a population size of 50 will converge in under 75 generations is 0.85. However, the probability that the population size of 50 will result in the best solution being the global optimum are between 0.60 and 0.70. About 80% of the time population size of 60 converged under 75 generations while generating the global optimum about 85% of the time.

- The optimal solution usually first appears in the population about half way through a run, typically between the 30th and 40th generation.

- It was seen that a population size larger than 60 leads to excessive computation time, without a proportionate increase in the probability of converging to the global optimum. A population size of 60 is used for both formulations.

Next we present the numerical results for formulation 1 and 2.

Formulation 1

In formulation 1, the improvement factor α for all the components in all the subsystems is 2.5. To solve the optimization problem we implement a classical GA. For each subsystem j, the number of parallel components m_j is coded as a 4-bit string. Each possible system design can then be described by a 16-bit string. For fitness evaluation for each coded string it is assumed that the failure rate constraint is active. Given the values of the m_j, this constraint equation is solved to give the time T_i and value of AAC_i. Since GAs are set up to maximize fitness hence the optimization model needs to be converted from maximization to a minimization one. This is done by subtracting each string's AAC from the largest AAC in the current population to give a positive fitness value.

Using the *five step methodology* proposed in section 5.5, we obtain the optimal system design, optimal PM intervals and system replacement time. Implementation of the *five step methodology* is shown in Table 5.2. It is found that optimal

Table 5.2: Implementation of the *Five Step Methodology* for Formulation 1

i	AAC_i^*	$m_j^{(i)}+1$	$AAC_{i+1}(m_j^{(i)})$	$AAC_{i+1}(m_j^{(i)}) > AAC_i^*$	*Action*
1	1985.015	7,3,2,2	1345.065	No	Next i
2	1345.065	7,3,2,2	1182.893	No	Next i
3	1182.893	7,3,2,2	1141.629	No	Next i
4	1141.629	7,3,2,2	1149.490	Yes	Stop
5	1149.490	7,3,2,2	1181.661	-	-
6	1181.661	7,3,2,2	1227.581	-	-

system design for minimum AAC under formulation 1 is 7, 3, 2 and 2 components in parallel for subsystems 1, 2, 3 and 4 respectively. For the obtained design, all the subsystems should be preventively maintained at $T_1 = 1.234$, $T_2 = 1.974$ and $T_3 = 2.418$ years. At $T_4 = 2.685$ years the system should be replaced. The AAC for the system life cycle is 1141.629 dollars.

Formulation 2

In formulation 2, the deterioration factors for each subsystem are given by equation (5.29) and (5.30). According to these deterioration factors, the components in subsystems 1 and 4 deteriorate slower than the components in subsystems 2 and 3 after PM action.

$$\theta_{1i} = \theta_{4i} = 1 + \sum_{k=0}^{i-1} \left(\frac{k}{k+1} \right) \tag{5.29}$$

$$\theta_{2i} = \theta_{3i} = 1 + \sum_{k=0}^{i-1} \left(\frac{3k}{2k+1} \right) \tag{5.30}$$

GA is implemented, with m_j coded as a 4-bit string and each possible system design represented by a 16-bit string. Fitness evaluation for each coded string assumes that the hazard rate constraint is active. Using our proposed five step methodology for formulation 2, we obtain optimal system design, optimal PM intervals and economic life of the given system. Implementation of the five step methodology is summarized in Table 5.3.

Table 5.3: Implementation of the *Five Step Methodology* for Formulation 2

i	AAC_i^*	$m_j^{(i)}+1$	$AAC_{i+1}(m_j^{(i)})$	$AAC_{i+1}(m_j^{(i)}) > AAC_i^*$	*Action*
1	1985.015	7,3,2,2	1234.047	No	Next i
2	1234.047	7,3,2,2	997.915	No	Next i
3	997.915	7,3,2,2	890.661	No	Next i
4	890.661	7,3,2,2	831.872	No	Next i
5	830.743	6,3,2,2	795.558	No	Next i
6	795.558	6,3,2,2	774.390	No	Next i
7	774.390	6,3,2,2	761.980	No	Next i
8	761.980	6,3,2,2	755.078	No	Next i
9	755.078	6,3,2,2	752.699	No	Next i
10	752.699	6,3,2,2	753.000	Yes	Stop
11	753.000	6,3,2,2	755.820	-	-
12	755.820	6,3,2,2	760.511	-	-

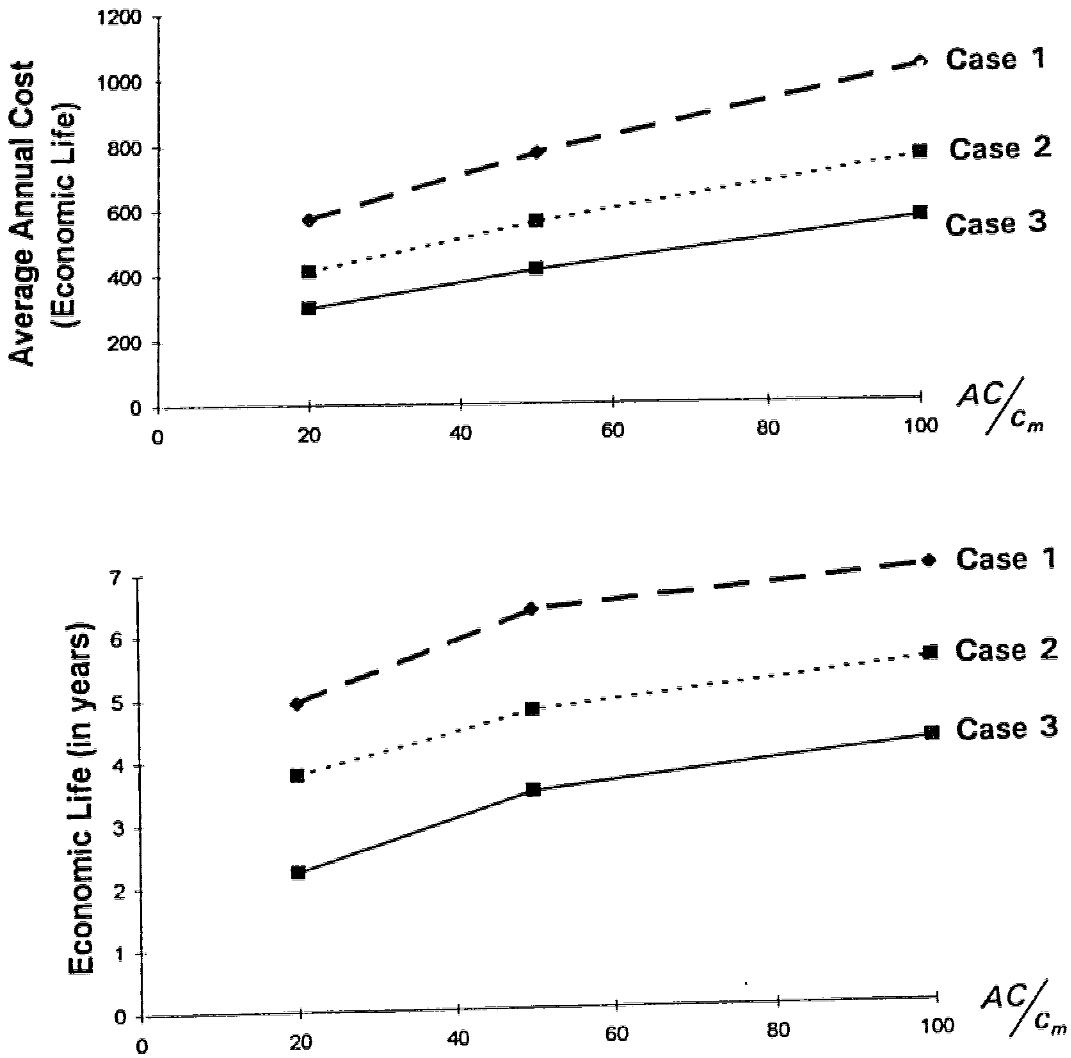
It can be seen that after seven intervals the AAC of the system starts increasing and it is not economical to keep the system. The optimal system design requires 6, 3, 2 and 2 components in parallel at subsystem 1, 2, 3 and 4 respectively. For the obtained system design, all the subsystems should be preventively maintained four times, i.e. at the end of 1.172, 2.049, 2.734, 3.294, 3.768, 4.180, 4.545, 4.875 and 5.179 years respectively. At the end of 5.454 years, the system should be replaced. This is also the economic life of the system with the proposed design and has an average annual cost of 752.699 dollars.

The trends for average annual cost for system's economic life ($AAC_{EconomicLife}$) for different ratios of AC/c_m and MC/c_m are shown in Figure 5.3. It can be seen that for a higher ratio of MC/c_m the $AAC_{EconomicLife}$ is higher. As AC/c_m increases the $AAC_{EconomicLife}$ and economic life of the system increases. The optimal system design for different ratios of AC/c_m and MC/c_m are shown in Table 5.4.

Table 5.4: System design ($m_j + 1$) for different cost ratios (Formulation 2)

MC/c_m	$AC/c_m = 20$	$AC/c_m = 50$	$AC/c_m = 100$
5	8,4,2,2	7,4,2,2	6,3,2,2
10	8,4,2,2	7,3,2,2	6,3,2,2
20	8,3,2,2	7,3,2,2	6,3,2,2

The five step methodology can be also used for formulation 1 to incorporate the case when the PM factors are different for each subsystem. The effects of the age reduction PM model are controlled by the improvement factor (α) due to PM. The effects of the hazard rate PM model are influenced by the failure rate deterioration factor (θ_{ji}). From the numerical example in this section, we have found that these two formulations can result in different optimal system designs, (7, 3, 2, 2) for formulation 1 and (6, 3, 2, 2) for formulation 2.



Case 1	$MC/c_m = 20$
Case 2	$MC/c_m = 10$
Case 3	$MC/c_m = 5$

Figure 5.3: Trends in $AAC_{\text{Economic Life}}$ and Economic Life for Formulation 2

It is important to distinguish between the level and shape of failure/hazard rate function as they relate system degradation with time (Canfield, 1986). The failure rate level reflects the extent of system degradation. Age reduction PM model is suited for systems where PM actions are routine inspections with replacement or repair of minor components which are worn or faulty. The hazard rate PM model is appropriate to model PM actions like lubrication, adjustment of tolerances and minor overhaul of components subject to wear. The reason is that: although such PM action brings the system back to operational stage, the rate of degradation increases due to cumulative effect of system degradation.

5.7 Concluding Remarks

This chapter presents a methodology to determine an optimal design of a system with deteriorative components. The algorithm provides users with optimal PM intervals and system replacement time. The proposed five step methodology allows the user to incorporate different PM factors for different subsystems and enhances the applicability of the presented formulations. Genetic algorithms are used as a tool to solve the traditional non-linear integer programming optimization problem.

In the next chapter we focus on RBD of systems using the condition when $h_{ji}(0) \neq 0$. This condition will allow us to model systems which have a non-zero failure rate at time equal to zero due to both externally and internally induced conditions. The model will also consider the effects of system salvage value on system design.

Chapter 6

Reliability Based Design Considering Maintenance and Salvage Value

6.1 Introduction

In chapter 5 it was shown that the concepts of minimal repair and PM can be incorporated into reliability based design (RBD) problems when the component reliability is expressed as a function of time. The formulations utilize two PM models proposed by Nakagawa (1988). A comprehensive *five step methodology* which produces optimal system design, optimal PM intervals, and optimal system replacement time, is also presented.

In this chapter we present a RBD model for a series-parallel system assuming that the failure rate is not zero at time zero. The system lifecycle cost function is further modified to include the salvage value of the system in addition to the acquisition, installation and maintenance costs. The effects of salvage value, preventive maintenance and minimal repair are incorporated to present a comprehensive methodology to evaluate and obtain the best system design over its life cycle. Genetic algorithms are used to perform constrained optimization of the system cost function using both active and non-active constraints. The results have important applications in the area of economic evaluation of automated manufacturing systems where high investment costs are involved to acquire a system which must perform below a given failure rate.

Section 6.2 discusses the system characteristics and presents the modified hazard rate PM model. Section 6.3 discusses the system costs and presents the salvage function for a system which is subject to preventive maintenance and physical depreciation. This section also presents the formulation of optimization problem followed by numerical illustration in section 6.4 and concluding remarks in section 6.5.

6.2 System Characteristics

In this chapter, we consider a system with similar characteristics as in Chapter 5, i.e., the system comprises of n subsystems in series. Subsystem j ($j = 1, ..., n$) consists of $(1 + m_j)$ identical components in active redundancy. All the components are statistically independent. Each subsystem acts as a 1-out-of-m:G configuration. All the system components have continuous and strictly increasing failure rates. It is required that the system should operate below a maximum allowed failure rate.

For the systems with increasing failure rates it is a common practice to perform PM whenever the system reaches the maximum allowed failure rate or minimum acceptable level of reliability (Lie and Chun, 1986; Malik, 1986; Jayabalan and Chaudhuri, 1992b). A PM policy for systems with increasing failure rates is one in which the PM is performed on the system at times when the system reaches the maximum allowed failure rate. If the system fails between these intervals, then minimal repairs are performed. Such a policy was proposed by Nagakawa

(1988) and further validated by Jayabalan and Chaudhuri (1992c) with a case study. Nakagawa (1988) presented two sequential PM models using *age reduction* and *hazard rate concepts.*

According to Nakagawa's *hazard rate model,* a PM restores the system to a working condition with a failure rate of zero. However, after each additional PM action, the slope of the hazard function increases. During the first interval, the component has undergone no PM and the component failure rate is $h_{j1}(t)$. This failure rate corresponds to the original failure rate of the component, that is, $h_{j1}(t) = h_j(t)$. At the end of the first interval the component undergoes PM and its failure rate is reduced instantly to zero but the slope of the new failure rate function is increased (See Figure 5.1). In general, the hazard rate of a component in the jth subsystem during the ith interval is $h_{ji}(t)$:

$$h_{ji}(t) = \theta_{ji} h_j(t) \tag{6.1}$$

where θ_{ji} is defined as *failure rate deterioration factor.* It was shown in Chapter 5 that depending on the effect of a PM action on the component, one can mathematically define θ_{ji} to satisfy the following two conditions:

1. $\theta_{j1} = 1$

2. $\theta_{j(i+1)} \geq \theta_{ji}$, where i=1,2,3,...

However, in many practical situations the failure rate of the system is non-zero after each PM action due to both externally and internally induced conditions. To model this more practical condition we modify the failure rate function (Equation (6.1)) by adding a constant λ_j to the variable (time), such that $\lambda_j \geq 0$. During the first interval ($i = 1$) when the component has undergone no PM, the component failure rate is $h_{j1}(t)$. This failure rate corresponds to the original failure rate of the component, i.e. $h_{j1}(t) = h_j(\lambda_j + t)$. At $t = 0$, $h_{j1}(0) = \xi' > 0$, At the end of the first interval, the component undergoes PM and its failure rate is now changed to $h_{j2}(t) = \theta_{j2} h_j(\lambda_j + t)$ with $h_{j2}(t = 0) = \theta_{j2} h_j(\lambda_j) = \xi''$, where $\xi'' \geq \xi'$ (See Figure 6.1). In general, the modified hazard rate function of a component in the jth subsystem during the ith interval can now be rewritten as:

$$h_{ji}(t) = \theta_{ji} h_j(\lambda_j + t) \tag{6.2}$$

For hazard rate deterioration factor, (θ_{ji}), we use equation (5.13) developed in Chapter 5. Similarly we can obtain the following equation for the cumulative hazard function:

$$H_{ji}(t) = \theta_{ji} H_j(\lambda_j + t) \tag{6.3}$$

The reliability of a component in subsystem j during the ith interval is:

$$r_{ji}(x_i) = e^{-\int_0^{x_i} h_{ji}(t)dt} = e^{-H_{ji}(x_i)} \tag{6.4}$$

where x_i is the length of interval i. The system reliability can be written as

$$R_{si}(x_i) = \prod_{j=1}^{n} [1 - [1 - r_{ji}(x_i)]^{(1+m_j)}] \tag{6.5}$$

while the corresponding system hazard rate is

$$h_{si}(x_i) = \frac{-R'_{si}(x_i)}{R_{si}(x_i)} \tag{6.6}$$

The system should always operate below a maximum allowable failure rate, which implies that equation (6.7) should always be satisfied:

$$h_{si}(x_i) \leq \xi \tag{6.7}$$

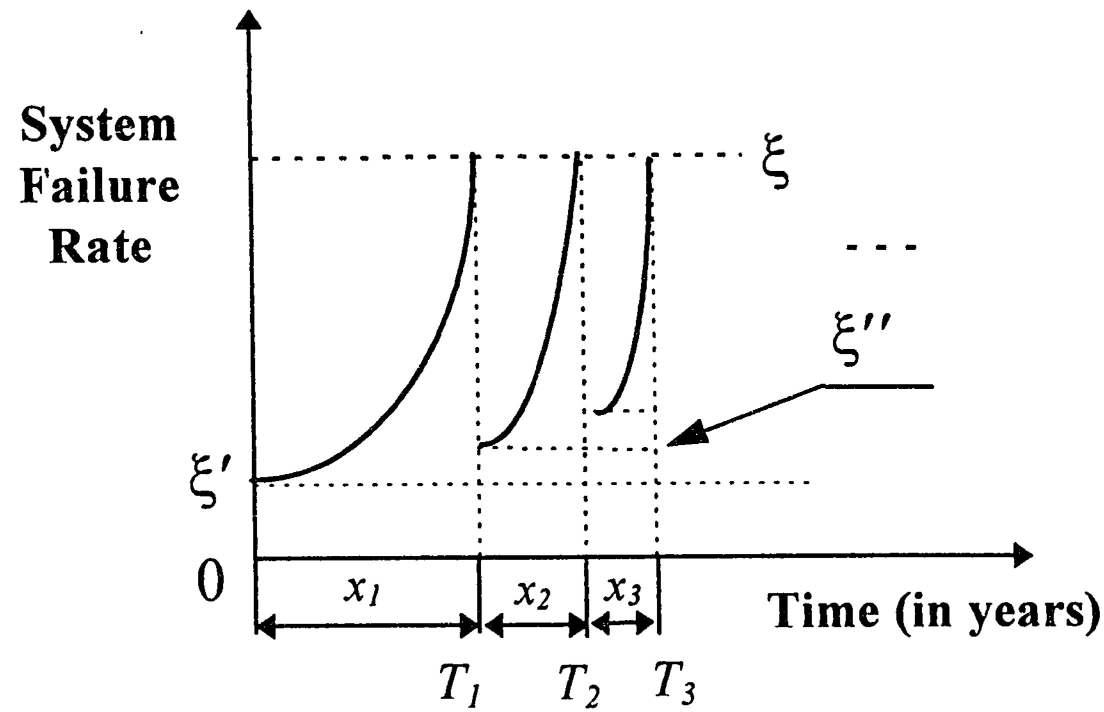


Figure 6.1: System Failure Rate and Preventive Maintenance Intervals (Using Modified PM Model)

6.3 System Cost and Problem Formulation

There are four main components of system cost in our formulation. These are: (a) net acquisition cost of the system, (b) installation cost of the system, (c) cost of minimal repairs, and, (d) cost of preventive maintenance. The acquisition cost includes design, development and production costs of the system. In this formulation the *net acquisition cost* of a component at a given time t incorporates both acquisition cost and salvage value. We propose to use equation (6.8) to represent the net acquisition cost of $(1 + m_j)$ components in subsystem j at the end of the ith interval.

$$NAC_{ji}(t) = \phi_j(1 + m_j)[AC_{ji} - SV_{ji}(t)] \tag{6.8}$$

where assembly coefficient ϕ_j accounts for the assembly costs of components in the jth subsystem and SV_{ji} is the salvage value of the component in the jth subsystem at the end of ith interval.

The salvage value is defined as market value of a component/system at the end of its life. It is the amount eventually recovered through sale, trade-in or salvage (Chan, Porteous, Sadler and Zuo, 1995). The salvage value of the system is estimated from a depreciation schedule established for a system. Depreciation can be classified into two categories, namely, physical and functional depreciation. Physical depreciation is defined as a reduction in a system's capacity to perform its intended service due to physical impairment. Physical depreciation can occur to any fixed asset in the form of deterioration from interaction with the environment, including such agents as corrosion, rotting, etc. It can also occur due to system wear and use. Physical depreciation leads to decline in performance and high maintenance costs. Functional depreciation occurs as a result of changes in the organization or in technology that decrease or eliminate the need for a system. Examples of functional depreciation include obsolescence due to advances in technology, a declining need for the services performed by a system, or the inability to meet increased quantity and/or quality demands.

Many researchers have used the concept of economic depreciation in various areas of research related to evaluation of manufacturing systems. Kaio and Osaki (1990) have used the economic depreciation to evaluate a one unit system, where each failed unit is scrapped without repair and each spare is provided through an order after a lead time. In their formulation they consider a non-linear increasing cost $s(t)$, which is suffered by the system for salvage at time t. Jones, Zydiak, Hopp and Wallace (1990) present an equipment replacement model for profit maximizing firm facing a demand curve. The firm's product is produced by machines whose capacity is a non-increasing function of age. The economic depreciation schedule is developed using the concepts from microeconomics. The objective of the firm is to maximize the sum of discounted profits over an infinite period. Gupta and Dai (1990) used the concept of salvage value for perishable product retailing systems like grocery and fashion clothing. The authors calculated the maximum profit ordering for a given salvage value. Cheng (1992) analyzed an optimal replacement problem of an aging equipment, where he defines overhaul and inspection costs as an increasing function of the inspection interval to model the equipment depreciation. The replacement cost is a decreasing function of the inspection interval. Total cost per unit time is minimized which minimizes the total cost of replacements and operating the equipment between the overhaul and inspection intervals.

In our formulation, we developed a salvage value function which incorporates the economic effects of system deterioration and PM. The salvage value function satisfies the following three conditions.

1. The salvage value function of a system with an increasing failure rate is a monotonically decreasing function during any given operational interval.

2. At the end of any interval, if a PM action is performed on the system then the salvage value of the system increases to a value such that

$$SV_{j(i+1)}(t=0) < SV_{ji}(t=0) \tag{6.9}$$

3. Functional depreciation of the system and the tax effects of system disposal are ignored.

To satisfy the above three conditions, we propose the following salvage value function of a component in subsystem j during the ith interval:

$$SV_{ji}(t) = \frac{AC_j}{\Gamma_i[\rho h_{ji}(t) + \beta]^t} \tag{6.10}$$

where Γ_i, ρ and β are the user defined constants which are market driven such that $\Gamma_{i+1} > \Gamma_i > 0$, $\rho > 0$ while $\beta \geq 0$. It will be shown through numerical illustration that $NAC_{ji}(t)$ plays an important role in system replacement decision making as it is an increasing cost function with respect to time.

The installation cost (IC) of the system is a one time cost during system life cycle and is independent of system design. Following the PM strategy for deteriorating system preventive maintenance (PM) is performed when the system reaches the maximum allowable failure rate. Minimal repairs are performed if the system fails within the scheduled PM intervals.

As presented in Chapter 5, the costs associated with minimal repairs depend on the frequency of failures in a system. According to equation (3.3), for our system, the expected minimal repair cost of subsystem j at the end of the first interval is given by:

$$\begin{aligned} C_{Mj1} &= c_{mj} \int_0^{x_1} h_{j1}(t)dt \\ &= c_{mj}\, \theta_{j1} \int_0^{x_1} h_j(\lambda_j + t)dt \end{aligned} \tag{6.11}$$

Since $\theta_{j1} = 1$ hence we can rewrite equation(6.11) as:

$$C_{Mj1} = c_{mj} \left[H_j(\lambda_j + x_1) - H_j(\lambda_j) \right] \tag{6.12}$$

The number of minimal repairs at the end of the second interval is given by:

$$\begin{aligned} C_{Mj2} &= C_{Mj1} + c_{mj} \int_0^{x_2} h_{j2}(t)dt \\ &= C_{Mj1} + c_{mj}\, \theta_{j2} \int_0^{x_2} h_j(\lambda_j + t)dt \\ &= C_{Mj1} + c_{mj}\, \theta_{j2} \left[H_j(\lambda_j + x_2) - H_j(\lambda_j) \right] \\ &= c_{mj} \left[H_j(\lambda_j + x_1) - H_j(\lambda_j) \right] + \theta_{j2} \left[H_j(\lambda_j + x_2) - H_j(\lambda_j) \right] \\ &= c_{mj} \sum_{k=1}^{2} \theta_{jk} \left[H_j(\lambda_j + x_k) - H_j(\lambda_j) \right] \end{aligned} \tag{6.13}$$

The minimal repair cost of subsystem j at the end of i intervals can be given by equation (6.14):

$$C_{Mji} = c_{mj} \sum_{k=1}^{i} \theta_{jk} \left[H_j(\lambda_j + x_k) - H_j(\lambda_j) \right] \tag{6.14}$$

When the system reaches the maximum allowed failure rate, one of the following decisions is made: (1) keep the system and perform PM, or (2) replace the system with an identical system. This decision is made by comparing the expected average annual costs (AAC) of the system and checking whether the system has reached its economic life or not.

If PM is performed on the system then all the components undergo PM. The cost due to PM is given by equation (5.9) Figure 6.1 shows the PM scheduling and the system failure rate behavior after each PM action. The AAC of the system at the end of the ith interval is given by equation (6.15).

$$AAC_i = \frac{IC + \sum_{j=1}^{n} [NAC_{ji}(t) + PM_{ji} + C_{Mji}]}{T_i} \tag{6.15}$$

where

$$T_i = \sum_{k=1}^{i} x_k \tag{6.16}$$

where,

NAC_j	Net Acquisition cost of the jth subsystem at the end of ith interval (Given by equation (6.8)
C_{Mji}	Minimal repair cost of jth subsystem at the end of the ith interval (Equation (6.14))
IC	Installation cost of the system
PM_{ji}	PM cost on subsystem j at the end of ith interval (Equation (5.9))
T_i	System calendar age at the end of ith interval

In addition, traditional RBD problems have constraints of weight, cost and volume. These constraints unlike the failure rate constraint (equation (6.7)) are not necessarily active. Equations (6.17), (6.18) and (6.19) represent the weight, investment/capital and volume constraints respectively which are usually encountered in RBD problem.

$$\sum_{j=1}^{n} W_j(1 + m_j) \leq W \tag{6.17}$$

$$\sum_{j=1}^{n} \phi_j AC_j(1+m_j) \leq C \qquad (6.18)$$

$$\sum_{j=1}^{n} V_j(1+m_j) \leq V \qquad (6.19)$$

To calculate the optimal system life cycle, its PM intervals, and system replacement time, we use the *five step methodology* proposed in Section 5.5. This *five step methodology* provides the user with optimal system design over its life cycle with increasing failure rates and allows the user to incorporate different deterioration factors for different subsystem. For this formulation we have additional resource constraints in step 2. The *five step methodology* for this formulation is as follows:

Step 1 Set $i = 1$

Step 2 Minimize the expected average annual costs AAC with respect to failure rate and resource constraints. The optimization problem is formulated as,

Minimize: AAC_i, subject to:

$h_{sk}(x_k) \leq \xi$, where $k = 1$ to i, and

$g_q(m_j) \leq 0$, where $q = 1$ to z. z is the number of resource constraints.

We then obtain AAC_i^* and $m_j^{(i)}$, where $m_j^{(i)}$ is the optimal values of m_j for the first i intervals.

Step 3 Calculate $AAC_{i+1}(m_j^{(i)})$, where $AAC_{i+1}(m_j^{(i)})$ is the expected average annual cost of the $(i+1)$ intervals for the system configuration $m_j^{(i)}$.

Step 4 Is $AAC_{i+1}(m_j^{(i)})$ greater than AAC_i^*?

If yes, then stop. $m_j^{(i)}$ is the optimal system configuration with T_i as the economic life of the system. If no, then go to Step 5.

Step 5 Set $i = i + 1$ and go to Step 2.

The optimization problem at Step 2 has a non-linear objective function with both linear and non-linear constraints and discrete variables. We use Genetic algorithms (GAs) to solve the optimization problem. In the next section we present the numerical illustration of the problem and implementation of GAs to minimize a constrained non-linear objective function.

6.4 Numerical Illustration

Consider a system with four subsystems connected in series. We need to determine the number of components in parallel at each subsystem. Assume all the components follow a Weibull distribution with shape parameter greater than one. The reliability functions of components in subsystems 1, 2, 3 and 4 are given by equations. (6.20), (6.21), (6.22) and (6.23) respectively.

$$r_1(t) = e^{-0.5(t+0.008)^2} \tag{6.20}$$

$$r_2(t) = e^{-0.15(t+0.005)^2} \tag{6.21}$$

$$r_3(t) = e^{-0.055(t+0.006)^{1.5}} \tag{6.22}$$

$$r_4(t) = e^{-0.095(t+0.003)^2} \tag{6.23}$$

Various costs associated with system components are presented in Table 6.1. It can be seen that the component which deteriorates faster costs less than the one which deteriorates more slowly. The installation costs of the system is 400 dollars and is independent of system design. The maximum allowed failure rate is 0.2 failures per year. The maximum number of components allowed in each sub system is equal to 15. To solve the optimization problem a classical GA is implemented.

In Chapter 5 we demonstrated the applications of GAs to solve a RBD problem with deteriorative components with an active failure rate constraint. In this

Table 6.1: Costs related to system components

j	AC_j	ϕ_j	MC_j	c_{mj}
1	90	1.11	10	1
2	125	1.2	15	1.5
3	150	1.33	20	2
4	225	1.11	25	2.5

formulation, we intend to generalize the RBD optimization problem by handling both active/non-active, linear and non-linear constraints using GAs. To handle the non-active resource constraints of the form $g_q(m_j) \leq 0$, where $q = 1, 2, 3, \ldots, z$, we incorporate the constraint violations as penalties (Goldberg, 1989) to the function we want to minimize.

$$F' = F + \sum_{q=1}^{z} P_q$$

where,

F' = Modified function to be minimized

F = Original function to be minimized

z = Number of constraints, and

P_q = Penalty for the violation of constraint q, such that

$$P_q = \begin{cases} 0, & \text{if } g_q(m_j) \leq 0 \\ |g_q(m_j)|, & \text{otherwise} \end{cases}$$

This penalty method to handle constraints was tested successfully to solve reliability related non-linear optimization problems with discrete variables from Tillman, Hwang and Kuo (1985).

For each subsystem j, the number of components m_j is coded as a 4-bit string. Each possible system design can then be described by a 16-bit string. For fitness evaluation for each coded string it is assumed that the failure rate constraint, i.e.

equation (6.7) is active. An additional resource constraint is considered. This resource constraint in this formulation is referred as the *investment constraint*, and is given by the equation (6.24).

$$\sum_{j=1}^{4} \phi_j AC_j(1 + m_j) \leq 2500 \tag{6.24}$$

Since the GAs are set up to maximize fitness, the optimization model is converted from a minimization problem to a maximization one. This is done by subtracting each string's AAC and constraint violation penalty from the largest in the current population to give a positive fitness value.

In the implementation of GAs, *linear scaling* is used to regulate the number of copies of extraordinary individuals within a population, using Goldberg's scaling routines: procedure *prescale*, function *scale* and procedure *scalepop*. We use *Stochastic remainder selection without replacement* and a linearly decreasing mutation rate for the first 40 generations with initial mutation probability as 0.05, decreasing to 0 at the end of the 40th generation. Mutation in early generations helps the algorithm to fully explore the search space. However, later on during the run the creation of an individual via mutation generally will be the one with lower fitness of that population. Hence, it is not recommended to have mutation after 40 generations as it might be counter productive in obtaining a global optimum. A convergence criterion is set up to stop a run when the average fitness in the population is within 0.5% of the maximum fitness. We performed an empirical study which showed that a population size of more than 60 leads to excessive computation time, without a proportionate increase in the probability of converging to the global optimum.

The deterioration factors for each subsystem are given by equation (6.25) and (6.26). According to these deterioration factors, the components in subsystems

1 and 4 deteriorate slower than the components in subsystems 2 and 3 after PM action.

$$\theta_{1i} = \theta_{4i} = 1 + \sum_{k=0}^{i-1} \left(\frac{k}{k+1} \right) \tag{6.25}$$

$$\theta_{2i} = \theta_{3i} = 1 + \sum_{k=0}^{i-1} \left(\frac{3k}{2k+1} \right) \tag{6.26}$$

The salvage value function defined by equation (6.10) was used with

$\rho = 2$

$\beta = 1.2$

$\Gamma_1 = 1$, $\Gamma_2 = 1.2$, $\Gamma_i = \Gamma_{i-1} + 0.1$ for i=3, 4, 5, ...

To calculate the optimal system design for minimal life cycle cost we follow the *five step methodology*. It is found that the optimal system design is 7,3,2 and 2 components in subsystem 1, 2, 3 and 4 respectively (Table 6.2). The system should undergo PM at 1.227, 2.136 and 2.849 years. At the end of 3.420 years the system should not be maintained but replaced. The average annual cost of the system for its economic life is 526.785 dollars.

Table 6.2: Implementation of *Five Step Methodology* considering salvage value

i	AAC_i^*	$m_j^{(i)}+1$	$AAC_{i+1}(m_j^{(i)})$	$AAC_{i+1}(m_j^{(i)}) > AAC_i^*$	*Action*
1	765.113	7,3,1,2	615.754	No	Next i
2	613.156	7,3,2,2	545.016	No	Next i
3	545.016	7,3,2,2	526.785	No	Next i
4	526.785	7,3,2,2	528.679	Yes	Stop
5	528.679	7,3,2,2	537.429	-	-
6	537.429	7,3,2,2	578.898	-	-

We then consider the same system but ignore the salvage value. The optimal design of 6,3,2 and 2 components for subsystems 1, 2, 3, and 4 respectively is obtained. For such a system, it is found that PM should be performed at 1.165, 2.036, 2.714, 3.269, 3.738, 4.145, 4.507, 4.833 and 5.127 years. The system should be replaced at the end of 5.399 years. The expected average annual cost of the system over its economic life is 760.477 dollars (Table 6.3). The economic life of the

Table 6.3: Implementation of *Five Step Methodology* when salvage value is ignored

i	AAC_i^*	$m_j^{(i)}+1$	$AAC_{i+1}(m_j^{(i)})$	$AAC_{i+1}(m_j^{(i)}) > AAC_i^*$	*Action*
1	1996.055	7,3,2,2	1241.570	No	Next i
2	1241.570	7,3,2,2	1004.051	No	Next i
3	1004.051	7,3,2,2	896.246	No	Next i
4	896.246	7,3,2,2	837.207	No	Next i
5	837.207	7,3,2,2	803.063	No	Next i
6	802.066	6,3,2,2	781.953	No	Next i
7	781.953	6,3,2,2	768.687	No	Next i
8	768.687	6,3,2,2	762.664	No	Next i
9	762.664	6,3,2,2	760.477	No	Next i
10	760.477	6,3,2,2	761.527	Yes	Stop
11	761.527	6,3,2,2	764.441	-	-
12	764.441	6,3,2,2	769.324	-	-

system is defined as the optimal length of time for which a system has a minimum average annual cost. In the case when the salvage value of the system is ignored, the net acquisition cost of the system (defined by equation (6.10)) is constant. The system cost increases with time due to increasing number of PM actions and minimal repairs. However, if the salvage value of the system is adjusted from the acquisition cost, the net acquisition cost of the system is no longer a constant but increases with time during an operational period. The net acquisition cost of the system decreases right after the PM action is performed (due to increase in the salvage value) and then increases with a steeper slope than in the preceding interval. In such a situation when the economic life of a system is calculated at the end of each interval, there exists an economic incentive of capturing the decreasing salvage value of the system. This decreasing salvage value of the system pushes the system to be replaced earlier, hence making the economic life of the system shorter than the case when the salvage value is ignored. In addition, the minimal AAC is smaller.

Whether the salvage value of the system should be included in the cost function of a design problem will depend on the type of system one is considering. If one is designing a system which has a custom defined function with a very limited market, then one is better off ignoring the salvage value of the system. The examples of such system are computer integrated manufacturing systems which are custom designed to perform a very specific task like assembly of an expensive defense equipment. However, if there exists a market application for a system then there exists a salvage value for which the system can be sold readily at any given point of time. For such products, it is justified to include the salvage value at the design stage to incorporate the economic impact of salvage value on system design.

It must be recognized that system design is very sensitive to system failure characteristics and its relation to the salvage value function. When developing the salvage value function for a given system, the user should carefully select various constants in the proposed salvage value function to reflect the actual salvage value of the system at a given point of time. For systems with short product life cycle, this would mean that the salvage value will be a very steep monotonically decreasing function. However, for systems with long product life cycle the salvage value will decrease faster at the beginning and more slowly later on.

6.5 Concluding Remarks

This formulation contributes towards effective economic evaluation of system design when the system has a given salvage value. A general salvage value function of a system with increasing failure rate is proposed. The salvage value function also incorporates the economic effects of PM. The modified PM modeling incorporates both externally and internally induced failure rates to give a non-zero failure rate at time equal to zero. Genetic algorithms are used as an optimization tool for non-linear constrained cost function with discrete variables. The inclusion of the salvage value while evaluating a system design can be more of a strategic management decision than an engineering decision. However, such decisions will be more significant if the system components are very expensive and will have a resale value at any given point of time.

In the next chapter we present a formulation to design an optimal series-parallel system over its life cycle including all the three characteristic periods of the failure rate bathtub curve of a system, namely, infant mortality, useful period and increasing failure rate period. The formulation gives an optimal system design, burn-in period, PM intervals and replacement time for a system for a given warranty period.

Chapter 7

Reliability Based Design Considering Maintenance and Warranty

7.1 Introduction

Product design deals with activities such as conceptual design, product development and testing. Manufacturing deals with the processes which ensure that the items produced conform to design specifications. Design, manufacturing and quality control decisions determine product characteristics such as reliability and maintainability. These characteristics can be further enhanced through burn-in after a product is manufactured. This reduces the expected cost of warranty and increases the burn-in cost. As a result, design and manufacturing decisions must be made considering not only design and manufacturing cost, but also the cost of burn-in and subsequent warranty. In Chapters 5 and 6, we developed reliability based design (RBD) formulations for systems with monotonically increasing failure rates. In this chapter we develop a RBD formulation for a series-parallel system with components which follow a bathtub shaped failure rate curve. The system cost include burn-in, warranty, installation, preventive maintenance and minimal repair. The formulation gives an optimal system design, burn-in period, preventive maintenance intervals and replacement time for a system which must perform below a given failure rate for the users. In section 7.2, we present a brief overview of various warranty policies, followed by a discussion of system cost both from the manufacturer's and the customer's perspective. In section 7.3 we develop an expression for system cost over the useful life of the system and formulate the optimization problem. Section 7.4 presents a numerical illustration of the formulation and analyzes three distinct types of failure rate bathtub curves to see the change in system design and burn-in period for different warranty periods.

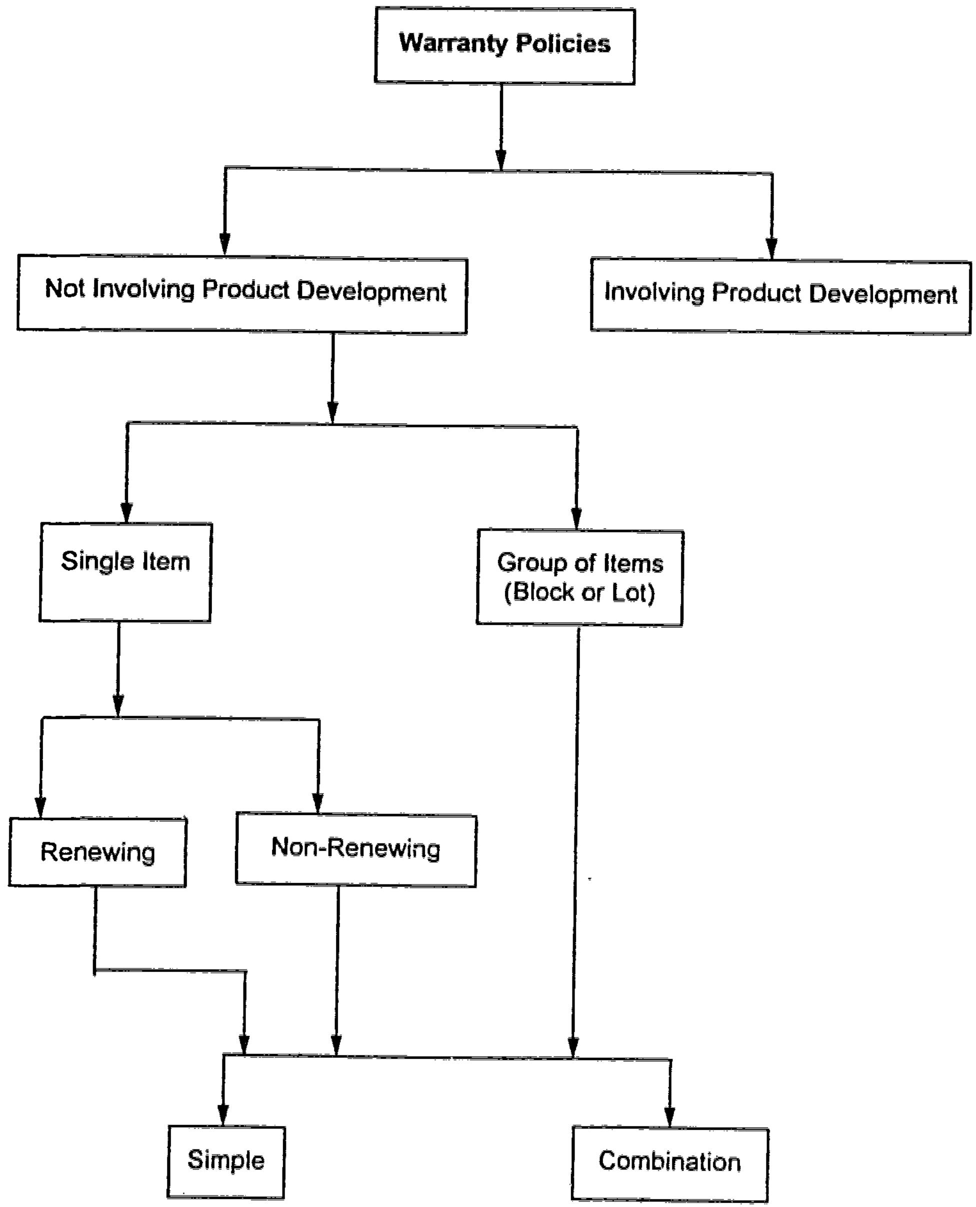
7.2 Warranty: A Brief Review

A warranty is a contractual obligation offered by the manufacturer (vendor or seller) in connection with the sale of the product (Berke and Zaino, 1991). The warranty contract is intended to assure the buyer (customer or consumer) that the product will perform its intended functions under the mutually agreed conditions for a specific period of time. If it fails to do so, the supplier will repair the product or provide replacements at no cost or at reduced cost to the buyer depending on the warranty terms. A warrantied product entails a greater cost to the supplier than that of an identical item sold without warranty. Logically, a buyer should be willing to pay more for a warrantied item than for an identical system sold without warranty (Blischke, 1990).

Customers need warranties to assure that the manufacturer assumes the responsibility/liability for its product for a specified time. Long warranties serve as an indicator of the reliability of the product and may increase sales. The length of the warranty period, however, is market driven and strongly influenced by the competitor's product. At the same time, warranty protects the manufacturer's interests by requiring that certain responsibilities in the part of the customer must be met (for example proper use of the product, adequate care and so on) and also explicitly restricting the liability of the manufacturer.

For new and innovative products, warranties serve an important additional role. Such products are often viewed with a degree of uncertainty by consumers at large. The uncertainty is reduced as more customers buy the product and information about the product performance is spread by consumer publications or through the word of mouth. This can often be a lengthy process, resulting in slow sales during the early stages following product introduction. Sales may be accelerated by signaling mechanism which conveys information to reduce the uncertainty or risk perceived by the customer. Warranty serves as one such signal. Better warranty terms convey that the risk is low and hence induce the consumer to buy the product. As a result, warranty is used as an advertising tool, similar to price or product performance, to compete with other manufacturers (Blischke and Murthy, 1992).

Warranty policies can be divided into two groups based on whether or not a policy involves product development after sale (Figure 7.1). Policies which do not involve product development can be further divided into two subgroups: Subgroup A comprises of policies applicable for single item sales while Subgroup B comprises of policies applicable only for the sale of a group of items (also called block or lot sales). Policies in Subgroup A can be subdivided into two further subgroups based on whether the policy is renewing or non-renewing.



Simple refers to Free Replacement Warranty (FRW) or Pro-Rata Warranty
Combination refers to a policy which consists of a combination of FRW and PRW

Figure 7.1: Classification of warranty policies (Blischke and Murthy, 1992)

In a renewing policy, whenever an item fails under warranty, it is replaced by a new item with a new warranty replacing the old one. In the case of a non-renewing policy, replacement of a failed item does not alter the original warranty. Thus, for renewing policies the warranty period begins anew with each replacement, while for non-renewing policies the replacement item assumes the remaining time of the item it replaced. The most common simple consumer warranties are the free replacement warranty (FRW), and the pro-rata warranty (PRW). Under FRW, the manufacturer agrees to repair or provide replacements for failed items free of charge up to a time w from the time of initial purchase, where w is called the warranty period. If a product is under a PRW then the replacements are provided under pro-rated cost to the consumer. Most of the warranty policies are combination of simple policies. These policies are specific for different products and are referred to as combination policies.

A common warranty policy utilized in commercial and government transactions, particularly in military acquisition of complex equipment, is the reliability improvement warranty. Warranties of this type are typically complex contractual agreements with many features unique to the particular acquisition. Salient features usually include guaranteed reliability terms, manufacturer's responsibility for field repair or replacement of failed items and incentive fees for demonstrated improvements in reliability (Trimble 1974). A detailed classification of various warranty policies is presented by Blischke and Murthy (1992).

For majority of consumer durables the most common warranty is non-renewing. The product can be either repairable or non repairable. After a failure, a failed product is restored to the operating condition by repair (for repairable products) or is replaced by a new one (for non-repairable products). We assume that repair and replacement times are negligible. For non-repairable products the sequence of failures with replacements constitute a renewal process, and the expected number of replacements in $[0, T]$, $M(T)$, is given by the following renewal equation (Barlow and Proschan, 1967):

$$M(T) = F(T) + \int_0^T M(T-t)dF(t) \tag{7.1}$$

where $F(t)$ is the cumulative failure distribution function.

For repairable products, it is assumed that the failure rate of the product remains unchanged after a repair, i.e. *minimal repair* or *bad as old* model (Nguyen and Murthy, 1982). This is a reasonable assumption for complex and expensive products, since the repair involves only a small part of the product. According to Barlow and Proschan (1967) the expected number of repairs in $[0, T]$, $E[N(T)]$, is given by:

$$E[N(T)] = \int_0^T h(t)dt \tag{7.2}$$

where $h(t)$ is the failure rate function.

As mentioned earlier, the study of product warranty is of importance to both manufacturers and customers, although the reasons and motivation for such study can be different for both groups. In the next section we discuss the two different perspectives and briefly review consumer and manufacturer cost models for non-renewing FRW for repairable systems.

7.2.1 Manufacturer's Perspective

All manufacturers desire to maximize profits. Offering a warranty results in additional cost due to service of the warranty and at the same time, if used properly as a marketing tool, increases sales. Warranty servicing cost depends on product characteristics and the usage patterns of consumers. If the extra revenue generated exceeds the warranty servicing cost, then it is more sensible to sell the product with warranty. As a result, manufacturers are interested in the study of warranty in order to seek answers to a variety of warranty related questions in context of manufacturing, marketing and servicing (Murthy and Blischke, 1992).

Models based on selecting warranty conditions so as to maximize profit have also been investigated by Glickman and Berger (1976). They deal with the selection of selling price c and warranty period w that maximize the profit to the seller. Optimal values are obtained assuming gamma-distributed lifetimes, constant cost of repair and a log-linear demand function.

The use of burn-in as a means of increasing reliability of items under warranty has been considered by Nguyen and Murthy (1982) for several warranty policies. Burn-in costs money and at the same time reduces warranty cost. Determination of the optimal burn-in period involves the trade-off between the burn-in cost and warranty cost. It is found that burn-in is cost effective if the initial failure rate is high and cost of repairs during warranty is high.

Renewal-theoretic results have been used to investigate other aspects of a seller's warranty cost as well (Nguyen and Murthy, 1984). In particular, if the sales rate as a function of time is known, then the expected warranty return rate and expected cash flow can be determined.

Nguyen and Murthy (1984) also consider a number of assumptions regarding the life distributions of repaired items and compare the results. As one would expect, average cost for good-as-new repair are identical to those for minimal repair if lifetimes are exponentially distributed (assuming the same cost structure for repair). Average warranty costs are less for good-as-new if the distribution is IFR (increasing failure rate) and more if it is DFR (decreasing failure rate). Mixed good-as-new and minimal repair is also considered.

Nguyen and Murthy (1986) suggest that in the case of repairable items, the seller/manufacturer often has the option of choosing between repairing a failed item and replacing it with a new one. This leads to further modifications of the cost models. One such policy has been investigated under the assumption that lifetimes of items have an increasing-failure-rate (IFR) distribution and that minimal repair is made. This is described as follows: if an item fails at an age less than or equal to a specified value σ, where $0 \leq \sigma \leq w$, then it is repaired and returned to stock for use only as replacement for another failed item. The failed item is replaced by a new item if it fails prior to a fixed time $(w - v)$, where $0 \leq v \leq w$, otherwise it is replaced by a repaired item. An optimal policy is to choose values of σ and v which minimize the expected warranty cost. The optimal choice is obtained as the solution to a nonlinear programming problem.

7.2.2 Consumer's Perspective

A consumer is usually faced with a decision of choosing between products with different characteristics and accompanying warranty policies. He or she would like to know if the warranty is worth the additional cost when the warranty is optional. This is important as there is a growing trend among manufacturers to offer extended-term warranties. These involve additional cost and the terms can vary considerably. For example, both labor and parts may be covered initially but only parts later on. The consumer needs to decide, often at the time of purchase and based on very limited information, whether to opt for an extended warranty and to determine the best extended terms for the situation when there are multiple options.

For users of industrial products, such cost can have a significant impact on their profits. Users of such products often have the skills and expertise and the bargaining power to demand relevant data from manufacturers to carry out such analyses and to negotiate terms on an individual basis. Here again, the most effective and appropriate approach is mathematically model the problem and solve it analytically when possible.

Let C be the cost to the buyer of individual items sold under a FRW and C_B be the total cost to the buyer over the period of use of the product, for non-renewing FRWs. In the analyses, it is assumed that failed items are repaired instantaneously. The buyer's total cost of a single item covered under a non-renewing FRW is presented next. This cost includes not only the original purchase price, but also installation cost, energy, maintenance and repair cost (except as covered by warranty) and many potential incidental cost, including possible legal costs, cost of invoking the warranty, unrecoverable incidental damages caused by failures, and, ultimately, the cost of disposal. All the formulations include installation cost as part of the purchase price and ignore disposal cost; other costs are included under *incidental.*

For a non-renewing FRW, the total time period involved may be written as $w + t_w$. where t_w is the residual life of the item in service at time w. Note also that under a FRW the buyer is responsible for maintenance and repair only after w, but is responsible for energy and most incidental costs for the entire period of its use. Based on these factors, the total cost of a single item under a FRW may be expressed as:

$$C_B = C + C_o(w + t_w) + C_M(w, w + t_w) + C_B(w + t_w) \tag{7.3}$$

where $C_o(w + t_w)$ is the total operating cost over the lifetime of the item and $C_M(w, w + t_w)$ is the cost of maintaining the item from time w to time $w + t_w$, including parts, service, shipping, possible overhaul and so forth (costs assumed to be covered by the warranty until time w), and $C_B(w + t_w)$ denotes incidental costs of ownership for the lifetime of the item (Blischke, 1990).

In a simplified form if only minimal repairs are performed on the system by the buyer, then one can easily write the total cost to the buyer for a time period of t_w after a warranty period of w as:

$$\begin{aligned} C_B(w, w + t_w) &= c_r E[N(w, w + t_w)] \\ &= c_r \int_0^{t_w} h(w + t)dt \end{aligned} \qquad (7.4)$$

where c_r is the repair cost during the warranty period and $E[N(w, w + t_w)]$ is the expected number of repairs in period $[w, w + t_w]$

In the recent years, the use of imperfect repair/Preventive Maintenance (PM) to improve the condition of the deteriorating system has become prevalent. Limited research has been done in the area of warranty management considering PM. Chun and Lee (1992) determine optimal replacement time for a system with PM under a modified warranty policy. The modified warranty policy is a mixed type of free and pro-rata warranty policy. The objective is to minimize the cost per unit time to the consumer. Numerical example using the Weibull distribution is presented.

Jack and Dagpunar (1994) consider a system with a monotonically increasing failure rate which is sold to a user with a warranty period of w. Under this warranty policy, manufacturer performs both minimal repairs and PM on the system during the warranty period. Each PM action makes the system younger, thus reducing the expected number of minimal repairs after the PM action. Optimal number of PMs is found by minimizing the expected cost of minimal repairs and PMs over the warranty period. Dagpunar and Jack (1994) further modified the policy under the assumption that the manufacturer has control over the amount of age reduction at each PM where the cost of PM depends on both the age reduction and age of the system. In the above formulations it is assumed that PM can be performed at anytime and there does not exist a prespecified maximum allowed failure rate below which the system must perform. The analysis is performed for a fixed system design with increasing failure rate only and hence the effects of burn-in are not incorporated.

In the next section we analyze the system over the three periods of the bathtub shaped , namely, infant mortality, useful period and increasing failure rate regions of the system. PM is performed only when the system reaches a maximum allowed failure rate, while minimal repairs are performed if the system fails at other times.

7.3 System Characteristics and Problem Formulation

In the earlier research performed in the area of warranty management, the cost models were developed separately from manufacturer's and customer's perspective. The manufacturer's costs includes: production cost, cost of burn-in and warranty, were calculated either for infant mortality and useful period (Nguyen and Murthy, 1982; 1988) or for increasing failure rate period of the system (Chun and Lee, 1992; Chun, 1992; Jack and Dagpunar, 1994; Dagpunar and Jack, 1994). From the consumer's perspective the analysis considered only the post warranty costs of the system, ignoring the effects of burn-in on the system failure characteristics after warranty (Blishcke, 1990). None of these formulations consider the complete bathtub shaped failure rate curve. Analyses are performed only on a part of a bathtub curve with a fixed system design. However, if the analysis is performed considering the whole failure rate bathtub curve, then one can see that length of burn-in and warranty period affects both the system costs during post warranty period and the system design.

The objective of our analysis is to find the optimal system design and burn-in period, which minimizes the total cost incurred on the system by customer and manufacturer over its useful life. The useful life of the system is defined as the time over which the system is used to sustain a process which directly or indirectly generates revenues. It is assumed that the useful life of the system begins when the system starts working and hence does not include the burn-in period for the system.

We consider a series-parallel system comprising of n subsystems in series. Subsystem j ($j = 1, ..., n$) consists of $(1 + m_j)$ identical components in active redundancy. All the components are statistically independent. Each subsystem acts as a 1-out-of-m:G configuration. All the component failure rates follow a bathtub shaped failure rate curve.

In our formulation, we assume that this analysis is performed after the components have been produced and its estimated failure rate is known. At this time we evaluate the system based on its failure characteristics to recommend burn-in or no-burn-in for a given warranty period to minimize its costs over the useful life. Since the design is also a variable in our formulation, the model will evaluate various configurations to provide an optimal design, optimal burn-in recommendation of b years where $b \leq 0$, optimal PM schedule and optimal replacement time for the system. All the costs incurred during the burn-in period are treated as additional costs due to burn-in at the beginning of the useful life. This is a fair assumption as burn-in costs are treated as part of manufacturing costs (Nguyen and Murthy, 1982) and hence can be treated as part of the acquisition cost of the system at the beginning of the useful life of the system.

It is required that over its useful life, the system should operate below a maximum allowed failure rate. In the earlier chapters we proposed a PM policy, where PM is performed when a system reaches the maximum allowed failure rate. If the system fails between these intervals, minimal repairs are performed. In the current formulation we use the same policy after the system is installed. As mentioned earlier, the system comes with a given warranty period. In the formulation we follow a simple non-renewable FRW policy for repairable systems. This means that if a failure occurs during the warranty period then the related costs will be borne by the manufacturer and these costs will be higher than the repair costs during burn-in. This is due to the fact that cost of repairs during warranty period also include various overheads like warranty administration and handling costs.

We divide total system costs into four categories: (a) Manufacturing Cost, (b) Installation and setup Cost, (c) Warranty Cost, and (d) Post Warranty Cost.

Manufacturing Cost

The manufacturing cost for components in subsystem j contains four costs (Nguyen and Murthy, 1982):

C_{0j} is the manufacturing cost per component without burn-in

C_{1j} is the setup cost of burn-in per component

C_{2j} is the cost per unit time of burn-in per component

C_{3j} is the repair cost of the subsystem per failure during burn-in

Let V_j be the expected manufacturing cost for subsystem j with $(1 + m_j)$ components and a burn-in time b. Using equation (7.2) we have:

$$V_j = (1 + m_j)(C_{0j} + C_{1j} + C_{2j}b) + C_{3j} \int_0^b h_j(t)dt \quad (7.5)$$

Installation and Setup Costs

Once the system completes the burn-in, it is installed and setup to be used by the customer. Depending on the type of system, the installation costs (IC) can be dependent or independent of design. In this formulation, we assume that it is a one time cost incurred at the beginning of the system's useful life and is independent of system design.

Warranty Cost

As mentioned earlier, the manufacturer is responsible for all repair or replacement costs during warranty period $[0, w]$. For this policy, the expected cost of repair of subsystem j, W_j, during the period $[b, b + w]$ can be written as:

$$W_j = (C_{3j} + C_{4j}) \int_0^w h_j(b + t)dt + \gamma \quad (7.6)$$

where C_{4j} is the additional cost that arises when a failure occurs during the warranty period (e.g. handling costs, warranty administration cost, etc.) and γ is a one time warranty implementation cost independent of number of failures during warranty.

Post Warranty Cost

Post warranty cost is incurred by the customer and include the cost due to preventive maintenance (PM) and cost of minimal repair after the warranty period.

Preventive maintenance is performed when the system reaches a maximum allowed failure rate. When a PM is performed on the system, all components undergo PM. Thus the cost of a PM action for a subsystem j with $(m_j + 1)$ components will be $MC_j(m_j + 1)$.

PM is modeled using the age reduction concept proposed by Nakagawa (1988). According to this concept the PM action reduces the effective age T_1 to T_1/α, where α is an improvement factor due to PM, such that, $1 \leq \alpha \leq \infty$. Assuming that the failure function form of the system does not change after PM and the improvement factor is the same for all the components in all the subsystems, then for a given system design, a closed form of T_i (for $i \geq 2$) can be expressed as a function of T_1 (See chapter 5):

$$T_i = T_1 \sum_{k=0}^{i-1} \left(\frac{\alpha - 1}{\alpha}\right)^k \tag{7.7}$$

where, $i = 2, 3, 4, \ldots$. Figure (7.2) shows the PM scheduling and system failure rate after each PM action. It is important that the value of α is chosen such that T_1/α will lie in the increasing failure rate region of the failure rate bathtub curve of the system. This is important because PM action on a system is only valid for the increasing failure rate region and in real life any PM action cannot improve the condition of a system to such an extent that it will follow a decreasing or a constant failure rate curve after the PM action.

Minimal repairs are performed if the system fails between the scheduled PM Intervals. The cost associated with minimal repairs depends on the frequency of failures in a system. The frequency of these failures is related to the individual subsystem's failure rate function. According to Boland (1982), the expected minimal repair cost of subsystem j in interval $[0, T]$ is C_{Mj}:

$$C_{Mj} = c_{mj} \int_0^T h_j(t)dt \tag{7.8}$$

where $h_j(t)$ is the failure rate function and c_{mj} is the cost of minimal repair of subsystem j. This equation can be rewritten in terms of reliability of the jth subsystem, $R_j(t)$, as:

$$\begin{aligned} C_{Mj} &= -c_{mj}\int_0^T \frac{R_j'(t)}{R_j(t)}\,dt \\ &= -c_{mj}\ln[R_j(T)] \end{aligned} \tag{7.9}$$

assuming $R_j(0) = 1$.

In our case the system is working for $(b+w)$ before it enters the post warranty period (See Figure 7.2). Hence the expected minimal repair costs of subsystem j during post warranty period is given by:

$$\begin{aligned} C_{Mj1} &= -c_{mj}\int_0^{x_1} \frac{R_j'(b+w+t)}{R_j(b+w+t)}\,dt \\ &= -c_{mj}\ln\left[R_j(b+w+x_1)\right] + c_{mj}\ln\left[R_j(b+w)\right] \end{aligned} \tag{7.10}$$

If $T_1 = b + w + x_1$, then T_1 is the first PM point. At T_1 the PM is performed and the effective age of the system becomes T_1/α instead of T_1. At the second maintenance point we can find the total expected minimal repair costs during the period $(x_1 + x_2)$, C_{mj2}:

$$\begin{aligned} C_{Mj2} &= -c_{mj}(\ln\left[R_j(b+w+x_1)\right] + \ln\left[R_j(b+w)\right] \\ &\quad - \ln[R_j(A_1+x_2)] + \ln[R_j(A_1)]) \end{aligned} \tag{7.11}$$

where A_1 is the effective age of subsystem j after the first PM action and is equal to T_1/α. Simplifying we get:

$$C_{Mj2} = c_{mj}\ln\left[\frac{R_j(b+w)\,R_j(A_1)}{R_j(T_1)\,R_j(A_1+x_2)}\right] \tag{7.12}$$

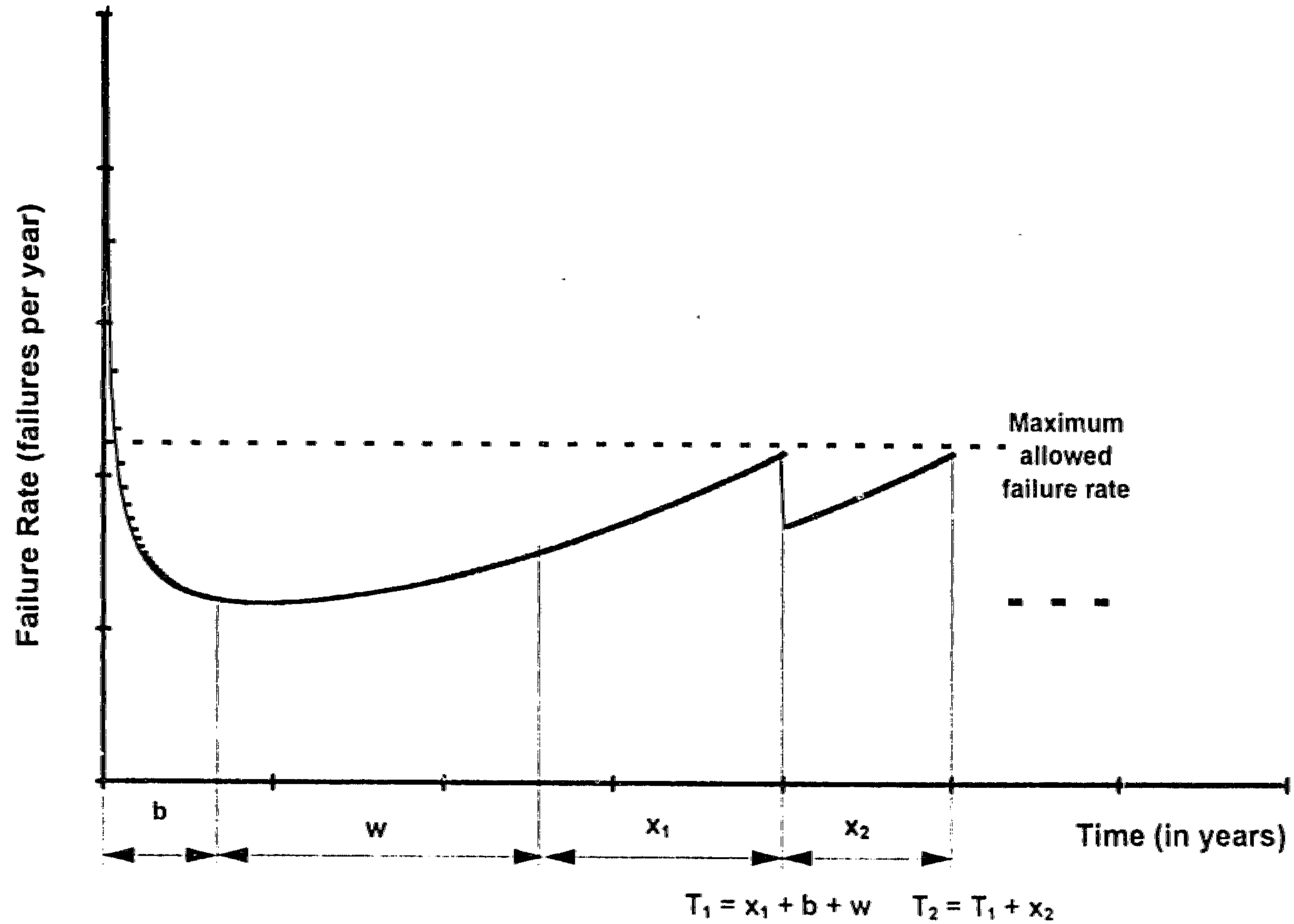


Figure 7.2: Failure rate bathtub curve and PM intervals

If $T_i = T_{i-1} + x_i$, then the expected cost of minimal repairs till T_i for subsystem j is given as:

$$C_{Mji} = c_{mj} \ln \left[\frac{\prod_{l=0}^{i-1} R_j(A_l)}{\prod_{l=0}^{i-1} R_j(A_l + x_{l+1})} \right] \tag{7.13}$$

where $A_l = T_l/\alpha$ and $A_0 = b + w$. Thus post warranty costs for subsystem j after a warranty period of w and i PM intervals is PW_{ji}:

$$PW_{ji} = (i-1)(m_j+1)MC_j + C_{Mji} \tag{7.14}$$

At T_i, the total cost of the system over its useful life is given by equation:

$$AAC_i = \frac{IC + \sum_{j=1}^{n} (V_j + W_j + PW_{ji})}{T_i - b} \tag{7.15}$$

where AAC_i is the average annual cost of the system at the end of ith interval. After the installation, the system should always perform below maximum allowed failure rate, ξ. This means that the following condition should be satisfied:

$$h_{si}(T_i) \leq \xi \tag{7.16}$$

where $h_{si}(T_i)$ is system failure rate in the ith interval. There can be additional resource constraints like weight, volume and capital which can be active or nonactive constraints on the system. These constraints are usually functions of system design. In this formulation, we will assume that these constraints are linear function of system design such that:

$$\sum_{j=1}^{n} g_j(1 + m_j) \leq G \tag{7.17}$$

where, g_j is the resource associated with a component in subsystem j and G is the maximum allowed resource.

As mentioned earlier, PM is performed when the system reaches a maximum allowed failure rate. At this point one of the following actions is selected, (a) either to keep the system and perform PM, or (b) replace the system with an identical system. This decision is made by comparing the average annual cost of the system at the end of each interval and checking whether the system has reached its economic life or not. The economic life of a system is also called the minimum cost life or optimal replacement time. If the values of m_j are known, then using the failure rate constraint (equation (7.16)) and equation (7.7) one can find the values of T_1, T_2, ..., T_i, and T_{i+1}. The corresponding values of AAC_i can be calculated using equation (7.15). We stop when $AAC_{i+1} \geq AAC_i$. The system is replaced at T_i which is the economic life of the system.

To calculate the optimal system design and burn-in period over system's life cycle, we follow the five step methodology proposed in chapter 5. To calculate the optimal values of m_j and b, we minimize the expected average annual costs AAC with respect to failure rate constraint for each value of i to obtain AAC_i^*. Once the optimal system configuration and burn-in is obtained for the first i intervals, we calculate AAC_{i+1} using the optimal configuration and optimal burn-in period obtained for i. If this AAC_{i+1} is less than AAC_i^* then it implies that for the given optimal configuration and burn-in period for intervals 1 to i, the corresponding value of T_i is not system's economic life. The system design and burn-in is then optimized for i+1 intervals. This process is repeated and we stop when AAC_{i+1} is greater than AAC_i^*. The basis behind this methodology is that if the system is replaced after the ith interval, the system cost should be minimum for T_i. The five step methodology for this formulation is as follows:

Step 1 Set $i = 1$

Step 2 Minimize the expected average annual costs AAC with respect to failure rate and resource constraints. The optimization problem is formulated as,

Minimize: AAC_i, subject to:

$h_{sk}(T_k) \leq \xi$, where $k = 1$ to i, and

$g_q(m_j) \leq 0$, where $q = 1$ to z. z is the number of resource constraints.

We then obtain AAC_i^* and $m_j^{(i)}$, $b^{(i)}$, where $m_j^{(i)}$ and $b^{(i)}$ are the optimal values of m_j and b for the first i intervals.

Step 3 Calculate $AAC_{i+1}(m_j^{(i)}, b^{(i)})$, where $AAC_{i+1}(m_j^{(i)}, b^{(i)})$ is the expected average annual cost of the $(i+1)$ intervals for the system configuration $m_j^{(i)}$ and burn-in period $b^{(i)}$.

Step 4 Is $AAC_{i+1}(m_j^{(i)}, b^{(i)})$ greater than AAC_i^*?

If yes, then stop. $m_j^{(i)}$ is the optimal system configuration and $b^{(i)}$ is the optimal burn-in period with T_i as the economic life of the system.

If no, then go to Step 5.

Step 5 Set $i = i + 1$ and go to Step 2.

The optimization problem in step 2 is a non-linear mixed integer programming problem with both linear and non-linear constraints. Genetic Algorithms are used to perform the optimization. In the next section we present the numerical illustration of the formulated problem.

7.4 Numerical Illustration

We consider a system with three subsystems in series. We need to determine the optimal number of parallel components in each subsystem and the optimal burn-in period for a given warranty period. All the components have a bathtub failure rate curve. Mathematical models of the reliability bathtub curves have been covered extensively by Kececioglu (1991). In this formulation we use Kececioglu's model 4 based on a bathtub function developed by Dhillon (1979). The model has five parameters, with the failure rate function being:

$$h(t) = kc\lambda t^{c-1} + (1-k)bt^{b-1}\beta e^{\beta t^b} \tag{7.18}$$

The reliability function for a component in subsystem j can be written as:

$$r_j(t) = e^{-\int_0^T h_j(t)dt} \tag{7.19}$$

For failure rate curves, we used $\beta = \lambda = 1$ and $k = 0.5$. The failure rate curves were developed for different values of b and c. Thus the reliability function used in this formulation for components in subsystem j is given by:

$$r_j(T) = e^{-0.5(T^{c_j} + e^{T^{b_j}} - 1)} \tag{7.20}$$

The various costs (in dollars) associated with system components are presented in Table 7.1. The installation cost of the system is 400 dollars. The one time

Table 7.1: Costs related to system components

j	C_0	C_1	C_2	C_3	C_4	MC	C_m
1	100	1	1	1	10	20	3
2	150	1	1	2	10	20	4
3	200	1	1	2.5	15	20	5

warranty implementation cost, γ, is equal to 10 dollars. The maximum allowed failure rate is 2 failures per year. The maximum number of components allowed in each subsystem is 15. A resource constraint is added with g_1=10, g_2=15, g_3=20, and G=200. The improvement factor (α) for all the components is 1.67.

To solve the optimization problem, Goldberg's classical GA is implemented. For each subsystem j, (m_j) is coded as a 4-bit string. We also use a 4-bit string for length of burn-in period, however we further discretize the burn-in period between minimum burn-in (*bmin*) and maximum burn-in period (*bmax*) using linear transformation. Thus for a user defined *bmin* and *bmax*, the burn-in period is calculated as:

$$\text{Burn-in period (b)} = \text{bmin} + \frac{\text{bmax} - \text{bmin}}{2^L - 1} X_4 \tag{7.21}$$

where L is the length of the string and $X_4 \in [0, 1, 2, ..., 2^L - 1]$

To handle the non-active resource constraints of the form $g_{mj} \leq 0$, we incorporate the constraint violations as penalties (Goldberg, 1989) to the function we want to minimize. Thus the modified objective function F' can be written as:

$$F' = AAC_i + P \tag{7.22}$$

where P = penalty for constraint violation such that

$$P = \begin{cases} 0, & \text{if } g(m_j) \leq 0 \\ |g(m_j)|, & \text{otherwise} \end{cases}$$

For fitness evaluation of each coded string it is assumed that the failure rate constraint (equation (7.16)) is active. For the given values of m_j and b, this constraint equation is solved to give the value of T_i and subsequently AAC_i. Since GAs are setup to maximize fitness hence the optimization model needs to be converted from a minimization to a maximization one. This is done by subtracting each string's AAC and constraint violation from the largest in the current population to give a positive fitness value. In the GA implementation we use linear scaling and stochastic remainder selection without replacement. A constant mutation rate of 0.005 is used. A convergence criteria is set up such that the algorithm will stop when the average fitness in the population is within 0.5% of the maximum fitness. A population size of 80 is used as a population size of higher than 80 leads to excessive computation time without a proportionate increase in the probability of converging to a global optimum.

The study was performed on three systems with the same cost parameters but three distinct failure characteristics (Figure 7.3). The failure rate parameters related to these cases are mentioned in discussion of each separate case. Discussion of the findings are presented next.

Case I

In this case, the initial failure rate is not too high. The slope of the DFR region is steep and the length of the DFR region is very short. This is followed by a small period of constant failure rate after which the system deteriorates very quickly. Such characteristics model a simple and a highly deteriorative system with short product life.

For failure rate parameters of $b_1 = 1.5, b_2 = 1.2, b_3 = 0.7, c_1 = 0.3, c_2 = 0.4, c_3 = 0.4$ for components in subsystems 1, 2 and 3 respectively, it is found that, if the warranty period is less than or equal to 613.2 hours, then the optimal system design is 5, 1 and 1 components in subsystem 1, 2, and 3 with an optimal burn-in period of 175.2 hours. For a warranty period greater than 613.2 hours the optimal design becomes more reliable with an additional component in subsystem 1. The components in subsystem 1, 2 and 3 are 6, 1 and 1 respectively and the optimal burn-in period is 140.16 hours. The optimal system design, optimal burn-in period, PM intervals and system economic life for $w = 0$ and 0.5 year are tabulated in Table 7.2.

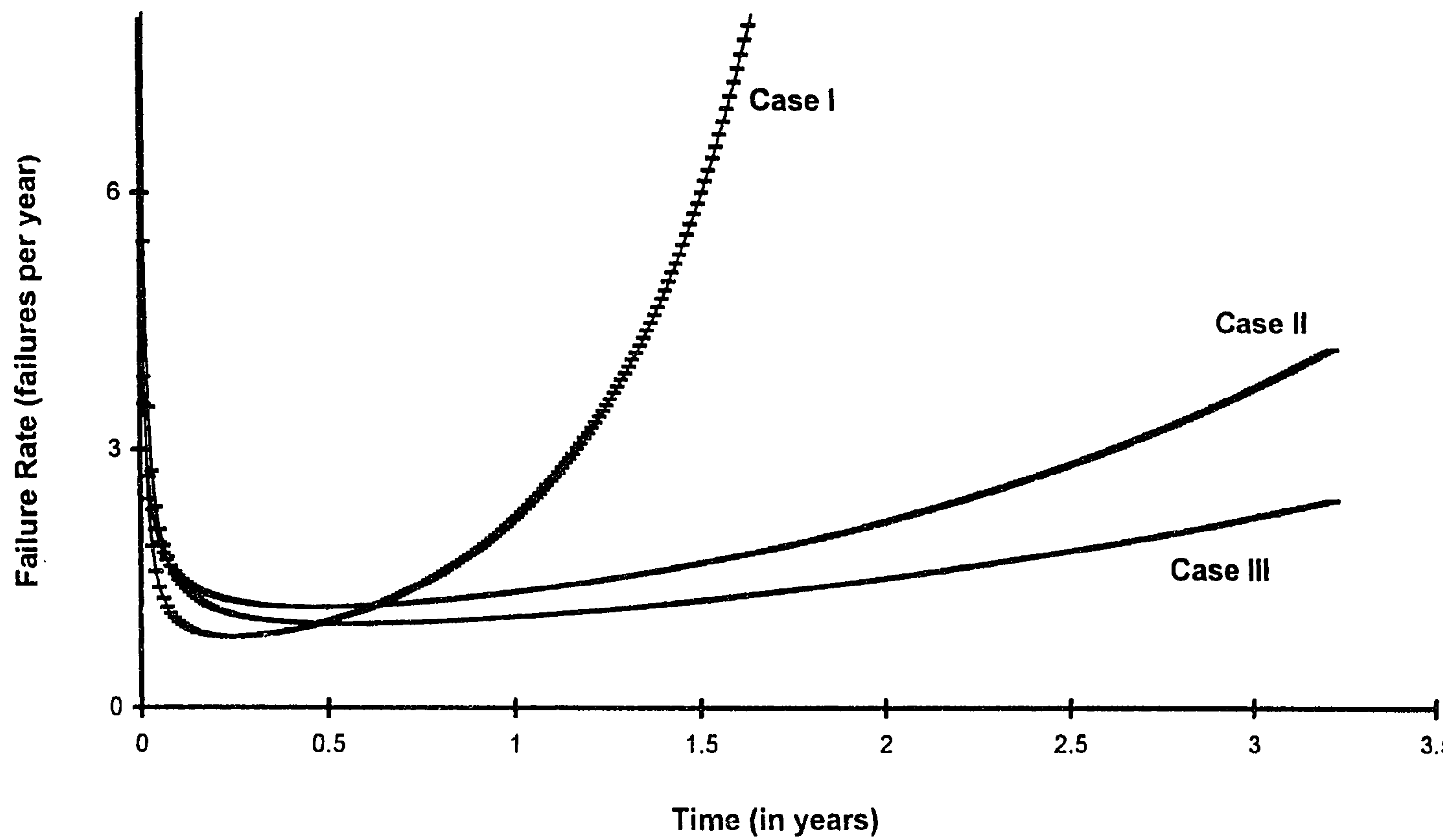


Figure 7.3: Failure characteristics of components in Case I - II - III

Table 7.2: System design,burn-in, PM intervals and economic life for Case I

w (in years)	$m_j + 1$	b (in hours)	1st PM (in years)	2nd PM (in years)	Economic Life (in years)	AAC (in dollars)
0	5, 1, 1	175.2	0.772	1.082	1.205	1303.795
0.5	6, 1, 1	140.16	0.828	1.159	1.269	1348.963

Case II

In this case, the initial failure rate is very high followed by a useful period which has a gradually increasing failure rate and merges with the faster increasing failure rate region. Such characteristics can be related to simple mechanical products which are prone to early failures and gradually deteriorate over time.

Simulating the above conditions using failure rate parameters $b_1 = 0.8, b_2 = 0.8, b_3 = 0.7, c_1 = 0.5, c_2 = 0.05, c_3 = 0.3$, it is found that the system design followed a similar trend as in Case I. For a warranty period less than 3942 hours (approximately 6 months), the optimal system design is recommended as 6,1, and 1 components in subsystem 1, 2, and 3 respectively with an optimal burn-in of 385.44 hours. The system design becomes more reliable with 7,1, and 1 components in subsystems 1, 2 and 3, for a warranty period greater than or equal to 3942 hours and a reduced burn-in of 350.4 hours is recommended. The optimal system design, optimal burn-in period, PM intervals and system economic life for $w = 0$ and 1 year are tabulated in Table 7.3.

Case III

This case represents the system for which the initial failure rate is very high and the system has a distinct DFR region which decreases gradually over longer period than the last two cases. The system's useful life is long and the system deteriorates very

Table 7.3: System design,burn-in, PM intervals and economic life for Case II

w (in years)	$m_j + 1$	b (in hours)	1st PM (in years)	2nd PM (in years)	Economic Life (in years)	AAC (in dollars)
0	6, 1, 1	385.44	1.086	1.521	1.695	1028.068
1	7, 1, 1	350.4	1.159	1.622	1.808	1059.033

gradually. Such characteristics model complex systems which have a high failure rate initially, because of the complexity. However, if given sufficient burn-in period which involves a lot of debugging, the system will perform under the maximum allowed failure rate for a longer time.

The failure rate parameters $b_1 = 0.2, b_2 = 0.4, b_3 = 0.1, c_1 = 0.7, c_2 = 0.7, c_3 = 0.6$ are used. For such systems it is found that although the system design becomes more reliable as the warranty increases, it is recommended to have a consistent burn-in period which is optimal for the given system characteristics. In this case for a warranty period less than 3854.4 hours, the optimal design is 6,1 and 1 components in subsystem 1, 2 and 3 respectively. When the warranty period is greater than 3854.4 hours, the optimal design is 7,1 and 1 components in subsystem 1, 2 and 3 respectively, while for both the designs a burn-in of 578.6 hours is recommended. The optimal system design, optimal burn-in period, PM intervals and system economic life for $w = 0$ and 1 year are tabulated in Table 7.4.

In general for all the three cases, it is found that the average annual cost of the system and the total number of components in the system increase as the warranty period increases. Higher average annual costs are due to the assumed higher repair costs during a warranty period. Thus as the warranty period increases the average

Table 7.4: System design,burn-in, PM intervals and economic life for Case III

w (in years)	$m_j + 1$	b (in hours)	1st PM (in years)	2nd PM (in years)	Economic Life (in years)	AAC (in dollars)
0	6, 1, 1	578.6	1.36	1.904	2.121	824.305
1	7, 1, 1	578.6	1.45	2.037	2.269	825.456

annual costs of the system increases. Also as the warranty period increases the model tries to minimize the number of failures during this period and hence the system goes towards a more reliable design (more number of parallel components). A more reliable design also gives a longer economic life for the system.

For all the three cases the system burn-in period decreases as the design becomes more reliable. This is because a better design will have less number of failures during its infant mortality phase and hence the need for a smaller burn-in period. In our formulation the constant failure rate period of system in Case I is the shortest, while Case III has the longest constant failure rate period. For Case III it is found that the length of burn-in period is not affected by the length of warranty period. It is also found that for a given warranty period, Case III has the lowest average annual cost, while Case I has the highest. This is due to the reason that for a system with longer constant failure rate period, the number of repairs performed during that period are lower and hence the costs are lower. Also longer constant failure rate period implies that system has an overall longer useful life which implies a lower average annual cost.

In all the cases, if only manufacturer's cost is minimized (Nguyen and Murthy, 1982), i.e. $\sum_{j=1}^{4}(V_j + IC + W_j)$ then it is found that optimal system design is 1,1 and 1 for subsystem 1,2, and 3 respectively for all the three cases. For Case I no burn-in is recommended due to low initial failure rate while for Case II and III, the burn-in increases to a maximum and then starts decreasing as the warranty period increases. These results match the conclusions of Nguyen and Murthy (1982), which states that for repairable systems, as the warranty period increases, the optimal burn-in period increases to a maximum value and then decreases.

One of the reason that higher number of system components are recommended in our formulation is that higher number of redundancies decrease the failure rate of the system during the wear-out period of the system. This extends the useful life of the system which in return decreases the average cost per unit time.

7.5 Concluding Remarks

In the previous research related to product warranty management, the systems have been economically evaluated for different warranty periods from either consumer's or manufacturer's perspective. Most of these analyses have been limited to the infant mortality and useful life period of the system. The issues of optimal reliability allocation for systems undergoing minimal repair considering warranty has been considered by Nguyen and Murthy (1988). However, they assume that system failure rate is constant over the warranty period. This research analyzes a system's cost over its useful life to incorporate all the costs incurred on the system during all the phases of system life cycle and gives a more comprehensive mathematical model for optimal system design with optimal burn-in period, optimal PM intervals, and optimal system replacement time.

Chapter 8

Conclusions

In the area of reliability based design (RBD), extensive research has been done considering systems with constant component reliability. This constant component reliability is the probability that the component will survive for a given mission period. Thus the system designs obtained in these formulations are valid only for the given mission period and can be applied only to non-repairable systems. In order to incorporate the effects of repair and maintenance during the system design phase, it is imperative that the system component reliability be modeled as a function of time. However, no research results are available for reliability based designs for such systems.

This research contributes to the area of reliability based design of series-parallel systems with components whose reliability is a function of time. The formulations presented in this research incorporate the concepts of preventive maintenance (PM), minimal repair, salvage value and warranty management to present a comprehensive model which calculates optimal system design, optimal preventive maintenance intervals and optimal system replacement time.

In Chapter 5, we have developed two RBD formulations based on PM models proposed by Nakagawa (1988) for systems with increasing failure rate. If the PM action involves routine checkup and replacement/repair of minor components, then in this case age reduction concept is applicable and formulation 1 is used. Hazard rate model used in formulation 2 models PM which involves actions like lubrication, adjustment of tolerances and minor overhaul of components subject to wear. A five step methodology is presented which provides the designer with optimal system design, optimal PM intervals and optimal replacement time for the system.

In Chapter 6, we proposed a modified PM model which adds more tunability to Nakagawa's hazard rate PM model and allows the user to model real life systems, which have non-zero failure rate at time zero. Based on system failure and maintenance characteristics, a salvage value function for deteriorating systems is proposed. The function incorporates the economic effects of failure rate deterioration and preventive maintenance. Using the modified PM model and salvage value function, we developed a RBD formulation and studied the effects of salvage value on system design for given failure, maintenance and cost characteristics.

Typically a product follows a characteristic bathtub shaped failure rate curve, after it has been produced. The early infant mortality period is represented by high initial failure rate decreases with time. The infant mortality period is later followed by a period of constant failure rate which represents the useful period of the system. The period of system deterioration and wear follows the system's useful period. This period is represented by the increasing failure rate. If a system is sold with a warranty, then to lower the costs incurred on the system during warranty, it is economical to recommend initial burn-in of the product. This reduces the expected cost of warranty but at the expense of increased burn-in costs and possibly shorter useful life of the product. In Chapter 7 we developed a comprehensive model which minimizes the total cost incurred on the system during its life and provides the designer with optimal burn-in period, optimal system design, optimal PM intervals and optimal replacement time for a given warranty period. This formulation is a significant contribution to the area of product warranty management.

In this thesis Genetic Algorithms (GAs) are used as an optimization tool to minimize system costs subject to failure rate and resource constraints. GAs were chosen due to their flexibility and ease to solve a complex nonlinear mixed integer programming problem like the one in our research.

The area of reliability based design is becoming strategically one of the most important areas in system design. In the last decade the use of various maintenance actions to improve system reliability has become increasingly popular. The focus of the system design has shifted accordingly. The designers who used to design systems for low production costs are now focusing on minimizing the system costs over its lifecycle. In order to achieve this objective, it is imperative to consider the effects of system design on the maintenance and operating costs of the system. The results presented in this research are one step towards the comprehensive analysis of reliability based design of the system which incorporate the effects of maintenance, repair, salvage value and warranty. These results can be applied to design high cost computer aided manufacturing systems which must perform below a maximum allowed failure rate.

Along the line of this thesis, further studies can be done in the following areas:

- **Improvement and Deterioration Factors**

  Our formulations are very sensitive to both improvement and deterioration factors used while modeling PM using age reduction hazard rate concept. These factors represent level and extent of system degradation after PM. Malik (1979) recommends expert judgment on determining the improvement factor. Lie and Chun (1986) propose using a set of cost curves to select the improvement factors. Empirical studies need to be done to determine what major factors affect the improvement factor of the system. The study should also be performed to recommend values of these factors for different types of PM actions on different types of equipment/systems.

- **Inspection of Deteriorating Systems**

  Our formulations are applicable only to systems which are continuously monitored. The costs of continuous monitoring of a system can be quite high. Such costs are justified for highly automated systems with high production rate or where failure of the system can result in loss of human lives. However, when designing systems which have a lower production rate, the cost of monitoring can be lowered considerably if the system is inspected regularly instead of being monitored continuously. In such models the failures can only be detected by an inspection and the objective is that the system should operate reliably between these inspection intervals. Much research has been done in the area of inspection of manufacturing systems. In a recent work by Mohandas, Chaudhuri and Rao (1992), the authors provide an optimal periodic replacement strategy for a deteriorating production system considering inspection and minimal repair. These inspection schedules can be combined into a reliability based design formulation where the users can define the maintenance strategy and select the option of continuous monitoring or inspection. This selection will be driven by the costs of each option and system requirements.

- **Warranty Policies**

  The reliability based formulations can be further extended to various type of warranty policies like pro-rata and combined warranty policies. The models can also be modified to incorporate the expected sales function of the system which will further dictate the system design according to the marketing needs.

In summary, this research introduces PM and minimal repair into reliability based design. It makes contributions to reliability based design over system life cycle and opens a fertile research area for optimal system designs.

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