Joint User and Antenna Selection for Multiple-antenna Wireless Relay Networks

Capstone Project Report

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Abstract

Multiple-antenna technology and relaying techniques are integral components of next generation wireless standards such as Long-Term Evolution-Advanced (LTE-A) and Worldwide Interoperability for Microwave Access (WiMAX). In this project, joint antenna and source selection strategies are designed and analyzed for multiple-antenna amplify-and-forward relay networks. To this end, the optimal transmit antenna pair at the best source and the optimal transmit-receive antenna pair at the relay are selected to minimize the overall outage probability of the multi-user multi-antenna relay networks. Under the aforementioned strategy, the end-to-end transmission scheme is first formulated and thereby the underlying transmission schemes are analyzed to obtain system specific performance metrics. To this end, the important performance metrics including the outage probability and average bit error rate are derived in closed-form. To obtain useful insights into practical system designs, numerical results pertinent to various system configurations are presented by using Matlab simulations. Here, the analytical lower bounds for the outage probability and average bit error rates are plotted by using the closed-form derivations while the corresponding exact performance metrics are plotted by using Monte-Carlo simulations. Valuable insights and intuitions, which are useful in designing practical multi-antenna relay networks, are then drawn by using these simulations results.

The potential impact of this project is as follows: Multiple-antenna terminals require multiple radio frequency (RF) chains to enable multiple data transmissions/receptions and thus resulting increased system complexity, and deployment cost. The joint transmit antenna and receive antenna selection at the relay terminal require only one RF chain and hence significantly reduces the network deployment cost. Moreover, the optimal joint user and antenna selection indeed improves the overall quality-of-service of the whole network. Thus, the proposed project outcomes would contribute to the evolution of wireless multi-antenna relay networks.

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Chapter 1

Introduction

1.1 Introduction and Motivation

Wireless communication technologies have become indispensable today. Various services provided by cellular networks, broadband networks, and personal area networks are been extensively utilized in accomplishing day-to-day activities. The demand for high speed and ubiquitous wireless access is increasing exponentially with the recent proliferation of data-centric smart phones, tablets, and portable computing devices. For example, as per CISCO, the global mobile data traffic has grown 70 percent in 2012. In fact, this data traffic demand has reached 885 petabytes per month at the end of 2012, which is an increase of 520 petabytes per month over that of at the end of 2011 [1]. CISCO has also predicted that by the end of 2013, the number of mobile-connected devices will exceed the number of people on earth, and there will be nearly 1.4 mobile devices per capita by 2017 [1]. Thus, it is evident that the future demand for wireless data rates, coverage, and link-reliability will be unprecedented. To meet this future demand, new wireless communication standards such as Long-Term Evolution-Advanced (3GPP LTE-A) [2], Worldwide Interoperability for Microwave Access (WiMAX - IEEE 802.16e) [3], and Wireless Fidelity (WiFi - IEEE 802.11n) [4], are being studied. Multiple-antenna and wireless relay technologies are integral building-blocks of the fourth generation (4G) wireless standards, including LTE-A and mobile-WiMAX [3]. In

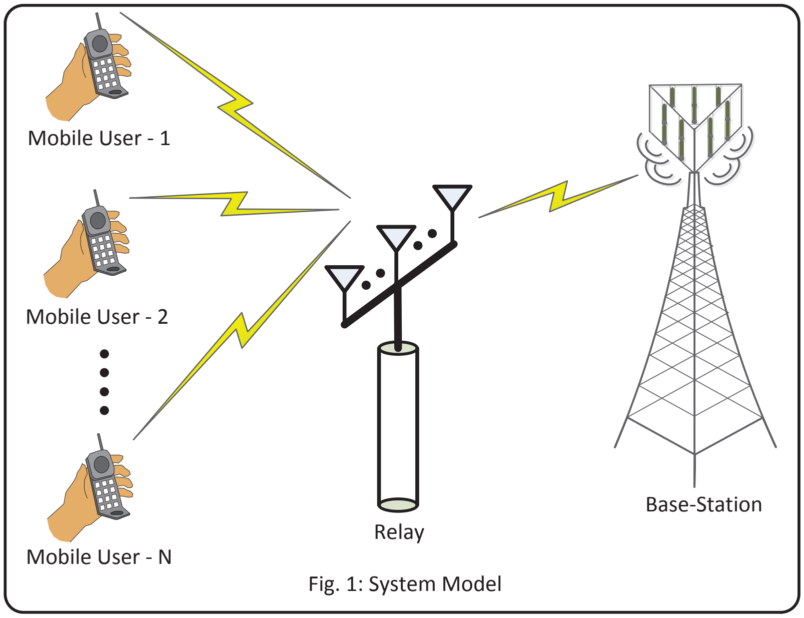

Figure 1.1: A practical system setup for MIMO relay networks

particular, multiple-antenna terminals can transmit and receive multiple data streams by exploiting the spatial dimension of the wireless channel to improve link-reliability and/or data rates [1]. Wireless relay terminals are far less expensive than traditional base-stations and can be deployed to extend coverage, improve power efficiency, and enhance link-reliability of traditional cellular and broadband wireless networks. To be more specific, wireless relays can be employed as a repeater in between mobile-users and base-station whenever the quality of the direct point-to-point communication is unusably low (see Fig.1.1). The main objective of this proposed project is to develop and analyze joint mobile-user and antenna selection strategies for multiple-antenna relay networks.

1.2 Prior Related Research

The prior related studies which analyze the impact of the joint user and antenna selection for multiple-antenna wireless relay networks are as follows: In [5], the impact of antenna correlation on the performance of dual-hop amplify-and-forward multiple-input multiple-output (MIMO) antenna relaying has been analyzed. Reference [6], studies the beam forming in dual-hop AF relay networks with the multiple antennas at the source and destination, but the relay is single-antenna only. Maximal ratio transmission (MRT) and maximal ratio combination (MRC) are used at the source and destination, respectively. The analysis of [6] is extended in [7] to study the effects of antenna correlation at the source and destination in dual hop systems. In [8], the performance of the dual-hop AF MIMO relaying with multiple-antenna source-destination (S-D) pair and a single antenna relay is analyzed. This analysis considers transmit antenna selection at the source and MRC at the destination in independent Rayleigh fading channels. In [9], the performance of the single-relay cooperative system where the source, relay and the destination terminals are equipped with multiple transmit/receive antenna using space-time block code is analyzed considering both AF and DF relays. Reference [10], provides the effects of feedback delays on the performance of multiple-input multiple-output antenna amplify-and-forward relay networks with the best transmit/receive antenna pair selection over Rayleigh fading. In [11], the outage probability and diversity order are analyzed for the decode and forward relay selection. In [12], the effect of feedback delays on the performance of the MRT in the down-link of the dual-hop AF relay networks over the Rayleigh fading channels is investigated. In [13], an analytical framework for the performance analysis of the three transmit antenna selection (TAS) strategies for dual hop MIMO channel-assisted amplify-and-forward (CA-AF) relay networks over Rayleigh fading is developed. In [14], the optimal SNR based TAS strategy (TAS_{opt}) is proposed for dual-hop MIMO AF cooperative relay networks. It is shown that TAS_{opt} achieves the full diversity order available in the MIMO relay channel but the search complexity of the TAS_{opt} strategy is relatively high. As a remedy, reference [15], proposes two suboptimal yet low-complexity TAS strategies ($TAS_{sub-opt_1}$ and $TAS_{sub-opt_2}$) for the same network. Although the performance of these two TAS strategies is presented by using Monte-Carlo simulations, without any analytical expressions. The other studies which take into account TAS for dual-hop MIMO relaying are [16] and [17]. In [16], three TAS strategies, which are optimal in terms of the outage probability, are developed for MIMO DF relaying. In [17], the performance of dual-hop AF relay network with a MIMO-enabled source-destination pair and a single-antenna relay is analyzed.

1.3 Our Problem Statement

The system model of interest consists of L multi-antenna mobile-users, a multiple-antenna relay terminal, and a multiple-antenna base-station. The wireless channel fading is modeled as Rayleigh fading, and the additive white noise follows the Gaussian distribution. Under this system and channel model, the optimal joint user and antenna selection algorithm will be developed by jointly selecting the best mobile-user and the best transmit/receive antenna pair of the relay terminal to maximize the end-to-end signal-to-noise ratio (SNR) and thereby to minimize the overall outage probability of the whole system. In particular, the SNR is the basic performance metric of wireless systems and is typically used to quantify the quality of the wireless communications links. Thus, by maximizing the end-to-end SNR, the overall quality-of-service of the relay network can be improved by minimizing the outage probability and average bit error rate. In order to quantify the system performance analytically, tight lower bounds for the outage probability and the average bit error rate are derived in closed form by using tools from the communication theory and probability and stochastic processes.

1.4 Objectives

The objectives of this project is to develop and design joint transmit-receive antenna pair selection and user selection for multi-user multi-antenna amplify-and-forward relay networks. The specific goals of this project can be next enumerated as follows:

1. To design user selection strategies for multi-user relay networks.

2. To design antenna selection strategies for multi-antenna relay terminals.

3. To develop joint user and antenna selection strategies for multi-user/multi-antenna relay networks.

4. To quantify/simulate the important performance metrics of wireless networks such as the outage probability and average bit error rate.

5. To compare and contrast the results with existing such techniques.

1.5 System Model

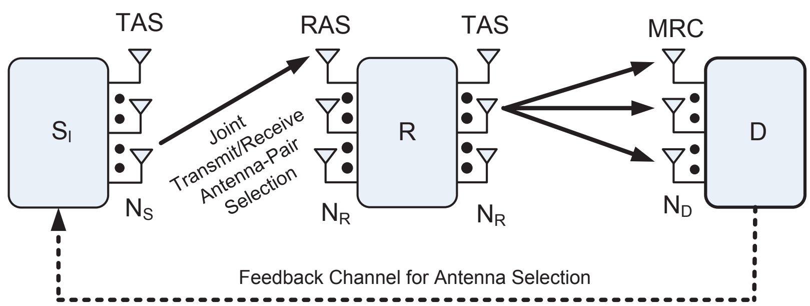

Figure 1.2: System model with single source.

In this section, the system model employed in this project is presented in detail. Specifically, the wireless channel models, receiver noise models and the corresponding signaling models are described. This specific system model will be utilized in subsequent sections of this report, specially for the analyzing the performance metrics such as the outage probability and average bit error rate in Chapter 2, and presenting the numerical results for obtaining insights onto practical MIMO relay designs in Chapter 3.

We first consider the dual-hop multiple-input multiple-output (MIMO) cooperative relay network depicted in Fig. 1.2. The source (S), relay (R) and destination (D) are MIMO-enabled and are equipped with N_s, N_r, and N_d antennas respectively. All terminals operates in the half duplex mode, and cooperation takes place in two time-slots. Perfect channel state information (CSI) is assumed at R and D. Further, the feedback channels for the transmit

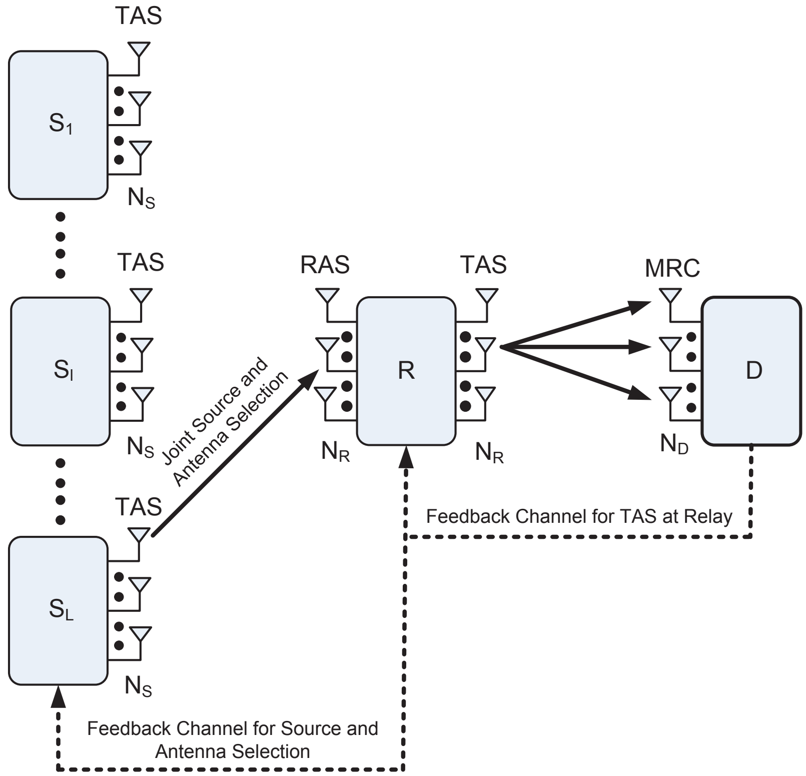

Figure 1.3: System model with multiple source.

antenna selection (TAS) at S and R are assumed to be perfect. The channel matrix from the terminal X to terminal Y, where $X \ \epsilon$ {S,R}, Y ϵ {R,D} and X≠Y, is denoted by H_{XY}. The elements of H_{XY} are independent and identically distributed as $h_{XY}^{i,j} \sim \mathrm{CN}(0,1)$. The channel vector from the j-th transmit antenna at X to Y is denoted by h_{XY}^{j}. Zero mean additive white Gaussian noise is assumed at each receiver. The transmission scheme designed for this system model in this project is as follows: In the first time-slot, the optimal transmit antenna at S and the optimal receive antenna at R are jointly selected to transmit the source signals to the relay. Then, in the second time-slot, the best transmit antenna at the relay is selected for transmitting the amplified signal by R toward D. The destination finally combines the signals received by R by using maximal ratio combining (MRC).

The aforementioned system model is then extended to treat multi-user MIMO relay networks as shown in Fig. 1.3. Here, L multi-antenna sources (S_l for $l \in \{1, \cdots L\}$) communicate with the multiple-antenna destination (D) via an intermediate multi-antenna relay R. The underlying transmission scheme for the multi-user multi-antenna relay network is as follows: In the first time slot, the optimal source, its optimal transit antenna and the optimal receive antenna at R are selected for source-to-relay transmission. In the second time-slot, the optimal transmit antenna at the relay is selected again to forward the amplified signal toward D, where the received signals are combined by using MRC.

In some practical cases, the relay-to-destination CSI would not be available at the destination due to practical and cost constraints. In such cases, an arbitrary transmit antenna at the relay is selected for the second time-slot in order to forward the amplified signal at R toward D. Such cases are also studied in this project.

Chapter 2

Signal Modeling and Performance Analysis

In this chapter, the system, channel and signal models utilized in this project are elaborated in detail. To begin with, the basic point-to-point wireless transmission system set-up and the basic dual-hop amplify-and-forward (AF) relay networks are studied by defining the pertinent channel and signal models. Then the joint transmit-receive antenna pair selection for signal-user MIMO relay networks are designed and the pertaining signal, channel and signal models are described. Finally, the joint user and antenna pair selection strategies for multi-user MIMO relay networks are developed and analyzed.

2.1 Simple point-to-point transmitter-receiver model

A simple point-to-point wireless transmitter-receiver model is shown in Fig. 2.1. The received signal at the destination is given by

$$Y_{RX} = hx + n, \tag{2.1}$$

where h is the channel fading coefficient between the transmitter and the receiver. Further, x is the transmitted signal by the source and n is the additive white Gaussian noise at the

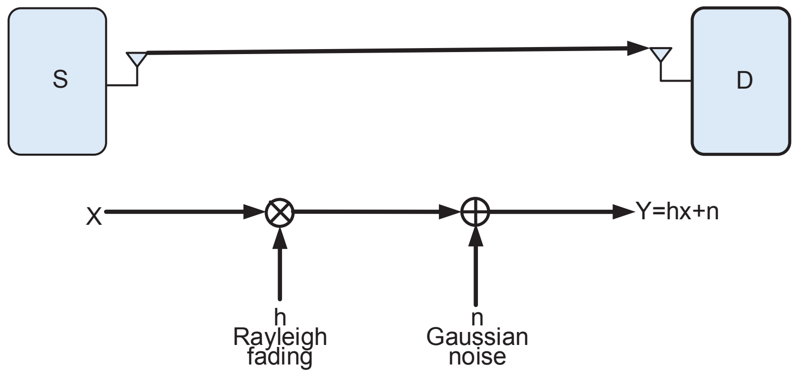

Figure 2.1: Simple transmitter receiver model.

destination receiver. The end to end SNR of a such transmitter-receiver system is defined as

$$\text{SNR} = \frac{\text{signal power}}{\text{noise power}}. \tag{2.2}$$

Received signal power at destination is denoted as

$$\text{Y}_{RX} \quad = \quad \frac{|h|^2 \mathcal{E}\{|x|^2\}}{\sigma_n^2}, \tag{2.3}$$

The channel fading coefficient between the transmitter and the receiver is modeled as $h \sim \mathcal{CN}(0, 1)$. Moreover, σ_n^2 is the variance of the additive Gaussian noise at the receiver and is modeled as $n \sim \mathcal{CN}(0, \sigma_n^2)$. Further, x is the transmitted signal by the transmitter and its average power is defined by $\mathcal{E}\{|x|^2\}$, where $\mathcal{E}\{\cdot\}$ denotes the expected value.

2.2 Simple dual-hop amplify-and-forward relay network

Fig. 2.2 shows the transmitter and receiver model with an intermediate relay. Here, the transmitted signal by the source is received by both the intermediate relay and the destination simultaneously due to the broadcast nature of the wireless channel. Since the relay has a copy of the source signal, it can also forward a pre-processed copy of this signal back to the destination in order to improve the overall quality of the end-to-end signal reception.

Let us next discuss the underlying signaling model pertinent to dual hop relaying. The

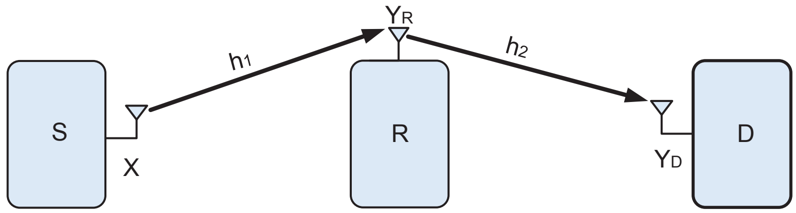

Figure 2.2: Transmitter receiver model with relay.

received signal at relay is given by

$$\mathrm{Y}_R = \sqrt{P_s} h_1 x + n_R. \tag{2.4}$$

where Y_R is received signal at relay, h_1 is a channel gain between transmitter and relay, x is transmitted signal, and n_R is additive white Gaussian noise at relay. Here, P_s is the transmit power at the source.

The relay amplifies the received signal Y_R with gain G and therefore the transmitted signal from relay toward the destination becomes GY_R. The channel gain between relay and destination is next denoted by h_2 and receiver noise at destination is denoted by n_D. Thus, the received signal at destination via the relay from the source is denoted as

$$\begin{aligned} \mathrm{Y}_D &= GY_R h_2 + n_D \\ &= G[\sqrt{P_s} h_1 x + n_R] h_2 + n_D \\ &= \sqrt{P_s} G h_1 h_2 x + G h_2 n_R + n_D. \end{aligned} \tag{2.5}$$

By using the definitions of (2.2) and (2.3), the end-to-end SNR at the destination can derived as follows:

$$\gamma_D = \frac{G^2 P_s |h_1|^2 |h_2|^2 \mathcal{E}\{|x|^2\}}{G^2 |h_2|^2 \sigma_R^2 + \sigma_D^2}. \tag{2.6}$$

By assuming the amplification gain of the relay is $G = \sqrt{\frac{P_r}{|h_1|^2 + \sigma_R^2}}$ [7], the end-to-end SNR at the destination becomes

$$\gamma_D = \frac{\frac{P_s |h_1|^2}{\sigma_R^2} \frac{P_r |h_2|^2}{\sigma_D^2}}{\frac{P_s |h_1|^2}{\sigma_R^2} + \frac{P_r |h_2|^2}{\sigma_D^2} + 1},$$

where P_r is the transmit power at the relay. In deriving (2.7), it is assumed that the signal power is normalized to one, i.e., $\mathcal{E}\{|x|^2\} = 1$. The average SNRs at the source-to-relay channel and relay-to-destination channel are defined as $\bar{\gamma}_1 = \frac{P_s}{\sigma_n^2}$ and $\bar{\gamma}_2 = \frac{P_r}{\sigma_D^2}$. Thus, the end-to-end SNR at the destination cab be derived as follows:

$$\begin{aligned}\gamma_D &= \frac{(\bar{\gamma}_1|h_1|^2)(\bar{\gamma}_2|h_2|^2)}{(\bar{\gamma}_1|h_1|^2) + (\bar{\gamma}_2|h_2|^2) + 1} \\ &= \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1},\end{aligned} \tag{2.7}$$

where γ_1 and γ_2 are the instantaneous SNRs at the first hop and the second hop, respectively.

2.3 Channel and signaling model for multi-user MIMO AF relay network with antenna selection

The system model for the single-user and multi-user MIMO AF relay network are depicted in Fig. 1.2 and Fig. 1.3, respectively. Here, let us now define the underlying channel and signal model for these two particular system set-ups.

2.3.1 Statistical channel modeling at the first hop under Rayleigh fading

At the first hop, there are two possibilities that only one source is transmitting at a time or can be multiple sources transmitting at the same time.

First hop with single source

From Fig. 2.3, the total number of channels between source and relay is $N_s \times N_r$ channels. Let us consider an arbitrary channel for the first hop (source-to-relay). The SNR of this arbitrary channel is denoted by $\gamma_{i,j}$, where $i \in \{1 \cdots N_s\}$ and $j \in \{1 \cdots N_r\}$. Then $\gamma_{i,j}$ is derived as follows:

$$\gamma_{i,j} = \frac{P_s|h_{i,j}|^2}{\sigma_R^2}, \tag{2.8}$$

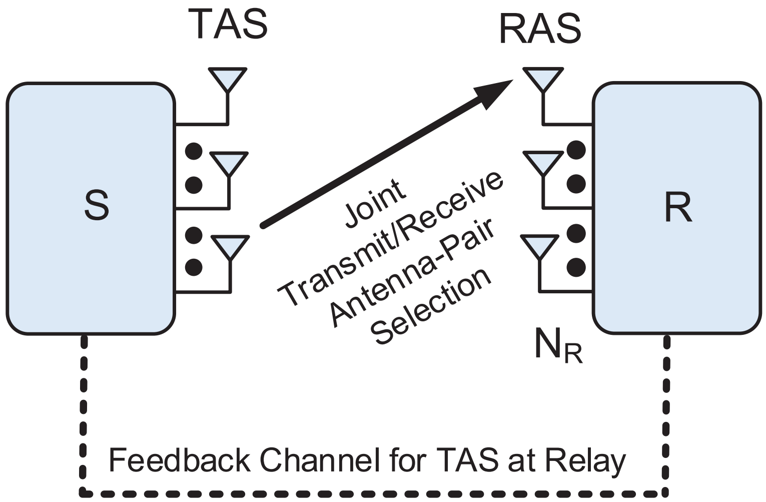

Figure 2.3: First hop with single source.

where $h_{i,j} \sim \mathcal{CN}(0,1)$.

Let us define the average SNR ($\gamma_{i,j}$) of the arbitrary channel of the first hop as $\bar{\gamma}_1$ and the instantaneous SNR is thus given by

$$\gamma_{i,j} = \bar{\gamma}_1 |h_{i,j}|^2, \quad \text{where} \quad \bar{\gamma}_1 = \frac{P_s}{\sigma_R^2}. \tag{2.9}$$

The probability density function (PDF) and the cumulative distribution function (CDF) of $\gamma_{i,j}$ are then given by

$$f_{\gamma_{i,j}}(x) = \frac{1}{\bar{\gamma}_1} e^{-\frac{x}{\bar{\gamma}_1}} \quad \text{and} \tag{2.10}$$

$$F_{\gamma_{i,j}}(x) = 1 - e^{-\frac{x}{\bar{\gamma}_1}}, \quad \text{respectively.} \tag{2.11}$$

For the optimal transmit-receive antenna pair selection at the first hop, the equivalent SNR is formulated to be the maximum from the all the possible SNR values. Thus, the equivalent SNR of the first hop is given by

$$\begin{aligned} \gamma_{eq_1} &= \max(\gamma_{1,1}, \gamma_{1,2}, \ldots \gamma_{i,j}, \ldots \gamma_{N_s,N_r}) \\ &= \max_{i \in \{1,\cdots,N_s\},\, j \in \{1,\cdots,N_r\}} (\gamma_{i,j}). \end{aligned} \tag{2.12}$$

Now, the CDF of γ_{eq_1} need to be derived. To this end, we utilize the theorem 1 from Appendix. The CDF of the equivalent SNR at the first of is thus derived as follows:

$$F_{\gamma_{eq_1}}(x) = \prod_{i=1}^{N_s}\prod_{j=1}^{N_r} F_{\gamma_{i,j}}(x). \tag{2.13}$$

By substituting (2.11) into (2.13), the CDF of γ_{eq_1} can be expanded as

$$\begin{aligned} F_{\gamma_{eq_1}}(x) &= \prod_{i=1}^{N_s}\prod_{j=1}^{N_r}[1 - e^{-\frac{x}{\bar{\gamma}_1}}] \\ &= \left[1 - e^{-\frac{x}{\bar{\gamma}_1}}\right]^{N_s N_r}. \end{aligned} \tag{2.14}$$

Next, by using Binomial theorem, (2.14) can be further expanded as follows:

$$F_{\gamma_{eq_1}}(x) = 1 + \sum_{i=1}^{N_s N_r} \binom{N_s N_r}{1} (-1)^i e^{-\frac{x_i}{\bar{\gamma}_1}}.$$

Now, the PDF of γ_{eq_1} is derived by differentiating the corresponding CDF as follows:

$$\begin{aligned} f_{\gamma_{eq_1}}(x) &= \frac{d}{dx}\left[1 - e^{-\frac{x}{\bar{\gamma}_1}}\right]^{N_s N_r} \\ &= N_s N_r (1 - e^{-\frac{x}{\bar{\gamma}_1}})^{N_s N_r - 1} \frac{d}{dx}(1 - e^{-\frac{x}{\bar{\gamma}_1}}) \\ &= N_s N_r (1 - e^{-\frac{x}{\bar{\gamma}_1}})^{N_s N_r - 1} \left(\frac{e^{-\frac{x}{\bar{\gamma}_1}}}{\bar{\gamma}_1}\right). \end{aligned} \tag{2.15}$$

First hop with multiple sources

Here, we have L sources having N_s antennas in each and the relay has total N_r antennas. Thus, the total number of channels between source and relay is $N_s \times L \times N_r$. This is illustrated in Fig. 2.4. Let us consider an arbitrary channel for the first hop (source-to-relay). We denote the SNR of this channel by $\gamma_{i,j,k}$, where, $i \in \{1, \cdots, N_s\}$, $j \in \{1, \cdots, N_r\}$ and $l \in \{1, \cdots, L\}$. Thus, $\gamma_{i,j,k}$ is written as follows:

$$\gamma_{i,j,l} = \frac{P_s |h_{i,j,l}|^2}{\sigma_R^2}, \tag{2.16}$$

where $h_{i,j,l} \sim \mathcal{CN}(0,1)$.

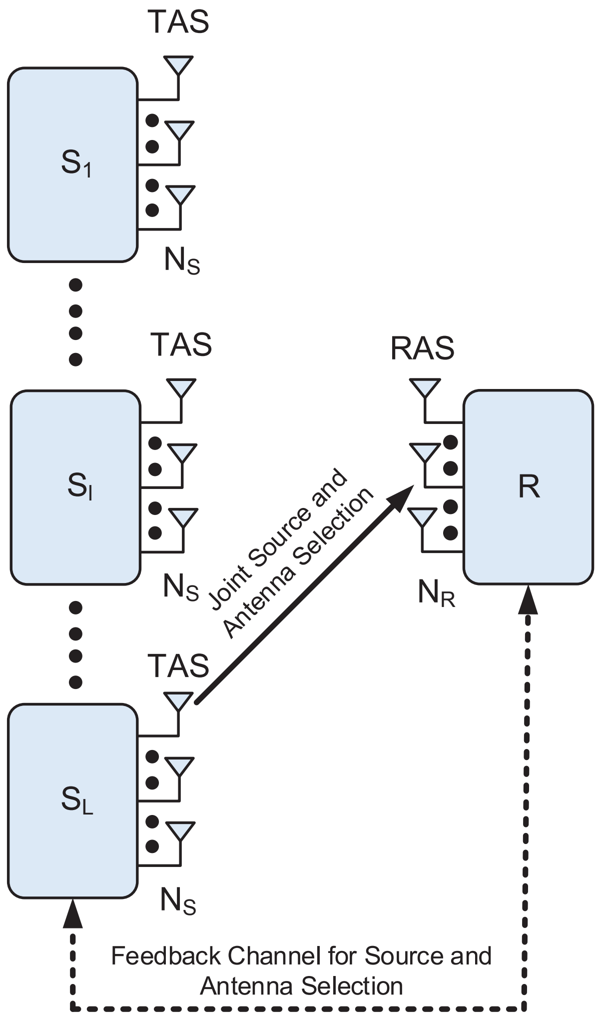

Figure 2.4: First hop with multiple sources.

For the first hop with multiple sources, the equivalent SNR is then given by

$$\gamma_{eq_1} = \max_{\substack{i\in\{1,\cdots,N_s\}\\ j\in\{1\cdots N_r\},\, l\in\{1\cdots L\}}} (\gamma_{i,j,l}). \tag{2.17}$$

Now, the CDF of γ_{eq_1} for multi-user selection is given by

$$\begin{aligned} F_{\gamma_{eq_1}}(x) &= \prod_{i=1}^{N_s}\prod_{j=1}^{N_r}\prod_{l=1}^{L}[1-e^{-\frac{x}{\bar{\gamma}_1}}] \\ &= \left[1-e^{-\frac{x}{\bar{\gamma}_1}}\right]^{N_sN_rL}. \end{aligned} \tag{2.18}$$

Using Binomial theorem will simplify the Eq. 2.18.

$$F_{\gamma_{eq_1}}(x) = 1 + \sum_{i=1}^{N_sN_rL}\binom{N_sN_rL}{i}(-1)^i e^{-\frac{x_i}{\bar{\gamma}_1}}.$$

Again, the PDF of γ_{eq_1} with multiple sources is given by

$$\begin{aligned} f_{\gamma_{eq_1}}(x) &= \frac{d}{dx}[1-e^{-\frac{x}{\bar{\gamma}_1}}]^{N_sN_rL} \\ &= N_sN_r(1-e^{-\frac{x}{\bar{\gamma}_1}})^{N_sN_rL-1}\frac{d}{dx}(1-e^{-\frac{x}{\bar{\gamma}_1}}) \\ &= N_sN_r(1-e^{-\frac{x}{\bar{\gamma}_1}})^{N_sN_rL-1}\left(\frac{e^{-\frac{x}{\bar{\gamma}_1}}}{\bar{\gamma}_1}\right). \end{aligned} \tag{2.19}$$

2.3.2 Statistical channel modeling at the second hop under Rayleigh fading

Arbitrary transmit antenna at the relay

Fig. 2.5 shows that relay has N_r antennas but only arbitrary antenna transmits the signal. On the other hand, the destination have N_d antennas and it does maximal ratio combination (MRC) of every received signal. Thus, the received signal at destination is given by

$$Y_D = x_1x_1^* + x_2x_2^* + ... + x_{N_d}x_{N_d}^*.$$

The SNR of the received signal at the destination (Y_D) is given by

$$\gamma_D = \frac{\sum_{n=1}^{N_d}|h_{n,i}|^2}{\sigma_D^2}, \tag{2.20}$$

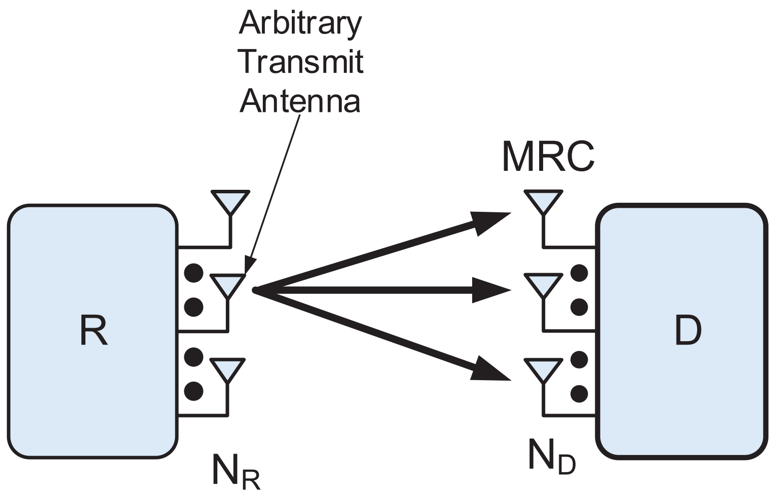

Figure 2.5: Second hop with arbitrary antenna at relay and MRC at destination.

where, i is index of the arbitrary transmit antenna at the relay.

The PDF of γ_D is given by

$$f_{\gamma_D}(x) = \frac{x^{N_d-1} e^{-\frac{x}{\bar{\gamma}_2}}}{(N_d-1)!(\bar{\gamma}_2)^{N}_{d}}. \tag{2.21}$$

Then the corresponding CDF of γ_D is written as follows:

$$F_{\gamma_D}(x) = 1 - e^{\frac{-x}{\bar{\gamma}_2}} \sum_{p=0}^{N_d-1} \frac{\left(\frac{x}{\bar{\gamma}_2}\right)^p}{p!}. \tag{2.22}$$

Selection of the optimal transmit antenna at the relay with feedback

Fig. 2.6 shows the relay-to-destination channel with the MRC at the destination. Additionally, the destination helps the relay in the selection of the optimal transmit antenna by virtue of channel state information via the feedback channel. In this case, the relay transmits from its best transmit antenna, which is selected by the destination by using a step-by-step MRC from all the relay antennas. Then, the destination receives the signal transmitted by the best transmit antenna at the relay and finally it combines the signals by using MRC. Therefore, the effective SNR of the second hop under transmit antenna selection at the relay

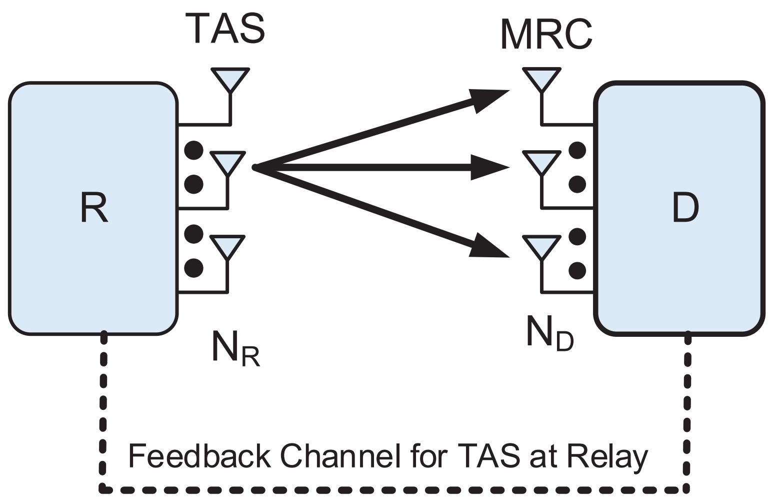

Figure 2.6: Second hop with feedback for selection of arbitrary antenna and MRC at destination.

is given by

$$\begin{aligned}\gamma_{eq_2} &= \max(\gamma_{D_1}, \gamma_{D_2}, \cdots, \gamma_{D_{N_r}}) \\ &= \max_{n\in\{1,\cdots,N_r\}}(\gamma_{D_n}).\end{aligned} \tag{2.23}$$

Then, the PDF of γ_{D_n} is written as

$$f_{\gamma_{\gamma_{D_n}}}(x) = \frac{x^{N_d-1}e^{-\frac{x}{\bar{\gamma}_2}}}{(N_d-1)!(\bar{\gamma}_2)^{N_d}}. \tag{2.24}$$

CDF of γ_{D_n} is

$$F_{\gamma_{\gamma_{D_n}}}(x) = 1 - e^{\frac{-x}{\bar{\gamma}_2}}\sum_{p=0}^{N_d-1}\frac{\left(\frac{x}{\bar{\gamma}_2}\right)^p}{p!}. \tag{2.25}$$

Now the CDF of γ_{eq_2} is derived by using the theorem in Appendix as follows:

$$\begin{aligned}F_{\gamma_{eq_2}}(x) &= \left[F_{\gamma_{D,n}}(x)\right]^{N_r} \\ &= \left[1 - e^{-\frac{x}{\bar{\gamma}_2}}\sum_{p=0}^{N_d-1}\frac{\left(\frac{x}{\bar{\gamma}_2}\right)^p}{p!}\right]^{N_r}.\end{aligned} \tag{2.26}$$

The CDF of $F_{\gamma_{eq_2}}(x)$ in (2.26) can be expanded as follows:

$$F_{\gamma_{eq_2}}(x) = \sum_{c=0}^{N_r} \sum_{d=0}^{c(N_d-1)} \frac{(-1)^c \binom{N_r}{c} \phi_{d,N_r,N_d}}{\bar{\gamma}_2^d} x^d \exp\left(-\frac{cx}{\bar{\gamma}_2}\right), \tag{2.27}$$

where $\phi_{k,N,L}$ is the coefficient of the expansion of $\left[\sum_{u=0}^{L-1} \frac{1}{u!}\left(\frac{x}{\bar{\gamma}}\right)^u\right]^N = \sum_{k=0}^{N(L-1)} \phi_{k,N,L} \left(\frac{x}{\bar{\gamma}}\right)^k$ and given by [18, Eq. (44)]

$$\phi_{k,N,L} = \sum_{i=k-L+1}^{k} \frac{\phi_{i,N-1,L}}{(k-i)!} I_{[0,(N-1)(L-1)]}(i), \tag{2.28}$$

$\phi_{0,0,L} = \phi_{0,N,L} = 1$, $\phi_{k,1,L} = 1/k!$, $\phi_{1,N,L} = N$ and $I_{[a,c]}(b) = \begin{cases} 1, & a \leq b \leq c \\ 0, & \text{otherwise} \end{cases}$.

2.3.3 Statistical characterization of the end-to-end SNR

In this subsection, the CDF of the end-to-end SNR with arbitrary transmit antenna selection at the relay is presented. From the first principles, the CDF of γ_{eq_1} is derived as follows:

$$\begin{aligned}
F_{\gamma_{eq_1}}(x) &= \Pr\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1} \leq \gamma_{th}\right) \\
&= \int_0^{\gamma_{th}} \Pr\left(\frac{\gamma_1 y}{\gamma_1 + y + C} \leq \gamma_{th}\right) f_{\gamma_2}(y)dy + \int_{\gamma_{th}}^{\infty} \Pr\left(\gamma_{th} \leq \frac{\gamma_{th}(y+C)}{y-\gamma_{th}}\right) f_{\gamma_2}(y)dy \\
&= \int_0^{\gamma_{th}} f_{\gamma_2}(y)dy + \int_{\gamma_{th}}^{\infty} \left(1 - \bar{F}_{\gamma_1}\left(\frac{\gamma_{th}(y+C)}{y-\gamma_{th}}\right)\right) f_{\gamma_2}(y)dy \\
&= 1 - \int_{\gamma_{th}}^{\infty} \bar{F}_{\gamma_1}\left(\frac{\gamma_{th}(y+C)}{y-\gamma_{th}}\right) f_{\gamma_2}(y)dy \\
&= 1 - \int_0^{\infty} \bar{F}_{\gamma_1}\left(\frac{\gamma_{th}(w+\gamma_{th}+C)}{w}\right) f_{\gamma_2}(w+\gamma_{th})dw.
\end{aligned}$$

where $w = y - \gamma_{th}$ and $F_x(x) = 1 - \bar{F}_x(x)$. The CDF of γ_{eq_1} can further be simplified as follows:

$$\begin{aligned} F_{\gamma_{eq_1}}(x) &= 1 - \int_0^\infty \frac{\sum_{i=1}^{MN} \binom{MN}{i} (-1)^i}{(L-1)!(\bar{\gamma}_2)^L} e^{-\left(\frac{\gamma_{th}(w+\gamma_{th}+C)}{w\bar{\gamma}_1}\right)} (w+\gamma_{th})^{L-1} e^{-\left(\frac{w+\gamma_{th}}{\bar{\gamma}_2}\right)} dw \\ &= 1 - \frac{\sum_{i=1}^{MN} \sum_{k=o}^{L-1} \binom{MN}{i} \binom{L-1}{k} (-1)^i}{(L-1)!(\bar{\gamma}_2)^L} \\ &\times \int_0^\infty (w+\gamma_{th})^{L-1} \exp\left(-\frac{\gamma_{th}(w+\gamma_{th}+C)i}{w\bar{\gamma}_1} - \frac{w+\gamma_{th}}{\bar{\gamma}_2} \right) dw. \end{aligned} \tag{2.29}$$

Then by using [19], the single-integral in (2.29) can be solved as follows:

$$\int_0^\infty x^V - e^{-\frac{\beta}{x} - \gamma x} dx = 2 \left(\frac{\beta}{\gamma} \right) K_V (2\sqrt{\beta\gamma}).$$

Suppose V=L-K, $\beta = \frac{\gamma_{th}(\gamma_{th}+C)i}{\bar{\gamma}_1}$, $\gamma = \frac{1}{\bar{\gamma}_2}$. Then the CDF of γ_{eq_1} is derived in closed form as

$$\begin{aligned} F_{\gamma_{eq_1}}(x) &= 1 - \frac{\sum_{i=1}^{MN} \sum_{k=0}^{L-1} \binom{MN}{i} \binom{L-1}{k} (-1)^i}{(L-1)!(\bar{\gamma}_2)^L} \exp\left(-\frac{\gamma_{th} i}{\gamma_1} - \frac{\gamma_{th}}{\gamma_2} \right) \\ &\times \left[2 \left(\frac{\gamma_{th}(\gamma_{th}+C)i\bar{\gamma}_2}{\bar{\gamma}_1} \right) K_V \left(2 \sqrt{\frac{\gamma_{th}(\gamma_{th}+C)i}{\bar{\gamma}_1\bar{\gamma}_2}} \right) \right]. \end{aligned} \tag{2.30}$$

2.4 Performance Analysis

In this section, the important performance metrics of joint user and antenna selection for multi-user relay networks are derived in closed-form. Specifically, the simple and tight lower bounds for the outage probability and the average bit error rate are derived by using tools from communication theory, and probability and stochastic theory.

2.5 Outage Probability

The outage probability is the end-to-end instantaneous SNR falls below the a predefined threshold γ_{th}. Hence, the outage probability is written as follows:

$$P_{out} = P_r(\gamma_{eq} \leq \gamma_{th}) = F_{\gamma_{eq}}(\gamma_{th}), \tag{2.31}$$

where $F_{\gamma_{eq}}(\gamma_{th})$ is the CDF of γ_{eq} evaluated at γ_{th}.

Next, an upper bound for the end-to-end SNR can be written as follows:

$$\gamma_{eq} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1} \leq \min(\gamma_1, \gamma_2) = \gamma_{eq}^{ub}. \tag{2.32}$$

The CDF of γ_{eq}^{ub} is then derived by using Theorem 2 in Appendix as follows:

$$\begin{aligned} F_{\gamma_{eq}^{ub}}(x) &= 1 - (1 - F_{\gamma_1}(x))(1 - F_{\gamma_2}(x)) \\ &= 1 - \prod_{i=1}^{2}[1 - F_{\gamma_i}(x)] \\ &= 1 - \bar{F}_{\gamma_1}(\gamma_x)\bar{F}_{\gamma_2}(\gamma_x), \end{aligned} \tag{2.33}$$

where $\bar{F}_{\gamma_i}(\gamma_x)$ is the complimentary cumulative distribution function (CCDF) of γ_i, and is defined as $\bar{F}_{\gamma_i}(\gamma_x) = 1 - F_{\gamma_i}(\gamma_x)$.

The lower bound for the outage probability can then be obtained by evaluating (2.33) at the threshold SNR as follows:

$$P_{out}^{lb} = 1 - \bar{F}_{\gamma_1}(\gamma_{\gamma_{th}})\bar{F}_{\gamma_2}(\gamma_{\gamma_{th}}). \tag{2.34}$$

2.5.1 Outage probability for multi-user MIMO relay network without transmit antenna selection at relay

In this subsection, the outage probability of the multi-user MIMO relay network without transmit antenna selection at relay is derived. To this end, the CCDF of γ_1 is given by

$$\bar{F}_{\gamma_1}(x) = \sum_{i=1}^{N_sN_rL} \binom{N_sN_rL}{i}(-1)^{i+1}e^{-\frac{x_i}{\bar{\gamma}_1}}. \tag{2.35}$$

Similarly, the CCDF of γ_2 is given by

$$\bar{F}_{\gamma_2}(x) = e^{\frac{-x}{\bar{\gamma}_2}} \sum_{p=0}^{N_d-1} \frac{\left(\frac{x}{\bar{\gamma}_2}\right)^p}{p!}. \tag{2.36}$$

Next, by substituting (2.35) and (2.36) into (2.34), a lower bound for the outage probability of multi-user MIMO relay networks without transmit antenna at the relay is derived as follows:

$$P_{out}^{lb} = 1 - \sum_{i=1}^{N_s N_r L} \sum_{p=0}^{N_d-1} \binom{N_s N_r L}{i} \frac{(-1)^{i+1}}{p!} \left(\frac{\gamma_{th}}{\bar{\gamma}_2}\right)^p \bar{\exp}\left(-\frac{i\gamma_{th}}{\bar{\gamma}_1} - \frac{\gamma_{th}}{\bar{\gamma}_2}\right). \tag{2.37}$$

2.5.2 Outage probability for multi-user MIMO relay network with optimal transmit antenna selection at relay

In this subsection, the outage probability of the multi-user MIMO relay network with optimal transmit antenna selection at relay is derived. To begin with, the CCDF of γ_1 is same as given in (2.35). However, the CCDF of γ_2 is different to (2.36) and is given by

$$\bar{F}_{\gamma_{eq_2}}(x) = \sum_{c=1}^{N_r} \sum_{d=0}^{c(N_d-1)} \frac{(-1)^{c+1}\binom{N_r}{c}\phi_{d,N_r,N_d}}{\bar{\gamma}_2^d} x^d \exp\left(-\frac{cx}{\bar{\gamma}_2}\right). \tag{2.38}$$

Next, by substituting (2.35) and (2.38) into (2.34), a lower bound for the outage probability of multi-user MIMO relay networks without transmit antenna at the relay is derived as follows:

$$\begin{aligned} P_{out}^{lb} &= 1 - \sum_{i=1}^{N_s N_r L} \sum_{c=1}^{N_r} \sum_{d=0}^{c(N_d-1)} (-1)^{c+i} \binom{N_s N_r L}{i} \binom{N_r}{c} \phi_{d,N_r,N_d} \left(\frac{\gamma_{th}}{\bar{\gamma}_2}\right)^d \\ &\times \exp\left(-\frac{c\gamma_{th}}{\bar{\gamma}_2} - \frac{i\gamma_{th}}{\bar{\gamma}_1}\right). \end{aligned} \tag{2.39}$$

2.6 Average Bit Error Rate

The average bit error rate is an important performance measure of wireless digital communication systems. In this section the average bit error rate of the multi-user MIMO relay network with joint user and antenna selection is derived.

To begin with, the conditional error probability (CEP) $P_e|\gamma$, which is valid for a wide range of modulation schemes, can be written as

$$P_{e|r}(x) = aQ(\sqrt{bx}), \tag{2.40}$$

where a and b are constants and depends on the modulation schemes. Moreover, $Q(x)$ is a Gaussian Q function. Then the average bit error rate is derived by integrating $P_e|\gamma$ over the PDF of the SNR as follows:

$$\begin{aligned}\bar{P}_e &= \int_0^\infty P_{e|r}(x) f_\gamma(x) dx \\ &= \int_0^\infty aQ(\sqrt{bx}) f_\gamma(x) dx \\ &= \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \int_0^\infty x^{\frac{-1}{2}} \exp\left(\frac{-bx}{2}\right) \bar{F}_{\gamma_{eq}^{ub}}(x) dx. \end{aligned} \tag{2.41}$$

2.6.1 Average bit error rate for multi-user MIMO relay network without transmit antenna selection at relay

The average bit error rate for multi-user MIMO relay network without transmit antenna selection at relay can then be derived by substituting the corresponding CCDF of γ_{eq}^{lb} into (2.41) as follows:

To begin with, the CCDF of γ_{eq}^{lb} is derived as

$$\bar{F}_{\gamma_{eq}^{ub}} = \sum_{i=1}^{N_s N_r L} \sum_{p=0}^{N_d - 1} \binom{N_s N_r L}{i} \frac{(-1)^{i+1}}{p!} \left(\frac{x}{\gamma_2}\right)^p \exp\left(-\frac{ix}{\bar{\gamma}_1} - \frac{x}{\bar{\gamma}_2}\right). \tag{2.42}$$

Next, by substituting (2.42) into (2.41), the average bit error rate lower bound of multi-user MIMO relay network without transmit antenna selection at relay can then be derived as follows:

$$\begin{aligned}\bar{P_e}^{lb} &= \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \int_0^\infty x^{\frac{-1}{2}} \exp\left(\frac{-bx}{2}\right) \\ &\times \left[\sum_{i=1}^{N_s N_r L} \sum_{p=0}^{N_d - 1} \binom{N_s N_r L}{i} \frac{(-1)^{i+1}}{p!} \left(\frac{x}{\gamma_2}\right)^p \exp\left(-\frac{ix}{\bar{\gamma}_1} - \frac{x}{\bar{\gamma}_2}\right)\right] \mathrm{d}x. \end{aligned} \tag{2.43}$$

The average bit error rate lower bound of (2.43) can be alternatively written as

$$\begin{aligned}\bar{P}_e^{lb} &= \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \sum_{i=1}^{N_sN_rL} \sum_{p=0}^{N_d-1} \binom{N_sN_rL}{i} \frac{(-1)^{i+1}}{p!\bar{\gamma}_2^p} \int_0^\infty x^{p-\frac{1}{2}} \exp\left(\frac{-bx}{2}\right) \\ &\times \left[\exp\left(-\frac{bx}{2} - \frac{ix}{\bar{\gamma}_1} - \frac{x}{\bar{\gamma}_2}\right)\right] \mathrm{d}x. \qquad (2.44)\end{aligned}$$

By evaluating the integral of (2.44) by using [19, Eq. (3.351.3)], a lower bound for the average bit error rate can be derived in closed-form as follows:

$$\bar{P}_e^{lb} = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \sum_{i=1}^{N_sN_rL} \sum_{p=0}^{N_d-1} \binom{N_sN_rL}{i} \frac{(-1)^{i+1}}{p!\bar{\gamma}_2^p} \Gamma\left(b+\frac{1}{2}\right) \left(\frac{b}{2} + \frac{i}{\bar{\gamma}_1} - \frac{1}{\bar{\gamma}_2}\right)^{-(b+0.5)}. \qquad (2.45)$$

2.6.2 Average bit error rate for multi-user MIMO relay network with optimal transmit antenna selection at relay

Similarly, the average bit error rate for multi-user MIMO relay network with optimal transmit antenna selection at relay can then be derived by substituting the corresponding CCDF of γ_{eq}^{lb} into (2.41) as follows:
To begin with, the CCDF of γ_{eq}^{lb} corresponding to optimal transmit antenna selection at the relay is derived as

$$\begin{aligned}\bar{F}_{\gamma_{eq}^{ub}} &= \sum_{i=1}^{N_sN_rL} \sum_{c=1}^{N_r} \sum_{d=0}^{c(N_d-1)} (-1)^{c+i} \binom{N_sN_rL}{i} \binom{N_r}{c} \phi_{d,N_r,N_d} \left(\frac{x}{\bar{\gamma}_2}\right)^d \\ &\times \exp\left(-\frac{cx}{\bar{\gamma}_2} - \frac{ix}{\bar{\gamma}_1}\right). \qquad (2.46)\end{aligned}$$

Thereafter, by substituting (2.46) into (2.41), the average bit error rate of multi-user MIMO relay network with optimal transmit antenna selection at relay can be derived as

$$\begin{aligned}\bar{P}_e &= \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \sum_{i=1}^{N_sN_rL} \sum_{c=1}^{N_r} \sum_{d=0}^{c(N_d-1)} (-1)^{c+i} \binom{N_sN_rL}{i} \binom{N_r}{c} \phi_{d,N_r,N_d} \left(\frac{1}{\bar{\gamma}_2}\right)^d \\ &\times \int_0^\infty x^{d-\frac{1}{2}} \exp\left(-\frac{bx}{2} - \frac{cx}{\bar{\gamma}_2} - \frac{ix}{\bar{\gamma}_1}\right) \mathrm{d}x. \qquad (2.47)\end{aligned}$$

The single-integral of 2.47 can then be evaluated by using [19, Eq. (3.351.3)] to obtain the average bit error rate lower bound in closed-form as follows:

$$\begin{aligned}\bar{P}_e &= \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \sum_{i=1}^{N_s N_r L} \sum_{c=1}^{N_r} \sum_{d=0}^{c(N_d-1)} (-1)^{c+i} \binom{N_s N_r L}{i} \binom{N_r}{c} \phi_{d,N_r,N_d} \left(\frac{1}{\bar{\gamma}_2}\right)^d \\ &\times \Gamma\left(d+\frac{1}{2}\right) \left(\frac{b}{2} + \frac{c}{\bar{\gamma}_2} + \frac{i}{\bar{\gamma}_1}\right)^{-(d+0.5)}. \end{aligned} \tag{2.48}$$

Chapter 3

Numerical Results

In this chapter, our numerical results are provided to obtain valuable insights from our analysis. To this end, the outage probability and the average bit error rate pertinent to several specific system configurations are studied by using analytical and simulation data by using Matlab. Specifically, the analytical lower bounds for the outage probability are plotted by using our analyses in (2.37) and (2.39). Moreover, the analytical lower bounds for the average bit error rate of binary phase shift keying (BPSK) are plotted by using (2.45) and (2.48). Whereas all the exact outage and average bit error rate curves are plotted by using Monte-Carlo simulations.

The outage and average bit error rate curves corresponding to two specific system set-ups are plotted. These two system set-ups are (i) multi-user MIMO relay networks without transmit antenna selection at the relay, and (ii) multi-user MIMO relay networks with transmit antenna selection at the relay. From our numerical results, we aim to demonstrate the benefits of several aspects of MIMO relay networks, namely (i) joint user and antenna-pair selection versus single-user antenna-pair selection, (ii) MRC combining at the multi-antenna destination versus single-antenna destinations, and (iii) arbitrary transmit antenna selection at the relay versus optimal transmit antenna selection at the relay.

3.1 Outage Probability

3.1.1 Single source relay networks without transmit antenna selection in the second hop

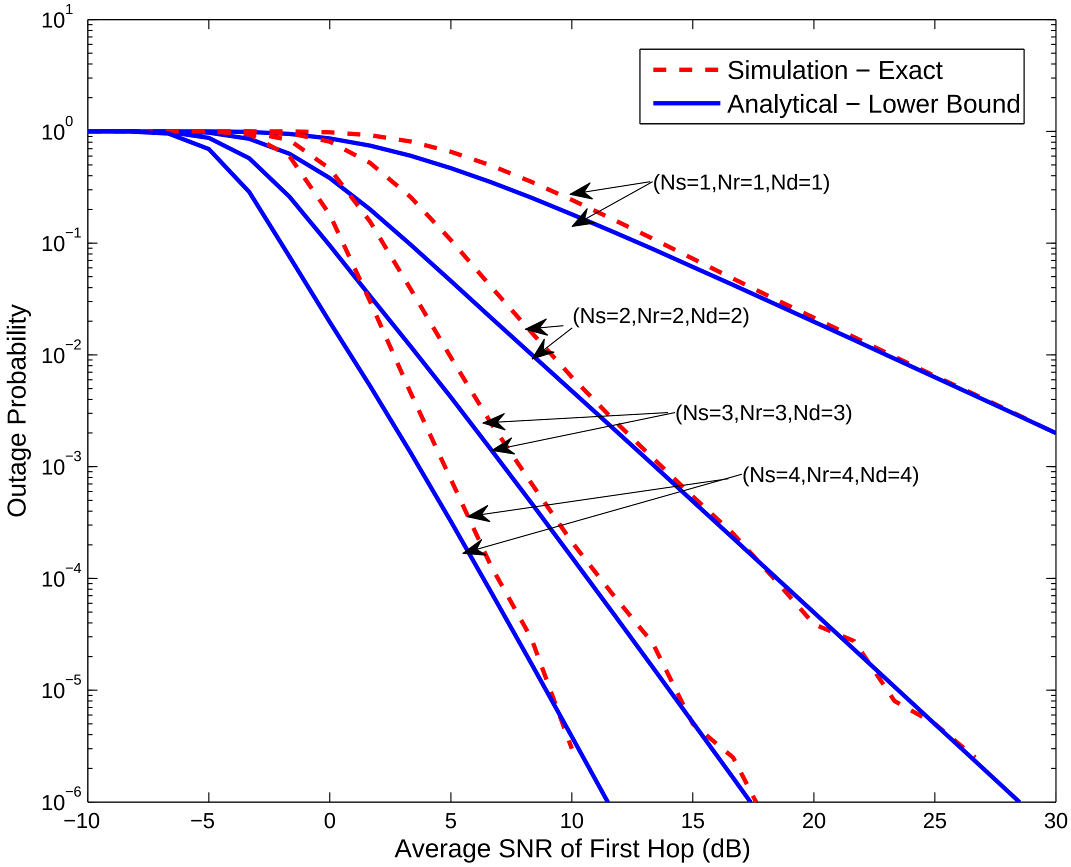

Figure 3.1: Outage probability for single source relay networks without transmit antenna selection in the second hop.

In Fig. 3.1, the outage probability of joint antenna selection for two-hop AF relay networks is plotted for several antenna configurations. Specifically, by changing the number of antennas at the source, relay and destination, four different outage curves are plotted. The exact outage curves are plotted by using Monte-Carlo simulation results. Whereas the outage lower bound is plotted by using the analysis in Eq. (2.45). For example, the outage curve corresponding to the single-antenna case ($N_s = 1, N_r = 1, N_d = 1$) is plotted for comparison purposes. Clearly, as the number of antennas at each source increases, the outage probability of the system of interest improves. For instance, by going from single-antenna nodes to quadruple-antenna nodes, almost 22 dB SNR gain can be obtained at an outage probability of 10^{-2}. Similarly, at an outage probability of 10^{-4}, triple-antenna system set-up provides 7 dB SNR gain over dual-antenna system set-up. Fig. 3.1 clearly reveals that our analytical outage lower bound is asymptotically exact at very high SNRs, and considerably tight to the exact outage curves at moderate-to-high SNR regime. Thus, our outage lower bound is useful as a benchmark in investigating the performance of MIMO relay networks with joint antenna selection.

3.1.2 Single source relay networks with transmit antenna selection in the second hop

Fig. 3.2 shows the outage probability of joint antenna selection for the two-hop AF relay network. Here, several outage probability curve are plotted by changing the antenna configuration at the relay for MIMO AF relay network with transmit antenna selection in the second hop. It can be clearly seen from the Fig. 3.2 that there is 19 dB improvement in SNR at the outage probability of 10^{-2} when the number of antennas at the source, relay and destination is doubled. Furthermore, 2 dB improvement in SNR at the outage probability of 10^{-5} can be seen when the number of antennas only at the relay is increased by 2. The outage probability curves obtained from the Monte-Carlo simulations agrees well with the results obtained analytically.

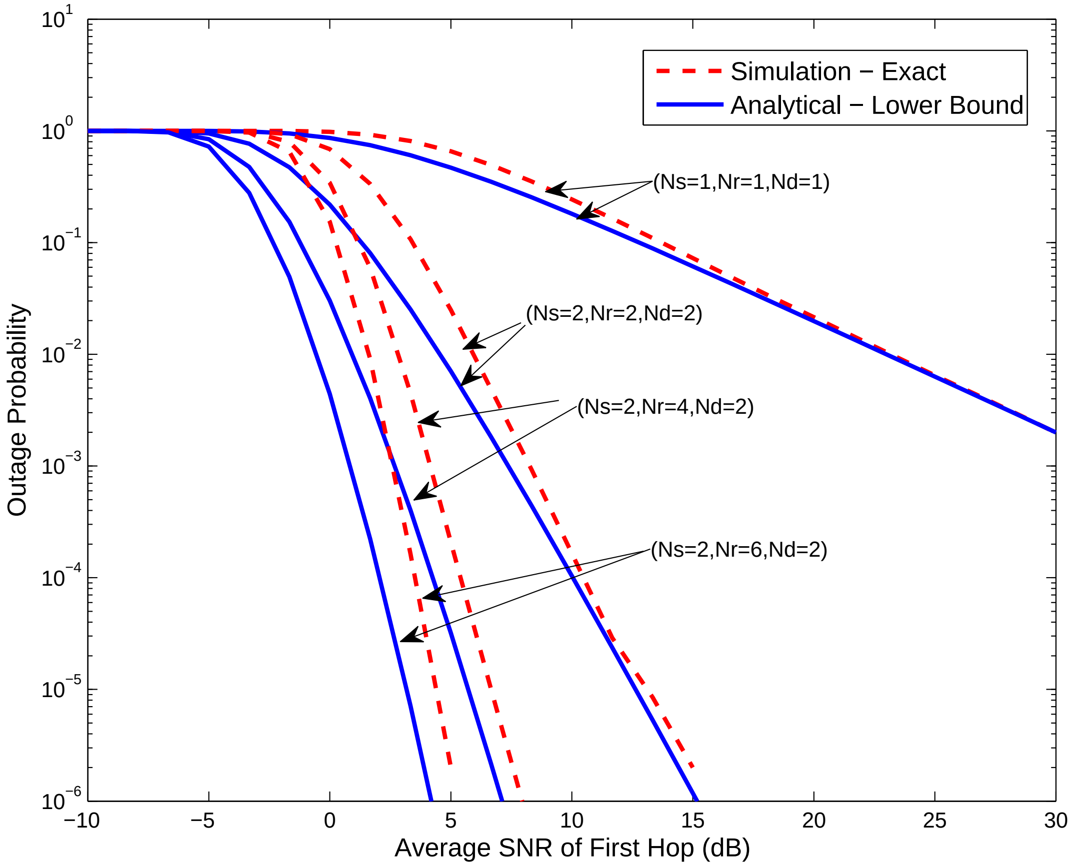

Figure 3.2: Outage probability for single source relay networks with transmit antenna selection in the second hop.

3.1.3 Multiple source relay networks without transmit antenna selection in the second hop

In Fig. 3.3 outage probability for the multiple sources in the first hop without the antenna selection in second hop is shown. Four different case are taken into consideration to study the effect of multiple sources. The number of antennas at the source, relay and destination were kept fixed while changing the number of sources for two different cases. The Fig. 3.3 clearly shows that there is a negotiable change in the SNR during the outage probability

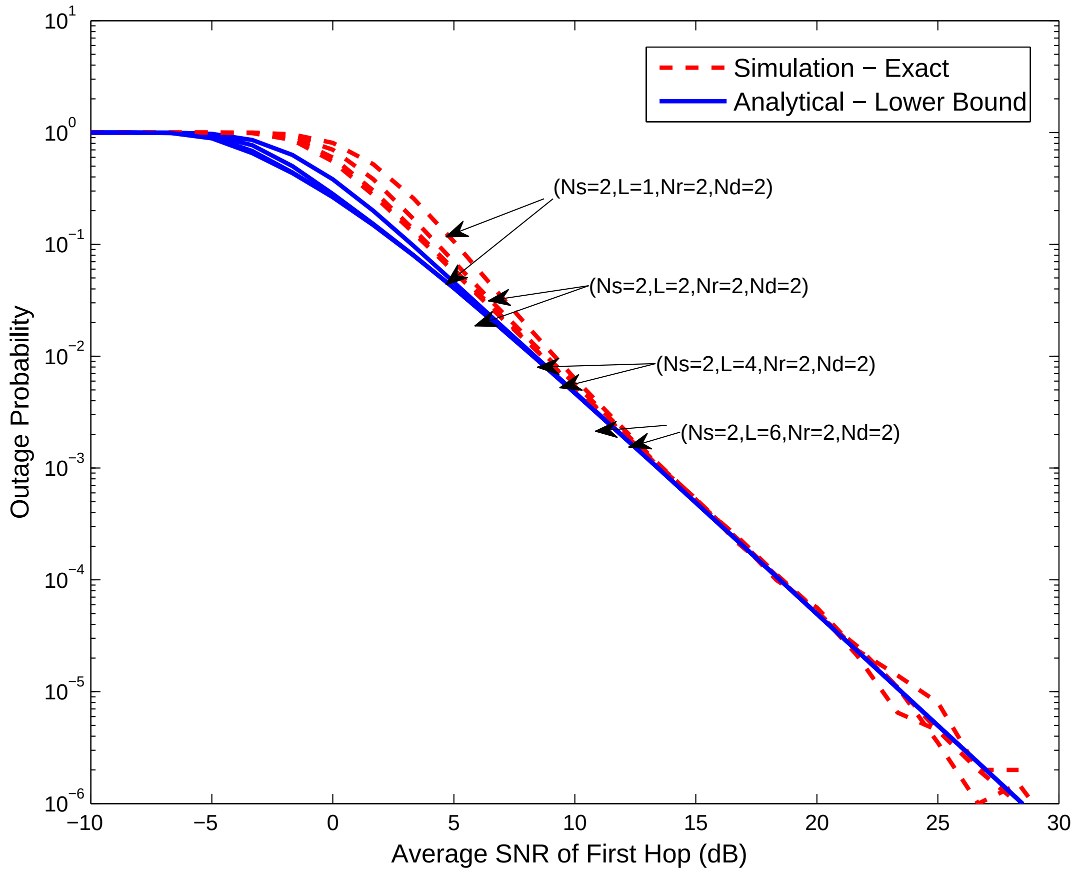

Figure 3.3: Outage probability for multiple source relay networks without transmit antenna selection in the second hop.

from 1 to 10^{-2} when the number of sources are increased from 1 to 6, whilst number of antenna at the source, relay and destination were fixed.

3.1.4 Multiple source relay networks with transmit antenna selection in the second hop

In Fig. 3.4, the effect of antenna selection in second hop whilst having multiple sources in first hop is analyzed. Significant improvement in SNR can be seen when the number of

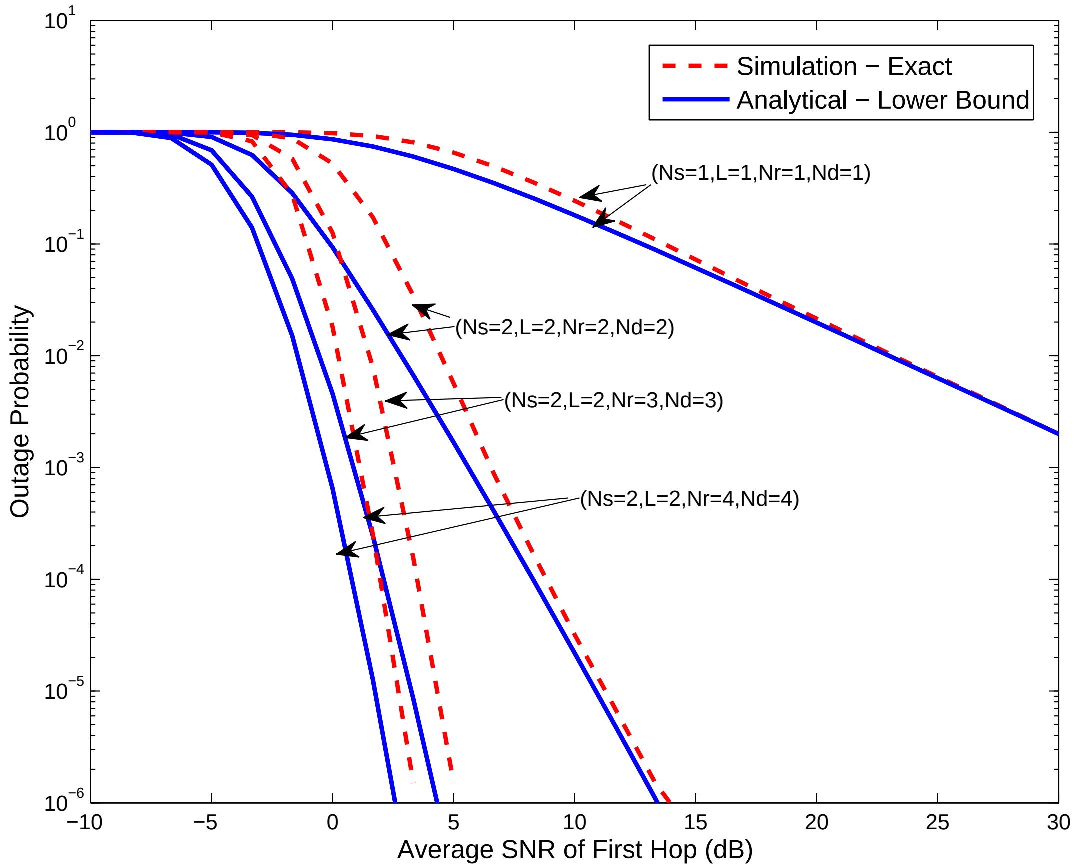

Figure 3.4: Outage probability for multiple source relay networks with transmit antenna selection in the second hop.

antenna at the relay and destination is increased. When the number of antenna at source, relay and destination is increased from 1 to 2 and the same for the number of source, 20 dB improvement in SNR can be achieved at the outage probability of 10^{-2}. Furthermore, when the values of N_s, L, N_r, and N_d is changed from [2,2,3,3] to [2,2,4,4], respectively, the minor 2 dB change in SNR can be seen at the outage probability of 10^{-5}.

3.2 Average Bit Error Rate

3.2.1 Single source relay networks without transmit antenna selection in the second hop

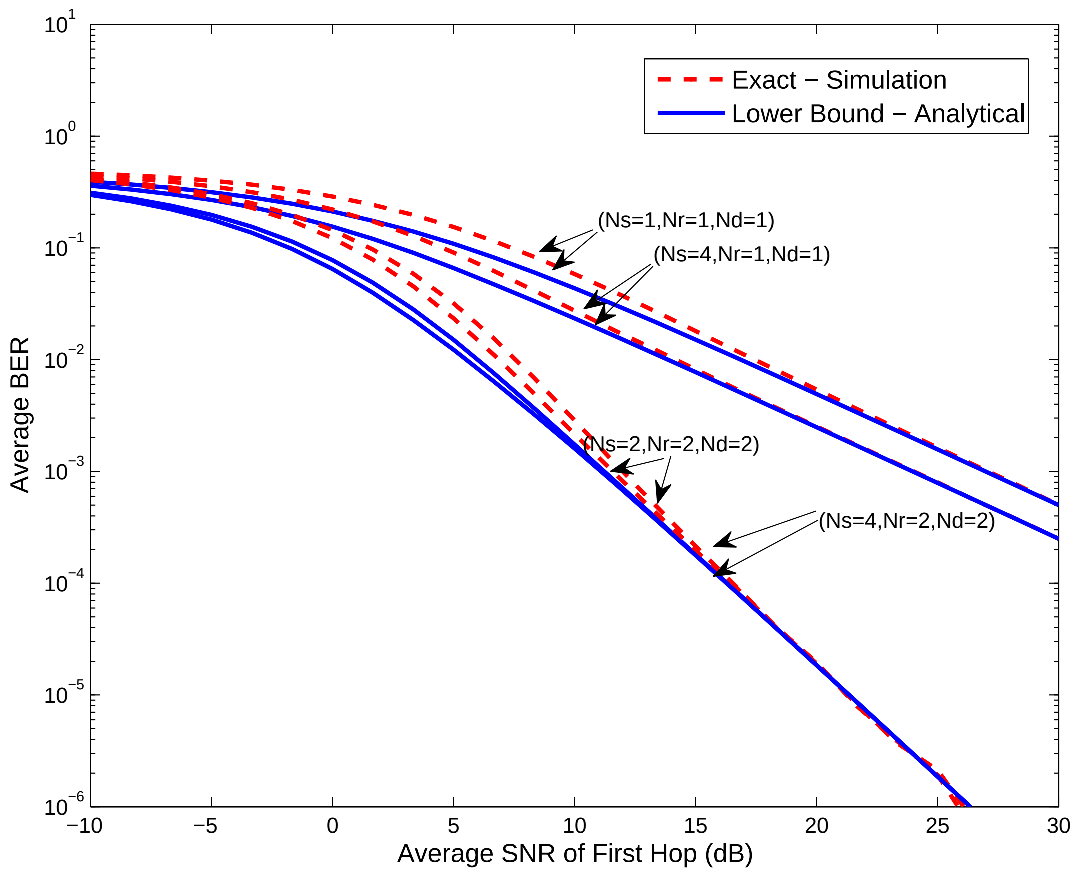

Figure 3.5: BER for single source relay networks without transmit antenna selection in the second hop.

Fig. 3.5 plots the average bit error rate of BPSK for the joint antenna selection for two-hop AF relay networks with several antenna configurations. In Fig. 3.5, four different BER curves are plotted by changing the number of antennas at the source, relay and destination. The exact BER curves are plotted by using Monte-Carlo simulation results for the comparison purposes. For example, the BER curve corresponding to the single-antenna case ($N_s = 1, N_r = 1, N_d = 1$) is plotted for comparison purposes. Clearly, as the number of antennas at each source increases, the BER of the system of interest improves. For instance, by going from single-antenna nodes to quadruple-antenna nodes, almost 3 dB SNR gain can be obtained at an outage probability of 10^{-3}. But, only minor change in SNR can be observed for the various values of BER when the number of antenna at the sources are increased from 2 to 4 and the number of antennas at the relay and destination kept unchanged. Fig. 3.5 clearly shows that the results obtained from simulation agrees with the results obtained from the Monte-Carlo curves.

3.2.2 Single source relay networks with transmit antenna selection in the second hop

In Fig. 3.6, four BER curves for different antenna configuration are plotted to see the effect of antenna selection on SNR at the various values of BER. The curve for the ($N_s = 1, N_r = 1, N_d = 1$) is also plotted as a benchmark for comparison purposes. Fig. 3.6 clearly shows that significant SNR gain can be achieved by using transmit antenna selection in second hop. When the number of antenna at the relay is changed from 1 to 3 whilst keeping the number of antenna at the source and destination fixed, about 17 dB gain in SNR at the BER of 10^{-3} can be achieved. Similarly, 3 dB SNR gain can be achieved when we change the values of (N_s, N_s, N_d) from (2,2,2) to (2,3,2) at the BER of 10^{-5}.

3.2.3 Multiple source relay networks without transmit antenna selection in the second hop

Fig. 3.7 shows the curves for different antenna configuration having multiple antenna at the source and no feedback for the antenna selection in second hop. The curve with ($N_s = 1, L = 1, N_r = 1, N_d = 1$) also plotted for as a reference. The number of source and number of antenna in each source are increased from 1 to 2 while the keeping the number of antenna

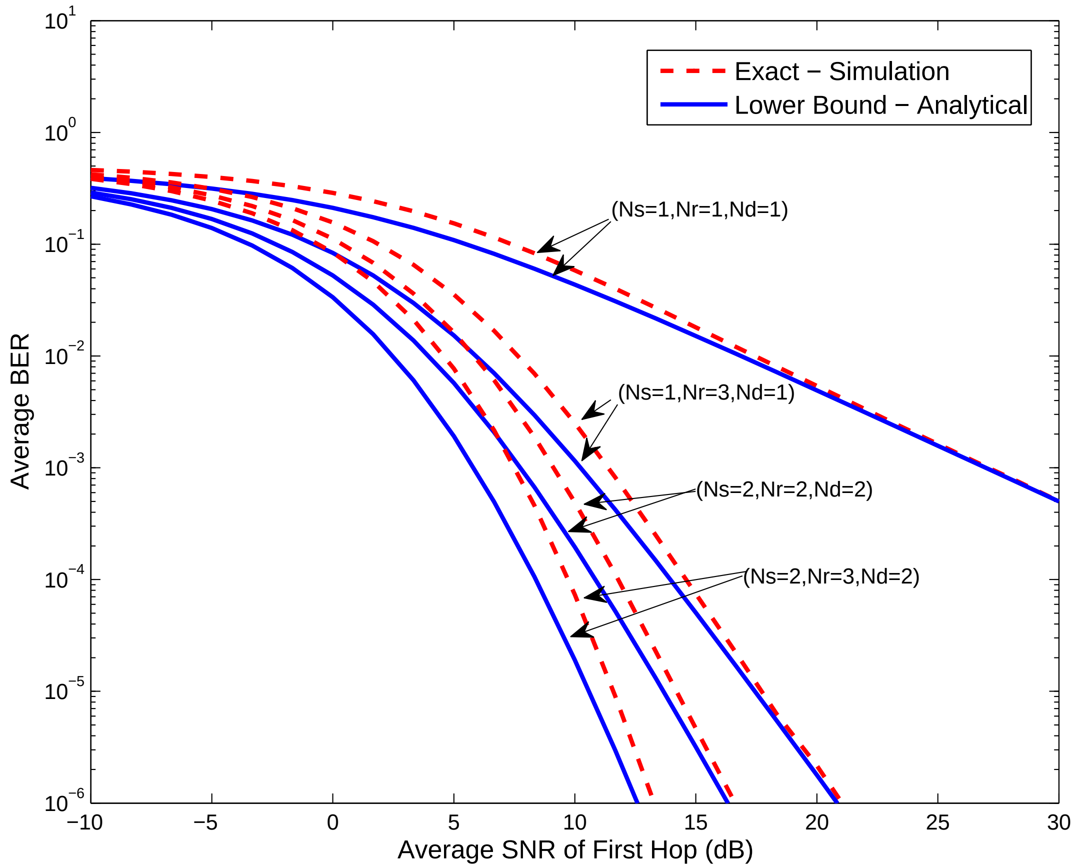

Figure 3.6: BER for single source relay networks with transmit antenna selection in the second hop.

fixed at relay and destination, 16dB gain can achieved at the BER of 10^{-3}. But no change can be seen while the number source and antenna in each source is shifted from 2 to 3 to 4. The exact curves obtained from Monte-Carlo simulation tightly agrees with analytical results.

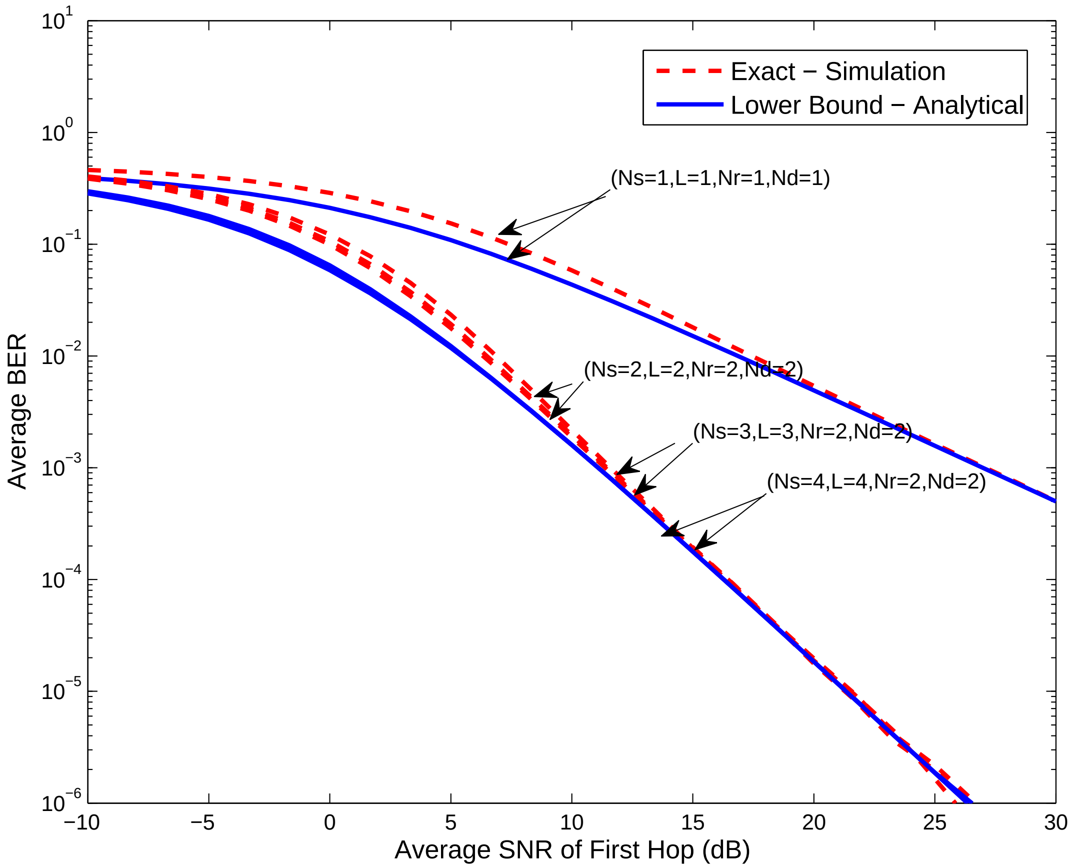

Figure 3.7: BER for single source relay networks without transmit antenna selection in the second hop.

3.2.4 Multiple source relay networks with transmit antenna selection in the second hop

Fig. 3.8 shows the four curves for the various antenna configuration at source, relay and destination having multiple source in first hop and feedback for antenna selection in second in second hop. It can be clearly seen that there is a 4.5 dB gain in SNR at the BER of 10^{-5} when the antenna configuration is changed from $(N_s = 2, L = 1, N_r = 2, N_d = 2)$ to $(N_s = 2, L = 2, N_r = 4, N_d = 2)$. Additionally to that 1 dB gain can be achieved at the

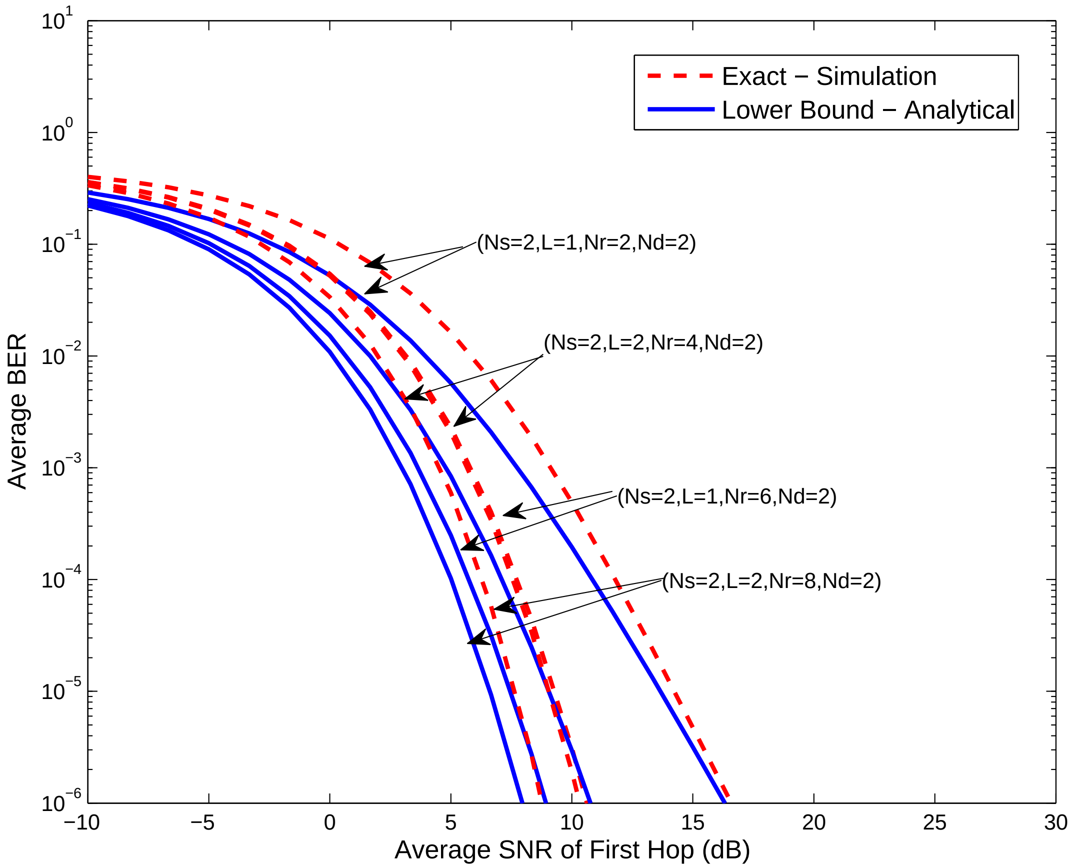

Figure 3.8: BER for single source relay networks with transmit antenna selection in the second hop.

same BER when we shift antenna configuration from $(N_s = 2, L = 1, N_r = 6, N_d = 2)$ to $(N_s = 2, L = 2, N_r = 8, N_d = 2)$. The exact curves plotted with Monte-Carlo simulations very well agrees with our results.

Chapter 4

Conclusions

In this project, the optimal joint source and transmit-receive pair selection strategies were developed and analyzed for MIMO AF relay networks. To this end, the best source, the best transmit antenna at the best source, and the optimal transmit-receive antenna pair at the relay were selected to maximize the end-to-end SNR and hence to minimize the overall outage probability. The problem statement of the underlying transmission strategy was first formulated and thereby the useful performance measures were derived in closed-form. To this end, tight lower bounds for the outage probability and the average bit error rate were derived in closed-form. Numerical results pertinent to specific system configurations were obtained by using Matlab simulations. For example, the analytical lower bounds for the outage probability and the average bit error rate were plotted by using the analysis and their corresponding exact curves were plotted by using Monte-Carlo simulations. Numerical results reveal that our analytical outage and average BER lower bounds are tight to the exact curves in the useful SNR regime. Thus, they can be used as benchmarks for designing practical MIMO relay networks. Moreover, system specific institutions and insights were obtained through the numerical results. These insights and conclusions drawn from our study render themselves useful as benchmarks for practical MIMO relay network designing.

Appendix A

Theorem 1

Let $X_1, X_2, \dots X_N$ be N independent random variables having CDFs as $F_{X_1}(x), F_{X_2}(x), \dots F_{X_N}(x)$, respectively. Let $Y = \max(X_1, X_2, \dots X_N)$, then the CDF of Y is given by [20]

$$\begin{aligned} F_Y(x) &= F_{X_1}(x).\, F_{X_2}(x) \dots F_{X_N}(x) \\ F_Y(x) &= \prod_{i=1}^{N} [F_{X_i}(x)]. \end{aligned}$$

Theorem 2

Let $X_1, X_2, \dots X_N$ be N independent random variables having CDFs $F_{X_1}(x), F_{X_2}(x), \dots F_{X_N}(x)$, respectively. Let $Y = \min(X_1, X_2, \dots X_N)$, then the CDF of Y is given by [20]

$$\begin{aligned} F_Y(x) &= 1 - \prod_{i=1}^{N} [1 - F_{X_i}(x)] \\ &= 1 - \prod_{i=1}^{N} [\bar{F}_{X_i}(x)], \end{aligned}$$

where $\bar{F}_{X_i}(x)$ is the complimentary cumulative distribution function of X_i.

The aforementioned two theorems are taken from [20] and are employed in deriving the analytical expressions for the outage probability and the average bit error rate.

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