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A New Ductility Exhaustion Model for High Temperature Low Cycle Fatigue Life Prediction of Turbine Disk Alloys

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Abstract. Based on ductility exhaustion theory and the generalized energy-based damage parameter, a new viscosity-based life prediction model is introduced to account for the mean strain/stress effects in the low cycle fatigue regime. The loading waveform parameters and cyclic hardening effects are also incorporated within this model. It is assumed that damage accrues by means of viscous flow and ductility consumption is only related to plastic strain and creep strain under high temperature low cycle fatigue conditions. In the developed model, dynamic viscosity is used to describe the flow behavior. This model provides a better prediction of Superalloy GH4133's fatigue behavior when compared to Goswami's ductility model and the generalized damage parameter. Under non-zero mean strain conditions, moreover, the proposed model provides more accurate predictions of Superalloy GH4133's fatigue behavior than that with zero mean strains.

Keywords. High Temperature Low Cycle Fatigue, Ductility, Life Prediction, Mean Strain, Viscosity.

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Nomenclature

A, B	model parameters
C_1, C_2, C_3	material constants representing the material energy absorption capacity
K'	cyclic strength coefficient
n'	cyclic strain hardening exponent
N_f	the number of cycles to failure
N_{ft}	the number of tested life cycles
N_{fp}	the number of predicted life cycles
R_ε	strain ratio
ΔW_p	plastic strain energy density (PSED)
ΔW_s	strain energy
W_f	fatigue toughness
α, β	model parameters
ε_a	strain amplitude
$\varepsilon_{\max}$	maximum strain
ε_m	mean strain
$\dot{\varepsilon}$	strain rate
ν_d	dynamic viscosity
$\Delta\varepsilon_t$	strain range
$\Delta\varepsilon_e, \Delta\varepsilon_p$	elastic strain range and plastic strain range
$\Delta\varepsilon_{\text{in}}$	inelastic strain range
σ_a	stress amplitude
$\sigma_{\max}, \sigma_{\min}$	maximum and minimum stress
σ_m	mean stress
σ_{sat}	saturated stress range
$\Delta\sigma$	stress range

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1 Introduction

The failure of a disk in an aircraft engine often leads to catastrophe failure. Therefore, disk reliability is classified as one of the most critical parts of flight safety. Low cycle fatigue (LCF) due to thermal transients under strain controlled conditions is the main failure mode of turbine disks. With higher performance requirements and thrust-to-weight ratios in modern aircraft engines, higher stresses and temperatures will be induced by these disks. At the same time, high aircraft engine reliability is required. All these factors generate new challenges to accurately predict aircraft turbine disk LCF life and develop experimental methodology for its assessment [1].

A clear understanding of LCF behavior at high temperature is very important for design, selection, and safety assessment of aircraft turbine disks. The disks often experience an extended period of steady loading in addition to dynamic loading caused by the start-up and shut-down phases. They may fail prematurely within a limited number of loading cycles, e.g. below about 10^4 cycles. This phenomenon is often called high temperature low cycle fatigue (HTLCF) [1]. HTLCF is an interactive mechanism arising from various processes such as time-independent plastic strain, time-dependent creep, dynamic strain ageing, and oxidation. Over the past several decades, the issue of accurately predicting HTLCF life of these hot section components has been an area of interest. Many life prediction methods have been reported [2–21]. However because of the complex damaging mechanisms involved, a unified model that can provide accurate life prediction for HTLCF does not exist [1].

The mechanisms of deformation and eventual failure of HTLCF are very complex. They depend on various test and material parameters, such as temperature, stress or strain rate, loading frequency and loading waveform. Several researchers have reported the influence of the cyclic frequency [2], hold period [3], strain range [4], loading waveform [5], strain rate [6, 7], heat treatment [8], testing temperature [9] and environment [10, 11] on the cyclic stress response, deformation mode, and fatigue life of superalloys during HTLCF. Increasing attention has been paid to the study of fatigue and creep interaction in either isothermal or thermal-mechanical fatigue conditions [12]. The reported methods for the HTLCF life prediction that have been developed [1, 13] include linear damage summation (LDS), frequency modified Manson–Coffin equation (FMMC), frequency separation technique (FS), strain range partition (SRP), strain energy partition (SEP), frequency modified damage function model (FMDF) and damage rate approach (DRA). These methods are prevalent for life prediction, but there are many difficulties in obtaining the various parameters and constants in their equations. In addition, it has also become apparent that fatigue life under HTLCF conditions depends not only on the testing temperature but also on the loading waveform due to creep damage. The transient effects such as cyclic hardening/softening have a significant effect on damage accumulation.

To improve the accuracy of HTLCF life prediction, researchers have presented several models [14–20]. For example, considering the dynamic viscosity as the damage parameter to describe the flow behavior, Goswami [14] proposed the Ductility Exhaustion (DE) approach for HTLCF life prediction which is based on extensive research of Cr-Mo steels. Recently, with the assumption that the creep effect of stress relaxation leads to the reduction of fatigue life, Jeong [3, 15] developed a stress relaxation range model (SRR). This model suggests that the relaxed stress range can be used as a creep-fatigue damage function and the Manson–Coffin curves under different conditions can be normalized to construct a master curve under strain controlled fatigue tests. Nam [16] has proposed another model for fatigue life prediction based on the nucleation and growth of grain boundary cavities of austenitic stainless steels. Based on DE theory and the effective stress concept, Fan et al. [17] presented a mean strain rate model under stress control, in which the mean strain rate is used as the main factor associated with the fracture life. When using this model, the predicted lives had good agreement with the observed ones in tests when DE was the dominant mechanism. Recently, Shang et al. [7, 18] developed two models for life prediction considering fatigue-creep interaction. One model uses the linear damage rule to predict the life of continuous cyclic loading at elevated temperature [18]. The second one is a time-dependent fatigue damage model without hold-time considering tension-compression strain rate and cycle period [7]. Zhu and Huang [19] proposed a Generalized Strain Energy Damage Function (GSEDF) model to account for the effects of different time-dependent damaging mechanisms on HTLCF life. This model has wider applications and is more precise to predict the life of fatigue-creep interaction. Nagode et al. [20] proposed a Damage Operator Approach (DOA) based on temperature-dependent cyclic damage calculation under low cycle thermo-mechanical fatigue. This approach focuses on fatigue damage while ignoring the effects of creep, viscoplasticity, or the interaction between them.

All reported models have shown some success on a limited number of alloys and loading conditions. Their limitations can be summarized as follows: (a) some cannot accurately describe the effects of different loading waveforms, for example, LDS, FMMC and FMDF; (b) most of them ignore the effects of mean strain or mean stress on the fatigue life, such as LDS, FS, DRA, SRR [3, 15], DE [14] and the models presented in [7, 18]; (c) many of the existing models are not applicable under stress controlled conditions, for example, SRP and SEP, DE [14] and SRR [3, 15]. The methods reported in [15–17] are limited to evaluation of HTLCF life under strain controlled conditions. In addition, many reported methods require specific types of tests to estimate the corresponding parameters or have too many "to be determined parameters or constants", thus restricting their usefulness, such as LDS, FS, SRP, SEP, DRA, GSEDF [19], models reported in [16] and DOA [20]. In summary, further improvement of HTLCF life prediction models is needed.

In this paper, first some background on the DE theory will be provided and then the concept of the generalized energy-based damage parameter for HTLCF life prediction as reported in [21] will be introduced. Then, based on the DE theory and the generalized energy-based damage parameter, a new life prediction model will be proposed that considers the effects of mean strain/stress on LCF lives at high temperature. It will be shown that by incorporating loading waveform parameters, such as strain rate, and mean stress, this model can describe the effects of different time-dependent damage mechanisms on the HTLCF life more accurately than those reported in the literature. Finally, to demonstrate the effectiveness of the proposed method, LCF life is assessed using experimental data of turbine disk material GH4133 taken from existing literature. The predicted lives were then compared with actual test data and a good agreement between them was observed.

2 Theoretical Background

2.1 Ductility Exhaustion Theory

Fatigue is a damage accumulation process in which the material property deteriorates continuously under cyclic loading. Goswami [14] developed a ductility model based on the assumption that deformation under HTLCF test conditions can be represented in terms of a viscous behavior. According to this viscosity-based model, the cyclic damage can be represented by viscous flow within a LCF test below the melting temperature of a material, T_m. Viscous flow occurs as loads are applied within the elastic and/or plastic regimes. Under cyclic loading, LCF is plastic strain dominated, but under high temperature conditions, the inelastic strain range dominates. The deformation caused by the loading is time dependent and is measured using the strain rate. With this method, dynamic viscosity is used as a damage parameter to represent the fatigue process. In a dwell fatigue cycle, the deformation depends upon the strain range and time and the rate of damage is in terms of the strain rate of the cycle. Therefore, dynamic viscosity can account for strain range effects and may be expressed as follows based on the fundamental viscosity concept [22]:

$$\text{Dynamic viscosity at failure} = \Delta\sigma\Delta\varepsilon_t/\dot{\varepsilon}. \qquad (1)$$

Cyclic and/or static loading triggers the initiation of the viscous flow accommodation process in a material. When the ability of a material to accommodate any further viscous flow ceases, as the dynamic viscosity reaches a critical value, failure occurs. The ability of a material to accommodate permanent deformation is reflected by the material toughness. It has been reported that material toughness is a more sensitive mechanical property parameter to the fatigue damage process than others [23]. The failure criterion is defined as when the dynamic viscosity becomes equal to the material toughness and can be expressed as

$$\sum f(\nu_d) = W_f, \qquad (2)$$

where W_f is the total cumulative energy dissipated by the material up to failure, also called the fatigue toughness. The toughness of a material is a product of its ductility and its cyclic strength [22],

$$\text{Material toughness} = \text{ductility} \times \text{strength}, \qquad (3)$$

where the strength can be indicated by the saturated stress range since strength in a cyclic fatigue test is in terms of the saturated stress range at a particular strain range. The continuous reduction of the material toughness during the fatigue process, or the continuous reduction of the material ductility, indicates the progressive exhaustion of the ability to absorb energy inherent in the material, which is directly associated with the increase of the material's internal energy resulting in the irreversible process of energy dissipation at fatigue failure [23]. The internal energy stored in the material by the formation of point defects such as vacancies and interstitials, the generation and rearrangement of dislocations and the formation of internal surfaces such as voids and cracks contributes to material damage in nature. This implies that the material toughness is a mechanical parameter that can be consistent with the fatigue damage physical mechanism.

Material ductility depends not only upon the test and material parameters, such as stress range, strain rate, and temperature, but also upon impurity content, grain size, relaxed stresses and other factors, such as cyclic hardening/softening [17]. Using the Edmund and White equation, ductility can be expressed as follows:

$$\text{Ductility} = \Delta\varepsilon_p N_f. \qquad (4)$$

Then, material toughness can be obtained by substituting (4) and the saturated stress range into (3):

$$\text{Toughness} = \Delta\varepsilon_p \cdot N_f \cdot \sigma_{\text{sat}}. \qquad (5)$$

In order to account for experimental conditions, curves for stress and strain rate with different hardness were constructed and showed a scaling relationship [14]. This behavior produced a linear equation with a slope of m between "cycle time" and cycles to failure. Since HTLCF life depends on test parameters, the scaling relationship may be obtained by plotting the ratio of the strain range over the strain rate (or cycle time) and the cycles to failure on a log-log scale.

The cyclic stress-strain response is an important material property [24]. It describes the relationship between flow stress and plastic strain amplitude under cyclic loading. Based on the Ramberg–Osgood relation [13], the cyclic stress-strain curve may be described by

$$\frac{\Delta\sigma}{2} = K'\left(\frac{\Delta\varepsilon_p}{2}\right)^{n'}, \qquad (6)$$

where $\Delta\sigma$ is the stress range with $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$. Based on (6), K' and n' can be obtained from the log-log linear regression analysis of the cyclic strain amplitudes and the corresponding cyclic stress amplitudes for fully reversed fatigue tests.

Based on the slope m, a ductility model for HTLCF life prediction was then given by [14]

$$\Delta\sigma \cdot (\Delta\varepsilon_t/\dot{\varepsilon})^m = A\Delta\varepsilon_p \cdot N_f \cdot \sigma_{\text{sat}}. \qquad (7)$$

Another expression of this model can be obtained by rearrangement of the parameters in (7) based on the cyclic stress-strain equation (6):

$$N_f = [K'(\Delta\varepsilon_p/2)^{n'-1}(\Delta\varepsilon_t/\dot{\varepsilon})^m]/(A \cdot \sigma_{\text{sat}}), \qquad (8)$$

where A is a model parameter which balances the units of the equation and is adjustable. The selection of A depends upon the test parameters employed, and some suggestions were given by Goswami [14].

In addition, in order to take dwell effects into consideration, a correction factor is introduced into the above equation, empirically, determined by data fitting

$$\dot{\varepsilon}^* = \dot{\varepsilon}/[1 + \log(t_H)], \tag{9}$$

where $\dot{\varepsilon}^*$ is the dwell time correction factor based on strain rate and t_H is the dwell time in second. Dwell effects were characterized by correction of strain rate with (9). In general, creep tests with dwell times are similar to tests under smaller strain rates with no dwell time. In the observed tests, longer dwell times corresponded to smaller corrected strain rates. The above factor was used only for the cycles that contained dwell times in (8).

It should be noted that (8) relates such parameters as stress, strain range and strain rate. However, HTLCF life prediction using this model is only applicable under continuous fatigue and slow-fast waveforms [14, 22]. Thus, the applicability of this model is restricted within narrow limits. Moreover, a certain degree of mean stress or mean strain exists in most fabricated components. Fully reversed cyclic loading is not representative of all potential applications of an engineering material. Hence, the mean stress or mean strain effects must be evaluated when developing fatigue life curves based on the results of fully reversed cyclic loading. Additionally, most engineering materials tend to exhibit a reduction in fatigue life with increasing mean stress when undergoing fatigue tests with a non-zero mean stress or mean strain. Consequently, mean stress plays an important role in determining the fatigue behavior of hot section components. The applicability of this model under other loading waveforms needs to be further improved.

2.2 A Generalized Energy-Based Fatigue Damage Parameter Considering the Mean Stress Effect

A certain quantity of energy is gradually dissipated due to cyclic fatigue and creep during HTLCF. Once the critical energy threshold is reached, fracture will occur. According to this relationship, the damage of materials can be characterized by the dissipated energy. Several energy criteria and corresponding assessments for fatigue life are based on this idea [14, 23, 25–27]. Under cyclic loading conditions, the relationship between the stress and the strain during deformation can be represented by hysteresis loops. The area within the hysteresis loop corresponds to the plastic strain energy input to the material [25]. Consequently, in terms of the stress-strain hysteresis loop under high temperature, various researchers have developed fatigue life curves by adopting an energy parameter to estimate the life of a material under different loading conditions.

It was found that the total plastic energy required for fatigue failure is not a constant but increases with a decrease in stress amplitude. This total energy depends on the stress or plastic strain amplitude via the cyclic stress-strain behavior [25]. This means that it is difficult or almost impossible to use this definition to measure fatigue damage directly in the vast majority of cases. Recent research indicates that it is better to use the plastic strain energy density (PSED) as the damage parameter rather than the plastic strain range for drastic hardening/softening conditions [25]. The stable PSED is commonly used as a damage parameter in predicting the fatigue life. Up to now, many attempts have been made to establish the fatigue criteria based on PSED [25–27]. By using the cyclic strain energy density parameter, Koh [26] investigated the fatigue life of high pressure tube steel under strain-controlled tests. The total cyclic strain energy density provided good prediction of the fatigue behavior of this steel. According to the mean strain effects in a LCF regime, Chiou [27] proposed a modified PSED energy parameter for life prediction. Since the magnitude of this damage parameter is half of that of PSED, there are no intrinsic differences in using either of these two parameters for life prediction. Recently, to account for the effect of temperature on fatigue life, Lee et al. [25] developed an energy-based life prediction model using PSED for different materials. The predicted lives by this model were within a factor of 2.5, which shows an overestimating tendency as temperatures increased.

According to the damage parameters reviewed above [25, 27], degradation mechanisms such as creep and mean stress effects have not been adequately addressed. These effects are considered to be important factors affecting the fatigue resistance at high temperatures [28]. In order to account for the effects of creep and mean stress on the LCF lives at high temperature accurately, in addition to including different loading waveforms, strain rate, and transient effects, a new damage parameter for life prediction based on PSED and FMDF methods has been developed [21].

Under cyclic loading, the plastic strain energy per cycle can be considered a measure of the amount of fatigue damage per cycle. The movement of dislocations is described by the cyclic plastic strain and the resistance against their motion is described by the cyclic stress [25]. In LCF, a considerable amount of plastic strain by the material and the hysteresis energy absorbed during the fatigue cycling has been postulated as a basis for failure analysis. Therefore, the fatigue resistance of material can be characterized in terms of its capacity to absorb and dissipate plastic strain energy. Morrow [13, 25] expressed the relation between PSED and the fatigue life as

$$\Delta W_p \cdot N_f^{\alpha} = C_1. \tag{10}$$

If a material satisfies Masing's hypothesis [1], the PSED absorbed per cycle is the inner area of the cyclic stress-strain hysteresis loop [25]:

$$\Delta W_p = \frac{1-n'}{1+n'} \cdot \Delta\sigma \cdot \Delta\varepsilon_p. \tag{11}$$

Materials for which the hysteresis loop can be described by magnifying the cyclic stress-strain curve by a factor of 2 are said to exhibit Masing behavior [29]. Some previous studies [30,31] revealed that material used in this study shows Masing behavior. Equation (11) can be written in terms of the cyclic plastic strain amplitude using the relationship given in (6).

$$\Delta W_p = 4K' \frac{1-n'}{1+n'} \cdot \left(\frac{\Delta\varepsilon_p}{2}\right)^{1+n'}. \tag{12}$$

The PSED can be measured from the half-life hysteresis loop and also calculated by using (12) and the LCF properties. In general, the measured plastic strain energy was in good agreement with the calculated strain energy for turbine disk alloys showing Masing behavior. The LCF life at high temperature is dependent on the test parameters. Due to time-dependent damaging mechanisms under high temperature such as creep and environmental corrosion, experimental results from [5–7] showed that both the shape and the size of the hysteresis loop are influenced by cyclic frequency, loading waveform, strain rate and cyclic hardening/softening. Enlightened by this characteristic, an attempt has been made to deduce a new damage parameter model for life prediction, in which the fatigue toughness is used as the control parameter, as explained below.

Similar to the material toughness calculated with (3), Ostergren proposed the strain energy damage function model [13]. The strain energy damage function ΔW_s can be expressed approximately by the multiplication of the inelastic strain range $\Delta\varepsilon_{\text{in}}$ and the maximum tension stress $\sigma_{\max}$, i.e.

$$\Delta W_s = \Delta\varepsilon_{\text{in}}\sigma_{\max}, \tag{13}$$

where $\Delta\varepsilon_{\text{in}}$ can be replaced by the plastic strain range $\Delta\varepsilon_p$ under the pure fatigue mode. The relationship between strain energy and fatigue life can be expressed by the power exponent function as

$$\Delta W_s N_f^{\beta} = C_2. \tag{14}$$

Under LCF conditions, $\Delta\varepsilon_{\text{in}}$ is approximated by $\Delta\varepsilon_p$. By integrating (12), a new expression to describe the process of HTLCF is obtained as

$$N_f = \left(\frac{C_2^{1+n'}}{2^{n'-1}\sigma_{\max}^{1+n'}\frac{1+n'}{(1-n')K'}\Delta W_p}\right)^{\frac{1}{\beta(1+n')}}. \tag{15}$$

Then, a new life evaluation relation can be described by the following equation (16) after rearranging various terms and further simplifying (15):

$$(\Delta W_p \sigma_{\max}^{1+n'}) \cdot N_f^{\beta(1+n')} = C_3, \tag{16}$$

$$C_3 = \frac{C_2^{1+n'}(1-n')K'}{2^{n'-1}(1+n')}. \tag{17}$$

In (16), it should be noted that $\Delta W_p \sigma_{\max}^{1+n'}$ follows a certain dependency with LCF life, which includes factors influencing fatigue and creep life and also takes into account the mean stresses. So $\Delta W_p \sigma_{\max}^{1+n'}$ was called a generalized energy-based fatigue-creep damage parameter.

The main factor influencing fatigue life is σ_a and the main factor influencing creep life is σ_m. It is worth noting that the generalized damage parameter ($\Delta W_p \sigma_{\max}^{1+n'}$) and the SWT parameter ($\sigma_{\max}\varepsilon_a$) [32] have similar forms even though they are derived from different theoretical backgrounds. They considered the effect of the mean stress on the predicted life through the maximum stress, where $\sigma_{\max} = \sigma_a + \sigma_m$. Combining (16)–(17), the mean stress effect on HTLCF life can be characterized by the proposed damage parameter. Moreover, the mechanism of cyclic hardening effect has been incorporated into the proposed damage parameter when using it for HTLCF life prediction.

To reflect the effects of mean stress/strain accurately, achieve higher utilization of test data, and expand the application of the model to different loading waveform conditions, a new model based on ductility exhaustion theory and the generalized energy-based fatigue-creep damage parameter has been developed in the next section.

3 A New Viscosity-Based Life Prediction Model for HTLCF

According to interaction mechanisms of failure at high temperature, a good life prediction method must consider not only the effects of mean strain/stress, but also creep factors such as frequency, strain rate and temperature. The main idea of DE theory is that the reduction of the fatigue ductility during the fatigue process indicates the progressive exhaustion of the ability to absorb energy inherent in the material due to fatigue damage evolution, which can be associated with the irreversible process of energy (ductility) dissipation leading to fatigue failure. In this section, using the proposed energy-based damage parameter to describe the relationship between the stress-strain and failure life and the main idea of DE theory, a new viscosity-based model is proposed.

The fatigue toughness that synthesizes both strength and plasticity of a material is a mechanical property parameter sensitive to the fatigue damage process. Based on the failure criterion of DE theory in (2), which assumes that the dynamic viscosity becomes equal to the fatigue toughness, the fatigue toughness can be determined using (16) as follows:

$$\text{Fatigue toughness} = (\Delta W_p \sigma_{\max}^{1+n'}) \cdot N_f^{\beta(1+n')}. \quad (18)$$

As discussed earlier, a new HTLCF life prediction equation can be derived equating these two terms, namely: dynamic viscosity equation (1) and fatigue toughness equation (18). Therefore, a new life prediction equation is derived, by rearranging the following equation:

$$\sum f(\nu_d) = (\Delta W_p \sigma_{\max}^{1+n'}) \cdot N_f^{\beta(1+n')}. \quad (19)$$

Similarly, a scaling relationship can be obtained by plotting the strain range to strain rate ratios and cycles to failure on a log-log scale. This behavior produced a linear equation with a slope of m. By using (1) and introducing the slope m, equation (19) can be rewritten as

$$B\Delta\sigma(\Delta\varepsilon_t/\dot{\varepsilon})^m = (\Delta W_p \sigma_{\max}^{1+n'}) \cdot N_f^{\beta(1+n')}, \quad (20)$$

where B is a material parameter which balances the units of the equation and is adjustable. Combining (20) with the cyclic stress-strain equation by substituting (12) into (20) results in the following equation:

$$2BK'\left(\frac{1+n'}{4K'(1-n')}\Delta W_p\right)^{\frac{n'}{1+n'}} \cdot \left(\frac{\Delta\varepsilon_t}{\dot{\varepsilon}}\right)^m = (\Delta W_p \sigma_{\max}^{1+n'}) \cdot N_f^{\beta(1+n')}. \quad (21)$$

Rearranging various terms in (21) leads to a new model for the life prediction of HTLCF:

$$N_f = C_4\left(\frac{K'}{\Delta W_p \sigma_{\max}^{(1+n')^2}}\right)^{\frac{1}{\beta(1+n')^2}} \left(\frac{\Delta\varepsilon_t}{\dot{\varepsilon}}\right)^{\frac{m}{\beta(1+n')}} \quad (22)$$

and

$$C_4 = 2^{\frac{1-n'}{\beta(1+n')^2}} B^{\frac{1}{\beta(1+n')}} \left(\frac{1+n'}{1-n'}\right)^{\frac{n'}{\beta(1+n')^2}}. \quad (23)$$

In (22), it is worth noting that the cyclic stress strain parameters K' and n' can be easily derived by appropriate data fitting for each material based on (6). The two material parameters (m and β) can be fitted from test data, which allows (22) to be easily used to predict the LCF life under high temperature.

It follows from the above analysis that the HTLCF life predicted in terms of the exhaustion of the fatigue ductility actually represents the degradation of both plasticity and strength of a material under cycling loading. Thus, this definition can contain some existing models that define the damage variable as the variation in the ductility and the viscosity separately [14, 23] as special cases. In addition, the development of this new model considers not only the cyclic hardening effects on the HTLCF life, but also the effects of mean strain/stress in this viscosity-based model under different loading waveform conditions. Validation of the proposed model to accurately predict the HTLCF life will be evaluated by analyses of LCF life prediction under high temperature with experimental data in the next section.

4 Validation of the New Life Prediction Model for HTLCF

To verify the validity and accuracy of the viscosity-based life prediction model for HTLCF, the proposed model will be evaluated using experimental results of turbine disk material GH4133 [33]. This alloy is a nickel-base Superalloy GH4133, which is used for the turbine disk material in jet engines. The heat treatment conditions of the alloy are as follows: austenitization (8 h at 1353.15 K, air-cooled) and tempering (16 h at 1023.15 K, air-cooled). In this section, three sets of experimental results are used to evaluate the model's ability to account for both the cyclic hardening effects and the mean strain/stress effects on the HTLCF life. The data used here is provided by Beijing Institute of Aeronautical Materials (China). Details on the mechanical properties of the materials, test conditions, parameters and strain-life data are reported in [33].

The main factor influencing fatigue life is σ_a and the main factor influencing creep life is σ_m. The tests were performed under axial total strain control with a triangular fully reversed waveform, using an axial extensometer placed on the specimen. Numerous tests were carried out with the following various conditions: mechanical strain range of 0.5%–1.4% for isothermal LCF at temperature 773.15 K, 673.15 K under strain ratio $R_\varepsilon = -1$ and mechanical strain range of 0.5%–1.2% for isothermal LCF at temperature 673.15 K under $R_\varepsilon = 0$, respectively. The test parameters and results are given in Tables 1–3.

According to the results of fully reversed fatigue tests, the Superalloy GH4133 exhibits a slender cyclic hardening characteristic at 773.15 K. By the least square method, the calculated values of K' and n' in (6) are 1716.5 MPa and 0.11068, respectively.

Life predictions of LCF were calculated using (22) and taking the experimental results and the parameters listed in Tables 1–2. Under different total strain control and strain ratio $R_\varepsilon = -1$, the fitted life prediction model for GH4133 at 773.15 K is given by

$$N_f = 1.44845 \cdot \left(\frac{1.71651 \times 10^9}{\Delta W_p \sigma_{\max}^{1.23361}}\right)^{0.57798} \cdot \left(\frac{\Delta\varepsilon_t}{\dot{\varepsilon}}\right)^{-1.3619\times 10^2}, \quad (24)$$

and the material constants in (22) are $\beta = 1.40251$ and $m = -2.12148 \times 10^2$.

The proposed model given in (24) can then be used for HTLCF life prediction based on DE theory. To reflect the capability of this new model and evaluate its applicability under the effects of mean stress/strain, two other methods, Goswami's ductility model and the generalized damage parameter $\Delta W_p \sigma_{\max}^{1+n'}$ [21], were employed for comparison respectively.

According to Goswami's ductility model in (8), the fatigue life can be approximately fitted as

$$N_f = \left[2.28161 \times 10^{-3} (\Delta\varepsilon_p)^{-0.88932} \cdot (\Delta\varepsilon_t/\dot{\varepsilon})^{-2.12148\times10^2}\right]/\sigma_{\text{sat}}, \quad (25)$$

where σ_{sat} is the saturated tension stress at half-life.

The coefficient and the exponent of the generalized damage parameter in (16) can be determined from a log-log linear regression analysis of fatigue life data. The fatigue life curve for GH4133 at 773.15 K can be expressed as

$$(\Delta W_p \sigma_{\max}^{1.11068})^{0.5825} N_f = 1.22885 \times 10^{14}. \quad (26)$$

Similarly at 673.15 K, the fitted life prediction model for GH4133 under strain ratio $R_\varepsilon = -1$ by the above-mentioned three methods can be written as follows:

Based on the proposed viscosity-based model,

$$N_f = 1.42022 \cdot \left(\frac{1.13596 \times 10^9}{\Delta W_p \sigma_{\max}^{1.26936}}\right)^{0.5502} \cdot \left(\frac{\Delta\varepsilon_t}{\dot{\varepsilon}}\right)^{-1.36785\times10^2}. \quad (27)$$

Based on Goswami's ductility model,

$$N_f = \left[8.61198 \times 10^{-4} (\Delta\varepsilon_p)^{-0.87334} \cdot (\Delta\varepsilon_t/\dot{\varepsilon})^{-2.20662\times10^2}\right]/\sigma_{\text{sat}}. \quad (28)$$

And based on the generalized damage parameter,

$$(\Delta W_p \sigma_{\max}^{1.12666})^{0.5583} N_f = 5.72095 \times 10^{13}. \quad (29)$$

The LCF lives predicted by these three methods are listed in Tables 1–2. The correlations between the experimental and the predicted fatigue lives are shown in Figures 1–2, in which comparisons between tests and prediction by Goswami's ductility model, the generalized damage parameter, and the new model are given. The dashed line in the graph corresponds to the ±1.5 factor indictors and the solid line for the ±2 factor indictors. Two evaluating parameters of life assessments were used: scatter band and standard deviation, the former describes the scatter of the predicted extreme point and the degree of deviation between test and prediction and the latter shows the degree of data points deviating from the average. The scatter band of the predicted life vs. the tested one is given by

$$\text{Scatter band} = \max(N_{fp}/N_{ft}, N_{ft}/N_{fp}). \quad (30)$$

The results show that all the predicted cyclic lives by the new model and the generalized damage parameter are within a factor of ±2 to the test ones and only about 85% of the values predicted by Goswami's ductility model are within a factor of ±2. From Tables 1–2, it should be noted that none of the three methods can consider the plastic strain because its range is so small (less than about 0.001). If the calculated plastic strain range is substituted into (12), the strain energy is negligible. Thus, the points with those small plastic strain ranges will be considered invalid data points when fitting the strain energy-fatigue life curve. However, when using the limited amount of test data, both the proposed method and the generalized damage parameter make a better prediction than Goswami's ductility model under zero mean strain conditions, with which 44 out of 55, 42 out of 55 and 34 out of 55 cyclic lives are predicted within a factor of ±1.5, respectively. Comparing the scatter band and the standard deviation of those methods, results indicate that the proposed model has a narrower band than the other two methods.

Moreover, in order to investigate the mean strain effect on the fatigue response of supperalloy GH4133 and verify the prediction effect of the new model developed in this paper, a series of strain-controlled LCF tests were performed at the mean strain levels of $\varepsilon_m = 0.6\%, 0.5\%, 0.4\%, 0.35\%, 0.3\%$ and 0.25%, with strain amplitudes ε_a ranging from 0.25% to 0.6%. The fatigue tests were carried out under strain-controlled conditions and a triangular waveform. The heat treatment conditions of the Superalloy GH4133 are the same as mentioned above. The experimental parameters and the results of the fatigue tests conducted with non-zero mean strains are summarized in Table 3.

Combining the experimental data and the parameters listed in Table 3, it can be shown that the fatigue life curve for GH4133 at 673.15 K under $R_\varepsilon = 0$ is given by the following:

Based on the proposed viscosity-based model,

$$N_f = 2.36582 \cdot \left(\frac{5.35289 \times 10^8}{\Delta W_p \sigma_{\max}^{1.48462}}\right)^{1.34818} \cdot \left(\frac{\Delta\varepsilon_t}{\dot{\varepsilon}}\right)^{-3.08518\times10^2}. \quad (31)$$

Based on Goswami's ductility model,

$$N_f = \left[0.9304 \cdot (\Delta\varepsilon_p)^{-0.78155} \cdot (\Delta\varepsilon_t/\dot{\varepsilon})^{-1.87813\times10^2}\right]/\sigma_{\text{sat}}. \quad (32)$$

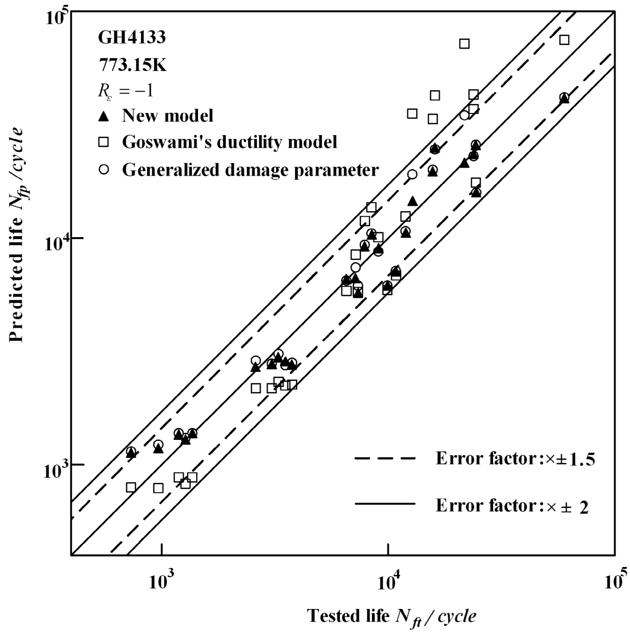

Figure 1. Comparison between lives predicted by new model, Goswami's ductility model, generalized damage parameter and lives tested for GH4133 at 773.15 K.

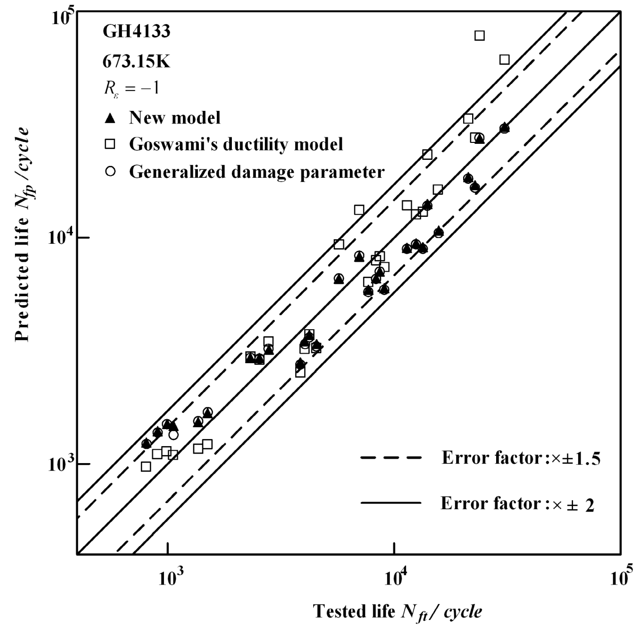

Figure 2. Comparison between lives predicted by new model, Goswami's ductility model, generalized damage parameter and lives tested for GH4133 at 673.15 K.

And based on the generalized damage parameter,

$$(\Delta W_p \sigma_{\max}^{1.21845})^{1.4011} N_f = 1.39628 \times 10^{30}. \qquad (33)$$

Comparisons between the experimental results and the theoretical predictions by Goswami's ductility model, the generalized damage parameter, and the new model under different mean strain level are shown in Figure 3.

Regarding the relationship between the fatigue life and the mean strain effect, the results of Tables 1–3 reveal that different combinations of mean strain and strain amplitude have different effects on the fatigue life. From experimental observation, it can be concluded that non-zero mean strain causes a change in the fatigue life. In general, a relaxation of the mean stress occurs in materials subjected to cyclic straining with a non-zero mean strain. If the mean stress had relaxed to zero, the imposed mean strain has no effect on the fatigue life. Moreover, a stable non-zero tensile mean stress often reduces the fatigue life but a compression one increases it due to mean strain action. Hence, it can be shown that the mean strain effect on the fatigue life of GH4133 Superalloy is mainly attributed to the presence of a stable non-zero mean stress. In other words, the mean stress is the main factor influencing life rather than the mean strain; the effects of mean strain on the fatigue life are not considerable unless they accompany the mean stresses [34]. Consequently, this paper develops a new viscosity-based model, which takes into account of the effects of mean strain/stress to develop life predictions.

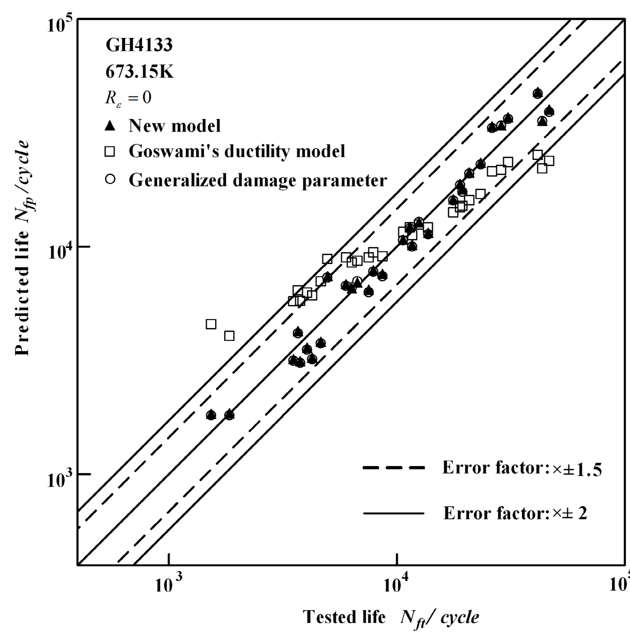

Figure 3. Comparison between lives predicted by new model, Goswami's ductility model, generalized damage parameter and lives tested for GH4133 under different mean strain levels at 673.15 K.

$\Delta\varepsilon_t/2$ (%)	$\Delta\varepsilon_e/2$ (%)	$\Delta\varepsilon_p/2$ (%)	$\Delta\sigma/2$ (MPa)	Cyclic life tested N_{ft} (cycle)	N_{fp} predicted by Goswami's ductility model (cycle)	N_{fp} predicted by the generalized damage parameter [21] (cycle)	N_{fp} predicted by the proposed viscosity-based model (cycle)
0.703	0.452	0.251	884	1382	807	1397	1373
0.695	0.440	0.255	874	1284	805	1403	1380
0.704	0.458	0.246	882	756	823	1417	1393
0.702	0.466	0.236	857	1226	879	1504	1480
0.702	0.444	0.258	883	993	788	1374	1352
0.500	0.400	0.100	792	3840	2042	2733	2693
0.498	0.407	0.091	759	3353	2317	3042	3004
0.498	0.400	0.098	778	2592	2116	2827	2787
0.501	0.407	0.094	788	3101	2168	2852	2809
0.502	0.406	0.096	792	3523	2117	2800	2757
0.401	0.378	0.023	714	7163	8368	7311	7197
0.398	0.384	0.014	732	7888	12693	9462	9286
0.400	0.370	0.030	711	10812	6635	6294	6207
0.402	0.366	0.036	732	7487	5480	5495	5380
0.401	0.370	0.031	703	7172	6518	6261	6180
0.400	0.360	0.040	691	9936	5286	5513	5453
0.350	0.347	0.003	650	16365	56246	26302	26369
0.350	0.346	0.004	664	15722	42631	21877	21724
0.349	0.336	0.013	649	8473	15291	11220	11321
0.349	0.328	0.021	643	9031	10075	8770	8684
0.351	0.347	0.004	674	12762	41999	19952	11309
0.350	0.334	0.016	643	12054	12831	10280	10162
0.301	0.297	0.004	565	24303	50101	26915	26758
0.300	0.298	0.002	548	22081	95679	39810	21550
0.301	0.288	0.013	555	24836	17881	13900	13856
0.300	0.295	0.005	582	24155	39883	22909	22638
0.251	0.247	0.003	481	59542	76007	39810	38897
0.251	0.251	0	484	51455	—	—	—

Table 1. Experimental parameters and life predictions of the new model, Goswami's ductility model, and the generalized damage parameter for Superalloy GH4133 under strain ratio $R_\varepsilon = -1$ and temperature 773.15 K.

The correlation between the mean stress effect and the fatigue life can be interpreted using the proposed model. By using this new model for life prediction under non-zero mean strains as shown in Table 3 and Figure 3, the results show that all the predicted cyclic lives both by the new model and the generalized damage parameter are within a factor of ± 1.5 to the test ones, which is better than Goswami's ductility model (only about 19 out of 31 cyclic lives are predicted within a factor of ± 1.5). It should also be noted that about 77% of cyclic lives predicted by the proposed model are within a factor of ± 1.25, while only 74% and 29% of the cyclic lives predicted by the generalized damage parameter and Goswami's ductility model, respectively, are within a factor of ± 1.25. Obviously, the proposed model and the generalized damage parameter are better life prediction models than Goswami's ductility model with or without an applied mean strain. As stated above, the effect of mean stress on the fatigue life has been incorporated into the new model and the damage parameter $\Delta W_p \sigma_{\max}^{1+n'}$ under mean strain conditions.

Based on the same theoretical backgrounds, the differences between the experimental and calculated LCF life by the proposed method and the generalized damage parameter are relatively small because these models consider the effect of mean stress. Moreover, the new model incorporates more loading waveform parameters rather than the generalized damage parameter. According to the theoretical derivation of the new model and comparisons of the prediction results by these models, it is better to use the proposed model rather than Goswami's ductility model for drastic hardening/softening and mean strain conditions. As the value of n' increases from 0.11068 at 773.15 K to 0.12666 at

$\Delta\varepsilon_t/2$ (%)	$\Delta\varepsilon_e/2$ (%)	$\Delta\varepsilon_p/2$ (%)	$\Delta\sigma/2$ (MPa)	Cyclic life tested N_{ft} (cycle)	N_{fp} predicted by Goswami's ductility model (cycle)	N_{fp} predicted by the generalized damage parameter [21] (cycle)	N_{fp} predicted by the proposed viscosity-based model (cycle)
0.696	0.487	0.209	964	1396	1207	1568	1566
0.705	0.451	0.254	894	1071	1098	1538	1546
0.702	0.477	0.225	931	1423	1172	1569	1571
0.703	0.448	0.245	901	916	1124	1555	1561
0.701	0.456	0.245	915	903	1107	1527	1532
0.699	0.459	0.240	914	975	1128	1546	1551
0.479	0.411	0.086	827	4066	3056	3089	3092
0.498	0.426	0.073	836	4091	3488	3342	3338
0.499	0.393	0.106	852	3962	2471	2653	2655
0.500	0.416	0.084	832	2556	3100	3108	3108
0.497	0.412	0.085	837	4538	3050	3066	3066
0.501	0.436	0.065	863	2820	3739	3434	3420
0.499	0.420	0.079	821	2389	3315	3267	3269
0.400	0.363	0.037	736	9180	7172	5682	5688
0.398	0.382	0.016	751	7123	14616	8858	8797
0.400	0.373	0.027	752	8842	9243	6604	6586
0.400	0.359	0.041	736	7775	6557	5365	5376
0.399	0.377	0.022	772	5721	10766	7177	7134
0.400	0.367	0.033	716	8722	8147	6258	6270
0.349	0.329	0.020	691	12720	13072	8634	8634
0.350	0.341	0.009	667	13941	27199	14062	14000
0.350	0.335	0.015	678	15913	17128	10369	10357
0.350	0.330	0.020	689	13577	13110	8663	8665
0.350	0.334	0.016	723	12239	15182	9267	9225
0.300	0.290	0.010	584	22705	28334	15524	15597
0.300	0.297	0.003	608	22712	77885	28985	28765
0.299	0.292	0.007	605	22200	37346	18167	18160
0.300	0.297	0.003	589	30816	80398	30099	29929
0.250	0.250	0	490	29084	—	—	—

Table 2. Same as Table 1 but for temperature 673.15 K.

673.15 K, the Superalloy GH4133 exhibits a more obvious cyclic hardening characteristic at 673.15 K than 773.15 K with zero mean strains. Compared with fatigue tests under the same temperature and zero mean strain conditions, it exhibits a more obvious cyclic hardening characteristic with non-zero mean strains, while the value of n' increases from 0.12666 with zero mean strains to 0.21845 with non-zero mean strains at 673.15 K. To a great extent, the mechanism of cyclic hardening effect has been incorporated into the proposed model when using (22) for HTLCF life prediction.

It can be concluded from Tables 1–3 and Figures 1–3 that accurate life prediction results for Superalloy GH4133 with zero or non-zero mean strains can be obtained by using both the proposed model and the generalized damage parameter. And this new model has higher precision of life prediction than the other models. Moreover, it has a higher accuracy of life prediction from results of GH4133 with non-zero mean strains than that zero mean strain conditions. For the practical applications of hot section components, a certain degree of mean stress/strain exists in these components. Thus it can be better used for life evaluation of these components in the actual loading under non-zero mean stress/strain conditions.

The new viscosity-based model can reflect the fundamentals of fatigue damage under strain control, and has the following advantages including simple form, limited number of life prediction parameters, considers mean stress/strain and loading waveforms effects, and has a higher precision of life prediction. This model is applicable to HTLCF life prediction under strain controlled conditions. The application of this model to other conditions

$\varepsilon_{t\,\max}$ (%)	$\varepsilon_{e\,\max}$ (%)	$\varepsilon_{p\,\max}$ (%)	$\sigma_{\max}$ (MPa)	Cyclic life tested N_{ft} (cycle)	N_{fp} predicted by Goswami's ductility model (cycle)	N_{fp} predicted by the generalized damage parameter [21] (cycle)	N_{fp} predicted by the proposed viscosity-based model (cycle)
1.200	0.433	0.767	863	1534	5240	2067	2072
			896	1871	5047	1840	1827
1.000	0.412	0.588	802	4244	6941	3769	3790
			834	3553	6674	3337	3325
			789	3576	7055	3965	4003
			837	4143	6651	3300	3285
			864	4329	6443	2990	2953
			830	4293	6707	3387	3379
0.800	0.367	0.433	780	7533	9064	6308	6284
			734	7778	9633	7620	7704
			757	6703	9340	6923	6947
			744	8800	9503	7306	7362
			743	4707	9516	7336	7395
			740	6249	9554	7430	7496
			761	641	9291	6811	6826
0.700	0.366	0.334	693	12619	12497	13107	13253
			730	13611	11864	11151	11134
			704	11457	12302	12481	12572
			735	10734	11783	10916	10882
			730	12285	11864	11150	11134
0.600	0.339	0.261	659	20852	15936	21651	21871
			701	19717	14981	17868	17783
			686	19411	15308	19110	19119
			714	18465	14708	16876	16721
			644	23660	16307	23257	23625
0.500	0.320	0.180	639	30727	21972	40103	40018
			651	26570	21567	37849	37599
			672	44070	20893	34293	33807
			659	29960	21305	36439	36092
			647	45090	21700	38582	38384
			599	41802	23439	49027	49692

Table 3. Experimental parameters and life predictions of the new model, Goswami's ductility model, and the generalized damage parameter at 673.15 K for Superalloy GH4133 under strain ratio $R_\varepsilon = 0$.

such as different loading waveform and strain rate will be further investigated.

5 Conclusions

Based on DE theory and the generalized energy-based damage parameter, a new viscosity-based life prediction model was developed to account for the effects of creep and mean stress on LCF life at high temperatures. The feasibility and validity of this proposed model was evaluated using the LCF test data of Superalloy GH4133. For comparison purposes, the generalized damage parameter and Goswami's ductility model were applied to predict the fatigue life. The prediction results obtained using these three models have been compared to the experimental data. Based on the results and analyses on LCF tests, the following conclusions can be drawn:

(1) Goswami's ductility model can not properly account for the mean stress effects on LCF life. The proposed model, on the other hand, considers not only the mechanisms of cyclic hardening effects, but also the effects of mean strain/stress on the LCF life. Based on DE theory, this model can transform the complicated relationship between N_f and loading waveform parameters ($\sigma_{\max}$, σ_m, σ_a, $\Delta\varepsilon_p$, strain rate) into a simple relation via the cyclic stress-strain behavior on a log-log scale.

(2) Both the proposed model and the generalized damage parameter yield more satisfactory life prediction results for Superalloy GH4133 than Goswami's ductility model. Moreover, all of the test data were within a factor of ± 2 and nearly 87.2% of the test data were within a factor of ± 1.5 of the predicted results by the new model. This is higher precision of life prediction than the other models. Under non-zero mean strain conditions, it provides more accurate predictions of GH4133 than that with zero mean strains, with the entire test data within a factor of ± 1.5 and nearly 77.4% within a factor of ± 1.25 of the predicted ones.

(3) It is worth suggesting that the life prediction model developed in this paper is mainly based on the facts of the progressive exhaustion of the ability to absorb energy (ductility) inherent in the material during fatigue failure process and the basic viewpoint of the irreversible process of energy (ductility) dissipation of fatigue failure. Thus, this model has a definite physical meaning and a strong experimental basis. The main characteristics of this model compared with some existing models for HTLCF life prediction based on ductility is that it takes the effects of mean stress/strain into account and can be applicable to cyclically non-stabilized materials under fatigue loading. Accordingly, it is more adapted to describe the dynamic deterioration process of various high temperature structural materials and hot section components even under thermal or thermal-mechanical fatigue.

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