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NUMERICAL ANALYSIS OF GENERAL SHELLS OF
REVOLUTION SUBJECTED TO ARBITRARY LOADING

by



AHMED MOHAMED EL-SHAZLY

A. THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
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The undersigned certify that they have read, and
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SHELLS OF REVOLUTION SUBJECTED TO ARBITRARY LOADING
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ABSTRACT

The governing partial differential equations of a classical shell theory are reduced to a set of eight first order ordinary differential equations. A forward numerical integration process is used to obtain influence coefficients and particular solutions for shells of revolution of general geometric configuration subjected to arbitrary types of loading.

Standard stiffness methods of structural analysis are employed to obtain displacements and stress resultants everywhere within a complex shell structure. A computer program is developed to perform the analysis based on the theory presented. Example problems are selected to test the accuracy of the method. Excellent results, when compared with known solutions, are achieved.

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NOMENCLATURE

- a = constant and equal to the throat radius of the hyperboloid shell of revolution (Eq. 4.9).
- [A] = flexibility matrix in Eq. 3.2.
- [A₁], [A₂], {A₃}
- = coefficient matrices defined in Table 2.1
- [A_(s)] = coefficient matrix relating the shell fundamental variables and their derivatives at the location s (Eq. 3.5).
- b = constant
- [B₁], [B₂], {B₃}
- = coefficient matrices defined in Tables 2.2.1 and 2.2.2.
- {B_(s)} = column vector of the loading terms in the basic set of equations (Eq. 3.5)
- {c} = column vector of the eight arbitrary constant of integration defined in Eq. 3.9.
- [C₁], [C₂], {C₃}
- = coefficient matrices defined in Tables 2.3.1 and 2.3.2.
- D = extensional rigidity of the shell defined in Eq. A.18.1
- {D} = column vector of the displacements defined in Eq. 2.19.1
- {D'} = derivative of {D} with respect to s (Eq. 2.19.2)
- E = modulus of elasticity.
- [Ec] = eccentricity transformation matrix defined in Eq. 3.31.
- {F_s} = column vector of primary stress resultants defined in Eq. 2.15.1.
- {F'_s} = derivative of {F_s} with respect to s (Eq. 2.15.2)

- $\{F_\theta\}$ = column vector of secondary stress resultants defined
 in Eq. 2.15.3.
- $\{E^0\}$ = column vector of the fixed end forces in Eq. 3.3.
- G = shear modulus defined in Eq. A.13.
- $\{h_{(s)}\}$ = column vector of the homogeneous solution of the eight
 fundamental equations (Eq. 3.6).
- $[H_{(s)}]$ = transfer matrix arises from integrating the inhomogeneous
 terms in the eight fundamental equations (Eq. 3.11).
- K = flexural rigidity of the shell defined in Eq. A.18.2.
- $[K]$ = stiffness matrix
- $[L]$ = transformation matrix from local to global coordinates
 defined in Eq. 3.29.
- M_s, M_θ = meridional and circumferential moments per unit
 length, respectively.
- $M_{s\theta}, M_{\theta s}$ = circumferential and meridional twisting moments per
 unit length, respectively.
- n = harmonic number
- N_s, N_θ = normal in-plane forces per meridional and circumferential
 unit length, respectively.
- $N_{s\theta}, N_{\theta s}$ = in-plane shear forces per meridional and circumferential
 unit length, respectively.
- P_s, P_z, P_θ = intensity of load components in the direction s , z and
 θ respectively.
- Q_s, Q_θ = transverse shear forces per meridional and circumferential
 unit length, respectively.

$\{Q_{(s)}\}$ = column vector arises from integrating the inhomogeneous terms in the eight fundamental equations (Eq. 3.14).

r = radius of curvature of parallel circles.

r_1 = radius of curvature of meridian.

r_2 = length of the normal between any point on the middle surface and the axis of revolution.

r_i = derivative of r_1 with respect to the coordinate s .

R = curvature of parallel circles.

R_1 = first principle curvature = $1/r_1$.

R_2 = second principle curvature = $1/r_2$.

s = coordinate measures the distance along the meridian of the shell.

S_s = effective transverse shearing force per circumferential unit length defined in Eq. 2.6.1.

t = thickness of the shell.

T = change in temperature from arbitrary level.

T_s = effective tangential shearing force per circumferential unit length defined in Eq. 2.6.2.

T_{o1}, T_{o2} = temperature terms defined in Eq. A.19.1.

T_{11}, T_{12} = temperature terms defined in Eq. A.19.2.

T^o, T^i = change in temperature from arbitrary level x measured at the exterior and interior face of the shell, respectively.

U = displacement component in the circumferential direction.

V = displacement component in the meridional direction.

- W = displacement component in the rotation direction.
 U_z, V_z, W_z = displacement components of a point at distance z from the middle surface.
 X = coordinate measures the distance along the axis of revolution.
 $\{y_{(s)}\}$ = column vector of the eight fundamental variables (Eq. 3.5).
 $\{y'_{(s)}\}$ = derivative of $\{y_{(s)}\}$ with respect to the coordinate s .
 α = coefficient of thermal expansion.
 β = rotation of the meridian due to deformation.
 γ = specific weight of the shell material.
 $\gamma_{s\theta}, \gamma_{\theta s}$ = shear strain.
 ϵ_s = meridional strain.
 ϵ_θ = hoop strain.
 θ = coordinate measures the angle in the circumferential direction.
 ν = Poisson's ratio.
 σ_s, σ_θ = normal stresses in the meridional and circumferential direction, respectively.
 $\tau_{s\theta}, \tau_{\theta s}$ = shearing stresses.
 ϕ = coordinate measures the angle between any point on the middle surface and the axis of revolution.
 $()'$ = $\frac{\partial ()}{\partial s}$
 $()''$ = $\frac{\partial ()}{\partial \theta}$

CHAPTER 1

INTRODUCTION

1.1 Introductory Remarks

Shells of revolution are important structural elements. Many structures such as storage tanks, pressure vessels, silos, chimneys and towers are composed of either a single shell unit or an assemblage consisting of different types of shells. Their behaviour allows the shell thickness to be reduced to a minimum and their advantageous shapes permit more modern architectural concepts.

Although the governing field equations have been known for many years, cases where analytical solutions can be obtained are relatively scarce and are restricted to simple forms of geometry, boundaries and loads. The determination of the forces and deformations in shells constitutes a difficult problem in the theory of elasticity, owing to the complexity of the mathematical equations involved. For conditions in which the analytical solution is complex, or is unknown, the application of numerical methods with the aid of a digital computer has proven to be useful and efficient. This approach allows the solution for generalized geometric configurations and loadings of shells of revolution.

1.2 Purpose of the Study

The objectives of this thesis are:

- 1) to develop a technique for evaluating the stiffness influence coefficients of any arbitrary element of a shell of revolution of general geometric configuration by means of a direct numerical integration method.
- 2) to employ the standard stiffness method of structural analysis to analyze assemblages of such elements.
- 3) to demonstrate the capability of the method to treat arbitrary surface loadings and thermal gradients, including line and concentrated loads.

1.3 Types of Shells of Revolution

The position of a point in a shell of revolution can be given by three, orthogonal coordinates s , θ and z (See Appendix A, Sect. A.1, for definitions). The shape of the shell is determined by specifying the two principal radii of curvature r_1 and r_2 of the middle surface and the thickness of the shell (Fig. A.1). Instead of r_2 it is sometimes convenient to use the distance r from a point on the middle surface to the axis of revolution (Fig. A.1) where

$$r = r_2 \sin\phi$$

1.1

in which ϕ is the angular distance of the point under consideration from the axis of revolution. The generating curve of the middle,

surface is defined by the equation

$$r = r(x) \quad 1.2$$

where $r(x)$ represent the radius r as a function of the distance measured along the axis of revolution, x . Therefore the principal radii of curvature can be determined by the following two equations

$$r_1 = \left[1 + \left(\frac{dr}{dx} \right)^2 \right]^{3/2} / \frac{d^2r}{dx^2} \quad 1.3.1$$

$$r_2 = r \left[1 + \left(\frac{dr}{dx} \right)^2 \right]^{1/2} \quad 1.3.2$$

The general shape of any type of shell of revolution (Fig. 1.1 to 1.8) is characterized by particular forms of Eqns. 1.3, as follows:

a) for plates (Fig. 1.1) $r_1 = \infty$ 1.4.1

$$r_2 = \infty \quad 1.4.2$$

$$\phi = 0 \quad 1.4.3$$

b) for spheres (Fig. 1.2) $r_1 = a$ 1.4.4

$$r_2 = a \quad 1.4.5$$

c) for cylinders (Fig. 1.3) $r_1 = \infty$ 1.4.6

$$r_2 = a \quad 1.4.7$$

$$\phi = \pi/2 \quad 1.4.8$$

d) for cones (Fig. 1.4) $r_1 = \infty$ 1.4.9

$$r_2 = \frac{r}{\sin\phi} \quad 1.4.10$$

$$\phi = \text{constant} \quad 1.4.11$$

e) for toroids (Fig. 1.5) $r_1 = a \quad 1.4.12$

$$r_2 = \frac{r}{\sin\phi} \quad 1.4.13$$

f) for ellipsoids (minor axis coincides with the axis of revolution) (Fig. 1.6)

$$r_1 = \frac{a^2 b^2}{(a^2 \sin^2\phi + b^2 \cos^2\phi)^{3/2}} \quad 1.4.14$$

$$r_2 = \frac{a^2}{(a^2 \sin^2\phi - b^2 \cos^2\phi)^{1/2}} \quad 1.4.15$$

g) for hyperboloids (Fig. 1.7),

$$r_1 = \frac{-a^2 b^2}{(a^2 \sin^2\phi - b^2 \cos^2\phi)^{3/2}} \quad 1.4.16$$

$$r_1 = \frac{a^2}{(a^2 \sin^2\phi - b^2 \cos^2\phi)^{1/2}} \quad 1.4.17$$

h) for arbitrary shell elements for which the form of the middle surface cannot be expressed as a closed form function (Fig. 1.8), one can describe r_1 , r_2 and r at discrete points along the meridian and interpolate numerically.

1.4 Loadings

Applied surface loads at any point of the shell can be resolved into three components in the three orthogonal directions s , θ and z . This load may vary in the direction along the meridian as well as in the circumferential direction of the shell.

Therefore a load component may be written as a function of the coordinates s and θ in the following form

$$P_i = F_{1i}(s, \theta) \quad 1.5.1$$

where P_i is the magnitude of the load at the point under consideration in the direction i ($i = s, \theta, z$) and F_{1i} is the function representing the applied load. For the special case of axisymmetric loading P_i is independent of θ , thus

$$P_i = F_{2i}(s) \quad 1.5.2$$

For non-axisymmetrical loading, the classical method of analysis is to expand this load into a Fourier series, analyze for each harmonic separately and superimpose the effects [10,27]. The number of terms considered in this series must be sufficient to give the desired degree of convergence.

Thermal loading can be considered in either case by algebraically adding the strain due to thermal expansion to the strain due to the surface loading in the stress-strain equations.

1.5 Shell Theory

Shell theories of various degrees of complexity may be derived, depending upon the degree to which the theory of linear elasticity is simplified.

In all cases one begins by reducing the three-dimensional shell problem to a two-dimensional problem expressed in terms of the deformation of the middle surface of the shell. Further simplifications establish various shell theories which may be classified into different categories [3]. Such categories are based on the terms that are retained in the strain and stress-

displacement equations with respect to the thickness coordinate.

The second order approximation theory for shells of revolution was presented in 1932 by Flugge [10] and based upon the following assumptions:

- 1) Consistent with the formulation of the classical theory of elasticity, strains and displacements under loads are small enough so that changes in geometry of the shell will not alter the equations of static equilibrium of the shell (i.e., equations of equilibrium are written in the undeformed configuration).
- 2) The components of stress normal to the middle surface are small compared to the other components of stress and may be neglected in the stress-strain relationships (i.e., the material may be considered to be in a plane stress condition).
- 3) Points on lines normal to the middle surface before deformation remain on straight lines normal to the middle surface after deformations (i.e., deformations of the shell due to the radial shears are neglected).

Contrary to other theories, Flugge's theory did not entirely neglect the ratio of the thickness to the radius of curvature (except for an occasional dropping of the fifth and higher order terms) in the stress resultant equations and in the strain-displacement relationships [10, pg. 320].

Applications of this theory have generally been restricted to circular cylindrical shapes, for which some solutions have been obtained [10,16,19] and are considered as standards against which other simplified theories may be compared [16,19].

The fundamental equations of Flugge's theory, upon which this study is based, are presented in Appendix A.

1.6 Methods of Analysis

A structure usually consists of an assemblage of many parts and tends to be complex in nature. Generally, the true structure must be replaced by an idealized approximation, or model, suitable for mathematical analysis. Structural analysis for shells may conveniently be carried out by matrix methods using influence coefficients.

In the literature, the analysis of symmetrically loaded shells of revolution is classically performed using flexibility influence coefficients [3,4,21]. These have been given, in an explicit manner, for very limited number of types of shells of revolution, such as, cylinders, spheres and cones, of uniform thickness.

For symmetrically loaded shells, the membrane solution, which represents the momentless state of stress in the shell, may approximate the particular solution which satisfies the general differential equation of the shell. Using the membrane

solution, a flexibility method of analysis can be performed by satisfying the continuity requirements at the joints at which the elements are connected. When the forces and displacements at these joints are known, the conditions within each element may be determined from the solution of the differential equation of the element.

For the case of arbitrary shells of revolutions, under arbitrary systems of loadings, the corresponding expressions for influence coefficients are unknowns. The analytical method of obtaining these expressions would involve the solution of eighth-order differential equations expressed in terms of the geometry of the shell surface and the physical constants. This method is difficult and complicated, even for a simple geometry such as a cylinder, and it is generally impossible for the case of an arbitrary shell under arbitrary loads. However, the latter case is the rule rather than the exception in modern architecture.

As a consequence of the availability of electronic computers, and the increasing familiarity of engineers with this computational tool, the application of numerical methods of analysis to shell problems has become more attractive.

Two numerical methods for the analysis of shells of revolution with arbitrary configurations have received extensive treatment in the technical literature. The first method is the finite difference method, which consists essentially of the direct replacement of the derivatives which appear in the governing

differential equations by finite difference approximations.

This method transforms the differential equations into a system of algebraic equations which may be solved by an iterative procedure [26] or by means of matrix methods [6]. The method is quite general in application. Replacement of the derivatives by finite difference approximations may be undertaken at any level in the basic formulation of the shell problem. However, it is difficult to introduce boundary conditions into the problem. It also becomes cumbersome when attempting to satisfy equations involving high order derivatives [9,12].

The second method is the finite element method. In this method the displacement of each element into which a shell is subdivided are represented by an approximation [9,11,13,20,24]. The most common practice is to represent each shell by a series of short conical shell elements of uniform thickness. The variation in thickness along the generator of the shell, can be accounted for by considering the average thickness of each element [20]. Once these short segments have been defined, the problem becomes one of analyzing a shell that is an assembly of many short conical shells. Stiffness influence coefficients are evaluated using energy methods. Conditions of continuity are then applied at the boundaries of each segment to evaluate the forces and deformations.

An alternative method of numerical analysis based on direct integration of the shell equations, as proposed by

Goldberg, et al [12] for spherical domes and by Tyer and Simmonds [17] for conical shells, is presented for general shells of revolution in this thesis.

In the application of this method to the analysis of shells of revolution under arbitrary loadings, the governing partial differential equations of a consistent shell theory are expanded by means of Fourier series. Differentiation with respect to the colatitude coordinate θ may be performed to transform the governing equations to ordinary differential equations. These equations are reduced to a set of eight first-order ordinary differential equations involving eight intrinsic dependent variables and their derivatives. The intrinsic variables are the three components of displacement at the middle surface, the rotation of the tangent to the meridian, the membrane normal force in the meridian direction, the moment acting on circumferential sections and two modified shear terms in the directions perpendicular and tangent to the meridian.

This system of equations can be integrated in a stepwise fashion across a given element. By performing matrix operations, which will be described in Chapter 3, the stiffness influence coefficients can be calculated. Fixed end forces can be obtained and stiffness analysis may then be performed to determine the conditions at the element boundaries. These conditions can be used to determine the forces and deformation everywhere within the shell element.

1.7 Outline of Contents

The governing equations of the classical theory of shells of revolution are reduced to a set of eight basic equations in Chapter 2. In Chapter 3, the solution technique for the basic equations is described. Standard stiffness methods for segmented shell structures are outlined. Two example problems are handled in Chapter 4. General types of loadings are considered. In Chapter 5, limitations of the technique are discussed. Conclusions are drawn and possible future developments are outlined in Chapter 6.

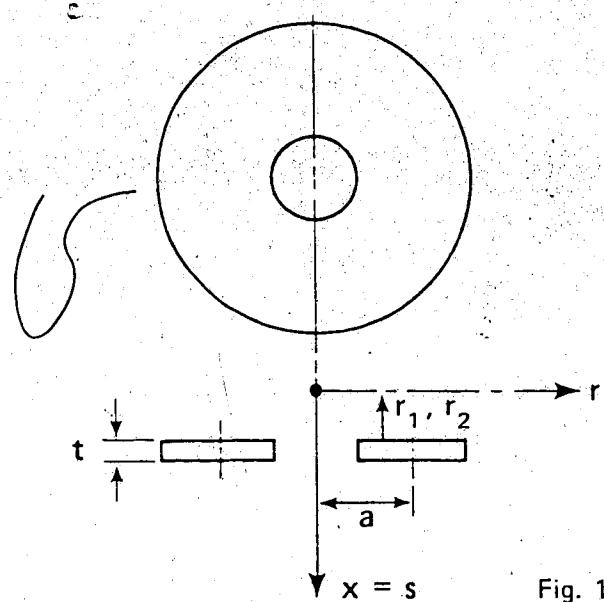


Fig. 1.1 Plate

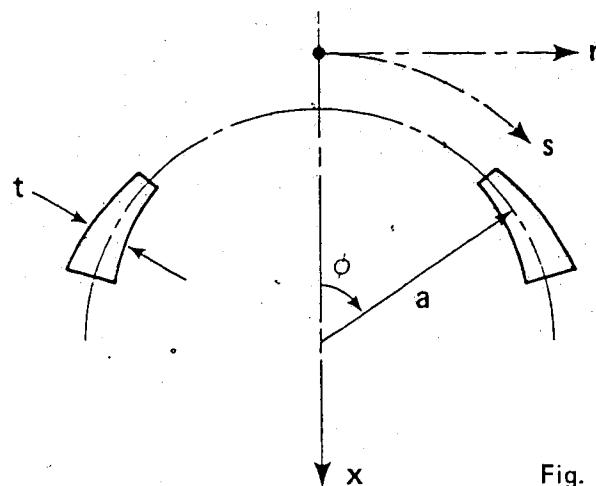


Fig. 1.2 Sphere

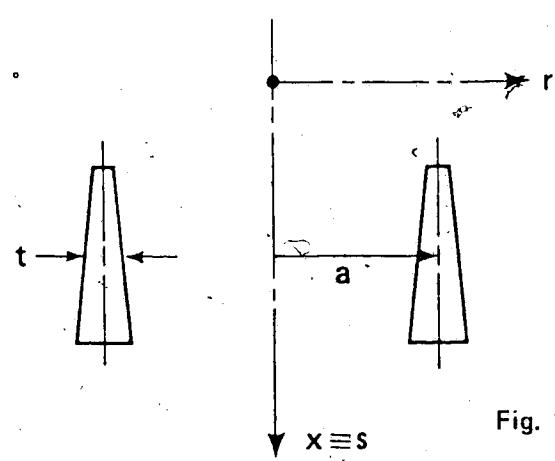


Fig. 1.3 Cylinder

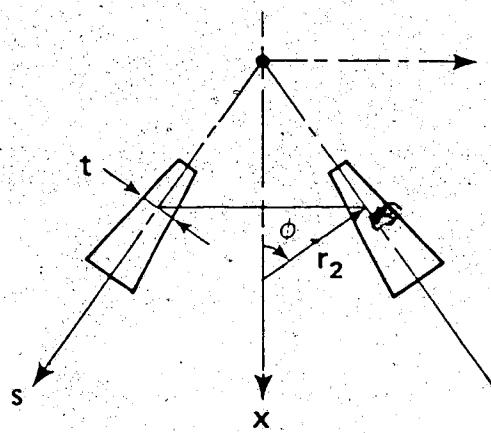


Fig. 1.4 Cone

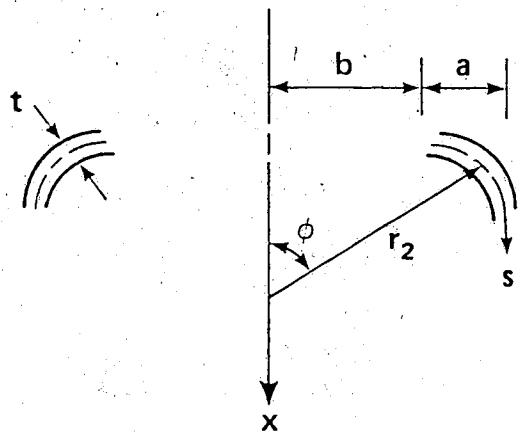


Fig. 1.5 Toroid

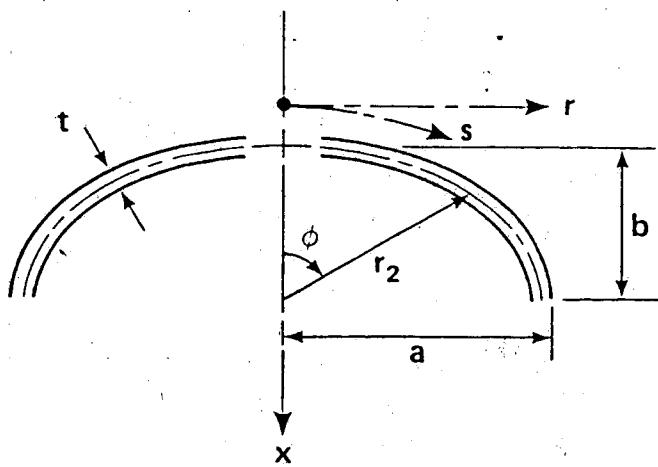


Fig. 1.6 Ellipsoid

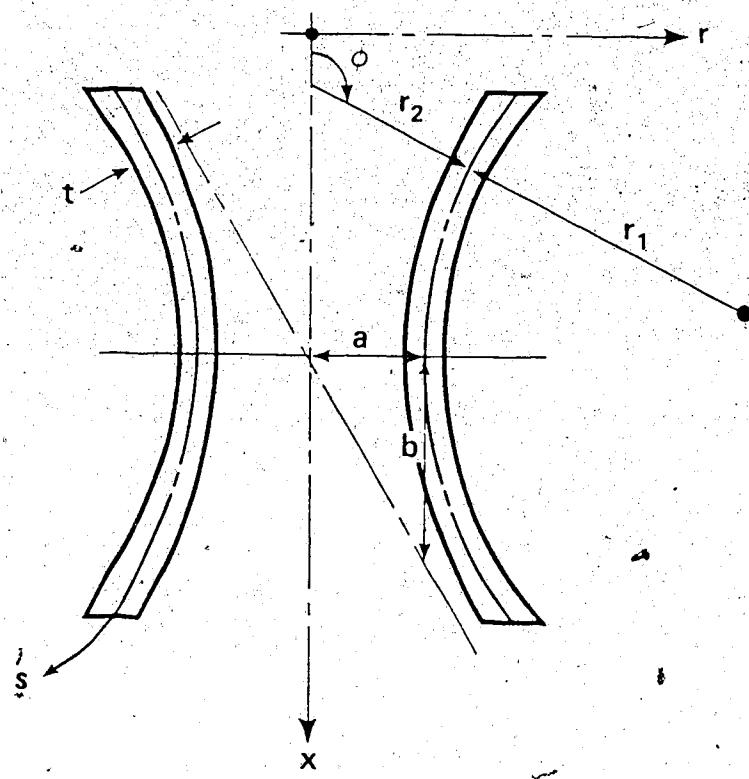


Fig. 1.7 Hyperboloid of Revolution

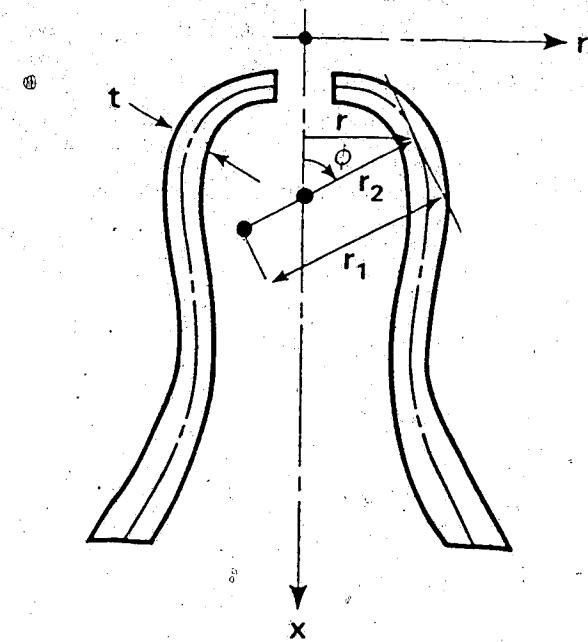


Fig. 1.8 Shell of Revolution of Arbitrary Shape

CHAPTER 2

BASIC EQUATIONS

2.1. Introduction

In this chapter the governing field equations of a shell of revolution, as derived in Appendix A, are expanded using Fourier series. Modified shear terms are introduced into the governing equations to eliminate the inplane shearing force in the circumferential direction and the meridional twisting moment, which appear at the boundaries $s = \text{constant}$. By performing matrix operations to eliminate the stress resultants in the circumferential direction (secondary stress resultants) the governing field equations are reduced to a set of eight first order equations involving four stress resultants (primary stress resultants) and four displacements and their derivatives.

2.2 Fourier Series

The load components and temperature, being arbitrary functions of s and θ , may always be represented in the form

$$P_s = \sum_{n=0}^{\infty} P_{sn}(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{P}_{sn}(s) \sin n\theta \quad 2.1.1$$

$$P_\theta = \sum_{n=0}^{\infty} \bar{P}_{\theta n}(s) \cos n\theta + \sum_{n=1}^{\infty} P_{\theta n}(s) \sin n\theta \quad 2.1.2$$

$$P_z = \sum_{n=0}^{\infty} P_{zn}(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{P}_{zn}(s) \sin n\theta \quad 2.1.3$$

$$T = \sum_{n=0}^{\infty} T_n(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{T}_n(s) \sin n\theta \quad 2.1.4$$

where P_s , P_θ and P_z are defined in Appendix A, T represents temperature, and the functions of s on the right hand side are Fourier coefficients [10,27]. The corresponding stress resultants and displacements may be expressed as

$$N_s = \sum_{n=0}^{\infty} N_{sn}(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{N}_{sn}(s) \sin n\theta \quad 2.2.1$$

$$N_\theta = \sum_{n=0}^{\infty} N_{\theta n}(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{N}_{\theta n}(s) \sin n\theta \quad 2.2.2$$

$$N_{s\theta} = \sum_{n=0}^{\infty} \bar{N}_{s\theta n}(s) \cos n\theta + \sum_{n=1}^{\infty} N_{s\theta n}(s) \sin n\theta \quad 2.2.3$$

$$N_{\theta s} = \sum_{n=0}^{\infty} \bar{N}_{\theta sn}(s) \cos n\theta + \sum_{n=1}^{\infty} N_{\theta sn}(s) \sin n\theta \quad 2.2.4$$

$$Q_s = \sum_{n=0}^{\infty} Q_{sn}(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{Q}_{sn}(s) \sin n\theta \quad 2.2.5$$

$$Q_\theta = \sum_{n=0}^{\infty} \bar{Q}_{\theta n}(s) \cos n\theta + \sum_{n=1}^{\infty} Q_{\theta n}(s) \sin n\theta \quad 2.2.6$$

$$M_s = \sum_{n=0}^{\infty} M_{sn}(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{M}_{sn}(s) \sin n\theta \quad 2.2.7$$

$$M_\theta = \sum_{n=0}^{\infty} M_{\theta n}(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{M}_{\theta n}(s) \sin n\theta \quad 2.2.8$$

$$M_{s\theta} = \sum_{n=0}^{\infty} \bar{M}_{s\theta n}(s) \cos n\theta + \sum_{n=1}^{\infty} M_{s\theta n}(s) \sin n\theta \quad 2.2.9$$

$$M_{\theta s} = \sum_{n=0}^{\infty} \bar{M}_{\theta sn}(s) \cos n\theta + \sum_{n=1}^{\infty} M_{\theta sn}(s) \sin n\theta \quad 2.2.10$$

$$U = \sum_{n=0}^{\infty} \bar{U}_n(s) \cos n\theta + \sum_{n=1}^{\infty} U_n(s) \sin n\theta \quad 2.2.11$$

$$V = \sum_{n=0}^{\infty} V_n(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{V}_n(s) \sin n\theta \quad 2.2.12$$

$$W = \sum_{n=0}^{\infty} W_n(s) \cos n\theta + \sum_{n=1}^{\infty} \bar{W}_n(s) \sin n\theta \quad 2.2.13$$

where the variables on the left hand side are defined in Appendix A, Sects. A.2 and A.3, and where the term (s) indicates that the variable coefficients with subscript n are functions of the coordinate s only and n is the harmonic number. The first and the second series in each expression represent the portions of the variables which are, respectively, symmetric and anti-symmetric with respect to the meridian passing through the line $\theta = 0$.

For an arbitrary applied load expressed as a Fourier series of the order N (Eqs. 2.1.1 to 2.1.3), there are $2N+1$ terms that represent each component of load; ($n = 0, 1, 2, \dots, N$) for the symmetric series and ($n = 1, 2, \dots, N$) for the anti-symmetric series. For each value of n the (s dependent) variables with the subscript n from each series can be entered in the governing equations of shells of revolution of Appendix A, because the sequences $\cos n\theta$ and $\sin n\theta$ are linearly independent. Differentiations with respect to θ can be performed and the terms grouped according to the common factors, $\cos n\theta$ or $\sin n\theta$. Since the coefficient of each of these factors must be zero, each

factor produces a separate equation. For example, Eqn. A.5.1 is

$$rN_s + \cos\phi N_{s\theta} + N'_{s\theta s} - \cos\phi N_{\theta\theta} - \frac{r}{r_1} Q_s + rP_s = 0 \quad 2.3.1$$

for which, for any n , the cosine terms become

$$\begin{aligned} & rN_{sn} \cos n\theta + \cos\phi N_{sn} \cos n\theta + nN_{\theta sn} \cos n\theta \\ & - \cos\phi N_{\theta n} \cos n\theta - \frac{r}{r_1} Q_{sn} \cos n\theta + rP_{sn} \cos n\theta = 0 \end{aligned} \quad 2.3.2$$

which, upon factoring out the common term, yields

$$\begin{aligned} & rN_{sn} + \cos\phi N_{sn} + nN_{\theta sn} - \cos\phi N_{\theta n} - \frac{r}{r_1} Q_{sn} \\ & + rP_{sn} = 0 \end{aligned} \quad 2.3.3$$

If the Fourier expansions of Eqs. 2.1 and 2.2 are truncated such that $0 \leq n \leq N$, the governing system of equations is replaced by $(2N+1)$ ordinary differential systems of equations, each equation of the type illustrated by Eq. 2.3.3, and an analysis is carried out to obtain the solutions for the dependent coefficients corresponding to each value of n . In the general case there are twenty-six variable coefficients associated with each $n > 0$ in Eqns. 2.2, thirteen associated with each of the factors.

Finally, since linear stress-displacement relations were assumed, the principle of superposition is valid and one can superimpose the $(2N+1)$ solutions so obtained.

The n^{th} set of equations can be written as follows.

The six equilibrium equations are obtained from Eqs. A.5 as

$$N_{sn} + R \cos \phi N_{s\theta n} \mp R_n N_{\theta sn} - R \cos \phi N_{\theta n} - R_1 Q_{sn} + P'_{sn} = 0 \quad 2.4.1$$

$$N_{s\theta n} + R \cos \phi N_{s\theta n} \mp R_n N_{\theta n} + R \cos \phi N_{\theta sn} - R_2 Q_{\theta n} + P_{\theta n} = 0 \quad 2.4.2$$

$$R_2 N_{\theta n} + R_1 N_{sn} \mp R_n Q_{\theta n} + Q_{sn} + R \cos \phi Q_{\theta n} - P_{zn} = 0 \quad 2.4.3$$

$$M_{sn} + R \cos \phi M_{s\theta n} \mp R_n M_{\theta sn} - R \cos \phi M_{\theta n} - Q_{sn} = 0 \quad 2.4.4$$

$$M_{s\theta n} + R \cos \phi M_{s\theta n} \mp R_n M_{\theta n} + R \cos \phi M_{\theta sn} - Q_{\theta n} = 0 \quad 2.4.5$$

$$N_{\theta sn} - N_{s\theta n} - R_2 M_{\theta sn} + R_1 M_{s\theta n} = 0 \quad 2.4.6$$

where R_1 and R_2 are the principal curvatures of the shell, defined as the reciprocals of the radii of curvature r_1 and r_2 , respectively, and R is the curvature of the parallel circle.

The eight stress-displacement equations are obtained from Eqs. A17 as

$$\begin{aligned} N_{sn} &= [K R_1 (R_1 - R_2) r'_1] \beta_n - [K(R_1 - R_2)] \beta'_n \\ &\quad + [D(R_1 + VR_2) + K R^2 r_1 (R_1 - R_2)] W_n \end{aligned}$$

$$\begin{aligned}
 & + [\nu DR \cos \phi - KR^2 (R_1 - R_2) r_1] V_n \\
 & + [D + KR_1 (R_1 - R_2)] V_n \pm [DRn] U_n \\
 & - [(1 + \nu) \alpha D] T_{o2n} \quad 2.5.1
 \end{aligned}$$

$$\begin{aligned}
 N_{\theta n} = & [KR (R_1 - R_2) \cos \phi] \beta_n \\
 & + [D(R_2 + \nu R_1) + K(R_1 - R_2)(R^2 n^2 - R^2 2)] W_n \\
 & + [DR \cos \phi - KR R_2 \cos \phi (R_1 - R_2)] V_n \\
 & + [\nu D] V_n \pm [DRn] U_n \\
 & - [(1 + \nu) \alpha D] T_{o1n} \quad 2.5.2
 \end{aligned}$$

$$\begin{aligned}
 N_{s\theta n} = & \frac{1 - \nu}{2} \{ \pm [KR (R_1 - R_2)n] \beta_n \\
 & \pm [KR^2 \cos \phi (R_1 - R_2)n] W_n \\
 & \mp [DRn + KR R_1 (R_1 - R_2)n] V_n \\
 & - [DR \cos \phi - KR \cos \phi (R_1 - R_2)^2] U_n \\
 & + [D + K (R_1 - R_2)^2] U_n \} \quad 2.5.3
 \end{aligned}$$

$$\begin{aligned}
 N_{\theta sn} = & \frac{1 - \nu}{2} \{ \pm [KR (R_1 - R_2)n] \beta_n \\
 & \mp [KR^2 \cos \phi (R_1 - R_2)n] W_n \\
 & \mp [DRn - KR R_2 (R_1 - R_2)n] V_n \\
 & - [DR \cos \phi] U_n + [D] U_n \} \quad 2.5.4
 \end{aligned}$$

$$M_{sn} = [KR_1 r_1 - \nu KR \cos \phi] \beta_n - [K] \beta_n$$

$$\begin{aligned}
 & + [K R_1 (R_1 - R_2) - \nu K R^2 n^2] W_n \\
 & - [K R^2 r_1] V_n + [K(R_1 - R_2)] V_n \\
 & + [\nu K R R_2 n] U_n + [(1 + \nu) \alpha K] T_{12n} \quad 2.5.5
 \end{aligned}$$

$$\begin{aligned}
 M_{\theta n} = & [\nu K R_1 r_1 - K R \cos \phi] \beta_n - [\nu K] \beta_n \\
 & - [K R^2 n^2 + K R_2 (R_1 - R_2)] W_n \\
 & - [\nu K R_1 r_1 + K R \cos \phi (R_1 - R_2)] V_n \\
 & + [K R R_1 n] U_n + [(1 + \nu) \alpha K] T_{11n} \quad 2.5.6
 \end{aligned}$$

$$\begin{aligned}
 M_{s\theta n} = & \frac{1 - \nu}{2} \{ \pm [2 K R n] \beta_n \pm [2 K R^2 n \cos \phi] W_n \\
 & \mp [K R R_1 n] V_n - [K R \cos \phi (R_1 - 2R_2)] U_n \\
 & + [K(R_1 - 2R_2)] U_n \} \quad 2.5.7
 \end{aligned}$$

$$\begin{aligned}
 M_{\theta sn} = & \frac{1 - \nu}{2} \{ \pm [2 K R n] \beta_n \pm [2 K R^2 n \cos \phi] W_n \\
 & \mp [K R R_2 n] V_n + [K R R_2 \cos \phi] U_n \\
 & - [K R_2] U_n \} \quad 2.5.8
 \end{aligned}$$

where Eqs. A.7 have been used to eliminate the first and second derivatives of the displacement component W .

Eqs. 2.4 and 2.5 are fourteen equations in terms of the thirteen unknown functions (the variable Fourier series coefficients). As such the system of equations is overspecified. In the theory developed herein, Eq. 2.4.6 is discarded and the

remaining thirteen equations allow the solution for the thirteen functions associated with each trigonometric functions of the Fourier series expansion.

2.3 Natural Boundary Conditions

According to the classical theory of shells, the quantities which appear in the natural boundary conditions on an edge $s = \text{constant}$ of a shell of revolution are the four displacements β, W, V, U and the corresponding four forces M_s, S_s, N_s, T_s .

The forces S_s, T_s are the transverse and tangential effective shears which are commonly known as Kirchhoff's shears [27]. The Kirchhoff shears are work-equivalent forces associated with the displacements W and U , and these effective shears replace the stress resultants $Q_s, N_{s\theta}$ and $M_{s\theta}$. Such replacement is essential to describe simple boundary conditions [10, 27]. Although the expressions for these forces may be derived rigorously from variational principles, the following physical interpretation, due to Kelvin and Tait [27, pg. 45], is more instructive. If the twisting moment $M_{s\theta}$ acting on an infinitesimally small element of the shell is replaced by a statically equivalent force as shown in Fig. 2.1, one can write from statics the expression for the effective transverse shearing force as [27]

$$S_s = Q_s + \frac{M' s\theta}{r}$$

2.6.1

and the effective tangential shearing force as

$$T_s = N_{s\theta} - \frac{M_{s\theta}}{r_2} \quad 2.6.2$$

Writing Eqs. 2.6.1 and 2.6.2 in Fourier series terms, and separating the linearly independent terms, we may define

$$S_{sn} = Q_{sn} \pm \frac{nM_{s\theta n}}{r} \quad 2.7.1$$

$$T_{sn} = N_{s\theta n} - \frac{M_{s\theta n}}{r_2} \quad 2.7.2$$

Using the geometrical relations in Eqs. A.2, the derivatives of these forces with respect to the coordinate s may be written as

$$S'_{sn} = Q'_{sn} \pm Rn M'_{s\theta n} + R^2 n \cos \phi M_{s\theta n} \quad 2.8.1$$

$$\begin{aligned} T'_{sn} &= N'_{s\theta n} - \frac{r_2 M'_{s\theta n} - M_{s\theta n} r'_2}{r^2_2} \\ &= M'_{s\theta n} - R_2 M'_{s\theta n} - R_2(R_1 - R_2) \cot \phi M_{s\theta n} \quad 2.8.2 \end{aligned}$$

2.4 Reduction of the Governing Equations

It can be seen that the stress resultants Q_θ , N_θ , M_θ , $M_{\theta s}$ and $N_{\theta s}$ do not enter into any boundary conditions on an edge for which $s = \text{constant}$. Therefore, it is preferable to

$$M_s = F_{11}(\beta, \beta^*, w, v, v^*, u, T_{12}) \quad 2.10.5$$

$$M_{\theta} = F_{12}(\beta, \beta^*, w, v, u, T_{11}) \quad 2.10.6$$

$$M_{s\theta} = F_{13}(\beta, w, v, u, u^*) \quad 2.10.7$$

$$M_{\theta s} = F_{14}(\beta, w, v, u, u^*) \quad 2.10.8$$

where the form of the functions F_1 to F_{14} is obtained from Eqs. 2.4 and 2.5. In addition, Eqs. A.7.1, 2.7 and 2.8, which will be called auxiliary equations, may be written in the following symbolic form,

$$\beta = F_{15}(w^*, v) \quad 2.11.1$$

$$S_s = F_{16}(Q_s, M_{s\theta}) \quad 2.11.2$$

$$T_s = F_{17}(N_{s\theta}, M_{s\theta}) \quad 2.11.3$$

$$S_s^* = F_{18}(Q_s^*, M_{s\theta}^*, M_{s\theta}) \quad 2.11.4$$

$$T_s^* = F_{19}(N_{s\theta}^*, M_{s\theta}^*, M_{s\theta}) \quad 2.11.5$$

It should be noted that for shells with equal radii of curvature (i.e., $r_1 = r_2$), the sixth equation of equilibrium Eq. 2.4.6 will be an identity as

$$N_{s\theta} = N_{\theta s}$$

$$M_{s\theta} = M_{\theta s}$$

Therefore, this equation will be discarded, as mentioned in

eliminate them in terms of the other stress resultants and evaluate them after the solution of the governing equations.

For convenience, the subscript n in Fourier coefficients will be omitted and the governing equations (Eqs. 2.4 and 2.5) can be written, for each set of equations, in the following symbolic form. The equilibrium equations (Eqs. 2.4) may be written symbolically as:

$$F_1 (N_s^*, N_s, N_{\theta s}, N_\theta, Q_s, P_s) = 0 \quad 2.9.1$$

$$F_2 (N_{s\theta}^*, N_{s\theta}, N_\theta, N_{\theta s}, Q_\theta, P_\theta) = 0 \quad 2.9.2$$

$$F_3 (N_\theta, N_s, Q_\theta, Q_s^*, Q_s, P_z) = 0 \quad 2.9.3$$

$$F_4 (M_s^*, M_s, M_{\theta s}, M_\theta, Q_s) = 0 \quad 2.9.4$$

$$F_5 (M_{s\theta}^*, M_{s\theta}, M_\theta, M_{\theta s}, Q_\theta) = 0 \quad 2.9.5$$

$$F_6 (N_{\theta s}, N_{s\theta}, M_{\theta s}, M_{s\theta}) = 0 \quad 2.9.6$$

The stress resultant-displacement equations (Eqs. 2.5) may be written symbolically as:

$$N_s = F_7(\beta, \beta^*, W, V, V^*, U, T_{o2}) \quad 2.10.1$$

$$N_\theta = F_8(\beta, W, V, V^*, U, T_{o1}) \quad 2.10.2$$

$$N_{s\theta} = F_9(\beta, W, V, U, U^*) \quad 2.10.3$$

$$M_{\theta s} = F_{10}(\beta, W, V, U, U^*) \quad 2.10.4$$

Appendix A, Sect. A.5 and as noted in Sect. 2.3.

In solving the above equations, the displacements which may be physically imposed on the boundary of a shell are β , W , V and U . The external forces associated with these displacements (in a work-equivalent sense) are M_s , S_s , N_s and T_s . The first set of variables are specified in the case of displacement boundary conditions while the latter set is specified in the case of mechanical or force boundary conditions. In general a combination of four of these eight quantities must be specified to properly define a boundary condition, provided that if one of the displacements is specified the associated force should not be specified and vice-versa. Since the remaining five of the thirteen variables cannot be determined at the boundary, it is desirable to eliminate them from the governing equations. The objective in the following is, therefore, to reduce the thirteen governing equations (Eqs. 2.9.1 to 2.9.5 and Eqs. 2.10) to a set of eight governing equations in terms of the eight variables which may arise in the specification of the boundary conditions. It is also desirable to keep the order of these differential equations to a minimum in order to facilitate the numerical solution. The final set of equations will be of first order.

Combining Eq. 2.9.1 and Eq. 2.11.2 to eliminate Q_s , one can get

$$N_s = R_1 S_s - R \cos \phi N_s + R R_1 M_{s\theta} + R \cos \phi N\theta + R_n N_{\theta s} - P_s$$

Using Eqs. 2.9.2 and 2.9.5 to eliminate Q_θ and then substituting for $M_{s\theta}$, $N_{s\theta}$, $N_{s\theta}$ by means of Eqs. 2.11.3 and 2.11.5, one can write

$$\begin{aligned} T_s^* &= -R \cos \phi T_s + RR_2 n M_\theta + RR_2 \cos \phi M_{\theta s} \\ &\quad - R \cos \phi (R_1 - R_2) M_{s\theta} \pm Rn N_\theta \\ &\quad - R \cos \phi N_{\theta s} - P_\theta \end{aligned} \quad 2.12.2$$

Eqs. 2.9.4 and 2.11.2, after eliminating Q_s are in the form

$$\begin{aligned} M_s^* &= -R \cos \phi M_s + S_s + R \cos \phi M_\theta \\ &\quad + Rn M_{\theta s} + Rn M_{s\theta} \end{aligned} \quad 2.12.3$$

Using Eq. 2.9.3, after eliminating Q_θ by means of Eq. 2.9.5, together with Eqs. 2.11.2 and 2.11.4 results in the equation

$$\begin{aligned} S_s^* &= -R \cos \phi S_s - R_1 N_s + R^2 n^2 M_\theta \mp R^2 n \cos \phi M_{\theta s} \\ &\quad \mp R^2 n \cos \phi M_{s\theta} - R_2 N_\theta + P_z \end{aligned} \quad 2.12.4$$

Eqs. 2.12, which represent the equilibrium equations, may now be written symbolically as

$$M_s^* = F_{20} (M_s, S_s, M_\theta, M_{\theta s}, M_{s\theta}) \quad 2.13.1$$

$$S^*_{\text{s}} = F_{21} (S_s, N_s, M_\theta, M_{\theta s}, M_{s\theta}, N_\theta, P_z) \quad 2.13.2$$

$$N^*_{\text{s}} = F_{22} (S_s, N_s, M_{s\theta}, N_\theta, N_{\theta s}, P_s) \quad 2.13.3$$

$$T^*_{\text{s}} = F_{23} (T_s, M_\theta, M_{\theta s}, M_{s\theta}, N_\theta, N_{\theta s}, P_\theta) \quad 2.13.4$$

or in matrix notation

$$\{F^*_{\text{s}}\} = [B_1 \quad B_2] \begin{Bmatrix} F_s \\ F_\theta \end{Bmatrix} + \{B_3\} \quad 2.14$$

where the loading terms in these equations are separated in the vector $\{B_3\}$, and

$$\langle F_s \rangle = \langle M_s \quad S_s \quad N_s \quad T_s \rangle \quad 2.15.1$$

$$\langle F^*_{\text{s}} \rangle = \langle M^*_{\text{s}} \quad S^*_{\text{s}} \quad N^*_{\text{s}} \quad T^*_{\text{s}} \rangle \quad 2.15.2$$

$$\langle F_\theta \rangle = \langle M_\theta \quad M_{\theta s} \quad M_{s\theta} \quad N_\theta \quad N_{\theta s} \rangle \quad 2.15.3$$

The coefficients of the matrices $[B_1]$, $[B_2]$ and $\{B_3\}$ are defined in Table 2.1

In Eq. 2.14, the stress resultants have been separated into the vector $\langle F_s \rangle$, whose components are desired in the final formulation, and the vector $\langle F_\theta \rangle$, which remains to be eliminated from the formulation.

Let us now turn our attention to the displacement variables. Eqs. 2.10.1 and 2.10.5, which are two equations in

β^* , V^* , can be converted to the following two equations

$$\begin{aligned} \beta^* &= \frac{1}{CA_2} \left\{ - \left[\frac{CA_1}{K} \right] M_s + [R_1 - R_2] N_s \right. \\ &\quad + [R_1 r_1 CA_2 - v R \cos \phi CA_1] \beta \\ &\quad - [v D R_2 (R_1 - R_2) + v R^2 n^2 CA_1] W \\ &\quad - [R_1^2 r_1 CA_2 + v D R \cos \phi (R_1 - R_2)] V \\ &\quad \left. + [v R R_1 n CA_2] U + (1 + v) \alpha [D(R_1 - R_2) T_{O2} + CA_1 T_{12}] \right\} \end{aligned} \quad 2.16.1$$

in which

$$CA_1 = D + K R_1 (R_1 - R_2) \quad 2.16.2$$

$$CA_2 = D + K R_2 (R_1 - R_2) \quad 2.16.3$$

and

$$\begin{aligned} V^* &= \frac{1}{CA_2} \left\{ -[R_1 - R_2] M_s + N_s \right. \\ &\quad - [v K R \cos \phi (R_1 - R_2)] \beta \\ &\quad - [CA_2 R_1 + v D R_2 + v K R^2 n^2 (R_1 - R_2)] W \\ &\quad - [v D R \cos \phi] V + [v R n CA_2] U \\ &\quad \left. + (1 + v) \alpha [DT_{O2} + K(R_1 - R_2) T_{12}] \right\} \end{aligned} \quad 2.16.4$$

Eq. 2.11.1 is in the form

$$W^* = -\beta + R_1 V \quad 2.16.5$$

Finally, Eq. 2.10.3, after eliminating $N_{s\theta}$ by means of Eq. 2.11.3 can be written as

$$\begin{aligned} U^* = & \frac{1}{CA_3} \left\{ \left[\frac{2}{1-v} \right] T_s \mp [K R_n (R_1 - 3R_2)] \beta \right. \\ & \mp [K R^2 n \cos \phi (R_1 - 3R_2)] W \pm [R_n CA_1 - K R R_1 R_2 n] V \\ & \left. + [R \cos \phi CA_3] U \right\} \end{aligned} \quad 2.16.6$$

in which

$$CA_3 = D + K(R^2 R_1 - 3R_1 R_2 + 3R^2 R_2) \quad 2.16.7$$

Eqs. 2.16.1, 2.16.4, 2.16.5 and 2.16.6 can be written in a symbolic form as

$$\beta^* = F_{24}(\beta, W, V, U, M_s, N_s, T_{O_2}, T_{12}) \quad 2.17.1$$

$$W^* = F_{25}(\beta, V) \quad 2.17.2$$

$$V^* = F_{26}(\beta, W, V, U, M_s, N_s, T_{O_2}, T_{12}) \quad 2.17.3$$

$$U^* = F_{27}(\beta, W, V, U, T_s) \quad 2.17.4$$

or can be written in matrix notation as

$$\{D^*\} = [A1 \quad A2] \begin{Bmatrix} D \\ F \\ S \end{Bmatrix} + \{A3\} \quad 2.18$$

where

$$\langle D \rangle = \langle \beta \quad W \quad V \quad U \rangle \quad 2.19.1$$

$$\langle D^* \rangle = \langle \beta^* \quad W^* \quad V^* \quad U^* \rangle \quad 2.19.2$$

The coefficients of the matrices $[A1]$, $[A2]$ and the column vector $\{A3\}$ are shown in Tables 2.2.

The stress resultants appearing in the vector $\langle F_\theta \rangle$ of Eq. 2.15.3 can be expressed in terms of the fundamental displacements by selecting Eqs. 2.10.2, 2.10.4, 2.10.6, 2.10.7 and 2.10.8 and writing them in matrix notation

$$\{F_\theta\} = [C1 \quad C2] \begin{Bmatrix} D^* \\ D \end{Bmatrix} + \{C3\} \quad 2.20$$

where $\{F_\theta\}$, $\{D\}$ and $\{D^*\}$ are defined in Eqs. 2.15.3, 2.19.1 and 2.19.2 respectively, and the coefficients of $[C1]$, $[C2]$ and $\{C3\}$ are defined in Tables 2.3.

The three sets of equations (Eqs. 2.14, 2.18 and 2.20) can now be used to form a set of eight first order differential equations relating the eight fundamental variables; four displacements β , W , V , U , and four corresponding forces M_s , S_s , N_s , T_s and their derivatives. By substituting Eq. 2.20 into Eq. 2.14, to replace $\{F_\theta\}$

$$\{F_s^*\} = [B1]\{F_s\} + [B2] \left\{ [C1]\{D^*\} + [C1]\{D\} + \{C3\} \right\} + \{B3\} \quad 2.21.1$$

and by substituting for $\{D^*\}$, from Eq. 2.18,

$$\begin{aligned} \{F_s^*\} &= [B1]\{F_s\} + [E1] \left\{ [A1]\{D\} + [A2]\{F_s\} + \{A3\} \right\} \\ &\quad + [E2]\{D\} + \{B3\} \end{aligned} \quad 2.21.2$$

or

$$\{F_s^*\} = [G1]\{D\} + [G2]\{F_s\} + \{G3\} \quad 2.22$$

where

$$[E1] = [B2][C1] \quad 2.23.1$$

$$[E2] = [B2][C2] \quad 2.23.2$$

$$\{E3\} = [B2]\{C3\} + \{B3\} \quad 2.23.3$$

$$[G1] = [E1][A1] + [E2] \quad 2.23.4$$

$$[G2] = [E1][A2] + [B1] \quad 2.23.5$$

$$\{G3\} = [E1]\{A3\} + \{E3\} \quad 2.23.6$$

Eq. 2.18 and Eq. 2.22, can be combined into one matrix equation as

$$\begin{aligned} \begin{Bmatrix} D^* \\ F_s^* \end{Bmatrix} &= \begin{bmatrix} A1 & A2 \\ G1 & G2 \end{bmatrix} \begin{Bmatrix} D \\ F_s \end{Bmatrix} + \begin{Bmatrix} A3 \\ G3 \end{Bmatrix} \\ &= [GA] \cdot \begin{Bmatrix} D \\ F_s \end{Bmatrix} + \{GB\} \end{aligned} \quad 2.24$$

Therefore, for the applied loads, stresses and displacements being periodic functions of the coordinate θ , the general governing partial differential equations, Eqs. A.5 and Eqs. A.17, have been transformed into a system of eight first order ordinary differential equations for each harmonic member n . These equations relate, at any point, the eight fundamental dependent variables, that appear in the natural boundary conditions of shells of revolution, and their derivatives with respect to the independent variable s .

As was seen, the reduction of the shell equation into this form involves only straight forward algebraic manipulations. The equations have been put in this form to facilitate numerical integration. They can be integrated with the aid of an appropriate integration technique, such as the Runge-Kutta process. Since a digital computer is needed to perform this integration, the reduction of the equations to the final form, as in Eq. 2.24, is not necessary and can be performed by feeding the three sets of equations (Eqs. 2.14, 2.18 and 2.20) into the computer.

It can be seen that the elements of the matrix GA in Eq. 2.24 are dependent only on the shell thickness, the physical constants and the coordinate s . It should be noted that in the case of shell of revolution under symmetrical loading conditions (i.e., $n = 0$) the system of the eight equations reduces naturally to a system of six first order equations as the displacement component U and the corresponding force T_s vanish due to the symmetry of the applied load.

$-R \cos \phi$			$R \cos \phi$	$\mp Rn$	$\mp Rn$		
$-R \cos \phi$	$-R_1$		$R^2 n^2$	$\mp R^2 n \cos \phi$	$\mp R^2 n \cos \phi$	$-R_2$	
R_1	$-R \cos \phi$				$\mp RR_1 n$	$R \cos \phi$	$\mp Rn$
			$\mp RR_2 n$	$RR_2 \cos \phi$	$-R \cos \phi$	$\pm Rn$	$-R \cos \phi$
					$(R_1 - R_2)$		$-P_0$

Matrix B1

Matrix B2

Vector B3

TABLE 2.1 Coefficients of Matrices B1, B2 and Load Vector B3 in Eq. 2.14

$R_1 r^* - \nu R \cos \phi$	$\frac{CA_1}{CA_2}$	$- DR_2 (R_1 - R_2)$	$- R^2 r^* 1$	$\mp \nu R R_1 n$
		$- \nu \frac{CA_1}{CA_2}$		
		$- \nu R^2 n^2 \frac{CA_1}{CA_2}$	$- \nu D \frac{R \cos \phi}{CA_2} (R_1 - R_2)$	
-1			R_1	
$-\nu K \frac{R \cos \phi}{CA_2} (R_1 - R_2)$	$- R_1 - \frac{\nu DR_2}{CA_2}$	$- \nu \frac{DR}{CA_2} \cos \phi$	$\mp \nu R n$	
		$- \nu K \frac{R^2 n^2}{CA_2} (R_1 - R_2)$		
$\mp K \frac{R n}{CA_3} (R_1 - 3R_2)$	$\mp K \frac{R^2 n \cos \phi}{CA_3} (R_1 - 3R_2)$	$+ Rn \frac{CA_1}{CA_3}$	$R \cos \phi$	
		$- K \frac{RR_1 R_2}{CA_3} n$		

TABLE 2.2.1 Coefficients of Matrix A1 in Eq. 2.18

$-\frac{CA_1}{K \cdot CA_2}$	$\frac{R_1 - R_2}{CA_2}$	$\frac{(1 + \nu)\alpha}{CA_2} \{D(R_1 - R_2) T_{02} + CA_1 T_{12}\}$
$-\frac{(R_1 - R_2)}{CA_2}$	$\frac{1}{CA_2}$	$\frac{(1 + \nu)\alpha}{CA_2} \{K(R_1 - R_2) T_{12} + DT_{02}\}$

Matrix A2

Vector A3

TABLE 2.2.2 Coefficient of Matrix A2 and Load Vector A3 in Eq. 2.18

$-\nu K$		$(1 + \nu) \alpha K T_{11}$
		$-(\frac{1 - \nu}{2}) K R_2$
		$(\frac{1 - \nu}{2}) \{K(R_1 - R_2)\}$
	νD	$-(1 + \nu) \alpha D T_{01}$
		$(\frac{1 - \nu}{2}) D$

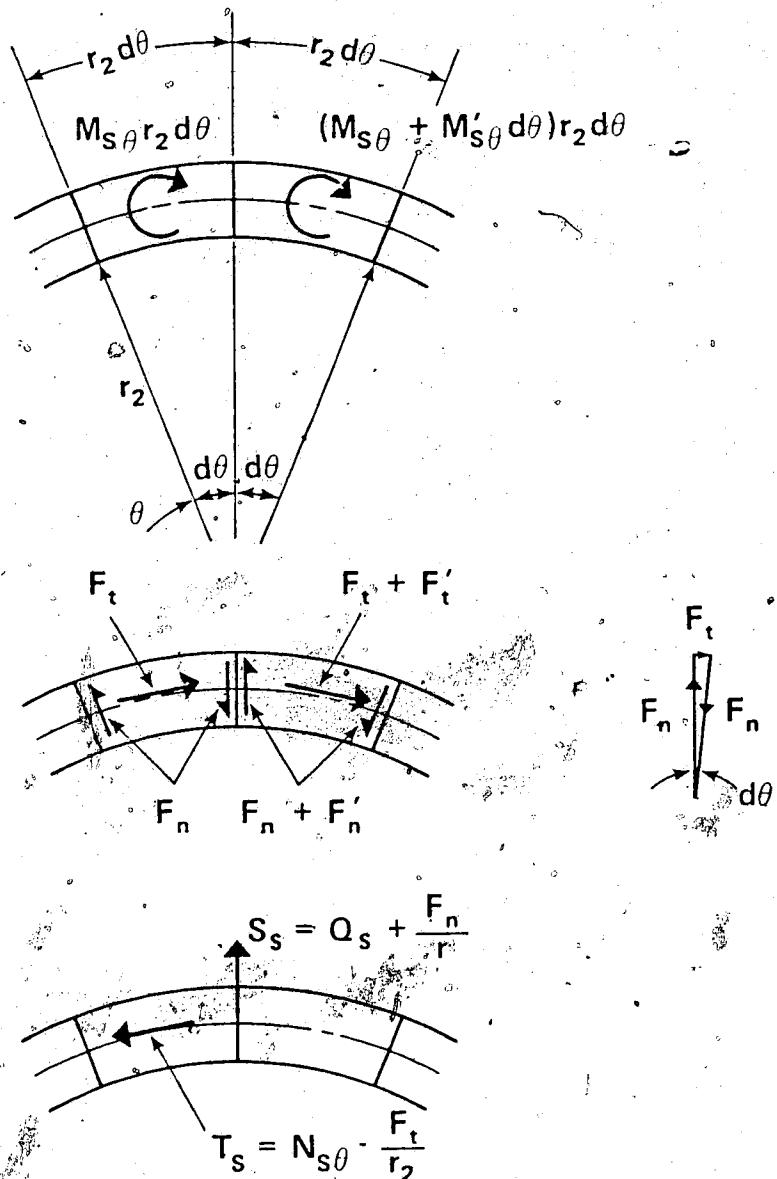
Matrix C1

Vector C3

TABLE 2.3.1 Coefficients of Matrix C1 and Load Vector C3 in
Eq. 2.20

$\nu K R_1 r_1 - K R \cos \phi$	$- K R^2 n^2 + K R_2 (R_1 - R_2)$	$- \gamma K R_1^2 r_1$ $- K R \cos \phi (R_1 - R_2)$	$- K R R_1 n$ $+ K R R_2 \cos \phi$
$\mp \left(\frac{1-\nu}{2}\right) (2K Rn)$	$\mp \left(\frac{1-\nu}{2}\right) (2K R^2 n \cos \phi)$	$\mp \left(\frac{1-\nu}{2}\right) (K R R_2 n)$	$\left(\frac{1-\nu}{2}\right) (K R R_2 \cos \phi)$
$\pm \left(\frac{1-\nu}{2}\right) (2K Rn)$	$\pm \left(\frac{1-\nu}{2}\right) (2K R^2 n \cos \phi)$	$\mp \left(\frac{1-\nu}{2}\right) (K R R_1 n)$	$\pm \left(\frac{1-\nu}{2}\right) (K R \cos \phi (R_1 - 2R_2))$
$K R \cos \phi (R_1 - R_2)$ $+ K(R_1 - R_2)(R^2 n^2 - R_2^2)$	$D(R_2 + \nu R_1)$ $+ K(R_1 - R_2)(R^2 n^2 - R_2^2)$	$R \cos \phi \{D - K R_2 (R_1 - R_2)\}$ $\pm D R n$	
$\mp \left(\frac{1-\nu}{2}\right) \{K R n (R_1 - R_2)\}$	$\mp \left(\frac{1-\nu}{2}\right) \{K R^2 n \cos \phi (R_1 - R_2)\}$	$\mp \left(\frac{1-\nu}{2}\right) R n [D - K (R_1 - R_2)^2]$ $- \left(\frac{1-\nu}{2}\right) D R \cos \phi$	

TABLE 2.3.2 Coefficients of Matrix C2 in Eq. 2.20



* F_t, F'_t are the Equivalent Static Forces of $M_{S\theta}$

Fig. 2.1 Effective Shearing Forces .

CHAPTER 3

STIFFNESS ANALYSIS

3.1 Introduction

This chapter describes how the standard methods of structural analysis can be employed to obtain a complete solution in terms of the stresses and the displacements at any point within a complex structure composed of many elements of any type of shell of revolution, using the eight fundamental equations derived in Chapter 2.

3.2 Influence Coefficients

If a perfectly elastic structural component, supported against rigid body motion, is acted upon by a set of forces $F_1, F_2, F_3, \dots, F_n$ at points 1, 2, 3, ..., n and if in addition to this set of forces, intermediate loads are applied simultaneously to the structural member, the induced deformation d_i at the point i due to the forces F_j ($j = 1, 2, 3, \dots, n$) can be expressed in the form

$$d_i = \sum_{j=1}^n a_{ij} + d_i^0$$

3.1

where d_i^0 is the additional deformation at point i due to the intermediate applied loads and a_{ij} represents the deflection d_i due to a unit value of F_j . The elastic constants a_{ij} are

independent of the magnitude of the applied forces F_j . Eq. 3.1, which represent the linear relation of deformations and forces, can be written in matrix form as

$$\{d\} = [A]\{F\} + \{d^0\} \quad 3.2$$

where the square matrix $[A]$ is known as the flexibility matrix.

For the same problem the relation given in Eq. 3.2 can also be written in a form which represents the equilibrium conditions for the structure as

$$\begin{aligned} \{F\} &= [K]\{d - d^0\} \\ &= [K]\{d\} + \{F^0\} \end{aligned} \quad 3.3$$

where the square matrix $[K]$ is defined the element stiffness matrix. The element K_{ij} is defined as the force F_j due to unit displacement at d_i . $\{F^0\}$ is the column vector $\{F^0_1, F^0_2, \dots, F^0_n\}$ which are the equivalent forces that replace the intermediate applied loads (equivalent in the sense that the work done during any incremental deformation approximates the work done by the actual applied load). These forces are numerically equal to the so-called fixed end forces.

As stated above, while establishing the flexibility matrix the structure is assumed to be supported against rigid body motion, a condition not necessary for the stiffness matrix.

For the latter case the structure can be free to move as a rigid

body when a set of nodal displacements is applied. The stiffness matrix, thus obtained, is called "the direct stiffness matrix" which is singular. The elements in any column represent a force system in equilibrium.

The well known advantage of the direct stiffness method is that for an assembly of structural elements, the total structural stiffness matrix can be easily formed by superposition of the individual stiffness matrices irrespective of the boundary conditions. The boundary conditions are considered only after assembly in the actual solution of the system of equations.

This permits the consideration of different boundary conditions while the total structure stiffness matrix remains unaltered.

3.3 Shell Element Influence Coefficients

By a procedure similar to that used for a beam element, the stiffness matrix for an element of shell of revolution can be obtained. Since the shell element is a rotationally symmetric element, the nodal points are replaced with nodal circles. (Fig. 3.1). The forces and displacements can be expressed as the amplitude of a harmonic number along the nodal circles.

Consider the shell element shown in Fig. 3.1. The eight fundamental stress resultants per unit length at the two edges of the element are shown acting in the positive sense according to the shell theory sign convention. Should the edge $s = a$ of the element be subjected to a unit value of displacement

in the direction of W while preventing any other displacement of the two ends of the element, the forces per unit length that would be required to maintain equilibrium of the element are the influence coefficients for a unit displacement β_a . If one writes the degrees of freedom of the element in the order $\beta_a \ W_a \ V_a \ U_a \ \beta_b \ W_b \ V_b \ U_b$, the element stiffness equation (Eq. 3.3) can be written as

$$\left\{ \begin{array}{l} M_{sa} \\ S_{sa} \\ N_{sa} \\ T_{sa} \\ M_{sb} \\ S_{sb} \\ N_{sb} \\ T_{sb} \end{array} \right\} = [K] \left\{ \begin{array}{l} \beta_a \\ W_a \\ V_a \\ U_a \\ \beta_b \\ W_b \\ V_b \\ U_b \end{array} \right\} + \left\{ \begin{array}{l} M_s^o \\ S_s^o \\ N_s^o \\ T_s^o \\ M_s^o \\ S_s^o \\ N_s^o \\ T_s^o \end{array} \right\} \quad 3.4$$

where $M_s^o, S_s^o, N_s^o, T_s^o$ are the fixed end stress resultants, at the ends a or b as subscripted, due to the applied loads or thermal gradients.

In order to establish the stiffness matrix and the fixed end stresses, the predetermination of the relationship between the edge displacements and the edge forces is necessary.

The stiffness coefficients and fixed end forces for a beam element can be evaluated by the well known method, such as consistent deformation, or solving the pertinent differential equations.

If an analytical solution of the relevant differential equations for a specific geometry of shells is known, both influence coefficients and fixed end forces due to the applied loads can be found. Such a solution is not available for an arbitrary element of shells of revolution with variable thickness and subjected to arbitrary loadings. However, the forces can be evaluated by solving the basic differential equations numerically.

3.4. Solution of the Governing System of Equations

For any given shell element with given geometry and applied load, the governing system of equations (Eq. 2.24) is in the form

$$\{y^*(s)\} = [A(s)]\{y(s)\} + \{B(s)\} \quad 3.5$$

where $\{y(s)\}$ is a vector of the eight dependent variables, four displacements and four corresponding forces, at a particular location $s = \text{constant}$; $\{y^*(s)\}$ is a vector of the derivatives of the eight variables with respect to the coordinate s , i.e., $d/ds\{y(s)\}$; $[A(s)]$ is the coefficient matrix relating the variables and their derivatives and consists only of functions of the shell thickness and the geometrical parameters of the location s ; and, $\{B(s)\}$ is a vector of the inhomogeneous terms in the equations and is a function of the applied loads and the location s .

The general solution of Eq. 3.5 consists of two parts:

- 1) The solution of the homogeneous part of the equations, i.e., the differential equations when all loading terms, $\{B_{(s)}\}$, are set equal to zero. This homogeneous solution involves the evaluation of eight constants of integration as the result of eight boundary conditions at the two discontinuous edges of the shell.
- 2) The particular solution of the equations in which all loading terms are considered. This solution does not depend upon the boundary conditions.

Therefore, the general solution that satisfies the governing system of equations together with the appropriate boundary conditions at the two edges of the shell element can be written as

$$\{y_{(s)}\} = \{h_{(s)}\} + \{P_{(s)}\} \quad 3.6$$

where $\{P_{(s)}\}$ represents the particular solution that satisfies the equation.

$$\{P^*(s)\} = [A(s)]\{P(s)\} + \{B(s)\} \quad 3.7$$

and, $\{h_{(s)}\}$ is the homogeneous solution that satisfies the equation.

$$\{h^*(s)\} = [A(s)]\{h(s)\} \quad 3.8$$

Consider now the solution of Eq. 3.8 for an element which spans the region $b > s \geq a$. Since Eq. 3.8 represents eight first order linear differential equations, the solution should contain eight arbitrary constants of integration. Let these arbitrary constants of integration be the eight (arbitrary), boundary values which can be imposed at edge "a", and denote these values by $\{c\}$. Then at edge "a"

$$\{h_{(a)}\} = \{c\} \quad 3.9$$

Substituting into Eq. 3.8 yields, for $s = a$

$$\{h^*(s)\}_{s=a} = [A(s)]_{s=a}\{c\} \quad 3.10$$

Integrating this numerically, as an initial value problem, allows the value of $h_{(s)}$ at any point $s > a$ to be determined as

$$\{h_{(s)}\} = [H(s)]\{c\} \quad 3.11$$

where $[H(s)]$ represents the matrix arising from the integration of the $[A(s)]$ matrix along the length of the element.

Since the matrix $[H_{(s)}]$ is independent of the initial conditions ($\{c\}$), it is a property of the element and may be interpreted as follows. As $\{c\}$ is arbitrary, assign a unit value to one component, say component c_j , and set the others equal to zero. Then the j^{th} column of $[H_{(s)}]$ represents the values of $\{h_{(s)}\}$ for a unit value of the j^{th} boundary condition at "a". It is apparent that when $s = a$, Eq. 3.11 must reduce to Eq. 3.9 in order to match the arbitrary boundary conditions, and hence $[H_{(a)}]$ must be the identity matrix, i.e.

$$[H_{(a)}] = [I] \quad 3.12$$

Eq. 3.12 may be considered to be a "boundary condition" on the numerical integration of the matrix $[H]$.

Turning now to the solution of Eq. 3.7, the equation at $s = a$ may be written as

$$\{P^*(s)\}_{s=a} = [A_{(s)}]_{s=a} \{c^*\} + \{B_{(s)}\} \quad 3.13$$

where $\{c^*\}$ represents an arbitrary set of initial values of $\{P_{(a)}\}$. Numerical integration of this equation yields

$$\{P(s)\} = [H_{(s)}] \{c^*\} + \{Q_{(s)}\} \quad 3.14$$

where $[H_{(s)}]$ is the matrix that arises in Eq. 3.11 and $\{Q_{(s)}\}$ is a vector arising from the integration of the inhomogeneous terms. Since the particular solution is any solution which

satisfies the inhomogeneous equations it is adequate to select

$$\{c^*\} = 0 \quad 3.15$$

in which case Eq. 3.14 reduces to

$$\{P(s)\} = \{Q(s)\} \quad 3.16$$

and consequently the general solution, Eq. 3.6, becomes

$$\{y(s)\} = [H(s)]\{c\} + \{Q(s)\} \quad 3.17$$

For $s = b$, Eq. 3.17 becomes

$$\{y(b)\} = [H(b)]\{y(a)\} + \{Q(b)\} \quad 3.18$$

in which each column vector of $[H]$ represents the variables at "b" corresponding to each unit variable applied at "a" in the absence of any applied loads. The vector $\{Q\}$ represents the variables at "b" corresponding to zero displacements and stresses resultants at "a" in the presence of the applied loads.

3.5 Shell Element Stiffness Matrix

The column vector $\{y(s)\}$ represents the vector of Eq. 2.24 consisting of the four displacements as defined by Eq. 2.15.1 and the four stress resultants as defined by Eq. 2.19.1.

Let

$$\{y_{(a)}\} = \begin{Bmatrix} D_a \\ F_a \end{Bmatrix} \quad 3.19$$

and

$$\{y_{(b)}\} = \begin{Bmatrix} D_b \\ F_b \end{Bmatrix} \quad 3.20$$

Eq. 3.18 can be written in partitioned form as

$$\begin{Bmatrix} D_b \\ F_b \end{Bmatrix} = \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \end{bmatrix} \begin{Bmatrix} D_a \\ F_a \end{Bmatrix} + \begin{Bmatrix} Q_d \\ Q_f \end{Bmatrix} \quad 3.21$$

where

D represents the four displacement variables,

F represents the four stress resultant variables, and

Q_d, Q_f are the displacement and the stress resultant parts of the particular solution respectively.

The total matrix appearing in Eq. 3.21 is usually referred to as a "transfer matrix" [23]. Eq. 3.21 can be expanded to form two equations as follow

$$\begin{Bmatrix} D_a \\ D_b \end{Bmatrix} = \begin{bmatrix} I & 0 \\ H_1 & H_2 \end{bmatrix} \begin{Bmatrix} D_a \\ F_a \end{Bmatrix} + \begin{Bmatrix} 0 \\ Q_d \end{Bmatrix}$$

$$= [Y_1] \begin{Bmatrix} D_a \\ F_a \end{Bmatrix} + \begin{Bmatrix} 0 \\ Q_d \end{Bmatrix} \quad 3.22$$

and

$$\begin{Bmatrix} F_a \\ F_b \end{Bmatrix} = \begin{bmatrix} 0 & I \\ H_3 & H_4 \end{bmatrix} \begin{Bmatrix} D_a \\ F_a \end{Bmatrix} + \begin{Bmatrix} 0 \\ Q_f \end{Bmatrix}$$

$$= [Y_2] \begin{Bmatrix} D_a \\ F_a \end{Bmatrix} + \begin{Bmatrix} 0 \\ Q_f \end{Bmatrix} \quad 3.23$$

Solving Eq. 3.22 for the vector $\langle D_a \ F_a \rangle^T$ and substituting
into Eq. 3.23, yields

$$\begin{Bmatrix} F_a \\ F_b \end{Bmatrix} = [Y_2] [Y_1]^{-1} \begin{Bmatrix} D_a \\ D_b - Q_d \end{Bmatrix} + \begin{Bmatrix} 0 \\ Q_f \end{Bmatrix} \quad 3.24$$

or

$$\begin{Bmatrix} F_a \\ F_b \end{Bmatrix} = [K] \begin{Bmatrix} D_a \\ D_b \end{Bmatrix} + \begin{Bmatrix} F^0_a \\ F^0_b \end{Bmatrix} \quad 3.25$$

It can be seen that each column of $[K]$ represents the stress resultants at each end for a unit displacement applied at one end while the other displacements are restrained and $\{F^0\}$ represents the stress resultants corresponding to the totally restrained boundaries.

3.6 Stiffness Matrix Sign Convention

In the derivation of the element stiffness matrix and the fixed end stresses, the sign convention used corresponds to that generally used in shell theory as given in Fig. 3.1. As a result, the stiffness matrix will have some negative elements on the main diagonal. This can be corrected by adapting the so called "stiffness matrix sign convention". This sign convention is shown in Fig. 3.2. It can be seen that the positive direction of the top normal in plane force N_s , the top tangential shearing force T_s , the bottom moment M_s and the bottom transverse shear S_s have been changed to the opposite direction. Therefore, the stiffness matrix and fixed end stresses in Eq. 3.25 are to be premultiplied by the diagonal matrix

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & 1 \\ & & & & -1 \\ & & & & & 1 \\ & & & & & & 1 \end{bmatrix}$$

3.7 Stress Resultants and Displacements at Intermediate Points

For a shell structure composed of a number of elements, the element stiffness matrices and fixed end forces are evaluated and assembled in the master stiffness equations of the structure. Boundary conditions are imposed, and the displacements at the boundaries of each element are obtained from the solution of the master equations. By substituting the final known boundary displacements of each element into the corresponding element stiffness equation (Eq. 3.25), the primary stress resultants at the element boundaries can be obtained. Thus the correct boundary conditions, displacements and primary stress resultants, for each element are known. In order to evaluate the displacements and the stress resultants at any desired number of intermediate points within the element, the correct boundary conditions at one end of the element, say the end at $s = a$, are used as initial conditions in integrating the governing set of equations (Eq. 3.5). At each intermediate point, the secondary stress resultants, (which were eliminated from the governing equations) can be evaluated, first by evaluating the derivatives of the displacements using Eq. 2.18.

$$\{D^*\} = [A1]\{D\} + [A2]\{F_s\} + \{A3\} \quad 3.26$$

and then substituting into Eq. 2.20

$$\{F_\theta\} = [C1]\{D^*\} + [C2]\{D\} + \{C3\} \quad 3.27$$

A simple check on the results of the integration is that the displacements and the primary stress resultants at the termination end of the element should agree exactly with the known boundary conditions at this end.

3.8 Transformation from Local to Global Coordinates

Displacements and stress resultants, at any point along the generator, are presented in the direction tangent to the meridian at this point and the direction perpendicular to it. Due to possible discontinuity of the meridian curve at a junction between two elements, it is necessary to transform the influence coefficients and the fixed end forces at this junction, to a new coordinates system. It is simplest to adopt the direction of the structure's axis of revolution, x , and the direction perpendicular to it, r . According to the stiffness matrix sign convention, the transformation equations may be written as follow

$$\{D_L\} = [L]\{D_G\} \quad 3.28.1$$

$$\{F_G\} = [L]^T\{F_L\} \quad 3.28.2$$

where

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin \phi_i & -\cos \phi_i \\ 0 & \cos \phi_i & \sin \phi_i \\ 0 & 0 & 0 \end{bmatrix} \quad 3.29$$

$\{D\}$ and $\{F\}$ represent the displacement and forces respectively and the subscripts L, G represent local and global coordinates.

ϕ_i is the angle measured from the global axis of rotation to the meridian at the point $s = i$ (Fig. 3.3).

3.9 Modification of Stiffness Coefficients to Account for Eccentricity

In many cases the middle surfaces of two elements which meet at a node do not coincide at the same point (Fig. 3.4). A transformation of the stiffness coefficients and fixed end stresses to a common reference point is then necessary before assembling the element stiffness matrices into the master stiffness matrix. Eqs. A.10 relate the displacement components of a point at a distance z from the middle surface to the displacement components of a point on the middle surface lying in the same plane. By expanding Eqs. A.10, by means of Fourier series, one can write the following equation

$$\begin{Bmatrix} \beta_z \\ w_z \\ v_z \\ u_z \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ z & 0 & 1 & 0 \\ 0 & \frac{nz}{r} & 0 & \frac{r_2+z}{r_2} \end{bmatrix} \begin{Bmatrix} \beta \\ w \\ v \\ u \end{Bmatrix} \quad 3.30$$

where Eq. A.7.1 has been used to eliminate the derivative of w with respect to s and the subscript z refers to the point at a

distance z from the middle surface. Inverting Eq. 3.30 results in

$$\begin{Bmatrix} \beta \\ W \\ V \\ U \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -z & 0 & 1 & 0 \\ 0 & \frac{-nr_2z}{r(r_2+z)} & 0 & \frac{r_2}{r_2+z} \end{bmatrix} \begin{Bmatrix} \beta_z \\ W_z \\ V_z \\ U_z \end{Bmatrix} \quad 3.31$$

or

$$\{D\} = [EC]\{D_z\} \quad 3.32$$

From the work equivalence requirements, the relation between the stresses at the two points can be written as

$$\{F_z\} = [EC]^T\{F\} \quad 3.33$$

Eq. 3.32 and 3.33 are used to transform the displacements and stresses of a point on the middle surface to a point at a distance z from the middle surface.

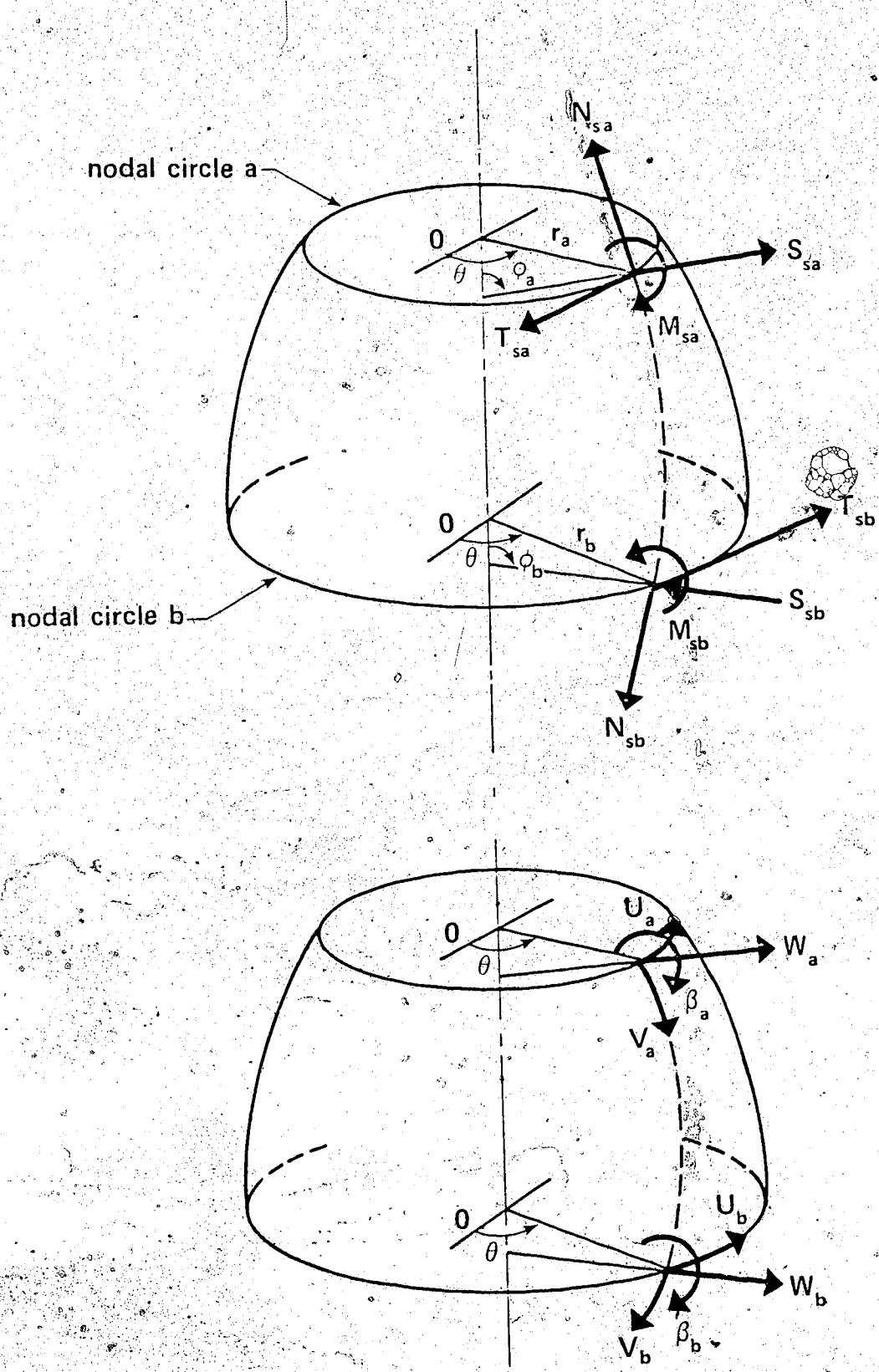


Fig. 3.1 Shell Theory Sign Convention for Stress Resultants and Displacements

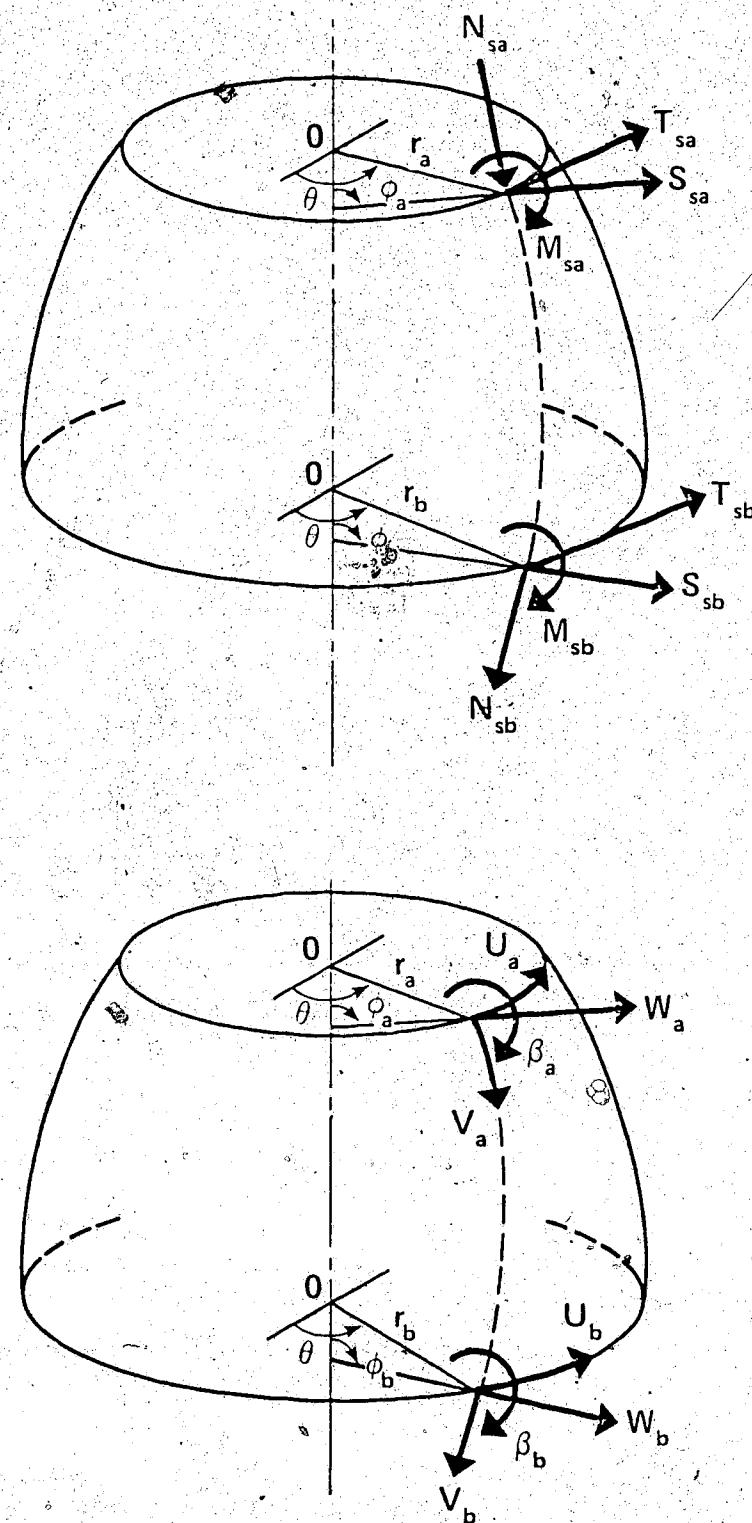


Fig. 3.2 Stiffness Matrix Sign Convention for Stress Resultants and Displacements.

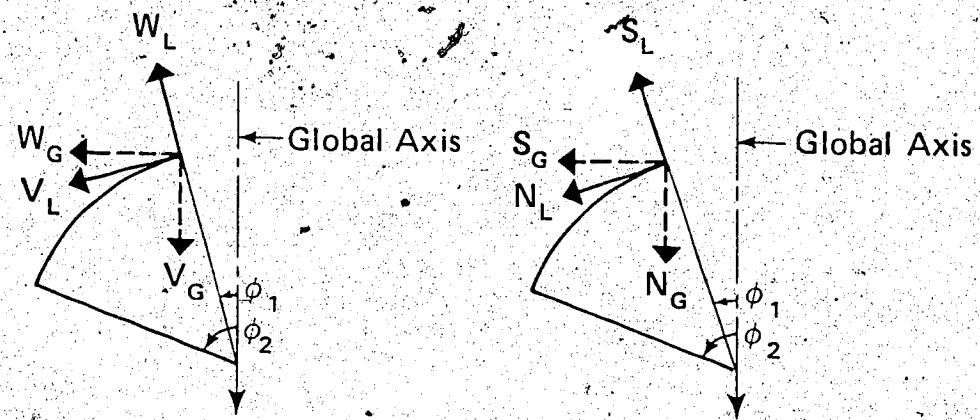


Fig. 3.3 Notation for Transformation of Stress Resultants and Displacements

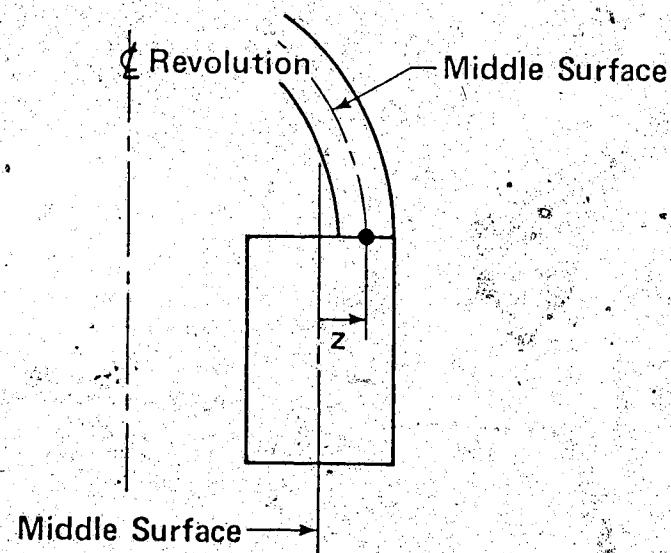


Fig. 3.4 Eccentricity at a Node

CHAPTER 4

EXAMPLE APPLICATIONS

4.1 Introduction

A computer program, named SASHELL, has been developed to perform the stiffness analysis of segmented shell structures based on the theory presented in the preceding chapters. The logic flow of SASHELL is outlined in Sect. 4.2 and listing of the program is included in Appendix C. The results of the analysis of two example applications, using SASHELL, are presented in this chapter.

The first example is the pinched cylinder. The exact analytical solution for a long cylinder pinched by a symmetrical circumferential line load (Fig. 4.1) is known [27, pp. 471; 10, pp. 280]. Finite element solutions, using (48×48) element stiffness matrices [11] and (24×24) element stiffness matrices [5,7], were obtained for the case of a cylindrical shell loaded by diametrically opposed concentrated loads (Fig. 4.2). This solution was compared [2] against an analytical solution based on the inextensional deformation theory (i.e. neglecting entirely the strain in the middle surface of the shell) [27, pp. 501-506].

The second example is the analysis of hyperboloid natural cooling tower under the action of wind load and

its own weight. Finite element analysis, using the computer program SORIII [14] and using conical shell elements to approximate the geometry [24] are known.

4.2 Logic Flow of SASHELL

In this section, the organization of the computer program SASHELL, which can serve as a summary for the solution technique of segmented shell structure as described in the preceding chapters, is outlined. Details of the required input are given in Appendix B.

- 1) The structure is divided into elements, each of which is a simple type of shell of revolution, and which are connected along nodal circles or "nodes". A concentrated load applied at a point along the meridional coordinate must be treated as a load acting on a node connecting two elements.
- 2) Nodal coordinates, system connectivity information and element types, properties and loading conditions are determined. The problem control parameters are established and input (Subroutine READIN and LOADIN).

The program now performs the following operations:

- 1) Each element is examined. If a coefficient which depends on the element geometric parameters (see the limitations in Sect. 5.3) exceeds a certain limit, the element is divided into subelements (segments) each of which satisfies this limit (subroutine SEGMENT).

- 2) The connectivity of the system is altered to include the new intermediate nodal points. Geometric parameters and loadings values at each segment boundary are calculated (subroutine SEGEOM).
- 3) The number of the structure's degrees of freedom are established.
- 4) If the external applied load on the structure is symmetric, the program discards step 5 and goes to step 6.
- 5) If the load is non-axisymmetric, the total number of points in the meridional and circumferential direction in the structure is determined and the "results" array is initialized.
- 6) The harmonic number, n , is set equal to zero.
- 7) The structure stiffness matrix and load vector are initialized. The nodal load coefficients, if any, which correspond to the harmonic n are added to the load vector.
- 8) The stiffness analysis starts by calculating the stiffness matrix and fixed end forces for each segment in the structure (subroutine STIFAN).
- 9) The geometric parameters and external applied loadings at the desired number of integration points in the segment under consideration are calculated. Dead weight of the segment, if required and if $n = 0$, is superimposed (subroutine PLSEG and DLSEG).

- 10) The initial conditions at the starting edge of the segment, as stated in Sect. 3.4 (Eqs. 3.12 and 3.15), are set. The governing equations (subroutine FLUGGE) are integrated, using a fourth order Runge-Kutta method, over the desired number of points to obtain the transfer matrix of the segment (subroutine RNGKT).
- 11) The segment stiffness matrix and fixed end forces are evaluated from the transfer matrix obtained in step 10, as described in Sect. 3.5 (subroutine STIFIX).
- 12) The results of step 11 are saved (subroutine STORE1).
- 13) The segment stiffness matrix and fixed end forces are modified to correspond to the stiffness matrix sign convention, as stated in Sect. 3.6.
- 14) The stiffness influence coefficients and fixed end forces are modified to account for nodal eccentricity, if any, as mentioned in Sect. 3.9. (subroutine ECCNTR).
- 15) If required, the segment stiffness influence coefficients and fixed end forces are transformed, due to discontinuity of the meridian at the node, to the structural global coordinates, as stated in Sect. 3.8 (subroutine GLTRAN).
- 16) The segment stiffness influence coefficients are assembled, with respect to the structure's degrees of freedom, into the master stiffness matrix. The segment fixed end forces are subtracted from the

corresponding values in the load vector (subroutine STORE).

- 17) Steps 9 to 16 are repeated for each segment in the structure.
- 18) The boundary conditions are imposed on the master stiffness equation (subroutine BOUNDC).
- 19) Segments edge displacements is obtained, using Gaussian elimination algorithm to solve the master stiffness equation (subroutine SOLVER).
- 20) The stiffness analysis, for harmonic number n, is completed. The displacements and stress resultants at the desired number of intermediate points in each segment are to be evaluated (subroutine SRADSP).
- 21) The segment edge displacements, obtained in step 19, are transformed, if necessary, to the segment local coordinates and to account for nodal eccentricity (subroutine GLTRAN and ECCNTR).
- 22) The known segment edge displacements are substituted in the corresponding segment stiffness equation, saved in step 12. The primary stress resultants at the segment boundaries are evaluated, as described in Sect. 3.7 (subroutine STORE1).
- 23) The shell equations are integrated, using the initial conditions calculated in step 22, in order to determine the displacements and primary and secondary stress resultants at the intermediate points within the segment, as described in Sect. 3.7 (subroutine RESULT).

- 24) If the load is symmetric (i.e., the required number of harmonics is zero), the results of step 23 are printed out and the program goes to step 26.
- 25) If the load is non-axisymmetric (i.e., the required number of harmonic is greater than zero), the displacements and stress resultants are calculated at the desired number of points in the circumferential direction. The results are superimposed in the "results" array (subroutine STORE2).
- 26) Steps 21 to 25 are repeated for each segment in the structure.
- 27) The harmonic number, n , is increased by one. Steps 7 to 26 are repeated until n is equal to the required number of harmonics.
- 28) The results saved in step 25 are printed out and the program stops.

4.3 Pinched Cylinder

The governing differential equation for a circular cylindrical element is

$$\frac{d^2W}{dx^2} (K \frac{d^2W}{dx^2}) + \frac{Et}{r^2} W = P_z \quad 4.1$$

where x measures the distance along the axis of the cylinder with the origin ($x = 0$) at midlength of the element. When the thickness of the shell is constant, this equation reduces to

$$\frac{d^4 W}{dx^2} + 4\lambda^4 W = \frac{P_z}{K} \quad 4.2$$

in which the flexural rigidity, K , is

$$K = \frac{Et^3}{12(1 - \nu^2)} \quad 4.3$$

and the coefficient λ is defined as

$$\lambda = \sqrt{\frac{3(1 - \nu^2)}{r^2 t^2}} \quad 4.4$$

For the special case of a long cylindrical shell pinched by a live load P uniformly distributed along a circular section (Fig. 4.1), the solution of Eq. 4.2 takes the form [27]

$$W = \frac{-P}{8\lambda^3} e^{-\lambda x} (\cos \lambda x + \sin \lambda x) \quad 4.5$$

The stress resultants may be expressed in terms of derivatives of the displacement W in the form

$$M_x = K \frac{d^2 W}{dx^2} \quad 4.6.1$$

$$Q_x = K \frac{d^3 W}{dx^3} \quad 4.6.2$$

$$M_\theta = \nu M_x \quad 4.6.3$$

$$N_\theta = \frac{Et}{r} W \quad 4.6.4$$

Numerical values of these expressions, at several points along a cylindrical element 20 feet long, 8 feet in diameter and with constant wall thickness equals to 1.24 inches, are shown in Table 4.1 for a live load $P = 1 \text{ kip/ft}$. Output of the analysis of the same element using SASHELL is included in Appendix D. The results are presented graphically in Fig. 4.3. The results of the two solutions are identical up to the number of significant figures contained in the output.

For the case of a cylindrical shell pinched by two concentrated loads as shown in Fig. 4.2 the concentrated load is approximated as a live load (see Sect. 5.4) that has a value of 1.0 kip/ft at the loaded points and zero at a short distance from the load points (Fig. 4.4.1). The loading function is expanded in Fourier coefficients and the results of the analysis of each harmonic are superimposed.

The results of this loading case are affected by the number of harmonics considered in the analysis, which will be discussed in Sect. 5.4.

If the circumference of the cylinder shown in Fig. 4.4 is divided to 36 intervals and the load is approximated as shown, the equivalent concentrated load is

$$\bar{P} = \frac{2\pi r}{36} P$$

$$= 0.698 \text{ kips}$$

$$L = 20' .0$$

$$E = 30 \times 10^3 \text{ ksi}$$

$$K = 436 \text{ K.ft}$$

$$r = 4'$$

$$\nu = 0.3$$

$$\lambda = 2.0$$

$$t = 1.24"$$

$$P = 1.0 \text{ K/ft}$$

X (ft)	λ_x	W (10^{-4} ft)	M_x	Q_x	M_θ	N_θ
0	0	-.3583	.1250	-.5000	+.0375	-3.997
0.25	0.5	-.2949	.0302	-.2661	+.0091	-3.290
0.5	1.0	-.1822	-.0138	-.0994	-.0041	-2.032
0.75	1.5	-.0854	-.0258	-.0079	-.0077	-0.953
1.0	2.0	-.0239	-.0224	+.0282	-.0067	-0.266
1.25	2.5	+.0059	-.0144	+.0329	-.0043	+0.066
1.5	3.0	+.0152	-.0070	+.0246	-.0021	+0.169
1.75	3.5	+.0139	-.0022	+.0142	-.0006	+0.155
2.0	4.0	+.0092	+.0002	+.0060	+.0001	+0.102
2.25	4.5	+.0047	+.0011	+.0012	+.0003	+0.052
2.5	5.0	+.0016	+.0011	-.0009	+.0003	+0.018
2.75	5.5	0	+.0007	-.0015	+.0002	0
3.0	6.0	-.0006	+.0004	-.0012	+.0001	-0.066

TABLE 4.1 Stresses and Radial Displacement in a Pinched Cylinder as per Eqs. 4.6

The analytical solution for a concentrated load, based on inextensional deformation theory [27, pg. 506], is of the form

$$W_{\phi} = \frac{Pr^3}{\pi KL} \sum_{n=2,4,6,\dots} \frac{1}{(n^2 - 1)^2} \cos n\theta \quad 4.8$$

where L is one-half the length of the cylinder. Eq. 4.8 yields

$$(W)_{\theta=0} = 0.382 \times 10^{-3} \text{ ft}$$

$$(W)_{\theta=\pi/2} = -0.351 \times 10^{-3} \text{ ft}$$

A comparison of these values with SASHELL values shown in Fig. 4.4.1 indicates good agreement for $\theta = \pi/2$ but considerable discrepancy for $\theta = 0$. The SASHELL solution predicts a displacement 17% larger than the inextensional solution at $\theta = 0$.

Now referring to Fig. 4.4.2, the cylindrical shell example as chosen by Cantin and Clough [7], is shown. For the same number of intervals (36), the equivalent concentrated load is

$$\bar{P} = 0.0721 \text{ kips}$$

The deflections for this case, from Eq. 4.8, are

$$(W)_{\theta=0} = 0.614 \times 10^{-2} \text{ ft}$$

$$(W)_{\theta=\pi/2} = -0.565 \times 10^{-2} \text{ ft}$$

The ratios of the SASHELL displacements are 1.10 and 1.14 to those of the inextensional theory for $\theta = \pi/2$ and $\theta = 0$, respectively.

4.4 Hyperboloid Natural Draft Cooling Tower

Large capacity power plants generate a substantial amount of operational heat that requires dissipation. One of the major structures in these power plants is the natural draft cooling tower in the form of the shell of revolution. The tower utilizes its height to create the necessary air flow in order to cool a large volume of water in a minimum land area. Hyperboloid cooling towers are the most preferable shape [14] when compared to conical or cylindrical shapes from the aerodynamic point of view.

4.4.1 Geometry of the Tower

The middle surface of a hyperboloid shell is shown in Fig. 1.7. The surface of a hyperboloid of revolution may be classified as a non-developable surface, which means that the surface will not tend to flatten out under load. This surface is generated by the rotation of a hyperbola about a vertical axis. The geometrical equation can be written as

$$\frac{r^2}{a^2} - \frac{x^2}{b^2} = 1$$

4.9

In which r is the horizontal radius, x is the vertical coordinate

measured from the origin at the throat of the shell, a is the throat radius at $x = 0$, and b is a constant in which the ratio b/a equals the slope of the asymptotes to the hyperbola.

The principal radii of curvature are given by

$$r_1 = - \frac{a^2 b^2}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{3/2}} \quad 4.10.1$$

$$r_2 = \frac{a^2}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{1/2}} \quad 4.10.2$$

where $\tan \phi = \frac{dx}{dr}$

$$= \frac{b}{a} \sqrt{\frac{r^2}{r^2 - a^2}} \quad 4.10.3$$

The expression for the derivative of r_1 with respect to the coordinate s (see Eq. 2.5.1) can be obtained by differentiating Eq. 4.10.1 to yield

$$r_{1s} = \frac{-3}{a^2 b^2} (b^4 r \cos \phi + a^4 x \sin \phi) \left(\frac{r^2}{a^4} + \frac{x^2}{b^4} \right)^{1/2} \quad 4.10.4$$

The middle surface of the hyperboloid tower considered in the following analysis is shown in Fig. 4.5. The shell is 355 ft high and is supported by columns evenly spaced on a circular base of 290 ft diameter. The throat of the tower is 165 ft in diameter and is located 60 ft below the top of the shell. The thickness varies from 30 inches at the bottom level

of the shell to 6 inches at 25 feet elevation from the bottom. In the top 10 feet of the shell, the thickness also varies from 6 inches to 24 inches. Other than the top and the bottom regions, the shell thickness remains constant at 6 inches. The increased thickness at the top provides a stiffening effect that reduces radial deformation under wind load [24]. The bottom ring at the base acts as an equivalent deep beam bridging between columns.

The constant b of Eqs. 4.10 can be calculated by substituting the values for $r = 145$ at $x = 295$ and $a = 82.5$ into Eq. 4.9, which yields $b = 204.1$ ft. For the purpose of comparing the results with References 14 and 24, the following concrete properties are used

$$\text{Young's Modulus } E = 4 \times 10^6 \text{ psi}$$

$$\text{Poisson's Ratio } \nu = 0.15$$

$$\text{Specific Weight } \gamma = 150 \text{ lb/ft}^3$$

4.4.2 Load Description

The tower is analyzed for gravity load and wind pressure load.

The gravity load, which is symmetric with respect to the coordinate θ , is determined from the following equations:

$$q_z = t \times Y \quad 4.11.1$$

$$P_z = -q \cos \phi \quad 4.11.2$$

$$P_s = q \sin \phi$$

4.11.3

where q is the intensity of weight per square area of the surface.

The wind load, which is the governing factor in the design of the cooling towers [14], is based on the ACI-ASCE Committee 334 recommendations [1]. The following equivalent static pressure distribution is used

$$q(H, \theta) = G C_\theta K_H q_{3.0}$$

4.12

where the factors in this equation are described as follows:

- a) $q(H, \theta)$ is the equivalent static normal pressure on the surface of the tower at a location defined by coordinates H, θ . H is the vertical distance measured from the ground level, θ is the circumferential angle measured from the windward meridian.
- b) G is the dynamic gust factor which accounts for the overstress due to the tower response to various time variations in the wind pressure.
- c) C_θ is the coefficient of wind pressure distribution in the circumferential direction.
- d) K_H is the exposure factor which establishes the vertical profile of wind pressure, which in turn, depends on wind speed and roughness of terrain. i.e., K_H can be evaluated from the equation

$$K_H = 2.64 \left(\frac{H}{Hg}\right)^2 \alpha$$

4.13

where H is the height measured from the ground level; Hg is the gradient height above which the wind velocity is assumed constant and ranges from 900 ft for open country to 1500 ft for center of large cities; α is a constant, depending on the terrain roughness, and ranges from $1/7$ for open country to $1/3$ for center of large cities.

- e) q_{30} is the basic wind pressure (psf) and is equal to the dynamic pressure of the free stream of wind at 30 ft above the ground level at a given site. It may be computed from

$$q_{30} = 0.00256 V_{30}^2$$

4.14

where V_{30} = wind velocity (m.p.h.) at 30 ft above the ground level.

The ACI-ASCE Committee 334 has suggested the normalized wind pressure distribution in the circumferential direction as shown in Fig. 4.6.1. As in Reference 24, assuming $Hg = 900$ ft, $\alpha = 1/7$ and $V_{30} = 100$ m.p.h., the variation of the wind pressure with the height can be taken as shown in Fig. 4.6.2.

4.4.3 Analysis Conclusions

The results of the analysis of the hyperboloid tower, loaded as described above, are represented graphically for

dead load in Figs. 4.7.1 to 4.7.3, and for wind load in Figs. 4.8.1 to 4.8.10. Since the actual condition at the bottom of the shell is only partially fixed [24], the results of the analysis for two boundary conditions, fixed and hinged at the bottom, are shown in these figures. Excellent agreement is observed between the displacements and stress resultants shown and the results of References 14 and 24.

The approximation of the geometry with a series of cones [24] does not affect the results for this particular geometry of the tower. However, the meridian curvature R_1 varies only from -0.002 ft^{-1} at the throat to -0.0003 ft^{-1} at the bottom of the hyperboloid shell, and is equal to zero for a conical element. For shells with larger R_1 curvature the conical segment approximation may not be as accurate.

4.5 Comments on Results

The examples of this chapter have been selected to test the ability of SASHELL to analyze different types of shells. The pinched cylinder solutions are common test problems because of the difficulty of achieving solutions for concentrated loads. The hyperboloid cooling tower is an illustration of a shell with negative Gaussian curvature under complex loading conditions. It may be concluded that SASHELL is capable of yielding good results on a wide variety of shell problems.

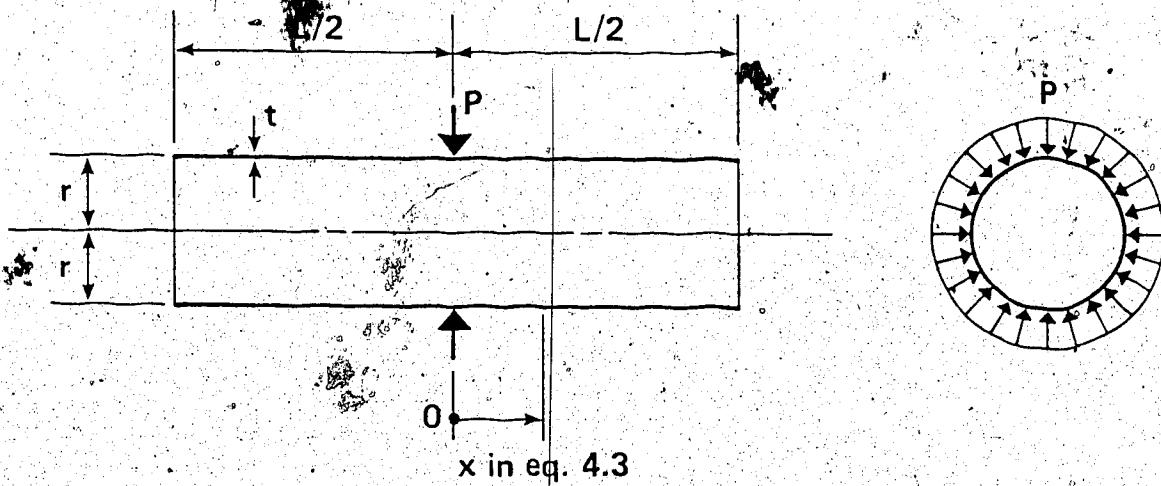


Fig. 4.1 "Pinched Cylinder" Circumferential Line Load

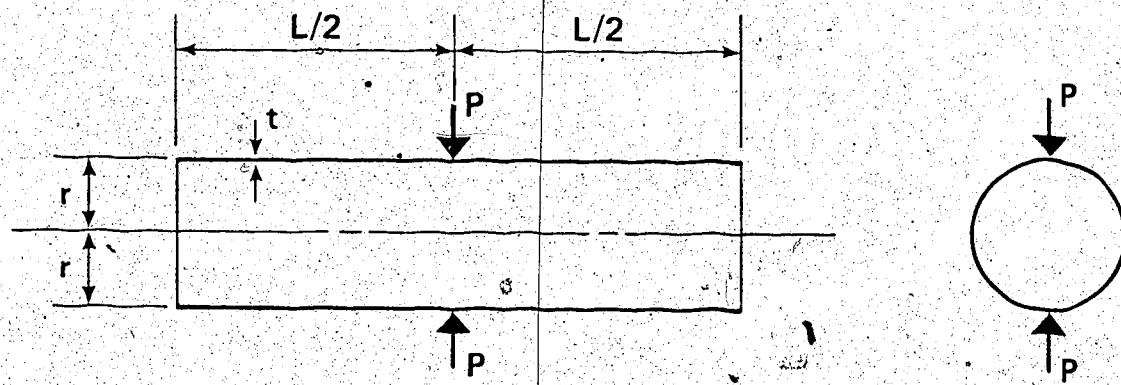


Fig. 4.2 "Pinched Cylinder" Two Concentrated Loads

Sashell Solution
Analytical Solution [27]

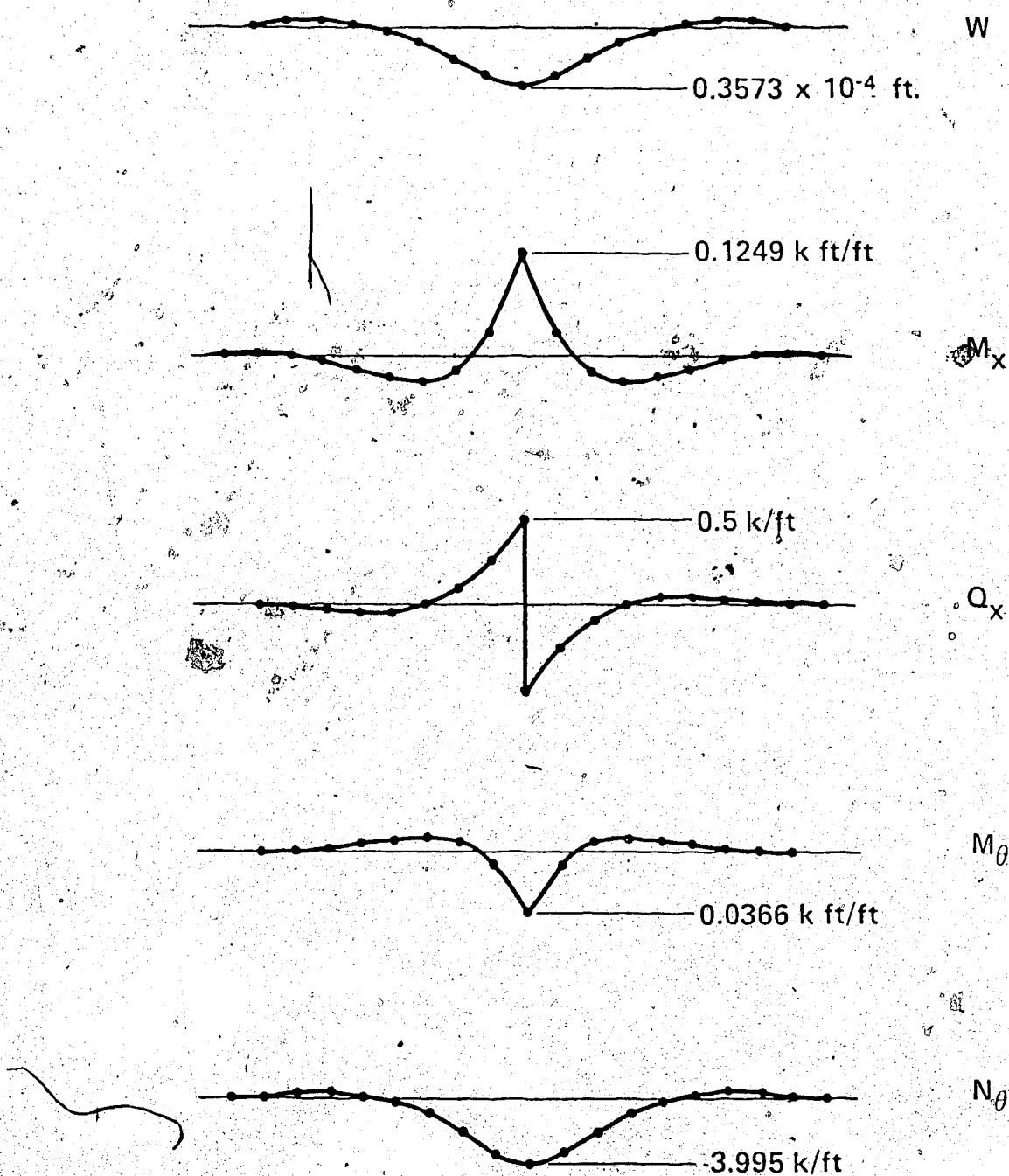


Fig. 4.3

"Pinched Cylinder" Circumferential Line Load Stress Resultants and Radial Displacement

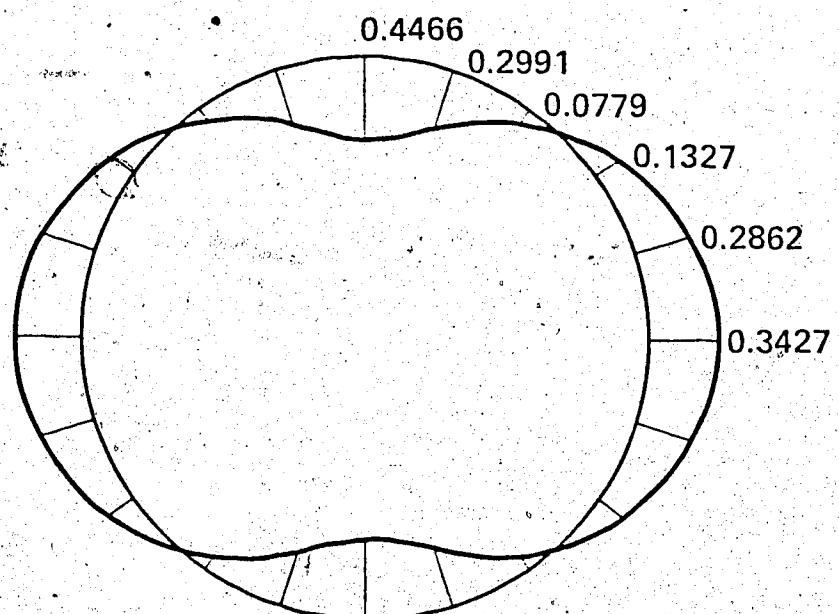
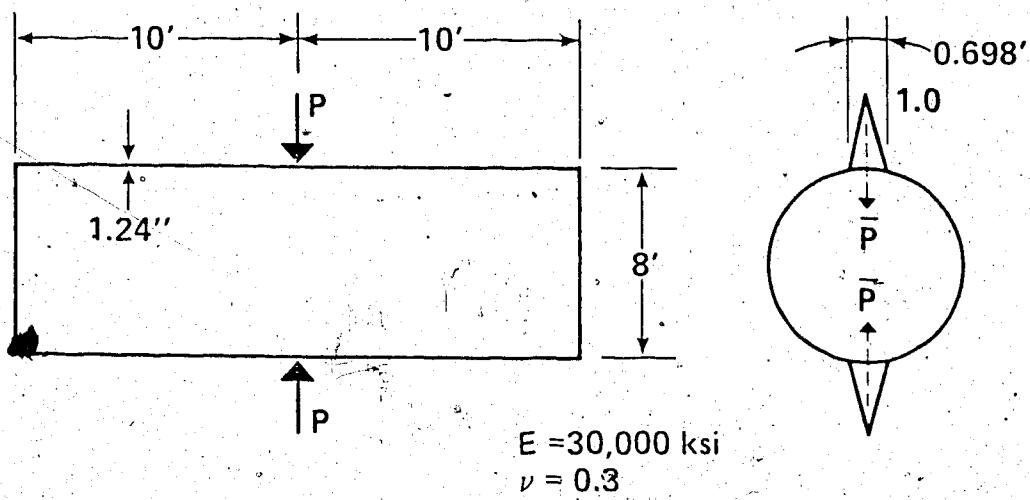
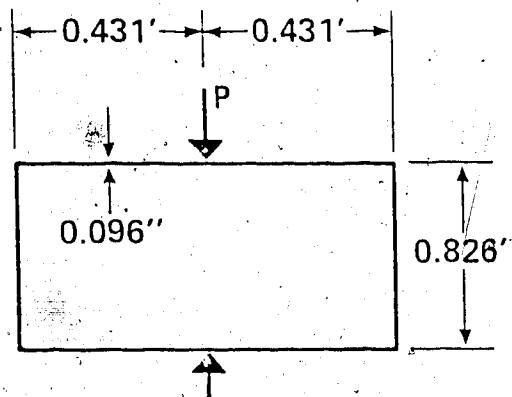
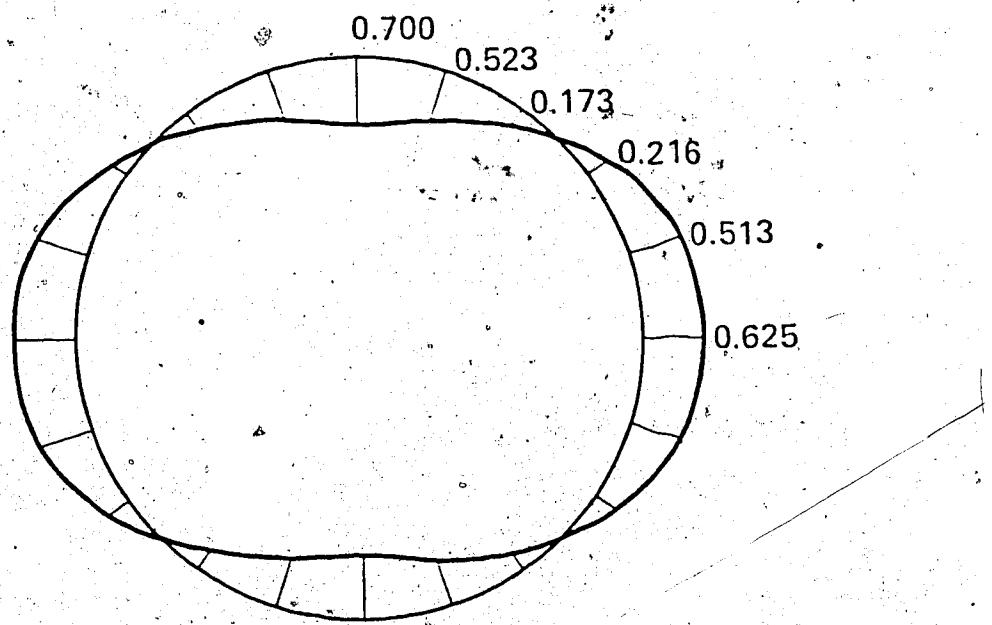
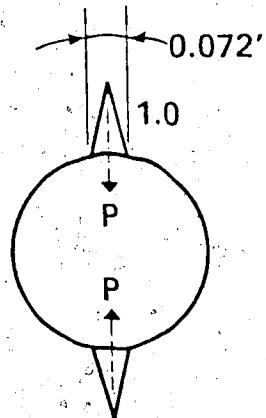


Fig. 4.4.1 "Pinched Cylinder" Two Concentrated Loads Radial Displacement ($r = 4.0 \text{ ft.}$)



$$E = 10,500 \text{ ksi}$$

$$\nu = 0.3125$$



"Pinched Cylinder" Two Concentrated Loads Radial Displacement ($r = 0.413 \text{ ft.}$)

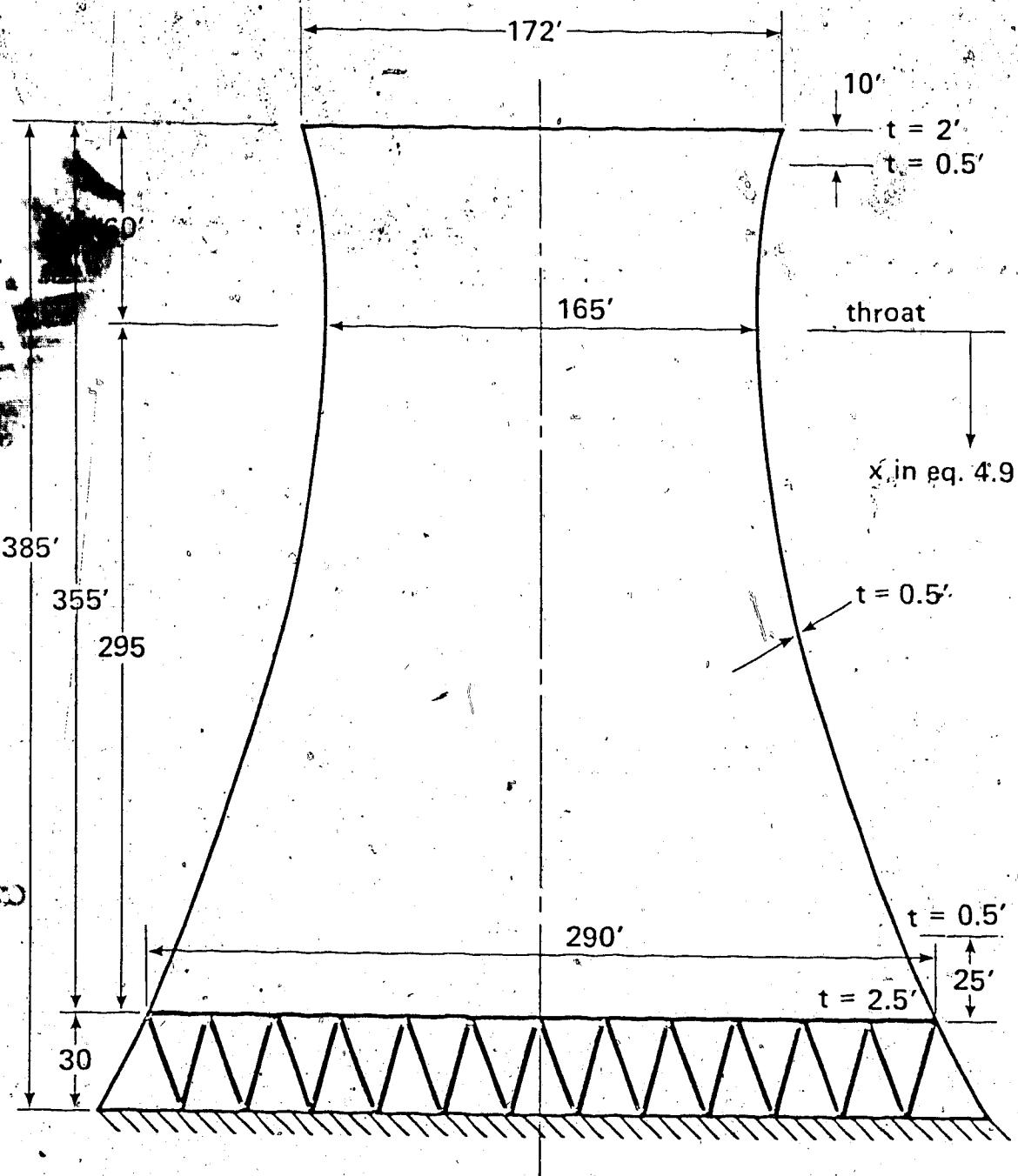


Fig. 4.5 Typical Hyperboloid Natural Draft Cooling Tower

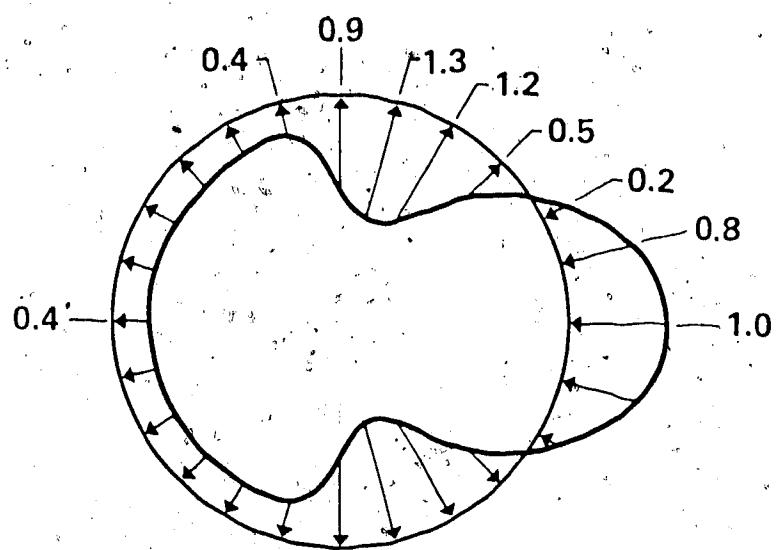


Fig. 4.6.1 Wind Pressure Coefficients C_θ

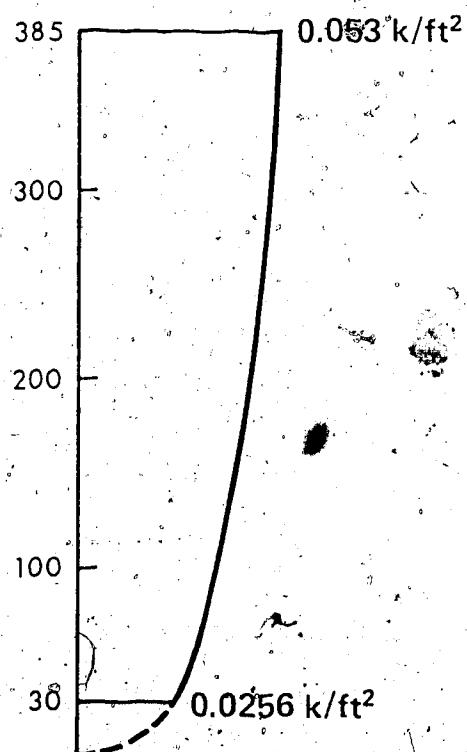


Fig. 4.6.2 Wind Pressure Profile ($\theta = 0$)

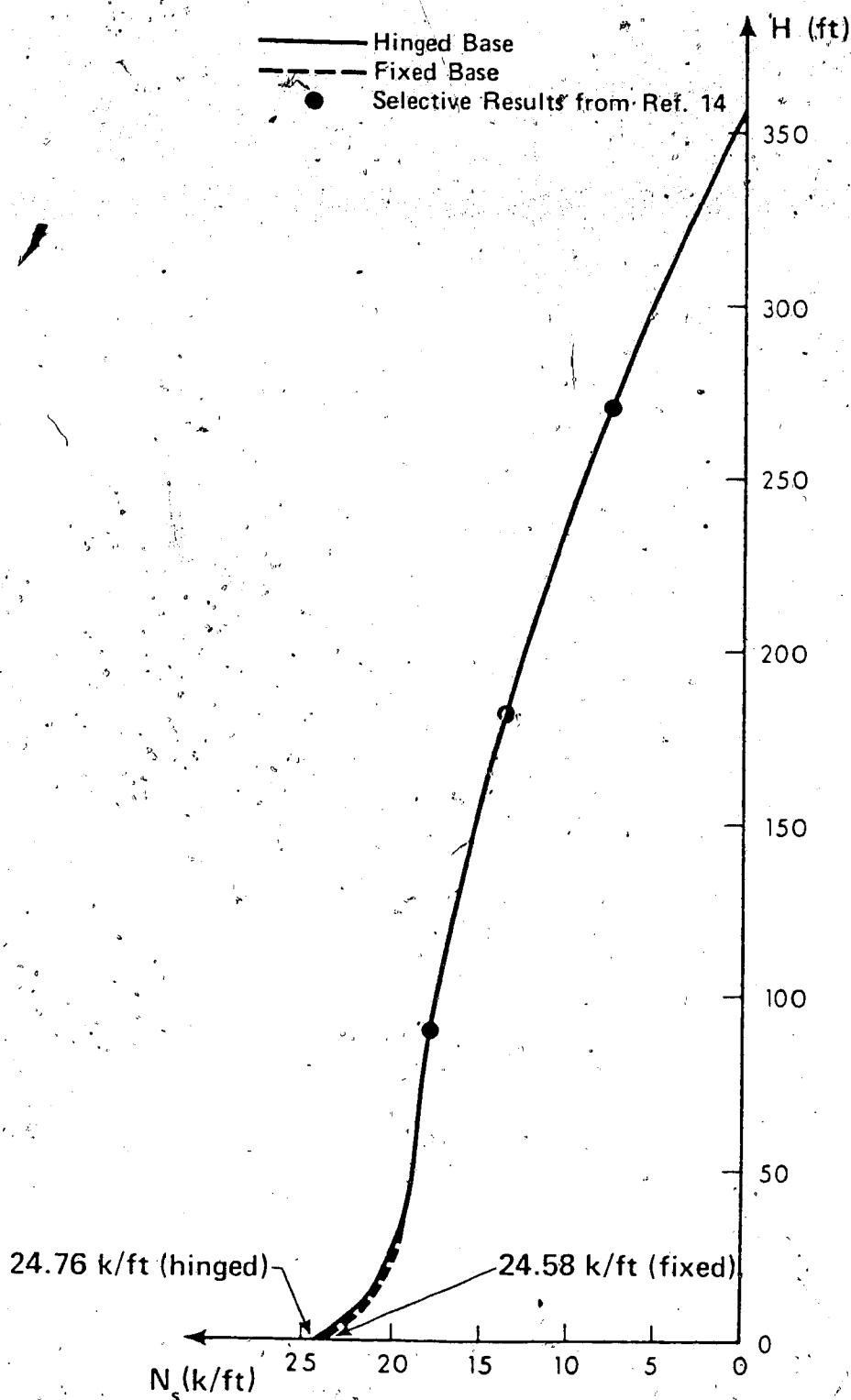


Fig. 4.7.1 Hyperboloid Tower Dead Load Membrane Force N_s

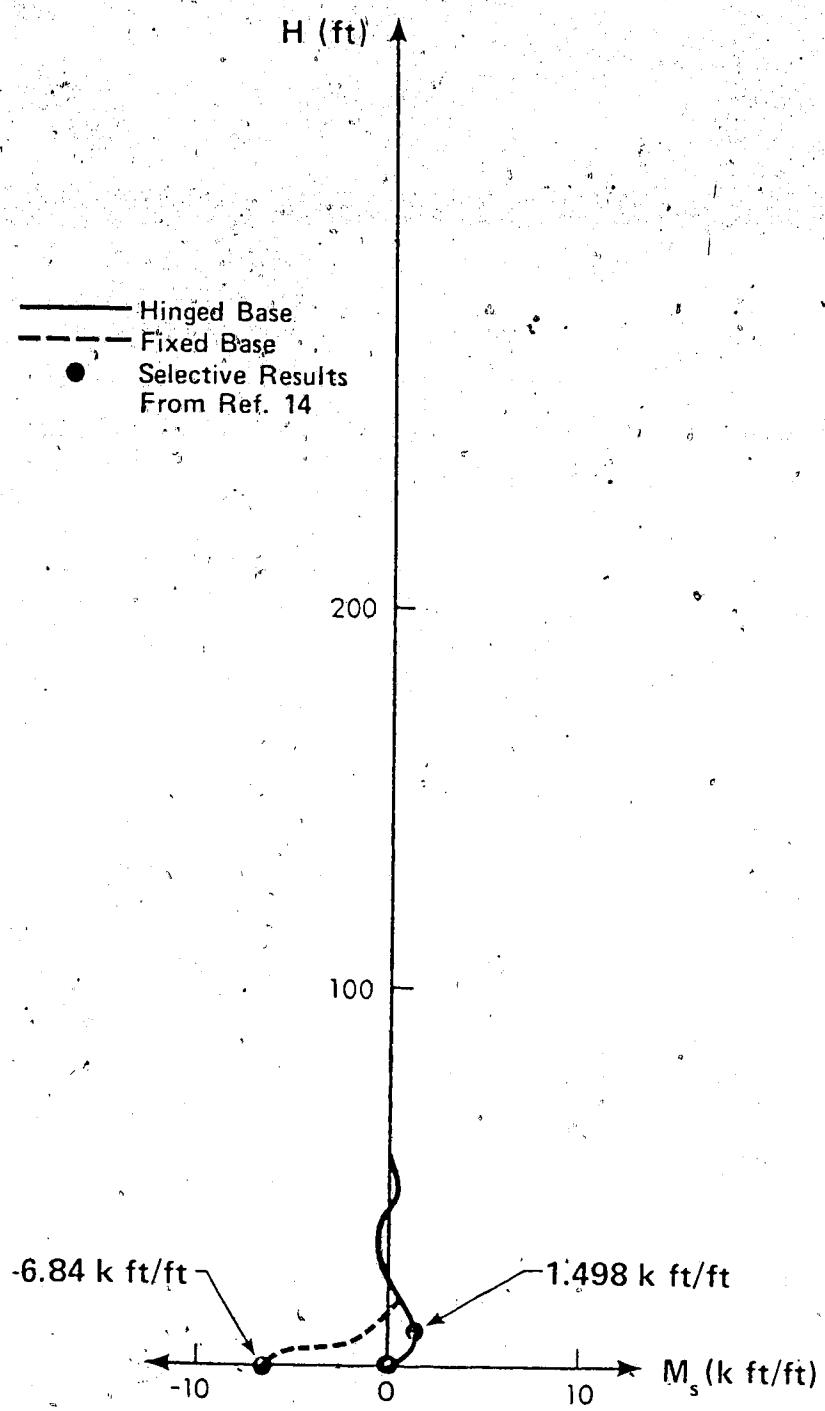


Fig. 4.7.2 Hyperboloid Tower, Dead Load Meridional Moment M_s

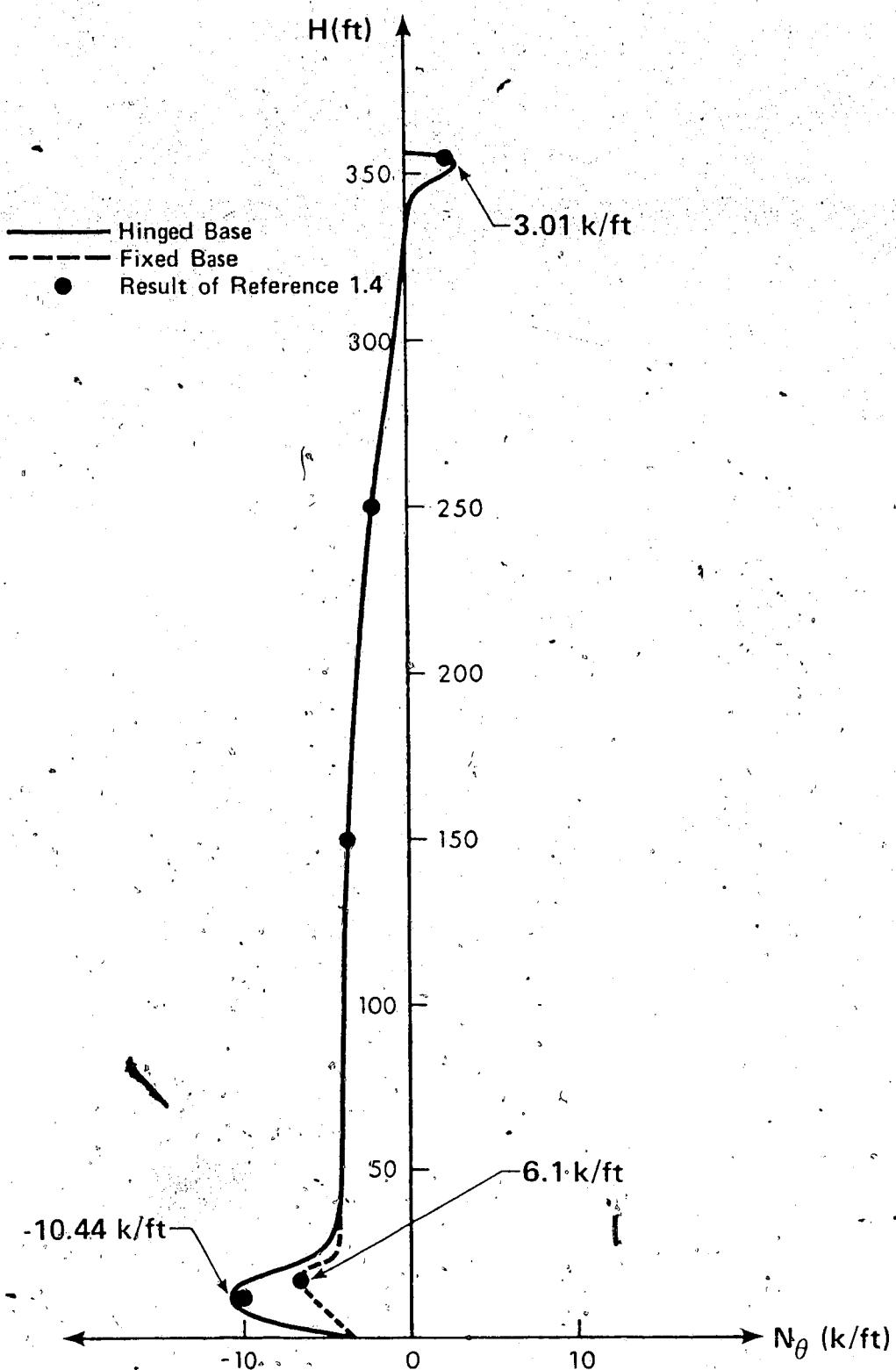


Fig. 4.7.3 Hyperboloid Tower Dead Load Membrane Force N_θ

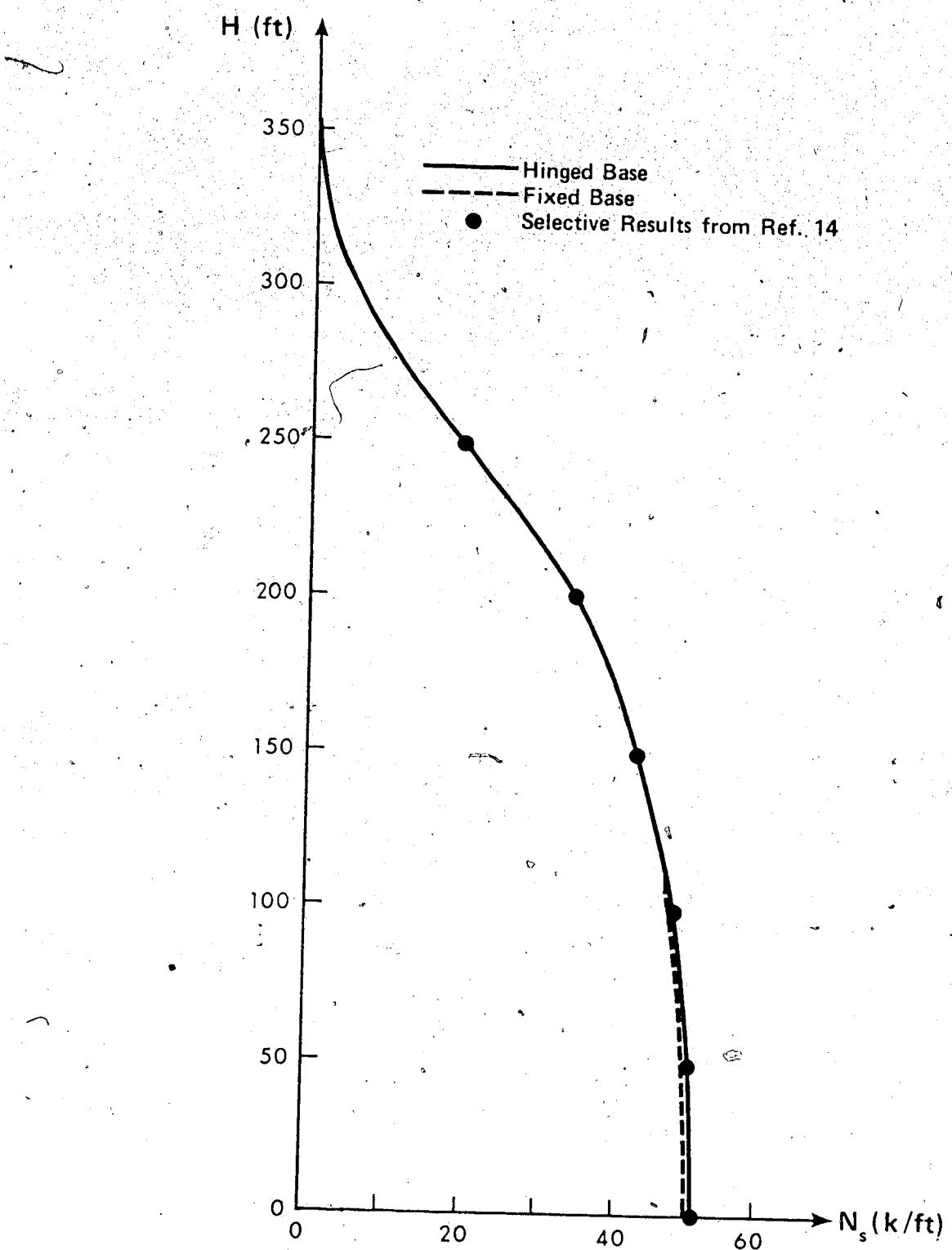


Fig. 4.8.1 Hyperboloid Tower Wind Load Membrane Force N_s ($\theta = 0$)

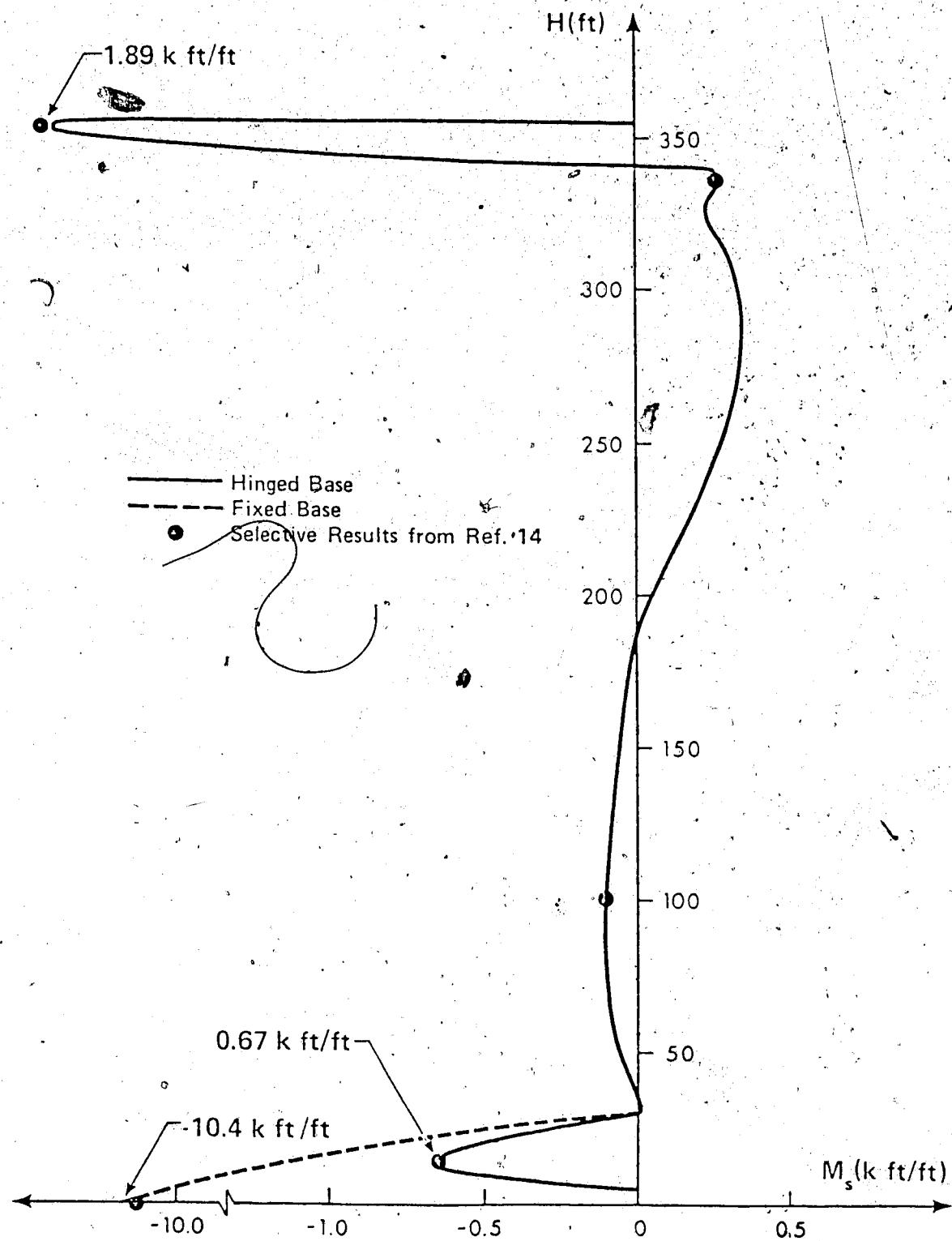


Fig. 4.8.2 Hyperboloid Tower Wind Load Meridional Moment M_s ($\theta = 0$)

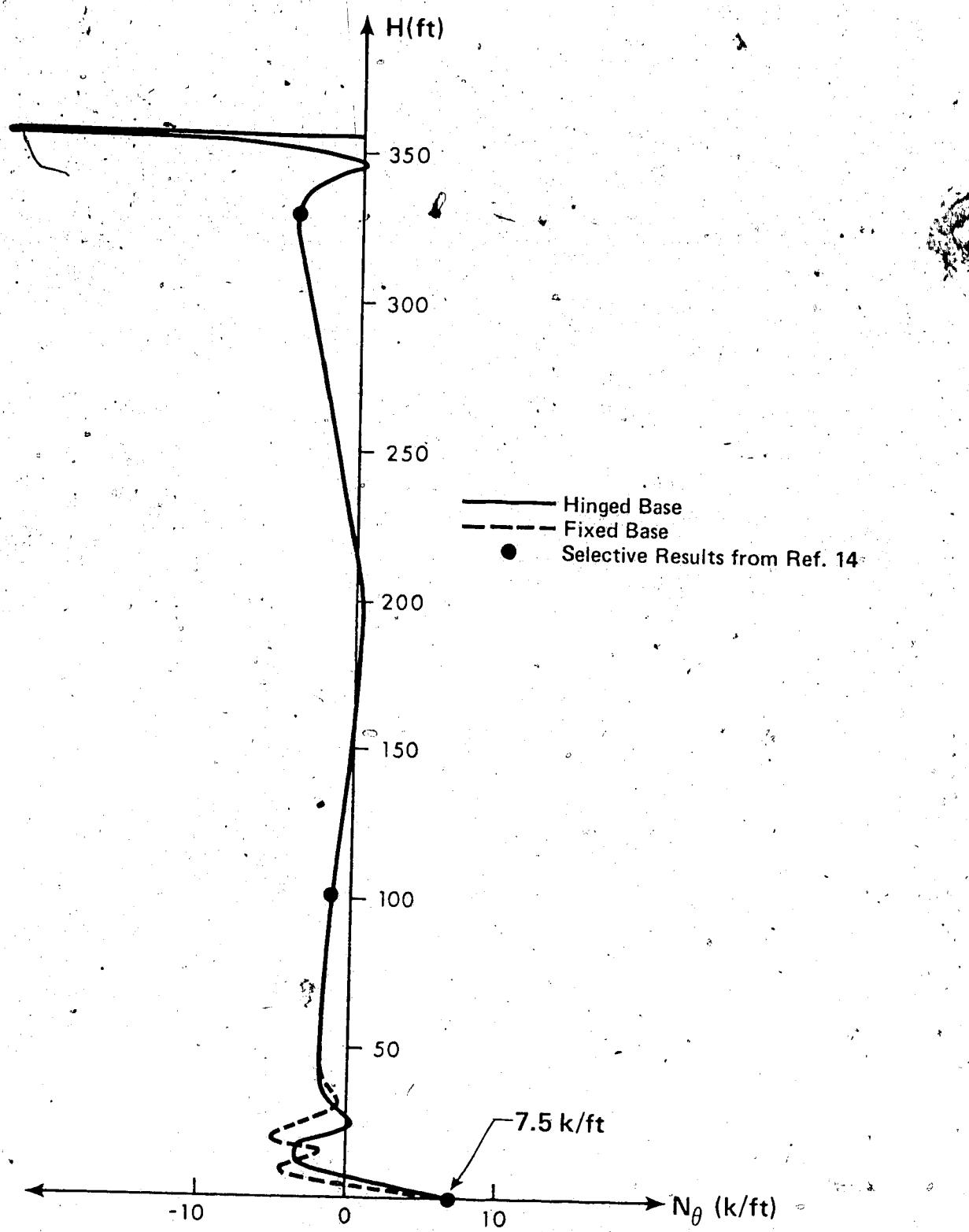


Fig. 4.8.3 Hyperboloid Tower Wind Load Membrane Force $N_\theta (\theta = 0)$

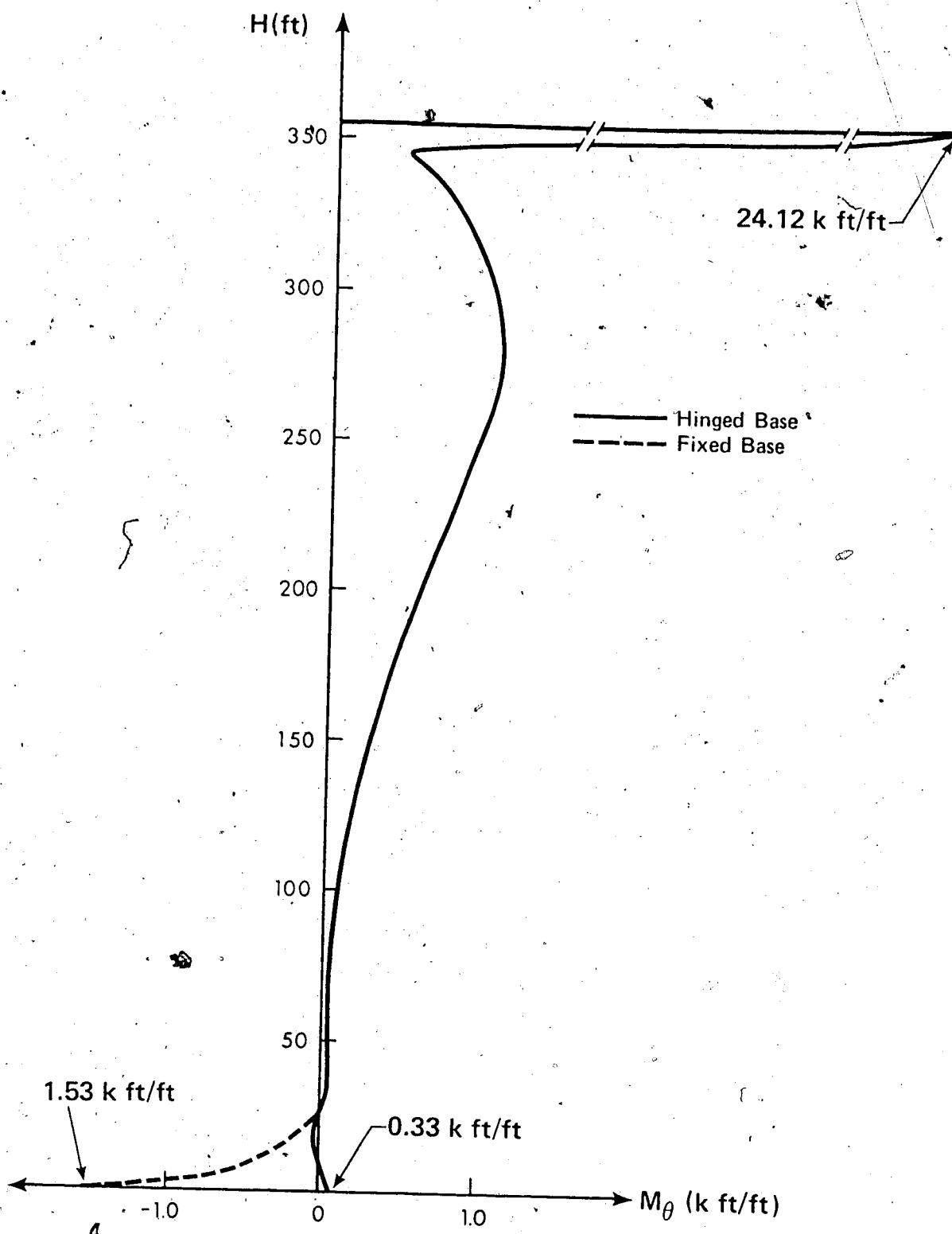


Fig. 4.8.4 Hyperboloid Tower Wind Load Circumferential Moment M_θ ($\theta = 0$)

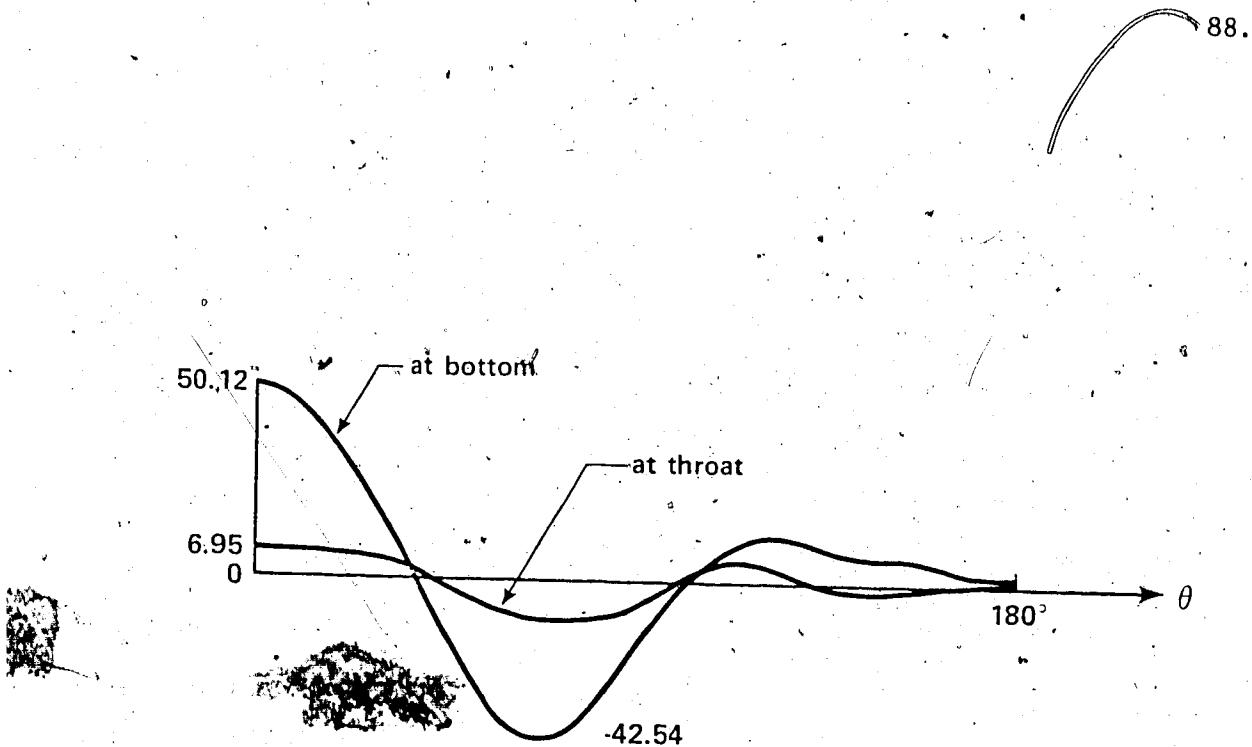


Fig. 4.8.5 Hyperboloid Tower Wind Load Circumferential Variation of N_s

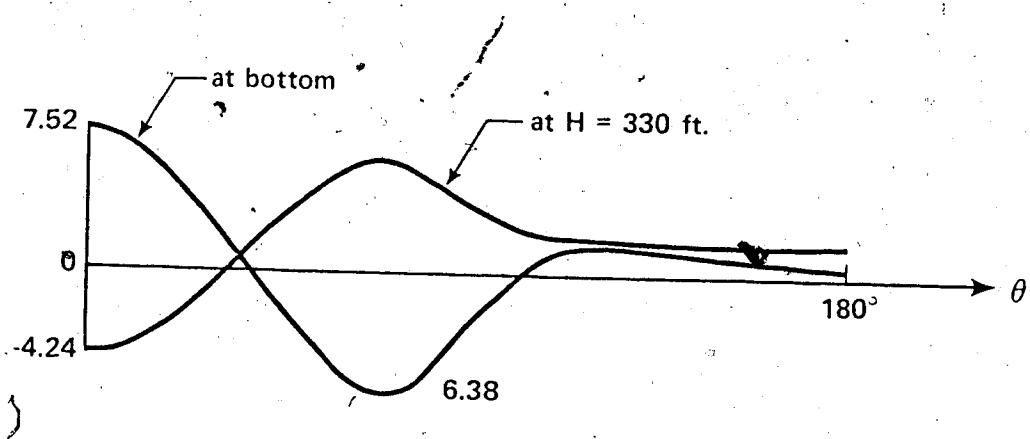
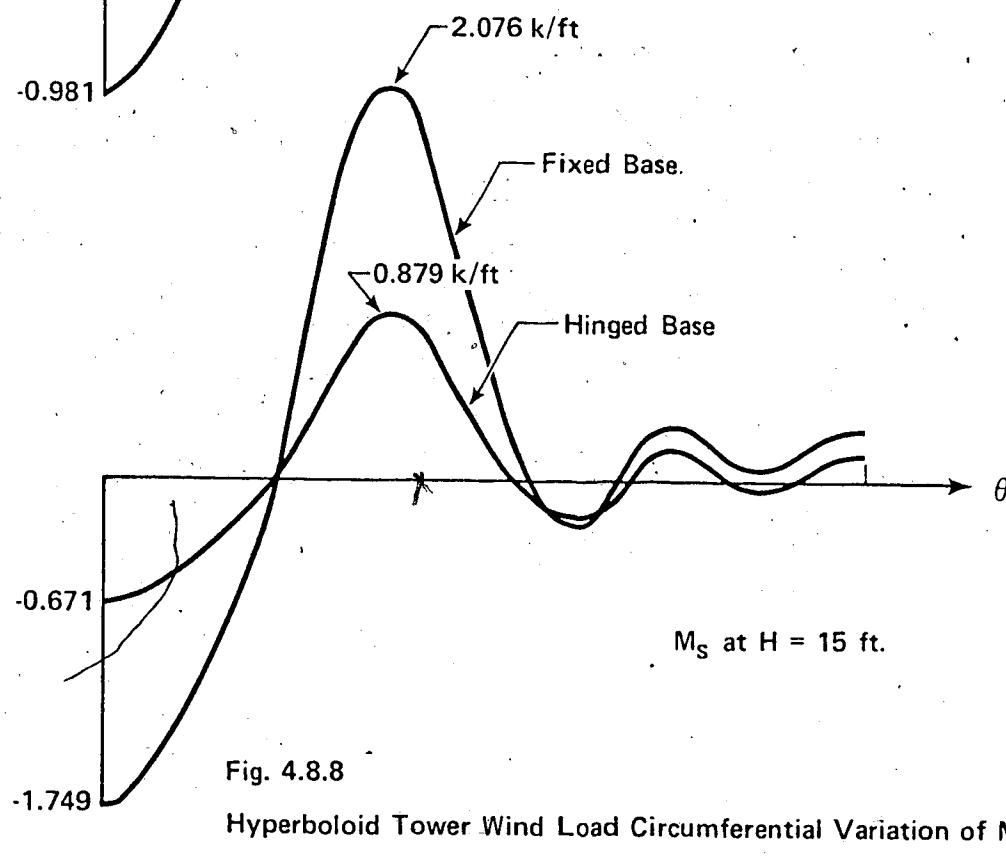
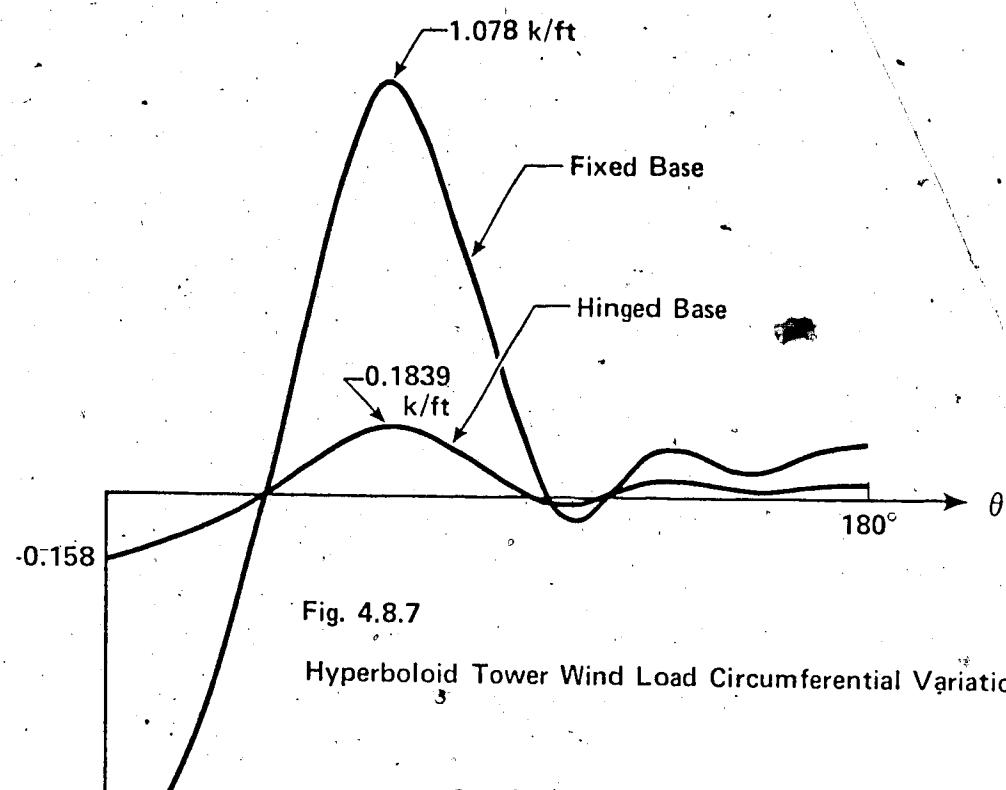


Fig. 4.8.6 Hyperboloid Tower Wind Load Circumferential Variation of N_θ



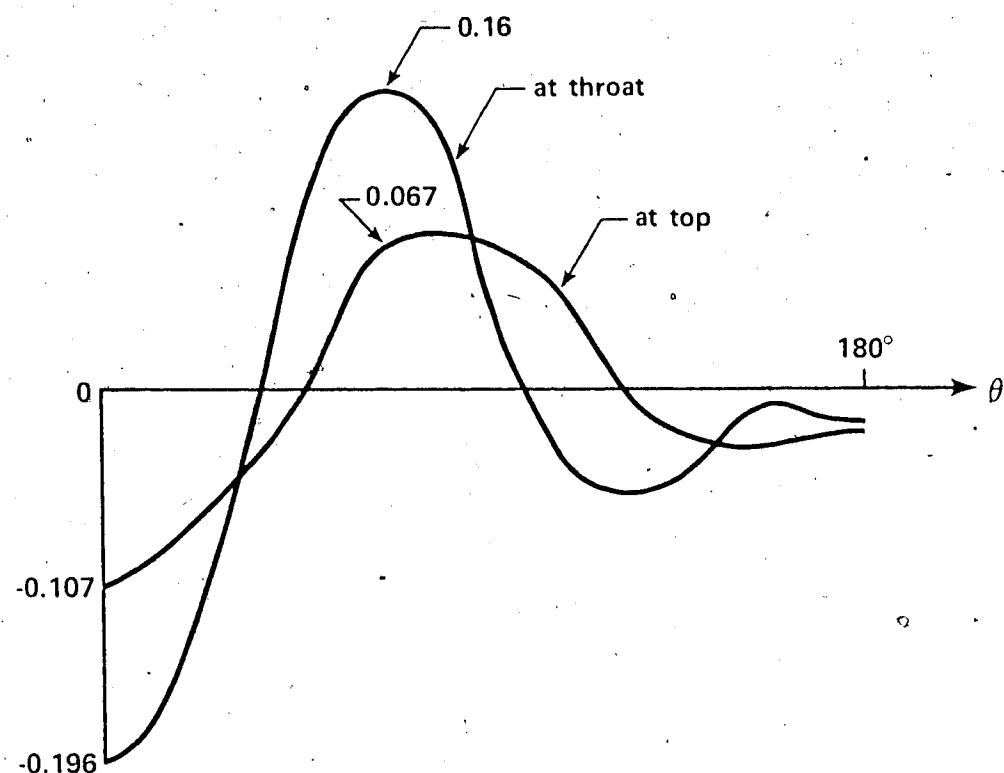


Fig. 4.8.9 Hyperboloid Tower Wind Load Circumferential Variation of W

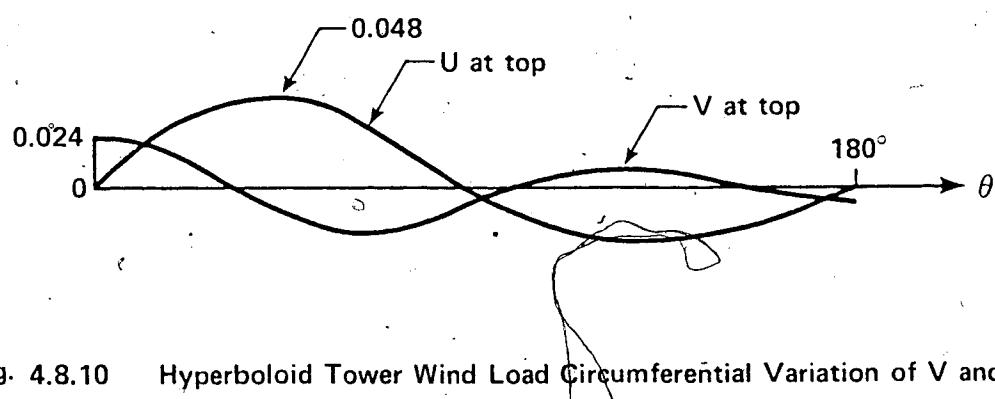


Fig. 4.8.10 Hyperboloid Tower Wind Load Circumferential Variation of V and U

CHAPTER 5

LIMITATIONS

5.1 Introduction

Three factors affect the solution technique presented in this thesis. These are summarized as follows.

- 1) Singularity of the governing equations at the apex.
- 2) Stability of the numerical integration process.
- 3) Convergence of Fourier expansions.

Each factor is discussed separately in the following sections.

5.2 Singularity of the Governing Equations at the Apex

If the shell has a pole (i.e., $r = 0$), coefficients in the governing sets of equations (Eqs. 2.14, 2.18 and 2.20) become singular. This is consistent with classical shell theory. A simple way to handle this situation is to choose the boundary, $s = 0$, not at the pole, but a very short distance away and then impose the boundary conditions at $s = 0$ as follows [6].

- 1) For harmonic number $n = 0$

$$\beta = V = U = S_s = 0$$

- 2) For harmonic number $n = 1$

$$W = M_s = N_s = T_s = 0$$

- 3) For harmonic number $n > 1$

$$W = V = U = M_s = 0$$

5.3 Stability of the Numerical Integration Process

Runge-Kutta fourth order integration method is used, in SASHELL, to integrate the basic set of equations (Eq. 2.24). This method is very well known and has the advantage of being self starting i.e., it needs only the information available at the preceding point. The method can be interpreted as follows:

- 1) The derivative is evaluated at the starting point of the interval.
- 2) The above derivative is used to obtain an approximate ordinate to determine an approximate derivative for the midpoint of the interval.
- 3) The above derivative is used to obtain a second approximation of the derivative at the midpoint of the interval.
- 4) The above derivative is used to obtain an approximate ordinate to determine an approximate derivative for the end point of the interval.
- 5) A weighted average of the above four derivatives is taken to determine a total increment in the function for the whole interval.

Analytically, this can be defined as follows:

$$Y_{i+1} = Y_i + \frac{\Delta h}{6} \{ f(Y_i, h_i) + 2f(Y^*_{i+1/2}, h_{i+1/2}) \}$$

$$+ 2f(Y^{**}_{i+1/2}, h_{i+1/2}) + f(Y^*_{i+1}, h_{i+1}) \} \quad 5.1$$

where

$$Y^*_{i+1/2} = Y_i + \frac{\Delta h}{2} f(Y_i, h_i) \quad 5.2.1$$

$$Y^{**}_{i+1/2} = Y_i + \frac{\Delta h}{2} f\left(Y^*_{i+1/2}, h_{i+1/2}\right) \quad 5.2.2$$

$$Y^*_{i+1} = Y_i + \Delta h f\left(Y^*_{i+1/2}, h_{i+1/2}\right) \quad 5.2.3$$

$$h_{i+1/2} = h_i + \frac{\Delta h}{2} \quad 5.2.4$$

$$h_{i+1} = h_i \Delta h \quad 5.2.5$$

in which $f(Y_i, h_i)$ is the value of the function at point i , and Δh is the step size of the interval.

It has been found [25] that the direct integration methods, when applied to a shell problem, suffer a complete loss of accuracy when the generator of the shell exceeds a critical length. The reason for this phenomenon is explained clearly in Reference 18. The general solution of the governing equations, Eq. 3.18, is of the form

$$\{y_{(b)}\} = [H_{(b)}]\{y_{(a)}\} + \{Q_{(b)}\} \quad 5.3$$

Because of the exponentially decaying behavior of the stresses and displacements, it is observed that the coefficients of $[H]$ increase in magnitude in such a way that if the length of the shell element is increased by any factor n , then these coefficients

increase in magnitude, approximately, exponentially with n .

For example, consider an element spans the region $a \leq s \leq b$.

For some prescribed edge conditions at "a", we expect the corresponding solution at "b" to become smaller and smaller when the element ab is increased in length. (i.e., $y_{(b)}$ small, $H_{(b)}$ large and $y_{(a)}$ has a prescribed value). The longer the element, the larger $[H_{(b)}]$ and the smaller $\{y_{(b)}\}$. The only way to get a small value for $\{y_{(b)}\}$ is for the elements of $[H_{(b)}]$ to subtract out and that is the reason that at some critical length of the element all significant digits of $[H_{(b)}]$, in Eq. 5.3, are lost and so is the accuracy.

The loss of accuracy cannot be avoided by choosing a fine mesh for the integration. By taking more steps "partial instability" arises [8]. This means that the numerical solution deviates from the actual solution as we take more steps.

In Reference 18, the critical meridian length is limited with a length factor $\lambda L \leq 3 - 5$, where

$$\lambda L^* = L^4 \sqrt{\frac{3(1 - \gamma^2)}{r^2 h^2}} \quad 5.4$$

in which

L is the length of the meridian of the shell, R is the minimum radius of curvature, and h is the thickness of the shell.

In the author's opinion, this is very conservative limitation. The loss of accuracy, using the computer program SASHELL, does not arise until the length factor λL exceeds 25.

To demonstrate this, Table 5.1 shows some results of the analysis of clamped pressurized cylindrical shells, as obtained by the computer program SASHELL, with various lengths and different number of steps of integration. From the symmetry of the problem, the end moments and the absolute values of the end shears should be equal. It is obvious, from Table 5.1, that the solution is not affected much by the number of steps adopted for the integration and it begins to break when

- 1) λL exceeds 20 and the number of steps is less than 20.
- 2) λL approaches 29 for any number of steps.

As a conservative limitation, the upper bound of λL shall not exceed 25 and the number of points of integration (NP), needed for convergence, is limited as follows:

$$NP \leq 21 \quad \text{for} \quad \lambda L < 20$$

$$NP \leq 31 \quad \text{for} \quad 20 < \lambda L < 25$$

This limitation does not affect the efficiency of this technique. It can be observed from Table 5.1, that the computation time (CPU) required for the analysis is proportional to the number of steps adopted.

5.4 Convergence of Fourier Expansions

The load, when varying with the circumferential coordinates is expanded in a Fourier series. The analysis is

carried out to determine the stresses and displacements everywhere within the structure. When the load is represented "exactly", the solution converges.

Theoretically, the number of harmonics required for an arbitrary periodic function to be represented exactly, by means of Fourier series, is infinity, i.e.,

$$f(\theta) = \sum_{n=0}^{\infty} A_n \cos n\theta + \sum_{n=1}^{\infty} b_n \sin n\theta \quad 5.5$$

However, the load can always be described at a sufficient number of points to satisfy our engineering judgement of representing the actual loading conditions. Therefore, our concern is to examine the convergence of a function known only at a set of discrete points. If these points are equally spaced, say $2N$ points, taken over the interval $0 \leq \theta \leq 2\pi$, the spacing is,

$$\theta_i = \frac{2\pi i}{2N} \quad \text{for } i = 0, 1, 2, \dots, 2N-1 \quad 5.6$$

If now an approximation is assumed in the form

$$f(\theta) \approx \sum_{n=0}^M A_n \cos n\theta + \sum_{n=1}^M B_n \sin n\theta \quad 5.7$$

where the coefficients are to be determined in such a way that the integrated squared error over the interval of length 2π is least, then the requirement is

$$\int_0^{2\pi} \left[f(\theta) - \sum_{n=0}^M A_n \cos n\theta - \sum_{n=1}^M B_n \sin n\theta \right]^2 d\theta = \min \quad 5.8$$

Since the function $f(\theta)$ is described only at the points defined by θ_i (Eq. 5.6), we have $2N$ independent data points which are sufficient to determine the coefficients of $2N$ terms of an approximation in the form of Eq. 5.7. When $M \leq N$, Eq. 5.8 can be rewritten as follows

$$\sum_{n=0}^{2N-1} \left[f(\theta_i) - \sum_{n=0}^M A_n \cos n\theta_i - \sum_{n=1}^M B_n \sin n\theta_i \right]^2 = \min \quad 5.9$$

where $f(\theta_i)$ is the value of the function $f(\theta)$ at the point i .

The solution is obtained when the partial derivatives of the left hand side of Eq. 5.9 with respect to A_n and B_n are equated to zero [15, pg. 446-457]. The coefficients are in the form

$$A_0 = \frac{1}{2N} \sum_{n=0}^{2N-1} f(\theta_i) \quad 5.10.1$$

$$A_n = \frac{1}{N} \sum_{n=0}^{2N-1} f(\theta_i) \cos n\theta_i \quad (n \neq 0, N) \quad 5.10.2$$

$$B_n = \frac{1}{N} \sum_{n=0}^{2N-1} f(\theta_i) \sin n\theta_i \quad 5.10.3$$

The calculation of each coefficient is independent of the calculation of the others and is independent of M as long as $M \leq N$. When $M = N$, the least-squares criterion (Eq. 5.9) becomes equivalent to the requirement that the "best" approximation of the function (Eq. 5.7) is obtained [15].

The loading cases of the examples discussed in Chapter 4 are shown in Fig. 5.1 and 5.2. The effect of the number of harmonics on the loading associated with these problems is considered in the following. Fourier coefficients of the wind pressure load on the hyperboloid tower are included in Table 5.2.1. Since the load is symmetric with respect to $\theta = 0$, the sine coefficients vanish. The results of superimposing the 12 coefficients, using Eq. 5.7, are shown in Table 5.2.2. It can be seen that these values approximate the load function to a very close agreement. Now examining the coefficients shown in Table 5.2.1. The 9th and the subsequent coefficients are small in comparison with the other coefficients. The magnitude of the 9th coefficient is in the same order of the 10th with opposite sign and so for the 11th and 12th. Therefore we expect that the approximation will be reasonably accurate if the series is terminated at the 8th harmonic. The results of an 8 harmonic approximation are shown in Table 5.2.2.

Fourier coefficients of the two concentrated loads of the pinched cylinder problem discussed in Sect. 4.3 are shown in Table 5.3.1. It can be seen that the odd cosine coefficients as well as the sine coefficients vanish for this case of symmetry

with respect to $\theta = 0$ and $\theta = \pi/2$. The approximation with 18 harmonics, for the loading function described at 36 points, represent the load more accurately when compared with the 12 harmonic approximation (Fig. 5.2 and Table 5.3.2). Therefore one can approximate this load by superimposing the results of $n = 0, 2, \dots, 16$.

As a general conclusion, one can consider N harmonics in the expansion of a load described at $2N$ points. A termination of the higher harmonics or exemption of a harmonic number in the series can be decided upon by examining the coefficient of each loading function as a special case.

$r = 10$ ft $E = 5000$ ksi $v = .15$ $h = 0.5$ ft $\lambda = 0.5852$

	L (ft)	Number of Elements	Number of Points	CPU Time (sec)	λL	$M_s(0)$ (K.ft/ft)	$M_s(L)$ (K.ft/ft)	$Q_s(0)$ (K/ft)	$Q_s(L)$ (K/ft)
1	10	1	11	0.34	15.85	1.442	1.438	-1.676	1.675
2	10	1	15	0.44	5.85	1.442	1.442	-1.676	1.676
3	20	1	21	0.59	11.71	1.430	1.430	-1.675	1.675
4	40	1	21	0.59	23.41	1.451	45.05	-1.697	28.65
5	40	1	31	0.83	23.41	1.427	1.427	-1.672	1.671
6	50	1	21	0.59	29.26	-0.139	-2446.0	0.167	-2173.0
7	50	1	31	0.82	29.26	1.426	1.424	-1.671	1.670
8	50	1	41	1.06	29.26	1.426	1.427	-1.671	1.671
9	50	1	51	1.32	29.26	1.426	1.428	-1.671	1.671
10	50	2	21	1.13	14.63	1.426	1.426	-1.671	1.671
11	60	1	31	0.84	35.11	1.388	121.8	-1.631	-62.74
12	60	1	41	1.08	35.11	1.426	0.876	-1.671	1.795
13	60	1	51	1.31	35.11	1.426	2.367	-1.670	2.271
14	60	1	81	2.10	35.11	1.426	0.511	-1.671	1.235
15	60	2	21	1.12	17.55	1.426	1.426	-1.670	1.670
16	60	2	31	1.60	17.55	1.426	1.426	-1.670	1.670
17	100	1	101	2.50	58.52	.075	-2.84x10 ⁸	-0.068	5.33x10 ⁹
18	100	2	31	1.61	29.41	1.425	1.422	-1.670	1.668
19	100	3	21	1.68	19.50	1.425	1.425	-1.670	1.670

Table 5.1 Effect of Length and Number of Integration Points on the Accuracy of the Solution

Harmonic Number	Cosine Coefficient
0	0.3833
1	-0.2792
2	-0.6198
3	-0.5093
4	-0.0917
5	0.1179
6	0.0333
7	-0.0447
8	-0.0083
9	0.0093
10	-0.0136
11	0.0060

TABLE 5.2.1
Fourier Coefficients
of Wind Pressure Load

θ	Load Value for 12 Harmonics	Load Value for 8 Harmonics
0	-1.016	-1.010
7.5	-0.955	-.952
15	-0.783	-0.787
22.5	-0.528	-0.535
30	-0.216	-0.219
37.5	0.136	0.141
45	0.516	0.522
52.5	0.888	0.887
60	1.183	1.178
67.5	1.335	1.335
75	1.316	1.319
82.5	1.146	1.144
90	0.883	0.878
97.5	0.610	0.613
105	0.416	0.429
112.5	0.352	0.355
120	0.383	0.366
127.5	0.423	0.406
135	0.416	0.427
142.5	0.387	0.416
150	0.383	0.391
157.5	0.407	0.378
165	0.416	0.388
172.5	0.398	0.409
180	0.383	0.420

TABLE 5.2.2 Effect of Number of
Harmonics on Representing
the Wind Pressure Load

Harmonic Number	Cosine Coefficient
0	0.0555
1	0
2	0.1111
3	0
4	0.1111
5	0
6	0.1111
7	0
8	0.1111
9	0
10	0.1111
11	0
12	0.1111
13	0
14	0.1111
15	0
16	0.1111
17	0

TABLE 5.3.1

Fourier Coefficients
of Two Diametrically
Opposed Concentrated
Loads

θ	Load Value for 18 Harmonics	Load Value for 12 Harmonics
0	0.994	0.611
5	0.635	0.522
10	0.055	0.301
15	-0.207	0.055
20	-0.055	-0.104
25	0.119	-0.131
30	0.055	-0.055
35	-0.079	0.041
40	-0.055	0.085
45	0.055	0.055
50	0.055	-0.012
55	-0.039	-0.061
60	-0.055	-0.055
65	0.026	-0.005
70	0.055	0.045
75	0.015	0.055
80	-0.055	0.019
85	0.005	-0.032
90	0.055	-0.055

TABLE 5.3.2 Effect of Number of
Harmonics on Representing
the Two Concentrated Loads

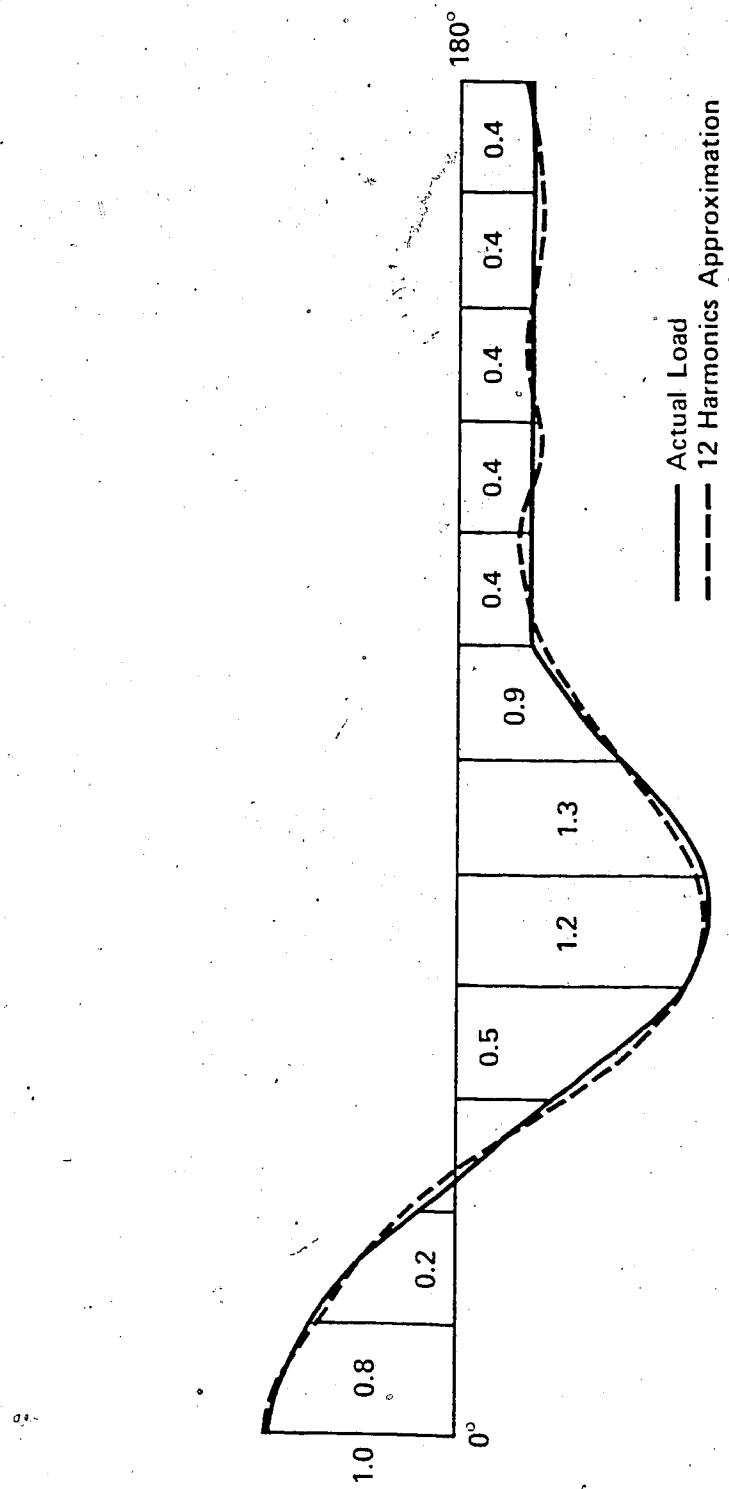


Fig. 5.1 Fourier Approximation for Wind Load

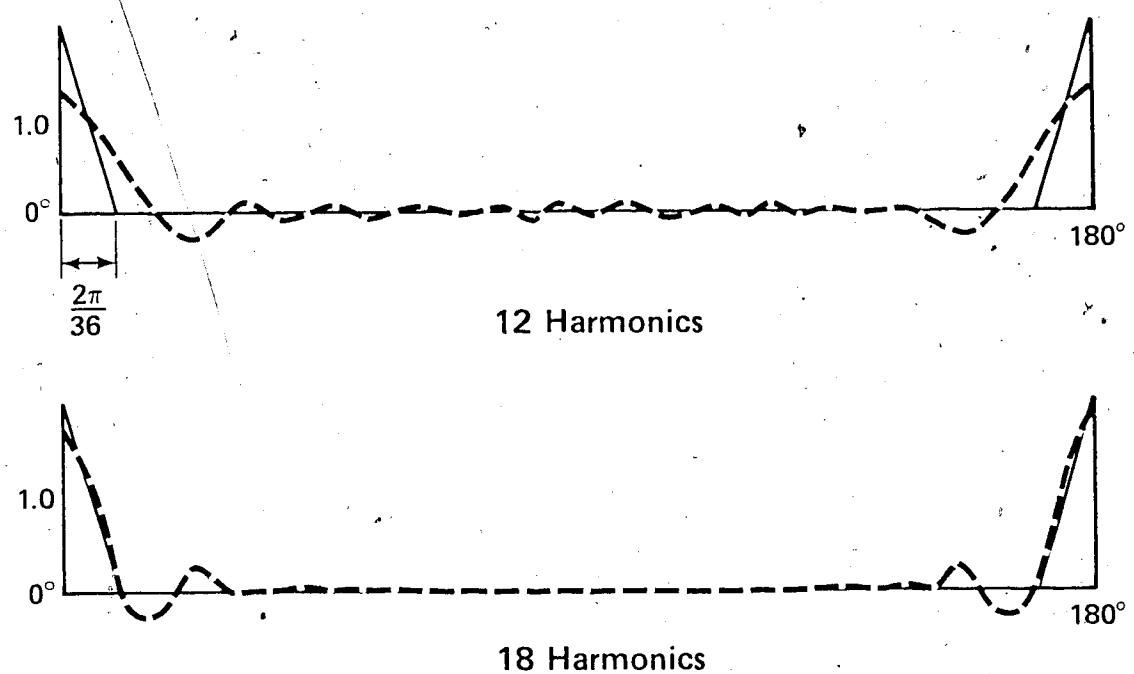


Fig. 5.2 Fourier Approximation for Two Diametrically Opposed Concentrated Loads

CHAPTER 6
SUMMARY AND CONCLUSIONS

In this study, a theory has been generalized. A computer program has been developed for the elastic analysis of axisymmetric segmented shell structure of general geometric configuration. General arbitrary loadings have been considered. Applications for a number of loading cases and elements have been presented.

The reduction of the governing partial differential equations of the classical shell theory to a set of eight first order ordinary differential equations involves only straight forward algebraic manipulations. The classical theory chosen as a foundation for the theory presented herein, Flügge's theory, is considered one of the most accurate theories available in the literature. The approximation in formulating the basic governing equations of this theory are such that the theory may be considered to be exact. The adaptation of the general form of these basic equations allows the geometry of any type of shell element to be considered, without any theoretical approximations which may be suitable for specific dimensions and not for others. Variation of the shell thickness along the meridian can be accounted for with accuracy comparable to that for constant thickness.

The loads, when approximated by means of Fourier series, may be represented with sufficient accuracy by including a number of harmonics, within practical limits. This approximation allows the consideration of general arbitrary types of loadings in a simple manner.

The excellent agreement between the results of the problems presented in this thesis and the results of other known solutions, analytical or numerical, demonstrate the accuracy of the solution technique and the reliability of the numerical integration process used.

It may be concluded that the solution technique presented in this study is simple and general for the elastic analysis of shells of revolution. The accuracy is consistent with a good shell theory. The geometric limitations are of no importance from the practical point of view.

Further development, by applying this method to the study of free vibration and elastic buckling criteria of shells of revolution is possible.

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APPENDIX A

SHELL THEORY

This appendix presents the analytical development of the basic equations, used in this study, as given by Flügge [10].

A.1 Geometry of Shells

The geometry of a shell is defined by specifying the form of the middle surface and the thickness of the shell at each point. The surface of a shell of revolution is generated by the rotation of a plane curve about an axis in its plane. This generating curve is called the "meridian". The intersection of the surface with planes perpendicular to the axis of revolution are "parallel circles". Two coordinates s, θ are required to describe any point on the middle surface of the shell:

- a) s measures the distance to the point along the meridian from the intersection of the middle surface with the axis of rotation, or from a datum parallel circle.
- b) θ is the angular "distance" of the point from a datum generator.

A third coordinate z is required to measure the distance along a normal to the middle surface.

The radii of curvature of a shell of revolution are:

- a) r is the radius of curvature of parallel circles
- b) r_1 is the radius of curvature of meridian

- c) r_2 is the length of the normal between any point on the middle surface and the axis of revolution.

The following fundamental geometrical relations can clearly be seen in Fig. A.1

$$r = r_2 \sin\phi \quad \text{A.1.1}$$

$$ds = r_1 d\phi \quad \text{A.1.2}$$

$$dr = ds \cos\phi \quad \text{A.1.3}$$

$$dx = ds \sin\phi \quad \text{A.1.4}$$

where ϕ represents the angular distance of the point under consideration from the axis of rotation.

From Eqs. A.1 one can write

$$\frac{dr}{ds} = \cos\phi \quad \text{A.2.1}$$

$$\frac{dr_2}{ds} = \frac{r_1 - r_2}{r_1} \cot\phi \quad \text{A.2.2}$$

For simplicity the derivatives with respect to s and θ will be indicated by dots and primes respectively, i.e.,

$$\frac{\partial}{\partial s} (\cdot) \equiv (\cdot) \quad \text{A.3.1}$$

$$\frac{\partial}{\partial \theta} (\cdot) \equiv (\cdot)' \quad \text{A.3.2}$$

A.2 Equations of Equilibrium

If a shell element is cut out by two meridians and two parallel circles, each pair infinitely close as seen in Fig. A.2, the element is stressed by ten stress resultant components which must be in equilibrium with the external applied load. These stress resultants are:

N_s, N_θ = normal in-plane forces per meridional and circumferential unit length, respectively.

$N_{s\theta}, N_{\theta s}$ = in-plane shear forces per meridional and circumferential unit length, respectively.

Q_s, Q_θ = transverse shear forces per meridional and circumferential unit lengths, respectively.

M_s, M_θ = meridional and circumferential moments per unit length, respectively.

$M_{s\theta}, M_{\theta s}$ = circumferential and meridional twisting moments per unit length, respectively.

The sign convention for the forces is shown in Fig. A.2

Referring to the three orthogonal axis's, θ , z , one can obtain six equations of equilibrium

$$\sum N_i = 0 \quad A.4.1$$

$$\sum M_i = 0 \quad A.4.2$$

where $\sum N_i$ is the sum of the forces in the i direction
 $(i = s, \theta, z)$.

$\sum M_i$ is the sum of the moments about the i axis

$$(i = s, \theta, z)$$

The six equilibrium equations in terms of the ten stress resultants are

$$r_1(rN_s) + r_1N'_{\theta s} - r_1N_{\theta} \cos\phi - rQ_s + rr_1P_s = 0 \quad A.5.1$$

$$r_1(rN_{s\theta}) + r_1N'_{\theta} + r_1N_{\theta s} \cos\phi - r_1Q_{\theta} \sin\phi + rr_1P_{\theta} = 0 \quad A.5.2$$

$$r_1N_{\theta} \sin\phi + rN_s + r_1Q'_{\theta} + r_1(rQ_s) - rr_1P_z = 0 \quad A.5.3$$

$$r_1(rM_s) + r_1M'_{\theta s} - r_1M_{\theta} \cos\phi - rr_1Q_s = 0 \quad A.5.4$$

$$r_1(rM_{s\theta}) + r_1M'_{\theta} + r_1M_{\theta s} \cos\phi - rr_1Q_{\theta} = 0 \quad A.5.5$$

$$rr_1N_{\theta s} - rr_1N_{s\theta} - r_1M_{\theta s} \sin\phi + rM_{s\theta} = 0 \quad A.5.6$$

where P_s, P_{θ}, P_z are the resolved components of the external applied load in the s, θ, z , respectively. If the subscript s , in the above equations, is replaced by ϕ , and the relations

$$\frac{1}{r_1} \frac{\partial}{\partial\phi} () = () \quad A.6$$

is observed. The above set of equations (Eqs. A.5) can be reduced to the equilibrium equations in Reference 10, pg. 318. Six

equations of equilibrium are not enough to determine the ten stress resultants. The shell element is four times internally statically indeterminate and therefore, an analysis of the deformation of the shell is required in order to obtain the solution.

A.3 Strain-Displacement Relations

The displacement vector of a point lying on the middle surface of an element may be described by its three orthogonal components, defined as:

W = the displacement component in the radial direction, positive when it points away from the centre of curvature.

V = the displacement component in the meridional direction, positive in the direction of increasing the coordinate s .

U = the displacement component in the direction of the tangent to the parallel circle, positive in the direction of increasing the coordinate θ .

In addition, the auxiliary variable β , which represents the angle by which an element of the meridian rotates during deformation may be expressed in terms of the displacement components, Fig. A.3, as [10].

$$\beta = -W + \frac{V}{r_1}$$

A.7.1

By differentiation, the change in slope of the meridian due to deformation, is

$$\beta^* = -w^{**} + \frac{r_1 V^* - r_1^* V}{r_1^2}$$

A.7.2

Referring to Fig. A.4, one can obtain the relationship between the strains and the displacement components of a point on the middle surface.

The meridional strain is

$$\begin{aligned}\varepsilon_s &= \frac{\text{Elongation of the line element } ds}{ds} \\ &= \frac{W}{r_1} + V^*\end{aligned}$$

A.8.1

The hoop strain is

$$\begin{aligned}\varepsilon_\theta &= \frac{\text{Elongation of the line element } rd\theta}{rd\theta} \\ &= \frac{U' + V \cos\phi + W \sin\phi}{r}\end{aligned}$$

A.8.2

The shear strain, which is the change of the right angle between the two line elements ds and $rd\theta$ (Fig. A.4.3), is equivalent to

$$\begin{aligned}\gamma_{s\theta} &= \gamma_1 + \gamma_2 = \\ &= \frac{V'}{r} + U^* + \frac{U}{r} \cos\phi\end{aligned}$$

A.8.3

Since the displacement is assumed to be very small in comparison with the principal radii of curvature, all products of two displacement components have been dropped in deriving the foregoing equations.

One may use the preceding equations to express the relationship between the strains and the displacement components of an arbitrary point at a distance z from the middle surface by simply replacing W , V , U with the displacement components of this point W_z , V_z , U_z , and replacing the radii r_1 , r_2 with $r_1 + z$ and $r_2 + z$, respectively.

Then

$$\epsilon_s = \frac{W}{r_1 + z} + V_z \quad A.9.1$$

$$\epsilon_\theta = U_z + \frac{V_2 \cos\phi + W_z \sin\phi}{(r_2 + z) \sin\phi} \quad A.9.2$$

$$\gamma_{s\theta} = U_z - \frac{U_z \cos\phi - V_z}{(r_2 + z) \sin\phi} \quad A.9.3$$

By introducing the assumption that lines normal to the middle surface before deformation remain normal after deformation, it can clearly be seen from Fig. A.5 that

$$W_z = W \quad A.10.1$$

$$V_z = V \frac{r_1 + z}{r_1} + W_z \quad A.10.2$$

$$U_z = U \frac{r_2 + z}{r_2} + W_z \quad A.10.3$$

Therefore, the strains at a distance z from the middle surface in terms of the displacements W , V , U at the middle surface can be obtained by substituting Eqs. A.10 into Eqs. A.9 to obtain

$$\begin{aligned}\varepsilon_s &= \frac{1}{r_1 + z} W + \frac{r_1 z}{r_1 + z} W' - \frac{r_1 z}{r_1 + z} W'' \\ &\quad + V' - \frac{r_1 z}{r_1(r_1 + z)} V\end{aligned}\quad \text{A.11.1}$$

$$\begin{aligned}\varepsilon_\theta &= \frac{1}{r_2 + z} W - \frac{z \cot\phi}{r_2 + z} W' - \frac{z}{r \sin\phi (r_2 + z)} W'' \\ &\quad + \frac{\cot\phi (r_1 + z)}{r_1(r_2 + z)} V + \frac{1}{r} U\end{aligned}\quad \text{A.11.2}$$

$$\begin{aligned}\gamma_{s\theta} &= \frac{r_1(r_2 + z)}{r_2(r_1 + z)} U' - \frac{r_1(r_2 + z)}{r_2^2(r_1 + z)} \cot\phi U \\ &\quad + \frac{(r_2 + z)}{r_1 \sin\phi(r_2 + z)} V' - \frac{z}{\sin\phi} \left(\frac{1}{r_2 + z} + \frac{r_1}{r_2(r_1 + z)} \right) W' \\ &\quad + \frac{\cot\phi}{r_2 \sin\phi} \left(\frac{z}{r_2 + z} + \frac{r_1 z}{r_2(r_1 + z)} \right) W\end{aligned}\quad \text{A.11.3}$$

A.4 Stress-Strain Relations

Hooke's law relates the strains to the corresponding stresses in linearized form, as long as the stresses remain within the elastic limit. If T is the change in temperature measured from arbitrary level. Hooke's law may be written, in index notation, as

$$E\varepsilon_i = \sigma_i - \nu(\sigma_j + \sigma_k) + E\alpha T \quad \text{A.12.1}$$

$$G\gamma_{ij} = \tau_{ij} \quad \text{A.12.2}$$

where

i, j, t take in turns the direction s, θ , z

σ_i is the normal stress in the i direction

τ_{ij} is the shearing stress in the i plane and
j direction

$$G = \frac{E}{2(1 + \nu)} \quad A.13$$

The modulus of elasticity, E, Poisson's ratio, ν , the coefficient of thermal expansion, α , and thus the shear modulus, G, are the material constants.

As in the theory of plates, except in the immediate vicinity of concentrated forces, the stresses in z direction are small in comparison with the stresses in s, θ directions and their influence in Hooke's law may be neglected, i.e., it is assumed that

$$\sigma_z = \gamma_{sz} = \gamma_{\theta z} = 0 \quad A.14$$

Therefore, the stress-strain relations can be written as

$$\sigma_s = \frac{E}{1 - \nu^2} \left[\epsilon_s + \nu \epsilon_\theta - (1 + \nu) \alpha T \right] \quad A.15.1$$

$$\sigma_\theta = \frac{E}{1 - \nu^2} \left[\epsilon_\theta + \nu \epsilon_s - (1 + \nu) \alpha T \right] \quad A.15.2$$

$$\tau_{s\theta} = \frac{E}{2(1+\nu)} \gamma_{s\theta}$$

A.15.3

A.5 Elastic Law

The internal stress resultants can be determined by integrating the stresses through the shell thickness (Fig. A.6). They are defined as

$$N_s = \int_{-t/2}^{t/2} \sigma_s \frac{r_2 + z}{r_2} dz \quad A.16.1$$

$$N_\theta = \int_{-t/2}^{t/2} \sigma_\theta \frac{r_1 + z}{r_1} dz \quad A.16.2$$

$$N_{s\theta} = \int_{-t/2}^{t/2} \tau_{s\theta} \frac{r_1 + z}{r_2} dz \quad A.16.3$$

$$N_{\theta s} = \int_{-t/2}^{t/2} \tau_{\theta s} \frac{r_1 + z}{r_1} dz \quad A.16.4$$

$$M_s = \int_{-t/2}^{t/2} \sigma_s \frac{r_2 + z}{r_2} zdz \quad A.16.5$$

$$M_\theta = - \int_{-t/2}^{t/2} \sigma_\theta \frac{r_1 + z}{r_1} zdz \quad A.16.6$$

$$M_{s\theta} = \int_{-t/2}^{t/2} \tau_{s\theta} \frac{r_2 + z}{r_2} zdz \quad A.16.7$$

$$M_{\theta s} = \int_{-t/2}^{t/2} \tau_{\theta s} \frac{r_1 + z}{r_1} zdz \quad A.16.8$$

It should be noted that the expressions for Q_s , Q_θ have been omitted as they are equal to the integral of the shearing stresses in z direction. Also the minus sign in these expressions correspond to the positive directions assumed for the stress resultants as shown in Fig. A.6.

The expressions for σ_s , σ_θ , $\tau_{s\theta}$ (Eqs. A.15) can be entered into Eqs. A.16. Then Eqs. A.11 are substituted for the displacements and the integration with respect to z is performed. The results in the stress-resultant-displacement relationships.

$$\begin{aligned} N_s &= D \left[V^* + \frac{W}{r_1} + v \frac{U' + V \cos\phi + W \sin\phi}{r} \right] \\ &\quad + \frac{K}{r_1^2} \frac{r_2 - r_1}{r_2} \left[\left(\frac{V}{r_1} - W^* \right) r_1 + r_1 W^* + \frac{W}{r_1} \right] \\ &\quad - (1 + v) \alpha D T_{o2} \end{aligned} \quad A.17.1$$

$$\begin{aligned} N_{\theta s} &= D \left[\frac{U' + V \cos\phi + W \sin\phi}{r} + v \left(V^* + \frac{W}{r_1} \right) \right] \\ &\quad - \frac{K}{rr_1} \frac{r_2 - r_1}{r_2} \left[\frac{V}{r_1} \left(\frac{r_1 - r_2}{r_2} \right) \cos\phi + \frac{W \sin\phi}{r_2} \right. \\ &\quad \left. + \frac{W''}{r} + W^* \cos\phi \right] - (1 + v) \alpha D T_{o1} \end{aligned} \quad A.17.2$$

$$N_{s0} = D \left(\frac{1 - v}{2} \right) \left[U^* + \frac{V'}{r} - \frac{U \cos\phi}{r} \right]$$

$$+ \frac{K}{r_1^2} \left(\frac{1 - v}{2} \right) \frac{r_2 - r_1}{r_2} \left[U^* \left(\frac{r_2 - r_1}{r_2} \right) \right]$$

$$+ U \frac{r_1 - r_2}{r_2} \frac{\cot\phi}{r_2} + W^* \cdot \frac{r_1}{r} - W' \frac{r_1 \cos\phi}{r^2}$$

A.17.3

$$N_{\theta s} = D \left(\frac{1 - v}{2} \right) \left[U^* + \frac{V'}{r} - \frac{U \cos\phi}{r} \right]$$

$$+ \frac{K}{rr_1} \left(\frac{1 - v}{2} \right) \frac{r_2 - r_1}{r_2} \left[V' \frac{r_2 - r_1}{r_1 r_2} - W^* \cdot + \frac{W' \cos\phi}{r} \right]$$

A.17.4

$$M_s = K \left[W^{**} - W^* \frac{r_1}{r_1} - W \frac{r_1 - r_2}{r_2} \frac{1}{r_1^2} - \frac{V^*}{r_2} + V \frac{r_1}{r_1^2} \right]$$

$$+ v \frac{W''}{r^2} + v \frac{W^* \cos\phi}{r} - v \frac{U'}{rr_2} - v \frac{V \cos\phi}{rr_1} \right]$$

$$+ (1 + v) \alpha K T_{12}$$

A.17.5

$$M_{\theta} = K \left[\frac{W''}{r^2} + \frac{W^* \cos\phi}{r} - \frac{W}{r_2^2} \frac{r_2 - r_1}{r_1} - \frac{U'}{rr_1} \right]$$

$$- \frac{V \cos\phi}{rr_1} \frac{2r_2 - r_1}{r_2} + v W^{**} - v W^* \frac{r_1}{r_1}$$

$$- v \frac{V^*}{r_1} + v \frac{V r_1}{r_1^2} \right] + (1 + v) \alpha K T_{11}$$

A.17.6

$$M_{s\theta} = K \frac{1 - \nu}{2} \left[\frac{2W'}{r} - \frac{2W' \cos\phi}{r^2} - \frac{U}{r_2} \frac{2r_1 - r_2}{r_2} \right]$$

$$+ \frac{U}{r_2^2} \frac{2r_1 - r_2}{r_1} \cot\phi - \frac{V'}{rr_1} \right], \quad A.17.7$$

$$M_{\theta s} = K \frac{1 - \nu}{2} \left[\frac{2W'}{r} - \frac{2W' \cos\phi}{r^2} - \frac{U}{r_2} \right]$$

$$+ \frac{U \cot\phi}{r_2^2} - \frac{V'}{rr_1} \frac{2r_2 - r_1}{r_2} \right], \quad A.17.8$$

Where the extensional rigidity, D, is defined as

$$D = \frac{Et}{1 - \nu^2} \quad A.18.1$$

and the flexural rigidity, K, is defined as

$$K = \frac{Et^3}{12(1 - \nu^2)} \quad A.18.2$$

The temperature terms T_{ok} and T_{1k} ($k = 1, 2$) are defined as follows:

$$T_{ok} = \frac{1}{t} \int_{-t/2}^{t/2} T dz + \frac{1}{tr_k} \int_{-t/2}^{t/2} T z dz \quad A.19.1$$

$$T_{1k} = \frac{12}{t^3} \int_{-t/2}^{t/2} T z dz + \frac{12}{t^3 r_k} \int_{-t/2}^{t/2} T z^2 dz \quad A.19.2$$

If a linear variation of the temperature T through the thickness is assumed, Eqs. A.19 can be integrated by parts to yield

$$T_{ok} = \left\{ \frac{T^o + T^i}{2} + (T^o - T^i) \cdot \frac{t}{12r_k} \right\} \quad A.20.1$$

$$T_{1k} = \left\{ \frac{T^o - T^i}{t} + \frac{T^o + T^i}{2r_k} \right\} \quad A.20.2$$

in which T^o and T^i are the temperature measured at the outer and inner face of the shell respectively. If the term t/r_k is neglected when compared with unity, the subscript k disappears from Eqs. A.20 and T_o , T_1 can be defined as the average temperature measured on the middle surface and the temperature gradient respectively.

By substituting Eq. A.6 into Eqs. A.17 and changing the subscript s to ϕ , Eqs. A.17 reduce to the elastic law in Reference 10, pg. 322.

Eqs. A.5 and A.17 are the governing equations for a shell of revolution. The sixth equation of equilibrium (Eq. A.5.6) is identically satisfied if $r_1 = r_2$. Therefore, the five remaining equations of equilibrium (Eqs. A.5) and the eight equations of the elastic law (Eqs. A.17) are 13 equations in 13 unknowns (the three displacement components and ten stress resultants). Theoretically, one can solve for the stresses and displacements at any point in the shell using these equations.

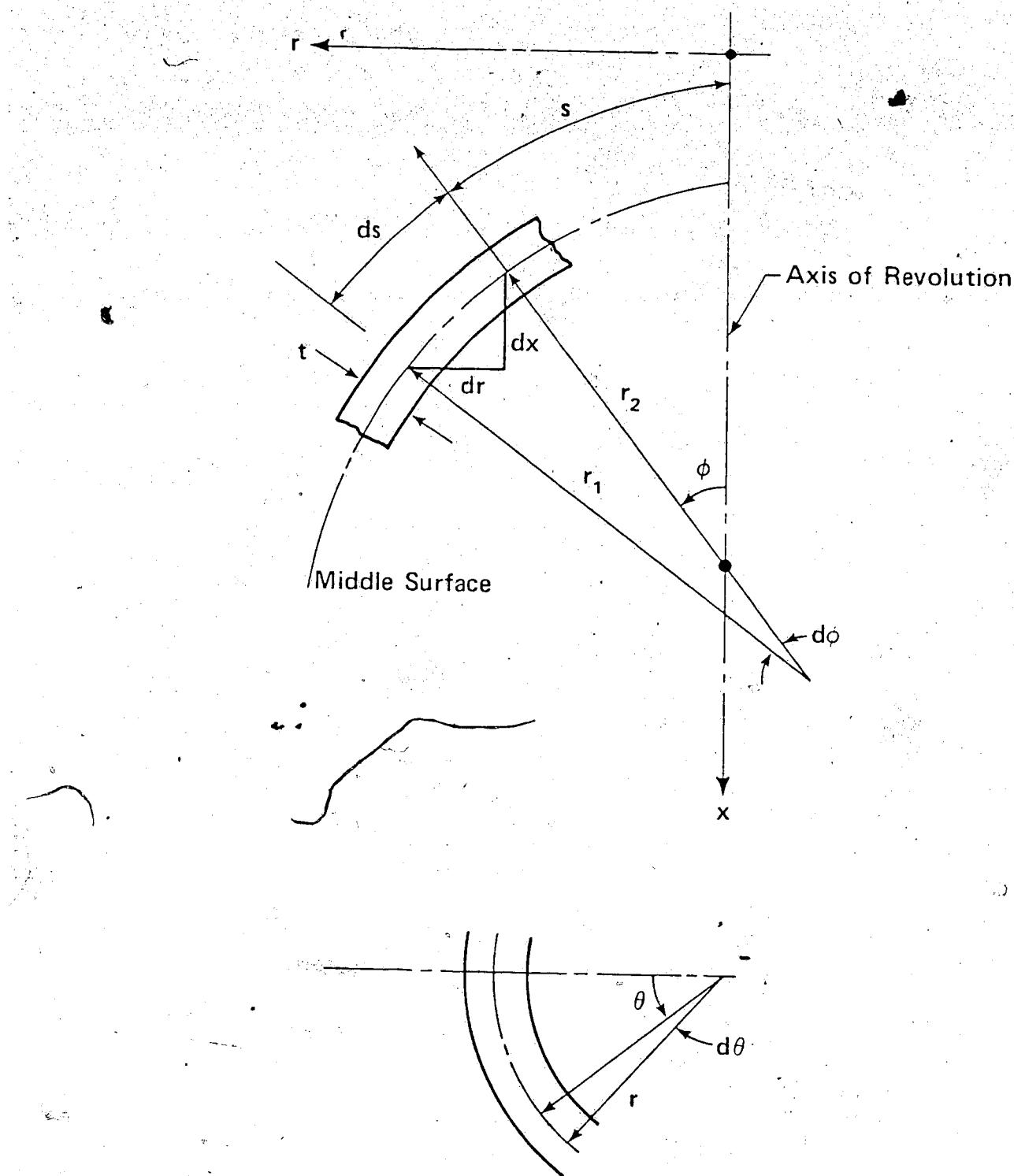


Fig. A.1 Meridian and Parallel Circle of a Shell of Revolution

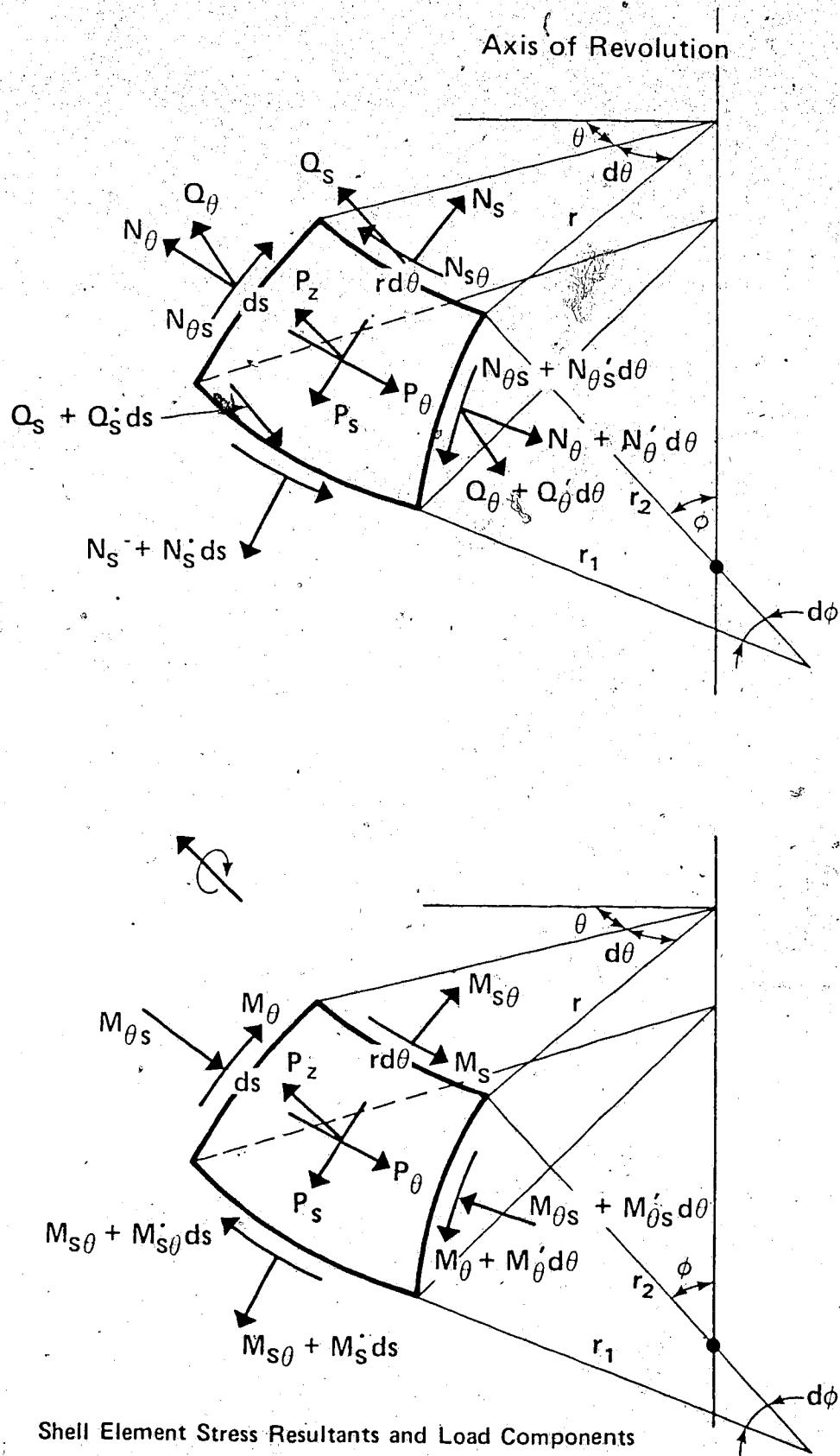


Fig. A.2 Shell Element Stress Resultants and Load Components

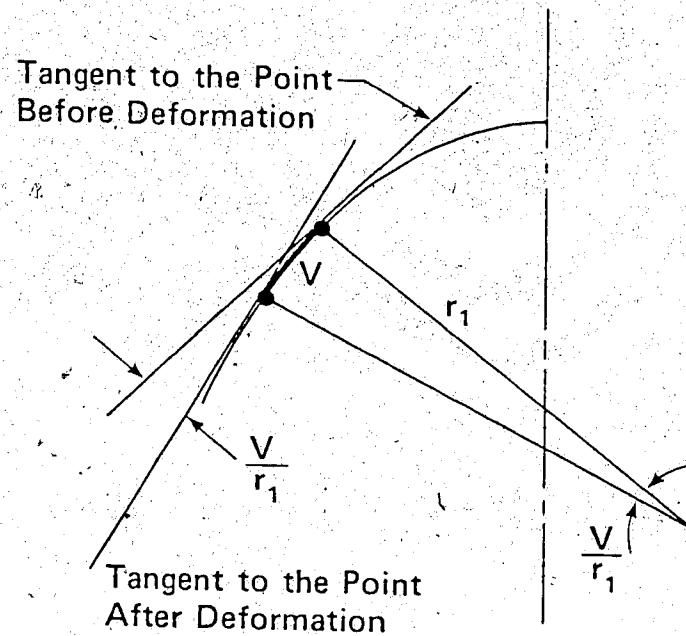


Fig. A.3.1 Meridional Rotation Due to Displacement V

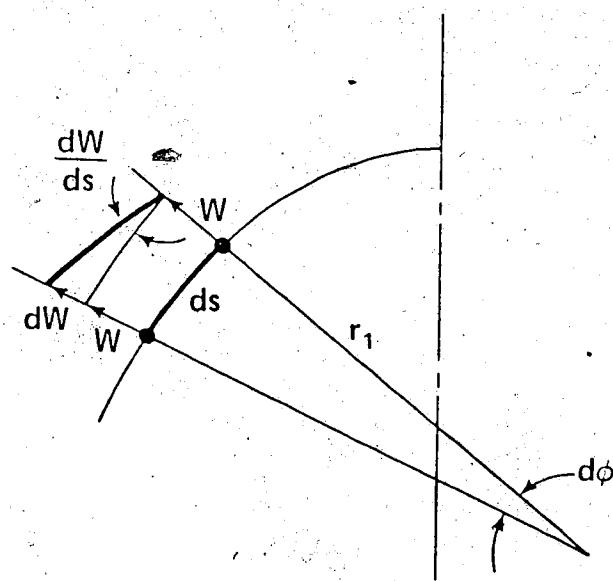


Fig. A.3.2 Meridional Rotation Due to Displacement W

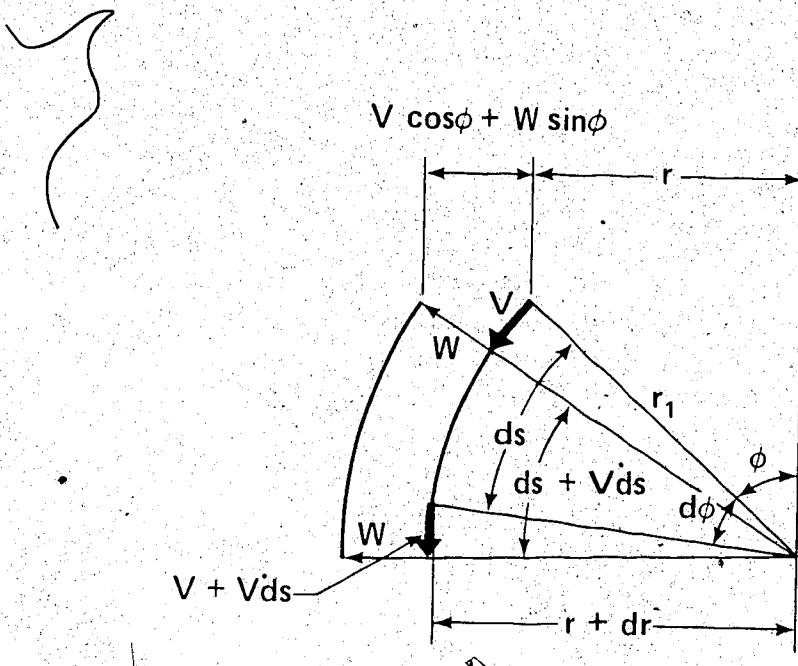


Fig. A.4.1 Meridian of a Shell Before and After Deformation

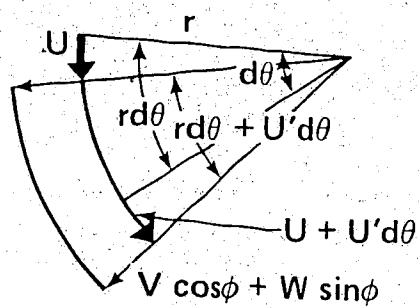


Fig. A.4.2 Parallel Circle Before and After Deformation

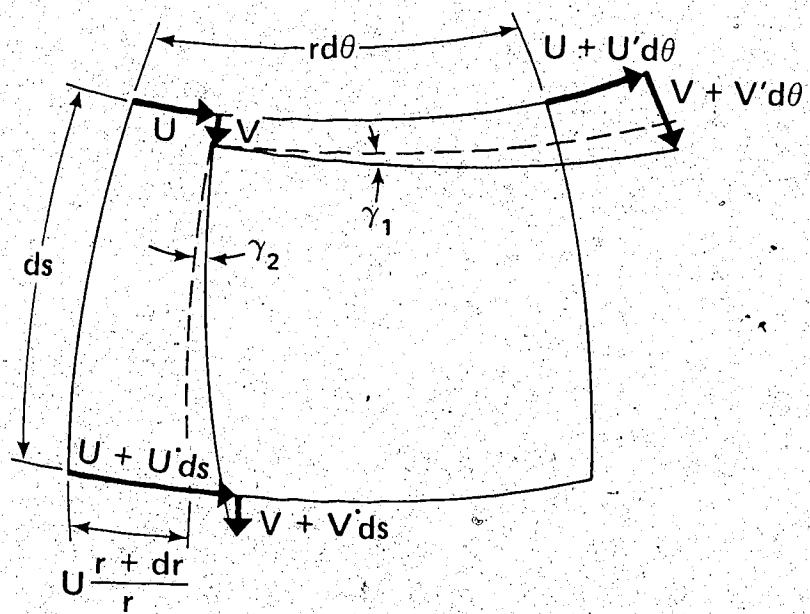


Fig. A.4.3 Change of the Right Angle Between Line Elements After Deformation

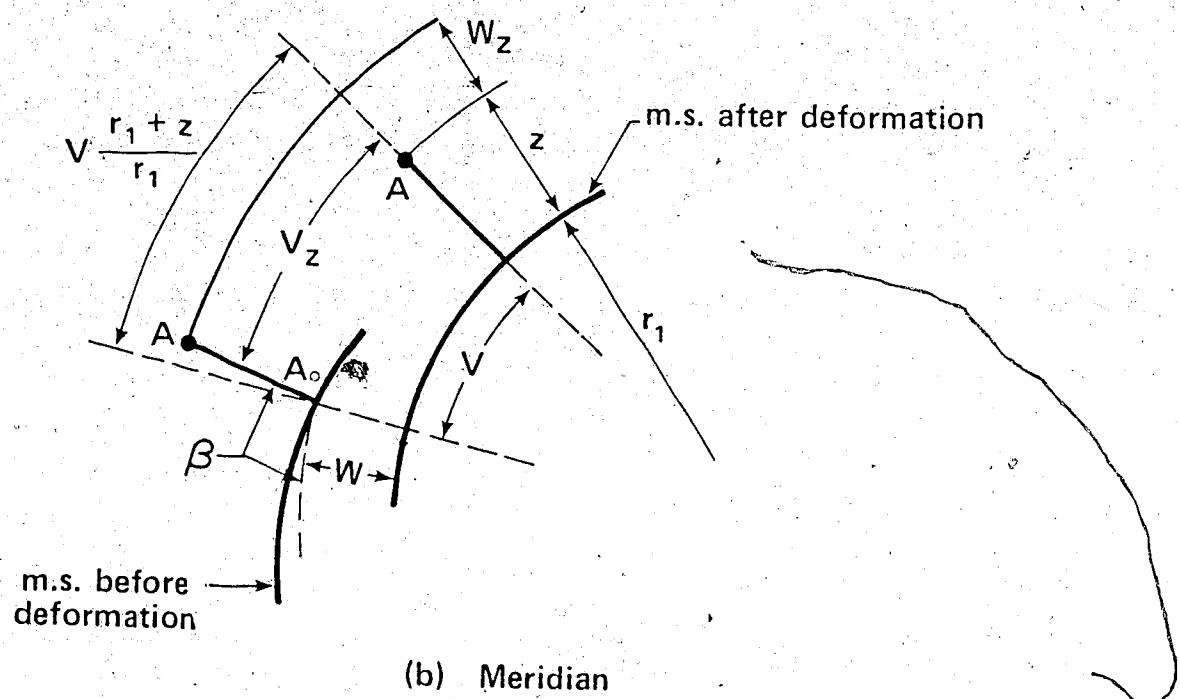
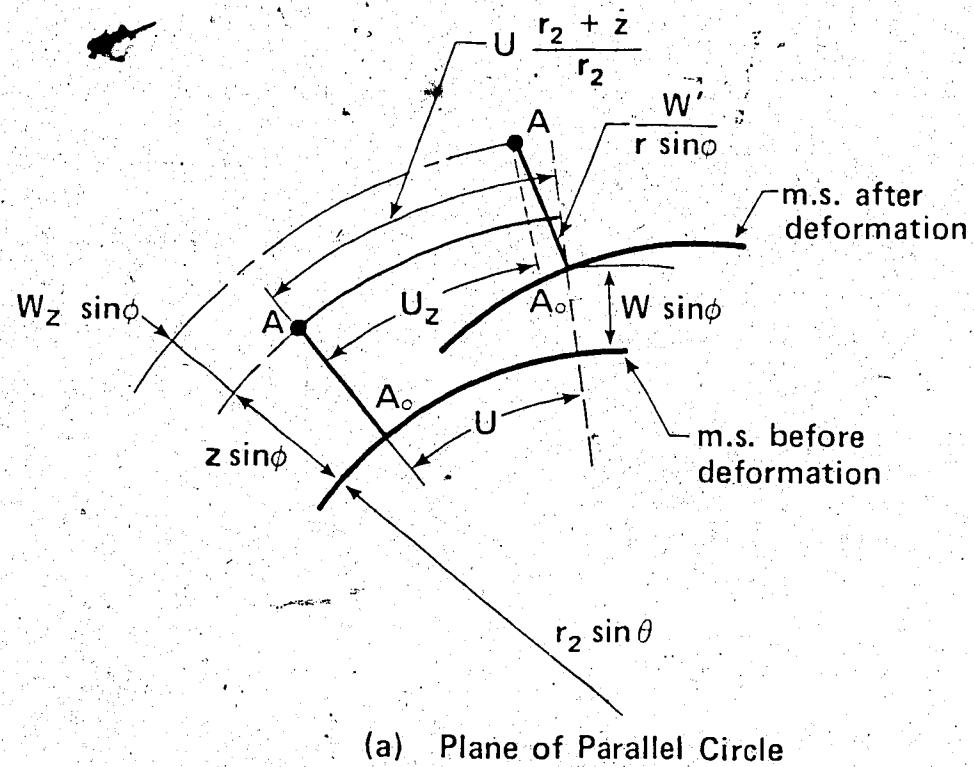


Fig. A.5 Relation Between Displacement
of Two Points on Line Normal to the Middle Surface

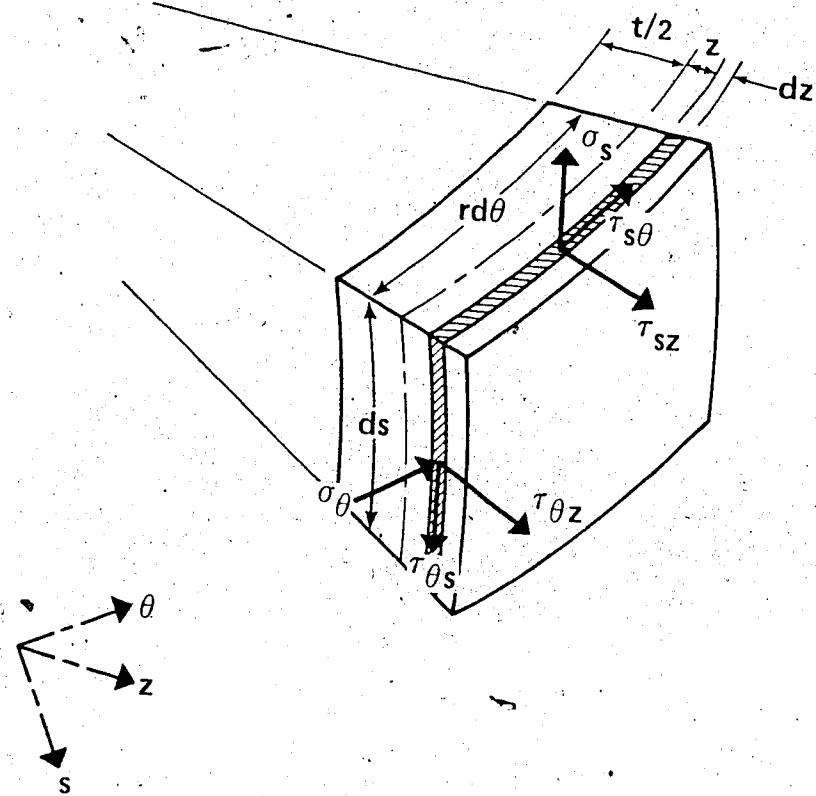


Fig. A.6 Stresses Acting on a Shell Element

APPENDIX B

USER'S MANUAL FOR PROGRAM SASHELL

Program SASHELL computes stress resultants and displacements for axisymmetric branched segmented shell structures due to their own weight, external applied loads and differential temperature variation (along the meridian or circumference).

In the present stage of development, the program is capable of analysing five types of shells of revolution of variable thickness. These are cylinders, circular plates, spheres, cones and hyperboloids of revolution. Loadings may be symmetric or non-axisymmetric with respect to the collatitude coordinate and may vary along the meridian.

The analysis procedure is based on the theory presented in this thesis and the program logic flow outlined in Sect. 4.2. A complete listing of the program is given in Appendix C. Input of the problems discussed in Chapter 4 and output of the pinched cylinder for a concentrated line load are given in Appendix D.

The input to SASHELL consists of several types of input cards. Certain card types may be repeated as required.

A typical explanation of a card type consists of the card type, a descriptive name indicating the nature of the data being entered and the format for the data on that card. This is followed by a symbolic line of input which, in turn, is followed by definitions of the input variables.

Limitations on SASHELL, due to the dimensions of the arrays in the program, are outlined following the explanation of the input cards.

TYPE 1: TITLE CARD (Format 10A8)

80	
AN IDENTIFIER STRING	

One card which contains any title for the problem

TYPE 2: ANALYSIS CONTROL CARD (Format 4I4, F7.0)

4	8	12	16	23
IPRINT	NP	NPCR	LDC	BETA

IPRINT : Print control parameter

If IPRINT = 0, the output will contain an echo check of the completed data. The loadings will be expanded in Fourier series, if required for the analysis, and the coefficients will be printed out.

If IPRINT = 1, the output will contain the echo check of the input data and the final results.

If IPRINT = 2, the output will contain the echo check of the input data, the results of the analysis for each harmonic and the superimposed final results.

If IPRINT = 3, full output including intermediate values will be printed out. (Used for checking purposes only.)

- NP : Number of points along the element meridian for which the Runge-Kutta integration process is used.
 NP should not be specified less than 21 (see Limitations).
- NPCR : Number of the circumferential points at which the final results are required.
 If the loads are symmetric or antisymmetric with respect to the meridian passing through $\theta = 0$, NPCR is the number of points along half the circumference $(0, \pi)$. If the loads vary randomly in the circumferential direction, NPCR is the number of points along the full circumference of the element $(0, 2\pi)$:
 If the loads are constants in the circumferential direction (symmetric), NPCR is equal to one.
- LDC : Dead load control parameter.
 If LDC = 0, dead load is excluded from the analysis.
 If LDC = 1, dead load is evaluated and superimposed on the external applied loadings.
- BETA : Maximum element length coefficient. BETA should not exceed 25 (see Limitations).

TYPE 3: STRUCTURE DATA CARD

(Format 2I3,4F12.0)

	3	6	18	30	42	54
NE	NJ	EG	PUG	GAMG	TKG	

NE : Number of elements

NJ : Number of junctions between elements (nodes).

EG : Global modulus of elasticity.

PUG : Global Poisson's ratio.

GAMG : Global specific weight.

TKG : Global coefficient of thermal expansion

NOTE: If the structure consists of elements of different materials, the global properties are to be omitted and the structural data card specifies the number of elements and nodes only.

TYPE 4: NODAL DATA CARDS (format I4,2F10.0,4I4)

One card is required for each node.

	4	14	24	28	32	36	40
I	XCOOR(I)	RCOOR(I)	IDF(I,1)	IDF(I,2)	IDF(I,3)	IDF(I,4)	

I : Node number

XCOOR(I) : Global X coordinate of node I along the axis of revolution directed downward from the top of the structure.

RCOOR(I) : Radius of the parallel circle passing through node I. RCOOR(I) should not be specified as zero (see Limitations).

IDF(I,J) : Identification of the jth degree of freedom at node I. J = 1,4 for the rotation of the meridian (β), the radial displacement component (W), the meridional displacement component (V) and the circumferential displacement component (U), respectively. When

$IDF(I,J) = 0$, the corresponding degree of freedom
is not restrained.

When $IDF(I,J) = 1$, the corresponding degree of
freedom is restrained.

TYPE 5: ELEMENT DATA CARDS

Two cards are required for each element.

First card Format (5I6,4F10.0)

6	12	18	24	30	40	50	60	70
I	IT(I)	NC(I,1)	NC(I,2)	NIP(I)	TH(I,1)	TH(I,2)	EC(I,1)	EC(I,2)

I : Element number

IT(I) : Element type

If $IT(I) = 1$, element I is a cylinder

If $IT(I) = 2$, element I is a cone or a
circular plate.

If $IT(I) = 3$, element I is a sphere of which $r_1 = r_2$.

If $IT(I) = 4$, element I is a sphere of which $r_1 \neq r_2$.

If $IT(I) = 5$, element I is a hyperboloid of revolution.

NC(I,1) : Node number at the top of element I.

NC(I,2) : Node number at the bottom of element I.

NIP(I) : Integer to indicate the number of intermediate points
in the element at which the final results are not
required. The number of equally spaced points at
which the final results will be printed out are

$\frac{NP-1}{NIP(I)} + 1$. A value of NP-1 will be assigned to NIP(I) when input is zero.

TH(I,1) : Element thickness at the top.

TH(I,2) : Element thickness at the bottom.

EC(I,1) : Eccentricity of the top node from the middle surface of the element at the top.

EC(I,2) : Eccentricity of the bottom node from the middle surface of the element at the bottom.

NOTE: The eccentricity is defined such that the radius of the parallel circle passes through the middle surface of the element is

$$r_{midsurface} = r_{node} - EC(I,J)$$

Thus, EC(I,J) is positive when directed inward from the node to the middle surface of the element.

Second card

(Format 7F10.0)

10	20	30	40	50	60	70
HPCN(1,I)	HPCN(2,I)	HPCN(3,I)	GAMA(I)	E(I)	P0(I)	TCOEF(I)

HPCN(1,I) : Radius of curvature of the meridian for spherical element of type 4 (i.e., $r_1 \neq r_2$), or throat radius of a hyperboloid element (type 5). For elements other than of type 4 and type 5, HPCN(1,I) is 0.0.

HPCN(2,I) : Angle in degrees measured from the axis of revolution to the top edge of the spherical element of type 4, or hyperboloid constant in which the ratio $\frac{HPCN(2,I)}{HPCN(1,I)}$

equals to the slope of the asymptotes of the hyperbola. For elements other than of type 4 and 5, HPCN(2,I) is 0.0.

HPCH(3,I) : Angle in degrees measured from the axis of revolution to the top edge of the spherical element of type 4, or global X coordinate of the throat of the hyperboloid element. HPCH(3,I) is equal to 0.0 for elements other than of type 4 and 5.

GAMA(I) : Element specific weight. Specified if different from the global specific weight, otherwise GAMA(I) = 0.0 or blank.

E(I) : Element modulus of elasticity. Specified if different from the global modulus of elasticity, otherwise E(I) = 0.0 or blank.

PU(I) : Element Poisson's ratio. Specified if different from the global Poisson's ratio, otherwise PU(I) = 0.0 or blank.

TCOEF(I) : Element coefficient of thermal expansion. Specified if different from the global coefficient otherwise TCOEF(I) = 0.0 or blank.

NOTE: For a segmented structure which does not include spherical or hyperboloid elements and for which the other elements of which the structure consists have the same material properties, this card is a blank card.

TYPE 6: LOADING SPECIFICATION CARD (Format (6I5))

	5	10	15	20	25	30
NEL	NJL	NHL	NTL	NHPL	NHIN	

NEL : Number of externally loaded elements in the structure.

NJL : Number of loaded nodes in the structure.

NHL : Maximum number of harmonics required for the analysis including the zero harmonic (i.e., NHL = maximum harmonic number +1).

NTL : Loading type character.

If NTL = 0, loading is symmetric and the analysis is required for the zero harmonic only.

If NTL = 1, loading is axisymmetric in the circumferential direction, input is provided at a number of discrete points along the circumference of the shell, and the analysis is required for the cosine coefficients of Fourier series only (i.e., loading is symmetric with respect to a meridian passes through $\theta = \text{constant}$).

If NTL = 2, loading is non-axisymmetric in the circumferential direction, input is provided at a number of discrete points along the circumference of the shell, and the analysis is required for both are cosine and sine coefficients of Fourier series.

If NTL = 3, loading is nonaxisymmetric along the

along the circumferential direction and input directly as cosine coefficients of Fourier expansion only.

NHPL : Number of points along the circumference of the shell at which the loads values are described.

These points are defined such that the circumferential coordinate is $\frac{2\pi i}{NHPL}$ where $i = 0, 1, 2, \dots, NHPL - 1$.

NHIN : Integer which defines the increment in the harmonics, starting from the zero harmonic, to be specified when the analysis is required for a harmonic number $(0, NHIN, 2 NHIN, \dots, NHL - 1)$. The program sets $NHIN = 1$ when it is specified as zero or blank.

NOTE: If the load is described at NHPL points, NHL can be specified as $\frac{NHPL}{2}$ and NHIN can be set equal to one. When IPRINT = 0, the Fourier coefficients of the input load are obtained with the echo check of the input data. Then, the user may decide, upon examining these coefficients, on the final values of NHL and NHIN (see Chapter 5, Sect. 5.2.3).

TYPE 7: ELEMENT LOADING CONTROL INPUT CARDS (Format 6I5)

One card for each loaded element

	5	10	15	20	25	30
LL(I)	ILOAD (LL(I), K),	K = 1,5				

I : Integer takes the value of 1 to NEL

LL(I) : Number of the loaded element.

ILOAD(LL(I),K)

: Identifier for loading type on the element LL(I)
in the order:

ILOAD(*,1) for loading in the direction tangent
to the meridian (s).

ILOAD(*,2) for loading in the direction tangent
to the parallel circle (θ).

ILOAD(*,3) for loading in the direction perpendicular
to the tangent to the meridian (ϕ_z).

ILOAD(*,4) for temperature at the shell exterior face.

ILOAD(*,5) for temperature at the shell interior face.

If ILOAD(*,K) = 0, no load of type K is applied.

If ILOAD(*,K) = 1, the applied load of type K is
constant along the meridian and to be specified at
one end of the element only.

If ILOAD(*,K) = 2, the applied load of type K varies
linearly along the meridian and is to be specified
at the two ends of the element.

If ILOAD(*,K) = 3, the applied load of type K
varies as a second degree function along the meridian
and is to be specified at the two ends of the element.

NOTE: Element loads are input in the element local coordinates.

TYPE 8: ELEMENT LOADING CARDS

This type of card depends upon NTL and is classified as TYPE 8-A, TYPE 8-B and TYPE 8-C for NTL equal to 0, 1 or 2, and 3 respectively.

TYPE 8-A: ELEMENT SYMMETRICAL LOADING CARDS (Format 2F10.0)

This type is required if NTL is equal to zero. Number of cards required, for each loaded element, is equal to, and input in the order consistent with, each non-zero term in the corresponding row in ILOAD array.

10	20
ACEL(K1)	ACEL(K2)

ACEL(K1) : Magnitude of the load at the element top.

ACEL(K2) : Magnitude of the load at the element bottom.

TYPE 8-B ELEMENT ASYMMETRIC TABULATED LOADING CARDS (Format 8F10.0)

This type is required if NTL is equal to 1 or 2.

Number of cards required, for each loaded element, is equal to NHPL 8 for, and input in the order consistent with, each non-zero term in the corresponding row in ILOAD array.

10	20	30	40	50	60	70	80
W(I),	I = 1, NHPL						

$W(I)$: Magnitude of the load, at the points defined by $\frac{2\pi i}{NHPL}$ ($i = 0, 1, 2, \dots, NHPL-1$) along the circumference of the element, and for which Fourier expansion is required.

TYPE 8-C ELEMENT LOADING FOURIER COEFFICIENTS CARDS

(Format 8F10.0)

This type is required if NTL is equal to 4. Number of cards required, for each loaded element, is equal to $\frac{NHL}{8}$ for, and input in the order consistent with, each non-zero term in the corresponding row in ILOAD array.

10	20	30	40	50	60	70	80
AL(K) , K = 1, NHL							

$AL(K)$: Cosine coefficients of Fourier expansion for Harmonics 0, 1, 2, ..., NHL-1.

TYPE 9: NODAL LOADING CONTROL INPUT CARDS (Format I5)

One card for each loaded node (NJL cards)

5

J

J : Number of loaded node.

TYPE 10: NODAL LOADING CARDS

This type of card depends upon NTL and is classified as TYPE 10-A, TYPE 10-B and TYPE 10-C for NTL equal to 0, 1 or 2, and 3, respectively.

TYPE 10-A: NODAL SYMMETRICAL LOADING CARDS (Format (4F10.0))

This type is required if NTL is equal to zero. One card is required for each loaded node.

10	20	30	40
ANJL (II,1)	,	II = 1,4	

ANJL(II,1): Magnitude of the nodal load, in the global structure coordinates, in the following order.

II = 1, meridional couple, M_s , positive when rotating in the clockwise direction.

II = 2, force normal to the tangent to meridian at the corresponding node, S_s , positive in the outward direction from the axis of revolution.

II = 3, force in the direction tangent to the meridian, N_s , positive when directed downward parallel to the axis of revolution.

II = 4, force in the direction tangent to the parallel circle, T_s , positive when directed in the anticlockwise rotating direction around the structure.

NOTE: The sign convention of the nodal loads as defined above is equivalent to the stiffness matrix sign convention as described in Sect. 3.7.

TYPE 10-B: NODAL ASYMMETRICAL TABULATED LOADING CARDS

This type is required if NTL is equal to 1 or 2 and consists of the following cards, for each loaded node.

- (1) Asymmetric nodal load control input card. Required to identify which to the nodal loads, classified as in Type 10-A cards (M_s , S_s , N_s , T_s), is applied and to be input. One card (Format 4I4) is required.

K : Integer which takes a value of 1 to 4 and represents the nodal forces, as defined in TYPE 10-A cards, in the order M_s , S_s , N_s and T_s , respectively.

KLD(K) : If equal to zero, nodal force of the type K
is not applied.

If equal to 1, nodal force of the type K is applied.

- (2) Nodal loading magnitude card (Format: 8F10.0)

Number of cards required is equal to $\frac{NHPL}{8}$ for, and
 input in the order consistent with, each non-zero term
 in KLD.

$W(I) \quad , \quad I = 1, \text{ NHPL}$

$W(I)$: Magnitude of the nodal load, at the points defined by $\frac{2\pi i}{NHL}$ ($i = 0, 1, 2, \dots, NHL - 1$) along the circumference of the nodal circle, and for which Fourier expansion is required.

TYPE 10-C: NODAL LOADING FOURIER COEFFICIENTS CARDS

This type is required if NTL is equal to 3 and NJL is greater than 0. It consists of the following cards, for each loaded node:

- (1) Asymmetric nodal load control input card of the same type described in TYPE 10-B.

4	8	12	20
KLD(K)	,	K = 1, 4	

- (2) Nodal loading magnitude card (Format 8F10.0)
 Number of cards required is equal to $\frac{NHL}{8}$ for, and
 input in the order consistent with, each non-zero
 term in KLD.

10	20	30	40	50	60	70	80
AL(N),	N = 1,	NHL					

AL(N) : Cosine coefficients of FOURIER expansion for harmonics 0, 1, 2, ... NHL - 1.

LIMITATIONS

Number of elements NE	* 20
Number of nodes NJ	* 21
Number of integration points NP	* 51
Number of circumferential points NPCR	* 11
Number of harmonics NHL	* 20
Number of circumferential points at which loadings is specified NMPL	* 40
Full band width NHB	* 80
Total number of Fourier coefficients in the problem for elements loading, associated with either cosine or sine factor and defined by NHL x NEL	* 200
Total number of Fourier coefficients in the problem for nodal loadings, associated with both cosine and sine factor and defined by 8 x NHL x NJL	* 200
Number of points in a non-axisymmetrically loaded structure at which the final results are required, and which can be calculated as	

$$\sum_{i=1}^{NE} NS_i \times \left\{ \frac{(NP-1)}{NIP_i} + 1 \right\} * 200$$

where NS_i = number of segments into which element i is subdivided and can be obtained by $\frac{L}{BETA} \sqrt{\frac{3(1 - \nu^2)}{r^2 t^2}}$ in which, L , is the element length, r , is the element radius, t , is the element thickness and ν is Poisson's ratio.

The above limitations are due to the dimension statements in the program SASHELL as listed in Appendix C. The dimension of the corresponding arrays may be modified to change these limitations.

Limitations on the theory employed in SASHELL are as follows (see Chapter 5).

- 1) RCOOR should not be specified as zero.
- 2) The length factor coefficient BETA should not be specified greater than 25.
- 3) The number of points of integration NP should not be less than 21 when BETA is specified less than 20, and not less than 31 when BETA is greater than 20 and less than 25.

APPENDIX C
PROGRAM LISTING

```

1 C DO 40 J=1,N
2 C TDATA(J)=DCS(J,J)
3 C DSN(J)=DSN(J,J)
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151.

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351      FA(2)=GECK(1,2)
352      FA(1,1)=S15(H1,1,3)
353      FA(1,2)=S15(H1,1,2)
354      FA(1,3)=S15(H1,1,4)
355      FA(1,4)=S15(H1,1,4)
356      FA(1,5)=S15(H1,1,5)
357      FA(1,6)=S15(H1,1,6)
358      FA(1,7)=S15(H1,1,7)
359      FA(1,8)=S15(H1,1,8)
360      FA(1,9)=S15(H1,1,9)  GO TO 93
361      FA(2,1)=S15(H2,1,1)
362      FA(2,2)=S15(H2,1,2)
363      FA(2,3)=S15(H2,1,3)
364      FA(2,4)=S15(H2,1,4)
365      FA(2,5)=S15(H2,1,5)
366      FA(2,6)=S15(H2,1,6)
367      FA(2,7)=S15(H2,1,7)
368      FA(2,8)=S15(H2,1,8)
369      FA(2,9)=S15(H2,1,9)  GO TO 93
370      FA(3,1)=S15(H3,1,1)
371      FA(3,2)=S15(H3,1,2)
372      FA(3,3)=S15(H3,1,3)
373      FA(3,4)=S15(H3,1,4)
374      FA(3,5)=S15(H3,1,5)
375      FA(3,6)=S15(H3,1,6)
376      FA(3,7)=S15(H3,1,7)
377      FA(3,8)=S15(H3,1,8)
378      FA(3,9)=S15(H3,1,9)  GO TO 93
379      FA(4,1)=S15(H4,1,1)
380      FA(4,2)=S15(H4,1,2)
381      FA(4,3)=S15(H4,1,3)
382      FA(4,4)=S15(H4,1,4)
383      FA(4,5)=S15(H4,1,5)
384      FA(4,6)=S15(H4,1,6)
385      FA(4,7)=S15(H4,1,7)
386      FA(4,8)=S15(H4,1,8)
387      FA(4,9)=S15(H4,1,9)  GO TO 93
388      FA(5,1)=S15(H5,1,1)
389      FA(5,2)=S15(H5,1,2)
390      FA(5,3)=S15(H5,1,3)
391      FA(5,4)=S15(H5,1,4)
392      FA(5,5)=S15(H5,1,5)
393      FA(5,6)=S15(H5,1,6)
394      FA(5,7)=S15(H5,1,7)
395      FA(5,8)=S15(H5,1,8)
396      FA(5,9)=S15(H5,1,9)  GO TO 93
397      FA(6,1)=S15(H6,1,1)
398      FA(6,2)=S15(H6,1,2)
399      FA(6,3)=S15(H6,1,3)
400      FA(6,4)=S15(H6,1,4)
401      FA(6,5)=S15(H6,1,5)
402      FA(6,6)=S15(H6,1,6)
403      FA(6,7)=S15(H6,1,7)
404      FA(6,8)=S15(H6,1,8)
405      FA(6,9)=S15(H6,1,9)  GO TO 93
406      FA(7,1)=S15(H7,1,1)
407      FA(7,2)=S15(H7,1,2)
408      FA(7,3)=S15(H7,1,3)
409      FA(7,4)=S15(H7,1,4)
410      FA(7,5)=S15(H7,1,5)
411      FA(7,6)=S15(H7,1,6)
412      FA(7,7)=S15(H7,1,7)
413      FA(7,8)=S15(H7,1,8)
414      FA(7,9)=S15(H7,1,9)  GO TO 93
415      FA(8,1)=S15(H8,1,1)
416      FA(8,2)=S15(H8,1,2)
417      FA(8,3)=S15(H8,1,3)
418      FA(8,4)=S15(H8,1,4)
419      FA(8,5)=S15(H8,1,5)
420      FA(8,6)=S15(H8,1,6)
421      FA(8,7)=S15(H8,1,7)
422      FA(8,8)=S15(H8,1,8)
423      FA(8,9)=S15(H8,1,9)  GO TO 93
424      FA(9,1)=S15(H9,1,1)
425      FA(9,2)=S15(H9,1,2)
426      FA(9,3)=S15(H9,1,3)
427      FA(9,4)=S15(H9,1,4)
428      FA(9,5)=S15(H9,1,5)
429      FA(9,6)=S15(H9,1,6)
430      FA(9,7)=S15(H9,1,7)
431      FA(9,8)=S15(H9,1,8)
432      FA(9,9)=S15(H9,1,9)  GO TO 93
433      FA(10,1)=S15(H10,1,1)
434      FA(10,2)=S15(H10,1,2)
435      FA(10,3)=S15(H10,1,3)
436      FA(10,4)=S15(H10,1,4)
437      FA(10,5)=S15(H10,1,5)
438      FA(10,6)=S15(H10,1,6)
439      FA(10,7)=S15(H10,1,7)
440      FA(10,8)=S15(H10,1,8)
441      FA(10,9)=S15(H10,1,9)  GO TO 93
442      FA(11,1)=S15(H11,1,1)
443      FA(11,2)=S15(H11,1,2)
444      FA(11,3)=S15(H11,1,3)
445      FA(11,4)=S15(H11,1,4)
446      FA(11,5)=S15(H11,1,5)
447      FA(11,6)=S15(H11,1,6)
448      FA(11,7)=S15(H11,1,7)
449      FA(11,8)=S15(H11,1,8)
450      FA(11,9)=S15(H11,1,9)  GO TO 93
451      FA(12,1)=S15(H12,1,1)
452      FA(12,2)=S15(H12,1,2)
453      FA(12,3)=S15(H12,1,3)
454      FA(12,4)=S15(H12,1,4)
455      FA(12,5)=S15(H12,1,5)
456      FA(12,6)=S15(H12,1,6)
457      FA(12,7)=S15(H12,1,7)
458      FA(12,8)=S15(H12,1,8)
459      FA(12,9)=S15(H12,1,9)  GO TO 93
460      FA(13,1)=S15(H13,1,1)
461      FA(13,2)=S15(H13,1,2)
462      FA(13,3)=S15(H13,1,3)
463      FA(13,4)=S15(H13,1,4)
464      FA(13,5)=S15(H13,1,5)
465      FA(13,6)=S15(H13,1,6)
466      FA(13,7)=S15(H13,1,7)
467      FA(13,8)=S15(H13,1,8)
468      FA(13,9)=S15(H13,1,9)  GO TO 93
469      FA(14,1)=S15(H14,1,1)
470      FA(14,2)=S15(H14,1,2)
471      FA(14,3)=S15(H14,1,3)
472      FA(14,4)=S15(H14,1,4)
473      FA(14,5)=S15(H14,1,5)
474      FA(14,6)=S15(H14,1,6)
475      FA(14,7)=S15(H14,1,7)
476      FA(14,8)=S15(H14,1,8)
477      FA(14,9)=S15(H14,1,9)  GO TO 93
478      FA(15,1)=S15(H15,1,1)
479      FA(15,2)=S15(H15,1,2)
480      FA(15,3)=S15(H15,1,3)
481      FA(15,4)=S15(H15,1,4)
482      FA(15,5)=S15(H15,1,5)
483      FA(15,6)=S15(H15,1,6)
484      FA(15,7)=S15(H15,1,7)
485      FA(15,8)=S15(H15,1,8)
486      FA(15,9)=S15(H15,1,9)  GO TO 93
487      FA(16,1)=S15(H16,1,1)
488      FA(16,2)=S15(H16,1,2)
489      FA(16,3)=S15(H16,1,3)
490      FA(16,4)=S15(H16,1,4)
491      FA(16,5)=S15(H16,1,5)
492      FA(16,6)=S15(H16,1,6)
493      FA(16,7)=S15(H16,1,7)
494      FA(16,8)=S15(H16,1,8)
495      FA(16,9)=S15(H16,1,9)  GO TO 93
496      FA(17,1)=S15(H17,1,1)
497      FA(17,2)=S15(H17,1,2)
498      FA(17,3)=S15(H17,1,3)
499      FA(17,4)=S15(H17,1,4)
500      FA(17,5)=S15(H17,1,5)
501      FA(17,6)=S15(H17,1,6)
502      FA(17,7)=S15(H17,1,7)
503      FA(17,8)=S15(H17,1,8)
504      FA(17,9)=S15(H17,1,9)  GO TO 93
505      FA(18,1)=S15(H18,1,1)
506      FA(18,2)=S15(H18,1,2)
507      FA(18,3)=S15(H18,1,3)
508      FA(18,4)=S15(H18,1,4)
509      FA(18,5)=S15(H18,1,5)
510      FA(18,6)=S15(H18,1,6)
511      FA(18,7)=S15(H18,1,7)
512      FA(18,8)=S15(H18,1,8)
513      FA(18,9)=S15(H18,1,9)  GO TO 93
514      FA(19,1)=S15(H19,1,1)
515      FA(19,2)=S15(H19,1,2)
516      FA(19,3)=S15(H19,1,3)
517      FA(19,4)=S15(H19,1,4)
518      FA(19,5)=S15(H19,1,5)
519      FA(19,6)=S15(H19,1,6)
520      FA(19,7)=S15(H19,1,7)
521      FA(19,8)=S15(H19,1,8)
522      FA(19,9)=S15(H19,1,9)  GO TO 93
523      FA(20,1)=S15(H20,1,1)
524      FA(20,2)=S15(H20,1,2)
525      FA(20,3)=S15(H20,1,3)
526      FA(20,4)=S15(H20,1,4)
527      FA(20,5)=S15(H20,1,5)
528      FA(20,6)=S15(H20,1,6)
529      FA(20,7)=S15(H20,1,7)
530      FA(20,8)=S15(H20,1,8)
531      FA(20,9)=S15(H20,1,9)  GO TO 93
532      FA(21,1)=S15(H21,1,1)
533      FA(21,2)=S15(H21,1,2)
534      FA(21,3)=S15(H21,1,3)
535      FA(21,4)=S15(H21,1,4)
536      FA(21,5)=S15(H21,1,5)
537      FA(21,6)=S15(H21,1,6)
538      FA(21,7)=S15(H21,1,7)
539      FA(21,8)=S15(H21,1,8)
540      FA(21,9)=S15(H21,1,9)  GO TO 93
541      FA(22,1)=S15(H22,1,1)
542      FA(22,2)=S15(H22,1,2)
543      FA(22,3)=S15(H22,1,3)
544      FA(22,4)=S15(H22,1,4)
545      FA(22,5)=S15(H22,1,5)
546      FA(22,6)=S15(H22,1,6)
547      FA(22,7)=S15(H22,1,7)
548      FA(22,8)=S15(H22,1,8)
549      FA(22,9)=S15(H22,1,9)  GO TO 93
550      FA(23,1)=S15(H23,1,1)
551      FA(23,2)=S15(H23,1,2)
552      FA(23,3)=S15(H23,1,3)
553      FA(23,4)=S15(H23,1,4)
554      FA(23,5)=S15(H23,1,5)
555      FA(23,6)=S15(H23,1,6)
556      FA(23,7)=S15(H23,1,7)
557      FA(23,8)=S15(H23,1,8)
558      FA(23,9)=S15(H23,1,9)  GO TO 93
559      FA(24,1)=S15(H24,1,1)
560      FA(24,2)=S15(H24,1,2)
561      FA(24,3)=S15(H24,1,3)
562      FA(24,4)=S15(H24,1,4)
563      FA(24,5)=S15(H24,1,5)
564      FA(24,6)=S15(H24,1,6)
565      FA(24,7)=S15(H24,1,7)
566      FA(24,8)=S15(H24,1,8)
567      FA(24,9)=S15(H24,1,9)  GO TO 93
568      FA(25,1)=S15(H25,1,1)
569      FA(25,2)=S15(H25,1,2)
570      FA(25,3)=S15(H25,1,3)
571      FA(25,4)=S15(H25,1,4)
572      FA(25,5)=S15(H25,1,5)
573      FA(25,6)=S15(H25,1,6)
574      FA(25,7)=S15(H25,1,7)
575      FA(25,8)=S15(H25,1,8)
576      FA(25,9)=S15(H25,1,9)  GO TO 93
577      FA(26,1)=S15(H26,1,1)
578      FA(26,2)=S15(H26,1,2)
579      FA(26,3)=S15(H26,1,3)
580      FA(26,4)=S15(H26,1,4)
581      FA(26,5)=S15(H26,1,5)
582      FA(26,6)=S15(H26,1,6)
583      FA(26,7)=S15(H26,1,7)
584      FA(26,8)=S15(H26,1,8)
585      FA(26,9)=S15(H26,1,9)  GO TO 93
586      FA(27,1)=S15(H27,1,1)
587      FA(27,2)=S15(H27,1,2)
588      FA(27,3)=S15(H27,1,3)
589      FA(27,4)=S15(H27,1,4)
590      FA(27,5)=S15(H27,1,5)
591      FA(27,6)=S15(H27,1,6)
592      FA(27,7)=S15(H27,1,7)
593      FA(27,8)=S15(H27,1,8)
594      FA(27,9)=S15(H27,1,9)  GO TO 93
595      FA(28,1)=S15(H28,1,1)
596      FA(28,2)=S15(H28,1,2)
597      FA(28,3)=S15(H28,1,3)
598      FA(28,4)=S15(H28,1,4)
599      FA(28,5)=S15(H28,1,5)
600      FA(28,6)=S15(H28,1,6)
601      FA(28,7)=S15(H28,1,7)
602      FA(28,8)=S15(H28,1,8)
603      FA(28,9)=S15(H28,1,9)  GO TO 93
604      FA(29,1)=S15(H29,1,1)
605      FA(29,2)=S15(H29,1,2)
606      FA(29,3)=S15(H29,1,3)
607      FA(29,4)=S15(H29,1,4)
608      FA(29,5)=S15(H29,1,5)
609      FA(29,6)=S15(H29,1,6)
610      FA(29,7)=S15(H29,1,7)
611      FA(29,8)=S15(H29,1,8)
612      FA(29,9)=S15(H29,1,9)  GO TO 93
613      FA(30,1)=S15(H30,1,1)
614      FA(30,2)=S15(H30,1,2)
615      FA(30,3)=S15(H30,1,3)
616      FA(30,4)=S15(H30,1,4)
617      FA(30,5)=S15(H30,1,5)
618      FA(30,6)=S15(H30,1,6)
619      FA(30,7)=S15(H30,1,7)
620      FA(30,8)=S15(H30,1,8)
621      FA(30,9)=S15(H30,1,9)  GO TO 93
622      FA(31,1)=S15(H31,1,1)
623      FA(31,2)=S15(H31,1,2)
624      FA(31,3)=S15(H31,1,3)
625      FA(31,4)=S15(H31,1,4)
626      FA(31,5)=S15(H31,1,5)
627      FA(31,6)=S15(H31,1,6)
628      FA(31,7)=S15(H31,1,7)
629      FA(31,8)=S15(H31,1,8)
630      FA(31,9)=S15(H31,1,9)  GO TO 93
631      FA(32,1)=S15(H32,1,1)
632      FA(32,2)=S15(H32,1,2)
633      FA(32,3)=S15(H32,1,3)
634      FA(32,4)=S15(H32,1,4)
635      FA(32,5)=S15(H32,1,5)
636      FA(32,6)=S15(H32,1,6)
637      FA(32,7)=S15(H32,1,7)
638      FA(32,8)=S15(H32,1,8)
639      FA(32,9)=S15(H32,1,9)  GO TO 93
640      FA(33,1)=S15(H33,1,1)
641      FA(33,2)=S15(H33,1,2)
642      FA(33,3)=S15(H33,1,3)
643      FA(33,4)=S15(H33,1,4)
644      FA(33,5)=S15(H33,1,5)
645      FA(33,6)=S15(H33,1,6)
646      FA(33,7)=S15(H33,1,7)
647      FA(33,8)=S15(H33,1,8)
648      FA(33,9)=S15(H33,1,9)  GO TO 93
649      FA(34,1)=S15(H34,1,1)
650      FA(34,2)=S15(H34,1,2)
651      FA(34,3)=S15(H34,1,3)
652      FA(34,4)=S15(H34,1,4)
653      FA(34,5)=S15(H34,1,5)
654      FA(34,6)=S15(H34,1,6)
655      FA(34,7)=S15(H34,1,7)
656      FA(34,8)=S15(H34,1,8)
657      FA(34,9)=S15(H34,1,9)  GO TO 93
658      FA(35,1)=S15(H35,1,1)
659      FA(35,2)=S15(H35,1,2)
660      FA(35,3)=S15(H35,1,3)
661      FA(35,4)=S15(H35,1,4)
662      FA(35,5)=S15(H35,1,5)
663      FA(35,6)=S15(H35,1,6)
664      FA(35,7)=S15(H35,1,7)
665      FA(35,8)=S15(H35,1,8)
666      FA(35,9)=S15(H35,1,9)  GO TO 93
667      FA(36,1)=S15(H36,1,1)
668      FA(36,2)=S15(H36,1,2)
669      FA(36,3)=S15(H36,1,3)
670      FA(36,4)=S15(H36,1,4)
671      FA(36,5)=S15(H36,1,5)
672      FA(36,6)=S15(H36,1,6)
673      FA(36,7)=S15(H36,1,7)
674      FA(36,8)=S15(H36,1,8)
675      FA(36,9)=S15(H36,1,9)  GO TO 93
676      FA(37,1)=S15(H37,1,1)
677      FA(37,2)=S15(H37,1,2)
678      FA(37,3)=S15(H37,1,3)
679      FA(37,4)=S15(H37,1,4)
680      FA(37,5)=S15(H37,1,5)
681      FA(37,6)=S15(H37,1,6)
682      FA(37,7)=S15(H37,1,7)
683      FA(37,8)=S15(H37,1,8)
684      FA(37,9)=S15(H37,1,9)  GO TO 93
685      FA(38,1)=S15(H38,1,1)
686      FA(38,2)=S15(H38,1,2)
687      FA(38,3)=S15(H38,1,3)
688      FA(38,4)=S15(H38,1,4)
689      FA(38,5)=S15(H38,1,5)
690      FA(38,6)=S15(H38,1,6)
691      FA(38,7)=S15(H38,1,7)
692      FA(38,8)=S15(H38,1,8)
693      FA(38,9)=S15(H38,1,9)  GO TO 93
694      FA(39,1)=S15(H39,1,1)
695      FA(39,2)=S15(H39,1,2)
696      FA(39,3)=S15(H39,1,3)
697      FA(39,4)=S15(H39,1,4)
698      FA(39,5)=S15(H39,1,5)
699      FA(39,6)=S15(H39,1,6)
700      FA(39,7)=S15(H39,1,7)
701      FA(39,8)=S15(H39,1,8)
702      FA(39,9)=S15(H39,1,9)  GO TO 93
703      FA(40,1)=S15(H40,1,1)
704      FA(40,2)=S15(H40,1,2)
705      FA(40,3)=S15(H40,1,3)
706      FA(40,4)=S15(H40,1,4)
707      FA(40,5)=S15(H40,1,5)
708      FA(40,6)=S15(H40,1,6)
709      FA(40,7)=S15(H40,1,7)
710      FA(40,8)=S15(H40,1,8)
711      FA(40,9)=S15(H40,1,9)  GO TO 93
712      FA(41,1)=S15(H41,1,1)
713      FA(41,2)=S15(H41,1,2)
714      FA(41,3)=S15(H41,1,3)
715      FA(41,4)=S15(H41,1,4)
716      FA(41,5)=S15(H41,1,5)
717      FA(41,6)=S15(H41,1,6)
718      FA(41,7)=S15(H41,1,7)
719      FA(41,8)=S15(H41,1,8)
720      FA(41,9)=S15(H41,1,9)  GO TO 93
721      FA(42,1)=S15(H42,1,1)
722      FA(42,2)=S15(H42,1,2)
723      FA(42,3)=S15(H42,1,3)
724      FA(42,4)=S15(H42,1,4)
725      FA(42,5)=S15(H42,1,5)
726      FA(42,6)=S15(H42,1,6)
727      FA(42,7)=S15(H42,1,7)
728      FA(42,8)=S15(H42,1,8)
729      FA(42,9)=S15(H42,1,9)  GO TO 93
730      FA(43,1)=S15(H43,1,1)
731      FA(43,2)=S15(H43,1,2)
732      FA(43,3)=S15(H43,1,3)
733      FA(43,4)=S15(H43,1,4)
734      FA(43,5)=S15(H43,1,5)
735      FA(43,6)=S15(H43,1,6)
736      FA(43,7)=S15(H43,1,7)
737      FA(43,8)=S15(H43,1,8)
738      FA(43,9)=S15(H43,1,9)  GO TO 93
739      FA(44,1)=S15(H44,1,1)
740      FA(44,2)=S15(H44,1,2)
741      FA(44,3)=S15(H44,1,3)
742      FA(44,4)=S15(H44,1,4)
743      FA(44,5)=S15(H44,1,5)
744      FA(44,6)=S15(H44,1,6)
745      FA(44,7)=S15(H44,1,7)
746      FA(44,8)=S15(H44,1,8)
747      FA(44,9)=S15(H44,1,9)  GO TO 93
748      FA(45,1)=S15(H45,1,1)
749      FA(45,2)=S15(H45,1,2)
750      FA(45,3)=S15(H45,1,3)
751      FA(45,4)=S15(H45,1,4)
752      FA(45,5)=S15(H45,1,5)
753      FA(45,6)=S15(H45,1,6)
754      FA(45,7)=S15(H45,1,7)
755      FA(45,8)=S15(H45,1,8)
756      FA(45,9)=S15(H45,1,9)  GO TO 93
757      FA(46,1)=S15(H46,1,1)
758      FA(46,2)=S15(H46,1,2)
759      FA(46,3)=S15(H46,1,3)
760      FA(46,4)=S15(H46,1,4)
761      FA(46,5)=S15(H46,1,5)
762      FA(46,6)=S15(H46,1,6)
763      FA(46,7)=S15(H46,1,7)
764      FA(46,8)=S15(H46,1,8)
765      FA(46,9)=S15(H46,1,9)  GO TO 93
766      FA(47,1)=S15(H47,1,1)
767      FA(47,2)=S15(H47,1,2)
768      FA(47,3)=S15(H47,1,3)
769      FA(47,4)=S15(H47,1,4)
770      FA(47,5)=S15(H47,1,5)
771      FA(47,6)=S15(H47,1,6)
772      FA(47,7)=S15(H47,1,7)
773      FA(47,8)=S15(H47,1,8)
774      FA(47,9)=S15(H47,1,9)  GO TO 93
775      FA(48,1)=S15(H48,1,1)
776      FA(48,2)=S15(H48,1,2)
777      FA(48,3)=S15(H48,1,3)
778      FA(48,4)=S15(H48,1,4)
779      FA(48,5)=S15(H48,1,5)
780      FA(48,6)=S15(H48,1,6)
781      FA(48,7)=S15(H48,1,7)
782      FA(48,8)=S15(H48,1,8)
783      FA(48,9)=S15(H48,1,9)  GO TO 93
784      FA(49,1)=S15(H49,1,1)
785      FA(49,2)=S15(H49,1,2)
786      FA(49,3)=S15(H49,1,3)
787      FA(49,4)=S15(H49,1,4)
788      FA(49,5)=S15(H49,1,5)
789      FA(49,6)=S15(H49,1,6)
790      FA(49,7)=S15(H49,1,7)
791      FA(49,8)=S15(H49,1,8)
792      FA(49,9)=S15(H49,1,9)  GO TO 93
793      FA(50,1)=S15(H50,1,1)
794      FA(50,2)=S15(H50,1,2)
795      FA(50,3)=S15(H50,1,3)
796      FA(50,4)=S15(H50,1,4)
797      FA(50,5)=S15(H50,1,5)
798      FA(50,6)=S15(H50,1,6)
799      FA(50,7)=S15(H50,1,7)
800      FA(50,8)=S15(H50,1,8)
801      FA(50,9)=S15(H50,1,9)  GO TO 93
802      FA(51,1)=S15(H51,1,1)
803      FA(51,2)=S15(H51,1,2)
804      FA(51,3)=S15(H51,1,3)
805      FA(51,4)=S15(H51,1,4)
806      FA(51,5)=S15(H51,1,5)
807      FA(51,6)=S15(H51,1,6)
808      FA(51,7)=S15(H51,1,7)
809      FA(51,8)=S15(H51,1,8)
810      FA(51,9)=S15(H51,1,9)  GO TO 93
811      FA(52,1)=S15(H52,1,1)
812      FA(52,2)=S15(H52,1,2)
813      FA(52,3)=S15(H52,1,3)
814      FA(52,4)=S15(H52,1,4)
815      FA(52,5)=S15(H52,1,5)
816      FA(52,6)=S15(H52,1,6)
817      FA(52,7)=S15(H52,1,7)
818      FA(52,8)=S15(H52,1,8)
819      FA(52,9)=S15(H52,1,9)  GO TO 93
820      FA(53,1)=S15(H53,1,1)
821      FA(53,2)=S15(H53,1,2)
822      FA(53,3)=S15(H53,1,3)
823      FA(53,4)=S15(H53,1,4)
824      FA(53,5)=S15(H53,1,5)
825      FA(53,6)=S15(H53,1,6)
826      FA(53,7)=S15(H53,1,7)
827      FA(53,8)=S15(H53,1,8)
828      FA(53,9)=S15(H53,1,9)  GO TO 93
829      FA(54,1)=S15(H54,1,1)
830      FA(54,2)=S15(H54,1,2)
831      FA(54,3)=S15(H54,1,3)
832      FA(54,4)=S15(H54,1,4)
833      FA(54,5)=S15(H54,1,5)
834      FA(54,6)=S15(H54,1,6)
835      FA(54,7)=S15(H54,1,7)
836      FA(54,8)=S15(H54,1,8)
837      FA(54,9)=S15(H54,1,9)  GO TO 93
838      FA(55,1)=S15(H55,1,1)
839      FA(55,2)=S15(H55,1,2)
840      FA(55,3)=S15(H55,1,3)
841      FA(55,4)=S15(H55,1,4)
842      FA(55,5)=S15(H55,1,5)
843      FA(55,6)=S15(H55,1,6)
844      FA(55,7)=S15(H55,1,7)
845      FA(55,8)=S15(H55,1,8)
846      FA(55,9)=S15(H55,1,9)  GO TO 93
847      FA(56,1)=S15(H56,1,1)
848      FA(56,2)=S15(H56,1,2)
849      FA(56,3)=S15(H56,1,3)
850      FA(56,4)=S15(H56,1,4)
851      FA(56,5)=S15(H56,1,5)
852      FA(56,6)=S15(H56,1,6)
853      FA(56,7)=S15(H56,1,7)
854      FA(56,8)=S15(H56,1,8)
855      FA(56,9)=S15(H56,1,9)  GO TO 93
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      20 10 Z=1652
10 Y(Z,K) = T5(I,J,I)*(1/6)*((AK(Z,K))**((AK(Z,K))**2 + BK(I,K)) +
C 2.0*(CK(I,K)*BK(I,K)))
      21 Z=27.125,E=.30,D=100J
      22 S1=(I,J,K)=T5(I,J,K)
      23 S2=(I,J,K)=(I5(I,K),K=1,38),I=1,K=1
      24 S3=(I,J,K)=(I5(I,K),K=1,38),I=1,K=4/7
      25 S4=(I,J,K)=(I5(I,K),K=1,38),I=10,I0=10,K=1,K=15,4/7
      26 G1=S1+S2+S3+S4
      27 G2=G1+G1
      28 G3=G2+G2
      29 G4=G3+G3
      30 G5=G4+G4
      31 G6=G5+G5
      32 G7=G6+G6
      33 G8=G7+G7
      34 G9=G8+G8
      35 G10=G9+G9
      36 G11=G10+G10
      37 G12=G11+G11
      38 G13=G12+G12
      39 G14=G13+G13
      40 G15=G14+G14
      41 G16=G15+G15
      42 G17=G16+G16
      43 G18=G17+G17
      44 G19=G18+G18
      45 G20=G19+G19
      46 G21=G20+G20
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      49 G24=G23+G23
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      51 G26=G25+G25
      52 G27=G26+G26
      53 G28=G27+G27
      54 G29=G28+G28
      55 G30=G29+G29
      56 G31=G30+G30
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      58 G33=G32+G32
      59 G34=G33+G33
      60 G35=G34+G34
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      74 G49=G48+G48
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      76 G51=G50+G50
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      88 G63=G62+G62
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      94 G69=G68+G68
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      604 G579=G578+G578
      605 G580=G579+G579
      606 G581=G580+G580
      607 G582=G581+G581
      608 G583=G582+G582
      609 G584=G583+G583
      610 G585=G584+G584
      611 G586=G585+G585
      612 G587=G586+G586
      613 G588=G587+G587
      614 G589=G588+G588
      615 G590=G589+G589
      616 G591=G590+G590
      617 G592=G591+G591
      618 G593=G592+G592
      619 G594=G593+G593
      620 G595=G594+G594
      621 G596=G595+G595
      622 G597=G596+G596
      623 G598=G597+G597
      624 G599=G598+G598
      625 G600=G599+G599
      626 G601=G600+G600
      627 G602=G601+G601
      628 G603=G602+G602
      629 G604=G603+G603
      630 G605=G604+G604
      631 G606=G605+G605
      632 G607=G606+G606
      633 G608=G607+G607
      634 G609=G608+G608
      635 G610=G609+G609
      636 G611=G610+G610
      637 G612=G611+G611
      638 G613=G612+G612
      639 G614=G613+G613
      640 G615=G614+G614
      641 G616=G615+G615
      642 G617=G616+G616
      643 G618=G617+G617
      644 G619=G618+G618
      645 G620=G619+G619
      646 G621=G620+G620
      647 G622=G621+G621
      648 G623=G622+G622
      649 G624=G623+G623
      650 G625=G624+G624
      651 G626=G625+G625
      652 G627=G626+G626
      653 G628=G627+G627
      654 G629=G628+G628
      655 G630=G629+G629
      656 G631=G630+G630
      657 G632=G631+G631
      658 G633=G632+G632
      659 G634=G633+G633
      660 G635=G634+G634
      661 G636=G635+G635
      662 G637=G636+G636
      663 G638=G637+G637
      664 G639=G638+G638
      665 G640=G639+G639
      666 G641=G640+G640
      667 G642=G641+G641
      668 G643=G642+G642
      669 G644=G643+G643
      670 G645=G644+G644
      671 G646=G645+G645
      672 G647=G646+G646
      673 G648=G647+G647
      674 G649=G648+G648
      675 G650=G649+G649
      676 G651=G650+G650
      677 G652=G651+G651
      678 G653=G652+G652
      679 G654=G653+G653
      680 G655=G654+G654
      681 G656=G655+G655
      682 G657=G656+G656
      683 G658=G657+G657
      684 G659=G658+G658
      685 G660=G659+G659
      686 G6
```



```

1741    74  CONTINUE
1742      DO 10 I=1,J
1743      J=J+1
1744      LR(I,I,J)=X(J,J)
1745      X(J,J)=X(J,J)
1746      X(J,J)=X(J,J)-BX(I,J)
1747      BX(I,J)=BX(I,J)+X(J,J)
1748      X(J,J)=X(J,J)
1749      BX(I,J)=BX(I,J)+X(J,J)
1750      X(J,J)=X(J,J)
1751      DO 10 I=1,J
1752      J=J+1
1753      X(J,J)=X(J,J)-BX(I,J)
1754      BX(I,J)=BX(I,J)+X(J,J)
1755      X(J,J)=X(J,J)
1756      BX(I,J)=BX(I,J)+X(J,J)
1757      X(J,J)=X(J,J)
1758      DO 40 I=1,J
1759      P(X,I)=P(I,J)
1760      CALL PLUG2(I,J,45,X(1,J),X(2,J),X(3,J),X(4,J))
1761      CALL APPROX(X(1,J),X(2,J),X(3,J),X(4,J),I,J)
1762      C
1763      57  PLD(I,J)=1.02*SEL(29,0) GO TO 55
1764      58  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1765      59  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1766      60  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1767      61  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1768      62  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1769      63  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1770      64  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1771      65  CALL APPROX(X(1,J),X(2,J),X(3,J),X(4,J),I,J)
1772      66  CALL PLUG2(I,J,55,X(1,J),X(2,J),X(3,J),X(4,J))
1773      67  DO 68 I=1,5
1774      68  SEL(I,J)=2.51(I,J)*252(I,J)+E3(I,J)
1775      69  DO 70 I=1,5
1776      70  SEL(I,J)=2.51(I,J)*252(I,J)+E3(I,J)
1777      71  DO 72 I=1,5
1778      72  SEL(I,J)=2.51(I,J)*252(I,J)+E3(I,J)
1779      73  DO 74 I=1,5
1780      74  SEL(I,J)=2.51(I,J)*252(I,J)+E3(I,J)
1781      75  DO 76 I=1,5
1782      76  SEL(I,J)=2.51(I,J)*252(I,J)+E3(I,J)
1783      77  DO 78 I=1,5
1784      78  SEL(I,J)=2.51(I,J)*252(I,J)+E3(I,J)
1785      79  DO 80 I=1,5
1786      80  SEL(I,J)=2.51(I,J)*252(I,J)+E3(I,J)
1787      81  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1788      82  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1789      83  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1790      84  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1791      85  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1792      86  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1793      87  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1794      88  CALL APPROX(X(1,J),X(2,J),X(3,J),X(4,J),I,J)
1795      89  CALL PLUG2(I,J,55,X(1,J),X(2,J),X(3,J),X(4,J))
1796      90  CALL APPROX(X(1,J),X(2,J),X(3,J),X(4,J),I,J)
1797      91  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1798      92  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1799      93  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)
1800      94  CALL PLD(I,J,PLD,I,J,PS,PLD,21X)

```


APPENDIX D

DATA FILES AND SAMPLE OUTPUT

OF EXAMPLE APPLICATIONS

NATURAL DRAFT HYPERBOLOID COOLING TOWER

45 1,21,0,0,25,0,
 46 37,0,40,0,0,0,15,15,0,0,
 47 1,0,0,0,0,0,0,0,0,0,
 48 1,0,0,0,0,0,0,0,0,0,
 49 2,1,0,0,0,0,0,0,0,0,
 50 5,1,0,0,0,0,0,0,0,0,
 51 7,1,0,0,0,0,0,0,0,0,
 52 7,3,0,0,0,0,0,0,0,0,
 53 8,335,0,0,145,0,0,0,1,1,1,
 54 1,2,0,1,2,2,2,0,0,0,0,
 55 32,3,0,204,1,60,0,0,0,0,
 56 2,37,2,0,0,0,0,0,0,0,
 57 32,5,204,1,60,0,0,0,0,0,
 58 2,3,0,3,4,2,0,5,0,0,0,
 59 32,3,204,1,60,0,0,0,0,0,
 60 4,5,0,3,0,2,0,0,0,0,0,
 61 32,3,204,1,60,0,0,0,0,0,
 62 3,5,0,3,2,0,0,0,0,0,
 63 32,3,204,1,60,0,0,0,0,0,
 64 3,3,0,3,2,0,0,0,0,0,
 65 1,7,0,0,2,0,0,0,0,0,
 66 2,7,0,0,2,0,0,0,0,0,
 67 3,0,0,2,0,0,0,0,0,0,
 68 4,0,0,2,0,0,0,0,0,0,
 69 5,0,0,2,0,0,0,0,0,0,
 70 ,0530,-,0424,-,0108,-,0265,-,0638,-,0687,-,1977,-,0212,-
 71 ,0212,-,0212,-,0212,-,0212,-,0212,-,0212,-,0212,-,0212,-
 72 ,0212,-,0212,-,0477,-,0689,-,0638,-,0265,-,0105,-,0124,-
 73 ,0516,-,0421,-,0105,-,0263,-,0632,-,0684,-,0474,-,0210,-
 ,0210,-,0210,-,0210,-,0210,-,0210,-,0210,-,0210,-,0210,-
 ,0210,-,0210,-,0474,-,0684,-,0632,-,0263,-,0105,-,0421,-
 ,0528,-,0421,-,0105,-,0263,-,0632,-,0684,-,0474,-,0210,-
 ,0210,-,0210,-,0210,-,0210,-,0210,-,0210,-,0210,-,0210,-
 ,0210,-,0210,-,0474,-,0684,-,0632,-,0263,-,0105,-,0421,-
 ,0473,-,0383,-,0076,-,0259,-,0574,-,0622,-,0430,-,0191,-
 ,0171,-,0191,-,0171,-,0191,-,0171,-,0171,-,0171,-,0171,-
 31 ,0171,-,0191,-,0430,-,0171,-,0171,-,0171,-,0171,-,0171,-
 32 ,0478,-,0533,-,0076,-,0207,-,0207,-,0321,-,0430,-,0171,-
 33 ,0171,-,0171,-,0191,-,0191,-,0171,-,0171,-,0171,-,0171,-
 34 ,0171,-,0171,-,0430,-,0621,-,0574,-,0239,-,0026,-,0383,-
 35 ,0414,-,0531,-,0083,-,0207,-,0477,-,0538,-,0372,-,0165,-
 36 ,0165,-,0165,-,0165,-,0165,-,0165,-,0165,-,0165,-,0165,-
 37 ,0165,-,0165,-,0372,-,0538,-,0477,-,0207,-,0933,-,0231,-
 38 ,0144,-,0531,-,0083,-,0207,-,0477,-,0538,-,0372,-,0165,-
 39 ,0165,-,0165,-,0144,-,0165,-,0165,-,0165,-,0165,-,0165,-
 40 ,0165,-,0165,-,0372,-,0538,-,0477,-,0207,-,0933,-,0331,-
 41 ,0324,-,0243,-,0081,-,0152,-,0265,-,0395,-,0273,-,0122,-
 42 ,0122,-,0122,-,0122,-,0122,-,0122,-,0122,-,0122,-,0122,-
 43 ,0122,-,0122,-,0273,-,0395,-,0365,-,0152,-,0061,-,0243,-
 44 ,0304,-,0243,-,0081,-,0152,-,0365,-,0395,-,0273,-,0122,-
 45 ,0122,-,0122,-,0122,-,0122,-,0122,-,0122,-,0122,-,0122,-
 46 ,0122,-,0122,-,0273,-,0395,-,0365,-,0152,-,0061,-,0243,-
 47 ,0256,-,0065,-,0051,-,0127,-,0307,-,0332,-,023,-,0102,-
 48 ,0102,-,0102,-,0102,-,0102,-,0102,-,0102,-,0102,-,0102,-
 49 ,0102,-,0102,-,0230,-,0332,-,0307,-,0127,-,0051,-,0202,-

NATURAL DRAFT HYPERBOLOID COOLING TOWER: BEAD LOAD

2,41,1,1,25,0,
 3,4,376000,0,,15,,15,0,0,
 4
 5
 2,10,0,34,74,
 6
 3,330,0,138,81,
 7
 4,335,0,145,0,0,1,1,1,
 8
 1,13,1,2,1,2,0,0,5,
 9
 32,3,204,1,60,0,0,0,
 10
 2,5,2,3,1,0,5,0,5,
 11
 32,3,204,1,60,0,0,0,
 12
 3,3,3,4,1,0,5,2,5,
 13
 32,3,104,1,60,0,0,0,
 14
 0,0,0,0,
 15 FINCHED CYLINDER: PLATE CIRCUMFERENTIAL LOAD
 16
 2,41,1,0,25,0,
 17
 2,3,4320000,0,3,,15,
 18
 1,90,0,4,0,1,1,1,1,
 19
 1,12,0,4,0,0,0,0,0,
 20
 3,20,0,4,1,1,1,1,
 21
 3,3,1,2,0,0,0,1033,
 22
 0,0,
 23
 1,1,2,3,1,0,1033,
 24
 0,0,
 25
 0,1,0,0,0,0,0,
 26
 1,2,
 27
 3,0,1,0,0,0,0,0,0,
 28 FINCHED CYLINDER: CONCENTRATED LOADS
 29
 2,41,1,0,25,0,
 30
 2,3,4320000,0,3,,15,
 31
 1,70,0,4,0,1,1,1,1,
 32
 1,12,0,3,0,0,0,0,0,
 33
 3,10,0,4,1,1,1,1,
 34
 1,1,1,2,1,0,1033,
 35
 0,0,
 36
 1,1,2,3,1,0,1033,
 37
 0,0,
 38
 0,1,10,2,38,2,
 39
 0,0,
 40
 0,1,7,0,0,
 41
 1,0,0,0,0,0,0,0,0,0,0,0,0,
 42
 0,0,0,0,0,0,0,0,0,0,0,0,0,
 43
 0,0,0,0,1,0,0,0,0,0,0,0,0,
 44
 0,0,0,0,0,0,0,0,0,0,0,0,0

WELDED CYLINDER LINE CIRCUMFERENTIAL LOAD

*** ANALYSIS DATA ***

H1101

41.6

0

= 25.000

*** STRUCTURAL DATA ***

NUMBER OF ELEMENTS = 2
 NUMBER OF NODES = 3
 LENGTH OF ELEMENT 1 = 0.4370E+07
 LENGTH OF ELEMENT 2 = 0.3000E+00
 TOTAL LENGTH = 0.1500E+00
 ELEMENT COEFFICIENT = 0.0

*** ELEMENT DATA ***

SEGMENT	COORD	R	W	Y	U
1	0.0	4.0000	1	1	1
2	15.000	4.0000	0	0	0
3	25.000	4.0000	1	1	1

*** ELEMENT DATA ***

ELEMENT NO	TH1	TH2	EC1	EC2	HE1	HE2	HE3	HE4	HE5	HE6	HE7	HE8	HE9	HE10	HE11	HE12	HE13	HE14	HE15
1	0.1033	0.1033	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
2	0.1033	0.1033	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

*** TABLING SPECIFICATIONS ***

NUMBER OF ELEMENTS = 0
 NUMBER OF LOADS = 0
 NUMBER OF FREQUENCIES TO FIT = 1
 NUMBER OF THE LOAD = 0
 NUMBER OF CONSTRAINT POINTS = 0
 NUMBER OF CIRCUMFERENTIAL INTEGRALS = 1

FOR SPECIAL LOADS ***

ONE D. NUMBER

NS SS TS

0.0 -0.1000E+01 0.0

*** CONNECTIVITY ARRAY ***

ELEMENT	SEGMENT	TOP NODE	BOT NODE
1	1	1	2
2	1	2	3
3	1	3	4

*** COMMUNICATIVE SYSTEM ***
ALL ELEMENTS OF SYSTEM

	X	R	TH	S	PHI	NS	SS	TS
1	0.0	4.000	-0.103	1.571	0.0	15.000	0.0	0.0
2	10.000	4.000	0.103	1.571	0.0	15.000	0.0	0.0
3	10.000	4.000	0.103	1.571	0.0	10.366	4.000	0.0
	20.000	4.000	0.103	1.571	0.0	20.000	4.000	0.0

NUMBER OF EQUATIONS = 12
NUMBER OF LOADS = 15

TOTAL LINES ASSEMBLED IN RHS LT= 1

0.0	0.0	0.0	0.0
0.0	-0.1000E+01	0.0	0.0
0.0	0.0	0.0	0.0

*** DISPLACEMENT AT NODES

0.0	0.0	0.0	0.0	
0.0	-0.14532E-11	-0.357279E-04	-0.0	-0.0
0.0	0.0	0.0	0.0	0.0

SEGMENT

```

SOLVER RESULTS FOR NISPL
-----  

    1.0E-03 0.0E+00 0.0E+00 0.0E+00 0.0E+00  

    0.0E+00 1.0E-03 0.0E+00 0.0E+00 0.0E+00  

    0.0E+00 0.0E+00 1.0E-03 0.0E+00 0.0E+00  

    0.0E+00 0.0E+00 0.0E+00 1.0E-03 0.0E+00  

    0.0E+00 0.0E+00 0.0E+00 0.0E+00 1.0E-03
    AT EIGENVALUES
    0.2455767E-02 -0.602625E-01 0.0
    0.124884E+00 0.500000E+00 -0.602625E-01 0.0
    0.124884E+00 0.500000E+00 -0.602625E-01 0.0
    0.575553E-03 -0.62567E-02 -0.602625E-01 0.0
    -0.0
    TOTAL CONDITIONS
    0.0

```

- 6 -

	DISPLACEMENT	X	Y	Z	
132	-0.0	-0.0	-0.0	-0.0	
133	-0.1831E-06	-0.1831E-07	-0.3092E-07	-0.0	
134	-0.2905E-06	-0.8049E-07	-0.6237E-07	0.0	
135	-0.1473E-06	-0.1473E-06	-0.9525E-07	0.0	
136	-0.7627E-07	-0.1523E-06	-0.1256E-06	0.0	
137	-0.2853E-07	-0.1651E-06	-0.1623E-06	0.0	
138	-0.1902E-03	0.1691E-03	-0.1926E-06	0.0	
139	-0.7569E-03	0.1482E-06	-0.2301E-06	0.0	
140	-0.9093E-06	0.1660E-06	-0.2640E-06	0.0	
141	-0.6731E-03	0.8164E-06	-0.2975E-06	0.0	
142	-0.3531E-03	0.1623E-06	-0.3316E-06	0.0	
143	-0.1627E-08	0.1520E-06	-0.3653E-06	0.0	
144	-0.13	-0.1617E-06	-0.3921E-06	0.0	
145	-0.14	-0.1614E-09	-0.1612E-06	-0.3229E-06	0.0
146	-0.15	-0.2457E-09	-0.1618E-06	-0.4366E-06	0.0
147	-0.16	-0.3243E-09	-0.1618E-06	-0.5004E-06	0.0
148	-0.17	-0.6310E-09	-0.1619E-06	-0.5341E-06	0.0
149	-0.18	-0.1341E-08	-0.1620E-06	-0.5679E-06	0.0
150	-0.15	-0.2454E-08	-0.1621E-06	-0.5017E-06	0.0
151	-0.16	-0.3534E-08	-0.1635E-06	-0.5554E-06	0.0
152	-0.17	-0.3748E-08	-0.1644E-06	-0.6592E-06	0.0
153	-0.18	-0.4117E-08	-0.1651E-06	-0.7000E-06	0.0
154	-0.19	-0.4672E-07	-0.1659E-06	-0.7369E-06	0.0
155	-0.20	-0.5293E-07	-0.1610E-06	-0.7707E-06	0.0
156	-0.21	-0.4792E-07	-0.1522E-06	-0.5043E-06	0.0
157	-0.22	-0.7804E-07	-0.1367E-06	-0.3788E-06	0.0
158	-0.23	-0.6855E-07	-0.1454E-06	-0.8709E-06	0.0
159	-0.24	-0.5233E-07	-0.9608E-07	-0.9036E-06	0.0
160	-0.25	-0.9140E-07	-0.9521E-07	-0.9340E-06	0.0
161	-0.26	-0.4673E-06	-0.1537E-06	-0.9637E-06	0.0
162	-0.27	-0.6232E-06	-0.1729E-06	-0.1044E-05	0.0
163	-0.28	-0.1573E-05	-0.6356E-06	-0.1043E-05	0.0
164	-0.29	-0.2027E-05	-0.1092E-05	-0.1090E-05	0.0
165	-0.30	-0.1535E-05	-0.1573E-05	-0.1143E-05	0.0
166	-0.31	-0.9246E-06	-0.1537E-05	-0.1202E-05	0.0
167	-0.32	-0.6725E-05	0.8076E-06	-0.1263E-05	0.0
168	-0.33	-0.1763E-04	-0.2175E-05	-0.1291E-05	0.0
169	-0.34	-0.3200E-04	-0.8349E-05	-0.1235E-05	0.0
170	-0.35	-0.2456E-04	-0.1807E-04	-0.1021E-05	0.0
171	-0.36	-0.4950E-04	-0.2939E-04	-0.5055E-06	0.0
172	-0.37	-0.11391E-11	-0.3573E-04	-0.2855E-11	0.0

PRIMARY STRESS RESULTANTS

TS

NS

C.

175	1.0	0.5726E+03	-0.2253E-02	-0.6027E-01	0.0			
176	1.0	0.1473E+03	-0.1264E-02	-0.6027E-01	0.0			
177	1.0	-0.5058E+04	-0.1983E-03	-0.6027E-01	0.0			
178	1.0	-0.1032E+03	-0.2544E-04	-0.6027E-01	0.0			
179	1.0	-0.8552E+04	-0.1324E-03	-0.6027E-01	0.0			
180	1.0	-0.4792E+04	-0.1484E-03	-0.6027E-01	0.0			
181	1.0	-0.1470E+04	-0.1080E-03	-0.6027E-01	0.0			
182	1.0	0.3421E-05	0.5715E-04	-0.6027E-01	0.0			
183	1.0	0.1633E+04	0.2355E-04	-0.6027E-01	0.0			
184	1.0	0.1577E+04	0.3590E-05	-0.6027E-01	0.0			
185	1.0	0.1929E+04	0.3344E-05	-0.6027E-01	0.0			
186	1.0	0.1766E+04	-0.3745E-05	-0.6027E-01	0.0			
187	1.0	0.1615E+04	-0.5053E-05	-0.6027E-01	0.0			
188	1.0	0.1515E+04	-0.2555E-05	-0.6027E-01	0.0			
189	1.0	0.1628E+04	-0.2266E-06	-0.6027E-01	0.0			
190	1.0	0.1499E+04	0.1655E-05	-0.6027E-01	0.0			
191	1.0	0.1575E+04	0.2914E-05	-0.6027E-01	0.0			
192	1.0	0.1635E+04	0.2991E-05	-0.6027E-01	0.0			
193	1.0	0.1657E+04	0.6562E-05	-0.6027E-01	0.0			
194	1.0	0.1631E+04	0.5768E-05	-0.6027E-01	0.0			
195	1.0	0.1335E+04	0.1876E-05	-0.6027E-01	0.0			
196	1.0	0.1583E+05	0.3772E-04	-0.6027E-01	0.0			
197	1.0	0.1665E+05	0.5791E-04	-0.6027E-01	0.0			
198	1.0	0.2119E+04	0.6133E-04	-0.6027E-01	0.0			
199	1.0	0.1453E+04	0.3610E-04	-0.6027E-01	0.0			
200	1.0	0.1293E+04	0.8602E-04	-0.6027E-01	0.0			
201	1.0	0.1119E+02	-0.1083E-02	-0.6027E-01	0.0			
202	1.0	0.1531E+03	0.7370E-03	-0.6027E-01	0.0			
203	1.0	0.1279E+03	0.1209E-02	-0.6027E-01	0.0			
204	1.0	0.7432E+03	0.1492E-02	-0.6027E-01	0.0			
205	1.0	0.1061E+02	0.1038E-02	-0.6027E-01	0.0			
206	1.0	0.1119E+02	-0.1083E-02	-0.6027E-01	0.0			
207	1.0	0.3033E+03	0.5910E-02	-0.6027E-01	0.0			
208	1.0	0.2134E+02	0.1414E-04	-0.6027E-01	0.0			
209	1.0	-0.8975E-02	-0.2460E-01	-0.6027E-01	0.0			
210	1.0	-0.1430E+01	-0.3325E-01	-0.6027E-01	0.0			
211	1.0	-0.2256E+01	-0.2324E-01	-0.6027E-01	0.0			
212	1.0	-0.2611E+01	0.7936E-02	-0.6027E-01	0.0			
213	1.0	-0.1441E+01	0.9891E-01	-0.6027E-01	0.0			
214	1.0	0.2990E+01	0.2661E+00	-0.6027E-01	0.0			
215	1.0	0.5000E+00	0.5000E+00	-0.6027E-01	0.0			
216	1.0	0.1249E+00	-0.6027E-01					
217	1.0	-0.1249E+00						

FILE

SECULARARY STRESS RESULTSANTS

AT.	HTS	HGT	NT	NTS	NET
1	0.1676E-03	0.0	0.0	-0.1804E-01	0.0
2	0.4320E-04	0.0	0.0	-0.1477E-01	0.0
3	0.1073E-04	0.0	0.0	-0.9037E-02	0.0
4	-0.3473E-04	0.0	0.0	-0.4149E-02	0.0
5	0.1533E-04	0.0	0.0	-0.1084E-02	0.0
6	0.1501E-04	0.0	0.0	0.3640E-03	0.0
7	0.4570E-05	0.0	0.0	0.7255E-03	0.0
8	0.4994E-05	0.0	0.0	0.6832E-03	0.0
9	0.5152E-05	0.0	0.0	0.4372E-03	0.0
10	0.6104E-05	0.0	0.0	0.2125E-03	0.0
11	0.5714E-05	0.0	0.0	0.6457E-04	0.0
12	0.4865E-05	0.0	0.0	0.1043E-04	0.0
13	0.4865E-05	0.0	0.0	0.3876E-04	0.0
14	0.5434E-05	0.0	0.0	0.3914E-04	0.0
15	0.4751E-05	0.0	0.0	0.3344E-04	0.0
16	0.4751E-05	0.0	0.0	0.2063E-04	0.0
17	0.4862E-05	0.0	0.0	0.1297E-04	0.0
18	0.5954E-05	0.0	0.0	0.1364E-04	0.0
19	0.5058E-05	0.0	0.0	0.6594E-04	0.0
20	0.4972E-05	0.0	0.0	0.1511E-04	0.0
21	0.4931E-05	0.0	0.0	0.2550E-03	0.0
22	0.1747E-05	0.0	0.0	0.3336E-05	0.0
23	0.1879E-05	0.0	0.0	0.2737E-03	0.0
24	0.6570E-05	0.0	0.0	0.1224E-03	0.0
25	0.1651E-04	0.0	0.0	0.4109E-02	0.0
26	0.3191E-05	0.0	0.0	0.2635E-02	0.0
27	0.3191E-05	0.0	0.0	0.5209E-02	0.0
28	0.1134E-04	0.0	0.0	0.7355E-02	0.0
29	0.3132E-03	0.0	0.0	0.7108E-02	0.0
30	0.3132E-03	0.0	0.0	0.5682E-03	0.0
31	0.3132E-03	0.0	0.0	0.1765E-03	0.0
32	0.3132E-03	0.0	0.0	0.5555E-01	0.0
33	0.1134E-03	0.0	0.0	0.1032E+00	0.0
34	0.6546E-03	0.0	0.0	0.1573E+00	0.0
35	0.6546E-03	0.0	0.0	0.1725E+00	0.0
36	0.6546E-03	0.0	0.0	0.2145E-01	0.0
37	0.6546E-03	0.0	0.0	0.2619E+00	0.0
38	0.6793E-02	0.0	0.0	0.9503E+00	0.0
39	0.5451E-02	0.0	0.0	0.1934E+01	0.0
40	0.5631E-02	0.0	0.0	0.3294E+01	0.0
41	0.6392E-01	0.0	0.0	0.3999E+01	0.0
42	0.6392E-01	0.0	0.0	0.3999E+01	0.0

***** SEGMENT 1 *****
***** SEGMENT 2 *****

DISPLACEMENT		U	W
3.01	1	0.1425E-11	-0.35735E-94
3.02	2	-0.4186E-04	-0.1927E-04
3.03	3	0.4440E-04	-0.1790E-04
3.04	4	-0.3152E-04	-0.3117E-05
3.05	5	0.1705E-04	-0.1991E-05
3.06	6	-0.6489E-05	-0.19008E-06
3.07	7	0.6414E-06	0.1719E-05
3.08	8	-0.1678E-05	0.1571E-05
3.09	9	0.1976E-05	0.1143E-05
3.10	10	-0.1490E-05	0.5682E-06
3.11	11	0.8424E-06	0.2395E-06
3.12	12	-0.3454E-06	0.1127E-06
3.13	13	0.5763E-07	0.9558E-07
3.14	14	-0.3462E-07	0.9833E-07
3.15	15	0.8361E-07	0.1561E-06
3.16	16	-0.3992E-07	0.1107E-06
3.17	17	0.4130E-07	0.1540E-06
3.18	18	-0.1807E-07	0.1622E-06
3.19	19	0.4024E-08	0.1448E-06
3.20	20	-0.2311E-08	0.1249E-06
3.21	21	0.3688E-09	0.1841E-06
3.22	22	-0.3252E-09	0.1631E-06
3.23	23	0.2040E-09	0.1625E-06
3.24	24	-0.1035E-08	0.1621E-06
3.25	25	0.4668E-09	0.1619E-06
3.26	26	-0.2690E-09	0.1618E-06
3.27	27	0.2358E-09	0.1617E-06
3.28	28	-0.2538E-10	0.1617E-06
3.29	29	0.5126E-09	0.1617E-06
3.30	30	-0.1888E-08	0.1620E-06
3.31	31	0.4204E-08	0.1628E-06
3.32	32	-0.7103E-08	0.1642E-06
3.33	33	0.9148E-09	0.1663E-06
3.34	34	-0.7144E-08	0.1684E-06
3.35	35	0.6419E-09	0.1690E-06
3.36	36	-0.3150E-07	0.1650E-06
3.37	37	0.5796E-09	0.1515E-06
3.38	38	-0.1443E-06	0.1236E-06
3.39	39	0.1612E-06	0.1974E-07
3.40	40	-0.1891E-06	0.2860E-07
3.41	41	0.1285E-11	-0.2959E-12
3.42			-0.1043E-12

END OF FILE

344 PRIMARY STRESS RESULTANTS
345 NS SS TS

347	1	0.1249E+00	-0.5000E+00	-0.6027E-01	0.0
348	2	-0.2981E+01	-0.2651E+00	-0.6027E-01	0.0
349	3	-0.1347E-01	-0.9719E-01	-0.6027E-01	0.0
350	4	-0.2669E-01	-0.5655E-02	-0.6027E-01	0.0
351	5	-0.2347E-01	0.2935E-01	-0.6027E-01	0.0
352	6	-0.1386E-01	0.7281E-01	-0.6027E-01	0.0
353	7	-0.5512E-02	0.2397E-01	-0.6027E-01	0.0
354	8	-0.1324E-02	0.1325E-01	-0.6027E-01	0.0
355	9	-0.4505E-03	0.5233E-02	-0.6027E-01	0.0
356	10	0.1139E-03	0.7653E-03	-0.6027E-01	0.0
357	11	0.1037E-02	-0.1155E-02	-0.6027E-01	0.0
358	12	0.6835E-03	0.1425E-02	-0.6027E-01	0.0
359	13	0.3464E-03	-0.1125E-02	-0.6027E-01	0.0
360	14	0.1209E-03	-0.6511E-03	-0.6027E-01	0.0
361	15	0.6121E-05	-0.2760E-03	-0.6027E-01	0.0
362	16	-0.3252E-04	-0.5405E-04	-0.6027E-01	0.0
363	17	-0.3213E-04	0.4325E-04	-0.6027E-01	0.0
364	18	-0.1730E-04	0.6564E-04	-0.6027E-01	0.0
365	19	-0.1930E-05	0.5258E-04	-0.6027E-01	0.0
366	20	0.8810E-05	0.3175E-04	-0.6027E-01	0.0
367	21	0.1454E-04	0.1624E-04	-0.6027E-01	0.0
368	22	0.1665E-04	0.3362E-05	-0.6027E-01	0.0
369	23	0.1672E-04	-0.1425E-05	-0.6027E-01	0.0
370	24	0.1603E-04	-0.3150E-05	-0.6027E-01	0.0
371	25	0.1533E-04	-0.2551E-05	-0.6027E-01	0.0
372	26	0.1434E-04	-0.1190E-05	-0.6027E-01	0.0
373	27	0.1479E-04	0.7758E-06	-0.6027E-01	0.0
374	28	0.1527E-04	0.3155E-05	-0.6027E-01	0.0
375	29	0.1635E-04	0.5302E-05	-0.6027E-01	0.0
376	30	0.1734E-04	0.6859E-05	-0.6027E-01	0.0
377	31	0.1950E-04	0.4155E-05	-0.6027E-01	0.0
378	32	0.1969E-04	-0.4802E-05	-0.6027E-01	0.0
379	33	0.1605E-04	-0.2165E-04	-0.6027E-01	0.0
380	34	0.5023E-05	-0.4393E-04	-0.6027E-01	0.0
381	35	0.1682E-04	-0.1120E-03	-0.6027E-01	0.0
382	36	0.5022E-04	0.1506E-03	-0.6027E-01	0.0
383	37	-0.8719E-04	-0.1228E-03	-0.6027E-01	0.0
384	38	-0.1031E-05	0.5327E-04	-0.6027E-01	0.0
385	39	-0.4941E-04	0.4426E-03	-0.6027E-01	0.0
386	40	0.1477E-03	0.1201E-02	-0.6027E-01	0.0
387	41	0.5783E-03	0.2258E-02	-0.6027E-01	0.0

END OF FILE

359

		SECONDARY STRESS, RESULTANTS	HTS	HTS	HST	NT	NTS	NST
390	391							
392	393							
394	1	0.3658E-01	0.0	0.0	-0.3995E+01	0.0	0.0	0.0
395	2	0.8612E-02	0.0	0.0	-0.3222E+01	0.0	0.0	0.0
396	3	-0.5173E-02	0.0	0.0	-0.2001E+01	0.0	0.0	0.0
397	4	-0.8671E-02	0.0	0.0	-0.9186E+00	0.0	0.0	0.0
398	5	-0.7249E-02	0.0	0.0	-0.2401E+00	0.0	0.0	0.0
399	6	-0.4475E-02	0.0	0.0	-0.8056E-01	0.0	0.0	0.0
400	7	-0.2075E-02	0.0	0.0	-0.1716E+00	0.0	0.0	0.0
401	8	-0.5571E-03	0.0	0.0	-0.1513E+00	0.0	0.0	0.0
402	9	0.1679E-03	0.0	0.0	0.9679E-01	0.0	0.0	0.0
403	10	0.3798E-03	0.0	0.0	0.4680E-01	0.0	0.0	0.0
404	11	0.3397E-03	0.0	0.0	0.1417E-01	0.0	0.0	0.0
405	12	0.2208E-03	0.0	0.0	-0.2080E-02	0.0	0.0	0.0
406	13	0.1103E-03	0.0	0.0	-0.7235E-02	0.0	0.0	0.0
407	14	0.3726E-04	0.0	0.0	-0.6940E-02	0.0	0.0	0.0
408	15	0.5599E-06	0.0	0.0	-0.4648E-02	0.0	0.0	0.0
409	16	-0.1115E-04	0.0	0.0	-0.2366E-02	0.0	0.0	0.0
410	17	-0.1093E-04	0.0	0.0	-0.8022E-03	0.0	0.0	0.0
411	18	-0.5957E-05	0.0	0.0	-0.1469E-04	0.0	0.0	0.0
412	19	-0.9122E-06	0.0	0.0	-0.3044E-03	0.0	0.0	0.0
413	20	0.2575E-05	0.0	0.0	0.3722E-03	0.0	0.0	0.0
414	21	0.4416E-05	0.0	0.0	0.2228E-03	0.0	0.0	0.0
415	22	0.5062E-05	0.0	0.0	0.1198E-03	0.0	0.0	0.0
416	23	0.5038E-05	0.0	0.0	0.4519E-04	0.0	0.0	0.0
417	24	0.4862E-05	0.0	0.0	0.2779E-05	0.0	0.0	0.0
418	25	0.4615E-05	0.0	0.0	-0.1738E-04	0.0	0.0	0.0
419	26	0.4455E-05	0.0	0.0	-0.2707E-04	0.0	0.0	0.0
420	27	0.4433E-05	0.0	0.0	-0.3402E-04	0.0	0.0	0.0
421	28	0.4544E-05	0.0	0.0	0.3907E-04	0.0	0.0	0.0
422	29	0.4600E-05	0.0	0.0	-0.3445E-04	0.0	0.0	0.0
423	30	0.5381E-05	0.0	0.0	-0.2917E-05	0.0	0.0	0.0
424	31	0.5883E-05	0.0	0.0	0.8019E-04	0.0	0.0	0.0
425	32	0.5832E-05	0.0	0.0	0.2373E-03	0.0	0.0	0.0
426	33	0.4932E-05	0.0	0.0	0.4691E-03	0.0	0.0	0.0
427	34	0.1577E-05	0.0	0.0	0.7105E-03	0.0	0.0	0.0
428	35	-0.11993E-05	0.0	0.0	0.2792E-03	0.0	0.0	0.0
429	36	-0.1506E-04	0.0	0.0	0.3227E-03	0.0	0.0	0.0
430	37	-0.2139E-04	0.0	0.0	-0.1105E-02	0.0	0.0	0.0
431	38	-0.3445E-04	0.0	0.0	-0.4291E-02	0.0	0.0	0.0
432	39	-0.1577E-04	0.0	0.0	-0.9185E-02	0.0	0.0	0.0
433	40	0.4342E-04	0.0	0.0	-0.1488E-01	0.0	0.0	0.0
434	41	0.1719E-03	0.0	0.0	-0.1803E-01	0.0	0.0	0.0