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Experiments on the flow behaviour of granular materials at high velocity in an open channel

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A series of laboratory flume experiments has been conducted to investigate the flow behaviour of dry sand at high velocities. The first incentive for this experimental programme was an attempt to explain the high mobility of apparently dry masses of rapidly moving broken rock during rock avalanches (stürzstroms). Test velocities of over 5 m/s and most other parameters of the tests were found to be dimensionally similar to those for a typical rock avalanche. Yet, the conclusion was negative: even the most rapid of the experiments indicated flow behaviour which is in conformity with the constant volume uniform Coulomb friction relationship as applicable to slow shearing. This conclusion was confirmed independently both by direct measurements of base friction and mean density of the flows and by detailed observations of flow uniformity, acceleration and velocity profiles. The surprising mobility of stürzstroms therefore requires a different explanation.

Une série de tests à canaux a été entreprise dans le laboratoire pour étudier l'écoulement du sable sec à de hautes vélocités. Le programme expérimental devait principalement expliquer la haute mobilité des masses apparemment sèches des rochers brisés en mouvement rapide pendant les avalanches de rochers (Sturzströme). Des vélocités expérimentales de plus de 5 m/s et la plupart des autres paramètres des tests étaient pareils en dimensions à une avalanche de rochers typique. La conclusion a été pourtant négative: même les expériences les plus rapides ont indiqué un écoulement qui est en conformité avec le rapport de frottement Coulomb uniforme à volume constant comme appliqué au cisaillement lent. Cette conclusion a été confirmée independamment par des mesures du frottement profond et de la densité moyenne des écoulements autant que par des observations detaillées de l'uniformité de l'écoulement, de l'accélération et des profils de vélocité. La mobilité surprenante des avalanches de rochers (Sturzströme) exige par conséquent une explication différente.

INTRODUCTION

Gravity flow of granular materials is an important geological process, playing a role in the dynamics of several types of slope movement. These include small scale sand and talus flows, subaqueous quick sand flows, debris flows, and avalanches and rock avalanches.‡ Besides being of geological significance, the process is also of direct practical interest in the industrial field of bulk material handling. Nevertheless, it is still poorly understood even in its simplest form, the flow of dry cohesionless grains. It is felt that little progress can be made in the understanding of the more complex two-phase flows (grains in clay slurry or water), until the behaviour of the granular phase alone can be explained satisfactorily.

Of particular interest for this study is the behaviour of granular materials during rapid shearing, such as occurs in major rock avalanches, when an unusual and so far unexplained increase in mobility can be observed.

This Paper describes laboratory experiments conducted at the University of Alberta (Hungr, 1981) with the purpose of studying the flow behaviour of granular materials at high velocities in an open flume. The research programme also included rapid ring shear experiments conducted at high normal stresses, which are reported in another Paper (Hungr & Morgenstern, 1984).

FRICTION FLOW OF GRANULAR MATERIAL

Slow flow of dry granular material with strong grains is well described by the Coulomb equation

$$\tau = \sigma \tan \phi_{cv} \qquad (1)$$

which relates the shear stress, τ, to normal stress, σ, by means of a material constant, the angle of friction at critical density, ϕ_{cv} (Casagrande, 1936).

This relationship implies several traits of grain flow which contrast with the behaviour of viscous fluids

(*a*) The flow is always non-uniform, except at a bed slope angle equal to ϕ_{cv}.

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‡ The landslide terminology used here is that of Varnes (1978).

(*b*) The velocity profile is indeterminate since, unlike the case for viscous fluids, the shear stress is independent of shear strain rate. The shearing action may therefore either be distributed throughout the depth of the flow, or concentrated at the base or at any intermediate level depending on some other conditions.

(*c*) There is no 'normal' depth, nor an associated normal velocity, since equation (1) is satisfied at any depth and velocity if only the bed slope is equal to the friction angle.

The validity of equation (1) in the sphere of static or quasi-static (slow) flow problems has been well established. It is the basis of soil mechanics design in granular materials, as well as flow analysis in the field of bulk material handling (e.g. Jenike, 1962; Roberts, 1969). Nevertheless, a scrutiny of relevant literature, particularly in the fields of sedimentology and engineering geology, indicates a lack of consensus regarding the universal validity of the Coulomb relationship and its implications during high velocity flow.

OTHER THEORIES OF DRY GRAIN FLOW

Bagnold (1954) extended his experimental results, obtained in the form of a torsional viscometer, to the flow of sand in an inclined flume. The extension involved an arbitrary assumption of a specific and uniform grain concentration, thus leading to the prediction of a normal velocity and a parabolic velocity profile. This was further elaborated by Lowe (1976). Hungr (1981, p. 192) presented an alternative analysis of Bagnold's data which abandons the assumption of uniform concentration, finding a velocity profile which is approximately linear with depth (for high velocities).

The grain flow theory of Goodman & Cowin (1971) incorporates a viscous relationship between shear stress and strain rate, without an experimental or conceptual justification. A revision of this theory due to Savage (1979) uses some of Bagnold's observations and results in a formula which seems to substantiate the notion of a normal depth at a variety of slope angles. Both the original (Goodman & Cowin, 1971) and extended (Savage, 1979) theories predict bulk density decrease with height above the base of an open channel flow, which seems intuitively questionable.

It is felt that a substantial amount of further experimental work is required to justify any one of these mutually conflicting theories.

HIGH VELOCITY GRAIN FLOW: ROCK AVALANCHES

Further exceptions to the general validity of the Coulomb relationship result from the observations of very mobile flows of granular material associated with major rapid rock avalanches. As pointed out by Heim (1932), large, apparently dry, rapidly moving masses of fragmented rock (stürzstroms) have the ability to maintain their velocities on slopes far flatter than any reasonably assumed dynamic friction angle. Furthermore, this observation appears to become more prominent with increasing volume of the rock avalanche, provided that the latter exceeds approximately 1×10^6 m^3 (Scheidegger, 1973).

A number of hypotheses have been advanced to explain this apparent abnormality, including mud lubrication (Heim, 1932), an entrapped air cushion (Shreve, 1968), air fluidization (Kent, 1966), vapour fluidization (Habib, 1975) and rock melting or dissociation (Erismann 1979). All of these are open to criticism, on the basis of circumstantial evidence (Hungr, 1981).

The recent finding of exceptionally mobile stürzstrom deposits on the moon (e.g. Howard, 1973) gave impetus to the development of an explanation which does not require the presence of air or water, i.e. a mechanistic theory. Allusions to a phenomenon which could be termed mechanical fluidization were made by Howard (1973), Scheidegger (1975), Hsü (1975), Körner (1977), McSaveney (1978) and others.

The premise of the mechanical fluidization hypothesis is the assumption that the Coulomb frictional relationship breaks down at very high rates of shearing. Intuitively, this would not seem surprising since rapid shearing involves a radically different type of particle contact from slow, quasi-static movement. Apart from this conceptual observation, however, no theoretical or experimental justification of the hypothesis has so far been presented. All the above authors quote Bagnold (1954), but unfairly, since his experimental results give little indication of other than a frictional character for granular dispersions, except where substantial viscous forces arise in an interstitial fluid (e.g. Bagnold, 1966).

One of the aims of the experimental work reported here was to explore the limits of validity of the frictional flow relationship at very high rates of shearing.

REVIEW OF PREVIOUS EXPERIMENTAL WORK USING OPEN CHANNELS

Experimental study of grain flow is considerably more difficult than that of a fluid. There is a geometrical scale effect determined by the grain size and distribution. The minimum experimental grain size is limited to fine sand, since finer sizes would be affected by electrostatic forces. Density is non-uniform, presenting an additional variable, and cannot be measured by sampling. The material is thixotropic, changing its bulk properties on passing from a state of rest to motion. The finite size of grains and the existence of static friction makes it impossible to use standpipes or submerged velocity measuring devices. Finally, the opacity of the material eliminates some conventional possibilities for optical measurements and generally limits observations to the surfaces of the flow.

As a result of these restrictions, both the number of reported experimental programmes and the scope of measurements within them, are relatively small. The following list is not quite complete, but nearly so.

Takahashi (1937) presented partial results from flume experiments with sand. Roberts (1969) studied flow of seed in a smooth-surfaced flume and obtained results confirming the frictional relationship. Suzuki & Tanaka (1971) derived a Bingham flow relationship from sand flume experiments, although their derivation entails uncertain assumptions. Augenstein & Hogg (1978) derived velocity and density profiles with respect to depth in a sand flow using an approximate method, finding a decrease in density with depth.

Savage (1979) observed uniform flow conditions in a flow of polystyrene beads at bed inclinations 8° to 15° greater than the estimated angle of repose of the material, taking this as a positive indication with respect to his previously mentioned theoretical results. Velocity profiles, measured using a sophisticated technique based on fibre optics, likewise generally confirmed predicted patterns, although they may have been distorted by side friction.

All the experimental programmes known to the Authors used flow thicknesses less than 15 mm and velocities less than 1·5 m/s.

OBJECTIVES OF THE EXPERIMENTAL PROGRAMME

An experimental programme was undertaken to study the behaviour of flowing sand at high velocities and thus to establish the possible limitations of the Coulomb relationship. In an attempt to achieve this as far as possible in an unambiguous manner, the following criteria were set for the experimental method.

(*a*) great velocity range: flow velocities of up to 6 m/s have been achieved

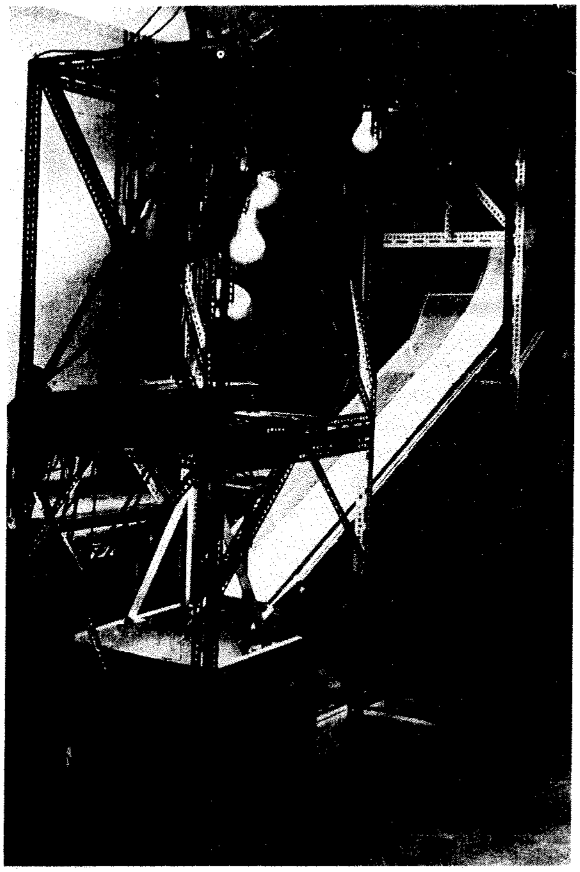

Fig. 1. An overall view of the flume apparatus

(*b*) large flow depth, to eliminate some of the possible scale effects: the typical flow depth (10 cm) was 100 times greater than the mean grain size*

(*c*) rough bed with characteristic roughness similar to the grain size: the bed was covered by a glued down layer of the test material

(*d*) a variety of bed slope angles

(*e*) a variety of test materials including rounded and angular sand, mixtures of sand and rock flour and polystyrene beads

(*f*) a comprehensive system of observations including the direct measurement of base shear force and material density during the flow.

THE TEST AND THE SYSTEM OF MEASUREMENTS

The test apparatus (Figs 1 and 2) consisted of an inclined measuring flume, a sand-supply hopper and chute and a receiving pan.

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* Concern about the low normal stress in these tests led the Authors to conduct a supplementary programme using ring shear tests (Hungr & Morgenstern, 1984).

The hopper was designed to provide a maximum mass flow rate of 200 kg/s, maintainable for 3 s, although most tests were carried out using lesser rates and durations of the order of 10 s. The flows were accelerated by free fall through a vertical chute up to 4 m long, deflected smoothly by a Teflon-lined knee and introduced on to the measuring flume.

The length of the measuring flume, 150 cm, was dictated by space restrictions; the overall height of the apparatus was 7 m. The relatively short length of the flume made it impossible to reach the steady-state flow profile in the faster tests. This was accepted, since the high local shear strain rates resulting from imperfectly developed plug flow would magnify the looked-for reduction in frictional resistance.

The flume was 20 cm wide, with transparent Perspex walls and changeable slope angle. The upper end of the flume was supported by a hinge. The lower end was suspended by a stiff tension load cell to permit continuous weighing of the flowing material.

The entire top surface as well as both sides of the flow were filmed by a high speed camera (400 frames/s). The sides were observed through the Perspex walls by means of two long mirrors mounted at 45° to the vertical (Fig. 2, inset). A random sampling of images from the camera records is shown in Fig. 3. The films were used to measure surface velocities and accelerations by observing the progress of coloured glass beads down the flume in subsequent frames, and to record the flow thicknesses at four positions.

The last 30 cm of the measuring flume were detached from the remainder and hung on suspension mountings allowing free longitudinal displacements. A stiff tension load cell was used to measure the total shear force applied by the sand to this section (both base and walls). The shear stress on the bed was calculated by correcting for the estimated value of the side friction. It was estimated by means of tilt tests of sand on Perspex, and amounted to less than 15% of the total resisting force. The receiving pan was suspended on a load cell, which continuously monitored its increase in weight. The mass flow rate out of the measuring flume in kg/s was obtained by differentiating this record.

Time was recorded separately for the camera and the gauge readings. The camera was equipped with a timer strobe with a 0·01 s period. The load cells were timed by the movement of a six-channel simultaneous chart recorder with a time marker. The two timing devices were syn-

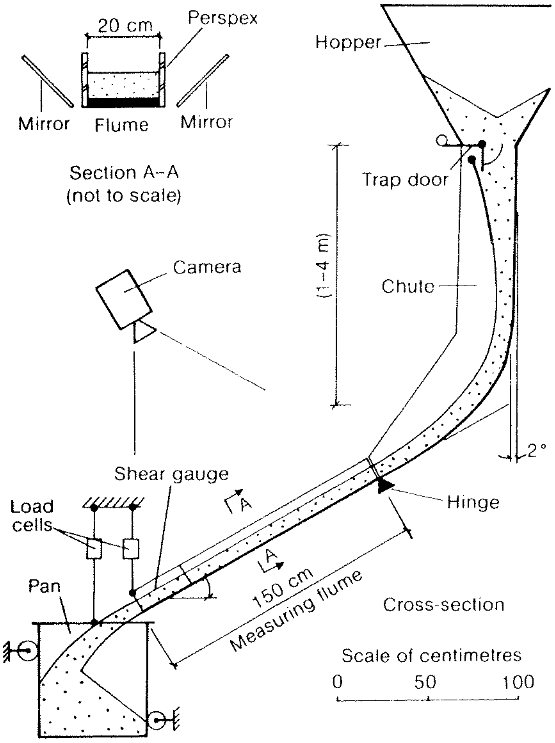

Fig. 2. Schematic diagram of the test flume

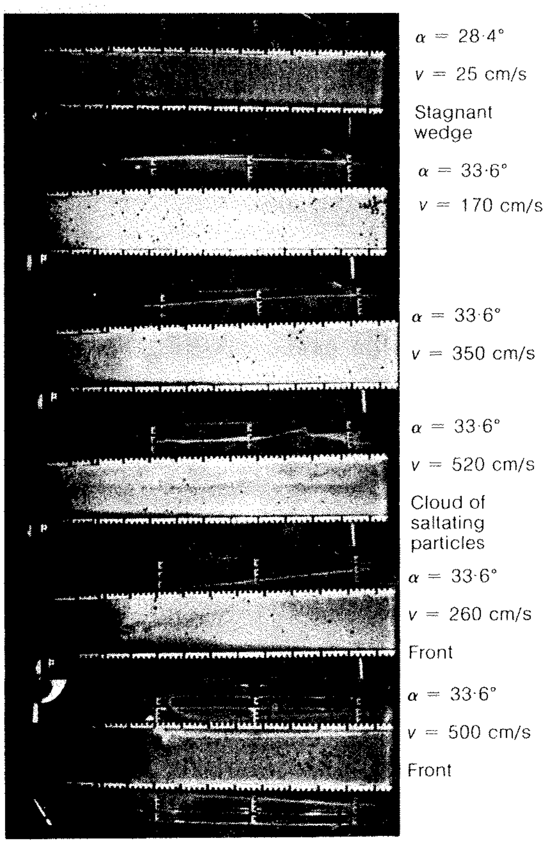

Fig. 3. Examples from the camera records (flow is from left to right)

chronized using a visible switch similar to the clapstick used in sound cinematography.

The complete record thus consisted of a film of the experiment consisting, typically, of 2000 to 3000 frames and a chart record containing the traces of the three load cell channels. These records were combined by calculations to obtain the following output parameters

(*a*) Surface velocity was obtained by differentiation of the time–displacement observations collected from the film.
(*b*) Surface acceleration was obtained by a second differentation of the same.
(*c*) Flow sheet thickness was measured directly from the film.
(*d*) Mean density of the flowing material on the flume was calculated by combining the flow thickness and flume weight records.
(*e*) Base shear stress was measured directly.
(*f*) Base normal stress was estimated by multiplying the flow thickness at the shear table by the mean unit weight. This of course neglected possible high frequency density changes (density waves) in the material and proved to be the source of significant random error in the measurements.
(g) Average velocity was calculated by combining the flow thickness, density and mass flow rate records.

A detailed error analysis of both the primary measurements and their combinations in data processing was conducted, indicating a probable error limit of ±8% for density and 11% for the ratio between shear and normal stress. Most of both surface and mean velocity measurements are considered to be within a 5% error margin.

RESULTS

Nature of the flow

Flow surfaces even in the fastest experiments were relatively smooth and quiet, free from turbulence or mixing. Glass beads were rarely seen to disappear beneath the surface of the flow, to emerge, or to move relatively to the neighbouring particles. The records of all three load cells were very smooth. Vibrations were registered at the opening of the hopper trap door but these always decayed rapidly. Except in the fastest experiments, the flow fronts had the smooth appearance of a straight taper, with a small cloud of saltating particles being pushed ahead (Fig. 3).

Density during flow

Figure 4 is a summary of mean density–velocity observations made in the course of 33 tests on both Ottawa sand and polystyrene beads with uniform diameters of 0·7 and 1·5 mm respectively. The data points are sorted with respect to the range of normal pressure, σ, as indicated in the legend to the figures. In both cases, a gradual decrease in mean density from that corresponding to the loose static piling is indicated. The polystyrene beads result is scattered within the range of error limits mentioned above. The Ottawa sand result shows a scatter which exceeds the estimated error limits approximately twice.

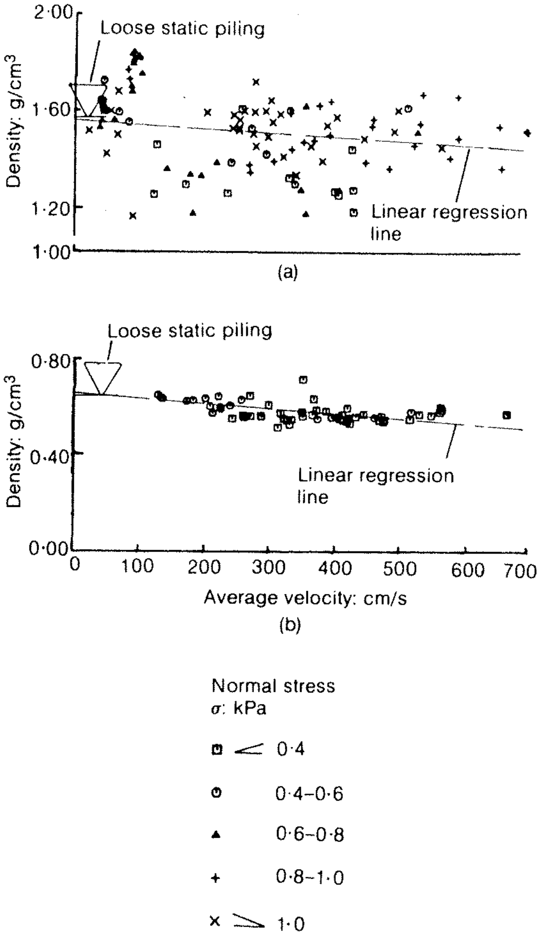

Fig. 4. Mean density–velocity diagrams: (a) Ottawa sand; (b) polystyrene beads

Shear stress/normal stress ratio

The dynamic friction coefficient, τ/σ, at the base of the flow, as measured instantaneously during the flow, is shown in Figs 5(a) and 6(a). This is either an equivalent or a lower bound for the internal friction coefficient at the higher speeds, which exhibited significant base slip. In the case of the slower tests, the boundary and internal frictions were similar, as witnessed by the smooth vertical velocity profiles observed (see below). In the case of both Ottawa sand and polystyrene beads, the dynamic friction coefficient is somewhat less than the pseudo-static equivalent measured as the angle of repose by a rotating drum experiment (Carrigy, 1970) and shows a gradual increase at greater speeds, contrary to the expectations of improving mobility which the Authors had prior to the start of this test programme.

The Ottawa sand results again indicate a scatter substantially exceeding the estimated error margins. The Authors have no explanation for this, except that both the flow density and the dynamic friction coefficient could be to a certain extent random in a sheet of flowing sand, although not in the more regularly shaped plastic beads. The correctness of the error estimate is defensible, among other things, by the satisfactory results of the polystyrene beads experiments. The angular sand and sand–rock flour experiments showed similar scatter as well as lack of trend to those with Ottawa sand.

A parallel derivation of the base dynamic friction coefficient, based on surface acceleration values measured by the camera, is shown in Figs 5(b) and 6(b). The shear stresses were calculated from the measured accelerations using the general equation of motion of unsteady, non-uniform flow (Chow, 1959), without recourse to the shear table measurements. This included a slight correction for side friction. The results are in agreement with the other method in terms of both trend and scatter. This can be conceived of as an independent check.

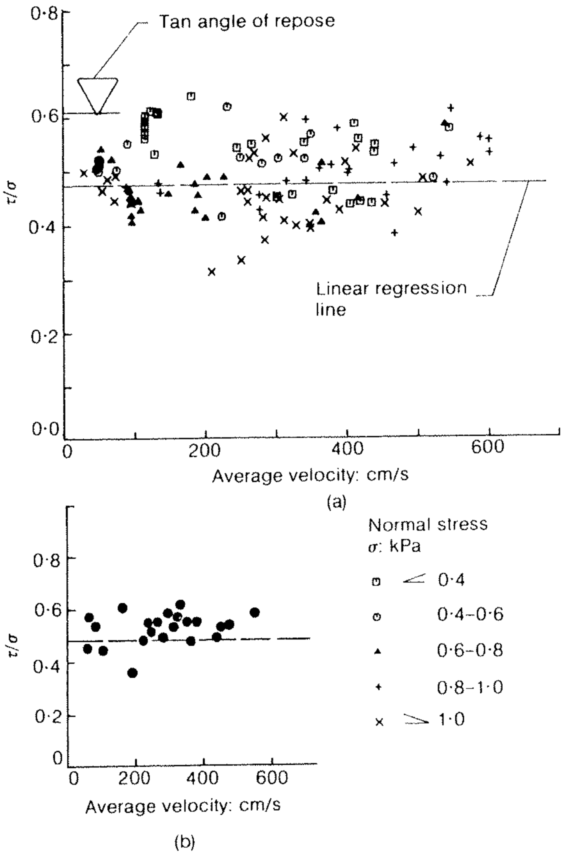

Fig. 5. Bed friction coefficient, Ottawa sand: (a) from direct measurement; (b) from accelerations

Flow uniformity

In order to provide a further indication of the dynamic force equilibrium at the boundaries of the test flows, a study of the flow uniformity was made. Four detailed examples of flow behaviour are indicated in Fig. 7, which summarizes the complete time history of four tests, carried out using Ottawa sand at four different slope inclinations and at approach velocities of the order of 1·5 to 2·5 m/s. These diagrams are derived entirely from the camera records. The left-hand side of each diagram shows the development of surface velocity along the flume length (the abscissa) in time (the ordinate). The velocity is shown in terms of bar diagrams, each of which shows the acceleration of a particular group of glass beads in its progress along the flume by means of the indicated velocity scale. Velocity contours, drawn in the time–distance plane, are also shown. The right-hand sides of the diagrams show the development of flow thickness, again in time and distance.

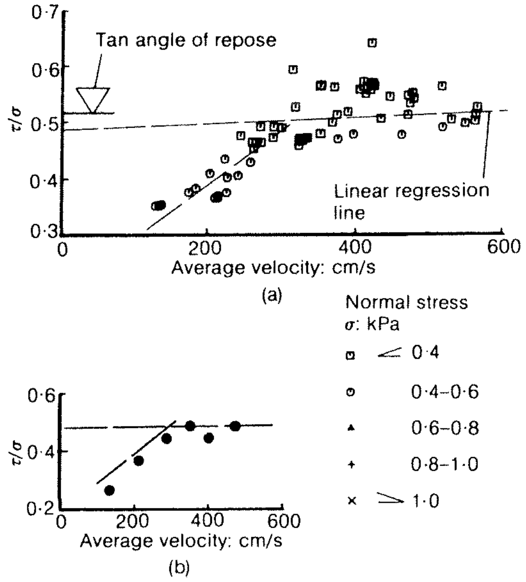

Fig. 6. Bed friction coefficient, polystyrene beads: (a) from direct measurement; (b) from accelerations

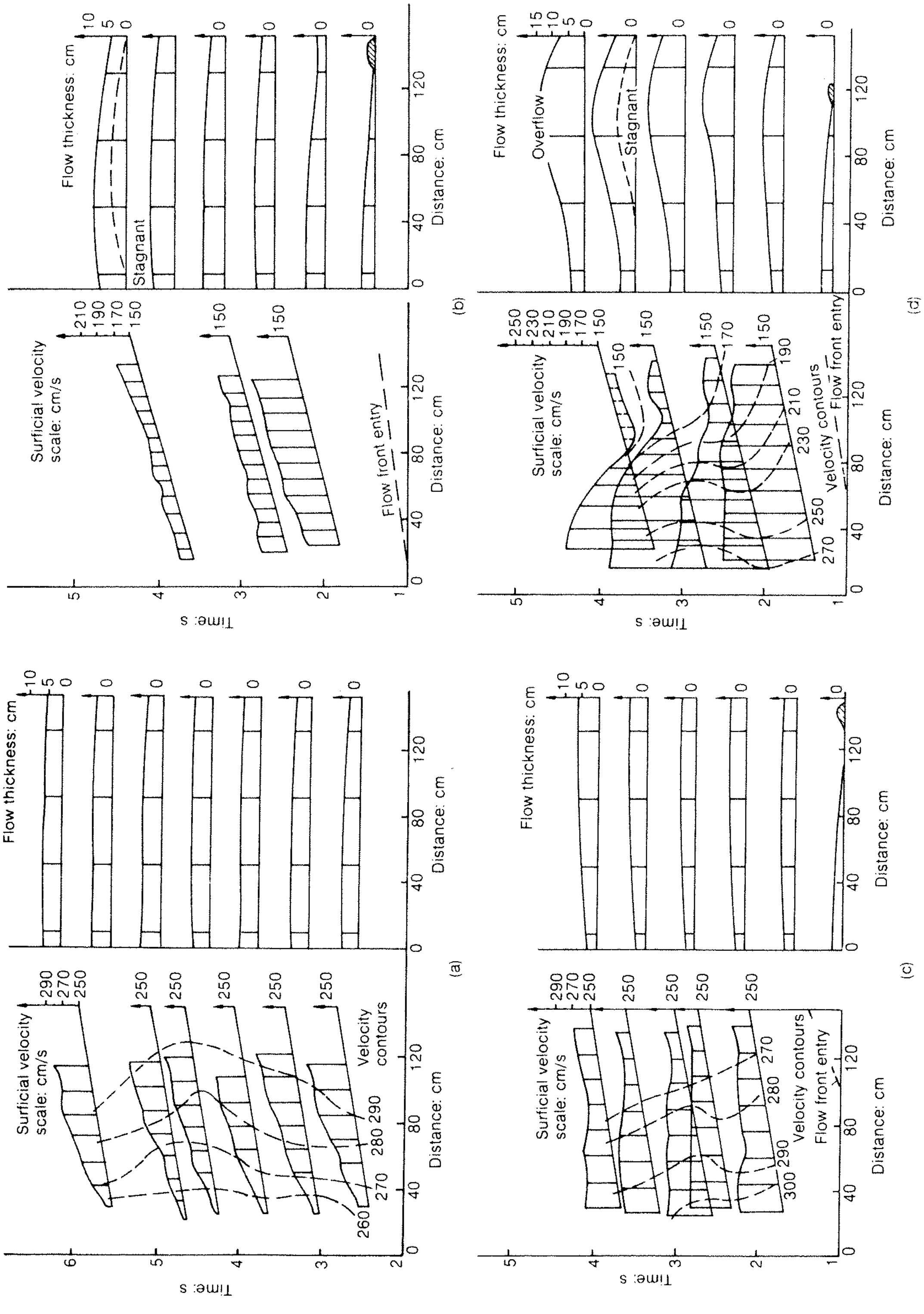

Fig. 7. Velocity and flow thickness against time and position on the flume at various bed slope angles: (a) 34·2°; (b) 29·0°; (c) 25·6°; (d) 20·2°

Test 9 (Fig. 7(a)) is accelerating, and thinning slightly. Test 10 (Fig. 7(b)), at a bed slope of 29°, is approximately uniform, although a stagnant bed is being deposited because of the low approach velocity. The remaining two tests are decelerating.

A comparison of 33 tests indicated that uniform flow conditions similar to those in test 10 could only be achieved at flume inclinations of approximately 28° to 29° in the case of Ottawa sand and 17° to 26° in the case of polystyrene beads, which is consistent with the observations shown in Figs 5 and 6.

An exception was observed in the case of flows thinner than about 30 mm, which tended to decelerate at angles considerably above the equilibrium values. Figs 5(a) and 6(a) indicate a trend towards increasing friction coefficients at the lowest limit of normal stresses. Fig. 4 shows a corresponding abnormal decrease in density. It is considered that very thin flows behave somewhat differently from more massive ones, possibly owing to a high degree of dispersion allowing each particle a large amount of movement freedom similar to the saltation movement mode. Such flows might therefore conceivably achieve the condition of uniform flow at an abnormally high bed slope. The uniform flow conditions observed by Savage (1979) may have occurred in this highly dispersed state, since his flows were less than 15 mm thick.

Velocity profiles

Comparison of surface and average velocity measurements indicated that plug flow (uniform velocity) conditions existed during the fastest trials, while approximately linear vertical velocity profiles developed during the slowest ones (approximately 1 m/s). Intermediate conditions, i.e. base slip combined with a velocity profile, were indicated at intermediate speeds. This behaviour is probably due to the imperfectly developed velocity profiles on the short flume. The approximate linearity of the vertical velocity profiles was confirmed by three experiments during which a second high-speed camera was stationed so as to take a close-up of the flume side, permitting observation of the movement of the coloured glass beads.

Observations of the base of the flows through small glass windows made in the bed of the flume confirmed the existence of base slip in the faster flows and its absence in flows slower than approximately 1·5 m/s. Very strong lateral variation in base slip velocity was also observed, due to arching in the flume corners. The surface velocity, on the other hand, was almost perfectly uniform across the flume width.

DIMENSIONAL ANALYSIS

A dimensional analysis of the flume experiments was carried out to ensure that a sufficiently wide range of physical parameters had been covered (Hungr, 1981). The main parameters of the flows were combined in several dimensionless ratios. The experimental range was found to be dimensionally similar to a typical rock avalanche event in terms of mean grain size and shape, velocity, density, intergranular friction and bed slope. Grain size uniformity was at least partly satisfied by the angular sand–rock flour mixture experiments. The main dimensional deficiency was found in the ratio between the grain deformation modulus and the normal stress, which was too high in the test. The Authors addressed themselves to this problem in the subsequent test programme using rapid ring shear experiments which permit higher normal stresses to be developed. (Hungr & Morgenstern, 1984).

CONCLUSIONS

The major finding of this test programme is that the internal friction angle of the various materials shows no systematic dependence on the shear strain rate. In other words, even during the most rapid flows, at mean velocities of over 5 m/s, the materials continued to behave as Coulomb frictional solids, with friction angles only slightly lower than the quasi-static constant volume angle. This conclusion is derived both from direct measurement of stresses at the base of the flow and from the fact that non-accelerating flows could only be established on slopes near this particular angle.

There was a slight reduction in bulk density with increasing flow speed, amounting to some 10 to 20% over the spectrum of velocities achieved. At any given velocity, the density was not affected by flow thickness, but only as long as the latter was not less than 30 mm.

The fully developed flows tended to assume nearly linear vertical velocity profiles. Slower, non-accelerating flows deposited stagnant layers, particularly in the corners of the flume. Faster, probably not fully developed, flows exhibited substantial base slips, approaching the condition of a plug flow. The reluctance of the flows to develop a specific velocity profile rapidly is consistent with the Coulomb equation.

The experiments run with Ottawa sand and with angular sand exhibited an unexplained high degree of random variation in both mean density and friction coefficient. The randomness is considered to be inherent in the flow behaviour of these natural materials.

In a final conclusion, these experiments indicated nothing to invalidate the Coulomb relationship represented by equation (1), or the consequences of it, even though the range of flow velocities has been significantly expanded over previous experiments. This is a negative indication with respect to the mechanical fluidization concept and it would seem that high rates of shearing inherent to rapid rock avalanches are not sufficient to explain their increased mobility. Another explanation should therefore be sought.

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