

Partial Zero Forcing for Multi-Way Relay Networks

by

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Abstract

The ever increasing demands for mobile network access have resulted a significant growth in bandwidth usage. By improving the system spectral efficiency, multi-way relay networks (MWRNs) provide promising approaches to address this challenge. In this thesis, we propose a novel linear beamforming design, namely partial zero-forcing (PZF), for MWRNs with a multiple-input-multiple-output (MIMO) relay. Compared to zero-forcing (ZF), PZF relaxes the constraints on the relay beamforming matrix such that only partial user-interference, instead of all, is canceled at the relay. The users eliminate the remaining interferences through self-interference and successive interference cancellation. A sum-rate maximization problem is formulated to exploits the extra degrees-of-freedom brought from PZF. In solving the optimization problem, a numerical method, called modified gradient-ascent method, is proposed. Simulation results show that the proposed PZF relay beamforming design achieves significantly higher network sum rates than existing linear beamforming designs.

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Table of Contents

1	Introduction	1
1.1	Requirements and Challenges for Future Wireless Networks	1
1.2	Techniques to Enhance Data-Rate	3
1.3	Contributions and Thesis Organization	4
1.4	Notation	6
2	Background	7
2.1	Wireless Channel	7
2.1.1	Large-Scale Fading and Small-Scale Fading	7
2.1.2	Small-Scale Fading Channel Models	9
2.2	MIMO Systems and Beamforming	10
2.2.1	MIMO Systems	10
2.2.2	Beamforming	11
2.2.2.1	Zero-Forcing	12
2.2.2.2	Minimum Mean Square Error	12
2.2.2.3	Matched Filter	13
2.3	Cooperative Relay Networks	14
2.4	Multi-Way Relay Networks	16
2.4.1	Single-Antenna MWRNs	19
2.4.2	Multiple-Antenna MWRNs	19
3	Partial Zero-Forcing for MWRN	21
3.1	System Model	22

3.2	Relay Beamforming Design Problem and Existing Beamforming Designs	26
3.2.1	Zero-Forcing Design	27
3.2.2	Minimum Mean Square Error Design	27
3.2.3	Matched Filter Design	28
3.3	PZF Relay Beamforming Design	28
3.3.1	PZF: Main Idea	28
3.3.2	PZF: Formulation	30
3.3.3	Joint Optimization of the Relay Beamforming Matrices . . .	32
3.3.3.1	Modified Gradient	33
3.3.3.2	Optimization Algorithm	34
3.3.4	Separate Optimization of the Relay Beamforming Matrices	34
3.4	Simulation Results	35
4	PZF with Hybrid Uni/Multicasting	38
4.1	Hybrid Uni/Multicasting Strategy	38
4.2	Performance Comparison	39
4.3	Scheduling of Detections	40
5	PZF with General Numbers of Users and Relay Antennas	43
5.1	MWRNs with the Number of Relay Antennas Larger than the Number of Users	43
5.2	MWRNs with the Number of Relay Antennas Equal to the Number of Users Minus One	45
6	Conclusion and Future Work	49
6.1	Conclusion	49
6.2	Future Work	49
6.2.1	Optimal Selection and Ordering of User Signals	50
6.2.2	Low Complexity Relay Beamforming Design	50
6.2.3	Massive MIMO MWRNs	50

List of Figures

2.1	MIMO communication system diagram.	11
2.2	Cooperative relay networks diagram.	15
2.3	An example on one-way relay networks.	17
2.4	An example on two-way relay networks.	17
2.5	General transmission model of MWRNs.	18
3.1	Transceiver protocol of MWRN.	23
3.2	A 3-user MWRN with unicasting strategy.	24
3.3	Sum-rates for a homogeneous 3-user MWRN. $P_R = 1$	37
3.4	Sum-rates for a heterogeneous 3-user MWRN. $P_R = 1$	37
4.1	Hybrid uni/multicasting strategy.	39
4.2	Sum-rates of unicasting strategy and hybrid uni/multicasting strategy for a 3-user MWRN.	40
4.3	Another scheduling of detections for hybrid uni/multicasting strategy.	41
4.4	Sum-rates of hybrid uni/multicasting strategy using different decoding schedule for a 3-user MWRN.	42
5.1	Sum-rates of PZF and ZF with unicasting strategy for different numbers of relay antennas, SNR=20 dB.	44
5.2	Sum-rates of PZF and ZF with unicasting strategy for different numbers of relay antennas in logarithmic scale, SNR=20 dB.	45
5.3	Sum-rates of PZF and ZF with unicasting strategy for a 3-user MWRN, $M=2, 3$	48

List of Abbreviations

AF	Amplify and forward
BC	Broadcast
CF	Compress and forward
CSI	Channel state information
DF	Decode and forward
i.i.d.	Independent and identically distributed
FDF	Functional decode forward
LOS	Line-of-sight
MAC	Multiple access
MF	Matched filter
MIMO	Multi-input-multi-output
MMSE	Minimum mean square error
MSE	Mean square error
MWRN	Multi-way relay network
PZF	Partial zero-forcing
RF	Radio frequency
SINR	Signal-to-interference-plus-noise-ratio
SISO	Single-input-single-output
SNR	Signal-to-noise-ratio
ZF	Zero-forcing

Chapter 1

Introduction

The last two decades have witnessed significant development and widespread success of wireless communication technology. For example cellphone is no longer just a calling and texting tool. With the explosive development of mobile systems and the support of increasingly powerful communication networks, a cellphone is now a personal portable communication terminal to receive and share information from anywhere to anyone at anytime. The forms of information include photos, music, videos, social network media, real-time game data and so on. Users' growing desire for better mobile system experiences is spurring the academia and industry to design and build innovative wireless networks that can meet future extreme capacity and performance requirements.

1.1 Requirements and Challenges for Future Wireless Networks

Due to its advantages in flexibility, wireless networks will progressively become the primary medium for network access for person-to-person, person-to-machine and machine-to-machine communications. To fulfill communication demands, they will need to match advances in wired networks in terms of data rate, reliability and security. In order to connect various applications with different characteristics and

requirements, future wireless networks must extend far beyond existing ones in the following aspects.

Higher and more stable data rate: Data rate improvement has always been a major technical specification of every generation of wireless communications. In the past, data rate was commonly represented by the peak data rate, defined as the fastest data transfer rate supported by a wireless communication link under ideal conditions. Nowadays, actual data rate is more important, which should be provided under practical conditions in various common scenarios. In addition to indoor and urban environments, data rate increase should be achieved in suburban and rural areas.

Lower energy consumption: As wireless traffic grows dramatically, the increase in energy consumption and its accompanied greenhouse gas emission may cause environmental damage. The development of low cost and low energy consumption mobile devices has been a key requirement since the early days of mobile communication. However, in order to support the huge number of new wireless devices in the near future, it becomes more urgent nowadays.

Lower latency: Lower latency has been energized by higher data rate. According to the protocols in the network layer, low latency at the physical layer can lead to high data rate of the communication system. Besides, lower latency will be galvanized by new applications. In recent years, a large number of new applications spring up. Some of them do not have high data rate requirements, but may have very strict requirements on the latency. These applications can be found in vehicle-to-vehicle communications, traffic safety control, wireless sensor communications and real-time online games.

Other challenges for future wireless network include but are not limited to massive access devices, reliability, communication safety, and device cost. In this thesis we will focus on the data rate challenge.

1.2 Techniques to Enhance Data-Rate

Data rate increase is still a key demand in the evolution of wireless communication systems. Bandwidth is the communication resource which has a direct relation with the data rate. According to Shannon-Hartley theorem [1], the theoretical tightest upper bound on the information rate is proportional to the bandwidth of the communication channel. This is why mobile network operators always desire as much bandwidth as possible to provide high data rate service for their customers. However, the spectrum range available for communications is limited, which makes it a rare resource. Nowadays, the popularity of bandwidth hungry applications, for example, live streaming and video telephone, forces the operators to acquire more bandwidth and use spectrum more efficiently. Below we will introduce three main approaches to exploit available spectral resource.

The first approach is millimeter wave communications. So far, due to its supportive propagation characteristics, almost all commercial wireless communications such as radio broadcast, cell phone system, satellite communications, GPS and Wi-Fi are below 3 GHz spectrum. To meet traffic demands in the future, spectrum above 3 GHz has the potential to be used. Commonly, 30-300 GHz spectrum is generally referred to as millimeter-wave bands [2]. Millimeter-wave communication systems will be different from systems at lower frequencies. In higher frequencies, the signal power attenuates much faster over the distance, and the scattering becomes weaker. The design of a system for such bands must overcome these propagation issues [3]. One way is to take advantage of large antenna arrays, made possible by short wavelength, to compensate pathloss with beamforming gain. Techniques following this way, such as hybrid beamforming, spatially sparse precoding, millimeter-wave lens antennas are proposed [4]–[7].

The second approach is cognitive radio. Up to now, spectra are licensed on a per-operator basis within a geographical area. An operator can only use its pre-licensed bands of spectrum. This pre-license regime may result in waste of spectrum because temporarily vacant bands of spectrum cannot be utilized by unlicensed

users. The US federal communications commission (FCC) has reported that a significant amount of the radio spectrum is underutilized during the day [8]. Some bands of spectrum are unoccupied most of the time; some other bands are partially occupied [9]. In cognitive radio [10], when licensed users do not use their licensed bands, they could lease the spectrum to unlicensed users. Essentially, a band is shared between network operators, rather than divided among them, making it easier for operators to cope with temporary peaks in demand. This leads to better spectrum utilization.

The third approach is spectral efficiency enhancement. Spectral efficiency refers to the information rate that can be transmitted over a given bandwidth. Researchers have been trying to design physical layer protocols which can efficiently utilize the limited frequency spectrum. Up to now, general techniques for spectral efficiency improvement include beamforming, precoding, interference management, power control, scheduling and many more. Multi-way communications is one of such techniques, where the main idea is to reduce the number of time slots needed for communications among multiple users.

Multi-way communications have potential applications in spectral efficiency improvement for many communication scenarios, such as device-to-device communications, wireless sensor networks, video conferences and multi-player games. We focus this thesis on the design of high data-rate multi-way communications. In specific, we work on multi-way relay networks (MWRN), where multiple single-antenna users communicate with each other via a multiple-antenna relay. The contributions of our work along with the thesis organization are provided in the next section.

1.3 Contributions and Thesis Organization

Our work aims at improving the sum-rate of MWRNs. We consider a MWRN where one relay equipped with multiple antennas helps multiple single-antenna users to receive the information from each other. To achieve the sum-rate improve-

ment goal, a novel relay beamforming design named partial zero-forcing (PZF) is proposed. In each relay broadcast (BC) transmission time slot, while zero-forcing (ZF) relay beamforming forces the interference from all interfering users to be zero [11], our proposed PZF only forces partial interference (the interference from a carefully designed subset of the interfering users) to be zero. Thus PZF allows more degrees-of-freedom in the relay beamforming design. Combined with self-interference cancellation and successive interference cancellation at the users, the proposed PZF relay beamforming allows each user in the MWRN to obtain interference-free observations of information from all other users.

Based on the PZF idea, we formulate the sum-rate maximization problem for the MWRN, which is a constrained multi-dimensional non-linear optimization problem. A numerical method, called modified gradient-ascent method, is proposed to find a joint solution of the PZF relay beamforming matrices for different BC time slots. In addition, to reduce the computational complexity, we propose another method to separately and alternatively optimize every relay beamforming matrix. Simulation results show that the proposed PZF relay beamforming design achieves significantly higher network sum rates than the existing ZF and minimum mean square error (MMSE) beamforming designs.

The rest of the thesis is organized as follows. Chapter 2 gives a brief review of the needed background, including wireless channels, beamforming in MIMO systems, cooperative relay networks, MWRNs and literature review on MWRNs. In Chapter 3, the system model of the non-regenerative MWRNs is described. We also illustrate PZF beamforming method and give the optimization problem relating to PZF beamforming design. A numerical method named gradient-ascent method is provided to solve the optimization problem. Simulation results show PZF outperforms ZF in sum-rate. In Chapter 4, The scenario that the relay uses hybrid uni/multicasting strategy is considered. Chapter 5 extends our work to MWRNs with general numbers of relay antenna. Conclusions are made in Chapter 6 together with possible future research directions.

1.4 Notation

In this thesis, bold upper case letters and bold lower case letters are used to denote matrices and vectors, respectively. For a matrix \mathbf{A} , its transpose, conjugate, Hermitian, inverse, pseudoinverse and trace are denoted by \mathbf{A}^T , \mathbf{A}^* , \mathbf{A}^H , \mathbf{A}^{-1} , \mathbf{A}^+ and $\text{tr}\{\mathbf{A}\}$. For a vector \mathbf{a} , $|\mathbf{a}|$ denotes its Euclidean norm. If \mathbf{a} is a one dimension vector, i.e., a number, $|\mathbf{a}|$ denotes its absolute value. For a complex number h , $\angle h$ is its argument. $\text{mod}_N(x)$ is the modulo N of x . \mathbf{I}_N is the $N \times N$ identity matrix, $\mathbb{E}(\cdot)$ denotes the expectation operator and \oplus denotes the exclusive or operation (i.e., XOR). Also, $\text{diag}\{a_1, \dots, a_N\}$ is the construction of a diagonal matrix whose diagonal entries starting from the upper left corner are a_1, \dots, a_N .

Chapter 2

Background

In this chapter, we provide the background needed to illustrate our relay beamforming design in MWRNs. First, we introduce wireless channel models, MIMO systems and cooperative relay communications. Then, the development of MWRNs as well as a literature review are given.

2.1 Wireless Channel

Wireless channels are much more complex than wired channels. For example, electromagnetic waves travel through different mechanisms: reflection, refraction and scattering, thus signals are transmitted and received through multiple paths over wireless channels. Also, wireless channels are time-varying, caused by the relative motion of the transmitter and the receiver and changes in the environment. Another distinctive feature of wireless channels is the interference of different signals being transmitted over a wireless medium. In the following we will illustrate commonly used fading channel models.

2.1.1 Large-Scale Fading and Small-Scale Fading

Large-scale fading is the result of signal power attenuation over distance due to path loss and shadowing [12]. Path loss is caused by dissipation of the power radiated

by the transmitter as well as effects of the propagation channel. If a long period of time is considered, the average power of the received signal, denoted by \bar{P}_r , can be written as

$$\bar{P}_r = \frac{b}{d^\nu} P_t, \quad (2.1)$$

where b is a constant, P_t is the transmit signal power level, d is the separation between the transmitter and the receiver, and ν is the path loss exponent. The path loss exponent is typically between two and six, depending on the specific environment. The constant b depends on a variety of factors. It is proportional to transmit and receive antenna gains and square of operating wavelength.

Shadowing is caused by obstacles between the transmitter and receiver that attenuate signal power through absorption, reflection, scattering, etc [13]. Instead of averaging over a very long period of time, if a smaller time window is used, say in the order of a few seconds or minutes, the average received signal strength will be a random variable. Denoting it by P_r , the received power at this scale can be modeled as

$$P_r \text{ dBm} = \bar{P}_r \text{ dBm} + X_\sigma \text{ dB}. \quad (2.2)$$

Powers are expressed in dBm, which indicates the ratio of the amount of power to 1mW in dBs. X_σ is a zero mean random variable. In the widely used log-normal shadowing model, X_σ is taken as a zero mean Gaussian random variable with standard deviation σ .

Small-scale fading, or fading is due to the constructive and destructive interference of the multiple signal paths between the transmitter and receiver [13]. This occurs at the spatial scale of the order of the carrier wavelength. Path loss and log-normal shadowing are average quantities. In fact, the actual received signal power in a wireless channel is a much more rapidly varying random quantity which needs to be characterized using statistical models. This is explained in detail in the following section.

2.1.2 Small-Scale Fading Channel Models

Fading channels are modeled as linear time-varying systems where the time variations are random [14]. Therefore, we can characterize them using a time-varying impulse response. Let us denote the transmitted baseband signal by $x(t)$. If the equivalent baseband channel impulse response is denoted by $c(\tau; t)$, the received signal can be written as

$$r(t) = \int_{-\infty}^{+\infty} c(\tau; t)x(t - \tau)d\tau. \quad (2.3)$$

According to central limit theorem, the channel impulse response $c(\tau; t)$ is a complex Gaussian random process.

Due to the effect of multipath delay spread, small-scale fading can be categorized into frequency flat fading and frequency selective fading [14]. The multipath structure can be characterized in the time domain using multipath intensity profile of wireless channels, which basically shows relative powers of the received signal through different delays. The multipath spread T_m is the time difference between the shortest and the longest paths that the transmitted signal goes through. The coherence bandwidth of the channel, B_C , is the range of non-negative values of the Fourier transform of the multipath intensity profile. Roughly speaking, $B_C \sim 1/T_m$. Two frequencies separated by less than the coherence bandwidth of the channel are affected in almost the same way by the channel. On the other hand, frequencies separated by more than the coherence bandwidth undergo different channel fades.

Consider digital modulation over a wireless channel, and assume that the bandwidth of the signal used in the transmission is W . If the signal bandwidth W is significantly smaller than the coherence bandwidth of the channel B_C , clearly, all the frequency components of the transmitted signal see the same effective channel. The channel is said to be frequency flat. The condition $W \ll B_C$ is equivalent to saying that the multipath spread of the channel is significantly smaller than the signal duration in the time, and therefore, there is no intersymbol interference between

consecutive transmitted symbols.

If the condition $W \ll B_C$, or equivalently, $T_m \ll T_S$ (where T_S is the symbol duration) is not satisfied, then different frequency components of the signal undergo different channel fades. In such a case, the channel is said to be frequency selective, and it causes intersymbol interference.

In this thesis frequency flat fading channel model is used. The channel gain can be denoted by a complex random variable h . Thus $|h|$ is the absolute value of the channel gain, and $\angle h$ is the channel phase. If there is no dominant propagation along a line-of-sight (LOS) between the transmitter and receiver, the channel is called Rayleigh fading channel [15]. For Rayleigh fading, h is a complex Gaussian random variable with zero-mean and $\angle h$ is uniformly distributed over $[0, 2\pi]$. Thus $|h|$ is Rayleigh distributed. Other popular fading channels include Rician fading channels [16] and Nakagami fading channels [17]. Rayleigh fading channel model is considered in this thesis.

2.2 MIMO Systems and Beamforming

In this section, we first talk about multi-input-multi-output (MIMO) system and an important signal processing technique in MIMO systems, beamforming. Later, an introduction to cooperative relay networks is given.

2.2.1 MIMO Systems

MIMO technology, the use of multiple antennas at the transmitter and receiver in wireless systems, have been extensively studied and applied to current wireless communications standards. Compared with the single-input-single-output (SISO) configuration, the MIMO configuration can significantly enhance the performance of wireless systems through multiplexing or diversity gain [18]–[22]. It provides higher data rates and lower bit error rates (BER). The MIMO system model is shown in Figure 2.1.

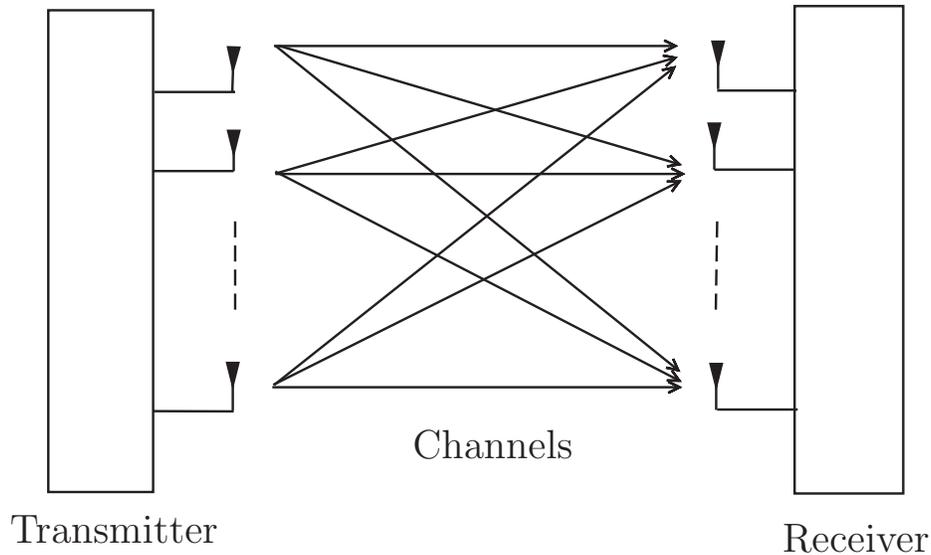


Fig. 2.1. MIMO communication system diagram.

The multiplexing gain of a MIMO system reflects the fact that a MIMO channel can be decomposed into a number of independent SISO channels. Independent data can be multiplexed on these independent SISO channels. In such a way, the overall data rate of the MIMO system is increased compared to the SISO system. This produces multiplexing gain. Diversity gain of MIMO systems results from the fact that it is unlikely that several antenna elements be in a fading dip simultaneously. A very robust channel can be obtained by coherently combining the channel gains. The probability for very low signal levels is thus decreased by the use of this combining. This produces diversity gain.

2.2.2 Beamforming

Beamforming is one of the many developed transmission and reception techniques to achieve the high performance provided by MIMO systems [23], [24]. By controlling the phase and relative amplitude of the signal at each antenna, a pattern of constructive and destructive interference in the wavefront is created. As a result, the signal-to-interference-and-noise ratio (SINR) of the communication link is

improved.

In the following we introduce three linear beamforming schemes: ZF, minimum mean square error (MMSE) and matched filter (MF) [25]. The vector of the information symbols sent by the transmitting antennas is denoted as \mathbf{s} . \mathbf{H} is the channel matrix between all transmitting antennas and receiving antennas. The received signal vector at the relay is

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{z}_R, \quad (2.4)$$

where \mathbf{z}_R is the noise vector at the receiver. After receiving the signal from the transmitter, the receiver applies beamforming to the received signal vector. The processed signal vector after receive beamforming is

$$\tilde{\mathbf{s}} = \mathbf{G}(\mathbf{H}\mathbf{s} + \mathbf{z}_R), \quad (2.5)$$

where \mathbf{G} is the receive beamforming matrix. Depending on different beamforming scheme, \mathbf{G} has different structure.

2.2.2.1 Zero-Forcing

ZF receive beamforming requires that $\tilde{\mathbf{s}}$ is an interference-free estimate of \mathbf{s} , i.e.,

$$\tilde{\mathbf{s}} = \mathbf{s} |_{\mathbf{z}_R=0}. \quad (2.6)$$

It means that the receiver can totally null the multi-antenna interferences from the transmitter. We can get the solution of (2.6) as follows:

$$\mathbf{G}_{ZF} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H. \quad (2.7)$$

2.2.2.2 Minimum Mean Square Error

The MMSE receive beamformer minimize the mean square error (MSE) of the signals,

$$\mathbf{G}_{MMSE} = \arg \min_{\mathbf{G}} \mathbb{E}\{|\mathbf{s} - \tilde{\mathbf{s}}|^2\}. \quad (2.8)$$

The objective function in (2.8) can be calculated as

$$\mathbb{E}\{|\mathbf{s} - \tilde{\mathbf{s}}|^2\} = \text{tr} \{ \mathbf{R}_s - (\mathbf{G}\mathbf{H}\mathbf{R}_s)^H - \mathbf{G}\mathbf{H}\mathbf{R}_s + \mathbf{H}^H\mathbf{G}^H\mathbf{R}_s\mathbf{G}\mathbf{H} + \mathbf{G}\mathbf{R}_{z_R}\mathbf{G}^H \}, \quad (2.9)$$

where \mathbf{R}_s is the covariance matrix of the transmitted signal and \mathbf{R}_{z_R} is the covariance matrix of the noise vector of all receiving antennas. By taking the derivative of (2.9) with respect to \mathbf{G} and setting it equal to zero, we have

$$\mathbf{G}_{\text{MMSE}} = (\mathbf{H}^H\mathbf{R}_{z_R}^{-1}\mathbf{H} + \mathbf{R}_s^{-1})^{-1}\mathbf{H}^H\mathbf{R}_{z_R}^{-1}. \quad (2.10)$$

2.2.2.3 Matched Filter

The MF beamforming is the optimal linear beamforming for maximizing the signal-to-noise-ratio (SNR) in the presence of additive noise. The MF beamforming design problem can be described as

$$\mathbf{G}_{\text{MF}} = \arg \max_{\mathbf{G}} \frac{\mathbb{E}\{|\mathbf{s}^H\tilde{\mathbf{s}}|^2\}}{\mathbb{E}\{|\mathbf{G}\mathbf{z}_R|^2\}}. \quad (2.11)$$

The objective function in (2.11) can be calculated to be

$$\frac{\mathbb{E}\{|\mathbf{s}^H\tilde{\mathbf{s}}|^2\}}{\mathbb{E}\{|\mathbf{G}\mathbf{z}_R|^2\}} = \frac{\text{tr}(\mathbf{G}\mathbf{H}\mathbf{R}_s)}{\text{tr}(\mathbf{G}\mathbf{R}_{z_R}\mathbf{G}^H)}. \quad (2.12)$$

By taking the derivative of (2.12) with respect to \mathbf{G} and setting it equal to zero, we have

$$\mathbf{G}_{\text{MF}} = \mathbf{R}_s\mathbf{H}^H\mathbf{R}_{z_R}^{-1}. \quad (2.13)$$

When the transmitter has the channel state information (CSI), transmit beamforming or precoding can also be used for the MIMO communications. The transmitted signal after transmit beamforming is

$$\mathbf{x} = \mathbf{T}\mathbf{s}, \quad (2.14)$$

where \mathbf{T} is the transmit beamforming matrix. Correspondingly the received signal

vector is

$$\tilde{\mathbf{s}} = \mathbf{H}\mathbf{T}\mathbf{s} + \mathbf{z}_R. \quad (2.15)$$

By following the same ideas of receive beamforming, the ZF, MMSE, and MF transmit beamforming matrices are

$$\mathbf{T}_{\text{ZF}} = \frac{1}{p_{\text{ZF}}} \mathbf{H}^H (\mathbf{H}\mathbf{H}^H)^{-1}, \quad (2.16)$$

$$\mathbf{T}_{\text{MMSE}} = \frac{1}{p_{\text{MMSE}}} \left(\mathbf{H}^H \mathbf{H} + \frac{\text{tr}(\mathbf{R}_{z_R})}{P_s} \mathbf{I} \right)^{-1} \mathbf{H}^H, \quad (2.17)$$

$$\mathbf{T}_{\text{MF}} = \frac{1}{p_{\text{MF}}} \mathbf{H}^H, \quad (2.18)$$

where P_s is the transmit power, p_{ZF} , p_{MMSE} and p_{MF} are used to fulfill the power constraints.

ZF, MMSE, MF are most common beamforming schemes. They can be used in MWRNs, which we will discuss in detail in Chapter 3.

2.3 Cooperative Relay Networks

Conventional MIMO systems require both transmit users and receive users to be equipped with multi-antennas. This is not practical for small users like cellphones since multi-antenna users need more RF circuits corresponding to each antenna and the signal processing procedure can be complex.

In relay networks, an illustration example is shown in Figure 2.2, one or more users (called relays) with single or multiple antennas help users to transmit or receive information [26]–[28]. Although the relays are separately distributed, their antennas can cooperate to form a multiple-antenna array. In this respect, relay networks can be seen as virtual MIMO systems [29], [30]. Besides, relays can also extend the coverage and enhance the performance of communication systems when the direct link between the communicating users is weak [26].

Depending on how the relays help the receiver and the transmitter in their communications, several kinds of relaying schemes have been proposed. They can

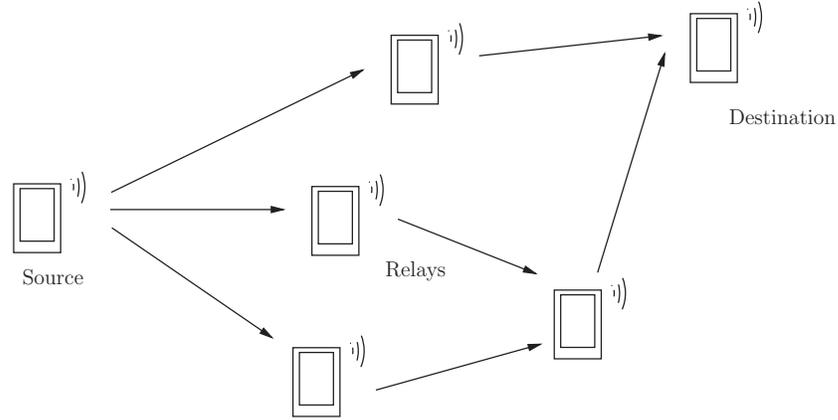


Fig. 2.2. Cooperative relay networks diagram.

be divided to two main categories. One is non-regenerative relaying. With non-regenerative relaying, the relays do not decode the information from the transmitter but only perform linear transformation or processing [31], [32]. Amplify-and-forward (AF) [33], linear-process-and-forward and non-linear-process-and-forward [34] belong to this category. The other one is regenerative relaying, where the relays decode the information and re-encodes it before forwarding [35], [36]. Regenerative relaying category includes decode-and-forward (DF) [33], [37], [38], compress-and-forward (CF) [39], [40], estimate-and-forward [41], [42], etc.

Each of the two categories of relaying schemes has its own advantages [43]. For non-regenerative relaying, no decoding error is propagated, no time delay is resulted from the decoding and re-encoding process. For regenerative relaying, noise from the relay is not propagated to the users. Besides, when the receive users decode the information, they only need to know the information of their own channel from the relay, thus the resources used on channel estimation are reduced.

Depending on whether the relays can transmit and receive simultaneously over the same frequency band, relays are divided into half-duplex relay and full-duplex relay. Traditional relays operate in half-duplex mode, in which the user-to-relay and relay-to-user communication links are kept orthogonal by time division multiplexing [44]. On the contrary, in the full-duplex mode, the user-to-relay and relay-

to-user links share a common time signal-space. Thus the relay can transmit and receive simultaneously. Generally, full-duplex relay has higher spectral efficiency. However, it is hard to implement in practice [45]. In this thesis, the relays are assumed to be non-regenerative and operate in the half-duplex mode.

2.4 Multi-Way Relay Networks

In recent years, a configuration called multi-way relay networks (MWRNs) [46] has been proposed. In an MWRN, multiple users exchange information with each other under the help of one cooperative relay node. By smartly leveraging user-interference, instead of completely avoiding it, MWRNs are able to achieve significantly improved spectral efficiency in wireless communication systems [47]. Possible applications of MWRNs cover a broad range from cellular communications to wireless sensor networks and satellite communications [48]. In the following part of this section, the research progress of MWRNs are given. We first discuss the development from one-way relaying to two-way relaying, and then MWRNs. After that, existing work on MWRNs and the relay beamforming designs are introduced.

In traditional relay networks, communications are one-way, where information is transmitted in one direction from data sources to data destinations. They are thus called one-way relay networks. But one-way relaying may not be efficient when two users want to share information with the help of one relay with each other. This can be shown by an example illustrated in Figure 2.3 and Figure 2.4. Assume that there is no direct link between the two users, thus the communications are to be completed with the help of a relay. With one-way relaying four time slots are required. Two time slots are used for the relay to receive data from user 1 and retransmit data to user 2, and the other two time slots are used for the reverse links.

With two-way relaying, only two communication time slots are needed. The scheme is shown in Figure 2.4. User 1 and User 2 transmit data, called s_1 and s_2 respectively, simultaneously to the relay in the first time slot. Employing the concept of network coding [49], the relay XORs s_1 and s_2 . In the second time slot,

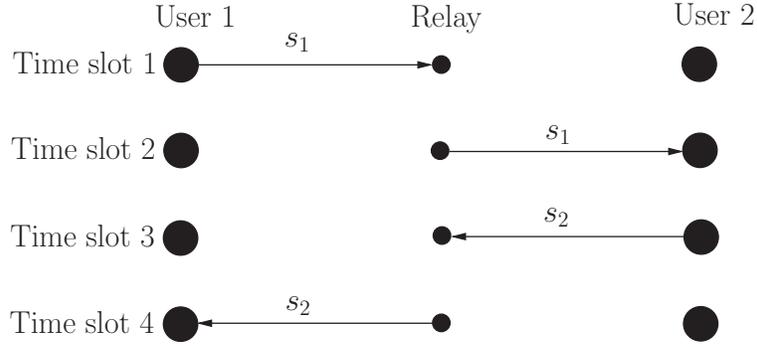


Fig. 2.3. An example on one-way relay networks.

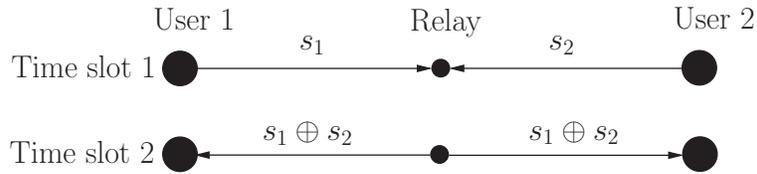
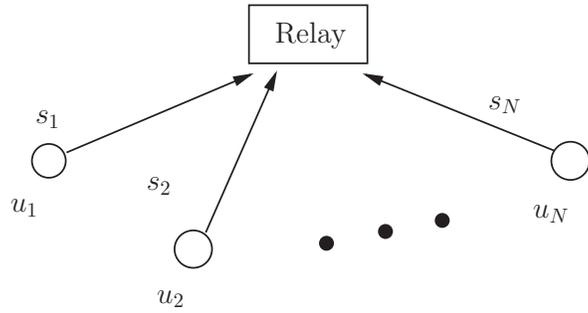


Fig. 2.4. An example on two-way relay networks.

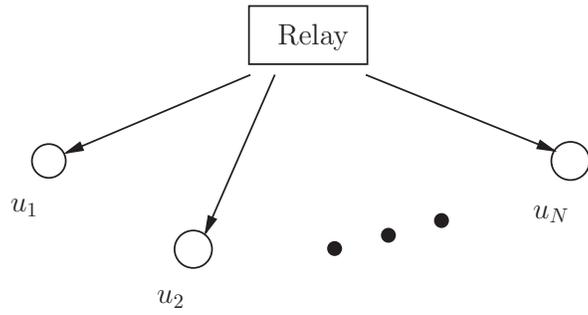
the relay broadcasts the XORed signal $s_1 \oplus s_2$ to both users. Knowing its own data, each user acquires the other user's data from the received XORed signal [50]. Since the transmission time is halved, two-way relaying has obvious advantage in spectral efficiency.

The applications of MIMO and cooperative communication techniques in two-way relaying design have been well studied [51]–[54]. [51] proposes optimal beamforming method for two-way multi-antenna relay channel. [52] proposes a semi-closed-form solution to optimal distributed beamforming for two-way relay networks. Multi-pair two-way relaying, an extension of one pair two-way relaying to have multiple pairs of users communicate via a relay, is considered in [53], [54].

MWRNs is a generalization of two-way relay networks. In MWRNs, multiple users, without direct communication links among them, share their data with all the other users via a relay. MWRNs can be applied to many scenarios, for instance, video conferences, wireless sensor networks and multi-player games. Communica-



(a) MAC phase.



(b) BC phase.

Fig. 2.5. General transmission model of MWRNs.

tions in MWRNs take two phases, one is the multiple access (MAC) phase, the other is the BC phase. Each phase can take one or multiple time slots. In the MAC phase, users transmit their own data to the relay. The relay process the received data based on different relaying strategies, say AF, DF and CF, etc. Then, in the BC phase, the relay transmits the processed data to the users. A general transmission model of MWRNs is shown in Figure 2.5. The numbers of time slots in the MAC phase and the BC phase depend on how many antennas the relay have. So, MWRNs can be classified into two types according the number of antennas the relay is equipped: single-antenna MWRNs and multiple-antenna MWRNs.

Research on MWRNs are limited. In what follows we give a literature review on single-antenna MWRNs and multiple-antenna MWRNs, respectively.

2.4.1 Single-Antenna MWRNs

In this section, we give a brief introduction of existing studies on single-antenna MWRNs. Gunduz *et al.* consider full-duplex single-antenna MWRNs in [46]. Full-duplex means both the users and relay can transmit and receive signals simultaneously. They provide upper bounds on the symmetric capacity of the symmetric Gaussian MWRNs and calculate the achievable symmetric rate for AF, DF and CF protocols. The calculations and simulations show that CF achieves a symmetric rate within a constant bit offset from the capacity. Except [46], most of the literatures related to single-antenna MWRNs treats half-duplex systems [48], [55]–[57]. In half-duplex single-antenna N -users MWRNs, $(N - 1)$ time slots are required in the MAC phase and $(N - 1)$ time slots are needed in the BC phase.

Ong *et al.* propose a novel relaying protocol called functional-decode-and-forward (FDF) in [56]. With FDF, the relay decodes smartly defined functions of the users' symbols. In the BC phase, the relay broadcasts the function back to all users. Following a certain order of decoding, the users will get all other users' data. FDF is shown to achieve the common-rate capacity of additive white Gaussian noise MWRN. [57] gives the gap between the achievable common rate and the capacity for AF, DF and FDF relaying protocols and show that the capacity gap of FDF is always less than $\frac{1}{2(N-1)}$ bit. The authors in [58] study MWRNs with unequal channel conditions. Pairwise transmission is considered, where users transmit their information symbols to the relay in pairs. They show that the achievable common rate of MWRNs depends on the order in which the users are paired. Optimal user pairing is given for both DF and FDF protocols.

2.4.2 Multiple-Antenna MWRNs

In multiple-antenna MWRNs, more than one antenna is equipped at the relay, resulting in multiple wireless channels between the relay and the users. Benefiting from these extra wireless channels, multiple-antenna MWRNs require fewer communication time slots than single-antenna MWRNs. In this section, we review related

literatures on multiple-antenna MWRNs.

The system models considered in existing literatures are similar, where multiple single-antenna users communicate with each other with the help of one half-duplex multiple-antenna relay. The number of antennas equipped at the relay is assumed to be equal to the number of users, which is denoted as N . In the MAC phase, all users transmit simultaneously to the relay. Only 1 time slot is needed because the multiple-antenna relay can separate the signals from different users. In the following BC phase, which takes $(N - 1)$ time slots, the relay sends to each user the intended $(N - 1)$ signals from the $(N - 1)$ other users.

In [59] a transceive strategy is proposed for regenerative MWRNs. They keep transmission rates in the BC phase equal to rates in the MAC phase and minimize the power cost at the relay. [60] investigates the situation when the CSI is not available at the relay. They present space-time analog network coding transmission and repetition transmission strategy for stationary channels and non-stationary channels. Another relaying scenario, namely superimposed uni-/multicasting, is reported in [61] that efficiently combines transceive processing at the relay with joint receive processing at the users. More specifically, an MMSE-based transceive filter is employed at the relay. Then, by carefully choosing the selection of uni-/multicast signals at the relay and the interference cancellation order at the users, the proposed transmit strategy improves the system sum-rate. While [59] and [60] focus on one group MWRNs. Multiple group MWRNs is considered in [62], [63]. For multi-group MWRNs, a half-duplex relay assists multiple groups of users to communicate with all the other users in their own group.

Depending on how the relay delivers information to the users in the BC time slots, three broadcast strategies are proposed in [64]. These strategies are called unicasting, multicasting, and hybrid uni/multicasting. Three linear relay transceive beamforming designs based on ZF, MMSE, and MF are also provided in [64]. Detailed explanation of these beamforming designs and performance comparison of our PZF design with these beamforming designs will be given in Chapters 3-5.

Chapter 3

Partial Zero-Forcing for MWRNs

This thesis aims at improving the sum-rate of MWRNs with new beamforming design at the multi-antenna relay. We consider a non-regenerative MWRN where N single-antenna users share information with each other via a half-duplex relay equipped with M ($M \geq N - 1$) antennas. To achieve the sum-rate improvement goal, a novel relay beamforming design, PZF, is proposed. In each relay BC transmission time slot, ZF relay beamforming forces the interference from all interfering users to be zero [11], while PZF only forces partial interference (the interference from a carefully designed subset of the interfering users) to be zero. Thus PZF allows more degrees-of-freedom in the relay beamforming design. Combined with self-interference cancellation and successive interference cancellation at the users, the proposed PZF relay beamforming allows each user in the MWRN to obtain interference-free observations of information from all other users.

Based on the PZF idea, we formulate the sum-rate maximization problem for the MWRN, which is a constrained multi-dimensional non-linear optimization problem. A numerical method, called modified gradient-ascent method, is proposed to find a joint solution of the PZF relay beamforming matrices for different BC time slots. In addition, to reduce the computational complexity, we propose another method to separately and alternatively optimize every relay beamforming matrix. Simulation results show that the proposed PZF relay beamforming design achieves significantly higher network sum-rate than the existing ZF, MMSE and MF beam-

forming designs.

This chapter explains the idea, formulation and performance of our PZF beamforming. In the network model, we assume that the number of antennas equipped at the relay M equals to the number of users N , to make fair comparison with existing beamforming designs. Also only uni-casting is considered for simplifying the discussion. The case of hybrid uni/multicasting strategy is considered in Chapter 4. And networks with general M and N ($M \geq N - 1$) is studied in Chapter 5.

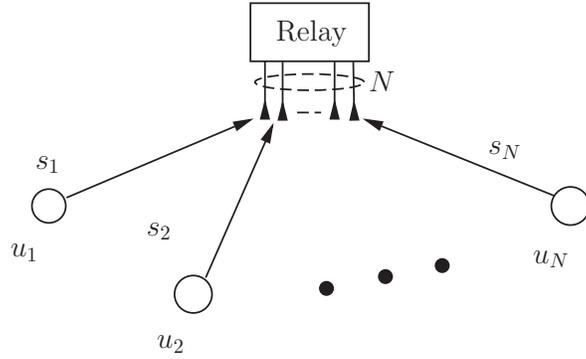
3.1 System Model

The system model of MWRNs includes two parts, the network model and the other is transceiver protocol. In network model, the physical parameters such as number of users, number of antennas and channel properties are clarified. For the transceiver protocol, we explain the unicasting strategy.

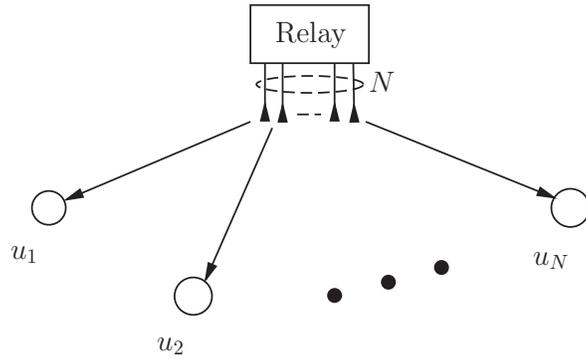
We consider an MWRN consisting of N users (called u_1, u_2, \dots, u_N) and one relay. Each user is equipped with one antenna, while the relay is equipped with N antennas. Both the users and the relay operate in half-duplex mode. There are no direct channels among the users and only the channels between the relay and the users are available. The users communicate with each other with the help of the relay.

Let $\mathbf{h}_i = (h_{i,1}, h_{i,2}, \dots, h_{i,N})^T$ for $i = 1, 2, \dots, N$, be the channel vector between u_i and the relay. Thus $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_N]$ is the $N \times N$ channel matrix between all users and the relay. The channels are assumed to follow independent frequency-flat Rayleigh fading, where $h_{i,n}$ follows $\mathcal{CN}(0, \sigma_i^2)$, the circularly symmetric complex Gaussian distribution whose mean is zero and whose variance is σ_i^2 . With this, we imply that the channels between the same user and different relay antennas have the same variance; while the channels between different users and the relay antennas can have different variances. Moreover, the channels are assumed to be reciprocal and keep unchanged in each communication block of N time slots.

For all users to send one symbol each to all other users, N time slots are needed,



(a) MAC phase.



(b) BC phase.

Fig. 3.1. Transceiver protocol of MWRN.

containing 1 MAC time slots and $N - 1$ BC time slots. In the MAC phase, as shown in Figure 3.1a, all users transmit their information symbols simultaneously to the relay. The $N \times 1$ received signal vector at the relay, \mathbf{r}_{RS} , is

$$\mathbf{r}_{\text{RS}} = \mathbf{H}\mathbf{s} + \mathbf{z}_{\text{RS}}, \quad (3.1)$$

where $\mathbf{s} = (s_1, s_2, \dots, s_N)^T$ is the vector of information symbols of the N users and \mathbf{z}_{RS} is the noise vector at the relay. The transmit power of u_i is denoted as P_i . Independent Gaussian codebook is used, where the information symbols are assumed to be independent and follow $\mathcal{CN}(0, P_i)$.

In the BC time slots, as shown in Figure 3.1b, the multi-antenna relay applies linear beamforming to its received signal vector \mathbf{r}_{RS} and broadcasts information to all users. For the n -th BC time slot where $n = 1, \dots, N - 1$, denote the $N \times N$

relay beamforming matrix as $\mathbf{G}^{(n)}$. Using unicasting, in every BC time slot, the relay transmits different information symbols to different users. Each symbol is intended only for one receiving user in each BC time slot. In other words, each user sees the symbols transmitted by the relay to the other users as interferences. The symbol transmitted from the relay to each user is changed in every BC time slot, such that within the $N - 1$ BC time slots, each user receives the information from all other users.

A 3-user MWRN using unicasting is shown in Figure 3.2. In the MAC phase, u_1 sends s_1 , u_2 sends s_2 and u_3 sends s_3 simultaneously to the relay. In the first time slot of the BC phase, u_1 decodes s_2 , u_2 decodes s_3 and u_3 decodes s_1 from the relay broadcast signal. In the second time slot of the BC phase, u_1 decodes s_3 , u_2 decodes s_1 and u_3 decodes s_2 . After the MAC phase and the BC phase, each user decodes the information symbols from all other users.

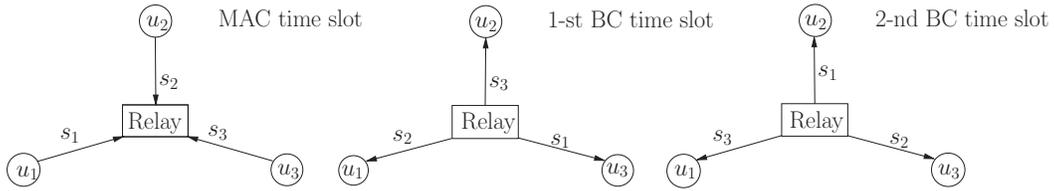


Fig. 3.2. A 3-user MWRN with unicasting strategy.

Now, we go back to the general N -user MWRNs and explain the BC phase model and the system sum-rate. Because the channels are reciprocal and stationary, the BC channel matrix from the relay to the users is simply the transpose of the MAC phase channel matrix \mathbf{H} . By using (3.1), the received signal vector of all users in the n -th BC time slot, $\mathbf{r}_{\text{users}}^{(n)}$, can be written as

$$\mathbf{r}_{\text{users}}^{(n)} = \mathbf{H}^T \mathbf{G}^{(n)} \mathbf{H} \mathbf{s} + \mathbf{H}^T \mathbf{G}^{(n)} \mathbf{z}_{\text{RS}} + \mathbf{z}_{\text{users}}^{(n)}, \quad (3.2)$$

where $\mathbf{z}_{\text{users}}^{(n)} = (z_1^{(n)}, \dots, z_N^{(n)})^T$ is the noise vector at the users in the n -th BC time slot. The additive noises at the relay and the users are modeled as independent circularly symmetric complex Gaussian random variables with zero-mean and unit

variance, i.e., $\mathcal{CN}(0, 1)$.

The transmit power of the relay for each BC time slot is

$$P_R = \mathbb{E}\{\text{tr}\{\mathbf{G}^{(n)}(\mathbf{H}\mathbf{s} + \mathbf{z}_{RS})[\mathbf{G}^{(n)}(\mathbf{H}\mathbf{s} + \mathbf{z}_{RS})]^H\}\}. \quad (3.3)$$

It can be further calculated as

$$\begin{aligned} P_R &= \mathbb{E}\left\{\text{tr}\left\{\mathbf{G}^{(n)}(\mathbf{H}\mathbf{s} + \mathbf{z}_{RS})(\mathbf{H}\mathbf{s} + \mathbf{z}_{RS})^H(\mathbf{G}^{(n)})^H\right\}\right\} \\ &= \mathbb{E}\left\{\text{tr}\left\{\mathbf{G}^{(n)}(\mathbf{H}\mathbf{s}\mathbf{s}^H\mathbf{H}^H + \mathbf{H}\mathbf{s}\mathbf{z}_{RS}^H + \mathbf{z}_{RS}\mathbf{s}^H\mathbf{H}^H + \mathbf{z}\mathbf{z}^H)(\mathbf{G}^{(n)})^H\right\}\right\} \\ &= \text{tr}\left\{\mathbf{G}^{(n)}(\mathbf{H}\mathbf{P}_s\mathbf{H}^H + \mathbf{I}_N)(\mathbf{G}^{(n)})^H\right\}, \end{aligned} \quad (3.4)$$

where $\mathbf{P}_s = \text{diag}\{P_1, P_2, \dots, P_N\}$.

After receiving the relay's signal in the n -th BC time slot, u_k decodes u_i 's information symbol, which is s_i . In this work, the communication scheme is designed to have the following relation among i, k, n :

$$i = \text{mod}_N(k + n - 1) + 1. \quad (3.5)$$

Accordingly, from (3.2), the received signal at u_k in the n -th BC time slot can be written as

$$r_k^{(n)} = \mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_i s_i + \sum_{j=1, j \neq i}^N \mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_j s_j + \mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{z}_{RS} + z_k^{(n)}. \quad (3.6)$$

In (3.6), the first term contains the useful signal, the second term contains the interferences from other users, the third term contains the noise propagated from the relay, and the last term is the noise at u_k . Thus, the signal-to-interference-plus-noise-ratio (SINR) for the communication from u_i to u_k , denoted as $\gamma_{k,i}$, can be calculated to be

$$\gamma_{k,i} = \frac{P_i |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_i|^2}{\sum_{j=1, j \neq i}^N P_j |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_j|^2 + |\mathbf{h}_k^T \mathbf{G}^{(n)}|^2 + 1}. \quad (3.7)$$

After each BC time slot, u_k performs interference cancellation by subtracting its self-interference and the interference of user symbols which have already been decoded in the previous BC time slots. The SINR after interference cancellation is

$$\gamma_{k,i} = \frac{P_i |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_i|^2}{\sum_{j=1, j \neq i, j \neq k, j \notin \mathbb{L}}^N P_j |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_j|^2 + |\mathbf{h}_k^T \mathbf{G}^{(n)}|^2 + 1}, \quad (3.8)$$

where $\mathbb{L} = \{\text{mod}_N(k + q - 1) + 1, q = 1, 2, \dots, n - 1\}$. The achievable rate from u_i to u_k , denoted as $R_{k,i}$ is thus

$$R_{k,i} = \log_2(1 + \gamma_{k,i}). \quad (3.9)$$

The common rate R_i that every user can reliably send to all other users is:

$$R_i = \min_{k \neq i} R_{k,i}. \quad (3.10)$$

The achievable sum-rate of the MWRN is [11]:

$$R_{\text{sum}} = \frac{N-1}{N} \sum_{i=1}^N R_i. \quad (3.11)$$

3.2 Relay Beamforming Design Problem and Existing Beamforming Designs

The sum-rate of the MWRN is given by (3.8)-(3.11). It is conceivable that the design of the relay beamforming matrices $\mathbf{G}^{(1)}, \mathbf{G}^{(2)}, \dots, \mathbf{G}^{(N-1)}$ are crucial for the sum-rate performance of the MWRN. In this section, a brief introduction of existing relay beamforming designs are given, including the ZF, MMSE, MF designs proposed in [11]. It should be noticed that the relay beamforming schemes for MWRNs serves both receive beamforming and transmit beamforming. Hence they are also called transceive beamforming.

3.2.1 Zero-Forcing Design

In ZF, $\mathbf{G}^{(n)}$ is designed such that the second term in (3.6) equals 0 for all $n = 1, \dots, N-1$. That is, at u_k , the interference from all other users except u_i is forced to zero. The relay beamforming matrix has the following closed-form expression:

$$\mathbf{G}_{\text{ZF}}^{(n)} = \mathbf{G}_{\text{TX}}^{(n)} \mathbf{P}^n \mathbf{G}_{\text{RX}}, \quad (3.12)$$

where \mathbf{G}_{RX} is the receive beamforming matrix and \mathbf{G}_{TX} is the transmit beamforming matrix defined as follows:

$$\begin{aligned} \mathbf{G}_{\text{RX}} &= (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H, \\ \mathbf{G}_{\text{TX}}^{(n)} &= \frac{1}{p_{\text{ZF}}^{(n)}} \mathbf{H}^* (\mathbf{H}^T \mathbf{H}^*)^{-1}. \end{aligned} \quad (3.13)$$

\mathbf{P} is the permutation matrix, obtained by shifting the columns of \mathbf{I}_N circularly to the right one time. \mathbf{P}^n is thus the permutation matrix to define the relationship among receiving user u_k , transmitting user u_i and the corresponding BC time slot n . $p_{\text{ZF}}^{(n)}$ is used to fulfill the power constraint.

3.2.2 Minimum Mean Square Error Design

MMSE beamforming minimizes the mean square error of the signal. For the MWRN, the MMSE receive beamforming matrix and transmit beamforming matrix are

$$\begin{aligned} \mathbf{G}_{\text{RX}} &= \mathbf{R}_s \mathbf{H}^H (\mathbf{H} \mathbf{R}_s \mathbf{H}^H + \mathbf{I}_N)^{-1}, \\ \mathbf{G}_{\text{TX}}^{(n)} &= \frac{1}{p_{\text{MMSE}}^{(n)}} (\mathbf{H}^H \mathbf{H} + \frac{\mathbf{I}_N}{P_R})^{-1} \mathbf{H}^H. \end{aligned} \quad (3.14)$$

$\mathbf{R}_s = \mathbb{E}\{\mathbf{s}\mathbf{s}^H\}$ is the covariance matrix of the transmitted signal. p_{MMSE} is used to fulfill the power constraint. Accordingly, the MMSE transceive beamforming matrix is given by

$$\mathbf{G}_{\text{MMSE}}^{(n)} = \mathbf{G}_{\text{TX}}^{(n)} \mathbf{P}^n \mathbf{G}_{\text{RX}}. \quad (3.15)$$

3.2.3 Matched Filter Design

MF beamforming is the optimal linear beamforming for maximizing the SNR in the presence of additive noise. The MF receive beamforming matrix and transmit beamforming matrix are

$$\begin{aligned}\mathbf{G}_{\text{RX}} &= \mathbf{R}_s \mathbf{H}^H (\mathbf{H} \mathbf{R}_s \mathbf{H}^H + \mathbf{I}_N)^{-1}, \\ \mathbf{G}_{\text{TX}}^{(n)} &= \frac{1}{p_{\text{MF}}^{(n)}} \mathbf{H}^H.\end{aligned}\quad (3.16)$$

p_{MF} is used to fulfill the power constraint. Correspondingly, the MF transceive beamforming matrix is given by

$$\mathbf{G}_{\text{MMSE}}^{(n)} = \mathbf{G}_{\text{TX}}^{(n)} \mathbf{P}^n \mathbf{G}_{\text{RX}}. \quad (3.17)$$

3.3 PZF Relay Beamforming Design

Based on the ZF relay beamforming design idea, we propose a novel relay beamforming design called PZF. In this section, we first explain the idea of PZF, then formulate the optimization problem of the PZF relay beamforming based on the sum-rate maximization. A numerical method called modified gradient-ascent method is proposed to solve the optimization problem. Finally, simulation results on the performance of PZF and the comparison with existing beamforming designs are given.

3.3.1 PZF: Main Idea

In the ZF relay beamforming design of [11], in all $N - 1$ BC transmission time slots, the relay beamforming matrices are designed such that at each user, the transmitted signals of all users, except the desired one, are forced to be zero. For instance, if u_k wants to receive u_i 's message in a BC time slot, all interference signals from $u_j, j \neq i$, (i.e. all terms in (3.6) containing $s_j, j \neq i$) are forced to zero by the

relay beamforming. This puts heavy constraints on the relay beamforming matrices $\mathbf{G}_{\text{ZF}}^{(1)}, \dots, \mathbf{G}_{\text{ZF}}^{(N-1)}$, i.e., for each $\mathbf{G}_{\text{ZF}}^{(n)}$, $N(N-1)$ entries of $\mathbf{H}^T \mathbf{G}_{\text{ZF}}^{(n)} \mathbf{H}$ must be zero, as can be seen in (3.12).

However, it is not necessary to force the interference from all users other than the desired one to zero to obtain interference-free observations at the users. Knowing its own information and the channel state information, every user can conduct self-interference cancellation. In addition, for the n -th BC time slot, since every user has already decoded the symbols of $n-1$ users in the previous $n-1$ BC time slots, it can cancel the interference from these users without the help of the relay. Thus, the relay beamforming matrix for the n -th BC time slot only needs to be designed to cancel the interference from the remaining $N-n-1$ users. This constraint relaxation, which we refer to as PZF, allows more degrees-of-freedom in the design of the relay beamforming matrices. The extra degrees-of-freedom can be used to improve the sum-rate.

In order to better illustrate the PZF design idea and to help the analysis later, we define

$$\mathbf{A}^{(n)} = \mathbf{H}^T \mathbf{G}^{(n)} \mathbf{H}. \quad (3.18)$$

With ZF, $\mathbf{A}^{(n)}$ should be equal to the permutation matrix $\mathbf{P}^{(n)}$ in (3.12), where $(N^2 - N)$ of the entries are zero and N of the entries are 1. But with PZF, only $(N-n-1)N$ of the entries are zero and other entries can take any complex number.

Take $N=3$ for an example. If ZF beamforming is used at the relay, $\mathbf{G}^{(1)}$ and $\mathbf{G}^{(2)}$ should be designed so that $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ have the following forms:

$$\mathbf{A}_{\text{ZF}}^{(1)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \mathbf{A}_{\text{ZF}}^{(2)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad (3.19)$$

Both $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ should have 6 zero-value entries, which means all the interference signals except the desired one are canceled through ZF relay beamforming.

If PZF beamforming is used at the relay, $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ are supposed to have the

following forms:

$$\mathbf{A}_{\text{PZF}}^{(1)} = \begin{pmatrix} * & * & 0 \\ 0 & * & * \\ * & 0 & * \end{pmatrix}, \mathbf{A}_{\text{PZF}}^{(2)} = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}, \quad (3.20)$$

where “*” means that the entry can take any complex number. The restrictions on $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ are reduced. Unlike ZF, “*” does not need to be 1. Besides Only 3 entries in $\mathbf{A}^{(1)}$ should be zero and all the entries in $\mathbf{A}^{(1)}$ can take any complex number. In the first BC time slot, the relay only needs to cancel part of the interferences, the rest part can be canceled through self-interference cancellation at the users. In the second BC time slot, the relay entirely leaves the interferences to the users. The users have knowledge of their own information and already decode the information symbols received in the first BC time slot. Thus all the interferences can be canceled by the users.

3.3.2 PZF: Formulation

In this section, we formulate the PZF idea in the relay beamforming design and specify the relay beamforming matrix optimization problem. From (3.8)-(3.11), the sum-rate maximization problem can be stated mathematically as

$$\max_{\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(N-1)}} \sum_{i=1}^N \min_{k \neq i} \left\{ \log_2 \left(1 + \frac{P_i |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_i|^2}{|\mathbf{h}_k^T \mathbf{G}^{(n)}|^2 + 1} \right) \right\} \quad (3.21)$$

$$\text{s.t.} \quad \text{tr} \left\{ \mathbf{G}^{(n)} (\mathbf{H} \mathbf{P}_s \mathbf{H}^H + \mathbf{I}) (\mathbf{G}^{(n)})^H \right\} \leq P_R, \quad (3.22)$$

$$\text{and} \quad \mathbf{H}^T \mathbf{G}^{(n)} \mathbf{H} = \mathbf{A}^{(n)}. \quad (3.23)$$

The non-linear constraints in (3.22) are due to the transmit power constraint of the relay. The linear constraints in (3.23) are for PZF.

This sum-rate maximization problem is a multi-dimensional non-linear optimization problem with linear and non-linear constraints. We first simplify the problem using transformation. The optimization variables are beamforming matrices

$\mathbf{G}^{(1)}, \dots, \mathbf{G}^{(N-1)}$. After applying the transformation in (3.18), the problem can be converted to an optimization over $\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(N-1)}$. The transformation between $\mathbf{G}^{(n)}$ and $\mathbf{A}^{(n)}$ are one to one correspondent. $\mathbf{G}^{(n)}$ can be calculated from $\mathbf{A}^{(n)}$ using

$$\mathbf{G}^{(n)} = (\mathbf{H}^T)^{-1} \mathbf{A}^{(n)} \mathbf{H}^{-1}. \quad (3.24)$$

This transformation eliminates the linear constraints thus simplifies the optimization problem. The constraints on $\mathbf{A}^{(n)}$ due to the PZF design are simpler and more direct than those on $\mathbf{G}^{(n)}$.

Denote the (i, j) -th element of $\mathbf{A}^{(n)}$ as $a_{ij}^{(n)}$. With the ZF design in [11], $\mathbf{A}^{(n)} = \mathbf{P}^n$. With the proposed PZF idea, the constraints on $\mathbf{A}^{(n)}$ can be relaxed. To express the PZF constraints on $\mathbf{A}^{(n)}$ clearly, a set of 3-tuples of indices are introduced. Define

$$\mathbb{A} = \left\{ (i, j, n) \left| \begin{array}{l} n = 1, 2, \dots, N-2; \\ i = 1, 2, \dots, N; \\ q = 1, 2, \dots, N-n-1 \\ j = \text{mod}_N(i+q+n-1)+1, \end{array} \right. \right\}, \quad (3.25)$$

which is a subset of the 3-tuples of the indices (i, j, n) representing the receiving user, the transmitting/interfering user, and the BC time slot. A tuple is an element of \mathbb{A} if in the n -th BC time slot, the interference of u_j to u_i needs to be canceled under the PZF design.

The sum-rate maximization problem is thus transformed as

$$\max_{\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(N-1)}} \sum_{i=1}^N \min_{k \neq i} \left\{ \log_2 \left(1 + \frac{P_i |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_i|^2}{|\mathbf{h}_k^T \mathbf{G}^{(n)}|^2 + 1} \right) \right\} \quad (3.26)$$

$$\text{s.t.} \quad \text{tr} \left\{ \mathbf{G}^{(n)} (\mathbf{H} \mathbf{P}_s \mathbf{H}^H + \mathbf{I}) (\mathbf{G}^{(n)})^H \right\} \leq P_R, \quad (3.27)$$

$$\text{and} \quad a_{ij}^{(n)} = 0, \text{ for } (i, j, n) \in \mathbb{A}. \quad (3.28)$$

The conditions in (3.28) are for PZF. If the non-zero entries in $\mathbf{A}^{(n)}$ only are used as optimization variables, the linear constraints in (3.28) are eliminated.

3.3.3 Joint Optimization of the Relay Beamforming Matrices

In this subsection, we provide a numerical method to jointly optimize all $\mathbf{A}^{(n)}$ matrices. We define

$$\mathbf{x}^{(n)} = [a_{11}^{(n)} \ a_{12}^{(n)} \ \cdots \ a_{ij}^{(n)} ((i, j, n) \notin \mathbb{A}) \ \cdots \ a_{N,N}^{(n)}], \quad (3.29)$$

$$\mathbf{x} = [\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(n)}, \dots, \mathbf{x}^{(N-1)}]. \quad (3.30)$$

The U_n -dimensional vectors $\mathbf{x}^{(n)}$, where

$$U_n = (n + 1)N, \text{ for } n = 1, 2, \dots, N - 2, \quad (3.31)$$

includes all the nonzero entries in $\mathbf{A}^{(n)}$. The W -dimensional vector \mathbf{x} , where

$$W = (N + 2)N(N - 1)/2, \quad (3.32)$$

is formed by concatenating all the vectors $\mathbf{x}^{(n)}$. It contains $a_{ij}^{(n)}$'s for $(i, j, n) \notin \mathbb{A}$.

The optimization problem in (3.26) can be written as an optimization problem over \mathbf{x} and the constraints in (3.28) are naturally eliminated. Since the objective function in (3.26) is non-convex and the constraints in (3.27) are non-linear, the solution is in general difficult to find. A common method for finding sub-optimal solutions for such problems is to use the gradient-ascent method. However conventional gradient-ascent method does not work well in our case because of the complicated non-linear constraint. By moving toward the gradient direction even with a small step size, the new \mathbf{x} -vector may violate the constraint. To avoid this, we propose a modification to the gradient-ascent method. Our *modified gradient-ascent method* updates the \mathbf{x} -vector toward the direction of the *modified gradient*, specified in what follows.

3.3.3.1 Modified Gradient

Denote the objective function of in (3.26) as $f(\mathbf{x})$ and denote the power constraint in (3.27) as $\phi(\mathbf{x}^{(n)}) \leq P_R$, where

$$\phi(\mathbf{x}^{(n)}) = \phi(\mathbf{A}^{(n)}) = \text{tr} \left\{ \mathbf{G}^{(n)} (\mathbf{H}\mathbf{P}_s\mathbf{H}^H + \mathbf{I}) (\mathbf{G}^{(n)})^H \right\}. \quad (3.33)$$

The optimization problem becomes

$$\max_{\mathbf{x}} f(\mathbf{x}) \quad \text{s.t.} \quad \phi(\mathbf{x}^{(n)}) \leq P_R. \quad (3.34)$$

Let \mathbf{e}_l be the l -th canonical basis vector. Define the power normalization factors as

$$\alpha_m^{\text{Re}} = \frac{\phi(\mathbf{x}^{(n)} + \epsilon \mathbf{e}_l)}{P_R} \quad \text{and} \quad \alpha_m^{\text{Im}} = \frac{\phi(\mathbf{x}^{(n)} + i\epsilon \mathbf{e}_l)}{P_R}. \quad (3.35)$$

Notice that from the definitions in (3.29)-(3.30) the m -th element in \mathbf{x} is the l -th element of $\mathbf{x}^{(n)}$ with the following relationship: $m = 2N + \dots + nN + l$. The modified partial derivative of f with respect to the m -th element of \mathbf{x} is given by

$$\begin{aligned} d(f, x_m) = & \lim_{\epsilon \rightarrow 0} \frac{f(\mathbf{x}^{(1)}, \dots, \frac{\mathbf{x}^{(n)} + \epsilon \mathbf{e}_l}{\alpha_m^{\text{Re}}}, \dots, \mathbf{x}^{(N-1)}) - f(\mathbf{x})}{\epsilon} \\ & + i \lim_{\epsilon \rightarrow 0} \frac{f(\mathbf{x}^{(1)}, \dots, \frac{\mathbf{x}^{(n)} + i\epsilon \mathbf{e}_l}{\alpha_m^{\text{Im}}}, \dots, \mathbf{x}^{(N-1)}) - f(\mathbf{x})}{\epsilon}. \end{aligned} \quad (3.36)$$

Compared with the definition of normal partial derivative

$$\begin{aligned} \frac{\partial f}{\partial x_m} = & \lim_{\epsilon \rightarrow 0} \frac{f(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(n)} + \epsilon \mathbf{e}_l, \dots, \mathbf{x}^{(N-1)}) - f(\mathbf{x})}{\epsilon} \\ & + i \lim_{\epsilon \rightarrow 0} \frac{f(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(n)} + i\epsilon \mathbf{e}_l, \dots, \mathbf{x}^{(N-1)}) - f(\mathbf{x})}{\epsilon}, \end{aligned} \quad (3.37)$$

this definition takes into account the non-linear constraint $\phi(\mathbf{x}^{(n)}) \leq P_R$. When $\mathbf{x}^{(n)}$ is modified to $\mathbf{x}^{(n)} + \epsilon \mathbf{e}_l$ or $\mathbf{x}^{(n)} + i\epsilon \mathbf{e}_l$, to make sure that the constraint is not violated, the vector is scaled by α_m^{Re} or α_m^{Im} , whose definition guarantees the constraint. Thus,

the modified gradient takes into account the effect of the change of one element of \mathbf{x} to all elements due to the constraint. The modified gradient of f is thus

$$D(f, \mathbf{x}) = [d(f, x_1) \cdots d(f, x_m) \cdots d(f, x_W)]. \quad (3.38)$$

3.3.3.2 Optimization Algorithm

In our numerical method, the \mathbf{x} -vector is updated toward the modified gradient with a step size α . Also, scaling is done at every iteration to guarantee each searched point satisfies the constraint. In other words, a new point is found by two moves. First, a move of \mathbf{x} is made proportional to the modified gradient. Second, constructed from \mathbf{x} , $\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(N-1)}$ are scaled to make the power constraint satisfied. \mathbf{x} is moved to the new point accordingly. Once a solution of \mathbf{x} is found, we can reconstruct $\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(N-1)}$ and from (3.24) calculate the solutions for $\mathbf{G}^{(1)}, \dots, \mathbf{G}^{(N-1)}$. It should be noted that similar to the gradient-ascent method, the proposed modified gradient-ascent method cannot guarantee the global optimality of the solution. However, we can use ZF relay beamforming matrices as the initial point to guarantee a better solution. The algorithm is described in Algorithm 1.

Algorithm 1 Joint optimization scheme.

- 1: Initialize $\alpha, tolerance, \mathbf{A}^{(n)}, \mathbf{x}$ and calculate $D(f, \mathbf{x})$.
 - 2: **while** $norm(D(f, \mathbf{x})) \geq tolerance$ **do**
 - 3: Update \mathbf{x} : $\mathbf{x} = \mathbf{x} + \alpha D(f, \mathbf{x})$.
 - 4: Construct $\mathbf{A}^{(n)}$ from \mathbf{x} .
 - 5: Scale $\mathbf{A}^{(n)}$ based on the constraint and construct \mathbf{x} .
 - 6: Calculate $D(f, \mathbf{x})$.
 - 7: **end while**
 - 8: Calculate $\mathbf{G}^{(1)}, \dots, \mathbf{G}^{(N-1)}$ using (3.24).
-

3.3.4 Separate Optimization of the Relay Beamforming Matrices

In the method described in Section 3.3.3, the matrices $\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(N-1)}$ are optimized jointly. Algorithm 1 can be computationally expensive for large MWRNs. In

this section, we propose to use separate optimization where the optimization over $\mathbf{A}^{(n)}$'s for $n = 1, \dots, N - 1$ is conducted separately and sequentially.

Notice that the relay beamforming matrix for the n -th BC time slot $\mathbf{G}^{(n)}$ directly affects the transmission rates $R_{k,i}$ during this phase, where k, i and n satisfy the relation in (3.5). It does not affect the transmission rates of the previous BC time slot or the later BC time slots if ideal source coding and detection are assumed. Thus we propose to optimize $\mathbf{G}^{(n)}$ or equivalently $\mathbf{A}^{(n)}$ by maximizing the sum-rate in the n -th BC time slot:

$$R_{\text{sum}}^{(n)} = \sum_{i=1}^N \log_2 \left(1 + \frac{P_i |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_i|^2}{|\mathbf{h}_k^T \mathbf{G}^{(n)}|^2 + 1} \right). \quad (3.39)$$

The optimization problem for $\mathbf{A}^{(n)}$ is thus

$$\max_{\mathbf{x}^{(n)}} R_{\text{sum}}^{(n)} \quad \text{s.t.} \quad \phi(\mathbf{x}^{(n)}) \leq P_R. \quad (3.40)$$

In solving the optimization problem in (3.40), the same modified gradient-ascent method is used.

Considering that the number of constraints on $\mathbf{A}^{(n)}$ decreases as n increases, we optimize $\mathbf{A}^{(n)}$'s sequentially with $\mathbf{A}^{(1)}$ being the first and $\mathbf{A}^{(N-1)}$ being the last. The algorithm for separate optimization is given in Algorithm 2. The separate optimization method has a lower computational complexity than the joint optimization method.

3.4 Simulation Results

In this section, we show simulation results on the MWRN sum-rate of our PZF design and existing designs [64]. Unicasting is applied in the relay. We choose $N = 3$, i.e. 3 single-antenna users communicate with each other with the help of a relay equipped with 3 antennas.

First we consider a homogeneous network where all channels follow i.i.d.

Algorithm 2 Separate optimization scheme.

- 1: Initialize α and *tolerance*.
 - 2: **for** $n = 1 : N - 1$ **do**
 - 3: Initialize $\mathbf{A}^{(n)}$, $\mathbf{x}^{(n)}$ and calculate $D(R_{\text{sum}}^{(n)}, \mathbf{x}^{(n)})$.
 - 4: **while** $\text{norm}(D(R_{\text{sum}}^{(n)}, \mathbf{x}^{(n)})) \geq \text{tolerance}$ **do**
 - 5: Update $\mathbf{x}^{(n)}$: $\mathbf{x}^{(n)} = \mathbf{x}^{(n)} + \alpha D(R_{\text{sum}}^{(n)}, \mathbf{x}^{(n)})$.
 - 6: Construct $\mathbf{A}^{(n)}$ from $\mathbf{x}^{(n)}$.
 - 7: Scale $\mathbf{A}^{(n)}$ and construct $\mathbf{x}^{(n)}$.
 - 8: Calculate $D(R_{\text{sum}}^{(n)}, \mathbf{x}^{(n)})$.
 - 9: **end while**
 - 10: **end for**
 - 11: Calculate $\mathbf{G}^{(1)}, \dots, \mathbf{G}^{(N-1)}$ using (3.24).
-

$\mathcal{CN}(0, \sigma_h^2)$. We set $P_R = P_1 = P_2 = P_3 = 1$, thus the signal-to-noise ratio (SNR) of each user's signal power to the noise power at the relay is σ_h^2 , i.e., $\text{SNR} = \sigma_h^2$. Figure 3.3 shows the sum-rate for different SNR values. We can see that the proposed PZF design has the best sum-rate performance for the whole SNR range. It can also be observed that for the proposed PZF scheme, the separate optimization and the joint optimization give very close sum-rate performance with the latter slightly better.

In addition, we consider a 3-user MWRN with non-identical fading channels due to different path-loss. Denote d_i as the distance from u_i to the relay. The channels between u_i and the N relay antennas $h_{i,n}$'s are assumed to follow $\mathcal{CN}(0, \sigma_i^2)$, where $\sigma_i^2 = (\phi/d_i)^\nu$, ϕ is a constant. In simulations, we set $d_3 = 2d_2 = 4d_1$ and assume $\nu = 2$. The x-axis, denoted as SNR, is u_1 's SNR at the relay, thus $\text{SNR} = \sigma_1^2 = \frac{1}{4}\sigma_2^2 = \frac{1}{16}\sigma_3^2$. Since the network is heterogeneous, the decoding order may affect the sum-rate. Thus, we consider 2 orderings: clockwise as defined in (3.5) and counter clockwise defined as $i = \text{mod}_N(k - n - 1) + 1$. We can see from Figure 3.4 that the proposed PZF design achieves a significantly higher sum-rate than ZF designs. For both clockwise and counter clockwise orderings, ZF provides exactly the same sum-rate performance. For PZF, clockwise and counter clockwise orderings offer slightly different performances.

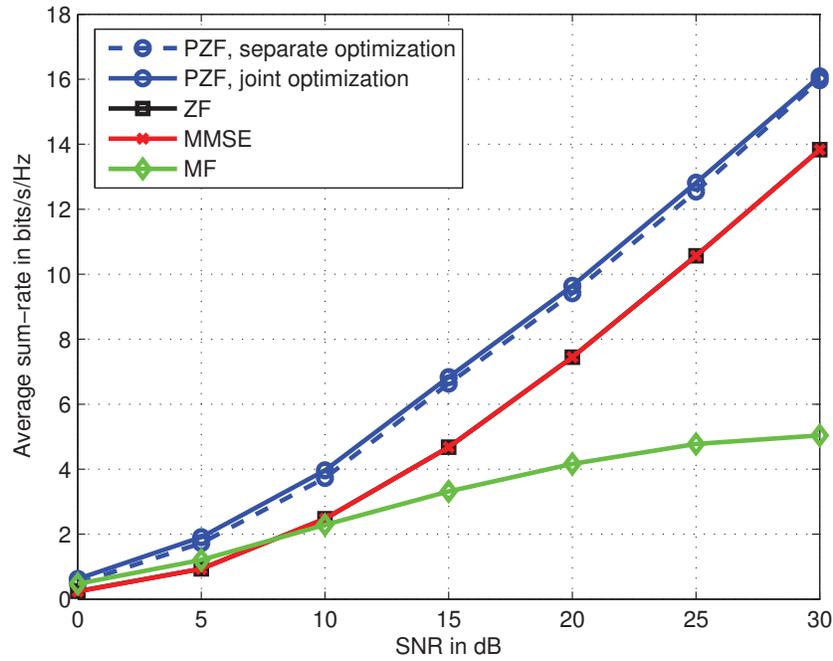


Fig. 3.3. Sum-rates for a homogeneous 3-user MWRN. $P_R = 1$.

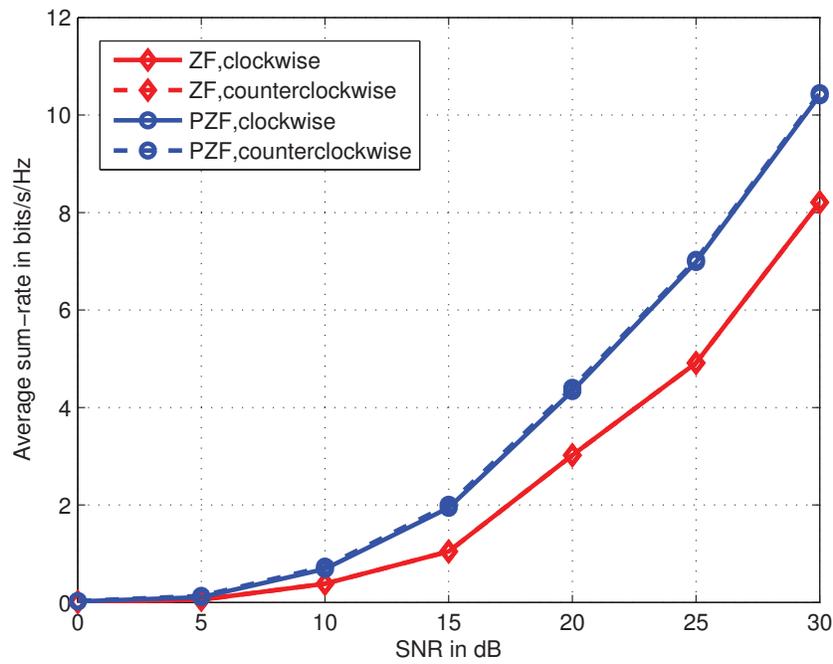


Fig. 3.4. Sum-rates for a heterogeneous 3-user MWRN. $P_R = 1$.

Chapter 4

PZF with Hybrid Uni/Multicasting

In Chapter 3, only unicasting is considered, where in each BC time slot, the relay transmits different information symbols to different users. Each information symbol is intended only for one receiving user. In this chapter the hybrid uni/multicasting strategy is considered. When the relay uses uni/multicasting strategy, PZF is still able to improve the sum-rate performance of MWRNs.

4.1 Hybrid Uni/Multicasting Strategy

Along with the unicasting strategy, hybrid uni/multicasting is also proposed in [62]. If hybrid uni/multicasting strategy is used, in each BC time slot, one information symbol is transmitted to one user exclusively (unicast transmission) and one information symbol is transmitted to the other $N - 1$ users (multicast transmission). The unicasted information symbol is fixed for all BC time slots. In different BC time slots, it is transmitted to a different user. While the multicasted information symbols are changed in different BC time slots. This hybrid uni/multicasting scheme ensures each user receives all other users' information within the $N - 1$ BC time slots.

A 3-user example is shown in Figure 4.1. In the MAC phase, u_1 sends s_1 , u_2 sends s_2 and u_3 sends s_3 simultaneously to the relay. In the BC phase, s_1 is chosen as the unicasting symbol, s_2 and s_3 are chosen as the multicasting symbols for the

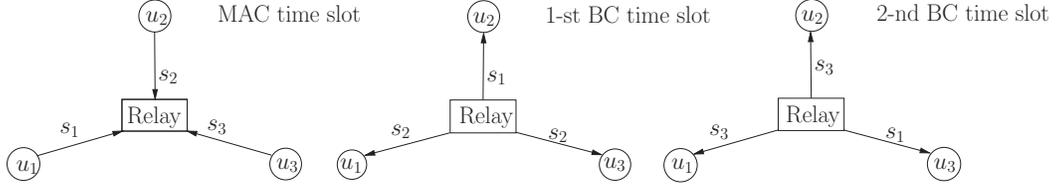


Fig. 4.1. Hybrid uni/multicasting strategy.

first and second time slots of the BC phase, respectively. In the first time slot of the BC phase, u_1 decodes s_2 , u_2 decodes s_1 and u_3 decodes s_2 from the relay broadcast signal. In the second time slot of the BC phase, u_1 decodes s_3 , u_2 decodes s_3 and u_3 decodes s_1 . After the MAC phase and the BC phase, each user decodes the information symbols from all other users.

PZF can naturally be extended to hybrid uni/multicasting strategy. The sum-rate maximization problem in this scenario can be solved by the modified gradient-ascent method proposed in Chapter 3. The only difference in problem formulation is that the structures of $\mathbf{A}^{(n)}$ matrices (or the location of zero entries in $\mathbf{A}^{(n)}$) need to be adjusted based on the uni/multicasting strategy. For example, for the aforementioned 3-user network, the $\mathbf{A}^{(n)}$ matrices for PZF should have the following structures.

$$\mathbf{A}_{\text{PZF}}^{(1)} = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & * & * \end{pmatrix}, \mathbf{A}_{\text{PZF}}^{(2)} = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}. \quad (4.1)$$

4.2 Performance Comparison

This section shows simulation results on the MWRN sum-rate of our PZF design and ZF design with hybrid uni/multicasting. $N = 3$ is chosen, i.e. 3 single-antenna users communicate with each other with the help of a relay equipped with 3 antennas. We consider a homogeneous network where all channels follow i.i.d. $\mathcal{CN}(0, \sigma_h^2)$. We set $P_R = P_1 = P_2 = P_3 = 1$. Figure 4.2 shows the sum-rate for different SNR values. We can see that the hybrid uni/multicasting PZF design has a better sum-rate performance than the hybrid uni/multicasting ZF design for

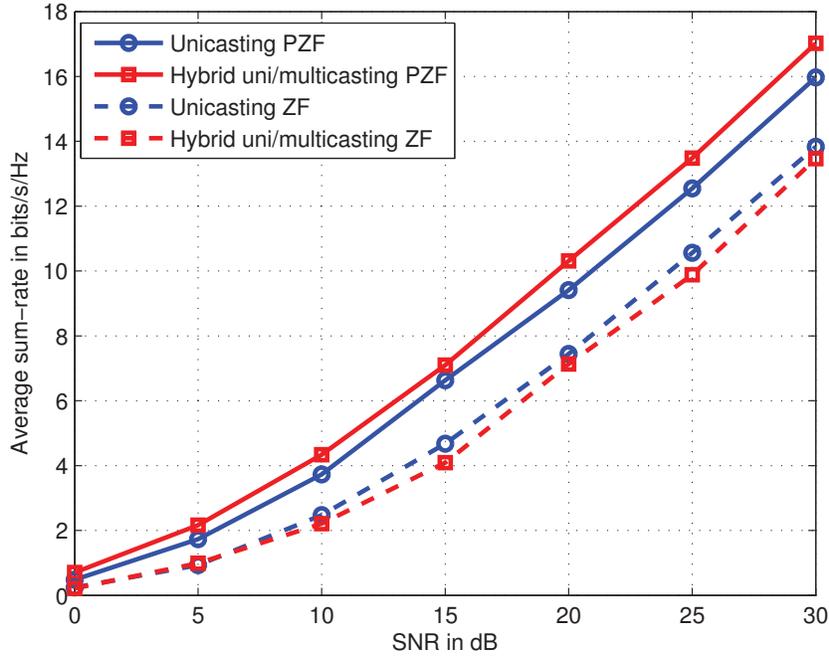


Fig. 4.2. Sum-rates of unicasting strategy and hybrid uni/multicasting strategy for a 3-user MWRN.

the whole SNR range. It can also be observed that when hybrid uni/multicasting is used, the sum-rate performance gap between PZF design and ZF design becomes larger.

4.3 Scheduling of Detections

Unlike unicast model, hybrid uni/multicast strategy brings some imbalance in different users' signals. For MWRNs with asymmetric channel conditions, the scheduling of detections in the BC phase, in other words the choices of signals for unicasting and multicasting in different BC time slots, may affect the sum-rate performance. In Figure 4.1, s_2 is chosen as the multicasted symbol in the first BC time slot, and s_3 is chosen as the multicasted symbol in the second BC time slot. Consequently u_1 decodes s_2 first and then s_3 . A different scheduling of detections is shown in Figure 4.3. s_3 is multicasted in the first BC time slot, s_2 is multicasted in the second BC time slot. Correspondingly, u_1 decodes s_3 first and then s_2 .

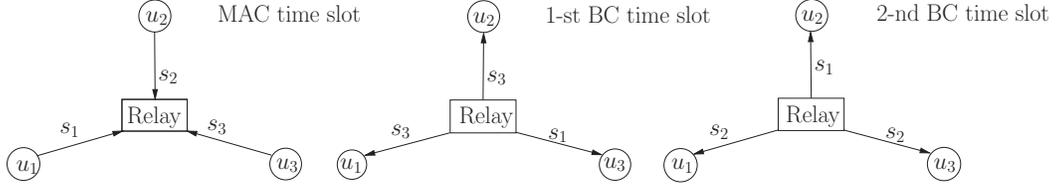


Fig. 4.3. Another scheduling of detections for hybrid uni/multicasting strategy.

Simulation results on different scheduling of detections are given. We consider a 3-user MWRN with non-identical fading channels due to different path-loss. Denote d_i as the distance from u_i to the relay. The channels between u_i and the N relay antennas $h_{i,n}$'s are assumed to follow $\mathcal{CN}(0, \sigma_i^2)$, where $\sigma_i^2 = (\phi/d_i)^\nu$ with ϕ a constant. In simulations, we set $d_3 = 2d_2 = 2d_1$ and assume $\nu = 2$. The x-axis, denoted as SNR, is u_1 's SNR at the relay, thus $\text{SNR} = \sigma_1^2 = \sigma_2^2 = \frac{1}{4}\sigma_3^2$. We consider 2 schedulings: one is described in Figure 4.1, it is denoted as hybrid uni/multicasting-1; the other is shown in Figure 4.3, it is denoted as hybrid uni/multicasting-2. From Figure 4.4, we can see that when PZF is applied at the relay, hybrid uni/multicasting-1 has higher sum-rate than hybrid uni/multicasting-2. An explanation is that in the simulation settings, the channel between u_3 and the relay is weaker than the other two. Hybrid uni/multicasting-1 chooses to decode the weakest s_3 in the last BC time slot.

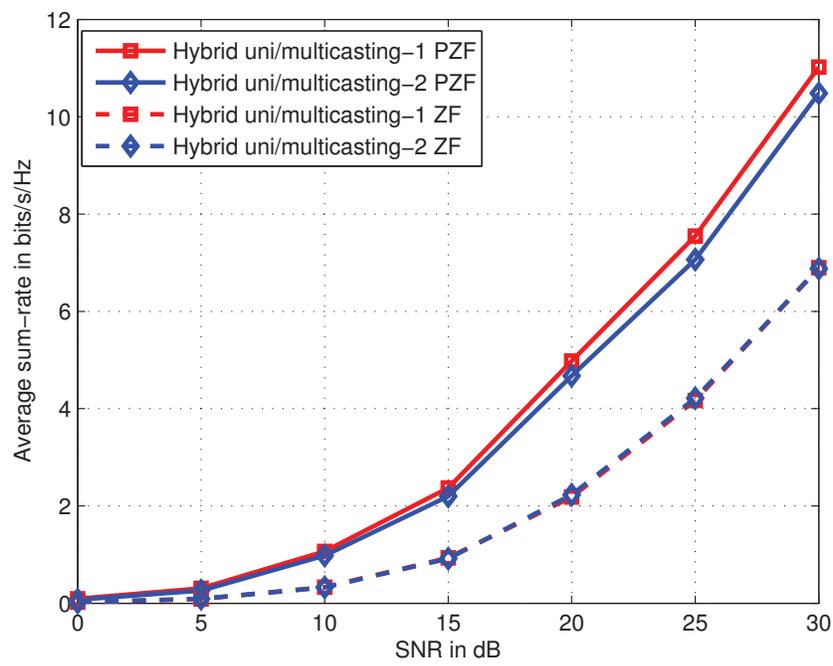


Fig. 4.4. Sum-rates of hybrid uni/multicasting strategy using different decoding schedule for a 3-user MWRN.

Chapter 5

PZF with General Numbers of Users and Relay Antennas

In ZF beamforming, there is a constraint that the number of relay antennas M is larger than or at least the same as the number of users N , i.e. $M \geq N$. Otherwise, there are not enough degrees-of-freedom to remove user interference [65]. In PZF, because the interference does not need to be fully canceled, the number of antennas at the relay can be reduced by one. That is, PZF can be used for MWRNs where $M \geq N - 1$. The case of $M = N$ has been considered in Chapters 3 and 4. In this chapter, we consider MWRNs where $M \geq N - 1$ but $M \neq N$. The case of $M > N$ is investigated in Section 5.1 while the case of $M = N - 1$ is investigated in Section 5.2.

5.1 MWRNs with the Number of Relay Antennas Larger than the Number of Users

In this section, we consider the case when number of relay antennas M is larger than the number of users N . The PZF sum-rate maximization problem in this scenario can be solved by the modified gradient-ascent method proposed in Chapter 3. However it should be noticed that in this case, \mathbf{H} is an M -by- N ($M > N$) matrix,

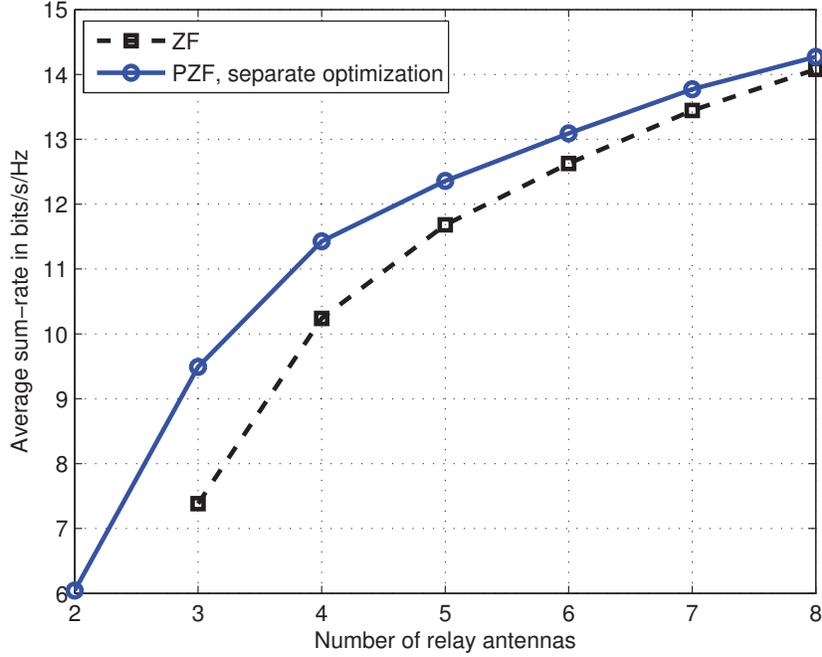


Fig. 5.1. Sum-rates of PZF and ZF with unicasting strategy for different numbers of relay antennas, SNR=20 dB.

thus both \mathbf{H} and \mathbf{H}^H are not invertible. Moore-Penrose pseudoinverse can be used to replace the inverse operation in (3.24). Using the pseudoinverse of \mathbf{H} and \mathbf{H}^H , $\mathbf{G}^{(n)}$ is given by

$$\mathbf{G}^{(n)} = (\mathbf{H}^T)^+ \mathbf{A}^{(n)} \mathbf{H}^+, \quad (5.1)$$

which satisfies (3.18).

Sum-rates of MWRNs with different number of relay antennas are simulated. Unicasting is applied at the relay. We set $P_R = P_1 = P_2 = P_3 = 1$ and SNR=20 dB. The channels are homogeneous and follow i.i.d. $\mathcal{CN}(0, \sigma_h^2)$. Figure 5.1 and Figure 5.2 show the relation between sum-rate and the number of antennas at the relay. In Figure 5.1, the numbers of relay antennas are in linear scale, while in Figure 5.2 they are in log-scale. From the figures, we can draw the conclusion that sum-rate increases logarithmically with respect to the increase of relay antenna numbers. Also, extra diversity gain provided by newly added antennas decreases the SINRs and then increases the sum-rate. It can be seen in Figure 5.1 that the gap

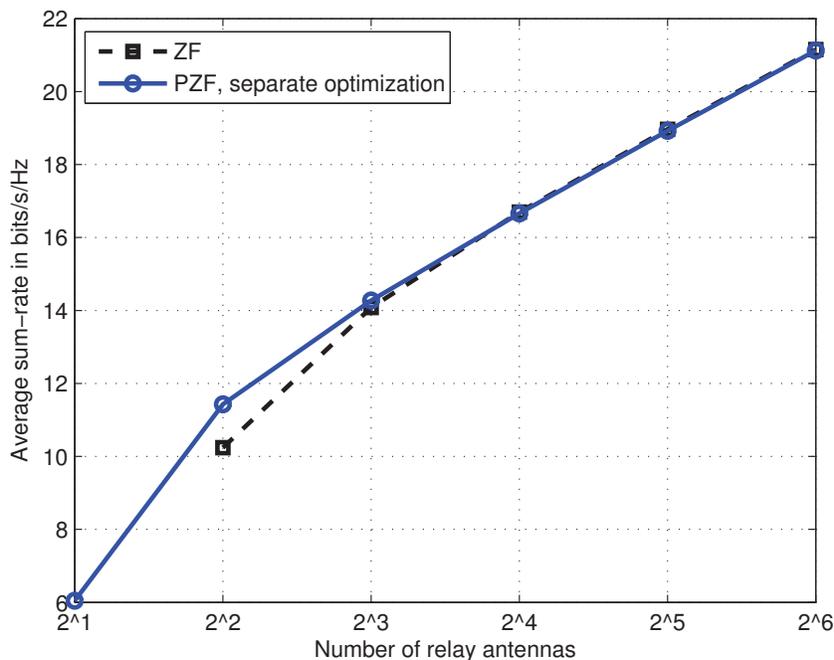


Fig. 5.2. Sum-rates of PZF and ZF with unicasting strategy for different numbers of relay antennas in logarithmic scale, SNR=20 dB.

between PZF and ZF gets smaller as the number of relay antennas increases.

5.2 MWRNs with the Number of Relay Antennas Equal to the Number of Users Minus One

This section is on MWRNs where the number of relay antennas is one less than the number of users, i.e, $M = N - 1$. The transceiver protocol is the same as before. Again, there are 1 MAC time slot and $N - 1$ BC time slots for the multi-way communications. With PZF in each BC time slot, for each user only partial interference (interference excluding self-interference and previously decoded signal-interference) needs to be canceled.

But the problem formulation of PZF relay beamforming design for the $M = N - 1$ case is largely different from the original one. Because the number of relay antennas is smaller than the number of users, the dimension of $\mathbf{G}^{(n)}$ is lower than the

dimension of $\mathbf{A}^{(n)}$. As a result, the map from $\mathbf{A}^{(n)}$ to $\mathbf{G}^{(n)}$ in either (3.24) or (5.1) does not apply. Thus we cannot transform the variables in the sum-rate optimization from $\mathbf{G}^{(n)}$ to $\mathbf{A}^{(n)}$. For this reason, the optimization needs to be conducted with respect to $\mathbf{G}^{(n)}$ directly. The optimization problem formulation is as follows.

$$\max_{\mathbf{G}^{(1)}, \dots, \mathbf{G}^{(N-1)}} \sum_{i=1}^N \min_{k \neq i} \left\{ \log_2 \left(1 + \frac{P_i |\mathbf{h}_k^T \mathbf{G}^{(n)} \mathbf{h}_i|^2}{|\mathbf{h}_k^T \mathbf{G}^{(n)}|^2 + 1} \right) \right\} \quad (5.2)$$

$$\text{s.t.} \quad \text{tr} \left\{ \mathbf{G}^{(n)} (\mathbf{H} \mathbf{P}_s \mathbf{H}^H + \mathbf{I}) (\mathbf{G}^{(n)})^H \right\} \leq P_R, \quad (5.3)$$

$$\text{and} \quad \mathbf{H}^T \mathbf{G}^{(n)} \mathbf{H} = \mathbf{A}^{(n)}. \quad (5.4)$$

According to (3.31), there are $(N - n - 1)N$, $n = 1, 2, \dots, N - 2$, zero valued entries in $\mathbf{A}^{(n)}$. The rest of the entries in $\mathbf{A}^{(n)}$ can take any complex value. Equation (5.4) can be written as $(N - n - 1)N$ linear homogeneous equations. We define vector $\mathbf{g}^{(n)}$ that contains all entries in $\mathbf{G}^{(n)}$,

$$\mathbf{g}^{(n)} = [g_{11}^{(n)} \quad g_{12}^{(n)} \quad \dots \quad g_{1,N-1}^{(n)} \quad g_{21}^{(n)} \quad \dots \quad g_{N-1,N-1}^{(n)}]. \quad (5.5)$$

Divide $\mathbf{g}^{(n)}$ into two vectors, $\mathbf{y}^{(n)}$ and $\mathbf{r}^{(n)}$, where $\mathbf{y}^{(n)}$ contains the first $(N - 1)^2 - (N - n - 1)N$ entries of $\mathbf{g}^{(n)}$ and $\mathbf{r}^{(n)}$ contains the rest $(N - n - 1)N$ entries of $\mathbf{g}^{(n)}$. Since the number of entries in $\mathbf{r}^{(n)}$ is equal to the number of linear equations in (5.4), $\mathbf{r}^{(n)}$ can be uniquely represented by $\mathbf{y}^{(n)}$ from (5.4). At the same time, the constraints in (5.4) are eliminated.

From the above discussion, the sum-rate maximization problem is transformed into an optimization with respect to $\mathbf{y}^{(n)}$ with the power constraint in (5.3) only. The proposed modified gradient ascent method can be used. The detailed algorithm is given below, where for the complexity consideration, separate optimization of the relay beamforming matrices are considered.

Next, we show simulation results on the MWRN sum-rate of our PZF design and compare with the ZF design. Unicasting is applied in the relay. We set $P_R = P_1 = P_2 = P_3 = 1$ and the channels are homogeneous and follow i.i.d. $\mathcal{CN}(0, \sigma_h^2)$. Figure 5.3 shows the sum-rate for PZF design when the relay has 2 antennas. We

Algorithm 3 Separate optimization scheme for MWRNs where $M = N - 1$.

- 1: Initialize α and *tolerance*.
- 2: **for** $n = 1 : N - 1$ **do**
- 3: Initialize $\mathbf{y}^{(n)}$ and construct $\mathbf{G}^{(n)}$ by solving (5.4).
- 4: Scale $\mathbf{G}^{(n)}$ to satisfy (5.3) and construct $\mathbf{y}^{(n)}$.
- 5: Calculate $D(R_{\text{sum}}^{(n)}, \mathbf{y}^{(n)})$.
- 6: **while** $\text{norm}(D(R_{\text{sum}}^{(n)}, \mathbf{y}^{(n)})) \geq \text{tolerance}$ **do**
- 7: Update $\mathbf{y}^{(n)}$: $\mathbf{y}^{(n)} = \mathbf{y}^{(n)} + \alpha D(R_{\text{sum}}^{(n)}, \mathbf{y}^{(n)})$.
- 8: Construct $\mathbf{G}^{(n)}$ from $\mathbf{y}^{(n)}$ by solving (5.4).
- 9: Scale $\mathbf{G}^{(n)}$ to satisfy (5.3) and construct $\mathbf{y}^{(n)}$.
- 10: **end while**
- 11: **end for**

can observe that PZF with 3-antennas relay has the highest sum-rate. The sum-rate of PZF with 2-antennas relay is lower than the PZF and ZF with 3-antennas relay.

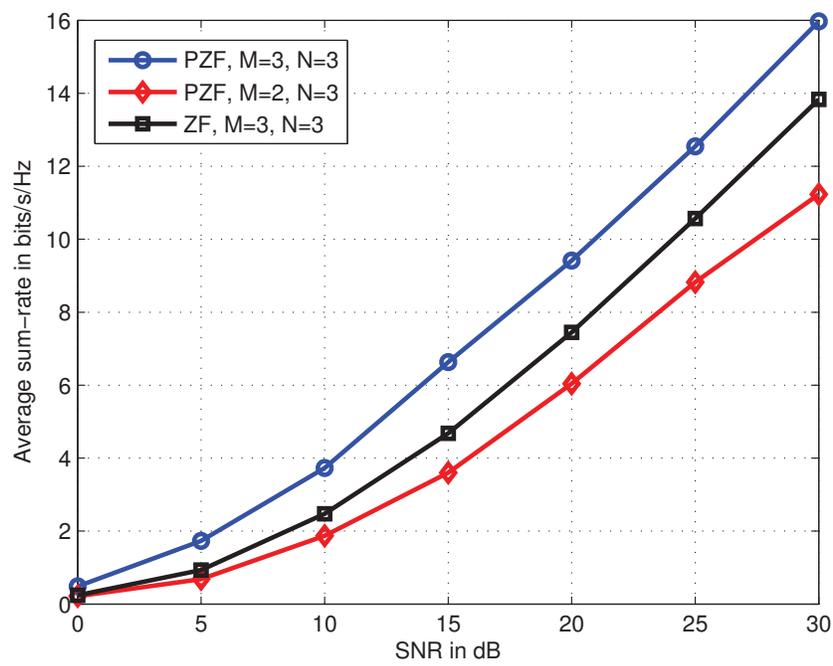


Fig. 5.3. Sum-rates of PZF and ZF with unicasting strategy for a 3-user MWRN, $M=2, 3$.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, a novel PZF relay beamforming design is proposed for MWRNs where single-antenna users communicate with each other with the help of one multiple-antenna relay. Compared with ZF relay beamforming, the proposed scheme allows more degrees-of-freedom in the beamforming optimization thus can improve the sum-rate. On the other hand, with the help of self-interference cancellation and successive interference cancellation, the proposed design enables interference-free communications. Modified gradient-ascent methods are proposed to solve the optimization problem jointly or separately. Simulation results show that the PZF design largely outperforms other existing designs in sum-rate.

6.2 Future Work

Since MWRNs is in its early stage of study, many aspects of them are still uninvestigated. For our PZF beamforming in MWRNs, there are some potential future research directions. Three of them are listed below.

6.2.1 Optimal Selection and Ordering of User Signals

In Chapter 4, we simulate the hybrid uni/multicasting relay MWRNs. The relay has the freedom to choose one from N users' signals as the unicasted signal. This selection may results in different SINR at users and leads to different sum-rate of the system. If the channels are asymmetric, the selection of unicasting signal and the order of multicasting signals will both influence the performance of PZF beamforming. Thus, based on the statistical and instantaneous channel conditions, how to choose the decoding order of the unicasting and multicasting signals are interesting problems.

6.2.2 Low Complexity Relay Beamforming Design

In this thesis, a numerical method is used to design the PZF beamforming matrix. Compared to the closed-form conventional linear beamforming method, PZF has more time complexity. Given by (3.32), the number of optimization variables in the PZF design is $W = (M + 2)M(M - 1)/2$, M is the number of antennas equipped at the relay. In consequence, situations are even worse when the number of relay antennas becomes larger. The beamforming design complexity may further leads to transmission delay and more power consumption. To address these problems, one may think of finding sub-optimal but low complexity or closed-form solutions for the PZF beamforming design problem.

6.2.3 Massive MIMO MWRNs

Massive MIMO is a promising technology for the next generation of cellular wireless networks. The basic idea is to set up a large number of antennas at base station such that the number of antennas is at least an order of magnitude larger than the number of served users. Extra antennas are used to focus energy within small angle of region. Therefore, the data rate and energy efficiency are dramatically improved. Other distinctive features of massive MIMO include enhanced reliability,

reduced interference, and can be built with low-complexity low-power amplifiers or RF chains.

In Section 5.2, we simulate the scenario when the number of relay antennas is larger than the number of users. Simulation results show that when the number of relay antennas becomes large, the sum-rate performance difference between PZF and ZF gets smaller. One may question whether the difference will still be noticeable if the relay in MWRNs is a massive MIMO station. Other design problems may come up if the relay antenna number becomes large. For example, finding low complexity PZF designs that are asymptotically optimal; designing new signal processing techniques and transmission strategies that result in better performance.

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