HETEROGENEOUS VEHICULAR PLATOONING WITH STABLE DECENTRALIZED LINEAR FEEDBACK CONTROL

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ABSTRACT

Platooning which is defined as controlling a group of autonomous vehicles (multiple followers and one leader) to have a desired distance between them while following a desired trajectory has caught on recently in the control engineering discipline. Platooning brings along promising advantages, namely, increasing highway capacity and safety, and reducing fuel consumption. In this paper, using linearized longitudinal dynamic models for each vehicle, we investigate the control problem of vehicular platooning to have all vehicles followed the leader under a constant spacing policy. Under decentralized linear feedback controllers and taking account of heterogeneity in the dynamic models and feedback information to the vehicles, a general dynamic representation for the platoon is obtained. Having this and the proposed controller, stability analysis is developed for any information flow topology (IFT) between vehicles and any number of vehicles. As a case study, a platoon with one leader and two followers is investigated through the proposed strategy, and its stability conditions are provided. Numerical simulations are provided in which the stability range of control gains and the effect of different IFTs on the performance of the platoon are discussed.

Index Terms— Autonomous vehicles, Platoon of vehicles, Stability, Heterogeneity, Information flow topology

1 Introduction

Intelligent transportation systems (ITS) leverage a high level of automation to provide an efficient and safe road transportation. Platooning, which corresponds to travel of a convoy of vehicles with an enforced desired spacing between them, can be subsumed under the ITS discipline. The promise of a reduction in vehicles' fuel consumption due to the decreased aerodynamic drag for back-to-back vehicles [1, 2], and an increased highway capacity and safety [3, 4, 5, 6, 7] warrant more research in this technology. Making sure that all platoon vehicles move at the same velocity as the leader vehicle while keeping a desired spacing among themselves underlies the platoon control problem.

Defining a desired inter-vehicle distance is specified by the spacing policy. Constant distance (CD) policy [8, 9] and constant time headway (CTH) policy [10] are the predominant policies studied in the literature. The CD policy, as its name implies, aims at maintaining a constant distance between consecutive vehicles. In the CTH policy, the spacing between vehicles is dependent on the velocity of the leader and thus no longer constant. Other policies are nonlinear distance policy [11] and delay-based distance policy [12].

From control perspective, dynamics of platoon is characterized by vehicle longitudinal dynamics, information flow topology (IFT), distributed controllers and the spacing policy of the platoon [13, 14]. See [15] to get a quick insight about these components. A platoon is called heterogeneous if the dynamics of the vehicles are not identical.

As linear feedback controllers (LFCs) are concerned, in [16] a decentralized LFC under identical control gains that benefit from position, velocity and acceleration measurements is proposed for a platoon of vehicles, under which the stability conditions for some certain IFTs are derived. In [17], a decentralized LFC is put forward that only utilizes position and velocity feedback signals, and the stability analysis is only applicable for bidirectional and bidirectional-leader IFTs. In [18], a distributed linear control under equal control gains that uses only position signals is devised for the IFT cases that was not addressed in the [16]. In this paper, we use a decentralized LFC with non-identical gains that position, velocity and acceleration of vehicles are fed back into the controllers. In this work and contrary to [16, 18], we incorporate the control gains and the way vehicles communicate with each other directly into the stability analysis of the overall platoon which, therefore, makes it applicable for any IFT, and can specify the stability ranges for the control gains. The adopted method can consider any IFT in the stability analysis, and is applicable for any number of vehicles.

2 Problem formulation

Figure 1 shows a platoon that has $N+1$ (not necessarily identical) vehicles such that the one designated by 0 is the leader

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vehicle and the others labeled by $1,\ldots,i,i{+}1,\ldots,N$ are the followers. The distance between the two consecutive vehicles i and $i{+}1$ is denoted by D_i^{i+1}, and L_i presents the length of the i^{th} follower vehicle. The x axis shows the position of the vehicles during their movement such that x_0 and x_i are the positions of the leader vehicle and the i^{th} follower, respectively. Generally speaking, longitudinal control of a platoon consists of 1) inner force/acceleration control loop, namely feedback linearization (FL) control that compensates for the nonlinear dynamics of the vehicles, and 2) an outer inter-vehicle distance control loop that is responsible for enforcing a desired spacing between the consecutive vehicles within the platoon according to the spacing policy. The FL control is based on the assumption that the vehicle dynamics and its parameters are fully known which means that a perfect nonlinear dynamics cancellation can be achieved. We assume that the FL part has already canceled the dynamics nonlinearities and therefore we will only focus on the inter-vehicle distance control loop. Consider that for platooning, and as far as the leader ve-

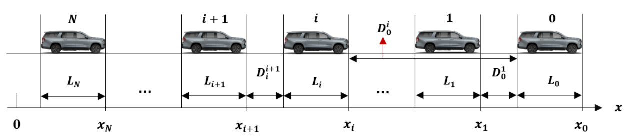

Fig. 1. A platoon with constant inter-vehicle spacing.

hicle is concerned, we only need its position, velocity and acceleration, and it does not undergo any control process. Given that, let the following formulation characterize the dynamics of the i^{th} follower vehicle [19]:

$$\dot{a}_i = f_i(v_i,a_i) + g_i(v_i)c_i \qquad i=1,\ldots,N \tag{1}$$

in which v_i and a_i are the velocity and acceleration of the i^{th} follower, and $f_i(v_i,a_i)$ and $g_i(v_i)$ are according to

$$\begin{aligned} f_i(v_i,a_i) &= -\frac{1}{\tau_i}\left(a_i + \frac{\sigma A_i C_{di} v_i^2}{2m_i} + \frac{d_{mi}}{m_i}\right) - \frac{\sigma A_i C_{di} v_i a_i}{m_i} \\ g_i(v_i) &= \frac{1}{\tau_i m_i} \end{aligned} \tag{2}$$

where c_i is the engine input. The parameters $\sigma, A_i, C_{di}, d_{mi}, m_i, \tau_i$ are specific mass of air, and vehicles' cross sectional area, drag coefficient, mechanical drag, mass, and engine time constant, respectively. Let the engine input c_i be governed by following FL controller:

$$c_i = u_i m_i + 0.5\sigma A_i C_{di} v_i^2 + d_{mi} + \tau_i \sigma A_i C_{di} v_i a_i \tag{3}$$

substituting which into (1) results in

$$\tau_i \dot{a}_i + a_i = u_i \tag{4}$$

in which u_i is an auxiliary input signal to be designed. Now, let $\mathbf{X}_i \triangleq [x_i, \dot{x}_i, \ddot{x}_i]$ denote the states of the i^{th} follower where $\dot{x}_i = v_i$ and $\ddot{x}_i = a_i$. Thus, given (4), the state-space model for the i^{th} follower can be written as

$$\dot{\mathbf{X}}_i = \mathbf{A}_i \mathbf{X}_i + \mathbf{B}_i u_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix} \mathbf{X}_i + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix} u_i \tag{5}$$

where both the vehicles' feedback-linearized dynamics (characterized by $\mathbf{A}_i, \mathbf{B}_i$ and τ_i) and the platoon's controllers (characterized by u_i) are nonidentical, meaning that they are not the same for all the follower vehicles, constituting a heterogeneous platoon. Therefore, the problem formulation and stability analysis will be developed with taking account of heterogeneity in the dynamic models and feedback information to the vehicles.

The objective of designing the controller u_i is to guarantee that when the leader has a constant steady velocity ($\triangleq v_0^s$), the followers' velocities track that leading velocity while desired constant distances ($\triangleq d_i^{i+1}$) are maintained between any two back-to-back vehicles within the platoon. In other words, for $\kappa = 1,\ldots,N-1$, the aim is to have

$$\begin{aligned} & v_i(t) = v_0^s(t) \\ & x_\kappa - x_{\kappa+1} = L_\kappa + d_\kappa^{\kappa+1} \qquad \equiv \qquad D_\kappa^{\kappa+1} = d_\kappa^{\kappa+1} \end{aligned} \tag{6}$$

and to ensure which, we design a distributed controller with non-identical gains as

$$\begin{aligned} u_i &= -\sum_{j\in\mathbb{I}_i} \left[k_i(x_i - x_j - d_{ij}) + b_i(\dot{x}_i - \dot{x}_j) + h_i(\ddot{x}_i - \ddot{x}_j)\right] \\ d_{ij} &\triangleq -sgn(i-j) \sum_{\kappa=\min(i,j)}^{\max(i,j)-1} \left[l_\kappa + d_\kappa^{\kappa+1}\right] \end{aligned} \tag{7}$$

where $\mathbb{I}_i \subset \{\{0,1,\ldots,N\} - \{i\}\}$ indicates the vehicles from which the vehicle i receives information. Please note that we develop the platooning formulation regardless of the type of communications between the vehicles such that all IFTs can suit properly in our problem development. Having d_i^{i+1} as the desired spacing between the consecutive vehicles and x_0 as the position of the leader vehicle, the desired position and velocity of the i^{th} follower can be defined accordingly as

$$x_i^* \triangleq x_0 - \sum_{\kappa=0}^{i-1} \left[l_\kappa + d_\kappa^{\kappa+1}\right], \qquad \dot{x}_i^* = v_0^s = \dot{x}_0^s \tag{8}$$

For conciseness in presentation and ease in later analysis, the state error of the i^{th} follower is defined as $\tilde{x}_i = x_i - x_i^*$ utilizing which readily results in $x_i - x_j = \tilde{x}_i - \tilde{x}_j + d_{ij}$, and subsequently substituting which into the controller (7) gives

$$u_i = -\sum_{j\in\mathbb{I}_i} \left[k_i(\tilde{x}_i - \tilde{x}_j) + b_i\left(\dot{\tilde{x}}_i - \dot{\tilde{x}}_j\right) + h_i\left(\ddot{\tilde{x}}_i - \ddot{\tilde{x}}_j\right)\right] \tag{9}$$

and plugging (9) in (4) yields

$$\begin{aligned} \dddot{\tilde{x}}_i = & -\frac{|\mathbb{I}_i| k_i}{\tau_i}\tilde{x}_i - \frac{|\mathbb{I}_i| b_i}{\tau_i}\dot{\tilde{x}}_i - \frac{1+|\mathbb{I}_i| h_i}{\tau_i}\ddot{\tilde{x}}_i \\ & + \frac{k_i}{\tau_i}\sum_{j\in\mathbb{I}_i}\tilde{x}_j + \frac{b_i}{\tau_i}\sum_{j\in\mathbb{I}_i}\dot{\tilde{x}}_j + \frac{h_i}{\tau_i}\sum_{j\in\mathbb{I}_i}\ddot{\tilde{x}}_j \end{aligned} \tag{10}$$

which obtained using the facts that $\ddot{x}_i = \ddot{\tilde{x}}_i$ and $\dddot{x}_i = \dddot{\tilde{x}}_i$. Note that $|\mathbb{I}_i|$ is the cardinality of the set $\mathbb{I}_i$. Considering (10), knowing $\tilde{x}_0 = \dot{\tilde{x}}_0 = \ddot{\tilde{x}}_0 = 0$, and defining the i^{th} vehicle control gains as $\mathbf{K}_i = [k_i,, b_i, h_i]^T$ and platoon state error as $\tilde{\mathbf{X}}_N \triangleq \left[\tilde{x}_1, \dot{\tilde{x}}_1, \ddot{\tilde{x}}_1, \ldots, \tilde{x}_N, \dot{\tilde{x}}_N, \ddot{\tilde{x}}_N\right]^T$, the platoon closed-loop state-space dynamics model can be characterized by

$$\dot{\tilde{\mathbf{X}}}_N=\tilde{\mathbf{A}}_N\tilde{\mathbf{X}}_N=\begin{bmatrix}\mathbf{A}^*_{11} & \mathbf{A}^*_{12} & \cdots & \mathbf{A}^*_{1N}\\ \mathbf{A}^*_{21} & \mathbf{A}^*_{22} & \cdots & \mathbf{A}^*_{2N}\\ \vdots & \dots & \ddots & \vdots\\ \mathbf{A}^*_{N1} & \mathbf{A}^*_{N2} & \cdots & \mathbf{A}^*_{NN}\end{bmatrix}\tilde{\mathbf{X}}_N \quad (11)$$

where $\tilde{\mathbf{A}}_N$ is overall closed-loop system matrix such that for a given follower i, we have $\mathbf{A}^*_{ii}\triangleq\mathbf{A}_i-|\mathbb{I}_i|\mathbf{B}_i\mathbf{K}_i^T$ and $\mathbf{A}^*_{ij}\triangleq\mathbf{B}_i\mathbf{K}_i^T$. Using $\tilde{\mathbf{A}}_N$, the determinant of the block matrix $s\mathbf{I}_N-\tilde{\mathbf{A}}_N$, which can be obtained analytically [20], will provide the characteristic polynomial of the platoon, using which the stability conditions with respect to the control gains can be obtained. Note that $\mathbf{I}_N$ is the identity matrix of size N, and the closed-loop system would be stable if all the eigenvalues of $\tilde{\mathbf{A}}_N$ are negative. In the rest of paper, we will consider stability conditions for an two-followers platoon.

Case study: stability analysis for $N=2$.
Considering $N=2$, (11) can be written as

$$\dot{\tilde{\mathbf{X}}}_2=\tilde{\mathbf{A}}_2\tilde{\mathbf{X}}_2=\begin{bmatrix}\mathbf{A}_1-|\mathbb{I}_1|\mathbf{B}_1\mathbf{K}_1^T & \mathbf{B}_1\mathbf{K}_1^T\\ \mathbf{B}_2\mathbf{K}_2^T & \mathbf{A}_2-|\mathbb{I}_2|\mathbf{B}_2\mathbf{K}_2^T\end{bmatrix}\tilde{\mathbf{X}}_2 \quad (12)$$

where the platoon would be asymptotically stable if and only if all the eigenvalues of the matrix $\tilde{\mathbf{A}}_2$ are negative. In this respect, the characteristic polynomial of matrix $\tilde{\mathbf{A}}_2$ can be derived by the following determinant:

$$\begin{aligned}&\left|\begin{bmatrix}s\mathbf{I}_3-\mathbf{A}^*_{11} & -\mathbf{A}^*_{12}\\ -\mathbf{A}^*_{21} & s\mathbf{I}_3-\mathbf{A}^*_{22}\end{bmatrix}\right|\\ &=|s\mathbf{I}_3-\mathbf{A}^*_{11}|\left|(s\mathbf{I}_3-\mathbf{A}^*_{22})-\mathbf{A}^*_{21}(s\mathbf{I}_3-\mathbf{A}^*_{11})^{-1}\mathbf{A}^*_{12}\right|\end{aligned} \quad (13)$$

deriving which presents the characteristic polynomial $as^6+bs^5+cs^4+ds^3+es^2+fs^1+g$ in which the coefficients are according to the following formulas.

$$\begin{aligned}&a=\tau_1\tau_2 \qquad b=\tau_1(1+h_2|\mathbb{I}_2|)+\tau_2(1+h_1|\mathbb{I}_1|)\\ &c=\tau_1 b_2|\mathbb{I}_2|+(1+h_1|\mathbb{I}_1|)(1+h_2|\mathbb{I}_2|)+\tau_2 b_1|\mathbb{I}_1|-h_1h_2\\ &d=\tau_1 k_2|\mathbb{I}_2|+b_2|\mathbb{I}_2|(1+h_1|\mathbb{I}_1|)+b_1|\mathbb{I}_1|(1+h_2|\mathbb{I}_2|)\\ &\quad+\tau_2 k_1|\mathbb{I}_1|-b_1h_2-b_2h_1\\ &e=k_2|\mathbb{I}_2|(1+h_1|\mathbb{I}_1|)+b_1|\mathbb{I}_1|b_2|\mathbb{I}_2|+k_1|\mathbb{I}_1|(1+h_2|\mathbb{I}_2|)\\ &\quad-k_2h_1-b_1b_2-h_2k_1\\ &f=b_1|\mathbb{I}_1|k_2|\mathbb{I}_2|+k_1|\mathbb{I}_1|b_2|\mathbb{I}|_2-k_2b_1-b_2k_1\\ &g=k_1|\mathbb{I}_1|k_2|\mathbb{I}_2|-k_1k_2\end{aligned} \quad (14)$$

and if the first follower does not receive information from the second follower, or vice versa, then we will have $\mathbf{A}^*_{12}=0$ or $\mathbf{A}^*_{21}=0$, respectively. Thus, the coefficients would be

$$\begin{aligned}&a=\tau_1\tau_2 \qquad b=\tau_1(1+h_2|\mathbb{I}_2|)+\tau_2(1+h_1|\mathbb{I}_1|)\\ &c=\tau_1 b_2|\mathbb{I}_2|+\tau_2 b_1|\mathbb{I}_1|+\tau_1(1+h_2|\mathbb{I}_2|)+\tau_2(1+h_1|\mathbb{I}_1|)\\ &d=\tau_1 k_2|\mathbb{I}_2|+b_2|\mathbb{I}_2|(1+h_1|\mathbb{I}_1|)+b_1|\mathbb{I}_1|(1+h_2|\mathbb{I}_2|)+\tau_2 k_1|\mathbb{I}_1|\\ &e=k_2|\mathbb{I}_2|(1+h_1|\mathbb{I}_1|)+b_1|\mathbb{I}_1|b_2|\mathbb{I}_2|+k_1|\mathbb{I}_1|(1+h_2|\mathbb{I}_2|)\\ &f=b_1|\mathbb{I}_1|k_2|\mathbb{I}_2|+k_1|\mathbb{I}_1|b_2|\mathbb{I}|_2 \qquad g=k_1|\mathbb{I}_1|k_2|\mathbb{I}_2|\end{aligned} \quad (15)$$

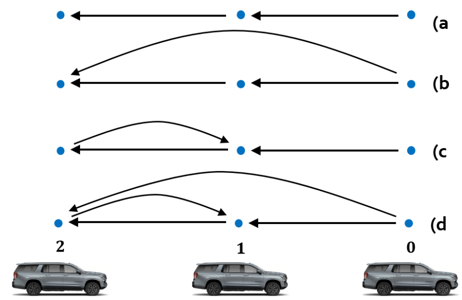

Fig. 2. Schematic of different IFTs between the vehicles in the one-leader-two-followers platoon.

Now, having (14)-(15) and using Routh–Hurwitz criterion, the stability conditions can be obtained as follows.

$$\begin{aligned}&1.\ a,b,c,d,e,f,g>0 \quad 2.\ ad-bc\leq 0 \quad 3.\ d(ad-bc)\leq b(af-be)\\ &4.\ (ad-bc)\left[b^2g+f(ad-bc)\right]\leq(af-be)[d(ad-bc)-b(af-be)]\\ &5.\ \left(b^2g+f(ad-bc)\right)\left[(ad-bc)\left[b^2g+f(ad-bc)\right]\right.\\ &\quad\left.-(af-be)[d(ad-bc)-b(af-be)]\right]\\ &\quad\geq bg[d(ad-bc)-b(af-be)]^2\end{aligned} \quad (16)$$

3 Simulation Results

In this section, simulation results are provided to evaluate the stability conditions for different IFTs that are depicted in Fig. (2). For simulations, we consider a velocity trajectory for the leader vehicle (see Fig. 4) and choose the vehicles' initial velocities and accelerations equal to zero. Also, the vehicles' length are the same and equal to 4 m, and vehicles' initial positions are selected as $x_0(0)=0$ m, $x_1(0)=-10$ m, and $x_2(0)=-20$ m. As you can see in the Fig. 4, the v_0^s velocities for the leader vehicle are 30 m/s (its maximum value) persisting for 12 s, and 0 m/s that is associated with the time the leader vehicle brakes and stands still. Furthermore, we choose $d_i^{i+1}=10$ m as the desired spacing between the vehicles.

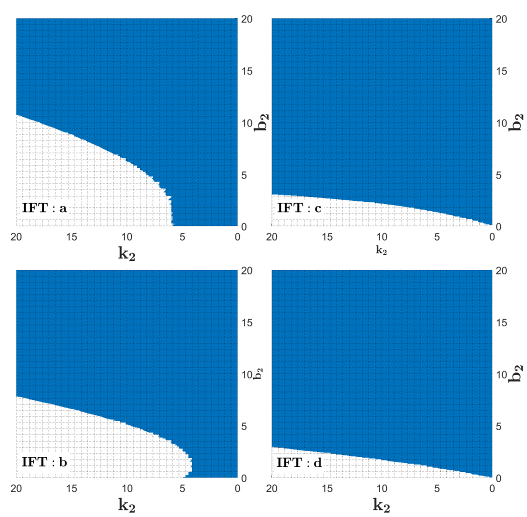

Fig. 3. The stability area (the blue area) with respect to the control gains k_2 and b_2 for different IFTs sketched in Fig. 2.

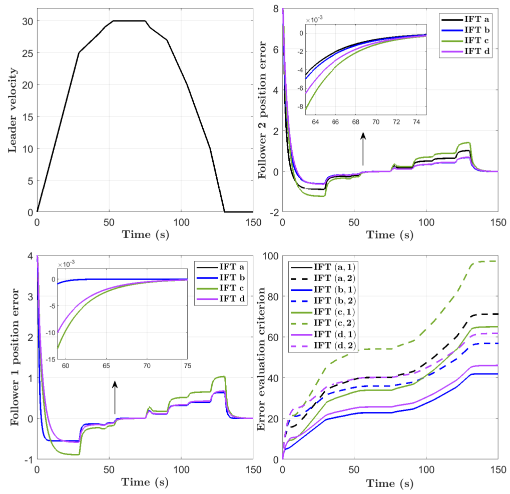

Fig. 4. Error signals of the followers for the different IFTs.

First, we assume that τ_1=τ_2=0.5 s, and the controller gains of all the vehicles are the same, i.e., k_1=k_2, b_1=b_2 and h_1=h_2=1. Based upon the stability conditions given in the work [16] and for IFT c illustrated in Fig. 2, we assign k_1=k_2=3, b_1=b_2=5, and h_1=h_2=1. Having k_1,b_1,h_1, we choose h_2=h_1 and let k_2 and b_2 to be selected within the stability conditions given in (16). Regarding (14)-(15), this time we will find stability conditions with respect to the control gains k_2 and b_2 and for the four IFTs in Fig. 2. The results for the different IFTs are depicted in Fig. 3. The stability areas are shown in Fig. 3. As you can see, by comparing the stability areas of IFTs a and b, or IFTs c and d, or IFTs a and c, and or IFTs b and d, an additional communication channel between the vehicles makes the stability area larger. The IFT a has the smallest stability area and the IFT d has the largest.

In order to draw an analogy between the controller performances in different IFTs, using root locus analysis for a given plausible k_2 or b_2 that belongs to all the stability areas of Fig. 3, we assign k_2=2.5 and b_2=10. Therefore, the control gains become k_1=3, k_2=2.5, b_1=5, b_2=10, and h_1=h_2=1. Note here τ_1=0.5 and τ_2=0.5 are chosen for engines time constants. So, using this controller, the results for the different IFTs are shown in Fig. 4 in which, for instance, IFT (a,2) indicates the position error for the second follower and implies that the controller is utilized within the IFT of case a represented in Fig. 2. Note that the position error for the i^{th} follower is defined as $e_i(t)$=$x_i(t)$−$x_i^*(t)$. Investigating the simulation results for the different IFTs, we can see that when the leader has a constant steady velocity, the followers' position errors asymptotically converge to zero. Also, in IFTs b and d, in which both the first and second followers receive information from the leader, the error signal exhibits better damping behavior that can come in handy when, for instance, we want to enforce small desired spacing between the vehicles. To shed more light on the damping behavior, let the following formula be defined as the *error evaluation criterion* (EEC) for the transient behavior of the error signals of the followers.

$$EEC_i \triangleq \int_0^t |e_i(t)| dt \tag{17}$$

regarding which the results for the followers within the given IFTs are shown in Fig. 4. It is possible to see that the IFTs b and d provide better performance for the platoon respecting EEC measure. Moreover, making a comparison between the IFTs b and d, we can see that the communication from the second follower to the first follower has increased the settling time and so the convergence occurs slower.

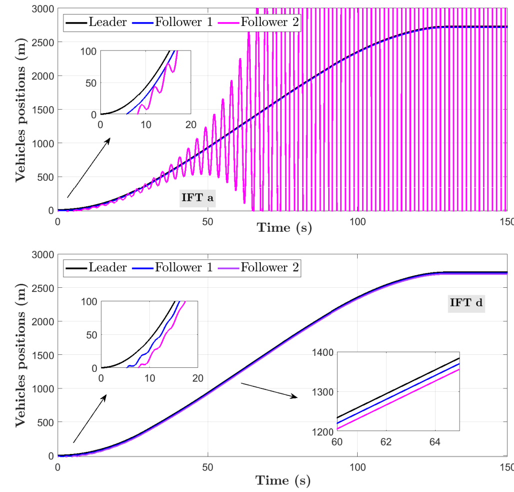

Fig. 5. Vehicles' positions using control gains k_1=3, b_1=5, h_1=1, k_2=10, b_2=2, and h_2=1, and IFTs a and d.

Fig. 5 shows the positions of vehicles for the given velocity of the leader and for the two IFTs a and d. As obvious from Fig. 3, for k_2=10 and b_2=2, the platoon of the IFT a would be unstable and the platoon of the IFT d would be stable. Accordingly, in Fig. 5, using the IFT d, the desired distances between the vehicles are maintained, however, in the IFT a the system is unstable and numerous collisions occur.

4 Conclusion

In this paper, using a decentralized linear feedback controller with non-identical gains, a state-space model for the heterogeneous platoon was obtained. We developed the problem in such a way as to could incorporate any IFT into the stability analysis. Thus, for any number of vehicles, using the characteristic polynomial of the closed-loop system, the Routh–Hurwitz criterion will present the stability conditions of the platoon. As a case study, the simulation results were provided for an two-followers platoon, and the effect of the different IFTs on the system performance were discussed. It was shown that, more communication between the vehicles can provide more flexibility in the selection of control gains that satisfy the stability conditions. The results also showed that using feedback signals of the leader in the both followers' controllers can offer better performance for the platoon.

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