University of Alberta

Essays in Corporate Finance and Fixed Income Analysis

by

Gary Robert Smith ©

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Abstract

This document presents three papers representing research from both corporate finance and investments. The second chapter is an investigation into the systematic variation of stock price reactions to corporate capital budget announcements. Within this chapter we use an event study methodology to measure the market's reaction to capital investment announcements. Cross sectional variation in these reactions is examined. We find that, on average, the market's reaction to both capital budget increases and decreases is statistically insignificant. We also find that there is support for the idea that agency problems affect the market's valuation of investment decisions and that there is support for the hypothesis of managerial entrenchment. We also find support for the notion that investment decisions made by managers with strong reputations are more highly valued than those made by other managers. We find no support for the notions that the stock market is myopic or that managers behave myopically nor do we find support for the idea that firms with high levels of cash flow invest inefficiently. The third chapter examines when a board of directors, working with a single-minded focus on value-maximization, will choose to replace an incumbent manager. We show that a board which is rational and acting exclusively in the interests of shareholders will not always replace managers who are of below average ability. The fourth chapter provides a brief description of the development of the structured note market in Canada and provides the tools for setting up a system to value a wide variety of these notes using nothing more than a standard spreadsheet and interest

rate swap rates. In brief, the valuation technique described involves decomposing a note with an imbedded option into a note, a forward swap, and an option on the forward swap. Techniques for valuing the final two components are described in the paper.

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Chapter 1

Introduction

This document presents three papers representing research from both corporate finance and investments. In the area of corporate finance, a study of market reactions to corporate investment announcements provides evidence on the factors the market values in analyzing the decisions made by managers. Another study on the replacement of managers by boards of directors provides theory that helps us to understand how a value maximizing board of directors will act. This assists us in understanding if a particular board is functioning well as we can only diagnose a problem if we properly understand health. Finally, the paper on investments provides a description of the Canadian structured note market, an emerging fixed income market segment in Canada, and shows how one can analyze a wide range of assets within this market segment. Brief overviews of each chapter are given below.

The second chapter, *A Study of Cross Sectional Variation in the Stock Market's Reaction to Corporate Investment Decisions*[1], is an investigation into the systematic variation of stock price reactions to corporate capital budget announcements. This chapter was motivated by the lack of existing empirical work on corporate investment and the factors that the market considers in evaluating investment decisions. Within this chapter we use an event study

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1 Co-authored with Mark Huson and Randall Morck, both of the University of Alberta, and Wayne Yu of the University of Lethbridge.

methodology to measure the market's reaction to capital investment announcements. These reactions are then regressed upon measures of agency problems and measures of intangible assets. We find that, on average, the market's reaction to both capital budget increases and decreases is statistically insignificant. We also find that there is support for the idea that agency problems affect the market's valuation of investment decisions. Furthermore, there is support for the hypothesis of managerial entrenchment. We also find support for the notion that investment decisions made by managers with strong reputations are more highly valued than those made by other managers. We find no support for the notions that the stock market is myopic or that managers behave myopically. There is also no support for the idea that firms with high levels of cash flow invest inefficiently.

The third chapter, *Rational, Value Maximizing Boards of Directors and the Replacement of Management*[2], was in part motivated by the fallout from the board actions to remove executives at General Motors and IBM in the early 1990s. At that time a number of commentators stated that the boards acted too late and that their slow action proved that boards are either ineffective or negligent. But this conclusion begs the question of what one would expect from a well-functioning board with respect to replacing managers. This chapter addresses this question by examining when a board, working with a single-minded focus on value-maximization, will choose to replace an incumbent

2 Co-authored with Greg MacKinnon of St. Mary's University and Jim Unterschultz of the University of Alberta.

manager. We show that a board which is rational and acting exclusively in the interests of shareholders will not always replace managers who are of below average ability. The intuition behind this finding is that, even if the board knows the managerial ability of the incumbent is below average, replacing the manager involves choosing a new, unknown manager from a pool of candidates. There is then the possibility of getting a replacement that is worse. Stated another way, there are conditions under which the value-maximizing decision of the board is to retain a manager they know to be below average.

The fourth chapter, *Canadian Structured Notes: A Framework for Analysis*, was motivated by my work in fixed income investments at Alberta Treasury. Within the last year, securities with highly flexible terms, imbedded options, and small size have frequently been offered to both retail and institutional investors. Almost always, these securities offer high yield spreads over benchmark bonds and, at first glance, appear very attractive. Through discussions with dealers and other investors I have learned that, in many cases, the analysis investors apply to the valuation of these securities is based solely on yield spread. This type of analysis does not capture all of the sources of value in these notes. This chapter provides a brief description of the development of the structured note market in Canada and provides the tools for setting up a system to value a wide variety of these notes using nothing more than a standard spreadsheet and interest rate swap rates. In brief, the valuation technique described involves decomposing a note with an imbedded option into a note, a forward swap, and an option on the forward swap. Techniques for valuing the final two components are described in the paper.

Chapter 2

A Study of Cross Sectional Variation in the Stock Market's Reaction to Corporate Investment Decisions[3]

1. Introduction

Normative corporate finance theory argues that a firm should undertake capital investment projects only if they have positive net present values. However, a large body of theoretical work, mainly stemming from Jensen and Meckling (1976), argues that agency problems cause some firms to undertake negative net present value projects that benefit insiders, and considerable empirical evidence supports this position. However, empirical studies have found that when firms announce increases in their capital budgets, their stock prices tend to rise (McConnell and Muscarella, 1985; Chen *et al.* 1988). In this paper, we construct a sample of such announcements using the criteria developed by McConnell and Muscarella (1985) and examine closely the cross sectional variation in such stock price reactions. We find evidence that the shareholders of firms likely to have agency problems do not welcome news of increased capital budgets.

In our sample, the average unconditional stock price reaction to both capital budget increases and decreases is zero. This differs from those of McConnell and Muscarella (1985) who report positive significant abnormal returns for firms announcing investment increases and negative significant

3 This chapter was co-authored with Mark Huson, Randall Morck, and Wayne Yu.

returns for firms announcing decreases. Our finding of no significant market reactions also appears inconsistent with stock market myopia, as argued by Porter (1990), and managerial myopia, as proposed by Stein (1989).

We use insider ownership variables to proxy for possible agency problems, and average Tobin's q to proxy for the quality of past managerial decisions, and find positive relationships between both and abnormal returns in regressions controlling for firm size and past capital budget growth. Average q is a general measure of the presence of intangible assets, and these may derive from superior investment opportunities, management, technology, marketing, etc., and we therefore consider alternative interpretations of our results. However, we argue that the positive relationship between abnormal returns and average q for both firms announcing capital budgeting increases and those announcing decreases suggests that average q is more appropriately viewed as a measure of intangible assets related to management reputation.

We also test for a relationship between cash flow levels and stock price reactions to changes in capital investment, and find no evidence of such a link. This fails to bolster arguments about the importance of free cash flow misinvestment as a pervasive agency problem, but mainly underscores the fact that free cash flow, as defined by Jensen (1986) and accounting cash flow are different, and that free cash flow is very difficult to measure.

2. Background and literature review

2.1 Theory

An economically efficient capital investment process should result in the maximization of shareholders' wealth. Elementary financial theory states that wealth will increase as long as the net present value (NPV) of the projects undertaken is positive where future cash flows are discounted at the appropriate risk adjusted discount rate. Thus, the optimal capital investment decision rule, as stated by Copeland and Weston (1988, p. 41), is that "managers should take projects with positive NPVs down to the point where the NPV of the last acceptable project is zero." There are two assumed conditions for this rule to lead to optimal investment: 1) managers and owners have the same information regarding investment opportunities and 2) shareholders are able to monitor managers to ensure that their actions are consistent with wealth maximization. It is unclear if either of these conditions hold in reality.

Porter (1992) addresses how the investment process might be affected by violations of the first condition. He describes investors as having short holding periods and lacking access to the proprietary information necessary for making informed decisions. In order to make inferences about the quality of management's decisions, investors are monitor some instrumental variable. In Porter's words, investors are driven "to focus on easily measurable company attributes, such as current earnings or patent approvals, as proxies of a company's value on which to base market timing choices" (p. 70). As current earnings, a key proxy, are negatively affected by increased capital investment in the short run, shareholders will react positively to decreases in capital investment outlays and negatively to increases. Thus, in this environment, asymmetric information leads managers to forego the longer term benefits of investment in order avoid putting downward pressure on current earnings. The stock market is said to be myopic in that its focus on current earnings leads to lower levels of investment.

Like Porter, Stein (1989) proposes that the information asymmetry between the market and managers causes the shareholders to use current earnings as a signal of the firm's long term prospects. This gives managers an incentive to manipulate earnings in order to generate a positive signal to the market. However, unlike Porter, in Stein's model the market is efficient: it conjectures such manipulation by managers of all firms. Managers interested in maximizing owners' wealth are therefore compelled to keep earnings high, even at the expense of foregoing positive NPV projects. This "prisoners' dilemma"-like outcome effectively translates into an unnecessarily high discount rate being used to compute NPV. Thus, in Stein's model, asymmetric information forces management to focus on keeping earnings high; therefore, management behaves myopically by foregoing investment in the interest of keeping earnings high. In this case, as in the Porter case, the information asymmetry leads to underinvestment.[4]

It is critical to note that while both versions of myopia lead to underinvestment, the market's reaction to capital investment will be quite different in the two cases. Recall that the market's negative view of investment induces underinvestment in the case of investor myopia. In the case of managerial myopia, investors should react to investment in the opposite way; the market will deduce that a project is quite valuable if it is acceptable when using an unnecessarily high discount rate. In this situation investors would react positively to the investment.

Violations of the second condition of the optimal investment rule result from the separation of corporate ownership from corporate control. Berle and Means (1932) indicate that for firms in which ownership is dispersed and managers own little equity, managers have the incentive to direct the firm's resources to undertakings that benefit themselves as opposed to the shareholders. Jensen and Meckling (1976) present a model that demonstrates that the smaller the proportion of management ownership, the more likely it is that corporate resources will be allocated to benefit the managers. Jensen and Meckling state that such benefits will take the form of both pecuniary returns and non-pecuniary aspects like "the physical appointments of the office, the

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[4] In this paper we investigate overinvestment and/or underinvestment that might be caused by the asymmetry of information between managers and shareholders. We do not investigate overinvestment or underinvestment resulting from the agency costs of debt. See Lyon (1995) for a survey of several models relating to this.

attractiveness of the secretarial staff, the level of employee discipline..." (p. 312). To their list we would add the indulgence of investing in pet projects. This follows because the managers, with only a fraction of the ownership, bear only a fraction of the cost of these misallocations, but receive all the benefits. As their proportion ownership increases, so too does the cost they pay for misallocating the firm's resources; therefore, the likelihood of such misallocation declines. Under this framework, the stock market should find investment decisions made by managers with large shareholdings more valuable than those made by managers with smaller stock positions as their interests will converge with those of outside shareholders.

Jensen and Meckling do not specify that the misallocation of resources they anticipate will take the form of overinvestment. Jensen (1986) offers a more specific notion of how the violation of the second condition for the optimal investment rule will affect corporate investment. He proposes that managers have incentives to overinvest as larger firms lead to more power for managers and more opportunities for the advancement of middle-managers within the firm. He posits that the degree to which they overinvest is an increasing function of the financial resources they have at their disposal. Jensen calls such resources *free cash flow*. Specifically, free cash flow is defined as "cash flow in excess of that required to fund all projects that have positive net present values when discounted at the relevant cost of capital" (p. 323).

A difficulty with empirically testing free cash flow theory is that the level of free cash flow is unobservable. Stulz (1990) addresses this problem. The Stulz model demonstrates that levels of cash flow are not in themselves sufficient indicators of the presence of a free cash flow problem. If the firm has a high quality investment opportunity set (i.e. high marginal product from corporate investment) then there may not be a free cash flow problem even though there is high cash flow; however, when high cash flow is accompanied by a low quality investment opportunity set, there is a high probability that management will invest resources in negative net present value projects. Thus, the requirements for a free cash flow problem are both high cash flow and a poor investment opportunity set.

2.2 Relevant empirical work

To date, the most direct test of the market's reaction to corporate capital budget announcements is provided by McConnell and Muscarella (1985). They find that, on average, capital spending increase announcements are accompanied by abnormally high stock returns while capital spending decrease announcements are greeted by abnormally low stock returns.[5] Their results are consistent with the notion that the market views capital investment as wealth-enhancing. In other words the market is not myopic in the sense of Porter

5 An exception to this general result occurs with firms in the oil industry. Here, capital spending increase announcements are greeted by abnormally low returns, a result consistent with the predictions of the free cash flow hypothesis of Jensen (1986).

(1992). We cannot, however, conclude that management is investing efficiently. Stein (1989) argues that the acceptance of projects by myopic managers signal that the projects have expected returns at least as great as a myopia-induced artificially high hurdle rate; therefore, the market will respond positively to a capital budget increase announcement. Similarly, a capital budget decrease may signal the elimination of a project because its expected return has dropped below the myopia-induced hurdle rate even though the expected return remains above the true cost of capital. Such an elimination will lead to a decrease in share price in an efficient market. Thus the McConnell and Muscarella results, while supportive of the notion that corporate investment is wealth-enhancing, are not necessarily consistent with the notion of investment is wealth-maximizing.

A test of the convergence-of-interest hypothesis is performed by Morck, Shleifer, and Vishny (1988). They find that for low levels of board ownership (0 to 5%) average *q* increases with board ownership, while intermediate levels of board ownership (5% to 25%) display a decreasing relationship. The increasing relationship resumes at levels of ownership greater than 25%. The authors' interpretation of this is that there is convergence-of-interest effect at work although it may be overshadowed by the entrenchment effect at intermediate levels of ownership. McConnell and Servaes (1990) confirm the existence of a nonlinear relationship of this type, but find evidence of entrenchment only at levels of managerial ownership in the 40% to 50% range, depending on the sample period examined.

A number of empirical tests of the free cash flow hypothesis have been undertaken in various contexts with mixed results. Lang and Litzenberger (1989) investigate the effect of significant dividend changes on market valuation. Using average *q* to differentiate between value-maximizing firms and suspected overinvestors, they find that dividend increases result in significantly higher returns among the suspected overinvestors than among value-maximizers, and conclude that the reaction is consistent with the free cash flow hypothesis. Lehn and Poulsen (1989) examine the relationship between levels of cash flow and going private transactions. Their results indicate the existence of a significant relationship between the level of undistributed cash flow and a firm's decision to go private. Furthermore, they show that premiums paid in the going private transactions are significantly related to the level of undistributed cash flow. They do not, however, use any measure of the corporate investment opportunity set in their analysis. Notwithstanding the omission of a direct test of the relationship between abnormal returns and cash flows in the Lang and Litzenberger paper and the lack of a measure of the investment opportunity set in the Lehn and Poulsen work, the results of both of these papers are consistent with the free cash flow theory.

Howe, He, and Kao (1992) follow a very similar methodology to that of Lang and Litzenberger (1989) with the principal difference being that the events analyzed in their paper are one-time cash flow announcements, namely share repurchases and special dividends. While the events are, in present value terms, similar to the Lang and Litzenberger event of dividend changes, the results are substantially different. Howe, He, and Kao find that the market's reaction to the one-time cash flow announcements is approximately the same for both high and low average *q* firms. Further, when the high cash flow-low average *q* sample is compared to the low cash flow-high average *q* sample, the comparison in which free cash flow effects should be most evident, there is still no significant difference in abnormal returns. Thus, the reaction to changing the cash flows of the firm through one-time cash flow announcements is, on average, invariant to both the level of average *q* and the level of cash flow, a result that runs counter to the free cash flow theory.

Lang, Stulz, and Walkling (1991) examine the free cash flow hypothesis in the context of bidder returns in tender offers. Following Stulz (1990), they condition the level of cash flow on average *q*, their measure of the quality of the firm's investment opportunity set. They partition their sample of tender offers into four groups: low cash flow-high average *q*, low cash flow-low average *q*, high cash flow-high average *q*, and high cash flow-low average *q*. Stulz's version of the free cash flow hypothesis predicts that the last of the four groups has the free cash flow problem. Consequently, this group should have the lowest average return. This is, in fact, what was discovered. They also find that the levels of returns are inversely related to levels of cash flow for firms with low average *q* values. Thus, there is more evidence consistent with the existence of an adjustment for free cash flow problems in tender offer situations.

It should be noted that the interpretation of average *q* varies across the studies. The Lang and Litzenberger study, for example, uses average *q* as a sorting variable to distinguish managers who invest inefficiently from efficient investors. This use of average *q* is consistent with the view of Veblen (1904) that "the value of any given block of capital...turns on its earning-capacity" (p. 152). Firms with high levels of average *q* have assets in place with values greater than their cost. That is, the investments made in the past by that firm's managers are good ones. Under this view, then, average *q* can be viewed as a measure of managerial "track record". Lang, Stulz, and Walkling (1991) take the view that average *q* is a measure of the quality of the firm's investment opportunities. While it is certainly true that Tobin's *q* is a measure the economic viability of investment opportunities, it is questionable if average *q* does so[6].

Overall, the studies referenced above provide some evidence consistent with the notion that managers, on average, invest in projects that have positive NPV. The studies also furnish some results consistent with agency problems in the sense of Jensen and Meckling (1976) and some contradictory results regarding the empirical validity of Jensen's free cash flow theory.

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6 Our empirical results indicate that it is not appropriate to interpret average q as Tobin's q for our sample.

The next section provides a description of the sample and the variables used in the analysis.

3. The Data

3.1 The sample

Our sample of announcements of changes in corporate capital budgets is constructed using the *Dow Jones News Retrieval Service* for the years 1984 through 1989 and the *Wall Street Journal* on CD-ROM for the years 1990 through 1993. The criteria we use to select observations follow those developed by McConnell and Muscarella (1985), namely:

1. The announcement must be about a change in the firm's total capital investment budget. Announcements about the undertaking or elimination of specific investment projects are not included. Announcements of a level of capital investment equal to that of the previous year are excluded. Also, announcements of mergers and acquisitions are excluded. This criterion avoids possible double counting, as specific announcements might be included in more general announcements.

2. The announcement must not include any other information, such as income or dividend announcements. Furthermore, the *Wall Street Journal Index* (hard-copy version) must refer to no other significant announcements about

the firm from one week prior to the event date to three days after it. The purpose of this restriction is to ensure that any abnormal stock returns occurring on the event date are due to the capital budget announcement.

3. The firm must be on the Centre for Research in Securities Prices (CRSP) daily returns tape. The firm's financial statements must be available from COMPUSTAT.

4. The firm must not be a regulated public utility. With this, we attempt to ensure that the market's reaction to the announcement is not colored by the prospect of public regulation. To eliminate such firms, we exclude firms whose two digit Standard Industrial Classification (SIC) code numbers are 48 (Communication industries) or 49 (Other utilities).

A total of ninety-six increase announcements and forty-seven decrease announcements meet our criteria.

The top panel of Table 2-1 shows a frequency distribution of events through time. Increase announcements are concentrated in 1988 through 1990, and decrease announcements are rare in these years. Decrease announcements are heavily concentrated in 1986, a year when increase announcements are at their lowest density.

The lower panel of Table 2-1 is a frequency distribution of increase and decrease events by two digit SIC codes. Both samples have heavy concentrations in oil and gas extraction (SIC 13) and petroleum refining (SIC

29). Increase announcements come from a broader range of industries than decrease announcements.

3.2 Abnormal Returns and Net Present Values

We begin by conducting a standard event study methodology, and examine stock returns for day -1 and day 0, where day 0 is the date of the announcement. Abnormal returns are calculated using the numeraire portfolio methodology of Long (1990). The cumulative abnormal return for firm *i* is

$$CAR_i = \sum_{t=-1}^{0}\left(\frac{1+R_{it}}{1+R_{mt}}-1\right)$$

where R_{it} is the return on stock *i* on date *t* and R_{mt} is the return on the CRSP value-weighted index on date *t*.

Comparing the market response to capital expenditure announcements by comparing abnormal stock returns may introduce a bias related to firm size, and therefore may not be entirely legitimate. For example, consider a project with an initial cost of \$10 and a net present value of +\$2. A firm with outstanding equity worth \$100 that announces it is undertaking this project should see its stock price rise of \$2/\$100 or 2%, whereas a firm with equity worth \$20 that announces the same project should experience a stock price rise of \$2/\$20 or 10%. To compensate for this, we calculate the market's expectation of the capital expenditure's net present value

$$E(NPV) = CAR \times V_E$$

where V_E is the market value of the firm's outstanding common equity on day zero. To make this comparable across firms, we scale by the dollar value of the announced change in the firm's capital budget, C_0. We define the ***profitability index***, *PI*, as:

$$\Pi = \frac{E(NPV)}{C_0} = \frac{CAR \times V_E}{C_0}$$

The profitability indices of both \$10 projects are 20%. The size of the firm undertaking it does not directly affect the project's profitability index.

3.3 Independent variables

Our basic methodology is to determine what variables are related to abnormal returns upon announcements of capital budget changes. The variables, the use of which we motivate below, are:

Firm Size: Our proxy here is the natural logarithm of the book value of net property plant and equipment.[7] This variable, *SIZE,* is included because it is harder for insiders to own a large stake in a larger firm. This creates a negative correlation between *SIZE* and α. Large firms may have more agency problems

7 .The natural log of COMPUSTAT item #8.

than small firms, however we need to distinguish size effects from ownership effects. Firm size may also be related to intangible assets.

Past Capital Expenditure Levels: This variable, *CAPEX*, is the average value of capital expenditures (net of acquisitions) per dollar of existing net property plant and equipment over the prior three years. We include it as a way of conditioning on past capital investment levels. Capital budget changes may signal different things in rapidly growing vs. stagnant firms.

Past Growth in Capital Expenditure Levels: This variable, $\Delta CAPEX$ is the annual fractional change in capital expenditures (net of acquisitions) averaged over the previous 3 years.

Average q: To proxy for this variable, we use the market value of equity plus the book value of debt (including short term liabilities) divided by the book value of net property plant and equipment and short term assets.[8] We call this variable q, and follow Tobin (1978) in interpreting it as a measure of the firm's intangible assets.

Research and Development Spending: This variable, *RD*, is the firm's annual R&D spending as a fraction of net property plant and equipment, averaged

8 COMPUSTAT items [(#24*#25/1000) + #19/(medium preferred stock dividend yield) + #9 + #5] / (#8+#4).

over the prior three years. We use this variable as a second proxy for intangible assets.

Advertising Spending: This variable, *ADV*, is the firm's annual advertising spending as a fraction of net property plant and equipment, averaged over the prior three years. We use this variable as a third proxy for intangible assets.

Cash Flow: We use the measure developed by Lang, Stulz, and Walkling (1991), total cash flows normalized by the book value of total assets.[9] We call this variable *CF*.

Insider ownership: This comes from proxy statements filed with the Securities and Exchange Commission (SEC) for the years 1984 through 1988, and from *US Disclosure CD-ROM* for 1989 through 1993.[10] We define "insiders" as officers and directors. We follow Jensen and Meckling (1976), and call this variable α. We interpret α as measuring the separation of ownership from control, and therefore as proxying for agency problems.

3.4 Univariate statistics for the independent variables

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9 COMPUSTAT items (#13-#15-#16-#19-#21+change in #35)/#8.

10 The primary source of information for the US Disclosure CD-ROM is SEC proxy statements.

Table 2-2 presents descriptive statistics for the independent variables in the capital expenditure increase and decrease samples. On average, companies that increase their capital budgets have more intangible assets, as measured by *q*, *R&D spending*, and *advertising spending*; and are smaller than those undertaking capital budget decreases. The cash flow and insider ownership levels of the two groups are statistically similar. This picture is robust to non-normality: equality of means is rejected wherever nonparametric tests also reject equality of medians.

4. Results

4.1 Unconditional and Conditional Abnormal Returns

Table 2-3 shows that firms announcing increases in their capital budgets see their share prices fall insignificantly, while firms announcing capital budget decreases see their share prices rise insignificantly. Adjusting these numbers to reflect profitability indices rather than stock returns does not alter their lack of significance.

McConnell and Muscarella (1985), studying a sample of similarly defined events in the period from 1975 to 1981, find that increase announcements associated with positive unconditional abnormal returns and decrease announcements associated with negative unconditional abnormal returns. These findings are consistent with the market viewing capital investment as wealth-enhancing, and inconsistent with claims by Porter (1992) and others that shareholders are myopic, and react negatively to long-term corporate investments. The apparent change in the market's reaction to corporate capital investment announcements revealed in our data bears closer investigation.

A simple comparison of univariate statistics may not take adequate account of how investors form their expectations about firms' investment plans. An announcement of a capital budget increase from a firm whose capital budget has been growing recently might convey less information that one from a firm with constant or shrinking past capital budgets. The absolute level of the capital budget might be important too, as a change in a small capital budget might convey more news than a change in a larger one. Firm size may also be important, above and beyond the size of prior years' capital budgets. Small firms, all else equal, can growth at higher rates than large firms.

To control for these factors, we use regressions containing these three variables:

$$\Pi = -0.921 + 12.847 CAPEX + 0.098 \Delta CAPEX - 0.21 SIZE + \varepsilon$$

(n=96, R^2=0.0542, p-value=0.1609)

for the increase subsample, and

$$\Pi = -0.516 - 1.367 CAPEX + 0.587 \Delta CAPEX + 0.106 SIZE + \varepsilon$$

(n=47, R^2=0.0414, p-value=0.6064)

for the decrease subsample.

Although the relatively high values of R^2 suggest that investors' expectations may be influenced by these variables as a group, the relationship lacks statistical power. The coefficient on previous levels of capital expenditure (*CAPEX*) in the increase sample is the only one that is statistically significant with a p-value of 0.0588. However, for reasons cited above, we include these control variables in all the regression models that we discuss in the next three sections. It is worthwhile to point out that our results are robust to the exclusion of these control variables.

4.2 Blurring or Clearing Vision?

Is the prevalence of myopia changing? Our insignificant results also fail to support the prediction by Porter's market myopia of a negative price reaction. Recall that Stein's managerial myopia predicts a positive reaction to capital budget increases and a negative reaction to decreases, as in McConnell and Muscarella. In this context, our negligible share price reactions point to less myopia, not more.[11] Does the marginal corporate capital investment now have a net present value closer to zero?

Stein's myopia should be more important in firms that are harder to value. We thus regress abnormal returns on indicators of intangible assets - *q*

11 Although new information, innovations, indivisibilities, and various market imperfections may keep the NPV of the marginal capital investment project away from zero, marginal NPVs closer to zero are arguably consistent with a more Pareto efficient economy.

ratios, R&D spending, and advertising spending. These regressions are displayed in Table 2-4.

In the capital budget increase subsample, the positive coefficients on *q* and a dummy variable for a *q* ratio greater than one suggest that firms with substantial intangible assets make marginal investments with higher NPVs, as Stein (1989) predicts. However, a high *q* ratio can reflect other things, such as good corporate governance or an attractive investment opportunity set. R&D spending and advertising spending, arguably more direct indicators of intangible assets, are insignificant. In the decrease subsample, *q* is insignificant, but low R&D spending predicts a stock price rise when capital budgets are cut. Although a decrease in managerial myopia is an *a priori* attractive hypothesis, our data do not speak clearly in its support.

4.3 Managerial Glaucoma?

Recall that Jensen (1986) argues that managers of firms with excess liquidity like to have big capital budgets, even if this means investing in value-decreasing projects. In his paper, Jensen singles out oil companies as especially susceptible. This view is supported by McConnell and Muscarella (1985): contrary to their evidence from other industries, they report that in the oil and gas industry, capital budget increases trigger share price declines.

Are free cash flow problems more important now than in previous years? A difficulty with empirically testing free cash flow arguments is that free cash flow as defined by Jensen is unobservable. Stulz (1990) points out that high cash flow must exist in conjunction with a dearth of positive NPV projects for a free cash flow problem to exist. Lang, Stulz, and Walkling (1991) operationalize this by studying low cash flow firms with high average q ratios, arguing that a component of the intangible assets average q measures is an attractive set of investment opportunities.

Table 2-5 contains regressions of abnormal returns on cash flow, q and their product. The logic of Lang, Stulz, and Walkling (1991) suggests that the coefficient of the interaction term should be negative. It is not.

To check if oil firms are more likely to suffer the free cash flow problem in our samples, we ran the regressions again including a dummy variable equal to one for oil companies and an interaction of it with the *Cash flow* variable. Neither is significant. Coefficients of these two variables are insignificant in every model, indicating there is no systematic difference in terms of market reactions between oil and non-oil firms. Furthermore, the inclusion of the variables does not qualitatively alter other coefficients in the models.

4.4 Willful Blindness?

There is substantial evidence in the corporate finance literature that managers often do not act in the interests of shareholders, as normative finance theory proscribes. Jensen and Meckling (1976) argue that such behavior is more likely when managers own little stock in the firm they run. Accordingly, Table 2-6 contains regressions of abnormal returns on managerial ownership and managerial ownership squared. We include the squared term in alternate regressions because Morck et al (1988) and McConnell and Servaes (1990) find that managers also appear to act suboptimally from public shareholders' perspective in very closely held firms. Stulz (1988) models managers in such firms as entrenched, and therefore subject to reduced shareholder pressure. The inclusion of firm size as a control variable incidentally insures that we are not accidentally using low insider ownership to proxy for large firm size, as the two are correlated.

Table 2-6 shows that abnormal returns are higher for firms with higher insider ownership up to a point, but that very high insider ownership is associated with lower abnormal returns. This suggests that misinvestment due to agency problems may be important in our sample.

To gauge the economic significance of our results, for a median sized firm with a median capital expenditure budget and change in past budget, if managerial ownership is between 4.9% and 71.3%, we predict a positive price reaction for the increase sample, indicating that investors view these firms as on average making positive NPV investments. Our results suggest that investors view firms with insider ownership below 4.9% or above 71.3% as typically making negative NPV investments.

The nature of this relationship differs from that of Morck, Shleifer, and Vishny (1988) and McConnell and Servaes (1990). In both of those papers the authors found that managerial entrenchment was evident at intermediate levels of insider ownership, but was dominated by the convergence of interest effect at high levels of insider ownership. Here we find that the entrenchment effect dominates only at very high levels of insider ownership. While the dependent variable in our study is akin to marginal Tobin's q and the dependent variable in the other studies is average q, it is puzzling why there are nature of the entrenchment effect differs across the studies.

4.5 Robustness

Our basic results are quite robust. Replacing q, RD, ADV, α, and CF by rank transformations gives similar results. Including industry and time dummies does not change the basic qualitative results. Heteroscedasticity is not significant in any regressions. Adding additional variables like leverage, and growth changes nothing.

4.6 Limitations of the study

In gathering the sample we implicitly assume that any variation in the economic environment across the observations is controlled for in the calculation of abnormal return. In fact, any number of firm-specific or industry-wide factors unrelated to the independent variables we use could influence the market's reaction to an announcement. Thus, in the construction of the sample we omit any notion of the specific context in which an investment decision was made. To the extent that these omitted variables are correlated with our independent variables, our estimates will be biased. Any interpretation of the statistics should be done with caution.

We have attempted to control for some of these "context" variables by including industry dummies and time dummies in the regressions. While their inclusion does not change our main results, it is possible that the tests of significance of some of the dummy coefficients are not very powerful due to the concentration of announcements in a small number of industries. One way of partially addressing the concern about getting annoucements in the proper context and thereby reducing bias due to omitted variables would be to gather a few more years worth of data and rerun the regressions.

Another possible source of bias is our reliance on the Wall Street Journal Index on CD and the Dow Jones News Retrieval Service in the sample selection process. We do not know what factors cause some firms to have their capital budget decisions noted in these media; therefore, we cannot be certain that drawing a sample from these media will yield a random sample of capital investment decisions. The only way to guard against any bias caused by the particular news media employed is to draw observations from a wider array of media. It is noteworthy that McConnell and Muscarella (1985) used trade journals and the Wall Street Journal paper index to construct a sample much larger than ours in a shorter sample period. We cannot explain this discrepancy.

Finally, while abnormal returns in the increase sample demonstrate a significant relationship to average *q* and to measures of insider ownership, there is a lack of explanatory power in all but one of the decrease sample regression models. This lack of support of the increase sample results by the decrease sample may be a result of a fundamental difference in the contexts of capital budget increases and decreases. Indeed, while an increase announcement has fairly clear implications for the use of corporate resources, the ultimate use of resources freed by a decrease announcement is unclear. The lack of support could also be due to the fact that the size of the decrease sample is less than half of that of the increase sample. Again, increasing the sample size would be helpful.

5. Conclusion

We find that firms' stock prices neither rise nor fall significantly when they announce changes in their capital budgets. This suggests that the markets reaction to such announcements has changed since the late 1970s, when such events are known to have triggered price increases.

We reject the hypothesis that this is due to myopia, as we can detect no significant negative stock price reaction either. We find that high market to book ratios predict positive price reactions to capital budget increases. Our results do not appear to be due to increased difficulty valuing firms, as investors react similarly to capital expenditure increases by firms with intangible assets related to R&D and by firms without such assets. Free cash flow problems are also an inadequate explanation, as investors react similarly to announcements by low market to book ratio firms with high cash flows as to those by other firms.

We do find that very low or very high managerial ownership does robustly predict negative price reactions, while median levels of insider ownership predict positive price reactions. We conclude that agency problems, stemming from a divergence of managers' interests form shareholders' interests in widely held firms (Jensen and Meckling, 1976), and from managerial entrenchment in closely held firms (Stulz, 1988; and others), may explain investors' lack of enthusiasm for some firm's investment plans.

Tables

Table 2-1: Frequency distribution of capital budget announcements

YEAR	Increase Sample		Decrease Sample	
	Frequency	Percent	Frequency	Percent
1984	10	10.4	3	6.4
1985	9	9.4	3	6.4
1986	6	6.3	16	34.0
1987	8	8.3	2	4.3
1988	10	10.4	1	2.1
1989	14	14.6	2	4.3
1990	19	19.8	3	6.4
1991	6	6.3	7	14.9
1992	7	7.3	8	17.0
1993	7	7.3	2	4.3
Total	96	100	47	100

Two-digit SIC	Industry	Increase Sample		Decrease Sample	
		Frequency	Percent	Frequency	Percent
13	Oil and gas extraction	36	37.5	19	40.4
24	Lumber and wood products			2	4.3
26	Paper and allied products	1	1.0	2	4.3
28	Chemicals and allied products	1	1.0		
29	Petroleum refining	16	16.7	13	27.7
30	Rubber and plastics	3	3.1	1	2.1
31	Leather and leather goods	1	1.0		
33	Primary metal industries	4	4.2	3	6.4
34	Fabricated metal products	4	4.2		
35	Machinery and computer equip.	3	3.1		
36	Electronic and electrical equip.	2	2.1		
37	Transportation equipment	3	3.1		
38	Measuring instruments	3	3.1		
40	Railroad transportation	1	1.0		
42	Motor freight transportation	1	1.0		
45	Transportation by air			1	2.1
50	Durable goods wholesale	1	1.0		
51	Non-durable goods wholesale	2	2.1	1	2.1
53	General merchandise stores	9	9.4	2	4.3
54	Food stores	1	1.0	1	2.1
56	Apparel and accessory stores	1	1.0		
58	Eating and drinking places	1	1.0		
75	Automotive repairs	1	1.0		
87	Professional services	1	1.0	2	4.3

Table 2-2: Descriptive statistics of the sample

q is the ratio of a firm's market value over its book value: ***q***=[Market value of the firm's common shares + (face value of preferred stocks)/(average medium-risk bond yield) + long-term debt] / [(net property plant and equipment) + (inventory)]; ***Cash flow*** is the cash that the firm has normalized by total assets: ***Cash flow***=(Operating income before depreciation - interest payments - income taxes - common dividends - preferred dividends + changes in deferred taxes) / (Total assets); **a** is the level of common stocks owned by a firm's officers and directors; ***Capex*** is the average value of capital expenditures (net of acquisitions) per dollar of existing net property plant and equipment over the prior three years; ***ΔCapex*** is the annual fractional change in capital expenditures (net of acquisitions) averaged over the previous three years; ***RD*** is the annual R&D spending as a fraction of net property plant and equipment averaged over the prior three years; ***ADV*** is the annual advertising spending as a fraction of net property plant and equipment averaged over the prior three years; ***Size*** is the natural logarithm of net property plant and equipment (in $ millions); ***%ΔCAPBUD*** is the announced % change in capital budget from its previous level. The number in square brackets is the p-value for the two-tailed significance test for the mean and is the p-value for the Signed-Rank test for the median. The number in round brackets is the p-value for the Wilcoxon test of equal medians.

	q	*Cash flow*	*α*	*RD*	*ADV*	*Capex*	*ΔCapex*	*Size*	*% ΔCAPBUD*
Increase sample									
Mean	1.611	0.071	0.074	0.028	0.032	0.172	0.058	7.594	24.64
Median	1.216	0.075	0.011	0	0	0.162	0.042	7.474	15.5
Std. dev.	1.326	0.048	0.170	0.052	0.068	0.068	0.194	1.376	27.45
Min.	0.690	-0.064	0.000	0	0	0	-0.378	3.946	1.1
Max	10.685	0.303	0.642	0.199	0.428	0.352	0.671	10.886	163.6
Decrease sample									
Mean	1.173	0.049	0.039	0.011	0.008	0.186	0.066	8.354	22.71
Median	1.098	0.067	0.008	0	0.012	0.177	0.053	8.380	23.00
Std. dev.	0.370	0.128	0.105	0.040	0.007	0.055	0.156	1.292	11.00
Min.	0.6443	-0.769	0.000	0	0	0.105	-0.204	5.732	50.00
Max	4.1724	0.170	0.631	0.217	0.077	0.355	0.448	10.784	3.00
Mean difference	**0.438** **[0.0279]**	-0.021 [0.1553]	0.035 [0.1969]	**0.016** **[0.0589]**	**0.024** **[0.0186]**	-0.014 [0.2179]	-0.008 [0.8087]	**-0.759** **[0.0019]**	
Median difference	**0.118** **(0.0525)**	0.008 (0.4082)	0.003 (0.2633)	**0.017** **(0.0224)**	-0.007 (0.7970)	-0.015 (0.1877)	-0.011 (0.5512)	**-0.906** **(0.0019)**	

Table 2-3: Descriptive statistics for CAR and PI and test of significance of means and medians.

CAR is the two-day (day -1 and day 0) cumulative abnormal returns in excess of the CRSP value-weighted market index returns. ***PI*** is defined as (***CAR*MKTCAP***) / |***ΔCAPBUD***|, where ***MKTCAP*** is the firm's equity market capitalization two days before the announcement, and ***ΔCAPBUD*** is the announced dollar change in capital budget from its previous level. The number in square brackets is the p-value for the two-tailed significance test for the mean and is the p-value for the Signed-Rank test for the median. The number in round brackets is the p-value for the Wilcoxon test of equal medians

	Mean	Median	Standard Deviation	Minimum	Maximum
PANEL A: *CAR*					
Increase Sample	-0.003 [0.3082]	-0.002 [0.5065]	0.024	-0.085	0.089
Decrease Sample	0.004 [0.7971]	0.002 [0.5108]	0.030	-0.089	0.079
Sample Differences	-0.007 [0.1990]	-0.004 (0.2854)			
PANEL B: *PI*					
Increase Sample	-0.309 [0.4820]	-0.042 [0.5469]	4.289	-30.294	21.609
Decrease Sample	0.156 [0.2518]	0.028 [0.2757]	0.919	-2.111	3.810
Sample Differences	-0.465 [0.4647]	-0.070 [0.2418]			

Table 2-4: Regression estimates of profitability index on average *q*, R&D Spending, and Advertising Spending

The dependent variable, *PI*, is defined as (*CAR***MKTCAP*) / |*ΔCAPBUD*|, where *CAR* is the two-day (day -1 and day 0) cumulative abnormal returns in excess of the CRSP value-weighted market index returns, *MKTCAP* is the firm's equity market capitalization two days before the announcement, and *ΔCAPBUD* is the announced dollar change in capital budget from its previous level; *q* is the ratio of a firm's market value over its book value: *q*=[Market value of the firm's common shares + (face value of preferred stocks)/(average medium-risk bond yield) + long-term debt] / [(net property plant and equipment) + (inventory)]; *High q* takes the value of 1 if *q* is greater than unity and the value of 0 otherwise; *RD* is the annual R&D spending as a fraction of net property plant and equipment averaged over the prior three years; *ADV* is the annual advertising spending as a fraction of net property plant and equipment averaged over the prior three years; *High RD* takes the value of 1 if the annual R&D spending as a fraction of net property plant and equipment averaged over the prior three years is greater than the sample median, and 0 otherwise; *High ADV* takes the value of 1 if the annual advertising spending as a fraction of net property plant and equipment averaged over the prior three years is greater than the sample median, and 0 otherwise; *Capex* is the average value of capital expenditures (net of acquisitions) per dollar of existing net property plant and equipment over the prior three years; *ΔCapex* is the annual fractional change in capital expenditures (net of acquisitions) averaged over the previous three years; *Size* is the natural logarithm of net property plant and equipment (in $ millions). The number in square brackets is the p-value for the two-tailed significance test.

	Increase sample				Decrease sample			
	(4.1)	(4.2)	(4.3)	(4.4)	(4.5)	(4.6)	(4.7)	(4.8)
Intercept	-2.346 [0.4745]	-3.035 [0.3664]	-0.872 [0.7941]	-0.483 [0.8913]	-1.463 [0.2928]	-1.057 [0.3808]	-0.310 [0.7984]	-1.264 [0.3558]
q	**0.785** **[0.0304]**				0.546 [0.2168]			
High *q*		**2.217** **[0.0306]**				0.462 [0.1384]		
RD			-2.789 [0.7557]				**-9.563** **[0.0054]**	
ADV			-2.933 [0.6640]				-3.830 [0.7582]	
High RD				-0.443 [0.6751]				**-1.076** **[0.0276]**
High ADV				0.288 [0.7794]				-0.324 [0.4227]
CAPEX	9.008 [0.1890]	**10.642** **[0.1132]**	**14.124** **[0.0516]**	**13.618** **[0.0603]**	-2.062 [0.4370]	-1.982 [0.4466]	-1.363 [0.5956]	-0.062 [0.9808]
ΔCAPEX	-1.274 [0.5981]	-1.163 [0.6287]	0.156 [0.9482]	-0.141 [0.9543]	0.241 [0.7957]	0.219 [0.8103]	1.062 [0.2469]	1.205 [0.1941]
SIZE	-0.092 [0.7901]	-0.079 [0.8189]	-0.223 [0.5317]	-0.286 [0.4894]	0.161 [0.1795]	0.153 [0.1825]	0.094 [0.4233]	0.197 [0.2111]
Adj. R^2	0.0624	0.0624	0.0051	0.0044	-0.0120	0.0042	0.1128	0.046
p-value	**0.0424**	**0.0425**	0.3679	0.3743	0.4936	0.3938	**0.0762**	0.2292

Table 2-5: Regression estimates of profitability index on average *q* and cash flows

The dependent variable, *PI*, is defined as (*CAR*MKTCAP*) / |*ΔCAPBUD*|, where *CAR* is the two-day (day -1 and day 0) cumulative abnormal returns in excess of the CRSP value-weighted market index returns, *MKTCAP* is the firm's equity market capitalization two days before the announcement, and *ΔCAPBUD* is the announced dollar change in capital budget from its previous level; *q* is the ratio of a firm's market value over its book value: *q*=[Market value of the firm's common shares + (face value of preferred stocks)/(average medium-risk bond yield) + long-term debt] / [(net property plant and equipment) + (inventory)]; *Low q* takes the value of 1 if *q* is less than unity and 0 otherwise; *Cash flow*=(Operating income before depreciation - interest payments - income taxes - common dividends - preferred dividends + changes in deferred taxes) / (Total assets); *High CF* takes the value of 1 if *Cash flow* is greater than the sample median and 0 otherwise; *Capex* is the average value of capital expenditures (net of acquisitions) per dollar of existing net property plant and equipment over the prior three years; *ΔCapex* is the annual fractional change in capital expenditures (net of acquisitions) averaged over the previous three years; *Size* is the natural logarithm of net property plant and equipment (in $ millions). The number in square brackets is the p-value for the two-tailed significance test.

	increase subsample (5.1)	decrease subsample (5.3)
Intercept	-1.136 [0.7252]	-0.332 [0.7881]
Low *q*	**-2.372** **[0.0786]**	-0.349 [0.4225]
High CF	1.180 [0.2591]	-0.368 [0.3069]
Low *q* ×High CF	0.477 [0.8007]	-0.084 [0.8861]
CAPEX	8.387 [0.2224]	-1.919 [0.4619]
ΔCAPEX	-1.135 [0.6373]	0.102 [0.9119]
SIZE	-0.066 [0.8482]	0.141 [0.2395]
Adj. R^2	0.0657	0.0082
p-value	**0.0593**	0.4005

Table 2-6: Regression estimates of profitability index on management ownership

The dependent variable, *PI*, is defined as (*CAR***MKTCAP*) / |*ΔCAPBUD*|, where *CAR* is the two-day (day -1 and day 0) cumulative abnormal returns in excess of the CRSP value-weighted market index returns, *MKTCAP* is the firm's equity market capitalization two days before the announcement, and *ΔCAPBUD* is the announced dollar change in capital budget from its previous level; a is the level of common stocks owned by a firm's officers and directors; *High α* takes the value of 1 if *α* is greater than the sample median and 0 otherwise; *Very High α* takes the value of 1 if *α* is greater than 0.381 (0.318) for the increase (decrease) sample and 0 otherwise [These points are chosen for respective samples because PI in models (6.3) and (6.7) as a function of α and α^2 is respectively maximized at those values]; *Capex* is the average value of capital expenditures (net of acquisitions) per dollar of existing net property plant and equipment over the prior three years; *ΔCapex* is the annual fractional change in capital expenditures (net of acquisitions) averaged over the previous three years; *Size* is the natural logarithm of net property plant and equipment (in $ millions). The number in square brackets is the p-value for the two-tailed significance test for the mean and is the p-value for the signed-rank test for the median. The number in round brackets is the p-value for the Wilcoxon test of equal medians.

	Increase Sample				Decrease Sample			
	(6.1)	(6.2)	(6.3)	(6.4)	(6.5)	(6.6)	(6.7)	(6.8)
Intercept	-3.807 [0.2779]	-3.227 [0.3757]	-5.691 [0.1167]	-3.709 [0.3068]	-0.776 [0.5430]	-2.447 [0.2222]	-1.327 [0.3131]	-2.947 [0.1618]
α	**5.751 [0.0375]**		**30.823 [0.0293]**		0.783 [0.6023]		8.603 [0.1183]	
α^2			**-40.442 [0.0696]**				-13.517 [0.1394]	
High α		1.321 [0.1545]		1.138 [0.2350]		0.546 [0.2361]		0.581 [0.2111]
Very High α				1.458 [0.4375]				0.620 [0.4043]
CAPEX	**17.128 [0.0148]**	**13.883 [0.0415]**	**15.211 [0.0297]**	**15.738 [0.0300]**	-1.245 [0.6369]	-0.505 [0.8509]	-2.304 [0.3938]	-0.083 [0.9758]
ΔCAPEX	1.463 [0.5468]	0.533 [0.8234]	1.510 [0.5289]	1.033 [0.6768]	0.768 [0.4280]	0.926 [0.3237]	0.966 [0.3176]	1.182 [0.2342]
SIZE	0.006 [0.9854]	-0.022 [0.9522]	0.234 [0.5322]	0.007 [0.9840]	0.130 [0.2859]	0.283 [0.1306]	0.201 [0.1223]	**0.327 [0.0956]**
Adj. R^2	0.0587	0.0345	0.0826	0.0303	-0.0430	-0.0150	-0.0124	-0.0220
p-value	**0.0493**	0.1264	**0.0250**	0.1698	0.7175	0.5135	0.4983	0.5550

Bibliography

Berle, A.A. and G.C. Means, 1932, *The modern corporation and private property*, MacMillan, New York.

Copeland, T.E., and J.F. Weston, 1988, *Financial Theory and Corporate Policy,* 3rd Ed. Addison-Wesley Publishing Co., Reading, Massachusetts.

Howe, Keith M., Jia He, and G. Wenchi Kao, 1992, One-time cash flow announcements and free cash-flow theory: Share repurchases and Special Dividends, *Journal of Finance* 47, 1963 - 1975.

Jensen, Michael C., 1986, Agency costs of free cash flow, corporate finance, and takeovers, *American Economic Review* 76, 323 - 329.

Jensen, Michael C., and W.H. Meckling, 1976, A theory of the firm: Managerial behavior, agency costs, and ownership structure, *Journal of Financial Economics* 3, 305 - 360.

Lang, Larry H.P., and Robert H. Litzenberger, 1989, Dividend announcements: Cash flow signaling vs. free cash flow hypothesis, *Journal of Financial Economics* 24, 181 - 191.

Lang, Larry H.P., René M. Stulz, and Ralph A. Walkling, 1991, A test of the free cash flow hypothesis, *Journal of Financial Economics* 29, 315 - 335.

Lehn, Kenneth and Annette Poulsen, 1989, Free cash flow and stockholder gains in going private transactions, *Journal of Finance* 44, 771 - 789.

Long, John B., 1990, The numeraire portfolio, *Journal of Financial Economics* 26, 29-69.

Lustig, Peter, Randall Morck, and Bernard Schwab, 1994, *Managerial Finance in a Canadian Setting*, 5th ed., John Wiley and Sons, Toronto.

Lyon, Andrew, 1995, Taxation, information asymmetries, and the financing choice of the firm. In Anwar Shaw (ed.) *Fiscal Incentives for Investment and Innovation,* Oxford University Press.

McConnell, John J. and Chris J. Muscarella, 1985, Corporate capital expenditure decisions and the market value of the firm, *Journal of Financial Economics* 14, 399 - 422.

McConnell, John J. and Henri Servaes, 1990, Additional evidence on equity ownership and corporate value, *Journal of Financial Economics*, 27, 595 - 612.

Morck, Randall, Andrei Shleifer, and Robert W. Vishny, 1988, Management ownership and market valuation: An empirical analysis, *Journal of Financial Economics* 20, 293 - 315.

Porter, Michael E., 1992, Capital Disadvantage: America s Failing Capital Investment System, *Harvard Business Review*, September-October, 65-82.

Stein, J.C., 1989, Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior, *Quarterly Journal of Economics*, November, 655-669.

Stulz, René M., 1990, Managerial discretion and optimal financing policies, *Journal of Financial Economics* 26, 3 - 27.

Tobin, James, 1978, Monetary Policies and the Economy: The Transmission Mechanism, *Southern Economic Journal*, January, 421-431.

Veblen, Thorstein, 1904, *The Theory of Business Enterprise*, Charles Scribner's Sons, New York.

Chapter 3

Rational, Value-Maximizing Boards of Directors and Replacement of Management[12]

1. Introduction

The modern form of the corporate entity has, at the peak of its hierarchical structure, the board of directors. In theory, the board is responsible directly to the shareholders of the firm and its duties include appointing and evaluating the management team that is responsible for running the operations of the firm. The board is designed to mitigate the agency costs associated with the separation of ownership and control in a world of less than perfect information. In line with this, the board is also responsible for monitoring the actions of management and replacing the management if they are not maximizing the value of the firm. Bruce Atwater, CEO of General Mills, states that "the most important function of directors is management selection, evaluation, compensation, and replacement...The board must regularly evaluate the CEO and evaluate his or her succession plan."

There has been much debate in the literature as to whether the board actually does fulfill this part of its mandate. Fama [1980] sees the board as part of a system that can effectively monitor and discipline managers. The board is, in turn, disciplined by the market for its members' services. Weisbach [1988] finds that the probability of CEO turnover is inversely related to firm

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12 This chapter was co-authored with Greg MacKinnon and Jim Unterschultz.

performance for firms with boards that are dominated by outside directors. This would indicate that boards, at least outsider dominated ones, are responsive to problems with management. This is in line with the findings of Coughlan and Schmidt [1985] who find that boards effectively link firm performance with manager compensation and turnover. Conversely, Mace [1971, p.182] concludes that "generally boards of directors do not do an effective job of evaluating or measuring the performance of the president." As well, whereas Jensen and Murphy [1990] find no "economically significant" link between firm performance and total manager wealth effects, Bentson [1985] finds that there is a significant effect when the change in compensation includes changes in the value of options and stock holdings for managers of large, conglomerate firms. Given that compensation is set by the board, these results represent conflicting evidence on the effectiveness of the board of directors at monitoring management. Perhaps most importantly though, there is a general feeling in the popular press that boards are not fulfilling their mandate and that "directors exist merely to ensure the smooth, uninterrupted reign of a CEO."[13]

While much of the debate has centered on the board's role in the turnover of top management (i.e. Coughlan and Schmidt [1985], Weisbach[1988]), there is also debate as to whether boards are serving shareholder interests when they do replace management. Jensen and Warner [1988] give a review of the literature's mixed results on the stock price reaction

13 The Globe and Mail. Sept. 10, 1993. page B1

to management turnover announcements. Mahajan and Lummer [1993] find that internally generated changes in top management results in negative stock price reactions, whereas externally generated changes result in positive reactions. They conclude that the decision of the board "may not be solely motivated by considerations of shareholder wealth maximization [p. 406]."

This gets to the heart of the debate about the effectiveness of the board of directors in replacing under-achieving managers. There seems to be some evidence (and a strong feeling in the popular press) that boards do not replace management that is "under-achieving" in some undefined sense. Many people have taken this as evidence that the board is either not rational or is not serving the objectives of shareholders exclusively. For example, Johnson [1990, pp.47-48] states that during the 1960's and 1970's, during a period of stability and prosperity, boards grew "fat, dumb and comfortable." Shleifer and Vishny (1988) suggest that boards are usually predisposed to give top managers the benefit of the doubt, rather than find fault with these managers. They further state that even when a board is value maximizing, it lacks sufficient information about the firm (and, by implication, the top manager) to maximize firm value. Shleifer and Vishny state that the board's information limitations are likely the most important impediments to value maximizing corporate governance. Ergo, hostile takeovers pre-empt boards as top management disciplining devices.

Evidence supporting value maximizing boards comes from Furtado and Rozeff (1987). In this work, the managerial labour markets are examined. They find that shareholders experience small but significantly positive wealth effects with appointments to the top managerial positions. Promotion to top management is more common than external hiring and Furtado and Rozeff explain this with the existence of firm specific human capital and the higher informational costs associated with external hiring.

One problem with the previous research in the area of managerial replacement is the lack of theory to provide a firm basis for interpreting the empirical results. Further theory on the nature of the replacement decision itself is necessary to give direction to future empirical work as well as to provide a framework within which to interpret past results. Previous theory on replacement of top management by the board of directors is limited to three papers. Hirshleifer and Thakor [1994] model the interaction between the information sets available to the board and to a raider in order to model how the decision to replace the CEO may change with a successful or unsuccessful takeover threat. Hermalin and Weisbach [1995] model the independence and effectiveness of boards as endogenous to a bargaining process between the CEO and the board. The model presented here differs from Hermalin and Weisbach in that their model assumes that board members derive disutility from monitoring while we assume that boards altruistically serve shareholders' interests. Finally, Parrino [1992] models the updating process on the board's estimate of the CEO's ability and finds that management turnover should be inversely related to the noisiness of the periodic signal received by the board. While the Parrino model is similar to this one, the key difference is that Parrino assumes that the current CEO will be replaced anytime the board's expectation of his managerial ability is below their expectation of the ability of a potential replacement. It is this point that we address here. We show that it is not necessarily an optimal replacement rule.

In particular, this paper addresses the question of when a rational board of directors will replace a manager. We show that a board of directors which is rational and acting exclusively in the interests of shareholders will not always replace managers who are of below average ability. The intuition behind this finding is that, even if the board knows the managerial ability of the incumbent is below average, replacing the manager involves choosing a new, unknown manager from a pool of candidates. Even though the incumbent is a "bad" manager, there is always a chance that any replacement may turn out to be worse. Furthermore, searching for a manager is costly and replacing the incumbent will result in a disruption of the firm's business. The model we present is simple in that it utilizes some well known principles of utility analysis. However, these principles are applied in an entirely new context.

Our basic premise is that many of the examples of the apparent ineffectiveness of boards at unseating incumbent management may not be the result of "lame-duck" boards nor of non-value maximizing boards. Rather, it is the uncertain environment in which boards make decisions that is responsible for the retention of below average managers.

We develop two models within this paper. The models are based on the idea that the value of a firm is a known function of the ability of the manager as well as a random state of the world. In the first model, it is assumed that the board of directors has perfect information regarding the ability of the incumbent manager. In the second model, this assumption is relaxed so that the board has only imperfect information regarding the incumbent[14]. Because the basic results of the two models are the same and the intuition of our findings is more clear in the perfect information case, we leave the imperfect information model to the end and concentrate our discussion on the first model.

The paper is set up as follows. In Section 2 we consider the rule that a rational board must use when deciding whether to replace management in a setting with perfect information about the incumbent manager's ability and show that some below average managers will be retained. In Section 3, we explicitly examine the maximization problem considered by the board of directors in considering the replacement of management. In Section 4 we repeat the process under the assumption that the board has only an imperfect signal about the ability of the incumbent manager. At no point do we make assumptions about the form of the distribution of managerial talent in the

14 We feel that presenting the two extreme cases, perfect knowledge of the incumbent versus a noisy signal of the incumbent's ability, demonstrates our results.

economy as a whole, and assume only that the firm's value function is concave in its arguments. Section 5 presents some brief conclusions.

In the perfect information model (Sections 2 and 3), it is assumed that manager ability is the only input into the firm value function. This perfect information case would correspond to the case where the board has been able to, over time, observe various states of the world and the resulting firm value. They can therefore determine the exact ability of the incumbent manager by using the observable firm value and the known functional form of firm value. To make the derivation simpler in this case we ignore the state of the world argument in the firm value function. This assumption is reasonable given that members of the board observe conditions in their particular industry and can filter out the effects of this random component when evaluating managers. Because the state of the world would not enter into the replacement decision process, we are justified in omitting it. It should be noted here that this omission presumes that management ability is an absolute measure and is independent of the state of the world. That is, the model does not allow a manager to be of one level of ability in one state and another level in another state (e.g. a "crisis" manager who is good at handling firms during bad times but not as good when the firm is more stable).

It should be noted at this point that what is developed here is a model of the board's replacement decision and, as a model, it is a simplification of the real world. Our purpose is not to argue that the world must follow the model as developed here, but rather to highlight a particular insight that must be considered when interpreting empirical evidence concerning the replacement of top management by the board of directors.

2. Theoretical Model

2.1 Perfect Information Model

The first case we consider is that in which the board of directors has perfect information about the managerial ability of the incumbent manager. We assume that all individuals are endowed with a certain level of managerial ability or talent that can be quantified by a cardinal number denoted a. Let the ability of the incumbent be denoted a_I. The assumption here is that the board knows the ability of the incumbent exactly because they may observe the value of the firm and simply invert the functional form in order to determine a_I. The value of the firm[15] is denoted:

$$V(a_I)$$
where:
$$V'(a_I) > 0 \quad (1)$$
$$V''(a_I) < 0$$

15 Alternatively, one can think of the V(.) function as denoting the utility of Board members. Assume that Board members' compensation (either pecuniary or in the form of increased value in the market for Board member services) is a function of firm value and firm value is, in turn some increasing function of managerial ability. As long as the Board members' utility functions are concave and there is no disutility to monitoring (i.e. Boards serve shareholders interests), then the results of the model all hold. However, we feel that taking V(.) to directly represent the value of the firm is a more intuitively appealing way in which to demonstrate the results of the model.

It is assumed that the value function has a lower bound of zero (i.e. the firm exhibits limited liability). The function V(.) is not adjusted for managerial compensation. Adjustments for the effect of compensation on total firm value will be made outside of the V(.) function[16]. Thus, V(.) may be thought of as a type of production function where the input is managerial ability. It is assumed that the firm is unlevered so that the value function represents the market value of equity and by maximizing V(.), the board is explicitly serving the interests of its shareholders. The assumption of no leverage can be relaxed with little difficulty, it is made for simplicity. The effect of considering a leveraged firm will be mentioned in the conclusions to the paper. Note that the assumption of a cardinal measure of managerial ability precludes the possibility of applying a monotonic transformation to the ability measure to render the value function non-concave.

The board will act to maximize the value of the firm and may wish to replace the incumbent manager in order to accomplish this. If the manager is replaced, the board randomly draws a replacement from the pool of available alternatives. Thus, while the board recognizes the ability level of the

16 We assume that, when there is a replacement of the CEO, the expected gains in gross firm value are divided between the firm and the new manager. Although we expect that the firm will get most of the gains because of competition on the supply side of the market for managers (i.e. the firm is effectively a monopsonist), this is not necessary. As long as the firm has some expected net gain when a CEO is replaced with a new manager of greater ability, then the results of the model hold. If this was not so (managers bargained to have all of the quasi-rent created by their ability accrue to them), then replacement of management would never be observed empirically.

incumbent, the ability of any potential replacement remains an unknown[17]. This assumption would seem reasonable for two reasons. First, the interview and "head-hunting" process is not perfect in that it certainly does not provide a perfect signal as to the ability of a particular potential manager. Second, signals of managerial ability taken from past performance are also very noisy because of the inherent differences between organizations. A manager who has been observed to be a good manager in one firm will not necessarily be a superior manager in another firm because of the different skills that may be required for some set of firm-specific problems. Furthermore, since the board of the firm that is considering replacing its incumbent manager does not know the reactions of another firm to particular states of the world as well as it knows its own, it may not be possible for that board to determine the ability of other managers from the performance of their current firm. Given these problems, we simply assume that the ability of any particular alternative manager is unknown to the board; however, we assume the board knows the distribution of the abilities of the alternative managers from which they will choose.

The seeming simplicity of the search is purposeful. This model is not meant to be a search model in the sense that we are looking for the optimal manner for the board to look for a new manager. Rather, we are modeling the uncertainty that will be present in any search. As long as there exists uncertainty of some type in the search process, our results will hold. We avoid

17 We recognize that candidates may attempt to signal their ability. For simplicity we assume that the cost of signaling is the same for all potential replacements, regardless of their ability; therefore, the signals are viewed by the Board as being meaningless. [Spence, 1973]

modeling the search procedure explicitly for simplicity. Our only assumption is that perfectly separating equilibria are not possible. That is, it is not possible for firms to offer compensation contracts that cause potential new managers to self select and reveal their ability. A secondary assumption that can justify this is simply to assume that managers possess only an unbiased expectation of their own ability. Thus, even if the firm could offer contracts designed to attract only "good" managers, there would still be some applicants for the job that only thought they were "good". Hence, there would still be some uncertainty about the true ability of the manager that the firm finally selected.

While the random drawing of a replacement may seem unrealistic, there are alternative interpretations of this drawing that may be more intuitive. For instance, assume that the board has gone through some type of optimal search procedure and now possesses a "short-list" of potential candidates. The board must now choose from among that short-list and this involves choosing a replacement from among a group, the same situation presented in our model. Alternatively, the board may be considering a single person as a possible replacement for the incumbent. In this case the distribution of managerial talent in the model can be viewed as the boards perceived distribution on the unknown ability of the single candidate.

The ability levels of managers in the economy is assumed to be distributed with the probability density function $f_a(\tilde{a})$. The density has support (ℓ, A). The entire "pool" of managers, (ℓ, A), is not necessarily available to the firm when choosing a replacement manager. The board of directors chooses from a truncated version of $f_a(\tilde{a})$ with upper bound a_u. We assume that $a_u \equiv a_u(c)$ where c is the cost of a search for an alternate manager and $a_u' > 0$. We define cost of search as the present value of all incremental expenses related to hiring a replacement. Thus, cost of search in this paper includes such expenses as advertising the position and interviewing candidates; but more importantly, it includes the present value of the compensation plan that is offered to potential replacements. The present value of the compensation plan presently being given to the incumbent is denoted as k; therefore it is possible within our model for the board to change compensation schemes when replacing management. Here, c will be the choice variable for the board. They decide how much to spend on searching for an alternative manager and the more that they spend, the greater the upper bound on the pool of managerial abilities that is available to them[18,19].

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18 If a "good" manager joins a "poor" firm, it may serve as a signal to the industry about the manager's ability. This will reduce the value of the manager's human capital. In order to be convinced to join the firm in question, the manager must be compensated for this loss through a large salary or incentives (or a signing bonus in the case of professional athletes). This is treated as part of the cost of search. Hence, a higher cost c means that the firm is willing to offer larger incentives to new managers and this will attract the interest of " a better class of managers". Therefore, the upper bound on the distribution is seen as an increasing function of c.

19 The model presented here is a one-shot search model. This omits the possibility that the Board can improve its signal about the alternative managers' ability by spending c* dollars on search, observing the pool available (through direct observation of the people involved, i.e. interviews), then spending c**>c* dollars on a second search and choosing from the new candidates who come forward under the higher search cost.

Thus, when choosing a replacement manager, the board of directors chooses randomly from a conditional density of the form:

$$f_a(\tilde{a}|\tilde{a} \leq a_u(c)) = \frac{f_a(\tilde{a})}{F_a(a_u(c))} \tag{2}$$

where $F_a(\cdot)$ is the cumulative distribution function of $\tilde{a}$.

The board will replace the incumbent if the expected change in firm value (given a search cost, c, and a current compensation plan, k) is positive[20]. That is, replace the incumbent if:

$$E[V(\tilde{a}) - c + k - d - V(a_I)] > 0 \tag{3}$$

where d represents the "disruption cost" of replacing management. This d includes such things as the cost of switching over an entire firm to a new management style, the cost of replacing lower levels of management and suppliers/buyers that may have been loyal to the previous manager, the loss of opportunities while the new manager "learns the ropes", and the present value of any retirement benefits or payouts to the manager who has been replaced. The disruption cost could be seen as a choice variable on the part of the incumbent by the entrenchment arguments of Shleifer and Vishny [1989], but, since our model involves board decisions only and is not of a game theoretic nature, we leave this to future research. Thus, the variable d is treated as a

20 It can be shown that if a Board decides to search it will automatically decide to replace the incumbent with the alternative that is found (given that the ability of the new manager is still unknown). Thus, we ignore the search/not search problem and look exclusively at the equivalent replace/do not replace decision.

known in this case and, as the current value of the firm is observable, we may rewrite (3) as:

$$G \equiv E\left[V(\tilde{a})\right] - c + k - d - V(a_I) > 0 \tag{4}$$

where G is the gain from replacement function. The board of directors will replace the incumbent manager if G>0 for their optimal choice of c.

We will assume, for the moment, that the board solves for their optimal $c=c^*$ and evaluates the gain from replacement function at that point. In Section II we will examine the actual first order condition that is generated.

2.1.1 An Average Incumbent Manager

Assume that the incumbent manager is of average ability with respect to the optimal pool of potential replacements. That is,

$$a_I = E^*\left[\tilde{a}\right]. \tag{5}$$

where the term $E^*[\cdot]$ refers to an expectation conditional on the optimum being chosen, i.e. $E^*[\cdot] = E\left[\cdot | a \le a_u(c^*)\right]$. Equation (4) then becomes:

$$E^*\left[V(\tilde{a})\right] - V(E^*\left[\tilde{a}\right]) > c^* - k + d \tag{6}$$

Equation (6) represents the condition under which the board of directors will choose to replace an incumbent manager that they know to be of average ability (with respect to the available pool of replacements). Equation (6) implies that if it is not expected value maximizing to replace an average manager, then it will also not be optimal to replace a manager who is above average.

Using the properties of the firm value function and Jensen's Inequality, it is apparent that:

$$V(E^*[\cdot]) > E^*[V(\cdot)]$$

Thus, the left hand side of equation (6) is always negative. This results in the conclusion that it is never rational for a value-maximizing board of directors to replace a manager who is of average (or above average) ability (as long *as* c^*-k+d is greater than or equal to zero, which will be discussed below). Note that the result holds *even if the costs of replacing the incumbent are zero* (i.e. the board's expenditure on searching for and hiring a replacement is equal to the current compensation being given to the incumbent and there is no disruption cost).

The inequality in (6) may hold if $c^*<k$. That is, (6) may hold if the optimal action for the board is to hire a replacement at a lower rate of compensation than his predecessor. Still, even in the case where $c^*<k$, as long as $d>k-c^*$ then the conclusion of non-replacement of average managers goes through. Intuitively, we feel that the disruption cost will be large as compared to salaries and search costs. Remembering that the disruption cost includes any termination payouts to the incumbent, it seems likely that any decrease in compensation for a new manager will be more than offset by the cost, *d*. Moreover, one may assume that the board considers the replacement decision during periodic evaluations of the manager. Since the current compensation of the incumbent, *k*, would have had to be an optimal contract at the time the current manager was hired, then barring a change in the firm since the incumbent was hired (e.g. a change in the shape of the value function), k should still be the optimal contract to offer a replacement. That is, without a change in the value function, $c^{*}=k$ (ignoring the direct costs of search momentarily) and inequality (6) will not hold for an average manager. Therefore, our results should go through in most (although admittedly not all) cases.

This conclusion is extended below to show that there is a range of abilities *below* the optimal mean for which a rational board will not replace the incumbent. The question of interest is how far below the mean of the optimal distribution must the incumbent's ability be before the board replaces.

2.1.2 A Below Average Incumbent Manager

Consider an incumbent manager who is of below average ability with respect to the optimal pool of potential replacements. That is,

$$\begin{aligned} a_I &= E^{*}\left[\tilde{a}\right] - \varepsilon \\ \varepsilon &> 0 \end{aligned} \tag{7}$$

The board of directors will replace the incumbent if:

$$E^{*}\left[V(\tilde{a})\right] - V(E^{*}\left[\tilde{a}\right] - \varepsilon) > c^{*} - k + d \tag{6a}$$

We would like to determine the level of managerial ability at which the board will be indifferent between replacing or keeping the incumbent, a_I^{Ind}.

The indifference point occurs where the left hand side and right hand side of (6a) are equal. Given that the firm value function is invertible, equality will obtain when:

$$\varepsilon = E^{*}[\tilde{a}] - V^{-1}\left[E^{*}[V(\tilde{a})] - c^{*} + k - d\right] \tag{8}$$

Therefore, the indifference point will be at the level of ability:

$$a_{I}^{ind} = E^{*}[\tilde{a}] - \varepsilon$$
$$a_{I}^{ind} = V^{-1}\left[E^{*}[V(\tilde{a})] - c^{*} + k - d\right]. \tag{9}$$

As long as d>*k*-*c**, then ε>0. Equation (9) says that the board of directors is indifferent to replacement when the ability of the incumbent is equal to the inverse of the expected value of the firm after replacement. The board will replace the incumbent when the manager's level of ability is less than a_{I}^{ind} from (9). Thus, there exists a range of below average abilities for which it is not rational for the board to replace.

Figure 3-1 shows the basic structure of the model when, for simplicity, it is assumed that d=*k*-*c**. One can easily see an alternative interpretation of a_{I}^{ind}. It is simply a "certainty equivalent manager". Given this interpretation, it can be shown that ε can also be represented by (the derivation is given Appendix 1):

$$\varepsilon = -\frac{1}{2}\sigma_{a}^{2}\frac{V''\left(E^{*}[\tilde{a}]\right)}{V'\left(E^{*}[\tilde{a}]\right)} + \frac{c^{*} - k + d}{V'\left(E^{*}[\tilde{a}]\right)} \tag{8a}$$

Again, it is apparent that as long as d>k-c^*, then ε>0. Of course, it is possible (although probably rare) to get the opposite situation. This would give the interesting, but perverse, situation of rational boards firing above average managers

This finding indicates that the apparent lack of disciplinary action on the part of boards when confronted with what are deemed "bad" managers is not necessarily the result of irrationality on the part of the board nor a lack of concern with the objectives of shareholders. Rather, there does exist discipline of the manager within the firm, but it simply does not pay to replace the manager in all circumstances. The result applies even if d=0 and $c^*=k$. This means that the following, seemingly counter-intuitive, situation is possible: assume that an incumbent manager falls within the range of below average managers for which replacement is not optimal as described above. Further, assume that d=0 and that $c^*=k$. This implies that the board could replace the incumbent with no disruption to the firm, could pay the new manager less than the incumbent (since c^* includes compensation as well as direct search costs) and would expect this "cheaper" manager to be better than the one they have now, yet the board would choose not to do this. Most importantly, the choice not to fire the incumbent is entirely rational and in the interests of shareholders.

3. Maximization Problem

We now turn to explicit consideration of the maximization process followed by a rational board of directors when considering the replacement of an incumbent manager. The optimal search cost is that which maximizes the gain from replacement and solves:

$$\underset{c}{Max}\, G = \underset{c}{Max}\left\{ \int_{\ell}^{a_u(c)} V(\tilde{a}) \frac{f_a(\tilde{a})}{F_a(a_u(c))} d\tilde{a} - c + k - d - V(a_I) \right\} \tag{10}$$

The first order condition for (10) is:

$$E^{*}\left[V(\tilde{a})\right] = V(a_u(c^{*})) - \frac{F_a(a_u(c^{*}))}{a_u'(c^{*}) f_a(a_u(c^{*}))} \tag{11}$$

Substituting equation (11) into the definition of the gain function from (4) gives the maximum expected gain possible if the board replaces the incumbent:

$$G^{*} = V(a_u(c^{*})) - \frac{F_a(a_u(c^{*}))}{a_u'(c^{*}) f_a(a_u(c^{*}))} - c^{*} + k - d - V(a_I) \tag{12}$$

The incumbent will be replaced if $G^{*} > 0$ which occurs if:

$$V(a_u(c^{*})) - V(a_I) > c^{*} - k + d + \frac{F_a(a_u(c^{*}))}{a_u'(c^{*}) f_a(a_u(c^{*}))} \tag{13}$$

Thus the replacement decision is dependent on the relative levels of the greatest *possible* gain in value given replacement and the costs associated with replacement. $F_a(a_u(c^{*}))$ is a measure of the size of the optimal pool with respect to the economy-wide pool. It can be viewed as a supply of relative managerial quality. The last term on the right hand side of (13) is the product of the inverse of the elasticity of this supply with respect to search costs and the search cost. That is, the supply elasticity of quality is $\eta = \left[\frac{\partial F}{\partial c}\right]\left[\frac{c}{F}\right]$. The last term in (13) is $\left[\frac{1}{\eta}\right]c^*$. The more elastic the supply, the smaller is this term since the board is choosing from a managerial pool with higher mean abilities.

Note that the explicit derivation of the maximization problem faced by the board of directors is not central to the main conclusion of the paper, that boards may rationally retain some below average managers. However, it is included because we feel that there exists substantial potential for its exploitation in deriving comparative statics across industries. Specifically, there may be differences in rates of retention of managers in industries characterized by different value functions (possibly expanding versus contracting industries), elasticity of managerial supply et cetera that may lead to empirically testable conclusions for the model. However, we leave this to future research.

4. Generalized Model

4.1 Noisy Model

Now assume that the firm's value is a function of managerial ability and a noise term. That is, the value function is now $V(\tilde{\theta})$ where $\tilde{\theta} = \tilde{a} + \tilde{\gamma}$. As before, $\tilde{a}$ is a random variable denoting managerial ability while $\tilde{\gamma}$ is a random variable representing noise. We assume that $\tilde{a}$ and $\tilde{\gamma}$ are independent. The probability density function for the noise component is $h(\tilde{\gamma})$. The value function retains all the properties described in the perfect information case.

We assume that the incumbent manager is an endowment of the firm, drawn from the full distribution $f_a(\tilde{a})$. As in the previous section, any management changes take place by drawing a manager from the truncated distribution $f_a(\tilde{a}|\tilde{a} \leq a_u(c))$.

Given the above, the board of directors can use the convolution formula to compute the marginal distribution of $\tilde{\theta}$. This yields:

$$f_\theta(\tilde{\theta}) = \int_\ell^A f_a(s)h(\tilde{\theta} - s)ds \tag{15}$$

With this information the board can use the observation of $\tilde{\theta}$ to formulate beliefs about the incumbent manager's ability. That is, they can make use of the conditional distribution function

$$f_{a/\theta}(\tilde{a}|\theta) = \frac{f_a(\tilde{a})h(\theta - \tilde{a})}{\int_\ell^A f_a(s)h(\theta - s)ds} \tag{16}$$

Using (16), the board formulates an expectation of the gain from replacing the incumbent manager. We assume that the board is rational and will follow an optimal search strategy. If the expected gain from a change of managers is positive, the board will replace; otherwise, they will retain the incumbent.

In our model the replacement decision involves the following steps:

(i) The board of directors observes current firm value and inverts the value function to extract the realization of θ at time 0.

(ii) Using information from step (i) and $f_{a|\theta}(\tilde{a}|\theta)$, the board calculates their belief of the incumbent manager's ability. This belief is $E_{a|\theta}(\tilde{a}|\theta)$. Denote this belief $\overline{a_I}$.

(iii) The board formulates an expectation of the value of the firm next period, period 1, under the incumbent. Denote this value of the firm in period 1 as V_1. In doing this we assume that they take their belief of the incumbent's ability as given and simply substitute this into the value function. That is, the board calculates $E_\gamma[V_1(\overline{a_I}+\tilde{\gamma})]$.

(iv) Using the above information the board solves:

$$\max_c \; G = E_{\theta|a\le a_u(c)}[V_1(\theta)] - E_\gamma[V_1(\overline{a_I}+\gamma)] - c + k - d$$

If the value of this function when maximized is positive, the board replaces the incumbent; otherwise they do not replace.

More formally, the objective function is:

$$\underset{c}{\text{Max}}\, G = \frac{1}{F_a[a_u(c)]}\int_\theta V_1(\tilde{\theta})\left[\int_\ell^{a_u(c)} f_a(s)h(\tilde{\theta}-s)ds\right]d\theta - \int_\upsilon V_1(\overline{a_I}+\tilde{\gamma})h(\tilde{\gamma})d\gamma - c + k - d \quad (17)$$

where υ is the range of γ.

Solving for the first order conditions yields:

$$E_\gamma[V_1(\tilde{\theta})|a = a_u(c^*)] - E_{\theta|a\le a_u(c^*)}[V_1(\tilde{\theta})] = \frac{c^*}{\eta} \quad (18)$$

where η is as defined in the previous section.

We assume that (18) can be solved for a positive valued c^* and that the second order conditions necessary for a maximum hold. We further assume that the solution is a unique global maximum.

Using (18), we can express the maximized gain function as:

$$G^{*} = E_{\theta|a \le a_u(c^{*})}[V_1(\theta)] - E_{\gamma}[V_1(\overline{a_l} + \gamma)] - c^{*} + k - d \qquad (19)$$

Henceforth, let the superscript * on an expectation operator denote that expectation conditional on $a \le a_u(c^{*})$.

4.1.1 Average Managers and Below Average Managers

When the board of directors observes the realization of θ they formulate a belief of the incumbent manager's ability. Suppose the board believes the incumbent is of average ability relative to the optimal truncated distribution of managers. That is,

$$\overline{a_l} = E_a^{*}[\tilde{a}]$$

Substituting this into (19) yields:

$$G^{*} = E_{\theta}^{*}[V_1(\tilde{\theta})] - E_{\gamma}[V_1(E_a^{*}[\tilde{a}] + \gamma)] - c^{*} + k - d$$

The board will replace the incumbent manager if:

$$E_{\theta}^{*}[V_1(\tilde{\theta})] - E_{\gamma}[V_1(E_a^{*}[\tilde{a}] + \gamma)] > c^{*} - k + d \qquad (20)$$

Use the law of iterated expectations to get:

$$E_{\theta}^{*}[V_1(\tilde{a} + \tilde{\gamma})] = E_{\gamma}\{E_{\theta|\gamma}^{*}[V_1(\tilde{a} + \tilde{\gamma})]\} = E_{\gamma}\{E_{\theta|\gamma}^{*}[V_1(\tilde{\theta})]\}$$

Also, $E_a^{*}[\tilde{a}] = E_{\theta|\gamma}^{*}[\tilde{\theta} - \tilde{\gamma}]$ as $\tilde{\theta} = \tilde{a} + \tilde{\gamma}$.

Using these facts (20) can be restated as

$$E_{\gamma}\left[E_{\theta|\gamma}^{*}[V_1(\tilde{\theta})]\right] - E_{\gamma}[V_1(E_{\theta|\gamma}^{*}[\tilde{\theta} - \tilde{\gamma}] + \tilde{\gamma})] > c^{*} - k + d \qquad (21)$$

This simplifies to

$$E_\gamma\left[E^*_{\theta|\gamma}[V_1(\tilde{\theta})]\right] - E_\gamma[V_1(E^*_{\theta|\gamma}[\tilde{\theta}])] > c^* - k + d \tag{21a}$$

But Jensen's Inequality states that

$$V_1(E^*_{\theta|\gamma}[\tilde{\theta}]) > E^*_{\theta|\gamma}\left[V_1(\tilde{\theta})\right]. \tag{22}$$

Thus, (22) implies that the left hand side of (21a) is non positive. Therefore, the board will <u>not</u> replace a manager whom they believe to be of average ability (again, under the assumption that d>k-c*)

Given the above result, the question arises as to what perceived level of ability will be necessary to induce the board to replace the incumbent. To answer this we suppose the board believes the incumbent to be of below average ability. That is,

$$\begin{aligned} \bar{a}_I &= E^*\left[\tilde{a}\right] - \varepsilon \\ \varepsilon &> 0 \end{aligned} \tag{23}$$

We define ε as the difference in perceived ability from the mean of the optimal distribution that leaves the board indifferent between replacing or retaining the incumbent. The replacement condition can therefore be written as:

$$E_\gamma[E^*_{\theta|\gamma}V_1(\tilde{a}+\tilde{\gamma}) - E_\gamma V_1(E^*_a[\tilde{a}]+\tilde{\gamma}-\varepsilon) = c^* - k + d \tag{24}$$

Solving this equation for ε will define the entire range of perceived below average managers who will not be replaced by a rational board.

Rewrite (24) as

$$E_\gamma\left[V_1(E_a^*[\tilde{a}]+\tilde{\gamma}-\varepsilon)\right]=E_\theta^*\left[V_1(\tilde{a}+\tilde{\gamma})\right]-c^*+k-d \qquad (25)$$

To avoid the non-linearity of the solving this for ε, we approximate this function by taking a first-order Taylor series expansion of the left hand side of (25) around $E_a^*[\tilde{a}]+\gamma$. This approximation yields:

$$E_\gamma\left[V_1(E_a^*[\tilde{a}]+\tilde{\gamma})-\varepsilon V_1'(E_a^*[\tilde{a}]+\tilde{\gamma})\right]\approx E_\theta^*[V_1(\tilde{a}+\tilde{\gamma})]-c^*+k-d \qquad (26)$$

Rearrange (26) to get

$$\varepsilon\approx\frac{E_\gamma\left[V_1(E_a^*[\tilde{a}]+\tilde{\gamma})-E_{\theta|\gamma}^*V_1(\tilde{a}+\tilde{\gamma})+c^*-k+d\right]}{E_\gamma V_1'(E_a^*[\tilde{a}]+\tilde{\gamma})} \qquad (27)$$

Again, $\varepsilon>0$ will hold under almost all circumstances.

5. Implications of the model

Our model is useful in interpreting some of the existing literature. Parrino [1992] observes that turnover is more likely in firms in homogeneous industries that those in less homogeneous industries. Homogeneous industries are defined as those comprised of firms with organizational structures, production technologies, and product markets similar to those of their industry cohorts. Parrino contends that this is due to the availability of less noisy performance signals and less valuable human capital for those in homogeneous industries. Our imperfect information model supports the idea that a less noisy signal will lead to a higher likelihood of turnover as a clearer signal of the abilities of potential replacements decreases the size of the range of below average managers who will be retained. Furthermore, the pool of candidates with the available to firms in homogeneous industries is richer than that available to less homogeneous industries. Our equation (13) shows that this will increase the probability of replacement as it increases the likelihood of getting a good manager from a search.

The results of Furtado and Rozeff (1987) are also consistent with our model. They observe that top management positions are more likely to be filled from within rather than through external hiring. We would explain this by saying that the distribution of ability within the firm is well-known to the board while the distribution outside is not. Replacement is more likely when drawing from a distribution with smaller variance; thus, replacements will tend to be drawn from the less noisy distribution.

Our model also helps put some structure on Shleifer and Vishny's (1988) contention that boards lack sufficient information about the firm to maximize firm value and will thus be pre-empted by hostile takeovers. A board with a lack of information about a firm cannot accurately gauge the abilities of either the incumbent or potential replacements. Thus, they are effectively drawing from a distribution with relatively high variance. There will therefore be a smaller likelihood of turnover. In our model a hostile takeover will take place when the outside observer has better information about a firm or better information about potential replacements.

Our model also shows that managers have an incentive to make it difficult for the board to extract a clear signal. By adding diverse assets to a firm a manager will succeed in making the board's job of gauging his or her ability and comparing it with potential replacements that much more difficult. Effectively, such moves increase the variance of the pool of potential replacements and increase the noisiness of the signal of the incumbent's ability. Both these factors reduce the likelihood of turnover. This is consistent with the empirical results of Morck, Shleifer, and Vishny (1990).

Comparative static analysis is not possible with our model in its current general form. In order to perform meaningful comparative statics specific functional forms must be assumed for the value function, the distribution function, and the responsiveness of available ability to increased expenditure. An attempt has been made to accomplish this by assuming a quadratic value function and a beta distribution of ability.[21] The first order conditions had no economically admissible solutions. We leave other attempts to find tractable models to future research.

5. Conclusions

There are many examples of top executives who are perceived to be performing poorly in their roles, either by shareholders, the business press, or other stakeholders. These groups observe that these "bad" managers are not

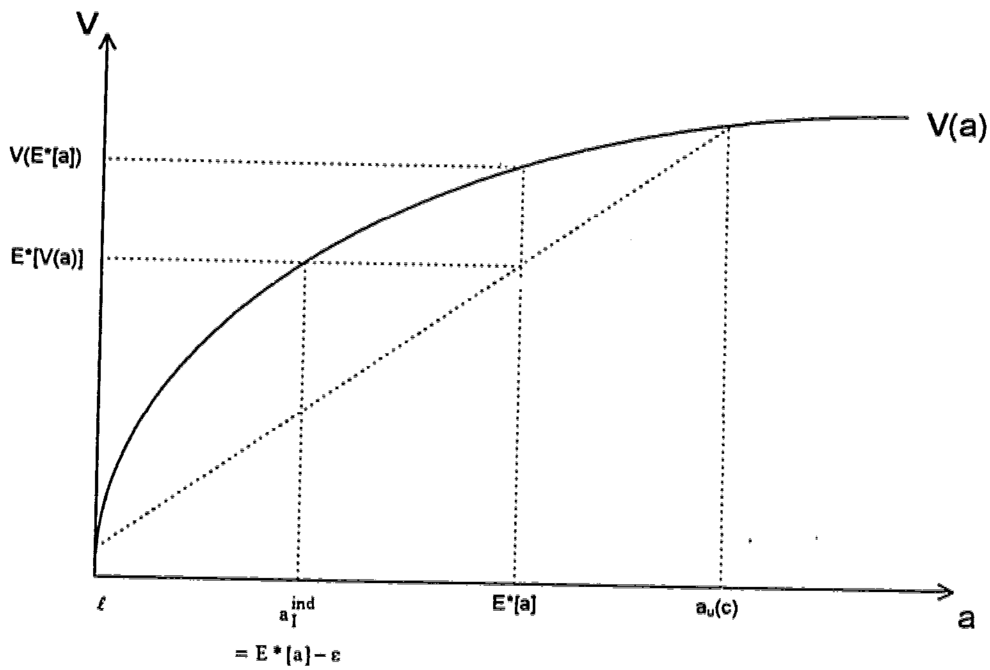
[21] The model attempted differed slightly from that in the body of the paper in that we assumed that increased expenditure increased a parameter value that simultaneously caused the mean of the distribution to increase while the variance decreased.

replaced by the boards of directors and take this as evidence that these boards are not serving the shareholders interests by monitoring and disciplining management. boards then get the reputation as being "yes men" to the senior officers of the firm.

This paper shows that these conclusions do not necessarily follow from the perceived evidence. A board of directors which is rational may be unwilling to replace a manager that is known to be of below average ability. The reason behind this is quite intuitive; by replacing the incumbent manager with a new unknown manager, the board runs the risk that the new manager may be worse than the one that they replaced. These conclusions hold in worlds where the board receives either perfect or noisy signals regarding the incumbent manager's ability.

Figure 3-1

A Simple Representation of the Model



Bibliography

Bentson, G.J. 1985. The Self-Serving Management Hypothesis: Some Evidence. *Journal of Accounting and Economics*. 7: 67-84.

Coughlan, A.T. and Schmidt, R.M. 1985. Executive Compensation, Managerial Turnover, and Firm Performance: An Empirical Investigation. *Journal of Accounting and Economics*. 7: 43-66.

Fama, E.F. 1980. Agency Problems and the Theory of the Firm. *Journal of Political Economy*. 88: 288-307

Furtado, E.P.H. and Rozeff, M.S. 1987. The Wealth Effects of Company Initiated Management Changes. *Journal of Financial Economics*. 18: 147-160.

Jensen, M.C. and Murphy, K.J. 1990. Performance Pay and Top-Management Incentives. *Journal of Political Economy*. 98: 225-264.

Jensen, J.C. and Ruback, R.S. 1983. The Market For Corporate Control: The Scientific Evidence. *Journal of Financial Economics*. 11: 5-50.

Jensen, M.C. and Warner, J.B. 1988. The Distribution of Power Among Corporate Managers, Shareholders and Directors. *Journal of Financial Economics* 20: 3-24

Johnson, E.W. 1990. An Insider's Call for Outside Direction. *Harvard Business Review*. 68, 2. 46-55.

Mace, M.L. 1971. *Directors, Myth and Reality*. Harvard Business School Press, Boston, MA.

Mahajan, A. and Lummer, S. 1993. Shareholder Wealth Effects Of Management Changes. *Journal of Business Finance and Accounting*. 20: 393-410..

Martin, K.J. and McConnell, J.J. 1991. Corporate Performance, Corporate Takeovers, and Management Turnover. *Journal of Finance*. 46: 671-687.

Morck, R., Shleifer, A. and Vishny, R.W. 1989. Alternative Mechanisms for Corporate Control. *American Economic Review*. 79: 842-852

Morck, R., Shleifer, A. and Vishny, R.W. 1990. Do managerial objectives drive bad acquisitions. *Journal of Finance*. 45: 31-48.

Parrino, R. 1992. CEO Turnover and Outside Succession: A Cross-Sectional Analysis. Unpublished Ph.D. Thesis. University of Rochester.

Shleifer, A. and Vishny, R.W. 1988. Value Maximization and the Acquisition Process. *Journal of Economic Perspectives*. 2: 7-20.

Shleifer, A. and Vishny, R.W. 1989. Managerial Entrenchment: The Case of Manager Specific Investments. *Journal of Financial Economics*. 25: 123-139.

Spence, A. M. 1973. Job Market Signalling. *Quarterly Journal of Economics*. 87: 355-374.

Weisbach, M.S. 1988. Outside Directors And CEO Turnover. *Journal of Financial Economics*. 20: 431-460.

Chapter 4

Canadian Structured Notes: A Framework for Analysis

1. Introduction

This paper describes a simple technique for evaluating a wide range of fixed income securities with atypical cash flow structures. Such securities are often referred to as structured notes. The aim is to provide both issuers and investors with a valuation method that can be easily implemented in a timely manner using nothing more than a spreadsheet program and current interest rate swap rates. This technique involves decomposing the security into more primitive components and using interest rate swap information to value the various components. The interest rate swap information is used as a proxy for the required returns on the issuer's instruments. As interest rate swap rates are, in general, different from the rates demanded of the structured note's issuer in the fixed income market, the value arrived at by the use of this technique is only an approximation to the "true" theoretical value of the security. Therefore, a second goal of this paper is to investigate how this approximation deviates from the true theoretical value of the security.

The technique described herein is suited to valuing structured notes that have imbedded options on the notes themselves and well-defined coupon payments (i.e. either fixed coupon or floating rate based on a commonly used reference rate).[22] In brief, the method involves decomposing the structured note into three components: a short dated straight fixed rate or floating rate security, a forward agreement to purchase a stream of cash flows beginning some time in the future (hereafter "forward bond"), and an option on the future stream of cash flows. The swap market information is used to value proxies for the latter two components, these proxies being a forward agreement to enter into a fixed-for-floating interest rate swap (hereafter "forward swap") and an option on the forward swap (hereafter "swaptions"). Each component can then be valued individually and the results summed to arrive at a proxy for the value for the security. This decomposition technique provides an alternative to option-adjusted spread (OAS) analysis for many structures while having the advantage of being easily adapted to a wider range of securities.

While other authors have discussed using swaptions to offset options on corporate bonds [e.g. Arditti (1996)], this paper adds to the existing literature in three ways. First, the paper describes a standard technique for decomposing complex instrument into instruments that are easy to value using widely available market information. This is important as many participants in the Canadian market have expressed a desire to know how to approach the valuation of structured notes.[23] Second, the paper describes the deficiencies in

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[22] None of the examples that follow analyze structured notes whose interest payments are in commodities or are based on complex formulas such as LIBOR-squared; however, the technique can be implemented when interest payments are in terms of any instrument upon which swap rates are quoted.

[23] Private discussions with fixed income managers and sales people across Canada have demonstrated to me that there is a lack of understanding as to how to value structured notes. Furthermore, analysis of term sheets has shown me that many of the instruments being offered

the decomposition technique and outlines the magnitude of the differences between the value arrived at via the decomposition technique and that arrived at assuming firm-specific characteristics are known. Third, the paper notes and offers a correction for an error in a previously published paper [i.e. Smith (1991)] on swaption valuation.

The remainder of the paper is organized as follows. The next section provides some background on the Canadian structured note market. The third section will use a case study to demonstrate the decomposition technique to value a simple structured note. Within this case study the details of using swap rates to value components of the note will be described in detail. A number of other cases will be valued using the technique. The fourth section provides an analysis of the problems that arise from the use of swap rates as a basis for valuation. The fifth and final section provides a summary.

2. Some background on the Canadian structured note market[24]

Canadian fixed income investors have long been offered securities whose cash flow configurations differ from that of the standard semi-annual coupon "bullet" bond.[25] For example, bonds with sinking funds and bonds with imbedded options (i.e. bonds callable by the issuer, putable by the investor, or convertible into other securities) have for years been issued by corporations,

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are being priced at less than fair value. Based on the anecdotal evidence, it is clear to me that there is a need for a straightforward discussion of the valuation of these notes.

24 Thanks to Derek Cook of RBC Dominion Securities and Linda Kennedy and Frederick Muller of the Province of Alberta for their perspective on the market's development.

25 The term "bullet bond" or "bullet" refers to an option-free semi-annual pay, fixed coupon bond with a balloon payment at maturity.

governments, and government agencies. In recent years, however, a new class of fixed income securities which combine imbedded options with irregular cash flow patterns has begun to be marketed. This class of securities is referred to as structured notes or structured products.

It is impossible to describe a generic structured note because they are usually constructed to suit the particular cash flow needs of a specific issuer or a specific investor; therefore, the general term "structured notes" encompasses an infinite number of possible cash flow patterns. Examples of such patterns include securities with coupon payments based on a commodity's price at some future date, securities with fixed coupon rates that vary over time (e.g. step-up coupons), and securities with floating rate coupon for part of their life followed by a fixed coupon paid periodically until maturity (known as fixed-floaters or floater-fixed). Additionally, these securities usually contain imbedded call or put options.

While there are examples of securities with atypical structures that date back many years, so-called structured notes took their place on the regular menu of choices for the fixed income investor only very recently.[26] Indeed, it is only since around 1993 that structured notes were offered to select investors in Canada and since 1995 that there has been wide distribution of these notes. Several factors have contributed to the development of the Canadian structured note market. Improved theoretical understanding of option valuation

26 An example of an atypical structure is that of gold loan taken in 1890 by the Imperial Government of Russia that is repayable in five different currencies.

and cheaper and more powerful computer technology has fostered the development of relatively liquid over-the-counter derivatives markets by providing investment dealers with the information they require to efficiently hedge options positions. Indeed, the interest rate swap market in Canada only began to develop seriously around 1988 to 1990 when electronic means of measuring exposures gave dealers the ability to warehouse swaps. Up to that time, swaps were only done on a matched basis. The new technology has also made it possible for dealers to make markets in a wide range of over-the-counter options, including swaptions.

The development of the interest rate swap market and the swaption market provided mechanisms by which cash flows could be manipulated into customized patterns to suit a clients' needs. However, until recently, the economics of the debt issuance process prevented issuers from doing small-sized issues in an economically efficient manner. This barrier fell in the early 1990s with the advent of medium term note (MTN) programs. These programs greatly streamlined the issuance process, allowing issuer's to do a series of smaller issues at times convenient to them under a common set of legal documents. Furthermore, the programs allowed investors to approach issuers with proposals. Concurrent with these developments was a compression of credit spreads that, according to several investment dealers, increased investors' desire to add yield-enhancements to their investments (Table 4-1 provides evidence of spread compression through the 1990s). The flexibility of

MTN programs combined with liquid swap and swaption markets opened the door for the dealer community to begin to create and propose small-sized issues with irregular cash flow streams.[27]

The drawback from the investor's (and issuer's) viewpoint is that the valuation of the irregular cash flow streams and imbedded options in a structured note presents a more complex problem than does the valuation of a bullet bond. Traditional evaluation of bullet bonds involves inspecting the difference in the bond's promised yield with that of a comparable federal government bond. This difference, or spread, is then compared to spreads offered by other bullet bonds of the same issuer or by the spreads offered by issuers of similar credit quality in order to gauge if the security offers adequate compensation for its inherent risk.[28] Structured notes do not always lend themselves to this sort of analysis. While some investors have developed heuristics for adjusting target spreads to include compensation for imbedded options, these rules can result in inconsistent decision making.[29] Theoretically sound valuation techniques such as OAS analysis have become more popular and easier to use through tools such as Bloomberg terminals; however, the implementation of a spread-based decision rule is impossible in some cases as

27 Crabbe (1995) has an good overview of MTN programs. While it does not directly address the Canadian market, it summarizes many of the characteristics of Canadian MTN programs.
28 The risk referred to here is credit risk. Investors will also require a few extra basis points in the spread to compensate for low liquidity if the issue being evaluated is small in size or has customized features that may make trade on the secondary market more difficult.
29 Later in the paper we will demonstrate that the perturbation of a parameter of a structured note may allow the note to meet or not to meet the heuristic adjustment without fundamentally changing the security's value.

the notes may not have a single yield from which to calculate a spread.[30] For example, one cannot readily state a single promised yield to maturity on a security that pays a floating rate coupon quarterly for half its life, followed by a period in which it pays a fixed rate semi-annual coupon. In order to properly evaluate structured notes, investors require a valuation technique that is robust to small changes in terms and is flexible enough to be adapted to a wide range of structures.

The need for an easily implemented valuation model is underscored by comments made by the individuals involved with the structured products departments of major investment dealers. When asked how institutional investors evaluate structured notes, one major Canadian dealer stated that a large number of investors simply look at the nominal spread of the yield to the call date and to the maturity date. Very little sophisticated analysis is used. With many of these notes the nominal spreads are often much larger than spreads on straight debt of the same issuer. A second investment dealer, when confronted on a proposal that upon proper decomposition revealed an enormous profit for him, stated that structured products desks are "arbitraging" the way clients look at credit when they construct deals.[31]

Such comments do not constitute proof that inefficiently priced deals are being consummated and that excess profits are being made. It is possible that a conservatively formed heuristic analysis is sufficient to filter out inefficiently

30 See Hayre (1995) for an overview of OAS analysis.
31 For obvious reasons, the investment dealers in question prefer to remain anonymous.

priced proposals. And, regardless of how many inefficiently priced proposals are made, there is nothing forcing investors or issuers to accept them. As structured notes are often small sized deals with no participants other than one issuer, one dealer, and one investor, there is little publicly available information regarding done deals that can be analyzed to provide concrete statistical evidence regarding pricing efficiency. What is clear is that information regarding how to efficiently price deals will be valuable to both issuers and investors as it will allow them to more quickly analyze "unusual" structures presented to them. It will also allow to be more proactive by giving them the tools to efficiently price their own proposals.

3. *The decomposition and valuation of a structured security*

3.1 The decomposition of a simple structure

In this section, the elements of the decomposition approach to valuing securities will be described in detail. A very simple yet common structure will be used as a vehicle to demonstrate the basic principles of the decomposition. Furthermore, the tools required to value the component parts will be described in detail. Later, three different structures will be used to demonstrate the technique.

To begin, assume that an investment dealer has contacted an issuer and has sent a term sheet out to an investor listing the following terms and conditions.

Issuer:	Issuer Corp.
Amount:	$20 million
Coupon:	9.5 percent, semi-annual pay
Term to maturity:	5 years
Imbedded option:	Callable by the issuer at par on the second anniversary date. Not callable after the second anniversary.
Offering price:	102.00

This type of structure is sometimes referred to in market parlance as a "five non-call two". Hereafter, the above note will be referred to as Note 1. Decomposition of this structure is straightforward. The three components of this structure are:

1. A two year Issuer Corp. note with a fixed coupon of 9.5% payable semi-annually, with a balloon payment of par payable in 2 years
2. A forward purchase agreement wherein the investor agrees to pay par to Issuer Corp. 2 years hence in exchange for payments over the following 3 years of 9.5% semi-annually plus repayment of par at the end of that 3 year period (i.e. 5 years after the original issue date).
3. A European call option written by the investor to Issuer Corp. under which Issuer Corp. has the right at the end of 2 years to purchase the 3 year note referred to above at the strike price of par.

Figure 4-1 provides a schematic of the decomposition.

In order to properly evaluate the proposed deal, the present value of the forward purchase agreement and the value of the option on that forward bond must be computed. With these values in hand, the value being asked for the two year bond alone can be found and a yield calculated. One can then judge if the yield on the two year bond represents fair value. For example, suppose that value of the option is 2 and value of the forward bond is 0.[32] As the option is being written by the investor, the price being asked for the two year bond is 104. Thus the offered yield on the two year bond is 7.314%. This is the yield that should be used in evaluating the deal.[33]

To compute the value of the forward purchase agreement and the option, one must be able to compute the size of the coupon interest payment that must be provided by the forward bond in order for it to sell at par two years hence. This will be referred to as the "on-market coupon rate". Furthermore, one must be able to compute the value of the payment that must be made today to compensate for those situations where the coupon payment to be made on the forward bond differs from the on-market coupon rate. This payment will be referred to as the "off-market adjustment". Finally, the valuation of the option requires that we know the volatility of the yields on the issuer's debt.

The required inputs for such a valuation exercise are the current yields on zero coupon bonds of the issuer of the structured note with maturities corresponding to the payment dates of the structured note. These zero coupon yields allow for computation of the discount factors that apply at each relevant date. In order to compute the discount factors implied by the yields on the

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32 These values are expressed in terms of percentage of par value. Some might express the net value as 200 "up-front" basis points.

33 An alternate way to analyze this is to set the spread at an appropriate level for such an issue, compute the option value that is implied by such a spread, and then compute the volatility that is implied by such an option value. This volatility can then be compared to the market volatility to see if the option is being offered at a fair price.

issuers outstanding debt, information on a broad spectrum of maturities is required. An active market in options underlied by the issuer's debt is required to get the appropriate volatility with which to value the option. Such a market will provide implied volatilities which can be used to value the option.

As a practical matter, few issuers have a broad enough maturity spectrum of debt or an active enough options market to provide the necessary information for valuing the components of the note. One can, however, use interest rate swap rates as proxies for the required yields on the issuer's credit. While the required yields on swaps will, in general, be different from those of the issuer, the swap market is quite liquid and has an active over-the-counter options market. Its set-up also lends itself to straightforward computation of implied zero coupon rates and discount factors. Thus, one can quite easily take swap rates and construct proxies for the values of the components of a structured note. Specifically, a forward swap agreement can be used as a proxy for the forward purchase agreement while a right-to-receive swaption can be used as a proxy for the call option on the forward bond. We now turn to the construction of these proxies.

3.2 Computing implied discount factors from the indicative swap rates

The first step in valuing the components of the structured note is to compute the implied zero coupon swap rates from the indicative swap pricing schedule. Swap pricing schedules in Canada provide the fixed rates at which a swap dealer will exchange semi-annual payments for a payment of the semi-annual banker's acceptance (BA) rate. These swap rates are referred to as par swap rates. Par swap rates are often quoted in terms of a spread over an equivalent maturity Government of Canada bond. Table 4-2 provides an example of how the bid side of an indicative swap pricing list. As an interest rate swap is akin to an exchange of a floating rate bond for a fixed rate bond, the swap rate for a given tenor (i.e. term of the swap) is the same as the yield and coupon on a par bond with a remaining term exactly equal to the tenor of the swap (see Galitz, 1995 for details).

The par bond nature of the swap curve makes computation of the implied discount factors straightforward. Suppose we know that, at a particular time, the quoted swap rates are as in table 4-3(a). In order to build proxies for the components of the semi-annual pay structured note, one must compute implied discount factors from the swap curve for maturities corresponding to each payment date. That is, implied discount factors are required for maturities at six month intervals. If one has the yields on a set of par bonds that mature at six month intervals, a bootstrap methodology can be employed to compute the implied discount factors [Bansal, Ellis, and Marshall (1993)]. One can obtain a set of par bond yields in six month intervals by applying linear interpolation to the rates given in table 4-3(a).[34] The result of this interpolation is given in 4-3(b).

34 Market convention dictates that rates for tenors one year or less are quoted as money market rates while tenors longer than one year are quoted as annual rates compounded semi-annually (i.e. bond equivalent yields). Thus, the one year rate below is based on annual compounding. It must be converted into "bond equivalent yield" terms before any further analysis can be done.

Because the indicative schedule is for semi-annual exchange of payments it is convenient to index the time period by the number of half-year periods of the swap. For example, a swap maturing in 0.5 years is said to have one period until maturity while a swap of 7.5 years matures in 15 periods. In computing the implied discount factors we take advantage of the fact that the discount factor for a one period (i.e. 6 month) swap is given by:

$$d_1 = \frac{1}{1+\frac{R_1}{2}}$$

where: d_1 is the discount factor for the first period (1A)

R_1 is the given 6 month swap rate quoted as a semi-annually compounded rate

For the remaining periods we can exploit the par bond nature of swap rates to compute the remaining discount factors. For each period i after the first the following is true:

$$100 = \frac{R_i}{2}\sum_{t=1}^{i-1} d_t + d_i\left(\frac{R_i}{2}+100\right) \text{ for } i>1 \qquad (1B)$$

Where R_i is quoted as a semi-annually compounded rate

d_i is the discount rate for the ith period

With d_1 in place we can proceed with calculating d_2, then d_3, and so on, by simply rearranging (1B) to get:

$$d_i = \frac{100-\frac{R_i}{2}\sum_{t=1}^{i-1} d_t}{\frac{R_i}{2}+100} \qquad (2)$$

The transformation of these discount factors into implied zero coupon or spot rates (quoted on a semi-annually compounded basis) can be done by employing the following equation:

$$s_i = 200 \times \left[\left(\frac{1}{d_i} \right)^{\frac{1}{i}} - 1 \right] \qquad (3)$$

where s_i is the spot swap rate (or zero coupon rate) for an exchange of payments at period i.

Table 4-4 summarizes the implied discount factors and implied zero coupon resulting from bootstrapping the swap rates given in table 4-3(a). Appendix 4-1 provides the necessary information for programming an Excel 5.0 spreadsheet for bootstrapping the swap curve.

3.3 Computing the forward swap rate

In order to build a proxy for the forward purchase agreement component of the structured note, one must use the implied discount factors to compute the forward swap rate. The on-market forward swap rate is the fixed rate at which a swapper would today agree to pay every six months in exchange for semi-annual payments of the banker's acceptance rate for a period of beginning sometime in the future. Equivalently, the forward swap rate is the coupon on a bond for which one would pay par at some future date. Suppose one wanted to initiate a swap which would last j semi-annual periods (i.e. $j/2$ years) beginning n periods from now. This is a forward swap. The rate on the forward swap will be denoted $F_{n,j}$. This value is set such that no payment need be made at the outset (i.e. the current economic value of the swap is zero). That is,

$$100d_n = \frac{F_{n,j}}{2}\sum_{t=1}^{j} d_{n+t} + 100d_{n+j} \tag{4}$$

To set this up such that it has a value of zero presently we exploit the discount factors. The forward swap rate is computed as the coupon rate on a bond that would be priced at par at period *n* with payments until period *n+j*. The formula for this is:

$$F_{n,j} = \frac{200(d_n - d_{n+j})}{\sum_{t=1}^{j} d_{n+t}} \tag{5}$$

This formula differs from that used by Smith (1991) to define the forward swap rate. In that paper Smith states that the forward swap rate is given by

$$200\left[\left(\frac{d_n}{d_{n+j}}\right)^{\frac{1}{j}} - 1\right].$$

While this quantity is the forward rate for a one-time cash flow, it is not appropriate rate for use as a proxy for a forward semi-annual series of payments. The correct number for that proxy is given by (5). While the numbers arrived at using Smith's formula will be relatively close to those given by (5) in economies where the swap curve is relatively flat, the errors can become significant in an economy with a relatively steep swap curve.

In many circumstances the decomposition of the structure will require that an off-market adjustment be made to account for the fact that the coupon to be paid or received on the forward purchase agreement differs from the forward swap rate. This off-market adjustment must be paid at the time the forward agreement is made. The value of this adjustment represents the present value of the difference between the actual amount of the payment to be made in the future from the forward swap rate. Let $C_{n,j}$ represent the semi-annual coupon payment that is actually going to be made for a period beginning at period n and ending at period j. (i.e. first payment to be made at period n+1, last payment at period j). The value of the off-market adjustment is therefore given by:

$$\text{off-market adjustment} = \left(\frac{C_{n,j} - F_{n,j}}{2}\right)\sum_{t=1}^{j} d_{n-t} \qquad (6)$$

The example structure, Note 1, described above, can be used as a first example of valuing the forward purchase agreement. Assume that the indicated swap rates at the time the proposal is received are as in table 4-3(a) and that the implied discount factors are as in table 4-4. The first step is to compute the on-market forward swap rate for a swap beginning two years from now and lasting 3 years.[35] The on-market swap rate is found by using equation (5). Substitution reveals:

35 Using the terminology of Smith (1991), this is a "five/three" swap.

$$F_{4,6} = \frac{200(d_4 - d_{10})}{\sum_{t=1}^{6} d_{4+t}} = \frac{200(0.88303 - 0.69245)}{4.626783} = 8.238$$

The on-market swap rate of 8.238 is less than the rate of 9.5 that will be received by the investor on the forward purchase agreement component of the structured note. Thus, the investor will have to pay an off-market adjustment today as compensation for the large coupon that will be received semi-annually for three years beginning in two years. The amount of this payment is given by (6):

$$\text{off-market adjustment} = \left(\frac{9.5 - 8.238}{2}\right) 4.626783 = 2.919$$

Thus, the value of the forward purchase agreement is 2.919 (i.e. 291.9 up-front basis points). This amount is owed by the investor to the issuer.

The last component of the structure, the option, must be valued to complete the analysis of the note. This is done by valuing an option on a swap or a swaption. The next section addresses this.

3.4 Valuing the imbedded option

The final component that needs to be addressed is the option on the forward par bond. This value is approximated by valuing an option on the forward swap that was used to evaluate the forward purchase agreement. That is, a swaption must be valued. Before proceeding, some terminology is required. First, define the party to a swap paying the fixed rate and receiving floating as being the "long" of the swap; therefore, the party receiving fixed and paying floating has a short position in the swap. A swaption that gives the holder the right to pay the fixed rate at some date in the future is called a "right-to-pay" swaption. This swaption gives the party the right to take a long position in a swap. A swaption that gives the holder the right to receive the fixed rate at some date in the future is called a "right-to-receive" swap; it gives the holder the right to take a short position in a swap.

There are many different interest rate models that have been proposed for valuing options on interest rate instruments such as swaps.[36] Many of these are quite versatile and can be applied to a number of different types of interest rate instruments; however, they are often cumbersome to use in that they require a dedicated computer program. A simple, fast, and easily implemented way to price options on generic semi-annual pay swaps is provided by Smith (1991). In that paper Smith proposes that the commodity futures option pricing model of Black (1976) can be modified to value swaptions. Indeed, Canadian industry standard has been to quote swaption values based on Black (1976) valuation.[37]

The Black (1976) model for valuing call options on commodity futures is given by

36 See Hull (1993, Ch. 15) for a brief summary of some of these models.
37 Discussions with representatives of RBC Dominion Securities and CIBC Wood Gundy indicated that, while all have more complex models for general valuation purposes, Black (1976) is the standard swaption valuation model. Indeed, swappers use Black (1976) values as the benchmark for calibrating their more complex models.

$$C = e^{-rn}\left[FN(\delta_1) - XN(\delta_2)\right]$$

Where:

$$\delta_1 = \frac{\ln\left(\frac{F}{X}\right) + \frac{\sigma^2(n)}{2}}{\sigma\sqrt{n}} \qquad (7)$$

$$\delta_2 = \delta_1 - \sigma\sqrt{n}$$

F is the futures price
X is the options exercise price
n is the number of years until option expiration
r is the non-stochastic risk-free interest rate
σ is the volatility (per year) of the futures price.
$N()$ is the cumulative normal distribution function.

In standard applications of this model, the inputs for F and X are dollar values while the volatility is a volatility of the dollar price; therefore, result of (7) is denominated in dollars. The Smith (1991) swaption application of the model is slightly different. The inputs are the forward swap rate and the exercise rate, and the volatility of the forward rate (i.e. the volatiiity of an interest rate). As the volatility will be different for underlying swaps of different tenors, the volatility is denoted $\sigma_{n,n+j}$. The discount rate is also the implied zero coupon swap rate for the period ending on the option maturity date. Using this model, the following valuation formulas result:

Black (1976) value for a right-to-pay swaption, expressed the using notation defined above:

$$C = d_n\left[F_{n,n+j}N(\delta_1) - XN(\delta_2)\right]$$

Where:

$$\delta_1 = \frac{\ln\left(\frac{F_{n,n+j}}{X}\right) + \frac{\sigma^2_{n,n+j}(n)}{2}}{\sigma_{n,n+j}\sqrt{n}} \tag{8}$$

$$\delta_2 = \delta_1 - \sigma_{n,n+j}\sqrt{n}$$

The value of a right-to-receive swaption is given by:

$$P = d_n\left[XN(-\delta_2) - F_{n,n+j}N(-\delta_1)\right] \tag{9}$$

In order to express this in terms of a number of up-front basis points, the result of (8) and (9) amount must be annuitized over the term of the underlying forward swap. That is, the result of (8) and (9) must be treated like a semi-annually compounded stream of payments where the payments are made at the same frequency as those of the underlying forward swap. The value of that stream of payments is the up-front value of the swaption.

The up-front value of the right-to-pay option (denoted *RTP*) on a swap beginning at period n and lasting j semi-annual periods is therefore given by:

$$RTP = \frac{\left[F_{n,n+j}N(\delta_1) - XN(\delta_2)\right]}{2}\left(\sum_{t=1}^{j} d_{n+t}\right) \tag{10}$$

The value of an equivalent right-to-receive option (denoted *RTR)* is given by

$$RTR = \frac{\left[XN(-\delta_2) - F_{n,n+j}N(-\delta_1)\right]}{2}\left(\sum_{t=1}^{j} d_{n+t}\right) \tag{11}$$

While the above model is very easy to program into a spreadsheet, it will be of no use if the user does not have an appropriate volatility to use as an input. Fortunately, swap dealers call markets on swaptions in terms of Black (1976) volatilities. Some publish matrices providing the appropriate volatility to use for swaptions for underlying forward swaps of a variety of different terms.

Note 1 provides an opportunity for using the above model. The first step is to note that holding the call in the structured note is equivalent to holding a right-to-receive option with an exercise rate of 9.5%. One must get the appropriate volatility for an option on a swap beginning in two years and lasting three years. In market parlance, this is referred to as "two into three" volatility. Suppose that this is quoted as 20%. Assuming that the swap rates hold as in table 4-4, the forward swap rate is 8.238. Substituting into equation (11) gives the value of the right-to-receive option. Specifically,

$$\frac{9.5N(0.645) - 8.238N(0.362)}{2}(4.626783) = 4.051$$

3.5 Putting it together

Two of the components of Note 1 have been valued using the information in table 4-4. The value of the forward purchase agreement, as approximated by the forward swap analysis, is 2.919. Recall that this was an amount owed by the investor to the issuer. The value of the call option, as approximated by a right-to-receive swaption, is 4.051. This is an amount that must be paid to the investor by the issuer. The net value of the two components is therefore $4.051 - 2.919 = 1.132$. As the offering price for Note 1 is 102 and the option and forward purchase agreement require that the issuer pay 1.132 to the investor, the two year bond component of the structured note is effectively being offered at 103.132. This translates to a yield to maturity on the two year note component of 7.779%. Using the information on the 2 year Government of Canada benchmark implied by tables 4-2 and 4-3, the two year note is priced at two year G/C + 156.9 bp. This is the spread that should be focused on in an evaluation of the proposal.

An alternate way of approaching this is to compute the implied volatility on the option assuming the 2 year component offers the appropriate spread for the credit quality. Assume that a spread of 150 basis points is thought to be appropriate for Issuer Corp. with two years until maturity. This implies that the two year note component of the structure should yield 7.71%. The price of the note would therefore be 103.26. As the price of the structured note is 102, one can infer that, at the required spread for the two year note component, the value of the forward and the swaption is 126 up-front basis points. With the value of the forward swap at 291.9 up-front basis points, the option value is therefore 417.9 up-front basis points. The volatility that is implied by such a price is 21.3%. The investor in the structured note is writing the swaption; therefore, they are receiving an implied volatility of 21.3% for the option. If the market on volatility is at this level the note is priced fairly.

3.6 Other examples

3.6.1 Note 2: A step-up callable note

A commonly seen structure is that of the step-up callable. This structure has an imbedded option and a coupon that changes on the call date. A sample term sheet that might be shown to an investor for a note being issued by a federal government agency would look something like the following (including the emphasis given below).

***Issuer*:**	Federal Government Agency
***Amount*:**	$20 million
***Issue date*:**	August 5, 1996
***Coupon*:**	6.71% (semi-annual) to August 5, 1998, then 8.24% (semi-annual) to August 5, 2001.
***Maturity*:**	August 5, 2001.
***Offering price*:**	100.00
***Imbedded option*:**	Callable in whole by Issuer Corp. at par on August 5, 1998 upon 14 days prior notice.
***Benchmarks*:**	G/C 2 year benchmark yield currently 6.21%. G/C 5 year benchmark yield currently 7.25%.
***Yield and spreads*:**	Blended yield to maturity 7.559%. Yield to call 6.71%. **Spread to maturity 30.9 bps** **Spread to call 50 bps.**

The first things that stands out when one looks at a term sheet such as this are the quoted spreads. As straight issues from federal government agencies tend to trade at very narrow spreads to government bonds (i.e. at time of writing, a spreads are less than 10 basis points), the indicated spreads look quite attractive. However, one must decompose the structure to see if it fairly priced.

The components of this note are very similar to the components of Note 1. They are a two year note with a coupon of 6.71%, a two year forward agreement to purchase at par a three year bond with a coupon of 8.24%, and a European option, held by the issuer, to purchase the forward bond at par in two years. As before, we can value proxies of these components using swap rates.

Assume that the swap rates that prevail at the time the above term sheet is received are as in table 4-4. Given this, the forward rate for a swap beginning in 2 years (4 semi-annual periods) and ending 3 years later can be found using equation (5):

$$F_{4,6} = \frac{200(d_4 - d_{10})}{\sum_{t=1}^{6} d_{4+t}} = \frac{200(0.88303 - 0.69245)}{4.626783} = 8.238$$

As the coupon that applies over this period is 8.24, the off-market adjustment, using equation (6), is:

$$\left(\frac{8.24 - 8.238}{2}\right) 4.626783 = 0.004$$

This half basis point is owed by the investor to the issuer.

The option can be viewed as a right-to-receive swaption held by the issuer. Its value can be found by using equation (11) with a strike rate of 8.24.

If we assume that the current market quote on 2 into 3 volatility is 15%, the value of the option is given by:

$$\frac{8.24N(0.107)-8.238N(-0.105)}{2}(4.626783)=1.612$$

Thus, the net value of the two components is $1.612-0.004=1.608$. This is the amount that should be paid by the issuer to the investor.

Given this value for the two components, the price that is effectively being asked for the two year note is 101.608. This translates to a yield on the two year note of 5.846% or a spread of -36.4 basis points to the two year benchmark yield of 6.21%. This is the spread that should be used in evaluating the deal. It is doubtful that one would find such a spread from a federal agency attractive.

To analyze this note by way of the implied volatility, suppose that the required spread on straight two year notes of the federal agency is 10 basis points. Thus, the required yield on the two year component of the structure is 6.31%. This translates into a price of 100.741; therefore, if the two year component has this price and the forward swap is worth 0.004, the price received for the option is 0.737 or 73.7 up-front basis points. The volatility implied by this price is 6.9%. Purchasing the structure involves selling an option at a volatility of 6.9% when the market for volatility is 15%. Clearly, this is an unattractive proposal.

3.6.1.1 A brief digression: The danger of using heuristics

In evaluating deals with imbedded options, some investors use heuristics that involve requiring a certain spread to the maturity date as compensation for the option. The above step-up callable note can be used as a vehicle to demonstrate that such rules are not necessarily useful in separating good value from bad.

Suppose that an investor believes that a straight 5 year issue of a federal government agency should provide a spread of 10 basis points over the five year government benchmark. Further suppose that the investor requires an additional 25 basis points in spread to the maturity date for a call option. Given this decision criterion and the information given above, the step-up callable described above will be rejected as it yield to maturity of 7.559% provides only an additional 20.9 basis points for the imbedded call option.

If the dealer reconstructs the above note and sets the coupon for the first two years at 6.6% and sets the coupon for the final 3 years at 8.41 percent, the blended yield on the note is 7.604% provides for a spread of 10 basis points plus an additional 25.4 basis points for the imbedded option. This structure would meet the investor's decision rule; however, the net value of the off-market coupon and the option is 1.44, assuming all the swap rates and the volatility are as above. Thus, the two year component of this note is effectively being offered for 101.44, providing a yield of 5.829% or, using the benchmark yield of 6.21%, a spread of -38.1 basis points. Thus, the structure has been massaged to meet the investor's heuristic decision rule, but the net value to the investor of the two year component has actually decreased. The implication is that investors must be extremely careful about relying on heuristic decision rules when evaluating investments as structures can be massaged to meet a heuristic even though their fundamental economic value remains unchanged.

3.6.2 Note 3: A floater-fixed note

Proposals for notes with a combination of fixed and floating rate coupons are often put forward. To provide an example of such a note, suppose a note is proposed that has the same structure as the step-up callable described in section 3.6.1 with the one exception. The terms of the coupon are as follows:

Coupon: 90-day BA plus 50 payable quarterly until August 5, 1998 then 8.24% (semi-annual) to August 5, 2001.

All other terms of the note are the same as described in the previous section. The only difference between the structure of this note and that of the step-up note is that there is one of the components is a two year, quarterly pay floating rate note rather than a semi-annual pay fixed rate note.

Assuming that the swap rates are as in table 4-4 and that the relevant volatility is 15%, it has been shown that the forward component and the right-to-receive swaption have a net value of 1.608. With floating rate notes it is easier to think of this amount being amortized over the two year life of the note rather than as an up-front payment. The question then becomes what number of basis points need be paid in quarterly payments to compensate for this 160.8 up-front basis point value. A quick method of approximating this follows.

The price value of a 100 basis points for a par bond is approximated by the modified duration of the bond. The modified duration of a semi-annual pay par bond is given by

$$ModDur_{parbond} = \frac{100}{C}\left[1-\left(1+\frac{C}{200}\right)^{-n}\right]$$

where C is the coupon rate (12)

n is the number of semi - annual periods until maturity

Using the data from table 4-4 and substituting into the above formula reveals that the modified duration of a 2 year bond is 1.8521. Thus, the price value of a basis point paid every six months is 1.8521 up-front basis points. To compute the price value of a basis point paid quarterly one first must determine the number of semi-annual basis points that is equivalent to a quarterly basis point. Time value mathematics reveals that:

$$R_s = 200\left(1+\frac{R_q}{400}\right)^2 - 200 \tag{13}$$

We can approximate the number of semi-annual basis points that is equivalent to a quarterly basis point by noting that

$$\frac{dR_s}{dR_q} = \left(1+\frac{R_q}{400}\right) \tag{14}$$

Therefore, the change in a semi-annually compounded rate that results from a one basis point increase in the equivalent quarterly compounded rate is

$$\Delta R_s \approx \left(1+\frac{R_q}{400}\right).01 \tag{15}$$

In this example, a semi-annual rate of 6.29 is equivalent to a quarterly rate of 6.2413. Substitution into (15) shows that one quarterly basis point is equivalent to 1.0156 semi-annual basis points. Thus, the price value of one quarterly basis point is approximately $\frac{1.8521}{1.0156} = 1.824$.

As the price value of one quarterly basis point is 1.881 up-front basis points and the total value of the option and the forward swap is 160.8 up-front basis points, the number of quarterly basis points that need be paid each period to compensate for the option is $\frac{160.8}{1.824} = 88.2$. As the note being offered is going to pay BA + 50 bps, the 2 year floating rate note component of the structure promises to pay BA - 38.2 on its own. This rate should be compared to the rate on a straight floating rate note for the issuer.

Valuing this proposal by way of the implied volatility requires that one compute the present value of the additional payment offered per period over the floating rate component of the note. This present value is the value of the swaption and the forward rate component of the structure. Assume that a straight quarterly-pay floating-rate note of this issuer would offer a yield of BA-5 bps. As the structure offers BA+50 bps, the additional payment is 55 bps per period. The present value of this can be found by using the price value of a quarterly basis point computed above. That is, the present value is

$.55 \times 1.824 = 1.003$ or 100.3 up-front basis points. Deducting the value of the forward agreement of 0.4 up-front basis points leaves a swaption value of 100.7. The volatility implied by this price is 9.4%, a level that does not compare favourably to the 15% market mentioned above.

3.6.3 Note 4: An extendible note

The last structure to be described is that of an extendible or putable note. The imbedded option on these notes gives the investor the right to extend the term of the note. Alternatively, one can say that the investor has the right to put the note back to the issuer at a specific date in the future. In the interest of economy of space, assume that all the terms of this note are identical to the terms of the step-up callable described in section 3.6.1 with the following exception:

Imbedded option: Putable by investor at par August 5, 1998 upon 14 days prior notice.

Again, the note can be viewed as three components: the two year note, a forward purchase agreement, and an option on the forward purchase agreement. In this case, however, the option is owned by the investor; therefore, the offering price of the note includes payment to the issuer for the option. This option can be viewed as a right-to-pay option with an exercise rate of 8.24. If exercised, the payments made by the investor under the terms of the option will exactly offset those received by the investor. Effectively, the investor will have put the bond back to the issuer.

Valuation of the right-to-pay swaption can be done using equation (10). Assuming the swap rates of table 4-4, the value of this swaption is 1.608 or 160.8 up-front basis points. The off-market adjustment is the same as before, 0.4 up-front basis points. Thus the net amount owed by the investor to the issuer for the option and the forward agreement is 161.2 up-front basis points. Therefore, the effective price of the 2 year note component is 98.388, implying a yield on the two year component of 7.594% or a spread of 138.4 bps. An investor should use this spread in evaluating the note.

4. *Analysis of the accuracy of the technique*

While the decomposition is quite straightforward, it suffers from a theoretical shortcoming of valuing only proxies for the components of the note. For a completely accurate analysis one should use the term structure of rates for the issuer in question to compute the forward rate for that issuer, the value of an option on that forward, and the value of the off-market payment required for having a coupon different from the forward rate. While such direct valuation may be difficult due to a lack of outstanding debt of the issuer to form such valuation, deviations of the swap-based valuation from the theoretical valuations using the correct basis are of interest. This section investigates how the decomposition technique's reliance on a different basis affects the valuation of these securities. We do this by examining the difference between the values of the various components when valued using swap rates from their value when one uses the issuer's discount rates.

The first step in the analysis is to describe the relationship between the swap rates and the issuer's rates. Assume that spot swap rates and spot issuer rates are related by the following:

$$r_i^S + \gamma_i = r_i^I$$

where:

r_i^S is the continuously compounded zero coupon swap rate maturing at period i.
r_i^I is the continuously compounded zero coupon issuer rate maturing at period i.

It follows that the discount factors for the swap rates and the issuer rates are then:

$$\begin{aligned} d_i^S &= e^{-ir_i^S} \\ d_i^I &= e^{-ir_i^I} \\ &= e^{-i(r_i^S+\gamma_i)} \\ &= d_i^S \eta_i \end{aligned}$$

where $\eta_i = e^{-i\gamma_i}$ is referred to as an issuer adjustment factor

Using the issuer's discount rates to compute a forward rate on a par bond for the issuer, a quantity modeled in the decomposition as the forward swap rate, we get:

$$F'_{n,J} = \frac{200(\eta_n d_n^s - \eta_{n+J} d_{n+J}^s)}{\hat{\eta}\sum_{t=1}^{J} d_{n+t}^s}$$

$$\text{where } \hat{\eta} = \frac{\sum_{t=1}^{J} \eta_{n+t} d_{n+t}^s}{\sum_{t=1}^{J} d_{n+t}^s}$$

The value of the off-market adjustment is given by:

$$\frac{C_{n,J} - F_{n,J}}{2} \hat{\eta}\sum_{t=1}^{J} d_{n+t}^s$$

The quantity $\hat{\eta}$ is a weighted average issuer adjustment factor. If the issuer adjustment factors are constant, then the forward rate implied by the issuer's discount factors is identical to the forward swap rate;[38] however, if the issuer adjustment factor is not constant, significant difference between the forward swap rate and true forward rate for the issuer can appear.

It is obvious that the forward swap rate will underestimate the forward rate appropriate for an issuer if the issuer's rates are greater that the swap rates. Similarly, the forward swap rate will overestimate the forward rate for issuer's with rates less than the swap rates. Thus, the off-market adjustment derived from swap rates overstates (understates) the true off-market adjustment for issuer's with higher (lower) rates than the swap rates.

The error in the forward rate feeds into the option valuation because the forward rate acts as the current market price in the option valuation model.

38 The assumption of constant issuer adjustment factors implies that the spreads between the spot swap rates and the spot issuer rates declines monotonically as the term increases.

Overestimating this rate will result in an overstatement of the value of right-to-pay options and an understatement of the value of right-to-receive options. Underestimating the forward rate will do the reverse.

As an example, assume that an issuer's implied zero coupon rates (continuously compounded) are 10 basis points higher than the implied zero coupon rates derived from swap rates for all maturities. Suppose the step-up callable described in section 3.6.1 is proposed. Using the swap rates from table 4-4, the forward swap rate is 8.238. Assuming a volatility of 15%, the net of the off-market adjustment for a 8.24 coupon and the right-to-receive swaption is 160.8 up-front basis points. Using the appropriate issuer rates, the forward rate is 8.342 and the off-market adjustment for the 8.24 coupon is -23.5 basis points. The value of the swaption is 149.9 up-front basis points for a net value of the two components of 173.4 (volatility is assumed to be the same). Thus, the net value of the two proxy components is less than the theoretical value of the true components. A similar analysis done for a credit with implied zero coupon rates 10 basis points below implied zero coupon swap rates reveals that the value of the two components is 148.6 up-front basis points. In this case, the proxies overstate the true theoretical value of the instruments. Analysis of the components of an putable note reveals that the swap rate proxies overstate (understate) the amount that the investor must pay for the forward and option if the issuer's yields are higher (lower) than spot swap rates.

Another important point to note is that the implicit assumption made thus far is that the volatility of the swap rates is the same as the volatility of the issuer's rates. This is not necessarily true. Assuming the forward rate is stated correctly, the size of the error can be approximated by:

$$(\sigma_I - \sigma_S)\frac{\partial B}{\partial \sigma_S} = (\sigma_I - \sigma_S)F\sqrt{n}N'(\delta_1)\hat{\eta}\sum_{t=1}^{J} d_{n+t}^{S}$$

where B is the Black option value.[39]

$$N'(x) = \frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}$$

All other notation is as before.

Thus, there are two sources of error in the option value: the difference of the forward swap rate from the issuer's forward rate and the possible error in the volatility.

The decomposition method outlined in this paper does not provide theoretically precise values for the components due to the use of swap rates for valuation rather than issuer rates. Indeed, for the notes of issuers who enjoy rates lower than swap rates, the method will overstate the theoretical potential for yield enhancement by taking on the derivative positions implied by callable or putable note. The method will understate the theoretical potential for yield enhancement by taking on the derivatives in notes of issuers with higher rates than swap rates. Clearly, the credit risk exposures through a structured note are different than the exposures taken on in the swap market and it is these differences that account for the error. In practice, however, the theoretical

39 There is no need to specify the type of option as this derivative is the same for both calls and puts. See Hull (1993) for details.

potentials are not necessarily achievable due to incomplete financial markets. Thus, the use of swap rates to value the components shows very clearly the value of the positions an issuer can use to offset or the investor can use to create the derivative positions imbedded in a structured note. While an investor must be mindful of the different credit exposures assumed by the decomposition technique, the values provided via the swap market provide an excellent means by which to evaluate the fairness of the pricing of a proposal.

5. Conclusion

This paper has described a very simple technique for analyzing a wide range of structured notes with imbedded options and nonstandard cash flow patterns. The technique involves decomposing the security into three components: a straight bond, a forward purchase agreement for a bond, and an option. Proxies for the derivative components can easily be valued using interest rate swap derivatives. While the technique does not provide theoretically precise valuations due to the difference in the credit risk of a swap counterparty from that of the issuer of the security, it provides the investor with a good measure for evaluating the fairness of a structured note proposal.

Tables

Table 4-1: Representative Canadian credit spreads 1990 - 96

This table shows spreads over equivalent maturity Government of Canada debt yields, in basis points, for the debt of the Province of Ontario, the bellwether Canadian provincial issuer, and for a representative sample of AA rated corporate issuers as at the beginning of June in each year shown. "Long Maturity" column gives the spread on maturities between 20 and 30 years.
Source: Montreal Bonds, Inc.

	Province of Ontario			**AA Corporate**		
Year	2 Year maturity	10 Year Maturity	Long Maturity	2 Year maturity	10 Year Maturity	Long Maturity
1990	24	43	42	35	71	79
1991	43	63	66	50	70	75
1992	30	68	64	40	80	77
1993	35	66	67	31	59	60
1994	21	55	65	28	63	73
1995	24	41	52	28	48	60
1996	13	23	30	19	40	47

Table 4-2: An example of indicative bid side swap quotes

This table provides an example of how swap dealers quote the fixed rate they would pay in exchange for receiving 6 month BA. There is an agreement amongst swappers as to which Government of Canada bonds are the benchmark bonds. Rates for shorter dated swaps are set based on current BA futures prices.

Tenor	**Bid side swap quote**
2 yr.	2 yr. G/C + 8bps
3 yr.	3 yr. G/C + 20bps
4 yr.	4 yr. G/C + 19bps
5 yr.	5 yr. G/C + 12bps
7 yr.	7 yr. G/C + 25bps
10 yr.	10 yr. G/C + 21bps

Table 4-3(a): Quoted swap rates

This table provides swap rates that apply at a particular point in time. The swap rates for tenors less than two years are computed based on BA futures prices [see, for example, Arditti (1996) or Marshall and Kapner (1993)]. Swap rates for tenors longer than one year are taken from an indicative pricing schedule as in table 4-2. Market convention dictates that rates for tenors one year or less are quoted as money market rates while tenors longer than one year are quoted as annual rates compounded semi-annually (i.e. bond equivalent yields). Thus, the one year rate below is based on annual compounding. It must be "decompounded" to get it into 'bond equivalent yield" terms before any further analysis can be done.

Tenor	Swap Rate
0.5 yr.	5.15
1 yr.	5.71
2 yr.	6.29
3 yr.	6.74
4 yr.	7.08
5 yr.	7.37
7 yr.	7.78
10 yr.	8.16

Table 4-3(b): Complete swap schedule

This table provides the schedule of par swap rates for all tenors that are multiples of 6 months, up to the 10 year tenor. Swap rates for those tenors which are not directly quoted by a swap dealer are calculated by linear interpolation. **All rates are quoted as semi-annually compounded rates.**

Tenor	Swap Rate
0.5 yr.	5.15
1 yr.	5.631
1.5 yr.	5.96
2 yr.	6.29
2.5 yr.	6.515
3 yr.	6.74
3.5 yr.	6.91
4 yr.	7.08
4.5 yr.	7.225
5 yr.	7.37
5.5 yr.	7.473
6 yr.	7.575
6.5 yr.	7.678
7 yr.	7.78
7.5 yr.	7.843
8 yr.	7.907
8.5 yr.	7.97
9 yr.	8.033
9.5 yr.	8.097
10 yr.	8.16

Table 4-4: Par swap rates and implied discount factors and zero coupon rates

This table provides the implied zero coupon or spot rates and the implied discount factors resulting from performing a bootstrap procedure on the par swap rates given in table 4-3(a). The implied zero coupon rate is shown in both semi-annually and continuously compounded forms.

Period i	Tenor (years)	Par Swap Rate	Implied Discount Factor	Implied Spot Rate (sa)	Implied Continously Compounded Spot Rate
1	0.5	5.150	0.97490	5.1500	5.0848
2	1.0	5.631	0.94592	5.6375	5.5595
3	1.5	5.960	0.91547	5.9751	5.8876
4	2.0	6.290	0.88303	6.3177	6.2200
5	2.5	6.515	0.85112	6.5533	6.4482
6	3.0	6.740	0.81840	6.7931	6.6803
7	3.5	6.910	0.78664	6.9757	6.8568
8	4.0	7.080	0.75467	7.1620	7.0368
9	4.5	7.225	0.72351	7.3228	7.1920
10	5.0	7.370	0.69245	7.4872	7.3505
11	5.5	7.473	0.66338	7.6028	7.4619
12	6.0	7.575	0.63473	7.7214	7.5760
13	6.5	7.678	0.60650	7.8429	7.6931
14	7.0	7.780	0.57873	7.9676	7.8130
15	7.5	7.843	0.55360	8.0418	7.8843
16	8.0	7.907	0.52907	8.1183	7.9578
17	8.5	7.970	0.50517	8.1972	8.0336
18	9.0	8.033	0.48188	8.2784	8.1117
19	9.5	8.097	0.45921	8.3622	8.1921
20	10.0	8.16	0.43714	8.4485	8.2750

Figure 4-1

The decomposition of a simple structured note

The entire note:

Time	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cash flow	-102	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	104.75

Component 1: The two year note

Time	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cash flow	-102	4.75	4.75	4.75	104.75	0	0	0	0	0	0

Component 2: Forward 3 year bond

Time	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cash flow	0	0	0	0	-100	4.75	4.75	4.75	4.75	4.75	104.75

Component 3: A European call option on component 2 with a strike price of par expiring 2 years hence. Will be exercised if issuer's 3 year rates are below 9.5 at expiration. The option is written by the investor and held by the issuer.

Bibliography

Arditti, F.D., 1996, *Derivatives*, Harvard Business School Press, Boston.

Bansal, V.K., M.E. Ellis, and J.F. Marshall, The spot swap yield curve: derivation and use, *Advances in Futures and Options Research* 6, 279 - 290.

Black, F., 1976, The pricing of commodity contracts, *Journal of Financial Economics* 3, 167 - 179.

Crabbe, L.E., 1995, Medium-Term Notes. In F.J. Fabozzi and T.D. Fabozzi (eds.) *The Handbook of Fixed Income* (4th ed.), Irwin, New York.

Galitz, L.C., 1995, *Financial Engineering: Tools and Techniques to Manage Financial Risk*, Irwin, New York.

Hayre, L.S., 1995, Option-Adjusted Spread Analysis. In F.J. Fabozzi and T.D. Fabozzi (eds.) *The Handbook of Fixed Income* (4th ed.), Irwin, New York.

Hull, J.C., 1993, *Options, Futures, and other Derivative Securities*

Marshall, J.F. and K.R. Kapner, 1993, *Understanding Swaps*, John Wiley and Sons, New York.

Smith, D.R., 1991, A simple method for pricing interest rate swaptions, *Financial Analysts Journal* 47, 72 - 76.

Appendix 1

Derivation of equation (8a):

The board will be indifferent to replacing the manager if:

$$V\left(E^{*}[\tilde{a}]-\varepsilon\right)=E^{*}\left[V(\tilde{a})\right]-c^{*}+k-d \qquad \text{(A1)}$$

Consider a second order Taylor expansion of $V(\tilde{a})$ at $E^{*}[\tilde{a}]$:

$$V(\tilde{a})\approx V\left(E^{*}[\tilde{a}]\right)+V'\left(E^{*}[\tilde{a}]\right)\left(\tilde{a}-E^{*}[\tilde{a}]\right)+\tfrac{1}{2}V''\left(E^{*}[\tilde{a}]\right)\left(\tilde{a}-E^{*}[\tilde{a}]\right)^{2} \qquad \text{(A2)}$$

Taking the expectation of (A2):

$$E^{*}\left[V(\tilde{a})\right]=V\left(E^{*}[\tilde{a}]\right)+\frac{\sigma_{a}^{2}}{2}V''\left(E^{*}[\tilde{a}]\right) \qquad \text{(A3)}$$

Now, take a first order Taylor expansion of $V\left(E[\tilde{a}]-\varepsilon\right)$ at $E^{*}[\tilde{a}]$:

$$V\left(E^{*}[\tilde{a}]-\varepsilon\right)=V\left(E^{*}[\tilde{a}]\right)+V'\left(E^{*}[\tilde{a}]\right)(-\varepsilon) \qquad \text{(A4)}$$

Combining A3, A4 and A1 and then solving yields:

$$\varepsilon=-\frac{1}{2}\sigma_{a}^{2}\frac{V''\left(E^{*}[\tilde{a}]\right)}{V'\left(E^{*}[\tilde{a}]\right)}+\frac{c^{*}-k+d}{V'\left(E^{*}[\tilde{a}]\right)}$$

Appendix 2

Excel Spreadsheet for Bootstrapping Swap Curve

	A	B	C	D	E	F
4						
5						
6	Term		Swap	Semi	Spot	
7	(years)	Benchmark	Spread	Swap Rate	Swap Rate	Discount Factor
8	0.5	5.15	0	=B8+C8	=D8	=(1+E8/200)^(-A8*2)
9	1	5.71	0	=((1+(B9+C9)/100)^0.5)*200-200	=(((100-D9/2*SUM($F$8:F8))/(100+D9/2))^(-1/(2*A9))-1)*200	=(1+E9/200)^(-A9*2)
10	1.5			=(D9+D11)/2	=(((100-D10/2*SUM($F$8:F9))/(100+D10/2))^(-1/(2*A10))-1)*200	=(1+E10/200)^(-A10*2)
11	2	6.21	0.08	=B11+C11	=(((100-D11/2*SUM($F$8:F10))/(100+D11/2))^(-1/(2*A11))-1)*200	=(1+E11/200)^(-A11*2)
12	2.5			=(D11+D13)/2	=(((100-D12/2*SUM($F$8:F11))/(100+D12/2))^(-1/(2*A12))-1)*200	=(1+E12/200)^(-A12*2)
13	3	6.54	0.2	=B13+C13	=(((100-D13/2*SUM($F$8:F12))/(100+D13/2))^(-1/(2*A13))-1)*200	=(1+E13/200)^(-A13*2)
14	3.5			=(D13+D15)/2	=(((100-D14/2*SUM($F$8:F13))/(100+D14/2))^(-1/(2*A14))-1)*200	=(1+E14/200)^(-A14*2)
15	4	6.89	0.19	=B15+C15	=(((100-D15/2*SUM($F$8:F14))/(100+D15/2))^(-1/(2*A15))-1)*200	=(1+E15/200)^(-A15*2)
16	4.5			=(D15+D17)/2	=(((100-D16/2*SUM($F$8:F15))/(100+D16/2))^(-1/(2*A16))-1)*200	=(1+E16/200)^(-A16*2)
17	5	7.25	0.12	=B17+C17	=(((100-D17/2*SUM($F$8:F16))/(100+D17/2))^(-1/(2*A17))-1)*200	=(1+E17/200)^(-A17*2)
18	5.5			=((D$21-D$17)/4)+D17	=(((100-D18/2*SUM($F$8:F17))/(100+D18/2))^(-1/(2*A18))-1)*200	=(1+E18/200)^(-A18*2)
19	6			=((D$21-D$17)/4)+D18	=(((100-D19/2*SUM($F$8:F18))/(100+D19/2))^(-1/(2*A19))-1)*200	=(1+E19/200)^(-A19*2)
20	6.5			=((D$21-D$17)/4)+D19	=(((100-D20/2*SUM($F$8:F19))/(100+D20/2))^(-1/(2*A20))-1)*200	=(1+E20/200)^(-A20*2)
21	7	7.53	0.25	=B21+C21	=(((100-D21/2*SUM($F$8:F20))/(100+D21/2))^(-1/(2*A21))-1)*200	=(1+E21/200)^(-A21*2)
22	7.5			=D21+(D$27-D$21)/6	=(((100-D22/2*SUM($F$8:F21))/(100+D22/2))^(-1/(2*A22))-1)*200	=(1+E22/200)^(-A22*2)
23	8			=D22+(D$27-D$21)/6	=(((100-D23/2*SUM($F$8:F22))/(100+D23/2))^(-1/(2*A23))-1)*200	=(1+E23/200)^(-A23*2)
24	8.5			=D23+(D$27-D$21)/6	=(((100-D24/2*SUM($F$8:F23))/(100+D24/2))^(-1/(2*A24))-1)*200	=(1+E24/200)^(-A24*2)
25	9			=D24+(D$27-D$21)/6	=(((100-D25/2*SUM($F$8:F24))/(100+D25/2))^(-1/(2*A25))-1)*200	=(1+E25/200)^(-A25*2)
26	9.5			=D25+(D$27-D$21)/6	=(((100-D26/2*SUM($F$8:F25))/(100+D26/2))^(-1/(2*A26))-1)*200	=(1+E26/200)^(-A26*2)
27	10	7.95	0.21	=B27+C27	=(((100-D27/2*SUM($F$8:F26))/(100+D27/2))^(-1/(2*A27))-1)*200	=(1+E27/200)^(-A27*2)