# Identifying non-invasible habitats for marine copepods using temperature dependent R<sub>0</sub>

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8

# 9 Abstract

10 If a non-indigenous species is to thrive and become invasive it must first 11 persist under its new set of environmental conditions. Net reproductive rate  $(R_0)$ 12 represents the average number of female offspring produced by a female over its 13 lifetime, and has been used as a metric of population persistence. We modeled  $R_0$  as a 14 function of ambient water temperature (T) for the invasive marine calanoid copepod 15 Pseudodiaptomus marinus, which was introduced to west coast of North America 16 from East Asia by ship ballast water. The model was based on temperature-dependent 17 stage-structured population dynamics given by a system of ordinary differential 18 equations. We proposed a methodology to identify habitats that are non-invasible for 19 *P. marinus* using the threshold of  $R_0(T) \le 1$  to identify potentially invasible habitats. 20 We parameterized the model using published data on P. marinus and applied  $R_0(T)$  to 21 identify the range of non-invasible habitats in a global scale based on sea surface 22 temperature data. Model predictions matched field evidence of species occurrence 23 well.

#### 24 Keywords

Net reproductive rate, invasive species, marine copepods, *Pseudodiaptomus marinus*, temperature, stage-structured population models, ordinary differential

27 equations, ecological modeling, habitat invasibility, habitat suitability

#### 1 Introduction

2

3 Assessment of habitat invasibility often relies on statistical matching of the external environmental variables in native and novel habitats via methods such as 4 ecological niche modeling (ENM) (Jeschke and Strayer 2008; Mercado-Silva et al. 5 6 2006). However, it is often the case that invasive species can tolerate environmental 7 conditions in novel habitats that are outside those found in their native habitats 8 (Broennimann et al. 2007; Elith and Leathwick 2009). This indicates that the absence 9 of a species in particular environments may not necessarily mean such environments 10 are unsuitable for the species. As an alternative to ENM, we can determine the 11 response of potential invaders to specific environmental conditions under controlled 12 laboratory settings. For example, we can measure the rates of mortality, offspring production, and stage durations under different environmental conditions. However, 13 14 we must still translate these measures into a habitat invasibility indicator or metric. 15 Will a population persist and grow under a given set of environmental conditions? To answer this question we can use the net reproductive rate  $R_0$  of a population as a 16 17 metric.  $R_0$  is a measure of a population's reproductive success (Ackleha and de-18 Leenheerb 2008), and therefore, is a population fitness trait, which represents the 19 average number of offspring produced by a female over its lifetime (de-Camino-Beck 20 and Lewis 2008). It has been used in evolutionary invasion analysis to predict long 21 term evolutionary outcomes (Hurford et al. 2010). When  $R_0 > 1$ , a population grows, 22 and when  $R_0 < 1$ , a population tends to decrease to extinction (Boldin 2006). Thus, we 23 can use  $R_0$  to decide which habitats are suitable or unsuitable for a species by 24 determining whether environmental parameters result in  $R_0 > 1$  or  $R_0 < 1$ . We derived  $R_0$ 25 from a mechanistic state-structured population model given by a system of ordinary differential equations and parameterized by data from laboratory experiments. This 26 27 method allows us to predict the range of habitats that are *non-invasible* or *potentially* 28 invasible for a species or strain. 29 Our model species, *Pseudodiaptomus marinus*, is an invasive marine calanoid

30 copepod that was introduced to the Pacific coast of North America (Fleminger and

1 Kramer 1998) and coastal waters in Southern Chile from its native habitat in East 2 Asia via ballast water (Bollens et al. 2002). It is a perennial egg-carrying calanoid copepod, spawns continuously throughout the year, and has multiple overlapping 3 4 generations (Uye et al. 1983). Its life-history traits such as fertility, mortality and maturation rates are known to be functions of temperature (Liang and Uye 1997a; Uye 5 6 et al. 1983). P. marinus has also been reported in many other oceanic habitats around 7 the world (Marine Planktonic Database) and has been expanding its range (Jiménez-8 Pérez and Castro-Longoria 2006). Despite high propagule pressure, P. marinus has 9 not been reported in the coastal ecosystems of Oregon and Washington (Cordell et al. 10 2009), or Vancouver Harbour (Piercey et al. 2000), indicating that it may be a 11 successful invader only in selected habitats. It has not been clear what environmental 12 factors limit its geographical distribution in terms of its physiological tolerance. 13 Here we modeled  $R_0$  of *P. marinus* as a function of temperature assuming 14 continuous time stage-structured population dynamics of the species based on a system of linear first order ordinary differential equations (ODEs). ODE transmission 15 16 models in epidemiology literature are commonly evaluated using  $R_0$ , although it is 17 less commonly used in stage-structured life-history dynamics. We parameterized the 18 model using previously published data from laboratory experiments and field surveys 19 (Liang and Uye 1997a; Uye et al. 1983). 20 The  $R_0$ -based approach to determining habitat invasibility, while appealing, is

21 necessarily limited by the range of environmental conditions under which the 22 laboratory experiments can produce parameters. When  $R_0$  is calculated using model 23 parameters that were estimated for a limited range of primary environmental variables 24 (e.g. temperature only), with other secondary environmental variables (e.g. salinity, 25 daylight levels) held at optimal levels in the laboratory, results are not likely to be 26 representative of what the species experiences in the field. In these cases, however, it 27 is possible to use the  $R_0$ -based approach to identify which habitats are non-invasible. If  $R_0 < 1$  when secondary variables are optimal it also should remain below one when 28 29 secondary variables are suboptimal. In this way we can identify temperature (T)30 thresholds for invasibility of the marine copepod P. marinus using  $R_0(T)$ .

The method we develop yields  $R_0(T)$  as a function of temperature, allowing us to predict the range of temperatures that inhibit the growth of *P. marinus*, and thereby to predict the range of habitats that are potentially invasible to *P. marinus*. This method can be generally applied to model  $R_0$  for other similar species. The results is complimentary to ENM and has a further advantage over ENM in terms of predicting species' potential spread over habitats that differ from their native habitats.

7

# 8 Methods

9

10 We modeled stage-structured population dynamics of *P. marinus* using a 11 system of first order linear ODEs assuming continuous year-round growth and 12 overlapping generations (Uye et al. 1983). We followed the methods in van den Driessche and Watmough (2002) to model the net reproductive rate  $R_0$  based on the 13 14 ODE model. Our model contains fertility, maturation, and mortality rate parameters. 15 Because stage based fertility, mortality, and maturation rates are temperaturedependent (Uye et al. 1983; Liang and Uye 1997a.), we modeled the rate parameters 16 17 as functions of temperature. This allowed us to calculate the temperature-dependent 18  $R_0$ .

19

20 Model

*P. marinus* has 12 life stages, consisting of eggs, five naupliar stages, five
copepodid stages, and one adult stage. We do not include naupliar stage 1 in the
model as data corresponding to this stage are not available due to difficulty in
measurement as it lasts only few minutes (Uye et al. 1983). However, the data on
naupliar stage 2 can be considered as an approximation, combining stage 1 and stage
2 into a single stage.

We define n(t) to be a vector representing the stage composition of the population at time *t*, and A(T) be a matrix of parameter space of vital rates (fertility, maturation, and mortality) that depend on temperature (*T*). Thus, we can write the rate of change of stage composition as follows:

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- 2
- 3 Where,

4 
$$n(t) = [n_1(t), n_2(t), ..., n_{12}(t)]^T$$

5

$$6 \quad A(T) = \begin{pmatrix} -\mu_1(T) - \gamma_1(T) & 0 & \vdots & 0 & q\beta(T) \\ \gamma_1(T) & -\mu_2(T) - \gamma_2(T) & \vdots & 0 & 0 \\ 0 & \gamma_2(T) & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & -\mu_{11}(T) - \gamma_{11}(T) & 0 \\ 0 & 0 & \vdots & \gamma_{11}(T) & -\mu_{12}(T) \end{pmatrix}$$

7

and  $\gamma_i(T)$ , are stage-dependent mortality, maturation, and 8 where, 9 fertility rates respectively, which are functions of temperature. Here, represents the number of eggs,  $n_2...n_6$  represents the number of individuals in the five naupliar 10 stages (excluding stage 1),  $n_7 \dots n_{12}$  represents the number of individuals in the five 11 12 copepodid stages,  $\beta(T)$  is the fertility rate (rate of egg production) in adult females as a functions of temperature. The constant q is the average proportion of ovigerous 13 females in the adult population, which is estimated to be 0.61 (Liang and Uye 1997b). 14 15 See Table 1 for all notations. We derived the net reproductive rate  $R_0$  for *P. marinus* based on the above model as described below. 16

17

#### 18 $R_0$ as a function of temperature

First, we wrote the matrix *A* as A = F - V where *F* is the matrix of fertility coefficients (non-negative and non-zero), and *V* is the matrix of transition coefficients (i.e. net maturation and mortality rates).  $R_0$  can then be written as  $R_0 = \rho[FV^{-1}]$ , where  $\rho$  is the spectral radius of the matrix (van den Driessche and Watmough 2002). That is  $\rho[FV^{-1}] = \max_{1 \le i \le n} |\lambda_i|$  where  $\lambda_1, \lambda_2, ..., \lambda_n$  are eigenvalues of the square

(1)

1 matrix . Note that the intrinsic growth rate defined as the maximum real 2 eigenvalue of the square matrix *A* has a non-linear relationship with net reproductive 3 rate  $R_0$  (Wallinga and Lipsitch M 2007). However, the intrinsic growth rate is positive 4 if and only if  $R_0 > 1$ .

5 We modified the model to express  $R_0$  as a function of temperature, such that 6  $R_0(T) = \rho[F(T)V(T)^{-1}]$ . Using the graph reduction method (de-Camino-Beck and

7 Lewis 2007) (see derivation in Appendix A), we can also write  $R_0$  as,

$$R_{0}(T) = \frac{q\beta(T)}{1 \frac{\mu}{2} \frac{\gamma}{7} \frac{q}{8}} \prod_{\substack{i=1\\a \text{ tabular}}}^{s-1} \left( \frac{\gamma_{i}(T)}{\frac{\mu}{2} \frac{q}{4} \frac{\gamma_{i}(T)}{4}} \right)$$
mortality rate at stage s

9 where s is the final stage (stage 12) for *P. marinus*. We modeled temperature
10 dependent parameters in the model as described in the next section.

11

8

12 *Fertility rates* 

13 Eggs are produced by adult females in stage 1  $(n_{11})$ . Fertility rate, , can be written as  $\beta(T) = f(T)/\Delta t$ , where f(T) is the number of eggs produced by an adult 14 female over time  $\Delta t$  at average temperature T. Uye et al. (1983) fitted a linear model 15 16 to parameterize . The linear model takes the form  $\beta(T) = 0.771T - 4.48$ , with  $R^2=0.84$ . Residual analyses of Uye's data, however, show that residuals are not 17 randomly distributed along the fitted line indicating that linearity may not be the 18 19 appropriate assumption. There is a depression in fertility rates at low temperatures. 20 Furthermore, the linear model assumes that fertility is unbounded with increasing 21 temperature, which is not a biologically valid assumption. We therefore refitted the 22 data with a sigmoidal curve, assuming log normally distributed errors. We 23 incorporated a lag parameter (b) to relax the assumption that the curve must otherwise 24 intercept the y-axis at the origin. The sigmoidal curve allows us to assume that 25 fertility rate has a maximum value. Biologically it is more appropriate to assume that 26 fertility rate is a bell-shaped curve, however we did not have the data to extend our

1 curve to the point were begins to decrease at high temperatures. Hence, our

2 model for fertility rate can be written as,

3

$$\beta(T) = f_m f_l e^{w(T-b)} / [f_m + f_l (e^{w(T-b)} - 1)]$$

Where, is the maximum rate of fertility, f<sub>l</sub> is fertility rate at the lowest temperature,
and w is a shape parameter that accounts for the depression in fertility at lower
temperatures. We compared the regression fit of linear model used in Uye et al. (1983)
with our sigmoidal model using residual sum of squares.

8

9 Maturation rates (T)

10 We solved the system of ODE's represented by Eq.1 analytically for initial 11 values corresponding to a single individual in stage 1,  $n_1(0) = 1$ , and  $n_i(0) = 0$  for 12 i=2,...,12. This allowed us to follow a single cohort over time with no additional 13 individuals being added to the system (Appendix B).

14 In experimental studies, maturation rates are commonly calculated using median development times, or the time it takes for 50% of the cohort to mature from 15 16 eggs past a given stage (e.g. Uve et al. 1983, Breteler et al 1994, Lee et al. 2003.) An assumption underlying such conventional calculation of maturation rate using 17 18 'proportions not yet past given stage' is that daily mortality rates of copepods are the 19 same across all stages for a cohort. It excludes the mortality rate parameter from the 20 equation and assumes that daily stage proportions are the result of individuals 21 maturing from one stage to another. We made the same assumption here in the 22 estimation of maturation rates from our model as P. marinus data are available only as 23 proportions of a cohort remains in each stage over time with the same assumption. 24 Thus, we normalized the stage size data  $n_a(t)$  for each time step (t) dividing it by total 25 remaining population of the cohort at that time step to give the proportion at each stage  $z_a(t)$ . This assumption made the proportion at each stage  $z_a(t)$  to be 26 27 independent from the mortality rates (Appendix C).

Using Eq.3 in Appendix C we can describe the proportion of individuals not
 having past stage *a*, i.e. , as,

3 . (4)

4	As shown by Cox (1967), this equation can also be derived from assuming the								
5	length of time that a copepod takes in a stage (stage duration time) as an exponentially								
6	distributed random variable, $d_a$ , such that the probability density function of $d_a$ is								
7	$\gamma_a e^{-\gamma_a t}$ and cumulative density function of $d_a$ is $(1 - e^{-\gamma_a t})$ , where $\gamma_a$ is the stage								
8	maturation rate, and $\mu_a = 0$ for all stages <i>a</i> . The length of mean time taken to exit stage								
9	<i>a</i> , i.e. stage development time, $D_a$ , becomes a random variable defined as $D_a = \sum_{i=1}^{a} d_i$ of								
10	which the cumulative density function of is . The quantity , thus								
11	yields the proportion of individuals not having past stage <i>a</i> .								
12	We fitted stage proportion data from Uye et al. (1983) to Eq.4 using nonlinear								
13	least squares regression to estimate $\gamma_a$ . The data used were collected for <i>P. marinus</i> at								
14	20 <sup>°</sup> C. We calculated the mean stage duration times $d_a$ as at 20 <sup>°</sup> C. This								
15	yielded from the fact that is an exponentially distributed random variable. We then								
16	used calculated for $20^{\circ}$ C to estimate the relationship between and temperature								
17	( $T$ ). We assumed the relationship given by Belehradek's function,								
18	(as used by Uye et al. (1983) for <i>P. marinus</i> ), where <i>T</i> is temperature in centigrade								
19	and is a constant that varies with stage <i>a</i> . Using calculated , we estimated the								
20	parameters for $\gamma_a(T)$ from the following equation derived from the above,								
21	for each stage $a$ at temperatures ( $T$ ). Here,								
22	As an advancement to the above model, we modified Eq.1 to assume that stage								

1 the earlier assumption on exponentially distributed times. That is, probability density

2 function of 
$$d_a$$
 now becomes  $\frac{\gamma^k}{\Gamma(k)} t^{k-1} e^{-\gamma_a t}$  where, , ,  $k > 0$ 

3 Mathematically this can be achieved by assuming that there are sub-stages  $(k_a)$  within 4 each stage *a* in Eq.1 given that duration times of sub-stages are exponentially distributed (see MacDonald 1978 for a full description). Here we assumed that 5 6 mortality and maturation rates of sub-stages were the same for each stage. Thus the 7 number of sub-stages, k, in Eq.1 is equivalent to assuming the shape parameter k in 8 the gamma distributed stage duration times. Here we assumed  $k_a$  to be the same for 9 all stages a as previous studies suggested for copepods (e.g., Breteler et al. 1994; Lee 10 et al. 2003). The method for fitting the model with multiple sub-stages is outlined in 11 Appendix D.

12

The mean stage duration times  $d_a$  become for the modified model for

13 gamma distributed  $d_a$ . We assumed  $k_a$  to be the same for all stages a (Breteler et al.

14 1994). Therefore

15

$$\gamma_a(T) = k(T-1)^{1.8} / (\alpha_a - \alpha_{a-1})$$
(5)

16 where . Note that the advanced model (see Appendix D through Eq.5) reduces 17 to simple model when k=1 and  $\varepsilon = 0$ . We compared the model fits for k=1, and k=2,318 using AIC and chi-squares test to determine which model assumption was the best to 19 estimate (*T*). We used the estimated stage duration times to calculate mortality rates 20 as shown in the next section.

21

# 22 *Mortality rates* $\mu(T)$

Liang and Uye (1997a) estimated the percent survival of nine generations of
the population for *P. marinus* from the west coast of Japan under different mean
temperatures. We used these data to estimate survival curves at different temperatures.
Because of their estimation procedure, Liang and Uye reported percent survival >100%
in some cases; these values were reduced to 100%. We fitted the function

1  $Sv = \exp(-\phi a^{\chi})$  for the proportion surviving from eggs to stage a, where  $\phi$  is a scale parameter and  $\chi$  is a shape parameter. We estimated  $\phi$  and  $\chi$  using non-linear least 2 squares regression. We calculated the proportion of individuals that died in each 3 stage with respect to the proportion of individuals that matured into the current stage 4 5 from the previous stage using Sv. We refer to Sv as a modified Wiebull function 6 because (1-Sv) is the cumulative density function of the Wiebull distribution (1951). 7 To obtain estimates of mortality rates for each stage *a*, we divided the 8 estimated proportions that died in each stage by the stage duration times, given by  $d_a = \frac{1}{v}$  for the exponential distributions (simple model), and  $d_a = \frac{k}{v}$  for gamma 9 distributions (advanced model) at the same temperatures. We pooled mortality rates 10 11 across stages so as to be consistent with our earlier assumption (in modeling stage 12 maturation rates using experimental data) that mortality rates across all stages are the same. We fitted a quadratic function  $\mu(T) = \kappa_2 T^2 + \kappa_1 T + \kappa_0$  for the pooled data 13 using nonlinear least squares regression. We did not use the survey measurement data 14 15 at 27.4°C in Uye et al. (1983) for above calculations as it yielded near zero daily 16 mortality rates at such a comparatively high temperature which resulted in a 17 biologically inexplainable pattern that contradicted the general trend, suggesting that 18 those data may be outliers.

We tested whether the assumption behind pooling data, i.e. mortality rates are
the same across all stages for a given temperature (as in Breteler et al. 1994; Uye et al.
1983) is a valid assumption for this species. To do this, we used the method of
positioning means within confidence intervals (Venables and Repley 2002).

Now we had  $\gamma_a(T)$  and  $\mu(T)$  modeled exclusively as functions of temperature to finally fit into  $R_0(T)$  model.

25

The model for  $R_0(T)$  for any k is as follows

26 
$$R_{0}(T) = \frac{q\beta(T)}{\mu_{s}(T)} \prod_{i=1}^{s-1} \left( \frac{\gamma_{i}(T)}{\mu_{i}(T) + \gamma_{i}(T)} \right)^{k}$$
(6)

27

(see derivation in Appendix A).

1

#### 2 Application and validation

We used the parameterized  $R_0(T)$  to predict the range of habitats that are noninvasible to *P. marinus* on a global scale, based on sea surface temperature data from NOAA Optimum Interpolation (OI) SST V2. The range of habitat temperatures where  $R_0(T) < 1$  is considered to be non-suitable for population persistence and hence noninvasible. We compared predictions with the known distribution of *P. marinus*.

8

### 9 **Results**

10

#### 11 Fertility rates

12 We found that the sigmoidal model for fertility rates fits the data better than 13 the linear model (Fig.1). The residual sum of squares (RSS) for the sigmoidal model 14 was 97.37, compared to 126.08 for the linear model. Parameters for the sigmoidal model were  $f_m = 13.89$ ,  $f_1 = 0.61$ , w=0.35, b=6.01°C. Using the sigmoidal model, fertility 15 16 rates started at zero near or slightly above 0°C, and tended to reach a maximum at 17 temperatures above 25°C. Intuitively, fertility rate should peak at some optimal 18 temperature, then decrease with increasing temperatures, which our sigmoidal model 19 does not recreate. However, we are more interested in predicting dynamics at lower 20 temperatures, so the sigmoidal model is sufficient. The results indicate that sigmoidal 21 model is a better statistical approximation as well as having a theoretically better 22 rationale than the linear model.

23

#### 24 Maturation rates

We estimated stage development times for cases k=1,2 and 3 in Eq.5 by fitting data from Uye et al. (1983) (Fig.2). We compared the fits using AIC and found that k=3 is the better statistical model than k=1,2 (Table 2). The model with k=3 gives the lowest AIC (Table 2). Note that *p*-values for chi-squares goodness of fit test for k=1and k=2 with respect to k=3 was <0.001. This suggests that model with k=3 is

1 significantly different from models with k=1 and k=2. Hence, we concluded that the 2 model with k=3 is the most reasonable.

3

4 Mortality rates

5	We estimated values of $\lambda$ and $\alpha$ for Wiebull model for different generations at								
6	different temperature regimes (Table 4). We also plotted mortality rates against								
7	temperatures based on the Wiebull model (Fig. 4) and in relation to at								
8	different temperature regimes. The parameters estimated for mortality rate were								
9	$\kappa_2$ =0.0022 /day, $\kappa_1$ =-0.0563 /^0C day, $\kappa_0$ =0.4211 /^0C^2 day. The assumption that								
10	mortality rates are the same across all stages was tested by examining confidence								
11	intervals. The mean values of the model coefficients fall within the confidence								
12	intervals of every other stage, indicating that the data can be pooled. Hence, our								
13	assumption that mortality rates are the same across all stages for a given temperature								
14	is valid for P. marinus.								
15									
16	Net reproductive rate								
17	We plotted $R_0(T)$ after incorporating the parameterized sub-models , $\gamma_a(T)$								
18	and $\mu(T)$ (Fig. 5). $R_0(T)$ tends to curve downwards at high temperatures due to								
19	increasing mortality rate (Fig.4) that suppresses the positive effect of increasing								
20	fertility rates at higher temperatures (Fig.1).								
21	We plotted $R_0(T)$ for the cases where $k=1$ and $k=3$ (Fig. 5). Relatively higher								
22	values of $R_0(T)$ for higher k suggest that the fitness of the population is reduced when								
23	k is low regardless of the temperature. The model $R_0(T)$ that best fits data was the one								
24	with parameter $k=3$ . The uncertainty associated with the estimates of $R_0(T)$ can not be								
25	calculated because parameters taken from the literature did not have confidence								
26	estimates (Uye et al (1983) and Liang and Uye (1997a)). We found that $R_0 > 1$								
27	between $11^{0}$ C and $23^{0}$ C, and this is therefore the range within which the habitats are								
28	potentially invasible. If other conditions in a habitat are ideal and temperature falls								
29	within this range, species could grow. At temperatures $<11^{\circ}$ C and $>23^{\circ}$ C, $R_{0}<1$ and								

- 1 habitats with these mean temperatures are non-invasible. If a habitat's temperature
- 2 fluctuates seasonally between these two limits, it is tolerable to *P. marinus*.
- 3
- 4 Application and validation

5 We mapped the range of habitats where yearly averaged sea surface 6 temperatures is between  $11^{\circ}$ C and  $23^{\circ}$ C (colored contours in Fig 6) where they are 7 potentially invasible to *P. marinus*. Hence, the area where there are no contour lines 8 ( $23^{\circ}$ C <T<11^{\circ}C) indicate the habitats where *P. marinus* is non-invasible. Field 9 sampling evidence depicted in Fig (6) suggests that our predictions fit well into 10 potentially invasible habitat range except for marginal deviations of few occurrences.

- 11
- 12 **Discussion**
- 13

14 Here we proposed a novel methodology to model net reproductive rate  $R_0$ , which is a population persistence metric, as a function of temperature (T) for invasive 15 marine copepod P. marinus based on the data from experiments. This approach can be 16 17 generally applied to model  $R_0$  for aquatic copepods that respond to environmental 18 parameters markedly, reproduce year-round, and have multiple overlapping 19 generations (species for e.g. as in Bonnet et al. 2009; Chen et al. 2006). Temperatures 20 giving  $R_0(T) > 1$  indicate habitats where the species can physiologically persist, 21 assuming that other environmental factors are suitable for its growth. Temperatures 22 resulting  $R_0(T) < 1$  indicate habitats where that the species cannot physiologically 23 persist regardless of the other environmental factors. Thus, our approach can 24 conservatively predict habitats which are non-invasible, and thereby habitats which 25 are potentially invasible. Note that we have not incorporated confidence intervals in the estimates due to unavailability of primary data to incorporate that. 26

The habitats that are potentially invasible to *P. marinus* as predicted by our model matched well with field evidence of species occurrences on a global scale except for few marginal deviations (miss-matches) on the borders limiting  $R_0(T)=1$ . In particular, we note that from Fig 6, Elliot bay, Puget Sound is on the border of non-

1 invasibility range limiting  $R_0(T)$ . It has been recorded in U.S Geological Surveys that 2 P. marinus has been sampled in that location by Cohen (2004). However, up to now, 3 there has been any indication that it has established in that location. Further 4 northwards, Piercey et al. (2000) found that there was a large propagule pressure of P. marinus on Vancouver harbor (in 25.4% ships sampled, and occurring in densities 5 6 from  $2 \sim 54 \text{m}^{-3}$ ). Our model predicts that Vancouver harbor is also located on the border where  $R_0(T)=1$ . We note that on the above locations, temperatures fluctuate 7 8 seasonally throughout the year (Lighthouse sea surface temperature data, DFO 9 Canada). To better predict non-invasibility in such habitats we need a model that 10 incorporates the effect of seasonal variation of temperatures. Furthermore, had we incorporated the survival data at 27 °C, then the upper 11 12 bound of  $R_0(T)=1$  would have shifted towards higher temperatures moving the 13 potentially invasible range towards the tropics. We did not incorporate those data as 14 they were inconsistent with the general trend in mortality rates with respect to 15 increasing temperatures and did not make sense biologically, as outlined in the 16 methods section. 17 The methodological basis adopted here in determining non-invasible habitats

18 is in contrast to that of ENM (Peterson 2003). ENM predicts habitat-suitability based 19 on a snapshot of environmental conditions and species occurrences (Herborg et al. 20 2007a; Peterson et al. 2007) by matching the range of environmental variables in 21 native habitats with that in novel habitats (Jeschke and Strayer 2008; Mercado-Silva 22 et al. 2006). For e.g. Genetic Algorithm for Rule-set Prediction (GARP) (Stockwell 23 and Peters 1999) in ENM has been commonly used to predict habitat suitability for 24 both terrestrial and aquatic invasive species (e.g. Herborg et al. 2007a; 2007b; 25 Peterson 2003; Peterson et al. 2007). The above methodology implicitly assumes that 26 the limit to phenotypic plasticity of population fitness traits is exhaustively 27 represented in the observed environmental set in their native habitats. This, in turn, 28 assumes that a species may only survive and reproduce in habitats those having 29 environmental sets similar to that in their native ranges. Often, species tolerate 30 environmental set beyond that is found in native habitats (Lockwood et al. 2006). For

1 example, a species distribution may be confined to a certain native range due to 2 natural barriers rather than environmental parameters (Lonhart 2009) suggesting that 3 absence is not necessarily indicative of a habitat's suitability. In such cases, ENM 4 may not be able to fully capture the potential range of the environmental set that a species may tolerate. For this reason, ENM can overlook habitats where a species can 5 6 potentially survive and reproduce, especially in cases where human-mediated 7 transport may facilitate jump dispersal (e.g. Broennimann et al. 2007). Our approach 8 avoids this particular limitation of ENM.

9 Our model is designed to quantify  $R_0$  at low introductory populations to 10 determine the species establishment potential. Hence, we did not explicitly account 11 for density dependence of the population considering high population levels. Further, 12 we disregarded Allee effects (Taylor and Hasting 2005; Courchamp et al. 2008; 13 Kramer et al. 2008) although it may be a factor that acts against species establishment at low population levels (Lockwood et al. 2005; Whitmann et al., accepted). In such 14 15 cases it is possible to have a *backward bifurcation*, where a species can persist even when  $R_0 < 1$ , and hence a different approach would be needed to analyze populations 16 17 with Allee effects. Biologically, inclusion of the Allee effect may further filter out a 18 subset of non-invasible habitats from potentially invasible habitats. This will 19 complement our predictions which were made without the case of Allee effect.

20 Sea surface temperature has been rising over the last few decades (Cane et al. 21 1997). Our model can be used as a tool to determine how climate change may affect 22 species range expansion. For *P. marinus*, the shape of  $R_0(T)$  curve suggests that with 23 increases in sea temperature, ranges may tend to shift towards currently cooler waters. 24 However, the effect of climate change on seasonal changes in sea surface temperature 25 may also be a critical factor in determining long term effects on niche shifts. For 26 example, temperature data from Racerock, B.C., spanning the years 1921-2008, 27 indicates that annual low temperatures have not increased as much as annual high 28 temperatures. The impact of such non-linear increases in temperatures may have non-29 linear effects on  $R_0$ . Hence, we may not be able to rescale the range of  $R_0$  by simply 30 adding the expected increment to mean sea surface temperature.

1 A proxy of using mean temperatures to characterize a habitat is appropriate in 2 cases where temperature forces  $R_0$  to be either strictly less than 1 or greater than 1. 3 Hence, our result is only applicable to habitats where all seasonal temperatures, were 4 they held constant or averaged, would force  $R_0(T)$  to be greater than 1 or less than 1 throughout years. However, in habitats where temperatures fluctuate seasonally, or 5 6 daily, forcing  $R_0(T) > 1$  in one period, and  $R_0(T) < 1$  in another period, we cannot make 7 clear predictions on habitat invasibility by metric  $R_0(T)$  alone. Yet, we could presume 8 that a habitat to be more unfavorable to a species when the seasonal fluctuations of a 9 factor forces  $R_0 < 1$  in longer period of the year, and *vice-versa*. It may be useful to 10 incorporate the effects of short term and seasonal temperature fluctuations on  $R_0$  (see 11 Bacaeer 2009; Bacaeer and Ouifki 2007, Wesley and Allen 2009). 12 An extension to our model would be to incorporate vital rates as functions of other environmental factors such as salinity. We can then calculate  $R_0$  in a two-13 14 dimensional environmental space. It may increase the non-invasible habitat set for the 15 species reducing the potentially invasible habitat set. Recent work towards modeling 16 the combined effect of temperature and salinity on population persistence is found in 17 Strasser et al. (in press). 18 19 Acknowledgements 20 21 Financial support for HR and CS came from the NSERC-funded Canadian 22 Aquatic Species Network (CAISN). HR also acknowledges the Department of 23 Biological Sciences, University of Alberta for providing financial support. MAL 24 gratefully acknowledges an NSERC Discovery Grant and a Canada Research Chair. 25 The authors thank Alex Potapov at the Centre for Mathematical Biology, University 26 of Alberta, and Claudio DiBacco at the Bedford Institute of Oceanography, Halifax, 27 for valuable suggestions.

28

29



29 yields the following diagraph with a single node.

$$-1 + \frac{q\beta}{\mu_{12}} \prod_{i=1}^{11} \left(\frac{\gamma_i}{\gamma_i + \mu_i}\right) \lambda^{-1}$$

3

6

1

2

4 Finally, we set the weight of this loop to zero giving and equation of lambda. The

5 smallest positive roots of this equation yielded  $R_0$ .

$$R_0 = \frac{q\beta}{\mu_{12}} \prod_{i=1}^{11} \left( \frac{\gamma_i}{\gamma_i + \mu_i} \right)$$

Furthermore, when there are 2 sub stages in each stage (that is k=2), the initial graph is given as follows:



18



17 Using Rule 1, this can be reduced as follows.

-γ<sub>11</sub> 19 -γ<sub>1</sub> **-**γ<sub>1</sub>  $-\gamma_2$  $-\gamma_{11}$  $(\gamma_1 + \mu_1)$   $(\gamma_2 + \mu_2)$   $(\gamma_{11} + \mu_{11})$   $(\gamma_{11} + \mu_{11})$  $(\gamma_1 + \mu_1)$ 20 21  $n_1$  $n_2$ n<sub>11</sub> n<sub>12</sub>  $n_1$ 22 23 24  $\left(\frac{q\beta\lambda^{-1}\gamma_{12}}{\mu_{12}} + q\beta\lambda^{-1}\right)\frac{1}{(\gamma_{12} + \mu_{12})} = \frac{q\beta\lambda^{-1}}{\mu_{12}}$ 25 26

27 It finally yields,

-1

1 
$$-1 + \frac{q\beta}{\mu_{12}} \prod_{i=1}^{11} \left(\frac{\gamma_i}{\gamma_i + \mu_i}\right)^2 \lambda^{-1} \quad \boxed{n_{12}}$$

2 Thus, 
$$R_0 = \frac{q\beta}{\mu_{12}} \prod_{i=1}^{11} \left(\frac{\gamma_i}{\gamma_i + \mu_i}\right)^2$$

Similarly, for any k sub stages, it yields,  $R_0 = \frac{q\beta}{\mu_{12}} \prod_{i=1}^{11} \left(\frac{\gamma_i}{\gamma_i + \mu_i}\right)^k$ 3

- The same result can be easily derived from  $R_0 = \rho [FV^{-1}]$  also. 4
- 5

#### 6 **Appendix B: General solution for**

7

12

where,

8 We obtained the following general solution for , the proportion of 9 individuals in a given stage *a* at time *t* in Eq.1:

such that

,

for any stage *i* and  $\sigma_{ij} = (\sigma_i - \sigma_j)$ , and is a row vector of dimension 1x(a-1) of the form  $\underline{b}_a = \prod_{j=1}^{a} B_j$ , j=1,...,a, where,  $B_j$ 13

and

14 matrices are non-square matrices such that,

15 
$$B_4 = \begin{bmatrix} \sigma_{41}^{-1} & 0 & \sigma_{43}^{-1} \\ 0 & \sigma_{42}^{-1} & \sigma_{43}^{-1} \end{bmatrix}$$
,  $B_5 = \begin{bmatrix} \sigma_{51}^{-1} & 0 & 0 & \sigma_{54}^{-1} \\ 0 & \sigma_{52}^{-1} & 0 & \sigma_{54}^{-1} \\ 0 & 0 & \sigma_{53}^{-1} & \sigma_{54}^{-1} \end{bmatrix}$ , and so on. The general formula for  $B_k$ 

16  $(k \ge 3)$  can be written as,

17 
$$B_{k} = \begin{bmatrix} \sigma_{k1}^{-1} & 0 & : & 0 & 0 & \sigma_{kk-1}^{-1} \\ 0 & \sigma_{k2}^{-1} & : & 0 & 0 & \sigma_{kk-1}^{-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & : & \sigma_{kk-3}^{-1} & 0 & \sigma_{kk-1}^{-1} \\ 0 & 0 & : & 0 & \sigma_{kk-2}^{-1} & \sigma_{kk-1}^{-1} \end{bmatrix}_{(k-2)\times(k-1)}$$

(2)

- 1 Note that due to the dimensions of the  $B_j$  matrices, the product  $\underline{b}_a = \prod_{j=1}^{a} B_j$  is a vector.
- 2 We define the vector to be a column vector of the form,

 $3 \qquad \underline{V}_{a} = \begin{bmatrix} e^{-\sigma_{1}t} - e^{-\sigma_{a}t} \\ e^{-\sigma_{2}t} - e^{-\sigma_{a}t} \\ e^{-\sigma_{3}t} - e^{-\sigma_{a}t} \\ \vdots \\ e^{-\sigma_{a-1}t} - e^{-\sigma_{a}t} \end{bmatrix}_{(a-1)x1}$ 

4

5

# Appendix C: Analysis of the case with constant mortality amongst stages

6

7 To see that the assumption of equal mortality at each stage cased the mortality 8 rates in Eq. 2 to cancel out mathematically, consider the case where each  $\mu_i$  is a 9 constant  $\mu$  in our solution Eq.2. Then note that in Eq.2, becomes independent of  $\mu$ , and as a result  $\underline{b}_a$  also becomes independent of  $\mu$ . Further, in  $\underline{\nu}_a$ ,  $(e^{-\sigma_t t} - e^{-\sigma_a t})$ 10 can be written as  $e^{-\mu t} (e^{-\gamma_i t} - e^{-\gamma_a t})$  for each element *i*. Thus, in the dot product 11  $(\underline{b}_a, \underline{v}_a)$  in the Eq.2, the term  $e^{-\mu t}$  can be separated out as a multiplier, and after 12 redefining,  $n_a(t) = e^{-\mu t} \left(\prod_{i=1}^{a-1} \gamma_i\right) (\overline{\underline{b}}_a \cdot \overline{\underline{v}}_a)$ , such that  $\operatorname{term} \left(\prod_{i=1}^{a-1} \gamma_i\right) (\overline{\underline{b}}_a \cdot \overline{\underline{v}}_a)$  becomes 13 independent of  $\mu$ . i.e.  $\underline{b}_a = \underline{b}_a$  and  $\underline{v}_a = \underline{v}_a$  for the special case where 14 for all 15 stages *i*. Now, we can write the proportion of each stage *a* that remains at time *t*,

16 
$$z_a(t)$$
, with respect to the total population at t

17 
$$z_a(t) = n_a(t) / \sum_{i=1}^s n_i(t) = \left(\prod_{i=1}^{a-1} \gamma_i\right) (\underline{\bar{b}}_a, \underline{\bar{v}}_a) / \sum_{j=1}^s \left(\prod_{i=1}^{j-1} \gamma_i\right) (\underline{\bar{b}}_j, \underline{\bar{v}}_j)$$

18 where, *s* is number of stages. Thus, this equation is independent of  $\mu$ . The numerator 19 of this equation is  $n_a(t)$  for the case where  $\mu_i = 0$  for all stages for any *t*. The

20 denominator is the solution to 
$$\sum_{i=1}^{s} n_i(t)$$
 for the special case where  $\mu_i = 0$  for all stages

at any *t* if the population starts from 1 egg, thus remains 1 at any *t*. Hence, this can be
simplified, so that,

- 3 (3)
- 4 which, is equivalent to  $z_a(t) = n_a(t)$  when  $\mu_i = 0$  for all stages at any *t*. Therefore,  $z_a(t)$ 5 can be equated with the stage sizes normalized at each time step *t* in experimental data 6 found in the literature which makes the assumption that  $\mu_i = \mu$  for all *i*=1 to *s*.
- 7

#### 8 Appendix D: Fitting Eq. 4 data using multiple substages

9

To derive solution to the modified system of equations in Eq.1 by adding *k*sub-stages to each stage required using Laplace transformations. It yielded a

12 complicated analytical result. Instead, we modified Eq. 4 to include sub-stages within

13 stages, by assuming small differences in maturation rates among sub-stages. However,

14 the solution in Eq.4 cannot be simply transformed into a general case for the system to

15 have multiple sub-stages, because in such case the denominator of the solution in Eq.4

16 becomes zero, mathematically, as when *i* and *j* were redefined for sub-stages

17 in each stage, such that . Therefore, we implemented the sub-stages for a

18 given stage *a* by adding and subtracting a small constant () to such that

19 For example, separating into three sub-stages would involve splitting among the

20 three sub-stages, such that maturation rates were . Then we

- estimated using the modified Eq.4 fitting to data from Uye et al. (1983) for small values of  $\varepsilon$ .
- 23

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# 1 <u>Tables</u>

2 3

4

# Table 1 Meaning of mathematical notations

# Notation Description

	number of eggs
<i>n</i> <sub>2</sub> <i>n</i> <sub>6</sub>	number of individuals in the five naupliar stages
$n_7n_{12}$	number of individuals in the five copepodid stages
	Fertility rate (rate of egg production) in adult females as a function of
	temperature
q	average proportion of ovigerous females in the adult population, assumed
	to be a constant value of 0.61 (Liang and Uye 1997b)
	rate of mortality in stage <i>i</i> as a function of temperature
	rate of maturation of individuals surviving to stage <i>i</i> as a function of
	temperature
A	12x12 linear matrix composed of maturation, mortality and fertility rates,
	such that $dn(t)/dt = An(t)$ , where <i>n</i> are vectors of stage classes
Т	temperature
$R_0$	net reproductive rate
$f_m$	maximum rate of fertility
$f_l$	fertility at the lowest temperature
W	shape parameter that accounts for the depression in fertility rate at low
	temperatures
b	lag parameter to relax the assumption that the fertility rate curve
	otherwise intercepts y-axis at the origin
$z_a(t)$	proportion of individuals at each stage a
$d_a$	stage (a) duration times random variable
$\overline{d}_{a}$	mean stage (a) duration times

$D_a$	stage (a) development time distribution
$\overline{D}_a$	mean stage (a) development times
	constant that varies with stage $a$ in maturation function of temperature
	where derived from Belehradek's function
$\phi$	scale parameter in $Sv = \exp(-\phi a^{\chi})$
χ	shape parameter in $Sv = \exp(-\phi a^{\chi})$
К	are parameters from mortality as a quadratic function of temperature
	$\mu(T) = \kappa_2 T^2 + \kappa_1 T + \kappa_0$

2
3 Table 2 Model comparisons for cases *k*=1, 2 and 3 in Eq.4.

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14

	Model	I RSS LL (I		(LL/LLmax)	Chi <sup>2</sup>	Deg	AIC	ΔΑΙΟ	<i>p</i> -of Chi <sup>2</sup>
-	k=3	0.57	120.56	0.00	0.00	14	-213.12	0.00	
	k=2	1.04	108.83	-11.73	23.45	13	-191.67	21.45	1.28E-06
	k=1	1.88	97.29	-23.27	46.54	12	-170.58	42.54	7.83E-11

4	**LL-Log likelihood, LLmax-Maximum Log likelihood
4	LL-Log Inkelilloou, LLillax-Maxillulli Log Inkelilloou
5	
•	

		Stage	Stage	
Stage		duration	Development	
		time	time	
		(days)	(days)	
e	3.64	0.27	-	55.01
n2	2.53	0.40	0.67	134.21
n3	1.05	0.96	1.63	325.81
n4	0.87	1.16	2.78	557.40
n5	0.65	1.53	4.31	864.01
n6	0.81	1.23	5.54	1110.7′
c1	0.54	1.84	7.39	1479.68
c2	0.58	1.73	9.12	1827.22
c3	0.60	1.66	10.78	2159.64
c4	0.40	2.48	13.26	2656.8
c5	0.29	3.48	16.74	3353.02
c6	-	4.84	21.57	4321.70

**Table 3** Stage maturation, duration, and development rates at  $20^{\circ}$ C, and coefficient 2 calculated for each stage *a* for *k*=1.

**Table 4** Estimation of  $\phi$  and  $\chi$  in  $Sv = \exp(-\phi a^{\chi})$  at different temperatures

Temp								
( <sup>0</sup> C)	10.60	14.30	16.70	20.20	21.50	22.30	25.60	27.40
$\phi$	0.02	0.01	0.00	0.00	0.13	0.53	0.00	0.10
χ	2.69	2.26	7.87	2.93	1.43	0.94	29.24	1.56
RSS	0.05	0.11	0.09	0.02	0.06	0.01	0.09	0.03

<sup>6 \*\*</sup>RSS-Residual sum of squares

# 1 Figures

2

3



4

5 b) Linear model



Fig. 1 Rate of fertility of adult females at different temperatures comparing sigmoidal
model with linear model by Uye et al. (1983). Dashed lines indicate 95% confidence
intervals.

a=12

a=1

Proportion not yet past a given stage



3 Fig. 2 Proportion of individuals in the population not yet past a given stage *a* 

4 obtained by fitting Eq.4 to data from Fig.2 in Uye et al (1983). Solid lines are the fits

```
5 for k=1, dashed lines are the fits for k=3.
```



# Temperature $(^{0}C)$

H Rajakaruna, CA Strasser and MA Lewis. 2012. Identifying non-invasible habitats for marine copepods using temperature-dependent R0. Biological Invasions 14:633-647. doi:10.1007/s10530-011-0104-x.





1

2 Fig. 4 Quadratic model of daily mortality rates as a function of temperature,

3 estimated for data where all stages are pooled. Parameter values for mortality rate

4 model are 
$$\kappa_2 = 0.0022$$
 /day,  $\kappa_1 = -0.0563$  /<sup>0</sup>C day,  $\kappa_0 = 0.4211$  /<sup>0</sup>C<sup>2</sup> day

5



6 7

8 Fig. 5  $R_0$  plotted as a function of temperature (T) for the cases where k=1

9 (exponentially distributed stage duration times), and k=3 (gamma distributed stage





2

**Fig. 6** Range of potentially invasible habitats [from  $11^{\circ}$ C to  $23^{\circ}$ C] by *P. marinus* as 3 predicted by our model based on  $R_0(T) > 1$  for sea surface temperature (T) data 4 5 averaged from year 1971-2000 through NOAA interactive database. Dots are the 6 habitats where P. marinus was collected or has established.

7

8 [References are from Fleminger and Kramer (1988) except \*: (A) West coast 9 of Hokkaido, Japan, Sato (1913), Sato Anraku (1953), Walter (1986b); (B) Qing-Chao and Shu-Zhen (1965); (C) Andaman Islands (Pillai 1980); (D) Mauritius (Grindley 10 and Grice 1969); (E) Moreton Bay, Queensland (Greenwood 1977); (F)\* Patagonian 11 12 Waters, Southern Chile (Jones, 1966; Grindley and Grice, 1969) from Hirakawa 13 (1986); (G) Oahu, Hawaii (Jones 1966) (Carlton 1985)\*; (H)\* San Francisco Bay, 14 California (Ruiz et al. 2000); (I) Peter the Great Bay (Brodsky 1948, 1950); (J) Chiba 15 (1956), Tanaka (1966), Tanaka and Huee (1966), Walter (1986b); (K) Brodsky (1948, 1950); (L)\* Elliot Bay, Puget Sound, Washington (Cohen 2004), USGS; (M) USGS; 16 17 (N) Shen and Lee (1963).]