

# CANADIAN THESES ON MICROFICHE

I.S.B.N.

# THESES CANADIENNES SUR MICROFICHE



National Library of Canada  
Collections Development Branch

Canadian Theses on  
Microfiche Service

Ottawa, Canada  
K1A 0N4

Bibliothèque nationale du Canada  
Direction du développement des collections

Service des thèses canadiennes  
sur microfiche

## NOTICE

The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us a poor photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this film is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30. Please read the authorization forms which accompany this thesis.

THIS DISSERTATION  
HAS BEEN MICROFILMED  
EXACTLY AS RECEIVED

## AVIS

La qualité de cette microfiche dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de mauvaise qualité.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, examens publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de ce microfilm est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30. Veuillez prendre connaissance des formules d'autorisation qui accompagnent cette thèse.

LA THÈSE A ÉTÉ  
MICROFILMÉE TELLE QUE  
NOUS L'AVONS REÇUE

0-315-06037-9



National Library of Canada

Bibliothèque nationale du Canada

Canadian Theses Division

Division des thèses canadiennes

Ottawa, Canada  
K1A 0N4

53957

**PERMISSION TO MICROFILM — AUTORISATION DE MICROFILMER**

• Please print or type — Écrire en lettres moulées ou dactylographies

Full Name of Author — Nom complet de l'auteur

JOEL OSCRETE KING

Date of Birth — Date de naissance

JULY 14, 1963

Country of Birth — Lieu de naissance

ST. VINCENT, WEST INDIES.

Permanent Address — Résidence fixe

RICHLAND PARK  
ST. VINCENT, WEST INDIES.

Title of Thesis — Titre de la thèse

POWER SYSTEM WORTH ANALYSIS - A PROPOSED  
METHOD.

University — Université

UNIVERSITY OF ALBERTA

Degree for which thesis was presented — Grade pour lequel cette thèse fut présentée

MASTER OF SCIENCE

Year this degree conferred — Année d'obtention de ce grade

1981

Name of Supervisor — Nom du directeur de thèse

R. E. RINK

Permission is hereby granted to the NATIONAL LIBRARY OF CANADA to microfilm this thesis and to lend or sell copies of the film.

L'autorisation est, par la présente, accordée à la BIBLIOTHÈQUE NATIONALE DU CANADA de microfilmer cette thèse et de prêter ou de vendre des exemplaires du film.

The author reserves other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without the author's written permission.

L'auteur se réserve les autres droits de publication; ni la thèse ni de longs extraits de celle-ci ne doivent être imprimés ou autrement reproduits sans l'autorisation écrite de l'auteur.

Date

July 9, 1981

Signature

Joel King

THE UNIVERSITY OF ALBERTA

Power System Worth Analysis: A Proposed Method

by



Joel Oscrete King

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF Master of Science

Department of Electrical Engineering

EDMONTON, ALBERTA

Fall, 1981



THE UNIVERSITY OF ALBERTA  
FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Power System Worth Analysis: A Proposed Method submitted by Joel Oscrete King in partial fulfilment of the requirements for the degree of Master of Science.

*R. E. Rink*.....

Supervisor

*Alan Kaval*.....

*J. Sprague*.....

Date... *June 16, 1981*.....

## Abstract

Properly fitting a power system to the society it serves is an exceedingly difficult and important problem. Service reliability is an important factor in this fit. It would be ideal if electricity service was never interrupted. But 100% reliability would be prohibitively expensive to attain, if indeed it could be attained, and utility rates would skyrocket. The objective of system planners is to achieve a balance between the cost of outages to consumers and the utility, and the cost of greater reliability.

~~It is relatively easy for the system planner to evaluate incremental costs of improvements. The social worth of such improvements, however, is difficult to evaluate.~~  
This thesis proposes a method for computing the worth to consumers of various levels of system reliability. An equation is derived relating social worth with an index of reliability, for a specific power system.

### Acknowledgements

The author wishes to gratefully acknowledge the encouragement and guidance given by Dr. R. Rink in preparation of this thesis, and to Dr. D. Koval for some useful discussions on the application of the concepts. The research was carried out in the Department of Electrical Engineering at the University of Alberta. The financial assistance provided by the department is gratefully acknowledged.

---

## Table of Contents

Chapter	Page
1. Introduction .....	1
1.1 The Reliability Problem .....	1
1.2 The High Cost of Construction .....	3
1.3 Consumer Problems .....	5
1.4 Fuel Supplies as a Constraint in System Planning ..	6
1.5 Need for Utility Management .....	7
1.6 The Cost-Benefit Problem .....	9
1.7 Value of Reliability .....	10
1.8 Summary of Outage Costs Obtained in the Past .....	16
1.9 Scope of the Thesis .....	20
2. Proposed Method for Reliability Worth Evaluation .....	21
2.1 Framework For Defining Social Worth .....	21
2.1.1 Indifference Curves [36] .....	21
2.1.2 Definition of Social Value of Electric Power Service .....	24
2.1.3 Definition of Reliability .....	27
2.2 Basic Assumptions .....	28
2.3 Distribution of Service Reliability over a Network .....	32
2.4 Evaluation of Marginal Reliability Costs .....	34
2.5 Construction of Marginal Social Value Curves .....	38
2.6 Final Social Value Curve .....	40
3. Reliability Model .....	42
3.1 Indices of reliability .....	42
3.2 Methods of Generation System Reliability Evaluation .....	43



3.2.1	Monte Carlo Techniques	43
3.2.2	Loss of Load Probability Approach	44
3.2.3	Frequency and Duration Methods	45
3.3	Transmission System Reliability Evaluation	45
3.4	Composite System Reliability Evaluation	47
3.5	Reliability model used in this Thesis	49
3.6	Assumptions and General Considerations	52
3.7	Mathematical Details	54
3.8	Digital Computer Program	57
3.8.1	Features of The Program	58
3.9	Limitations of the Program	60
3.10	Application to System Models	63
4.	System Application	70
4.1	System Description	70
4.2	Marginal Social Value Curves	73
4.3	Final Social Value Function	77
5.	Comments and Conclusions	80
5.1	Analysis of Results	80
5.1.1	Social Value Curve	81
5.1.2	Social Worth in \$/kw	83
5.2	Advantages of the Proposed Method	84
5.2.1	Advantages over "Direct Methods"	85
5.2.2	Advantages over the use of GNP	86
5.3	Projections for Further Research	88
5.4	Summary	92
	Bibliography	94
	Appendix	102

Appendix II. ....105  
Appendix III. ....109

## List of Tables

Table		page
1-1	Cost of Interruptions to Residential Consumers in Sweden	17
1-2	Average cost of Interruptions to Industrial plants in U.S.A. and Canada	17
2-1	Long-Run Elasticities of Electricity Demand	27
3-1	Reliability Indices for System 1	64
3-2	Reliability Indices for System 2	65
4-1	Outage Data	73
4-2	Effect of improvements	73
4-3	Values of S.V.	79
III-1	Detailed Outage Data for System 2	109

## List of Figures

Figure		Page
1-1	Value of Electric Utility Stocks for the years 1965 to 1974	5
1-2(a) and (b)	Diagrams showing the results of Shipley et al	12
1-3	Ontario Hydro user estimates of interruption costs - large users (5mw+)	18
1-4	Summary of interruption duration versus costs for various classes of consumers	19
2-1	Typical consumer indifference curves	22
2-2	Budget line of typical consumer	24
2-3	Possible shape of the reliability distribution curve	33
2-4	Possible effect of an improvement	35
2-5	Possible shape of curve showing marginal cost as a function of per unit power level	36
2-6	Possible shape of curve showing marginal cost <i>per unit power flow</i> as a function of power level.	39
2-7	Possible shape of curve showing marginal cost per unit power flow, as a function of <i>failure probability</i> .	41
3-1	Result of reliability analysis of system 1 (IEEE 14-bus test system)	68

3-2	Result of reliability analysis of system 2.	69
4-1	Schematic diagram of 37-bus system	71
4-2	Load duration curve of 37-bus system	72
4-3	Plot of $ (\Delta S/\Delta F.P.)/\Delta f $ as a function of $f$	74
4-4	Plot, of $ (\Delta S/\Delta F.P.)/\Delta f $ as a function of failure probability	76
4-5	Social value of electric service reliability as a function of failure probability	78
5-1	Diagram showing how several power systems might possibly have social value curves lying within two bounds.	90
5-2	Diagram showing possible deviation of the social value curve of a system from within acceptable bounds.	91

## 1. Introduction

### 1.1 The Reliability Problem

Power utilities are finding themselves in a position where they are called upon to provide a high quality product upon which the consumer is becoming increasingly dependent[26]. At the same time, an unprecedented array of serious problems have combined to make the job of the long-term utility planning engineer an extremely difficult one. By 1975, increased costs of new generating capacity, double digit inflation, delays in the regulatory process, forced conversion to coal-fired boilers, shortages of fuel, siting delays, new environmental regulations, rising cost of debt, falling stock prices and sinking bond ratings have forced the utilities into their worst position in forty years. The situation has become so bad that the popular press carried several articles with titles such as: "UTILITIES: WEAK POINT IN THE ENERGY FUTURE" [26], and "FOR THE UTILITIES, IT'S A FIGHT FOR SURVIVAL"[27]. Business Week reported:

"Pressed to the wall by an ever-tightening cash squeeze, power companies are hacking away at both operating and construction budgets to find a way out. However unavoidable, these expediences are laying the groundwork for an even worse problem later this decade-a chronic power shortage that may completely overshadow the sporadic blackouts and brownouts of recent years, with devastating effects on the whole economy." [27]

Although the industry's prospects have improved somewhat[38], analysts are still warning that the industry still is not yet "out of the woods". As the industrialized countries attempt to achieve some degree of energy self-sufficiency, electricity will have to take on a bigger and bigger role. Even though conservation measures and high prices may dig sharply into the growth of electricity demand, the power industry in the United States must still bring on line in the nineteen eighties as much generating capacity as it has in the last decade[27]. Most energy planners say that the demand for electricity will grow faster than energy demand as a whole, averaging perhaps 4-6% [27]. In the short run, they point out, electricity is the best way to distribute the energy locked in natural reserves of coal and uranium, and thus lessen dependence on foreign oil. In the long run it will be the chief means of distributing energy from solar, wind, geothermal and nuclear fusion plants. The U.S. Federal Power Commission predicts that by the year 2001, more than half of all energy consumed by end-users will be in the form of electricity[26]. Major segments of the economy now fueled chiefly by petroleum, such as transportation and space heating, will more and more be taken over by electricity.

During the hard times of the seventies, the utilities had severely slowed down or postponed construction of new facilities. The Virginia Electric & Power Company predicted that by the early 1980's the company would have only a 5%.

capacity reserve, which is about one-third the back-up that is considered necessary to avoid shortages. Says Allen B. Wilson, financial vice president of Georgia Power: "The greatest blessing now would be to be in a state where demand is static"[27].

## 1.2 The High Cost of Construction

Most of the blame for this sad state of affairs has been attributed to the high inflation rate in construction wages, which has steadily raised expenses relative to income. Assuming a moderate growth rate of 6%, the utility would have to double plant facilities every twelve years. For any industry this would be expensive, but the utilities are in a particularly difficult position. They have always been capital intensive, and therefore catch the full effects of inflation and the blown-up interest rates that go with it. They have been severely hit by the rampant inflation in construction wages.

Beyond that, too, the utilities have been locked into heavy spending to protect the environment[27]. And before the Three-Mile-Island incident they had made a big move toward nuclear facilities, which involve very high capital costs (though low operating costs). The industry today is the most capital-intensive around, getting \$1 of revenue for every \$4 of investment, compared with \$1 of investment for the steel industry, which itself is considered highly



capital intensive. This means that, with 6% growth over the next fifteen years, the industry would have to somehow muster \$650 billion for construction, compared to the \$145 billion spent from 1960 to 1975[27].

For most investor-owned utilities, profits are a fixed percentage of the amount of capital invested in power plants and equipment [27]. But due to the lag between the time when utilities spend money on new plant and the day when rate increases are granted by regulatory agencies, utilities find more and more of their capital tied up and not yet returning income. Several state regulatory commissions in the United States allowed the companies to show income from capital tied up in construction, but this only improved the balance sheet without doing anything for real earnings or cash flow. Accelerated depreciation and investment tax credits helped to bolster earnings reports, but served to disguise the utilities' very real cash flow problems. When these ills could no longer be hidden, investor confidence plummeted, until, at one time, some companies were no longer able to obtain long-term investments to finance construction. Figure 1-1 shows the decline in market price of electric utility stocks compared with their book value[27].

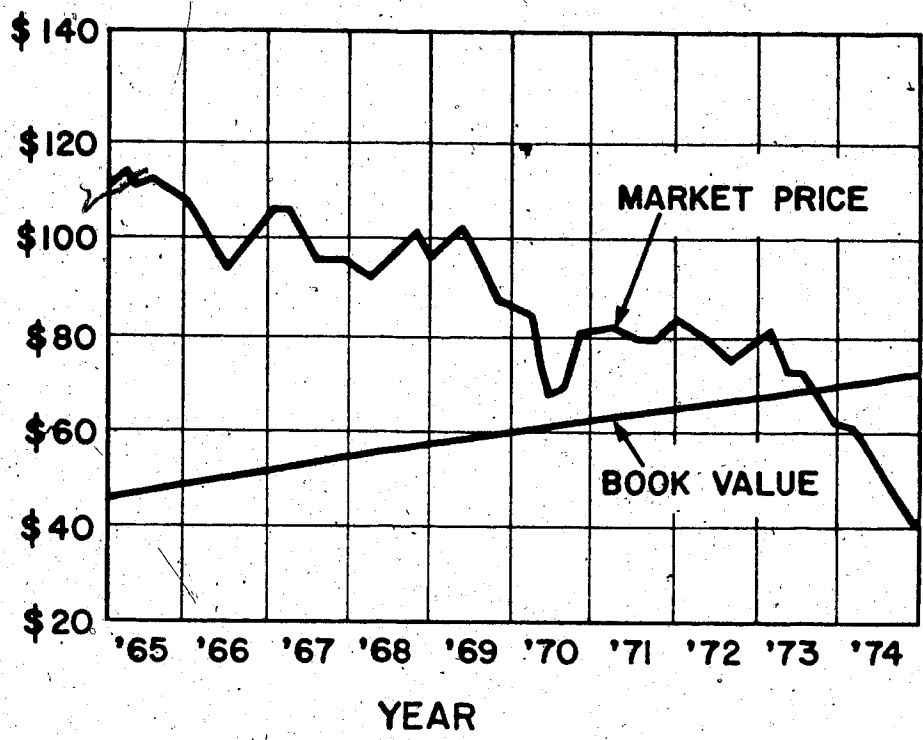


Figure 1-1. Value of Electric Utility Stocks for the years 1965 to 1974.

(Source: see reference 27)

### 1.3 Consumer Problems

When utilities drop plans for new base nuclear or coal-fired plants, they usually use gas turbines to meet the demand in three or four years. Gas turbines are cheaper to build but use natural gas as fuel. The result is high fuel costs, which the utilities count on passing on to consumers through automatic fuel adjustment clauses. However, consumers can "flex a lot of muscle" at rate hearings. They usually regard a rate increase as a "rip-off". Facing such public pressure, regulatory commissioners, who in most

countries are elected officials or political appointees, sometimes trim rate requests beyond reason. In California, where utilities sell more kilowatt-hours than in any other state, the pressure is particularly great. In 1975, all of the utilities were forced to sell stocks at prices below book value, and the rate commission was not able to come up with rate increases to correct the situation. Vernon Sturgeon, president of the state commission, says that consumers employ "every device they know how to use to keep rates down - and there are some pretty smart guys out there"[27].

Since the Three-Mile-Island nuclear incident, anti-nuclear groups have found renewed strength in voicing their concerns over problems associated with transportation of nuclear fuel, construction and operation of nuclear reactors and spent fuel storage and disposal. If such opposition continues, then the relative contribution of this energy source might decline in the future[39].

#### 1.4 Fuel Supplies as a Constraint in System Planning

An overall view of the nuclear fuel cycle indicates that there are no problems in the immediate future concerning nuclear fuel supplies to utilities. However, due to the effect of the Three-Mile-Island incident on public opinion, the importance of this fuel source may decline in the future. Oil and natural gas would be in even more

limited use due to high prices and restrictions in the use of natural gas. The major source remaining is coal.

Coal is the largest energy source known in North America. Despite its abundance, the contributions of coal in the energy economy have been limited. Mining and transportation of coal to utilities are the predominant barriers for coal supplies (i.e. mines remote from markets). The environmental problem posed by sulphur dioxide in stack emissions from coal-fired boilers is also a serious one, and with the assured increase in the use of this fuel, problems with acid rain and the resulting pollution of rivers and lakes, along with federal pressure, are bound to play an important part in the decisions of utility planning engineers, executives and regulators.

### 1.5 Need for Utility Management

The problems that have been highlighted above have produced fears of major power shortages in the 1980's. Even though these fears may not be realized, it is virtually certain that projections of electricity demand and generating capacity will exceed or fall short of actual demand and capacity. In either case, costs are incurred both by consumers (in outage costs or higher rates), and the utility (in excess expenditure or loss of sales), which can perhaps be avoided by more sophisticated planning techniques. Many analysts are now realizing the need for

more careful utility management, and the National Association of Regulatory Utility Commissioners in the U.S. is now backing a study of utility management[26].

When a utility plans to have a supply of power available to meet the load, it does so in three time frames, namely, a short time frame (daily and next day of dispatching), a mid-time frame (seasonal and annual), and a long time frame (from two to as long as fifteen or twenty years). It is this long-term planning that we are concerned with here, and it is at this level that decisions affecting the construction of new generating capacity are made. It is here also that decisions are made that affect the overall financial picture of the utility and the prospects for possible capacity shortages in the future.

Long-range planners have at their disposal sophisticated models of the utility which are used to plan capacity additions based on a load forecast and a measure of system reliability. A description of some of these reliability models is given in Chapter 3. Despite the sophisticated models, capacity reserve margins (i.e. reserve generation capacity, fuel supplies, and transmission/distribution capacity) are still to a certain extent based on rule-of-thumb criteria. Comparisons between different system configurations are possible, but the problem of fitting a power system to the needs of its customers is still an unsolved one. In fact, present electricity service to certain customers may actually be more reliable than

necessary. The schedule of additions to the system must therefore be optimized in the sense of cost versus benefit.

### 1.6 The Cost-Benefit Problem

Probability methods have gained widespread use in generation system planning. However, despite refinements to loss of load measures by recognition of additional indices such as frequency and duration of occurrence, and magnitude of lost load or lost energy, there is a hesitancy to use these techniques in overall system expansion planning[40]. Comparisons between different system configurations may be possible, but the problem of reliability optimization has still not been adequately addressed. A typical question in system planning is: "Why did you select a loss of load probability of one day in ten years for planning your generation reserve?". Answers to questions like this are so far based mainly on experience[40].

Selection of an optimum standard of service reliability requires evaluation of: (1) the economic value to the service area of the increase in the average amount of annual energy that will be supplied as the result of system improvements, and (2) the associated increases in the annual costs. The optimum reliability level for a load point occurs when the incremental value to the service area of a marginal improvement in reliability of supply is exactly equal to the incremental annual cost to the utility company of providing

the improvement. This is the most meaningful standard of service reliability obtainable.

### 1.7 Value of Reliability

It would be most helpful to be able to claim an equivalent dollar value for a specific improvement in load point reliability. It would be possible to determine the economic merit of proposed design or operating procedure changes which are intended to improve reliability performance. This would permit optimization of reliability performance based on obtaining minimum total (capital plus operating) cost plus cost of service interruptions.

Even though the worth may be extremely difficult to determine, system reliability does have an economic value. Several ways of approaching the problem are possible. One approach considers the cost of interruptions from the viewpoint of the power customer. The cost of interruption is likely to vary widely depending on the type of consumer. Industrial consumers may suffer losses due to lost production, spoiled product, equipment damage, idle production facilities and labour, etc. Domestic consumers may suffer losses which depend on the frequency and duration of the outage. Other losses to the society in general may be in the form of discomfort, inconvenience, lack of safety and disruption of public order. Some of these losses may be easy to evaluate, whilst others are intangible losses which are

extremely difficult to quantify. The dollar value of these losses is taken as an indication of the worth of the corresponding level of reliability.

A second approach involves the viewpoint of the electric utility. It considers the effect of service interruptions or of total system reliability performance on public goodwill and regulatory or legislative bodies. The utility may also suffer losses in the form of loss of revenue from load not served and loss of potential sales due to adverse consumer reaction[11]. The response of the utility to this reaction takes the form of preventive steps such as further expenditures for maintenance or extra service restoration efforts, and/ or additional equipment and lines. The amount of utility effort depends to a certain extent on public response[37], hence evaluation of the costs of these preventive steps provides a means of quantifying the worth of service reliability.

In this thesis the problem is looked at from a viewpoint which is different from both the above approaches, and involves considerations of both the utility and consumers. Chapter 2 describes this approach in detail, and provides the arguments used to support it.

Several Worth Analyses have been conducted so far, the most popular being that described by Shipley, Patton and Denison[11]. In this method, the value of unserved energy is calculated on the basis of an empirical relationship between the reduction in Gross National Product and the



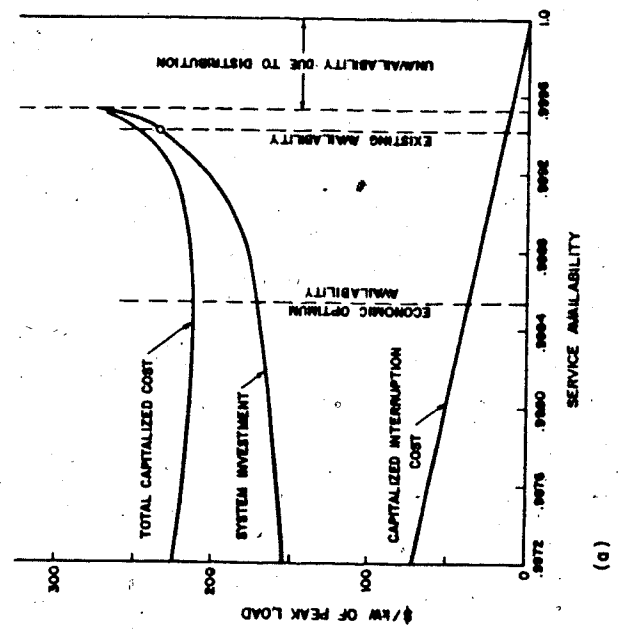
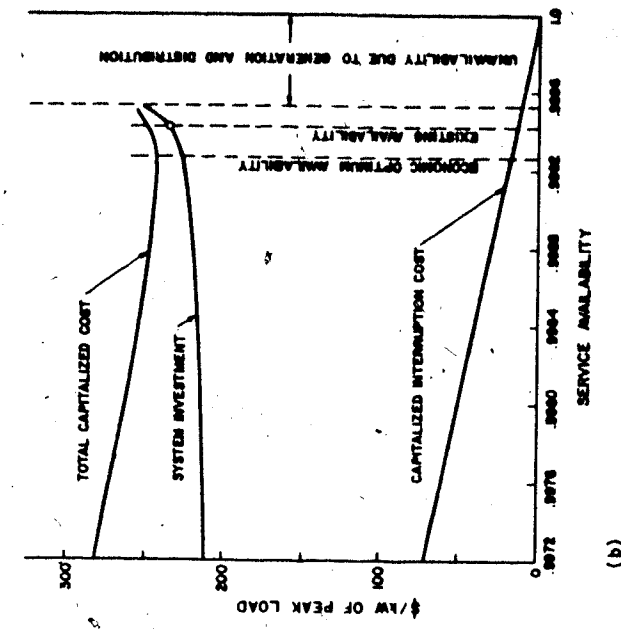


Figure 1-2 (a) and (b). Diagrams showing the results of Shipley et al. (Source: see reference 11).

corresponding kwh interrupted. The assumption is made that the GNP is not only a measure of the total output of goods and services, but that it also reflects the country's comfort, convenience and safety. Using this approach, the capitalized interruption cost is summed with the power-system yearly owning and operating costs, and the total plotted as a function of service availability. Figures 1-2 (a) and (b) show the results for two different ways of calculating system investment.

The economic optimal level of service availability is shown here as the point at which the total capitalized cost is a minimum. This is consistent with what has been stated in a previous section as the most meaningful standard of service reliability obtainable. At the optimum point in Figure 1-2 (a), or (b), it is in fact true that "the incremental value to the service area of a marginal improvement in the reliability of supply is exactly equal to the incremental annual cost to the utility of providing the improvement". Both criteria are identical.

The idea of combining system investment and outage costs into a total cost as a function of unreliability, which can then be used in reliability optimization, seems to be a sound one. The problem, however, lies in obtaining true values for outage costs. It was admitted by the authors of Reference 11 that their approach was quite rough in the way the costs were estimated. The approach was also severely criticized by others as being inadequate. Billinton had this

to say about it: "the use of the GNP to provide an average cost figure does not take into account the individual nature of the system, the region within it, and finally the customer himself. The only redeemable feature about the use of the GNP is that it is easy to find and to place in a formula". (See discussion at end of Reference 11.)

The reliability index used in the method has also been criticized as being unsuitable, and Billinton proposed the use of three indices at the customer bus, namely, average frequency and average duration of outages, and availability of service. He stated that the "cost of an outage to a particular customer is related to all three, and not to just the kilowatt-hour loss aspect of unavailability". (Ibid.) He also pointed out that the technique used in the reliability evaluation of the system was not responsive to the actual factors that influenced reliability, since all outages were not equally likely.

Other studies that have estimated the value of reliability have been similarly criticized. Nearly all of them have relied upon easily obtainable social cost indicators such as Gross Regional or National Product, wages and salaries, or value added by the manufacture[22]. They calculated an aggregate social value of reliability that is insufficient from a policy evaluation standpoint. This is so because of the wide diversity in service areas and associated consumer mixes, and because of wide variation in consumer losses resulting from power interruptions.

Several researchers have used so-called "direct methods" in appraising the worth of reliability to a customer. These methods consist of listing the various losses suffered by consumers during a power outage and valuing them. The major disadvantage with this approach is that it involves asking a consumer to evaluate his losses. Industrial and commercial consumers may be able to evaluate their losses due to loss of production, idle labour, material spoilage, clean-up, etc., quite accurately, and residential customers may be able to quantify their actual out-of-pocket expenses caused by the outage. However, there are certain intangible losses to residential consumers such as fear, inconvenience, and lack of safety, which are extremely difficult to quantify. As Turvey put it:

"The difficulties involved can be readily seen by considering the kind of research which would have to be undertaken in order to provide an answer. It would involve knocking at people's doors and saying 'Give me 1.0 (i.e. a certain amount of money) or I will cut off your electricity'. Then either the money would be taken or the threat carried out! The same procedure would be used for other sub-samples who would be asked for other sums of money. Given a proper sampling design, determination, and total indifference to consumer relations, an estimate of the demand for reliability could be constructed!" [29]

The method that is proposed in this thesis seeks to solve some of these problems. Chapter 5 explains certain advantages which may be obtained by use of the proposed method.

### 1.8 Summary of Outage Costs Obtained in the Past

One of the first comprehensive surveys of consumer losses due to system outages was conducted in Sweden in 1969[30]. The estimates of the costs of interruptions were obtained by direct questioning of various consumer groups. A similar method was used by the Reliability Subcommittee of the Industrial and Commercial Power Systems Committee of the I.E.E.E. in 1973 to estimate the costs of supply interruptions to industrial plants[31]. In 1974 another survey was conducted by Ontario Hydro[42]. The results of these three surveys are given in Tables 1-1 and 1-2, and in Figure 1-3. Figure 1-4 shows the results of these and some other studies that were conducted using direct survey methods.[32] The majority of published studies have been directed at evaluating industrial losses. Residential and commercial loss studies are still being conducted by various governmental agencies in North America and Europe.

Most of these studies have established that the cost of outages is related to the peak capacity and energy demand of the customers, and to the duration of the outage. The cost estimates obtained by the studies were then expressed in terms of two costing components related to the reliability indices of the customer. The first component was proportional to the frequency and magnitude of the load interrupted, i.e. dollars per kw interrupted. The second component was related to the duration of the interruption, and expressed in dollars per kwh.

Table 1-1.  
Cost of Interruptions to Residential Consumers in Sweden

IF THE INTERRUPTION LASTS FOR	FIXED COST AT START OF RANGE	VARIABLE COST WITHIN RANGE
(hours)	(\$US/kw)	(\$US/kwh)
0 - 1	0.00	0.80
1 - 2	0.80	0.80
2 - 8	1.60	0.70
8 - 24	5.80	0.50
24 - 48	13.80	1.20

Table 1-2.  
Average cost of Interruptions to Industrial Plants in U.S.A.  
and Canada

PLANTS	\$/kw	\$/kwh
All Plants	1.89	2.68
Plants with Maximum demand < 1000kw	1.05	0.94
Plants with maximum demand > 1000 kw	4.59	8.11.

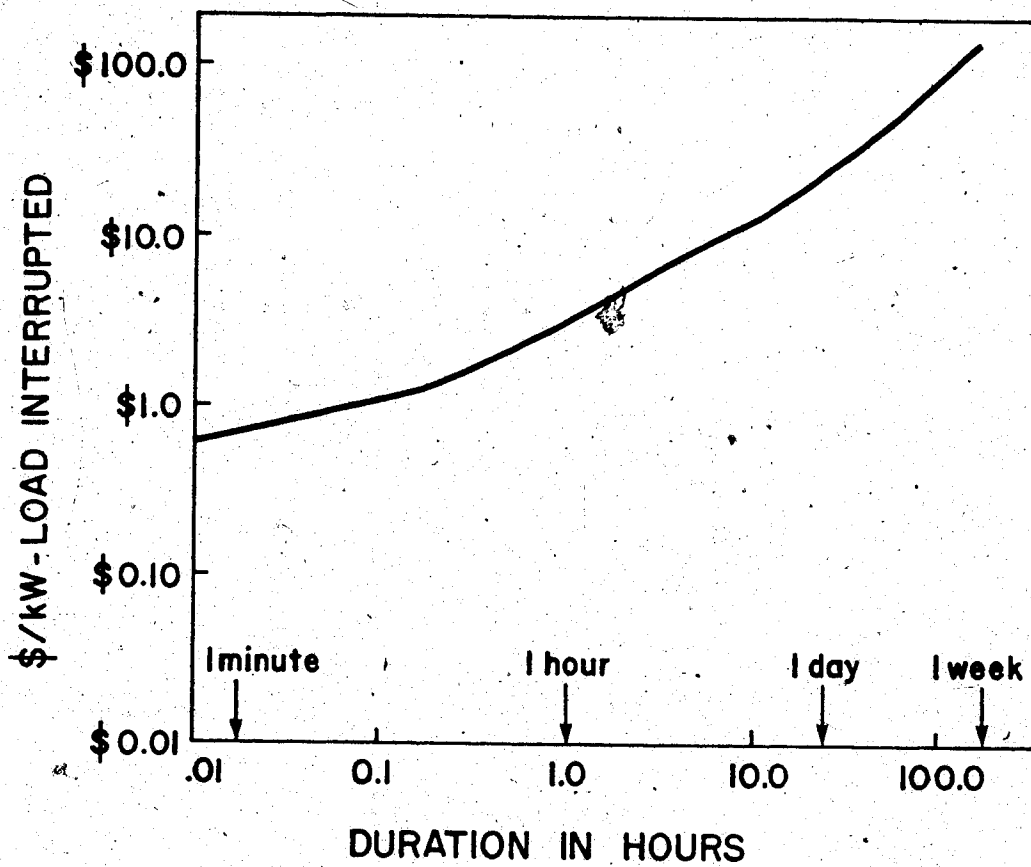


Figure 1-3. Ontario Hydro user estimates of interruption costs-large users (5mw+)  
(Source: see reference 42).

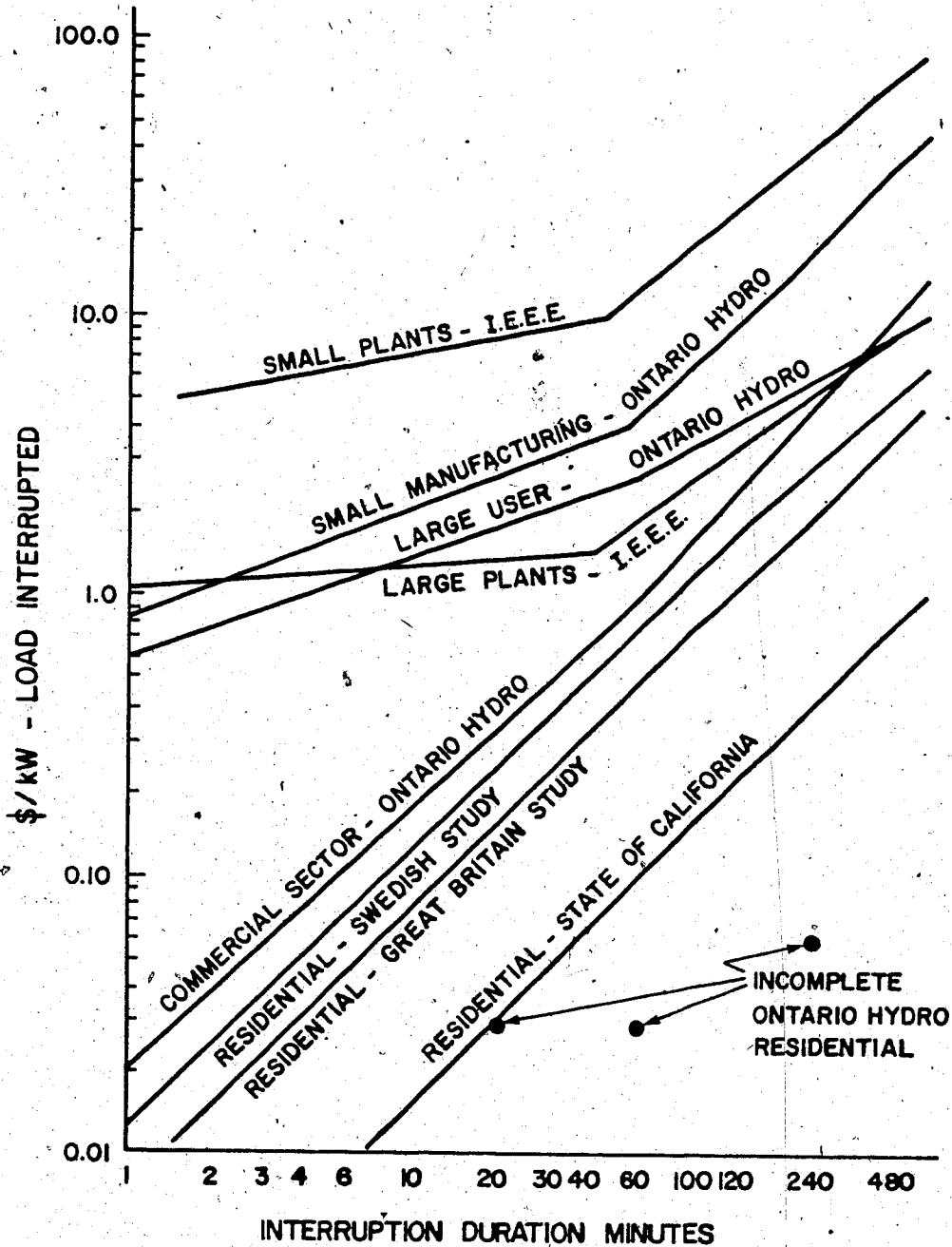


Figure 1-4. Summary of interruption duration versus interruption costs for various classes of consumers. (Source: see reference 32)



### 1.9 Scope of the Thesis

Most power system planners agree that a function to relate reliability or unreliability to true economic worth is sorely needed. A method is described for constructing such a function. This function is in the form of a curve giving social value on the ordinate and unreliability on the abscissa.

It should be noted that the work contained in this thesis is exploratory in nature, and it would be inadvisable at this time to use any specific results obtained here in any cost/benefit analysis.

A complete investigation of the applicability of this idea to power systems in general, and a complete description of the application of the social value function to optimization of a system design is beyond the scope of this thesis. The objective is limited to an examination of the concept of the construction of the social value curve. The natural extension of this research is the use of the social value function in reaching a trade-off between reliability worth and design cost.

## 2. Proposed Method for Reliability Worth Evaluation

### 2.1 Framework For Defining Social Worth

#### 2.1.1 Indifference Curves [36]

In economic utility theory the term "utility" refers to some benefit that accrues to a consumer (i.e. user of the commodity in question), or group of consumers, as a result of the consumption of some good (i.e. the commodity), or set of goods. A given collection of goods is associated with a given level of utility. A collection with a higher level of utility is considered to be always preferable to a collection with a lower level of utility. Level of utility is related to the collection of goods by a "utility function". Figure 2-1 shows curves of constant utility,  $U$ , as a function of two goods, "good1" and "good2". This functional relationship is written in the usual way, as follows:

$$U = F(\text{good1}, \text{good2})$$

A curve of constant utility is usually referred to as an "indifference curve". Any combination of the different goods which produces the same utility is considered

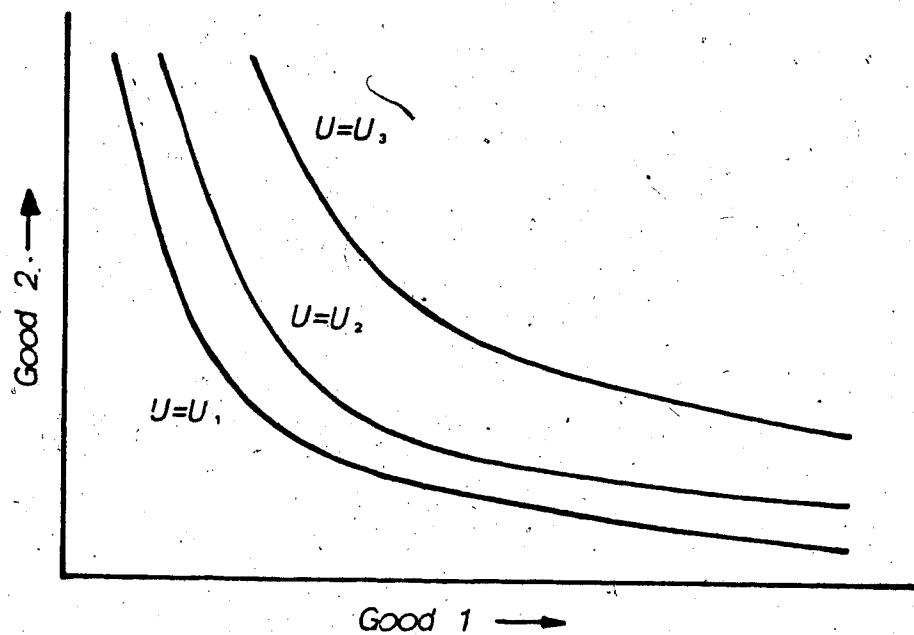


Figure 2-1. Typical consumer indifference curves;  
 $U$  refers to the consumer's "utility".

acceptable to the consumer, hence the term indifference curve. An indifference curve usually slopes downward to the right. An increase in utility,  $U$ , causes it to shift outwards; for example,  $U_3$  is greater than  $U_1$  in Figure 2-1. A set of indifference curves for a particular consumer is referred to as his "indifference map". A consumer's preferences in relation to a set of goods is usually taken as completely described by his indifference map. Once this is available, one can give the consumer's relative preferences between any two combination of goods by simply

noting which one lies on the higher indifference curve.

A consumer is usually considered as being able to choose any collection of goods attainable by him within his budget constraint. Furthermore, the assumption is usually made that he will choose a collection that maximizes his utility with respect to the set of goods in question. In Figure 2-2 the line AB represents the set of combinations of two goods which can be obtained if the consumer's total budget is spent. In other words, it represents the line  $M = p_1q_1 + p_2q_2$ , where M is the total budget,  $p_1$  and  $p_2$  are the prices of the two goods, and  $q_1$  and  $q_2$  are the corresponding quantities of the goods consumed. The particular combination of goods represented by the point C will be chosen, since it is the only point within the consumer's budget that gives him maximum utility.

The line AB is called the budget line of the consumer, and its slope is given by the price ratio between the two goods. If the price ratio were to change, then the point at which the consumer's utility is maximized will also change, and may now lie on a new indifference curve. This is due to the consumer ideally substituting more of the lower-priced good for the higher-priced good to fill his budget and at the same time maximize his utility.

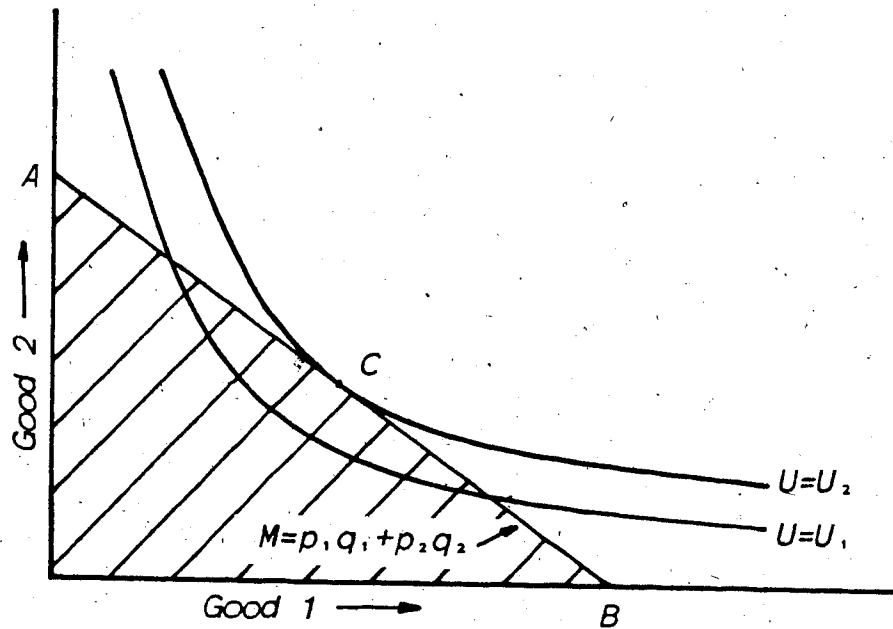


Figure 2-2. Budget line of typical consumer

### 2.1.2 Definition of Social Value of Electric Power Service

Electric service is viewed in this thesis as competing with other forms of energy to satisfy the consumer's total energy-derived utility. Other forms of energy include natural gas for space heating, fuel oil for powering machines run by internal combustion engines, etc. The consumer's utility might be in the form of revenue from an industrial or commercial operation, or in the form of comfort, convenience and safety brought about by use of elevators, street-lighting, room heating or cooling,

cooking, etc.

In this view of the energy market, a consumer's total utility is derived from the use of a combination of the several sources of energy available to him. This aggregate is composed of *contributions* from all the available energy sources. In particular, there is a contribution that is derived from the consumer's use of electrical energy. The magnitude of this contribution may depend on the relative price of electricity, and perhaps on the reliability of service experienced. We therefore define a consumer's social value of his electrical service as the contribution to his total energy-related utility made over a period of time (taken as a year in this thesis) by his electrical service. Social value is measured in dollars per kilowatt of peak capacity demanded, or in terms of dollars per kilowatt-hour of energy consumed.

This thesis uses a "revealed preference" technique to quantitatively estimate the relationship between a consumer's social value of electricity service and the reliability of that service.

The view of the energy service market as one in which different forms of energy compete against each other is supported by two studies into the long-term price-elasticity of electricity demand. The first was done by Nelson[34], and the second by Chapman, Mount and Tyrrell[35]. Nelson made a study of thirty-three Nebraska communities using information obtained over fifteen years. They concluded that there was

an elastic relationship between the average electricity rates to consumers and demand. There was also an increase in total electricity consumption over the fifteen years which was attributed to the increase in electricity-using appliances such as television sets, air conditioning units, and home food freezers which occurred as a function of time. However, this was accounted for in the regression analysis, so that it did not interfere with the determination of the elastic relationship between average electricity rates and demand.

The second study used data accumulated over about twenty years and for 77 cities in nine regions of the U.S. Their results are shown in Table 2-1. They indicate that average price has been the most important determining factor of demand, and that price elasticities range from -1.3 to -1.7. In other words, for residential customers who have a price elasticity of -1.3, a one percent increase in average electricity price to them would in the long run cause demand to be 1.3 percent less than it would otherwise have been.

This price elasticity of demand is possibly due to substitution of other forms of energy for electricity, for example gas for space heating instead of electrical heating. Another explanation is offered by the view that consumers have a range of potential uses of electricity[28]. The lower the projected future price of the commodity, the more it is likely to be used. At high prices the electricity consumption may be restricted to satisfy higher valued uses

Table 2-1.  
Long-Run Elasticities of Electricity Demand

Factor	Residential	Commercial	Industrial
Average- electricity- price	-1.3	-1.5	-1.7
Population	+0.9	+1.0	+1.1
Income	+0.3	+0.9	+0.5
Average gas price	+0.15	+0.15	+0.15

only. In the long-run, consumers might adjust the number of their electricity-using equipment to reflect the value of their uses, or perhaps install on-site generating capacity of their own. In this view the total energy related utility to the consumer may decrease, since his overall energy consumption is limited by his budget constraints.

### 2.1.3 Definition of Reliability

The demand for electricity can be viewed as the *desire* for a certain capacity (measured in kw), and energy (measured in kwh), delivered within specified voltage and frequency tolerances.

It therefore seems appropriate to define the term "reliability" to mean a given combination of capacity and energy supplied when required, and at the specified voltage and frequency. This is justified by the fact that the supply of electricity to a customer is affected by changes in any



one of these three attributes. Inadequate kilo-watt capacity may result in overloading of supply circuits if all electricity-using appliances are in use at the same time, or there might be inconvenience to the customer due to restrictions on the number of appliances he may be able to have in use together. Energy shortfalls affect the length of time the customer may use his appliances, and inadequate voltage and frequency can result in inadequate performance or destruction to equipment, or can even cause service interruptions. Outages or equipment failures can be caused by inadequate capacity and energy or by voltage and frequency variations outside nominal values. Hence, from the customer perspective, the word unreliability is used to indicate any deviation from the requirement of any of the three attributes of capacity, energy and quality of service. In the following pages the term reliability is used in this context.

## 2.2 Basic Assumptions

Regardless of the current quality of service at a given bus, if the system planner concludes that the cost of an incremental improvement there would be *more than* recovered in value to the company, perhaps in the form of public satisfaction leading to long-range market growth, then he would recommend such an improvement. Over time, then, the planner's decisions would cause his system to evolve toward

the point where further incremental capital investments can be barely recovered.

Consumers, on the other hand, may decide to make incremental changes in their ownership of electrical capital equipment or consumer durables if they perceive that the cost of such investment could be *more than* recovered in value, given the quality of service they receive. Over time, then, their decisions cause bus loads to evolve toward the point where further investment can barely be recovered.

From these considerations we see that the notion of "acceptable quality of service" can be looked at from the points of view of both the utility and its customers, and evolution of the system must always be towards a compromise or trade-off that is acceptable to both. This leads to the conclusion that, if we examine a system that has evolved slowly over a long period of time, the levels of service quality at the various buses may be close to the point where both consumers and utility are satisfied. Since these levels will differ at different buses, the costs of incremental improvements at different buses may provide approximations to the slope of the social value curve at various reliability levels.

There is some indication that there does exist a point in the electric service market where both supplier and consumer are satisfied. Moreover, it appears that utilities aim at this point by design, as indicated in the recent Calgary Power report "Quality of Electric Power Service

Study"[33]. The authors state that a reduction in Calgary Power's reserve capacity from 25% to zero would reduce rates by about 15%, but would render the system incapable of meeting the demands of its customers, and create an intolerable reduction in quality of service. On the other hand, increasing the reserve from 25% to 40% would not be justified by the small increase in reliability level that would result. "Between these two extremes", they state, "lies a range of continuity of service levels which is in balance with considerations of productivity... While it is not possible to determine the limits of that range in precise mathematical terms, it is believed, on the basis of the comparative performance of Calgary Power and other Canadian utilities, that Calgary Power is operating within this reasonable range". [33]

The view that many power systems operate at reliability levels where both supplier and consumer are satisfied is further enhanced by the practice of utilities to monitor customer complaints. According to the Calgary Power report just mentioned, "...all written enquires and complaints received at the company's head office are recorded and analyzed monthly". According to them, recent complaints related to electric power supply have numbered about 60 each year. Presumably, if complaints about any section of their system were to increase sharply, they may take action to correct the situation where it is economically feasible. On the other hand, if fewer than 60 complaints were received,

then perhaps that section of the system would receive only normal attention, so that with load growth and age of equipment the reliability may in time fall to the level where improvement may again be required.

The following quote from Gangel and Ringlee further indicates the effect of consumer complaints on utility effort:

"There is little doubt that the effect of service interruptions is to affect public reaction toward the utility company. Depending upon degree of concern, this reaction may range from quiet dissatisfaction through mild complaints to vigorous public reaction which could affect the entire business climate.

Electric utility response to this reaction takes the form of positive preventive steps, such as further expenditures for maintenance as well as service restoration efforts and additional expenditures for equipment and lines." [37]

The view that in the long-run the power system may evolve toward the state where both supplier and customer maximize their utility benefits is also supported by Gangel and Ringlee in the following quote:

"The amount of utility effort is proportional to public response. As a competitive factor with other energy sources, expenditures for reliability may well lead public response. This is a dynamic situation which at equilibrium logically tends to equalize incremental costs of preventive and restorative steps with the customer's unquantified incremental measure of current competitive worth of system reliability" [37]

### 2.3 Distribution of Service Reliability over a Network

One would expect to find significant differences in the reliability of the various buses of a network, especially if the network serves loads with very different magnitudes and characteristics, and scattered over a physically wide area. This concept is of fundamental importance to the technique of worth evaluation proposed in this thesis, and the results obtained in later chapters seem to justify this assumption.

Power systems are "hierarchical", i.e. power flow is from a few generator buses through more numerous bulk-power buses, and finally to many load buses. A bus which supplies a larger proportion of the total load is likely to have a higher reliability than a bus which supplies power to a smaller proportion of the total load. If a reliability index is calculated for each bus in a network, and the indices are then plotted as a function of the relative magnitude of the power which flows through the buses, the curve might resemble Figure 2-3. This follows from the above discussion. Note that the quantity on the abscissa is the fraction,  $f$ , of total power flow handled by each bus, so that buses which handle a large part of the total system power flow would have values of  $f$  close to one, whereas buses which handle a small part of total power flow would have values of  $f$  close to zero. The quantity on the vertical axis is a measure of unreliability.

The reliability index used in Figure 2-3 is that calculated by the "conditional probability approach" which

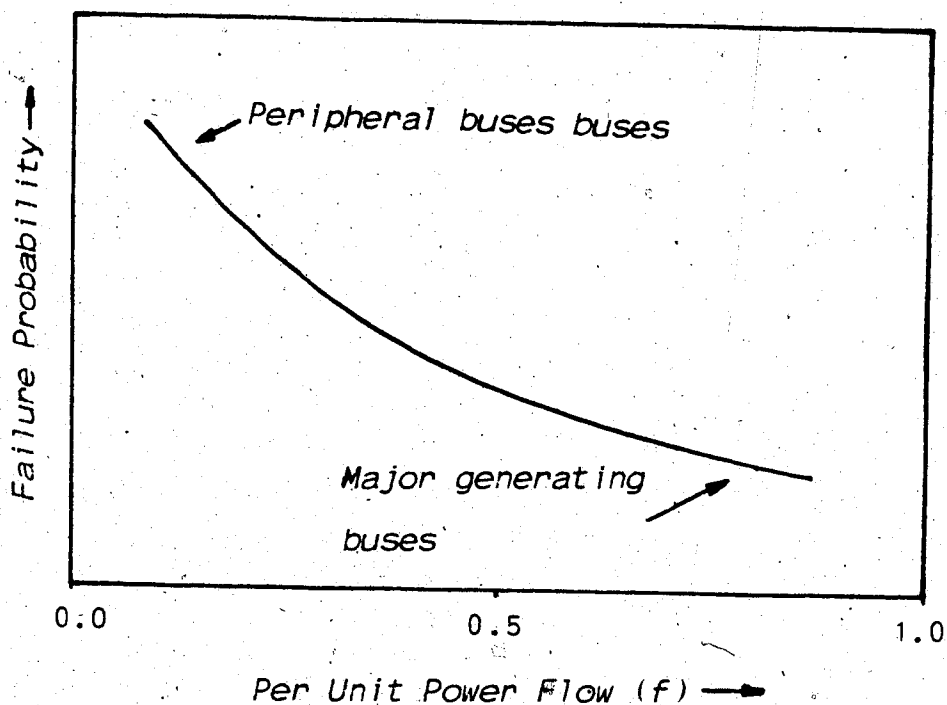


Figure 2-3. Possible shape of the reliability distribution curve.

is described in Chapter 3. It is the "probability of failure of a bus". Results obtained for the IEEE 14 - bus test system and for a 37-bus model of part of the Alberta Interconnected Network are shown later in Figures 3-1 and 3-2. These diagrams show that there is indeed a decrease in Failure Probability as  $f$  increases. (See Chapter 3 for the details of how these curves were obtained.)

## 2.4 Evaluation of Marginal Reliability Costs

Using the assumption that each point in a practical system is approximately at its optimum reliability point in that the cost of a marginal increase in reliability is exactly offset by the corresponding increase in social value to those affected, we can obtain the minimum cost for a marginal increase in the reliability of each point in the system, and a curve of minimum cost divided by marginal reliability increase can then be plotted as a function of the different power levels ( $f$ ) present in the system. This is done as follows:

Suppose Figure 2-3 shows the distribution of reliability for a particular system. A small addition of extra generation and/or transmission/distribution equipment at selected points in the network may improve the reliability of supply to a set of customers indicated by their location on the horizontal axis of the curve. Depending on the specific improvement that is made, the effect may be felt by a few customers only, or by all customers. The reliability distribution curve would show the effect of the improvement and the position on the horizontal axis of the buses affected. Figure 2-4 is one possible result, where the buses affected are those having the lowest values of  $f$ . By simulating many improvements, a curve of  $[\Delta\$/\Delta\text{Failure Probability}]$ , versus  $f$  can be constructed. This curve will show the variation, over  $f$ , of the investment that would be required in order to effect an improvement in

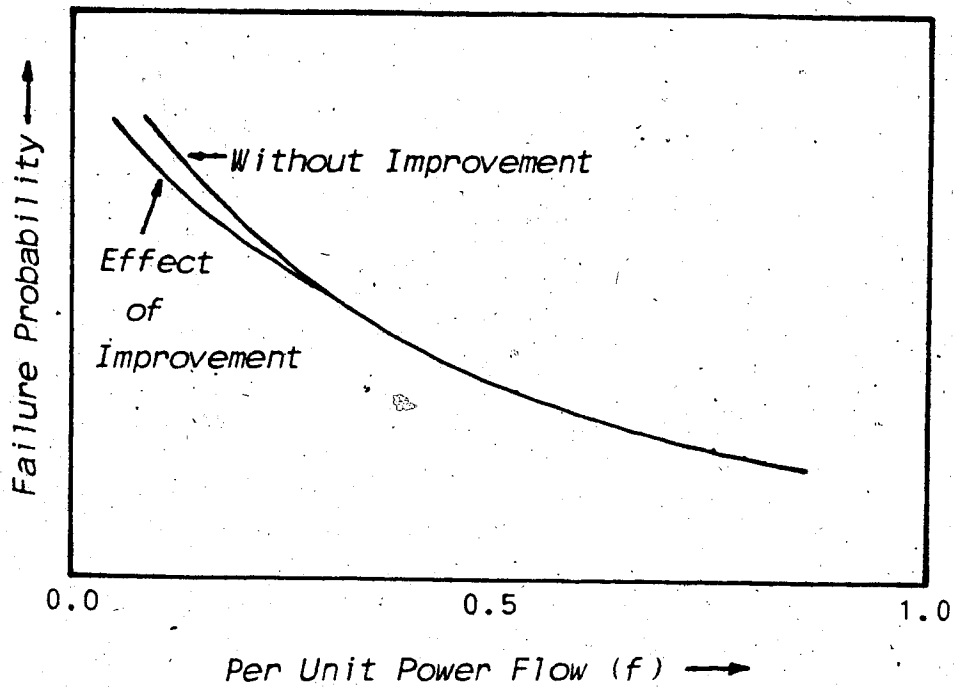


Figure 2-4. Possible effect of an improvement.

service reliability to specific customers by a fixed amount. Figure 2-5 shows how this curve might look for a specific system design. The curve is arbitrarily sketched and in reality is unlikely to be as smooth as is indicated in the diagram.

It is not the practice of utilities to make additions to a power system for the sole purpose of maintaining or improving reliability. An addition of a new line might be made to provide added transmission capacity to supply increased demand by existing customers, or to supply new



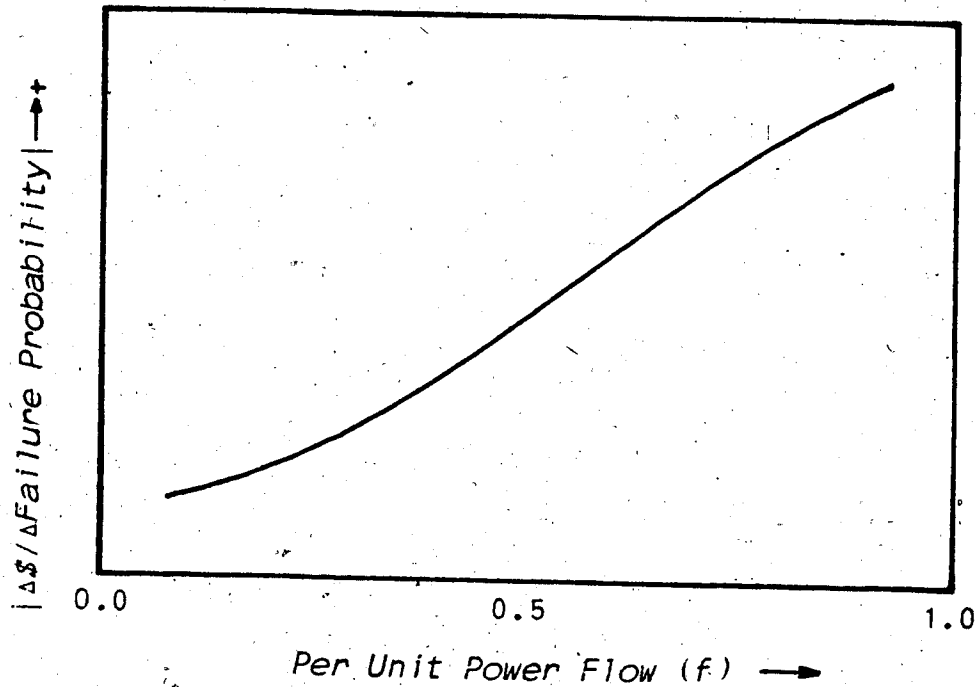


Figure 2-5. Possible shape of curve showing marginal cost as a function of per unit power level.

customers. However, in designing and constructing new power lines, a portion of the cost is included to improve reliability. As pointed by Gangel and Ringlee[37], this portion of the cost may go toward "use of spacings which may necessitate taller poles or longer cross-arms, use of shield wires, separate pole lines rather than multiple-circuit pole lines". At the distribution level costs which can be attributed to improving reliability include that for spare transformer capacity, a good portion of the total spent on distribution system protective devices of all types, and

that for network equipment in excess of that required to serve the same load by means of a conventional radial distribution system. Certain maintenance costs also go towards maintaining reliability.

It is difficult to specify exactly what percentage of an addition to the system goes towards improving reliability. Gangel and Ringlee estimate that this may amount to as much as 40% of the total cost of installing protective equipment. Calgary Power [33] estimates that the percentage of the total cost of service aimed at maintaining the existing level of service continuity is probably less than 15%. Since the system studied in Chapter 4 has characteristics which are similar to the Calgary Power system (they both form part of the Alberta Interconnected System, and are designed using the same set of standards), the latter figure is used. The cost figure used in constructing Figure 2-3 is that which is attributed to reliability improvement. In Chapter 4 it is taken as 15% of the total capital cost of the improvement.

It is conceivable that a reliability improvement to a specific set of customers may be effected in several ways. It should be noted that the above curve must be drawn by selecting the alternative with the minimum overall (i.e. capital plus operating) cost which provides the same or better level of reliability. The selection would be based on the knowledge of the system characteristics acquired over a period of time.

Note also that the index used in Figure 2-3 to measure reliability is in fact a measure of unreliability. An improvement in reliability is therefore indicated by a decrease in failure probability (F.P.), that is,  $\Delta F.P.$  is negative. Hence  $[\Delta \$ / \Delta F.P.]$  is negative. Figure 2-5 shows the magnitude,  $|\Delta \$ / \Delta F.P.|$ , on the vertical axis. The sign is introduced later.

Figure 2-5 implies that the cost of a marginal improvement in reliability for affected buses is small for buses with low  $f$  and larger for buses with higher values of  $f$ . This may or may not be true, but it seems more probable that the cost *per unit*  $f$  for the marginal improvement is greater at low values of  $f$  than it is at higher levels. Figure 2-6 shows a possible result for  $|\Delta \$ / \Delta F.P.|$  *per unit*  $f$ , as a function of  $f$ . This would be computed by dividing  $|\Delta \$ / \Delta F.P.|$  by the range of  $f$  ( $\Delta f$ ) over which the improvement is felt.

## 2.5 Construction of Marginal Social Value Curves

Using the original assumption that each section of the system network is at its optimum reliability point, the deduction is now made that the marginal investment *per unit decrease* in Failure Probability is equal to the marginal Social Value of that unit decrease in Failure Probability. That is,

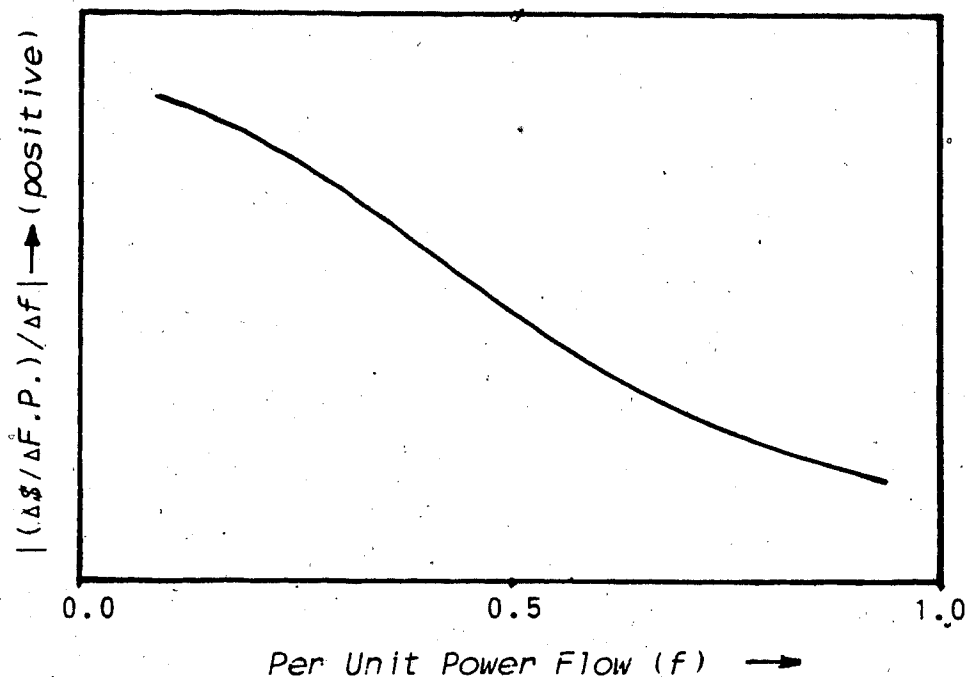


Figure 2-6. Possible shape of curve showing marginal cost *per unit power flow* as a function of power level.

$$[\Delta\$/\Delta F.P.] = [\Delta \text{Social Value}/\Delta F.P.]$$

Hence Figure 2-5 is also a curve of  $|\Delta \text{Social Value}/\Delta F.P.|$ , and Figure 2-6 also shows  $|\Delta S.V./\Delta F.P.)/\Delta f|$  as a function of power-level. The diagram indicates that the marginal Social Value *per unit*  $f$  of an increase in supply reliability is greater for persons living in rural districts (low  $f$ ) than for persons living in densely populated areas (high  $f$ ). In other words, the optimal trade-off for rural districts is at a point on the social-value-of-reliability curve where

the slope is steeper and the reliability is lower.

The next step is the construction of a curve showing marginal Social Value per customer as a function of unreliability (Failure Probability). This is a straightforward procedure using the relationship between Failure Probability and  $f$  shown in Figure 2-3. The values of  $f$  on the horizontal axis of Figure 2-6 are replaced by the corresponding values of F.P. as obtained from Figure 2-3, and these rearranged in ascending order. The result, judging from the shapes of Figures 2-3 and 2-6, would likely be of the form shown in Figure 2-7.

This is a very important relationship. It gives marginal Social Value per unit F.P., per unit  $f$ , as a function of unreliability. Presumably, integration of this curve with respect to Failure Probability would give the desired Social Value function.

## 2.6 Final Social Value Curve

The integration can be performed by computing the area under small enough segments of the curve of Figure 2-7, thus giving a set of points from which the Social Value curve can be constructed. An alternative approach is to fit a polynomial to Figure 2-7 and perform the integration analytically. In either case, a suitable constant of integration must be found. The application of the method in Chapter 4 performs the integration analytically, and an

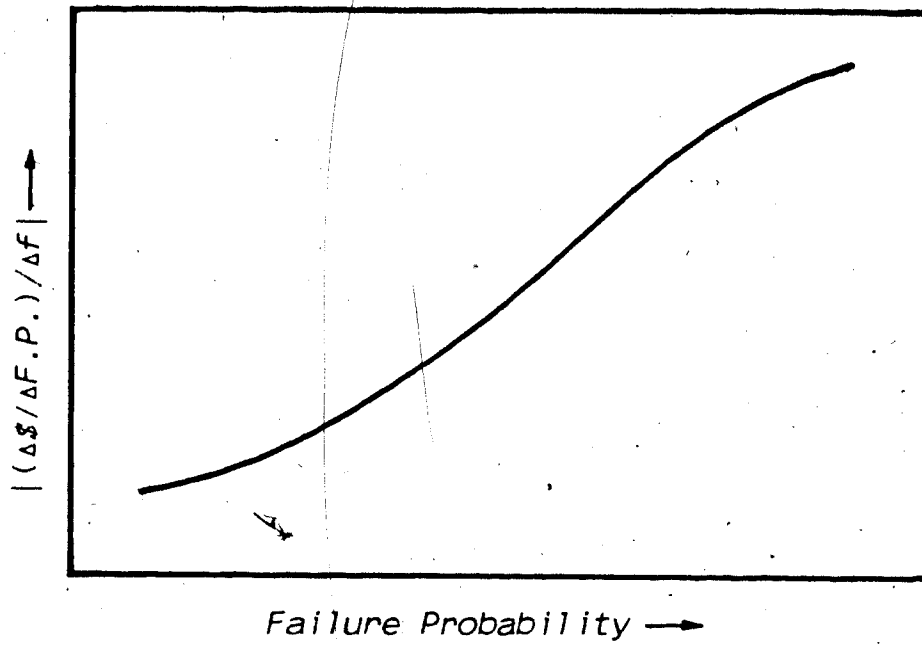


Figure 2-7. Possible shape of curve showing marginal cost per unit power flow, as a function of *Failure Probability*.

arbitrary method used to select the constant of integration.

### 3. Reliability Model

#### 3.1 Indices of reliability

The past two decades have seen considerable emphasis placed on the development of quantitative indices which respond meaningfully to the factors that actually influence system reliability. This chapter assesses the performance of some methods proposed by researchers for evaluation of steady state reliability indices for Electric Power Systems. Some basic concepts in System Reliability Modelling are given in Appendix 1. It is usual in the literature to define reliability indices in terms of system success or failure. Many systems, however, have several levels of failure. It is therefore better to define the indices in terms of a set which may contain any number of system states. Several steady state indices are given in Appendix 1. Three of them are accepted as being the most meaningful. They are: (1) average number of interruptions per customer per year, (2) average interruption time per customer per year and (3) average and maximum expected duration of a customer interruption.

### 3.2 Methods of Generation System Reliability Evaluation

The techniques used for generation system reliability evaluation can be broadly classified into three groups: (1) Monte Carlo Simulation techniques, (2) Loss of Load Probability (LOLP) and Loss of Energy Probability (LOEP) methods, and (3) Frequency and Duration methods. Each class can be adapted for application to the different sub-systems of a power network. The choice of a suitable method depends, among other things, upon the complexity of the system network, the degree of accuracy required, the computational complexity, and the various failure modes of the system. All three classes of methods employ mathematical models which reasonably idealize the physical system, and are amenable to calculation. The first class, however, performs sampling experiments on the model whereas the last two calculate the reliability measures by manipulation of the model.

#### 3.2.1 Monte Carlo Techniques.

In 1964 a paper was published by C.F. De Sieno and L.L. Stine[1], describing a method for transmission system reliability evaluation based on the Monte Carlo technique. The method was criticized by many because of the long simulation period and large number of trials required for reasonably accurate results. Noferi and Paris[2] cited a computer simulation for a relatively small system which required 100,000 Monte Carlo trials. It took slightly less than an hour on a large high-speed computing system (the



Univac 1108). Other criticisms were that the reliability indices calculated by this method were not meaningful when applied to power systems, and that considerations of various failure modes of the individual components could not be efficiently included in the technique. R. J. Ringlee and A. J. Wood, in discussing the technique (see the discussion at end of reference 2), stated that as far as they were concerned, "the best Monte Carlo" is the "least Monte Carlo". This emphasizes the high cost and inefficiency of sampling techniques.

### 3.2.2 Loss of Load Probability Approach

The approaches used by the LOLP and Frequency and Duration methods are superior to the simulation approach. De Sieno and Stine had in their paper developed some ideas relating to the Markov treatment of a system of components, but they abruptly dropped them in favour of simulation techniques. These ideas were further developed by others, and today they form the basis of the probabilistic models employed in the LOLP and Frequency and Duration approaches.

The LOLP method is simple to appreciate and apply. For medium sized systems, computer techniques have been developed to deal with the complex outage tables. This method is widely used in generation system planning; a criterion of one day in ten years is often taken as the maximum acceptable loss of load probability. The Loss of Load Probability index, however, is restrictive in scope. It

gives only the frequency of outage but does not give its mean cycle time, or its duration. The complexity of outage tables obtained when the method is applied to large systems is another serious disadvantage. A Normal approximation to the discrete method is used in certain circumstances when the law of large numbers is applicable[3].

### 3.2.3 Frequency and Duration Methods

Frequency and Duration methods provide perhaps the most meaningful reliability indices of all the methods mentioned here. At the same time, the computations involved are lengthy. For accurate predictions to be made, the system model must include all possible failure modes. This of course is quite unfeasible even for medium sized systems. Proper choice of the failure modes to include in the model is therefore an important part of system evaluation using this approach.

### 3.3 Transmission System Reliability Evaluation

There have been numerous papers written on the application of F&D methods to generation system reliability evaluation. This statement, however, is not true of the other subsystems. Two papers published in 1964 were among the first to apply the methods to transmission systems. The first, by Z. G. Todd[4], describes a technique which determines the probability of a forced outage of specified

minimum duration, and then calculates the customer interruption rate and the number of interruptions exceeding a specified minimum duration. It assumes a two state component model (up or down), independent failures, and complete redundancy of parallel paths. The approach is quite straightforward and can be extended to fairly large systems, but because of the assumption of complete redundancy of parallel components, it measures the continuity of service only and gives no evaluation of the quality of service at the load point. It therefore cannot be extended to systems that are not fully redundant.

The second paper, written by Gaver, Montmeat and Patton[5], also assumes redundancy of parallel paths but predicts both outage duration and frequency. The most important aspect of this approach is the introduction of a varying environmental condition associated with the operating component. Outage rates for parallel facilities are developed which include the bunching effect of storm associated failures. An attempt is also made to include the probability of a component successfully carrying contingency loads of given durations. More meaningful predictions can be made using this approach. However, in its complete form, the method is difficult to apply to a complicated system, and becomes more of an approximation as additional parallel elements are combined.

The above method can be thought of as an approximation to the more accurate 'Markov Approach'[6]. The indices

calculated are the same but the Markov approach attempts to include all failure states. It becomes extremely cumbersome to attempt to obtain general expressions even for very small systems.

### 3.4 Composite System Reliability Evaluation

The techniques of reliability evaluation that have been described so far are usually used to calculate a reliability index for the generation system by itself, or for certain isolated sections of the transmission/distribution network. The reliability of service at a load point, however, is dependent not only upon the system configuration in the vicinity of the load point, but also upon the performance of the rest of the system during an outage condition. Methods used for composite system reliability evaluation take into account the outage probabilities of the generating units, and of all other system components which affect the loss of load probability at the load point. Of course there is always the disadvantage that including all possible outage states can increase the computational complexity beyond feasibility, but in many cases, certain simplifying approximations can be made. Those outage states which have small probabilities of occurrence can often be ignored without seriously degrading the usefulness of the final results.

A program for determining outage probabilities of the bulk supply points in a medium sized network has been developed at the University of Saskatchewan [7],[8],[9]. The technique recognizes that power system networks are not usually designed using a simple continuity criterion at the load points. Instead, even when there is continuity between the load and source points under an outage state, there might be a finite probability of loss of load. This probability is essentially a function of the load distribution at the load point and of the capability of the network to supply load with low voltages and without overloading supply circuits. The method evaluates the network capability under various outage states by load flow analysis at several load levels. Conditional probabilities are calculated and then combined with generation outage probabilities to obtain risk measures for all bulk supply points.

The F&D approach as applied to composite system reliability evaluation was illustrated by Mallard and Thomas[10]. The reliabilities of the transmission supply to several substations were calculated. Included in the analysis was the effect on system reliability of performing scheduled outages according to specified policies.

### 3.5 Reliability model used in this Thesis

Central to this research scheme is the use of a technique which would compute indices at each load point. It is mainly for this reason that the reliability model to be described in this chapter was chosen. Although frequency and duration methods could be used to compute indices at each bus of a system made up of generation and transmission equipment [10], they are more difficult to apply, especially when considerations of voltage quality at a bus and branch overloads are as important as they are in this study. The technique described in this chapter does not assume complete redundancy of parallel components; instead it takes into account the possibility of a branch overload (i.e. power flow greater than some fixed maximum value) occurring when a parallel branch is on outage. Voltage quality is also part of the failure criterion at each bus.

The reliability model was developed at the University of Saskatchewan by Billinton, Medicherla, Sachdev and others [7],[8],[9]. It calculates the probability of failure of a bus to supply its load continuously, within a given service quality criterion. Two methods of computing this index are described; only one method is employed here. This method is the less accurate of the two, but requires less computation. A computer program, written in FORTRAN, is developed and used to compute the reliability indices for two systems: the IEEE 14-bus test system and a 37-bus network which forms part of the Alberta Interconnected

System.

The approach considers generating capacity outages as affecting the whole system. Outage states consist of outages of individual lines and transformers in combination with other lines and transformers. Each outage state has a probability of existence and a frequency of occurrence. Under each outage state there is a maximum load at each bus that can be supplied without lowering the quality of service at the bus in question below acceptable limits. The probability that the load will exceed this maximum is a conditional probability, since the maximum load is determined given that the system is experiencing a certain outage state. This conditional probability is combined with the probability of occurrence of the outage state in order to calculate the reliability index. Generation and transmission outages are considered as independent events; simultaneous multiple outages are also considered as occurring independently.

The equation used to compute the reliability index is:

Probability of failure at bus k

$$= \sum_j P(B_j) [PG_j + PL_{jk} - PG_j PL_{jk}] \quad (3-1)$$

where:

$B_j$  is an outage condition in the network,

$P(B_j)$  is the probability of existence of  $B_j$ ,

$PG_j$  is the cumulative probability of the generating

capacity outage exceeding the reserve, and  $PL_{jk}$  is the probability of the load at bus  $k$  exceeding the maximum load that can be supplied at that bus without failure under outage state  $B_j$ .

As was mentioned in chapter 1, it is necessary in principle to consider all possible outage contingencies when determining reliability indices at the load buses. In a power system network, a tremendous number of outage combinations of lines, transformers and generating units exist. It is not computationally feasible to attempt to simulate all of them. Network planners normally assess the adequacy of a given composite generation and transmission system by testing the system performance under a series of "credible" outages. These outages are used in evaluation of the two systems. Billinton et al[9] have shown that single outages usually give sufficiently accurate results. The number of simultaneous outages is limited to a maximum of two.

A useful method for selecting the outage states to be simulated is demonstrated by Marks[12]. Those contingencies for which the probability of occurrence is very small are eliminated. This is done by establishing a cut-off value below which contingencies are considered unlikely to happen, and are ignored. Contingencies with probabilities larger than the cut-off value are termed "probable" occurrences, and those with smaller values are referred to as "improbable" occurrences.



### 3.6 Assumptions and General Considerations

The reliability evaluation proceeds by simulating outage conditions chosen consecutively from the list of "probable outages", and, for each outage state, performing a series of load flow runs at several load levels in order to calculate the maximum load that can be supplied by each bus without violating the service quality criterion. (Service quality criterion explained below.) The probability of this load level occurring is obtained from the yearly load duration curve. In order to reduce the number of load flow runs required, the assumption is made that if a contingency does not cause loss of service at the peak load level, it will not cause any loss of service at lower load levels. A load flow run is performed for the peak load level first, and checks are made at each bus for any violation of the service quality criterion. If there are any violations, it becomes necessary to rerun the load flow at lower load levels. Each case is rechecked to see if service failures still persist, and the procedure repeated until either there are no service failures remaining or until the minimum load level has been reached.

The computation involved in the approach is also dependent on the number of load levels selected. The greater the number of load levels, the more accurate would be the determination of the maximum load at the bus which can be supplied without failure of that bus. However, the number of computations also increases. The procedure used for the

14-bus system employs ten load levels, and has been found to give reasonable results[9].

The determination of the maximum load that can be supplied at each bus without failure under an outage condition is based on the following two assumptions:

1. The load variation at each bus is represented by a single yearly load duration curve, with different peak loads at each bus. The individual bus load levels used in each load flow run are those corresponding to a specified probability of being exceeded.
2. Under each outage condition  $B_j$ , if a bus fails at any of the decreasing load levels,  $PL_{jk}$  in Equation 3-1 is taken as the average value of the load level at which the failure just disappeared and the previous higher level.  $PL_{jk}$  is zero if the bus does not fail at the peak load level, and one if it fails at the lowest level.

The bulk of the computation time is utilized in load flow analysis of each outage condition. Accurate load flow methods like the Gauss-Seidel and Newton-Raphson techniques are computationally expensive. On the other hand, approximate methods like DC load flow are fast but only provide estimates of line power flows. They do not provide information about voltage levels, which are essential to the reliability evaluation procedure. The Fast Decoupled Load Flow technique [14] is chosen as a good compromise between

DC and accurate AC load flow methods. It possesses reasonable computer storage requirements, and provides fairly accurate information about bus voltages and line power and reactive flows.

Service failure at a bus is defined as follows[7]:

1. The voltage at the bus is less than a specified minimum value.
2. A line or transformer supplying power to the bus is beyond its emergency maximum power rating.
3. The generating capacity required to meet the total load plus losses exceeds the available capacity. When this occurs, the total system is considered as having failed.

It has been pointed out[13] that this list does not include all the possible modes of load-point failure. The recognition and classification of how a system can fail is perhaps the most difficult part of any reliability analysis. This analysis does not include transient responses of the system; the failure modes noted above are the most frequent steady state ones

### 3.7 Mathematical Details

An outage state  $B_j$  is any state of the system in which one or more components normally in service are temporarily out of service. This may or may not result in loss of load at any load point. The requirement of independence of the

individual component outage probabilities is necessary, so that the following equations for single contingency outage states apply:

$$P(B_j) = \bar{A}_x \cdot \prod_{\substack{i=1 \\ i \neq x}}^n A_i \quad (3-2)$$

$$u(B_j) = u_x + \sum_{\substack{i=1 \\ i \neq x}}^n \lambda_i \quad (3-3)$$

where:

$P(B_j)$  = Probability of the outage state  $B_j$  occurring.

$u(B_j)$  = Departure rate from this outage state  $B_j$ .

$A_i$  = Probability of component  $i$  being in service.  
(availability)

$\bar{A}_x$  = Probability of component  $x$  being out of service.  
(forced outage rate).

$u_x$  = Restoration rate for component  $x$ .

$\lambda_i$  = Failure rate of component of  $i$ .

$n$  = Total number of components represented as being in service on the base case load flow.

For double contingencies, the equations are:

$$P(B_j) = \bar{A}_x \cdot \bar{A}_y \cdot \prod_{\substack{i=1 \\ i \neq x, i \neq y}}^n A_i \quad (3-4)$$

$$u(B_j) = u_x + u_y + \sum_{\substack{i=1 \\ i \neq x, i \neq y}}^n \lambda_i \quad (3-5)$$

These equations, together with the conditional probabilities, provide sufficient information for the computation of the reliability index, that is, the probability of bus failure. As pointed by Marks[12], other indices can be calculated using the above procedures.

It should be noted here that each component is assumed to be in one of two states - in service or out of service. However, the method of reliability evaluation used in this thesis can conceptually be extended to incorporate components with more than one failure mode. The above equations may still apply, but with the appropriate values for the parameters of each derated component. The outage state would have to be simulated in some appropriate way, in order to make load flow runs on the derated system. Since the purpose of this thesis is to illustrate a methodology for evaluating social worth, such detailed reliability analysis is not considered necessary, so the two state component is taken as satisfactory.

For large systems, it is computationally inefficient to re-calculate the product and summation terms of Equations (3-2) and (3-3) for each outage state. The cumulative values of availabilities and failure rates are calculated only once:

$$A_{\Omega} = \prod_{i=1}^n A_i, \quad \lambda_{\Omega} = \sum_{i=1}^n \lambda_i \quad (3-6)$$

Then for each single outage state, we have:

$$P(B_j) = \bar{A}_x \cdot A_\Omega / A_x \quad (3-7)$$

$$u(B_j) = u_x + \lambda_\Omega - \lambda_x \quad (3-8)$$

For multiple contingencies, these equations hold, where now:

$$\bar{A}_x = \prod_{j=1}^k \bar{A}_j; \quad A_x = \prod_{j=1}^k A_j \quad (3-9)$$

$$u_x = \sum_{j=1}^k u_j; \quad \lambda_x = \sum_{j=1}^k \lambda_j \quad (3-10)$$

and "j" indicates those components whose combined outages caused the state  $B_j$ . "k" is the number of components in the list.

### 3.8 Digital Computer Program

The computer program is patterned after that described in Reference 9. It uses the Fast Decoupled Load Flow (FDLF) technique [14] which is a simple, reliable and fast method of solution, and which usually gives accurate results to within 0.01 MW/MVAR tolerance in four to seven iterations. If programmed correctly, each iteration is equal to about 1/5 of a Newton iteration. Storage requirements are similar to those of the Gauss-Seidel method. The technique has been tested by Stott and Alsac on several networks ranging in size from 13 to 1080 buses, each time giving good results in no more than 11 iterations. Its reliability in terms of

convergence is considered better than that of the conventional Newton-Raphson technique.

### 3.8.1 Features of The Program

Details of the load flow procedure are given in Appendix II. This section is a summary of the pertinent features of the reliability analysis.

1. *Branch Outage Simulation:* If the outage of one or two branches does not cause isolation of a bus, or a system split, the outage is reflected in the appropriate elements of  $[G]$  and  $[B]$  (see Appendix II). A bus isolation requires renumbering the buses, and the corresponding row and column of  $[G]$  and  $[B]$  are deleted. If a system split occurs, each subsystem is analysed separately. A check is made for the existence of at least one generator bus in each subsystem.

This method requires retriangulation of the matrices  $B'$  and  $B''$  after each outage, and is therefore less efficient than the technique recommended by Stott and Alsac (see appendix 4 of reference 14), but since generator Q-limit enforcement was performed by retriangulation of  $B''$ , Stott and Alsac's technique cannot be used here.

The program is written assuming a maximum of two simultaneous independent outages.

2. *Other Features:* (i) The use of Sparse Matrix techniques in triangulating the matrices  $B'$  and  $B''$  (see appendix II), and in solving the load-flow equations. The

techniques used are described by Sato and Tinney[16] and by Tinney and Walker[17]. However, only an inferior hand pre-ordering scheme was used. (For a description of ordering schemes see Stott and Hobson[18]).

(i) Loads are represented as constant power loads.

(iii) If an outage condition results in the isolation of a bus, the conditional probability of failure for that bus is set equal to one. Two simultaneous single outages may result in the isolation of two separate buses. A bus isolation is detected by subtracting the branch series and shunt admittances from the diagonal terms corresponding to the near and far buses, and testing for zero. System split is detected by checking bus connections.

(iv) The outages to be studied, and the number of load steps, are inputs to the program and can be specified by the user.

(v) The load model used is the yearly load-duration curve.

(vi) The maximum load carrying capability of each line and transformer is taken as fixed under all conditions. When an overload occurs, the bus to which the line supplies power is considered as having failed, but it is not isolated from the network.

(vii) Loadflow is performed for the peak load level first, and if any bus fails, for decreasing load levels until either no bus failure remains, or the lowest load level is reached.



### 3.9 Limitations of the Program

As with most of the reliability models available to date, there are serious limitations to the use of the computer program just described. These are discussed below:

(i) The load model used in the calculation of load-level probabilities is the yearly load duration curve. Since frequency and duration of service failures are being calculated for line or generator outages, a preferred load model would be one that can represent frequency and duration of hourly load variations. When these are averaged over a period of a year, as is done in constructing the yearly load duration curve, the effects of load peaks of relatively short durations may be ignored. For example, a transmission outage of eight hours duration could result in a single service failure at a bus, lasting eight hours, or two service failures of shorter durations. The latter could occur if there were two high enough load peaks during the eight hour period. A simple probability type of load model like the yearly load duration curve would give no insight into the frequency and duration of such occurrences.

This problem can be partially remedied by performing the calculations using monthly or even daily load duration curves, then combining the results for the whole year. This approach obviously increases quite considerably the number of computer runs required.

(ii) The assumption that transmission lines have fixed maximum load carrying capabilities is convenient, but in

practice this is usually not the case. Utilities generally specify a normal maximum loading, and an emergency maximum loading which can be carried for a period of time without tripping the protective equipment. In the present analysis the emergency maximum loading is used as the limit of line capacity.

(iii) Weather effects have been ignored. Environmental exposure has been cited as the the major cause of transmission line failures. The technique used by Gaver et al[5] for including weather effects into transmission line failure rates can be incorporated quite easily into the model used in this thesis. However, apart from the unavailability of sufficient data, it was considered that, since this thesis is a proposal for worth evaluation, weather effects can be left for further development and refinement of the methodology.

(iv) The dynamic aspects of a fault in the transmission system have not been included. The reliability indices are all steady state indices, and do not indicate whether, after a fault occurs, the system would break up or stay in synchronism. A frequency-quality criterion should also be included in the calculation of reliability indices, but this is extremely difficult, since emergency analysis of power systems is not well understood by researchers at this time. A load-shedding schedule could be added to guarantee stability of the reduced system under certain outage conditions.

(v) Overlapping outages were assumed to be a combination of independent outages. As noted by Fong, Le Reverend and Neudorf[19], results obtained using this assumption can be overly optimistic. Overlapping outages are often a result of common mode and/or adverse weather outages. Including such considerations in the computer program[20] can make an already complicated model even more difficult to program.

(vi) There is some concern as to the validity of Equation of 3-1. The calculation of  $PG_j$ , the probability that the availability system generating capacity is insufficient to meet the load, is determined from a capacity outage table without regard to transmission system and generator transformer limitations. Ku and Sulzberger (see discussion of Reference 7) have pointed out that, with  $PG_j$  calculated this way, the equations may be inaccurate. The second method for calculating the reliability indices[7] is more accurate; it considers the effect of individual generating unit outages on voltage levels. However, it requires much more computation time, and the difference in the results from the two methods may not be significant.

(vii) The program as implemented here allows for the operation of lines with an overloaded condition. This assumption results in highly optimistic indices. An alternative solution is to remove the overloaded element, and continue the analysis until no other line is overloaded, or until all elements have been removed. This second technique results in pessimistic indices. The third

technique, and the one which most closely represents actual operating practice, [41] is to alleviate the line overloads by either rescheduling of generation, or by curtailment of interruptible loads. One method of doing this is described by Billinton, Medicherla and Sachdev [21], and is compatible with the Fast Decoupled Load Flow.

### 3.10 Application to System Models.

System model 1. is the 14-bus IEEE test system. Line data is obtained from Reference 23, and outage data from Reference 9. A ten step load variation is used, with a straight-line load duration curve with base load equal to sixty percent of peak load. Table 3-1 shows the reliability indices for this system, in increasing order of power-level ( $f$ ).

System model 2. is a 37-bus section of the Alberta Interconnected Network. A full description of this system is given in Chapter 4 and Appendix II. Table 3-2 shows the reliability indices calculated for this system, in increasing order of power-level.

It will be observed that there are wide variations in the reliability indices at different buses for both systems. In order to extract a meaningful relationship between Failure Probability and power-level, straight lines have been fitted to the data in Tables 3-1 and 3-2. This procedure generates analytic functions which are plotted in

Table 3-1. Reliability Indices for System 1.

Bus No.	$f$	F.P.
8	0.000	0.00148981
11	0.026	0.00148981
12	0.029	0.00311616
10	0.035	0.00054029
14	0.057	0.01118844
13	0.073	0.00231923
7	0.110	0.00000302
6	0.167	0.00148981
9	0.174	0.00000302
3	0.364	0.00196695
5	0.407	0.00000302
4	0.437	0.00196695
2	0.801	0.00000302
1	0.891	0.00000302

Table 3-1. Reliability Indices for System 2.

Bus No.	$f$	F.P.
444	0.006	0.00740672
480	0.008	0.00825422
1039	0.008	0.01071701
428	0.008	0.00637926
418	0.009	0.00906870
479	0.010	0.00746793
414	0.010	0.00670912
448	0.011	0.00914616
441	0.013	0.00624820
425	0.014	0.00624820
432	0.019	0.00618494
478	0.022	0.00746793
452	0.029	0.00895183
468	0.032	0.00882077
462	0.044	0.01045942
464	0.047	0.01032836
436	0.050	0.00618494
482	0.052	0.00746793
473	0.055	0.01275365
472	0.056	0.00978496
484	0.059	0.00746793
438	0.062	0.00618494
422	0.064	0.00727360
410	0.074	0.00727360

477	0.079	0.00746793
412	0.082	0.00618494
404	0.086	0.00644253
403	0.086	0.00644253
440	0.099	0.00605388
424	0.115	0.00605388
402	0.172	0.00727360
960	0.203	0.00605388
476	0.260	0.00605388
400	0.265	0.00605388
405	0.431	0.00644253
406	0.431	0.00644253
401	0.873	0.00605388

Figures 3-1 and 3-2. For system 1 we have:

$$\begin{aligned} \text{Failure Probability} = \\ -0.003409f + 0.002697 \end{aligned} \quad (3-11)$$

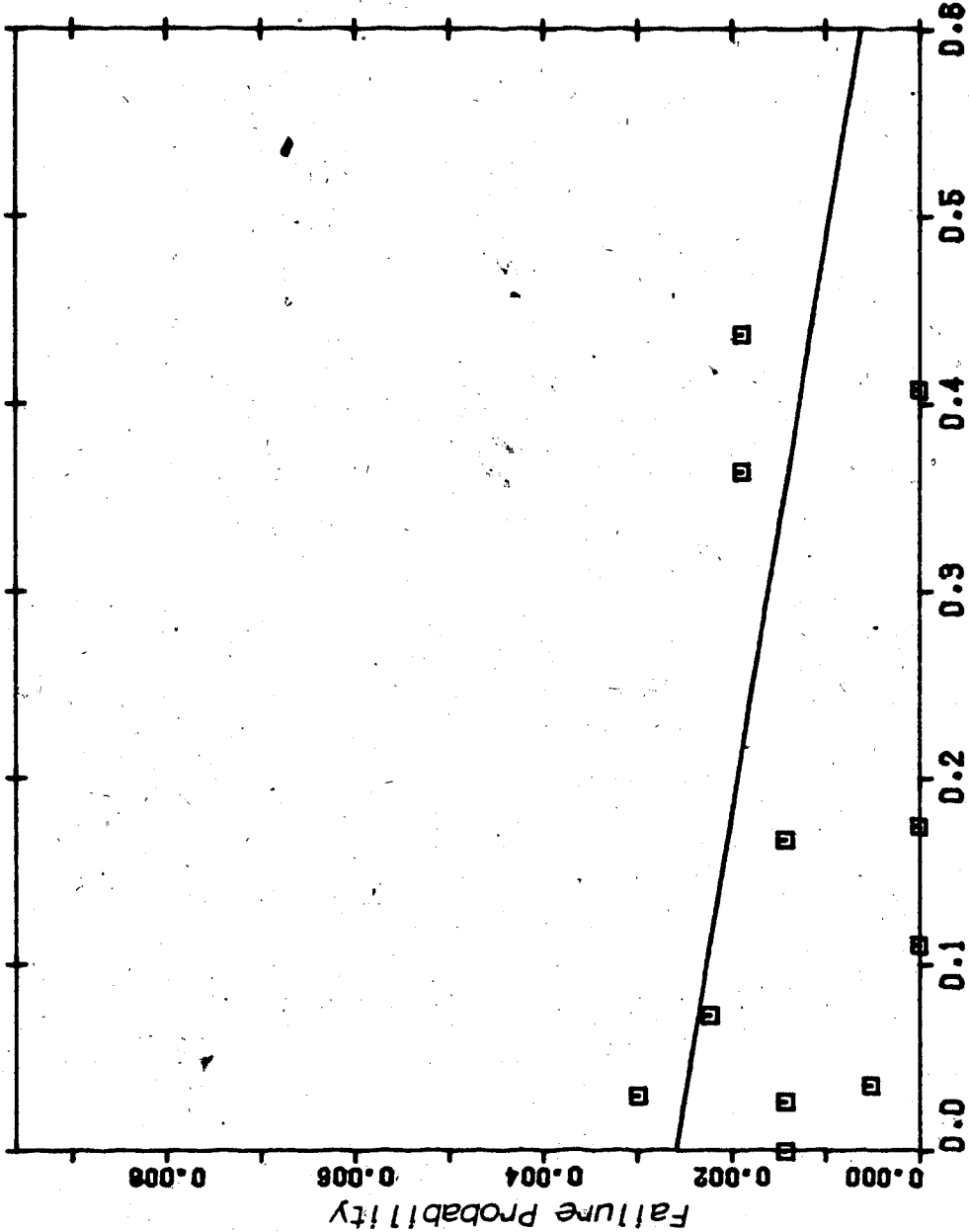
and for system 2 we have:

$$\begin{aligned} \text{Failure Probability} = \\ -0.003247f + 0.00784 \end{aligned} \quad (3-12)$$

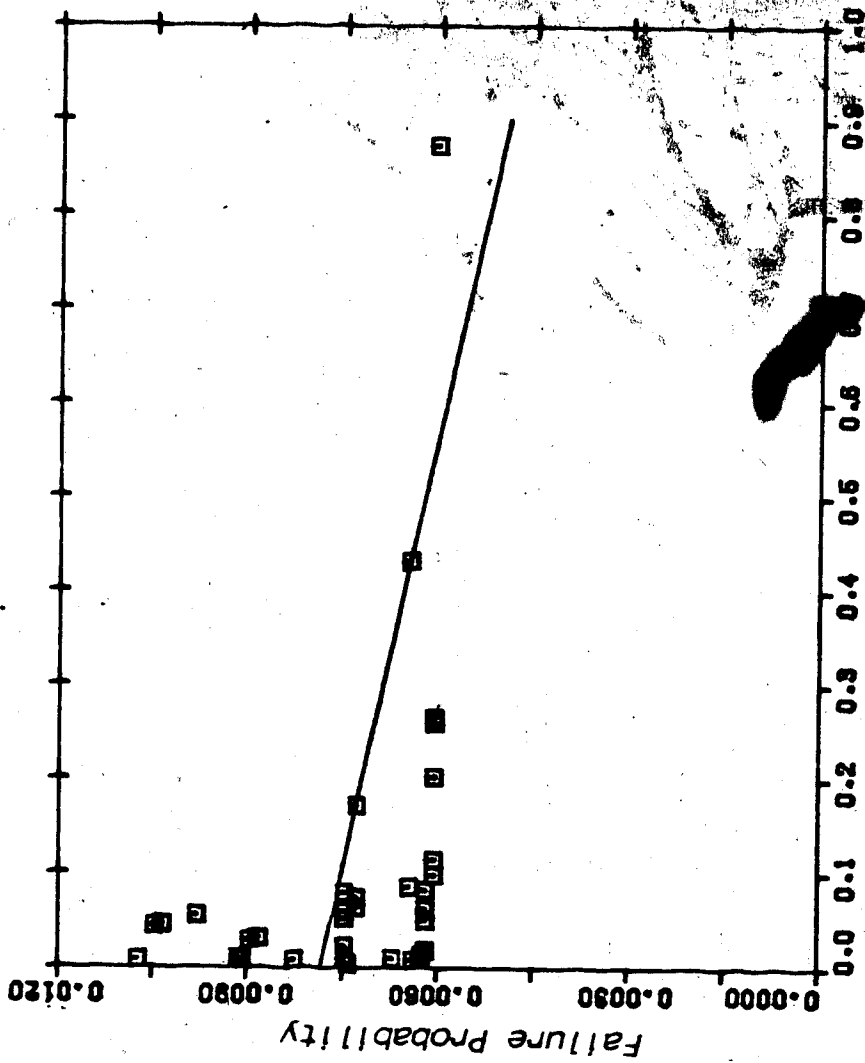
where  $f$  is per-unit power level.

Note that for both systems there is, as predicted in Chapter 2, an increase in Failure Probability as Load-Level decreases. Both lines have almost the same slope, but System 2 has a larger intercept, which translates into larger failure probabilities. This is due to the small number of generators in System 2 compared with System 1.





Per Unit Power Flow (f)  
Figure 3-1. Result of reliability analysis  
of system 1 (IEEE 14-bus test system).



Per Unit Power Flow  
Figure 3-2. Result of reliability analysis  
of system 2.

## 4. System Application

### 4.1 System Description

The proposed method for worth analysis is tested on a reduced model of a power system, i.e. a 37-bus model of part of the Alberta Interconnected Network. System data was obtained from Alberta Power Limited. The single line diagram is shown in Figure 4-1. It has 4 generator-buses, 14 transformers, and 28 transmission lines. Bus voltages range from 14kv to 240kv. Detailed system data are given in Appendix II.

The system is spread over an area of some 2,500 square miles, and serves distribution circuit loads ranging in size from 15mw to 1mw. It also supplies 246.5mw of power to the Calgary Power System over four interconnections, the biggest single transfer being 139mw.

Outage data were not available for the system. Instead, the data shown in Table 4-1, and taken from Reference 9, were used. Detailed outage data are given in Appendix II.

Only single outages were considered, and the load-duration curve was approximated to a straight line. The yearly load duration is shown in Figure 4-2.

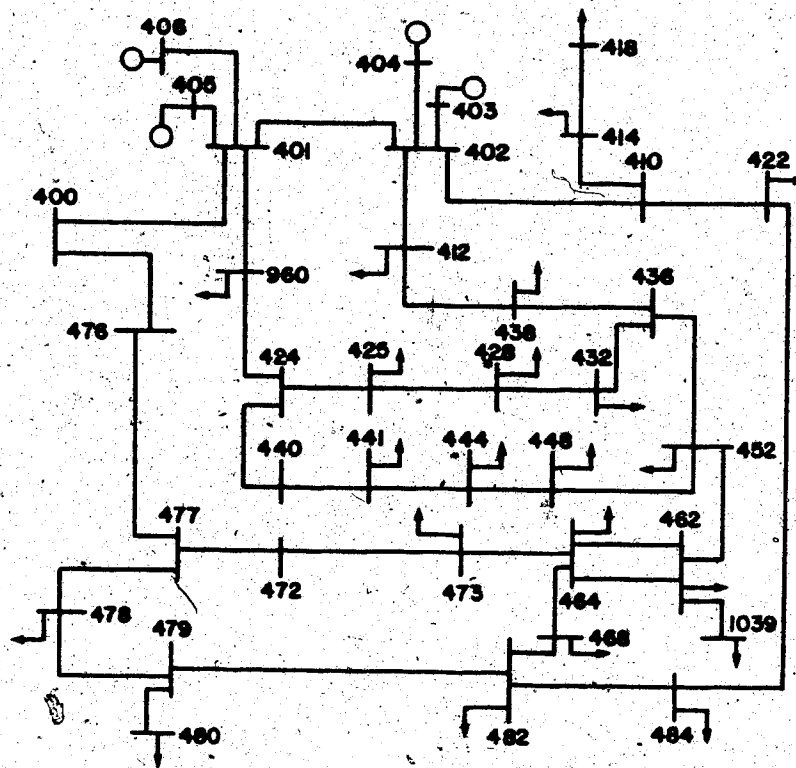


Figure 4-1. Schematic diagram of 37-bus system.

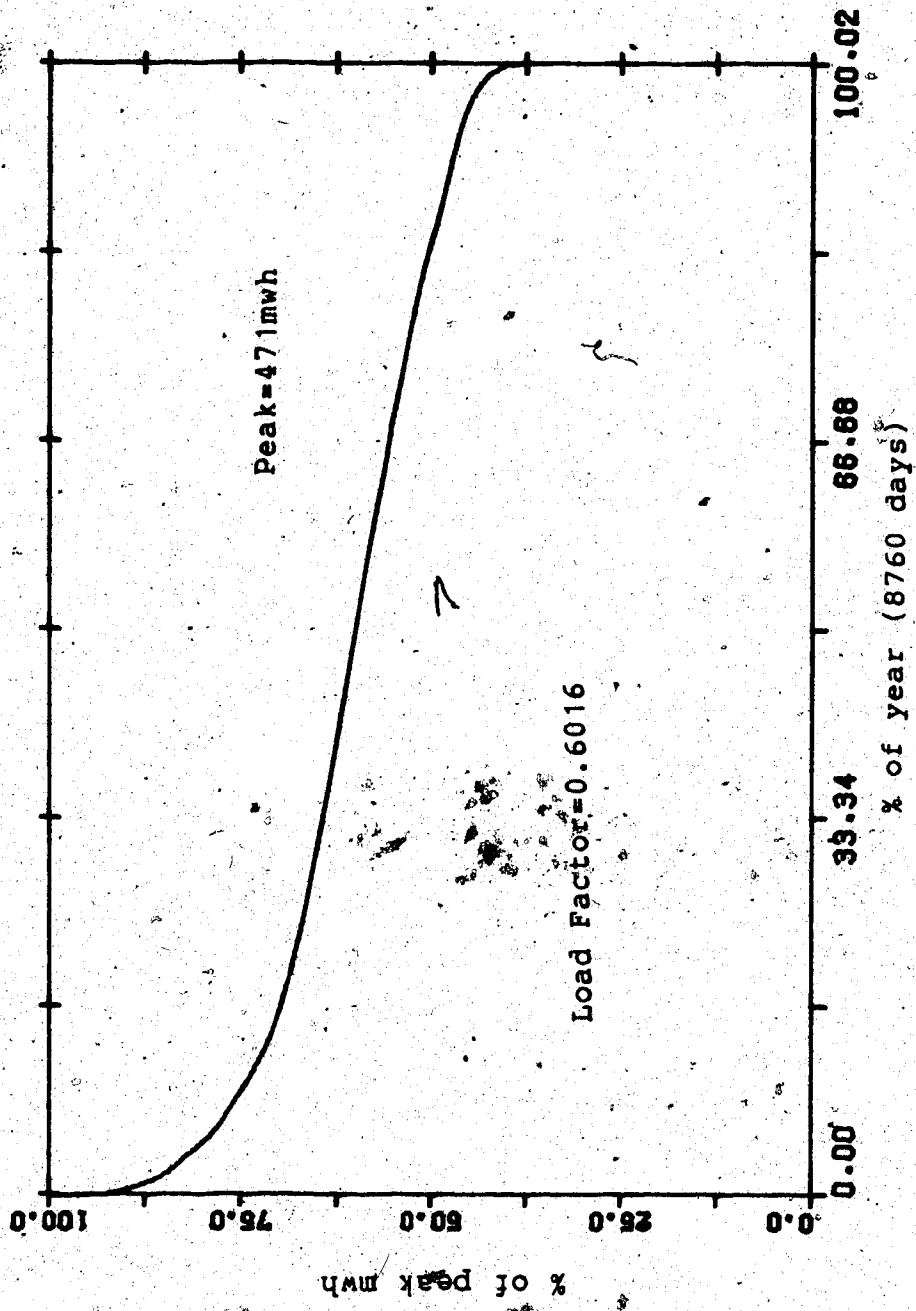


Figure 4-2. Load Duration curve of 37-bus system. (Source: Alberta Power Limited).

Table 4-1. Outage data.

Line KV	Outages/mi/yr	Ave. Repair Time(hrs)
240	.02545	20.39
138	.02787	13.69
72	.13600	10.00
Transformer	.02400	168

Table 4-2. Effect of Improvements.

Load-Level Affected	$\Delta F.P.$	Cost, $\Delta \$$ (Dollars)
0.0472	0.0008233	$2.682 \times 10^4$
0.0500	0.0011619	$2.437 \times 10^5$
0.0862	0.0003559	$3.540 \times 10^4$
0.1718	0.0011870	$6.435 \times 10^4$
0.4226	0.0002588	$1.889 \times 10^5$
0.8670	0.0020560	$2.025 \times 10^6$

#### 4.2 Marginal Social Value Curves

Table 4-2 shows the Load-Levels affected by the improvements to the system that have been simulated, and the corresponding costs and decreases in Failure Probability ( $\Delta F.P.$ ). Appendix II lists the nature of these improvements. Values of  $|(\Delta \$ / \Delta F.P.) / \Delta f|$  are calculated from this table, and used to fit a quadratic function which is plotted in Figure 4-3. This diagram corresponds to Figure 2-6. Note that it does show a decrease in  $|(\Delta \$ / \Delta F.P.) / \Delta f|$  for values

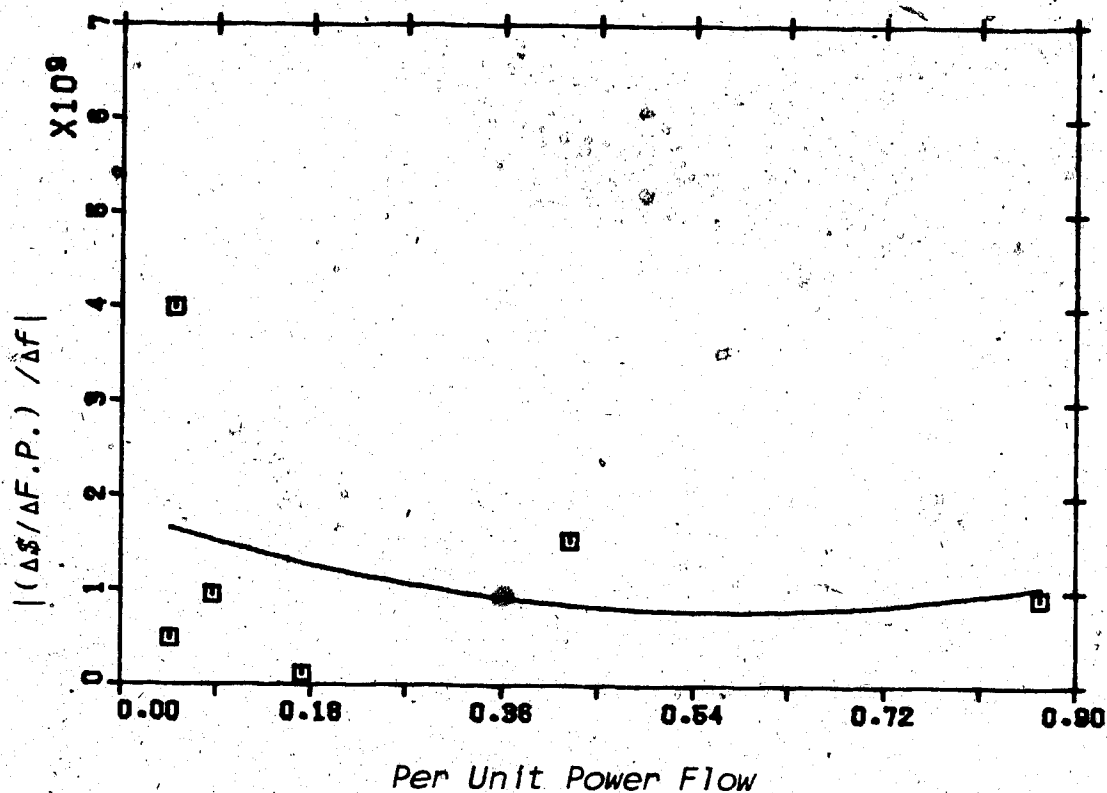


Figure 4-3. Plot of  $|(\Delta S / \Delta F.P.) / \Delta f|$  as a function of  $f$ .

of  $f$  from 0.0 to 0.5, but that there is an increase as  $f$  increases from 0.5 to 0.9. This is in contradiction to the suspected shape of this curve as drawn in Chapter 2, which showed a strict decrease for all values of  $f$ . The increase seems to indicate that although the number of customers served by sections of the system represented by this part of the curve is large, the cost of an improvement is also very large. In fact, the cost is so large that the cost per customer of an improvement that affects 90% of the total system load is greater than the cost of providing a similar

improvement that affects only 60% of the load. Perhaps this level of investment results from the need at high levels of utilization to bring higher cost plants on stream.

The equation is:

$$\begin{aligned} |(\Delta S.V./\Delta F.P.)/\Delta f| \\ = 3.152 \times 10^3 (f^2) - 3.619 \times 10^3 (f) \\ + 2.019 \times 10^3 \end{aligned}$$

\$/unit load. (4-1)

So far in the derivation, the unit of load has been taken as the total power demand on the system. A more meaningful way of expressing Social Value is in terms of \$/KW. Equation (4-1) can be expressed in terms of \$/KW by dividing it by the total load, which is equal to 350,000KW. Equation (4-1) now becomes:

$$\begin{aligned} |(\Delta S.V./\Delta F.P.)/\Delta f| \\ = 9.004 \times 10^3 (f^2) - 1.034 \times 10^4 (f) \\ + 5.769 \times 10^3 \text{ \$/kw} \end{aligned}$$

(4-2)

The next step in the procedure is the formulation of a function, either in analytic or graphical form, relating  $|(\Delta \text{Social Value}/\Delta F.P.)/\Delta f|$  and Failure Probability. The analytic procedure is chosen since it results in a more systematic formulation of the Social Value function. Equations 4-2 and 3-12 are used to generate values of



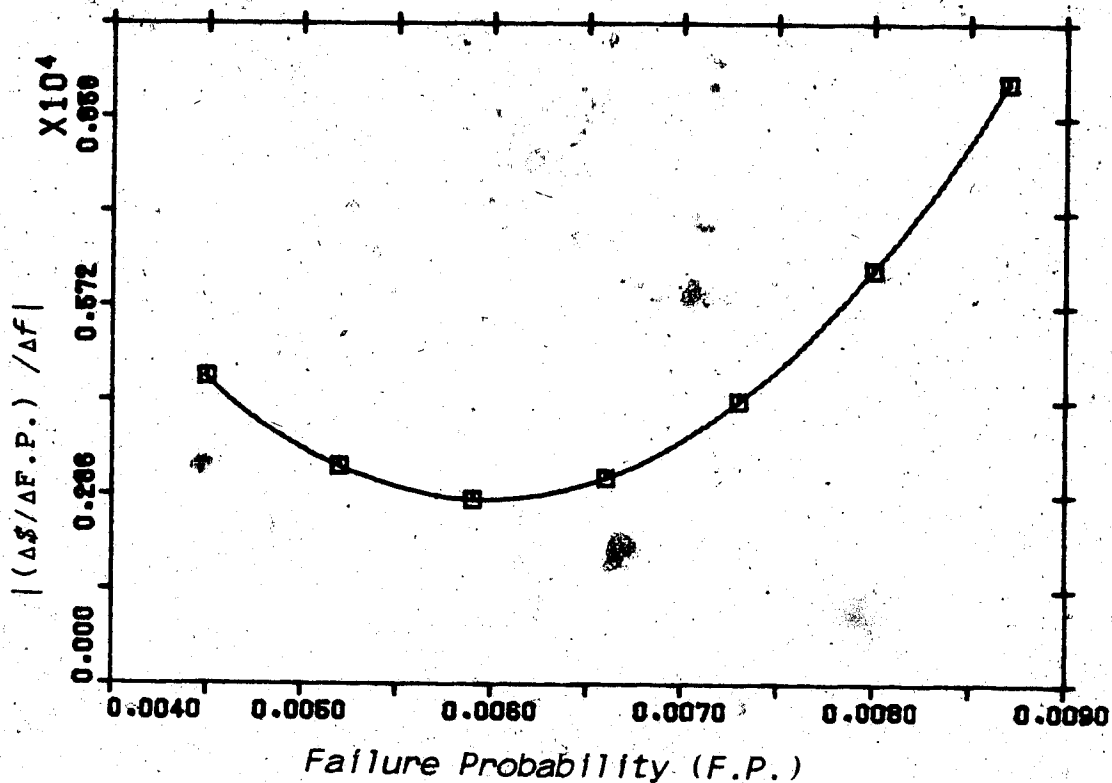


Figure 4-4. Plot of  $|(\Delta S / \Delta F.P.) / \Delta f|$  as a function of failure probability.

$|(\Delta S.V. / \Delta F.P.) / \Delta f|$  vs. F.P. for several values of  $f$ . Note the assumption is made that Equations (3-12) and (4-2) not just to the points shown in Table 4-2, but also to intervals between these points. Again, a quadratic curve (using apolynomial regression routine) has been fitted to these values and the resulting function is shown in Figure 4-4. The resulting equation is:

$$|(\Delta S.V. / \Delta F.P.) / \Delta f| = 8.540 \times 10^8 (F.P.)^2 - 1.021 \times 10^7 (F.P.)$$

$$+ 3.330 \times 10^4 (\$/KW)$$

(4-3).

It will be recalled that, since  $\Delta F.P.$  is negative, then  $[(\Delta S.V./\Delta F.P.)/\Delta F]$  is actually negative. Re-introduction of the negative sign and subsequent integration of Equation 4-3, with a suitable constant of integration, would yield the required Social Value function.

#### 4.3 Final Social Value Function

The integration yields the third order polynomial:

$$\begin{aligned} S.V.(F.P.) = & -2.847 \times 10^4 (F.P.^3) \\ & + 5.105 \times 10^4 (F.P.^2) - 3.330 \times 10^4 (F.P.) + A (\$/KW) \end{aligned} \quad (4-4).$$

where:  $S.V.(F.P.)$  indicates that  $S.V.$  is a function of  $F.P.$ ,  
 $A$  is the constant of integration.

The problem of choosing a suitable constant of integration must now be addressed.

Equation 4-4 is valid for  $F.P.$  in the range (0.0045, 0.0087), and the Social Value must be positive over this interval. Setting  $S.V.(0.0087)=0$  ensures positive Social Values in the given range. This results in  $A=90.79$ . The Social Value function is therefore:

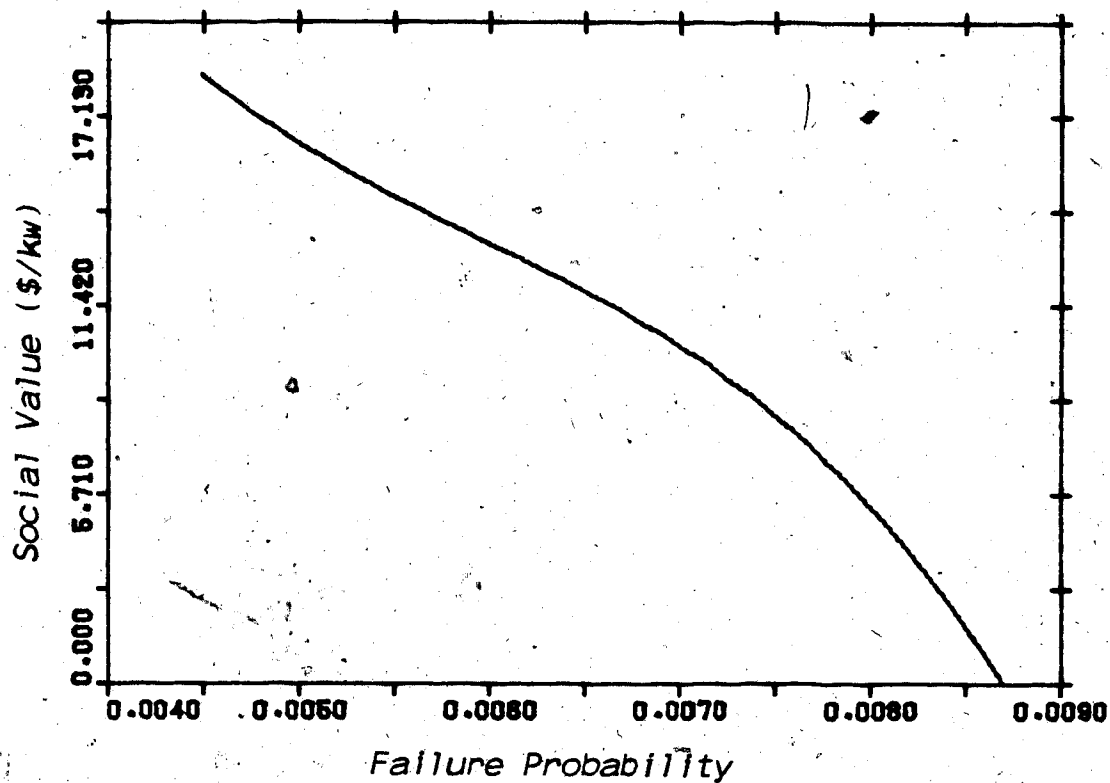


Figure 4-5. Social value of electric service reliability as a function of failure probability.

$$S.V.(F.P.) = -2.847 \times 10^4 (F.P.^2) + 5.105 \times 10^4 (F.P.) - 3.330 \times 10^4 + 90.79 \text{ (\$/kw)} \quad (4-5)$$

Values of S.V. are plotted in Figure 4-5. Table 4-3 shows these values.

Table 4-3. Values of S.V.

F.P.	Social Value. \$/Kw
0.004500	18.37
0.004850	16.88
0.005200	15.63
0.005550	14.54
0.005900	13.55
0.006250	12.57
0.006600	11.53
0.006950	10.36
0.007300	8.99
0.007650	7.34
0.008000	5.34
0.008350	2.91
0.008700	0.00

## 5. Comments and Conclusions

### 5.1 Analysis of Results

In Chapter 1 the reader was cautioned not to take the results obtained in Chapter 4 too seriously. Indeed it should be emphasized here again that due to the unavailability of sufficient and suitable information regarding system improvements and their costs, the results of Chapter 4 may not be very reliable.

It is important to the methodology described in this thesis that the simulated improvements be those that would be chosen by someone who knows the system very well. An improvement made by someone who is not thoroughly familiar with the system, and who does not have a feeling for the desirability of a specific improvement as perceived by the customer, is likely to cost more than its dollar value to the customer, hence the assumption that the cost of any improvement that would actually be implemented by the power utility is exactly offset by the value of this improvement to the customer, would not hold. If the improvements are small enough to be truly "marginal", then the improvements could be chosen arbitrarily. The additions simulated in Chapter 4 were arbitrarily chosen, but there was no way of determining whether they were "small enough" to qualify as marginal.

### 5.1.1 Social Value Curve

Despite the difficulty experienced in choosing improvements to the system, it still seems possible to draw certain conclusions from the Social Value curve. First, it does show a decrease in social worth as reliability decreases. This can perhaps be explained by the view that industrial and commercial customers who habitually enjoy only minor power interruptions may have over the years acquired a relatively large number of electricity-using equipment, so that these users are likely to value their electricity service rather highly. This may not be true of residential customers, since they are more likely to acquire home-electrical appliances whether their reliability of service is good or not, as long as they can afford the appliances (and sometimes when they cannot, too). On the other hand, industrial and commercial customers who receive a poor quality service may not have come to depend so heavily on electricity as a major source of their energy requirements. To these customers, the value of electric power is likely to be low.

The second conclusion that can be drawn from the Social Value curve concerns its slope at the various reliability levels represented in the diagram (Figure 4-5). At the two extremes of very high and very low reliability the curve is steep, indicating that these two sets of customers would value highly a rather small change in their present reliability levels. Perhaps this could be explained by the

concept that users who receive a high reliability have become so dependent on this resource that even a small drop in their reliability level would cause inconvenience. If this lower reliability were to continue, then perhaps it might eventually lead to a lesser dependence on electricity to supply the customers' energy needs.

At the other extreme of the reliability curve, customers might be influenced by a relatively small improvement in reliability, to invest in a larger proportion of electricity-using equipment, and hence they might value this reliability improvement quite highly.

Between these two extremes of very high and very low reliability levels, there are customers who receive some intermediate level. These customers are represented by a section of the Social Value curve which is fairly straight, and has a low slope. This seems to indicate that these customers have, over a period of time, established a balance between their dependence on electricity and on alternate sources of energy (e.g. gas for heating instead of electrical heating). Such customers are not seriously affected by a small increase or decrease in their reliability supplies. The change would have to be large, and the customer would have to be aware of his new level before he decides to make any changes in his energy balance.

It is difficult at this point to make any definite claims as to the correct shape of the social value curve. Further research would be necessary. This would require more

accurate data collection, and the application of the methodology to many systems. Perhaps some pattern would emerge that could be accepted as truly representing the Social Value curve. On the other hand, one might find significant differences between the social value curves of different societies.

### 5.1.2 Social Worth in \$/kw

The method proposed in this thesis computes the worth of a specific level of reliability in \$/kw. The actual values obtained in Chapter 4 seem to be within the same order of magnitude as values obtained by other methods (see Chapter 1). Failure probabilities obtained for the system ranged from 0.0040 to 0.0088. An examination of Figure 4-5 shows that values obtained for S.V. over this range were between 18.37\$/kw and zero. The Ontario Hydro study of large consumers (Figure 1-3) estimated social value by collecting data on outage costs, and representing them as a function of outage duration. Their values ranged from 0.01\$/kw of load interrupted for a duration of 0.01 hours to about 60\$/kw of load interrupted for 100 hours. It is difficult to relate the reliability index used in their study to failure probability (F.P), hence one cannot make a close comparison between their values and those obtained in Chapter 4. The average duration of an interruption of service to customers served by the 37-bus system must be less than 10 hours, since this is the average down-time of the system's



components, and since a component outage may not result in inadequate service to any customer. The Ontario Hydro value of outage cost for an interruption lasting ten hours is about 10\$/kw, which is within the range estimated in Chapter 4.

Figure 1-4 (Chapter 1) does not show outage costs for an interruption lasting ten hours. However, it is interesting to compare it with Figure 4-5. Values of outage costs shown there range from 0.01\$/kw to 100\$/kw for durations lasting from one minute to 480 minutes. If a straight line is drawn vertically through the 20-minute point on the horizontal axis of Figure 1-4, it will be seen that higher values of S.V. along this line are incurred by industrial and commercial consumers. It seems likely that industrial and commercial consumers lie at points on the social value curve of Figure 4-5 which have low failure probability. This is in agreement with Figure 1-4 in indicating that these customers do indeed have comparatively high values of S.V.

## 5.2 Advantages of the Proposed Method

In Chapter 1 a discussion was presented of several methods that have been used in the past for evaluating social worth. These included direct survey methods, the use of the GNP, and wages, and of the value of manufacture, to obtain estimates of outages costs. These techniques have

been shown to be inadequate in certain ways. This section explains the advantages over these methods which can be obtained by the method proposed in this thesis.

### 5.2.1 Advantages over "Direct Methods"

Direct methods involve listing the various losses suffered by consumers during a power outage, and valuing them. The major disadvantage with this approach is that it requires the customer to evaluate in dollars his intangible losses such as lack of safety, fear, and inconvenience. This is extremely difficult, if not impossible, for him to do. As the quote from Turvey in Chapter 1 graphically points out, even extremely skillful questioning may not be sufficient, and one may want to resort to the techniques he suggested. Unfortunately, such drastic procedures are not feasible, and would certainly do more harm than good.

The method proposed in this thesis avoids having to construct the demand, or "desirability" curve for electric power. The assumption that most power systems have evolved over the years where the long-term market equilibrium has been maintained implicitly includes an evaluation of this "desirability" curve. Unless the electric power market is experiencing serious upsetting forces as it was in the mid-1970's in the U.S., this assumption is usually true. In fact, most utilities place great emphasis on maintaining good customer relations, which involves keeping an attentive ear to customer complaints and rectifying any problem that

may arise in a manner acceptable to both customer and utility. Utilities usually monitor customer complaints, and may initiate corrective action when it is economically feasible to do so.

The regulating agency is seen as complementary to the forces acting to carry the market towards the point where reliability demand (desirability) is in equilibrium with supplied levels of reliability. At the equilibrium point the total capitalized system investment (measured in terms of \$/kw of peak load) is balanced by the capitalized consumer desirability, hence direct questioning of customers is not necessary.

#### 5.2.2 Advantages over the use of GNP

It has already been noted that the GNP technique does not take into account the individual nature of the system. This difficulty occurs because there are usually tens, or perhaps hundreds of individual power networks within a country. Even if the Regional Product were used instead of the National Product, there still might be more than one power network within the region. For even the smallest possible region for which data is available, the Gross Regional Product still does not take into account the individual characteristics of the customer himself.

The method proposed in this thesis evaluates marginal social worth at different load points within a single network. Each load point has its own characteristics

depending on the requirements of the customer, so that if the utility were to consider a marginal improvement to a set of customers, the characteristics of those customers may influence the manner in which the improvement is effected, and also its cost. Ideally, then the model of the network used in the analysis should include details of the individual distribution system loads. Although the model of the network used in Chapter 4 included only bulk supply points, the concepts remain the same, and indeed can just as easily be applied to a network which includes details almost down to the individual customer himself.

The method described in this thesis computes a relation between social value and reliability level, as a function of reliability level. The GNP, on the other hand, allows only the computation of a single aggregate social value on the assumption that all customers receive the same reliability. This is not true in practice, nor is it desirable to have all customers experiencing the same reliability. Power system planners require knowledge of the social values as perceived by customers with different reliability levels, so that their supply targets can be set to cope with their projections of new reliability levels as customers increase or decrease their desirability for electricity.

### 5.3 Projections for Further Research

An accurate Social-Value-of-Reliability curve is a planning tool that might be useful to the long-range power system planner. Capacity reserve margins might no longer have to be based on rule-of-thumb criteria. Instead, should such a curve be available for his system, the planning engineer or executive would then be able to balance utility investment with social worth to consumers. Should it be determined that there is a trend among a specific group of customers towards a higher reliability demand, perhaps due to the movement to higher income levels, the planning engineer would then be able to determine what level of system investment is required to adequately supply the needs of this group of customers. The framework discussed in this thesis for the evaluation of the social worth of reliability offers a way out of some of the difficulties faced by other methods. Further research must therefore include acquiring more accurate system cost data, and perhaps investigation of another method for evaluating the constant of integration,  $A$ , in Equation 4-5.

It may be argued that the methodology proposed in this thesis does not provide objective estimates of the true value of reliability. Be that as it may, the social value curve so constructed can nonetheless be a great asset to the normal system planning procedures. For this to be possible, several systems that have evolved using similar design standards, and which serve similar classes of customers,

would have to be analyzed. Social value curves would be obtained for all the systems. One might find that most of these curves lie close to each other, perhaps an acceptable band, as shown in Figure 5-1. A curve that lies outside this band would therefore be considered sub-optimal, indicating either an under or an over invested system. The long-range planner of such a system might then plan certain mid- or long-range improvements, or in the case of an over invested system, might delay certain planned investments, so that over a period of time his social value curve would move towards the others which were considered optimal. Further research must therefore also include analysis of many systems in order to determine whether their social value curves do indeed lie close to each other.

Suppose the social value curve for a particular system is constructed and is considered by its planners to be acceptable. Over time, certain parts of the system may change and result in distortion of the social value curve to an extent that is not considered acceptable. The curve may now resemble the dotted curve shown in Figure 5-2. The system planner would most likely want to take corrective action that would, over a period of time, result in a return to optimality of the affected sections of the system. Curves such as those that have been constructed in Chapters 3 and 4 can be used to provide guidelines as to what reliability targets should be aimed at. This can be done in two ways, as follows:

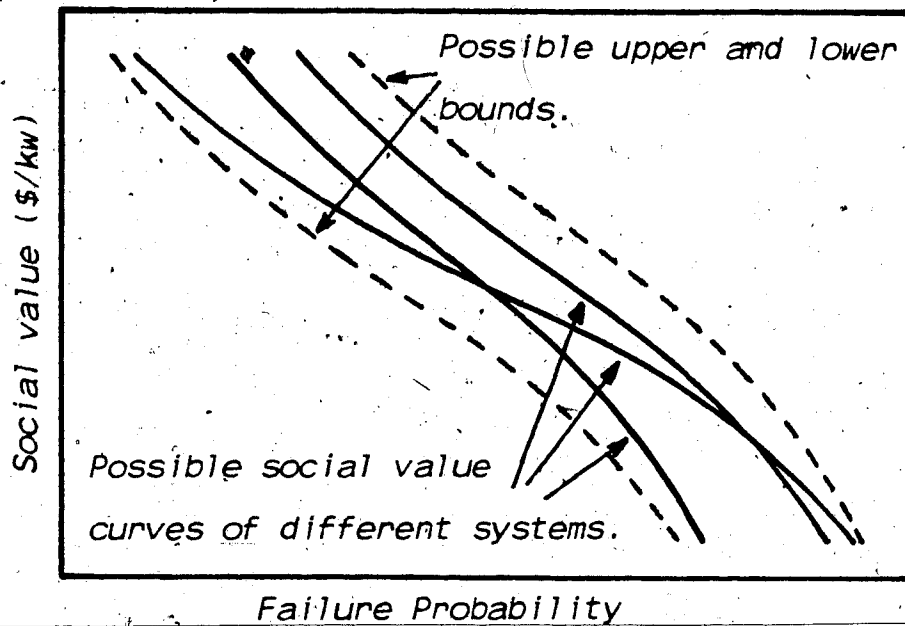


Figure 5-1. Diagram showing how several power systems might possibly have social value curves lying within two bounds.

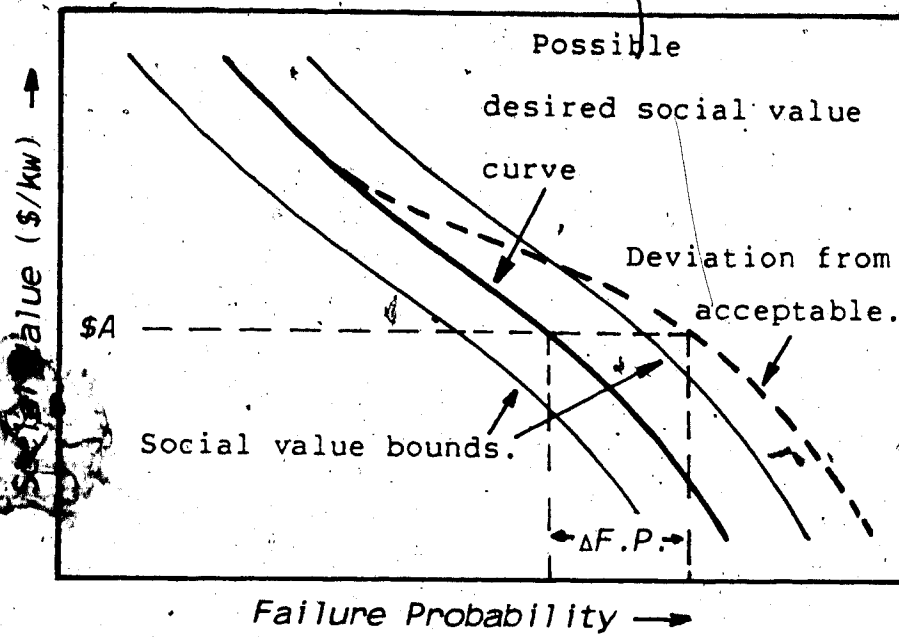


Figure 5-2. Diagram showing possible deviation of the social value curve of a system from within acceptable bounds.



Suppose the dotted curve in Figure 5-2 represents the system as it presently exists, and the solid curve represents the desired social value curve. Customers that had placed a value of \$A on their service reliability are now experiencing a lower reliability. Their reliability has shifted by an amount  $\Delta F.P.$  The utility can embark on two courses of action. One, it can implement a set of additions to bring the reliability back to the original level. The cost of doing this, per kw of load, can be obtained from Figure 4-4. This can then be translated by the utility into a desired rate increase in order to recover this expenditure.

The second alternative is to let the reliability remain at its present level. Customers might then be asked to reduce their dependence on electricity, perhaps by appeals for lower consumption, or by the use of load management devices. The utility may have more difficulty implementing this second alternative because of customer complaints. More research will have to be carried out to determine if the social value curve can indeed be used in this manner.

#### 5.4 Summary

A method was proposed for the derivation of an empirical formula for evaluating the Social Worth of Reliability of a specific power system, as a function of reliability. The method uses a voltage criterion, in

addition to the standard continuity criterion, to compute reliability indices for various sections of the system. The results show that it is unwise to compute a single value to represent the social worth of a whole power system as other studies have done.

**Bibliography.**

1. De Sieno C.F. and Stine L.L.: "A Probability Method for Determining the Reliability of Electric Power Systems"; *I.E.E.E. Transactions on Power Apparatus and Systems*, vol. 83, Feb. 1964, pp174-181.
2. Nofeni P.L. and Paris L.: "Quantitative Evaluation of Power System Reliability in Planning Studies"; *I.E.E.E. Transactions on Power Apparatus and Systems*, 1972.
3. Allan R.N., Takeddine F.N.: "Generation Modelling in Power System Reliability Evaluation"; *I.E.E. Conf. Pub. No. 148: "Reliability of Power Supply Systems"*, pp47-50, 1977.
4. Todd Z.G.: "A Probability Method for Transmission and Distribution Outage Calculations"; *I.E.E.E. Transactions on Power Apparatus and Systems*, vol. 83, 1964, pp695-701.
5. Gaver D.P., Montmeat F.E. and Patton A.D.: "Power System Reliability I-Measures of Reliability and Methods of Calculations"; *I.E.E.E.*

*Transactions on Power Apparatus and Systems*,  
vol.84, July 1964, pp636-643.

6. Billinton R. and Bollinger K.E.: "Transmission System Reliability Evaluation using Markov Processes"; *I.E.E.E. Transactions on Power Apparatus and Systems*, Feb. 1968, pp538-547.
7. Billinton R.: "Composite System Reliability Evaluation"; *I.E.E.E. Transactions on Power Apparatus and Systems*, 68-TP-76-PWR, I.E.E.E. Meeting, Jan. 1968, New York.
8. Billinton R. and Bhavaraju M.P.: "Transmission Planning using a Reliability Criterion, Part 1: A Reliability Criterion"; *I.E.E.E. Transactions on Power Apparatus and Systems*, vol. PAS-89, No.1, Jan. 1970.
9. Billinton R., Medicherla T.K.P. and Sachdev M.S.: "Composite Generation and Transmission System Reliability Evaluation"; Paper No. A 78 237-0, presented at the I.E.E.E. PES Winter Meeting, New York, Jan. 1978.
10. Mallard S.A. and Thomas V.C.: "A Method for Calculating Transmission System Reliability"; *I.E.E.E.*

*Transactions on Power Apparatus and Systems*,  
vol. PAS-87, No.3, March 1968, pp824-834.

11. Shipley R.B., Patton A.D. and Denison J.S.: "Power Reliability Cost vs Worth"; *I.E.E.E. Transactions on Power Apparatus and Systems*, vol. PAS-91, Sept.-Oct., 1972, pp2204-2212.
12. Marks G.E.: "A Method of Combining High-Speed Contingency Load Flow Analysis with Stochastic Probability Methods to Calculate A Quantitative Overall Measure of Power System Reliability"; Paper A 78 053-1 presented at the I.E.E.E. PES Winter Meeting, New York, Jan.-Feb., 1978.
13. Billinton R., Ringlee R.J. and Wood A.J.: "Power System Reliability Calculations"; (Book), M.I.T. Press, 1973.
14. Stott B. and Alsac O.: "Fast Decoupled Load Flow"; *I.E.E.E. Transactions on Power Apparatus and Systems*, vol. PAS-93, May/June, 1974. pp859-869.
15. Tinney W.F. and Hart C.E.: "Power Flow Solution by Newton's Method"; Presented at I.E.E.E.

*Winter Power Meeting, New York, 1967.*

16. Sato N. and Tinney W.F.: "Techniques for Exploiting the Sparsity of the Network Admittance Matrix"; *I.E.E.E. Transactions on Power Apparatus and Systems*, vol. 82, pp944-950, Dec. 1963.
17. Tinney W.F. and Walker J.W.: "Direct Solutions of Sparse Network Equations by Optimally Ordered Triangular Factorization"; *Proc. I.E.E.E.*, vol. 55, No. 11, Nov. 1969, pp1801-1809.
18. Stott B. and Hobson E.: "Solution of Large Power-System Networks by Ordered Elimination: A Comparison of Ordering Schemes"; *Proc. I.E.E.E.*, vol. 118, No. 1, Jan. 1971, pp125-134.
19. Fong C.C., Le Reverend B.K. and Neudorf E.G.: "Observations on Transmission Circuit Common Mode Dependent and Adverse Weather Outages"; *Ontario Hydro Report, Toronto, Ontario, 1979.*
20. Billinton R., Medicherla T.K.P. and Sachdev M.S.: "Application of Common Cause Outage Models in Composite System Reliability Evaluation"; *I.E.E.E. Summer Power Meeting, Paper No. A 79 461-5, 1979.*

21. Billinton R., Medicherla T.K.P. and Sachdev M.S.:  
"Adequacy Indices for Composite Generation and Transmission System Reliability Evaluation"; Paper A 79 024-1, presented at the I.E.E.E. PES Winter Meeting, New York, Feb. 4-9, 1979.
22. Samsa M.E., Hub K.A. and Krohm G.C.: "Electrical Service Reliability: The Customer Perspective"; Prepared for the U.S. Dept. of Energy under contract W-31-109-Eng-38 by Argonne National Laboratory, Energy and Environmental Systems Division, Sept. 1968.
23. Ferris L.L. and Sasson A.M.: "Investigating the Load Flow Problem"; Proc. I.E.E.E., vol. 115, No. 10, Oct. 1969, pp1458-1470.
24. Singh C. and Billinton R.: "System Reliability Modelling and Evaluation"; (Book), Hutchinson, 1977.
25. Billinton R.: "Power System Reliability"; (Book), Gordon and Breach, 1970.
26. Business Week: "UTILITIES: WEAK POINT IN THE ENERGY FUTURE"; January 20, 1975, pp46-54.

27. Fortune: "FOR THE UTILITIES, IT'S A FIGHT FOR SURVIVAL";  
*March, 1975, pp97-101.*
28. Sanghvil A.P.: "When is an Electricity "Shortage" a  
Shortage"; *Working Report, ICF Incorporated,  
Washington, D.C., 1980.*
29. Turvey, R.: "Economic Analysis and Public Enterprise",  
*Reading, Mass. (George Allen and Unwin,  
1971).*
30. "Costs of Interruptions in Electricity Supply": *Swedish  
Report From Committee on Supply Interruption  
Costs (English Edition), Sept., 1969.*
31. "Report on Reliability Survey of Industrial Plants, Part  
2 - Cost of Power Outages, Plant Restart  
Time, Critical Service Loss Duration Time,  
and Type of Loss vs Time of Power Outages":  
*Report by Reliability IEEE Subcommittee on  
Industrial and Commercial Power Systems, May,  
1973.*
32. Koval D.O.: "Evaluation of Distribution System  
Reliability"; *Ph.D thesis, Dept. of  
Electrical Engineering, University of  
Saskatchewan, Saskatoon, Saskatchewan, June,*



1978.

33. Calgary Power Limited: "Quality of Electric Power Service Study"; November, 1978.

34. Nelson D.C.: "A Study of the Elasticity of Demand for Electricity by Residential Consumers: Sample Markets in Nebraska"; *Land Economics*, February, 1965, pp92-96.

35. Chapman D., Mount T, and Tyrrell T.: "Modelling Electricity Demand Growth": *Energy Modelling- Art, Science, Ppractice; Working Papers for a Seminar on Energy Modelling, January 25-26, 1973, Washington, D.C. Milton F Searle, Editor. Resources for the Future, Inc.*

36. Lancaster K.: "Introduction to Modern Micro-economics"; *Rand Mc Nally College Publishing Company, Chicago. 1974.*

37. Gangel M.W. and Ringlee R.J.: "Distribution System Reliability Performance"; *I.E.E.E. Transactions on Power Apparatus and Systems*, vol. PAS-87, July, 1968. pp1657-1665.

38. Ford A.: "Simulating the Impact of Regulatory Changes on

electric Utilities"; University of California  
Los Alamos Scientific Laboratory, Los Alamos,  
New Mexico. 1978.

39. SRI Project Report No. 6841. "Large Scale System Effectiveness Analysis Sub-Problem 3. A Conceptual Framework for System Cost"; Prepared for U.S. department of Energy by Myers D.R., Lee W.C. and Yabroff I.W., Stanford Research Institute, Menlo Park, April 20, 1979.
40. Bhavaraju M.: "Reliability Measures for System Planning"; Electric Power Research Institute (EPRI) Journal, December, 1978, pp6-13.
41. Medicherla T.K.P, Billinton R. and Sachdev M.S.: "Generation Rescheduling and Load Shedding to Alleviate Line Overloads-Analysis"; Paper No. F78 686-8, presented at the IEEE PES Summer Meeting, Los Angeles, July, 1978.
42. Berk L.H., Mackay E.M.: "Ontario Hydro Survey on Supply Reliability: Viewpoint of Large Users"; CEA, March 21, 1977.

## Appendix I.

### Some Basic Concepts in System Reliability Modelling.[24]

A widely accepted definition of reliability is: "Reliability is the probability of a device performing its purpose adequately for the period of time intended, under the operating conditions encountered". This definition, although useful when applied to electronic components, is not directly applicable to power system reliability evaluation. Power systems do not have definite mission times, but are essentially required to operate forever. To define reliability as the probability of successful operation for a specified time period may be meaningful when applied to electronic or military systems, but is of no importance when applied to power systems. It is more appropriate to speak of quantitative measures which, when compared with reference indices, indicate expected consistency with, or deviation from, the required performance.

Measures of reliability may be time-dependent, (i.e. they change over a small period of time, perhaps a few seconds); these are required when the transient behaviour of the system is the subject of concern. Steady-state indices are of more importance to the economic planners of power systems. Meaningful steady-state measures of system "goodness" are: (1) average number of interruptions per

customer per year, (2) average interruption duration per customer per year, and (3) average and maximum expected duration of customer interruptions.

It is usual in the literature to define reliability indices in terms of success or failure. Many complex systems have, however, several levels of failure. For example, in the case where two generating units feed the load through two paralleled lines, the loss of one line may not result in the loss of generation of one unit, but the load may be limited to the emergency capacity of the remaining line. It is therefore appropriate to define the calculated reliability measures in terms of a subset  $X$  which may contain any number of system states. In particular applications  $X$  may be referred to as success, failure, or by some other appropriate name.

Steady State Measures of reliability are: (See Ref. 24)

(a) *Steady State Availability of Set X*. This can be interpreted in two ways: first as the probability of being in a state contained in  $X$ , and second as the time spent in  $X$  as a fraction of the total time  $(0, T)$ , as  $T$  tends to be very large.

(b) *Steady State Frequency of Encountering X*. This can also be defined in two ways. The first is the mean rate at which  $X$  is being encountered at some point in time remote from the origin. The second is the average number of encounters of  $X$ , considered over a very large time interval (over 20 years).

(c) *Mean Cycle Time*. This is time between two successive

encounters of X. It is the reciprocal of the Steady State Frequency. If X is system failure, then the Mean Cycle Time is called the Mean Time Before Failure, MTBF.

(d) *Mean Duration of X*. This is the expected time of residence in X in one cycle. If X is system failure, then the Mean Duration of X is the repair (or replacement) time of the system.

(e) *Mean Passage Time*. If X is system failure, the Mean Passage Time is called the Mean Time To failure, MTTF. It differs from the MTBF by the repair time of the system.

## Appendix II.

### Formulation of the FDLF.

The polar power-mismatch in Newton's method is taken as the starting point of the formulation. The real sparse Jacobian matrix equation is :

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V/V \end{bmatrix} \quad (\text{II-1})$$

where:

$\Delta P, \Delta Q$  = real and reactive bus power mismatch vectors  
 $\Delta \theta, \Delta V$  = bus voltage angle and magnitude correction vectors.

The MW- /MVAR-V decoupling principle is applied by first neglecting the coupling submatrices N and J, since these terms are usually very small. This leaves two separated equations which may be solved independently:

$$[\Delta P] = [H] [\Delta \theta] \quad (\text{II-2})$$

$$[\Delta Q] = [L] [\Delta V/V] \quad (\text{II-3})$$

where:

H is a square matrix of dimension  $p \times p$ ,

L is a square matrix of dimension  $q \times q$ ,

$p$  = total number of buses less one (the swing bus),

$q$  = number of P-Q buses,

$$H_{ij} = L_{ij} = V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$

for  $i \neq j$

$$H_{ij} = -B_{ij} V_i^2 - Q_i,$$

$$L_{ij} = -B_{ij} V_i^2 + Q_i,$$

for  $i = j$ ,

$G_{ij} + jB_{ij} = (i, j)$ th element of bus admittance matrix  $[G]$   
 $+ j[B]$ .

$\theta_i, V_i =$  voltage angle, magnitude at bus  $i$ .

$$\theta_{ij} = \theta_i - \theta_j$$

$$Q_i = V_i \sum_{j \in j} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$

$i \in j$  signifies that  $i$  is connected to  $j$ .

Further physically justifiable simplifications are made using the following assumptions:

$$\cos \theta_{ij} \sim 1; G_{ij} \sin \theta_{ij} \ll B_{ij};$$

$$Q_i \ll B_{ii} V_i^2.$$

Equations II-2 and II-3 can now be written as follows:

$$[\Delta P] = [V \cdot B' \cdot V] [\Delta \theta] \quad (\text{II-4}).$$

$$[\Delta Q] = [V \cdot B'' \cdot V] [\Delta V/V] \quad (\text{II-5}).$$

The elements of  $B'$  and  $B''$  are elements of  $[B]$ , the imaginary part of the bus admittance matrix. The final algorithm is completed by making the following modifications to equations II-4 and II-5:

1. Omit from  $[B']$  in (4) network components that affect MVAR flow, e.g. shunt reactances and off nominal in-phase transformer taps.
2. Omit from  $[B'']$  in (II-5) the angle shifting effects of phase shifting transformers.
3. The left-hand  $V$  terms in (II-4) and (II-5) are taken over to the left hand sides of the equations,

and then in (II-4), the influence of MVAR flows is removed by setting all V terms on the right hand side to 1.0 per unit.

4. Series resistances are neglected in calculating the elements of [B'], which now becomes the DC approximation load flow matrix.

The final form of the Fast Decoupled Load Flow equations are now:

$$[\Delta P/V] = [B'] [\Delta \theta] \quad (\text{II-6}).$$

$$[\Delta Q/V] = [B''] [\Delta V] \quad (\text{II-7}).$$

Equations II-6 and II-7 are solved alternately, with the correction produced by one applied to the other before it is solved. This is referred to by Stott and Alsac[14] as the (1 $\theta$ , 1V) iteration scheme.

#### Generator Q-Limits.

The method described by Stott and Alsac for correcting violations to P-V bus Q-limits without retriangulation of B'' was not considered satisfactory, due to its oscillatory nature. Instead, the violating buses are explicitly converted to P-Q type, and the MVAR output is held at the limiting value. This requires retriangulation of the matrix B'', but takes less iterations to converge to an accurate solution.



### On-Load Transformer Tap Changing.

The conventional technique of representing a transformer by an equivalent pi model is compatible with the FDLF method. The adjusting algorithm for in-phase off nominal taps  $t$  is:

$$t_i(\text{new}) - t_i(\text{old}) = \alpha (V_i - V_i^{\text{sp}}),$$

where:

$\alpha$  = error feedback factor, = 1,

$V_i^{\text{sp}}$  = controlled-bus specified voltage.

This has been found to give results identical to those obtained when the algorithm is used with the straight Newton method, and in a similar number of iterations. The minimum tap size is taken as 0.005. Of course a tap change requires adjustment of the appropriate elements in the bus admittance matrix.

## Appendix III.

## 1. Table III-1.

## Detailed Outage Data for System 2.

From Bus	To Bus	Failures per year	Duration (hours)
400	401	0.024	168
400	476	1.0435	20.39
401	402	0.024	168
401	405	0.024	168
401	406	0.024	168
960	401	0.024	168
402	403	0.024	168
402	404	0.024	168
402	410	0.136	10.0
402	412	0.272	10.0
410	414	0.680	10.0
410	422	0.272	10.0
412	438	1.360	10.0
414	418	2.448	10.0
422	484	2.448	10.0
424	425	0.024	168
424	440	0.024	168
960	424	0.024	168
425	428	3.672	10.0
428	432	2.448	10.0

432	436	3.128	10.0
436	438	1.904	10.0
436	452	3.762	10.0
440	441	0.024	168
441	444	6.256	10.0
444	448	3.808	10.0
448	452	2.176	10.0
452	462	4.896	10.0
462	464	2.448	10.0
462	464	2.448	10.0
1039	462	0.024	168
464	468	5.576	10.0
464	473	3.128	10.0
468	482	0.952	10.0
472	473	0.024	168
472	477	1.7558	13.69
476	477	0.024	168
477	478	0.024	168
478	479	0.68	10.0
479	480	0.816	10.0
479	482	2.448	10.0
482	484	1.224	10.0

## 2. Improvements to System.

- (i). Addition of Capacitors to buses 448, 452, 464, 432. Effect is the improvement of voltage at some buses under certain outage conditions.
- (ii) Addition of Parallel Transformer between buses 401 and 402. Effect is the alleviation of overloaded condition of existing transformer under certain outage states.
- (iii) Addition of transformer between buses 406 and 401. Effect is to prevent loss of generation under outage of existing transformer.
- (iv) Transformer added between buses 472 and 473--to prevent overload of existing transformer.
- (v) Addition of extra transformer between buses 403 and 402.
- (vi) Addition of 30mw of reserve generation.

