



# Stochastic Simulation of Tailings Consolidation Process

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# Agenda



- Background
- Introduction to Causal Loop Diagrams (CLD)
- Re-interpretation of Consolidation using CLD
- Stochastic Setup
- Concluding Remarks

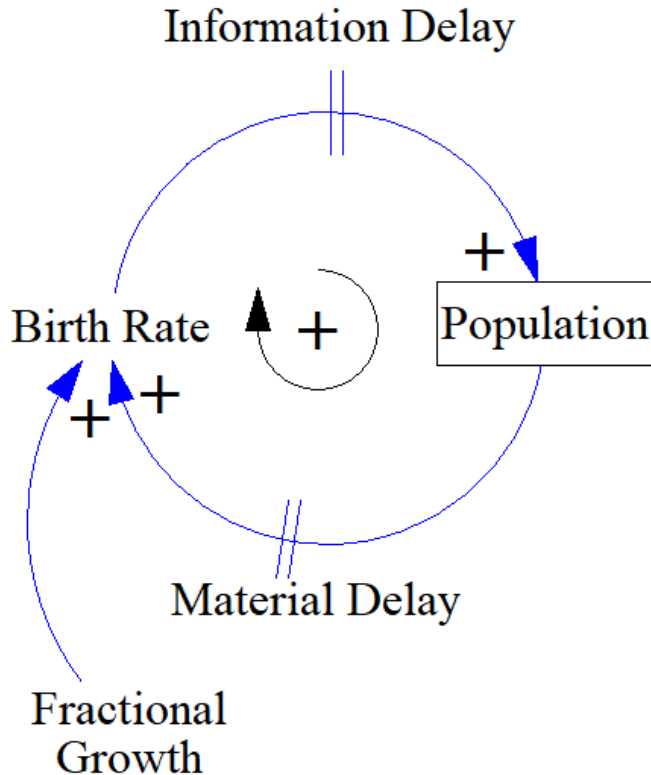


## Why Care About Tailings Consolidation ?

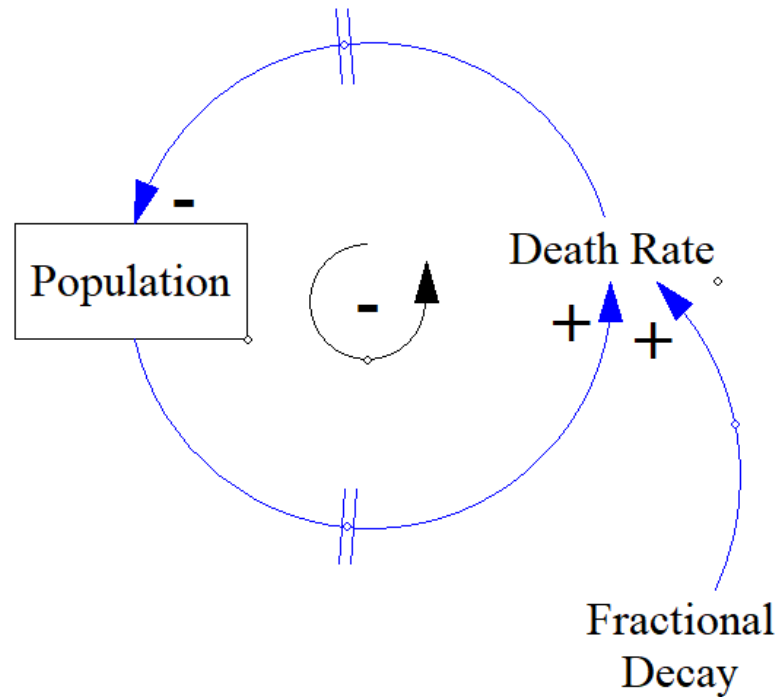
- Reclamation Schedule and Cost
- Regulatory Pressure
- Long-Term Ecological Impact

# Causal Loop Diagram (CLD)

**Positive Feedback = Reinforcing**



**Negative Feedback = Balancing**



Rate = Fraction (% per time step) \* Population

$$\text{Stock}(t) = \text{Stock}(t_0) + \int_{t_0}^t [\text{Inflow}(s) - \text{Outflow}(s)] ds$$

# Finite Difference Method (Cargill, 1982)

Governing Equation:

$$\left\{ \gamma_c \beta(e) + \frac{\partial}{\partial z} [\alpha(e)] \right\} \frac{\partial e}{\partial z} + \alpha(e) \frac{\partial^2 e}{\partial z^2} + \gamma_w \frac{\partial e}{\partial t} = 0$$

$$\alpha(e) = \frac{k(e)}{1+e} \frac{d\sigma'}{de} \quad \beta(e) = \frac{d}{de} \left[ \frac{k(e)}{1+e} \right]$$

Solution:

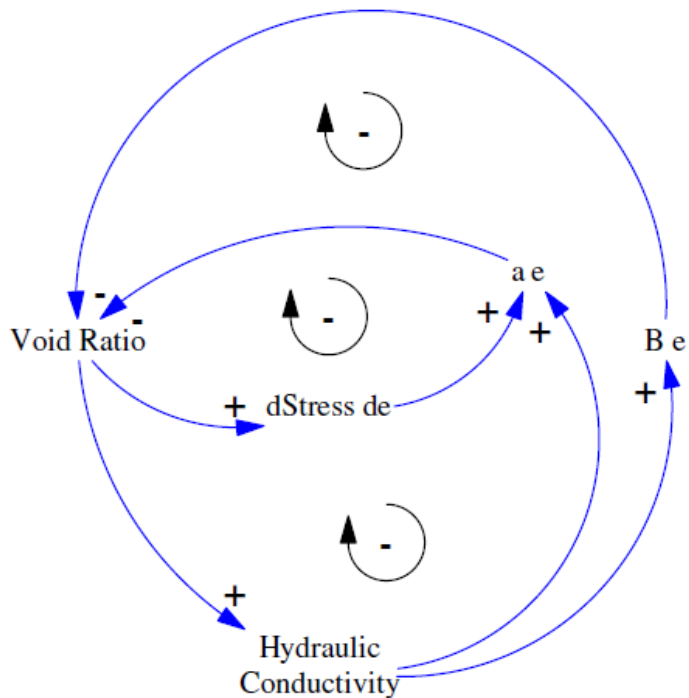
$$e_{i,j+1} = e_{i,j} - \frac{\tau}{\gamma_w} \left( \left\{ \gamma_c \beta(e_{i,j}) + \left[ \frac{\alpha(e_{i+1,j}) - \alpha(e_{i-1,j})}{2\delta} \right] \right\} \right)$$

$$\left[ \frac{e_{i+1,j} - e_{i-1,j}}{2\delta} \right] + \alpha(e_{i,j}) \left[ \frac{e_{i+1,j} - 2e_{i,j} + e_{i-1,j}}{\delta^2} \right]$$

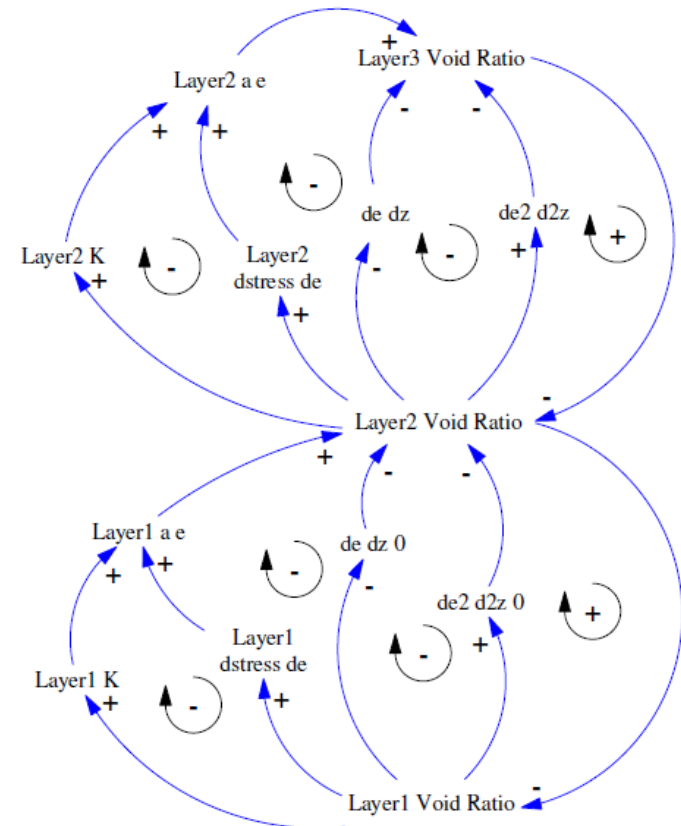
# Re-interpretation of Cargill (1982)

## 9 Negative Feedback Loops and 2 Positive Feedback Loops

### Temporal Feedback

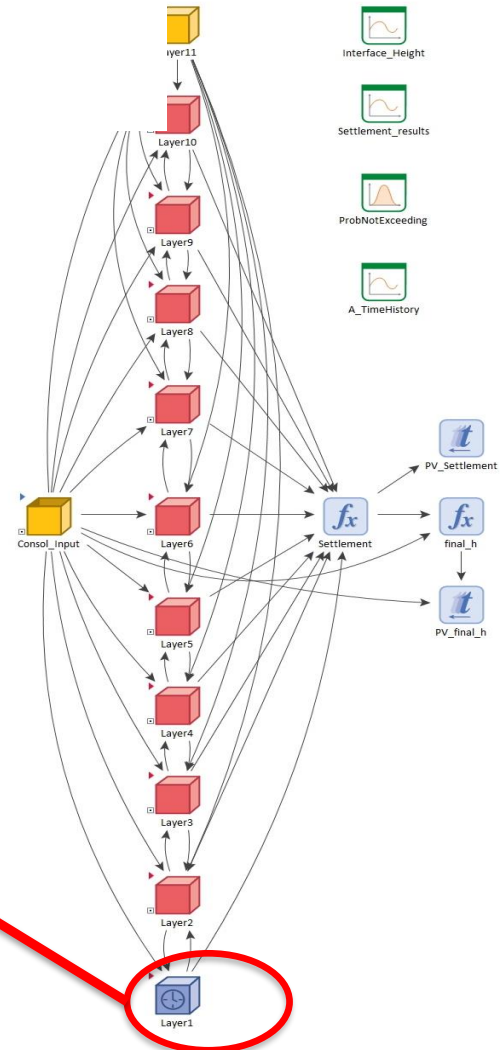
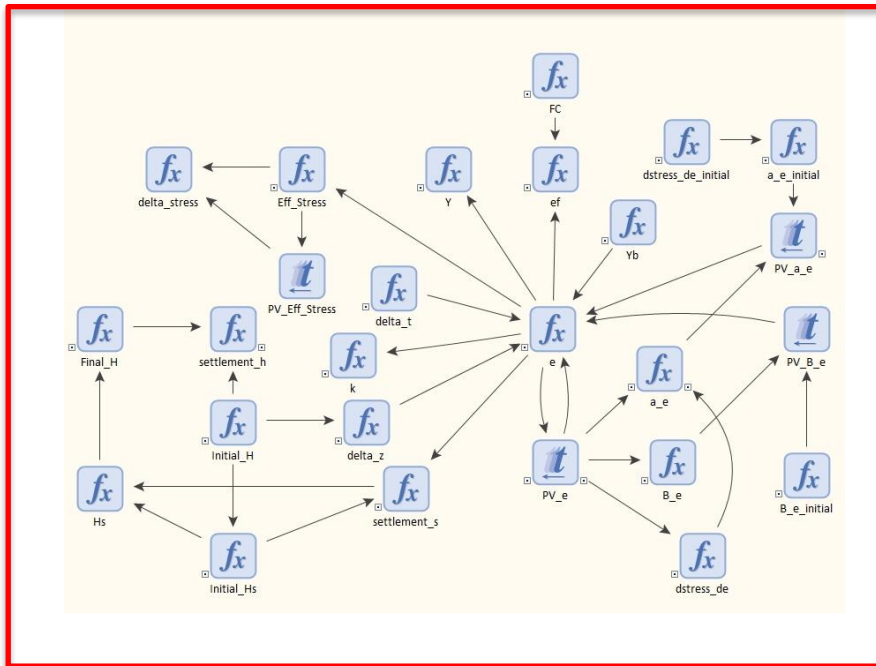


### Spatial Feedback



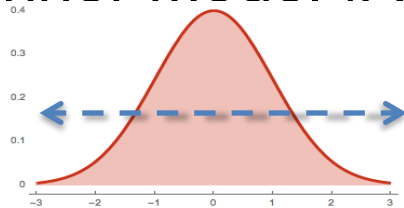
# Methodology

## Key Stochastic Input: Compressibility and Hydraulic Conductivity Fitting Parameters

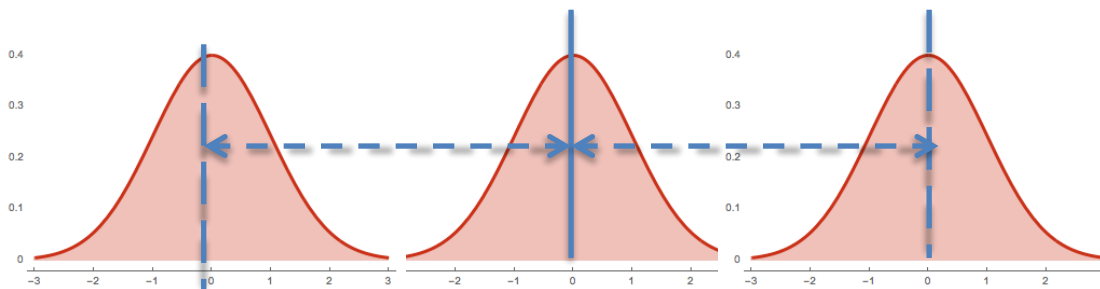


# Stochastic Model Setup

- Aleatory Uncertainty (“Inherent” Randomness)
- Epistemic Uncertainty (Lack of Knowledge)
- **Nested** Monte Carlo (Distribution of Distribution)
- Dynamic Inner Model (Aleatory)



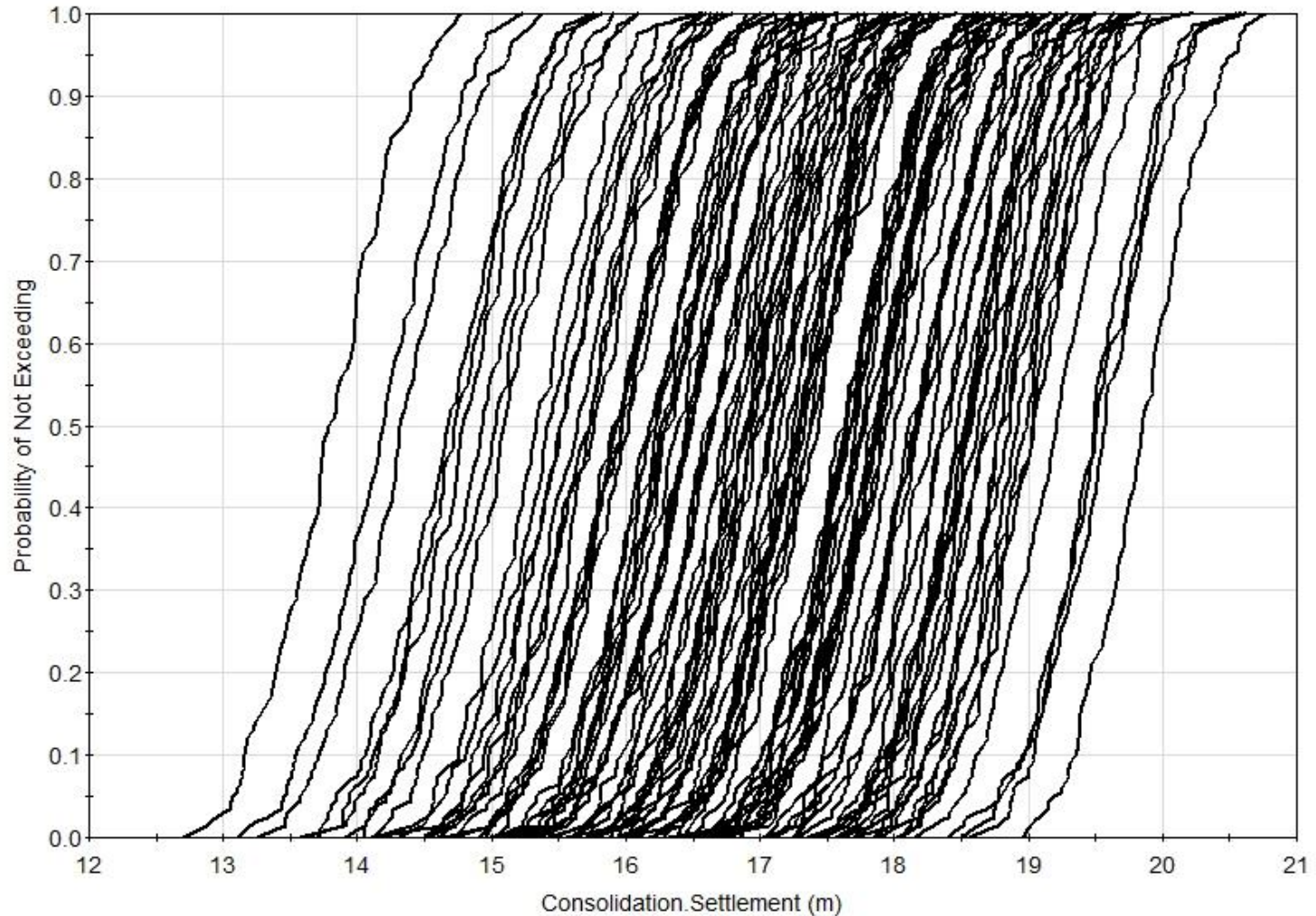
- Static Outer Model (Epistemic)



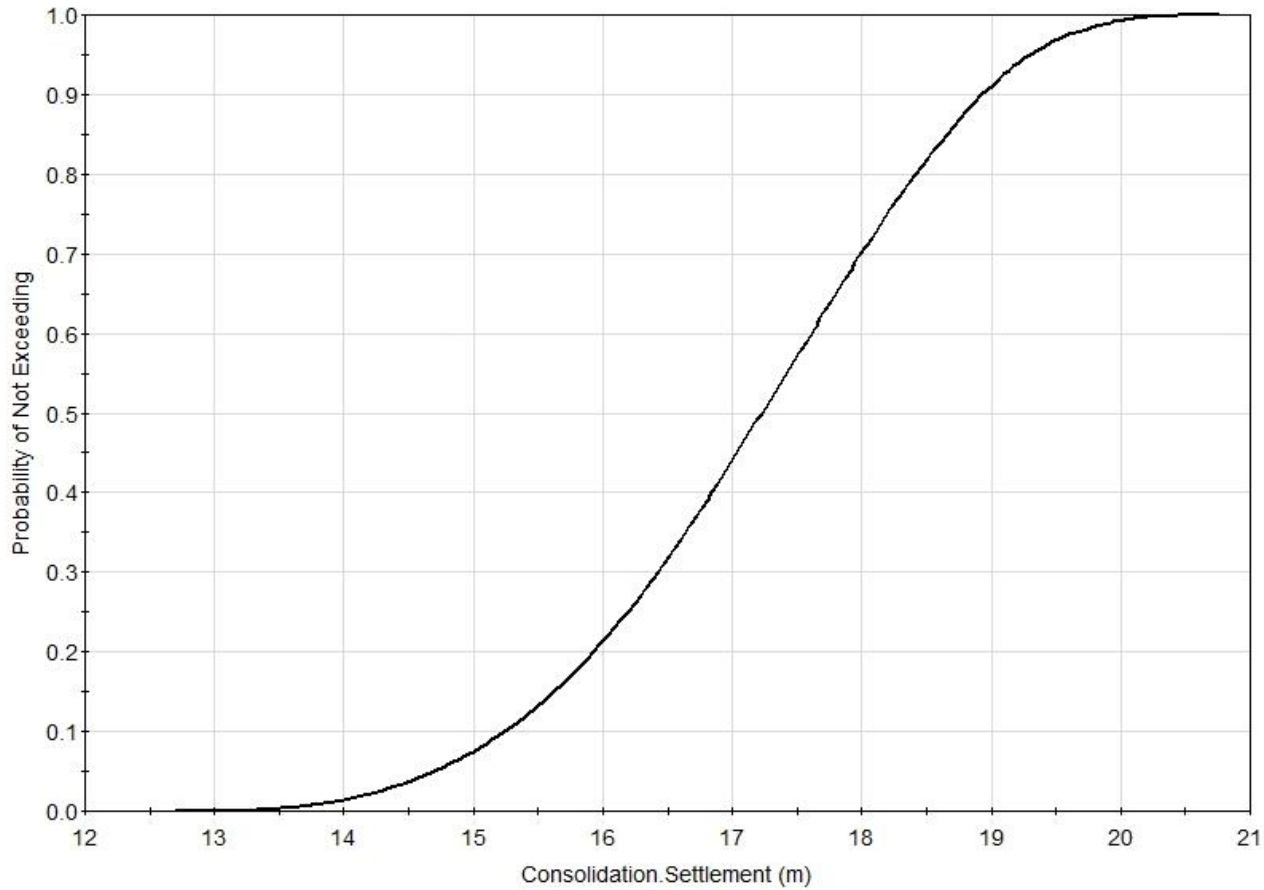


# Stochastic Model Setup

100 X 100 Realizations = 10,000 Monte Carlo runs



# Stochastic Model Setup



# Concluding Remarks

- Simplicity is found in complexity using Causal Loop Diagrams (CLD).
- CLD coupled with Monte Carlo techniques can be a useful qualitative and quantitative tool in Risk Management Modeling
- Separation of Aleatory and Epistemic uncertainties (nested Monte Carlo) demands intensive computing power

# Acknowledgements

- Dr Nicholas Beier
- NSERC / COSIA Industrial Research Chair (IRC) program and Alberta Innovates Energy and Environment Solutions (AIEES)
- Software Support: GoldSim, Vensim and FSCA (Dr Silawat Jeeravipoolvarn)