Stochastic Simulation of Tailings Consolidation Process

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ABSTRACT
A system dynamics (SD) model was developed in the Tailings Management Simulator (TMSim) to simulate consolidation processes under quiescent conditions. A top-down iterative approach guides the overall philosophy of the modelling process. The GoldSim software was used as the main simulation environment to implement various stock-flow relationships and causal loop diagrams. For quiescent consolidation, an explicit finite difference scheme was used to solve the governing equation for one-dimensional large-strain consolidation process. The SD consolidation model was calibrated with a commercial software FSCA under a variety of tailings parameter input and deposit geometry. The model was also validated against past case histories and experimental data. Simulation results demonstrated that the SD model is capable of preserving key physics of the large-strain consolidation process while exposing important variables in a simplified and transparent manner. Once the deterministic base case is successfully simulated, stochastic processes are incorporated. Uncertainties are addressed by assigning probability distributions to selected input parameters. Nested Monte Carlo techniques will be used to explicitly model the two types of uncertainties: those due to inherent randomness (i.e. fines content) and those due to lack of knowledge (i.e. insufficient data collection or ignorance).

Un modèle de dynamique des systèmes (SD) a été développé dans le simulateur de gestion des résidus (TMSim) pour simuler les processus de consolidation des résidus dans des conditions de repos. Une approche itérative descendante guide la philosophie globale du processus de modélisation. Le logiciel GoldSim a été utilisé comme l' environnement principal de simulation pour implémenter différentes relations stock-flux et diagrammes de boucles simples. Pour la consolidation au repos, un schéma de différences finies explicite a été utilisé pour résoudre l'équation régissant le processus de consolidation unidimensionnel à grande contrainte. Le modèle de consolidation SD a été calibré avec un logiciel commercial FSCA sous une variété de paramètres d'entrée des résidus et de géométrie de dépôt. Le modèle a également été validé par rapport aux antécédents de cas et aux données expérimentales. Les résultats de la simulation ont démontré que le modèle SD est capable de préserver les principales caractéristiques du processus de consolidation à grandes contraintes tout en exposant les variables importantes de manière simplifiée et transparente. Une fois le cas de base déterministe simulé avec succès, les processus stochastiques sont incorporés. Les incertitudes sont adressées en affectant des distributions de probabilité aux paramètres d'entrée sélectionnés. Les techniques de mise à jour de Monte Carlo imbriquées seront utilisées pour modéliser explicitement les deux types d'incertitudes: celles dues au caractère aléatoire inhérent et celles dues au manque de connaissance.

1   INTRODUCTION

In the oil sands mining industry, the Tailings Management System (TMS) is a complex process that involves dynamic interaction among distinct but inter-related sub-systems. Various research has focused, in isolation, on the individual components of TMS at a micro scale (Beier et al, 2014). Therefore, a system dynamic simulation tool, TMSim, was developed to address this lack of integration and bring together different disciplines on a common platform. GoldSim software was chosen as the primary simulation engine for its graphical user-friendly interface, object-oriented programming environment and ability to provide insights in system behavior under data-poor scenarios frequently encountered in mining operations.

Current tailings regulatory framework (Directive 085) in Alberta emphasizes the importance of fluid tailings management through volumetric reduction of both legacy Mature Fine Tailings (MFT) and future growth of fluid fine tailings. The implementation of new dewatering technologies, some borrowed from metals mining industry while others still in the pilot stage, poses significant risks when the extended mine life of oil sands mines is taken into account. To effectively communicate risks, it is useful to first classify uncertainties into two separate categories: those due to lack of knowledge or ignorance (epistemic) and those due to inherent variabilities (aleatory).

For the deterministic model, a simplified quiescent or self-weight consolidation module was created in TMSim based on the causal loop diagram in Figure 1 and an explicit finite difference numerical scheme described in Section 3.3.2. For the Monte Carlo simulation, compressibility (e-σ') and permeability (e-k) relationships are assigned probabilistic distributions. Nested Monte Carlo technique is used to explicitly model epistemic and aleatory uncertainties. A simple case study based on deposition of Thickened Tailings (TT) is demonstrated here.
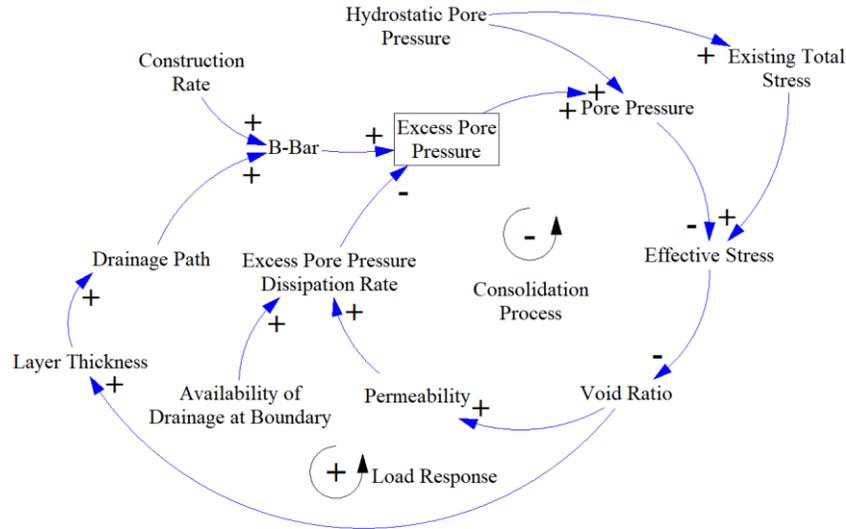
Figure 1: Conceptual causal loop diagram prior to detailed model development

2   OBJECTIVES

The objectives of this paper are developed sequentially during the modelling process and are summarized below:

   a) Implement a consolidation module based on large-strain consolidation theory in a SD-based environment.
   b) Using the base case model developed above, explicitly model uncertainties due to lack of knowledge and those due to inherent variabilities using nested Monte Carlo techniques.
   c) Demonstrate the value of probabilistic approach in communicating uncertainties to regulators and internal stakeholders.

3   DETERMINISTIC MODEL

3.1   Causal Loop Diagram

The first step in SD-based model formulation is to identify feedback loop structures by creating a causal loop diagram. Causal loop diagram is an effective tool of communication during early conceptual development stage (Richardson, 1986). In this modelling exercise, causal loop diagram is used for conceptualization purposes only.

   The dynamics of consolidation process is conceptualized as a causal loop diagram drawn in the Vensim software (Figure 1). Stock variables are represented by labels inside a rectangle. Flow rate and converters that explain the flow rates are simply labelled in plain text. The positive sign at the arrow head indicates a positive relationship, that is an increase in one variable will cause increases in another. The negative sign denotes a negative or inverse relationship, that is an increase in one variable will cause decreases in another. In a closed loop,

odd number of negative signs indicate negative feedback structure while even number of negative signs indicate positive feedback structure. Negative feedback brings the system to equilibrium state while positive feedback amplifies growth and cause run-away behaviors (Ford, 2010).

   In Figure 1, excess pore pressure is modelled as a stock element with construction rate as inflow and dissipation rate as outflow. Assuming that the Darcy's Law is valid and that the principle stress in the foundation soil can be approximated by the vertical stress (i.e. no principle stress rotation), an increase in construction rate will trigger a series of chain reactions and lead to a negative feedback structure, which makes sense since the consolidation process brings the system back into balance. The counter-clockwise loop symbol is also given a name to communicate the major theme of the feedback structure.

   This paper will focus on modelling the consolidation feedback loop in Figure 1.

3.2   Hydraulic Conductivity and Compressibility Constitutive Relationships

To solve the governing equation of large-strain consolidation, two key relationships are required: compressibility or effective stress-void ratio and saturated hydraulic conductivity-void ratio (Jeerivalpoolvarn, 2010). Constitutive relationships are derived from experimental data and most commonly curve-fitted to a power law function below:

$$e = A\sigma'^{B} \qquad [1]$$

$$k = Ce^{D} \qquad [2]$$

Where e is the average void ratio of the soil layer and k is the hydraulic conductivity in m/day; A, B, C and D are curve-fitting coefficients.

Other forms of equations, such as Weibull functions for hydraulic conductivity, have been proposed to handle different types of tailings and deposition conditions. For demonstration purposes and simplicity, the power law functions are used in the TMSim module. If required, users can easily define customary constitutive relationships in TMSim.

3.3 Model Formulation

3.3.1 Governing Equation

The one-dimensional finite strain consolidation theory (Gibson et al, 1967) has been the theoretical basis for modelling consolidation behavior of soft soil and tailing slurry. The theory assumes that Darcy's Law is valid, properties of soil skeleton is not time-dependent and there is no lateral consolidation strain. The governing equation is derived from satisfying both material equilibrium and fluid continuity equations and expressed in terms of void ratio below:

$$\pm \left(\frac{\rho_s}{\rho_f} - 1\right)\frac{d\left[\frac{k(e)}{1+e}\right]}{de}\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[\frac{k(e)}{\rho_f(1+e)}\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right] + \frac{\partial e}{\partial t} = 0 \qquad [3]$$

Where ρ_s is the solids density; ρ_f is the fluid density; e is the void ratio; k is the hydraulic conductivity expressed as function of void ratio; σ' is the effective stress also expressed as function of void ratio; t is the time step; and z is the material coordinate.

3.3.2 Numerical Solution

An analytical solution to equation [3] is not possible due to non-linearity of its coefficients. Finite difference methods based on either explicit or implicit scheme are used to solve equation [1]. Bromwell (1984) and Pollock (1988) showed that explicit scheme produced similar and sometimes better results than the implicit scheme provided that stability and convergence issues are properly addressed. Additionally, the ability to reduce spatial discretization in the implicit scheme is difficult to achieve in SD-based software. Implicit scheme also involves solving systems of equations at each time step, requiring external solvers to be linked to the SD platform.

Therefore, an explicit numerical scheme is chosen for its simplicity, non-iterative nature and ease of implementation in a SD environment. For the TMSim module, an explicit backward time and central difference space numerical scheme formulated by Cargill (1982) is used to solve the governing equation [3]. The solution is written as:

$$e_{i,j+1} = e_{i,j} - \left(\frac{\Delta t}{\gamma_w}\right)\left[\left(\gamma_b \beta(e_{i,j}) + \frac{a(e_{i+1,j}) - a(e_{i-1,j})}{2\Delta z}\right) * \frac{e_{i+1,j} - e_{i-1,j}}{2\Delta z} + a(e_{i,j}) * \left(\frac{e_{i+1,j} - 2e_{i,j} + e_{i-1,j}}{(\Delta z)^2}\right)\right] \qquad [4]$$

Where γ_w is the unit weight of water; γ_b is the buoyant unit weight of solids; γ_s is the unit weight of solids; i is the spatial increment and j is the time increment; Δz is the mesh discretization in material coordinate and Δt is the time discretization. $a(e_{i,j})$ and $\beta(e_{i,j})$ are re-formulated in terms of void ratio and power law curve-fitting coefficients:

$$a(e_{i,j}) = \left(\frac{Ce^D}{1+e}\right) * \left[\left(\frac{1}{A}\right)^{\frac{1}{B}}\left(\frac{1}{B}\right)e^{\frac{1}{B}-1}\right] \qquad [5]$$

$$\beta(e_{i,j}) = \frac{CDe^{D-1}}{1+e} - \frac{Ce^D}{(1+e)^2} \qquad [6]$$

$$\gamma_b = \gamma_s - \gamma_w \qquad [7]$$

For impermeable boundaries, an imaginary mesh point below the bottom boundary is used:

$$e_{0,j} = e_{2,j} + 2\Delta z \left(\frac{de}{d\sigma'}\right)_{e_{i,j}} * \gamma_b \qquad [8]$$

Where $\frac{de}{d\sigma'}$ is determined from compressibility relationship at $e_{1,j}$. Once the fictitious $e_{0,j}$ is calculated at time step j, $e_{1,j+1}$ can be determined from equation [4]. Then the entire process is repeated at each time step.

3.3.3 Model Calibration

Calibration of the model involves optimization of spatial discretization and time stepping at the bottom boundary. The SD module was initially developed with only three layers of spatial discretization. Once the simulated behaviors qualitatively matched the expected behaviors, the number of discretization was increased and kept at ten layers as a compromise between numerical accuracy and ease of model maintenance and communication.

Further discretization of spatial and time step variable, denoted by an integer value of N, is required for equation [4] and [8] since void ratio is highly sensitive to changes in effective stress at the initial stage of consolidation. For example, to model a deposit thickness of 1.5 m, a time step of 1 second and spatial discretization of 100 at the bottom boundary are required. However, as general rule of thumb, the value of N shall be kept as small as possible to avoid truncation errors due to abrupt transition from finer mesh to coarser mesh. Very fine mesh also requires Δt to be extremely small, significantly increasing computing time. This manipulation of Δz and Δt is only applied at the

bottom boundary whenever large values of $\frac{de}{d\sigma'}$ are expected. The rest of spatial and time discretization scheme remains the same during simulation.

3.3.4 Model Visibility and Scalability

Model organization takes advantage of the object-oriented programming environment in GoldSim. In the model, each discretized layer is organized in containers (Figure 2). All elements and relationships of the numerical solution are exposed and visible to the user (Figure 3), facilitating communication of the numerical process. With the exception of the bottom layer where special manipulation in spatial discretization and time step is required, elements in all other containers share the same functional relationships and naming conventions. Therefore, by making the model scalable and repeatable, additional layer discretization can be easily added or removed with minimum amount of work. The benefit of this modelling best practice cannot be under-estimated when SD model becomes large and complex.
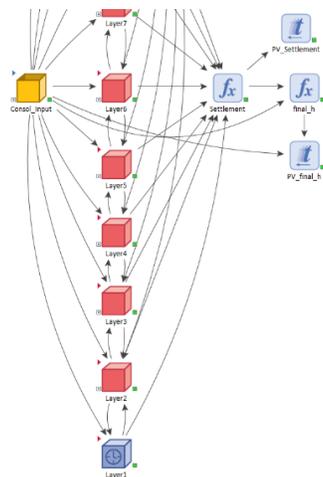
Figure 2. Layer setup using containers
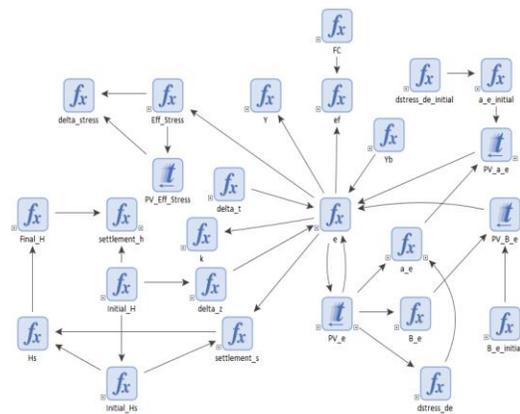
Figure 3: Exposed elements of the numerical solution inside each container

4   VALIDATION OF DETERMINISTIC MODEL

Three sets of scenarios were executed to simulate the quiescent consolidation of tailings with different compressibility and permeability constitutive relationships. Single drainage condition is assumed for all three scenarios. Three types of tailings were used in the validation runs: un-treated raw Mature Fine Tailings (MFT), Thickened Tailings (TT) and Phosphate Tailings (PT). A wide range of initial deposit heights and solids content were used to test the robustness of the model, particularly under extreme input parameters such as small initial deposit thickness and low initial solids content. Settlement over time, effective stress and void ratio profile are the primary performance indicators used to validate the model.

Table 1 and Table 2 listed input parameters and initial boundary conditions for various validation runs respectively. Only single drainage conditions are shown since bottom drainage is typically less than 10% of the total drainage for most deep cohesive low-permeability tailings (Jeeravipoolvarn et al, 2014). In all three validation scenarios, a uniform initial solids content profile is assumed for the entire depth of the deposit. Unit for compressibility parameter A is in kPa. Unit for hydraulic conductivity parameter C is in m/day. It should be noted that for thickened tailings, key performance indicators were validated against results from a commercial software FSCA developed by Dr Jeeravipoolvarn. Results from the validation runs are shown in Figure 4 to 6.

Table 1. Input parameters for validation runs

Deposit Type	A	B	C	D	Height (m)
MFT[1]	3.40	-0.31	7.0E-06	3.82	10
TT[2]	1.62	-0.26	3.5E-04	3.45	70
PT[3]	7.72	-0.22	2.5E-07	4.65	9.6

[1]From Pollock, 1988

[2] Total TT laboratory data from COSIA, 2012

[3] Phosphate Tailings from Townsend and McVay, 1990

Table 2. Boundary conditions for validation runs

Deposit Type	Drainage	Initial Solids Content (%)	Gs
MFT	Single	32	2.27
TT	Single	60	2.44[1]
PT	Single	16	2.82

[1]From Scott et al, 2008b

5   DISCUSSION

Scenarios with varying deposit thickness, tailings properties and consolidation duration were conducted to calibrate and validate the model against a commercial finite strain consolidation analysis software FSCA.

For the un-treated MFT material in the 10 m column settling test, TMSim consolidation model showed agreement with the settlement output from numerical model based on the implicit scheme. In his thesis, Pollock (1988) attributed the discrepancy between settlement predicted by the numerical model and measured data to thixotropy and sensitivity of settlement output to minor changes in hydraulic conductivity. Figure 4a indicated that consolidation occurred throughout the entire depth of the column while numerical results showed no consolidation above 5 m depth. Differences in solids content may be due to errors in sampling and thixotropy not being modelled.

Similar to the model validation done by Jeeravipolvarn (2010), benchmark cases from Townsend and McVay (1990) were used to assess the performance of TMSim consolidation module. TMSim under-estimated the percentage of settlement in terms of the initial thickness by approximately 11% at the end of the analysis period (Figure 5a). The difference between predicted and measured data gradually increased starting from the one-year mark of consolidation and became greater at the end of the analysis period. Figure 5b indicated good agreement in terms of void ratio profile except at the very top where such difference may be due to the spill-over effect of differences in settlement prediction.

Figure 4b. Void ratio profile for raw MFT during 10 m column settling test

Results based on a TT deposit thickness of 70 m and initial solids content of 60% indicated that TMSim is in good agreement with FSCA in terms of interface height and void ratio profile as shown in Figure 6a and 6b.

For deposit thickness less than 10 m and initial solids content less than 20%, TMSim may not be a suitable tool if greater accuracy at this data range is required. Predictions of void ratio profiles by TMSim near the surface of the deposit are poor based on results from Figure 4b and Figure 5b. However, the model performance near the surface may be attributed to the spill-over effect of differences in settlement prediction.

For deep deposits with thickness greater than 20 m, TMSim consolidation module is a screening-level tool for basic prediction of consolidation behavior. It should be noted that the sedimentation process is assumed to have been completed in TMSim thus is not explicitly modelled. It is also worth noting is that spatial variability of consolidation behavior in three dimensions is not captured by TMSim as the numerical model is one-dimensional. Settlement prediction and void ratio profiles are assumed to be representative of the entire deposit area. These assumptions do not reflect the reality. Therefore, the SD-based consolidation module is only suitable as a high-level screening tool. Finite strain analysis based on more rigorous numerical scheme such as the implicit method or finite element is required for detailed evaluations.

Figure 4a. Settlement of raw MFT during 10 m column test
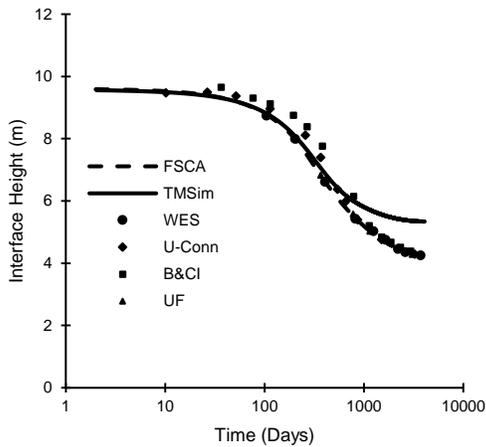
Figure 5a. Interface height for Phosphate Tailings
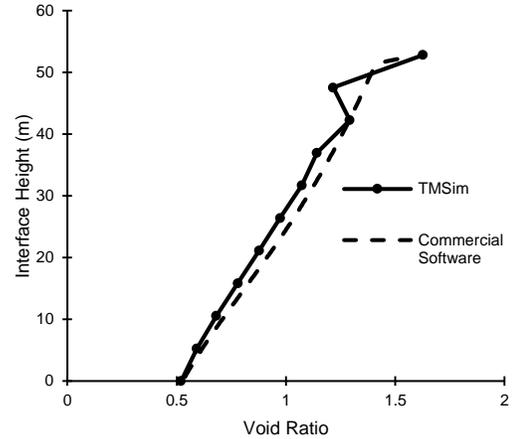
Figure 6b. End-of-consolidation void ratio profile for Thickened Tailings
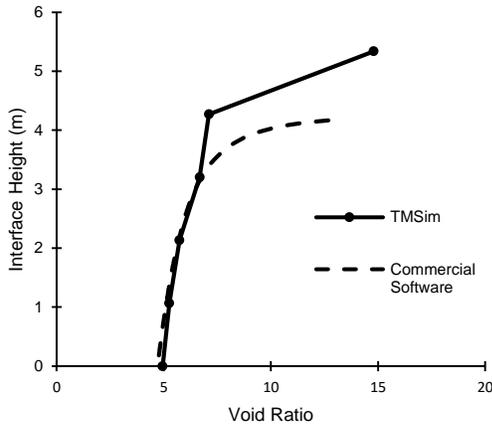
Figure 5b. End-of-consolidation void ratio profile for Phosphate Tailings
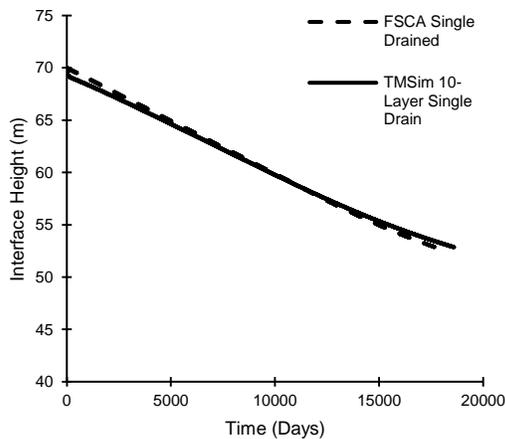
Figure 6a. Interface height for Thickened Tailings

6   STOCHASTIC MODEL FORMULATION

One of the main advantages in adopting SD-based approach is the ability to simulate multiple scenarios using Monte Carlo techniques. Once the deterministic model has been calibrated and validated, stochastic simulation can be easily set up by assigning probabilistic distributions to controlling parameters and slightly modifying the model structure. In stochastic simulation models, it is useful to separately model uncertain or epistemic parameters as well as variable or aleatory parameters (Baecher, 2016). Uncertainty in epistemic parameters represents lack of knowledge that can be reduced through additional investigation or research. This lack of knowledge can come from insufficient laboratory and field investigation, theoretical simplifications and assumptions. On the other hand, aleatory parameters represent inherent randomness or uncertainties that cannot be reduced or eliminated.

A simple case study involving quiescent consolidation of a 75 m thick Thickened Tailings (TT) deposit is demonstrated below. The main objective of the case study is for demonstration purposes. Therefore,

Input parameters controlling the quiescent consolidation process can be divided into epistemic and aleatory category. This is accomplished by using a nested Monte Carlo technique. In a nested or two-dimensional Monte Carlo set-up, probabilistic input in the outer model represents epistemic uncertainty due to lack of knowledge while those in the inner model represents aleatory uncertainty due to inherent uncertainties in the system.

In the simulation, the inner model represents variation of compressibility (A2 and B2) and permeability (C2 and D2) due to aleatory uncertainty which, by definition, cannot be reduced or eliminated. The outer model represents lack of experimental and field data. Since both the inner and outer model carry out Monte Carlo simulation, running the outer model multiple times creates a distribution of distribution for any output from the inner model.

For aleatory uncertainties, it is important to distinguish between natural randomness and inherent uncertainties. There is no natural randomness in the tailings deposit since the deposition process, which is within operator's control, has already taken place. Thus, the aleatory uncertainty in this case is assumed to arise from the limitation of site characterization. In reality, financial and logistical limitations restrict minimum borehole spacing and maximum coverage of the testing program. The probabilistic input for the inner model captures the inherent uncertainty, not the natural randomness.

TT from Total's experimental program is used as a simple case study to demonstrate the stochastic approach. Table 3 shows the probabilistic distribution input assigned to the curve fitting parameters in the outer model. Table 4 shows the probabilistic distribution input assigned to curve fitting parameters in the inner model. Beta-PERT distribution is used for compressibility parameter A and permeability parameter C since there is lack of statistical data and insufficient sample size from which statistically significant inference can be drawn. Uniform distribution is used for compressibility parameter B and permeability parameter D since there is no significant variation in B and D based on large strain consolidation test data conducted on different types of TT (COSIA, 2012). Assignment of probabilistic distributions to each input can be data-driven or subjective based on expert opinions (Vick, 2002). In this case, epistemic uncertainty is assumed to deviate 30% from the most likely value. As mentioned before, aleatory or inherent uncertainty is assumed to be based on limitation of site characterization of the tailings deposit. Therefore, parameters in the inner model are assumed to deviate only 10% from values sampled from the outer distribution (Table 4). This is a reasonable assumption since variation in tailings properties is well delineated during the operational phase (CNRL, 2016 and Kearl, 2013).

In the nested Monte Carlo simulation, the outer model is set up as a static model without incorporating time duration and time stepping while the deterministic model is converted to a stochastic inner model. As a simple demonstration, a total of 50 realizations for both inner and outer model were run using Latin Hypercube Sampling method due to its balanced sampling of probability space. (McKay et al, 1979).

Table 3. Probabilistic input for the outer model (epistemic uncertainty)

Parameter	Distribution	Most Likely	Minimum	Maximum
A1	Beta-PERT	1.62	1.14	2.11
B1	Uniform	N/A	-0.29	-0.24
C1	Beta-PERT	3.46E-04	2.42E-04	4.50E-04
D1	Uniform	N/A	3.11	3.69

Table 4. Probabilistic input for the inner model (aleatory uncertainty)

Parameter	Distribution	Most Likely	Minimum	Maximum
A2	Beta-PERT	A1	0.9*A1	1.1*A1
B2	Beta-PERT	B1	0.9*B1	1.1*B1
C2	Beta-PERT	C1	0.9*C1	1.1*C1
D2	Beta-PERT	D1	0.9*D1	1.1*D1

7 RESULTS AND DISCUSSION

By creating a distribution result element in the outer model, the probability of not exceeding certain magnitude of settlement is known. A probability-based settlement prediction will provide further input to future risk assessment exercises. Figure 7 shows multiple Complementary Cumulative Distribution Functions (CCDF) in the outer model as the result of epistemic uncertainty due to lack of knowledge. The most useful output is the CCDF in Figure 8, which considers both epistemic and aleatory uncertainty. In this case, the CCDF was constructed from aggregating a total of 2500 Monte Carlo runs which consisted of 50 realizations in the inner model and 50 realizations in the outer model. The total model run-time was approximately four hours.

Based on the CCDF in Figure 8, there is a 30% probability that the total settlement will exceed 18 m. Alternatively, the probability of total settlement not exceeding 18 m is 70% based on the Cumulative Distribution Function (CDF) instead.
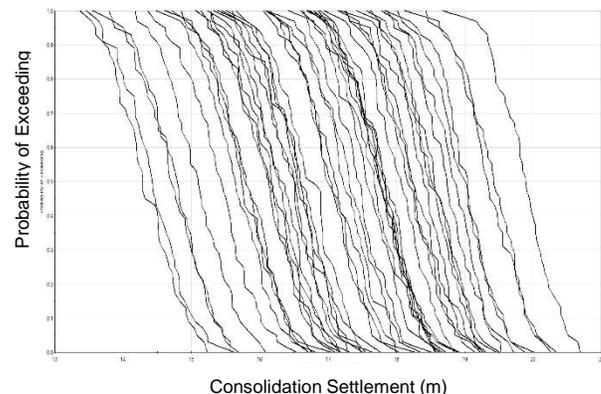
Figure 7. CCDFs from inner and outer model.
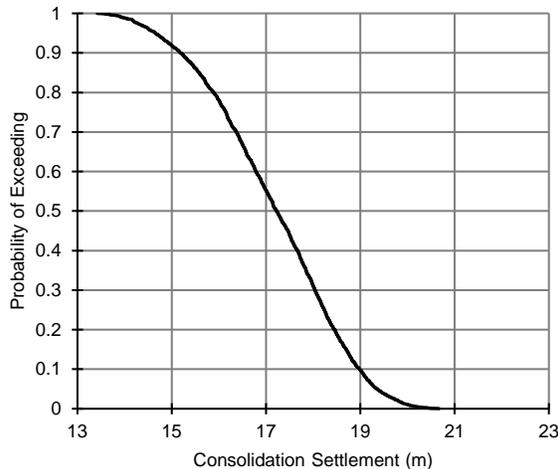
Figure 8. Combined CCDF incorporating both epistemic and aleatory uncertainties.

8   CONCLUSIONS

Consolidation of tailings deposit plays an important role in closure planning. A system-dynamics based model is developed and tested in this paper as the foundation for further stochastic simulation of tailings consolidation process. SD-based methods made all components of the numerical solution visible and easy to modify. This transparency also makes SD-based tools an ideal platform for communication and education. The developed model shows that system-dynamics is a viable approach provided that spatial variability and constitutive relationship can be reasonably simplified. The SD-based consolidation module in TMSim demonstrated successful simulation of deep cohesive tailings deposit. However, the model is still in its early stage of development. Therefore, the TMSim consolidation module should only be used as a high-level screening tool due to simplification of spatial variability in the model. Any detailed design or further analyses should be handled by more rigorous tools based on the large strain consolidation theory. Additional modification is required to extend TMSim's applicability in partially-drained and un-saturated conditions. The calibration process can also be automated to reduce computing time and increase numerical stability.

ACKNOWLEDGEMENTS

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