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THE UNIVERSITY OF ALBERTA

A MULTIPLICATIVE MODEL OF PRICES AND ASSESSMENTS FOR SINGLE-FAMILY RESIDENCES

by

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A THESIS

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "A Multiplicative Model of Prices and Assessments for Single-Family Residences" submitted by Douglas G. MacLean in partial fulfilment of the requirements for the degree of Master of Business Administration.

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ABSTRACT

The study is an analysis of estimates of single family residence property values made by assessors, real estate agents, and the market of buyers and sellers. The objective is a better understanding of underlying theoretical relationships.

Multiple regression techniques with multiplicative models are applied to actual data for 116 properties in Southwest Edmonton. The housing, neighbourhood, realtor, timing and financing variables used to estimate property value are identified from regression equations. Variables which explain the difference between the total assessment and sale price, and the difference between the real estate agent's list price and the sale price, are identified. The effect of each significant variable is quantified. The influence of broker selection on sale and list price is analyzed and discussed with comparisons and references to similar results in other investigations.

The analysis shows that a number of variables not included in some major previous studies are significant. These variables include 'traffic', 'corner lot', 'trees', 'undesirable proximity', 'park area', 'open roof-beam construction', and 'curb appeal'. It is found that land and building assessment can be used to provide a reasonable estimate of the sale price, provided the month of sale is included as a variable in the regression equation. Also, there appears to be more random influence on land assessment than on either building or total assessment.

As well as providing statistical evidence for some well-known intuitive beliefs and less well understood phenomena in real estate, a practical framework is provided as a basis for further work toward a better understanding of the factors affecting property value. In particular, there appears to be significant potential for the use of certain additional, a priori land assessment variables, and the methods detailed here, in the study of land assessment. Any improvement in the land assessment regression results is expected to result in improved sale and list price regression equations as well. An approach is suggested to quantify major interaction effects, thereby more accurately determining the coefficient for each independent variable. Finally, there appears to be scope for the use of ridge regression or other techniques in combination with the methods employed in this paper to control multicollinearity.

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Chapter I INTRODUCTION

The purpose of this study is to provide a better understanding of the underlying theoretical relationships which affect the estimates of single family property values made by assessors, real estate agents, and the market of buyers and sellers. In particular, property characteristics that appear to be used by assessors in arriving at the land, building and total assessment of single family residences for property taxation purposes are determined.

An attempt is made to find empirical evidence that the requirements and guidelines for determining assessment in the Assessment Manual (1967) are being followed by assessors in Edmonton, Alberta. Similarly, property characteristics which appear to be used by the real estate agent (listing broker) for determining the list price are investigated.

Also of considerable interest is the valuation done by the market, as indicated by property sale prices.[1] The assessor's and listing broker's property valuations are compared with the market's valuation. Since assessment should be a portion of market value, evidence is presented on how well this portion is maintained. Similarly, evidence of the difference in variable use by brokers and the market is presented.

For the investigation of assessment practice three main multiplicative regression equations are developed.[2] First, a regression is done with total assessment as the dependent variable to select significant independent variables. Then a regression is done with sale price as the dependent variable to select significant independent variables. Finally, a regression is done with the ratio of total assessment to sale price as the dependent variable to compare the variables used by assessors and the market to estimate property value.

Three main regression equations, similar to those used for assessment analysis, are developed to investigate listing practice. This time, however, the dependent variables are 'list price', 'sale price' and 'ratio of list price to sale price'.

The approach just described is adapted from that used by Berry and Bednarz (1975) to investigate the effects of a proposed change in the assessment system in Chicago. In this study, emphasis is placed on investigation of assessment procedures and the variables used, rather than on identification of assessment inequities as in the Berry and Bednarz paper. Thus assessment practice is studied to determine areas in which assessment compares with what would be expected and where there a e discrepancies. Listing practice is also investigated using the same methods, an area not touched in the Berry and Bednarz study. This analysis gives practical insight into broker and market behaviour.

In chapter II is a literature review of relevant past studies. Following this is a description of Multiplicative Models (chapter III), Data and Variables (chapter IV) and Methodology (chapter V). After development of the method, the List Price and Assessment Analyses are presented (chapters VI and VII). Chapter VIII is the Summary and Conclusions. The Appendix provides further detail on multiplicative models with some practical examples of typical calculations.

Chapter II LITERATURE REVIEW

Significant results from important previous work are taken as a starting point for the present study and are, in part, a justification for the approach used. For example, the Oldman and Aaron (1965) study, the oldest article reviewed, illustrates the difficulties in obtaining good results that can occur with any technique when too few variables are used, even with a large sample. In the present study, several variables are included to avoid this shortcoming. The Smith (1971) and Janssen (1977) studies are examples of previous work which used conventional linear regression models and the results which may be expected with these models. Goldberger (1968), Teekens (1972) and Teekens and Koertz (1972) provide considerable theoretical background on the multiplicative model which is a non-linear regression model. These studies are included partly for the background information they provide, and partly to illustrate the potential of multiplicative models which to date have been largely overlooked in real estate, even though the techniques have been widely applied in econometric research. Finally, the paper by Berry and Bednarz (1975), which applies the multiplicative model to real estate, is reviewed in some detail. The Berry and Bednarz study is a natural outgrowth of the work that had been done previously and illustrates the advantages of the techniques using the multiplicative model in a practical application. The present study uses the methods employed by Berry and Bednarz, with some refinements.

In addition to the summaries, highlights, and comparisons from an extensive literature review, there are footnotes in the text, and a bibliography indicating other sources of relevant information for the reader wishing to investigate a particular point, apply the results, or do further research. Also, results from two studies using factor analysis, Janssen and Jobson (1978) and (1980), are discussed briefly in conjunction with the results from this study.

The literature review discusses the studies by Oldman and Aaron (1965), Smith (1971), Janssen (1977), Goldberger (1968), Teekens (1972), Teekens and Koertz (1972) and Berry and Bednarz (1975) in that general chronological order, to reflect theoretical development in the subject area of this study.

One objective of a rather early assessment study by Oldman and Aaron (1965) was to analyze the deviation of property tax assessment in Boston from the legally established norm of uniform assessment relative to market value.[3] Another was to describe the variation in assessment within particular groupings of properties. To do this, Oldman and Aaron tabulated assessment - sale price ratios by price range and locations and also by price range and type (single-family residence, apartment, commercial property, etc.) for 13,769 properties.

Through examination of the standard deviations of assessment - sale price ratios, these authors found that single family residences in moderate price classes ($10,000 to $50,000) were fairly consistently assessed, compared to other types of property, at thirty-four percent of market value. Possibly since there were only a small number of transactions (twenty-six) no trend in vacant land assessment relative to price was found. This result indicates a possible tendancy towards greater variation in land assessment than building assessment.

Through analysis of variance on the tabulated assessment - sale price ratios, Oldman and Aaron achieved R^2 values no greater than 0.29, even when interaction effects were included. They concluded that these poor results were due in part to "factors other than those considered in this study that must also have influenced the assessment - sales ratio" and the possibility that "property assessment and the bargaining process through which sale prices are reached are rife with arbitrary or chance elements".[4]

No attempt was made in the Oldman and Aaron study to quantify the sources of the variation in assessment or sale price for comparable properties or to investigate the listing and selling process for possible random influences. Since analysis of variance was used, only three variables could be considered at one time. Additional variables which might have explained more of the variance in the assessment - sale price ratios could not be considered as was done in later regression studies. The results are an indication of the early difficulties encountered in analyzing real estate data using only a few variables even when the sample size was very large.

Smith (1971) used a linear regression model for sale price to investigate "worthwhile considerations of the use of regression in the appraisal process".[5] As an example of the benefits of regression he noted that it was quite helpful in indicating which variables appeared to be taken into consideration by the market. This variable selection approach is, of course, justified provided only reasonable a priori variables are available, or there are not too many spurious correlations between the dependent variable and unrelated independent variables, or among the independent variables themselves.

Smith (1971) obtained very good results in his regression for sale price (R^2=0.94, $\overline{R}^2$=0.94) with a sample of 309 sales and eleven variables. These exceptional results with few variables suggest the data were somewhat homogeneous and this was admitted by Smith.[6] Smith (1971) concluded optimistically that "... it does appear that well specified multiple regression equations can be extremely valuable to the appraiser".[7]

Janssen (1977) followed an approach similar to that of Smith, and used linear regression models to estimate the sale price of single family residential properties. He attempted to limit the amount of collinearity among the independent variables in order to have stable and statistically significant coefficients. His best equation, with all ten variables significant at the 5 percent level had an R^2 of 0.73 ($\overline{R}$=0.71).

As Smith (1971) had indicated, Janssen found the regression technique suitable to identify and isolate the influence of different property characteristics on sale price. He also found that selection of the listing broker had an influence on sale price and concluded that regression analysis could be used to gain further insight into the selling process.

In comparing the Smith and Janssen studies, it is noticeable that both developed equations in which no more than eleven variables are significant. It is also apparent that there can be large differences in the R^2 obtained, depending on the homogeneity of the data and the variables selected. This variation indicates that more analysis into underlying processes may be profitable. For this, alternative approaches may be beneficial. The alternative approach selected in this paper is to use multiplicative models rather than linear models. These models have desirable analytical features, but have previously been avoided because they are biased estimators. Goldberger (1968), however, studied this aspect of multiplicative models and showed that:

> For empirical implementation of the Cobb-Douglas function [a multiplicative model] it is customary to append a multiplicative lognormal disturbance and fit a linear regression in the logarithmic variables. When this is done, attention is shifted (apparently unwittingly) to the conditional median [of the dependent variable] from conditional mean which is ordinarily the prime target of study. The customary procedure, however, may be modified to provide minimum variance unbiased estimation of the conditional median or condition mean.[8]

Goldberger gave a numerical example and concluded:

> For a given feature of the conditional distribution of the dependent variable in a Cobb-Douglas model, the minimum variance unbiased estimator may not, in practice, differ detectably from the somewhat simpler alternative estimators considered in the course of the analysis...It would appear desirable, in any case, for researchers to report the value of s^2 obtained when linear logarithmic regressions are run, so that readers may adjust results if they so desire.[9]

Goldberger's results indicate that selection of a regression model can be based on theoretical and analytical considerations, rather than a concern about a small and correctable bias in the estimator.

After the Goldberger study, considerable work on multiplicative models remained undone. As J. Koerts stated in the foreward to Teekens (1972):

> Multiplicative relationships are often used to describe economic relationships. This means that the multiplicative model is an important tool in econometric research...However, despite its importance in economics there is no systematic study available about the correct statistical treatment of problems arising in the multiplicative model.
>
> The reason for this unsatisfactory state of affairs may be that this model can so easily be transformed into a linear model. This procedure, however, has serious drawbacks and cannot be considered an appropriate way of dealing with the problems.[10]

The drawbacks referred to by Koertz include: uncertainty as to whether the transformed problem is identical to the original one, the danger that assumptions may no longer hold for the transformed problem, and inconsistencies in the estimate of the expectation of the dependent variable using the alternative models. These problems are tackled by Teekens in his study.

The Teekens and Koertz (1972) study is written in the same vein as the Teekens (1972) monograph. It "set[s] out the implications of the log transformation on the stochastic properties of the model, which are postulated in the original multiplicative relationship".[11]

Teekens (1972) and Teekens and Koerts (1972) contain some of the most recent work on the theory of multiplicative models. The literature search done for this study, however, revealed that considerable research is still being done on multiplicative models.[12] No applied studies were found, however, which compare multiplicative and linear models in a routine way. It thus seems likely that methods to compare multiplicative and linear models or select which model is more appropriate could be developed further.[13] There is also an apparent need for more definition of methods to limit multicollinearity. Use of methods such as ridge regression, for example, are being actively researched.[14]

Having given some background on theoretical work on multiplicative models, it is appropriate to review the actual practical application by Berry and Bednarz (1975) which was used as the point of departure for the present study.

Berry and Bednarz (1975) used multiplicative regression models to assist them in determining who would benefit from a proposed change in Chicago's assessment method through reduced taxes and who would pay more taxes. Their technique was

> to develop a hedonic price index for house values, and then to repeat the method of index construction for land, improvement and total property assessments and for the assessment - price ratio. This procedure enables direct comparison of the factors contributing to the variance of prices with those related to assessments, and from this to note significant differences that produce variance in the assessment - price ratio.
>
> The underlying rationale is that any commodity (such as a house) can be effectively disaggregated into a bundle of separately measurable characteristics, of which the relative contribution of each of these to price can be determined....
>
> Since tax assessments are...based upon characteristics of the improvements (house) and value of the land, spatial variations in the assessment - price ratio, then can be attributed to: (a) factors influencing assessors' decisions that play no role in market price determination; (b) factors affecting

> market prices that are not considered by the assessors; or (c) factors valued differently by assessors and the market....Chicago's proposed change from traditional assessment practice to a constant percentage of market values can be interpreted as a change in tax burdens and benefits—those favored by the assessors but not by the market will lose; those favored by the market but not by the assessors will gain.[15]

The technique Berry and Bednarz developed was applied to independent variables divided into six subsets: housing characteristics, housing improvements, neighbourhood characteristics, racial and ethnic variables, environmental pollution and accessibility. Seventeen variables were selected from a larger set of possible variables based on contribution to explanation of the variance.[16] Stepwise entry of each of these sets of variables into each of the models for sale price, land, building and total assessment was forced.

The results of the regressions were much as expected. In the regression for sale price all the groups of variables were significant with the exception of environmental pollution which was significant only some of the time. Sale price increased with both house and lot size and decreased with age. Housing improvements added to the sale price. House prices were greater the higher the income level of the neighbourhood and the greater the portion of apartments (possibly reflecting competition for land). One unexpected coefficient was the positive coefficient for distance from the Central Business District. It was suggested that the amenity associated with the greater distance from downtown Chicago outweighed the disamenity of the additional transportation cost.

An approach similar to that just described was also used to investigate the equations for land, building and total assessment. Effort was devoted to explaining the sign of coefficients and assessment inequity related to racial effects. Of most interest were the differences in assessment and market value found through examination of the regression using the total assessment - sale price ratio as the dependent variable.

Inspection of coefficients in the ratio equation is useful because it permits analysis of differences in assessors' and the market's valuation of variables affecting property value. In particular, a positive coefficient indicates a heavier assessment with respect to that factor than what the sale price would indicate. A negative coefficient indicates the opposite. A zero coefficient indicates no difference in the assessors' and market's valuation of a particular factor.

Berry and Bednarz found that:

> ...assessments tend to run ahead of market prices for larger units...the assessor depreciates property more rapidly than does the market... [and] ... blacks [are] assessed at higher rates...[17]

Hence Berry and Bednarz were able to conclude that the proposed change in Chicago's assessment practice to make assessment a percentage of market value would benefit certain groups more than others. For example, taxes paid by the owners of larger or newer houses would be proportionately lower than taxes paid by those owning smaller or older houses. The former would gain from the change and the latter would lose.

R^2 values as high as 0.79 were obtained in the regression equations with a sample of 275 houses. However some insignificant variables were included. This happened because entry of the same variables was forced into the equations for sale price, land, building and total assessment. The same variables would not normally be expected to affect both sale price and land, building and total assessment. For example, 'number of baths' and 'garage' should be insignificant in the land assessment equation. Although this can be a problem with the method used by Berry and Bednarz, it did not affect their major conclusions.

An important contribution of Berry and Bednarz was to show that multiplicative models can be applied to real estate questions, and that statistically significant, practical and credible results can be obtained.

In this chapter, selected articles relevant to the present study have been reviewed. These studies include examples of past practical applications using linear regression models, theoretical analysis of non-linear (multiplicative) models, and finally, an application of the multiplicative model by Berry and Bednarz. In the next chapter, the multiplicative model is described in more detail.

Chapter III MULTIPLICATIVE MODELS

Multiplicative models are that class of equations which may be written in the form:

$$Y = b_o X_1^{b_1} X_2^{b_2} \ldots X_i^{b_i} \ldots X_k^{b_k} e^u$$ (here a multiplicative error is assumed)

where:

X_i, i = 1,2,...k are the independent variables, $X_i \neq 0$

b_i, i = 1,2,...k are the coefficients

e^u is the error term

or, taking the natural logarithm of both sides yields:

$$\ln Y = \ln b_o + b_1 \ln X_1 + b_2 \ln X_2 \ldots + b_i \ln X_i \ldots + b_k \ln X_k + u$$

To estimate the exponents of the variables in the multiplicative model, the logarithmic form of the variables are regressed using standard computer programs. Variables which may take the value of zero are entered in the form $\ln(X_i+1)$ in order to avoid taking the logarithm of zero which is undefined.

The conditional expectation of the dependent variable, Y is as follows:

$$E(Y/X) = b_o X_1^{b_1} X_2^{b_2} \ldots X_i^{b_i} \ldots X_k^{b_k} e^{\sigma^2/2}$$

where the right-hand side of the equation without the term $e^{\sigma^2/2}$ represents the conditional median.[18]

Estimates of functions of the type below can be readily compared:

$$P = b_o X_1^{b_1} X_2^{b_2} \ldots X_k^{b_k}$$

and

$$A = c_o X_1^{c_1} X_2^{c_2} \ldots X_k^{c_k}$$

where

P and A are the dependent variables

$X_1 X_2 \ldots X_k \neq 0$

b_o and c_o are constants, and

b_i and c_i, $i = 1,2,\ldots k$ are exponents

The relative influence of the independent variables on the dependent variable can be compared by first developing a ratio equation as follows:

$$A/P = (c_o X_1^{c_1} X_2^{c_2} \ldots X_k^{c_k})/(b_o X_1^{b_1} X_2^{b_2} \ldots X_k^{b_k})$$

or

$$A/P = (c_o/b_o) X_1^{c_1-b_1} X_2^{c_2-b_2} \ldots X_k^{c_k-b_k}$$

Thus the coefficients of the independent variables in the equation for the ratio A/P are the differences between the values of the coefficients for the same variables in the equations for A and P.

Independent variables, X_i, which have the same influence on A as P will not appear in the ratio equation since $X_i^{c_i-b_i}$ will equal 1. Variables which have a greater positive influence on A than P will appear in the ratio equation with the coefficient $c_i - b_i > 0$, that is, A tends to "run ahead" of P as X_i increases. A negative coefficient in the ratio equaion indicates the opposite, that is, P outpaces A as X_i increases.

A derivation of this result is given in the Appendix.

Taking the logarithm of both sides of the ratio equation:

$$\ln A - \ln P = (\ln c_0 - \ln b_0) + (c_1 - b_1)\ln X_1 + (c_2 - b_2)\ln X_2 + \ldots + (c_k - b_k)\ln X_k$$

Thus the values of the coefficients of the independent variables in the regression for the log ratio ln(A/P) are the differences between the values of the exponents for the same variables in the equations for A and P.

The coefficients in the regression equation for the A/P ratio may not be exactly as expected from the equations for A and P. This occurs because a variable significant in the equations for A or P or both is not necessarily significant in the ratio equation. Hence, a regression with the A/P ratio as the dependent variable is necessary in order to compare the relative influence of the independent variables on A and P.

Assumptions

There are a number of assumptions for the use of multiplicative models that should be recognized. These assumptions, listed below, are generally analogous to those for linear (additive) models.

a) X_i are positive nonstochastic variables;

b) b_i are unknown parameters;

c) the expected value of the random error term, e^u, equals one, that is, $E(e^u) = 1$;

d) the random error terms are distributed independently and identically with bounded variance;

e) the random error terms have a lognormal probability density function. Assumptions d) and e) may be relaxed in some circumstances.[19]

Dummy Variables

In multiplicative models,

if $\ln P = \ln b_o + b_1 \ln X_1 + b_2 \ln X_2 + \ldots + b_k \ln X_k + \ln a \cdot Z$

where Z is a dummy variable, having the value 0 or 1 such that:

if $Z = 0 \quad \ln P = \ln b_o + b_1 \ln X_1 + b_2 \ln X_2 + \ldots + b_k \ln X_k$

if $Z = 1 \quad \ln P = \ln b_o + b_1 \ln X_1 + b_2 \ln X_2 + \ldots + b_k \ln X_k + \ln a$

then the equivalent multiplicative model format is:

if $Z = 0 \quad P = b_o X_1^{b_1} X_2^{b_2} \cdot \ldots \cdot X_k^{b_k}$

if $Z = 1 \quad P = b_o X_1^{b_1} X_2^{b_2} \cdot \ldots \cdot X_k^{b_k} \cdot a$

or, in a more general format:

$P = b_o X_1^{b_1} X_2^{b_2} \cdot \ldots \cdot X_k^{b_k} e^{Z(\ln a)}$

A number of dummy variables were used in the present study.

Interaction Effects

An interaction effect is a change in the dependent variable produced by two variables jointly which cannot be attributed to either acting alone. It is also called the 'synergistic' effect of two variables acting together. When using a linear model, interaction effects can be handled by creating a new variable. For example:

Let $Y = X_1 X_2$

where Y is the interaction term and X_1, X_2 are the interacting variables.

Inclusion of interaction terms in linear models increases problems of multicollinearity due to the high correlation of the interaction terms with the interacting variables already in the equation. In multiplicative models this is not a problem as the following proof demonstrates.

Let $P = b_o X_1^c X_2^d (X_1 X_2)^e \ldots X_k^{b_k}$ be a multiplicative model, with the interaction term $(X_1 X_2)^e$

In logarithmic form:

$$\ln P = \ln b_o + c\ln X_1 + d\ln X_2 + e\ln(X_1 X_2) + \ldots + b_k \ln X_k$$

where $\ln(X_1 X_2)$ is an interaction term, c, d, e are constants.

The equivalent multiplicative model format may be simplified as follows:

$$P = b_o X_1^c X_2^d (X_1 X_2)^e \cdot \ldots \cdot X_k^{b_k}$$

or $$P = b_o X_1^{c+e} X_2^{d+e} \cdot \ldots \cdot X_k^{b_k}$$

if $$b_1 = c + e, \; b_2 = d + e$$

then $$P = b_o X_1^{b_1} X_2^{b_2} \cdot \ldots \cdot X_k^{b_k}$$

This last equation is no different in form from the multiplicative model without interaction effects. Hence, this proof shows that when using the multiplicative model it is not necessary to compensate for interaction by adding interaction terms as in a linear model. The multiplicative model "automatically" adjusts for interaction effects, an attractive feature of this approach. The result is a potential gain in explanatory accuracy and predictive power. The term "combined effect" may be used in order to distinguish between the traditional definition of interaction effects and interaction effects as used with the multiplicative model. An example showing how the combined effect of two variables may be calculated is given in the Appendix.

If a term for a combined effect (X_1X_2) is included in the multiplicative model, one of the original variables (X_1 or X_2) may not be significant. For example if $(X_1X_2)^e$ is significant, X_2 may be insignificant and not enter the regression equation, that is d = 0. Also since $b_1 = c + e$, the exponent of X_1, c, is lower than b_1 when the term $(X_1X_2)^e$ for combined effects is included in the equation. This means that the 'true' values of the coefficients for the variables are not known until the combined effects have been accounted for. By analogy, this is also true for linear models (as is well known). In recognition of this, the conclusions from the analysis are tempered with the caution that no separate terms for combined effects are included.

Multiplicative Models in Real Estate

Multiplicative models have certain characteristics which make them appropriate for application in real estate. For example, the exponents of the variables derived using regression methods are estimates of the price elasticities.[20] That is, percentage changes in the explained variable are related by a constant factor to percentage changes in the independent variables.

Diminishing marginal returns are accounted for directly and can be calculated. Marginal analysis is thus facilitated. As an example of diminishing marginal returns a larger house would be expected to sell for more than a smaller house, ceteris paribus, but the change in sale price may be less with each equal increase in floor area. This decreasing price per square foot for a marginal increase in floor area is known as a diminishing marginal return.

The combined effect of factors is 'automatically' accounted for since all the variables are multiplied together.[21] The combined effect of adding a garage and increasing floor area, for example, can be calculated. Additive models, in contrast, assume that the effect of each variable is additive, independent of the effect of other variables in the equation. The linear (additive) model must be adjusted using interaction terms to account for interdependencies among the independent variables. This increases problems with multicollinearity due to the high correlation of the interaction terms with the interacting variables already in the equation.

Chapter IV DATA AND VARIABLES

The data were based on a sample of 116 properties from Janssen (1977) comprising all bungalows that were MLS listed and sold in 1974 in three Southwest Edmonton neighbourhoods.[22,23] These MLS sales, according to Janssen, represented approximately two-thirds of all sales in the areas studied.[24] The MLS statistics gave particulars on a number of factors for each property including list price, floor area, number of rooms, age of house and first mortgage rate. A sample listing is shown in fig 1.

A total of thirty-eight variables were considered in this study: twenty-three from the Janssen study and fifteen new variables. The Janssen variables and the fifteen new variables are defined in Tables 1 and 2 respectively.[25,26]

Supplementing the Janssen variables with fifteen new ones gave more assurance that important factors had not been excluded. Thus there was less chance of correlated variables acting as surrogates for missing variables. Further, the number of variables that can be used for assessment analysis was more than doubled from ten to twenty-three permitting better statistical treatment of assessment methods.

The fifteen new variables consisted of nine physical variables, three neighbourhood variables, and land, building and total assessment.[27] All of the new physical variables were obtained by exterior inspection of each property.[28] The three new neighbourhood variables ('traffic', 'park area' and 'undesirable proximity') were obtained by inspecting the

INFORMATION HEREIN IS BELIEVED TO BE ACCURATE BUT NOT WARRANTED TO BE SO

Address of Property 4144 - 113 Street
Owner's Name Mr. and Mrs. H. Owner
Owner's Address 4144 - 113 Street
Owner's Res. Phone 435-4325 Bus. Phone 472-5561
Lot 64 Blk. 35 Plan 1184 NY Subdiv. Petrolia

No. Rooms 6
No. Bedrooms 3
Showing Ins. Phone first
Offers thru L.B.

M.L.S. No. R-4843-14
Expires 12-C-74
Price $ 52,900
Cash Down $ 40,300
Poss Aug 15/74
Code

1st FLOOR	2nd FLOOR	BASEMENT	GARAGE	GROUNDS	DWELLING
L.R. 19'6 x 12	B.R. x	R.R. 20' x 14'	Size 24 x 24	Lot Size 50x120	Style Bungalow
D.R. 8 x 9'2	B.R. x	B.R. x	Heated No	Walks Yes	Size 30 x 40.7
KIT. 15'1 x 10	B.R. x	B.R. x	Floor Concrete	Shrubs Yes	Sq. Ft. 1221
BR. 11'10 x 13'4	B.R. x	BATH 2 pce	Wired Yes	Fenced Yes	Ext. Fin. S/S
B.R. 9 x 10'6	DEN x	Storage room	Ext. Fin. Stucco	Lawns In Yes	Age 7 years
B.R. 7'10 x 11'6	BATH	HEATING	Permission to Place Sign Yes		Taxes $ 489.16 19
DEN x	BATH	F.A. Yes			
BATH 4 pce; 2 pce		Grav			
		Thermostat Yes			
		Open Fireplace No			

Distance To Bus Stores
Prot. Elem. School Jr. High Sr. High
Cath. Elem. School Jr. High Sr. High

1st Mtge or A/S to C.M.H.C. at $ 164.00 mo int 6 3/4 % Amt $ 12,600
2nd Mtge or A/S to at $ mo int % Amt $
Vendor will carry at $ mo int % Amt $
Are taxes included in monthly payment? (Yes) ~~No~~ Total monthly payment $ 164 IPT including taxes Total $ 12,600
1st Mortgage renewal Date 2nd Mortgage Renewal Date

CHATTELS INCLUDED All securely attached fixtures and

Listing Broker Edmonton Real Estate Ltd. Salesman Jim Mundo
Branch Office 19 Fairway Drive Office Phone 436-5181 Residence Phone 466-4268
REMARKS: Very clean property.

FIG. 1.—Sample multiple listing service (MLS) listing. Information pertains to an actual property in the sample. Owner, address, and broker are, however, fictitious. The sale statistics include, in addition to the above information, selling price, down payment, date of sale, number of days to sell, and selling broker.*

* From Janssen (1977:64).

TABLE 1

DEFINITION OF VARIABLES IN JANSSEN (1977) STUDY *

LNLP, LNSP....List Price and Sale Price, respectively

Physical Features

LNSQF.........Floor area on main floors, excluding basement measured in square feet

LNR, LNBR.....Rooms, bedrooms: number of rooms and bedrooms on main floors

LNB...........Baths: number of pieces on main floors (the standard four-piece bath would thus be coded 4)

LNX...........Extras: 0 if no basement development; 1 if roughed-in double plumbing or similar start on development; 2 if fire-place, basement room or basement bath; 3 if two or more basement rooms, or fireplaces, etc.

LNA...........Age in years

LNG...........Garage: 0 if none; 1 if single; 2 if double, 3 if triple

LNC...........Chattels: number of chattels (stove, refrigerator, freezer, dishwasher, washer, dryer, drapes, garage opener, etc.)

L.............Lot: 0 if standard; 1 if better (e.g. crescent location, larger or pie shaped)

Neighbourhood Characteristics

AL............Area Lendrum: 1 if Lendrum; 0 otherwise

AD............Area Duggan: 1 if Duggan; 0 otherwise

Realtor Effects

M,R,U,W,......L and S** preceding the letters listed refer to listing and selling brokers, respectively. Listing brokers, LM, LR, LU and LW listed 16, 13, 13 and 12 properties respectively, of the 116 properties in the sample. The remaining houses were listed by brokers who did not list a sufficient number of properties to permit separate analysis, they were simply lumped together as "others".

Timing Effects

LNM...........Month sold: 1 if January,..., 12 if December (year = 1974)

LND**.........Days: number of days to sell

VA............Vacant: 1 if vacant, 0 otherwise

Financing Variables

LNFM, LNSM....Existing first and second mortgage, respectively, in dollar amount

LNFMR, LNSMR..Percent rate of interest on first and second mortgage respectively

LNDP**........Down payment: actual down payment of the sale

*'LN' in front of any variable means the log-form of the variable is being used. For example, LNSQF = ln(SQF). Only additive models were used in the Janssen (1977) study and the log-form of variables was not used. Hence LNSQF was used in this study while SQF was used in Janssen (1977).

** Indicates variables from Janssen (1977) not used in this study.

TABLE 2

DEFINITION OF VARIABLES ADDED IN THIS STUDY*

LNLAND.......Land Assessment
LNBLDG.......Building Assessment
LNTOT.......Total Assessment (Land plus Building Assessment)

Physical Features

TRE..........Mature trees: 1 if mature trees (some at least seven feet tall); 0 otherwise
CO...........Corner lot: 1 if corner lot; 0 otherwise
LAN..........Lane: 1 if lane or walkway by house; 0 otherwise
LNFDA........Front drive attached garage: 1 if one car front drive attached garage; 2 if two car front drive attached garage; 0 otherwise
SD...........Side drive garage: 1 if side drive garage (i.e. a separate garage behind the house accessible from the street in front of the house); 0 otherwise
CURG.........Curb appeal, attractive appearance when viewed from the street in front of the house: 1 if "Curb Appeal"; 0 otherwise
BEAM.........Open roof-beam construction: 1 if open roof beam; 0 otherwise
LNSID........Siding: 0 if no aluminum siding, cedar, brick or stone veneer. Only stucco or wood: 1 if siding or veneer on the front half of the house; 2 if aluminum siding, brick or stone veneer, or spanish stucco all around the house
BAY..........Bay window: 1 if bay window; 0 otherwise

Neighbourhood Characteristics

LNBAC........Undesirable proximity: 0 if no undesirable area nearby; 1 if major road with traffic noise separated from back of house lot only by open space. High density housing, condominiums, rental housing, school or church directly adjacent to the house; 2 if freeway behind house
LNTR.........Traffic on street in front of house: 0 if cul-de-sac or short crescent; 1 if street with no cross streets; 2 if longer street with cross streets; 3 if major road or bus route; 4 if four lane or larger through street connecting two areas
LNPAR........Park area: 0 if no open space nearby; 1 if small park, pipe line easement or quiet school ground near house; 2 if large park near house

*'LN' in front of any variable means the log-form of the variable is being used. For example, LNLAND = ln(LAND).

immediate neighbourhood of each property. The land, building and total assessments were obtained from the assessment records of the City of Edmonton.

The variable 'garage' from Janssen (1977), was refined with the addition of two new variables: 'front drive attached garage' and 'side drive' which refers to those garages which are accessible from the front, but located behind the house.[29]

Location-related and financing variables incorporated from the Janssen study were 'area Lendrum', 'area Duggan', 'lot quality' as well as first and second mortgage amounts and rates. Location-related variables added in this study included 'corner lot', 'trees', 'traffic', 'undesirable proximity', 'park area', 'land assessment' and 'total assessment'.[30]

All the variables in Tables 1 and 2 are market variables which are defined as those which influence sale price. The market variables are physical features such as 'floor area' and 'number of rooms'; neighbourhood characteristics such as 'undesirable proximity' and 'traffic'; timing effects such as 'month of sale'; 'listing and selling realtor effects'; and financing.[31] Market variables may be sub-classified as list or assessment variables or both.

List variables are the subset of market variables which are known at the time that a house is listed for sale and useable by the listing broker for arriving at the list price, that is, all physical features, neighbourhood characteristics, land, building and total assessment and some realtor, timing and financing variables. List variables are all the variables in Tables 1 and 2 with the exception of selling broker (a realtor effect), number of days to sell (a timing effect), actual down payment (a financing variable) and sale and list price. These variables can only be known after a house is sold and hence are not list variables.

Assessment variables are the subset of market and list variables which are used, or available to be used, by the assessor to arrive at the land and building assessment. These variables include all physical features, except chattels, which are not assessed, and neighbourhood characteristics as shown in Tables 1 and 2. Not included are sale and list price, land, building and total assessment, all realtor, timing and financing variables.

In this study, sale price was used as a market estimate of property value. However, sale price and market value are not strictly synonymous. The market value cannot be observed and may be defined as the mean or median of the potential sale price distribution.[32]

List price was taken as the broker's estimate of the market value of each house and was discussed as if it were set by the listing broker. It is recognized that the broker may be influenced by his client when setting the list price, but this should not detract from the analysis. Some strategy on the part of the listing broker in terms of a bargaining allowance is involved in setting a list price. This was not accounted for in this study. Also, the sample consisted only of properties sold through MLS during the one year period of the study. Hence only the list prices of these houses were analyzed.

Chapter V METHODOLOGY

The approach in this paper is first to determine the variables which affect estimates, made by assessors, listing brokers and the market, of single family property values. These estimates are then compared. The study is divided into two parts, List Price Analysis and Assessment Analysis. In the first part, List Price Analysis (chapter VI), listing brokers' and the market's estimates of the value of factors which affect property value are determined and compared. In the second part, Assessment Analysis (chapter VII), assessors' and the market's estimates of factors which affect property value are determined and compared.

The basic technique is to develop multiplicative models for the dependent variables and then compare them using a ratio equation. For the List Price Analysis the equations are:

List Price = f(list variables)

Sale Price = f(list variables)

List Price/Sale Price = f(list variables)

where the list variables are as defined in Chapter III. They are listed in this chapter in Table 3. Only list variables are included in the List Price Analysis since these are the variables which are available at the time of listing for the listing broker to arrive at the list price and for the market to arrive at a sale price.

One of the most important variables for explaining the sale or list price is floor area, which is highly correlated with the variables for land, building and total assessment.

TABLE 3

PRELIMINARY CLASSIFICATION OF VARIABLES

Variable (Definitions are given in Tables 1 and 2)	Variables from Tables 1 and 2: Form Used in Equations	List Variables	Assessment Variables: For Land Assessment	Assessment Variables: For Building Assessment
List Price	LNLP			
Sale Price	LNSP			
Land Assessment	LNLAND*	LNLAND*		
Building Assessment	LNBLDG*	LNBLDG*		
Total Assessment	LNTOT*	LNTOT*		
	Physical Features			
Floor Area	LNSQF	LNSQF		LNSQF
Rooms	LNR	LNR		LNR
Bedrooms	LNBR	LNBR		LNBR
Baths	LNB	LNB		LNB
Extras	LNX	LNX		LNX
Age	LNA	LNA		LNA
Garage Space	LNG	LNG		LNG
Chattels	LNC	LNC		
Lot Quality	L	L	L	
Trees	TRE*	TRE*	TRE*	
Corner Lot	CO*	CO*	CO*	
Lane	LAN*	LAN*	LAN*	
Front Drive Attached Garage	LNFDA*	LNFDA*		LNFDA*
Side Drive	SD*	SD*		SD*
Curb Appeal	CURG*	CURG*		CURG*
Open Roof Beam Construction	BEAM*	BEAM*		BEAM*
Siding	LNSID*	LNSID*		LNSID*
Bay Window	BAY*	BAY*		BAY*
	Neighbourhood Characteristics			
Area Lendrum	AL	AL	AL	
Area Duggan	AD	AD	AD	
Undesirable Proximity	LNBAC*	LNBAC*	LNBAC*	
Traffic	LNTR*	LNTR*	LNTR*	
Park Area	LNPAR*	LNPAR*	LNPAR*	
	Realtor Effects			
Listing, Selling Broker 'M'	LM, SM**	LM		
Listing, Selling Broker 'R'	LR, SR**	LR		
Listing, Selling Broker 'U'	LU, SU**	LU		
Listing, Selling Broker 'W'	LW, SW**	LW		
	Timing Effects			
Month	LNM	LM		
Vacant	VA	VA		
Days to Sell	LND**			
	Financing Variables			
First Mortgage	LNFM	LNFM		
Second Mortgage	LNSM	LNSM		
First Mortgage Rate	LNFMR	LNFMR		
Second Mortgage Rate	LNSMR	LNSMR		
Down Payment	LNDP			

*/ Indicates additions to the variables used by Janssen (1977)
** Indicates variables from Janssen (1977) not used in this study
Variables are defined in Tables 1 and 2

In order to avoid having major variables in the equation which are highly correlated, 'land assessment', 'building assessment' and 'total assessment' are not included as factors in the List Price Analysis.

The first model, for list price, permits analysis of how much weight the broker puts on each factor affecting property value. The second model, for sale price, permits analysis of how much weight the market puts on each factor affecting property value. The third model, for the list-sale price ratio, permits analysis of differences in the brokers' and market's valuations.

For the Assessment Analysis, the equations are:

Land Assessment = f(assessment variables)

Building Assessment = f(assessment variables)

Total Assessment = f(assessment variables)

Sale Price = f(assessment variables)

Total Assessment/Sale Price = f(assessment variables)

where the assessment variables are as defined in Chapter III. The assessment variables are classified as a priori land or building assessment variables in Table 3. Only assessment variables are included in the assessment analysis as only these are available to the assessor when computing the land and building assessment, or their sum, the total assessment.

The models for land, building and total assessment permit analysis of how much weight the assessor puts on each factor affecting property value.[33] The model for sale price permits analysis of how much weight the market puts on each factor affecting property value, using only assessment variables. The total assessment - sale price ratio model permits analysis of differences in the assessors' and market's valuations.

Stepwise regression with forward selection is used as the method of selecting variables to derive the "best" regression equation. Identical results are obtained using stepwise regression.[34] Only those variables which are significant in explaining the dependent variable are included, the criterion of significance being an F-value greater than or equal to 3.0.[35] An F-value of 3.0 for inclusion of variables is used as a matter of convention and convenience. Significance at the 5 percent level is equivalent to an F-value of 3.94 for a two-tailed and 2.76 for a one-tailed test using 1 and 105 degrees of freedom. The corresponding table values at the 1 percent level of significance are 6.91 and 5.60 respectively. A one-tailed test is used only when a coefficient is expected to have a certain sign. In the absence of such an a priori reason, a two-tailed test is used. Most variables are significant at the 1 percent level. The interested reader can calculate the actual F-value for an [illegible] coefficient by dividing the coefficient by the coefficient standard error and squaring the result.

The method of analysis is similar to that used by Berry and Bednarz (1975) described in the literature review (chapter II), but there are some basic differences. First, the variables used in the land and building assessment equations are not identical. Berry and Bednarz used the same variables for both. This change in the method obviates the need to interpret a number of variables as surrogates. It also permits a more detailed analysis of land and building assessment practice by using land

and building assessment variables respectively. Second, only significant variables are included in the equations. Berry and Bednarz forced entry of sets of variables and thus included some insignificant ones. This change in method makes it possible to draw stronger conclusions about statistically significant variables. More theoretical background on the method is given in the Appendix.

Chapter VI LIST PRICE ANALYSIS

The purpose of this chapter is to study and compare in detail the effect of market factors on the list and sale price. First, an attempt is made to identify factors which appear to be used by brokers to arrive at the list price for a property, and factors which appear to be used by the market in arriving at the sale price. This is done by running regressions with all list variables using first 'list price' and then 'sale price' as the dependent variable. Then an attempt is made to identify the factors which appear to be evaluated differently by brokers and the market. This is done by running a regression with all list variables using the list price—sale price ratio as the dependent variable.

For ease of comparison, the list price, sale price and ratio equations are presented together in Tables 4a and 4b. The results are as follows:

TABLE 4a

SALE PRICE AND LIST PRICE:
SUMMARY OF REGRESSION RESULTS WITH LIST VARIABLES
Significance Criterion: $F \geq 3.0$

Dependent Variable	LNSQF	LNM	LNX	CURG	LNG	LNSM	LNSMR	LR	TRE	LNFM	LNFMR	LNBAC	LW	LNBR	CO	LNTR	LNC
LNLP	1	1	1	1	1	-1	NS	1	1	-1	NS	-1	-1	-1	NS	NS	NS
LNSP	1	1	NS	1	1	NS	-1	1	1	-1	NS	NS	-1	-1	-1	NS	NS
LN(LP/SP)	NS	1	1	1	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	1	1

Legend:

NS = Not Significant

1 = Variable is significant in the equation with a positive coefficient

-1 = Variable is significant in the equation with a negative coefficient

TABLE 4b

SALE PRICE AND LIST PRICE:
REGRESSION RESULTS WITH LIST VARIABLES
Significance Criterion: $F \geq 3.0$

Dependent Variable	Constant													R^2	SE	F
		LNSQF	LNM	LNX	CURG	LNG	LNSM	LR	TRE	LNFM	LNBAC	LW	LNBR			
LNLP	4.402	0.915 (0.070)	0.082 (0.008)	0.033 (0.014)	0.063 (0.022)	0.042 (0.012)	-0.009 (0.003)	0.059 (0.018)	0.056 (0.016)	-0.007 (0.002)	-0.035 (0.017)	-0.046 (0.021)	-0.198 (0.108)	0.821	0.060	39.3
		LNSQF	LNM	LNG	LR	LNSMR	LNFM	CURG	TRE	LW	CO	LNBR				
LNSP	4.606	0.889 (0.071)	0.074 (0.008)	0.054 (0.012)	0.069 (0.018)	-0.028 (0.012)	-0.009 (0.002)	0.050 (0.022)	0.053 (0.016)	-0.057 (0.021)	-0.062 (0.026)	-0.232 (0.109)		0.793	0.061	36.3
		CURG	LNX	LNTR	LNC	LNM										
LN(LP/SP)	0.012	0.022 (0.009)	0.011 (0.006)	0.013 (0.006)	0.011 (0.005)	0.008 (0.004)								0.204	0.028	5.6

List Price with List Variables

Twelve variables were found significant in the equation for list price: 'floor area', 'month', 'extras', 'curb appeal', 'garage', 'second mortgage', 'listing broker R', 'trees', 'first mortgage', 'undesirable proximity', 'listing broker W', and 'bedrooms', suggesting that these variables are being used by listing brokers in arriving at the list price. R^2 was 0.82 ($\overline{R}^2 = 0.80$), and the equation as a whole was significant beyond the one percent level.[36]

The entry of the variables 'floor area', 'month', 'extras', 'curb appeal', 'garage', 'broker R', 'first mortgage', and 'broker W' could be expected due to their correlation of $|\pm 0.15|$ or higher with list price. (Correlation coefficients are given in Table 5). This leaves only four variables in the equation which have a lower correlation with list price ('second mortgage', 'trees', 'undesirable proximity' and 'bedrooms')

The variables, 'corner lot', (with a negative coefficient) 'front drive attached garage', 'lot quality', and 'area Duggan' all entered the equation in the order listed using the minimum standard error criterion. Hence these factors tended to have some effect on list price at a lower significance level.

List price tended to increase with 'floor area', 'month', 'extras', 'curb appeal', 'garage space', 'listing broker R', and 'trees'. List price tended to decrease with 'second mortgage', 'first mortgage', 'undesirable proximity', 'listing broker W' and 'bedrooms'. These relationships are much as would be expected. Desirable features tend to increase the list price. Undesirable features tend to lower it. The

TABLE 5

CORRELATION COEFFICIENTS

	LNLP	LNSP	LNSQF	LNR	LNBR	LNB	LNX	LNA	LNG	LNC	L	AL	AD	LM	LR	LU	LW	LNM	VA
LNLP	1.0	.97	.63	.23	-.10	.45	.28	-.06	.24	.07	.22	-.11	.02	.01	.22	-.12	.17	.47	.04
LNSP		1.0	.62	.24	-.11	.43	.23	-.09	.26	.01	.20	-.10	.05	.00	.23	-.14	.16	.45	.04
LNSQF			1.0	.49	.19	.59	.01	-.42	.02	.14	.11	-.32	.28	-.15	.04	.01	.27	-.01	.05
LNR				1.0	.47	.19	-.11	-.22	-.01	.19	.15	-.22	.14	-.06	-.01	-.07	.22	.02	.14
LNBR					1.0	.10	.01	-.03	-.04	.11	.05	-.12	.01	-.08	-.08	.06	.07	-.16	.03
LNB						1.0	.10	-.25	.04	.13	.14	-.23	.08	-.03	.01	.14	.11	-.03	.15
LNX							1.0	.43	.26	.16	.01	.18	-.36	.09	.16	.05	-.05	.07	.05
LNA								1.0	.25	-.03	.08	.53	-.82	.13	.04	-.05	.12	.28	-.02
LNG									1.0	.02	.04	.09	-.26	.02	.00	-.01	.14	.03	.04
LNC										1.0	-.25	-.23	.05	-.01	-.05	.09	.07	-.01	.09
L											1.0	.14	-.07	-.15	.04	-.05	.13	.11	.07
AL												1.0	-.46	.11	-.02	-.15	-.01	.07	.09
AD													1.0	-.16	.00	.06	-.15	-.25	-.07
LM														1.0	-.14	-.14	-.14	.24	.05
LR															1.0	-.13	-.12	.03	.01
LU																1.0	-.12	-.05	.01
LW																	1.0	.11	.02
LNM																		1.0	.14
VA																			1.0

TABLE 5

CORRELATION COEFFICIENTS (continued)

	LNFM	LNSM	LNFMR	LNSMR	LNLAND	LNBLDG	LNTOT	TRE	CO	LAN	LNFDA	SD	CURG	BEAM	LNSID	BAY	LNBAC	LNTR	LNPAR
LNLP	-.15	-.11	-.15	-.11	.35	.62	.66	.12	.04	-.08	.21	.01	.49	.13	.26	-.12	-.06	-.10	-.01
LNSP	-.17	-.10	-.17	-.10	.35	.62	.66	.10	.01	-.07	.23	.02	.45	.16	.26	-.11	-.07	-.15	-.00
LNSQF	.03	.10	.05	.11	.29	.78	.77	-.03	.16	-.15	.36	.12	.40	.26	.43	.00	.09	-.08	-.05
LNR	.13	.06	.13	.07	.06	.28	.26	-.01	.08	-.11	.14	.04	.05	-.10	.21	.00	.02	-.08	-.06
LNBR	.06	.38	.05	.38	.10	.03	.06	.12	-.05	-.04	-.08	.09	-.07	-.11	.15	.10	.06	-.04	-.09
LNB	.09	.13	.09	.13	.23	.48	.48	.06	-.00	-.17	.12	.06	.19	.14	.28	.14	-.03	-.06	-.21
LNX	-.12	.11	-.15	.11	.09	.12	.12	.14	.05	-.09	-.15	-.09	.14	-.08	-.08	-.13	-.05	.08	-.14
LNA	-.09	.10	-.16	.10	.08	-.55	-.46	.31	-.13	.13	-.43	-.33	-.13	-.12	-.36	-.20	-.09	.10	.06
LNG	-.15	.06	-.16	.06	.08	.15	.16	.08	.05	-.08	.16	.01	.12	.09	-.00	-.08	-.06	.06	-.07
LNC	.09	-.03	.10	-.03	.08	.14	.15	-.03	.07	.10	.11	.03	.11	-.06	.10	.13	.02	-.03	-.02
L	-.01	-.08	-.01	-.08	.23	-.02	.06	.17	-.09	-.07	-.06	-.02	.06	-.00	-.01	-.01	-.03	-.10	.07
AL	-.01	-.17	.19	-.04	-.08	-.47	-.43	-.34	.14	.10	-.16	-.11	-.19	.03	-.29	-.17	.14	.04	.20
AD	-.01	-.17	.06	-.17	-.15	.44	.34	-.37	.14	-.15	.36	.33	.11	.15	.29	.17	.05	-.08	.03
LM	.09	-.08	.06	-.08	.18	-.20	-.11	-.06	-.09	.06	.00	-.12	.04	-.09	-.15	-.02	-.05	-.04	-.02
LR	.06	.04	.06	.05	.11	.18	.11	-.03	.04	-.07	.04	-.11	.07	.01	.10	-.00	-.07	-.07	-.15
LU	.12	.21	.14	.20	-.03	.03	.01	-.17	.04	-.07	.04	-.11	-.11	.01	.04	.10	-.07	.15	.01
LW	-.16	.06	-.18	.07	-.05	.15	.11	.28	.05	-.06	.05	-.10	.08	.10	.13	-.10	.09	-.02	-.06
LNM	-.05	-.09	-.08	-.09	.16	-.09	-.03	-.00	-.04	-.03	-.23	-.06	.07	-.26	-.05	-.20	-.00	-.04	.07
VA	.10	.03	.08	.04	-.07	.02	-.00	-.19	-.10	-.08	.07	-.13	-.05	.02	.09	-.12	.11	.01	.03
LNFM	1.0	.03	.99	.04	.08	-.09	-.05	-.02	-.10	-.16	.03	-.02	-.03	-.04	-.06	.09	-.02	.09	-.09
LNSM		1.0	.02	1.0	.02	-.03	-.02	.10	-.05	-.04	-.08	-.06	-.07	-.11	.15	-.06	-.02	.07	-.09
LNFMR			1.0	.02	.07	-.05	-.02	-.06	-.08	-.17	.06	.01	-.02	-.04	-.05	.11	-.03	.08	-.08
LNSMR				1.0	.01	-.03	-.02	.11	-.05	-.04	-.08	-.06	-.07	-.11	.15	-.06	-.02	.07	-.09
LNLAND					1.0	.23	.53	-.02	.09	.06	-.05	.05	.20	-.01	.12	.15	.06	-.42	.07
LNBLDG						1.0	.94	-.11	.14	-.18	.42	.12	.45	.30	.38	.00	.09	-.13	-.02
LNTOT							1.0	-.11	.15	-.13	.36	.13	.46	.26	.37	.04	.10	-.26	.01
TRE								1.0	-.11	.03	-.10	-.06	.00	-.03	-.01	-.05	.04	-.00	-.07
CO									1.0	-.04	.15	.21	.06	-.12	.11	-.07	.16	.14	-.10
LAN										1.0	-.07	-.06	-.06	-.10	.01	-.05	-.10	-.10	.06
LNFDA											1.0	.11	.24	.26	.18	-.01	.09	.05	-.08
SD												1.0	.00	-.01	.15	.14	.00	-.10	-.04
CURG													1.0	.26	.21	-.09	-.07	-.01	-.05
BEAM														1.0	.05	-.15	.01	-.01	-.04
LNSID															1.0	.17	-.01	-.08	-.09
BAY																1.0	.02	-.15	.06
LNBAC																	1.0	-.14	.02
LNTR																		1.0	-.13
LNPAR																			1.0

high significance of the variable 'month' suggests a gradual price increase during the year. The negative coefficient of the variable 'first mortgage' is somewhat puzzling. A possible explanation is that 'first mortgage' is a surrogate for 'first mortgage rate', explaining the negative coefficient, as a high mortgage rate on an existing mortgage generally makes a property less attractive financially. The same holds for 'second mortgage'. The negative coefficient of the variable 'bedrooms' is as expected. For houses with comparable floor areas more bedrooms generally mean smaller room sizes which listing brokers tend to view negatively.[37]

Broker effects were quite significant. Controlling for other variables in the equation, 'broker R' tended to list at six percent above the average list price for all brokers while broker W tended to list at more than four percent below the average. For example, on a property listed at $49,924, the average in the sample, the list price could differ by as much as $5,021 depending on the listing broker selected.

Variables not in the equation were 'area Duggan', 'area Lendrum', 'side drive', 'front drive attached garage', 'park', 'siding', 'bay window', 'lane', 'corner lot', 'rooms', 'age', 'lot quality', 'baths', 'chattels', 'traffic', 'open roof beam construction', 'listing brokers M and U', 'vacant', and 'first and second mortgage rates' suggesting that these variables are not being used by listing brokers for arriving at the list price, or possibly that they are correlated with other variables already in the equation.

The variable 'age' would be expected to be an important variable in explaining list price, but its correlation with list price is only 0.06. 'Age' has very high correlations with 'floor area' (-0.433) and 'trees' (0.311) both of which are in the equation. This suggests that floor area and trees already account for most of the variance explained by age in the sample since older houses tend to be smaller and older houses also tend to be located in areas with more mature trees.

Some of the other excluded variables might have been expected to enter the equation especially if they have high correlations (greater than $|\pm 0.15|$) with list price. Table 6 shows variables which have correlation coefficients greater than $|\pm 0.09|$ with list price and which did not appear in the equation, along with their correlations with selected variables. The tolerance level of each of these variables is also included. The variables for the first and second mortgage rate did not appear in the equation due to the high correlation with the other two financial variables which were already in the equation. The remaining variables which did not appear in the equation were also generally highly correlated with independent variables already in the equation which acted as surrogates.

In some cases, ('side drive', 'park', 'lane', 'chattels', and 'vacant') the correlation with the dependent variable, 'list price', was low. Hence, the variables which entered the equation were much as expected a priori from examination of correlation coefficients and a posteriori from examination of tolerances. It cannot be claimed, however, that the coefficients represented only the effect of the

TABLE 6

SOME LIST VARIABLES NOT IN THE EQUATION FOR LIST PRICE:
MAJOR CORRELATIONS AND TOLERANCES
(Only list variables not in the equation with a correlation coefficient of greater than |±0.09| with LNLP are shown)

Variable	Correlation Coefficient	Correlated Variable Already in Equation	Correlation with LNLP	Tolerance*
AL	-0.322 0.336	LNSQF TRE	-0.111	0.707
LNFDA	0.365 -0.233 0.241	LNSQF LNM CURG	0.212	0.697
LNSID	0.430 0.209 0.154	LNSQF CURG LNBR	0.257	0.767
BAY	-0.203	LNM	-0.124	0.915
LNR	0.489 0.469 0.219	LNSQF LNBR LW	0.232	0.542
L	0.169	TRE	0.215	0.920
LNB	0.586 0.190	LNSQF CURG	0.449	0.609
LNTR			-0.0972	0.937
BEAM	0.256 0.260	LNSQF CURG	0.133	0.774
LU	0.121 0.211	LNFM LNSM	-0.122	0.852
LNFMR	0.988	LNFM	-0.152	0.019
LNSMR	0.999	LNSM	-0.108	0.001

*The tolerance of an independent variable being considered for inclusion in the regression equation is the portion of the variance of that variable not explained by the independent variables already in the regression equation.

corresponding variables due to the presence of multicollinearity. Nor can it be said with certainty that a variable did not enter the equation to compensate for over or under representation of another variable or variables with which it is correlated. It is believed, though, that the coefficients represented predominantly the effect of the associated variable.

Sale Price with List Variables

Eleven variables were found significant in the equation for sale price namely 'floor area', 'month', 'garage', 'broker R', 'second mortgage rate', 'first mortgage', 'curb appeal', 'trees', 'broker W', 'corner lot' and 'bedrooms', suggesting that these variables are being used by the market in arriving at the sale price. R^2 was 0.79 ($\bar{R}^2$=0.77), and the equation as a whole was significant beyond the one percent level.[38]

The entry of the variables 'floor area', 'month', 'garage', 'broker R', 'first mortgage', 'curb appeal' and 'broker W' could be expected due to their correlation of $|\pm 0.15|$ or higher with sale price. (Correlation coefficients are given in Table 5). This leaves only four variables in the equation which have a lower correlation with sale price ('second mortgage rate', 'trees', 'corner lot' and 'bedrooms').

The variables 'undesirable proximity', 'traffic', (negative coefficients), 'extras', 'area Duggan', 'chattels' (negative coefficient), 'front drive attached garage', 'second mortgage', 'broker U' (negative coefficient), and 'lane' all entered the equation in the order listed using the minimum standard error criterion. Hence these factors tended to have some effect on sale price at a lower significance level.

The signs of the coefficients were all as expected, and the magnitudes were all plausible in relation to construction cost and other relevant information.

The regression results indicated that sale price tended to increase with 'floor area', 'month', 'garage space', 'listing broker R', 'curb appeal', and 'trees'. As examples, the regression equation indicated a sale price increase of 15% between January and June, and one and two garages yielded increases of 2.5 and 4 percent in sale price, respectively. Thus a second bay tended to add marginally less to the value of a property than a first bay, other things equal.[39]

Factors which had a negative impact on sale price were 'second mortgage rate', 'first mortgage', 'bedrooms', 'corner lot', and 'listing broker W'. These relationships are much as would be expected.[37] Desirable features tend to increase the sale price. Undesirable features tend to lower it. Again, as in the list price equation, it is believed that 'first mortgage' is a surrogate for 'first mortgage rate', explaining the negative coefficient.

The order of entry of variables into the sale price equation changed from the order of entry in the list price equation. While 'floor area' and 'month' remained the two most important variables, 'garage' became more important than 'extras' and 'listing broker R' became more important than 'curb appeal'.

Controlling for other variables in the equation, properties listed by 'broker R' tended to sell seven percent above the average sale price for comparable houses, while those listed by broker W tended to sell almost six percent lower than average. For example on a property sold for $47,441, the average in the sample, the sale price could vary as much as $6,167 depending on the listing broker selected.[40] This implies that the choice of listing broker is important with respect to the price realized for a property. In the previous section it was found that brokers R and W also tended to list higher and lower than average, respectively. Hence, houses that were listed higher by broker R generally sold higher and houses that were listed lower by broker W generally sold for less than other comparable houses.[41]

All the possible reasons for the differences among brokers were not controlled. For example, no test was done to see if properties listed by broker R were different in some way. A paper by Janssen and Jobson (1978) using factor analysis on the same data with the original variables in Table 1 found that one of the reasons for the stronger performance on the part of broker R was a tendancy to list larger houses at relatively higher prices.[42] They also found that broker R was able to realize higher sale prices for the properties they listed. Janssen and Jobson speculated on two other explanations for the success of broker R. First, broker R handled a number of executive transfers and sold many of its own listings. These buyers may be less knowledgeable about local conditions and less sensitive to price. Second, broker R also had access to secondary financing at special rates. Secondary financing is often needed to bridge the gap between the sale price and the down payment plus the existing first mortgage. It is well known that the availability of secondary financing is an important factor in making a sale and special rates could provide a significant inducement.

Variables not in the equation were 'area Duggan', 'area Lendrum', 'side drive', 'front drive attached garage', 'undesirable proximity', 'park', 'siding', 'bay window', 'side lane', 'rooms', 'age', 'lot quality', 'extras', 'baths', 'chattels', 'traffic', 'open roof beam construction', 'brokers M and U', 'vacant', 'first mortgage rate' and 'second mortgage'.

Some of the excluded variables might have been expected to enter the equation especially if they had high correlations (greater than $|\pm 0.15|$) with sale price. An example is 'front drive attached garage' which has a correlation with 'sale price' of 0.228. However, it is correlated with 'floor area' (0.365) and garage (0.160). Thus 'front drive attached garage' is an important variable, as is recognized by brokers as well as the market, but its high correlation with variables already in the equation prevents it from entering.

Table 7 shows variables which have correlation coefficients of greater than $|\pm 0.09|$ with sale price, and which did not appear in the equation, along with their correlations with selected variables. The tolerance level of each of these variables is also included. The variables which did not appear in the equation are generally highly correlated with one or more independent variables already in the equation. In some cases ('area Duggan', 'side drive', 'undesirable proximity', 'park', 'side lane', 'chattels', 'broker M', 'vacant'), the correlation with the dependent variable 'sale price' was low. Hence, the variables which entered the equation are much as expected a priori from examination of correlation coefficients and a posteriori from examination of tolerances.

TABLE 7

SOME LIST VARIABLES NOT IN THE EQUATION FOR SALE PRICE: MAJOR CORRELATIONS AND TOLERANCES

(Only list variables not in the equation with a correlation coefficient of greater than |±0.09| with LNSP are shown)

Variable	Correlation Coefficient	Correlated Variable Already in Equation	Correlation with LNSP	Tolerance*
AL	[illegible].322 [illegible]191 [illegible]336	LNSQF CURG TRE	-0.105	0.735
LNFDA	0.[illegible]365 0.[illegible]60	LNSQF LNG	0.228	0.730
LNSID	0.[illegible] 0.2[illegible]	LNSQF CURG	0.265	0.781
BAY	0.00[illegible]	LNSQF	-0.109	0.916
LNR	0.48[illegible] 0.46[illegible] 0.21[illegible]	LNSQF LNBR LW	0.236	0.553
LNA	-0.423 0.311 0.283	LNSQF TRE LNM	-0.0923	0.571
L	0.169 0.112	TRE LNSQF	0.204	0.915
LNX	0.265 0.164	LNG LR	0.232	0.804
LNB	0.586 0.190	LNSQF CURG	0.427	0.615
LNTR	0.141	CO	-0.148	0.933
BEAM	0.256 0.260 -0.257	LNSQF CURG LNM	0.159	0.738
LU	0.198 -0.167	LNSMR TRE	-0.137	0.870
LNFMR	0.995	LNFM	-0.174	0.020
LNSM	0.999	LNSMR	-0.103	0.001

*See footnote on Table 6

Ratio of List Price to Sale Price with List Variables

The regression of 'list price' and 'sale price' with list variables was done as a prelude to the regression of the list price - sale price ratio with listing variables. If listing brokers and the market value a property in the same way then the list price and sale price equations should be similar. The same factors should thus influence both the numerator and denominator of the list price - sale price ratio equation and have similar coefficients. The coefficients for each factor in the numerator and denominator would then nullify each other. Variables with significant coefficients in the ratio equation therefore explain differences between list price, and sale price relationships.

Many of the significant variables in the list and sale price equations did not appear in the ratio equation. The variables which entered the ratio equation were 'curb appeal', 'extras', 'traffic', 'chattels', and 'month' suggesting that these variables explain the difference between the broker's valuation (list price) and the market's valuation (sale price). All the variables had positive coefficients. R^2=0.20, ($\bar{R}^2$=0.16) was lower than R^2 for both the list and sale price equations, but the equation as a whole was significant beyond the one percent level.[43] See Table 4a.

The appearance of 'extras' in the equation makes sense as it appeared in the list price equation, but not in the sale price equation. The appearance of 'curb appeal' and 'month' in the ratio equation even though they appear in both the list and sale price equations implied that these were valued differently by listing brokers and the market.

The appearance of 'traffic' and 'chattels' in the ratio equation, even though they appeared in neither the list nor sale price equations, implied that they were not factors in the list or sale price models, but became important in explaining the difference between list and sale price.

The positive coefficient for 'curb appeal' indicated that listing brokers tended to overestimate the effect of this factor relative to the market when setting the list price.[44] The positive coefficient of 'month' indicated that listing brokers anticipated a greater upward trend in prices than was supported by actual market price experience.

Similarly, listing brokers tended to value extras and chattels even though these items tend to have no effect on the sale price. These results verify the view in the real estate industry that a finished basement does not in general increase the sale price often because the development is only suited to the taste of the present owner. Chattels are not generally believed to bring their worth if sold with the house. This latter result agrees with the result previously obtained by Janssen (1977) of no price effect.

The coefficient of 'traffic' deserves comment. This factor would certainly be viewed negatively by both real estate agents and the market. The positive sign indicated that listing brokers tended to view the negative effect of traffic as being less significant than the market. Perhaps, for the listing broker, the negative aspects are offset by the fact that more traffic means greater exposure for the 'For Sale' sign and potentially more prospective buyers.

Variables not in the equation were 'floor area', 'rooms', 'bedrooms', 'baths', 'age', 'garage', 'lot', 'trees', 'corner lot', 'side lane', 'front drive attached garage', 'side drive', 'open roof beam construction', 'siding', 'bay window', 'area Lendrum', 'area Duggan', 'undesirable proximity', 'park', 'listing brokers M, R, U and W', 'vacant', 'first and second mortgage', 'first and second mortgage rates'. This implies that these factors tend to be valued equally by both listing brokers and the market or that they are unimportant in explaining the difference between list and sale price.

These results were not unexpected in light of previous results. Many of the variables which did not appear in the ratio equation had very similar coefficients in both the list and sale price equations. Since the coefficient for a variable in the ratio equation is approximately equal to the difference in the coefficients in the list and sale price equations such variables would be expected to have a coefficient not significantly different from zero in the ratio equation. Variables in this category were: 'floor area', 'garage', 'broker R', 'trees', 'first mortgage', 'broker W', and 'bedrooms'. Other variables were not significant in either the list or sale price equations and hence would not be expected to appear in the ratio equation. Variables in this category were 'rooms', 'baths', 'age', 'lot quality', 'side lane', 'front drive attached garage', 'side drive', 'open roof beam construction', 'siding', 'bay window', 'area Lendrum', 'area Duggan', 'park', 'brokers M and U', 'vacant', and 'first mortgage rate'.

Chapter VII ASSESSMENT ANALYSIS

The purpose of this chapter is to study and compare the effect of market factors on assessed values and sale price. First, an attempt is made to identify factors which appear to be used by assessors to arrive at the land, building and total assessment for a property. This is done by running regressions with all assessment variables using 'land assessment', 'building assessment' and 'total assessment' in turn as the dependent variable. Then an attempt is made to identify variables which could be used by assessors to arrive at an estimate of market value. This is done by running a regression with all assessment variables using 'sale price' as the dependent variable.

Once the assessment and sale price equations are developed, an attempt is made to identify which assessment variables are valued differently by assessors and the market. This is done by running a regression with all assessment variables using the total assessment - sale price ratio as the dependent variable.

Assessors are supposed to determine assessment on land and buildings ("improvements") such that it is a percentage of the market value in a given previous year. The province of Alberta has such requirements for assessment of both land and buildings. For land assessment, according to Alberta Regulation 353/73 (1973), the requirement is as follows:

> The fair actual value of land...in urban municipalities...assessed pursuant to the 1967 Provincial Assessment Manual shall be 65% of its market value in the year of general reassessment.[45]

"The fair actual value of land" is interpreted here as, "land assessment". "Market value" may be interpreted as "the mean sale price for actual sales of comparable properties without buildings". The last "year of general reassessment" for assessing land prior to this study was 1970. The exact date in 1970 on which the land assessment is based is not defined in the Municipal Taxation Act or its regulations. For the purpose of this study it is assumed that land assessment is based on the market value of the land on January 1st, 1970. During periods of significant inflation, the market value would not remain constant during the year and it is then necessary to be more specific regarding the date to which the market value refers.

The assessor's job of assessing land at a percentage of market value is difficult, since the market value is not known, but can only be estimated. This also makes it difficult to compare two similar properties to ensure that they are "equitably assessed". Details of the different methods used by assessors for estimating market value are not given here. Instead, an attempt is made to identify factors affecting firstly land value and secondly building value that could be used in regression models for land and building assessment. It must be recognized that if certain market variables are not used the resulting market value estimates will be more uncertain than if they were included.[46]

Building assessment according to Alberta Regulation 250/73 (1973) and the Assessment Manual (1967) is as follows:

> The percentage rates applicable to the fair actual value of improvements which are located in a municipality where improvements are valued pursuant to the provisions

of the 1967 Provincial Assessment Manual, First Edition, shall be...45% of the fair value of all improvements that are assessable...[47]

The Assessment Manual, 1967 First Edition adopts 1963 normal cost in the City of Edmonton and surrounding area as a benchmark....The resulting assessment, if properly determined, will then relate uniformly to market value in terms of the location and base year for which the Assessment Manual is determined.[48]

The "fair actual value of improvements" and "fair value of all improvements that are assessable" may be interpreted as the mean sale price for actual sales of comparable properties exclusive of the cost of the land. The "base year" for purposes of assessing buildings prior to this study was 1963. The exact date in 1963 on which building assessment is based is not defined in the Municipal Taxation Act or its regulations. For the purpose of this study it is assumed that building assessment is based on replacement cost of the building(s) on January 1st, 1963. The total of the replacement costs for a house based on the Assessment Manual is assumed in the definition to be a realistic estimate of the market value in 1963.

House assessment is based both on the regulations established in the Alberta Gazette and on the Assessment Manual (1967). Not all the information in the Assessment Manual is gazetted. Some is included to assist assessors and is not obligatory.

The assessed value of a property is determined by first estimating the market value of the land as though vacant. Land value is estimated through a market survey by comparing site sales and offers to purchase. 65% of the estimated land value in 1970 is added to 45% of the cost of constructing a new house on the land, based on replacement costs in 1963, less accrued depreciation. Building replacement costs are determined by using data in the Assessment Manual (1967). Depreciation, based on physical deterioration and functional obsolescence is as defined in the Assessment Manual. This is known as the cost approach to estimating house market value. An "Example Improvement (House) Assessment" from the Assessment Manual is shown in Fig. 2.

Thus, in the assessment procedure there is no market comparison with respect to building assessment, but only with respect to land assessment. The building assessment is based entirely on construction costs and accrued depreciation. It must be recognized that building market value may well differ from replacement costs. For instance, if demand for houses is high relative to supply, market values may be higher than costs. Similarly, if demand is low relative to supply market values may be lower than building replacement costs. Such swings in demand and supply are a normal occurrence in the market for houses.

Furthermore, disproportionate changes in construction costs may occur over time so that, for instance, plumbing features experience relatively greater cost increases. Then, even if the building market value in 1974 was identical to construction costs in 1974, these would not be a constant factor times 1963 construction costs, but would differ by the above-average increase in plumbing costs. Hence reliance on 1963 construction costs presumes that price increases have been equivalent for all cost components.

FIG. 2:

EXAMPLE IMPROVEMENT (HOUSE) ASSESSMENT*

An example improvement assessment would have a class 5-D (1½ Storey Dwelling with full basement) assessed as follows:

BASIC COST

Area 1,000 square feet @ a Basic Rate of $12.65 per square foot equals a Cost of ------------------------------------	$12,650.00
Should this residence have a 300 square foot one storey addition of the same type of construction and also with a full basement the class 3-D **"Extension Rate"** would be utilized to add this portion e.g.	
Area 300 square feet at an Extension rate of $6.19 per square foot equals a Cost of ------------------------------------	$ 1,857.00
Basic Cost	$14,507.00

SHAPE

Assuming our total property has a perimeter of 154' then no addition may be made for Shape. At 1,300 square feet the perimeter of subject property must be at least 155' before a percentage adjustment may be made for roof framing and number of corners.

VARIATIONS

If subject improvement has:

(a) 16' of **kitchen cabinets**, minus for 3' @ 29.80 equals ---- Subject property has a finished area of 1,900 sq. ft. and will therefore contain 19' of kitchen cabinets.	$ 89.00
(b) 400 square feet of **F** quality up-finish — deduct $1,915 — $715 equals ------------------------------------ (See Section 12, Page 2). If up windows are more or less than 6% of ground floor area a further adjustment must be made; **heat cost** should also be considered.	$ 1,200.00
(c) Modern gravity warm air, deduct -------------------- deduct $80 for 1½ storey portion and $120 — 90 = $30 for 1 storey portion. This adjustment assumes that no previous variation was necessary for heat.	$ 110.00
(d) Checkrail windows, deduct -------------------------- deduct — for 1½ storey portion $210 and for 1 storey portion $170 — $130 = $40.	$ 250.00
Variation	$ 1,649.00
Total equals ------------------------------------	$12,858.00

* From: Assessment Manual (1967:Sec 1, p.22)

Note: This assessment example is for a 1½ storey house. Unfortunately, there is no assessment example for a bungalow in the Assessment Manual so this one was used instead. As the example may be somewhat difficult to follow, the interested reader is referred to the Assessment Manual for more detailed information.

There are difficulties also in the assessment of land, particularly the lack of comparable sales of land without buildings. Once a house has been erected it is not really possible to disaggregate the market value into a land and building component. Nor is it possible to break down the market value estimate, that is, the sale price in this way.

Hence, even if assessors were doing an absolutely superb job of assessing land and buildings it would not be expected that total assessment should equal a constant proportion of the sale price. It is possible, none-the-less to identify the factors which contribute to the variation observed.

The relationships, firstly between land assessment and secondly between building assessment and the assessment variables are studied before analyzing total assessment and sale price.

Land Assessment With Assessment Variables

Land assessment is basically the assessor's estimate of land market value in a given previous year (Land assessment should be 65 percent of land market value). It is thus expected that the following are possible variables which may be significant in explaining land assessment: 'traffic', 'area Duggan', 'lot quality', 'corner lot', 'trees', 'park', 'area Lendrum', 'undesirable proximity' and 'lane'. These variables are physical features and neighbourhood characteristics known at the time of assessment.

These a priori land assessment variables are tabulated in Table 3 to permit comparison with variables which would not be considered by the land assessor, either because they are not known at the time of assessment, or are not used for assessment purposes.

'Land assessment' was regressed against all a priori assessment variables listed in Table 3. The regression procedure was thus used as a land assessment variable selection method.[49]

Six variables were found significant in the equation for land assessment: 'traffic', 'floor area', 'area Duggan', 'lot quality', 'corner lot' and 'trees' suggesting that these variables are being used by the assessor to arrive at land assessment. R^2 was 0.37 ($\bar{R}^2$=0.34), and the equation as a whole was significant beyond the one percent level.[50] See Tables 8a and 8b.

Land assessment tended to increase with 'floor area', 'lot quality', and for a 'corner lot'. It tended to decrease with 'traffic', 'area Duggan' and 'trees'.

Traffic, the first variable which entered the equation, as expected from its high negative correlation coefficient of -0.424 with 'land assessment', tended to have a significant negative impact on the assessed value of residential land. In contrast to this, traffic has a negative correlation of only -0.134 with 'building assessment'. The Assessment Manual (1967) lists "character of street traffic" as an important factor in "the process of estimating values by comparison for (land) assessment purposes.[51] Further, "Residential properties (here, read buildings) may suffer if located on truck routes, main arterial roads, etc".[52] The Assessment Manual considers traffic as a factor which also results in

TABLE 8a

SALE PRICE AND ASSESSMENT
SUMMARY OF REGRESSION RESULTS WITH ASSESSMENT VARIABLES
Significance Criterion: $F \geq 3.0$

Dependent Variable	LNSQF	LNTR	AL	LNX	LNA	LNG	CURG	LNPAR	LNR	LNBR	L	LNBAC
LNTOT	1	-1	-1	1	-1	1	1	1	-1	NS	NS	NS
LNSP	1	-1	NS	1	NS	1	1	NS	NS	-1	1	NS
$LN(\frac{TOT}{SP})$	-1	NS	-1	NS	-1	NS	NS	NS	-1	1	NS	1

Legend:

NS = Not Significant

1 = Variable is in the equation with a positive coefficient.

-1 = Variable is in the equation with a negative coefficient.

TABLE 8b

SALE PRICE AND ASSESSMENT
REGRESSION RESULTS WITH ASSESSMENT VARIABLES
Significance Criterion: $F \geq 3.0$

Dependent Variable	Constant										R^2	SE	F
		LNTR	LNSQF	AD	L	CO	TRE						
LNLAND	7.39	-0.060	0.190	-0.041	0.032	0.039	-0.024				0.37	0.050	10.7
		(0.011)	(0.052)	(0.011)	(0.015)	(0.022)	(0.013)						
		LNSQF	LNA	LNX	LNG	AL	BEAM	LNPAR	LNBR				
LNBLDG	2.65	0.890	-0.095	0.106	0.057	-0.072	0.048	0.078	-0.210		0.812	0.072	57.9
		(0.087)	(0.014)	(0.017)	(0.015)	(0.019)	(0.017)	(0.029)	(0.120)				
		LNSQF	LNTR	AL	LNX	LNA	LNG	CURG	LNPAR	LNR			
LNTOT	5.44	0.594	-0.042	-0.038	0.042	-0.033	0.032	0.037	0.039	-0.131	0.779	0.045	41.5
		(0.062)	(0.010)	(0.012)	(0.011)	(0.009)	(0.009)	(0.016)	(0.018)	(0.062)			
		LNSQF	LNG	LNBR	LNX	CURG	L	LNTR					
LNSP	5.80	0.766	0.047	-0.468	0.046	0.067	0.047	-0.034			0.580	0.085	21.3
		(0.094)	(0.017)	(0.014)	(0.018)	(0.030)	(0.024)	(0.018)					
		LNTOT	LNA	LNSOF	LNBR	L	LNBAC						
LNSP (W/LNTOT)	1.80	0.676	0.053	0.529	-0.446	0.045	-0.041				0.602	0.082	27.5
		(0.135)	(0.013)	(0.133)	(0.135)	(0.024)	(0.022)						
		LNA	LNSQF	LNBR	LNBAC	AL	LNR						
$LN(\frac{TOT}{SP})$	0.525	-0.049	-0.276	0.478	0.055	-0.048	-0.244				0.295	0.083	7.6
		(0.015)	(0.101)	(0.150)	(0.023)	(0.022)	(0.122)						

what it calls "economic obsolescence" of residential houses.[53] The Manual thus states that traffic is a factor which affects both land and building assessment. The results of the regression analysis agree with this in that traffic is firstly a factor affecting land assessment and its impact on building assessment is much more minor, as will be shown later.[54]

The presence of 'floor area' in the equation with a positive coefficient is believed due to its correlation with lot size. Larger houses tend to be on larger lots. Hence 'floor area' is interpreted as a surrogate for "lot area" in the equation for land assessment.

The presence of 'area Duggan' in the equation with a negative coefficient reflects the influence of location on land assessment. This conclusion is based on the assumption that other assessment variables such as lot area, lot quality and traffic have similar distributions in each of the three areas studied. Lower assessed values in Duggan may be explained by the fact that in 1974 this subdivision was just in the process of development.

As expected, lot quality tended to increase the land assessment. A larger well located or pie-shaped lot is usually more desirable and hence more heavily assessed.

Corner lots (positive coefficient) tended to be assessed more heavily because of a higher proportion of lot frontage. The Assessment Manual (1967) states:

> If applicable at all, maximum residential corner influence is 10% of the value the lot would normally have if it was an inside lot and is restricted to the first 50 feet of the corner lot. Corner influence should not be applied to lots bordering on main arterial roads, truck routes, dead end streets, etc.[55]

Land assessment is thus based on a 'dollars-per-front-foot' approach. Corner residential lots may be assessed an extra amount up to ten percent for the first fifty feet from the corner. Corner influence, however, is not normally applied to lots bordering on arterial roads because of the assumed reduction in value due to traffic noise. Dead end streets also sometimes result in reduced access so that no premium for corner influence is added. The Assessment Manual (1967) does not give specific rules for calculating the value of a residential corner lot, but does give tables and a sample calculation showing how to calculate the value of a commercial or industrial corner lot.

The Assessment Manual (1967) points out that "Corner influence is a subject which commands a considerable amount of dispute among experts on urban land appraisals" and "since the factors determining the degree of corner influence are local in effect and origin, no specific table of corner influence factors is recommended for use on a Provincial scale".[55]

'Trees' has a negative coefficient and appears to have entered the equation in order to compensate for over-inclusion of the variable 'area Duggan'. That is, there is multicollinearity among the variables in the equation. This belief is confirmed by:

- 'Trees' high negative correlation with 'area Duggan' (-0.37) and the large negative change in the coefficient for 'area Duggan' when 'trees' is added to the equation,
- the low correlation of 'trees' with 'land assessment' (-0.024), such that 'trees' would not be expected to enter the equation,

- the low tolerance value of 'trees' relative to other variables which implies high correlation with one or more other variables in the equation.[56]

'Trees' high positive correlation with 'age' (0.31) which is not in the equation suggests that 'trees' may be a surrogate for older neighbourhoods and that land assessment should perhaps include a factor for the age of a neighbourhood.

The correlation of the a priori assessment variables 'undesirable proximity', 'lane', and 'park' with land assessment is low. The absence of these variables from the equation suggests that they are not important in land assessment. The Assessment Manual (1967) vaguely refers to these neighbourhood factors as "character of surrounding improvements" and "recreational facilities" but does not say how they are to be incorporated.[57] 'Area Lendrum' also has a low correlation with land assessment, so that it is not surprising that this a priori land assessment variable did not enter the regression. The relatively low R^2 (0.37) suggests that there are large random influences on land assessment, or that some land assessment variables are missing. Known missing land assessment variables are 'lot area', for which 'floor area' which appeared in the land regression acts as a surrogate, or 'lot frontage' and 'lot depth', 'lot shape', if "pie-shaped", 'average income level of neighbourhood', 'age of neighbourhood', a dummy variable for houses located on the west or south sides of the street, and 'neighbourhood vacancy rate'.[58] Variables for individual assessors could be included to determine if this factor has a significant effect on the assessment, similar to that of the listing broker on list price. These variables were not included as the data was based on MLS statistics and external inspection of the house and surrounding neighbourhood.

Building Assessment With Assessment Variables

If building assessment is based on replacement cost figures and the procedure in the Assessment Manual (1967), it is expected that the following variables may be significant in explaining building assessment: 'floor area', 'age', 'extras', 'garage', 'open roof beam construction', 'front drive attached garage', 'siding', 'side drive', 'bay window', 'baths', 'rooms', 'bedrooms' and 'curb appeal' and to some extent 'area Duggan' and 'area Lendrum'. These a priori building assessment variables are tabulated in Table 3 so as to permit comparison with variables which would not be considered by the building assessor.

'Building Assessment' was regressed against all physical features and neighbourhood characteristics as listed in Table 3. The regression procedure was thus used as a building assessment variable selection method.

The results were largely as expected. Eight variables were found significant in the equation for building assessment: 'floor area', 'age', 'extras', 'garage', 'area Lendrum', 'open roof beam construction', 'park' and 'bedrooms' suggesting that these variables are being used by the assessor to arrive at building assessment. R^2 was 0.81 ($\bar{R}^2$=0.80) and the equation as a whole was significant beyond the one percent level.[59]

Building assessment tended to increase with 'floor area', 'extras', 'garage', 'open roof beam construction' and 'park'. These variables have positive coefficients and positive correlations with 'building assessment' as shown in Table 5 except 'park' which has a small negative correlation (-0.025).

Building assessment tended to decrease with 'age', 'area Lendrum' and 'bedrooms'. The negative coefficient of 'age' is as expected since assessors include a factor for depreciation related to the age of the house. The negative coefficient of 'bedrooms' might also be expected from the results of the list price analysis.[37] The Assessment Manual (1967), however, does not say that the assessor should deduct for small room sizes so that this result is somewhat surprising. It suggests that small bedroom sizes were taken into account by assessors.

There is no evidence that the assessor assigns a higher value to an attached garage than a separate garage, as 'front drive attached garage' did not appear in the equation. The Assessment Manual (1967), however, indicates that attached garages should be assessed at a higher rate than detached garages.[60] This result may be due to the correlation of 'front drive attached garage' of 0.275 with 'floor area' which is in the regression equation and preventing 'front drive attached garage' from entering. Further work is required to isolate the effects of garage type. This could possibly be achieved through a revised definition of 'front drive attached garage', 'garage', and 'side drive', or the inclusion of a term for the combined effect of floor area and garage, or possibly the use of factor analysis to reduce collinearity among the variables in the equation.

'Baths' did not appear in the equation even though baths are generally considered by the assessor. This is likely due to the high correlations with 'floor area' and 'bedrooms' which were both in the equation acting as surrogates or proxy variables.[61] The a priori building assessment variables 'rooms', 'siding' and 'curb appeal' also did not appear in the equation most likely because of their high correlations with 'floor area'. 'Side drive' and 'bay window' have low correlations with building assessment. Hence it is not surprising that these a priori building assessment variables did not enter the regression.

It was expected that assessors would include 'park' in the land valuations. Results indicate, however, that nearby park area tends to influence the building assessed value and not the land assessment as this variable entered the building, but not the land assessment equation. The location variable 'area Lendrum' was expected to affect primarily the land assessment, even though the Assessment Manual (1967), states that "building assessment...will...relate uniformly to market value in terms of location...".[62] This is a significant difference from the situation faced by Berry and Bednarz in Chicago where:

> ...identical physical properties [improvements] in significantly different neighbourhoods should still have identical [building] assessments...If the assessor's method is carried out correctly, neighbourhood characteristics, racial or ethnic variables, and environmental externalities should have no

> effect on the assessment figures for improvements... land is supposed to be assessed on the basis of market value, and it is land that expresses the benefits and disbenefits of environment and location to urban property.[63]

Although the R^2 for the equation for building assessment was quite high (0.81), there may still be significant variables which could be included in any further work, for example 'house type', based on the Assessment Manual (1967) classifications, 'window area', 'heating cost' and 'insulation R-value'.

Total Assessment With Assessment Variables

Nine variables were found significant using total assessment as the dependent variable in a regression with the assessment variables. They were 'floor area', 'traffic', 'area Lendrum', 'extras', 'age', 'garage', 'curb appeal', 'park' and 'rooms' suggesting that these variables are being used by the assessor for arriving at the total assessment.

The appearance of most of the variables in the total assessment equation could be predicted based on their significance in the land and building assessment equations. R^2 was 0.78 ($\bar{R}^2$=0.76) and the equation as a whole was significant beyond the one percent level.[64]

Total assessment tended to increase with 'floor area', 'extras', 'garage', 'curb appeal', and 'park'. It tended to decrease with 'traffic', 'area Lendrum', 'age', and 'number of rooms'. The signs of the coefficients were as expected from the land and building assessment equations with the negative coefficient for 'rooms' being analogous to the negative coefficient for 'bedrooms'.[37] Since 'floor area', a major building assessment variable, also acts as a surrogate for lot area, an important land assessment variable, it is not possible to comment on the relative significance of lot and floor area in the total assessment equation.

Sale Price Estimates Using Land, Building and Total Assessment

Suppose that the assessment procedure was accurate enough to achieve a land assessment for each property exactly equal to 65 percent of the market value for land in 1970, and a building assessment of exactly 45 percent of the market value for the building in 1963. Suppose further that the land value increased from 1970 to 1974 by a factor B_1, and the building value similarly increased from 1963 to 1974 by a factor B_2. Then the market value in 1974, denoted by MV_{74} should be given by the following relationship:

$$MV_{74} = B_1 \text{ (LAND/0.65)} + B_2 \text{ (BLDG/0.45)}$$

which under the assumptions made should hold exactly.

Using sale price as a proxy for market value, it should be possible to approximate the above relationship with the following additive multiple linear regression model:

$$SP = a_1 + b_1 \text{ (LAND/0.65)} + b_2 \text{ (BLDG/0.45)}$$

where b_1 and b_2 should approximate B_1 and B_2, respectively. The intercept, a_1, is included to permit the fitted regression equation not to go through the origin.

If sale price is a good proxy for market value and assessors do a good job of assessing land and building at their respective proportions we would expect a_1 to be close to zero and b_1 and b_2 to be good approximations of B_1 and B_2, respectively.

It is recognized that there may be errors of measurement in land and building assessment, that is, a violation of one of the fundamental assumptions underlying the Ordinary Least Squares model.[65] This would be expected to lead to some downward bias in the slope coefficient estimates and as well to a larger intercept term.

The results in Table 9 show that $\hat{a}_1$ was not zero, but was not particularly large either. The land coefficient was 2.33 which corresponds to an average annual compound rate of growth of $r_1=(2.33)^{\frac{1}{4}}-1=23.6$ percent. The building coefficient was 1.68 corresponding to an average annual compound rate of growth of $r_2=(1.68)^{\frac{1}{4}}-1=4.83$ percent. The coefficients as well as the equation as a whole were significant beyond the one percent level.

The estimate of B_2 is perhaps believable, but the estimate of B_1 is certainly exaggerated. This could be due to one factor not yet accounted for. If real estate values increase gradually during the year, it is not sufficient to refer to the market value in 1974. A time within the year needs to be specified as well. An assumption is made that assessed values relate to the market value as of January first. Then 'month' is added as an independent variable in the equation.[66] The model fitted then becomes:

$$SP = a_1 + b_1\ (LAND/0.65) + b_2\ (BLDG/0.45) + b_3M$$

The result is given in Table 9. The coefficients as well as the equation as a whole were significant beyond the one percent level.

$\hat{a}_1$ was not equal to zero, but on the other hand it was not large. $\hat{b}_1$ being 1.71 implies an average annual compound growth rate in the

TABLE 9

SALE PRICE AND ASSESSMENT: REGRESSION RESULTS WITH MARKET VARIABLES (LAND AND BUILDING ASSESSMENT ARE DIVIDED BY THE ASSESSMENT – MARKET VALUE RATIO LAND1=LAND/0.65, BLDG1=BLDG/0.45, TOT1=LAND1 + BLDG1)

Dependent Variable	Constant	BLDG1	LAND1	R^2	SE	F
SP	4270	1.68 (0.203)	2.33 (0.760)	0.456	4581*	47.3

Dependent Variable	Constant	BLDG1	M	LAND1	R^2	SE	F
SP	3062	1.82 (0.171)	812. (114.8)	1.71 (0.641)	0.624	3825*	61.9

	Constant	LNTOT1	LNM	AL	LNBAC	LNSQF	LR	LNG	TRE	LNSMR	LNFM	LW	R^2	SE	F
LNSP	1.85	0.522 (0.096)	0.081 (0.007)	0.032 (0.015)	-0.053 (0.016)	0.504 (0.098)	0.059 (0.017)	0.037 (0.012)	0.052 (0.015)	-0.027 (0.011)	-.005 (0.002)	-0.038 (0.020)	0.822	0.056	43.6

* This regression is a linear model

market value of land since 1970 of $r_1=(1.71)^{\frac{1}{4}}-1=14.4$ percent. $\hat{b}_2$ being 1.82 similarly implies an average annual compound growth rate since 1963 of $r_2=(1.82)^{\frac{1}{11}}-1=5.6$ percent.

The fact that 'month' was significant indicates that assessors are not adjusting their assessments to January first. If house prices are going up by over $800 per month as the coefficient of 'month' indicates, it is important that assessments be based on a particular date during the year.

If other variables are found significant in an equation for sale price in which the market value of land and buildings, as determined by the assessor, is accounted for, this would indicate that these variables are insufficiently accounted for in total assessment. One would expect particularly that important market variables which are not assessment variables would be significant. The third equation in Table 9 confirms these expectations. Ten other variables were found significant at the 5 percent level and the equation as a whole was significant beyond the one percent level. Five of these are market variables not considered by the assessor ('month, 'first mortgage', 'broker R', 'second mortgage rate' and 'broker W').[67] The fact that five market variables not considered by the assessor entered the equation accentuates the differences in the market's and assessor's valuations. The remaining five variables ('area Lendrum', 'undesirable proximity', 'floor area', 'garage', and 'trees') perhaps tend to be evaluated differently by assessors and the market. It may be concluded that assessors are using these variables incorrectly in their estimates of market value.[68]

Sale Price With Assessment Variables and Total Assessment

Sale Price was regressed against total assessment and the assessment variables to determine how well these variables reflect the market value of a property. Six variables were found significant in this equation for sale price namely, 'total assessment', 'age', 'floor area', 'number of bedrooms', 'lot quality', and 'undesirable proximity'. Sale price thus tended to increase with 'total assessment', 'age', 'floor area' and 'lot quality', and decrease with 'number of bedrooms' and 'undesirable proximity'.

The relatively low R^2 (=0.60, $\bar{R}^2$=0.58) in comparison to the previous sale price equation with market variables (R^2=0.79, $\bar{R}^2$=0.77) is due to the exclusion of market-related factors such as month of sale, listing broker, and financial variables (See Tables 8a and 8b). The equation as a whole was, none-the-less, significant beyond the one percent level.[69]

'Total Assessment' was the first variable to enter the equation, indicating that it is the best single explanatory factor for the sale price of a house, better even than floor area. This is an indication that assessors are reasonably effective in determining market value. Previous results indicate that this is due mainly to success in estimating building market value. The fact that other assessment factors appear in the equation indicates some weakness in the assessment procedure.

'Age' had a positive coefficient and appears to have entered the equation in order to compensate for underinclusion of the variable 'total assessment'. This belief is confirmed by

- the high negative correlation of 'age' with total assessment (-0.456), and the large increase in the coefficient for 'total assessment' when 'age' enters the equation,
- the fact that 'age' does not enter the equation when 'total assessment' is excluded from the regression.
- the low correlation of 'age' with sale price (-0.092), and
- the low tolerance value of 'age' relative to other variables.

With this evidence it cannot be concluded that older houses (and lots) tend to have higher sale prices. These results suggest that the assessment method does not adequately account for the depreciation (or appreciation) of houses.

The coefficients for 'undesirable proximity', 'number of bedrooms', and 'lot quality' were as expected from the previous analysis.[37]

'Month of sale', was not permitted in this regression as it is not an assessment variable. This variable explained over 15 percent of the variance in sale price in the previous regression for sale price in the List Price Analysis (Chapter VI) using market variables known at the time of listing. One solution would be to make an adjustment for date of sale. This approach was apparently not used by assessment officials. Assessment is not calculated to reflect prices at a certain date during the year. Similarly, existing financing is not considered by assessors even though found to be significant in Table 4b and generally believed to significantly affect property value in the real estate industry, particularly in periods when mortgage rates are high. These results indicate that the assessor's estimate of the sale price could be improved if these market variables were considered.

Sale Price With Assessment Variables

When 'total assessment' was excluded from the regression, sale price was found to increase with 'floor area', 'garage', 'extras', 'curb appeal' and 'lot quality', and to decrease with 'number of bedrooms' and 'traffic'. These results were consistent with those discussed previously and with what would be expected ($\overline{R}^2$=0.55). The equation as a whole was significant beyond the one percent level.[70]

In the previous section, 'total assessment' was the first variable to enter the sale price equation giving an R^2 change of 0.43. In this section, with 'total assessment' excluded, 'floor area' was the first variable to enter the sale price equation giving an R^2 change of 0.39. These results confirm that total assessment is the best single explanatory factor for sale price, better even than floor area. The small difference in the R^2 values, however, is an indication that assessment is based heavily on floor area (and also on lot area since floor and lot area are usually highly correlated). The results are another indication that assessors overall are doing a reasonable job in assessing property value.

Ratio of Total Assessment to Sale Price With Assessment Variables

The regression of sale price and total assessment with assessment variables was done as a prelude to the regression of the total assessment - sale price ratio with the assessment variables. If assessment valuation kept pace with market price movements one would expect roughly similar statistical relationships in the models that were formulated to explain variations in sale price and in total assessment of single-family homes. Hence one would expect the independent variables to have very little explanatory power in the model of the assessment - sale price ratio, that is, the same factors, if used in the same way by assessors and the market, would influence both the numerator and denominator and tend to cancel. The same logic requires that variables which do appear in either the total assessment or sale price equation but not both should tend to appear in the ratio equation.

The results confirmed these expectations. Many variables significant in the total assessment and sale price equations did not appear in the ratio equation and many variables significant in either the total assessment or sale price equation but not both did appear in the ratio equation. R^2 equalled 0.30 ($\bar{R}^2$=0.26) for the ratio equation which was lower than R^2 for both the 'sale price' and 'total assessment' equations. The equation as a whole was, however, significant beyond the one percent level.[71]

The variables which entered the ratio equation were 'age', 'floor area', 'bedrooms', 'undesirable proximity', 'area Lendrum' and 'rooms' suggesting that these variables explain the difference between the assessors' valuation (total assessment) and the market's valuation (sale price). Only 'bedrooms' and 'undesirable proximity' had positive coefficients.

Variables not in the ratio equation were 'area Duggan', 'side drive', 'front drive attached garage', 'curb appeal', 'trees', 'park', 'siding', 'bay window', 'lane', 'corner lot', 'garage', 'lot quality', 'extras', 'baths', 'traffic' and 'open roof beam construction'. This implies that these factors were either valued equally by both the assessor and the market or that they were unimportant.

From inspection of Table 8a the appearance of 'bedrooms' in the equation makes sense as it appeared in the sale price equation but not in the total assessment equation. The appearance of 'age', 'rooms' and 'area Lendrum' makes sense because they appeared in the total assessment equation but not in the sale price equation. The appearance of 'undesirable proximity' in the ratio equation, even though it appeared in neither the sale price nor assessment equations, implies that it is not a major factor in the sale price or assessment models but it does become important in explaining the difference between the sale price and the total assessed value.

The appearance of 'floor area' in the ratio equation even though it appeared in both the total assessment and sale price equations implies that floor area is valued differently by assessors and the market.

It is somewhat surprising that 'rooms' entered the ratio equation which already included 'floor area' and 'bedrooms' as 'rooms' is highly correlated with both these variables. 'Rooms' and 'bedrooms' did not enter the 'total assessment' or 'sale price' equations together. 'Rooms' is also the least significant variable in the ratio equation. Since 'rooms' and 'bedrooms' may represent the same effect, interpretation of this variable is not entirely clear.[37]

The negative coefficient for 'floor area' and 'area Lendrum' indicates that assessors tended to underestimate the value of house size and location in Lendrum relative to the market.[44,72] Assessors also tended to depreciate a house more rapidly than the market as 'age' has a negative coefficient. This is likely due to assessors depreciating houses at a time when house values were rising. This appreciation in house values was indicated by the positive coefficient for 'month' in the sale price equation in the List Price Analysis (Chapter VI). Expressed differently, larger, or older houses, or those located in Lendrum tended to be underassessed relative to the target percentage of market value. Conversely, properties with more bedrooms or adjacent to high density housing, condominiums, rental housing, freeways, schools or churches appeared overassessed relative to their market value, as indicated by the positive coefficients for 'bedrooms' and 'undesirable proximity'.[37]

Chapter VIII SUMMARY AND CONCLUSIONS

In this study, multiple regression techniques with multiplicative models and market variables were used to analyze estimates of single family residence property values made by assessors, real estate agents and the market. Certain properties of multiplicative models facilitated the analysis.

The techniques were similar to those employed by Berry and Bednarz (1975) in a real estate application with the modification that only significant variables were included in any equation, that is, entry of variables was not forced. This made it possible to interpret the coefficients of all variables. Further, only land assessment variables were used to analyze land assessment; building assessment variables were used to study building assessment; et cetera, obviating the need to interpret a number of variables as surrogates. The method was also applied to list price analysis, something not done by Berry and Bednarz.

The data was from Janssen (1977) with the addition of several new variables. A number of these new variables were found significant: 'traffic', 'corner lot', 'trees', 'undesirable proximity', 'park area', 'open roof beam construction' and 'curb appeal'. These variables were not included in a number of the previous studies reviewed.

It was found that a number of variables tended to be valued higher by listing brokers than the market, including: the appearance of the house, ('curb appeal'), extras such as a developed basement or fireplace, traffic, chattels, such as stoves and refrigerators, and the monthly upward trend in prices. All other items tended to be valued equally by brokers and the market.

The importance of selecting the right listing broker to get the maximum sale price for a property, found by Janssen (1977), was confirmed here. Broker R tended to list at six percent above the average list price for comparable properties while broker W tended to list at more than four percent lower than average. Houses that were listed higher by broker R generally sold seven percent higher and houses that were listed lower by broker W generally sold almost six percent lower than comparable houses.

Compared to sale prices, larger or older properties or those located in Lendrum tended to be underassessed. Conversely, properties with more bedrooms or those adjacent to high density housing, condominiums, rental housing, freeways, schools or churches appeared overassessed relative to their market value.

Total assessment was found to be the best single explanatory factor for the sale price of a property, better even than floor area. This was an indication that assessors are reasonably effective in determining market value. The fact that other assessment variables entered the equation for sale price along with total assessment, however, indicated some weakness in the assessment procedure. Further, the R^2 for the sale price equation using assessment variables was low compared to the R^2 for the sale price equation using additional market-related factors such as 'listing broker' and financing variables. Hence the assessor's estimate of sale price tended to be more uncertain than if these market variables had been included. None-the-less, it was found that land and building assessment can be used to provide a reasonable estimate of the sale price, provided the month of sale is included as a variable in the regression equation.

Considerable random influence was found on land assessment compared to building assessment. The errors in estimating building and total assessment tended to be lower, which implied that errors in land and building assessment tended to be compensating when added to arrive at the total assessment. The success in estimating total assessment thus appeared to be due mainly to obtaining reasonable accuracy in estimating building assessment. These results indicate that inclusion of more land assessment variables, such as 'lot frontage', 'lot depth', 'average income level of neighbourhood', 'age of neighbourhood', 'neighbourhood vacancy rates', a dummy variable for properties located on the west or south sides of the street, and dummy variables for individual assessor effects may be relatively important in any further assessment analysis. If the inclusion of these variables results in a better land assessment equation, improvement in the list and sale price equations should also be possible since assessment variables are considered to be a subset of list variables.

A definition for the combined effect of variables in multiplicative models was proposed which makes it possible to quantify major interaction effects, thereby more accurately determining variable coefficients. Use of techniques based on this definition with multiplicative models may also facilitate prediction of major interaction effects in linear models.

There appears to be scope for the use of ridge regression or other techniques such as backward elimination regression, or regression using all possible subsets of variables, to control multicollinearity and obtain more significant results. Results in this study had to be qualified because of collinearity among major variables.

Based on the improved understanding of real estate processes obtained in this study it is believed that multiple regression with multiplicative models has the potential to become a practical tool for widespread use in real estate. As computer costs continue to drop, the use of such models in real estate applications should become increasingly attractive.

FOOTNOTES

[1] The Appraisal Terminology and Handbook (1962) defines sale price, market value and the difference between them as follows:

> SALE PRICE - The price paid for a property; the amount of money that must be given or which may be obtained at the market in exchange under the immediate conditions existing at a certain date. The price paid for a property regardless of pressures, motives, or intelligence. To be distinguished from market value.
> MARKET VALUE
> 1. As defined by the courts, the highest price estimate in terms of money which a property will bring if exposed for sale in the open market allowing a reasonable time to find a purchaser who buys with knowledge of all the uses to which it is adapted and for which it is capable of being used.
> 2. Frequently, it is referred to as the price at which a willing seller would sell and willing buyer would buy, neither being under abnormal pressure.
> 3. It is the price expectable if a reasonable time is allowed to find a purchaser and if both seller and prospective buyer are fully informed.
>
> The essential difference between sale price and market value, as above defined, lies in the premises of intelligence, knowledge, and willingness, all of which are contemplated in market value but not in [sale price]. Stated differently, at any given moment of time market value connotes what a property is actually worth and [sale price] what it may be sold for.

In this study, sale price is defined as the price for which the property sold. Market value is regarded as the mean (or median) of the potential sale price distribution.

[2] For more information on multivariate regression, see Draper and Smith (1967), Johnston (1972), and Goldfeld and Quandt (1972).

[3] The Department of Municipal Affairs of the Government of Ontario has also conducted a number of assessment studies in recent years. See, for example, the Ontario Report on Taxation (1967).

[4] Oldman and Aaron (1965:47)

[5] Smith (1971:277)

[6] Smith (1971:281).

[7] Smith (1971:284)

[8] See Goldberger (1968:464) for a transformation and discussion of theory.

[9] Goldberger (1968:472)

[10] Teekens (1972:Foreword)

[11] Teekens and Koertz (1972:793)

[12] See, for example, Goldfeld and Quandt (1972) and (1976) which contain many references to the work of other authors.

[13] Trippi (1974) compares the R^2 of multiplicative and additive models with limited success and without discussion of some of the underlying theoretical difficulties in comparisons of this type.

[14] Churchill (1975:686) using actual property data found that "ridge estimates were closer to the true values, on the average, for each coefficient and for each sample size, that is, for all 78 comparisons". See also Smith (1976) which describes some present limitations with the method.

[15] Berry and Bednarz (1975:23)

[16] The market variables used by Berry and Bednarz were 'floor area', 'age of dwelling', 'lot area', 'air conditioning', 'garage', 'improved attic', 'improved basement', 'number of baths', 'median family

income', 'multiple family dwellings', 'migration', '% Blacks', '% Cubans/ Mexicans', '% Irish', 'Sulphur Dioxide', 'particulates', and 'Distance to Central Business District'.

[17] Berry and Bednarz (1975:38)

[18] See Goldfeld and Quandt (1972) for a detailed discussion.

[19] From Teekens (1972:22 and Appendix A). See also Goldfeld and Quandt (1972).

[20] The price elasticity, E, is defined as:

$$E = (dP/P)/(dX_i/X_i)$$

where $P = b_o X_1^{b_1} X_2^{b_2} \ldots X_1^{b_1} \ldots X_k^{b_k}$

therefore

$$E = (b_o X_1^{b_1} X_2^{b_2} \ldots b_i X_i^{b_i - 1} \ldots X_n^{b_k} dX_i / b_o X_1^{b_1} X_2^{b_2} \ldots X_i^{b_i} \ldots X_k^{b_k})/(dX_i/X_i)$$

$$= b_i$$

Hence the elasticity with respect to X_i is the exponent of X_i, that is b_i.

[21] See <u>Interaction Effects</u> in Chapter III for a derivation of this result.

[22] MLS is a service whereby a number of realty firms co-operate in the sale of a property. A member firm may sell listed properties and list properties to be sold by others. The data were taken from the monthly MLS Sales Summaries published by the Edmonton Real Estate Board.

[23] The three neighbourhoods were Petrolia/Royal Garden, Duggan and Malmo/Lendrum. Bungalows are one-storey single-family dwellings with basements (in the present case). About seventy percent of the houses in the areas studied are bungalows.

24 Janssen (1977:66)

25 The mortgage variables refer to existing financing which may change upon transfer of the property.

26 Regressions show the derived variables used by Janssen:

F = FM + SM,

FR = (FMR(FM) + SMR(SM))/F,

and AA = SQF(A)

do not improve the regression results as they did in his study. This is most likely the result of higher correlation of the derived variables with the new variables.

27 Some of the new variables may have changed since the original Janssen (1977) study was done. For example, if there are large trees on the property in 1978, it was assumed that they were there at the time the original data were collected in 1974. Similarly, if a house has a bay window, it was assumed that it was not added recently. Hence the possibility of errors from changes in the data is recognized, but is considered small.

28 As the houses in the sample had been sold before this study was started, it was impractical to inspect them inside in order to add interior variables.

29 Certain variables used in this study were not easily coded. These include curb appeal, siding and undesirable proximity.

Classification of houses based on curb appeal should be possible. The subjective nature of this variable, however, may leave room for disagreement on the coding for a particular property. The coding which, among a few alternative possibilities, gave the highest R^2 value in simple linear regressions with sale price was selected. The coding presented in Table 2 reflects the best variable classification found for curb appeal.

Similar issues arose in coding the variables siding and undesirable proximity and were resolved in the same way. Regressions with sale price indicated that the market considers aluminum siding superior to stucco or wood. Houses which had siding on half or the entire front wall of the house were grouped together. The coding for siding and undesirable proximity in Table 2 was selected as the best based on regression results.

[30] Use of the locational dummy variables (area Duggan and area Lendrum) is in keeping with the recommendation of Straszheim (1974) on geographic disaggregation.

[31] The average selling period is 34 days with a standard deviation of 20 days. Hence the month of sale tends to follow that of listing by one month and is used as a surrogate for month of listing.

[32] See Ratcliff (1965) for a more detailed definition and Colwel (1979) for a number of statistically oriented definitions of market value. The mean, median or mode of a potential sale price distribution may be used as Colwel indicates. As shown in Goldberger (1968) a transformation from the median to the mean is possible when using multiplicative models for a given set of conditions.

[33] The analysis was performed using the SPSS computer programs. See Nie et al (1975).

[34] Durbin-Watson tests found no autocorrelation in the error terms.

[35] As there are twelve variables or less in each regression, a reduction in the number of variables was not considered important. Some assessment systems (such as in California) have included over one hundred variables, many of them insignificant. See Ratcliff (1968:79) for details.

[36] F=39.3 for the equation compared to a table value at the one percent level of 2.38 for 12 and 103 degrees of freedom.

[37] The partial correlation of sale price and list price with both 'bedrooms' and 'rooms' is negative when controlling for 'floor area'. For houses with comparable floor areas more bedrooms or more rooms means smaller room sizes which listing brokers (and the market) tend to view negatively.

[38] F=36.3 for the equation compared to a table value at the one percent level of 2.44 for 11 and 104 degrees of freedom.

[39] Sample calculations are given in the Appendix.

[40] Similar differences between brokers had been found previously using factor analysis. See Janssen and Jobson (1978), (1980).

[41] The fact that the variable for broker R is significant is evidence that the choice of listing broker can increase the sale price of a property by a percentage amount and not just by a fixed amount as found with Janssen (1977)'s additive model.

[42] Janssen and Jobson (1978:15-16)

[43] F=5.6 for the equation compared to a table value at the one percent level of 3.20 for 5 and 110 degrees of freedom.

[44] See the discussion in Chapter III and <u>Interpretation of the Signs of Coefficients in Ratio Equations</u> in the Appendix for a proof.

45 Alberta Regulation 353/73 (1973:982).

46 Further analysis could be done by including all list variables in the sale price analysis. This would indicate differences in the market's and assessors' property valuations. This was not attempted.

47 Alberta Regulation 250/73 (1973:787-788).

48 Assessment Manual (1967:Sec.1, p.4).

49 Regression variable selection, however, could not cope with spurious correlations with the dependent variable. Therefore, the variables 'chattels' and 'bay window' which appeared in preliminary 'land assessment' regressions and could not be logically explained were excluded from the final regression.

50 F=10.7 for the equation compared to a table value at the one percent level of 2.99 for 6 and 109 degrees of freedom.

51 Assessment Manual (1967:Sec.1, p.24).

52 Assessment Manual (1967:Sec.1, p.52).

53 Assessment Manual (1967:Sec.1, p.51).

54 This conclusion is reinforced in the building assessment regression which follows. Traffic is not a significant variable in the 'building assessment' equation.

55 Assessment Manual (1967:Sec.1, p.32).

56 See <u>Tolerance</u> in the Appendix for a definition.

57 Assessment Manual (1967:Sec.1, p.24).

58 'Average income level of neighbourhood' is recommended as a variable because it influences land value according to the Assessment Manual (1967) and a similar variable 'median family income' was significant in the Berry and Bednarz (1975) study of assessment practice

in Chicago. 'Neighbourhood vacancy rate' is recommended as a variable which would reflect market demand for property in the neighbourhood relative to other neighbourhoods and is similar to the variable 'population growth or decline' suggested by the Assessment Manual (1967:Sec.1, p.24).

59 F=57.9 for the equation compared to a table value at the one percent level of 2.69 for 8 and 107 degrees of freedom.

60 Assessment Manual (1967:Sec.4, p.48).

61 The Assessment Manual (1967:Sec.1, p.8) indicates 'baths' may be considered a "trended cost", that is, their value is correlated with floor area. The correlation of 'baths' with 'floor area' (0.59) confirms this.

62 Assessment Manual (1967:Sec.1, p.4).

63 Berry and Bednarz (1975:25-26).

64 F=41.5 for the equation compared to a table value at the one percent level of 2.60 for 9 and 106 degrees of freedom.

65 For a discussion of the errors of measurement problem see Johnston (1972:281-291)

66 An alternative would have been to deflate sale prices by a monthly increase found from a simple linear regression of sale price with 'month'. The approach selected, however, was considered more direct.

67 Very similar results were obtained using LAND1 and BLDG1 instead of TOT1 and are not reported here.

68 The possibility exists that there has been a shift in the market's valuation of these variables.

69 F=27.5 for the equation compared to a table value at the one percent level of 2.99 for 6 and 109 degrees of freedom.

70 F=21.3 for the equation compared to a table value at the one percent level of 2.82 for 7 and 108 degrees of freedom.

71 F=7.6 for the equation compared to a table value at the one percent level of 2.99 for 6 and 109 degrees of freedom.

72 Interpretation of this result is clouded, however, as 'floor area' possibly also acts as a surrogate for lot area which was not included in the analysis.

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APPENDIX

Hedonic Price Functions

The models used in this study may be recognized by the economist as being of the same form as the Cobb-Douglas type or the gravitational trade flow equations. These equations are multiplicative models developed using regression methods. Justification for using this type of model in the application in this paper can be partly based on the hedonic price method or 'characteristics of goods' approach to consumer theory. This approach assumes that a good can be disaggregated into a bundle of separately identifiable features whose contribution to the value of the good can be measured. That is, each good is valued for its utility which is derived from the utility of each of its characteristics. Hedonic prices are the implicit prices for each characteristics, attribute or quality of an 'economic good' which a consumer may buy. Satisfaction of wants is accomplished as members of households search for and purchase combinations of market commodities yielding the mix of desired characteristics which give the highest utility. Constraints such as that of a budget or the availability of the goods desired may be assumed.

It can be shown that a hedonic price function or index can be related to supply and demand functions given certain assumptions on the behaviour of producers, consumers, and the market. Hedonic price functions are thus regression equations of the general class:

$$SP = SP\ (X_1, X_2 \ldots X_k;\ e)$$

where SP is the sale price

X_i, i = 1,2,...k are the characteristics.

e is a random error term.

In this study, prices (or assessments which are related to price) are used as the dependent variables in the regressions. The independent variables are property characteristics including physical features such as floor area, number of rooms, extras, and age; neighbourhood characteristics such as area of the city, street traffic, and nearby park area; realtor effects; timing effects and financing.

Interpretation of the Signs of Coefficients in Ratio Equations

Given a ratio equation of the form:

$$A/P = aX_1^{b_1}X_2^{b_2}\ldots X_i^{b_i}\ldots X_k^{b_k}e^u$$

where $X_i > 0$, i = 1,2,...k

and e^u is the error term

Taking the partial derivatives with respect to X_i:

$$\partial(A/P)/\partial X_i = ab_iX_i^{b_i-1}\ldots X_j^{b_j}\ldots X_k^{b_k}$$

Where j = 1,2,...k, $j \neq i$

If $b_i = 0$ then $\partial(A/P)/\partial X_i = 0$ and the slope of the regression equation with respect to X_i is zero

If $b_i > 0$ then $\partial(A/P)/\partial X_i = ab_iX_i^{b_i-1}\ldots X_j^{b_j}\ldots X_k^{b_k}$

and the slope is positive if a is positive.

If $b_i < 0$ then $\partial(A/P)/\partial X_i = (ab_iX_j^{b_j}\ldots X_k^{b_k})/X_i^{|b_i-1|}$

and the slope is negative if a is positive.

A positive sign of a coefficient, b_i, in the A - P ratio equation indicates that the ratio of A to P increases as the variable X_i

increases. In other words, A tends to "run ahead" of P as X_i increases. A negative coefficient indicates the opposite, that is, P outpaces A as X_i increases, and a zero coefficient indicates that the variable has no effect. A positive coefficient for 'curb appeal', for example, in the list price - sale price ratio equation would indicate that listing brokers tend to overestimate the effect of 'curb appeal', relative to the market.

Effect of Individual Variables

The following is an example showing how the effect of a variable on sale price can be calculated.

Let SP = $c(M^{0.080})$ be the estimated relationship where c represents the effect of other variables held constant and M is the month. Hence the increase in sale price from January (month 1) to June (month 6) is $c(6^{0.08} - 1^{0.08}) = 0.15$ c or 15 percent. For the average ($47,441) property, this implies and increase in sale price of $7,300.

The following is an example showing how the effect of a dummy variable on sale price can be calculated.

Let SP = $c(0.948^{CO})$ be the estimated relationship where c represents the effect of other variables held constant and CO is a dummy variable for 'corner lot'. When CO=1 the sale price is $ .948c. When CO=0 the sale price is $c. Thus when a house is on a corner lot the sale price is reduced by approximately 5%. The same approach may be used for other dummy variables such as 'curb appeal', 'side lane', and 'listing broker R'.

Combined Effect of Variables

The following is an example showing how the combined effect of two variables can be calculated if special terms for interaction effects are not included in the equation. The combined effect of floor area and garage space can be measured by multiplying the magnitudes of the effect of each variable i.e. ($SQF^{0.5196}(G+1)^{0.03642}$) where $SP = c(SQF^{0.5196})(G+1)^{0.03642}$ is the estimated relationship, c represents the effect of other variables, held constant, SQF is floor area and G is the number of bays in the garage.

For a 1,000 sq. ft. house and no garage, sale price equals 36.21c.
For a 1,000 sq. ft. house and a one car garage, sale price equals 37.13c.
For a 1,500 sq. ft. house and no garage, sale price equals 44.70c.
For a 1,500 sq. ft. house and a one car garage, sale price equals 45.84c.

Adding a garage to a 1,000 sq. ft. house increases sale price (37.13 - 36.21)c	=	0.92c
Increasing floor area from 1,000 to 1,500 sq. ft. increases sale price (44.70 - 36.21)c	=	8.49c
Total		9.41c
Increasing floor area from 1,000 to 1,500 sq. ft. and adding a garage increases sale price (45.84 - 36.21)c	=	9.63c
Adding a garage to a 1,500 sq. ft. house increases sale price (45.84 - 44.70)c	=	1.14c

The combined effect of adding a garage and increasing floor area (\$9.63c) is greater than the sum of the individual effects (\$9.41c). Adding a garage to a large house increases sale price more (\$1.14c) than adding the same garage to a small house (\$0.92c). These results indicate that a garage has a greater effect on the sale price the larger the house, ceteris paribus.

The combined effect of dummy variables is also easy to calculate. The combined effect of 'trees' and 'broker R' can be measured by multiplying the magnitudes of the effects of each variable, i.e. $((1.057^{TRE})(1.054^{LR})-1)100 = 11.4\%$. where $SP = c(1.057^{TRE})(1.054^{LR})$ is the estimated relationship, 'c' represents the effect of other variables, held constant, and the dummy variables for 'trees' (TRE) and 'broker R' (LR) equal 1 or 0. From the equation it can be seen that 'trees' and 'broker R' increase the sale price by 5.7% and 5.4% respectively. The combined effect (11.4%) is greater than the sum of the individual effects (5.7 + 5.4 = 11.1%)

Diminishing Marginal Returns and Marginal Analysis

The multiplicative model is appropriate where percentage changes in the explained variable are related to percentage changes in the independent variables by a constant factor. For example, a house with more floor area will normally sell for more than a house with less floor area, all other things being equal. However, the increase in sale price is less with each equal increase in floor area, as illustrated below:

$$\text{Let } SP = c(SQF^{0.5196})$$

where SP = the sale price in dollars

SQF = the floor area

c = all other factors in the equation held constant

For a 1,000 sq. ft. house this expression equals \$36.21c
For a 1,250 sq. ft. house this expression equals \$40.66c
For a 1,500 sq. ft. house this expression equals \$44.70c
For a 1,562.5 sq. ft. house this expression equals \$45.66c

Sale Price ($)	Floor Area (Square Feet)	% Increase in Floor Area	% Increase in Sale Price
36.21c	1000.0		
		25%	12.3% ($=\frac{(40.66-36.21)c(100)}{36.21c}$)
40.66c	1250.0		
		25%	12.3% ($=\frac{(45.66-40.66)c(100)}{40.66c}$)
45.66c	1562.5		

Hence, in this equation, percentage changes in sale price are related to percentage changes in area by a constant factor.

Similarly, equal absolute increases in floor area result in progressively smaller increases in sale price due to diminishing marginal returns. Below is an example, using figures from the previous illustration.

Sale Price ($)	Floor Area (Square Feet)	Increase in Floor Area (Square Feet)	% Increase in Sale Price
36.21c	1000		
		250	12.3% ($=\frac{(40.66-36.21)c(100)}{36.21c}$)
40.66c	1250		
		250	9.9% ($=\frac{(44.70-40.66)c(100)}{40.66c}$)
44.70c	1500		

Letting e = 1000 (c can be determined from the regression equation), increasing house size from 1000 to 1250 square feet increases the sale price at the rate of $(\frac{40.66-36.21}{250})1000$ = \$17.80 per square foot. Similarly, increasing house size from 1250 to 1500 square feet increases the sale price at the rate of \$16.20 per square foot.

More accurate results can be obtained by solving the equation for the derivative of sale price with respect to floor area ($\partial SP = 519.6\ SQF^{-.4804}\partial SQF$). These results can also be verified by plotting a curve of sale price versus floor area assuming any arbitrary value for c as shown in fig. 3. Note that any exponent greater than zero and less than one implies diminishing marginal returns.

<u>Tolerance</u>

The tolerance of an independent variable being considered for inclusion in the regression equation is the portion of the variance of that variable not explained by the independent variables already in the regression equation. Tolerance has a range of 0 to 1. A tolerance of 0 indicates that the variable is a linear combination of the independent variables already in the equation. A tolerance of 1 indicates that the variable is uncorrelated with the independent variables already in the equation. An intermediate tolerance of 0.8 indicates that 80 percent of the variance of the variable is unexplained by the independent variables already in the equation.

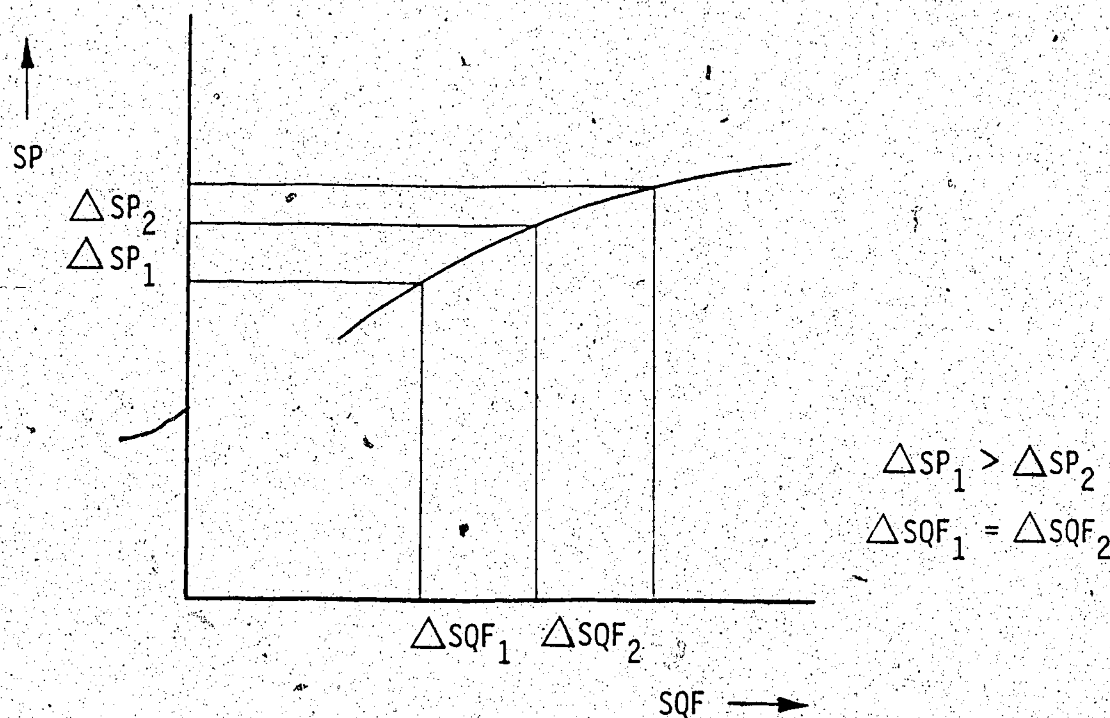

Fig. 3 Illustration of Diminishing Marginal Returns