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THE UNIVERSITY OF ALBERTA

ADAPTIVE CONTROL USING A KALMAN FILTER

BY

MICHAEL W. FOLEY

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

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IN

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled ADAPTIVE CONTROL USING A KALMAN FILTER submitted by MICHAEL W. FOLEY in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE IN PROCESS CONTROL.

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Date: September 2, 1988

To Albert Holtforster

Without whom this work would not have been probable

Abstract

Adaptive controllers are typically designed with the primary emphasis on servo response. In realtime chemical engineering applications, however, the chief task of most control schemes is regulation. This thesis analyzes the regulatory behaviour of a number of single-step and multi-step control strategies with the objective of unifying their various approaches to the problem of predicting future outputs, especially in the presence of nonstationary stochastic disturbances.

Work in the 1970's on the equivalence of optimal stochastic controllers implemented in the state space and transfer function domains culminated in a general proof of equivalence for minimum variance control of plants subject to stationary stochastic disturbances. This thesis presents a simpler approach to the problem by analogy with the Smith predictor, and extends the result to include plants having nonstationary disturbances. The correspondence that exists between the Generalized Minimum Variance forms of these controllers is also shown.

Adaptive long-range predictive control strategies have found a wide range of application in chemical process control because they have demonstrated a greater degree of robustness than single-point strategies in the presence of model/plant mismatch. Once again, methods have been proposed that may be implemented in the state space domain

using a Kalman filter, or in the transfer function domain by recursion of a Diophantine equation. Two such algorithms, Multistep Adaptive Predictive Control (MAPC) and Generalized Predictive Control are shown to provide asymptotically equal control of time-invariant plants subject to time delay and nonzero-mean disturbances, by an extension of the equivalence result derived for the minimum variance case. A comparison between these techniques and two nonparametric LRPC schemes, Dynamic Matrix Control and the Multivariable, Optimal, Constrained Control Algorithm is also presented.

The thesis concludes with an experimental evaluation of Multistep Adaptive Predictive Control on the double-effect evaporator at the University of Alberta. The MAPC scheme is shown to provide excellent servo/regulatory performance using a minimum of *a priori* knowledge regarding the plant dynamics.

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Nomenclature

Technical Abbreviations

AMKFP	Adaptive Modified Kalman Filter Predictor
ARMA	Autoregressive, Moving Average
ARIMA	Autoregressive, Integrated, Moving Average
CM	Covariance Modification
DARMA	Deterministic, Autoregressive, Moving Average
DMC	Dynamic Matrix Control
ELS	Extended Least Squares
FF	Feedforward
GMV	Generalized Minimum Variance
GPP	Generalized Pole Placement
GPC	Generalized Predictive Control
ID	Identification
ILS	Improved Least Squares
ILSP	Integrating Least Squares Predictor
IO	Input/Output
ISTC	Integrating Self-Tuning Control
KF	Kalman Filter
KFP	Kalman Filter Predictor
LQG	Linear Quadratic Gaussian
LRPC	Long-Range Predictive Control
LSP	Least Squares Predictor
MAPC	Multistep Adaptive Predictive Control
MISO	Multi-input, Single-output
MIMO	Multi-input, Multi-output
MKFP	Modified Kalman Filter Predictor

MOCCA	Multivariable Optimal Constrained Control Algorithm
MPM	Model/Plant Mismatch
MV	Minimum Variance
PDG	Polynomial Disturbance Generator
RLS	Recursive Least Squares
SISO	Single-input, Single-output
SP	Smith Predictor
SSF	Single Series Forecasting
STC	Self-Tuning Control
TEG	Triethylene Glycol

Alphabetic

$A(z^{-1})$	Polynomial corresponding to the process output
$a_1,\ldots,a_n$	Coefficients of $A(z^{-1})$
$B(z^{-1})$	Polynomial corresponding to the process input
$b_1,\ldots,b_n$	Coefficients of $B(z^{-1})$
$C(z^{-1})$	Polynomial corresponding to the stochastic disturbance
d	Physical time delay of the process
$E\{\cdot\}$	Statistical expectation operator
$E(z^{-1})$	Diophantine polynomial of degree (k-1)
$F(z^{-1})$	Diophantine polynomial corresponding to the remainder term
$G(z^{-1})$	Product of $E(z^{-1})$ and $B(z^{-1})$; also actual plant transfer function
$G_F(z^{-1})$	Residual filter transfer function
$G_m(z^{-1})$	Model of plant with delay
$G_p(z^{-1})$	Model of plant without delay
H	Output coefficient vector
J	Quadratic cost index
$K_1(z^{-1})$	Polynomial of Kalman gains, order n-1)

$K_2(z^{-1})$	Polynomial of Kalman gains, order (d-1)
$K_3(z^{-1})$	Diophantine polynomial of order (k-1)
$K_4(z^{-1})$	Diophantine polynomial corresponding to the remainder term
$K_5(z^{-1})$	Diophantine polynomial corresponding to the remainder term
k	Time delay of process plus hold device
L	Kalman gain vector
$L_0, \ldots, L_{n+d}$	Elements of Kalman gain vector
N_1	Lower limit of output weighting horizon
N_2	Upper limit of output weighting horizon
N_u	Control weighting horizon
N_y	Output weighting horizon
n	Order of $A(z^{-1})$
$n_1(t)$	Process noise
$n_2(t)$	Measurement noise
$P(z^{-1})$	Output weighting transfer function
$Q(z^{-1})$	Input weighting transfer function
$R(z^{-1})$	Setpoint weighting transfer function
R_1	Process noise covariance
R_2	Measurement noise covariance
$u(t)$	Process input
$x(t)$	State vector
$y(t)$	Process output
z	Forward shift operator

<u>Greek</u>

Δ	Differencing operator
δ	Order of polynomial
Φ	State coefficient matrix
Γ	Process noise coefficient vector
$\gamma_1, \ldots, \gamma_n$	First n elements of Γ
Λ	Input coefficient vector

λ	RLS forgetting factor; also input weighting coefficient
$\omega(t)$	Kalman filter innovations sequence

<u>Superscripts</u>

F	Filtered value
$\hat{}$	Estimated value
$\tilde{}$	Augmented variable

<u>Subscripts</u>

SP	Setpoint
SS	Steady state

Introduction

Optimal stochastic control strategies may be divided into two major categories, *i.e.* those that use a Kalman filter to predict future outputs, and those that accomplish this by solution of a Diophantine identity. The original objective of this thesis was to compare two such algorithms, the integrating Self-Tuning Controller of Tuffs and Clarke (1985) and a control scheme derived by Lu (1986) for use with the Modified Kalman Filter Predictor of Walgama et al (1988). This work resulted in a new general proof of equivalence for state space and transfer function controller designs.

However, as the work progressed, it became apparent that these arguments could be extended to the multistep case to include a comparison of Generalized Predictive Control (Clarke et al, 1987) and the Multistep Adaptive Predictive Control algorithm of Sripada (1988). Moreover, a generalized block diagram developed for this analysis was found to provide a convenient framework for comparison of these algorithms with "nonparametric" control techniques such as Dynamic Matrix Control (Cutler and Ramaker, 1980) and the Multivariable Optimal Constrained Control Algorithm (Sripada and Fisher, 1985).

Chapters 2 to 5 of this thesis follow approximately the chronological order of these developments, *i.e.*

1) Single-step predictive control of known systems

2) Single-step adaptive predictive control

3) Multistep predictive control of known systems

4) Multistep adaptive predictive control

In addition, a series of experimental trials conducted using the double effect evaporator at the University of Alberta has been included in Chapter 6, which represents a feasibility study of the MAPC algorithm when applied to a real plant.

Each of these chapters builds on the results of previous chapters, and numerous cross references have been included. However, key results from earlier chapters have been repeated to make them relatively independent, in acccordance with the "paper format" option for theses at the University of Alberta. The overall conclusions and some recommendations for future research are included in Chapter 7 to provide a larger perspective on the contributions of this thesis.

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2

Single-Step Predictive Control

2.1 Introduction

The equivalence of the Kalman Filter Predictor and the Least Squares Predictor was a topic of considerable interest in the mid-1970's, following the advent of optimal stochastic controllers that could be implemented in the transfer function domain, *i.e.* without resort to conventional state space design techniques. The equivalence of the two approaches was formally demonstrated by Caines (1972) for the unity delay case. This result was subsequently strengthened by Hughes (1973) to include a general delay term, subject to the restriction $A(z^{-1}) = 1$ (*cf.* Eqn. (2.18) below). This final requirement was in turn removed by Watson (1976) to complete the proof for the general case.

The strategy used by these authors to establish the equivalency principle consisted of two steps. First, it was demonstrated that the innovation sequences for the state space and transfer function methods are asymptotically equal. An expression was then constructed for each technique relating the control action to the innovation sequence, so that the two control laws could be compared on a term-by-term basis. This step proved to be somewhat tedious, despite the fact that the (nonminimal) state space realization was chosen to lead to a block diagonal state transition matrix. Furthermore, it is difficult to generalize this result to state space controllers which do not possess this special property.

This chapter will present a novel derivation of the principle by analogy with the Smith Predictor, and will extend the result slightly to demonstrate the equivalence of the imum variance forms of the integrating Self-Tuning Controller of Tuffs and Clarke (1985) and the Modified Kalman Filter Predictor of Walgama (1986). In addition, the correspondence of the Generalized Minimum Variance forms of these schemes will be discussed. The chapter concludes with a simulation example demonstrating this relationship for minimum variance control of a second order plant having time delay and nonstationary stochastic disturbances.

2.2 Smith Predictor

It is well known that stability considerations often require very low proportional constants to be used in the application of conventional PID control to plants having significant time delay, which results in sluggish servo and regulatory performance. For this reason, Smith (1957, 1959) developed a time delay compensation scheme for deterministic systems to eliminate dead time from the closed loop characteristic equation. The development of the discrete Smith Predictor (SP) begins by considering the DARMA plant model:

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) \tag{2.1}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \ldots + b_n z^{-n+1}$$

and the plant dead time $k \geq$ includes the unit delay due to discretization. The essential idea is to feed to the controller the output of the plant minus the delay, or equivalently $\hat{y}(t+k\mid t)$ using the following expression:

$$\hat{y}(t+k\mid t)=\frac{B(z^{-1})}{A(z^{-1})}u(t)$$
$$+\left[y(t)-z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right] \qquad (2.2)$$

or

$$\hat{y}(t+k\mid t)=G_P(z^{-1})u(t)+G_F(z^{-1})[y(t)-G_M(z^{-1})u(t)] \qquad (2.3)$$

where for the Smith Predictor,

$$G_P(z^{-1})=\frac{B(z^{-1})}{A(z^{-1})},\quad G_M(z^{-1})=z^{-k}\frac{B(z^{-1})}{A(z^{-1})}$$

and $G_F(z^{-1})=1$.

G_F may be interpreted as an ad hoc noise filter useful when applying the Smith Predictor to stochastic plants, but is included here primarily for later reference. Equation (2.3) (in closed loop form) is represented schematically in Fig. 2.1.

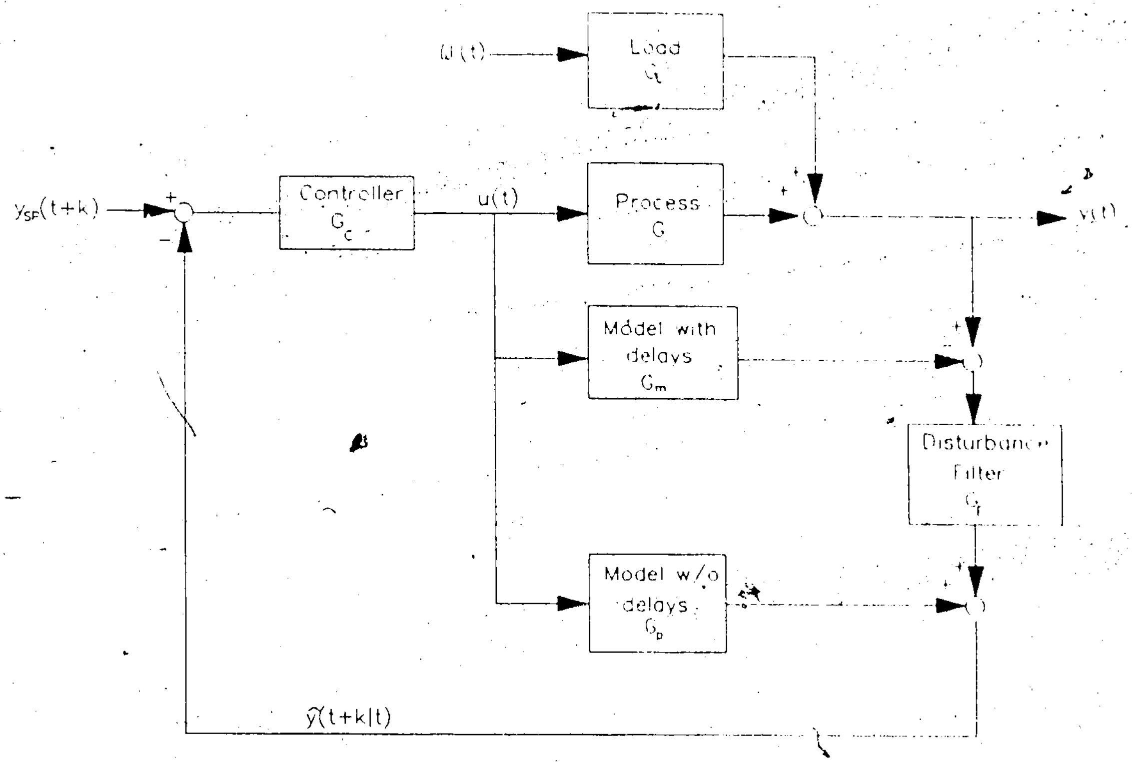

Figure 2.1 Time delay compensation using the Smith Predictor

Note that if no disturbances are present ($\omega(t)=0$) and there is no model/process mismatch (MPM), then the error term or residual is zero, and $\hat{y}(t+k|t)=G_P u(t)$.

2.3 Kalman Filter Predictor

The optimal control law derived by Lu (1986) for the Kalman Filter Predictor (KFP) minimizes the performance index

$$J_{KFP}=[R(z^{-1})y_{SP}(t+k)-P(z^{-1})E\{y(t+k)|t\}]^2 +[Q(z^{-1})u(t)]^2 \qquad (2.4)$$

where $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ are transfer functions chosen to tailor closed loop performance to the user's specifications. (Note that $Q(1)=0$ and $P(1)=R(1)$ are necessary conditions for zero controller offset.) The k-step-ahead output prediction $E\{y(t+k)|t\} = \hat{y}(t+k|t)$ is obtained using the Kalman Filter Predictor of Bialkowski (1978, 1983).

Consider the discrete state space representation

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \Gamma n_1(t) \qquad (2.5)$$

$$y(t) = H x(t) + n_2(t) \qquad (2.6)$$

where for $d = k-1$,

$$x(t) = [x_1(t), x_2(t), \ldots, x_n(t), x_{n+1}(t), \ldots, x_{n+d}(t)]^T$$

$$\Phi = \begin{bmatrix} 0 & 0 & \ldots & 0 & -a_n & 0 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & -a_{n-1} & 0 & \ldots & 0 & 0 \\ . & . & & . & . & . & & . & . \\ . & . & & . & . & . & & . & . \\ . & . & & . & . & . & & . & . \\ 0 & 0 & \ldots & 1 & -a_1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & 0 \\ . & . & & . & . & . & & . & . \\ . & . & & . & . & . & & . & . \\ . & . & & . & . & . & & . & . \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}_{(n+d)\times(n+d)}$$

$$\Lambda = [b_n, b_{n-1}, \ldots, b_1, 0, \ldots, 0]^T_{1\times(n+d)}$$

$$\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n, 0, \ldots, 0]^T_{1\times(n+d)}$$

and

$$H = [\;\;, 0, \ldots, 0, 1]_{1\times(n+d)}$$

$n_1(t)$ and $n_2(t)$ are uncorrelated Gaussian noise sequences, with covariances R_1 and R_2.

As mentioned above, the optimal prediction $\hat{y}(t+k|t)$ can be obtained by applying the standard Kalman filter

(Appendix A) to the state equations (2.5), (2.6) as follows. A one-step-ahead state estimate conditioned upon dat up to and including time t is given by

$$\hat{x}(t+1|t) = \Phi\hat{x}(t) + \Lambda u(t)$$
$$\hat{y}(t+1|t) = H\hat{x}(t+1|t)$$

which in turn implies that

$$\begin{aligned}\hat{y}(t+k|t) &= H\hat{x}(t+k|t)\\ &= H\Phi^{k}\hat{x}(t) + \sum_{j=t}^{t+d} H\Phi^{t+d-j}\Lambda u(j)\\ &= \hat{x}_{n-1}(t) - a_1\hat{x}_n(t) + b_1 u(t)\end{aligned} \tag{2.7}$$

A minimum variance control law design using the KFP is obtained by setting $P(z^{-1}) = R(z^{-1}) = 1$ and $Q(z^{-1}) = 0$ in (2.4), leading to the modified cost functional:

$$J_{MV} = [y_{SP}(t+k) - \hat{y}(t+k|t)]^2 \tag{2.8}$$

J_{MV} is clearly minimized by choosing $u(t)$ in (2.7) such that $\hat{y}(t+k|t) = y_{SP}(t+k)$, *i.e.*,

$$u(t) = \frac{1}{b_1}[y_{SP}(t+k) - \hat{x}_{n-1}(t) + a_1\hat{x}_n(t)] \tag{2.9}$$

Innovations analysis is a useful tool in the interpretation of the Kalman Filter Predictor, as it enables the KFP to be represented in terms of Fig. 2.1. The development begins by writing the steady state Kalman filter equations corresponding to Eqns. (2.5) and (2.6):

$$\hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L\omega(t+1) \tag{2.10}$$

$$\begin{aligned}\omega(t) &= y(t) - \hat{y}(t|t-1)\\ &= y(t) - H\hat{x}(t|t-1)\\ &= y(t) - H\Phi\hat{x}(t-1) - H\Lambda u(t-1)\end{aligned} \tag{2.11}$$

The Kalman gain vector L is given by

$$L^T = [L_1, L_2, \ldots, L_{n+d}]$$

and $\omega(t)$ is henceforth assumed to be a zero-mean random sequence. Using successive forward substitution for states 1 to n,

$$\begin{aligned}\hat{x}_n(t) &= [1 - A(z^{-1})]\hat{x}_n(t) + B(z^{-1})u(t-1) \\ &\quad + [L_n + L_{n-1}z^{-1} + \ldots + L_1 z^{-n+1}]\omega(t) \\ &= [1 - A(z^{-1})]\hat{x}_n(t) + B(z^{-1})u(t-1) + K_1(z^{-1})\omega(t)\end{aligned} \tag{2.12}$$

where

$$K_1(z^{-1}) = L_n + L_{n-1}z^{-1} + \ldots + L_1 z^{-n+1} \tag{2.13}$$

Similarly, for states $n+1$ to $n+d-1$,

$$\begin{aligned}\hat{x}_{n+d-1}(t) &= z^{-d+1}\hat{x}_n(t) \\ &\quad + [L_{n+d-1}z^{-1} + \ldots + L_{n+1}z^{-d+1}]\omega(t+1)\end{aligned}$$

or

$$\hat{x}_{n+d-1}(t-1) = z^{-d}\hat{x}_n(t) + K_2(z^{-1})\omega(t) \tag{2.14}$$

with

$$K_2(z^{-1}) = L_{n+d-1}z^{-1} + \ldots + L_{n+1}z^{-d+1} \tag{2.15}$$

quation (2.12) implies that

$$\hat{x}_n(t) = A^{-1}(z^{-1})B(z^{-1})u(t-1) + A^{-1}(z^{-1})K_1(z^{-1})\omega(t) \tag{2.16}$$

so that

$$\begin{aligned}\hat{x}_{n+d-1}(t-1) &= z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1) \\ &\quad + z^{-d}A^{-1}(z^{-1})K_1(z^{-1})\omega(t) + K_2(z^{-1})\omega(t)\end{aligned} \tag{2.17}$$

From (2.11),

$$\begin{aligned}y(t) &= \hat{y}(t\,|\,t-1) + \omega(t) \\ &= H\Phi\hat{x}(t-1) + H\Lambda u(t-1) + \omega(t)\end{aligned}$$

Note that for $d \geq 1$,

$$H\Phi = [0, 0, \ldots, 1, 0]_{1 \times (n+d)}$$

and $H\Lambda = 0$. Therefore,

$$\begin{aligned} y(t) &= \hat{y}(t \mid t-1) + \omega(t) \\ &= \hat{x}_{n+d-1}(t-1) + \omega(t) \\ &= z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1) + z^{-d}A^{-1}(z^{-1})K_1(z^{-1})\omega(t) \\ &\quad + A^{-1}(z^{-1} \quad 1(z^{-1})(1 + K_2(z^{-1}))\omega(t) \end{aligned}$$

or

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})}u(t-k) + \frac{C(z^{-1})}{A(z^{-1})}\omega(t) \tag{2.18}$$

where

$$C(z^{-1}) = A(z^{-1})(1 + K_2(z^{-1})) + z^{-d}K_1(z^{-1}) \tag{2.19}$$

Equation (2.18) is an ARMA representation equivalent to e state space model (2.5), (2.6). $A(z^{-1})$, $B(z^{-1})$ and the delay k are as defined in Eqn. (2.1); $C(z^{-1})$ as defined above is a monic polynomial of degree $n+d-1$, where d is the time delay exclusive of the hold device.

A k-step-ahead minimum variance control strategy requires an estimate of the future output $y(t+k)$ (or equivalently, $y(t+d+1)$). For the Kalman Filter Predictor, it was demonstrated in (2.7) above that

$$\begin{aligned} E\{y(t+d+1)\} &= \hat{y}(t+d+1 \mid t) \\ &= H\hat{x}(t+d+1 \mid t) \\ &= \hat{x}_{n-1}(t) - a_1\hat{x}_n(t) + b_1u(t) \end{aligned}$$

But from (2.10),

$$\hat{x}_n(t+1) = \hat{x}_{n-1}(t) - a_1\hat{x}_n(t) + b_1u(t) + L_n\omega(t+1)$$

Thus,

$$\hat{y}(t+d+1\,|\,t) = \hat{x}_n(t+1) - L_n\omega(t+1) \tag{2.20}$$

Now, (2.16) implies that

$$\hat{x}_n(t+1) = A^{-1}(z^{-1})B(z^{-1})u(t) + A^{-1}(z^{-1})K_1(z^{-1})\omega(t+1)$$

Therefore,

$$\begin{aligned}\hat{y}(t+d+1\,|\,t) &= A^{-1}(z^{-1})B(z^{-1})u(t) \\ &\quad + A^{-1}(z^{-1})K_1(z^{-1})\omega(t+1) \\ &\quad - A^{-1}(z^{-1})A(z^{-1})L_n\omega(t+1) \\ &= \frac{B(z^{-1})}{A(z^{-1})}u(t) \\ &\quad + \frac{[K_1(z^{-1}) - L_nA(z^{-1})]}{A(z^{-1})}z^{+1}\omega(t)\end{aligned} \tag{2.21}$$

Rewriting (2.18) in terms of ω $)/A(z^{-1})$ and substituting into (2.21),

$$\begin{aligned}\hat{y}(t+d+1\,|\,t) &= \frac{B(z^{-1})}{A(z^{-1})}u(t) \\ &\quad + z^{+1}\frac{[K_1(z^{-1}) - L_nA(z^{-1})]}{C(z^{-1})}\left[y(t) - z^{-d-1}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right]\end{aligned}$$

or

$$\hat{y}(t+k\,|\,t) = G_P(z^{-1})u(t) + G_F(z^{-1})[y(t) - G_M(z^{-1})u(t)] \tag{2.22}$$

where $G_P(z^{-1})$ and $G_M(z^{-1})$ are as defined for the Smith Predictor, and

$$G_F(z^{-1}) = z^{+1}\frac{(K_1(z^{-1}) - L_nA(z^{-1}))}{C(z^{-1})} \tag{2.23}$$

Returning to the definition of $C(z^{-1})$ Eqn. (2.19),

$$C(z^{-1}) = A(z^{-1})(1 + K_2(z^{-1})) + z^{-d}K_1(z^{-1})$$

$$= A(z^{-1})(1 + K_2(z^{-1})) + L_n z^{-d} + z^{-d}(K_1(z^{-1}) - L_n)$$

$$= A(z^{-1})(1 + K_2(z^{-1}) + L_n z^{-d})$$

$$+ (1 - A(z^{-1}))L_n z^{-d} + z^{-d}(K_1(z^{-1}) - L_n)$$

$$= A(z^{-1})(1 + K_2(z^{-1}) + L_n z^{-d})$$

$$+ z^{-d-1}z^{+1}(K_1(z^{-1}) - L_n A(z^{-1}))$$

or

$$\frac{C(z^{-1})}{A(z^{-1})} = K_3(z^{-1}) + z^{-k}\frac{K_4(z^{-1})}{A(z^{-1})} \qquad (2.24)$$

where

$$K_3(z^{-1}) = 1 + K_2(z^{-1}) + L_n z^{-k+1}$$

$$K_4(z^{-1}) = z^{+1}[K_1(z^{-1}) - L_n A(z^{-1})]$$

It is clear that the KFP as given by Eqn. (2.22) may also be represented by Fig. 2.1, with G_F defined as

$$G_F(z^{-1}) = \frac{K_4(z^{-1})}{C(z^{-1})} \qquad (2.25)$$

The KFP may therefore by interpreted as a Smith Predictor with an optimal disturbance filter G_F. The KFP in fact reduces to the SP when $\omega(t) = 0$ and $G_M = G$. As discussed in Walgama et al (1988), $G_F(1) = 1$ is required to prevent prediction offset in the presence of nonzero-mean $\omega(t)$. This is equivalent to requiring that $K_4(1) = C(1)$, which is not true in general, according to Eqn. (2.24).

2.4 Least Squares Predictor

The Self-Tuning Controller (STC) of Clarke and Gawthrop (1975, 1979) minimizes the cost functional

$$J_{STC} = [R(z^{-1})y_{SP}(t+k) - E\{P(z^{-1})y(t+k)\mid t\}]^2 + [Q(z^{-1})u(t)]^2 \tag{2.26}$$

It is clear that any strategy for which $P(z^{-1}) = 1$ will make the performance index in (2.26) equal to that of (2.4). In particular, this is true for minimum variance control, where $P(z^{-1}) = R(z^{-1}) = 1$ and $Q(z^{-1}) = 0$, which leads to Eqn. (2.8) above:

$$J_{MV} = [y_{SP}(t+k) - \hat{y}(t+k\mid t)]^2 \tag{2.8}$$

For this case, the k-step-ahead prediction of the process output, $\hat{y}(t+k\mid t) = E\{y(t+k)\mid t\}$ is obtained from the Least Squares Predictor (LSP), which is formulated as follows.

Consider the ARMA plant given by Eqn. (2.18):

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\omega(t) \tag{2.27}$$

It is evident from (2.19) that the polynomial $C(z^{-1})$ is monic and is of order $\delta C = \max(n, n+k-2)$. In addition, $C(z^{-1})$ is assumed to have all its roots inside the unit circle. (This requirement is equivalent to requiring that all eigenvalues of the state transition matrix of the KFP are stable, which was shown to be true by Walgama (1986).) Multiplication of (2.27) by $E(z^{-1})$ and rearrangement yields

$$C(z^{-1})y(t+k) = F(z^{-1})y(t) + G(z^{-1})u(t) + C(z^{-1})E(z^{-1})\omega(t+k) \tag{2.28}$$

where the polynomials $E(z^{-1})$ (of order $k-1$) and $F(z^{-1})$ (of order $n-1$) are obtained from the Diophantine identity

$$\frac{C(z^{-1})}{A(z^{-1})} = E(z^{-1}) + z^{-k}\frac{F(z^{-1})}{A(z^{-1})} \tag{2.29}$$

and $G(z^{-1}) = E(z^{-1})B(z^{-1})$. Equation (2.28) can be rewritten as

$$C(z^{-1})y(t+k) = C(z^{-1})\hat{y}(t+k\,|\,t) + C(z^{-1})E(z^{-1})\omega(t+k)$$

where

$$\hat{y}(t+k\,|\,t) = \frac{F(z^{-1})}{C(z^{-1})}y(t) + \frac{G(z^{-1})}{C(z^{-1})}u(t) \tag{2.30}$$

This implies that

$$y(t+k) = \hat{y}(t+k\,|\,t) + E(z^{-1})\omega(t+k)$$

It is seen that $\hat{y}(t+k\,|\,t)$ represents an optimal prediction of $y(t+k)$ in the least squares sense as it is uncorrelated with the future noise terms $E(z^{-1})\omega(t+k)$. Once again, it is obvious that J_{MV} is minimized by choosing $u(t)$ in (2.30) such that $\hat{y}(t+k\,|\,t) = y_{SP}(t+k)$, *i.e.*

$$u(t) = \frac{C(z^{-1})}{G(z^{-1})}y_{SP}(t+k) - \frac{F(z^{-1})}{G(z^{-1})}y(t) \tag{2.31}$$

Following the development of Gawthrop (1977), (2.30) can be expanded to give

$$\hat{y}(t+k\,|\,t) = \frac{F(z^{-1})}{C(z^{-1})}y(t) + \frac{B(z^{-1})}{C(z^{-1})}\left[\frac{C(z^{-1})}{A(z^{-1})} - z^{-k}\frac{F(z^{-1})}{A(z^{-1})}\right]u(t)$$

$$= \frac{F(z^{-1})}{C(z^{-1})}y(t) + \frac{B(z^{-1})}{A(z^{-1})}u(t) - \frac{F(z^{-1})}{C(z^{-1})}z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)$$

$$= \frac{B(z^{-1})}{A(z^{-1})}u(t) + \frac{F(z^{-1})}{C(z^{-1})}\left[y(t) - z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right]$$

or

$$\hat{y}(t+k\mid t)=G_P(z^{-1})u(t) + G_F(z^{-1})[y(t)-G_M(z^{-1})u(t)] \qquad (2.32)$$

where G_P and G_M are as defined above and

$$G_F(z^{-1})=\frac{F(z^{-1})}{C(z^{-1})}$$

Referring to Fig. 2.1, it is clear that the LSP can be interpreted as a SP with an optimal noise filter G_F. The LSP in fact reduces to the SP when $\omega(t)=0$ and $G_M=G$. It is also clear from Eqn. (2.25) that $G_F(1)$ must equal unity for no prediction offset in the presence of nonzero-mean disturbances. This is equivalent to assuming that $F(1)=C(1)$, which is not in general true in Eqn. (2.29).

2.5 KFP versus LSP-based Minimum Variance Control

Since both the KFP and the LSP can be represented by Fig. 2.1, it is natural to ask whether they are equivalent; that is, whether G_F for the KFP is identical to G_F for the LSP. Equations (2.25) and (2.32) state that

$$(G_F)^{KFP}=\frac{K_4(z^{-1})}{C(z^{-1})}$$

$$(G_F)^{LSP}=\frac{F(z^{-1})}{C(z^{-1})}$$

Thus, proving the equivalence of the two schemes requires showing that $K_4(z^{-1})=F(z^{-1})$. To see that this is true, consider Eqns. (2.24) and (2.29):

$$\frac{C(z^{-1})}{A(z^{-1})} = K_3(z^{-1}) + z^{-k}\frac{K_4(z^{-1})}{A(z^{-1})}$$

$$= E(z^{-1}) + z^{-k}\frac{F(z^{-1})}{A(z^{-1})}$$

But $\delta K_3(z^{-1}) = \delta E(z^{-1}) = k-1$, so $K_4(z^{-1}) = F(z^{-1})$ because the expansion $C(z^{-1})/A(z^{-1})$ is unique.

Now, a minimum variance control strategy for either scheme sets $\hat{y}(t+k\,|\,t)$ to $y_d(t+k)$ if the setpoint is known k steps in advance (*cf*. Eqns. (2.9) and (2.31)). Since each predictor scheme is represented by (2.3), this would imply

$$y_{SP}(t+k) = \frac{B(z^{-1})}{A(z^{-1})}u(t) + G_F(z^{-1})\left[\, y(t) - \frac{B(z^{-1})}{A(z^{-1})}u(t-k)\right]$$

and that

$$u(t) = \frac{A(z^{-1})}{B(z^{-1})}[\,y_{SP}(t+k) - G_F(z^{-1})\left(y(t) - \frac{B(z^{-1})}{A(z^{-1})}u(t-k)\right)]$$

All terms on the RHS of the above expression are known and identical for both methods; hence the MV control action obtained using the LSP and the KFP are asymptotically equal (*i.e.* when the Kalman gains have converged).

2.6 Modified Kalman Filter Predictor

As noted in the final paragraphs of Sections 2.3 and 2.4, both the Kalman Filter Predictor and the Least Squares Predictor will exhibit offset in the presence of unmeasured deterministic disturbances. This phenomenon has received a good deal of attention recently in the self-tuning literature with the result that the algorithms have been modified to ensure zero prediction offset. In particular, the integrating Self-Tuning Controller (ISTC) of Tuffs and Clarke (1985) and the Modified Kalman Filter Predictor (MKFP) of Walgama et al (1988) have this property. The next two sections demonstrate the equivalence of the new schemes for known, time-invariant stochastic processes.

In order to guarantee zero steady state prediction offset, the KFP was modified by Walgama et al (1988) to model the process noise $n_1(t)$ in Eqn. (2.5) as a random walk. The state space model (2.5), (2.6) can then be augmented with an additional state $x_p(t)$ having an integrator to represent the disturbance dynamics:

$$x(t+1) = \begin{bmatrix} 1 & 0 \\ \Gamma & \Phi \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \Lambda \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} n_2(t) \tag{2.33}$$

$$y(t) = [0 \quad H] x(t) + n_2(t) \tag{2.34}$$

where

$$x(t) = [x_p(t), x_1(t), \ldots, x_{n+d}(t)]^T$$

A minimum variance control law analogous to that derived above for the KFP is obtained by noting that the k-step-ahead prediction $\hat{y}(t+k\,|\,t)$ of the MKFP is given by

$$\begin{aligned} \hat{y}(t+k\,|\,t) &= H\hat{x}(t+k\,|\,t) \\ &= \gamma_n \hat{x}_p(t) + \hat{x}_{n-1}(t) - a_1 \hat{x}_n(t) + b_1 u(t) \end{aligned}$$

It is again apparent that J_{MV} in Eqn. (2.8) is minimized by the optimal control policy

$$u(t) = \frac{1}{b_1}[y_{SP}(t+k) - \gamma_n \hat{x}_p(t) - \hat{x}_{n-1}(t) + a_1 \hat{x}_n(t)] \tag{2.35}$$

Using an innovations analysis similar to that of Section 2.3 (see Appendix B), (2.33) and (2.34) may be represented in the equivalent ARIMA form

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})}u(t-k) + \frac{C(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta}, \tag{B.7}$$

where

$$C(z^{-1}) = [A(z^{-1})(1 + K_2(z^{-1})) + z^{-d}K_1(z^{-1})]\Delta + z^{-d}L_p[\gamma_n z^{-1} + \ldots + \gamma_1 z^{-n}] \tag{2.36}$$

Note that $C(z^{-1})$ is now defined to be of order $n+d$. The k-step-ahead prediction is once again given by

$$\hat{y}(t+k|t) = G_P(z^{-1})u(t) + G_F(z^{-1})[y(t) - G_M(z^{-1})u(t)] \tag{B.12}$$

where

$$G_F(z^{-1}) = \frac{K_5(z^{-1})}{C(z^{-1})} \tag{B.13}$$

$$K_5(z^{-1}) = z^{+1}[(K_1(z^{-1}) - L_n A(z^{-1}))\Delta + D(z^{-1})] \tag{B.11}$$

As discussed in Section 2.3, zero prediction offset requires that $G_F(1) = 1$ or $K_5(1) = C(1)$. Equation (B.14) states that

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = K_3(z^{-1}) + z^{-k}\frac{K_5(z^{-1})}{A(z^{-1})\Delta} \tag{B.14}$$

from which it is apparent that $G_F(1) = 1$.

2.7 Integrating Least Squares Predictor

Consider the ARIMA process representation given by (B.7)

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})}u(t-k) + \frac{C(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \quad \text{(B.7)}$$

Multiplication of (B.7) by $E(z^{-1})\Delta$, where $\delta E(z^{-1}) = k-1$ and rearrangement yields

$$C(z^{-1})y(t+k) = F(z^{-1})y(t) + G(z^{-1})u(t) + C(z^{-1})E(z^{-1})\omega(t+k) \quad (2.37)$$

where $E(z^{-1})$ and $F(z^{-1})$ are obtained from the Diophantine identity

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = E(z^{-1}) + z^{-k}\frac{F(z^{-1})}{A(z^{-1})\Delta} \quad (2.38)$$

and $G(z^{-1}) = E(z^{-1})B(z^{-1})$. (Note that $E(z^{-1})$ and $F(z^{-1})$ are different from E and F defined in Section 2.4.) It is evident from (2.37) that the least squares prediction of $y(t+k)$ is given by

$$\hat{y}(t+k \mid t) = \frac{F(z^{-1})}{C(z^{-1})}y(t) + \frac{G(z^{-1})}{C(z^{-1})}\Delta u(t) \quad (2.39)$$

A minimum variance control law based on the ILSP is given by

$$u(t) = u(t-1) + \frac{C(z^{-1})}{G(z^{-1})}y_{SP}(t+k) - \frac{F(z^{-1})}{G(z^{-1})}y(t) \quad (2.40)$$

which sets J_{MV} to its minimum value of zero. Using the identity (2.38), Eqn. (2.39) may be written as

$$\hat{y}(t+k\,|\,t) = \frac{F(z^{-1})}{C(z^{-1})}y(t) + \frac{B(z^{-1})}{C(z^{-1})}\left[\frac{C(z^{-1})}{A(z^{-1})} - z^{-k}\frac{F(z^{-1})}{A(z^{-1})}\right]u(t)$$

$$= \frac{B(z^{-1})}{A(z^{-1})}u(t) + \frac{F(z^{-1})}{C(z^{-1})}\left[y(t) - z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right]$$

or

$$\hat{y}(t+k\,|\,t) = G_P(z^{-1})u(t) + G_F(z^{-1})[y(t) - G_M(z^{-1})u(t)] \qquad (2.32)$$

where

$$G_F(z^{-1}) = \frac{F(z^{-1})}{C(z^{-1})}$$

and

$$C(z^{-1}) = [A(z^{-1})(1 + K_2(z^{-1})) + z^{-d}K_1(z^{-1})]\Delta + z^{-d}L_p[\gamma_n z^{-1} + \ldots + \gamma_1 z^{-n}] \qquad (2.41)$$

From (2.38) it is seen that $F(1) = C(1)$; hence the integrating LSP exhibits zero steady state offset in the presence of nonzero-mean disturbances.

2.8 MKFP versus ILSP-based Minimum Variance Control

To demonstrate the equivalence of the MKFP and the ILSP, it is sufficient to show, as in Section 2.5, that $G_F(z^{-1})$ in Fig. 2.1 is the same for both methods. A comparison of Eqns, (B.13) and (2.41) indicates that this is indeed true if $K_5(z^{-1}) = F(z^{-1})$. But from (B.14) and (2.38) it is seen that

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = K_3(z^{-1}) + z^{-k}\frac{K_5(z^{-1})}{A(z^{-1})\Delta}$$

$$= E(z^{-1}) + z^{-k}\frac{F(z^{-1})}{A(z^{-1})\Delta}$$

$K_3(z^{-1}) = E(z^{-1})$ because $\delta K_3(z^{-1}) = \delta E(z^{-1}) = k-1$; hence $(G_F)^{MKFP} = (G_F)^{ILSP}$ and the two schemes are equivalent. Using an argument similar to that of Section 2.5, it may be concluded that the minimum variance forms of the integrating Self-Tuning Controller and the MKFP-based scheme provide asymptotically equal closed loop performance for stochastic plants having time-invariant dynamics.

2.9 Extension to Generalized Minimum Variance Control

In the previous section, the equivalence of minimum variance control schemes based on the Modified Kalman Filter Predictor and the integrating Self-Tuning Controller was discussed. However, as seen in (2.4) and (2.26), both methods minimize cost functionals containing rational weighting polynomials $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$; hence it is natural to ask whether this equivalence holds for general P, Q and R.

Rewriting (2.4) and (2.26) for ease of reference,

$$J_{KFP} = [R(z^{-1})y_{SP}(t+k) - P(z^{-1})E\{y(t+k)\,|\,t\}]^2 + [Q(z^{-1})u(t)]^2 \tag{2.4}$$

$$J_{STC} = [R(z^{-1})y_{SP}(t+k) - E\{P(z^{-1})y(t+k)\,|\,t\}]^2 + [Q(z^{-1})u(t)]^2 \tag{2.26}$$

The obvious difference between these expressions is that in J_{KFP} the quantity $P(z^{-1})E\{y(t+k)\,|\,t\}$ is considered, while in J_{STC} it is $E\{P(z^{-1})y(t+k)\,|\,t\}$. The case of general $Q(z^{-1})$, $R(z^{-1})$ and scalar $P(z^{-1})$, i.e. $P(z^{-1}) = p_0$ will be examined first. For this case it is apparent that $J_{KFP} = J_{STC}$ because $E\{p_0 y(t+k)|t\} = p_0 E\{y(t+k)|t\}$. This implies that

$$J = J_{KFP} = J_{STC}$$

$$= [R(z^{-1}) y_{SP}(t+k) - p_0 E\{y(t+k)|t\}]^2 + [Q(z^{-1}) u(t)]^2 \quad (2.42)$$

However, as pointed out earlier, both prediction schemes may be represented by

$$\hat{y}(t+k|t) = G_P(z^{-1}) u(t) + G_F(z^{-1})[y(t) - G_M(z^{-1}) u(t)] \quad (2.32)$$

so that

$$p_0 \hat{y}(t+k|t) = p_0 \frac{B(z^{-1})}{A(z^{-1})} u(t) + p_0 G_F(z^{-1}) \left[y(t) - \frac{B(z^{-1})}{A(z^{-1})} u(t-k) \right] \quad (2.43)$$

Differentiating w.r.t. $u(t)$,

$$\frac{\partial J}{\partial u(t)} = 2[R(z^{-1}) y_{SP}(t+k) - p_0 \hat{y}(t+k|t)] \frac{\partial [p_0 \hat{y}(t+k|t)]}{\partial u(t)} + 2[Q(z^{-1}) u(t)] \frac{\partial [Q(z^{-1}) u(t)]}{\partial u(t)} \quad (2.44)$$

But it is easily seen from (2.43) that

$$\frac{\partial [p_0 \hat{y}(t+k|t)]}{\partial u(t)} = p_0 b_1$$

$$\frac{\partial [Q(z^{-1}) u(t)]}{\partial u(t)} = q_0$$

where q_0 is defined as the first coefficient in the expansion of $Q_n(z^{-1})/Q_d(z^{-1})$. Setting $\partial J/\partial u(t)$ in (2.44) to zero yields, upon rearrangement,

$$u(t) = \frac{p_0 b_1}{(p_0 b_1)^2 - q_0^2}[R(z^{-1})y_{SP}(t+k) - p_0\left[\frac{B(z^{-1})}{A(z^{-1})} - b_1\right]u(t)$$
$$- p_0 G_F(z^{-1})[y(t) - G_M(z^{-1})u(t)]]$$
$$+ \frac{q_0}{(p_0 b_1)^2 - q_0^2}[Q(z^{-1}) - q_0]u(t) \tag{2.45}$$

All terms on the RHS of (2.45) are known and (asymptotically) equal for both methods; hence t two control strategies are equivalent for general $Q(z^{-1})$, $R(z^{-1})$ and scalar $P(z^{-1})$.

The optimal control law that results from minimization of (2.4) for general $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ will now be formulated. Differentiating (2.4) w.r.t. $u(t)$,

$$\frac{\partial J_{KFP}}{\partial u(t)} = 2[R(z^{-1})y_{SP}(t+k) - P(z^{-1})\hat{y}(t+k\,|\,t)]$$
$$\cdot \frac{\partial[P(z^{-1})\hat{y}(t+k\,|\,t)]}{\partial u(t)} + 2[Q(z^{-1})u(t)]\frac{\partial[Q(z^{-1})u(t)]}{\partial u(t)} \tag{2.46}$$

From (2.32),

$$P(z^{-1})\hat{y}(t+k\,|\,t) = \frac{(p_{n0} + p_{n1}z^{-1} + \ldots)(b_1 + b_2 z^{-1} + \ldots)}{(p_{d0} + p_{d1}z^{-1} + \ldots)(1 + a_1 z^{-1} + \ldots)}u(t)$$
$$+ P(z^{-1})G_F(z^{-1})\left[y(t) - \frac{B(z^{-1})}{A(z^{-1})}u(t-k)\right]$$

from which it is evident that

$$\frac{\partial[P(z^{-1})\hat{y}(t+k\,|\,t)]}{\partial u(t)} = \frac{p_{n0}b_1}{p_{d0}} = p_0 b_1 \tag{2.47}$$

where $p_0 = p_{no}/p_{do}$ is the first coefficient of the $P_n(z^{-1})/P_d(z^{-1})$ expansion. Setting $\partial J_{KFP}/\partial u(t) = 0$ in (2.46) yields

$$u(t) = \frac{p_0 b_1}{(p_0 b_1)^2 - q_0^2}[R(z^{-1})y_{SP}(t+k)$$

$$-\left[\frac{P(z^{-1})B(z^{-1})}{A(z^{-1})} - p_0 b_1\right]u(t)$$

$$-\frac{P_n(z^{-1})G_F(z^{-1})}{P_d(z^{-1})}[y(t) - G_M(z^{-1})u(t)]]$$

$$+\frac{q_0}{(p_0 b_1)^2 - q_0^2}[Q(z^{-1}) - q_0]u(t) \qquad (2.48)$$

The formulation of the integrating STC for general P, Q, R weighting will now be treated. Recall the ARIMA plant representation given by (B.7):

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})}u(t-k) + \frac{C(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \qquad (B.7)$$

(B.7) may once again be multiplied by $E(z^{-1})\Delta$ and rearranged to obtain

$$C(z^{-1})P(z^{-1})y(t+k) = \frac{F(z^{-1})}{P_D(z^{-1})}y(t) + G(z^{-1})\Delta u(t)$$

$$+C(z^{-1})E(z^{-1})\omega(t+k) \qquad (2.49)$$

where $E(z^{-1})$ and $F(z^{-1})$ are obtained from the Diophantine identity

$$\frac{P_n(z^{-1})}{P_d(z^{-1})}\frac{C(z^{-1})}{A(z^{-1})\Delta} = E(z^{-1}) + \frac{z^{-k}F(z^{-1})}{P_d(z^{-1})A(z^{-1})\Delta} \qquad (2.50)$$

and $G(z^{-1}) = E(z^{-1})B(z^{-1})$. After defining

$$\psi(t+k) = P(z^{-1})y(t+k), \quad y'(t) = \frac{y(t)}{P_d(z^{-1})}$$

and substituting in (2.49) one obtains

$$\psi(t+k)=\frac{F(z^{-1})}{C(z^{-1})}y'(t)+\frac{G(z^{-1})}{C(z^{-1})}\Delta u(t)$$

$$+E(z^{-1})\omega(t+k)$$

Thus, a least squares prediction of $\psi(t+k)$ is given by

$$E\{P(z^{-1})y(t+k)\,|\,t\}$$

$$=\hat{\psi}(t+k\,|\,t)$$

$$=\frac{F(z^{-1})}{C(z^{-1})}y'(t)+\frac{G(z^{-1})}{C(z^{-1})}\Delta u(t)$$

$$=\frac{F(z^{-1})}{C(z^{-1})}y'(t)+\frac{B(z^{-1})}{C(z^{-1})}\left[\frac{P(z^{-1})C(z^{-1})}{A(z^{-1})}-\frac{z^{-k}F(z^{-1})}{P_d(z^{-1})A(z^{-1})}\right]u(t)$$

$$=\frac{B(z^{-1})P(z^{-1})}{A(z^{-1})}u(t)+\frac{F(z^{-1})}{C(z^{-1})P_d(z^{-1})}\left[y(t)-z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right]$$

or

$$\hat{\psi}(t+k\,|\,t)=P(z^{-1})G_P(z^{-1})u(t)$$

$$+\frac{G_F(z^{-1})}{P_D(z^{-1})}[y(t)-G_M(z^{-1})u(t)] \tag{2.51}$$

where G_P, G_F and G_M are as defined in section 2.7.

Differentiating J_{STC} in Eqn. (2.26) by $u(t)$,

$$\frac{\partial J_{STC}}{\partial u(t)}=2[R(z^{-1})y_{SP}(t+k)-\hat{\psi}(t+k\,|\,t)]\frac{\partial\hat{\psi}(t+k\,|\,t)}{\partial u(t)}$$

$$+2[Q(z^{-1})u(t)]\frac{\partial[Q(z^{-1})u(t)]}{\partial u(t)} \tag{2.52}$$

But from (2.51),

$$\hat{\psi}(t+k\,|\,t)=\frac{(p_{n0}+p_{n1}z^{-1}+\ldots)(b_1+b_2z^{-1}+\ldots)}{(p_{d0}+p_{d1}z^{-1}+\ldots)(1+a_1z^{-1}+\ldots)}u(t)$$

$$+\frac{G_F(z^{-1})}{P_d(z^{-1})}\left[y(t)-\frac{B(z^{-1})}{A(z^{-1})}u(t-k)\right]$$

so that

$$\frac{\partial \hat{\psi}(t+k\,|\,t)}{\partial u(t)} = \frac{p_{n0} b_1}{p_{d0}} = p_0 b_1 \tag{2.53}$$

where p_0 is the leading coefficient of $P(z^{-1})$. Setting $\partial J_{STC} / \partial u(t) = 0$ in (2.52) leads to

$$\begin{aligned} u(t) = \frac{p_0 b_1}{(p_0 b_1)^2 - q_0^2} [R(z^{-1}) y_{SP}(t+k) \\ - \left[\frac{P(z^{-1}) B(z^{-1})}{A(z^{-1})} - p_0 b_1 \right] u(t) \\ - \frac{G_F(z^{-1})}{P_d(z^{-1})} [y(t) - G_M(z^{-1}) u(t)]] \\ + \frac{q_0}{(p_0 b_1)^2 - q_0^2} [Q(z^{-1}) - q_0] u(t) \end{aligned} \tag{2.54}$$

which is clearly different from Eqn. (2.48). Note in particular that $(G_F)^{STC}$ in (2.54) is different than $(G_F)^{KFP}$ in (2.48) because the Diophantine equations (2.50) and (B.14) are now different.

Hence, the two schemes are **not** equivalent when nonscalar $P(z^{-1})$ weighting is used. This raises the question of whether one cost functional is better than the other and if so, how might one ensure the equivalence of the schemes for general $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$? The answer to this question lies in the fact that J_{KFP} and J_{STC} are both physically reasonable strategies, hence there would appear to be no benefit to, for instance, modifying Eqn. (2.4) to include $E\{P(z^{-1})y(t+k)\,|\,t\}$ rather than

$P(z^{-1})E\{y(t+k)|t\}$. (This could be achieved if necessary by basing the design of the Modified Kalman Filter Predictor upon an augmented DARMA process model such as

$$\tilde{A}(z^{-1})y(t)=\tilde{B}(z^{-1})u(t-k) \tag{2.55}$$

where $\tilde{A}(z^{-1})=P(z^{-1})A(z^{-1})$ and $\tilde{B}(z^{-1})=P(z^{-1})B(z^{-1})$.)

2.10 Simulation Example

In this section the equivalence of the minimum variance control schemes based on the Modified Kalman Filter Predictor (*cf*. Eqn. (2.35)) and the integrating Least Squares Predictor (*cf*. Eqn. (2.40)) is demonstrated using the underdamped second order state space model

$$\begin{bmatrix} x_\rho(t+1) \\ x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -0.7 & 0 & 0 \\ 1 & 1 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_\rho(t) \\ x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} n_1(t)$$

$$y(t)=[0\ \ 0\ \ 0\ \ 0\ \ 1]x(t)+n_2(t)$$

where $n_1(t)$ and $n_2(t)$ are independent Gaussian noise sequences of variance 10^{-4}. (Note that the DARMA equivalent of the process is

$$(1-1.5z^{-1}+0.7z^{-2})y(t)=z^{-2}(1+0.5z^{-1})u(t-1))$$

The plant was first controlled using the minimum variance control law (Eqn. (2.35)) based on the Modified Kalman Filter Predictor (see Fig. 2.2a). The MKFP was

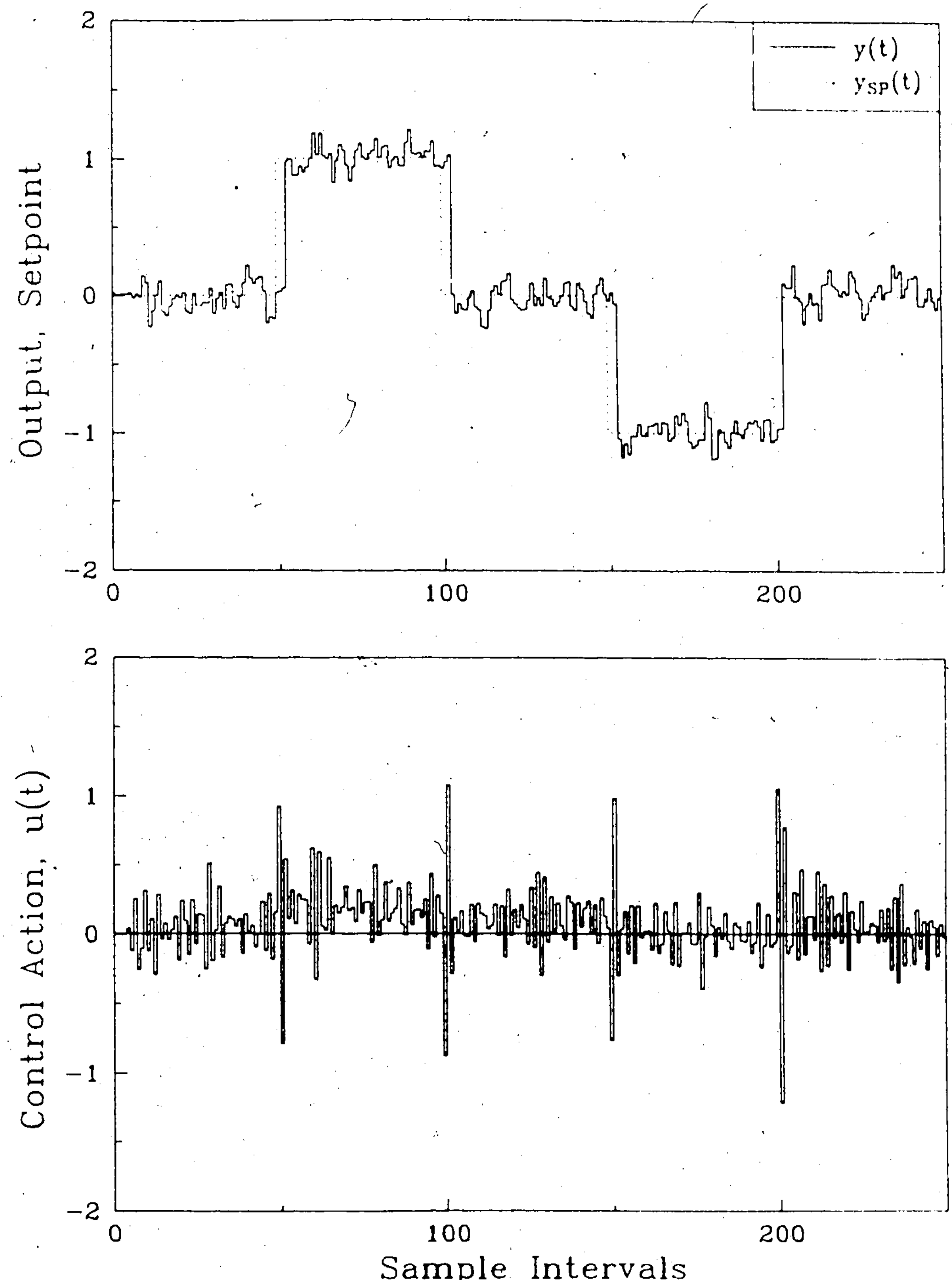

Figure 2.2a Minimum variance control using the Modified Kalman Filter Predictor

equipped with a perfect model of the plant, and the noise covariances R_1 and R_2 were set to their true values (10^{-4}). The control error variance obtained for this run was 0.0548. Notice that offset-free regulation was achieved despite the nonstationary nature of the disturbance. Figure 2.2b shows that the Kalman gain trajectories converge very quickly to their steady state values, *i.e.*

$$L_{ss} = \begin{bmatrix} L_\rho \\ L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix} = \begin{bmatrix} 0.3274 \\ -0.9710 \\ 2.8120 \\ 1.8548 \\ 0.8928 \end{bmatrix}$$

Before conducting a comparable trial using the integrating Self-Tuning Controller, it was necessary to derive an equivalent observer polynomial $C(z^{-1})$ using Eqn. (2.36):

$$\begin{aligned} C(z^{-1}) = [A(z^{-1})(1 + K_2(z^{-1})) + z^{-d}K_1(z^{-1})]\Delta \\ + z^{-d}L_\rho[\gamma_n z^{-1} + \ldots + \gamma_1 z^{-n}] \end{aligned} \qquad (2.36)$$

$K_1(z^{-1})$ and $K_2(z^{-1})$ are as defined in (2.13) and (2.15), respectively. For this example, $n = d = 2$ and $\gamma_2 = \gamma_1 = 1$. Hence,

$$K_1(z^{-1}) = L_2 + L_1 z^{-1}$$

$$= 2.8120 - 0.9710 z^{-1}$$

$$K_2(z^{-1}) = L_3 z^{-1} = 1.8548 z^{-1}$$

$$L_\rho[\gamma_2 z^{-1} + \gamma_1 z^{-2}] = 0.3274 z^{-1} + 0.3274 z^{-2}$$

and

$$A(z^{-1}) = 1 - 1.5 z^{-1} + 0.7 z^{-2}$$

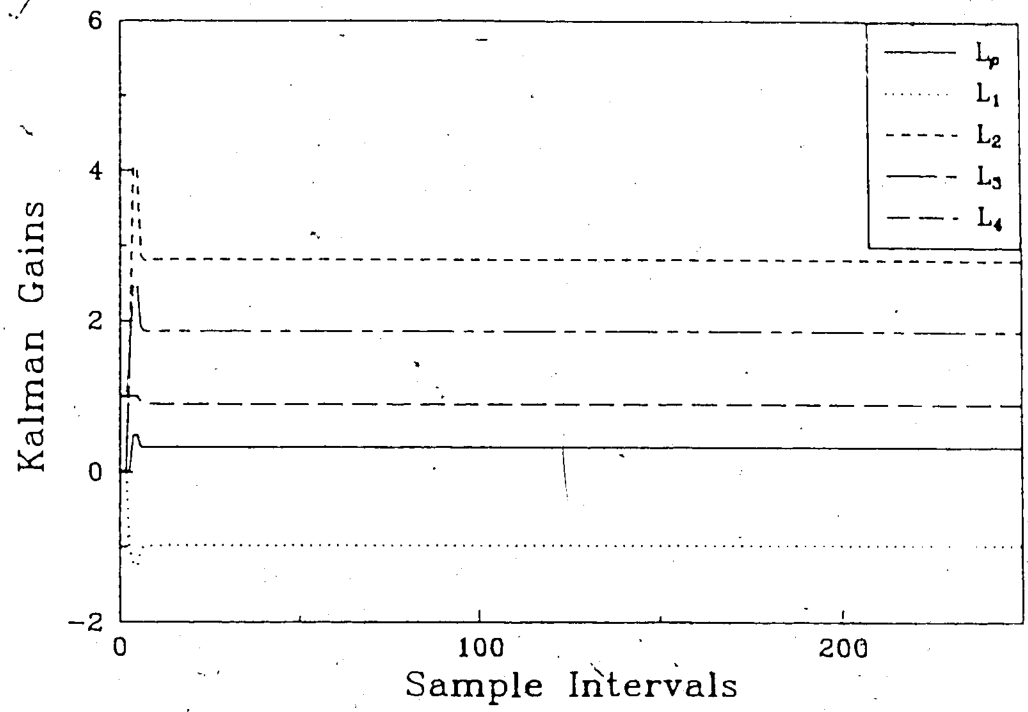

Figure 2.2b Kalman gain trajectories

Inserting these values into (2.36) yields, upon rearrangement,

$$C(z^{-1}) = 1 - 0.6452z^{-1} + 0.3750z^{-2} - 0.075z^{-3} + 0.00004z^{-4}$$
$$\cong 1 - 0.6452z^{-1} + 0.3750z^{-2} - 0.075z^{-3} \qquad (2.56)$$

The above values for $C(z^{-1})$ and $A(z^{-1})$ were then substituted into the Diophantine equation (2.38), leading to the solution

$$E(z^{-1}) = 1 + 1.8548z^{-1} + 2.7895z^{-2}$$
$$F(z^{-1}) = 3.538 - 4.8448z^{-1} + 1.9527z^{-2}$$
$$G(z^{-1}) = (1 + 1.8548z^{-1} + 2.7895z^{-2})(1 + 0.5z^{-1})$$
$$= 1 + 2.3458z^{-1} + 3.7124z^{-2} + 1.3948z^{-3}$$

It is interesting to note that $G_F(z^{-1})$ for this example is given by

$$G_F(z^{-1}) = \frac{F(z^{-1})}{C(z^{-1})} = \frac{K_5(z^{-1})}{C(z^{-1})}$$
$$= \frac{3.538 - 4.8448z^{-1} + 1.9527z^{-2}}{1 - 0.6452z^{-1} + 0.3750z^{-2} - 0.075z^{-3}}$$

which implies that

$$G_F(z) = \frac{3.538z(z - 0.6847 \pm 0.2883j)}{(z - 0.1885 \pm 0.4847j)(z - 0.2773)}$$

The calculated $C(z^{-1})$, $F(z^{-1})$ and $G(z^{-1})$ were subsequently used in the minimum variance control law Eqn. (2.40). The results are shown in Fig. 2.2c and are seen to be identical to those obtained using the MKFP after the first 15 sample intervals, which from Fig. 2.2b is

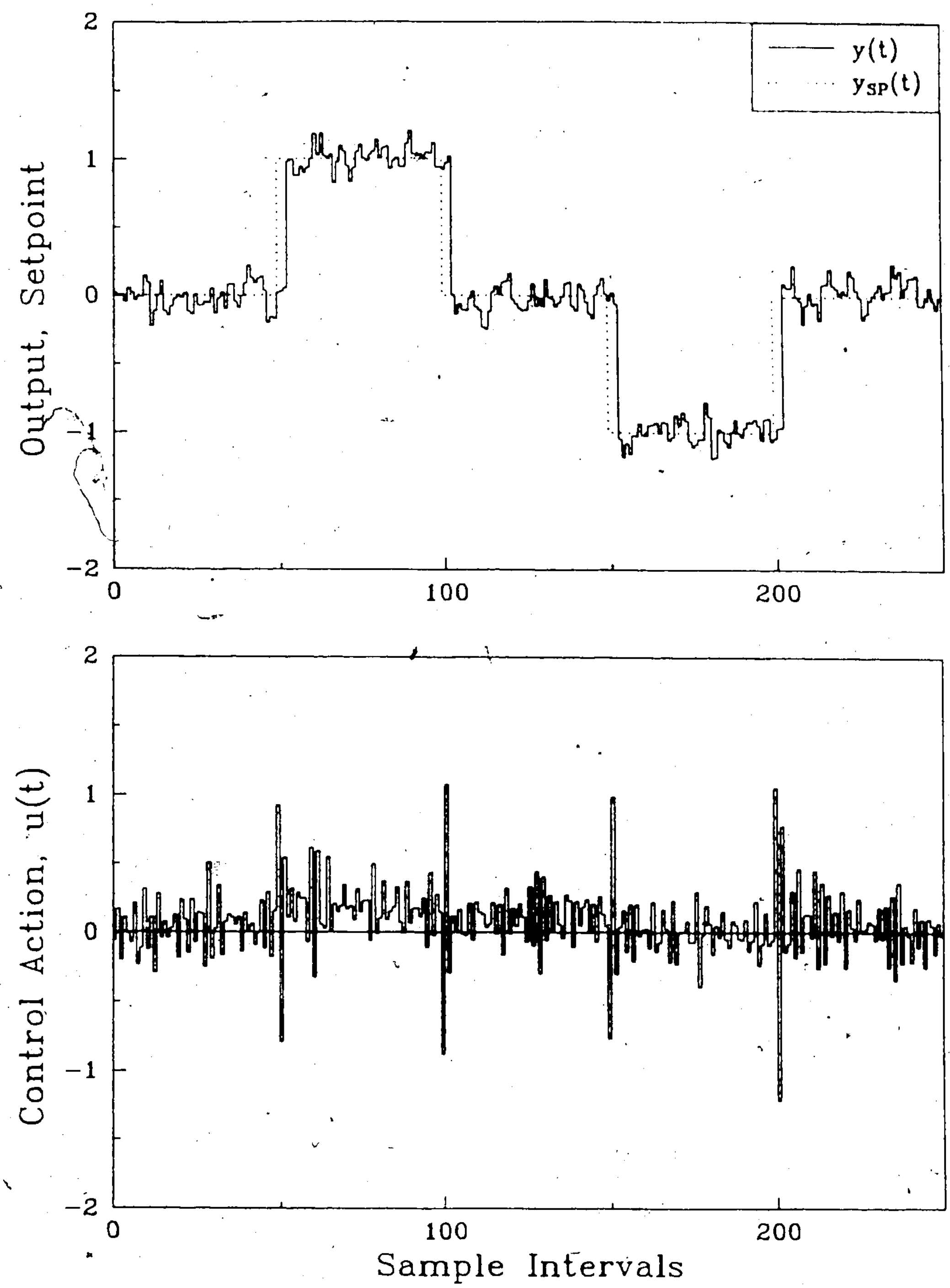

Figure 2.2c Minimum variance control using the integrating Self-Tuning Controller

precisely the time during which the Kalman gains are converging to their final values. Moreover, the control error variance obtained for this run was identical to that of the MKFP example, *i.e.* 0.0548.

2.11 Conclusions

The main results of this chapter may be seen as a reprise on the Box, Jenkins - Åström and Kalman linear regulator equivalence problem, which was a topic of some interest in the mid - 1970's. The equivalence has been demonstrated here in a new way by analogy with the Smith Predictor, and the method is readily applied to other state space realizations. The result was strengthened slightly to include plants having nonstationary disturbances and has been demonstrated via simulation example. The extension to Generalized Minimum Variance - type control schemes has also been discussed.

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3

Single-Step Adaptive Predictive Control

3.1 Introduction

In Chapter 2, a comparison was made of several single-step predictive control strategies for the control of linear time-invariant plants. In this chapter, the extension of these techniques to plants having time-varying dynamics and/or mild nonlinear characteristics will be presented. This extension is accomplished by estimating plant parameters online using a recursive parameter estimation algorithm such as Recursive Least Squares (RLS).

Several variants of the basic RLS technique are discussed with particular emphasis on an application of the Kalman filter to parameter identification. The Kalman Filter approach is shown to be equal to RLS with covariance modification (Goodwin and Sin, 1984). The Improved Least Squares (ILS) algorithm of Sripada and Fisher (1987) is also discussed and a correction is made to the variable forgetting factor calculation. The method of removing the d.c. bias from the I/O data by independently estimating the mean levels of y(t) and u(t) will be shown to be equivalent to incremental ID with a first order T-filter (Clarke et al, 1987). Adaptive versions of the Modified Kalman Filter Predictor of Walgama et al (1988) and the integrating Self-Tuning Controller of Tuffs and Clarke (1985) are then formulated and illustrated by means of a simulation study.

3.2 Parameter Identification

Consider the n'th order ARIMA process representation

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\frac{\omega(t)}{\Delta} \tag{3.1}$$

where $y(t)$ and $u(t)$ are the process output and input, respectively, and

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \ldots b_n z^{-n+1}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \ldots + c_{n+d} z^{-n-d}$$

k is the total process time delay, *i.e.* the sum of the physical delay (d) and the unit delay due to discretization, and $\omega(t)$ is a zero-mean Gaussian noise sequence. Multiplying Eqn. (3.1) by $\Delta = 1 - z^{-1}$ and rearranging yields the compact form

$$\Delta y(t) = \phi^T(t)\theta + \omega(t) \tag{3.2}$$

where

$$\phi(t) = [-\Delta y(t-1), \ldots, -\Delta y(t-n), \Delta u(t-k), \ldots,$$
$$\Delta u(t-k-n+1), \omega(t-1), \ldots, \omega(t-d-n)]^T$$
$$\theta = [a_1, \ldots, a_n, b_1, \ldots, b_n, c_1, \ldots, c_{n+d}]^T \tag{3.3}$$

If the process were time-invariant, *i.e.* if θ was constant, then the batch least squares technique described in Appendix A could be applied to get an optimal or least squares estimate of θ (provided that $\omega(\cdot)$ was measurable). This is achieved by minimizing the least squares cost index

$$J = \frac{1}{2}\sum_{t=1}^{N}(\Delta y(t) - \phi^T(t)\theta)^2 \qquad (3.4)$$

However, for online identification of the system parameters it is necessary to use the Recursive Least Squares algorithm, also derived in Appendix A:

a) Gain Calculation

$$K(t-1) = P(t)\phi(t) = \frac{P(t-1)\phi(t)}{1 + \phi^T(t)P(t-1)\phi(t)} \qquad (A.23)$$

b) Parameter Update

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t-1)[\Delta y(t) - \phi^T(t)\hat{\theta}(t-1)] \qquad (A.18)$$

c) Covariance Update

$$P(t) = P(t-1) - \frac{P(t-1)\phi(t)\phi^T(t)P(t-1)}{1 + \phi^T(t)P(t-1)\phi(t)} \qquad (A.22)$$

This scheme is known in the literature as Pseudolinear or Extended Least Squares (ELS) because of the presence of the disturbance mode $C(z^{-1})\omega(t)/\Delta$ in Eqn. (3.1). Since $\omega(t)$ is unmeasurable, it must be proxied, *e.g.* by

$$\hat{\omega}(t-1) = \Delta y(t-1) - \phi^T(t-1)\hat{\theta}(t-1) \qquad (3.5)$$

An attractive alternative to ELS is the "regressor filtering" or "Tuff's T-Filter" approach of Clarke et al. (1987). In the context of their Generalized Predictive Control algorithm, the authors point out that $C(z^{-1})$ is very difficult to identify because $\omega(t)$ is not directly measurable; that is, it must be proxied by Eqn. (3.5). For this reason the authors recommend filtering the input/output (I/O) data by a fixed low-pass filter $1/T(z^{-1})$, where $T(z^{-1})$ is typically chosen as

$T(z^{-1}) = 1 - 0.8z^{-1}$

From Eqn. (3.1), this implies that

$$A(z^{-1})\frac{y(t)}{T(z^{-1})} = B(z^{-1})\frac{u(t)}{T(z^{-1})} + \frac{C(z^{-1})}{T(z^{-1})}\frac{\omega(t)}{\Delta} \tag{3.6}$$

or

$$\Delta y^F(t) = \phi^T(t)\theta + \frac{C(z^{-1})}{T(z^{-1})}\omega(t) \tag{3.7}$$

where

$$\phi(t) = [-\Delta y^F(t-1), \ldots, -\Delta y^F(t-n), \Delta u^F(t-k), \ldots, \Delta u^F(t-k-n+1)]^T$$

$$\theta = [a_1, \ldots, a_n, b_1, \ldots b_n]^T \tag{3.8}$$

and the superscript "F" denotes division by $T(z^{-1})$. For the special case $T(z^{-1}) = C(z^{-1})$, it is seen that application of standard RLS to (3.7) will result in unbiased estimates of $A(z^{-1})$ and $B(z^{-1})$ as the disturbance term will be uncorrelated with the regressor $\phi(t)$. The form of Eqn. (3.2) is called "incremental" because of the presence of the Δ term operating on $y(t)$ and on the regressor vector $\phi(t)$.

It is apparent from Eqn. (A.22) that the covariance matrix $P(t)$ will tend to zero for large t because the second term on the RHS of (A.22) is positive definite. Hence, as t becomes large, the Kalman gain vector $K(t) = P(t)\phi(t)$ becomes small due to the decay of $P(t)$. This fact makes Eqns. (A.22) and (A.23) unsuitable for tracking changes in θ because little updating of (A.18) will occur even if the innovation is large. For this reason, the RLS algorithm is said to "go to sleep".

One approach to overcoming this difficulty is to introduce exponential data forgetting into the cost functional of Eqn. (3.4) to yield

$$J = \frac{1}{2}\sum_{t=1}^{N} \lambda^{N-t}(\Delta y(t) - \phi^T(t)\theta)^2 \tag{A.24}$$

which implies that (A.22) and (A.23) are to be replaced by

$$K(t-1) = \frac{P(t-1)\phi(t)}{\lambda + \phi^T(t)P(t-1)\phi(t)} \tag{A.25}$$

$$P(t) = \left[P(t-1) - \frac{P(t-1)\phi(t)\phi^T(t)P(t-1)}{\lambda + \phi^T(t)P(t-1)\phi(t)} \right]\frac{1}{\lambda} \tag{A.26}$$

The function of the forgetting factor, $\lambda \in (0, 1]$ is to weigh out past data; hence it is useful to compute the "asymptotic sample length" α (Clarke and Gawthrop, 1975) given by

$$\alpha = \sum_{t=0}^{\infty} \lambda^t = \frac{1}{1-\lambda}$$

which shows that λ must be close to unity so that fluctuations in the data are not excessively reflected in the estimates.

The main difficulty with this method arises from the fact that (A.26) reduces to

$$P(t) = \frac{P(t-1)}{\lambda}$$

during periods of low excitation. Since $0 < \lambda \leq 1$, $P(t)$ is seen to increase exponentially, giving rise to the term "covariance blowup." This phenomenon can lead to bursting in the output because of the large parameter changes that may be caused by relatively small values of $\phi(t)$.

Several methods have been proposed which maintain the alertness of the algorithm while preventing covariance blowup, generally at the expense of an increase in computational complexity of the least squares solution (Shah and Cluett, 1987). One such technique involves use of the Kalman filter for parameter estimation; this is the subject of the following subsection.

3.2.1 Parameter Identification using a Kalman Filter

The similarity that exists between the RLS and KF algorithms is well known (Goodwin and Sin, 1984; Ljung and Soderstrom, 1983), particularly in the context of simultaneous estimation of the state and ARMA model parameters in the Extended Kalman filter. It has also been noted in the literature that because of the duality that exists between the Kalman filter and the matrix Ricatti equation, it is sometimes possible when implementing Linear Quadratic Gaussian (LQG) control to use the same subroutine for parameter identification, state estimation and calculation of the optimal control action (see Clarke et al, 1985). Here it is of interest to determine whether the KF approach will be subject to the difficulties discussed in the previous section.

It is possible to model the time-varying parameters of Eqn. (3.2) by the Markov process (Åström and Wittenmark, 1984):

$$\theta(t+1) = \Phi\theta(t) + v(t) \tag{3.9}$$

$$\Delta y(t) = \phi^T(t)\theta(t) + \omega(t) \tag{3.10}$$

where $v(t)$, the process noise, is a zero-mean Gaussian sequence which is assumed to be uncorrelated with the measurement noise, $\omega(t)$. From Appendix A, it is apparent that the KF equations written for this system are

a) Gain Calculation

$$L(t) = \frac{M(t)\phi(t)}{R_2 + \phi^T(t)M(t)\phi(t)} \tag{3.11}$$

b) Measurement Update

i) A Posteriori State Update

$$\hat{\theta}(t) = \bar{\theta}(t) + L(t)[\Delta y(t) - \phi^T(t)\hat{\theta}(t-1)] \tag{3.12}$$

ii) A Posteriori Covariance Update

$$P(t) = M(t) - L(t)\phi^T(t)M(t) \tag{3.13}$$

c) Time Update

i) A Priori State Update

$$\bar{\theta}(t+1) = \Phi\bar{\theta}(t) \tag{3.14}$$

ii) A Priori Covariance Update

$$M(t+1) = \Phi P(t)\Phi^T + R_1 \tag{3.15}$$

where $R_1 = E\{v(t)v^T(t)\}$ and $R_2 = E\{\omega(t)\omega^T(t)\}$. R_2 may be chosen equal to unity in the above expressions without loss of generality. Further, let $\Phi = I$; this implies that the system parameters vary as a random walk. Equations (3.11) through (3.15) may now be combined to give the one-step Kalman filter:

a) Gain Calculation

$$L(t) = \frac{P(t-1)\phi(t)}{1+\phi^T(t)P(t-1)\phi(t)} \tag{3.16}$$

b) A Posteriori State Update

$$\hat{\theta}(t) = \hat{\theta}(t-1) + L(t)[\Delta y(t) - \phi^T(t)\hat{\theta}(t-1)] \tag{3.17}$$

c) A Posteriori Covariance Update

$$P(t) = P(t-1) - \frac{P(t-1)\phi(t)\phi^T(t)P(t-1)}{1+\phi^T(t)P(t-1)\phi(t)} + R_1 \tag{3.18}$$

where the prior covariance $M(t)$ has been replaced by $P(t-1)$.

Comparing these equations with Eqn. (A.18), (A.22) and (A.23) reveals that parameter estimation with a Kalman filter is actually equal to RLS with covariance modification (Goodwin and Sin, 1984). It is therefore interesting to analyze this algorithm from a state space point of view. Clearly, the RLS covariance matrix decays to zero because the method implicitly assumes that the process dynamics are time-invariant (*i.e.* $R_1 = 0$ implies $v(t) = 0$ in (3.9)). But this is exactly the "filter divergence" problem found in the Kalman filter literature. From consideration of this phenomenon has evolved the practice of assuming a nonzero value of R_1 even if it is felt that no process noise is present in the system. This ensures that the covariance matrix $P(\cdot)$ converges to a nonzero final value which in turn implies a nonzero Kalman gain vector. Thus the algorithm remains "alert" and is capable of tracking changes in the parameter vector θ.

The matrix R_1 may alternately be considered as a "tuning knob" which allows the size of $P(t)$ to be varied to reflect *a priori* knowledge regarding the process noise covariance. (Note that virtually all variants of the simple RLS technique assume, explicitly or otherwise, some degree of knowledge concerning the variance of the measurement noise.) Typically, $\hat{R}_1 = kI$, where k is a constant value, *e.g.* unity. However, it should be noted here that covariance windup may also occur using this method, since Eqn. (3.18) reduces to $P(t) = P(t-1) + R_1$ during periods of low excitation, *i.e.* $P(t)$ behaves as a ramp function. This phenomenon will be further illustrated by means of a simulation example in Section 3.5.

3.2.1.1 Observability

Note that the observability matrix for the system (3.10) is given by

$$W_o = \begin{bmatrix} \phi^T(t) \\ \phi^T(t+1) \\ \cdot \\ \cdot \\ \cdot \\ \phi^T(t+m) \end{bmatrix}$$

where $m = 3n + d$. The observability requirement that Φ_0 be of full rank is seen to be a requirement that the regressor vector $\phi(\cdot)$ be persistently exciting.

3.2.1.2 Stability

The parameter update law for the Kalman filter is given by Eqn. (3.17):

$$\hat{\theta}(t) = \hat{\theta}(t-1) + L(t)[\Delta y(t) - \phi^T(t)\hat{\theta}(t-1)]$$

or, using (3.2),

$$\begin{aligned}\tilde{\theta}(t) &= \tilde{\theta}(t-1) - L(t)\phi^T(t)\tilde{\theta}(t-1) + L(t)\omega(t) \\ &= [I - L(t)\phi^T(t)]\tilde{\theta}(t-1) + L(t)\omega(t) \qquad (3.19)\end{aligned}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$. From Eqn. (3.19) it is apparent that the stability of the estimation scheme is contingent upon the location of the eigenvalues of the matrix

$$\begin{aligned}&I - L(t)\phi^T(t) \\ &= I - \frac{P(t-1)\phi(t)\phi^T(t)}{1 + \phi^T(t)P(t-1)\phi(t)} \qquad (3.20)\end{aligned}$$

from (3.16). (The time argument will henceforth be dropped from $\phi(t)$ for convenience.)

Lemma 3.1

The matrix $P(t-1)\phi\phi^T$ is of rank ≤ 1 and its only (potentially) nonzero eigenvalue is given by

$$\lambda_{max} = \phi^T P(t-1)\phi \qquad (3.21)$$

Proof:

First, recall that $r(A \cdot B) \le \min(r(A), r(B))$ for arbitrary $m \times m$ matrices A and B. Therefore, the rank of $P(t-1)\phi\phi^T$ is at most unity since $r(\phi\phi^T) = 1$. This implies $\lambda = tr(P(t-1)\phi\phi^T)$ because $tr(A) = \sum_i \lambda_i(A) \quad \forall A_{m \times m}$.

Observe that

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \phi_m \end{bmatrix}, \quad P(t-1) = \begin{bmatrix} p_1 & p_{12} & \cdot & \cdot & \cdot & p_{1m} \\ p_{12} & p_2 & \cdot & \cdot & \cdot & p_{2m} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ p_{1m} & p_{2m} & \cdot & \cdot & \cdot & p_m \end{bmatrix}$$

$$\phi^T = [\phi_1, \phi_2, \ldots, \phi_m]$$

and so

$$\phi\phi^T = \begin{bmatrix} \phi_1^2 & \phi_1\phi_2 & \cdot & \cdot & \cdot & \phi_1\phi_m \\ \phi_1\phi_2 & \phi_2^2 & \cdot & \cdot & \cdot & \phi_2\phi_m \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \phi_1\phi_m & \phi_2\phi_m & \cdot & \cdot & \cdot & \phi_m^2 \end{bmatrix}$$

Hence, the diagonal elements of $\phi\phi^T P(t-1)$ are given by

$$\phi_1^2 + p_{12}\phi_1\phi_2 + \ldots + p_{1m}\phi [illegible]$$

$$[illegible]\phi_1\phi_2 + [illegible]\phi_2^2 + \ldots + p_{2m}\phi_2 [illegible]$$

$$\cdot$$

$$\cdot$$

$$[illegible]_m\phi_1\phi_m + p_{2m}\phi_2\phi_m + \ldots [illegible]\phi_m^2$$

or

$$p_1\phi_1^2 + \sum_{j=1}^{m} p_{1j}\phi_1\phi_j$$

$$p_2\phi_2^2 + \sum_{j=1}^{m} p_{2j}\phi_2\phi_j$$

.

.

$$p_m\phi_m^2 + \sum_{j=1}^{m} p_{mj}\phi_m\phi_j$$

Therefore,

$$\lambda(\phi\phi^T P(t-1)) = p_1\phi_1^2 + \sum_{j=1}^{m} p_{1j}\phi_1\phi_j + \ldots + p_m\phi_m^2 + \sum_{j=1}^{m} p_{mj}\phi_m\phi_j \qquad (3.22)$$

It is also apparent that

$$P(t-1)\phi = \begin{bmatrix} p_1\phi_1 + \sum_{j=1}^{m} p_{1j}\phi_j \\ \cdot \\ \cdot \\ \cdot \\ p_m\phi_m + \sum_{j=1}^{m} p_{mj}\phi_j \end{bmatrix}$$

which implies that

$$\begin{aligned}\phi^T P(t-1)\phi &= p_1\phi_1^2 + \sum_{j=1}^{m} p_{1j}\phi_1\phi_j + \ldots + p_m\phi_m^2 \\ &\quad + \sum_{j=1}^{m} p_{mj}\phi_m\phi_j \\ &= \lambda(\phi\phi^T P(t-1))\end{aligned} \qquad (3.23)$$

However, since $P(t-1)$ is positive semidefinite, $\phi^T P(t-1)\phi \geq 0$ and so $\lambda = \lambda_{max}(\phi\phi^T P(t-1))$.

Q.E.D.

Using the above lemma, it is clear that

$$\frac{P(t-1)\phi\phi^T}{1+\phi^T P(t-1)\phi}$$

has (m-1) eigenvalues at zero and one at

$$\lambda_{max} = \frac{\phi^T P(t-1)\phi}{1+\phi^T P(t-1)\phi}$$

But the stability of (3.17) is dependent on the eigenvalues of

$$I - \frac{P(t-1)\phi\phi^T}{1+\phi^T P(t-1)\phi}$$

so that it is of interest to develop a relationship between the eigenvalues of an arbitrary matrix A and the matrix $(I-A)$. The following two lemmas will prove useful in this respect.

Lemma 3.2

If μ_i, $i \in [1,m]$ are the eigenvalues of an arbitrary $m \times m$ matrix A, and ν_i, $i \in [1,m]$ are the eigenvalues of $-A$, then $\mu_i = -\nu_i$, $i \in [1,m]$.

Proof:

If the μ_i are the eigenvalues of A, then they satisfy the characteristic equation

$$|\mu I - A| = 0$$

Also, since the ν_i are eigenvalues of $-A$,

$$|\nu I + A| = 0$$
$$= |\nu I + A|(-1)^m$$
$$= |-\nu I - A|$$
$$= |(-\nu)I - A|$$
$$= 0$$

Hence, the $-\nu_i$ are the eigenvalues of A, so $\mu_i = -\nu_i$.

Q.E.D.

Lemma 3.3

If a matrix $A_{m\times m}$ has (m-1) eigenvalues at 0 and one eigenvalue at $\bar{\lambda}$, then $I - A$ has (m-1) eigenvalues at 1 and one at $1 - \bar{\lambda}$.

Proof:

For convenience, define $B = I - A$ and assume that the m eigenvalues of A are given by

$$\lambda_i(A) = 0, \ldots, 0, \bar{\lambda}$$

Now, the eigenvalues of B satisfy

$$|\lambda I - B| = |\lambda I + I - A|$$
$$= |(\lambda + 1)I - A|$$
$$= |\mu I - A|$$
$$= 0$$

so that μ_i, $i \in [1, m]$ are the eigenvalues of A, *i.e.*

$$\mu_i = 0, \ldots, 0, \bar{\lambda}$$

But $\mu_i = \lambda_i + 1$, which implies

$$\lambda_i = -1, \ldots, -1, \bar{\lambda} - 1 \qquad (3.24)$$

The eigenvalues of $A-I$ are therefore given by (3.24). Using Lemma 3.2, the eigenvalues of $-(A-I)=I-A$ are in turn given by

$$-\lambda_i = 1, \ldots, 1, 1-\bar{\lambda}$$

i.e., (m-1) eigenvalues at 1 and one at $1-\bar{\lambda}$.

Q.E.D.

Using Lemmas 3.1 and 3.3, it is apparent that the matrix

$$I - \frac{P(t-1)\phi\phi^T}{1+\phi^T P(t-1)\phi}$$

has (m-1) eigenvalues at 1 and one at

$$\lambda_{min} = 1 - \frac{P(t-1)\phi\phi^T}{1+\phi^T P(t-1)\phi}$$

$$= \frac{1}{1+\phi^T P(t-1)\phi}$$

so that $0<\lambda_{min}\le 1$. This would imply that the parameter update equation (3.17) is marginally stable, although it is difficult to predict the time domain behaviour of $\hat{\theta}(t)$ since the regressor vector $\phi(t)$ is time-varying and is generally not known *a priori*.

3.2.1.3 Positional Formulation

In Eqn. (3.1) the ARIMA plant model

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\frac{\omega(t)}{\Delta} \tag{3.1}$$

was seen to lead to the incremental input/output relationship

$$\Delta y(t) = \phi^T(t)\theta + \omega(t) \tag{3.2}$$

Since $\omega(t)$ is uncorrelated with the regressor $\phi(t)$, application of the RLS scheme described above to Eqn. (3.2) will result in an asymptotically unbiased $\hat{\theta}$.

The difficulty that arises from this approach in practice is that Δ is a high pass filter. This means that high frequency effects due to MPM and/or measurement noise will tend to be amplified at the expense of the low frequency signal content which is typically of greater concern in chemical process control. An alternate process model that has been proposed in the past (*e.g.* Clarke and Gawthrop, 1975) is given by

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\omega(t) + d \tag{3.25}$$

or

$$y(t) = \phi^T(t)\theta + \omega(t) \tag{3.26}$$

where

$$\begin{aligned} \phi(t) &= [-y(t-1),\ldots,-y(t-n),u(t-k),\ldots,u(t-k-n+1), \\ &\qquad \omega(t-1),\ldots,\omega(t-d-n),1]^T \\ \theta &= [a_1,\ldots,a_n,b_1,\ldots,b_n,c_1,\ldots,c_{n+d},d]^T \end{aligned} \tag{3.27}$$

Equation (3.25) now employs a stationary noise model in contrast to the nonstationary noise model of Eqn. (1). The d.c. bias d may be considered to be constant or slowly time-varying.

This is known as the "1-in-the-data-vector" method because the final element of $\phi(t)$ is unity. Note that the regressor is now full-valued or positional, as opposed to incremental or deviational. The approach appears to have been abandoned because of difficulties associated with estimating d, especially if it varies with operating region. These difficulties arise from the inherently poor conditioning of the problem due to the fact that the element "1" in the regressor is obviously not persistently exciting.

It was pointed out by Åström and Wittenmark (1984), during a discussion of the frequency domain properties of the steady-state Kalman filter, that in order to design a Kalman filter that blocks certain frequencies (*i.e.* a notch filter), it is necessary to incorporate into the filter a noise model with poles at those frequencies. Here it is desired to block the effect of the d.c. bias on $\hat{\theta}(t)$, so a noise model must be chosen with a pole at zero frequency, *i.e.* $z = e^{j\omega T} = e^0 = 1$. In order to determine, then, whether the KF approach offers any advantages over the "1-in-the-data-vector" method, the state space model will be partitioned into process and disturbance subsystems as follows:

$$\begin{bmatrix} \theta(t+1) \\ e(t+1) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} v(t) \\ q(t) \end{bmatrix} \tag{3.28}$$

$$y(t) = [\phi^T(t) \quad 1] \begin{bmatrix} \theta(t) \\ e(t) \end{bmatrix} + \omega(t) \tag{3.29}$$

where $v(\cdot)$, $q(\cdot)$, and $\omega(\cdot)$ are sequences of uncorrelated zero-mean random variables. Note that $\phi(\cdot)$ and θ have been redefined once more as

$$\phi(t) = [-y(t-1), \ldots, -y(t-n), u(t-k), \ldots, u(t-k-n+1), \omega(t-1), \ldots, \omega(t-d-n)]^T$$

$$\theta = [a_1, \ldots, a_n, b_1, \ldots, b_n, c_1, \ldots, c_{n+d}]^T \quad (3.30)$$

The augmented system may be represented as

$$z(t+1) = z(t) + v_1(t) \quad (3.31)$$

$$y(t) = H(t)z(t) + \omega(t) \quad (3.32)$$

This leads to the one-step Kalman filter (with R_2 again set to unity):

a) Gain Calculation

$$L(t) = \frac{P(t-1)H(t)}{1 + H(t)P(t-1)H^T(t)} \quad (3.33)$$

b) A Posteriori State Update

$$\hat{z}(t+1) = \hat{z}(t) + L(t)[y(t+1) - H(t+1)\hat{z}(t)] \quad (3.34)$$

c) A Posteriori Covariance Update

$$P(t) = P(t-1) - \frac{P(t-1)H^T(t)H(t)P(t-1)}{1 + H(t)P(t-1)H^T(t)} + R_1 \quad (3.35)$$

But it is obvious that these equations are identical to those obtained by the "1-in-the-data-vector" approach, *i.e.* by redefining the regressor vector $\phi(t)$ as

$$\phi'(t) = H^T(t) = \begin{bmatrix} \phi(t) \\ 1 \end{bmatrix}$$

where $\phi(t)$ is given by Eqn. (3.30). Hence, it is apparent that a Kalman filter design utilizing frequency domain considerations to eliminate parameter offset due to the d.c. bias term results in the well-known (and poorly conditioned) "1-in-the-data-vector" technique.

3.2.1.4 Conclusions re KF Parameter Identification

It has been demonstrated in this section that application of a Kalman filter to the identification of ARMA model parameters is in fact equal to the "RLS with covariance modification" technique. Although this method prevents "turnoff" by ensuring that the covariance matrix does not decay to zero, it was shown to alter the geometry of the parameter update in an ad hoc fashion and may be subject to covariance blowup when the plant is operating near steady state. In Section 3.2.2 a method due to Sripada and Fisher (1987) will be presented which maintains alertness by keeping the trace of the covariance matrix constant without altering the least squares properties of the algorithm.

3.2.2 Improved Least Squares

In order to maintain optimal control of time-varying systems, a candidate algorithm must be able to adaptively update the parameters of Eqn. (3.1), while avoiding the estimation difficulties associated with standard Recursive Least Squares. The method to be described in this section is the Improved Least Squares (ILS) of Sripada and Fisher (1987) which minimizes the exponentially weighted least squares cost functional:

$$J = \sum_{t=0}^{N} \lambda^{N-t}(t)[y(t) - \phi^T(t)\hat{\theta}(t-1)]^2 \tag{3.36}$$

The algorithm is characterized by five modifications to the standard least squares procedure which are described in the following subsections.

3.2.2.1 Normalization

The regressor vector $\phi(\cdot)$ and the output $y(\cdot)$ are normalized by a factor n, *i.e.*

$$y^n(t) = y(t)/n$$

$$\phi_n(t) = \phi(t)/n$$

where $n = \max(1, \|\phi(t)\|)$.

3.2.2.2 Scaling

In order to improve the numerical conditioning of the least squares problem, scaling is introduced to minimize the condition number of the scaled covariance matrix. The parameter update law becomes

$$\hat{\theta}(t) = \hat{\theta}(t-1) + S(t)^{-1} P_s(t) \phi_{ns}(t) [y^n(t) - \phi_n(t)^T \hat{\theta}(t-1)] \qquad (3.37)$$

with $P_s(t) = S(t)P(t)S(t)$ and $\phi_{ns}(t) = S^{-1}(t)\phi_n(t)$.

$S(t)$ is a diagonal scaling matrix chosen to minimize $C\{S(t)Q(t)\}$, where $P(t) = Q(t)Q^T(t)$, *i.e.* a Cholesky factorization of $P(t)$. The S_{ii} are chosen as the absolute row sums of $Q(t)$. It is important to note that the geometry of the update (3.37) is unaffected by the scaling procedure. Although the authors recommend scaling only when $C\{P_s(\cdot)\} > C_{\max}$ (see 3.2.2.4 below), it has been observed that scaling should be carried out at each sampling interval since $P_s(\cdot)$ can become negative definite if the number of intervals between scaling becomes too large.

3.2.2.3 Constant Trace

The covariance update law corresponding to Eqn. (3.37) is

$$P_s(t) = \left(I - \frac{P_s(t-1)\phi_{ns}(t)\phi_{ns}(t)^T}{\lambda(t) + \phi_{ns}(t)^T P_s(t-1)\phi_{ns}(t)} \right) \frac{P_s(t-1)}{\lambda(t)} \tag{3.38}$$

In order to maintain constant $tr\{P_s(\cdot)\}$, one may set $tr\{P_s(t)\} = tr\{P_s(t-1)\}$ and rearrange to obtain

$$\lambda(t) = 1 - \frac{1}{2}\left[g(t) - \left\{ g(t)^2 - \frac{4\,||P_s(t-1)\phi_{ns}(t)||^2}{tr\,P_s(t-1)} \right\}^{\frac{1}{2}} \right] \tag{3.39}$$

where $g(t) = 1 + \phi_{ns}(t)^T P_s(t-1)\phi_{ns}(t)$. The trace of $P_s(\cdot)$ is chosen by the user when $P_s(0)$ is specified.

It was found by experience, however, that this method does not actually result in a constant $tr\{P_s(\cdot)\}$ because the trace of $P_s(\cdot)$ will change each time the matrix is rescaled. Therefore, it was decided to rewrite (3.39) in terms of the unscaled covariance matrix $P(\cdot)$, because the true gain of the update (3.37) is

$$S^{-1}(t)S(t)P(t)S(t)S^{-1}(t)\phi_n(t) = P(t)\phi_n(t)$$

so that it is actually $tr\{P(\cdot)\}$ that should be kept constant and not $tr\{P_s(\cdot)\}$. Rewriting the expression for the forgetting factor $\lambda(t)$ with $tr\{P(t)\} = tr\{P(t-1)\}$,

$$\lambda(t) = 1 - \frac{1}{2}\left[g(t) - \left\{ g(t)^2 - \frac{4\,||P(t-1)\phi_n(t)||^2}{tr\,P(t-1)} \right\}^{\frac{1}{2}} \right] \tag{3.40}$$

where $g(t) = 1 + \phi_n(t)^T P(t-1)\phi_n(t)$. Again, $tr\{P(0)\}$ must be set by the user and should be chosen carefully to balance *a priori* knowledge regarding the variance of the plant dynamics against the noise level of the process.

3.2.2.4 On/off Criteria

In order to prevent drift of the parameter estimates during periods of low excitation, the algorithm is designed to shut off if either

1) $||P_s(t)\phi_{ns}(t)|| < \Delta$,

or 2) $C\{P_s(t)\} > C_{max}$

where Δ and C_{max} are user-specified constants.

3.2.2.5 Mean Deviational Data

The relative advantages/disadvantages of incremental vs. positional data for recursive parameter estimation have been discussed above. However, a third option exists for the identification of systems of the form:

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\frac{\omega(t)}{\Delta} + d \qquad (3.41)$$

which involves removal of the d.c. bias by independent estimation of the mean levels ($\bar{y}$, $\bar{u}$) of $y(t)$ and $u(t)$, *i.e.* since $d = A\bar{y} - B\bar{u}$. The mechanism for doing this is problematic; Sripada and Fisher (1987) recommend using the first order filters

$$\begin{aligned} \bar{y}(t) &= \lambda_y \bar{y}(t-1) + (1-\lambda_y)y(t) \\ \bar{u}(t) &= \lambda_u \bar{u}(t-1) + (1-\lambda_u)u(t) \end{aligned} \qquad (3.42)$$

where λ_y and λ_u are chosen in an ad hoc fashion subject to $0 < (\lambda_y, \lambda_u) \leq 1$. For the choice $\lambda_y = \lambda_u = \lambda$,

$$\frac{\bar{y}(t)}{y(t)} = \frac{1-\lambda}{1-\lambda z^{-1}}, \quad \frac{\bar{u}(t)}{u(t)} = \frac{1-\lambda}{1-\lambda z^{-1}}$$

or

$$\bar{y}(t) = \frac{y(t)}{T'(z^{-1})}, \quad \bar{u}(t) = \frac{u(t)}{T'(z^{-1})}$$

with

$$T'(z^{-1}) = \frac{1-\lambda z^{-1}}{1-\lambda}.$$

(Note that λ here is generally different from the forgetting factor of Eqn. (3.40).) Dropping the argument z^{-1} for convenience, the effect of using deviational variables on the LHS of (3.41) is seen to be

$$A(y(t)-\bar{y}(t)) = A\left(y(t) - \frac{1-\lambda}{1-\lambda z^{-1}} y(t)\right)$$

$$= A\left(\frac{\lambda \Delta y(t)}{1-\lambda z^{-1}}\right) = A\frac{\Delta y(t)}{T} \qquad (3.43)$$

where

$$T = T(z^{-1}) = \frac{1}{\lambda} - z^{-1} \qquad (3.44)$$

Subtracting $(A(z^{-1})\bar{y}(t) - z^{-k}B(z^{-1})\bar{u}(t))$ from both sides of (3.41),

$$Ay(t) - (A\bar{y}(t) - z^{-k}B\bar{u}(t))$$

$$= z^{-k}Bu(t) - (A\bar{y}(t) - z^{-k}B\bar{u}(t)) + C\frac{\omega(t)}{\Delta} + d$$

$$A(y(t)-\bar{y}(t))$$
$$= z^{-k}B(u(t)-\bar{u}(t)) + C\frac{\omega(t)}{\Delta} + d - (A\bar{y}(t) - z^{-k}B\bar{u}(t)) \quad (3.45)$$

But,

$$A\bar{y}(t) - B\bar{u}(t)$$
$$= \frac{1}{T'}(Ay(t) - z^{-k}Bu(t))$$
$$= \frac{1}{T'}\left(C\frac{\omega(t)}{\Delta} + d\right)$$

Therefore, (3.45) becomes

$$A\left(y(t) - \frac{y(t)}{T'}\right)$$
$$= z^{-k}B\left(u(t) - \frac{u(t)}{T'}\right) + C\frac{\omega(t)}{\Delta} + d - \frac{1}{T'}\left(C\frac{\omega(t)}{\Delta} + d\right)$$
$$= z^{-k}B\left(u(t) - \frac{u(t)}{T'}\right) + \frac{C}{\Delta}\left(\omega(t) - \frac{\omega(t)}{T'}\right) + \left(d - \frac{d}{T'}\right)$$

Using Eqn. (3.43), this in turn becomes

$$A\frac{\Delta y(t)}{T} = B\frac{\Delta u(t-k)}{T} + \frac{C}{\Delta}\frac{\Delta\omega(t)}{T} + \frac{\Delta d}{T}$$

or

$$A(z^{-1})\Delta y^F(t) = B(z^{-1})\Delta u^F(t-k) + \frac{C(z^{-1})}{T(z^{-1})}\omega(t)$$

which would imply that

$$\Delta y^F(t) = \phi^T(t)\theta + \frac{C(z^{-1})}{T(z^{-1})}\omega(t) \quad (3.7)$$

where the superscript "F" denotes division by $T(z^{-1})$. This scheme is clearly equal to that of Clarke et al (1987), *i.e.* incremental ID with prefiltering of the regressor using "Tuff's T-Filter". Note once again that unbiased estimates of θ will be obtained only when $T(z^{-1}) = 1/\lambda - z^{-1} = C(z^{-1})$ and is designed to be used in place of, rather than in conjunction with Extended Least Squares. Notice also that using $\lambda_y \neq \lambda_u$ is equivalent to multiplying the LHS and RHS of (3.41) by different $T(z^{-1})$ polynomials, which will result in biased parameter estimation.

3.3 The Adaptive Kalman Filter Predictor

In Chapter 2 of this thesis, a generalized k-step-ahead control law (Lu, 1986) was introduced to be used with the Modified Kalman Filter Predictor (MKFP) of Walgama et al (1988). In summary, the cost functional to be minimized was given by

$$J_{KFP} = [R(z^{-1})y_{SP}(t+k) - P(z^{-1})E\{y(t+k)|t\}]^2 + [Q(z^{-1})u(t)]^2 \tag{3.46}$$

where $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ are rational weighting polynomials subject to $P(1) = R(1)$ and $Q(1) = 0$ for zero controller offset.

The state space model of the process was given by

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \Gamma n_1(t) \tag{3.47}$$

$$y(t) = Hx(t) + n_2(t) \tag{3.48}$$

where for $d = k-1$,

$$x(t)=[x_0(t),x_1(t),\ldots,x_n(t),x_{n+1}(t),\ldots,x_{n+d}(t)]^T$$

$$\Phi=\begin{bmatrix} 1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\ \gamma_1 & 0 & \ldots & 0 & -a_n & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \gamma_n & 0 & \ldots & 1 & -a_1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}_{(n+k)\times(n+k)}$$

$$\Lambda=[0,b_n,\ldots,b_1,0,\ldots,0]^T$$

$$\Gamma=[1,0,\ldots,0,0,\ldots,0]^T$$

$$H=[0,0,\ldots,0,0,\ldots,1]$$

$n_1(\cdot)$ and $n_2(\cdot)$ are uncorrelated Gaussian random variables having covariances R_1 and R_2 respectively. The state vector is not directly measurable and so is reconstructed by means of the Kalman filter update (see Appendix A):

$$\hat{x}(t+1\mid t+1)=\hat{x}(t+1)=\Phi\hat{x}(t)+\Lambda u(t)+L(t+1)\omega(t+1) \qquad (3.49)$$

where the innovations sequence $\omega(\cdot)$ is defined as

$$\omega(t+1)=y(t+1)-H\hat{x}(t+1\mid t) \qquad (3.50)$$

By innovations analysis for the steady state KF, it was found that an equivalent ARIMA representation for (3.47), (3.48) is given by

$$A(z^{-1})y(t)=B(z^{-1})u(t-k)+C(z^{-1})\frac{\omega(t)}{\Delta} \qquad (3.1)$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 + b \; z^{-1} + \ldots + b_n z^{-n+1}$$

$$C(z^{-1}) = A(z^{-1})\Delta[1 + z^{-1}K_2(z^{-1}) + z^{-d}A^{-1}(z^{-1})K_1(z^{-1})]$$

$$+ z^{-d}D(z^{-1}) \tag{3.51}$$

and

$$K_1(z^{-1}) = L_n + L_{n-1}z^{-1} + \ldots + L_1 z^{-n+1}$$

$$K_2(z^{-1}) = L_{n+d-1} + L_{n+d-2}z^{-1} + \ldots + L_{n+1}z^{-d+2}$$

$$D(z^{-1}) = L_p[\gamma_n z^{-1} + \ldots + \gamma_z z^{-n}]$$

(The L_i in Eqn. (3.51) are elements of the steady state Kalman gain vector L.)

It was also pointed out in Chapter 2 that the k-step-ahead KF prediction of the process output is given by

$$E\{y(t+k)\,|\,t\} = \hat{y}(t+k\,|\,t)$$

$$= H\hat{x}(t+k\,|\,t)$$

$$= H\Phi^k\hat{x}(t) + \sum_{j=t}^{t+d} H\Phi^{t+d-j}u(j)$$

$$= \gamma_n\hat{x}_p(t) + \hat{x}_{n-1}(t) - a_1\hat{x}_n(t) + b_1u(t) \tag{3.52}$$

Differentiating (3.46) with respect to $u(t)$ yields

$$\frac{\partial J_{KFP}}{\partial u(t)} = 2[R(z^{-1})y_{SP}(t+k) - P(z^{-1})\hat{y}(t+k\,|\,t)]$$

$$\cdot\frac{\partial[P(z^{-1})\hat{y}(t+k\,|\,t)]}{\partial u(t)} + 2[Q(z^{-1})u(t)]\frac{\partial[Q(z^{-1})u(t)]}{\partial u(t)} \tag{3.53}$$

But (3.52) implies that

$$P(z^{-1})\hat{y}(t+k\,|\,t) = P(z^{-1})[\gamma_n\hat{x}_p(t) + \hat{x}_{n-1}(t) - a_1\hat{x}_n(t)]$$

$$+ \frac{(p_{n0} + p_{n1}z^{-1} + \ldots)}{(p_{d0} + p_{d1}z^{-1} + \ldots)}b_1u(t)$$

so that

$$\frac{\partial[P(z^{-1})\hat{y}(t+k\,|\,t)]}{\partial u(t)} = \frac{p_{n0}}{p_{d0}} b_1 = p_0 b_1 \tag{3.54}$$

where p_0 is the first coefficient in the expansion $P_n(z^{-1})/P_d(z^{-1})$. Similarly,

$$\frac{\partial[Q(z^{-1})u(t)]}{\partial u(t)} = \frac{q_{n0}}{q_{d0}} = q_0 \tag{3.55}$$

Substituting (3.54) and (3.55) into (3.53) and setting $\partial J_{KFP}/\partial u(t) = 0$ leads to the certainty equivalence control law

$$u(t) = \frac{\hat{b}_1}{\hat{b}_1^2 P(z^{-1}) + q_{n0}Q(z^{-1})}[R(z^{-1})y_{SP}(t+k) - P(z^{-1})\gamma_n \hat{x}_p(t) - P(z^{-1})\hat{x}_{n-1}(t) + \hat{a}_1 P(z^{-1})\hat{x}_n(t)] \tag{3.56}$$

Note that a_1 and b_1 in Eqn. (3.52) have been replaced by $\hat{a}_1$ and $\hat{b}_1$ to reflect the fact that they are to be estimated online using the Improved Least Squares algorithm described in Section 3.2.2 above. Combining a recursive estimation scheme such as ILS with the MKFP results in the Adaptive Modified Kalman Filter Predictor (AMKFP) of Walgama (1986), which is illustrated in Fig. 3.1. From this diagram, it is evident that the proposed control scheme is explicit or indirect. This implies that the parameter estimation step and the controller design step are accomplished separately. (It is possible, however, to design an implicit or direct scheme wherein the parameter estimation and state estimation steps are carried out simultaneously, *e.g.* by use of the Extended Kalman filter (Walgama, 1986)).

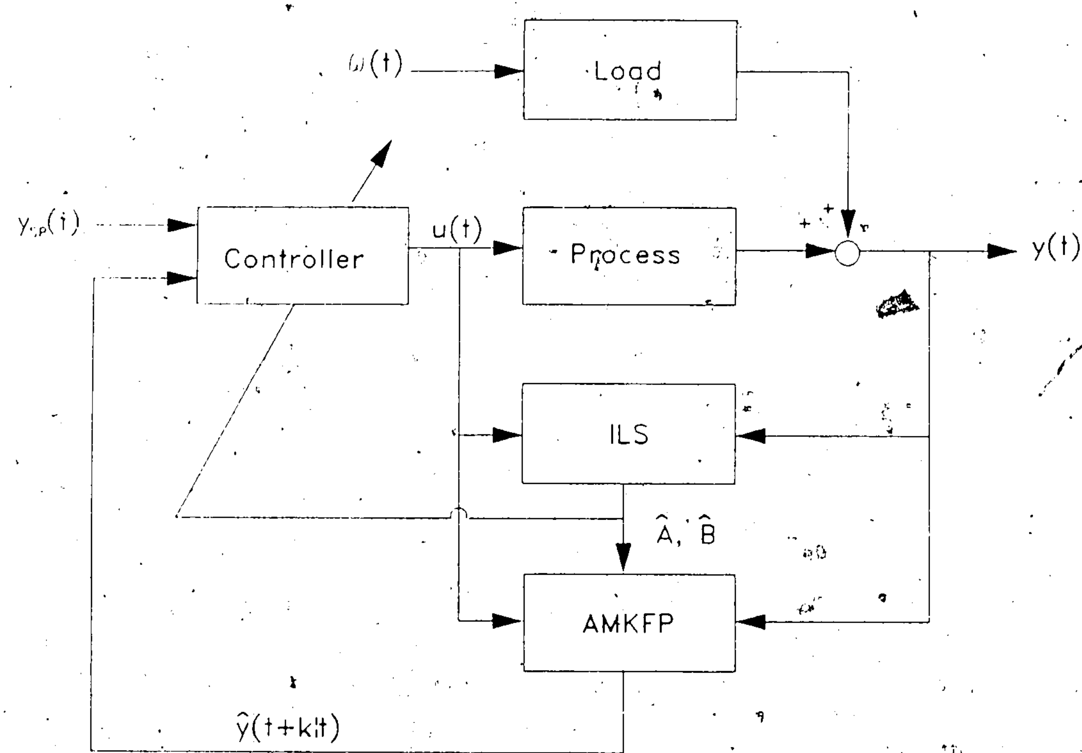

Figure 3.1 - Structure of the Adaptive Kalman Filter Control Scheme

As a final note, recall that the independent mean estimation described in Section 3.2.2 effectively employs a first order $T(z^{-1})$ filter in an effort to cancel $C(z^{-1})$ in (3.51), which is of order $\delta C(z^{-1}) = n + d$. It is apparent, therefore, that some degree of correlation will normally exist between the regressor vector $\phi(t)$ and the disturbance term $C(z^{-1})\omega(t)/T(z^{-1})$ in Eqn. (3.7) unless $T(z^{-1})$ is exactly equal to $C(z^{-1})$.

3.4 Integrating Self-Tuning Control

As described in Chapter 2 in the context of optimal control of time-invariant processes, the integrating form of the Self-Tuning Controller (Tuffs and Clarke, 1985) minimizes the performance index

$$J_{STC} = [R(z^{-1})y_{SP}(t+k) - E\{P(z^{-1})y(t+k)\,|\,t\}]^2 + [Q(z^{-1})u(t)]^2 \tag{3.57}$$

where $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ are user-specified rational weighting polynomials. Once again, $P(1) = R(1)$ and $Q(z^{-1}) = 0$ are seen to be necessary conditions for zero control offset. The predicted k-step-ahead auxiliary output $\hat{\psi}(t+k\,|\,t) = E\{P(z^{-1})y(t+k)\,|\,t\}$ is formed using the ARIMA plant model

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\frac{\omega(t)}{\Delta} \tag{3.1}$$

where $A(z^{-1})$, $B(z^{-1})$ and $C(z^{-1})$ are as defined in Eqn. (3.51) above. Multiplication of (3.1) by $E(z^{-1})\Delta$, where $\delta E(z^{-1}) = k-1$ and rearrangement yields

$$\psi(t+k) = P(z^{-1})y(t+k)$$
$$= \frac{F(z^{-1})}{P_d(z^{-1})C(z^{-1})}y(t) + \frac{G(z^{-1})}{C(z^{-1})}\Delta \hat{u}(t) + E(z^{-1})\omega(t+k) \tag{3.58}$$

where $E(z^{-1})$ and $F(z^{-1})$ are obtained from the Diophantine identity

$$\frac{P_n(z^{-1})C(z^{-1})}{P_d(z^{-1})A(z^{-1})\Delta} = E(z^{-1}) + z^{-k}\frac{F(z^{-1})}{P_d(z^{-1})A(z^{-1})\Delta} \tag{3.59}$$

and $G(z^{-1}) = E(z^{-1})B(z^{-1})$. It is evident that a least squares prediction of the auxiliary output $\psi(t+k)$ is given by

$$\hat{\psi}(t+k\mid t) = \frac{F(z^{-1})}{P_d(z^{-1})C(z^{-1})}y(t) + \frac{G(z^{-1})}{C(z^{-1})}\Delta u(t) \tag{3.60}$$

Differentiating J_{STC} in Eqn. (3.57) w.r.t. $u(t)$,

$$\frac{\partial J_{STC}}{\partial u(t)} = 2[R(z^{-1})y_{SP}(t+k) - \hat{\psi}(t+k\mid t)]\frac{\partial\hat{\psi}(t+k\mid t)}{\partial u(t)}$$

$$+2[Q(z^{-1})u(t)]\frac{\partial[Q(z^{-1})u(t)]}{\partial u(t)} \tag{3.61}$$

But from (3.60),

$$\hat{\psi}(t+k\mid t) = \frac{F(z^{-1})}{P_d(z^{-1})C(z^{-1})}y(t) + g_0\Delta u(t)$$

$$+\left(\frac{G(z^{-1})}{C(z^{-1})} - g_0\right)\Delta u(t) \tag{3.62}$$

because $C(z^{-1})$ is monic. Equation (3.62) implies that

$$\frac{\partial\hat{\psi}(t+k\mid t)}{\partial u(t)} = g_0$$

and it may be recalled from (3.55) that

$$\frac{\partial[Q(z^{-1})u(t)]}{\partial u(t)} = \frac{q_{n0}}{q_{d0}} = q_0 \tag{3.55}$$

Substituting these relationships into (3.61) and setting $\partial J_{STC}/\partial u(t) = 0$ leads to the optimal control law

$$u(t) = \frac{\hat{g}_0}{\hat{g}_0^2 - q_0 Q(z^{-1})}[R(z^{-1})y_{SP}(t+k) - \frac{F(z^{-1})}{P_d(z^{-1})\hat{C}(z^{-1})}y(t)$$

$$+u(t-1) + \left(\hat{g}_0 - \frac{\hat{G}(z^{-1})}{\hat{C}(z^{-1})}\right)\Delta u(t)] \tag{3.63}$$

The polynomials F, G and C have been replaced in (3.63) by their estimated values, which must be obtained ia online parameter identification in order to maintain optimal control of plants having time-varying dynamics. This can be accomplished in an implicit or direct fashion by directly estimating these polynomials as illustrated in Fig. 3.2, *i.e.* the parameter estimation and controller design steps have been combined. Equation (3.58) may be rearranged to obtain

$$\psi(t) = (1 - C(z^{-1}))\psi(t) + F(z^{-1})y'(t-k)$$
$$+ G(z^{-1})\Delta u(t-k) + E(z^{-1})\omega(t)$$

or

$$\psi(t) = \phi^T(t)\theta + E(z^{-1})\omega(t) \qquad (3.64)$$

where

$$\phi(t) = [-\psi(t-1), \ldots, -\psi(t-n-d), y'(t-k), \ldots, y'(t-\delta F),$$
$$\Delta u(t-k), \ldots, \Delta u(t-2k-n+2)]^T$$
$$\theta = [c_1, \ldots, c_{n+d}, f_0, \ldots, f_{\delta F}, g_0, \ldots, g_{k+n-2}]^T$$
$$(3.65)$$

$y'(t)$ is defined as $y'(t) = y(t)/P_d(z^{-1})$ and the order of $F(z^{-1})$ is given by $\delta F(z^{-1}) = \max(\delta P_n + n - 1, \delta P_d + n)$. Application of a recursive parameter estimation scheme such as ILS will produce a LS estimate of θ, since the noise term $E(z^{-1})\omega(t+k)$ is independent of $\phi(t)$. Once again, $C(z^{-1})$ could be replaced by $T(z^{-1})$ in Eqn. (3.64), thereby reducing the dimension of the estimation problem considerably.

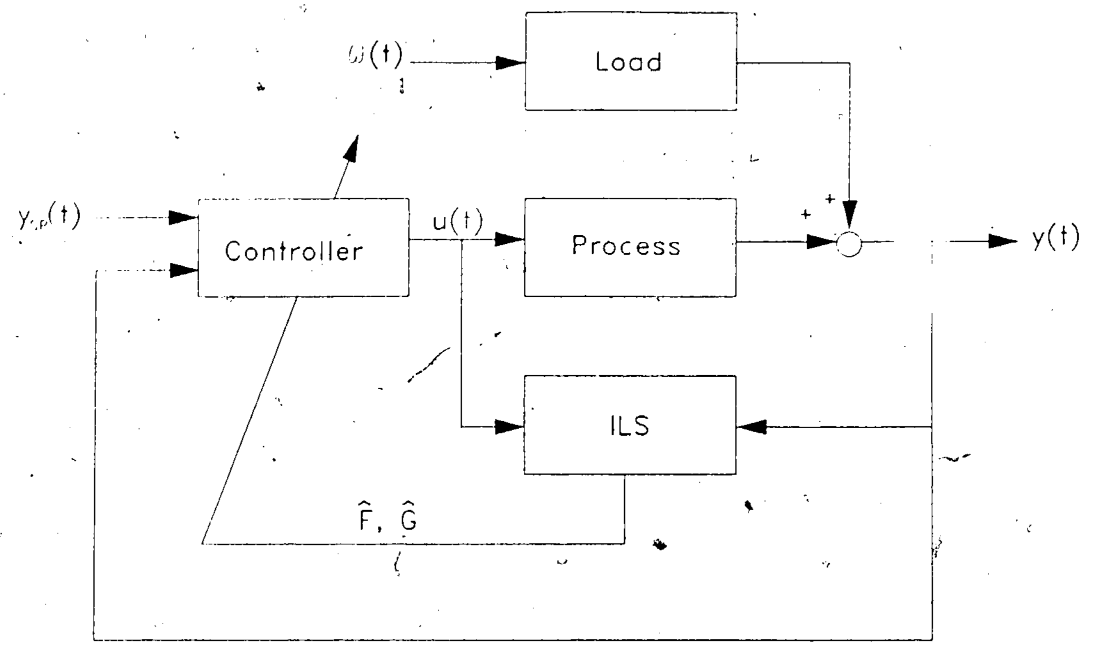

Figure 3.2 Structure of the Implicit Self-Tuning Controller

3.5 Simulation Results

The adaptive versions of the MKFP and ISTC algorithms introduced in Chapter 2 were first tested on the same plant described in the simulation example of that chapter, *i.e.*

$$\begin{bmatrix} x_p(t+1) \\ x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -0.7 & 0 & 0 \\ 1 & 1 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0.5 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} n_1(t)$$

$$y(t) = [0 \quad 0 \quad 0 \quad 0 \quad 1]x(t) + n_2(t) \tag{3.66}$$

$n_1(t)$ and $n_2(t)$ are independent Gaussian noise sequences of variance 10^{-4}. Note once again that an equivalent DARMA representation of the process is given by

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1)$$

$$(1 - 1.5z^{-1} + 0.7z^{-2})y(t) = z^{-2}(1 + 0.5z^{-1})u(t-1)$$

The performance of the minimum variance form of the AMKFP-based controller (*i.e.* Eqn. (3.56) with $P(z^{-1}) = R(z^{-1}) = 1$ and $Q(z^{-1}) = 0$) is illustrated in Fig. 3.3a. The ratio of the noise covariances R_1/R_2 was set to its correct value of unity. The $A(z^{-1})$ and $B(z^{-1})$ parameters were identified using ILS with $tr\ P(\cdot) = 4$, $\Delta = 10^{-6}$ and $C_{max} = 10^{-6}$, *i.e.* the algorithm was on at all times throughout the run. The estimated parameter vector $\hat{\theta}(t)$ was initialized to $\theta(0) = [1 \quad 1 \quad 1 \quad 1]^T$ and a first order T-filter, $T(z^{-1}) = 1 - 0.8z^{-1}$, was used to reduce the effect of correlated noise on the estimates (*cf.* Eqn. (3.7)). It is evident from Fig. 3.3b that the parameters converged to "good" values after approximately 75 samples. This

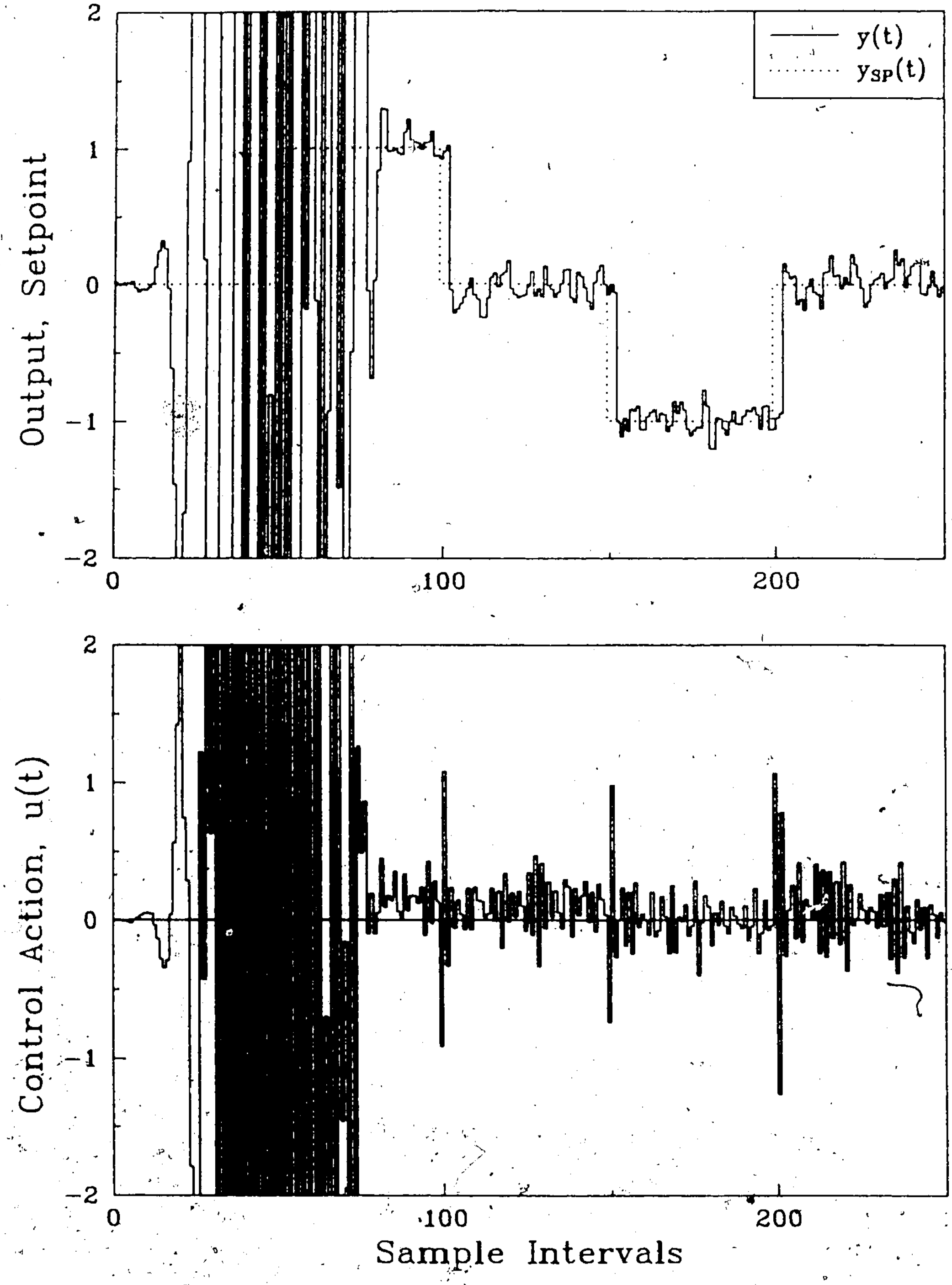

Figure 3.3a Adaptive minimum variance control using the AMKFP

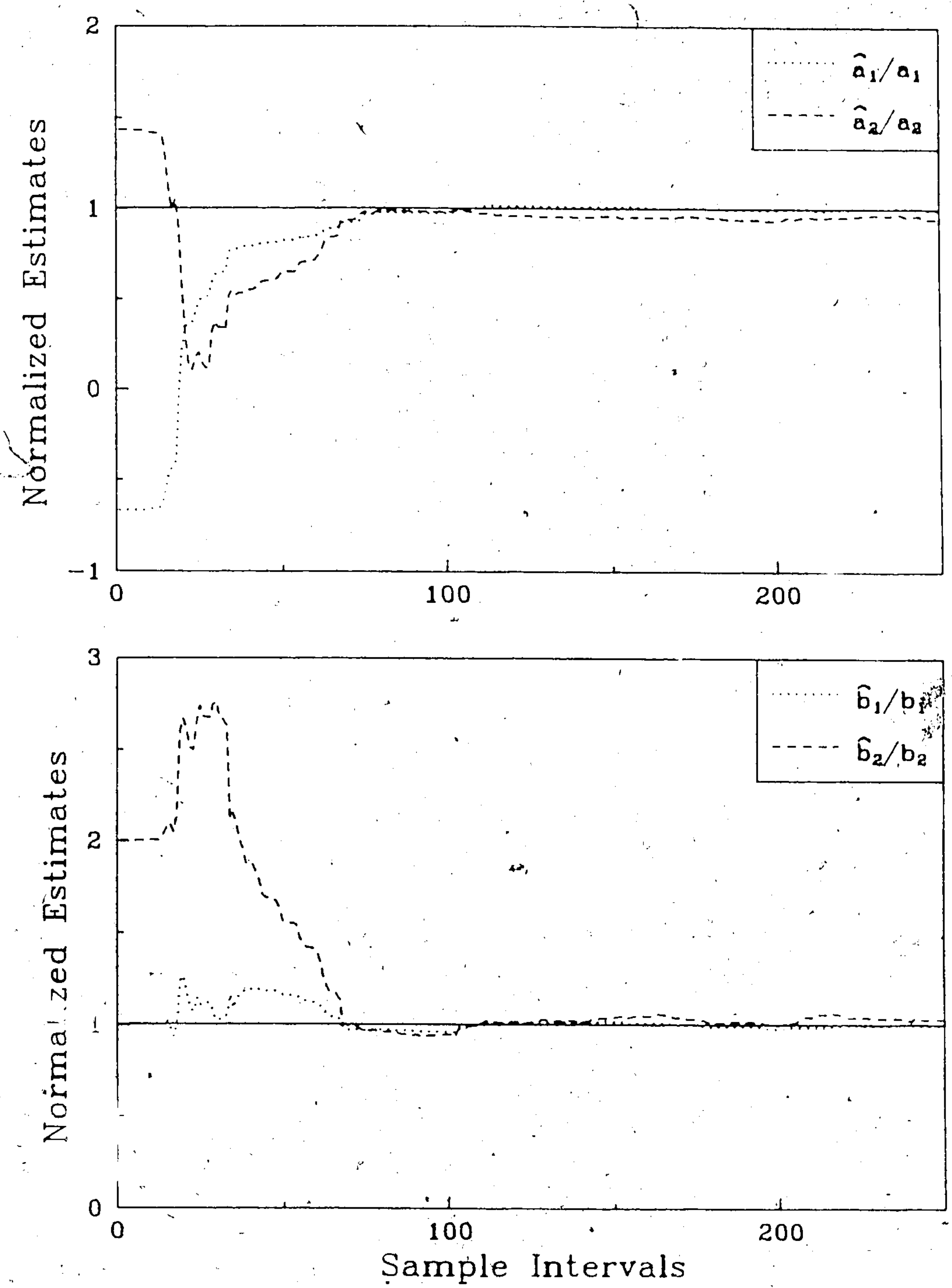

Figure 3.3b Parameter estimation using ILS (w P(·)=

corresponds roughly to the time at which the plant output was stabilized in Fig. 3.3a, after which the performance was very close to that obtained for the fixed parameter case (*cf*. Fig. 2.2a).

In Figs. 3.3c and 3.3d the behaviour of the minimum variance ISTC (Eqn. (3.63) with $P(z^{-1}) = R(z^{-1}) = 1$ and $Q(z^{-1}) = 0$) is demonstrated with initial parameter estimates $\hat{\theta}(0) = [1 \ \ 1 \ \ 1 \ \ 1 \ \ 1 \ \ 1]^T$. The regressor vector was filtered by $T(z^{-1}) = 1 - 0.8z^{-1}$, rather than trying to estimate $C(z^{-1})$ online as described in Section 2.4. Figure 3.3d indicates that parameter convergence for the ISTC was generally poorer than that obtained in Fig. 3.3b for the AMKFP, despite the fact that the trace of the covariance matrix was kept at a much higher level. This is due to the fact that more parameters were estimated in the implicit ISTC scheme (seven) than in the explicit AMKFP (four). Note also that even after the output was stabilized at t=80, the performance of the ISTC was significantly worse than that observed for the fixed parameter case (*cf*. Fig. 2.2c).

Several identification techniques were discussed in Section 3.2, with particular emphasis on Improved Least Squares and Kalman filter parameter ID; the latter method was shown to be equivalent to the well-known "RLS-with-covariance-modification" method. In Fig. 3.4 the behaviour of these algorithms was compared for minimum variance control of a time-varying ARMA plant given by

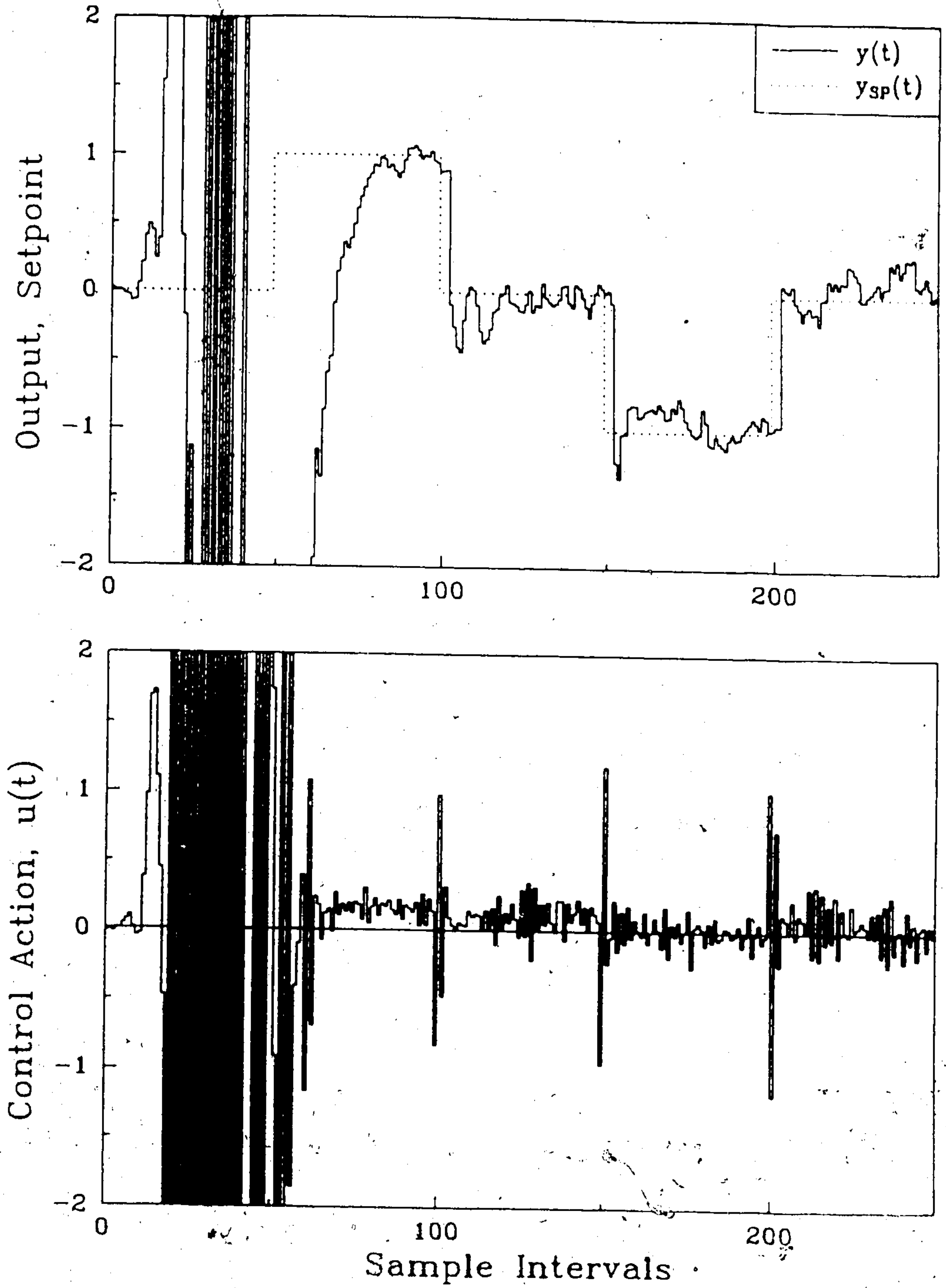

Figure 3.3c Adaptive minimum variance control using the ISTC

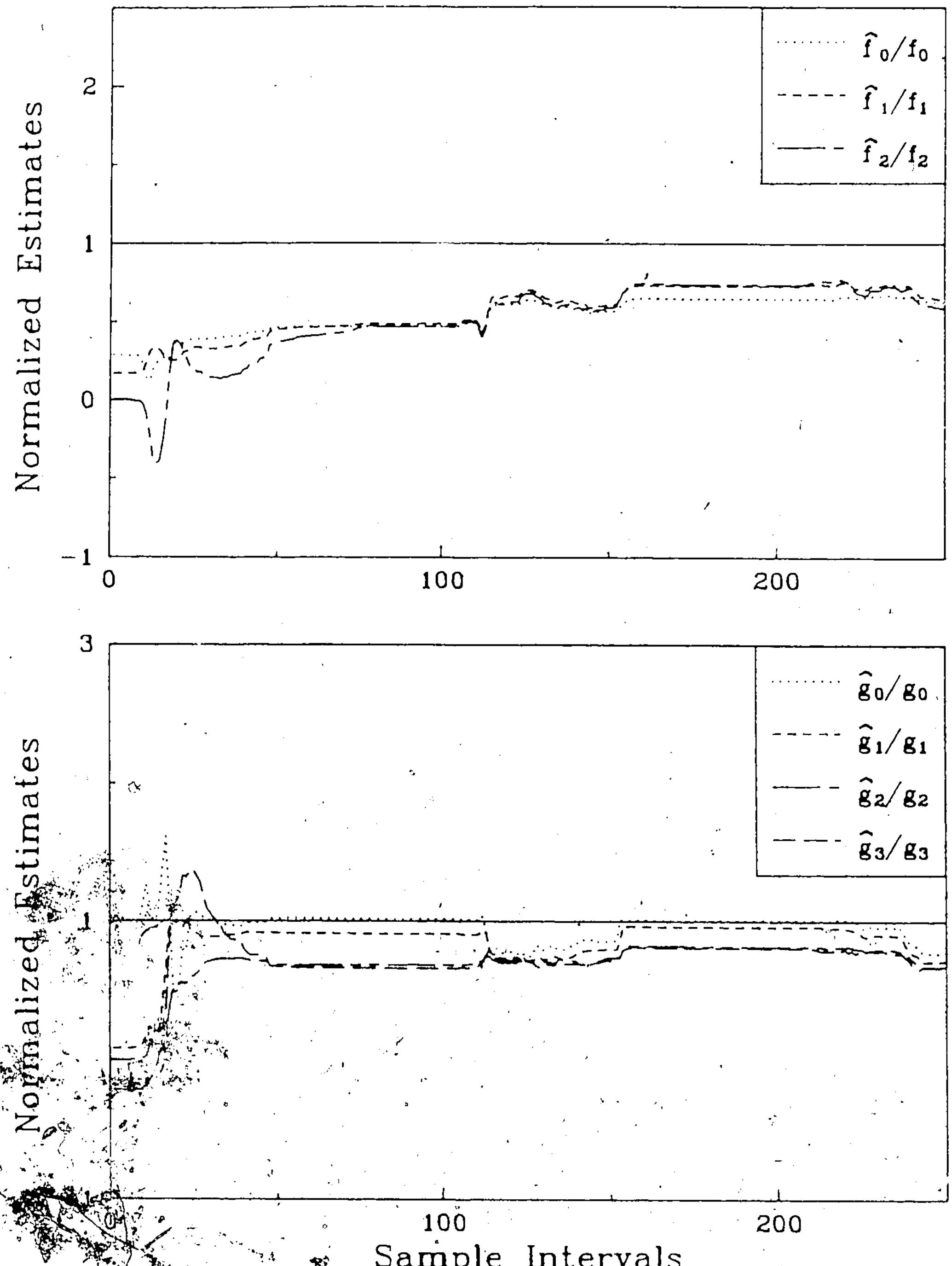

Figure 3.3d Parameter estimation using ILS ($tr\ P(\cdot) = 70$)

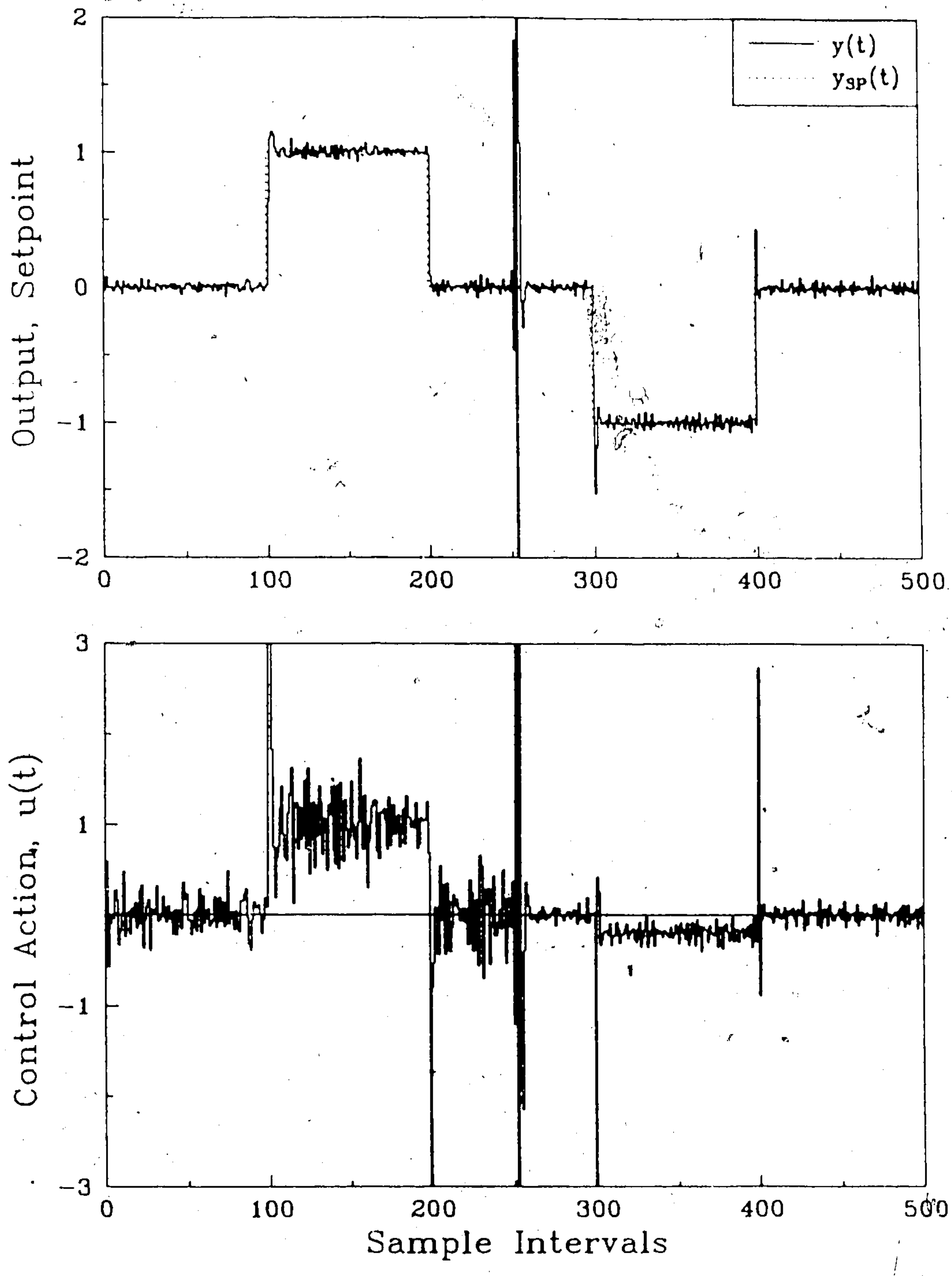

Figure 3.4a Adaptive control of a time-varying plant using the AMKFP plus Recursive Least Squares

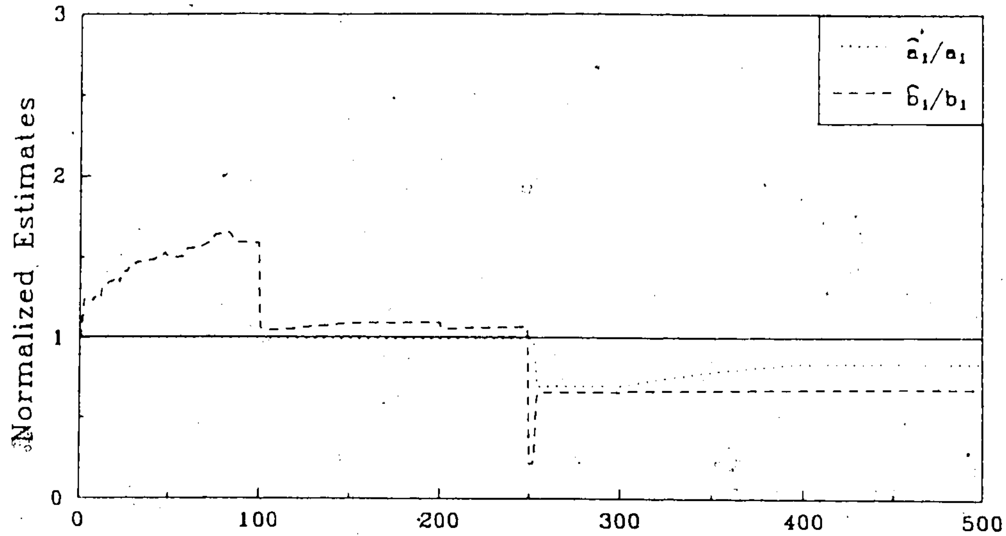

Figure 3.4b Parameter estimation using RLS

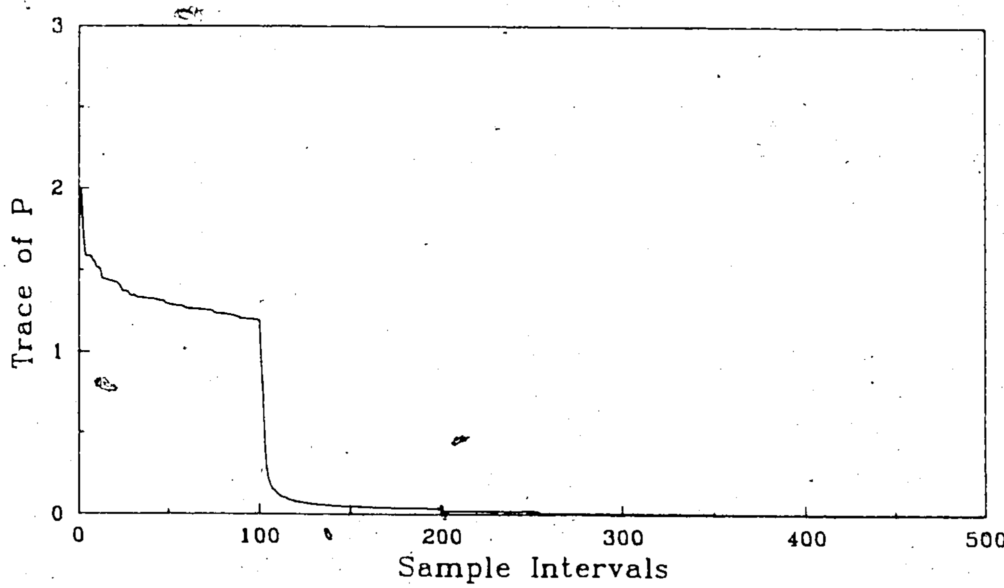

Figure 3.4c Trace of the covariance matrix for RLS

$$(1+a_1 z^{-1})y(t) = b_1 u(t-1) + \omega(t) \tag{3.67}$$

$$a_1 = -0.9$$

$$b_1 = 0.1, \quad \forall t \in [1, 250]$$

$$= 0.5, \quad \forall t > 250$$

where $\omega(t)$ is a zero-mean Gaussian sequence of variance 4×10^{-4}. All simulations were carried out using the AMKFP with $R_1/R_2 = 1$, correct initial parameter estimates, *i.e.* $\theta(0) = [-0.9 \;\; 0.1]^T$ and initial covariance matrix $P(0) = I$. (Note that a <u>positional</u> formulation was used in this example since the disturbance term in Eqn. (3.67) is stationary.)

Figure 3.4a sh[illegible]formance obtained using Recursive Least[illegible]timate a_1 and b_1. Minimum variance co[illegible] the first 250 samples, but the cha[illegible]used a period of instability in the [illegible] algorithm was able to recover event[illegible]ervo behaviour was significantly worse than that observed prior to the parameter change. The RLS approach was unable to track the change in b_1 (see Fig. 3.4b) because the trace of the covariance matrix had decayed to a very small value, as evidenced by Fig. 3.4c. (Note also the parameter drift that occurred during the period of steady state operation from t=0 to t=100, when $tr\ P(t)$ was still relatively large.)

Fig. 3.5a illustrates the results obtained using RLS with covariance modification (CM) for the choice $R_1 = I_2$ in Eqn. (3.18). There was a marked improvement over the

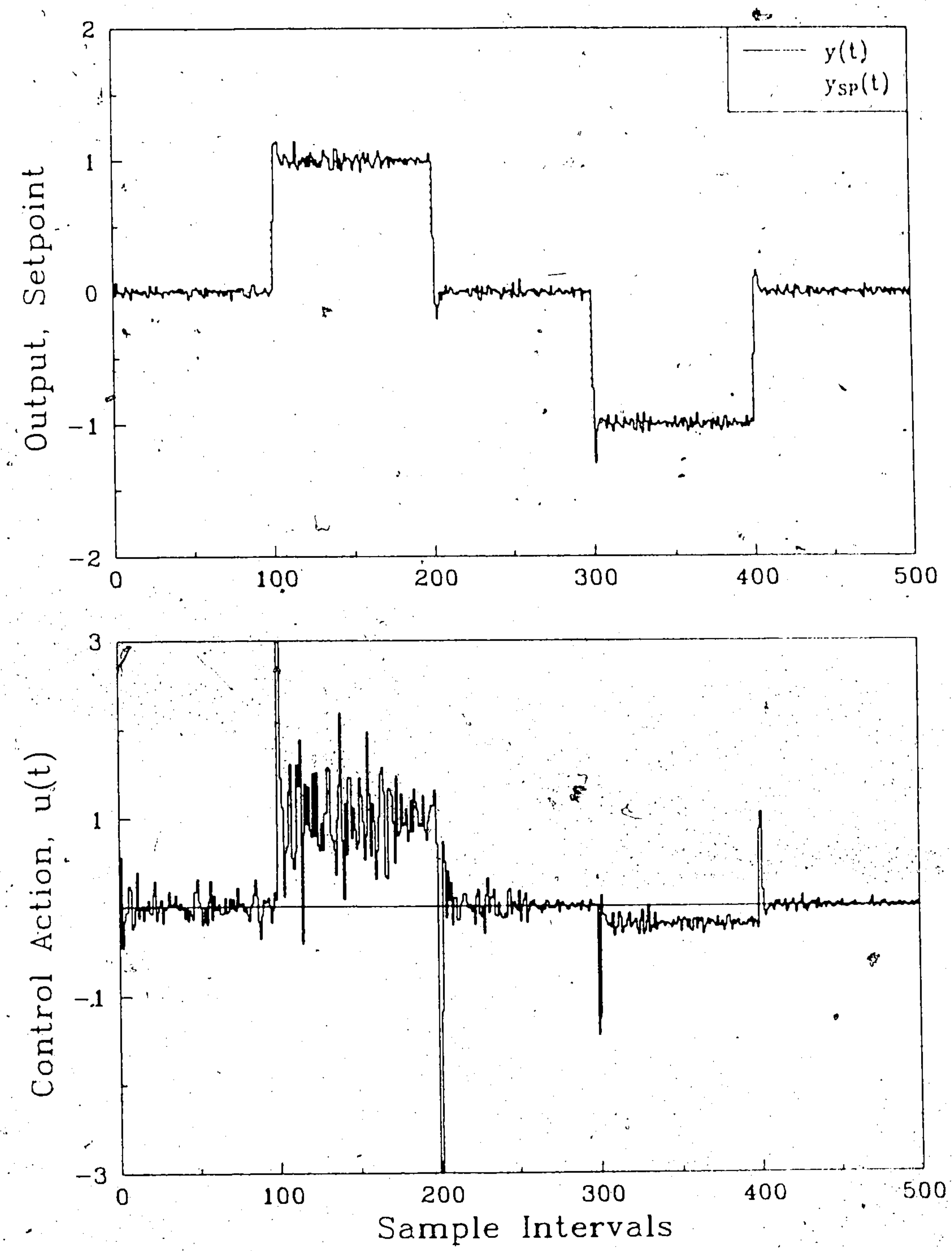

Figure 3.5a Adaptive control of a time-varying plant using the AMKFP plus RLS with covariance modification

performance obtained using simple RLS, as the parameter change at time 250 was immediately tracked by the identification algorithm (see Fig. 3.5b). This is because the trace of $P(t)$ was much higher in Fig. 3.5c than in Fig. 3.4c, which also led to a greater variance in the parameter estimates. Indeed, $tr\ P(t)$ was seen to increase in a roughly linear fashion during periods of steady operation; this caused significant drift in the estimates which could lead to bursting in the output.

In Fig. 3.6, however, the simulation was repeated using Improved Least Squares to maintain the trace of the covariance matrix at its initial value of 2. The behaviour of the algorithm can be said to lie somewhere in between that observed for simple RLS and RLS with CM in the sense that although there was some some movement in the output at time 250, it was much less than that observed using RLS. As seen in Fig. 3.6b, the parameter change was tracked fairly well using ILS, particularly after the setpoint change at time 300. This is because the trace of the covariance matrix was kept at its initial value. This in turn resulted in a much smaller amount of parameter drift than observed for RLS with CM, which lessens the possiblity of bursting. Indeed, it may be argued that the servo reponse obtained using ILS was superior to that of Fig. 3.5a after the setpoint change at t=300.

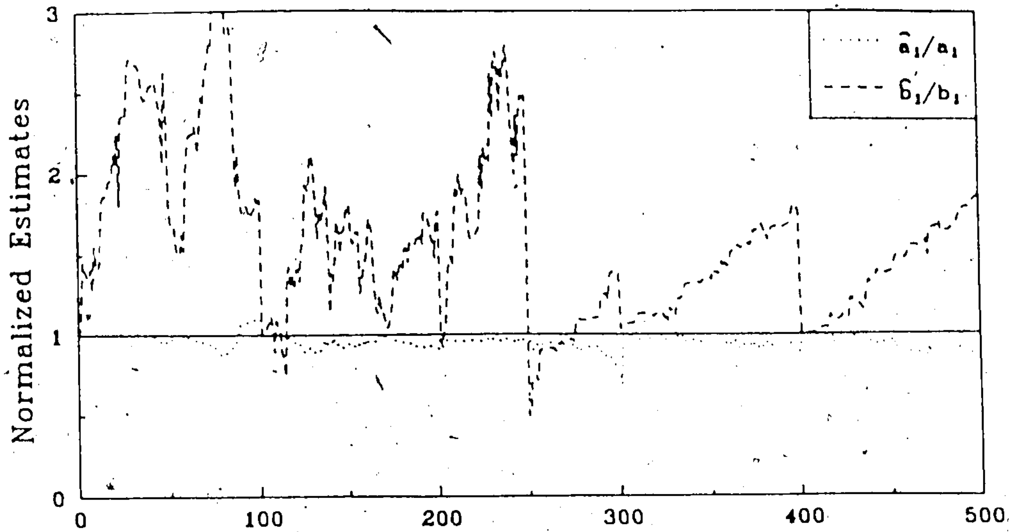

Figure 3.5b Parameter estimation using RLS with covariance modification

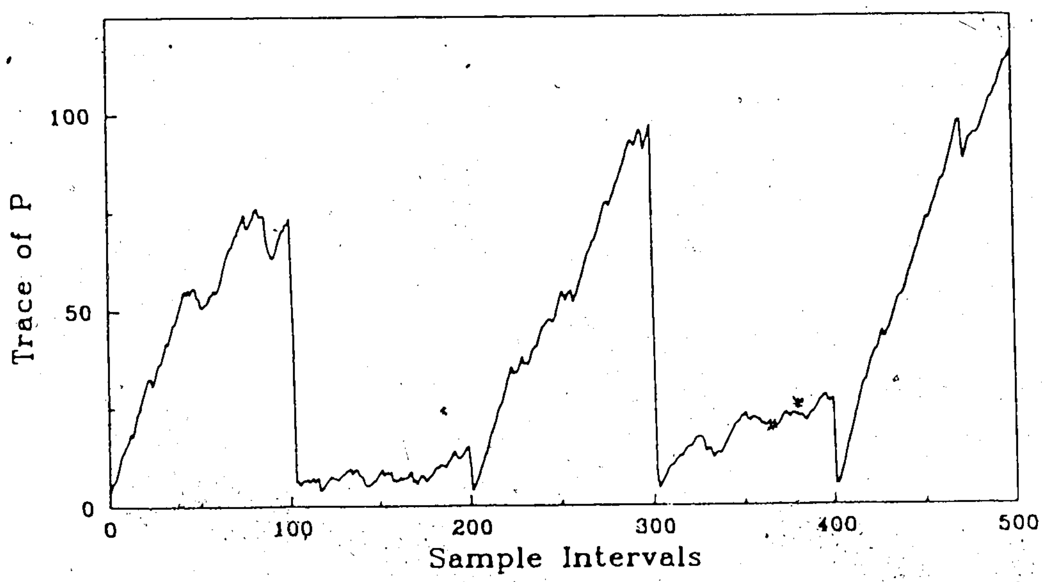

Figure 3.5c Trace of the covariance matrix for RLS with covariance modification

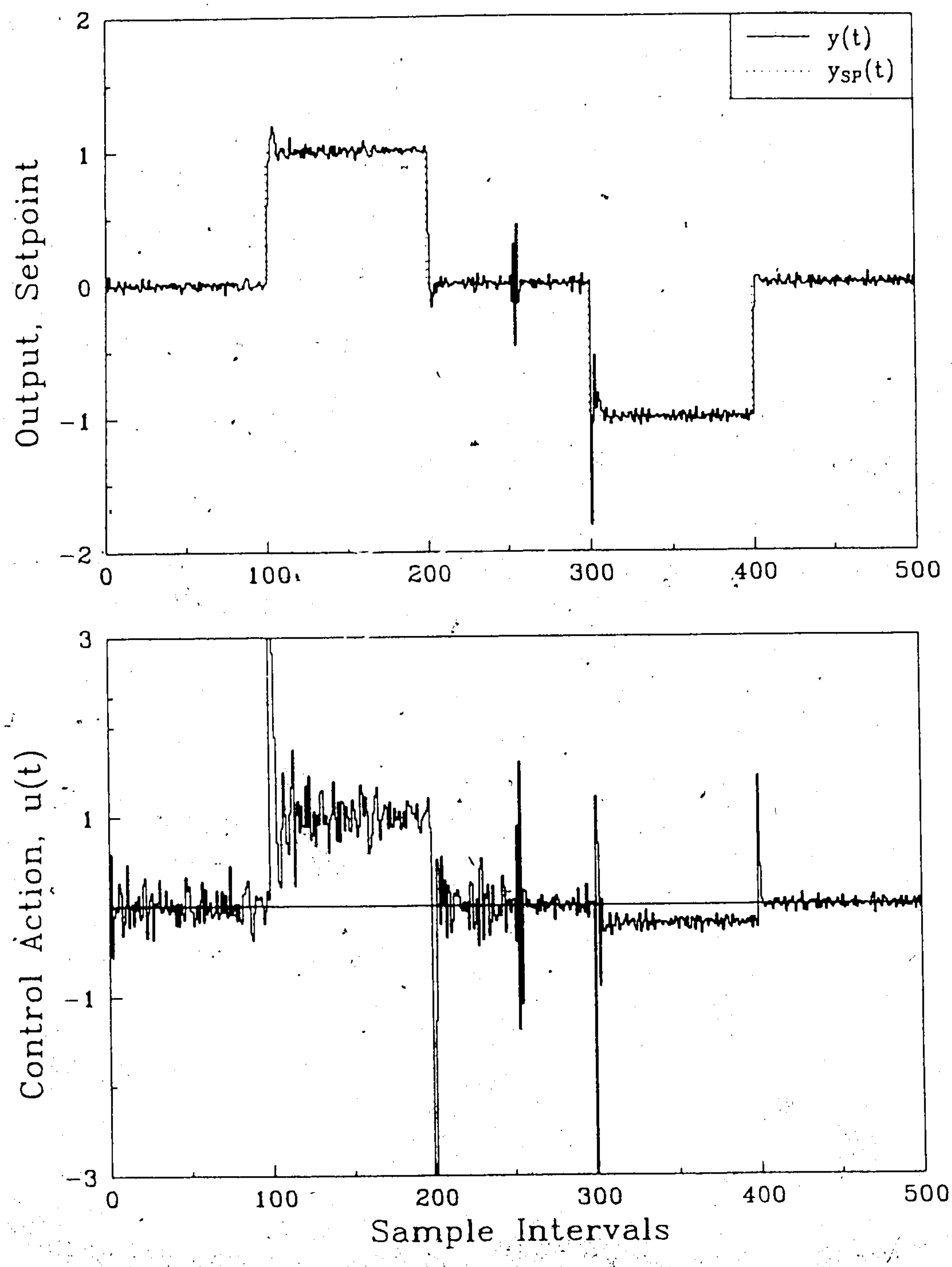

Figure 3.6a Adaptive control of a time-varying plant using the AMKFP plus ILS

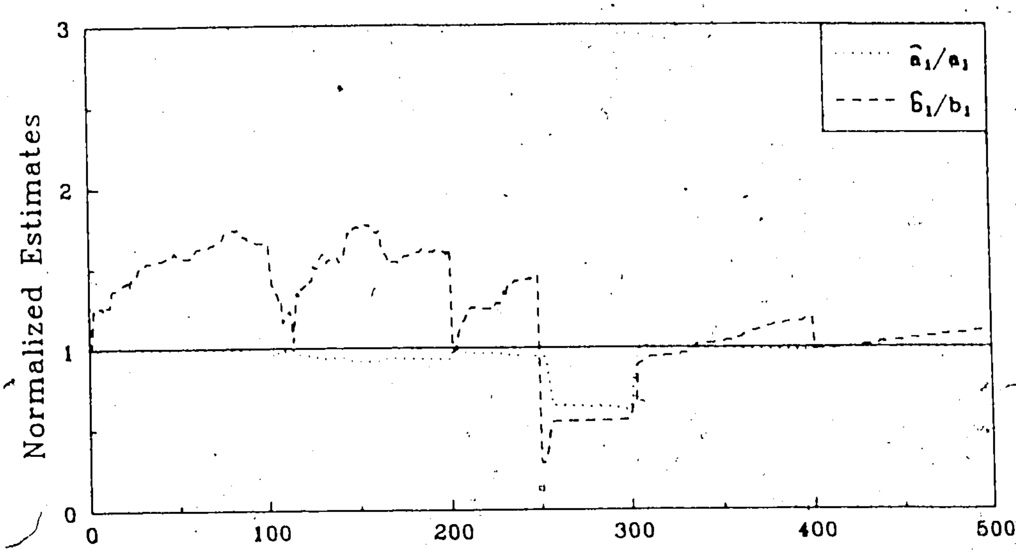

Figure 3.6b Parameter estimation using ILS

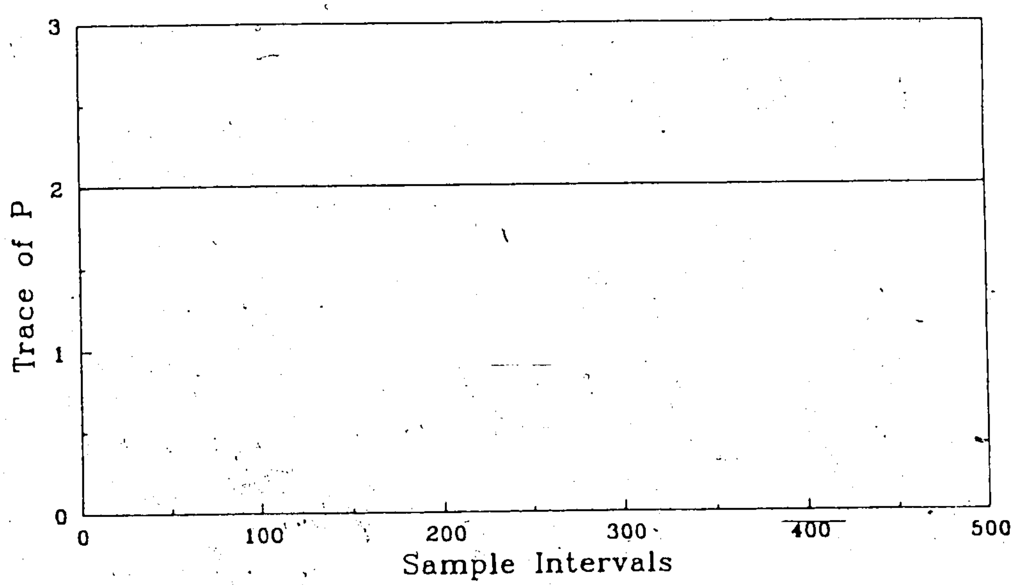

Figure 3.6c Trace of the (unscaled) covariance matrix for ILS

Finally, the behaviour of the AMKFP and ISTC algorithms was illustrated in the control of the nonminimum phase state space plant

$$\begin{bmatrix} x_p(t+1) \\ x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -0.8981 & 0 & 0 & 0 \\ 1 & 1 & 1.8954 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0.9 \\ 0.7975 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} n_1(t)$$

$$y(t) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]\,x(t) + n_2(t) \tag{3.68}$$

where $n_1(t)$ and $n_2(t)$ are independent random sequences of variance 10^{-6}. This process has the deterministic equivalent

$$(1 - 1.8954z^{-1} + 0.8981z^{-2})y(t)$$
$$= z^{-3}(0.7975 + 0.9z^{-1})u(t-1)$$

which is clearly nonminimum phase due to the zero at $z = -1.1285$.

As expected, the minimum variance forms of the AMKFP and ISTC controllers were observed to be unstable (see Figs. 3.7a, 3.7c), despite the fact that "good" estimates of the plant parameters were obtained in both cases (*cf*. Figs. 3.7b, 3.7d). (All parameters estimates were ini-

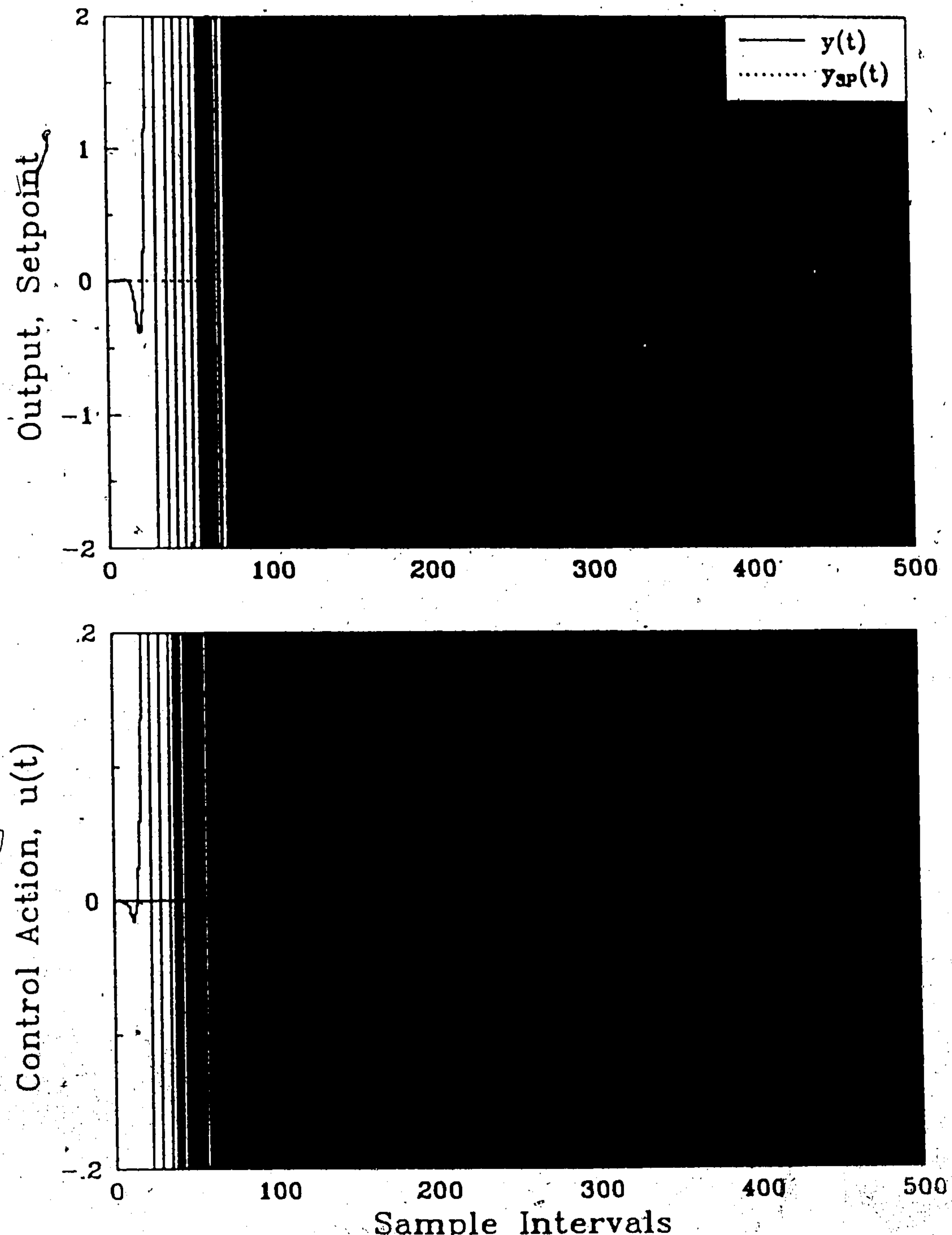

Figure 3.7a Adaptive MV control of a nonminimum phase plant using the AMKFP

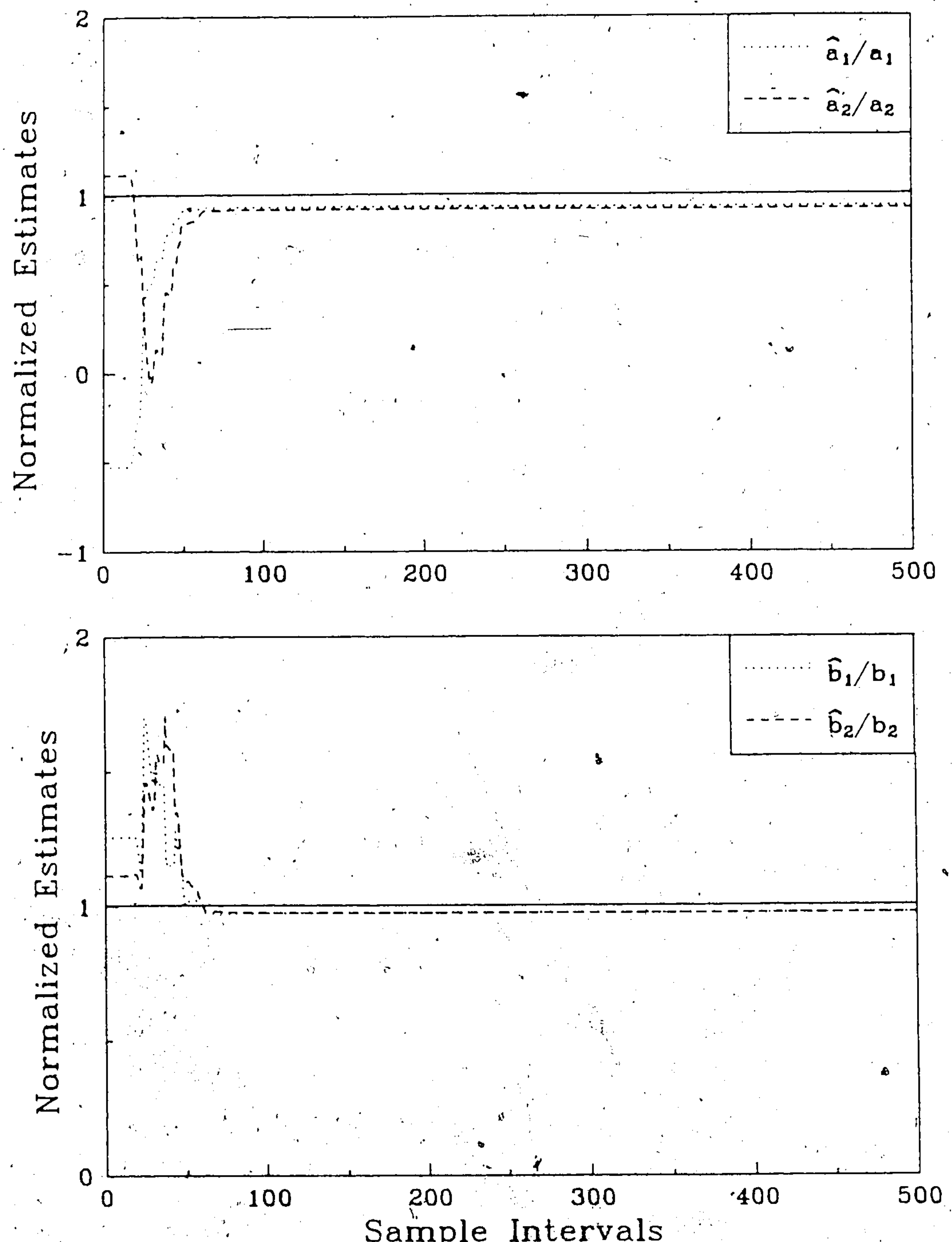

Figure 3.7b Parameter estimation using ILS ($tr.\ P(\cdot) = 4$)

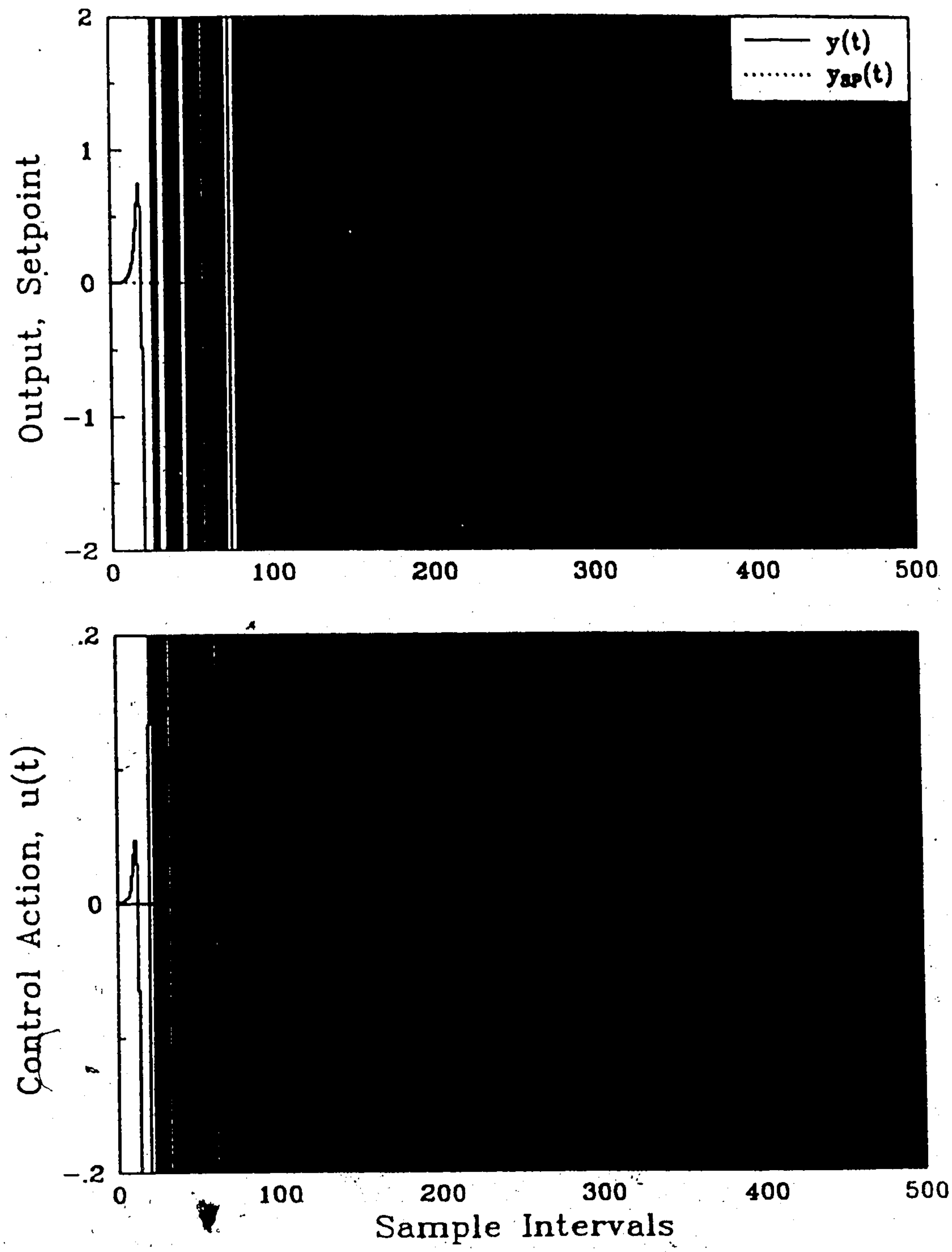

Figure 3.7c Adaptive MV control of a nonminimum phase plant using the ISTC

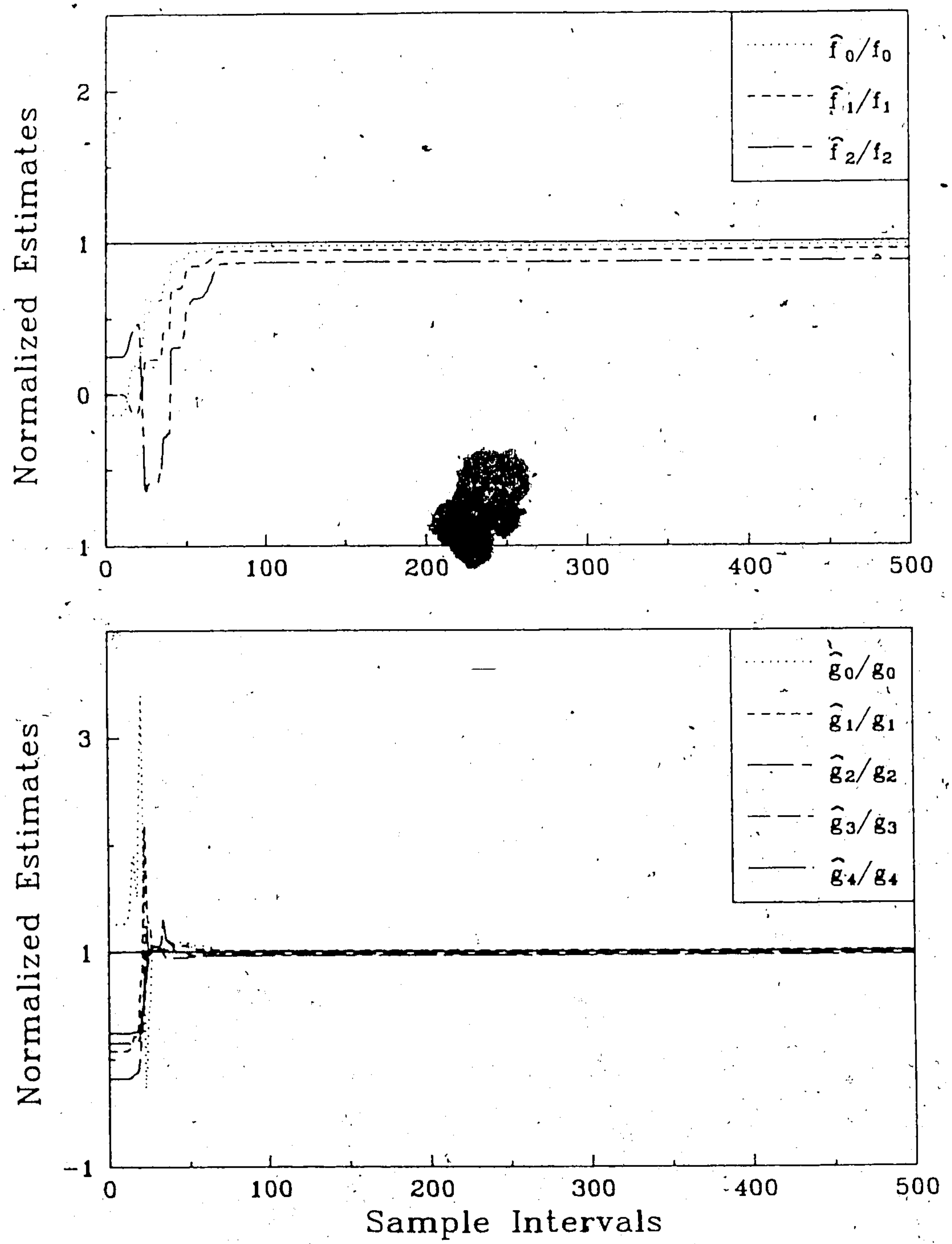

Figure 3.7d Parameter estimation using ILS ($tr\ P(\cdot) = 800$)

tialized to unity and $T(z^{-1})$ was chosen as $1-0.8z^{-1}$.) This is because a model inverse strategy results in an unstable control law if the plant zeros are unstable.

It is well known, however, that the addition of suitably chosen control weighting to the minimum variance cost functional will move the poles of the controller inside the unit circle, resulting in a stable configuration. In this example, $Q(z^{-1})$ was chosen as

$$Q(z^{-1}) = \frac{\Delta}{1-0.95z^{-1}}$$

which resulted in stable control of the plant (see Figs. 3.8a, 3.8c). It was observed that the ISTC took longer to stabilize, possibly due to the larger number of parameters to be estimated (*i.e.* eight in Fig. 3.8d versus four in Fig. 3.8b). It should be noted, however, that the choice of $Q(z^{-1})$ is very difficult to make without *a priori* knowledge of the plant; in Chapter 4, multistep versions of these algorithms will be introduced that generally provide stable control of nonminimum phase systems using default controller settings.

3.6 Conclusions

This chapter has outlined the extension of the Modified Kalman Filter Predictor of Walgama et al (1988) and the integrating Self-Tuning Controller of Tuffs and Clarke (1985) to the control of plants having time-varying dynamics and/or mild nonlinearities. This has been accomplished by combining the fixed parameter

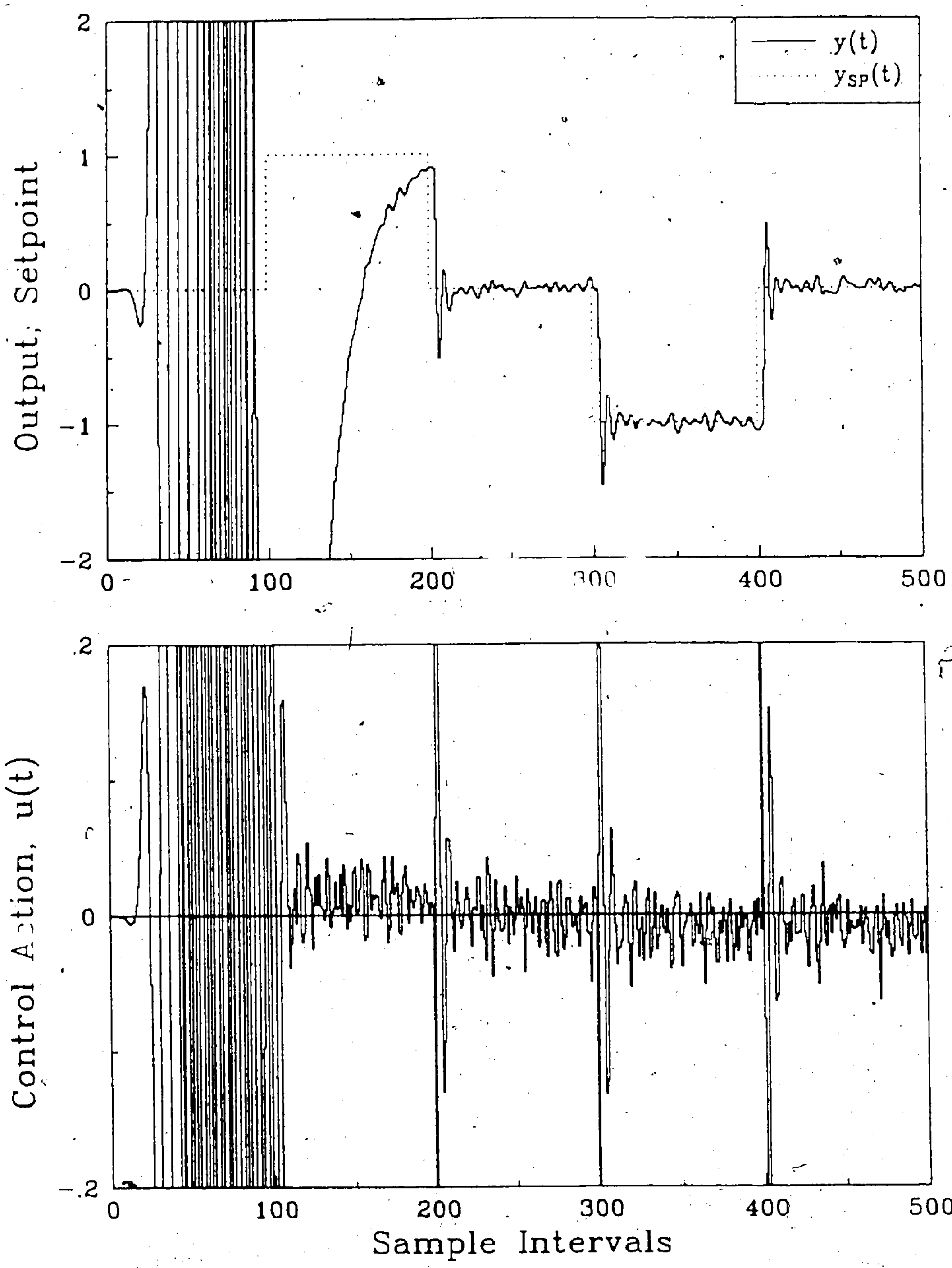

Figure 3.8a Adaptive GMV control of a nonminimum phase plant using the A[illegible]P

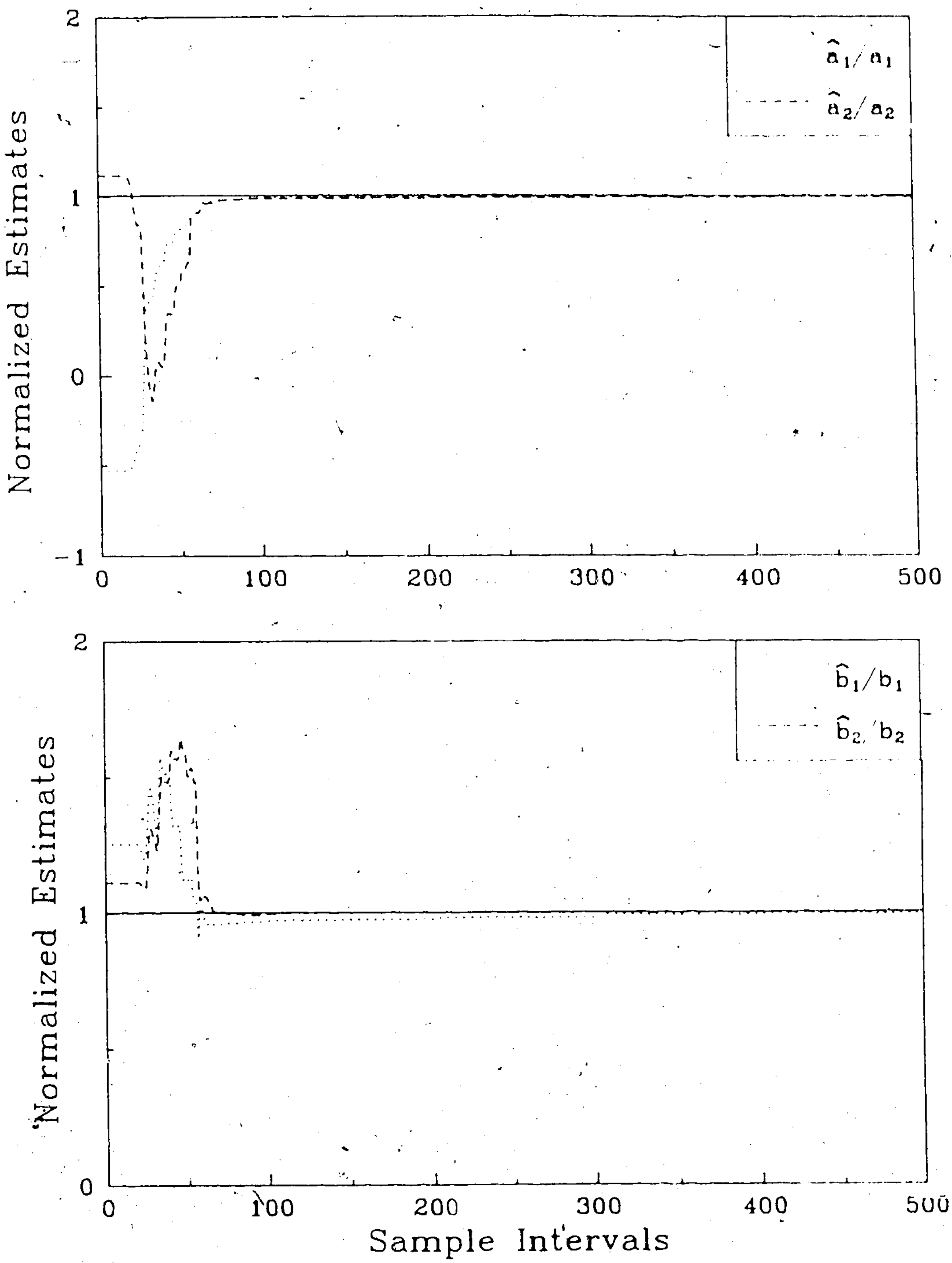

Figure 3.8b Parameter estimation using ILS ($tr\ P(\cdot)=4$)

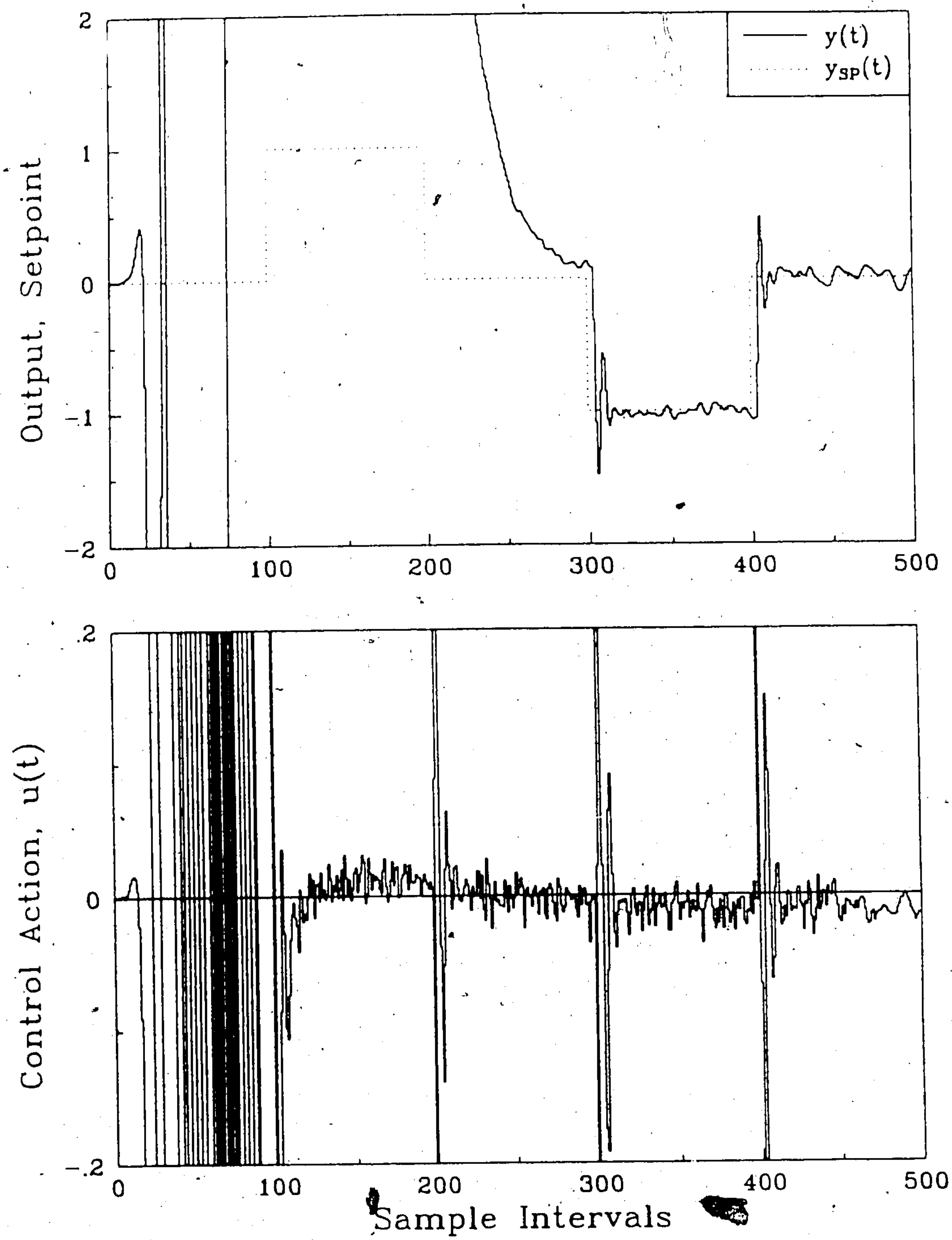

Figure 3.8c Adaptive GMV control of a nonminimum phase plant using the ISTC

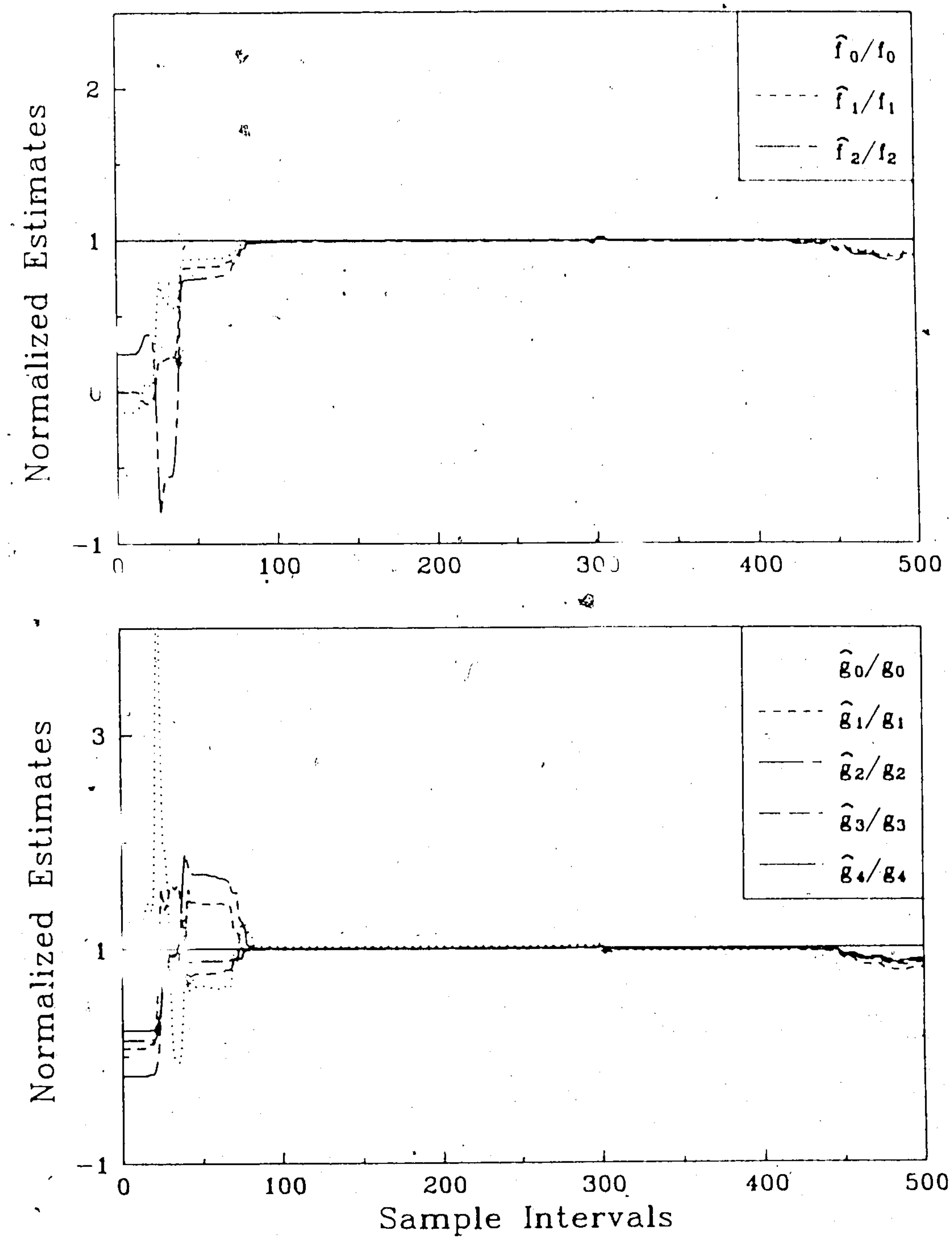

Figure 3.8d Parameter estimation using ILS ($tr\ P(\cdot) = 800$)

versions of these methods with a recursive parameter estimation algorithm for online identification of the process parameters.

In this connection, it was shown that use of the Kalman filter to recursively estimate the parameters of an ARMA process model is equivalent to "RLS-with-covariance-modification" (Goodwin and Sin, 1984). It was found, however, that this technique was subject to covariance windup during periods of low excitation, which may lead to bursting in the output. The Improved Least Squares algorithm of Sripada and Fisher (1987) was also introduced to overcome this bursting problem. A correction was made to the calculation of the variable forgetting factor in ILS.

A series of simulation examples demonstrated that both the ISTC and the AMKFP-based control strategies perform well on unknown or time-varying plants subject to time delays and nonstationary stochastic disturbances. However, it was observed that the number of parameters to be estimated in the ISTC may become prohibitively large due to the implicit nature of the algorithm. This leads to generally slower convergence than that observed using the (explicit) AMKFP.

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4

Multistep Predictive Control

4.1 Introduction

This chapter examines the similarities that exist between the designs of certain long-range predictive control (LRPC) strategies. In particular, Multistep Adaptive Predictive Control (Sripada, 1988), Generalized Predictive Control (Clarke et al, 1987) and the Multivariable, Optimal, Constrained Control Algorithm (Sripada and Fisher, 1985; Li et al, 1988; Navratil et al, 1988) are examined for their approach to the control of known, time-invariant SISO plants by analogy with the Smith Predictor.

The Multistep Adaptive Predictive Control (MAPC) scheme of Sripada (1988) will be extensively analyzed in Chapter 5, hence only a summary of the approach will be given here. Specifically, it will be shown in Chapter 5 that the proposed method of partitioning the state space formulation into "u-y" and disturbance subsystems is infeasible when the system parameters are unknown and/or time-varying. It was therefore decided to use the Modified Kalman Filter Predictor (MKFP) of Walgama et al (1988) to formulate the output trajectory, which will guarantee asymptotically zero prediction offset in the presence of nonstationary disturbances.

The Generalized Predictive Control (GPC) algorithm of Clarke et al (1987) is a generalization of the single-point or "k-step-ahead" Generalized Minimum Variance (GMV) scheme of Clarke and Gawthrop (1975, 1979). The primary motivation for this development was the rather poor performance yielded by GMV when applied to plants with unknown and/or variable time delay. Additionally, it was found through experience that the weighting polynomials $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ often required a good deal of online tuning. GPC, on the other hand, has been observed to work well on a wide variety of applications using a "default" controller configuration, *i.e.* $N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 0$ (*cf.* Eqn. (4.1) below).

The Multivariable, Optimal, Constrained Control Algorithm or MOCCA (Sripada and Fisher, 1985; Li et al, 1988; Navratil et al, 1988) is in essence a refinement of the industrially successful Dynamic Matrix Control (DMC) of Cutler and Ramaker (1980). Both methods employ a "nonparametric" model of the process to form a trajectory of future control errors to be minimized in a multistep predictive control strategy. The models are called nonparametric since they use step response data (*e.g.* the process reaction curve) directly, rather than an ARMA or state space representation of the process. This method of modelling the process, unlike the MAPC and GPC schemes, requires no *a priori* assumptions regarding the model order or time delay. (The time delay can in fact be obtained from the reaction curve by counting the number of leading zero step response coefficients.) On the other hand, the DMC or MOCCA approach is limited to time-invariant processes. A further disadvantage arises from the fact that the process representation is non-minimal as it will generally result in a less computationally efficient implementation that those of the ARMA or state space methods.

In a recent review article comparing Generalized Predictive Control with various representative Model Predictive Control (MPC) schemes (of which DMC and MOCCA are two examples), Clarke and Mohtadi (1986) appear to have claimed that this class of algorithms is strictly limited to open loop stable processes. This is untrue insofar as virtually any unstable or marginally stable system can be stabilized by the addition of a proportional controller with a suitably chosen gain. Step response data may then be obtained from the augmented plant by issuing a step change in setpoint to the proportional controller. The analysis which follows will, however, assume that the plant is open loop stable for purposes of comparison with the MAPC and GPC schemes discussed earlier as these algorithms would surely be different from MOCCA operating on an augmented process. Furthermore, the assumption of a known and time-invariant plant implies that the step response data are uncorrupted by unmeasured disturbances.

4.2 Multistep Adaptive Predictive Control

The Multistep Adaptive Predictive Controller of Sripada (1988) minimizes a cost functional of the form

$$J = \sum_{j=N_1}^{N_2} \{y_{SP}(t+j) - \hat{y}(t+j \mid t)\}^2 + \sum_{j=1}^{N_u} \lambda \{\Delta u(t+j-1)\}^2 \tag{4.1}$$

As mentioned in the introduction, the trajectory of predicted outputs $\{\hat{y}(t+j|t),\ j\in[N_1,N_2]\}$ is formed using the Modified Kalman Filter Predictor (Walgama et al, 1988), which is based upon the following state space process model:

$$x(t+1)=\Phi x(t)+\Lambda u(t)+\Gamma n_1(t) \tag{4.2}$$

$$y(t)=Hx(t)+n_2(t) \tag{4.3}$$

where

$$x(t)=[x_p(t),x_1(t),\ldots,x_n(t),x_{n+1}(t),\ldots,x_{n+d}(t)]^T$$

$$\Phi=\begin{bmatrix} 1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\ \gamma_1 & 0 & \ldots & 0 & -a_n & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \gamma_n & 0 & \ldots & 1 & -a_1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}_{(n+k)\times(n+k)}$$

$$\Lambda=[0,b_n,\ldots b_1,0,\ldots,0]^T$$

$$\Gamma=[1,0,\ldots,0,0,\ldots,0]^T \tag{4.4}$$

$$H=[0,0,\ldots,0,0,\ldots,1]$$

and d is the time delay excluding that due to discretization. In Appendix B an ARMA representation for the Kalman filter prediction $\hat{y}(t+k|t)$ was derived using innovations analysis, where $k=d+1$, *i.e.* the total time delay of the process. The derivation was carried out for the steady state Kalman filter, assuming no model/plant mismatch (MPM). A similar exercise will now be performed for the KF prediction $\hat{y}(t+j|t)$, where j is an arbitrary integer such that $N_1 = k \leq j \leq N_2$. Note that N_1 in Eqn. (4.1) will be set equal to k, as it is useless to penalize outputs which cannot be affected by $\Delta u(t)$. (This assumption is consistent with the overall requirement of a perfect process model.)

Equation (B.7) indicates that an equivalent ARIMA representation for Eqns. (4.2), (4.3) is given by

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t-k) + \frac{C(z^{-1})}{A(z^{-1})} \frac{\omega(t)}{\Delta} \tag{4.5}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \ldots + b_n z^{-n+1}$$

$$C(z^{-1}) = [A(z^{-1})(1 + K_2(z^{-1})) + z^{-d} K_1(z^{-1})]\Delta + z^{-d} D(z^{-1}) \tag{4.6}$$

and

$$K_1(z^{-1}) = L_n + L_{n-1} z^{-1} + \ldots + L_1 z^{-n+1}$$

$$K_2(z^{-1}) = L_{n+d-1} z^{-1} + \ldots + L_{n+1} z^{-d+1}$$

$$D(z^{-1}) = L_\rho [\gamma_n z^{-1} + \ldots + \gamma_1 z^{-n}]$$

The L_i are steady state Kalman gains taken from the Kalman filter state update (see Appendix A):

$$\begin{aligned} E\{x(t+1) \mid t+1\} &= \hat{x}(t+1) \\ &= \Phi \hat{x}(t) + \Lambda u(t) + L\omega(t+1) \end{aligned} \tag{4.7}$$

and $\omega(t)$, the innovations sequence is defined as

$$\begin{aligned} \omega(t) &= y(t) - \hat{y}(t \mid t-1) \\ &= y(t) - H\Phi\hat{x}(t-1) - H\Lambda u(t-1) \end{aligned} \tag{4.8}$$

Recall that $\omega(t)$ becomes a zero-mean random sequence when the Kalman gain vector L has converged to its final solution. To obtain an optimal j-step-ahead output prediction, consider the state update (4.7) written for a forward shift of j sample intervals:

$$\hat{x}(t+j\,|\,t+j) = \Phi\hat{x}(t+j-1\,|\,t+j-1) + \Lambda u(t+j-1) + L\omega(t+j) \tag{4.9}$$

Using successive backward substitution to time t, (4.9) may be written as

$$\hat{x}(t+j\,|\,t+j) = \Phi^{j}\hat{x}(t\,|\,t) + \Lambda u(t+j-1) + \Phi\Lambda u(t+j-2) + \ldots + \Phi^{j-1}\Lambda u(t) + L\omega(t+j) + \Phi L\omega(t+j-1) + \ldots + \Phi^{j-1}L\omega(t+1) \tag{4.10}$$

To obtain an optimal prediction of $x(t+j)$ using data available at time t+j-d,

$$\hat{x}(t+j\,|\,t+j-d) = \Phi^{j}\hat{x}(t\,|\,t) + \Lambda u(t+j-1) + \Phi\Lambda u(t+j-2) + \ldots + \Phi^{j-1}\Lambda u(t) + \Phi^{d}L\omega(t+j-d) + \ldots + \Phi^{j-1}L\omega(t+1) \tag{4.11}$$

which is obtained by setting the future innovations $\omega(t+j),\ldots,\omega(t+j-d+1)$ to their expected value of zero. (Note that it has been assumed that the future control actions $u(t+j-1),\ldots,u(t+j-d)$ are known at time t+j-d.) Similarly, the prediction conditioned upon data up to and including time t is

$$\hat{x}(t+j\,|\,t) = \Phi^{j}\hat{x}(t\,|\,t) + \Lambda u(t+j-1) + \Phi\Lambda u(t+j-2) + \ldots + \Phi^{j-1}\Lambda u(t) \tag{4.12}$$

Subtraction of (4.12) from (4.11) leads to

$$\hat{x}(t+j\,|\,t+j-d) = \hat{x}(t+j\,|\,t) + \Phi^{d}L\omega(t+j-d) + \ldots + \Phi^{j-1}L\omega(t+1) \tag{4.13}$$

Hence, the optimal prediction of the future output $y(t+j)$ at time t+j-d is given by

$$\begin{aligned}\hat{y}(t+j\,|\,t+j-d) &= H\hat{x}(t+j\,|\,t+j-d)\\ &= \hat{y}(t+j\,|\,t) + H\Phi^{d}L\omega(t+j-d)\\ &\quad + \ldots + H\Phi^{j-1}L\omega(t+1)\end{aligned} \tag{4.14}$$

At this point, note that a d-step-ahead prediction of y at time t can be obtained from Eqn. (4.12) by setting j=d, *i.e.*

$$\hat{y}(t+d\,|\,t) = H\Phi^{d}\hat{x}(t\,|\,t) + H\Lambda u(t+d-1) + H\Phi\Lambda u(t+d-2) + \ldots + H\Phi^{d-1}\Lambda u(t) \tag{4.15}$$

But the scalars $H\Lambda,\ldots,H\Phi^{d-1}\Lambda$ are the first d coefficients of $z^{-k}B(z^{-1})/A(z^{-1})$, *i.e.* he first d impulse response coefficients of the process, which are zero because k=d+1. (This may also be verified from the state coefficient matrices given in (4.4).) It may be further verified that

$$H\Phi^{d} = [0,\ldots,0,1,0,\ldots,0]$$

with the unity element occurring in the n+1'st position. Therefore,

$$\hat{y}(t+d\,|\,t) = \hat{x}_{n}(t\,|\,t)$$

which implies that

$$\hat{y}(t+j\,|\,t+j-d) = \hat{x}_{n}(t+j-d\,|\,t+j-d) \tag{4.16}$$

Using (4.14) and (4.16), then,

$$\hat{y}(t+j\,|\,t) = \hat{x}_{n}(t+j-d) - K'_{3,j}(z^{-1})\omega(t+j-d) \tag{4.17}$$

where

$$K'_{3,j}(z^{-1}) = k_{d+1} + k_{d+2}z^{-1} + \ldots + k_j z^{-j+d+1} \tag{4.18}$$

and

$$k_i = H\Phi^{i-1}L \tag{4.19}$$

However, Eqn. (B.5) indicates that

$$\hat{x}_n(t+j-d) = A^{-1}(z^{-1})B(z^{-1})u(t+j-d-1) + A^{-1}(z^{-1})K_1(z^{-1})\omega(t+j-d) + A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t+j-d)}{\Delta}$$

Therefore,

$$\begin{aligned}\hat{y}(t+j|t) &= A^{-1}(z^{-1})B(z^{-1})u(t+j-d-1) \\ &+ A^{-1}(z^{-1})K_1(z^{-1})\omega(t+j-d) \\ &+ A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t+j-d)}{\Delta} \\ &- A^{-1}(z^{-1})A(z^{-1})K'_{3,j}(z^{-1})\omega(t+j-d) \\ &= \frac{B(z^{-1})}{A(z^{-1})}u(t+j-k) \\ &+ \frac{[(K_1(z^{-1}) - K'_{3,j}(z^{-1})A(z^{-1}))\Delta + D(z^{-1})]}{A(z^{-1})} \\ &\cdot z^{j-d}\frac{\omega(t)}{\Delta}\end{aligned}$$

or

$$\hat{y}(t+j|t) = \frac{B(z^{-1})}{A(z^{-1})}u(t+j-k) + \frac{K_{4,j}(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \tag{4.20}$$

with

$$K_{4,j}(z^{-1}) = z^{j-d}[(K_1(z^{-1}) - K'_{3,j}(z^{-1})A(z^{-1}))\Delta + D(z^{-1})] \tag{4.21}$$

Rewriting (4.5) in terms of $\omega(t)/A(z^{-1})/\Delta$ and substituting in (4.20),

$$\hat{y}(t+j\,|\,t) = \frac{B(z^{-1})}{A(z^{-1})}u(t+j-k)$$
$$+\frac{K_{4,j}(z^{-1})}{C(z^{-1})}\left[y(t) - z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right]$$

or

$$\hat{y}(t+j\,|\,t) = G_P(z^{-1})u(t+j-k)$$
$$+G_{F,j}(z^{-1})[y(t) - G_M(z^{-1})u(t)] \tag{4.22}$$

where

$$G_P(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}, \quad G_M(z^{-1}) = z^{-k}\frac{B(z^{-1})}{A(z^{-1})} \tag{4.23}$$

and

$$G_{F,j}(z^{-1}) = \frac{K_{4,j}(z^{-1})}{C(z^{-1})} \tag{4.24}$$

Notice that the j-step-ahead prediction that would be given by the Smith Predictor (Smith, 1957, 1959) is obtained from (4.22) by setting $G_F(z^{-1}) = 1$. Returning to the definition of $C(z^{-1})$ in Eqn. (4.6),

$$C(z^{-1}) = [A(z^{-1})(1+K_2(z^{-1})) + z^{-d}K_1(z^{-1})]\Delta + z^{-d}D(z^{-1})$$
$$= A(z^{-1})(1+K_2(z^{-1}))\Delta + z^{-d}[K_1(z^{-1})\Delta + D(z^{-1})]$$

or

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = (1+K_2(z^{-1})) + z^{-d}\frac{[K_1(z^{-1})\Delta + D(z^{-1})]}{A(z^{-1})\Delta}$$
$$= (1+K_2(z^{-1})) + z^{-d}\frac{[A(z^{-1})K'_{3,j}(z^{-1})\Delta]}{A(z^{-1})\Delta}$$
$$+z^{-d}\frac{[(K_1(z^{-1}) - K'_{3,j}(z^{-1})A(z^{-1}))\Delta + D(z^{-1})]}{A(z^{-1})\Delta}$$

$$= (1 + K_2(z^{-1}) + z^{-d}K'_{3,j}(z^{-1})) + z^{-j}\frac{\{z^{j-d}[(K_1(z^{-1}) - K'_{3,j}(z^{-1})A(z^{-1}))\Delta + D(z^{-1})]\}}{A(z^{-1})\Delta}$$

This may be written as

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = K_{3,j}(z^{-1}) + z^{-j}\frac{K_{4,j}(z^{-1})}{A(z^{-1})\Delta} \tag{4.25}$$

where

$$K_{3,j}(z^{-1}) = 1 + K_2(z^{-1}) + z^{-d}K'_{3,j}(z^{-1})$$

and $K_{3,j}(z^{-1})$ is of order $\delta K_{3,j} = j - 1$.

From Eqn. (4.22), it is seen that the trajectory of future outputs $\{\hat{y}(t+j\,|\,t),\ N_1 = k \leq j \leq N_2\}$ may be represented by the block diagram of Fig. 4.1. Hence the predictor structure of MAPC may be interpreted as a network of parallel, optimal Smith Predictors. It is seen from the diagram that $G_{F,j}(1) = 1,\ j \in [k, N_2]$ is required to eliminate steady state prediction offset in the presence of nonzero-mean disturbances. From Eqn. (4.24) the condition $G_{F,j}(1) = 1$ implies $K_{4,j}(1) = C(1)$, which from (4.25) is true for all j. Thus, the multistep KF prediction scheme exhibits zero offset in the presence of nonstationary load processes.

Figure 4.1 may be modified slightly using Eqn. (4.24) and redrawn as in Fig. 4.2., in order that certain other interpretations of the predictor structure can be made. Consider first the limiting case of zero model/plant mismatch. In this instance, the residual signal $r(t)$ will exactly represent the process disturbances. Man (1984)

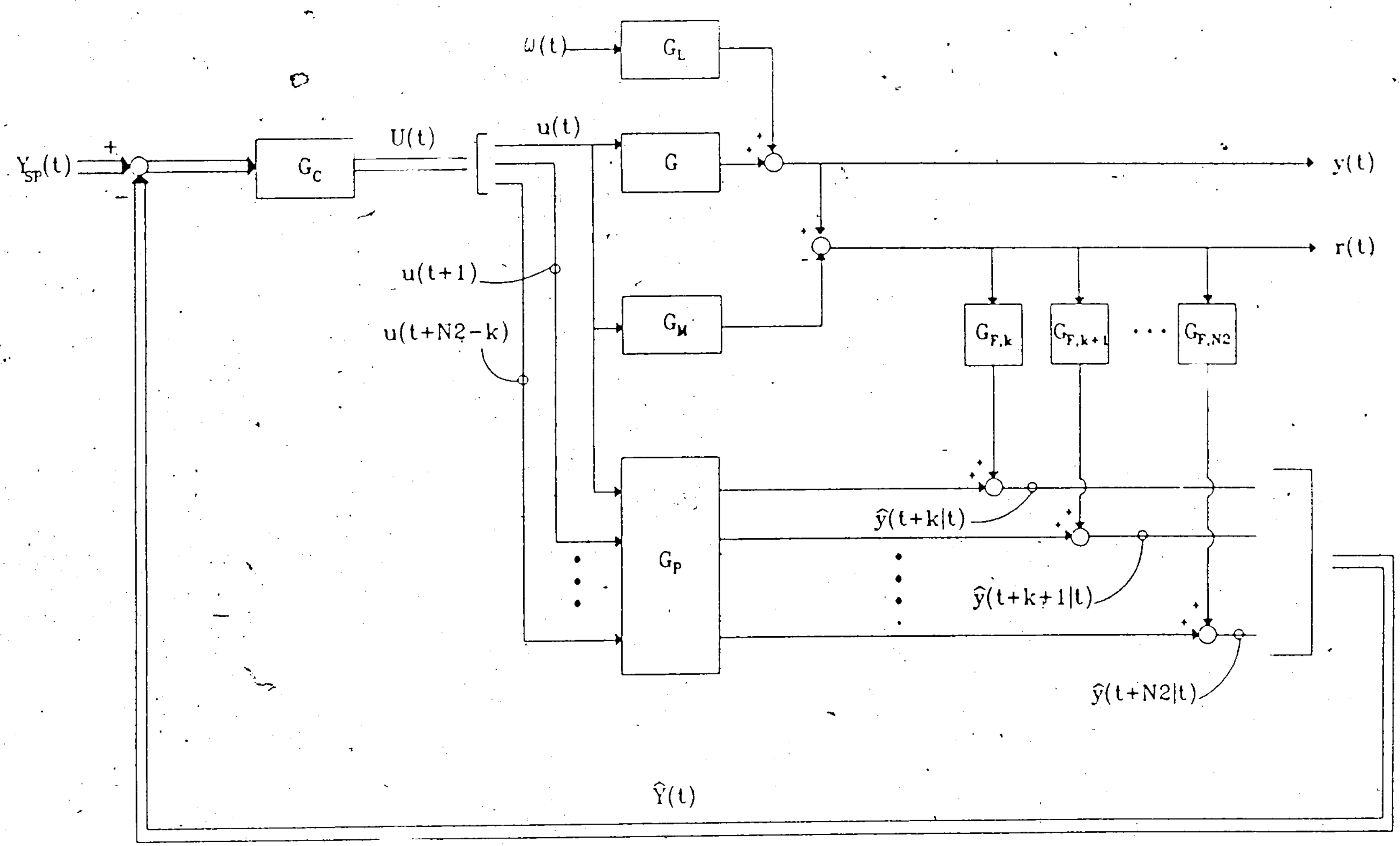

Figure 4.1 Generalized block diagram for Long Range Predictive Control

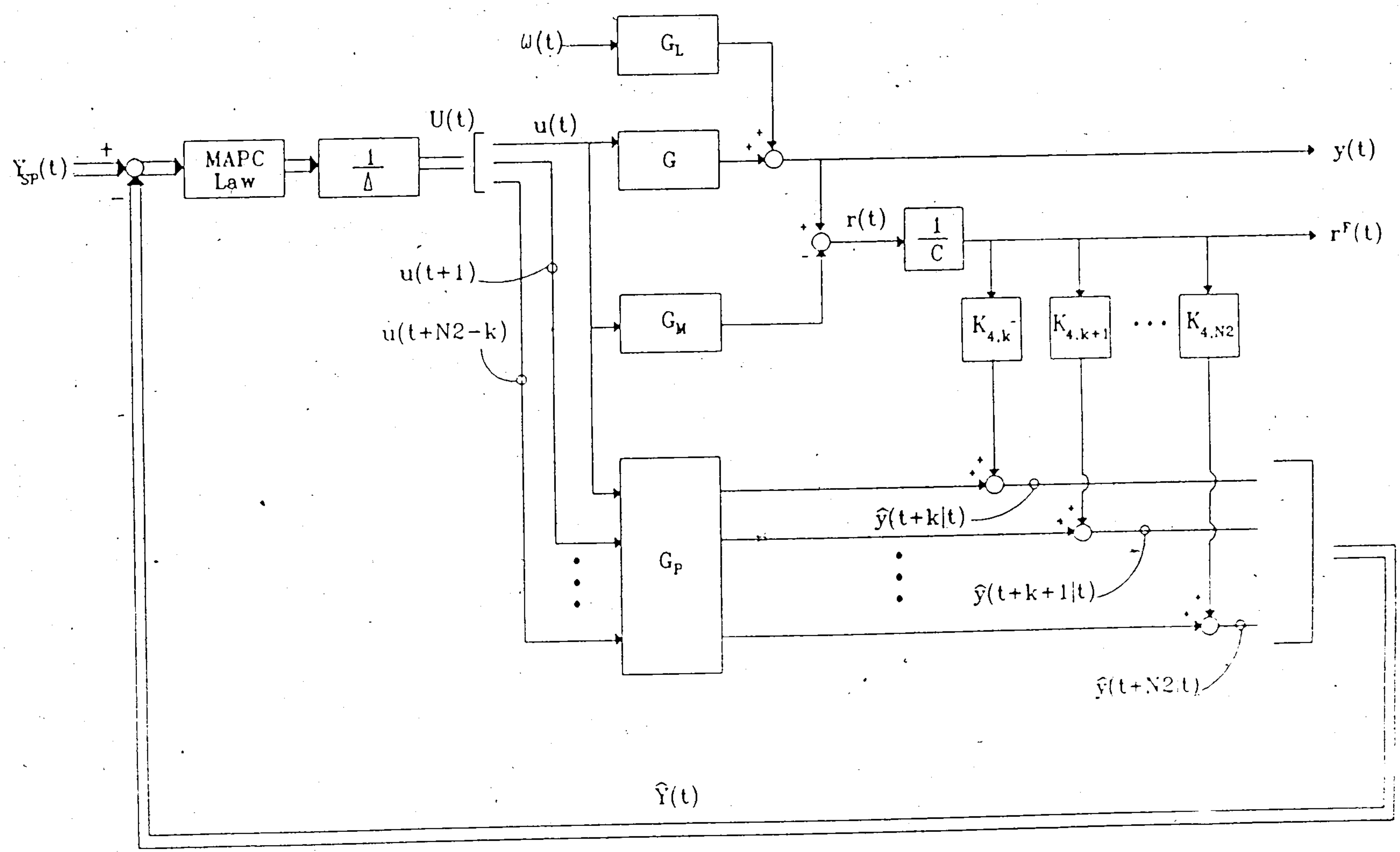

Figure 4.2 Predictor structure of Multistep Adaptive Predictive Control

describes a method of disturbance modelling called "Single Series Forecasting (SSF)" in which the residual term is prefiltered, typically using a first-order low pass filter. The filtered residual, $r^F(t)$, is then forecast j steps into the future using an autoregressive model which is identified online by a recursive parameter estimation technique such as Improved Least Squares (Sripada and Fisher, 1987), which was described in Chapter 3. The structure of the autoregressive forecaster was generally chosen in an ad hoc manner. From Fig. 4.2, it is apparent that the optimal choices for the residual filter and forecaster are $1/C(z^{-1})$ and $K_{4,j}(z^{-1})$, respectively.

It should also be noted that the ratio of the noise covariances, R_1/R_2 may be tuned to alter the roots of $C(z^{-1})$. A decrease in this ratio will cause the roots of $C(z^{-1})$ to move toward the unit circle, which results in a more conservative disturbance prediction scheme. Hence, the MAPC scheme is a "two-degrees-of-freedom" controller as it allows the servo and regulatory performances to be specified independently for the limiting case of no MPM.

The second case of interest involves MPM with no disturbance term. Here again, $C(z^{-1})$ can be chosen via R_1/R_2 to control the MPM contribution to the predicted outputs. The Smith Predictor lacks this extra degree of freedom (*i.e.* $G_{F,j}(z^{-1}) = 1$) and is known to perform very poorly in the presence of MPM.

4.2.1 Long-Range Predictive Control Strategy

The state space control law to be derived subsequently in Chapter 5 (*i.e.* Eqn. (5.51)) is the one actually implemented in practice. However, for the purposes of analysis an equivalent ARMA domain expression can be obtained that minimizes (4.1) by use of the block diagram of Fig. 4.1. Equation (4.22) implies that

$$\hat{y}(t+j\,|\,t) = G_P(z^{-1})u(t+j-k) + G_{F,j}(z^{-1})r(t) \qquad \forall j \in [k, N_2]$$

or

$$\begin{aligned}\hat{y}(t+j\,|\,t) &= \frac{B(z^{-1})}{A(z^{-1})\Delta}\Delta u(t+j-k) + G_{F,j}(z^{-1})r(t)\\ &= g_1\Delta u(t+j-k) + g_2\Delta u(t+j-k-1) + \ldots\\ &\quad + g_{j-k+1}\Delta u(t) + g_{j-k+2}\Delta u(t-1) + \ldots\\ &\quad + G_{F,j}(z^{-1})r(t)\end{aligned}$$

where the g_i are the step response coefficients of the open loop plant. It is clear that this expression is composed of terms dependent upon future controls and on present and past controls and residuals. Grouping the latter terms together leads to

$$\hat{y}(t+j\,|\,t) = g_1\Delta u(t+j-k) + \ldots + g_{j-k+1}\Delta u(t) + y^{*}(t+j\,|\,t) \tag{4.26}$$

where

$$y^{*}(t+j\,|\,t) = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots + \frac{K_{4,j}(z^{-1})}{C(z^{-1})}r(t) \tag{4.27}$$

from which it is apparent that $y^*(t+j|t)$ represents the output j steps into the future that would result if no further changes were made to the manipulated variable. Writing Eqn. (4.26) for the entire trajectory leads to

$$\hat{Y}(t) = Y^*(t) + A\Delta U(t) \tag{4.28}$$

where $\hat{Y}(t) = \{\hat{y}(t+j|t),\ \ j\in[k,N_2]\}$, $Y^*(t) = \{y^*(t+j|t),\ \ j\in[k,N_2]\}$, $\Delta U(t) = \{\Delta u(t+j-1),\ \ j\in[1,N_u]\}$ and

$$A = \begin{bmatrix} g_1 & 0 & \dots & 0 \\ g_2 & g_1 & \dots & 0 \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ g_{N_u} & g_{N_u-1} & \dots & g_1 \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ g_{N_y} & g_{N_y-1} & \dots & g_{N_y-N_u+1} \end{bmatrix} \tag{4.29}$$

with $N_y = N_2 - k + 1$ and $\Delta u(t+j) = 0,\ \ j \geq N_u$. Expressing (4.1) in vector/matrix form yields

$$J = [Y_{sp}(t) - \hat{Y}(t)]^T[Y_{sp}(t) - \hat{Y}(t)] + (\Delta U(t))^T \lambda I \Delta U(t) \tag{4.30}$$

where $Y_{SP}(t) = \{y_{SP}(t+j),\ \ j\in[k,N_2]\}$. Differentiating (4.30) with respect to $\Delta U(t)$ and setting $\partial J/\partial\Delta U(t) = 0$, results in the optimal control policy

$$\Delta U(t) = A^*\{Y_{sp}(t) - Y^*(t)\} \tag{4.31}$$

where

$$A^* = (A^T A + \lambda I)^{-1} A^T \tag{4.32}$$

The control action is implemented in a receding horizon fashion, hence only the first row of $\mathbf{L}$ has to be calculated at each control interval.

4.3 Generalized Predictive Control

Generalized Predictive Control (Clarke et al 1987a, 1987b) is an ARMA domain predictive controller which minimizes the cost functional of Eqn. (4.1):

$$J = \sum_{j=N_1}^{N_2} \{y_{SP}(t+j) - \hat{y}(t+j\,|\,t)\}^2 + \sum_{j=1}^{N_u} \lambda \{\Delta u(t+j-1)\}^2 \tag{4.1}$$

where $\hat{y}(t+j\,|\,t) = E\{y(t+j)\,|\,t\}$. (Note that the cost index given in the 1987 papers by Clarke et al was

$$J = E\left\{ \sum_{j=N_1}^{N_2} (y_{SP}(t+j) - y(t+j))^2 + \sum_{j=1}^{N_u} \lambda (\Delta u(t+j-1))^2 \right\}$$

Using arguments similar to those presented by MacGregor (1977) in a discussion of the Clarke and Gawthrop (1975) GMV paper, it is evident that the control policy derived by the authors actually minimizes the related cost functional (4.1).) The trajectory of future outputs is formed using the ARIMA process model given by

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t-k) + \frac{C(z^{-1})}{A(z^{-1})} \frac{\omega(t)}{\Delta} \tag{4.5}$$

The development which follows assumes that the plant is known and time-invariant; in addition, N_1 in Eqn. (4.1) will be set equal to k to avoid penalizing outputs which cannot be affected by $\Delta u(t)$.

Multiplication of (4.5) by $E_j(z^{-1})\Delta$, where $\delta E_j = j-1$ yields, upon rearrangement,

$$C(z^{-1})y(t+j) = F_j(z^{-1})y(t) + G_j(z^{-1})\Delta u(t+j-k) + C_j(z^{-1})E(z^{-1})\omega(t+j) \qquad (4.33)$$

where $E_j(z^{-1})$ and $F_j(z^{-1})$ are obtained from the Diophantine identity

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = E_j(z^{-1}) + z^{-j}\frac{F_j(z^{-1})}{A(z^{-1})\Delta} \qquad (4.34)$$

and $G_j(z^{-1}) = E_j(z^{-1})B(z^{-1})$. (Note that it is possible to write a recursive formula for calculating E_{j+1} and F_{j+1} given E_j and F_j, which speeds online computation of these polynomials considerably (see Clarke et al, 1987a).) The optimal or least squares j-step-ahead prediction of $y(t)$ is given by

$$\hat{y}(t+j|t) = \frac{F_j(z^{-1})}{C(z^{-1})}y(t) + \frac{G_j(z^{-1})}{C(z^{-1})}\Delta u(t+j-k)$$

$$= \frac{F_j(z^{-1})}{C(z^{-1})}y(t) + \frac{B(z^{-1})}{C(z^{-1})}\left[\frac{C(z^{-1})}{A(z^{-1})} - z^{-j}\frac{F_j(z^{-1})}{A(z^{-1})}\right]u(t+j-k)$$

$$= \frac{B(z^{-1})}{A(z^{-1})}u(t+j-k) + \frac{F_j(z^{-1})}{C(z^{-1})}\left[y(t) - z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right]$$

or

$$\hat{y}(t+j|t) = G_P(z^{-1})u(t+j-k) + G_{F,j}(z^{-1})[y(t) - G_M(z^{-1})u(t)] \qquad (4.22)$$

where

$$G_{F,j}(z^{-1}) = \frac{F_j(z^{-1})}{C(z^{-1})} \qquad (4.35)$$

Hence, the GPC predictor structure can also be represented by Fig. 4.1, and (using (4.35)) by the block diagram of Fig. 4.3. Now $G_{F,j}(1) = 1$ or equivalently $F_j(1) = C(1)$ is a necessary condition for zero steady state prediction error in the presence of nonstationary disturbances (see Figs. 4.1, 4.3). It is evident from Eqn. (4.34) that this condition is indeed satisfied in the GPC scheme.

Equation (4.22) indicates that the interpretations given earlier for the KF predictor structure (*e.g.* optimal Smith Predictor, optimal single series forecaster, etc.) apply equally to GPC. It appears from Fig. 4.3 that GPC lacks the extra degree of freedom necessary to design servo and regulatory response independently. This is in fact the case when the polynomial $C(z^{-1})$ is estimated online, *e.g.* by Extended Least Squares (see Chapter 3). However, this is seldom done in practice since the parameters of $C(z^{-1})$ are known to be slow to converge. Instead, an ad hoc observer polynomial $T(z^{-1})$ (see Clarke et al, 1987b) is specified, and is typically

$$T(z^{-1}) = 1 - 0.8z^{-1}$$

so that $1/T(z^{-1})$ may serve as a low-pass residual filter (see Fig. 4.3). This filter enables the user to specify setpoint following and disturbance rejection characteristics separately. For example, disturbance forecasting may be made more cautious by moving the root of $T(z^{-1})$ toward the

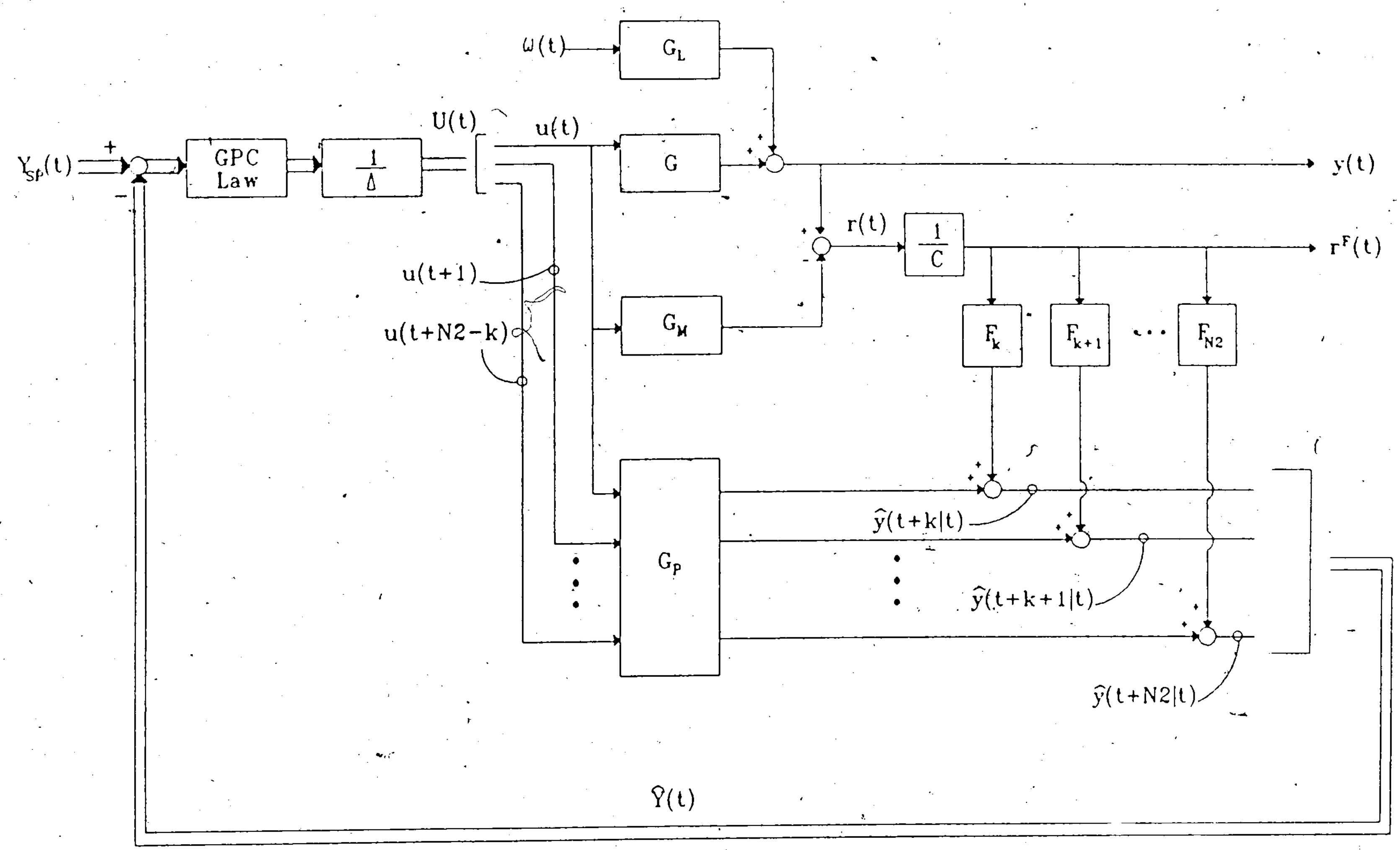

Figure 4.3 Predictor structure of Generalized Predictive Control

unit circle; this is analogous to decreasing the ratio R_1/R_2 in the MAPC approach. However, if the true plant is given by Eqn. (4.5), then this extra design feature is achieved at the cost of suboptimal prediction of $y(t+j)$.

4.3.1 Long-Range Predictive Control Strategy

As in Section 4.2.1, it may be written using Fig. 4.1 (or Eqn. (4.22)),

$$\hat{y}(t+j\,|\,t) = G_P(z^{-1})u(t+j-k) + G_{F,j}(z^{-1})r(t)$$
$$\forall\, j \in [k, N_2]$$

or

$$\hat{y}(t+j\,|\,t) = \frac{B(z^{-1})}{A(z^{-1})\Delta}\Delta u(t+j-k) + G_{F,j}(z^{-1})r(t)$$
$$= g_1\Delta u(t+j-k) + \ldots + g_{j-k+1}\Delta u(t) + y^*(t+j\,|\,t) \qquad (4.26)$$

where

$$y^*(t+j\,|\,t) = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots + \frac{F_j(z^{-1})}{C(z^{-1})}r(t) \qquad (4.36)$$

Equation (4.26) may once again be cast in the vector/matrix form of Eqn (4.28), *i.e.*

$$\hat{Y}(t) = Y^*(t) + A\Delta U(t) \qquad (4.28)$$

Minimization of (4.1) with respect to $\Delta U(t)$ leads to the optimal input trajectory

$$\Delta U(t) = A^*\{Y_{sp}(t) - Y^*(t)\} \qquad (4.31)$$

where $\Delta U(t)$, A^* and $Y_{SP}(t)$ are as defined for the MAP controller above.

4.3.2 GPC versus MAPC

The optimal GPC law

$$\Delta U(t) = \Lambda^{*}\{Y_{sp}(t) - Y^{*}(t)\} \tag{4.31}$$

was derived in the previous section and is seen to be identical in form to that derived using innovations analysis for the MAPC algorithm (Section 4.2.3). Since Λ^{*} and $Y_{SP}(t)$ are independent of the type of observer used, the two schemes can be shown to provide equal control action in the steady state if it can be demonstrated that $(Y^{*}(t))_{GPC} = (Y^{*}(t))_{MAPC}$. The $Y^{*}(t)$ vector in either case is defined as $\{y^{*}(t+j\,|\,t),\ j \in [k, N_2]\}$, where

$$y^{*}(t+j\,|\,t) = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots + \frac{F_j(z^{-1})}{C(z^{-1})} r(t) \tag{4.36}$$

$$= g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots + \frac{K_{4,j}(z^{-1})}{C(z^{-1})} r(t) \tag{4.27}$$

So it is apparent that the elements of the $Y^{*}(t)$ vector will be equal if $F_j(z^{-1}) = K_{4,j}(z^{-1}),\ \forall j \in [k, N_2]$. To see that this is indeed the case, consider Eqns. (4.34) and (4.25):

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = E_j(z^{-1}) + z^{-j}\frac{F_j(z^{-1})}{A(z^{-1})\Delta}$$

$$= K_{3,j}(z^{-1}) + z^{-j}\frac{K_{4,j}(z^{-1})}{A(z^{-1})\Delta}$$

But $\delta E_j(z^{-1}) = \delta K_{3,j}(z^{-1}) = j-1$, so $F_j(z^{-1}) = K_{4,j}(z^{-1})$ and the overall GPC and MAPC schemes will provide asymptotically equal control of plants with known, time-invariant dynamics.

4.4 Multivariable Optimal Constrained Control Algorithm

The Multivariable Optimal Constrained Control Algorithm also minimizes a performance index of the form of Eqn. (4.1):

$$J = \sum_{j=N_1}^{N_2} \{y_{SP}(t+j) - \hat{y}(t+j|t)\}^2 + \sum_{j=1}^{N_u} \lambda\{\Delta u(t+j-1)\}^2 \tag{4.1}$$

where N_1 will be set equal to k, as the time delay is known by virtue of the perfect plant model assumption. The trajectory of future outputs is affected by both measured inputs and unmeasured inputs or disturbances. There are several different techniques (*i.e.* observers) available for incorporating the effect of disturbances into the prediction of future outputs. In this connection, MOCCA has been formulated using both an ARMA domain disturbance forecaster (Sripada and Fisher, 1985) and a Kalman filter (Li et al, 1988; Navratil et al, 1988). The former method will be described first.

4.4.1 MOCCA with a Polynomial Disturbance Generator

The derivation begins by considering the ARIMA process model of Eqn. (4.5):

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})}u(t-k) + \frac{C(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \tag{4.5}$$

where $y(\cdot)$, $u(\cdot)$ and $\omega(\cdot)$ are the process output, input and unmeasured random disturbance, respectively. As mentioned above, the future outputs of the plant will depend upon both measured (*e.g.* $u(t)$) and unmeasured (*e.g.* $\omega(t)$) process inputs. Consider first the deterministic part of (4.5):

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t-k) \tag{4.37}$$

Expansion of the transfer function $B(z^{-1})/A(z^{-1})$ yields the impulse response coefficients of the plant. Note that one definition of (bounded input, bounded output) stability of a linear system (Ludeman, 1986) is the requirement that the impulse response of the plant is bounded; that is,

$$\sum_{i=1}^{\infty} |h_i| < \infty$$

This implies that Eqn. (4.37) may be written (for stable systems) in the form

$$y(t) = \sum_{i=1}^{N} h_i u(t-i-d) \tag{4.38}$$

where d=k-1, *i.e.* the physical time delay of the system. The h_i are the impulse response coefficients and $h_i = 0, \ \forall i > N$, where N is the settling time of the plant. Equation (4.37) may be written in terms of control increments $\Delta u(t)$ (or equivalently step changes in $u(t)$) as

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})\Delta} \Delta u(t-k) \tag{4.39}$$

Clearly, the transfer function $B(z^{-1})/A(z^{-1})/\Delta$ now represents the step response or process reaction curve of the plant. Equation (4.39) can also be written as the discrete convolution sum

$$y(t) = \sum_{i=1}^{N} g_i \Delta u(t-i-d) + \sum_{i=N+1}^{\infty} g_i \Delta u(t-i-d) \tag{4.40}$$

where the g_i are now the step response coefficients of the plant. It is obvious from a comparison of Eqns. (4.37) and (4.39) that $g_1 = h_1$ and $\Delta g_i = h_i$, $\forall i > 1$. Hence, $h_i = 0$, $\forall i > N$ implies $g_i = g_{ss}$, $\forall i > N$. This means that (4.40) can be written as

$$y(t) = \sum_{i=1}^{N} g_i \Delta u(t-i-d) + g_{SS} u(t-d-N-1) + g_{SS} u(t-\infty)$$

$$y(t) = \sum_{i=1}^{N} g_i \Delta u(t-i-d) + g_{SS} u(t-d-N-1) \tag{4.41}$$

if it can be assumed that the process input was at its steady state value at $t = -\infty$. In order to develop a trajectory of future outputs, the time arguments in (4.41) can be shifted forward by an arbitrary number of samples (j) giving

$$\hat{y}(t+j|t) = \sum_{i=1}^{N} g_i \Delta u(+j-i-d) + g_{SS} u(t+j-d-N-1)$$

or

$$\hat{y}(t+j|t) = g_1 \Delta u(t+j-k) + \ldots + g_{j-k+1} \Delta u(t) + y^{*}(t+j|t) \tag{4.26}$$

where

$$y^{*}(t+j|t) = g_{j-k+2} \Delta u(t-1) + g_{j-k+3} \Delta u(t-2) + \ldots$$
$$+ g_N \Delta u(t+j-N-k+1) + g_{SS} u(t+j-N-k) \tag{4.42}$$

At this point, it becomes necessary to consider the design of the observer mechanism to account for the presence of the disturbance mode in Eqn. (4.5). Sripada and Fisher (1985) recommend that disturbance prediction be achieved using a polynomial "disturbance generator" (PDG) of the form

$$r(t) = \frac{C(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \tag{4.43}$$

where $A(z^{-1})$, $C(z^{-1})$ and $\omega(t)$ are as defined in Eqn. (4.5) and $r(t)$ is as illustrated in Fig. 4.1, *i.e.*

$$r(t) = y(t) - \frac{B(z^{-1})}{A(z^{-1})}u(t)$$

Since $\omega(.)$ is unmeasurable, it is difficult if not impossible to obtain step response data relating $r(t)$ to $\omega(t)$. It is therefore necessary to obtain $C(z^{-1})$ and $A(z^{-1})$ by a least squares fit of $\Delta r(t)$ and $\omega(t)$, *e.g.* using some variant of Recursive Least Squares (RLS) as described in Appendix A. Recalling that both $A(z^{-1})$ and $C(z^{-1})$ are monic (*i.e.* $A(0) = C(0) = 1$), (4.43) may be written as

$$A(z^{-1})\Delta r(t) = (C(z^{-1}) - 1)\omega(t) + \omega(t) \tag{4.44}$$

The first term on the RHS represents past noise and must be "proxied" by

$$\hat{\omega}(t-1) = A(z^{-1})\Delta r(t-1) - (C(z^{-1}) - 1)\hat{\omega}(t-1)$$

The second term on the RHS of (4.44) will be uncorrelated with the regressor as $\omega(t)$ is a zero-mean uncorrelated noise sequence.

Assuming that perfect estimates of $A(z^{-1})$ and $C(z^{-1})$ have been obtained by whatever means necessary, Eqn. (4.43) may be rewritten as

$$r(t+j) = E_j(z^{-1})\omega(t+j) + \frac{F_j(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta}$$

where $E_j(z^{-1})$ is of order j-1 and is obtained by solution of the Diophantine identity

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = E_j(z^{-1}) + z^{-j}\frac{F_j(z^{-1})}{A(z^{-1})\Delta} \qquad (4.34)$$

Note that E_j and F_j can be calculated efficiently using the Diophantine recursion of Clarke et al (1987a). It is evident that an optimal or least squares prediction is achieved by setting the future noise $E_j(z^{-1})\omega(t+j)$ to its expected value of zero, leading to the expression

$$\hat{r}(t+j\,|\,t) = \frac{F_j(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta}$$

which becomes, using (4.43),

$$\hat{r}(t+j\,|\,t) = \frac{F_j(z^{-1})}{C(z^{-1})}r(t)$$

$$= G_{F,j}(z^{-1})r(t) \qquad (4.45)$$

where $G_{F,j}(z^{-1})$ is as defined in (4.35), *i.e.*

$$G_{F,j}(z^{-1}) = \frac{F_j(z^{-1})}{C(z^{-1})} \qquad (4.35)$$

Having obtained the j-step-ahead disturbance forecast in this manner, it is simply added to $y^*(t+j\,|\,t)$ in Eqn. (4.42); that is,

$$y^*(t+j\,|\,t) = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots$$
$$+ g_N\Delta u(t+j-N-k+1) + g_{SS}u(t+j-N-k)$$
$$+ \frac{F_j(z^{-1})}{C(z^{-1})}r(t) \qquad (4.46)$$

It is clear that MOCCA with a polynomial disturbance generator can also be represented by Figs. 4.1 and 4.4, with the attendant interpretations (optimal Smith Predictor, optimal SSF, etc.). The block diagram of Fig. 4.4 is seen to be identical to that of Fig. 4.3 presented above for GPC. It is natural, then, to ask whether this form of MOCCA is equal to GPC - this is the subject of a later section.

4.4.2 MOCCA with a Kalman Filter Predictor

In an effort to simplify the disturbance forecasting scheme discussed in the previous section, Li et al (1988) and Navratil et al (1988) have presented a r v observer scheme which uses a Kalman Filter Predictor (KFP) to forecast residual effects. This method obviates the need for least squares identification of the load dynamics and subsequent recursion of a Diophantine equation as described above.

The new approach was facilitated by Li et al (1988) when the authors noted that Eqn. (4.42) lends itself naturally to a (recursive) state space form:

$$x(t) = \Phi x(t-1) + \Lambda u(t-1) \tag{4.47}$$

$$y(t) = H x(t) \tag{4.48}$$

where

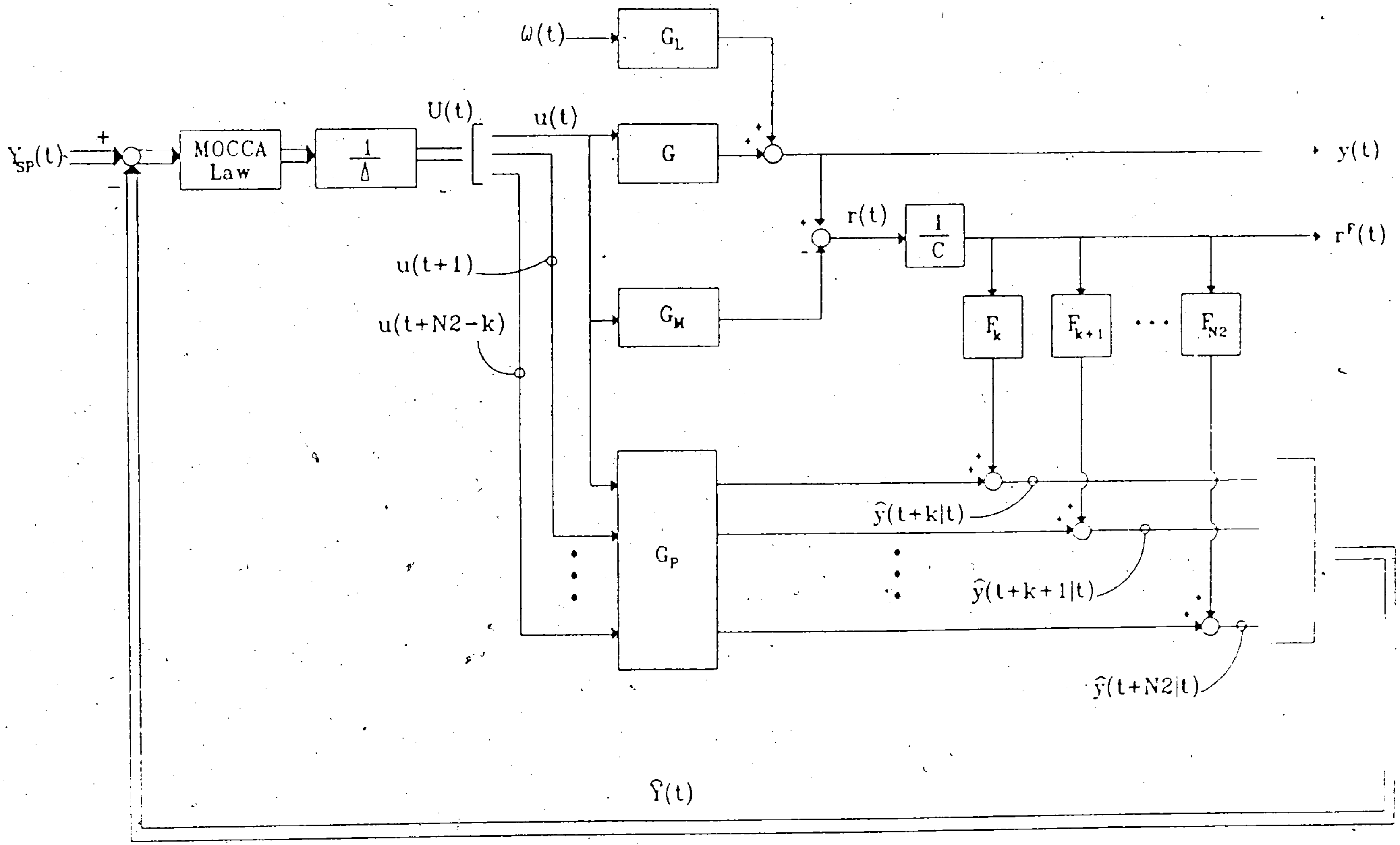

Figure 4.4 Predictor structure of MOCCA with ARMA domain disturbance forecasting

$$x(t) = [y^*(t|t), y^*(t+1|t), \dots, y^*(t+d|t),$$
$$y^*(t+d+1|t), \dots, y^*(t+d+N|t)]^T_{1\times(N+k)}$$

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}_{(N+k)\times(N+k)}$$

$$\Lambda = [0, 0, \dots, g_1, g_2, \dots, g_N, g_{SS}]$$

$$H = [1, 0, \dots, 0]_{1\times(N+k)} \qquad (4.49)$$

It is evident that the first d states are due to the physical time delay of the plant. It is important to note that by convention, z^{-1} is assumed to operate only on the second argument of each element of the state vector, *i.e.*

$$x(t-1) = [y^*(t|t-1), y^*(t+1|t-1), \dots, y^*(t+d|t-1),$$
$$y^*(t+d+1|t-1), \dots, y^*(t+d+N|t-1)]^T_{1\times(N+k)}$$

To see how Eqns. (4.47), (4.48) are developed, assume that a change in the input variable $\Delta u(\cdot)$ is made at time t-1. It is of interest to estimate the new output trajectory, *i.e.* to add the effect of $\Delta u(t-1)$ to the known prediction. From Eqn. (4.41) it is apparent that

$$y^*(t|t) = y^*(t+1|t-1)$$

$$\vdots$$

$$y^*(t+d-1|t) = y^*(t+d|t-1)$$

$$y^*(t+d|t) = y^*(t+d+1|t-1) + g_1\Delta u(t-1)$$

$$y^*(t+d+1|t) = y^*(t+d+2|t-1) + g_2\Delta u(t-1)$$

$$\vdots$$

$$y^*(t+d+N-1|t) = y^*(t+d+N|t-1) + g_N\Delta u(t-1)$$

$$y^*(t+d+N|t) = y^*(t+d+N+1|t-1) + g_{SS}\Delta u(t-1) \tag{4.50}$$

But Eqn. (4.41) implies that

$$y^*(t+d+N|t-1) = y^*(t+d+N+1|t-1)$$

i.e. the output settles to a new steady state value in N steps. With this substitution Eqn. (4.50) becomes

$$y^*(t+d+N|t) = y^*(t+d+N|t-1) + g_{SS}\Delta u(t-1) \tag{4.51}$$

and the state space (4.47), (4.48) is established.

4.4.2.1 Reduced Order Model

Recall from Eqn. (4.1) that it is necessary to predict only N_2 steps into the future, where $N_2 \leq N$. In an effort to reduce the computational load associated with Eqns. (4.47), (4.48), a reduced order state space model was also proposed by Li et al (1988). The derivation begins as before by writing

$$y^{*}(t|t) = y^{*}(t+1|t-1)$$

$$y^{*}(t+d-1|t) = y^{*}(t+d|t-1)$$
$$y^{*}(t+d|t) = y^{*}(t+d+1|t-1) + g_{1}\Delta u(t-1)$$

$$y^{*}(t+N_{2}-1|t) = y^{*}(t+N_{2}|t-1) + g_{N_{2}-d}\Delta u(t-1)$$
$$y^{*}(t+N_{2}|t) = y^{*}(t+N_{2}+1|t-1) + g_{N_{2}-d+1}\Delta u(t-1) \tag{4.52}$$

But it is again apparent from (4.41) that

$$\begin{aligned} y^{*}(t+N_{2}|t-1) = {} & g_{N_{2}-d+1}\Delta u(t-2) + g_{N_{2}-d+2}\Delta u(t-3) + \dots \\ & + g_{N}\Delta u(t-N-d-1+N_{2}) \\ & + g_{ss}u(t-N-d-2+N_{2}) \end{aligned} \tag{4.53}$$

and

$$\begin{aligned} y^{*}(t+N_{2}+1|t-1) = {} & g_{N_{2}-d+2}\Delta u(t-2) + g_{N_{2}-d+3}\Delta u(t-3) + \dots \\ & + g_{N}\Delta u(t-N-d+N_{2}) \\ & + g_{ss}u(t-N-d-1+N_{2}) \end{aligned} \tag{4.54}$$

Subtracting (4.53) from (4.54) and collecting like terms leads to

$$\begin{aligned} y^{*}(t+N_{2}+1|t-1) = {} & y^{*}(t+N_{2}|t-1) \\ & + (g_{N_{2}-d+2} - g_{N_{2}-d+1})\Delta u(t-2) \\ & + (g_{N_{2}-d+3} - g_{N_{2}-d+2})\Delta u(t-3) + \dots \\ & + (g_{ss} - g_{N})\Delta u(t-N-d-1+N_{2}) \\ & + g_{ss}u(t-N-d-2+N_{2}) \\ & - g_{ss}u(t-N-d-2+N_{2}) \end{aligned}$$

As noted previously, the difference in step response coefficients can be replaced by impulse response coefficients to give

$$y^{*}(t+N_2+1\,|\,t-1)=y^{*}(t+N_2\,|\,t-1)+h_{N_2-d+2}\Delta u(t-2)$$
$$+h_{N_2-d+3}\Delta u(t-3)$$
$$+\ldots+h_{SS}\Delta u(t-N-d-1+N_2) \tag{4.55}$$

Substituting (4.55) for $y^{*}(t+N_2+1\,|\,t-1)$ in Eqn. (4.52) leads to

$$x(t)=\Phi x(t-1)+\Lambda\Delta U(t-1) \tag{4.56}$$
$$y(t)=Hx(t) \tag{4.57}$$

where

$$x(t)=[y^{*}(t\,|\,t),\ldots,y^{*}(t+d\,|\,t),y^{*}(t+d+1\,|\,t),$$
$$\ldots,y^{*}(t+N_2\,|\,t)]^{T}_{1\times(N_2+1)}$$

$$\Phi=\begin{bmatrix}0&1&0&\ldots&0&0\\0&0&1&\ldots&0&0\\ \cdot&\cdot&\cdot&&\cdot&\cdot\\ \cdot&\cdot&\cdot&&\cdot&\cdot\\ \cdot&\cdot&\cdot&&\cdot&\cdot\\0&0&0&\ldots&0&1\\0&0&0&\ldots&0&1\end{bmatrix}_{(N_2+1)\times(N_2+1)}$$

$$\Lambda = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \dots & 0 & g_1 \\ 0 & 0 & \dots & 0 & g_2 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \dots & 0 & g_{N_2-d} \\ h_{SS} & h_N & \dots & h_{N_2-d+2} & g_{N_2-d+1} \end{bmatrix}_{(N_2+1)\times(N+k-N_2)}$$

$$\Delta U(t-1) = [\Delta u(t-N-d-1+N_2), \dots, \Delta u(t-1)]^T_{(N+k-N_2)\times 1} \quad (4.58)$$

Note that the input term in (4.56) becomes a matrix/vector product rather than a vector/scalar product (*cf.* (4.47)) and that the model order has been reduced to N_2+1.

In order to account for the effects of unmeasured stochastic disturbances on the future outputs, (4.56), (4.57) may be modified to include process ($n_1(\cdot)$) and measurement ($n_2(\cdot)$) noises as follows:

$$x(t) = \Phi x(t-1) + \Lambda \Delta U(t-1) + \Gamma n_1(t-1) \quad (4.59)$$

$$y(t) = H x(t) + n_2(t) \quad (4.60)$$

$n_1(\cdot)$ and $n_2(\cdot)$ are zero-mean uncorrelated noise sequences having covariances R_1 and R_2 respectively. It has been assumed for the sake of simplicity that

$$\Gamma = [0, \dots, 1, \dots, 1]^T_{(N_2+1)\times 1}$$

but Γ could be replaced by disturbance mode impulse response coefficients if $n_1(t)$ was found to be measurable (*cf.* feed-forward control). Note also that no process noise is added to the delay states.

A minimum variance estimate of the state vector may be obtained using the two-step Kalman filter described in Appendix A. Since MOCCA is to be applied to time-invariant plants only, the Matrix Ricatti Equation (Eqns. (A.33) and (A.36)) may be iterated upon until a final solution is reached, yielding a steady-state Kalman gain vector. Hence the state updates may be written as

a) A Posteriori State Update

$$\hat{x}(t) = \bar{x}(t) + L\omega(t) \tag{A.35}$$

b) A Priori State Update

$$\bar{x}(t+1) = \Phi\hat{x}(t) + \Lambda\Delta U(t) \tag{A.30}$$

where by convention, $\hat{x}(t) = \hat{x}(t|t)$ and $\bar{x}(t) = \hat{x}(t|t-1)$. The innovations sequence $\omega(t)$ is defined as

$$\begin{aligned}\omega(t) &= y(t) - \hat{y}(t|t-1) \\ &= y(t) - H\bar{x}(t)\end{aligned} \tag{4.61}$$

which will be a zero-mean random sequence when the Kalman filter has converged. Using this approach to obtain $\hat{x}(t)$, one is in fact obtaining optimal or minimum variance estimates of $y^*(t+j|t)$, $j \in [k, N_2]$ which may be added to the effects of future inputs to form the trajectory of predicted outputs $\hat{y}(t+j|t)$, $j \in [k, N_2]$ (see Eqn. (4.26)).

4.4.2.2 Derivation of an Equivalent Polynomial Observer

In practice, the $y^*(t+j|t)$ are indeed computed using (A.35) and (A.30). However, for purposes of comparing MOCCA with a Kalman Filter Predictor versus the LRPC schemes mentioned earlier, it is advantageous to bring these equations into the ARMA domain where they can be represented in the form of Fig. 4.1.

Using successive forward substitution for states 1 to d,

$$\bar{x}_1(t) = \hat{x}_2(t-1)$$

$$\hat{x}_2(t) = \hat{x}_3(t-1) + L_2\omega(t)$$

$$\bar{x}_1(t) = \hat{x}_3(t-2) + L_2\omega(t-1)$$

$$\bar{x}_1(t) = \hat{x}_{d+1}(t-d) + (L_2 z^{-1} + \ldots + L_d z^{-d+1})\omega(t) \tag{4.62}$$

For states $d+1$ to N_2,

$$\hat{x}_{d+1}(t) = \hat{x}_{d+2}(t-1) + g_1\Delta u(t-1) + L_{d+1}\omega(t)$$

$$\hat{x}_{d+2}(t) = \hat{x}_{d+3}(t-1) + g_2\Delta u(t-1) + L_{d+2}\omega(t)$$

$$\hat{x}_{d+1}(t) = \hat{x}_{d+3}(t-2) + g_1\Delta u(t-1) + g_2\Delta u(t-2) + (L_{d+1} + L_{d+2}z^{-1})\omega(t)$$

$$\hat{x}_{d+1}(t) = \hat{x}_{N_2+1}(t-N_2+d) + g_1\Delta u(t-1) + \ldots + g_{N_2-d}\Delta u(t-N_2+d) + \left(L_{d+1} + \ldots + L_{N_2}z^{-N_2+d+1}\right)\omega(t) \tag{4.63}$$

For state N_2+1,

$$\hat{x}_{N_2+1}(t) = \hat{x}_{N_2+1}(t-1) + g_{N_2-d+1}\Delta u(t-1) + h_{N_2-d+2}\Delta u(t-2) + \ldots + h_{ss}\Delta u(t-N-d-1+N_2) + L_{N_2+1}\omega(t)$$

$$\hat{x}_{N_2+1}(t) = g_{N_2-d+1}u(t-1) + h_{N_2-d+2}u(t-2) + \ldots + h_{ss}u(t-N-d-1+N_2) + L_{N_2+1}\frac{\omega(t)}{\Delta} \tag{4.64}$$

Substituting (4.64) into (4.63),

$$\hat{x}_{d+1}(t) = g_1 \Delta u(t-1) + \ldots + g_{N_2-d} \Delta u(t - N_2 + d)$$
$$+ g_{N_2-d+1} u(t - N_2 + d - 1) + h_{N_2-d+2} u(t - N + d - 2)$$
$$+ \ldots + h_{SS} u(t - N - 1) + \left(L_{d+1} + \ldots + L_{N_2} z^{-N_2+d+1} \right) \omega(t)$$
$$+ L_{N_2+1} z^{-N_2+d} \frac{\omega(t)}{\Delta} \tag{4.65}$$

Continuing in this fashion, (4.65) is inserted into (4.62) to obtain

$$\overline{x}_1(t) = g_1 \Delta u(t - d - 1) + \ldots + g_{N_2-d} \Delta u(t - N_2)$$
$$+ g_{N_2-d+1} u(t - N_2 - 1) + h_{N_2-d+2} u(t - N - 2)$$
$$+ \ldots + h_{SS} u(t - N - d - 1)$$
$$+ \left(L_2 z^{-1} + \ldots + L_{N_2} z^{-N_2+1} \right) \omega(t) + L_{N_2+1} z^{-N_2} \frac{\omega(t)}{\Delta},$$

$$\overline{x}_1(t) = h_1 u(t - d - 1) + \ldots + h_{N_2-d} u(t - N_2)$$
$$+ h_{N_2-d+1} u(t - N_2 - 1) + \ldots + h_{SS} u(t - N - d - 1)$$
$$+ \left[\left(L_2 z^{-1} + \ldots + L_{N_2} z^{-N_2+1} \right) \Delta + L_{N_2+1} z^{-N_2} \right] \frac{\omega(t)}{\Delta}, \tag{4.66}$$

But, from (4.61),

$$\overline{x}_1(t) = \hat{y}(t \mid t-1)$$
$$= y(t) - \omega(t)$$

Therefore,

$$y(t) = h_1 u(t - d - 1) + \ldots + h_{N_2-d} u(t - N_2)$$
$$+ h_{N_2-d+1} u(t - N_2 - 1) + \ldots + h_{SS} u(t - N - d - 1)$$
$$+ \left[\left(1 + \ldots + L_{N_2+1} z^{-N_2+1} \right) \Delta + L_{N_2+1} z^{-N_2} \right] \frac{\omega(t)}{\Delta}$$

or, if the truncation error is sufficiently small (*i.e.* if $h_i = 0,\ i > N + 1$), then this expression becomes

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})}u(t-k) + \frac{C(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \tag{4.5}$$

where

$$C(z^{-1}) = \left[\left(1 + \ldots + L_{N_2} z^{-N_2+1}\right)\Delta + L_{N_2+1} z^{-N_2}\right] A(z^{-1}) \tag{4.67}$$

so that $C(z^{-1})$ is a monic polynomial of degree $\delta C(z^{-1}) = N_2 + n$, with $n = \delta A(z^{-1})$. Note that the $C(z^{-1})$ polynomial defined here is in general different than that derived for MAPC in Section 4.2.

Note at this point that (4.62) may be written as

$$y(t) = \hat{x}_{d+1}(t-d \mid t-d) + (1 + L_2 z^{-1} + \ldots + L_d z^{-d+1})\omega(t) \tag{4.68}$$

Now, $\hat{y}(t+j \mid t+j) = H\hat{x}(t+j \mid t+j)$ and

$$\hat{x}(t+j \mid t+j) = \Phi^j \hat{x}(t \mid t) + \sum_{i=1}^{j} \Phi^{j-i} \Lambda \Delta U(t+i-1) + \sum_{i=1}^{j} \Phi^{j-i} L\omega(t+i) \tag{4.69}$$

Further,

$$\hat{x}(t+j \mid t+j-d) = \Phi^j \hat{x}(t \mid t) + \sum_{i=1}^{j} \Phi^{j-i} \Lambda \Delta U(t+i-1) + \sum_{i=1}^{j-d} \Phi^{j-i} L\omega(t+i) \tag{4.70}$$

and

$$\hat{x}(t+j \mid t) = \Phi^j \hat{x}(t \mid t) + \sum_{i=1}^{j} \Phi^{j-i} \Lambda \Delta U(t+i-1) \tag{4.71}$$

Subtracting (4.71) from (4.70) gives

$$\hat{x}(t+j \mid t+j-d) = \hat{x}(t+j \mid t) + \sum_{i=1}^{j-d} \Phi^{j-i} L\omega(t+i) \tag{4.72}$$

or, using (4.68) and (4.72),

$$\begin{aligned}\hat{y}(t+j\mid t+j-d) &= H\hat{x}(t+j\mid t+j-d)\\ &= \hat{x}_{d+1}(t+j-d\mid t+j-d)\\ &= \hat{y}(t+j\mid t) + H\Phi^{d}L\omega(t+j-d)\\ &\quad + H\Phi^{d+1}L\omega(t+j-d-1)+\ldots\\ &= \hat{y}(t+j\mid t) + K'_{3,j}(z^{-1})\omega(t+j-d)\end{aligned}$$

where

$$K'_{3,j}(z^{-1}) = L_{d+1} + \ldots + L_j z^{-j+d+1} \tag{4.73}$$

which implies that

$$\hat{y}(t+j\mid t) = \hat{x}_{d+1}(t+j-d) - K'_{3,j}(z^{-1})\omega(t+j-d) \tag{4.74}$$

But from (4.65),

$$\begin{aligned}\hat{x}_{d+1}(t) &= \frac{B(z^{-1})}{A(z^{-1})}u(t-1)\\ &\quad + \left[\left(L_{d+1} + \ldots + L_{N_2} z^{-N_2+d+1}\right)\Delta + L_{N_2+1} z^{-N_2+d}\right]\frac{\omega(t)}{\Delta}\end{aligned}$$

Therefore,

$$\begin{aligned}\hat{y}(t+j\mid t) &= \frac{B(z^{-1})}{A(z^{-1})}u(t+j-d-1)\\ &\quad + \left[\left(L_{d+1} z^{j-d} + \ldots + L_{N_2} z^{j-N_2+1}\right)\Delta + L_{N_2+1} z^{j-N_2}\right]\frac{\omega(t)}{\Delta}\\ &\quad - (L_{d+1} z^{j-d} + \ldots + L_j z^{+1})\Delta\frac{\omega(t)}{\Delta}\\ &= \frac{B(z^{-1})}{A(z^{-1})}u(t+j-k)\\ &\quad + \left[\left(L_{j+1} + \ldots + L_{N_2} z^{j-N_2+1}\right)\Delta + L_{N_2+1} z^{j-N_2}\right]\frac{\omega(t)}{\Delta}\end{aligned}$$

or

$$\hat{y}(t+j\,|\,t) = \frac{B(z^{-1})}{A(z^{-1})}u(t+j-k)$$
$$+\left[\left(L_{j+1}+\ldots+L_{N_2}z^{j-N_2+1}\right)\Delta + L_{N_2+1}z^{j-N_2}\right]$$
$$\cdot A(z^{-1})\frac{\omega(t)}{A(z^{-1})\Delta} \tag{4.75}$$

Rewriting (4.5) in terms of $\omega(t)/A(z^{-1})/\Delta$ and substituting in (4.75),

$$\hat{y}(t+j\,|\,t) = \frac{B(z^{-1})}{A(z^{-1})}u(t+j-k)$$
$$+\frac{K_{4,j}(z^{-1})}{C(z^{-1})}\left[y(t) - z^{-k}\frac{B(z^{-1})}{A(z^{-1})}u(t)\right]$$

or

$$\hat{y}(t+j\,|\,t) = G_P(z^{-1})u(t+j-k)$$
$$+G_{F,j}(z^{-1})[y(t) - G_M(z^{-1})u(t)] \tag{4.22}$$

where

$$G_{F,j}(z^{-1}) = \frac{K_{4,j}(z^{-1})}{C(z^{-1})} \tag{4.24}$$

and

$$K_{4,j}(z^{-1}) = \left[\left(L_{j+1}+\ldots+L_{N_2}z^{j-N_2+1}\right)\Delta - L_{N_2+1}z^{j-N_2}\right]A(z^{-1})$$

$K_{4,j}(z^{-1})$ satisfies the identity

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = K_{3,j}(z^{-1}) + z^{-j}\frac{K_{4,j}(z^{-1})}{A(z^{-1})\Delta} \tag{4.25}$$

where

$$K_{3,j}(z^{-1}) = 1 + L_2 z^{-1} + \ldots + L_j z^{-j+1}$$

from which $\delta K_{3,j}(z^{-1}) = j-1$.

From Eqn. (4.22), it is seen that the trajectory of future outputs $\{\hat{y}(t+j\,|\,t),\ N_1 = k \le j \le N_2\}$ may be represented by the block diagrams of Figs. 4.1 and 4.5. Once again, MOCCA with a KFP invites the various interpretations given to the previous schemes (optimal Smith Predictor, optimal SSF, etc.). Note again that Eqn. (4.25) guarantees that $K_{4,j}(1) = C(1)$, hence the prediction scheme will exhibit asymptotically zero prediction offset in the presence of nonzero-mean disturbances. Furthermore, Eqn. (4.22) may be written in the form

$$\hat{y}(t+j\,|\,t) = g_1 \Delta u(t+j-k) + \ldots + g_{j-k+1} \Delta u(t) + y^*(t+j\,|\,t) \tag{4.26}$$

where

$$\begin{aligned} y^*(t+j\,|\,t) = {} & g_{j-k+2} \Delta u(t-1) + g_{j-k+3} \Delta u(t-2) + \ldots \\ & + g_N \Delta u(t+j-N-k+1) + g_{SS} u(t+j-N-k) \\ & + \frac{K_{4,j}(z^{-1})}{C(z^{-1})} r(t) \end{aligned} \tag{4.76}$$

4.4.3 Long-Range Predictive Control Strategy

The multistep cost functional Eqn. (4.1) is of course identical for both MOCCA predictors. Equation (4.26) implies that the vector equation

$$\hat{Y}(t) = Y^*(t) + A \Delta U(t) \tag{4.28}$$

is valid for both schemes, with $\hat{Y}(t)$, A and $\Delta U(t)$ as defined in Eqn. (4.29). Minimization of (4.1) with respect to $\Delta U(t)$ leads to the by now familiar control strategy

$$\Delta U(t) = A^* \{Y_{sp}(t) - Y^*(t)\} \tag{4.31}$$

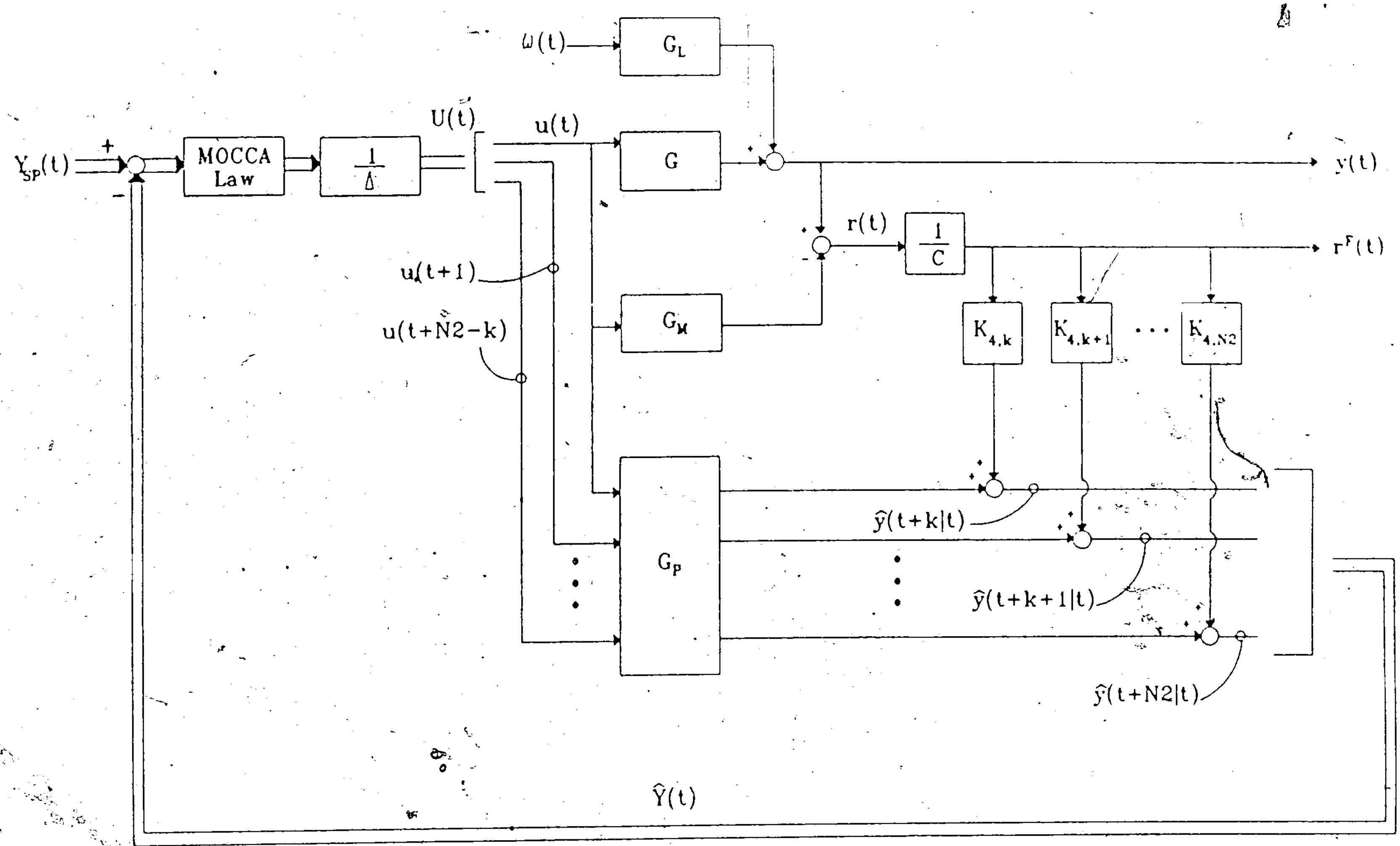

Figure 4.5 Predictor structure of MOCCA with a Kalman Filter Predictor

$Y^*(t)$ here represents the effects of past inputs and disturbances on the vector of predicted outputs $\hat{Y}(t)$, so to compare the two forms of MOCCA one must compare Eqns. (4.46) and (4.76):

$$\begin{aligned} y^*_{PDC}(t+j\,|\,t) = {} & g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots \\ & + g_N \Delta u(t+j-N-k+1) + g_{SS} u(t+j-N-k) \\ & + \frac{F_j(z^{-1})}{C(z^{-1})} r(t) \end{aligned} \qquad (4.46)$$

$$\begin{aligned} y^*_{KFP}(t+j\,|\,t) = {} & g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots \\ & + g_N \Delta u(t+j-N-k+1) + g_{SS} u(t+j-N-k) \\ & + \frac{K_{4,j}(z^{-1})}{C(z^{-1})} r(t) \end{aligned} \qquad (4.76)$$

where, for both schemes,

$$\begin{aligned} r(t) = {} & y(t) - g_1 \Delta u(t-k) - g_2 \Delta u(t-k-1) - \ldots \\ & - g_N \Delta u(t-k-N+1) - g_{SS} u(t-N-k) \end{aligned} \qquad (4.77)$$

Hence, it is apparent that $y^*_{PDC}(t+j\,|\,t) = y^*_{KFP}(t+j\,|\,t)$ if $F_j(z^{-1}) = K_{4,j}(z^{-1})$. But from (4.34) and (4.25),

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = E_j(z^{-1}) + z^{-j}\frac{F_j(z^{-1})}{A(z^{-1})\Delta}$$

$$= K_{3,j}(z^{-1}) + z^{-j}\frac{K_{4,j}(z^{-1})}{A(z^{-1})\Delta}$$

Therefore, $F_j(z^{-1}) = K_{4,j}(z^{-1})$ since $\delta E_j(z^{-1}) = \delta K_{3,j}(z^{-1}) = j-1$ implying that Eqns. (4.46) and (4.76) are equivalent in the steady state; that is, after the Kalman gains have converged to their final values. Consequently, the vector $Y^*(t)$ in Eqn. (4.28) is the same for both schemes, which in turn implies that the two forms of MOCCA, *i.e.* MOCCA with a polynomial disturbance generator (Sripada and Fisher, 1985) and MOCCA with a KFP (Li et al, 1988; Navratil et al, 1988) will provide asymptotically equal control of stable, time-invariant plants of the form of Eqn. (4.5), with $C(z^{-1})$ as defined in (4.67).

4.4.4 MOCCA versus GPC versus MAPC

The optimal MOCCA control law derived in the previous section was seen to be identical to that derived earlier for GPC and MAPC, *i.e.*

$$\Delta U(t) = A^{*}\{Y_{sp}(t) - Y^{*}(t)\} \tag{4.31}$$

The correspondence that exists between GPC and MOCCA with a polynomial disturbance generator will first be examined. Since A^{*} and $Y_{SP}(t)$ in (4.31) are independent of the predictor mechanism, demonstrating that $(Y^{*}(t))_{GPC} = (Y^{*}(t))_{MOCCA}$ would prove the equality of the two schemes. Rewriting Eqns. (4.36) and (4.46),

$$(y^{*}(t+j\,|\,t))_{GPC} = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots + \frac{F_j(z^{-1})}{C(z^{-1})}r(t) \tag{4.36}$$

$$(y^{*}(t+j\,|\,t))_{MOCCA} = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots + g_N \Delta u(t+j-N-k+1) + g_{SS} u(t+j-N-k) + \frac{F_j(z^{-1})}{C(z^{-1})}r(t) \tag{4.46}$$

from which it is evident that MOCCA with a PDG becomes <u>equal</u> to GPC (and therefore <u>equivalent</u> to MAPC) if N in Eqn. (4.46) is sufficiently large, *i.e.* if the assumption

$$\sum_{i=N+1}^{\infty} g_i \Delta u(t-i-d) = g_{SS} u(t-N-k)$$

is valid.

It was demonstrated in the previous section that MOCCA with a Kalman Filter Predictor is equivalent to MOCCA with a PDG; hence MOCCA with a KFP is equivalent to GPC if the entire process reaction curve is captured when the step or impulse response data are collected. To examine the relationship that exists between this algorithm and Multistep Adaptive Predictive Control, consider Eqns. (4.76) and (4.27):

$$(y^*(t+j|t))_{MOCCA} = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots$$
$$+ g_N \Delta u(t+j-N-k+1)$$
$$+ g_{SS} u(t+j-N-k) + \frac{K_{4,j}(z^{-1})}{C(z^{-1})} r(t) \tag{4.76}$$

$$(y^*(t+j|t))_{MAPC} = g_{j-k+2}\Delta u(t-1) + g_{j-k+3}\Delta u(t-2) + \ldots$$
$$+ \frac{K_{4,j}(z^{-1})}{C(z^{-1})} r(t) \tag{4.27}$$

from which it would appear that $(Y^*(t))_{MOCCA} = (Y^*(t))_{MAPC}$ as $N \to \infty$. This is in fact not the case because as mentioned in Section 4.4.2.2, the definition of $C(z^{-1})$ for MOCCA with a KFP is generally different from that of MAPC, i.e.

$$(C(z^{-1}))_{MOCCA} = \left[\left(1 + \ldots + L_{N_2} z^{-N_2+1}\right)\Delta + L_{N_2+1} z^{-N_2}\right] A(z^{-1}) \tag{4.67}$$

$$(C(z^{-1}))_{MAPC} = [A(z^{-1})(1 + K_2(z^{-1})) + z^{-d} K_1(z^{-1})]\Delta$$
$$+ z^{-d} D(z^{-1}) \tag{4.6}$$

Hence, MOCCA with a KFP is not equivalent to MAPC because the two methods differ in their assumptions regarding the dynamics of the disturbance mode in Eqn. (4.5).

4.4.5 MOCCA versus DMC

As mentioned earlier, MOCCA may be regarded as a refinement of the Dynamic Matrix Control (DMC) algorithm of Cutler and Ramaker (1980) in the sense that the cost functional (4.1) was formally applied by Sripada and Fisher (1985) to the DMC approach. In particular, Sripada and Fisher first presented the idea of an output horizon $N_y = N_2 - N_1 + 1$, and introduced the idea of control weighting (*cf*. λ in Eqn. (4.1)). Perhaps the most important difference between the two schemes, however, is their repective methods for disturbance forecasting. As seen above, MOCCA incorporates the disturbance transfer function $C(z^{-1})/A(z^{-1})/\Delta$ into the predictor design and "splits" the projected disturbance into future and past terms using the Diophantine equation (4.34). The future terms are then set to zero to provide an optimal or least squares forecast of the disturbance.

DMC, on the other hand, computes the residual $r(t)$ and simply adds it to the open loop prediction for each value of $j \in [k, N_2]$. In other words, $F_j(z^{-1})/C(z^{-1}) = G_{F,j}(z^{-1})$ in Eqn. (4.46) is set to unity (*cf*. Fig. 4.1). But this strategy is exactly that of the Smith Predictor (*cf*. Section 2.2) written for an arbitrary forward shift of j sample periods. Hence, DMC will provide suboptimal control of plants subject to stochastic disturbances.

4.5 Conclusions

It has been established in this chapter that for stable, known, time-invariant plants,

1. GPC (Clarke et al, 1987) is equivalent to MAPC (Sripada, 1988) in the steady state, *i.e.* when the Kalman gains have converged. (This statement does not require the assumption of open loop stability.)
2. MOCCA with a polynomial observer or disturbance generator (Sripada and Fisher, 1985) becomes <u>equal</u> to GPC when N approaches ∞ (*i.e.* as the truncation or modelling error becomes zero). Hence, MOCCA with a PDG may be considered equivalent to MAPC in the steady state when the MOCCA truncation error is small.
3. MOCCA with a Kalman filter predictor (Li et al, 1988; Navratil et al, 1988) and MOCCA with a PDG provide asymptotically equal control action. Therefore, GPC and MOCCA with a KFP will also become equal in the steady state if the truncation error associated with the MOCCA approach is small.
4. Although MAPC and MOCCA with a KFP can both be considered equivalent to GPC and MOCCA with a PDG, they are not generally equivalent to each other. This is because these methods implicitly assume different $C(z^{-1})$ polynomials (*cf.* Eqns. (4.66) and (4.67)). The other methods, (*i.e.* GPC and MOCCA with a PDG) can be shown to be equivalent to both of these schemes because the choice of $C(z^{-1})$ in either scheme is arbitrary.

4.6 References

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5

Multistep Adaptive Predictive Control

5.1 Introduction

Adaptive long-range predictive control strategies such as Generalized Predictive Control (Clarke et al 1987), Generalized Pole Placement (Lelic and Zarrop, 1987; Lelic and Wellstead, 1987) and Linear Quadratic Gaussian control (Clarke et al, 1985) are currently an area of considerable research interest. This is due primarily to the fact that single point strategies have been found to lack robustness in the presence of unknown and/or variable time delays. With this experience in mind, Sripada (1988) proposed a generalization of the Adaptive Modified Kalman Filter Predictor (AMKFP) of Walgama (1986) to include output and control horizons. In addition, the integrator disturbance model of the AMKFP was extended in the Multistep Adaptive Predictive Control (MAPC) scheme to allow disturbance models of arbitrary structure to be included in the state space representation.

In this chapter the univariate form of MAPC is derived and analyzed with particular emphasis on residual modelling as a technique for improving the transient characteristics of disturbance rejection. It is shown that the proposed scheme for partitioning the state space formulation into "u-y" and disturbance subsystems is infeasible when the system parameters are unknown and/or time-varying; hence an alternative approach is formulated based on the Modified

Kalman Filter Predictor of Walgama et al (1988). A series of simulation examples has been included to demonstrate the robustness of the MAPC scheme in the presence of model/plant mismatch, variable time delay and nonstationary stochastic disturbances.

5.2 The State Space Process Model

Consider the n'th order DARMA process model

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1) \tag{5.1}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \dots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \dots + b_n z^{-n+1}$$

and d is the process time delay excluding the unit delay. An equivalent state space representation of (5.1) is given by

$$x'(t+1) = \Phi_1 x'(t) + \Lambda_1 u(t) \tag{5.2}$$

$$y(t) = H_1 x'(t) \tag{5.3}$$

or

$$\begin{bmatrix} x'_1(t+1) \\ x'_2(t+1) \\ \cdot \\ \cdot \\ \cdot \\ x'_n(t+1) \\ \cdot \\ \cdot \\ \cdot \\ x'_{n+d}(t+1) \end{bmatrix} = \begin{bmatrix} 0 & \dots & 0 & -a_n & \dots & 0 & 0 \\ 1 & \dots & 0 & -a_{n-1} & \dots & 0 & 0 \\ \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot & & \cdot & \cdot \\ 0 & \dots & 1 & -a_1 & \dots & 0 & 0 \\ \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot & & \cdot & \cdot \\ 0 & \dots & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} x'_1(t) \\ x'_2(t) \\ \cdot \\ \cdot \\ \cdot \\ x'_n(t) \\ \cdot \\ \cdot \\ \cdot \\ x'_{n+d}(t) \end{bmatrix}$$

$$+\begin{bmatrix} b_n \\ b_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ b_1 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [0, 0, \ldots, 0, 0, \ldots, 1]x(t)$$

For plants having process and measurement noise (5.2) and (5.3) can be written as

$$x'(t+1) = \Phi_1 x'(t) + \Lambda_1 u(t) + \Gamma_1 n_1(t) + \Gamma_2 r(t) \qquad (5.4)$$

$$y(t) = H_1 x'(t) + H_3 r(t) + n_2(t) \qquad (5.5)$$

where $\Gamma_1^T = [\gamma_1^1, \ldots, \gamma_n^1, 0, \ldots, 0]$. $n_1($ and $n_2(t)$ are zero-mean Gaussian noise sequences known as the process and measurement noise, respectively. It is assumed that the process noise affects only the process states, *i.e.* $x'_1(t)$ to $x'_n(t)$. The term $r(t)$ is included in (5.4) and (5.5) as a disturbance term generated by the linear system:

$$\xi(t+1) = \Phi_2 \xi(t) + \Gamma_3 n_3(t) \qquad (5.6)$$

$$r(t) = H_2 \xi(t) \qquad (5.7)$$

$n_3(t)$ is typically a Poisson distributed noise sequence, *i.e.* it assumes non-zero values only at isolated instants in time. It will henceforth be assumed for simplicity that $r(t)$ is additive at the output only, *i.e.* $\Gamma_2 = 0$ and $H_3 = 1$. Equations (5.4) through (5.7) can be combined to form the augmented system

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \Gamma q(t) \tag{5.8}$$

$$y(t) = H x(t) + v(t) \tag{5.9}$$

$$\Phi = \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \Lambda_1 \\ 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_3 \end{bmatrix}$$

$$H = [H_1 \quad H_2], \quad x(t) = [x'(t)^T \quad \xi(t)^T]^T$$

$$q(t) = [n_1(t) \quad n_3(t)]^T, \quad v(t) = n_2(t)$$

5.3 The Kalman Filter Predictor

A minimum variance estimate of the state vector $x(t)$ in Eqn. (5.8) above may be obtained using the standard two-step Kalman filter (see Appendix A) summarized below:

a) Gain Calculation

$$L(t) = M(t) H^T [H M(t) H^T + R_v]^{-1} \tag{5.10}$$

b) Measurement Update

i) A Posteriori State Update

$$\hat{x}(t|t) = \hat{x}(t|t-1) + L(t)\omega(t) \tag{5.11}$$

ii) A Posteriori Covariance Update

$$P(t) = M(t) - L(t) H M(t) \tag{5.12}$$

c) Time Update

i) A Priori State Update

$$\hat{x}(t+1|t) = \Phi \hat{x}(t|t) + \Lambda u(t) \tag{5.13}$$

ii) A Priori Covariance Update

$$M(t+1) = \Phi P(t) \Phi^T + Q_1 \tag{5.14}$$

where $Q_1 = \Gamma R_q \Gamma^T$. The innovations sequence, $\omega(t)$, is defined as

$$\begin{aligned} \omega(t) &= y(t) - \hat{y}(t|t-1) \\ &= y(t) - H\hat{x}(t|t-1) \end{aligned} \tag{5.15}$$

Application of Eqns (5.10) through (5.15) to the state model (5.8), (5.9) is known in the literature as the Kalman Filter Predictor (KFP) (see Walgama et al, 1988). For time-invariant systems, the Kalman gain $L(t)$ will tend toward a unique steady state solution if the plant is completely observable and controllable (Watson, 1976). For this case, a steady state $L(t)$ may be precomputed offline by iteration of Eqns. (5.10), (5.12) and (5.14). Hence only Eqns. (5.11) and (5.13) remain for online computation; this is known as the steady state Kalman filter. (Note that it is a property of the Kalman filter that the innovations sequence $\omega(t)$ becomes a zero-mean uncorrelated sequence upon convergence of $L(t)$ to its final value).

5.4 Innovations Analysis for the KFP

In this section, an ARMA domain realization of (5.8), (5.9) is derived by innovations analysis. The discussion will assume time-invariant dynamics, no model/plant mismatch (MPM) and will employ the steady state Kalman filter described above. The development begins by assuming the following structure for the disturbance model (5.6), (5.7):

$$\begin{bmatrix} \xi_1(t+1) \\ \xi_2(t+1) \\ \cdot \\ \cdot \\ \cdot \\ \xi_m(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 & -f_m \\ 1 & 0 & \dots & 0 & -f_{m-1} \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \dots & 1 & -f_1 \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \xi_m(t) \end{bmatrix} + \begin{bmatrix} \gamma_1^3 \\ \gamma_2^3 \\ \cdot \\ \cdot \\ \cdot \\ \gamma_m^3 \end{bmatrix} n_3(t) \tag{5.16}$$

$$r(t) = [0, 0, \dots, 1]\xi(t) \tag{5.17}$$

Denoting $\hat{x}(t) = \hat{x}(t|t)$, Eqn. (5.11) may be written for the plant model defined by (5.8) and (5.9) as

$$\begin{bmatrix} \hat{x}_1(t+1) \\ \cdot \\ \cdot \\ \hat{x}_n(t+1) \\ \cdot \\ \cdot \\ \hat{x}_{n+d+1}(t+1) \\ \cdot \\ \cdot \\ \hat{x}_{n+d+m}(t+1) \end{bmatrix} = \begin{bmatrix} 0 & \dots & -a_n & \dots & 0 & 0 \\ \cdot & & \cdot & & \cdot & \cdot \\ \cdot & & \cdot & & \cdot & \cdot \\ 0 & \dots & -a_1 & \dots & 0 & 0 \\ \cdot & & \cdot & & \cdot & \cdot \\ \cdot & & \cdot & & \cdot & \cdot \\ 0 & \dots & 0 & \dots & 0 & -f_m \\ \cdot & & \cdot & & \cdot & \cdot \\ \cdot & & \cdot & & \cdot & \cdot \\ 0 & \dots & 0 & \dots & 1 & -f_1 \end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\ \cdot \\ \cdot \\ \hat{x}_n(t) \\ \cdot \\ \cdot \\ \hat{x}_{n+d+1}(t) \\ \cdot \\ \cdot \\ \hat{x}_{n+d+m}(t) \end{bmatrix}$$

$$+ \begin{bmatrix} b_n \\ \cdot \\ \cdot \\ \cdot \\ b_1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} L_1 \\ \cdot \\ \cdot \\ \cdot \\ L_n \\ L_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ L_{n+d} \\ L_{n+d+1} \\ \cdot \\ \cdot \\ \cdot \\ L_{n+d+m} \end{bmatrix} \omega(t+1) \qquad (5.18)$$

Using successive forward substitution for states 1 to n,

$$\hat{x}_1(t+1) = -a_n \hat{x}_n(t) + b_n u(t) + L_1 \omega(t+1)$$

$$\hat{x}_1(t) = -a_n z^{-1}\hat{x}_n(t) + b_n z^{-1}u(t) + L_1 z^{-1}\omega(t+1)$$

$$\hat{x}_n(t) = [1 - A(z^{-1})]\hat{x}_n(t) + B(z^{-1})u(t-1)$$
$$+ [L_n + L_{n-1}z^{-1} + \ldots + L_1 z^{-n+1}]\omega(t)$$
$$= [1 - A(z^{-1})]\hat{x}_n(t) + B(z^{-1})u(t-1) + K_1(z^{-1})\omega(t) \quad (5.19)$$

where $A(z^{-1})$ and $B(z^{-1})$ are as defined in (5.1) and

$$K_1(z^{-1}) = L_n + L_{n-1}z^{-1} + \ldots + L_1 z^{-n+1}$$

For states n+1 to n+d-1,

$$\hat{x}_{n+1}(t+1) = \hat{x}_n(t) + L_{n+1}\omega(t+1)$$

$$\hat{x}_{n+1}(t) = z^{-1}\hat{x}_n(t) + L_{n+1}z^{-1}\omega(t+1)$$

$$\hat{x}_{n+d-1}(t) = z^{-d+1}\hat{x}_n(t) + [L_{n+d-1}z^{-1} + \ldots + L_{n+1}z^{-d+1}]\omega(t+1)$$
$$= z^{-d}\hat{x}_n(t) + K_2(z^{-1})\omega(t) \quad (5.20)$$

where

$$K_2(z^{-1}) = L_{n+d-1} + \ldots + L_{n+1}z^{-d+1}$$

Note that from (5.19),

$$\hat{x}_n(t) = A^{-1}(z^{-1})B(z^{-1})u(t-1) + A^{-1}(z^{-1})K_1(z^{-1})\omega(t) \quad (5.21)$$

$$\hat{x}_{n+d-1}(t) = z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1)$$
$$+ z^{-d}A^{-1}(z^{-1})K_1(z^{-1})\omega(t) + K_2(z^{-1})\omega(t) \quad (5.22)$$

For states n+d+1 to n+d+m,

$$\hat{x}_{n+d+1}(t+1) = -f_m\hat{x}_{n+d+m}(t) + L_{n+d+1}\omega(t+1)$$

$$\hat{x}_{n+d+1}(t) = -f_m z^{-1}\hat{x}_{n+d+m}(t) + L_{n+d+1}z^{-1}\omega(t+1)$$

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$$\hat{x}_{n+d+m}(t) = [f_1 z^{-1} + \ldots + f_m z^{-m}]\hat{x}_{n+d+m}(t)$$
$$+[L_{n+d+m}z^{-1} + \ldots + L_{n+d+1}z^{-m}]\omega(t+1)$$
$$= F^{-1}(z^{-1})K_3(z^{-1})\omega(t) \qquad (5.23)$$

where

$$F(z^{-1}) = 1 + f_1 z^{-1} + \ldots + f_m z^{-m}$$

$$K_3(z^{-1}) = L_{n+d+m} + \ldots + L_{n+d+1}z^{-n+1}$$

Recall from Eqn. (5.15) that

$$\omega(t) = y(t) - \hat{y}(t|t-1)$$
$$= y(t) - \theta\Phi\hat{x}(t-1) - \theta\Lambda u(t-1)$$

Since for $d \geq 1$ and $m \geq 1$

$$\theta\Phi = [00 \ldots 010 \ldots 1 - f_1]$$

and $\theta\Lambda = [0]$, then from (5.18), (5.22) and (5.23),

$$\hat{y}(t|t-1) = \hat{x}_{n+d-1}(t-1) + \hat{x}_{n+d+m-1}(t-1) - f_1\hat{x}_{n+d+m}(t-1)$$
$$= \hat{x}_{n+d-1}(t-1) + \hat{x}_{n+d+m}(t) - L_{n+d+m}\omega(t)$$
$$= z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1)$$
$$+ z^{-d}A^{-1}(z^{-1})K_1(z^{-1})\omega(t) + K_2(z^{-1})\omega(t)$$
$$+ F^{-1}(z^{-1})K_3(z^{-1})\omega(t) - L_{n+d+m}\omega(t)$$

But $y(t) = \omega(t) - \hat{y}(t|t-1)$. Therefore,

$$y(t) = z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1) + A^{-1}(z^{-1})z^{-d}K_1(z^{-1})\omega(t)$$
$$+ A^{-1}(z^{-1})A(z^{-1})K_2(z^{-1})\omega(t)$$
$$+ A^{-1}(z^{-1})A(z^{-1})(1 - L_{n+d+m})\omega(t) + F^{-1}(z^{-1})K_3(z^{-1})\omega(t)$$

or

$$y(t) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(t-1) + \frac{C(z^{-1})}{A(z^{-1})}\omega(t) + r(t) \qquad (5.24)$$

where

$$r(t) = \frac{K_3(z^{-1})}{F(z^{-1})}\omega(t) \qquad (5.25)$$

and $C(z^{-1})$ is a polynomial of order = max (n,n+d-1), defined as

$$C(z^{-1}) = (1 - L_{n+d+m})A(z^{-1}) + A(z^{-1})K_2(z^{-1}) + z^{-d}K_1(z^{-1}) \qquad (5.26)$$

Thus, it has been shown that the state space model given by Eqns. (5.8) and (5.9) has an equivalent ARMA realization given by (5.24). In order that $r(t)$ may represent step-like changes in load, $F(z^{-1})$ will henceforth be assumed to have a factor $\Delta = 1 - z^{-1}$, *i.e.* $F(z^{-1}) = D(z^{-1})\Delta$, where

$$D(z^{-1}) = 1 + d_1 z^{-1} + \ldots + d_{m-1} z^{-m+1} \qquad (5.27)$$

It is proved in the following section that the inclusion of Δ in $F(z^{-1})$ guarantees zero steady state prediction offset for the KFP in the presence of nonzero-mean disturbances.

5.5 Analogy with the Smith Predictor

In order to simplify the discussion of the asymptotic properties of the KFP and the analogy that exists between the KFP and the Smith Predictor (Smith 1957, 1959), $D(z^{-1})$ in Eqn. (5.27) will be set to unity, *i.e.* $F(z^{-1}) = \Delta$.

A minimum variance d-step-ahead prediction of the state vector $x(t)$ in Eqns (5.8) and (5.9) is obtained by setting the future innovations $\{\omega(t+j),\ j=1,\ldots,d\}$ to their expected value, *i.e.* zero. This results in the expression given by Sripada (1988):

$$\hat{y}(t+d\,|\,t) = H\hat{x}(t+d\,|\,t)$$

$$= H\Phi^{d}\hat{x}(t\,|\,t) + \sum_{j=t}^{t+d-1} H\Phi^{t+d-1-j}\Lambda u(j) \tag{5.28}$$

For the special case $F(z^{-1}) = 1/\Delta$, Eqn. (5.28) becomes

$$\hat{y}(t+d\,|\,t) = \hat{x}_{n}(t) + \hat{x}_{n+d+1}(t) = \hat{x}_{n}(t) + L_{n+d+1}\frac{\omega(t)}{\Delta}$$

Using (5.21), this becomes

$$\hat{y}(t+d\,|\,t) = A^{-1}(z^{-1})B(z^{-1})u(t-1)$$

$$+ A^{-1}(z^{-1})K_{1}(z^{-1})\omega(t) + L_{n+d+1}\frac{\omega(t)}{\Delta}$$

$$= \frac{B(z^{-1})}{A(z^{-1})}u(t-1) + \frac{[K_{1}(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]}{A(z^{-1})}\frac{\omega(t)}{\Delta} \tag{5.29}$$

But Eqn. (5.24) may be written as

$$y(t) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(t-1) + \frac{[C(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]}{A(z^{-1})}\frac{\omega(t)}{\Delta}$$

which implies that

$$\frac{1}{A(z^{-1})}\frac{\omega(t)}{\Delta} = \frac{1}{[C(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]}$$

$$\cdot\left[y(t) - z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(t-1)\right]$$

Hence,

$$\frac{[K_1(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]}{A(z^{-1})}\frac{\omega(t)}{\Delta}$$

$$= \frac{[K_1(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]}{[C(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]}\cdot\left[y(t) - z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(t-1)\right]$$

$$= G_F(z^{-1})\cdot[y(t) - G_M(z^{-1})u(t-1)] \tag{5.30}$$

where

$$G_M(z^{-1}) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}$$

and

$$G_F(z^{-1}) = \frac{[K_1(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]}{[C(z^{-1})\Delta + L_{n+d+1}A(z^{-1})]} \tag{5.31}$$

Substituting (5.30) in (5.29),

$$\hat{y}(t+d\,|\,t) = G_P(z^{-1})u(t-1) + G_F(z^{-1})[y(t) - G_M(z^{-1})u(t-1)] \tag{5.32}$$

where

$$G_P(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}$$

Equation (5.32) is illustrated in block diagram form in Fig. 5.1, where it is seen that the KFP may be interpreted as a Smith Predictor with an optimal disturbance filter $G_F(z^{-1})$. (Recall from Chapter 2 that the SP is obtained from (5.32) when $G_F(z^{-1}) = 1$). Indeed, it is apparent also that the KFP reduces to the SP when $\xi(t) = 0$ (*i.e.* no disturbances present). Furthermore, the disturbance filter is capable of reducing the effects of model/plant mismatch which limit the applicability of the SP and reintroduce the time delay into the closed loop characteristic equation (see

Gawthrop (1977) and Walgama et al (1988) for additional discussion of these points). Finally, Eqn. (5.31) indicates that $G_F(1)=1$, which is a necessary condition for zero steady state prediction offset if $\xi(t)$ has nonzero mean.

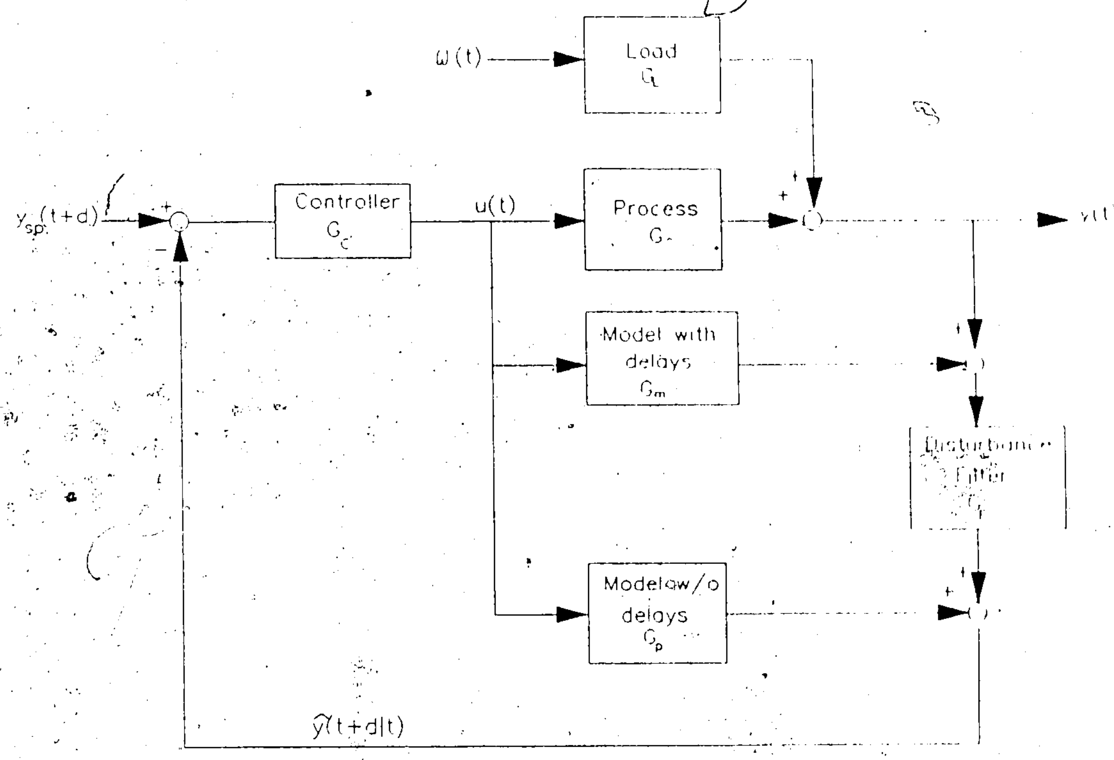


Figure 5.1 Structure of the Kalman Filter Predictor

5.6 Improved Least Squares

In order to maintain optimal control of time-varying systems, an adaptive controller must be able to update the parameters of the Kalman filter estimate in Eqn. (5.18), or equivalently the polynomials $A(z^{-1})$, $B(z^{-1})$ and $F(z^{-1})$ in Eqns. (5.24) and (5.25). The method selected for use in this thesis was the Improved Least Squares of Sripada and

Fisher (1987) which was discussed in detail in Chapter 3. In summary, this estimation scheme minimizes the exponentially weighted least squares cost functional:

$$J = \sum_{t=0}^{N} \lambda^{N-t}(t)[y(t) - \phi^T(t)\hat{\theta}(t-1)]^2 \tag{5.33}$$

The algorithm is characterized by five modifications to the standard least squares procedure resulting from the minimization of (5.33):

a) Normalization

The regressor vector $\phi(\cdot)$ and the output $y(\cdot)$ are normalized by a factor n, *i.e.*

$$y^n(t) = y(t)/n$$

$$\phi_n(t) = \phi(t)/n$$

where $n = \max(1, ||\phi(t)||)$.

b) Scaling

In order to improve the numerical conditioning of the least squares problem, scaling is introduced to minimize the condition number of the scaled covariance matrix. The parameter update law becomes

$$\hat{\theta}(t) = \hat{\theta}(t-1) + S(t)^{-1}P_s(t)\phi_{ns}(t)[y^n(t) - \phi_n(t)^T\hat{\theta}(t-1)] \tag{5.34}$$

with $P_s(t) = S(t)P(t)S(t)$ and $\phi_{ns}(t) = S^{-1}(t)\phi_n(t)$.

$S(t)$ is a diagonal scaling matrix chosen to minimize $C\{S(t)Q(t)\}$, where $P(t) = Q(t)Q^T(t)$, *i.e.* a Cholesky factorization of $P(t)$. The S_{ii} are chosen as the absolute row sums of $Q(t)$.

c) Constant Trace

The covariance update law corresponding to Eqn. (5.34) is

$$P_s(t) = \left(I - \frac{P_s(t-1)\phi_{ns}(t)\phi_{ns}(t)^T}{\lambda(t) + \phi_{ns}(t)^T P_s(t-1)\phi_{ns}(t)} \right) \frac{P_s(t-1)}{\lambda(t)} \tag{5.35}$$

A constant $tr\{P(\cdot)\}$ may be maintained by setting $tr\{P(t)\}$ equal to $tr\{P(t-1)\}$ and rearranging to obtain

$$\lambda(t) = 1 - \frac{1}{2}\left[g(t) - \left\{ g(t)^2 - \frac{4\,||P(t-1)\phi_n(t)||^2}{tr\,P(t-1)} \right\}^{\frac{1}{2}} \right] \tag{5.36}$$

where $g(t) = 1 + \phi_n(t)^T P(t-1)\phi_n(t)$. The trace of $P(\cdot)$ is chosen by the user when $P(0)$ is specified.

d) <u>On/off Criteria</u>

To prevent drift of the parameter estimates during periods of low excitation, the algorithm is designed to shut off if either

1) $||P_s(t)\phi_{ns}(t)|| < \Delta$,

or 2) $C\{P_s(t)\} > C_{max}$

where Δ and C_{max} are user-specified constants.

e) <u>Mean-Deviational Data</u>

In order to prevent parameter offset due to the presence of a nonzero d.c. bias term, the mean levels of y and u ($\bar{y}$, $\bar{u}$) are subtracted from the current values; *i.e.* the regresssor becomes mean-deviational. $\bar{y}$ and $\bar{u}$ are updated using ad hoc exponential filters given by

$$\begin{aligned} \bar{y}(t) &= \lambda\bar{y}(t-1) + (1-\lambda)y(t) \\ \bar{u}(t) &= \lambda\bar{u}(t-1) + (1-\lambda)u(t) \end{aligned} \tag{5.37}$$

As shown in Chapter 3, this scheme is equivalent to incremental ID combined with a first order "T-filter" (see Clarke et al, 1987).

5.7 Difficulties with the Formulation

This section treats the difficulties that arise when one tries to adaptively estimate the coefficients of the state space model (5.8), (5.9) (*i.e.* the coefficients of $A(z^{-1})$, $B(z^{-1})$, and $F(z^{-1})$). As discussed in Section 5.4, an equivalent ARMA representation is given by

$$y(t) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(t-1) + \frac{C(z^{-1})}{A(z^{-1})}\omega(t) + r(t)$$

or

$$y(t) = \phi_{uy}(t)^T\theta_{uy}(t) + \omega(t) + r(t) \tag{5.38}$$

where

$$\phi_{uy}(t)^T = [-y(t-1),\ldots,-y(t-n),u(t-d-1),\ldots,$$
$$u(t-d-n),\omega(t-1),\ldots,\omega(t-n-d+1)]$$
$$\theta_{uy}(t)^T = [a_1,\ldots,a_n,\ldots b_1,\ldots,b_n,c_1,\ldots c_{n+d-1}]$$

Also, from (5.25),

$$r(t) = \frac{K_3(z^{-1})}{F(z^{-1})}\omega(t) = \frac{K_3(z^{-1})}{\Delta D(z^{-1})}\omega(t)$$

$$\Delta r(t) = \phi'_r(t)^T\theta'_r(t) + L_{n+d+m}\omega(t) \tag{5.39}$$

with

$$\phi'_r(t)^T = [-\Delta r(t-1),\ldots,-\Delta r(t-m+1),\omega(t-1),\ldots,$$
$$\omega(t-m+1)]$$
$$\theta'_r(t)^T = [d_1,\ldots,d_{m-1},L_{n+d+m-1},\ldots,L_{n+d+1}]$$

But the Kalman gain elements $L_{n+d+m-1}, \ldots, L_{n+d+1}$ are deterministic; that is, they can be taken directly from the Kalman gain vector. So, for the purposes of least squares (LS) identification, Eqn. (5.39) can be written as

$$y_{res}(t) = \phi_r(t)^T \theta_r(t) + L_{n+d+m}\omega(t) \tag{5.40}$$

where

$$y_{res}(t) = \Delta r(t) - L_{n+d+m-1}\omega(t-1) - \ldots - L_{n+d+1}\omega(t-m+1)$$

The question that one is immediately led to ask regarding the ID procedure is this: "When should the u-y and residual identification be on or off, and will they interact if they are both on at the same time?". Sripada (1988) suggested that the two schemes can run in parallel despite the interaction that will exist between them, and proposed the following on/off criteria:

<u>u-y Model Identification</u>

1. Switch on if setpoint changes or external excitation is added.

2. Switch off if disturbance is detected (*e.g.* by CUSUM testing on the residuals). Switch on when y returns to y_{sp}, *i.e.* when the disturbance has been removed.

3. Switch off when the ILS criteria for the u-y model have been violated.

<u>Residual Model Identification</u>

1. Switch off when the ILS on/off criteria for the residual model have been violated.

The difficulty with this approach arises from the fact that $r(t)$ is a time-varying nonstationary disturbance, which means that unbiased estimates of $A(z^{-1})$, and $B(z^{-1})$ (*i.e.* θ_{uy}) will never be attained using Eqn. (5.38). So although the Kalman filter update can be written as (5.18) when the $A(z^{-1})$, $B(z^{-1})$ and $F(z^{-1})$ polynomials are known, this model cannot be realized in practice if the system dynamics are unknown and/or time-varying.

Since it appears to be at best difficult to estimate the parameters of the state model (5.8), (5.9) online, the simplest alternative would seem to be to choose the disturbance model as an integrator, as discussed above in the Smith Predictor section. This will not guarantee good transient disturbance rejection but will ensure zero steady state predictor offset. However, if this assumption is to be made regarding the disturbances, then rather than proceed as above it is convenient to use the Modified Kalman Filter Predictor (MKFP) of Walgama et al (1988). In addition, Walgama (1986) proved that for multivariable systems having a diagonal interactor matrix, the MIMO MKFP decomposes into p MISO subsytems, where p is the number of outputs. This property would become important in any multivariable extension of the MAPC technique.

5.8 The Modified Kalman Filter Predictor

The MKFP ensures zero steady state prediction offset by modelling the process noise term as integrated white noise.

This implies that the state space model (5.4) can be augmented with an additional state $x_p(t)$ having an integrator to represent the disturbance dynamics:

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \Gamma n_1(t) \tag{5.41}$$

$$y(t) = H x(t) + n_2(t) \tag{5.42}$$

where

$$\Phi = \begin{bmatrix} 1 & 0 \\ \Gamma_1 & \Phi_1 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 0 \\ \Lambda_1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x(t) = [x_p, x_1, x_2, \ldots, x_n, x_{n+1}, \ldots, x_{n+d}]^T$$

Using an innovations analysis similar to that in Sections 5.2.4 and 5.2.5, Walgama et al (1988) derived an ARIMA expression equivalent to (5.41) and (5.42) and analogous to Eqn. (5.24), which was derived for the full disturbance model:

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1) + C(z^{-1})\frac{\omega(t)}{\Delta} \tag{5.43}$$

where

$$C(z^{-1}) = A(z^{-1})\Delta[1 + z^{-1}K_2(z^{-1}) + z^{-d}A^{-1}(z^{-1})K_1(z^{-1})] + z^{-d}D(z^{-1})$$

$$K_1(z^{-1}) = L_n + L_{n-1}z^{-1} + \ldots + L_1 z^{-n+1}$$

$$K_2(z^{-1}) = L_{n+d-1} + L_{n+d-1}z^{-1} \ldots + L_{n+1} z^{-d+2}$$

$$D(z^{-1}) = L_p[\gamma_n z^{-1} + \ldots + \gamma_1 z^{-n}]$$

The optimal d-step-ahead estimate of the process output obtained from the MKFP may also be represented by Fig. 5.1 (or equivalently by Eqn. (5.32)) where

$$G_F(z^{-1}) = \frac{[\Delta K_1(z^{-1}) + D(z^{-1})]}{C(z^{-1})} \tag{5.44}$$

and it is seen that $G_f(1) = 1$.

5.9 Long-Range Predictive Control Strategy

The MAPC algorithm calculates a vector of future outputs $\{\Delta u(t+i),\ i \in [1, N_u]\}$ (where N_u is the control horizon), to minimize the following multistep quadratic cost functional:

$$J = \sum_{i=N_1}^{N_2} \{y_{sp}(t+i) - \hat{y}(t+i|t)\}^2 \gamma_{yi}(t) + \sum_{i=1}^{N_u} \{\Delta u(t+i-1)\}^2 \gamma_{ui}(t) \quad (5.45)$$

where $\{\hat{y}(t+i|t),\ i \in [N_1, N_2]\}$ is the trajectory of optimal i-step-ahead predictions of the process output. The $\gamma_{yi}(t)$ and $\gamma_{ui}(t)$ represent weights on the output deviations and changes in the input variable, respectively. (Note that this cost index is of a slightly more general nature than that presented in Chapter 4 (*cf*. Eqn. (4.1)) in the sense that (5.45) allows the user to weight individual elements of the output and control horizons differently.) To establish the output trajectory, consider Eqn. (5.28) written for an arbitrary forward shift of i sample intervals:

$$\hat{y}(t+i|t) = H\hat{x}(t+i|t) = H\Phi^i \hat{x}(t|t) + \sum_{j=t}^{t+i-1} H\Phi^{t+i-1-j} \Delta u(j) \quad (5.46)$$

In vector-matrix notation, this becomes

$$\{\hat{y}(t+i|t);\ i \in [N_1, N_2]\} = \{H\Phi^i \hat{x}(t|t),\ i \in [N_1, N_2]\} + A'\{u(t+i-1),\ i \in [1, N_2]\} \quad (5.47)$$

where

$$A' = \begin{bmatrix} H\Phi^{N_1-1}\Lambda & H\Phi^{N_1-2}\Lambda & \dots & H\Lambda & \dots & 0 \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ H\Phi^{N_2-1}\Lambda & H\Phi^{N_2-2}\Lambda & \dots & H\Phi^{N_2-N_1}\Lambda & \dots & H\Lambda \end{bmatrix}$$

and $N_y = N_2 - N_1 + 1$. Equation (5.47) can be written in terms of control increments $\Delta u(t)$ as follows:

$$\{\hat{y}(t+i|t),\ i\in[N_1,N_2]\} = \{y^*(t+i|t),\ i\in[N_1,N_2]\} + A\{\Delta u(t+i-1),\ i\in[1,N_u]\} \tag{5.48}$$

with

$$A = A'S, \quad S = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & 1 & \dots & 1 \end{bmatrix}_{N_2\times N_u}$$

$$\{y^*(t+i|t),\ i\in[N_1,N_2]\}$$
$$= \{H\Phi^i\hat{x}(t|t),\ i\in[N_1,N_2]\}$$
$$+\left\{\left(\sum_{j=1}^{i} a'_{i-N_1+1,j}\right)u(t-1),\ i\in[N_1,N_2]\right\}$$

It is seen that the $y^*(t+i|t)$ are terms containing the effects of past (known) inputs on $y(t+i|t)$. The $\Delta u(t+i-1)$ are the current and future inputs to be calculated. Defining $\hat{Y}(t) = \{\hat{y}(t+i|t),\ i\in[N_1,N_2]\}$, $Y^*(t) = \{y^*(t+i|t),\ i\in[N_1,N_2]\}$ and $\Delta U(t) = \{\Delta u(t+i-1),\ i\in[1,N_u]\}$, Eqn. (5.48) can be reexpressed as

$$\hat{Y}(t) = Y^*(t) + A\Delta U(t) \tag{5.49}$$

which is obtained by setting $\Delta u(t+i-1)=0$ for $i > N_u$, with $N_u \leq N_y$. Writing (5.45) in vector-matrix form leads to

$$J = [Y_{sp}(t) - \hat{Y}(t)]^T \Gamma_y(t)[Y_{sp}(t) - \hat{Y}(t)] + \Delta U(t)^T \Gamma_u(t) \Delta U(t) \tag{5.50}$$

where

$Y_{sp}(t) = \{y_{sp}(t+i|t),\ i \in [N_1, N_2]\}$, $\Gamma_y(t) = diag\{\gamma_{yi}(t),\ i \in [N_1, N_2]\}$ and $\Gamma_u(t) = diag\{\gamma_{ui}(t),\ i \in [1, N_u]\}$. Differentiating (5.49) with respect to $\Delta U(t)$ and setting $\partial J / \partial \Delta U(t) = 0$,

$$\Delta U(t) = A^*\{Y_{sp}(t) - Y^*(t)\} \tag{5.51}$$

where

$$A^* = (A^T \Gamma_y A + \Gamma_u)^{-1} A^T \Gamma_y$$

The control action is implemented in a receding horizon fashion, hence only the first row of A^* has to be calculated at each control interval.

Note that although the process time delay d does not appear explicitly in (5.51), it is common practice to set $N_2 \geq N_1 \geq (d+1)$, which will make the matrix A nonsingular (*i.e.* by assigning no penalty to future output deviations that cannot be affected by $u(t)$). Refer to Sripada (1988) and McIntosh (1988) for discussions regarding the choice of the control parameters N_1, N_2, N_u, Γ_y, Γ_u and the various special cases (*e.g.* minimum variance control, mean level control, etc.) as well as an interpretation of MAPC as an Internal Model Control (Garcia and Morari, 1982) scheme that is optimal for stochastic processes.

5.10 Feedforward Control

The addition of a measured feedforward mode to the MAPC strategy is relatively straightforward. Denoting the manipulated variable by $u_1(t)$ and the feedforward variable by $u_2(t)$, Eqns. (5.41) and (5.42) become

$$x(t+1) = \Phi x(t) + \Lambda_1 u_1(t) + \Lambda_2 u_2(t-q) + \Gamma n_1(t) \tag{5.52}$$

$$y(t) = Hx(t) + n_2(t) \tag{5.53}$$

where $q = d_2 - d_1$, *i.e.* the difference between the physical delays associated with the feedforward and manipulated variables, respectively. (Note that $d_2 \geq d_1$ is a necessary condition for Eqns. (5.52), (5.53) to be a causal realization.) Λ_1 and Λ_2 are $(n+d_1+1) \times 1$ vectors defined as

$$\Lambda_1 = \begin{bmatrix} 0 \\ b_{1n} \\ \cdot \\ \cdot \\ \cdot \\ b_{11} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0 \\ b_{2n} \\ \cdot \\ \cdot \\ \cdot \\ b_{21} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

The coefficient matrices Φ, H and Γ are as defined in (5.41), (5.42). It is evident that an equivalent DARMA representation of Eqns. (5.52) and (5.53) is given by

$$A(z^{-1}) = z^{-d_1} B_1(z^{-1}) u_1(t-1) + z^{-d_2} B_2(z^{-1}) u_2(t-1) \tag{5.54}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B_1(z^{-1}) = b_{11} + b_{12} z^{-1} + \ldots + b_{1n} z^{-n+1}$$

$$B_2(z^{-1}) = b_{21} + b_{22} z^{-1} + \ldots + b_{2n} z^{-n+1}$$

The trajectory of future outputs is formed in a fashion analogous to the development of Section 5.9, *i.e.*

$$\begin{aligned}\hat{y}(t+i\,|\,t) &= H\hat{x}(t+i\,|\,t) \\ &= H\Phi^i \hat{x}(t\,|\,t) + \sum_{j=t}^{t+i-1} H\Phi^{t+i-1-j}[\Lambda_1 u_1(j) + \Lambda_2 u_2(j-q)]\end{aligned} \quad (5.55)$$

where $\hat{x}(t|t)$ is obtained from application of the MISO Kalman filter to (5.52), (5.53). In vector-matrix notation, Eqn. (5.55) may be written as

$$\begin{aligned}&\{\hat{y}(t+i\,|\,t),\ \ i\in[N_1,N_2]\} \\ &= \{H\Phi^i\hat{x}(t\,|\,t),\ \ i\in[N_1,N_2]\} + A_1\{u_1(t+i-1),\ \ i\in[1,N_2]\} \\ &\quad + A_2\{u_2(t+i-1-q),\ \ i\in[1,N_2]\}\end{aligned}$$

where

$$A_1 = \begin{bmatrix} H\Phi^{N_1-1}\Lambda_1 & H\Phi^{N_1-2}\Lambda_1 & \ldots & H\Lambda_1 & \ldots & 0 \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ H\Phi^{N_2-1}\Lambda_1 & H\Phi^{N_2-2}\Lambda_1 & \ldots & H\Phi^{N_2-N_1}\Lambda_1 & \ldots & H\Lambda_1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} H\Phi^{N_1-1}\Lambda_2 & H\Phi^{N_1-2}\Lambda_2 & \ldots & H\Lambda_2 & \ldots & 0 \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ H\Phi^{N_2-1}\Lambda_2 & H\Phi^{N_2-2}\Lambda_2 & \ldots & H\Phi^{N_2-N_1}\Lambda_2 & \ldots & H\Lambda_2 \end{bmatrix}$$

This can be written in terms of Eqn. (5.49) as

$$Y(t) = Y^*(t) + A\Delta U_1(t) \quad (5.56)$$

where $A = A_1 S$, $\Delta U_1(t) = \{\Delta u_1(t+i-1),\ i \in [1, N_u]\}$, and

$$
\begin{aligned}
&\{y^*(t+i\,|\,t),\ i \in [N_1, N_2]\} \\
&= \{H\Phi^i \hat{x}(t\,|\,t),\ i \in [N_1, N_2]\} \\
&\quad + \left\{ \left(\sum_{j=1}^{i} a^1_{i-N_1+1,j} \right) u_1(t-1),\ i \in [N_1, N_2] \right\} \\
&\quad + A_2\{u_2(t+i-1-q),\ i \in [1, N_2]\}
\end{aligned}
\tag{5.57}
$$

(The $a^1_{i,j}$ in (5.7) denote elements of the A_1 matrix.)

The remainder of the derivation is identical to that presented in 5.9 for the single-input, single-output case. Note also that $u_2(t+i-1-q) = u_2$ [illegible] $q+1$, i.e., future values of the feedforward variable are [illegible] equal to the current measurement, $u_2(t)$.

5.11 Simulation Examples

The MAPC approach was tested on an underdamped second order state space process given by

$$
\begin{bmatrix} x_p(t+1) \\ x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -0.7 & 0 & 0 \\ 1 & 1 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} x_p(t) \\ x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0.5 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t)
+
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} n_1(t)
$$

$$
y(t) = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 1]\,x(t) + n_2(t) \tag{5.58}
$$

where $n_1(t)$ and $n_2(t)$ are independent, zero-mean Gaussian noise sequences of variance 10^{-4}. Note that the process has a deterministic equivalent

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1)$$
$$(1 - 1.5z^{-1} + 0.7z^{-2})y(t) = z^{-2}(1 + 0.5z^{-1})u(t-1) \tag{5.59}$$

This simulated plant was used in Chapter 3 of this thesis to demonstrate the self-tuning behaviour of minimum variance control schemes based on the MKFP and the integrating Self-Tuning Controller of Tuffs and Clarke (1985) (*cf*. Fig. 3.3). An identical run was carried out using MAPC and is shown in Fig. 5.2a. As in the earlier example, the $A(z^{-1})$ and $B(z^{-1})$ parameters were estimated recursively using (incremental) ILS with $\hat{\theta}(0) = [1 \quad 1 \quad 1 \quad 1]^T$, $P(0) = I_2$, $T(z^{-1}) = 1 - 0.8z^{-1}$, $\Delta = 10^{-6}$ and $C_{max} = 10^6$. The controller parameters were chosen as $N_1 = N_u = 1$, $N_2 = 10$, $\Gamma_y = I_{10}$, $\Gamma_u = \lambda I = 0$ (default settings) and $R_1 = R_2 = 10^{-4}$. The performance using this default configuration was satisfactory, if somewhat oscillatory in comparison to the MV controllers of Chapter 3. Convergence of the parameter estimates in Fig. 5.2b was also more sluggish because control action in this example was much less vigorous than that of Fig. 3.3a.

It may also be recalled from Chapter 3 that the AMKFP and ISTC strategies were applied to the nonminimum phase state space plant

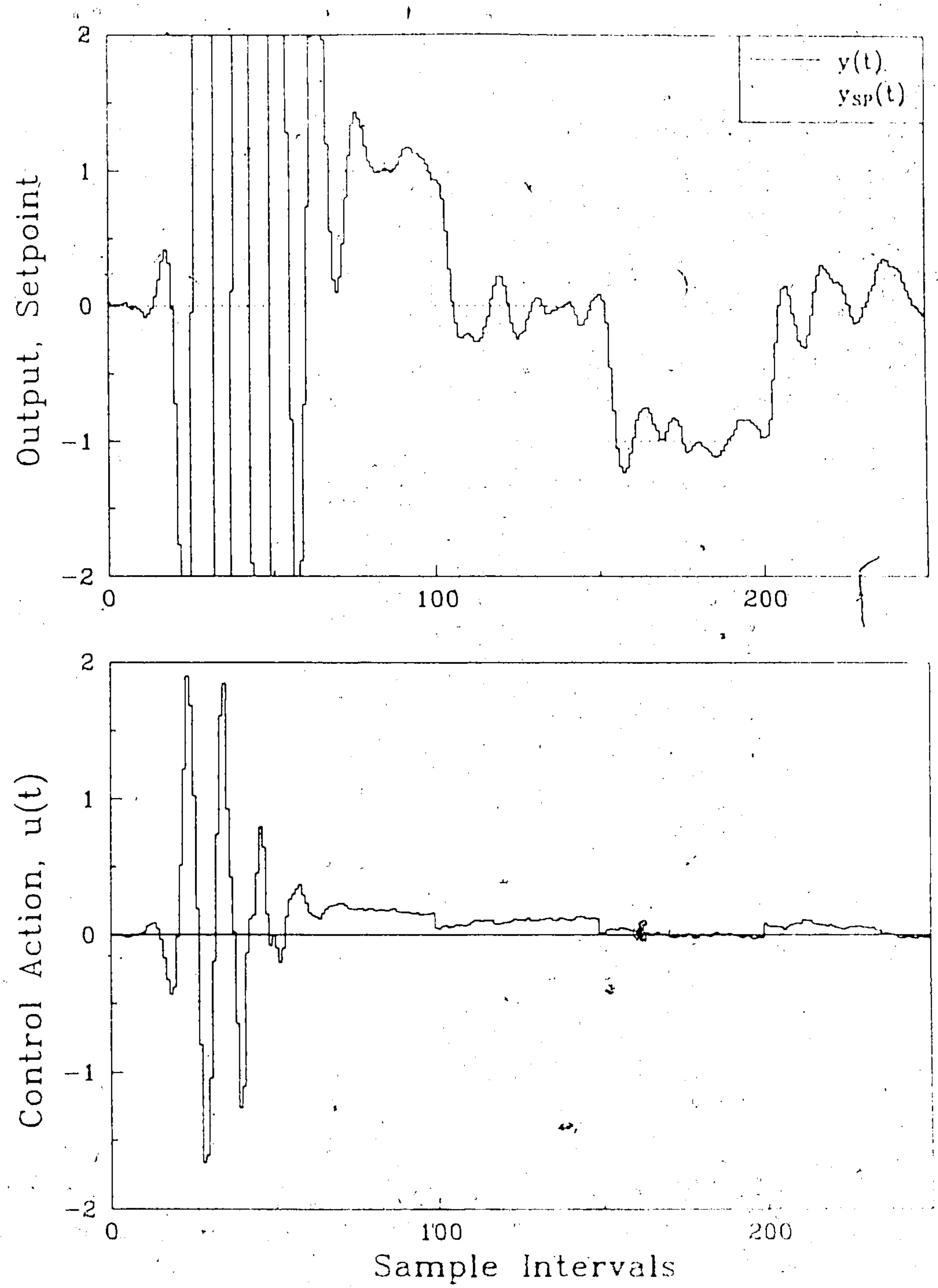


Figure 5. a Multistep Adaptive Predictive Control

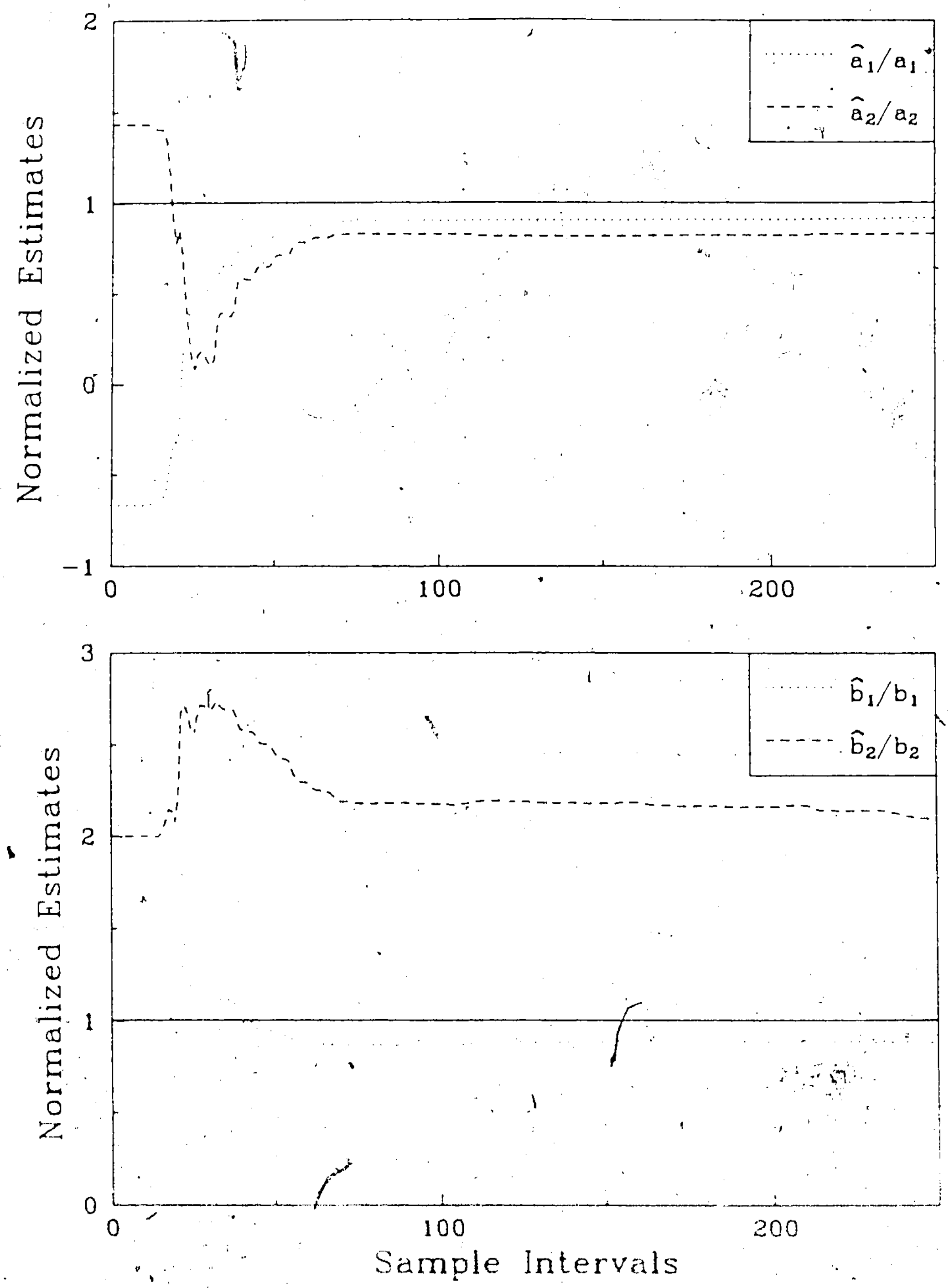


Figure 5.2b Parameter estimation using ILS

$$\begin{bmatrix} x_p(t+1) \\ x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -0.8981 & 0 & 0 & 0 \\ 1 & 1 & 1.8954 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0.9 \\ 0.7975 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} n_1(t)$$

$$y(t) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1] x(t) + n_2(t) \qquad (5.60)$$

$n_1(t)$ and $n_2(t)$ were zero-mean Gaussian sequences of variance 10^{-6}. It was observed that the minimum variance forms of these controllers could be stabilized by the addition of suitably chosen control weighting $Q(z^{-1})$ to the MV cost functional. However, the choice of $Q(z^{-1})$ becomes very difficult in systems where little *a priori* knowledge of the plant is available. In Fig. 5.3, the MAPC approach was seen to provide stable control of the plant using the <u>default</u> control settings listed above. This is due to the presence of the output horizon N_y and the control horizon $N_u \le N_y$, which move the poles of the controller inside the unit circle.

To investigate the robustness of the MAPC design philosophy, the controller was applied using a reduced-order model of the second order plant

(5.61)

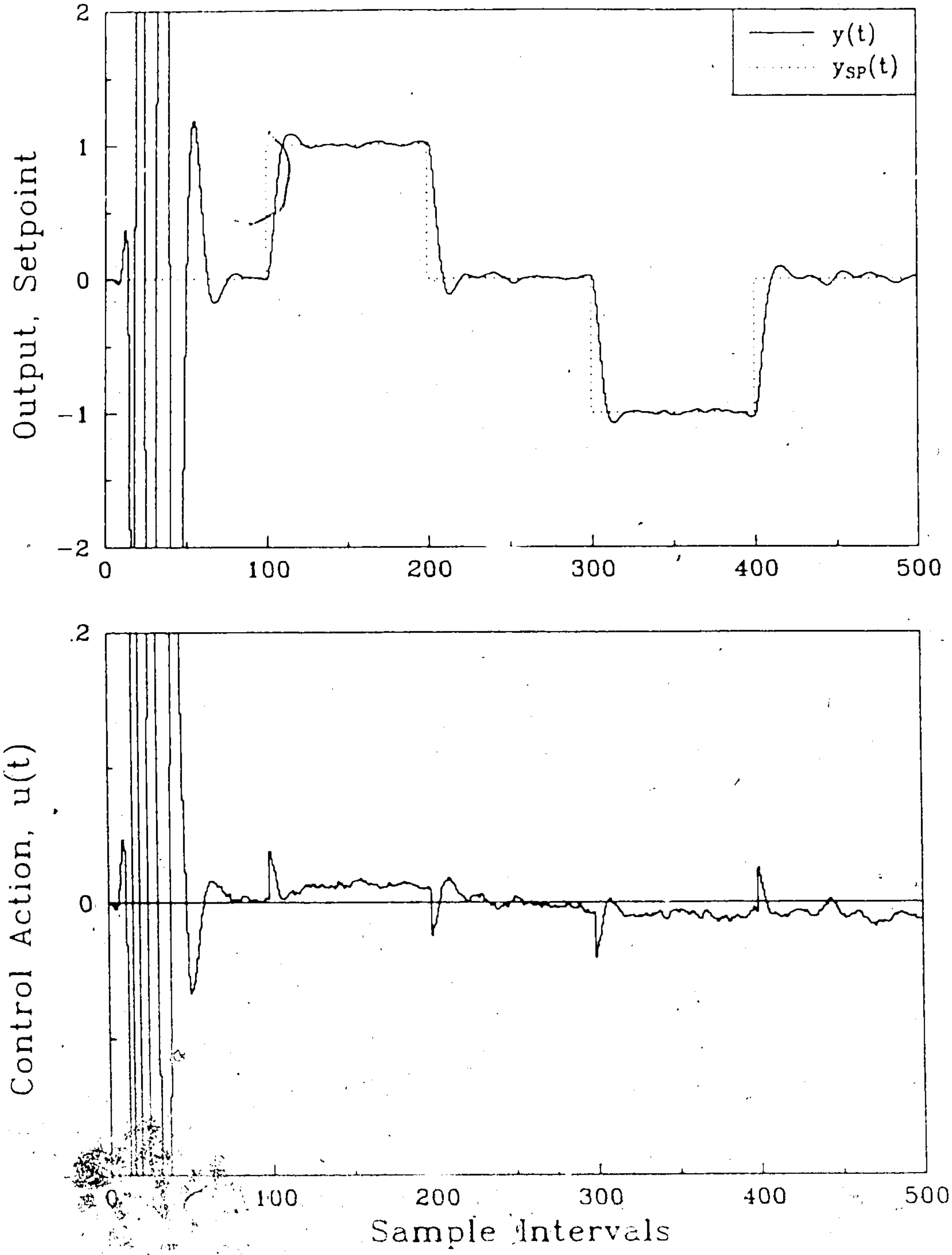


Figure 5.14 Multistep Adaptive Predictive Control of a non-minimum phase plant

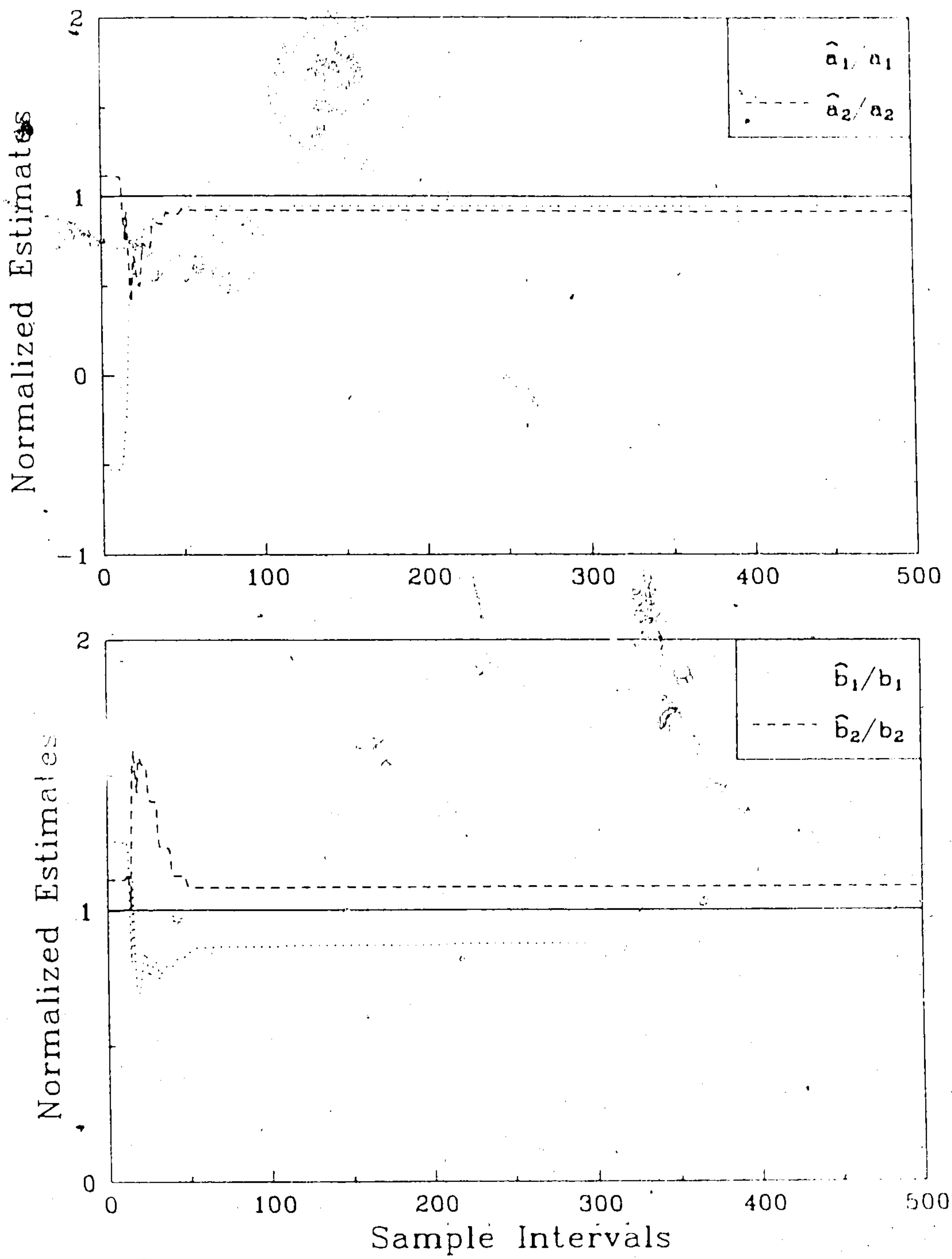


Figure 5.3b Parameter estimation using ILS

$A(z^{-1})$ in this example has poles at $z = 0.8320$ and $z = 0.4422$ so that neglecting the faster mode should still result in a realistic approximation of the true plant. Indeed, the unit step response of Fig. 5.4a looks very much like that of a typical first order plant with delay.

However, as seen in Fig. 5.4b, the control action was unstable for $T(z^{-1}) = 1$, $R_1/R_2 = 0.1$, $P(0) = I_2$ and default values of N_1, N_2, N_u and λ. The trajectories of estimated a_1 and b_1 are shown in Fig. 5.4c, where it is evident that they did not converge to constant values.

It was observed that stable control of the process could be obtained by detuning the controller in any of several possible ways, *e.g.* by increasing N_2 or λ, adding T-filtering, or decreasing R_1/R_2 or $tr\ P(t)$. All of these cases are illustrated in the remaining graphs of Fig. 5.4. Figures 5.4d through 5.4g show that stable, offset-free control was achieved by increasing the output horizon N_2 to 11 or by the addition of the control weighting $\lambda = 100$. In both examples, the estimated parameters were seen to converge to constant values (see Figs. 5.4e and 5.4g).

Prefiltering the regressor vector using the T-filter $T(z^{-1}) = 1 - 0.8z^{-1}$ also resulted in satisfactory control (Figs. 5.4h, 5.4i), as did a reduction in the ratio of the process and measurement noise covariances R_1/R_2 from 0.1 to 0.01 (see Figs. 5.4j, 5.4k). This latter modification had the effect of moving the poles of $G(z^{-1})$ in Fig. 5.1 toward the unit circle, which is equivalent to low-pass filtering the

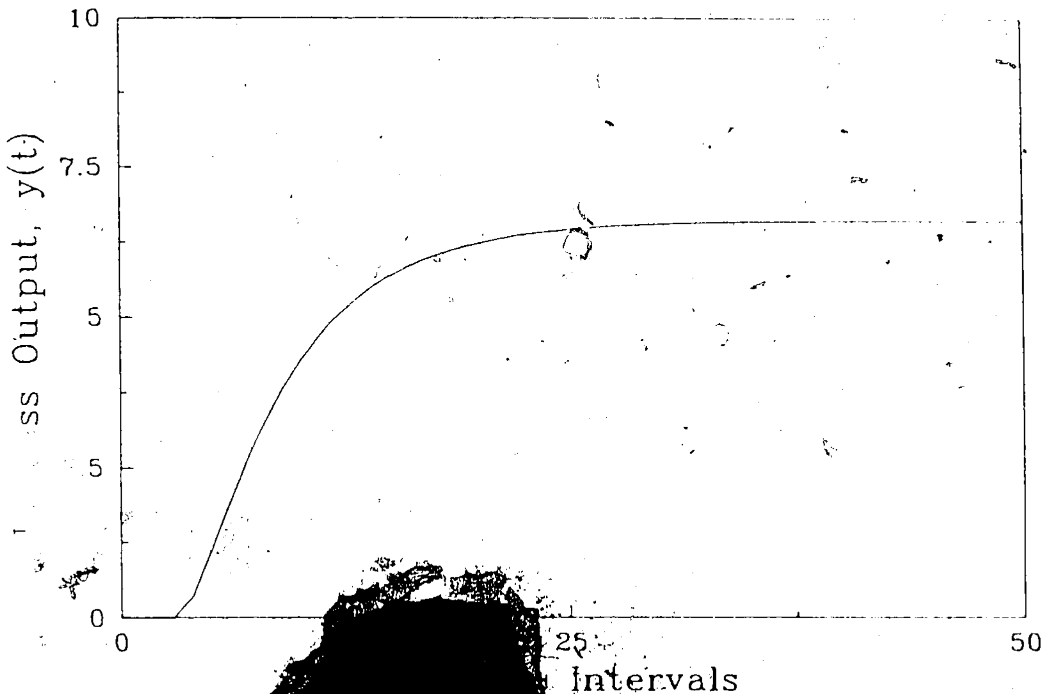


Figure 5.4a [illegible] of a second order plant

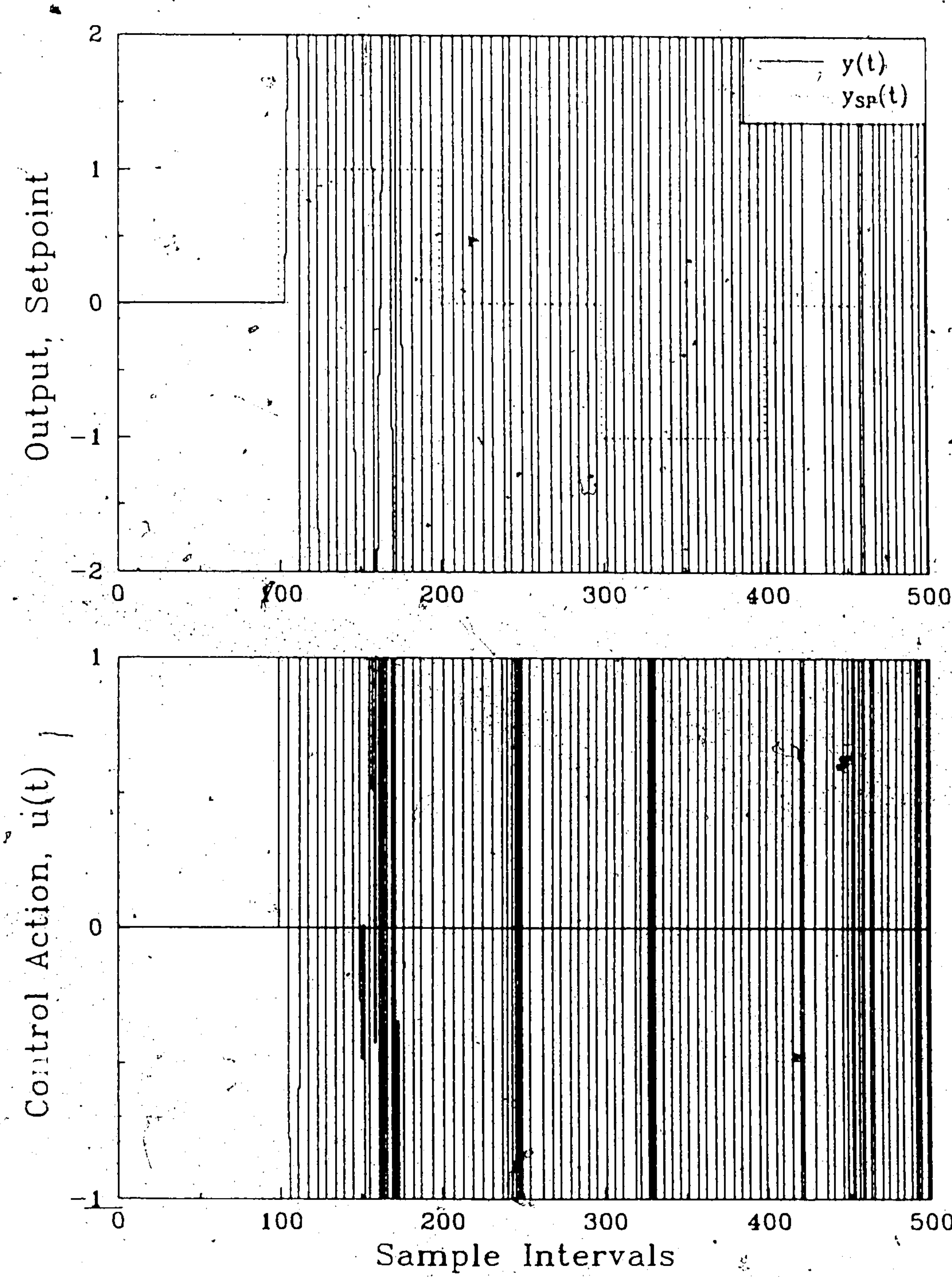


Figure 5.4b Multistep Adaptive Predictive Control with model order mismatch ($N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 0$, $T = 1$, $R_1/R_2 = 0.1$, $tr\ P(\cdot) = 2$)

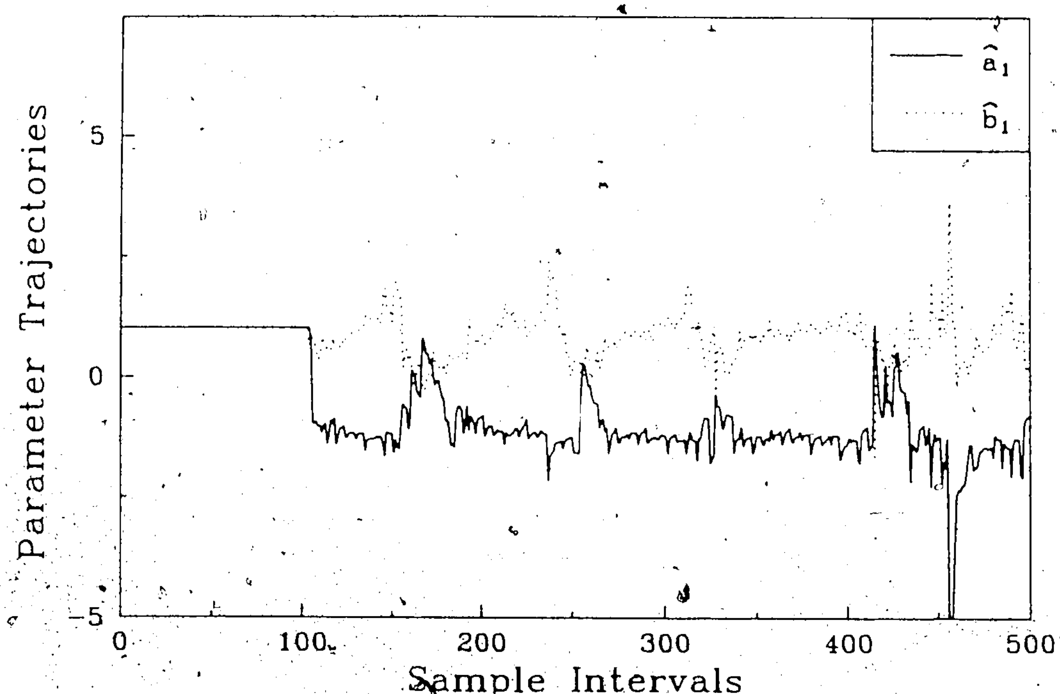


Figure 5.4c Parameter estimation using ILS

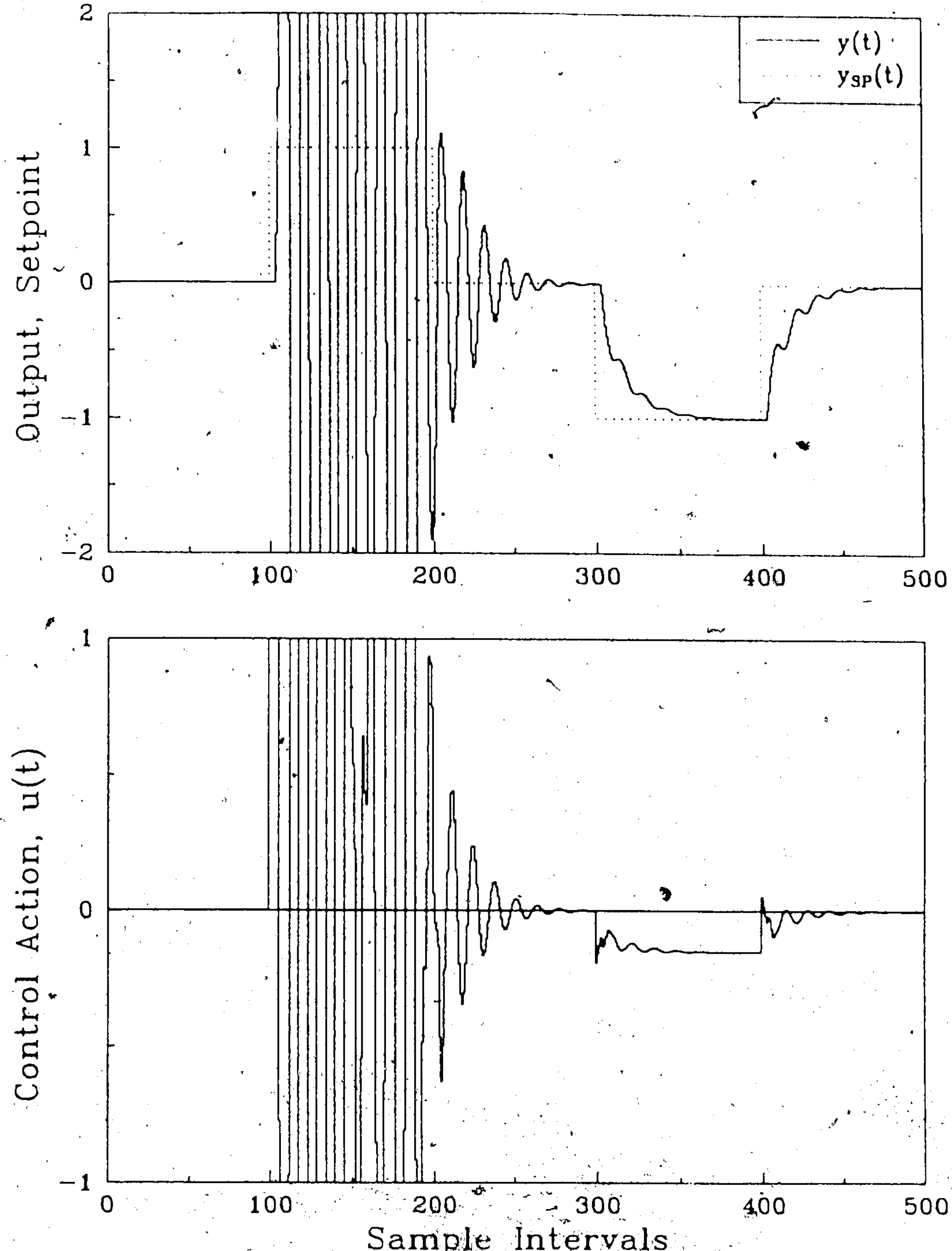


Figure 5.4d Multistep Adaptive Predictive Control with model order mismatch ($N_1 = N_u = 1$, $N_2 = 11$, $\lambda = 0$, $T = 1$, $R_1/R_2 = 0.1$, $tr\ P(\cdot) = 2$)

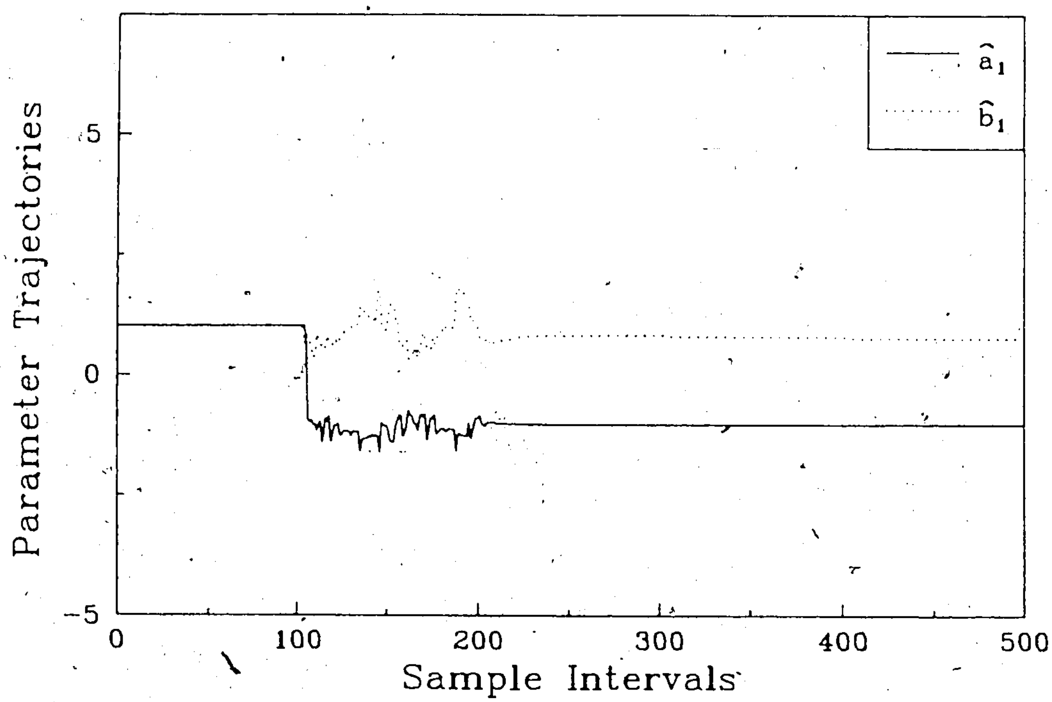


Figure 5.4e Parameter estimation using ILS

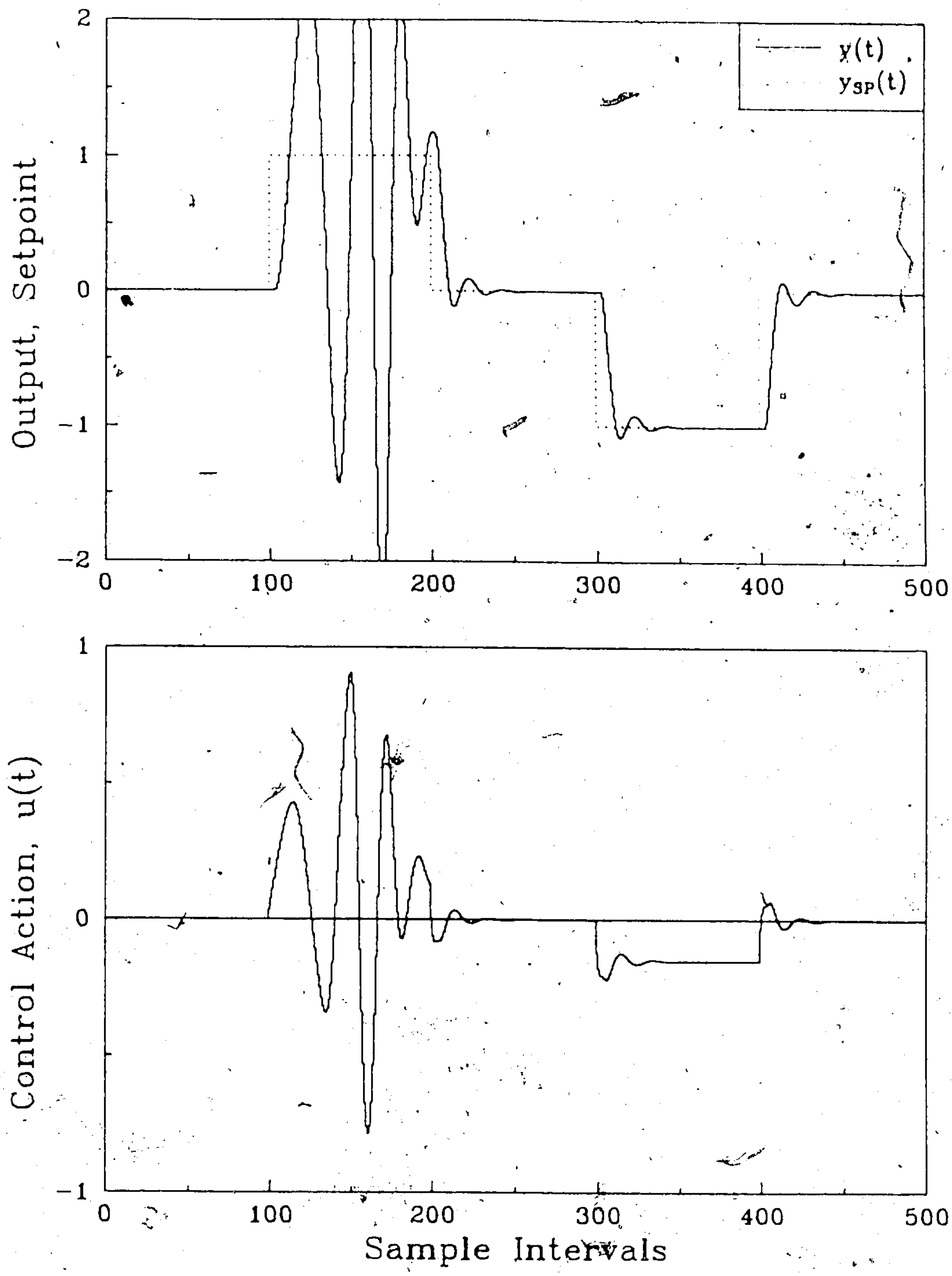


Figure 5.4f Multistep Adaptive Predictive Control with model order mismatch ($N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 100$, $T = 1$, $R_1/R_2 = 0.1$, $tr\ P(\cdot) = 2$)

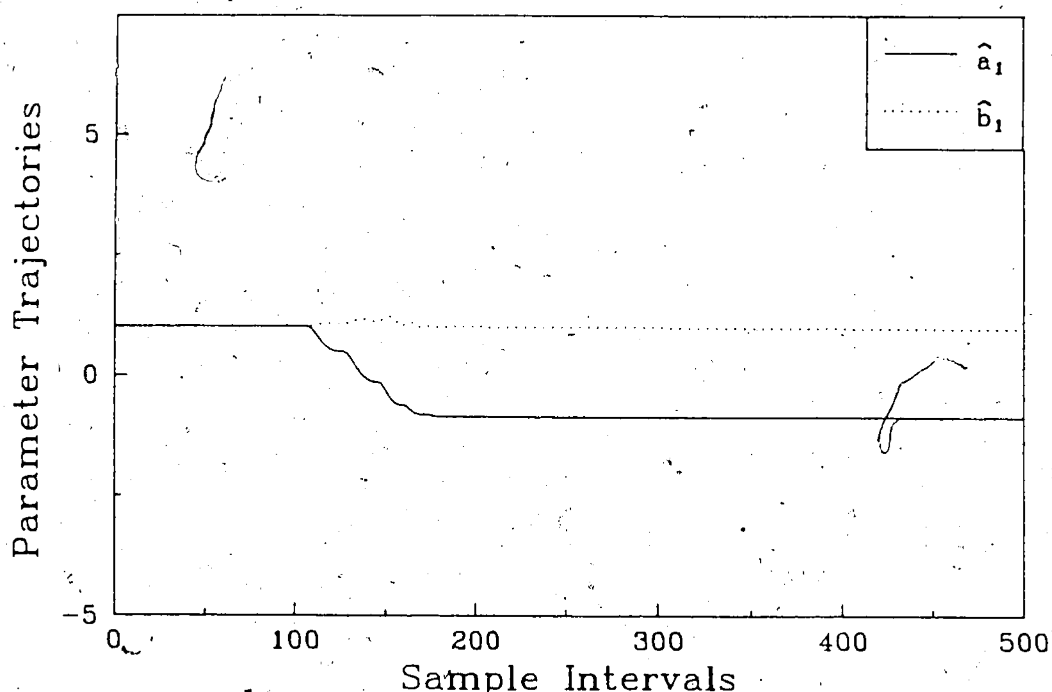


Figure 5.4g Parameter estimation using ILS

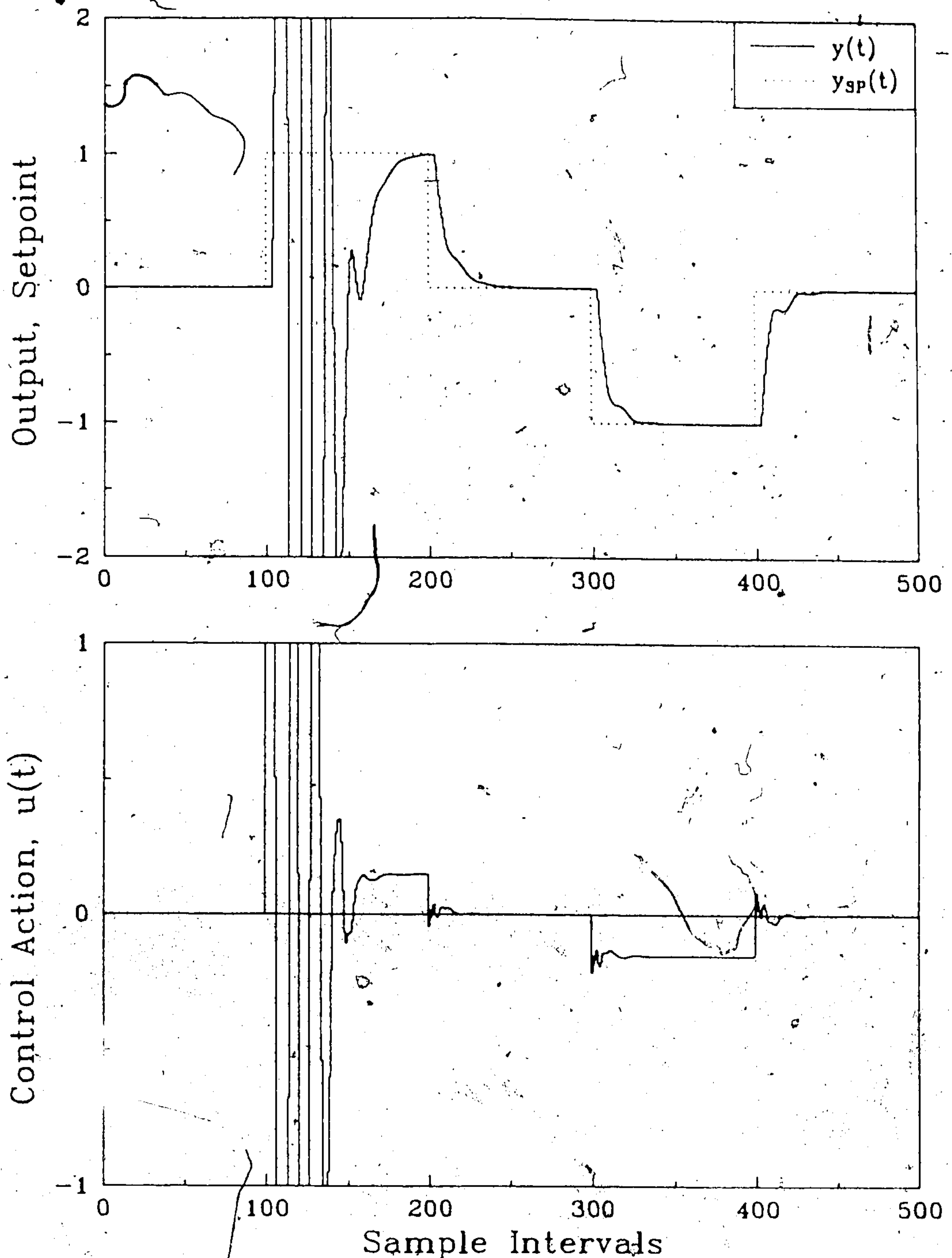


Figure 5.4h Multistep Adaptive Predictive Control with model order mismatch ($N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 0$, $T = 1 - 0.8z^{-1}$, $R_1/R_2 = 0.1$, u $P(\cdot) \neq 2$)

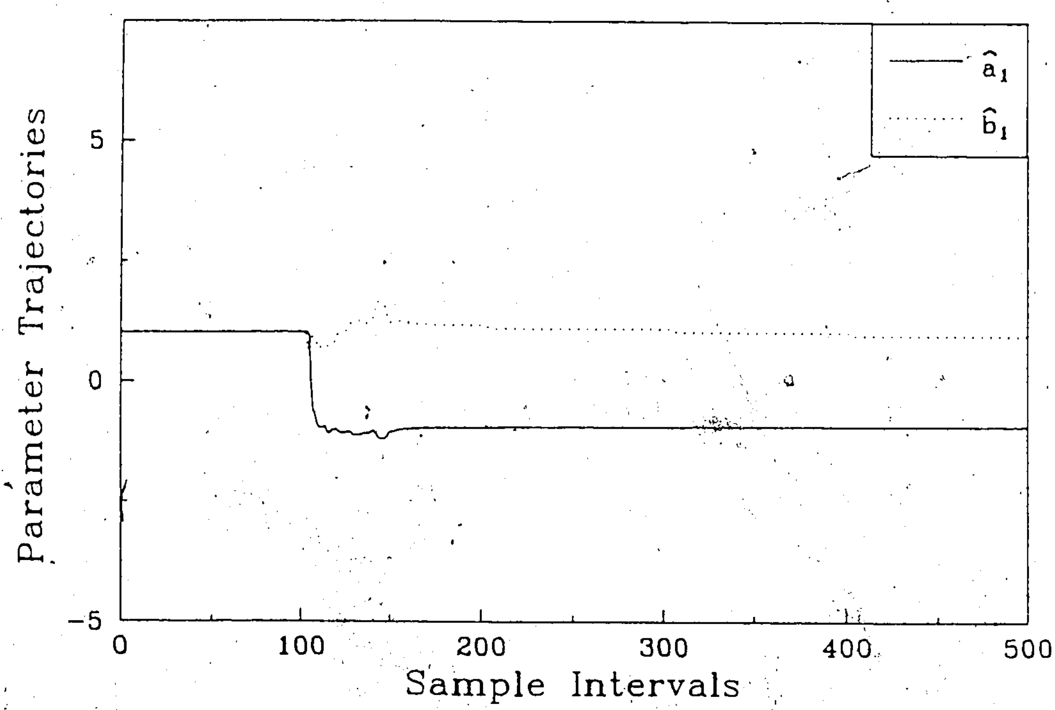


Figure 5.4i Parameter estimation using ILS

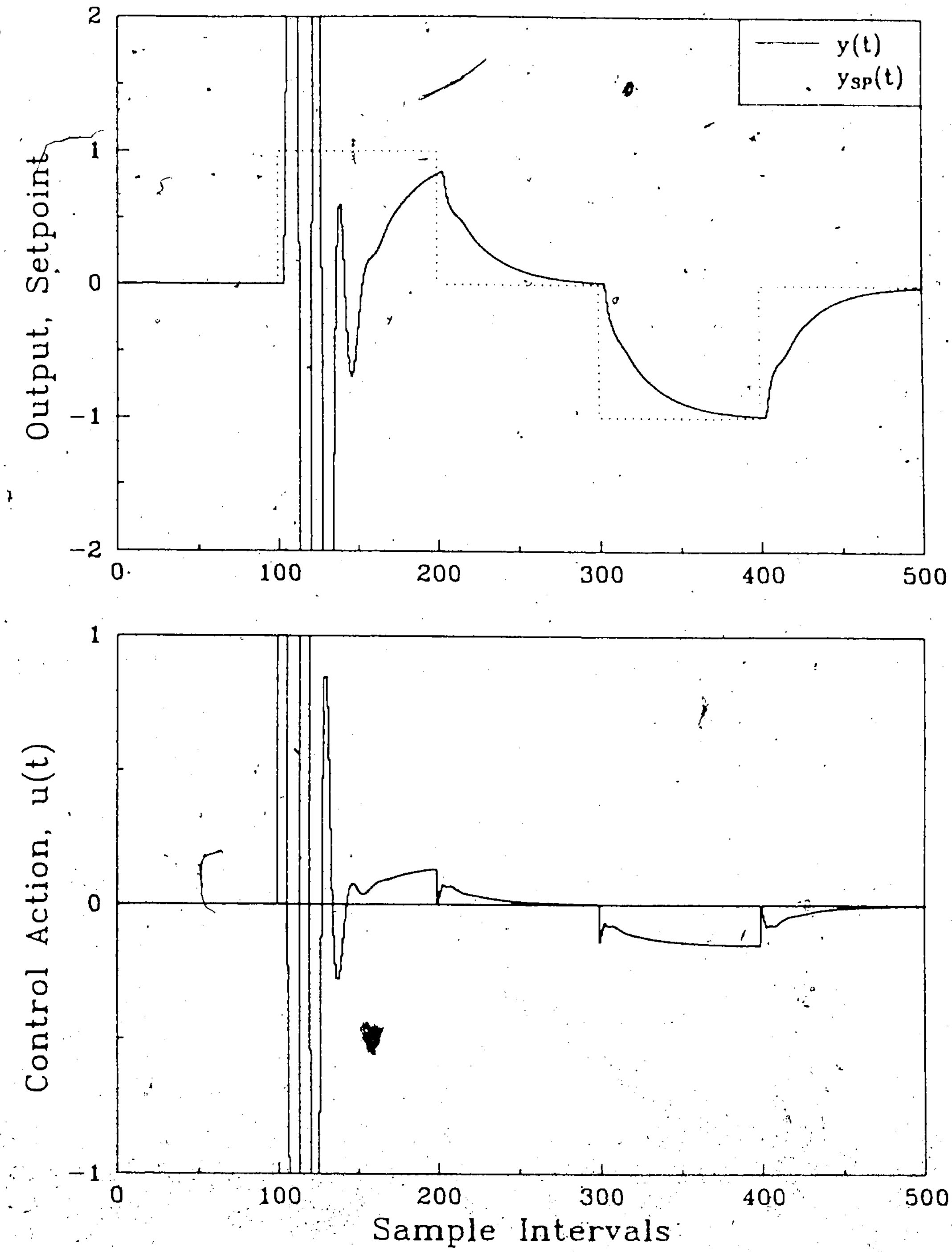


Figure 5.4j Multistep Adaptive Predictive Control with model order mismatch ($N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 0$, $T = 1$, $R_1/R_2 = 0.01$, $tr\ P(\cdot) = 2$)

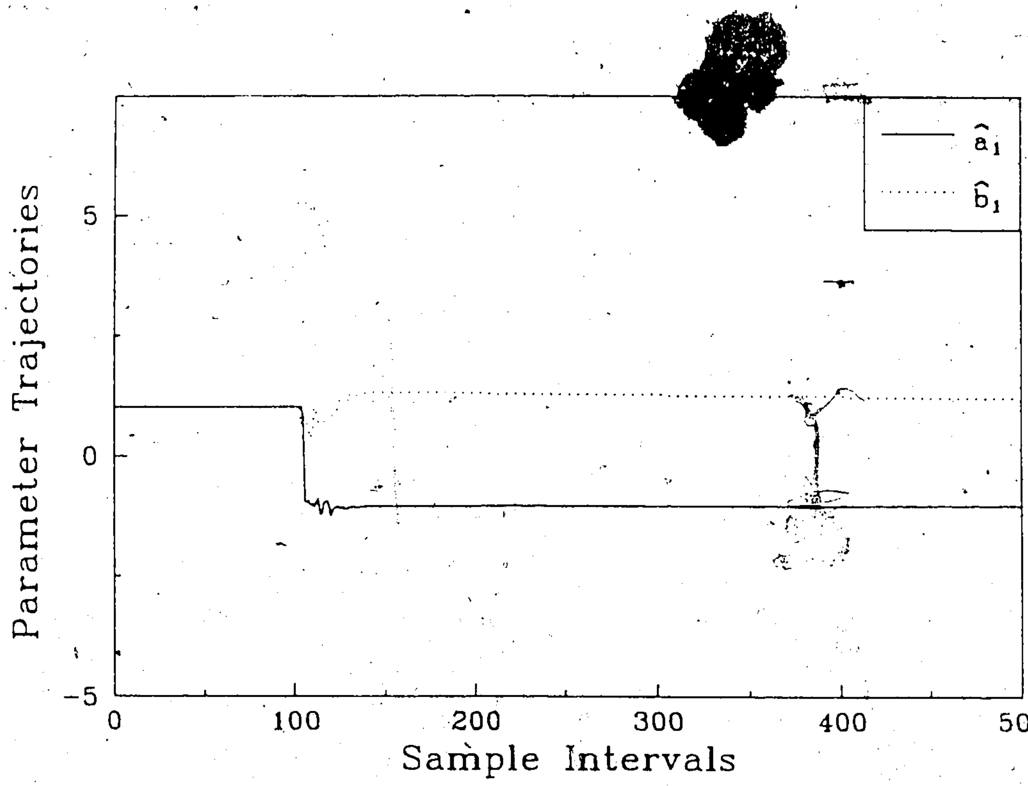


Figure 5.4k Parameter estimation using ILS

residual due to MPM, *i.e.* $y(t) - G_m(z^{-1})u(t)$. Finally, in Figs. 5.4l and 5.4m it was shown that detuning the estimation by reducing the trace of the covariance matrix from 2 to 0.2 also enabled the default MAPC configuration to provide stable control of the process.

One of the primary applications for LRPC strategies such as MAPC is in processes with unknown and/or variable time delay. Figures 5.5a to 5.5d show the behaviour of the minimum variance AMKFP and ISTC-based schemes for servo control of the DARMA plant (5.59) with a variable time delay, *i.e.*

$$
\begin{aligned}
d &= 4 \quad \forall t < 150, \quad t \in [250, 500] \\
&= 0 \quad \forall t \in [150, 250] \\
&= 8 \quad \forall t > 500
\end{aligned}
$$

The process parameters were initialized to the true values, *i.e.* $\hat{\theta}(0) = [-1.5 \;\; 0.7 \;\; 1.0 \;\; 0.5]^T$, and the covariance matrix was initialized to $P(0) = I_4$ for the AMKFP and $P(0) = 100 I_9$ for the ISTC. No T-filtering was used ($T(z^{-1}) = 1$), and the ratio of the covariances R_1/R_2 was set to unity.

In Figs. 5.5a, 5.5b it is seen that the KF-based controller provided deadbeat setpoint following for $t < 150$ because the AMKFP was designed assuming ~~a~~ time delay of four intervals. However, the controller was observed to become unstable when the delay decreased to 0 at time 150, and was never able to regain control of the process. Likewise, the ISTC approach illustrated in Figs. 5.5c, 5.5d was observed

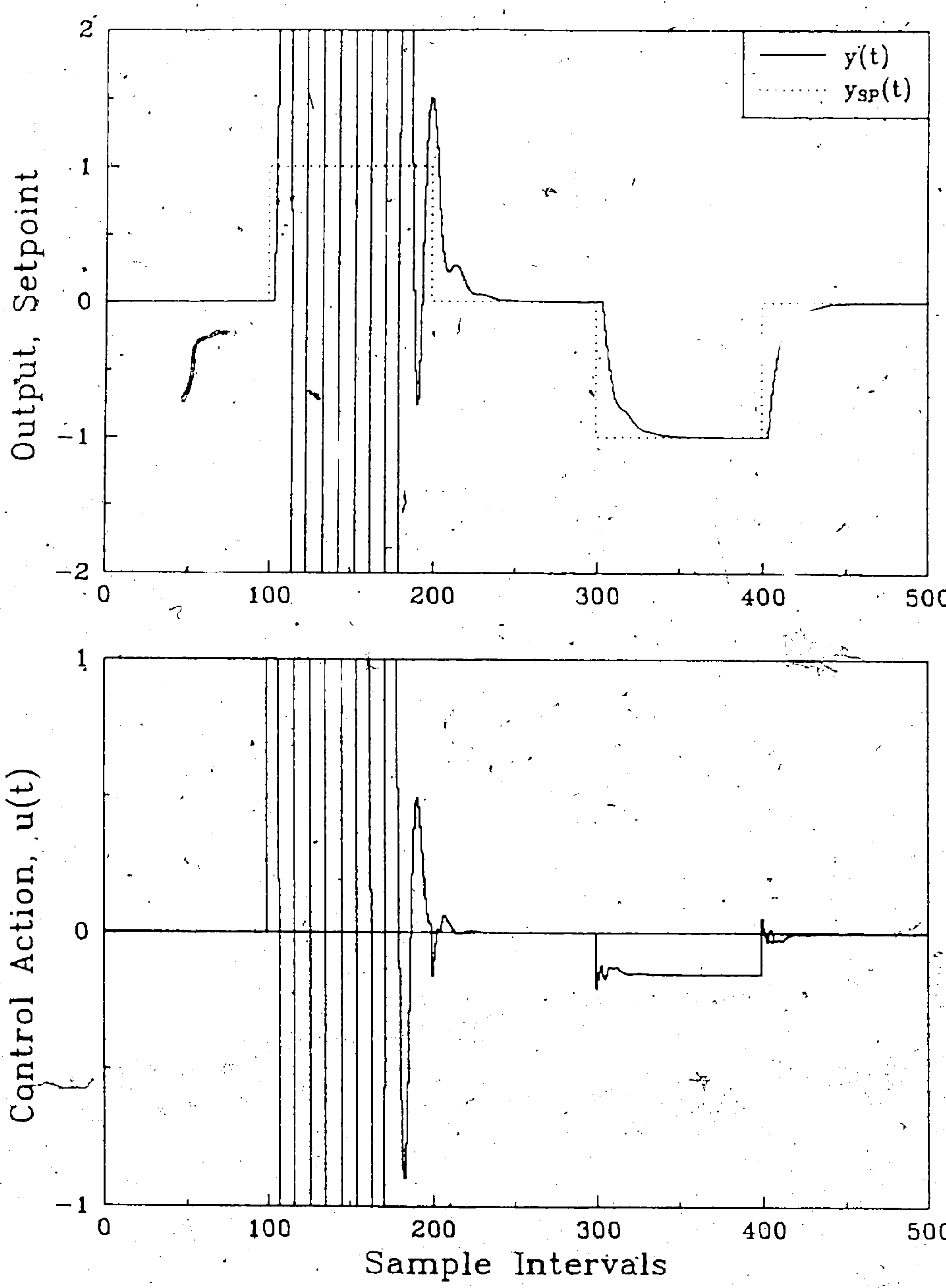


Figure 5.41 Multistep Adaptive Predictive Control with model order mismatch ($N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 0$, $T = 1$, $R_1/R_2 = 0.1$, $tr\ P(\cdot) = 0.2$)

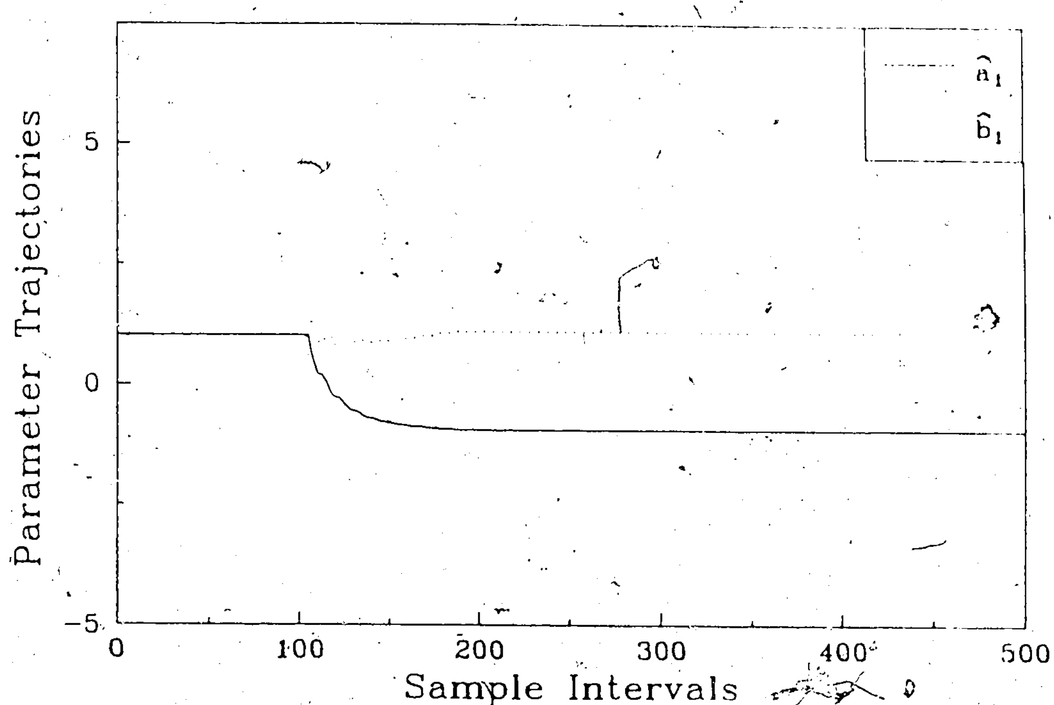


Figure 5.4m Parameter estimation using ILS

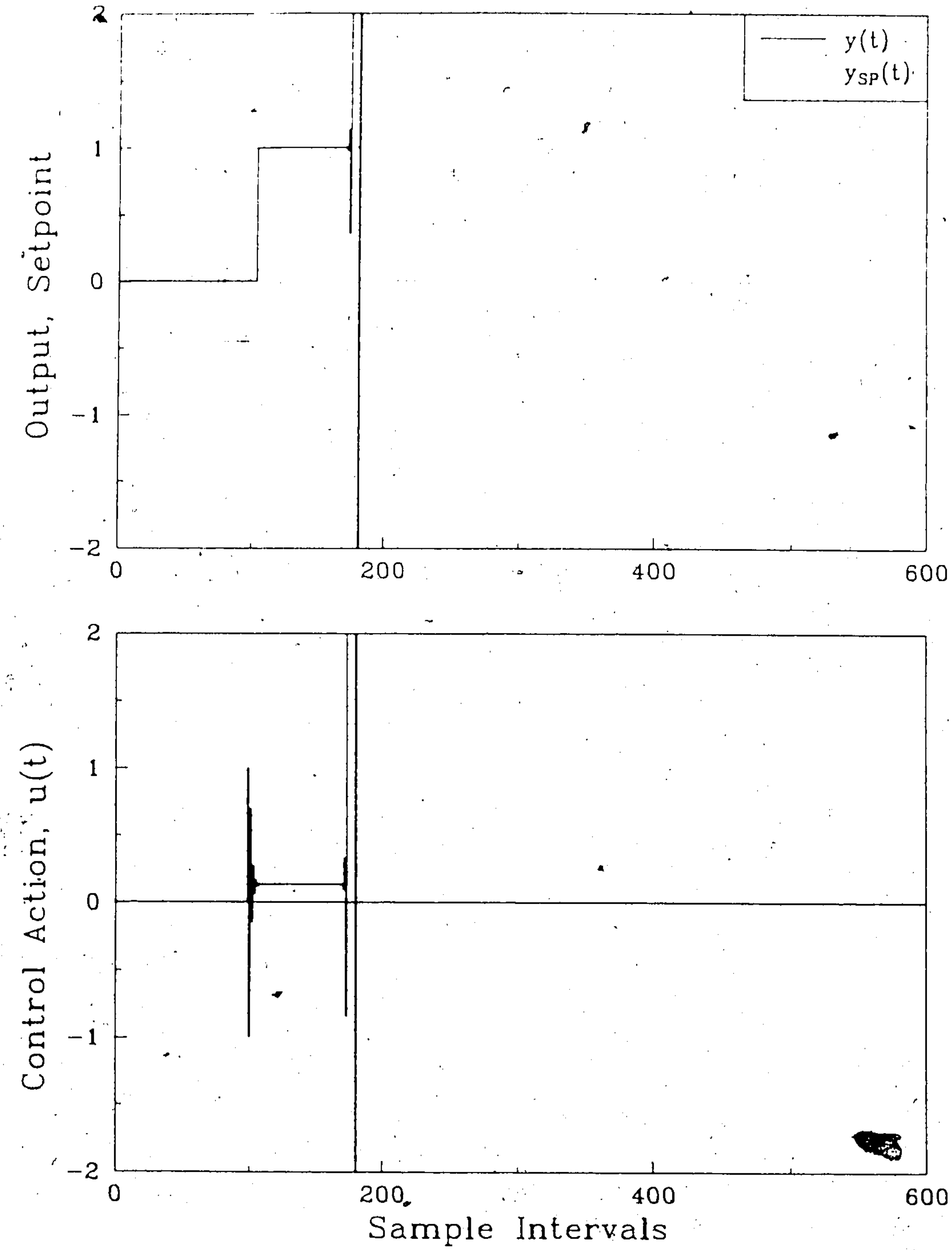


Figure 5.5a Adaptive MV control of a variable time delay plant using the AMKFP

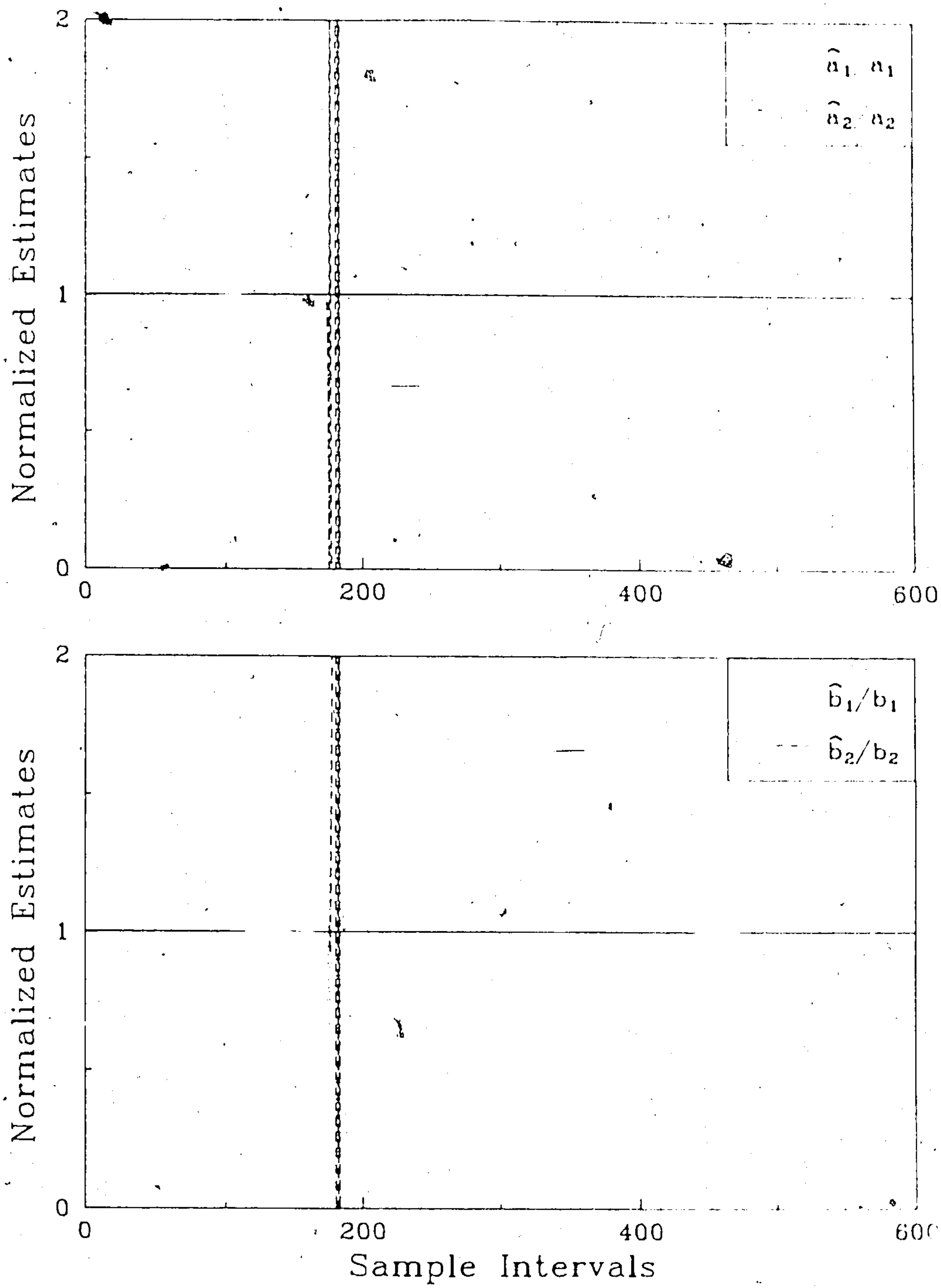


Figure 5.5b Parameter estimation using ILS ($tr\ P(\cdot) = 4$)

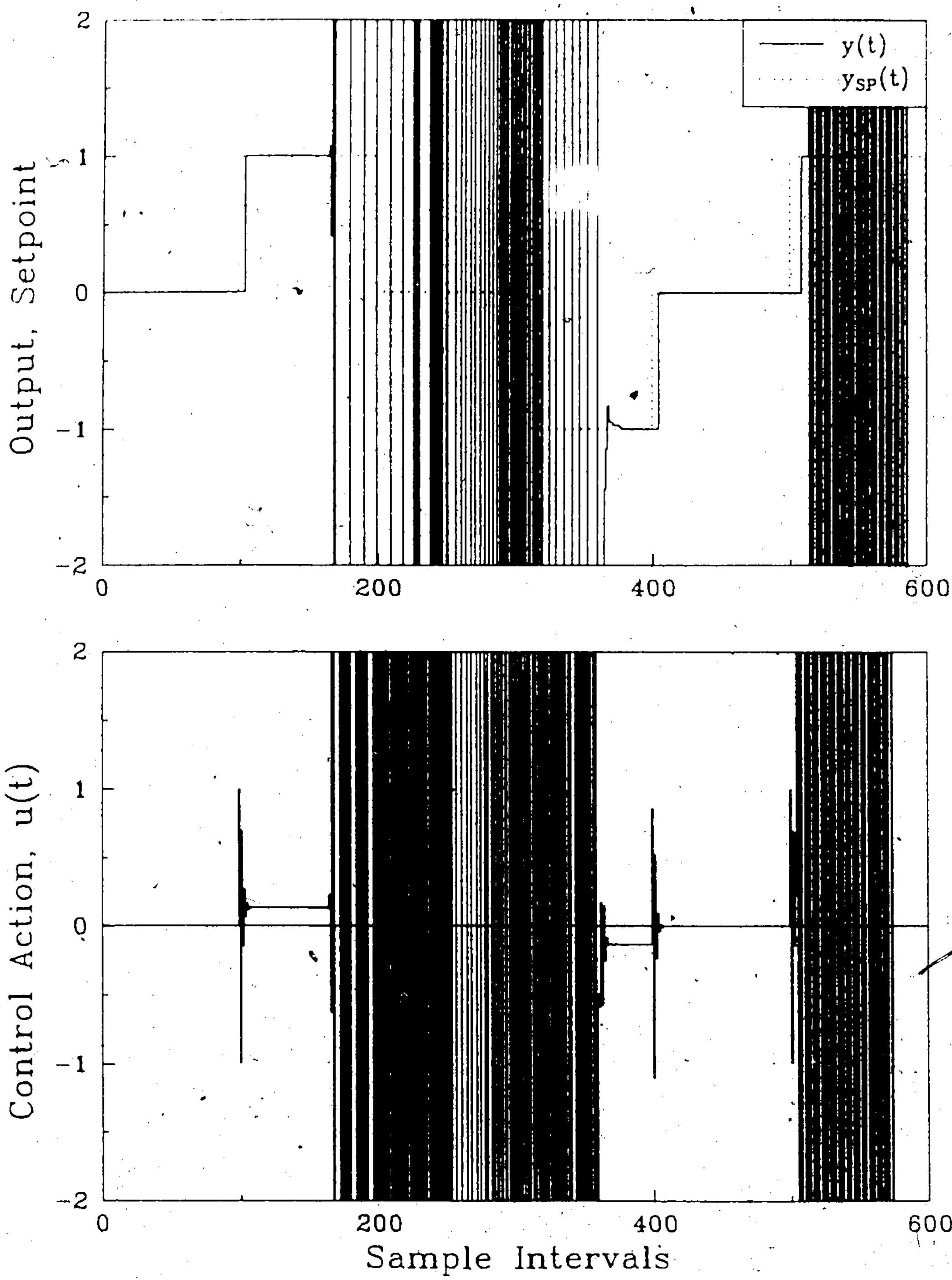


Figure 5.5c Adaptive MV control of a variable time delay plant using the ISTC

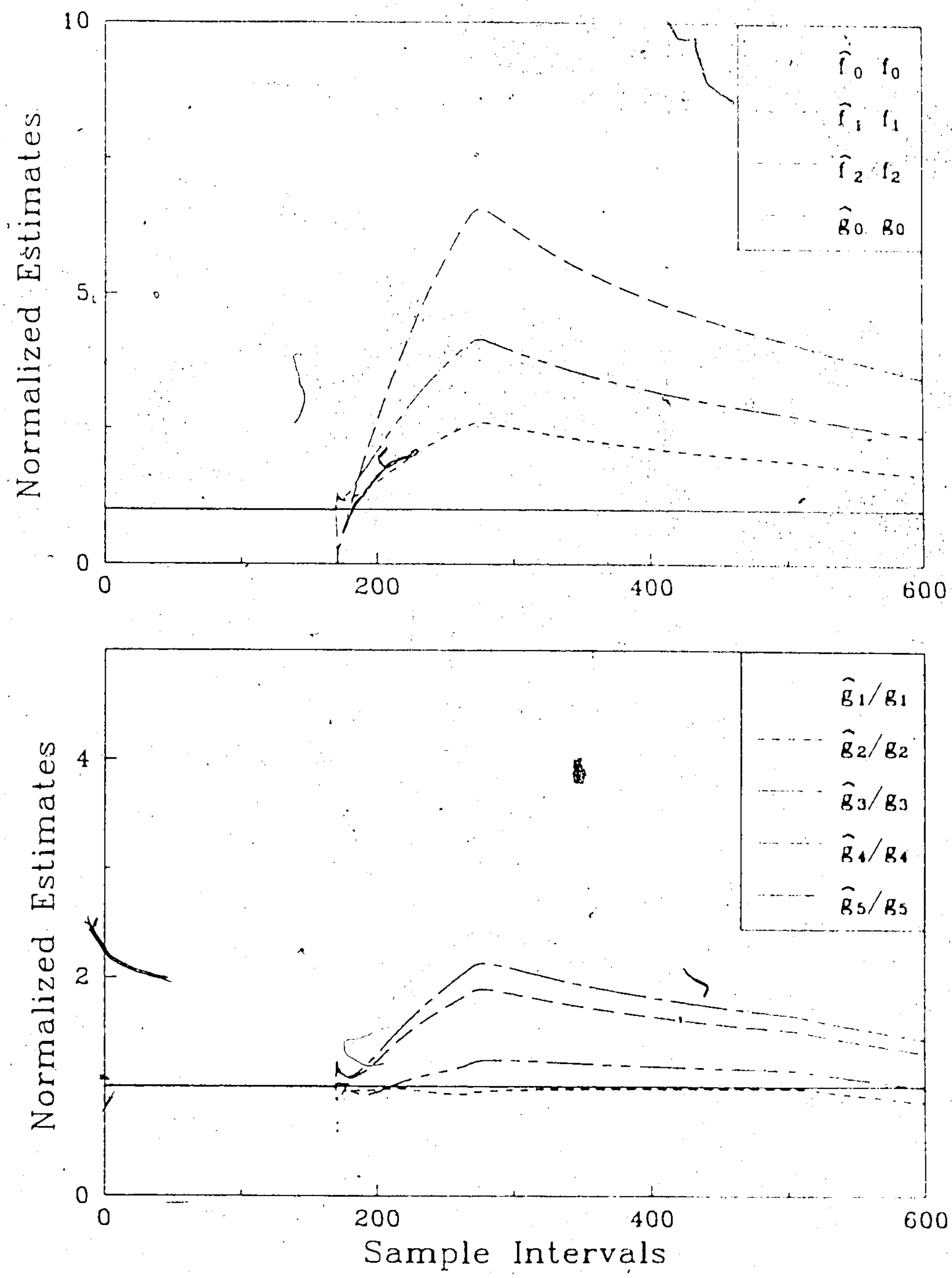


Figure 5.5d Parameter estimation using ILS (μ $P(\cdot)$ 900)

to become unstable both when the time delay was overestimated during the period [150,250] and when it was underestimated, *i.e.* $\forall t > 500$. This typifies the sensitivity of single-point control strategies to time delay mismatch.

Figures 5.5e and 5.5f show the improved perfomance obtained with MAPC using the default design $N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 0$. It is apparent that the long-range strategy was able to provide satisfactory servo performance while the delay was overestimated, but not when the delay exceeded its assumed value, despite the choice of $N_2 > d_{max} + 1$. In fact, stable results could only be achieved in this example by detuning the Kalman filter, *i.e.* by reducing the value of R_1/R_2 from unity to 10^{-4} (see Figs. 5.5g and 5.5h). This is analogous to moving the root(s) of the observer polynomial closer to the unit circle in the GPC approach described in Chapter 4.

5.12 Conclusions

This chapter presents a study of the theoretical properties and performance characteristics of the Multistep Adaptive Predictive Controller of Sripada (1988). It was demonstrated that the prediction mechanism, *i.e.* the Modified Kalman Filter Predictor can be interpreted as a Smith Predictor with an optimal disturbance filter. The long-range predictive control approach that MAPC shares with several other techniques (eg. GPC, GPP, LQG, MOCCA, etc.) was seen to be capable of providing stable control of plants in the presence of MPM, *e.g.* model order or time delay mismatch,

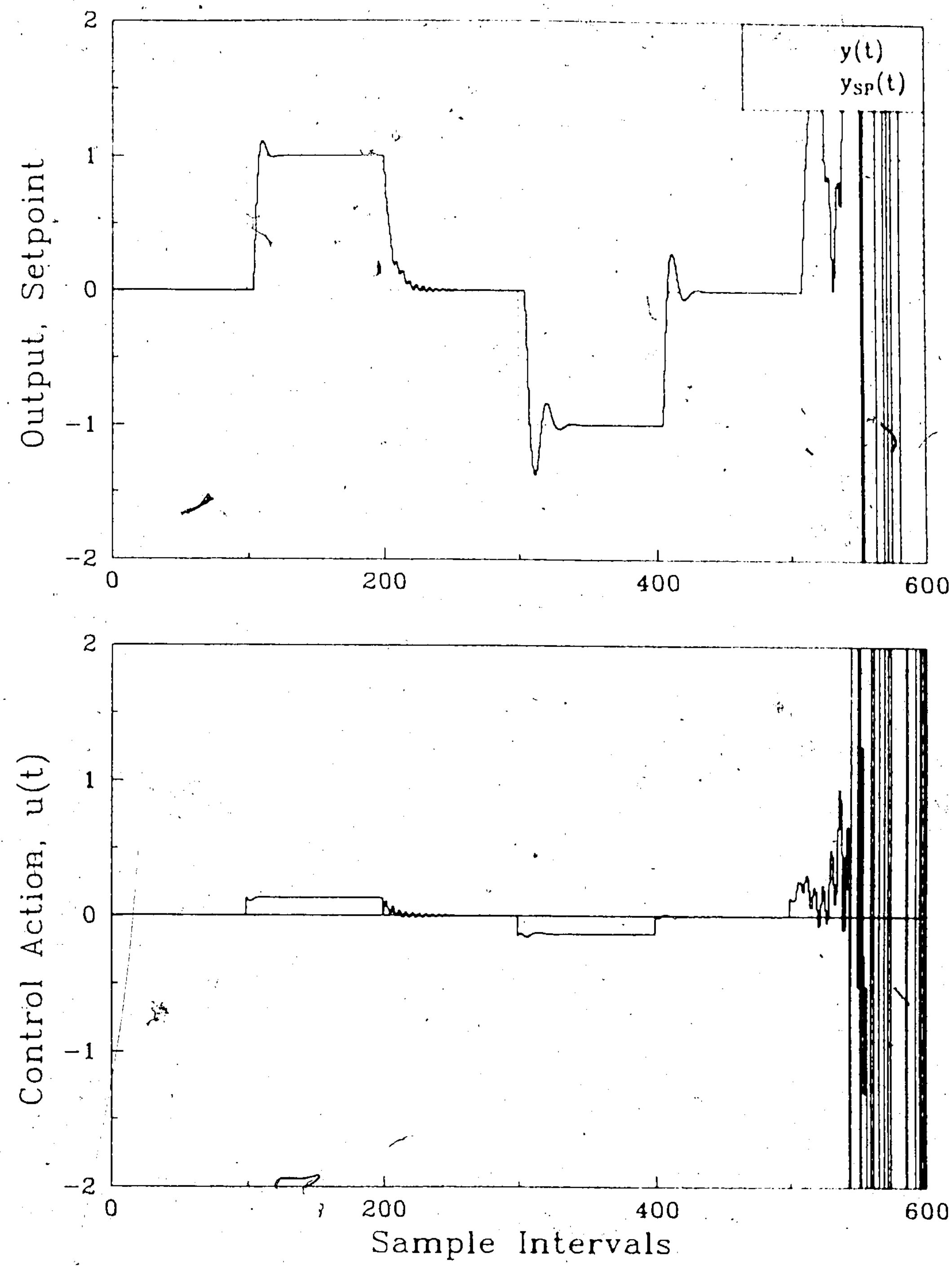


Figure 5.5e Multistep Adaptive Predictive Control of a variable time delay plant ($R_1/R_2 = 1$)

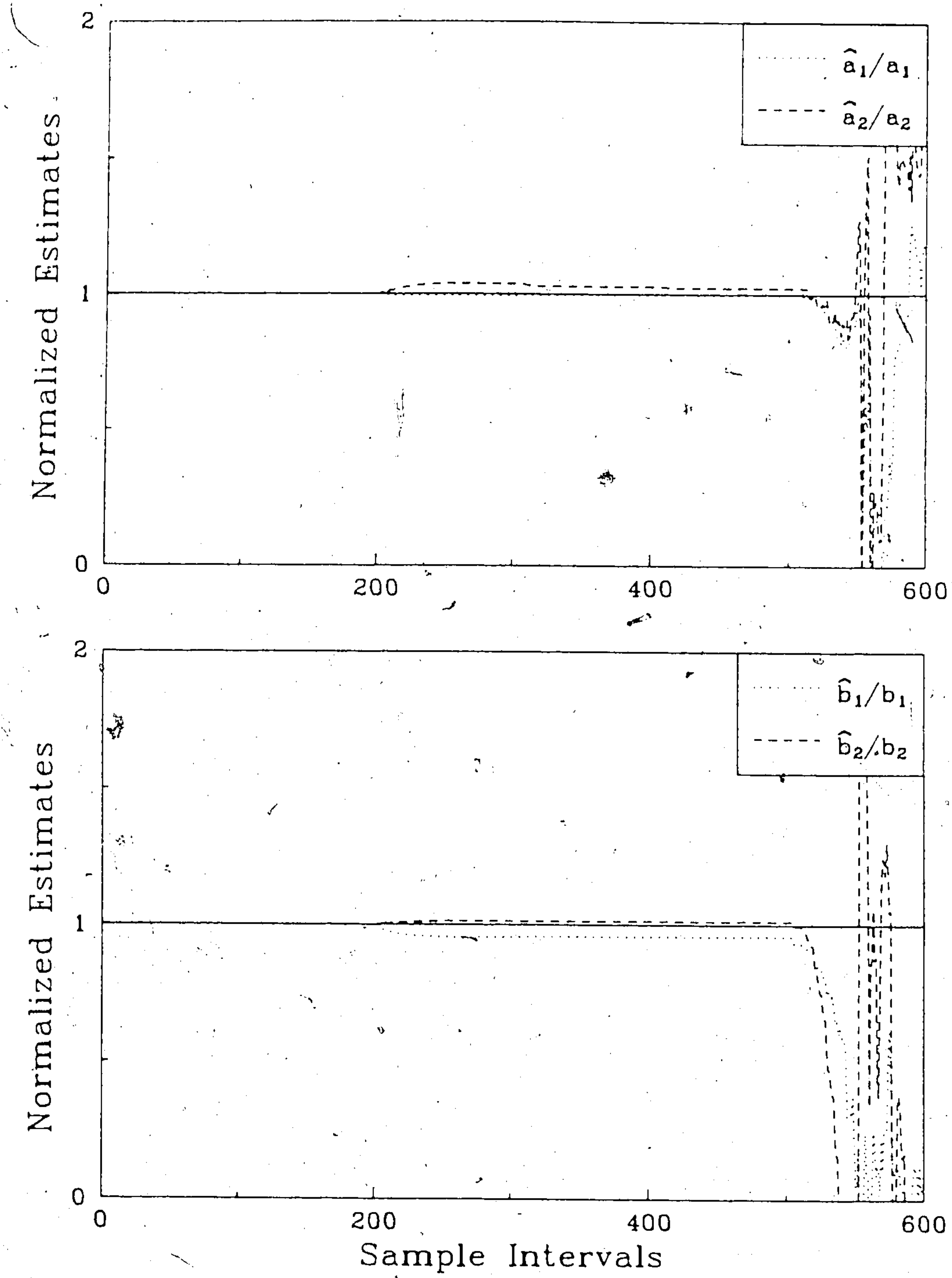


Figure 5.5f Parameter estimation using ILS ($tr\ \ P(\cdot) = 4$)

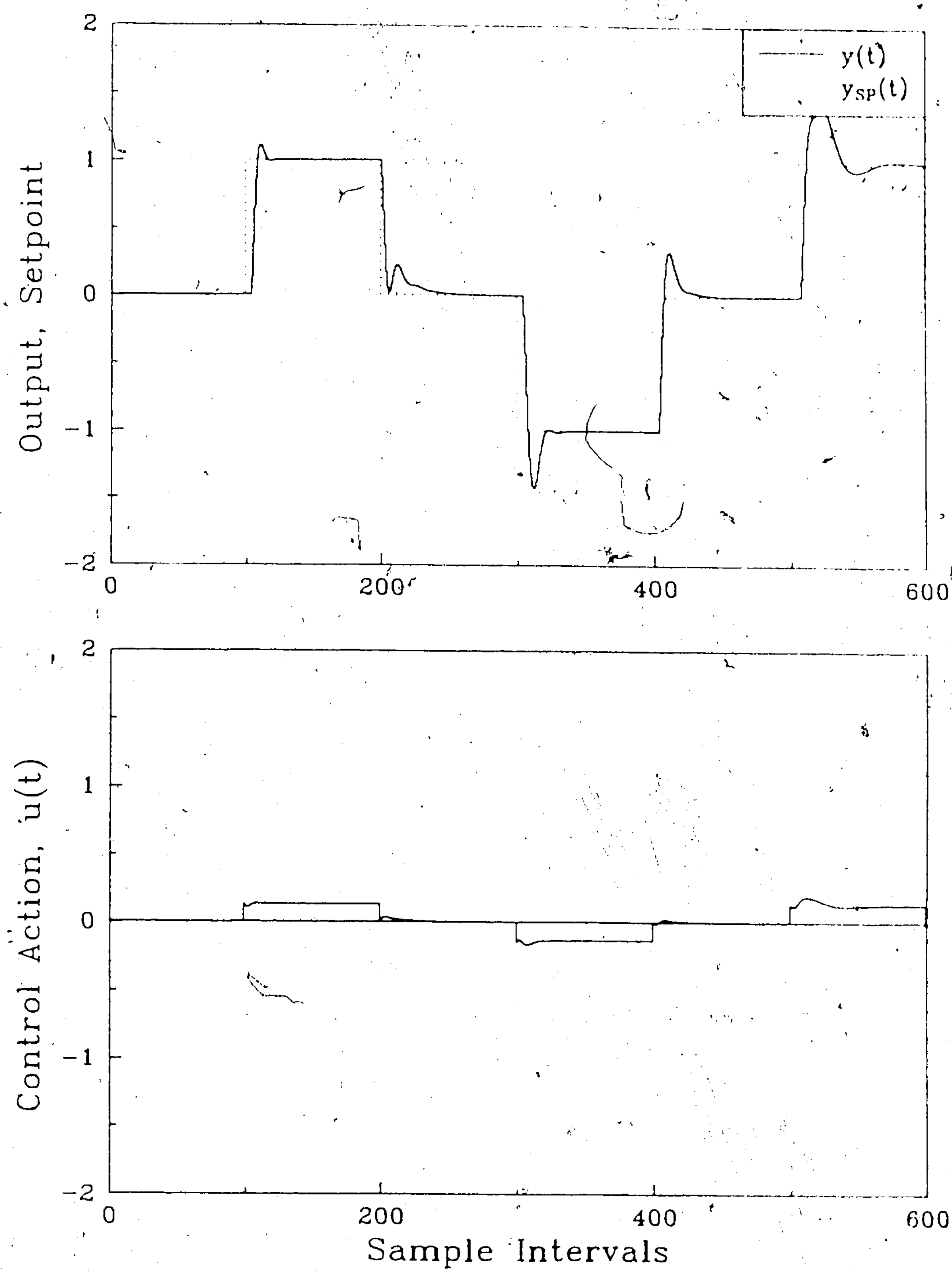
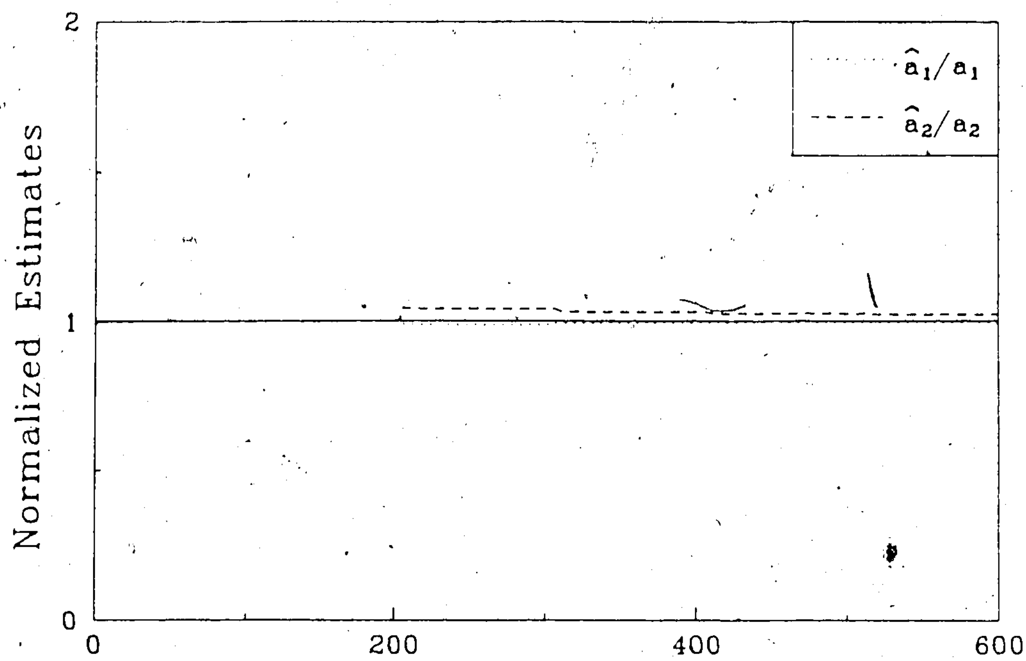


Figure 5.5g Multistep Adaptive Predictive Control of a variable time delay plant ($R_1/R_2 = 10^{-4}$)



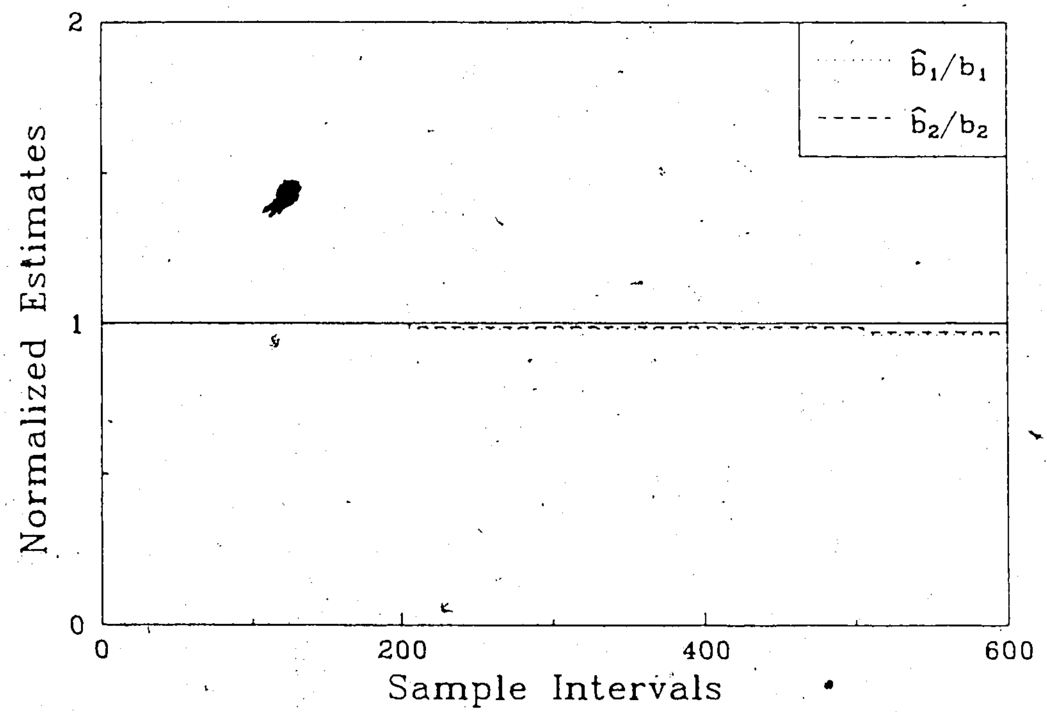


Figure 5.5h Parameter estimation using ILS ($tr\ P(\cdot) = 4$)

provided that the controller parameters are chosen in a conservative manner. It was shown that the MAPC algorithm in combination with an 'alert' estimation scheme such as Improved Least Squares can provide optimal control of plants with unknown and/or time-varying system dynamics. Additionally, the scheme proposed by Sripada (1988) for simultaneous online identification of the 'u-y' and residual models was demonstrated to be infeasible in its present form.

5.13 References

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6

Adaptive Control of a Double Effect Evaporator

6.1 Introduction

In Chapter 5, the Multistep Adaptive Predictive Control (MAPC) algorithm of Sripada (1988) was discussed with emphasis on its proposed technique for online identification of disturbance dynamic. Specifically, it was demonstrated that biased parameter estimation will generally result from the application of this scheme to systems having unknown and/or time-varying dynamics. The algorithm was therefore reformulated using the Adaptive Modified Kalman Filter Predictor of Walgama (1988), which does not involve online estimation of the disturbance mode.

This chapter presents an application of the reformulated MAPC to control of a pilot plant double effect evaporator at the University of Alberta. The algorithm is seen to provide stable control of the plant using a default configuration, provided that a "good" set of initial parameters is available.

6.2 Description of Equipment

The double effect pilot plant evaporator has been describe in detail elsewhere (see Fisher and Seborg, 1976), hence a cursory description will be given here. The evaporator is illustrated in the schematic diagram of Fig. 6.1 and a list of typical steady state operating values has been included in Appendix C.

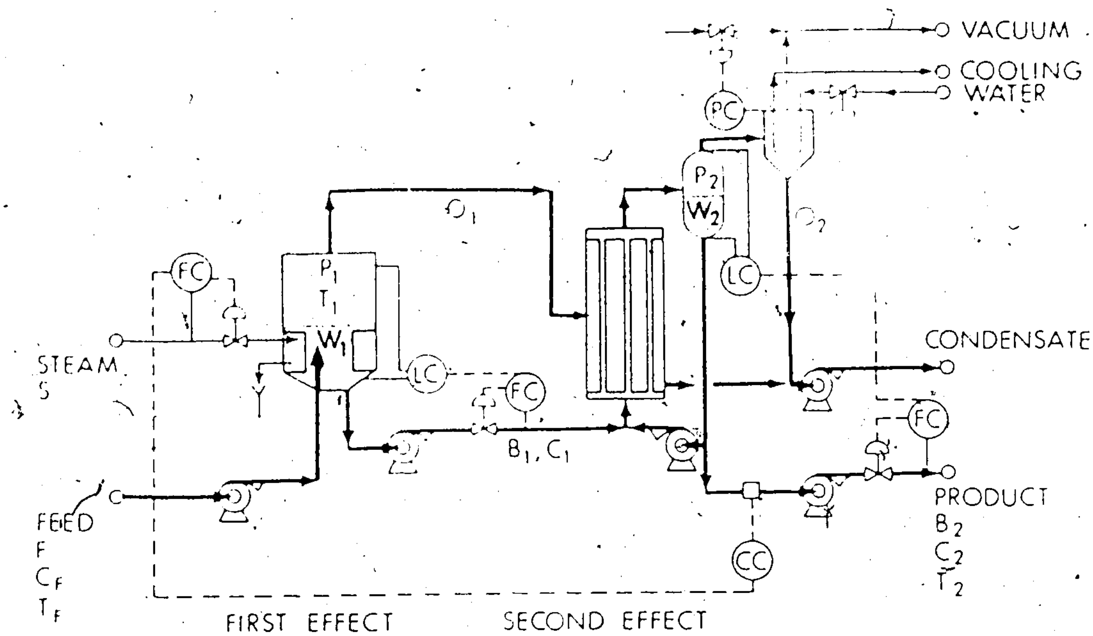

Figure 6.1 Schematic diagram of the double effect evaporator

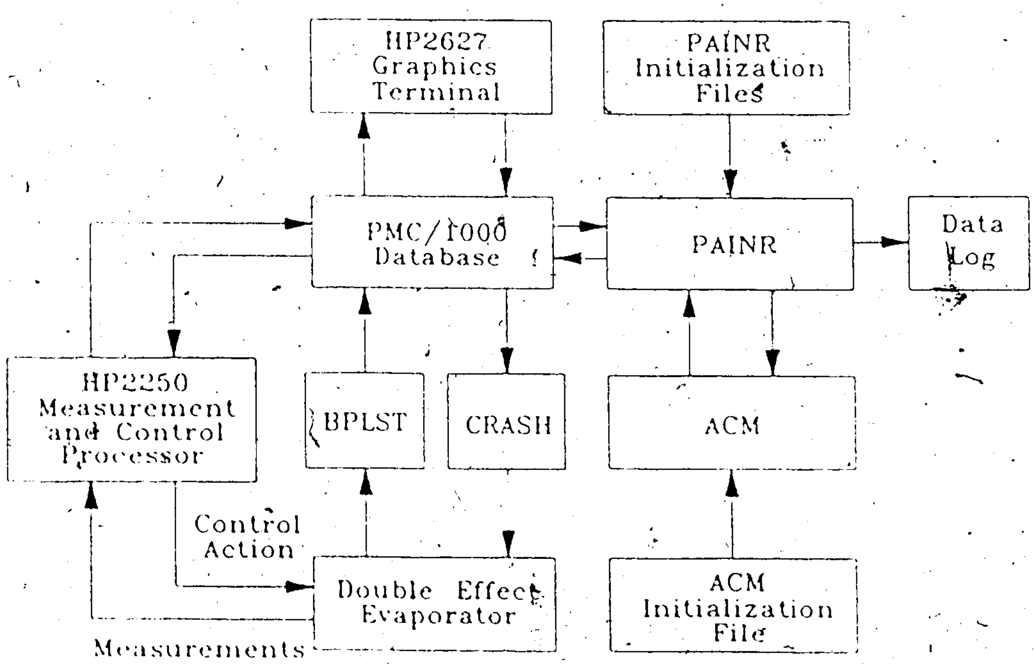

Figure 6.2 Structure of the realtime interface

The first effect of the evaporator is a natural circulation calandria-type unit with thirty-two 3/4 inch OD tubes, 18 inches long. Triethylene glycol (TEG) of known concentration is fed to the first effect and process steam is used to heat the solution. The concentrated product solution is then fed to the second effect. The second effect is a forced circulation evaporator with three 6 feet long, 1 inch OD tubes. It is operated under vacuum and is heated by the overhead vapour from the first effect.

The primary controlled variable is the second effect or product concentration C2, so a typical SISO control stategy (such as the one described in Section 6.5 below) will manipulate the steam flowrate S to control this concentration. Other variables of interest are the first (W1) and second (W2) effect mass holdups, which are cascaded to the bottoms flowrates B1 and B2, respectively.

The evaporator is fully instrumented and can be controlled either by conventional PID electronic controllers (as shown in Fig. 6.1) or under direct digital control (DDC) using a Hewlett Packard 1000/A700 computer. A process control software package, PMC (Process Monitoring and Control)/1000 supplied by Hewlett Packard is used to interface user-written FORTRAN control subroutines called Advanced Control Modules - or ACM's - to the HP2250 Measurement and Control Processor (see Fig. 6.2). The PMC database is accessed by program PAINR and is linked together with the ACM to allow pilot plant variables to be measured and/or manipulated online. User-selected variables are logged to a datafile by PAINR for later anlaysis. In addition, key process variables are displayed by PMC on the HP2267 graphics terminal (and by several smaller HP150 microcomputers) using preformatted colour displays. System parameters such as PID controller constants (or flags to be used as input to the ACM) can then be changed online via keyboard input.

Bumpless transfer from the local control panel to DDC is achieved using program BPLST, which initializes the set-points of all slave loops to the current measured values. Program CRASH is used to monitor critical process variables such as the first and second effect holdups and shuts the evaporator down if these are found to "persistently" exceed predefined upper or lower limits.

6.3 Evaporator Model

Before applying adaptive control to a real plant, it is often useful to carry out some offline analysis of plant operating data in order to obtain an initial model of the process. Figure 6.3 shows the response of TEG concentration to step changes of ±10% in steam and feed mass flowrates.

The second order ARMA model

$$(1 + a_1 z^{-1} + a_2 z^{-2}) C2(t) = (b_{11} + b_{12} z^{-1}) S(t-1) + (b_{21} + b_{22} z^{-1}) F(t-1) \tag{6.1}$$

was fit to the data using batch least squares (see Appendix A), which resulted in the estimated model

$$(1 - 0.8091 z^{-1} - 0.1565 z^{-2}) C2(t) = (2.622 + 3.821 z^{-1}) S(t-1) + (-0.9884 - 1.434 z^{-1}) F(t-1) \tag{6.2}$$

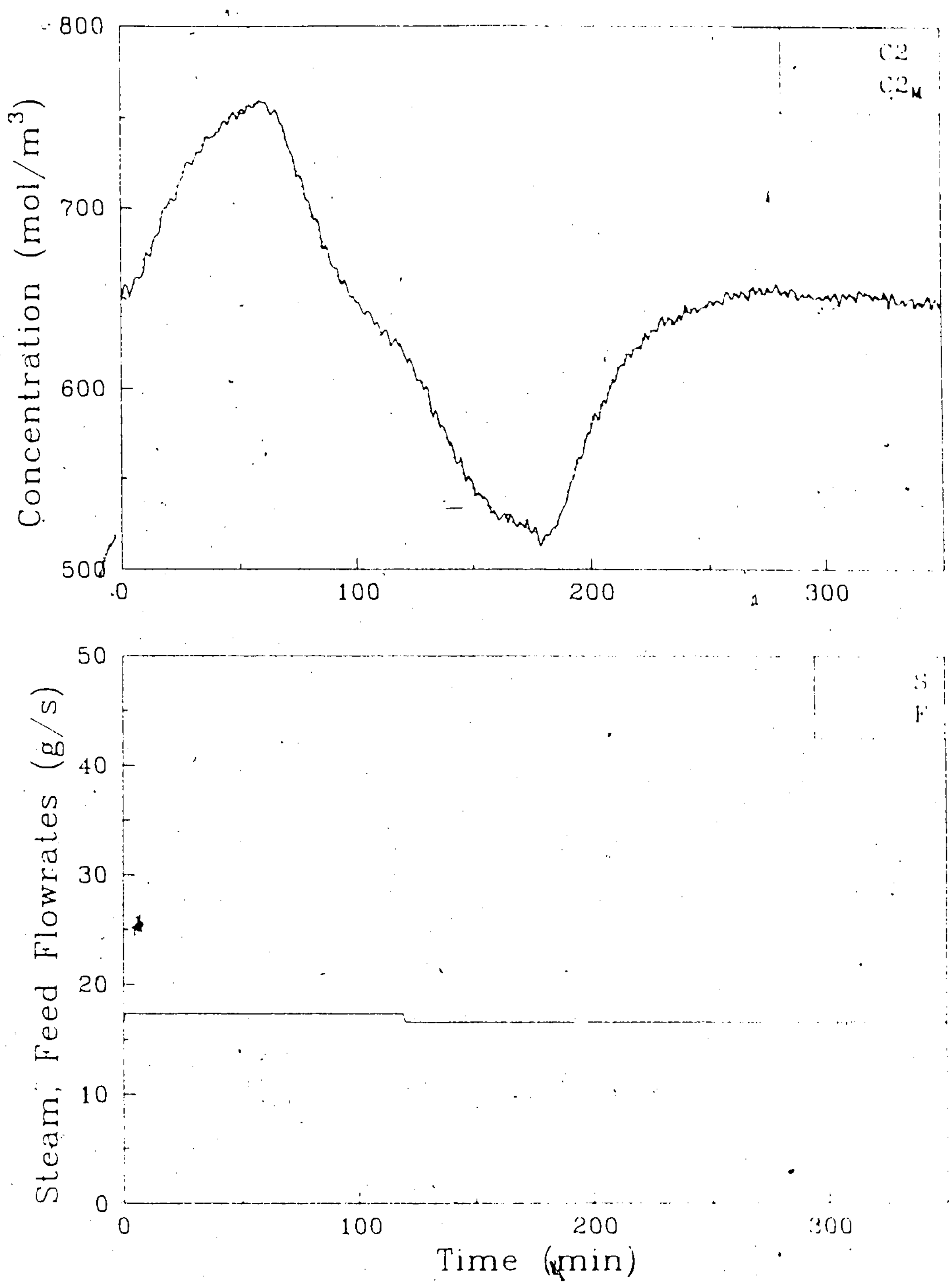

Figure 6.3 Actual and predicted C2 step response data

Note that the relationship between C2 and S is nonminimum phase due to the presence of the unstable zero at $z=-1.457$. The integrity of the model was tested by comparing the actual C2 with that generated using Eqn. (6.2), *i.e.*

$$\begin{aligned} C2_M(t) = {} & 0.8091C2(t-1)+0.1565C2(t-2) \\ & +2.622S(t-1)+3.821S(t-2) \\ & -0.9884F(t-1)-1.434F(t-2) \end{aligned} \tag{6.3}$$

which is also plotted in Fig. 6.3. It is seen from the figure that these values are essentially coincident with those observed experimentally.

6.4 Multistep Adaptive Predictive Control

This section presents a brief summary of the Multistep Adaptive Predictive Control scheme of Sripada (1988), which has been discussed extensively in earlier chapters. The algorithm minimizes the following multistep quadratic cost functional:

$$\begin{aligned} J = {} & \sum_{i=N_1}^{N_2} \{y_{sp}(t+i)-\hat{y}(t+i|t)\}^2 \gamma_{yi}(t) \\ & + \sum_{i=1}^{N_u} \{\Delta u(t+i-1)\}^2 \gamma_{ui}(t) \end{aligned} \tag{6.4}$$

where $\{\hat{y}(t+i|t),\ i\in[N_1,N_2]\}$ is the trajectory of predicted future outputs. The $\gamma_{yi}(t)$ and $\gamma_{ui}(t)$ represent weights on the output deviations and changes in the input variable, respectively. For convenience in the sequel, $\gamma_{yi}(t)$ will be set equal to unity and $\gamma_{ui}(t)$ to an arbitary constant λ.

The state space model of the process is given by

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \Gamma n_1(t) \tag{6.5}$$

$$y(t) = Hx(t) + n_2(t) \tag{6.6}$$

where for $d = k-1$, *i.e.* the physical delay of the process,

$$x(t) = [x_p(t), x_1(t), \ldots, x_n(t), x_{n+1}(t), \ldots, x_{n+d}(t)]^T$$

$$\Phi = \begin{bmatrix} 1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\ \gamma_1 & 0 & \ldots & 0 & -a_n & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \gamma_n & 0 & \ldots & 1 & -a_1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}_{(n+k)\times(n+k)}$$

$$\Lambda = [0, b_n, \ldots, b_1, 0, \ldots, 0]^T$$

$$\Gamma = [1, 0, \ldots, 0, 0, \ldots, 0]^T$$

$$H = [0, 0, \ldots, 0, 0, \ldots, 1]$$

$n_1(\cdot)$ and $n_2(\cdot)$ are uncorrelated zero-mean Gaussian noise sequences having covariances R_1 and R_2 respectively. The state vector is not directly measurable and so is reconstructed by means of the Kalman filter update (see Appendix A):

$$\hat{x}(t+1 \mid t+1) = \hat{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) + L(t+1)\omega(t+1) \tag{6.7}$$

where the innovations sequence $\omega(\cdot)$ is defined as

$$\omega(t+1) = y(t+1) - H\hat{x}(t+1 \mid t) \tag{6.8}$$

By innovations analysis for the steady state KF (Appendix B), it was found that an equivalent ARIMA representation for (6.5), (6.6) is given by

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\frac{\omega(t)}{\Delta} \tag{6.9}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \ldots + b_n z^{-n+1}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \ldots + c_{n+d} z^{-n-d}$$

(The coefficients c_1 to c_{n+d} in (6.9) are complex functions of $A(z^{-1})$ and the steady state Kalman gain vector, L.)

Equation (6.9) can be rewritten in the compact form

$$\Delta y^F(t) = \phi^T(t)\theta + \omega(t) \tag{6.10}$$

where

$$\phi(t) = [-\Delta y^F(t-1), \ldots, -\Delta y^F(t-n), \Delta u^F(t-k), \ldots,$$
$$\Delta u^F(t-k-n+1)]^T$$

$$\theta = [a_1, \ldots, a_n, b_1, \ldots, b_n]^T$$

and the superscript "F" denotes division by $C(\quad)$. A least squares estimate of θ may then be obtained online using the constant trace Improved Least Squares technique of Sripada and Fisher (1987), which was analyzed in Chapter 3. Note that as $C(z^{-1})$ is generally unknown and/or time-varying, it may be replaced by the *ad hoc* observer polynomial $T(z^{-1})$ (Clarke et al, 1987). Combination of a recursive parameter estimation algorithm such as ILS with the Kalman filter state update given by Eqn. (6.7) results in the Adaptive Modified Kalman Filter Predictor of Walgama (1986).

Equation (6.4) can be written in vector/matrix form as follows:

$$J = [Y_{sp}(t) - \hat{Y}(t)]^T [Y_{sp}(t) - \hat{Y}(t)] + \lambda \Delta U(t) \Delta U(t) \quad (6.11)$$

where $Y_{sp}(t) = \{y_{sp}(t+i|t),\ i \in [N_1, N_2]\}$,

$\hat{Y}(t) = \{\hat{y}(t+i|t),\ i \in [N_1, N_2]\}$ and $\Delta U(t) = \{\Delta u(t+i-1),\ i \in [1, N_u]\}$.

Differentiating (6.11) with respect to $\Delta U(t)$ and setting $\partial J / \partial \Delta U(t) = 0$ leads to the optimal control trajectory

$$\Delta U(t) = A^{*}\{Y_{sp}(t) - Y^{*}(t)\} \quad (6.12)$$

where

$$A^{*} = (A^T A + \lambda I)^{-1} A^T \quad (6.13)$$

and the vector $Y^{*}(t)$ is given by

$$Y^{*}(t) = \{y^{*}(t+i|t),\ i \in [N_1, N_2]\}$$
$$= \{H \Phi^{i} \hat{x}(t|t),\ i \in [N_1, N_2]\}$$
$$+ \left\{ \left(\sum_{j=1}^{i} a'_{i-N_1+1, j} \right) u(t-1),\ i \in [N_1, N_2] \right\}$$

The $a'_{i,j}$ are elements of the matrix A', which is generally a lower triangular matrix consisting of the first $N = N_2 - N_1 + 1$ impulse response coefficients of the process, which are in turn obtained by deconvolution of $\hat{B}(z_o^{-1})/\hat{A}(z^{-1})$ in (6.9). The matrix A in (6.13) is then given by $A = A'S$, where S is an $N_2 \times N_u$ matrix defined by

$$s_{ij} = 0, \quad i < j$$
$$s_{ij} = 1, \quad i \geq j$$

The control action is implemented in a receding horizon fashion, hence only the first row of A^{*} has to be calculated at each control interval.

Refer to McIntosh (1988) for a full discussion regarding choice of the parameters N_1, N_2, N_u and λ in Generalized Predictive Control (GPC). These guidelines apply equally well to the MAPC scheme since it was shown in Chapter 4 that MAPC and GPC provide asymptotically equal control of known, time-invariant processes of the form of Eqn. (6.9).

6.5 Experimental Results

The fixed parameter version of the MAPC algorithm was first used to control second effect concentration by manipulating the steam flowrate (*i.e.* by replacing Y with C2 and U with S in (6.12)). This implies that the initial C2 to S relationship given in Eqn. (6.2) was used throughout the duration of the run. The results obtained using the default MAPC configuration ($N_1 = N_u = 1$, $N_2 = 10$, $\lambda = 0$) with $R_1/R_2 = 0.1$ are shown in Fig. 6.4. The setpoint was varied as a square wave of amplitude ±10%, and a step decrease in feed flow was introduced at time 130. The servo/regulatory behaviour in this example is seen to be stable, if somewhat sluggish. Furthermore, it is apparent that offset-free regulation of the process was achieved despite the nonstationary nature of the (unmeasured) feed disturbance.

Figure 6.5 illustrates the effect of incorporating feed-forward (FF) control of the feed flowrate (*cf.* Section 5.10) to the control strategy of Fig. 6.4 using the estimated model in Eqn. (6.2). It is evident that both servo and regulatory behaviour were improved by the addition of the

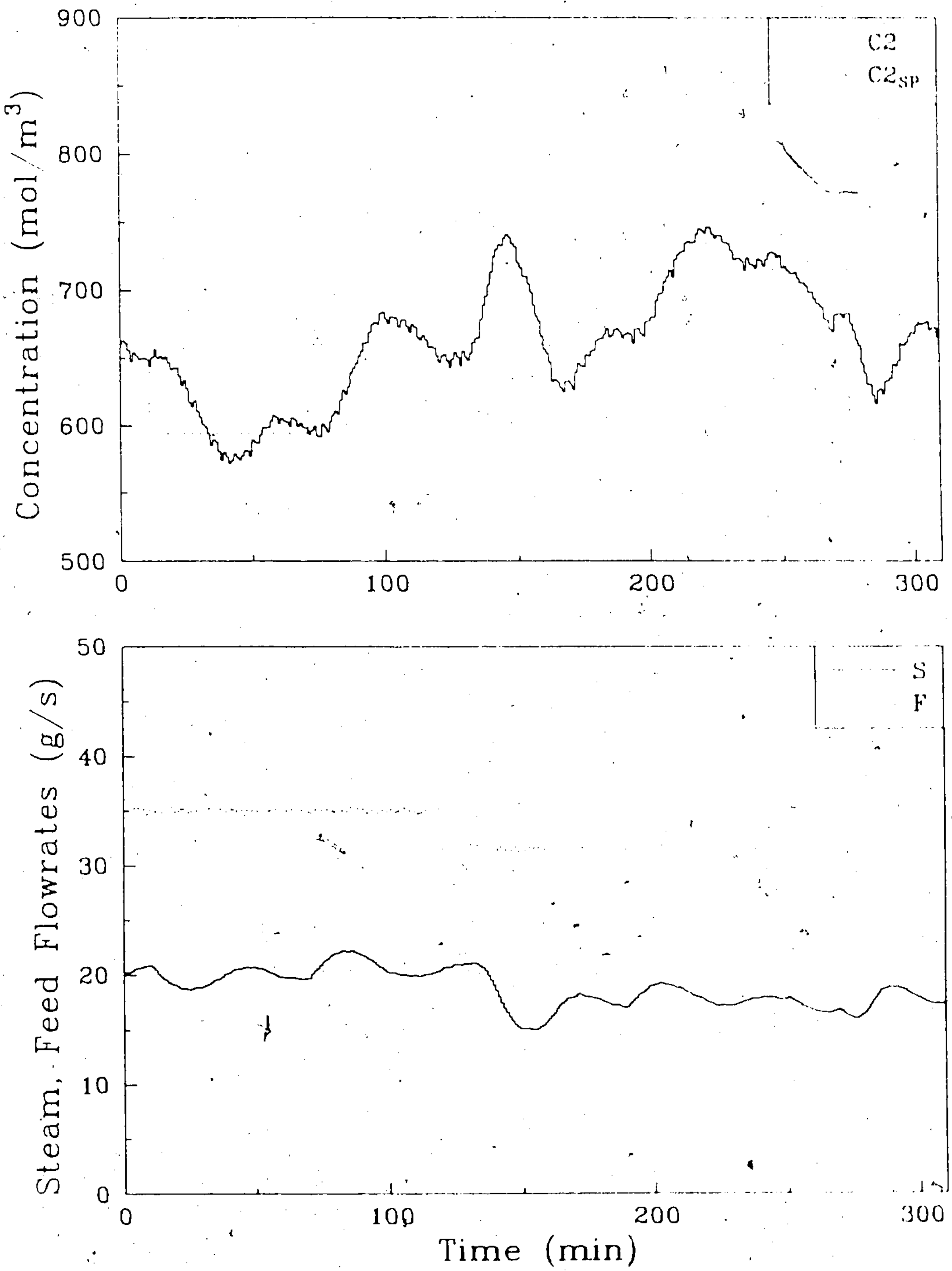

Figure 6.4 Fixed parameter Multistep Adaptive Predictive Control (default settings)

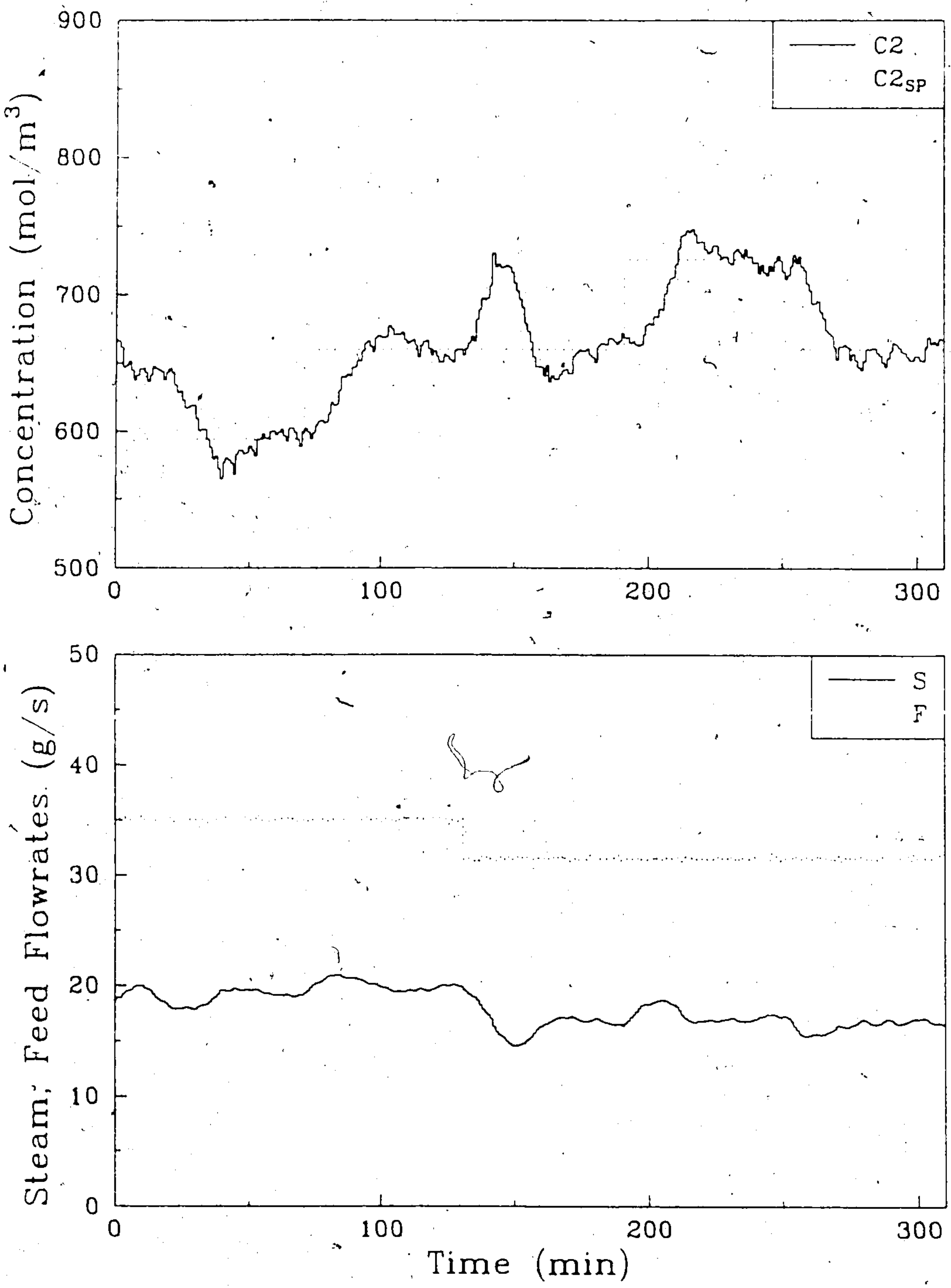

Figure 6.5a Fixed parameter MAPC with feedforward control of the feed flowrate

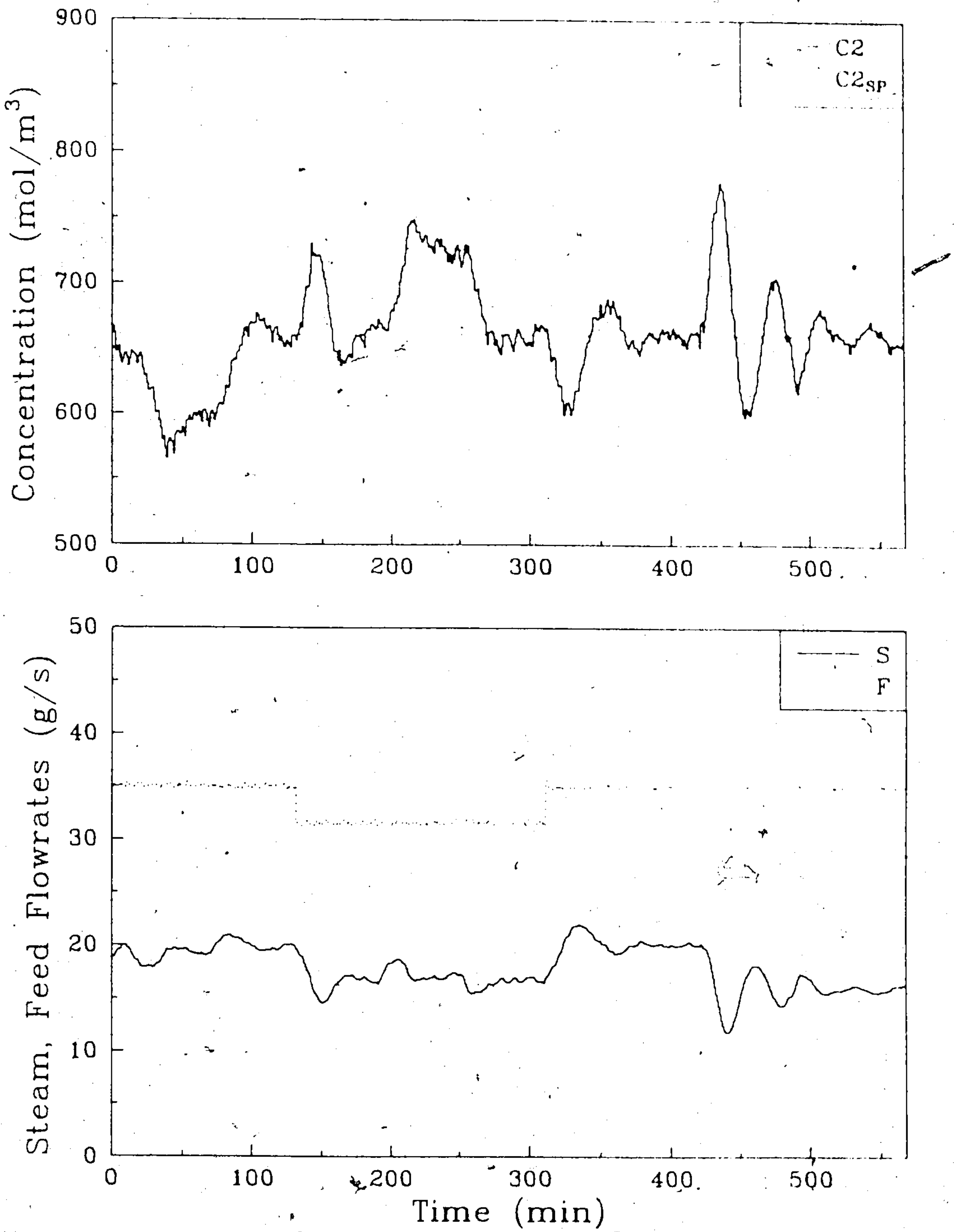

Figure 6.5b Fixed parameter MAPC regulatory response to change in steam supply pressure

feedforward mode. A smaller deviation in C2 from the setpoint is observed when the step change occurs in F because the controller anticipates an increase in C2. That is, the steam flowrate in Fig. 6.5a is seen to decrease immediately after the change in feed, as opposed to Fig. 6.4 where the controller waits several minutes before reacting, *i.e.* until it observes an increase in C2.

Figure 6.5b is a continuation of Fig. 6.5a, and has been included to demonstrate the response of MAPC to a very realistic type of plant disturbance. The first 310 minutes of operation are identical to the results of Fig. 6.5a; at time 310, the feed flowrate is increased back to its steady state level of 35 g/s, producing a response similar in shape to that caused by the step decrease after 130 min. At time 425, the steam supply pressure was increased by approximately 30%, which was seen to change the gain of the process considerably (*cf.* steady state steam levels before and after t=425). The fixed parameter MAPC scheme was seen to retain control of the process despite the presence of significant model/process mismatch.

In Fig. 6.6, the adaptive version of MAPC (with FF) was tested on the process using the initial controller parameters given above. The parameters of Eqn. (6.1) were updated using Improved Least Squares with $P(0)=0.1I_6$, and $T(z^{-1})=1$ *i.e.* no T-filtering was used. In addition, the on/off parameters Δ and C_{max} (*cf.* Section 3.2.2) were set to 10^{-6} and 10^{6}, respectively, so that the estimation algortihm was on at all

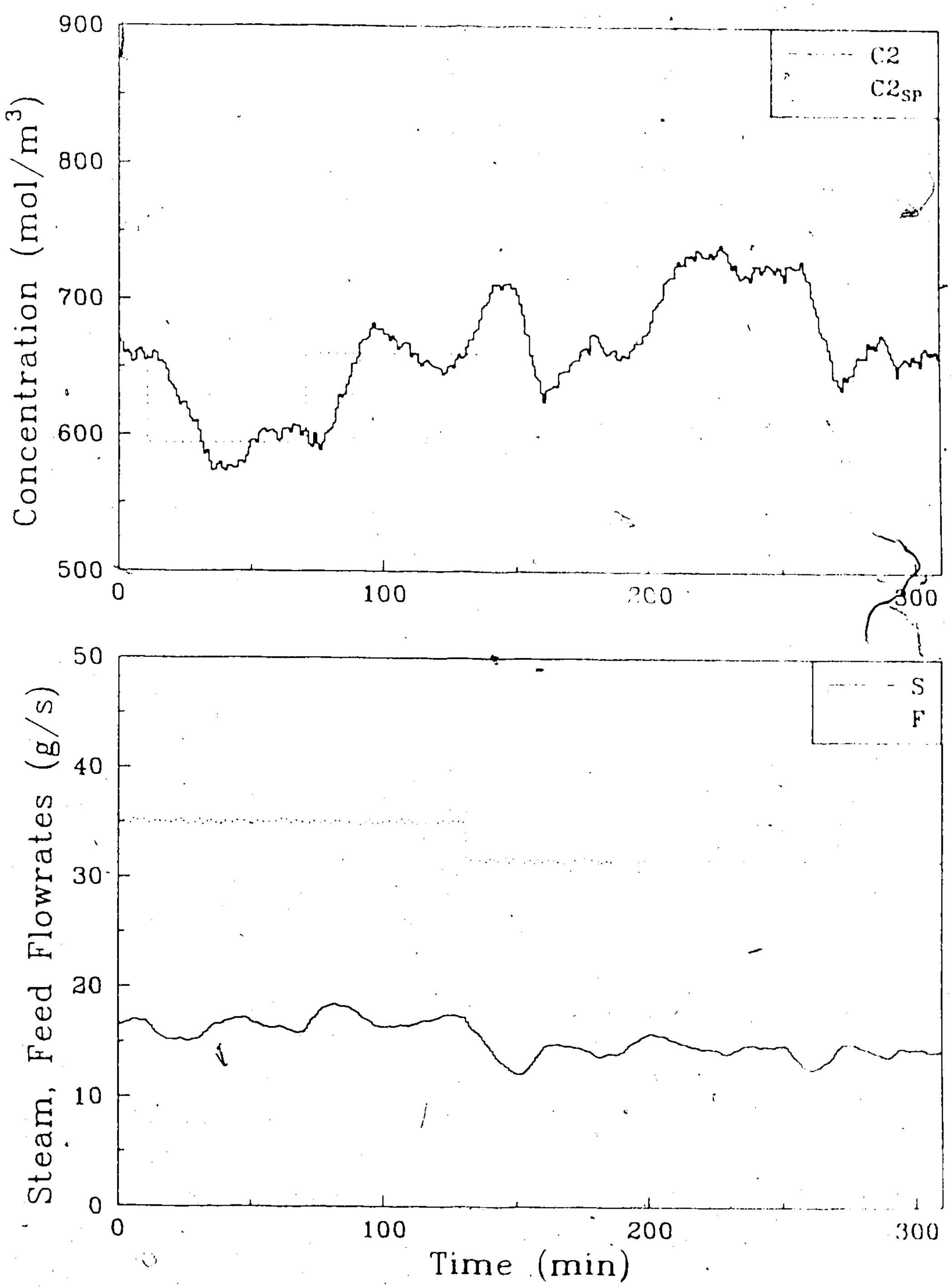

Figure 6.6a Multistep Adaptive Predictive Control
($tr\ P(\cdot) = 0.6$)

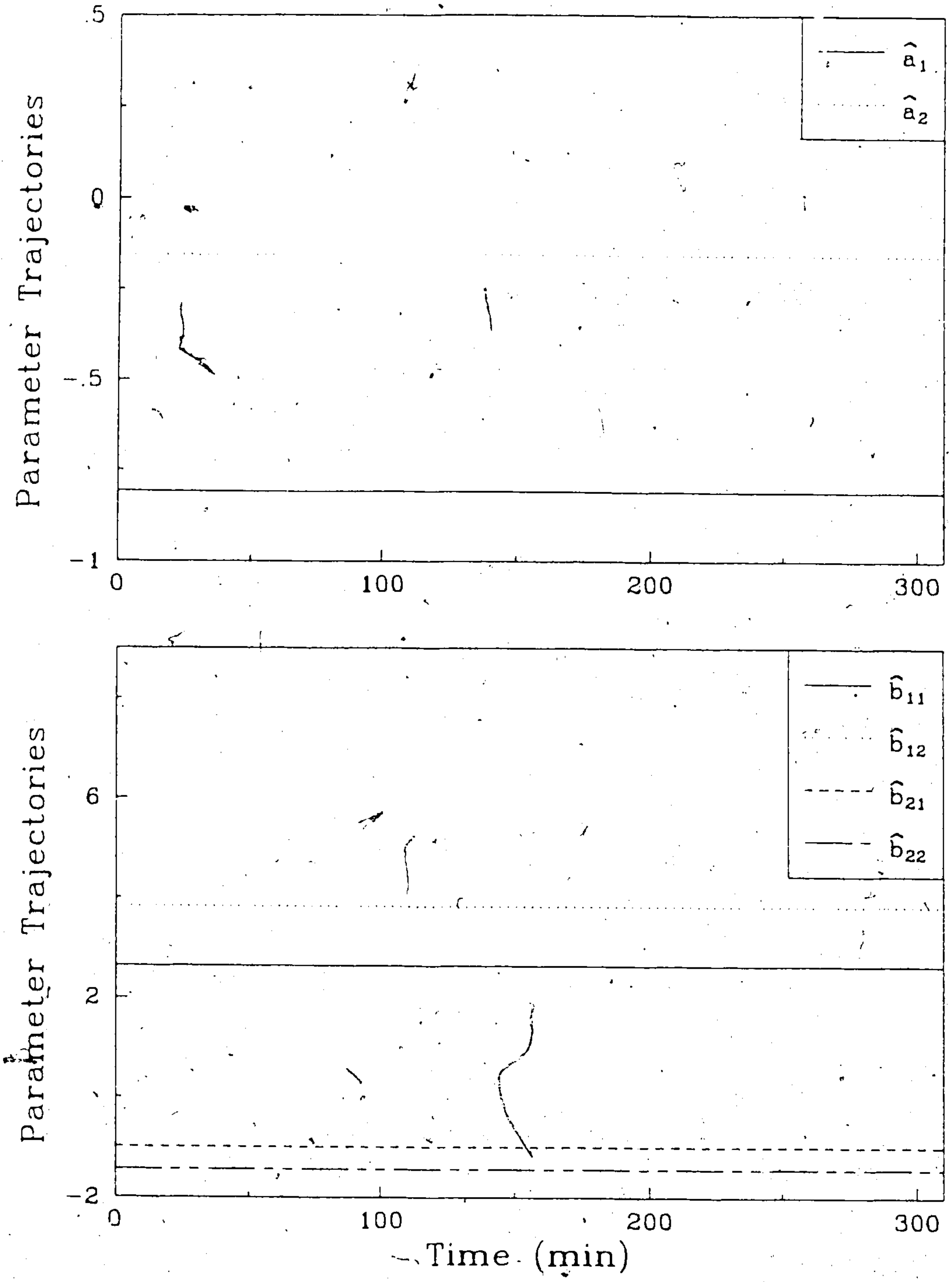

Figure 6.6b Parameter estimation using ILS

times. The results of Fig. 6.6a are seen to be very similar to those of Fig. 6.5a, which is reasonable given the fact that the parameters in Fig. 6.6b barely deviate from their initial values.

To see whether a larger estimator gain would yield an improvement in performance, the covariance matrix was initialized in Fig. 6.7 to $P(0) = 10I_6$, *i.e.* $tr\ P(t) = 60\ \forall t$. This change seemed to have very little effect on C2 control, as the parameters remained essentially constant despite the increase of $tr\ P(t)$ by a factor of 100. This would seem to indicate that the initial parameters obtained from offline analysis were indeed representative of the true process dynamics.

To test the self-tuning capability of the MAPC scheme on a real plant, the controller was implemented using the above default settings with the initial parameter vector $\theta(0) = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1]^T$ (*cf.* Eqn. (6.10)). Once again, the trace of the covariance matrix was maintained at 60 for the duration of the run. The control action was held constant for the first seven minutes of operation in order that the regressor vector, *etc.* would contain "good" values when the controller was turned on. From Fig. 6.8a, one can see that the immediate reaction of the algorithm upon closing the loop was to manipulate the steam flowrate in an unstable manner, despite a good deal of movement in the estimated parameters (see Fig. 6.8b). The steam was observed to saturate at its upper limit for t >23 min, which eventually

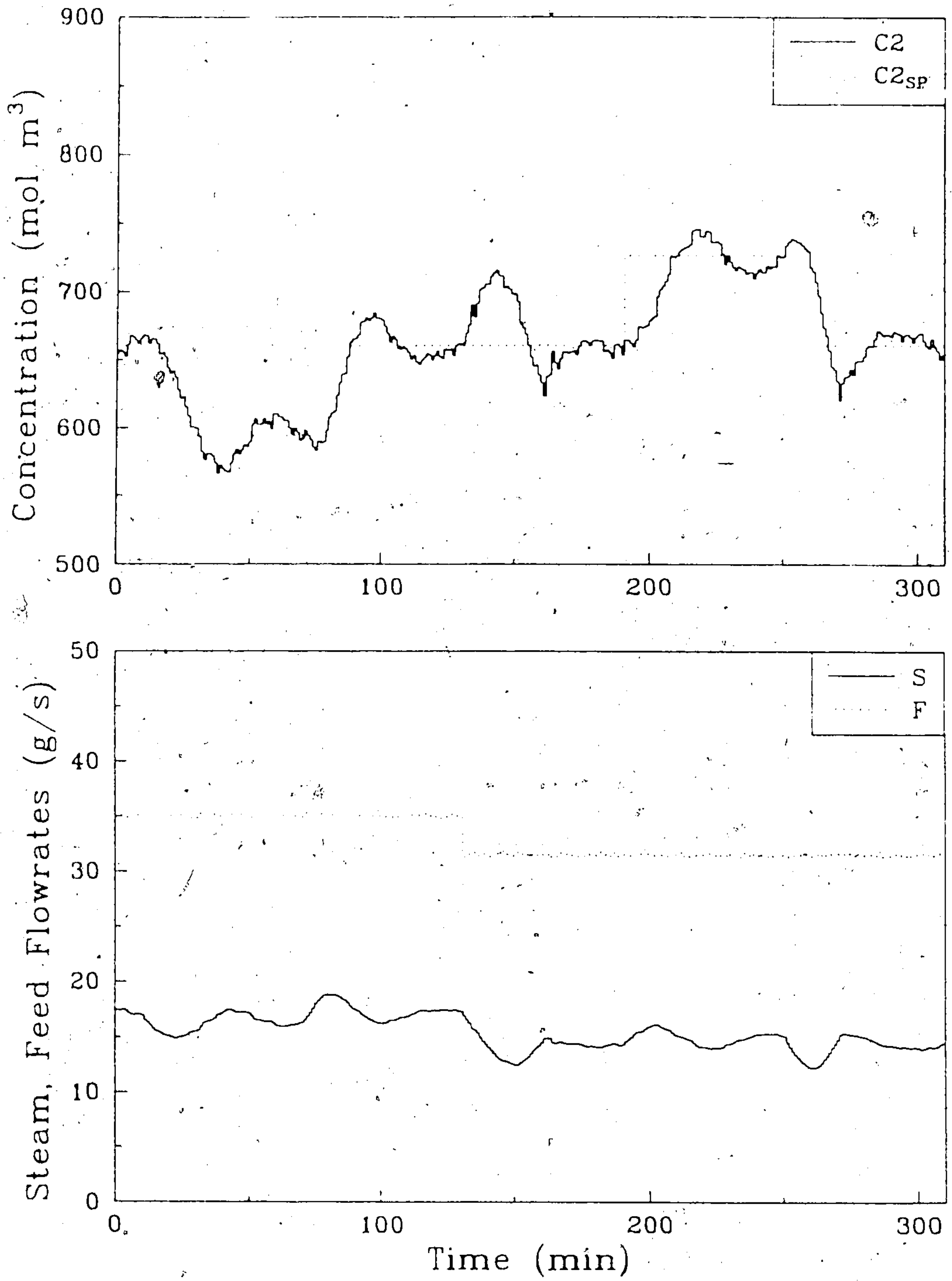

Figure 6.7a Multistep Adaptive Predictive Control

$(tr\ P(\cdot) = 60)$

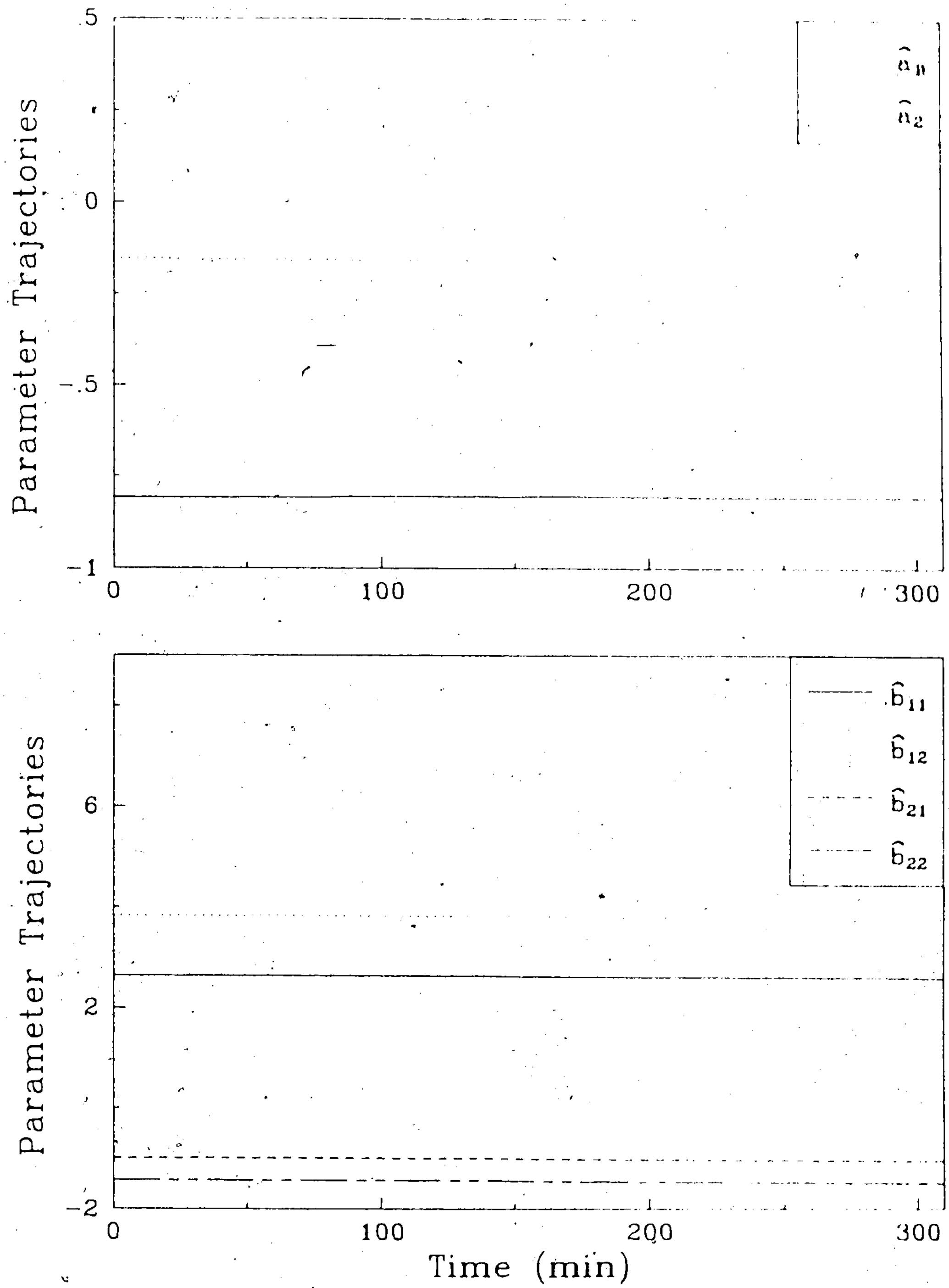

Figure 6.7b Parameter estimation using ILS

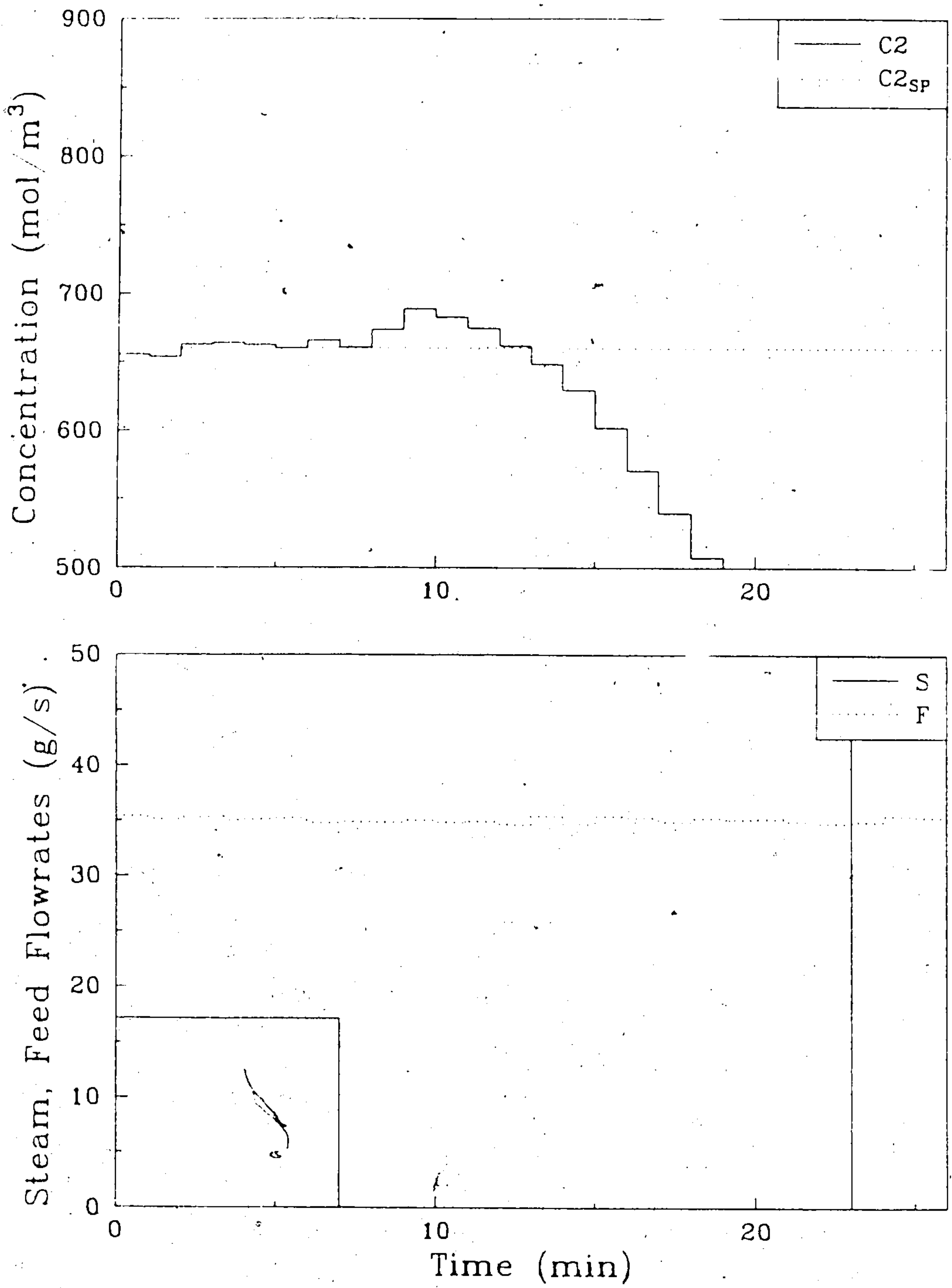

Figure 6.8a Multistep Adaptive Predictive Control
($\hat{\theta}(0) = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1]^T$)

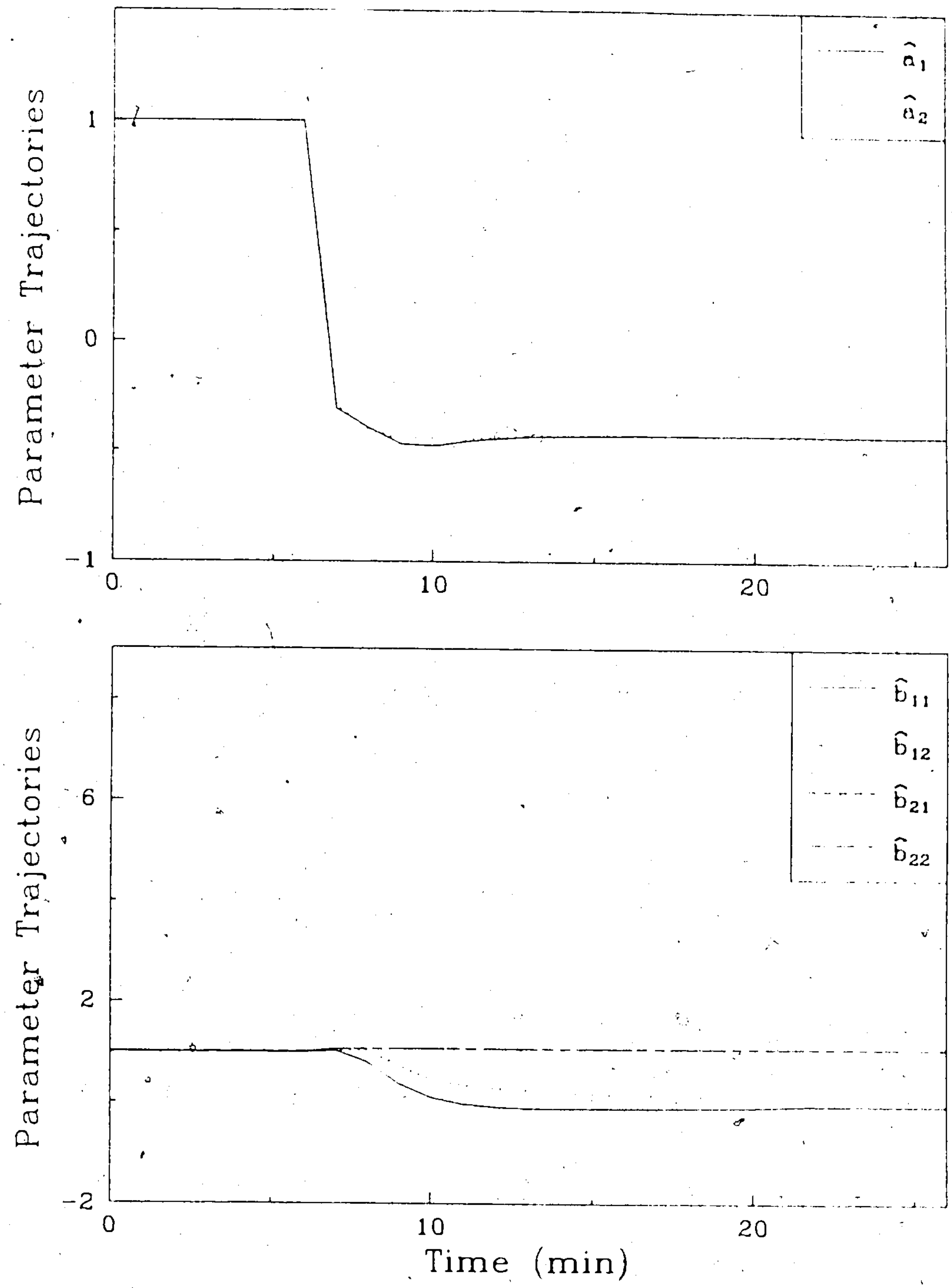

Figure 6.8b Parameter estimation using ILS

caused the pressure relief valve in the first effect to vent to the atmosphere, requiring shutdown of the evaporator. Thus, the controller was unable to bring the process under control with no *a priori* knowledge of the plant parameters.

Finally, an alternate controller design was employed in an effort to improve upon the performance obtained above for the default configuration. Figure 6.9 illustrates the results for $N_1 = N_2 = 10$, $N_u = 1$, $\lambda = 0$ and $R_1/R_2 = 0.1$. (Note that this strategy is analogous to the Extended Horizon Adaptive Controller of Ydstie (1984).) The servo response is seen to be much tighter than that of Figs. 6.4 through 6.7 and is in fact comparable to that which has been obtained using well-tuned PI. The advantage of the MAPC approach in this respect is that these design parameters were obtained with a minimum of effort, while tuning the PI controller online involves a 4-5 day trial and error procedure. Furthermore, the effect of the feed disturbance is almost entirely cancelled in Fig. 6.9a, due to the less conservative (and consequently less robust) nature of this design.

6.6 Conclusions

This chapter has documented the application of the Multistep Adaptive Predictive Control technique of Sripada (1988) to control of the product concentration of the double effect evaporator at the University of Alberta. Stable, offset-free control of the process was obtained using a

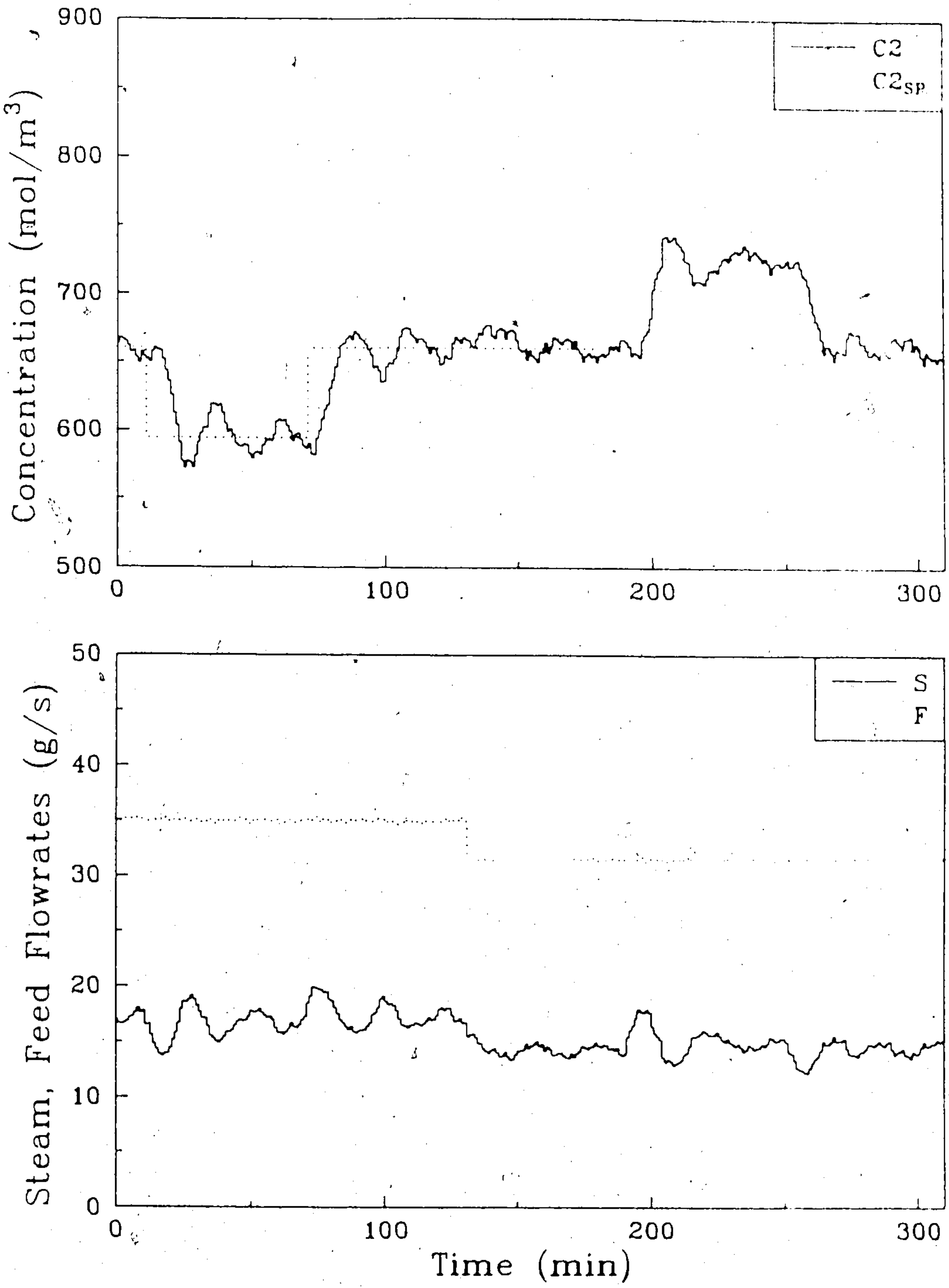

Figure 6.9a Multistep Adaptive Predictive Control
($N_1 = N_2 = 10$)

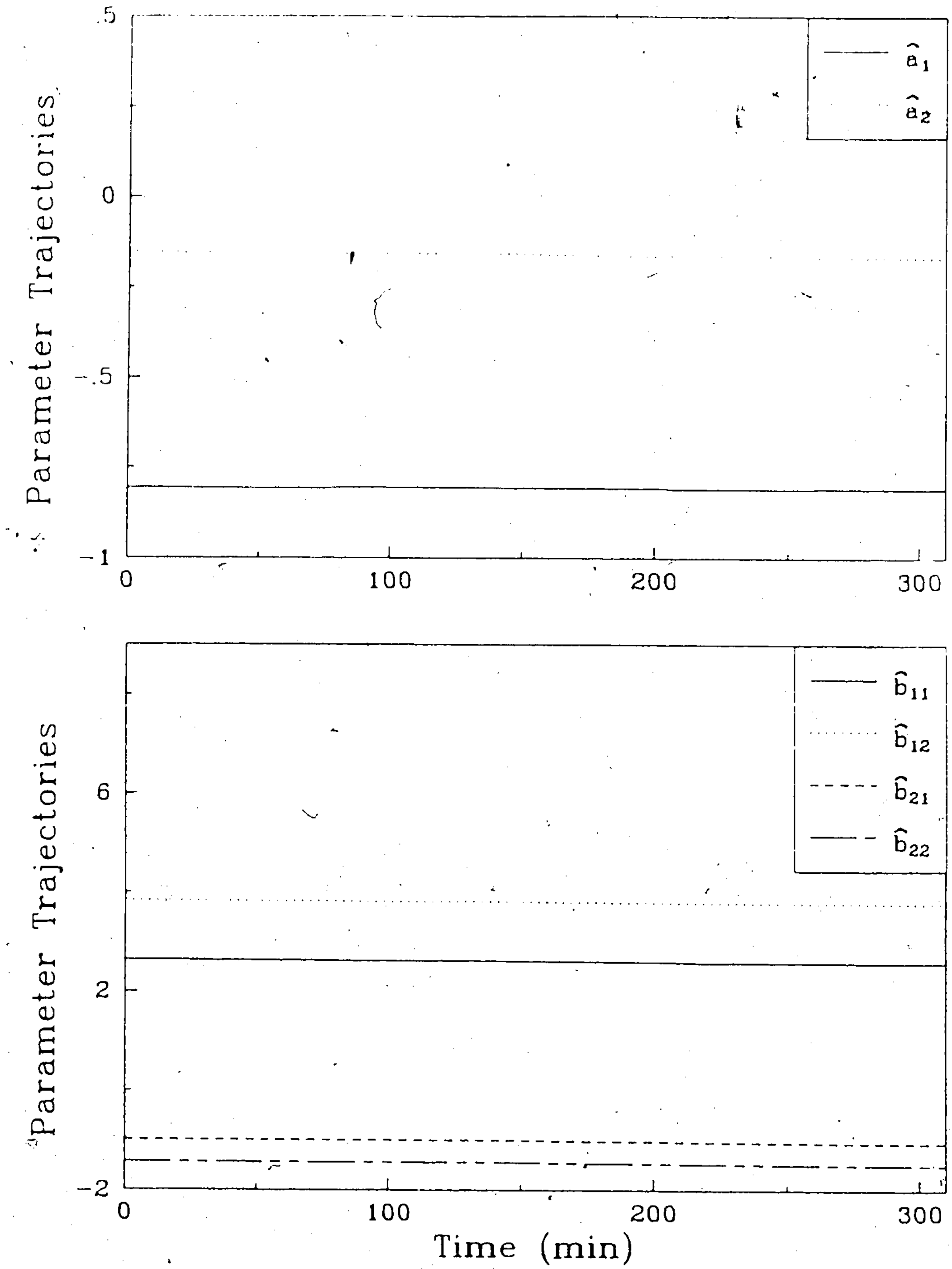

Figure 6.9b Parameter estimation using ILS

default controller configuration in the presence of nonstationary disturbances, nonlinearities and model/plant mismatch. It was found, however, that it may be unrealistic to commission adaptive controllers without some *a priori* knowledge of the plant (*e.g.* from a least squares analysis of step response data) since the initial period of instability will cause most industrial plants to shut down before a stable set of parameters can be obtained by the estimation algorithm.

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7

Overall Conclusions and Future Work

7.1 Conclusions

The main contribution of this thesis is in the analysis and proof of equivalence of several estimation and/or adaptive control algorithms based on state space techniques versus those based on transfer function design methods. For example, it was shown that minimum variance controllers implemented using a Kalman filter are asymptotically equal to those based on a least squares approach, *e.g.* via solution of a Diophantine equation. Although the result was previously proved by Watson (1976) for the general case, his derivation was cumbersome and difficult to generalize to other state space realizations.

Other contributions made during the course of this work are listed below:

1. The equivalence result was extended to include plants subject to nonstationary stochastic disturbances.
2. A Generalized Minimum Variance-type cost functional derived by Lu (1986) was presented for use with the Modified Kalman Filter Predictor of Walgama et al (1988). The equivalence of this scheme with the integrating Self-Tuning Controller of Tuffs and Clarke (1985) was demonstrated for general $Q(z^{-1})$, $R(z^{-1})$ and scalar $P(z^{-1})$. When a nonscalar choice of $P(z^{-1})$ was made, the two schemes were shown to minimize different cost functionals.

3. An adaptive control law based on the MKFP was introduced by combining the MKFP and the GMV-type control law of Lu (1986) with the Improved Least Squares algorithm of Sripada and Fisher (1987). A correction was made to the ILS forgetting factor calculation to ensure that the trace of the unscaled covariance matrix remains constant.
4. The Multistep Adaptive Predictive Controller (MAPC) proposed by Sripada (1988) is a long-range predictive control strategy based on a Kalman Filter Predictor that incorporates full disturbance modelling. It was found, however, that it is difficult to obtain unbiased estimates of the "u-y" model parameters in the presence of disturbances; hence an alternative approach was proposed based on the MKFP of Walgama et al (1988).
5. The equivalence of the MAPC scheme (based on the MKFP) with GPC (Clarke et al, 1987) was demonstrated for time-invariant plants subject to nonstationary stochastic disturbances, by a further extension of the minimum variance result. The discussion was then broadened somewhat to include DMC (Cutler and Ramaker, 1980) and MOCCA, two nonparametric LRPC techniques. It was shown that MOCCA with a polynomial disturbance generator (Sripada and Fisher, 1985) is <u>equal</u> to GPC (and, by association, equivalent to MAPC) provided that the entire reaction curve of the plant is captured when the step or impulse response data are collected. MOCCA with a KFP (Li et al, 1988; Navratil et al, 1988), on the other

hand, was found to be equivalent to GPC, and different from MAPC because the two algorithms implicitly differ in their assumptions regarding the dynamics of the plant disturbances.

6. The robustness of the MAPC approach was demonstrated by a series of experimental trials on the double-effect evaporator at the University of Alberta. The algorithm provided stable offset-free control in the presence of both unmeasured and measured disturbances using a default controller configuration.

7.2 Future Work

1. It is important for MAPC to be compared with GPC on an experimental basis, vis a vis ease of commissioning, sensitivity of controller parameters, robustness in the presence of nonlinearities and model/plant mismatch, etc.
2. The robustness of the Kalman filter approach should be further analyzed on a theoretical level as well, in order to determine whether this design offers any benefit over the conceptually (and computationally) simpler transfer function techniques.
3. It may be interesting to recast the equivalence analyses presented in Chapters 2 and 4 in the form of the Internal Model Control approach of Garcia and Morari (1982). This would enable the MAPC and GPC approaches to be unified with the larger class of algorithms known as Model Predictive Control.

4. The success of the experimental evaluation of Chapter 6 seems to justify an extension of MAPC to the multivariable case. This topic was discussed briefly by Sripada (1988) for the KFP with full disturbance modelling, but the revised MAPC should be reformulated for MIMO systems with a view to implementation on a real plant.

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Appendix A

Optimal Parameter Estimation

The Kalman Filter was originally presented by Kalman (1960, 1963) and Kalman and Bucy (1961) as a method of obtaining an optimal estimate of the state vector in the presence of process and measurement noise. This appendix will follow the example of Franklin and Powell (1980) and derive the Kalman filter (KF) by generalization of the Recursive Least Squares (RLS) parameter estimation technique. Alternative derivations of the KF can be found in for example, Åström and Wittenmark (1984) and Goodwin and Sin (1984).

A.1 Batch Least Squares

Consider the n'th order ARMA process representation

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + \omega(t) \tag{A.1}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \ldots + b_n z^{-n+1}$$

$y(\cdot)$ and $u(\cdot)$ are the plant output and input, respectively. The plant time delay k is the sum of the physical plant delay (d) and the unit delay due to discretization, *i.e.* k=d+1. Equation (A.1) may be written in a more compact form as

$$y(t) = \phi^T(t)\theta + \omega(t) \tag{A.2}$$

where

$$\phi(t) = [-y(t-1),\ldots,-y(t-n),u(t-k),\ldots,u(t-k-n+1)]^T$$
$$\theta = [a_1,\ldots,a_n,b_1,\ldots,b_n] \tag{A.3}$$

Define further

$$\hat{y}(t) = \phi^T(t)\theta \tag{A.4}$$

The least squares problem requires that θ be chosen in such a way that the cost index

$$J(\theta) = \frac{1}{2}\sum_{t=1}^{N}\epsilon(t)^2 \tag{A.5}$$

is minimized, where

$$\epsilon(t) = y(t) - \hat{y}(t), \quad t = 1,\ldots,N \tag{A.6}$$

Equation (A.5) can be written in vector notation as

$$J(\theta) = \frac{1}{2}\epsilon^T\epsilon \tag{A.7}$$

where

$$\epsilon = [\epsilon(1)\ldots\epsilon(N)]^T$$

In addition, define

$$Y = [y(1)\ldots y(N)]^T$$

and

$$\hat{Y} = \Phi\theta \tag{A.8}$$

with

$$\Phi = \begin{bmatrix} \phi^T(1) \\ \cdot \\ \cdot \\ \cdot \\ \phi^T(N) \end{bmatrix}$$

Using this notation, (A.7) can be written as (Åström and Wittenmark, 1984)

$$2J(\theta) = Y^T Y - Y^T \Phi \theta - \theta^T \Phi^T Y + \theta^T \Phi^T \Phi \theta \qquad (A.9)$$

Before proceeding further, a method of minimizing functions of the type given in Eqn. (A.9) will be established.

Theorem A.1 - Completing the Square

Consider the following general cost function (Åstrom and Wittenmark, 1984):

$$J(u) = u^T S u + r^T u + u^T r \qquad (A.10)$$

where S is a symmetric positive definite matrix of order $n \times n$ and u and r are n vectors. The minimum of J with respect to u can be obtained by setting

$$u = -S^{-1} r \qquad (A.11)$$

Proof:

Equation (A.10) may be rewritten as

$$\begin{aligned} J(u) &= u^T S u + r^T u + u^T r \\ &= u^T S u + r^T u + u^T r + r^T S^{-1} r - r^T S^{-1} r \\ &= (u + S^{-1} r)^T S (u + S^{-1} r) - r^T S^{-1} r \end{aligned}$$

The first term is always nonnegative; thus the minimum is obtained for

$$u = -S^{-1} r \qquad (A.11)$$

and the minimum value of $J(u)$ is

$$J_{min} = -r^T S^{-1} r \qquad (A.12)$$

Q.E.D.

Equation (A.9) is easily cast into the form of (A.10) by defining

$$u = 0$$

$$r = -\Phi^T Y$$

$$S = \Phi^T \Phi$$

and ignoring $Y^T Y$ as it is a constant unaffected by the choice of θ. Hence, the least squares cost functional (A.5) is minimized by setting

$$\begin{aligned} \theta &= (\Phi^T \Phi)^{-1} \Phi^T Y \\ &= \Phi^{+} Y \end{aligned} \tag{A.13}$$

where $\Phi^{+} = (\Phi^T \Phi)^{-1} \Phi^T$ is known as the extended inverse or pseudoinverse of the matrix Φ.

A.2 Recursive Least Squares

It was shown in Section A.1 that $\hat{\theta}$ as given by (A.9) minimizes the least squares cost fuctional (A.5). Note, however, that the dimension of the pseudeoinverse is $N \times N$, where N is the number of observations. If θ is to be identified online, the computational requirements are clearly prohibitive. Hence a recursive form of (A.13) has been derived and is known in the literature as Recursive Least Squares (RLS). The derivation presented here closely follows that of Åström and Wittenmark (1984) and begins by defining

$$\Phi(N) = \begin{bmatrix} \phi^T(1) \\ \cdot \\ \cdot \\ \cdot \\ \phi^T(N) \end{bmatrix}, \quad Y(N) = \begin{bmatrix} y(1) \\ \cdot \\ \cdot \\ \cdot \\ y(N) \end{bmatrix} . \tag{A.14}$$

When an additional measurement is obtained,

$$\Phi(N+1)=\begin{bmatrix}\Phi(N)\\ \phi^T(N+1)\end{bmatrix},\quad Y(N+1)=\begin{bmatrix}Y(N)\\ y(N+1)\end{bmatrix}$$

It is apparent from (` 3) that

$$\hat{\theta}(N+1)=[\Phi^T(N+1)\Phi(N+1)]^{-1}\Phi^T(N+1)Y(N+1)$$
$$=[\Phi^T(N)\Phi(N)+\phi(N+1)\phi^T(N+1)]^{-1}$$
$$\cdot[\Phi^T(N)Y(N)+\phi^T(N+1)y(N+1)] \tag{A.15}$$

Dropping for convenience the argument N ... (N) and $Y(N)$ and the argument $N+1$ of $\phi(N+1)$, (A.15) can be written

$$\hat{\theta}(N+1)=[\Phi^T\Phi+\phi\phi^T]^{-1}[\Phi^TY+\phi y(N+1)]$$
$$=(\Phi^T\Phi)^{-1}\Phi^TY+[(\Phi^T\Phi+\phi\phi^T)^{-1})-(\Phi^T\Phi)^{-1}]\Phi^TY$$
$$+(\Phi^T\Phi+\phi\phi^T)^{-1}\phi y(N+1) \tag{A.16}$$

But from (A.12),

$$\theta(N)=(\Phi^T\Phi)^{-1}\Phi^TY \tag{A.17}$$

Also,

$$[(\Phi^T\Phi+\phi\phi^T)^{-1}-(\Phi^T\Phi)^{-1}]\Phi^TY$$
$$=(\Phi^T\Phi+\phi\phi^T)^{-1}(\Phi^T\Phi-\Phi^T\Phi-\phi\phi^T)(\Phi^T\Phi)^{-1}\Phi^TY$$
$$=-(\Phi^T\Phi+\phi\phi^T)^{-1}\phi\phi^T(\Phi^T\Phi)^{-1}\Phi^TY$$
$$=-(\Phi^T\Phi+\phi\phi^T)^{-1}\phi\phi^T\hat{\theta}(N)$$

from (A.17). This implies that Eqn. (A.16) can be put in the form

$$\hat{\theta}(N+1)=\hat{\theta}(N)+K(N)[y(N+1)-\phi^T(N+1)\hat{\theta}(N)] \tag{A.18}$$

where $K(N)$, the Kalman gain vector is given by

$$K(N) = [\Phi^T(N)\Phi(N) + \phi(N+1)\phi^T(N+1)]^{-1}\phi(N+1)$$
$$= [\Phi^T(N+1)\Phi(N+1)]^{-1}\phi(N+1)$$
$$= P(N+1)\phi(N+1) \tag{A.19}$$

and

$$P(N+1) = [\Phi^T(N+1)\Phi(N+1)]^{-1} \tag{A.20}$$

$P(\cdot)$ is called the covariance matrix as it may be shown to be proportional to the variance of $\hat{\theta}(\cdot)$ (Åström and Wittenmark, 1984).

At this point it is necessary to apply the Matrix Inversion Lemma (MIL):

$$[A + BCD]^{-1} = A^{-1} - A^{-1}B[C^{-1} + DA^{-1}B]^{-1}DA^{-1} \tag{A.21}$$

where A, C and $C^{-1} + DA^{-1}B$ are nonsingular square matrices. Setting $A = \Phi\Phi^T$, $B = \phi$, $C = 1$, and $D = \phi^T$, (A.20) can be rearranged to give

$$P(N+1) = [\Phi^T\Phi + \phi\phi^T]^{-1}$$
$$= (\Phi^T\Phi)^{-1} - (\Phi^T\Phi)^{-1}\phi(1 + \phi^T(\Phi^T\Phi)^{-1}\phi)^{-1}\phi^T(\Phi^T\Phi)^{-1}$$

or

$$P(N+1) = P(N) - \frac{P(N)\phi(N+1)\phi^T(N+1)P(N)}{1 + \phi^T(N+1)P(N)\phi(N+1)} \tag{A.22}$$

which leads to

$$K(N) = P(N)\phi(N+1) - \frac{P(N)\phi(N+1)\phi^T(N+1)P(N)}{1+\phi^T(N+1)P(N)\phi(N+1)}$$

$$= \frac{P(N)\phi(N+1)}{1+\phi^T(N+1)P(N)\phi(N+1)}[1+\phi^T(N+1)P(N)\phi(N+1)$$

$$-\phi^T(N+1)P(N)\phi(N+1)]$$

$$K(N) = \frac{P(N)\phi(N+1)}{1+\phi^T(N+1)P(N)\phi(N+1)} \tag{A.23}$$

Hence, the sequence of calculations for computer implementation at time t=N+1 is

1. Calculate $K(N)$ using (A.23).
2. Update $\hat{\theta}(N+1)$ using (A.18).
3. Update $P(N+1)$ using (A.22).

Note that the algorithm requires the user to specify $\hat{\theta}(0)$ and $P(0)$. $P(0)$ is generally chosen to reflect the degree of certainty regarding $\hat{\theta}(0)$.

The RLS cost functional Eqn. (A.5) weights all data equally. This approach leads to sluggish identification if θ is time-varying. Therefore, (A.5) can be modified to include "exponential data forgetting" as follows:

$$J(\theta) = \frac{1}{2}\sum_{t=1}^{N}\lambda^{N-t}\epsilon(t)^2 \tag{A.24}$$

where $0 < \lambda \leq 1$. λ is called the "forgetting factor" and it is seen that the choice $\lambda < 1$ places more weight on newer data points. A parallel development to that given above for (A.5) leads to the modified gain and covariance updates:

$$K(N) = \frac{P(N)\phi(N+1)}{\lambda+\phi^T(N+1)P(N)\phi(N+1)} \tag{A.25}$$

$$P(N+1) = \left[P(N) - \frac{P(N)\phi(N+1)\phi^T(N+1)P(N)}{\lambda + \phi^T(N+1)P(N)\phi(N+1)} \right] \frac{1}{\lambda} \quad \text{(A.26)}$$

A.3 The Kalman Filter

The Kalman filter is designed to provide a minimum variance estimate of the process states given the model:

$$x(t+1) = \Phi x(t) + \Lambda u(t) + \Gamma n_1(t) \quad \text{(A.27)}$$

$$y(t) = H x(t) + n_2(t) \quad \text{(A.28)}$$

The process noise $n_1(\cdot)$ and the measurement noise $n_2(\cdot)$ are random sequences with zero mean, *i.e.*

$$E\{n_1(t)\} = E\{n_2(t)\} = 0,$$

have no time correlation or are "white noise", *i.e.*

$$E\{n_1(i)n_1(j)\} = E\{n_2(i)n_2(j)\} = 0, \quad \forall i \neq j$$

and have covariances or "noise levels" defined by

$$E\{n_1(t)n_1^T(t)\} = R_1$$

$$E\{n_2(t)n_2^T(t)\} = R_2$$

It is proposed to design an estimator of the form

$$\hat{x}(t) = \bar{x}(t) + L(t)(y(t) - H\bar{x}(t)) \quad \text{(A.29)}$$

where

$$\bar{x}(t+1) = \Phi \hat{x}(t) + \Lambda u(t) \quad \text{(A.30)}$$

The Kalman gain vector $L(t)$ is to be chosen such that the estimate $\hat{x}(t)$ is optimal. Equation (A.29) is referred to as the "measurement update"; (A.30) is called the estimator "time update".

Comparing this problem to the identification problem in Section A.2, it is evident that the measurement equation (A.28) is similar in form to Eqn. (A.2); thus the optimal state estimation solution is given by Eqns. (A.18), (A.22), and (A.23) with $\lambda = R_2$. With modifications for notation, the solution equations are

$$\hat{x}(t) = \bar{x}(t) + M(t)H^T(HM(t)H^T + R_2)^{-1}(y(t) - H\bar{x}(t)) \tag{A.31}$$

$$P(t) = M(t) - M(t)H^T(HM(t)H^T + R_2)^{-1}HM(t) \tag{A.32}$$

after making the replacements

$$\hat{\theta}(N+1) \to \hat{x}(t),\quad \hat{\theta}(N) \to \bar{x}(t),\quad \phi^T(t) \to H;$$

$$P(N) \to M(t),\quad P(N+1) \to P(t),\quad \lambda \to R_2$$

It is noted that $M(\cdot)$ represents the estimation error covariance before measurement; $P(\cdot)$ is the error covariance after measurement. Subtracting (A.30) from (A.27),

$$x(t+1) - \bar{x}(t+1) = \Phi(x(t) - \hat{x}(t)) + \Gamma n_1(t)$$

so the error covariance <u>before</u> measurement is apparently equal to

$$E\{(x(t+1) - \bar{x}(t+1))(x(t+1) - \bar{x}(t+1))^T\}$$

$$= E\{[\Phi(x(t) - \hat{x}(t)) + \Gamma n_1(t)]\cdot[\Phi(x(t) - \hat{x}(t)) + \Gamma n_1(t)]^T\}$$

If $n_1(t)$ and $n_2(t)$ are uncorrelated, and $x(t)$ and $n_1(t)$ are uncorrelated, the cross product terms vanish, leaving

$$E\{(x(t+1) - \bar{x}(t+1))(x(t+1) - \bar{x}(t+1))^T\}$$

$$= E\{\Phi(x(t) - \hat{x}(t))(x(t) - \hat{x}(t))^T\Phi^T + \Gamma n_2(t)n_2^T(t)\Gamma^T\}$$

or

$$M(t+1) = \Phi P(t)\Phi^T + \Gamma R_1\Gamma^T \tag{A.33}$$

where

$$M(t) = E\{(x(t) - \bar{x}(t))(x(t) - \bar{x}(t))^T\}$$

$$P(t) = E\{(x(t) - \hat{x}(t))(x(t) - \hat{x}(t))^T\}$$

The Kalman filter algorithm can therefore be summarized as

a) Gain Calculation

$$L(t) = M(t)H^T(HM(t)H^T + R_2)^{-1} \quad \text{(A.34)}$$

b) Measurement Update

i) A Posteriori State Update

$$\hat{x}(t) = \bar{x}(t) + L(t)(y(t) - H\bar{x}(t)) \quad \text{(A.35)}$$

ii) A Posteriori Covariance Update

$$P(t) = M(t) - L(t)HM(t) \quad \text{(A.36)}$$

c) Time Update

i) A Priori State Update

$$\bar{x}(t+1) = \Phi\hat{x}(t) + \Lambda u(t) \quad \text{(A.30)}$$

ii) A Priori Covariance Update

$$M(t+1) = \Phi P(t)\Phi^T + \Gamma R_1 \Gamma^T \quad \text{(A.33)}$$

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Appendix B

Innovations Analysis for the Modified KFP

Writing the Kalman filter update (A.35) for (2.33) and (2.34),

$$\hat{x}(t+1) = \Phi_1 \hat{x}(t) + \Lambda_1 u(t) + L\omega(t+1) \quad \text{(B.1)}$$

$$\omega(t) = y(t) - \hat{y}(t|t-1)$$

$$= y(t) - H_1 \Phi_1 \hat{x}(t-1) - H_1 \Lambda_1 u(t-1) \quad \text{(B.2)}$$

where

$$\Phi_1 = \begin{bmatrix} 1 & 0 \\ \Gamma & \Phi \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 0 \\ \Lambda \end{bmatrix}, \quad H_1 = [0 \quad H]$$

and the Kalman gain vector

$$L = [L_\rho, L_1, \ldots, L_{n+d}]$$

Using successive forward substitution for states ρ to n,

$$\hat{x}_n(t) = [1 - A(z^{-1})]\hat{x}_n(t) + B(z^{-1})u(t-1) + K_1(z^{-1})\omega(t) + D(z^{-1})\frac{\omega(t)}{\Delta} \quad \text{(B.3)}$$

where

$$D(z^{-1}) = L_\rho[\gamma_n z^{-1} + \ldots + \gamma_1 z^{-n}] \quad \text{(B.4)}$$

and $K_1(z^{-1})$ is defined by Eqn. (2.13).

Similarly, for states $n+1$ to $n+d-1$,

$$\hat{x}_{n+d-1}(t) = z^{-d}\hat{x}_n(t) + K_2(z^{-1})\omega(t) \quad \text{(2.12)}$$

where $K_2(z^{-1})$ is defined by (2.15). Equation (B.3) implies that

$$\hat{x}_n(t) = A^{-1}(z^{-1})B(z^{-1})u(t-1) + A^{-1}(z^{-1})K_1(z^{-1})\omega(t) + A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t)}{\Delta} \quad \text{(B.5)}$$

so that

$$\hat{x}_{n+d-1}(t-1) = z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1)$$
$$+ z^{-d}A^{-1}(z^{-1})K_1(z^{-1})\omega(t) + K_2(z^{-1})\omega(t)$$
$$+ z^{-d}A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t)}{\Delta} \qquad \text{(B.6)}$$

From (B.2),

$$y(t) = \hat{y}(t \mid t-1) + \omega(t)$$
$$= H_1\Phi_1\hat{x}(t-1) + H_1\Lambda_1 u(t-1) + \omega(t)$$

Note that for $d \geq 1$,

$$H_1\Phi_1 = [0, 0, \ldots, 1, 0]_{1\times(n+d+1)}$$

and $H_1\Lambda_1 = 0$. Therefore,

$$y(t) = \hat{y}(t \mid t-1) + \omega(t)$$
$$= \hat{x}_{n+d-1}(t-1) + \omega(t)$$
$$= z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1) + z^{-d}A^{-1}(z^{-1})K_1(z^{-1})\omega(t)$$
$$+ A^{-1}(z^{-1})A(z^{-1})(1 + K_2(z^{-1}))\omega(t)$$
$$+ z^{-d}A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t)}{\Delta}$$

$$= z^{-d}A^{-1}(z^{-1})B(z^{-1})u(t-1)$$
$$+ z^{-d}A^{-1}(z^{-1})K_1(z^{-1})\Delta\frac{\omega(t)}{\Delta}$$
$$+ A^{-1}(z^{-1})A(z^{-1})(1 + K_2(z^{-1}))\Delta\frac{\omega(t)}{\Delta}$$
$$+ z^{-d}A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t)}{\Delta}$$

or

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})}u(t-k) + \frac{C(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \qquad \text{(B.7)}$$

where

$$C(z^{-1}) = [A(z^{-1})(1 + K_2 \quad + z^{-d}K_1(z^{-1}))]\Delta + z^{-d}D(z^{-1}) \quad \text{(B.8)}$$

Equation (B.7) is an ARIMA representation equivalent to the state space model (2.33), (2.34). $C(z^{-1})$ is now defined as a monic polynomial of order n+d.

To obtain an optimal k-step-ahead estimate of y(t+d+1),

$$\begin{aligned} E\{y(t+d+1)\} &= \hat{y}(t+d+1 \mid t) \\ &= H_1\hat{x}(t+d+1 \mid t) \\ &= H_1\Phi_1^{d+1}\hat{x}(t) + \sum_{j=t}^{t+d} H_1\Phi_1^{t+d-j}\Lambda_1 u(j) \\ &= \gamma_n x_p(t) + \hat{x}_{n-1}(t) - a_1\hat{x}_n(t) + b_1 u(t) \end{aligned}$$

But from (B.1),

$$\hat{x}_n(t+1) = \gamma_n x_p(t) + \hat{x}_{n-1}(t) - a_1\hat{x}_n(t) + b_1 u(t) + L_n\omega(t+1)$$

Thus,

$$\hat{y}(t+d+1 \mid t) = \hat{x}_n(t+1) - L_n\omega(t+1) \quad \text{(B.9)}$$

But (B.5) implies that

$$\begin{aligned} \hat{x}_n(t+1) = {} & A^{-1}(z^{-1})B(z^{-1})u(t) + A^{-1}(z^{-1})K_1(z^{-1})\omega(t+1) \\ & + A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t+1)}{\Delta} \end{aligned}$$

Therefore,

$$\begin{aligned} \hat{y}(t+d+1 \mid t) = {} & A^{-1}(z^{-1})B(z^{-1})u(t) \\ & + A^{-1}(z^{-1})K_1(z^{-1})\omega(t+1) \\ & + A^{-1}(z^{-1})D(z^{-1})\frac{\omega(t+1)}{\Delta} \\ & - A^{-1}(z^{-1})A(z^{-1})L_n\omega(t+1) \end{aligned}$$

$$= \frac{B(z^{-1})}{A(z^{-1})}u(t)$$

$$+ \frac{[(K_1(z^{-1}) - L_n A(z^{-1}))\Delta + D(z^{-1})]}{A(z^{-1})} z^{-1} \frac{\omega(t)}{\Delta}$$

$$\hat{y}(t+d+1\,|\,t) = \frac{B(z^{-1})}{A(z^{-1})}u(t) + \frac{K_5(z^{-1})}{A(z^{-1})}\frac{\omega(t)}{\Delta} \qquad \text{(B.10)}$$

where

$$K_5(z^{-1}) = z^{-1}[(K_1(z^{-1}) - L_n A(z^{-1}))\Delta + D(z^{-1})] \qquad \text{(B.11)}$$

Rewriting (B.7) in terms of $\omega(t)/A(z^{-1})/\Delta$ and substituting in (B.10),

$$\hat{y}(t+d+1\,|\,t) = \frac{B(z^{-1})}{A(z^{-1})}u(t)$$

$$+ \frac{K_5(z^{-1})}{C(z^{-1})}\left[y(t) - z^{-d-1}\frac{B(z^{-1})}{A(z^{-1})}u(t) \right]$$

or

$$\hat{y}(t+k\,|\,t) = G_P(z^{-1})u(t) + G_F(z^{-1})[y(t) - G_M(z^{-1})u(t)] \qquad \text{(B.12)}$$

$G_P(z^{-1})$ and $G_M(z^{-1})$ are as given in (2.3), and

$$G_F(z^{-1}) = \frac{K_5(z^{-1})}{C(z^{-1})} \qquad \text{(B.13)}$$

Returning to Eqn. (B.8),

$$C(z^{-1}) = [A(z^{-1})(1 + K_2(z^{-1})) + z^{-d}K_1(z^{-1})]\Delta + z^{-d}D(z^{-1})$$

$$= [A(z^{-1})(1 + K_2(z^{-1})) + z^{-d}K_1(z^{-1})]$$

$$-[z^{-1}A(z^{-1})(1 + K_2(z^{-1})) + z^{-d-1}K_1(z^{-1})]$$

$$+ z^{-d}D(z^{-1})$$

$$= \{A(z^{-1})(1 + K_2(z^{-1}) + L_n z^{-d})$$

$$- z^{-1}A(z^{-1})(1 + K_2(z^{-1}))\}$$

$$+ \{z^{-d-1}z^{+1}(K_1(z^{-1}) - L_n A(z^{-1}))$$

$$- z^{-d-1}K_1(z^{-1}) + z^{-d-1}z^{+1}D(z^{-1})\}$$

$$= A(z^{-1})[(1 + K_2(z^{-1}))\Delta + L_n z^{-d}]$$

$$+ z^{-d-1}[z^{+1}(K_1(z^{-1})\Delta - L_n A(z^{-1}) + D(z^{-1}))]$$

$$= A(z^{-1})\Delta[(1 + K_2(z^{-1})) + L_n z^{-d}] + A(z^{-1})L_n z^{-d-1}$$

$$+ z^{-d-1}[z^{+1}(K_1(z^{-1})\Delta - L_n A(z^{-1}) + D(z^{-1}))]$$

$$= A(z^{-1})\Delta[(1 + K_2(z^{-1})) + L_n z^{-d}]$$

$$+ z^{-d-1}[z^{+1}(K_1(z^{-1})\Delta - L_n A(z^{-1})\Delta + D(z^{-1}))]$$

$$= A(z^{-1})\Delta[(1 + K_2(z^{-1})) + L_n z^{-d}]$$

$$+ z^{-d-1}[z^{+1}(K_1(z^{-1}) - L_n A(z^{-1}))\Delta + D(z^{-1})]$$

So it is apparent that (B.8) can be rearranged to give an expression of the form:

$$\frac{C(z^{-1})}{A(z^{-1})\Delta} = K_3(z^{-1}) + z^{-k}\frac{K_5(z^{-1})}{A(z^{-1})\Delta} \tag{B.14}$$

where $K_3(z^{-1})$ is as defined following Eqn. (2.24).

Appendix C

Steady State Operating Data for the Evaporator

The following list of steady state operating data for the double effect evaporator was adapted from Wilson (1974).

Symbol	Description	Steady State
T_S	Steam temperature	177.8 ^oC
T_{w1}	First effect tube wall temperature	108.3 ^oC
W_1	First effect holdup	20.64 kg
C_1	First effect concentration	305 mol/m^3
H_1	First effect enthalpy	440.1 kJ/kg
T_{w2}	Second effect tube wall temperature	82.8 ^oC
W_2	Second effect holdup	18.82 kg
C_2	Second effect concentration	675 mol/m^3
H_2	Second effect enthalpy	311.9 kJ/kg
T_{w3}	Condenser tube wall temperature	42.2 ^oC
S	Steam flowrate	0.0151 kg/s
B_1	First effect bottoms flowrate	0.0263 kg/s
B_2	Second effect bottoms flowrate	0.0120 kg/s
F	Feed flowrate	0.0378 kg/s
C_F	Feed concentration	211 mol/m^3
H_F	Feed enthalpy	364.9 kJ/kg

References

Wilson R.G. (1974). Model Reduction and Reduced Order Control Law Design. Ph.D. Thesis, University of Alberta, Dept. of Chemical Engineering.