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Energy Efficient Relay Network Design Using Power-Normalized SNR

by

Yichen Hao

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Dedicate to my beloved family

Abstract

Relay network designs have been widely studied in recent years. It is known that cooperative relay network can achieve cooperative diversity with the help of relays and improve the data rate and/or the reliability of the network. On the other hand, green communication design has also attracted significant attention due to the drastic increase in energy consumption. We are going to investigate green communication designs in relay network in our work.

In this thesis, we adopt a novel efficiency measure, the power-normalized received signal to noise ratio (PN-SNR) in relay network design for several scenarios and analyze the performance of the proposed designs. In single-relay network and multi-relay network with a sum relay power constraint, the PN-SNR maximization problem is formulated and solved. In multi-relay network with individual power constraint on each relay, we investigate both the basic PN-SNR maximization problem and the quality of service (QoS)-constrained PN-SNR maximization problem. Performance of the proposed designs is compared with the fixed relay power scheme and the SNR-maximization scheme analytically and numerically via simulation. Our results show that with the same average relay transmit power, the PN-SNR maximizing scheme is superior to the fixed relay power scheme not only in the PN-SNR but also in the outage probability for both single and multi-relay networks. Compared with SNR-maximizing scheme, it is significantly superior in PN-SNR with moderate degradation in outage probability. Our results reveal the potential of PN-SNR as efficiency measure in relay network design.

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Chapter 1

Introduction

Wireless communication has experienced remarkable development in the past decades. With the deployment of commercial wireless systems and the booming of wireless users, wireless communication networks are faced with higher demands for throughput and reliability.

Compared with wired communication, wireless communication is more challenging in the sense that the propagation environment of radio signals is much more complicated [1]. Radio signals usually experience three mechanisms: reflection, diffraction, and scattering. As a result of these three mechanisms, radio propagation in wireless environment can be severely influenced by three phenomenons: path loss, shadowing, and multipath fading. The first two phenomenons are known as large scale fading while the third one is usually called small scale fading. Specifically, path loss is the attenuation of an electromagnetic wave as it propagates through space. Shadowing is introduced due to the presence of obstacles such as buildings, hills and valleys. Shadowing may result in temporary failure of communication due to the severe drop in the channel signal-to-noise ratio (SNR). Multipath fading is caused when plane waves arrive from different directions with random amplitudes and phases. It will result in rapid variations in the envelope of the received signal and will also introduce time dispersion. In the presence of these phenomenons, the throughput and reliability for wireless communication networks are limited.

1.1 Multipath Fading Channel Model

In mobile communication, large scale fading is relevant to cell site planning while small scale fading is relevant to the design of reliable and efficient communication system. We focus on multipath fading channel in this thesis. A continuous-time multipath channel under additive white Gaussian noise (AWGN) can be typically modeled as a linear time-variant system [2–4]. The equivalent baseband model can be written as

$$y(t) = \sum_{i=1}^{L} a_i(t)s(t - \tau_i(t)) + n(t),$$

where $a_i(t)$ represents the time-variant attenuation factor for the ith path and $\tau_i(t)$ is the corresponding time delay. In the equivalent baseband model, $a_i(t)$ is a complex factor that can be expressed as $a_i(t) = |a_i(t)|e^{-j2\pi f_c \tau_i(t)}$ with the carrier frequency f_c. $n(t)$ is the additive Gaussian noise, which is a complex Gaussian random process. We can see that the received signal is consist of L multipath components of the transmitted signal $s(t)$, where the ith is attenuated by $a_i(t)$ and delayed by $\tau_i(t)$.

Usually, $a_i(t)$ and $\tau_i(t)$ are time-variant due to the movement between the transmitter and the receiver. On special occasions when the transmitter, the receiver and the channel are stationary, the attenuation factors and delays do not depend on time t. Thus, the channel model can be reduced to a time-invariant system as

$$y(t) = \sum_{i=1}^{L} a_i s(t - \tau_i) + n(t). \tag{1.1}$$

The multipath propagation will stretche the signal in time domain and it may introduce intersymbol interference (ISI). We define the multipath delay spread T_d as the difference in propagation time between the longest and the shortest path, counting only the path with significant energy [4], i.e.,

$$T_d = \max_{i,j}(\tau_i - \tau_j).$$

Then, we can introduce the concept of channel coherent bandwidth B_c and roughly establish a relationship between B_c and T_d as

$$B_c \approx \frac{1}{T_d}.$$

The equation points out that the coherence bandwidth is reciprocal to the delay spread. In frequency domain, coherent bandwidth represents the frequency interval over which different frequency components of a signal are likely to experience correlated fading. In other word, when the bandwidth of the input signal is considerably less than B_c, all frequency components of the signal will experience the same magnitude of fading and the channel is said to be flat fading channel. In time domain, the delay spread T_d is considerably less than the symbol interval T_s when the channel is flat fading. Thus, flat fading channel will introduce very little ISI.

The discrete-time baseband model for flat fading channel can be obtained from (1.1) by sampling as

$$y(k) = \sum_{i=1}^{L} a_i s(k) + n(k) = hs(k) + n(k), \tag{1.2}$$

where $h = \sum_{i=1}^{L} a_i$ can be viewed as the channel coefficient and the sampled noise can be proved to be complex Gaussian random variable [4]. Statistically, we assume that the attenuation factors are independently distributed random variables. When the number of multipath L is sufficient large, h is the sum of many independent random variables, and by the Central Limit Theorem, it can be reasonably modeled as circular symmetric complex Gaussian random variable.

In this thesis, we focus on such flat fading channels. In the absence of line of sight, h is complex Gaussian random variable with zero-mean. We further assume that h has unit-variance, i.e., $h \sim \mathcal{CN}(0, 1)$. It can be shown that the magnitude of h is Rayleigh distributed and the phase is uniform distributed over $[0, 2\pi]$. The transceiver equation of such Rayleigh fading channels can be expressed as

$$y = hs + n, \tag{1.3}$$

where s and y are the transmitted signal and the received signal, respectively. h is the channel coefficient whose amplitude has the following probability density function (pdf)

$$f(x) = 2xe^{-x^2}, \quad x \geq 0. \tag{1.4}$$

and the noise n is complex Gaussian noise with zero-mean and unit-variance.

1.2 Diversity Techniques and MIMO System

Several techniques have been proposed to combat multipath fading. Among these, diversity techniques play an important role [1,4,5]. It can improve the quality of communication by using several communication channels with different characteristics. Different paths may experience different levels of fading. Thus, we are able to acquire independently faded replicas of transmitted data at the receiver by sending the signals with the same information through different paths. Due to the independence of these replicas, the probability that all replicas have poor quality is low and we can achieve more reliable detection by properly combining these replicas at the receiver. Common combining schemes include equal gain combining, maximum ratio combining, selection combining, and switched combining [6,7]. In the presence of channel information, maximum ratio combining weights and rotates the replicas according to the phase and strength of the channels, yielding a coherent summation of the signal-to-noise ratio (SNR) from each replica at the receiver. It maximizes the SNR at the receiver and is proved to be the optimal combining scheme in independent AWGN channels. According to the domain where diversity is introduced, diversity techniques are classified into time, frequency and space diversity. Most of the wireless communication systems involve multiple diversity techniques.

Time diversity is achieved by sending the signals carrying the same information in different time intervals spaced more than the coherence time of the channel. Error control coding and interleaving are usually used in time diversity techniques, which may result in decoding delays and make it intolerable for delay sensitive applications.

Frequency diversity is achieved by sending the signals carrying the same information in different frequency bands separated by the coherence bandwidth of the channel. The major disadvantage of frequency diversity lies on the inefficient use of frequency resources.

Unlike time diversity schemes and frequency diversity schemes, spatial diversity can be achieved with no extra cost on time and frequency resources. Thus, it attracts significant attentions in recent years. It relies on the fact that signals will experience independent fading in geographically separated paths. Spatial diversity schemes can thus be implemented by employing multiple antennas at the transmitter and the receiver. This multiple-input-multiple-output (MIMO) system, or multiple-antenna system, has evolved into the most vibrant research areas in the past two decades [8–13]. It is shown that MIMO techniques can be used to combat fading and improve the throughput and reliability of the network. Beamforming and space-time coding are among the most successful techniques in multiple-antenna system [14–16]. Beamforming is implemented by controlling the phase and amplitude of the signal sent by each transmit antenna in a way that signals at particular angles experience constructive interference while others experience destructive interference. At the receiver, the signals are recombined so that the expected pattern is preferentially observed. Space-time coding is based on introducing joint correlation in transmitted signals in both the space and time domains. Through this approach, simultaneous diversity and coding gains can be obtained.

Due to these appealing characteristics, MIMO techniques soon become key techniques in modern wireless communication and has been commercialized as wireless communication standards. For example, MIMO is employed in wireless communication standards such as 3rd Generation Partnership Project (3GPP), High-Speed Packet Access plus (HSPA+), IEEE 802.11 (Wi-Fi) and Long Term Evolution (LTE).

Even though multiple-antenna system has been prosperously developed, in some situations, it may not be practical to implement multiple antennas. As we mentioned before, it is desirable that the multiple faded replicas experience independent fading in order to achieve spatial diversity. However, it is shown that sufficient spatial independence can be obtained by spacing the antenna elements at least a half wavelength apart. For instance, we consider a wireless communication system with its carrier frequency 900 MHz. A roughly 16 cm separation between antennas is required to achieve the independence of fading.

This separation between antennas is not possible for some users, especially for small size wireless mobile devices.

1.3 Cooperative Communication and Relay Networks

To overcome this disadvantage of multiple-antenna system, the concept of cooperative communication and relay network has been proposed [17–20]. Note that wireless channel is broadcast in nature, which indicates that users within a region are able to receive the information from the source user and can help relay the information if needed. The basic idea of cooperative communication is that users in a wireless network can help each other cooperatively to send signals to the destination, i.e., users in cooperative communication are responsible for transmitting not only their own information but also the information from some other users. Cooperative network can be thought of as a generalized MIMO system in the sense that it can generate independent MIMO-like channel links between source and destination and thus achieves spatial diversity. Relay network refers to a class of network topologies where the source and destination are interconnected by relay nodes. It is an implementation of cooperative communication network where users that help in transmission can be viewed as relays and they serve the source and the destination.

1.3.1 Brief History of Cooperative Communication

The basic analysis of relay channel in communication can be traced back to [21, 22] in which a three-node network consisting of a source, a destination and a relay was investigated. Later, the bounds on capacity of the three-node relay network discovered by van der Meulen was significantly improved in [23]. In [24,25], the authors generalized the capacity derivations with multiple relay network and investigated deterministic relay network with no interference. These works are viewed as the most prominent information results on relay network to date. However, the interest in cooperative communication and relay network once diminished after these works due to the difficulty in finding new theoretical results and the technology challenges.

Until the end of 20th century, the remarkable developments in the area of wireless communication techniques, such as the discovery of multi-antenna systems, the advancement in understanding of fading channels and the research on channel codings, have set another wave of research on cooperative communication and relay network. Some of the contributive works that help to draw attention to cooperative communication in recent years include [26–30]. In [26, 27], the authors considered user-cooperation as a form of diversity in a mobile uplink channel, then proposed the implementation and analyzed the performance. The work points out that cooperation can not only extend the communication distance but also achieve spatial diversity. In [28–30], the authors studied the network performance under different transmission protocols in fading channels. In parallel with these works were numerous literature on extension of the basic cooperative relay network model [31–33]. Relay network with more than one relay and no direct source to destination link was considered in [31, 32]. While the author in [33] discovered the coding design and the power control strategies in relay networks. Later on, some researchers combined cooperative communication with multi-antenna system and proposed to use space-time coding in relay network [30, 34, 35].

With remarkable advances in wireless communications over the last two decades, the promise of cooperative communication and relay network is very real. Numerous studies are currently geared toward developing practical cooperative communication network. In recent years, cooperative communication is being applied to many aspects such as sensor network, cellular network, and wireless local area network (WLAN).

1.3.2 Review on Related Works

Literature on relay network designs can be categorized in different manners according to the network topologies, the transmission protocols or the assumptions on the networks.

There are many different network topologies and system models for relay network. The basic one is the three-node relay network with one transmitter, one receiver, and one relay. In many applications, however, this basic three-node relay network is far from enough in achieving satisfactory performance. Thus, the general multi-relay network model is studied in a large amount of literature [31,32,36–39]. Moreover, the relays are likely to serve more than one pair of users in practice. In these system models, the inter-user interference and the user fairness are of great concern. Many researchers try to overcome the difficulties in multi-user relay network [40–42]. The authors of [39,43–45] also discovered the two-way network topology and study the transmission schemes that are bandwidth efficient in bidirectional communications.

Even if the network topology is determined, the relay network can be different when different transmission schemes are used. For example, the relays process the received signals in different manners according to the transmission protocols. Common protocols are amplify-and-forward (AF) protocol [29], decode-and-forward (DF) protocol [46] and compress-and-forward (CF) protocol [47]. In AF protocol, the relays adjust the amplitude and phase of the received source transmission signals before forwarding them to the destination. In DF protocol, the relays forward the decoded source transmission signals after possibly compressing or adding redundancy. In CF protocol, the relays may not be able to decode the source transmission signals, but they quantize the signals and provide independent observations of the source transmission signals at the destination. In multi-relay network, whether orthogonal channels are used among relays also matters in network design. In some works, relays are using orthogonal channels [38,48]. Channel orthogonality can be in time domain or in frequency domain, where there is no interference between relays. Even though the orthogonality among relay channels largely simplifies the analysis, it is less bandwidth efficient compared with the design where all relays share the band [36,37].

Finally, literature on relay network assume different knowledge of channel information. In the absence of channel information, space-time coding is usually used for diversity [34]. In the presence of perfect or partial channel information, performance can be further improved through beamforming or selection since it takes advantage of the channel information on relays to obtain higher received SNR [36, 37, 49, 50].

In many applications of cooperative relay network, there are constraints on the power consumed on users due to the limited power supply or the inter-user interference. Thus, power control is of great importance in cooperative relay network design to improve the network performance and there are numerous results on this topic [38, 51–55]. The aim of these works is to optimize the network performance by adaptive relay power allocation under different constraints. Commonly used performance measure includes received SNR, capacity or throughput, and outage probability or error rate. For example, [36, 37] involved performance maximization in single user and multi-relay networks where the received SNR of the network was optimized under sum relay power constraint and separate relay power constraint, respectively. In [50, 54] and [38], the same model was considered while the goal was to maximize the capacity of the network. The authors of [39, 43, 45] focused on two-way single user networks in which either the minimum received SNR or the sum rate (throughput) was optimized. Throughput as a measure was also considered in [42] where the relays serve more than one pair of users and the minimum throughput among all users was maximized. In [55], the outage probability was viewed as the objective function to be minimized.

1.4 Green Communication

1.4.1 Motivation and Background

As the wireless traffic rapidly multiplies, the increase in energy consumption is dramatic, which leads to the increase of greenhouse gas emission that causes severe environmental depredation.

According to the survey from [56, 57], wireless communications technology usage has grown at a staggering rate worldwide with an estimated 6 billion subscriptions in 2010. Every year, 120,000 new base stations are deployed serving 400 million new mobile users around the world. It indicates that the mobile subscription in developed regions increases by about 200%, whereas that in developing regions increases by about 1300% within the past decade.

Mobile communications growth may have an alarming effect on carbon usage and energy costs due to the inefficient use of energy sources. As a result, the cellular networks have become the largest factor contributing to the mobile industry's environmental impact with the emissions estimated at between 0.5% and 1% of the entire world's carbon footprint. The growth trend of carbon footprint will remain within future years.

Thus, the reduction of greenhouse gas emission and the efficient use of power is becoming a practical problem. And green communication designs have attracted significant attention in recent years [58–60]. Green communications is defined as the striving to reduce energy costs while still maintaining quality of service (QoS) in terms of coverage needs, capacity and user needs.

In traditional network designs, efficiency measure includes spectral efficiency and energy efficiency. Spectral efficiency is defined as the achievable transmission bit-rate and its maximization guarantees the highest amount of information flow for fixed transmit power [54]. Energy efficiency, defined as the number of transmission bits per unit energy or power, was first proposed and analyzed in [61]. The energy efficiency was used in MIMO beamforming design in recent works [62, 63]. The author in [64] derived the bound on energy efficiency in relay network. In [65–68], energy efficiency was used in relay network designs.

1.4.2 Power-Normalized SNR

Inspired by the advantages of cooperative relay networks and the call of green communications, we are going to investigate power-efficient power control strategies in relay network design. In this thesis, we propose to used a new efficiency measure in network design. We first review the limitations on previous measures.

We can see that spectral efficiency is related to the capacity and the received SNR, but the maximization over spectral efficiency does not consider how efficient the transmit power is used. Energy efficiency takes into account how efficient the power is used and is a natural measure. But for most communication systems, energy efficiency is maximized when the transmit power approaches 0, i.e., when the system works in low SNR regimes. To see this, we consider the simple point-to-point single-antenna system with transmit power P, unit-variance noise, and channel gain λ. The energy efficiency of the system is $[\log(1+\lambda P)]/P$, which takes its maximum λ when $P \to 0$. Hence, the energy-efficiency-optimal scheme will trap the system in the low SNR regime, where the actual bit-rate and reliability can be low and it is not in accordance with the concept of green communication.

The aforementioned limitations of spectral efficiency and energy efficiency measures have inspired us to study a new efficiency metric, namely SNR-per-unit-power or power-normalized SNR (PN-SNR), to design network beamforming algorithms and to evaluate the network efficiency. For a single-user network, the PN-SNR is defined as

$$\eta \triangleq \frac{\text{SNR}}{P_{\text{total}}} \tag{1.5}$$

where SNR is the end-to-end received SNR calculated at the destination and P_{total} is the total power used for signal transmission in the network. The parameter η can be viewed as the power efficiency of the network. Without loss of generality, we have assumed that the equivalent channel noise between the transmitter and the receiver has unit-variance. Thus, P_{total} can also be seen as the transmit SNR. In this sense, η represents the received SNR the system provides per unit transmit SNR.

The PN-SNR was first used as an efficiency measure in [53], where it was called power efficiency. It was later used in [39, 69], and [70], where the term PN-SNR was introduced. While the PN-SNR was employed for numerical performance evaluation in [39, 53, 69], its properties and optimal designs using this measure were not investigated. For asynchronous two-way multi-relay network, [70] investigated the joint subcarrier transceiver power loading and relay beamforming optimization that maximizes the minimum SNR of the two users across all subcarriers, where, on each subcarrier, the subproblem of the SNR-maximization for given subcarrier power vectors was proved to result in PN-SNR optimization. But this work focused on SNR optimization and PN-SNR was not considered as an energy efficiency measure.

1.5 Contributions and Thesis Organization

In this thesis, we discover the potential of the PN-SNR as an efficiency measure in network design. Our goal is to design the beamforming scheme that maximize the PN-SNR in relay network. We formulate and solve the PN-SNR maximization problem for several relay network scenarios and compare the performance of the proposed design with existing schemes. We find that PN-SNR maximizing designs are more power efficient with moderate degradation in other performance compared with existing schemes, which means PN-SNR can be used as efficiency measure in green communication designs.

In Chapter 2, design using PN-SNR in single-relay network is considered. We formulate the PN-SNR maximization problem and find the optimal relay power that maximizes the PN-SNR for arbitrarily given transmitter power in close-form. We also study the average received SNR, the outage probability, and the average relay power offered by the proposed design; and compare them with those of the SNR-maximizing scheme and the fixed relay power scheme. Our results indicate that the proposed PN-SNR-maximizing scheme is more power efficient in single-relay network compared with the other two schemes. Meanwhile, with the same power resource, the proposed scheme has comparable performance in the average SNR and is better in outage probability compared with fixed relay power scheme. Also, the proposed scheme has considerably higher PN-SNR with moderate degradation in outage probability compared with the SNR-maximizing scheme.

In Chapter 3, design using PN-SNR in multi-relay network with a sum power constraint on the relays is investigated. We formulate the PN-SNR maximization problem and simplify it into a one-dimensional problem in finding the optimal sum relay power. Then, we prove that the simplified problem has a unique maximum, thus the globally optimal solution can be found with gradient-ascent algorithm. We simulate the performance of the proposed scheme and compare it with the SNR-maximizing scheme and the fixed relay power scheme. Similar results with single-relay network are observed in multi-relay network with a sum relay power constraint.

In Chapter 4, design using PN-SNR in multi-relay network with individual constraint on each relay is investigated. We propose a numerical algorithm for the global optimal solution and a low complexity suboptimal numerical solution. Performance of the proposed designs is simulated and compared with the SNR-maximizing scheme and the all maximum scheme. We observe through simulation that the proposed PN-SNR maximization scheme achieves the highest PN-SNR among the three schemes and the average PN-SNR increases with the number of relays. The proposed scheme is also comparable in average received SNR and outage probability with the SNR maximization scheme. In this chapter, we also consider the problem with one more constraint on the received SNR to maintain the quality of service (QoS) and propose a low complexity algorithm for a suboptimal solution. The average PN-SNR and average throughput of the proposed design are simulated and compared with existing designs. We observe that the proposed design is more power efficient than existing ones with comparable performance in the average throughput. When the transmitter power is small, with higher QoS constraint, the probability of no transmission gets higher; the average throughput decreases, but the the PN-SNR of the network increases.

In Chapter 5, we conclude the thesis and outline possible extensions for this thesis in future works. We also formulate the PN-SNR maximization problem in multi-relay network with relay selection and present the preliminary simulation results of such design. The simulation results imply that PN-SNR is also potential in relay selection design, which can be subsequent works of this thesis. The involved proof is provided in the appendix.

1.6 Notation

For a matrix $\mathbf{A}$, $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$. For a complex scalar α, $|\alpha|$ and $\angle\alpha$ denote the amplitude and phase of α, respectively. For a vector $\mathbf{a}$, $\|\mathbf{a}\|$ denotes its Euclidean norm. For two vectors $\mathbf{a}$ and $\mathbf{b}$ of the same dimension, $\mathbf{a} \circ \mathbf{b}$ is the Schur-Hadamard product of the two vectors. In this thesis, $\text{erf}(\cdot)$ is the error function, $\tan^{-1}(\cdot)$ is the inverse tangent function, and $K_1(\cdot)$ is the first order modified Bessel function of second kind. $\ln(\cdot)$ and $\log(\cdot)$ denote the natural logarithm function and common logarithm function, respectively. The term "subject to" in problem formulations is abbreviated to "s. t." in this thesis.

Chapter 2

Single-Relay Network Design Using PN-SNR

In this chapter, we investigate the power control design using PN-SNR in single-relay network. In Section 2.1, the single-relay system model is introduced. In Section 2.2, we formulate the PN-SNR maximization problem and provide a close-form solution. The performance of the proposed scheme is evaluated in Section 2.3. In Section 2.4, the performance of the proposed scheme is analytically compared with existing schemes. Section 2.5 presents the simulation results. Section 2.6 summarizes the contributions and observations of the chapter.

2.1 System Model

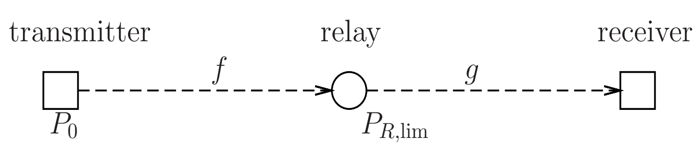

Fig. 2.1: Single-relay network.

In this chapter, we consider the basic three-node relay network with one transmitter, one receiver and one relay, as depicted in Fig. 2.1. The relay has only one single antenna which can be used for both transmission and reception. We denote the channel from the transmitter to the relay as f and the channel from the relay to the receiver as g. There is no direct link between the transmitter and the receiver. We assume that f and g are AWGN Rayleigh flat fading channels as depicted in Chapter 1, i.e., f and g are i.i.d. complex Gaussian with zero-mean and unit-variance. We further assume that the relay and the receiver have perfect knowledge of the channel f and g. We denote the power constraint on the relay as $P_{R,\lim}$, and denote the actual transmit power of the relay as P. Thus, $P \leq P_{R,\lim}$.

We consider a two-step AF protocol for relay transmission. During the first step, the transmitter sends $\sqrt{P_0}s$, where the information symbol s is randomly selected from the codebook $\mathcal{S}$. We assume that s in the codebook are normalized as $\mathbb{E}\{|s|^2\} = 1$. Thus, the power used at the transmitter is P_0. The signal received at the relay can be represented as

$$x = \sqrt{P_0} f s + n_1, \tag{2.1}$$

where n_1 is the noise in the first step and is a Gaussian random variable with zero-mean and unit-variance.

During the second step, the relay adjusts the magnitude and phase of the received signal in (2.1) and retransmits the processed signal with power P. The processed signal can be expressed as

$$t = \sqrt{\frac{P}{1 + |f|^2 P_0}} x e^{j\theta}, \tag{2.2}$$

and the received signal at the receiver is

$$y = tg + n_2, \tag{2.3}$$

where n_2 is the noise in the second step. We assume that n_2 is independent with n_1 and is a Gaussian random variable with zero-mean and unit-variance. With (2.1) and (2.2), the received signal in (2.3) can be further rewritten as

$$y = \sqrt{\frac{PP_0}{1 + |f|^2 P_0}} f g e^{j\theta} s + \sqrt{\frac{P}{1 + |f|^2 P_0}} g e^{j\theta} n_1 + n_2. \tag{2.4}$$

It can be seen that the first term in (2.4) corresponds to the information symbol and the last two terms are the noise. With the fact that $\mathbb{E}\{|s|^2\} = 1$

and n_1 and n_2 are independent Gaussian random variables with unit-variance, the end-to-end received SNR can be expressed as

$$\mathrm{SNR} = \frac{\left|\sqrt{\frac{PP_0}{1+|f|^2P_0}} fge^{j\theta}s\right|^2}{\left|\sqrt{\frac{P}{1+|f|^2P_0}} ge^{j\theta}\right|^2 + 1} = \frac{|fg|^2PP_0}{1+|f|^2P_0+|g|^2P} \approx \frac{|fg|^2PP_0}{|f|^2P_0+|g|^2P}. \quad (2.5)$$

In the third step in (2.5), we have used an approximation which has been shown to be tight in the high SNR regime [71].

According to the definition in (1.5), the PN-SNR of the network is thus

$$\eta = \frac{|fg|^2PP_0}{(1+|f|^2P_0+|g|^2P)(P+P_0)} \approx \frac{|fg|^2PP_0}{(|f|^2P_0+|g|^2P)(P+P_0)}. \quad (2.6)$$

2.2 Problem Formulation and Solution

Using (2.6), our PN-SNR maximization problem can be formulated as

$$\max_{0\leq P\leq P_{R,\lim}} \frac{|fg|^2PP_0}{(1+|f|^2P_0+|g|^2P)(P+P_0)}. \quad (2.7)$$

We first consider the ideal case that the relay power is unlimited, i.e., $P_{R,\lim} = \infty$, then consider the practical case of finite $P_{R,\lim}$. Infinite power constraint is of course ideal, as any device has a finite power limit. We consider this ideal case to better understand the behavior of the PN-SNR efficiency measure. Studying this ideal case also helps us to find the solution to the finite power constraint case.

The PN-SNR maximization problem with unlimited relay power is given as

$$\max_{P\geq 0} \frac{|fg|^2PP_0}{(1+|f|^2P_0+|g|^2P)(P+P_0)}. \quad (2.8)$$

Differentiating the objective function in (2.8) with respect to P and equating it to zero, the optimal relay power, denoted as P^*, can be obtained as

$$P^* = \frac{\sqrt{P_0(1+|f|^2P_0)}}{|g|}. \quad (2.9)$$

When the transmitter power is high ($P_0 \gg 1$), this solution can be approximated as

$$P^* \approx P^*_{\text{approx}} = \frac{|f|}{|g|}P_0. \quad (2.10)$$

The same result can be obtained if the approximate SNR formula in (2.5) is used in the PN-SNR formula.

From (2.9) and (2.10), we can see that although the relay power constraint is assumed to be infinity, for the highest PN-SNR, the relay should only use a finite amount of power. This is different from the SNR-maximizing scheme, where the optimal solution is easily seen to be $P = P_{R,\lim} = \infty$. Also, we can see that the optimal relay power in (2.9) or (2.10) is channel dependent, meaning that for the highest PN-SNR, the relay should adjust its transmit power according to the channel qualities. When the ratio of the transmitter-relay channel quality ($|f|$) to the relay-receiver channel quality ($|g|$) is larger, the relay should use more power; and vice versa. This property complies with the relay noise suppression idea. When this ratio is high, the transmitter-relay channel has a better quality than the relay-receiver channel, the received signal at the relay contains a small noise component and it should use a relatively large amount of power to forward. On the other hand, when the ratio is low, the transmitter-relay channel has lower quality than the relay-receiver channel, the received signal at the relay is highly noisy and the relay should use low power to suppress relay noise amplification.

Now, we consider the practical case that $P_{R,\lim}$ is finite, i.e., $P_{R,\lim} < \infty$. It is straightforward to show that $\frac{d\eta}{dP} > 0$ when $P \leq \frac{|f|}{|g|}P_0$ and $\frac{d\eta}{dP} < 0$ when $P \geq \frac{|f|}{|g|}P_0$. Thus, the PN-SNR increases with P when $P \leq \frac{|f|}{|g|}P_0$ and decreases with P when $P \geq \frac{|f|}{|g|}P_0$. So for the finite power constraint case, the PN-SNR maximizing solution to (2.7) can be easily extended from (2.10) as

$$P_{\text{opt}} \approx \min\left(\frac{|f|}{|g|}P_0, P_{R,\lim}\right). \tag{2.11}$$

In the following subsection, performance of the proposed power control in (2.11) is evaluated.

2.3 Performance Evaluation

In this section, we analyze the network performance under the proposed PN-SNR-maximizing solution to further understand the adopted PN-SNR measure. For the performance, we consider the average relay transmit power, the average PN-SNR, the average end-to-end received SNR, and the outage probability.

We summarize our performance analysis results in the following theorem.

Theorem 1 *With the relay power design in (2.11), the average power consumed by the relay is*

$$P_{\text{ave}} = P_0 \tan^{-1}\left(\frac{P_{R,\text{lim}}}{P_0}\right). \tag{2.12}$$

Define $\xi \triangleq \frac{P_{R,\text{lim}}}{P_0}$. *When* $P_0 \gg 1$ *and using the SNR approximation in (2.5), the average PN-SNR of the network is given as*

$$\eta_{\text{ave}} \approx \frac{3}{8}\pi - \frac{3}{4}\tan^{-1}\left(\frac{1}{\xi}\right) - \frac{4\xi^3 - 7\xi^2 - \xi}{4(\xi+1)(\xi-1)^2} + \frac{2\xi^2 \ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3(\xi+1)}, \tag{2.13}$$

and the corresponding average end-to-end received SNR is

$$\text{SNR}_{\text{ave}} \approx \tag{2.14}$$
$$P_0\left[\frac{\pi}{8} - \frac{1}{4}\tan^{-1}\left(\frac{1}{\xi}\right) - \frac{3\xi^3+5\xi}{4(\xi-1)^2(\xi^2+1)} - \frac{1}{4}\ln\left(\frac{(\xi+1)^2}{\xi^2+1}\right) + \frac{2\xi^2\ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3}\right].$$

Also, with SNR threshold γ_{th}, *the outage probability, denoted as* O, *can be bounded as*

$$\left(1+\frac{1}{\xi}\right)\frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^2}\right) \lesssim O \lesssim \left(1+\frac{1}{\xi}\right)\frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^{4/3}}\right). \tag{2.15}$$

Proof. See Appendix A. ■

First, from (2.12) in Theorem 1, we can see that under the PN-SNR-maximizing design, the average relay power is non-decreasing in P_0. But it is always finite, regardless of the relay power constraint. From (2.13) and (2.14), we can also show that η_{ave} and $\frac{\text{SNR}_{\text{ave}}}{P_0}$ are continuous and non-decreasing in ξ (it can be shown that $\xi = 1$ is not a singular point). We can conclude from Theorem 1 that increasing the ratio of the maximum relay power and the transmitter power (ξ) improves the average PN-SNR, the average received SNR, and the outage probability. For a given P_0, larger ξ means that more power is available at the relay, which results in better performance. However, the performance is bounded by the extreme case where ξ is infinity, i.e., the relay power constraint $P_{R,\text{lim}}$ is unlimited. The performance of the extreme case can be summarized in the following corollary.

Corollary 1 *When $\xi = \infty$, or equivalently, $P_{R,\text{lim}} = \infty$, with the relay power design in (2.11) and using the SNR approximation in (2.5), the average power consumed on the relay is*

$$P_{\text{ave}} = \frac{\pi}{2} P_0, \tag{2.16}$$

the average PN-SNR of the network is

$$\eta_{\text{ave}} \approx \frac{3}{8}\pi - 1, \tag{2.17}$$

the corresponding average end-to-end received SNR is

$$\text{SNR}_{\text{ave}} \approx \frac{\pi}{8} P_0, \tag{2.18}$$

and the outage probability with SNR threshold γ_{th} can be bounded as

$$\frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^2}\right) \lesssim O \lesssim \frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^{4/3}}\right). \tag{2.19}$$

Proof. This corollary can be easily obtained from Theorem 1 by setting $\xi = \infty$. It is also derived in our paper [72]. ■

2.4 Comparison with Existing Schemes

In this section, we compare the performance of the proposed scheme with existing schemes. The first is the fixed relay power scheme, where the relay power P is fixed regardless of the channel quality. The network performance in fixed relay power scheme can be summarized as follows.

Lemma 1 *When $P_0 \gg 1$, with the relay transmit power fixed as P for each transmission, the average PN-SNR of the network is*

$$\eta_{\text{ave\_fix}} \approx \frac{PP_0}{(P-P_0)^2} - \frac{2P^2P_0^2}{(P-P_0)^3(P+P_0)} \ln\left(\frac{P}{P_0}\right), \tag{2.20}$$

the corresponding average end-to-end received SNR is

$$\text{SNR}_{\text{ave\_fix}} \approx \frac{PP_0(P+P_0)}{(P-P_0)^2} - \frac{2P^2P_0^2}{(P-P_0)^3} \ln\left(\frac{P}{P_0}\right). \tag{2.21}$$

If P has the same scaling as P_0, i.e., $P = \zeta P_0$, the outage probability with SNR threshold γ_{th} is

$$O_{\text{fix}} \approx \left(1 + \frac{1}{\zeta}\right) \frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{\ln P_0}{P_0^2}\right). \tag{2.22}$$

Proof. See Appendix B. ■

For a given P_0, by equaling the first order derivative of (2.20) to 0, we can observe that the channel-independent optimal relay power that maximizes the average PN-SNR is P_0, i.e., the relay power should be the same as the transmitter power for the highest $\eta_{\text{ave\_fix}}$.

In what follows, we compare the performance of the proposed PN-SNR maximizing scheme with that of the fixed relay power scheme. For fairness, we set the average relay power used in the two schemes to be the same, i.e., they have the same power resource. Thus, for the fixed relay power scheme, we have $P = P_{\text{ave}} = P_0 \tan^{-1}(P_{R,\text{lim}}/P_0)$. Recalling that $\xi = P_{R,\text{lim}}/P_0$, we simplify the average PN-SNR, the average SNR, and the outage probability in (2.20), (2.21), and (2.22), respectively, as

$$\eta_{\text{ave\_fix}} \approx \frac{\tan^{-1}(\xi)}{[\tan^{-1}(\xi) - 1]^2} - \frac{2\tan^{-1}(\xi)^2 \ln[\tan^{-1}(\xi)]}{[\tan^{-1}(\xi) - 1]^3[\tan^{-1}(\xi) + 1]}, \tag{2.23}$$

$$\text{SNR}_{\text{ave\_fix}} \approx \left[\frac{\tan^{-1}(\xi)[\tan^{-1}(\xi) + 1]}{[\tan^{-1}(\xi) - 1]^2} - \frac{2\tan^{-1}(\xi)^2 \ln[\tan^{-1}(\xi)]}{(\tan^{-1}(\xi) - 1)^3}\right] P_0, \tag{2.24}$$

and

$$O_{\text{fix}} \approx \left[1 + \frac{1}{\tan^{-1}(\xi)}\right] \frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{\ln P_0}{P_0^2}\right). \tag{2.25}$$

Comparing (2.13) with (2.23), we discover that the average PN-SNR in the proposed scheme is always higher than that of the fixed relay power scheme. For the extreme case of $P_{R,\text{lim}} = \infty$, our scheme is 11.3% better in the average PN-SNR. With respect to the outage probability, comparing (2.15) with (2.25), the proposed scheme has the following gain in dB:

$$\text{G}_{\text{outage}} \triangleq 10 \log \frac{O_{\text{fix}}}{O} = 10 \log \left[\frac{\xi \tan^{-1}(\xi) + \xi}{\xi \tan^{-1}(\xi) + \tan^{-1}(\xi)}\right]. \tag{2.26}$$

Note that as $\xi \geq \tan^{-1}(\xi)$ for $\xi \geq 0$, the numerator in (2.26) is larger than the denominator, meaning that G_{outage} is always non-negative. Thus, our scheme outperforms the fixed relay power scheme in outage probability. It can also be shown that $\mathrm{G}_{\text{outage}} \leq 2.14$ dB with equality when $P_{R,\lim} = \infty$.

We can conclude that our proposed scheme is more power efficient than the fixed relay power scheme. Furthermore, it also outperforms the fixed relay power scheme in outage probability with the same relay power consumption. These advantages are due to the PN-SNR measure used in our scheme, leading to a channel-dependent relay power control.

Another existing scheme is the SNR-maximizing scheme, where the relay uses its maximum power for the highest received SNR, i.e., $P = P_{R,\lim}$. In fact, the SNR-maximizing scheme can be viewed as a fixed relay power scheme and its performance can be obtained from (2.20) to (2.22) by setting $P = P_{R,\lim}$ and $\zeta = \xi$. Compared to our proposed method, the SNR-maximizing scheme is expected to have better average received SNR but significantly lower average PN-SNR. Its outage probability (given in (2.25) with $\zeta = \xi$) has the same dominant term as that of the proposed method, indicating that for high P_0, the two schemes have the same outage probability. Thus, the proposed scheme achieves significantly better efficiency in terms of the PN-SNR with about the same outage probability.

2.5 Simulation

In this section, we present the simulated performance of our proposed PN-SNR-maximizing scheme. We also compare the proposed scheme with the fixed relay power scheme and the SNR-maximizing scheme. Channels are randomly generated as i.i.d. circularly symmetric complex Gaussian with zero-mean and unit-variance in our simulation. The main criterion we use to evaluate the network is the average PN-SNR. Meanwhile, we also simulate the average end-to-end received SNR and the outage probability as alternative criteria for the performance evaluation.

We simulate the average PN-SNR, end-to-end received SNR and outage probability with threshold $\gamma_{\text{th}} = 0$ dB for the proposed PN-SNR-maximizing scheme (denoted as "Proposed"), the SNR-maximizing scheme (denoted as

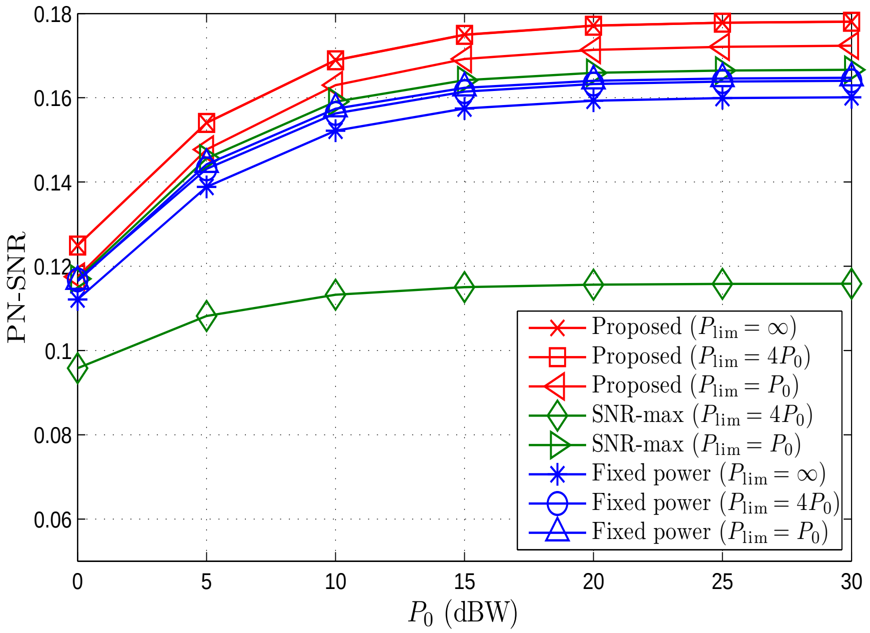

Fig. 2.2: Average PN-SNR versus P_0 for a single-relay network.

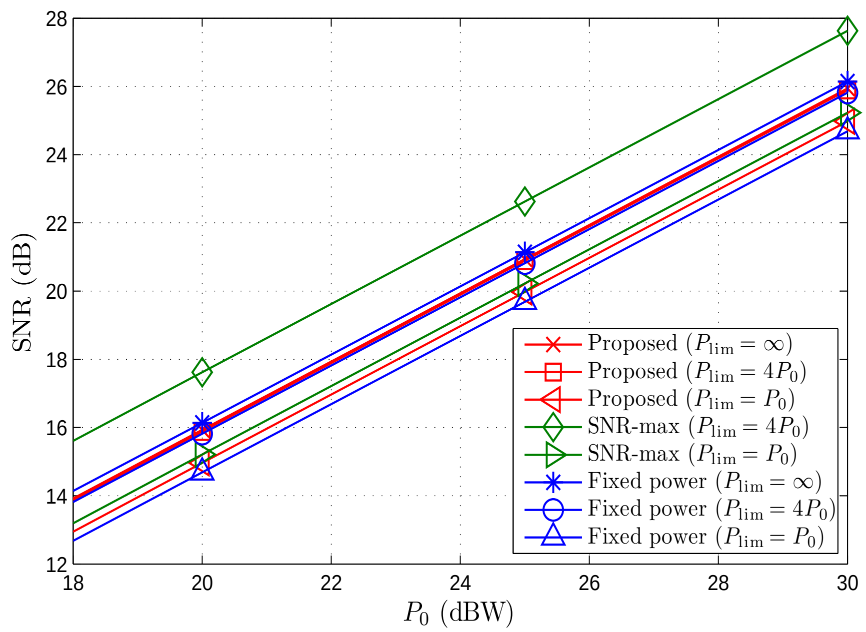

Fig. 2.3: Average received SNR versus P_0 for a single-relay network.

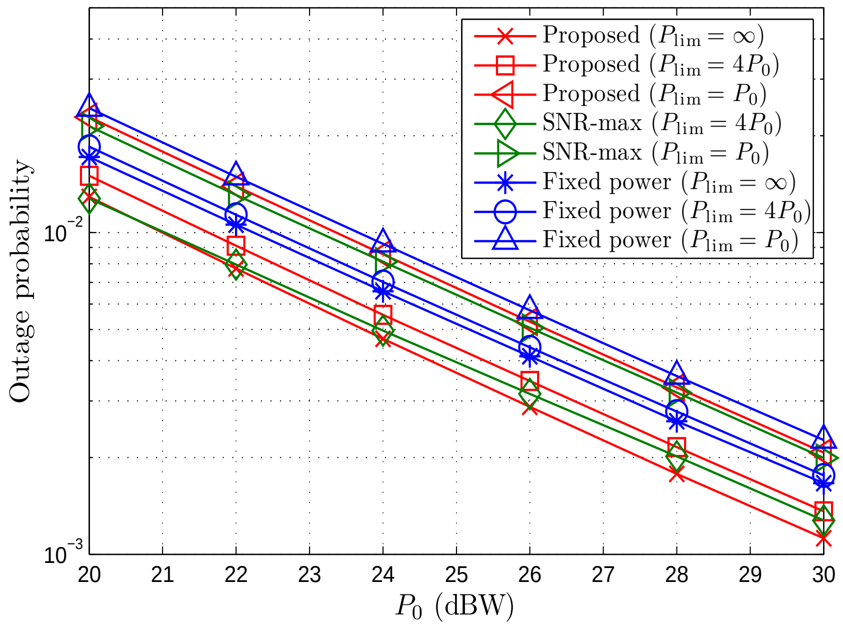

Fig. 2.4: Outage probability versus P_0 for a single-relay network.

"SNR-max") and the fixed relay power scheme (denoted as "Fixed power"). For the relay power constraint, three cases are considered: 1) $P_{R,\text{lim}} = \infty$, 2) $P_{R,\text{lim}} = 4P_0$, and 3) $P_{R,\text{lim}} = P_0$. For fair comparison, in the fixed relay power scheme, the relay power is set to be P_{ave} provided in (2.12). So the proposed and the fixed relay power schemes use the same amount of power resource. In the SNR-maximizing scheme, the relay always uses its maximum power $P_{R,\text{lim}}$, and thus, it consumes more relay power than the other two schemes. Also, the case $P_{R,\text{lim}} = \infty$ does not apply to the SNR-maximizing scheme since it is impractical for the relay to transmit with infinite power.

Fig. 2.2 shows the average PN-SNR versus P_0 for the three schemes. We can see that in the proposed PN-SNR-maximizing scheme, the PN-SNR is non-decreasing as $P_{R,\text{lim}}$ increases. In the SNR-maximizing scheme, however, PN-SNR decreases sharply as $P_{R,\text{lim}}$ increases. It can be shown that the PN-SNR decreases to 0 as $P_{R,\text{lim}}$ tends to infinity. In the fixed relay power scheme, the PN-SNR slowly decreases as $P_{R,\text{lim}}$ increases. Among the three schemes, our proposed scheme always achieves the highest PN-SNR. When $P_{R,\text{lim}} = P_0$, the proposed scheme is 5.4% better than the fixed relay power scheme and 4.2%

better than the SNR-maximizing scheme at $P_0 = 30$ dBW. When $P_{R,\text{lim}} = 4P_0$, it is 9.2% better than the fixed relay power scheme and 53% better than the SNR-maximizing scheme at $P_0 = 30$ dBW. These observations comply with the theoretical analysis in previous sections.

Fig. 2.3 shows the average end-to-end received SNR versus P_0 for the three schemes. We can see that in the SNR-maximizing scheme, the average SNR increases rapidly as $P_{R,\text{lim}}$ increases, and grows out of bound when $P_{R,\text{lim}}$ approaches infinity. But in both the proposed scheme and the fixed relay power scheme, the average SNR increases but saturates quickly. This is because for these two schemes, only partial relay power is used. When $P_{R,\text{lim}} = P_0$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is 0.25 dB better than the fixed relay power scheme but 0.2 dB worse than the SNR-maximizing scheme. When $P_{R,\text{lim}} = 4P_0$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is 0.1 dB better than the fixed relay power scheme, while about 2 dB worse than the SNR-maximizing scheme. When $P_{R,\text{lim}} = \infty$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is inferior to the fixed relay power scheme by 0.2 dB. The simulation results comply with the analysis in previous sections and indicate that our proposed scheme has comparable performance in average SNR with the fixed relay power scheme but it is inferior to the SNR-maximizing scheme.

Fig. 2.4 shows the outage probability versus P_0 for the three schemes. We can see that as $P_{R,\text{lim}}$ increases, all three schemes have better performance in outage probability. When $P_{R,\text{lim}} = P_0$, our proposed scheme is 0.5 dB superior to the fixed relay power scheme but is 0.1 dB inferior to the SNR-maximizing scheme. When $P_{R,\text{lim}} = 4P_0$, our proposed scheme is 1.2 dB superior to the fixed relay power scheme but is inferior, by 0.25 dB, to the SNR-maximizing scheme. For the extreme case $P_{R,\text{lim}} = \infty$, our proposed scheme is superior by about 1.8 dB to the fixed relay power scheme. The two curves in our scheme and the SNR-maximizing scheme become closer to each other as P_0 increases. These observations are in accordance with the analysis in previous sections. We can conclude that the proposed PN-SNR-maximizing scheme outperforms the fixed relay power scheme in outage probability and is comparable to the

SNR-maximizing scheme.

2.6 Summary

In this chapter, we formulated and solved the PN-SNR maximization problem in single-relay network with relay power constraint. Optimal relay power was provided in close-form and the network performance was compared analytically and numerically with the fixed relay power scheme and the SNR-maximizing scheme.

We can see from the derivations in Section 2.4 and the simulation results in Section 2.5 that the proposed PN-SNR-maximizing scheme is more efficient in single-relay network compared with the other two schemes. Meanwhile, with the same power resource, the proposed scheme has comparable performance in the average SNR and is better in outage probability compared with the fixed relay power scheme. Naturally, the SNR-maximizing scheme achieves higher SNR than the proposed scheme. But its advantage in outage probability is small and negligible in the high SNR regime. And it has significant lower PN-SNR than the proposed scheme, implying that the power is not efficiently used in this method.

The observations in this chapter reveal the potential of PN-SNR in relay network design and encourage us to discover the multi-relay network design using PN-SNR, which is the major problem in the next two chapters.

Chapter 3

Multi-Relay Network Design Using PN-SNR Under Sum Relay Power Constraint

In this chapter, we investigate the power control design using PN-SNR in multi-relay network with a sum power constraint on all relays. In Section 3.1, we introduce the system model of multi-relay network and derive the expression of PN-SNR in such networks. In Section 3.2, we formulate the PN-SNR maximization problem in multi-relay network with a sum power constraint on relays and propose a low complexity algorithm for the optimal solution. In Section 3.3, we simulate the performance of the proposed scheme and compare it with existing schemes. In Section 3.4, we conclude the results and observations from previous sections.

3.1 System Model

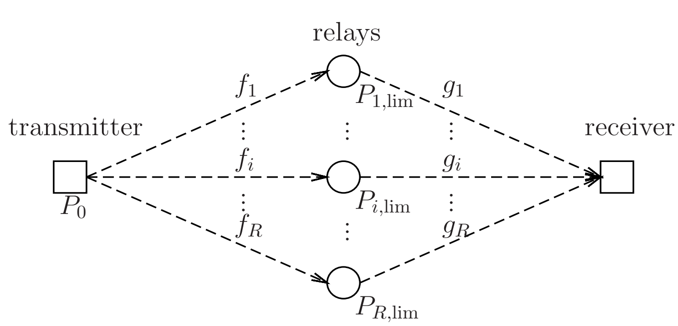

Fig. 3.1: Multi-relay network.

In this chapter, we consider a general distributed network with one transmitter, one receiver, and R relays, as depicted in Fig. 3.1. Each relay has only one single antenna which can be used for both transmission and reception. We denote the channel from the transmitter to the ith relay as f_i and the channel from the ith relay to the receiver as g_i. We assume that there is no direct link between the transmitter and the receiver. We assume that f_i and g_i are i.i.d. complex Gaussian random variable with zero-mean and unit-variance, so the channel magnitudes follow Rayleigh distribution. All channels are assumed to be flat-fading channels. We also assume that each relay knows its own channels, i.e., the ith relay knows f_i and g_i, and the receiver knows all channels. The required channel state information at the receiver can be obtained via channel estimation and feedback [49, 73–75]. The ith relay can obtain f_i by training and g_i by feedback. Let

$$\mathbf{f} \triangleq [f_1 \ f_2 \ \ldots \ f_R]^T,$$

and

$$\mathbf{g} \triangleq [g_1 \ g_2 \ \ldots \ g_R]^T,$$

which are the transmitter-relay and relay-receiver channel vectors. We define the effective end-to-end channel vector between the transmitter and receiver as

$$\mathbf{h} \triangleq \mathbf{f} \circ \mathbf{g} = [f_1 g_1 \ f_2 g_2 \ \ldots \ f_R g_R]^T.$$

We herein consider a two-step AF protocol with relay beamforming, where the relays adjust the amplitudes and the phases of their received signals before forwarding them [37]. During the first step, the transmitter sends $\sqrt{P_0}s$, where the information symbol s is randomly selected from the codebook $\mathcal{S}$. We assume that s in the codebook are normalized as $\mathbb{E}\{|s|^2\} = 1$. Thus, the average power used at the transmitter is P_0. The signals received at the relays can be represented as

$$\mathbf{x} = \sqrt{P_0}\mathbf{f}s + \mathbf{z}, \tag{3.1}$$

where $\mathbf{x}$ is the $R \times 1$ complex vector of the signals received by relays and x_i corresponds to the received signal on the ith relay. $\mathbf{z}$ is the $R \times 1$ complex vector of the relay noises. We assume that all noises are i.i.d. complex Gaussian random variables with zero-mean and unit-variance.

In the second step, the ith relay multiplies its received signal by a complex weight w_i to adjust the phase and magnitude of the signal and transmits the adjusted signal. All relays share the same channel and are assumed to be perfectly synchronized. The $R \times 1$ complex vector $\mathbf{t}$ of the transmitted signals of all relays can then be expressed as

$$\mathbf{t} = \mathbf{w} \circ \mathbf{x}, \tag{3.2}$$

where $\mathbf{w} \triangleq [w_1 \; w_2 \; \ldots \; w_R]^T$ is referred to as the relay beamforming vector. Denoting the ith entry of $\mathbf{t}$ as t_i, the power consumed on the ith relay, denoted as P_i, can be calculated, using (3.2), as

$$P_i = \mathbb{E}\{|t_i|^2\} = (1 + P_0|f_i|^2)|w_i|^2, \tag{3.3}$$

The signal received at the receiver, denoted as y, can be written as

$$\begin{aligned} y &= \mathbf{t}^T\mathbf{g} + n = (\mathbf{w} \circ \mathbf{x})^T\mathbf{g} + n \\ &= (\mathbf{f} \circ \mathbf{w})^T\mathbf{g}\sqrt{P_0}s + (\mathbf{w} \circ \mathbf{z})^T\mathbf{g} + n \\ &= \sqrt{P_0}s\mathbf{w}^T\mathbf{h} + \mathbf{w}^T(\mathbf{g} \circ \mathbf{z}) + n \end{aligned} \tag{3.4}$$

where the noise n at the receiver is assumed to be independent of $z_1, \ldots, z_R$ and is Gaussian distributed with zero-mean and unit-variance. Note that the first term in (3.4) corresponds to the information symbol and the last two terms are the noise. With the fact that $\mathbb{E}\{|s|^2\} = 1$ and all noises are i.i.d. Gaussian random variable with zero-mean and unit-variance, the end-to-end received SNR can be expressed as

$$\text{SNR} = \frac{\left|\sqrt{P_0}s\mathbf{w}^T\mathbf{h}\right|^2}{\left|\mathbf{w}^T(\mathbf{g} \circ \mathbf{z})\right|^2 + 1} = \frac{P_0\left|\mathbf{w}^T\mathbf{h}\right|^2}{1 + \|\mathbf{w} \circ \mathbf{g}\|^2}. \tag{3.5}$$

Recall the power consumed by the ith relay is given as in (3.3). The total transmit power consumed on all relays is

$$\sum_{i=1}^{R}(1 + P_0|f_i|^2)|w_i|^2 = P_0\|\mathbf{w} \circ \mathbf{a}\|^2,$$

where

$$\mathbf{a} \triangleq \left[\sqrt{\frac{1}{P_0} + |f_1|^2} \quad \cdots \quad \sqrt{\frac{1}{P_0} + |f_R|^2} \right].$$

The total transmit power consumed in the whole network is thus

$$P_T = P_0 + P_0 \|\mathbf{w} \circ \mathbf{a}\|^2.$$

According to our definition in (1.5), the PN-SNR of the relay network is

$$\eta \triangleq \frac{\text{SNR}}{P_T} = \frac{P_0 \left|\mathbf{w}^T \mathbf{h}\right|^2}{(1 + \|\mathbf{w} \circ \mathbf{g}\|^2)(P_0 + P_0 \|\mathbf{w} \circ \mathbf{a}\|^2)}$$
$$= \frac{\left|\mathbf{w}^T \mathbf{h}\right|^2}{(1 + \|\mathbf{w} \circ \mathbf{g}\|^2)(1 + \|\mathbf{w} \circ \mathbf{a}\|^2)}. \tag{3.6}$$

Denote the amplitude and the phase of w_i as α_i and θ_i, respectively, i.e., $w_i = \alpha_i e^{j\theta_i}$. Let

$$\boldsymbol{\alpha} \triangleq [\alpha_1 \ \cdots \ \alpha_R]^T,$$

and

$$\boldsymbol{\theta} \triangleq [\theta_1 \ \cdots \ \theta_R]^T.$$

Note that both $\|\mathbf{w} \circ \mathbf{g}\|^2$ and $\|\mathbf{w} \circ \mathbf{a}\|^2$ are independent of the phase vector $\boldsymbol{\theta}$. Thus, the denominator of η given in (3.6) is independent of $\boldsymbol{\theta}$. It is obvious that the numerator is maximized when $\theta_i = -\angle h_i$ for any given $\boldsymbol{\alpha}$, where $h_i = f_i g_i$ is the ith entry of $\mathbf{h}$. That is, the ith relay should adjust to cancel the phase of its channels during the second step.

With the optimal phase adjustment at the relays, the end-to-end received SNR in (3.5) reduces to

$$\text{SNR} = \frac{P_0 (\boldsymbol{\alpha}^T \mathbf{b})^2}{1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2}, \tag{3.7}$$

and the PN-SNR in (3.6) reduces to

$$\eta = \frac{(\boldsymbol{\alpha}^T \mathbf{b})^2}{(1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2)(1 + \|\boldsymbol{\alpha} \circ \mathbf{a}\|^2)}, \tag{3.8}$$

where

$$\mathbf{b} \triangleq [|f_1 g_1| \ \cdots \ |f_R g_R|],$$

$$\mathbf{d} \triangleq [|g_1|, \cdots, |g_R|],$$

and

$$\mathbf{a} = \left[\sqrt{\frac{1}{P_0} + |f_1|^2} \quad \cdots \quad \sqrt{\frac{1}{P_0} + |f_R|^2} \right].$$

3.2 Problem Formulation and Solution

In this section, we formulate the PN-SNR maximization problem in multi-relay network with a sum power constraint on relays, where the total power consumed by all relays, denoted as P, is no larger than $P_{R,\text{lim}}$, i.e., $P = \sum_{i=1}^{R} P_i \leq P_{R,\text{lim}}$. This sum-power constraint model has been widely used in the literature, e.g., [36, 45, 50].

From (3.3) and (3.8), the PN-SNR maximization problem can be expressed as

$$\begin{aligned} \max_{\boldsymbol{\alpha} \succeq 0} \quad & \frac{(\boldsymbol{\alpha}^T \mathbf{b})^2}{(1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2)(1 + \|\boldsymbol{\alpha} \circ \mathbf{a}\|^2)} \\ \text{s. t.} \quad & \sum_{i=1}^{R} (1 + P_0 |f_i|^2) |\alpha_i|^2 \leq P_{R,\text{lim}}. \end{aligned} \tag{3.9}$$

The problem in (3.9) is a non-convex optimization problem since the objective function is non-convex. In this section, we first simplify the problem into a one-dimensional problem using the results in [36], then prove that the maximum of the simplified problem is unique. Thus, we propose to use a gradient-ascent algorithm to find the optimal solution.

To simplify the problem, we can rewrite (3.9) as follow:

$$\begin{aligned} \max_{P} \quad & \frac{1}{P_0 + P} \left(\max_{\boldsymbol{\alpha} \succeq 0} \frac{P_0 (\boldsymbol{\alpha}^T \mathbf{b})^2}{1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2} \right) \\ \text{s. t.} \quad & 0 \leq P \leq P_{R,\text{lim}} \,, P = P_0 \|\boldsymbol{\alpha} \circ \mathbf{a}\|^2, \end{aligned} \tag{3.10}$$

With any fixed sum relay power P, the inner problem in (3.10) is an SNR optimization problem with a sum relay power constraint. This problem is solved in [36] where the optimal power coefficient of the ith relay is

$$\alpha_i = \frac{|f_i g_i|}{|f_i|^2 P_0 + |g_i|^2 P + 1} \sqrt{\frac{P}{\sum_{i=1}^{R} \frac{|f_i|^2 |g_i|^2 (|f_i|^2 P_0 + 1)}{(|f_i|^2 P_0 + |g_i|^2 P + 1)^2}}}, \tag{3.11}$$

and the corresponding maximum end-to-end received SNR is

$$\text{SNR}_{\max}(P) = \max_{\boldsymbol{\alpha} \succeq 0} \frac{P_0 (\boldsymbol{\alpha}^T \mathbf{b})^2}{1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2} = \sum_{i=1}^{R} \frac{|f_i|^2 |g_i|^2 P_0 P}{|f_i|^2 P_0 + |g_i|^2 P + 1}. \tag{3.12}$$

Substituting (3.12) into (3.10), our PN-SNR maximization problem is reduced to the following one-dimensional problem of finding the optimal sum power P consumed on all relays:

$$\max_{0 \le P \le P_{R,\lim}} \sum_{i=1}^{R} \frac{|f_i|^2 |g_i|^2 P_0 P}{(|f_i|^2 P_0 + |g_i|^2 P + 1)(P + P_0)}. \tag{3.13}$$

The second order derivative of the objective function can be calculated as

$$\frac{d^2\eta}{dP^2} = \tag{3.14}$$
$$\sum_{i=1}^{R} \frac{2|f_i|^2|g_i|^2 P_0 (|g_i|^4 P^3 - |g_i|^2 P_0 (|f_i|^2 P_0^2 + 1)(3P + P_0) - P_0 (|f_i|^2 P_0^2 + 1)^2)}{(P + P_0)^3 (|f_i|^2 P_0 + |g_i|^2 P + 1)^3}.$$

It can be seen from (3.14) that $\frac{d^2\eta}{dP^2}$ is not always negative, so the objective function in (3.13) is not concave in general.

To solve (3.13), we first consider the case when the sum power constraint on relays is unlimited, i.e., $0 \le P < \infty$. The following lemma is proved.

Lemma 2 *The objective function in (3.13) is a semi-strictly quasi-concave function and has only one maximum for* $0 \le P < \infty$*.*

Proof. The objective function in (3.13) can be expressed as $\frac{\text{SNR}_{\max}(P)}{P+P_0}$ where the nominator is provided in (3.12). It is easy to verify that

$$\frac{d^2 \text{SNR}_{\max}}{dP^2} = -\sum_{i=1}^{R} \frac{2|f_i|^2 |g_i|^4 P_0 (|f_i|^2 P_0 + 1)}{(|f_i|^2 P_0 + |g_i|^2 P + 1)^3} < 0.$$

In other words, the numerator is a strict concave function of P. It is obvious that $P + P_0$ is a convex function and both $\text{SNR}_{\max}$ and $P + P_0$ are positive for $P \ge 0$. According to Theorem 2.3.8 in [76], the objective function is a semi-strictly quasi-concave function. Moreover, it is shown in [77] that a semi-strictly quasi-concave function has a unique maximum if the numerator is strictly concave. Thus, the maximum of (3.13) is unique for $P \ge 0$. ■

Denote the optimal sum power as P^*. To find P^*, we propose to use a gradient-ascent algorithm. It has been shown in [78] that by proper step size selection, gradient-ascent algorithm will converge to a stationary point that satisfies $\frac{d\eta}{dP} = 0$. According to Lemma 2, this is also the only stationary point for $P \geq 0$. Thus, gradient-ascent algorithm will converge to the optimal solution. The complexity of such algorithm is low. Newton's algorithm, for example, has quadratic convergence. In each iteration, Newton's algorithm only needs to calculate the second order derivative in (3.14).

We now consider the case when $P_{R,\lim}$ is finite. Since the objective function in (3.13) has a unique maximum at P^*, it is non-decreasing when $P \leq P^*$ and non-increasing when $P \geq P^*$. Thus, the optimal sum relay power in this case can be expressed as

$$P_{\text{opt}} = \min(P^*, P_{R,\lim}). \tag{3.15}$$

Finally, the optimal power control coefficient for each relay under sum relay power constraint can be easily obtained by using (3.15) in (3.11).

Unfortunately, we are unable to analytically investigate the network performance in this case since the optimal solution can only be found numerically. Numerical simulation results of the network performance will be shown in Section 3.3.

3.3 Simulation

In this section, we present the simulation results for multi-relay network with a sum relay power constraint on all relays. We also compare the proposed scheme with the fixed relay power scheme and the SNR-maximizing scheme.

Channels are randomly generated as i.i.d. circularly symmetric complex Gaussian with zero-mean and unit-variance in our simulation. The main criterion we use to evaluate the network is the average PN-SNR. Meanwhile, we also simulate the average end-to-end received SNR and the outage probability as alternative criteria for the performance evaluation.

We simulate the average PN-SNR, the average received SNR and the outage probability with threshold $\gamma_{\text{th}} = 0$ dB for the proposed PN-SNR-maximizing scheme, the SNR-maximizing scheme, and the fixed relay power scheme. In the fixed relay power scheme, the sum power on relays is fixed for each transmission regardless of the channel quality. For fair comparisons, this fixed power is set to be the average sum relay power P in the proposed scheme. In the SNR-

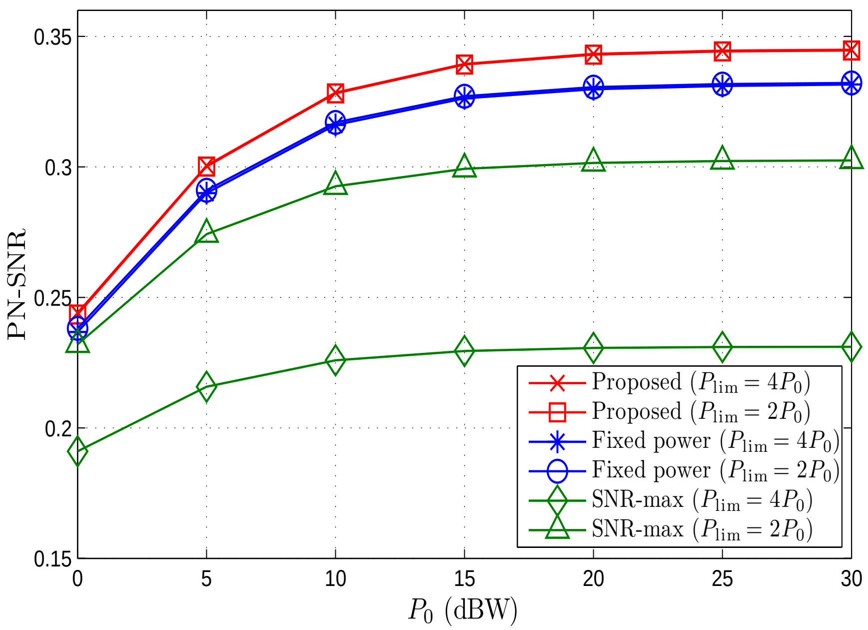

Fig. 3.2: Average PN-SNR versus P_0 for a two-relay network with sum relay power constraint.

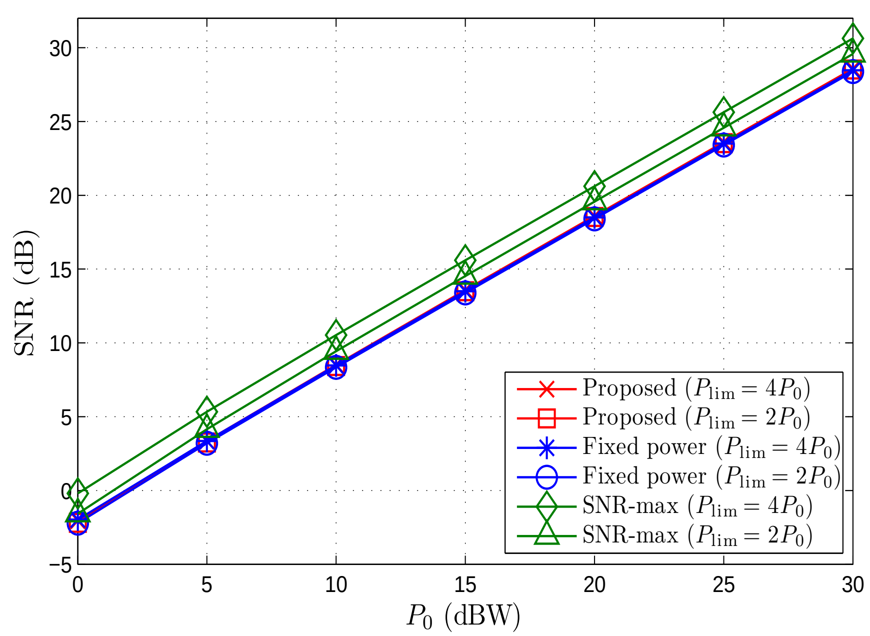

Fig. 3.3: Average SNR versus P_0 for a two-relay network with sum relay power constraint.

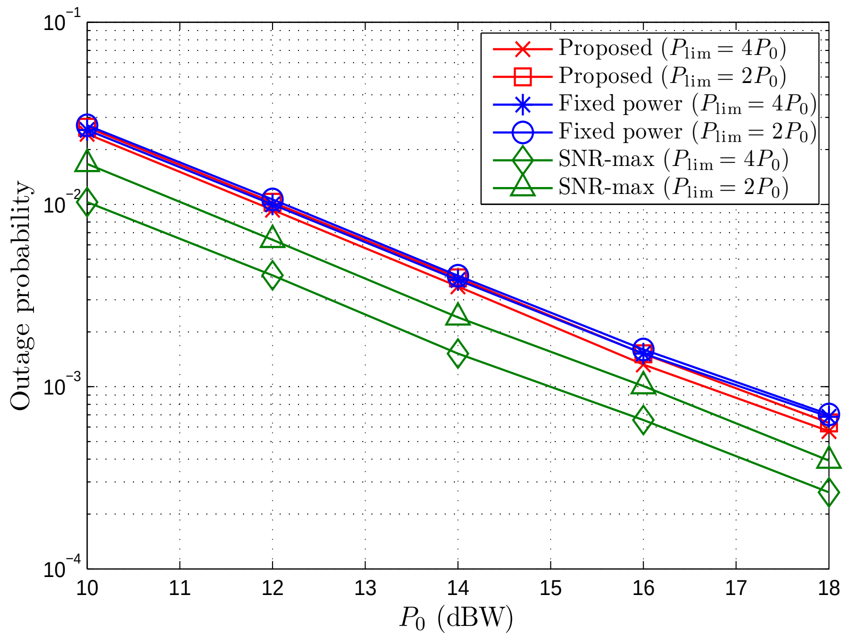

Fig. 3.4: Outage probability versus P_0 for a two-relay network with sum relay power constraint.

maximizing scheme, the relays always use the maximum sum power $P_{R,\lim}$ to achieve the maximum SNR. The power control coefficient for each relay is obtained according to (3.11). We simulate a two-relay network with the sum power constraint $P_{R,\lim} = 2P_0$ and $P_{R,\lim} = 4P_0$.

Fig. 3.2 shows the average PN-SNR versus P_0 for the three schemes. In the PN-SNR-maximizing scheme, the average PN-SNR slightly increases as $P_{R,\lim}$ changes from $2P_0$ to $4P_0$. In the fixed relay power scheme, the average PN-SNR slightly decreases as $P_{R,\lim}$ increases. In the SNR-maximizing scheme, the average PN-SNR sharply decreases as $P_{R,\lim}$ increases. This is the same trend as in single-relay networks. Among the three schemes, the proposed scheme always achieves the highest PN-SNR. When $P_{R,\lim} = 2P_0$ and $P_0 = 30$ dBW, the proposed scheme outperforms the fixed relay power scheme and the SNR-maximizing scheme in term of PN-SNR by 3.8% and 14%, respectively, and the percentage turns to 4% and 48% when $P_{R,\lim} = 4P_0$.

Fig. 3.3 shows the average end-to-end received SNR versus P_0 for the three schemes. We can see that in the SNR-maximizing scheme, the average SNR increases significantly as $P_{R,\lim}$ increases from $2P_0$ to $4P_0$. But in both the proposed scheme and the fixed relay power scheme, the average SNR increases but saturates quickly. When $P_{R,\lim} = 2P_0$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is 0.2 dB better than the fixed relay power scheme but 1 dB worse than the SNR-maximizing scheme. When $P_{R,\lim} = 4P_0$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is 0.15 dB better than the fixed relay power scheme, while about 2 dB worse than the SNR-maximizing scheme. The simulation results indicate that the proposed scheme is better than the fixed relay power scheme where the same power source is used while it is inferior compared with the SNR-maximizing scheme.

Fig. 3.4 shows the outage probability versus P_0 for the three schemes. Note that for all three schemes, the outage probabilities decreases as $P_{R,\lim}$ grows from $2P_0$ to $4P_0$. When $P_{R,\lim} = 2P_0$, our proposed scheme outperforms the fixed relay power scheme by about 0.4 dB, but it is 1 dB inferior to the SNR-maximizing scheme. When $P_{R,\lim} = 4P_0$, our proposed scheme is 0.6 dB superior to the fixed relay power scheme but is about 1.8 dB inferior to the SNR-maximizing scheme. We can see that the gap between the proposed scheme and the SNR-maximizing scheme grows larger as $P_{R,\lim}$ increases.

3.4 Summary

In this chapter, we investigated the PN-SNR maximization problem in multi-relay network design under a sum power constraint on the relays. The problem was simplified into a one-dimensional problem and proved to have unique maximum. Then, gradient-ascent algorithm was used as an optimal numerical solution and the performance of the proposed design was compared with the fixed relay power scheme and the SNR-maximizing scheme.

We can conclude from the derivations and the simulation results that the PN-SNR-maximizing scheme is more power efficient than the other two schemes. Compared with the fixed relay power scheme with the same power resource, the proposed scheme also outperforms in outage probability. In the SNR-maximizing scheme, more power is used to achieve a better outage probability compared with our proposed scheme. But the efficiency of consumed power is low in the sense of producing received SNR. Our results indicate that there is a tradeoff between PN-SNR and received SNR.

Chapter 4

Multi-Relay Network Design Using PN-SNR Under Individual Relay Power Constraints

Even though the sum relay power constraint model in Chapter 3 has been widely studied in literature, it is not practical in many wireless applications in the sense that relays may not be able to share power in most distributed networks. In this chapter, we investigate the power control design using PN-SNR in multi-relay network, where each relay has an individual power constraint. We use the same network topology as depicted in Fig. 3.1 in Chapter 3, but different power constraints are considered. In Section 4.1, we consider the basic PN-SNR maximization problem where each relay has its own power constraint. In addition to the basic PN-SNR maximization problem in Section 4.1, we also consider the QoS-constrained PN-SNR maximization problem under individual power constraint on each relay in Section 4.2. Section 4.3 concludes the works in this chapter.

4.1 Basic PN-SNR Maximization

In this section, we formulate and solve the basic PN-SNR maximization problem in multi-relay network with separate relay power constraints. The same network topology in Fig. 3.1 in Chapter 3 is considered. But in this chapter, we assume that the ith relay has its own power constraint denoted as $P_{i,\lim}$.

4.1.1 Problem Formulation

Since the same transceiver design in Chapter 3 is considered, the SNR formula in (3.7) and the PN-SNR formula in (3.8) can be used in this Chapter. Recall the power consumed on the ith relay in (3.3), our PN-SNR maximization problem under separate power constraints can be described as

$$\begin{aligned} \max_{\boldsymbol{\alpha} \succeq 0} \quad & \frac{(\boldsymbol{\alpha}^T \mathbf{b})^2}{(1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2)(1 + \|\boldsymbol{\alpha} \circ \mathbf{a}\|^2)} \\ \text{s. t.} \quad & 0 \leq \alpha_i \leq \sqrt{\frac{P_{i,\lim}}{1 + P_0 |f_i|^2}}, \ \text{for } i = 1, \cdots, R, \end{aligned} \tag{4.1}$$

where $\mathbf{a} = \left[\sqrt{\frac{1}{P_0} + |f_1|^2} \ \cdots \ \sqrt{\frac{1}{P_0} + |f_R|^2} \right]$ as defined in Section 3.1.

This is a non-convex optimization problem in which finding the globally optimal solution is usually sophisticated. We first propose a numerical algorithm to obtain the optimal solution. Next, we provide a low-complexity algorithm to find a suboptimal solution for the problem. The performance of the suboptimal solution is simulated and compared with that of the optimal solution.

4.1.2 Optimal Solution

We first examine the Karush-Kuhn-Tucker (KKT) conditions for (4.1) to better understand the problem. In general, the KKT conditions are not sufficient optimality conditions for non-convex problems. With linear constraint, however, the KKT conditions are necessary optimality conditions. With straightforward calculations, KKT conditions of the problem in (4.1) can be derived as

$$\alpha_i \left(\alpha_i - \sqrt{\frac{P_{i,\lim}}{1 + P_0 |f_i|^2}} \right) \frac{\partial \eta}{\partial \alpha_i} = 0.$$

The optimal solution will either be an inner point of the feasible set satisfying $\nabla_{\boldsymbol{\alpha}} \eta = 0$ or be a boundary point meaning that there exists at least one i, such that $\alpha_i = 0$ or $\alpha_i = \sqrt{\frac{P_{i,\lim}}{1+P_0|f_i|^2}}$. If the optimal solution is an inner point, we will show later with Lemma 3 that it can be easily found by gradient-ascent algorithm. However, if it is on the boundary, gradient-ascent algorithm can only converge to a stationary point which may not even be locally optimal [78]. The uniqueness of locally optimal solution of (4.1) is not guaranteed either.

When the optimal solution is on the boundary, the constraints in (4.1) are satisfied with equality for some i, which means some relays will transmit with zero or maximum power. The difficulty lies in determining which relay transmit with zero or maximum power. Exhaustive search for these relays has exponential complexity in the number of relays, and thus, it is obviously impractical. The same problem is encountered in [37] and [53], where optimal and suboptimal relay ordering criteria are proposed to reduce the complexity. In our problem, however, optimal relay ordering criteria may not exist.

Thus, we combine sequential quadratic programming (SQP) algorithm [79] with scatter search to obtain the globally optimal solution. The former algorithm is guaranteed to converge to a locally optimal solution [80] and the latter search starts SQP algorithm from different randomly selected initial points for a number of times to find the globally optimal solution.

SQP algorithm is widely used in solving nonlinear optimization problems whose main idea is to solve a non-convex problem by successive convex approximation [81]. It is also a gradient based iterative algorithm. At each major iteration, a Taylor series approximation of the objective function (or Lagrangian function if nonlinear constraints are involved) at a local iteration point is made. Then, an approximation of the Hessian matrix of the objective function is used to generate a convex quadratic programming (QP) subproblem whose solution is used to form a direction for the next iteration. With properly selected step size, SQP algorithm will converge to a local optimum in finite iterations for arbitrarily small error tolerance. In our simulations, we use Matlab's optimization toolbox to implement the SQP algorithm.

This SQP algorithm is more computationally complex compared with the gradient-ascent algorithm. According to [80], the rate of convergence of SQP algorithm is at best super-linear. Meanwhile, a QP subproblem is involved in each iteration and the SQP algorithm is run several times to find the globally optimal solution. Due to the disadvantage in complexity, the optimal solution proposed in this subsection is mainly used as a benchmark for performance evaluation.

4.1.3 Suboptimal Solution

In this subsection, we will discover a computationally more affordable algorithm to find a suboptimal solution. Recall that in single-relay and multi-relay networks with a sum relay power constraint, while solving the PN-SNR maximization problems, we first find the optimal solutions without any power constraint then project the optimal solution into the feasible set. In both cases, it is either a one-dimensional problem or it can be simplified into a one-dimensional problem in which the projection preserves the optimality. In the PN-SNR maximization problem with separate power constraints on relays, however, projection no longer preserves optimality. Nevertheless, the same methodology can be used to obtain a suboptimal solution. We first ignore the power constraints in (4.1) and focus on the following problem

$$\max_{\boldsymbol{\alpha}\succeq 0} \frac{(\boldsymbol{\alpha}^T\mathbf{b})^2}{(1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2)(1+\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2)}. \tag{4.2}$$

The following property for the objective function in (4.2) is proved.

Lemma 3 *The objective function in (4.2) has unique maximum for* $\boldsymbol{\alpha}\succeq 0$.

Proof. The problem in (4.2) and (3.9) have the same objective function while the constraint in (3.9) is $P_0\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2 < P_{R,\lim}$. Thus, (4.2) can be viewed as a special case of (3.9) when $P_{R,\lim}$ is infinity, where the sum relay power constraint is eliminated. According to Lemma 2, the problem in (3.9) has a unique maximum for all $P_{R,\lim}$. Thus, the maximum for (4.2) is unique. ■

With Lemma 3, the globally optimal solution for problem (4.2) can be easily located with the gradient-ascent algorithm used in Section 3.2. We denote the optimal solution for (4.2) as $\boldsymbol{\alpha}^*$. Our suboptimal solution for (4.1), denoted as $\boldsymbol{\alpha}^{\text{sub}}$, is obtained by truncating those entries of $\boldsymbol{\alpha}^*$ whose amplitudes exceed $\sqrt{\frac{P_{i,\lim}}{1+P_0|f_i|^2}}$, i.e.,

$$\alpha_i^{\text{sub}} = \min\left(\alpha_i^*, \sqrt{\frac{P_{i,\lim}}{1+P_0|f_i|^2}}\right) \text{ for } i=1,\cdots,R.$$

In fact, we know from previous discussion that $\boldsymbol{\alpha}^{\text{sub}}$ is the optimal solution if it is an inner point of the feasible set. If it is a boundary point, $\boldsymbol{\alpha}^{\text{sub}}$ is suboptimal. We will see in the next section that this suboptimal solution actually has close-to-optimal performance.

4.1.4 Simulation

In this subsection, we investigate the performance of a multi-relay network with an individual power constraint on each relay. We simulate the average PN-SNR, the average received SNR and the outage probability with threshold $\gamma_{\text{th}} = 0$ dB for the PN-SNR-maximizing scheme (denoted as "Proposed") and compare them with the SNR-maximizing scheme (denoted as"SNR-max") and the all maximum scheme (denoted as "All-max"). In the SNR-maximizing scheme, the optimal beamforming design proposed in [37] is employed to maximize the end-to-end received SNR. In the all maximum scheme, all relays transmit with their maximum power.

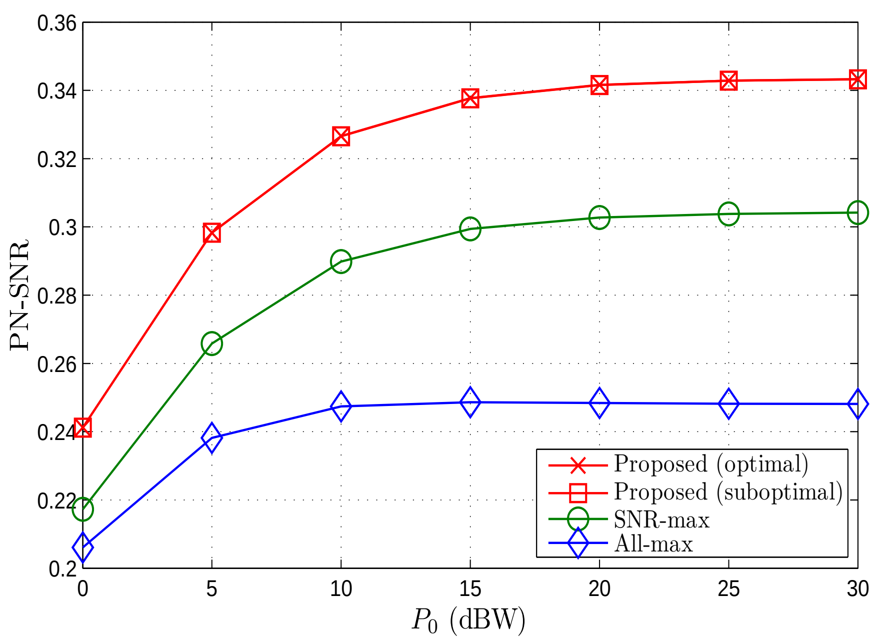

Fig. 4.1: Average PN-SNR versus P_0 for a two-relay network with separate relay power constraints.

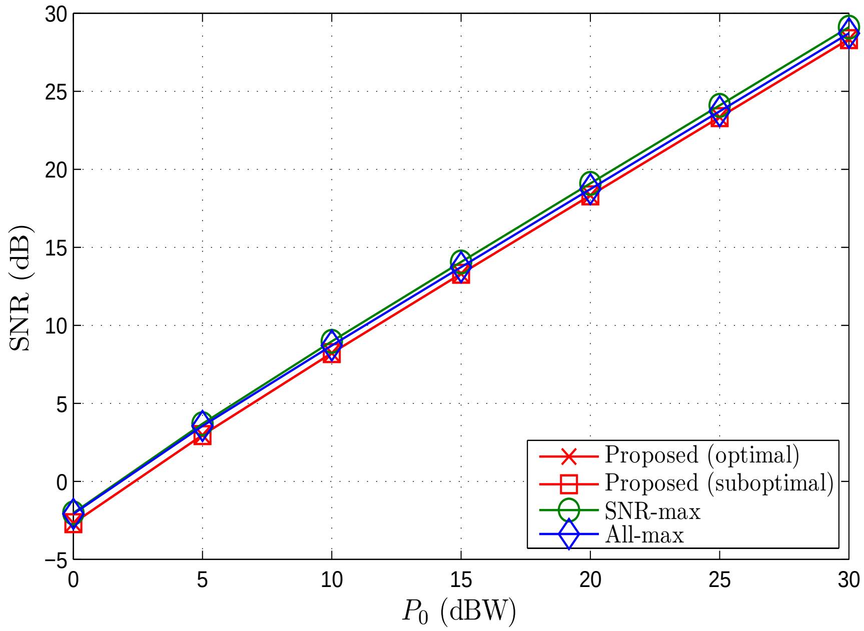

Fig. 4.2: Average SNR versus P_0 for a two-relay network with separate relay power constraints.

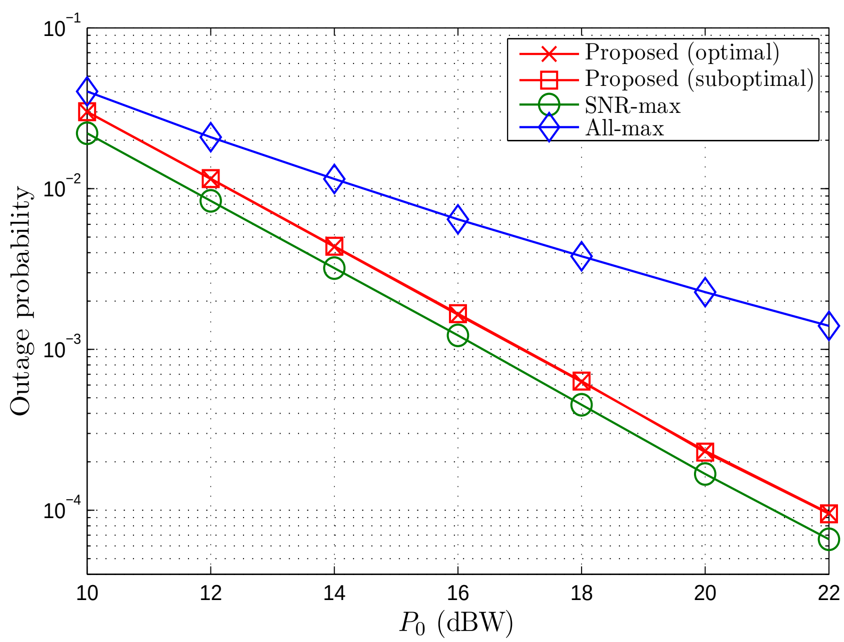

Fig. 4.3: Outage probability versus P_0 for a two-relay network with separate relay power constraints.

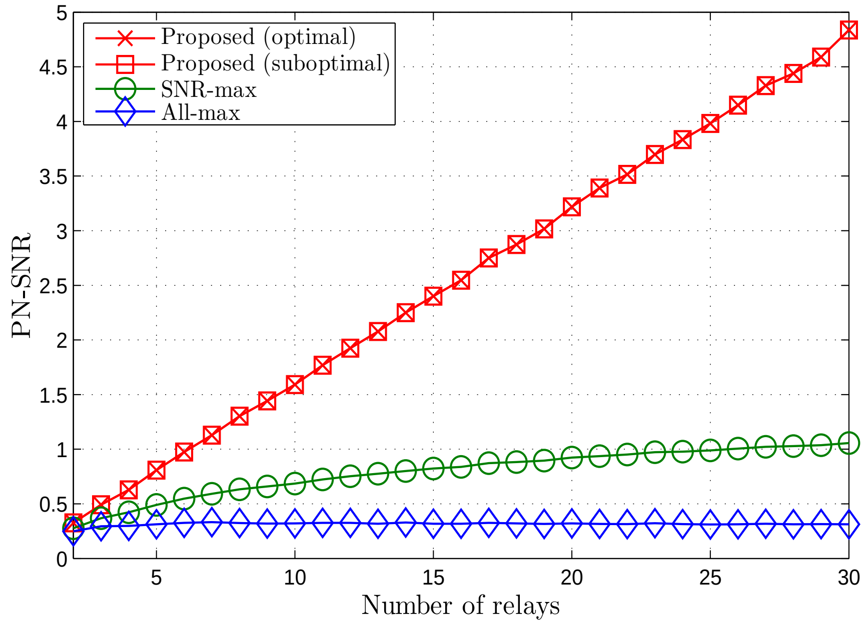

Fig. 4.4: Average PN-SNR versus number of relays for networks with separate relay power constraints.

We first simulate a two-relay network and assume that all nodes have the same power constraint i.e, $P_{i,\text{lim}} = P_0$ for $i = 1, 2$. Next, the average PN-SNR in networks with more that two relays is also simulated.

Fig. 4.1 shows the average PN-SNR versus P_0 for the three schemes in two-relay networks. First, we can see that the PN-SNR of the suboptimal solution is almost the same as the optimal solution. Also, the proposed PN-SNR-maximizing scheme outperforms the other two schemes in terms of the PN-SNR. When $P_0 = 30$ dBW, we can read from the plot that our proposed scheme is superior by 20.4% and 40% compared with the other two schemes.

Fig. 4.2 shows the average received SNR versus P_0 for the three schemes in two-relay networks. We observe that the proposed scheme is comparable in average received SNR with the other two scheme. When $P_0 = 30$ dBW, the proposed scheme is inferior by about 0.8 dB with the SNR-maximizing scheme and by 0.4 dB with the all maximum scheme. It is noteworthy that even though the all maximum scheme achieves better average received SNR, it dose not perform well in outage probability.

Fig. 4.3 shows the outage probability versus P_0 for the three schemes in two-relay networks. We can see that the proposed scheme is 0.7 dB worse in outage probability than the SNR-maximizing scheme. By reading from the slopes of the outage curves, we can also see that the all-maximum scheme loses diversity order while the other two schemes achieve full diversity.

Fig. 4.4 shows the average PN-SNR in networks with different numbers of relays. The transmit power on transmitter and each relay is set to be 10 dBW. We can first see that our suboptimal solution performs as well as the optimal solution. In the proposed PN-SNR-maximizing scheme, the average PN-SNR increases linearly with the number of the relays. In the SNR-maximizing scheme, however, the average PN-SNR increases with a significant smaller rate and saturates as the number of the relays increases. For the all-maximum scheme, the average PN-SNR remains unchanged as the number of the relays increases.

4.2 QoS-Constrained PN-SNR Maximization

In this section, we consider the QoS-constrained PN-SNR maximization problem under individual power constraint on each relay as an extension of the problem in the last section. In the problem formulation, we consider one more constraint on the QoS in order to maintain the SNR at the receiver.

4.2.1 Problem Formulation

According to the basic problem in (4.1) and the expression of received SNR in (3.7), the QoS constrained PN-SNR maximization problem under individual power constraint on each relay can be formulated as

$$\max_{\boldsymbol{\alpha}\succeq 0} \quad \frac{(\boldsymbol{\alpha}^T\mathbf{b})^2}{(1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2)(1+\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2)} \tag{4.3}$$

$$\text{s. t.} \quad 0 \le \alpha_i \le \sqrt{\frac{P_{i,\lim}}{1+P_0|f_i|^2}}, \text{ for } i=1,\cdots,R, \tag{4.4}$$

$$\mathrm{SNR} = \frac{P_0(\boldsymbol{\alpha}^T\mathbf{b})^2}{1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2} \ge \mathrm{SNR}_{\mathrm{th}}. \tag{4.5}$$

In (4.3) to (4.5), we try to maximize the PN-SNR while maintaining service quality. The first R constraints in (4.4) are equivalent to $P_i \leq P_{i,\text{lim}}$, which are the individual power constraints for the relays. The second constraint in (4.5) is introduced to guarantee the QoS at the receiver.

Note that the problem is not concave in general and finding the globally optimum is usually sophisticated. We herein propose a low complexity suboptimal solution.

4.2.2 Suboptimal Solution

We first ignore the separate power constraints in (4.4) and focus on the following problem

$$\max_{\boldsymbol{\alpha}\succeq 0} \quad \frac{(\boldsymbol{\alpha}^T\mathbf{b})^2}{(1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2)(1+\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2)} \tag{4.6}$$

$$\text{s. t.} \quad \text{SNR} \geq \text{SNR}_{\text{th}}. \tag{4.7}$$

We introduce the symbol P for the sum relay power and rewrite (4.6) to (4.7) as

$$\max_{P} \quad \frac{1}{P_0+P}\left(\max_{\boldsymbol{\alpha}\succeq 0} \frac{P_0(\boldsymbol{\alpha}^T\mathbf{b})^2}{1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2}\right) \tag{4.8}$$

$$\text{s. t.} \quad P = P_0\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2\ ,0\leq P<\infty, \tag{4.9}$$

$$\text{SNR} \geq \text{SNR}_{\text{th}}. \tag{4.10}$$

The inner problem in (4.8) to (4.10) can be solved with the results in Chapter 3 and the optimal power control coefficient for each relay is in (3.11).

With the results in (3.11), (4.8) to (4.10) can be reduced to a one-dimensional problem in finding P as follow:

$$\max_{P} \quad \sum_{i=1}^{R} \frac{|f_i|^2|g_i|^2P_0P}{(|f_i|^2P_0+|g_i|^2P+1)(P+P_0)} \tag{4.11}$$

$$\text{s. t.} \quad P = P_0\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2\ ,0\leq P<\infty, \tag{4.12}$$

$$\text{SNR} = \sum_{i=1}^{R} \frac{|f_i|^2|g_i|^2P_0P}{|f_i|^2P_0+|g_i|^2P+1} \geq \text{SNR}_{\text{th}}. \tag{4.13}$$

The left-hand-side of (4.13) is the received SNR of the network. We can observe that it is an increasing function of P. Thus, (4.11) to (4.13) can be equivalently expressed as

$$\max_{P} \quad \sum_{i=1}^{R} \frac{|f_i|^2|g_i|^2 P_0 P}{(|f_i|^2 P_0 + |g_i|^2 P + 1)(P + P_0)} \tag{4.14}$$

$$\text{s. t.} \quad P_{\text{th}} \leq P < \infty, \tag{4.15}$$

where P_{th} is the unique solution of the following equation:

$$\sum_{i=1}^{R} \frac{|f_i|^2|g_i|^2 P_0 P}{|f_i|^2 P_0 + |g_i|^2 P + 1} - \text{SNR}_{\text{th}} = 0\ . \tag{4.16}$$

It is the minimum total relay power needed to satisfy the SNR constraint.

To solve (4.14) to (4.15), it is shown in Lemma 3 that the objective function in (4.14) has a unique maximum, denoted as P^*. In addition, we can also show that the objective function is non-decreasing when $P \leq P^*$ and non-increasing when $P \geq P^*$. Thus, the optimal P for (4.14) to (4.15), and equivalently (4.6) to (4.7), can be expressed as

$$P = \max\left(P^*, P_{\text{th}}\right). \tag{4.17}$$

In summary, the solution of the problem in (4.6) to (4.7) is given by (4.17) and (3.11). Note that the problem in (4.6) to (4.7) is a relaxation of our original problem in (4.3) to (4.5) by ignoring the separate relay power constraints. So the derived solution may not satisfy the constraints in (4.4). To find a solution that satisfies the constraints in (4.4), we truncate those entries of $\boldsymbol{\alpha}^*$ that violate the constraints as follows:

$$\alpha_i^{\text{sub}} = \min\left(\alpha_i^*, \sqrt{\frac{P_{i,\text{lim}}}{1 + P_0|f_i|^2}}\right) \quad \text{for } i = 1, \cdots, R. \tag{4.18}$$

By this truncation, the solution in (4.18) satisfies all separate constraints and can be a suboptimal solution for the problem in (4.3) to (4.5).

It is noteworthy that the suboptimal solution guarantees to satisfy the power constraints of the relays but may not satisfy the SNR constraint. If the SNR constraint is not satisfied, we claim that no solution is found and no

Algorithm 1 Suboptimal solution.

1. For the given P_0, find the unique maximum P^* of the objective function in (4.11) using a gradient-ascent algorithm.
2. Obtain the solution of (4.6) to (4.7).
   Calculate P_{th} by solving (4.16) using Newton's algorithm.
   if (4.16) has no solution, **then** the problem is infeasible.
   else $P = \max(P^*, P_{\text{th}})$ and obtain $\boldsymbol{\alpha}^*$ according to (3.11).
3. Truncate $\boldsymbol{\alpha}^*$ according to (4.18) and obtain $\boldsymbol{\alpha}^{\text{sub}}$.
4. Calculate the corresponding received SNR.
   if $\text{SNR} < \text{SNR}_{\text{th}}$, **then** claim that no solution is found.

transmission can be made. There are two scenarios that can lead to no solution being found: 1) Problem (4.3) to (4.5) is infeasible, which means that the power constraints and the SNR constraint cannot be satisfied simultaneously; and 2) the problem is feasible but the truncation in (4.18) reduces the received SNR and makes it fall below the threshold. We summarize the suboptimal solution as Algorithm 1.

The suboptimal solution only involves two gradient-ascent numerical algorithms, one for the search of P_{th} and the other for the search of P^*. Thus, the overall complexity is low. When $\text{SNR}_{\text{th}} = -\infty$ dB, problem (4.3) to (4.5) reduces to the PN-SNR maximization under separate relay power constraints without the QoS constraint discussed in previous section. On this occasion, Algorithm 1 also applies.

4.2.3 Simulation

In this section, we present the simulated performance of our proposed PN-SNR maximizing scheme with a constraint on the received SNR. We also compare the proposed scheme with the SNR-maximizing scheme (denoted as "SNR-max") and the all maximum scheme (denoted as "All-max"). In the SNR-maximizing scheme, the beamforming design of [37] is used for the highest received SNR. In the all maximum scheme, all relays transmit with full power.

We simulate and compare the average PN-SNR and the average throughput for the three schemes in a two-relay network. In our simulations, we assume that all nodes have the same power constraint, i.e., $P_{1,\text{lim}} = P_{2,\text{lim}} = P_0$. For

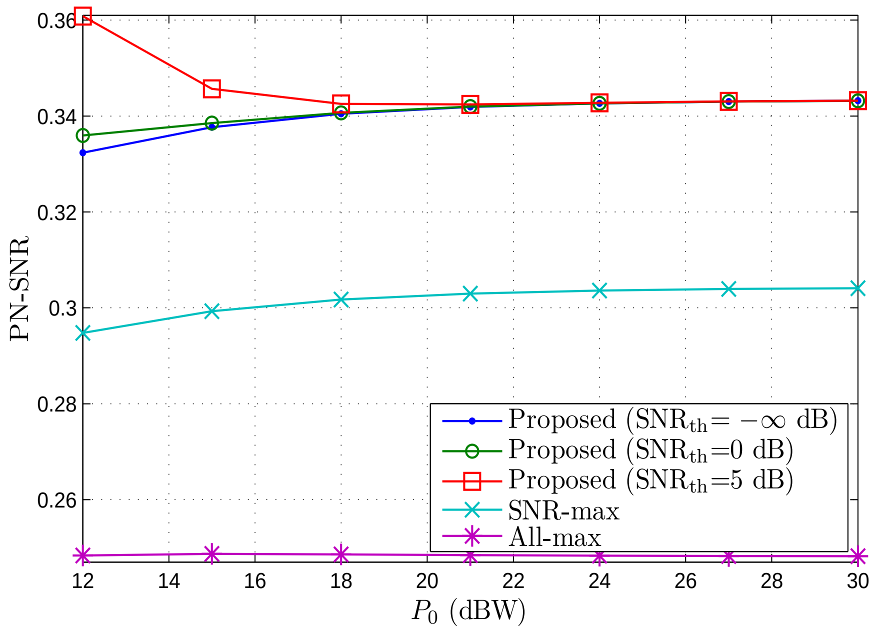

Fig. 4.5: Average PN-SNR versus P_0 for a two-relay network.

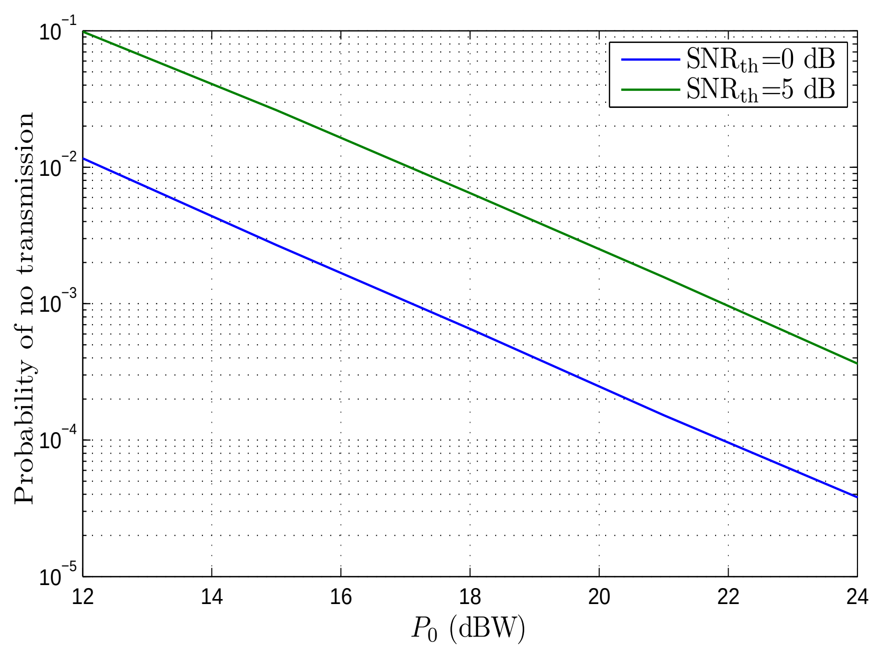

Fig. 4.6: Probability of no transmission versus P_0 for a two-relay network.

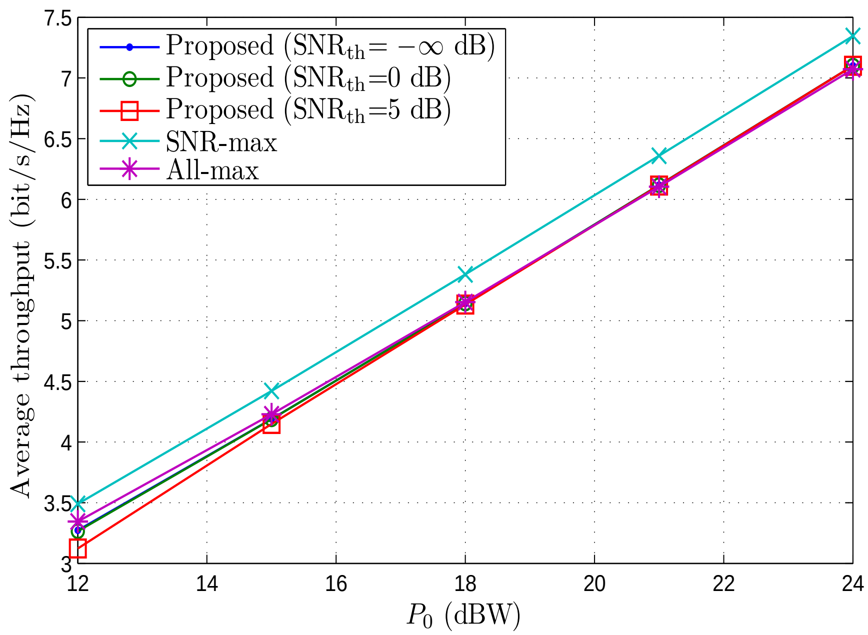

Fig. 4.7: Average throughput versus P_0 for a two-relay network.

the proposed scheme, we consider three SNR thresholds, $\text{SNR}_{\text{th}} = -\infty$ dB, 0 dB, and 5 dB. We assume no transmission if no solution is found. In this case, the throughput is 0 but the PN-SNR is not applicable, i.e., the PN-SNR is the average over cases where suboptimal solutions can be found. We also simulate the probability of no transmission of the proposed scheme.

Fig. 4.5 shows the average PN-SNR versus P_0 for the three schemes. For the proposed scheme, we can see that when P_0 is smaller than 18 dBW, the curve with larger SNR_{th} achieves higher PN-SNR. This is because when P_0 is smaller than 18 dBW, the probability of no transmission is not negligible and the average PN-SNR value is based on scenarios where the SNR constraint is satisfied. On the other hand, when P_0 is higher than 18 dBW, the probability of no transmission is very small. In this case, we observe that the average PN-SNR for different SNR_{th} values are about the same and keeps increasing as P_0 increases. Compared with the other two schemes, our proposed scheme is significantly superior in average PN-SNR. When $P_0 = 30$ dBW, we can read from the plot that our proposed scheme outperforms the SNR maximizing scheme by about 13% and outperforms the all maximum scheme by about

38%.

Fig. 4.6 shows the probability that no solution is found or no transmission can be made. We see that the probability of no transmission decreases quickly as the transmitter power increases. With $\text{SNR}_{\text{th}} = 5$ dB, when P_0 is 18 dBW, the probability of no transmission is 0.45%. When P_0 increases to 22 dBW, the probability decreases to 0.1%. The same trend is observed when $\text{SNR}_{\text{th}} = 0$ dB.

Fig. 4.7 shows the average throughput versus P_0 for the three schemes. For the proposed scheme, we observe that when P_0 is smaller than 18 dBW, the average throughput is lower for larger SNR_{th}. As P_0 increases, the average throughput with different SNR_{th} values are about the same. The average throughput of the proposed scheme is about 0.25 bit/s/Hz inferior compared with the SNR-maximizing scheme for P_0 greater than 18 dBW. Compared with the all maximum scheme, it achieves the same average throughput.

We can conclude from Fig. 4.5 to Fig. 4.7 that the proposed scheme achieves higher PN-SNR and comparable average throughput compared with the other two schemes. The probability of no transmission decreases quickly as the transmitter power increases. When the transmitter power is small, with higher QoS constraint, the probability of no transmission gets higher; the average throughput decreases, but the the PN-SNR of the network increases. It makes sense in AF relay networks. When the transmitter power is small, the received signals at the relays are highly noisy and the noises are amplified in the second step. In this case, we cannot acquire a satisfactory received SNR and no transmission will help save power.

4.3 Summary

In this chapter, we first investigated the basic PN-SNR maximization problem in multi-relay network with separate power constraints. Then, the QoS-constrained PN-SNR maximization problem in multi-relay network with separate power constraints is considered as an extension. For the basic PN-SNR maximization problem, we studied the optimal solution, which functions as performance benchmark. Then, we proposed a low complexity suboptimal solution for the problem. The suboptimal solution was numerically shown to have close-to optimal performance. For the QoS-constrained PN-SNR maximization problem, we proposed a low complexity suboptimal solution based on the basic PN-SNR maximization problem.

We can conclude from Section 4.1 that our PN-SNR-maximizing scheme is more efficient than the other two schemes in using transmit power to provide the received SNR. This scheme also has comparable network performance with the SNR-maximizing scheme in two-relay networks with $P_{i,\mathrm{lim}} = P_0$ for $i = 1, 2$. Even though there is a trade-off between the PN-SNR and the received SNR, the PN-SNR can be a promising measure in designing energy efficient networks.

We can conclude from Section 4.2 that the proposed scheme achieves higher PN-SNR with comparable performance in the average throughput compared with the other two schemes. And the probability of no transmission decreases quickly as the transmitter power increases. When the transmitter power is small, with higher QoS constraint, the probability of no transmission gets higher; the average throughput decreases, but the the PN-SNR of the network increases.

Chapter 5

Conclusion and Future Work

5.1 Thesis Summary

In this thesis, we adopted PN-SNR in relay network beamforming design and investigated the potential of PN-SNR in efficiency measure. Specifically, PN-SNR maximization problems were formulated and solved in relay networks with different configurations. Network performance of such PN-SNR maximizing scheme (average PN-SNR, average received SNR, outage probability, and throughput) was analyzed or simulated.

In single-relay network, we found the optimal relay power shcmem in close-form and evaluated the network performance under such scheme. The performance was analytically and numerically compared with existing schemes. Our observations showed that the proposed scheme achieves better PN-SNR. The proposed scheme has comparable average received SNR with existing schemes. Meanwhile, it has asymptotic optimal outage performance compared with the SNR-maximizing scheme.

In multi-relay network with a sum power constraint on relays, we simplified the PN-SNR maximization problem into a one-dimensional problem in finding the optimal sum power and proved that the problem has unique maximum. The network performance was numerically simulated using gradient-ascent algorithm and compared with existing schemes. We observed that with the same power resource, the proposed scheme has comparable performance in the average SNR and is better in outage probability compared with fixed relay power scheme. Also, the proposed scheme has considerably higher PN-SNR with moderate degradation in outage probability compared with the SNR-maximizing scheme.

In multi-relay network with individual power constraint on each relay, we investigated the basic PN-SNR maximization problem and the QoS constraint PN-SNR maximization problem. For the former problem, we proposed an optimal solution and a low complexity suboptimal solution. For the latter problem, a low complexity suboptimal solution was proposed. Network performance was numerically simulated and compared with existing schemes. We observed that the proposed scheme achieves better PN-SNR than the SNR-maximizing scheme and the all-maximum scheme. The proposed scheme is also comparable in average received SNR and outage probability with the SNR maximization scheme.

The derivations and analysis in this thesis indicated that PN-SNR can be used as a new efficiency measure in relay network beamforming design.

5.2 Future Work

There are several possible directions for future research on PN-SNR. A few are listed below:

1. PN-SNR maximization in multi-relay network with relay selection

We have worked on PN-SNR design for multi-relay network under beamforming. While beamforming can significantly improve the network performance, it requires channel state information (CSI) at the relays. Meanwhile, synchronization among relays is also hard to implement. Another widely used scheme in multi-relay network is relay selection. Relay selection can be implemented with partial CSI at the relays and it simplifies the synchronization at relays. Thus, relay selection design using PN-SNR can be studied in the future. We will present some preliminary results on relay selection design using PN-SNR later.

2. PN-SNR maximization with DF protocol

We have considered AF transmission protocol in this thesis. A natural extension is using PN-SNR in relay network design under DF transmission protocol.

3. PN-SNR maximization for multi-user and multi-relay network

In this thesis, network with one transmitter and one receiver have been considered. As a generalization, multi-user multi-relay network design using PN-SNR can be studied in the future. In these designs, the pairwise PN-SNR can be defined and the problem can be formulated as maximizing the minimum pairwise PN-SNR.

5.2.1 Preliminary Results on Relay Selection Design Using PN-SNR

In this subsection, we present some preliminary results on relay selection design using PN-SNR in multi-relay network.

Different from the beamforming design where all relays participate into cooperation, only partial relays are selected to help forward the information in the relay selection design. Relay selection design has been widely studied in literature [38, 49, 53, 82]. Some of these works considered the single-relay selection scheme where only one relay is selected to cooperate [38, 49, 82]. It has been proved that the capacity/realibility optimal single-relay selection scheme is to choose the relay whose path has the maximum received SNR. The author in [53] investigated the multi-relay selection design and proposed several suboptimal schemes. Most of the aforementioned selecting schemes are based on the received SNR and the selected relay transmit with its maximum power. However, such designs may not be power efficient.

In our design, we consider the single-relay selection scheme where the path with the maximum PN-SNR is selected. The selected relay may not transmit with its maximum power but adjust the power for the highest PN-SNR. We focus on the same system model as depicted in Fig. 3.1 in Chapter 3. A two-step AF protocol is used for relay transmission. Recall the derivations in Chapter 2, the received SNR for the ith path can be expressed as

$$\mathrm{SNR}_i = \frac{|f_i g_i|^2 P_i P_0}{1 + |f_i|^2 P_0 + |g_i|^2 P_i} \approx \frac{|f_i g_i|^2 P_i P_0}{|f_i|^2 P_0 + |g_i|^2 P_i}, \tag{5.1}$$

and the corresponding PN-SNR for the ith path is

$$\eta_i = \frac{|f_i g_i|^2 P_i P_0}{(1+|f_i|^2 P_0 + |g_i|^2 P_i)(P_i + P_0)} \approx \frac{|f_i g_i|^2 P_i P_0}{(|f_i|^2 P_0 + |g_i|^2 P_i)(P_i + P_0)}, \quad (5.2)$$

where P_0 is transmitter power and P_i is the power consumed on the ith relay.

Thus, our PN-SNR maximization problem in multi-relay network with single-relay selection can be formulated as

$$\begin{aligned} \max_{i} \ \max_{P_i} & \quad \frac{|f_i g_i|^2 P_i P_0}{(1+|f_i|^2 P_0 + |g_i|^2 P_i)(P_i + P_0)} \\ \text{s. t.} & \quad 0 \le P_i \le P_{i\_\lim}, \ \text{for } i = 1, \cdots, R, \end{aligned} \quad (5.3)$$

Note that the inner problem in (5.3) aims at finding the optimal relay power for the ith path. Then, the path with maximum PN-SNR is chosen for transmission. The inner problem has been solved in Chapter 2 with the optimal solution for the ith path

$$P_{\text{opt\_i}} = \min\left(\frac{\sqrt{P_0(1+|f_i|^2 P_0)}}{|g_i|}, P_{i\_\lim}\right) \approx \min\left(\frac{|f_i|}{|g_i|} P_0, P_{i\_\lim}\right). \quad (5.4)$$

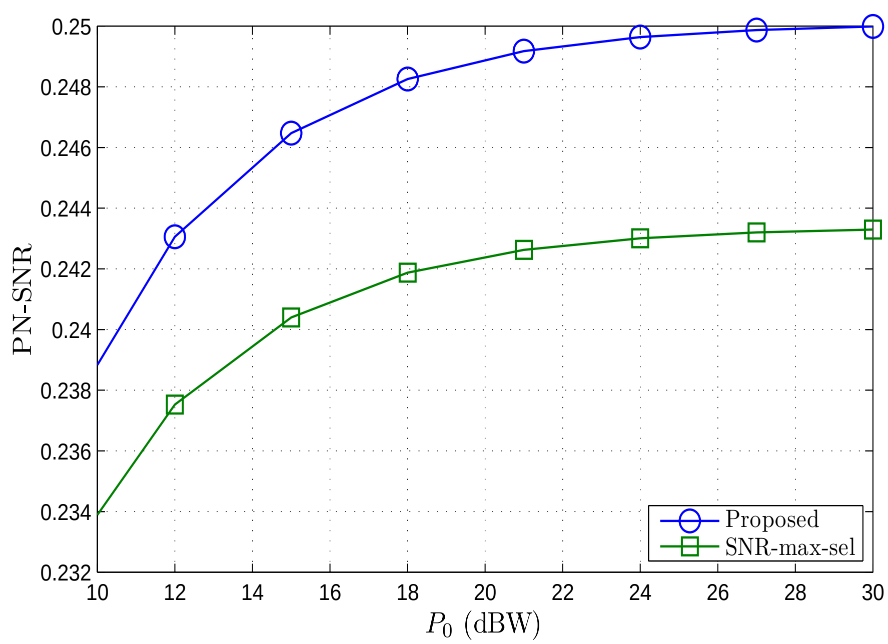

Fig. 5.1: Average PN-SNR versus P_0 for relay selection.

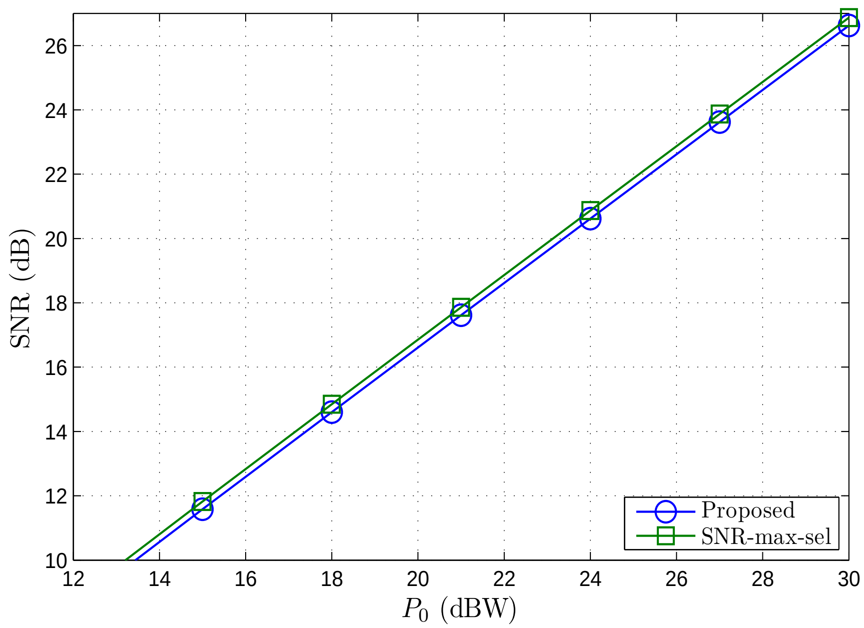

Fig. 5.2: Average received SNR versus P_0 for relay selection.

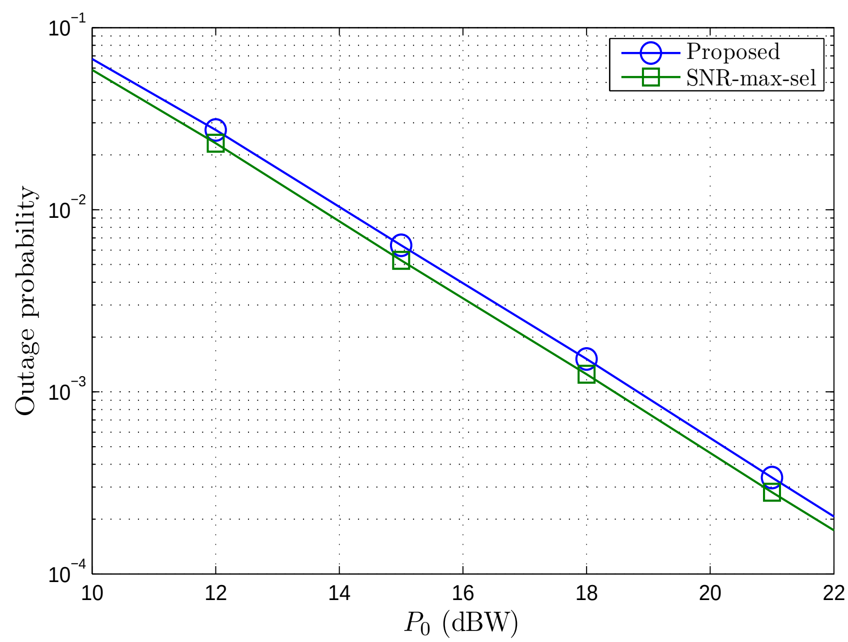

Fig. 5.3: Outage probability versus P_0 for relay selection.

We numerically simulate the performance of this selection design. Specifically, We simulate the average PN-SNR, the average received SNR and the outage probability with threshold $\gamma_{\text{th}} = 0$ dB for the proposed PN-SNR-maximizing selection scheme (denoted as "Proposed") and compare the performance with the SNR-maximizing selection scheme (denoted as "SNR-max-sel") mentioned in [53]. In our simulation, we assume that there are two relays and all nodes have the same power constraint, i.e, $P_{i,\text{lim}} = P_0$ for $i = 1, 2$.

We observe from Fig. 5.1 to Fig. 5.3 that the proposed PN-SNR maximizing selection scheme achieves better PN-SNR than the SNR maximizing selection scheme. Meanwhile, the proposed scheme has comparable performance in average received SNR and outage probability with the SNR maximizing selection scheme. It achieves the same diversity order as the SNR maximizing selection scheme, which is proved to have full diversity. The simulation results indicate that PN-SNR is also potential in the design of multi-relay network with relay selection.

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Appendix A

Proof of Theorem 1

We herein provide the proof of Theorem 1. Define $X \triangleq |f|$ and $Y \triangleq |g|$, then X and Y are Rayleigh distributed whose probability density function (pdf) is (1.4). The average of the relay power in (2.11) can be calculated as

$$
\begin{aligned}
&\mathbb{E}\{P_{\text{approx}}\} \\
&= \int_0^{+\infty}\int_0^{+\infty} \min\left(\frac{x}{y}P_0, P_{R,\lim}\right) 4xy \cdot e^{-(x^2+y^2)} dxdy \\
&= \int_0^{+\infty}\left[\int_0^{\frac{P_{R,\lim}}{P_0}y} 4P_0 x^2 e^{-(x^2+y^2)}dx + \int_{\frac{P_{R,\lim}y}{P_0}}^{+\infty} 4P_{R,\lim} xye^{-(x^2+y^2)}dx\right] dy \\
&= \int_0^{+\infty}\left\{2P_0 e^{-y^2}\left[\frac{\sqrt{\pi}}{2}\text{erf}\left(\frac{P_{R,\lim}}{P_0}y\right) - \frac{P_{R,\lim}}{P_0} ye^{-\left(\frac{P_{R,\lim}y}{P_0}\right)^2}\right]\right. \\
&\left. +2P_{R,\lim} ye^{-y^2}e^{-\left(\frac{P_{R,\lim}}{P_0}y\right)^2}\right\} dy \\
&= \int_0^{+\infty} \sqrt{\pi}P_0 e^{-y^2}\text{erf}\left(\frac{P_{R,\lim}}{P_0}y\right) dy = P_0 \tan^{-1}\left(\frac{P_{R,\lim}}{P_0}\right).
\end{aligned}
$$

Recall the expression of the PN-SNR in (2.6) and also $\xi = \frac{P_{R,\lim}}{P_0}$. With the relay power design in (2.11), the average PN-SNR can be calculated as

$$
\begin{aligned}
&\eta_{\text{ave}} \\
&\approx \int_0^{+\infty}\int_0^{+\infty} \frac{x^2y^2 \min\left(\frac{x}{y}P_0, P_{R,\lim}\right) 4xye^{-(x^2+y^2)}}{\left[x^2P_0 + y^2\min\left(\frac{x}{y}P_0, P_{R,\lim}\right)\right]\left[P_0 + \min\left(\frac{x}{y}P_0, P_{R,\lim}\right)\right]} dxdy \\
&= 4\int_0^{+\infty}\int_0^{\xi y} \frac{x^3y^3}{(x+y)^2} e^{-(x^2+y^2)}dxdy + 4\frac{\xi}{1+\xi}\int_0^{+\infty}\int_{\xi y}^{+\infty} \frac{x^3y^3}{x^2+\xi y^2} e^{-(x^2+y^2)}dxdy \\
&= 4\int_{\tan^{-1}\left(\frac{1}{\xi}\right)}^{\frac{\pi}{2}} \frac{\cos^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} d\theta + \frac{4\xi}{1+\xi}\int_0^{\tan^{-1}\left(\frac{1}{\xi}\right)} \frac{\cos^3\theta\sin^3\theta}{\cos^2\theta + \xi\sin^2\theta} d\theta
\end{aligned}
$$

$$= \frac{3}{8}\pi - \frac{3}{4}\tan^{-1}\left(\frac{1}{\xi}\right) - \frac{4\xi^3 - 7\xi^2 - \xi}{4(\xi+1)(\xi-1)^2} + \frac{2\xi^2 \ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3(\xi+1)}.$$

In the third step, we use the polar coordinate system and the integral that $\int_0^{+\infty} r^5 e^{-r^2} dr = 1$. From (2.5), the corresponding average SNR can be obtained in the same way as

$$\begin{aligned}
&\mathrm{SNR}_{\mathrm{ave}} \\
&\approx \int_0^{+\infty}\int_0^{+\infty} \frac{x^2y^2 \min\left(\frac{x}{y}P_0, P_{R,\mathrm{lim}}\right)}{x^2P_0 + y^2\min\left(\frac{x}{y}P_0, P_{R,\mathrm{lim}}\right)} 4xye^{-(x^2+y^2)}dxdy \\
&= 4P_0\int_0^{+\infty}\int_0^{\xi y} \frac{x^3y^2}{x+y}e^{-(x^2+y^2)}dxdy + 4P_0\xi\int_0^{+\infty}\int_{\xi y}^{+\infty} \frac{x^3y^3}{x^2+\xi y^2}e^{-(x^2+y^2)}dxdy \\
&= 4P_0\int_{\tan^{-1}\left(\frac{1}{\xi}\right)}^{\frac{\pi}{2}} \frac{\cos^3\theta\sin^2\theta}{\cos\theta+\sin\theta}d\theta + 4P_0\xi\int_0^{\tan^{-1}\left(\frac{1}{\xi}\right)} \frac{\cos^3\theta\sin^3\theta}{\cos^2\theta+\xi\sin^2\theta}d\theta \\
&= P_0\left[\frac{\pi}{8} - \frac{1}{4}\tan^{-1}\left(\frac{1}{\xi}\right) - \frac{3\xi^3+5\xi}{4(\xi-1)^2(\xi^2+1)} - \frac{1}{4}\ln\left(\frac{(\xi+1)^2}{\xi^2+1}\right) + \frac{2\xi^2\ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3}\right].
\end{aligned}$$

Defining $A \triangleq |f|^2$ and $B \triangleq |g|^2$, the outage probability can be expressed as

$$\begin{aligned}
&O = \mathbb{P}\left(\mathrm{SNR} \le \gamma_{\mathrm{th}}\right) = \mathbb{P}\left(\mathrm{SNR} \le \gamma_{\mathrm{th}} \cap X \le \xi Y\right) + \mathbb{P}\left(\mathrm{SNR} \le \gamma_{\mathrm{th}} \cap X \ge \xi Y\right) \\
&\approx \mathbb{P}\left(\frac{X^2YP_0}{X+Y} \le \gamma_{\mathrm{th}} \cap X \le \xi Y\right) + \mathbb{P}\left(\frac{ABP_{R,\mathrm{lim}}}{A+\xi B} \le \gamma_{\mathrm{th}} \cap A \ge \xi^2 B\right) \\
&= \mathbb{P}\left(Y \le \frac{\gamma_{\mathrm{th}}X}{P_0X^2-\gamma_{\mathrm{th}}} \cap Y \ge \frac{X}{\xi} \cap P_0X^2 \ge \gamma_{\mathrm{th}}\right) + \mathbb{P}\left(X \le \xi Y \cap P_0X^2 \le \gamma_{\mathrm{th}}\right) \\
&+\mathbb{P}\left(A \le \frac{\gamma_{\mathrm{th}}\xi B}{P_{R,\mathrm{lim}}B-\gamma_{\mathrm{th}}} \cap A \ge \xi^2 B \cap B \ge \frac{\gamma_{\mathrm{th}}}{P_{R,\mathrm{lim}}}\right) + \mathbb{P}\left(A \ge \xi^2 B \cap B \le \frac{\gamma_{\mathrm{th}}}{P_{R,\mathrm{lim}}}\right) \\
&= \int_{\sqrt{\frac{\gamma_{\mathrm{th}}}{P_0}}}^{\sqrt{\frac{(1+\xi)\gamma_{\mathrm{th}}}{P_0}}} f_X(x)dx\left(\int_{\frac{x}{\xi}}^{\frac{\gamma_{\mathrm{th}}x}{P_0x^2-\gamma_{\mathrm{th}}}} f_Y(y)dy\right) + \int_0^{\sqrt{\frac{\gamma_{\mathrm{th}}}{P_0}}} f_X(x)dx\left(\int_{\frac{x}{\xi}}^{+\infty} f_Y(y)dy\right) \\
&+ \int_{\frac{\gamma_{\mathrm{th}}}{P_{R,\mathrm{lim}}}}^{\frac{\gamma_{\mathrm{th}}}{P_{R,\mathrm{lim}}}\left(1+\frac{1}{\xi}\right)} f_B(b)db\left(\int_{\xi^2 b}^{\frac{\gamma_{\mathrm{th}}\xi b}{P_{R,\mathrm{lim}}b-\gamma_{\mathrm{th}}}} f_A(a)da\right) + \int_0^{\frac{\gamma_{\mathrm{th}}}{P_{R,\mathrm{lim}}}} f_B(b)db\left(\int_{\xi^2 b}^{+\infty} f_A(a)da\right) \\
&= \frac{\xi^2}{\xi^2+1}\left[1 - e^{-\left(1+\frac{1}{\xi^2}\right)(1+\xi)\frac{\gamma_{\mathrm{th}}}{P_0}}\right] - e^{\frac{\gamma_{\mathrm{th}}}{P_0}}\int_0^{\frac{\gamma_{\mathrm{th}}\xi}{P_0}} e^{-\frac{\gamma_{\mathrm{th}}^3}{P_0^3u_1^2} - \frac{\gamma_{\mathrm{th}}^2}{P_0^2u_1} - u_1}du_1 \qquad \text{(A.1)} \\
&+ \frac{1}{\xi^2+1}\left[1 - e^{-\left(1+\frac{1}{\xi}\right)(1+\xi^2)\frac{\gamma_{\mathrm{th}}}{\xi P_0}}\right] - e^{\left(1+\frac{1}{\xi}\right)\frac{\gamma_{\mathrm{th}}}{P_0}}\int_0^{\frac{\gamma_{\mathrm{th}}}{\xi^2P_0}} e^{-\frac{\gamma_{\mathrm{th}}^2}{\xi P_0^2u_2} - u_2}du_2. \qquad \text{(A.2)}
\end{aligned}$$

The integral in (A.1) can be upper bounded by

$$\int_0^{\frac{\gamma_{\rm th}\xi}{P_0}} e^{-\frac{\gamma_{\rm th}^3}{P_0^3 u_1^2}-\frac{\gamma_{\rm th}^2}{P_0^2 u_1}-u_1} du_1 \leq \int_0^{\frac{\gamma_{\rm th}\xi}{P_0}} e^{-u_1} du_1 = 1-e^{-\frac{\gamma_{\rm th}\xi}{P_0}} = \frac{\gamma_{\rm th}\xi}{P_0}+\mathcal{O}\left(\frac{1}{P_0^2}\right).$$

It can also be lower bounded by

$$\begin{aligned}
&\int_0^{\frac{\gamma_{\rm th}\xi}{P_0}} e^{-\frac{\gamma_{\rm th}^3}{P_0^3 u_1^2}-\frac{\gamma_{\rm th}^2}{P_0^2 u_1}-u_1} du_1 \\
&\geq \int_{P_0^{-4/3}}^{\frac{\gamma_{\rm th}\xi}{P_0}} e^{-\frac{\gamma_{\rm th}^3}{P_0^3 u_1^2}-\frac{\gamma_{\rm th}^2}{P_0^2 u_1}-u_1} du_1 \geq e^{-\frac{\gamma_{\rm th}^3}{P_0^3\left(P_0^{-4/3}\right)^2}-\frac{\gamma_{\rm th}^2}{P_0^2 P_0^{-4/3}}} \int_{P_0^{-4/3}}^{\frac{\gamma_{\rm th}\xi}{P_0}} e^{-u_1} du_1 \\
&= \left(1-\frac{\gamma_{\rm th}^3}{P_0^3\left(P_0^{-4/3}\right)^2}-\frac{\gamma_{\rm th}^2}{P_0^2 P_0^{-4/3}}\right)\left(\frac{\gamma_{\rm th}\xi}{P_0}-P_0^{-4/3}\right)+\mathcal{O}\left(\frac{1}{P_0^{4/3}}\right) \\
&= \frac{\gamma_{\rm th}\xi}{P_0}+\mathcal{O}\left(\frac{1}{P_0^{4/3}}\right).
\end{aligned}$$

We can see that the dominant terms are the same for the upper and the lower bound. For the second integral, it can be shown in the same manner that

$$\frac{\gamma_{\rm th}}{\xi^2 P_0}+\mathcal{O}\left(\frac{1}{P_0^{3/2}}\right) \leq \int_0^{\frac{\gamma_{\rm th}}{\xi^2 P_0}} e^{-\frac{\gamma_{\rm th}^2}{\xi P_0^2 u_2}-u_2} du_2 \leq \frac{\gamma_{\rm th}}{\xi^2 P_0}+\mathcal{O}\left(\frac{1}{P_0^2}\right)$$

By using Taylor series expansion for large P_0, the outage probability can be bounded as (2.15).

Appendix B

Proof of Lemma 1

With the relay transmit power fixed as P for each transmission, the PN-SNR and the end-to-end received SNR can be expressed as

$$\eta_{\text{fix}} \approx \frac{|fg|^2 P P_0}{(|f|^2 P_0 + |g|^2 P)(P_0 + P)}, \quad \text{SNR}_{\text{fix}} \approx \frac{|fg|^2 P P_0}{|f|^2 P_0 + |g|^2 P}.$$

The average PN-SNR thus is

$$\begin{aligned}
\eta_{\text{ave_fix}} &\approx \frac{P}{P+P_0} \int_0^{+\infty} \int_0^{+\infty} \frac{P_0 x^2 y^2}{P_0 x^2 + P y^2} 4xy \cdot e^{-(x^2+y^2)} dx dy \\
&= 4 \frac{P}{P+P_0} \int_0^{+\infty} \int_0^{+\infty} \frac{x^3 y^3}{x^2 + P/P_0 y^2} e^{-(x^2+y^2)} dx dy \\
&= 4 \frac{P}{P+P_0} \int_0^{\frac{\pi}{2}} \frac{\cos^3\theta \sin^3\theta}{\cos^2\theta + P/P_0 \sin^2\theta} \left(\int_0^{+\infty} r^5 e^{-r^2} dr \right) d\theta \\
&= \frac{P P_0}{(P-P_0)^2} - \frac{2P^2 P_0^2}{(P-P_0)^3 (P+P_0)} \ln\left(\frac{P}{P_0}\right).
\end{aligned}$$

The average end-to-end received SNR can be easily derived as

$$\text{SNR}_{\text{ave\_fix}} \approx \eta_{\text{ave\_fix}}(P_0 + P) = \frac{P P_0 (P + P_0)}{(P-P_0)^2} - \frac{2P^2 P_0^2}{(P-P_0)^3} \ln\left(\frac{P}{P_0}\right).$$

The outage probability with SNR threshold γ_{th} can be derived as [49]

$$\begin{aligned}
O_{\text{fix}} &\approx 1 - e^{-\frac{\gamma_{\text{th}}}{P}} e^{-\frac{\gamma_{\text{th}}}{P_0}} 2\frac{\gamma_{\text{th}}}{P}\sqrt{\frac{P}{P_0}} K_1\left(2\frac{\gamma_{\text{th}}}{P}\sqrt{\frac{P}{P_0}}\right) \\
&= 1 - e^{-\frac{\gamma_{\text{th}}}{P} - \frac{\gamma_{\text{th}}}{P_0}} \left[1 - \mathcal{O}\left(\frac{\ln(PP_0)}{PP_0}\right)\right] = \left(1 + \frac{1}{\zeta}\right)\frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{\ln(P_0)}{P_0^2}\right).
\end{aligned}$$

This ends the proof.