Data-driven approaches to defining material property and performance relationships of armor ceramics undergoing dynamic loading

by

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Abstract

This thesis utilizes a combined numerical and machine learning approach to explore the performance of an alumina ceramic tile undergoing high-velocity impact. The finite element model is established by incorporating a user-defined Johnson-Holmquist-Beissel (JHB) material model within the framework of smoothed particle hydrodynamics (SPH) in LS-DYNA finite element software. The computational framework is validated across a range of conditions by matching the simulation results from both plate impact experiments and ballistic testing from the literature. Once validated, the model is used to generate training data sets for an artificial neural network (ANN) to predict the residual velocity and projectile erosion of an alumina ceramic tile undergoing high-velocity impact in the SPH framework. The ANN is then used to perform a sensitivity analysis involving exploring the effect of mechanical properties (e.g., strength and shear modulus) and impact simulation geometries (e.g., the thickness of ceramic tile) on material performance (i.e., residual projectile velocity and erosion). Overall, this study shows the capability of the hybrid FEM-ANN approach in studying the high-velocity impact on ceramic tiles and is applicable to guide the structural-scale design of ceramic-based protection systems

Preface

This thesis titled: "Data-driven approaches to inform quantification of property and performance relationship for armor ceramics material under dynamic loading" is originally written by Alex Yang.

- The work of Chapter 2 in this thesis has been published in *Ceramic International* as **Alex Yang**, Dan Romanyk, and J.D. Hogan, "High-velocity impact study of an advanced ceramic using finite element coupling with a machine learning approach". I was responsible for performing the simulations in this study and the manuscript composition. Dan Romanyk was the co-supervisory author and contributed to the manuscript composition. J.D. Hogan was the supervisory author and was involved with concept formation and manuscript composition.

"You are what you think; you become what you believe; you turn out to be what you have always been."

- Matshona Dhliwayo

Acknowledgments

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List of Symbols

Greek

$\bar{\sigma}$ Equivalent von Mises stress,Johnson-Cook model

σ_m Mean stress

σ_y Yield stress

A Material constant,Johnson-Cook model

B Material constant,Johnson-Cook model

C Material constant, Johnson-Cook model

C_P Specific heat

D_1^* Material damage constant,Johnson-Cook model

D_2^* Material damage constant,Johnson-Cook model

D_3^* Material damage constant,Johnson-Cook model

D_4^* Material damage constant,Johnson-Cook model

D_5^* Material damage constant,Johnson-Cook model

T Current temperature of material

T_0 Reference temperature

T_{melt} Melting temperature of material

γ_0 Gruneisen gamma

a	First order volume correction to Gruneisen gamma
C	Intercept to the cubic shock-velocity versus particle velocity curve
$S_{\mathrm{MG},i}$	Coefficient i to the cubic shock-velocity versus particle velocity curve (i=1,2,3)

$\Delta\varepsilon^p$	Incremental equivalent plastic strain
$\dot{\varepsilon}$	Equivalent strain rate
σ_1	Intact strength constant
σ_2	Failed material strength constant
σ_{maxf}	Maximum failed strength of the material (compressive)
σ_{max}	Maximum intact strength of the material (compressive)
$\widetilde{c}$	Strain rate effect coefficient
D_1	Material damage constant, Johnson-Holmquist-Beissel model
G	Shear modulus of elasticity
k_i	Bulk modulus (i=1); ith pressure coefficient (i=2,3)
n	Material damage constant, Johnson-Holmquist-Beissel model
P^*	Dimensionless pressure
P_1	Intact strength constant
P_2	Failed material strength constant
T^*	Dimensionless hydrostatic pressure
U_f	Internal energy of the failed material
U_i	Internal energy of the intact material
β	The fraction of the internal (deviator) energy loss converted to potential hydrostatic energy

δ^L	The error term for output layer
δ_j^L	The error term for hidden layer
η	Learning rate factor
$\theta_{0j}^{\,l}$	The bias in lth layer
$\theta_{ij}^{\,l}$	The weight that is used to send the input to the ith neuron, from the jth neuron in layer l
$^{(i)}$	Labels the vector of actual values
a_i^l	The activation of the ith neuron in the lth layer,
J	Cost function
m^*	The number of training samples

α	Space index
α^*	Artificial viscosity constant
β^*	Artificial viscosity constant
Ω	Support domain of the smoothing function
ρ	Density of particles
ζ	Distance between particles x and x$'$
d	Number of space dimensions
h	Smooth length
m	Mass of particles
N	Number of neighboring particles
r	Distance between particles x and x$'$

W Smoothing function

x A particle

Abbreviations & Acronyms

2D Two-dimensional.

3D Three-dimensional.

ANN Artificial neural network.

CSLH Appled constant to smooth length.

EOS Equation of state.

FEM Finite element method.

HEL Hugoniot Elastic Limit.

JC Johnson-Cook.

JHB Johnson-Holmquist-Beissel.

ML Machine learning.

MLP Multilayer perceptron.

MSE Mean squared error.

RMS-prop Root mean squared propagation.

SPH Smoothed particle hydrodynamic.

Chapter 1

Introduction

1.1 Motivation

Advanced ceramics, such as alumina, have been incorporated into the design of various armor systems as frontal layers, mainly owing to their relatively high strength, hardness, and low cost-to-performance ratio [1–3]. To make efforts towards designing and improving armor systems, many experimental and numerical studies have sought to understand the role of mechanical properties (e.g., strength and modulus), geometries, and governing failure physics (e.g., shock physics [4, 5] and granular physics [6]) on the dynamic ballistic performance of ceramics [7–10]. Compared to experimental approaches, numerical approaches enable a wider range of material constants and design parameters to be explored, with improved temporal and spatial resolutions, especially under extreme loading conditions where experimentation and field testing are difficult and costly (e.g., ballistic impact [11], laser shock [12]). Hence, future design strategies and materials development will be largely guided by advancements in numerical approaches after careful verification and validations [13–16].

Numerical simulations informed and validated by experiments form a powerful engineering tool for the optimization and design of structures subjected to complex loading conditions (e.g., impact loads [17]). The choice of the material model and computational framework are important because they influence the accuracy of predictive results. The material model has constitutive relations that allow describing material behaviors with consideration of how the material strengths soften from an intact state to a failed state. For example, the Johnson-Holmquist-Beissel material model enables a better characterization of a catastrophic failure of ceramics by constituting a piece-wise strength-pressure and damage-pressure envelope compared to other JH models, leading to a more realistic representation of the response of ceramics subject to impact loading [18, 19]. An appropriate physics-based numerical framework is crucial because it can better approximate the system solution and reasonably simulate the response of ceramics under high-velocity impact [16, 20]. For example, smoothed particle hydrodynamic (SPH) method provides an alternative to the traditional finite element method (FEM) for ballistic impact problems [17, 21, 22] as they are able to overcome limitations caused by element distortion existing in FEM and better simulate the fragmentation process of ceramics under dynamic loading [23–25]. In addition, given the considerable computational cost associated with the increasingly sophisticated numerical models and boundary conditions, machine learning (ML) techniques have been employed recently to improve the computational efficiency and, more importantly, enable the possibility of statistical analysis of the behavior of materials across large amounts of input conditions [26, 27]; To the authors' best knowledge, there have been limited impact-related studies to date where artificial computer analysis techniques are used to develop statistical models for describing the performance and properties of materials [28, 29]. This thesis study will pursue those efforts.

1.2 Thesis objectives

Understanding the influences of the mechanical properties of alumina ceramic on its impact performance under dynamic loading is crucial for designing a ceramic-based protection system. Specifically, to address this, the data-driven approaches (e.g., machine learning-assisted computational modeling) are utilized to inform on the relationship between property and performance and understand the behaviors of ceramic material under dynamic loading. The data-driven approaches rely on a robust computational framework (material model and numerical methods) and machine learning algorithms. Specifically, the computational framework is developed by incorporating the Johnson-Holmquist material models within the framework of SPH in LS-DYNA modeling software. The developed numerical models are validated against experimental data from literature and then used to generate training data sets to train the machine learning algorithms that can inform us of the performance behavior of materials across large amounts of input conditions. Overall, one vision is that the data-driven approaches enable an accelerated identification and quantification of the property-performance relationships of ceramic material to support rapid design and optimization of ceramic materials in protection system design.

The outcome of this thesis will be important to: (i) provide valued contributions to our fundamental understanding of dynamic behaviors of ceramic under impact loading and serve as a foundation for further development of multi-scale numerical models, (ii) provide insights into factors (e.g., mechanical properties) that govern advanced brittle materials performance under ballistic impact loading and guidance for future structural scale design and optimization of armor systems.

1.3 Thesis goals

The objectives of this thesis will be accomplished by completing the following research goals:

1. Implement a user-defined subroutine of the JHB material model within the framework of SPH in LS-DYNA to simulate the response of alumina under impact loading. The subroutine is programmed within the FORTRAN environment in LS-DYNA modeling software. A test simulation of a single element is carried out to verify the implementation of the JHB model into finite element code of LS-DYNA [30]. The subroutine enables the finite element code of the

JHB material model to run within the LS-DYNA platform.

2. Validate the developed computational model in predicting outcomes of alumina subjected to plate and ballistic impact, focusing on particle velocity profile for plate impact and the damage patterns, residual mass, and residual velocity of the projectile as a measure of ballistic performance for high-velocity impact [19, 31].

3. Develop a multi-layer perceptron (MLP) model to statistically explore the non-linear relationships between inputs (e.g., shear modulus, material strength, tile thickness, and impact velocity) and performance (i.e., residual velocity and mass of the projectile) in the situation of a projectile impacting a single alumina tile. The developed MLP aims to ultimately achieve an accelerated prediction and optimization of the performance.

   Altogether, the investigations inform the design ideas and provide guidance for future structural scale design and optimization of armor systems.

1.4 Thesis contributions

The contributions from this thesis are summarized below:

1. Made novel contributions to better physics-based modeling and predicting the outcomes of ballistic impact for alumina-based ceramics. The developed computational framework has well simulated and captured the failure mechanisms of ceramics during the impact penetration process, enabling a better understanding of their role in contributing to impact performance.

2. Made novel contributions in informing the designer of the large-scale fracture behaviors of ceramics and providing guidance for future structural scale design and optimization of ceramic-based armor systems. The ANN model provides

insights into ballistic impact problems from a statistical view, achieving high-throughput identification and quantification of property and performance relationships. The developed model enables further structural scale design and optimization of armor systems.

1.5 Thesis structure

The ultimate objective of the thesis work is to investigate the influences of mechanical properties of alumina ceramic on its impact performance under dynamic loading and utilize the investigations to inform the design ideas of the protection system. The thesis' structure is organized as follows:

1. Chapter 1 introduces the motivation, objectives, goals, contributions, and structures of this thesis study.

2. Chapter 2 focuses on studying the influence of mechanical properties and geometries of intact ceramic tile under the high-velocity impact through a combing SPH and machine learning approach.

3. Chapter 3 ends with concluding remarks on the research as well as recommendations for future research.

Chapter 2

High-velocity impact study of an advanced ceramic using finite element coupling with a machine learning approach

Part of this Chapter was published as **Alex Yang**; Dan Romanyk; and James Hogan. *High-velocity impact study of an advanced ceramic using a combined finite element and machine learning approach.* Ceramic International. (2022)

Author	Contributions
Alex Yang	Conceptualization, Methodology, Simulation, Validation, Formal analysis, Writing - Original Draft, Visualization
Dan Romanyk	Writing - Review & Editing, Supervision
James Hogan	Principal investigator, Conceptualization, Writing - Review & Editing, Supervision

2.1 Introduction

Advanced ceramics, such as alumina, have been incorporated into the design of various armor systems as frontal layers, mainly owing to their relatively high strength, hardness, and low cost-to-performance ratio [1–3]. To make efforts towards designing and improving armor systems, many experimental and numerical studies have sought to understand the role of mechanical properties and geometries on the dynamic, ballistic performance of ceramics [7–10]. Comparing with experimental approaches, numerical approaches enable a wider range of material constants and design parameters to be explored, with improved temporal and spatial resolutions, especially under extreme loading conditions where experimentation and field testing are difficult and costly (e.g., ballistic impact [11], laser shock [12]). For example, ballistic testing in the literature are often conducted within a rather narrow impact velocity range [24], which limits the systematic study of both ballistic (e.g., dwell and penetration [32, 33]) and material responses (e.g., change of mechanisms). Hence, future design strategies and materials development will be largely guided by advancements in numerical approaches after careful verification and validations [13–16], and these will be pursued in this study.

Numerical simulations informed and validated by experiments is a powerful engineering tool for the optimization and design of structures subjected to complex loading conditions (e.g., impact loads [17]). The choices of the material model and the numerical framework plays a key role in the accuracy of predictive results [16]. In the literature, phenomenological models have been extensively implemented to study the behavior of ceramics under the high-velocity impact, such as the Johnson-Holmquist models which considers the strain rate, pressure, bulking, and phase change effects (JH1, JH2, and JHB) [30, 34, 35]. A recent study conducted by Islam et al.[18] compared these three models for silicon carbide under ballistic simulations, and it was found that the JHB model resulted in a better prediction of the response of the ceramic than two others (e.g., crack propagation and the cone fracture zone). The improved accuracy by using the JHB model stems from two perspectives: 1. it combines the characteristics of the JH1 and JH2 models in describing material behaviors with consideration of how the material strengths softens from intact state to failed state [30];2. it enables a better characterization on catastrophic failure of ceramics by constituting a piece-wise strength-pressure and damage-pressure envelope leading to a more realistic representation of the response of ceramics subject to impact loading [18, 19]. Accordingly, to better simulate the failure and catastrophic response of ceramics within a computational scheme, this study implements the JHB material model as a user-defined subroutine into the finite element code of LS-DYNA in literature to simulate the response of alumina ceramic tiles under high-velocity impact.

Next, selecting an appropriate physics-based numerical framework is crucial because it can better approximate the system solution and reasonably simulate the crack initiation, propagation, and coalescence in ceramics [16, 20]. Mesh-free methods provide an alternative to the traditional finite element method (FEM) for ballistic impact problems, and theses have been implemented by many researchers in the literature [17, 21, 22]. For example, smoothed particle hydrodynamics (SPH) is suited for large deformation problems as they are able to overcome limitations caused by element distortion existing in FEM due to their mesh-less discretization feature [23–25]. In conjunction to the proper numerical framework, a comprehensive parametric study and sensitivity analysis of numerical settings in SPH is critical to enhance the simulation accuracy when modeling impacts of brittle solids [36–38]. Large sensitivity of the SPH parameters on impact simulation results have been noted before but not comprehensively studied [18, 36, 39], such as particle spacing [17], artificial viscosity coefficients [36], and constant applied to smooth length [21]. In this work, parametric studies of these SPH settings are conducted.

In addition, given the considerable computational cost associated with the increasingly sophisticated numerical models and boundary conditions, machine learning

(ML) techniques have been employed recently to improve the computational efficiency, and more importantly, enable the possibility of statistical analysis of the behavior of materials across large amounts of input conditions [26, 27]. To the authors' best knowledge, there have been limited impact-related studies to date where artificial computer analysis techniques are used to develop statistical models for describing the performance and properties of materials [28, 29]. Among various techniques (e.g., Bayesian's regression [40] and deep learning [41]), the multilayer perceptron (MLP) approach is the most commonly applied neural network in the field of mechanics [42, 43]. More recently, in the field of impact mechanics, Liu et al. [44] used MLP in combination with a conjugate gradient method to optimize the design of functionally graded metal/ceramic materials. They showed that the neural network possessed good capacity in describing and handling the non-linearity between the design parameters and objective optimization parameter (e.g., depth of penetration) [44]. In a separate study, Bobbili et al. [43] developed a predictive tool based on the MLP method to determine the residual velocity of a projectile impacting an aluminum 1100-H12 thin plate, and they found a good agreement between the experimental and MLP results. Motivated by these limited studies are numerically studying and linking the property and geometrical variables to material performance [43, 44], the present work explores the use of a predictive MLP model coupled with SPH impact simulations to inform the effect of mechanical properties and geometries on impact performance of alumina ceramic tiles, which can then serve as a computationally-efficient tool for material and system design.

In the present study, we first develop a computational framework by combining the user-defined JHB material model and SPH method in LS-DYNA. The computational code was verified through single-element simulation, and the computational framework was simulated by comparing with the experiment's results of plate impacts and ballistic impacts for an alumina ceramic tile [19, 45]. Then, the sensitivity of numerical settings of the SPH method on predicted results (e.g., particle velocity, residual velocity and mass of projectile) are investigated through parametric studies. The results are then used to guide the selection of parameters values for the fully validated and verified models. Lastly, we train an artificial neural network (ANN) with the training datasets obtained from ballistic simulations, which is then applied to study the ballistic performance (e.g., residual projectile velocity) of single alumina ceramic tiles considering both material variation (e.g., strength) and geometry (e.g., tile thickness). The contributions of this work are re-articulated within the following sections: 1. A comprehensive and robust implementation of the JHB material model within the SPH framework in LS-DYNA and the determination of corresponding material constants for alumina are demonstrated (Section 2.2), followed by verifying the model implementation with a single-element test simulation (see Appendix B). As far as we are aware, this is the first time in the literature where the JHB has been implemented via user subroutine in LS-DYDA. We will make the sub-routine accessible in the supplementary files, thus contributing to future usage [24, 31, 46] and modification in the LS-DYNA solver [17, 24, 47]. 2. Parametric studies on the SPH numerical settings reveal the sensitivities of the settings on key model performance metrics (e.g., residual velocity) (Section 2.3.1). 3. Structural-scale simulation cases, including plate impact experiments, [19] and ballistic testing [45] are conducted and shown to be in good agreement with the literature (Section 2.3.2). Finally, an MLP algorithm is then constructed and coupled with the JHB material model to investigate the sensitivity of both material properties and geometries on the ballistic performance of alumina tiles undergoing high-velocity impact (Section 2.4), followed by discussions of the implications for the current study (Section 2.5).

2.2 Determinations of material constants

The detailed descriptions of the used Johnson-Holmquist-Beissel (JHB) material model is provided in the Supplementary Material (see Appendix). Table 2.1 summarizes all the JHB model parameters for the alumina material used in this work. Specifically, the model constants for alumina are obtained based on the existing experimental data in the literature [48, 49], and calibrated against the shock and ballistic impact validation data (Section 2.3.2). The initial density, shear modulus, and bulk modulus are obtained from Scazzosi et al. [24], Alexander et al. [48], and Simons et al. [50]. For pressure constants, Alexander et al. [48] examined the dynamic response of alumina under shock loading and generated the test data of pressure vs. relative volume, as shown in Figure 2.1 (a). The fitted pressure parameters k_1= 265.5 GPa, k_2=181.6 GPa, and k_3= 171.4 GPa are extracted from the fitted curves of the experimental data following the equation noted in the sub-figure and from Equation (A.23). According to the description in Johnson et al. [30], materials that exhibit phase change shows three distinct response regions under shock loading in their pressure-volume relationship, where the phase transition manifests at a relatively low-pressure state. Figure 2.1 (a) indicates that the alumina ceramic does not undergo the phase change subjected to high-shocked pressures up to 100 GPa, and as noted by Alexander et al. [48]. As an outcome, the phase change effects are not considered in this work.

For strength constants, Subhash, et al. [49] provided the testing data on a variety of brittle materials that employed a wide range of confinement conditions beyond the HEL (i.e., shock, triaxial compression, and impact experiments). The test data of alumina ceramic are extracted from their work [48] and re-fitted with the constitutive law of the JHB model in Figure 2.1 (b) using the gradient decent algorithm in Matlab. The values of the intact strength model parameters are determined from the fitted curve: T= 0.20 GPa, σ_i=1.82 GPa, P_i=2.23 GPa, and σ_{max}= 6.83 GPa. The values of the failed strength model parameters are directly extracted from the JH2 curve provided by Bavdekar et al. [49] and then calibrated as: P_f=1.35 GPa, σ_f=1.35 GPa, and $\sigma^f_{\max}$= 2.7 GPa. The damage constants are provided by Toussaint et al.[17] and then calibrated as D_1=0.03 and n=1. More importantly, to illustrate the improvements in the JHB model, the JH2 intact strength model [34] is also re-plotted in Figure 2.1 (b). It is observed that the gradually increasing JH2 strength model (i.e., assuming the plot begins at T= 0.20 GPa) deviates from the data points when the stress exceeds the HEL, while the JHB model traces the data points in much better agreement. In summary, the selection of the JHB model describes the three stages of material strength-pressure response with a single curve: 1. linearly at low pressure; 2. non-linearly at higher pressures up to the HEL; and 3. pressure-independent beyond the HEL.

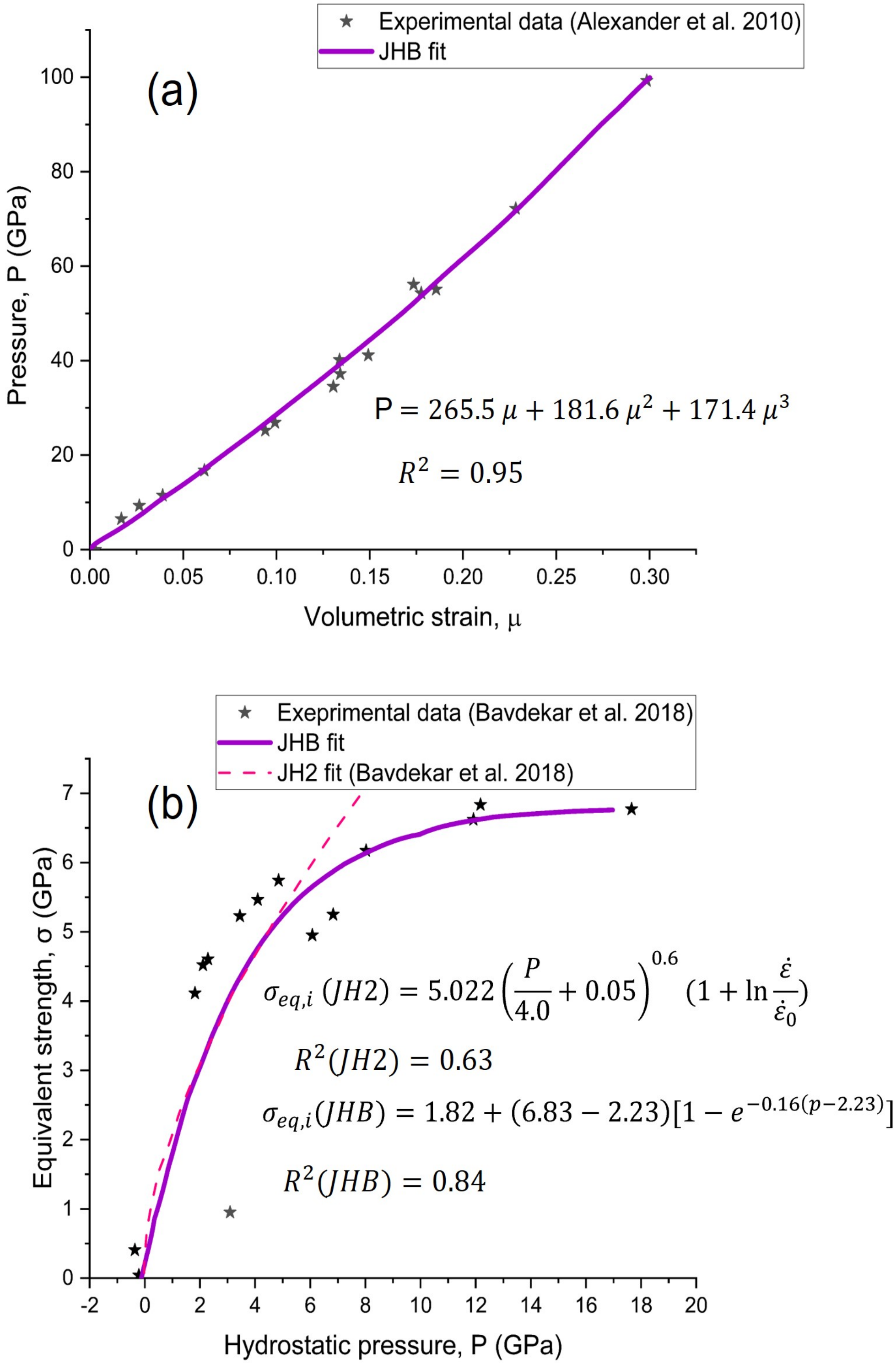


Figure 2.1: Experimental data from literature [48, 49] with noted curve fits for obtaining the pressure and strength constants of alumina ceramic for the JHB model. (a) Pressure vs. volumetric strain with curve fits for JHB model parametrizations (solid purple) [49], and (b) Equivalent strength vs. hydrostatic pressure with JHB (solid purple) and JH2 (dashed pink) curves fit, demonstrating noted differences between the two models [48].

Table 2.1: Johnson-Holmquist-Beissel material constants for alumina ceramic.

Model Parameters	Notation	Value
Density (kg/m^3)	ρ	3890 [51]
Shear modulus (GPa)	G	152 [24]
Bulk modulus (GPa)	K	265 [51]
Elastic modulus (GPa)	E	360 [50]
Hydrostatic tensile strength (GPa)	T	0.2 [17]
Intact strength constant (GPa)	σ_i	1.816 [48]
Intact pressure constant (GPa)	P_i	2.228 [48]
Max intact strength	σ_{max}	6.83 [48]
Strain rate coefficient (s^{-1})	C	0.0665 [50]
Failure strength constant (GPa)	σ_f	1.35 [48]
Failure pressure constant (GPa)	P_f	1.35 [48]
Max failure strength	$\sigma^f_{\max}$	2.7 [48]
Reference strain rate (s^{-1})	ϵ_0	1 [52]
Bulking factor	B	1 [52]
Elastic bulk modulus (GPa)	K_1	265 [48]
Coefficient for 2nd degree term in EOS (GPa)	K_2	181.6 [48]
Coefficient for 3rd degree term in EOS (GPa)	K_3	171.4 [48]
Damage coefficient	D_1	0.03 [52]
Damage exponent	n	1 [52]

2.3 Simulation results and discussions

This section provides the parametric studies on the effects of the numerical settings of SPH within the context of both plate impact (see Figure 2.2) and ballistic (see Figure 2.3) testing cases, including the particle spacing, applied constant to smooth length, and artificial viscosity parameters. A better evaluation of these parameters is important given their noted sensitivities in the literature to simulations of various problems [18, 36], and our desire to guide other researchers in the future. The best combinations of SPH parameters for each testing case are identified by matching experimental results in the literature [19, 45], the detailed discussions on a quantitative and qualitative comparison with the experimental data provided in Section 2.3.2.

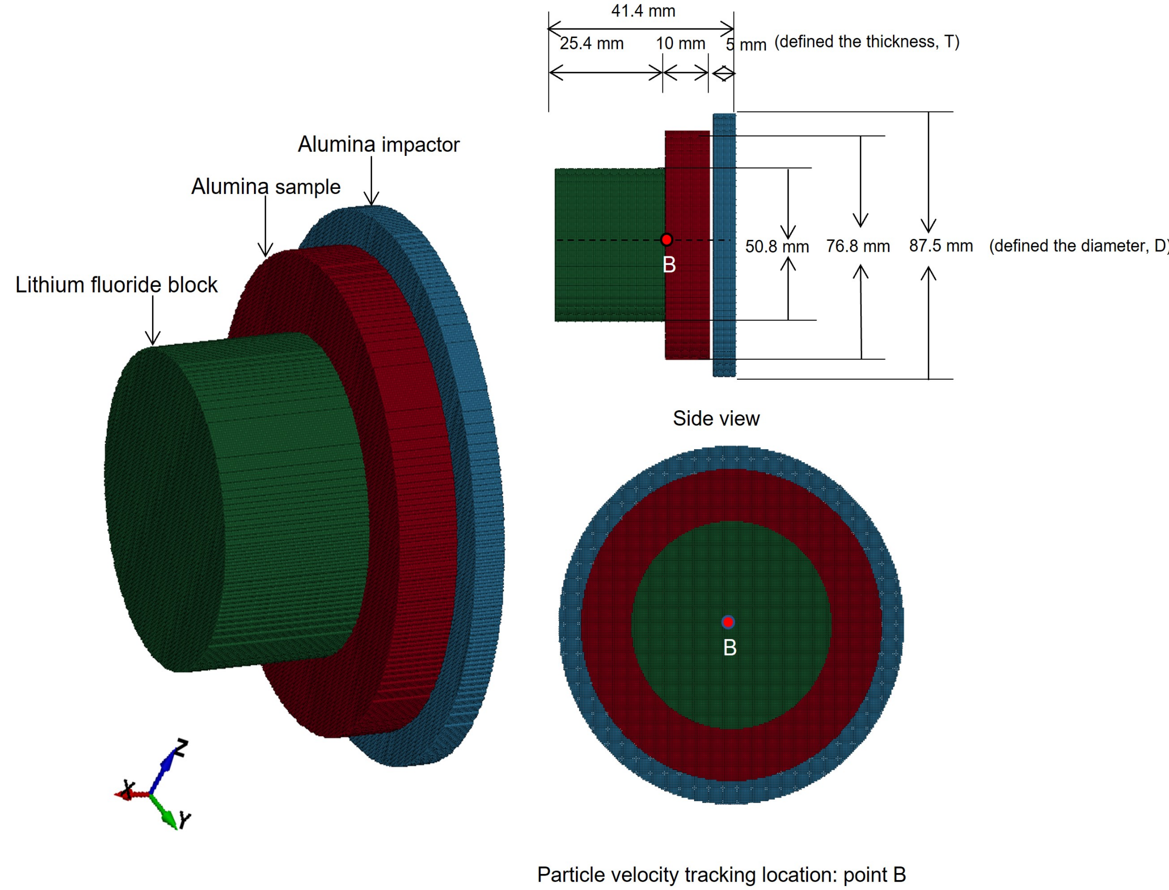


Figure 2.2: Configuration of the compressive plate impact experiment with an alumina plate impacting on the alumina sample backed by a lithium fluoride block. This configuration is used to conduct parametric studies of SPH numerical settings and parameterize the Johnson-Holmquist-Beissel model by comparing the shock response measured at point B at the center of the back surface of sample. The geometry in this study follows the work by Grady and Moody [19]. Note that the dimensions of the impactor, sample, and block is varied in our simulations based on the ones reported in Grady and Moody [19].

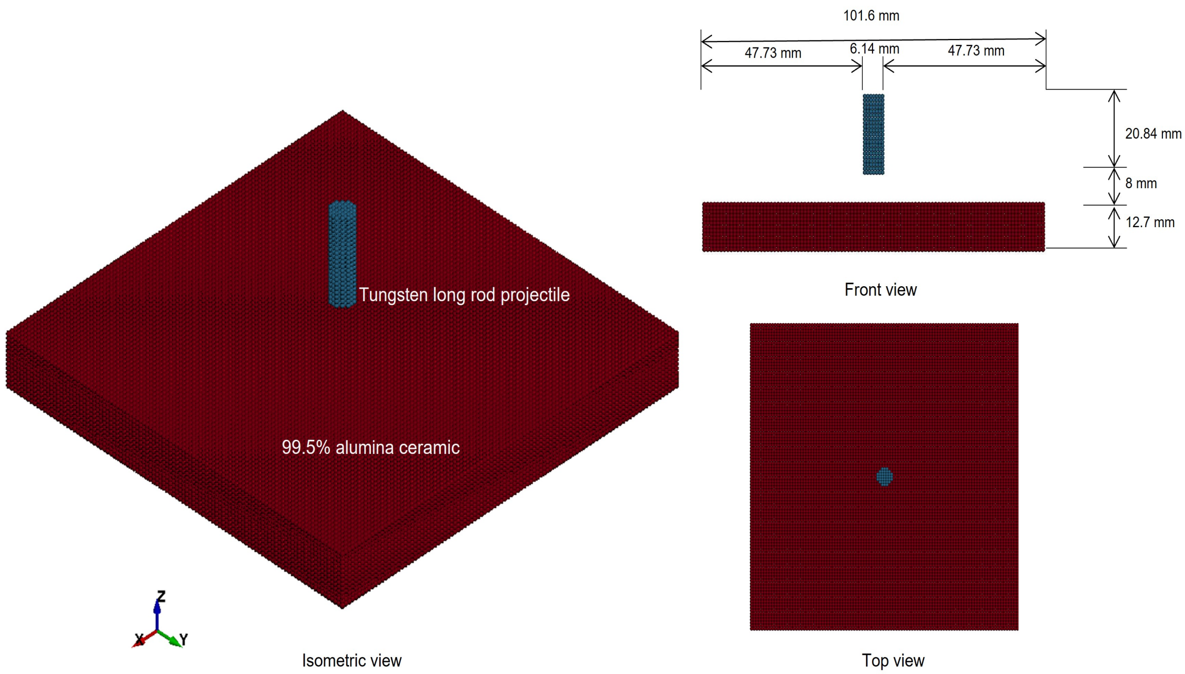


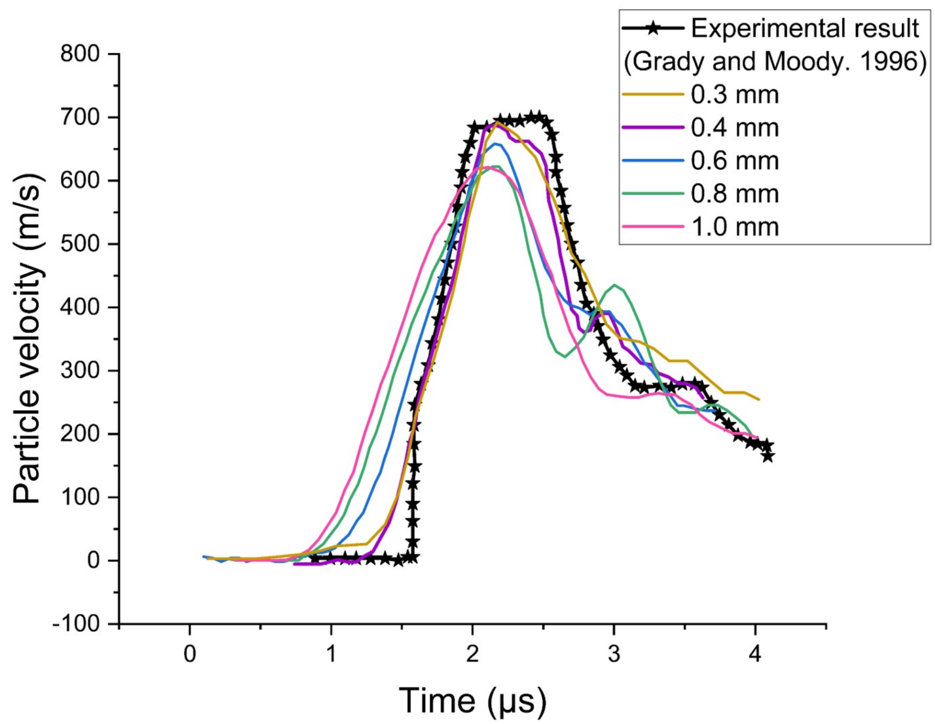
Figure 2.3: Configuration of the simulation setup of a tungsten long rod impacting on an alumina ceramic tile. The geometries and dimensions of the setup follow the study by Nemat-Nasser et al. [45].

2.3.1 Parametric studies of the smoothed particle hydrodynamics numerical settings

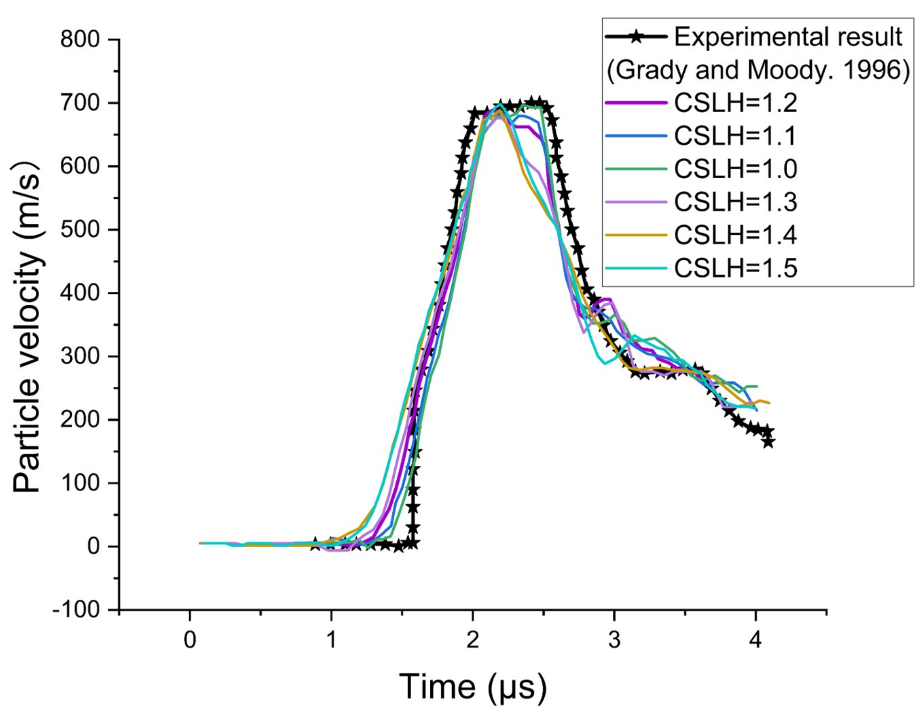
The effects of particle spacing

Known from literature [23], the shock profile predicted by SPH (particle velocity) is affected by the particle spacing when the applied constant to smooth length (CSLH) is fixed because the history variables at a particle (e.g., stress, strain, pressure, and particle velocity) are averaged based on the particle approximation. Figure 2.4 (a) shows the effects of particle spacing (p_c) on the particle velocity profile, with p_c selected between 0.3 and 1.0 mm. The minimum particle spacing is limited to 0.3 mm considering the exponentially increase in computational time when using a particle spacing of 0.2 mm or smaller. In Figure 2.4 (a), the particle spacing of 0.4 mm shows the closest prediction to the experimental measurement when compared to the lower and higher values. The maximum deviations at the peak velocity for p_c = 0.3 mm, p_c = 0.4 mm, p_c = 0.6 mm, p_c = 0.8 mm and p_c = 1 mm from the experimental result, are

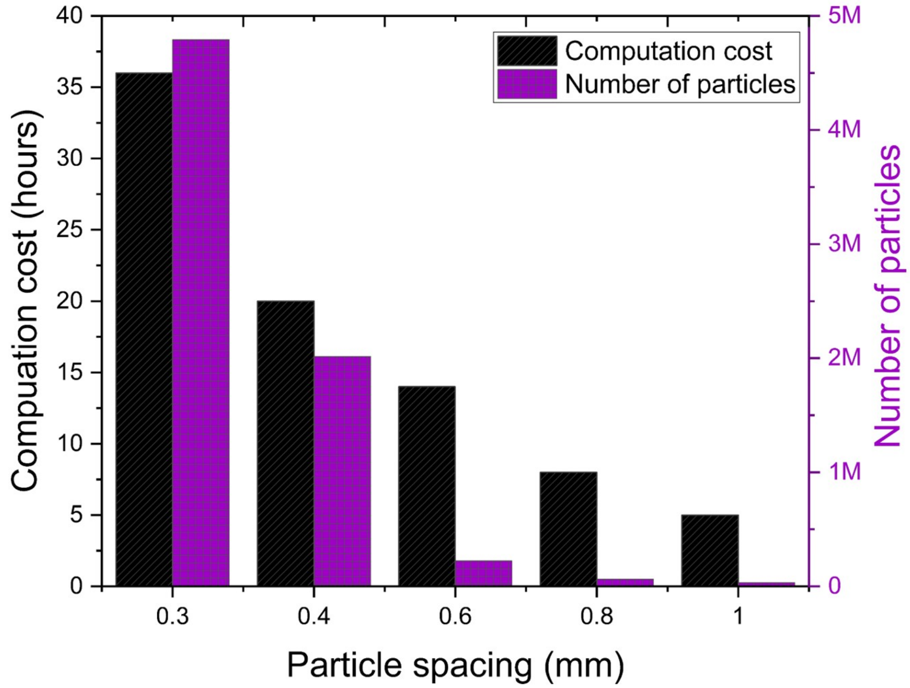
0.5%. 0.1%, 7%, 17%, and 18%, respectively. Without considering the computational efficiency, a finer particle spacing trend results in a more accurate prediction of the particle velocity, which is consistent with other studies [18]. From Figure 2.4 (a), it is also observed that a larger particle spacing tends to shift the overall shock velocity profile towards the left, indicating a delay of the particle response. As a result, the obtained particle velocity is affected by the size of particle spacing with a fixed CSLH. To better illustrate the influence of the CSLH on the shock profile response, the observed trend for CSLH is shown in Figure 2.4 (b) with a fixed particle spacing of 0.4 mm. Figure 2.4 (b) shows the shock profile response is slightly affected by the CSLH at the Hugoniot state when the values of smooth length is less than 1.2. Lastly, it is noted that there is a seven times increase in computational time when the particle spacing decreases from 1 mm to 0.3 mm, as shown in Figure 2.4 (c).


Experimental result
(Grady and Moody. 1996)
0.3 mm
0.4 mm
0.6 mm
0.8 mm
1.0 mm
Particle velocity (m/s)
Time (μs)
800
700
600
500
400
300
200
100
0
-100
0
1
2
3
4

(a)


Experimental result
(Grady and Moody. 1996)
CSLH=1.2
CSLH=1.1
CSLH=1.0
CSLH=1.3
CSLH=1.4
CSLH=1.5
Particle velocity (m/s)
Time (μs)
800
700
600
500
400
300
200
100
0
-100
0
1
2
3
4

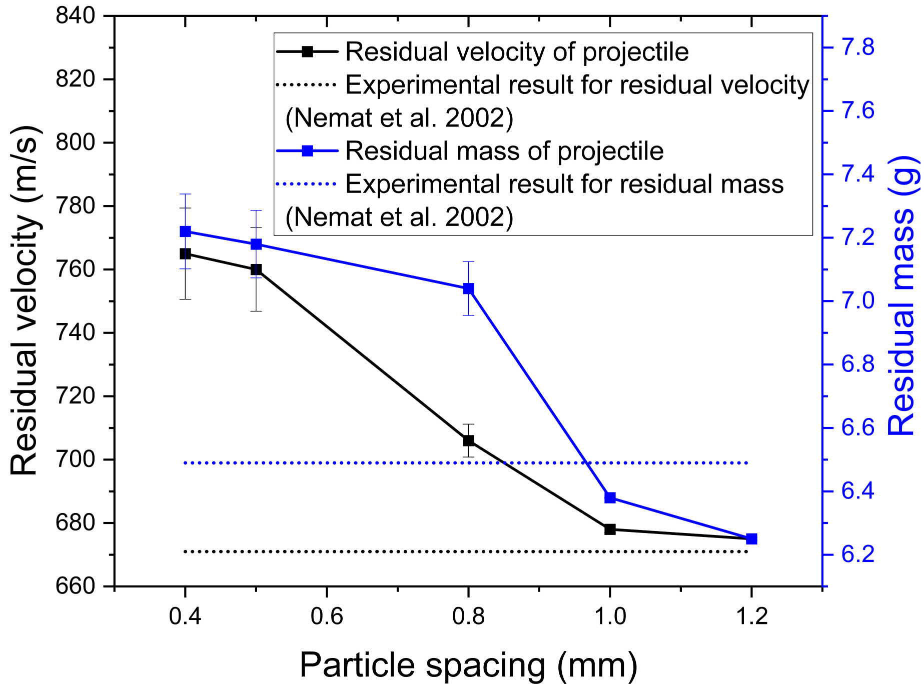
(b)



(c)

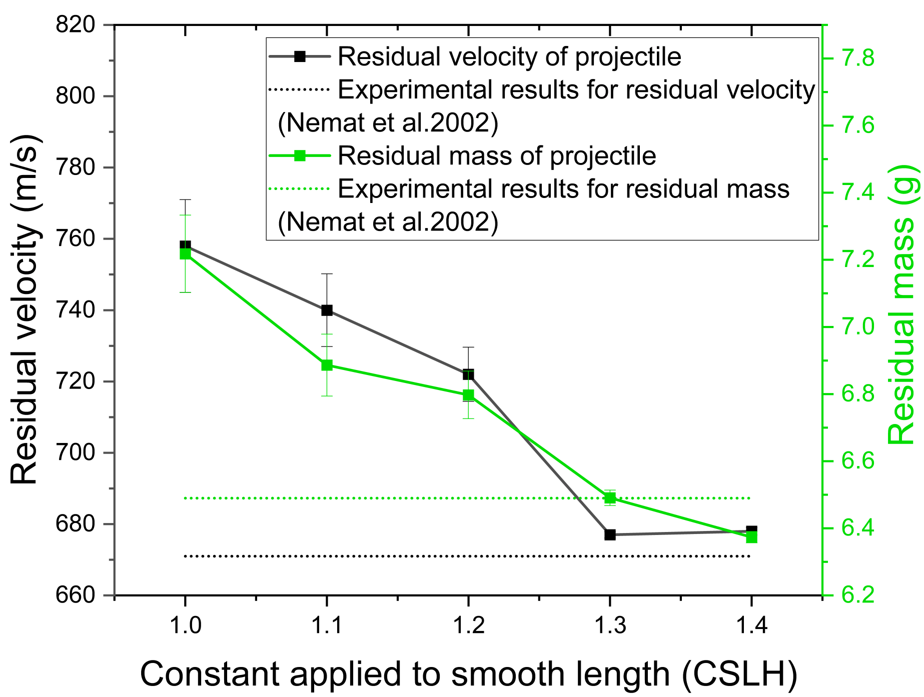
Figure 2.4: Sensitivity analysis of particle spacing and constant applied to smooth length and computational cost analysis of particle spacing used in the SPH framework for the impact configuration of Figure 3 at the striking plate velocity of 1070 m/s. (a) Sensitivity analysis on particle spacing with fixed CSLH=1.2: 0.3 0.4, 0.6, 0.8, and 1 mm. (b) Sensitivity analysis of constant applied to smooth length. (c) Computational cost for varying particle spacing with fixed CSLH=1.2. Experimental results are taken from Grady and Moody [19].

To assess the effects of particle spacing on the predicted results of ballistic simulations, a series of particle spacing values covering the recommended range of 0.4 mm to 1.2 mm [17] are used with a fixed CSLH of 1.2. Figure 2.5 (a) shows the effect of particle spacing on the predicted residual mass and velocity of the projectile in ballistic impact simulations. In Figure 2.5 (a), the associated error bar at each particle spacing reflects the difference between the simulated and experimental results. Figure 2.5 (a) confirms that a reasonable prediction of residual velocity and mass of projectile can be reached when using a particle spacing between 0.4 mm and 1.2 mm, where a particle spacing of 1.0 mm gives the most accurate prediction with an associated error bar of 0.2%. The maximum deviations of the velocity for $p_c = 0.4$ mm $p_c = 0.5$ mm, $p_c = 0.8$ mm, $p_c = 1$ mm, and $p_c = 1.2$ mm from the experimental results, are 15.2% 13.5%, 5.4%, 0.2%, and 0.8% for the residual velocity prediction, respectively, and 11%, 10%, 8%, 0.3% and 0.7% for residual mass prediction, respectively. Generally, the trend demonstrates that the predicted projectile residual velocity and mass decrease with increasing particle spacing, and this trend is consistent with observations in other FEM simulations from the literature [17]. In previous simulations performed in an FEM framework, the mesh sensitivity has been mainly attributed the strain softening behavior which is widely observed in brittle materials [53, 54]. Such strain softening behavior leads to the strain and damage localization in a reduced volume after the mesh is refined [35]. In the SPH framework, the non-local effect is reduced with decreasing particle spacing, which leads to the strain and damage becoming more localized, resulting in less global damage and resistance of the projectile penetration. This is evident in Figure 2.5 (a) where decreasing the particle spacing results in an increased residual velocity and mass of the projectile. A similar trend is observed for the CSLH where a decrease in the CSLH constant results in an increased residual velocity and mass of the projectile by fixing the particle spacing at 1.0 mm (see Figure 2.5 (b)). Finally, a trade-off should be often sought between computational efficiency and accuracy when choosing an appropriate particle spacing. For example, a twenty times increase in computational time is recorded when the particle spacing decreases from 1.2 mm to 0.4 mm (see Figure 2.5 (c)).



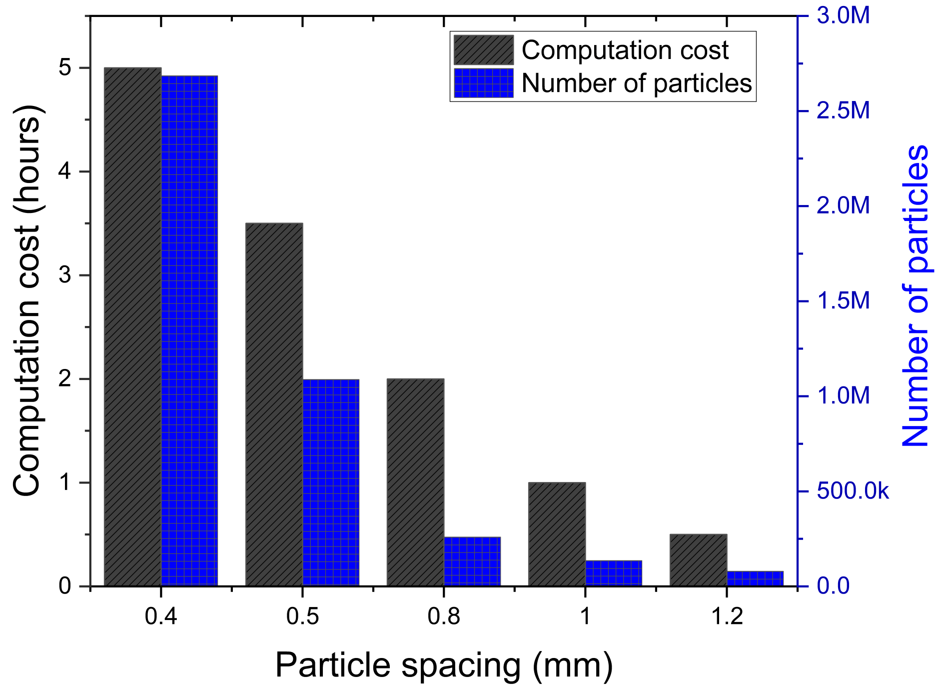
Residual velocity of projectile
Experimental result for residual velocity (Nemat et al. 2002)
Residual mass of projectile
Experimental result for residual mass (Nemat et al. 2002)
Residual velocity (m/s)
Residual mass (g)
Particle spacing (mm)

(a)



Residual velocity of projectile
Experimental results for residual velocity (Nemat et al.2002)
Residual mass of projectile
Experimental results for residual mass (Nemat et al.2002)
Residual velocity (m/s)
Residual mass (g)
Constant applied to smooth length (CSLH)

(b)

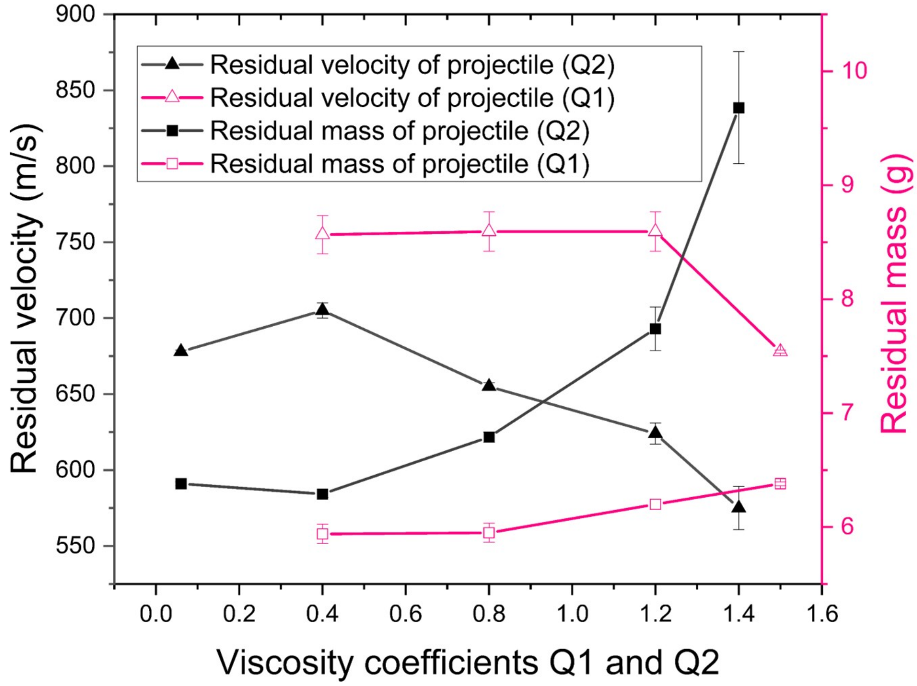


(c)

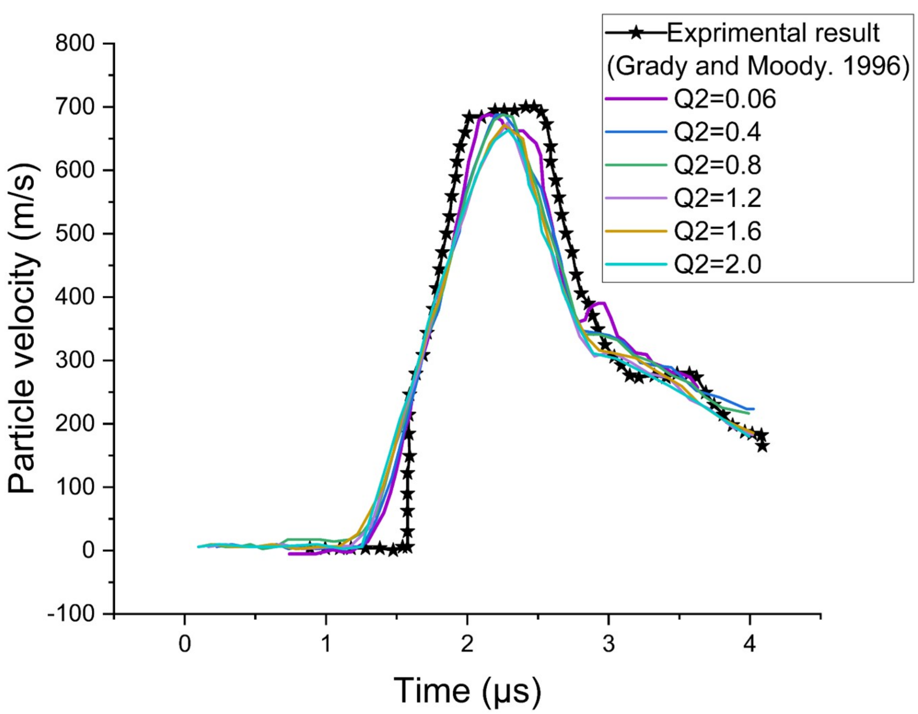
Figure 2.5: Sensitivity analysis of particle spacing and constant applied to smooth length and computation cost analysis of particle spacing used in the SPH framework for the impact configuration of Figure 4 at the projectile at an impact velocity of 901 m/s. (a) Sensitivity analysis on particle spacing with fixed CSLH=1.2: 0.4 0.5, 0.8, 1.0, and 1.2 mm. (b) Sensitivity analysis of constant applied to smooth length with fixed particle spacing of 0.4 mm=1.0, 1.2, 1.1, 1.3, 1.4, and 1.5. (c) Computational cost for varying particle spacing with fixed CSLH=1.2. Experimental results are taken from Nemat e t al . 2002 [45] .

The effects of artificial viscosity

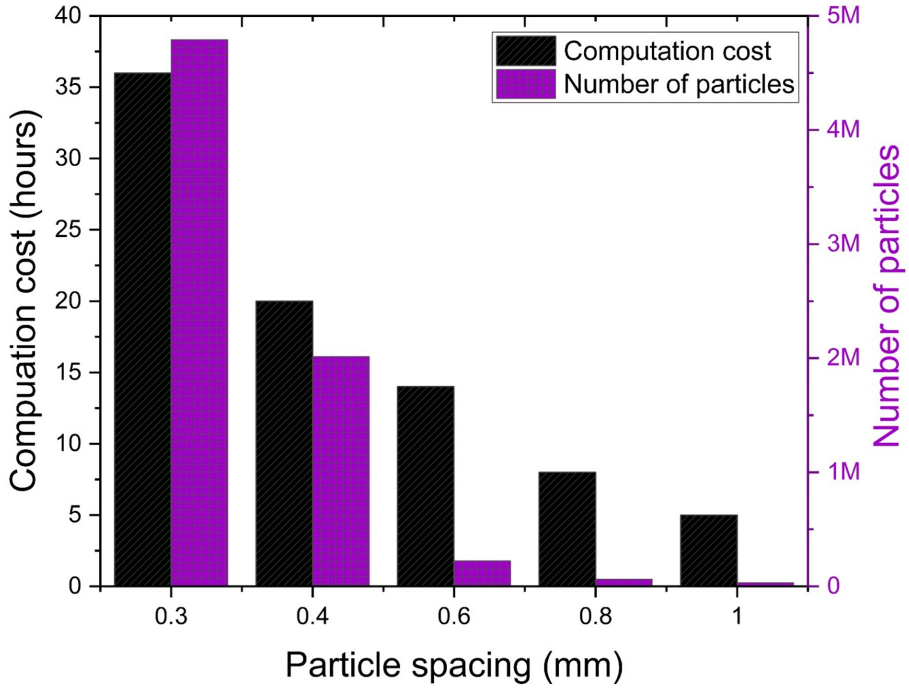
Known from literature [23, 36], the predicted results are effected when strong discontinuities occur under the shock loading process. In SPH framework, artificial viscosity terms are introduced into the momentum and energy governing equations to prevent large unphysical oscillations and numerical instability under shock loading conditions [23]. Thus, the magnitude of artificial viscosity can affect the final simulation solution. In the current study, the effect of artificial viscosity on the numerical solution stability is investigated in the context of the plate impact simulations, with experimental data taken from Grady and Moody [19]. In the SPH framework, the artificial viscosity is defined as two terms: (1) the quadratic artificial viscosity term, Q1, which is primarily introduced to handle shocks generated at high Mach numbers [36], and (2) the linear artificial viscosity term, Q2, which is used to handle low gradient regions in the SPH simulations [36].


Residual velocity of projectile (Q2)
Residual velocity of projectile (Q1)
Residual mass of projectile (Q2)
Residual mass of projectile (Q1)
Residual velocity (m/s)
Residual mass (g)
Viscosity coefficients Q1 and Q2

(a)


Exprimental result
(Grady and Moody. 1996)
Q2=0.06
Q2=0.4
Q2=0.8
Q2=1.2
Q2=1.6
Q2=2.0
Particle velocity (m/s)
Time (μs)

(b)

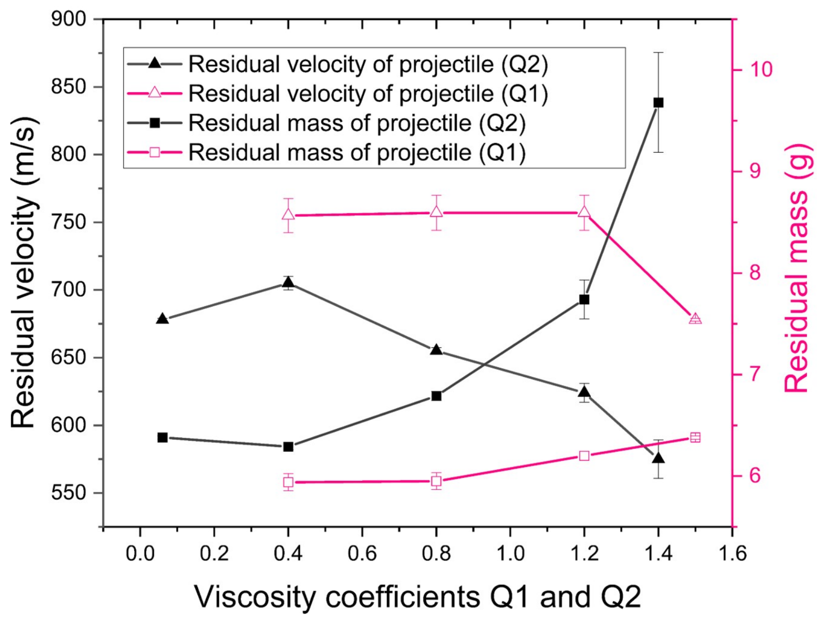


(c)

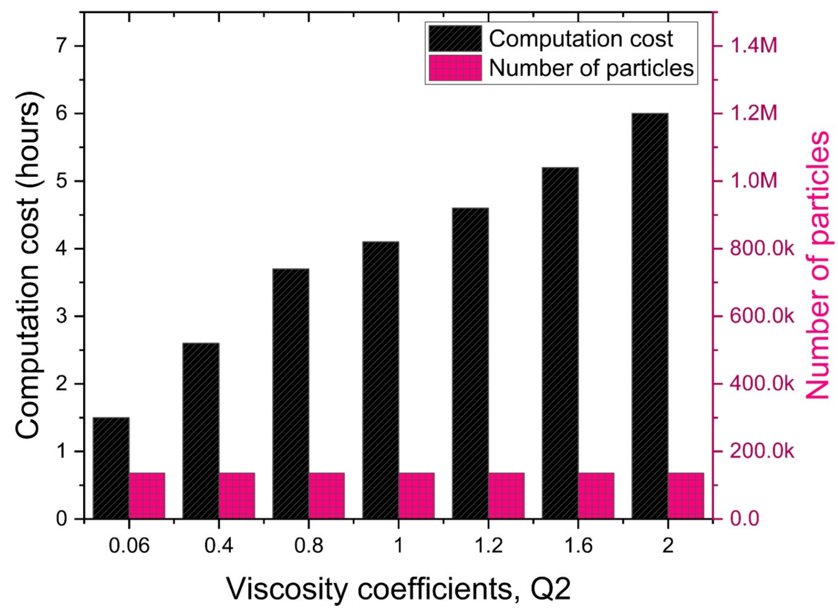
Figure 2.6: Sensitivity and computation cost analysis of artificial viscosity terms used in the SPH framework for the impact configuration of Manuscript Figure 3 at the striking plate velocity of 1070 m/s. (a) Sensitivity study on artificial viscosity Q1 parameters: 0.8, 1.0, 1.2, 1.5, 1.6, and 1.8 with fixing Q2 at 0.06 (default value in LS-DYNA). (b) Artificial viscosity Q2 parameters: 0.4, 0.8, 1.2, 1.5 with a fixed Q1 of 1.5 (default value in LS-DYNA). (c) Computation cost for varying Q2 with fixed Q1=1.5.Experimental results are taken from Nemat et al . 2002 [45]

In this work, various combinations of Q1 and Q2 around the default LS-DYNA values (i.e., Q1 =1.5 and Q2 =0.06) are explored by fixing one value while varying the other to examine their effects on the plate impact simulations. Figure 2.6 (a) and (b) show the effects of Q1 and Q2 on the simulated shock profiles. It is observed that the simulated particle velocity is more sensitive to the Q2 values. The Q1 values only become dominant when the shock velocity becomes significantly large when compared to sound speed in material, where the current simulations have a peak velocity range from 800 m/s to 1300 m/s in comparison with the sound speed of 10740 m/s [55, 56]. It is also noted that the computational time increase three times when changing Q2 from 0.06 to 2 (with Q1 fixed), and hence, a trade-off must be sought between computational efficiency and accuracy (see Figure 2.6 (c)).

A similar approach is used to examine the sensitivity of the artificial viscosity terms for the ballistic impact simulations (see Figure 2.7a). It is observed that the results for an impact velocity of 901 m/s have less than 4.4% of difference when varying Q1 from 0.4 to 1.5, where varying Q2 from 0.06 to 1.4 results in a difference up to 26.6% for both residual velocity and mass prediction. This confirms that the effect of artificial viscosity on both residual velocity and mass is greater affected by the linear artificial viscosity term Q2. For residual velocity, the simulated projectile residual velocity tends to decrease as Q2 increases (black curve with triangle on it in Figure 2.7a), where Q2 of 0.06 gives the most accurate prediction when compared to the experimental result [45] with an associated error of 0.5%. This trend is attributable to a enhancement of strength when Q2 increases, leading to more energy been dissipated, and resulting in a decreasing residual velocity [23]. For residual mass, the projectile residual mass increases with increasing Q2, and this is consistent with the trend observed in Xiao et al. [21], where the residual mass tends to increase as Q2 increases. Overall, these observed trends demonstrate that projectile erosion is a complicated process and the ability for the material to erode the projectile may be related with fragment morphology [57, 58], friction between projectile and plate [57], and material strength [59, 60], and some of these link to energy dissipation. Finally, the computational time increase four times when changing Q2 from 0.06 to 2 (with Q1 fixed), and hence, a trade-off must be sought between computational efficiency and accuracy (see Figure 2.7b).



(a)



(b)

Figure 2.7: Sensitivity and computation cost analysis of artificial viscosity terms used in the SPH framework for the impact configuration of Figure 4 at the projectile at an impact velocity of 901 m/s. (a) Sensitivity study on artificial viscosity Q1 parameters: 0.8, 1.0, 1.2, 1.5, 1.6, and 1.8 with fixing Q2 at 0.06 (default value in LS-DYNA). (b) Artificial viscosity Q2 parameters: 0.4, 0.8, 1.2, 1.5 with a fixed Q1 of 1.5 (default value in LS-DYNA). (c) Computation cost for varying Q2 with fixed Q1=1.5.Experimental results are taken from Nemat et al . 2002 [45].

2.3.2 Plate impact and ballistic impact simulations

The plate impact test is an experimental technique used to study the shock response of ceramics and inform the values of the Hugoniot elastic limit (HEL), spallation strength, and equation of state (EOS) parameters [48, 55, 61, 62]. Figure 2.2 shows the simulation setup of a typical plate impact test, following the configuration provided in Grady and Moody [19], with dimensions varying in our simulations according to Grady and Moody [19] in order to generate different shock velocities. In this study, a particle spacing of 0.4 mm is used in our simulations (e.g., 2015254 particles involved when impact velocity is 1070 m/s), where this value is chosen based on the results from sensitivity studies (see Section 2.3.1 for details). Here, the responses of the alumina impactor and target are characterized using the same material constants of the JHB model as shown previously in Table 2.1. The response of the lithium fluoride (LiF) block is defined using a Steinberg-Guinan material model and Mie-Grüneisen EOS [15]. In the Steinberg-Guinan model, the reference yield strength (A_s) and the shear modulus (G_s) of the lithium floride block was chosen as A_s=0.36 GPa, and G_s=49 GPa at $T_s = 300$ K, $P_s = 0$ GPa, and $\epsilon_{pls} = 0$, where the subscript S is added because of repeated notations. The other material constants of LiF and computational parameters are provided by Sukanta et al.[63], and these are summarized in Table 2.2.

Table 2.2: Steinberg-Guinan material model constants for lithium fluoride window block in plate impact simulation [63].

Model Parameters	Notation	Value
Density (kg/m^3)	ρ_l	3890
Bulk modulus (GPa)	K_l	265
Strength parameter (GPa)	dG/dp	2.45
Strength parameter (GPa)	dG/dT	0.0303
Melting temperature (K)	T_{ml}	1480
EOS Parameters		
Gruneisen coefficient	G_c	1
Linear Hugoniot slope coefficient	S_1	0.005
Bulk speed of sound (m/s)	C_1	5150

Figure 2.8 shows the comparisons between the experimental and simulated plate impact results with striking plate velocities of 1070 m/s, 1551 m/s, 1573 m/s, and 1911 m/s. The plate impact experimental results are provided by Grady and Moody [19], and three different geometric configurations of the experimental setup are investigated in this study (configurations denoted in the caption of Figure 2.8). Figure 2.8 (a) labels the critical stages of the typical material response during a plate impact experiment: (1-2) elastic response up to HEL; (3-4) inelastic response up to the Hugoniot state, where the Hugoniot state describes the locus of all final shocked states (pressure–volume relationship) in a material for various maximum pressure values [4]; and (4-5) the shock release undergoes unloading where tensile stress are built up during the elastic unloading, leading to spalling of the material [4]. From Figure 2.8 (a)-(d), it is observed that the model reasonably captures the shock profile from Grady and Moody [19], with some differences noted in the stage (1-2) up to HEL and the plateau at peak velocity, where differences likely stem from the numerically-introduced artificial viscosity in the SPH framework [25]. The experimental results

are reasonably validated when compared with other such approaches made in the literature [18], where it has been observed that impact velocity does not greatly affect the HEL [64, 65], and increasing impact velocity results in greater amplitude and slope of the plastic front [64, 65].

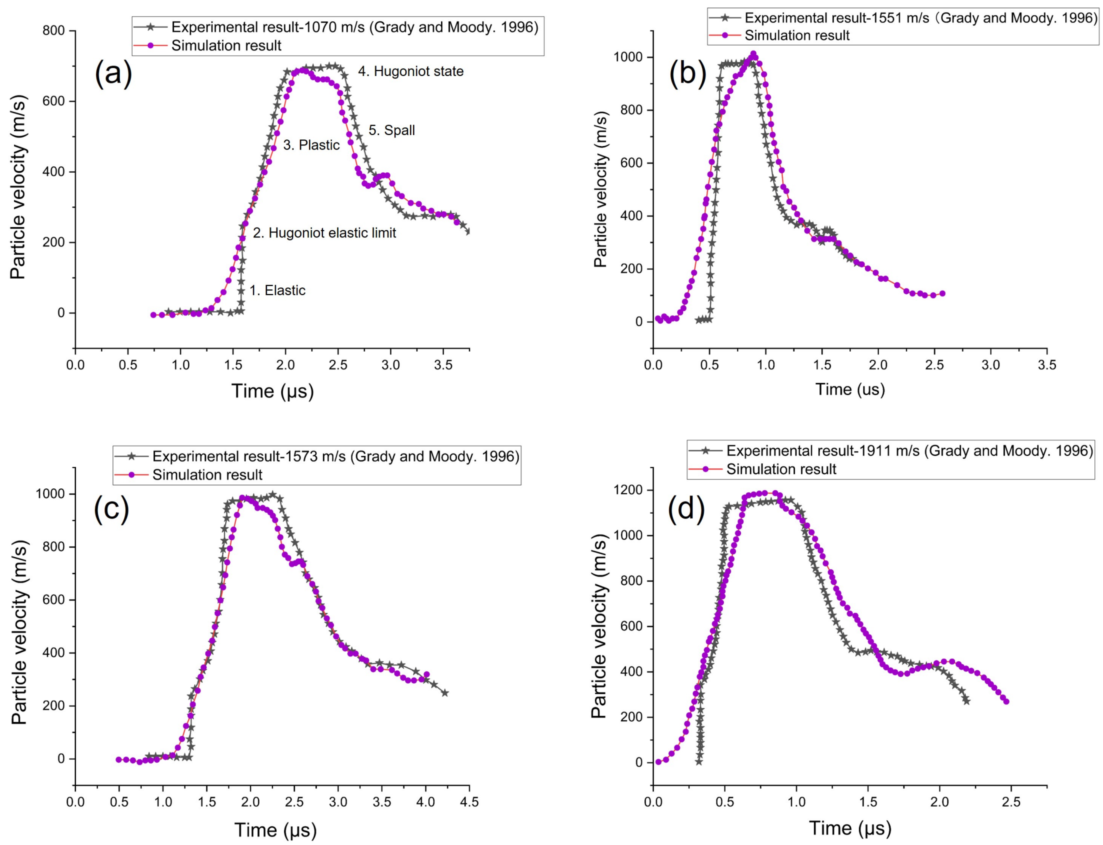


Figure 2.8: Comparisons between experimental and simulated plate impact results : histories of the particle velocity captured at Point B (see Figure 2.2) for the striking plate velocity = 1070 m/s (a), 1551m/s (b), 1573 m/s (c), and 1911 m/s (d) based on JHB model. The experimental data is taken from Grady and Moody [19]. (a,b): alumina sample D: 76.2 mm, T: 10 mm, alumina impactor D: 87.5 mm, T: 5 mm, lithium fluoride window block D: 50.8 mm, T: 25.4 mm, (c): alumina sample D: 76.2 mm, T: 4.762 mm, alumina impactor D: 76.31 mm, T: 2.475 mm, lithium fluoride window block D: 38.1mm, T: 25.4 mm. (d): alumina sample D: 76.3 mm, T: 2.478 mm, alumina impactor D: 76.2 mm, T: 2.477 mm, lithium fluoride window block D: 38.1mm, T: 25.4 mm.

In addition, the implementation of the Johnson-Holmquist-Beissel model along with the best combination of SPH parameters is also quantitatively and qualitatively matched and compared with ballistic impact data from the literature [45]. Specifically, the quantitative data is in the analysis of the residual velocity and mass of the projectile after penetration, while the qualitative comparisons are performed by matching time resolved X-ray images provided in Nemat-Nasser et al. [45]. Figure 2.3 shows the configuration of the SPH-based model corresponding to the ballistic experiment performed by Nemat-Nasser et al. [45] involving a tungsten alloy long rod impacting on a single alumina tile. In this setup, the ceramic tile is made of 99.5% purity alumina and has dimensions of 101.6 mm$\times$101.6 mm$\times$12.7 mm. The tungsten heavy alloy projectile has a diameter of 6.14 mm and a length of 20.86 mm (see the front view in Figure 2.3), and the impact velocities of the projectile are 901 and 904 m/s. The alumina tile is described using the same JHB model parameters provided in Table 2.1. The Johnson-Cook strength and damage model constants of the projectile are provided in Table A.1 [31] (See Appendix).

Table 2.3 summarizes the results of the simulated and experimental impact data, including the residual velocities of projectile, the mass, and relative error between the simulation results and experimental measurements. The residual velocity of the projectile in the simulation is taken at the resultant rear center node of the projectile. The residual mass of the projectile in the simulation is calculated as the difference between the initial projectile mass and mass of the fully damaged projectile particles upon penetration. The damaged projectile particles are simulated by setting (`EROD=2`) in the (`*CONTROL_SPH*`) card of LS-DYNA. Here, the “smooth” option is selected in LS-DYNA PrePost to distinguish between the activated and deactivated particles. From Table 2.3, for both impacting velocities, the model predictions show good agreements with the experimental results by Nemat-Nasser et al. [45], with relative errors within 6% for both residual velocities and masses. The proposed model performs significantly better than those reported in the literature for similar impact conditions [31], where most of them showed relative errors of greater than 20% for the residual mass prediction [31]. For example, Bresciani et al. [31] showed 33% error against experimental results with a cohesive model for the same impact case.

Table 2.3: Comparisons between experimental results from Nemat-Nasser et al. [45] and simulated results in predicting the residual velocity and residual mass for the projectile impacting an alumina tile with the implemented Johnson-Holmquist-Beissel model [30]

Experiments	Impact velocity (m/s)	Residual velocity of the projectile (m/s)	Residual mass of the projectile (g)
Experiment 1 [45]	901	671	6.49
LS-DYNA	901	686	6.11
Error (%)		2.23	5.84
Experiment 2 [45]	904	682	6.42
LS-DYNA	904	689	6.1
Error (%)		1.02	4.98

Furthermore, the nature of the mesh-less feature of the SPH method allows to simulate severe deformations and fragmentation of both the projectile and ceramic tile without defining element erosion [31, 50] and the use of cohesive elements [31]. In turn, these advantages of the SPH method allow more accurate qualitative replication of the ballistic events (e.g., debris cloud [50] and back-face spallation [66]). Figure 2.9 (a) shows the simulated residual velocity vs. time curve for the impact condition of initial velocity of 901 m/s with times denoting selection of still frames from experiments by Nemat-Nasser et al. [45](X-ray images) and models shown in: Figure 2.9(b) for 7 μs with a residual velocity of 840 m/s, and Figure 2.9(c) for 15 μs with a residual velocity of 779 m/s. Overall, the computational framework implemented in this work can reasonably capture the damage evolution process and failure modes when the projectile penetrates the ceramic target. For example, deformation and erosion of the projectile, target spallation, material pulverization into fine powder, and ejection of the debris cloud occurs from both front and rear target surfaces. More simulated damage evolution images of ceramic tile and projectile are provided in Figure A.2 of the Appendix.

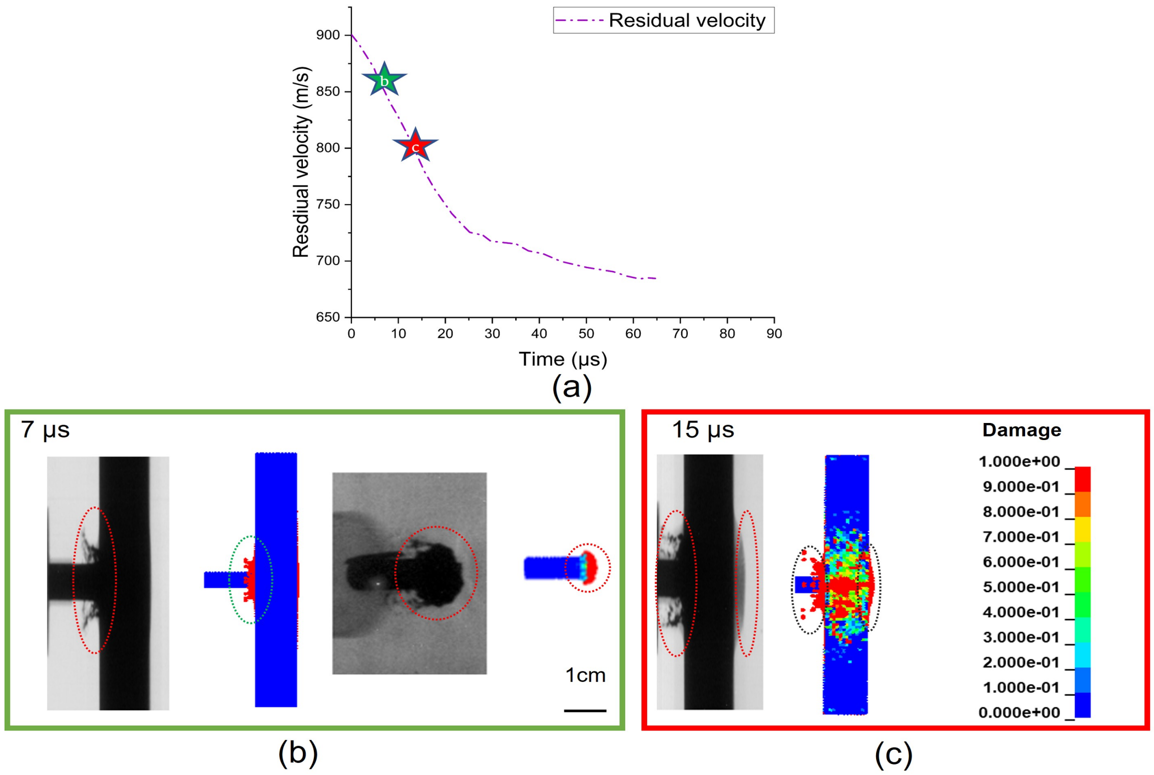


Figure 2.9: Comparisons between the simulated results and time-resolved experimental flash X-ray images [45] during a long rod impact at 901 m/s. Plot of residual velocity of projectile vs. time with noted time points identified for comparing simulation and experimental results of the tile and projectile are compared at: (b) 7 μs and (c) 15 μs.

2.4 Multi-layer perceptron model and sensitivity analysis

In this section, a multi-layer perceptron (MLP) model has been developed to statistically explore the non-linear relationships between inputs (e.g., shear modulus, material strength, thickness, and impact velocity) and performance (i.e., residual velocity and mass of the projectile) in the situation of a projectile impacting a single alumina tile. Here, the residual mass and velocity are two common performance metrics used in literature [24, 31, 46, 67]. Other performance metrics such as dwell and penetration may also be used, although the literature is much more limited [59, 68].

2.4.1 Architecture of the neural network

The topology of the proposed MLP model (see Figure 2.10) is characterized by grouping neurons in the input layer (1), hidden layers (4), and output layer (1) following the work by Parsazadeh et al.[69]. The input layer consists of the most influential variables which are thought to have significant effects on the material performance during ballistic impact events (e.g., tensile strength), and this will help gain a better understanding of the physical phenomenon during impact. Specifically, important characteristics such as the thickness of the ceramic tile, impact velocity of the projectile, and material strength parameters for both ceramic and projectile that may affect the ballistic performances (i.e., residual mass and velocity) are included in the training of the MLP model. For geometries (e.g., thickness of ceramic tile, impact velocity of projectile), these represent simple and standard considerations based on literature [68, 70, 71]. Additional geometric variables could be considered for the target (e.g., hexagonal geometries [72], spatial arrangements [10], lattice structures [73, 74]) and projectile (e.g., nose and fin geometries [75], sphere [76] vs. rod [31]) in future works. For mechanical properties (e.g., compressive and tensile strength of ceramic, shear modulus), these represent simple and physically significant choices that have been studied previously in the literature [70, 71, 77]. Additional properties could be considered (e.g., Poisson's ratio, damage constants [30, 35]), including those that are dependent on the FEM scheme of choice (e.g., fracture toughness in discrete-element framework [78], defect populations and crack speeds in micro-mechanical models [79]). Each input parameter has a range associated with them, which is surveyed from literature and demonstrates both the material variability in properties (e.g., max intact strength) and typical configurations of ballistic impact experiments (e.g., impact velocity) [80]. Figure 2.10 illustrates a complete list of the inputs and Table 2.4 summarizes the references for the ranges. Four hidden layers with 32 neurons at each layer have been determined through the trial-and-error approach given the

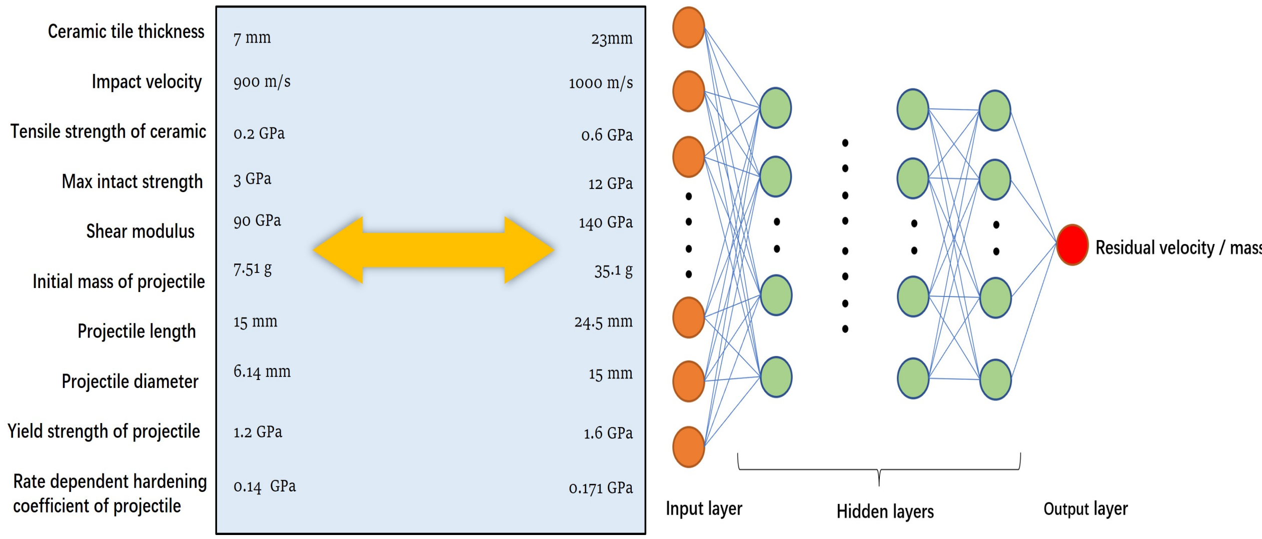


Figure 2.10: Architecture schematics of multiple perceptron model showing the all the input variables considered for ballistic impact case (left) associated with identified parameter range from literature (middle), and construction of the MLP with output of residual velocity and mass of the projectile (right).

accurate prediction and then assigned to the ML model [69]. In this study, the MLP model is trained using the forward and backward propagation algorithms. The Relu function is adopted in the forward algorithm as the activation function to output the non-linearity at each layer. The Relu function is chosen because of the nature of its differentiation form which allows the computation to overcome the vanishing gradient problem by keeping high computational efficiency [81]. In conjunction with forward propagation, the Bayesian regularization back-propagation is used for computing the gradient in the weight space of the MLP model with respect to a loss function [81]. Finally, the RMS-propagation algorithm is then used to minimize the error by updating the weight and bias values of the MLP model [82]. Relevant equations for the MLP were described previous in Appendix A.

Table 2.4: Input variables for MLP model associated with a range that is identified from the literature.

Model Parameters	Notation	Value Range
Ceramic thickness (mm)	t	7 to 23 [17, 51]
Impact velocity (m/s)	V_i	700 to 1000 [24]
Tensile strength of ceramic (GPa)	T	0.2 to 0.6 [66, 83]
Maximum intact strength (GPa)	σ_{max}	3 to 12 [50, 84]
Shear modulus (GPa)	G_p	90 to 140 [17, 66]
Initial mass of projectile (g)	m_i	7.51 to 35.1 [31]
Projectile length (mm)	l	15 to 24.5 [31]
Projectile diameter (mm)	D_p	6.14 to 15 [31]
Yield strength of projectile (GPa)	A	1.2 to 1.6 [21, 85]
Rate dependent hardening coefficient of projectile (GPa)	B_p	0.14 to 0.171 [21, 85]

2.4.2 Training, validation, and testing of the multi-layer perceptron neural networks

Next, we train, validate and test the MLP model within the Python environment. The sample size covers the ranges shown in Table 2.4. The required number of samples for training, validation, and testing is determined through a trial-and-error approach until the prediction variation is less than 10% for convergence. The total required sample size for training, validation, and testing started with 188 samples with bad prediction performance for interesting ranges of input parameters. We gradually increased the sample size to 320 and a good prediction performance was obtained because of the generalization of sample size. The sample size of 320 is generated to train, validate and testing the MLP model using the validated SPH model by randomly varying any input parameters while keeping others fixed within the defined ranges shown in Figure 2.10. This approach is common in literature [42, 43, 86]. Then, the generated data are randomly assigned into training, validation and testing sets by following the training-validation-testing split method with a ratio of 80:10:10 [86]. The training-validation process uses the Mini-Batch RMS-propagation algorithm [87] to achieve the best training stability and generalization performance with normalized inputs and output. The batch size is defined as 32 [88], along with epochs of 200. A batch size of 32 is commonly recommended in literature because it is practically efficient in computing the matrix-matrix products over matrix-vector products [88, 89]. The RMS-propagation algorithm run with default values of learning rate = 0.001, gradient moving average decay factor (rho) = 0.9 [88]. The training and validation performance is evaluated using the mean square error loss function [88]. An early stopping criterion is also considered during the training process for stopping the ML model overfitting. The model will stop for training if a bad degraded performance is observed (mean sqaure error goes up) during the validation process [88]. Figure 2.11 shows the results of training, validation, and testing capabilities of the proposed MLP model. Figure 2.11 (a) and (b) shows the mean squared error vs. epochs plot for training and validating the MLP model, with a continuous and rapid decrease in the mean square error (MSE) close to zero for the predicted residual velocity and residual mass, respectively. Both of these plots indicate a well trained and validated MLP model has been achieved [42]. Figure 2.11 (c) and (d) are predicted values vs. actual values plots showing the comparable accuracy of the MLP model to SPH simulation in predicting the residual velocity and mass of projectile based on testing-split data (experimental and numerical data). In the figures, the center diagonal line indicates a perfect match between the predicted and true values, and the more points close to the diagonal line indicates a better prediction. From the figure, it is observed that the proposed MLP model can predict both residual mass and residual velocity with less than 7% of absolute percentage error compared to the SPH model (purple dots), with simulation results of experimental data shown in green (see Figure 2.11 (c) and (d)). The computational time for MLP prediction is only 800 ms, and this is compared to approximately of two hours for the SPH simulations. The observed outliers in the plot (e.g., two points in Figure 2.11 (c) and three points in Figure 2.11 (d)) suggests further detailed error analysis on data-noise and model architecture needs to be performed to improve model prediction. Altogether, the MLP model in the current work showcases a clear path that can be used to develop such efficient machine learning models for generating accurate predictions for a specific loading case encompassing wide ranges of conditions.

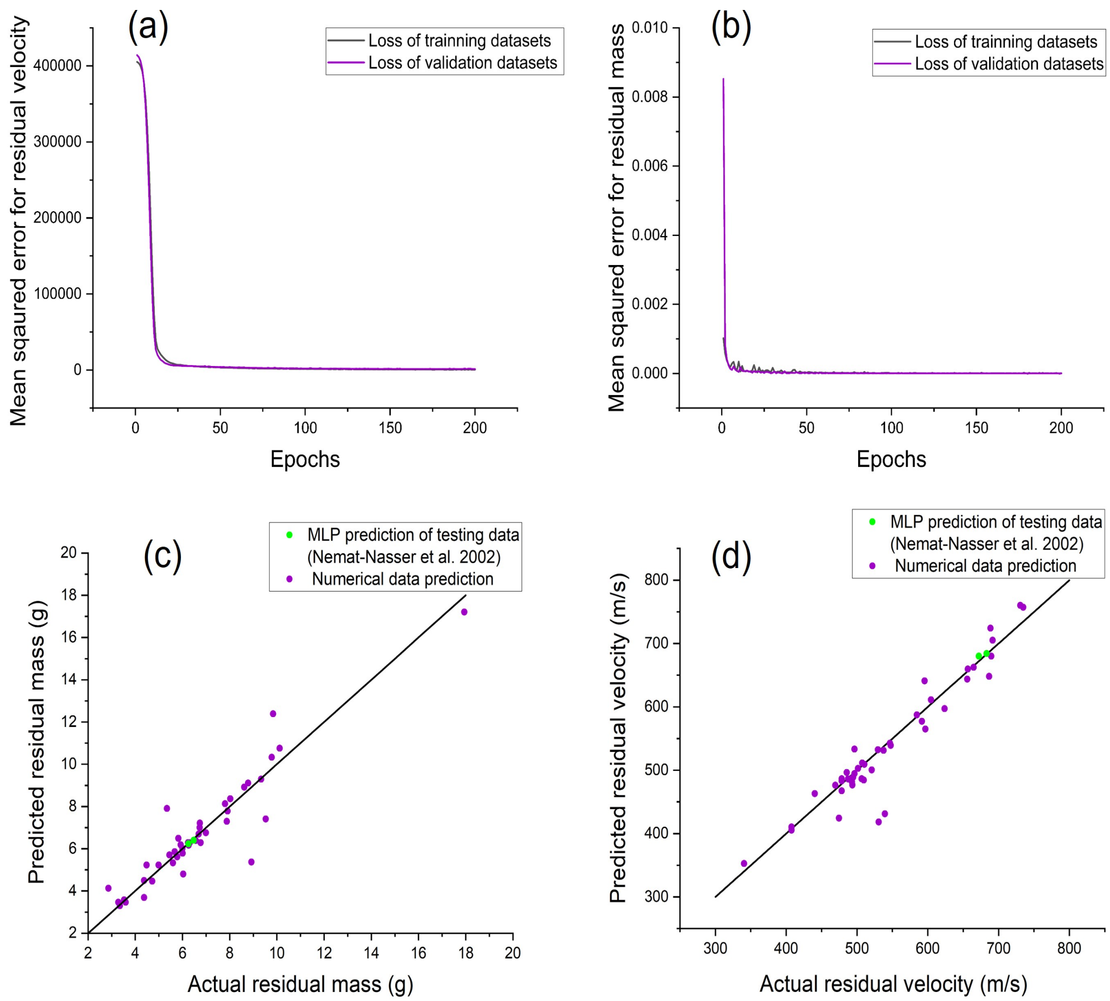


Figure 2.11: Training, validation, and testing of the multi-layer perceptron model. (a) The mean squared error plot shows the well-trained and validated multi-layer perceptron model for the residual velocity. The model is trained and validated through train-validation-split datasets with a ratio of 80:10:10. (b) The mean square error plot shows the well-trained and validated multi-layer perceptron model for residual mass. (c) A prediction vs. actual plot for the residual velocity shows the capability of the model in predicting the residual velocity of projectile based on the testing-split dataset. (d) A prediction vs. actual plot for the residual mass shows the capability of the model in predicting the residual mass of projectile based on the testing-split dataset (the diagonal line represents an excellent match).

2.4.3 Sensitivity analysis using multi-layer perceptron

Next, parametric studies are carried out to investigate the effects of mechanical properties (e.g., strength and shear modulus) and geometries (e.g., tile thickness) of the alumina ceramic tile on its ballistic performance by using the well-trained MLP model. Typical parameters, including impact velocity of the projectile, thickness, tensile strength [90], maximum intact strength [59, 60], and shear modulus [90] of the alumina ceramic tile are selected to be analysed in this work because of their noted influences on ballistic performance [59, 71].

Shown in Figure 2.12 are the results of the parametric studies on residual velocity (a) and residual mass (b) of projectile by varying the corresponding parameters over the assigned range (see Table A.1). Note that the inputs are normalized on the x-axis based on the average of values for the ease of comparison. Data points are plotted as averages with standard deviations from five simulations using the MLP model for given parameters of interest. Trends in Figure 2.12 (a) demonstrates that the residual velocity decreases with the increase of thickness, maximum intact strength, and tensile strength of the ceramic tile, with a higher sensitivity for the intact maximum strength and thickness parameters. These trends and sensitivities are consistent with the experimental results from literature (i.e, tensile strength [70, 77], intact maximum strength [71], and thickness [90]). For example, the intact maximum strength (i.e., compressive strength) often plays an important role in the ballistic performance of ceramics because of the compressive and shock waves generated failure during impact [4]. Similarly, Figure 2.12 (b) shows the sensitivity of residual mass to thickness, impact velocity, maximum intact strength, shear modulus, and tensile strength. Residual mass is a metric of projectile defeat through erosion [91]. From the plot, a clear trend of decreasing residual mass is seen for an increasing impact velocity and thickness, which is consistent with the literature (i.e. impact velocity [68] and thickness [68]. Note that a similar trend for the tensile strength is observed for the residual velocity (see Figure 2.12 (a)) and residual mass (see Figure 2.12 (a)), and further investigations are needed to identify the roles of possible physical phenomena (e.g., the commonly denoted fracturing from the reflected tensile waves at the free surface of the ceramic tile [84]) and numerical effects. Regardless, the residual mass appears to be less sensitive to tensile strength with no previous literature available to confirm this observation. Finally, the shear modulus exhibits non-linear decreasing-increasing trends for both the residual velocity (Figure 2.12 (a)) and mass (Figure 2.12 (b)), which is either: (1) associated with the interplay between impact physics (e.g., dwell [32, 33], wave propagation [4], and damage accumulation [52, 91, 92]); (2) the constitutive model construction (i.e., see Equation (A.26) in the JHB material model). Further experimental data and explicit failure modeling of impact phenomena [93–95] are needed to unravel individual effects of these input parameters.

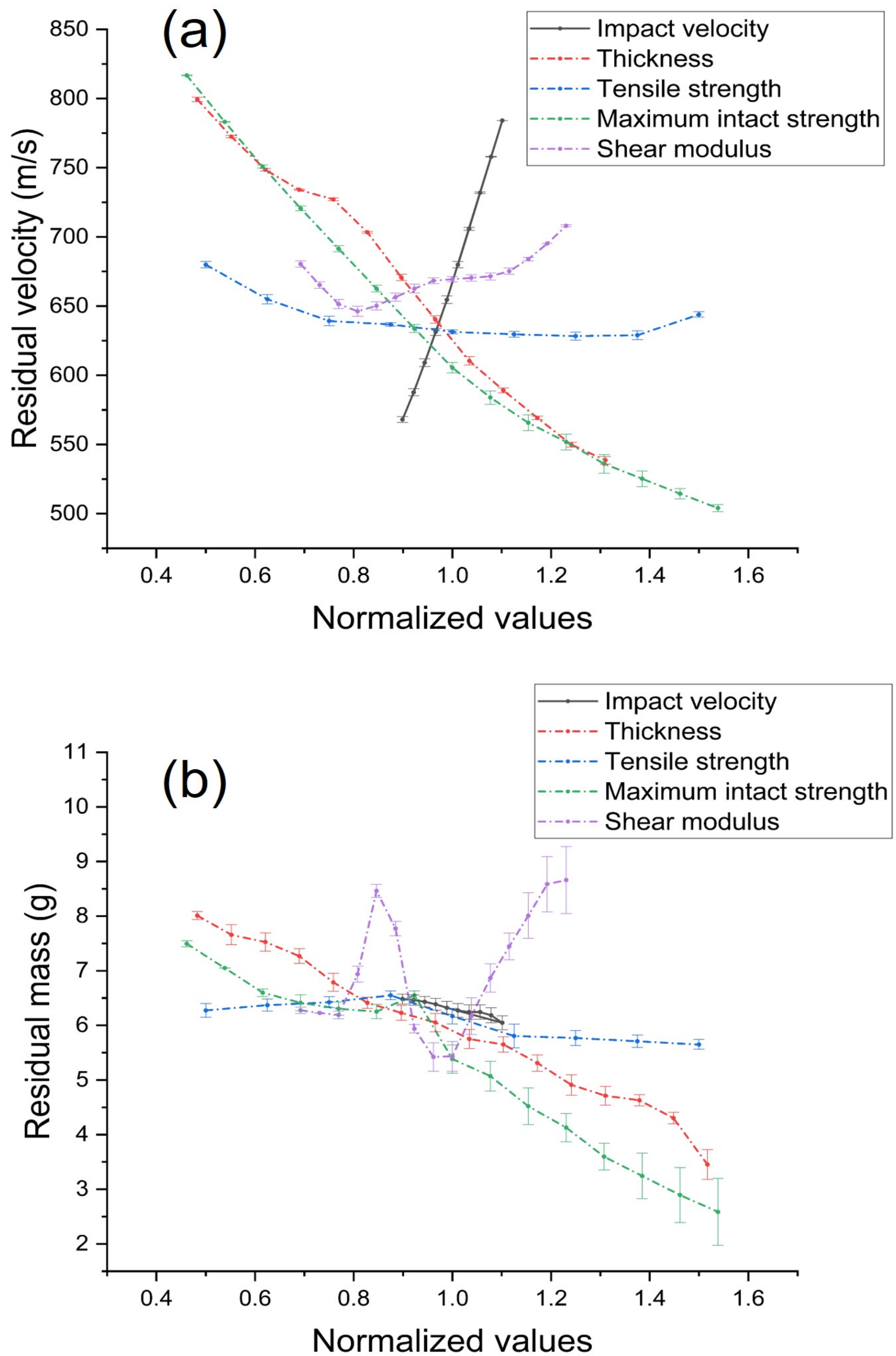


Figure 2.12: Sensitivity analysis for studying the effects of mechanical properties (tensile strength, maximum intact strength, and shear modulus) of the alumina ceramic tile and geometries (thickness and impact velocity) on the: (a) residual velocity and (b) mass of the projectile by using the multilayer perceptron model.

2.5 Conclusions

This study has developed a combined computational framework based on the smoothed particle hydrodynamics (SPH) and machine learning algorithms for investigating impact responses of an alumina ceramic, in which the Johnson-Holmquist-Beissel (JHB) material model was implemented into the LS-DYNA through a user-defined subroutine to account for high pressure, high strain rate, and damaged behavior during plate impact and ballistic loading. The implementation of the JHB model was verified by comparing the equivalent stress-pressure plots through a single element simulation test. The JHB model material constants were inferred and calibrated based on data from the literature. The developed computational framework demonstrated a good agreement between the numerical and experimental results, both quantitatively (e.g., particle velocity signal, residual velocity, and residual mass of the projectile) and qualitatively (e.g., debris, spall, cone, mushrooming deformation, and erosion of the projectile). A comprehensive sensitivity analysis on the SPH numerical settings was then conducted and revealed that the change of particle spacing could result in 12% and 17.2% change in residual velocity and mass of projectile, respectively, but with a six times increase in computational time. Lastly, the developed neural network model as an alternative to SPH model demonstrated that an accelerated prediction and optimization of the performance characteristic for ballistic impact case with reasonable accuracy. Overall, the proposed combined SPH-MLP approach and the associated analysis provide an alternative path for high throughput identification and insights into the property and performance relationships, which is applicable to structural-scale design of ceramic-based protection systems.

Chapter 3

Conclusion and future works

3.1 Implications

This thesis explored the relationships between material properties, geometries, and dynamic impact performance of alumina through a data-driven approach.

The main contributions of this thesis are summarized below:

1. This thesis simulates an alumina ceramic tile undergoing the ballistic and plate impact loading using the user-defined JHB model in LS-DYNA software. The JHB model enables better characterization of catastrophic failure of ceramics by constituting a piece-wise strength-pressure and damage-pressure envelope compared to other JH models, leading to a more realistic representation of the response of ceramics subject to impact loading. As far as we are aware, this is the first time in the literature that the JHB has been implemented via user subroutine in LS-DYDA to simulate ceramic material behavior.

2. This thesis presents a novel SPH-based computational framework that enables improved physics-based modeling and predicting the outcomes of the plate and ballistic impact. The developed computational framework can accurately simulate the response of ceramic (e.g., residual velocity and mass of projectile) and capture the failure mechanisms of ceramics (e.g., target spallation, pulverization, and ejected debris) during the impact penetration process, enabling a

better understanding of their role in contributing to impact performance.

3. The sensitivity and computational cost analysis on the SPH numerical settings were conducted. The evaluation of these parameters is important given their noted sensitivities in the literature to simulations of various problems [18, 36, 39]. The outcomes of sensitivity and computational cost analysis can be used to guide other researchers who study the impact problems in the future.

4. This thesis has developed a hybrid numerical approach by coupling SPH with a machine-learning approach. Such a hybrid approach enables efficient high-throughput identification and quantification of property and performance relationships using a machine learning model. Such a computationally efficient framework can be applied to structural scale design and optimization of armor systems.

3.2 Future work and recommendations

To build upon the current work of this thesis, possible research directions in future work may include:

- *Alternative numerical method for simulating crack initiation and propagation of ceramic* - The current SPH framework implemented in this thesis can robustly simulate the response of ceramics (e.g., spallation and debris) under impact loading. However, some inherent drawbacks of the SPH method may result in an unrealistic simulated response of ceramic as the SPH applies the kernel functions to control particle separations (e.g., crack initiation and propagation) without linking it with material properties. To improve the model, the finite discrete element method (F-DEM) can serve as an alternative numerical method to SPH for simulating the response of ceramic material (e.g., fragmentation) to better account for crack initiation and propagation under dynamic loading [78, 96].

- *Modifying the rate-dependent functional forms of the JHB model* - The current JHB model considers strain rate dependency on the material strength, and the material strength is updated through a defined linear formulation by fitting the experimental data in the literature [52, 97]. A new functional formulation (e.g., mixed linear and power law formulations) can be proposed while a wider range of strain rate data is available under quasi-static and dynamic loading conditions. On the other hand, recent data from our group demonstrates the tensile strength of alumina ceramic is strain rate sensitive, where the tensile strength is currently defined as constant in the JHB model. Therefore, a modification on the hydrostatic pressure tensile strength constant needs to be made to account for the strain rate effect, and this could be achieved through a user-defined subroutine.

- *Considering the stress-state dependency of JHB model* - Developing a better understanding of the stress-state dependency behavior of ceramic material strength and damage accumulation is important to computationally design better-performing ceramic structures. In the current JHB model, the plastic behavior of the material is described based on the von Mises criteria, where the material strength and damage evolution are defined in terms of the equivalent stress and plastic strain, respectively. Thus, there are no constitute laws in the JHB framework that enable distinguishing plastic behavior of the material under different stress states (e.g., compressive, tensile, and shear). The Lode angle parameter as a function of the third deviatoric stress invariant- J3 proposed by Polanco-Loria et al. [98] and Liu et al.[1] can be introduced into the strength and damage constitutive law of the JHB model to incorporate the effect of shear behavior between the tensile and compressive meridians. With that we may decompose the strength and damage constitutive law as:

$$\sigma_{eq,i} = \begin{cases} f\left\{D_t, r'(\theta,\psi)\right\}, P < 0 \\ f\left\{D_t D_C, r'(\theta,\psi)\right\},\}, 0 < P < P_1 \\ f\left\{D_t D_C, r'(\theta,\psi)\right\},\}, P_1 < P \end{cases}$$
$$D = f\left\{D_C, D_t\right) \tag{3.1}$$

where $\sigma_{eq,i}$ is equivalent stress, D_t is damage accumulation under tensile loading, D_C is damage accumulation under compressive loading, P is hydrostatic pressure, P_1 is hydrostatic pressure constant, r' is denoted to describe the third invariant effect on the yielding and damage surface of the JHB model, θ is lode angle, and ψ is denoted to describe the ratio between tensile and compressive meridians.

- *Stress-state constitutive laws of JHB model informed by coupling experimental and numerical techniques*- Recent efforts in the literature studying the stress-dependent behavior (e.g., strength and damage) of ceramics under dynamic loading mainly rely on the controlled experiments under simplified loading conditions (e.g., uni-axial compression [99, 100], tensile, and shear-compression experiments) coupled with imaging techniques. However, experimental techniques are costly, and difficult to directly measure the damage of ceramics under different stress-states. A numerical framework validated against experimental data can serve as an alternative to experimental techniques for direct quantitative measurement of damage inside the material. Such a validated framework also enables damage parameters (e.g., cracks volume) to be explored with improved temporal and spatial resolutions. A rate-dependent finite element discrete model (F-DEM) is currently being developed and validated against experimental data by the authors for compressive, indirect tension, and compression-shear stress states. The developed framework will be then used to provide insights into the stress-state-dependent crack damage evolution behavior of ceramics. The generated data and insights will be served as important inputs to develop strain rate and stress-state-dependent constitutive laws for the damage evolution of ceramics.

- *Granular physics in ballistic impact*- Shockey et al.'s [101] work demonstrates that granular flow of material generates significant penetration resistance and often results in penetrator rebound during ballistic impact (resulting from frictional flow characteristics of fine fragments and fragment abrasiveness). The penetration resistance could depend upon the fragment morphology and friction between the target and projectile [101]. However, simulating fragmentation in terms of crack branching and coalescence is challenging in computational mechanics at the structural scale [1]. Therefore, more effort must be devoted to developing a computational framework that can adequately simulate the fragmentation process of the ceramic during ballistic impact. A rate-dependent F-DEM framework is currently being developed by authors at the mesoscale and validated against compressive, indirect tension, and compression-shear experimental data. In the F-DEM framework, Cohesive Zone Modelling (CZM) combined with finite discrete element (F-DEM) algorithm showed a capability of accounting for both physical continuities of material properties and discontinuity of cracks [78] can successfully simulate the fragmentation of ceramic sample under dynamic loading. The successful implementation of such a framework at the mesoscale will be further linked to the structural scale to simulate ceramic fragmentation during the ballistic impact process.

- *Informing constitutive law of material through deep learning neural networks* - A deep learning neural network model can be used to learn the nonlinear constitutive laws (e.g., damage evolution law) for brittle materials. The training data can be generated based on experimental data, material properties, and the developed numerical model to incorporate more physics. Such a neural network-based framework will provide a convenient and general methodology for constitutive modeling.

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Appendix A: Appendices for Chapter 2

A.1 Numerical methods

In this work, a finite element model has been implemented in LS-DYNA software within the framework of smoothed particle hydrodynamics (SPH) by adopting the Johnson-Holmquist-Beissel (JHB) constitutive model for ceramics, the Johnson-Cook (JC) material model, and Gruneisen equation of state for a tungsten long rod projectile. LS-DYNA software has been chosen in the present study as it is better suited for large-scale dynamic simulations with consideration of inertia effects for dynamic impact problems [102]. Even though LS-DYNA has a large variety of material models (e.g., explicit dynamic models) and contact algorithms (e.g., eroding contacts and tied surfaces contacts) [102], the JHB model is currently not available in the LS-DYNA software and has to be written through a user-defined sub-routine. Hence, the proper implementation and verification of the subroutine is essential before carrying out any impact simulations. A flow chart used for defining the implemented computer code for the present study is included (see Figure A.1). Lastly, this section presents the structure of an artificial neural network tool based on multi-layer perceptron (MLP) method for predicting and investigating the effect of mechanical properties and geometries on material performance.

Smoothed particle hydrodynamics method

The SPH method utilizes a set of particles to represent a system [23]. In solving the partial differential equations of solid mechanics, the SPH method approximates the partial differential equations by discretizing a physical domain into the particles. In particular, a function at a particle is approximated with the average of the function's values at all neighboring particles in the support domain weighted by the kernel functions [21]. The SPH method has been widely adopted in the literature [17, 36, 66] to solve problems that are large-scale, highly-deformed, and dynamic in nature, given it is a mesh-free approach and described in a Lagrangian frame of reference [23, 24]. To obtain the particle approximation of a continuous field function $u(\mathrm{x})$, the kernel approximation of the field function is firstly defined as [21, 23]:

$$\prod u(x)^h = \int_{\Omega} u(x')W(x - x', h)dx' \tag{A.1}$$

where x represents a particle, W is the smoothing function, h is the smoothing length, and Ω is the support domain of the smoothing function.

The most widely used smoothing function is the cubic B-spline function, which is defined as [21, 23]:

$$W(x - x', h) = \frac{C}{h^d} \begin{cases} 1 - \frac{3}{2} * \zeta^2 + \frac{3}{4} * \zeta^3 & x \leq 0 \\ \frac{1}{4} * (2 - \zeta)^3 & 0 \leq x \leq 100 \\ 0 & 100 \leq x \end{cases} \tag{A.2}$$

where d is the number of space dimensions, $\zeta = \frac{r}{h}$, and $r = x - x'$ is the distance between particles x and x'. C is a constant that depends on the space dimension (n = 1, 2, or 3), which is defined as [21, 23]:

$$C = \begin{cases} \frac{2}{3}, & n = 1 \\ \frac{10}{7\pi}, & n = 2 \\ \frac{1}{10\pi}, & n = 3 \end{cases} \tag{A.3}$$

For the particle approximation of a specific particle x_i, the field function is obtained as [21, 23]:

$$\prod u(x_i)^h = \sum_{n=1}^{N} \frac{m_j}{\rho_j} u(x' W(x - x', h) \tag{A.4}$$

where N is the number of neighboring particles, m is the mass, and ρ is the density.

Through a similar procedure, the particle approximation of the partial derivative of the field function $u(x)$ is [21, 23]:

$$\prod \frac{\partial u x_i}{\partial x^\alpha} = \sum_{n=1}^{N} \frac{m_j}{\rho_j} u(x' (A_{ij})^\alpha \tag{A.5}$$

where α is the space index, ${A_{ij}}^\alpha$ is the α^{th} component of the vector A_{ij}, that is given by [21, 23]:

$$A_{ij} = A(x_i, x_j) = \frac{x_i - x_j}{r_{ij}} \frac{\partial w(x_i - x_j, h)}{\partial r_{ij}} \tag{A.6}$$

The commonly used SPH equations for the conversion of mass, momentum, and energy are expressed as [21, 23]:

$$\begin{aligned}
\frac{d\rho(x_{i)}}{dt} &= \sum_{j=1}^{N} m_j \; ({u_i}^\alpha - {u_i}^\alpha) {A_{ij}}^\alpha \\
\frac{dU^\alpha(x_i)}{dt} &= \sum_{j=1}^{N} m_j [\frac{\sigma^{\alpha\beta}\,(x_i)}{{\rho_i}^2} {A_{ij}}^\beta - \frac{\sigma^{\alpha\beta}\,(x_j)}{{\rho_j}^2} {A_{ij}}^\beta] \\
\frac{dE(x_i)}{dt} &= \sum_{j=1}^{N} m_j U_{ij} [\frac{\sigma^{\alpha\beta}\,(x_i)}{{\rho_i}^2} {A_{ij}}^\beta - \frac{\sigma^{\alpha\beta}\,(x_j)}{{\rho_j}^2} {A_{ij}}^\beta]
\end{aligned} \tag{A.7}$$

To reduce the instability of the SPH-based model, an artificial viscosity term is introduced into the energy and momentum equations, and this results in:

$$\begin{aligned}
\frac{dU^\alpha(x_i)}{dt} &= \sum_{j=1}^{N} m_j [\frac{\sigma^{\alpha\beta}\,(x_i)}{{\rho_i}^2} {A_{ij}}^\beta - \frac{\sigma^{\alpha\beta}\,(x_j)}{{\rho_j}^2} {A_{ij}}^\beta - \Pi_{ij} I] \\
\frac{dE(x_i)}{dt} &= \sum_{j=1}^{N} m_j U_{ij} [\frac{\sigma^{\alpha\beta}\,(x_i)}{{\rho_i}^2} {A_{ij}}^\beta - \frac{\sigma^{\alpha\beta}\,(x_j)}{{\rho_j}^2} {A_{ij}}^\beta - \Pi_{ij}]
\end{aligned} \tag{A.8}$$

In this research, the artificial viscosity Π_{ij} term introduced by Monaghan [103] is used as:

$$\Pi_{ij} = \begin{cases} \frac{-\alpha^*_{cj} \lambda_{ij} + {\beta^*_{ij}}^2}{\rho_{ij}} & \boldsymbol{u}_{ij} \cdot x_{ij} < 0 \\ 0 & \boldsymbol{u}_{ij} \cdot x_{ij} \geq 0 \end{cases} \tag{A.9}$$

$$\lambda_{ij} = \frac{h_{ij}\boldsymbol{u}_{ij} \cdot \boldsymbol{x}_{ij}}{|\boldsymbol{x}_{ij}|^2 + 0.01h^2} \tag{A.10}$$

$$\bar{c}_{ij} = \frac{1}{2}(c_i + c_j) \tag{A.11}$$

$$\bar{\rho}_{ij} = \frac{1}{2}(\rho_i + \rho_j) \tag{A.12}$$

$$h_{ij} = \frac{1}{2}(h_i + h_j) \tag{A.13}$$

$$\boldsymbol{u}_{ij} = (\boldsymbol{u}_i - \boldsymbol{u}_j) \tag{A.14}$$

$$\boldsymbol{x}_{ij} = (\boldsymbol{x}_i - \boldsymbol{x}_j) \tag{A.15}$$

where the parameters α^* and β^* are artificial viscosity constants and they vary with applications [36]. The quadratic viscosity term α^* is only dominant in high gradient regions such as shock fronts, while the linear viscosity term β^* dominates in low gradient regions [36].

Johnson-Holmquist-Beissel model for alumina ceramic

The Johnson-Holmquist-Beissel model (JHB) [30] has been selected to describe the mechanical responses of the alumina ceramic in this work. The JHB model consists of three main components: (1) a strength material model, (2) a damage model, and (3) an equation of state model that considers the phase change of materials [18, 30]. When compared to the previous JH models [34, 35], the differences of the JHB model are [30]: (1) a piece-wise strength-pressure and damage-pressure envelope is included, (2) the pressure and strain-rate independent response beyond the Hugoniot elastic limit (HEL) is included, and (3) phase change effects are considered. Experimental data for alumina ceramics reported in the literature [84, 104–109] indicated that the material strength softening formulation in the JHB model is more representative of experiments when compared to the JH1 and JH2 models by demonstrating a linear response at low pressures, nonlinear response at higher pressures up to HEL, and pressure and strain-rate independent response when the pressure level drove the material over the its HEL (see Figure 2.1(b)). All these improvements over the previous JH models are important when modelling shock and impact conditions, as we do in this study. A brief summary of the JHB model is presented below, and a detailed explanation of parameterization is described later in Section 2.3. The associated parameters and units of the JHB model are shown in Table 2.1, and this will be revisited later.

*JHB strength model

In the JHB model, the material strength is dependent on hydrostatic pressure, equivalent strain rate, and the accumulation of damage in the material. In the JHB strength model, the strength of the material is represented by a linear curve up to a pressure value of P_1 (intact strength constant), where the corresponding strength is σ_1. Prior to P_1, the equivalent material strength is $\sigma = 0$ at $P = 0$. After the P_1, the intact strength of the material is expressed in von Mises equivalent stress as [30]:

$$\sigma_{eq,i} = \sigma_1 + (\sigma_{max} - \sigma_1)\left[1 - e^{-\alpha_1(p-p_1)}\right] \tag{A.16}$$

$$\alpha_1 = \frac{\sigma_1}{(\sigma_{max} - \sigma_1)(P_1 + T)} \tag{A.17}$$

where σ_{max} is the maximum intact strength of the material (i.e., compressive) and T is the hydrostatic tensile strength. Similarly, the strength of the failed material (when D=1) is expressed as [30]:

$$\sigma_{eq,f} = \sigma_2 + \left(\sigma^{maxf} - \sigma_2\right)\left[1 - e^{-\alpha_2(p-p_2)}\right] \tag{A.18}$$

$$\alpha_2 = \frac{\sigma_1}{P_2(\sigma^{maxf} - \sigma_2)} \tag{A.19}$$

Considering the strain rate effect on strength, the strength of the material is expressed as [30]:

$$\sigma_{eq} = \sigma_{eq,0}[1 + \widetilde{c}ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)] \tag{A.20}$$

where $\widetilde{c}$ is the strain rate effect coefficient, $\dot{\varepsilon}$ is the equivalent strain rate, and $\dot{\varepsilon}_0 = 1$ s^{-1} is the reference strain rate.

* JHB damage model

The accumulation of damage in the material is represented by a form that is similar to the Johnson-Cook fracture model [30]:

$$D = \sum \Delta\varepsilon^p / \varepsilon_f^p \tag{A.21}$$

where $\Delta\varepsilon^p$ is the incremental equivalent plastic strain during a computational cycle, and $\varepsilon_f^p = f(p)$ is the plastic strain at fracture and is defined as [30]:

$$\varepsilon_f^p = D_1 \left(P^* + T^*\right)^n \tag{A.22}$$

where both D_1 and n are material constants. P^* is the dimensionless pressure and $P^* = P/\sigma_{max}$. T^* is the dimensionless hydrostatic pressure and $T^* = T/\sigma_{max}$. The ε_f^p increases as P^* increases, and the material does not undergo any plastic strain at $P^* = -T^*$.

*JHB pressure-volume relationship with bulking

It is worth noting that the phase change effect of alumina ceramic is not considered in this work given the lack of proof in the literature of such phenomenon in alumina [48] (see Section 2.3 for details). Without considering the phase change effect, the hydrostatic pressure begins to accumulate before failure and is defined as [30]:

$$P = k_1\mu + k_2\mu^2 + k_3\mu^3 \tag{A.23}$$

$$\mu = \frac{\rho}{\rho_0} - 1 \tag{A.24}$$

where k_1 (bulk modulus), k_2, and k_3 are material constants. ρ and ρ_0 is the current density and reference density, respectively. For tensile stress states ($\mu < 0$), Equation (A.23) is replaced by $P = k_1\mu$.

The model [30] also considers the bulking effect when the material fails ($D = 1$) [30]. The effect of bulking results in an increase in the pressure and volume [110]. The bulking effect is represented by having an incremental pressure ΔP adding to Equation (A.23) [30]:

$$P = k_1\mu + k_2\mu^2 + k_3\mu^3 + \Delta P \tag{A.25}$$

The bulking-induced pressure increment is determined from energy considerations. When the material fails, the material strength decreases, and this corresponds to a decrease in the deviatoric stresses, further resulting in a decrease in the incremental internal elastic energy. The loss of incremental internal elastic energy is converted to potential hydrostatic energy by incrementally increasing ΔP. The general expression for the elastic internal energy is [30]:

$$U = \frac{\sigma^2}{6G} \tag{A.26}$$

where G is the shear modulus of elasticity.

The incremental energy loss is computed as [30]:

$$\Delta U = U_i - U_f \tag{A.27}$$

where U_i is the internal energy of the intact material before failure and U_f is the internal energy of the material when it is failed. The conversion between the pressure and elastic internal energy is [30]:

$$\Delta P \mu_f + \Delta P^2 / (2k_1) = \beta \Delta U \tag{A.28}$$

where μ_f is the value of μ when the material is failed, and β is the fraction ($0 \le \beta \le 1$) of the internal (deviator) energy loss converted to potential hydrostatic energy. The first term ($\Delta P \mu_f$) is the approximate potential energy for $\mu > 0$, and the second term ($\Delta P^2 / (2k_1)$) is the corresponding potential energy for $\mu < 0$. The ΔP is given by [30]:

$$\Delta P = -k_1 \mu_f + \sqrt{(k_1 \mu_f)^2 + 2\beta k_1 \Delta U} \tag{A.29}$$

The bulking pressure is computed only for failure under compression ($\mu_f > 0$). Note that $\Delta P = 0$ for $\beta = 0$ and that ΔP increases as ΔU increases and/or μ_f decreases.

Johnson-Cook material model for tungsten alloy projectile

In this study, the Johnson-Cook (JC) plasticity model is selected to define the material behavior of the tungsten alloy projectile [31] (see Figure 2.3). The JC model reasonably captures the material response when subjected to high strain rate loading [31, 50]. In addition, the JC model is commonly used in ballistic impact simulations due to its uncoupled approach in calibrating material parameters [17, 111]. In this section, a brief summary of the model is provided. *Johnson-Cook strength model The flow stress-equivalent plastic strain relation of the JC model is given as [17]:

$$\sigma_y = (A + B(\varepsilon^p)^n)(1 + C \ln \dot{\varepsilon}_p{}^*)(1 - \ {T_{JC}}^m) \tag{A.30}$$

where σ_y is the yield stress, ε^p is the equivalent plastic strain, $\dot{\varepsilon}_p$ is the equivalent plastic strain rate, and A, B, and C are the material constants. The $\dot{\varepsilon}_p^*$ and T_{JC} are obtained from:

$$\dot{\varepsilon}_p{}^* = \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \tag{A.31}$$

$$T_{JC} = \frac{T - T_0}{T_{melt} - T_0} \tag{A.32}$$

where $\dot{\varepsilon}_0$ is the reference strain rate, T is the current temperature, T_{melt} is the melting temperature, and T_0 is the reference temperature.

*Johnson-Cook damage model

Similar to the JH models, the JC fracture model is used to describe the fracture behavior of the projectile material. The fracture criterion in the JC model is based on the accumulation of effective plastic strain [17]:

$$D^* = \sum (\Delta\varepsilon^{p*} / \varepsilon_f^{p*}) \tag{A.33}$$

where $\Delta\varepsilon^{p*}$ is the incremental equivalent plastic strain during a computational cycle, and $\varepsilon_f^{p*} = f(p)$ is the effective plastic strain at fracture, which is expressed as [17]

$$\varepsilon_f^{p*} = \left[D_1^* + D_2^* e^{D_3^* \frac{\sigma_m}{\bar{\sigma}}} \right] \left[1 + D_4^* \ln \dot{\varepsilon}^* \right] \left[1 + D_5^* T^* \right] \tag{A.34}$$

where σ_m is the mean stress, $\bar{\sigma}$ is the equivalent von Mises stress, and D_1^* to D_5^* are the material damage constants.

Multi-layer perceptron neural networks

The multilayer perceptron (MLP) approach is a useful tool in solving non-linear classification and regression problems [112], and it has been employed in this work to develop statistical models for impact performance optimization. The MLPs in this study are trained using forward and backward propagation algorithms. In the forward algorithm, a linear activation function is used to map the weighted inputs to the output of each neuron, and each layer in the MLP is described mathematically as [113, 114]:

$$a_i^{(l)} = f\left(\sum_{j=1}^{n} a_i^{(l)} \theta_{ij}^{(l)} + \theta_{0,j}^{(l)}\right), 1 \leq l \leq L \tag{A.35}$$

where the $a_i^{(l)}$ is the activation of the $\imath^{th}$ neuron in the l^{th} layer, $\theta_{ij}^{(l)}$ represents the weight that is used to send the input to the i^{th} neuron, from the j^{th} neuron in layer l, and $\theta_{0,j}^{(l)}$ represents the bias in l^{th} layer.

The Relu function is chosen for nonlinear activation for this work, which is described as [113, 114]:

$$f(x) = \max(0, x) \tag{A.36}$$

For the back-propagation training, all the weights and thresholds are updated using the root mean squared propagation (RMS-prop) algorithm [113, 114]:

$$\theta_i := \theta_i + \Delta\theta_i \tag{A.37}$$

$$\Delta\theta_i = -\eta \frac{\partial J(\theta)}{\partial \theta_i} \tag{A.38}$$

where η is the learning rate factor, and $\frac{\partial J(\theta)}{\partial \theta_i}$ is the partial derivatives of the cost function with respect to weights. The partial derivatives of the cost function with respect to all of the parameters that feed into the current layer and the output layer $\delta(L-1)$ are computed as [113, 114]:

$$\frac{\partial J(\theta)}{\partial \theta_{ij}^{(l)}} = \left(\delta^{(l+1)}\right)^T a^{(l)} \tag{A.39}$$

$$\frac{\partial J(\theta)}{\partial \theta_{ij}^{(L-1)}} = \left(\delta^{(L)}\right)^T a^{(L-1)} \tag{A.40}$$

The error term δ for the output layer and the hidden layers are computed as [113, 114]:

$$\delta^{(L)} = \frac{1}{m}\left(y - a^{(L)}\right) f'\left(a^{(L)}\right) \tag{A.41}$$

$$\delta_j^{(l)} = f'\left(a^{(l)}\right) \sum_{i=1}^{n} \delta_i^{(l+1)} \theta_{ij}^{(l)} \tag{A.42}$$

Finally, the overall performance of the MLP is measured by the mean squared error (MSE) which is expressed by [113, 114]:

$$J(\theta) = \frac{1}{2m^*} \sum_{i=1}^{m^*} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 \tag{A.43}$$

where m^* is the number of training samples, $h_\theta(x^{(i)})$ is the vector of predicted values based on the training samples, and $y^{(i)}$ labels the vector of actual values.

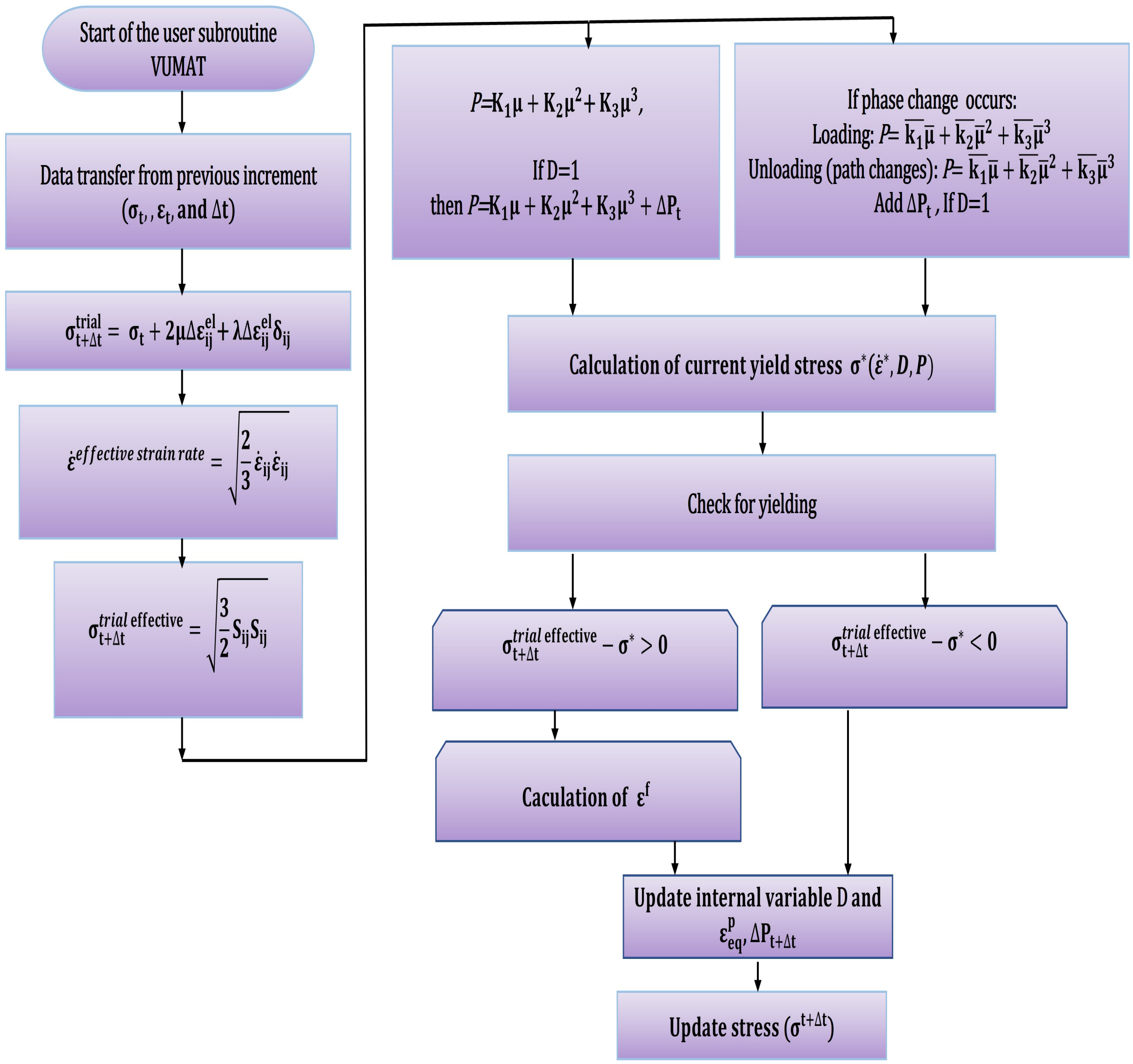

Figure A.1: The flowchart for writing the subroutine code of the JHB material model in LS-DYNA.

Table A.1: Johnson-Cook material constants of the tungsten alloy long rod projectile that is taken from Bresciani et al. [31].

Model Parameters	Notation	Value
Density (kg/m^3)	ρ_p	17600
Shear modulus (GPa)	G_p	152
Elastic modulus (GPa)	E_p	314
Quasi-static tensile yield strength (GPa)	A	1.6
Hardening exponent (GPa)	B_p	0.1765
Thermal softening exponent	C_p	0.016
Strain rate sensitivity coefficient	N	0.12
Temperature exponent	M	1
Melting temperature (K)	T_{melt}	3695
Room temperature (K)	T_0	291
Heat capacity (J/Kg*K)	c_p	384
Damage Parameters		
Damage coefficient 1	D_1^*	0
Damage coefficient 2	D_2^*	1
Damage coefficient 3	D_3^*	-1.5
Damage coefficient 4	D_4^*	0.042
Damage coefficient 5	D_5^*	0
Equation of State Parameters		
Gruneisen coefficient	G_2	1.67
Linear Hugoniot slope coefficient	S_1	1.237
Bulk speed of sound (m/s)	C_1	4030

A.2 Ballistic impact simulation results

Figure A.2 shows the damage evolution images for both the alumina ceramic tile and the projectile under a ballistic impact velocity of 901 m/s. In Figure A.2, the conical damage starts to form on the front surface of the ceramic as it is impacted by projectile at 8 µs. Next, the fractured cone starts to develop in the ceramic from the contact surface and propagates towards the back surface between 8 µs and 30 µs, and the completed perforation and erosion of projectile is observed at 72 µs.

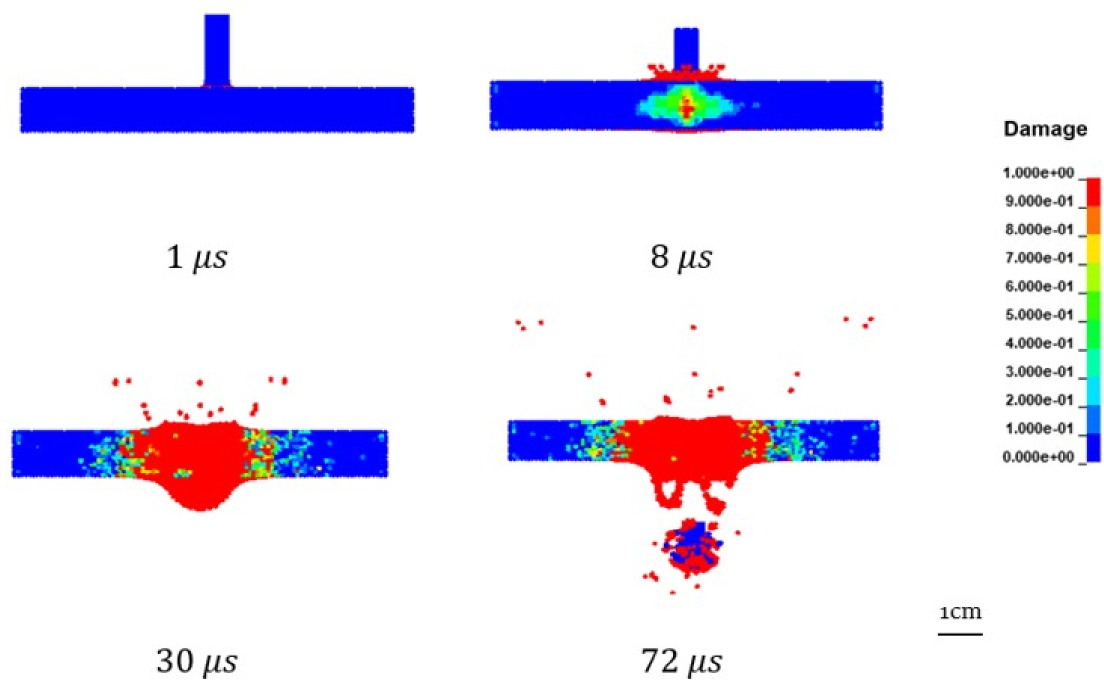

Figure A.2: Damage evolution in alumina ceramic tile undergoing penetration of tungsten long rod projectile at time of 1 µs, 8 µs, 30 µs, and 72 µs under ballistic impact.

A.3 Single element verification

The JHB material model has been implemented into the finite element code of LS-DYNA through a user-defined subroutine. To demonstrate the accuracy of the code, a single-element model has been developed and utilized to verify the implementation of the JHB material model in LS-DYNA following the approach outlined by Johnson et al. [30], where the loading history of the element interrogates high deviatoric and hydro-static pressures in this verification. The material constants used in the single element simulations are based on the material constants provided in the original paper by Johnson et al. [30] for aluminum nitride. The material constants for the alumina studied here are validated in Section 2.3 for plate and ballistic impact cases that exhibit the high strain rate and high-pressure behavior. The single-element model is established in LS-DYNA with the dimension of 1 m $\times$ 1 m $\times$ 1 m, as shown in Figure A.3 (a). The confined boundary conditions are employed to constrain the element to displace at four sides and bottom (fixed), and thus, the element is only allowed to displace vertically along the z-direction. Next, the element is subjected to external load via displacement control on the top surface. For each simulation, the element is compressed to a strain of nearly 14%, then unloaded to allow recovery to its initial length. The verification of the subroutine codes is achieved by comparing the equivalent stress-pressure curve provided in Johnson et al. [30]. Figure A.3 (b) shows the equivalent stress vs. pressure plot of the single element under loading and unloading conditions in comparison with the response reported in Johnson et al. [30]. The main critical stages of the material under compressive loading followed by tensile loading are identified on the plot with inserted numbers (Figure A.3 (b)): (1-3) the material undergoes elastic deformation up to yielding followed by plastic deformation with damage accumulation; (3-4) the material failure lead to an abrupt increase in the pressure due to bulking, where the bulking phenomenon as a consequence of the decrease in axial stress due to the decrease in the deviatoric stress (i.e., degradation in material strength); and (4-8) the material undergoes phase transformation and reversal of loading. Overall, it is observed that the resultant curves generated from the implemented code in this work is in reasonable agreement with the one reported in the literature [30], as demonstrated in Figure A.3(b), with slight deviations occurring at the end of the first and third stages. These deviations are likely caused by the differences in software and numerical algorithms between the studies.

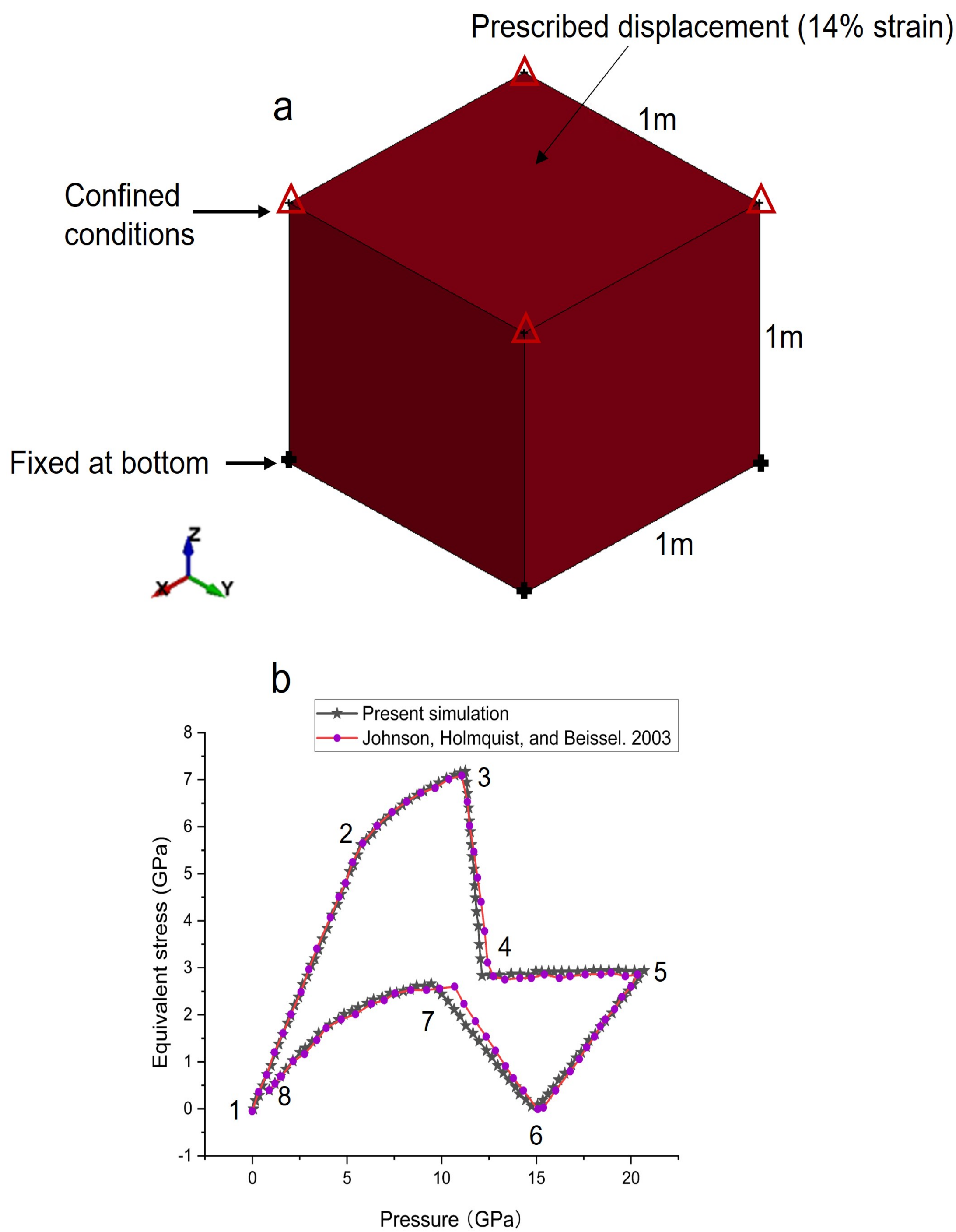

Figure A.3: Verification of the implemented user-defined sub-routine of the Johnson-Holmquist-Beissel model through a single element simulation under uniaxial loading-unloading condition, where the element is constrained from displacing of sides and bottom with an external prescribed displacement acts on the top surface. (a) Single element model configuration.(b) Predicted equivalent stress vs. pressure plot compared to the numerical result published by Johnson et al. [30].